ESSAYS ON THE ALLOCATION OF TALENT, SKILLS AND INEQUALITY, AND LIFE-CYCLE EFFECTS OF HEALTH RISK

by

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Abstract

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This dissertation consists of three essays. The first essay studies how health risk affects individuals’ economic decisions due to changes in productivity, required medical expenditures, available time and survival probabilities implied by changes in health status. It assesses the role of these four channels in determining labour supply, asset accumulation and welfare using a life-cycle model calibrated to the U.S. economy. I find that all channels and the interactions between them have large implications for the macroeconomic variables studied. Health has larger effects for the non-college than college educated, explaining a significant fraction of the difference in labour supply, degree of reliance on government transfers and asset accumulation across education groups. Improving non-college health outcomes to approach those of college graduates results in large welfare gains, higher labour supply, and significantly lower reliance on government welfare programs.

The second essay studies the evolution of wage inequality in the United States between 1980 and 2002 in a framework that accounts for changes in the employment of physical and cognitive skills and their returns. I find that within education-gender groups, average employed cognitive skills have remained constant, while average physical skills have declined. The returns to high levels of cognitive skills have increased dramatically, while returns to low levels of cognitive skills and physical skills have remained approxi-
mately constant. Skills account for approximately half of the increase in the college wage premium, and for a small but growing fraction of residual wage inequality.

The final essay studies the sorting decisions of students with different levels of analytical and verbal skills into college fields of study. I build a model where each field tests and perfectly reveals to potential future employers only the students’ skill that is intensively required in that field. Students’ expected wages after graduation are a function of their revealed skill levels and firms’ expectations of the unrevealed skills given the chosen field. I show how the size of each field and the average talent it attracts depend on the average skill levels, on skill dispersion and on the degree of correlation between skills in the student population.
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Chapter 1

Life-cycle Effects of Health Risk

1.1 Introduction

Health care system reform is a contentious topic of debate highlighting the need for a better understanding of how health affects individuals and their economics decisions. Current statistics suggest that health risk is a major component of overall risk and a big determinant of welfare with potential large macroeconomic implications. For example, “using a conservative definition, 62.1% of all bankruptcies in 2007 were medical” (Himmelstein et al., 2009).¹ Recent and upcoming changes in the U.S. health care system will undoubtedly have large impacts on the type and degree of health risk faced by individuals. It is therefore extremely important to understand the different channels through which health affects individuals and the economic consequences of different health effects.

This paper studies four channels through which health affects individuals: (1) productivity, (2) medical expenditures, (3) available time and (4) survival probabilities, and assesses their roles in determining labor supply, asset accumulation and welfare. An important feature of this paper is to study these health effects over the entire life-cycle in a unified framework. Lower earnings caused by bad health occur at the same time when

¹Bankruptcies are classified as “medical” based on debtors’ stated reasons for filing, income loss due to illness, and the magnitude of their medical debts.
required medical expenses increase. A diminished productivity and a decrease in available time due to bad health could force individuals to take time off work, losing income and employer sponsored health insurance precisely when they need it most. The joint occurrences of these effects suggests that it is important to consider how they interact.

The contribution of my paper is to assess the relative importance of each channel and quantify the interactions between them and between how they operate at different stages of the life-cycle. This evaluation is crucial for designing and predicting the outcomes of health care reforms. First, I show that health has large implications for the macroeconomic variables studied through all four channels, and that due to significant interactions between them, they need to be studied within a unified framework over the entire life-cycle. Second, I find that health affects education groups differently, with larger effects for non-college individuals, explaining a significant fraction of the difference in labour supply, degree of reliance on government transfers and asset accumulation across groups. Finally, when accounting for all heath effects, I show that differences in health outcomes across education groups are very important and improving non-college health closer to college levels results in large welfare gains, higher labour supply, and significantly lower reliance on government welfare programs. Policies such as Medicaid expansion and increased insurance coverage have relatively small benefits.

The framework used is a standard Bewley (1986) life-cycle model with incomplete markets and uninsurable income risk augmented with health shocks as previously done in French and Jones (2007). To make the model realistic and useful for policy evaluation, I also model partial insurance through a consumption floor, Medicaid and Medicare programs, and employer sponsored health insurance, and study how these mediate the transmission of health effects to individuals. I calibrate the model to the U.S. using data on males, separately for college and non-college education groups in order to explicitly study differences between them. The model gives a very good fit of earnings and labor supply over the life-cycle and successfully matches statistics on government transfers.
French and Jones (2007) is the only existing paper to model all four channels through which health affects individuals. However, the object of interest in their paper is retirement behavior, and while they conduct a policy experiment related to Medicare in this framework, they only consider the effects on the labor supply around retirement age. Different from their paper, I study each channel separately and analyze health effects at different stages of the life-cycle. I find that the risk implied by productivity and available time effects generates strong precautionary saving motives accounting for 10% of asset accumulation before the age of 60 for the non-college educated. In bad health states, lower average incomes and time endowments imply very large drops in utility. The risk of entering these extremely low utility states generates a strong incentive to self insure through asset accumulation against the loss of income and possible bankruptcy. Productivity and time effects also significantly lower labor supply. On the other hand, lower survival rates associated with bad health greatly discourage asset accumulation at all stages of the life-cycle.

The results show that even though the probability of bad health before retirement age is relatively small, the presence of health risk still has large consequences. Papers such as De Nardi, French and Jones (2006) and Kopecky and Koreshkova (2009) study the importance of individual health effects, but only after retirement age, and are therefore unable to capture some of the largest health effects. Of the few papers that study health effects before retirement, none looks at the importance of individual effects.

In addition, I show that due to interactions between health effects at different stages of the life-cycle, papers that model health risk only after retirement cannot accurately estimate their importance in isolation. Medical expenditures are a good example: for non-college graduates, the presence of retirement age medical expenditures increases asset accumulation before the age of 60 by 10%, but this is entirely reversed by the presence of medical expenditures before retirement that lowers disposable incomes. In a model that abstracts from medical expenditures before retirement, the effect of old age medical
expenses on asset accumulation is over predicted by between 8% and 22%, depending on the stage of the life-cycle. In the presence of medical expenses before retirement, individuals have fewer resources to save out of for old age medical expenditures. Kopecky and Koreshkova (2009) who study health effects only after retirement likely overestimate these effects.

I also quantify significant interactions between different types of health effects. Papers that model only a few of the channels studied in this paper may wrongly estimate the importance of a particular health effect by missing the interaction effects. For example, I estimate that the effect of medical expenditures before retirement on asset accumulation before the age of 60 is underestimated by 16% and 51% in the absence of productivity effects for non-college and college, respectively. The joint occurrence of these adverse health effects makes them more painful. Therefore, Hubbard, Skinner, and Zeldes (1995) who model only medical expenses and survival risk would likely underestimate the importance of health expenses on asset accumulation.

I consistently find larger health effects for the non-college educated. One reason is that they face higher probabilities of bad health at any age. Also, bad health lowers non-college productivity to a greater extent. Finally, at the lower income levels of the non-college, fluctuations in incomes and time endowments caused by health imply larger utility changes, making health risk more important. Overall, health accounts for 53% of the observed difference in labor supply between education groups and for 40% of the observed difference in the fraction of government transfer recipients. Depending on the stage of life-cycle, it also accounts for between 5% and 17% of the percentage difference in asset accumulation in the model between groups.

I use the model to conduct several counterfactual experiments that shed light on the importance of different model environment aspects: health transition probabilities,

\footnote{For example, French (2005) does not model medical expenditures, Attanasio, Kitao and Violante (2010) do not model the time endowment effect, and Hubbard, Skinner, and Zeldes (1995) do not model the productivity effect nor the time endowment effect.}
efficiency of the health care system (reflected in the time and medical costs incurred), and degree of health insurance coverage through employers and Medicaid. The results suggest that due to the presence of many channels, improvements in non-college health transition probabilities have very large benefits. For example, when the probabilities of transitioning to bad health of the non-college are decreased to the levels of college graduates, non-college welfare improves by 6.3% in terms of CEV. The percentage of the total population relying on social insurance programs drops by 20%, lowering Medicaid government expenses by 41% and other welfare program expenses by 14%. The benefits associated with improvements in the other aspects of the environment are lower both in terms of welfare gains and reductions in government expenditures because they only diminish the effects of health through one or two of the transmission channels studied, implying relatively small effects when other important channels remain operative.

1.2 Model

The model of this paper is a life-cycle model with idiosyncratic labor earnings risk and health status risk, where health effects are modeled based on French and Jones (2007) and Attanasio, Kitao and Violante (2010). An individual’s health status is either good (G), average (A) or poor (P) in any given period. Average and poor health states affect individuals by (1) lowering their productivity, (2) increasing their medical expenditures, (3) decreasing their time endowments and (4) lowering their survival probabilities relative to those in good health. Hence, health status risk adds uncertainty through these four channels. I study non-college and college educated individuals separately, allowing all health effects to differ between these groups. The model is solved in partial equilibrium.

3 Most previous models modeling health assume only two health states (good and bad). Only Low and Pistaferri (2010) distinguish between moderate and severe work limitations (disabilities). I find important differences between the effects of average and poor health states.

4 Related papers that also study health separately by education groups are Attanasio, Kitao and Violante (2010), Hubbard, Skinner and Zeldes (1995) and Low and Pistaferri (2010).
assuming a small open economy with a fixed interest rate of 1.04%.

**Demographics**  The age of entry into the labor force is 18 for the non-college group and 22 for college degree holders. Everybody retires at the age of 65. Individuals can live to a maximum of 100 years, however starting at the age of 55, they face survival uncertainty. The probability of surviving to the next period depends on age \( (j) \), health status \( (h) \) and education \( (e) \), and is given by the function \( s(j, h, e) \). The variation in survival probabilities captures the following facts observed in the data: people die at faster rates as they age; they die at slower rates if they are in good health; and the college educated group lives on average to an older age.\(^5\)

An exogenous retirement age could be problematic: for example, individuals might retire at older ages if they expect high medical expenditures late in life. However, these effects are likely to be small. French and Jones (2007) find a significant but small effect of health insurance on retirement.\(^6\) Early retirement is allowed in the model since individuals can simply exit the labor force any time, however, social security cannot be collected until the age of 65.

**Health Status and Medical Costs**  Health status \( (h) \) evolves stochastically according to the transition function \( \Lambda_{e,j}(h, h') \): the probability of a given health state next period depends on the individual’s age, education, and current health state. Transition probabilities are exogenous, so it is assumed that individuals cannot invest time in preventive activities or buy medical goods and services in order to improve the probability of good health. Medical expenditures are modeled as negative income shocks, assuming they must be incurred in every period in order to survive to the next period but have no

---

\(^5\)De Nardi, French and Jones (2006) found that mortality rates vary significantly with sex, permanent income and health status. They find that allowing for differences in life expectancy leads to a noticeable effect on asset decumulation for retirees, especially at the top end of the permanent income distribution.

\(^6\)Specifically, they find that raising the Medicare eligibility age from 65 to 67 leads to an increase in labor force participation for those aged 60-67 by only .07 years.
effect on future health status. Medical expenditures $m(j, h)$ depend on age and health status.\(^7\)

In reality, health outcomes are determined by both exogenous factors such as genes, environment and random events, and choice variables such as lifestyles, time spent exercising and health care expenditures. Since existing literature is inconclusive in assessing the relative importance of these factors, I follow De Nardi, French and Jones (2006) and Attanasio, Kitao and Violante (2010) among others in making the simplifying assumption that health transition probabilities are exogenous. The model results will best approximate reality when exogenous factors dominate in relative importance. If choice variables played a large role, the model would overstate the amount of health risk faced by individuals as they can in fact invest resources to lower the probabilities of adverse shocks. Moreover, the ability to invest in health would depend on available resources such as income and available time, so the model would fail to capture the tradeoffs between allocating these resources to health production and allocating resources to work, leisure, asset accumulation and consumption.\(^8\)

All medical expense uncertainty comes from health uncertainty. Several previous models (e.g. De Nardi, French and Jones (2006), Hubbard, Skinner and Zeldes (1994) and Kopecky and Koreshkova (2009)) also model medical expense variation around the deterministic component. However, they find that shutting down out-of-pocket medical expense risk while keeping average medical expenditure constant (conditional on all of the relevant state variables) has only a small effect. Due to this finding and the fact that there are already many channels through which health generates uncertainty in my model, I abstract from this feature.

---

\(^7\)In reality, many health expenditures such as annual checkups, blood pressure medication and weight loss programs may be entirely of a preventive nature. Also, low income individuals without insurance often choose not to undergo expensive treatments. Feng (2009) is a recent paper modeling endogenous medical expenditures.

\(^8\)For example, individuals in bad health might work less in order to devote more time to taking care of their health, but they will also have incentives to work more in order to have income for better medical treatments.

Employer Provided Health Insurance:  An exogenous fraction of workers has employer-sponsored health insurance covering $k^m$ percent of total medical expenditures. If workers become unemployed, they no longer hold this insurance. For a fraction of these workers, the employer-sponsored health insurance coverage extends into retirement, covering $k^{ret}$ percent of expenditures when they are retired. Let $i \in \{0, 1, 2\}$ denote the insurance type with $i = 0$ indicating no coverage, $i = 1$ indicating employer-sponsored coverage only when working, and $i = 2$ indicating coverage extending into retirement. Each individual’s type $i$ is determined when he enters the labor force according to a random draw from the distribution $\Omega_e(i)$. This distribution depends on education in order to capture the fact that college educated individuals are more likely to have jobs that offer more comprehensive health insurance plans.\(^9\) Workers of type $i = 1$ or $i = 2$ pay a premium $p^w$ deducted from their earnings, and not subject to income tax. Individuals of type $i = 2$ pay a premium $p^{ret}$ when retired.

Medicare  The second form of health insurance is provided by the government through Medicare: starting at the age of 65, all individuals are covered by Medicare with coverage rate $k^{med}$ and premium $p^{med}$. The government finances the system through the collection of premiums and a proportional payroll tax, $\tau_{Med}$.

Medicaid  The Medicaid program covers those who cannot afford required medical services. I model this form of insurance through the inclusion of a consumption floor,

\(^9\)French and Jones (2007) show that individuals with strong preferences for leisure self-select into jobs that provide health insurance coverage after retirement. Properly accounting for this self-selection is important for their results, changing the estimated effects of policies such as increasing the Medicare eligibility age. I abstract from modeling this in my paper.
discussed below.

**Social Security and Social Insurance** The government runs a social assistance program which guarantees a minimum level of consumption $\bar{c}$ to every individual. When disposable income (net of required medical expenditures) falls below $\bar{c}$, the person receives a transfer $tr$ that compensates for the difference. The Medicaid program pays for the medical expenditures of those receiving the consumption floor.\(^{10}\)

Finally, retirees receive social security payments $SS_t$, which vary with education but are independent of earning histories, and which are financed by proportional payroll taxes $\tau_{SS}$ paid up to an income threshold $\bar{y}$, set to 2.5 times average earnings.

**Preferences** Individual preferences are given by:

$$U(c, n, h) = \frac{1}{1 - \sigma} [c^\alpha (1 - n - \theta I_{n>0} - \Phi_1 I_{h=A} - \Phi_2 I_{h=P})^{(1-\alpha)}]^{1-\sigma},$$

where $c$ denotes consumption of non-medical goods; $n$ is the number of hours worked; $I_{n>0}$ is an indicator equal to 1 if the individual is working and 0 otherwise; $\theta$ captures the fixed time cost associated with going to work; $I_{h=A}$ and $I_{h=P}$ are indicator functions equal to 1 if the individual is in average or poor health, respectively; and $\Phi_1$ and $\Phi_2$ capture the time costs associated with average and poor health states. I follow French (2005) and French and Jones (2007) in modeling the effect of health on utility as a time cost.\(^{11}\) For simplicity, I model only the extensive margin of labor supply, so the number

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\(^{10}\)Under the Medicaid eligibility rules prior to 2010, not all individuals below the poverty level were covered by Medicaid. For example, low income males with no disabilities and no children were unlikely to be covered. For simplicity, I do not model the uncertainty of receiving Medicaid. Feng (2009) is a paper that models this. However, the minimum consumption floor in my model is much lower than the federal poverty level (approximately 55% lower), increasing the likelihood that individuals with such low incomes are covered by Medicaid in reality as well.

\(^{11}\)Another way of modeling the effect of health on utility could be by lowering the utility received from consumption when health is bad, as in De Nardi, French and Jones (2006). However, using the NLSY, I find supporting evidence that those in average and poor health states spend a significant amount of time on health care activities: those in average health spend 39 minutes per day and those in poor health spend 82 minutes per day on average, i.e. 4.55% and 9.57% of total leisure time, respectively.
of hours worked per week is either 0 or 40 (0.4 in the model). Results in previous papers indicate that modeling the extensive margin alone captures the most important effect of health on labor supply. French (2005) documents small effects of health on hours worked but relatively larger effects on participation.

**Labor Productivity** Labor productivity is modeled as the sum of a deterministic component $w$ which is a function of health $h$ and age $j$, an individual fixed effect $\mu$ determined at birth, an idiosyncratic transitory shock $\lambda$, and an idiosyncratic shock $u$ assumed to follow an AR(1) process with innovation $\eta$. All components are allowed to vary with education, but the notation is suppressed.

$$
\ln W = w(h, j) + \mu + \lambda + u, \text{ where}
$$

$$
w(h, j) = \beta_0 + \beta_1 j + \beta_2 j^2 + \beta_3 j^3 + \beta_4 I_{h=G} + \beta_5 I_{h=G} \times j
$$

$$
\mu \sim N(0, \sigma^2_{\mu})
$$

$$
\lambda \sim N(0, \sigma^2_{\lambda})
$$

$$
u = \rho u_{-1} + \eta, \ \eta \sim N(0, \sigma^2_{\eta}).
$$

The deterministic productivity component is described in more detail in section 1.4.2. This component is likely to differ across education groups since (1) college educated individuals receive higher returns to experience and (2) non-college workers in bad health are likely to suffer larger declines in productivity since their jobs are more likely to require physical ability and health as argued in Attanasio, Kitao and Violante (2010). French (2005) and French and Jones (2007) who also model the effect of health on productivity estimate significant effects, but they do not consider differences by education groups.
1.2.1 Individual’s Problem

In each time period, the state of the individual is summarized by the following variables: education attainment $e$, type of health insurance $i$, age $j$, health status $h$, the realizations of the idiosyncratic labor income shock $u$ and of the transitory shock $\lambda$ for non-retired individuals, and assets $a$. In each period, an individual maximizes the expected discounted lifetime utility by choosing the period consumption level $c$ and making a labor force participation decision, $n$ (when younger than 65). The problem of a non-retired individual is summarized below.

$$V(e, i, j, h, \mu, u, \lambda, a) = \max_{(c, n)} \{U(c, n, h) + \beta s(j, h, e)EV(e, i, j+1, h', \mu, u', \lambda', a')\}$$

subject to

$$a' = [1 + (1 - \tau^r)c + tr + (1 - \tau^w)(nW - p^w I_{(1|i>0)}) - 0.5(\tau_{SS} + \tau_{Med})\min\{nW, \bar{y}\} - (1 - k^w I_{(1|i>0)})m(j, h)$$

$$tr = \max\{0, (1 + \tau^c)c - [1 + (1 - \tau^r)c + tr + (1 - \tau^w)(nW - p^w I_{(1|i>0)}) - 0.5(\tau_{SS} + \tau_{Med})\min\{nW, \bar{y}\} + (1 - k^w I_{(1|i>0)})m(j, h)\}$$

$$c \leq \frac{1}{1 + \tau^c}[tr + [1 + (1 - \tau^r)c + tr + (1 - \tau^w)(nW - p^w I_{(1|i>0)}) - 0.5(\tau_{SS} + \tau_{Med})\min\{nW, \bar{y}\} - (1 - k^w I_{(1|i>0)})m(j, h)]$$

$$\ln W = w(h, j) + \mu + \lambda + u$$

$$h' \sim \Lambda_{e,j}(h, h')$$

The first constraint summarizes the evolution of assets. Next period assets are equal to current period assets plus interest income (subject to a capital income tax $\tau^c$), plus a government transfer, less consumption (subject to a tax $\tau^c$) plus labor income (net of labor income, social security and Medicare taxes and insurance premiums) minus medical
Chapter 1. Health Risk

1.2 Health Risk Expenditures. \( I_{i>0} \) is an indicator equal to 1 for those with employer provided health insurance \((i>0)\), and equal to zero otherwise, so the premium \( p^w \) and medical benefits \( k^w \) apply only to those with employer health insurance. The second constraint describes the government transfer \( tr \) that guarantees a minimum consumption level \( \bar{c} \). The third constraint is a zero borrowing constraint. The final constraints describing wage income and health transitions have been explained previously.

When retired, individuals face a similar problem summarized below.

\[
V_r(e, i, j, h, a) = \max_c \{ U(c, h) + \beta s(j, h, e) EV_r(e, i, j + 1, h', a') \}
\]

subject to

\[
a' = [1 + (1 - \tau^r) r]a - (1 + \tau^c)c + tr + SS - [1 - k^\text{med} - k^\text{ret} \cdot I_{i=2}]m(j, h)
\]

\[
- p^\text{med} - p^\text{ret} \cdot I_{i=2}
\]

\[
tr = \max \{0, (1 + \tau^c) \bar{c} - [1 + (1 - \tau^r) r]a - SS + [1 - k^\text{med} - k^\text{ret} \cdot I_{i=2}]m(j, h)
\]

\[
+ p^\text{med} + p^\text{ret} \cdot I_{i=2} \}
\]

\[
c \leq \frac{1}{1 + \tau^c}[tr + [1 + (1 - \tau^r)r]a + SS - [1 - k^\text{med} - k^\text{ret} \cdot I_{i=2}]m(j, h) - p^\text{med}
\]

\[
- p^\text{ret} \cdot I_{i=2} \]
\]

\[
h' \sim \Lambda_{e,j}(h, h')
\]

1.3 Data

An ideal data set for this study would be a representative panel of individuals observed over several years containing information on health status, medical expenditures (total and out-of-pocket), insurance, earnings and assets, employment status, education, etc. Unfortunately, no such comprehensive survey exists, so I utilize several data sets which
together enable me to estimate the required parameters: (1) Health and Retirement Survey (HRS), (2) Medical Expenditure Panel Survey (MEPS), (3) Current Population Survey (CPS), (4) National Longitudinal Survey of Youth 1997 (NLSY), (5) Survey of Consumer Finances (SCF) and (6) Panel Study of Income Dynamics (PSID).

While most of these surveys extract some information related to health, the HRS and MEPS surveys contain the most detailed variables on health, limitations, disability, insurance and medical expenditures for individuals over time. They have been extensively used and described in previous literature. An advantage of the MEPS survey is that it includes individuals of all adult ages, however, there are relatively few observations for ages above 70. Therefore, I use as a complement the HRS which is a national panel survey of individuals aged 51 and above containing detailed statistics on the elderly. Another disadvantage of MEPS is that a new panel of sample households is selected each year, and data for each panel is collected for only two calendar years. Data from the PSID is used when estimation requires a longer panel dimension. I use the CPS for aggregate statistics that do not require a panel study due to its large sample size. The SCF is used because it is one of the few data sets containing detailed information on wealth accumulation, and finally, the NLSY is the only survey with information on individual time spent on health care activities. The time period used from most data sets is between 1992 and 2006.

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1.4 Parameter Values and Calibration

Many parameters can be estimated directly from the data sets described above. These are the parameters summarizing health insurance coverage and premiums, health status transitions, medical expenditures, and survival probabilities. In their estimation, I use data on civilian males only and all statistics are calculated using sample weights. I define the non-college group as those with a high-school degree only and no years of college. The college group includes all those with four or more years of college. Full time workers are defined as workers who are not self-employed, with hourly earnings greater than 5$ per hour, and who work more than 30 hours per week and at least 48 weeks per year. Those working less hours or weeks are considered as not working. All dollar amounts are CPI adjusted to 2006 U.S. dollars.

Other parameters such as most utility function and earnings process parameters are calibrated to match statistics on saving rates, labor supply, and earnings observed in the data. Finally, Table 1.1 provides a summary of the demographic structure, tax and social security environment, and fixed parameters taken as given in the model. The consumption floor is fixed at 10% of average earnings.\footnote{As noted in Hubbard, Skinner, and Zeldes (1995), “measuring the means-tested consumption floor is difficult since potential payments from social insurance programs differ dramatically according to the number of children, marital status, age, and even the recipient’s state or city.” However, a consumption floor of 10% of average earnings (4,576 2006 $) is consistent with previous estimates in the literature: for example, 4,118 1998 $ in French and Jones (2007) who consider childless households. Attanasio, Kitao and Violante (2010) also assume a 10% consumption floor.}

1.4.1 Parameters estimated directly from the data

Health Insurance Table 1.2 reports the fraction of the working male population by employer health insurance type, estimated from the CPS, and reports average premiums and insurance coverage rates. The college group contains fewer uninsured workers and a significantly higher percentage of workers whose insurance extends into retirement. The annual Medicare premium is set to 779, obtained from an average of CPI adjusted

\[\text{As noted in Hubbard, Skinner, and Zeldes (1995), “measuring the means-tested consumption floor is difficult since potential payments from social insurance programs differ dramatically according to the number of children, marital status, age, and even the recipient’s state or city.” However, a consumption floor of 10\% of average earnings (4,576 2006 $) is consistent with previous estimates in the literature: for example, 4,118 1998 $ in French and Jones (2007) who consider childless households. Attanasio, Kitao and Violante (2010) also assume a 10\% consumption floor.}\]
premiums over the sample period. The annual single coverage employer-sponsored health insurance premium is 3,852 for an active worker, and 3,497 for a retired worker 65 and over. The employee share of this premium is 18 percent for active workers, and 45 percent for retirees. These numbers are calculated according to estimates from Buchmueller et al. (2007).\textsuperscript{15} I take the health insurance coverage rates from Attanasio, Kitao and Violante (2010) who estimate $k^{med} = 0.5$, $k^{w} = 0.7$ and $k^{ret} = 0.3$.

**Health Status Transitions** I use the MEPS and HRS data sets to estimate health transition probabilities: MEPS for age groups younger than 60 and HRS for age groups older than 60. In both data sets, respondents report their perceived health status, on a scale from 1 to 5. I group these five states into three: the good health state (G) corresponds to a self-reported health status of excellent or very good; the average health state (A) corresponds to a self-reported health status of good or fair; and the poor health state (P) corresponds to a self-reported health status of poor.\textsuperscript{16} I estimate education and age specific health transition probabilities using a logistic regression model that includes age, age squared and age cubed. Figure 1.1 shows selected health transition probability profiles. As expected, the probability of declining health increases with age and the college educated group is less likely to transition to bad health states at any given age.

The self-reported health measure is imperfect since respondents could hold different views of what good and bad health states entail (e.g. individuals with long term disabilities may classify themselves in good health if they do not suffer from any illnesses). To test the validity of this measure, I use data on functional and activity limitations avail-

\textsuperscript{15}Buchmueller et al. (2007) estimate average premiums and employee shares for 2003 using the MEPS data set. I adjust the premiums to 2006 dollars. The data from the MEPS Insurance Component shows that the employee share of the premium while actively working has been very steady over time at around 17 to 18 percent, thus not creating a problem when averaging over the sample period. However, premiums have increased much faster than inflation, more than doubling over time from 1,992 in 1996 to 4,118 in 2006. This is therefore be a problem when averaging over the sample period.

\textsuperscript{16}The self reported health variable is consistent across the surveys used, having the same ranking and description of health states.
able in MEPS for 2006 only. The functional limitations variable is coded "yes" if the respondent has "difficulties walking, climbing stairs, grasping objects, reaching overhead, lifting, bending or stooping, or standing for long periods of time," and "no" otherwise. The activity limitations variable is coded "yes" if the respondent has limitation in work, housework, or school, and "no" otherwise. I find that 61% of those with functional and activity limitations report fair or poor health status, and only 11% of those without such limitations report average or poor health status. This indicates that the self reported health status is indeed imperfect: we should observe 100% of those with limitations reporting fair or poor health states. However, the correlation between these variables provides some confidence in the self reported health measure.

**Medical Expenditures**  
Average total health expenditures by age and health status are estimated from MEPS and are reported in Table 1.3. These expenditures include the out-of-pocket expenditures plus what is covered by insurance, but they do not include nursing home costs and insurance premiums since these are accounted for separately. The sample size of individuals over 70 is relatively small in MEPS, so this requires relatively broader age groups in order to have enough individuals. The HRS data contains only information on out-of-pocket expenditures, so unfortunately it cannot be used to directly estimate total costs.

My paper takes the view that nursing homes are luxury goods, so nursing home costs are not included in medical expenditures. They are simply part of regular consumption. This view is consistent with De Nardi, French and Jones (2006) who argue that medical expenditures after the age of 85 are luxury goods based on the observation that there are huge differences in out of pocket expenditures between the top and bottom quintiles.

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17 These variables are available for MEPS Panel 10, rounds 3, 4 and 5, and Panel 11, rounds 1, 2 and 3.
18 The validity of the self-reported health status measure has been discussed extensively in previous literature. Examples include Bentez-Silva and Ni (2008); Crossley and Kennedy (2002), Baker, Stabile and Deri (2004), and Hurd and McGarry (1995).
19 Kopecky and Koreshkova (2009) calibrate a medical expenditures process using the HRS.
after this age. The top 10% of spenders account for 57% of total health expenditures and the top 1% for 25% when nursing home costs are included. Since in my model I want to capture the role played by required medical expenses (as much as the data allows), I exclude nursing home costs. However, my model could endogenously explain the high consumption of luxury goods in poor health states by the rich old people because the marginal utility of consumption is higher in bad health states due to the decrease in leisure time. This feature also leads individuals to save more for old age when health is likely to deteriorate so they can smooth utility by consuming more in bad health states. This is in contrast to Kopecky and Koreshkova (2009) who treat nursing home expenditures as exogenous.

Survival Probabilities I use the HRS to estimate mortality probabilities by education group, health status, and age. The HRS reports whenever an individual misses a survey wave due to death. I first calculate a raw measure of mortality probabilities by dividing the number of deaths reported for a given (education, health, age) group by the number of total respondents in that group (dead or alive). Since some groups have zero or very few observations, this measure leads to very irregular mortality age profiles. To correct this, I run a regression of log raw mortality probabilities on a constant, age, and age squared and obtain the fitted values, imposing that death at the age of 100 occurs with certainty. These are shown in Figure 1.2. As expected, the estimated probabilities reveal that as health declines, mortality rates increase significantly. Mortality probabilities increase with age, but for any health-age group, they are lower for the college educated.

20 The existing literature is inconclusive with respect to the effect of health on the marginal utility of consumption. Lillard and Weiss (1997) estimate that the marginal utility of consumption increases after a health shock. Edwards (2008) also finds that patterns in the data are consistent with a negative cross partial for individuals. However, Finkelstein et al. (2008) estimate that the marginal utility of consumption decreases with bad health.
1.4.2 Calibrated Parameters

I jointly calibrate the time discount factor $\beta$, the earnings process parameters and the parameters in the utility function associated with the time costs of work and bad health. I use the method of indirect inference first introduced by Smith (1990, 1993) and extended by Gouriéroux et al. (1993) and Gallant and Tauchen (1996). All these parameters together affect labor supply, observed average earnings and savings rates. A major issue with observed earnings in the data is selection bias: we observe only the earnings of those who choose to work. The data reveals a strong selection effect into the labor force by education, age and health status: on average, we observe that the college group, the healthy groups and the age groups 30-50 supply the most labor to the market (Figure 1.4). Therefore, we cannot directly estimate the true earnings process parameters through a regression on actual data. Suppose we run the following regression in the data, separately by education, where $w$ represents hourly earnings, $j$ represents age, and $I_{h=G}$ is an indicator equal to one for those in good health:

$$
\log w = \beta_0 + \beta_1 j + \beta_2 j^2 + \beta_3 j^3 + \beta_4 I_{h=G} + \beta_5 I_{h=G} \ast j + \varepsilon.
$$

It is very likely that only workers with high earnings choose to work in average and poor health states, implying a negative bias on $\beta_4$. All other coefficients will be biased for similar reasons. The goal is to infer the unobserved parameters driving the earnings process. I assume that the bias in the earnings profiles of workers is the same in both actual and simulated data. The goal is to find parameter values that when fed into the model generate the same earnings profiles in the model (post-selection into the labor force) as those observed in the data. Since the utility function parameters $\theta$, $\Phi_1$ and $\Phi_2$

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$^{21}$Gouriéroux and Monfort (1996) provide a survey of indirect inference.

