ON MESH CONVERGENCE AND ACCURACY BEHAVIOUR FOR CFD APPLICATIONS

Master of Applied Science

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Abstract

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Computational Fluid Dynamics (CFD) is a main field that contributes to the development of high efficiency aircraft. CFD accuracy depends on the flow solver and the meshing of the geometry, and while it is doable to determine why a certain solver is more accurate than another, it is much more difficult to discern why two meshes produce differently accurate solutions. A framework is presented to evaluate the performance or “goodness” of a mesh and to compare meshes. The framework is composed of quantifiable mesh parameters which define a mesh, and three performance measures: functional accuracy, their order of convergence, and their behaviour under the adjoint correction method. Although it seems that the relationships between parameters and results are not trivial, there are trends from which optimal mesh parameters are deduced. The H topology performs best, and the most important parameters are related to spacings and cell quality around the aerofoil leading edge.
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Chapter 1

INTRODUCTION

1.1 Environment and Air Transportation

As we enter into the second decade of the twenty-first century, our need for proactive efforts to reduce the human impact on climate change and the environment has reached an unprecedented level. This impact, mainly caused by our industries such as the oil and transportation industries, is currently growing, and there are no strong signs of it declining soon. Let alone that a peak demand for oil is predicted to occur within ten years from now [1]. The impact is mainly due to CO$_2$ and NO$_x$ contributions. The civil aviation industry, even though contributing only 2% of global CO$_2$ emissions [2], sees the unstoppable growth in air transport demand and the fact that the full climate impact of aviation is between two to four times that of CO$_2$ [3]. On the other hand, the global economic status has been unstable since the 2008 economic crisis, and the cost of oil, and thus fuel, is higher than ever. Therefore the aviation research and industry communities have a duty to work hard to cope with the difficult challenge of maintaining environmentally responsible and economically feasible aircraft. In other words, there is a dire need for highly efficient aircraft, to ensure that aviation continues to be one of the cleanest industries contributing towards a better future for mankind.

1.2 Unconventional Aircraft Design

One way to increase aircraft efficiency is to reduce drag. Reducing drag results in high fuel efficiency and therefore less demand for fuel and increases the feasibility of using alternative cleaner sources of fuel such as bio-fuel. However, related research done while dealing with the conventional fuselage-wing aircraft body configuration has been producing diminishing returns. On the other hand, unconventional configurations such as the blended wing body (BWB) shown in Figure 1.1 appear to have the potential to reduce drag to lower levels in comparison to conventional configurations. This pushes us to use computational fluid
1.3 Literature Review

Figure 1.1: Blended wing body configuration [10]

dynamics (CFD) to aid in the design of new unconventional aircraft shapes.

Exploring novel aircraft configurations using algorithms for aerodynamic shape optimization, requires accurate CFD solutions. Therefore, it is necessary to study accuracy improvement methods, taking into account that meshes change during the optimization process. One intuitive step that first comes to mind is implementing a solution-based adaptive mesh refinement algorithm into the CFD solution and shape design processes. A more preliminary yet wise step is to study mesh generation on a more basic level, the manual level. In other words, we should investigate and explore correlations between mesh properties and flow solution accuracy. Mesh generation in 2D provides excellent results at the present time due to the sheer experience the CFD community possesses related to 2D mesh generation. Nevertheless, the literature remains very scarce in terms of systematic mesh accuracy studies in 2D, let alone 3D. Therefore, I attempt through this thesis to present a systematic way of studying meshes and their accuracy behaviour.

1.3 Literature Review

“We know embarrassingly little about how the mesh affects the CFD solution”, these are the words of Professor Carl Ollivier-Gooch of the University of British Columbia [20]. This corresponds with the fact that there is no consensus on what constitutes a “good” mesh. As mentioned in Section 1.2, accurate CFD solutions are becoming more and more essential for efficient aircraft design. On the other hand, the CFD solution depends heavily on the mesh provided for the solver, i.e. a powerful solver and an optimal mesh are both required for maximum accuracy. Mesh convergence is the classical way to measure the overall mesh accuracy, which is mathematically represented as an error between the numerical solution and the exact solution. When it comes to mesh solution, this could be a fluid flow variable such as density at a certain point in the mesh [21,23,28] or an aerodynamic functional computed as a function of the mesh solution such as drag and lift coefficients $C_D, C_L$ [22]. The exact solution could be synthesized using the method of manufactured solutions [9,21,24,25] in
which case integer or non-integer mesh refinement can be used [26]. In these cases, the exact
solution already exists and the order is obtained from the slope of a log-log plot of error
against mesh resolution. The exact solution can also be estimated using a few methods
such as Richardson extrapolation method [27] which would entail using integer refinement
yielding three meshes to form one convergence study. In these cases, the exact solution
is taken as the grid converged estimate and the order is computed by solving a non-linear
algebraic equation. However in most of the literature, mesh convergence plays the role of
validating the correctness of a solver by applying the solver to a family of meshes which is
known through experience to be a good order tester. Little attention is given to why these
meshes yield almost exact results in terms of expected convergence order or why they yield
the error which they yield. Most of these meshes are created to certain standards acquired
through years of CFD experience; when it comes to 2D meshes, the tester meshes (used for
mesh convergence studies) are probably optimal, but the lack of an underlying foundation
of why this is the case is a main reason we are behind in terms of 3D.

The drag prediction workshop (DPW) is one of the main organizations that has a large focus
on tracking CFD errors and their relationship to meshes; in particular qualities of meshes,
and what quantifies a good mesh through these qualities. Which qualities are significant and
which are not? And before that, what set of qualities is sufficient to predict the performance
of a mesh and under which flow conditions, how can these qualities be put in a concrete
form to be metrics [29-33]. Metrics such as the far-field boundary location, mesh spacing
and clustering at critical regions such as leading and trailing edges, and mesh refinement
scaling, are discussed and their effects on mesh convergence and drag solution accuracy is
investigated. A subset of these metrics is the mesh resolution and at which parts of the
mesh; this is really the area of adaptive mesh refinement [34]. In the DPWs, having a large
number of nodes for reasonable accuracy in 3D cases (flow over wings) is deemed necessary.
It is also deemed necessary to have proper mesh convergence studies because it takes high
resolution meshes to be in the asymptotic range in the whole physical domain and resolve
all the physics. As for mesh qualities or metrics, relative distribution of the mesh points is
a key element. Moreover, topology plays an important role, aspect ratios, element types,
and orthogonality [35]. Important observations such as the discrepancy between the result
of two very fine meshes in 3D in the asymptotic range call for the need to do manual and
incremental mesh behaviour investigation.

Fortunately for the CFD community, the method known as the adjoint method [36,37,40] is
available to help get around the limitations that we have with the current knowledge about
mesh optimization. The adjoint method originally developed to reduce cost of computing
1.4 Thesis Outline and Summary of Objectives

In this section, I will summarize the objectives of this thesis, and then give an outline of the thesis structure.

There are four main objectives for this thesis:

▶ Investigate and explore correlations between aerofoil and wing mesh parameters and the corresponding CFD solutions. This will contribute towards improving accuracy of CFD solutions and therefore aircraft design. Furthermore, this will serve as a basis for future mesh convergence and mesh adaptation studies;
▶ provide a framework for studying mesh convergence;
▶ provide guidance related to work in the CFD lab in order to accelerate research progress of future students;
▶ provide more insight on manual mesh generation with emphasis on using the ANSYS-CFD software package.

As for the thesis layout, Chapter 2 will present work done in laying the framework for mesh convergence, accuracy, and comparison studies. Chapter 3 is concerned with the CFD aspect of the research. This includes a basic description of the Diablo solver, a description of the studies, and a description of adjoint error estimation and aerodynamic functional correction. Chapter 4 presents the main contribution of the thesis, entailing selected mesh convergence studies with graphical illustrations, then showing conclusions inferred related to generating meshes that produce low numerical error followed by post-analysis mesh generation and adjoint error estimation studies verifying the conclusions. The main body of the thesis ends with the conclusions in Chapter 5. The appendices provide more technical details of the work done in this project.
Chapter 2

Mesh Generation

2.1 Geometry and Mesh Topology

As explained by Lomax et al. [4], the first step of a CFD problem is geometry definition and mesh generation, which is the main topic of this chapter. The main geometry parameter that I choose to discuss is the aerofoil in the 2D case or wing in the 3D case. There are three main identifiable aspects of an aerofoil: geometry profile, smoothness of the profile, which is related to how the profile is constructed, and tip properties such as being sharp, rounded, or blunt (flat with sharp corners). A fourth aspect of the profile is added in the 3D case which is the wing span, denoted by $L$. Each aerofoil has a profile which is a curve defined by known equations [5]. The aerofoil geometry is constructed with a CFD mesh generation tool through generating points which lie on the profile theoretical curve, i.e. through sampling from the equations and then fitting curves through these points, the curves connecting the points make the external physical surface of the aerofoil i.e. a solid wall in fluid dynamics terms. For example a NACA0012 aerofoil profile with a sharp trailing [19] edge is defined by

$$y = \pm 0.6 \left( 0.2969\sqrt{x} - 0.1260x - 0.3516x^2 + 0.2843x^3 - 0.1036x^4 \right).$$

(2.1)

Figure 2.1 shows the NACA0012 aerofoil profile, and Figure 2.2 shows the RAE2822 aerofoil profile.

