RATIONAL MODELING OF ARCHING ACTION IN LATERALLY RESTRAINED BEAMS

by

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Abstract

It is well known that arching action in reinforced concrete slabs resulting from surrounding restraining elements is responsible for much greater collapse loads than those estimated considering flexural effects only. However, the subject needs to be better understood and simplified if it is to be reliably applied in broader practice. This thesis presents a rational treatment of the problem. By limiting the scope of investigation to one-way slab systems, for the first time an explicit method of calculating the load-carrying capacity of elastic-plastic slab strips with a laterally rigidly restrained boundary condition is derived. Application of the proposed model to specimens selected from four experiment programs proves its reliability in ultimate strength calculations. The proposed model is then employed in a parametric study of structural responses of deck slab strips. The parametric study shows that a longer span, lightly reinforced deck slab system is still adequate in strength if it is cast in higher strength concrete and sufficient lateral restraint is available.
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# Table of Contents

Chapter 1. Introduction, 1

1.1 Overview, 1

1.2 Arching Action in Concrete Beams and Slabs, 1
   1.2.1 An Overview of the Behavior, 1
   1.2.2 Literature Review - Field and Experimental Investigations, 4
   1.2.3 Literature Review - Existing Analytical Models, 5

1.3 Design Applications and Current CHBDC Requirements and Practice, 13

1.4 Scope and Objective of Current Research, 15

Chapter 2. The Proposed Analytical Model, 17

2.1 Overview, 17

2.2 Fundamental Assumptions and Simplifications, 18

2.3 Material Properties, 19
   2.3.1 Stress-Strain Response of Reinforcement, 19
   2.3.2 Concrete Compressive Stress-Strain Relationship, 20
   2.3.3 Concrete Confinement, 20

2.4 Equilibrium Conditions, 23

2.5 Compatibility Condition under Full Lateral Restraint, 23

2.6 The Modeling of Forces under Full Lateral Restraint, 24

2.7 Evaluation of Section Forces and the Moment-Curvature Response, 26

2.8 Predicting Response of Concrete Members under Full Lateral Restraint, 30

2.9 Numerical Integration - Sensitivity Analysis, 32

2.10 Conclusion, 34

Chapter 3. Validation and Evaluation of the Proposed Analytical Model, 35
3.1 Overview, 35

3.2 S-Series Specimens, 37
   3.2.1 General Arrangement, 37
   3.2.2 Modeling of the Specimens, 39
   3.2.3 Results and Discussion, 41

3.3 W-Series Specimens, 49
   3.3.1 General Arrangement, 49
   3.3.2 Modeling of the Specimens, 50
   3.3.3 Results and Discussion, 51

3.4 T-Series Specimens, 59
   3.4.1 General Arrangement, 60
   3.4.2 Modeling of the Specimens, 61
   3.4.3 Results and Discussion, 61

3.5 M-Series Specimens, 67
   3.5.1 General Arrangement, 67
   3.5.2 Modeling of the Specimens, 69
   3.5.3 Results and Discussion, 70

3.6 Conclusion, 72

Chapter 4. Parametric Studies on Bridge Deck Slabs, 74

4.1 Overview of the Study Parameters, 74

4.2 Deck Slab Strip Modeling, 75
   4.2.1 General Arrangement, 75
   4.2.2 Bridge Deck Slab Strip Modeling Assumptions and Simplifications, 77

4.3 Analytical Results and Discussion, 77
   4.3.1 Midspan Loading Cases, 77
   4.3.2 Two-point Loading Cases, 86
Chapter 5. Conclusions and Recommendations, 90

5.1 Conclusions, 90

5.2 Limitations, 91

5.3 Recommendations for Future Investigations, 92
   5.3.1 Experiment Proposal, 92
   5.3.2 Recommendations for Improving the Proposed Model, 92

References, 94

Appendices, 96

   A. Proposed Analytical Model - Sample Calculation, 97
   B. Plastic Collapse Mechanism Approach - Sample Calculation, 104
   C. Experiment Proposal, 106
List of Figures

Figure 1-1. Arching action, 2

Figure 1-2. Load-deflection relationship for reinforced concrete slabs with edges restrained against lateral movement, 2

Figure 1-3. Ultimate moment-axial force interaction of symmetrically reinforced concrete sections, 3

Figure 1-4. Idealized load-deflection behavior of laterally restrained slabs for two-part methods, 5

Figure 1-5. Horizontal forces acting at support and at midspan in a laterally restrained beam, 6

Figure 1-6. Ultimate strength of a beam under arching action - Christiansen's method, 7

Figure 1-7. Rankin's equivalent three-hinged arch model for rigidly restrained slab strips, 8

Figure 1-8. Rankin's model: idealized geometry of deformation of half span of laterally restrained slab strip, 8

Figure 1-9. Park's model: collapse mechanism of a strip with ends restrained against rotation and translation, 10

Figure 1-10. Park's model: internal actions at yield sections of the end portion of a strip, 10

Figure 1-11. Park's model: conditions at a positive moment yield section, 11

Figure 1-12. Comparison of theoretical versus experimental load-central deflection behavior of Roberts' specimen, Strip RB18, 13

Figure 1-13. Typical bridge deck slab reinforcement layout (CHBDC, 2006), 14

Figure 1-14. An increase in girder spacing reduces the number of girders required for a composite deck slab system, resulting in substantial material and labor cost, 16

Figure 2-1. Typical bilinear steel reinforcement stress-strain relationship, 19

Figure 2-2. Typical bilinear concrete stress-strain relationship (adapted from fib MC2010), 20

Figure 2-3. Effectively confined core for rectangular hoop reinforcement (Mander et al., 1988), 21

Figure 2-4. Confined strength determination from lateral confining stresses for rectangular sections (Mander et al., 1988), 22

Figure 2-5. Comparison between confined and unconfined concrete ultimate compressive strengths and strains, 23

Figure 2-6. Idealization of the lateral restraint as an in-plane axial force, 24

Figure 2-7. Moment distribution and the corresponding top surface and bottom surface strain distribution profiles of a laterally restrained beam under midspan loading, 25
Figure 2-8. Relative position of the midspan- and end- moments when viewed from the context of the moment-curvature curve, 25

Figure 2-9. Sectional properties of a typical reinforced concrete slab strip with $N$ applied at an unknown eccentricity $e$, 26

Figure 2-10. Steps for determining the internal stress resultants, 26

Figure 2-11. Typical moment-curvature curve (positive bending shown), 28

Figure 2-12. A family of moment-curvature response curves for a given cross-section with axial force acting at different magnitudes, 28

Figure 2-13. Arching action development as reflected in the moment-curvature response curve of the midspan section in a laterally rigidly restrained beam, 29

Figure 2-14. The iterative cycle for calculating the load capacity of a structure under full lateral restraint, 30

Figure 2-15. Discretization of strain distribution profiles for deformation or elongation calculations, 31

Figure 2-16. Sensitivity analysis revealed that the loss of precision is insignificant when reduced the number of numerical integration slices from 200 to 20, 33

Figure 3-1. Plan, elevation and typical cross-section views of S-series specimens, 37

Figure 3-2. Schematic illustration of S-series specimens test setup, 37

Figure 3-3. Correlation between experimental and analytical results of the S-series specimens, 42

Figure 3-4. Structural responses of specimens S1, S2 and S3, 43

Figure 3-5. Structural responses of specimens S4, S5 and S6, 44

Figure 3-6. Structural responses of specimens S7 and S8, 45

Figure 3-7. Typical strain distribution profiles for A) specimens that are doubly-reinforced with equal amounts in the top and bottom faces, and B) specimens that are asymmetrically reinforced, 46

Figure 3-8. General structure structural arrangement of W-series specimens, 49

Figure 3-9. Load and support system of W-series specimens: (a) Elevation view; (b) Plan view, 49

Figure 3-10. Loading arrangement and the corresponding internal moment distribution for the W-series specimens, 51

Figure 3-11. Correlation between experimental and analytical results of the W-series specimens, 52

Figure 3-12. Structural responses of specimens W3 and W4, 55

Figure 3-13. Structural response of specimens W1 and W4, 56
Figure 3-14. Structural responses of specimens W4, W5 and W6, 57
Figure 3-15. Structural responses of specimens W3 and W7, 59
Figure 3-16. General arrangement, cross-section dimension and reinforcement position of T-series specimens, 60
Figure 3-17. Correlation between experimental and analytical results of the T-series specimens, 62
Figure 3-18. Structural responses of specimens T1 to T5, 64
Figure 3-19. Structural responses of specimens T5, T6 and T9, 65
Figure 3-20. Structural responses of specimens T7 and T8, 66
Figure 3-21. General arrangement of M-series specimens, 68
Figure 3-22. Simplified loading condition and the corresponding internal moment distribution of the W-series specimens, 69
Figure 3-23. Correlation between experimental and analytical results of M-series specimens, 70
Figure 3-24. Variation of load enhancement with percentage of reinforcement, 71
Figure 4-1. Midspan-loading cases with uniform-depth cross-sections; percentage of reinforcement varied from: a) 0.22% top and bottom, b) 0.33% top and bottom, and c) 0.44% center, 76
Figure 4-2. Two-point loading cases; all beams have a 6m span length, and 300mm2 in top and bottom, 76
Figure 4-3. Effect of concrete strength in the load capacity of laterally restrained slab strips, 80
Figure 4-4. Effect of concrete strength as reflected in the load enhancement factor, 80
Figure 4-5. Effect of span length in load carrying capacity of laterally restrained slab strips, 80
Figure 4-6. Arching action development as reflected in the midspan moment-curvature and applied-load versus lateral restraining reaction graphs for the 2m span models, 82
Figure 4-7. Typical midspan-loading analytical results; note the symmetry in terms of magnitudes of midspan- and end-moments, 83
Figure 4-8. Typical strain profiles of laterally rigidly slab strip with loading at midspan, 84
Figure 4-9. Ultimate loads of two-point loading models in different cross-sectional properties, 87
Figure 4-10. Strain profiles of uniform depth versus variable depth laterally rigidly slab strips with two-point loading, 87
Figure C-1. Reinforcement layout and cross-sectional design of Specimen #1, 108
Figure C-2. Load and reaction forces acting on the specimen and the resulting internal moment distribution, 110
Figure C-3. A schematic drawing of test rig for the proposed experiment, 111

Figure C-4. Instrumentation layout for the proposed experiment, 112

Figure C-5. Relative midspan deflections of the proposed specimens, 113
List of Tables

Table 3-1. Geometric and material properties, and reinforcement layouts of S-series specimens, 38
Table 3-2. Comparison of experimental and analytical ultimate loads of S-series specimens, 41
Table 3-3. Geometric and material properties, and reinforcement layouts of W-series specimens, 50
Table 3-4. Comparison of experimental and analytical results of W-series specimens, 52
Table 3-5. Properties of T-series specimens, 61
Table 3-6. Comparison of experimental and analytical results of T-series specimens, 62
Table 3-7. Geometric and material properties of M-series specimens, 68
Table 3-8. Comparison of experimental and analytical results of M-series specimens, 70
Table 4-1. Summary of study parameters, 75
Table 4-2. Analytical results - load carrying capacity of slab strips with 0.22% reinforcements in the top and bottom, 78
Table 4-3. Analytical results - load carrying capacity of slab strips with 0.33% reinforcements in the top and bottom, 79
Table 4-4. Analytical results - load carrying capacity of slab strips with 0.44% reinforcements in the mid-depth, 79
Table 4-5. Analytical results of the two-point loading cases, 86
List of Symbols

\(a\)  
leaver arm of the lateral restraining force couple

\(a_1\)  
depth available for arching

\(A_c\)  
area of concrete in a beam section

\(A_s\)  
area of steel in a beam section

\(A_{xc}\)  
the total area of transverse bars running in the \(x\) direction

\(A_{y}\)  
the total area of transverse bars running in the \(y\) direction

\(b\)  
width of strip

\(b_c\)  
core dimension to centerlines of perimeter hoop in \(x\) direction

\(C\)  
compressive force due to arching

\(C_{oe}/C'_c\)  
concrete compressive force

\(C_e\)  
force in compressive reinforcement

\(d\)  
depth of bottom reinforcement in Park's model

\(d\)  
effective depth of section

\(d'\)  
depth of top reinforcements in Park's model

\(d_t\)  
half the depth of arching section

\(d_c\)  
core dimension to centerlines of perimeter hoop in \(y\) directions

\(e\)  
eccentricity

\(E_c\)  
concrete elastic modulus

\(E_s\)  
Young's modulus for steel reinforcement

\(f'_{cc}\)  
ultimate compressive strength of confined concrete

\(f'_{co}\)  
unconfined ultimate concrete compressive strength

\(f'_{ls}\)  
the effective lateral confining stresses in the \(x\) direction

\(f'_{ly}\)  
the effective lateral confining stresses in the \(y\) direction

\(f_{cu}\)  
concrete cube compressive strength

\(f_s\)  
stress in the reinforcing steel

\(f_{sl}\)  
stress in longitudinal reinforcement

\(f_y\)  
yield strength of steel reinforcement

\(f'_{c}\)  
concrete cylinder compressive strength
$f_c$  
compressive stress in the concrete

$h$  
overall depth of section

$k$  
ratio of the outward movement of the support to the elastic shortening of a beam

$k_e$  
confinement effectiveness coefficient for rectangular hoops

$Kr$  
stiffness of restraint (kN/mm) for the T-series specimens

$K_s$  
stiffness of slab (kN/mm) for the T-series specimens

$L$  
span length

$l_n$  
the clear span length of S-series specimens

$L_r$  
half of the span of a rigidly restrained slab strip in Rankin’s model

$M$  
moment

$m'_{u}$  
negative resisting moment at a yield section summed about the mid-depth axis in Park's model

$M_r$  
arching moment ratio in Rankin's model

$m_u$  
positive resisting moment at a yield section summed about the mid-depth axis in Park's model

$N$  
in-plane axial force

$n_u$  
the membrane force acting at mid-depth of a yield section in Park’s model

$N_{ult}$  
the maximum possible axial force that a laterally restrained structure can sustain

$P$  
applied load

$P_a$  
component of total load carried by arching

$P_b$  
component of total load carried by bending

$P_{ult}$  
ultimate load/ ultimate load predicted by the proposed analytical model

$P_{ult, p}$  
ultimate load predicted by Johansen's yield line method

$P_{ult,e}$  
ultimate load measured from experiment

$R$  
geometric and material property parameter for arching in Rankin's model

$s$  
center to center spacing of hoop reinforcement

$t$  
the outward lateral displacement at each boundary in Park's model

$T/ T'/ T1/ T2$  
force in tensile reinforcement

$u$  
arching deflection parameter in Rankin’s model

$w$  
midspan deflection in Rankin's model
\( w'_i \) the \( i \)th clear distance between adjacent longitudinal bars

\( \beta_1 \) the ratio of the depth of the equivalent stress block to the neutral-axis depth, as defined in ACI 318-77

\( \delta \) the midspan deflection in Park's model

\( \varepsilon \) the sum of the elastic, creep, and shrinkage strain in Park's model

\( \varepsilon_b \) concrete bottom surface strain

\( \varepsilon_c \) plastic strain of concrete in Rankin's model

\( \varepsilon_{c3} \) concrete yielding compressive strain, from the CEB-FIB Draft Model Code 2010

\( \varepsilon_{cu} \) ultimate compressive strength of confined concrete

\( \varepsilon_{cu3} \) ultimate concrete compressive strain, from the CEB-FIB Draft Model Code 2010

\( \varepsilon_s \) strain in the reinforcing steel

\( \varepsilon_{xy} \) yield strain of steel reinforcement

\( \varepsilon_t \) concrete top surface strain

\( \varepsilon_u \) ultimate compressive strain of concrete in Rankin's model

\( \theta \) plastic hinge rotation

\( \rho_{cc} \) ratio of area of longitudinal reinforcement to area of core of section

\( \rho_s \) ratio of the volume of transverse confining steel to the volume of confined concrete core

\( \phi \) curvature

\( \Delta/\Delta_m \) deflection at midspan

\( \Delta_e \) elastic deformation

\( \Delta_p \) deflection due to rotation at the hinges
Chapter 1. Introduction

1.1 Overview

This thesis presents a rational approach to modeling the structural response of laterally restrained reinforced concrete slabs. The focus of interest lies in improving the design of bridge deck slabs. Specifically, the research study consists of five parts: (1) a brief review of field and laboratory investigations conducted and existing analytical models for laterally restrained beams and slabs, and the current empirical bridge deck design method prescribed in the Canadian Highway Bridge Design Code (CHBDC, 2006), (2) a detailed presentation on the proposed model and method of calculation for analyzing the structural response of laterally rigidly restrained slab strips, (3) a validation of the proposed model based on four laboratory experiments done in the past, (4) an application and parametric study of the proposed model to predict the one-way structural response of bridge deck slabs, and (5) recommendations for future studies to extend the application of the proposed model to two-way slabs and partially laterally restrained slabs.

1.2 Arching Action in Concrete Beams and Slabs

1.2.1 An Overview of the Behavior

For a reinforced concrete slab or beam member under flexure, as a result of the great difference between the tensile and compressive strengths of concrete, the member deflects and the concrete on the tensile face cracks, causing an upward migration of the neutral axis level and resulting in an outward expansion of the member at the supports. If the tendency to expand is restrained by some stiff boundary elements, compressive in-plane forces will be developed in the slab. The structural response due to these in-plane
axial forces is referred to as arching action in beam members, or compressive membrane action in two-way slabs (Figure 1-1).

![Figure 1-1. Arching action](image)

Park (1964) depicted the influence of lateral restraint in the load-central deflection relationship of a uniformly loaded reinforced concrete slab as the curve shown in Figure 1-2. As the load increases from A, the yield line pattern develops with the help of compressive membrane forces until the slab reaches its enhanced ultimate load at B. The induced compressive membrane force in the slab results in an enhancement of the flexural strength of the slab sections, as typical moment-axial force interaction diagrams show (Figure 1-3). In the slab the compressive membrane forces should be substantial but never great enough for the tension reinforcement not to yield so an increase in the ultimate moment of resistance can be achieved. Park also pointed out that the enhancement of moment due to a given compressive membrane force will be particularly high in the case of lightly reinforced slabs.

![Figure 1-2. Load-deflection relationship for reinforced concrete slabs with edges restrained against lateral movement](image)

As the deflection increases beyond B, the load carried by the slab decreases rapidly because of a reduction in the compressive membrane force, caused by concrete crushing at critical regions. As C is approached,
the membrane forces in the central region of the slab change from compression to tension. At more advanced stages of deflection beyond C, for slabs with rigid boundary conditions, the reinforcements can act as a tensile net and the slab continues to carry further load until at D the reinforcements begin to fracture. This second-stage behavior, from C to D, is called catenary action and it is beyond the scope of the current research.

Figure 1-3. Ultimate moment-axial force interaction of symmetrically reinforced concrete sections

It should be noted that the compressive membrane forces developed at small deflections are a result of cracking in the slab. In a clamped elastic slab, there is no tendency for the ends of the span to move outward as the neutral axis always remaining at its original neutral level of the cross-section.

