TEACHERS WRITING ABOUT MATH: EXPLORING INQUIRY IN AN ONLINE COMMUNITY

by

Brenda McLoughlin

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Teachers Writing About Math: Exploring inquiry in an online community

Brenda McLoughlin
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Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education of the University of Toronto

Abstract

This study followed three elementary-school teachers as they engaged in online discussions about inquiry-based mathematics teaching, and wrote and tested inquiry lessons for their own classrooms. In an inquiry lesson, students bring their own knowledge to open-ended problem situations, and build on that knowledge as they try out solutions and share their ideas with others.

Evidence from the study suggests that teachers may turn to inquiry as an antidote to the way they learned about mathematics as schoolchildren, and that participating in an online community is a way for teachers to gain new mathematical and pedagogical knowledge and to change their conceptual understanding of inquiry-based teaching. The study results indicate that online professional development can help teachers improve their practice, but that care must be taken to build social ties within the group, and to structure tasks in a way that encourages collaboration and constructive criticism.
Acknowledgements

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Chapter One: Introduction

1.1: Introduction

The purpose of this thesis is to examine changes in how elementary mathematics teachers perceived their role as teachers as they explored inquiry-based mathematics teaching through online discussion and writing activities. I chose this topic because, in my career as an editor of teaching materials for elementary mathematics, I travel a path that is constantly informing and changing my thinking about how and why we teach mathematics.

When I was a kid, I hated math. I recall sitting in silent dread as my teachers demonstrated complicated algorithms at the chalkboard, knowing that I would be expected to reproduce them exactly. Math class was a time when I struggled with endless practice exercises, often with no real understanding of what I was doing or why I was doing it. As an adult, I love math! I love the way it challenges me, the way it frequently surprises me, and the feeling of accomplishment I have whenever a new idea falls into place.

As a teacher and editor, I gradually discovered that the best way for children to learn about mathematics with understanding is for them to be actively engaged in building their own knowledge through inquiry (National Council of Teachers of Mathematics, 2000). This is a very different approach to learning from the more traditional methods that dominated the classrooms where I learned as a child.

Today we know that children benefit from experimenting with different ways to solve problems, and from interacting with others as they do so (Carpenter & Lehrer, 1999; National Center for Research on Teacher Learning, 1994). Nevertheless, it is still very common to enter a mathematics classroom and see the teacher solving a problem while the children watch passively from their seats, or to see children completing worksheets by rote.
An abundance of research has been done about active learning in mathematics (Ross, Hogaboam-Gray, & McDougall, 2002), but this research has generally focused on benefits of active learning for the students (Boaler, 1993, 1998; Fennema, Franke, & Carpenter, 1993; Villasenor & Kepner, 1993) or on strategies for overcoming barriers to implementation (Anderson & Piazza, 1996; Graue & Smith, 1996; Keiser & Lambdin, 1996; Remillard, 2000; Spillane, 2000), rather than on the personal journey a teacher takes along the road from traditional teacher-centred methods to less familiar student-centred approaches. The goal of my qualitative research was to observe a group of elementary-school mathematics teachers as they engaged in online discussion about mathematics teaching and worked collaboratively to create and share their own inquiry-based mathematics lessons.

1.2 Research Context

Inquiry-based mathematics teaching is not easy, and is quite different from the way many teachers learned about mathematics at school (Anderson & Piazza, 1996). In a traditional classroom, problems and exercises are isolated from students’ real-life experience, and there is usually one correct answer to be obtained in one conventional way. Mathematical knowledge is taught as a collection of disconnected facts and procedures, and learning flows from the teacher to the students.

In an inquiry-based classroom, tasks are more complex and open-ended, and are embedded in multiple strands of mathematics and in real-life contexts. Students bring their own knowledge to problem situations, and build on that knowledge as they try to find their own solutions. The teacher gives students time to think about problems and talk about them with peers, and then guides a group discussion about the process in a way that helps students consolidate new concepts and clarify their understanding. Mathematics is taught not as a
collection of disconnected facts, but as a connected whole that is constantly growing (National Council of Teachers of Mathematics, 2000).

The curricula in mathematics that Canadian teachers are expected to follow are constructed around this view of mathematics as a set of interconnected ideas that can be constructed through problem solving: in fact, an inquiry-based approach. The Ontario mathematics curriculum says:

Problem solving is central to learning mathematics. By learning to solve problems and by learning through problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding. Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction. (Ontario Ministry of Education, 2005, p. 11)

The curriculum framework for the Western and Northern Canadian Protocol says:

Problem solving is a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers. (Western and Northern Canadian Protocol for Collaboration in Education, 2006, p. 8)

In order to teach through inquiry, a teacher needs to adopt a set of beliefs about what mathematics is and how it should be taught. The teacher must also build an extensive knowledge of the subject. This subject knowledge makes it possible for teachers to connect students’ responses to one another, and to use key ideas to help the students move forward in their thinking.

For teachers who are accustomed to a more traditional approach, this can be a very difficult change to make. It requires substantial professional-development support (Ross, Hogaboam-Gray, & McDougall, 2002). Without this support, changes to the mathematics curriculum and the provision of new teaching materials from publishers seem to have little impact on the methods of instruction teachers use in their classrooms (Price, Ball, & Luks, 1995).
Online communities have proven to be a useful way to support teachers through the process of developing their teaching practice. Such communities can provide effective ongoing professional supports such as coaching, online learning, teacher networking, and teacher partnerships, and have proven to be more beneficial to teachers than one-shot professional-development workshops (Hunzicker, 2011; Lock, 2006; Scher & O’Reilly, 2009). An example of an online learning environment for teachers is the Inquiry Learning Forum, designed to “support a virtual community of in-service and preservice mathematics and science teachers, sharing, improving, and creating inquiry-based pedagogical practice.” (Barab, MaKinster, & Scheckler, 2003, p. 240). The Inquiry Learning Forum is a large-scale project that encourages teachers around the world to share inquiry-based lesson ideas and upload videos of their classrooms. The site also provides links to useful information and allows teachers to set up online discussion groups.

For this thesis project, I sought to determine how participating in an online community similar to the Inquiry Learning Forum, but much smaller in scale, could benefit mathematics teachers as they developed plans for inquiry-based lessons and looked for ways to incorporate more inquiry-based teaching into their classrooms.

1.3 Research Questions

In this study, I worked with three elementary-school mathematics teachers who wanted to learn more about inquiry-based methods. I used observation and interviews to explore how their thinking about mathematics teaching changed as they exchanged ideas in an online discussion forum and worked collaboratively to create and test inquiry-based lesson plans. The research questions were:

1. What benefits did the teachers identify in inquiry-based mathematics teaching?
2. What kinds of obstacles and difficulties did the teachers encounter as they implemented inquiry-based teaching in their classrooms?
3. What benefits did the teachers perceive in online collaboration with other teachers?
4. What kinds of obstacles and difficulties did these teachers encounter as they engaged in online collaboration?
5. How did each teacher’s understanding of inquiry-based mathematics teaching change over the course of the project?
6. What insights and observations did this process yield about how to design other professional-development initiatives like this one?

My goal was to find out how different people experience the process of sharing ideas in an online community as they work to improve their understanding of inquiry-based mathematics teaching.

1.4 Significance of the Study

This study is significant because the experiences of these teachers may give administrators and university educators insights into how to plan online professional-development activities that will help more teachers adopt inquiry-based methods of instruction. This, in turn, will ultimately provide the teachers with strategies they can use to help their students develop a better conceptual understanding of mathematics and a more positive disposition towards the subject.

While mathematics curricula and mathematics programs created by educational publishers are designed to support an inquiry-based approach, many teachers are still relying on traditional methods of mathematics teaching. I believe this is because the shift to inquiry-based methods can be very difficult to make, and many teachers do not yet recognize the need to make it.

Despite the concerted effort of many classroom teachers, administrators, teacher-leaders, curriculum developers, teacher educators, mathematicians, and policymakers, the portrayal of mathematics teaching and learning in Principles and Standards is not the reality in the vast majority of classrooms, schools, and districts. (National Council of Teachers of Mathematics, 2000, p. 5)
This study explores what happened when a workshop about inquiry-based teaching was followed by the opportunity to write inquiry-based lessons and discuss them online with other teachers. The goal was to find out whether this type of collaboration is an effective way to help teachers improve their knowledge of mathematics and better understand the differences between transmissive and inquiry-based methods, and to determine how future professional-development experiences could be structured to provide a wider range of participants with the greatest possible benefit.

1.5 Background of the Researcher

This research interested me because I began my teaching career many years ago with the same traditional ideas about teaching mathematics that many teachers continue to adhere to today. In the 1980s, I was a primary-level teacher in both public and private schools for several years. My understanding of how to teach mathematics was shaped partly by what I had learned in my B.Ed. program and partly by what I saw other teachers doing in their classrooms, but mostly by my own experiences as an elementary and secondary student. For the most part, I taught my students the same way I had been taught. I found it a struggle to engage my students and to figure out how best to help them move forward in their mathematical understanding.

In 1988, I left teaching to become a mathematics editor for a publishing company. The early 1990s were an exciting time for educational publishers because of the release of the first National Council of Teachers of Mathematics (NCTM) Standards document in 1989. This document, which outlined what students should learn in mathematics and how to measure their learning, had a huge impact on mathematics curricula across North America, and on mathematics teaching in general. Many publishers, including the one I worked for, began developing
elementary mathematics programs that were very different from any that had gone before. These new programs valued an inquiry-based approach over more traditional instruction.

Over the next few years, many of my ideas about what constituted effective mathematics teaching, and even effective teaching in general, were challenged and reshaped by my work with authors who were top-notch teachers and teacher educators, and who had a fluent understanding of then-current research in mathematics. Gradually, through much trial and error, I came to understand that good teaching is based on three things:

1. a thorough knowledge of the subject matter,
2. a thorough understanding of how children grow and learn, and
3. a teaching approach that is founded on asking rather than telling.

The transition was not easy. It took a long time for me to come to understand not just the wording of the Standards but the spirit behind them, and I meet teachers all the time who are still struggling with the enormity of the changes that the Standards writers envisioned. These teachers feel baffled by the curriculum they are required to teach and by the published resources they are required to use.

Over the years, I have helped to edit more than 100 books for teachers, and I have learned something about good teaching from every one of them. One of the driving forces of my work in publishing has always been that I want to help children learn what I did not learn in school, so they will experience success with mathematics and, as a result, will have a wider array of career options open to them.

For me, a move into the publishing industry was the catalyst for a new system of beliefs about mathematics and about teaching. I developed these beliefs gradually, through the process of collaborating with others to create learning materials that aligned with the latest research about how children learn best. What motivated my study was a desire to help struggling teachers
in the same way other people have helped me over the years. I was also curious to find out how the experiences of other teachers are like or different from my own.

It has been said that the best way to learn is to teach someone else. Over the course of a career in publishing, I have had a great many personal opportunities to learn through teaching. Each new lesson, new book, or new textbook series represents an opportunity for me to help children and teachers learn new ideas, and also for me to learn new ideas myself.

With this research, I had the opportunity to explore whether and how this process of learning through the creation of learning materials can work for other teachers too.

1.6 Plan of the Thesis

The five chapters in the thesis have been organized to provide an overview of the research project. Chapter One identifies the research questions and provides information about the background of the study and its significance. Chapter Two is a review of the themes in the literature that relate to the study. Chapter Three describes the processes that were used to collect and analyse the data, and the ethical considerations involved. It describes the setting for the study, introduces the participants, and explains how the study was organized.

Chapter Four is the heart of the thesis, following each participant’s individual journey through the project. Chapter Five ties information from Chapter Four back to the research questions from Chapter One, identifying the main themes that emerged from the data analysis and concluding with suggestions for further research.
Chapter Two: Literature Review

2.1 Introduction

Chapter One outlined the need for an in-depth study framed around this question: “How do different teachers experience the process of sharing ideas in an online community as they work to improve their understanding of inquiry-based mathematics teaching?” To provide a thorough context for this investigation, this chapter reviews the research literature relating to the benefits and barriers associated with inquiry-based teaching, values and beliefs that guide teacher decision-making, and professional-development strategies that have proven effective for bringing about changes in teachers’ values, beliefs, and practices.

2.2 Benefits of Inquiry-Based Teaching

The key difference between inquiry-based teaching and more traditional methods is where the knowledge is situated (Leighton, Rogers, & Maguire, 1999). In the traditional mathematics classroom, the teacher has knowledge to impart to the students. The students receive the knowledge from the teacher and then apply it in practice situations. In the inquiry-based classroom, the teacher poses a problem, and students bring their own knowledge to bear in solving it. As they struggle to solve the problem, they add new knowledge to what they initially brought. The teacher’s role is that of a co-learner and problem creator, rather than a solution-demonstrator (Ross, Hogaboam-Gray, & McDougall, 2002).

In general, students who learn through inquiry tend to be more engaged than students who learn from a more traditional approach. They also tend to demonstrate better recall. Heafner and Friedman (2008) studied two sections of a history class taught by the same teacher. One section was taught using traditional methods, while students in the other group used inquiry-based methods. During the study, the class that used the inquiry approach proved to be far more
engaged by learning than the control group. Eight months later, the researchers interviewed students from each group to find out what learning students had retained. Overwhelmingly, students from the learning-through-inquiry class gave much more accurate and detailed answers than students from the control class. (Heafner & Friedman, 2008).

In mathematics, students who learn through inquiry also demonstrate a deeper and longer-lasting understanding of mathematical concepts. In a study by Fennema, Franke, and Carpenter (1993), Grade 1 students who used an inquiry approach solved more complex problems than students in a control group, used higher-level strategies, and adapted their procedures to deal with new situations. They showed more persistence and a stronger ability to communicate their thinking (Fennema et al., 1993).

Inquiry also allows for differentiated learning. The right open-ended problem makes it possible for all students to apply and add to their existing knowledge, no matter how narrow or broad that knowledge may be. Silver and Stein (1996) found that inquiry benefits disadvantaged students, while Mann (2006) found that it is also beneficial for gifted students.

Small (2010) presents two strategies for differentiation that are founded in inquiry-based learning: open questions and parallel tasks. Open questions are mathematics problems that can engage students with a wide range of ability and prior knowledge. Parallel tasks are problems that can be adapted up or down by making a slight change. Both types of problems allow for all students to engage with the same task or similar tasks at the same time, while still learning in ways that meet their individual needs (Small, 2010).

2.3 Barriers to Adopting Inquiry-Based Teaching

While inquiry is increasingly recognized as an effective tool for helping students learn about mathematics, there are many practical barriers that can make it difficult for teachers to
implement (Ross, Hogaboam-Gray, & McDougall, 2002). Some of these barriers have to do with teachers’ knowledge of mathematics. Traditional teaching is fairly predictable, but teachers who use inquiry must be ready to react to student ideas that can be wide-ranging and unpredictable. In order to respond appropriately, these teachers need a solid understanding of mathematics as a whole body of knowledge. This understanding must reach well beyond the “outcomes” or “expectations” laid out in the curriculum for a particular grade (Ball, 1990, 1996; Henningsen & Stein, 1997; Lloyd & Wilson, 1998; Ma, 1999; Spillane, 2000).

Other practical barriers include opposition from parents (Graue & Smith, 1996; Lehrer & Shumow, 1997), clashes with assessment programs that measure computational speed and accuracy (Firestone, Mayrowetz, & Fairman, 1998), and time limitations that require teachers to cover an extensive list of curriculum expectations in a limited number of teaching days (Keiser & Lambdin, 1996).

2.4 Values and Beliefs that Guide Teacher Decision-Making

People’s beliefs influence their decision-making and their actions. In order for a teacher to move from a traditional teaching approach to a more inquiry-based approach, that teacher must change and reprioritize existing beliefs. For example, a teacher who believes that he or she is doing an excellent job if all the students are listening attentively during the lesson phase and answering questions correctly during the practice phase might have a great deal of difficulty accepting that she or he is doing a better job if the children are actively and noisily experimenting with manipulatives as they try to solve a problem that has no single correct answer.

Kennedy (2004) conducted a study to find out what key values teachers think about as they are planning and developing lessons. She videotaped teachers as they taught, and then had
the teachers watch the tapes to look for moments they thought were significant in some way.

From their comments and observations, Kennedy concluded that the key values for the teachers in her study were:

- avoid distractions and ensure lesson momentum,
- cover all content required for the next grade level,
- use teaching strategies that foster student learning,
- affirm all students at all times,
- maintain a persona that will promote an appropriate classroom community, and
- reduce personal strain (Kennedy, 2004).

Kennedy (2004) also concluded that “The problem reformers face may not be one of persuading teachers of their ideals, but instead one of persuading teachers to weigh different areas of concern differently as they make moment-by-moment tradeoffs” (p. 28). The idea that teachers can change their conceptions of teaching only if they can align new conceptions with their underlying values and beliefs is consistent with conceptual change theory, developed by science educators (Posner, Strike, Hewson, & Gertzog, 1982). Conceptual change theory suggests that, in order for a learner to change a fundamental misconception, the learner must first recognize that the misconception is unsatisfactory in some way, and then recognize that the new conception is intelligible, plausible, and fruitful (productive). As applied to my study, this theory holds that teachers will move from a traditional concept of teaching to an inquiry-based concept only if they can see that inquiry-based teaching is more intelligible (easy to understand), more plausible (capable of solving problems left unsolved by previous ideas), and more fruitful (likely to expand potential areas of inquiry).

