Control and Protection of Multi-DER Microgrids

by

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This dissertation proposes a power management and control strategy for islanded microgrids, which consist of multiple electronically-interfaced distributed energy resource (DER) units, to achieve a prescribed load sharing scheme. This strategy provides i) a power management system to specify voltage set points based on a classical power flow analysis; 2) DER local controllers, designed based on a robust, decentralized, servomechanism approach, to track the set points; and 3) a frequency control and synchronization scheme. This strategy is then generalized to incorporate both power-controlled and voltage-controlled DER units.

Since the voltage-controlled DER units do not use inner current control loops, they are vulnerable to overcurrent/overload transients subsequent to system severe disturbances, e.g., faults and overloading conditions. To prevent DER unit trip-out or damage under these conditions, an overcurrent/overload protection scheme is proposed that detects microgrid abnormal conditions, modifies the terminal voltage of the corresponding VSC to limit DER unit output current/power within the permissible range, and restores voltage controllers subsequently.

Under certain circumstances, e.g., microgrid islanding and communication failure, there is a need to switch from an active to a latent microgrid controller. To minimize the resultant transients, control transition should be performed smoothly. For the aforementioned two circumstances, two smooth control transition techniques, based on 1) an observer and 2) an auxiliary tracking controller, are proposed to achieve a smooth control transition.

A typical microgrid system that adopts the proposed strategy is investigated. The microgrid dynamics are investigated based on eigenvalue sensitivity and robust analysis studies to evaluate the performance of the closed-loop linearized microgrid. Extensive case studies,
based on time-domain simulations in the PSCAD/EMTDC platform, are performed to evaluate performance of the proposed controllers when the microgrid is subject to various disturbances, e.g., load change, DER abrupt outage, configuration change, faults, and overloading conditions. Real-time hardware-in-the-loop case studies, using an RTDS system and NI-cRIO industrial controllers, are also conducted to demonstrate ease of hardware implementation, validate controller performance, and demonstrate its insensitivity to hardware implementation issues, e.g., noise, PWM nonidealities, A/D and D/A conversion errors and delays.
Acknowledgements

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Amir Hossein Etemadi
University of Toronto
August 2012
To Maryam
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Nomenclature

List of Abbreviations

CB  Circuit Breaker
DER  Distributed Energy Resource
DFM  Decentralized Fixed Mode
DRSP  Decentralized Robust Servomechanism Problem
FPGA  Field-Programmable Gate Array
GPS  Global Positioning System
HIL  Hardware-In-the-Loop
IEEE  Institute of Electrical and Electronics Engineers
IGBT  Insulated Gate Bipolar Transistor
LC  Local Controller
LCL  Inductive(L)-Capacitive(C)-Inductive(L)
LTI  Linear Time-Varying
MIMO  Multi-Input Multi-Output
NI-cRIO  National Instrument-Compact Reconfigurable Input Output
PC-DER  Power-Controlled DER
PCC  Point of Common Coupling
PID  Proportional, Integral, Derivative
PLL  Phase-Locked Loop
PMS  Power Management System
PWM  Pulse-Width Modulation
RL  Resistive(R)-Inductive(L)
RLC  Resistive(R)-Inductive(L)-Capacitive(C)
RMS  Root Mean Square
RTDS  Real-Time Digital Simulator
SISO  Single-Input Single-Output
List of Symbols

\( \alpha \)
Scaling factor

\( \beta_i \)
Stabilizing compensator states of \( i \)th control agent

\( X \)
Dynamic phasor corresponding to \( X \)

\( \Delta I_{\text{tol}} \)
Tolerance level for restoring the original controller

\( \Delta \)
Real perturbation matrix

\( \delta_i \)
Set point for PCC\(_i\) voltage angle

\( \delta_{ik} \)
Difference in voltage angle between PCC\(_i\) and PCC\(_k\)

\( \epsilon \)
Controller design parameter

\( \eta_i \)
Servo-compensator states of \( i \)th control agent

\( \gamma \)
Voltage restoration level used for fault clearance determination

\( \Lambda \)
Solution of the Lyapunov matrix equation

\( C_g \)
Left-half complex plain

\( \mathcal{C}_{1,i}, \mathcal{C}_{2,i} \)
Controller transfer functions corresponding to Subsystem\(_i\)

\( A, B \)
State-space matrices of the stabilizing compensator

\( K_i \)
Controller gains

\( \nu \)
Number of decentralized control agents

\( \omega_0 \)
Nominal power angular frequency

\( \bar{\beta} \)
Input time-delay tolerance

\( \partial C_g \)
Imaginary axis

\( \theta_0 \)
Initial phase-angle

\( \theta_i \)
Phase-angle waveform of DER\(_i\)

\( A \)
State matrix of the open-loop system

\( A_c \)
State matrix of controller

\( A_o \)
State matrix of observer

\( A_{\text{close}} \)
State matrix of the closed-loop system

\( B \)
Input gain matrix of the open-loop system

\( B_c \)
Input gain matrix of controller

\( B_i \)
Input gain matrix of Subsystem\(_i\)

\( B_o \)
Input gain matrix of observer

\( B_{ik} \)
Imaginary part of the element \( Y_{ik} \) of \( Y_{\text{Bus}} \)
C  Output gain matrix of the open-loop system
$C_A$  Active controller
$C_c$  Output gain matrix of controller
$C_i$  Output gain matrix of Subsystem$_i$
$C_L$  Latent controller
$c_{tran}$  Constant coefficient to account for current transients
$D_c$  Feedforward matrix of controller
$E$  Unknown disturbance gain matrix
$e$  Error signal
$E_i(s)$  Error signal of Subsystem$_i$ in frequency domain
$F$  Unknown output noise matrix
$f_0$  Nominal power frequency
$G$  Plant
$G_{ik}$  Real part of the element $Y_{ik}$ of $Y_{Bus}$
$I$  Identity matrix
$I_{max}$  Maximum permissible output current
$I_{i,ref}$  Set point for the $d$-component of DER$_i$ current
$i_{i,abc}$  Three-phase output current of DER$_i$
$I_{i,dq}$  $d$ and $q$ components of output current of DER$_i$
$i_{Li,abc}$  Three-phase current of the reactive branch of Load$_i$
$I_{Li,dq}$  $d$ and $q$ components of current of the reactive branch of Load$_i$
$I_{pf}$  Prefault current
$I_{qi,ref}$  Set point for the $q$-component of DER$_i$ current
$I_{rs}$  Output current for restoration process
$I_{ii,dq}$  $d$ and $q$ components of current of Line$_i$
$I_{ii,dq}$  $d$ and $q$ components of current of Line$_i$
$J$  Optimal control cost function
$M, N$  Scaling matrices for real stability radius computation
$m_i$  Number of outputs of the $i$th control agent
$n$  Order of the open-loop system
$P_I$  Real power injected at PCC$_i$
$P_{DG,i}$  Real power output of DER$_i$
$p_{ki}$  Participation factor relating the $i$th eigenvalue to the $k$th state variable
$P_{Loss}$  Total real power loss
$P_{ref,i}$  Set point for the output real power of DER$_i$ current
$Q_i$  Reactive power injected at PCC$_i$
Q_{DG,i} \quad \text{Reactive power output of DER}_i \\
Q_{Loss} \quad \text{Total reactive power loss} \\
Q_{ref,i} \quad \text{Set point for the output reactive power of DER}_i \text{ current} \\
R_g \quad \text{Resistance of the main grid Thevenin equivalent} \\
R_i \quad \text{Resistive branch of Load}_i \\
r_i \quad \text{Number of outputs of Subsystem}_i \\
r_R \quad \text{Real stability radius} \\
R_{fi} \quad \text{Resistance of DER}_i \text{ filter} \\
R_{li} \quad \text{Series resistance of the reactive branch of Load}_i \\
S \quad \text{DER unit output MVA} \\
S_{base} \quad \text{Base MVA} \\
S_{max} \quad \text{DER unit MVA capacity} \\
T_L \quad \text{Auxiliary tracking controller} \\
u \quad \text{Controller output} \\
u_i \quad \text{Controller output to Subsystem}_i \\
U_i(s) \quad \text{Controller output to Subsystem}_i \text{ in frequency domain} \\
V_i \quad \text{Voltage of PCC}_i \\
v_{ri,wi} \quad \text{Right and left eigenvectors associated with the } i\text{th eigenvalue} \\
V_{base,high} \quad \text{Base voltage for the high-voltage side of the transformer} \\
V_{base,low} \quad \text{Base voltage for the low-voltage side of the transformer} \\
V_{d,ref,i} \quad \text{Set point for the } d\text{-component of PCC}_i \text{ voltage} \\
V_{dc} \quad \text{Ideal DC source representing the prime mover} \\
v_{i,abc} \quad \text{Three-phase voltage of PCC}_i \\
V_{i,dq} \quad d \text{ and } q \text{ components of voltage of PCC}_i \\
V_{q,ref,i} \quad \text{Set point for the } d\text{-component of PCC}_i \text{ voltage} \\
v_{i,abc,j,HV} \quad \text{VSC}_j \text{ terminal voltage referred to the high-voltage side} \\
v_{i,abc,j,LV} \quad \text{VSC}_j \text{ terminal voltage referred to the low-voltage side} \\
V_{id,i} \quad d\text{-component of the voltage terminal of VSC}_i \\
V_{id,mod} \quad \text{Modified VSC terminal voltage, } d\text{-component} \\
v_{i,abc} \quad \text{Three-phase voltage of VSC}_i \text{ terminal} \\
V_{ii,dq} \quad d \text{ and } q \text{ components of voltage of VSC}_i \text{ terminal} \\
V_{iq,i} \quad q\text{-component of the voltage terminal of VSC}_i \\
V_{iq,mod} \quad \text{Modified VSC terminal voltage, } q\text{-component} \\
x \quad \text{State variable} \\
X_g \quad \text{Reactance of the main grid Thevenin equivalent} \\
X_T \quad \text{Transformer leakage reactance}
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<tr>
<td>$X_{fi}$</td>
<td>Reactance of DER$_i$ filter</td>
</tr>
<tr>
<td>$X_{Li}$</td>
<td>Reactive branch of Load$_i$</td>
</tr>
<tr>
<td>$y$</td>
<td>Output of the open-loop system</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Open-loop system output of Subsystem$_i$</td>
</tr>
<tr>
<td>$Y_i(s)$</td>
<td>Output of Subsystem$_i$ in frequency domain</td>
</tr>
<tr>
<td>$Y_{Bus}$</td>
<td>Bus admittance matrix</td>
</tr>
<tr>
<td>$Y_{ik}$</td>
<td>$ik$th element of the bus admittance matrix</td>
</tr>
<tr>
<td>$y_{obs}$</td>
<td>Observer output</td>
</tr>
<tr>
<td>$y^i_{ref}$</td>
<td>Set points of Subsystem$_i$</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Background

TECHNICAL and economical viability of the distributed energy resource (DER) technologies for distribution voltage class applications have resulted in the rapid deployment of DER units at distribution voltage classes. This increasing interest is due to the following benefits: local response to load growth, reduction of environmental impacts, low capital investment and construction time of DER systems, deferral of transmission and distribution expansion, reduction of distribution and transmission losses, and power quality/reliability enhancement by generation augmentation. Difficulties arise, however, due to the interconnection of DER units in the absence of appropriate control and power management, e.g., variation in voltage profile as well as power flow direction, increase in short circuit level, lack of coordination or malfunction of the conventional protective devices, instability issues, and potential adverse impact on reliability and power quality [1].

To fully realize the emerging potential of DER technologies and avoid their drawbacks, a system approach can be taken which views distributed generation and associated loads as a microgrid [2–4]. A microgrid is a group of DER units and electrical loads, served by a distribution system, that acts as a single controllable entity with respect to the main grid. A microgrid should be able to operate in grid-connected mode, islanded mode, and the transition between these two [5].

In grid-connected mode, the microgrid voltage and frequency are predominantly dictated by the main grid. The microgrid control strategy ensures that electrical loads within the microgrid are supplied by the DER units and the excess (shortage) of power can be managed by exporting (importing) power to (from) the main grid. In grid-connected mode of operation, DER units are generally required to generate a constant amount of power with a prespecified range of power factor, e.g., a power factor greater than 0.95 [6,7].
1.2 Literature Review

In the islanded mode of operation, which was not permitted until the approval of IEEE Standard 1547.4 [8], the microgrid operates as an autonomous power system and its control strategy must regulate voltage/frequency of the system and create a balance between microgrid net demand and generation. Controlling a microgrid, especially in the islanded mode of operation, is inherently more complicated as compared with a conventional power system due to the following reasons [9]: close geographical/electrical proximity of DER units that closely couple; fast dynamics and short response time of DER units, particularly electronically-coupled units; low energy storage capacity and lack of inertia due to the increasing penetration and/or dominance of electronically-interfaced DER units; nondispatchable nature of certain DER technologies such as wind and solar photovoltaic; and the high degree of uncertainty in microgrid load composition and parameters.

This research work seeks to devise a control strategy that maintains voltage and frequency of an islanded microgrid that consists of multiple electronically-interfaced DER units and enables it to ride through transients caused by system disturbances such as load change, DER unit switching, fault conditions, distribution line outage, and other disturbances.

The rest of this chapter is organized as follows. Section 1.2 reviews the technical literature concerning the existing microgrid control methods and identifies their merits/drawbacks. Section 1.3 presents the statement of the problem and research objectives. The proposed methodology to achieve research objectives is presented in Section 1.4. Section 1.5 provides the dissertation layout.

1.2 Literature Review

The existing microgrid control schemes can be divided into droop-based and non-droop-based approaches. Controlling DER units based on droop characteristics is the ubiquitous method in the literature [10–17]. The droop-based approach originates from the principle of power balance of synchronous generators in large interconnected power systems. That is, an imbalance between the input mechanical power of the generator and its output electric real power causes a change in the rotor speed which is translated to a deviation of the frequency. Likewise, output reactive power variation results in deviation of voltage magnitude. The same principle is artificially employed for electronically-interfaced DER units of a microgrid as well. Opposite droop control, i.e., using real power/voltage and reactive power/frequency droop characteristics, has also been applied for low voltage microgrids in view of their low $X/R$ ratios [18, 19].

The main advantage of a droop-based approach is that it obviates the need for communication since the control action is performed merely based on local measurements. This feature
gives droop control a significant flexibility in that as long as a balance between generation and demand can be maintained, there is no interdependency between the DER unit local controllers. However, a droop-based approach inherently exhibits a number of limitations [20–23]:

- It suffers from poor transient performance or instability issues due to the use of average values of active and reactive power over a cycle.
- It generally does not take into account the load dynamics and consequently, may fail or not properly respond after a large or a fast load change.
- It cannot initiate a black start and certain provisions are required for system restoration.
- It exhibits poor performance when adopted for distribution networks due to their low $X/R$ ratios.
- It fails to provide accurate power sharing among the DER units due to output impedance uncertainties.
- It is not suitable for nonlinear loads since it fails to account for harmonic currents.
- It fails to fully decouple active and reactive power components.
- It fails to provide a fixed frequency and frequency restoration for distribution system is not a fully resolved issue.

Droop-based methods have been modified in an attempt to overcome these drawbacks by including a frequency restoration loop [24]; introducing virtual output impedance for more precise power sharing [21, 25–28]; proposing an adaptive droop function to improve transient performance [29], including an adaptive feedforward compensation to improve microgrid stability [20]; including a $V_{DC}/V_{AC}$ droop characteristics to account for DC-link voltage fluctuations [19]; drooping the inverter output voltage angle rather than its frequency using the GPS system to achieve a better transient performance [30]; and including an internal controller to address nonlinearity and imbalance of microgrid loads [31–34]. Small signal stability and eigenvalue analysis of converter-fed microgrids that are controlled based on droop characteristics are also reported in the literature [26, 35–37] and conclude that the dominant modes of the closed-loop system are determined by droop controllers, and the overall stability of the system depends on droop control gains, system loading conditions, and network parameters.

The second category of the existing microgrid control methods in the technical literature are non-droop-based approaches which can be further divided into schemes that are suitable for (i) a single-DER and (ii) a multi-DER microgrid. The main objective of the former schemes is to control the voltage/frequency of the microgrid within a permissible range, whereas the latter schemes should also provide appropriate load sharing among the DER units. The following schemes have been proposed for single-DER microgrids: a voltage controller, designed using an $H_{\infty}$ approach and repetitive control technique, to mitigate voltage harmonics of the point of common coupling (PCC) [38]; a robust control scheme for a microgrid designed based on an
$H_{\infty}$ approach to provide a robust performance [39]; and a robust servomechanism approach for PCC voltage control [40]. These methods are, however, only applicable to single-DER microgrids.

The reported control schemes for an islanded multi-DER microgrid are as follows. A centralized controller is proposed in [35] where a central controller determines the contribution of each DER unit and an outer voltage control loop regulates the voltage of the system. This method results in fast mitigation of transients; however, it relies on the availability of a voltage-controlled DER unit and a high-bandwidth communication link. It is suggested in [41] that a master/slave control strategy be utilized where a dominant DER unit regulates microgrid voltage and other units supply the load. This method is flexible in terms of connection and disconnection of DER units, however the presence of the dominant DER is crucial. A voltage control and power sharing method for multiple parallel DER units in a microgrid is proposed in [42, 43] where a low-bandwidth communication link is used to achieve load sharing and voltage is controlled through a central controller. This method is only applicable to a single-bus microgrid whose DER units operate in parallel.

Although a great deal of research exists on the development of microgrid control strategies, none fully address the following requirements: robustness to topological and parametric uncertainties, satisfactory transient response of the controllers, obviating the need for a complex communication infrastructure, improving fault ride-through capabilities, and developing smooth control transition schemes. This research is an attempt to develop a microgrid control strategy to address the drawbacks/limitations of the existing approaches and meet the above requirements.

1.3 Statement of the Problem and Research Objectives

As it was presented in the previous section, each of the existing control schemes suffers from one or more of the following limitations/weaknesses:

- lack of adequate robustness and inability to accommodate microgrid uncertainties,
- poor transient performance,
- inability to initiate a black start after system collapse,
- dependency on specific microgrid configurations,
- coupled real/reactive output power components of DER units,
- relying on a dominant DER unit to regulate microgrid voltage/frequency,
- the need for a high-bandwidth and uneconomical communication link,
- lack of a back-up control scheme in case of communication failure, and
- the need to modify a central controller after each DER unit switching.
This research work seeks to develop a microgrid control strategy that addresses the limitations of the existing methods with the following objectives:

1. To develop a voltage control method to maintain the stable operation of an islanded microgrid consisting of electronically-interfaced DER units;
2. To develop a power management system to specify set points for DER local controllers;
3. To generalize the control scheme such that DER units can control either their output power or the corresponding PCC voltage;
4. To develop an overcurrent/overload protection scheme, as a part of the proposed controller, to prevent damage and trip-out of DER units due to overcurrent/overload, subsequent to disturbances, e.g., faults and overloading conditions;
5. To devise a back-up control in case of communication failure and provide smooth transition from the main to the backup controllers;
6. To provide a smooth control transition scheme in case of microgrid transition from grid-connected to islanded mode of operation.

To achieve these objectives, an innovative high-performance, MIMO\(^1\), robust\(^2\), decentralized\(^3\) control strategy is proposed. This control scheme is suitable for the problem of controlling a multi-DER islanded microgrid in view of the following considerations.

- A MIMO controller can inherently account for multiple input/output control channels. The proposed control strategy is performed in a \(dq\) frame of reference, and as a result, each controller has at least two inputs and outputs that are accounted for through a MIMO control design.
- A robust control strategy is preferred due to microgrid uncertainties, e.g., load changes, DER units switchings, and configuration changes. These uncertainties can cause oscillations and/or instability due to the lack of enough energy storage and inertia in a microgrid. However, a robust microgrid controller guarantees a satisfactory performance despite microgrid unmeasurable disturbances/uncertainties.
- A decentralized control is well suited for a microgrid for the following reasons.
  - The DER units are not necessarily in geographical proximity. Therefore, employing local decentralized controllers for each DER unit obviates the need for communicating time-varying signals to a distant central control system which needs a

---

\(^1\)Multi-Input Multi-Output

\(^2\)Robustness means that the control scheme is capable of stabilizing and maintaining viable operation of the system despite parametric or structural uncertainties, i.e., the parameters of the plant and its configuration can change in the practical range of variation while controllers perform satisfactorily despite these plant perturbations.

\(^3\)Decentralized control is a type of control scheme which splits a central controller into several local controllers that can perform the same control action without communicating with other peer controllers. Decentralized control is desirable for complex industrial systems with high dimensionality, information structure constraints, and uncertainty.
A decentralized controller can meet its requirements despite unplanned outage or abrupt connection of other DER units, thus eliminating the interdependency of DER local controllers.

- The computational burden of a high order system, e.g., a microgrid, is overcome by splitting the system into several subsystems and implementing the controller in a decentralized manner.

The above considerations conclude that the proposed control strategy is a suitable solution for the problem of islanded microgrid control.

### 1.4 Methodology

In order to achieve the aforementioned dissertation objectives, the following methodology is employed:

**Linear model development:** In this dissertation, the control scheme is devised based on a linear state-space model of a microgrid. The equations describing the microgrid are first derived in natural (abc) reference frame and then transformed to a synchronous (dq) reference frame based on which the robust decentralized controllers are designed. MATLAB/Simulink environment is used to verify the derived linear model, design and analyze the proposed control strategy, and evaluate its robustness.

**Time-domain simulation:** The linear model of the system is unable to represent certain effects that exist in the actual system such as harmonics and nonlinearities. To evaluate and validate the performance of the proposed control strategy and to investigate the envisioned controller behavior under the aforementioned unmodeled effects, a time-domain model of the system, including the proposed control system, is developed in PSCAD/EMTDC environment. Dynamic behavior of the system is investigated through time-domain simulations and the performance of the proposed controller is verified thereby.

**Hardware implementation:** To demonstrate the feasibility of hardware implementation, and to validate the performance of the proposed control strategies despite real-world implementation issues, the controllers are implemented and tested in a real-time hard-ware-in-the-loop (HIL) simulation environment.

### 1.5 Dissertation Layout

The rest of this dissertation is organized as follows.
Chapter 2 proposes a power management and control strategy for an islanded three-DER microgrid. The control objective is to regulate the voltage of DER PCCs to achieve a prespecified load sharing among the DER units. To this end, a PMS specifies voltage set points for the PCCs based on a classical power flow. The set points are communicated to local robust decentralized controllers which are designed using a decentralized, robust, servomechanisms problem (DRSP) approach, based on a linear mathematical model of the microgrid. Frequency control and DER synchronization is achieved using DER internal oscillators which are synchronized based on a time-reference signal received from the GPS system. Microgrid dynamics are investigated based on eigenvalue and robustness analysis, and performance of the proposed strategy is evaluated based on both offline time-domain and HIL simulation test cases.

Chapter 3 proposes an overcurrent and overload protection scheme for DER units which directly control their PCC voltage and consequently are vulnerable to microgrid severe disturbances, e.g., faults and overloading conditions. Offline time-domain and HIL simulations verify the performance of the proposed scheme.

Chapter 4 generalizes the voltage control scheme of Chapter 2 and applies it to a more elaborate microgrid system. In Chapter 2, each DER unit controls the voltage of its PCC to indirectly regulate its power output. In Chapter 4, however, the microgrid operator can assign a number of DER units as voltage-controlled to guarantee an acceptable microgrid voltage profile, and other units operate as power-controlled to generate specified amounts of real/reactive power. The assignment scheme depends on microgrid configuration, DER unit locations, load level, and other possible considerations. Analysis of microgrid dynamics and offline/HIL simulations demonstrate the desirable performance of the proposed generalized microgrid control strategy.