Arching action also exists in bridge deck slabs. For instance, in slab-on-girder bridge decks, the longitudinal beams, adjacent slabs, the surrounding slab area and the diaphragms provide lateral restraints, allowing arching action to develop both longitudinally and transversely (Hon et al., 2005)

Conventionally, the determination of bending moments and shear forces and the design of reinforced concrete slabs involves plasticity analysis using Johansen’s yield line method (Park, 1999) or Hillerborg’s strip method (Hillerborg, 1996). The former approach is based on the upper bound theorem of plasticity that requires a stress state in a structure to produce sufficient yielding, so a kinematically admissible mechanism of elements satisfying equilibrium can be formed. The latter method is based on the lower bound theorem of plasticity that requires the satisfaction of equilibrium at all locations and the stress state does not exceed yielding. Both procedures calculate bending moments without the consideration of in-plane forces due to lateral restraints. Hence, they may considerably underestimate the ultimate strength of slabs with stiff boundary conditions that can induce significant compressive membrane stress.
1.2.2 Literature Review - Field and Experimental Investigations

The existence of arching action or compressive membrane action in laterally restrained, reinforced concrete members was first recognized in the early twentieth century (Westgaard and Slater, 1921). Many field investigations as well as experimental studies have been reported in this area since the 1950s.

One of the earliest documented field tests was reported by Ockleston in 1955. The author intentionally load-tested three interior panels of floor slabs to destruction in a 10-year-old, three-storey reinforced concrete building in South Africa, and found the collapse loads were more than twice the loads predicted by the yield-line method. Ockleston attributed this enhancement of strength to the presence of compressive membrane forces. It was found that the lateral restraint provided by the stiffness of the surrounding beams and panels was sufficient to induce pronounced membrane action in the loaded panels.

In 1969, Roberts tested the load-carrying capacity of twenty-seven slab strips restrained against longitudinal expansion and reported the ultimate loads to be in factors of 2.87 to 17.24 higher than those estimated based on theoretical limit-analysis considering failure by bending only. The load enhancement effect was less pronounced in those slab strips with high percentage of reinforcement (0.926%) but became quite significant in slabs strips with only 0.231% of reinforcement.

In 1978, a collapsed warehouse building in Niagara Falls, Canada revealed that part of this reinforced concrete structure with flat-slab floors supported on columns carried a total load estimated to be in excess of 48kN/m² at the time of collapse, while exposed to an explosion and a major fire. The floor system demonstrated a safety factor of 4.5 relative to the total design loads (10.8kN/m²). Investigations conducted by Vecchio and Collins (1990) surmised that this high strength reserve was developed primarily through the influence of compressive membrane action in the floor slabs. Subsequently, an experiment program was undertaken (Vecchio and Tang, 1990) that involved the testing of two slab strip specimens, loaded statically at the midspan. One of the slab strips was simply supported while the other one was laterally restrained against longitudinal expansion. The slab thickness and span dimensions represented a half-scale model of the collapsed warehouse floor. The results of the experimental program confirmed the existence of considerable reserve strength in concrete slabs due to membrane action that the load-carrying capacity of the restrained slab was increased by about 30-40% relative to the unrestrained scenario.

The research on arching action continues to the present day. Latest laboratory investigations include arching action in high-strength concrete and fiber reinforced concrete slabs (Taylor et al., 2001), and progressive collapse resistance of axially restrained frame beams (Su et al., 2009).
1.2.3 Literature Review - Existing Analytical Models

Since the 1950s there have been several attempts to include arching action in ultimate load predictions. The approaches can be generally classified into two categories (Eyre, 1985):

![Figure 1-4. Idealized load-deflection behavior of laterally restrained slabs for two-part methods; the ultimate capacity is predicted by summing the bending and arching load capacities.](image)

1. Two-part methods in which the ultimate load is considered as the interaction of bending and arching action (Figure 1-4). Load-carrying capacities in bending and arching action are calculated separately and then sum up for the ultimate load.

2. Methods based upon the plastic theory of plates using a criterion of yield described by a moment-membrane force interaction curve.

One of the earliest rational treatments of arching action in reinforced concrete beams that can be categorized as a two-part method was carried out by Christiansen in 1963. The part of loading carried by arching action is calculated based on a lateral elongation compatibility condition. With an elastic-plastic assumption made for a concrete beam that has fully developed plastic hinges at the supports and at midspan, the unit-width horizontal forces and their relative positions in those regions are simplified as shown in Figure 1-5:
Figure 1-5. Horizontal forces acting at support and at midspan in a laterally restrained beam

The distribution of compressive stress in the concrete at the hinges is assumed to be uniform with a constant average stress, $f_c$. The depth of the compression becomes $(T_1+C)/f_c$ at the supporting region and $(T_2+C)/f_c$ at the midspan region, where $T_1$ and $T_2$ are equal to the force per unit width in the reinforcements and $C$ is the additional compressive force due to arching.

The deflection at midspan, $\Delta$, is assumed to be the sum of a deflection due to elastic deformations of the member, $\Delta_e$, and a deflection due to rotation at the hinges, $\Delta_p$.

To satisfy compatibility, Christiansen imposed that the total outward movement of the support is equal to the elongation due to rotation of the plastic hinges minus the elastic and plastic deformations of the member. In other words, expansion of a beam at each of the supports is equal to the elongation due to the increasing deflection of the member, $\delta \Delta$, subtract the elastic shortening of the member due to an increasing lateral restraining force, $\delta C$, and subtract the shortening of crack widths at the support and midspan, which is also caused by $\delta C$. Mathematically, this compatibility criterion can be expressed in the following equation:

$$
k \frac{\delta CL}{2Ech} = \frac{a_1}{L} \frac{2\delta \Delta}{L} - \frac{\delta CL}{2Ech} - \frac{4\Delta_p \delta C}{L f_c}
$$

note that,

$k$ is the ratio of the outward movement of the support to the elastic shortening of a beam of L/2 and depth h, and with a modulus of elasticity of $E_c$. Under a rigidly restrained condition, $k = 0$.

$a_1$ is defined as the "depth available for arching". It is the clear difference in level between the compression zones and approximately equal to the difference in level of centers of rotation. From Figure 1-5 it can be seen that:

$$a_1 = h - \Delta - (T_1 + T_2 + 2C)/f_c.$$
Upon certain algebraic manipulations, Christiansen's method implicitly, in the form of a quartic equation, derives the maximum arching moment, $C_a$, where $a$ is the lever arm of the lateral restraining force couple, $C$. This maximum arching moment is then added on top of the maximum bending moment to get the total moment capacity at ultimate conditions. Using equilibrium equations the load carrying capacity can be then calculated. Due to the level of complexity the algebraic equations involved, solutions in graphical forms was provided as the following:

![Diagram](image-url)

Figure 1-6. Ultimate strength of a beam under arching action - Christiansen's method
Two-part methods similar to Christiansen's, based on “three-hinge arch” models and elastic-plastic concrete behaviors, are commonly used to estimate the ultimate strengths due to the combined effects of bending and membrane stresses up to present day.

Figure 1-7. Rankin's equivalent three-hinged arch model for rigidly restrained slab strips

Figure 1-8. Rankin's model: idealized geometry of deformation of half span of laterally restrained slab strip

Among those methods one of the commonly cited was developed at Queen’s University of Belfast in UK by Rankin (1982). A series of subsequent articles had been published to validate or examine the applicability of Rankin's model (Rankin and Long 1997; Taylor et al. 2001; Ruddle et al. 2003). Rankin extended the research of McDowell et al. (1956) in laterally restrained masonry walls. The similarities in the material properties of masonry and concrete provided a justifiable basis for Rankin’s extension of their theory to restrained reinforced concrete slab strips. Based on an idealized geometry of deformation for a laterally restrained slab strip (Figure 1-8) and by assuming a three-hinge arch model (Figure 1-7) with linear variation in the strain between the end support and the central contact zone, analytical forms for the stress distributions were derived in terms of two parameters, defined as:

\[ R = \frac{\varepsilon L^2}{4d^2} \]

\[ u = \frac{w}{2d} \]
By converting the parabolic stress-strain curve for concrete to an equivalent trapezoidal (elastic-plastic) stress-strain relationship, and by utilizing the conventional parabolic and rectangular stress block parameters (Honegstad et al., 1955), a stress-strain relationship for the concrete plastic strain was defined as:

\[ \varepsilon_p = 2\varepsilon_c (1 - \beta) = (-400 + 60f' / \sigma - 0.36f' / \sigma) \times 10^{-6} \]

The idealized geometry, in combination with the elastic-plastic stress-strain relationship, was then applied to derive the arching moment of resistance in two scenarios:

(i) \( R > 0.26 \),

\[ M_r = \frac{0.3615}{R} \]

(ii) \( 0 < R < 0.26 \)

\[ M_r = 4.3 - 16.1\sqrt{3.3 \times 10^{-4} + 0.1243R} \]

Finally, the maximum arching moment of resistance per unit width, \( M_a \), becomes:

\[ M_a = \frac{M_r \cdot 0.85f' d_i^2}{4} \]

and the corresponding arching deflection parameter, which can be applied to find ultimate deflection of a slab strip due to arcing, becomes:

(i) \( R > 0.26 \),

\[ u = 0.31 \]

(ii) \( 0 < R < 0.26 \)

\[ 6u - 8 + \frac{8R_r^2}{3u^2} = 0 \]

note that:

\( R \) is the geometric and material property parameter for arching
\( u \) is the arching deflection parameter
\( \varepsilon_p \) is the plastic strain of concrete
\( \varepsilon_u \) is the ultimate compressive strain of concrete
\( f'_c \) is the concrete cylinder compressive strength (0.8 x cube strength)
\( L_r \) is half of the span of a rigidly restrained slab strip
\( d_i \) is half the depth of arching section
\( w \) is the midspan deflection
\( M_r \) is the arching moment ratio

Both the methods of Christiansen and Rankin can be described as intuitive approaches where the load is assumed to be carried in part by bending and in part by arch action. As pointed out by Chattopadhyay
(1981), these methods do not reflect the real situation that there is a combined stress state of moment and axial force acting over the depth of a beam or slab. At the same time, the complete set of equations for compatibility, equilibrium and stress strain relationships are not developed for the combined set of stress resultants under bending and membrane force, and no discussion is presented about the criterion of yield. As the satisfaction of force equilibrium and compatibility of deformations in accordance with material stress-strain relationships is the basis of all structural analysis, a rational method should take all of the above into consideration. When complications arise due to the true nature of the constitutive relationships, idealizations can be made in this area to simplify the problem instead of ignoring it.

![Diagram](Figure 1-9. Park's model: collapse mechanism of a strip with ends restrained against rotation and translation)

![Diagram](Figure 1-10. Park's model: internal actions at yield sections of the end portion of a strip)

Park (1964) introduced another approach to include the effect of arching action in load carrying capacity calculation that was based on rigid-perfectly plastic analysis. As a contrast to the previous two methods, Park generated united expressions for bending and arching action from the equilibrium of forces acting on a yielding section, and then applied the load-deflection behavior of a restrained strip to virtual work formulations from plastic theory to implicitly obtained the load carrying capacity. Park's model (Figure 1-
9 and Figure 1-10) was developed to make applicable for a restrained slab strip based on the following assumptions:

- Plastic hinges at critical locations have developed and at each plastic hinge the tension steel has yielded.
- The compressed concrete has reached its strength with the stress distribution as defined by ACI 318-77 (1977) equivalent rectangular stress block (Park and Gamble, 1999).
- The tensile strength of the concrete can be neglected.
- The slab strip has a uniform cross-section with constant percentage of top and bottom reinforcements along the entire length.

Figure 1-11. Park’s model: conditions at a positive moment yield section

This model suggests that the strain and stress distributions at a positive moment yield section should be similar to the one shown in Figure 1-11 (at a negative moment yield section conditions would be similar but opposite). From static equilibrium for the stress resultants at the section, the following internal moment relationship for a strip of unit width is derived:

\[ m_u + m_u - n_u \delta = 0.85f'c(0.5-h/2) + \frac{\delta}{4}(\beta_1 - 3) \]

\[ + \frac{\beta_1 h^2}{4h}(\beta_1 - 1)(\varepsilon + \frac{2l}{l}) + \frac{\delta^2}{8h}(2 - \beta_1) \]

\[ + \frac{\beta_1 h^2}{4h}(1 - \beta_1)(\varepsilon + \frac{2l}{l}) - \frac{\beta_1 \delta^2 h^2}{16h^2}(\varepsilon + \frac{2l}{l})^2 \]

\[ - \frac{1}{3.4f_c}(T' - T - C' + C_s)^2 + (C' + C_s)(\frac{h}{2} - d' + \frac{\delta}{2}) \]

\[ (T' + T)(d - \frac{h}{2} + \frac{\delta}{2}) \]

where,

- \( n_u \) is the membrane force acting at mid-depth of a yield section
- \( m_u \) is positive resisting moment at a yield section summed about the mid-depth axis
$m'_u$ is negative resisting moment at a yield section summed about the mid-depth axis

$f'_c$ is the concrete cylinder compressive strength

$h$ is the thickness of strip

$\beta_1$ is the ratio of the depth of the equivalent stress block to the neutral-axis depth, as defined in ACI 318-77

$\delta$ is the midspan deflection

$t$ is the outward lateral displacement at each boundary

$\varepsilon$ is the sum of the elastic, creep, and shrinkage strain

$l$ is the total length of strip

$T$ and $T'$ are the steel tensile force acting on sections 1 and 2, respectively

$Cc$ and $C'c$ are the concrete compressive forces acting on sections 1 and 2, respectively

$d'$ and $d$ are the depth of the top and bottom reinforcements, respectively

Then, by equating the virtual work done by internal moment, that is $(m'_u + m_n - n_a \delta) \theta$, where $\theta$ is the plastic hinge rotation, to the external work done by loading on the strip in undergoing the virtual displacement, an equation can be obtained which relates the deflection of the strip to the load carried.

Note that solutions provided by Park's method is implicit because deflection must be increased gradually to determine the peak value of loading.

Also note that the load-deflection relationship generated assumes that the critical sections have reached their yielding strength from the onset of deflection. Because of this assumed plastic behavior, the initial part of the load-deflection curve plotted from such relationship will not be accurate. In other words, Park's method cannot be applied to predict the structural response prior to yielding.

At the same time, although Park claimed that the derived load-deflection relationship can be expected to apply accurately when sufficient deformation has occurred to allow full plasticity to develop at the critical sections, application of the model to a fixed-end strip tested by Roberts (1969) revealed that the model underestimated both the load carrying capacity and the corresponding central deflection for this particular beam strip (Figure 1-12). Park's method also described a stiffer load-deflection behavior for the prior peak load region. Despite such shortcomings, Park's model, modifications of Park model, or models based on a
yield criterion (Wood, 1961), has been widely used as a foundation by many of the other studies up to present day (Taylor et al., 2001; Muthu et al., 2006; Su et al., 2009).

![Figure 1-12. Comparison of theoretical versus experimental load-central deflection behavior of Roberts' specimen, Strip RB18](image)

1.3 Design Applications and Current CHBDC Requirements and Practice

Though researchers have recognized arching action for decades, the complexities of the analytical approaches to the problem, the difficulty in evaluating the level of surrounding rigidity, and the variable degree of accuracy as demonstrated in the previous section are not particularly helpful in conveying the general acceptance.

The design of bridge deck slabs in rectangular reinforced concrete has traditionally been based on methods developed by Westergaard (1930) or Pucher (1977) in which it is assumed that a wheel load is distributed over a prescribed effective width and the slab is designed for bending effects only. There have been scanty design and assessment codes acknowledged the benefits of lateral restraining forces in deck slabs. In Europe, the Department of Regional Development of Northern Ireland's "Design Specification for Bridge Decks" (1986), and the recent UK Highways Agency Standard BD81/02 (2002) are the only two that allowed practical applications of arching action in the design of deck slabs by requiring much less than the amount of reinforcement calculated using conventional flexural methods. In the NI code, for
instance, it is suggested the use of 0.6%, top and bottom, transverse reinforcement in the standard 160mm thick deck slab of M-beam type bridges with a main beam spacing of 2m or less. This reinforcement ratio imposes a fairly high safety margin but has halved the amount of reinforcement comparing to that predicted by the use of Pucher’s or Westgaard’s methods.

In Canada, model studies of compressive membrane action in reinforced concrete bridge decks were conducted at Queen’s University commencing in the late 1960s (Batchelor, 1987). With initial theoretical and laboratory investigations that later supplemented by field tests on a full-scale experimental bridge in Conestoga, Ontario in 1975 (Dorton et al, 1977), their research confirmed that:

- Deck slabs take load by an internal arching action rather than flexural action.
- The effect of membrane action can be predicted quite closely for a bridge deck slab system of nearly uniform thickness and bounded by supporting girders.

These findings eventually led to the establishment of the empirical design method prescribed in the first edition of the former Ontario Highway Bridge Design Code (1979). These were the first design code provisions that took full advantage of membrane action in bridge slab systems. Presently it constitutes the basis of Section 8.18: Special provisions for deck slabs in the latest Canadian Highway Bridge Design Code (2006). Typically, for cast-in-place composite bridge deck slabs, two orthogonal assemblies near the top and bottom of the slab, with a minimum of 0.3% reinforcement, is prescribed. That is tantamount to four layers of reinforcement, with each layer being at least 0.3% of the effective cross-sectional area of the concrete.

![Figure 1-13. Typical bridge deck slab reinforcement layout (CHBDC, 2006)](image)

The savings from the empirical design method are considerable, primarily from the reduction of reinforcement required in deck slabs. Bakht and Markovic (1985) have reported savings of 35-40% in slab reinforcement, depending on girder spacing. However, application of this empirical design method requires certain stringent geometric specifications on top of diaphragms and edge stiffening requirements, including:
1. Girder spacing must not exceed 4m. Note that this value is not derived theoretically but rather an empirical decision because no full-scale tests done previously exceeded this transverse span limit.

2. The slab thickness shall not be less than 175mm, and for reinforcement corrosion protection the clear distance between the top and bottom transverse reinforcement must be at least 55mm.

3. The ratio of the spacing of the girders to the thickness of the slab must be less or equal to 18.0.

Slabs that span above this imposed limit must be designed using the conventional methods, which would result in a densely reinforced solution, making it economically and technically unfavorable. The geometrical restrictions also make designers become reluctant to use transverse post-tensioning or larger girder spacing for bridge deck slabs.

1.4 Scope and Objective of Current Research

Although the current CHBDC utilizes some advantages of arching or compressive membrane action in bridge deck slabs design, the nature of the method prescribed thwarts the freedom for a structural designer to come up more efficient solutions for bridge deck slab systems. Indeed, the 0.3% isotropic reinforcement specification is 50% greater than the amount of reinforcement (0.2%) recommend by Batchelor et. al (1985) in their pioneer studies. Unless there is a more direct and rational approach to evaluate the serviceability and load carrying capacity of bridge deck slabs, it is unlikely that any revolutionary movements can be accomplished.

