Finite Element Analyses of Failure Mechanisms and Structure-Property Relationships in Microtruss Materials

by

Eral Bele

A thesis submitted in conformity with the requirements for the degree of
Doctor of Philosophy
Graduate Department of Materials Science and Engineering
University of Toronto

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Abstract

Microtruss materials are assemblies of struts or columns arranged periodically in space. The majority of past research efforts have focused on the key issue of microtruss architectural optimization. By contrast, this study focuses on the internal material structure at the level of the individual struts. Microstructural, geometrical, and material design techniques are used to improve their mechanical properties.

The finite element method is used to verify and create predictive analytical models, explain the dependence of strut properties on geometry, material properties and failure mechanisms, and extend the strut design analysis into suggestions for the improvement of fabrication methods. Three strut design methods are considered. First, microstructural design is performed by considering the influence of strut geometry on the strain energy imparted during stretch bending. By using the perforation geometry to modify the location and magnitude of this strain energy, microtruss materials with lower density and higher strength can be fabricated. Second, structural sleeves of aluminum oxide and electrodeposited nanocrystalline nickel are used to reinforce architecturally optimized aluminum alloy microtruss assemblies, creating hybrid materials with high weight-specific strength. The mechanical properties are controlled by the interaction between material and mechanical failure; this interaction is studied through finite element analyses and a proposed analytical relationship to provide suggestions for further improvements. Finally,
hollow cylindrical struts are fabricated from electrodeposited nanocrystalline nickel. The high strength to weight ratio achieved in these struts is due to the microstructural and cross-sectional efficiency of the material.
Acknowledgements

The work of this thesis and my experience throughout its completion would not have been possible, stimulating, and enjoyable without the mentorship, guidance, and optimism of my supervisor, Prof. Glenn Hibbard.

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<td>Cross sectional area of the sleeve, core, column, microtruss, strut envelope</td>
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<td>$b$</td>
<td>Thickness of a column or strut</td>
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<td>$B$</td>
<td>Ratio of core width to length</td>
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| $c$    | 1. Number of struts in a unit cell (Chapter 1)  
           2. Fitting parameter for Hooputra’s model (Section 2.3.1) |
| $C$    | Material fitting parameter for Voce model                                    |
| $D$    | Area envelope of a strut                                                     |
| $D^p$  | Plastic strain rate tensor                                                   |
| $d$    | Compressive displacement                                                     |
| $d_0$, $d_1$ | Fitting parameters for Hooputra’s model (Section 2.3.1)                        |
| $E$, $E_C$, $E_S$ | Young’s modulus, Young’s modulus of core, Young’s modulus of sleeve                                      |
| $E_T$, $E_{T,C}$, $E_{T,S}$ | Tangent modulus, tangent modulus of core, tangent modulus of sleeve |
| $f$, $f_C$, $f_N$, $f_F$ | Void volume fraction, critical void volume fraction, nucleated void fraction, final void volume fraction at failure |
| $f_S$  | Sleeve area fraction                                                         |
| $\dot{f}$, $\dot{f}_{gr}$, $\dot{f}_{nucl}$ | Rate of change of void volume fraction, void growth rate, void nucleation rate |
| $F$    | Functional requirements                                                     |
| $G_D$  | Envelope geometry                                                           |
| $G_f$  | Fracture energy                                                             |
| $H$    | Height of microtruss unit cell                                               |
\[ I, I_C, I_S, I_D \]  
Second moment of area, second moment of area of core, second moment of area of sleeve, second moment of strut envelope

\[ j \]  
Number of joints in a unit cell

\[ k \]  
Constant that describes the end constraints of a column

\[ K \]  
One of the Hollomon fitting parameters

\[ L \]  
Length of a column or strut

\[ m \]  
Number of mechanisms

\[ M \]  
Moment, Material parameters (Section 2.2.3)

\[ n \]  
One of the Hollomon fitting parameters

\[ N \]  
1. Number of struts contained in a unit cell  
2. Load at buckling (Section 3.3.1)

\[ P_{CR}, P_{COL} \]  
Critical load, critical load of a column

\[ q_1, q_2, q_3 \]  
Material fitting parameters for GTN model

\[ r \]  
1. Radius of gyration of a column  
2. Lankford ratio (Section 4.1)

\[ r_C \]  
Radius of gyration of the core

\[ R \]  
Radius of a cylinder

\[ s \]  
States of self-stress

\[ s_N \]  
Standard deviation of equivalent plastic strain distribution (Section 2.3.2)

\[ S \]  
Shape of a strut

\[ S_A \]  
Electroplated surface area

\[ t \]  
1. Sleeve thickness (Section 4.2.)  
2. Core thickness (Section 4.3.)  
3. Wall thickness of a hollow cylinder (Section 4.4.)

\[ t_S \]  
Sleeve thickness (Section 4.3.)
Ratio of sleeve thickness to length

Volume of a column, volume of microtruss unit cell

Width of a column or strut, strut width in a conventional perforation geometry (Section 4.1.)

Mean value of equivalent plastic strain distribution (Section 2.3.2)

Plastic strain

Equivalent plastic strain, equivalent plastic strain at fracture

Equivalent plastic strain rate

Strain at peak compressive stress

Stress triaxiality

Wavelength of XRD source

Poisson’s ratio

Density of core, sleeve, column, microtruss assembly, hybrid column

Relative density

Stress at bifurcation (Section 4.2.)

Stress in core, sleeve

Critical stress, critical stress of a column, critical stress of the microtruss assembly, critical stress of a hybrid column, critical stress of the core in a hybrid column

Von Mises stress

Equivalent flow stress

Stress at fracture (Section 4.2.)

Macroscopic mean stress

Stress normal to sleeve-core interface (Section 4.2.)
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<td>$\sigma_Y$</td>
<td>Yield stress</td>
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<tr>
<td>$\sigma_{PK}, \sigma_{PK}^{COL}, \sigma_{PK}^{MT}$</td>
<td>Peak compressive stress, peak compressive stress of a column, peak compressive stress of the microtruss</td>
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<tr>
<td>$\bar{\sigma}$</td>
<td>Strength/density ratio</td>
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<tr>
<td>$\sigma_\infty$</td>
<td>Material fitting parameter for Voce model</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Open area fraction</td>
</tr>
<tr>
<td>$\psi_A, \psi_I$</td>
<td>Area and second moment of area shape transformers</td>
</tr>
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<td>$\omega$</td>
<td>Strut angle</td>
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1. INTRODUCTION

This chapter introduces microtruss architectures as a subset of cellular materials. The distinction and advantages are outlined in Section 1.1. The scope of this thesis, the objectives, and the main contributions are summarized in Section 1.2.

1.1. Microtruss Architectures

Cellular materials are hybrids of solids and voids [1], whose low relative densities (e.g. less than 10% of the parent metal) are achieved through an internal void structure which can be either unconnected (open cell) or connected (closed cell). Examples are metallic foams and honeycomb assemblies, whose structural benefits are well established [2]. Due to their high weight-specific strength, they are typically used as cores for sandwich panels where their role is to provide high specific strength and stiffness at low densities [1, 3]. Since they are hybrids of air and metal, they can occupy new regions of material property space that are inaccessible by monolithic materials; an example is shown in Figure 1.1.1 [4].

Microtruss materials are a special class of cellular solids that have recently attracted considerable interest, having been proven to be light, strong, stiff, and multifunctional structures [5-7]. They can be thought of as a frame of beams, columns or wires arranged in periodic, three-dimensional architectures, such as the ones shown in Figure 1.1.2 [8].

Recent studies have shown that the microtruss architectures significantly improve the strength-to-weight and stiffness-to-weight ratios over conventional metal foams, because the material mass is reduced by retaining only that which has a high load bearing efficiency [1]. This is achieved by selecting unit cells where the applied external load is resolved axially in the constituent struts. Thus, in contrast with conventional foams, where
cell collapse is resisted by bending of the cell walls or ligaments (bending-dominated deformation), the deformation in microtruss materials is resisted by tension or compression of the constituent struts (stretch-dominated deformation) [1]. In order for a unit cell to be stretch-dominated, its strut connectivity must satisfy Maxwell’s stability criterion [9], which in three dimensions is given by [10]:

$$c - 3j + 6 = s - m \quad (1.1.1)$$

where $c$ and $j$ are the number of struts and joints in a unit cell, and $s$ and $m$ are the states of self-stress and mechanisms respectively.

Figure 1.1.1: Material property space, showing the achievable Young’s modulus and density of engineering materials, and guidelines for minimum mass design [4].
This change in the deformation mechanism can yield significant benefits in structural properties. For instance, theoretical calculations of simplified unit cells show that the relationship between the relative stiffness (stiffness of cellular material divided by stiffness of solid, $E/E_s$) and relative density (density of cellular material divided by density of solid, $\rho/\rho_s$) is $E/E_s \propto \rho/\rho_s$ in stretch-dominated lattices and $E/E_s \propto (\rho/\rho_s)^2$ in bending-dominated structures [11]. Examples of the material property space occupied by the two sets of cellular solids are shown in Figure 1.1.1. Thus, microtruss materials can fill holes in material property space by providing higher weight-specific strength and stiffness than bending-dominated counterparts.

1.2. Scope of This Thesis

The majority of studies on microtruss materials have concentrated on optimizing the architecture of the unit cell or sandwich construction; see [12-14] for representative examples. By contrast, this thesis is focused on optimization of the material and geometry of the building blocks (i.e. individual struts) of the cellular lattice. The finite element (FE) method is used as a tool to understand experimental results, create and verify new analytical models, and predict structural properties by investigating phenomena (e.g. stress...
distribution, buckling loads, failure limits) that occur during the fabrication and mechanical testing of these materials.

The content of this thesis has been largely published in References [15-19]. The author’s main contribution is the development of finite element models to interpret experimental observations that were obtained by him and other members of the Hybrid Materials Design Group at the University of Toronto.

Chapter 2 provides a summary of the relevant literature on microtruss strengthening methods, material fracture mechanisms, and the implementation of failure models in finite element analyses. Since one of critical functions of a microtruss strut is to support internally resolved compressive loads, Chapter 3 provides an overview of the relevant analytical models and the results of simple FE models that can only dissipate energy through structural instability. These FE models are used to illustrate the assumptions behind the predictions of the various closed-form expressions, and to illustrate the stress evolution in the buckling and post-buckling regime. The systems of experimental investigations are presented in Chapter 4. Section 4.1 is a study on strut geometry modification for the purpose of controlling the strain distribution imparted during stretch-bending fabrication in such a way that lighter and stronger microtruss architectures can be produced. Section 4.2 provides a discussion on mechanical properties and failure mechanisms of nanocrystalline-reinforced hybrid struts. These struts possess three levels of structural efficiency: microstructural (grain size refinement), architectural (the load resolution is axial), and sectional (the reinforcement is positioned ideally away from the bending axis), resulting in significant benefits in the achieved strength-density material space. The mechanical properties and failure modes of ceramic/metal hybrid struts are discussed in Section 4.3, showing an example of using what has previously been considered a surface treatment to
increase strength at little to no weight penalty. Section 4.4 discusses the relationship between material properties, geometry, and failure mechanisms of nanocrystalline hollow struts, showing the influence of each on microtruss properties. Finally, Chapter 5 provides a guide to material selection for the design of hybrid microtruss materials, and Chapter 6 presents a summary of this thesis.

### 1.3. References


2. BACKGROUND: APPLICATIONS, STRUT DESIGN STRATEGIES AND MATERIAL FRACTURE MODELS

This chapter provides a background on the current state of the art in the field. An overview of the applications and typical loading conditions of microtruss materials is provided in Section 2.1. Two branches of research are particularly important to this thesis. The techniques used to access new regions of material property space are related to the experimental fabrication methods described in Chapter 4, and are reviewed in Section 2.2. Material failure occurs both during fabrication (deformation forming process), and as an additional failure mode during compression testing. A review of material fracture models is provided in Section 2.3.

2.1. Applications and Loading Conditions

Like other cellular materials (e.g. honeycombs and foams), microtruss assemblies can be used as cores in sandwich panels; the latter are particularly attractive in weight-sensitive applications, e.g. in the aerospace, automotive and defence industries [1, 2]. In these cases their function is the same as that of conventional core materials, i.e. to provide separation of the facesheets at low weight. In addition, some new and multifunctional applications have been proposed to take advantage of the open cellular architecture and unit cell periodicity [3-15].

In one example, microtruss architectures have shown promise as components in armor technology, where they can modify the effects of the localized impact loading during blasts [3]. Experimental studies [4, 5] have shown that in this type of loading, the impact is carried primarily by the core of a sandwich panel (see Figure 2.1.1a), thus the back face deflection can be significantly lower than that seen on solid plates with equivalent areal
density [6-11]. This benefit has been assigned to the reduction in the impulse of the front face due to the fluid-structure interaction, and the higher flexural stiffness and strength of the sandwich [3]. In these applications, the energy of the blast is absorbed by yielding and buckling of the front facesheet, and the buckling of the microtruss core struts.

Multifunctional applications can be achieved by exploiting the mechanical properties of the microtruss material and the topology of the unit cells for an additional functionality, e.g. as conduits for wiring and piping, fuel storage, or as scaffolds for biological tissue growth [1, 12-14]. An example of a multifunctional application is the active cooling system in an aircraft wing skin, shown in Figure 2.1.1b. Experimental studies [15] have shown that in this heat exchanger system, microtruss arrangements perform as well as open cell Cu foams, because both have large internal surface areas. However, due to unit cell periodicity, the required power to pump the fluid is lower in microtruss architectures.

![Figure 2.1.1: Deformation of a sandwich panel with a pyramidal steel microtruss core after a localized impulse of magnitude 2.3 kPa-s [3] (a), and conceptualization of a microtruss sandwich panel for active heat exchange in a wing skin [14] (b).](image)
In structural applications, extended microtruss cores can be subjected to a variety of loading conditions e.g. compression/tension, bending, shear and fatigue. When used as cores for sandwich panels, flexural testing is most typical. In this test, the sandwich panel can fail by five competing failure mechanisms, dictated by the material and geometrical properties of the core, facesheet and the adhesive material (see Figure 2.1.2): facesheet yielding, buckling or indentation, core failure (e.g. shear or crushing) and facesheet-core delamination [16]. Of the above failure mechanisms, the most relevant for the microtruss materials studied in this thesis is core failure. On a strut level, this failure can occur by compressive or tensile yielding, or by strut buckling [17]. Since the buckling strength of a slender column in compression will typically be substantially lower than the tensile strength of the material, the overall mechanical properties of the microtruss are largely determined by the initiation of buckling instability.

Figure 2.1.2: Failure modes of a sandwich panel in bending: facesheet yield, facesheet buckling, facesheet indentation, and core failure [16].
2.2. Strut Design Strategies

The architecture of the unit cell and geometry of the constituent struts are critical parameters in the determination of the structural properties of a microtruss material, to such an extent that architecture-property has been described as a research frontier to the field of cellular metals [13]. The effect of these parameters on compressive strength and density, for instance, is explained in Section 3.1. However, the three techniques mentioned below can be just as important to expand the accessible material property space.

2.2.1. Microstructural design

Stretch bending is a comparatively simple microtruss fabrication technique [18-21]. A starting metal sheet with hexagonal or square perforations is placed in a forming press, whose pins displace the connective nodes of the ligaments above and below the plane of the sheet (Figure 2.2.1a). The resultant three-dimensional structure is a single microtruss layer with tetrahedral or pyramidal unit cell geometry (Figure 2.2.1b), which can subsequently be joined to facesheets or other microtruss layers to create a sandwich panel (Figure 2.2.1c).

During this process, the ligaments and connective nodes of the precursor sheet experience uniaxial tensile deformation and plastic bending around the forming pin, respectively. The tensile plastic deformation imparted to the struts has recently been shown to be a useful in-situ strengthening mechanism for microtruss struts [19, 22]. Hardness profiles of microtruss architectures fabricated from AA3003 aluminum alloy initially in the softened O temper were found to have the hardness of an H14 (half-hard) temper in the part of the strut that had bent around the forming pin curvature (i.e. the hinge), and the hardness of an H12 (quarter-hard) temper in the straight portion of the strut [19] (see Figure 2.2.2). This work hardening had a significant effect on the inelastic buckling resistance of the
struts, providing an increase of nearly a factor of two in peak compressive strength [19] (See Figure 2.2.3). However, an effort to design the struts with the intent of exploiting this microstructural evolution (work hardening and grain size reduction) has not been made.

Figure 2.2.1: A precursor sheet with hexagonal perforation in a forming press (a), resultant tetrahedral microtruss layer (b), and the node of a microtruss unit cell brazed to the facesheet of a sandwich panel (c) [18].

Figure 2.2.2: Microhardness profiles (a) and values (b) along the connective strut region, and hinge and straight portion of the strut (Regions I, II, and III respectively in (a)) [19].
2.2.2. Strut hybridization

Recent studies have shown that the strength of microtruss struts that fail by buckling can be increased significantly by the electrodeposition of a thin sleeve of nanocrystalline metal [23-25]. There are three distinct levels of structural hierarchy controlling the strength of these materials. First, there is the length scale of the crystalline building blocks making up the nanocrystalline reinforcement. Nanocrystalline materials are physically distinct from conventional polycrystalline metals because a large volume fraction of atoms exists within the grain boundaries (i.e. the interconnected network of interfaces between crystalline building blocks). Grain boundaries act as barriers to dislocation motion, thus decreasing the grain size to below 100 nm can result in very large strength increases (e.g. see reviews in [26-28]). For example, there is an order of magnitude increase in tensile strength (see Figure 2.2.4 [29]) when the grain size of conventional polycrystalline nickel is reduced from 10 µm to 10 nm. This level of structure controls the yield strength, ductility, and strain hardening ability of the electrodeposited sleeves [e.g. 26-28]. Second, there is the strut or...
ligament connectivity within the cellular architecture of the starting pre-form. This level of structure determines whether externally applied loads are resolved axially or transversely to the constituent struts or ligaments [e.g. see reviews in 30, 31]. When the connectivity is such that Maxwell’s stability criterion is not satisfied [e.g. 30], failure occurs by bending-dominated mechanisms and the specific strength increase is much lower than what can be obtained if the conformal network of nanocrystalline material satisfies the stability criterion and deformation is stretch-dominated [32]. Finally, there is a level of structure intermediate to the first two, which describes the internal shape factor of the nanocrystalline tubes. Since the electrodeposited material is optimally positioned away from the neutral bending axis of the starting pre-form micro-truss struts, even very small amounts of nanocrystalline material can have a large effect on the overall hybrid performance because of its large second moment of area [23-25].

![Tensile stress-strain curves of electrodeposited Ni alloys with grain sizes between 10 nm and 100 µm](image)

**Figure 2.2.4**: Tensile stress-strain curves of electrodeposited Ni alloys with grain sizes between 10 nm and 100 µm [29].

Electrodeposition is particularly well suited as a method for reinforcing microtruss cellular materials. It is a technologically and economically viable non-line-of-sight deposition technique; coatings or freestanding materials can be produced in virtually any
geometry having a wide range of internal nanostructures (e.g. [33]). For the case of microtruss reinforcement, a conformal shell of ultrahigh strength nanocrystalline material is electrodeposited around a conventional microtruss core, which is used as the starting cathodic template. The mobility and concentration of the depositing metal adatoms on the cathode surface can be controlled such that the nucleation of new crystalline building blocks (grains) is favoured over the growth of existing grains (e.g. [34]), thus decreasing the grain size to the nm range. Since the electrodeposited material is being optimally positioned away from the neutral bending axis of the microtruss struts, very little material is needed to provide a significant strength increase. For example, corrosion protection in the form of a 50 µm thick coating of nanocrystalline Ni (see Figure 2.2.5a [24]) was sufficient to more than double the inelastic buckling resistance of 0.6 x 1.1 x 5.8 mm plain carbon steel microtruss struts (see Figure 2.2.5b [24]).

![Figure 2.2.5](image)

*Figure 2.2.5: Cross-sectional image of a plain carbon steel strut coated with a 60 µm sleeve of nanocrystalline Ni (a), and compressive load-displacement curves of n-Ni/steel hybrids (the peak load of the uncoated strut is ≈ 4.5 kN) (b) [24].*

Since the surface area of microtruss architectures is high and the cross-sectional dimensions of microtruss struts are small, it is possible to create structurally efficient
reinforcing sleeves through processes that are typically considered surface treatments. Previously, chemical anodizing [35] and oxidizing heat treatments [36] have been applied to closed-cell aluminum foams to transform the cell wall surfaces into aluminum oxide layers. In the case of [35], an increase in compressive strength from ~7.2 MPa to ~9.8 MPa was observed after the application of the anodizing treatment, followed by an aging period of 60 days, on an aluminum foam of density 0.3 Mg/m$^3$ (relative density of ~0.1). The oxidizing heat treatment on an ALCAN foam of the same density produced an oxide thickness of 1-2 $\mu$m; the compressive curves of the oxidized samples were reported to be essentially the same as those of non-oxidized samples [36]. In addition, plasma electrolytic oxidation has been used to create oxide coatings on the ligaments of open-cell aluminum foams [37, 38]. In the case of [37], porous sleeves were created on an AA6061 Duocel foam of density 0.25 Mg/m$^3$ (relative density of 0.09); the oxidized specimens showed nearly the same compressive strength as the untreated specimens. Alternatively, in [38], a pure aluminum foam of density ~1.2 Mg/m$^3$ (relative density of ~0.4) showed an increase in compressive yield strength from ~2MPa to ~12 MPa, after the formation of an oxide layer with thickness 241 $\mu$m. Finally, chemical anodizing has been used to reinforce 3003 aluminum alloy honeycombs [39]. In this case, a compressive strength increase of 110% was reported for honeycombs of web thickness of 40 $\mu$m and oxide coating thickness of 10 $\mu$m (density of 0.06 Mg/m$^3$). This technique has not yet been used on microtruss architectures.

In both cases there is a complex relationship between the strut geometry, material properties of the two phases and their interfaces, failure mechanisms, and microtruss properties. No studies have attempted to map the synthesis-structure-architecture-properties of these hybrid materials.
2.2.3. Cross-sectional design

A third method of increasing the weight-specific properties of microtruss unit cells is to consider the cross-sectional efficiency of their strut members. Pasini et al. [40, 41] have developed a systematic method to characterize the cross-sectional efficiency of microtruss struts. By starting with an area envelope D (a rectangular area defined by the extremities of the cross-section), and a shape S (e.g. hexagonal, diamond, or circular; see Figure 2.2.6), the area and second moment of area efficiencies of the shape can be defined by the shape transformers $\psi_A = A/A_D$ and $\psi_I = I/I_D$ respectively, where $A$, $I$, $A_D$, and $I_D$ are the area and second moment of area of the shape and envelope respectively. These indices make it possible to characterize the effect of the shape efficiency of the strut on structural performance parameters (e.g. mass, stiffness, buckling load). The latter are equations of mechanics (E.M.), and can be described as separable functions of functional requirements ($F$), material properties ($M$), shape transformers ($S$), and envelope geometry ($G_D$), i.e. $E.M. = F \times M \times S \times G_D$, thus making it possible to perform shape selection analyses in mechanical properties of struts. Figure 2.2.7 shows an example of a stiffness/mass tradeoff chart for several shapes [40].

![Figure 2.2.6: Envelope (D), shape (S) and material (M) of hexagonal (a), diamond (b), and hollow hexagonal (c) cross-sections [40].](image-url)
Figure 2.2.7: Stiffness-mass property space of cross-sectional shapes [40].

The efficiency of hollow columns to resist bending is well known; for instance the improvement in bending strength exhibited by a hollow cylinder with radius $r$ and thickness $t$ compared to a solid cylinder of the same area scales with $r/t$ [16]. This sectional efficiency improvement has been exploited in several experimental studies, where hollow struts have been used as the building blocks of microtruss unit cells. For instance, a collinear layup of stainless steel cylinders was used to create microtruss architecture with a textile layup (See Figure 2.2.8a) [42] of relative densities 3%-23%. The compressive strength was found to be higher than that of pyramidal and honeycomb structures of the same density by a factor of 2.4 and 2.6 respectively. Lattices with pyramidal unit cells have been constructed by
deformation forming initially flat tubular assemblies [43], and arranging and subsequently brazing hollow stainless steel cylinders in a template [44]; see Figure 2.2.8b. The achieved relative densities were on the order of 1%-6% and provided a compressive strength improvement of factors of 2-5 compared to solid lattices of the same relative density (see Figure 2.2.8c). Similar mechanical property improvements have been achieved in hollow strut lattices created with two other synthesis methods: by casting a carbon fibre fabric around silicone rubber molds [45], and by electrodepositing a Ni sleeve around a polymer precursor [46]. However, in order to use this method to expand the achievable material property space, it is important to understand the relationship between the cross-sectional strut design (geometry), material properties (strength/ductility tradeoffs), and failure mechanisms.

Figure 2.2.8: Hollow strut microtruss architectures fabricated by a collinear arrangement of cylinders [42] (a), deformation forming of an initially flat precursor [43] (b), and compressive stress-strain curves of pyramidal architectures with hollow and solid truss members [43].
2.3. Modeling of Material Fracture

The strength of microtruss architectures fabricated with the three design strategies described above is controlled by several failure mechanisms. In this thesis, three types of models are used within the FE analyses to predict material fracture. This section provides a brief description of each, and details of their implementation within the FE code.

2.3.1. Phenomenological model of ductile fracture

The fracture of a ductile metal in forming can occur due to two main mechanisms: void nucleation, growth and coalescence, and shear fracture due to shear band localization [47]. The commercial finite element analysis software ABAQUS provides an embedded criterion to predict ductile failure, based on the integral criterion proposed by Komolgorov [48]. The condition is met when:

\[
\int \frac{d\varepsilon_{p}^{EQ}}{\varepsilon_{p,F}^{EQ}(\eta, \dot{\varepsilon}_{p})} = 1 \quad (2.3.1)
\]

where \( \varepsilon_{p}^{EQ} \) is the equivalent plastic strain and \( \varepsilon_{p,F}^{EQ}(\eta, \dot{\varepsilon}_{p}) \) is the relationship between plastic strain at fracture (\( \varepsilon_{p,F}^{EQ} \)), stress triaxiality (\( \eta \)), and plastic strain rate (\( \dot{\varepsilon}_{p}^{EQ} \)). This is a phenomenological failure model; it requires a fracture diagram (e.g. a forming limit curve) to provide the relationship between the plastic strain at fracture and stress triaxiality. This relationship can be determined through experimental failure tests at numerous stress triaxialities and strain rates (e.g. ASTM E2218 [49]), however Hooputra et al. [47] have proposed a simplified correlation of the type:

\[
\varepsilon_{p,F}^{EQ} = d_0 \exp(-c\eta) + d_1 \exp(c\eta) \quad (2.3.2)
\]

where \( d_0, d_1, \) and \( c \) are material constants that can be obtained through a smaller number of simpler standard tests (e.g. uniaxial or equibiaxial tension).
2.3.2. Microscopic model of ductile fracture

Unlike phenomenological models, microscopic damage models are based on microstructural interactions within materials that contain grains, precipitates and voids. Due to this, they are more suitable to model fracture in inhomogeneous materials. Gurson first introduced such a model to predict the influence of voids in the plastic flow of a material [50]; Tvergaard and Needleman modified it further to account for the effects of void growth [51]. The collective model is known as the Gurson-Tvergaard-Needleman (GTN) model, and is embedded within ABAQUS as a porous metal failure criterion.

The evolution of the yield surface is given by:

$$
\left( \frac{\sigma^2}{\sigma_{EQ}^2} \right) + 2f^* q_1 \cosh \left( \frac{-3q_2 \sigma_H}{2\sigma} \right) - \left( 1 + q_3 f^2 \right) = 0 \quad (2.3.3)
$$

where \( \sigma_e \) is the macroscopic von Mises stress, \( \sigma_{EQ} \) is the equivalent stress in the undamaged material, \( \sigma_H \) is the macroscopic mean stress, \( f \) is the void volume fraction, \( q_1 \), \( q_2 \), and \( q_3 \) are material parameters that are introduced to improve the agreement with numerical studies of materials containing periodically distributed circular cylindrical or spherical voids [51-53], and \( f^* \) represents the loss of load carrying capacity that accompanies coalescence. The later has the form:

$$
f^* = \begin{cases} 
  f, & \text{for } f \leq f_C \\
  f_C + \frac{f_F^* - f_C}{f_F - f_C} (f - f_C), & \text{for } f_C < f < f_F \\
  f_F^*, & \text{for } f > f_F 
\end{cases} \quad (2.3.4)
$$

where \( f_C \) is the critical void volume fraction at which coalescence starts, \( f_F \) is the final void volume fraction at fracture, and \( f_F^* = q_1 + \sqrt{q_1^2 - q_3} \). The macroscopic stress-strain
behaviour of a porous material with yield stress $\sigma_y$ is compared with the behaviour of the behaviour of the elastic-plastic matrix material in Figure 2.3.1. The porous material hardens in compression due to void closure and softens in tension due to void growth and nucleation.

In the porous metal failure criterion implemented in ABAQUS, the total change in void volume fraction ($\dot{f}$) is given in the proposed by the original Gurson model [50]:

$$\dot{f} = \dot{f}_{gr} + \dot{f}_{nuc} \quad (2.3.5)$$

where $\dot{f}_{gr}$ is the change due to the growth of existing voids, and $\dot{f}_{nuc}$ is the void nucleation rate. The change due to growth of existing voids is based on conservation of mass, and is related to the void volume fraction ($f$), and the plastic strain rate tensor $\mathbf{D}^p$ by:

$$\dot{f}_{gr} = (1 - f) \mathbf{D}^p : \mathbf{I} \quad (2.3.6)$$

The void nucleation rate ($\dot{f}_{nuc}$) is given by the strain-controlled relationship:

$$\dot{f}_{nuc} = A\dot{\varepsilon}_P^{EQ} \quad (2.3.7)$$
where $\varepsilon^{EQ}_p$ is the equivalent plastic strain in the material, and the variable $A$ is dependent on the volume fraction of nucleated voids ($f_N$) has the form:

$$A = \frac{f_N}{s_N \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \frac{\varepsilon^{EQ}_p - \varepsilon_N}{s_N} \right]$$  \hspace{1cm} (2.3.8)

The nucleation function $A/f_N$ has a normal distribution with respect to the equivalent plastic strain ($\varepsilon^{EQ}_p$); the mean value and standard deviation of this distribution is given by $\varepsilon_N$ and $s_N$ respectively (see Figure 2.3.2).