Chapter 5 proposes two smooth control transition schemes in the event of 1) microgrid transition from grid-connected to islanded mode of operation, and 2) communication failure. These control transitions should be performed smoothly while the resultant microgrid transients do not cause instability or trigger the protection system. To this end, two smooth control transition schemes, based on 1) an observer (or state estimator) and 2) an auxiliary tracking controller are proposed for the above two circumstances, respectively. Simulation results demonstrate feasibility of these smooth control transition schemes.

Chapter 6 summarizes the contributions of the dissertation, presents its conclusions, and recommends future research directions.
2 Robust Decentralized Microgrid Control

2.1 Introduction

This chapter presents a power management system (PMS) and a control strategy for an islanded multi-DER microgrid. Based on the proposed strategy (i) the PMS specifies voltage set points for each voltage-controlled bus based on a power flow analysis, (ii) local voltage controllers (LCs) provide tracking of the voltage set points, and (iii) an open-loop frequency control and synchronization scheme maintains system frequency.

The prominent features of the proposed strategy are the following: (i) The PMS precisely controls power flow of the system and achieves a prescribed load sharing among the DER units, (ii) LCs track voltage set points and rapidly reject disturbances, (iii) LCs are highly robust to parametric, topological, and unmodeled uncertainties of the microgrid, (iv) LCs are implemented in a decentralized manner; this obviates the need for a high-bandwidth communication medium to feed system’s information to a central authority and makes it scalable for larger number of DER units, (v) LCs enable the system to sudden connection/disconnection of DER units, and (vi) frequency of the system is fixed and cannot deviate due to transients. The proposed approach requires low-bandwidth communication for both synchronization and PMS data transmission. However, temporary failure of communication will not lead to system collapse, provided it recovers within a reasonable period of time, i.e., prior to significant changes in the microgrid operating point.

Local controllers, which are the main focus in this chapter, are developed based on a decentralized robust servomechanism approach, i.e., a decentralized controller is devised so that outputs of the system asymptotically track constant reference inputs independent of (i) constant disturbances to which the microgrid is subjected, and (ii) variations in the plant parameters and gains of the control system [44]. The robustness and the decentralized nature of the controller are highly desirable for a microgrid since
2.1 Introduction

- a centralized controller which requires all inputs to be communicated to a control center is uneconomical due to the complexity and cost of the required high-bandwidth communication infrastructure, and

- a robust controller overcomes the uncertainty issues of the microgrid structure/parameters.

In a decentralized scheme, a plant often has a number of control agents. Each control agent has a number of local control inputs and outputs, and a separate and distinct controller is applied to each control agent. In particular, there exists a solution to the problem of stabilizing a plant based on a decentralized control system if and only if the plant has no unstable decentralized fixed modes (DFM) and certain rank conditions of the plant data hold true [45].

The notion of robustness leads to the problem of constructing a controller for a plant such that the resultant closed-loop system satisfies a given robustness constraint, e.g., having a comparable real stability radius for both open-loop and closed-loop systems [46, 47]. This can be done by a decentralized controller design based on the approach of [44, 48–50], subject to a robustness constraint. The principles of application of this decentralized control scheme to conventional power systems is provided in [51–55].

In this chapter, the decentralized control strategy is applied to a microgrid system. To ensure robust performance of the controller, the decentralized controller optimization is carried out subject to a robustness constraint. To solve the robust servomechanism problem and to achieve further improvements in microgrid performance, the normal optimal control performance index, \( J = \int_{0}^{\infty} (x'Qx + u'Ru)d\tau \), which is used for obtaining optimal controllers to reject impulse disturbances, is replaced with the performance index \( J = \int_{0}^{\infty} (e'e + \epsilon \dot{u}'\dot{u})d\tau \), where \( \epsilon > 0 \) is a small weighting parameter. The resultant optimal controller obtained is an optimal servomechanism controller [56]. The microgrid model, existence conditions of the controller, design procedure, and the properties of the closed-loop system, including eigenvalue sensitivity and robustness analyses are discussed.

To study different performance aspects of the proposed strategy, it is applied to a three-DER microgrid and the results, based on both off-line and real-time simulations, are presented. A set of comprehensive digital time-domain simulation studies, in the PSCAD/EMTDC platform, validates the desired performance of the proposed PMS/control strategy in response to load change, set point tracking, nonlinear load (induction motor) energization, and changes in the microgrid topology. To demonstrate the feasibility of the power management and the control algorithms for digital implementation and also to evaluate the impact of nonidealities, e.g., noise, A/D and D/A conversion process, discretization error/delay, and PWM errors, performance of the control system was also examined in a hardware-in-the-loop (HIL) environment.
2.2 Study Microgrid System

A schematic diagram of a typical radial distribution feeder that is adopted as the study microgrid system is illustrated in Fig. 2.1(a). The microgrid includes three dispatchable DER units with the voltage rating of 0.6 kV and power ratings of 1.6, 1.2, and 0.8 MVA. It also includes three local loads, and two 13.8-kV distribution line segments. Each DER unit is represented by a 1.5-kV DC voltage source, a voltage sourced converter (VSC), and a series RL filter. DER units are interfaced to the grid through a 0.6-kV/13.8-kV step-up transformer (with the same power rating as the corresponding DER unit) at the point of common coupling (PCC). The main utility grid is represented by an AC voltage source behind series R and L elements. The microgrid can be operated in the grid-connected or the islanded modes based on the status of circuit breaker $\text{CB}_g$. The system parameters are given in Table 2.1.
2.3 Power Management and Control Strategy

Table 2.1
PARAMETERS OF THE MICROGRID

<table>
<thead>
<tr>
<th>Base Values</th>
<th>Transformer Parameters</th>
<th>Load Parameters</th>
<th>Line Parameters</th>
<th>Filter Parameters (based on DER_i Ratings)</th>
<th>Grid Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{\text{base}} = 1.6 \text{ MVA}$</td>
<td>$V_{\text{base, low}} = 0.6 \text{ kV}$</td>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
<td>$0.6/13.8 \text{ kV}$</td>
<td>$\Delta/Y_g$</td>
<td>$X_T = 8%$</td>
</tr>
<tr>
<td>$s_{\text{base}}$</td>
<td>$\Delta/Y_g$</td>
<td>$X_T = 8%$</td>
<td>$R_1$ 350 $\Omega/2.94 \text{ pu}$</td>
<td>$R_2$ 375 $\Omega/3.15 \text{ pu}$</td>
<td>$R_3$ 400 $\Omega/3.36 \text{ pu}$</td>
</tr>
<tr>
<td>$1.6 \text{ MVA}$</td>
<td>$\Delta/Y_g$</td>
<td>$X_T = 8%$</td>
<td>$X_{L1}$ 41.8 $\Omega/0.35 \text{ pu}$</td>
<td>$X_{L2}$ 37.7 $\Omega/0.32 \text{ pu}$</td>
<td>$X_{L3}$ 45.2 $\Omega/0.38 \text{ pu}$</td>
</tr>
<tr>
<td>$V_{\text{base, low}} = 0.6 \text{ kV}$</td>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
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<td>$R_3$ 400 $\Omega/3.36 \text{ pu}$</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
<td>$X_{C1}$ 44.2 $\Omega/0.37 \text{ pu}$</td>
<td>$X_{C2}$ 40.8 $\Omega/0.34 \text{ pu}$</td>
<td>$X_{C3}$ 48.2 $\Omega/0.41 \text{ pu}$</td>
<td></td>
</tr>
<tr>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
<td>$V_{\text{base, high}} = 13.8 \text{ kV}$</td>
<td>$X_{L1}$ 41.8 $\Omega/0.35 \text{ pu}$</td>
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<td>$X_{C3}$ 48.2 $\Omega/0.41 \text{ pu}$</td>
<td>$R_{l1}$ 2.09 $\Omega/0.02 \text{ pu}$</td>
<td>$R_{l2}$ 1.89 $\Omega/0.02 \text{ pu}$</td>
<td>$R_{l3}$ 2.26 $\Omega/0.02 \text{ pu}$</td>
</tr>
<tr>
<td>$X_{C1}$ 44.2 $\Omega/0.37 \text{ pu}$</td>
<td>$X_{C2}$ 40.8 $\Omega/0.34 \text{ pu}$</td>
<td>$X_{C3}$ 48.2 $\Omega/0.41 \text{ pu}$</td>
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<td>$R_{l3}$ 2.26 $\Omega/0.02 \text{ pu}$</td>
</tr>
</tbody>
</table>

2.3 Power Management and Control Strategy

Fig. 2.1(b) shows a schematic diagram of the proposed control strategy which is composed of (i) a power management system (PMS), (ii) local controllers (LCs) of each DER unit, and (iii) microgrid frequency control and synchronization scheme. The instantaneous real/reactive power of DER units and loads are measured and communicated to the PMS through a low-bandwidth communication system. Based on the total real/reactive power demand of the microgrid and a prescribed load sharing strategy, the PMS determines $P_{DG,i}$ and $Q_{DG,i}$ set points for DER_i, and determines the voltage set points (magnitude and angle) for the PCCs. The set points are then transmitted to DER LCs. Each LC measures the voltage of its corresponding PCC and provides voltage tracking based on the received reference set point. Owing to its robust design, each LC also rejects disturbances due to microgrid parameter and configuration variations.

The LCs are formulated in the $dq$-frame of reference, and there is a need for (i) a reference
power management and control strategy 12

2.3 Power Management and Control Strategy

The reference phase-angle for each LC is generated by the internal oscillator of each DER unit and the three phase-angle signals are synchronized based on a common time-reference signal provided by the GPS, Fig. 2.1(b). These entities are further described in Sections 2.3.1, 2.3.2, and 2.3.3.

2.3.1 Power Management System

The main function of the PMS is to provide load sharing among DER units based on either a cost function associated with each DER unit or a market signal. After determining the required real/reactive output power from each DER unit, and measuring the load connected to each PCC, power flow analysis yields voltage angle $\delta$ and magnitude $|V|$ of each PCC. Power flow equations are

$$P_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}), \quad (2.1)$$

$$Q_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\sin\delta_{ik} + B_{ik}\cos\delta_{ik}), \quad (2.2)$$

where $P_i/Q_i$ is the net real/reactive power injected at PCC$_i$, $G_{ik}/B_{ik}$ is the real/imaginary part of the element $Y_{ik}$ of the bus admittance matrix $Y_{Bus}$, and $\delta_{ik}$ is the difference in voltage angle between PCC$_i$ and PCC$_k$. Equations (2.1) and (2.2) indicate that power flow of the system is determined based on the voltage magnitude and angle of PCC$_1$, PCC$_2$, and PCC$_3$. Load and DER buses measure their power demand and generation and subsequent to a major change, they notify the PMS to update the set points to maintain an optimal operating point for the microgrid.

2.3.2 Frequency Control and Synchronization

The microgrid frequency is controlled in an open-loop manner. The LC of DER$_i$ includes an oscillator which generates a 60 Hz sawtooth waveform $\theta_i(t) = \int_0^t \omega_0 d\tau + \theta_0$, where $\omega_0 = 2\pi f_0$, and $f_0$ is the nominal power frequency of the microgrid. Fig. 2.2 illustrates the angle waveform deduced from the oscillator of LC$_i$ which is used for the $abc(dq)$ to $dq(abc)$ transformation of the DER$_i$ mathematical model.

Based on the proposed control strategy, all DER units are synchronized by a global synchronization signal that is communicated to the oscillators of DER units through the GPS [57]. The global synchronization signal is communicated at relatively large time intervals, e.g., one pulse per second, and is used (i) to prevent drift among local oscillators, and (ii) to initialize incoming DER units. Crystal oscillators with high accuracies, e.g., an error of $2 \times 10^{-6}$ to
2.3 Power Management and Control Strategy

\[ \theta_i(\text{rad}) \]
\[ \theta_0 \]
\[ \frac{2\pi}{t(s)} \]

One 60 Hz Cycle

Figure 2.2: The phase-angle waveform generated by the internal oscillator of DER\(_i\).

2 \times 10^{-11} \text{ seconds per year} \text{ are currently available at relatively low costs} [58]. All LCs can be synchronized with a high degree of reliability of a common time-reference signal of the GPS radio clock, e.g., with a theoretical accuracy of 1 µs [59]. Although there are 6–10 satellites visible to each area at all times, one can rely on the accuracy of crystal oscillators in case of unavailability of the synchronizing signal.

### 2.3.3 Local Controllers

The LC of DER\(_i\) tracks the set points specified by the PMS and rejects disturbances. LC\(_i\) measures the voltage of the corresponding PCC, and transforms the three-phase voltage to the dq frame based on the phase-angle signal \(\theta_i\) generated by its internal oscillator and synchronized with the global time-reference signal received from the GPS. The voltage magnitude and angle set points received from the PMS, \(|V_{\text{ref},i}| \angle \delta_i\), are also transformed to the dq frame to generate dq-based reference values, i.e., \(V_{d,\text{ref},i} = |V_{\text{ref},i}| \cos \delta_i\) and \(V_{q,\text{ref},i} = |V_{\text{ref},i}| \sin \delta_i\). The measured and reference values are provided to LC\(_i\) to determine the dq voltage components of the terminal of the corresponding VSC unit, i.e., \(V_{d,i}\) and \(V_{q,i}\) and generate the terminal voltage \(v_{\text{t},abc,i,HV}\) at the high-voltage side of the transformer. \(v_{\text{t},abc,i,HV}\) is divided by the turn ratio of the transformer and shifted if necessary to obtain \(v_{\text{t},abc,i,LV}\) corresponding to the low-voltage side of the transformer. \(v_{\text{t},abc,i,LV}\) is then fed to the PWM signal generator of the interface VSC of DER\(_i\). It should be noted that although the realistic and theoretical turn ratio and phase shift of the transformer are slightly different, the controller can compensate for the mismatch. Fig. 4.2 illustrates a block diagram of LC\(_i\) in which the measured and reference voltages are transformed to the dq frame of reference. After performing the control action, the outputs are transformed to the abc frame to generate the PWM switching signals of the \(i\)th interface VSC.

Section 2.4 develops a mathematical model of the microgrid, based on which the decentralized LCs will be devised in Section 2.5. It should be noted that although the controllers are designed based on RLC load models, other types of loads, e.g., motor loads, can also be handled due to the robustness of the controllers.
2.4 Mathematical Model of the Microgrid

The proposed decentralized control is developed based on a linearized model of the microgrid of Fig. 2.1(a) in a synchronously rotating \(dq\)-frame. Fig. 2.4 shows a one-line diagram of the microgrid model. Each DER unit is represented by a three-phase controlled voltage source and a series, three-phase RL branch. Dynamics of the source-side of the interface VSC of each DER unit have secondary effect on the performance of the controller and have not been accounted for this modeling. Each load is modeled by an equivalent three-phase parallel RLC network. Each distribution line is represented by lumped, series, three-phase RL elements. The controller is designed based on the fundamental frequency component of the system of Fig. 2.4.

The microgrid of Fig. 2.4 is virtually partitioned into three subsystems. The mathematical
model of Subsystem$_1$, in the $abc$ frame, is

$$
\begin{align*}
   i_{1,abc} &= i_{1,abc} + C_1 \frac{di_{1,abc}}{dt} + i_{L1,abc} + \frac{v_{1,abc}}{R_1}, \\
   v_{t1,abc} &= Lf_{1} \frac{di_{1,abc}}{dt} + Rf_{1}i_{1,abc} + v_{1,abc}, \\
   v_{1,abc} &= L_1 \frac{di_{1,abc}}{dt} + R_1i_{1,abc}, \\
   v_{1,abc} &= L_t \frac{di_{1,abc}}{dt} + R_t i_{1,abc} + v_{2,abc},
\end{align*}
$$

(2.3)

where $x_{abc}$ is a $3 \times 1$ vector. The mathematical model associated with other subsystems are provided in Appendix A. Assuming a three-wire system, (2.3) is transformed to the synchronously rotating $dq$-frame of reference, as described in section 2.3.2 by [60]

$$
f_{dq} = \frac{2}{3} \begin{pmatrix}
   \cos \theta & \cos(\theta - \frac{2}{3}\pi) & \cos(\theta - \frac{4}{3}\pi) \\
   -\sin \theta & -\sin(\theta - \frac{2}{3}\pi) & -\sin(\theta - \frac{4}{3}\pi) \\
   \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix} f_{abc},
$$

(2.4)

where $\theta(t)$ is the phase-angle with the frequency of the oscillator internal to DER$_1$. Based on (2.3) and (2.4), the mathematical model of Subsystem$_1$ in the $dq$-frame is

$$
\begin{align*}
   \frac{dV_{1,dl}}{dt} &= \frac{1}{L_1} V_{1,dl} - \frac{1}{L_1} I_{1,dl} - \frac{1}{L_1} I_{L1,dl} - \frac{1}{R_1 L_1} V_{1,dl} - j\omega V_{1,dl}, \\
   \frac{dI_{1,dl}}{dt} &= \frac{1}{L_1} V_{1,dl} - \frac{R_1}{L_1} I_{1,dl} - \frac{1}{L_1} V_{1,dl} - j\omega I_{1,dl}, \\
   \frac{dI_{L1,dl}}{dt} &= \frac{1}{L_1} V_{1,dl} - \frac{R_1}{L_1} I_{L1,dl} - \frac{1}{L_1} V_{1,dl} - j\omega I_{L1,dl},
\end{align*}
$$

(2.5)

Similarly, the $dq$-frame based models of Subsystem$_2$ and Subsystem$_3$, are also developed (see Appendix A), and used to construct the state-space model of the overall system

$$
\begin{align*}
   \dot{x} &= Ax + Bu, \\
   y &= Cx,
\end{align*}
$$

(2.6)

where

$$
x^T = (V_{1,dl}, V_{1,q}, I_{1,dl}, I_{1,q}, I_{L1,dl}, I_{L1,q}, I_{11,dl}, I_{11,q}, V_{2,dl}, V_{2,q}, I_{2,dl}, I_{2,q}, I_{L2,dl}, I_{L2,q}, I_{12,dl}, I_{12,q}, V_{3,dl}, V_{3,q}, I_{3,dl}, I_{3,q}, I_{L3,dl}, I_{L3,q}),
$$

$$
u^T = (V_{1,dl}, V_{1,q}, V_{2,dl}, V_{2,q}, V_{3,dl}, V_{3,q}),
$$

$$
y^T = (V_{1,dl}, V_{1,q}, V_{2,dl}, V_{2,q}, V_{3,dl}, V_{3,q}),
$$

and $A \subset \mathbb{R}^{22 \times 22}$, $B \subset \mathbb{R}^{22 \times 6}$ and $C \subset \mathbb{R}^{6 \times 22}$ are the state matrices are given in Appendix A.

The system (2.6) can alternatively be written as

$$
\begin{align*}
   \dot{x} &= Ax + B_1 u_1 + B_2 u_2 + B_3 u_3, \\
   y_1 &= C_1 x, \\
   y_2 &= C_2 x, \\
   y_3 &= C_3 x,
\end{align*}
$$

(2.7)
where
\[ y_i = (V_{i,d}, V_{i,q}), i = 1, 2, 3, \]
\[ u_i = (V_{ti,d}, V_{ti,q}), i = 1, 2, 3, \]
and a decentralized controller
\[ U_i(s) = C_{1,i}(s)E_i(s) + C_{2,i}Y_i(s), i = 1, 2, 3, \] (2.8)
is to be found, where \( E_i(s) \) denotes the system error, \( U_i(s) \) denotes the input, and the controller transfer function, \( \{C_{1,i}(s), C_{2,i}(s)\} \), is restricted to being a proper transfer function, \( i = 1, 2, 3. \)

### 2.5 Decentralized Control Strategy

In this section, the proposed robust decentralized servomechanism controller is designed with an imposed robustness constraint. To ensure robust performance, the decentralized controller will be found so that the real stability radius of the final closed-loop system is approximately the same as that of the open-loop system, i.e., the closed-loop system should have a robustness index which is not less than 50% of the robustness index of the open-loop system. Throughout the development, the norm \( \| \cdot \| \) is assumed to be the spectral norm and a square real matrix is said to be asymptotically stable if the eigenvalues of the matrix are contained in the open left-half complex plane.

An extended LTI model of the microgrid based on (2.6) is given by
\[ \dot{x} = Ax + Bu + Ew, \]
\[ y = Cx + Fw, \]
\[ e = y - y_{ref}, \] (2.9)
where \( x \in \mathbb{R}^n \) is the state, \( u \in \mathbb{R}^m \) is the input, \( y \in \mathbb{R}^r \) is the output, \( \omega \in \mathbb{R}^\Omega \) belongs to the class of unmeasurable constant disturbances, \( y_{ref} \in \mathbb{R}^r \) is the desired constant set point for the system, and \( e \in \mathbb{R}^r \) is the measured error signal.

The system must contain \( \nu = 3 \) control agents, each corresponding to one of the three virtual subsystems of Fig. 2.4, and (2.9) is rewritten as
\[ \dot{x} = Ax + \sum_{i=1}^{\nu} B_iu_i + Ew, \]
\[ y_i = C_ix + F_iw, \]
\[ e_i = y_i - y_{i,ref}, \] (2.10)
where \( u_i \) and \( y_i \) are the inputs and outputs of LC_i, and \( e_i \) is the measured error signal, \( i = 1, 2, \ldots, \nu. \) The open-loop eigenvalues and transmission zeros [61] of (2.9), corresponding to
Table 2.2
OPEN-LOOP PLANT EIGENVALUES AND TRANSMISSION ZEROS

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Transmission Zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>-570.79 ± 4074.8i</td>
<td>-1116.71 ± 377.0i</td>
</tr>
<tr>
<td>-570.79 ± 3320.8i</td>
<td>-1127.93 ± 377.0i</td>
</tr>
<tr>
<td>-547.12 ± 2519.7i</td>
<td>-18.85 ± 377.0i</td>
</tr>
<tr>
<td>-547.12 ± 1765.7i</td>
<td>-18.85 ± 377.0i</td>
</tr>
<tr>
<td>-29.33 ± 946.1i</td>
<td>-18.85 ± 377.0i</td>
</tr>
<tr>
<td>-29.33 ± 192.1i</td>
<td></td>
</tr>
<tr>
<td>-84.85 ± 377.0i</td>
<td>-18.85 ± 377.0i</td>
</tr>
<tr>
<td>-39.85 ± 377.0i</td>
<td></td>
</tr>
<tr>
<td>-14.07 ± 377.0i</td>
<td></td>
</tr>
<tr>
<td>-12.66 ± 377.0i</td>
<td></td>
</tr>
<tr>
<td>-11.98 ± 377.0i</td>
<td></td>
</tr>
</tbody>
</table>

the system of Fig. 2.4, are given in Table 2.2, which indicates that the open-loop system is stable and minimum phase.

2.5.1 Controller Design Requirements

A decentralized controller for the plant (2.10) should provide the following features:

1. The closed-loop system is asymptotically stable.

2. Steady-state asymptotic tracking and disturbance regulation occurs for (i) all constant set points $y_{ref}^1, y_{ref}^2, y_{ref}^3$, and (ii) all constant disturbances $w$, i.e., $\lim_{t \to \infty} e_i(t) = 0$, $i = 1, 2, 3$, for all constant disturbances and set points.

3. The controller is robust, i.e., Condition 2 should hold for practical perturbations of the plant model (2.10), including dynamic perturbations which do not destabilize the perturbed closed-loop system.

4. The controller should be “fast” with smooth non-peaking transients, e.g., it should respond to constant set points and constant disturbance changes, e.g., within about three cycles of 60 Hz.