$^{22}$I estimate the parameters in the data using male full time workers. Observations are weighted using sample weights. Hourly earnings are calculated as annual earnings divided by hours worked.
are also major determinants of labor force participation through the effect on the period time endowment, these parameters need to be calibrated simultaneously.

Table 1.4 lists the targets used, the source of different targets, and the model results. I estimate $\beta_4$ and $\beta_5$ using the MEPS data set since this is the only large survey containing information on earnings and detailed health status in the same time period. Unfortunately, there are very few individuals in the poor health state who are observed working, making it impossible to accurately estimate the effect of poor health on wages. Therefore, I assume that the productivity effect of poor health is the same as that of average health. The model parameters $\beta_0$, $\beta_1$, $\beta_2$ and $\beta_3$ are identified by targeting the average hourly wages of four different equally spread and narrowly defined age groups.

The time discount factor $\beta$ is calibrated for the non-college group by targeting the average asset to income ratio of males between 30 and 50 years of age observed in the SCF data. One of the interests of this paper is to estimate the degree to which health can explain differences in asset accumulation between education groups. Therefore, I do not want to target this difference in the calibration. I assign the calibrated non-college value of $\beta$ to the college group. Note that differences in the effective discount rates still exist between education groups due to different survival probabilities. For the remaining parameters, the calibrations are performed separately for the two education groups.

The calibrated parameters are summarized in Table 1.5. Here, we can note that the time cost of poor health is very large, approximately 1/4 of total available time for both education groups. The effect of bad health (either average or poor) on productivity is very large: at age 45, a non-college individual in good health has 17.8% higher productivity than a similar individual in bad health, and a college individual has 6.3% higher productivity.

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23 The targeted average asset to income ratio in the SCF is obtained after excluding individuals in the top 5% of the wealth distribution, and using only financial wealth as the measure for assets.
1.5 Results

1.5.1 Model Fit

The calibrated model matches the targets closely and performs well in matching other statistics in the data. The earnings age profiles in the model are an almost perfect fit of the data (Figure 1.3) and the labor force participation rates by age, health status and education are very close to the data for ages 30 to 55 (Figure 1.4). It is extremely important for the model to closely match these profiles since a main interest of the paper is to evaluate how different channels affect labor supply for different education groups. The key calibrated parameters that enable the model to account for the observed differences in labor supply are the fixed time costs of work, the time costs of average and poor health states, and the productivity costs associated with average and poor health. A large time cost of poor health is required to match the very low participation levels of those in poor health. Productivity costs account for most differences in labor supply between average and good health states, so the calibrated time costs of average health are low (only 2% and 4% of available time for non-college and college). The productivity costs of bad health and the time cost associated with poor health states are higher for the non-college group, enabling the model to capture the observed differences in labor supply and wage age profiles across education groups (Table 1.5).

The model does not replicate the sharp exit from the labor force observed in the data after the age of 55. In reality, individuals can apply for and start receiving social security benefits at the age of 62, so a large fraction of workers retires at this age. Also, the model abstracts from private pensions and does not model pension accrual or the progressive taxation of pension income. French (2005) establishes that the tax structures of Social Security and pensions are very important in explaining the sharp exit from labor force while health plays only a minor role. It is therefore not surprising that the model fails to replicate the data in this respect. The implication is that the model might overpredict
the degree of health risk at ages just prior to 65 since it does not allow individuals to withdraw social security benefits early in the advent of bad health that makes working more painful.

An important aspect of the model is the degree of social insurance available to individuals through the consumption floor because this provides partial insurance against health effects. Therefore, it is crucial to generate the same degree of reliance on government transfers in the model as in the data. Table 1.6 shows that the model approximates the data very well in general with respect to the percentage of people receiving transfers, the average amounts received and the health states of those receiving them. The model performs particularly well for the non-college group, perfectly matching the fraction of low income individuals receiving a transfer between ages 30-50 in the CPS of 59%. Also, we observe that 64% of non-college and 43% of college individuals in average or poor health states receive government transfers other than Medicaid, indicating that indeed, health is an important reason for bankruptcy. The model captures this fact very well by approximating these statistics (72% and 45% for non-college and college, respectively).24

The model produces a reasonable hump shaped consumption-age profile (Figure 1.5). The profile peaks at the age of 53, just one year later than in the data as found in Hansen and İmrohorolu (2008). Average consumption (excluding medical expenditures) increases by 36.8% between the ages of 25 and 50. Aguiar and Hurst (2008) estimate a similar consumption profile for core nondurables and housing services; however, the profile of core nondurables alone is lower, with only approximately 29% consumption growth between the ages of 25 and 50. Since there is no mechanism for the accumulation of durables in the model, it is likely that the model overpredicts consumption growth. The hump shaped profile is generated by non-separable utility, borrowing constraints, uncertain lifetimes in the absence of annuity markets and income uncertainty.25

24 The success of the model in matching these statistics supports the assumption of a 10% consumption floor level as a fraction of average earnings.

25 For example, Heckman (1974) and Bullard and Feigenbaum (2003) have studied the effects of non-
Finally, I compare median asset accumulation in the model with median financial asset profiles from the SCF (Figure 1.6). The model approximates well the college profile until the age of 55, after which asset accumulation in the data is much higher, even after excluding the wealthiest top 5% in the SCF. A possible explanation is that the return on assets for the college educated is likely to be much higher than the interest rate of 1.04% since they hold more stocks. The model in general overestimates asset accumulation for the non-college.

1.5.2 Health Effects on Labor Supply, Asset Accumulation, and Welfare

I first determine the importance of health effects for aggregate labor supply, asset accumulation and welfare. To do so, I study how these macroeconomic variables change in the absence of health effects through a series of experiments: (1) first, I study each effect’s individual outcome by eliminating each one separately (for example, all individuals are given the productivity of those in good health); (2) second, I consider the importance of the effects at different stages of the life cycle (for example, I eliminate medical expenditures only before or after retirement); (3) third, I study the interactions between effects by removing a combination of them simultaneously and (4) finally, I estimate the total effect of health by considering a model where everyone is in good health with certainty for the entire lifetime. In each experiment, I compare the results to those of the benchmark separability on consumption profiles; Hansen and Imrohoroglu (2008) have studied the role of uncertain lifetimes in the absence of annuity markets; Thurow (1969) has suggested that borrowing constraints may contribute to the hump shaped profile; and Attanasio et al. (1999) and Gourinchas and Parker (2002) argue that the hump shape is explained in part by the fact that individuals face income uncertainty and must die with non-negative assets.

26 I use data on only financial wealth (and exclude non-financial wealth) because there is no mechanism in my model for non-financial wealth accumulation.

27 Health is also related to portfolio choice: previous literature has found strong evidence that higher health risk leads to safer portfolio choice. Since non-college individuals have higher probabilities of bad health (higher health induced risk), they hold safer, lower return assets. (See for example Edwards (2008), Guiso et al. (1996), Rosen and Wu (2004), Heaton and Lucas (2000), Yogo (2009) and Hugonnier et al. (2009).)
model, which for the remaining of the analysis is defined as the calibrated model where all four health channels operate. The results are summarized in Tables 1.7, 1.8 and 1.9. Each effect considered separately is important for at least one variable. Lower productivity and available time associated with poor and average health states greatly decrease non-college labor supply, especially for those in poor health. In the absence of the productivity effect, non-college labor supply would be 6.6% higher, and in the absence of the time endowment effect, 5.3% higher. On the other hand, the presence of medical expenditures leads to higher labor supply because workers with employer provided health insurance are better off working when faced with high medical expenditures. In the Benchmark economy, the average labor supply of non-college workers with tied employer insurance is 4.8 percentage points higher than that of workers with no insurance. Eliminating the medical expenditure effect leads to 2.4% lower labor supply for non-college, generated mostly by a decrease of 40% and 3.8% in the labor supply of those in poor and average health states, respectively. The welfare costs implied by health effects are large, particularly those generated through the time endowment and productivity channels: in their absence, welfare would be 6.8% and 7.7% higher in terms of CEV, respectively.

Asset accumulation is influenced by all effects. Lower survival rates associated with bad health imply a higher future discount rate which significantly reduces average asset accumulation especially after the age of 60: if all individuals had the survival rates of

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28The demographic structure of the population in the model does not take into account population growth. It assumes that a constant number of individuals is born every period.  
29Welfare is defined as in Conesa, Kitao and Krueger (2009) to equal the ex-ante expected lifetime utility of a new-born agent. It is assumed that all individuals start with zero assets and an average level of labor productivity. Welfare changes are measured in terms of consumption equivalent variation (CEV): switching from a consumption-labor allocation of \((c_0; l_0)\) to \((c^*; l^*)\) results in a welfare change given by \(CEV = \left[\frac{W(c^*,l^*)}{W(c_0,l_0)}\right]^{\frac{1}{1-\sigma}} - 1\). Intuitively, this measures the permanent percentage change in consumption which is equivalent for an agent in expected utility terms to changes in initial conditions that lead to the different consumption-labor allocation. Note that this measure cannot be used to evaluate the welfare effects of changes in survival probabilities since period utility is negative and higher survival rates would lead to a lower expected lifetime utility and a negative CEV. For methods that enable the estimation of the value of changes in longevity, see Usher (1973), Rosen (1988), Murphy and Topel (2003) and Becker et al. (2005).
the healthy, asset accumulation would be 55% higher for non-college and 39% higher for college after the age of 60 (Table 1.8). Productivity effects imply lower average asset accumulation (e.g. in their absence, average non-college savings before the age of 60 increase by 11%), while time costs and medical expenditures lead to higher average savings (e.g. in their absence, non-college savings before 60 increase by 9.3% and 1.6%, respectively). These results capture two different effects: first, the elimination of health risk (elimination of fluctuations around the mean), and second, a level effect from higher average earnings, more available time on average, and higher average disposable incomes when the productivity, time and medical expenditures channels are shut down, respectively. In section 1.5.3, I study the effect of health risk independent of level effects in order to better explain these results.

I find that it is very important to model health effects over the entire life-cycle. To illustrate this, I take the example of medical expenditures and consider what would happen if we eliminated medical expenditures only before retirement, only after retirement, and for the entire life-cycle.\(^{30}\) For non-college graduates, the elimination of old age medical expenditures decreases asset accumulation before the age of 60 by 10%, but this is almost entirely reversed by the elimination of medical expenditures before retirement that increases disposable incomes (Table 1.8). In addition, in a model that abstracts from medical expenditures before 65, the effect of old age medical expenses on asset accumulation is overpredicted by between 8% and 22%, depending on the stage of the life-cycle (Table 1.10). In the presence of medical expenses before retirement, individuals have fewer resources to save out of for old age medical expenditures.\(^{31}\) For example, Kopecky and Koreshkova (2009) study health effects only after retirement age and find they go a long way in accounting for differences in asset holdings across permanent in-

\(^{30}\)Note that in the previous experiment with medical expenditures, I assigned all individuals the medical expenditures of the healthy, whereas here I am eliminating them entirely for the ages considered.

\(^{31}\)If we considered the productivity effect that further reduces incomes, the effect of medical expenditures after retirement on asset accumulation during working life would be even smaller.
come groups; however, they likely overestimate these effects because they do not account for the interactions mentioned above.

The interactions between different types of health effects are important for similar reasons. For example, I estimate that the effect of medical expenditures before retirement on asset accumulation before the age of 60 is underestimated by 16% and 51% in the absence of productivity effects for non-college and college, respectively (Table 1.10). The joint occurrence of these adverse health effects makes them more painful: not only do poor health individuals have to pay high medical costs, but these expenses are incurred at a time when average earnings are lower due to the productivity cost of bad health. Therefore, utility decreases more sharply because the change in disposable income caused by medical expenditures occurs at lower income levels where marginal utility is high. Hubbard, Skinner, and Zeldes (1995) who model only medical expenses and survival risk likely underestimate the importance of health expenses on asset accumulation for this reason.

Finally, in an economy where all individuals are healthy, or where health has no effect, all three macroeconomic variables, labor supply, average assets and welfare increase substantially. The welfare consequences are extremely large, especially for the non-college whose CEV is 18.3% in the absence of productivity, time and medical costs associated with bad health. The above experiments have revealed consistently that health effects are much larger for the non-college group, resulting in larger percentage changes relative to the Benchmark.

Therefore, I estimate the extent to which health can account for differences in various macroeconomics variables between education groups. There are several reasons why the removal of health effects has greater impact on the non-college. One reason for this is that they face higher probabilities of bad health at any age. Also, bad health lowers

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32 This welfare gain would be even greater when accounting for the extra years of life gained when health has no effect on survival. As mentioned previously, this cannot be captured by the CEV measure.
non-college productivity to a greater extent. Finally, at the lower income levels of the non-college, fluctuations in incomes and time endowments caused by health imply larger utility changes, making health risk relatively more important. Overall, I find that health accounts for 53% of the 11 percentage point difference in labor supply observed in the data between education groups. The productivity and leisure effects play the biggest roles since together, they decrease non-college labor supply by 8.4 percentage points while lowering college labor supply by only 2.4 percentage points. Next, I find in the data that the fraction of non-college educated receiving a government transfer is 9 percentage points higher than for college. Health accounts for 40% of this difference.\footnote{Health accounts for 58% of the difference observed in the model, but because the model slightly overestimates the fraction of non-college receiving transfers and slightly underestimates the fraction for college, of the total observed difference in the data, health accounts for only 40%.

Finally, the average assets accumulated by the college group in the model before the age of 60 are 394 percent higher than the non-college. After the age of 60, this statistic is 275 percent. Of these differences, health accounts for 5% and 17%, respectively.

1.5.3 The Importance of Health Risk

In the previous section, eliminating productivity, time and medical expenditures effects entails both (1) an elimination of health risk (elimination of fluctuations around mean earnings, mean available time and mean medical expenditures across individuals of different health states), and (2) a level effect from higher average earnings, more available time on average, and higher average disposable incomes when the productivity, time and medical expenditures channels are shut down, respectively. In order to assess the role of health risk, this section considers similar experiments as the previous section, but this time only the variation around an average effect is eliminated. For example, I eliminate the productivity risk implied by changes in health status by giving all individuals the average wages across health groups, weighted by the fraction of people in good, average and poor health at each age. I eliminate the time endowment risk by giving all individ-

\footnote{Health accounts for 58% of the difference observed in the model, but because the model slightly overestimates the fraction of non-college receiving transfers and slightly underestimates the fraction for college, of the total observed difference in the data, health accounts for only 40%.

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uals the average leisure cost across health groups at every age. Since average earnings are the same as in the Benchmark, this enables me to isolate the effect of health induced productivity risk.\footnote{The average is taken over wages before selection into the labor force takes place. Post-selection, average earnings may not be the same in this experiment as in the Benchmark.}

To get an idea about the magnitude of health risk, I first study how the variance of earnings and disposable income changes relative to the Benchmark when health risk transmitted through each channel is eliminated (Table 1.11).\footnote{Disposable income is defined as the sum of after tax assets, after tax earnings, social security and government transfers minus health insurance premiums and medical expenditures.} Because of the ability to self insure through savings and due to the presence of government insurance programs, the variance of disposable income need not necessarily change in these experiments. Indeed, we see that each channel considered separately contributes only a very small fraction to overall income inequality. For example, the variance of non-college disposable income declines by only 1.3% in the absence of health induced productivity risk. However, any individual’s ability to self insure against health risk transmitted through multiple channels is lower, so when considering all channels together, health risk accounts for 13% of disposable income inequality in the model, for non-college. For the college educated, it accounts for a much smaller fraction (1.5%) because the risk of bad health is lower and health effects are in general smaller, so this group’s ability to self insure is greater. As a reference, I also calculate the contribution of transitory earnings shocks to inequality and find it to be much larger: transitory shocks account for 36% and 66% of non-college and college disposable income inequality.

I find that health risk accounts for 10% of non-college average asset accumulation before the age of 60, and for 6% after 60 (Table 1.13). Time endowment risk is particularly important for asset accumulation before 60: the loss of leisure is very painful for those in poor health who suffer a decrease of about 1/4 in their available time. If they choose to work, they are left with very little leisure, and if they take time off work, they suffer from
a large income drop. In either case, the drop in utility is so large that even though the probability of poor health is very low, this effect generates large precautionary savings. The consumption floor is not as effective in insuring against this type of risk because the decrease in leisure implies very low utility when consumption is at the minimum level.

Comparing the results of eliminating health effects as conducted in section 1.5.2 with the results of eliminating health risk alone, we see the distinction between level effects associated with the four channels and the effects of risk. For example, tables 1.8 and 1.13 reveal that health induced productivity risk is an important precautionary savings motive for the non-college, accounting for 4.4% of average asset accumulation before the age of 60; however, productivity costs associated with bad health overall decrease asset accumulation because average earnings are lower (in their absence, asset accumulation would increase by 10.8%).

It is important to note that people self insure through asset accumulation for a variety of reasons. Transitory earnings shocks are important to model because in their presence, people already accumulate a large amount of assets to self insure. Since the timing of adverse health shocks need not coincide with negative transitory shocks, people are already insured to some extent against health shock. Models that do not include earnings shocks (other than those generated by health status) such as in Attanasio, Kitao and Violante (2010) or who exclude all transitory variation in earnings such as Hubbard, Skinner, and Zeldes (1995) likely overestimate the effects of health risk on asset accumulation: the removal of all health risk in the absence of transitory shocks leads to a 40% larger decline in average asset accumulation for non-college before the age of 60 than in the presence of shocks.

The simultaneous elimination of productivity, available time and medical expenditure risk leads to a welfare increase of 1.5% in terms of CEV relative to the Benchmark for non-college, and .9% for college (Table 1.12). The time endowment risk has the largest effect on welfare (1.4% and .89% for non-college and college), followed by medical expense
risk (.58% and .36% for non-college and college, respectively). As argued before, the time endowment risk is very important due to the large drops in utility suffered in poor health states as a result of big leisure time reductions.

An important factor in the assessment of health risk is the consumption floor and Medicaid programs which provide some degree of insurance against bad health shocks. Hubbard, Skinner, and Zeldes (1995) argue that in the presence of asset based means-tested social insurance, medical expenditure, survival, and earnings risks together explain why low income groups save less than high income groups: the means-tested program effectively taxes savings 100% in the advent of bad health or large negative earnings shocks that force individuals to rely on the consumption floor. However, their paper studies all effects together, without a decomposition exercise that shows which type of risk is the driving force. My results indicate that actually, in the presence of health risk (independent of level effects), asset accumulation is higher even for low income groups. On the other hand, the level effect may lead to lower asset accumulation if individuals have on average lower incomes from which to save in the presence of health effects.

1.5.4 Model Counterfactuals and Health Policy

I use the calibrated model to conduct several counterfactual experiments in order to evaluate the importance of various features in the model environment. The first set of counterfactual experiments considers the effects of improved non-college health outcomes, shedding light on the importance of health transition probabilities in the model. The second set of experiments analyzes the role played by health care system efficiency by considering different environments where medical and time costs associated with bad health are lower. Since there is large uncertainty regarding the type of policies and associated costs involved in making improvements in health outcomes and health care system efficiency, I abstract from conducting full policy evaluations that account for costs involved and transition effects. However, the counterfactual experiments are useful in evaluat-
ing the importance of these aspects of the model and estimating the benefits associated with different degrees of improvements in these areas.\textsuperscript{36} The third experiment considers the effects of mandated employer provided health insurance, and the final experiment is a public finance experiment of expanding Medicaid insurance, where the additional government expenditures are financed through lump sum taxes.

These four experiments are relevant since the 2010 Patient Protection and Affordable Care Act includes provisions targeted at improving general health, increasing health care system efficiency, encouraging individuals to buy health insurance and redistributing health care costs across individuals of different incomes. Understanding how the model reacts to changes in the environment that embody these goals will shed light on potential long run benefits.

\textbf{Counterfactual experiments: improving non-college health outcomes} The data reveals that the non-college educated have significantly higher probabilities of bad health at any given age compared to college graduates (Figure 1.1). Many previous papers have studied the reasons for this disparity.\textsuperscript{37} Differences in incomes and occupational choice as well as differences in behavior across education groups such as smoking, drinking, diet/exercise, use of illegal drugs, household safety, use of preventive medical care, and care for hypertension and diabetes have been found to be very important. Differential access to care may also be responsible for differences in behavior. In addition, those predisposed to illness when young are less likely to graduate from college, as shown in Gan and Gong (2007) resulting in a selection effect.

I consider the effects of narrowing the gap in health status across education groups in order to understand the role played by health transition probabilities in the model for

\textsuperscript{36}The approach of evaluating only one side of the welfare equation had been previously used: for example, Hopenhayn and Rogerson (1993) focus entirely on the costs associated with policies that affect firm-level decisions about labor force adjustment, ignoring the benefits.

the non-college. Also, while highly uncertain, various policies might succeed in closing this gap, making these results relevant for future policy evaluation.\textsuperscript{38} I conduct several experiments where the health transition probabilities of the non-college approach those of the college. I consider a best case scenario where non-college health is improved to match that of college graduates, as well as intermediate scenarios where non-college probabilities approach those of the college by 25\%, 50\% and 75\%. Survival rates conditional on education, health and age are left unchanged.

Tables 1.14 and 1.15 show the effects of improving non-college health outcomes. When non-college health transition probabilities equal those of college, non-college labor supply and asset accumulation increase, and welfare increases by 6.3\% in terms of CEV relative to the Benchmark. The most striking results are that the percentage of non-college graduates receiving the consumption floor declines from 16.1\% to 12.7\% and that the average Medicaid amount spent per living non-college person declines by 45\% from 633$ to only 351$.\textsuperscript{39} This is very significant considering that Medicaid expenditures were 1.4\% of GDP in 2008. This result also indicates that the number of bankruptcies due to medical reasons would decline dramatically. Because I model all four channels through which health affects individuals, an improvement in average health status leads to large benefits because all effects and interactions are accounted for. Models that do not account for all four channels would underestimate the benefits of improving health outcomes.\textsuperscript{40}

\textsuperscript{38}For example, provisions of the 2010 Act target behavioral factors and increase access to health education and preventive care for those with lower incomes. In 2010, the Act creates the National Prevention, Health Promotion and Public Health Council to develop a national prevention, health promotion and public health strategy. New health plans are required to provide at a minimum coverage without cost-sharing for preventive services, recommended immunizations, preventive care for infants, children, and adolescents, and additional preventive care and screenings for women. In 2011, it eliminates cost-sharing for Medicare-covered preventive services and authorizes Medicare coverage for a personalized prevention plan, including a comprehensive health risk assessment. Medicaid enrollees will be given incentives to participate in comprehensive health lifestyle programs and meet certain health behavior targets. Small employers are encouraged to establish wellness programs. Disclosure of nutritional content will also be improved. A list of provisions can be found in the “Implementation Timeline” available on the Henry J. Kaiser Family Foundation website, http://www.kff.org/.

\textsuperscript{39}The level of 633$ of Medicaid expenditures per person in the Benchmark is very close to the actual expenditure per person in U.S..

\textsuperscript{40}For example, in a model abstracting from health induced productivity risk, non-college welfare
Tables 1.14 and 1.15 also reveal that even more modest improvements in non-college health outcomes lead to very large benefits, for example increasing welfare by 3.2% when probabilities approach those of the college half way.

**Improving Efficiency: lowering medical costs and time costs** The efficiency cost of the U.S. health system has been estimated at between 20 and 30% of health care spending, or 3 to 5% of GDP (Fisher et al. (2003); Skinner, Fisher, and Wennberg (2005)). Garber and Skinner (2008) show that most evidence indicates that the U.S. lags behind other wealthy countries in terms of productive and allocative efficiency. The U.S. is productively inefficient due to its fragmented care, high administrative costs, and heterogeneity in treatment because of race, income, and geography.\(^{41}\) It is allocatively inefficient relative to other countries because it is more likely to pay for diagnostic tests, treatments, and other forms of care before effectiveness is established and with little consideration of the value they provide.\(^{42}\) It also has poorly restrained incentives for overutilization and pays high prices for inputs (e.g. U.S. physicians’ salaries are significantly higher than in most other countries). These inefficiencies imply not only higher medical costs but also wasted time: just half of recommended care is provided in a typical primary care visit (McGlynn et al., 2003), increasing the need for subsequent visits.

Several provisions of the 2010 Patient Protection and Affordable Care Act target these inefficiencies. For example, effective as of 2012, the Centers for Medicare and Medicaid Services will begin tracking hospital readmission rates and provide financial increases by only 3.4% and the average Medicaid amount per non-college person declines by only 39% when the non-college are given the health transition probabilities of the college. (These results are calculated relative to a new benchmark where the productivity risk is also absent.)

\(^{41}\)See for example Cebul et al. (2008) who show that the U.S. fragmented organizational structures lead to disrupted relationships, poor information flows, and misaligned incentives that combine to degrade care quality and increase costs. Also, Woolhandler, Campbell and Himmelstein (2003) show that administrative costs comprise almost twice as much of health care spending in U.S. than in Canada.

\(^{42}\)Garber and Skinner (2008) give the example of ezetimibe, an expensive component of the controversial cholesterol-reducing drug Vytorin, that had never been recommended as a first-line treatment because of a lack of direct evidence that it was effective in reducing cardiovascular disease. Nevertheless, ezetimibe accounted for 15 percent of U.S. cholesterol-lowering drug sales in the U.S., but only 3 percent in Canada according to Jackevicius et al. (2008).
incentives to reduce preventable readmissions. As of 2013, a national pilot program will be established for Medicare on payment bundling to encourage doctors, hospitals and other care providers to better coordinate patient care. In 2015, Medicare will create a physician payment program aimed at rewarding quality of care rather than volume of services.

To capture the potential benefits of these reforms, I conduct several experiments where medical costs and time costs associated with bad health states decline by between 10 and 30% (Tables 1.16 and 1.17). In a very optimistic scenario in which both medical and time costs decline by 30%, this policy results in only a small change in labor supply (0.5% for non-college) since people have more time to allocate to work when in bad health but less incentive to keep working to keep the employer insurance when medical costs are lower, slightly lower asset accumulation (-5.1% for non-college before the age of 60) since poor health states are not as painful in terms of utility when time costs are lower and lower medical costs decrease the need to save, and greater welfare (4.1% in terms of CEV for non-college). The effects on the college educated are similar, but quantitatively smaller. The percentage of government consumption floor recipients declines by only 11.9% on average (fewer individuals go bankrupt due to bad health, but receiving the consumption floor is more attractive when time costs are lower), and the average Medicaid amount spent per person declines by 50%. However, the benefits are lower in more plausible scenarios where these costs decline by smaller fractions.

**Expanding Medicaid to Low Income Groups and Increasing Employer Sponsored Insurance** One of the provisions of the 2010 Patient Protection and Affordable Care Act is extending Medicaid to individuals below 158% of the poverty level, and subsidizing health insurance premiums and medical expenditures for those up to 400% of the poverty level, where the poverty level in 2010 is 10,800 current $. In addition, effective in 2014, employers with 50 or more workers will be required to provide health insurance
plans to employees or pay a fine.

I first study the effects of increasing the percentage of workers covered by employer sponsored health insurance. In the Benchmark model, individuals with employer health insurance are significantly better off (Table 1.20). I conduct an experiment where all individuals are given employer health insurance, keeping constant the fractions of those with tied insurance and with insurance that continues into retirement. In this experiment, the cost of insurance (measured as the percentage difference between premiums and expected benefits) is the same as in the Benchmark economy.\textsuperscript{43} Results show that there are only small welfare gains and small declines in the percentage of those receiving government transfers (Tables 1.18 and 1.19). The effects are small because only 18% of non-college and 8% of college workers lack employer health insurance in the Benchmark economy. Also, since poor health greatly increases the likelihood of unemployment due to time and productivity costs and since the unemployed lose the employer health insurance, only a small fraction of those in poor health benefits from this reform. Therefore, individuals with the largest medical expenditures who need insurance the most are least likely to benefit.

Finally, I study the effects of mandated employer provided health insurance together with an expansion of Medicaid to all individuals with incomes below 150% of the current poverty level.\textsuperscript{44} Note that now, all unemployed individuals have their medical expendi-

\textsuperscript{43}The cost of insurance is very high in the model, at 54% of premiums paid. In reality the cost of insurance is only approximately 12%. The difference arises partly because individuals in the model pay premiums for single coverage, which are about 44% higher than the average per enrollee premiums in a family health insurance plan. Also, in reality, moral hazard leads to higher health care utilization of those insured. The model does not capture this, and therefore health care expenditures are lower in the model for those with insurance. Despite the high costs, individuals in the model are better off with insurance because the employer is assumed to pay for a large fraction of premiums, as is the case in reality. With mandated employer health insurance, we might observe a small decline in average wages since employers are likely to transfer some of the costs to workers by offering them health benefits but lower wages. I do not account for this in this experiment. I leave these issues to be studied in future work.

\textsuperscript{44}In reality, this policy could go a long way in improving the average health of the low income groups by encouraging them to seek treatment. However, it is very hard to predict to what extent this would happen since there are many other factors that determine health. In this exercise, I capture only the direct effect of expanding insurance coverage.
tasures covered since Medicaid coverage is a function of income. In the Benchmark model, only those receiving the consumption floor (equal to 46% of the current poverty level) were covered by Medicaid. However, the 2010 Act changed Medicaid eligibility rules so it is no longer an asset based means-tested program. Therefore, many individuals with enough accumulated assets will receive Medicaid but not the consumption floor.

I first study only the benefit side of this reform, leaving taxes unchanged. The labor supply decreases substantially for both college groups, by 9% and 7% for non-college and college respectively, and asset accumulation increases since the average disposable income increases. The percentage of those receiving the consumption floor decreases by 15.5% on average, decreasing expenses on the consumption floor program by 14% (Table 1.19). However, the number of Medicaid recipients increases by 112%, increasing Medicaid expenditures by 129%. Of this increase, 11.3% is offset by lower expenditures on the consumption floor program. Welfare increases by 2.4% and 1.3% for non-college and college, respectively. Since the costs of expanding Medicaid can be calculated explicitly in the model, I also conduct a public finance experiment where the additional government expenditures are financed through lump sum taxes. With a government budget balancing tax of 646$ per person (placed on non-college and college), the percentage of those receiving the consumption floor decreases by only 5.4%, and welfare decreases by .4% in terms of CEV relative to the Benchmark.

Discussion Tables 1.18 and 1.19 compare the outcomes in different model environments, assuming intermediate levels of success in improving non-college health outcomes and health care system efficiency. It is clear that health outcomes and system efficiency play very important roles as improvements in these areas generate large increases in welfare and lower government expenditures. Also, the benefits of improving non-college health outcomes rise rapidly as health transition probabilities near those of the college, especially in terms of welfare. Better health diminishes the importance of all four health
effects studied. Since one of the most important channels is the effect of health on the productivity of the non-college group, improving health outcomes has very large welfare benefits because it is also able to raise overall productivity. Changing other features of the environment such as the degree of health insurance has smaller effects as the role of only one of the four channels is diminished.