![Figure 2.1: NACA0012 aerofoil](image1)

![Figure 2.2: RAE2822 aerofoil](image2)
2.1. Geometry and Mesh Topology

(a) 2D aerofoil terminology

(b) Wing terminology

Figure 2.3: Terminology

(a) Rough NACA0012

(b) Smooth NACA0012

Figure 2.4: NACA0012 with differing smoothness levels

Before continuing, it is beneficial to look at Figure 2.3 to become familiar with aerofoil terminology. Smoothness refers to the graphical smoothness of the curves (formed by the mesh generation tool) making the aerofoil profile. These curves are formed through an interpolation between the points selected along the theoretical curve profile. ICEM-CFD software provides different types of interpolation. When the software is asked to interpolate through all the points in one step, the result is low smoothness which is $C^0$ around the leading edge, as shown in Figure 2.4a. The figure shows a NACA0012 aerofoil formed with few points and a one-step interpolation. Figure 2.4b shows a NACA0012 aerofoil formed with more points and a two-step interpolation where the aerofoil profile is split into two, and there is an interpolation through the points making the leading edge and a separate interpolation through the rest of the points. The result is high smoothness that is at least $C^1$. Tips on editing surface smoothness using ICEM-CFD are included in Appendix B.
2.1. Geometry and Mesh Topology

The third aspect of geometry includes the trailing edge and wing tip properties. Figures 2.1 and 2.2 show aerofoils with sharp trailing edges. Figure 2.5(a) shows the NACA0012 aerofoil with a blunt trailing edge, which has a profile equation identical to equation 2.1 except for the last term, which is changed from $0.1036x^4$ to $0.1015x^4$ [19]. Figure 2.5(b) shows an extruded 3D NACA0012 with a sharp wing tip. Figure 2.5(c) shows an ONERA-M6 wing with a rounded wing tip.

The first, and very important, parameter which defines our CFD mesh is the mesh topology. The mesh topology is a macroscopic property of a CFD mesh. The far-field shape thus changes with topology. The topology dictates the spatial distribution of the mesh elements and nodes. The mesh topology is a measure of how the nodes are distributed in space or how the mesh engulfs the aerofoil. The following are some CFD topologies which were used in this project. Note that the figures used for illustration show very good and high quality meshes, i.e. there are many other differing shapes of a mesh which would classify under the same topology but differ in other mesh qualities such as skewness or orthogonality.

2D Topologies: C Topology

A mesh with C topology, or a C-mesh, is a mesh where the mesh lines follow the shape of the aerofoil in a $C$ shape. The streamwise lines forming the mesh in a $C$ shape trace the body of the aerofoil from the trailing edge to the leading edge and back to the trailing edge. Figure 2.6 illustrates this topology.
Figure 2.6: C topology mesh
**O Topology**

A mesh with O topology, or an O-mesh, is a mesh where the mesh lines follow the shape of the aerofoil in an $O$ shape. The streamwise lines forming the mesh in an $O$ shape trace the body of the aerofoil from the trailing edge to the leading edge and back to the trailing edge. Figure 2.7 illustrates the topology. The O topology is characterized by high cell skewness at the trailing edge.

![Figure 2.7: O topology mesh](image-url)
**H Topology**

An H topology mesh is a mesh that is constructed as if it was a square grid with square cells and then a hole is introduced to the mesh. The hole takes the shape of the aerofoil and the rest of the cells move to accommodate such a shape. Figure 2.8 illustrates this topology.

**Figure 2.8:** H topology mesh
3D Topologies: HH Topology

An HH mesh is a 3D mesh and is simply an H mesh that is extruded in a third dimension. Therefore, in addition to normal cell clustering, the extruded part (wing) has its own clustering. The 2D aerofoil is extruded a certain *span* in the same dimension or it could be in the direction of a vector for a swept wing. This is the same for the next topology, CH. Figure 2.9 illustrates an HH mesh.

*Figure 2.9: HH topology mesh*
CH-Topology

A CH mesh is a 3D mesh and is a C mesh extruded in a third dimension. Figure 2.10 illustrates.

Figure 2.10: CH topology mesh
2.2 Mesh Parameters

In order to study a geometric system such as a CFD mesh, we need to find a good way through which the particular geometric system, i.e. the mesh, becomes defined. This is similar to the idea of representing mechanical systems governed by many variables as elements of an $\mathbb{R}^n$ space in differential geometry. In this section, variables which are identified to be of significance are defined and presented. Such parameters are by no means the only important ones since there are lots of microscopic and macroscopic parameters which can be identified within meshes, especially in 3D. The first parameter in defining a mesh is the topology, which is a macroscopic parameter or characteristic. It was included in the previous section separately due to its extra importance.

**Blocking** refers to how the computational space is divided graphically. Blocking, even though not directly related to spacing parameters, affects accuracy measures. Blocking could be customized for the sake of improving other mesh qualities or for gaining computational advantages. In general we use multi-block meshes for two reasons: 1) to accommodate different geometries and to assist in dividing the mesh into domains whose spacings are to be monitored; and 2) to gain computational advantages by using parallel solution techniques. With the current diablo solver, adjacent blocks are required to be conforming. Figure 2.11 shows the difference between conforming and non-conforming blocks. The thick blue line is the block interface; in Figure 2.11b the green block becomes non-conforming with the blue block.

![Figure 2.11: Conforming vs. non-conforming blocks](image)

(a) Conforming blocks  (b) Non-conforming blocks
2.2. Mesh Parameters

(a) Schematic of leading edge and parameters

(b) $L_r = 10\%, \phi = 62^\circ, \theta = 78^\circ$

(c) $L_r = 3\%, \phi = 45^\circ, \theta = 57^\circ$

Figure 2.12: Illustration of mesh parameters

Leading Edge Skewness Factor, $\phi$, is a parameter defined to measure the overall skewness of mesh cells and in particular around the leading edge in C and O mesh types. $\phi$ is defined as the angle between the leading edge block interface and the horizontal. As this angle decreases from $90^\circ$, the cells around the leading edge become compressed and the cells around the rest of the aerofoil become tilted and thus more skewed as shown in Figure 2.12c. As $\phi$ gets closer to $90^\circ$, the cells become better aligned around the aerofoil such as Figure 2.12b.

Leading Edge Region, $L_r$, is a quantification of what constitutes the leading edge of an aerofoil. It is defined as a percentage of the aerofoil surface length which will be termed the leading edge.

Leading Edge Spacing, $L_s$, refers to the streamwise spacing of nodes along the leading
edge region $L_r$ starting from the leading edge of the aerofoil to the end of the leading edge region. A hyperbolic spacing is denoted by $h$, and a uniform spacing is denoted by $u$.

**Trailing Edge Spacing**, $T_s$, refers to the streamwise spacing of nodes along the remainder of the aerofoil after the leading edge region i.e. starting from the leading edge region boundary and ending at the trailing edge. This parameter is named after the trailing edge because the stream-wise spacing along the rest of the aerofoil surface i.e. apart from the leading edge is defined in the results section by the spacing at the trailing edge.

**Off-wall Spacing**, $O_s$ refers to the spacing between the aerofoil surface and the first node in the normal direction.

**Far-Field Spacing**, $F_s$, refers to the streamwise and normal spacing at the the far-field boundary.

**Leading edge Orthogonality Factor**, $\theta$, is defined to measure orthogonality of the mesh lines at the aerofoil/wing surface, and is defined as the angle between a certain mesh line which is generally a block interface at the end of the leading edge zone and the tangent at the aerofoil surface point intersecting with the mesh line. Figure 2.12a shows a schematic. Figures 2.12b and 2.12c show two examples illustrating $L_r$, $\phi$ and $\theta$ on a NACA0012 aerofoil with C topology and 5 blocks.

3D meshes involve all these parameters with the addition of two more: the **Root Spacing**, $W_r$, and the **Tip Spacing**, $W_t$. These are illustrated in Figure 2.13.

![Figure 2.13: Wing spacings](image)
2.2.1 Mesh Comparison

Mesh comparison here means topology comparison. In order to have a fair comparison between meshes of different topologies, it is essential that we hold mesh parameters the same or very close in both meshes. This is done by duplicating the leading edge and trailing edge regions, and duplicating the mesh spacings. Figure 2.14 shows an example of meshes of three different topologies used in one of the studies.

![Mesh Comparison Diagram](image-url)

**Figure 2.14:** 2D meshes for comparison
Chapter 3

MESH ASSESSMENT FRAMEWORK

3.1 Flow Solver

The flow solver used was developed by Hicken and Zingg [6] and is known as Diablo. This is a second-order finite-difference solver that uses the Newton-Krylov approach to solve the non-linear algebraic equations governing inviscid flow, i.e. the Euler equations, or with the Spalart-Allmaras turbulence model to solve the Reynolds-Averaged Navier Stokes RANS equations. The solver uses scalar or matrix numerical dissipation (scalar dissipation was used in this project). The non-linear system resulting from the spatial discretization is solved using an inexact Newton-Krylov approach [7]. All of the present studies were done using the main second-order version of the code in contrast to higher-order methods [8], and most of the studies in this work are of subsonic inviscid flow; therefore a brief summary of the Euler equations is given below.