5. Low interaction should occur among the output channels of the $\nu$ control agents, and among the outputs contained in each of the $\nu$ control agents, for both tracking and regulation [62].
6. It is required that the above conditions be satisfied for as wide as possible range of the parameters $R, L, C$ of each load.

These conditions will be achieved by a decentralized controller based on the solution of the decentralized robust servomechanism problem (DRSP) [44, 48].

### 2.5.2 Existence Conditions

The following existence conditions for a solution to the DRSP, such that the above conditions 1–3 hold, are obtained from [44]. Given plant (2.10), let

$$
C_m := [C_1^T, C_2^T, \cdots, C_v^T]^T, 
$$

where $C_1^* := \begin{bmatrix} C_1 & 0 & 0 & \cdots & 0 \\ 0 & I_r & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & I_r \end{bmatrix}$, $C_2^* := \begin{bmatrix} C_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & I_r & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $\cdots$, $C_v^* := \begin{bmatrix} C_v & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & I_r \end{bmatrix}$, and let $B := [B_1, B_2, \cdots, B_v]$, and $C := [C_1^T, C_2^T, \cdots, C_v^T]^T$.

**Theorem 1** [44]: Given the system (2.10), there exists a solution to DRSP such that conditions 1–3 all hold, if and only if the following conditions are all satisfied:

1. The system (2.10) has no unstable DFM.

2. $m_i \geq r_i$, where $m_i$ is the number of outputs of the $i$th control agent and $r_i$ is the number of outputs of Subsystem$_i$, $i = 1, 2, \cdots, v$.

3. The system $\{C_m, [A \ 0] \ C, B\}$ has no DFM = 0.

**Remark 1**: If $m_i = r_i, i = 1, 2, \cdots, v$, Condition 3 becomes

$$
\text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + r_1 + r_2 + \cdots + r_v.
$$

For the microgrid of Fig. 2.4, it can be verified that the existence conditions of Theorem 1 are all satisfied. In particular, 1) plant (2.10) has no decentralized fixed modes, 2) it has $m_i = r_i = 2, i = 1, 2, 3$, and 3) the rank of the matrix $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$, given by (2.6), is equal to $n + r_1 + r_2 + r_3 = 22 + 2 + 2 + 2 = 28$.

**Remark 2**: In the plant model (2.9), the three load parameters $R, L, C$ of each subsystem, Fig. 2.4, can vary and result in structural uncertainty in the plant’s nominal model. It is also observed in (2.9) that the load parameters affect neither the output gain matrix $C$, nor the input control matrix $B$. Therefore, only the $A$ matrix of (2.9) is affected by changing load parameters. We use this observation to design a controller with the desirable robustness properties.

**Remark 3**: The following analysis shows that Condition 3 of Theorem 1 always holds true for the studied microgrid system regardless of its numerical values. The determinant of the
2.5 Decentralized Control Strategy

matrix \( \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \) is given by \( \rho_1 \) where
\[
\rho_1 = (R_{11}^2 + \omega^2 L_1^2) (R_{12}^2 + \omega^2 L_2^2) (R_{13}^2 + \omega^2 L_3^2) (R_{11}^2 + \omega^2 L_{11}^2) (R_{12}^2 + \omega^2 L_{12}^2),
\]
\[
\rho_2 = (C_1 L_1 L_{f1} L_{11} C_2 L_2 L_{f2} L_{12} C_3 L_3 L_{f3})^2.
\]
Clearly, the determinant is always non-zero which implies \( \text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = 28 \) for any numerical value of microgrid parameters and thus the third condition of Theorem 1 always holds.

2.5.3 Real Stability Radius Constraint

To evaluate the robustness of a control scheme, the following definition is used [47].

Given a real \( n \times n \) matrix \( A \) which is asymptotically stable, assume that \( A \) is subject to a real perturbation \( A \rightarrow A + M \Delta N \), where \( M \in \mathbb{R}^{n \times m} \) and \( N \in \mathbb{R}^{p \times n} \) are known scaling matrices defining the structure and scale of the perturbation [63], and \( \Delta \) is a real matrix of uncertain parameters. Then it is desired to find \( r_{stab} > 0 \), such that (i) \( A + M \Delta N \) is asymptotically stable for all real perturbations \( \Delta \) with the property that \( \| \Delta \| < r_{stab} \) and (ii) there exists a perturbation \( \Delta^* \) with the property that \( \| \Delta^* \| = r_{stab} \), such that \( A + M \Delta^* N \) is unstable. In this case, \( r_{stab} \) is called the real stability radius of \( \{A,M,N\} \).

Real stability radius is the solution of the following linear algebra optimization problem [47]:
\[
r_R(A,M,N) = \left\{ \sup_{s \in \partial \mathbb{C}_g} \mu_R \left( N(sI - A)^{-1} M \right) \right\}^{-1},
\]
where \( \mathbb{C}_g \) is the open left-half complex plain, \( \partial \mathbb{C}_g \) denotes the boundary of \( \mathbb{C}_g \), which is the imaginary axis for continuous-time systems, and
\[
\mu_R(Q) = \inf_{\gamma \in (0,1]} \sigma_2 \left( \begin{pmatrix} \Re Q & -\gamma \Im Q \\ \gamma^{-1} \Im Q & \Re Q \end{pmatrix} \right),
\]
where \( \Re Q \) and \( \Im Q \) denote, respectively, the real and imaginary parts of a complex matrix \( Q \), and \( \sigma_2(P) \) denotes the second largest singular value of the real matrix \( P \).

2.5.4 Controller Design Procedure

Given the plant (2.10) with \( \nu = 3 \), to solve the DRSP it is necessary [44] that the decentralized controller include the decentralized servo-compensator
\[
\eta_i = 0 \eta_i + (y_i - y_{i \text{ref}}), i = 1, 2, 3,
\]
where \( \eta_i \in \mathbb{R}^2, i = 1, 2, 3 \), together with a decentralized stabilizing compensator which will be assumed to have the structure
\[
\dot{\beta} = A \beta + By,
\]
\[
u = K_1 y + K_2 \eta + K_3 \beta,
\]
where

\[
A = \begin{bmatrix}
A_1 & 0 & 0 \\
0 & A_2 & 0 \\
0 & 0 & A_3
\end{bmatrix},
B = \begin{bmatrix}
B_1 & 0 & 0 \\
0 & B_2 & 0 \\
0 & 0 & B_3
\end{bmatrix},
\mathcal{K}_1 = \begin{bmatrix}
\mathcal{K}_1^1 & 0 & 0 \\
0 & \mathcal{K}_2^1 & 0 \\
0 & 0 & \mathcal{K}_3^1
\end{bmatrix},
\mathcal{K}_2 = \begin{bmatrix}
\mathcal{K}_1^2 & 0 & 0 \\
0 & \mathcal{K}_2^2 & 0 \\
0 & 0 & \mathcal{K}_3^2
\end{bmatrix},
\mathcal{K}_3 = \begin{bmatrix}
\mathcal{K}_3^1 & 0 & 0 \\
0 & \mathcal{K}_3^2 & 0 \\
0 & 0 & \mathcal{K}_3^3
\end{bmatrix},
\]

so that the controlled closed-loop system is described by

\[
\begin{bmatrix}
\dot{x} \\
\dot{\eta} \\
\dot{\beta}
\end{bmatrix} =
\begin{bmatrix}
A + BK_1C & BK_2 & BK_3 \\
C & 0 & 0 \\
BC & 0 & A
\end{bmatrix}
\begin{bmatrix}
x \\
\eta \\
\beta
\end{bmatrix} +
\begin{bmatrix}
0 \\
-I \\
0
\end{bmatrix} y_{ref} +
\begin{bmatrix}
BK_1F + E \\
F \\
BF
\end{bmatrix} w,
\]

(2.16)

\[
y = \begin{bmatrix}
C & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\eta \\
\beta
\end{bmatrix} + Fw.
\]

In this case, the controller parameters (2.15) are obtained by applying optimal controller design method [64], [65] to minimize the expected value of the performance index \( \int_0^\infty (e' e + \epsilon \dot{u}' \dot{u}) d\tau \) given by

\[
J = \text{trace}(\Gamma),
\]

(2.17)

where \( \Gamma > 0 \) is obtained by solving the Lyapunov matrix equation corresponding to

\[
\int_0^\infty (e' e + \epsilon \dot{u}' \dot{u}) d\tau = \begin{bmatrix}
\dot{x}(0) \\
\dot{e}(0) \\
\dot{\beta}(0)
\end{bmatrix}' \Gamma \begin{bmatrix}
\dot{x}(0) \\
\dot{e}(0) \\
\dot{\beta}(0)
\end{bmatrix},
\]

(2.18)

where \( \epsilon = 10^{-6} \), subject to the conditions that

1. the resultant closed-loop system (2.16) is asymptotically stable,

2. the real stability radius of the closed-loop system, \( r_R(A_{close}, \begin{bmatrix} A^* & 0 \\ 0 & 0 \end{bmatrix}, [I, 0, 0]) \) should be greater than or equal to half of the real stability radius of the open-loop system, \( r_R(A, A, I) \), where

\[
A_{close} = \begin{bmatrix}
A + BK_1C & BK_2 & BK_3 \\
C & 0 & 0 \\
BC & 0 & A
\end{bmatrix}.
\]
Substituting for \( e \) and \( \dot{u} \) from (2.16) in (2.18) provides a closed-form expression for (2.18)

\[
A_{\text{close}}^{T} \Gamma + \Gamma A_{\text{close}} = - \left( \begin{bmatrix} 0 & I \\ 0 & I \end{bmatrix} \right)^{T} \left( \begin{bmatrix} 0 & I \\ 0 & I \end{bmatrix} + \epsilon \mathcal{K}^{*T} \mathcal{K}^{*} \right),
\]

(2.19)

where \( \mathcal{K}^{*} = [\mathcal{K}_{1} C, \mathcal{K}_{2}, \mathcal{K}_{3}] \). The control design problem becomes, therefore, a constrained parameter optimization problem [66] whose solution yields the control parameters \( A, B, K_{1}, K_{2}, \) and \( K_{3} \).

The reason to impose the robust constraint above is that the load parameters \( R_{i}, L_{i}, C_{i}, i = 1, 2, 3 \) may vary, but since they affect only the \( A \) matrix and not the \( B_{1}, B_{2}, B_{3}, C_{1}, C_{2}, \) and \( C_{3} \) matrices of (2.10), then it is desired to minimize the effect of load parameter perturbations by keeping the closed-loop system robustness index, as measured by the real stability radius, as “close” as possible to the open-loop robustness index. The constraint imposed is such that the closed-loop robustness index is not less than half of the open-loop robustness index.

### 2.5.5 Obtained Decentralized Controller

On carrying out the optimization of the controller (2.15) as measured by the performance index (2.17), the following decentralized controller is obtained for the microgrid of Fig. 2.4:

\[
\begin{align*}
\dot{\eta} &= y - y_{\text{ref}}, \\
\dot{\beta} &= A \beta + B y, \\
u &= K_{1} y + K_{2} \eta + K_{3} \beta,
\end{align*}
\]

(2.20)

where

\[
\begin{align*}
u &= \begin{bmatrix} u_{1} \\ u_{2} \\ u_{3} \end{bmatrix}, \\
y &= \begin{bmatrix} y_{1} \\ y_{2} \\ y_{3} \end{bmatrix},
\end{align*}
\]

\[
\mathcal{K}_{1} = \begin{bmatrix}
-15.83 & 3.37 & 0 & 0 & 0 & 0 \\
-7.97 & -110.14 & 0 & 0 & 0 & 0 \\
0 & 0 & -5.11 & 104.77 & 0 & 0 \\
0 & 0 & -10.38 & -132.44 & 0 & 0 \\
0 & 0 & 0 & 0 & -5.51 & 6.81 \\
0 & 0 & 0 & 0 & -15.39 & -118.91 \\
\end{bmatrix}, \\
\mathcal{K}_{2} = \begin{bmatrix}
-1047 & 649 & 0 & 0 & 0 & 0 \\
-834 & -1103 & 0 & 0 & 0 & 0 \\
0 & 0 & -496 & 970 & 0 & 0 \\
0 & 0 & -1072 & -839 & 0 & 0 \\
0 & 0 & 0 & 0 & -636 & 793 \\
0 & 0 & 0 & 0 & -1002 & -1039 \\
\end{bmatrix}, \\
\mathcal{K}_{3} = \begin{bmatrix}
0.18 & 0 & 0 \\
881.47 & 0 & 0 \\
0 & 0 & -3668.6 \\
0 & 4495.3 & 0 \\
0 & 0 & -1639.3 \\
0 & 0 & 57906 \\
\end{bmatrix}, \\
B = \begin{bmatrix}
1 & 579.02 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 205.21 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 12.754 \\
\end{bmatrix}.
\]
A transfer function representation of the control agents of the decentralized controller (4.3) is given by

\[
\begin{bmatrix}
U_1(s) \\
U_2(s)
\end{bmatrix} = \frac{1}{s} \begin{bmatrix}
-1047 & 649.5 \\
-834.4 & -1103.3
\end{bmatrix} \begin{bmatrix}
E_1(s) \\
E_2(s)
\end{bmatrix} + \frac{1}{s + 5313} \begin{bmatrix}
-15.83s - 8.4 \times 10^4 & 3.37s + 1.8 \times 10^4 \\
-7.97s - 4.1 \times 10^4 & -110.1s - 7.5 \times 10^4
\end{bmatrix} \begin{bmatrix}
Y_1(s) \\
Y_2(s)
\end{bmatrix},
\]

\[
\begin{bmatrix}
U_3(s) \\
U_4(s)
\end{bmatrix} = \frac{1}{s} \begin{bmatrix}
-495.6 & 969.7 \\
-1072 & -838.6
\end{bmatrix} \begin{bmatrix}
E_3(s) \\
E_4(s)
\end{bmatrix} + \frac{1}{s + 7702} \begin{bmatrix}
-5.11s - 4.3 \times 10^4 & 104.8s + 5.4 \times 10^4 \\
-10.38s - 7.5 \times 10^4 & -132.4s - 9.8 \times 10^4
\end{bmatrix} \begin{bmatrix}
Y_3(s) \\
Y_4(s)
\end{bmatrix},
\]

\[
\begin{bmatrix}
U_5(s) \\
U_6(s)
\end{bmatrix} = \frac{1}{s} \begin{bmatrix}
-635.9 & 793 \\
-1002 & -1039
\end{bmatrix} \begin{bmatrix}
E_5(s) \\
E_6(s)
\end{bmatrix} + \frac{1}{s + 6913} \begin{bmatrix}
-5.51s - 4.0 \times 10^4 & 6.8s + 2.6 \times 10^4 \\
-15.39s - 4.9 \times 10^4 & -118.9s - 8.3 \times 10^4
\end{bmatrix} \begin{bmatrix}
Y_5(s) \\
Y_6(s)
\end{bmatrix},
\]

where \( U_i(s), E_i(s), Y_i(s), i = 1, 2, \ldots, 6, \) are the outputs and errors of controllers, and plant’s outputs, respectively. Poles of the decentralized controller are 0 (repeated 6 times), -7702, -6913, and -5313.

### 2.6 Analysis of Microgrid Dynamics

To investigate the dynamic behavior of the studied microgrid, eigen analysis including eigenvalue calculation, participation factor analysis, and a set of eigenvalue sensitivity analyses are carried out and results are presented in this section.

#### 2.6.1 Eigenvalues and Participation Factors

The eigenvalues of the closed-loop system, or alternatively, the eigenvalues of the matrix \( A_{close} \in \mathbb{R}^{31 \times 31} \) are listed in Table 4.2. Table 4.2 shows that the closed-loop system (2.16) has 14 pairs of complex conjugate and 3 real eigenvalues. Dominant eigenvalues, i.e., eigenvalues 1–8, are highly damped, which indicates the proper design of the controllers. To identify the dominant eigenvalues, i.e., the closest eigenvalues to the imaginary axis, the eigenvalues are sorted by their real-part absolute value in an ascending order. To identify the state variables that dominantly influence an eigenvalue, the normalized participation factors associated with each eigenvalue are calculated using [67]

\[
p_{ki} = \frac{v_{ik}w_{ki}}{\sum_{k=1}^{n} v_{ik}w_{ki}},
\]

where \( v_i \) and \( w_i \) are the right and left eigenvectors associated with the \( i \)th eigenvalue, and \( p_{ki} \) is the participation factor relating the \( i \)th eigenvalue to the \( k \)th state variable. State variables include 22 open-loop state variables given in (2.6) and 9 state variables that correspond to the three LCs. LC\(_i\) has three states, two of which, \( \eta_i \in \mathbb{R}^2 \) in Table 4.2, participate in regulating the
Table 2.3
EIGENVALUES OF THE CLOSED-LOOP SYSTEM (2.16) AND THE ASSOCIATED PARTICIPATING STATES

<table>
<thead>
<tr>
<th>Number</th>
<th>Eigenvalue</th>
<th>Participating States</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>-17.594 ± 377.75i</td>
<td>(\eta_3, \eta_3)</td>
</tr>
<tr>
<td>3,4</td>
<td>-18.133 ± 377.52i</td>
<td>(\eta_1, \eta_2)</td>
</tr>
<tr>
<td>5,6</td>
<td>-18.391 ± 377.14i</td>
<td>(\eta_1, I_{Ldq1})</td>
</tr>
<tr>
<td>7,8</td>
<td>-31.809 ± 5.3436i</td>
<td>(I_{dq3}, I_{Ldq3})</td>
</tr>
<tr>
<td>9,10</td>
<td>-60.465 ± 4.0087i</td>
<td>(I_{q2}, I_{Lq2})</td>
</tr>
<tr>
<td>11,12</td>
<td>-91.223 ± 29.715i</td>
<td>(I_{dq1}, I_{Ldq1})</td>
</tr>
<tr>
<td>13,14</td>
<td>-134.73 ± 425.55i</td>
<td>(I_{dq2})</td>
</tr>
<tr>
<td>15,16</td>
<td>-284.21 ± 431.66i</td>
<td>(I_{q1}, I_{d3})</td>
</tr>
<tr>
<td>17,18</td>
<td>-311.67 ± 1648.9i</td>
<td>(V_{d3}, I_{d2})</td>
</tr>
<tr>
<td>19,20</td>
<td>-519.53 ± 2529.7i</td>
<td>(V_{d3}, I_{d2})</td>
</tr>
<tr>
<td>21</td>
<td>-829.98</td>
<td>(\beta_{1,2,3})</td>
</tr>
<tr>
<td>22,23</td>
<td>-857.42 ± 4033.4i</td>
<td>(V_{d2}, I_{d1})</td>
</tr>
<tr>
<td>24,25</td>
<td>-1373.4 ± 5144.2i</td>
<td>(V_{q1,2}, I_{lq1,2})</td>
</tr>
<tr>
<td>26,27</td>
<td>-1440.0 ± 3666i</td>
<td>(V_{q2,3}, I_{lq2})</td>
</tr>
<tr>
<td>28,29</td>
<td>-1847.0 ± 2055.9i</td>
<td>(V_{d3}, \beta_{2,3})</td>
</tr>
<tr>
<td>30</td>
<td>-4539.8</td>
<td>(V_{q3}, \beta_{2,3})</td>
</tr>
<tr>
<td>31</td>
<td>-5463.8</td>
<td>(V_{q2, \beta_2})</td>
</tr>
</tbody>
</table>

\(d\) and \(q\) components of \(PCC_i\) and ensure tracking the reference set points with zero steady-state error. The third state of the controller corresponds to a stabilizing compensator, \(\beta_i \in \mathbb{R}\) in Table 4.2. Results of participation factor analysis are given in Table 4.2, which shows that

- most dominant eigenvalues of the closed-loop system (2.16), i.e., eigenvalues 1–6, are predominantly affected by the controller regulating states,

- state variables corresponding to PCC voltages are associated with the eigenvalues which are distant from the imaginary axis and thus, have fast dynamics,

- the stability compensator states have very fast dynamics which is desirable,

- state variables corresponding to VSC output currents and load currents are associated with relatively slow modes which is expectable due to the large time-constant of the corresponding inductors, and

- state variables corresponding to distribution line currents are associated with faster modes due to the low \(X/R\) ratio of the distribution line, which results in a small time-constant.
2.6 Analysis of Microgrid Dynamics

(a) (b)

![Graph](image)

**Figure 2.5:** Trajectories of eigenpairs (1,2), (3,4), and (5,6) as load inductance and capacitance increase, (a) $L_i \rightarrow \alpha L_i$, (b) $C_i \rightarrow \alpha C_i$, $0.1 < \alpha < 10$.

The above observations suggest that the optimal control design procedure of [68, 69] is a promising solution for islanded microgrid voltage control problem.

### 2.6.2 Eigenvalue Sensitivity Analysis

To study the sensitivity of the eigenvalues to microgrid parameter and controller gain perturbations, a set of eigenvalue sensitivity analyses are carried out by plotting the trajectories of sensitive eigenvalues as a parameter changes.

Fig. 2.5 illustrates the trajectories of closed-loop system eigenvalues as $L_i$ and $C_i$, load inductance and capacitance, change to $\alpha L_i$ and $\alpha C_i$, where $0.1 < \alpha < 10$. This was also performed for the load resistive branch, $R_i$, whose results are not presented here. Results of this study show that the eigenvalues move insignificantly as $C_i$ and $R_i$ change, which demonstrates the robustness of the controllers as load parameters change. As load inductance increases, eigenvalues 1–6 move towards the imaginary axis and become less damped. However, the closed-loop system remains stable for any arbitrarily large values of $\alpha$, which indicates robustness of the system to load inductance perturbations.

Fig. 2.6(a) shows the trajectories of eigenvalues 7–10 as the VSC filter impedance changes from 0.05 pu to 0.3 pu. VSC filter impedance is generally selected from this range in practice to achieve a desirable level of harmonics, and reduce the overall cost of the system. Fig. 2.6(a) shows that the sensitive eigenvalues are initially real and as the impedance increases, they form a complex conjugate pair and move towards the imaginary axis, which makes the system more oscillatory. The reason for this is that a larger filter impedance decreases the coupling between
2.6 Analysis of Microgrid Dynamics

Figure 2.6: (a) Trajectories of eigenpairs (7,8) and (9,10) as VSC filter impedance changes from 0.05 pu to 0.3 pu, (b) trajectories of eigenpairs (7,8), (9,10), and (11,12) as the controller gain $K_2$ is scaled with a factor of $0.1 < \alpha < 8$.

the VSC terminal and PCC voltage, which in turn, makes the controller less influential.

A similar eigenvalue sensitivity analysis is conducted when distribution line length changes and results show that as the length of distribution line increases, sensitive eigenvalues become more damped and the dominant ones do not move noticeably. This indicates that controller performance improves as distribution line length increases. This is due to two main reasons. First, since the $X/R$ ratio of the line is less than 1, a longer distribution line corresponds to more resistance and thus, more damping in the closed-loop system. Second, longer distribution line decreases the interaction between PCC voltages, and thereby, oscillations of a PCC voltage will have a less significant effect on others.

Fig. 2.6(b) illustrates eigenvalue trajectories as the controller parameter $K_2$ is scaled with a factor of $0.1 < \alpha < 8$. It can be seen that as $K_2$ increases, the eigenpair (17,18) moves towards the imaginary axis and eventually departs to the right-half complex plain when $\alpha = 7.26$. Eigenvalues 7–12 move away from the imaginary axis and dominant eigenvalues 1–6 move insignificantly. A similar study was performed for controller gains $K_1$ and $K_3$ and it was observed that the closed-loop system is stable for $\alpha K_1$ and $\alpha K_3$, where $0.09 < \alpha < 1.13$. 