However, as reviewed in Section 1.2.3, most existing analytical models focus primarily on ultimate load calculations with the assumption that plastic hinges have been fully developed in the slab. In addition to disadvantages such as being mostly intuition based and requiring intensive calculations, they cannot be applied to the service limit states of bridge deck slabs when reinforcements are still far away from yielding, and only very isolated cracking has occurred in the concrete. Hence, the objective of the current research is to:

a) propose a more rational, theoretical model for the structural response of laterally restrained reinforced concrete slab strips

b) develop a method of calculation for predicting the load carrying capacity of concrete slab strips, applicable to any given combination of longitudinal reinforcement, concrete strength, span-to-depth ratio and loading arrangement
No comparable models have been reported in existing literature. To minimize the level of complexity and number of parameters to be investigated, the scope of the current research is limited to rigidly, perfectly restrained beams or one-way slab systems. The term "rigidly restrained" means that the boundary elements are infinitely stiff so under gravity loading the restrained slab will not exhibit any outward movement. In addition, the influence from creep and shrinkage will not be considered. It is optimistic that the method of calculation proposed can be extended to two-way slabs and slabs with variable level of lateral restraints once it is augmented with a reliable model to quantify the lateral restraint inherent in the structural systems.

The current research is motivated by the hope that the proposed model can lead to a more practical design approach, resulting in more efficient and economic designs of bridge deck slab systems, especially in terms of span-to-depth ratio or girder spacing. As shown below, an increase in girder spacing reduces the number of I-girders required for a slab-on-girder composite bridge. This will not only reduce material cost but can also bring substantial savings from girder erection and other labor costs.

![Figure 1-14](image)

Figure 1-14. An increase in girder spacing reduces the number of girders required for a composite deck slab system, resulting in substantial material and labor cost

Another motivation for deriving a rational arching action model is driven by the intention to further extend its application to other deck slab designs, such as box girders or double-T systems to yield in even greater beneficial consequences. Also, bridge deck slabs previously assessed as being inadequate in strength may be proved to have sufficient load carrying capacity, thus reducing the need for strengthening or replacement of these slabs.

In the next three chapters, detailed descriptions of the proposed model and method of calculation, validation procedures and results, and application of the model to strength calculations of bridge deck slab strips with different geometric and material properties will be presented.
Chapter 2. The Proposed Analytical Model

2.1 Overview

The purpose of this chapter is to present an analytical model to calculate the load capacity of laterally rigidly restrained concrete beams or slab strips. Unlike the existing models presented in the preceding chapter, the proposed model takes equilibrium conditions, compatibility requirements and material nonlinearity into consideration, with the rigid boundaries being idealized as in-plane, compressive forces exerted to the restrained member.

The method of calculation is similar to the evaluation of structural response of a concrete member subjected to combined flexural and axial loads. The sectional forces are determined based on equilibrium and the assumption that "plain section remains plain". However, there is one additional compatibility requirement that is unique to the proposed model:

*For a laterally rigidly restrained slab strip under gravity loading, there will be no lateral expansion at the supports; hence, the integrals of the top and bottom surface strains along the length of the span are equal to zero.*

In the development of a suitable method of calculation for the proposed model, an iterative, numerical integration procedure is employed to determine the unique top and bottom surface strain distribution profiles which will give the solution, in terms of a unique applied load ($P$) and idealized axial force combination ($N$) that satisfies both force equilibrium and strain compatibility. Following the iterative numerical procedure, the ultimate strength of a laterally rigidly restrained concrete member is reached when at critical point(s) the concrete compressive strain reaches its ultimate value. The corresponding $P$-$N$ force combination represents the failure state of the structure.
2.2 Fundamental Assumptions and Simplifications

The proposed model is built upon the traditional flexure theory for reinforced concrete. It is based on the three fundamental assumptions (MacGregor and Wight, 2005) for calculating the moment resistance and load-carrying capacity of a beam:

1. Plane sections perpendicular to the axis of bending remain plane after bending.
2. The strain in the reinforcement is equal to the strain in the concrete at the same level.
3. The stresses in the concrete and reinforcement can be computed from the strains by using stress-strain relationships for concrete and reinforcement.

The first of these is the conventional "plain sections remain plane" assumption that forms the basis of engineering beam theory. This geometric assumption enables the concrete strain in a beam to be defined by just two variables. In the method of calculation developed for the proposed model (to be discussed later in the chapter), the two variables chosen to define the linear strain distribution are the strain at the top-surface, $\varepsilon_t$, and curvature, $\phi$. This assumption also implies that cracks due to loading will be evenly distributed (i.e. a smeared crack model).

The second assumption implies a perfect bond between the concrete and reinforcement, and they act together to carry load.

The third assumption makes it possible that if the strain across the section is known, then stress-strain relationships can be used to find the distribution of stress across the section. Subsequently, internal moment and axial force acting on the section can be determined from equilibrium equations.

These three assumptions stated are sufficient to allow calculation of the strength and behavior of reinforced concrete elements. For computational purposes, however, three additional assumptions are introduced to simplify the problem, with relatively little loss of accuracy:

1. The tensile strength of concrete is neglected.
2. Concrete is assumed to crush when its compressive strain reaches a limiting value defined in the chosen constitutive relationship.
3. The compressive stress-strain relationship for concrete may be based on any reasonable stress-strain curves or may be assumed to be trapezoidal, parabolic, or any other shapes provided that they adequately predict the actual responses.
The tensile strength of concrete is roughly one-tenth of the compressive strength, and the tensile force in the concrete below the neutral axis is small compared with the tensile force in the steel reinforcement. Hence, it can be neglected to simplify flexural sectional forces calculation.

The maximum compressive strain value is limited by a numbers of factors, including the concrete compressive strength and the presence of confinement. When at any point along a concrete member the predefined maximum allowable compressive strain is reached, the member is considered reaching failure. (Note: another possible failure state is the fracture of reinforcement, but it is unlikely in laterally rigidly restrained concrete beams, as demonstrated in laboratory and field tests done in the past (Batchelor, 1987)).

2.3 Material Properties

Material stress-strain relationship is the key to link deformations in a structure to the resultant internal forces. Since the proposed model involves the assessment of compatibility of reinforced concrete structures, stress-strain relationships must be properly predefined and then applied to the sectional force calculation part of the analytical procedure.

2.3.1 Stress-Strain Response of Reinforcement

The scope of the proposed model is confined to concrete beams or slab strips with non-prestressed, ductile steel reinforcement only. The relationship between the stress in the reinforcing steel, \( f_s \), and the strain caused by this stress, \( \varepsilon_s \), is assumed to be bilinear, as shown in Figure 2-1. The same relationship is assumed to be valid for both reinforcements in tension and compression. The Young's modulus for steel reinforcement, \( E_s \), is taken as 200,000MPa unless specified otherwise.

![Figure 2-1. Typical bilinear steel reinforcement stress-strain relationship](image-url)
2.3.2 Concrete Compressive Stress-Strain Relationship

The bilinear stress-strain relationship from the CEB-FIB Draft Model Code (2010), as shown in Figure 2-2, was used for developing the method of calculation, and applied in the parametric studies and model validation.

\[ f_c \]

\[ f'_c \]

\[ E_c \]

\[ E_{c3} \]

\[ E_{cu3} \]

Figure 2-2. Typical bilinear concrete stress-strain relationship (adapted from fib MC2010)

In order to predict the relationship between stress and strain in concrete, this constitutive model requires only three constants, namely the compressive strength, \( f'_c \), the yielding compressive strain \( \varepsilon_{c3} \) and the ultimate compressive strain \( \varepsilon_{cu3} \). It helps to reduce the amount of computations to a minimum while relatively accurate results can still be obtained.

2.3.3 Concrete Confinement

When selecting appropriate specimens tested previously from literature for validating the proposed model, slab strips with a considerable amount of transverse reinforcements were frequently encountered (see Section 3.2.1). Tests have shown that the confinement of concrete by suitable arrangements of transverse reinforcement can result in a significant increase in both the flexural strength and ductility of compressed concrete. To determine the enhancement in compressive strength of confined concrete, \( f'_{cws} \) and corresponding ultimate compressive strain, \( \varepsilon_{cuw} \), the theoretical stress-strain model for confined concrete developed by Mander, Priestley and Park (1988) was employed when applicable.
For concrete confined with rectangular hoop reinforcements, $f'_{cc}$ is determined from the following procedure:

1. Determine the geometric variables $b_c, d_c, w', s$ and $s'$ as labeled in Figure 2-3:

2. Compute $k_e$, the confinement effectiveness coefficient for rectangular hoops, and subsequently, the effective lateral confining stresses in the x- and y-directions, $f'_{lx}$ and $f'_{ly}$:

\[
k_e = \left(1 - \sum_{i=1}^{n} \left(\frac{w_i}{6b, d_c}\right) \left(1 - \frac{s'_i}{2b_c}\right) \left(1 - \frac{s'_i}{2d_c}\right)\right) \left(1 - \rho_{ct}\right)
\]

\[
f'_{lx} = k_e \frac{A_{x}}{s d_c} f_{th}
\]

\[
f'_{ly} = k_e \frac{A_{y}}{s b_c} f_{th}
\]

where

$\rho_{ct} =$ ratio of area of longitudinal reinforcement to area of core of section

$A_{x}$ and $A_{y} =$ the total area of transverse reinforcements running in the x- and y-directions, respectively.
3. Obtain the confined strength ratio from Figure 2-4 and calculate $f'_cc$. Note $f'co$ is the unconfined ultimate compressive strength of concrete.

![Figure 2-4. Confined strength determination from lateral confining stresses for rectangular sections (Mander et al., 1988)](image)

The ultimate compressive strain, $\varepsilon_{cu}$, is defined as the strain causing hoop reinforcement fracture and is determined from a strain energy equation that the ultimate strain energy capacity of the confining reinforcement per unit volume of concrete core ($U_{sh}$) is equal to the difference in area between the confined ($U_{cc}$) and the unconfined ($U_{co}$) concrete stress-strain curves, plus additional energy required to maintain concrete compressive strain corresponding to hoop fracture:

$$U_{sh} = U_{cc} + U_{co} - U_{co}$$

which expands to,

$$\rho A_{e} \int_{0}^{\varepsilon_{cu}} f d\varepsilon_{e} = A_{e} \cdot \int_{0}^{\varepsilon_{cu}} f d\varepsilon_{e} + \rho A_{e} \int_{0}^{\varepsilon_{cu}} f_{d} d\varepsilon_{e} - A_{e} \cdot \int_{0}^{\varepsilon_{cu}} f d\varepsilon_{e}$$

By assuming $\int_{0}^{\varepsilon_{cu}} f_{d} d\varepsilon_{e} = 110MJ/m^3$ and $\int_{0}^{\varepsilon_{cu}} f_{d} d\varepsilon_{e} = 0.017\sqrt{f'_{co}}MJ/m^3$, the final formula for calculating $\varepsilon_{cu}$ becomes:

$$110\rho_{s} = \int_{0}^{\varepsilon_{cu}} f_{d} d\varepsilon_{e} + \int_{0}^{\varepsilon_{cu}} f_{d} d\varepsilon_{e} - 0.017\sqrt{f'_{co}}MJ/m^3$$

where

- $\rho_{s}$ = ratio of the volume of transverse confining steel to the volume of confined concrete core
- $f_{d}$ = stress in longitudinal reinforcement
The following graph, taken from the calculation of S-series specimen S-1 (to be discussed in Section 3.2), illustrates how the concrete ultimate compressive strength and strain will be enhanced by the transverse reinforcement confinement effects:

![Graph showing comparison between confined and unconfined concrete ultimate compressive strengths and strains](image)

Figure 2-5. Comparison between confined and unconfined concrete ultimate compressive strengths and strains

### 2.4 Equilibrium Conditions

With the "plane-section remains plane" assumption, for a given strain distribution across the section, a stress-strain relationship can be used to find the distribution of stresses across the section. If the stresses are known, then the moment and section forces acting at the section can be determined. However, in order to attain equilibrium, at any section the internal stresses, when integrated over the entire section, must add up to the required sectional forces $M$ and $N$:

\[
\int_A f_c dA_c + \int_A f_t dA_t = N
\]

and

\[
\int_A f_c y dA_c + \int_A f_t y dA_t = -M
\]

In these equations, tensile stresses are taken as positive and compressive stresses are taken as negative. The axial load, $N$, is positive if tensile and negative if compressive. The moment, $M$, is positive if it causes tensile stresses on the bottom face.

### 2.5 Compatibility Condition under Full Lateral Restraint

In this analytical model, it is assumed that a concrete member subject to full lateral restraint is surrounded by perfectly rigid boundaries at both ends, so there is no tendency for the member to elongate outward.
upon gravity loading and cracking of the member. In other words, the net elongation of the member always maintains at zero. Also, cracks developed in the member are assumed to be evenly distributed as delineated in a smeared crack model. Mathematically, this assumption is equivalent to a compatibility condition that when integrating the top surface strain ($\varepsilon_t$) and the bottom surface strain ($\varepsilon_b$) along the length of the span, the sums must equal to zero:

\[
\int_0^l \varepsilon_t \, dx = 0 \quad \text{and} \quad \int_0^l \varepsilon_b \, dx = 0
\]

In the subsequent studies, these two equations will be referred as the "strain compatibility criteria".

2.6 The Modeling of Forces under Full Lateral Restraint

The proposed analytical model is based on the following assumptions and simplifications to simulate the development of arching action in a laterally rigidly restrained beam or slab strip:

1. For each sustainable gravity load ($P$) applied, there will be a corresponding lateral restraint exerted by the rigid surroundings to each end of the beam. The restraining reaction can be idealized as an in-plane axial force ($N$) imposed on the beam, with a specific magnitude acting at a specific depth of the beam section.

   ![](image)

   Figure 2-6. Idealization of the lateral restraint as an in-plane axial force

2. The applied load and axial force ($P-N$) combination produces a unique internal moment distribution which satisfies the equilibrium criteria stated in Section 2.4 at every vertical section of the beam. At the same time, the resultant strain distributions satisfy the compatibility criteria proposed in Section
2.5, producing a net elongation of zero when integrating along the length of the beam for both the top and bottom surfaces (Figure 2-7).

Figure 2-7. Moment distribution and the corresponding top surface and bottom surface strain distribution profiles of a laterally restrained beam under midspan loading

Figure 2-8. Relative position of the midspan- and end- moments when viewed from the context of the moment-curvature curve

3. As $P$ increases, the magnitude of $N$ also increases (though maybe at different rates) until the beam reaches a failure state, which would be dominated by crushing of the concrete at critical location(s).

Once a failure state is reached, the compatibility requirements can no longer be fulfilled.

However, the determination of $P-N$ responses in a laterally retained beam requires an iterative cycle of calculations. In the coming sections, a method of calculation will be developed for this iterative procedure, with detailed discussions on how to incorporate material nonlinearity in section forces analysis, and how to perform strain integral iterations to meet the compatibility criteria. The method of
calculation proposed tries to solve for the load-carrying capacity \((P_{ult})\) by monotonically increasing the magnitude of \(N\) until a failure state occurs.

### 2.7 Evaluation of Section Forces and the Moment-Curvature Response

In order to obtain the strains present in a loaded structural member for performing the strain integral calculations, the first step is to perform sectional force analysis. Based on the "plain sections remain plain" assumption, the response of a reinforced concrete section under combined action of flexural and axial forces can be predicted by using the equilibrium equations accompanied with the material stress-strain relationships described previously. The proposed method of calculation adopts a numerical, iterative approach for determining section forces, which can be easily executed in spreadsheets.

For a given axial force \((N)\) acting on a section with geometric and material properties known, evaluation of section forces can be proceeded according to the following steps:

1. Start with a value of top surface concrete strain \((\varepsilon_t)\).

2. Propose a curvature \((\varphi)\) and determine the corresponding strain distribution of the section.
3. Evaluate the compressive stress in concrete by idealizing the portion in the compressive zone (e.g. from the edge to the neutral axis) as a series of rectangular layers, assuming the strain in each layer is uniform and equal to the strain at the center of that layer (Figure 2-10). In all calculations executed for validation and parametric studies, a total of ten slices were used. Instead of rigidly dividing the entire section into equally spaced intervals, this approach ensures that there are always ten slices for numerical integration the compressive zone, so a high degree of accuracy can be obtained, especially when a simple constitutive relationship is used. No discretization is required for concrete in tension because of the zero concrete tensile strength assumption.

4. Calculate the resultant force in each layer by multiplying the stress in the layer by the area of the layer. The sum of forces represents the total compressive force in concrete ($C_c$).

5. Calculate the resulting force in reinforcement ($C_r$ & $T$), assuming stress of each reinforcing bar is equal to the stress at the centroid of the bar.

6. Through an iterative procedure (e.g. using Solver Function in Microsoft Excel), repeat steps 2 to 5 by varying $\phi$ until the relationship $C_c + C_r + T = N$ for force equilibrium is satisfied.

7. Calculate the corresponding internal moment ($M$) acting about the mid-depth on the section. From $M$, the eccentricity ($e$) can be found using the relationship $M = N \cdot e$.

8. From the resultant strain distribution, determine the bottom surface strain ($\varepsilon_b$) using the equation $\varepsilon_b = \varepsilon_c + \phi h$.

Similar to the process of predicting the sectional response of flexural members, by repeating these steps with different values of $\varepsilon_c$, a complete moment-curvature curve can be obtained (Figure 2-11). At the same time, the strain distribution of the section corresponding to each point at the curve can also be recorded.
Following the same approach, by varying the magnitude of $N$, a family of moment-curvature response curves can be obtained (Figure 2-12).

![Figure 2-11. Typical moment-curvature curve (positive bending shown)](image)

![Figure 2-12. A family of moment-curvature response curves for a given cross-section with axial force acting at different magnitudes](image)
From the preceding chapter, it has been shown that the arching action phenomenon can be attributed to the axial compression-flexure interaction effects (Figure 1-3). Hence, in investigating the arching action response of a laterally rigidly restrained beam, as an initiation, it is helpful to develop a set of moment-curvature responses of the beam section under different magnitudes of axial force before proceeding to the iterative cycle of finding their corresponding applied loads (i.e. the P-N combination). This set of moment-curvature diagrams is useful in visualizing the strength enhancement due to arching action. At the same time, it is useful for estimating the level of axial force that the structure can sustain and the approximate ultimate load and moment capacity that the structure can support. Indeed, when superimposing $M_m$ (the midspan-moment corresponding to each $P-N$ pair; for a midspan loading arrangement, it governs the load-carrying capacity) to the moment-curvature curves, a clear arching action development trend becomes apparent, as illustrated in Figure 2-13. Under arching action with $f'c = 70\text{MPa}$, for the particular section shown, the moment capacity is almost four times higher than the unrestrained, simply supported condition (i.e. $N = 0\text{kN}$). The sectional response also becomes much more stiffer that the "yield plateau" for $N = 0\text{kN}$ is receded to a more linear curve (highlighted with red dots) for the rigidly restrained condition.