1.6 Conclusion

My paper shows that health affects individuals’ economic decisions significantly through productivity, available time, medical expenditures and survival probability effects. Since bad health impacts people simultaneously through all these channels, I find large interactions between them, underscoring the importance of using a unified framework accounting for all effects when evaluating policy. Health is found to have large effects on the non-college group, and can account for significant differences in macroeconomic variables between college and non-college groups. Due to the presence of many health effects, differences in health outcomes across education groups are very important and improving non-college health closer to college levels results in large welfare gains, higher labour supply, and significantly lower reliance on government welfare programs.
### 1.7 Tables

Table 1.1: Model Parameters

<table>
<thead>
<tr>
<th>Demographics</th>
<th>Non-college</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of entry into labor force</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Age of retirement</td>
<td>65</td>
<td>65</td>
</tr>
<tr>
<td>Average lifespan</td>
<td>75</td>
<td>82</td>
</tr>
<tr>
<td>Measure of education types</td>
<td>70%</td>
<td>30%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Taxes (%)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital income tax</td>
<td>40.0</td>
<td></td>
</tr>
<tr>
<td>Consumption tax</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>Labor income tax</td>
<td>16.0</td>
<td>19.0</td>
</tr>
<tr>
<td>Social Security tax</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>Medicare tax</td>
<td>1.45</td>
<td></td>
</tr>
</tbody>
</table>

| Interest rate | 1.04 |
| Utility function | |
| \( \alpha \) | 0.4 |
| \( \sigma \) | 3.0 |

| Social Security and Insurance | |
| Consumption floor | 10% of average wages |
| Consumption floor ($) | 4576 |
| Social Security income when retired | 12804 | 14069 |
Table 1.2: Insurance, premiums and coverage rates

% workers by Employer Provided Health Insurance (EPhI)

<table>
<thead>
<tr>
<th></th>
<th>Non-college</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td>While working only</td>
<td>54</td>
<td>58</td>
</tr>
<tr>
<td>Extends into retirement</td>
<td>28</td>
<td>35</td>
</tr>
</tbody>
</table>

Insurance premiums (annual amount for single coverage, 2006 dollars)

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Employee Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medicare premium (part B)</td>
<td>779</td>
<td>779</td>
</tr>
<tr>
<td>EPHI Premium</td>
<td>3852</td>
<td>693 (18%)</td>
</tr>
<tr>
<td>EPHI Premium (coverage into retirement)</td>
<td>3497</td>
<td>1574 (45%)</td>
</tr>
</tbody>
</table>

Insurance coverage rates of total medical expenditures

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Medicare</td>
<td>50%</td>
</tr>
<tr>
<td>Employer Provided, age&lt;65</td>
<td>70%</td>
</tr>
<tr>
<td>Employer Provided, age&gt;=65</td>
<td>30%</td>
</tr>
</tbody>
</table>

Table 1.3: Total Health Expenditures, MEPS

<table>
<thead>
<tr>
<th>Age group</th>
<th>Health Poor</th>
<th>Health Average</th>
<th>Health Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-29</td>
<td>4,577</td>
<td>1,174</td>
<td>607</td>
</tr>
<tr>
<td>30-39</td>
<td>8,373</td>
<td>1,775</td>
<td>874</td>
</tr>
<tr>
<td>40-49</td>
<td>11,286</td>
<td>3,024</td>
<td>1,353</td>
</tr>
<tr>
<td>50-59</td>
<td>11,597</td>
<td>5,433</td>
<td>2,172</td>
</tr>
<tr>
<td>60-64</td>
<td>16,293</td>
<td>7,492</td>
<td>3,205</td>
</tr>
<tr>
<td>65-74</td>
<td>18,578</td>
<td>8,329</td>
<td>4,327</td>
</tr>
<tr>
<td>75-82</td>
<td>18,489</td>
<td>10,233</td>
<td>5,550</td>
</tr>
<tr>
<td>83+</td>
<td>16,454</td>
<td>9,582</td>
<td>5,943</td>
</tr>
</tbody>
</table>

Note: 2006 US dollars
### Table 1.4: Calibration Targets and Model Results

<table>
<thead>
<tr>
<th></th>
<th>Non-College Targets</th>
<th>College Targets</th>
<th>Source of Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>% P health working, ages 30-50</td>
<td>17.1</td>
<td>33.7</td>
<td>CPS</td>
</tr>
<tr>
<td>% A health working, ages 30-50</td>
<td>72.4</td>
<td>82.6</td>
<td>CPS</td>
</tr>
<tr>
<td>% G health working, ages 30-50</td>
<td>86.5</td>
<td>89.6</td>
<td>CPS</td>
</tr>
<tr>
<td>Health (≥1 if good)</td>
<td>0.05290</td>
<td>0.00000</td>
<td>MEPS regression</td>
</tr>
<tr>
<td>Health * age</td>
<td>0.00044</td>
<td>0.00000</td>
<td>MEPS regression</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_\mu$</td>
<td>0.09640</td>
<td>0.06991</td>
<td>PSID regression</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_\lambda$</td>
<td>0.08534</td>
<td>0.12380</td>
<td>PSID regression</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.94292</td>
<td>0.97254</td>
<td>PSID regression</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_0$</td>
<td>0.01819</td>
<td>0.01846</td>
<td>PSID regression</td>
</tr>
<tr>
<td>Av hourly wages, ages 23-26</td>
<td>14.6</td>
<td>21.3</td>
<td>CPS</td>
</tr>
<tr>
<td>Av hourly wages, ages 33-36</td>
<td>18.4</td>
<td>30.3</td>
<td>CPS</td>
</tr>
<tr>
<td>Av hourly wages, ages 43-46</td>
<td>20.7</td>
<td>33.9</td>
<td>CPS</td>
</tr>
<tr>
<td>Av hourly wages, ages 53-56</td>
<td>21.3</td>
<td>34.7</td>
<td>CPS</td>
</tr>
<tr>
<td>Av assets/income, ages 30-50</td>
<td>1.7</td>
<td>1.5</td>
<td>SCF</td>
</tr>
</tbody>
</table>

### Table 1.5: Calibrated Parameters

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$ (time cost of A health)</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Phi_2$ (time cost of P health)</td>
<td>0.24</td>
<td>0.224</td>
</tr>
<tr>
<td>$\theta$ (fixed time cost of work)</td>
<td>0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>Constant</td>
<td>0.98</td>
<td>1.43</td>
</tr>
<tr>
<td>Age</td>
<td>0.0781</td>
<td>0.0862</td>
</tr>
<tr>
<td>Age squared</td>
<td>-0.001</td>
<td>-0.0012</td>
</tr>
<tr>
<td>Age cubed</td>
<td>0.0000024</td>
<td>0.0000035</td>
</tr>
<tr>
<td>Health (≥1 if good)</td>
<td>0.14</td>
<td>0.04</td>
</tr>
<tr>
<td>Health * age</td>
<td>0.00085</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.964</td>
<td>0.9772</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_\mu$</td>
<td>0.0113</td>
<td>0.0131</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_\eta$</td>
<td>0.045</td>
<td>0.039</td>
</tr>
<tr>
<td>$\hat{\sigma}^2_\lambda$</td>
<td>0.07</td>
<td>0.11</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.975</td>
<td>0.975</td>
</tr>
</tbody>
</table>
### Table 1.6: Government Transfers, Model and Data, ages 30-50

<table>
<thead>
<tr>
<th>% receiving transfers</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>College</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% of low income receiving transfer (&lt;10,000$)</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
<td>59</td>
<td>59</td>
</tr>
<tr>
<td>College</td>
<td>15</td>
<td>42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% in Poor or Average Health if receiving transfer</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
<td>72</td>
<td>64</td>
</tr>
<tr>
<td>College</td>
<td>45</td>
<td>43</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average government transfer per recipient</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
<td>4576</td>
<td>5029</td>
</tr>
<tr>
<td>College</td>
<td>4576</td>
<td>5965</td>
</tr>
</tbody>
</table>

Note: data estimates are obtained using the CPS for years 1996-2006.

### Table 1.7: Health Effects and Labor Supply

Percentage Change in Average Labor Supply Relative to Benchmark

<table>
<thead>
<tr>
<th>Health Effect Removed</th>
<th>Non-College</th>
<th>College</th>
<th>Non-College, by Health</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Poor</td>
<td>Average</td>
<td>Good</td>
</tr>
<tr>
<td>All</td>
<td>10.3</td>
<td>1.7</td>
<td>-0.3</td>
</tr>
<tr>
<td>Medical Expenditure</td>
<td>-2.4</td>
<td>-2.4</td>
<td>-40.1</td>
</tr>
<tr>
<td>Survival</td>
<td>1.1</td>
<td>1.1</td>
<td>8.5</td>
</tr>
<tr>
<td>Productivity</td>
<td>6.6</td>
<td>0.8</td>
<td>41.7</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>5.3</td>
<td>2.6</td>
<td>340.8</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>11.9</td>
<td>3.0</td>
<td>395.1</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>4.4</td>
<td>-1.4</td>
<td>-12.4</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &lt; 65</td>
<td>-2.0</td>
<td>-4.7</td>
<td>-47.6</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &gt; 65</td>
<td>-1.3</td>
<td>-1.1</td>
<td>-3.5</td>
</tr>
<tr>
<td>All Medical Expenditures, all, all ages</td>
<td>-3.7</td>
<td>-6.0</td>
<td>-48.5</td>
</tr>
</tbody>
</table>
### Table 1.8: Health Effects and Asset Accumulation

**Percentage Change in Average Assets Relative to Benchmark**

<table>
<thead>
<tr>
<th>Health Effect Removed</th>
<th>Ages &lt; 60</th>
<th>Ages &gt; 60</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-College</td>
<td>College</td>
</tr>
<tr>
<td>All</td>
<td>8.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Medical Expenditure</td>
<td>-1.6</td>
<td>-1.5</td>
</tr>
<tr>
<td>Survival</td>
<td>10.7</td>
<td>8.3</td>
</tr>
<tr>
<td>Productivity</td>
<td>10.8</td>
<td>2.2</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>-9.3</td>
<td>-3.7</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>0.6</td>
<td>-1.3</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>8.3</td>
<td>-1.2</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &lt; 65</td>
<td>10.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &gt; 65</td>
<td>-9.7</td>
<td>-6.9</td>
</tr>
<tr>
<td>All Medical Expenditures, all, all ages</td>
<td>-0.2</td>
<td>-7.0</td>
</tr>
</tbody>
</table>

### Table 1.9: Effects of Health on Welfare

**Consumption Equivalent Variation (CEV) Relative to Benchmark (%)**

<table>
<thead>
<tr>
<th>Health Effect Removed</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Expenditure</td>
<td>3.51</td>
<td>1.91</td>
</tr>
<tr>
<td>Productivity</td>
<td>7.70</td>
<td>1.81</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>6.79</td>
<td>5.16</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>15.32</td>
<td>7.10</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>11.23</td>
<td>3.66</td>
</tr>
<tr>
<td>Productivity, Leisure and Med. Exp.</td>
<td>18.31</td>
<td>8.69</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &lt; 65</td>
<td>6.66</td>
<td>4.12</td>
</tr>
<tr>
<td>Medical Expenditure, all, ages &gt; 65</td>
<td>2.17</td>
<td>2.22</td>
</tr>
<tr>
<td>All Medical Expenditures, all, all ages</td>
<td>8.86</td>
<td>6.39</td>
</tr>
</tbody>
</table>
### Table 1.10: Importance of Interactions for Asset Accumulation

<table>
<thead>
<tr>
<th>Health Effect Removed</th>
<th>Ages &lt; 60</th>
<th>Ages &gt; 60</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-College</td>
<td>College</td>
<td>Non-College</td>
<td>College</td>
</tr>
<tr>
<td>M.E., ages &gt; 65 (M.E. ages &lt; 65 present)</td>
<td>-9.7</td>
<td>-6.9</td>
<td>-40.0</td>
<td>-20.9</td>
</tr>
<tr>
<td>M.E., ages &gt; 65 (M.E. ages &lt; 65 absent)</td>
<td>-10.4</td>
<td>-7.9</td>
<td>-44.3</td>
<td>-25.5</td>
</tr>
<tr>
<td>% difference if not accounting for M.E. &lt;65</td>
<td>7.5</td>
<td>14.5</td>
<td>10.8</td>
<td>22.2</td>
</tr>
<tr>
<td>M.E. of Healthy</td>
<td>-1.6</td>
<td>-1.5</td>
<td>-20.0</td>
<td>-7.0</td>
</tr>
<tr>
<td>M.E.of Healthy, Productivity Effect Absent</td>
<td>-2.6</td>
<td>-3.4</td>
<td>-21.0</td>
<td>-8.2</td>
</tr>
<tr>
<td>% change</td>
<td>65.2</td>
<td>121.0</td>
<td>4.9</td>
<td>16.1</td>
</tr>
<tr>
<td>M.E.of Healthy, Prod. and Time Effect Absent</td>
<td>-2.2</td>
<td>-2.5</td>
<td>-16.7</td>
<td>-6.6</td>
</tr>
<tr>
<td>% change</td>
<td>43.5</td>
<td>64.4</td>
<td>-16.6</td>
<td>-5.7</td>
</tr>
<tr>
<td>ME, ages &lt; 65, Productivity Effect Present</td>
<td>10.2</td>
<td>0.9</td>
<td>9.5</td>
<td>5.2</td>
</tr>
<tr>
<td>ME, ages &lt; 65, Productivity Effect Absent</td>
<td>8.6</td>
<td>0.5</td>
<td>16.7</td>
<td>0.1</td>
</tr>
<tr>
<td>% change</td>
<td>-15.8</td>
<td>-50.9</td>
<td>75.0</td>
<td>-98.9</td>
</tr>
</tbody>
</table>

Note: M.E. = Medical Expenditures
Table 1.11: Health Risk Effects on Earnings and Income Inequality

% Change in Average Earnings and Income Variance Relative to Benchmark

<table>
<thead>
<tr>
<th>Health Risks Removed</th>
<th>Observed Wages</th>
<th>Disposable Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-College</td>
<td>College</td>
</tr>
<tr>
<td>All</td>
<td>-3.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Medical Expenditure</td>
<td>0.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Survival</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Productivity</td>
<td>-0.3</td>
<td>0.0</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>-1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>-2.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>-1.8</td>
<td>1.6</td>
</tr>
<tr>
<td>Productivity, Leisure and Med. Exp.</td>
<td>-3.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>Transitory earnings shocks</td>
<td>-25.3</td>
<td>-24.9</td>
</tr>
</tbody>
</table>

Table 1.12: Effects of Health Risk on Welfare

% Consumption Equivalent Variation (CEV) Relative to Benchmark

<table>
<thead>
<tr>
<th>Health Risks Removed</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Expenditure</td>
<td>0.58</td>
<td>0.36</td>
</tr>
<tr>
<td>Productivity</td>
<td>-0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>1.40</td>
<td>0.89</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>1.49</td>
<td>0.86</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>0.43</td>
<td>0.26</td>
</tr>
<tr>
<td>Productivity, Leisure, and Med. Exp.</td>
<td>1.49</td>
<td>0.87</td>
</tr>
<tr>
<td>Medical Expenditure &gt; 65</td>
<td>0.13</td>
<td>0.11</td>
</tr>
<tr>
<td>Medical Expenditure &lt; 65</td>
<td>0.44</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Table 1.13: Effects of Health Risk on Asset Accumulation

% Change in Average Assets Relative to Benchmark

<table>
<thead>
<tr>
<th>Health Risks Removed</th>
<th>Ages $&lt; 60$</th>
<th></th>
<th>Ages $&gt; 60$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-College</td>
<td>College</td>
<td>Non-College</td>
<td>College</td>
</tr>
<tr>
<td>All</td>
<td>-9.9</td>
<td>1.1</td>
<td>-6.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Medical Expenditure</td>
<td>0.5</td>
<td>-1.3</td>
<td>-6.8</td>
<td>-4.6</td>
</tr>
<tr>
<td>Survival</td>
<td>1.7</td>
<td>3.2</td>
<td>6.0</td>
<td>7.7</td>
</tr>
<tr>
<td>Productivity</td>
<td>-4.4</td>
<td>-0.6</td>
<td>5.0</td>
<td>4.4</td>
</tr>
<tr>
<td>Leisure Endowment</td>
<td>-6.9</td>
<td>-2.0</td>
<td>-9.6</td>
<td>-4.0</td>
</tr>
<tr>
<td>Productivity and Leisure</td>
<td>-9.8</td>
<td>-1.7</td>
<td>-6.9</td>
<td>-1.0</td>
</tr>
<tr>
<td>Productivity and Medical Expenditures</td>
<td>-5.0</td>
<td>-1.7</td>
<td>-4.6</td>
<td>-4.2</td>
</tr>
<tr>
<td>Productivity, Leisure and Med. Exp.</td>
<td>-10.6</td>
<td>-2.7</td>
<td>-7.4</td>
<td>-2.3</td>
</tr>
<tr>
<td>Medical Expenditure $&lt; 65$</td>
<td>1.3</td>
<td>-0.5</td>
<td>1.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>Medical Expenditure $&gt; 65$</td>
<td>-0.4</td>
<td>0.6</td>
<td>-10.7</td>
<td>-4.4</td>
</tr>
<tr>
<td>All (No Transitory Earnings Shocks)</td>
<td>-14.0</td>
<td>-4.0</td>
<td>-6.3</td>
<td>1.9</td>
</tr>
<tr>
<td>All (Consumption floor is 5%)</td>
<td>-20.3</td>
<td>0.2</td>
<td>-16.0</td>
<td>7.0</td>
</tr>
</tbody>
</table>
Table 1.14: Improving Non-College Health Outcomes: Effects on Labor Supply, Asset Accumulation and Welfare

<table>
<thead>
<tr>
<th>% Change in Labor Supply Relative to Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
</tr>
<tr>
<td>25% closer to College</td>
</tr>
<tr>
<td>50% closer to College</td>
</tr>
<tr>
<td>75% closer to College</td>
</tr>
<tr>
<td>College levels</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% Change in Average Assets Relative to Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
</tr>
<tr>
<td>Ages&lt;60</td>
</tr>
<tr>
<td>25% closer to College</td>
</tr>
<tr>
<td>50% closer to College</td>
</tr>
<tr>
<td>75% closer to College</td>
</tr>
<tr>
<td>College levels</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% CEV Relative to Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-College</td>
</tr>
<tr>
<td>25% closer to College</td>
</tr>
<tr>
<td>50% closer to College</td>
</tr>
<tr>
<td>75% closer to College</td>
</tr>
<tr>
<td>College levels</td>
</tr>
</tbody>
</table>
### Table 1.15: Improving Non-College Health Outcomes: Effect on Government Transfers for Non-College

<table>
<thead>
<tr>
<th></th>
<th>% Receiving Government Transfer (excluding Medicaid)</th>
<th>Average Transfer Amount Per Person ($) (excluding Medicaid)</th>
<th>Average Medicaid Amount Per Person ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark</strong></td>
<td>16.1</td>
<td>648</td>
<td>633</td>
</tr>
<tr>
<td>25% closer to College</td>
<td>15.1</td>
<td>621</td>
<td>532</td>
</tr>
<tr>
<td>50% closer to College</td>
<td>14.3</td>
<td>599</td>
<td>474</td>
</tr>
<tr>
<td>75% closer to College</td>
<td>13.4</td>
<td>575</td>
<td>407</td>
</tr>
<tr>
<td>College levels</td>
<td>12.7</td>
<td>552</td>
<td>351</td>
</tr>
</tbody>
</table>
Table 1.16: Effects of Greater Efficiency on Labor Supply, Asset Accumulation and Welfare

### % Change in Labor Supply Relative to Benchmark

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>1.5</td>
<td>0.3</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>-0.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>0.4</td>
<td>-0.6</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>1.3</td>
<td>-0.3</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>0.5</td>
<td>-0.9</td>
</tr>
</tbody>
</table>

### % Change in Average Assets Relative to Benchmark

#### Ages < 60

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>-4.6</td>
<td>-1.8</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>-2.2</td>
<td>-1.6</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>-3.1</td>
<td>-2.1</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>-5.1</td>
<td>-2.6</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>-5.1</td>
<td>-3.3</td>
</tr>
</tbody>
</table>

#### Ages > 60

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>-1.5</td>
<td>-0.9</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>-6.3</td>
<td>-2.4</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>-3.3</td>
<td>-2.7</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>-10.1</td>
<td>-2.9</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>-8.8</td>
<td>-4.8</td>
</tr>
</tbody>
</table>

### % CEV Relative to Benchmark

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>2.8</td>
<td>2.1</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>2.7</td>
<td>2.1</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>3.4</td>
<td>2.6</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>4.1</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Table 1.17: Effects of Greater Efficiency on Government Transfers

<table>
<thead>
<tr>
<th>% Receiving Government Transfer (excluding Medicaid)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>16.1</td>
<td>2.1</td>
</tr>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>15.3</td>
<td>1.9</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>14.9</td>
<td>1.8</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>15.0</td>
<td>1.8</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>14.6</td>
<td>1.8</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>14.3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Transfer Amount Per Person ($) (excluding Medicaid)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>648</td>
<td>87</td>
</tr>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>635</td>
<td>88</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>665</td>
<td>87</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>669</td>
<td>86</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>653</td>
<td>86</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>683</td>
<td>83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Medicaid Amount Per Person ($)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>633</td>
<td>84</td>
</tr>
<tr>
<td>10% lower medical and 30% lower time costs</td>
<td>512</td>
<td>58</td>
</tr>
<tr>
<td>20% lower medical and 10% lower time costs</td>
<td>421</td>
<td>52</td>
</tr>
<tr>
<td>20% lower medical and 20% lower time costs</td>
<td>422</td>
<td>50</td>
</tr>
<tr>
<td>20% lower medical and 30% lower time costs</td>
<td>404</td>
<td>50</td>
</tr>
<tr>
<td>30% lower medical and 30% lower time costs</td>
<td>316</td>
<td>42</td>
</tr>
</tbody>
</table>
**Table 1.18: Policy Effects on Labor Supply, Asset Accumulation and Welfare**

### % Change in Labor Supply Relative to Benchmark

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>2.5</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>-0.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>1.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>-8.6</td>
<td>-6.7</td>
</tr>
</tbody>
</table>

### % Change in Average Assets Relative to Benchmark

#### Ages < 60

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>2.1</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>-2.2</td>
<td>-1.6</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>-0.8</td>
<td>-0.4</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>6.4</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

#### Ages > 60

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>9.5</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>-6.3</td>
<td>-2.4</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>5.6</td>
<td>-2.9</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>3.7</td>
<td>-2.9</td>
</tr>
</tbody>
</table>

### % CEV Relative to Benchmark

<table>
<thead>
<tr>
<th></th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>3.2</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>2.4</td>
<td>1.3</td>
</tr>
</tbody>
</table>

*Note: EPHI=Employer Provided Health Insurance*
### Table 1.19: Policy Reform and Government Transfers

<table>
<thead>
<tr>
<th>% Receiving Government Transfer (excluding Medicaid)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>16.1</td>
<td>2.1</td>
</tr>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>14.3</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>14.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>15.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>13.7</td>
<td>1.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Transfer Amount Per Person ($) (excluding Medicaid)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>648.0</td>
<td>86.6</td>
</tr>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>599.3</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>665.3</td>
<td>87.2</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>630.5</td>
<td>85.8</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>558.4</td>
<td>69.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Average Medicaid Amount Per Person ($)</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>632.6</td>
<td>83.8</td>
</tr>
<tr>
<td>Improved Health (50% closer to college)</td>
<td>474.2</td>
<td>-</td>
</tr>
<tr>
<td>Greater Efficiency (lower 20% med, 10% time costs)</td>
<td>421.6</td>
<td>52.1</td>
</tr>
<tr>
<td>Mandated EPHI</td>
<td>596.0</td>
<td>78.1</td>
</tr>
<tr>
<td>Medicaid Expansion and Mandated EPHI</td>
<td>1284.3</td>
<td>562.5</td>
</tr>
</tbody>
</table>

### Table 1.20: Welfare Benefits of Employer Provided Insurance

<table>
<thead>
<tr>
<th>% CEV relative to no Employer Health Insurance in Benchmark Model</th>
<th>Non-College</th>
<th>College</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health Insurance, Working Only</td>
<td>2.10</td>
<td>1.44</td>
</tr>
<tr>
<td>Health Insurance, Continues into Retirement</td>
<td>2.82</td>
<td>2.06</td>
</tr>
</tbody>
</table>
1.8 Figures

Figure 1.1: Health Transitions

- Probability of transitioning from Good to Good Health by Education
- Probability of transitioning from Good to Average Health by Education
- Probability of transitioning from Average to Poor Health by Education
- Probability of transitioning from Average to Good Health by Education
Figure 1.2: Probability of Death by Education and Health Status

![Probability of Death by Health Status and Education](image1)

Figure 1.3: Wage Age Profiles - Model and Data

![Wage Age Profiles by Education](image2)
**Figure 1.4:** Labor Force Participation - Model and Data

Note 1: Profiles are smoothed by constructing age groups combining 4 ages, and taking averages.

Note 2: The 95% Confidence Intervals are well within 1 percentage point of the estimated means, with only a few exceptions.

**Figure 1.5:** Average Consumption over the Life-Cycle
Figure 1.6: Asset Accumulation, Model and Data

Note: 95% Confidence Intervals are very close to the estimated SCF medians and are not shown.
Chapter 2

The Value of Specific Skills and the Evolution of Wage Inequality

2.1 Introduction

The college wage premium in the United States increased dramatically between 1980 and 2002, from 23% to 46%. Residual wage inequality also increased rapidly during this time period, by 18% and 32% within high school and college educated groups, respectively (Figure 2.1). In this paper, I study trends in the employment of cognitive and physical skills over time for different education groups and estimate the returns to these skills. I then evaluate the extent to which quantity and price changes in skills can account for the observed evolution of wage inequality.

The rise in wage inequality and the changing nature of inequality have been docu-

\footnote{These figures are obtained using CPS data on males working at least 40 hours per week, 50 weeks per year, with hourly wages greater than 3$. The college wage premium is the coefficient on college from a regression of log wages on the usual controls (region and race dummies, age, age squared and experience). Residual wage inequality is measured as the log 90/10 percentiles ratio of the residual from the same regression, estimated separately by college. The college group includes all workers with 4 or more completed years of college, and the high school group includes all those with high school diplomas or GED equivalent who have not attended any college. The estimated trends in inequality are consistent with those reported in previous literature. See for example Katz and Murphy (1992), Card and DiNardo (2002), Autor, Katz and Kearney (2005), and Lemieux (2008).}
mented and studied extensively in previous literature (e.g. Katz and Murphy (1992), Juhn, Murphy and Pierce (1993), Acemoglu (2002), Card and DiNardo (2002), Beaudry and Green (2005), and Lemieux (2008)). Returns to education increased steeply in the 1980s and continued to rise at a slower rate in the 1990s (Card and DiNardo (2002) and Beaudry and Green (2005)). Lemieux (2006) finds that residual wage inequality growth was also concentrated in the 1980s after controlling for composition effects. However, while the increase in inequality prior to 1989 was pervasive in the sense that wage differentials increased at all points of the distribution, after 1989 inequality growth became concentrated in the top end of the wage distribution (Juhn, Murphy and Pierce (1993) and Lemieux (2006)).

To explain the sharp increase in inequality in the 1980s, the early literature focused on skill-biased technical change (SBTC): the computer revolution led to a widespread increase in the demand and price of skills in the 1980s, changing the wage structure and raising inequality (e.g. Bound and Johnson (1992), Katz and Murphy (1992), and Juhn, Murphy and Pierce (1993)). However, since the SBTC explanation is hard to reconcile with the slowing inequality growth in the 1990s and the fact that inequality growth continued only for the top half of the wage distribution, alternative explanations were proposed, including de-unionization, the increased prevalence of pay for performance, and changes in the relative demand for the types of tasks performed by workers in high-paying occupations (Card and DiNardo (2002) and Lemieux (2007)).

One difficulty in assessing the validity of the SBTC explanation has been in defining and measuring workers’ skills. Education and experience, the two commonly used measures in the 1990s literature, may not be adequate measures of the underlying skills affected by technical change, skills that may vary significantly within education and experience categories. More recent literature has constructed alternative measures of skills (e.g. cognitive, motor, clerical, routine and non-routine cognitive and manual skills) based on observed occupation characteristics available in the Dictionary of Occupational
Titles (DOT) (e.g. Autor, Levy and Murnane (2003), Wolff (2003), Ingram and Neumann (2006), and Bacolod and Blum (2010)). The DOT data set contains information on detailed occupation characteristics, providing measures of 44 different skills required to perform more than 12,000 occupations.

These new measures have allowed for richer models studying the relationship between technical change and specific skills (e.g. Autor, Levy and Murnane (2003)), and led to a re-examination of wage inequality using empirical models that account for more precise skill measures (Autor, Levy and Murnane (2003), Ingram and Neumann (2006), and Bacolod and Blum (2010)). I build on this literature by studying how changes in prices and changes in quantities of employed cognitive and physical skills contribute to the rising college wage premium and rising residual wage inequality over the time period 1980 to 2002.

The main findings are as follows: (1) there has been a strong shift in employment away from occupations requiring physical skills towards those intensive in cognitive skills and most of the growth in the employment of cognitive skills can be accounted for by the increase in the fraction of college educated women in the labor force; (2) returns to high levels of cognitive skills have increased dramatically, while returns to low levels of cognitive skills have remained approximately constant; (3) differences in average employed cognitive skills between college and high school groups account for a large fraction (approximately half) of the college wage premium, and changes in returns to skills account for approximately one half of the growth in the college wage premium; and (4) while only a small fraction (at most 9%) of residual wage inequality is accounted for by skills, this fraction has grown over time, and skills explain approximately 12% and 16% of the increase in residual wage inequality for high school and college graduates, respectively, with larger fractions for inequality in the bottom half of the residual wage distribution.

I construct measures of cognitive and physical skills for each worker in the CPS by matching the occupation characteristics from the DOT data set to occupations held by
workers in the CPS, and using data reduction methods on selected characteristics to estimate the physical and cognitive skill requirements of Census occupations.\(^2\) I assume that workers (whose individual skills are not observed) are perfectly matched to occupations that require precisely their skill levels.\(^3\)

Consistent with previous literature, I find that there has been a large increase in average employed cognitive skills and a decline in average employed physical skills.\(^4\) However, new from existing papers, I show that these trends are driven by labor force composition effects. Within education-gender groups, average employed cognitive skills have at best remained constant, declining slightly for high school educated men, indicating that workers with similar characteristics have not switched into occupations more intensive in cognitive skills over time.\(^5\) Almost the entire increase in the demand for cognitive skills has been matched by an increase in the supply of college workers, the great majority of which are women.

I also find that the returns to cognitive skills have increased on average. For example, the wage premium associated with a 1 standard deviation increase in cognitive skills above the 1990 mean increases from 5% in 1980 to 17% in 2002. This is consistent with previous findings: Murnane, Willett and Levy (1995) find that basic cognitive skills (measured by mathematics test scores) had larger impacts on wages six years after high school graduation in 1986 than in 1978, Ingram and Neumann (2006) find that the returns to mathematical and verbal ability have doubled between 1980 and 1998, and Bacolod and Blum (2010) find large increases in the returns to cognitive and people skills from

---

\(^2\)This approach has been used in Autor, Levy and Murnane (2003), Ingram and Neumann (2006) and Bacolod and Blum (2010).

\(^3\)Ingram and Neumann (2006) have noted that constructing skill measures using the DOT is sensible only if the matching of workers to jobs is fairly tight, as predicted by theories of search and matching found in Albrecht and Vroman (2002) and Wong (2003).

\(^4\)For example, Ingram and Neumann (2006) show consistent with my paper that median employed intelligence has increased from 1985 to 1998. Bacolod and Blum (2010) show that average employed cognitive skills have increased between 1968-1990.

\(^5\)Bacolod and Blum (2010) report average skills by education groups, but do not decompose them further by gender.
1968 to 1990. New from this literature, I show that the increase in these returns has not been uniform: the returns at high levels of cognitive skills have increased very sharply while the returns at low levels have remained roughly constant.

Accounting for workers’ employed skills explains approximately half of the increase in the male college wage premium between 1980 and 2002. Previous papers have also studied how controlling for skills affects the estimated returns to education: Murnane, Willett and Levy (1995) find that basic cognitive skills (measured by mathematics test scores) explain all of the increase in the returns to college attendance between 1978 and 1986, and Ingram and Neumann (2006) find that the return to an additional year of education falls and shows less growth from 1970 to 1998 when they control for skill factors. Different from these papers, I determine the relative importance of skill quantity and skill price changes in explaining the college premium using the decomposition approach of Juhn, Murphy and Pierce (1993). Changes in cognitive skill prices are by far the most important in explaining the rising premium: the difference in average employed cognitive skills between college and high school graduates has remained approximately constant, but the returns to high levels of cognitive skills supplied by college graduates have increased more rapidly over time than returns to low skill levels supplied by high school graduates.

Finally, I explore the extent to which skills can account for the increase in residual wage inequality. Similar to Ingram and Neumann (2006), I find that skills play a small role in explaining residual inequality (accounting for only up to 9% in any given year), but have become increasingly important over time. Of the total increase in residual inequality from 1980 to 2002, skills account for 12% of this growth for high school and 16% for college graduates. Skills play a more important role in the determination of within group inequality over time since (1) the dispersion in employed cognitive skills within

---

6 The college wage premium is measured as the percentage difference in average hourly wages between those with college degrees (4+ years of college) and high school graduates (excluding college drop-outs).
education groups has increased by 17% for college and 12% for high school graduates, and (2) returns to cognitive skills have increased disproportionally, much more rapidly at higher skill levels. New from previous literature, I also check whether skills can explain the different patterns that emerge in the 1990s in the evolution of top and bottom wage distribution inequality: while inequality at the top of the distribution rises sharply, it stagnates at the bottom half of the distribution. Skills explain 6.7% of the increase in residual inequality in the top half of the wage distribution in the 1990s for high school graduates, and 7.5% for college graduates. While these numbers are not very large, they show that skills could be one of the many contributing factors to growing top end inequality.