2.7 Robustness Properties of the Microgrid

2.7.1 Robustness of the Nominal Closed-loop System

The closed-loop system of plant (2.10) and controller (4.3) is highly robust with respect to changes in the load parameters $R_i, L_i, C_i, i = 1, 2, 3$, of the microgrid. In particular assume that in the open-loop system (2.9), the matrix $A$ is subject to uncertainty, e.g., due to changes in the load parameters and perturbations in the elements of the matrix $A$. Also assume that $A \rightarrow A(I + \Delta)$ where $\Delta$ is an unknown real perturbation matrix, and where $A$ is an asymptotically stable matrix. The largest perturbations $\Delta$ for which the system remains stable is determined from the real stability radius of $A$.

The open-loop perturbed plant $\dot{x} = A(I + \Delta)x$ remains asymptotically stable for all real $\Delta$ matrix perturbations with $\|\Delta\| < rstab$ if and only if $rstab < rstab^*$, where $rstab^* = 1.495 \times 10^{-3}$.

Consider now the closed-loop system obtained by applying the controller (4.3) to (2.10), and assume that the elements of the controller are fixed and that the $B_i$ and $C_i, i = 1, 2, 3$, matrices of the plant are fixed, but that due to load changes in $R_i, L_i, C_i$ (and other perturbations) the elements of $A$ are allowed to perturb. We also assume in the closed-loop plant (2.16) that $A \rightarrow A(I + \Delta)$, where $\Delta$ is an unknown perturbation matrix. The perturbed closed-loop system remains asymptotically stable for all real $\Delta$ matrix perturbations with $\|\Delta\| < rstab$, if and only if $rstab < rstab^*$, where $rstab^* = 7.752 \times 10^{-2}$.

Thus the stability robustness index of the closed-loop system $rstab^* = 7.752 \times 10^{-2}$ with respect to perturbations of the $A$ matrix is larger than the open-loop system given by $rstab^* = 1.495 \times 10^{-3}$, and so there is no deterioration in robustness of the closed-loop system as compared to the open-loop system with respect to perturbations of the elements of $A$. A bode plot of the closed-loop system is illustrated in Fig. 2.7, which shows that the closed-loop transfer function from $V_{d,ref1}$ to $V_{d1}$ of LC1 has a gain margin of 29.5 dB at the frequency of 372 Hz (as denoted by a dotted line) and a phase margin of -180° which clearly highlights the robust stability of the closed-loop system.

2.7.2 Robustness Sensitivity Analysis

In this section, the effect of microgrid parameter and controller gain perturbations on the real stability radius, $r_R$, of the closed-loop system is investigated. To reflect microgrid load uncertainties, the matrix triple $\{A_{closer}, [A, A, 0]^T, [I, 0, 0]\}$ is considered as the argument of the function $r_R(.)$.

Fig. 2.8(a) illustrates variations of $r_R$ when each of the parameters $R_i, L_i$, and $C_i$ are scaled
2.7 Robustness Properties of the Microgrid

Figure 2.7: Bode plot of the closed-loop system, corresponding to LC\(_1\) (associated with input \(V_{d,\text{ref}1}\) and output \(V_{d1}\)).

by a coefficient of \(0.5 < \alpha < 2\). \(r_R\) increases as load resistance increases and it decreases as load inductance and capacitance increase. The reason for this is that more resistance corresponds to more damping and more inductance and capacitance correspond to more oscillations.

Fig. 2.8(b) illustrates variations of \(r_R\) as the VSC filter impedance and the length of distribution feeder change with a coefficient of \(0.2 < \alpha < 1.6\). It is observed that larger VSC impedances degrade the robustness of the closed-loop system and a longer distribution feeder will improve the robustness. Similar results were obtained through eigenvalue sensitivity analysis of Section 2.6.2.

Fig. 2.9 illustrates variations of \(r_R\) and closed-loop system maximum settling time as the controller gain \(\mathcal{K}_2\) changes with a coefficient of \(0.1 < \alpha < 1\). It can be seen that the increase of \(\mathcal{K}_2\) degrades closed-loop system robustness. However, it reduces the maximum settling time of the system, which corresponds to a faster control performance. Thus, there is a trade-off between the speed and robustness of the controller. To verify this observation, the bode plots of the transfer function \(V_{d1}(s)/V_{d,\text{ref}1}(s)\) are depicted in Fig. 4.5 for two coefficients of \(\alpha = 1\) and \(\alpha = 0.5\). The gain margin corresponding to a smaller controller gain is larger, which signifies more robustness, and verifies the conclusion of Fig. 2.9.

2.7.3 Input and Output Gain-Margins

The following definition is an extension of the classical SISO gain-margin to multivariable systems [70].
Figure 2.8: (a) Effect of load parameter perturbations on the robustness of the control scheme, (b) effect of VSC filter impedance and distribution line length on the closed-loop system robustness.

Definition: Given a plant \( \dot{x} = Ax + Bu, y = Cx \) controlled by a controller \( \dot{\eta} = A_c \eta + B_c y, u = C_c \eta \), assume that the closed-loop system is asymptotically stable. Let \( y = Cx \) be replaced by \( y = (I + K_y)Cx \), where \( K_y \) is a constant gain matrix; then if the closed-loop system remains stable \( \forall K_y \) with \( ||K_y|| < l_y \), the system has an output gain-margin of \( (1 + l_y) \). Also let \( Bu \) be replaced by \( B(I + K_u)u \); then if the closed-loop system remains stable \( \forall K_u \) with \( ||K_u|| < l_u \), the system is said to have an input gain margin of \( (1 + l_u) \). The gain margin can be calculated from the real stability radius of the system. The closed-loop system (2.10), (4.3) has output and input gain-margins of 1.38 and 1.28, respectively, which are quite satisfactory [71].

2.7.4 Input Time-Delay Tolerance

Often a system may have time delays which are ignored in its model, and it is important that the controlled system be robust to such unmodelled effects. The following definition is used to describe such a robust property [72].

Definition: Given a plant \( \dot{x} = Ax + Bu, y = Cx \) controlled by the controller \( \dot{\eta} = A_c \eta + B_c y, u = C_c \eta \), assume that the closed-loop system is asymptotically stable, and let \( u(t) \) be replaced by \( u(t - \beta) \), corresponding to a time delay of \( \beta \) s. Then if there exists \( \beta > 0 \) such that the closed-loop system remains stable \( \forall \beta \in [0, \beta] \), the closed-loop system has an input time-delay tolerance of \( \beta \). In this case the closed-loop system (2.10), (4.3) has an input time-delay tolerance of \( 1.58 \times 10^{-4} \) s. This is a lower bound time-delay tolerance and the actual system may be able to longer delays.
2.8 Test Cases

A set of test cases, based on both off-line and real-time simulations, are carried out to evaluate performance of the microgrid of Fig. 2.1(a), in an islanded condition, based on the proposed control strategy and power management system.

2.8.1 Off-Line Time-Domain Simulation Test Cases

A comprehensive set of test cases, based on digital time-domain simulation studies in the PSCAD/EMTDC environment, are carried out to evaluate the control performance subject to (i) a significant load change, (ii) control set point change, (iii) induction motor energization, (iv) accidental DER outage, and (v) changes in the microgrid topology. The component models for the test cases are as follows and parameter values are given in Table 2.1:

- Each DER unit is represented by (i) a constant DC voltage source, (ii) a two-level, IGBT-based VSC, and (iii) a three-phase series filter. The VSC uses a SPWM technique at the frequency of 6 kHz. The VSC filter is modeled as a series, three-phase $RL$ branch.

- The interface transformer for each DER unit is represented as a linear, three-phase, $\Delta/Y_g$ configuration.

- Each three-phase load is represented by a parallel RLC branch at each phase.
2.8.1.1 Case-1: Load Change

The objective of this case study is to demonstrate robust performance of the LCs subject to parametric uncertainties of the microgrid. In this case study, the uncertain parameter is the resistive branch of Load 2, given by $R_2$ in Fig. 2.4. Prior to $t = 1$ s, the $R_2$ branch is open and at $t = 1$ s, $R_2 = 1.61$ pu is connected to the system. This change in the value of $R_2$ corresponds to an increase in the real power demand of Load 2 from zero to 0.8 pu, which is equal to the full generation capacity of DER 2. Fig. 2.11 shows the impact of this load change on the system voltages, power flow, and the modulation index of DER 2.

Prior to the load change, the voltages of the three buses are regulated at the prespecified values of $1 \angle 0^\circ$. DER 2 is under no-load condition at a modulation index of 0.66. Fig. 2.11(a)–(e) illustrates the $d$ and $q$ components, RMS value, phase-angle, and instantaneous voltages of PCC 2 and indicates the robustness of the LCs in response to transients after the load change. The microgrid control effectively controls the voltage within 15 ms. Fig. 2.11(e) shows that the transients of the PCC 2 instantaneous voltages are practically insignificant.
Figure 2.11: Increase in real power of Load\textsubscript{2} at $t = 1$ s from 0 to 0.8 pu: (a) and (b) $d$ and $q$ components of PCC\textsubscript{2} voltage, (c) and (d) RMS value and phase-angle of PCC\textsubscript{2} voltage, (e) instantaneous voltages of PCC\textsubscript{2}, (f) output real power of the DER units, (g) output reactive power of the DER units, (h) real power transfer of the distribution lines, (i) modulation index of the VSC of DER\textsubscript{2}.

Fig. 2.11(f) and (g) depict output real and reactive power components of the three DER units. Output real power of DER\textsubscript{1} and DER\textsubscript{3} remain the same as the predisturbance values after riding through the load-change transients. However, the output real power of DER\textsubscript{2} increases from 0 to 0.8 pu in response to the increased power demand of Load\textsubscript{2}. The real power transferred on the distribution lines are depicted in Fig. 2.11(h). Since each DER unit supplies its dedicated load prior to the change of Load\textsubscript{2}, and since the change is within the supply range of DER\textsubscript{2}, the disturbance does not change the steady-state power flow.

Fig. 2.11(i) illustrates the modulation index, $m_2$, of the interface converter of DER\textsubscript{2} which changes from 0.66 to 0.69. The reason for modulation index rise is that the output current of DER\textsubscript{2} increases due to the load increase, and as a result, the voltage drop on the filter impedance increases. The controller increases the modulation index of DER\textsubscript{2} to compensate for the voltage drop on the filter impedance.

This case study demonstrates the robust performance of the LCs in maintaining the stability and operational integrity of the system despite the significant parametric perturbation of the nominal plant. It also indicates fast response of the controllers in regulating PCC\textsubscript{2} voltage. The instantaneous voltages of PCC\textsubscript{2} exhibit no observable transients despite the considerable load...
increase.

### 2.8.1.2 Case-2: Set Point Tracking and Power Flow Regulation

The objective of this case study is to demonstrate the capability of LCs in voltage set point tracking, and as a result, their power flow regulation capability. The adopted load sharing scheme requires that each DER unit contribute to the net microgrid load, proportional to its maximum generation capacity, and supply reactive power locally, i.e.,

$$P_{DER,i} = \frac{S_{Max,DER,i}}{\sum_j S_{Max,DER,j}} \sum_i P_{Load,i},$$

where $S_{Max,DER,i}$ denotes the maximum apparent power generation capacity of DER$_i$. This case study follows the previous scenario in which the output real power of DER$_2$ increases in response to the change in Load$_2$. To share the total power demand among the three DER units, based on the aforementioned strategy, the PMS assigns the following new set point for the three LCs, respectively: $1.01\angle0.14^\circ$, $1.00\angle0^\circ$, and $1.00\angle-0.06^\circ$. At $t = 1.5$ s, LCs receive the new set points for tracking. Fig. 2.12 shows transients of the microgrid due to this set point update. Fig. 2.12(a)–(d) shows the $d$ and $q$ components, RMS value, and phase-angle of PCC$_1$ voltage, and illustrates that LC$_2$ manages to reach the new set points in less than 50 ms with zero steady-state error. Fig. 2.12(e) shows the instantaneous voltages of PCC$_1$. The output real and reactive power of the three DER units are illustrated in Fig. 2.12(f) and (g). Prior to $t = 1.5$ s, DER$_2$ generates more real power than the other two DER units. After $t = 1.5$ s, however, the total real power demand is divided among the DER units proportional to their rated generation capacities and DER units keep their output reactive power component unchanged. Fig. 2.12(h) shows real power flows of Line$_1$ and Line$_2$, and indicates changes from zero to 0.25 pu and 0.06 pu, respectively, which are contributions of DER$_2$ and DER$_3$ to the nonlocal load, Load$_2$.

This case study shows the capability of the LCs in tracking the voltage set points specified by the PMS. It also concludes that power flow can be effectively controlled by specifying appropriate voltage set points for LCs.

To evaluate performance of the controllers when the PMS specifies set points based on inaccurate microgrid data, the power flow analysis is performed based on the inaccurate set of data given in Table 2.4. The results of this study show the microgrid stable operation is maintained despite the incorrect power flow results. However, the prescribed load sharing has an error of $\pm10\%$. 

2.8 Test Cases

Figure 2.12: Reference set point tracking and power flow control: (a) and (b) $d$ and $q$ components of PCC$_1$ voltage, (c) and (d) RMS value and phase-angle of PCC$_1$ voltage, (e) instantaneous voltages of PCC$_1$, (f) output real power of three DER units, (g) output reactive power of the DER units, (h) real power flow of the distribution lines.

Table 2.4

<table>
<thead>
<tr>
<th>Microgrid Inaccurate Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Net Load</td>
</tr>
<tr>
<td>Line Length</td>
</tr>
<tr>
<td>Line Length</td>
</tr>
</tbody>
</table>

2.8.1.3 Case-3: Induction Motor Energization

The objective of this case study is to illustrate the robust performance of the LCs when the microgrid is subject to a nonlinear load, i.e., an induction motor energization. The motor is rated at 500 hp, 2.3 kV and connected to PCC$_1$ through a three-phase, 0.5-MW, 13.8-kV/2.3-kV, $Yg/\Delta$ transformer with a leakage reactance of 8%. The induction motor data are given in Table 2.5 [60]. The induction motor is represented by a 6$^{th}$-order state-space model in the $dq$ frame.

Prior to the motor start-up, Line$_1$ (Line$_2$) transfers 0.13 pu (0.12 pu) real power, and DER units generate 0.60 pu, 0.47 pu, and 0.30 pu real power, respectively. At $t = 1$ s the motor is energized under full-load condition. Fig. 4.3(a)–(d) shows the $d$ and $q$ components, RMS value, and phase-angle of PCC$_1$ voltage that experience transients until the motor reaches its steady-state. Fig. 4.3(a)–(d) shows that despite the severe disturbance due to the motor start-up, the DER control maintains the voltage deviation at PCC$_1$ within a negligible range. The real and reactive
output power of DER\textsubscript{1} is depicted in Fig. 4.3(e) and (f) which are increased to supply real power demand of the motor.

Phase-A instantaneous current of DER\textsubscript{1} is illustrated in Fig. 4.3(g), which shows the base load current and the inrush current of the motor. The current drawn by the motor is per-unitized based on the motor ratings. Fig. 4.3(h) indicates the inrush current of the motor reaches to a peak value of 4.5 pu. Fig. 4.3(i) illustrates the transients of the motor electric torque.

This case study demonstrates robust performance of LCs in the event of a severe disturbance, i.e., energization of a 500 hp induction motor. Although the motor behaves as a nonlinear and time-varying system during its start-up period, and the LCs are designed based on an LTI system, the robustness of the LCs guarantees stability and a desirable microgrid performance subject to such a disturbance.

### 2.8.1.4 Case-4: Accidental Outage of DER\textsubscript{2}

The objective of this case study is to demonstrate that the proposed controller can maintain stability and operational integrity of the microgrid after a sudden accidental outage of a DER unit, provided the remaining DER units can meet the power demand.

Prior to DER\textsubscript{2} outage, 0.18 pu real power is transferred by Line\textsubscript{1} from PCC\textsubscript{2} to PCC\textsubscript{1} and there is no power transfer by Line\textsubscript{2}. At \( t = 0.5 \) s, DER\textsubscript{2} is disconnected and Fig. 2.14 shows the corresponding system transients. After the DER\textsubscript{2} disconnection, PCC\textsubscript{2} voltage magnitude drops by 2% since it is not directly controlled. However, since the other two DER units manage to control the voltage of their corresponding PCCs, the voltage of PCC\textsubscript{2} is also indirectly controlled and remains within the permissible range. Moreover, Fig. 2.14(f) shows that power flow of the
2.8 Test Cases

Figure 2.13: Induction motor energization: (a) and (b) \( d \) and \( q \) components of PCC\(_1\) voltage, (c) and (d) RMS value and phase-angle of PCC\(_1\) voltage, (e) output real power of DER\(_1\), (f) output reactive power of DER\(_1\), (g) phase-A instantaneous current of DER\(_1\), (h) phase-A instantaneous current of the motor, (i) electric torque of the motor.

distribution lines changes such that the on-line DER units compensate for the lack of generation at PCC\(_2\). This case study demonstrates the robustness of the proposed decentralized control scheme in case of deactivation of one of its agents.

2.8.1.5 Case-5: Change in Microgrid Configuration

The objective of this case study is to demonstrate the robust performance of the LCs despite major topological uncertainties. To conduct this case study, the microgrid of Fig. 2.1(a) is extended to the microgrid of Fig. 2.15. The added distribution line sections have the same per kilometer parameters as given in Table 2.1 and their lengths are specified on Fig. 2.15. The new loads \( L_4 \), \( L_5 \), and \( L_6 \), are resistive and rated at \( P_{L4} = 0.12 \) pu, \( P_{L5} = 0.4 \) pu, and \( P_{L6} = 0.13 \) pu. Prior to \( t = 0.5 \) s, the circuit breakers CB\(_i\), \( i = 1, 2, 3 \), are open and the DER units generate 0.63 pu, 0.45 pu, and 0.31 pu real power, respectively. Line\(_1\) (Line\(_2\)) transfers 0.14 pu (0.11 pu) real power.

At \( t = 0.5 \) s, all circuit breakers are simultaneously closed and the microgrid radial configuration of Fig. 2.1(a) changes to the meshed configuration of Fig. 2.15. The microgrid transients
due to this switching event are illustrated in Fig. 2.16. Fig. 2.16(a)–(d) shows the \(d\) and \(q\) components, RMS value, and phase-angle of PCC\(_1\) voltage. The specified set points for this voltage are faithfully tracked by LC\(_1\) despite the drastic change in the configuration. Fig. 2.16(e) and (f) shows real and reactive power generated by each DER unit. The real power transfer of Line\(_1\) is depicted in Fig. 2.16(g) and shows an increase in the steady-state power flow of this line due to the change in the microgrid configuration. Fig. 2.16(h) shows the real power flow through CB\(_1\) and the corresponding line is increased to 0.21 pu.

This case study reveals the robust performance of the LCs subject to a major topological uncertainty. Although the control scheme is designed based on the radial system of Fig. 2.1(a), the LCs manage to maintain the stability of the microgrid after a significant configuration change.
2.8 Test Cases

The test cases reported in the previous section demonstrate the feasibility of the proposed power management scheme, the control strategy, and the open-loop frequency control for governing dynamics of the microgrid of Fig. 2.1(a), subject to various types of uncertainties in the system. This section presents the results of real-time simulation of the proposed control design. The objective of this study is to verify that

- the proposed control/PMS strategy and corresponding algorithms can be digitally implemented in an industrial hardware platform, and

- practical issues that do not manifest themselves in off-line simulation studies can be readily addressed. These practical issues include noise, sampling rate of the digital controllers, nonidealities of the SPWM signals, and nonidealities associated with A/D and D/A conversion process.

Fig. 2.17(a) shows a schematic representation of the hardware-in-the-loop (HIL) environment for closed-loop performance evaluation of the microgrid of Fig. 2.1(a). The system of Fig. 2.17(a) is composed of

- an RTDS® [73] rack that simulates the power circuitry of Fig. 2.1(a), including the interface VSCs of the DER units, with a simulation time-step of 50 µs (3 µs) for large (small) time-step components,
Figure 2.17: Real-time simulation test bed, (a) schematic diagram, and (b) picture.

- I/O interface for signal transfer between the simulator and control platform of the DER units,

- three NI-cRIO real-time control platforms where each NI-cRIO unit imbeds the control algorithms of one DER unit, and

- a function generator that sends once every second a synchronization pulse to each NI-cRIO platform to synchronize the independently generated 60 Hz phase-angle waveforms of the DER units.

Fig. 2.17(b) shows a picture of the real-time simulation environment. Each NI-cRIO features a real-time processor communicating with an FPGA chip via the back-plain. The FPGA chips are used to generate the SPWM signals with the switching frequency of 6 kHz. The proposed controller is discretized with a sampling frequency of 6 kHz and implemented in the real-time processors using C-programming language. The 60 Hz phase-angle required for \(dq\) transformation is generated in each NI-cRIO separately and synchronized using the external digital signal generator that emulates a GPS. A computer emulates the PMS by performing power flow analysis and communicating the reference set points to NI-cRIOs via Ethernet.
2.8 Test Cases

2.8.2.1 Case 6: Sudden Load Increase

In this case study, the three DER units initially deliver 0.87 pu, 0 pu, and 0.4 pu real power, respectively. The real power flow in Line1 and Line2 is 0.1 pu. The resistive component of Load2 is suddenly increased from 0 to 0.6 pu at the time-instant denoted by “Switching Instant” in Fig. 2.18(a). Fig. 2.18(a) illustrates the $d$ and $q$ components and the instantaneous voltages of

**Figure 2.18**: Microgrid transients due to a load change: (a) $d$, $q$, and instantaneous components of PCC$_2$ voltage, and output reactive power of DER$_2$ prior to, during, and after load change at PCC$_2$, (b) output real power of the three DER units and output current of DER$_2$ in response to the load change.
PCC$_2$ and the output reactive power of the three DER units. It is observed that voltage signals experience practically insignificant transients due to the effective and robust performance of the DER controls. The output real power of the three DER units and the output current of DER$_2$ are depicted in Fig. 2.18(b). The output real power of DER$_2$ increases to respond to the increased real power demand, and those of the other DER units remain practically unchanged. This scenario demonstrates the robustness of the proposed control scheme despite microgrid parametric uncertainties.

2.8.2.2 Case 7: Accidental DER$_2$ Outage

In this case study, the response of the control system to an accidental outage of DER$_2$ is studied. Prior to disconnection of DER$_2$, the DER units generate 0.74 pu, 0.46 pu, and 0.15 pu real power, respectively. The real power flow on Line$_1$ (Line$_2$) are 0.05 pu (0.12 pu). Fig. 2.19 illustrates the transients in voltage of PCC$_2$ and output real and reactive power of the three DER units. Instantaneous voltages of PCC$_2$ exhibit no significant transients since the controllers of DER$_1$ and DER$_3$ rapidly increase the corresponding power generation to compensate for the generation loss due to DER$_2$ outage. This case study verifies the robust stability of the decentralized control system in response to the loss of one of the decentralized control agents. It should also be noted that during all off-line and real-time case studies, the voltage and frequency of the system remain well within the permissible limits defined by standards and grid codes such as IEEE 1547 [7], UL1740 [74], and the Nordic Grid Code [75].