Gill (2006) listened to teacher conversations during weekly collaborative planning sessions for elementary-school mathematics teachers, and found that collaborative planning discourse provides a unique opportunity to gain insight into rationales for decision making, and perhaps to challenge those rationales. Another related study found that preservice teachers’
beliefs about the nature of mathematics knowledge and teaching can be challenged through a process of bringing those beliefs to the surface (via a survey), and then explicitly presenting and discussing opposing views (Gill, Ashton, & Algina, 2004). This idea is supported by Wideen, Mayer-Smith, and Moon (1998), who found that the most productive approach to promoting learning in novice teachers involved creating a teacher-education program that builds on the beliefs they hold when they enter the program, and beginning the program by having them examine those beliefs.

2.5 Teacher Knowledge

Few elementary-school teachers have degrees in mathematics. Most rely on a combination of what they learned as students, what they learned during their approximately 40 hours of teacher training in mathematics instruction, and what they learn through classroom experience and professional development.

A study of 64 preservice teachers at California State University showed that many preservice teachers have difficulty transferring their “learner knowledge,” the knowledge gained from their experiences as students, into “teacher knowledge,” the knowledge that allows them to assess what their students understand and use this as a foundation for planning (Menon, 2009). The preservice teachers, all of whom had studied some mathematics at the college level and were preparing to teach junior- or intermediate-level math, were given three mathematics problems. They were asked to solve each problem, to construct a context, and finally to explain how they would teach the concept to someone else. For a simple multiplication problem, 95% of teachers were able to give the correct answer, but only 75% were able to provide an appropriate contextual problem, and only 18% were able to explain how they would teach multiplication in a way that emphasized conceptual understanding over procedural steps (Menon, 2009). This would
indicate that even people with a substantial background in mathematics need professional training in order to become effective mathematics teachers. For people without a mathematics background, this need is even more intense.

2.6 Effective Forms of Professional Development

In order to overcome barriers imposed by teacher beliefs, teacher knowledge, conflicting expectations, and practical constraints, teachers need strong professional-development support. Effective professional development provides models of inquiry-based teaching to follow, along with opportunities to talk about new ideas with colleagues.

Especially important is public and private reflection. Sharing professional experiences is such an essential element of professional growth that it has become axiomatic that inservice events should provide opportunities for participants to describe their experiences, reflect on the meanings of personal practice, and exchange interpretations with colleagues. (Ross, Hogaboam-Gray, & McDougall, 2002, p. 129)

Ginsburg, Gray, and Levine (2004) provide a strategic review of the quality of various approaches to online professional development in the United States and compare online approaches with traditional face-to-face approaches. They conclude that effective professional development has four characteristics:

- learner-centred (addresses the teacher’s perspective with respect to pedagogical and content needs and shows awareness that most teachers have limited time),
- knowledge-centred (imparts research-based professional knowledge),
- community-centred (provides teachers with community learning opportunities), and
- assessment-centred (assesses and provides feedback on teacher performance).

(Ginsburg, Gray, & Levine, 2004)

This contrasts with the traditional approach that generally involves one-day or two-day workshops during which teachers are bombarded with information and left with little opportunity to challenge their own preconceptions or to reflect on and try out new ideas (Ginsburg, Gray, & Levin, 2004; Hunzicker, 2011). Workshops tend to be “heavily loaded with generalities about the teaching process and light on mathematical content. Most teachers do not change their
instructional practices after workshops, and the training has little or no impact on students’ mathematical outcomes” (Ginsburg, Gray, & Levin, 2004, p. 2).

Teachers tend to be more engaged by professional development that is supportive, job-embedded, instructionally focused, collaborative, and ongoing (Hunzicker, 2011). In one professional-development study, Lynch (1997) documents the training of 25 beginning science teachers. Her student teachers were required to design a 10-day teaching unit to reflect the goals of science reform and then develop a set of criteria, based on reform principles, for evaluating their own work. Lynch concluded that her participants had a unique opportunity to learn and to think about how they were learning. The process of evaluating their own work through a reform lens was especially valuable.

A related body of literature is about lesson study, an approach to professional development that originally developed in Japan (Lewis, Perry, & Hurd, 2009). The four phases of lesson study are:

1. investigation (studying a new concept or recent research),
2. developing a research lesson,
3. trying out the research lesson while team members observe and collect data, and
4. reflecting in writing on what was learned and revising/reteaching as necessary. (Lewis, Perry, & Hurd, 2009)

Lesson study has been shown to be an effective way to help teachers improve their practice, and to change their beliefs about teaching (Lewis, Perry, & Hurd, 2009).

2.7 Challenges of Online Professional Development

Online environments are an area of rapidly expanding interest for those seeking to establish professional-development programs for teachers (Lock, 2006). The Internet gives teachers access to professional support at any time and from any place. However, accessibility alone is not enough to ensure that online professional development will be successful. Other
factors required for success include a sense of community, similar needs that are shared by all the participants, and tasks that foster true collaboration.

Wenger (1999) proposed that what he called “community of practice” could be bound together by one of four dimensions of learning:

- learning as doing (engaging in practice),
- learning as becoming (developing an identity),
- learning as experience (finding meaning), and
- learning as belonging (becoming part of a community).

Recent research in the area of e-learning has focused on how to construct online spaces in a way that supports communities of practice (Vrasidas & Zembylas, 2004). “What characterizes communities of practice is a shared commitment for a particular practice that creates an interactional network that enables and promotes knowledge sharing and professional development” (Vrasidas & Zembylas, 2004, p. 328).

Dillenbourg and Schneider (1995) explored the difference between cooperation, which occurs wherever people share a task, and true collaboration, in which people interact to jointly solve a problem. With cooperation, people can combine their strengths to reach a goal, but they do not necessarily learn from one another during the process. With collaboration, everyone learns as the group exchanges ideas and looks for ways to resolve conflicts.

Kreijns, Kirchner, and Jochems (2003) posited that there are two factors that cause collaborative learning environments to fail: taking it for granted that online collaboration will occur just because it is possible, and neglecting the social dimension of the group. They contend that group learning in asynchronous online environments depends on social interaction, which must be intentionally designed into the environment (Kreijns, Kirchner, & Jochems, 2003). For true collaboration to develop, participants need to have a shared history and a shared need. When
true collaboration does occur, it leads to deeper learning, more critical thinking, shared understanding, and longer-term retention of learning.

If students are to offer their tentative ideas to their peers, if they are to critique the ideas of their peers, and if they are to interpret others’ critiques as valuable rather than as personal affronts, certain conditions must exist. Students need to trust each other, feel a sense of warmth and belonging, and feel close to each other before they will engage willfully in collaboration and recognize the collaboration as a valuable experience. (Rourke, as cited in Kreijns, Kirchner, & Jochems, 2003, p. 341)

2.8 Summary

The literature shows that students who learn through inquiry tend to be more engaged than students who learn through a more traditional approach, and also tend to demonstrate a deeper and longer-lasting understanding of mathematical concepts (Heafner & Friedman, 2008; Fennema et al., 1993). Inquiry-based methods also help teachers accommodate a variety of learning needs (Small, 2010; Silver & Stein, 1996).

While inquiry-based teaching methods have proven to be effective, there are barriers that can discourage teachers from adopting them. Some of these barriers are knowledge-related: teachers cannot teach what they do not understand themselves (Ball, 1990, 1996; Henningsen & Stein, 1997; Lloyd & Wilson, 1998; Ma, 1999; Spillane, 2000). Other barriers are practical in nature, requiring teachers to deal with time limitations (Keiser & Lambdin, 1996), opposition from parents (Graue & Smith, 1996; Lehrer & Shumow, 1997), and clashes with assessment programs that measure computational speed and accuracy (Firestone, Mayrowetz, & Fairman, 1998).

In order for teachers to adopt new practices, they must recognize two things: firstly that the new practices have more value than the existing ones, and secondly that the new practices will be practical to implement (Kennedy, 2004; Posner, Strike, Hewson, & Gertzog, 1982).
Teacher decision-making can also be influenced by peer collaboration and discussion (Gill, 2006; Gill, Ashton, & Algina, 2004).

Changes in teaching practice happen more easily when they are supported by effective professional development. To be effective, professional-development experiences need to provide opportunities for participants to describe and reflect on their experiences and share them with others (Ross, Hogaboam-Gray, & McDougall, 2002). Effective professional development is learner-centred, knowledge-centred, community-centred, and assessment-centred; and longer-term professional development experiences tend to offer more for teachers than one-day workshops (Ginsburg, Gray, & Levin, 2004; Lynch, 1997). One approach that works well is the lesson study approach, which involves developing and trying out a research lesson while team members observe (Lewis, Perry, & Hurd, 2009).

The Internet is an effective medium for providing professional development for teachers, but there are challenges. In order to engage teachers, online environments must include a social component to foster a sense of community. When participants share similar needs, and are engaged in tasks that foster true collaboration, they are more likely to benefit from the experience (Wenger, 1999; Dillenbourg & Schneider, 1995; Kreijns, Kirchner, & Jochems, 2003; Vrasidas & Zembylas, 2004).
Chapter Three: Methodology

3.1 Introduction

This thesis describes the experience of three elementary-school teachers as they exchanged ideas about inquiry-based mathematics teaching and developed lesson plans.

3.2 Research Overview

This study is a qualitative case study. The intent was to explore changes in the thinking and practice of a small group of teachers by collecting detailed information over time using a variety of data collection procedures.

At the beginning of the study, a meeting was held to introduce the participants to the structure of an inquiry-based lesson (see Appendix A) and to the research that supports the benefits of this type of teaching. Over the course of the three-month study, teachers engaged in asynchronous online discussions about inquiry-based instruction, using Knowledge e-Commons as a discussion forum (see Appendix D for discussion questions). They also created lesson outlines for their own inquiry-based lessons, tested them in their classrooms, and posted them in the discussion forum for peer review.

Along with the lessons teachers developed, interviews and surveys were used at the beginning and end of the project to gauge the growth in the teachers’ understanding of inquiry-based teaching. Some items on the survey are based, in part, on ideas from the CGI (Cognitively Guided Instruction) survey developed by Peterson, Fennema, Carpenter, and Loef (1989).

3.3 Participants

The participants in the study were three elementary-school mathematics teachers in the greater Toronto area who shared an interest in learning about inquiry-based instruction. Their names were suggested by mathematics consultants. The teachers were contacted by email, and
then invited to attend an in-person meeting to learn more about the project. The teachers were different ages, had different levels of teaching experience, and taught at different grade levels. They also had different levels of comfort with mathematics. The one thing they shared in common was a strong desire to improve their mathematics teaching and a willingness to exchange ideas with other teachers.

3.4 Setting

This study was conducted mainly online, with teachers engaging in asynchronous online discussion about their teaching and exchanging lessons with other people involved via the online forum. There were two opportunities for participants to meet in person—at a workshop on inquiry-based learning held at the beginning of the study, and again at a dinner held at the conclusion of the project.

3.5 Timeline

The study was conducted over four months, beginning with the project-launch meeting in August and ending when the final versions of the lessons were posted online in early December. Figure 1 shows how the project developed over time.

![Figure 1](image-url)
Data was collected from various sources during the course of the project. First, surveys were administered early in the project, and then again near the end, to collect data about teachers’ experiences with and attitudes to different teaching strategies (see Appendices B and E). Telephone interviews were also conducted with each participant in the first week of the project, and then again during the last week (see Appendix C).

The main venue for communication during the project was an online forum set up on OISE’s Knowledge e-Commons website. Weekly discussion questions were posted, and the teachers responded in writing.

The final source of data for the project was the lessons that the teachers developed over several months. Early in the project, the teachers posted their lesson outlines online and invited comments from the group. Later, they tried out their lesson ideas in their classrooms and reported to the group about the results. Finally, each teacher posted a finished lesson plan at the end of the project.

3.6 Data Analysis

All interviews were transcribed and the transcripts were reviewed against the recordings to ensure accuracy. The survey data was collated, with written comments recorded in the collated file in their entirety.

The data analysis for this project followed a series of coding cycles. The first step involved reading interview transcripts, survey results, and online comments and highlighting key ideas. These key ideas were given labels, and, in the second step, pieces of text from surveys, interview transcripts, and online postings were attached to each label.

The following key ideas were identified: links between teachers’ practices and their own experiences as elementary-school students; links between inquiry-based teaching and higher
levels of student and teacher engagement; growth in mathematical knowledge; growth in pedagogical knowledge; teacher beliefs about inquiry-based learning; evidence of online collaboration; evidence of lack of collaboration in lesson planning; comments related to constructive criticism. Once these categories emerged, I returned to the literature to look for information that might help me better understand what I discovered and identify ways that the data might be used to frame future professional-development initiatives.

3.7 Ethical Considerations

This project posed no physical or social risk to the participants, who were all volunteers, and did not involve sensitive emotional issues. An ethics review was conducted through the University of Toronto to ensure that the project conformed to university ethics standards. Pseudonyms for teachers were used to ensure confidentiality, and specific details about the locations of their schools were omitted.

Participants consented to be part of the study via email, and then signed a formal consent letter to confirm their participation. All participants knew that they could participate as much or as little as they wanted to, and could stop at any time.
Chapter Four: Findings

4.1 Introduction

Each of the three study participants, Jessica, Kaitlyn, and Joan, completed pre-project and post-project surveys and interviews, and engaged in online discussions throughout the fall of 2011. Participants submitted lesson outlines, test-taught the lessons in their classrooms, reported on the results for the group, and then developed a final lesson draft, incorporating what was learned at each previous stage. (Final lesson plans can be found in Appendices F through H.)

This chapter describes each person’s experience in the project, based on evidence from survey responses, comments in interviews, and writing posted online. Surveys and interviews were conducted in August 2011, and then again in December 2011. Online comments are referenced by number, reflecting the order in which they were posted. A total of 138 comments were posted by the end of the project.

The chapter begins with the participants’ memories about how they learned mathematics when they were children, and how these memories influence their teaching as adults. The next section deals with the project-launch meeting, and the initial thinking about inquiry-based mathematics that emerged as the teachers watched and commented on a videotaped lesson. Individual participant profiles are next, and the chapter ends with a final summary.

4.2 Influence of Teachers’ Memories of Elementary-School Mathematics

In the initial survey for the project, all three participants said their memories of elementary-school mathematics include completing worksheets or exercises from the blackboard or from a mathematics textbook, playing games, and doing timed drills. No one remembers learning about more than one way to do mathematics operations. Only the youngest teacher in
the group mentioned learning about how people use mathematics at work or going on mathematics-related field trips.

Two participants said they were dissatisfied or very dissatisfied with the quality of their mathematics education, and one responded that she was undecided. All three described negative feelings and experiences associated with math. Jessica, who still has a very wary relationship with numbers, described her feelings this way.

I remember standing in front of the class with chalk in my hand and feeling like I did not even know where to begin. I felt stupid and like I was going to cry. It was horrible!… I wish I could understand math better and I know that if I could go back and do it all again with the right teacher who understood how to teach math the right way, I might actually be good at it, not scared! (Jessica, survey response)

Joan showed more anger than fear in the way she described her elementary-school experiences with mathematics:

There was an insistence on getting correct answers quickly and we were only taught the regular algorithms, which made almost no sense to me. Math was a strange language and I had no idea why it was relevant. It became learning a sequence of steps that I repeated over and over again to get the right answers to number problems…. We did drill when learning our times tables and the teacher walked up and down the rows of desks listening to us saying out loud $2 \times 4 = 8$, $3 \times 4 = 12$. If you were not speaking for fear of being wrong, or if you said something incorrectly, you had to run around the playing field in 4 minutes. It was a scary place and under those conditions learning math was not the least bit enjoyable. (Joan, survey response)

All of the participants expressed the belief that the way teachers are taught as schoolchildren has a strong influence on their practice as adults:

It is perhaps the only way they know how to teach the material because it is all they know—it is how they learned! (Jessica, survey response)

The participants also considered the idea that teachers who have a difficult time with mathematics as schoolchildren sometimes make excellent mathematics teachers because they understand what it is like to struggle, and because they work hard to ensure that their students will have a better experience than they did.
I was terrible at in school, and I have given it a lot of thought since then. So I made that case for, so, if you are not very good at math and you overcome that, you can come up with a number of different ways of approaching a problem. As I say, I think that might describe me. I hope so. (Joan, interview 1)

Not everyone agreed with this point of view. Jessica felt that, while some people may be able to overcome their difficulties with mathematics as adults, others are doomed to failure by their own intellectual limitations.

I have heard many teachers say that they are better math teachers because they struggled so badly with math in their past, and I believe this is true to a point. Yes, they may take the time to really sit down one to one to help a confused student understand because they have walked in those shoes before. But I do believe there comes a point or limit in one’s own ability to understand based on one’s own brain ability and genetic make-up that nurturing cannot always accommodate for. (Jessica, survey response)

Still, Jessica recognized that, as an adult, she has learned to master some of the mathematical skills that defeated her as a child. This experience has increased her confidence that there are ways to help children understand—she just needs to find out what those ways are.

I did not understand subtraction with regrouping until I was in teacher’s college. I just learned that with 0, you cross it out and put the 9 on top, and go onto the next one over. And that is what you did. …And then when we had to use the hundred blocks and the tens rods and the ones, and you had to break that ten rod down into ones, I just, honestly, it just flipped me out. I was like, “Oh my goodness. I had no clue that that is what subtraction with regrouping was all about!” (Jessica, interview 1).