![Figure 2-13. Arching action development as reflected in the moment-curvature response curve of the mid-span section in a laterally rigidly restrained beam](image)

An iterative, numerical approach to obtain these solutions will be presented in the next section.
2.8 Predicting Response of Concrete Members under Full Lateral Restraint

Figure 2-14. The iterative cycle for calculating the load capacity of a structure under full lateral restraint
The proposed method of calculation utilizes numerical integrations with an iterative procedure for predicting the structural response of a laterally rigidly restrained beam or slab strip. The flowchart shown in the previous page illustrates the process of finding the load carrying capacity of such structure.

As shown in the flowchart, the procedure of solving for load capacity involves two levels of iterations. The first level (i.e. the inner-loop) deals with finding the equilibrium conditions that satisfy the compatibility criteria under a given magnitude of axial force. Each inner-loop iteration produces a $P$-$N$ combination that satisfies the strain compatibility criteria. Then, there is a second level of iteration (i.e. the outer-loop) which is necessary for finding the maximum possible axial force, $N_{ult}$, that the structure can sustain. The equilibrium conditions derived from $N_{ult}$ represent the ultimate states of the beam. Once $N_{ult}$ is determined, the load-carrying capacity of the beam, $P_{ult}$, can be derived.

Specifically, for a typical uniform-depth reinforced concrete beam with midspan loading, the top and bottom surface strain integrals can be calculated thorough a discretization procedure as shown in the figure below:

![Figure 2-15. Discretization of strain distribution profiles for deformation or elongation calculations. Taking advantage of symmetry, only half of the entire span is required to execute the calculations.](image_url)

The strain in each slice is assumed to be uniform and equal to the strain at the center of that layer. To ensure a higher degree of precision, it is advisable to use finer slices for regions in the strain curve where
nonlinearity is more pronounced. From the discretized strain distribution profiles for top and bottom surface strains, the resultant deformation or elongation simply becomes:

\[ \Delta_l = \sum_{i=1}^{i=k} \varepsilon_i x_i \]

where

\[ \Delta_l = \text{the resultant deformation or elongation} \]

\[ k = \text{number of slices presented in the member} \]

To satisfy the strain compatibility criteria, \( \Delta_l \) from the above numerical integration must equal to zero.

It is advisable to start the iterative procedure with a conservative initial input of \( N \) and then gradually increase its magnitude until the failure state occurs, as demonstrated in the previous section. This approach enables the proposed model to produce a complete set of structural responses for a given structure, including load-central deflection diagram, and moment-curvature response at the critical location(s). They can be useful for design and service limit states analysis purposes.

On the other hand, this iterative method of calculation also makes the proposed model unique comparing with those discussed in Chapter One that in those methods only the ultimate load is concerned.

A step-by-step sample calculation is presented in Appendix A to demonstrate how this method of calculation is executed.

### 2.9 Numerical Integration - Sensitivity Analysis

Since the proposed method involves numerical integrations, sensitivity analysis was carried out to investigate its level of precision. Based on calculations done for a 6m span bridge deck slab strip in the parametric study (refer to Chapter 4 and Appendix A), it was found that even when reducing the number of slices from 200 to 20, the loss in precision remains insignificant (Figure 2-16). With 200 slices for numerical integration, the load-carrying capacity, \( P_{ulb} \), was calculated to be 135.3kN, at \( N_{ulb} = 910 \text{kN} \).

When only 20 slices were used, \( P_{ulb} \) increased slightly to 139.0kN, at \( N_{ulb} = 950 \text{kN} \). Only marginal variations were detected between the two diagrams, and they were shown up primarily in the midspan and end regions of the beam where concrete had been cracked and nonlinearity in the strain curves were more pronounced. Therefore, the proposed method of calculation can be adapted for quick load-capacity predictions and design estimations when necessary.
Part A) Strain distribution profiles generated using 200 slices

Part B) Strain distribution profiles generated using 20 slices

Part C) Typical stress states of a beam at the ultimate limit states under aching action and loading at midspan

Figure 2-16. Sensitivity analysis revealed that the loss of precision is insignificant when reducing the number of numerical integration slices from 200 to 20
2.10 Conclusion

An analytical model is proposed in this chapter for predicting the structural response of a laterally rigidly restrained beam. This model is based on strain compatibility criteria that for such slab strip under gravity loading, the integrals of the top and bottom surface strains along the length of the span must equal to zero. A method of calculation is also presented in this chapter to execute this model. Using an iterative, numerical approach, the method of calculation proposed produces $P-N$ combinations that satisfy the strain compatibility criteria. Through two loops of iterations, the proposed method of calculation finds the load-carrying capacity of a restrained beam by searching for the maximum possible axial forces that the beam can sustain before a failure state occurs and the compatibility criteria can no longer be fulfilled. In the next chapter, the proposed analytical model and method of calculation will be validated using experimental data available in literature.
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

3.1 Overview
In order to examine the capability of the proposed analytical method in modeling arching action development and its accuracy in predicting the ultimate load in laterally rigidly restrained reinforced concrete beams and slab strips, four series of specimens chosen from related literature were studied and analyzed. These specimens were chosen primarily because of their relatively straightforward geometry and loading arrangement. Other relevant investigations that have discussed previously in Chapter 1, such as the slab-on-columns experiment conducted by Vecchio and Tang (1990), had more sophisticated general arrangement and boundary conditions that would require further simplifications and assumptions when applying the proposed model. In order to obtain more direct results and to reduce the amount of computations to a minimum, they were not chosen for validation studies.

The selected test series include:

• Ten reinforced concrete frame beams, selected from the twelve specimens tested by Su et al. (2010) at the Hebei Polytechnic University in Tangshan, China (S-series)

• Seven of the twelve one-way slab specimens restrained by shear-walls investigated by Wang et al. (2011) at the Dalian University of Technology in Dalian, China (W-series)

• Nine of the fifteen concrete slab strips typical of a bridge deck slab from the experiment done by Taylor et al. (2000) at the Queen's University of Belfast in United Kingdom (T-series)
• All ten laterally restrained reinforced concrete slab strips investigated by Muthu et al. (2006) from Ramaiah Institute of Technology in India (M-series)

From sections 3.2 to 3.5, detailed descriptions of the geometric and material properties, boundary and loading conditions, and comparisons of theoretical and experiment results for these four series of specimens will be presented.

Other than the analytical results obtained using the proposed model, for comparison purposes, ultimate bending capacity of each specimen according to the conventional plastic analysis approach \( (P_{ult,p}) \), determined at the formation of a collapse mechanism due to hinging at the midspan and the supports will also be presented (see Appendix B for a sample calculation). This plastic collapse mechanism approach is commonly referred as the Johansen's yield line method in bending capacity calculations for reinforced concrete slabs (Park and Gamble, 1999). As an upper bound method of plasticity analysis, this approach gives the highest possible bending capacity of each specimen when arching action is not considered.

When evaluating the degree of accuracy of the proposed model, in addition to the experimental results, analytical results calculated by the authors using existing arching action models, such as the Park's model discussed in Chapter 1, will also be considered.
3.2 S-Series Specimens

The experimental program that involved the S-series specimens was documented in an article published in the September-October Issue of ACI Structural Journal in 2009, titled "Progressive collapse resistance of axially-restrained frame beams". All relevant data, including material properties, specimen dimensions and experiment results were referenced from this journal article.

3.2.1 General Arrangement

Figure 3-1 shows the structure general geometry and reinforcement arrangement, Figure 3-2 cites the schematic illustration of the test setup from the journal article published that documented this experiment program, and Table 3-1 summarizes the specimen properties of the S-series specimens.

![Figure 3-1](image1.png)

(a) Plan view

(b) Elevation view

Typical Section View

Figure 3-1. Plan, elevation and typical cross-section views of S-series specimens

![Figure 3-2](image2.png)

Figure 3-2. Schematic illustration of S-series specimens test setup
Table 3-1. Geometric and material properties, and reinforcement layouts of S-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Original specimen label</th>
<th>$b \times h$ (mm)</th>
<th>$l_e$ (mm)</th>
<th>$l_e/h$</th>
<th>$f_{cw}$ (Mpa)</th>
<th>Longitudinal reinforcement and ratio</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>A1</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>32.3</td>
<td>2φ12, ρ = 0.55%</td>
<td>φ8 at 100</td>
</tr>
<tr>
<td>S2</td>
<td>A2</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>35.3</td>
<td>3φ12, ρ = 0.83%</td>
<td>φ8 at 80</td>
</tr>
<tr>
<td>S3</td>
<td>A3</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>39.0</td>
<td>3φ14, ρ = 0.113%</td>
<td>φ8 at 80</td>
</tr>
<tr>
<td>S4</td>
<td>A4</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>28.8</td>
<td>2φ12, ρ = 0.55%</td>
<td>φ8 at 100</td>
</tr>
<tr>
<td>S5</td>
<td>A5</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>33.1</td>
<td>3φ12, ρ = 0.83%</td>
<td>φ8 at 80</td>
</tr>
<tr>
<td>S6</td>
<td>A6</td>
<td>150 x 300</td>
<td>1225</td>
<td>4.08</td>
<td>35.8</td>
<td>3φ14, ρ = 1.13%</td>
<td>φ8 at 80</td>
</tr>
<tr>
<td>S7</td>
<td>B1</td>
<td>150 x 300</td>
<td>1975</td>
<td>6.58</td>
<td>23.2</td>
<td>3φ14, ρ = 1.13%</td>
<td>φ8 at 100</td>
</tr>
<tr>
<td>S8</td>
<td>B2</td>
<td>150 x 300</td>
<td>2725</td>
<td>9.08</td>
<td>24.1</td>
<td>3φ14, ρ = 1.13%</td>
<td>φ8 at 120</td>
</tr>
<tr>
<td>S9</td>
<td>B3</td>
<td>150 x 300</td>
<td>2725</td>
<td>9.08</td>
<td>26.4</td>
<td>3φ14, ρ = 1.13%</td>
<td>φ8 at 120</td>
</tr>
<tr>
<td>S10</td>
<td>C1</td>
<td>100 x 200</td>
<td>1225</td>
<td>6.12</td>
<td>19.9</td>
<td>2φ12, ρ = 1.30%</td>
<td>φ8 at 80</td>
</tr>
</tbody>
</table>

The initiative of this laboratory test was to investigate the axial restraining effects on gravity load-carrying capacity of reinforced concrete frame beams against progressive collapse. Each specimen represented a two-bay beam resulting from the removal of a supporting column in the midspan. Each test assembly consisted of two doubly-reinforced beams connected by a column stub in the center and two short columns at the edges where the rotational and longitudinal restraints on beams were applied. The center column represented the removed column and had a 250mm x 250mm cross-section for all specimens. The edge columns had an enlarged size for anchorage of the beams into the steel sockets, which were connected with the rigid supports on a steel test bed by a pin located 150mm from each beam end. The sockets were also connected by a vertical strut and a horizontal strut to achieve axial and rotational restraints, so arching action can be developed in the beam. Gravity loading was simulated by applying downward displacement at the center column stub through a servo-controlled actuator reacting against a loading frame. According to the authors, the test assembly was able to achieve a horizontal rigidity of 1000kN/mm and a rotational stiffness of 17 500kNm/rad.
This series of specimens can be divided into three groups:

- **Group A** beams, originally labeled as A1 to A6, varied in flexural reinforcement ratios and each had a width of 150mm, depth of 300mm, and measured 1225mm long for the clear span, $l_c$.

- **Group B** beams, originally labeled as B1 to B3, had identical cross-sections as the Group A beams but varied in clear span lengths.

- **A group C** beam, originally labeled as C1, had a smaller cross-section. It was measured 100mm wide, 200mm deep, and 1225mm long in clear span.

The concrete cube compressive strengths, $f_{cu}$, of these specimens varied from 19.9MPa to 39MPa. Deformed steel bars in diameters of 12 and 14mm, and yield at 350MPa and 340MPa, respectively, were used for flexural reinforcement. The longitudinal reinforcements were anchored into the edge columns with hooks that met the ACI 318-05 code requirements for development length. Smooth bars in diameter of 8mm were used for ties. All specimens had a concrete clear cover of 20mm for the longitudinal bars.

### 3.2.2 Modeling of the Specimens

Two sets of modeling were carried out for this series of specimens.

The first set neglected the confinement effects from the transverse reinforcements and used the bilinear stress-strain relationship from the fib MC2010 final draft (Figure 2-2) to model concrete in compression, with the cube concrete strengths ($f_{cu}$) given in reference article converted to cylinder compression strengths ($f'_c$) by a multiple of 0.8. The bilinear stress-strain relationship for steel reinforcements (Figure 2-1) was used to model the longitudinal reinforcements. Strain hardening was not considered for the longitudinal reinforcements. This was a legitimate simplification because as the level of axial forces increases, the restrained beams behaved in stiffer modes (Figure 2-12). Extensive simulations done for the current research revealed that the reinforcements usually do not reach to the strain-hardening stage at the ultimate states.

The second set of calculations incorporated confinement effects due to transverse reinforcements. By considering the confinement effectiveness of rectangular concrete sections confined by rectangular hoop reinforcements, the theoretical stress-strain formulations for confined concrete proposed by Mander et al. (Section 2.3.3) were used to compute the ultimate compressive strength ($f'_{cu}$) of confined concrete, and the corresponding ultimate concrete compression strain ($\varepsilon_{cu}$).
In sectional force analysis, numerical integration was used to find the stress resultants acting on the section. This was done by dividing the compressive zone of concrete into 10 layers of equal thickness, calculating the force in each individual layer, and then summing the forces up (Figure 2-10).

For each specimen, the steel framed surrounding was assumed to be perfectly rigid. As the vertically applied midspan loading \( (P) \) increases, the thrust exerted by the rigid surroundings to prevent lateral expansion of the specimen was idealized as an axial force \( (N) \) applied at each end. To attain force equilibrium, it was expected that as the loading at midspan increases, the magnitude of the axial forces would also increase. In order to capture the complete response of each specimen modeled, a complete \( P-N \) response curve, instead of only the ultimate loading condition, was modeled using the proposed method of calculation. As discussed in the preceding chapter, this procedure was done by gradually increasing \( N \) from a relatively low level of magnitude to the ultimate level that was defined by the crushing of concrete at critical location(s). A moment distribution that satisfies the strain compatibility criteria was obtained first, followed by calculating the corresponding \( P \) at each cycle of iteration based on equilibrium conditions. Simultaneously, central deflection at each loading stage could be determined using a numerical form of the integral:

\[
\int_{0}^{l} \varphi_{1} x \, dx
\]

Taking advantage of symmetry, only half of the span of each specimen was modeled.

In addition, the presence of the center column-stud with densely placed hoop reinforcements indicated that the strength of it would be much higher than that of the clear span region of the specimen. Experimental results revealed that concrete crushing and severe cracking occurred at regions immediately next to the center column-stud and the edge columns. Therefore, it was assumed that the center column-stud was a rigid region, and only the clear span region, \( l_{n} \), was modeled.

Bending effects due to the self-weight of specimens were ignored in the calculations.

Because the current research focused on the modeling of arching action, the post concrete-crushing behavior of the specimens, including the transition from the compressive arch action to the tensile catenary action, and the development of catenary action were not included in the analysis.
3.2.3 Results and Discussion

Table 3-2 summarizes the experimental and analytical loading capacities and their corresponding horizontal reactions within the compressive arch action state for each specimen modeled. Both results from the confined- and unconfined- concrete models are presented. For comparison purposes, the loading capacity of each specimen calculated according to the conventional plastic analysis approach ($P_{ult,p}$) is also list in Part B of this table.

Table 3-2. Comparison of experimental and analytical ultimate loads of S-series specimens

**Part A) Experimental results versus analytical results assuming no concrete confinement**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experiment results</th>
<th>Proposed model, unconfined case</th>
<th>Accuracy, proposed model, unconfined case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{ult,e}$ (kN)</td>
<td>$N_{ult,e}$ (kN)</td>
<td>$P_{ult,uc}$ (kN)</td>
</tr>
<tr>
<td>S1</td>
<td>168</td>
<td>281</td>
<td>147.6</td>
</tr>
<tr>
<td>S2</td>
<td>221</td>
<td>318</td>
<td>189.0</td>
</tr>
<tr>
<td>S3</td>
<td>246</td>
<td>296</td>
<td>228.5</td>
</tr>
<tr>
<td>S4</td>
<td>147</td>
<td>309</td>
<td>121.1</td>
</tr>
<tr>
<td>S5</td>
<td>198</td>
<td>340</td>
<td>159.0</td>
</tr>
<tr>
<td>S6</td>
<td>226</td>
<td>177</td>
<td>189.1</td>
</tr>
<tr>
<td>S7</td>
<td>125</td>
<td>211</td>
<td>117.7</td>
</tr>
<tr>
<td>S8</td>
<td>82.9</td>
<td>190</td>
<td>85.9</td>
</tr>
<tr>
<td>S9</td>
<td>74.7</td>
<td>172</td>
<td>74.1</td>
</tr>
<tr>
<td>S10</td>
<td>60.9</td>
<td>91.6</td>
<td>55.2</td>
</tr>
</tbody>
</table>

|          | Average            | 0.90                        | 0.78          |
|          | Standard deviation | 0.08                        | 0.19          |

**Part B) Experimental results versus analytical results with concrete confinement taken into consideration; also shown in this table are the ultimate loads calculated using plastic analysis**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experiment results</th>
<th>Proposed model, confined case</th>
<th>Accuracy, proposed model, confined case</th>
<th>Plastic analysis</th>
<th>Accuracy, plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{ult,e}$ (kN)</td>
<td>$N_{ult,e}$ (kN)</td>
<td>$P_{ult,c}$ (kN)</td>
<td>$N_{ult,c}$ (kN)</td>
<td>$P_{ult,c}/P_{ult,e}$</td>
</tr>
<tr>
<td>S1</td>
<td>168</td>
<td>281</td>
<td>156.3</td>
<td>227</td>
<td>0.93</td>
</tr>
<tr>
<td>S2</td>
<td>221</td>
<td>318</td>
<td>204.0</td>
<td>275</td>
<td>0.92</td>
</tr>
<tr>
<td>S3</td>
<td>246</td>
<td>296</td>
<td>252.1</td>
<td>310</td>
<td>1.02</td>
</tr>
<tr>
<td>S4</td>
<td>147</td>
<td>309</td>
<td>132.9</td>
<td>197</td>
<td>0.90</td>
</tr>
<tr>
<td>S5</td>
<td>198</td>
<td>340</td>
<td>180.4</td>
<td>250</td>
<td>0.91</td>
</tr>
<tr>
<td>S6</td>
<td>226</td>
<td>177</td>
<td>214.8</td>
<td>275</td>
<td>0.95</td>
</tr>
<tr>
<td>S7</td>
<td>125</td>
<td>211</td>
<td>124.6</td>
<td>182</td>
<td>1.00</td>
</tr>
<tr>
<td>S8</td>
<td>82.9</td>
<td>190</td>
<td>89.5</td>
<td>177</td>
<td>1.08</td>
</tr>
<tr>
<td>S9</td>
<td>74.7</td>
<td>172</td>
<td>78.7</td>
<td>180</td>
<td>1.05</td>
</tr>
<tr>
<td>S10</td>
<td>60.9</td>
<td>91.6</td>
<td>62.3</td>
<td>92</td>
<td>1.02</td>
</tr>
</tbody>
</table>

|          | Average            | 0.98                        | 0.95          |
|          | Standard deviation | 0.06                        | 0.25          |

Chapter 3. Validation and Evaluation of the Proposed Analytical Model
The significant discrepancy between $P_{ult,e}$ and $P_{ult,p}$ indicates the remarkable underestimation in load carrying capacities of the specimens based on the conventional plastic analysis approach.