2.9 Conclusions

This chapter presents a power management and a control strategy for an autonomous, multi-DER microgrid. The envisioned strategy provides (i) a power management system for the microgrid, (ii) an open-loop frequency control and synchronization, and (iii) a local, robust, decentralized control for each DER unit. The power management system, based on the classical power flow analysis, determines the terminal voltage set points for DER units. Frequency of the system is controlled in an open-loop manner by utilizing an internal crystal oscillator for each DER unit that also generates the phase-angle waveform required for $dq/abc$ ($abc/dq$) transformations. Synchronization of DER units is achieved by exploiting a GPS-based time-reference signal. The local control of each DER unit, which is the main focus of the chapter, is designed using a new multivariable decentralized robust control approach based on a linear state-space model of the microgrid. Various attributes of the controller, e.g., the existence conditions, gain-margins, robustness, and tolerance to delays are analytically discussed and the design procedures are outlined.
Analysis of microgrid dynamics highlights the robust performance of the proposed microgrid control scheme. Participation factors that associate closed-loop system modes and state variables indicate that the proposed control design is a viable solution for microgrid applications. Eigenvalue sensitivity analysis results show that the closed-loop system remains stable for a wide range of load parametric perturbations. Robustness analysis, using the notion of real stability radius, indicates that there is a compromise between robustness and speed of the controllers and that the microgrid operator can tune control parameters based on microgrid operating conditions to achieve a more robust or faster performance.
This chapter next presents the study results corresponding to the application of the proposed power management and control system to a three-DER microgrid system. The reported results are obtained from (i) time-domain simulation studies in the PSCAD/EMTDC platform, and (ii) an RTDS-based hardware-in-the-loop environment. The simulation studies validate the desired performance of the microgrid subject to load changes, voltage set point variations, nonlinear load energization process, accidental outage of DER units, and major changes in the microgrid topology. The test cases in the HIL environment are based on digital implementation of the controller of each DER unit in an NI-cRIO industrial control platform and verify (i) ease of hardware realization of the proposed power management and DER controller, and (ii) lack of sensitivity to hardware environment issues, e.g., noise, A/D-D/A conversion process, discretization error/delay, and PWM errors.
3 Overcurrent and Overload Protection

3.1 Introduction

One of the microgrid control objectives, particularly in the islanded mode of operation, is to control the PCC voltage of prespecified DER units to guarantee an overall acceptable voltage profile. The voltage control of an electronically-coupled DER unit can be realized either directly or through an inner current controller. The advantage of the inner current controller \([10,31,32,41,42]\) is that the DER unit is largely protected against fault currents since the reference set point of the current controller restricts the phase currents within permissible values. Thus, the DER unit can ride through faults without trip-out or damage to the unit. Likewise, the inner current controller can limit the DER output power and prevent overloading conditions. However, it 1) limits the speed of the overall controller and degrades its robustness due to the need for multiple feedback loops and 2) requires both voltage and current measurements.

A direct voltage control method \([15,38,40,68,69,76–78]\), without the inner current control loop, can provide a faster and a more robust performance, however, is unable to limit the current of the voltage-controlled DER (VC-DER) unit during abnormal conditions. Thus a fault condition can either trip out the VC-DER unit or damage its power electronic components. A VC-DER unit is also prone to dynamic overload conditions since it inherently lacks the ability to transiently limit the output power.

A direct voltage control method was introduced in Chapter 2. This chapter introduces two control add-on features for this control strategy to overcome the above drawbacks of a direct voltage control scheme:

- The first feature is an overcurrent protection scheme that operates solely based on voltage measurements and includes 1) fault detection, 2) fault current limitation, 3) fault clearance detection, and 4) smooth voltage control restoration mechanisms.
• The second feature is an overload protection scheme to dynamically limit the VC-DER output power. This scheme detects overload conditions based on voltage measurements, and limits the output power by assigning appropriate voltages to the terminals of the interface-VSC of the DER unit.

A set of comprehensive digital time-domain simulation studies, in the PSCAD/EMTDC platform, are performed to demonstrate performance of the proposed overcurrent and overload protection schemes under various system transient scenarios. To verify the feasibility of the proposed strategies, the corresponding algorithms are digitally implemented in an industrial hardware platform, i.e., NI-cRIO controllers, and tested in the RTDS real-time simulation environment based on the hardware-in-the-loop (HIL) approach.

### 3.2 Voltage Control Scheme

Fig. 3.1 illustrates a VC-DER unit, represented by a dc voltage source, a voltage-sourced converter (VSC), and a series RL filter. The unit is interfaced to the microgrid at the point of common connection (PCC in Fig. 3.1) through a step-up transformer. The block denoted by “Remainder of Microgrid” in Fig. 3.1 may include current- and/or power-controlled DER units and other components, e.g., distribution lines, loads, and capacitor banks.

A block diagram of a the direct voltage control scheme, applied to the local controller (LC) of each VC-DER unit as described in Chapter 2, is illustrated in Fig. 3.2. The control objective is to maintain the PCC voltage at its set point with zero steady-state error. The control function is performed in the \(dq\) frame of reference. Three-phase instantaneous voltages of the PCC, \(v_{abc}\), are measured and transformed to a \(dq\) frame of reference, \(V_{dq}\), and provided to the control block. The controller outputs, \(V_{t,dq}\), are transformed to \(abc\) frame of reference taking into account the turns ratio and phase shift of the interface transformer. Then the PWM signals are generated based on the controller outputs, \(v_{t,abc}\), to synthesize the voltage at the terminal of the interface VSC. Since the control scheme includes no inner current control loop, the VC-DER unit is subject to overcurrent and overload during system abnormal conditions, e.g., faults.
3.3 Overcurrent Protection Scheme

The objectives of the overcurrent protection scheme are to 1) limit the output current of the VC-DER units during fault conditions by modifying voltage controller outputs and 2) restore the controller after fault clearance. The overcurrent protection scheme is first described for a microgrid with a single VC-DER unit and then generalized for a microgrid with multiple VC-DER units. Both series L and LCL ac-side filter configurations are considered.

3.3.1 Microgrid with a Single VC-DER Unit

The proposed overcurrent protection scheme consists of the following.

3.3.1.1 Fault Detection

An overcurrent condition is detected based on the dynamic phasor analysis [79] by calculating the magnitude of the VC-DER output current dynamic phasor

\[
|I| = \left| \frac{V_t - V}{R_f + jX_f} \right| = \frac{\Delta V}{R_f + jX_f},
\]

where\(^1\) \(V = V_d + jV_q\) is the dynamic phasor of the PCC voltage and \(V_t = V_{td} + jV_{tq}\) is the voltage controller output, Fig. 3.1.

The PCC voltage drops during a fault condition and the voltage controller, in response to the drop, increases \(V_t\) in Fig. 3.3, which in turn increases \(|I|\). The fault is detected when the output current magnitude of the DER unit, calculated based on (3.1), reaches to its maximum limit of \(I_{max}\).

\(^1\)Bold math symbols indicate dynamic phasors.
3.3 Overcurrent Protection Scheme

Figure 3.3: Dynamic phasors of a VC-DER unit, relating VSC terminal and PCC voltages.

3.3.1.2 Fault Current Limiting Scheme

To prevent an excessive output current, the outputs of the controller, $V_{\text{t,dlq}}$ in Fig. 3.2, are overridden when a fault is detected and the VSC terminal voltage is set to $V_{\text{t,mod}}$, Fig. 3.3. The circle in Fig. 3.3 encompasses all VSC terminal voltage values that result in acceptable output current magnitudes for a given PCC voltage $V$. $V_{\text{t,mod}}$ is calculated based on 1) the measured PCC dynamic voltage phasor, $V$; 2) the impedance between the converter terminal and the PCC, $R_f + jX_f$; and 3) the monitored prefault output current of the VC-DER unit, $I_{pf}$. Thus

$$V_{\text{t,mod}} = V + (R_f + jX_f) \frac{I_{pf}}{c_{\text{tran}}}. \quad (3.2)$$

$I_{pf}$ is to keep the system operating point as close as possible to the prefault conditions and minimize microgrid transients after the fault clearance. Constant $c_{\text{tran}} > 1$ is to compensate for the decaying dc component of the instantaneous current which is not considered by the dynamic phasor analysis. Based on a large number of case studies, $c_{\text{tran}} = 3$ is selected which results in satisfactory performance even under worst-case scenarios.

During the time period that the overcurrent protection scheme is active, the PCC voltage can significantly deviate from the reference set point in which case the voltage controller accumulates error in its integrator states which is known as “integral windup” [80]. This phenomenon can cause excessive overshoots and consequently an unacceptable transient response after the voltage controller is restored. To avoid this situation, the integrator states of the voltage controller are frozen when the overcurrent protection scheme is active and released subsequent to the fault clearance.

3.3.1.3 Fault Clearance Determination and Controller Restoration

After the fault clearance, the PCC voltage increases, $|I|$ drops below the limit $I_{\text{max}}$, and the fault clearance is acknowledged. Then, the voltage controller is reactivated to restore voltage...
tracking.

### 3.3.2 Microgrid with Multiple VC-DER Units

This section generalizes the proposed overcurrent protection scheme for a microgrid with multiple VC-DER units. The fault detection and the fault current limitation schemes described in Section 3.3.1 are equally applicable to the VC-DER units of the microgrid. However, the fault clearance determination and controller restoration stages of each VC-DER unit require further enhancement. The reason is that the current magnitude of the distribution line is sensitive to the voltage difference between its two end buses, due to the distribution line small impedance. Thus, if the controllers fail to maintain the voltage of the corresponding PCC at the set point, the line current can exceed the DER output current limit.

In a microgrid with multiple VC-DER units, PCC voltages can significantly drift from their set point values during a fault and thus voltage controllers need additional time to restore the PCC voltages to the prefault values. Meantime, current magnitude can transiently exceed the permissible value. To avoid an inadvertent fault detection as a result of current overshoots, subsequent to a fault clearance, the voltage controllers should be smoothly restored to minimize current transients. The following describes the fault clearance determination and smooth controller restoration for a microgrid with multiple VC-DER units.

#### 3.3.2.1 Fault Clearance Determination

Fault clearance instant is determined based on the PCC voltage magnitude,

\[
V_{\text{mag}} = \sqrt{V_d^2 + V_q^2},
\]

(3.3)

which drops during the fault and starts increasing immediately after the fault clearance. Fault clearance is acknowledged when \(V_{\text{mag}}\) has increased to a predefined level. It should be noted that the instantaneous voltage magnitude, calculated based on (3.3), is subject to oscillatory components during unbalanced faults. Hence, the lower envelope of \(V_{\text{mag}}\) is monitored and the fault clearance is acknowledged when this envelope has increased to a predefined value.

Fig. 3.4 shows a typical PCC voltage magnitude and its lower envelope for a single-phase to ground fault, where \(V_{\text{mag}}\) exhibits double-frequency oscillations. To construct \(V_{\text{env}}\), the minimum of each double frequency cycle is identified and the envelope is set to the minimum for the next double-frequency cycle. The fault clearance is acknowledged when

\[
\frac{V_{\text{env}} - V_{\text{env,min}}}{\sqrt{V_{d,\text{ref}}^2 + V_{q,\text{ref}}^2}} \times 100 > \gamma,
\]

(3.4)
3.3 Overcurrent Protection Scheme

Figure 3.4: PCC voltage magnitude during a single-phase to ground fault condition and its lower envelop.

where $V_{env,\text{min}}$ is the global minimum of the envelope and $\gamma$ is the percentage increase of $V_{env}$ with respect to the reference voltage set point. The reference voltage set point is selected for calculation of $\gamma$ since the actual voltage of a faulty PCC can be close to zero. Selecting higher values for $\gamma$ ensures reliable fault clearance determination; however, it reduces the restoration speed.

3.3.2.2 Voltage Control Restoration

Restoration procedure begins after the fault clearance. This procedure is an integral part of the overcurrent protection scheme for a microgrid with multiple VC-DER units to ensure the controllers are smoothly restored and current transients are limited within the permissible range. The procedure depends on the relative difference between the prefault and postfault operating points of the VC-DER unit and two conditions can occur:

A1) The postfault operating point can be approximately the same as the prefault point. This is the case when the fault 1) is temporary and clears itself without any line or generation outage, i.e., 80–90% of the faults in distribution systems [81], 2) is permanent but its clearance does not significantly change the VC-DER unit output power.

A2) The postfault operating point can significantly deviate from the prefault operating point. This is the case when the fault is permanent and its clearance entails a significant change in the VC-DER unit output power.

To determine which one of the above has occurred, the output current of the VC-DER unit, which was limited to $I_{pf}/c_{\text{tran}}$ during fault based on (3.2), is gradually increased until the magnitude of the corresponding PCC voltage is almost the same as the prefault value. The corresponding VC-DER unit output current is $I_{rs}$, and the relative difference between $I_{rs}$ and $I_{pf}$ is

$$\Delta I = \frac{\sqrt{(\Re(I_{rs}) - \Re(I_{pf}))^2 + (\Im(I_{rs}) - \Im(I_{pf}))^2}}{|I_{rs}|}.$$  (3.5)
3.3 Overcurrent Protection Scheme

\( \Delta I \) is compared with a tolerance level, \( \Delta I_{tol} \), to determine which one of the above conditions holds. \( \Delta I_{tol} \) depends on the system and the controllers parameters. Based on extensive case studies, \( \Delta I_{tol} = 0.1 \) is selected for the reported studies.

To accommodate reclosure operations, the overcurrent protection scheme waits in restoration state until the microgrid protection scheme acknowledges the reclosure operation has been completed. Depending on which one of \( A1 \) or \( A2 \) conditions holds, the overcurrent protection scheme proceeds to restore the normal voltage control as follows.

\( A1 \) holds) In this case, \( V_t \) has been already modified such that output current of the VC-DER unit is \( I_{pf} \) and the system should be given enough time to reach its steady state. Since the postfault operating point is almost identical to the prefault operating point and the state variables of the controllers were frozen during the fault and the restoration stages, the controller is smoothly restored.

\( A2 \) holds) In this case the controller restoration may result in excessive overshoot of the VC-DER output current which, in turn, can be mistakenly interpreted as a fault condition. To avoid this scenario and achieve a controller smooth restoration with minimum current transients, the VC-DER whose operating point has significantly changed is temporarily disconnected and then the following two steps are taken:

1. The voltage reference for the controller is set at the current PCC voltage.
2. The state variables of the controller are initialized such that the the outputs of the controller before and after its restoration are equal. This is achieved by a state observer which is designed based on the state-space parameters of the voltage controller \( A_c, B_c, C_c, \) and \( D_c \) in Fig. 3.2. The observer inputs are (i) \( V_{td} \) and \( V_{tq} \) at the end of restoration stage, (ii) the measured PCC voltage, and (iii) the reference set points. The output of the observer is the state vector \( x \) that determines the initial conditions for the state variables of the voltage controller. Details of designing such an observer are described in Chapter 5.

The voltage controller then receives updated set points from the power management system and provides tracking. The VC-DER unit can be subsequently reconnected to the microgrid based on a command from the power management system. Fig. 3.5 illustrates a flowchart of the voltage control scheme, including the overcurrent protection mechanism.

### 3.3.3 VC-DER Units with LCL Filters

The interface VSC filter is either a series RL filter, Fig. 3.1, or a T-shape LCL filter, Fig. 3.6. The proposed overcurrent protection scheme of Section 3.3.1 can be readily applied to a DER unit.
3.3 Overcurrent Protection Scheme

![Flowchart of the voltage control scheme including the overcurrent protection.](image)

**Figure 3.5:** Flowchart of the voltage control scheme including the overcurrent protection.

![Schematic single-line diagram of a DER unit equipped with an LCL filter.](image)

**Figure 3.6:** Schematic single-line diagram of a DER unit equipped with an LCL filter.

with a T-shape LCL filter by replacing (3.1) and (3.2) with

\[
|I_{lin}| = \left| \frac{V_t \left( 1 + \frac{X_{f,L2}}{X_{f,C}} \right) - V}{X_{f,L1} + X_{f,L2} + \frac{X_{f,L1}X_{f,L2}}{X_{f,C}}} \right|, 
\]

(3.6)

and

\[
V_{t,mod} = \frac{I_{pf,tran} \left( X_{f,L1} + X_{f,L2} + \frac{X_{f,L1}X_{f,L2}}{X_{f,C}} \right) + V}{1 + \frac{X_{f,L2}}{X_{f,C}}}, 
\]

(3.7)

respectively, where (3.6) and (3.7) are derived based on dynamic phasor analysis of the circuit of Fig. 3.6. Since the series impedances are highly inductive, the resistive components are ignored in the analysis.
3.4 Overload Protection

The objective of the overload protection scheme is to limit the output power of a VC-DER unit whose controller does not include an inner current control loop. The instantaneous three-phase apparent output power of the DER unit of Fig. 3.1, based on dynamic phasors, is expressed as

\[
S = 3VI^* = 3V\left|\frac{V_t - V}{R_f + jX_f}\right|^* = 3V\left|\frac{\Delta V}{R_f + jX_f}\right|^*.
\]  

(3.8)

Based on the maximum generation capacity of the unit, \(S_{\text{max}}\), the impedance \(R_f + jX_f\), and the PCC voltage \(V\), the maximum magnitude of the voltage difference, \(\Delta V_{\text{max}}\), that corresponds to the maximum output power is

\[
\Delta V_{\text{max}} = \frac{S_{\text{max}} |R_f + jX_f|}{3|V|}.
\]  

(3.9)

Thus the VC-DER unit is overloaded if \(|\Delta V| > \Delta V_{\text{max}}\) and leads to the following overload protection strategy. The magnitude

\[
|\Delta V| = \sqrt{(V_{td} - V_d)^2 + (V_{tq} - V_q)^2},
\]  

(3.10)

is compared with \(\Delta V_{\text{max}}\). If \(|\Delta V| > \Delta V_{\text{max}}\), the output of the voltage controller is ignored and the terminal voltage, in the \(dq\) reference frame, is assigned based on (3.9) and the measured PCC voltage \(V_{dq}\). The area within the circle of Fig. 3.3 identifies all \(|\Delta V|\) that result in an acceptable amount of output power. To maximize the output power of the unit, \(\Delta V\) with the magnitude of \(\Delta V_{\text{max}}\) is selected. To operate as close as possible to the operating point that the voltage controller has specified, the angle of \(\Delta V\) is selected such that the distance between \(V_{t,\text{mod}}\) and \(V_t\), Fig. 3.3, is minimized, i.e.,

\[
V_{td,\text{mod}} = V_d + \Delta V_{\text{max}} \cos \left(\tan^{-1}\frac{V_{tq} - V_q}{V_{td} - V_d}\right),
\]

\[
V_{tq,\text{mod}} = V_q + \Delta V_{\text{max}} \sin \left(\tan^{-1}\frac{V_{tq} - V_q}{V_{td} - V_d}\right).
\]  

(3.11)

This will limit the output power of the DER unit to \(S_{\text{max}}\). When the output power is limited, the integrator states of the voltage controller are kept unchanged to avoid integral windup. The microgrid power management system resolves the overloading condition of the DER unit as fast as possible, e.g., by either load shedding or power flow alteration. The voltage controller can then be restored and since the integral windup has been avoided, the system continues operating with minimum transients. This scheme is an integral part of the voltage controllers of all VC-DER units of the microgrid.
3.5 Performance Evaluation

This section presents the results of offline time-domain and real-time HIL simulation case studies to demonstrate performance and feasibility of the proposed overcurrent and overload protection algorithms.

3.5.1 Offline Time-domain Simulation Studies

A set of test cases, based on digital time-domain simulation studies in the PSCAD/EMTDC environment, were carried out to evaluate performance of the proposed overcurrent/overload protection strategies. The component models and parameter values are the same as those of Chapter 2. The overcurrent/overload protection feature for each LC is coded in FORTRAN and imbedded in the FORTRAN-code implementation of the corresponding controller.

3.5.1.1 Case 1: Overcurrent Protection – Temporary Fault

This case study demonstrates 1) performance of the overcurrent protection scheme in limiting the VC-DER output current when the microgrid is subject to a three-phase temporary fault and 2) restoration of the normal control action subsequent to the fault clearance. Before fault inception, the three DER units deliver 1 pu, 0.75 pu, and 0.35 pu apparent power at a 0.94, 0.96, and 0.92 lagging power factor, respectively. The real power transfers of Line 1 and Line 2 are 0.1 pu and 0.15 pu, respectively. At $t = 0.6$ s, a bolted temporary three-phase to ground fault at PCC1 occurs and lasts for 5 cycles. The fault at PCC1 results in the most severe overcurrent with respect to DER1. Fig. 3.7 shows the output current of DER1 prior to, during, and after the fault while the overcurrent protection scheme is disabled. Provided DER1 could withstand the overcurrent, the peak current reaches 18 pu. In practice, this output current magnitude either damages the power electronic switches or trips out the DER unit.

Fig. 3.8 illustrates the microgrid transients prior to, during and subsequent to the same fault while the overcurrent protection scheme is in service. Fig. 3.8(a) and (b) shows that PCC1
3.5 Performance Evaluation

Figure 3.8: Performance of the overcurrent protection scheme for an L-L-L-G fault at PCC1: (a) and (b) three-phase voltages of PCC1 and PCC2, (c) and (d) three-phase output currents of DER1 and DER2.

and PCC2 voltages significantly decrease during the fault and are smoothly restored to their prefault reference set points subsequent to the fault clearance. Fig. 3.8(c) and (d) shows that the instantaneous three-phase output currents of DER1 and DER2 are effectively limited during the fault (compare with Fig. 3.7) and increase to their prefault steady-state values after the fault clearance. The maximum permissible current magnitude for each DER unit is assumed to be 2 pu, based on the unit ratings.

Similar test cases were carried out when the DER units were equipped with LCL filters, and the modified overcurrent protection scheme of Section 3.3.3 was in service and results similar to Fig. 3.8 were obtained. Extensive case studies for different fault types and locations were conducted which demonstrated the ability of the overcurrent protection method in 1) detecting a fault condition, 2) limiting the output current magnitudes of DER units, and 3) restoring the microgrid to its prefault operating conditions.
3.5 Performance Evaluation

3.5.1.2 Case 2: Overcurrent Protection – Successful Reclosure

The objective of this case study is to demonstrate performance of the overcurrent protection for a temporary fault and its subsequent successful reclosure event. Prior to the fault, the three DER units deliver 0.71 pu, 0.43 pu, and 0.54 pu apparent power at 0.93, 1.00, and 0.92 lagging power factor, respectively, and 0.25 pu (0.1 pu) real power is transferred by Line1 (Line2). At $t = 0.6 \text{s}$, a single-phase to ground temporary fault occurs at the middle of Line1. The breakers at both ends of Line1 are opened after 5 cycles at $t = 0.68 \text{s}$ and remain open for 15 cycles. The temporary fault is self-cleared prior to the reclosing attempt at $t = 0.93 \text{s}$. Since the reclosure is successful, the microgrid continues its operation at an operating point identical to its prefault conditions.

Fig. 3.9 illustrates instantaneous three-phase PCC voltages and output currents corresponding to the three DER units, prior to, during and subsequent to the fault and its subsequent switching events. Fig. 3.9 shows that the fault is detected within about 500 µs, the output currents of the DER units are effectively limited during the fault to less than 1.2 pu, the original controllers are restored at about $t = 1 \text{s}$, and the PCC voltages are maintained at the prefault values.

3.5.1.3 Case 3: Overcurrent Protection – Unsuccessful Reclosure

The prefault system operating condition and the fault scenario are the same as those of previous case study. However, the fault is permanent and the reclosure at $t = 0.93 \text{s}$ is unsuccessful. Thus, the breakers are reopened after 5 cycles at $t = 1.01 \text{s}$ and are locked out subsequently. The microgrid operation continues subsequent to fault clearance while Line1 is open, and the three DER units generate 0.48 pu, 0.62 pu, and 0.46 pu apparent power at a 0.83, 1.00, and 0.94 lagging power factor, respectively. Since the power transfer on Line1 is interrupted due to line outage, the operating point of DER2 is significantly changed, and based on the discussion of Section 3.3.2.2, DER2 is disconnected by opening Line2. After the restoration of LC2, Line2 can be closed to connect the unit to the system.