Jessica also highlighted the value of learning mathematics in a familiar context when she commented on her perception that her own teachers had never helped her connect mathematics to subjects she cared about. Jessica, who loves animals, said:

If math had been embedded in something I could relate to and cared about, such as how much of the rainforest is being destroyed or how many animals will be endangered by a certain year based on current levels of habitat destruction, I would have wanted to spend the time to figure out the problem. (Jessica, note 86)

Most of the participants said they recalled learning mathematics by rote, rather than truly learning to understand what they were doing and why.
What I do recall, as I worried about memorizing my times tables and how to do an algorithm back in the 1960s, is that I had no idea what I was actually doing except going for the right answer. As an adult, math started to make sense for me as I figured out why I used that operation and what the answer was actually telling me. In other words I could probably have benefitted from seeing what we were really doing when manipulating numbers and why it was useful. (Joan, note 81)

The participants’ unsatisfactory experiences with their rote learning of mathematics in elementary school heightened their interest in exploring inquiry-based approaches. All wanted to broaden their understanding of how mathematics works, and to find ways of teaching that would be more productive, and more fun, than what they recalled from their student days.

Children come to school thinking that math is a foreign language, and those can be difficult to learn and understand. If they then meet a teacher who also feels that math is difficult and teaches “one answer for every question” type stuff, then the child’s perception is affirmed. If a child comes to school and meets a teacher who gets visibly excited about math, … then they tend to open their minds to the math experience and even start to enjoy it…. I wonder how many teachers see math teaching this way, and how many think they are simply teaching rules that must be memorized for success. (Joan, survey response)

Conceptual change theory (Posner et al., 1982) says that learners need to recognize that the old way of doing something is unsatisfactory before they will be ready to try something new. While many mathematics teachers base their practice on what they remember from how they were taught in school (Anderson & Piazza, 1996), it is evident that all three teachers in this study regard the way they studied mathematics as unsatisfactory.
4.3 Concepts of Inquiry-Based Teaching at the Project Launch

All three participants were familiar with the structure of an inquiry-based lesson at the outset of the project. The participants’ term for this type of lesson is the “three-part lesson.” This term is used in documents from the Ontario Ministry of Education (Literacy and Numeracy Secretariat, 2009; Ontario Ministry of Education, 2006), and in professional-development activities in the participants’ Boards.

In mid-August 2011, the participants attended a project-launch meeting. The goal of the meeting was to give the participants and the researcher a chance to get to know each other, to explain how the project would work, and to assess the participants’ initial understanding of inquiry-based teaching. At the meeting, the participants viewed and discussed a video of an inquiry-based lesson in action. The video (from Boaler & Humphreys, 2005) was chosen, in part, because of its capacity to promote discussion about the basics of this way of teaching.

The video showed a class of seventh-grade students working on this problem: “How can you show what \(1 \div \frac{2}{3}\) means in a way that makes sense?” Some students said that \(1 \div \frac{2}{3} = 6\), because there are 3 thirds in 1 and, and \(2 \times 3 = 6\). Others said that \(1 \div \frac{2}{3} = 1 \frac{1}{2}\) because that is the result of multiplying \(1 \times \frac{3}{2}\), but could not explain why this made sense. The teacher then asked, “Which answer makes sense? Why?” Students discussed the questions for few minutes in groups, and then began to share their thoughts.

During this final stage of the lesson, the class proposed several different representations of \(1 \div \frac{2}{3}\), including diagrams, number lines, and calculations, as shown in Figure 2. The teacher
prompted discussion using questions such as “How could you convince a skeptic that this makes sense?” and “What made you change your mind?”

An inquiry lesson is framed around one central problem for students to solve. It is difficult to overemphasize the importance of choosing the right wording for this question when planning the lesson, and this is a task that can be difficult for teachers.

When the study teachers watched the video, not everyone found it easy to identify the teacher’s key question, and not everyone agreed that it was an appropriate question for inquiry. Some of the participants expressed surprise that the students in the video were using pictures and numbers rather than manipulatives, revealing a preconception that the process of inquiry is tied to the use of manipulatives rather than to the structure of the problem. Others expressed concern that the problem was not presented in a real-life context, even though the students in the video
were clearly engaged by it. This indicates a preconception that inquiry is tied to problem context, rather than to problem structure.

A real-life context is not the sole hallmark of a good inquiry question, although it can certainly be a factor. Asking “How many lengths of board $\frac{2}{3}$ of a foot long can a carpenter cut from a board 1 ft long?” may add a context, but it yields exactly the same limited response as “What is $1 \div \frac{2}{3}$?”

To foster a situation in which students with varying levels of prior knowledge can all construct new learning, the teacher needs to choose a question that allows for either multiple responses, or multiple paths to a response. Asking for ways to show $1 \div \frac{2}{3}$ in a way that makes sense is this kind of question, while asking for the answer to $1 \div \frac{2}{3}$ is not, even if the problem is presented in context, with manipulatives.

The topic of choosing the right question came up again later in the study, as teachers struggled with developing inquiry-based lessons of their own. Kaitlyn and Joan found it especially difficult to anticipate how students would respond to their questions, and what prerequisite knowledge would need to be in place to make the question work. For example, Joan observed that one of the challenges in choosing the right question was working backwards from what she wanted her students to learn.

The hardest thing for me was coming up with an idea that I thought would be workable…. I thought I would do patterning, and then I realized that to get there I would have to do other stuff, and that headed me towards what I needed for equalities. Once I got started, it created a pathway and I used the different strategies. It helped route me there. You were asking for something up front that was going to happen down the road, and it is unusual for me to plan that way. (Joan, interview 2)
In addition, all the teachers were surprised that more time was spent on the consolidation phase of the lesson than on the independent-work phase. This is an idea that came up later on as teachers began working on their lesson plans.

I think, given the video you showed us, I am going to change the structure in terms of the time that I give to my three-part lessons to allow more time for discussion afterwards. (Joan, interview 1)

After the discussion, we watched a follow-up video in which three students from the first video met with a researcher several weeks later to share their thoughts about inquiry-based teaching versus a more traditional approach. The study participants all showed a high level of interest in what the students had to say. Even though we were short of time at the end of the meeting, they wanted to watch the video through to the end and discuss it afterwards.

At one point in the interview, a student in the video indicated that a really good math lesson is “one where we are all in a conversation, where we are all disagreeing on it.” (Interview 1 video from Boaler & Humphreys, 2005)

This response caught the interest of my study participants because their perception was that teachers tend to view disagreement as something to be avoided, rather than as something to be actively sought out as an opportunity for learning. This was an interesting perception, in view of the way things worked out later in the study. When the participants were asked to critique each other’s lessons, they were reluctant to do so. Some comments indicated that they were uncomfortable with the idea of criticizing someone else’s work, even when the criticism was intended to be constructive.

Although we read each other’s lessons, maybe we did not feel comfortable critiquing other peoples’ lessons, especially if they took a lot of time to put their lesson together to present it to the group. (Jessica, survey 2)
4.4 The Cases

4.4.1 Jessica’s Story

Jessica is a 37-year-old kindergarten teacher who has been teaching for 13 years. Jessica is an enthusiastic teacher who loves her work in kindergarten. When asked to describe the essence of great mathematics teaching at the outset of the project, she said, “The possibilities to explore and solve are endless!”

Jessica had a very difficult time with mathematics as a student, and feels most comfortable teaching at the kindergarten level because she is unsure of her ability to handle mathematics at a higher grade level. At the initial workshop, Jessica described how she never truly understood two-digit subtraction with regrouping until she was a practice teacher and the host teacher showed her how to do it with manipulatives. When asked to describe her goal for the project, Jessica said:

I am hoping to learn how to be a better math teacher, and to learn more about inquiry and how I can use that to become a better teacher, and to engage all the different students that I have in the learning and create lessons where it is not just me doing the talking. (Jessica, interview 1)

Jessica also expressed her belief in professional development as a way to help teachers learn new and better ways to do things, especially in view of the number of subjects teachers need to master, and the pace of change in each subject area. She expressed frustration with the way Boards expect teachers to find ways to learn about these changes on their own, rather than providing support through training courses offered at work.

Not every teacher cares or has time to become a better teacher, and Boards usually only disseminate new documents in hopes that teachers will read them, understand them, and implement them. This is usually not the case, and many documents are not even read and remain hidden in the back of a cupboard. (Jessica, survey response)
4.4.1.1 Thoughts About Inquiry-Based Teaching

A major challenge for teachers is the fact that their students represent a wide range of abilities and interests. Jessica described the range of ability she typically faces in her kindergarten teaching.

So you have some children who come in and, I had one student last year in kindergarten …he could do computations, mental math in kindergarten, things that when I taught Grade 2 a lot of kids struggled with. We have children like that who seem to really understand math and see math and shapes in the world all around them. And then you have other children who do not even recognize a number 1. It is, you know, when you do number identification and maybe you say, ‘What number is that?’ And the child will say, ‘Stick.’ (Jessica, interview 1)

For Jessica, inquiry-based teaching is a way to facilitate learning for all of her students, no matter how much mathematical knowledge they bring to the task.

I believe students come to their own understanding about what they are learning and assimilate what they are doing with their previous knowledge in a way that makes sense to them. (Jessica, survey response)

When Jessica test-taught an inquiry-based sorting lesson in her classroom, she reported that her students gained a deeper understanding of sorting from the experience of sharing their different approaches to the sorting task with one another:

I gathered the students on the carpet and invited some to come up to show some of the different ways they came up with sorting. I really saw some of the other students’ eyes light up as they saw how others had sorted. They could see with their own eyes how the same buttons could be sorted in so many ways. (Jessica, note 132)

Because of Jessica’s own negative experiences with mathematics in school, she was especially interested in the benefits of inquiry for students who have math difficulties. In a survey response, Jessica expressed her sense that inquiry-based approaches would have been helpful to her when she was an elementary-school student, and might perhaps have led to more success, and to a more positive disposition towards mathematics:
I believe that if I had been taught another way using manipulatives and that there is usually more than one way to get to an answer I may not have the attitude I hold or the avoidance tactics I use when faced with math problems in my everyday life right now. I wish I could understand math better and I know that if I could go back and do it all again with the right teacher who understood how to teach math the right way, I may be actually good at it, not scared! (Jessica, survey response)

When the group watched the videotaped demonstration lesson at the project-launch meeting, Jessica commented that she liked seeing the different methods students were using to divide fractions. Many junior- and intermediate-level teachers find fraction division to be a challenging concept to teach, and yet Jessica, who lacked confidence in her own understanding of fraction operations, felt reassured by the way the teacher in the video was inviting various responses and responding to them in a non-judgmental way.

I liked how the video was showing ‘inquiry in progress’ because many different students discussed how to tackle the question and came to the board to demonstrate their understanding and explain their reasoning. Then at the end the teacher explained her reasoning and confirmed the different ways that the answer was correct. She invited students to discuss whether they agreed or disagreed with other students’ responses and there was no judgment on her part. That meant the students felt they could take a risk and share their answers. (Jessica, note 44)

The idea that inquiry-based teaching builds confidence and encourages risk-taking was another benefit that sometimes came up Jessica’s comments during the project. Jessica believes that, when students recognize that there are multiple routes to an answer, they are more willing to tackle unfamiliar problems:

[Students] are not afraid to take risks with this kind of teaching and are able to see that there can be several different ways to solve the same problem. How great is that? Think of what our world would be like if people always thought there was only one right answer or only one way to do things. (Jessica, survey response)

Another benefit of inquiry-based teaching that Jessica mentioned was the fact that information flows in multiple directions—from teacher to student, from student to student, and
even from student to teacher. In one instance, Jessica described a situation in which a
kindergarten child suggested a sorting rule that Jessica herself had not considered.

So we were talking about sorting and I had some children come up and they were
demonstrating how they could sort these in different ways. And this little girl came up
and did this crazy sort and I was like ‘Oh, oh. She’s having difficulty with understanding
the concept of sorting.’ And I cannot remember what it was, but it was something that I
completely did not even see. And, quite plain as day she sort of turned to me and said,
‘Well it is their belt buckle!’ …And she was completely right! (Jessica, interview 1)

A challenge of teaching through inquiry is that teachers need a solid base of both
mathematical and pedagogical knowledge in order to develop rich problems for inquiry-based
lessons, and then to facilitate productive discussion as students solve the problem and share
solutions. While Jessica felt comfortable teaching mathematics at the kindergarten level, her
unhappy experience as a mathematics student left her with the feeling that she would have
difficulty teaching a higher grade because of her limited mathematical knowledge. This is
unfortunate, since Jessica is a very capable teacher in other respects.

I do not feel threatened with [kindergarten,] but certainly, if I were teaching a higher
grade… I just wish that I had the knowledge base and the learning myself of maybe the
newer methods of teaching…. I think had I maybe not learned that there was a right way
and a wrong way, and that there were different ways of coming up with the answer. And
showing your work, explaining your work, explaining how you know, hearing other
children explaining how they know so maybe you have that ‘Aha!’ moment…. I think
maybe if I had had that, maybe I would not feel so threatened by teaching, you know,
Grade 6 math or something like that. Because the thought of it just scares the pants off
me! (Jessica, interview 1)

Another challenge of inquiry-based teaching is time. Inquiry-based lessons do not have to
be long, but they do generally tend to take longer than direct-instruction lessons because of the
need to follow the different paths students discovered while solving the problem. Presenting
multiple solution strategies usually takes much longer than simply identifying a right answer.

Jessica commented that inquiry-based lessons can be more time-efficient than direct
instruction, even though more time is spent on exploration and discussion.
Some teachers feel there is a rush to get through lessons, and so by teaching through direct instruction, they believe they are getting more done—and done in the right way! They feel they can teach the material by telling and showing the students everything that they need to know, rather than by them just haphazardly finding a solution. But really, if a good inquiry lesson is planned, it can cover everything a direct teaching lesson can and more: because the students’ exploration can open the door to new questions and deeper understanding of concepts – not just a tell-all that will be forgotten in a few weeks’ time. (Jessica, survey response)

When asked if inquiry-based teaching is more difficult for teachers than direct instruction, Jessica summarized her thinking this way:

I think it is just a different way of posing the problem. It means maybe relinquishing some of your responsibility in terms of taking over the lesson, and allowing the lesson to go into the hands of the students. For them to kind of see where things go. It is just maybe changing your wording when it comes to posing the question, and allowing students more responsibility. … I definitely do not think it is something that is really far-fetched…. You just need to think about what it is that your goal of the lesson is, and then think of a different way of wording it and allow the students a chance to give it a go. (Jessica, interview 1)

4.4.1.2 Jessica’s Kindergarten Lesson

As a kindergarten teacher, Jessica uses inquiry in her classroom all the time. For the project, she chose to plan a lesson she could use as a summative assessment after a sorting unit. The central question she chose for her lesson was: What different ways can you sort these buttons?

Jessica was the most experienced teacher in the group, and has often worked with sorting before. However, this lesson gave her an opportunity to reflect on the difference between two-way sorting and sorting into multiple groups.

In a two-way sort, students simply decide if an object does or does not have a given characteristic (e.g., red/not red). In a multiple-group sort, students create more than two groups. An example might be sorting toy animals into groups that are blue/red/yellow/black/brown. The challenge of a multiple-group sort is that you sometimes get objects that fit into more than one
group. For example, it is difficult to know what to do with an animal that has both black and brown on it.

Jessica’s initial lesson plan used multiple sorting. She planned to demonstrate sorting with a multiple-group laundry sort (e.g., T-shirts/pants/socks, and then use sorting language to describe buttons (e.g., “This button is small, black, and shiny”), and then ask students to sort the buttons. After discussing her idea with me, she re-focused on two-way sorting in her final lesson:

When I look at this button, I see that it is small. But I also see that it is black, oh… and shiny too. It has three different things I noticed about it! Small, black, and shiny! Then pick out another button. And this one is big and I can also see that it is red. It has two things I noticed about it. Hey, I think I can sort these buttons into groups based on things that are the same about them. I can also make a pile of buttons that do not have that feature to show the difference between the two groups. (Jessica, note 132)

In the inquiry phase of her lesson, Jessica invited her students to sort the buttons any way they chose. Her description of the results indicates that most students intuitively chose two-way sorts:

Most of the SKs and a few of the JKS [Senior and Junior Kindergarten students, respectively] were able to come up with sorting based on size, which most called “big and little.” I had quite a few SKs and a few JKS (again) sort buttons that had a ball shape (sphere-like) and ones that were flat. I had other students come up with other ways to sort such as shiny/not shiny, pattern on it/no pattern, holes/no holes, two holes and four holes, even an edge on the button and no edge… (Jessica, note 132)

Writing this lesson helped Jessica learn something about sorting that she had not articulated for herself before—the difference between multiple-group and two-group sorting.