3.2 Euler Equations

The Euler equations represent the mathematical model of fluid flow with zero viscosity and heat conduction. Equation 3.1 shows the Euler equations in tensor form:

\[
\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}_i}{\partial x_i} = 0 \quad \forall \mathbf{x} \in \Omega,
\]

where \( \Omega \) is the physical domain, \( \mathbf{x} \) is the Cartesian coordinates, and the Einstein summation convention is used over subscript \( i \). \( \mathbf{Q} \) and \( \mathbf{F}_i \) are given by

\[
\mathbf{Q} = \begin{bmatrix}
\rho \\
\rho u_1 \\
\rho u_2 \\
\rho u_3 \\
e
\end{bmatrix}, \quad \mathbf{F}_i = \begin{bmatrix}
\rho u_i \\
\rho u_1 u_i + p \delta_{1i} \\
\rho u_2 u_i + p \delta_{2i} \\
\rho u_3 u_i + p \delta_{3i} \\
(e + p)u_i
\end{bmatrix},
\]

where \( \delta_{ij} \) is the Kronecker delta, \( \rho \) is density, \( \rho u_i \) is \( i \)-direction momentum per unit volume, \( e \) is total energy per unit volume. The pressure \( p \) is given by:
\[ p = (\gamma - 1)(e - \frac{1}{2}\rho \mathbf{u} \cdot \mathbf{u}), \]  
(3.2)

where \( \gamma = 1.4 \). For more details on the equations and transforming them into a computational space designed for multi-block structured meshes see [13,6].

### 3.3 Generalized Curvilinear Coordinate System

This section presents some details of the transformation from the physical space \((x, y, z)\) on the meshes to the computational space \((\xi, \eta, \zeta)\). These coordinates can be parallel and normal and across the span of the wing as shown in Figure 3.1 which shows a CH topology, but their general direction depends on the topology.

![Curvilinear coordinates for wing](image)

**Figure 3.1:** Curvilinear coordinates for wing

These generalized coordinates are then given by :

\[ \xi = \xi(x, y, z) \]  
\[ \eta = \eta(x, y, z) \]  
\[ \zeta = \zeta(x, y, z), \]  
(3.3)

(3.4)

(3.5)

which yield the following metric relationships :

\[ \xi_x = J(y_\eta z_\zeta - y_\zeta z_\eta) \]  
\[ \xi_y = J(z_\eta x_\zeta - z_\zeta x_\eta) \]  
\[ \xi_z = J(x_\eta y_\zeta - y_\eta x_\zeta) \]  
(3.6)

\[ \eta_x = J(z_\xi y_\zeta - y_\xi z_\zeta) \]  
\[ \eta_y = J(x_\xi z_\zeta - z_\xi x_\zeta) \]  
\[ \eta_z = J(y_\xi x_\zeta - y_\xi x_\eta) \]  
(3.7)

\[ \zeta_x = J(y_\xi z_\eta - z_\xi y_\eta) \]  
\[ \zeta_y = J(z_\xi x_\eta - x_\xi z_\eta) \]  
\[ \zeta_z = J(x_\xi y_\eta - y_\xi x_\eta) \]  
(3.8)
where $J$ is the Jacobian given by

$$J = x_\xi y_\eta z_\zeta + x_\zeta y_\xi z_\eta + x_\eta y_\zeta z_\xi - x_\xi y_\zeta z_\eta - x_\eta y_\zeta z_\xi - x_\zeta y_\eta z_\xi,$$

and $x_\xi = \frac{\partial x}{\partial \xi}$ and so forth. Combining the results of these transformations and substituting them into the Euler equations results in the following transformed Euler equations in terms of the new coordinate system:

$$\frac{\partial \hat{Q}}{\partial t} + \frac{\partial \hat{F}_i}{\partial x_i} = 0,$$

where $(\xi_1, \xi_2, \xi_3)^T = (\xi, \eta, \zeta)^T$,

$$\hat{Q} = \frac{1}{J} \begin{bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ e \end{bmatrix}, \quad \hat{F}_i = \frac{1}{J} \begin{bmatrix} \rho U_i \\ \rho u_1 U_i + p \partial_x \xi_i \\ \rho u_2 U_i + p \partial_y \xi_i \\ \rho u_3 U_i + p \partial_z \xi_i \\ (e + p) U_i \end{bmatrix},$$

where $U_i \equiv u_j \partial_{x_j} \xi_i$. Details about the Navier-Stokes equations and their transformations can be found in [44].

### 3.4 Aerodynamic Functionals

As engineers, we are often interested in dimensionless quantities to quantify a certain performance of a system. In aerodynamics, we are mainly interested in the drag coefficient, $C_D$, and the lift coefficient, $C_L$. $C_D$ and $C_L$ are defined as

$$C_D = \frac{F_D}{\rho v^2 A}, \quad C_L = \frac{F_L}{\rho v^2 A},$$

where $F_D$ and $F_L$ are drag and lift forces respectively, $v$ is the speed of the aerofoil with respect to the fluid, and $A$ is a reference area for wings and a characteristic length for aerofoils. In the Diablo solver, for inviscid flow the coefficients are computed through surface pressure integration between the top and bottom of an aerofoil or wing. We present here the concept in physical space. For further details and the friction contributions to $C_D$ and $C_L$, see [18]. Computationally, the functionals are computed for inviscid flows as follows:

$$C_L = \oint p n \cdot k \, dS$$  \hspace{1cm} (3.9)

$$C_D = \oint p n \cdot i \, dS$$  \hspace{1cm} (3.10)
3.5 Order of Accuracy Estimation Methodology

For every mesh topology, a family consisting of three meshes (coarse, medium, fine) is constructed through the following steps:

1. The finest mesh is generated using ICEM CFD based on the required mesh parameters. The number of nodes on every side of every block, denoted by \( N \), must satisfy the following
   - \( N \) is odd.
   - \( \frac{N+1}{2} \) is odd.

2. The `coarsen_grid` grid utility is used to construct the medium mesh from the fine one by removing every second node in all directions in 2D and 3D.

3. Step 2 is repeated on the medium mesh to generate the coarse mesh.

4. For specific flow conditions, the three meshes (fine, medium, coarse) are used to solve the steady fluid flow problem using *Diablo*, and \( C_L \) and \( C_D \) are computed.

5. Using a generalized form of the Richardson extrapolation method [11], four quantities are estimated:
   - estimate of grid converged coefficient of lift denoted by \( C_L^* \)
   - estimate of grid converged coefficient of drag denoted by \( C_D^* \)
   - estimated order of convergence for lift denoted by \( P_{C_L} \)
   - estimated order of convergence for drag denoted by \( P_{C_D} \)

The method provides an estimate for the order of convergence \( P \) for any aerodynamic functional \( \mathcal{F} \) on a family of three consecutive meshes as the solution to the following equation:

\[
\frac{\mathcal{F}_f - \mathcal{F}_m}{\mathcal{F}_m - \mathcal{F}_c} = \frac{R_1^P - 1}{R_2^P - R_1^P},
\]
where the subscripts $f, m, c$ indicate fine, medium and coarse respectively. Furthermore, $R_1$ is the ratio between the spacing in the medium mesh and the coarse mesh, which in the basic scenario (i.e. family of meshes constructed by removing every second node) is 2. Similarly, $R_2$ is the ratio of spacing between the fine mesh and the coarse mesh, which in the basic scenario is 4. Newton’s method is applied as follows to find $P$:

$$P^{n+1} = P^n - \frac{\mathcal{G}(P^n)}{\mathcal{G}_P(P^n)},$$

(3.12)

where

$$\mathcal{G}(P) = (R_2^P - R_1^P)\mathcal{R} - R_1^P + 1 = 0,$$

$$\mathcal{G}_P(P) = \ln(R_2)R_2^P\mathcal{R} - \ln(R_1)R_1^P(\mathcal{R} + 1),$$

and

$$\mathcal{R} = \frac{\mathcal{F}_f - \mathcal{F}_m}{\mathcal{F}_m - \mathcal{F}_c}.$$ 

Once the Newton iterations have converged, the estimated exact functional value $\mathcal{F}^*$ is computed using the following equation:

$$\mathcal{F}^* = \mathcal{F}_f + \frac{\mathcal{F}_f - \mathcal{F}_m}{R_1^P - 1}$$

(3.13)

### 3.6 Adjoint Error Estimation

We present here a method that is used to estimate aerodynamic functionals on a fine mesh given the value of the functionals on a coarser mesh. It utilizes interpolation methods, Taylor series expansion, and the concept of adjoint variables. As shown in (3.12) and (3.13), $C_D$ and $C_L$ are formed using pressure integration, and therefore we can write $\mathcal{F} = \mathcal{F}(Q)$, where $\mathcal{F}$ is either $C_D$ or $C_L$ or any other scalar functional. In this section $Q$ is the vector containing the flow solution for every node in the mesh. Furthermore, the finite-difference discretization of the Euler and Navier-Stokes equations results in a residual vector denoted by $\mathbf{R} = \mathbf{R}(Q)$. Let us then say we have two meshes, a fine mesh and a coarser one. Since we can think of the functional as a function of the mesh, we perform a first-order Taylor series expansion for the functional on the fine mesh around the coarse one and perform a similar expansion of the residual, i.e. we have:

$$\mathcal{F}_h \approx \mathcal{F}_H^h + \frac{\partial \mathcal{F}_H^h}{\partial Q_h} (Q_h - Q_H^h),$$