On the other hand, good agreement is achieved between the experimental results and the analytical results obtained using the current model, even in the case in which the beneficial effects from concrete confinement were neglected. In the "unconfined" case, the average ratio of calculated to measured capacity is 0.90 with a standard deviation of 0.08. When concrete confinement was considered, the results improve to an average ratio of 0.98, with a standard deviation of 0.06. These analytical results are slightly better than those calculated by the authors, which were based on Park's arching action model (Section 1.2.3, Figure 1-9). Their average ratio of calculated to measured capacity is 0.95 with a standard deviation of 0.04. Figure 3-3 illustrates the relatively high level of consistency and accuracy of the analytical results from the "confined" case when comparing with those based on the plastic collapse mechanism approach.

Arguably Park's model is sufficient for estimating the ultimate capacities of laterally restrained concrete slabs; however, as explained in Chapter 1, it cannot be exploited to predict their structural response from the initial loading stage to the ultimate states like the proposed model. Some representable structural response curves taken from this series of analysis are listed in the following four sets of figures.
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-4. Structural responses of specimens S1, S2 and S3

Notes:

- Each dot in the graphs represents an iteration done in the calculation that satisfied compatibility and equilibrium requirements. A complete curve was obtained by gradually increasing N from a low magnitude to its ultimate possible value.

- The last dot of each curve represents the ultimate conditions in which concrete confinement was taken into consideration. The second-last dot of each curve represents the ultimate conditions in which concrete confinement was not considered. The experimental results shown represent the maximum loadings recorded.
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-5. Structural responses of specimens S4, S5 and S6
Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-6. Structural responses of specimens S7 and S8
Part A) Top and bottom strain profiles of specimen S1 (half span shown)

Part B) Top and bottom strain profiles of specimen S5 (half span shown)

Figure 3-7. Typical strain distribution profiles for A) specimens that are doubly-reinforced with equal amounts in the top and bottom faces, and B) specimens that are asymmetrically reinforced; Note the strain distributions are biased to the midspan for S5, a longitudinally asymmetrically reinforced specimen.

Because the tensile strength of concrete was ignored in the analysis, the pre-flexural-cracking stage of the response of the specimens when "the horizontal reaction forces were generally negligible" was not observed from the analytical P-N and M-φ response curves. However, because of the presence of axial restraint, concrete cracking should not cause a significant weakening in stiffness. Instead, the curves show an approximately linear behavior until the onset of yielding of the longitudinal reinforcements.

According to the authors' observations, first yielding of the longitudinal reinforcements occurred at the midspan of the specimens due to positive bending and then at the supports due to negative bending, which
resulted in pronounced stiffness degradation. The analytical response does show the significance of stiffness degradation in the post-yielding stage; however, the moment distributions and strain profiles for those symmetrically reinforced specimens reveal that yielding of the top layer reinforcements at the supports and the bottom layer reinforcements at midspan occurred at the same time. In other words, throughout loading at the midspan, the negative moments at the ends and the positive moment at midspan maintained equal in magnitude (see Part B of Figure 3-4 and Figure 3-6). This discrepancy can be attributed to the fact that the proposed model bases all calculation on the original geometry of the structure and does not take second-order effects into account. Even with only a slight amount of deflection, P-Δ effect will impose a greater moment demand at the midspan. Subsequently, yielding will occur at midspan instead of happening concurrently at both the ends and the midspan. Nonetheless, due to the stiffness of laterally rigidly restrained beams, P-Δ effect should be insignificant and therefore good correlation is achieved between the analytical and experimental results.

Comparing between the structural responses of symmetrically and asymmetrically reinforced specimens (Figure 3-4 versus Figure 3-5, and Figure 3-7 Part A versus Part B), it becomes apparent how the strain compatibility criteria of the proposed model are taken into effect. The curvatures and the strains biased to the weaker side to maintain compatibility of the entire specimen. Since the bottom face of those specimens had less reinforcement, the midspan became the critical region and the ultimate compressive strain of the midspan determined the load carrying capacity of the specimen.

Comparisons between the structural responses of specimens S7 and S8 revealed another characteristic of laterally rigidly restrained beams. These two specimens had identical cross-sections but differed in span lengths. The span-to-depth ratio of S7 and S8 were 13.2 and 18.2, respectively. From Part A of Figure 3-6, it is clear that the longer one, S8, gave a significantly lower ultimate load. But when looking at their moment-curvature responses (Part B of Figure 3-6), the moment distributions of these two specimens at each stage of loading were almost identical. In other words, for the same sectional properties, arching action is almost equally effective in slab strips or beams of longer span. The drop in strength enhancement is only about 2%, from a strength enhancement (i.e. \((P_{ult,c} - P_{ult,p})/P_{ult,p}\)) of 63% for S7 to 61% for S8.

In addition, by taking concrete confinement into consideration, \(P_{ult}\) was increased by an average of 8%. However, analysis from the confined concrete case evidenced that the main contribution of concrete confinement was not so much in the enhancement of concrete compressive strength but mainly in the extension of the ultimate compressive strain, which increased the crushing limit of the concrete so the specimens could sustain more axial restraints at the ultimate stage. For instance, in specimen S2, by
considering confinement, the ultimate compressive strain increased from \(-3.50 \times 10^{-3}\) to \(-5.58 \times 10^{-3}\), but the ultimate compressive stress only slightly increased from 29.4MPa to 32.5MPa. Nevertheless, the ultimate load of S2 raised from 189kN to 204kN.

Although the calculation of load-carrying capacity for each specimen is in good corroboration with that recorded from the actual tests, the load-deflection response for each specimen did demonstrate discrepancies from that observed during loading tests. The specimens generally exhibited a much more ductile behavior than the analytical expectations. Based on the current theory these axially restrained beams should be stiff and crushing at low deflections upon yielding of reinforcements. But in reality all specimens exhibited a pronounced post-yielding plateau until concrete crushed. Even after the crush of concrete at midspan, the reinforcements were able to hold the structure together and onset the development of catenary action. Perhaps this can be ascribed to the fact that these specimens were densely reinforced transversely so the crushed concrete could still be compacted together. In addition, since the experiment setup was not really perfectly rigid as assumed in the proposed model, it was reasonable that the specimens experienced greater deflection during loading.

Lastly, except for Specimen A5, \(N_{ult}\) calculated were generally lower than the actual values from the experiment, with a ratio of 0.95 and a standard deviation of 0.25 for the confined case. However, the determination of \(N_{ult}\) was actually ambiguous in the experiment. As explained in the reference article, following the yielding at the supports, the peak load was achieved at a normalized deflection (deflection/depth of beam) ranging between 0.16 and 0.34 but the maximum compressive reaction force was reached when concrete crushed at the midspan at a value of deflection ratio between 0.29 and 0.50. In other words, the horizontal reaction force kept increasing even though the peak load had been reached. Without further details provided, it is difficult to conclude the main reasons that caused this discrepancy.
3.3 W-Series Specimens

The experimental program that involved the W-series specimens was documented in an article published in the Journal of Central South University of Technology in 2011, titled "Membrane action in lateral restraint reinforced concrete slabs". All relevant data, including material properties, specimen dimensions and experimental results were taken from this journal article.

3.3.1 General Arrangement

W-Series Specimens consist of seven of the twelve reinforced concrete slab strips made to investigate the load-carrying capacity under arching action. These specimens were designed to simulate the middle span of continuous slab in shear-wall structures. Figure 3-8 shows general geometry and reinforcement arrangement, Figure 3-9 cites the test rig setup from the journal article published that documented this experiment program, and Table 3-3 summarizes the specimens properties of the W-series specimens.

Figure 3-8. General structure structural arrangement of W-series specimens

Figure 3-9. Load and support system of W-series specimens: (a) Elevation view; (b) Plan view
All selected specimens had a 2400mm clear center span supported on shear-wall studs measured 100mm in width and 250mm in height. The width of all selected specimens was 300mm, but their depth varied from 80mm to 120mm, so the influence of span-to-depth ratio in ultimate load could be investigated. Except for specimen W5, which was reinforced with 6.5mm diameter steel bars that yield at 241MPa, 8mm diameter reinforcing bars with a yielding strength of 305MPa were used for longitudinal and transverse reinforcements. All selected specimens were cast in normal strength concrete that had cube compressive strength ($f_{cu}$) of either 32.6MPa or 41.2MPa. A cantilever end section was attached to each end of every specimen, with metal rods inserted to connect the end section to the horizontal support. The horizontal support, which was composed of metal beams and metal poles, was used to provide lateral restraint for the specimens. It could generate a lateral restraining stiffness of 16.7kN/mm.

Unlike the S-series specimens, in which load was applied only at the midspan, a four-point-loading system was utilized for this series of specimens to mimic a uniformly distributed loading condition.

### 3.3.2 Modeling of the Specimens

The modeling of W-series specimens were similar to that of the S-series, with the following assumptions and simplifications made:

- All longitudinal reinforcements had a 10mm clear cover.
• The top reinforcements would be able to develop full strength at the hooked ends 800mm from the shear-walls and the supports.

• The supports provided perfectly rigid lateral restraints to the specimens.

• Taking advantage of the symmetrical loading arrangement and the symmetric sectional properties, only half of the clear span was modeled for each specimen.

• The four-point loading system was expected to produce a bending moment distribution in following shape:

![Moment Distribution Diagram](image)

Figure 3-10. Loading arrangement and the corresponding internal moment distribution for the W-series specimens

### 3.3.3 Results and Discussion

Table 3-4 summarizes the experimental and analytical results of the W-series specimens. For comparison purposes, analytical load-carrying capacities calculated using a plastic collapse mechanism approach were also presented here. See also the correlation graph (Figure 3-11) to visualize the differences between experimental and analytical results.
Table 3-4. Comparison of experimental and analytical results of W-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experiment results</th>
<th>Proposed's model</th>
<th>Accuracy, Proposed's model</th>
<th>Plastic analysis</th>
<th>Accuracy, plastic analysis</th>
<th>Experiment/plastic analysis</th>
<th>Proposed/plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{ult,e}$ (kN)</td>
<td>$P_{ult}$ (kN)</td>
<td>$N_{ult}$ (kN)</td>
<td>$P_{ult}/P_{ult,e}$</td>
<td>$P_{ult,p}$ (kN)</td>
<td>$P_{ult,p}/P_{ult,e}$</td>
<td>$P_{ult,e}/P_{ult,p}$</td>
</tr>
<tr>
<td>W1</td>
<td>20.3</td>
<td>26.8</td>
<td>77.0</td>
<td>1.32</td>
<td>15.9</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>W2</td>
<td>36.9</td>
<td>45.8</td>
<td>85.0</td>
<td>1.24</td>
<td>25.4</td>
<td>0.69</td>
<td>1.45</td>
</tr>
<tr>
<td>W3</td>
<td>24.6</td>
<td>34.1</td>
<td>72.0</td>
<td>1.39</td>
<td>23.3</td>
<td>0.95</td>
<td>1.06</td>
</tr>
<tr>
<td>W4</td>
<td>28.4</td>
<td>38.4</td>
<td>90.0</td>
<td>1.35</td>
<td>21.0</td>
<td>0.74</td>
<td>1.35</td>
</tr>
<tr>
<td>W5</td>
<td>24.0</td>
<td>34.3</td>
<td>95.0</td>
<td>1.43</td>
<td>16.1</td>
<td>0.67</td>
<td>1.49</td>
</tr>
<tr>
<td>W6</td>
<td>32.6</td>
<td>42.0</td>
<td>90.0</td>
<td>1.29</td>
<td>26.9</td>
<td>0.83</td>
<td>1.21</td>
</tr>
<tr>
<td>W7</td>
<td>20.4</td>
<td>20.6</td>
<td>75.0</td>
<td>1.01</td>
<td>9.9</td>
<td>0.49</td>
<td>2.06</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>1.34</td>
<td></td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-11. Correlation between experimental and analytical results of the W-series specimens

Because this set of specimens was restrained by a relatively weak lateral support (16.7kN/mm), in the actual tests the resulting enhancement in load-carrying capacity was not as significant as that of the S-series, in which the lateral restraining stiffness reached up to 1000kN/mm. Consequently, the average experimental load-carrying capacity was only 38.3% higher than that obtained by plastic analysis. On the other hand, for the analytical results produced from the proposed model, because a perfectly rigid lateral restraining system was assumed, as expected the ultimate loads predicted were consistently higher than the experimental records for six out of the seven specimens studied.
At this stage it is difficult to quantitatively judge whether the results produced by the proposed model using the proposed method of calculation actually represent the perfectly, rigidly restrained states of these specimens. However, the fact that the proposed model was able to consistently overestimate most of the ultimate loads must be recognized and credited. Indeed, several insightful points can be drawn from the comparisons between the analytical and experimental results.

1. As seen from the moment-curvature (Part B) graphs of Figure 3-12 to Figure 3-14, for a four-point loading arrangement, the structural response is different from a midspan loading condition as previously shown in the S-series. In this series, the requirement of a zero elongation for fulfilling compatibility imposes a high moment demand in the negative bending region, making the ends the critical regions that govern the load-carrying capacity of each specimen.

2. Concrete compressive strength is a key factor in the additional load capacity that arching action enhances. Comparing between specimens W3, which had an $f_{cu}$ of 32.6MPa, and W4, which $f_{cu}$ was 41.2MPa, the experimental results, although were obtained from load tests with mild lateral restraint, showed an enhancement of 6% and 35%, respectively. It indicates that laterally restrained structures made of stronger concrete have better load enhancement potentials. The proposed model predicted a similar trend, estimating that if rigidly restrained, $P_{ult}$ of W3 and W4 will be 46% and 83% higher than those predicted by plastic analysis, respectively (Figure 3-12). Hence, in contrast to simply supported structures, in which the load capacity is mainly dependent on reinforcement percentage, the ultimate capacity of concrete slabs restrained against longitudinal expansion is also dependent on concrete compressive strength.

3. Slenderness is another factor that affects the significance of load enhancement from arching action. A decrease in slab thickness would result in a reduction in strength enhancement. The experimental results showed that for specimen W4, which measured 100mm in depth, $P_{ult,o}$ was 35% higher than that obtained from plastic analysis. On the other hand, specimen W1, which had the same $f_{cu}$ and clear span as W4 but a slender cross-section ($h = 80mm$), the enhancement ratio reduced to 28%. The proposed model predicted the same trend, expecting an increase in load carrying capacity of 83% and 69% for W4 and W1, respectively. Figure 3-13 shows the load and moment-curvature responses of these two specimens. Also note that Specimens W2 and W3, with a same $f_{cu}$ of 32.6MPa but varied in thickness (120mm and 100mm, respectively), exhibited a similar trend.

4. Lightly reinforced slabs performed better from the perspective of load capacity enhancement under arching action. This can be observed from the comparison of load carrying capacities between
specimens W4, W5 and W6 (Figure 3-14). These specimens had identical cross-sectional geometry and concrete strength but varied in reinforcement percentages. Using the results obtained from plastic analysis as the base of comparison, the experimental results showed that the load capacity enhancement dropped from 49% for W5 to only 21% for W6. Similarly, analytical results from the proposed model predicted that if laterally rigidly restrained, the load capacity of W5 would be more than twice (2.13) of that obtained from plastic analysis. This factor dropped to 1.83 for W4 and further reduced to 1.56 for W6.

5. W7, the only specimen in this series that had no top layer reinforcement, was the solely case that did not show a correlative trend. The proposed model produced an ultimate load very close to the experimental value. In other words, the model might underestimate the enhancement due to arching action for structures unreinforced in the negative bending region. From the moment-curvature responses and strain distribution profiles (Figure 3-15), it can be seen that the failure is due to the sudden crush of concrete at the ends of the specimen. The proposed model expected that without flexural reinforcement provided for negative bending, W7 had low negative moment capacity and so the load capacity should be lower than a doubly-reinforced design. This might not be the case when looking at the overall behavior of the specimen. The end regions may have been disturbed by the stress from the supports and from the axial restraints, thus behaving like a D-region. Under this condition the "plane-section remains plane" assumption, which the proposed model is built upon, is no longer valid. Nonetheless, this is unlikely to occur in bridge deck slabs or general slab-on-columns designs of which reinforcements are isotropically or orthotropically continuous or spliced, and negative bending strength is provided. The plane-section assumption remains valid in those scenarios.
Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-12. Structural responses of specimens W3 and W4
Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-13. Structural responses of specimens W1 and W4
Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-14. Structural responses of specimens W4, W5 and W6
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions
Part C) Top-surface and bottom-surface strain distribution profiles of W3 and W7 (half span shown); note the difference in strain distribution at the end-regions

Figure 3-15. Structural responses of specimens W3 and W7; note without reinforcement for negative bending, W7 had less strain reserve and behaved much weaker

3.4 T-Series Specimens

The experimental program that involved the T-series specimens was documented in an article published in the Proceedings of the Institute of Civil Engineers: Structures & Buildings in 2001, titled "Arching Action in high-strength concrete slabs". All relevant data, including material properties, specimen dimensions and experimental results were taken from this journal article.

The previous two series of specimens all dealt with normal strength concrete. Since the results of those have demonstrated that the strength of a laterally restrained slab is sensitive to both the degree of external restraint and concrete compressive strength, it follows that the study of arching action in high-strength concrete beams would be beneficial and hence the T-series specimens were included as part of the validation process.
3.4.1 General Arrangement

The T-series specimens were slab strips representative of a typical section from a bridge deck slab at full-scale. The original experimental program consisted of two phases of testing. The eight slab strips from Phase 1 had $f_{cu}$ varying between 30MPa and 100MPa, and longitudinal reinforcement of 0.68% in top and bottom. For the Phase 1 slabs, the ratio of external lateral restraint stiffness to the test slab stiffness ($K_r/K_s$) was calculated to be 0.12:1 for the 30MPa specimen, decreasing with increase in concrete compressive strength to a ratio of 0.08:1 for the 100MPa specimens. Seven additional slabs with concrete strengths varying between 30MPa to 100MPa were carried out in the Phase 2 tests using an increased external lateral restraint stiffness, 0.29:1 for the 40MPa specimen and decreasing with increase in concrete compressive strength to a ratio of 0.17 for the 97MPa specimen. Test parameters in the phase 2 tests including loading arrangement, addition of steel fibers, and reinforcement percentage and position. Nine specimens that were relevant to the current research objective and scope were chosen among the fifteen for investigation. Figure 3-16 shows the general test arrangement and Table 3-5 summarizes the geometric and material properties and boundary conditions for the chosen specimens.