Conceptual change about inquiry-based teaching was also indicated by the increased time and value that Jessica attributed to the consolidation phase of the lesson—an idea that grew out of the video we watched at the launch meeting and the ensuing online discussion about the video. When asked to describe what she gained from her participation in the project, Jessica indicated
that she had developed a greater appreciation for the capacity of inquiry-based teaching to help students learn:

> With the button activity, we had done prior sorting activities and by seeing not me, but actually learning from their peers…I was surprised at how well they did in different categories they came up with and they were able to freely explore with a lot of the manipulatives. … Each child came up with something, either by their own way or whether they learned from observing and doing hands-on. The beauty was it was their way, and they came to their own understanding and learned through play and were able to see different attributes because they had explored other ways. (Jessica, interview 2)

### 4.4.1.3 Thoughts About Online Collaboration

Every teacher in the study group identified time as barrier to participating in the online group. When asked about the challenges involved in the study, Jessica had this to say:

> For me, as the school year progressed, there seemed to be more school-based and personal life responsibilities to deal with. My time constraints increased and my life got almost too busy for me to deal with at times. I am taking two night-time courses during the week and am so busy my other nights (laundry, cleaning, shopping, etc.) that I sometimes did not have time to log on and read, let alone write the in-depth answers I wanted to explain. Add in the reality of planning for other lessons to teach each day at school, prepping for those classes, report cards, ALPs, IEPs, and completing all sorts of paperwork on individual children, and voila – I began to lag behind. (Jessica, survey response)

Despite the time requirements of the study, Jessica found it useful to be able to log onto the website at any time to talk about what was happening in her classroom:

> I think online collaboration can be good for teachers because you can be far distances away or it can be any hour of the day and you can log on and access information. You can pose questions, ask for advice, and chances are someone else has done a similar lesson to what you’re looking for before, and can share insight as to what worked and what did not work with you. (Jessica, survey response)

Jessica also found it helpful to read about what the other teachers in the group were doing. Kaitlyn, who teaches Grade 4, planned a lesson about graphs for the project. When Jessica read Kaitlyn’s lesson, it gave her an idea for something new to try in her own classroom:

> Having looked at Kaitlyn’s lesson, where she had the graphing activity and there were no titles,… I kind of took Kaitlyn’s idea. I had the three pictures of our three castanet creatures (ladybug, frog, duck) and I asked the kids to come up with what my graph
question might have been. They each got to come up and they knew exactly what the question would be: ‘Which castanet do you like the best?’ Then another said, ‘Which castanet did you use?’ And I had one little boy who said, ‘I was not here so I only got to use the frog one. So it could have been, Which one did you use?’ Sometimes if you write just Yes or No and ask what the question could be, you get something interesting. I had one little girl who asked if children liked mushrooms. And then someone else said, ‘Do you have laces in your shoes?’… This all came from reading Kaitlyn’s idea online and incorporating that into what I’m doing. (Jessica, interview 2)

4.4.1.4 Conceptual Changes Demonstrated in Survey Responses

All three teachers completed a Likert-scale question about their beliefs about mathematics teaching on the initial survey, and then completed the same question again on the final survey (see Appendix E). The items were loosely based, in part, on ideas from the CGI (Cognitively Guided Instruction) survey developed by Peterson, Fennema, Carpenter, and Loef (1989).

Jessica’s responses on the initial survey indicated that she holds beliefs that are generally consistent with an inquiry-based philosophy. For example, Jessica believes that there are multiple ways to solve mathematics problems, and that most children can figure out ways to solve unfamiliar problems if the problems are set at an appropriate level.

When the survey was repeated at the end of the project, most of Jessica’s responses remained unchanged. There were, however, a few differences. On Survey 2, Jessica expressed a stronger belief that children find problems easier to solve when they relate to everyday life. At the same time, she expressed a slight decrease in the belief that children needed to be shown how to perform procedures before they could perform them on their own. She also showed a decrease in the belief that children learn best from teacher demonstrations and explanations.

4.4.1.5 Summary: Jessica

Jessica’s goal for the project was to learn more about inquiry-based teaching, and especially about how to use it to help students who arrive in her classroom with very different
levels of ability. Over the course of the project, Jessica successfully developed an inquiry-based sorting lesson and reported on several benefits for her students, including the opportunity to learn from peers and an increased willingness to take risks. She also reported some challenges, such as her lack of confidence in her own mathematical knowledge and the difficulty she experienced finding time for the project while teaching, taking courses, and fulfilling responsibilities in her personal life.

Jessica’s main gains during the project were in the area of mathematical knowledge. She learned about two-group and multi-group sorting as she developed her trial lesson, and also gained some new understandings about addition, subtraction, and fraction operations in the course of our group discussions.

In her closing interview, Jessica said that the project had also helped her become more acutely aware of the importance of waiting for students to discover solutions on their own, rather than showing them the answer too quickly.

I think it is natural for the teacher to want to jump in and show. And what I have learned is that you need to take a step back and allow the kids to explore freely and see what they come up with. I find that inquiry is all around, and for me, trying to stop yourself when you want to tell kids the answer is the hardest. You have to pull back on your own reins. (Jessica, interview 2)
4.4.2 Joan’s Story

Joan is a 55-year-old teacher in a Grade 3 class. She came to teaching after working at other careers and is in her seventh year. Joan is very comfortable with the idea of inquiry-based teaching, especially in science, and has been invited to teach lessons for other teachers to observe.

Joan explained that she initially learned about inquiry-based teaching from professional reading, and then furthered her understanding by taking Additional Qualification courses in science.

When I first started teaching, I happened to come across, for some reason, this Ministry booklet [that] had been put in my mail slot. It was part of a series of booklets the Ministry put out about five or six years ago, and it was teaching using a four-part lesson plan. And the book that got put into my mail slot was actually using this structure and teaching through problem solving. And I read it and it was just serendipity that I got this book. But I thought, ‘This makes a lot of sense. I like this stuff.’ So I started using it quite a number of years ago. (Joan, interview 1)

Joan and Jessica are part of the same teaching “hub.” They both work with a consultant who uses demonstration lessons as a way to help teachers from different schools share their knowledge about inquiry-based teaching. Each month, teachers from seven schools get together and one teacher acts as a host. This teacher co-plans a lesson with the numeracy coach and the teacher and coach teach the lesson collaboratively while the visiting teachers observe.

Joan has also had some experience co-planning lessons with other teachers. She described the experience of co-planning with a colleague who works at another school:

I tend to be what I think of as an intuitive teacher and she’s one of those teachers who is very systematic, very planned. She even pre-plans the questions she is going to ask. She’s that detailed. Whereas I do not. I think I have developed my questioning skills, but she has very set questions where I generally formulate questions in response to what is happening in the classroom and what students are saying to me. So we have two different approaches, both of which I think are valid and work for each one of us. But it is nice now working with someone who is very different to me, very strong but very different, and I think she feels the same way. (Joan, interview 1)
When asked about her goal for this project, Joan said:

It will give me an opportunity to work with something and really hone it. Because what I find is that I do a lesson and then I sit back and go, ‘OK, next time I’ll do this, this, and this.’ And then I very rarely get to the next time because we have moved on. So I think this will slow me down and make me think a lot more carefully about exactly what I’m planning, what I think about how the children will react, and plan for covering…a lot of the responses to whatever we are teaching. (Joan, interview 1)

For Joan, the project was an opportunity to “really think through a lesson very carefully in terms of structuring it” (Joan, interview 1). In fact, Joan’s engagement with the project enabled her to plan not just a lesson, but an entire unit. In the process, she discovered that inquiry can be a lot of fun, and that it can easily be interwoven with other approaches.

### 4.4.2.1 Thoughts About Inquiry-Based Teaching

Joan clearly enjoys teaching mathematics through inquiry, and this was evident from her reports about her previous classroom experiences with this way of teaching.

I had a situation once a few years ago where I had given a problem to a bunch of Grade 1s. We had not done anything like it, but it was a problem where there were 35 rabbits and a child was going to build rabbit hutches. And each rabbit hutch was going to sleep 6 rabbits. How many rabbit hutches did he need? So I put out the cube-a-links and I explained the problem and I let the children get into it. And they all started by doing things with the cube-a-links and trying to sort it out. And of course it is a division problem, but there was a little bit of…they are wondering and you can see what they are thinking. Then one kid made a chain of 35 rabbits, and then he started breaking it off into 6s. And he knew that those last 5 rabbits had to have their own hutch, even when there was a spare bed… And the others are like, ‘Oh yeah!’ And it was wonderful and it was must a small thing perhaps, but it just cemented for me that using problems and working through things like that, children are able to figure out things that are workable and correct, and other children get excited about it…. And I was not up at the front going, ‘First do this, and then this and this.’ It came from them. They are all like, ‘Oh yeah, we can do this.’ And that was exciting. It was exciting because the kids were saying, ‘Yay! I love math!’ (Joan, interview 1)

Over the course of the study, Joan often commented on her students’ engagement with inquiry. On a note posted online, Joan said, “Inquiry should be enjoyable. It taps into learning for
the sheer pleasure of learning and allows everyone to approach the problem or situation from a position of strength” (Joan, note 61).

Along with student engagement, inquiry-based teaching can also lead to improved knowledge and understanding of mathematics. As Joan developed her unit plan for the project, her goal was to help her students develop a body of interconnected knowledge about number—a key benefit of the inquiry-based approach.

In an online follow-up discussion about the videotaped demonstration lesson from the project launch meeting, Joan noted the importance of developing true understanding of a mathematical idea rather than simply parroting something learned by rote:

What I found interesting about the video and something I think the teacher wanted to explore were students who had learned the algorithm or rule and applied it without presenting the underlying thinking as to why that rule worked. She was making them explain why they used the reciprocal instead of just accepting that this is what you do. How often do we hear children say the answer but have difficulty explaining how they understood what to do to solve the problem? To be able to articulate how you reached a conclusion is a valuable skill and it solidifies your understanding of what you've done. (Joan, note 59)

Like Jessica, Joan identified time and teacher knowledge as two of the main challenges teachers face as they try to incorporate more inquiry-based teaching into their practice. And, like Jessica, Joan saw inquiry not just as the cause of the problems, but also as a potential solution.

While agreeing that finding time for inquiry can be difficult for teachers, Joan commented that inquiry can be liberating because it allows you to think beyond the preconception that mathematics lessons should last about 40 minutes to 1 hour.

I am one of those teachers that allow myself the flexibility in my timetable that, if we are doing a math lesson and I have scheduled two periods for it, it is quite obvious at the end of two periods that the children are really engaged. If they want to continue, I let them. So I have been known to have three periods of math in a day. We will have done two periods, it is lunch, and they come back and we finish up and we look at it all. And that is the luxury of primary. (Joan, interview 1)
Joan also saw inquiry as a vehicle for improving teacher knowledge. In a classroom where people are solving problems together, the teacher can become a learner, and the students can become teachers.

Inquiry is about learning and that includes my learning just as much as the students’…. I like being able to say ‘I do not know, how do we test that?’ instead of saying ‘That is right!’ or “That is wrong!”… Interestingly, that can lead to greater effort on behalf of the students because their work has become an authentic adventure into the unknown. (Joan, note 61)

4.4.2.2 Joan’s Grade 3 Lesson

Joan’s initial goal for the project was to slow herself down so she could think through the planning phase for a lesson. As Joan began thinking more about inquiry-based approaches, she began to see ways to incorporate inquiry into activities she was already planning for her classroom.

Initially, Joan expressed doubt about how to plan ahead when she did not know what else would be happening in her classroom when it was time to test-teach the lesson.

Coming up with a lesson is proving to be extremely difficult presumably because right now there is not a context for it to fit into. In the past, my ideas have been related to what is happening in the classroom and are usually in response to what students are doing right then and there. Perhaps that is why I am not a huge fan of math book ideas; the chances of seeing something that ‘fits the bill’ of being immediately usable are low. I feel like I am trying to create a specific lesson in a vacuum instead of in relationship to something that is already happening. (Joan, note 78)

After considering several possible ideas, Joan identified an area of difficulty for her students, and this proved to be the starting point for a series of lessons.

In grade 2 children are expected to know: $2 + 4 = 6$ is related to: $6 - 2 = 4$ etc. What I have found is that grade 2 and 3 children do not retain this tidbit of information, and they do not use it to solve for missing numbers. The EQAO on the other hand always include missing-number questions so obviously the Ministry thinks grade 3 students should at least have memorized this. I tried various ways to explain this information last year and yet some children just could not grasp it. I wonder if having them “find” the math equations themselves would help them see and better internal the relationship. During
Joan realized that having a good grasp of number lines would help students solve for missing numbers. Rather than simply demonstrating how a number line works, she decided to see if she could get the students to introduce the idea themselves through inquiry.

Tomorrow I have a lesson set up: Show me the relationship between: 80, 96, 150, 24, 5, 45 and some other numbers. For those students who are familiar with number lines this should not take too long but hey then we can move on to use number lines to add and subtract. Finally we are starting number sense. (Joan, note 101)

The lesson was a great success! At first, students tried comparing pairs of numbers. Then, with some prompting from Joan, they tried to find a way to organize all the numbers in order. Finally, they came up with the idea of ordering the numbers along a line.

I had so much fun I cannot wait to return to this lesson tomorrow. Oh, I think the students had fun too… At the end of the class – lunch was looming – they did a gallery walk type thing to look at each other's work. Tomorrow I will introduce some anchor numbers that have the same increment and ask them to add these numbers to the mix. No doubt they will slip them in where they fit in terms of value but I will question if that is accurate and how could they make it look accurate. Anyhow today was an exciting math day. (Joan, note 105)

As Joan continued to develop her unit, she continued to report regularly. Her own growing enthusiasm, and her students’, was very evident in her writing.

What started as a slightly bizarre idea has opened up the floodgates of teaching and learning opportunities. My students are not always fully engaged (I would like to be that good but gosh I am human) however since we started this adventure they’ve been pretty close to it. (Joan, note 113)

The lesson Joan formally developed for the project addressed her initial idea about the relationship between addition and subtraction. She provided paper, markers, and cube-a-links and posed this question: “What is the relationship between the numbers 6, 17, and 23?” Her students created cube models and number lines and discovered on their own that they could
create related addition and subtraction sentences from their models. Then they did a gallery walk, moving from table to table, and followed with a consolidation discussion.

One student noticed that there were small, medium and large numbers in all of the equations. This led nicely into ‘boxing’ the numbers using specific coloured markers (red for 6 or small; yellow for 17 or medium; and blue for 23 or large)….With the numbers “boxed” we used the equations and their colour codes to build the same relationships with the pre-cut coloured squares. Students (still in their working groups) were told to add in the appropriate math symbols +, −, =. They were pretty diligent about putting the coloured and differently-sized boxes into the right order. We ended up with 6 sheets with the arrangement of boxes representing related number sentences. The ‘abstract’ representation of the math sentences allowed us to focus in on what was happening in the sentences. Children noticed that the biggest number was at one side of the equation when it said + and at the opposite end when it said −. They noticed that the biggest number was not in the “middle” location (in between an operational sign and =). After a bit more looking they realized that the yellow and red boxes simply changed places whether it was around a + sign or the = sign. (Joan, note 119)

Joan was excited to discover how natural it felt to move back and forth between inquiry-based teaching and direct instruction and practice. After the initial inquiry, Joan demonstrated how to use boxes and operations signs to create equations, and then gave students an opportunity to explore the different equations they could make with 6, 17, and 23. Joan then followed up with a practice opportunity to create equations with other sets of related numbers (e.g., 12, 23, and 35). Soon, the students were asking for “harder” sets of numbers (e.g., 27, 39, and 66). Soon, they moved on to use x instead of a coloured box to represent the unknown number. (Joan, note 119).

Reflecting on the transition later, Joan expressed the belief that inquiry was most useful for exploring new ideas, and that balancing inquiry with direct instruction and practice gave her a way to draw as much learning as possible from class activities.
4.4.2.3 Thoughts About Online Collaboration

Perhaps one of the reasons why Joan became so engaged with this project was that the online group provided a connection with other teachers who shared her professional interests. Joan related her experience in the project to a similar experience with an online quilting group.

[The quilting group] has become a very important conversation for me as I articulate art principles and constructively analyze what we’re doing well and what we need to work on and receive feedback on my own work. In teaching, it is hard to find people who are willing to invest the time and are at the same or a similar stage in their understanding of teaching whatever subject. (Joan, survey response)

The online nature of the project meant that most of the group’s conversations were written rather than oral. In her end-of-project survey, Joan commented on how the need to articulate ideas in writing helped her focus her thinking:

[Posting on the website] means that ideas have to be articulated in writing, which generally requires the writer to think more carefully about what they want to express, further consolidating the ideas for themselves as well as for others. (Joan, survey response)

Like Jessica, Joan found that talking to others about what was happening in her classroom helped her build her own knowledge of mathematics. For example, Joan’s goal for her trial lesson was to help her students discover the inverse relationship between addition and subtraction, and many did. But one group of students took things in quite a different direction. They began looking for one number they could count by to reach all three numbers (that is, a common factor). They tried 2s, 3s, and 4s, but none worked for 17 or 23. At this point, the group decided to take a different tack (Joan, note 138). In discussing the lesson after the fact, Joan realized that this might have been a good opportunity to explore prime numbers, a topic not usually covered in Grade 3.
At the end of the project, Joan expressed frustration with the fact that teachers often seem to feel more competitive than collaborative, perhaps because they feel that they are constantly being evaluated by administrators, parents, students, and the media.

When the new curriculum came in, there was so much resentment, and there was EQAO and OCT and it all came in at the same time. Teachers are not feeling like a cohesive collegial unit. We are not collaborating and sharing. There is a lot of isolation within a staff. When I was a beginner teacher, nobody came to my door to say, “How can I help?” Or just came in to check up on me…. A teacher who was away came back and apparently she asked another teacher for some unit plans, and that teacher said no, because she had put the work into it, so why should she give them away? What do you get from doing that? I do not know how the education profession is going to change itself because it does need to shake itself up. It has teachers pitted against administration because of evaluation and strict rules that prevent a lot of professional development from happening. Restrictions on being able to enable teachers are incredible. So I do not know how you go about changing it. So far, it only changes for individuals because it has to be a bottom-up thing. Teachers look for someone to work with. If you cannot work on your own in teaching, you’re going to have a hard time. (Joan, interview 2)

Joan also commented on the fact that she sees teaching as a “culture of evaluation” and that this shapes the way teachers interact in professional-development situations.