2.2 Data Description

2.2.1 The Dictionary of Occupational Titles

I use data from the Dictionary of Occupational Titles (DOT) 1977 Fourth Edition, DOT 1991 Revised Fourth Edition and the Current Population Survey (CPS). The Dictionary of Occupational Titles was developed in the 1930’s by the U.S. Department of Labor with the goal of facilitating job placement activities. Occupational analysts collected data on detailed job characteristics in order to help job interviewers compare and match the specifications of employer job openings with the qualifications of applicants. There are four editions of the DOT published in 1939, 1949, 1965, and 1977. Changes in occupational content and job characteristics due to technological advancement occurred at a rapid pace after the publication of the 1977 version, so later efforts were focused on studying selected industries in order to document the jobs that have undergone the most significant occupational changes. The resulting additions and changes led to the publication of a revised fourth edition of the DOT in 1991. The DOT is now one of the fundamental tools of career guidance counselors assisting individuals in making occupa-
The Data Appendix contains more information on the differences between DOT editions. Throughout the body of the paper, I use information only from the DOT 1991 Revised Fourth Edition since this data set contains the most up to date information on occupation characteristics. Since the time frame studied in this paper is 1980-2002, the DOT 1991 which contains data updated in the 1980s is the most appropriate. However, the Data Appendix also presents results obtained using information from the DOT 1977 version.

The 1991 DOT contains data on 12,741 detailed occupations. Each occupation is described in terms of several dimensions that summarize the job requirements and conditions: the degree of interaction with Data, People and Things, General Educational Development level, Specific Vocational Preparation, Physical Demands, Environmental Conditions, Aptitudes, Temperaments, and the Materials, Products, Subject Matter and Services related to the job. Each of these characteristics is recorded in one of three ways: either as a number on a scale (e.g. General Learning Aptitude can range from 1 to 5, where 1 represents a level of ability present in the top 10% of the distribution and 5 is associated with the lowest 10%); a letter indicating the level of an activity (e.g. "S" for "Sedentary" and "V" for "Very Heavy" in the description of Strength); or an indicator that appears when the characteristic applies to the job (e.g. the Temperament "Dealing with People" is indicated by the letter D when this characteristic is present).

The DOT occupation characteristics data has some important limitations. In all cases, there is some degree of subjectivity in assessing occupation requirements. Also, we can only infer ordinal rankings of requirements, not cardinal measures. In addition, the DOT only records the minimum requirements of each occupation. In reality, expectations may be much higher in some occupations, and the difference between actual expectations and minimum requirements may systematically vary with the type of occupation. Miller et al. (1980) and Spenner (1983) are two papers that study the limitations of the DOT.
2.2.2 Construction of Physical and Cognitive Skill Measures

I match the DOT 1991 occupation characteristics with occupations held by respondents in the CPS. Unfortunately, this is not straightforward since the CPS employs a different occupation coding system than the DOT. The Census occupations are much less detailed, so in any given year there are only between 412 and 540 occupation codes. Also, the Census occupation codes change over time, with different classifications used between 1971-1982, 1983-1991, and 1992-2002.

I match Census occupations to the DOT 1991 occupations and their corresponding characteristics using available crosswalks and correspondences, described in detail in the Data Appendix. Each Census occupation is usually associated with many detailed DOT occupations, so I take the average of the DOT characteristics over each Census occupation. In the Data Appendix, I show that results are not sensitive to the weighing of DOT occupations when constructing mean skill requirements for Census occupations.

Many variables reported by the DOT measure the same broad skills. Existing literature has explored various skill measures constructed using data reduction methods combining similar characteristics into broader skill categories. For example, Ingram and Neumann (2006) use factor analysis to obtain four skill dimensions: intelligence, fine motor skills, coordination and strength. Bacolod and Blum (2010) also use factor analysis to construct measures of cognitive skills, fine motor skills, people skills, and physical strength. Autor, Levy and Murnane (2003) focus on routine and non-routine tasks by averaging over a few chosen variables thought to embody these.

In this paper, I focus on two skills, physical and cognitive, constructed using factor analysis on selected DOT characteristics that are likely to be correlated with these skill measures. Table 2.1 lists the DOT variables used. Ingram and Neumann (2006) and Poletaev and Robinson (2008) describe in detail the factor analysis method and how it can be used to estimate skill measures from DOT variables. To summarize, factor analysis assumes that the variations in observed variables reflect variations in a few
unobserved variables, called “factors.” For example, the variation across occupations in their requirements of reasoning ability, numerical aptitudes and verbal skills reflects variation in general cognitive skills required. The observed DOT variables are modeled as linear combinations of the unobserved “factors” plus error terms. The correlation coefficients between the observed variables and factors are called factor loadings. Higher factor loadings indicate that a higher percentage of the variance in DOT variables is explained by the factors.\footnote{A general rule of thumb is that the loadings should be .7 or higher for DOT variables to be represented by a particular factor, so at least half of the variance in the variable should be explained by the factor.}

I perform the factor analysis using data on all workers in the CPS in 1990, both men and women. Next, I use the coefficients from this estimation to predict the cognitive and physical skill factors for all remaining years from 1980 to 2002. Table 2.1 presents the factor loadings. The constructed cognitive skill factor explains most of the variation in the Reasoning Development, Language Development, General Intelligence, Verbal Aptitude and Data Complexity variables. The physical skill factor explains a high percentage of variation in Climbing, Kneeling and Crouching requirements. In the Data Appendix, I show that results are not sensitive to the year selected for the factor analysis. I also discuss differences by gender and show that results are not sensitive to the gender group used.

Factor analysis produces physical and cognitive skill factors with a mean of zero and standard deviation of one in the year used for the estimation (1990). When conducting the factor analysis, I weigh each census occupation in 1990 by the full-time equivalent hours of labor supply observed in that occupation, so the factor analysis procedure places more weight on the characteristics of occupations with relatively higher employment levels. As noted by Poletaev and Robinson (2008), an important consequence of using this weighing procedure is that the units of the derived factors represent standard deviations of the factors in the employed population in 1990. For example, a difference of one unit in
cognitive skills between one census occupation and another represents a move across the distribution of cognitive skills in the 1990 employee population of one standard deviation.

2.2.3 Data Sample Restrictions and Measurement Issues

After constructing measures of employed cognitive and physical skills for each occupation in the CPS, I make some restrictions on the sample of individuals used to obtain the results. I exclude high school and college drop-outs and define college graduates as all individuals who have completed at least 4 years of college, and high school graduates as those with a high school degree or GED equivalent who have not attended any college. I keep individuals 18-64 years old who are not self-employed. Wage rates are constructed by dividing total annual earnings in the previous year by hours of work. As many previous studies in the wage inequality literature show, measurement error in earnings and hours reported in the CPS is a major cause of concern (e.g. Autor, Katz and Kearney (2005a) and Lemieux (2006)). Therefore, I eliminate outliers by restricting the sample to workers with hourly wages greater than 3$. Earnings are top coded in the CPS, so there are no high earnings outliers, however, top coding leads to an understatement of earnings for the 1-5 percent of individuals with censored data. As in much of the existing wage inequality literature, I also restrict the sample to full time, full year workers (working at least 40 hours per week, 50 weeks per year) in order to minimize problems associated with measurement error in the number of hours worked.\footnote{8For evidence and analysis of measurement error in the CPS, see for example Bound and Krueger (1991) and Lemieux (2006).}
2.3 Results

2.3.1 Trends in the Employment of Skills

I start by documenting trends in the quantities of cognitive and physical skills embodied in the labor force. These quantities may change over time for two reasons: (1) the distribution of workers across occupations requiring different skills changes, for example if cognitive intensive occupations expanded faster (extensive margin shifts), and (2) occupations change their skill requirements over time, for example the occupation “secretary” could require more cognitive skills in 2002 than 1980 (intensive margin shifts). The constructed data set allows me to capture only the extensive margin changes since occupation characteristics are fixed to levels captured by the 1991 DOT. In the Data Appendix, I use differences between the 1977 DOT and the 1991 DOT occupation requirements to identify intensive margin shifts and show differences in these across education groups. Switching between DOT editions in a given year leads to large discontinuities in estimated average skills. Therefore, I use only the 1991 DOT characteristics and focus on studying extensive margin shifts only.\(^9\)

I calculate average employed cognitive and physical skills in each year by taking a weighted average of these skills across workers in the CPS, using sample weights multiplied by the number of hours worked as weights. Note that while factor analysis generates skills with a mean of zero and standard deviation of one in 1990, due to the sample restrictions made after estimation (e.g., keeping only full time workers), the new mean and standard deviation in 1990 may differ.\(^10\) Figure 2.2 shows the evolution of average employed skills over time, for both men and women. From 1980 to 2002, average employed

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\(^9\)The constructed data set only accounts for characteristics of occupations that existed in the 1991 DOT. Additional extensive margin shifts could arise from a movement of workers into newly created occupations after 1991 that are not captured by the data.

\(^10\)However, we still interpret differences in skill levels according to the distribution of skills across workers in the CPS before making these restrictions. (Recall that a difference of one unit in skills represents a move across the distribution of that skill in the 1990 employee population of one standard deviation.)
cognitive skills have increased steadily by .15 standard deviation points. On the other hand, average physical skills have declined by .1 standard deviation point. These results indicate a strong shift in employment away from occupations requiring physical skills toward those intensive in cognitive skills. These shifts are consistent with skill-biased technical change that increases the demand for cognitive skills.

To evaluate how these shifts are linked to wage inequality, it is useful to determine whether these shifts occurred within education and gender groups, or whether they occurred due to the relative expansion of a particular group whose average skills have not changed, or a combination of the two. Figure 2.3 plots average skills separately for men and women, and Figure 2.4 plots cognitive skills by gender and education groups. Finally, Figure 2.5 shows the changing composition of the labor force. These figures reveal that most of the increase in the employment of cognitive skills can be accounted for by the growing fraction of college educated women in the labor force.

First, we observe that surprisingly, average employed cognitive skills have remained roughly constant over time for college educated men and women and high school educated women, and declined slightly for high school educated men. At the same time, the fractions of college educated men and women in the labor force have increased over time, and more so for college women whose fraction doubled from 11% to 22% compared to an increase of only 4% for men. Therefore, it appears that college educated workers have not switched into occupations more intensive in cognitive skills over time, but that the entire increase in the demand for cognitive skills has been matched by an increase in the supply of college workers, the great majority of which are women. (Overall, women experienced an increase of .26 standard deviation point in average employed cognitive skills, while the increase for males was only .08 from 1980 to 2002 (Figure 2.3).)

\[11\] While it is not the focus of this paper, it is interesting to note that college educated men and women do not differ in terms of their average employed cognitive skill levels and differ only slightly in terms of physical skills. However, there are large differences in average skills employed between high school men and women: women’s occupations employ much higher levels of cognitive skills and men’s occupations employ much more physical skills.
Looking at physical skills employed, we observe that high school males have switched into occupations that are slightly more intensive in physical skills, while college women have moved away from such occupations. (In both cases, the shifts are very small, but statistically significant at the 95% confidence level.) Changes in the composition of the labor force contribute the most to the overall declining trend in physical skills since the fraction of high school workers in the labor force has declined, and since high school educated workers supply most physical skills.

My results are new since previous literature documents trends in the employment of skills over time but without studying the relationship between these and labor force composition changes. Ingram and Neumann (2006) report the median levels of employed intelligence, clerical skills, gross motor skills, and strength from 1972 to 1998, and show consistent with my paper that median employed intelligence has increased after 1985. Bacolod and Blum (2010) estimate average cognitive skills, fine motor skills, people skills, and physical strength skills employed over the period 1968-1990, by different education groups, but without separating the genders. Rendall (2010) shows trends in brain and brawn (calculated similarly to cognitive and physical skills) from 1950 to 2005, by gender, but not by education.

I consider the possibility that the quality of college graduates might have declined as more individuals graduate from college over time. For example, the more talented college graduates could be moving into occupations requiring higher cognitive skill levels while more recent, less talented graduates might be entering occupations requiring low

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12 The confidence intervals around the mean skills are not shown in the figures, however, they are very close to the means, being different from them only at the 3 digit level. (The standard errors are very small, always under 0.00025.)

13 They do however calculate the cognitive to motor skills ratio of males and females for different education groups at different points in time, and these ratios are consistent with my findings, remaining fairly constant from 1980 to 1990.

14 Rendall (2010) finds similar trends in skills by gender as in Figure 2.3. While the paper suggests that the upward trend in female supplied brain skills could be linked with increased educational investment, it does not show that indeed, it is the increase in the fraction of college educated women in the labor force that accounts for the growth in the employment of cognitive skills (overall and within the female group).
levels of cognitive skills. To test this hypothesis, I calculate average employed cognitive skills for recent graduates, within 5 years of degree completion (Figure 2.6). I find no evidence that recent graduates enter occupations requiring lower levels of cognitive skills. However, if matching between occupation requirements and workers’ skills is imperfect, it is possible that while new positions are created in occupations requiring high cognitive skills, these positions are filled by individuals lacking these skills.

Finally, I study the degree of inequality in employed cognitive skills within groups. Figure 2.7 plots the variance of cognitive skills within education-gender groups over time. First, we observe that the variance for high school workers is much higher than for college. Also, the variance has increased over time within all groups, but especially for high school educated women. The variance of cognitive skills for college men has increased by 17% from 1980 to 2002, and 12% for high school, whereas for women it increased by 13% and 31% for college and high school, respectively.

### 2.3.2 Returns to Skills

Several previous studies have estimated the returns to specific skills over time (e.g. Murnane, Willett and Levy (1995), Ingram and Neumann (2006), and Bacolod and Blum (2010)). I also estimate the returns to cognitive and physical skills by year, and use the results on returns and quantities of skills employed to address questions regarding wage inequality. Since this paper focuses on explaining inequality for males, I estimate skill returns using data on males only.\(^{15}\) I follow previous literature in estimating skill returns through a regression of log hourly earnings on the usual controls (education, experience, geographical region, race, age, age squared) and on measures of employed cognitive and

\(^{15}\)Bacolod and Blum (2010) note that if labor markets are segmented across genders, then men and women would face different returns to skills. Also, they note that unobserved worker heterogeneity that is correlated with gender would also lead to different estimated returns to skills. Using only data on males to estimate returns to skills eliminates these concerns.
physical skills and their squared terms.\textsuperscript{16} This is estimated separately by year. The estimated coefficients of interest from these regressions are presented in Table 2.2. Almost all coefficients on skill measures are statistically significant at the 1\% confidence level.

In interpreting these results, recall that skills are scaled such that one unit represents a move of one standard deviation across the distribution of the skills in the employee population in 1990. The coefficients on skills tell us the percentage change in wages associated with skill levels one standard deviation point above their 1990 means.\textsuperscript{17} Figure 2.8 plots estimated returns to skills over time.

We observe a high rate of growth in the wage premium associated with cognitive skills one standard deviation point higher than the 1990 mean, from 5\% in 1980 to 17\% in 2002. However, the returns to physical skills remained roughly constant around zero. These findings are consistent with previous literature: Murnane, Willett and Levy (1995) find that basic cognitive skills (measured by mathematics test scores) had larger impacts on wages in 1986 than in 1978 six years after high school graduation; Ingram and Neumann (2006) find that the return to mathematical and verbal ability has increased dramatically since the early 1980s, essentially doubling between 1980 and 1998, and the return to manual skill, while positive, has declined steadily; and Bacolod and Blum (2010) find that the wage returns to cognitive and people skills have increased while the wage return to motor skills declined greatly from 1968 to 1990.

Different from this literature, I allow returns to skills to vary at different levels of cognitive and physical skills by including quadratic terms in the estimation and I find that their coefficients are statistically significant. There may be increasing (or diminishing) returns to cognitive skills, so a worker with skill level 2 standard deviation points above

\textsuperscript{16}The literature mentioned does not include the squared terms of skill measures. Bacolod and Blum (2010) find that these are not statistically significant, so they do not report results for these specifications. My finding differ in this respect.

\textsuperscript{17}More generally, for example, the percentage change in wages associated with a one s.d. point increase above some level of cognitive skill \(c\) is equal to: coefficient on cognitive skills + \(2^*(c)^*(\text{coefficient on cognitive}^2) + \text{coefficient on cognitive}^2\).
the mean may be more (less) productive than two workers with skills of one s.d. point above the mean. Figure 2.9 plots the returns to skills when they increase by one s.d. point above levels of -2, -1 and 1. (Figure 2.8 plots the returns when skills increase by 1 s.d. point above zero.) The confidence intervals (not shown) are very small, never exceeding 0.5% around the calculated returns. These figures reveal that the returns to low levels of cognitive skills are generally higher, especially at the beginning of the time period. For example, in 1980 an increase in cognitive skills from -2 to -1 s.d. points raised wages by 13%, from -1 to 0 raised wages by 10%, from 0 to 1 raised wages by 5%, and from 1 to 2 raised wages by only 2%. However, the striking finding is that over time, the returns to high levels of cognitive skills (1 to 2 s.d. points) have increased dramatically, from 2% to 21%, while returns at low levels (-2 to -1 s.d. points) have increased only slightly (2%).

One explanation for the steep increase in estimated returns to high levels of cognitive skills may be the movement toward performance paying schemes in the 1980s and 1990s documented by Lemieux, MacLeod, and Parent (2007). Since the DOT reports the minimum requirements for each occupation, it is likely that skills supplied by workers in performance-pay jobs are higher than those captured by the DOT. Workers on performance-pay jobs tend to be more educated and work in professional and managerial occupations that require high levels of cognitive skills even as a minimum. Therefore, it is possible that the earnings of workers with high cognitive skills increased because pay schemes have changed to increasingly reward skill levels above minimum occupation requirements, and more so for occupations intensive in cognitive skills.

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18Note that since the DOT characteristics used to construct the skill factors are not cardinal measures, we should be cautious in drawing any conclusion regarding the presence of increasing or diminishing returns to skills. The most relevant finding is how these returns change over time.

19They find that the fraction of U.S. male workers on performance-pay jobs increased from about 30 percent to over 40 percent in the 1980s and 1990s.

20Annual earnings in the CPS include tips, bonuses and commissions, so they capture well the performance based earnings.

21Bacolod and Blum (2010) have also noted that in general, the returns to skill estimates would be biased if any excess skills above the minimum requirements are both rewarded and also somehow
Skill-biased technical change could also favor workers with high cognitive skills relatively more if the complementarity between computer capital and cognitive skills is higher at high skill levels. However, this is hard to reconcile with the observation that among high-skill occupations, the only ones that experience negative relative wage changes are precisely the ones most closely linked to the computer revolution. Computer programmers and engineers experience negative relative wage changes while lawyers and professors who are less linked to the computer revolution do better (Lemieux (2007)).

Another possibility is that occupations with above average cognitive skill requirements in the 1991 DOT have further increased their cognitive skill requirements over time. In this case, since the data does not capture intensive margin shifts (changes within occupations), cognitive skills in the top half of the distribution would be underestimated. This measurement error would introduce a positive bias in estimated returns to high levels of cognitive skills, and this bias would grow over time as measurement error becomes more pronounced with ongoing changes in skill requirements within occupations. The Data Appendix provides evidence of intensive margin shifts by comparing DOT 1977 and DOT 1991 average requirements and shows that indeed, the occupations of college educated workers with high levels of cognitive skills have increased their cognitive skill requirements, and occupations dominated by high school workers with low cognitive skills have actually decreased their cognitive skill requirements.

There are several other potential problems in estimating returns to skills. Bacolod and Blum (2010) note that the skill requirements of occupations may vary within the three-digit occupational categories used in the Census. If such variation is systematically correlated with the measured DOT requirements, the parameter estimates will be biased. Estimates would also be biased if workers have unobserved characteristics that are correlated with their measured DOT skills. To address these issues, they use panel data from the NLSY79 to conduct several robustness checks and they find that the main
patterns and results are robust to different specifications.

### 2.3.3 The College Premium

Figure 2.10 plots the college wage premium over time for males: on average, in 1980, college graduates earned 27% higher wages than high school graduates, while by 2002 this gap had increased to 52%. The same figure shows that accounting for the usual control variables (age, age squared, experience, region, race) leads to a lower estimated college wage premium in every year.\(^{22}\) However, the premium still increases from 21% to 47% over the time period, revealing that changes in quantities and prices of these control variables cannot account for any of the 25% increase in the college premium from 1980 to 2002.

In order to evaluate the extent to which skills can account for this sharp increase, I re-estimate the college wage premium using the same regression of log hourly earnings on the usual controls but also adding cognitive and physical skills and their quadratic terms. Figure 2.10 shows that accounting for these skills explains about half of the increase in the premium observed from 1980 to 2002: the estimated premium now increases from 13% to 25%, by only 12% compared to 25% observed without controls.\(^{23}\) Results in previous literature also show similar effects of accounting for skills: Murnane, Willett and Levy (1995) find that basic cognitive skills (measured by mathematics test scores) explain all of the increase in the return to college attendance between 1978 and 1986, and Ingram and Neumann (2006) find that once they control for skill factors, the return to an additional year of education falls and shows less growth from 1970 to 1998.

There are two ways in which skills can contribute to the growth in the college wage premium: through a skill quantity effect and a skill price effect. For example, if college

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\(^{22}\)The college wage premium reported in the figure is the estimated coefficient on the college dummy (*100) from a log hourly earnings regression on the usual explanatory variables.

\(^{23}\)The 95% confidence intervals around the estimated returns to college are very narrow around the estimates, and are therefore not shown in the figure as they would almost coincide with the estimates.
graduates moved into occupations that employed higher levels of cognitive skills over time while high school graduates remained in the same type of occupations, then the college group’s wages would increase relative to high school graduates since cognitive skills are highly valued. On the other hand, the college group’s wages could increase relative to high school when employed skills remain constant and returns increase more for levels of skills supplied by college graduates than for levels supplied by high school workers.

I find that it is the increase in the return to high levels of cognitive skills (the price effect) that helps explain the increase in the college premium, not quantity effects. This should not be surprising considering Figure 2.4 shows that the average employed cognitive skills have remained approximately constant for the college and high school groups. On the other hand, as Figure 2.8 shows, the return to cognitive skills has increased dramatically after 1980, and to a much greater extent for those with high skill levels such as the college educated.

To illustrate the importance of price effects, Figure 2.11 plots the estimated college premium from a regression of log hourly earnings on all variables, including skills, but keeping skill prices fixed at the 1980 level. As expected, we observe only a level effect in the college premium, where the level change is approximately equal to the coefficient on cognitive skills in 1980 times the difference in mean cognitive skills between college and high school. Differences in physical skills between education groups do not significantly impact the college premium. Since the difference in cognitive skills remains roughly constant, the college premium shifts down by a constant. The increase in the estimated premium is 27% between 1980 and 2002.

To formalize this, I also use the approach of Juhn, Murphy and Pierce (1993) to decompose the total change in the college wage premium over time into components capturing changes in the distribution of individual characteristics (quantity effect), changes in the prices of observable characteristics (price effects), and changes in the distribution
of residuals. I do this for the time periods 1980-1990, 1990-2002, and 1980-2002. Table 2.3 reports the results of the decomposition when only the usual controls are included in the log hourly earnings regression, and also when skills are included. First, the inclusion of the usual control variables helps explain only 6% of the total increase in the college premium in the 1980s, and none of the increase after 1990. However, with the inclusion of skills in the regression, observables now account for 78% of the premium increase in the 1980s and 57% of the increase from 1990-2002. The prices of observables alone account for 65% and 56% of the increase in the premium in these time periods.

In interpreting the estimated college premium from the regressions including skills, it is important to note that the coefficient on college does not measure the true return to college if college attendance produces higher cognitive (or physical) skills, as noted by Ingram and Neumann (2006). If for example cognitive skills increased due to college attendance, the true return to college would equal the coefficient on college plus the return to the cognitive skills acquired during college. Ingram and Neumann (2006) find using NLSY79 data that indeed years of education have a noticeable effect on the constructed “intelligence” factor after controlling for other characteristics. They estimate that an additional year of schooling leads to an increase in “intelligence” of 14.7 percent of a standard deviation point in 1986 and 16.6 percent in 1996.

If cognitive skills developed through college increased over time, or if their returns increased, then the true return to college would increase by more than the change in the estimated coefficient on college (12%). We cannot infer from the estimates in Ingram

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24 Juhn, Murphy and Pierce (1993)’s approach has been extensively used and explained in the wage inequality literature. See for example Autor, Katz and Kearney (2005b), Lemieux (2006), and Bacolod and Blum (2010).
25 In fact, the inclusion of these controls leads to a steeper college premium growth in the 1990s, reflected by the negative sign in the “Change Explained” column in Table 2.3.
26 The prices of the skills alone actually account for slightly more than this since the price of the other control variables work in the opposite way, accentuating the increase in the premium in both time periods.
27 Heckman and Vytlacil (2001) argue that ability and schooling are so strongly dependent that it is not possible to independently vary these two variables and estimate their separate impacts.
28 Ingram and Neumann (2006) calculate the adjusted returns to an additional year of education and
and Neumann (2006) whether there is a significant difference in skills acquired through college over time. However, the large increase in returns to cognitive skills likely accounts for a considerable fraction of the increase in the true returns to college.\footnote{Skills developed through college are most likely just a fraction of the observed difference in mean skills between college and non-college. If all the difference in mean skills came from learned skills, than the true return to college would increase by the full 25% estimated through the regression excluding skills. (The increase in the college premium from the regression without skills is equal to the change in the coefficient on college (from the regressions including skills) + the change in the difference between mean college and non-college skills times the corresponding skill prices.)}

\section*{2.3.4 Residual Earnings Inequality}

Figure 2.12 shows the evolution of earnings inequality for males, for the entire sample and by education groups, and Figure 2.13 plots residual earnings inequality after controlling for the usual characteristics (age, race, region, etc.). Inequality is calculated as the 90-10 percentile log hourly earnings differential, and residual inequality is calculated as the 90-10 percentile log hourly residual earnings differential from the log hourly earnings regression on the usual controls. I also plot the 90-50 and 50-10 differentials since previous literature has shown that inequality has evolved differently for the top and bottom ends of the wage distribution (e.g. Autor, Katz and Kearney (2005b), and Lemieux (2006)). As in this previous literature, I find that overall inequality increased in the 1980s, but continued to rise in the 1990s only for the top half of the wage distribution group. Residual earnings inequality follows approximately the same pattern, with a very sharp increase in the 90-50 differential in the 1990s and a flattening of the 50-10 differential. Overall inequality and residual inequality are in general much higher among college graduates than high school graduates, and also increase much faster for the college group, especially after 1990.

While these trends have been studied in previous literature, no single explanation emerged dominant in accounting for them. For example, the SBTC explanation is hard
to reconcile with the slowing inequality growth in the 1990s and with various patterns in inequality between groups (e.g. Card and DiNardo (2002) and Lemieux (2006)). Ingram and Neumann (2006) and Bacolod and Blum (2010) have studied the contribution of various skill measures to residual inequality growth from 1970-1998 and 1969-1989, respectively. Following this literature, I study the contribution of physical and cognitive skills to residual inequality, and in addition, their contribution to the diverging inequality patterns in top and bottom ends of the wage distribution observed after 1990.

As in previous literature, I run two log hourly earnings regressions, including only the usual control variables in the first, and adding skills in the second, and I estimate residual earnings inequality from both regressions. Figure 2.14 presents these results. Accounting for skills leads to lower residual earnings inequality in every year, especially in the 50-10 residual differential. To illustrate the effects of adding skills more clearly, Figure 2.15 shows the percentage of residual inequality that is accounted for by skills for the two education groups over time. The inclusion of skills lowers 90-10 residual inequality by approximately only 1.5% in 1980 for both college and high school groups. However, even though it remains quantitatively small, the percentage explained increases over time to 3% in 2002 for high school and 5% for college. Since the trend in the 90-10 differential explained is irregular, I re-do the exercise measuring inequality as the variance of the residuals. Using this measure, we obtain that the percentage explained by skills increases from 3% in 1980 to 6% in 2002 for high school and 9% for college. This indicates that while skills do not play a big role in explaining residual inequality, they are becoming more important over time.30 Of the total increase in 90-10 residual inequality from 1980 to 2002, skills account for 12% of this growth for high school and 16% for college graduates (Table 2.4).

I also study whether skills can account for the divergent growth patterns observed

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30Ingram and Neumann (2006) find slightly larger effects: for high school educated workers, their skill factors account for about 4% of the differential in any year, and for college educated workers, variations in skill account for 4% in 1980, but nearly 10% in 1992.
Chapter 2. Skills and Wage Inequality

after 1990 in the 90-50 and 50-10 percentile differentials. From Figure 2.15, we see that skills account for small fractions of the 90-50 percentile differential within both education groups. Table 2.4 shows that skills still explain 6.7% of the increase in 90-50 residual inequality for high school graduates from 1990 to 2002, and 7.5% for college graduates.\textsuperscript{31} While these numbers are not very large, they are significant and show that skills could be one of the many contributing factors to growing top end inequality.

Finally, I test whether skills can explain residual inequality better for workers who are not protected by unions. I run the same regressions and estimate residual inequality that is explained by skills first for high school educated workers who are not part of a union and second for those who are union members.\textsuperscript{32} I find that skills account for a higher percentage of residual earnings inequality for non-unionized workers (Figure 2.16). This is the case especially in the early 1990s when the difference is about 4%. This indicates that unions could prevent wages from adjusting to reflect returns to skills; for example, minimum wages protected by unions could be higher than those predicted by returns to skills.

Skills have become more important determinants of residual inequality over time partly because the variance of cognitive skills within education groups has increased from 1980 to 2002, by 17% for college and 12% for high school graduates (Figure 2.7) and partly because the increase in returns to cognitive skills disproportionately favors those with high skill levels. The importance of skills has grown faster for college graduates since their skill dispersion has increased faster and since there is a larger fraction of high cognitive skill individuals within the college group whose skill returns have grown disproportionally more.

\textsuperscript{31}Note however that this result is sensitive to the last year considered for comparison. Figure 2.14 shows an irregular pattern in the growth of the 90-50 residual differential from the regression including skills relative to the differential from the regression without skills.

\textsuperscript{32}Data on union membership is only available after 1990.
2.4 Conclusion

In this paper, I have shown that skills can account for a large fraction of the increase in the college premium and a smaller fraction of the increase in residual wage inequality. There has been a strong shift in employment away from occupations requiring physical skills towards those intensive in cognitive skills, but most of the growth in the employment of cognitive skills can be accounted for by the increase in the fraction of college educated women in the labor force, not by shifts within education-gender groups. Within these groups, average employed cognitive skills have remained approximately constant. However, the returns to high levels of cognitive skills have increased dramatically over time while returns to low skill levels have remained approximately constant. Since college graduates supply a high fraction of employed cognitive skills whose returns have increased more than those of high school, skills can account for about half of the increase in the college wage premium. Finally, while only a small fraction (at most 9%) of residual wage inequality is accounted for by skills, there has been an increasing trend in the fraction of residual inequality that is explained by skills over time, and skills explain approximately 12% and 16% of the increase in residual wage inequality for high school and college graduates, respectively. The dispersion in average employed cognitive skills within education groups has increased over time, and since returns to high levels of skills have increased disproportionally more than at low levels, skills have become increasingly important in the determination of within-group inequality. One limitation in this paper is that the constructed data can only capture changes in employed skills over time that arise from changes in the distribution of employment across different occupations. However, it does not account for within-occupation changes in skill requirements or for shifts into new occupations that were created after 1991. In the future, as data becomes available, it would be interesting to study how these changes contribute to wage inequality.
### 2.5 Tables

**Table 2.1:** Factor Loadings, factor analysis in 1990

#### Constructing the Cognitive Skill Measure

<table>
<thead>
<tr>
<th>Construct</th>
<th>Factor Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reasoning Development</td>
<td>0.981</td>
</tr>
<tr>
<td>Mathematical Development</td>
<td>0.931</td>
</tr>
<tr>
<td>Language Development</td>
<td>0.975</td>
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<tr>
<td>General Intelligence</td>
<td>0.960</td>
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<tr>
<td>Verbal Aptitude</td>
<td>0.966</td>
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<tr>
<td>Numerical Aptitude</td>
<td>0.902</td>
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<tr>
<td>Clerical Perception</td>
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<tr>
<td>Data Complexity</td>
<td>0.934</td>
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<tr>
<td>People Complexity</td>
<td>0.777</td>
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<tr>
<td>Direction and Planning Ability</td>
<td>0.702</td>
</tr>
<tr>
<td>Interpretation Ability</td>
<td>0.174</td>
</tr>
<tr>
<td>Influencing People</td>
<td>0.444</td>
</tr>
<tr>
<td>Making Judgments (judgemental criteria)</td>
<td>0.747</td>
</tr>
</tbody>
</table>

**Eigenvalue** 8.75  
**% of variance explained** 67.33%

#### Constructing the Physical Skill Measure

<table>
<thead>
<tr>
<th>Skill Measure</th>
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<tbody>
<tr>
<td>Manual Dexterity</td>
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<tr>
<td>Strength</td>
<td>0.794</td>
</tr>
<tr>
<td>Climbing</td>
<td>0.890</td>
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<tr>
<td>Stooping</td>
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<tr>
<td>Kneeling</td>
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<tr>
<td>Balancing</td>
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</tr>
<tr>
<td>Crouching</td>
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<tr>
<td>Crowling</td>
<td>0.656</td>
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<tr>
<td>Difficult Environmental Conditions</td>
<td>0.742</td>
</tr>
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</table>

**Eigenvalue** 5.81  
**% of variance explained** 64.52%
Table 2.2: Coefficient Estimates from Log-wage Regression Including Skills

<table>
<thead>
<tr>
<th>Year</th>
<th>College</th>
<th>Cognitive</th>
<th>Physical</th>
<th>Cognitive^2</th>
<th>Physical^2</th>
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<td>1980</td>
<td>0.1293</td>
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<td>(0.0001)</td>
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<td>1982</td>
<td>0.1442</td>
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<td>1989</td>
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Standard errors in parentheses.