Fig. 3.10 illustrates the microgrid transients, i.e., instantaneous three-phase PCC voltages, output currents of the three DER units, and power transfer on Line1 in response to the fault and its subsequent switching events. Fig. 3.10 shows that the overcurrent protection scheme effectively limits DER output currents both at the inception of fault and at the unsuccessful reclosure instant. Fig. 3.10 indicates 1) the current transients are restricted within the permissible limits, 2) the restoration procedure successfully restores the controllers to track the voltage set points after the fault clearance, and 3) the current transients are within the acceptable range during the restoration process.
3.5 Performance Evaluation

![Figure 3.9: Performance of the overcurrent protection scheme for a temporary L-G fault at the middle of Line1 and its subsequent single-pole successful reclosure: (a) – (f) three-phase PCC voltages and output currents of the three DER units.](image)

3.5.1.4 Case 4: Overload Protection

This case study demonstrates the effectiveness of the proposed overload protection scheme, Section 3.4. Initially, the DER units supply the microgrid net load proportional to their maximum generation capacities. At $t = 0.5$ s, real component of Load$_1$ increases to 0.9 pu, which is beyond the generation capacity of DER$_1$. Fig. 3.11 shows that output power of DER$_1$ is limited to its maximum generation capacity by the overload protection method and the other DER units automatically increase their output power to meet the demand. At $t = 1$ s, the PMS updates the voltage reference set points to alter the power flow within the microgrid such that the output power of DER$_1$ does not exceed its permissible limit. Fig. 3.12 shows the instantaneous three-phase voltages of PCC$_1$ and PCC$_2$ for the same period of time and indicates acceptable voltage waveforms for the entire period.

To illustrate the effect of integral windup, the same test case is carried out while the integrator states of the LC are not frozen when the overload protection scheme is active. Fig. 3.13 shows excessive overshoots at $t = 1$ s when the LCs are restored. A comparison between
3.5 Performance Evaluation

Figure 3.10: Performance of the overcurrent protection scheme for a permanent L-G fault at the middle of Line1: (a) – (f) three-phase PCC voltages and output currents of the DER units, and (g) real and reactive power transfer of Line1.

The corresponding transients of Figs. 3.11 and 3.13 reveals the necessity of the anti-windup scheme for a smooth restoration process.

3.5.2 Hardware-in-the-loop Simulation Test Case

The objective of this study is to verify that the proposed overcurrent/overload protection schemes can be digitally implemented in an industrial hardware platform despite hardware implementation issues, e.g., noise, sampling rate of the digital controllers, sample and hold of SPWM signals, delay and nonzero bias of analog inputs and outputs, and discretization errors that do not manifest themselves in off-line simulation studies. HIL environment, component models, and parameter values are the same as those of Chapter 2, as described in Section 2.8.2. Overcurrent/overload protection scheme of each VC-DER unit is imbedded in the C-code implementation of the corresponding voltage controller.
3.5 Performance Evaluation

**Figure 3.11:** Apparent output power of the three DER units when Load\(_1\) increases beyond the maximum generation capacity of DER\(_1\).

**Figure 3.12:** Three-phase voltages of PCC\(_1\) and PCC\(_2\) while overload protection scheme in service.

### 3.5.2.1 Case 5: Overcurrent Protection – Temporary Fault

Fig. 3.14 shows the system response, i.e., DER\(_1\) instantaneous voltages and currents, to a three-phase temporary fault at PCC\(_1\). Prior to the fault inception, the microgrid supplies the loads and LCs maintain voltages of PCC at the prespecified set points. Fig. 3.14 shows that voltage drops to zero due to the fault and the overcurrent protection scheme effectively limits the output current magnitude. After the fault clearance, the voltages are restored to prefault steady-state value.

Fig. 3.15 illustrates the fault inception and fault detection instants. Fig. 3.15 indicates

**Figure 3.13:** Power transients subsequent to restoration of LC\(_1\) while the anti-windup scheme is disabled.
3.6 Conclusions

This chapter presents two new concepts for overcurrent and overload protection to enable direct voltage control of electronically-coupled DER units in an islanded microgrid. The overcurrent protection scheme detects the fault, limits the DER unit output current magnitude within its permissible range, e.g., 1.2 pu, and restores the microgrid to its normal operating condition subsequent to the fault clearance. The fault detection scheme can detect a fault in 200 µs; thus that the fault detection scheme, based on instantaneous voltage measurements, can detect the three-phase to ground fault in less than 200 µs. This is significantly faster than a conventional overcurrent relay whose response time is in the range of 15 ms to 50 ms. Due to its speed, the proposed fault detection scheme can be implemented for applications that require fast fault detection.

Figure 3.14: Transients of PCC\textsubscript{1} voltages and DER\textsubscript{1} output currents due to a L-L-L-G fault at PCC\textsubscript{1}.

Figure 3.15: Fault detection speed of the proposed overcurrent protection scheme.
it can be implemented in protective relays where speed is a concern. The overload protection scheme limits the output power of the DER unit to its rated MVA when excessive power demand is imposed on the DER unit. An extensive set of time-domain simulation studies in the PSCAD/EMTDC platform and an RTDS-based hardware-in-the-loop environment were performed to evaluate and verify the feasibility of the proposed schemes.

The study results demonstrate the desired performance of the add-on features when the microgrid is subject to various types of faults and the subsequent fault overcurrent/overload conditions. The real-time case study demonstrates 1) ease of hardware realization of the proposed methods, and 2) lack of sensitivity to noise, A/D-D/A conversion process, discretization error/delay, and PWM errors. In the reported studies, the proposed schemes are used to limit the 1) overcurrent to 1.2 pu and 2) overload to the DER unit maximum generation capacity, in a microgrid with three DER units.
4 Generalized Microgrid Control

4.1 Introduction

A MICROGRID control strategy is proposed in Chapter 2 and applied to a three-DER microgrid where each DER unit controls the voltage of its point of common coupling (PCC). This chapter generalizes the approach of Chapter 2, based on the same robust decentralized control scheme, such that 1) each DER unit can now be designated as a voltage-controlled (VC-DER) or a power-controlled (PC-DER) unit depending on microgrid configuration, DER unit location, load information, and other possible considerations; 2) more than one DER unit can contribute to voltage regulation if the distribution feeder is long and/or heavily loaded; and 3) none of the DER units is assumed to be dominant.

The strategy proposes that (i) a PMS should specify voltage and power set points for the corresponding DER units, (ii) local controllers (LCs) should provide tracking of the set points, and (iii) an open-loop frequency control and synchronization scheme should maintain the system frequency. This method provides prescribed load sharing among DER units, precise power flow regulation, robust tracking of set points, operation with no need for high-bandwidth communication, and a fixed microgrid frequency. The proposed approach requires low-bandwidth communication for both synchronization and PMS data transmission. However, temporary failure of communication does not lead to system collapse, provided it recovers prior to significant changes in the microgrid operating point.

A set of eigenvalue and robustness studies are performed to signify the properties of the proposed controllers. A set of comprehensive digital time-domain simulation studies, in the PSCAD/EMTDC platform, are performed to demonstrate performance of the proposed scheme. To verify performance of the hardware-implemented strategy, its algorithms are digitally implemented in an industrial platform and tested in a real-time simulation environment based on the hardware-in-the-loop (HIL) approach.
4.2 Study System

A schematic diagram of a distribution system that is adopted as the study microgrid system is illustrated in Fig. 4.1(a). The distribution system of Fig. 4.1(a) is selected since it includes a relatively large number of buses, feeder branches, and electric loads and it provides a more realistic environment to evaluate performance of the controllers. The microgrid includes three 0.6 kV dispatchable electronically-interfaced DER units and eleven loads which are connected to a 13.8 kV distribution system. Each DER unit is represented by a 1.5 kV DC voltage source, a two-level voltage-sourced converter (VSC), and a series RL filter, and is interfaced to the distribution feeder by a 0.6 kV / 13.8 kV step-up transformer at its PCC, Fig. 4.1(b). The system data are given in Table 4.1. The microgrid is operated in islanded mode by opening the circuit breaker CB_g.
### 4.3 Power Management and Control Strategy

The proposed strategy is composed of (i) a power management system (PMS), (ii) a microgrid frequency control and the associated synchronization scheme, and (iii) local controllers (LCs) of DER units.

#### 4.3.1 Power Management System

DER$_1$ and DER$_3$ are designated as power-controlled DER (PC-DER) units, i.e., their real/reactive output power components are controlled, and DER$_2$ is a voltage-controlled DER (VC-DER) unit, i.e., it regulates the voltage of its PCC, and as a result, the voltage profile of the entire microgrid. DER$_2$ is designated as a VC-DER unit due to its location in the studied microgrid system, Fig. 4.1(a). In general, to improve the overall voltage profile of the microgrid, depending on the distribution feeder length, configuration, and load information, multiple DER units can be assigned as VC-DER units and others can operate as PC-DER units. To determine appropriate real/reactive power set points for PC-DER units, the microgrid is continuously monitored, its net load is measured, and the collected data is communicated to the central PMS through a low-bandwidth communication link. Based on the measured total real/reactive power components of the microgrid load and a prescribed load sharing scheme, the PMS determines the real/reactive power set points for PC-DER units, $P_{\text{ref},i}$ and $Q_{\text{ref},i}, i = 1,3$. The PMS also specifies an appropriate voltage set point, e.g., $1 \angle 0^\circ$, for the VC-DER unit (DER$_2$). By maintaining the voltage of its PCC, DER$_2$ delivers

$$
P_{\text{DER},2} = P_{\text{Load}} + P_{\text{Loss}} - P_{\text{DER},1} - P_{\text{DER},2},$$

$$
Q_{\text{DER},2} = Q_{\text{Load}} + Q_{\text{Loss}} - Q_{\text{DER},1} - Q_{\text{DER},2}.
$$

#### Table 4.1

**MICROGRID DATA, $S_{\text{base}} = 1.6$ MVA, $V_{\text{base}} = 13.8$ kV**

<table>
<thead>
<tr>
<th>Size of DER Units</th>
<th>Load Data: $L_i$(MVA-power factor)</th>
<th>Line Impedance: $(0.35 + j0.31)$ Ω km$^{-1}$</th>
<th>Transformer Reactance: 8%</th>
<th>RL filter: $Z =15%$, Quality factor = 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>DER$_1$: 1.6 MVA</td>
<td>$L_1(0.08-0.94)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DER$_2$: 1.2 MVA</td>
<td>$L_2(0.16-0.98)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DER$_3$: 0.8 MVA</td>
<td>$L_3(0.16-0.90)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L_4(0.17-0.94)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L_5(0.25-0.96)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3 Power Management and Control Strategy

where $P_{\text{Load}}/Q_{\text{Load}}$ are the total real/reactive power demand of microgrid net load and $P_{\text{Loss}}/Q_{\text{Loss}}$ are the total real/reactive power loss of the distribution feeder. Thus, the PMS regulates the power flow of the microgrid. The power management process should be performed frequently to maintain the prespecified load sharing scheme among the DER units as the microgrid operating point changes, e.g., due to load and generation changes.

4.3.2 Frequency Control and Synchronization

The frequency control and synchronization scheme of Section 2.3.2 is equally applicable for this control strategy and adopted for the reported studies of this chapter.

4.3.3 Local Controllers

The power and voltage set points are transmitted from the PMS to each DER LC. Since LCs perform the control in a $dq$ frame of reference, the set points should be first transformed to the $dq$ frame based on the globally synchronized phase-angle waveform $\theta(t)$. For the VC-DER unit, the set point $V_{\delta}$ is expressed as $V_{d,\text{ref}} = V \cos \delta$ and $V_{q,\text{ref}} = V \sin \delta$ in the $dq$ frame. For the PC-DER units, since the LCs provide tracking of their output current components $I_d$ and $I_q$, the real/reactive power set points should be first expressed in terms of current set points $I_{d,\text{ref}}$ and $I_{q,\text{ref}}$ using

\[
I_{d,\text{ref}} = \frac{2}{3} \left( \frac{V_{d,\text{ref},i} + V_{q,\text{ref},i}}{V_{d,i}^2 + V_{q,i}^2} \right), \\
I_{q,\text{ref}} = \frac{2}{3} \left( \frac{V_{q,\text{ref},i} - V_{d,\text{ref},i}}{V_{d,i}^2 + V_{q,i}^2} \right),
\]

where $V_{d,i}$ and $V_{q,i}$ are the dynamic $d$ and $q$ voltage components of PCC$_i$. The measured and reference signals are then provided to LC$_i$, whose outputs $V_{d,i}$ and $V_{q,i}$ are the $d$ and $q$ voltage components of DER$_i$ VSC interface-terminal with respect to the high-voltage side of the transformer. $V_{d,i}$ and $V_{q,i}$ are then transformed to $abc$ frame, divided by the turn ratio of the transformer and shifted, if necessary, to obtain $v_{t,abc,i,\text{LV}}$ corresponding to the low-voltage side of the transformer. $v_{t,abc,i,\text{LV}}$ is then fed to the PWM signal generator of the interface VSC. Fig. 4.2 illustrates a block diagram of LC$_i$ which applies for both voltage and current control loops.

To design parameters of the three LCs, a mathematical model of the open-loop microgrid is required. A microgrid generally includes a large number of loads, distribution line segments, and DER units which drastically increase the order of its mathematical model. Hence, it is desirable to develop a low order model of the system for control design. Due to the robustness of the proposed control scheme, potential discrepancies between the low order model and the detailed higher-order model cannot significantly impact on controller performance or cause
instability. To develop a low order equivalent linear model, the microgrid of Fig. 4.1(a) is approximated by a radial distribution feeder with the three DER units interconnected through two distribution line segments while their electrical distance is preserved. Loads electrically close to each unit are combined and connected to the corresponding PCC. A single-line diagram of the simplified microgrid is shown in Fig. 2.4. Each DER unit is represented by a three-phase controlled voltage source and a series RL branch. Each load is modeled by an equivalent parallel three-phase RLC network. Each distribution line is represented by lumped series three-phase RL elements. Parameters of the model are given in Table 2.1. The controllers are designed based on the fundamental frequency component of the microgrid model, Fig. 2.4. Thus, the mathematical model of Section 2.4 is equally applicable in this case as well, except for the following: 1) $y^T$ in (2.6) is modified to $y^T = (I_{1,d}, I_{1,q}, V_{2,d}, V_{2,q}, I_{3,d}, I_{3,q})$; 2) $y_i$ in (2.7) is modified to $y_i = (I_{i,d}, I_{i,q}), i = 1, 3$; and 3) the output matrix $C'$ in Appendix A, instead of $C$, is used.

Carrying out the optimization of the controller (2.15), as measured by the performance index (2.17), yields the following decentralized controller for the microgrid of Fig. 4.1(a):

$$
\dot{\eta} = y - y_{ref},
$$
$$
\dot{\beta} = A\beta + B y,
$$
$$
u = K_1 y + K_2 \eta + K_3 \beta,
$$

where
4.4 Analysis of Microgrid Dynamics

To investigate the dynamic behavior of the closed-loop microgrid, a set of studies including eigen and robustness analyses are carried out whose results are presented in this section. Eigenanalysis includes eigenvalue, participation factor, and eigenvalue sensitivity analyses. Robustness analysis evaluates the variation of the real stability radius as controller parameters change.

4.4.1 Eigenvalue and Participation Factor Studies

Closed-loop system poles, i.e., eigenvalues of the closed-loop $A$ matrix, $A_{\text{close}} \in \mathbb{R}^{31 \times 31}$, are listed in Table 4.2 and show the closed-loop system (2.16) has 13 pairs of complex conjugate and 5 real eigenvalues. The dominant eigenvalues, i.e., the closest eigenvalues to the imaginary axis, are highly damped, which indicates proper design of the controllers. To identify the state variables that dominantly influence an eigenvalue, the normalized participation factors associated with each eigenvalue are calculated based on (2.21). State variables include 22 open-loop state variables of the open-loop microgrid and 9 states corresponding to the three LCs. LC$_i$ has three states, two of which, designated by $\eta_i \in \mathbb{R}^2$ in Table 4.2, participate in regulating the $d$ and $q$ components of the controlled variables and ensure tracking the set points with zero steady-state error. The third state of LC$_i$, designated by $\beta_i \in \mathbb{R}$ in Table 4.2, corresponds to a stabilizing compensator. The results of participation factor analysis indicate
\begin{itemize}
\item all closed-loop eigenvalues are contained in the open left-half complex plain and are far from the imaginary axis, thus the closed-loop system is stable and well-damped,
\item dominant eigenvalues of the closed-loop system, i.e., eigenvalues 1–8, are predominantly affected by the controller regulating states,
\item state variables corresponding to $V_{dq2}$ are associated with the eigenvalues which are far from the imaginary axis and have fast dynamics,
\item stabilizing compensator states have very fast dynamics which is desirable,
\item state variables corresponding to DER output and load currents are associated with relatively slow modes which is expectable due to the large time-constant of the corresponding inductors, and
\item state variables corresponding to distribution line currents are associated with faster modes due to the low $X/R$ ratio of the distribution line.
\end{itemize}

These observations suggest that the optimal control design procedure meets requirements for a generalized islanded microgrid control problem.

In another study, the controller parameters $K_i$ are scaled by parameter $\alpha_i$ and corresponding eigenvalue trajectories are illustrated in Fig. 4.3. Fig. 4.3(a) shows that eigenpairs (5,6) and (18,19) depart to the right-hand-side as $\alpha_1$ is decreased from 1 to 0.8 and the departure to the right-half complex plane occurs at $\alpha_1 = 0.92$. It is also observed the closed-loop system becomes unstable for $\alpha_1 > 144$. Fig. 4.3(b) shows that eigenpairs (1,2) and (5,6) move towards the imaginary axis as $\alpha_2$ is increased in the range $0.1 < \alpha_2 < 2$ and depart to the right-hand-side when $\alpha_2 = 1.61$. Fig. 4.3(c) shows that the eigenpair (11,12) exhibits higher damping as $\alpha_3$ is increased in the range $0.8 < \alpha_3 < 1.2$ and departs to the right-hand-side at $\alpha_3 = 1.09$. Similar results are obtained for $K_4$. Fig. 4.3(d) shows that eigenpairs (11,12) and (5,6) move further away from the imaginary axis and become more damped as $\alpha_5$ is increased in the range $0.8 < \alpha_5 < 1.2$ and the closed-loop system is unstable for $\alpha_5 < 0.91$. This study shows the range of variations of controller parameters which results in a stable closed-loop system. Similar eigenvalue sensitivity analysis was carried out for load parameter perturbations. It was observed that for a wide range of load parameter perturbations, the closed-loop eigenvalues remain in open left-half complex plane which highlights robust performance of the controllers. Next section presents robustness analysis studies of the closed-loop system.
4.4 Analysis of Microgrid Dynamics

Table 4.2

EIGENVALUES OF THE CLOSED-LOOP SYSTEM AND THE ASSOCIATED PARTICIPATING STATES

<table>
<thead>
<tr>
<th>Number</th>
<th>Eigenvalue</th>
<th>Participating States</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>-18.01 ± 377.75i</td>
<td>$I_{d1}$, $\eta_1$</td>
</tr>
<tr>
<td>3,4</td>
<td>-18.54 ± 377.13i</td>
<td>$\eta_2$, $\eta_3$</td>
</tr>
<tr>
<td>5,6</td>
<td>-22.22 ± 371.74i</td>
<td>$\eta_3$, $I_{d3}$</td>
</tr>
<tr>
<td>7,8</td>
<td>-43.45 ± 6.22i</td>
<td>$\eta_1$</td>
</tr>
<tr>
<td>9</td>
<td>-55.87</td>
<td>$I_{q2}$, $I_{Lq2}$</td>
</tr>
<tr>
<td>10</td>
<td>-65.45</td>
<td>$I_{d3}$, $\eta_3$</td>
</tr>
<tr>
<td>11,12</td>
<td>-81.19 ± 375.4i</td>
<td>$I_{d1}$, $I_{d3}$</td>
</tr>
<tr>
<td>13,14</td>
<td>-87.54 ± 369.34i</td>
<td>$I_{d1}$</td>
</tr>
<tr>
<td>15</td>
<td>-128.09</td>
<td>$I_{Ld}$, $I_{d3}$</td>
</tr>
<tr>
<td>16,17</td>
<td>-346.34 ± 1755i</td>
<td>$I_{q3}$, $\beta_3$</td>
</tr>
<tr>
<td>18,19</td>
<td>-445.41 ± 344.4i</td>
<td>$\eta_2$, $\beta_2$</td>
</tr>
<tr>
<td>20,21</td>
<td>-488.95 ± 4317i</td>
<td>$V_{d2}$, $I_{d1}$</td>
</tr>
<tr>
<td>22,23</td>
<td>-547.8 ± 2560i</td>
<td>$V_{q1}$, $I_{q1}$</td>
</tr>
<tr>
<td>24,25</td>
<td>-610.97 ± 1792.6i</td>
<td>$V_{d3}$</td>
</tr>
<tr>
<td>26,27</td>
<td>-806.45 ± 1048.7i</td>
<td>$I_{q2}$, $\beta_2$</td>
</tr>
<tr>
<td>28</td>
<td>-1846.4</td>
<td>$I_{q2}$, $\beta_2$</td>
</tr>
<tr>
<td>29,30</td>
<td>-20763. ± 6130i</td>
<td>$V_{q2}$, $I_{q2}$</td>
</tr>
<tr>
<td>31</td>
<td>-89282</td>
<td>$\beta_1$</td>
</tr>
</tbody>
</table>

4.4.2 Robustness Analysis

Subsequent to a control design, it is required the closed-loop system should be robust, i.e., the state matrix of the closed-loop system should preserve its stability property when its elements are perturbed. The notion of real stability radius, defined in Section 2.5.3, is used to evaluate the robustness of the closed-loop system as control parameters are perturbed.

In this part, the effect of controller parameter variations on the real stability radius, $r_R$, of the closed-loop system state matrix, $A_{close}$, is studied. The matrix triple $\{A_{close}, [A, A, 0]^T, [I, 0, 0]\}$ is considered as the argument of function $r_R(.)$ in (2.12) to reflect microgrid load uncertainties. Fig. 4.4 illustrates the variations of $r_R$ and closed-loop system maximum settling time as the controller parameter $\mathcal{K}_2$ changes to $\alpha \mathcal{K}_2$ for $0.1 < \alpha < 1$, and indicates an increase of $\mathcal{K}_2$ degrades closed-loop system robustness. However, it reduces the maximum settling time of the system, which corresponds to a faster control performance and points to a compromise between controller speed and robustness. To verify this observation, the bode plots of the transfer function $V_{d1}(s)/V_{dref,1}(s)$ are depicted in Fig. 4.5 for the two coefficients $\alpha = 1$ and $\alpha = 0.5$. The gain margin corresponding to a smaller controller gain is larger, which verifies the observation of Fig. 4.4. A similar study for other controller parameters reveals the following relationship
between the controller parameter $K_i$ and closed-loop real stability radius $r_R$: $K_1 \uparrow \Rightarrow r_R \uparrow$, $K_3 \uparrow \Rightarrow r_R \downarrow$, $K_4 \uparrow \Rightarrow r_R \downarrow$, and $K_5 \uparrow \Rightarrow r_R \uparrow$.