![General arrangement, cross-section dimension and reinforcement position of T-series specimens](image)

Figure 3-16. General arrangement, cross-section dimension and reinforcement position of T-series specimens

Load was applied at the midspan of each specimen. End-restraint was provided by a self-straining stiff steel frame. The steel reinforcements were deformed high yield 12mm diameter bars with a yield strength of approximately 500MPa. Except T6, in which there was only reinforcement in the bottom, T9, in which reinforcement was placed in the center of the slab, and T8, in which there was no reinforcement provided, all specimens were reinforced in the top and bottom (T & B) and each had a 40mm concrete cover.
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Table 3-5. Properties of T-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Original specimen label</th>
<th>Longitudinal reinforcement and ratio</th>
<th>$f_{cu}$(Mpa)</th>
<th>Stiffness ratio $K_r/K_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>S1</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>31.2</td>
<td>0.12</td>
</tr>
<tr>
<td>T2</td>
<td>S2</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>40.8</td>
<td>0.11</td>
</tr>
<tr>
<td>T3</td>
<td>S3</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>64.5</td>
<td>0.09</td>
</tr>
<tr>
<td>T4</td>
<td>S4</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>82.2</td>
<td>0.08</td>
</tr>
<tr>
<td>T5</td>
<td>S5</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>101.1</td>
<td>0.08</td>
</tr>
<tr>
<td>T6</td>
<td>S8</td>
<td>-</td>
<td>100.1</td>
<td>0.08</td>
</tr>
<tr>
<td>T7</td>
<td>S9</td>
<td>$3\phi 12, \rho = 0.68%$</td>
<td>89.3</td>
<td>0.18</td>
</tr>
<tr>
<td>T8</td>
<td>S10</td>
<td>-</td>
<td>90.5</td>
<td>0.18</td>
</tr>
<tr>
<td>T9</td>
<td>S11</td>
<td>$3\phi 12, \rho = 0.68%, center$</td>
<td>96.8</td>
<td>0.17</td>
</tr>
</tbody>
</table>

3.4.2 Modeling of the Specimens

Modeling of the T-series was similar to those discussed previously. For high strength concrete stress-strain relationship modeling (i.e. $f'_{c} > 50$MPa), $\varepsilon_{c3}$ and $e_{cu3}$ were modified accordingly based on the formulations provided in the MC2010 draft code:

$$\varepsilon_{c3}(\%_0) = 1.75 + 0.55[(f'_{c} - 50)/40]$$

$$\varepsilon_{cu3}(\%_0) = 2.6 + 35[(90 - f'_{c})/100]$$

Since the $K_r/K_s$ ratio of these specimens were low (ranging from 0.08 to 0.18), it was expected that the proposed method would produce overestimated results. The amount of overestimation was expected to be proportional to the degree of lateral restraint that the specimens possessed in the loadings tests.

3.4.3 Results and Discussion

Analytical results from this series of investigation further proved that concrete strength plays a significant role in laterally restrained concrete structures. Also, comparing between the experimental and analytical results, it is promising that the proposed model works equally well for high strength concrete slab strips.
Table 3-6. Comparison of experimental and analytical results of T-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experiment results</th>
<th>Proposed's model</th>
<th>Accuracy, Proposed's model</th>
<th>Plastic analysis</th>
<th>Accuracy, plastic analysis</th>
<th>Experiment/plastic analysis</th>
<th>Proposed/plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{ult,e}$ (kN)</td>
<td>$P_{ult}$ (kN)</td>
<td>$N_{ult}$ (kN)</td>
<td>$P_{ult}/P_{ult,e}$</td>
<td>$P_{ult,p}$ (kN)</td>
<td>$P_{ult,e}/P_{ult,p}$</td>
<td>$P_{ult}/P_{ult,p}$</td>
</tr>
<tr>
<td>T1</td>
<td>135</td>
<td>182.1</td>
<td>365</td>
<td>1.35</td>
<td>91.5</td>
<td>0.68</td>
<td>1.48</td>
</tr>
<tr>
<td>T2</td>
<td>145</td>
<td>217.3</td>
<td>460</td>
<td>1.50</td>
<td>93.4</td>
<td>0.64</td>
<td>1.55</td>
</tr>
<tr>
<td>T3</td>
<td>175</td>
<td>303.4</td>
<td>700</td>
<td>1.73</td>
<td>94.7</td>
<td>0.54</td>
<td>1.85</td>
</tr>
<tr>
<td>T4</td>
<td>187</td>
<td>338.2</td>
<td>780</td>
<td>1.81</td>
<td>94.7</td>
<td>0.51</td>
<td>1.97</td>
</tr>
<tr>
<td>T5</td>
<td>192</td>
<td>367.5</td>
<td>850</td>
<td>1.91</td>
<td>94.7</td>
<td>0.49</td>
<td>2.03</td>
</tr>
<tr>
<td>T6</td>
<td>183</td>
<td>281.2</td>
<td>720</td>
<td>1.54</td>
<td>64.6</td>
<td>0.35</td>
<td>2.83</td>
</tr>
<tr>
<td>T7</td>
<td>252</td>
<td>359.8</td>
<td>840</td>
<td>1.43</td>
<td>94.7</td>
<td>0.38</td>
<td>2.66</td>
</tr>
<tr>
<td>T8</td>
<td>200</td>
<td>247.1</td>
<td>690</td>
<td>1.24</td>
<td>0.0</td>
<td>0.00</td>
<td>-</td>
</tr>
<tr>
<td>T9</td>
<td>223</td>
<td>316.8</td>
<td>820</td>
<td>1.42</td>
<td>68.3</td>
<td>0.31</td>
<td>3.27</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td>182.5</td>
<td>365</td>
<td>1.35</td>
<td>91.5</td>
<td>0.68</td>
<td>1.48</td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td>0.23</td>
<td>25.0</td>
<td>1.35</td>
<td>91.5</td>
<td>0.68</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Figure 3-17. Correlation between experimental and analytical results of the T-series specimens

Table 3-6 summarizes and Figure 3-17 illustrates the correlation between the measured and the predicted ultimate loads. Similar to the W-series experiment setup, these slab strips were not rigidly restrained at the supports and hence the predicted results were proportionally higher. For specimens T1 to T6, in which the degree of restraint (i.e. $K/K_s$ ratio) ranged from only 0.08 to 0.12, the proposed model predicted ultimate loads that were 35 to 91% higher than the experimental results. On the other hand, for specimens T9 to T11, in which the degree of restraint improved to between 0.17 and 0.18, the overestimation amount
dropped to between 24% and 43%. These results demonstrate that even for mildly restrained concrete slabs, the load enhancement due to arching action can still be significant.

Figure 3-18 plots the analytical results for T1 to T5. These specimens had identical cross-sectional geometry, span length and percentage of reinforcement but varied in concrete strengths. It illustrates that the load-carrying capacity increases with increase in concrete compressive strength. Also, the rate of increase reduces for higher strength concretes (i.e. T4 and T5) due to the fact that they crush at lower ultimate compressive strains when $f'_c$ exceeds 50Mpa.

An intriguing observation from this series of investigation is that reinforcement position also substantially influences the structural response of laterally restrained concrete slabs. As seen from the comparison of midspan-loading versus lateral restraining force between T5, T6 and T9 (Figure 3-19), the predictions from the proposed model conformed to the experimental results on the fact that concrete slabs reinforced in the center can sustain a much higher level of gravity loading than those that are reinforced near the bottom, and can almost compete with those that are reinforced in both top and bottom. This finding coincides with the results from an investigation done by Taylor et al. (2007) on the serviceability of bridge deck slabs under arching action. They found that deck slabs with center reinforcement had strengths far in excess of the design ultimate loads and behaved in a similar manner to those with top and bottom reinforcements up to a high level of applied loads. This finding indicates that substantial economies in the amount of reinforcement can be made. Due to the increase in the concrete cover, this also provides an enhancement in the overall durability of bridge deck slabs without affecting its serviceability capacity.

In addition, specimen T8 proved the salient load capacity of unreinforced but laterally restrained high strength concrete slabs. With a $K_r/K_s$ Ratio of 0.18, T8 could be loaded up to 200kN. This value is 79.4% of the ultimate load of T7, which had about the same level of concrete strength but had 0.68% of reinforcements in the top and bottom. Also, the proposed model predicted that if rigidly restrained, this specimen could reach an ultimate load of 247kN.
**Part A** Arching action development as reflected in lateral restraining force (N) versus applied load (P)

**Part B** Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-18. Structural response of specimens T1 to T5
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-19. Structural responses of specimens T5, T6 and T9
Chapter 3. Validation and Evaluation of the Proposed Analytical Model

Part A) Arching action development as reflected in lateral restraining force (N) versus applied load (P)

Part B) Arching action development as reflected in midspan and end-of-beam moment-curvature distributions

Figure 3-20. Structural responses of specimens T7 and T8
3.5 M-Series Specimens

The experimental program that involved the M-series specimens was documented in an article published in the Structural Concrete Journal in 2006, titled "Load-deflection behavior of restrained RC slab strips". All relevant data, including material properties, specimen dimensions and experimental results were taken from this journal article.

The M-series specimens were included as part of the validation process mainly because the authors claimed a highly rigid lateral support was provided for the load tests. Since the previous two series of specimens were tested under mild lateral restraints, it is valuable to have one more set of data to validate the accuracy of the proposed model. Also, this series of specimens had a four-point loading arrangement that was not examined in the preceding studies.

3.5.1 General Arrangement

The experimental program consisted of casting and testing ten restrained slab strips. The main study parameters were percentage of reinforcement and span-to-depth ratio. Figure 3-21 shows the general arrangement of the slabs. All specimens were tested under four-point loads to simulate a uniformly distributed load. Table 3-7 summarizes the geometric and material properties of the specimens. Each specimen had a 910mm clear span, and a width of 500mm. Specimens M1 to M5, with thicknesses of 50mm, gave a span-to-depth ratio of 18.2; the rest, measured 65mm in depth, had a span-to-depth ratio of 14. The reinforcement used in all the slabs was 5mm Torkari steel with a yield strength of 550MPa. The restraint at the supports was achieved by clamping the slabs to a steel frame with five bolts in each end of the slabs.
Table 3-7. Geometric and material properties of M-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Original specimen label</th>
<th>Geometric dimensions</th>
<th>Longitudinal reinforcement and ratio</th>
<th>$f_{cu}$ (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>S1</td>
<td>b (mm) 500 h (mm) 50 l (mm) 910</td>
<td>$l/h$ 18.2</td>
<td>$3\phi5,$ $\rho = 0.35%$ $3\phi5,$ $\rho = 0.35%$ 40</td>
</tr>
<tr>
<td>M2</td>
<td>S2</td>
<td>b (mm) 500 h (mm) 50 l (mm) 910</td>
<td>$l/h$ 18.2</td>
<td>$4\phi5,$ $\rho = 0.44%$ $4\phi5,$ $\rho = 0.44%$ 40</td>
</tr>
<tr>
<td>M3</td>
<td>S3</td>
<td>b (mm) 500 h (mm) 50 l (mm) 910</td>
<td>$l/h$ 18.2</td>
<td>$3\phi5,$ $\rho = 0.52%$ $3\phi5,$ $\rho = 0.52%$ 40</td>
</tr>
<tr>
<td>M4</td>
<td>S4</td>
<td>b (mm) 500 h (mm) 50 l (mm) 910</td>
<td>$l/h$ 18.2</td>
<td>$4\phi5,$ $\rho = 0.61%$ $4\phi5,$ $\rho = 0.61%$ 50.37</td>
</tr>
<tr>
<td>M5</td>
<td>S5</td>
<td>b (mm) 500 h (mm) 50 l (mm) 910</td>
<td>$l/h$ 18.2</td>
<td>$3\phi5,$ $\rho = 0.70%$ $3\phi5,$ $\rho = 0.70%$ 50.37</td>
</tr>
<tr>
<td>M6</td>
<td>S6</td>
<td>b (mm) 500 h (mm) 65 l (mm) 910</td>
<td>$l/h$ 14.0</td>
<td>$3\phi5,$ $\rho = 0.27%$ $3\phi5,$ $\rho = 0.27%$ 47.41</td>
</tr>
<tr>
<td>M7</td>
<td>S7</td>
<td>b (mm) 500 h (mm) 65 l (mm) 910</td>
<td>$l/h$ 14.0</td>
<td>$4\phi5,$ $\rho = 0.34%$ $4\phi5,$ $\rho = 0.34%$ 44.74</td>
</tr>
<tr>
<td>M8</td>
<td>S8</td>
<td>b (mm) 500 h (mm) 65 l (mm) 910</td>
<td>$l/h$ 14.0</td>
<td>$3\phi5,$ $\rho = 0.40%$ $3\phi5,$ $\rho = 0.40%$ 44.74</td>
</tr>
<tr>
<td>M9</td>
<td>S9</td>
<td>b (mm) 500 h (mm) 65 l (mm) 910</td>
<td>$l/h$ 14.0</td>
<td>$4\phi5,$ $\rho = 0.47%$ $4\phi5,$ $\rho = 0.47%$ 44.74</td>
</tr>
<tr>
<td>M10</td>
<td>S10</td>
<td>b (mm) 500 h (mm) 65 l (mm) 910</td>
<td>$l/h$ 14.0</td>
<td>$3\phi5,$ $\rho = 0.54%$ $3\phi5,$ $\rho = 0.54%$ 44.74</td>
</tr>
</tbody>
</table>
3.5.2 Modeling of the Specimens

Several assumptions and simplifications were made in the modeling of M-series specimens:

1. The top longitudinal reinforcements were assumed to be continuous across the entire span, and were able to develop negative moments to the maximum possible amount at the supports. Detailed dimensioning was not provided in the reference article.

2. Effective depth of the slab section, d, was assumed to be 45mm for the 50mm thick slabs, and 60mm for the 65mm slabs. This assumption coincided with the settings the authors used for parametric studies of their specimens. Actual effective depths were not given in the reference article.

3. The bolts connecting to the two end-supports were able to achieve perfectly rigid restraint. This was a valid assumption because according to the authors no rotational or in-plane deformations at the supports were observed during the tests.

4. The cylinder compressive strength of concrete ($f'_c$) was assumed to be 80% of the cube strength ($f'_{cu}$) reported in the article.

5. Because the proposed method is still at a two dimensional stage, two line-loads of uniformly distributed forces were assumed instead of the four point loading system used in the actual test. The resultant internal moment diagram was expected to be similar to the one shown below:

![Internal Moment Diagram](image)

Figure 3-22. Simplified loading condition and the corresponding internal moment distribution of the W-series specimens
3.5.3 Results and Discussion

Table 3-8 lists the comparison between analytical and experimental ultimate loads of the M-series specimens.

Table 3-8. Comparison of experimental and analytical results of M-series specimens

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Experiment results</th>
<th>Proposed's model</th>
<th>Accuracy, Proposed's model</th>
<th>Plastic analysis</th>
<th>Accuracy, plastic analysis</th>
<th>Experiment/plastic analysis</th>
<th>Proposed/plastic analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{ult,e}$ (kN)</td>
<td>$P_{ult}$ (kN)</td>
<td>$N_{ult}$ (kN)</td>
<td>$P_{ult}/P_{ult,e}$</td>
<td>$P_{ult,p}$ (kN)</td>
<td>$P_{ult,p}/P_{ult,e}$</td>
<td>$P_{ult,e}/P_{ult,p}$</td>
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<td>M1</td>
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<td>45.6</td>
<td>105</td>
<td>1.10</td>
<td>32.9</td>
<td>0.79</td>
<td>1.26</td>
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<td>110</td>
<td>0.89</td>
<td>40.8</td>
<td>0.71</td>
<td>1.41</td>
</tr>
<tr>
<td>M3</td>
<td>80.0</td>
<td>56.4</td>
<td>115</td>
<td>0.70</td>
<td>48.5</td>
<td>0.61</td>
<td>1.65</td>
</tr>
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<td>M4</td>
<td>73.6</td>
<td>67.8</td>
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<td>0.92</td>
<td>56.8</td>
<td>0.77</td>
<td>1.30</td>
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<tr>
<td>M5</td>
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<td>73.2</td>
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<td>0.91</td>
<td>64.5</td>
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<td>89.8</td>
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<td>64.2</td>
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<td>1.20</td>
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<td>M10</td>
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<td>105.9</td>
<td>165</td>
<td>0.93</td>
<td>84.5</td>
<td>0.74</td>
<td>1.35</td>
</tr>
</tbody>
</table>

|          | Average 1.04   | Average 0.77   | Standard deviation 0.25 | Standard deviation 0.08 |

Figure 3-23. Correlation between experimental and analytical results of M-series specimens

The ratios of computed ultimate load to the experimental ultimate load ($P_{ult}/P_{ult,e}$) range from 0.70 to 1.62. The average ratio of $P_{ult}/P_{ult,e}$ is 1.04 with a standard deviation coefficient of 0.25. These results are about
the same level of accuracy than those computed using the deflection-based method proposed by the authors. With an average $P_{ult}/P_{ult,e}$ ratio of 0.94 and a standard deviation of 0.24, their method produced $P_{ult}/P_{ult,e}$ ratios in the range between 0.65 and 1.49.

Probations were carried out for the extreme cases in which there existed relatively significant discrepancy between the experimental and computed ultimate loads. These included specimens M3 and M6.

It was noted that for specimen M3, the proposed method produced an ultimate load of 56.4kN, which was only 70% of the experimental result (80kN). However, comparisons between the material properties and experimental ultimate loads of M3, M4 and M5 revealed that even for specimens in the same geometry but cast in higher strength concretes and had more reinforcement in the longitudinal direction, the ultimate loads were below 80kN. Further investigations on this discrepancy would be beneficial but without detailed data available it was difficult to draw any conclusions.

The proposed model also over estimated the ultimate load capacity of specimens M6 by a factor of 1.62. The method introduced by the authors, based on idealized load-deflection behaviors, also overestimated the load capacity of M6 substantially to a factor of 1.49. However, since the experimental ultimate load reported was only 10.6% higher than the theoretical value obtained from plastic analysis, it remains dubious whether the inconformity was due to malfunction of the restraining devices or the proposed model failed to capture the behavior properly when based on the simplifications noted in Section 3.5.2.

Other than M3 and M6, the rest of the specimens show relatively agreeable results, with an average $P_{ult}/P_{ult,e}$ ratio of 1.01 and a standard deviation of 0.12.

The variation in load enhancement versus percentage of reinforcement for specimens M6 to M10 obtained using the proposed method is plotted in Figure 3-24. The load enhancement factor decreases with increase in percentage of reinforcement. In other words, the lesser the quantity of steel for a given section, the more load a restrained beam can sustain. This trend agrees quite well with other investigations reported in literature. For instance, in Robert's (1969) experiment of load-carrying capacity of slab strips restrained against longitudinal expansion, it was found that when percentage of reinforcement dropped from 0.92% to 0.23%, the ratio of $P_{ult}/P_{ult,e}$ increased from less than 5 to over 17.
Lastly, comparison of the ultimate loads between the first five and last five specimens also confirms that a decrease in slab thickness reduces the load enhancement factor of laterally restrained slab strips. For the first five specimens, whose slab thickness were 50mm, the load enhancement ranges from 10% to 38%. For the last five specimens, whose slab thickness were 65mm, the load enhancement effect improved, ranging from 25% to 0.79%.