(3.14)

$$\mathbf{R}_h (Q_h) = 0 \approx \mathbf{R}_h (Q_H^h) + \frac{\partial \mathbf{R}_h (Q_H^h)}{\partial Q_h} (Q_h - Q_H^h),$$

(3.15)
where the subscript $h$ indicates the fine mesh, $Q^H_h$ indicates the flow solution projected onto the fine mesh by interpolating from the coarse mesh, i.e. without running the fine mesh through the flow solver and is a column vector, $R_h(Q^H_h)$ is the residual computed on the fine mesh using an interpolated solution from the coarser mesh, $F^H_h = F_h(Q^H_h)$, $\frac{\partial F^H_h}{\partial Q_h}$ is a row vector which represents the functional sensitivities of the fine mesh evaluated at the interpolated flow solution, and $\frac{\partial R_h(Q^H_h)}{\partial Q_h}$ is the Jacobian of the fine mesh residual evaluated at $Q^H_h$ and is a matrix. We define the discrete adjoint solution [16] on the fine mesh $\psi_h|Q^H_h$ as the solution to the following matrix equation :

\[
\left( \frac{\partial R_h(Q^H_h)}{\partial Q_h} \right)^T \psi_h|Q^H_h = \left( \frac{\partial F^H_h}{\partial Q_h} \right)^T,
\]

(3.16)

Now by substituting the transpose of (3.16) into (3.14), we get

\[
F_h \approx F^H_h + (\psi_h|Q^H_h)^T \left( \frac{\partial R_h(Q^H_h)}{\partial Q_h} \right) (Q_h - Q^H_h),
\]

and from (3.15) we already have

\[
\frac{\partial R_h(Q^H_h)}{\partial Q_h} (Q_h - Q^H_h) = -R_h(Q_h),
\]

so we substitute this residual to get

\[
F_h \approx F^H_h - \psi_h|Q^H_h R_h(Q^H_h).
\]

Therefore we reach a measure of the error between the fine mesh functional computed using an interpolated solution from the coarser mesh and a full solution on the fine mesh, this is given by:

\[
E_F = F_h - F^H_h \approx - (\psi_h|Q^H_h)^T R_h(Q^H_h)
\]

(3.17)

Equation (3.20) dictates that we solve the adjoint problem on the fine mesh to determine $\psi_h|Q^H_h$ which contradicts the point of finding an error measure to be used for predicting the functional value on a finer mesh without solving equations on the finer mesh. Therefore, we solve the adjoint problem on the coarser mesh and interpolate the values onto the finer mesh using the same interpolation operator, i.e. we solve for $\psi_H$ in the following equation :

\[
\left( \frac{\partial R(Q^H_H)}{\partial Q^H_H} \right)^T \psi_H = \left( \frac{\partial F^H_H}{\partial Q^H_H} \right)^T,
\]

(3.18)

and obtain an interpolation of $\psi_H$ onto the fine mesh denoted by $\psi^H_h$, so we get

\[
E^H_F = F_h - F^H_h \approx - (\psi^H_h)^T R_h(Q^H_h)
\]

(3.19)

\[
F_h \approx F^H_h + E^H_F.
\]

(3.20)
3.6. Adjoint Error Estimation

Figure 3.2: Adjoint weighted error map for bicubic interpolation of drag coefficient and flow solution for inviscid flow around a NACA0012 aerofoil with $\alpha = 1.5^\circ$ and $Ma = 0.3$.

The work in this project focuses on how the accuracy of this error estimate depends on mesh properties. It is also helpful here to introduce the idea of adjoint error maps. We see $E^H_H$ as a scalar, but it is still a function of the flow and therefore it would be very helpful to get a visualization of how this function is distributed through a mesh. Figure 3.2 shows such an error map on a C-mesh. The figure shows which areas of the mesh contribute most to the total error.
Chapter 4

RESULTS

4.1 Results Layout

In this chapter, I start by presenting a collection of mesh convergence and adjoint correction studies, both 2D and 3D, in Sections 4.1 and 4.2. There were more studies conducted than presented here. However, the studies presented are representative of the overall trends. Each study below represents a category of studies embodying certain mesh qualities and parameters. In some studies, a family of meshes of a certain topology was made erroneously with different mesh parameters than the study uses, these families were not omitted but their topology is written in italicized font. These erroneous studies were still included because they are erroneous in terms of providing fair comparison between topologies but still represent mesh convergence studies with known mesh parameters which can be used in determining what are the “good” mesh parameters. Unless otherwise stated, the studies presented are conducted with the flow conditions and computational parameters given in Table 4.1. The 2D inviscid studies are studies in Section 4.2, the 3D inviscid are in Section 4.3, and the turbulent study is Study 7 in Section 4.4.

Each 2D study comprises three families of meshes, each with a certain topology, namely C, H, and O. Each study is presented first with a table that shows the mesh parameters that define the fine mesh in each family. As shown in Section 2.2, $\phi$, $L_r$, and $\theta$ indicate orthogonality and skewness of the mesh, $L_s$, $T_s$, and $O_s$ indicate spacings close to the aerofoil surface. $F_d$ and $F_s$ indicate far-field distance and spacing respectively, and $N$ represents the number

<table>
<thead>
<tr>
<th>Condition</th>
<th>2D-Inviscid</th>
<th>2D-Turbulent</th>
<th>3D-Inviscid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Governing Equations</td>
<td>2D Euler</td>
<td>RANS</td>
<td>3D Euler</td>
</tr>
<tr>
<td>Geometry</td>
<td>NACA0012</td>
<td>RAE2822</td>
<td>NACA0012/ONERA-M6</td>
</tr>
<tr>
<td>Ma</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Re</td>
<td>500</td>
<td>$2.5 \times 10^6$</td>
<td>500</td>
</tr>
<tr>
<td>$\alpha/\circ$</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\kappa_4$</td>
<td>0.04</td>
<td>0.08</td>
<td>0.04</td>
</tr>
</tbody>
</table>
of nodes. Images of meshes not shown in this section are found in Appendix C. Next is a table showing the functionals $C_D$ and $C_L$ from the flow solution on each mesh, $P_{C_D}$ and $P_{C_L}$, the estimated orders of convergence for $C_D$ and $C_L$ from (3.12), and $C_D^*$ and $C_L^*$, the estimated grid-converged functionals obtained from (3.13). Two plots follow showing the error between the functionals on each member of each mesh family and the estimated grid-converged functional. A table showing the results of the adjoint error correction method is presented, where $F^i$ is the functional computed using an interpolated flow solution from a coarser mesh to a finer mesh i.e. $F^i$ represents $F^H_i$. $F^+$ represents the corrected functional after adding $E^H_i$ to $F^i$, and $\Delta$ gives a measure of how good the correction is and is given by

$$\Delta = \frac{F^+ - F_{\text{coarse}}}{F_{\text{fine}} - F_{\text{coarse}}},$$

where $\Delta = 1.00$ is a perfect correction. Four plots are then presented showing raw data of $C_D$ and $C_L$, the interpolated functionals, the corrected functionals, and the corresponding error plots. All the interpolations used in this project are bicubic interpolations for 2D and trilinear for 3D [17]. Finally a table is presented showing rankings of the families in the evaluation. Three methods were used to evaluate the meshes. First, accuracy, which is measured by how close the computed functional on a mesh is to the estimated grid converged value of the functional. The error plots show the errors based upon each topology’s grid converged functional estimate. In the ranking table the first place is given to the topology with the lowest error between computed functional and its grid-converged estimate. This accuracy measure is denoted $E_F$. Instead of including the numerical data for this measure for each mesh in each family, the error plots included in each study provide an excellent presentation of the errors of mesh convergence and adjoint correction in all of the meshes. It should be noted though that in our inviscid case, the trend is that as we increase number of nodes and thus accuracy, $C_D$ decreases. However this does not mean that the grid converged solutions of $C_D$ and $C_L$ are zero; this is because we are applying boundary conditions at a finite distance from the aerofoil. Therefore, we cannot take the topologies with the lowest grid-converged estimate $C_D^*$ to be most accurate because it could be lower than the actual grid-converged solution. The second method is how close the estimated order of convergence is to the theoretical value, which is 2 in our case, since the second-order version of the code was used. First place is given to the topology which yields a convergence order closest to 2.00. This measure is denoted by $O_F$, and it is mathematically represented as $O_F = |P_F - 2|$. The third method is how well does the adjoint error correction method work in estimating functionals on finer grids. First place is given to the topology where the average $\Delta$ on all the functionals and along all the corrections is closest to 1.00, i.e. the topology with which
the adjoint error estimation is most accurate. For 2D studies with the Euler case defined by $\alpha = 1.5^\circ$ and $Ma = 0.5$, the trend is that $C_D$ decreases as the mesh resolution increases since the grid converged value of $C_D$ in inviscid flow is close to zero. When an order or estimated grid convergence functional is 0 or $\pm\infty$, this means the Richardson method did not converge or the sequence of functionals on the family of meshes is not monotone.