Bold indicates statistical significance at 1% level.
Table 5 continued...

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<tr>
<th>Year</th>
<th>College</th>
<th>Cognitive</th>
<th>Physical</th>
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<th>Physical^2</th>
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<td>1997</td>
<td>0.1899</td>
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</table>

Standard errors in parentheses. Bold indicates statistical significance at 1% level.

Table 2.3: Components of Changes in the College Wage Premium

Regression of log wages on usual control variables

<table>
<thead>
<tr>
<th>% Change in College Prem</th>
<th>Change Explained by Observables (P &amp; Q)</th>
<th>Change explained by Quantities (Q)</th>
<th>Change explained by Prices (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-1990: 14.8</td>
<td>0.9 (5.9%)</td>
<td>1.1 (7.7%)</td>
<td>-0.3 (-1.8%)</td>
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<tr>
<td>1990-2002: 10.4</td>
<td>-2.4 (-23.4%)</td>
<td>-1.5 (-14.4%)</td>
<td>-0.9 (-9.0%)</td>
</tr>
<tr>
<td>1980-2002: 25.3</td>
<td>-1.6 (-6.2%)</td>
<td>-0.6 (-2.4%)</td>
<td>-1.0 (-3.8%)</td>
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</tbody>
</table>

Regression of log wages on usual control variables and skills

<table>
<thead>
<tr>
<th>% Change in College Prem</th>
<th>Change Explained by Observables (P &amp; Q)</th>
<th>Change explained by Quantities (Q)</th>
<th>Change explained by Prices (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-1990: 14.8</td>
<td>11.6 (78.1%)</td>
<td>2.0 (13.3%)</td>
<td>9.6 (64.8%)</td>
</tr>
<tr>
<td>1990-2002: 10.4</td>
<td>6.0 (57.1%)</td>
<td>0.1 (5.5%)</td>
<td>5.9 (56.6%)</td>
</tr>
<tr>
<td>1980-2002: 25.3</td>
<td>17.5 (69.4%)</td>
<td>1.4 (5.5%)</td>
<td>16.2 (64.0%)</td>
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</tbody>
</table>
Table 2.4: % of Total Increase in Residual Wage Inequality Explained By Skills

### High School

<table>
<thead>
<tr>
<th>Period</th>
<th>% Explained in log 90/10 ratio</th>
<th>% Explained in log 90/50 ratio</th>
<th>% Explained in log 50/10 ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-1990</td>
<td>16.9</td>
<td>12.0</td>
<td>24.4</td>
</tr>
<tr>
<td>1990-2002</td>
<td>7.2</td>
<td>6.7</td>
<td>8.5</td>
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<tr>
<td>1980-2002</td>
<td>12.2</td>
<td>9.2</td>
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### College

<table>
<thead>
<tr>
<th>Period</th>
<th>% Explained in log 90/10 ratio</th>
<th>% Explained in log 90/50 ratio</th>
<th>% Explained in log 50/10 ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-1990</td>
<td>13.5</td>
<td>27.2</td>
<td>10.3</td>
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<tr>
<td>1990-2002</td>
<td>17.4</td>
<td>7.5</td>
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<td>1980-2002</td>
<td>16.1</td>
<td>9.4</td>
<td>27.3</td>
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</tbody>
</table>
Chapter 2. Skills and Wage Inequality

2.6 Figures

**Figure 2.1:** Trends in Wage Inequality

![Graph showing trends in wage inequality.](image)

**Figure 2.2:** Average Employed Skills

![Graph showing average skills over time.](image)
Figure 2.3: Average Employed Skills by Gender

Figure 2.4: Average Cognitive Skills by Education
**Figure 2.5:** Labor Force Composition

**Figure 2.6:** Cognitive Skills when Entering Labor Force
Figure 2.7: Variance of Cognitive Skills

Figure 2.8: Returns to Skills, from 0 to 1 standard deviations
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Figure 2.9: Returns to Skills, Different Levels

Figure 2.10: College Wage Premium
Figure 2.11: College Wage Premium, Skill Prices Fixed at 1980 Level
Figure 2.12: Trends in Hourly Wage Inequality
Figure 2.13: Trends in Residual Hourly Wage Inequality
Figure 2.14: Effect of Skills on Residual Hourly Wage Inequality
Figure 2.15: Percentage of Residual Wage Inequality Explained by Skills

Figure 2.16: Residual Inequality Explained by Skills, by Union Membership
Figure 2.17: Mean Skills, Factor Analysis in Different Years

Figure 2.18: Mean Skills, DOT 1977 and DOT 1991
Figure 2.19: Mean Skills, DOT 1977 and DOT 1991
2.7 Data Appendix

Matching DOT data to the CPS

This section describes in more detail how the data was constructed. The method is based on the approach used in Autor, Levy and Murnane (2003). I construct two data sets: first, I match workers in the CPS with characteristics from the 1977 DOT Fourth Edition, and second I match CPS workers with characteristics from the 1991 DOT Revised Fourth Edition. The second is the data set used throughout the paper. The main two reasons for constructing these two versions of the data set are: (1) to estimate shifts in employed skills within occupations (i.e. intensive margin shifts) by comparing estimated skills across data sets and (2) to conduct robustness checks on how the main data set used in the paper was constructed since better crosswalks between the DOT 1977 and CPS occupations allow for a more accurate matching of CPS workers to DOT characteristics.

Differences between the DOT 1977 and 1991 editions. The DOT 1991 Revised Fourth Edition contains updated information collected in the 1980s. Autor, Levy and Murnane (2003) report that a total of 2,452 occupations were reviewed, updated, or added: 646 nominal titles were revised, 136 titles were combined, and 75 were deleted. The revisions were conducted for occupations that were observed to have most substantively changed between 1977 and 1991, predominantly those affected by technological advancements. Bacolod and Blum (2010) report that 761 new occupations were added, mostly computer-related jobs.

Matching the 1977 DOT Fourth Edition to the CPS. As in the case of the 1991 DOT, the 1977 edition contains data on very detailed occupations (approximately 12,000). On the other hand, the CPS occupation classification is much less detailed: there are only 412 occupations in the 1970 Census classification, and 540 in 2000. First,
I match DOT occupations to those in the CPS, and then obtain characteristics for each Census occupation by taking the average of the DOT task measures within each Census occupation.

In order to construct accurate Census occupation characteristics, it is important to know the weight of each DOT occupation comprising a given Census occupation. For example, the census occupation “Policemen and Detectives” contains many DOT occupations including “Traffic Sergeant” and “Commanding Officer in the Homicide Squad” that may have very different skill requirements. Fortunately, constructing accurate weights is facilitated by the existence of one data set that matches CPS workers in 1971 directly to DOT 1977 occupations and characteristics. The Committee on Occupational Classification and Analysis of the National Academy of Sciences acquired a selection of variables from the April 1971 CPS that were gathered from a sample of households which yielded 60,441 workers in the experienced civilian labor force and augmented this sample with DOT 1977 characteristics, for each worker in the survey. Therefore, by looking at the number of workers in each DOT occupation, their sampling weights and hours worked, it is possible to calculate accurate weights for each occupation.\(^{33}\) Specifically, I take a weighted average of the DOT occupation characteristics over each 1971 Census occupation, where each observation is weighed by the full-time equivalent hours of labor supply, calculated as the product of the individual CPS sampling weight and hours of work in the past week.

This accurate weighing is not possible in the case of the DOT 1991 version since no data set exists matching the DOT 1991 occupations directly to workers in the CPS.\(^{34}\) In the case of the DOT 1991, I take simple averages of DOT occupation characteristics when aggregating up to the census occupation level. Therefore, one advantage of constructing

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33 This file contains 412 1970 Census occupations matched to 3,585 DOT occupations.

34 Autor, Levy and Murnane (2003) use a crosswalk between the 1977 and 1991 DOT occupation codes to match DOT 1991 task variables to the 1971 CPS file. I have not done this in my paper. Instead, I simply show that the weighing of observations does not affect the results.
the data set using the DOT 1977 version is that it allows me to conduct a robustness check, seeing how results are affected by the weighing of DOT occupations when assigning occupation characteristics to census occupations.

The next step is assigning DOT occupation characteristics to 1980s and 1990s census occupations. The occupation classification used in the CPS changes in 1983, 1992 and 2003. There are substantial differences between the 1970s and 1980s classifications, relatively small differences between the 1980s and 1990s classifications, and major differences between the 1990s and the 2000s. I use available crosswalks from the U.S. Census Bureau, NOICC Crosswalk and Data Center and the National Crosswalk Service Center as well as a file prepared by the Committee on Occupational Classification and Analysis chaired by Donald Treiman that contains 122,141 observations from the 1980 CPS that are individually dual coded with both 1970 and 1980 census occupation codes (known as the “Treiman file”) to match DOT characteristics to 1980s and 1990s Census occupations. These files allow for some weighing of the more detailed occupations when constructing DOT task averages for less detailed Census occupations.

Matching the 1991 DOT Revised Fourth Edition to the CPS. I first use the available crosswalk from National Crosswalk Service Center (NCSC) that matches the Census occupation codes from the 1990s classification to the 1991 DOT. To obtain occupation characteristics for these 1990s census occupations, I take the simple average of the DOT occupation characteristics comprising each census occupation. Next, I use available crosswalk between the census occupations in the 1990’s and census occupations in the 1980’s to assign characteristics to 1980s census occupation codes. This results in a very high match rate since the 1980s and 1990s classifications are very similar. Finally, I assign occupation characteristics to 1970s census codes using the concordance between the 1980’s and 1970’s classifications constructed by the U.S. Bureau of the Census which assigns weights to occupations that do not uniquely map into one another across classi-
factor analysis robustness checks: using different years and different samples (men and women). After matching DOT characteristics to workers in the CPS from 1971 to 2002, I use factor analysis to construct cognitive and physical skill variables. The paper presents the results obtained using skill factors constructed by performing the factor analysis in 1990 and using the coefficients from this estimation to calculate the predicted values of the skill factors for the remaining years in the data set. I test whether the results are robust when using a different year for the estimation of the factor loadings and when the estimation is performed using only men or women.

The availability of the 1971 CPS augmented with DOT characteristics makes 1971 the best year for the estimation of the factor loadings since factor analysis can be conducted using variation across 3,585 DOT occupations that appear in this file. Each observation is weighed by the sample weight times labor hours. For any year other than 1971, the estimation can use only the variation in characteristics across less detailed Census occupations, numbering at most 540 in a given year. Figure 2.17 shows a comparison of trends in average skills estimated using factor analysis conducted in 1971 and 1990. In both cases, factors are normalized to have a mean of zero and standard deviation of one in 1990. The figure shows that we cannot visually distinguish between the two estimates as they are extremely close to one another. Plots of mean skills by education-gender subgroups (not shown) also reveal that differences are extremely small.

Another source of concern is that estimated factor loadings might differ for men and women, implying that separately estimated coefficients should be used to predict skills of men and women. For example, in 1971 the census occupation "Policemen and Detectives" is composed of many police officers, chiefs, sergeants and detectives for men, but is

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composed of many school-crossing guards and parking enforcement officers for women. The interdependencies between observed DOT characteristics used in factor analysis may differ systematically across genders when men and women work in different occupations. Since my paper aims to explain male wage inequality, I verify that estimated skill factors remain unchanged when using only men in the estimation. I find that even though the factor loadings change slightly, the re-estimated employed cognitive and physical skills are almost identical to those found in the paper.36

**Comparison of results using the DOT 1977 and DOT 1991.** I check whether the accurate weighing of DOT occupations in constructing Census occupation characteristics leads to any differences in estimated skills. I compare mean skills obtained using the accurate weighing of DOT occupations in 1971 with mean skills obtained using simple averages of DOT characteristics in 1971. I find no significant difference in estimated skills, and both methods produce the same trends in average skills over time. Taking simple averages of DOT characteristics as done in the construction of the main data set used in the paper should not affect the results.37

I check whether the two constructed data sets (CPS matched to DOT 1977 and DOT 1991) lead to different estimated trends in average employed skills. I perform the factor analysis in 1990 in both data sets, and predict cognitive and physical skills for the remaining years. In both data sets, skills are normalized to have a mean of zero and standard deviation of one in 1990. Figure 2.18 shows average employed skills estimated using these data sets. We observe that estimated average cognitive skills using DOT 1991 characteristics increase slightly faster over time than those estimated using DOT 1977 characteristics, and physical skills decline slightly faster. This indicates that the CPS matched to the 1991 DOT data set captures to a greater extent the extensive margin

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36 I find that the factor loadings of physical characteristics are smaller for women than men. However, the predicted skills are not changed significantly by this difference.

37 The importance of the accurate weighing of DOT occupations may change over time. However, 1971 is the only year that allows for comparisons.
shifts into cognitive intensive occupations and away from physical intensive occupations since it contains updated information on requirements as well as new occupations created in the 1980s.

**Estimating intensive margin shifts.** Comparison across these data sets also allows us to estimate how average employed skills have changed due to intensive margin shifts (changes in skill requirements within occupation) that occurred in the 1980 when the DOT requirements were updated for the 1991 DOT Revised Edition. To allow for a comparison, we need to use the same factor analysis coefficients to predict cognitive and physical skills in both data sets. I create a new data set containing the CPS matched to the DOT 1977 up to year 1991 and the CPS matched to the DOT 1991 after 1991, and estimate skill factors using the same factor analysis coefficients from 1990. Figure 2.19 shows that average skills are greatly influenced by the updated information in the 1991 DOT. Intensive margin shifts lead to a sharp decline in employed physical skills and a smaller increase in employed cognitive skills, observed in 1992 as we switch from the DOT 1977 to the DOT 1991 occupation characteristics. The right hand side of Figure 2.19 shows changes within education groups: interestingly, only average college cognitive skills increase in 1992, and high school cognitive skills actually decline slightly. Average physical skills decline for both groups, but to a greater extent for high school graduates. Therefore, only occupations dominated by college workers have increased their cognitive skill requirements, perhaps due to a high complementarity between computer capital and cognitive skills. Occupations dominated by high school workers have lowered their cognitive skill requirements, perhaps because computer capital substitutes for low levels of cognitive skills.

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38 Even though new occupations were introduced in the 1991 DOT, since many DOT occupations comprise each Census occupation, their introduction affects the Census occupation requirements. Therefore, their introduction still leads to “intensive” margin shifts in requirements at the Census occupation level.

39 In the previous exercise, we could not compare the levels of average skills employed across the data sets because they were both normalized to have mean zero in 1990.
To conclude, using the more recent DOT 1991 edition instead of the DOT 1977 does not lead to large differences in results. Both data sets lead to similar trends in employed skills over time, both aggregate and by education-gender groups (not shown). Since skill levels are normalized, their estimated returns also do not differ significantly. However, using either data set does not allow us to capture the large intensive margin skill shifts that occurred in the 1980s. Ideally, we would like to observe the gradual change that occurred within occupations over the years in order to estimate the importance of intensive margin changes and their contribution to wage inequality. Since changes within occupations and the creation of new occupations seem to have increased college employed cognitive skills and decreased those of high school, the growing difference in cognitive skills across education groups contributes to the increasing college wage premium. Had we accounted for intensive margin shifts, the estimated quantity effects in Section 2.3.3 would have been relatively more important. Not only have cognitive skill prices increased over time, and more so at high skill levels, but also changes in occupation requirements have led to an even higher employment of cognitive skills for college and lower for high school.
Chapter 3

A Model of the Distribution of Talent Across Education Fields

3.1 Introduction

The main question of this paper is how talent is allocated across education fields in universities. The paper analyzes the sorting decision of students into two different fields of study. It is assumed that university education does not influence students’ skill levels. Skills are developed only prior to university entry and students are assumed to have perfect knowledge of their own skill levels, but this information is private and cannot be observed by potential employers.\(^1\) Students attend colleges or universities in order to signal their talents. The model developed in this paper assumes that students are characterized by two types of skills, analytical and verbal, and that university fields of study are characterized by their ability to test and reveal each skill through grades. For

\(^1\)To justify the assumption of privately observed talent levels at the end of high school, I argue that high school grades are a very noisy measure of talent, so employers cannot use them to infer talent levels. High school projects are often graded based on student effort and time spent on them rather than actual talent levels. However, since students know the effort level they exert and are also aware of their knowledge and the quality of education offered in their own high school, they can use this information to perfectly infer their talent levels.
example, Math reveals students’ analytical skill levels, while English Literature reveals verbal skill levels. In one version of the model, the students’ expected payoffs after university graduation are exogenous functions of the chosen field, the revealed skill level, and the mean unobserved skill in the chosen field. In the second version of the model, employers become active players who face an inference problem about students’ unrevealed skills, and wages are determined competitively. The sorting decision into fields of study depends crucially on the relative weights of the observed and unobserved skill levels in determining post-graduation wages.

The paper first studies the effects of changing average skill levels in the student population. For example, improving the quality of high school Math teachers might result in higher average analytical skills, and I study how this affects the allocation of students into university fields of study. As a result of the change, university enrollment in the Math field could increase or decrease, depending on the relative weight of the unrevealed skill in determining students’ wages after graduation. When the revealed skill is valued relatively more than the unrevealed skill, enrollment in the Math field increases, and when it is valued less, Math enrollment decreases. Next, I look at the effects of changing the variance of the skills in the population. This is useful for predicting students’ sorting decisions when for example the quality of Math or English high school teachers becomes more or less equal across schools. I find that higher analytical skills variance leads to an increase in the number of students entering the Math field as students around the middle of the analytical skill distribution find it increasingly in their interest to distinguish themselves from the very low analytical types that enter the English field. Finally, I study the effects of changing the degree of correlation between the two skills in the student population. For example, the correlation would be high if high schools were equally good or bad in teaching both skills. I find that everything else equal, positive correlation between skills results in two stable equilibria, where either the Math or the English field attracts more students. An increase in the degree of correlation leads to
higher enrollment in the larger field.

The paper focuses on the case of uniformly distributed skills so the model can be solved numerically. In general, only a few results can be generalized for larger classes of distributions. However, I find that simulations conducted using normally distributed skills generate the same qualitative results as when skills are uniformly distributed.

3.2 Previous Literature

While there is a large literature on education signaling models, there is no model that directly addresses the questions of this paper. Quinzii and Rochet (1985) extended the Spence (1974) model of market signaling to the case of multidimensional characteristics. Workers have privately observed characteristics and choose from available signals. Their productivities and costs of signaling depend both on their characteristics and their signals, however, wages depend only on signals. The resulting signaling equilibrium is separating in the sense that agents with the same productivity levels choose the same signals.

Damiano, Li, and Suen (2005) model sorting choices of agents into two organizations where there are two effects: the peer effect and the pecking order effect. Unlike my paper, agents are characterized by a one-dimensional type. Also, while my paper uses the concept of Nash Equilibrium, Damiano, Li, and Suen (2005) use the concept of a “priority equilibrium,” defined as an allocation of types such that no agent whose type is higher than the lowest type in the other organization wishes to move. The sorting then results in an overlapping interval structure in the type space.

A related empirical paper is Turner and Bowen (1999). While their paper is concerned primarily with differences in field majors between men and women, the relevance to my paper lies in their estimates of the relationship between verbal and analytical abilities (as measured by the SAT Math and verbal scores) and fields chosen. Their study finds that students with high verbal scores and low quantitative scores have a high probability of
choosing Humanities; students with high Math scores have a high probability of choosing Life Sciences and Economics, and a higher probability of choosing Physical Sciences and Engineering; the highest probability of choosing Math and Physical Sciences is associated with the top Math and verbal scores; and finally there is no relationship between the probability of choosing Engineering and verbal scores.

Other papers have highlighted the importance of studying the sorting decisions of students into fields of study. In “Microeconomics of College Choice, Careers, and Wages,” Behrman et al. (1998) discuss reasons why better models of education field choice are needed. The paper looks particularly at the field of Science and Engineering since this field is one of the most important for growth. Personal and family background characteristics, actual and expected market conditions, and economic incentives are all factors that interact in determining the choice of major, and the paper argues that career choice should be viewed as a sequential process and differences in background, aptitude and circumstances should be emphasized. The authors discuss for example how changes in the wages of microbiologists affect the relative supply of students in microbiology and their abilities. On the one hand, the most talented students in microbiology might stay in the field since they have a relative advantage in microbiology, while the less talented will not. On the other hand, the top students might have very good alternatives and could change fields relatively easier, leaving the less talented to stay in the field.

My paper builds on this literature by studying the allocation of students into fields of study using a simple framework in which students have different talents in two skill dimensions, and fields differ in their abilities to test and reveal these skills. The rest of paper is organized as follows. First, I describe the environment of the model and distinguish between the two model versions analyzed in this paper, discussing the differences between them. Second, I consider Model 1 in detail, define an equilibrium, and prove that an equilibrium exists. Then, I explicitly solve an example where students’ skill levels are uniformly distributed, and wage functions are linear and symmetric. Comparative
static exercises are performed in this environment. I generalize some results to other
classes of distributions. Next, I present simulation results for normally distributed skill
types. The final section analyzes Model 2 and compares the results of the two versions
of the model.

3.3 Model

3.3.1 Environment

This section describes the environment of Models 1 and 2. All features of the environ-
ment are common to both models unless otherwise noted. There is a continuum of mass
one of agents to be allocated into two fields, A and B. The two fields are for example
Math and English. There are no capacity constraints for the two fields. There are two
skills, x and y, for example analytical and verbal skills. The talent levels in these two
skills of an individual i are given by the pair $(x_i, y_i)$. Each student’s (from here on called
an agent) talent level is private information. The supports of the skills x and y are $[x, \bar{x}]$,
and $[y, \bar{y}]$.

The distribution of skills in the population is given by the cumulative distribution
$F(x, y)$, assumed to be strictly increasing in both $x$ and $y$, everywhere on the type
support. The associated probability density is $f(x, y)$, assumed to be continuous. Model
1 assumes that $F(x, y)$ is not observed; only the population means of $x$ and $y$ denoted
by $m_x$ and $m_y$ are assumed to be known by all agents. Model 2 assumes that $F(x, y)$ is
observed. This is one of the differences between the two models’ environments.

All agents are assumed to enter one of the two fields, A or B. Agents in field A are
subject to a test that fully reveals their individual levels of skill $x$, but does not reveal

---

2 The assumption of Model 1 that $F(x, y)$ is not observable is justifiable: agents know the average
skill levels in a population, but not how these skills are distributed. It is a much weaker assumption
than observable talent distribution. It also brings the benefit of greater model tractability, as will be
seen later.
y. Similarly, agents in field B are subject to a test that fully reveals the individual levels of y, but does not reveal x.

For example, agents in the Math field are subject to exams that evaluate analytical ability alone. The grades received are a direct indication of their analytical levels. So, I assume that test scores are not rescaled in any way, making it possible for the actual talent levels to be observed instead of just relative rankings.\(^3\)

Let \(F^j(x, y)\) be the distribution of types in field \(j\), where \(j \in \{A, B\}\). A **feasible allocation** is defined as a pair of distributions \((F^A(x, y), F^B(x, y))\) such that \(F^A(x, y) + F^B(x, y) = F(x, y), \forall (x, y)\). Also, let \(T_j\) be the support set of field \(j\), defined as the closure of the set types at which \(F^j(x, y)\) is strictly increasing in the x or y direction, or both.

The second difference between the two models is the agents’ payoffs. In Model 1, the payoffs to joining fields A and B of an agent with type \((x, y)\) are given by:

\[
W^A(x, y) = w^A(x, m_{y,A}), \text{ and } W^B(x, y) = w^B(m_{x,B}, y),
\]

where \(m_{i,j}\) is the mean of skill \(i\) in field \(j\). The payoff of an agent going to field A depends on the talent revealed by field A, x, and the average of the unrevealed talent in field A, \(m_{y,A}\). Similarly, the payoff of an agent going to field B depends on the talent revealed by field B, y, and the average of the unrevealed talent in field B, \(m_{x,B}\).

In Model 2, it is assumed that after completing a degree in a certain field, agents are hired by firms. Firms belong to one of two sectors, and each sector hires graduates from one field. Firms observe the chosen field of study of all agents and their talent levels

\(^3\)In reality, we observe that test scores are often rescaled according to a bell curve. This means that the talent levels are not actually observed, but only relative rankings. So, it would seem more appropriate to use a model where only the relative ranks of agents in field A according to \(x\) are revealed, and only the relative ranks of agents in field B according to \(y\) are revealed. In such a model, if \(F(x, y)\) is not observed, it is not possible to infer the actual skill levels of each agent unless it is know precisely how the grades are scaled. Therefore, in order not to have uncertainty in both talent dimensions in Model 1, it is necessary to make the assumption that the actual skill levels are revealed. This assumption is not necessary in Model 2.
revealed by that field. Firms additionally observe \( F(x, y) \). Wages are determined in a competitive labor market.

The wages of an agent with a degree in fields A and B and type \((x, y)\) are given by:

\[
W^A(x, y) = w^A(x, E[y|x, A]),
\]

\[
W^B(x, y) = w^B(E[x|y, B], y).
\]

The payoff of an agent going to field A depends on the talent revealed by field A, \( x \), and on the expected level of the unrevealed type. Similarly, the payoff of an agent going to field B depends on the talent revealed by field B, \( y \), and on the expected level of the unrevealed type. Since \( F(x, y) \) is assumed to be observable in Model 2, the expected unrevealed talent level is conditional on both the field chosen, and the revealed type.

The wage function \( w^i : R^2_+ \to R_+ \) is assumed to be separable and linear in its two arguments, continuous and differentiable everywhere, and strictly increasing in its two arguments.\(^4\) Also, the wage function itself depends on the field chosen. In Model 2 this means that agents from field A will be hired by a sector where the returns to talent may be different from those in the sector that hires agents from field B.

While the differences between Models 1 and 2 seem significant, the final section of the paper shows that the results obtained in the more simple Model 1 are robust when we allow for a more complex environment as in Model 2.

### 3.3.2 Model 1

The following section describes Model 1 in more detail.

Let \( m_{i,j} \) be the mean of skill \( i \) in field \( j \). \( m_{x,A} \) and \( m_{y,B} \) are the means of the revealed talents in A and B, and are therefore observed. Since the population means \( m_x \) and \( m_y \)

\(^4\)I define the wage function for any types in \( R^2_+ \) so the payoffs of any hypothetical types of agents (with positive talent levels) outside the support are also known. I also assume that payoffs cannot be negative.
and the masses of each field are also known, agents and firms can calculate the means of the unrevealed types \( m_{y,A} \) and \( m_{x,B} \): 

\[
m_{y,A} = \frac{m_y - \text{(mass } B \text{)mass } A}{\text{mass } A}, \quad \text{and } m_{x,B} = \frac{m_x - \text{(mass } A \text{)mass } B}{\text{mass } B}.
\]

The payoffs of an agent of type \((x, y)\) in A and B are:

\[
W^A(x, y) = w^A(x, m_{y,A}), \quad \text{and } W^B(x, y) = w^B(m_{x,B}, y).
\]

The problem of each agent is to choose the field that maximizes the payoff given by the wage function. Define 

\[
m \equiv (m_{x,B}, m_{y,A}) \in \mathcal{M}_x \times \mathcal{M}_y,
\]

where \( \mathcal{M}_y \) is the set of feasible values of \( m_{y,A} \) and \( \mathcal{M}_x \) is the set of feasible values of \( m_{x,B} \).

Agents hold a belief \( \mu(m) \) about \( m \). Given \( \mu(m) \), and their individual talent levels, agents will sort themselves into field A or B. In order to maximize their payoffs, agents with types such that \( W^A(x, y) > W^B(x, y) \) choose field A, and agents with types such that \( W^A(x, y) < W^B(x, y) \) choose field B. The types of agents indifferent between the two fields are given by all \((x, y)\) that solve:

\[
w^A(x, m_{y,A}) = w^B(m_{x,B}, y).
\]

It is assumed that the indifferent types go to either field with some unknown positive probability.\(^5\) In equilibrium, the agents’ belief about \( m \) is equal to the true value of \( m \).

The indifference curve: The indifference curve between the two fields will be re-defined in a more convenient way and its properties will be analyzed. The original indifference condition is \( w^A(x, m_{y,A}) = w^B(m_{x,B}, y) \). A final assumption is made about the wage functions: for any feasible \( m \), for any fixed level of \( y \), as \( x \to \infty \), \( w^A - w^B \to \infty \), and for any fixed level of \( x \), as \( y \to \infty \), \( w^A - w^B \to -\infty \). This ensures that there exists a type in the space \( \mathbb{R}_+^2 \) that is a solution to the indifference condition.

---

\(^5\)It does not matter what assumption is made about the field chosen by the indifferent types when the population density function is continuous. As will be seen later, the indifference curve between the two fields is a single valued function when the probability density \( f(x, y) \) is continuous, so the mass of indifferent types is zero.
This assumption together with the assumption that \( w^j() \) is continuous and strictly increasing guarantees that for any given \( y \) and \( m \), there exists a unique level of \( x \) such that \( w^A(x, m_y, A) = w^B(m_x, B, y) \). (\( w^B(m_x, B, y) \) is a constant for fixed \( m \) and \( y \), while \( w^A(x, m_y, A) \) is strictly increasing in \( x \) by assumption, so the two wage functions are equal at a unique level of \( x \).) Similarly, for any given \( x \) and \( m \), there exists a unique level of \( y \) such that the indifference condition is satisfied.

Now, we can define the following functions which are known to exist by the above argument:

\[
x^* : [y, \bar{y}] \times \mathcal{M}_x \times \mathcal{M}_y \to \mathbb{R} \text{ where } x^*(y, m) \text{ solves } w^A(x^*(y, m), m_y, A) = w^B(m_x, B, y),
\]
and

\[
y^* : [x, \bar{x}] \times \mathcal{M}_x \times \mathcal{M}_y \to \mathbb{R} \text{ where } y^*(x, m) \text{ solves } w^A(x, m_y, A) = w^B(m_x, B, y^*(x, m)).
\]

For any given \( m \) and \( y \), the function \( x^*(y, m) \) gives the type \( x^* \) such that the agent with type \((x^*, y)\) is indifferent between the two fields. Note that \( x^* \) may not be in the support of \( x \). In this case, \( x^* \) gives the hypothetical value that \( x \) would have to take so that the agent with a given \( y \) is indifferent between the two fields. The function \( y^* \) is defined similarly.

\( x^*(y, m) \) and \( y^*(x, m) \) can be shown to be differentiable and continuous by application of the implicit function theorem. Using implicit differentiation, it can be shown that \( x^* \) is strictly increasing in \( y \) and \( m_x, B \), and strictly decreasing in \( m_y, A \).

The types of agents for whom \( W^A(x, y) > W^B(x, y) \) will satisfy \( y < y^*(x, m) \), so all agents below the indifference curve (in \((x, y)\) space) will maximize their wages by going to field A. The types of agents for whom \( W^A(x, y) < W^B(x, y) \) will satisfy \( y > y^*(x, m) \), so all agents above the indifference curve will maximize their wages by going to field B. Figure 3.1 shows an example of an allocation consistent with \( m \).

**The means of unobserved skills:** For an allocation consistent with \( m \), agents can calculate the unobserved talent means in each field: \( m_y, A = \frac{m_y - (\text{mass}_B m_y, B)}{\text{mass}_A} \), and \( m_x, B = \)
Figure 3.1: Example of indifference curve

\[
\frac{m_x}{\text{mass } B}. \text{ These means can also be theoretically calculated using the distributions of types in A and B.}^6
\]

The distributions of types in the two fields in an allocation consistent with \( m \) are given by the following density functions:\(^7\)

\[
f^A(x, y)(m) \equiv \begin{cases} 
0 & \text{if } y \geq y^*(x, m) \\
\frac{\int_{x^*(y, m)}^{x} \int_{y}^{y^*(x, m)} f(x, y) \, dy \, dx}{\int_{x^*(y, m)}^{x} \int_{y}^{y^*(x, m)} f(x, y) \, dy \, dx} & \text{otherwise}
\end{cases}
\]

\[
f^B(x, y)(m) \equiv \begin{cases} 
0 & \text{if } y < y^*(x, m) \\
\frac{\int_{x^*(y, m)}^{x} \int_{y}^{y^*(x, m)} f(x, y) \, dy \, dx}{\int_{x^*(y, m)}^{x} \int_{y}^{y^*(x, m)} f(x, y) \, dy \, dx} & \text{otherwise}
\end{cases}
\]

In an allocation consistent with \( m \), there is a zero probability that an agent with type \((x, y)\) chooses field A when \( y \geq y^*(x, m)\) since choosing field A gives a lower expected payoff than B. On the other hand, all agents with types such that \( y < y^*(x, m)\) will choose field A, so the probability of this type being in field A is given by the population probability density divided by the mass of field A.