![Figure 2.3.2: Distribution of void nucleation function $A/f_N$ with respect to the equivalent plastic strain $\varepsilon^{EQ}_p$ [54].](image)

The implementation of this model requires 8 material variables to define: a) the yield surface ($q_1, q_2, q_3$), b) the void nucleation ($\varepsilon_N, s_N, f_N$), and c) failure ($f_C, f_F$). The values recommended by Tveergard and Needleman to represent most metals are $q_1 = 1.5$, $q_2 = 1$, $q_3 = 2.25$, $\varepsilon_N = 0.3$, $s_N = 0.1$, $f_N = 0.04$, $f_C = 0.15$, $f_F = 0.25$ [51]. However, in order for the model to be representative of experimental failure these values must first be calibrated for the material and loading condition; a large variation exists among studies that have used this model [e.g. review in [55]].
2.3.3. Model for ceramic fracture

In order to explain fracture initiation and propagation in ceramic materials (Section 4.3), the Hilleborg failure model [56] can be used within the ABAQUS code. This model is used to predict cracking in brittle materials which a) can support large compressive stresses but have limited ductility in tension, b) display only elastic behavior in tension, and c) have crack initiation in Mode I (uniaxial fracture), although crack propagation in Mode II (shear) is taken into account through degradation of shear stiffness [57]. For this reason, this model is good for fracture in concrete [58] and ceramics [57, 59].

The crack initiation and evolution behavior in Mode I is modeled by defining the relationship between crack opening displacement (COD) and tensile stress. In the context of this model, the COD is defined as the length ahead of the true crack, where cohesive stresses are present in the matrix (see Figure 2.3.3a [57]). The shape of the stress degradation – COD has been investigated in [58]; a bilinear relationship minimizes mesh sensitivity.

![Figure 2.3.3: Crack opening displacement and fracture process zone in the Hilleborg model (a), and relationship between the degraded stress ($s_1$), tensile failure stress ($f'_t$), fracture energy $G_F$, and COD values $w_1$ and $w_2$ (b) [57].](image)

The values $\sigma_1$, $w_1$ and $w_2$ in Figure 2.3.3b are determined from the tensile stress $\sigma_T$ and the fracture energy $G_F$ by the relationships: $\sigma_1 = \frac{\sigma_T}{4}$, $w_1 = 0.75 \frac{G_F}{\sigma_T}$, $w_2 = 5 \frac{G_F}{\sigma_T}$ [58].

In ABAQUS, this model also supports a definition of shear stiffness degradation in Mode II,
and element removal when the element’s tensile stress or shear stiffness degrades to zero (Mode I and II respectively).

2.3.4. Comparison of failure models

The failure models outlined in this section are used in the finite element studies of Chapter 4 to predict the initiation and propagation of material fracture during the stretch-bend forming fabrication of microtruss materials, and during the compression of metal/metal and metal/ceramic architectures.

The phenomenological ductile failure model is a relatively simple criterion, based on the state and rate of change of macroscopic field variables (in this case, stress and strain tensors). Its advantage is that its calibration can be performed from experimental observations of material failure under a range of stress triaxialities (e.g. a forming limit diagram (FLD)). For this reason it is widely used in to predict failure in sheet metal forming processes. One of its disadvantages is that plasticity evolution and damage are independent, resulting in an oversimplified post-failure behavior of the material, characterized by a sudden drop in stress-strain response. A further disadvantage is that a large number of experimental observations are required to calibrate the model. In this thesis, this model is used to predict failure in the stretch-bend forming process (Section 4.1.). Since failure initiation is used as a comparative formability criterion among samples with different perforation geometries, the shortcomings in predicting failure evolution have minimal consequences.

The microscopic ductile failure model is based on failure evolution due to microstructural phenomena, e.g. growth, nucleation and coalescence of voids. In it, plasticity evolution and damage propagation are coupled, thus it is ideally suited for studies where fracture propagation must be modeled accurately. However, calibration of this model
requires extensive microscopic examination of void volume fraction evolution during failure initiation and propagation. In the absence of this information, calibration of the model can be done by comparing experimental stress-strain curves of specimens of various stress-triaxialities with simulated results; the range of appropriate values for the calibration variables is thereby found by best-fitting studies. However, since the number of calibration variables is large, the calibration procedure can be computationally expensive. In this thesis, the microscopic failure model is used to simulate failure initiation and propagation during the local buckling of hollow nanocrystalline cylinders (Section 4.4.).

Finally, the ceramic failure model is used to predict fracture in the Al₂O₃ reinforcing sleeves of ceramic/metal microtruss cores. Its advantage is that it is a very simple model to calibrate; it requires only two readily obtainable material parameters: the tensile stress and fracture energy of the material. Its disadvantage is that the accuracy of the crack propagation path is highly dependent on mesh refinement.

2.4. References

3. ANALYTICAL AND FINITE ELEMENT MODELS

The purpose of this chapter is to clarify the following three issues:

1) The influence of the geometry of the column and material properties on buckling regimes (column curves or maps).

2) The connection between macroscopic strength and local stress distribution in typical microtruss strut geometries, where material nonlinearities must be taken into account.

3) The relationship between the analytically predicted instability stress and observed strength of a strut.

Section 3.1 provides the relationship between the structural properties (strength, density) of the microtruss and those of the individual struts, providing a connection between the analysis presented in this chapter and the overarching subject of this thesis. The Shanley-Engesser inelastic buckling theory is used to predict the buckling strength and define the elastic/inelastic buckling regimes of solid columns in Section 3.2. This theory is used to derive the critical buckling strength relationship of a hybrid column, consisting of a rectangular core coated with a reinforcing outer sleeve. Based on it, a map is constructed to relate non-dimensional geometrical parameters to stress state (elastic/inelastic) and ensuing buckling regimes. Finally, the local buckling of the reinforcing sleeve as a hollow column is discussed in Section 3.3. In the last two sections, finite element analyses are used to illustrate stress evolution in the presence of material nonlinearities, connecting it to the assumptions of the analytical relationships.
3.1. From Microtruss Cellular Architectures to Individual Struts

Figure 3.1.1 shows a schematic diagram of a typical microtruss core; the constituent struts are periodically arranged in pyramidal units. The mechanical properties of the extended strut assembly are controlled by those of the pyramidal unit cells, which are in turn dictated by the material and geometric properties of the constituent struts. Likewise, the density of the microtruss material is controlled by the volume fraction of open space in the unit cell and the density of the strut material. Thus, an explicit relationship can be established between the critical properties of the overall microtruss assembly and those of the individual struts.

![Figure 3.1.1: Schematic diagram of a pyramidal microtruss core.](image)

When strut buckling is the dominant failure mechanism, the critical strength of the extended microtruss assembly ($\sigma_{CR}^{MT}$) is determined by the load supported by each strut or column at instability ($P_{CR}^{COL}$), the strut angle ($\omega$), the number of struts contained in a unit cell ($N$) and the area of the microtruss unit cell ($A_{MT}$). Assuming that all struts are identical and buckle simultaneously, the internal load resolution gives the following general relationship:
\[ \sigma_{CR}^{MT} = \frac{N \left( \frac{P_{CR}^{COL}}{A_{MT}^{COL}} \right)}{A_{MT}^{COL}} = \left( N \sin \omega \right) \frac{A^{COL}}{A_{MT}^{COL}} \sigma_{CR}^{COL} \]  

(3.1.1)

where \( A^{COL} \) is the cross-sectional area of an internal strut and \( \sigma_{CR}^{COL} \) is its critical buckling strength. Figure 3.1.2 shows two typical microtruss unit cells. Their areas are proportional to the projection of a strut member on the plane raised to a power of 2, i.e. \( A^{MT} = C_i L^2 \cos^2 \omega \), where \( C_i \) is a proportionality constant that describes the geometry of the unit cell; for instance in pyramidal and tetrahedral unit cells, \( C_i = 2 \) and \( C_i = \frac{3\sqrt{3}}{2} \) respectively (see Figures 3.1.2a and 3.1.2b). Thus, the critical stress of the microtruss assumes the form:

\[ \sigma_{CR}^{MT} = \frac{N \sin \omega}{C_i \cos^2 \omega} \frac{A_{MT}^{COL}}{L^2} \sigma_{CR}^{COL} \]  

(3.1.2).

**Figure 3.1.2:** Schematic representation of two typical microtruss unit cells, pyramidal (a) and tetrahedral (b), showing the truss angle \( \omega \), strut length \( L \), strut width \( w \), strut thickness \( b \), unit cell dimension \( u \), and height \( H = L \sin \omega \). The unit cell size of the pyramidal unit cell is \( u = \sqrt{2} L \cos \omega \), whereas the tetrahedral unit cell is characterized by \( u = \sqrt{3} L \cos \omega \).

The weight of the microtruss architecture, characterized by its density \( \rho^{MT} \), is similarly related to the number of struts in a unit cell \( (N) \), the mass of each column strut
\( m_{\text{COL}} \), and the volume of the unit cell \((V_{\text{MT}})\). The general relationship between these variables is:

\[
\rho_{\text{MT}} = \frac{Nm_{\text{COL}}}{V_{\text{MT}}} = N \frac{V_{\text{COL}}}{V_{\text{MT}}} \rho_{\text{COL}} \quad (3.1.3)
\]

where \( V_{\text{COL}} \) is the volume of a column strut and \( \rho_{\text{COL}} \) is its density. The unit cell volume is the product of the unit cell area \( A_{\text{MT}} \) and height \( H \). The latter is the projection of a strut member in the vertical plane, i.e. \( H = L \sin \omega \) (see Figure 3.1.2), thus the relationship between the density of the microtruss and that of the constituent strut becomes:

\[
\rho_{\text{MT}} = \frac{N}{C_1 \cos^2 \omega \sin \omega} \frac{A_{\text{COL}}}{L^2} \rho_{\text{COL}} \quad (3.1.4)
\]

where \( C_1 \) is the constant of proportionality that describes the geometry of the unit cell mentioned earlier. Finally, the weight-specific strength of the microtruss is described by the ratio of critical strength to density, i.e.:

\[
\left( \frac{\sigma_{\text{CR}}}{\rho} \right)_{\text{MT}}^M = \sin^2 \omega \left( \frac{\sigma_{\text{CR}}}{\rho} \right)_{\text{COL}}^{\text{COL}} \quad (3.1.5).
\]

These relationships show that the structural properties of the microtruss are determined by two sets of variables: the architecture of the unit cell (i.e. parameters \( N, C_1 \) and \( \omega \)) and the characteristics of the constituent struts. The latter affect the properties of the microtruss directly, e.g. through the factor \( \frac{A_{\text{COL}}}{L^2} \), and indirectly through the variables \( \sigma_{\text{CR}}^{\text{COL}} \) and \( \rho_{\text{COL}} \), which depend both on material and geometrical characteristics.
3.2. Inelastic Buckling Failure of Conventional and Hybrid Columns

Microtruss assemblies are typically used as cores in sandwich panels. When metals are used to fabricate the components of the sandwich panel (i.e. core and facesheets), one of these three failure modes is predominant: facesheet wrinkling and yielding, core failure and core-facesheet shear [1]. Since this thesis is focused on struts within microtruss cores, core failure is the most relevant type of failure mechanism. The struts of microtruss cores generally fail in one of two ways: tensile failure or compressive failure in buckling [2]. Since the buckling strength of a slender column in compression will typically be substantially lower than the tensile strength of the material, it is the critical buckling stress of the composite strut that will largely determine the overall mechanical properties.

3.2.1. Buckling instability of conventional columns

The stability relationship for an initially straight column subjected to an axial load $P$ is:

$$M = Py \quad (3.2.1)$$

where $y(x)$ is the lateral distance from the neutral axis, and $M$ is the moment applied at a point $(x, y)$ (Figure 3.2.1a). In the elastic material region, the moment is related to the Young’s modulus ($E$) by $M = -E \frac{d^2 y}{dx^2} \int y^2 \, dA$, thus for long, slender columns, the critical bending load ($P_{CR}$) is related to material and geometrical properties by the well-known Euler equation of elastic buckling:

$$P_{CR} = \frac{k^2 \pi^2 EI}{L^2} \quad (3.2.2)$$

where $I$ and $L$ are the column’s second moment of area and length respectively. The parameter $k$ is a constant that is equal to the number of half-sine waves present in the
buckling mode and is determined by the constraint conditions of the column ends, e.g. \( k = 2 \) for rigid end conditions and \( k = 1 \) for pin end conditions. The critical stress of the column is:

\[
\sigma_{CR} = \frac{k^2 \pi^2 EI}{AL^2} = \frac{k^2 \pi^2 E}{(L/r)^2} \quad (3.2.2)
\]

where \( A \) is the cross-sectional area, \( r \) is the radius of gyration, and \( L/r \) is the slenderness ratio of the column. For a material with a given elastic modulus \( E \), long, thin columns with high slenderness ratios will buckle elastically at low \( \sigma_{CR} \). As the slenderness ratio of the column decreases, the critical stress can increase beyond the proportional limit stress \( \sigma_p \) of the material. Using Equation 3.2.3 in these cases will overestimate the critical buckling stress, because increments of stress and strain are now related by a proportionality constant that is smaller than the elastic modulus [3]. Thus, the proportional limit stress of the material marks the transition from elastic to inelastic buckling. The critical buckling stress predicted by Equation 3.2.3 is only valid in the elastic buckling regime, which is defined by the conditions \( 0 < \sigma_{CR} < \sigma_p \) and \( \frac{L}{r} > \left( \frac{L}{r} \right)_{MIN} \), where the minimum slenderness ratio \( \left( \frac{L}{r} \right)_{MIN} \) is determined by:

\[
\left( \frac{L}{r} \right)_{MIN} = k \pi \sqrt{\frac{E}{\sigma_p}} \quad (3.2.4).
\]

Due to size limitations (e.g. short length and relatively large cross-sectional area for satisfactory mechanical performance), the struts of microtruss cores typically do not fail by elastic buckling. The inelastic buckling theory proposed by Shanley [3] can be used to predict the strength of columns with intermediate or low slenderness ratios. According to this analysis [3, 4], the stress applied on the column remains uniformly distributed up to the onset of instability, at which point the column starts to bend. Continued compression causes
the stress on opposite sides of the neutral plane to be different from the mean, i.e. the stress increases faster on the concave side than on the convex side (Figure 3.2.1b). The column stability equation provides the following relationship for the critical buckling stress of an inelastic column:

\[
\sigma_{CR} = \frac{k^2 \pi^2 E_T}{(L/r)^2} \quad (3.2.5)
\]

where \( E_T \) is the value of the tangent modulus (i.e. the slope of the stress-strain curve of the material) at the stress \( \sigma = \sigma_{CR} \).

For a given constitutive behavior and column slenderness ratio, the critical stress can be calculated by finding the stress at which the two sides of Equation 3.2.5 are equal. In a work hardening material, the tangent modulus is a function of stress and can be determined from standard tensile tests [4, 5]. When the relationship \( E_T = f(\sigma) \) between the tangent modulus and stress is determined, Equation 3.2.5 provides a complete relationship between the slenderness ratio and critical stress, known as the column curve of

**Figure 3.2.1**: Schematic diagram of a straight column subjected to an axial load \( P \) (a) and schematic stress evolution during inelastic buckling (b).
the material. In many cases, it is convenient to represent the stress-strain relationship of the material as an analytical relationship; e.g. a power law of the type \( \sigma = K \varepsilon_p^n \), where \( K \) is a material constant, \( \varepsilon_p \) is the plastic strain and \( n \) is the work-hardening coefficient [6]. In this case, there exists an explicit relationship between tangent modulus and stress:

\[
E_T = \left( \frac{d \varepsilon}{d \sigma} \right)^{-1} = \left( E^{-1} + n^{-1} K \varepsilon^{-1} \sigma^{-n} \right)^{-1}
\]

(3.2.6)

thus Equation 3.2.5 can be evaluated analytically. For instance, Figure 3.2.2a shows the engineering stress-strain curve of an AA3003 aluminum alloy in the fully annealed O-temper, obtained from [7], and fitted to a power law relationship with elastic modulus \( E = 69 \) GPa, proportional limit stress \( \sigma_p = 18.9 \) MPa and parameters \( K = 201 \) MPa and \( n = 0.266 \). Also shown is the right side of Equation 3.2.5 for columns with pin end constraints \((k=1)\), and slenderness ratios \( L/r = 10, 25 \) and \( 200 \). The critical stresses in these cases are \( 76.2 \) MPa, \( 46.4 \) MPa, and \( 11.5 \) MPa respectively. The column curve of this material is presented in Figure 3.2.2b, showing that the elastic buckling regime \( (\sigma_{CR} < \sigma_p) \) occurs for \( \frac{L}{r} > \left( \frac{L}{r} \right)_{MIN} = 190 \). Columns with lower slenderness ratios buckle inelastically.

![Figure 3.2.2: Computation of the critical stress of AA3003 aluminum alloy columns with slenderness ratios of 10, 25 and 200 (a), and column curve of this alloy (b).](image-url)
3.2.2. Stress evolution and postbuckling behaviour of conventional columns during uniaxial compression

The critical stress of a column does not represent the maximum stress that can be supported by the column; rather, it is the stress at which the column starts to bend. According to the Shanley-Engesser theory [3], the initiation of bending does not imply a decrease in stress (i.e. stress reversal) in any region of the column; the stress can still increase while the column is bending (see Figure 3.2.1b). The stress evolution from buckling instability to the peak compressive load can be illustrated with the use of finite element modeling of an initially straight column.

The geometry of a column with the typical dimensions of strut members in a micro-truss assembly [8, 9] is shown in Figure 3.2.3. The column has width $w = 1.20$ mm, thickness $b = 0.72$ mm, and length $L = 5.18$ mm, giving it a slenderness ratio $L/r = \sqrt{12L/b} = 25$. The stress-strain curve was modeled after the AA3003 aluminum alloy shown in Figure 3.2.2a. The column was meshed with linear hexahedral elements with reduced integration, enhanced hourglass control, and average length of 0.05 mm (biased to produce a finer through-thickness mesh in the outer surfaces of the bending plane where the first divergence from uniform stress distribution is expected - see Figure 3.2.3); convergence studies showed that this mesh refinement was satisfactory. The upper and lower surfaces of this column were connected with tie constraints to analytical rigid surfaces, which have no rotational stiffness about the Z-axis, representing pin-jointed end conditions ($k = 1$). In order to simulate compression, the upper surface is prescribed a displacement of -0.15 mm and the bottom surface is constrained in the Y direction. To shorten the solution time, an explicit analysis with a time scale of 0.001s was performed using the commercial ABAQUS Explicit package. This procedure yielded no change in the
load-displacement curve relative to implicit analyses, and produced a low kinetic to internal energy ratio ($<10^{-4}$), thus showing that dynamic effects do not influence the results. Further details of the FE model details are provided in Appendix A.

Figure 3.2.4a shows the nominal compressive stress-strain curve of this column; the vertical (Y direction) stresses $\sigma_A$ and $\sigma_B$, determined at the centre of the concave and convex surfaces respectively (points A and B in Figure 3.2.3) are shown in Figure 3.2.4b. Initially the deformation is elastic, i.e. $\sigma_A = \sigma_B < 18.9$ MPa for compressive strains $\varepsilon < \varepsilon_1 = 2.7 \times 10^{-4}$. When the proportional limit stress of the material is exceeded, the compressive stress continues to remain uniformly distributed ($\sigma_A = \sigma_B$) throughout the cross-sectional area up to the point of critical stress: in this case $\sigma_{CR} = 48.1$ MPa at $\varepsilon_{CR} = \varepsilon_2 = 0.005$, in good agreement with the analytical model of Equation 3.2.5 ($\sigma_{CR} = 46.4$ MPa; $\varepsilon_{CR} = 0.005$). Beyond this point, there is stress bifurcation on the outer surfaces of the column in the bending plane and while the stress in both surfaces continues to increase, the rate of change is higher on the concave side (i.e. $\sigma_A > \sigma_B$ and $\frac{d\sigma_A}{d\varepsilon} > \frac{d\sigma_B}{d\varepsilon} > 0$ for $0.005 < \varepsilon < \varepsilon_3 = 0.008$). Subsequently, stress reversal occurs in the convex side when the stress ceases to increase (in this case $\frac{d\sigma_B}{d\varepsilon} < 0$ for $\varepsilon > \varepsilon_3 = 0.008$), resulting in the eventual development of tensile stresses in this surface ($\sigma_B < 0$ for $\varepsilon > \varepsilon_4 = 0.012$). Throughout this process, the average stress of the column continues to increase, i.e. the compressive stress-strain curve has a positive rate of change (see Figure 3.2.4a). A peak strength is eventually reached ($\sigma_{PK} = 59.4$ MPa at $\varepsilon_5 = 0.015$), after
which, continued deformation results in a softening behavior. Notice that the peak load-carrying capacity of the column is 1.24 times greater than the critical inelastic buckling strength. A summary of these observations is presented in Appendix B, Table B.1.

![Figure 3.2.3: Geometry and mesh of a column used for the FEA compression model, showing the width w, thickness b and length l.](image)

![Figure 3.2.4: Finite element analysis compressive stress-strain (σ-ε) curve of an AA3003 column with slenderness ratio 25 (a), and stress in the outer surfaces in the bending plane (σ_A and σ_B, corresponding to points A and B in Figure 3.2.3) (b), showing the significant strains ε₁-ε₅.](image)
3.2.3. Buckling instability of hybrid columns

The Shanley-Engesser inelastic analysis can also be extended to determine the critical buckling stress of nanocrystalline metal reinforced composites. Three assumptions are used to define the geometry of the sleeve and interfacial behavior of the two phases. First, the deposited sleeve has uniform thickness and rectangular cross-section, thus the neutral axis of the composite column is the same as that of the uncoated column. Second, it is assumed that there is sufficient interfacial adhesion to prevent debonding (i.e. isostrain conditions prevail at the sleeve-core interface in any cross-section that is normal to the applied axial stress). Finally, it is assumed that the materials of both phases have sufficient ductility to prevent crack formation, such that the failure of the reinforced strut is determined only by buckling instability.

The critical stress of this composite column can be obtained by solving Equation 3.2.1. The moment associated with the applied stress \( \sigma_s \) in this column is:

\[
M = - \int_{TOT} \sigma_s y \, dA = - \int_{CORE} \sigma_{s,C} y \, dA - \int_{SLEEVE} \sigma_{s,S} y \, dA \tag{3.2.7}
\]

where \( \sigma_{s,C} \) and \( \sigma_{s,S} \) are the stresses applied to the core and sleeve respectively, and \( dA \) is an incremental section of the cross-sectional area perpendicular to the applied stress. The applied vertical stress in the area \( dA \) is \( \sigma_s = \sigma_0 + d\sigma_s \), where \( \sigma_0 \) is the mean stress applied to the sleeve or core cross-section and \( d\sigma_s \) is positive in the concave portion of the area and negative in the convex portion (See Figure 3.2.1b). The moment then becomes:

\[
M = - \int_{CORE} (d\sigma_{s,C}) y \, dA - \int_{SLEEVE} (d\sigma_{s,S}) y \, dA \tag{3.2.8}.
\]

In a material with work hardening, the stress in the inelastic regime is related to the strain by the instantaneous or tangent modulus \( E_T \), i.e. \( d\sigma = E_T d\varepsilon \). Furthermore, according to the
small displacement theory, the strain is related to the lateral position \( y \) and bending curvature \( \frac{d^2 y}{dx^2} \) by:

\[
\varepsilon = y \frac{d^2 y}{dx^2} \quad (3.2.9)
\]

The stability relationship thus becomes:

\[
\frac{d^2 y}{dx^2} \left( E_{T,C} \int_{\text{CORE}} y^2 \, dA + E_{T,S} \int_{\text{SLEEVE}} y^2 \, dA \right) + Py = 0 \quad (3.2.10)
\]

where \( E_{T,C} \) and \( E_{T,S} \) are the tangent moduli of the core and sleeve respectively and \( \int y^2 \, dA \) represents the second moment of area \((I)\) of the cross-section of the integral. The solution of this equation provides the following relationship for the critical load:

\[
P_{CR} = \frac{k^2 \pi^2 (E_{T,S} I_S + E_{T,C} I_C)}{L^2} \quad (3.2.11).
\]

Following the sleeve/core isostrain assumption, the strain is uniformly distributed over the cross-sectional area up to the initiation of bending. As a result, the total stress is related to the stress in the sleeve and core by the rule of mixtures, and the critical stress can be found by solving the system of equations:

\[
\begin{align*}
\sigma_{CR} &= \frac{\sigma_S A_S + \sigma_C A_C}{A_S + A_C} \quad (3.2.12a) \\
\sigma_{CR} &= \frac{k^2 \pi^2 (E_{T,S} I_S + E_{T,C} I_C)}{(A_S + A_C) L^2} \quad (3.2.12b)
\end{align*}
\]

Thus, the critical stress of the coated column is determined by three groups of variables: the stress in the sleeve and core \((\sigma_S \text{ and } \sigma_C)\) at the critical strain, the stress-strain rate of change of the sleeve and core at the critical strain \((i.e. \ E_{T,S} \text{ and } E_{T,C}\), determined by the stress values and work-hardening properties of the two materials), and the geometry of the
column (i.e. cross-sectional dimensions of the core, thickness of the sleeve, and column length). Knowing the geometric characteristics of the core and sleeve (i.e. $A_S, A_C, I_S, I_C$ and $L$), and the stress-strain properties of the two materials (i.e. $E_{T,S} = f(\sigma_S)$ and $E_{T,C} = f(\sigma_C)$), Equation 3.2.12 contains two unknowns: $\sigma_S$ and $\sigma_C$. The second relationship that is used to provide a unique solution is the isostrain assumption, i.e. $\sigma_S$ and $\sigma_C$ occur at the same total strain.

Figure 3.2.5 illustrates the evaluation of the critical stress for a composite column with the same AA3003 core discussed in the previous section ($L/r = 25$, $L = 5.18$ mm, $w = 1.20$ mm, $b = 0.72$ mm) and sleeve with thicknesses $t = 0.050$ mm and $t = 0.150$ mm. The material properties of the sleeve were modeled after an electrodeposited nanocrystalline nickel (n-Ni) alloy (details in [8, 9]) and are described by $E = 145$ GPa, $\sigma_p = 446$ MPa, $K = 2355$ MPa and $n = 0.161$. Equation 3.2.12a expresses the stress of the column at any strain (i.e. stress-strain curve of the composite material) as an area fraction weighted average of the stresses in the two phases. Equation 3.2.12b evaluates the combination of the tangent moduli as a function of strain. Notice that this equation is an area fraction weighted average of the factors $\frac{k^2 \pi^2 E_T I}{AL^2}$ for the sleeve and core. It is thus subject to the sleeve and core properties independently, not on the stress-strain property of the composite material. The critical stress is determined at the strain where the two functions coincide.
3.2.4. Stress evolution and postbuckling behaviour of hybrid columns during uniaxial compression

Finite element models can also be used to illustrate the stress evolution in this composite column. The analysis method, geometry, meshing and material properties of the core column were as described previously. Reinforcing sleeves with thickness 0.010-0.050 mm were meshed with shell elements with one planar and five through-thickness integration points and had an identical mesh size to the surface of the column, shown in Figure 3.2.3. The sleeve-core interfacial surfaces were connected with tie constraints (all degrees of freedom are identical), simulating isostrain conditions. Finally, the material properties of the sleeve were based on the Hollomon constants for n-Ni.

The compressive stress-strain curve of a column with a 0.050 mm thick n-Ni sleeve, obtained from the FE simulation, is shown in Figure 3.2.6. The compressive stresses in the outer surfaces of the sleeve and the core in the bending plane are shown in Figure 3.2.7.
The deformation is elastic in both components for compressive strains $\varepsilon < \varepsilon_1 = 2.7 \times 10^{-4}$, after which the core starts to yield. Stress bifurcation (initiation of bending) occurs simultaneously in the sleeve and the core, at strain $\varepsilon = \varepsilon_2 = 0.011$. Similarly to the uncoated core, the stresses on all four surfaces continue to increase after this point; however the rate of increase is lower on the convex surfaces. Stress reversal happens simultaneously in both components at strain $\varepsilon = \varepsilon_3 = 0.012$. The peak stress is reached at a strain of $\varepsilon = \varepsilon_4 = 0.016$, at which point tensile stresses are developed in the core. Finally, tensile stresses appear in the sleeve at strain $\varepsilon = \varepsilon_5 = 0.025$. In this case, the peak compressive strength is larger than the critical stress by a factor of 1.08. A summary of these observations is presented in Appendix B, Table B.2.

In this composite column, the geometry was such that the stress of the core and sleeve at bifurcation was in the inelastic regime of their respective materials. In general, the elastic-inelastic buckling transition depends on the material properties of the two phases; the connection between geometrical properties and buckling mode can be made using an analogous treatment to the uncoated column. The material properties of the nanocrystalline Ni and AA3003 alloys used in this example define three regimes. For

$0 < \varepsilon_{CR} < \frac{\sigma_{p,AA3003}}{E_{AA3003}} = 2.7 \times 10^{-4}$, the stresses in both materials are within the proportional limit, thus the composite column fails by elastic buckling. For

$2.7 \times 10^{-4} < \varepsilon_{CR} < \frac{\sigma_{p,Ni}}{E_{Ni}} = 3.1 \times 10^{-3}$, the stresses are elastic in the sleeve and inelastic in the core. Finally, for $\varepsilon_{CR} > 3.1 \times 10^{-3}$ the stresses in both components are inelastic, and the column fails by inelastic buckling.
Figure 3.2.6: Stress-strain ($\sigma$-$\varepsilon$) compressive curve of a n-Ni/Al composite column with sleeve thickness $t = 0.050$ mm showing the significant strains $\varepsilon_1$-$\varepsilon_5$.

Figure 3.2.7: Compressive stress on the concave and convex surfaces of the core column ($\sigma_{A,C}$ and $\sigma_{B,C}$ respectively) and sleeve ($\sigma_{A,S}$ and $\sigma_{B,S}$ respectively) showing the significant strains $\varepsilon_1$-$\varepsilon_5$. 
3.2.5. Influence of sleeve and core geometry on the buckling regime

The buckling regime is determined by four geometrical parameters: the sleeve thickness $t$, the strut width $w$, the strut thickness $b$ and the column length $L$. The influence of the sleeve and core geometry on the buckling regime can be analyzed more systematically through the following modification of free variables:

1) The analysis can be simplified considerably by considering core struts with square cross-sections (i.e. $w=b$). In addition to reducing the number of variables needed to characterize the cross-sectional dimensions of the core, this practice makes sense from a mechanical performance point of view; columns with square cross-sections possess higher strength-limiting radii of gyration $r$ (defined as $r = \sqrt{\frac{I}{A}}$, where $I$ is the minimum second moment of area and $A$ is the cross-sectional area of the column).