4.2 2D Mesh Convergence Studies

Study 1

In this study, a small leading edge is defined, spanning 3% of the aerofoil, with uniform streamwise spacing of $2.5 \times 10^{-4}$. The aspect ratio has an average of 25 in the region since the off-wall spacing is $0.1 \times 10^{-4}$. The number of nodes for the fine meshes was between 200 and 250 thousand as shown in Table 4.2. The far-field spacing is smaller than normally used. The way the C and O meshes were generated with ICEM-CFD ignored optimizing the skewness, smoothness of mesh lines through block interfaces, and orthogonality, which can be seen by the values of $\phi$ and $\theta$. The O mesh has the highest skewness, then the C mesh, then the H mesh, which has excellent skewness and orthogonality. For the three topologies, mesh convergence data resulted in an overshoot in the convergence order for $C_D$, with $P_{C_D}$ around 3.0 as shown in Table 4.3. Values for $P_{C_L}$ were obtained from the C and H meshes: 2.2 and 2.6 respectively. As for $C_L$ accuracy, H and C clearly win against O with lower and similar errors on the fine meshes, we may infer that skewness and orthogonality, which are the measures at which the H mesh clearly wins against the C and O meshes, result in higher accuracy and estimated order of convergence for $C_L$, but at this point this would be a guess. As we can see from Figures 4.1 and 4.2, C and O meshes win against H for $C_D$ accuracy, especially on the coarse meshes. As for functional adjoint correction, H performed best followed by C and then O, where none of the results here were wrong in the sense that the corrections were in the proper direction i.e. $C_D$ decreased and $C_L$ increased, as shown in Table 4.4. All the performance scores are shown in Table 4.5. An odd observation is that the adjoint correction method predicts the medium mesh functional using the coarse mesh solution better than predicting the fine mesh functional using the medium mesh solution i.e. $\Delta_{c\rightarrow m}$ is usually closer to 1.00 than $\Delta_{m\rightarrow f}$. 
### Table 4.2: Study 1 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>45°</td>
<td>3%</td>
<td>$2.5 \times 10^{-4}$u</td>
<td>$0.1 \times 10^{-4}$h</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>0.5</td>
<td>57°</td>
<td>262,397</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>45°</td>
<td>10%</td>
<td>$1.3 \times 10^{-4}$u</td>
<td>$1.0 \times 10^{-4}$h</td>
<td>$1.0 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>57°</td>
<td>198,147</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>45°</td>
<td>3%</td>
<td>$2.5 \times 10^{-4}$u</td>
<td>$0.1 \times 10^{-4}$h</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>0.5</td>
<td>57°</td>
<td>198,147</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>N/A</td>
<td>N/A</td>
<td>$2.5 \times 10^{-4}$u</td>
<td>$0.1 \times 10^{-4}$h</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>0.5</td>
<td>N/A</td>
<td>264,710</td>
</tr>
</tbody>
</table>

### Table 4.3: Study 1 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.29300$\times 10^{-3}$</td>
<td>$0.11448 \times 10^{-3}$</td>
<td>3.0471</td>
<td>0.21076</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.13637$\times 10^{-3}$</td>
<td>$0.11426 \times 10^{-3}$</td>
<td>0.21178</td>
<td>0.11448</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11720$\times 10^{-3}$</td>
<td>$0.21153 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11734</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.11234$\times 10^{-3}$</td>
<td>$0.10246 \times 10^{-3}$</td>
<td>$\infty$</td>
<td>0.21178</td>
<td>0.21199</td>
<td>$-\infty$</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.066754$\times 10^{-3}$</td>
<td>$0.10246 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11448</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11720$\times 10^{-3}$</td>
<td>$0.21153 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11734</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.35190$\times 10^{-3}$</td>
<td>$0.10246 \times 10^{-3}$</td>
<td>3.1677</td>
<td>0.21113</td>
<td>0.21145</td>
<td>0.2119</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.13836$\times 10^{-3}$</td>
<td>$0.11434 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11448</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11434$\times 10^{-3}$</td>
<td>$0.21153 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11734</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.70418$\times 10^{-3}$</td>
<td>$0.10619 \times 10^{-3}$</td>
<td>2.9710</td>
<td>0.21031</td>
<td>0.21159</td>
<td>0.2118</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.17457$\times 10^{-3}$</td>
<td>$0.095860 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11734</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.10619$\times 10^{-3}$</td>
<td>$0.21153 \times 10^{-3}$</td>
<td>0.21125</td>
<td>0.11734</td>
<td>0.21139</td>
<td>0.2116</td>
</tr>
</tbody>
</table>

### Table 4.4: Study 1 adjoint error correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_i$</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$\Delta$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.25412$\times 10^{-3}$</td>
<td>0.11542$\times 10^{-3}$</td>
<td>1.13</td>
<td>0.21055</td>
<td>0.21134</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.13980$\times 10^{-3}$</td>
<td>0.12531$\times 10^{-3}$</td>
<td>0.58</td>
<td>0.21133</td>
<td>0.21143</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Coarse → Medium</td>
<td>0.30618$\times 10^{-3}$</td>
<td>0.13541$\times 10^{-3}$</td>
<td>1.01</td>
<td>0.21091</td>
<td>0.21137</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.13875$\times 10^{-3}$</td>
<td>0.12375$\times 10^{-3}$</td>
<td>0.61</td>
<td>0.21139</td>
<td>0.21150</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Coarse → Medium</td>
<td>0.51097$\times 10^{-3}$</td>
<td>0.16818$\times 10^{-3}$</td>
<td>1.01</td>
<td>0.20932</td>
<td>0.21161</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.13148$\times 10^{-3}$</td>
<td>0.094570$\times 10^{-3}$</td>
<td>1.17</td>
<td>0.21134</td>
<td>0.21178</td>
<td>0.90</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.5: Study 1 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>O</td>
<td>H</td>
<td>H</td>
<td>C</td>
<td>C</td>
<td>H</td>
</tr>
<tr>
<td>Second place</td>
<td>C</td>
<td>C</td>
<td>O</td>
<td>H</td>
<td>H</td>
<td>C</td>
</tr>
<tr>
<td>Third place</td>
<td>H</td>
<td>O</td>
<td>C</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

![Figure 4.1: Study 1 interpolation and adjoint correction plots](image-url)
4.2. 2D Mesh Convergence Studies

Figure 4.2: Study 1 interpolation and adjoint correction error plots
4.2. 2D Mesh Convergence Studies

Study 2

In this study the leading edge spacing is decreased two and a half times while keeping the same off-wall spacing for the C and H meshes. The far-field spacing is tripled to become in the normally used range as shown in Table 4.6. The resulting meshes are denser than in study 1, having between 300 and 400 thousand nodes, for further investigation of meshes refer to Appendix C. Note that the u/h in Table 4.6 indicates a nodal spacing between full uniform and full hyperbolic on the leading edge of the H mesh; this means we have higher aspect ratios than the C mesh. Orthogonality and skewness are similar to study 1, i.e. the main difference in this study is the aspect ratio in the leading edge region being lower and close to 10. This resulted in lower $C_D$ and higher $C_L$ values and grid-converged estimates and with lower errors, i.e. higher accuracy, as Figures 4.3 and 4.4 show, especially for the H mesh. The $C_D$ and $C_L$ orders went down for the C mesh, and went up for the H mesh, as in Table 4.7, probably because of the not fully uniform spacing. Two variant families of O topologies are included in this study. O is considered unfair to compare with the C mesh and the H mesh because O has higher aspect ratio and skewness along the leading edge region, which in turn is lengthened. Both O families yielded similar and overshooting orders of accuracy for $C_D$ and $C_L$. Improving the H mesh leading edge spacing and aspect ratio from study 1 resulted in more accurate functionals but the orders are the worst for $C_D$.

This study leads us to think that skewness and orthogonality and smooth spacing improve accuracy while better aspect ratios improve orders. This is because when all the spacings were improved and the number of nodes increased, H won in accuracy even though it had a worse aspect ratio of $\approx 14$ at the leading edge, followed by O and then C. And even though the number of nodes yielded by the parameters was more, the errors at the medium H mesh is almost the same at the errors at the fine C and O meshes. Table 4.8 shows the adjoint corrections, which were best for H then O then C. This suggests the spacing variation has something to do with accuracy as well. It is notable from Figure 4.4 that the interpolated solution for the O mesh is not very accurate even though a bicubic interpolation is used; for more information about the interpolation refer to Appendix B. Another note regarding Figure 4.4 is that the medium to fine adjoint correction error point does not show on the lift coefficient plot because the adjoint correction yielded a value identical to the grid converged $C_L$ and so the error is theoretically $10^{-\infty}$. 
### Table 4.6: Study 2 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>45°</td>
<td>3%</td>
<td>$1.0\times10^{-4}u$</td>
<td>$1.0\times10^{-4}h$</td>
<td>$0.1\times10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>330,245</td>
</tr>
<tr>
<td>O</td>
<td>45°</td>
<td>10%</td>
<td>$1.0\times10^{-4}u$</td>
<td>$1.0\times10^{-4}h$</td>
<td>$0.1\times10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>341,039</td>
</tr>
<tr>
<td>O</td>
<td>45°</td>
<td>3%</td>
<td>$1.0\times10^{-4}u$</td>
<td>$1.0\times10^{-4}h$</td>
<td>$0.1\times10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>299,739</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>$1.0\times10^{-4}u/h$</td>
<td>$0.1\times10^{-4}h$</td>
<td>$0.1\times10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>N/A</td>
<td>396,294</td>
</tr>
</tbody>
</table>