The eigen and robustness analysis study results can be used to fine tune the parameters of the optimal controllers. The range of controller parameter perturbation that result in a stable system were obtained by the eigenvalue sensitivity analysis and the effect of these changes on the robustness of the closed-loop system were studied in this section. The microgrid operator can tune controller parameters based on these studies to obtain a faster or a more robust control performance, if necessary.

### 4.5 Performance Evaluation

This section describes the results of offline time-domain and real-time HIL simulation case studies to demonstrate performance of the proposed microgrid control and power management strategy.
4.5 Performance Evaluation

![Figure 4.4: Effect of the controller parameter $K_2$ on the robustness and settling time of the closed-loop system.](image)

4.5.1 Offline Time-domain Simulation Studies

A set of case studies, based on digital time-domain simulations in the PSCAD/EMTDC environment, were conducted to evaluate performance of the microgrid control strategy. The component models and parameter values for the test cases are given in Section 2.8.1 and Table 2.1.

4.5.1.1 Case 1: Load Change

The objective of this case study is to demonstrate the robust performance of the proposed control strategy when a significant portion of the microgrid load is suddenly disconnected. Prior to $t = 0.8 \, \text{s}$, the three DER units supply microgrid net load proportional to their sizes, i.e., they deliver 0.70, 0.53, and 0.35 pu apparent power at a 0.95 lagging power factor. At $t = 0.8 \, \text{s}$, CB$_1$ in Fig. 4.1(a) is opened and its downstream load is disconnected from the system. Fig. 4.6 shows microgrid transients due to this switching event. PC-DER units, i.e., DER$_1$ and DER$_3$, continue to deliver the same real/reactive power since they operate in power-controlled mode. However, the VC-DER unit (DER$_2$) reduces its apparent output power to 0.02 pu at a 0.98 leading power factor, as illustrated in Fig. 4.6(c), to prevent the microgrid voltage rise due to the demand reduction. Therefore, PCC$_2$ voltage is maintained at 1.00 pu as shown in Fig. 4.6(a).

This case study demonstrates the robust performance of the LCs at the event of a significant load curtailment. It also shows the mechanism of microgrid power flow regulation by assigning a DER unit as a VC-DER. This case study also shows the effectiveness of the open-loop frequency control, described in Section 4.3.2, in maintaining a fixed microgrid frequency irrespective of its dynamics and transients.
4.5 Performance Evaluation

Figure 4.5: Bode plots of the transfer function $V_{d1}(s)/V_{dref,1}(s)$ for the control parameter change: $\mathcal{K}_2 \rightarrow \alpha \mathcal{K}_2$, (a) $\alpha = 1$, (b) $\alpha = 0.5$.

4.5.1.2 Case 2: Load Sharing by the PMS

This case study demonstrates the capability of the PMS in enforcing a prescribed load sharing scheme among the DER units. Subsequent to the switching of CB$_1$ described in Case 1, the prescribed load sharing is disrupted and the output power of each DER unit does not remain proportional to its size. To restore the prescribed load sharing, the PMS updates DER$_1$ and DER$_3$ output power set points to 0.46 pu and 0.23 pu apparent power at a 0.95 lagging power factor, respectively. The output power of DER$_2$ is automatically adjusted through regulating PCC$_2$ voltage at $1\angle 0^\circ$. PC-DER units receive the new power set points at $t = 1.5$ s and provide tracking of them. Fig. 4.7 indicates the PC-DER units track their new power set points and the VC-DER unit automatically adjusts its power output to 0.32 pu apparent power at a 0.95 lagging power factor. Fig. 4.7 shows that the power output of the three DER units is proportional to their maximum generation capacity subsequent to the power set point update and PCC$_2$ voltage is regulated at 1.00 pu during this process. This case study demonstrates the effectiveness of
4.5.1.3 Case 3: DER\textsubscript{2} Overload Protection

The objective of this case study is to demonstrate the necessity and performance of the overload protection scheme of Section 3.4 when the VC-DER is subject to an overload. Subsequent to the power set point update of Case 2, CB\textsubscript{1} which was opened in Case 1, is closed at $t = 3$ s. Fig. 4.8 shows that since the low output power of the PC-DER units cannot be changed without a command from the PMS, DER\textsubscript{2} undertakes supplying the increased power demand of the microgrid. However, since it reaches its maximum generation capacity, the overload protection scheme is triggered and limits the apparent output power of DER\textsubscript{2} to its maximum capacity of 0.75 pu, Fig. 4.8(c). Since the three DER units fail to meet microgrid net load demand, and the voltage controller of the VC-DER is disabled to limit its output power, PCC\textsubscript{2} voltage drops to 0.90 pu, Fig. 4.8(a). Fig. 4.8(b) and (d) shows that the output power of the PC-DER units remain unchanged during this process. This case study demonstrates that the proposed overload protection scheme for the VC-DER unit, which inherently lacks a power limiting capability, can effectively limit the output power in case of overloading conditions.

To restore the voltage of the system and the prescribed load sharing scheme, the PMS updates power set points for the PC-DER units to increase their output power. The new set points of the PMS balance the power demand and generation of the microgrid and restore the
4.5 Performance Evaluation

Figure 4.7: Achieving a prescribed load sharing scheme by the PMS: (a) PCC$_2$ voltage, (b)–(d) real/reactive output power of the three DER units, $P$: _____ $Q$: _____.

voltage to 1.00 pu, Fig. 4.9(a). Fig. 4.9(b)–(d) shows that, subsequent to this power set point update, microgrid operating condition will be restored to that of Case 1, prior to the switching of CB$_1$.

4.5.2 Hardware-in-the-loop Simulation Test Case

This section presents the results of real-time simulation of the proposed control strategy based on the HIL concept. The objective of this study is to verify that the proposed scheme can be digitally implemented in an industrial hardware platform despite hardware implementation issues. Real-time simulation setup, equipment, and component models are the same as those of Section 2.8.2.

4.5.2.1 Case 4: Induction Motor Energization

To verify the performance of the proposed control strategy, an induction motor energization test case is carried out while the controllers are implemented in the HIL environment. This case study demonstrates the robustness of LCs when the microgrid is subject to a motor energization, which is a nonlinear, time-varying disturbance. Prior to motor start-up, the three DER units deliver 0.75 pu(0.25 pu), 0.22 pu(0.19 pu), and 0.44 pu(0.19 pu) real/reactive power. Three induction motors are simultaneously connected to the bus labeled as B$_m$ in Fig.
4.6 Conclusions

This chapter presents a generalized power management and control strategy for an autonomous, multi-DER microgrid. The envisioned strategy designates DER units as power-controlled and voltage-controlled and assumes no dominant DER unit. It provides (i) power management for the overall microgrid, (ii) an open-loop frequency control and synchronization, and (iii) a local,
4.6 Conclusions

Figure 4.9: Resolving the overload condition by assigning new power set points for PC-DER units: (a) PCC$_2$ voltage, (b)–(d) apparent/real/reactive output power of the three DER units, $P$: ____ $Q$: ____ $S$: ____

robust, decentralized controller for each DER unit. The power management system determines output power set points for the PC-DER units and a voltage set point for the VC-DER unit to enforce a prescribed load sharing scheme among the DER units and regulate the overall microgrid voltage profile within a permissible range. Frequency of the system is controlled in an open-loop manner by utilizing an internal crystal oscillator for each DER unit that also generates the angle waveform required for $dq/abc$ ($abc/dq$) transformations. Synchronization of DER units is achieved by exploiting a GPS-based common time-reference signal. The local control of each DER unit is designed based on a new multivariable decentralized robust controller approach which utilizes a linear state-space model of the microgrid. Eigenanalysis shows that the closed-loop system is stable and well-damped and the controllers are appropriately designed. The controller parameter range of variations that result in a stable closed-loop system are reported and the relationship between controller robustness and its parameters are determined.

The reported results are obtained from (i) time-domain simulation studies in the PSCAD/-EMTDC platform, and (ii) an RTDS-based hardware-in-the-loop environment. Various simulation studies validate the desired performance of the microgrid. The test cases in the HIL environment are based on digital implementation of the controller of each DER unit in an NI-cRIO industrial control platform and verify (i) ease of hardware realization of the proposed power management and DER controller, (ii) lack of sensitivity to hardware environment issues.
Table 4.3
NUMERICAL PARAMETERS OF THE INDUCTION MACHINES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Power</td>
<td>180 hp</td>
</tr>
<tr>
<td>Rated Voltage</td>
<td>0.4 kV</td>
</tr>
<tr>
<td>Rotor $H$ Constant</td>
<td>0.6 s</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td>0.0425 pu</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td>0.0870 pu</td>
</tr>
<tr>
<td>First Rotor Resistance</td>
<td>0.0300 pu</td>
</tr>
<tr>
<td>First Rotor Leakage Inductance</td>
<td>0.0658 pu</td>
</tr>
<tr>
<td>Second Rotor Resistance</td>
<td>0.0500 pu</td>
</tr>
<tr>
<td>Second Rotor Leakage Inductance</td>
<td>0.0739 pu</td>
</tr>
<tr>
<td>Magnetizing Inductance</td>
<td>2.97 pu</td>
</tr>
</tbody>
</table>

Figure 4.10: Microgrid transients due to simultaneous energization of three induction motors.
5 Smooth Microgrid Control Transition

5.1 Introduction

Chapter 2 introduced a voltage control scheme for an islanded multi-DER microgrid. The control method is also required to perform satisfactorily under the following two circumstances:

1. **Transition to an islanded mode of operation**: In the grid-connected mode of operation, microgrid voltage is predominantly dictated by the main grid and DER units control their output real/reactive power components. This is achieved based on the well-established inner current control loop [82] which is briefly described in Appendix B. Should a transition to an islanded mode of operation occur, due to either a disturbance in the main grid or other considerations, the microgrid control should switch from the power tracking mode to the voltage tracking mode to regulate microgrid voltage profile within the permissible range.

2. **Communication failure**: The microgrid voltage control scheme relies on a low-bandwidth communication link for 1) synchronizing the DER units and 2) communicating set points from the power management system (PMS) to local controllers (LCs). Loss of neither of these communication signals will cause immediate microgrid instability. However, if communication failure lasts long enough so that DER units step out of synchronization due to the oscillator inaccuracy or significant microgrid operating point changes, instability issues can arise. Hence there is a need for a backup control scheme in case of communication failure.

In the above two cases, there is a need to switch from an “active” to a “latent” controller. In Case 1, the active (latent) controller tracks DER output power (the corresponding PCC voltage). In Case 2, the active (latent) controller is the voltage controller (a droop-based backup
controller). This transition should be performed smoothly while a “bumpless” control signal is retained [80], otherwise the closed-loop system may experience transients that can result in protection system activation and/or instability. This chapter proposes smooth transition schemes for the above two scenarios and verifies the effectiveness of the proposed schemes based on simulation case studies.

5.2 Transition to Islanded Mode of Operation

The existing microgrid control methods can be categorized based on employing 1) the same control scheme for both grid-connected and islanded modes of operation [23,83], and 2) two different controllers one corresponding to each mode of operation. The latter needs various methods to minimize the transients, e.g., use of communication [43], a special PLL design [84], admittance compensation [85], and indirect current control [86]. In this section, we propose a new observer-based method. The focus is on the transition from the grid-connected to the islanded mode since it can accompany severe transients and consequently result in instability, especially when a significant amount of power is exchanged between the main grid and the microgrid immediately prior to the transition. To minimize the transients due to the switching between the two controllers corresponding to the two modes of operation, a smooth control signal should be retained prior and subsequent to the transition, i.e., \( u_1(t - SW) = u_2(t + SW) \), where \( t_{SW} \) is the instant of switching between the grid-connected mode and the islanded mode control schemes, and \( u_1 \) and \( u_2 \) are the active and the latent control outputs, respectively. Smooth transition can be achieved by appropriately initializing the voltage controller state variables. Since the controller states are available owing to its digital implementation, an observer-based method is proposed.

5.2.1 Smooth Transition Scheme Based on a State Observer

In the grid-connected mode of operation, DER units control their output real/reactive power components based on the approach described in Appendix B. In the islanded mode of operation, DER units control their PCC voltages. Each DER LC can be described by Fig. 5.1, based on the control method introduced in Chapter 2. In the schematic diagram of Fig. 5.1, the measured PCC voltage is transformed to a \( dq \) frame of reference and compared with voltage set points, then the control action is performed and necessary VSC switching signals are generated to track the set points. To provide a smooth transition from the power controller, Appendix B, to the voltage controller of Fig. 5.1, the control signal should remain continuous at the instant of switching, i.e., \( V_{ld1}(t_{SW}) = V_{ld2}(t_{SW}) \) and \( V_{lq1}(t_{SW}) = V_{lq2}(t_{SW}) \), Fig. 5.2. To achieve this, the voltage controller is augmented with a state observer (or estimator) to properly initialize its
5.2 Transition to Islanded Mode of Operation

Figure 5.1: A block diagram of the voltage control scheme.

state variables. The observer is designed based on the state-space parameters of the voltage controller, $A_c$, $B_c$, $C_c$, $D_c$, as specified in Fig. 5.1. The observer inputs are the outputs of the current controller, and the measured PCC voltage and reference set points, Fig. 5.2. The output of the observer is the state vector $x$ that determines the initial conditions of the state variables of the voltage controller. To further reduce transients, the voltage reference set points of the voltage controller are selected to be equal to the measured PCC voltage.

Each observer is designed based on the Kalman estimation process [87] and is of the form

$$\dot{x} = A_o \hat{x} + B_o \hat{u},$$

$$y_{obs} = \hat{x},$$

where $\hat{x}$ is the estimated state variable, $\hat{u} = [V_{d1}, V_{q1}, V_{d,ref}, V_{q,ref}, V_{d1}, V_{q1}]^T$, $y_{obs}$ is the output of the observer which is equal to the estimated states, and $F$ and $G$ are the outputs of the Kalman estimation process.

5.2.2 Performance Evaluation

To verify performance of the proposed smooth transition scheme from the grid-connected mode controllers to islanded mode controllers, two simulation case studies in PSCAD/EMTDC platform are conducted. Component models and parameters are the same as those of Chapter 2. Each observer is designed using the kalman function of Matlab Control Toolbox [87], discritized with a sampling rate of 6 kHz, coded in FORTRAN, and embedded in the FORTRAN code of voltage controllers. The observer design process is performed for each LC and the following $F_i$ and $G_i$, corresponding to LC$_i$, $i = 1, 2, 3$, are obtained:

$$A_o1 = \begin{bmatrix} 0.73 & -0.40 & -0.66 \\ 0.01 & 0.90 & 0.05 \\ 0.02 & -0.05 & 0.99 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -20.16 & 127.11 & 0.20 & 0.33 & 0.38 & -3.37 \\ -7.81 & -2.83 & -0.95 & -0.02 & -0.52 & -0.05 \\ -3.53 & -10.45 & 0.02 & -0.99 & -0.14 & -0.14 \end{bmatrix}. $$
5.2 Transition to Islanded Mode of Operation

\[ A_o^2 = \begin{bmatrix} 0.74 & -0.46 & -0.65 \\ 0.01 & 0.95 & 0.00 \\ 0.01 & 0.09 & 0.91 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -25.31 & 119.71 & 0.23 & 0.32 & 0.11 & -3.48 \\ -2.59 & -6.47 & -0.97 & -0.01 & -0.18 & -0.08 \\ 9.20 & -7.73 & -0.04 & -0.95 & 0.61 & -0.08 \end{bmatrix}, \]

\[ A_o^3 = \begin{bmatrix} 0.74 & -0.46 & -0.65 \\ 0.01 & 0.95 & 0.00 \\ 0.01 & 0.09 & 0.91 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -25.31 & 119.71 & 0.23 & 0.32 & 0.11 & -3.48 \\ -2.59 & -6.47 & -0.97 & -0.00 & -0.18 & -0.08 \\ 9.20 & -7.73 & -0.04 & -0.95 & 0.61 & -0.08 \end{bmatrix}. \]

5.2.2.1 Planned Islanding

Initially, the microgrid is importing 0.28/0.39 pu real/reactive power from the main grid which is a significant amount of power mismatch and can cause transients and potentially instability issues, if necessary measures are not taken before islanding. The three DER units deliver 0.56/0.00, 0.69/0.13, and 0.38/0.00 pu real/reactive power, respectively, and Line\(_1\) (Line\(_2\)) transfers 0.42(0.00) pu real power. At \( t = 1 \) s, CB\(_g\) is opened and the microgrid is expected to continue its operation in the islanded mode. After the switching event, the three DER units deliver 0.68/0.09, 0.79/0.41, and 0.44/0.63 pu real/reactive power, respectively, and Line\(_1\) (Line\(_2\)) transfers \(-0.15\)(\(-0.10\)) pu real power. The resultant microgrid transients for the case that the

![Diagram of the proposed observer-based smooth transition scheme.](image-url)
5.2 Transition to Islanded Mode of Operation

smooth transition scheme is not in service are illustrated in Fig. 5.3(a–c). Fig. 5.3(a) and (c) shows that three-phase voltages of PCC\(_1\) are distorted and the voltage magnitude transiently drops to 0.2 pu. Fig. 5.3(b) indicates that real/reactive output power components of DER\(_1\) exhibit excessive transients that can damage either the prime-mover or the power electronic components of the DER unit. However, since the voltage controllers are highly robust, they manage to bring the microgrid to a steady-state normal operating point.

The same study is repeated while the smooth transition scheme is in service and the results are shown in Fig. 5.3(d–f). Fig. 5.3(d) and (f) show that PCC\(_1\) voltage is well regulated and experiences insignificant transients. DER\(_1\) output power components also exhibit less severe transient responses while the significant power exchange between the microgrid and the main grid is abruptly interrupted due to the islanding event. Fig. 5.3 demonstrates the effectiveness of the proposed smooth transition scheme.

5.2.2.2 Unintentional Islanding

The initial conditions of this case study are the same as those of the previous one. However, in this case the islanding is due to a single-phase to ground fault which occurs at \(t = 0.95\) s in the main grid. The fault location is at a node which divides the impedance \(R_g + jX_g\), Fig. 2.1(a), into two equal parts. 50 ms after the fault occurrence, CB\(_g\) is opened to isolate the microgrid from the faulty main grid. This switching event is communicated to LCs of DER units to switch
their modes of operation from the power tracking mode to the voltage tracking mode while the smooth transition scheme is in service. Fig. 5.4 shows microgrid transients due to this sequence of events. Fig. 5.4 indicates that although the microgrid is isolated under a severe disturbance, 1) a smooth transition is achieved and 2) the latent voltage controllers manage to restabilize the microgrid and reach a new feasible steady-state operating condition. If the smooth transition scheme were not in service, the excessive postfault transients exacerbated with control transfer transients could trigger microgrid protection system and trip out DER units.

5.3 Control Transfer due to Communication Failure

Since the operation of voltage controllers of Chapter 2 depends on the availability of two low-bandwidth communication links to 1) synchronize the DER units and 2) communicate set points to local controllers, there is a need to provide a backup control scheme in case of communication failure. Switching to the backup scheme is initiated depending on the type of communication failure as described below:

- In case of communication failure of the DER synchronizing signals, which is considered unlikely due to the reliability of the GPS system, the impacted local controller notifies the PMS, Fig. 2.1(b), which in turn, notifies all local controllers to switch to the backup control scheme.

- In case of communication failure associated with set point transmitting signals, the impacted local controller switches to the backup scheme immediately and the PMS, that has also detected the loss of communication, notifies the other two LCs to switch to the backup control scheme.
A droop-based control (Appendix C) is adapted to operate as the backup control. This section also proposes a smooth control transition scheme for transition between the voltage controllers of Chapter 2 and the droop-based backup control scheme of Appendix C. The reasons for introducing this smooth transition scheme, rather than using that of Section 5.2.1, are the following:

- If states of the latent controller, i.e., the droop-based backup control, are not accessible or modifiable, e.g., when the latent controller is analog, the observer-based smooth transition scheme will not be applicable.

- This new scheme does not require an observer and thus simplifies its implementation process.

This smooth control transition scheme uses an auxiliary controller and is described in the next section.

### 5.3.1 Smooth Transition Based on an Auxiliary Controller

Fig. 5.5 illustrates a schematic diagram of the smooth transition scheme based on an auxiliary controller. Initially, the system is in closed-loop condition while the active controller $C_A$ is controlling the plant $G$ and switch $S$ is closed. In this case, the latent controller $C_L$ is considered as a dynamical system whose output $u_L$ is forced to track the output of the active controller, $u_A$. The auxiliary tracking controller $T_L$ is used to use the output of the active controller as its set point and make the latent controller generate the same output as the active controller. While $u_L$ is tracking $u_A$, smooth transition to the latent controller can be achieved by opening switch $S$ and eliminating the effect of the auxiliary controller $T_L$. 

**Figure 5.5:** Smooth control transfer scheme based on an auxiliary tracking controller.
5.3 Control Transfer due to Communication Failure

Fig. 5.6 shows the modified droop-based backup control of Fig. C.2 which enables a smooth control transition. The droop-based control of each DER unit includes two controllers: 1) the controller of Fig. 5.6(a) which specifies the angle of VSC terminal voltage and 2) the controller of Fig. 5.6(b) which specifies the amplitude of VSC terminal voltage. Thus, while the voltage controllers are active, the angle and the amplitude of VSC terminal voltage, $|V_{t,A}|$ and $\angle V_{t,A}$, are measured and provided to the auxiliary PI controllers as highlighted by the bold lines in Fig. 5.6. These auxiliary controllers force the backup droop-based controller to track the VSC terminal voltage prior to control transfer instant. At the time of switching, $t_{SW}$, the Sample/Hold block is activated so that the outputs of the auxiliary PI controllers are frozen and the backup droop-based controllers undertake control of the corresponding DER unit thereafter.

5.3.2 Performance Evaluation

To verify performance of the proposed smooth transition scheme based on the auxiliary controller, two simulation case studies in PSCAD/EMTDC platform are conducted. Component models and parameters are the same as those of Chapter 2. The droop-based backup controllers are augmented with the auxiliary tracking controllers and included in the simulation model. Parameters of the droop-based backup control of Fig. 5.6 are $H = 0.5 \text{s}$, $D_P = 0.05 \text{pu}$, $D_Q = 0.05 \text{pu}$, and $K_D = 30$. 

**Figure 5.6:** Droop-based backup control scheme that provides smooth transition, $L$: Latent, $A$: Active.
5.3 Control Transfer due to Communication Failure

5.3.2.1 Simultaneous Transfer to Backup Controller

In this case study, transition from the main controllers to the backup controllers of the three DER units occurs simultaneously. Initially, the microgrid is operating in an islanded mode based on the voltage control scheme of Chapter 2 and the three DER units deliver 0.84/0.25, 0.64/0.19, and 0.43/0.13 pu real/reactive power, respectively. Line1 (Line2) transfers 0.00 (0.13) pu real power, respectively. At \( t = 3 \) s, due to a communication failure, the LCs are notified to switch to the backup scheme. Microgrid transients due to this switching event are illustrated in Figs. 5.7(a)–(d)/5.7(e)–(h) when the smooth transition scheme is inactive/in service. In both cases, subsequent to the control transfer, the same steady-state operating point is achieved. However, Fig. 5.7 clearly reveals that the microgrid transient response is substantially improved due to the proposed smooth control transition scheme.