2) The geometry of the core and sleeve components can be described through the use of non-dimensional parameters. Traditionally, the slenderness ratio ($L/r$) of a column has been chosen as a non-dimensional parameter that represents the collective effect of geometrical characteristics on the buckling strength, producing a representative column curve of the material. In the case of reinforced columns, the slenderness ratios of the core column and sleeve are $\left( \frac{L}{r} \right)_c = \frac{\sqrt{12}}{b}L$ and $\left( \frac{L}{r} \right)_s = \frac{12}{\sqrt{(b+2t)^2+b^2}}L$ respectively. Because these parameters are interdependent, they are not suitable to explain the individual effects of core and sleeve geometry on the structural properties of the hybrid. For instance, a reduced sleeve slenderness ratio is not necessarily due to an increase in sleeve thickness; it could also be due to an increase in core width. In this analysis we choose the following
two variables: the width to length ratio of the core, \( B = \frac{b}{L} \) and the thickness to length ratio of the sleeve \( T = \frac{t}{L} \). These parameters are non-dimensional, independent, and convenient to characterize the entire geometry of the two components. They are inversely proportional to the core and sleeve slenderness ratios by the following relationships:

\[
\left( \frac{L}{r} \right)_c = \frac{\sqrt{12}}{B}
\]

and

\[
\left( \frac{L}{r} \right)_s = \frac{12}{\sqrt{(B + 2T)^2 + B^2}}.
\]

Figure 3.2.8a shows the value of the critical strain at instability for n-Ni-AA3003 composite columns with a range of normalized core width \( B \) and sleeve thickness \( T \); the three dimensional grid is formed by plotting lines of constant normalized core widths and sleeve thicknesses. Long, slender columns (low \( B \)) with thin coatings (low \( T \)) fail by elastic buckling (\( \epsilon_{cr} < 2.7 \times 10^{-4} \)). In this regime, the maximum coating thickness to length ratio \( T \) for a core with thickness to length ratio \( B \) is determined by solving the system of Equations 3.2.12a and 3.2.12b with \( \sigma_c = \sigma_{P,AA3003} = 18.9 \text{ MPa} \), \( \sigma_s(\epsilon = 2.7 \times 10^{-4}) = 39.8 \text{ MPa} \), \( E_{T,s} = E_s = 145 \text{ GPa} \), and \( E_{T,c} = E_C = 69 \text{ GPa} \). The solution to this system of equations defines the region in \( T-B \) space, shown in Figure 3.2.8b, where buckling is elastic in both phases. Similarly, the boundary between the elastic-inelastic and fully inelastic buckling regions, occurring at \( \epsilon_{cr} = 3.1 \times 10^{-3} \), can be determined by solving Equations 3.2.12a and 3.2.12b with \( \sigma_c(\epsilon = 3.1 \times 10^{-3}) = 40.8 \text{ MPa} \), \( \sigma_s = 446 \text{ MPa} \), \( E_{T,s} = E_s = 145 \text{ GPa} \), and \( E_{T,c}(\epsilon = 3.1 \times 10^{-3}) = 4.10 \text{ GPa} \). The boundaries of the three buckling regimes, shown in Figure 3.2.8b, indicate that in most
practical cases, microtruss struts will fail in the inelastic regime. For instance, in the experimental research published to date [8-11] the thickness to length ratio of microtruss constituent struts was in the range $0.07<B<0.1$ and the coating thickness to strut length ratio was in the range $0.003<T<0.04$.

As a complement this discussion, the influence of the geometrical parameters $B$ and $T$ on the critical strength, density and strength/density ratio of a column is provided in Appendix C.

Figure 3.2.8: Critical strain in a n-Ni/AA3003 composite column with normalized core width $B = b/L$ and sleeve thickness $T = t/L$ (a) and the three buckling regimes defined by the magnitude of the critical strain (b).
3.3. Buckling Failure of Hollow Cylindrical Columns

3.3.1. Local buckling instability of hollow columns

Hollow cylindrical shells under uniaxial compression tend to fail by the progressive buckling of local folds, as reported in [12, 13]. Starting from the pre-buckled, fully compressed state, the equilibrium equations are [14]:

\[
\begin{align*}
\frac{d^2 M_x}{dx^2} + N \frac{d^2 v_x}{dx^2} - \frac{\dot{N}_\theta}{R} &= 0 \quad (3.3.1a) \\
\frac{d}{dx} \left[ \dot{N}_x + \frac{\dot{v}_x}{R} N \right] &= 0 \quad (3.3.1b)
\end{align*}
\]

where \( M_x, \dot{N}_x, \dot{N}_\theta \) are the rates of stress resultants shown in Figure 3.3.1, \( N \) is the load at buckling, and \( R \) is the radius of the cylinder, defined as shown in Figure 3.3.1. For simply supported edge boundary conditions (fixed radial displacement, free rotation), and in the elastic material regime, the solution of the above equations provides the following value of critical buckling stress [15]:

\[
\sigma_{CR} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R} \quad (3.3.2)
\]

where \( E \) and \( \nu \) are the elastic modulus and Poisson’s ratio of the material, and \( t \) is the cylinder’s wall thickness. When the compressive stress in the prebuckled initial state is in the inelastic regime of the material, the critical stress becomes [14]:

\[
\sigma_{CR} = \frac{2Et}{R} \left\{ 3 \left[ (5-4\nu) \frac{E}{E_T} - (1-2\nu)^2 \right] \right\}^{1/2} \quad (3.3.3)
\]

where \( E_T \) is the tangent modulus of the material at \( \sigma = \sigma_{CR} \). In the case of free edge (free radial displacement and rotation) boundary conditions, the critical inelastic buckling stress becomes [16]:

47
Since in experimental tests the cylinder and compressing surfaces have a frictional coefficient of finite value, the boundary conditions are intermediate to the simply supported and free edge assumptions, thus equations 3.3.3 and 3.3.4 provide an upper and lower predictive boundary [16].

![Figure 3.3.1: Planar (a) and through-thickness (b) stress and geometrical notation for the variables of Equation 3.3.1 [16].](image)

The transition between elastic and inelastic buckling regimes can be determined from one non-dimensional geometric variable \( \frac{R}{t} \), and the material properties. Figure 3.3.2 shows the dependence of the critical stress on the \( R/t \) ratio for hollow tubes made of the n-Ni material discussed previously in this chapter. The elastic-inelastic buckling regime boundary occurs at \( \sigma_{CR} = \sigma_p = 446 \text{ MPa} \); achieved at \( R/t = 199 \) and 99.5 for simply supported and free edge boundary conditions respectively.
3.3.2. Stress evolution and postbuckling behaviour of cylindrical columns

Similarly to solid rectangular columns, the stress evolution of a cylindrical column during uniaxial compression can be illustrated with the use of FE analyses. Figure 3.3.3a shows the geometry of a hollow tube, modeled after an experimental specimen with radius $R = 3.5$ mm, thickness $t = 0.4$ mm, and length $L = 20$ mm. The model was meshed with linear hexahedral elements with reduced integration and enhanced hourglass control. The mesh, shown in Figure 3.3.3a contained five through-thickness elements and was biased in plane to produce a finer mesh size close to the upper and lower edges where local buckling occurs (average element length ~100 µm). Mesh convergence studies indicated that this level of refinement is adequate. Compression was simulated by positioning the tube between two analytical rigid surfaces; the bottom surface remains stationary, whereas the top surface is prescribed a displacement of 0.05 mm in the $-Y$ global direction. A coefficient of friction $\mu = 0.6$, typical of steel/Ni interfaces [17] was used to model the
interaction between the rigid surfaces and the tube. Finally, the tube was assigned the material properties of n-Ni described in section 3.2.3, and the problem was solved using ABAQUS Explicit with a step time of 0.015 s. Further details on the FE model are provided in Appendix A.

Figure 3.3.3: FE model and mesh of a n-Ni hollow cylindrical column with radius $R = 3.5$ mm, thickness $t = 0.4$ mm, length $L = 20$ mm (a), and local buckling mode at a nominal strain $\varepsilon = 0.05$, showing the four probe nodes A-D (b).

The model showed an axisymmetric shell buckling mechanism, shown in Figure 3.3.3b. The nominal stress-strain curve of this model, and the stress evolution in four representative nodes (labeled A-D in Figure 3.3.3b) are shown in Figures 3.3.4a and 3.3.4b respectively. The stress is initially elastic and uniformly distributed. At a nominal strain $\varepsilon = \varepsilon_1 = 0.7 \times 10^{-3}$, bending in the outer surfaces of the cylinder starts to occur ($\sigma_C < \sigma_B < \sigma_A$). The elastic limit stress is exceeded first at node A at a strain...
\( \varepsilon = \varepsilon_2 = 2.8 \times 10^{-3} \); at this stage, the middle (reference) surface of the cylinder still has uniform stress distribution \( (\sigma_B = \sigma_D) \). The critical stress is defined as the stress in which bending occurs in the middle surface \( (\sigma_B > \sigma_D) \); the latter occurs at \( \varepsilon = \varepsilon_3 = 0.021 \). As predicted by Equations 3.3.2-3.3.4, the instability of a cylinder with this geometry \( (R/t = 9) \) will be in the inelastic regime of the material; the critical stress \( (\sigma_{CR} = 1371 \text{ MPa}) \) is in good agreement with the boundaries predicted by Equations 3.3 and 3.3.5 (1448 MPa and 1368 MPa respectively). Stress reversal first occurs in the outer surface of the cylinder (node C) at a strain value \( \varepsilon = \varepsilon_4 = 0.023 \). Finally, the peak stress and first tensile strain in the outer surface of the cylinder occur at strains \( \varepsilon = \varepsilon_5 = 0.031 \) and \( \varepsilon = \varepsilon_6 = 0.041 \) respectively. Notice that similarly to the case of solid columns, the critical stress does not signify a reduction in load carrying capacity, but the initiation of bending in the reference plane; the ratio of peak to critical stress in this case is 1.06. A summary of these results is shown in Appendix B, Table B.3.

![Figure 3.3.4: Nominal stress-strain (\( \sigma-\varepsilon \)) curve of the FE model shown in Figure 3.13 (a), and evolution of longitudinal stress in nodes A-D (b), showing the characteristic strains \( \varepsilon_1-\varepsilon_6 \).](image-url)
3.4 Summary

This chapter introduced the inelastic buckling failure mechanism in solid and hollow cylindrical struts through a combination of theoretical predictions and FE results. During the initial compression stage of a column, the compressive stress is evenly distributed in the cross-sectional area, up to a critical strain, which signifies the initiation of bending. In long, slender columns with thin coatings, this strain can be within the elastic regime of the material; in this case the column fails by elastic buckling, and the critical stress is determined by the elastic moduli of the constituent materials. In stockier columns with thicker sleeves, the critical strain can be in the inelastic regime of one or both materials; in these cases the column fails by inelastic buckling. The proportional limits of the constituent materials define ranges of dimensions where buckling is elastic in both phases, elastic in only one phase, or inelastic in both phases. Columns with the dimensions of typical microtruss struts often fail by inelastic buckling, i.e. the critical stress is not within the elastic regime of the involved materials. In this case, the critical stress is dependent on the complete stress-strain evolution of the two materials (i.e. stress and tangent modulus at a particular strain) and the geometry of the sleeve and core phases (i.e. core width, sleeve thickness and column length).

The critical stress can be predicted by the relationships developed in this chapter; however it does not signify a reduction in load-carrying capacity. At continued compression, the mean compressive stress over the cross-sectional area of the column continues to increase, however the rate of increase is higher in the concave surfaces. Subsequently, stress reversal occurs in the convex surface. Subsequently, the column exhibits a reduction in load-carrying capacity (i.e. there exists a peak stress in the global
compressive stress-strain curve). Finally, tensile stresses are developed in the convex surface of the column.

The finite element models of this chapter showed that the peak stress is larger than the analytically predicted critical stress by a factor of 1.24, 1.08, and 1.06 in the uncoated Al column, hybrid n-Ni/Al column, and hollow n-Ni column respectively. This is an important consideration, since it is the peak stress that will be measured experimentally, but it is the critical stress that will be predicted analytically. In addition, material and geometrical nonlinearities and the ductility limits of the involved materials can introduce failure modes that compete with inelastic buckling and can determine the strength of the experimental microtruss. Four such cases are discussed in Chapter 4.
3.5. References


4. EXPERIMENTAL SYSTEMS

This chapter provides finite element and theoretical analyses of four experimental examples of strut design. In Section 4.1, the work hardening imparted to the struts during fabrication is exploited for microstructural design. By modifying the strut geometry, mechanical property improvements are achieved both in density and strength. Strut hybridization is considered in Section 4.2 by adding a strengthening sleeve of electrodeposited nanocrystalline Ni to precursor struts. In this case, the architectural and material efficiency of the sleeve is shown to improve both the absolute and weight-specific strength of the initial microtruss. In Section 4.3 Al₂O₃/Al hybrid struts are shown to increase the strength of the precursor Al microtruss at virtually no weight penalty. Finally, the mechanical properties of geometrically efficient hollow columns fabricated from electrodeposited nanocrystalline Ni alloys are discussed in Section 4.4. These struts show a significant improvement in weight-specific strengths compared to microtruss architectures fabricated to date. In each case, finite element models are used to study the interactions between failure modes, determine how each influences the structural properties, and provide guidelines for improving the fabrication methods.
4.1. Effect of Work Hardening on the Buckling Resistance of Perforation Stretched Microtruss Cores

Previous studies on microtruss architectures fabricated by stretch bending have used either square [1, 2] or hexagonal [3, 4] starting sheet perforations, meaning that each strut had a constant cross-section along its length. In this section, alternate perforation geometries for the stretch bending fabrication method are considered, in order to create a strut design analysis with the following objectives:

1) Improve the geometrical efficiency of the unit cell (i.e. lower core density and higher truss angle), by increasing the formability of the precursor sheet.

2) Increase the microstructural benefits of the struts by concentrating the plastic deformation imparted during fabrication to the mid-strut where it is needed most to resist the bending moment during inelastic buckling collapse.

4.1.1. Experimental and FE model details

Two sets of aluminum alloy 3003-H14 (half-hard) perforated metal sheets (initial sheet thickness 0.80 ± 0.01 mm) were purchased from Woven Metal products Inc. (Alvin, TX). The first set of sheets was perforated in a square pattern (Figure 4.1.1a) and was used to experimentally validate forming failure under the pin, whereas the second set was perforated in a rounded-square pattern (Figure 4.1.1b) and was used to validate mid-ligament tensile failure. The square perforated precursor sheet had unit cell length \( L_U = 12.7 \text{ mm} \), uniform cross-sectional area ligament length \( l_{LS}^{SO} = 9.5 \text{ mm} \), and ligament width \( w_0 = 3.2 \text{ mm} \) (Figure 4.1.1a). The geometry of the forming press dictates that rounded-square perforations have diagonal distances \( l_D = \sqrt{2} l_{LS}^{SO} \) and unit cell length \( L_U \) identical to the square perforations, thus making the curvature radius \( R \), ligament width \( w \), and ligament...
length $l_L$ mutually dependent. Rounded square geometries are thus produced by reducing the mid-ligament width; a larger reduction in width corresponds to a larger corner curvature. Figure 4.1.1b shows a geometry with $w = 0.5w_0$. Note that the introduction of the corner curvature shortens the length of ligament having uniform cross-sectional area ($l_L^{SQ} = 5.6$ mm for $w = 0.5w_0$).

![Figure 4.1.1: Perforated sheet geometries: square ($w = w_0$) (a), and rounded square with $w = 0.5w_0$ (b), showing the unit cell length $L_U$, diagonal distance $l_D$, ligament length $l_L$, corner radius $R$, and ligament width $w$.](image)

The as-received sheets were strain-relief annealed prior to forming (600°C for 1 hour [1]) in order to increase the forming limit. Pyramidal microtruss cores were fabricated by deforming alternate nodes above and below the starting plane of the sheet, following the perforation-forming process first described by Sypeck and Wadley [3]. The forming limit was determined by the forming pin displacement at maximum load, at which point visible strut fracture developed in the blank. Vickers microhardness indentation measurements (0.49 N load) were made on longitudinal strut cross-sections.

Uniaxial compression tests were used to measure the mechanical properties of truss cores fabricated from square and rounded-square precursors, pressed to 95% of their
forming limit. A subset of each sample group was annealed to 600°C for 1 hr. to remove
the fabrication-induced work hardening and return the core material to the fully softened
state [1]. All four sets of micro-truss cores were tested using confinement plates, which
prevent lateral displacement of truss nodes in order to simulate their behaviour in a
sandwich panel (details in [2, 4]). The peak strength was taken from the maximum stress
following the initial loading slope, measured using an overall specimen compression area
of 29.1 cm² (2 × 2 pyramidal cells).

The formability and mechanical properties of a range of square and rounded-square
perforation geometries having struts widths ranging from 0.5w₀ to w₀ were studied through
finite element (FE) simulations. A schematic of the FE assembly with a square perforated
blank is shown in Figure 4.1.2a. To simulate the forming process, pins P2 and P4 were
fixed in space, whereas pins P1 and P3 were displaced in the –Y direction. Faces F1 and F3
had zero displacement in the X direction and zero rotation about the Y and Z axes, whereas
faces F2 and F4 had zero displacement in the Z direction and zero rotation about the X and
Y axes. A frictional coefficient of 0.4, typical of aluminum-steel interfaces [5], was
prescribed to model tangential interaction between the pins and the blank. The forming
simulations were solved with the commercial ABAQUS Standard package. The sheet was
meshed with linear hexahedral elements with reduced integration and enhanced hourglass
control (element C3D8R in the ABAQUS library). Figures 4.1.2b and 4.1.2c show the
element size under the pin and at mid-strut; a mesh convergence study (provided in
Appendix D) showed that this level of refinement was satisfactory.

The uniaxial plastic behaviour of the starting material was modeled by fitting
published stress-strain data of this alloy [6] to a power law of the type \( \sigma_p = K \varepsilon_p^n \), where
\( \sigma_P \) and \( \varepsilon_P \) are the true plastic stress and strain respectively, and \( K \) and \( n \) are the Hollomon fitting parameters (for the AA3003 alloy in the fully softened O-temper, \( K = 201.2 \) MPa and \( n = 0.266 \)). Anisotropic behaviour was modeled using Hill’s quadratic yield criterion [7]; the sheet was assumed to have planar isotropy with an average through-thickness Lankford ratio \( r = \frac{d\varepsilon_{yy}}{d\varepsilon_{zz}} = \frac{d\varepsilon_{yy}}{d\varepsilon_{xx}} = 0.75 \) [8].

![Diagram of the square perforated blank and mesh refinement](image)

**Figure 4.1.2**: Assembly of the square perforated blank in the FE model (a), and mesh refinement in the bending region under the pin (b) and mid-strut (c).

To investigate the mechanical properties of the fabricated blanks, the forming model at the initiation of instability was first imported to a springback analysis, where elastic strains were recovered. The results were exported to a third analysis, where the model was compressed between two analytical rigid surfaces, preserving the symmetry conditions of unit cell faces. The compressive properties of the work-hardened strut were investigated by importing both the geometry and stress/strain state to the compression simulation; the compressive properties of annealed struts were simulated by importing only
the geometry to the compression simulation. Further details on the FE model are provided in Appendix A.

4.1.2. Forming: Failure under the pin

In forming FE simulations, instability was predicted using the phenomenological ductile failure criterion described in Section 2.3.1. Due to the fact that the deformation forming process is used as a fabrication method for micro-truss structures, it is more desirable to calculate the initiation of diffuse necking rather than ultimate fracture. A reasonable estimate can therefore be taken from the theoretical predictions for the onset of instability in the metal sheet in order to determine the material constants \((d_0, d_1, c)\) of Equation 2.3.2. Instability criteria in three loading conditions were employed: uniaxial tension \((\eta = \frac{1}{3}; \varepsilon^{EQ}_{\nu,F} = n)\), plane strain \((\eta = \frac{\sqrt{3}}{3}; \varepsilon^{EQ}_{\nu,F} = \frac{2n}{3})\), and equibiaxial tension \((\eta = \frac{2}{3}; \varepsilon^{EQ}_{\nu,F} = 2n)\); these criteria provide the following estimates for the AA3003 material constants of Equation 4.1.2: \(d_0 = 2.403, d_1 = 0.004, c = 7.182\).

The experimental and simulated forming load-displacement curves of square-perforated blanks are shown in Figure 4.1.3. Both curves show an initial zone where deformation is dominated by bending of the blank around the forming pin, followed by a longer near-linear region where stretching in the unsupported region of the blank is dominant [10]. The prolonged transition between these two regions in the experimental forming curve may be due to the less stringent experimental boundary conditions; bending around the pins can occur more readily in the edge unit cells due to the loss of symmetry. The FE-predicted instability (at \(d = 6.08\) mm) occurs before the experimentally observed fracture (at \(d = 7.06 \pm 0.16\) mm); this is expected because fracture contains additional non-uniform deformation beyond the point of instability.
Figure 4.1.3: Simulated and experimental forming load (F) - displacement (d) curves of the square perforated blank.

The deformation forming process induces combined bending and stretching strains in the region of the blank under the pin, and tensile strains in the unsupported strut region. Figure 4.1.4 shows the distribution of equivalent plastic strain in the FE model of the square perforated blank at the onset of instability. The first incidence of instability occurs in the region of the blank under the pin; this failure location is in agreement with experimental observations (Figure 4.1.4 inset).

The location and magnitude of strain concentration regions can be indicated by the distribution of equivalent plastic strain across the strut surface in the symmetry cross-section (Figure 4.1.5a). The two regions of the graph (peak due to bending around the pin, and uniform distribution in the stretched region) are also observed in microhardness profiles of as-formed samples (Figure 4.1.5b). Microhardness values of strain-hardening materials can be related to their yield strength by [11]:

- Instability Initiation ($d_c = 6.08$ mm)
- Experimental Fracture ($d_c = 7.06 \pm 0.16$ mm)
where $HV$ is the microhardness value, $\sigma_Y$ is the yield strength, and $n$ is the Hollomon strain hardening coefficient (for work-hardened tempers of AA3003, $n \sim 0.06$ [6]). Assuming no path reversal effects on the flow curve of the material, the yield strength can be estimated from the flow stress:

$$\sigma_Y = \sigma_{EQ}^{0.3} = 201.2 \cdot \left(\epsilon_p^{EQ}\right)^{0.266} \text{ (MPa)}$$  (4.1.4)

where $\sigma_{EQ}$ is the equivalent flow stress, and $\epsilon_p^{EQ}$ is the equivalent plastic strain. The maximum plastic strain observed in the bending region under the pin is 0.39; this would correspond to a microhardness value of 55 HV, in good agreement with the maximum hardness value observed in experimental samples (56 HV). Similarly, the average plastic strain in the uniaxially stretched portion of the blank is $0.121 \pm 0.006$, which corresponds to a predicted hardness value of $40 \pm 1$ HV, in reasonable agreement with the experimentally observed value ($44 \pm 1$ HV).

A critical issue is that conventional perforation geometries localize the majority of deformation to the region under the pin. This limits both the architecture and microstructure of the fabricated truss core. For example, the uniaxial tensile strain in the unsupported region of the strut ($\epsilon_{UNI} = 0.121 \pm 0.006$) is significantly lower than the strain required to cause instability in tension ($\epsilon_{UNI} = n = 0.266$). Thus, the formability of the blank (and by consequence, the relative density of the resultant truss core) stands to be improved if more strain is shifted to the mid-strut.
Figure 4.1.4: Distribution of equivalent stress at instability in the square perforated blank. The inset shows the location of failure in an experimental specimen.

Figure 4.1.5: Distribution of the FE equivalent plastic strain ($\varepsilon^E_Q$) across the strut (highlighted line in the inset) (a), and experimental microhardness values of as-formed specimens along the same line (b). The dashed line in (b) is meant to guide the eye.

4.1.3. Forming: Failure in the unsupported strut

Figure 4.1.6 shows the strain distribution in the FE model of the rounded-square specimen at the onset of instability. In this case, the FE simulated instability initiation was
6.86 mm, while the experimental fracture limit was 7.47 ± 0.05 mm. Note that while the rounded-square geometry considered here exhibits only a minor improvement in formability relative to the square geometry (FE instability displacement improved by $\Delta d_I = 0.78$ mm; experimental fracture displacement improved by $\Delta d_F = 0.41 ± 0.21$ mm), the introduction of the corner curvature shifts the concentration of maximum plastic strain to mid-strut, where failure ultimately occurs (see inset in Figure 4.1.6). The distribution of plastic strain and microhardness profile across the strut is shown in Figure 4.1.7. The plastic strain concentration zones in the FE model agree with the microhardness distribution in experimental samples: the mid-strut zone has approximately the same plastic strain and hardness as the zone that bends around the pin. A quantitative comparison can be made in a similar way as for the square perforated geometry. The maximum equivalent plastic strain in the region that bends around the pin is $\varepsilon_p^{EQ} = 0.25$, corresponding to a flow stress of $\sigma_{flow}^{EQ} = 139$ MPa. Using Equation 4.1.3, this stress value would produce a hardness of 49 HV, in good agreement with the maximum hardness observed in this zone (50 HV). Similarly, the average plastic strain along the portion of the strut having constant cross-section is $\varepsilon_p^{EQ} = 0.20 ± 0.09$, corresponding to a flow stress of $\sigma_{flow}^{EQ} = 131±16$ MPa, and hardness of 46 ± 6 HV; in good agreement with the experimentally observed mid-strut hardness (47 ± 2 HV).

Finally, Figures 4.1.5a and 4.1.7a indicate distinct differences in the magnitude and distribution of plastic strain throughout the truss ligaments. First, the distribution of plastic strain in the rounded-square blank has improved: the difference between the maximum strain under the pin and at mid-strut is 0.013 in the rounded square blank and 0.342 in the square perforated blank. More importantly, however, the modified perforation geometry
has allowed the mid-strut region to achieve the expected upper limit of plastic strain, i.e. the tensile instability criterion at $\varepsilon_P^{EQ} = n = 0.266$.

Figure 4.1.6: Distribution of equivalent plastic strain ($\varepsilon_P^{EQ}$) at instability in the rounded square perforated blank. The inset shows the location of failure in an experimental specimen.

Figure 4.1.7: Distribution of the FE equivalent plastic strain ($\varepsilon_P^{EQ}$) in the rounded square perforated blank on the surface in contact with the pin at the symmetry cross section (a), and experimental microhardness values of as-formed specimens along the same line (b).
4.1.4. Forming: Effect of perforation geometry on the achievable microtruss density and imparted plastic strain

The effect of varying the perforation geometry between the \( w = w_0 \) and \( w = 0.5w_0 \) experimental limits was investigated by FE simulations. Figure 4.1.8a shows instability and fracture displacements for the entire range of rounded-square perforation geometries considered here. There are two distinct failure regimes. First, failure occurs at mid-strut for rounded square geometries having \( w \leq 0.8w_0 \). In this regime, the corner curvature is effective in improving the strain distribution across the blank and shifting the failure location to the mid-strut. This phenomenon is analogous to the formability trend that is reported in stretch-bending forming processes of conventional metal sheets: when the punch curvature is kept constant and the instability mechanism is stretch-dominated, formability improves with increasing sheet thickness [12]. For geometries with \( w \geq 0.85w_0 \), failure occurs under the pin. In this regime, the corner curvature is not effective in improving the strain distribution across the blank. Formability decreases with increasing strut width, due to the fact that the decreasing corner curvature causes the localized strain under the punch to reach the failure strain earlier.

The effect of the rounded-square perforation geometries on the limiting achievable architecture of the resulting micro-truss assembly is shown in Figure 4.1.8b. The minimum relative density (i.e. ratio of the density of truss core to the density of the parent material, \( \bar{\rho} = \frac{\rho}{\rho_{Al}} \)) is achieved for rounded-square perforations with \( w = 0.8w_0 \) (\( \bar{\rho} = 0.035 \)), showing a 32% relative decrease in density compared to conventional square-perforated geometries. Although this improvement is significant, it is only one of the benefits of modifying the perforation geometry.
Figure 4.1.8: Maximum forming displacement ($d_{\text{MAX}}$) of the range of perforation geometries considered here (a), and relative density ($\bar{\rho}$) of the resultant microtruss structures (b).

The equivalent plastic strain profiles in blanks with three perforation geometries are shown in Figure 4.1.9a. Of practical interest is the maximum plastic strain induced in the hinge (bending-dominated region) and mid-strut (tension-dominated region); this is shown in Figure 4.1.9b for the range of geometries considered in this study. The location of instability dictates the maximum plastic strain imparted in that region: perforations with $w \leq 0.8w_0$ have the same value of plastic strain at the mid-strut instability ($\varepsilon_{\text{EQ}}^p = 0.266$), and perforations with $w > 0.8w_0$ have a narrow range of plastic strain imparted to the hinge region ($\varepsilon_{\text{EQ}}^p = 0.39$ to $0.44$). The largest combination of induced plastic strain is produced in the perforation geometry that also maximizes the architectural efficiency ($w = 0.8w_0$). Here the mid-strut plastic strain is equal to the tensile instability strain, and the maximum plastic strain in the hinge exceeds that of the conventional square-perforated geometry.

From a microstructural design standpoint, the increased strain energy imparted to the strut is of considerable interest. The following section investigates the potential for exploiting the increased mid-strut plastic strain energy as a strengthening mechanism.
4.1.5. Inelastic buckling resistance

The mechanical properties of the micro-truss cores were investigated experimentally and by FE simulations; Figure 4.1.10 presents compressive stress-strain curves for $w = w_0$ and $w = 0.5w_0$ samples that were tested in the as-fabricated condition and after post-fabrication annealing. For all four sample conditions there was an initial linear stress increase followed by a transition to a peak stress $\sigma_{PK}$, at which point the individual struts failed by an inelastic buckling mechanism (see the FE and experimental insets in Figure 4.1.10). While the FE models are able to capture the overall buckling mode, they tend to over predict the experimental peak buckling stress by $\sim$20%. Part of this discrepancy likely occurs because the FE simulations do not incorporate the effect of strut imperfections on the initiation of buckling. For example, all four struts in the FE model collapse simultaneously, whereas in experimental test samples the strut properties possess statistical variations [13], which can result in gradual failure rather than collective bifurcation of the micro-truss [14, 15]. However, the more important issue concerns the
relative effect of architecture and microstructure on the mechanical performance and here there was good agreement between the FE and experimental results. For instance, the FE simulated peak strength of square perforated structures was 1.79 times greater than the strength of structures fabricated from perforations with \( w = 0.5w_0 \), which agrees well with the experimentally measured factor of 1.71 ± 0.08. In addition, the FE simulated strength ratio of work hardened to annealed cores fabricated from square precursors is 1.78, in good agreement with the experimentally observed ratio of 1.85 ± 0.07.

![Figure 4.1.10: Simulated (a) and experimental (b) compressive stress (\( \sigma \))–strain (\( \varepsilon \)) curves of annealed and work hardened (WH) structures fabricated from square (\( w = w_0 \)) and rounded square (\( w = 0.5w_0 \)) perforation geometries.](image)

Both architectural and microstructural effects therefore contribute to the range of mechanical behaviours seen in Figure 4.1.10. Architectural effects originate from the forming limits, determining the minimum relative density of the core, the strut dimensions and the truss angle, while the microstructural attributes are determined by the distribution of plastic strain imparted to the struts during fabrication. Architectural effects can be isolated by investigating the simulated compressive properties of annealed truss cores having strut widths ranging from 0.5\( w_0 \) to \( w_0 \) (Figure 4.1.11a). These specimens have the material properties of the fully softened O temper, but have a range of strut geometries and
strut angles. As would be expected, there is a general trend of decreasing peak stress with decreasing strut width (Figure 4.1.11b).