### Table 4.7: Study 2 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^+$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^+$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.24229 $\times 10^{-3}$</td>
<td>0.21088</td>
<td>0.21133</td>
<td>0.21151</td>
<td>1.3244</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.13212 $\times 10^{-3}$</td>
<td>0.11048</td>
<td>0.21101</td>
<td>0.21151</td>
<td>0.3438</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11407 $\times 10^{-3}$</td>
<td>0.21151</td>
<td>0.21193</td>
<td>0.21194</td>
<td>2.8499</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.44175 $\times 10^{-3}$</td>
<td>0.21131</td>
<td>0.21156</td>
<td>0.21179</td>
<td>2.8499</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.15663 $\times 10^{-3}$</td>
<td>0.21129</td>
<td>0.21151</td>
<td>0.21179</td>
<td>2.8499</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.10176 $\times 10^{-3}$</td>
<td>0.21129</td>
<td>0.21156</td>
<td>0.21179</td>
<td>2.8499</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.12798 $\times 10^{-3}$</td>
<td>0.10923</td>
<td>0.11048</td>
<td>0.21133</td>
<td>0.3438</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.10176 $\times 10^{-3}$</td>
<td>0.11048</td>
<td>0.21133</td>
<td>0.3438</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.12798 $\times 10^{-3}$</td>
<td>0.11048</td>
<td>0.21133</td>
<td>0.3438</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.060046 $\times 10^{-3}$</td>
<td>0.055852 $\times 10^{-3}$</td>
<td>0.05556 $\times 10^{-3}$</td>
<td>0.21136</td>
<td>2.8499</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.055852 $\times 10^{-3}$</td>
<td>0.05556 $\times 10^{-3}$</td>
<td>0.21136</td>
<td>2.8499</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.8: Study 2 adjoint error correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^0$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L^0$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.22322 $\times 10^{-3}$</td>
<td>0.11947 $\times 10^{-3}$</td>
<td>0.42</td>
<td>0.21074</td>
<td>0.21112</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.14496 $\times 10^{-3}$</td>
<td>0.13291 $\times 10^{-3}$</td>
<td>-0.04</td>
<td>0.21129</td>
<td>0.21136</td>
<td>0.17</td>
</tr>
<tr>
<td>O</td>
<td>Coarse → Medium</td>
<td>0.44344 $\times 10^{-3}$</td>
<td>0.17029 $\times 10^{-3}$</td>
<td>0.95</td>
<td>0.21094</td>
<td>0.21114</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.16550 $\times 10^{-3}$</td>
<td>0.11462 $\times 10^{-3}$</td>
<td>0.77</td>
<td>0.21127</td>
<td>0.21141</td>
<td>0.55</td>
</tr>
<tr>
<td>H</td>
<td>Coarse → Medium</td>
<td>0.031126 $\times 10^{-3}$</td>
<td>0.060674 $\times 10^{-3}$</td>
<td>0.99</td>
<td>0.21125</td>
<td>0.21190</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.036366 $\times 10^{-3}$</td>
<td>0.056039 $\times 10^{-3}$</td>
<td>0.96</td>
<td>0.21183</td>
<td>0.21194</td>
<td>1.14</td>
</tr>
</tbody>
</table>

### Table 4.9: Study 2 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{C_D}$</th>
<th>$O_{C_D}$</th>
<th>$A_{C_D}$</th>
<th>$E_{C_L}$</th>
<th>$O_{C_L}$</th>
<th>$A_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>H</td>
<td>C</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Second place</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Third place</td>
<td>C</td>
<td>H</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

Figure 4.3: Study 2 interpolation and adjoint correction plots
4.2. 2D Mesh Convergence Studies

Figure 4.4: Study 2 interpolation and adjoint correction error plots
4.2. 2D Mesh Convergence Studies

Study 3

In this study, Table 4.10 the offwall spacing $O_s$ is increased tenfold to yield square cells in the leading edge and trailing edge areas. This is the main modification in this study. No significant changes occurred in terms of relative accuracy performance, i.e. H is first then O then C for both functionals, which can be seen in Table 4.11 and Figure 4.6. However in terms of actual accuracy represented by the error, the three topologies yielded higher errors than study 2 for $C_D$. As for $C_L$, the H and C meshes yielded almost the same errors as in study 2, while the O mesh yielded a slightly higher error. In terms of $O_{C_D}$, O and H meshes have improved with lower values closer to 2.00; the C mesh order has significantly been lower but with the same error margin from 2.00. As for $O_{C_L}$, the H mesh improved, while the C and O meshes are still far from 2.00. When it comes to adjoint error correction, the H mesh performs best with almost perfect corrections from coarse to medium and from medium to fine as shown in Table 4.12. The C and O meshes both performed well in the coarse to medium correction but performed very poorly from medium to fine corrections with very little or completely wrong corrections with $-0.33 \leq \Delta \leq 0.20$. Table 4.13 shows the performance scores.

Table 4.10: Study 3 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>45°</td>
<td>3%</td>
<td>1.0×10^{-4}u</td>
<td>1.0×10^{-4}h</td>
<td>1.0×10^{-4}h</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>351,897</td>
</tr>
<tr>
<td>O</td>
<td>45°</td>
<td>3%</td>
<td>1.0×10^{-4}u</td>
<td>1.0×10^{-4}h</td>
<td>1.0×10^{-4}h</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>292,383</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>1.0×10^{-4}u/h</td>
<td>1.0×10^{-4}h</td>
<td>1.0×10^{-4}h</td>
<td>40</td>
<td>3</td>
<td>N/A</td>
<td>311,902</td>
</tr>
</tbody>
</table>

Table 4.11: Study 3 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.18224×10^{-3}</td>
<td>0.093613×10^{-3}</td>
<td>1.3553</td>
<td>0.21130</td>
<td>0.2119</td>
<td>1.2241</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.12843×10^{-3}</td>
<td>0.010736×10^{-3}</td>
<td>0.21165</td>
<td>0.21180</td>
<td>0.2119</td>
<td>1.2241</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.10594×10^{-3}</td>
<td>0.09613×10^{-3}</td>
<td>0.21170</td>
<td>0.21176</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.22478×10^{-3}</td>
<td>0.10594×10^{-3}</td>
<td>0.21152</td>
<td>0.21164</td>
<td>0.2117</td>
<td>1.0000</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.13184×10^{-3}</td>
<td>0.11163×10^{-3}</td>
<td>0.21170</td>
<td>0.21176</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11163×10^{-3}</td>
<td>0.10594×10^{-3}</td>
<td>0.21170</td>
<td>0.21176</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.13785×10^{-3}</td>
<td>0.056423×10^{-3}</td>
<td>3.3821</td>
<td>0.21131</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.064352×10^{-3}</td>
<td>0.056423×10^{-3}</td>
<td>3.3821</td>
<td>0.21131</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.057207×10^{-3}</td>
<td>0.056423×10^{-3}</td>
<td>3.3821</td>
<td>0.21131</td>
<td>0.2119</td>
<td>2.5152</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

Table 4.12: Study 3 adjoint error correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^{+}$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^{+}$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.16652×10^{-3}</td>
<td>0.11316×10^{-3}</td>
<td>1.28</td>
<td>0.21118</td>
<td>0.21167</td>
<td>1.06</td>
</tr>
<tr>
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<td>Medium → Fine</td>
<td>0.14108×10^{-3}</td>
<td>0.12988×10^{-3}</td>
<td>-0.07</td>
<td>0.21162</td>
<td>0.21168</td>
<td>0.20</td>
</tr>
<tr>
<td>O</td>
<td>Coarse → Medium</td>
<td>0.20608×10^{-3}</td>
<td>0.12880×10^{-3}</td>
<td>1.03</td>
<td>0.21139</td>
<td>0.21164</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.14155×10^{-3}</td>
<td>0.13107×10^{-3}</td>
<td>0.04</td>
<td>0.21161</td>
<td>0.21162</td>
<td>-0.33</td>
</tr>
<tr>
<td>H</td>
<td>Coarse → Medium</td>
<td>0.059609×10^{-3}</td>
<td>0.064606×10^{-3}</td>
<td>1.00</td>
<td>0.21121</td>
<td>0.21183</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.044606×10^{-3}</td>
<td>0.057389×10^{-3}</td>
<td>0.97</td>
<td>0.21179</td>
<td>0.21191</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 4.13: Study 3 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>H</td>
<td>O</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Second place</td>
<td>O</td>
<td>C</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Third place</td>
<td>C</td>
<td>H</td>
<td>C</td>
<td>C</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>
Figure 4.5: Study 3 interpolation and adjoint correction plots
Figure 4.6: Study 3 interpolation and adjoint correction error plots


**Study 4**

In study 4 the minimum normally used offwall spacing was of $0.1 \times 10^{-4}$, and the leading edge is set to have hyperbolic spacing with the same limit of $0.1 \times 10^{-4}$; this means that the average streamwise spacing is $2.0 \times 10^{-4}$, as shown in Table 4.14, i.e. towards the horizontal center-line of the aerofoil the cells have an aspect ratio of 1.00, then this increases to reach a maximum of 20.00 and then decreases towards 1.00 at the edge of the leading edge zone. The parameters in this study resulted in a larger number of nodes. The C and O meshes yielded much better $C_L$ convergence order estimates of 1.93 and 2.33 respectively as in Table 4.15 combined with lower errors than studies 2 and 3 as in Figure 4.7. The odd result in this study is the non-monotone $C_D$ and $C_L$ sequences obtained from the H mesh which resulted in a lack of estimated order and a grid-converged estimate for the functionals. As for $C_D$, the C and O meshes had lower errors than the previous studies but with off order estimates. The adjoint correction worked better with the O mesh than the C mesh but with poor performance on correcting from medium to fine, but a better correction from medium to fine was obtained for $C_L$ on the C mesh as shown in Table 4.16. Furthermore, the adjoint correction for the H mesh from medium to coarse lead to values that follow a monotone sequence which suggests that the adjoint correction method cannot accurately predict breaks in a monotone sequence. Table 4.17 shows the performance results.