For all that we are told to “share” in teacher’s college, amazingly in the field that practice is rare as teachers hold onto ownership of their unit and lesson plans and their experiences. I’ve mentioned the teaching profession being a culture of evaluation and I’m becoming increasingly convinced that teachers are constantly scared of being judged. With that pervasive attitude and the limitations of time, it is difficult for teachers to engage in an ongoing conversation. (Joan, survey response)

**4.4.2.4 Conceptual Changes Demonstrated in Survey Responses**

Like Jessica’s, Joan’s responses to Survey 1 indicated beliefs that are consisted with an inquiry-based philosophy. About half of Joan’s responses changed on Survey 2, generally indicating increased support for inquiry-friendly ideas.

On Survey 2, Joan showed more support for the idea that most children can figure out how to solve an unfamiliar math problem at their level. Joan also showed a significant decrease in support for the idea that children get distracted if they talk to each other while working on
math problems. This may reflect her observations of the learning that resulted from student
conversations during the trial lesson.

On Survey 2, Joan also showed less support for the idea that children benefit from using
concrete materials to solve problems. While inquiry is often linked to manipulatives, this could
reflect Joan’s experience with a trial lesson that began with cube-a-links, but moved swiftly to
more abstract representations of number relationships.

One surprising result was that Joan increased her support for the idea that there’s always
one best way to get the right answer. Since Joan was very engaged with inquiry during the course
of the project, this is perhaps indicative of her belief that sorting through multiple ways to do
something helps students find the one that works best. It seems unlikely, in view of her work on
the project, that she intended to suggest teaching only one way to do things.

4.4.2.5 Summary: Joan

Joan’s involvement in the research project helped her discover more about how ideas we
often teach separately in mathematics are connected (e.g., number patterns and
addition/subtraction), and also how she can incorporate inquiry as a more regular part of her
classroom activities, rather than as a special event. When asked to describe what she gained from
her participation in the project, Joan said:

I think I have gotten a better idea of how to use inquiry and dovetail it with other teaching
techniques in order to get the math across…. I had things on the board, and I was
questioning. It was not me telling them, but because I knew where I wanted to go, I used
questions. I was shocked at the kids’ response. They would do an inquiry and be excited
working in groups and then I would build with them sitting with clipboards, continuing,
demonstrating, practising, and embedding, and giving them various skills or tools. This
term it became me going from inquiry into teaching and coming back out and then going
back and forth, and the student response was fantastic. They would complain about
lunchtime because they would rather do math. With our hubs, the big question was
always what do we do next. Now I feel I have a better idea how this all works. (Joan,
interview 2)
4.4.3 Kaitlyn’s Story

Kaitlyn is a 27-year-old Grade 4 teacher with three years of classroom experience. She is very involved in professional-development activities, and is very interested in learning as much as she can that will help her excel in teaching.

As a new teacher, Kaitlyn sometimes faces challenges as she tries to figure out how to order and present new concepts in a way that makes sense to her students.

Because I am pulling from so many different places, I struggle with how to organize my unit properly in a way that makes sense, where I am moving from something simple to something that is more complex. I also struggle with knowing that I have a gifted student in my class, and making sure that he is challenged, and that he is not just given additional tasks…[So I struggle with] how to challenge them, and also making my lower kids, my identified kids, not overwhelmed with what is going on. (Kaitlyn, interview 1)

Kaitlyn learned about inquiry-based teaching mainly through professional reading and professional-development activities. She is confident in her own mathematical knowledge, and feels that studying inquiry-based methods has helped to develop a solid conceptual understanding: “Math just clicks for me, and I have so much more of an interest in it now that I understand why we follow those mathematical rules that we do.”

In addition to Kaitlyn’s involvement in this project, she is also involved in a project called iCLIPPS. Teachers in the iCLIPPS program work with a numeracy coach and a partner teacher to plan and teach inquiry-based lessons.

When asked about her goal for the study, Kaitlyn said:

I hope to have a greater confidence in my abilities to mix traditional methods of teaching with more problem solving, and just put problem solving and inquiry-based learning into my units at appropriate times. That way I know that they have some explicit instruction, but then they also have some opportunities to explore and figure out answers on their own… Once I hear from other people, I like to borrow ideas and try them out and see how they work. (Kaitlyn, interview 1)
Kaitlyn feels that teacher’s college did not adequately prepare her for teaching mathematics. In her end-of-project interview, she described how she seeks out professional-development activities in mathematics because she has found that these are a good way to fill the gaps left by teacher’s college:

When I graduated teacher’s college, I really do not feel like I was prepared for teaching mathematics. The teacher who taught my mathematics course was, well, she liked manipulatives but other than that she was very traditional about a lot of things. We did not know about the 3-part lesson. There were not any well-known names in math that were brought to my attention. I did not know who Marian Small or Van de Walle or anyone like that was until I got into a classroom and I had to look into it myself. (Kaitlyn, interview 2)

4.4.3.1 Thoughts About Inquiry-Based Teaching

Like Jessica, Kaitlyn was interested in the value of inquiry-based teaching for differentiated learning. In a survey response, Kaitlyn identified this as a key benefit of inquiry:

Students with a wide variety of abilities can work to complete these problems. We can provide multiple entry points to engage various learners. (Kaitlyn, survey response)

Kaitlyn has found that the most effective way to group students for inquiry is by similar ability level.

I kind of got to a point where I did put them in more equal-ability groupings, as opposed to at the beginning of the year, where I put them in mixed-ability groupings. So you would have your higher kids sort of write the answer to the question and your lower kid just kind of staring at them and pretending that they knew what was going on when they did not. And once I started putting them more with kids who were more on par with them, it was when they actually had those conversations and really took those risks. (Kaitlyn, interview 1)

Kaitlyn finds that the way her students respond to inquiry varies from year to year. In an interview, she described her experience with two classes that responded quite differently:

[My class last year] loved worksheets and workbooks. They wanted to have pages. They would love to have pages and pages of work to do… They just liked to keep busy and feel they were on task and getting through that. So I guess it was the satisfaction of being successful and getting to do that. But I can already tell that my class this year is the opposite. They will want nothing to do with that kind of stuff. (Kaitlyn, interview 1)
Inquiry lessons can be daunting for students who like rules and algorithms that help them get the right answer with a minimum of effort. For children who are not accustomed to inquiry, understanding mathematics may have less value than simply getting the job done. Kaitlyn described a time when this idea came up in her class:

One day last year, we were learning to multiply by 10s and I had all the base ten blocks and I was trying to model it for them, and show them what happens when I multiply a number by 1 and when I multiply a number by 10 and I had this one girl stop me and she said, ‘Oh now Miss T., you’re making it too difficult. When you multiply by 10, you just add a 0 to the end.’ And all the other kids were so excited. ‘Yeah, that is what we learned in Grade 3. You’re making it too hard…’ They loved that rule and it just works. (Kaitlyn, interview 1)

4.4.3.2 Kaitlyn’s Grade 4 Lesson

Kaitlyn’s lesson plan was to present two graphs that showed the same data—one with a gap in the scale and one without. She used unlabelled graphs because she felt that this would be a good way to create an opportunity for open-ended problem solving and multiple answers. She asked her students to compare the graphs (in order to see if they would notice that the data was the same) and to identify how a business might use each graph to advance its interests.

Kaitlyn ran into some difficulty when she tried out her lesson in her classroom. Her students had limited experience with business and found it hard to think of ways that a business might use a graph.

When they had to think about businesses, most groups had a difficult time. When looking at the ‘growing’ chart, some groups decided that it showed how much money the business made each year, and that they would like to share this with their customers, so they could see that their business was doing well. All students had difficulty explaining why someone might want to show the graph where all the bars looked approximately equal. I started to give students some prompting about a possible topic for the graphs, such as the environment, to stir up ideas. One group was able to tell me at this point that Graph B might be showing something that you would not want to see grow, like the amount of pollution that they make. Other groups found this part challenging. (Kaitlyn, note 126)
Kaitlyn solved the problem by inviting suggestions for other possible contexts from the class, and expanded beyond business to areas such as sports (which her students love) and the environment (which they were studying in science). In her final write-up of the lesson, Kaitlyn noted that, if she used the lesson again, she would use a context other than business for the graphs.

Kaitlyn is still struggling to balance her own need for control with her understanding that when students are given more leeway, the lesson may be enriched by unexpected new perspectives. Kaitlyn’s lesson was constructed around a series of step-by-step teacher-guided questions, rather than a single open-ended question that allowed for multiple student entry points. Her guiding questions were:

- What conclusions can you make about Graph A?
- What business would want to share this data with its customers? What might this data represent?
- How are Graphs A and B similar?
- What does the scale tell you about your graph?
- What business does Graph A belong to? Graph B? How do you know this?
- If you were working for a large business, when might you like to use Graph A? When might you like to use Graph B?
- Label each graph with a title and a legend to show how the data might be used in an advantageous way.

(Kaitlyn, note 126)

This series of questions illustrates Kaitlin’s need to direct the lesson, rather than simply posing a problem and then stepping back to allow the students to follow their own paths. In her
final written version of her lesson plan, Kaitlyn made an interesting choice. She decided to bold
the final question about labelling the graph. This would seem to indicate that Kaitlyn grasps
intuitively that this instruction is the true heart of her central question, which could have been
more simply presented as: How can you label each graph with a title and a legend to show how
someone might use it to prove something?

With a more focused central question, there would have been no need to ask explicitly
about similarities and differences, or about the scale, or about possible contexts, since this
information would likely have flowed more naturally from the students during the inquiry and
consolidation phases of the lesson.

A simple, straightforward central question often works better than one with multiple
prongs. Focusing students on the idea of adding a title and legend might have been a simpler way
to prompt a discussion of the differences between the two graphs, without asking explicitly.
Finally, leaving the context open might have helped students come up with more ideas, because
they could have drawn on their own interests rather than being constrained by their limited
knowledge of business—a problem Kaitlyn identified when she described what happened when
she tried out her lesson. Interestingly, none of the more experienced teachers in the group
commented on Kaitlyn’s choice of central question, even when invited to offer suggestions.

In the consolidation phase of her lesson, Kaitlyn asked, “Should the scale of all graphs
start at 0?” Most students thought it was fairer to start at 0 all the time, but a handful felt that the
people making the graphs should be able to choose. Kaitlyn concluded her lesson by having the
students create an anchor chart about graphs called “Good Mathematicians know…” and add it
to their mathematics journals.
Kaitlyn’s experience with writing, testing, and revising the lesson helped her learn more about data analysis, and also about how to structure a successful inquiry-based lesson. When asked to describe what she gained from her participation in the project, Kaitlyn said:

I feel like now that it is my third year teaching the same grade, I sort of have a better understanding of the specific expectations of the curriculum more…So I felt more comfortable with the curriculum expectations, that I could take those and then find problem-solving opportunities for them, so, to kind of look deeper into the curriculum. This project also just kind of gave me that drive, that energy, to want to kind of do a little more with that, to make sure that I was incorporating problem solving more… I was just putting in more of an effort into really finding those opportunities and to really look deeper into the specific expectations that we had, to see how I could really challenge some of the kids. (Kaitlyn, interview 2)

4.4.3.3 Thoughts About Online Collaboration

For Kaitlyn, online exchanges are a good way to build a repertoire of lesson ideas.

I’ve worked with other teachers online for AQ courses in mathematics, so I always like to hear other people’s ideas and what they are saying, and just kind of if they can relate to a similar lesson they have taught, or just kind of know that helpful little tip. I have always found it useful to have them, so I could look to different ideas or suggestions, and also see the lessons people were putting forward and kind of borrow from some of those ideas. (Kaitlyn, interview 2)

Like Jessica and Joan, Kaitlyn found the project fairly demanding in terms of time. She commented that she sometimes found it difficult to remember to log in when things got busy.

Because the comments need to be accessed directly by using the weblink and you cannot see otherwise if you have messages, I found it difficult to remember to login and check regularly. With online AQ courses, because you were working with specific deadlines, and a Pass/Fail option, you remember to log in more often. (Kaitlyn, survey response)

Perhaps a weekly email sent when the discussion questions were posted might have encouraged Kaitlyn to check in more often. Kaitlyn also noted that, at her school, the project’s website was initially blocked, so she could only access it at home. Together, these factors meant that Kaitlyn spent significantly less time online than the other participants in the group.

I tried to [access the website] at school, but this site was blocked…. They seem to restrict so many different sites and you never know why, because a lot of them are math games
for kids. So you have it at home, and then you go to school and the site’s blocked. So I requested that they unblock it so I can access it at school too. (Kaitlyn, interview 1)

4.4.3.4 Conceptual Changes Demonstrated in Survey Responses

Like Joan and Jessica, Kaitlyn responded to the belief statements in ways that were generally consistent with an inquiry-based philosophy. However, there were a few areas where Kaitlyn’s responses differed from those of her colleagues. While Joan and Jessica indicated a belief that children are comfortable with problems that have more than one answer, Kaitlyn thought children would find these problems confusing. Also, while Joan and Jessica believed that children can figure out mathematical procedures on their own, Kaitlyn said that teachers have to show children first.

On Survey 2, Kaitlyn expressed less support for the idea that basic facts are the building blocks of mathematics, and that there’s always one best way to get the right answer. She expressed a stronger belief that children can figure out ways to solve unfamiliar problems at their level, and that children find problems easier to solve when they relate to everyday life. She was also more accepting of the idea that children can handle problems with more than one answer.

At the end of Survey 2, each teacher was asked if her view of the essence of great mathematics teaching had changed since the beginning of the project. While Joan’s and Jessica’s responses reiterated ideas from Survey 1, Kaitlyn’s response indicated a shift from viewing herself as a mathematics learner (“Math just clicks for me…”) to viewing herself as a mathematics teacher. She expressed the hope that, as a teacher, she would be willing to take risks, would provide tasks with different entry points to engage a variety of learners, would provide opportunities for students to practise and apply new learning, and would ensure that her students conceptually understand what they are doing.
4.4.3.5 Summary: Kaitlyn

Kaitlyn’s goal for the project was to learn how to mix inquiry-based methods and traditional methods in an appropriate way. In her second interview, Kaitlyn mentioned that she felt the project had helped her move in this direction. However, her interview comments also indicated that she still saw inquiry more as a way to challenge her students than as a staple of everyday practice.

For Kaitlyn, changes in thinking about inquiry were demonstrated mainly through survey responses. Initially, Kaitlyn’s responses indicated a slightly more traditional view than those of the other participants. She was wary of problems with more than one answer, and felt that teachers needed to show children how to perform new procedures before the children could do them on their own. At the end of the project, Kaitlyn indicated a more inquiry-supportive view. She expressed less support for the idea that there’s always one best way to get the answer, and a stronger belief in children’s ability to figure out how to solve problems and perform procedures without being shown how to do so first. It seems fair to conclude that these changes could have been due, at least in part, to Kaitlyn’s experiences in the project.

4.5 Summary

Each of the three participants began the project with a strong understanding of the basics of inquiry-based teaching, and some professional-development experience in mathematics. All three regularly commit a great deal of time and effort to their own professional learning. They all spend time outside the classroom doing professional reading and online research to find ideas they can use in their mathematics classrooms, and they have all taken courses and workshops designed to improve their teaching.
The teachers’ involvement in this project provided an opportunity for longer-term professional development based on their individual strengths and needs. It also gave them an opportunity to reflect on their own experiences as students, and how those experiences shaped the way they teach in a reactive way: all three are determined to provide their students with a better experience than the one they remember.

For the teachers in the study group, online discussion and collaboration provided a source of affirmation. Simply belonging to a group that was brought together by a shared value for inquiry gave everyone more confidence that they were making the right choices in the classroom. Group members expressed support for one another in online comments.

All three study participants believe that student engagement is an important ingredient in learning, and that teacher engagement can stimulate student engagement. As the participants tried out their lessons in their classrooms, all three discovered that an inquiry-based approach can help to build this engagement.

While the teachers all demonstrated gains in mathematical knowledge, pedagogical knowledge, and facility with inquiry-based teaching during the project, they did so more through interaction with me and with their own students than with each other. Over time, it became evident that collaboration among participants does not happen naturally in an online learning environment. Instead, it must be fostered deliberately.

The project also revealed a reluctance on the part of the participants to offer constructive criticism about each other’s lesson ideas. Even when encouraged to make suggestions, these teachers did not do so. Since collaboration often leads to better results than individual efforts alone, it would be worthwhile to do further research to find out more about whether teachers in general behave this way, why this happens, and what can be done to change it.
Chapter Five: Discussion and Interpretation of Findings

5.1 Introduction

This chapter revisits the research questions posed in Chapter 1 and explores how data from the case study answers those questions. It also reviews the major findings from the study, and suggests areas for further research in the area of online professional development for mathematics teachers.

5.2 The Research Questions

This section focuses on the research questions posed in Chapter 1. Those questions were:

1. What benefits did the teachers identify in inquiry-based mathematics teaching?
2. What kinds of obstacles and difficulties did the teachers encounter as they implemented inquiry-based teaching in their classrooms?
3. What benefits did the teachers perceive in online collaboration with other teachers?
4. What kinds of obstacles and difficulties did these teachers encounter as they engaged in online collaboration?
5. How did each teacher’s understanding of inquiry-based mathematics teaching change over the course of the project?
6. What insights and observations did this process yield about how to design other professional-development initiatives like this one?