![Figure 4.1.11: Compressive stress (σ) – strain (ε) curves of annealed truss cores (a), and peak compressive strength of architectures fabricated from rounded square precursors (σ_{PK}^{RSQ}) normalized by the peak strength of structures fabricated from square precursors (σ_{PK}^{SQ}), predicted by FE models and the analytical relationship of Equations 3.1.2 and 3.2.5 (b). The inset in (b) shows the truss thickness t, length l and angle ω.](image)

For weight-sensitive applications, however, the specific strength (i.e. peak stress divided by density, \( \bar{\sigma}_{PK} = \frac{\sigma_{PK}}{\rho} \)) is a more meaningful structural performance index. In this case, the density decrease that is due to the improved formability and reduced perforation width becomes an important parameter. The highest specific strength is achieved at a strut width of \( w = 0.8w_0 \). The specific strength of these structures improves by 21.5% and 90.2% relative to cores fabricated from square and rounded blanks with \( w = w_0 \) and \( w = 0.5w_0 \), respectively.

The effect of architectural parameters on the relationship between the peak strength and strut width of annealed architectures can be extracted from Equation 3.1.2. It becomes:

\[
\frac{\sigma_{CR}^{MT,RSQ}}{\sigma_{CR}^{MT,SQ}} = \frac{\sin \omega^{RSQ}}{\sin \omega^{SQ}} \frac{A^{COL,RSQ}}{A^{COL,SQ}} \frac{\sigma_{CR}^{COL,RSQ}}{\sigma_{CR}^{COL,SQ}} \quad (4.1.5).
\]
A first approximation that neglects changes to the rotational stiffness of the joint provides a reasonable account for the effect of the strut geometry on its buckling resistance. Equation 3.2.5 becomes:

$$\frac{\sigma_{CR}^{COL,RSQ}}{\sigma_{CR}^{COL,SQ}} = \frac{E_T^{RSQ}}{E_T^{SQ}} \left( \frac{t/L}{t/L} \right)^{RSQ}$$

While the geometry and material properties (i.e. $E_T$) of the struts are not independent, the relative significance of each factor in Equations 4.1.5 and 4.1.6 can provide some insight into their effect on peak strength. On the microtruss level (Equation 4.1.5), the decreasing strut width monotonically decreases the column’s buckling critical stress and area fraction, but the strut angle assumes a maximum at $w = 0.8w_0$ (see Figure 4.1.12a). On the strut level (Equation 4.1.6), the decreasing perforation width generally decreases the slenderness ratio and increases the tangent modulus ratio (minima/maxima for variable respectively are seen for $w = 0.8w_0$ due to the improved formability; see Figure 4.1.12b), and the multiplicative effect of these parameters is a monotonic decrease in the strut’s critical stress. The cumulative effect of all geometrical and material parameters on the analytically predicted critical stress of the microtruss is shown in Figure 4.1.11b; there is a good agreement with the FE observations.

The relative amount of work hardening imparted to the micro-truss can be estimated from the distribution of plastic strain at the end of the forming process (i.e. at instability, Figure 4.1.9). Generally, perforation geometries with mid-strut failure have more cold work introduced in the strut, however the hinge region is less work hardened compared to perforation geometries where failure occurs under the punch. The effectiveness of work hardening as a strengthening mechanism is shown for all perforation geometries in Figure
4.1.13a. Note that architectures where the work hardening is concentrated in the strut are most effective in using this strengthening mechanism to increase the inelastic buckling resistance. The combined effect of architecture and microstructure on the mechanical performance of rounded square perforation geometries can be seen by plotting the weight-specific compressive strength of as-fabricated rounded square geometries, normalized by the specific strength of conventional square perforations (Figure 4.1.13b). In all but two of the rounded square perforations considered here, the benefits of low relative density and improved work hardening distribution outweigh the decrease in strut cross-sectional area and increase in strut length. The maximum relative increase in specific strength occurs for structures fabricated from perforations with $w = 0.8w_0 \left(\frac{\bar{\sigma}_{PK}}{\sigma_{PK}^{SQUARE}} = 1.53\right)$, showing that the introduction of corner curvatures can be significant in improving the architecture and microstructure of microtrusses fabricated by deformation forming.

![Figure 4.1.12: Effect of material and geometrical parameters on the buckling stress ratio of the microtruss (a) and strut column (b) as a function of perforation width reduction ($w/w_0$).](image)
Figure 4.1.13: Ratio of work-hardened to annealed compressive peak strength ($\sigma_{pk}^{WH}/\sigma_{pk}^{ANN}$) in the geometries considered in this study (a), and specific strength of work-hardened truss cores fabricated from rounded-square perforation geometries ($\sigma_{pk}^{RSQ}$), normalized by the specific strength of the square geometry ($\sigma_{pk}^{SQ}$) (b).

From a microstructural design standpoint, the increased strain energy imparted to the struts is of considerable interest. In addition to the work hardening described above, it may be possible to use the fabrication-induced strain energy to enhance the second phase precipitation in heat-treatable alloys in order to achieve greater strength than that which can be achieved by age-hardening alone (e.g. a T8 temper, as opposed to an age hardened T6 temper [6]). For example, this type of thermo-mechanical processing schedule can eliminate the time and energy required for a post-fabrication solutionizing treatment stage in AA6061 micro-truss cores [16]. It may also be possible to exploit this fabrication-induced strain energy in low stacking fault energy FCC alloys in order to increase the fraction of low-$\Sigma$ coincidence site lattice (CSL) grain boundaries. Grain boundary engineering is a thermo-mechanical processing methodology for enhancing the fraction of ‘special’ grain boundaries in engineering alloys; the formation of annealing twins are used to generate other low-$\Sigma$ CSL's in their wake [17, 18]. Grain boundary engineered micro-trusses would be expected to have greater resistance to intergranular degradation processes.
(e.g. fatigue, creep-cracking and corrosion), which are often the causes of premature and unpredictable failure.

4.1.6. Summary

The overall properties of micro-truss structures are controlled by both architectural and microstructural variables. On the architectural side this can include the truss angle and the core relative density. On the microstructural side, this includes the grain size distribution, precipitate dispersion, and dislocation density. In this study, modifying conventional square perforation geometries improved the formability and changed the failure location during fabrication from under the pin to the unsupported strut. As a result, it was possible to fabricate truss cores with lower overall relative densities and higher specific strength. However, if the work hardening introduced during fabrication is to be used as a strengthening mechanism, the set of architectural and microstructural variables are not independent. Given a perforation and press geometry, the extent of the induced cold work and the distribution of the work hardened zones depend solely on the forming displacement, which in turn determines the maximum truss angle and the minimum relative density. In other words, shape limits microstructure and microstructure limits shape.

4.1.7. Directions for future research

The objectives of this research are twofold: 1) to shift the failure location from the region under the pin to the mid-strut in order to improve the formability of the blank and decrease the density of the microtruss, and 2) to concentrate plastic strain to the mid-strut in order to improve the mechanical properties. In this study, this was achieved by introducing simple quarter-circular curvatures to the corners of the square perforation geometry. The continuity between the arc and straight portion of the strut is a G1 continuity, i.e. the arc is tangent to the straight line at the meeting point. A more sophisticated method to improve
the uniformity of the plastic strain distribution is to use G2 continuities, where different portions of the blank are both tangent to each other and have the same radius of curvature at their meeting points. Neuber [19] has shown that these types of continuities eliminate the local stress concentration that can occur at the meeting point of G1 continuities. Khalil Abad et al. [20] have used this principle to redesign the planar geometry of Nitinol stent-grafts; they have shown that it can be used to eliminate the stress concentrations that occur at the corners during the initial expansion of the stent in the artery and throughout its service life.

However, since a large stress concentration exists under the forming pin, due to the bending of the blank around its outer edges (see Figures 4.1.4 - 4.1.7), a far more successful strategy would be to redesign the geometry of the forming pin. A change in curvature can eliminate the severe stress concentration that exists due to the sharp bending of the blank around the edge. Furthermore, a change in pin radius can be used to control the location of the maximum strain concentration in the rest of the strut. Figure 4.1.14 shows an initial result of the stress distribution that can be obtained through this direction of research [21]. In general, both the formability and uniformity of stress distribution can be improved even in blanks with square perforation geometries.

The formability of perforated sheets in the stretch-bend forming process depends on three factors: the material properties of the sheet, its geometry (e.g. thickness and perforation geometry), and the geometry of the forming pin (e.g. radius, and radius of curvature). Forming maps have been constructed for the stretch-bending process of continuous metal sheets (e.g. [12]); the formability limit and location of failure is plotted as a function of the ratio of pin radius to sheet thickness for each material. Ultimately, it would be practical to construct similar maps for perforated metal sheets.
Figure 4.1.14: Distribution of von Mises stress in a perforated blank when the forming pins have hemispherical profiles [21].

4.1.8 References

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4.2. Failure Mechanisms in Metal/Metal Microtruss Architectures

Nanocrystalline electrodeposition can be used to reinforce micro-truss cellular metals, creating new types of metal/metal nanocrystalline hybrids [1-3]. For each of the hybrid systems studied to date, the metal/metal hybrids failed during uniaxial compression by a combination of inelastic buckling collapse and nanocrystalline sleeve fracture mechanisms [1-3]. In the case of aluminum micro-trusses reinforced with nanocrystalline Ni-Fe [1] or nanocrystalline Ni [2] (n-NiFe/Al and n-Ni/Al hybrids, respectively), sleeve fracture occurred before the peak inelastic buckling strength of the micro-truss cellular architecture was reached. On the other hand, there was no evidence of crack formation until well after the peak stress in plain carbon steel micro-trusses reinforced with nanocrystalline Ni (n-Ni/steel hybrid) [3]. The extent to which sleeve fracture or strut buckling controls the overall strength is not yet clear, nor is there a good model for predicting the strength of these new hybrid materials. This study addresses these issues by employing experimental and finite element methods to understand the stress evolution during compression testing, and its effect on failure mechanisms and achievable strength.

4.2.1. Experimental and FE methods

The n-Ni/Al micro-trusses were fabricated from aluminum alloy AA3003 (thickness $t_o = 0.76$ mm) precursor sheets having the same starting perforation sheet geometry as n-Ni/steel micro-trusses studied previously [3]. The 25.81 mm$^2$ square perforations were punched on a two-dimensional square lattice (unit cell size of 6.35 mm), creating an open area fraction of $\phi = 0.64$. The pyramidal cores (strut geometry shown in Figure 4.2.1a) were fabricated by deforming alternating nodes above and below the starting plane using a modified stretch-bending process, details in [10]. The resulting pyramidal architecture had
a density of 0.184 ± 0.003 Mg/m$^3$ (relative density of 6.4 %) and truss angle of $\omega = 35^\circ$.
The as-formed struts had a width $w = 1.19 \pm 0.02$ mm, thickness $t = 0.71 \pm 0.01$ mm, and length $L = 5.75 \pm 0.08$ mm (over 10 measurements each).

The as-fabricated samples were subsequently pulse current electroplated with n-Ni, after [11, 12]. The n-Ni coating thicknesses and deposition conditions were selected to match those used for the n-Ni/steel hybrids [3]. The average nanocrystalline coating thickness was determined by assuming a uniform thickness of nickel over the entire surface area of the micro-truss, i.e. $t_{Ni} = m/S_{A}\rho_{Ni}$, where $m$ is the electrodeposited mass, $S_{A}$ is the electroplated surface area, and $\rho_{Ni}$ is the density of nickel. The nominal n-Ni sleeve thicknesses $t_{Ni}$ ranged from $17.1 \pm 1.7$ µm to $47.7 \pm 1.6$ µm. The Ni nanostructure was characterized by X-ray diffraction (XRD) using Co-K$_\alpha$ radiation ($\lambda = 1.79$ nm). An average grain size of 16 nm was measured from the diffraction peak broadening, which is typical of n-Ni produced by pulse current electrodeposition [e.g. 13, 14]. Microhardness measurements (0.49 N applied load and 10 second dwell time) were made on strut cross-sections that had been mounted in epoxy and prepared using standard metallographic methods; the n-Ni sleeves had a hardness of $473 \pm 11$ HV, which is typical of pulse current deposited n-Ni [13, 14].

Mechanical properties of the Al and n-Ni/Al hybrid micro-trusses were evaluated in uniaxial compression at a crosshead displacement rate of 1 mm/min. Micro-truss cores were tested using confinement plates (i.e. recessed channels in steel compression plates that rigidly lock the truss-core nodes in place – details in Refs. [15, 16]). This test method can be used to simulate the mechanical performance that truss cores would exhibit in a sandwich structure [15, 16]. Recorded displacements, adjusted for machine compliance
based on the measured load, were used to estimate the compressive strain. Failure mechanisms in the electroformed microtruss were investigated by scanning electron microscopy (SEM) characterization of samples pre-loaded to strains of up to $\varepsilon = 0.45$.

Finite element (FE) analysis was conducted using the ABAQUS commercial package. Nanocrystalline Ni sleeves having uniform thicknesses of 10, 20, 30, 40 and 50 $\mu$m were modeled on a single strut of the AA3003 truss core having the geometry of the present study and 1006 plain carbon steel strut cores having the geometry of the n-Ni/steel micro-trusses studied previously [3]. Figure 4.2.1b illustrates the geometry, mesh and boundary conditions of the 3D hybrid strut model. Compression platens were modeled as analytical rigid surfaces; platen P1 was stationary, whereas platen P2 underwent prescribed displacement in the $-Y$ direction. Symmetry in the X-direction (i.e. zero displacement in the X-axis, and zero rotation about the Y- and Z-axes) was imposed on faces F1 and F2, and no vertical displacement was allowed for face F1. The Al and steel truss cores were meshed with linear hexahedral elements with reduced integration and enhanced hourglass control (element C3D8R in the ABAQUS library); the n-Ni sleeve was meshed with shell elements having five through-thickness integration points and one in-plane integration point. Tie constraints were used to connect the sleeve/core interface, preventing relative displacement between the two surfaces. A set of FE simulations was also conducted for the case when the sleeve and core were left untied, i.e. in the limiting case of a fully debonded sleeve-core interface. In all cases, the sleeve-platen interface was modeled with a Coulombic contact using a frictional coefficient of 0.6, typical of Ni-steel surfaces [17]. Material properties of plain carbon steel and aluminum strut core were modeled by fitting ASM published data to a power law of the type $\sigma_p = K\varepsilon_p^n$, where $\sigma_p$ and $\varepsilon_p$ are the true
plastic stress and strain respectively, and $K$ and $n$ are the Hollomon fitting parameters ($K = 201$ MPa and $n = 0.27$ for AA3003; $K = 428$ MPa and $n = 0.08$ for 1006 plain carbon steel).

A reference tensile curve for freestanding nanocrystalline Ni (electrodeposited using the same deposition conditions and having the same hardness [18]) was used to model the material properties of the sleeve, producing $K = 2355$ MPa and $n = 0.16$. The isotropic von Mises yield function was used in all simulations. To reduce the analysis time, a quasi-static explicit analysis was performed using ABAQUS Explicit. An artificial increase of the compression velocity to 1500 mm/s was found to be satisfactory, producing no change in the load-displacement curve compared to static analyses, and maintaining a negligible ratio of kinetic to internal energy (i.e., $<10^{-6}$). Further details on the FE model are provided in Appendix A.

Figure 4.2.1: Schematic diagram of the strut cross-section showing the strut length $l$, thickness $t$, height $h$ and angle $\omega$ (a), and mesh of the finite element (FE) model (b). The inset in (b) shows the FE strut assembly.
4.2.2. Experimental and FE results

The mechanical properties of the n-Ni/Al hybrid micro-trusses were evaluated in uniaxial compression. Figure 4.2.2 presents typical stress-strain curves for samples in the as-fabricated (uncoated) condition and after electrodeposition. The uncoated samples collapsed by inelastic buckling of the micro-truss struts at a peak strength of $\sigma = 2.01 \pm 0.03$ MPa. This failure mode is typical of aluminum alloy micro-truss materials having struts with intermediate slenderness ratios [19]. The peak strength increased quickly with increasing n-Ni sleeve thickness: at $t_{Ni} = 27$ µm the peak strength had increased by more than 100% to 4.52 MPa, while at $t_{Ni} = 48$ µm the peak strength increased by ~200% to 6.30 MPa. However the failure mechanism of the composite struts also changed over this thickness range. There was no evidence of mid-strut inelastic buckling for the n-Ni/Al hybrids having the thickest sleeves (e.g. $t_{Ni} = 48$ µm, Figure 4.2.3a). Instead, failure in these samples occurred by a combination of wrinkling and fracture at the hinge region of the struts. This is very different from the case of the n-Ni/steel hybrids where the peak strength was controlled by the same inelastic buckling failure mechanism as the starting (uncoated) micro-truss [3]. An intermediate case was seen for n-Ni/Al hybrids with thinner n-Ni sleeves (e.g. $t_{Ni} = 17$ µm, Figure 4.2.3b), which exhibited both inelastic buckling and plastic wrinkling/sleeve fracture at the hinge. In order to study the onset of this new hinge-dominated failure mechanism, samples were pre-loaded to the peak compressive strength and characterized by SEM. Figure 4.2.4 presents a SEM micrograph of a $t_{Ni} = 48$ µm sample pre-loaded to a compressive strain of $\epsilon = 0.22$; the onset of plastic wrinkling on the strut faces and the start of sleeve fracture at the strut edge can be seen.
Figure 4.2.2: Typical experimental compressive stress ($\sigma$)-strain ($\varepsilon$) curves of n-Ni/Al hybrids.

Figure 4.2.3: SEM characterization of n-Ni/Al hybrids with sleeve thicknesses of $t_{Ni} = 48$ $\mu$m (a) and $t_{Ni} = 17$ $\mu$m (b) preloaded to a strain $\varepsilon = 0.45$ (i.e. beyond the peak compressive load).

Figure 4.2.4: Plastic wrinkling and sleeve fracture in the hinges of a n-Ni/Al hybrid with a 48 $\mu$m thick sleeve compressed to the peak strength ($\varepsilon = 0.22$).
Figure 4.2.5 plots the weight-specific peak strength (σ/ρ) as a function of the sleeve thickness for the n-Ni/Al and n-Ni/steel hybrids. In both cases, the strength increase provided by the n-Ni sleeve exceeded the associated weight penalty. While the uncoated steel micro-truss started at a higher specific strength, the n-Ni/steel hybrids exhibited a more gradual specific strength increase with increasing coating thickness. For example, a 60 µm n-Ni coating on the steel micro-truss raised the specific strength to 17.6 MPa/(Mg/m³) (an increase of 40%), while only \( t_{Ni} = 48 \) µm was needed to increase the specific strength of the n-Ni/Al hybrid to 22.8 MPa/(Mg/m³) (an increase of 81%). Despite the higher starting specific strength of the steel micro-truss and the transition from mid-strut inelastic buckling failure to hinge-dominated wrinkling and fracture, the nanocrystalline Ni sleeves were more effective at increasing the specific strength in the n-Ni/Al hybrids than the n-Ni/steel hybrids.

![Figure 4.2.5: Experimental specific peak strength (σ/ρ) as a function of nanocrystalline sleeve thickness (t_{Ni}).](image)

FE simulated compressive stress-strain curves of the n-Ni/Al hybrids are shown in Figure 4.2.6. The starting (uncoated) Al and steel samples collapsed at peak strengths of σ
\(2.25\) MPa and \(6.61\) MPa respectively, which represents an overestimation of the experimental peak stress by a factor of \(1.12 \pm 0.02\) for the Al core and \(1.11 \pm 0.01\) for the steel core. Similar overestimation factors have been reported in studies of conventional micro-truss materials [20] and have been attributed to the fact that the FE simulation is conducted on an idealized strut model that is not subject to the imperfections that would be present in an experimental sample [21]. The FE model therefore effectively assumes that all struts in a given sample would collapse simultaneously, instead of the more gradual progression of strut collapse seen in experimental samples [22-24].

\[
\begin{align*}
\text{Figure 4.2.6: Uniaxial FE-simulated compressive stress (}\sigma\text{)-strain (}\varepsilon\text{) curves for n-Ni/Al micro-truss hybrids.}
\end{align*}
\]

The peak strength of the n-Ni/Al and n-Ni/steel hybrids increased quickly with increasing n-Ni sleeve thickness and in all cases the composite struts failed by inelastic buckling; Figure 4.2.7 presents a FE contour plot showing the axial stress distribution in a \(t_{Ni} = 50\) \(\mu\)m n-Ni/Al strut loaded to a strain of \(\varepsilon = 0.38\). The same over-prediction factor seen for the uncoated Al and steel cores was seen for the n-Ni/steel hybrids (\(1.13 \pm 0.02\)). However, the presence of the hinge-dominated failure mechanism in the n-Ni/Al hybrids
meant that these FE predictions were significantly greater than the experimentally measured peak strength. At $t_{\text{Ni}} = 17 \, \mu\text{m}$ the FE simulation over-predicted the peak strength by a factor of 1.32, while at $t_{\text{Ni}} = 47 \, \mu\text{m}$ the FE simulations over-predicted the peak strength by a factor of 1.51. The larger overestimation factor for the thicker sample is consistent with the experimentally observed transition from buckling-dominated to hinge-dominated failure mechanisms. As such, the FE simulations provide a measure of the idealized strength that the composite n-Ni/Al struts would exhibit in the absence of hinge failure, thus representing an upper bound strength for the materials and architectures of the present study.

![Figure 4.2.7: FE contour plots showing the distribution of strut longitudinal stress in the sleeve (a) and core (b) of a n-Ni/Al hybrid with sleeve thickness $t_{\text{Ni}} = 50 \, \mu\text{m}$ at a strain of $\varepsilon = 0.38$.](image)

A stress-strain curve for the case of a $t_{\text{Ni}} = 50 \, \mu\text{m}$ n-Ni/Al hybrid where the n-Ni sleeve was not connected to the Al core by tie constraints is also shown in Figure 4.2.6. This simulation allowed free tangential motion at the sleeve-core interface; plastic wrinkling in the FE simulation occurred at the same near hinge location as seen in the experimental n-Ni/Al hybrids. However, the FE composite strut still underwent inelastic buckling failure and the peak strength was only 12% lower than the strength of an
equivalent strut where the sleeve and core were tied rigidly together. Wrinkling alone is therefore not able to account for the reduced peak strength or change in overall failure mechanism seen for the thicker n-Ni/Al struts. Sleeve fracture and the concomitant loss of a continuous n-Ni sleeve load path is consequently a critical factor in reducing the peak strength from the upper limiting inelastic buckling failure mechanism.

4.2.3. Inelastic buckling failure of composite struts

The evolution in resolved axial stress along a given strut forms the basis for understanding inelastic buckling in the composite nanocrystalline struts. Similarly to the case of an uncoated column under uniaxial stress (see Section 3.2.2), in the uncoated aluminum micro-truss, there is uniform axial stress over the strut cross-section during the initial period of elastic loading. The axial stress eventually exceeds the proportional limit of the parent metal, but remains uniformly distributed until the point of stress bifurcation, i.e. the point at which the stresses at the outermost surfaces on the bending plane begin to deviate. The distribution of the compressive stress across the thickness of an uncoated Al strut beyond this point is shown in Figure 4.2.8a. As deformation continues, the overall axial stress increases, but the amount of increase is smaller on the side that will develop convex curvature and eventually tensile axial stresses. In the case of the n-Ni/Al hybrids, the sleeve axial stress is initially higher than the core axial stress by the Young’s modulus ratio of the two components. The stress remains uniform in each component until the point of bifurcation and the onset of mid-strut curvature (Figure 4.2.8b). Beyond this point both the sleeve and core undergo a similar process of stress reversal and eventually the development of tensile axial stresses on the convex side of the buckled strut.

Equation 3.2.12 can be used to evaluate the strength increase that can be expected in these struts due to the addition of the reinforcing sleeve. Figure 4.2.9a plots the increase in
critical inelastic buckling stress as a function of n-Ni coating thickness for the limiting cases of pin \((k = 1)\) and rigid \((k = 2)\) strut connections. The FE predicted bifurcation stresses, determined by tracking the stress across the mid-plane strut cross-section, are also plotted in Figure 4.2.9a. The FE bifurcation stresses fall between the \(k = 1\) and \(k = 2\) limits, which is consistent with the expectation that the actual end constraint of a micro-truss strut is intermediate to pin and rigid end conditions \([27]\). By fitting the FE bifurcation stress to the analytical model and solving for \(k\), an estimate of the effective strut end constraint can be obtained. The presence of the nanocrystalline sleeves resulted in little relative change to the effective rotational resistance of the strut ends, with only a small increase in the end constraint from the uncoated micro-trusses \((k_{Al} = 1.43\) and \(k_{steel} = 1.36\)) to the n-Ni coated micro-trusses \(k_{n-Ni/Al} = 1.48 \pm 0.03\) and \(k_{n-Ni/steel} = 1.38 \pm 0.02\). Using the respective \(k\) values for the uncoated cores therefore results in good agreement to the FE model bifurcation stress of the n-Ni/Al and n-Ni/steel hybrids (Figure 4.2.9a).

The stress at bifurcation within each component of the composite strut can also be considered. Figure 4.2.9b plots the FE bifurcation stresses within the sleeve and core as a function of coating thickness. Also shown are the predicted analytical values using the end constraints determined for the respective uncoated samples. The critical buckling stress of the core is higher in the n-Ni/steel hybrids than the n-Ni/Al hybrids, but neither stress is significantly affected by the thickness of the nanocrystalline sleeves. On the other hand, the critical stress of the n-Ni sleeves does increase with increasing sleeve thickness. This occurs because even though the tangent modulus at instability in the nanocrystalline sleeve decreases with increasing sleeve thickness (e.g. a decrease of 27% in the n-Ni/Al hybrids
from $E_{T,S} = 21$ GPa at $t_{Ni} = 10 \, \mu$m to $E_{T,S} = 15$ GPa at $t_{Ni} = 50 \, \mu$m), the sleeve moment of inertia increases much faster than the sleeve cross-sectional area. There is also a higher sleeve stress at instability when it is deposited on aluminum core rather than steel. This can be explained by considering the bifurcation strain in the composite columns. For a 50 µm thick coating, stress bifurcation occurs at a total strain of 0.010 in the n-Ni/steel hybrid and 0.018 in the n-Ni/Al hybrid. As a consequence, the longitudinal sleeve stress at bifurcation is expected to be lower in the steel core (945 MPa) than the aluminum core (1130 MPa). This means that even though it is the same electrodeposited n-Ni and essentially the same cellular architecture, the effective specific strength provided by the n-Ni sleeve is higher on the aluminum core than on the steel core (e.g. at $t_{Ni} = 50 \, \mu$m the specific strength increase is 14.1 MPa/(Mg/m$^3$) for n-Ni/Al and 6.0 MPa/(Mg/m$^3$) for n-Ni/steel).

Figure 4.2.8: Distribution of the strut longitudinal stress through its thickness (see inset) in an Al strut (a) and n-Ni/Al hybrid strut with sleeve thickness $t_{Ni} = 50 \, \mu$m (b) at the instance of stress bifurcation, peak stress and first manifestation of tensile stresses. These instances correspond to micro-truss compressive strains of 0.03, 0.15, and 0.16 for the Al core and 0.08, 0.17, and 0.32 for the n-Ni/Al hybrid (see Figure 4.2.6).
Figure 4.2.9: Analytical predictions and FE results for the average axial bifurcation stress of the composite strut (a) and bifurcation stress of the individual sleeve and core components (b) in n-Ni/Al and n-Ni/steel hybrids. The end rotational stiffness values of uncoated cores, $k = 1.36$ and $k = 1.43$ provide good agreement for coated hybrids.

Finally, although the bifurcation stress is an important parameter in that it marks the onset of buckling instability, measuring this value experimentally is somewhat difficult. On the other hand, the maximum load carrying capacity of the strut (initial peak stress) is a very simple parameter to measure experimentally and is the one typically reported in studies of micro-truss materials [e.g. 8, 16, 19]. The FE simulations can be used to gain some insight into how both the peak stress and bifurcation stress are changed by the presence of the electrodeposited nanocrystalline sleeves. Figure 4.2.10 plots the nominal bifurcation and peak stresses for the micro-truss (i.e. the vertical load supported by a single strut divided by the horizontal projected area of the cellular architecture that it supports) as a function of the n-Ni sleeve thickness for both the n-Ni/Al and n-Ni/steel hybrids. While the peak stress is always higher than the analytical bifurcation stress (by $\sim 16\%$), the increase in peak stress with n-Ni sleeve thickness is almost the same as the increase in bifurcation stress (within $\sim 2\%$). This has important consequences for simplifying the analysis of the composite micro-truss cellular materials. For example, a simple approximation was taken to model the performance of the n-Ni/steel hybrids where the
strength increase provided by the nanocrystalline reinforcement was modeled in terms of the predicted strength of a hollow tube n-Ni micro-truss undergoing inelastic buckling failure [3]. Reasonable approximations are possible using this approach because the stress supported by the core at bifurcation is not changed significantly by the presence of the sleeve (Figure 4.2.9b) and the magnitude of the peak strength increase is approximately the same as the bifurcation stress increase (Figure 4.2.10).

![Figure 4.2.10: The effect of nanocrystalline Ni sleeve thickness on the FE simulated values of micro-truss stress at bifurcation and peak.](image)

**4.2.4. Hinge dominated failure in n-Ni/Al hybrids**

Some insight into the transition from inelastic buckling to hinge-dominated failure can be seen from FE stress profiles of the near hinge region. Figure 4.2.11 plots the normal stress (i.e. the stress required to maintain adhesion) at the sleeve/core interface of a n-Ni/Al hybrid with 50 µm sleeve thickness as a function of position for micro-truss compressive strains of \( \varepsilon = \varepsilon_{PK} = 0.169 \) (i.e. strain at peak stress) and \( 0.5\varepsilon_{PK} = 0.084 \). While negligible normal stresses are developed along the straight portion of the strut, significant tensile
stresses are established at the sleeve/core interface of the hinge well before the inelastic bifurcation stress is reached, suggesting that normal separation will occur if the Ni/Al adhesive strength is insufficient to hold the surfaces together. Note that the regions of tensile stress in Figure 4.2.11 are the same locations where plastic wrinkling was seen in the experimental n-Ni/Al hybrids (Figure 4.2.4). The maximum tensile normal stress developed in the hinge at bifurcation provides some indication towards the likelihood of sleeve wrinkling prior to inelastic buckling; the maximum normal stresses increased quickly with increasing sleeve thickness: values of 29 MPa, 49 MPa, 73 MPa, 94 MPa, and 111 MPa were seen for sleeve thicknesses of 10 µm, 20 µm, 30 µm, 40 µm, and 50 µm, respectively. The potential for wrinkling is therefore highest for the n-Ni/Al hybrids having the thickest n-Ni sleeves and is one factor in contributing to the change in strut failure mechanism with increasing n-Ni sleeve thickness. Sleeve wrinkling on its own, however, is not able to account for the transition to hinge-dominated failure. FE simulations for the limiting case of a fully debonded interface showed that while sleeve/core separation occurred in the same region as the experimental samples, the peak strength was still largely determined by the inelastic buckling of the n-Ni sleeve (Figure 4.2.6). This is perhaps not unexpected, since in the absence of fracture the n-Ni sleeve would still represent a continuous path along which the nanocrystalline tube is able to transmit load.