**Table 4.14:** Study 4 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>45°</td>
<td>3%</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>353,262</td>
</tr>
<tr>
<td>O</td>
<td>45°</td>
<td>3%</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>57°</td>
<td>293,748</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>$0.1 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>N/A</td>
<td>298,606</td>
</tr>
</tbody>
</table>
### Table 4.15: Study 4 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{CD}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.23967 × 10(^{-3})</td>
<td>0.011382 × 10(^{-3})</td>
<td>3.4079</td>
<td>0.21090</td>
<td>0.21162</td>
<td>1.930</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.12585 × 10(^{-3})</td>
<td>0.011499 × 10(^{-3})</td>
<td>0.21143</td>
<td>0.21143</td>
<td>0.21157</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11499 × 10(^{-3})</td>
<td>0.21162</td>
<td>1.930</td>
<td>0.21162</td>
<td>0.21157</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.38568 × 10(^{-3})</td>
<td>0.11427 × 10(^{-3})</td>
<td>3.8931</td>
<td>0.21120</td>
<td>0.21151</td>
<td>2.330</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.13277 × 10(^{-3})</td>
<td>0.11427 × 10(^{-3})</td>
<td>3.8931</td>
<td>0.21120</td>
<td>0.21151</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11555 × 10(^{-3})</td>
<td>0.21151</td>
<td>2.330</td>
<td>0.21151</td>
<td>0.21151</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.27305 × 10(^{-3})</td>
<td>N/A</td>
<td>0.21132</td>
<td>0.21215</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.11976 × 10(^{-3})</td>
<td>N/A</td>
<td>0.21132</td>
<td>0.21215</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.14722 × 10(^{-3})</td>
<td>N/A</td>
<td>0.21132</td>
<td>0.21215</td>
<td>N/A</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.16: Study 4 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C^i_D$</th>
<th>$C^+_D$</th>
<th>$\Delta$</th>
<th>$C^i_L$</th>
<th>$C^+_L$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.20344 × 10(^{-3})</td>
<td>0.10933 × 10(^{-3})</td>
<td>1.15</td>
<td>0.21072</td>
<td>0.21150</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.13181 × 10(^{-3})</td>
<td>0.12513 × 10(^{-3})</td>
<td>0.07</td>
<td>0.21138</td>
<td>0.21149</td>
<td>0.43</td>
</tr>
<tr>
<td>O</td>
<td>Coarse → Medium</td>
<td>0.34316 × 10(^{-3})</td>
<td>0.13123 × 10(^{-3})</td>
<td>1.01</td>
<td>0.21095</td>
<td>0.21144</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.13426 × 10(^{-3})</td>
<td>0.11967 × 10(^{-3})</td>
<td>0.76</td>
<td>0.21139</td>
<td>0.21145</td>
<td>0.00</td>
</tr>
<tr>
<td>H</td>
<td>Coarse → Medium</td>
<td>0.14297 × 10(^{-3})</td>
<td>0.079354 × 10(^{-3})</td>
<td>1.26</td>
<td>0.21113</td>
<td>0.21221</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.077616 × 10(^{-3})</td>
<td>0.067434 × 10(^{-3})</td>
<td>-1.91</td>
<td>0.21210</td>
<td>0.21238</td>
<td>-2.88</td>
</tr>
</tbody>
</table>

### Table 4.17: Study 4 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>O</td>
<td>C</td>
<td>O</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Second place</td>
<td>C</td>
<td>O</td>
<td>C</td>
<td>O</td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td>Third place</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

Figure 4.7: Study 4 interpolation and adjoint correction plots
4.2. 2D Mesh Convergence Studies

Figure 4.8: Study 4 interpolation and adjoint correction error plots
Study 5

In this study, the leading edge region is extended to span 20% of the aerofoil surface length. This is done by moving the leading edge region block interface ahead in the streamwise direction. This results in left tilted cells midway in the leading edge region in the C and O meshes; this is a consequence of ANSYS mesh generation. This is shown in the Figure 4.9. More nodes were used in this study: around half a million. The leading edge spacing starts small at $0.1 \times 10^{-3}$ and increases hyperbolically to the region’s end as in Table 4.18. Small offwall spacing was used, making fairly high aspect ratio cells at the leading edge. Cells around the trailing edge are, like previous studies, mostly square. The grid convergence results for $C_L$ in Table 4.19 were very erroneous in that the three topologies did not yield monotone sequences. As for $C_D$, the H mesh did not yield monotone sequence as shown in Figure 4.10 and thus no grid converged estimate was obtained, this is reflected in all the broken error lined in Figure 4.11. Therefore accuracy cannot be fairly judged in this study for $C_L$ and $C_D$ for the H mesh. Where grid converged estimates were yielded, the C mesh performed better than the O was. Similarly in terms of orders of convergence, the C mesh performed better than the O mesh with an order of 2.3 for $C_D$. The adjoint correction results were erroneous as well, with either a wrong or a very slight correction, with the exception of the correction of $C_L$ for the C mesh.

![Figure 4.9: Illustration of smooth cell variation through block interfaces](image-url)
4.2. 2D Mesh Convergence Studies

Table 4.18: Study 5 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>( \phi )</th>
<th>( L_r )</th>
<th>( L_s )</th>
<th>( T_s )</th>
<th>( O_s )</th>
<th>( F_d )</th>
<th>( F_s )</th>
<th>( \theta )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90°</td>
<td>20%</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} )</td>
<td>40</td>
<td>3</td>
<td>90°</td>
<td>473,060</td>
</tr>
<tr>
<td>O</td>
<td>90°</td>
<td>20%</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} )</td>
<td>40</td>
<td>3</td>
<td>90°</td>
<td>479,020</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} h )</td>
<td>( 0.1 \times 10^{-4} )</td>
<td>40</td>
<td>3</td>
<td>N/A</td>
<td>478,550</td>
</tr>
</tbody>
</table>

Table 4.19: Study 5 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>( C_D )</th>
<th>( C_D^* )</th>
<th>( P_{CD} )</th>
<th>( C_L )</th>
<th>( C_L^* )</th>
<th>( P_{CL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.3076 \times 10^{-3}</td>
<td>0.25166 \times 10^{-3}</td>
<td>2.3059</td>
<td>0.2100</td>
<td>0.2115</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.2631 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>2.50</td>
<td>2.50</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.2540 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>4.00</td>
<td>4.00</td>
<td>4.00</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.6238 \times 10^{-3}</td>
<td>0.15777 \times 10^{-3}</td>
<td>3.4642</td>
<td>0.2110</td>
<td>0.2113</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.2000 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>5.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.13031 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>6.00</td>
<td>6.00</td>
<td>6.00</td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.2958 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>7.00</td>
<td>7.00</td>
<td>7.00</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.1926 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>8.00</td>
<td>8.00</td>
<td>8.00</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.3117 \times 10^{-3}</td>
<td>( -\infty )</td>
<td>0</td>
<td>9.00</td>
<td>9.00</td>
<td>9.00</td>
</tr>
</tbody>
</table>

Table 4.20: Study 5 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>( C_D^i )</th>
<th>( C_D^+ )</th>
<th>( \Delta )</th>
<th>( C_L^i )</th>
<th>( C_L^+ )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.90541 \times 10^{-3}</td>
<td>0.19181 \times 10^{-3}</td>
<td>2.60</td>
<td>0.20987</td>
<td>0.21136</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.21301 \times 10^{-3}</td>
<td>0.13228 \times 10^{-3}</td>
<td>14.4</td>
<td>0.21145</td>
<td>0.21137</td>
<td>0.26</td>
</tr>
<tr>
<td>O</td>
<td>Coarse → Medium</td>
<td>0.51342 \times 10^{-3}</td>
<td>0.14798 \times 10^{-3}</td>
<td>1.12</td>
<td>0.21092</td>
<td>0.21176</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.20455 \times 10^{-3}</td>
<td>0.16866 \times 10^{-3}</td>
<td>0.82</td>
<td>0.21132</td>
<td>0.21115</td>
<td>0.50</td>
</tr>
<tr>
<td>H</td>
<td>Coarse → Medium</td>
<td>0.10954 \times 10^{-3}</td>
<td>0.072866 \times 10^{-3}</td>
<td>2.16</td>
<td>0.21099</td>
<td>0.21217</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.11731 \times 10^{-3}</td>
<td>0.057267 \times 10^{-3}</td>
<td>-1.13</td>
<td>0.21179</td>
<td>0.21242</td>
<td>-0.66</td>
</tr>
</tbody>
</table>