Also, define the following distribution functions:

---

^6In practice, these distributions are not observable, but the two methods of calculating the unobserved means produce identical results.

^7Note that \( f(x, y) \) is assumed to be equal to zero for all values of \((x, y)\) outside the type support.
\[ F^A(x, y)(m) \equiv \int_{\mathcal{X}} \int_{\mathcal{Y}} f^A(t, s)(m) dt ds * \int_y^{y^*(\pi, m)} \int_{x^*(y, m)}^x f(x, y) dx dy \]

\[ F^B(x, y)(m) \equiv \int_{\mathcal{X}} \int_{\mathcal{Y}} f^B(t, s)(m) dt ds * \int_x^{x^*(y, m)} \int_y^{y^*(x, m)} f(x, y) dy dx \]

These distribution functions are equal to the cumulative distribution functions in each field multiplied by the mass of each field, so by construction, \( F^A(x, y)(m) + F^B(x, y)(m) = F(x, y) \). The pair of distribution functions \((F^A(x, y)(m), F^B(x, y)(m))\) describes the distribution of agents in fields A and B resulted from the allocation of agents consistent with a given \( m \).

For a given \( m \), if \( \exists \) an open set of types \( O \subset [\mathcal{X}, \mathcal{Y}] \times [\mathcal{Y}, \mathcal{Y}] \) such that \( \forall (x, y) \in O, y < y^*(x, m) \), then field A has positive mass. If such a subset does not exist, then field A has zero mass. Similarly, if \( \exists \) an open set of types \( P \subset [\mathcal{X}, \mathcal{X}] \times [\mathcal{Y}, \mathcal{Y}] \) such that \( \forall (x, y) \in P, y > y^*(x, m) \), then field B has positive mass.

Let \( m_{y, A}(m) \) be the mean of \( y \) in field A in an allocation consistent with \( m \). When field A has positive mass, this mean can be calculated using the marginal p.d.f. in A with respect to \( y \) denoted by \( f^A_y(x, y)(m) \). If field A contains all agents, \( m_{y, A}(m) \) equals the population mean of \( y \). When field A has zero mass, \( m_{y, A}(m) \) is defined to equal the lowest feasible level of \( y \) in the population. That is, if one agent makes a mistake and goes to field A when it is optimal for all agents including himself to go to B, then the belief of everyone else about his type \( y \) is that it is the lowest possible level. To summarize,

\[ m_{y, A}(m) = \begin{cases} \int_y^{y^*(\pi, m)} y f^A_y(x, y)(m) dy & \text{if field A has positive mass} \\ y & \text{otherwise.} \end{cases} \]

\( m_{x, B}(m) \) is defined in a similar way. Finally, define \( G(m) : \mathcal{M}_x \times \mathcal{M}_y \rightarrow \mathcal{M}_x \times \mathcal{M}_y \) as the mapping from any given \( m \) to a new \( m \): \( G(m) = (m_{x, B}(m), m_{y, A}(m)) \).

**Proposition 1.** \( G(m) \) is a continuous function of \( m \).

**Proof:** See appendix. □
Equilibrium

**Definition 1.** A feasible allocation \((F^A(x,y), F^B(x,y))\) is an equilibrium if for \(m = (E_{FB}(x), E_{FA}(y))\), for each \(i,j = A,B, i \neq j\), if \((x,y) \in T_i\), then \(W^i(x,y) \geq W^j(x,y)\).

In equilibrium, given the field choice of all other agents, no single agent has an incentive to deviate. Also, the agents’ belief about \(m\) is consistent with the value of \(m\) calculated after the sorting of agents takes place. This equilibrium corresponds to a fixed point of the function \(G(m)\). It can be characterized by the fixed point \(m^* = G(m^*)\). The equilibrium allocation of agents is given by \((F^A(x,y)(m^*), F^B(x,y)(m^*))\).

**Existence of equilibrium:**

The set \(M_x \times M_y\) is non-empty, compact and convex, and \(G(m) : M_x \times M_y \rightarrow M_x \times M_y\) is continuous. Using Brauwer’s Fixed Point Theorem, \(\exists m^* \in M_x \times M_y\) such that \(G(m^*) = m^*\), so the function \(G(m)\) has at least one fixed point.

**Example with Specific Linear Wage Function and Uniform Distribution:**

This section considers the environment where the wage functions are given by:

\[
\begin{align*}
   w^A &= x + \alpha m_{y,A} \\
   w^B &= y + \alpha m_{x,B}, \text{ where } \alpha \in [0, \infty).
\end{align*}
\]

It is also assumed that the types of agents are uniformly and independently distributed over the support of types, so \(f(x,y) = \frac{1}{(y-x)(y-x)}\), if \((x,y)\) is in the support of types, and \(= 0\) otherwise.

The indifferent types between the two fields are given by \((x,y)\) that solve: \(y = x - \alpha(m_{x,B} - m_{y,A})\). For any pair of means \(m\), let \(d = m_{x,B} - m_{y,A}\). The set of feasible differences in means is denoted by \(D\). The indifference line between the two fields can be rewritten in terms of \(d\) as: \(y = x - \alpha d\). With linear and symmetric wage functions, \(d\) is sufficient information for agents to be able to choose the field that maximizes their payoffs. Writing \(x - \alpha d\) in place of \(y^*(\cdot)\) and \(y + \alpha d\) in place of \(x^*(\cdot)\) everywhere in the definitions
Figure 3.2: Examples of indifference curves for different values of $d$.

of $m_{y,A}(m)$ and $m_{x,B}(m)$, we obtain these as functions of $d$: $m_{y,A}(d)$ and $m_{x,B}(d)$. Define the function $M(d) : D \rightarrow D$ as $M(d) = m_{x,B}(d) - m_{y,A}(d)$. The function $M(d)$ equals the new difference in the means resulted from the incentive compatible allocation of agents into fields A and B consistent with $d$. $m_{y,A}(d)$ and $m_{x,B}(d)$ are continuous functions of $d$, so their difference is also a continuous function. Figure 3.2 shows three examples of indifference lines for different values of $d$.

The set $D$ is compact and convex. $M(d) : D \rightarrow D$ is a continuous function, and by Brauwer’s Fixed Point theorem, the function $M(d)$ has at least one fixed point. The equilibrium can now be characterized as a value $d^*$ such that $M(d^*) = d^*$. From now on, $d^*$ denotes any equilibrium value of $d$.

**Benchmark Case:** $m_x = m_y$, $\text{var}(x) = \text{var}(y)$

In the benchmark case the two skills are uncorrelated, distributed uniformly, the population means of the two skills are equal, and the variances of the two skills are equal. This implies that $\overline{x} = \overline{y}$ and $\overline{x} - \overline{x} = \overline{y} - \overline{y}$.

Let $z \equiv \overline{x} - \overline{x} = \overline{y} - \overline{y}$. Also, let $c(d) = \alpha d + y - \overline{x}$. $c$ is a useful construct in calculating

---

$^8$I do not include a proof of why these are continuous functions since it is the same as the proof of continuity for $m_{y,A}(m)$ and $m_{x,B}(m)$. 
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$M(d)$, so in what follows, for simplicity, most equations will be expressed as functions of $c(d)$ instead of $d$. In the benchmark case, $c(d) = \alpha d$.

In this case $D = [-z/2, z/2]$ since \( \min(m_{x,B} - m_{y,A}) = x - m_y = x - y - z/2 = -z/2 \), and \( \max(m_{x,B} - m_{y,A}) = m_x - y = x + z/2 - y = z/2 \).\(^9\)

For $c(d) \in [-z, z]$, the mass of field $A = \frac{(z-c(d))^2}{2}$, and the mass of field $B = z^2 - \frac{(z-c(d))^2}{2}$. Note that the mass of field $A$ is strictly decreasing in $c()$, and the mass of field $B$ is strictly increasing in $c()$ for $c(d) \in [-z, z]$. For any other value of $c()$, the masses of these fields are constant, and one of them equals one, and the other equals zero.

The function $M(d)$ is given by:

\[
M(d) = \left\{ \begin{array}{ll}
\frac{z}{2} & \text{if } c(d) > z \\
\frac{z+2c(d)(z^2-(z-c(d))^2)}{4z^2-(z-c(d))^2} & \text{if } c(d) \in [0, z] \\
\frac{z+c(d)}{3} - \frac{z^3}{2z^2-(z+c(d))^2} & \text{if } c(d) \in [-z, 0) \\
-\frac{z}{2} & \text{if } c(d) < -z,
\end{array} \right.
\]

For $c(d) \in [-z, z]$, $\frac{\partial M(d)}{\partial d} > 0$, so $M(d)$ is strictly increasing. For any $c(d) \in [0, z]$, $\frac{\partial^2 M(d)}{\partial d^2} < 0$, so $M(d)$ is strictly concave in this interval. For any $c(d) \in [-z, 0]$, $\frac{\partial^2 M(d)}{\partial d^2} > 0$, so $M(d)$ is strictly convex in this interval. For $c(d) > z$ field $A$ has zero mass and for $c(d) < -z$ field $B$ has zero mass, so in both cases $M(d)$ is a constant. Figure 3.3 illustrates three variations of $M(d)$ functions for different values of $\alpha$.

The function $M(d)$ is continuously differentiable for all values of $c()$, except $c(d) = z$, and $c(d) = -z$. Also, $\frac{\partial m_{x,B}}{\partial d} > 0, \forall c \in [-z, z]$ and $= 0$ otherwise, so the maximum value of $m_{x,B}$ is achieved when $c(d) = z$, i.e. when all agents are in field $B$, so $\max(m_{x,B}) = m_x$. Similarly it can be shown that $\max(m_{y,A}) = m_y$.

**Characterization of Equilibrium:** The number of equilibria and their properties in the benchmark case depend on $\alpha$. $\alpha$ is the relative weight of the unobserved skill relative to the observed skill in the wage function. When this weight is close to zero, agents’

\(^9\)It will be verified later that $\max(m_{x,B}) = m_x$ and $\max(m_{y,A}) = m_y$, for any number $d \in R$, so the set $D$ is specified correctly.
sorting decision will be almost independent of the unobserved means, so we expect agents with $x > y$ to go to field A, and those with $x < y$ to go to field B. Therefore, we expect a unique equilibrium where the masses of the two fields are equal. For higher values of $\alpha$, we expect the sorting of agents to become increasingly sensitive to the difference in the unobserved means. Agents will have a higher incentive to go to the field with the higher unobserved mean. Whichever this field is, it will have an advantage over the other field and it will attract more agents. For $\alpha$ great enough, we expect this advantage to be so high that all agents will go to only one field. Notice that $\alpha$ also determines the size of the externality that each agent entering a field has on the payoffs of the other agents in that field. For any positive mass of agents entering a field, the unobserved talent mean will change. For large $\alpha$, the payoffs of the others change more as a result of this than if $\alpha$ was small. Proposition 2 states this precisely.

**Definition 2.** A **stable equilibrium** is defined as an equilibrium such that if a small mass of agents deviates once from it and goes to the field where its agents obtain a lower payoff, then the unobserved means in each field change in such a way that this mass of agents will find it optimal to use their old equilibrium strategies, and go to the field they were supposed to go in the original equilibrium. In a stable equilibrium, after a small
deviation, all agents will find it optimal to return to their original strategies, and the original equilibrium is restored.

**Proposition 2.**

1. When $\alpha = 0$, $\exists$ a unique equilibrium in which all agents with types such that $x > y$ go to field $A$ and the rest go to field $B$. This is true not only in the benchmark case, but also when $m_y \neq m_x$ and/or $\text{var}(x) \neq \text{var}(y)$.

2. When $\alpha \in (0, \frac{3}{2}]$, $d^* = 0$ is the unique equilibrium. This equilibrium is stable.

3. When $\alpha \in (\frac{3}{2}, 2)$, $\exists$ three equilibria in which both fields have a positive mass of agents: one symmetric unstable equilibrium $d^* = 0$, and two stable non-symmetric equilibria in which both fields have positive masses, and one field is larger than the other.

4. When $\alpha \in [2, \infty)$, $\exists$ three equilibria: one symmetric unstable equilibrium at $d^* = 0$, and two stable non-symmetric equilibria where one of the fields has mass zero.

**Proof:** See appendix. □

**Comparative statics with respect to $\alpha$ in the benchmark case:** This section considers how the equilibria change in response to a change in $\alpha$.

**Proposition 3.** The function $M(d)$ is pointwise increasing in $\alpha$ on the interval $(0, z/2]$, and pointwise decreasing in $\alpha$ on the interval $[-z/2, 0)$.

**Proof:** See appendix. □

**Proposition 4.** As $\alpha$ increases on the interval $(\frac{3}{2}, 2)$, the positive non-symmetric equilibrium value of $d$ increases continuously, and the negative non-symmetric equilibrium value of $d$ decreases continuously. The field with relatively more agents gets increasingly bigger in equilibrium as $\alpha$ increases on this interval.
**Proof:** See appendix. ■

**Discussion:** Consider the equilibrium with a positive difference in means. In such an equilibrium, field B has a higher mass of agents than field A, and a higher mean of the unobserved skill than A. If the return to unobserved skills increases, then it means that agents who were just indifferent between the two fields before will now strictly prefer going to the field with the higher mean of the unobserved type. So agents will leave field A and enter field B, causing $m_{x,B}$ to increase and $m_{y,A}$ to decrease. This leads to an even greater advantage of field B over field A, so it will attract even more agents. This process continues until the difference of means is again such that no agent wants to deviate. It is certain that a new equilibrium is reached because $m_{x,B}(d)$ is increasing and is concave in $d$, while $m_{y,A}(d)$ is decreasing and linear in $d$, so as $d$ increases, $M(d)$ increases by less and less, attracting less and less people into B, until eventually no one wants to deviate anymore. This is because as $d$ increases, each additional agent entering field B will have a smaller positive externality on the payoff of others in B.

If $\alpha$ is very high (i.e. the payoff from the unobserved skill is at least twice as big as the payoff from the observed skill in a field), then the field with the higher mean of the unobserved type will have a sufficiently great advantage over the other field so agents will keep entering until they are all in field B. This is because the externality of each additional agent leaving A and entering B on the payoffs of others is larger when $\alpha$ is high.

Note that in the benchmark case, it is reasonable to assume that $\alpha < 1$. This is because agents who graduate from field $i$ are likely to be hired by firms whose production functions are more intensive in the skill perfectly observed in field $i$, so the return to the revealed talent in $i$ should be higher than the return to the unobservable talent. For example, Math graduates are hired by firms whose output depends more on analytical than verbal skills, so analytical skills have higher returns.
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Figure 3.4: Examples of indifference curves, different values of d

Changes in the population mean of $y$ relative to the population mean of $x$: This section considers the effects of an increase in the population mean of $y$ relative to the mean of $x$ when the variances of the two skills are equal and types are uniformly and independently distributed. To model this effect, I increase $y$ and $\bar{y}$ by equal amounts.

Now, $c(d) = \alpha d + y - \bar{x}$, and $D = [\bar{x} - y - \frac{\bar{x}}{2}, \bar{x} - y + \frac{\bar{x}}{2}]$. Note that $D$ changes as $y$ changes. Also, $m_x - m_y = \frac{\bar{x} + \bar{y}}{2} - \frac{y + \bar{y}}{2} = \frac{2\bar{x} + x - 2y + y}{2} = \bar{x} - y$.

The relevance of $c(d)$: It is particularly useful in this case to write everything in terms of $c(d)$. By construction, when $c(d) = 0$ the indifference line divides the type space into two equal halves along the diagonal. All agents with a higher rank in the population with respect to $x$ will be in field A, and the rest will be in field B. The masses of agents in each field are equal. When $c(d) > 0$, the indifference line is below the diagonal, and field B has a larger mass than field A. Finally, when $c(d) < 0$, the indifference line is above the diagonal, and field A has a larger mass than field B. The masses of agents in each field depend only on $c(d)$. The value of $d$ at which $c(d) = 0$ is equal to $\hat{d} \equiv \frac{\bar{x} - \bar{y}}{\alpha}$. Figure 3.4 shows examples of indifference lines for different values of $d$.

The size of each field is given by the same functions of $c(d)$ as in the benchmark case.
The explicit function $M(d)$ is given in the Appendix of Equations.

Differentiating, I find that $\frac{\partial M(d)}{\partial d} \geq 0$, with equality only for $c(d) > z$ and $c(d) < -z$, so $M(d)$ is weakly increasing. Also, $\frac{\partial^2 M(d)}{\partial d^2} \geq 0$ for $c(d) \in [0, z]$, and $\leq 0$ for $c(d) \in [-z, 0)$. Therefore $M(d)$ is convex for $d < \hat{d}$, and concave for $d > \hat{d}$.

**Characterization of equilibrium:** As in the benchmark case, the characteristics of equilibria depend on $\alpha$. In this case however, the difference in population means $m_y - m_x = \underline{y} - \underline{x} \equiv e$ also plays a role.

**Proposition 5.** If $m_x \leq m_y$, then

1. when $\alpha \in [0, \frac{e - z}{e - z/2}]$ and $e > z$, $\exists$ a unique equilibrium $d^* > \hat{d}$ in which all agents are in field B.

2. when $e > z$ and $\alpha \in (\frac{e - z}{e - z/2}, 1)$ or when $e < z$ and $\alpha \in [0, 1)$, $\exists$ a unique equilibrium $d^* > \hat{d}$, in which both fields have positive masses, but the mass of field B is greater.

3. when $\alpha = 1$, $\exists$ a unique symmetric equilibrium $d^* = \hat{d}$.

4. when $\alpha \in (1, \min\{3/2, \frac{e + z}{e - z/2}\})$, $\exists$ a unique equilibrium $d^* < \hat{d}$ in which both fields have positive masses, but the mass of field A is greater.

5. when $\alpha \in [\frac{e + z}{e + z/2}, 3/2)$, $\exists$ a unique equilibrium $d^* < \hat{d}$ in which all agents go to field A.

6. when $\alpha \in [3/2, \frac{e - z}{e - z/2}]$, multiple equilibria may exist, but none in which all agents are in one field.

7. if $e > \frac{z}{2}$ and $\alpha \in [3/2, \frac{e}{e - z/2}]$, multiple equilibria may exist, one of which is such that all agents are in field A.

8. if $e > \frac{z}{2}$ and $\alpha > \frac{e}{e - z/2}$, $\exists$ a unique equilibrium in which all agents go to field A.
9. if \( e < \frac{z}{2} \) and \( \alpha \in \left[\frac{e+\frac{z}{2}}{e+z/2}, \frac{z-e}{z/2-e}\right] \), multiple equilibria may exist, one of which is such that all agents are in field A.

10. if \( e < \frac{z}{2} \) and \( \alpha > \frac{z-e}{z/2-e} \), \( \exists \) multiple equilibria, two of which are such that all the agents are in one of the two fields.

**Proof:** See appendix. ■

**Comparative statics with respect to the relative population means:** The following analyzes the effects of an increase in the population mean of \( y \) relative to the population mean of \( x \).

**Proposition 6.** For any \( m_y \geq m_x \), as \( m_y \) increases relative to \( m_x \), the function \( M(d) \) decreases pointwise.

**Proof:** See appendix. ■

**Proposition 7.** For any \( m_y \geq m_x \), as \( m_y \) increases relative to \( m_x \), any stable equilibrium with a positive mass of agents in A and B changes as follows: (a) if \( \alpha = 1 \), \( \hat{d} \) remains the unique stable equilibrium and the equilibrium masses of agents in each field remain unchanged; (b) if \( \alpha < 1 \), the equilibrium mass of agents in field A decreases, and the equilibrium mass of field B increases; (c) if \( \alpha > 1 \), the equilibrium mass of agents in field A increases, and the equilibrium mass of field B decreases.

**Proof:** See appendix. ■

**Intuition of Proposition 7:** Let \( S \) be the set of types \( (x, y) \) that are indifferent between the two fields in a stable equilibrium in which both fields have positive masses of agents. These agents’ types satisfy: \( x = y + \alpha(m_{x,B} - m_{y,A}) \). Now, let \( m_y \) increase relative to \( m_x \). This means that every agent’s talent level of \( y \) increases by an amount \( \delta \). If no agent changes fields, then \( m_{y,A} \) increases by \( \delta \) while \( m_{x,B} \) remains constant, so the difference in means decreases by \( \delta \). Agents in set \( S \) will now obtain a payoff from going
to field A equal to \( w^A = x + \alpha(m_y,A + \delta) \), and a payoff from going to field B equal to \( w^B = y + \delta + \alpha m_{x,B} \).

(a) We know from Proposition 5 that when \( \alpha = 1 \), there exists a unique stable equilibrium. From the above, we can see that when \( \alpha = 1 \), the agents in set S will remain indifferent between the two fields since their payoffs from going to A and B remain equal, so in this case no agents have an incentive to change fields as \( m_y \) increases.

(b) If \( \alpha < 1 \), then for agents in set S, \( w^A < w^B \), so they have an incentive to enter B and leave A. When they do so, \( m_{y,A} \) decreases while \( m_{x,B} \) increases, so the difference in means \( d \) increases. This is the externality that agents switching fields have on the payoffs of the other agents. This will attract even more agents in field B. This process continues until equilibrium is restored. In the new equilibrium, the mass of field A will be smaller and the mass of field B will be bigger than previously.

(c) If \( \alpha > 1 \), then for agents in set S, \( w^A > w^B \), so they will enter A and leave B. As a result, \( m_{y,A} \) further increases, while \( m_{x,B} \) decreases. Since the difference in means \( d \) has decreased further, even more agents have an incentive to switch from B to A. This process continues until equilibrium is restored. In the new equilibrium, the mass of field A will be bigger and the mass of field B will be smaller than previously.

**Introducing correlation of types in the population when \( m_x = m_y \) and \( var(x) = var(y) \)**

This section models the effect of introducing some degree of correlation between the two dimensions of skills. The distribution of types in the population is now assumed to be as follows. For \( \beta \) percentage of the population, the rank in the population with respect to \( x \) is equal to the rank with respect to \( y \), so the two types are perfectly correlated. These agents are uniformly distributed on the diagonal of the population support: in other words, the talent level \( y \) is distributed uniformly over the interval \([y, \bar{y}]\), and the talent \( x \) for these agents is given by \( x = y \). The rest \( 1 - \beta \) percentage are distributed
uniformly on the type support. Note that when $\beta > 0$, the population p.d.f. is no longer continuous, and there is a mass of $\beta$ individuals along the diagonal of the support. As will be seen later, this leads to some different results than obtained so far. The explicit function $M(d)$ is given in the appendix of equation. Figure 3.5 shows examples of the function $M(d)$.

$M(d)$ is still a well defined continuous function for $d \in (0, \frac{z}{2})$ and $d \in [-\frac{z}{2}, 0)$. However, for $d = 0$, $M(d)$ is a correspondence. In this case, the indifference line corresponds to the diagonal of the population support where there is a mass $\beta$ of agents. So, the two unobserved means depend on how these indifferent agents sort themselves. When all indifferent agents go to field B, $M(0) = lim_{d \to 0} \frac{z^3 - \frac{1-\beta}{2z^2 - (1-\beta)(z-c(d))^2} \frac{1}{3}(z-c(d))}{\frac{\beta z}{3(1+\beta)}}$.

On the other hand, when all indifferent agents go to field A, $M(0) = lim_{d \to 0} \frac{z + c(d)}{3} - \frac{z^3 - \frac{1-\beta}{2z^2 - (1-\beta)(z+c(d))^2} \frac{1}{3}(z+c(d))}{\frac{\beta z}{3(1+\beta)}} = -\frac{\beta z}{3(1+\beta)}$. Any values in between these are feasible when only a fraction of the indifferent agents go to one field.

As in the benchmark case, for any $\beta \in [0, 1)$, for $\forall d \in (0, min\{z/\alpha, z/2\})$, $M(d)$ is strictly increasing and strictly concave, and for $\forall d \in [max\{-z/\alpha, -z/2\}, 0)$, $M(d)$ is strictly increasing and strictly convex.

**Characterization of equilibrium:** The properties of equilibria depend on $\alpha$ and $\beta$.

**Proposition 8.** 1. When $\beta \in (0,1)$, for $\forall \alpha \in (0, \infty)$, $\exists$ three equilibria: two stable
equilibria where one field has a larger mass of agents than the other, and one non-stable equilibrium where the two fields have equal masses.

2. When $\alpha = 0$ and $\beta \in (0, 1)$, $\exists$ a continuum of equilibria where all agents with types $x > y$ are in field A, all agents with $x < y$ are in field B, and indifferent agents go to either field with a given probability.

3. When $\alpha \in (0, 2)$ and $\beta \in (0, 1)$, the two non-symmetric equilibria are such that there is a positive mass of agents in both fields.

4. When $\alpha \in [2, \infty)$ and $\beta \in [0, 1)$, the two non-symmetric equilibria are such that one field has mass zero.

5. When $\beta = 0$, all the results of the benchmark case apply, and when $\beta = 1$, there are two stable equilibria such that all agents go to one field, and one unstable symmetric equilibrium.

**Proof:** See appendix. ■

**Comparative statics with respect to $\beta$** The following analyzes the effects of increasing the degree of correlation between types in the population.

**Proposition 9.** As $\beta$ increases on the interval $(0, 1)$, for $\alpha \in (0, 2)$, $M(d)$ strictly increases pointwise $\forall d > \hat{d}$, and it strictly decreases pointwise $\forall d < \hat{d}$. As a result, the equilibrium value $d^* > 0$ strictly increases, and the equilibrium $d^* < 0$ strictly decreases, both continuously. The mass of the larger field in a stable equilibrium increases. For $\alpha \geq 2$, the equilibria do not change as $\beta$ increases on the interval $(0, 1)$.

**Proof:** See appendix. ■

**Effects of increasing the variance of $y$ relative to the variance of $x$ when $m_x = m_y$:** This section considers the effects of an increase in the variance of $y$ relative
Figure 3.6: Increasing the variance of $y$.

to the benchmark case. To model this, I increase the length of the support of $y$ relative to $z$. Let $t = \overline{y} - y - z$, so the support of $y$ is greater than $z$ by $t$. In order to leave $m_y$ unchanged, this means that $y = \overline{x} - t/2$, and $\overline{y} = \overline{x} + t/2$. An increase in the the variance of $y$ is now equivalent to an increase in $t$. Figure 3.6 shows this graphically.

\[ D = [x - m_y, m_x - (x - t/2)] = [-\frac{z}{2}, \frac{z + t}{2}] \]

The support of $d$ changes when $t$ changes: the lower bound is not affected because the population means remain unchanged and because $m_{x,B}(d)$ is increasing in $d$ while $m_{y,A}(d)$ is decreasing in $d$ for all $d \in D$, so the minimum feasible difference in means remains $-z/2$. However, the upper bound increases since the lowest feasible $m_{y,A}$ decreases in $t$.

Also, $c(d) = \alpha d + y - x = \alpha d - t/2$. As $t$ increases, the value of $d$ that corresponds to $c(d) = 0$ increases and the value of $d$ that corresponds to $c(d) = -t$ decreases. Note that $c(d) = 0$ no longer corresponds to the indifference line that divides the type space in two equal masses. The explicit function $M(d)$ is given in the Appendix of Equations.

When $c(d) \in [-t, z]$, $\frac{\partial M(d)}{\partial d} > 0$ and $\frac{\partial^2 M(d)}{\partial d^2} < 0$, so $M(d)$ is strictly increasing and strictly concave in this interval, $\forall t$. When $c(d) \in [-(z + t), -t]$, $\frac{\partial M(d)}{\partial d} > 0$ and $\frac{\partial^2 M(d)}{\partial d^2} > 0$, so $M(d)$ is strictly increasing and strictly convex. Also, it was verified that $M(d)$ is continuously differentiable everywhere except when $c(d) = z$ and when $c(d) = -(z + t)$. 


Figure 3.7: Examples of $M(d)$ functions

When $c(d) > z$ and when $c(d) < -(z + t)$, $M(d)$ is constant and one of the fields has a mass of zero.

Characterization of equilibrium: The properties of equilibria depend on $\alpha$ and $t$.

Proposition 10. 1. When $\alpha \in (0, \max\{\frac{3}{2}, \frac{t}{z}\}]$, $\exists$ a unique stable equilibrium.

2. When $\alpha \in [\max\{\frac{3}{2}, \frac{t}{z}\}, \infty)$, multiple equilibria may exist.

3. When $\alpha \geq (2z + t)/(z + t)$, $\exists$ a stable equilibrium where field A is empty.

4. When $\alpha \geq 2 + t/z$, $\exists$ a stable equilibrium where field B is empty.

Proof: See appendix. ■

Effect of increasing $t$:

Proposition 11. As $t$ increases on the interval $(0, 2x)$, the function $M(d)$ increases pointwise $\forall d \in D$.

Proof: See appendix. ■
Proposition 12. As $t$ increases on the interval $(0, 2x)$, any stable equilibrium with positive masses of agents in both fields changes as follows: the equilibrium mass of agents in field A decreases, and the equilibrium mass of agents in field B increases. The equilibrium difference in unobserved means $d^*$ associated with this equilibrium increases.

Proof: See appendix. ■

Discussion: Refer to Figure 3.6. To see the intuition for this result, start from the benchmark case and consider any stable equilibrium. Let $S$ be the set of types in this population that are indifferent between the two fields.

Now, increase the population by adding two equal masses of agents uniformly and independently distributed, the first over the support $[\frac{x}{y-t}, y]$, and the second over support $[\frac{x}{y+\frac{1}{2}}]$. The effect of adding these new agents to the population is that the variance of $y$ increases, while the mean of $y$ in the population remains constant.

As the variance of $y$ increases, the first order effect is that for any given indifference line between the two fields, $m_{x,B} - m_{y,A}$ increases. $m_{x,B}$ always increases as the new agents enter field B. This is because field B attracts more new agents in the set with high $y$ who also have an average of $x$ above the initial $m_{x,B}$. $m_{y,A}$ always decreases as new agents enter field A because field A attracts more new agents in the set with very low levels of $y$. As a result of the change in $m_{x,B} - m_{y,A}$, agents in set $S$ will be strictly better off in field B, so they leave field A and enter field B. As a result, $m_{y,A}$ decreases even more, while $m_{x,B}$ increases even more. This is the second order effect. This process continues until equilibrium is restored.

An interpretation for this is that since the variance of the talent perfectly revealed in field B increases relative to the variance of the other talent, field B becomes more useful in the sense that it reveals the skill for which there is more uncertainty. When agents with very low levels of $y$ enter field A, they have a negative externality on the payoffs of the rest of agents in field A since they decrease the expected level of $y$ in A. So, agents in A with the highest levels of $y$ will find it optimal to distinguish themselves from the
rest by entering field B and revealing their $y$.

For example, if the population experienced an inflow of foreign students, some with very high verbal skills, and some with very low, the model predicts that more domestic students will join verbal intensive fields to distinguish themselves from the rest. It becomes less important for domestic students to distinguish themselves based on analytical ability, since everybody’s analytical skills are relatively tightly distributed around the mean of analytical skills of people in the verbal intensive fields.

**Note:** So far, we do not know the effect of increasing the variance of $y$ on the relative sizes of the two fields. Even though agents from the initial population leave field A and enter field B, if enough new agents enter field A, then its relative size may increase. So, in the example above, the analytical intensive field might have a higher percentage of the new total population after most foreign students with low verbal skills have entered it.

We can also interpret the increase in variance as a spread of the existing agents over a wider interval. In this case, we have to first take into account that as we spread the agents more thinly over a bigger support, some agents will have their types changed in such a way such that for a given $d$, they will switch fields. Then, as the equilibrium $d$ increases, some of these agents, (or all of them and even more) will switch back, so it is ambiguous if the overall effect is that people join or leave field A. To find the answer to this question, we need to consider what happens to the relative sizes of the two fields in equilibrium.