In samples pre-loaded to the peak strength, sleeve fracture was most significant adjacent to the plastic wrinkles (e.g. Figure 4.2.4). Plotting the slope of the tangent to the stress-strain curve as a function of strain can provide an indication of when sleeve fracture occurs (Figure 4.2.12). A series of deep valleys in the tangent modulus plot (corresponding to sudden load drops) occurred in all n-Ni/Al hybrids, but was not seen for the uncoated
cores or the n-Ni/steel hybrids [3]. Furthermore, the onset of sleeve fracture occurred well before the analytical model predicted bifurcation stress. For example, the first tangent modulus indication of sleeve fracture in the $t_{Ni} = 48 \mu$m n-Ni/Al hybrid samples began at a strut axial stress of $114 \pm 7$ MPa, which is less than half of the 261 MPa predicted bifurcation stress for this sleeve thickness. Sleeve fracture would cause a reduction in the rotational resistance of the composite struts that would become proportionately more significant as the sleeve thickness increases. The present results suggest that fracture of the hinge sleeves in the thickest samples provides enough of a reduction to hinge rotation that the inelastic buckling mechanism is short-circuited altogether and failure happens instead by deformation at the strut ends rather than buckling at the strut middle.

Figure 4.2.11: Normal stress ($\sigma_N$) in the sleeve-core interface of a n-Ni/Al truss with a 50 $\mu$m thick sleeve. The stress is probed along the highlighted line shown in the inset (as a function of length $l$ over unit cell length $L_u$) at two compression strains: peak ($\varepsilon = \varepsilon_p = 0.169$), at $\varepsilon = 0.5\varepsilon_p = 0.084$. 
Figure 4.2.12: Instant modulus \(\frac{d\sigma}{d\varepsilon}\) of the uniaxial compression stress-strain curve as a function of strain \(\varepsilon\) for an uncoated Al core and \(n\)-Ni/Al hybrids with sleeve thickness \(t_{Ni} = 17\ \mu m\) and \(48\ \mu m\). The peak stress occurs when the modulus is zero.

The effect that the transition from mid-strut inelastic buckling failure to hinge-dominated sleeve fracture has on the peak strength of the micro-truss hybrids can be seen by plotting the experimentally measured peak compressive strength divided by the predicted inelastic buckling stress as a function of nanocrystalline sleeve thickness in Figure 4.2.13. The peak strength of the \(n\)-Ni/steel hybrids were all within \(~5\%\) of the predicted bifurcation stress, which is consistent with the fact that there was no evidence of tangent modulus perturbations in these materials [3]. While sleeve fracture eventually occurred in the \(n\)-Ni/steel hybrids, it did not happen until well after the composite strut had buckled and would therefore not have contributed to the experimentally measured peak compressive strength. On the other hand, the \(t_{Ni} = 17\ \mu m\) \(n\)-Ni/Al sample (undergoing both hinge and buckling failure) was \(~10\%\) below the analytical prediction and the \(t_{Ni} = 48\ \mu m\) \(n\)-Ni/Al sample (which only exhibited hinge-dominated failure) was \(~30\%\) below the inelastic buckling model predicted value. This transition is mirrored by the relative onset
point of sleeve fracture (i.e. the stress at first load drop divided by the predicted bifurcation stress). Sleeve fracture occurs before the point of buckling instability in all the n-Ni/Al hybrids. It happens at \( \sim 70\% \) of the bifurcation stress when there is still a component of inelastic buckling within the overall micro-truss failure mechanism (e.g. for \( t_{Ni} = 17 \) \( \mu \)m) but at only \( \sim 44\% \) of the bifurcation stress when inelastic buckling is not present (e.g. for \( t_{Ni} = 48 \) \( \mu \)m).

There are several materials design strategies that could be undertaken to delay the onset of sleeve fracture in the n-Ni/Al hybrids, allowing them to reach the mechanical performance predicted by the inelastic buckling model. For example, improving the adhesion at the sleeve/core interface may be one way of preventing plastic wrinkling at the hinges. While there are many factors controlling the adhesion of electrodeposited coatings, the presence of an oxide layer at the substrate/coating interface is one factor that can limit the adhesion [28, 29] and aluminum alloy substrates are generally more susceptible to this issue than plain carbon steel substrates [e.g. 28]. Strategies such as surface roughening, intermediate strike coatings, and displacement films could be used to improve the coating/sleeve adhesion [28]. For example, a zincate treatment was found to improve the adhesion between an aluminum foam core and an electrodeposited n-NiW sleeve [30]. A second design approach would be to improve the ductility of the n-Ni sleeve itself, such that the sleeve could accommodate more plastic deformation before fracture. This could be achieved by modifying the grain size distribution within the electrodeposited sleeves. For example, fine grain sized nanocrystalline Ni deposits with average grain size 10-30 nm typically result in yield strengths of 800 to 950 MPa, with tensile elongations to fracture on the order of \( \sim 5\% \) [31]. On the other hand, a much broader grain size distribution (e.g. of \( \sim 20 \) to \( \sim 200 \) nm with average of \( \sim 70 \) nm) increases the tensile elongation to the range of 15\%
to 20% at a comparatively small reduction in yield strength (~700 MPa) [31, 32]. The issues of coating adhesion and sleeve ductility will be important considerations in the development of hybrid nanocrystalline micro-truss cellular materials.

4.2.5. Summary

A complex set of failure mechanisms is involved in the structural collapse of the interconnected network of nanocrystalline tubes reinforcing conventional micro-truss cores during uniaxial compression. For the n-Ni/Al hybrids of the present study, failure included inelastic strut buckling, plastic sleeve wrinkling, delamination at the metal/metal interface, and nanocrystalline sleeve fracture. There was a transition from inelastic buckling dominated failure for the thinnest n-Ni sleeves to hinge-dominated failure without buckling for the thickest n-Ni sleeves. FE simulations showed that tensile stresses normal to the Ni/Al interface were highest in the hinge region of the struts, where plastic wrinkling (and debonding) of the sleeve/core interface was seen in the experimental samples; these stresses
were most significant for the n-Ni/Al hybrids having the thickest n-Ni sleeves. Finally, it was found that a composite inelastic column buckling approach could be used to predict the onset of structural instability in the hybrid struts and provides an estimate for the upper limiting strength of the n-Ni/Al hybrids.

4.2.6. Directions for future research

This work showed that reinforcing microtruss materials with ultra-high strength sleeves of nanocrystalline metals has the potential of increasing the weight-specific strength of the substrate and make new regions of material property space available for mechanical design. The highest strength is achieved in the absence of material fracture and sleeve/core delamination. In order to determine the conditions that are needed to avoid these failure modes, it would be useful to model their initiation and evolution. For instance, by connecting the material and geometrical properties of the sleeve with the interfacial stress state at the initiation of delamination, it will be possible to propose safe design guidelines to eliminate this failure mechanism.

Sleeve-core delamination can be modeled by introducing cohesive elements in the interface. The behavior of the cohesive elements is defined by their stiffness in the tangential and normal direction, the stress condition for initiation of relative sliding, and the traction-separation relationship after failure initiation has occurred. The stiffness of the interface can be acquired either by numerical simulations (molecular dynamics or density functional theory), or through experimental observations (e.g. nanoindentation). The conditions for the initiation and evolution of delamination can be acquired by experimental tests. One option is to use a four point bending specimen of a substrate coated with a nanocrystalline metal. The stress and strain field on the compressive side (where delamination is bound to occur) can be analyzed by digital image correlation (DIC)
methods, which are recently being developed in the Hybrid Materials Design Group. The stress field during the ductile failure on the tensile side can be used to provide experimental information to calibrate microscopic ductile failure criteria (e.g. the GTN model described in section 2.3.2). The latter can also be incorporated in the finite element code to model the sleeve fracture, and its interaction with interfacial delamination.

4.2.7. References

[18] Integran Technologies Inc., unpublished research.


4.3. Failure Mechanisms in Ceramic/Metal Microtruss Architectures

In this study, an electrochemical anodizing process is used to oxidize the surface of 3003 aluminum alloy microtruss structures, creating a reinforcing sleeve of aluminum oxide and resulting in a ceramic/metal cellular composite. The aluminum oxide provides nearly the same strength increase per unit coating thickness to the starting microtruss as was seen in the case of nanocrystalline Ni [1]. However, unlike nanocrystalline electrodeposition, the strength increase in the present case is accomplished at virtually no additional weight penalty.

4.3.1. Experimental and FE methods

The metal-ceramic composite microtruss materials were fabricated from a perforated aluminum alloy (AA3003) sheet (initial thickness $t_o = 0.78 \pm 0.01$ mm). The $25.81 \text{ mm}^2$ square perforations were punched on a two-dimensional square lattice of unit cell size $6.35 \text{ mm} \times 6.35 \text{ mm}$, creating an open area fraction of $\phi = 0.64$ (same as in [2]). The pyramidal cores were fabricated by deforming alternating nodes above and below the starting plane using a modified stretch-bending process, details in [2, 3]. A fixed out-of-plane forming displacement of 3.50 mm was used, leading to a truss height $H$ of 4.13 ± 0.06 mm, strut length $L$ of 5.30 ± 0.06 mm, strut width $w$ of 1.06 ± 0.03 mm, and strut thickness $t$ of 0.68 ± 0.05 mm, which corresponded to a truss angle $\omega$ of 33° and relative density of ~6.6%. The aluminum oxide surface layer was created using a four-step process. The first three (surface cleaning, etching, and de-smutting) were used to prepare the microtruss surfaces for anodizing. Of the three, the etching step (2 minutes in 5 wt% NaOH at 55-60 °C) was the only one that removed a significant amount of the starting microtruss substrate; the strut width and thickness after this step were 1.03 ± 0.02 mm and 0.66 ± 0.02
mm respectively. The final anodizing step was performed after the Hard Anodize Alumilite 225/226 Method [4], with a DC current density of 28 mA/cm². Anodizing times were set at 10-minute intervals for up to 60 minutes.

For each anodizing time, the thickness of the sleeve was measured from strut cross-sections using standard metallographic techniques and scanning electron microscopy (SEM). The mechanical properties of the anodic coating were evaluated by Vickers microhardness measurements at a load of 0.25 N. The Al₂O₃/Al microtruss cores were tested in compression using confinement plates, i.e. recessed channels in steel plates that rigidly lock the truss nodes in place – details in Refs. [5, 6]. This test method can be used to simulate the mechanical performance that microtruss sandwich cores would exhibit in a sandwich structure [5, 6]. Failure mechanisms in the composite microtruss cores were investigated by pre-loading samples to characteristic strain values and examining them by SEM.

The failure mechanisms and strengthening potential of the aluminum oxide coating were also investigated by finite element analyses. Al₂O₃ sleeves with uniform thicknesses of 0.01, 0.02, 0.03, 0.04 and 0.05 mm were modeled on a single strut with the geometry of the pyramidal truss cores studied here. A schematic of the assembly is shown in Figure 4.3.1a. The compression platens were modeled as analytical rigid surfaces; platen P2 was constrained in all degrees of freedom of the global XYZ coordinate system, whereas platen P1 was prescribed a vertical displacement in the –Y direction to simulate compression. To reproduce the symmetry conditions of the pyramidal unit cell, faces F1 and F2 of the strut and coating were constrained from displacement in the global X-axis and rotation about the Y and Z axes; furthermore, no vertical displacement was allowed for face F2.
The truss core was modeled with linear hexahedral elements with reduced integration and enhanced hourglass control, whereas shell elements having one in-plane and five through-thickness integration points were used to model the Al$_2$O$_3$ sleeves. The mesh of the strut and sleeve is shown in Figure 4.3.1b; mesh convergence studies showed that this level of refinement was satisfactory. To simulate the interfacial conditions between the Al$_2$O$_3$ sleeve and underlying AA3003 truss core, tie constraints were used to connect the contacting sleeve-truss surface nodes, preventing relative displacement. The commercial ABAQUS package was used to execute the FE simulations. Due to the large number of elements (5.2×10$^4$ in the core and 1.1×10$^4$ in the sleeve) a quasi-static explicit analysis was performed to speed up the analysis. An artificial increase of the compression velocity to 1500 mm/s was found to be satisfactory, producing no change in the load-displacement curve compared to static analyses, and maintaining a negligible ratio of kinetic to internal energy (i.e., <10$^{-6}$).

Figure 4.3.1: Finite element model assembly (a) and strut mesh (b), showing platens P1, P2, strut faces F1, F2, global and local coordination systems (XYZ and 123 respectively), and mid-strut locations A, B.
To model the material properties of the core, an ASM published stress-strain curve of AA3003 aluminum alloy in the fully softened O-temper [7] was fitted to the Hollomon power law $\sigma = K \varepsilon_p^n$, where $\sigma$ and $\varepsilon_p$ represent the true stress and plastic strain respectively, and $K$ and $n$ are the Hollomon fitting parameters (here $K = 201$ MPa and $n = 0.27$). In order to examine the effect of coating stiffness, the sleeve material was modeled to have elastic modulus values in the range of 50 to 130 GPa (typical values of anodized Al$_2$O$_3$ coatings [8-14]). Local coordinate systems that follow the deformation of elements were defined to capture the stress state of the model during buckling. The local coordinate system of the Al$_2$O$_3$ sleeve is shown in Figure 4.3.1a: axis 3 is normal to the surface of shell elements, axis 2 follows the outline of the length of the strut, and axis 1 follows the circumference of the sleeve.

To illustrate material failure in the sleeve, fracture was simulated using Hilleborg’s model [15]. Typical tensile strengths of Al$_2$O$_3$ coatings fabricated using the same processing methods as the present study are in the range 108-228 MPa [14]; a value of 150 MPa was chosen for the present study. In the bilinear stress-COD stress degradation model [16], a fracture energy of 100 J/m$^2$ was used to specify the area under the stress-COD curve, which is typical of concretes and Al$_2$O$_3$ composites [17]. Mode II crack propagation was defined by a linear degradation of shear stiffness to zero at a fracture strain of $w_2/50\mu m$, where $w_2$ is the crack opening displacement when the uniaxial stress is degraded to zero, and 50 $\mu m$ is the average element size in the hinge. Finally, element removal was specified when the stiffness degraded to zero from the stress degradation criterion of Mode I. Further details on the FE model are provided in Appendix A.
4.3.2. Characterization and mechanical performance of Al$_2$O$_3$/Al experimental specimens

The Al$_2$O$_3$ sleeve thickness ($t_s$) and AA3003 core thickness ($t$) are shown as a function of anodizing time in Figure 4.3.2a. The anodic film thickness increased from $t = 4.8 \pm 0.4 \mu m$ after 10 minutes of anodizing to $t = 38.5 \pm 2.7 \mu m$ after 60 minutes (an average rate of $0.62 \pm 0.08 \mu m/min$ over all samples). The cross-section of a typical microtruss strut anodized by 50 minutes ($t = 35.3 \pm 2.5 \mu m$) is shown in Figure 4.3.2b. An average microhardness of $410 \pm 10$ HV over 10 measurements was obtained in the anodic film; while the hardness depends greatly on the anodizing parameters, e.g. [19], the average obtained here falls within the range of 350 HV to 600 HV typically seen for anodic Al$_2$O$_3$ [e.g. 19-22]. The coating discontinuity at the corners of the cross-section corresponded to cracks running along the length of the strut edges. These cracks form because the volume expansion associated with anodization at the Al$_2$O$_3$/Al interface results in tensile stresses building up at regions of large convex curvature – an issue that becomes more significant with greater anodic layer thickness [20]. It is worth noting that this non-uniformity in structural reinforcement is different than that found during microtruss reinforcement by electrodeposition, where the amount of electrodeposited metal is greatest at the strut corners, since the rate of metal ion reduction is highest at regions of large convex curvature [23].

Despite the longitudinal cracks in the metal/ceramic composite struts, significant strength and modulus increases were seen with increasing coating thickness. Typical compression stress-strain curves of the as-formed, pre-anodized, and anodized microtrusses are shown in Figure 4.3.3. It is important to note that the peak compressive strength of the pre-anodized microtrusses (i.e. after the surface preparation steps, but without the ceramic
reinforcement) was 9.4% lower than the peak compressive strength of the as-formed starting structure. This decrease in strength is due to the removal of material during the etching stage: approximately 2.5% of the aluminum microtruss weight was lost after 2 minutes in the NaOH solution. The cross-sectional dimensions of the strut after the etching stage correspond to a ~10% reduction in the second moment of area $I$, which is consistent with the 9.4% reduction in peak compressive strength. Despite the material loss during the pre-anodizing stages, the strength of the starting microtruss was completely recovered with an anodic sleeve thickness of less than 4.8 $\mu$m. With greater coating thickness the microtruss strength and modulus also increased: at a thickness of 12.4 $\mu$m, the compressive strength was approximately 60% greater than the as-formed microtruss; at the greatest thickness, 38.5 $\mu$m, the compressive strength was approximately 140% greater than the as-formed microtruss. Over this same range, the compressive modulus increased by approximately 40% over that of the as-formed microtruss.

![Graph and cross-section image](image)

Figure 4.3.2: Sleeve thickness ($t_S$) and core thickness ($t$) as a function of anodizing time (a), and cross-section of an $\text{Al}_2\text{O}_3/\text{Al}$ composite microtruss strut with a sleeve thickness $t_S = 35.3 \pm 2.5 \mu m$ (b).
Figure 4.3.3: Typical uniaxial compression stress ($\sigma^{MT}$)-strain ($\varepsilon$) curves of as-formed, pre-anodized, and anodized microtruss cores with sleeve thickness $4.8 \, \mu m \leq t_S \leq 38.5 \, \mu m$.

Figure 4.3.4a presents an SEM micrograph of a typical microtruss strut (pre-anodized, loaded past the peak strength at $\varepsilon = 0.36$), exhibiting a mid-strut plastic hinge due to inelastic buckling failure. This failure mechanism was also observed for ceramic/metal microtrusses with $t_S = 4.8$ and 12.4 µm. Note that for these samples, sleeve fracture was eventually observed at comparatively large post-buckling strains (e.g. $\varepsilon > 0.04$) in the convex side of the buckled strut as a result of the induced tensile stresses. At greater coating thicknesses, however, the failure mechanism changed: there was no evidence of buckling failure, and collapse was localized to the strut ends. An example of a strut undergoing this failure mechanism is shown in Figure 4.3.4b (sample with sleeve thickness $t_S = 38.5$ µm, compressed to $\varepsilon = 0.36$). Progressive failure was observed in the form of sleeve fracture (Figure 4.3.4c) that spread along the strut length in a series of waves; the onset of sleeve fracture and the accompanying loss of load carrying capacity that each of these failure events indicates is the likely cause of the serrated stress-strain curve. From an energy absorption perspective, this failure mechanism may be desirable since the failure zone
moves progressively along the strut, akin to a ‘travelling knuckle’ mechanism [e.g. 24], and a large amount of new surface area is created within the fragmented Al₂O₃ coating.

![Figure 4.3.4: Low magnification SEM micrographs showing the overall strut failure mode for a pre-anodized aluminum core (a) and Al₂O₃/Al core with sleeve thickness tₜ = 38.5 µm (b) loaded to just after the initial peak stress. The higher magnification SEM image (c) of the Al₂O₃/Al core shows the progression of coating fracture above the hinge-front.](image)

The difference between the two strut failure mechanisms can also be seen when the slope of the stress-strain curves is plotted as a function of strain (Figure 4.3.5). The thinnest coatings exhibited a smooth transition from the initial elastic region through a peak buckling stress, which was comparable to what is seen in conventional aluminum alloy microtruss materials. On the other hand, for Al₂O₃ coating thicknesses of ~20 µm and greater, a series of small load drops could be seen in the stress-strain curve (Figure 4.3.3), which correspond to sharp valleys in the slope-strain plot of Figure 4.3.5.
4.3.3. FE analyses of failure and fracture in Al$_2$O$_3$/Al microtruss architectures

FE simulations of individual struts having the same geometry as the experimental microtrusses can provide insight into the overall failure mechanisms of the ceramic/metal microtruss. Upper and lower bound estimates for the elastic modulus of the oxide layer can be taken from previous studies of anodized Al$_2$O$_3$, which typically fall in the range of 30 to 150 GPa [8-14]. FE simulations of strut collapse for coating thicknesses of 10, 20, 30, 40 and 50 µm with elastic modulus values of 50, 90, and 130 GPa were studied. Figure 4.3.6 shows compression stress-strain curves of hybrid struts with a sleeve modulus of 50 GPa and thicknesses between 10 and 50 µm. The stress-strain curves exhibit an initial elastic region followed by non-linear deformation when the stress in the aluminum core exceeds the proportional limit. At continued deformation, strut bending is initiated at the onset of buckling instability. The strut continues to support increasing compressive loads as it starts to bend before finally exhibiting a peak strength in the overall stress-strain curve. In the
final stage of compression, the models display a softening behaviour, which is caused by a loss in load carrying capability due to continued buckling.

The onset point of buckling (critical or bifurcation stress) can be determined in FE models by monitoring the longitudinal stress on opposing surfaces of the sleeve on the bending plane, i.e. points A and B in Figure 4.3.1a. In the initial stage of compression, the strut remains straight and the longitudinal stress is uniformly distributed across a cross-sectional area perpendicular to the length of the strut, i.e. \( \sigma_{2,A} = \sigma_{2,B} \), where subscript 2 refers to the direction of the local coordinate system aligned with the length of the strut (see Figure 4.3.1a). The critical buckling strength represents the initiation of bending: at continued deformation, the stress perpendicular to the cross-section increases faster on the concave side than the convex side, i.e. \( \sigma_{2,B} > \sigma_{2,A} \). The critical stress and strain in the FE model can thus be observed by detecting the instant at which stress bifurcates on the opposite surfaces of a cross-sectional area located at mid-length. The simulated critical

![Figure 4.3.6: FE simulated stress (\( \sigma^{MT} \))-strain (\( \varepsilon \)) curves for the starting Al strut and Al\(_2\)O\(_3\)/Al composite struts with sleeve elastic modulus \( E_S = 50 \) GPa and thickness \( 10 \) µm \( \leq t_S \leq 50 \) µm.](image)
buckling stress for the present microtruss strut geometry is plotted as a function of sleeve thickness in Figure 4.3.7. For each simulated modulus value, the critical buckling stress increased with Al₂O₃ coating thickness: for instance, at a thickness of 10 µm the critical buckling stress of the hybrid increased over that of the uncoated model by factors of 2.8, 5.9, and 6.9 for sleeve modulus values of 50 GPa, 90 GPa, 130 GPa, respectively. The increased critical buckling stress is not unexpected for $E_S = 90$ GPa and 130 GPa since these values are higher than the Young’s modulus of the core material (69 GPa). However, at first it might be somewhat unexpected that a sleeve with modulus of $E_S = 50$ GPa, i.e. 27 % smaller than that of the core material, could also increase the critical buckling stress.

![Figure 4.3.7: FE predicted critical microtruss stresses ($\sigma_{CR}^{MT}$) for Al₂O₃/Al composite struts with sleeve elastic moduli ($E_S$) of 50, 90 and 130 GPa and sleeve thicknesses $0 \mu m \leq t_S \leq 50 \mu m$ (the dashed lines are meant to guide the eye).](image)

The mechanism by which a sleeve with a lower elastic modulus can increase the critical buckling stress of a strut can be understood by examining the values of tangent modulus and stress of each material at the critical strain. The dimensions of the composite struts (slenderness ratio of 25.6) are such that buckling occurs inelastically. For the present
uncoated strut, $E_{T,C}$ is calculated as 1.7 GPa. As the sleeve thickness ($t_S$) increases, the tangent modulus of the core at buckling ($E_{T,C}$) further decreases below the elastic modulus of the sleeve; from $E_{T,C} = 591$ MPa to $E_{T,C} = 360$ MPa for $t_S = 10 \, \mu\text{m}$ and $50 \, \mu\text{m}$ respectively. Furthermore, the addition of the sleeve increases the critical strain in the aluminum core component, also raising its stress above that possible in the absence of the sleeve; e.g. from 56.0 MPa (uncoated core) to 84.8 MPa and 102.2 MPa ($t_S = 10 \, \mu\text{m}$ and 50 \, \mu\text{m}, respectively). As a result of these stress magnitudes, anodized sleeves can provide a reinforcing role in $\text{Al}_2\text{O}_3$/Al composite struts, even if the sleeve modulus is at the low end of the range of reported values.

This critical buckling strain (and therefore the geometries over which a lower modulus anodized coating might strengthen an aluminum alloy microtruss) can be predicted analytically by using the composite strut buckling Equation 3.2.12, with $E_{T,S} = E_S$ (elastic modulus of the ceramic sleeve). The critical strength of the microtruss, obtained from Equation 3.1.1, is shown in Figure 4.3.8, using the theoretical limits $k = 1$ and $k = 2$ and sleeve elastic modulus $E_S = 50$ GPa. As expected, these $k$ limits bracket the critical stress obtained by the FE simulations. The actual end constraint of the starting Al strut can be estimated by using the critical buckling stress from the FE model to solve for $k$ in Equation 3.2.5, giving a value of $k = 1.4$. There is reasonable agreement between the analytical and FE-predicted critical buckling stresses of the $\text{Al}_2\text{O}_3$/Al composite struts using $k = 1.4$ (see Figure 4.3.8), but this agreement decreases with increasing sleeve thickness.

The effectiveness of Equation 3.2.12 breaks down at higher sleeve thicknesses because local shell buckling is initiated in the hinge regions of the struts. Shell buckling occurs during the uniaxial compression of thin-walled hollow columns when energy loss
can occur more readily (i.e. at lower stresses) by local bending of the section’s faces rather than by global bending of the column [25-28]. If the sleeves that are being considered here were compressed in the absence of the aluminum core, the material properties and dimensions would dictate local buckling as the preferred failure mechanism for all thicknesses. However, the Al core acts as a medium that provides resistance to the creation of local folds. Thus, the critical strength of the struts is ultimately determined by a complex interaction between the global bending of the composite strut and the local folding deformation near the hinge.

![Analytical model predictions of the microtruss stress (σ_MT) at bifurcation in an Al2O3/Al hybrid with sleeve modulus E_s = 50 GPa, for strut end constraint values k = 1.0, 1.4 and 2.0. Also shown are the FE predictions of the critical stress, and the value of the microtruss stress at the first appearance of longitudinal tensile stresses in the sleeve (σ_MT at σ_t: FE).](image)

Localized folds are regions of concentrated deformation where longitudinal tensile and compressive stresses create a high stress gradient. Figure 4.3.9 shows the distribution of longitudinal stresses in the outer shell surface of a FE model with a sleeve thickness t = 30 µm and elastic modulus E_s = 50 GPa, at the stage of compression when tensile stresses
are first developed in the sleeve. Notice that the location of the maximum tensile stress is in the same region that fracture and failure of the Al₂O₃ coating was seen experimentally (Figure 4.3.4b). These tensile stresses are developed at the hinge due to the development of local buckling folds, thus they can appear before or after stress bifurcation (i.e. global buckling) occurs in the strut.

![Figure 4.3.9: Distribution of longitudinal stress (σ_{22}; values in Pa) in the outer surface of shell elements representing an Al₂O₃ sleeve with thickness tₕ = 30 µm and elastic modulus Eₕ = 50 GPa, at the first appearance of tensile stresses (positive in this coordinate system).](image)

Some insight into the initiation of fracture in the Al₂O₃ sleeves can be obtained from the FE model. The microtruss stress at the stage of deformation in which tensile stresses are first observed in the hinge region of the sleeve is plotted as a function of coating thickness in Figure 4.3.8. Among the simulations of composite struts with Eₕ = 50 GPa, models with sleeve thicknesses of 10 and 20 µm showed tensile stresses only after the critical inelastic buckling stress had been achieved (i.e. global buckling had been initiated), whereas in all other models tensile stresses were developed in the hinge before the initiation of global buckling. In the latter models, distinct folds were observed in the hinge,
and their progression led to the sudden drops in the microtruss stress-strain curves of Figure 4.3.6.

Of these two competing failure mechanisms, the dominant one will be that which is activated at a lower stress. The interaction between buckling and sleeve fracture thus defines two failure regimes. At low sleeve thicknesses (e.g. \( t \leq 0.02 \text{ mm} \) in these simulations), the stress and strain required for inelastic buckling is small, and tensile stresses in the hinge are developed after the critical strain is reached. Thus, in this regime strut buckling is the predominant failure mode, and the critical stress and strain of the composite strut is modeled well by the analytical inelastic buckling model (see Figure 4.3.8). As the coating thickness increases (e.g. \( t > 0.02 \text{ mm} \) in these simulations), the stress and strain required for strut buckling also increases. In this regime, local buckling in the hinge can be activated before the necessary deformation for global buckling is reached (see Figure 4.3.8).

In experimental samples however, the sleeves are not capable of supporting large tensile stresses, due to the limited ductility of the ceramic coating. As a result, local buckling and the resultant formation of tensile stresses coincide with the development of cracks in the hinge region; subsequent deformation occurs by way of progressive crack propagation. An illustration of this failure propagation mechanism can be provided by the FE simulation of a strut with a 50 \( \mu \text{m} \) sleeve thickness, where the sleeve elements are assigned crack initiation and propagation properties according to the Hilleborg model [15].

The compressive stress-strain curve of this model (Figure 4.3.10) exhibits numerous load drops before and after the peak strength, similar to the experimental curves of Figure 4.3.3. These load drops correspond to element deletions, which occur when stiffness is lost due to the stress degradation from the opening of the simulated cracks, resulting in an
overall peak strength reduction of 42% when compared to the FE model without a fracture criterion. Notice also that due to the element deletion, the peak strain is 0.14, i.e. 0.53% lower than the corresponding peak strain of the model without a fracture criterion.