5.3.2.2 Nonsimultaneous Transfer to Backup Controllers

Since the transfer to the backup controller occurs due to a communication failure, it may well be the case that the three LCs are transferred to the backup scheme at different time instants.
This case study illustrates the response of the closed-loop microgrid when the transfers are not simultaneous. The pretransition operating point is the same as that of the previous case study. At $t = 3$ s, a loss of communication is detected by LC$_2$ and the backup controller is activated immediately. LC$_1$ and LC$_3$ transfer to the backup scheme with 1 s and 0.5 s delays, respectively. Fig. 5.8 shows microgrid transients due to this sequence of controller transfers. Fig. 5.8 signifies that although the microgrid exhibits insignificant oscillatory dynamics due to the transfer delays, it maintains stability and continues its normal operation after all LCs switch to the backup scheme.

5.4 Conclusions

This chapter presents two microgrid smooth control transition schemes for the following two scenarios:

1. In case of microgrid islanding, there is a need to transfer from the power-tracking to the voltage-tracking controllers.
2. In case of communication failure, there is a need to transfer from the voltage controllers to the backup droop-based controllers.

Controller transfer should be performed smoothly to avoid severe transients due to the switching event. To achieve this, two different smooth control transition schemes based on 1) an observer, and 2) an auxiliary tracking controller are proposed for the above two scenarios, respectively. The observer-based method initializes states of the voltage controller properly such that its outputs are identical to those of the power controller at the instant of switching and thereby a smooth transition is achieved. The second scheme employs an auxiliary tracking controller that makes the backup droop-based controller track the output of the voltage controller so that at the switching time instant a bumpless control output is retained.

The performance of the proposed smooth transition schemes are verified based on time-domain simulations. Results show that employing the proposed smooth transition schemes substantially improves the transient response of the microgrid. In case of the observer-based scheme, it was shown that subsequent to islanding, the microgrid continues its normal operation smoothly, even when a significant power mismatch between the microgrid and the main grid exists, and also when the transition occurs due to a fault in the main grid. This is mainly due to the robustness of the voltage controllers and effectiveness of the smooth transition scheme. In case of smooth transition to droop-based backup control, it was shown that the smooth transition scheme responded as expected, even when the transfer instants of different DER units are not simultaneous.
6 Conclusions

6.1 Summary & Conclusions

This dissertation proposes a power management and a control strategy for an islanded microgrid that includes multiple electronically-interfaced DER units. The envisioned strategy consists of

1. a microgrid power management system that, based on a classical power flow analysis, determines the terminal voltage set points for DER units;
2. an open-loop frequency control and synchronization scheme that controls the frequency of the system in an open-loop manner, by utilizing an internal crystal oscillator for each DER unit, and synchronizes the DER units by exploiting a GPS-based time-reference signal; and
3. a local, robust, decentralized controller for each DER unit which is designed taking a novel multivariable robust decentralized servomechanism approach, based on a linear state-space model of the microgrid.

The controller design procedure is outlined and various attributes of the controller, i.e., the existence conditions, gain-margins, robustness, and tolerance to input delays are analytically discussed. This control strategy is then generalized by considering both voltage-controlled and power-controlled DER units and applied to a more elaborate microgrid system. Eigenvalue sensitivity and robustness analyses are performed to study dynamic behavior of the closed-loop system.

Since the proposed voltage control method lacks an inner current control loop, it subjects the corresponding DER unit to overcurrent/overload conditions due to system severe disturbances, e.g., faults and switching events. Hence, this dissertation proposes control-imbedded overcurrent and overload protection schemes to prevent DER unit from trip-out or damage in case of faults and overload conditions. The proposed protection schemes detect the abnormal
condition, disable the main voltage controllers, and modify the terminal voltage of the interface VSC to limit the output current and/or power of the DER unit within the permissible range.

This dissertation also devises smooth control transition schemes for 1) microgrid islanding, and 2) communication failure events. These schemes operate based on 1) an observer and 2) an auxiliary tracking controller, and are integrated with the microgrid active controllers to provide a smooth control transition when switching to a latent controller is necessary.

The following qualitative and quantitative conclusions can be drawn from the dissertation.

6.1.1 Qualitative Conclusions

- A robust, decentralized control strategy is a viable solution for control of an islanded microgrid since 1) a microgrid exhibits a high level of uncertainty and a robust controller performs its satisfactory operation regardless of the uncertainties, and 2) time-varying signals cannot be transmitted to a central control system due to communication limitations and a decentralized controller obviates such a need and also facilitates unplanned connection/disconnection of DER units.

- Eigenvalues and participation factor analysis indicates that the dominant poles of the closed-loop system are predominantly influenced by controllers’ states and signifies the appropriate design of the controllers thereby (Sections 2.6.1 and 4.4.1).

- Eigenvalue sensitivity analysis of the microgrid indicates that the closed-loop system remains stable for practical perturbations of load parameters. However, reducing the resistance and increasing the inductance/capacitance of loads result in a more oscillatory system. Likewise, a larger (smaller) filter (distribution line) impedance renders the system less damped (Section 2.6.2).

- Robustness analysis verifies the results obtained from eigenvalue sensitivity analysis and also concludes that a higher control gain results in a faster control response while reducing its robustness (Section 2.7.2 and 4.4.2).

- Offline time-domain simulations in PSCAD/EMTDC platform demonstrate the robust performance of the controllers subject to various system disturbances, e.g., load change, set point change, microgrid configuration variation, abrupt outage of DER units, and even induction motor energization which exhibits a nonlinear and time-varying behavior in the start-up phase. The simulation results also suggest that the proposed PMS can successfully control the power flow of the microgrid and the proposed open-loop frequency control and synchronization scheme maintains microgrid frequency irrespective of its transients.

- Real-time HIL simulation studies, using an RTDS system and NI-cRIO industrial controllers, validate offline time-domain simulation results, signify ease of hardware im-
6.1 Summary & Conclusions

- The proposed strategy can be generalized to cover both voltage-controlled and power-controlled DER units to achieve an optimal operating condition.
- The overcurrent/overload protection schemes can effectively limit the DER unit output current/power in case of faults and overloading conditions. Furthermore, an anti-windup scheme can be used to minimize the controller restoration transients, subsequent to the operation of the overcurrent/overload protection.
- The fault clearance determination scheme performs successfully for all types of faults, including those that generate double-frequency oscillations in $d$ and $q$ components of the PCC voltage.
- The observer-based smooth control transition scheme can effectively minimize the transients caused by control switching due to microgrid islanding. The smooth control transition scheme, based on an auxiliary tracking controller, can effectively minimize microgrid transients caused by switching from the main voltage controllers to the droop-based backup controllers in case of communication failure. It was also observed that the stability of the microgrid is preserved even if the backup controllers of DER units are not activated simultaneously (Section 5.3.2.2).

6.1.2 Quantitative Conclusions

- Analysis of the closed-loop linearized microgrid dynamics reveals that the system has an input/output gain margin of 1.28/1.38 which is quite satisfactory, and it is insensitive to input delays of at least $1.58 \times 10^{-4}$ s.
- Robustness index of the closed-loop system ($7.752 \times 10^{-2}$) is greater than that of the open-loop system ($1.495 \times 10^{-3}$), which signifies there is no deterioration in the robustness of the system after augmenting the controllers.
- Eigenvalue sensitivity analysis determines the range of control parameter variations that result in a stable closed-loop system (Sections 2.7.2 and 4.4.2).
- Offline time-domain simulation test cases conclude that controller settling time is less than 50 ms and PCC voltages remain within the permissible range as defined by standards and grid codes, in all case studies.
- The overcurrent protection scheme can limit the DER unit output current to about 1.2 pu in the worst-case fault scenarios. In addition, the overload protection scheme can limit the DER unit output power to its rated MVA under overloading conditions.
- The fault detection algorithm can rapidly detect fault conditions, e.g., within 200 µs
(Section 3.5.2.1) and has the potential to be implemented in industrial protective relays where speed is a major concern.

6.2 Contributions

The salient contributions of this thesis are as follows.

- It proposes i) a power management system to control the load sharing among DER units by defining appropriate set points, ii) robust, decentralized DER local controllers to track the specified set points and reject disturbances, and iii) an open-loop frequency control and DER synchronization technique based on a time-reference signal received from a GPS.

- An overcurrent protection scheme, including fault detection, current limitation, fault clearance determination, and controller restoration processes, is suggested to prevent trip-out or damage to DER units in case of faults. Moreover, an overload protection scheme is proposed that limits the output power of the microgrid constituent DER units within the permissible range.

The dissertation also

- introduces the notion of real stability radius, for the first time in the area of power systems, to quantify control scheme robustness,

- integrates an anti-windup scheme with the overcurrent/overload protection algorithms, which freezes/releases controller states when the main controller is disabled/in service, to minimize microgrid transients subsequent to restoration of the main controllers,

- presents an observer-based smooth control transition scheme to minimize transients caused by microgrid islanding, which is only applicable to controllers with modifiable state variables, i.e., digital controllers,

- presents a smooth control transition scheme, based on an auxiliary tracking controller to minimize transients caused by switching from the main to the droop-based backup controllers. This scheme is applicable to controllers whose state variables are not accessible/modifiable, i.e., analog controllers.

6.3 Future Research Directions

Further research in continuation of this work includes the following:

- The power management system can be elaborated to include grid resynchronization, state estimation, optimal power flow, unit commitment, contingency analysis, and data mining functions.
6.3 Future Research Directions

- Source-side dynamics, i.e., dynamics of the DC-side of the interface VSC, have a secondary effect on the AC-side and are not included in the reported studies. Future research should therefore include source-side dynamics as well. Since DC-side modeling involves nonlinearity, control development may necessitate using tools from nonlinear control theory.

- The control system proposed in this dissertation does not actively compensate PCC voltage imbalance and distortions caused by unbalanced and nonlinear loads. To ensure a balanced PCC voltage, it is suggested that robust decentralized controllers be developed in the $abc$ frame of reference to track sinusoidal reference signals and, as a result, eliminate the imbalance. To ensure an acceptable total harmonic distortion of the PCC voltage, controllers should include imaginary poles corresponding to significant harmonic frequencies.

- It is suggested that the control strategy be modified for microgrid applications where the DER units are nondispachable, e.g., wind and photovoltaic units. Integration of an energy storage system, e.g., a battery bank, would be an option in this case to absorb DER unit intermittency.

- It is also suggested that a microgrid protection scheme be devised and coordinated with the proposed control and overcurrent/overload protection algorithms. In this case, some level of interaction between the control and protection systems is anticipated that may degrade microgrid performance and should be considered in the coordination procedure.
A Mathematical Model
Details

A.1 Equations in abc Frame

Equations describing the dynamics of Subsystem 1 of Fig. 2.4 are

\[
\begin{align*}
    i_{1,abc} &= i_{11,abc} + C_{1} \frac{dv_{1,abc}}{dt} + i_{L1,abc} + \frac{v_{1,abc}}{R_1}, \\
    v_{1,abc} &= L_{f1} \frac{di_{1,abc}}{dt} + R_{f1} i_{1,abc} + v_{1,abc}, \\
    v_{1,abc} &= L_{1} \frac{di_{1,abc}}{dt} + R_{1} i_{1,abc}, \\
    v_{1,abc} &= L_{t1} \frac{di_{1,abc}}{dt} + R_{t1} i_{1,abc} + v_{2,abc},
\end{align*}
\]

(A.1)

and those of Subsystem 2 and Subsystem 3 are

\[
\begin{align*}
    i_{2,abc} &= i_{2,abc} + C_{2} \frac{dv_{2,abc}}{dt} + i_{L2,abc} + \frac{v_{2,abc}}{R_2}, \\
    v_{2,abc} &= L_{f2} \frac{di_{2,abc}}{dt} + R_{f2} i_{2,abc} + v_{2,abc}, \\
    v_{2,abc} &= L_{2} \frac{di_{2,abc}}{dt} + R_{2} i_{2,abc} + v_{3,abc}, \\
    v_{2,abc} &= L_{t2} \frac{di_{2,abc}}{dt} + R_{t2} i_{2,abc} + v_{3,abc}.
\end{align*}
\]

(A.2)

and

\[
\begin{align*}
    i_{3,abc} &= i_{3,abc} + C_{3} \frac{dv_{3,abc}}{dt} + i_{L3,abc} + \frac{v_{3,abc}}{R_3}, \\
    v_{3,abc} &= L_{f3} \frac{di_{3,abc}}{dt} + R_{f3} i_{3,abc} + v_{3,abc}, \\
    v_{3,abc} &= L_{3} \frac{di_{3,abc}}{dt} + R_{3} i_{3,abc},
\end{align*}
\]

(A.3)

A.2 Equations in dq Frame

The above equations are then transformed to a dq frame of reference, yielding the following:

\[
\begin{align*}
    \frac{dV_{1,q}}{dt} &= \frac{1}{C_{1}} I_{1,q} - \frac{1}{L_{r1}} V_{1,q} - \frac{1}{L_{r1}} F_{1,q} - \frac{1}{R_{r1}} V_{1,dq} - j\omega V_{1,dq}, \\
    \frac{dI_{1,q}}{dt} &= \frac{1}{L_{r1}} V_{1,q} - \frac{R_{f1}}{L_{r1}} I_{1,q} - \frac{1}{L_{r1}} V_{1,dq} - j\omega I_{1,dq} \\
    \frac{dI_{1,dq}}{dt} &= \frac{1}{L_{r1}} V_{1,q} - \frac{R_{f1}}{L_{r1}} I_{1,dq} - j\omega I_{1,dq}, \\
    \frac{dI_{1,dq}}{dt} &= \frac{1}{L_{r1}} V_{1,q} - \frac{R_{f1}}{L_{r1}} I_{1,dq} - j\omega I_{1,dq}.
\end{align*}
\]

(A.4)
Subsystems:

\[
\begin{align*}
\text{Subsystem 1:} & \quad \frac{dV_{2,d}}{dt} = \frac{1}{C_2} l_2, dq - \frac{1}{C_2} l_2, dq - \frac{1}{C_2} l_2, dq - \frac{1}{C_2} V_{2,d}, q - j\omega V_{2,d}, q, \\
& \quad \frac{dI_{2}}{dt} = \frac{1}{L_2} V_{2,d}, q - \frac{R_2}{L_2} I_{2}, dq - \frac{1}{L_2} V_{2,d}, q - j\omega I_{2}, dq, \\
& \quad \frac{dI_{2,d}}{dt} = \frac{1}{L_2} V_{2,d}, q - \frac{R_2}{L_2} I_{2}, dq - \frac{1}{L_2} V_{2,d}, q - j\omega I_{2,d}, q. 
\end{align*}
\]

\[
\begin{align*}
\text{Subsystem 2:} & \quad \frac{dV_{3,d}}{dt} = \frac{1}{C_3} l_3, dq - \frac{1}{C_3} l_3, dq - \frac{1}{C_3} l_3, dq - \frac{1}{C_3} V_{3,d}, q - j\omega V_{3,d}, q, \\
& \quad \frac{dI_{3}}{dt} = \frac{1}{L_3} V_{3,d}, q - \frac{R_3}{L_3} I_{3}, dq - \frac{1}{L_3} V_{3,d}, q - j\omega I_{3}, dq, \\
& \quad \frac{dI_{3,d}}{dt} = \frac{1}{L_3} V_{3,d}, q - \frac{R_3}{L_3} I_{3}, dq - j\omega I_{3,d}, dq. 
\end{align*}
\]

A.3 State-space Equations

The above equations are used to construct the state-space model of the overall system

\[
\begin{align*}
\dot{x} &= Ax + Bu, \\
y &= Cx, 
\end{align*}
\]

where

\[
x^T = (V_{1,d}, V_{1,q}, I_{1,d}, I_{1,q}, I_{11,d}, I_{11,q}, I_{111,d}, I_{111,q}, V_{2,d}, V_{2,q}, I_{2,d}, I_{2,q}, I_{12,d}, I_{12,q}, I_{2,q}, I_{2,q}, I_{3,d}, I_{3,q}, I_{31,d}, I_{31,q}, V_{3,d}, V_{3,q}, I_{3,q}, I_{31,q}) \],
\]

\[
u^T = (V_{1,d}, V_{1,q}, V_{2,d}, V_{2,q}, V_{3,d}, V_{3,q}),
\]

\[
y^T = (V_{1,d}, V_{1,q}, V_{2,d}, V_{2,q}, V_{3,d}, V_{3,q}),
\]

and the \(A\)-matrix of (A.7), \(A \in \mathbb{R}^{22 \times 22}\), is \(A = \text{blockdiag} \{ A_1, A_2, A_3 \} \) where

\[
A_1 = \begin{bmatrix}
-\frac{1}{R_1 C_1} & -\omega & \frac{1}{C_1} & 0 & -\frac{1}{C_1} & 0 & -\frac{1}{C_1} & 0 \\
-\omega & -\frac{1}{R_1 C_1} & 0 & \frac{1}{C_1} & 0 & -\frac{1}{C_1} & 0 & -\frac{1}{C_1} \\
-\frac{1}{L_1} & 0 & -\frac{R_1}{L_1} & \omega & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{L_1} & -\omega & -\frac{R_1}{L_1} & 0 & 0 & 0 & 0 \\
\frac{1}{L_1} & 0 & 0 & 0 & -\frac{R_1}{L_1} & -\omega & 0 & 0 \\
0 & \frac{1}{L_1} & 0 & 0 & -\omega & -\frac{R_1}{L_1} & 0 & 0 \\
\frac{1}{L_1} & 0 & 0 & 0 & 0 & -\omega & -\frac{R_1}{L_1} & -\omega \\
0 & \frac{1}{L_1} & 0 & 0 & 0 & 0 & -\omega & -\frac{R_1}{L_1} \\
\end{bmatrix},
\]
\[ A_2 = \begin{bmatrix} \frac{1}{R_2 C_2} & -\omega & \frac{1}{C_2} & 0 & \frac{-1}{C_2} & 0 & \frac{1}{C_2} & 0 \\ -\omega & \frac{-1}{R_2 C_2} & 0 & \frac{1}{C_2} & 0 & \frac{-1}{C_2} & 0 & \frac{-1}{C_2} \\ -\frac{1}{L_2} & 0 & -\frac{R_2}{L_2} & \omega & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{L_2} & -\omega & -\frac{R_2}{L_2} & 0 & 0 & 0 & 0 \\ \frac{1}{L_2} & 0 & 0 & 0 & -\frac{R_2}{L_2} & -\omega & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & 0 & -\omega & \frac{-R_3}{L_3} & 0 & 0 \\ \frac{1}{L_2} & 0 & 0 & 0 & 0 & 0 & \frac{-R_2}{L_2} & -\omega \\ 0 & \frac{1}{L_2} & 0 & 0 & 0 & 0 & -\omega & \frac{-R_2}{L_2} \end{bmatrix} , \]

and

\[ A_3 = \begin{bmatrix} \frac{-1}{R_3 C_3} & -\omega & \frac{1}{C_3} & 0 & \frac{-1}{C_3} & 0 \\ -\omega & -\frac{1}{R_3 C_3} & 0 & \frac{1}{C_3} & 0 & \frac{-1}{C_3} \\ -\frac{1}{L_3} & 0 & -\frac{R_3}{L_3} & \omega & 0 & 0 \\ 0 & -\frac{1}{L_3} & -\omega & -\frac{R_3}{L_3} & 0 & 0 \\ \frac{1}{L_3} & 0 & 0 & 0 & -\frac{R_3}{L_3} & -\omega \\ 0 & \frac{1}{L_3} & 0 & 0 & -\omega & \frac{-R_3}{L_3} \end{bmatrix} . \]

Other entries of \( A \) are zero except for the following: \( A_{7,9} = A_{8,10} = -1/L_1 \), \( A_{9,7} = A_{10,8} = 1/C_2 \), \( A_{15,17} = A_{16,18} = -1/L_2 \), and \( A_{17,15} = A_{18,16} = 1/C_3 \).

The \( B \)- and \( C \)-matrices of (A.7) are
Mathematical model of the microgrid of Fig. 4.1, Chapter 4, is the same as the above model except for the following: \( y^T = (I_{1,d}, I_{1,q}, V_{2,d}, V_{2,q}, I_{3,d}, I_{3,q}) \), \( y_i = (I_{i,d}, I_{i,q}) \), \( i = 1, 3 \), and the
output matrix $C$ should be replaced with $C'$ where

$$
C' = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$
A SINGLE-LINE block diagram of a VSC-interfaced DER unit is illustrated in Fig. B.1(a).

This appendix describes how the DER unit controls its output real/reactive power components in grid-connected mode of operation. To achieve this, the well-established inner current control loop of Fig. B.1(b) [82] is used. The set points for real/reactive power components are translated into the set points for the $d$ and $q$ components of DER unit output current by

\[
I_{d,\text{ref}} = \frac{2}{3} \left( \frac{V_d P_{\text{ref}} + V_q Q_{\text{ref}}}{V_d^2 + V_q^2} \right),
\]

\[
I_{q,\text{ref}} = \frac{2}{3} \left( \frac{V_q P_{\text{ref}} - V_d Q_{\text{ref}}}{V_d^2 + V_q^2} \right),
\]

(B.1)

where $V_d$ and $V_q$ are the $d$ and $q$ components of the PCC voltage. Current set points are then fed into the current control loop of Fig. B.1(b) which provides tracking of these set points and indirectly controls the output real/reactive power of the DER unit at the PCC.

Figure B.1: (a) Schematic diagram of a DER unit, (b) block diagram of the current control loop.
This appendix describes the droop-based control of [15] which is used as the backup scheme in Chapter 5. The configuration of the DER unit is first explained and then the frequency/real power \( f/P \) and voltage/reactive power \( V/Q \) droop-based controls are described.

### C.1 DER Unit Configuration

Fig. C.1 illustrates a DER unit, represented by a dc voltage source, a voltage-sourced converter (VSC), and a series RL filter, which is interfaced to the microgrid at the point of common coupling (PCC in Fig. C.1) through a step-up transformer. The DER unit is interfaced to the remainder of the microgrid at the PCC.

### C.2 Frequency Control

This control scheme is analogous to that of a synchronous machine. A virtual inertia constant \( H \) for the VSC unit of Fig. C.1 is assumed such that angular frequency of the voltage of the VSC terminal, \( \omega \), is governed by

\[
2H \frac{d\omega}{dt} = P_m - P_{out} \tag{C.1}
\]

**Figure C.1:** Schematic diagram a DER unit which is interfaced to the remainder of microgrid at the PCC.
C.3 Voltage Control

where $P_{out}$ is the DER unit output power at the PCC and $P_m$ is the DER unit input power. $P_m$ is expressed as a droop function of system angular frequency by

$$P_m - P_{ref} = \frac{1}{D_P} (\omega_{ref} - \omega_{PCC}), \quad (C.2)$$

where $\omega_{PCC}$ is the angular frequency of PCC voltage, $D_P$ is the $f/P$ droop coefficient, and $\omega_{ref}$ and $P_{ref}$ are reference set points for the PCC voltage angular frequency and DER output real power. Augmenting the term corresponding to the viscous damping of the rotating shaft of a synchronous machine with (C.1) results in the following

$$2H \frac{d\omega}{dt} = P_m - P_{out} - K_D(\omega - \omega_{PCC}), \quad (C.3)$$

where $K_D$ is the damping constant. The VSC terminal voltage angular frequency $\omega$ and phase-angle $\delta$ are related based on

$$\frac{d\delta}{dt} = \omega. \quad (C.4)$$

The frequency control developed above is represented by the block diagram of Fig. C.2.

C.3 Voltage Control

The voltage controller of the DER unit of Fig. C.1 is based on $V/Q$ droop characteristic given by

$$V_t = V_{t,ref} - D_Q Q, \quad (C.5)$$

where $Q$ is the measured DER unit output reactive power, $D_Q$ is the $V/Q$ droop coefficient, and $V_{t,ref}$ is the reference value of the VSC terminal voltage.
References