When $c(d) > 0$, the proportion of agents in field A is $\frac{(z-c(d))^2/2}{z(z+t)}$. Using implicit differentiation and denoting the level of $c(d)$ that leaves this ratio constant when $t$ changes by $\hat{c}(d,t)$, we obtain $\frac{\partial \hat{c}(d,t)}{\partial t} = \frac{(z-c(d))}{2(z+t)}$. When $-t < c(d) < 0$, the proportion of agents in field A is $\frac{-c(d)z+z^2/2}{z(z+t)}$, and $\frac{\partial \hat{c}(d,t)}{\partial t} = \frac{c(d)-z/2}{z+t}$.

Because in equilibrium $c^*(d,t) = \alpha d^*(t) - \frac{1}{2} t$, then $\frac{\partial c^*(d,t)}{\partial t} = \alpha \frac{\partial d^*(t)}{\partial t} - \frac{1}{2}$. If this is

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10I am only considering the equilibrium where $d^* > 0$, so I am not interested in the case in which $c(d) < -t$. 
larger than \( \frac{\partial c(d,t)}{\partial t} \), then the fraction of agents in field A decreases, and if it is smaller, than the fraction of agents in field A increases. It is uncertain which is the case in general. \(^{11}\)

**Proposition 13.** When \( \alpha \in (3/2, \frac{2z+t}{z+t}) \) which is non empty if \( t < 3z \), as the variance of \( y \) increases, then the stable equilibrium proportion of agents in field A decreases.

**Proof:** See appendix. ■

**Proposition 14.** For any \( \text{var}(y) > 0 \), as \( \text{var}(x) \rightarrow 0 \), the equilibrium mass of agents in field B approaches 1 iff \( \alpha \geq 1 \).

**Proof:** See appendix. ■

**Proposition 15.** If \( \text{var}(y) > 0 \) and \( \alpha < 1 \), then as \( \text{var}(x) \rightarrow 0 \), the equilibrium mass of agents in field A approaches \( \frac{1-\alpha}{2-\alpha} \).

**Proof:** See appendix. ■

**General Results**

The results obtained using the uniform distribution example are difficult to generalize for any other distributions. However, maintaining the linear symmetric wage functions used in the example above, we can obtain some results for more general distributions.

**Proposition 16.** For any continuous and bounded distribution of types, if \( \alpha = 0 \), there exists a unique equilibrium allocation of agents in which all agents with types \( (x,y) \) such that \( x > y \) are in field A, and the rest are in field B. This equilibrium is stable.

**Proof:** The payoffs are given by: \( W^A(x,y) = x \) and \( W^B(x,y) = y \). Comparing agents’ payoffs from going to each field, the result stated in the proposition is obvious.

\(^{11}\) We can calculate \( \frac{\partial c^*(d,t)}{\partial t} \), but it does not simplify to something that we can directly compare with \( \frac{\partial c(d,t)}{\partial t} \).
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Proposition 17. For any symmetric bounded distribution \( f(x, y) \) over a support of types such that \( x = y \), and \( \bar{x} = \bar{y} \), \( \exists \) a symmetric equilibrium in which all agents with types such that \( x > y \) go to field A, and the rest go to field B.\(^{12}\)

Proof: See appendix. ■

Proposition 18. For any distribution of types, when \( \frac{x}{y} > 1 \) and \( \alpha \leq \frac{\bar{x} - \bar{y}}{\bar{x} - \bar{y}} < 1 \) or when \( \frac{y}{x} > 1 \) and \( \alpha \geq \frac{\bar{y} - \bar{x}}{\bar{y} - \bar{x}} > 1 \), \( \exists \) a unique equilibrium in which field A contains all agents. When \( \frac{y}{x} > 1 \) and \( \alpha \leq \frac{\bar{y} - \bar{x}}{\bar{y} - \bar{x}} < 1 \) or when \( \frac{x}{y} > 1 \) and \( \alpha \geq \frac{\bar{x} - \bar{y}}{\bar{x} - \bar{y}} > 1 \), \( \exists \) a unique equilibrium in which field B contains all agents.

Proof: See appendix. ■

Proposition 19. For any symmetric finite type support, as \( \alpha \to \infty \), the only stable equilibria are such that one field contains all agents.

Proof: See appendix. ■

Proposition 20. When \( \alpha = 1 \), any equilibrium allocation of agents remains unchanged when all agents’ types increase by a constant in one of the two dimensions.

Proof: The proof is identical to the proof of Proposition 7(a).

Simulation Results: Normally Distributed Types

This section considers the same effects studied before when skills are normally distributed. The simulation results indicate that the effects of changes in \( \alpha \), changes in relative population means, and changes in the relative population variances of the two talent dimensions are the same as when skills are uniformly distributed.

For equal population type means, equal variances, and no correlation, Figure 3.8 shows the function \( M(d) \) for three different values of \( \alpha \). Note that as in the uniform

\(^{12}\)It is assumed that \( f(x, y) = 0 \) outside the type support
distribution case, $M(d)$ is convex for $d < 0$, and concave for $d > 0$. As $\alpha$ increases, $M(d)$ decreases pointwise for $d < 0$, and increases pointwise for $d > 0$. For low values of $\alpha$, there exists a unique equilibrium, while for higher values, multiple equilibria will exist.

![Figure 3.8: Simulated $M(d)$ for different values of $\alpha$](image)

Figure 3.8 shows the effects of increasing the population mean of $y$, when $\alpha = 1$, variances are equal, and there is no correlation between skills. For this value of $\alpha$, there exists a unique equilibrium given the rest of the parameters chosen for this simulation. Similar to the uniform distribution case, as the mean of $y$ increases, the unique equilibrium value of $d$ decreases.

Figure 3.9 shows the change in the equilibrium value of $d$ as the variance of $y$ increases, for equal population means, $\alpha = 1$, no correlation between skills, and fixed variance of $x$. As in the uniform distribution case, the unique equilibrium value of $d$ increases as the variance of $y$ increases. Figure 3.11 plots the relative sizes of fields A and B as the variance increases, showing that the mass of field A decreases. This result remains robust even for very low values of $\alpha$. 
Figure 3.9: Equilibrium value of $d$ as the mean of $y$ increases

Figure 3.10: Equilibrium value of $d$ as the variance of $y$ increases
3.3.3 Model 2

In this section, I consider the implications of altering the environment so that firms become active players and face an inference problem regarding agents’ unobserved skills, and the population talent distribution, \( F(x, y) \) is now observable. Firms hire all agents after they complete a degree. Firms belong to one of two sectors, and each sector hires graduates from one field. Firms observe the chosen field of study of all agents and their talent levels revealed by that field. Wages are determined in a competitive labor market.

The wages of agents with degrees in fields A and B and types \((x, y)\) are given by:

\[
W^A(x, y) = w^A(x, E[y|x, A]),
\]
\[
W^B(x, y) = w^B(E[x|y, B], y).
\]

The payoff of an agent going to field A depends on the skill revealed by field A, \(x\), and on the expected level of the unrevealed skill, \(y\). Similarly, the payoff of an agent going to field B depends on the revealed skill, \(y\), and on the expected level of the unrevealed skill,
x. Since $F(x, y)$ is assumed to be observable in Model 2, the expected unrevealed talent level is conditional on both the field chosen and the revealed type.

Maintaining the assumption of linear wages, the payoff of an agent with type $(x, y)$ in A and B can be rewritten as:

\[
W^A(x, y) = x + \alpha E[y|x, A], \\
W^B(x, y) = y + \alpha E[x|y, B].
\]

First, I consider the equilibrium properties. Then I compare the comparative statics results in this environment with the previously obtained results.

**Environment with Uniform Distribution:**

**Equilibrium** In this section, I only consider Nash Equilibria of the type such that there exists a monotone indifference curve between the two fields. As before, the indifference curve is allowed to take values outside the support of types.

Similar to the previous model, let $x^*(y)$ and $y^*(x)$ be the functions of the indifference curve. For any level $y$, the function $x^*(y)$ tells us the type $x^*$ such that the agent with type $(x^*, y)$ is indifferent between the two fields.

When this function is monotone, the Nash equilibrium strategies are: all agents with type $x \in [x, x^*(y)]$ go to one field, while the agents with type $x \in [x^*(y), \bar{x}]$ go to the other field. Given these strategies, when types are uniformly and independently distributed, the payoff functions of an agent with type $(x, y)$ are:

\[
W^A(x, y) = x + \alpha \left[\frac{y + y^*(x)}{2}\right], \\
W^B(x, y) = y + \alpha \left[\frac{x + x^*(y)}{2}\right].
\]

The types of agents $(x^*, y^*)$ that are indifferent between the two fields satisfy the indifference condition:

\[
x^* + \alpha \left[\frac{y + y^*(x^*)}{2}\right] = y^* + \alpha \left[\frac{x + x^*(y^*)}{2}\right] \Leftrightarrow \\
(1 - \frac{\alpha}{2})x^* = (1 - \frac{\alpha}{2})y^* - \frac{\alpha}{2}(y - \bar{x})
\]
Proposition 21. In any Nash Equilibrium with monotone indifference curve, the equilibrium strategies are: all agents with types \((x, y)\) such that \(x < x^*(y)\) go to field B, and the rest go to field A, where \(x^*(y) = [(1 - \frac{\alpha}{2}) y - \frac{\alpha}{2} (y - x)] / (1 - \frac{\alpha}{2})\).

Proof: See appendix. ■

Characterization of equilibrium:

Proposition 22. 1. If \(\alpha \neq 2\), and if \(\alpha [\frac{x + y}{2} - y] < \overline{x} - y\) and \(\alpha [\frac{y + x}{2} - x] < \overline{y} - x\), there exists a unique Nash Equilibrium with monotone indifference curve in which both fields have positive masses. Equilibria where either field has zero mass do not exist.

2. If \(\alpha \neq 2\), and if \(\alpha [\frac{x + y}{2} - y] \geq \overline{x} - y\), there exists a Nash Equilibrium in which all agents go to field B. If \(\alpha [\frac{y + x}{2} - x] \geq \overline{y} - x\), there exists a Nash Equilibrium in which all agents go to field A.

3. If \(\alpha = 2\), and if \(\overline{y} = \overline{x}\), there is an infinite number of Nash Equilibria with monotone indifference curve where both fields have positive mass, and also all agents going to one field is a Nash Equilibrium.

4. If \(\alpha = 2\), and if \(\overline{y} \neq \overline{x}\), then there does not exist an equilibrium with monotone indifference curve where fields have positive masses; if \(y > \overline{x}\), then all agents going to field A is an equilibrium, and if \(y < \overline{x}\), then all agents going to field B is an equilibrium.

Proof: See appendix. ■

Benchmark Case: uniform distribution, \(m_x = m_y\), \(\text{var}(x) = \text{var}(y)\)

In this section, I reconsider the benchmark case environment. Since \(\overline{y} = \overline{x}\), when \(\alpha \neq 2\) the indifference curve equation simplifies to \(x^*(y) = y\). The indifference curve is the 45 degree line, which implies that the two fields have equal masses of agents.
Changes in $\alpha$: The unique monotone equilibrium indifference curve does not depend on the value of $\alpha$, as long as $\alpha \neq 2$. This differs from Model 1 where multiple equilibria with monotone indifference curves existed for values of $\alpha \in (\frac{3}{2}, 2)$. The conditions for extreme equilibria to exist can be simplified for the benchmark case to $\alpha \geq 2$. As in Model 1, when $\alpha \in (2, \infty)$, there exist at least three equilibria: the symmetric equilibrium where $x^*(y) = y$, and two extreme equilibria where all agents go to one of the two fields. When $\alpha \in (0, 2)$, no extreme equilibria exist, as in Model 1.

Changes in the population mean of $y$ relative to the population mean of $x$

Proposition 23. For any $m_y \geq m_x$, as $m_y$ increases relative to $m_x$, then in the Nash Equilibrium with monotone indifference curve: (1) if $\alpha \in [0, 1)$, the equilibrium mass of field A decreases while the equilibrium mass of field B increases; (2) if $\alpha = 1$, the equilibrium allocation of agents remains unchanged; (3) if $\alpha \in (1, 2)$, the equilibrium mass of field A increases while the equilibrium mass of field B decreases; (4) if $\alpha = 2$, no such equilibrium exists; and (5) if $\alpha \in (2, \infty)$, the equilibrium mass of field A decreases.

Proof: See appendix. ■

Discussion: Cases (1), (2) and (3) predict the same effects as Model 1, and the intuition remains the same. However, when $\alpha \geq 2$, this model leads to very different results. There is a discontinuity at $\alpha = 2$, and when $\alpha > 2$, while Model 1 predicted that the equilibrium mass of field A increases as $m_y$ increases relative to $m_x$, Model 2 predicts the opposite.

Effects of increasing the variance of $y$ relative to the variance of $x$ when $m_x = m_y$: I let $t = \bar{y} - \bar{y} - z$, and I model the increase in variance as an increase in $t$. As before, $\bar{y} = \bar{x} - t/2$, and $\bar{y} = \bar{x} + t/2$. The function giving the indifferent types simplifies to:

$$(1 - \frac{\alpha}{2})x^*(y) = (1 - \frac{\alpha}{2})y + \frac{\alpha}{4}.$$
When $\alpha \neq 2$, this further simplifies to $x^*(y) = y + \frac{\alpha - 1}{2 - \alpha} t$.

**Proposition 24.** When $\alpha \in (0, 2)$, as $t$ increases in the interval $(0, 2\bar{x})$, the equilibrium indifference curve shifts down, and the equilibrium mass of agents in field $A$ decreases. When $\alpha \in (2, \infty)$, as $t$ increases in the interval $(0, 2\bar{x})$, the equilibrium indifference curve shifts up, and the equilibrium mass of agents in field $A$ increases.

**Proof:** See appendix. ■

The result obtained for $\alpha \in (0, 2)$ is the same as in Model 1. However, the results reverse for values of $\alpha \in (2, \infty)$.

To summarize, the results obtained in this version of the model are qualitatively identical to those obtained in Model 1 for $\alpha < 2$. Since it is reasonable to assume that $\alpha < 2$, this strengthens the results of the paper.

**More General Distribution**

In order to see whether the equilibrium properties found for the uniform distribution case in this model apply to more general distributions, I first explore the case in which the two skills are independently and identically distributed with probability density $g()$.

The payoffs of an agent with type $(x, y)$ in the two fields are:

$$W^A(x, y) = x + \alpha \frac{\int_{y}^{x^*} y g(y) dy}{\int_{y}^{x^*} g(y) dy},$$

$$W^B(x, y) = y + \alpha \frac{\int_{y}^{x^*} x g(x) dx}{\int_{x}^{x^*} g(x) dx}.$$

The indifference condition is:

$$x^* + \alpha \frac{\int_{y}^{x^*} y g(y) dy}{\int_{y}^{x^*} g(y) dy} = y^* + \alpha \frac{\int_{x}^{x^*} x g(x) dx}{\int_{x}^{x^*} g(x) dx}$$

From this, it is clear that when $y = x$, one equilibrium indifference function is $x^*(y) = y$. This equilibrium is the same as in the uniform distribution benchmark case.
Proposition 25. With the types of distributions specified above, for $\alpha \in (0, 1]$, there exists at most one Nash Equilibrium with monotone indifference curve. For $\alpha > 2$, more than one such Nash Equilibrium may exist.

Proof: See appendix. ■

The equilibrium properties of the model with more general distributions of talents differ from those in the uniform case. While there exists a symmetric Nash Equilibrium when the two talent dimensions are independent of each other and have the same distribution, this equilibrium may not be unique, depending on the value of $\alpha$.

3.4 Conclusion

In conclusion, this paper provides a simple signaling model where the allocation of talent into two fields of study can be analyzed. The paper presents results for two model versions, first assuming that the distribution of skills is unobservable, and second assuming that this distribution is observed, allowing firms to better predict students’ unrevealed skills in each field. I show how the size of each field and the average talent it attracts depend on the average skill levels, on skill dispersion and on the degree of correlation between skills in the student population. Most analytical results are specific to the uniform distribution example studied, however, simulations performed using normally distributed skill types generate similar qualitative results. While this paper is limited to the study of two fields and two skills, the same intuition of the main results is likely to apply to a more realistic model with more than two fields of study where each field reveals some information about both skills to different degrees. Future empirical research studying the sorting decisions of students could be used to test some general predictions.
3.5 Appendices

3.5.1 Appendix of Equations

The function $M(d)$ in Model 1, benchmark case:

$$M(d) = \begin{cases} \frac{z^2}{2} & \text{if } c(d) > z \\ \frac{z^2 - (z - c(d))^2}{2z^2 - (z - c(d))^2} & \text{if } c(d) \in [0, z] \\ \frac{z + c(d)}{3} - \frac{z^3 - \frac{1}{3}(2z - c(d))(z + c(d))^2}{2z^2 - (z + c(d))^2} & \text{if } c(d) \in [-z, 0] \\ -\frac{z^2}{2} & \text{if } c(d) < -z, \end{cases}$$

where:

$$\begin{align*}
&c(d) > z \Rightarrow d \in (z/\alpha, z/2] \\
&c(d) \in [0, z] \Rightarrow d \in [0, \min\{z/\alpha, z/2\}] \\
&c(d) \in [-z, 0] \Rightarrow d \in [\max\{-z/\alpha, -z/2\}, 0) \\
&c(d) < -z \Rightarrow d \in [-z/2, -z/\alpha) 
\end{align*}$$

The function $M(d)$ in Model 1, $m_y > m_x$:

$$M(d) = \begin{cases} x - y + \frac{z}{2} & \text{if } c(d) > z \\ x - y + \frac{z^2 - (z - c(d))^2}{2z^2 - (z - c(d))^2} & \text{if } c(d) \in [0, z] \\ x - y + \frac{z + c(d)}{3} - \frac{z^3 - \frac{1}{3}(2z - c(d))(z + c(d))^2}{2z^2 - (z + c(d))^2} & \text{if } c(d) \in [-z, 0] \\ x - y - \frac{z}{2} & \text{if } c(d) < -z, \end{cases}$$

where,

$$\begin{align*}
&c(d) > z \Rightarrow d \in [(z + x - y)/\alpha, x - y + z/2] \equiv D_1; \\
&c(d) \in [0, z] \Rightarrow d \in [\frac{x - y}{\alpha}, \min\{x - y + \frac{z}{2}, \frac{x - y + z}{\alpha}\}] \equiv D_2; \\
&c(d) \in [-z, 0] \Rightarrow d \in [\max\{x - y - \frac{z}{2}, \frac{-z + x - y}{\alpha}, \frac{x - y}{\alpha}\}, \frac{x - y}{\alpha}] \equiv D_3;
\end{align*}$$
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\[ c(d) < -z \Rightarrow d \in \left[ x - y - \frac{z}{2}, -\frac{z + x - y}{\alpha} \right] \equiv D4. \]

Note that depending on \( \alpha \) and \( x - y \), some of these sets may be empty.\(^\text{13}\)

The function \( M(d) \) for Model 1, positive correlation:

For \( \beta \in [0, 1) \), \( M(d) \) is given by:

\[
M(d) = \begin{cases} 
\frac{z}{2} & \text{if } c(d) > z \\
\frac{z^3 - \frac{1}{2}(z+c(d))(z-c(d))^2}{2z^2 - (1-\beta)(z-c(d))^2} - \frac{1}{3}(z - c(d)) & \text{if } c(d) \in [0, z] \\
\frac{z+c(d)}{3} & \text{if } c(d) \in [-z, 0) \\
\frac{-z}{2} & \text{if } c(d) < -z,
\end{cases}
\]

where the intervals of \( d \) corresponding to each of these intervals are the same as in
the benchmark case.

The function \( M(d) \) in Model 1, \( var(y) > var(x) \):

\[
M(d) = \begin{cases} 
\frac{z + t}{2} & \text{if } c(d) > z \\
\frac{z^2(z+t)-(2z+c(d))(z-c(d))^2/3}{2z(z+t)-(z-c(d))^2} + \frac{t}{2} - \frac{z-c(d)}{3} & \text{if } c(d) \in [0, z] \\
\frac{z(t+c(d))+z^2/3}{z+2(t+c(d))} + \frac{t}{2} - \frac{(c(d)^2+z^2/3-zc(d))}{zt-2c(d)t} & \text{if } c(d) \in (-t, 0) \\
\frac{1}{3}(z + t + c(d)) + \frac{t}{2} - \frac{z^2(z+t)-\frac{1}{2}(z+t+c(d))^2(2z+2t-c(d))}{2z^2+2zt-(z+t+c(d))^2} & \text{if } c(d) \in [-z + t), -t] \\
\frac{-z}{2} & \text{if } c(d) < -(z + t),
\end{cases}
\]

where

\[ c(d) > z \Rightarrow d \in \left[ \frac{z + t/2}{\alpha}, \frac{z + t}{2} \right] \equiv E1 \]

\[ c(d) \in [0, z] \Rightarrow d \in \left[ \frac{t}{2\alpha}, \min\left\{ \frac{z + t/2}{\alpha}, \frac{z + t}{2} \right\} \right] \equiv E2 \]

\[ c(d) \in (-t, 0) \Rightarrow d \in \left( \max\left\{ -\frac{t}{2\alpha}, -\frac{z}{2} \right\}, \min\left\{ \frac{t}{2\alpha}, \frac{z + t}{2} \right\} \right) \equiv E3 \]

\(^{13}\)The values of \( d \) corresponding to each bound of the set of \( c(d) \) are found using the definition of \( c(d) \).
\[ c(d) \in [-(z + t), -t] \Rightarrow d \in \left[ \max\left\{ -\frac{z + t/2}{\alpha}, -\frac{z}{2} \right\}, -\frac{t}{2\alpha} \right] \equiv E4 \]

\[ c(d) < -(z + t) \Rightarrow d \in \left[ -\frac{z}{2}, -\frac{z + t/2}{\alpha} \right] \equiv E5. \]

### 3.5.2 Appendix of Proofs

**Proof of Proposition 1.** I show that \( m_{x,B}(m) \) and \( m_{y,A}(m) \) are continuous in \( m \).

Fix \( y = \bar{y} \) and \( x = \bar{x} \), so these become parameters in the indifference condition. Since \( w^j \) is a differentiable function and has non-negative partial derivatives, the inverse function \( (w^j)^{-1} \) exists. So, taking the inverse, obtain the inverse function: \( m^*_{x,B}(m_{y,A}) = (w^B)^{-1}(w^A(m_{y,A}; \bar{x}); \bar{y}) \). This function gives all the pairs of means such that the agent with type \((\bar{x}, \bar{y})\) is indifferent between the two fields. Since \( w^j \) is continuous and increasing, the inverse, \((w^j)^{-1}\) is also continuous and increasing in its argument.

For any \( m \) such that: (1) \( m_{x,B} < m^*_{x,B}(m_{y,A}), w^A(m_{y,A}; \bar{x}) > w^B(m_{x,B}; \bar{y}) \) \( \Rightarrow \) the agent with type \((\bar{x}, \bar{y})\) obtains a strictly greater payoff when going to field A; (2) \( m_{x,B} > m^*_{x,B}(m_{y,A}), w^A(m_{y,A}; \bar{x}) < w^B(m_{x,B}; \bar{y}) \) \( \Rightarrow \) the agent with type \((\bar{x}, \bar{y})\) obtains a strictly greater payoff when going to field B; (3) \( m_{x,B} = m^*_{x,B}(m_{y,A}), w^A(m_{y,A}; \bar{x}) = w^B(m_{x,B}; \bar{y}) \) \( \Rightarrow \) the agent with type \((\bar{x}, \bar{y})\) is indifferent between field A and B.

Let \( \mathcal{M}_1 \) be the set of all \( m \) such that (1) holds, \( \mathcal{M}_2 \) be the set of all \( m \) such that (2) holds, and \( \mathcal{M}_3 \) be the set of all \( m \) such that (3) holds. For any \( m \in \mathcal{M}_1 \), field A has a positive mass since \( \exists \) an open interval around \((\bar{x}, \bar{y})\) such that the payoffs of these agents are greater in field A. So, \( m_{y,A}(m) \) is calculated as the mean of \( y \) for these agents. This is continuous in \( m \) since the functions \( x^* \) and \( y^* \) are continuous and since \( f(x, y) \) is assumed to be continuous. For any \( m \in \mathcal{M}_2 \), \( m_{y,A}(m) = \bar{y} \). For \( m_{y,A}(m) \) to be continuous for all \( m \), it remains to be shown that it is continuous for all \( m \in \mathcal{M}_3 \). Let \( \hat{m} \) be any element of \( \mathcal{M}_3 \).

Any \( m \) close to \( \hat{m} \) falls in one of the three sets: \( \mathcal{M}_1, \mathcal{M}_2 \) or \( \mathcal{M}_3 \). I will show that given \( \epsilon > 0, \exists \Delta > 0 \) such that \( |m - \hat{m}| < \Delta \Rightarrow |m_{y,A}(m) - m_{y,A}(\hat{m})| < \epsilon \).
∀ m ∈ M3 and ∀ m ∈ M2, |m_y,A(m) − m_y,A(∗m)| = y − 2y = 0 < ϵ.

∀m ∈ M1, we can simply take the limit as m → ∗m: lim_{m→∗m} \int_y y f_y(x,y)(m)dy = y.  
This implies that ∃δ > 0 such that |m − ∗m| < δ ⇒ |m_y,A(m) − m_y,A(∗m)| < ϵ. This completes the proof that m_y,A(m) is continuous.

Similarly, it can be shown that m_x,B(m) is continuous. From this, it follows that G(m) is continuous in m. ■

Proof of Proposition 2.

(1) When α = 0, W^A(x,y) = x and W^B(x,y) = y. Clearly, all agents with types such that x > y obtain a greater payoff when choosing field A, while the rest obtain a greater payoff when choosing field B. In the special benchmark case, this equilibrium is characterized by d^* = 0.

(2) M(0) = 0, ∀α ∈ (0, ∞), so for any value of α one fixed point of M(d) is d^* = 0. This is the symmetric equilibrium in which all agents with types x > y go to field A, and all agents with types x < y go to field B.

\frac{∂M(d)}{∂d}|_{d=0} = \frac{2}{3}α. This is ≤ 1 if α ≤ \frac{3}{2}, in which case M(d) crosses the 45 degree line from above at d = 0. It can be verified that M(d) < d for any d > 0, and M(d) > d for any d < 0. Hence, d^* = 0 is a unique equilibrium when α ∈ (0, \frac{3}{2}].

Also, it is a stable equilibrium. M(d) crosses the 45 degree line from above when α ∈ (0, \frac{3}{2}). So, if a small mass of agents deviates once from the equilibrium, and goes to field A when it is optimal to go to field B, the equilibrium d^* = 0 will be restored. This is because the difference in means will increase or decrease as a result of the deviation. Supposing it increases to d > 0, then even more agents will go to field B. But M(d) < d, so the new difference in means will be smaller. As a result, agents will exit field B and enter A, causing the difference in means to decrease. This process continues until d = 0 and equilibrium is restored.

14 This is found by applying L’Hopital’s rule twice.
(3) For $\alpha \geq \frac{3}{2}$, $\exists$ three equilibria. $\frac{\partial M(d)}{\partial d}|_{d=0} - 1 = \frac{2}{\beta} \alpha - 1 > 0$ when $\alpha > \frac{3}{2}$, so the equilibrium $d^* = 0$ is no longer stable. In this case, $M(d)$ crosses the 45 degree line at $d^* = 0$ from below.

I now show that $\exists$ a unique stable fixed point of $M(d)$ in $(0, \frac{z}{2}]$. Since $M(d)$ is strictly increasing and it crosses the 45 degree line from below at $d = 0$, $\exists \epsilon > 0$ where $\epsilon$ can be very small, such that $M(\epsilon) > \epsilon$, and since $M(d)$ is increasing, this implies that $\forall d \in (0, \frac{z}{2}]$, $M(d) \in (0, \frac{z}{2}]$. Since $M(d)$ is continuous on this interval, Brauer's Fixed Point Theorem implies that $\exists$ a fixed point of $M(d)$ in $(0, \frac{z}{2}]$. Since $M(\epsilon) > \epsilon$, for $\epsilon$ very small, $M(d)$ crosses the 45 degree line from above at least once in $(0, \frac{z}{2}]$, so at least one equilibrium is stable. If $\alpha \leq 2$, $M(d)$ is strictly concave on $(0, \frac{z}{2}]$, so it can cross the 45 degree line only once.

If $\alpha < 2$, all equilibria are such there are positive masses of agents in both fields. If one field has zero mass, then $M(d) = \frac{z}{2}$ or $-\frac{z}{2}$, so if $d^*$ does not equal these, then there is a positive mass of agents in both fields. It can be shown that $M(z/2) < z/2$, which implies that for all $d \in (0, z/2)$, $M(d) \in (0, z/2)$. Applying Brauer's Fixed Point theorem to this interval, we obtain that there exist a fixed point in $(0, z/2)$, so the fixed point is smaller than $z/2$. This means that the non-symmetric equilibrium $d^* > 0$ is such that there is a positive mass of agents in both fields.

(4) When $\alpha > 2$, the sets $[z/\alpha, z/2]$ and $[-z/2, -z/\alpha]$ are not empty. For values of $d$ in these intervals, there is no type $(x, y)$ on the indifference line that is in the interior of the type support. This means that all agents will find it optimal to be in only one of the two fields. For any such values of $d$, $M(d)$ is constant. These equilibria are $d^* = \frac{z}{2}$ in which field A has zero mass, and $d^* = -\frac{z}{2}$ in which field B has zero mass. There are no other equilibria on $(0, \frac{z}{2}]$ since $M(d)$ is strictly concave on $(0, \frac{z}{\alpha})$ and constant on $[\frac{z}{2}, \frac{z}{2}]$, so $M(d)$ can cross the 45 degree line only once. The same can be shown for the interval $[\frac{z}{2}, 0)$. ■
Proof of Proposition 3. $\frac{\partial M(d)}{\partial \alpha} = d \frac{\partial M(d)}{\partial c(d)}$.

When $\alpha \in (0, 2)$, $\frac{\partial M(d)}{\partial c} > 0, \forall d \in D \Rightarrow \frac{\partial M(d)}{\partial \alpha} > 0$ for $d > 0$, and $< 0$ for $d < 0$. When $\alpha \in (2, \infty)$, $\frac{\partial M(d)}{\partial c} > 0, \forall d \in (-\frac{z}{\alpha}, \frac{z}{\alpha})$ and $= 0$ otherwise. So, $\frac{\partial M(d)}{\partial \alpha} \geq 0$ for $d > 0$, and $\leq 0$ for $d < 0$. So, whenever the function $M(d)$ is not a constant, as $\alpha$ increases, it shifts up for $d > 0$, and it shifts down for $d < 0$.

Proof of Proposition 4. For $\alpha \in (\frac{3}{2}, 2)$, $M(d)$ has a fixed point on the interval $(0, z/2)$. This fixed point can be solved for explicitly: $d^* = 1/2 \left( \frac{3\alpha - 6 + \sqrt{25\alpha^2 - 84\alpha + 72}}{2\alpha - 3} \right) \frac{z}{\alpha}$.

Also, $\frac{\partial d^*}{\partial \alpha}$ can be solved explicitly, and it is clear it is continuous in $\alpha$.

The following proves that $d^* > 0$ is increasing in $\alpha$. Suppose that for a given $\alpha$, the positive equilibrium value of $d$ is $d^*(\alpha)$. For any $\alpha + \epsilon$, $M(d^*(\alpha)) > d^*(\alpha)$ since $M(d)$ has increased pointwise. Apply Brauwer's Fixed Point theorem to the interval $(d^*(\alpha), z/2)$, and conclude that $\exists$ a fixed point in this interval. This implies that $d^*(\alpha + \epsilon) > d^*(\alpha)$, so the equilibrium $d^* > 0$ is increasing in $\alpha$. Hence, the equilibrium indifference curve shifts down continuously as $\alpha$ increases, so the equilibrium mass of field A decreases.

Similarly, we can solve for the negative equilibrium $d^*$ and show that it is decreasing continuously in $\alpha$. In this case the equilibrium indifference curve shifts up continuously as $\alpha$ increases, so the equilibrium mass of field B decreases.

Proof of Proposition 5. The different types of equilibria for different values of $\alpha$ and $e$ are shown in Figure 3.12.

1. At $\hat{d} = 0$, $M(d)$ turns from convex to concave. Hence, the maximum value of $\frac{\partial M(d)}{\partial d}$ is at $\hat{d}$ (where $c(d) = 0$). $\frac{\partial M(d)}{\partial d} |_{c(d)=0} = \alpha \frac{2}{3}$, so for any $\alpha \leq 3/2$, the slope of $M(d)$ does not exceed one, and is strictly smaller than one for all $d$ other than $\hat{d}$. So when $\alpha \leq 3/2$, $M(d)$ can only cross the 45 degree line exactly once, so a unique equilibrium exists. Since
the slope at this equilibrium point is less than one, the equilibrium is stable.