![Figure 4.3.10: Compressive stress ($\sigma^{MT}$) – strain ($\varepsilon$) curve of an FE simulation with a sleeve thickness of 50 µm and activated material fracture model. Points 1-4 correspond to the stress states illustrated in Figure 4.3.11.](image)

The fracture propagation mechanism can be illustrated through sequential plots of the maximum principal stress distribution in the sleeve elements (Figure 4.3.11). The principal stress first exceeds the prescribed tensile strength of the sleeve at a strain of 0.016 (Figure 4.3.11a, corresponding to point 1 in Figure 4.3.10), due to the tendency of the sleeve to fail by the formation of localized folds. Because the hinge rotates about its centre of curvature, these maximum tensile stresses are first formed on the convex surface. Elements with a maximum principal stress equal to the prescribed tensile strength of the material (150 MPa) satisfy the condition for the coalescence of microcracks ahead of the crack tip, and their load-carrying capacity progressively degrades. Figure 4.3.11b shows the stress state of the sleeve at a strain of 0.07 (point 2 in Figure 4.3.10). At this stage, the first
elements have been removed because of a complete loss of stiffness (i.e. degradation of the principal stress to a value of zero). The distribution of the maximum principal stress at the point of peak strength is shown in Figure 4.3.11c (strain of 0.14; point 3 in Figure 4.3.10), and Figure 4.3.11d shows the stress state at a post-peak strain of 0.25 (point 4 in Figure 4.3.10). The removed elements have been predominantly localized in the near-hinge region; it is significant to note that when the axial stresses were tracked at the mid-section of the strut (points A and B in Figure 4.3.1a), there was no indication of stress bifurcation either before or after the peak stress had been reached (i.e. no appreciable buckling had occurred). By contrast, in the FE model without the sleeve fracture criterion, stress bifurcation was observed at a strain of 0.28; this strain was smaller than the peak strain by 0.01. The FE models therefore suggest that the peak strength of these composite microtruss cores is controlled by localized material fracture in the hinge rather than global buckling instability, in agreement with the experimental micrographs of Figures 4.3.4b-c.

The activation and evolution of failure within the sleeve elements and its propagation down the length of the strut can be tracked by plotting the maximum principal stress as a function of nominal compressive strain in a series of elements along the length of the strut (I-V in Figure 4.3.11a). Figure 4.3.12 shows that elements within the strut connective region (such as element I) never achieve the maximum tensile stress of the material, thus the failure criterion is not activated. By contrast, elements located in the hinge and its proximity (elements II-V), satisfy the fracture initiation criterion at successively larger deformation. Stress degradation corresponding to microcrack coalescence begins at the elements that are closest to the hinge and progressively moves down the length of the strut as the overall compressive strain increases ($\varepsilon = 0.033, 0.041, 0.050$ and 0.057 for elements II, III, IV and V respectively).
Figure 4.3.11: Maximum principal stress state at strains 0.016 (a), 0.06 (b), 0.14 (c) and 0.25 (d) (points 1-4 in Figure 4.3.10). For clarity, only half of the model is shown. Elements in grey have failed and experience brief instabilities before removal.

A transition in microtruss strut failure mechanism was also seen in nanocrystalline nickel (n-Ni) reinforced Al microtrusses. In the case of n-Ni/Al microtruss cores with the same starting geometry as that used in the present study, the compressive peak strength of the hybrid was determined by global strut buckling at small coating thicknesses ($t_S = 17.1$ µm), a combination of strut buckling and hinge failure at intermediate coating thicknesses (27.2 µm and 36.8 µm), and only hinge failure at large coating thicknesses ($t_S \geq 48$ µm). However, there is an important difference in the onset of these failure mechanisms. In the
n-Ni/Al composites, hinge failure started as delamination of the sleeve from the aluminum substrate, which resulted in comparatively few larger cracks within the coating near the hinge. Because of the intrinsic ductility of the n-Ni material, the hinge-dominated failure was only possible due to the imperfect adhesion between the two components [1]. By contrast, the Al$_2$O$_3$ sleeves are well-adhered to the underlying Al substrate, but their limited ductility gives rise to crack initiation almost as soon as tensile stresses are developed in the newly-created fold, and crack propagation occurs in the form of a moving fracture front along the strut length.

![Figure 4.3.12: Maximum principal stress ($\sigma_1$) as a function of strain ($\varepsilon$) in elements I to V (see Figure 4.3.11a) of the FE model with a sleeve thickness of 50 $\mu$m and activated material fracture model.](image)

One measure of the effectiveness of a structural coating is the strength increase provided per unit of layer thickness; Figure 4.3.13a plots the n-Ni/Al and Al$_2$O$_3$/Al microtruss peak strengths as a function of their respective coating thickness. For this particular type of architecture, n-Ni is more effective than Al$_2$O$_3$ at increasing the strength: the measured rates are $\sim$90 kPa/$\mu$m for n-Ni and $\sim$70 kPa/$\mu$m for Al$_2$O$_3$. However, on a
weight-specific basis the aluminum oxide sleeve provides a significantly greater property enhancement; this can be seen in Figure 4.3.13b, which plots the overall microtruss strength ($\sigma_{PK}^{MT}$) against density ($\rho$) for the Al$_2$O$_3$/Al and n-Ni/Al hybrids. For example, at a n-Ni coating thickness of 37 µm, the n-Ni/Al trusses in [9] possessed a strength of $\sim$5.2 MPa at a density of 0.25 Mg/m$^3$ (specific strength of $\sim$21 MPa/(Mg/m$^3$)). By contrast, the microtruss reinforced with a 38 µm thick anodized sleeve had a strength of $\sim$4.6 MPa at a density of 0.17 Mg/m$^3$ (specific strength of $\sim$27 MPa/(Mg/m$^3$)); the density of this metal-ceramic composite was only 3% higher than the density of the starting core.

![Figure 4.3.13](image)

*Figure 4.3.13: Experimentally measured compressive peak strength ($\sigma_{PK}^{MT}$) as a function of coating thickness ($t_S$) for n-Ni/Al [9] and Al$_2$O$_3$/Al hybrids (a) and strength ($\sigma_{PK}^{MT}$)-density ($\rho$) property map (b).*

### 4.3.4. Summary

Metal-ceramic composite microtruss materials were created by anodizing a structural aluminum oxide coating around an AA3003 aluminum alloy starting microtruss core. This approach increased the compressive strength and elastic modulus of the cellular material considerably. Compared to the case of electrodepositing high strength nanocrystalline Ni sleeves on the same type of aluminum microtruss core, the strength
increase per unit sleeve thickness provided by anodizing was lower. However, because anodizing is a transformative surface treatment, the Al$_2$O$_3$ coating was able to achieve these performance increases with little overall weight penalty. With this processing approach, a nearly vertical path can be traced upwards through the strength-density material property space.

Scanning electron microscopy characterization revealed that the peak compressive strength of structures with very thin (<20 µm) ceramic sleeves was determined by inelastic strut buckling; at larger thicknesses, however, the struts exhibited progressive fracture of the ceramic sleeve near the hinge region. The interaction between these competing failure mechanisms and their effect on compressive strength was elucidated with the use of finite element analyses. The preferred failure mechanism of the high-strength Al$_2$O$_3$ sleeve is local shell buckling. However, the encapsulated aluminum substrate within the microtruss hybrids inhibits the cross-sectional deformation required for local folding, thus the composite strut instead fails by global buckling. At higher sleeve thicknesses, the deformation required for global buckling increases rapidly, thus localized folding at the hinges can short-circuit global buckling as the strut failure mechanism. Due to the limited tensile ductility of the ceramic component, the formation of a local fold is associated with the initiation of tensile cracks in the hinge and the propagation of cracks in the form of a moving fracture front along the length of the strut.

4.3.5. Directions for future research

The cohesive zone model used in this study to model ceramic failure is adequate to capture the initiation of fracture and locate the regions that are most susceptible to localized sleeve failure. Its formulation with the bilinear stress degradation – COD relationship has also been shown to minimize mesh sensitivity [16], thus making it adequate to model stress
propagation. However, if the crack path must be modeled accurately, it is necessary to introduce two improvements. First, it is recommended to introduce adaptive remeshing techniques to avoid the approximations in failure stress prediction and crack path evolution that are inherent through the use of a static mesh. Second, continuum elements can be used to model the ceramic sleeve in order to predict the through-thickness evolution of the fracture surface.

4.3.6. References

4.4. Structural and Material Failure in Hollow Cylindrical Columns

In this section, strut design is discussed for columns of hollow circular cross-section. Two electrodeposited Ni alloys, with average grain size of 20 nm and 100 nm, are used to fabricate cylinders with wall thickness between ~50 µm and ~400 µm. Finite element models are used to relate material and geometrical properties to the observed failure mechanisms (local buckling and fracture), and compressive strength. Finally, composite and hollow struts with nanocrystalline sleeves are compared on a basis of architectural efficiency.

4.4.1. Experimental and FE model details

Polymeric precursor cylinders with outer radius of 3.5 mm were pulse current electroplated; details of the procedure can be found in References [1, 2]. Subsequently, the polymeric precursor was removed, resulting in standalone Ni cylinders with an inner radius \( R = 3.5 \) mm. The microstructure of the Ni sleeve was analyzed by X-ray diffraction using Co K\(_\alpha\) radiation (\( \lambda = 1.79 \) mm); the average grain sizes of the two sets of samples were found to be 20 nm and 100 nm, which is typical of Ni electroplated with these conditions [3, 4]. The cross-sectional wall thickness \( t \) was determined by sectioning the cylinders and measuring the thickness of the sleeve through optical microscopy; the thickness values are provided in Table 4.4.1. Subsequently, samples of length \( L = 20 \) mm were tested in uniaxial compression using a cross-head displacement rate of 1 mm/min.

The geometry and mesh of FE models are shown in Figure 4.4.1. The dimensions of these models were identical to those of the experimental samples; details of the mesh, element type, boundary conditions, surface interaction properties, and analysis method are explained in Section 3.3.2. The material properties were modeled after the reference tensile
curves presented in [5]. The 20 nm grain size material was fit with a Voce hardening law 
\( \sigma = \sigma_x - (\sigma_x - \sigma_y) \exp(-C\varepsilon) \) with \( E = 167 \) GPa, \( \sigma_x = 1480 \) MPa, \( \sigma_y = 104 \) MPa, and \( C = 90 \) (R\(^2 = 0.991\)), whereas the 100 nm grain size material was fit by a Hollomon power law 
\( \sigma = K\varepsilon^p \) with \( E = 124 \) GPa, \( K = 951 \) MPa, and \( n = 0.101 \) (R\(^2 = 0.992\)). Further details on

Figure 4.4.1: FE assembly, showing rigid compressive surfaces, and finite element mesh of
the cylindrical model.

Ductile failure due to nucleation and coalescence of voids was modeled using the
Gurson-Tveergard-Needleman (GTN) model [6, 7]. Simulations of a standard tensile
testing coupon with the dimensions of ASTM E8 [8] were carried out to calibrate the
parameters of this model (see Equations 2.3.3-2.3.5); details of the models and calibration
results are provided in Appendix E. The parameters that best represent the post-necking
portion of the engineering stress-strain curves are \( q_1 = 1.5, q_2 = 1, q_3 = 2.25, \varepsilon_N = 0.3, S_N = 0.1, f_N = 0.14, f_{CR} = 0.15, f_F = 0.25 \) for the material with a grain size of 20 nm, and \( q_1 = 1.5, \)
\( q_2 = 1, q_3 = 2.25, \varepsilon_N = 0.3, S_N = 0.1, f_N = 0.02, f_{CR} = 0.15, f_F = 0.25 \) for the material with a grain size of 100 nm.

<table>
<thead>
<tr>
<th>Experimental</th>
<th>FE</th>
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<tbody>
<tr>
<td>( t (\mu m) )</td>
<td>( \sigma_{PK} ) (MPa)</td>
</tr>
<tr>
<td>GS = 20 nm</td>
<td></td>
</tr>
<tr>
<td>59 ± 2</td>
<td>375 ± 162</td>
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<tr>
<td>96 ± 1</td>
<td>711 ± 129</td>
</tr>
<tr>
<td>251 ± 6</td>
<td>997 ± 119</td>
</tr>
<tr>
<td>382 ± 3</td>
<td>1294 ± 93</td>
</tr>
<tr>
<td>GS = 100 nm</td>
<td></td>
</tr>
<tr>
<td>57 ± 2</td>
<td>267 ± 74</td>
</tr>
<tr>
<td>104 ± 2</td>
<td>410 ± 42</td>
</tr>
<tr>
<td>166 ± 2</td>
<td>561 ± 27</td>
</tr>
<tr>
<td>372 ± 3</td>
<td>654 ± 21</td>
</tr>
</tbody>
</table>

Table 4.4.1: Wall thickness (\( t \)), peak compressive strength (\( \sigma_{PK} \)), and failure mechanisms in experimental samples and FE models of cylinders with grain size (GS) of 20 nm and 100 nm.

4.4.2. Local buckling failure

The nominal compressive stress (\( \sigma \))-strain (\( \varepsilon \)) curves of experimental samples with a grain size of 20 nm and 100 nm are shown in Figure 4.4.2. The peak strength and local buckling modes are shown in Table 4.4.1. The post-buckling mechanisms followed the classical local shell buckling shapes exhibited by cylindrical shells [9]; the number of lobes in each fold decreased from four to zero (denoting an axisymmetric fold) as the wall thickness increased from \( \approx 57 \, \mu m \) to \( \approx 380 \, \mu m \). In samples with a grain size of 20 nm and thicknesses \( t = 251 \pm 6 \, \mu m \) and \( 382 \pm 3 \, \mu m \) fracture occurred before the symmetry of the first lobe could be detected, and further deformation proceeded by opening of the fracture surfaces instead of progressive local buckling. Figure 4.4.3a shows the four fold local buckling mechanism in a sample with a grain size of 20 nm and wall thickness of 59 \( \mu m \) at a strain of \( \approx 0.14 \).
The symmetry of the local buckling modes developed in FE simulations was similar to that observed in experimental tests (See Table 4.4.1). Figure 4.4.3b shows the deformation of a model with the material properties of n-Ni with a grain size of 20 nm, and wall thickness of 59 nm. Figure 4.4.4 shows the nominal compressive stress-strain curves of FE simulations with the same thickness as those of experimental samples. These curves show the same general characteristics as the experimental curves: there is a peak strength followed by a softening regime that occurs due to local buckling.

![Figure 4.4.2: Experimental nominal stress (σ)-strain (ε) curves of cylinders with wall thickness 57µm ≤ t ≤ 382 µm and grain size of 20 nm (a) and 100 nm (b).](image1)

*Figure 4.4.2: Experimental nominal stress (σ)-strain (ε) curves of cylinders with wall thickness 57µm ≤ t ≤ 382 µm and grain size of 20 nm (a) and 100 nm (b).*

![Figure 4.4.3: Characteristic 4-fold diamond local buckling mechanism of an experimental specimen with grain size of 20 nm and wall thickness t = 59 µm (a), and deformation of the corresponding FE model (b), both at a nominal strain ε ≈ 0.14.](image2)

*Figure 4.4.3: Characteristic 4-fold diamond local buckling mechanism of an experimental specimen with grain size of 20 nm and wall thickness t = 59 µm (a), and deformation of the corresponding FE model (b), both at a nominal strain ε ≈ 0.14.*
Figure 4.4.4: FE nominal stress (σ)-strain (ε) curves of cylinders with wall thickness $57\mu m \leq t \leq 382 \mu m$ and material properties of n-Ni with a grain size of 20 nm (a) and 100 nm (b). The instance of the first element removal due to the activation of the porous failure mechanism is shown in (a).

The critical local buckling strength ($\sigma_{CR}$) of a cylindrical shell is given by Equations 3.3.3 and 3.3.4 for simply supported and free edge boundary conditions respectively. Since the frictional coefficient between the platen and tube edges is finite (in this case $\mu_{Ni/Steel}=0.6$), it is expected that the actual critical strength will be between these upper and lower boundaries. Figure 3.4.5a shows the analytical boundaries, and the FE and experimental peak strengths. The agreement between the FE and analytical results is satisfactory. At low thicknesses, the frictional coefficient is sufficiently large to prevent lateral translation of the edges before local buckling occurs, hence the FE peak strength is nearly identical to the one predicted by the simply supported edge assumption. As the thickness increases, some lateral translation is possible before buckling occurs, thus the peak strength is intermediate between the two boundaries.

Both the analytical and FE models overestimate the experimental peak strength, particularly in the low thickness regime. This discrepancy has been reported elsewhere [10-12] and has been attributed to the sensitivity of shell buckling to axisymmetric (through
thickness) imperfections. In this case, this mechanism is particularly relevant, because the thickness uniformity of electroplated sleeves is adversely affected by preferred nucleation sites, electroplating additives and substrate non-uniformities [e.g. 13]. Cylinders with higher $R/t$ ratios are more sensitive to thickness variations [12], as shown by the large standard deviation of the peak strengths of cylinders with thickness of 57 and 59 µm. Notice, however, that the variation of the peak strength discrepancy factor ($\sigma_{PK,FE}/\sigma_{PK,Exp.}$) with wall thickness (Figure 3.4.5b) for the two materials considered here is similar, indicating that it is more sensitive to geometrical properties (i.e. $R/t$ ratio) than material properties or failure mechanism (i.e. buckling or fracture).

![Figure 4.4.5: Experimental, FE, and analytically predicted nominal peak strength ($\sigma_{PK}$) for free and simply supported edge conditions (a), and ratio of FE to experimental peak strength ($\sigma_{PK,FE}/\sigma_{PK,Exp.}$) (b) as a function of wall thickness (t).](image)

4.4.3. Material fracture

In electrodeposited materials, the yield strength increase that results from grain size refinement is also accompanied by a reduction in ductility. For instance, the uniaxial tensile fracture strain of the materials used in this study is 0.06 and 0.16 (average grain size of 20 nm and 100 nm respectively). Due to this reduction in ductility, cracks were observed in all post-buckled cylinders with grain size of 20 nm. In specimens with relatively low wall
thicknesses (59 ± 2 µm and 96 ± 1 µm) fracture surfaces were observed alongside local folds, and were not noticed to have penetrated the thickness of the cylinder at the end of the test (strain of 0.2). However, as the thickness increased (251 ± 6 µm and 382 ± 3 µm) compressive deformation occurred exclusively by crack propagation (see Table 4.1.1). On the other hand, no cracks or fracture were observed in any specimens with a grain size of 100 nm.

Since the initiation and propagation of fracture is dependent on the multiaxial stress state that is developed during the local folding mechanism, the FE models can be useful to illustrate the relationship between the two. The activation of material failure through the porous failure mechanism occurs when the void volume fraction in an element reaches the critical value (in this case $f_{CR} = 0.15$); this corresponds to a macroscopic crack initiation. Crack propagation is modeled through the loss of stiffness and removal of elements where the void volume fraction reaches the failure value (in this case $f_F = 0.25$) in all integration points.

Among the models with the material properties of the 100 nm grain size n-Ni, the maximum void volume fraction at a nominal strain of 0.25 was 0.05, 0.09, 0.03, and 0.06 for models with wall thickness of 57 µm, 104 µm, 165.5 µm, and 371.5 µm respectively. In the models with the two smallest thicknesses, the elements with the largest volume fraction were located in the corners of the four- and three-fold buckling folds, due to the strain concentration and localized deformation in these regions. Conversely, in the models with the two largest thicknesses, these elements were located in the axisymmetric fold; due to the absence of strain concentration sites in these models, the values of the maximum void volume fraction are smaller than those observed in the thinner specimens. Notice also that the maximum void volume fraction is well below the critical value, showing that, in
agreement with experimental observations, no crack initiation is expected in these specimens.

Among the simulations with the material properties of n-Ni with a grain size of 20 nm, the maximum void volume fraction at a nominal strain of 0.2 exceeded the critical void fraction. In the case of a model with a wall thickness of 59 µm, this value was 0.23, indicating that cracks can initiate around the corners of the four-fold buckling pattern, but the strain is insufficient to allow their propagation. Conversely, the failure void volume fraction was exceeded in models with larger wall thicknesses. Figure 4.4.6 shows the mesh state and distribution of void volume fraction at a nominal strain of 0.25 in models with wall thickness of 96 µm and 251 µm. Notice that the location of the removed elements in the model with thickness of 96 µm is in relatively concentrated regions around the corners of the fold, where tensile strain concentrations exist. By contrast, a circumferential band of removed elements is found in the model with a thickness of 251 µm, due to the symmetry of the fold.

It is worth mentioning that crack propagation is mesh sensitive, thus the static mesh employed in these simulations can only approximate the crack path. For instance, crack propagation through the thickness of the model is not possible, since the inner surface of the cylinder is in compression thus those elements cannot be removed because not all integration points can satisfy the failure criterion. However, the fraction of the removed elements and their location can be used to determine the extent of fracture. For instance, the fraction of removed elements in simulations with wall thicknesses of 96 µm, 251 µm, and 382 µm is $0.2 \times 10^{-3}$, $2.0 \times 10^{-3}$, and $2.5 \times 10^{-3}$ respectively, showing a significantly higher probability that deformation is controlled by fracture propagation in the last two models. Figure 4.4.7 shows a micrograph of the fracture surface of an experimental sample with a
wall thickness of 382 µm, and corresponding state of the FE model at a strain of 0.2. Notice that the circumferential fracture path is developed in both, and that at this stage the through thickness extent of element removal in the FE model has reached 60%.

Figure 4.4.6: Element removal and distribution of the porous volume fraction (VVF) among the remaining elements for models with material properties of n-Ni with GS=20nm, and wall thickness of 96 µm (a), and 251µm (b) at a strain $\varepsilon \approx 0.2$.

Figure 4.4.7: Fracture in an experimental specimen with a grain size of 20nm, and wall thickness of 382 µm (a), and element removal in the corresponding FE model (b) at a strain $\varepsilon_N \approx 0.2$. 
An important issue in this analysis is the interaction between the two failure mechanisms (local buckling and fracture), and the influence of each on compressive strength. Figure 4.4.4a shows the strain at which the first element removal occurs in the models with the properties of the material with a grain size of 20 nm. In all cases, these strains are larger than the peak strain. This is to be expected, because ductile failure due to the coalescence and growth of microvoids occur in a tensile stress state, which, in a cylinder that is under uniform uniaxial compression, can only be introduced after the first local buckling fold has occurred. More specifically, the first element removal occurs in the second peak of the compressive stress-strain curve, which, macroscopically corresponds to the formation of the second half of the full fold, i.e. when all stress states (longitudinal, hoop, and through-thickness) reach the maximum tensile value. Therefore, local buckling failure can be expected to precede material fracture and determine the maximum compressive strength of the hollow column.

From a strut design perspective this can be advantageous. If the cylindrical component is designed to not deform past the peak compressive load (e.g. in microtruss materials that are used to support static loads), grain size reduction can provide only strength benefits: the accompanying reduction in ductility is not an issue since fracture is not an active failure mechanism. However, in many cases cylindrical components and microtruss materials are used in energy absorption applications, where the magnitude of plateau stress is more important than the peak stress. In these cases, material fracture will be a relevant failure mechanism and will reduce the plateau stress.
4.4.4. Architectural efficiency of hollow and composite n-Ni struts

The architectural efficiency of an electrodeposited sleeve, such as the one considered here and in Section 4.4.2, stems from two factors: the inherent strength of the nanocrystalline material, and its position away from the bending axis of the strut. The latter is related to the large second moment of area of the sleeve; it is well known that hollow sections have a larger shape efficiency factor than solid sections of the same cross-sectional area [e.g. 14]. This condition is common for the sleeves of all three n-Ni hybrids discussed in this chapter (n-Ni/Al, n-Ni/steel, and n-Ni/air); furthermore, they have the same range of values of the shape efficiency factor for elastic bending ($\approx 9-70$), defined as the ratio of the second moment of areas of a hollow and solid section of the same area [14].

Figure 4.4.8a shows the experimental strut density and peak compressive strength (the load normalized by total strut area) achieved in these studies. As expected, hollow struts occupy the low-density region of the graph, due to the lack of core material. The presence of the local buckling mechanism generally decreases the peak stress compared to what could be achieved by a strut of the same external dimensions in global buckling, thus the compressive strength is comparable to that of n-Ni/Al struts. However, the weight savings provided by the hollow strut result in significant improvement in weight-specific strength (Figure 3.4.8b); for instance a specific strength of 45 MPa-cm$^3$/g can be achieved at a density that is lower by a factor of 7 and 16 compared to n-Ni/Al and n-Ni/steel struts respectively.
Figure 4.4.8: Experimental peak strength ($\sigma_{PK}^{\text{COL}}$) - density ($\rho^{\text{COL}}$) of n-Ni/Al, n-Ni/steel n-Ni hollow columns of grain size 20 nm and 100 nm (a), and specific strength ($\left(\sigma_{PK} / \rho\right)^{\text{COL}}$) – density of the same columns (b).

4.4.5. Summary

Hollow cylindrical struts created by nanocrystalline electrodeposition can support high compressive strengths, due to the increased yield strength that results from the grain size reduction. Two failure mechanisms were observed: inelastic shell buckling and material fracture. In spite of the reduced ductility and presence of material fracture, cylinders fabricated with a grain size of 20 nm showed significantly higher peak strengths compared to their 100 nm grain size counterparts of the same dimensions. The reason for the insensitivity of peak compressive strength to the reduction in ductility and presence of fracture is due to the fact that fracture and local buckling are not competing failure mechanisms. Ductile fracture can only be initiated after the development of a tensile stress state, which in turn, is created as a result of the formation of the first local buckling fold.

The absence of a core material means that hollow struts can occupy a lower density region in material property space compared to hybrid struts with nanocrystalline sleeves. Since the peak strength is dictated by local buckling, the compressive strength achieved by
the struts of this study is comparable to that of n-Ni/Al and n-Ni/steel struts. The above factors mean that hollow struts are desirable in weight-sensitive applications, when maximizing the weight-specific strength is a more important design objective than the improvement of the absolute strength.

4.4.6. Directions for future research

The microstructural failure model used in this study is based on the evolution of void volume fraction under a stress field. By definition, the stress field must be tensile in order to have stress degradation and eventual removal of elements within the FE model (see Figure 2.3.1). During the local buckling of a hollow cylinder, the surface of a local fold is typically under a compressive axial and hoop stress field, thus the propagation of fracture in the through-thickness direction is highly sensitive to the mesh refinement in that direction. For this reason, it is recommended that adaptive remeshing rules be incorporated to update the mesh refinement around the failure zones, in order to predict the crack propagation path. The extended finite element (XFEM) method has also been used successfully to model the strong discontinuities that exist around crack tips.

A second recommendation is to obtain information from microstructural characterization of the specimens on the types of material and geometrical defects that exist within the electrodeposited n-Ni sleeve. These defects can be incorporated within the FE model as inhomogeneous regions with different material properties than the rest of the model (material imperfections), or as perturbations in the local buckling modes (geometrical imperfections). These techniques can be used to improve the agreement between the FE and experimental peak strength, and to give a quantitative measure of the effect of material fracture on the reduction of the mechanical performance (e.g. energy absorption) of the cylindrical component.
4.4.7. References

5. MATERIAL SELECTION IN HYBRID STRUTS

This chapter presents a material selection guide for the design of hybrid struts. Section 5.1 considers the simplified case of an elastic-plastic buckling mechanism. With this simplification the material inputs are reduced to two single-valued parameters, resulting in material selection criteria to screen and rank core materials for a given sleeve reinforcement. Section 5.2 discusses the effect of material and geometrical for the more realistic case of inelastic buckling; accuracy is gained at a loss of generality. FE and experimental results of the n-Ni/steel and n-Ni/Al material systems are used as comparative examples.

5.1. Material Selection for Composite Strut Cores Undergoing Elastic and Plastic Buckling

The analysis of Section 3.2 showed that the strength of a hybrid column is dependent on the entire stress-strain curve of the two materials and their densities, i.e. on the five material parameters: \( E, \sigma_P, K, n \) and \( \rho \). These properties are seldom available before the material has been selected, thus conventional selection techniques simplify the material properties to an elastic-plastic behavior in order to reduce the number of variables [1].

The simplified elastic-plastic buckling assumption is used in this section to provide a starting point for a material selection design guide. The material variables are thus reduced to the elastic modulus \( E \), the yield strength \( \sigma_Y \) and the density \( \rho \) of each material. This guide has two objectives: first, to identify acceptable core materials for a coating material (screening), and second, to select core materials based on the mechanical efficiency of the resultant composite (ranking). An analogous analysis can be prepared to select acceptable sleeve materials for a precursor strut.
The screening criteria are:

1) The compressive strength of the hybrid strut is higher than the compressive strength of the uncoated strut, i.e. \( \sigma_{CR}^{HYB} > \sigma_{CR}^{CORE} \), which is equivalent to \( \Delta P_{CR}^H/\Delta A > \sigma_{CR}^{CORE} \).

2) The compressive strength of the hybrid strut increases with increasing normalized sleeve thickness \( (T=t/L) \), i.e. \( \frac{\sigma_{CR}^{HYB}}{dT} > 0 \).

3) The specific compressive strength of the hybrid strut is higher than the compressive strength of the uncoated strut in both the elastic and plastic buckling regime, i.e.

\[
\frac{\sigma_{CR}^{HYB}}{\rho_{HYB}} > \frac{\sigma_{CR}^{CORE}}{\rho^{CORE}},
\]

which is equivalent to \( \Delta \sigma/\Delta \rho > \sigma_{CR}^{CORE}/\rho^{CORE} \).

4) The specific compressive strength of the hybrid strut increases with an increasing normalized sleeve thickness, i.e. \( \frac{d(\sigma/\rho)^{HYB}}{dT} > 0 \).

5.1.1. Core material in elastic buckling

In the elastic regime, the critical stress of the core is \( \sigma_{CR}^{CORE} = \frac{\pi^2}{12} B^2 E_C \) and the critical stress of the hybrid column, for end constraints represented by \( k=1 \), is:

\[
\sigma_{CR}^{HYB} = \frac{\pi^2}{12} E_S \left( B + 2T \right)^4 - \frac{B^4}{(B + 2T)^2} + \frac{\pi^2}{12} E_C \frac{B^4}{(B + 2T)^2} \quad (5.1.1)
\]

where \( B \) and \( T \) are the normalized core and sleeve thickness respectively \( (B=b/L \text{ and } T=t/L) \).

The density of the hybrid column is:

\[
\rho_{HYB} = \rho_S \left[ 1 - \left( \frac{B}{B + 2T} \right)^2 \right] + \rho_C \left( \frac{B}{B + 2T} \right)^2 \quad (5.1.2).
\]
The first condition is satisfied when \( E_C < E_S \left[ \left( 1 + \frac{2T}{B} \right)^2 + 1 \right] \). A conservative limit can be taken for \( T<<B \), in which case, the second condition becomes \( E_C < 2E_S \). Note that this condition defines a very broad region of the material space, indicating that acceptable core materials can be stiffer than the sleeve material. Although counterintuitive, this selection is reasonable: the buckling-resistant force added by the area of a less stiff sleeve can be sufficient to increase the mean critical stress of the composite column if this first condition is satisfied.