3.6 Conclusion

The four series of validation and evaluation studies show that the proposed analytical model, except under some isolated cases, can relatively accurately predict the load-carrying capacity of one-way, laterally restrained slab strips.

In the S-series and M-series specimens, for which rigid boundary conditions were available, the analytical results based on the proposed model agreed well with the experiment records. The average analytical to experimental load ratios were 0.98 and 1.04 for these two series, respectively. Conversely, the plastic collapse mechanism approach substantially underestimated the ultimate loads of these restrained beams and one-way slabs, giving mean ratios of predicted to test load of 0.52 and 0.77, respectively.

In the W-series and T-series specimens, for which the boundary conditions were less rigid, analytical results produced using the proposed model generally overestimated the load-carrying capacity of each specimen. However, this is not due to drawbacks of the proposed model but the fact that at current stage the strain compatibility criteria implied in the proposed model accounts for perfectly rigid boundary
conditions only. In addition, although the proposed model overestimated the ultimate loads, it performed reasonably well in predicting the structural response trend of partially rigidly restrained beams. Therefore, it is promising that once the model is augmented with strain compatibility criteria for variable degree of lateral restraint, better correlation with test results will be obtained.

Generally, the following observations can be drawn from the valuation studies:

• For laterally restrained beams having identical cross-sectional geometry and span length, the load enhancement factor decreases as the percentage of reinforcement increases.

• The load enhancement factor increases with increase in concrete compressive strength.

• The load enhancement factor increases slightly when concrete is properly confined with transverse reinforcements.

• The load enhancement factor decreases with decrease in thickness of slab strips.

• The proposed model works equally adequately in normal strength and high strength concrete beams of different arrangements of loading, reinforcement percentages and span lengths.

• Slab strips with reinforcement in the mid-depth level have significant strength enhancements and behave in a similar manner to those with top and bottom reinforcements.
Chapter 4. Parametric Studies on Bridge Deck Slabs

4.1 Overview of the Study Parameters

The analytical work from the preceding chapter verifies that load-carrying capacity of laterally rigidly restrained beams and slab strips can be predicted by the proposed model with sufficient accuracy. Moreover, the proposed model gives valuable insights on how reinforcement percentage and position, concrete strength and confinement, loading arrangement, and slenderness of slab geometry affect the strength reserve from arching action. Hence, the proposed model can be applied confidently to the study of bridge deck slabs.

In order to investigate the load capacity enhancement of bridge deck slabs due to arching action, a systematic study was carried out with concrete strength, reinforcement ratio and position, slenderness and loading conditions being the key analysis parameters. Table 4-1 in the next page summarizes the details of these study parameters. These parameters were chosen because they are the main variables to be considered in the design of bridge deck slabs. Also, from the preceding chapter, it has been shown that they would affect the load enhancement factor to a great extend.

Concrete strength of these slab strips ranged from 30MPa to 90MPa with a 20MPa increment for each set of analysis.

Currently, the empirical design method prescribed in the Canadian Highway Bridge Design Code (CHBDC) limits the span of deck slabs to be less than 4m. In this investigation, this limit was pushed to 6m to verify the viability of expanding the deck span while maintaining adequate load-carrying capacity and serviceability.
The percentage and position of reinforcement of these slab strips can be defined in three categories: (1) 0.22% in each face (top and bottom faces of the slab strip), (2) 0.44% in the middle and, (3) 0.33% in each face. These percentages of reinforcement are defined as the ratio of nominal area of reinforcements to the gross cross-sectional area of the concrete. If the percentages of reinforcement are calculated based on the effective cross-sectional area of the concrete, they are 0.29%, 0.44% and 0.89%, respectively. These reinforcement percentages correspond to 200mm² ductile steel reinforcements in each of the top and bottom face, 400mm² reinforcements in the center layer, and 300mm² reinforcements in each of the top and bottom face of the concrete.

Gravity loading was applied either at midspan or at locations measured one-quarter span from each end (i.e. two-point loading).

Table 4-1. Summary of study parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
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<tr>
<td>Concrete compressive strength (Mpa)</td>
<td>30  50  70  90</td>
</tr>
<tr>
<td>Span (m)</td>
<td>2  4  6  6 with variable depth</td>
</tr>
<tr>
<td>Percentage of reinforcement (%)</td>
<td>0.22  0.33  0.44</td>
</tr>
<tr>
<td>Position of reinforcement</td>
<td>top and bottom</td>
</tr>
<tr>
<td>Loading arrangement</td>
<td>midspan</td>
</tr>
</tbody>
</table>

4.2 Deck Slab Strip Modeling

4.2.1 General Arrangement

For the uniform-depth slab strips, a consistent bridge deck cross-section was used throughout the analysis (Figure 4-1). This proposed cross-section was designed with reference from the specifications outlined in the latest CHBDC (2006). With a constant width \( b \) of 400mm and a uniform depth \( h \) of 225mm, the longitudinal reinforcements were assumed to have concrete covers of 50mm (except for the cases which reinforcements were concentrated in the mid-depth).
Figure 4-1. Midspan-loading cases with uniform-depth cross-sections; percentage of reinforcement varied from: a) 0.22% top and bottom, b) 0.33% top and bottom, and c) 0.44% center.

Figure 4-2. Two-point loading cases; all beams have a 6m span length, and 300mm² in top and bottom.

Figure 4-2 gives a schematic drawing of the general arrangement of two-point loading cases. A 6m span slab strip with variable-depth cross-section was included as part of the parametric study. This model was proposed because the validation studies revealed that for laterally rigidly restrained slab strips with two-point loads, there is a high demand of negative moments at the supports. A variable depth model would fulfill the demand while reducing the unnecessary moment capacity in the middle region, and improving
the overall efficiency of the member. The slab strip measured 300mm in depth at each end and tapered to 200mm at a distance measured 1.5m from the supports. The middle region had a uniform thickness of 200mm. In order to make the results comparable to those of 6m span, uniform-depth slab strips, the variable-depth models had the same volume of concrete as those in uniform-depth and measured 225mm in thickness. At the same time, they both had the same amount of top and bottom longitudinal reinforcements. In addition, two 6m span slab strips in uniform thickness of 300mm were also analyzed for comparison purposes. Concretes in the 50MPa and 70Mpa classes with 0.33% reinforcement (3-10M bars, 300mm$^2$) in the top and bottom were the primary parameters used for this set of investigation.

4.2.2 Bridge Deck Slab Strip Modeling Assumptions and Simplifications

A total of 42 slab strips were modeled. The first 36 had the gravity loading applied at the midspan. The midspan-loading cases were relatively simple to calculate and convenient for load enhancement comparison but they could hardly represent the actual load distribution in bridge deck slabs. To address this drawback, the last 7 slab strips (included the variable-depth models) had two-point loads, each applied at the quarter-span location.

The modeling of the bridge deck slab strips was no different than the validation investigations done in the previous chapter, with the following assumptions and simplifications made:

- Taking advantage of symmetry, only half of the span length of each slab strip was modeled.
- All reinforcements were assumed to yield at 400MPa.
- The slab strips were not shear-critical.
- Theoretical ultimate loads based from the plastic collapse mechanism approach were employed as the comparison base to evaluate the load enhancement due to arching action.
- For the variable depth models, the strain compatibility criteria was assumed to be the integral of strains along the top surface equals to zero, and the integral of strains along the bottom surface (vary in depths) equals to zero.

4.3 Analytical Results and Discussion

4.3.1 Midspan Loading Cases

Table 4-2 to Table 4-4 summarize the theoretical ultimate loads and lateral restraining forces of the 36 bridge deck slab strip models simulated with midspan-loading. Also included are the corresponding
midspan moments and deflections in the ultimate states. Ultimate loads calculated from plastic analysis are presented in the tables as the common denominator for the load enhancement factor calculation. The tables are categorized based on the amount of reinforcement in the slab strips. All figures listed are first-order analysis results based on the undeformed structure.

To visualize the analytical results for ultimate loads and load enhancement factors, three set of graphs are plotted and presented in page 80.

Several arching moment development curves and strain profile graphs are listed in pages 74 to 77 for the coming detailed discussions.

Table 4-2. Analytical results - load carrying capacity of slab strips with 0.22% reinforcements in the top and bottom

<table>
<thead>
<tr>
<th>Concrete Strength (MPa)</th>
<th>Span (m)</th>
<th>Plastic Analysis, Pult,p (kN)</th>
<th>Proposed Model</th>
<th>Ultimate Midspan Moment, Mm,ult (kNm)</th>
<th>Midspan Deflection (mm)</th>
<th>Load Enhancement Factor, (Pult,Pult,p)/Pult,p</th>
</tr>
</thead>
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<td></td>
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<td>450</td>
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</table>
Table 4.3. Analytical results - load carrying capacity of slab strips with 0.33% reinforcements in the top and bottom

<table>
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<tr>
<th>Concrete Strength (MPa)</th>
<th>Span (m)</th>
<th>Plastic Analysis, P_{ult,p} (kN)</th>
<th>Proposed Model</th>
<th>Ultimate Midspan Moment, M_{ult} (kNm)</th>
<th>Midspan Deflection (mm)</th>
<th>Load Enhancement Factor, (P_{ult} - P_{ult,p})/P_{ult,p}</th>
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Table 4.4. Analytical results - load carrying capacity of slab strips with 0.44% reinforcements in the mid-depth

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<th>Concrete Strength (MPa)</th>
<th>Span (m)</th>
<th>Plastic Analysis, P_{ult,p} (kN)</th>
<th>Proposed Model</th>
<th>Ultimate Midspan Moment, M_{ult} (kNm)</th>
<th>Midspan Deflection (mm)</th>
<th>Load Enhancement Factor, (P_{ult} - P_{ult,p})/P_{ult,p}</th>
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<td>14.11</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>23.44</td>
<td>157.23</td>
<td>1150</td>
<td>117.93</td>
<td>31.75</td>
</tr>
</tbody>
</table>
Chapter 4. Parametric Studies on Bridge Deck Slabs

Figure 4-3. Effect of concrete strength in the load capacity of laterally restrained slab strips

Figure 4-4. Effect of concrete strength as reflected in the load enhancement factor for the 2m span models

Figure 4-5. Effect of span length in load carrying capacity of laterally restrained slab strips
Figure 4-6. Arching action development as reflected in the midspan moment-curvature and applied-load versus lateral restraining reaction graphs for the 2m span models.
Part A) Applied load versus lateral restraining forces

Part B) Midspan- and end-moment versus curvature

Figure 4-7. Typical midspan-loading analytical results; note the symmetry in terms of magnitudes of midspan- and end-moments, also note each dot represents an iteration done for obtaining the full response. At $N = 300\text{kN}$ and $910\text{kN}$, the corresponding strain distribution profiles are shown in the next figure.
Valuable implications can be drawn from these analytical results:

1. Concrete strength is the key contributor in strength and load enhancement of restrained slab strips. For instance, in deck slab strips with 0.22% reinforcement, regardless of span length, the load enhancement factor increases from around 2.9 for slab strips in 30MPa concrete to around 7.1 for slab strips in 90MPa concrete (Figure 4-4 part a). Although only results for the 2m span slab strips are plotted in Figure 4-4, as seen in the tables, slab strips in 4m and 6m spans have almost identical load enhancement factors. Hence, from the perspective of load enhancement, the proposed model predicts that span length does not affect the development of arching action in laterally rigidly restrained slab strips. The longer span ones are equally capable of developing high levels of lateral restraining force at ultimate limit states, resulting in almost identical ultimate midspan moments.

2. In addition, from Figure 4-5, it can be seen that the load capacities of the 6m span models in 70MPa and 90Mpa concrete are higher than the 4m span ones in 30MPa concrete, regardless of their percentage of reinforcement. In other words, if cast in higher strength concrete, it is possible to extend the lateral span of bridge deck slabs to be longer than 4m.

3. Conversely, an increment of reinforcement percentage does not result in a commensurate enhancement in load-carrying capacity as expected in conventional analysis. Given a 50% increment of the percentage of reinforcement (from 0.22% to 0.33%), plastic analysis estimated
that the ultimate load would raise from 53.14kN to 78.77kN for a 2m slab strip cast in 30MPa concrete. However, under a rigidly restrained scenario, the proposed model expected only a marginal enhancement in load capacity, raising from 207.93kN to 227.24kN. These analytical results match with those from the extensive investigations done by Batchelor et al. (1987) on load studies of isotropically reinforced bridge deck slabs. Their studies concluded that slabs with isotropic reinforcement as low as 0.2% (of effective cross-section of slab) is satisfactory for the load ranges that typical bridge deck slabs in Ontario are subjected to. In addition, the load carrying capacities of slab strips with concrete strengths of 50MPa or higher and 0.44% center reinforcements marginally outperformed (by about 3%) those of slab strips with 0.33% reinforcement in top and bottom. Figure 4-5 illustrates how steel reinforcement, especially for bridge decks in which the amount of reinforcement is low, plays a much less significant role in load capacity than concrete strength. Comparing between Table 4-3 and Table 4-4, it can be seen that the load enhancement factors even dropped for slab strips containing more reinforcement.

4. When looking at the entire arching action development response for slab strips shown in Figure 4-6, an almost linear profile of applied load versus lateral restraining force is obtained for every case, regardless of span lengths, percentage of reinforcement, and concrete strength. When juxtaposing these response graphs one next to the other, it can be seen that increasing the concrete strength simply extends the load capacity of these slab strips linearly. At the same time, the moment-curvature responses become stiffer for those with higher concrete strengths.

5. For the midspan loading cases with sectional forces calculated based on the undeformed geometry, the failure mode is always simultaneous crushing of concrete at midspan and at the supports (Figure 4-7). The resulting internal moment-curvature distribution is hence similar to the S-series specimens, S1 to S3, examined in the preceding chapter (Figure 3-4). Furthermore, the iterations for each stage of loading show that the midspan- and end-moments always maintain equal in magnitude. Because of the strain compatibility criteria imposed in the proposed method, this is a reasonable moment distribution as they produce top and bottom surface strain distributions that integrate to zero (Figure 4-8). Theses strain distribution profiles also illustrate that crushing of concrete only occurs at the very ends of the slab strips and right at the midspan. Strain distribution in between the extreme regions is almost linear and low in magnitude. A considerable length of the span between the end and the midspan even has compressive stresses across the entire depth, since
both the top and bottom surfaces in that region have negative strains. It indicates that even at the ultimate states cracking would occur only in isolated regions and the remaining is intact.

6. The deflection-over-span ratios at ultimate states remain low for all slab strips. For instance, in slab strips with 0.22%, double-layered reinforcement (Table 4-2), the ratio raised from less than 1/600 (~3mm) for a 2m span to about 1/200 (~28mm) for a 6m span. The calculated midspan deflections are low as compared to those obtained from the unrestrained condition at \( N = 0 \text{kN} \), which are 16.3mm and 54.7mm for the 50MPa class concrete in 2m and 6m spans, respectively (refer to Appendix A for a sample midspan deflection calculation). Because of the low deflections obtained, the proposed model expects that for rigidly restrained slab strips, second-order effects caused by deflections would not be significant.

### 4.3.2 Two-point Loading Cases

Table 4-5. Analytical results of the two-point loading cases

**Part A) Ultimate loads and midspan deflections**

<table>
<thead>
<tr>
<th>Span (m)</th>
<th>Thickness, ( h ) (mm)</th>
<th>( f'c ) (MPa)</th>
<th>Reinforcement (mm²)</th>
<th>Proposed's model</th>
<th>Plastic Analysis</th>
<th>Proposed/Plastic Midspan Deflection (mm)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>T: top; B: bottom</td>
<td>( P_{ult} ) (kN)</td>
<td>( N_{ult} ) (kN)</td>
<td>( P_{ult,p} ) (kN)</td>
</tr>
<tr>
<td>6</td>
<td>225</td>
<td>30</td>
<td>300 T &amp; B</td>
<td>95.5</td>
<td>315.0</td>
<td>105.0</td>
</tr>
<tr>
<td>6</td>
<td>225</td>
<td>50</td>
<td>300 T &amp; B</td>
<td>140.0</td>
<td>510.0</td>
<td>106.5</td>
</tr>
<tr>
<td>6</td>
<td>225</td>
<td>70</td>
<td>300 T &amp; B</td>
<td>163.7</td>
<td>610.0</td>
<td>107.2</td>
</tr>
<tr>
<td>6</td>
<td>Variable</td>
<td>50</td>
<td>300 T &amp; B</td>
<td>202.2</td>
<td>650.0</td>
<td>122.5</td>
</tr>
<tr>
<td>6</td>
<td>Variable</td>
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<td>300 T &amp; B</td>
<td>242.2</td>
<td>800.0</td>
<td>123.2</td>
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<td>6</td>
<td>300</td>
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<td>300 T &amp; B</td>
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<td>300 T &amp; B</td>
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<td>745.0</td>
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**Part B) Ultimate moments**

<table>
<thead>
<tr>
<th>Span (m)</th>
<th>Thickness, ( h ) (mm)</th>
<th>( f'c ) (MPa)</th>
<th>Proposed's model</th>
<th>( M_{m,ult} ) (kN( \cdot )m)</th>
<th>( M_{e,ult} ) (kN( \cdot )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>225</td>
<td>30</td>
<td>23.7</td>
<td>-48.0</td>
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<tr>
<td>6</td>
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<td>50</td>
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<td>Variable</td>
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<td>300</td>
<td>70</td>
<td>71.0</td>
<td>-126.6</td>
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</table>
Table 4-5 shows the loads, end moments, midspan moments, midspan deflections and load enhancement factors at the failure states for the seven slab strips modeled with two-point loading. Figure 4-9 offers a visualized comparison of load-carrying capacities for the models studied that had 50MPa or 70MPa concrete strength. Figure 4-10 compares the subtle difference in strain distribution between a uniform-depth slab strip and a variable-depth one. Several insightful points can be drawn from the analytical results:

1. For the same amount of concrete volume used, under two-point loading, a variable-depth cross-section slab strip is more efficient in terms of load-carrying capacity. The variable-depth models
have 44% and 47% more load capacity than models in 225mm uniform thickness. Moreover, both variable-depth models have ultimate loads that are over 90% of those of in 300mm uniform thickness. In terms of concrete volume, the former designs are only 75% of those of the latter. Thus, under a laterally rigidly restrained condition, a variable-depth design is a more efficient and cost-effective solution for longer span slabs.

2. The two-point loads cases demand a much higher magnitude of negative moment at the supports than the positive moment in the midspan region between the quarter-span points.

3. The variable-depth cross-section design has a much higher load enhancement factor because it increases the negative moment capacity while reducing the unnecessary positive moment capacity. The corresponding strain distribution profile (Figure 4-10) shows the overall strain increases for the variable-depth slab strip; it indicates a more efficient design.

4. However, because of the reduced thickness in the midspan region, the variable depth models have higher ultimate deflections. Further investigations are required to validate if that level of deflection will affect the serviceability of bridge deck slabs.