5.3 Discussion of Each Research Question

5.3.1 What benefits did the teachers perceive in inquiry-based teaching?

The study participants identified three main benefits of inquiry-based teaching: improved understanding of mathematics, heightened student engagement, and suitability for differentiated learning.
5.3.1.1 Improved Understanding

Inquiry-based teaching has its roots in constructivism, a theory that learning is a process driven by the learner, who discovers new ideas through practical activity and discussion with others (Vygotsky, 1978). When children explore mathematics by solving problems with others, they add new ideas onto their existing framework. This helps each child develop a broader understanding that is more thorough and longer lasting than when learning occurs simply through watching and listening.

Jessica found that her students learned more from sharing ideas about sorting with one another than they did by watching her examples. Joan found that her students showed an improved understanding of number relationships, and were able to use their new knowledge to readily solve algebra problems with missing numbers. Kaitlyn’s students learned to read graph scales with a more critical eye, and discovered some real-life applications for the mathematics they were learning about at school.

5.3.1.2 Increased Student Engagement

Student engagement in mathematics is a strong predictor of success. In an EQAO survey, only about 50% of Ontario students in Grade 3 and Grade 6 reported that they like mathematics most of the time, and that they feel that they are good at mathematics most of the time (EQAO Provincial Results for Student Questionnaire, 2009-2010).

All three teachers in the study reported high levels of student engagement with the inquiry-based lessons they designed for the project. Joan’s class, in particular, became very highly engaged with a series of lessons on number relationships. This supports Zion and Sadeh (2007) who found that students are more motivated if they are allowed to be curious about open-ended situations.
Also evident in Joan’s class was the fact that student engagement and teacher engagement go hand in hand. Joan’s students recognized their teacher’s interest, and became more engaged with her lessons as a result. The students’ obvious enthusiasm then encouraged Joan to try more experiments with inquiry, and the results were positive for everyone. Joan reported that her students sometimes complained about having to stop for lunch, because they wanted to continue with math (Joan, interview 2).

Both Kaitlyn and Jessica also reported that their students became engaged by inquiry-based approaches. However, Jessica also demonstrated an increased level of engagement with mathematics as a teacher. At the beginning of the project, Jessica commented in a survey that mathematics made her feel “scared” and “stupid,” because of her unpleasant memories of mathematics as a student in elementary and high school. She also expressed the belief that some people just cannot do mathematics because of their “brain ability and genetic make-up.” In her final interview, Jessica reported that she felt that the project had helped her become a better mathematics teacher because she had learned to allow the students to discover their “own rules and ways” for doing things. Her online comments showed an increased belief that she might have had more success with mathematics as a student if she had had the opportunity to find her own ways of doing things.

5.3.1.3 Suitability for Differentiated Learning

One of the reasons why inquiry engages students is that inquiry problems are designed with multiple solution strategies in mind, so students with very different levels of mathematical understanding can approach them in developmentally appropriate ways. Traditional teaching tends to reach the students in the middle ability range, while students on the outskirts are either left out entirely, or are taught separately from the rest of the group. Inquiry makes it possible for
a single whole-class activity to address a wide range of needs. Students bring what they know to the problem, share ideas with one another, and add new learning to what they brought with them.

This supports the approach suggested by Small (2009), who proposes ways for teachers learn to create open-ended problems that are “helpful to the many children who come into classrooms with highly differentiated mathematical preparation, skill, and confidence” (Small, 2009, p. ix).

5.3.2 What kinds of obstacles and difficulties did the teachers encounter as they implemented inquiry-based instruction in their classrooms?

The study participants identified four areas of difficulty they faced as they tried to implement inquiry-based instruction: finding time for inquiry, overcoming teacher and student preconceptions, coming up against the limits of their own mathematical knowledge and training, and balancing the demands of multiple subjects as they tried to prepare their students for next year’s teacher and this year’s EQAO tests.

5.3.2.1 Finding Time for Inquiry

Teachers fight a constant battle against time. The curriculum requires them to address a long list of learning expectations in many subject areas over the course of a school year with about 190 instructional days. In Ontario, the Grade 3 curriculum has 65 expectations for mathematics alone, and many of these take far longer than a single mathematics period to address.

Teachers must find time not only for teaching, but also for planning, record keeping, parent meetings, staff meetings, recess and lunch supervision, and organizing extracurricular activities. A typical school day lasts about 6 hours, with interruptions for morning recess, afternoon recess, and lunch. Children are constantly coming and going because of their activities
with other teachers or education assistants, or their involvement with clubs, teams, choirs, or bands. Time must often be found for special events such as field trips, concerts, plays, sports tournaments, and school assemblies that, for all their advantages, take precious hours away from day-to-day learning.

The teachers in the study mentioned two difficulties with respect to time: difficulty finding enough time to plan inquiry-based lessons, and difficulty fitting inquiry-based lessons into the rhythm of the classroom. This supports Keiser and Lambdin (1996), who found that using a problem-based approach requires more time for discussion than a more traditional approach. Several teachers also mentioned the temptation to skimp on the consolidation stage of an inquiry lesson because they find they are running out of time. As Jessica noted, however, students who learn through inquiry retain more of what they learn. As a result, the increased time required for discussion can be balanced by less time spent on practice activities.

**5.3.2.2 Overcoming Teacher and Student Preconceptions**

Peterson, Fennema, Carpenter, and Loef (1989) found that teachers’ beliefs about mathematics and instruction were reflected in their teaching methods. The study found that teacher beliefs fell into one of two groups. Teachers with a cognitively based perspective believed that children should construct their own mathematics knowledge through problem solving, and that instruction should be sequenced in a way that builds on children’s developmental understanding and facilitates the construction of new knowledge. Teachers with a less cognitively based perspective believed that children should receive mathematics knowledge from others, that mathematics skills should be taught in isolation from problem solving, and that instruction should be sequenced to reflect the structure of mathematics, rather than on students’ individual understandings (Peterson et al., 1989).
In order to teach through inquiry, teachers need to adopt a cognitively based perspective. To teachers with a less cognitively based perspective, this can seem like an abdication of teacher responsibility and an abandonment of classroom control. Several participants in the study mentioned that teachers may feel they are taking a risk when they let the children take the lead in a mathematics lesson rather than directing it to where they want to go.

Teaching through inquiry is challenging. Even experienced teachers sometimes find that a lesson has not gone the way they expected it to go. Kennedy (2004) found that teachers would not adopt a new teaching strategy that left them feeling less in control, unless the loss of control was outweighed by gains in other areas, such as student learning and affirmation. All of the teachers involved in the study were supporters of an inquiry-based approach from the outset. And yet these teachers, especially Kaitlyn, still struggled with the issue of how much control to give over to the students.

Teachers are not the only people who find that inquiry-based teaching conflicts with their preconceptions about what goes on in a classroom. Kaitlyn and Joan both mentioned that they sometimes face opposition from students who prefer direct instruction over inquiry. This supports Prestie and Smith (2010), who found that, while many students prefer learning through inquiry, some continue to express a preference for learning “just one way to do it” (Prestie & Smith, 2010, p. 11).

5.3.2.3 Limits of Teacher Mathematical Knowledge and Training

In order to teach successfully through inquiry, teachers require a solid foundation of mathematical knowledge to build on, a desire to learn more, and confidence in their ability to understand and integrate new mathematical ideas as they encounter them. Time spent in teacher training is not enough, on its own, to provide new teachers with a solid knowledge base about
mathematics (Menon, 2009; Ball, 1990), or a positive disposition towards new learning in mathematics (Ball, 1990). Instead, ongoing professional development is needed to help teachers improve their mathematical knowledge, as well as their pedagogical knowledge of how children learn about mathematics.

The teachers in the study benefitted from the opportunity to exchange mathematical and pedagogical knowledge over the course of the study. This began at the project launch meeting, where Jessica’s story about learning to do subtraction with regrouping as an adult led to an exchange of ideas about other subtraction methods. The video lesson about fraction operations also led to an exchange of ideas about algorithms for dividing with fractions.

Each teacher also identified new learning that took place during the lesson-writing process. Jessica, for example, mentioned her increased understanding about the difference between two-way sorts and multiple-group sorts, while Joan mentioned new learning about how exploring number relationships on a number line can help students solve addition and subtraction problems with missing numbers. Kaitlyn commented on the need to focus the key question in an inquiry-based lesson, and the importance of considering students’ prior knowledge and experiences (in this case, about business) when planning a lesson context.

5.3.2.4 Meeting the Expectations of Others

In order to meet the expectations of others, teachers need to balance the demands of teaching mathematics with those of teaching a wide range of other subjects, including language arts, social studies, science, music, visual arts, French, dance, media studies, and more. Several teachers in the study mentioned their feeling that they are being judged by other teachers according to how well their students perform in subsequent years.
Kennedy (2004) identified teachers’ obligation to cover all of the content needed to prepare students for the next grade level as something that shapes their instructional choices. Kennedy found that teachers will not adopt new teaching strategies unless it becomes clear to them that these strategies will make it more likely, not less likely, that future teachers will judge them positively.

5.3.3 What benefits did the teachers perceive in online collaboration with other teachers?

Because teachers spend much more time each day with their students than with each other, teaching can be a very isolating profession (Lock, 2006). Simply finding time to talk with colleagues about lesson ideas can be difficult to manage. The teachers found that being part of an online group provided a valuable opportunity to share ideas in an environment where there was no professional competition, and where there were no administrators or parents judging their comments.

The participants particularly enjoyed the asynchronous nature of the project website because it allowed them to post their ideas whenever it was convenient for them. Joan, in particular, found that having to compose her thoughts in writing helped her reflect on and consolidate her ideas.

The teachers also enjoyed sharing lesson ideas, and sharing links to websites with additional ideas. At the conclusion of the project, all the participants asked if it would be possible to keep the website active, so they could explore the wealth of ideas through the summer months when they had more leisure time.

While some participants expressed the feeling that the project would have been more helpful if all the group members were teaching the same grade, it was interesting that idea-
sharing sometimes crossed grade boundaries in unexpected ways. This was illustrated when Jessica adapted Kaitlyn’s Grade 4 graphing lesson for her kindergarten class. Although the lesson context was different, the fundamental idea of generating labels for an unlabelled graph worked well for two very different grade levels.

5.3.4 What kinds of obstacles and difficulties did the teachers encounter as they engaged in online collaboration?

The study participants identified three obstacles they encountered as they engaged in online collaboration over the length of the project: finding the time and motivation to get online, overcoming difficulties with technology, and a feeling of concern about being judged, and about judging someone else.

5.3.4.1 Finding Time and Motivation to Get Online

For teachers who stayed with the project from beginning to end, the total time spent online varied from about 3.5 hours to about 12 hours. This had an impact on the number of notes the participants read. One read 50%, another read 72%, and the third read 82%. The fact that one participant, who was just as busy as the rest with teaching and other commitments, found time to spend 12 hours online and read 82% of the notes suggests that motivation is a key factor. Teachers who are highly motivated to get online will do so more often, and will become more engaged with the group activity.

Time spent online also had an impact on the number of notes written, and on the length of the notes. The two participants who read the most notes also wrote the most. One wrote 17 notes totalling about 6000 words, and the other wrote 16 notes, or about 5700 words. The third participant wrote 10 notes, or about 4000 words.
There was also a considerable time commitment involved in writing and test-teaching lessons for the project. Jessica reflected on this in an online post near the end of the project:

The one thing I think is tricky for teachers, and which may affect whether [online collaboration] does work or not, is the time it would take to post and share lessons on a frequent or regular basis. It means taking the time to type, edit, proofread and reflect on not just your own, but others’ lessons and ideas. ... But if you are the only teacher for [your] grade in a small school, there is no chance for reflection with a grade-team partner, so an online forum is a wonderful idea for bouncing ideas off other teachers. (Jessica, note 114)

Kaitlyn commented that she sometimes found it difficult to remember to log in when things got busy. Since the teachers were not given academic credit on their work for the project, the main motivation for logging in was the opportunity to chat with other teachers in a social setting, and the availability of information and lesson ideas that could help in the classroom.

One simple way to motivate teachers to log in regularly is to send an email each week when new discussion questions are posted. However, motivation and engagement depend on more than just reminders. Kreijns, Kirschner, and Jochems (2003) found that social interaction is a key factor for success in asynchronous online learning environments. Several teachers in the study commented on how much they appreciated the opportunity to meet each other at the beginning of the study. The teachers met again at the end for a celebratory dinner, and the conversation at that time was animated and productive.

It would have been helpful to bring the teachers together socially much earlier in the project, perhaps around the end of September. The bonds formed at a social event earlier on would have helped to motivate the teachers to participate more actively, and might have encouraged them to overcome their diffidence about offering each other constructive criticism.
In situations where face-to-face social get-togethers are not possible, there are other options. These include scheduling online chats or exchanging videos, so the participants may have an opportunity to get to know each other better and form social bonds.

5.3.4.2 Difficulties with Technology

Online collaboration depends on having reliable access to the Internet. One thing this project showed was that this was more of a challenge than I expected. Early in the project, one participant had to withdraw from the project because of technical problems with both her home computer and her school computer. This was a significant loss for the group, because the participant had a lot of knowledge and experience to contribute. However, the combined challenge of the time commitment and the technological issues proved too difficult to overcome.

Almost every participant had some sort of technology-related problem during the project. Joan had problems with her school computer, while Kaitlyn and Jessica both found that they could not initially log onto the website from their school computers because the site was blocked.

While all the teachers in the study were comfortable with using computers to access the website and post comments, Joan noted that this might not be the case for everyone:

In our school, there are a number of teachers who feel like they are computer illiterate. They know how to do Word and email and Google, but when it comes to anything over and above that, they are stymied. They do not try, ‘What if I click this button?’ (Joan, interview 2)

5.3.4.3 Concerns about Judgment

It takes a measure of courage to post your thoughts in writing, online, where they will be read by other teachers, and by a university researcher. It takes even more courage to shine a light on what is happening in your classroom as you share a lesson plan you have written, and then discuss what went wrong, as well as what went right, when you test-taught your lesson. Concerns about being judged are only to be expected.
However, the participants in this project not only expressed concerns about being judged themselves, but also demonstrated a reluctance to be seen to be judging one another. In Week 4, the teachers in the study were asked to post their outlines so people in the group could offer suggestions to each other. Everyone posted an outline, but no one posted comments except me.

This was unexpected. Working in publishing, I am accustomed to having editors, reviewers, and others read lesson plans and offer suggestions. Thinking that the problem might have to do with time or technological know-how, I gave the teachers an extra week to post comments and offered some how-to instructions about using the edit function of the software. I also posted the sample lesson that I gave the participants at the launch meeting and invited their suggestions about possible improvements.

I also posted this online note, encouraging people to respond to each other’s lessons:

[In publishing], whenever we develop a textbook chapter, we always send it out to a team of reviewers who add notes to the file. You would think that we would like getting reviews back that say everything is perfect, but those reviews are not actually very helpful. What really helps is when people suggest new questions to ask or tell us where they find things unclear or confusing. We also really like hearing about different people's classroom experiences that have a bearing on the chapter. What I am asking all of you to do with the lesson outlines is to be reviewers. The person who wrote the outline is still in charge of the lesson and can decide which, if any, review ideas to use.

The reason why we are doing this is that I am trying to find out through my research if this kind of online collaboration is a viable way for teachers to exchange ideas and learn from each other. (And if it is not, I want to find that out too – so if you do not feel comfortable commenting on lesson outlines, please tell me what you're thinking!) (Researcher, note 97)

All these efforts yielded exactly one suggestion from Joan about Kaitlyn’s lesson plan:

Something we have been working on with grade 3 data management was using information presented in a graph leading to an action or policy decision. So another question could be along the lines of ‘What decision about . . . would you make given what you are learning from the data in the graph?’ (Joan, note 102)
In the end-of-project survey, the participants were asked why there was so little discussion about people’s lesson ideas. One explanation was that the participants said they saw me as someone who had mathematical knowledge to share, so they were comfortable with my suggestions about ways to improve their lesson plans. They considered the other participants peers, and so did not feel qualified to make suggestions to one another.

I liked the conversation that we had about my lesson plan, however you came at it as an ‘outsider’ with an overview of math-related things. It did not feel threatening and your input was constructive. As teachers, we do not get any experience of actually doing this with colleagues, and our credentials are all the same, so why is what I say better or more appropriate than what the teacher is suggesting? You present as having a wealth of math textbook development experience. You are different, so you can therefore enter into the lesson plan development from a different place. You were safe. (Joan, survey response)

Perhaps it might have been better for me to wait a little longer to post comments, or to post them privately. Since participants saw me as an expert “outsider,” they may have been reluctant to add to or contradict my suggestions.

Another explanation was that participants simply did not have the time to read other people’s lesson plans carefully and respond to them thoughtfully.

I think everyone was crazy busy with their own teaching/personal lives and perhaps did not have time to answer in depth or even think about the other people’s lessons in depth to offer suggestions. (Jessica, survey response)

A third explanation was the teachers might have been more willing to exchange suggestions if they had all been teaching the same grade.