The second condition is satisfied when \( E_C < 3E_S \left[ \left( 1 + \frac{2T}{B} \right)^4 + 1 \right] \). A conservative limit can be taken for \( T<<B \), in which case, the second condition becomes \( E_C < 6E_S \). Note that the materials that satisfy the first condition are a subset of materials that satisfy the second condition, thus the criterion \( E_C < 2E_S \) can be taken as the limiting condition to satisfy the first two criteria.

The third condition is satisfied for \( \frac{E_C}{\rho_C} < \frac{E_S}{\rho_S} \left[ \left( 1 + \frac{2T}{B} \right)^2 + 1 \right] \). A conservative limit can again be taken for \( T<<B \), in which case, this condition becomes \( \frac{E_C}{\rho_C} < 2 \frac{E_S}{\rho_S} \). Finally, the fourth condition imposes the limit \( \frac{E_C}{\rho_C} < 2 \frac{E_S}{\rho_S} \left( 1 + \frac{2T}{B} \right)^3 \frac{E_S}{\rho_S} \left[ \left( 1 + \frac{2T}{B} \right)^2 - 1 \right] \). This condition is satisfied by all materials which satisfy the third condition, thus the two material selection criteria for elastic buckling are \( E_C < 2E_S \) and \( \frac{E_C}{\rho_C} < 2 \frac{E_S}{\rho_S} \).

Software packages are available for visual representation of material selection analyses; an example is the commercially available CES Selector [2], which is used here.
Figure 5.1.1 shows the region of material property space that contains suitable core materials for the n-Ni sleeves used in the examples of this chapter. The accessible space for core materials is large due to the high elastic modulus of the n-Ni electrodeposit, indicating its suitability as a reinforcing phase; only a few materials do not satisfy these criteria, namely tungsten alloys and certain glasses.

The available core materials can furthermore be ranked for mechanical performance criteria. The geometry of the two phases influences the properties of the composite, and is usually given as a constraint or free variable [1]. Furthermore, the objective of the design (e.g. minimization of mass, or maximization of strength or specific strength) determines the materials performance index. For instance, the strength and density of a composite strut with a n-Ni sleeve of normalized thickness $T = 0.001$ and core width $B = 0.01$ are represented by the functions $\sigma_{CR}^{COL} = 8.9 + 5.7 \times 10^{-5} E_C$ (GPa) and $\rho^{COL} = 2713 + 0.7 \rho_C$ (kg/m$^3$) respectively.

Acceptable metallic cores and their effect on the mechanical performance of the composite are shown in Figure 5.1.2. Isostress (horizontal) lines represent core materials that provide equal buckling strength performance of the composite strut (e.g. Al and Zn alloys). Core materials for composite struts with high buckling strength include steels and Ni superalloys; for instance the strength of a n-Ni/steel strut is expected higher than that of a n-Ni/Al strut by a factor of approximately 1.4. In the experimental study of Section 4.2, an even higher strength increase was observed, e.g. for struts with $B \approx 0.135$ and $T \approx 0.003$, the strength of the n-Ni/steel hybrids was higher than that of n-Ni/Al hybrids by a factor of 2.2. The presence of material fracture, inelastic buckling, and different geometrical parameters is therefore expected to affect the magnitude of the relative improvement in
material performance. Similarly, vertical lines represent core materials with equal density performance: the density of a n-Ni/Mg strut is expected to be lower than that of a n-Ni/Al strut by a factor of 1.2. Finally, the specific strength performance of these core materials when coated with a n-Ni sleeve is represented by a family of lines with slope 1. Figure 5.1.2 shows iso-strength/density lines in the range $3.0 \times 10^{-3} < \left( \frac{\sigma_{cr}}{\rho} \right)^{COL} < 8.9 \times 10^{-4}$. For this coating material, the most beneficial specific strength performance is obtained when Al alloys and Mg alloys serve as core materials; for instance the specific strength of a n-Ni/Al hybrid can be expected to be better than that of a n-Ni/steel hybrid by a factor of 1.2. Notice that this is close to what was observed experimentally in the hybrid struts of Section 4.2 ($B \approx 0.135$), where for a sleeve with thickness $T \approx 0.003$, the specific strength of n-Ni/Al hybrids was higher than that of n-Ni/steel hybrids by a factor of 1.18.

Figure 5.1.1: Accessible core material space for elastic buckling of composite columns with a n-Ni sleeve, showing material families and the design region defined by the four selection criteria.
5.1.2. Core material in plastic buckling

In the case of yielding, the critical stress of the core column is $\sigma_{CR}^{\text{CORE}} = \sigma_{Y,C}$, and the critical stress of the composite column is

$$\sigma_{CR}^{\text{COL}} = \sigma_{Y,S} \left[ 1 - \left( \frac{B}{B + 2T} \right)^2 \right] + \sigma_{Y,C} \left( \frac{B}{B + 2T} \right)^2 \quad (5.1.3).$$

The first and second conditions are satisfied when $\sigma_{Y,C} < \sigma_{Y,S}$, and the third and fourth conditions are satisfied when $\frac{\sigma_{C}}{\rho_{C}} < \frac{\sigma_{S}}{\rho_{S}}$.

Figure 5.1.3 shows a material property chart that maps the yield strength and acceptable specific strength for core materials for the n-Ni sleeve (0.2% offset yield...
strength of 825 MPa [3]) that has been used in the examples of this chapter. Most materials (with the exception of some glasses) fit these criteria.

Figure 5.1.3: Accessible core material space for plastic buckling of composite columns with a n-Ni sleeve, showing material families and the design region defined by the four selection criteria.

A quantitative comparison of the performance of these core materials can be made similarly to the case of elastic buckling. For example, in the case of a composite strut with normalized core width \( B = 0.1 \) and n-Ni sleeve of normalized thickness \( T = 0.01 \), the buckling strength and density are given by \( \sigma_{COL}^{CR} = 256 + 0.7\sigma_y \) (MPa) and \( \rho^{COL} = 2713 + 0.7\rho_c \) (kg/m\(^3\)) respectively. Figure 5.1.4 shows the available core materials and the performance of the composite. The buckling strength performance is maximized by selecting W alloys, steels and Ni alloys as core materials \( (\sigma_{Y,C} \equiv \sigma_{Y,Ni}) \). Alternatively, a high specific strength performance is achieved by using Al or Mg alloys as core materials \( (\sigma_{CR}/\rho)^{COL} \approx 87 \times 10^{-3} \) MPa/(kg/m\(^3\)).
Figure 5.1.4: Core material selection for the plastic buckling of a composite column with normalized core width $B = 0.1$ and $n$-Ni sleeve with normalized thickness $T = 0.01$, showing the strength and density of the column ($\sigma_{CR}^{COL}$ and $\rho_{COL}$ respectively), and family of lines representing the specific strength of the column.

5.1.3 References

5.2. Material Performance in Inelastic Buckling

In the case of inelastic buckling, the mechanical performance of the hybrid is dictated by a complex interaction of geometrical properties of the two phases and the stress-strain relationship of both materials. An additional complication is the discrepancy between the predicted critical buckling stress and the peak compressive stress of the microtruss. In this section, two hybrid systems (n-Ni/Al and n-Ni/steel) are discussed to compare the role of material properties in the mechanical performance of columns and microtruss architectures. Furthermore, the FE results of Section 4.2 are used to illustrate the discrepancy between the critical stress and the observed peak strength.

5.2.1 Role of the reinforcing sleeve on the mechanical performance of a hybrid column

In order to study the efficiency of the sleeve in improving the mechanical properties of a starting core, it is beneficial to characterize the precursor strut by its slenderness ratio \((L/r_c)\), and the sleeve by its area fraction \(f_s\). The column inelastic buckling and density equations assume the forms:

\[
\sigma_{cr}^{COL} = \frac{k^2 \pi^2 E_{T,S} \left(1 - f_s \right) \left(1 - f_s \right) E_{T,C} \left(1 - f_s \right)}{(L/r_c)^2} \quad (5.2.1),
\]

\[
\rho^{COL} = \rho_S f_s + \rho_C (1 - f_s) \quad (5.2.2)
\]

respectively. The material performance parameters mentioned above can then be represented as analytical functions of geometric properties, material properties, and the end constraint constant.

An envelope of achievable material performance can therefore be defined by the \(k = 1\) and \(k = 2\) boundary conditions for hybrid struts. Microtruss struts fabricated to date have
had slenderness ratios on the order of 15 to 55 [1-14]; Figure 5.2.1 illustrates the strengthening envelopes for two low carbon steel and 3003 aluminum alloy cores ($L/r_C = 15$ and 55) reinforced with nanocrystalline Ni sleeves of $0 \leq f_S \leq 0.9$ with the material properties described in Section 4.2.

Two material performance parameters that can be used to determine the efficiency of reinforcement by nanocrystalline electrodeposition are the buckling strength increase per unit density increase ($\Delta \sigma/\Delta \rho$) and the change in specific strength ($\Delta (\sigma/\rho)$) of the composite core relative to the uncoated core. The strength increase per unit density indicates the reinforcing efficiency of the sleeve material; in a $\sigma$-$\rho$ material property space, it indicates the relative coordinate change between the starting (uncoated) and hybrid (coated) microtrusses and is equal to the initial slope of the envelopes shown in Figure 5.2.1. On the other hand, the change in specific strength indicates the capacity of the reinforcing material to improve the strength to weight ratio of the precursor. It is shown by the increase in slope of lines that intersect the origin and the $(\sigma, \rho)$ positions of the hybrid and precursor.

The effect of the slenderness ratio of the starting column on the strength increase per unit density change ($\Delta \sigma/\Delta \rho$) is shown in Figure 5.2.2a. For both starting materials, the strength increase per unit density is higher at low slenderness ratios, because the largest difference in the stress values of the reinforcing and core material at the critical strain is observed at large values of critical strains, achieved in stocky columns. The effect of the sleeve area fraction on ($\Delta \sigma/\Delta \rho$) is shown in Figure 5.2.2b. High values of $\Delta \sigma/\Delta \rho$ are observed at high area fractions, showing the effectiveness of the sleeve in improving the mechanical performance of the precursor. Notice that the strength increase per unit density
is higher when the sleeve is deposited on steel precursors, because the additional strength is associated with a relatively small increase in density. Notice however that, due to the uncertainty in the end rotational constraint $k$, this trend can be reversed at large $L/r_C$ or small $f_S$.

**Figure 5.2.1:** Critical stress ($\sigma_{CR}$)-density ($\rho$) envelopes of $n$-Ni/Al and $n$-Ni/steel columns with core slenderness ratio $L/r_C=15$ and 55 and $0 \leq f_S \leq 0.9$, showing the $k=1$ and $k=2$ lower and upper bounds, and critical stress where the compressive stress in the sleeve is lower than the tensile stress of the material (solid lines).

**Figure 5.2.2:** Structural performance index $\frac{\Delta \sigma}{\Delta \rho}$ envelopes as a function of $L/r_C$ for $n$-Ni/steel and $n$-Ni/Al columns with $f_S = 0.1$ and 0.25 (a), and a function of $f_S$ for $L/r_C = 25$ and 50 (b).
The effects of the starting strut slenderness ratio and sleeve area fraction on the change in specific strength ($\Delta(\sigma/\rho)$) are shown in Figures 5.2.3a and 5.2.3b respectively. For the same reasons described above, the sleeve is more efficient in reinforcing the starting strut at a small slenderness ratio and large area fraction. Notice however, that for a given slenderness ratio and sleeve area fraction, the sleeve is more efficient at increasing the specific strength of the aluminum alloy core, because the same amount of strength increase per unit density is more effective in increasing the strength to density ratio of the material with the lowest density. Similar analyses have also been performed for two casting alloys that have been used in microtruss materials (Cu-Be alloy and Si-Cu alloy, with properties from [3, 6, 7]). The results, shown in Appendix F, indicate that the sleeve provides the largest strength increase per unit density change for CuBe cores, and the largest specific strength increase for Al cores.

![Figure 5.2.3: Structural performance index envelopes as a function of L/r_C for n-Ni/steel and n-Ni/Al columns with f_S = 0.1 and 0.25 (a), and a function of f_S for L/r_C = 25 and 50 (b).]
5.2.2. Role of the reinforcing sleeve on the mechanical performance of the microtruss

The strength and density of the microtruss ($\sigma_{CR}^{MT}$ and $\rho^{MT}$ respectively) are related to the critical strength and density of the hybrid column by Equations 3.1.1 and 3.1.3. Figure 5.2.4 shows the strength-density space that can be occupied by pyramidal architectures with $\omega = 45^\circ$, $k = 1$, $15 \leq L/r_c \leq 55$, and $0 \leq f_S \leq 0.8$. The strength increase per unit density follows the slope of the lines of constant core slenderness ratio, exhibiting the same trend seen for individual struts: a higher strength increase per unit density is possible for the steel core composites. Note that moving from the properties of the strut to those of the microtruss means that it is possible for cellular composites of different core slenderness ratios and sleeve area fractions to have not just the same values of specific strength, but also the same values of strength and density. For instance, the $L/r_c = 15$ and 55 lines of the n-Ni/Al strength envelope intersect at the coordinates 0.39 g/cm$^3$, 34 MPa, corresponding to structures of $L/r_c = 55$, $f_S = 0.78$ and $L/r_c = 15$, $f_S = 0.22$. Finally, lines of constant specific strength are also plotted in Figure 5.2.4. Notice that while the uncoated Al hybrids possess a lower specific strength than their steel equivalents (e.g. 7.5 MPa/(g/cm$^3$) and 19.3 MPa/(g/cm$^3$) respectively, at $L/r_c = 55$), the rate of increase in specific strength is larger when the sleeve is deposited around Al precursors (e.g. $\Delta(\sigma/\rho) = 46$ MPa/(g/cm$^3$) and 26 MPa/(g/cm$^3$), respectively, at $L/r_c = 55$ and $f_S = 0.5$).
5.2.3 Material effects on the discrepancy between peak and critical strength

In addition to the critical buckling stress, there are several other factors that need to be considered when predicting the strength of composite microtruss materials. First, the critical buckling stress defines the point at which a column starts to bend; the overall load carrying capacity in the post-buckling regime can continue to increase before eventually reaching a peak stress, $\sigma_{PK}$. Figure 5.2.5a shows simulated nominal microtruss stress ($\sigma_{MT}$) – strain ($\varepsilon$) curves of n-Ni/steel and n-Ni/Al models of Section 4.2 ($L/r_C = 25, f_S = 0.04$). Also shown is the strain evolution of axial stresses for elements in the nanocrystalline sleeve on the convex (stress eventually becomes tensile), $\sigma_T$, and concave (stress remains compressive), $\sigma_C$, sides of the bending strut. The critical stress marks the divergence of stress values on opposite surfaces of the column in the bending plane, while the microtruss peak strength occurs just after the $\sigma_T$ stress reversal. Figure 4b plots the ratio of peak strength to critical strength for n-Ni/Al and n-Ni/steel hybrid FE models. The $\sigma_{PK}^{FE} / \sigma_{CR}^{FE}$
ratios were higher for the n-Ni/Al hybrids (1.21 ± 0.02) than the n-Ni/steel hybrids (1.09 ± 0.02), which can be largely explained by the difference in work hardening capability of the two core materials (Hollomon strain hardening exponent of 0.27 for Al and 0.08 for steel).

Figure 5.2.5: Nominal microtruss stress ($\sigma_{MT}$)-strain ($\varepsilon$) and normal stress ($\sigma_N$)-strain curves (the subscripts C and T denote the stress in the surface that eventually becomes compressive and tensile respectively) in n-Ni/steel and n-Ni/Al FE models (a), and ratio of FE peak to critical stress ($\sigma_{PK}^{FE}/\sigma_{CR}^{FE}$) and experimental to FE peak stress ($\sigma_{PK}^{EXP}/\sigma_{PK}^{FE}$) as a function of sleeve area fraction $f_S$ (b).

However, of greater significance in determining the peak load carrying capacity of a composite microtruss is the validity of the sleeve/core isostrain assumption. Figure 5.2.5b plots the experimentally measured peak strengths of n-Ni/Al and n-Ni/steel hybrids normalized by their respective single strut FE peak strengths, $\sigma_{PK}^{EXP}/\sigma_{PK}^{FE}$. Because of the rigid boundary conditions and lack of statistically distributed defects in the FE-modeled struts, the uncoated ($f_S = 0$) experimental peak strengths were lower than the FE peak strengths by a factor of 0.9. The n-Ni/steel hybrids exhibited this same peak strength ratio over all sample types tested ($\sigma_{PK}^{EXP}/\sigma_{PK}^{FE} = 0.89 ± 0.01$ for $0.06 \leq f_S \leq 0.16$). By contrast, the peak strength ratio of the n-Ni/Al hybrids decreased from 0.78 at $f_S = 0.6$ to 0.66 at $f_S = 0.18$, because of the gradual breakdown in the sleeve/core isostrain condition as sleeve
delamination, wrinkling and fracture become progressively more important with increasing coating thickness.

5.2.4. References

5.3. Summary

The mechanical performance of microtruss cores reinforced with high strength sleeves can be evaluated by the buckling strength and density of the hybrid struts. While material properties in composite struts that undergo inelastic buckling depend on geometrical properties, in the simplified case of elastic-plastic buckling they can be considered constant. In this case the number of free variables in a material selection analysis is reduced significantly. Due to its elastic modulus and yield strength, the n-Ni sleeve material is suitable for reinforcing most core materials, except W alloys and some glasses. The highest strength and specific strength is provided for steels and Al or Mg alloys respectively.

The buckling strength increase per unit density and the increase in specific strength were identified as material performance parameters that can be used to evaluate the suitability of a given core material for nanocrystalline sleeve reinforcement. An envelope of achievable material performance values can be determined based on the limiting end constraint conditions for given values of sleeve area fraction and core slenderness ratio. When comparing the performance of nanocrystalline Ni coatings on low carbon steel and aluminum alloy columns, the strength increase per unit density increase was higher on a steel core, but the increase in specific strength was higher in an aluminum core. Although the uncoated Al cores possess a lower specific strength compared to steel cores, the sleeve is more effective in increasing the specific strength of the former. The major factors complicating material selection can be grouped as follows: the requirement for knowledge of the entire stress-strain curves of both materials, the interdependence of geometrical and material parameters, uncertainty in the end constraint value, the difference between the
analytically predicted critical strength and experimental peak strength, and the demand for adequate interfacial adhesion to prevent competing methods of failure.
6. CONCLUSIONS

6.1. Geometrical Design for Microstructural Reinforcement

The usefulness of microtruss cores can be improved if their weight-specific strength is increased. This objective can be achieved by modifying the architectural parameters (which control both density and strength) or material properties (which control strength). In this thesis, design of strut geometry was done to control the microstructural evolution during fabrication, which in turn was exploited to increase the achievable strength and decrease the microtruss density.

In the study of Section 4.1, stretch forming was used as a fabrication mechanism. By introducing curvatures in the corners of the perforation geometry of the starting blanks, the failure location was shifted to the mid-strut. In the blanks studied in this thesis, all geometries with \( w \leq 0.8w_0 \) exhibited mid-strut failure. This change produced two results:

a) The forming limit and relative density of the fabricated microtruss improved by as much as 32% (at \( w=0.8w_0 \)) relative to the conventional geometry (\( w=w_0 \)).

b) The location of maximum imparted strain energy shifted from the node to the mid-strut, where it is more useful to resist compressive loads during compression.

c) The strain energy imparted to the mid-strut increased relative to the conventional geometry by as much as 90%. Furthermore, this value is achieved for all specimens that fail in the mid strut (i.e. the magnitude is independent of width reduction), due to the uniform distribution of plastic strain.

Since shape and microstructure are interdependent, finite element analyses of the forming and compression process were used sequentially to: a) map the process-structure-
property relationship, and b) improve the geometry of the perforation pattern. Within the constraints of the problem, the most suitable perforation geometry was determined to be the one that satisfies criteria a)-c) with minimal truss width reduction \((w=0.8w_0)\). For the material and geometrical constraints of this study, both the density and specific strength of structures fabricated from this geometry were improved by a factor of \(\approx 1.5\) relative to the respective properties of structures fabricated from the conventional geometry.

### 6.2. Sectional and Material Efficiency Design in Hybrid Struts

Strengthening a precursor microtruss core with a reinforcement sleeve is advantageous for structural properties, because the sleeve possesses material, sectional, and architectural efficiencies. In this thesis, an analytical model was created to predict the critical buckling strength of the constituent hybrid struts. FE models were used to verify its validity based on the stress distribution assumptions, and identify the reasons for the discrepancy between the predicted critical strength and observed peak buckling stress.

The first study considered aluminum and low carbon steel microtruss structures reinforced with nanocrystalline metal sleeves; the n-Ni/Al hybrid architectures showed strength and specific strength improvements by factors as large as 3.0 and 1.8 respectively, relative to their precursor counterparts. FE analyses were used to determine the relationship between the observed failure mechanisms (buckling and material fracture), and mechanical strength. By analyzing stress evolution during bending, the precursor/sleeve interface in the hinge region was identified as the critical region for the initiation of material fracture. The normal interfacial stress was found to increase with coating thickness; in n-Ni/Al hybrids its magnitude is sufficient to allow delamination and sleeve fracture well before the critical buckling strength. In the material and architectural systems considered in Section 4.2, the
results of this premature failure can be significant; the peak strength can be reduced by as much as a factor of 2.2 relative to what would be expected in the absence of material fracture. These conclusions led to fabrication recommendations along two directions: improving the ductility of the sleeve, and improving the sleeve/core interfacial adhesion.

The second study considered aluminum microtruss structures reinforced with ceramic sleeves. These types of hybrids displayed nearly the same range of strength increase as their nanocrystalline-reinforced counterparts (factor of \(\approx 2.0\)); however this strength increase was achieved at little to no weight penalty. FE analyses were used to connect the two failure mechanisms (buckling and sleeve fracture) to strength, and showed that the sleeve undergoes brittle fracture due to the development of a tensile stress state. The embedded failure model showed that material failure is initiated in the hinge, and propagates progressively down the length of the strut. The stress state evolution that dictates the initiation and propagation of brittle fracture is dependent on the thickness of the sleeve; at large thicknesses (>20 \(\mu\)m in the system of Section 4.3), this material fracture can shortcut buckling instability to an extent that deformation proceeds exclusively by fracture propagation, resulting in a peak strength decrease of a factor as large as 1.3.

The third study discussed the compressive properties of nanocrystalline hollow cylinders. Experimental results showed that they can achieve densities that are lower than those of n-Ni/Al and n-Ni/steel struts by a factor of 5 and 16 respectively, resulting in a specific peak strength increase of a factor of 4. The FE analyses were used to connect geometry to stress state and fracture mechanism (local buckling or material fracture). It was concluded that material fracture does occur when the n-Ni alloy has low ductility (fracture strain of \(\approx 0.06\) in the system of Section 4.4), and the stress state that leads to it occurs more readily in specimens with low radius to thickness ratios. However, this type of failure is
developed in a tensile stress state, which only occurs after the peak strain, therefore it is not a competitive failure mechanism with buckling when the strut is not expected to deform past the local buckling strain. However, in energy absorbers, material fracture of low-ductility nanocrystalline alloys will be the dominant failure propagation mechanism and is expected to decrease the average absorbed energy.

6.3. Guide for Material Selection Design

Four screening criteria were identified in this thesis to select effective material combinations for hybrid struts: the strength and specific strength must increase relative to the uncoated strut, and the increase must happen monotonically with sleeve thickness. Due to the interdependence of material and geometrical properties, a general material selection study was completed by using the simplifying assumption of elastic-plastic buckling. Due to its elastic modulus and yield strength, the n-Ni sleeve material is suitable for reinforcing most core materials, except W alloys and some glasses. The highest strength and specific strength is provided for steels and Al or Mg alloys respectively.

At a loss of generality, inelastic buckling strength was used to compare the performance of two material systems (n-Ni/steel and n-Ni/Al): the strength increase per unit density increase was found to be higher when the sleeve material is used on a steel core, but the increase in specific strength was higher in an aluminum core. FE analyses were used to illustrate the factors that cause discrepancies between predictive and experimental properties in material selection design. They can be grouped as follows: the requirement for knowledge of the entire stress-strain curves of both materials, the interdependence of geometrical and material parameters, uncertainty in the end constraint value, the difference
between the analytically predicted critical strength and experimental peak strength, and the
demand for adequate interfacial adhesion to prevent competing methods of failure.
APPENDIX A: Details of FE Analyses

A.1. Hybrid Column in Uniaxial Compression (Sections 3.1. and 3.2.)

i) Geometry and Mesh

ii) Material Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>ρ (g/cm³)</th>
<th>σₚ (MPa)</th>
<th>Hardening: ( \sigma = K(\varepsilon_p)^N )</th>
<th>Exp. Source</th>
<th>R² of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3003</td>
<td>69.0</td>
<td>2.7</td>
<td>18.9</td>
<td>201.2</td>
<td>[1]</td>
<td>0.986</td>
</tr>
<tr>
<td>1006 Low Carbon Steel</td>
<td>210.0</td>
<td>8.9</td>
<td>165.0</td>
<td>428.0</td>
<td>[1]</td>
<td>0.967</td>
</tr>
<tr>
<td>n-Ni</td>
<td>145.3</td>
<td>8.0</td>
<td>445.9</td>
<td>2354.6</td>
<td>[2]</td>
<td>0.955</td>
</tr>
</tbody>
</table>
### iii) Simulation Details

| 2. Sleeve-Core Interaction | Tie constraints |
| 3. Element Type | Core: Hexahedral with reduced integration and enhanced hourglass control (C3D8R in Abaqus library).  
|                  | Sleeve: Shell with 5 through-thickness integration point, reduced planar integration and enhanced hourglass control (S4R in Abaqus library). |
| 4. Analysis Method | Dynamic, Explicit  
|                  | Abaqus Explicit solver  
|                  | Step time: 0.001 s |

### A.2. Stretch-Forming of Perforated Blanks (Section 4.1.)

i) Geometry and Mesh

![Diagram (a)](image1)

![Diagram (b)](image2)

![Diagram (c)](image3)

<table>
<thead>
<tr>
<th>E (GPa)</th>
<th>ρ (g/cm³)</th>
<th>σₚ (MPa)</th>
<th>Hardening: σ = K(εₚ)^N</th>
<th>K (MPa)</th>
<th>N</th>
<th>R^2 of fit</th>
<th>Lankford Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.0</td>
<td>2.7</td>
<td>18.9</td>
<td></td>
<td>201.2</td>
<td>0.986</td>
<td>0.986</td>
<td>0.75</td>
</tr>
</tbody>
</table>

iii) Simulation Details

1. Boundary Conditions
   • Pins P2, P4: constrained in all degrees of freedom
   • Pins P1, P3: Prescribed displacement in negative Y-axis.
   • Faces F2, F4: Symmetry about YZ-plane
   • Faces F1, F3: Symmetry about XY-Plane

2. Interaction Properties
   • Pin-Blank: Static frictional coefficient of 0.4

3. Element Type
   • Blank: Hexahedral with reduced integration and enhanced hourglass control (C3D8R in Abaqus library).
   • Pins: Analytical surfaces

4. Analysis Method
   • Static
   • Abaqus Standard solver

5. Failure Model
   • Phenomenological ductile failure (Section 2.3.1)
   • Evaluation basis: strain

A.3. Compression of n-Ni/steel and n-Ni/Al Microtruss Struts (Section 4.2.)

i) Geometry and Mesh
ii) Material Properties: Same as in A.1.

iii) Simulation Details

| 1. Boundary Conditions | • Platen P2: constrained in all degrees of freedom  
• Platen P1: Prescribed displacement of – 2mm in Y-axis; constrained in all other degrees of freedom  
• Faces F1, F2: Symmetry about YZ-plane  
• Face F1: Constrained from displacement in Y-Axis |
|------------------------|--------------------------------------------------------------------------------------------------|
| 2. Interaction Properties | • Platen-Sleeve: Static frictional coefficient of 0.6  
• Sleeve-Core: Tie constraints (No constraints and frictionless in simulations of unbounded conditions) |
| 3. Element Type | • Core: Hexahedral with reduced integration and enhanced hourglass control (C3D8R in Abaqus library).  
• Sleeve: Shell with 5 through-thickness integration points, reduced planar integration and enhanced hourglass control (S4R in Abaqus library).  
• Platens: Analytical surfaces |
| 4. Analysis Method | • Explicit, Dynamic  
• Abaqus explicit solver  
• Time step: 0.001s |

A.4. Compression of Al₂O₃/Al Microtruss Struts (Section 4.3.)

i) Geometry and Mesh: Same as in A.3.

ii) Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>ρ (g/cm³)</th>
<th>σₚ (MPa)</th>
<th>K (MPa)</th>
<th>N</th>
<th>Exp. Source</th>
<th>R² of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA3003</td>
<td>69</td>
<td>2.7</td>
<td>18.9</td>
<td>201.2</td>
<td>0.266</td>
<td>[1]</td>
<td>0.986</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>50, 90, 130</td>
<td>3.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[3]</td>
<td>-</td>
</tr>
</tbody>
</table>

iii) Simulation Details

| 1. Boundary Conditions | • Platen P2: constrained in all degrees of freedom  
• Platen P1: Prescribed displacement of – 2mm in Y-axis; constrained in all other degrees of freedom  
• Faces F1, F2: Symmetry about YZ-plane  
• Face F1: Constrained from displacement in Y-Axis |

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| 2. Interaction Properties | • Platen-Sleeve: Frictionless  
<table>
<thead>
<tr>
<th></th>
<th>• Sleeve-Core: Tie constraints</th>
</tr>
</thead>
</table>
| 3. Element Type          | • Core: Hexahedral with reduced integration and enhanced hourglass control (C3D8R in Abaqus library).  
|                         | • Sleeve: Shell with 5 through-thickness integration points, reduced planar integration and enhanced hourglass control (S4R in Abaqus library).  
|                         | • Platens: Analytical surfaces |
| 4. Analysis Method       | • Explicit, Dynamic  
|                         | • Abaqus explicit solver  
|                         | • Time step: 0.001s |
| 5. Failure Model         | • Hilleborg model (Section 2.3.3); “Brittle Cracking” in Abaqus  
|                         | • Evaluation basis: stress (initiation); energy (propagation) |

A.5. Compression of Hollow n-Ni Cylinders (Sections 3.3. and 4.4.)

i) Geometry and Mesh:
ii) Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>E (GPa)</th>
<th>ρ (g/cm³)</th>
<th>σₚ (MPa)</th>
<th>Hardening</th>
<th>Exp. Source</th>
<th>R² of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>GS = 20nm</td>
<td>166.8</td>
<td>8.9</td>
<td>104</td>
<td>$\sigma = \sigma_{\infty} - (\sigma_{\infty} - \sigma_Y)\exp(-C\varepsilon)$</td>
<td>[4]</td>
<td>0.991</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{\infty} = 1480$ MPa, $\sigma_Y = 104$ MPa $C = 90$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GS = 100nm</td>
<td>123.8</td>
<td>8.9</td>
<td>390.0</td>
<td>$\sigma = K(\varepsilon_p)^N$</td>
<td>[4]</td>
<td>0.992</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$K = 950.8$ MPa $N = 0.101$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iii) Simulation Details

1. Boundary Conditions
   - Bottom platen: constrained in all degrees of freedom
   - Top platen: Prescribed displacement of – 5mm in Y-axis; constrained in all other degrees of freedom

2. Interaction Properties
   - Platen-Cylinder: Static frictional coefficient of 0.6

3. Element Type
   - Cylinder: Hexahedral with reduced integration and enhanced hourglass control (C3D8R in Abaqus library).