When $e > z$ and $\alpha \in [0, \frac{e-z}{e-z/2}]$, the set $\mathcal{D}1$ is non-empty. Hence, $M(x - y + \frac{z}{2}) = x - y + \frac{z}{2}$, so $d^* = x - y + \frac{z}{2}$ is an equilibrium. Since $\frac{e-z}{e-z/2} < 1$ for $e > z$, this equilibrium is unique. This value of $d$ is feasible only when all agents go to field B.

2. It can be shown that the set $\mathcal{D}1$ is empty for $\alpha \in (\frac{e-z}{e-z/2}, 1)$ when $e > z$. Also, the set $\mathcal{D}1$ is empty for $\alpha \in [0, 1)$ when $e < z$. When $\mathcal{D}1$ is empty, there does not exist an equilibrium in which all agents go to field B. When $\alpha < 1$, $\hat{d} = \frac{e-z}{\alpha} < -e = M(\hat{d})$. Since $M(d)$ is continuous, increasing and concave over the interval $(\hat{d}, -e + z/2]$, and since for any $d$ in this interval, $M(d)$ also belongs in this interval, we can apply Brauwer’s Fixed Point Theorem to $(\hat{d}, -e + z/2]$, and conclude that there exists a fixed point in this interval. This equilibrium value of $d$ is greater than $\hat{d}$. For any such equilibrium, there are less agents in field A than in field B. Again, this equilibrium is unique since all values of $\alpha$ in the interval considered are less than $3/2$.

3. $M(\hat{d}) = -e$. When $\alpha = 1$, for any difference in means, $M(\hat{d}) = \hat{d}$, so $\hat{d} = -e$ is a fixed point. Since $\alpha = 1 < 3/2$, this fixed point is unique. Also, since the equilibrium value of $d$ is $\hat{d}$, the two fields have equal masses of agents.
4. When $3/2 > \alpha > 1$, $\hat{d} = \frac{e^\alpha - e}{\alpha} > -e = M(\hat{d})$. Since $M(d)$ is continuous, increasing and convex over the interval $[-e - z/2, \hat{d}]$, and since for any $d$ in this interval, $M(d)$ also belongs in this interval, we can apply Brauwer’s Fixed Point Theorem to $[-e - z/2, \hat{d}]$ and conclude that there exists a fixed point in this interval, so the equilibrium is $d^* < \hat{d}$. In any such equilibrium, there are more agents in field A than in field B. When $\alpha < \frac{e + z}{e + z/2}$, the set $D_4$ is empty, so the equilibrium is not such that all agents are in field A. This implies that the masses of both fields are positive in the unique equilibrium that exists when $\alpha \in (1, \min\{3/2, \frac{e + z}{e + z/2}\})$.

5. When $\alpha \in \left[\frac{e + z}{e + z/2}, 3/2\right)$, there exists a unique equilibrium $d^* < \hat{d}$ in which all agents go to field A. Since $\frac{e + z}{e + z/2} > 1$, the values of $\alpha$ considered here are all greater than 1. We can apply the same existence of equilibrium argument as above to conclude that there exists a unique equilibrium $d^* < \hat{d}$. However, when $\alpha > \frac{e + z}{e + z/2}$, the set $D_4$ is non-empty, so the equilibrium is such that all agents are in field A.

6. When $\alpha \in [3/2, \frac{e + z}{e + z/2}]$, multiple equilibria may exist, but none of them such that all agents are in one field. This is because when $\alpha > 3/2$, the slope of $M(d)$ at $\hat{d}$ is greater than one, so it is possible to have more than one equilibrium. However, when the population means are different, this is no longer a sufficient condition for multiple equilibria because $\hat{d}$ may not be an equilibrium, so even though the slope at this point is greater than one, it does not imply that multiple equilibria exist. It can be shown that for $\alpha \in [3/2, \frac{e + z}{e + z/2}]$, the sets $D_1$ and $D_4$ are both empty, so there are no extreme equilibria in which all agents go to one field.

7. and 8. When $\alpha > \frac{e + z}{e + z/2}$, the set $D_4$ is not empty, so there exists an equilibrium in which all agents are in field A. So, when $\alpha > \max\{3/2, \frac{e + z}{e + z/2}\}$, multiple equilibria may exist, one of which is such that all agents are in field A. For $e > \frac{z}{2}$, $\max\{3/2, \frac{e + z}{e + z/2}\} = 3/2$. When $e > \frac{z}{2}$, and $\alpha > \frac{e}{e - z/2}$, the sets $D_1$ and $D_2$ are both empty. This implies that $M(d)$ is convex $\forall d \in D$ because $\hat{d} > -e + z/2$, and therefore, the equilibrium has to be unique. So when $e > \frac{z}{2}$ and $\alpha > \frac{e}{e - z/2}$, there exists a unique equilibrium in which all agents go to field A.
9. and 10. As before, when \( \alpha > \frac{e+z}{e+z/2} \), the set \( D4 \) is not empty, so there exists an equilibrium in which all agents are in field A. So, when \( \alpha > \max\{3/2, \frac{e+z}{e+z/2}\} \), multiple equilibria may exist, one of which is such that all agents are in field A. For \( e < \frac{z}{2} \), \( \max\{3/2, \frac{e+z}{e+z/2}\} = \frac{e+z}{e+z/2} \). Also, for \( e < \frac{z}{2} \), when \( \alpha > \frac{e-z}{e-z/2} \), the set \( D1 \) is not empty, so there also exists an equilibrium where all agents go field B. So, if \( e < \frac{z}{2} \) and \( \alpha > \frac{e-z}{e-z/2} \), \( \exists \) multiple equilibria, two of which are such that all the agents are in one of the two fields. ■

**Proof of Proposition 6.**

An increase in \( m_y \) of \( \delta \) can be modeled as an increase in \( y \) and \( y \) of \( \delta \). Calculating \( \frac{\partial M(d)}{\partial y} \), it is found that it is strictly negative for all \( d \in D \).

For \( c(d) \in (0, z) \):

\[
\frac{\partial M(d)}{\partial y} = \frac{z^2 - c(d)^2 - 2(m_{x,B}^c(d) - x)(z - c(d))}{z^2 - c(d)^2 + 2zc(d)} - \frac{2}{3}
\]

This is negative if \( z^2 - c(d)^2 - 6(m_{x,B}^c(d) - x) - (z - c(d)) - 4zc(d) < 0 \), which has to be the case since \( m_{x,B}^c(d) - x \in [z/3, z/2] \).

For \( c(d) \in [-z, 0) \):

\[
\frac{\partial M(d)}{\partial y} = \frac{z^2 - c(d)^2 + 4zc - 6(z + c(d))(m_{y,A}^c(d) - y)}{3(2z^2 - (z + c(d))^2)}
\]

Again, \( m_{y,A}^c(d) - y \in [z/3, z/2] \) implies that this is negative. For all other values of \( c(d) \), the derivative is equal to \(-1\). ■

**Proof of Proposition 7.** (a) From Proposition 5.3, we know that when \( \alpha = 1 \), \( \exists \) a unique equilibrium \( d^* = \hat{d} \), and this equilibrium is symmetric. This means that the equilibrium indifference line corresponding to \( \hat{d} \) divides the type space into two equal halves along the diagonal of the support. So, for any \( m_y > m_x \), the allocation of agents is the same, and the equilibrium is unique and symmetric.
(b) The equilibrium mass of field $A = \frac{(z-c(d^*))^2}{2}$, where $c(d^*) = \alpha d^* + y - \underline{x}$. Suppose one original stable equilibrium is $d_1^*$, and now let $m_y$ increase by $\delta$. (The support of agents shifts up by $\delta$.) As a result, we know from Proposition 6 that $M(d)$ decreases pointwise from $M_1(d)$ to $M_2(d)$, as shown in Figure 3.13. The new equilibrium value of $d$ is a value $d_2^* < d_1^*$. In order for the equilibrium masses of agents to remain unchanged, $c(d^*)$ has to remain unchanged. Hence, it is required that $\alpha \Delta d^* + \Delta y = 0$, i.e. $\Delta d^* = -\frac{\delta}{\alpha}$. If $\Delta d^* > -\frac{\delta}{\alpha}$, then $c(d_2^*) > c(d_1^*)$, so the equilibrium mass of A in the new equilibrium is lower relative to the original equilibrium. (This is determined using the formula for calculating the mass of A.) If $\Delta d^* < -\frac{\delta}{\alpha}$, then $c(d_2^*) < c(d_1^*)$, and the new equilibrium mass of A is higher.

In order to determine what happens to the equilibrium masses of agents, we need to determine whether the new equilibrium $d_2^*$ is smaller or bigger than $d_1^* - \frac{\delta}{\alpha}$. Consider the new allocation (after $m_y$ has increased) corresponding to a difference in means equal to $d_1^* - \frac{\delta}{\alpha}$. In this allocation, $m_{x,B}$ is the same as in the allocation before the change in mean corresponding to $d_1^*$. $m_{y,A}$ is $\delta$ higher than in the allocation before the change in mean corresponding to $d_1^*$. Hence, $M_1(d_1^*) - M_2(d_1^* - \frac{\delta}{\alpha}) = \delta$. This is shown in Figure 3.13.

When $\alpha < 1$, $\frac{\delta}{\alpha} > \delta$, so $M_2(d_1^* - \frac{\delta}{\alpha}) > d_1^* - \frac{\delta}{\alpha}$. Applying Brauwer’s Fixed Point Theorem to the function $M_2(d)$ over the interval $(d_1^* - \frac{\delta}{\alpha}, d_1^*)$, we conclude that $\exists$ an
equilibrium in this interval. Hence, \( d_2^* > d_1^* - \frac{\delta}{\alpha} \), which implies that \( \Delta d^* > -\frac{\delta}{\alpha} \). So \( c(d_2^*) > c(d_1^*) \), which means that the equilibrium mass of A in the new equilibrium is lower relative to the original equilibrium.

(c) Apply a similar argument as above. However, now when \( \alpha > 1, \frac{\delta}{\alpha} < \delta \), so \( M_2(d_1^* - \frac{\delta}{\alpha}) < d_1^* - \frac{\delta}{\alpha} \). Applying Brauwer’s Fixed Point Theorem to the function \( M_2(d) \) over a small interval left of \( d_1^* - \frac{\delta}{\alpha} \), we conclude that \( \exists \) an equilibrium in this interval. Therefore, \( d_2^* < d_1^* - \frac{\delta}{\alpha} \), so \( \Delta d^* < -\frac{\delta}{\alpha} \), and \( c(d_2^*) < c(d_1^*) \). As a result, the equilibrium mass of A in the new equilibrium is greater relative to the original equilibrium. ■

Proof of Proposition 8. \( \lim_{d \to 0^-} M(d) = \frac{\beta z}{3(1+\beta)} \). This is strictly bigger than zero if \( \beta \in (0, 1) \), and \( = 0 \) if \( \beta = 0 \). So, for any \( \beta \in (0, 1) \), and \( \epsilon > 0 \) sufficiently small, \( M(\epsilon) > \epsilon \). \( M(d) \) is increasing, so \( \forall d \in (0, \frac{z}{2}] \), \( M(d) \in (0, \frac{z}{2}] \). \( M(d) \) is also continuous over this interval. Apply Brauwer’s Fixed Point Theorem to the interval \( (0, \frac{z}{2}] \), and obtain that \( \exists \) a fixed point in this interval. This fixed point is unique and stable. For \( \alpha < 2 \), the equilibrium is such that there is a positive mass of agents in both fields, and for \( \alpha \geq 2 \), field A has zero mass in equilibrium. \(^{16}\) Similarly, it can be shown that \( \exists \) a unique stable fixed point on \( [-\frac{z}{2}, 0) \). As \( \beta \to 0, M(\epsilon) \to 0, \) for \( \epsilon \) very small.

When \( \alpha = 0 \), agents’ payoffs are independent of the unobserved means, so all agents with types \( x > y \) will maximize their payoffs by going to field A, all agents with \( x < y \) will maximize their payoffs by going to field B. The indifferent agents can go to either field, so the difference in means in an incentive compatible allocation can take any values in the set \([ -\frac{\beta z}{3(1+\beta)}, \frac{\beta z}{3(1+\beta)} ] \). (The upper and lower bounds are found by allowing all indifferent agents to go to only one field.) So, if indifferent agents go to one field with a fixed probability, then any \( \forall d \in [-\frac{\beta z}{3(1+\beta)}, \frac{\beta z}{3(1+\beta)}] \) is an equilibrium.

When \( \beta = 1 \), \( M(d) = \frac{z}{2}, \forall d \in (0, \frac{z}{2}] \) and \( = -\frac{z}{2}, \forall d \in [-\frac{z}{2}, 0) \). At \( d = 0 \), \( M(d) \) can take any value in \( D \). So, the two stable equilibria are such that all agents go to one field.

\(^{16}\)The proof uses the fact that \( M(d) \) is concave and crosses the 45 degree line from above, and is the same as in the benchmark case.
Such an equilibrium is a pure strategy Nash Equilibrium. The other equilibrium is at $d = 0$. If all agents go to either field with probability $1/2$, then $d = 0$ is a mixed strategy Nash Equilibrium. ■

**Proof of Proposition 9.** Differentiating $M(d)$ with respect to $\beta$, it is found that:

$$
\frac{\partial M(d)}{\partial \beta} = \begin{cases} 
0 & \text{if } c(d) > z \\
> 0 & \text{if } c(d) \in (0, z] \\
< 0 & \text{if } c(d) \in [-z, 0) \\
= 0 & \text{if } c(d) < -z.
\end{cases}
$$

This is continuous in $\beta$ and $d$ for $c(d) \in (0, z]$ and $c(d) \in [-z, 0)$, so $M(d)$ shifts continuously. It is known from before that if $\alpha < 2$, then the highest equilibrium value of $d$ is smaller than $z/2$. For any $\beta \in [0, 1)$, let $d^*(\beta)$ be the corresponding positive equilibrium value of $d$. Fix an interval $[d^*(\beta), d^*(\beta) + \epsilon)$. Now, let $\beta$ increase by $\delta > 0$. Since $M(d)$ shifted up, $M(d^*(\beta)) > d^*(\beta)$. So, by Brauwer’s Fixed Point theorem, $\exists$ an equilibrium in $(d^*(\beta), z/2]$, so $d^*(\beta + \delta) > d^*(\beta)$. Since $M(d)$ shifts continuously and is continuous in $d$, $\exists \delta > 0$ such that $d^*(\beta + \delta) < d^*(\beta + \epsilon)$. Since $\epsilon$ can be made very small, $\frac{\partial d^*(\beta)}{\partial \beta}$ is continuous. When $\alpha \geq 2$, the positive equilibrium value of $d$ is $z/2$, and all agents are in field A. When $\beta$ changes, this remains true, so there is no change in equilibrium. Similarly, it can be shown that the negative equilibrium value of $d$ is decreasing and continuous in $\beta$. ■

**Proof of Proposition 10.** Refer to Figure 3.7. 1. We know from Proposition 11 that $M(d)$ increases pointwise in $t$. We know that when $\alpha < 3/2$, in the benchmark case $M(d) > d \ \forall d < 0$. Now, when $t$ increases, since $M(d)$ shifts up, it will still be the case that $M(d) > d \ \forall d \leq 0$. Hence, there cannot exist an equilibrium $d^* < 0$. $M(d)$ is continuous over the interval $(0, \frac{z + t}{2}]$, and for any $d$ in this interval, $M(d)$ is also in this interval since $M(0) > 0$ for any $t > 0$. So we can apply Brauwer’s Fixed Point theorem to $(0, \frac{z + t}{2}]$, and conclude that $\exists$ at least one fixed point of $M(d)$ in this interval. Further, there is exactly one fixed point since $M(d)$ is strictly concave everywhere on the interval.
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(0, \min \{\frac{z+t}{\alpha}, \frac{z+t}{2}\}) and is constant on \([\frac{z+t}{\alpha}, \frac{z+t}{2}]\), so \(M(d)\) crosses the 45 degree line only once.

When \(\alpha \in (0, \frac{t}{z}]\), the sets \(E4\) and \(E5\) are empty, so \(M(d)\) is concave for \(\forall d \in D\), and \(M(-z/2) > -z/2\). This implies that there exists a unique stable equilibrium.

2. When \(\alpha \in [\max \{\frac{3}{2}, \frac{t}{z}\}, \infty)\), multiple equilibria may exist because \(M(d)\) turns from being convex to concave at a value of \(d\) interior to \(D\), and the slope at this point may be greater than one, depending on \(t\), so \(M(d)\) could cross the 45 degree line from below.

3. The set \(E1\) is non-empty when \(\alpha \geq \frac{(2z+t)}{z+t}\). If this is satisfied, \(M(\frac{z+t}{2}) = \frac{z+t}{2}\), so \(d = \frac{z+t}{2}\) is an equilibrium. This value of \(d\) is feasible only when field A has zero mass. Note that \(\frac{(2z+t)}{z+t}\) is strictly decreasing in \(t\), so a sufficient condition for there to exist an equilibrium with field A empty is that \(\alpha > 2\).

4. The set \(E5\) is non-empty when \(\alpha \geq 2 + t/z\). If this holds, \(d = -\frac{z}{2}\) is an equilibrium. In this case, field B has zero mass. ■

Proof of Proposition 11. The set \((0, 2x)\) is the set of feasible values of \(t\) such that the variance of \(y\) is greater than the variance of \(x\). The upper bound is there since it is assumed \(y \geq 0\), so there cannot be negative talent levels. Solving for \(\frac{\partial M(d)}{\partial t}\), we find that it is \(\geq 0\) for all \(d\), so \(M(d)\) increases pointwise in \(t\). ■

Proof of Proposition 12.

Take any stable equilibrium with positive masses of agents in both fields. For such an equilibrium, let the equilibrium value of \(d\) corresponding to a given \(t\) be \(d^*(t)\). We want to show that \(\frac{\partial d^*(t)}{\partial t} \geq 0\). Suppose that \(d^*(t) < \frac{z+t}{2}\). Then for any \(t + \epsilon\), \(M(d^*(t)) > d^*(t)\) since the function \(M(d)\) shifts up. Hence, we can apply Brauwer’s Fixed Point theorem to the interval \((d^*(t), \frac{z+t}{2}]\), and obtain that there must be at least one fixed point in this interval, which means that \(d^*(t + \epsilon) > d^*(t)\). So, \(d^*(t)\) is increasing in \(t\). Also, \(\frac{\partial M(d)}{\partial t}\) is continuous in \(d\), so the function \(M(d)\) continuously increases pointwise in response to
increases in $t$, so $d^*(t)$ must also increase continuously up to the point $\frac{z+t}{2}$. Since the equilibrium difference in means increases in $t$, it follows that the equilibrium indifference line between the two fields shifts down as $t$ increases, so in the new equilibrium, field B will be bigger and field A will be smaller. ■

**Proof of Proposition 13.** Consider the first order effect of the increase in variance, i.e. the increase in $M(d)$ for any given $d$. When both fields have positive masses of agents, the minimum value of $\frac{\partial M(d)}{\partial t}$ is $1/3$. When $t$ increases, $m_{x,B}$ increases by less and less, and in the limit, it does not change. However, $\frac{\partial m_{y,A}}{\partial t} = -1/3$ always. Now, because the function $M(d)$ is increasing and concave, it must be the case that $\frac{\partial d^*(t)}{\partial t} > \frac{\partial M(d)}{\partial t} > 1/3$. This is because the total change in $d^*$ consists of the first order effect plus a second order effect that is also positive. (The second order effect is caused by agents changing fields as a result of changes in means). Therefore, the total effect, $\frac{\partial c^*(d,t)}{\partial t}$ must be greater than the first order effect $\frac{\partial M(d)}{\partial t}$. So, $\frac{\partial c^*(d,t)}{\partial t} = \alpha \frac{\partial d^*(t)}{\partial t} - \frac{1}{2} > \alpha \frac{1}{3} - \frac{1}{2}$. This is greater than $\frac{\partial c(d,t)}{\partial t}$ when $c(d) \in (0, z]$ if $\alpha > \frac{3(t+c(d))}{2(z+t)}$, and when $c(d) \in (-t, 0)$ if $\alpha > \frac{3(t+2c(d))}{2(z+t)}$. Both these terms increase in $t$, and their limit as $t \to \infty$ is $3/2$. So, a sufficient condition for the size of field A to decrease is that $\alpha > 3/2$.

The intuition for this result is that as $m_{y,A}$ decreases when more people with very low $y$ levels enter field A, the negative externality of these people on the payoffs of the rest of agents in A will be bigger the greater $\alpha$ is. So, for $\alpha$ high enough, the negative effect will be big enough to result in enough people leaving field A such that its relative size decreases. (A large enough second order effect).

For the extreme case where $\alpha = 0$, $\frac{\partial c^*(d,t)}{\partial t} = \frac{\partial c(d,t)}{\partial t} = -1/2$, so the fraction of agents in each field does not change. If we start from the symmetric stable equilibrium in the benchmark case and $t$ increases, then the size of field A is also guaranteed to decrease. This is because in the symmetric equilibrium in the benchmark case, the sizes of the two fields are equal. If $t$ increases slightly, then the difference in means increases, so
the indifference line shifts up. As a result, less than $1/2$ of the total mass will be in field A. Also, for $\alpha \geq 2$, in the equilibrium $d^* > 0$, field B will contain all agents, and as $t$ increases, this remains the case, so again the number of agents in each field stays constant.

**Proof of Proposition 14.** Fix the the length of the support of $y$ to $\bar{y} - y \equiv h$, and let $z \to 0$. So, $\frac{\text{var}(y)}{\text{var}(x)} \to \infty$. Proposition 10.2 can be restated as: "when $\alpha \in \left[\max\{\frac{3}{2}, \frac{\bar{y} - y}{z} - 1\}, \infty\right]$, multiple equilibria may exist." As $z \to 0$, for $\bar{y} - y > 0$, $\frac{\bar{y} - y}{z} \to \infty$, so the value of $\alpha$ necessary for multiple equilibria to possibly exist goes to infinity. Hence, as $z \to 0$, $\exists$ a unique equilibrium for $\alpha < \infty$. In order for this equilibrium to be such that all agents are in field B, it is required that $c(d^*) \geq z$. So, when $z = 0$, it is required that $\alpha d^* + y - x \geq 0$. The value of $d$ corresponding to an allocation in which all agents are in field B is $x - y$. So, if $\alpha(x - y) + y - x \geq 0$, then this allocation is an equilibrium. This condition holds when $\alpha \geq 1$. If $\alpha < 1$, then the maximum feasible value of $c(d)$ approaches a negative number as $z \to 0$, so the equilibrium mass of B does not approach $1$. ■

**Proof of Proposition 15.** When $z = 0$, all agents' type $x$ is equal to a constant $\ddot{x}$. Then, for a fixed $h \equiv \bar{y} - y > 0$, and $\alpha < 1$, the equilibrium is such that there are positive masses of agents in both fields. Let $\ddot{y}$ be the value of $y$ such that all agents with types $y > \ddot{y}$ go to field B, and the rest go to field A in equilibrium. The type of agent that is indifferent in equilibrium between the two fields has type $(\ddot{x}, \ddot{y})$ such that:

$$\ddot{x} + \alpha(\frac{\ddot{y} + \ddot{y}}{2}) = \ddot{y} + \alpha \ddot{x}, \text{ i.e. } \ddot{y} = \frac{(1-\alpha)\ddot{x} + \frac{1}{2} \ddot{y}}{1-\frac{\alpha}{2}}.$$ 

This is the threshold value of $y$ in equilibrium when $z = 0$. In this equilibrium, the fraction of agents in field A is equal to $\frac{\ddot{y} - y}{h}$ which simplifies to $\frac{1}{2 - \alpha}$. ■

**Proof of Proposition 17.** Any equilibrium $d^*$ satisfies:
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\[ d^* = M(d^*) = \frac{\int_{x}^{\bar{y} + \alpha d^*} x \int_{x - \alpha d^*}^{\bar{y}} f(x, y) dy dx}{\int_{x}^{\bar{y} + \alpha d^*} \int_{x - \alpha d^*}^{\bar{y}} f(x, y) dy dx} - \frac{\int_{y}^{\bar{x} - \alpha d^*} y \int_{y + \alpha d^*}^{\bar{x}} f(x, y) dx dy}{\int_{y}^{\bar{x} - \alpha d^*} \int_{y + \alpha d^*}^{\bar{x}} f(x, y) dx dy} \]

For \( d = 0 \):

\[ M(0) = \frac{\int_{x}^{\bar{y}} x \int_{x}^{\bar{y}} f(x, y) dy dx}{\int_{x}^{\bar{y}} \int_{x}^{\bar{y}} f(x, y) dy dx} - \frac{\int_{y}^{\bar{x}} y \int_{y}^{\bar{x}} f(x, y) dx dy}{\int_{y}^{\bar{x}} \int_{y}^{\bar{x}} f(x, y) dx dy} \]

If \( \bar{x} = y, \bar{y} = \bar{y} \), and \( f(x, y) = f(y, x) \), then it can be seen from the above that the two terms are equal, so \( M(0) = 0 \), and \( d^* = 0 \) is an equilibrium. In this equilibrium, the indifference line coincides with the diagonal of the type support.

\[ \square \]

**Proof of Proposition 18.** When \( \bar{y} \leq \bar{x} - \alpha (\max(m_{x,B} - m_{y,A})) \), the minimum feasible indifference line is everywhere above the support of types, so no agent will ever find it optimal to go to field B. Since \( \max(m_{x,B} - m_{y,A}) \leq \bar{x} - y \), we can substitute this in, and derive the conditions in the claim.

Similarly, when \( \bar{y} + \alpha (\max(m_{x,B} - m_{y,A})) \geq \bar{x} \), the maximum feasible indifference line is everywhere below the support of types, so no agent will ever find it optimal to go to field A.

\[ \square \]

**Proof of Proposition 19.** The indifference condition is \( y = x - \alpha d \). With finite type support, the last agent to exit field A is the agent with type \((\bar{x}, \bar{y})\), and the last agent to leave field B is the agent with type \((\bar{x}, \bar{y})\). For any \( d > 0 \), \( \exists \alpha \) such that type \((\bar{x}, \bar{y})\) will find it optimal to go to field B. This is given by \( \hat{\alpha} = \frac{\bar{x} - \bar{y}}{d} + \epsilon \), for any finite \( \epsilon > 0 \). Since \( d \) is not equal to zero, \( \hat{\alpha} \) is finite. As \( d \to 0^+ \), \( \hat{\alpha} \to \infty \). So as \( \alpha \to \infty \), for any \( d > 0 \), all agents will sort themselves in field B. Similarly, for \( d < 0 \), for \( \alpha \) large enough,
all agents will sort themselves in field A. These are the two stable equilibria. $d = 0$ is not a stable equilibrium because for any slight deviation from it, all agents will go to one of the two fields and stay there. ■

**Proof of Proposition 21.** Given the specified strategies, no single agent has an incentive to deviate, for any $\alpha$. Consider the agent with a type $(x, y)$, where $x > x^*(y)$. This agent’s strategy is to go to field A, where the payoff is $x + \alpha\left[\frac{y + y^*(x)}{2}\right]$. If instead this type goes to field B, the payoff is $y + \alpha\left[\frac{x + x^*(y)}{2}\right]$. Since $x + \alpha\left[\frac{y + y^*(x)}{2}\right] > x^*(y) + \alpha\left[\frac{y^*(x^*)}{2}\right] = y + \alpha\left[\frac{x + x^*(y)}{2}\right]$, the agent is better off in A. Similarly, it can be shown that no agent supposed to go to B is better off by going to A. ■

**Proof of Proposition 22.** 1. The indifference condition when $\alpha \neq 2$ simplifies to $x^*(y) = y - \frac{\alpha}{2-\alpha}(y - x)$. Clearly, for any talent level $y$, there exists a unique level $x^*$ such that the agent with this type is indifferent between the two fields. The conditions in the first part of the claim ensure that there exists a $x^*$ in the support of $x^*$, and hence that the indifference line goes through the interior of the type support.

2. To obtain a condition under which all agents going to B is an equilibrium, we need that when all agents go to field B, the agent with type $(y, x)$ has no incentive to deviate to field A. The payoff of this agent when going to field A is $x + \alpha\left[\frac{y + y^*(x)}{2}\right]$, and the payoff from going to field B is $y + \alpha\frac{x + x^*}{2}$. Comparing these, we can see that this agent is better off in field B as long as the condition $\alpha\left[\frac{x + x^*}{2} - y\right] \geq x - y$ holds.

Similarly, it can be shown that the agent with type $(y, x)$ does not have an incentive to deviate when all agents go to field A when $\alpha\left[\frac{y + y^*}{2} - x\right] \geq y - x$. Since when these conditions do not hold, the agents considered above find it optimal to deviate, all agents going to one field cannot be an equilibrium in this case.

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17It is assumed as before that if one agent deviates to a different field, the expected unobserved talent is the lowest possible level.
3. When $\alpha = 2$ and $\underline{y} = \underline{x}$, any type $(x, y)$ satisfies the indifference equation. So, there is an infinite number of monotone equilibrium indifference curves.

4. When $\alpha = 2$ and $\underline{y} \neq \underline{x}$, there does not exist a type $(x, y)$ that satisfies the indifference equation. Hence, no Nash equilibrium with monotone indifference curve where both fields have positive masses exists. If $\underline{y} > \underline{x}$, then all agents going to field A is an equilibrium because no agent has an incentive to deviate. Similarly, if $\underline{y} < \underline{x}$, if all agents go to field B, no agent has an incentive to deviate to A, and hence all agents going to B is a Nash Equilibrium. ■

**Proof of Proposition 23.** When $\alpha \neq 2$, the indifference condition is:

$$y^*(x) = x + \frac{\alpha}{2-\alpha} (\underline{y} - \underline{x}).$$

As before, when the population mean of $y$ increases relative to the population mean of $x$ by $\delta$, all agents’ talent level $y$ increases by $\delta$.

(1) By looking at the above indifference condition, we can see that if $\alpha \in (0, 1)$, then as $(\underline{y} - \underline{x})$ increases by $\delta$, $y^*(x)$ increases by less than $\delta$. This means that the indifference curve shifts up by less than the increase in the support of $y$. As a result, field B has a greater mass of agents in the new equilibrium.

(2) If $\alpha = 1$, then as $(\underline{y} - \underline{x})$ increases by $\delta$, $y^*(x)$ increases by exactly $\delta$. Since the indifference curve shifts up by the same amount as the increase in the support of $y$, no agent changes field.

(3) If $\alpha \in (1, 2)$, then as $(\underline{y} - \underline{x})$ increases by $\delta$, $y^*(x)$ increases by more than $\delta$. This means that the indifference curve shifts up by more than the increase in the support of $y$. As a result, field A has a greater mass of agents in the new equilibrium.

(4) For $\alpha = 2$, it was previously argued that there does not exist a type that is indifferent between the two fields when $\underline{y} \neq \underline{x}$.

(5) For $\alpha \in (2, \infty)$, as $(\underline{y} - \underline{x})$ increases by $\delta$, $y^*(x)$ decreases by more than $\delta$. So, as the talent support shifts up, the indifference curve shifts down. Clearly, the equilibrium
mass of agents in field A decreases. ■

**Proof of Proposition 24.**

This claim follows directly from the indifference curve function $x^*(y) = y + \frac{\alpha}{2-\alpha}t$. When $\frac{\alpha}{2-\alpha}$ is positive, the indifference curve shifts down as a result of an increase in $t$, and when it is negative, it shifts up. ■

**Proof of Proposition 25.** Take an agent with type $(\hat{x}, y)$ where $\hat{x} > x^*(y)$, and let $\delta = \hat{x} - x^*(y)$. We know that if $\delta = 0$, this agent satisfies the indifference condition. If there exists $\delta \neq 0$ such that this agent is indifferent when others follow their strategies, then another Nash Equilibrium with monotone indifference curve exists. $W^A(\hat{x}, y) = \delta + W^A(x^*(y), y)$. For this agent to be indifferent, it is needed that $W^B(\hat{x}, y) = \delta + W^B(x^*(y), y)$. This requires that:

$$\alpha \left[ \frac{\int_{x^*(y)+\delta} xg(x)dx}{\int_{x^*(y)+\delta} g(x)dx} - \frac{\int_{x^*(y)} xg(x)dx}{\int_{x^*(y)} g(x)dx} \right] = \delta.$$

Since the term in square brackets is smaller than $\delta$, then if $\alpha \leq 1$, the above equality can never hold for any $\delta > 0$. However, if $\alpha > 1$, then it is possible that there exists a positive value of $\delta$ for which the equality holds.

The same argument can be applied to a type $(\tilde{x}, y)$ where $\tilde{x} < x^*(y)$. ■
Bibliography


Bibliography