I admit I did not feel comfortable making suggestions to the kindergarten teacher. I have absolutely no experience teaching kindergarten kids so it would have felt presumptuous to suggest something. (Joan, survey response)

The explanation that perhaps comes closest to the heart of the matter is that, while teachers spend much of their day giving constructive criticism to students, they do not feel
comfortable offering constructive suggestions to other teachers. Several participants supported this idea with their explanations:

Although we read each other’s lessons, maybe we did not feel comfortable critiquing other people’s work, especially if they took a lot of time to put their lesson together to present it to the group. Personally, I read and liked my peers’ lessons and felt they were really well thought out and worded. I did not have any suggestions for them as I thought they did a very good job! (Jessica, survey response)

Joan suggested that teachers may be uncomfortable with conflict, and that the line between a personal conflict and a productive exchange of conflicting ideas is not always clear.

I find that online people are very careful about what they say. They do not want to get into any kind of argument. Argument has gotten a bad name. It is like being politically correct. Instead of having the classic argument, it has been taken as having bad feelings, and people seem to get very tied to what they say. I am on the journey of being able to detach myself from something I have said and see it in the public domain as something that is open for discussion. I was not like that in my 20s and 30s. (Joan, interview 2)

It is unfortunate that, for whatever reason, the participants in the study were apparently unable, or unwilling, to respond to each other’s lessons in more depth. As Joan noted, an exchange of ideas can be a productive way to move forward in your understanding of mathematics:

One of the refreshing things about all this is that you pointed out all this stuff and I thought, wow, we are going to move forward here. Instead of just, they think I am doing a great job. People are always saying that. If we are always hearing ‘That is terrific,’ is it really true? Do they really mean it? If you do not get some more constructive feedback, then you cannot really judge. If it is constructive, it builds on what you already have. If you do not have a good base, then you’ll just get basic stuff back. (Joan, interview 2)

The lack of collaboration that occurred within the study group supports Kreijns, Kirchner, and Jochems (2003), who found that true collaboration in an asynchronous online learning environment does not happen unless it is deliberately fostered by features designed into the environment to nurture the social dimension of the group. In future projects like this one, it will
be important to place a greater emphasis on the development of this social dimension, and to structure group tasks in a way that explicitly encourages collaboration.

**5.3.5 How did each teacher’s understanding of inquiry-based mathematics teaching change over the course of the project?**

All three teachers in the study began with a strong understanding of inquiry-based mathematics teaching and an interest in learning more about how to apply this teaching approach in their classroom. Each teacher also demonstrated changes in thinking over the course of the project.

Jessica’s involvement in discussions about inquiry helped her gain confidence in her own ability to understand mathematics. She began to realize that her difficulties with mathematics stemmed from the way she had been taught, rather than from her own mathematical ability. In her end-of-project survey, Jessica showed an increase in her belief that children could learn from figuring out ways to solve on their own. In her final interview, Jessica described her discovery that it was important to wait for students to come up with ideas on their own, rather than jumping in too quickly to help.

Joan’s view of inquiry also changed over the course of the project. At the beginning, she saw inquiry and direct instruction as two approaches to be used separately. As Joan experimented with inquiry in her classroom, she discovered that she could comfortably move back and forth from one approach to the other during the course of a lesson. In her final interview, Joan described her learning in terms of having a better grasp of “what to do next.”

In her end-of-project survey, Joan showed increased support for the idea that children could solve unfamiliar math problems on their own, and a decrease in support for the idea that talk is distracting while students are solving problems. Joan also discovered that, while she has a
flair for coming up with ideas for class activities on the fly, she is also pretty good at longer-term planning. In spite of some initial difficulties, Joan was able to identify an area of need for her students—their difficulty dealing with missing-number problems in addition/subtraction situations. Joan was then able to develop a series of inquiry lessons that met this need by helping her students build a deeper understanding of addition/subtraction relationships.

Kaitlyn also demonstrated changes in her thinking about inquiry over the course of the project. Kaitlyn’s lesson provided an opportunity for her to hone her problem-development skills. Her test lessons demonstrated the importance of grounding an inquiry question not just in the mathematics you want students to learn, but also in the mathematics they already know, and in contexts that connect to their real-life experiences.

Kaitlyn’s survey responses demonstrated more significant changes in attitude than those of the other two teachers, changing for almost every question. The most significant change was in her support for the statement that teachers have to show children how to perform procedures before children can perform them on their own, which decreased from 4 to 2. Other changes included a stronger belief that children can figure out how to solve unfamiliar problems, and that they can handle problems with more than one answer.

Kaitlyn’s response to a survey question about the essence of great mathematics teaching indicates another way her thinking changed. At the beginning of the project, Kaitlyn’s answer to this question was focused on her own understanding of mathematics. By the end, her focus had shifted from mathematical content to teaching strategies, with an emphasis on risk-taking, differentiation, and the development of conceptual understanding—all benefits of an inquiry-based approach.
5.3.6 What insights and observations did this process yield about how to design other professional-development initiatives like this one?

A goal of this study was to set up a collaborative situation in which participants could learn more about inquiry-based teaching from me and from one another. In some ways, the study succeeded in meeting this goal. The Knowledge eCommons website provided a good starting point for online learning. Teachers were able to visit at their convenience and communicate in a variety of ways. Tools on the website made it possible to search for information of interest and link to external information.

All three participants were sufficiently committed to the project to visit the website regularly over the course of four months, and to draft, test-teach, and finalize inquiry-based lesson plans. All three engaged in online discussion and commented on each other’s online posts, and all three demonstrated gains in both engagement in inquiry-based teaching and mathematical knowledge.

The project was less successful in fostering collaborative lesson-planning. While there was an exchange of lesson ideas flowing between me and each participant, there were few such exchanges among the participants. Even when the teachers were explicitly invited to make suggestions about other people’s lesson plans, they remained silent.

In hindsight, it might have been better to choose participants who taught at the same grade level. This supports Kreijns, Kirchner, and Jochems (2003), who found that true collaboration is unlikely to develop unless the participants have shared interests and a shared need. The teachers in the study felt that they would have had more interest in one another’s lesson plans had they been working with a same-grade group. In response to an interview question, Joan said:
I really was not interested in the kindergarten thing and I am not going to teach kindergarten. I thought, if I was having a discussion with other Grade 3 teachers, I would probably find it more relevant and there would be more back-and-forth. We were all different grade levels and that was perhaps a problem. (Joan, interview 2)

Kaitlyn, who teaches Grade 4, suggested that she would be most interested in working with teachers within a closer grade range, but not necessarily in the same grade.

I think I always like my grade and a little bit up. I think it is kind of interesting to see Grade 3 teachers and what they are doing with EQAO. Am I continuing with those good practices? Making sure that it is not like an isolated thing, that that is just good teaching? I also like to see what Grades 5 and 6 are doing, so that way I have a better understanding of where I really need to get my kids, or where the Grade 5 teachers are expecting my kids to be. (Kaitlyn, interview 2)

At the same time, there can be value in exchanging ideas with teachers who work with other grades. For example, after reading Kaitlyn’s Grade 4 lesson on graphing, Jessica decided to adapt the core of Kaitlyn’s idea for use in her kindergarten class.

At the conclusion of the study, the participants were asked who they thought might benefit most from participating in a project like this one. In their responses, they identified two separate groups: new teachers, and more experienced teachers who were accustomed to using direct instruction but interested in trying something else. Joan made the point that the people who would benefit most are people who are just beginning to toy with the idea of inquiry. “They have just got a toe in the pool, so they are interested. They need a reason to plunge in and refine and have a safe place to talk about it” (Joan, interview 2).

It might also have been better to incorporate more opportunities for social interaction into the project. While the teachers had the opportunity to meet in person at the project launch meeting, there was really no opportunity for a social get-together until the project was over. Several studies indicate that social interaction among participants, either in person or online, is a key requirement for fostering the sense of trust and belonging that is necessary for collaboration
For future projects like this one, it will be important to schedule a social event for the participants earlier in the project. It might also be beneficial to schedule online chats from time to time.

Finally, it would have been beneficial to structure collaboration directly into the lesson-planning task. Although one of the goals for this project was collaboration, the structure of the task was not set up in a way that required teachers to work together. Each person wrote and taught a lesson in isolation.

Visiting one another’s classrooms might have increased the likelihood that the teachers would exchange ideas about each other’s work. If exchanges could not be arranged in person, another option would be to share videotapes of the lessons. Perhaps the teachers would have been more forthcoming with suggestions about a lesson they could see in action than they were about other people’s written lesson plans.

Another option might be to present the teachers with a description of a fictional classroom and have them work together to develop lessons for that class, instead of for their own. A group of Grade 3/4/5 teachers, for example, could work in pairs to create lessons for a hypothetical Grade 4 classroom where a hypothetical teacher faces a series of challenges outlined in a case-study-style problem. This would require the teachers to collaborate, and might reduce people’s discomfort with critiquing others’ ideas, because they would not be commenting on one another’s professional practice.

Joan suggested that it might also be beneficial to involve the participants in more decision-making about how the project will work. The important thing is that the project will be most successful if it can be set up in a way that creates a true community of practice (Wenger, 1999), where the participants share common interests and common needs, where social ties
develop between the participants, and where true collaboration occurs because it is structured into the task.

5.4 Major Findings

This study examined a group of elementary-school teachers and how their thinking about inquiry-based teaching changed as they participated in an online discussion group about their teaching and developed inquiry-based lessons for their classrooms. The study showed that this kind of online professional development can be an effective way to help teachers improve their practice, but that care must be taken to build social ties within the group, and to structure tasks in a way that encourages collaboration and constructive criticism.

The major findings of the case study can be summarized as follows:

1. Teachers’ thinking about inquiry-based mathematics instruction is strongly influenced by their own experiences as students in elementary school.
2. An inquiry-based approach can help both students and teachers become more engaged with mathematics.
3. Participation in an online community can help teachers build new mathematical and pedagogical knowledge.
4. Participation in an online community can improve teachers’ conceptual understanding of inquiry-based teaching.
5. Online collaboration does not happen naturally, but must be fostered deliberately.
6. Teachers may be hesitant to provide constructive criticism to one another in an online forum.

5.5 Implications for Further Research

This study shows what one small group of teachers learned from participating in online discussion about inquiry-based teaching and exchanging ideas for inquiry-based lesson plans. The findings that emerged are specific to this group of teachers. Much more information must be collected to create a more general picture of how to structure effective online professional development for other groups hoping to incorporate more inquiry-based teaching into their
practice. One possible avenue for future research would be to conduct a similar study that follows some of the recommendations from the study—grouping teachers by grade interest, offering more opportunities for social engagement, adding a classroom-exchange or video-exchange component to the study, and/or restructuring the lesson-writing task so teamwork is required.

The study also revealed a disinclination on the part of teachers to make constructive suggestions about each other’s practice. In view of the fact that peer training and peer review both play an important role in the business world, it might be interesting to further investigate what can be done to encourage peer exchanges in teaching. Why do teachers feel uncomfortable critiquing one another’s work? What can be done to make them feel more comfortable? And how can online learning environments be adjusted to make this happen?

This study has provided some information about how online collaboration helped a small group of teachers to improve their practice. Further research in this area could shed light on how to structure online professional learning activities so they can provide greater benefits to a much larger group of teachers.

5.6 Final thoughts

In my career as an educational textbook editor, I have mainly worked with teachers and students at one remove—incorporating suggestions received from anonymous reviewers into manuscripts, and then passing my ideas along to production to be turned into books that will eventually make their way into classrooms. This project gave me an opportunity to work directly with teachers, and I found the experience both exhilarating and frustrating.
It was exhilarating because I could see the teachers’ knowledge and enthusiasm increasing day by day. I loved reading their accounts of what was happening in their classrooms, and often wished I could be there in person to see what was happening.

My frustration came from my participants’ unexpected unwillingness to engage with me in the process of writing, reviewing, and rewriting that is an integral part of my work in publishing. And yet I found their unwillingness to critique each other’s lessons as intriguing as it was frustrating. Even when given more time, and explicit instructions about how to use the web tools to comment directly on lessons or post links, the teachers still declined to respond to the lessons the others had written.

When asked why this happened, some participants mentioned the disparity in grade levels—they simply were not interested in what teachers at a different grade level were doing in their classrooms. Joan, however, caught my interest when she mentioned her perception that giving constructive criticism to peers is a practice that is actively discouraged in schools.

We’re conditioned not to ‘evaluate’ each other and making suggestions could be construed as an evaluation. It is in the union rules: ‘Thou shalt not evaluate another teacher.’ (Joan, survey response)

In a conversation at the end of the project, Joan described how what happened with the project reminded her of an incident at her school. Like several of the participants, Joan is sometimes involved in teaching and observing demonstration lessons. On one occasion, she was part of a group of teachers invited to observe a language-arts lesson being taught by a colleague. At the conclusion of the demonstration lesson, Joan had made a page of notes with ideas for ways to improve the lesson. However, in the sharing circle that followed the lesson, the principal began by praising the lesson as a “Great job!” without offering any constructive suggestions. The other teachers in the circle followed suit, and when Joan’s turn came, she felt uncomfortable
about offering criticism where others had offered only praise. As a result, Joan opted to keep her ideas to herself rather than share them, even though the information would have been beneficial for the demonstrating teacher.

In many fields other than education, peer evaluation is a routine part of everyday practice. In publishing, for example, manuscripts are reviewed and edited by many people before they see print. In business, the same thing happens with reports and publications, and people often work as teams to accomplish tasks. Even this thesis is the product of collaboration with my faculty advisors.

Wenger (1999) talks about the value of belonging to a community of practice, which is a group of people who “share a concern or passion for something they do, and who learn how to do it better as they interact regularly” (Wenger, 2006). If education is truly a culture that discourages teachers from interacting in a way that helps them learn to “do it better,” then it is no wonder that changes in classroom practice tend to happen at a glacial pace. Teamwork is recognized in business as a proven way to improve practice and provide professional development for individual team members, and yet teachers seem somehow resistant to the process. Perhaps further research could help us understand more about whether the education business really does operate differently from other businesses in this respect, and, if so, why this happens.
References


Appendix A
The Inquiry-Based Lesson (information for participants)

In a traditional mathematics lesson, the teacher shows the students how to do the mathematics and the students practise what they were shown. In an inquiry-based lesson, the students actively build their own understanding of the mathematics by solving a problem. For example, in a traditional lesson, a teacher might show students how to add 35 + 28 by grouping the ones, trading, and then grouping the tens. In an inquiry-based lesson, a teacher might pose a problem such as, “There are 35 children in our class and 28 children in the class next door. If both classes go to see a play together, how many tickets will we need?” The teacher would then give the students time to figure out their own strategies for solving the problem. After this, the teacher would conduct a follow-up discussion and invite different students to demonstrate their strategies and answer questions about them. If the regrouping strategy did not come up naturally, the teacher might present it now as another possible strategy for solving the problem.

The advantage of the inquiry-based approach is that students are more likely to understand and remember what they’ve learned when they construct that learning for themselves. They are also more likely to come to understand mathematics as a body of connected knowledge rather than as a set of procedures to be memorized.

Before: Introduction
Set the stage for the lesson with a few questions or a short activity to activate the students’ prior knowledge about the lesson topic.

During: Teaching and Learning
Pose a problem for students to work on alone, with partners, or in groups. Depending on the situation, you might also want to provide a selection of materials students could use for manipulative support. Move around the classroom as the students work, watching and listening. Be supportive, but try to give students time to work through roadblocks on their own rather than showing them how to get solutions or prompting them with hints. Encourage students to record how they solved the problem (younger students can use pictures) so they can share their ideas at the consolidation stage.

After: Consolidation
Invite several students to explain to the class how they solved the problem. Look for relationships among solutions and ask questions to draw out key ideas about the math. Ask,
• Will your way of solving the problem always work or will it only work sometimes? How do you know?
• What did you learn about mathematics from solving this problem?
• Can you make up another problem that is like the one you solved? What is the solution?
Appendix B
Beginning-of-Project Survey

1. Please complete the following information.

Name:
School:
Mailing Address:

Phone number:
Email:
Gender:
Age:
Grade taught:
Undergraduate major or teachable subjects:
Years of teaching experience:

2. When you were in elementary school, which of these activities do you remember doing in math class? (If you’re doing this electronically, just bold your choice.)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completing worksheets</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Solving problems with partners or groups</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Using a math textbook</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Doing timed drills</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Playing games</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Keeping a math journal or writing about math ideas</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Learning more than one way to do math operations</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Learning about how people use math at work</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Doing math projects</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Going on field trips</td>
<td>Yes</td>
<td>No</td>
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<tr>
<td>Having guest speakers</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Other (please specify):</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. When you were in elementary school, did you like math?  
Yes  No  Undecided

4. How satisfied do you feel about the quality of your math education in elementary school?
very satisfied
satisfied
neither satisfied nor unsatisfied
unsatisfied
very unsatisfied

5. Describe why you feel that way.

6. Please rate your level of agreement with each of the following statements.

<table>
<thead>
<tr>
<th>Statement</th>
<th>completely disagree</th>
<th>completely agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic facts (+, −, ×, ÷) are the building blocks of mathematics.</td>
<td>1 2 3 4 5</td>
<td></td>
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<tr>
<td>Children find problems easier to solve when the problems relate to their everyday life.</td>
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</tr>
<tr>
<td>Problems with more than one answer are confusing for children.</td>
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<tr>
<td>Children benefit from using concrete materials to solve problems.</td>
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<tr>
<td>Teachers have to show children how to perform procedures before children can perform them on their own.</td>
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<td>Children learn mathematics best from teacher demonstrations and explanations.</td>
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<td></td>
</tr>
<tr>
<td>Canadian schools are doing a better job of teaching math today than they did when I went to school.</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>
7. If you had to choose one word, phrase, or image to describe the essence of great math teaching, what would it be? Give reasons for your choice. (You can type as much as you want and the box will expand to fit it.)