4. Analysis Method
   - Explicit, Dynamic
   - Abaqus explicit solver
   - Time step: 0.015s

5. Failure Model
   - GTN model (Section 2.3.2); “Porous Metal Plasticity” in Abaqus
   - Evaluation basis: strain

References

Appendix B

Table B.1.: Characteristic strain intervals from the onset of elastic deformation to the post-peak softening regime during the compression of an AA3003 aluminum alloy column with a slenderness ratio of 25, showing the relationship between stresses $\sigma_A$ and $\sigma_B$ in points A and B respectively in Figure 3.2.3, their derivative with respect to the column’s compressive strain, and the rate of change of the column’s compressive stress with strain ($\frac{d\sigma}{d\varepsilon}$).

<table>
<thead>
<tr>
<th>Strain Range</th>
<th>$\sigma_A$; $\sigma_B$</th>
<th>$\frac{d\sigma_A}{d\varepsilon}$; $\frac{d\sigma_B}{d\varepsilon}$</th>
<th>$\frac{d\sigma}{d\varepsilon}$</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \varepsilon &lt; 2.7 \times 10^{-3}$</td>
<td>$\sigma_A = \sigma_B &lt; \sigma_p$</td>
<td></td>
<td>$\frac{d\sigma_A}{d\varepsilon} = \frac{d\sigma_B}{d\varepsilon} &gt; 0$</td>
<td>Elastic Compression</td>
</tr>
<tr>
<td>$\varepsilon = 2.7 \times 10^{-4}$</td>
<td>$\sigma_A = \sigma_B = \sigma_p$</td>
<td></td>
<td>$\frac{d\sigma}{d\varepsilon} &gt; 0$</td>
<td>End of elastic compression</td>
</tr>
<tr>
<td>$2.7 \times 10^{-4} &lt; \varepsilon &lt; 0.005$</td>
<td>$\sigma_A = \sigma_B &gt; \sigma_p$</td>
<td></td>
<td>$\frac{d\sigma_A}{d\varepsilon} &gt; \frac{d\sigma_B}{d\varepsilon} \geq 0$</td>
<td>Inelastic Compression</td>
</tr>
<tr>
<td>$\varepsilon = 0.005$</td>
<td>$\sigma_A = \sigma_B = \sigma_{CR}$</td>
<td></td>
<td></td>
<td>Critical (bifurcation) stress ($\sigma = \sigma_{CR}$)</td>
</tr>
<tr>
<td>$0.005 &lt; \varepsilon &lt; 0.008$</td>
<td>$\sigma_A &gt; \sigma_B &gt; 0$</td>
<td></td>
<td></td>
<td>Stress bifurcation; Column bending</td>
</tr>
<tr>
<td>$0.008 &lt; \varepsilon &lt; 0.012$</td>
<td></td>
<td></td>
<td></td>
<td>Stress reversal</td>
</tr>
<tr>
<td>$\varepsilon &gt; 0.012$</td>
<td>$\sigma_A &gt; 0$; $\sigma_B &lt; 0$</td>
<td></td>
<td>$\frac{d\sigma_A}{d\varepsilon} &gt; 0$; $\frac{d\sigma_B}{d\varepsilon} &lt; 0$</td>
<td>Peak compressive stress ($\sigma = \sigma_{PK}$)</td>
</tr>
<tr>
<td>$\varepsilon = 0.015$</td>
<td></td>
<td></td>
<td>$\frac{d\sigma}{d\varepsilon} = 0$</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon &gt; 0.015$</td>
<td></td>
<td></td>
<td>$\frac{d\sigma}{d\varepsilon} &lt; 0$</td>
<td>Softening compressive behavior</td>
</tr>
</tbody>
</table>
Table B.2.: Characteristic strain intervals from the onset of elastic deformation to the post-peak softening regime during the compression of a n-Ni/AA3003 composite column with a slenderness ratio of 25, showing the relationship between stresses $\sigma_A$ and $\sigma_B$ in points A and B of the core (subscript C) and sleeve (subscript S) respectively in Figure 3.2.3, their derivative with respect to the column’s compressive strain, and the rate of change of the column’s compressive stress with strain ($\frac{d\sigma}{de}$).

<table>
<thead>
<tr>
<th>Strain Range</th>
<th>$\sigma_{AC}$; $\sigma_{BC}$</th>
<th>$\sigma_{AS}$; $\sigma_{BS}$</th>
<th>$\frac{d\sigma_{AC}}{de}$; $\frac{d\sigma_{BC}}{de}$</th>
<th>$\frac{d\sigma_{AS}}{de}$; $\frac{d\sigma_{BS}}{de}$</th>
<th>$\frac{d\sigma}{de}$</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; \varepsilon &lt; 2.7 \times 10^{-4}$</td>
<td>$\sigma_{AC} = \sigma_{BC} &lt; \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} &lt; \sigma_{PS}$</td>
<td>$\frac{d\sigma_{AC}}{de} = \frac{d\sigma_{BC}}{de} &gt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} = \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} &gt; 0$</td>
<td>Elastic compression in both phases</td>
</tr>
<tr>
<td>$\varepsilon = 2.7 \times 10^{-3}$</td>
<td>$\sigma_{AC} = \sigma_{BC} = \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} = \sigma_{PS}$</td>
<td></td>
<td></td>
<td>$\frac{d\sigma}{de} = 0$</td>
<td>End of elastic compression in core</td>
</tr>
<tr>
<td>$2.7 \times 10^{-3} &lt; \varepsilon &lt; 3.1 \times 10^{-3}$</td>
<td>$\sigma_{AC} = \sigma_{BC} &gt; \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} &lt; \sigma_{PS}$</td>
<td>$\frac{d\sigma_{AC}}{de} = \frac{d\sigma_{BC}}{de} &gt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} = \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} &gt; 0$</td>
<td>Elastic compression in sleeve, inelastic compression in core</td>
</tr>
<tr>
<td>$\varepsilon = 3.1 \times 10^{-3}$</td>
<td>$\sigma_{AC} = \sigma_{BC} = \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} = \sigma_{PS}$</td>
<td></td>
<td></td>
<td>$\frac{d\sigma}{de} = 0$</td>
<td>End of elastic compression in sleeve</td>
</tr>
<tr>
<td>$3.1 \times 10^{-3} &lt; \varepsilon &lt; 0.011$</td>
<td>$\sigma_{AC} = \sigma_{BC} &gt; \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} &gt; \sigma_{PS}$</td>
<td>$\frac{d\sigma_{AC}}{de} = \frac{d\sigma_{BC}}{de} &gt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} = \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} &gt; 0$</td>
<td>Inelastic compression in both phases</td>
</tr>
<tr>
<td>$\varepsilon = 0.011$</td>
<td>$\sigma_{AC} = \sigma_{BC} = \sigma_{PC}$</td>
<td>$\sigma_{AS} = \sigma_{BS} = \sigma_{PS}$</td>
<td></td>
<td></td>
<td>$\frac{d\sigma}{de} = 0$</td>
<td>Critical (bifurcation) stress ($\sigma = \sigma_{CR}$)</td>
</tr>
<tr>
<td>$0.011 &lt; \varepsilon &lt; 0.012$</td>
<td>$\sigma_{AC} &gt; \sigma_{BC} &gt; 0$</td>
<td>$\sigma_{AS} &gt; \sigma_{BS} &gt; 0$</td>
<td>$\frac{d\sigma_{AC}}{de} &gt; \frac{d\sigma_{BC}}{de} &gt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} &gt; \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} &gt; 0$</td>
<td>Stress bifurcation; Column bending</td>
</tr>
<tr>
<td>$0.012 &lt; \varepsilon &lt; 0.016$</td>
<td>$\sigma_{AC} &gt; 0; \sigma_{BC} = 0$</td>
<td>$\sigma_{AS} &gt; \sigma_{BS} \geq 0$</td>
<td>$\frac{d\sigma_{AC}}{de} &gt; 0; \frac{d\sigma_{BC}}{de} = 0$</td>
<td>$\frac{d\sigma_{AS}}{de} &gt; \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} = 0$</td>
<td>Stress reversal in sleeve and core</td>
</tr>
<tr>
<td>$\varepsilon = 0.016$</td>
<td>$\sigma_{AC} &gt; 0; \sigma_{BC} = 0$</td>
<td>$\sigma_{AS} &gt; \sigma_{BS} \geq 0$</td>
<td>$\frac{d\sigma_{AC}}{de} &gt; 0; \frac{d\sigma_{BC}}{de} = 0$</td>
<td>$\frac{d\sigma_{AS}}{de} &gt; \frac{d\sigma_{BS}}{de} &gt; 0$</td>
<td>$\frac{d\sigma}{de} = 0$</td>
<td>Peak column stress ($\sigma = \sigma_{PK}$)</td>
</tr>
<tr>
<td>$0.016 &lt; \varepsilon &lt; 0.025$</td>
<td>$\sigma_{AC} &gt; 0; \sigma_{BC} &lt; 0$</td>
<td>$\sigma_{AS} &gt; 0; \sigma_{BS} &lt; 0$</td>
<td>$\frac{d\sigma_{AC}}{de} &gt; 0; \frac{d\sigma_{BC}}{de} &lt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} &gt; \frac{d\sigma_{BS}}{de} &lt; 0$</td>
<td>$\frac{d\sigma}{de} &lt; 0$</td>
<td>Softening compressive behavior, Tensile stresses in core</td>
</tr>
<tr>
<td>$\varepsilon &gt; 0.025$</td>
<td>$\sigma_{AC} &gt; 0; \sigma_{BC} &lt; 0$</td>
<td>$\sigma_{AS} &gt; 0; \sigma_{BS} &lt; 0$</td>
<td>$\frac{d\sigma_{AC}}{de} &gt; 0; \frac{d\sigma_{BC}}{de} &lt; 0$</td>
<td>$\frac{d\sigma_{AS}}{de} &gt; \frac{d\sigma_{BS}}{de} &lt; 0$</td>
<td>$\frac{d\sigma}{de} &lt; 0$</td>
<td>Tensile stresses in sleeve</td>
</tr>
</tbody>
</table>
Table B.3.: Characteristic strain intervals during the compression of an $n$-Ni hollow cylindrical column with a slenderness ratio of 9, showing the relationship between stresses $\sigma_A$, $\sigma_D$ in points $A$-$D$ respectively in Figure 3.3.1b, their derivative with respect to the column’s compressive strain ($\dot{\sigma}_A - \dot{\sigma}_D$), and the rate of change of the column’s compressive stress with strain ($d\sigma/d\epsilon$).

<table>
<thead>
<tr>
<th>Strain Range</th>
<th>$\sigma_A, \sigma_B, \sigma_C, \sigma_D$</th>
<th>$\dot{\sigma}_A, \dot{\sigma}_B, \dot{\sigma}_C, \dot{\sigma}_D$</th>
<th>$d\sigma/d\epsilon$</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0&lt;\epsilon&lt;0.7 \times 10^{-3}$</td>
<td>$\sigma_A=\sigma_B=\sigma_D=\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A = \dot{\sigma}_B = \dot{\sigma}_D = \dot{\sigma}_C &gt; 0$</td>
<td></td>
<td>Uniform Elastic Compression</td>
</tr>
<tr>
<td>$\epsilon=0.7 \times 10^{-3}$</td>
<td>$\sigma_A&gt;\sigma_B=\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B = \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$</td>
<td></td>
<td>Initiation of Bending in Outside Surfaces</td>
</tr>
<tr>
<td>$0.7 \times 10^{-3}&lt;\epsilon&lt;2.8 \times 10^{-3}$</td>
<td>$\sigma_P &gt; \sigma_A &gt; \sigma_B = \sigma_D &gt; \sigma_C &gt; 0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$</td>
<td>$d\sigma/d\epsilon &gt; 0$</td>
<td>Elastic Deformation</td>
</tr>
<tr>
<td>$\epsilon=2.8 \times 10^{-3}$</td>
<td>$\sigma_A&gt;\sigma_B=\sigma_D&gt;\sigma_C&gt;0$; $\sigma_A&gt;\sigma_P$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$</td>
<td></td>
<td>Initiation of Inelastic Deformation</td>
</tr>
<tr>
<td>$2.8 \times 10^{-3}&lt;\epsilon&lt;2.2 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B=\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$</td>
<td></td>
<td>Uniform Stress Distribution in Middle Surface</td>
</tr>
<tr>
<td>$\epsilon=2.2 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$</td>
<td></td>
<td>Initiation of Bending in Middle Surface; Nominal Critical Stress</td>
</tr>
<tr>
<td>$2.2 \times 10^{-2}&lt;\epsilon&lt;2.3 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$; $\dot{\sigma}_C = 0$</td>
<td></td>
<td>Longitudinal Stress Increase in All Surfaces</td>
</tr>
<tr>
<td>$\epsilon=2.3 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$; $\dot{\sigma}_C = 0$</td>
<td></td>
<td>Stress Reversal in Outside Surface</td>
</tr>
<tr>
<td>$2.3 \times 10^{-2}&lt;\epsilon&lt;3.1 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$; $\dot{\sigma}_C = 0$</td>
<td></td>
<td>Nominal Stress Increase</td>
</tr>
<tr>
<td>$\epsilon=3.1 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &gt; 0$; $\dot{\sigma}_C = 0$</td>
<td>$d\sigma/d\epsilon = 0$</td>
<td>Peak Strength</td>
</tr>
<tr>
<td>$3.1 \times 10^{-2}&lt;\epsilon&lt;4.1 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&gt;0$; $\sigma_C&lt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &lt; 0$</td>
<td>$d\sigma/d\epsilon &lt; 0$</td>
<td>Decrease of Load Carrying Capacity</td>
</tr>
<tr>
<td>$\epsilon=4.1 \times 10^{-2}$</td>
<td>$\sigma_A&gt;\sigma_B&gt;\sigma_D&gt;\sigma_C&lt;0$</td>
<td>$\dot{\sigma}_A &gt; \dot{\sigma}_B &gt; \dot{\sigma}_D &gt; \dot{\sigma}_C &lt; 0$</td>
<td>$d\sigma/d\epsilon &lt; 0$</td>
<td>Tensile Stresses in Outer Surface</td>
</tr>
</tbody>
</table>
APPENDIX C: Effect of Sleeve and Core Geometry on Mechanical Performance

The addition of a reinforcing sleeve has two consequences on the mechanical performance of the precursor strut. First, the critical buckling stress increases as a result of the addition of a high strength, geometrically efficient component. Second, the density of the column can also increase as a result of the addition of a high-density component. In the case of elastic buckling, the effect of core and sleeve geometry on the increase in buckling strength and density is straightforward to evaluate and predict; the critical strength can be evaluated using Equation 3.2.12b with $E_{T,C} = E_C$ and $E_{T,S} = E_S$. The density of the column is a weighted area fraction of the density of the core material ($\rho_C$) and that of the sleeve material ($\rho_S$). In this case, geometrical and material properties are independent; furthermore material properties are characterized by the four constants $E_S$, $E_C$, $\rho_S$ and $\rho_C$. However, when the composite struts buckle inelastically, the material properties are not constant (i.e. stress and tangent modulus components in Equations 3.2.12a and 3.2.12b are a function of strain), and not independent of the geometrical variables (i.e. the stress and tangent modulus values that satisfy Equations 3.2.12a and 3.2.12b depend on the column’s geometrical characteristics). This section discusses the dependence of strength increase, weight penalty and strength to weight ratio on the geometrical properties of a composite columns that fail by inelastic buckling.

C.1. Effect of core and sleeve geometry on critical strength

The geometrical properties of the hybrid column can be represented by the non-dimensional core width $B = b/L$ and sleeve thickness $T = t/L$ introduced in Section 3.2.5. The critical strength relationship presented in Equation 3.2.12 assumes the form:
\[
\begin{align*}
\sigma_{CR}^{\text{COL}} &= \sigma_S \left[ 1 - \left( \frac{B}{B+2T} \right)^2 \right] + \sigma_C \left( \frac{B}{B+2T} \right)^2 \tag{C.1a} \\
\sigma_{CR}^{\text{COL}} &= \frac{\pi^2}{12} E_{T,S} \left( \frac{B+2T}{B+2T} \right)^4 - B^4 + E_{T,C} \left( \frac{B}{B+2T} \right)^2 \tag{C.1b}
\end{align*}
\]

where \( \sigma_s, \sigma_C, E_{T,S} \) and \( E_{T,C} \) are functions of strain \( \varepsilon \), and are evaluated at the critical strain \( \varepsilon = \varepsilon_{CR} \). The solutions of this system for a range of column widths and sleeve thicknesses in a n-Ni/AA3003 composite column are shown in Figure C.1.

\( \text{Figure C.1: Effect of normalized core width (B=b/L) and sleeve thickness (T=t/L) on the critical stress (} \sigma_{CR}^{\text{COL}} \text{) of an n-Ni/AA3003 composite column.} \)

The effect of increasing the normalized sleeve thickness at a constant core width is revealed in Figure C.2a, which shows the relationship \( \sigma_{CR}^{\text{COL}} (\varepsilon) \), obtained from Equations C.1a and C.1b for a reference composite column with core width \( B = 0.05 \) and sleeve thicknesses \( T = 0.03, 0.05 \) and \( 0.20 \). A higher sleeve thickness increases the area fraction of
the stronger phase, i.e. the coefficient \(1 - \left(\frac{B}{B + 2T}\right)^2\) of the variable \(\sigma_s(\varepsilon)\), thus increasing the stress of the composite material at any given strain (i.e. \(\sigma_{COL}^{\varepsilon}(\varepsilon)\) obtained by Equation C.1a). Similarly, there is an increase in the \(\sigma_{CR}^{COL} (\varepsilon)\) relationship obtained from the combination of the tangent moduli of Equation B.1b, due to an increase in the bending resistance the sleeve (i.e. the coefficient \(\frac{(B + 2T)^4 - B^4}{(B + 2T)^2}\) of the variable \(E_{T,S}(\varepsilon)\)). As a consequence of the increase of the stress at a given strain in both curves, their intersection (i.e. the critical stress) increases with \(T\) as shown in Figure C.2a. The geometry of the sleeve thus determines the critical stress through a combination of tangent modulus and stress values in the sleeve and core materials at the critical strain.

![Figure C.2: Effect of increasing n-Ni sleeve thickness (T=t/L) on the critical stress of an AA3003 core column with width B=b/L=0.05 (a). Shown are the stress-strain relationships obtained by Equations 18a and 18b for three normalized sleeve thicknesses: T= 0.03, 0.05 and 0.20. Effect of normalized column width (B) on the critical stress of a n-Ni/AA3003 column with a limiting sleeve thickness T=0.01 (b). Shown are the stress-strain relationships obtained by evaluating Equations 18a and 18b for core widths B=0.02, 0.06 and 0.20.](image-url)
The lines of constant $T$ in Figure C.1 show the effect of a variable core width at a limiting sleeve thickness; a maximum achievable critical stress is associated with each value of $T$. The presence of this upper limiting stress can be explained by considering the effect of strut width $B$ on $\sigma_{CR}^{COL} (\varepsilon)$ in Equations C.1a and C.1b. Figure C.2b shows this effect for a reference n-Ni/AA3003 hybrid column with limiting sleeve thickness $T=0.01$ and core widths $B =0.02, 0.06$ and $0.20$. At a constant sleeve thickness, an increasing strut width decreases the stress of the hybrid column at any given strain (Equation C.1a), because the area fraction of the weaker core $\left(\frac{B}{B + 2T}\right)^2$ increases at the expense of the area fraction of the sleeve $\left(1 - \frac{B}{B + 2T}\right)^2$. However, a higher core width increases the coefficient of the sleeve and core tangent moduli in Equation C.1b, due to an increase in second moment of area in both phases. As a result, the stress at any given strain determined by Equation C.1b, increases as shown in Figure C.2b. Consequently the dependence of the critical stress on column width follows two regimes. Initially the critical stress of the composite column increases with increasing $B$, because the decrease in composite stress is offset by the increase in tangent modulus stress. Subsequently the critical stress decreases with increasing core width because the decrease in composite stress at a given strain cannot be offset by the increase in tangent modulus stress.

C.2. Effect of core and sleeve geometry on density

While strengthening the composite column, the addition of the reinforcing nanocrystalline sleeve also introduces weight penalties. The density of the composite
column is an area fraction weighted average of the sleeve and core densities. Its dependence on the normalized core width $B$ and sleeve thickness $T$ is given by:

$$\rho_{\text{COL}} = \rho_S \left[ 1 - \left( \frac{B}{B + 2T} \right)^2 \right] + \rho_C \left( \frac{B}{B + 2T} \right)^2$$  
(C.2)

where $\rho_S$ and $\rho_C$ are the densities of the sleeve and core respectively. Figure B.3 plots the density of n-Ni/AA3003 composite columns as a function of the normalized core width and sleeve thickness; the density of composite columns is bracketed by the densities of the two starting materials, in this case $\rho_S = 8.9 \text{ g/cm}^3$, and $\rho_C = 2.7 \text{ g/cm}^3$. The factors $\left( \frac{B}{B + 2T} \right)^2$ and $\left[ 1 - \left( \frac{B}{B + 2T} \right)^2 \right]$ are the area fractions of the core and sleeve respectively, thus the density of the column changes monotonically with $B$ and $T$ between the density limits imposed by the two phases. Similar trends will always be seen when the sleeve is the denser phase.

Figure C.3: Effect of normalized core width ($B=b/L$) and sleeve thickness ($T=t/L$) on the density ($\rho_{\text{COL}}$) of an n-Ni/AA3003 composite column
C.3. Effect of core and sleeve geometry on specific strength

The specific buckling strength of the composite column is the ratio of critical strength to density, i.e. \( \frac{\sigma_{cr}}{\rho} \). Since the specific strength of the microtruss is proportional to the specific strength of the constituent column (see Equation 3.1.5), the strength to density ratio of the column contains all effects of strut geometry on the specific strength of the microtruss. The effect of \( B \) and \( T \) on the specific strength of a composite n-Ni/AA3003 column is shown in Figure C.4. The effect of sleeve thickness on specific strength is dependent on the rate of change of critical strength and density with thickness \( T \). Figure C.4 shows that for n-Ni/AA3003 composite columns, the strength benefits outweigh the weight disadvantages, i.e. all composite cores have higher specific strengths than their uncoated precursors, and the value of the specific strength increases with increasing sleeve thickness. Notice, however, that at a constant sleeve thickness there is an upper bound in the achievable specific strength. At higher core widths, the specific strength decreases as a consequence of the decreasing critical stress. While the density of the column decreases at high core widths, the rate of change is not sufficient to offset the rate of decrease of the critical strength. Therefore, the maximum achievable specific strength and the core width associated with it depend on the rate of change of the critical strength and density with \( B \), i.e. on the solution of Equations C.1a and C.1b. Note that the core width associated with the maximum specific strength of a composite column with a constant sleeve thickness is slightly higher than the core width that produces the upper bound of the critical strength, because the column’s density decreases monotonically with increasing core width.
Figure C.4: Effect of normalized core width ($B=b/L$) and sleeve thickness ($T=t/L$) on the specific strength ($\left(\sigma_{CR}/\rho\right)_{COL}$) of a n-Ni/AA3003 composite column.

C.4. Influence of geometry on material properties

This analysis on n-Ni/AA3003 composite columns can be used to propose guidelines for the selection of core and sleeve materials that can produce a structurally effective hybrid column. First, the strength of the hybrid must be higher than the strength of the uncoated column, i.e. $\sigma_{HYB}^{CR} > \sigma_{CORE}^{CR}$. If the critical load of the uncoated core is $P_{CR}^{CORE}$ and its area is $A_{CORE}$, this condition becomes $\frac{P_{CR}^{CORE} + \Delta P_{CR}^{CORE}}{A_{CORE} + \Delta A} > \frac{P_{CR}^{CORE}}{A_{CORE}}$, which can be further simplified into:

$$\frac{\Delta P_{CR}^{CORE}}{\Delta A} > \sigma_{CR}^{CORE} \quad (C.3).$$

Thus, the critical load increase per added unit area contributed by the reinforcing sleeve must be higher than the critical stress of the core. The solution of Equations C.1a and C.1b,
shown in Figure C.1 indicates that n-Ni/AA3003 composite columns satisfies this criterion.

Second, the specific strength of the hybrid column for any sleeve thickness must be higher than the specific strength of the uncoated column, or \( \frac{\sigma_{\text{CR}}^{\text{HYB}}}{\rho^{\text{HYB}}} > \frac{\sigma_{\text{CR}}^{\text{CORE}}}{\rho^{\text{CORE}}} \), where \( \sigma_{\text{CR}}^{\text{HYB}} \) and \( \rho^{\text{HYB}} \) are the critical stress and density of the hybrid column and \( \sigma_{\text{CR}}^{\text{CORE}} \) and \( \rho^{\text{CORE}} \) are the critical stress and density of the core column. This condition can be further decomposed into \( \frac{\sigma_{\text{CR}}^{\text{CORE}} + \Delta \sigma}{\rho^{\text{CORE}} + \Delta \rho} > \frac{\sigma_{\text{CR}}^{\text{CORE}}}{\rho^{\text{CORE}}} \), where \( \Delta \sigma \) and \( \Delta \rho \) are the strength and density increments of the hybrid column relative to the core column. When the sleeve is the denser phase, the first condition becomes:

\[
\frac{\Delta \sigma}{\Delta \rho} > \frac{\sigma_{\text{CR}}^{\text{CORE}}}{\rho^{\text{CORE}}} \tag{C.4}
\]

This condition indicates that the reinforcing sleeve must provide an increase in critical strength per unit change in density that is at least as large as the specific critical strength of the initial core column. The values of \( \frac{\Delta \sigma}{\Delta \rho} \) and \( \frac{\sigma_{\text{CR}}^{\text{CORE}}}{\rho^{\text{CORE}}} \) for the n-Ni/AA3003 columns of this study are shown in Figure C.5. Notice that the extent of increase in critical strength per unit density is due to both the dimensions of the core and those of the sleeve. For instance, at a constant core width, the addition of the sleeve increases the second moment of area of the reinforcing phase and the width (i.e. the stockiness) of the composite column, and increases the area fraction of the denser phase. Since the strength benefits (\( \Delta \sigma \)) outweigh the weight penalties (\( \Delta \rho \)), the strength increase per unit density increases at larger sleeve thicknesses \( T \). Similarly, the core width \( B \) determines the second moment of area of both the sleeve and the core, the stockiness of the composite column and the area fraction of the
two phases. The strength increase due to the higher second moment of areas and column stockiness dominates the ratio $\frac{\Delta \sigma}{\Delta \rho}$, thus higher values of strength to weight ratios are obtained for starting columns with larger width.

*Figure C.5: Strength increase per unit density ($\frac{\Delta \sigma}{\Delta \rho}$) and critical strength of the uncoated core ($\frac{\sigma_{CR}}{\rho}$) for n-Ni/AA3003 composite columns.*

The design criteria established in this section can be used to impose limits on the materials that can be used as effective reinforcing sleeves, given a proposed core material. Furthermore, Equations C.1 and CS.2 can be used to predict the range of the achievable strength, density and specific strength for any core-sleeve material combination. The results presented in this section have shown that when the composite columns fail by inelastic buckling, this material selection analysis will demand knowledge of the complete stress-strain properties and density of the involved materials.
APPENDIX D: Mesh convergence study in the stretch-forming model of Section 4.1.

Figure D.1: Mesh convergence results of the forming of a square perforated blank (See Figure 4.1.2) at an initial (a), intermediate (b) and near-necking (c) stage of forming, and corresponding results for a rounded-square perforated blank at an initial (d), intermediate (e) and near-necking (f) stage (compression depth of 1.0 mm, 4.0 mm and 7.0 mm respectively). In each figure the values of the load (F), Mises stress ($\sigma_M$), and equivalent plastic strain ($\varepsilon_P$) are normalized by the values of the coarsest mesh.
APPENDIX E: Calibration of Gurson-Tvergaard-Needleman model for ductile failure

This Appendix shows the calibration of the GTN model parameters. A tensile test model was used for this purpose; the geometry and is shown in Figure E.1a. This geometry models one quarter of the tensile gage section (geometry details in standard ASTM E8). The deformed state and void volume fraction (VVF) distribution close to the initiation of failure (VVF = 0.15) is shown in Figure E.1b. Due to the symmetry condition, the corner elements (corresponding to the centre of the gage section) are the ones that show the first failure initiation.

Of the 8 material parameters of the GTN model (see Section 2.3.2) only one was changed from the values recommended by Tveergard and Needleman: the volume fraction

Figure E.1: Geometry (a) and distribution of void volume fraction (VVF) close to failure initiation (b) of the FE model used for calibration of GTN model parameters.
of nucleated voids $f_N$. The resultant stress-strain curves for models with the material properties of nanocrystalline Ni with grain size 20 nm and 100 nm are shown in Figures E.2a and E.2b respectively. The values that best reproduce the experimental failure strain are $f_N = 0.12$ and $f_N = 0.04$ for the materials with grain size of 20 nm and 100 nm respectively.

\[ \begin{align*}
\text{Figure E.2: Calibration of GTN model parameter } f_N, \text{ showing the resultant stress-strain curves for material properties of nanocrystalline Ni with grain size 20 nm (a) and 100 nm (b).}
\end{align*} \]
APPENDIX F: Structural performance indices of struts reinforced with nanocrystalline Ni sleeves

The figures of this Appendix extend the material performance analysis of Section 5.2.1 to two other core materials, a Cu-Be alloy and a Si-Cu alloy, with properties given in Refs. [3, 6, 7] of Section 5.2.1. These envelopes were calculated using Equations 5.2.1 and 5.2.2.

Figure F.1: Structural performance index envelopes $\Delta \sigma / \Delta \rho$ (a) and $\Delta (\sigma / \rho)$ (b) as a function of the core slenderness ratio $L/r_C$ for columns coated with n-Ni sleeves with area fraction $f_S = 0.01$.

Figure F.2: Structural performance index envelopes $\Delta \sigma / \Delta \rho$ (a) and $\Delta (\sigma / \rho)$ (b) as a function of the sleeve area fraction $f_S$ for columns coated with n-Ni sleeves with core slenderness ratio $L/r_C = 25$. 