4.4 Concluding Remarks and Comments on Practical Applications

In this chapter, the theoretical results of applying the proposed model to calculate load-carrying capacities of one-way slab strips with dimensions comparable to CHBDC prescribed bridge deck slabs are presented and discussed. This parametric study confirms the possibility of a longer span, lightly reinforced bridge deck slab design under certain circumstances. Based on the results obtained, the following conclusions and recommendations can be made on the design of bridge deck slabs:

1. Given a predetermined span length and cross-section geometry, concrete strength is the primary factor that will control the load-carrying capacity of a bridge deck slab that has sufficient lateral restraints. Therefore, when adequate lateral rigidity is available, it is recommended to cast concrete deck slabs with higher strength concretes. By using 50MPa concrete instead of 30MPa, for instance, the load capacity will be increased by more than 50% under the fully restrained condition.

2. Given an equal amount of reinforcement, centrally reinforced slab strips show higher load-carrying capacities than those that are doubly-reinforced. For a cost-effective bridge deck construction and to reduce cost for maintenance, it is advisable to orthotropically reinforce bridge deck slabs at the mid-depth level with one layer of reinforcement in each direction.
3. The analytical results confirm the viability of expanding the span of bridge deck slabs up to 6m while retaining the percentage of reinforcement at the current CHDBC prescribed level. This can be achieved by utilizing a variable-depth deck lateral cross-section design and use high strength concrete (50MPa and above). By increasing the lateral span, the number of girders required for a slab-on-girders bridge will be reduced, eventually reducing the overall bridge construction cost.

Nevertheless, these are only commentaries drawn from the exhaustive but still limited analysis conducted using the proposed model. To confidently apply them to the design of bridge deck slabs, detailed studies on the variable degree of lateral restraints, load-deflection behavior, and a rational application of the proposed model to compressive membrane action response in two-way slabs are necessary before any practical applications can be made.
Chapter 5. Conclusions and Recommendations

5.1 Conclusions

The current research reviewed the behavior of reinforced concrete slab strips under arcing action, examined the currently existing theories to model this behavior, and subsequently, a new analytical model for laterally rigidly restrained one-way concrete slabs was proposed.

Unlike the existing theories, most of which rely on plastic-elastic material models with plastic-hinges assumed at critical locations, the proposed model takes material nonlinearity, force equilibrium relationships, and strain compatibility into considerations. The method of calculation developed accommodates these different aspects through two loops of iteration cycles.

Four series of laboratory specimens were selected from recent experiment programs done for related subjects. These specimens represented a wide range of structural properties in terms of loading arrangement, slenderness, concrete strength, degree of restraint, percentage and position of reinforcement, etc. The calculated results proved that the proposed model predicts relatively reliable ultimate loads, and the expected structural behavior generally matches with other experimental and field observations reported in literature.

The proposed model was then applied to a parametric study of bridge deck slab strips, with concrete strength, span length, and amount and position of reinforcement being the key investigation variables. Both midspan- and two-point- loading conditions were examined. The analytical results suggest that if cast in higher strength concrete, it is possible to extend the lateral span limit of bridge deck slabs to be over 4m. The analytical results also suggest that one layer of orthogonal reinforcement, positioned in the
mid-depth of bridge deck slabs, may be sufficient in fulfilling the moment demand for both positive and negative bending.

Besides accuracy, another advantage about the proposed model is its ability in producing solutions not only for the ultimate states but a complete of structural response curves can be obtained. These include applied load-axial reaction response, moment-curvature response, strain distribution profiles, and load-deflection relationships. If the proposed model is to be applied to the design of reinforced concrete slabs, these structural response curves would be useful for analyzing the behavior of concrete slabs in both the service limits states (SLS) and ultimate limit states (ULS).

On the other hand, since the proposed model is built upon material stress-strain relationships and does not rely on the assumption of plastic hinges, its application to unconventionally reinforced concrete slabs, such as precast concrete slabs reinforced in glass fiber reinforced polymer (GFRP, which does not exhibit a yield plateau like steel reinforcing bars) is possible if reliable constitutive relationships are available for these materials.

5.2 Limitations

At current stage, the proposed analytical model is still in a rudimentary form, with limitations come from three different aspects:

- Although the strain compatibility criteria proposed for laterally restrained concrete slabs is theoretically also applicable to two-way slabs, the method of calculation developed can only be utilized for two-dimensional, one-way slabs.

- The zero-elongation strain compatibility criteria of the proposed model is applicable to laterally rigidly restrained structures only. In reality, bridge deck slabs, similar to other slab-on-columns structures, are only partially restrained. The degree of restraint depends on the rigidity of the surrounding deck slab, the girders underneath, and the diaphragms nearby.

- Since the proposed model assumes a perfectly rigid boundary condition, the deflections obtained from first-order analysis are low and the overall structure always behaves in a stiffer manner comparing to the unrestrained or partially-restrained experimental observations. This behavior needs to be verified. As a start, an experiment program is proposed as part of the current research to fulfill the purpose. Details of the experiment proposal are included in Appendix C and will be discussed in the next session.
5.3 Recommendations for Future Investigations

5.3.1 Experiment Proposal
This research study aims for improving the design of bridge deck slabs. Throughout the past decades, although numerous theoretical studies and laboratory tests that had been carried out were directly related to arching action in reinforced concrete beams or slabs, each of these investigations had its limitations and many specimens tested and reported in literature were not representable as "scaled models" for bridge deck slabs in longer spans. As seen in Chapter 3, most specimens reported were small in scale and had span-to-depth ratios below 20, whereas a 225mm thick deck slab that spans 6m has a span-to-depth ratio of 26.7. Some experiments were designed with insufficient amount of lateral restraint, or with sufficient lateral restraint but the corresponding magnitude of lateral restraining forces were neither measured nor reported. At current stage, although a number of models for accommodating variable degree of lateral restraint in concrete slabs are available (Rankin, 1997), the level of accuracy is still questionable. Hence, it is imperative to design and carry out a series of experiments that are representable of bridge deck slabs in terms of scale and material properties, and that relevant statistics can be obtained comprehensively for further investigations.

A brief examination on the available instrumentation devices, equipment and floor-layout of the structural testing lab at the University of Toronto revealed that the demand on a rational experimental investigation with comprehensive instrumentation for bridge deck slabs under arching action can be fulfilled. A brief experimental proposal is presented in Appendix C for verifying the accuracy of the proposed model on predicting the response of bridge deck slabs under arching action.

5.3.2 Recommendations for Improving the Proposed Model
To confidently apply the proposed model to the design of bridge deck slabs, detailed studies on the rigidity of lateral restraints, load-deflection behavior, and a rational approach of adopting the proposed model to the analysis of compressive membrane action in two-way slabs are necessary.

The following recommendations are proposed for future investigations of arching action in bridge deck slabs:

- Conduct detailed studies on the degree of lateral restraint exists in bridge deck slabs of different systems.
• Develop formulations that can accurately delineate the variable degree of lateral restrains exist in different bridge deck slab systems. Integrate the formulations, in a form of strain compatibility criteria, to the proposed arching action model, and then incorporate the compatibility criteria to develop a method to calculate the load capacity and structural response of partially restrained slabs.

• Expand the proposed model from a two-dimensional, one-way slab stage to a three-dimensional one that can calculate the structural behavior of laterally restrained two-way slabs. A possible approach would be:

   1. Distribute the applied loads acting on a two-way slab in the x- and y-directions based on the available level of stiffness in each direction.

   2. Calculate the load-carrying capacity in each direction using the proposed model.

   3. Apply principle of superposition, summing up the load-carrying capacity from each direction to the get total load.

• Develop a protocol that defines the service limit states and ultimate limit states of bridge decks designed based on the proposed model.

• Simplify the proposed model to suit hand calculations for design estimations, or as an alternative, integrate the proposed model to nonlinear concrete structure analysis programs.

It is promising that the proposed analytical model for arching action can eventually help to develop more efficient and cost-effective reinforced concrete slab systems.
References


Eyre, J.R. “Strength Enhancement in Reinforced Concrete Slabs Due to Compressive Membrane Action.” University College London (University of London).


Hognestad, E., N.W. Hanson, and D. McHenry. 1955. Concrete Stress Distribution in Ultimate Strength Design.


Appendices
Appendix A: Proposed Analytical Model - Sample Calculation
A. Proposed Analytical Model - Sample Calculation

Note:

The following sample calculation was taken from the parametric studies done on bridge deck slabs (Chapter 4). To obtain the complete structural response, the iterative procedure proposed in Chapter 2 was utilized, as illustrated in Figure 2-14, Page 30.

1. Input structure geometric and material properties and loading arrangement:

![Concentric loading at midspan](image)

![Cross-section](image)

<table>
<thead>
<tr>
<th>Concrete Properties</th>
<th>Reinforcement Properties</th>
<th>Sectional Properties</th>
<th>Member Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$ (Mpa)</td>
<td>$f_y$ (MPa)</td>
<td>$b$ (mm)</td>
<td>$L$ (m)</td>
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<td>40</td>
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<td>400</td>
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<td>$E_s$ (MPa)</td>
<td>$h$ (mm)</td>
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<td>225</td>
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<td>$\varepsilon_{cu}$</td>
<td>$\varepsilon_y$</td>
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<td>$A's$ (mm$^2$)</td>
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<td>55</td>
<td>$d$ (mm)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$As$ (mm$^2$)</td>
</tr>
</tbody>
</table>

1. Propose an axial load, $N$ (start with a smaller force): $N = 100$ kN

2. Use the proposed concrete and reinforcement stress-strain relationships to perform sectional-force analysis:
3. Pick an end-moment ($M_e$) and a midspan-moment ($M_m$) to establish an internal moment distribution for the structure:

$$M_e = -17.0 \text{kN}, \quad M_m = 17.0 \text{kN}$$
5. Generate the corresponding top-surface and bottom-surface strain distribution profiles:

![Strain Distribution Profiles](image)

6. Perform numerical integrations of the top and bottom surface strains over length of the structure to obtain the resultant structure deformation/elongation:

<table>
<thead>
<tr>
<th>Elongation (m)</th>
<th>Top</th>
<th>Bottom</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.000261452</td>
<td>0.000270858</td>
</tr>
</tbody>
</table>

7. Compare top-surface and bottom-surface strain elongations, are both next axial elongations equal to zero? No.

8. Pick another $M_m$ and $M_e$, and repeat steps 4 to 6:
9. Compare top-surface and bottom-surface strain elongations, are both next axial elongations equal to zero? Yes, very close!

10. Calculate the corresponding applied load: \( P = 4(M_s + M_a) / L = 15.18\,\text{kN} \)

11. If desired, derive the corresponding midspan deflection by generate a curvature diagram and then perform an integration using a numerical form of the formula:

\[
\Delta_m = \int_{0}^{L/2} \varphi \cdot x \, dx
\]

At \( N = 100\,\text{kN} \) and \( P = 15.18\,\text{kN} \), the curvature diagram looks like the following one:

Numerically calculate deflection, \( \Delta_m = 3.1\,\text{mm} \).

12. Check stress conditions at critical regions. Has the structure reached a failure state? No.
13. Increase $N$ and repeat the iterative procedure...

By bringing $N$ up from 100kN to 910kN, concrete at midspan and the supporting regions reach the ultimate compressive state. Stop Iteration.
The following two graphs illustrate the structural response of the beam from $N = 100\text{kN}$ to $N = 910\text{kN}$. The first one is the overall member P-N response. The second one is the sectional response at midspan and the ends.

![Graphs illustrating structural response](image)

Finally, the table below summarizes the calculation results:

<table>
<thead>
<tr>
<th>$N$ (kN)</th>
<th>$P$ (kN)</th>
<th>$M_m$ (kNm)</th>
<th>$\phi_m$</th>
<th>$M_e$ (kNm)</th>
<th>$\phi_e$</th>
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<td>100</td>
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<td>-4.67E-05</td>
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Appendix B: Plastic Collapse Mechanism Approach - Sample Calculation
Using the same structure geometric and material properties and loading arrangement as for the calculation done in Appendix A:

![Diagram of concentric loading at midspan]

<table>
<thead>
<tr>
<th>Concrete Properties</th>
<th>Reinforcement Properties</th>
<th>Sectional Properties</th>
<th>Member Properties</th>
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<tr>
<td>$E_c$ (Mpa)</td>
<td>$E_s$ (MPa)</td>
<td>$h$ (mm)</td>
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</tr>
<tr>
<td>35000</td>
<td>200000</td>
<td>225</td>
<td>Loading Condition</td>
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<td>170</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$A's$ (mm$^2$)</td>
<td>300</td>
</tr>
</tbody>
</table>

The collapse mechanism happens when plastic hinges formed at the midspan and at both ends. At those locations, the ultimate unit bending moment resistance, without the inclusion of partial safety factors, may be most accurately taken as that given by Mattock et al.'s (1961) formula:

$$m_b = \rho f'_c d^3 \left(1 - 0.59 \frac{\rho f_s}{f_c'}\right)$$

This formula can be manipulated to:

$$M_b = A_s f_s \left(d - 0.59 \frac{A_s f_c}{f_c'}\right)$$

So the full bending moment capacity can be calculated.

For this beam, $M_b = 20.10$ kNm.

For midspan loading:

$$P = 4M_b / L$$

Therefore, the load-carrying capacity is 26.8 kN.
Appendix C: Experiment Proposal
C.1 General Arrangement

The proposed experiment is assumed to be taken place in the Mark Huggins Structures Laboratory in the basement of Galbraith Building at the University of Toronto.

Three reinforced concrete slab strips with cross-sectional dimensions and reinforcement ratio that are similar to those empirically designed bridge deck slabs given in the latest CHBDC are to be constructed and tested for this proposed experiment program.

The dimensions of the proposed specimens are 225mm height and 400mm in width. The total lengths vary from 4 876mm to 11 582mm, equivalent to 16 feet, 26 feet and 37 feet, respectively. These particular total lengths of the proposed specimens are predetermined by the layout of the strong floor in the testing lab. General arrangement drawings for the first, 16 feet specimen are shown in Figure C-1.

The two ends of each specimen are restrained against both horizontal and vertical displacements. Simply supported conditions are provided at locations measured 1/3 of the total length from each end. This arrangement, with the restrained conditions at the ends, will be able to develop negative moments at the support locations. In addition, it divides each specimen into three regions: center-spans (the main span) measure in 2 286mm, 3 963mm and 5 639mm, respectively, leaving an overhanging half of the length of the center-span at each end.

All specimens have the same amount and arrangement of longitudinal reinforcement, doubly-reinforced with the longitudinal reinforcement bars placed symmetrically over the depth of the specimens. This arrangement enables the intention of having the same structural properties in both positive and negative bending. CSA Standard G30.18 10M bars, with a 100mm$^2$ cross-sectional area and a 11.4mm nominal diameter are used as longitudinal reinforcements. With three bars placed at the bottom and three at the top, spanning continuously over the entire length of each specimen, and with a 30mm clear cover, the longitudinal reinforcement ratio for the specimens is maintained constantly at 0.33%. Closed stirrups are made from U.S. Standard D-7 deformed wire with a 38.7mm$^2$ cross-sectional area and a 7.56m diameter to serve the purpose of holding the steel caging together. They are widely spaced at 600mm except at the ends so their influence can be minimized. The stirrups are closely spaced at the ends of each specimen to hold and spread evenly the horizontal forces exerted from the restraining devices.

All specimens are to be casted in concrete of 50Mpa nominal strength.
Figure C-1. Reinforcement layout and cross-sectional design of Specimen #1
**C.2 Test Setup and Instrumentation**

Figure C-3 provides a schematic drawing of the rest rig, and Figure C-4 illustrates the instrumentation layout for the proposed experiment #1. The test setup can be divided into two categories: (a) the vertical load application and reaction system setup, and (b) the lateral restraining system setup.

The vertical load application and reaction system includes three components:

1. Vertical loads applying at the midspan of each specimen by a 1000kN capacity servo-controlled actuator. The actuator reacts against a loading frame that is connected to the strong floor.

2. The center-span supporting on pin-roller assemblies that simulate the simply supported conditions.

3. Two 25mm diameter steel tie-rods bolted on a HSS beam on top and at each end of the specimen and anchored another HSS beam that is connected to the strong floor. The purpose of this installment is to restrain the overhanging portion against any vertical displacement, so the forces exerted from the lateral restraining system can be always acting at the mid-depth level of each specimen. Consequently, no internal moments will be developed at the ends.

As the strain compatibility criteria of the proposed model suggests, in order to develop full lateral restraint, there should be no net elongation along length of the span. Hence, the goal of the test rig is to maintain the relative horizontal displacement of the main span of the specimens at zero. This can be achieved by setting up a rigidly framed system, with a high capacity (2000kN) servo-controlled force actuator bearing on the edge face at one end of the specimen. The actuator is then mounted horizontally to a reaction A-frame that is connected to the strong floor. At the other end of the specimen, horizontal movement can be restricted by another A-frame that is also connected to the strong floor. Using a displacement-controlled mode, this actuator, with its built-in load-cell system, can be utilized to maintain zero net elongation of the specimens while recording the magnitude of lateral forces exerted on the specimens during testing. Similar experiment setup has been done at the University of Toronto in the investigation of membrane action in reinforced concrete floor slabs of a warehouse structure (Vecchio and Tang, 1990).
When acting together, the entire load-application and reaction system will produce a moment diagram that is similar to the one shown below:

![Moment Diagram](image)

**Figure C-2.** Load and reaction forces acting on the specimen and the resulting internal moment distribution

Instrumentation of this proposed experiment consists of data acquisition devices for force, strain and displacement. Throughout each load test, the applied loads and reaction forces are to be monitored by load cells built into the actuators. In addition, load cells are installed on each of the tie-rods that provide vertical restraint at the ends of each specimen. Linear variable differential transformers (LVDTs), placed at various points along the length of each specimen, are to be used to monitor vertical deflections or uplifting. LVDTs are also placed at the specimen ends and at the supports to record lateral expansions during loading tests. Strain measurements will be taken both internally and externally. Internally, electrical strain gauges are to be mounted on the reinforcements at critical locations. Externally, strains measurements can be obtained through a CLR scanner (MV 224/246 Coherent Laser Radar System) that records the relative positions of tooling balls glued at regular intervals along the one of the vertical concrete surfaces of each specimen.
Figure C-3. A schematic drawing of test rig for the proposed experiment
Figure C.4: Instrumentation layout for the proposed experiment
C.3 Expected Behaviors

Based on the calculated structural responses of the similar slab strip models presented in Chapter 4, the following structural responses are expected to be observed during loading tests:

1. Relatively stiff sectional responses and linear responses of applied load versus lateral restraining reaction force will be produced at the earlier stages of loading until yielding of the bottom layer reinforcements at the midspan and of the upper layer reinforcements at the supports (refer to Figure 4-6).

2. Yielding of the reinforcements demands greater concrete compressive strains at the midspan and supports. Consequently, the strains at those critical locations will increase at a much greater rate, diverging the strain profile of the center span from an almost linear distribution to a curvilinear one (refer to Figure 4-8).

3. As the compressive strains of concrete at the critical location (considering geometric effects, it will be the midspan) reaching to the limiting value, crushing will occur at the upper side of the midspan region.

4. Much larger displacements are expected in longer specimens. The figure shown below represents the expected relative midspan displacements for the specimens proposed.

![Relative midspan deflections of the proposed specimens](Figure C-5)