8. Do you think mathematics education in Canadian elementary schools is better, worse, or about the same as when you went to elementary school? Give reasons for your choice.


9. If you have any thoughts, concerns, or questions about this study, please record them here.


Appendix C
Questions for Telephone Interviews

Questions for initial interview:
• What do you hope to gain from participating in this project?
• How would you describe your current approach to teaching math?
• Tell me about a time when something really great happened in your classroom.
• What do you find difficult about teaching math?
• What would you like to change about your mathematics teaching? Why?
• Do you think it is difficult for teachers to use inquiry-based methods? Why or why not?

Questions for final interview:
• In our last telephone interview, you told me that something you hoped to gain from participating in this project was [_________]. Now that the project is nearly finished, do you feel that you have gained what you expected to gain? Is there anything else that has changed for you as a result of the experience?
• What did you find difficult about writing and/or teaching your lesson? How did you deal with the problems?
• How would you describe the experience of working with other teachers online?
• What did you like or not like about the way the website worked?
• What do you plan to try next in your classroom, and how do you plan to work inquiry into it?
Appendix D

Questions for Online Discussion

• Week 1:
  What do you do at the beginning of the year to get your math class up and running?

• Week 2:
  What do you know about inquiry-based teaching that you think might help others in the group as they’re planning their lessons for the project?
  What are some examples of good open questions, and what makes them good?

• Week 3:
  How do you differentiate instruction in your class?

• Week 4
  No discussion topic. Instead, teachers were asked to review one another’s lesson outlines.

• Week 5:
  Is online collaboration a good way to help elementary-school teachers learn more about teaching math? If so, what makes it work? If not, what makes it not work? and what would work better?
  What’s one great new math thing you’ve heard about recently?

• Week 6
  No discussion topic. Instead, teachers were asked to try out their lesson ideas in their classrooms and report to the group.

• Week 7:
  As you try out your lessons, you’re probably noticing that inquiry-based lessons come with their own set of benefits and challenges. What benefits have you noticed for your students? What challenges make it difficult to use this kind of teaching more often?

• Week 8:
  What happened when you tried test-teaching your lesson? What did you notice about your own teaching or about your kids while you were working on the lesson?

• Week 9:
  Report-writing week. No discussion question.

• Week 10:
  What advice would you give to a teacher who wants to build more inquiry-based teaching into his/her classroom? What are some effective ways for administrators (and university researchers) to help teachers who want to do this?
Weeks 11 and 12
Write final versions of your lessons and post them for the group. What changes did you make to the questions you asked? the materials you used? something else?
Appendix E
End-of-Project Survey

Some of the questions on this survey are similar to some that were on the survey you completed at the beginning of this project. One purpose of this second survey is to find out if and how any of your ideas about teaching have changed as a result of your involvement in the project.

1. Please complete the following information.

Name:

2. Please rate your level of agreement with each of the following statements. (If you are completing this electronically, you can shade or bold your choices.)

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
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<th>3</th>
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<tr>
<td>Canadian schools are doing a better job of teaching math today than they did when I went to school.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
3. In the previous survey, you wrote a description of the essence of great math teaching. Now that the project is complete, is there anything you would like to add to what you wrote before?

4. What do you think are the main benefits of inquiry-based teaching?

5. What do you think are the main obstacles that make teachers reluctant to incorporate inquiry-based teaching into their mathematics programs?

6. If another teacher working at your grade level came to you for advice about how to do more inquiry-based math teaching, what would you recommend?

7. Much of this project was conducted online. In general, what thoughts do you have about the usefulness of online collaboration for teachers? Are online groups a useful way for teachers to help each other? What drawbacks are there?

8. Part of this project involved opportunities for teachers to offer each other suggestions about their inquiry-based lesson plans, but this part of the project didn’t work very well. Everyone wrote a lesson plan, but not many people made suggestions. Why do you think this happened?

9. How would you describe your experience in this research project? If we conduct a similar project in the future, what should we do differently?

10. Is there anything else you’d like to tell me about any aspect of this project?

Thank you very much for all the time you took to help me with my research. It’s been a great experience for me, and I’ve learned a lot from every one of you.
Appendix F
Joan’s final lesson

Grade Three: Related Math Sentences

Central Question: What is the relationship between 6, 17, and 23? Justify your answer.

Background Information:

Lesson Goal: To understand the relationship between addition and subtraction number equations

Grade 3 Curriculum:
Pattern and Algebra
- Determine, through investigation, the inverse relationship between addition and subtraction (e.g., since 4 + 5 = 9, then 9 – 5 = 4; since 16 – 9 = 7, then 7 + 9 = 16)
- Determine, the missing number in equations involving addition and subtraction of one- and two-digit numbers, using a variety of tools and strategies (e.g., modeling with concrete materials, using guess and check with and without the aid of a calculator) (sample problem in the curriculum: what is the missing number in the equation 25 – 4 = 15 + □? However first students need to know: 12 + □ = 18)

Key Ideas:
- Addition and subtraction are opposite operations but they are related to each other
- Addition means joining to make more
- Subtraction means taking something away to make less or find a difference
- Addition can be used to find the solution to a subtraction question
- An equal sign is a way to show that two amounts have the same value
- We can use the same 3 numbers to write four related math equations
- We can use our knowledge of how related sentences work to answer missing number questions

Prerequisite Knowledge:
- Count to 100
- Solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials and student-generated algorithms
- How to use a number line

Materials:
- Number lines
- Cube-a-links
- Paper, pencils, rulers, markers, large sheets of grid paper

Differentiation for Individual Needs:
- Groups with students of like ability
- Additional teacher support for students who are having difficulty
- Provision of ‘easier’ meaning smaller numbers ie: 2, 5, 7
- Promote a variety of ways for showing their thinking (words, numbers, pictures/diagrams or models with spoken explanations/scribing)
- Require only one way of ‘showing thinking’

Stage 1 – Activation (review of related prior knowledge/ addition and subtraction): whole-class discussion
- Tell me about the signs +, -, and =? What do they do? What do they mean?
- How do we use them?
- Provide examples of how they’re used.

Stage 2 – Inquiry: students work in pairs or small groups. Students having difficulty (ELL or academically low) sit with the teacher to get started, ensuring understanding of the task and provision of ‘easier’ numbers if necessary
Central Question: What is the relationship between 6, 17 and 23? Justify your answer.

Questions to ask while students are working:
- Show me another relationship between these numbers?
- Can you show that on a number line?
- How could knowing this relationship be useful if a number was missing?

Note: questioning is used to steer thinking, to clarify understanding on both the teacher’s and student’s part, and to promote new ideas. It has to be relevant to what the student is doing and therefore spontaneous. Questions have to reflect the student’s readiness, students who can make thinking leaps can usually handle more complex questions; others have to take baby steps and need ‘step-by-step’ questions to help them navigate their way to a big idea. Having pre-planned questions is all right but they may or may not get asked depending upon what students are doing.

Stage 3 – Making connections: Consolidate student understandings – focus on related sentences. Class discussions, put up student ideas, and look for connections between student groups.
Discuss how understanding related sentences could help us answer other questions where we have two numbers either an addition or subtraction sign and an equal sign (e.g., 35 – 14 = □ which can become 14 + □ = 35) using strategies such as drawing number lines to see what is actually occurring.
Move to math equations such as: 25 – 4 = 15 + □ and figure out, knowing how related sentences work, how to solve for □.
Please note: As per our discussion when developing this lesson we did as a class use the idea of coloured squares of different sizes. After discussing the relationship between the three numbers and coming up with 4 related sentences I assigned the largest number a large blue square. The other two numbers were assigned smaller red and yellow squares (slightly different sizes but not as large as the blue square). Students were asked to use their squares (I’d cut them out of construction paper and given each group a set of 4 of each colour) with the math signs +, -, = to create the four math sentences. Some students had difficulty but most groups got the idea. We put these ‘templates’ up during the equalities block.
Appendix G
Jessica’s final lesson

Kindergarten Lesson Plan: Sorting

Central Question: What different ways can you sort these buttons?

Lesson Goal: For children to demonstrate and explain multiple ways they can sort buttons.

Kindergarten Curriculum:
DM5.1 sort, classify, and compare objects and describe the attributes used.

Key Ideas:
a variety of buttons with similar attributes are used for children to freely explore and manipulate
students may focus on putting like buttons together based on attributes they see which depends on their level of understanding of sorting and what attributes are the same

Prerequisite Knowledge:
understand what sorting is (putting objects with something in common together)
understand what an attribute is, understand that one object can have several attributes (e.g., colour, size, shape, texture, pattern etc.) and that that same object can be sorted in many different ways depending on the sorting rule chosen
experience with sorting a variety of items as a large group, with a partner and during free-activity time given in class
experience with doing a 2-way sort (e.g., objects that do have the required attribute go in one pile and objects that do not have that attribute go in the other pile)

Materials:
bin with a variety of buttons in it
several paper plates to use as sorting trays
paper plates to save sorted sets for display at the Sorting Station
folded tent paper to display sorting rule

Differentiation for Individual Needs:
the variety of buttons allows each individual to sort in their own way and to show a variety of ways based on the attributes they choose to sort by
I might reword my question to, “Show me some buttons that are the same in some way” and I might demonstrate an example using blocks how I could sort by an attribute like colour

Stage 1-Activation of Prior Knowledge: 2

Bring in a laundry basket filled with a variety of appropriate clothes such as socks, T-shirts, long sleeved shirts, shorts, pants, sweatshirts etc.
Tell the students you were so busy on the weekend you did not have time to deal with
the laundry and that you may need their help with this chore. Start by acting out how
you pull out one item, label it by name, and put it in a pile on the floor. Then pull out
another item, name it (for those ELL and exceptional students) and model self talk.

“Hmmm, a T-shirt, where should I put this? Does it go with the pants in the pants
pile?” Encourage the children to put their hands up and share where they think it goes
and why. Invite other students up and ask them to select an item and share why they
have put it where they did. Ask the students if they know the mathematical name of
what they are doing. If they do not, introduce the word sorting and explain that we sort
many things in our everyday lives. Can they think of some things at home that they sort
(i.e., forks, spoons, knives; Lego bits go away with Lego etc.) Then, take the individual
piles we’ve sorted as a group and place them back into a large pile. Ask the students,
“Can any of you think of another way we could sort these clothes based on something
that is the same about them? See if students sort by colour, by pattern or by other
attributes such as where on the body the clothes are worn or for which season……If
students are stuck, begin modeling a sort based on an attribute and then see if anyone
can guess your sorting rule. Perhaps then, alternate ways of thinking about the clothes
will be sparked.

Stage 2-Inquiry
Show children the bin of buttons. Stress to the kids that they are all different in some
way. Pick one out, show it to the students and model out loud how to describe what you
see e.g.,) “When I look at this button, I see that it is small. But, I also see that it is black,
oh… and it is shiny too. It has three different things I noticed about it! Small, black and
shiny!” Then pick out another button. “And this one is big and I can also see that it is
red. It has two things I noticed about it. Hey, I think I can sort these buttons into groups
based on things that are the same about them. I can also make a pile of buttons that do
not have that feature to show the difference between the two groups.” Tell the children
that today you’re going to invite them to come up and sort these buttons into
groups/piles, just like they did with the laundry.

Invite students up, one at a time, to freely explore the buttons in the bin. Encourage
them to talk about (describe) what different buttons they see. Ask them to sort the
buttons in whatever way they would like to. Once they have sorted a few into a pile, ask
them to explain to you why they chose to put those buttons together. Can they explain
their reasoning to you? After they have finished, ask them if there is another way they
can sort more of the buttons in the bin and repeat the process. Observe the student as
they sort the buttons and note their ability to sort and level of reasoning for assessment
purposes. To have a product that other children can observe you can write their name
on a paper plate and then create a tent card with the sorting rule on it to be displayed at
a “Sorting Station.”

Stage 3-Making Connections
Bring the students back to the carpet and invite someone to recall what the name was
of putting similar things into groups. Review the word “sorting” and revisit the things in
their everyday lives that they may sort at home and within the classroom daily. Then
invite a few students to “Show and Tell” the class their plate with the buttons on it and to
tell their sorting rule based on common attributes. What were some of the new and
different ways that students sorted? Make a chart with attributes they commonly came
up with like shape, size and colour. Jot down new and interesting attributes some
(hopefully) came up with like dull, shiny, bumpy, even the number of holes on the
button. Encourage students to visit the Sorting Station to sort in some of these new
ways. Add other items that can be sorted based on several attributes such as rocks,
shells, even everyday things like Lego. Let them have fun!
Appendix H
Kaitlyn’s final lesson

Sample Lesson Outline: Grade 4 Data Management

Central Question:

How are these two graphs similar and how are they different? What businesses would prefer to use each set of data, and why?

Background information:

Lesson Goal: Read and interpret graphs, drawing appropriate conclusions.

[Processes addressed: Problem Solving, Reasoning/Proving, Reflecting, Connecting, Representing, Communicating]

Grade 4 Curriculum:

Data Management and Probability:

- collect and organize discrete primary data and display the data in charts, tables, and graphs (including stem-and-leaf plots and double bar graphs) that have appropriate titles, labels (e.g., appropriate units marked on the axes and scales (e.g., with appropriate increments) that suit the range and distribution of the data, using a variety of tools (e.g., graph paper, simple spreadsheets, dynamic statistical software).

- read, interpret, and draw conclusions from primary data (e.g., survey results, measurements, observations) and from secondary data (e.g., temperature data in the newspaper, data from the Internet about endangered species), presented in charts, tables and graphs (including stem-and-leaf plots and double bar graphs);

Key Ideas:

- There are different ways to present data such as using a table, graph or chart. Similar conclusions can be drawn from each, but different mediums are more useful at different times.

- We must understand the scale of the graph to make sense of the data.

- Scales can use a variety of start points and end points, and we need to be aware of these to understand the data in a sophisticated way

- Graphs can be skewed to promote certain ideas
- A graph can be vertical or horizontal
- We can make inferences based off of data provided on a graph to provide a deeper understanding of the data

**Prerequisite knowledge:**
- able to draw conclusions based off of single bar graphs
- understand the parts of a graph including a title, scale, and axes
- count by common multiples including 2s, 3s, 5s, and 10s
- able to understand how to read a T-Chart

**Materials:**
- simple unlabeled bar graph drawn on a large chart paper
- paper, pencils, crayons
- Handout of two graphs showing the same data, but with much different scales. One graph can show 5 years, and the numbers reached for each year (such as 4950, 4970, 4980, 4995, 5010), with a scale starting at 0, and reaching above the largest number of 5010. The next graph can display the same data, but have the scale start at 4900, and finish at 5100, rather than begin at 0. This will show a steep rise in data, rather than a consistent line.

**Differentiation for Individual Needs:**
- The size of numbers can be varied, to meet some students' needs. Rather than working with a scale that uses numbers past 4000, students could use a scale reaching numbers past 400 instead.
- Some students might work better on their own or with an EA or older student

**Lesson outline (3 stages)**

**Stage 1—Activation (review of related prior knowledge): Whole-class discussion**
Show a simple vertical bar graph. Ask students to work in their table groups of 4 to record what they know about the graph. Students can make simple observations about the graph (i.e., This is a Bar Graph, There are 4 bars, The green bar is the longest), or they could make more complex observations (The red bar is slightly longer than the purple bar, The green bar is greater than any other bar, and therefore it was the most popular, etc.)
Have the groups share their conclusions with the class, and record the ideas of the students down on the board for future use.

*Next time, I might show a horizontal bar graph, to have the conversation (or refresh their knowledge) that bar graphs can be either horizontal or vertical.*

**Stage 2—Inquiry:** *Students work on the problem in pairs or small groups.*

**Central Question:** What is the same about these two graphs, what is different about them. If you were working for a large business, when might you like to use Graph A? When might you like to use Graph B? **Label each graph with a title, and a legend to show how the data might be used in an advantageous way.**

Questions to ask while students are working:

- Tell me what conclusions you can make about Graph A.
- What business would want to share this data with its customers? What might this data represent?

*Next time- I would give students specific topics, such as the environment, rather than being too open ended like I had originally about businesses. I would also provide more than one set of graphs to partners, in case they have multiple ideas.*

- How are Graph A and Graph B similar?
- Tell me what you did?
- How did you use your graph to help you to make these conclusions?
- What does the scale tell you about your graph?
- Why does this make sense?

- I see that both your graphs are about _____________________ (the environment, hockey, etc). Can you label another graph to focus on a different topic? Can you make a new graph that shows different data, that the business might also be interested in?

**Stage 3—Making Connections:** *Each pair or group presents one example of how each graph could be advantageous to a business. They will show the graph, and had added in a title, and a legend to show what the data represents.*

Questions to ask when students are presenting their work:

- Why might a business want to share these results with their customers? Why might they not want to share this data with their customers?
- What does the scale tell you about the graph?
- Should the scale of all graphs start at zero? Why or why not?
- Do you agree with the chosen scale? Why or why not?
- How are these two graphs connected to each other?
- How did you decide what scale to use?

Next Time - Add the Anchor Chart, “Good Mathematicians Know…”, to help summarize their new knowledge. Students could have a copy of this, and keep it in their Math Journals.