Table 4.21: Study 5 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>( E_{C_D} )</th>
<th>( O_{C_D} )</th>
<th>( A_{C_D} )</th>
<th>( E_{C_L} )</th>
<th>( O_{C_L} )</th>
<th>( A_{C_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>C</td>
<td>C</td>
<td>O</td>
<td>N/A</td>
<td>N/A</td>
<td>C</td>
</tr>
<tr>
<td>Second place</td>
<td>O</td>
<td>O</td>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>O</td>
</tr>
<tr>
<td>Third place</td>
<td>H</td>
<td>H</td>
<td>C</td>
<td>N/A</td>
<td>N/A</td>
<td>H</td>
</tr>
</tbody>
</table>
Figure 4.10: Study 5 interpolation and adjoint correction plots
4.2. 2D Mesh Convergence Studies

Figure 4.11: Study 5 interpolation and adjoint correction error plots
Study 6

This study represents a brute force experiment where we use a very large (beyond necessary) number of nodes to see what are the best results we can get by simply increasing the number of nodes. Increasing the number of nodes is guaranteed to increase accuracy but what about convergence order and adjoint correction? In this study, the fine meshes range from a million to one and a half million nodes with $0.05 \times 10^{-4}$ hyperbolic spacing around the leading edge, with the rest of the parameters in Table 4.22. Table 4.23 shows that the results for the three meshes were anomalous; a non-monotone sequence of $C_D$ was obtained for both meshes, causing non-convergence of the Richardson method and therefore no $C'_D$ or $P_{C_D}$ were obtained. The case is the same with $C_L$ for both meshes. Due to the poor mesh convergence performance, the adjoint correction method was omitted and saved for better studies, and no scores were given in Table 4.24. From looking at the results of studies 1 to 8, one is lead to say that the leading edge region chosen of 3% is not enough and should be increased. The following studies, 9 and 10, use a larger leading edge region spanning 10% of the aerofoil surface length. One last note regarding this study is that the large number of nodes used yielding a non-monotonic sequence suggests that surpassing the asymptotic range is not suitable for mesh convergence studies. Furthermore, this also suggests that there might be much more complex relations between spacings and number of nodes underlying the obtained results. Moreover, it could be because the Richardson method has a limitation at high resolution meshes because it fails to estimate orders with the small difference of functionals along each mesh family. Figure 4.12 shows the data for the three topologies.

<table>
<thead>
<tr>
<th>Grid</th>
<th>Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>62°</td>
<td>3%</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>113°</td>
<td>1,407,190</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>90°</td>
<td>20%</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>90°</td>
<td>1,091,684</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>62°</td>
<td>3%</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>90°</td>
<td>1,091,684</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}h$</td>
<td>$0.05 \times 10^{-4}$</td>
<td>40</td>
<td>3</td>
<td>N/A</td>
<td>1,242,038</td>
<td></td>
</tr>
</tbody>
</table>
### Table 4.23: Study 6 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>$0.10636 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21131</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.11001 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21134</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.094617 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21069</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>$0.12564 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21139</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.11028 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21150</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.090115 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21167</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>$0.11836 \times 10^{-3}$</td>
<td>$0.11290 \times 10^{-3}$</td>
<td>2.36</td>
<td>0.21213</td>
<td>$0.20990$</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.15135 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21202</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.23150 \times 10^{-3}$</td>
<td>$-\infty$</td>
<td>0</td>
<td>0.21154</td>
<td>$-\infty$</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 4.24: Study 6 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>N/A</td>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>H</td>
<td>N/A</td>
</tr>
<tr>
<td>Second place</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Third place</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>
Figure 4.12: Study 6 adjoint correction results
Study 7

In this study, we combine nice square cells around a large leading edge region with a large number of nodes and good far-field spacing and square cells around the trailing edge as well, this is shown in Table 4.25. All meshes yielded acceptable orders except for the O mesh with $C_L$ where the order was low. This can be shown through the relative slopes of the error plots in Figure 4.14 and the data in Table 4.26. The reader also notices that the H mesh is most accurate for $C_D$, followed by the C and then the O meshes. Figure 4.13 shows that the C and H meshes have quite a resemblance in how much the values of the functionals change as the number of nodes increase. The O mesh is most accurate for $C_L$, followed by the H and then the C meshes. The adjoint correction results on the other hand were completely erroneous yielding corrections that pushed the coarse values even farther from the medium, and the medium from the fine ones.

Table 4.25: Study 7 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>62°</td>
<td>10%</td>
<td>$1.0 \times 10^{-4} u$</td>
<td>$1.0 \times 10^{-4} h$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>78°</td>
<td>1,393,308</td>
</tr>
<tr>
<td>O</td>
<td>62°</td>
<td>10%</td>
<td>$1.0 \times 10^{-4} u$</td>
<td>$1.0 \times 10^{-4} h$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>78°</td>
<td>1,393,614</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>$1.0 \times 10^{-4} u$</td>
<td>$1.0 \times 10^{-4} h$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>N/A</td>
<td>1,399,740</td>
</tr>
</tbody>
</table>

Table 4.26: Study 7 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_L^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.14421 $\times 10^{-3}$</td>
<td>0.079011 $\times 10^{-3}$</td>
<td>1.6453</td>
<td>0.21158</td>
<td>0.21207</td>
<td>1.5869</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.099907 $\times 10^{-3}$</td>
<td>0.085736 $\times 10^{-3}$</td>
<td>1.5869</td>
<td>0.21185</td>
<td>0.21197</td>
<td>1.5289</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.085736 $\times 10^{-3}$</td>
<td>0.085736 $\times 10^{-3}$</td>
<td>1.5869</td>
<td>0.21185</td>
<td>0.21197</td>
<td>1.5289</td>
</tr>
<tr>
<td>O</td>
<td>Coarse</td>
<td>0.21129 $\times 10^{-3}$</td>
<td>0.086911 $\times 10^{-3}$</td>
<td>2.1118</td>
<td>0.21170</td>
<td>0.21182</td>
<td>1.2231</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.11579 $\times 10^{-3}$</td>
<td>0.093637 $\times 10^{-3}$</td>
<td>2.1118</td>
<td>0.21177</td>
<td>0.21180</td>
<td>1.2231</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.093637 $\times 10^{-3}$</td>
<td>0.093637 $\times 10^{-3}$</td>
<td>2.1118</td>
<td>0.21177</td>
<td>0.21180</td>
<td>1.2231</td>
</tr>
<tr>
<td>H</td>
<td>Coarse</td>
<td>0.11189 $\times 10^{-3}$</td>
<td>0.066233 $\times 10^{-3}$</td>
<td>1.7099</td>
<td>0.21205</td>
<td>0.21240</td>
<td>1.5618</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.080142 $\times 10^{-3}$</td>
<td>0.066233 $\times 10^{-3}$</td>
<td>1.7099</td>
<td>0.21227</td>
<td>0.21240</td>
<td>1.5618</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.070547 $\times 10^{-3}$</td>
<td>0.066233 $\times 10^{-3}$</td>
<td>1.7099</td>
<td>0.21235</td>
<td>0.21240</td>
<td>1.5618</td>
</tr>
</tbody>
</table>
### Table 4.27: Study 7 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^{+i}$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^{+i}$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse $\rightarrow$ Medium</td>
<td>$0.32952 \times 10^{-3}$</td>
<td>$0.15575 \times 10^{-3}$</td>
<td>-0.26</td>
<td>$0.21149$</td>
<td>$0.21130$</td>
<td>-1.04</td>
</tr>
<tr>
<td></td>
<td>Medium $\rightarrow$ Fine</td>
<td>$0.19471 \times 10^{-3}$</td>
<td>$0.13515 \times 10^{-3}$</td>
<td>-2.49</td>
<td>$0.21183$</td>
<td>$0.21154$</td>
<td>-2.59</td>
</tr>
<tr>
<td>O</td>
<td>Coarse $\rightarrow$ Medium</td>
<td>$0.39432 \times 10^{-3}$</td>
<td>$0.18052 \times 10^{-3}$</td>
<td>0.32</td>
<td>$0.21161$</td>
<td>$0.21118$</td>
<td>-7.49</td>
</tr>
<tr>
<td></td>
<td>Medium $\rightarrow$ Fine</td>
<td>$0.21018 \times 10^{-3}$</td>
<td>$0.14197 \times 10^{-3}$</td>
<td>-1.18</td>
<td>$0.21175$</td>
<td>$0.21139$</td>
<td>-12.69</td>
</tr>
<tr>
<td>H</td>
<td>Coarse $\rightarrow$ Medium</td>
<td>$0.23833 \times 10^{-3}$</td>
<td>$0.10792 \times 10^{-3}$</td>
<td>0.13</td>
<td>$0.21197$</td>
<td>$0.21197$</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td>Medium $\rightarrow$ Fine</td>
<td>$0.13592 \times 10^{-3}$</td>
<td>$0.095258 \times 10^{-3}$</td>
<td>-1.58</td>
<td>$0.21225$</td>
<td>$0.21209$</td>
<td>-2.25</td>
</tr>
</tbody>
</table>

### Table 4.28: Study 7 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First place</td>
<td>H</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>C</td>
<td>H</td>
</tr>
<tr>
<td>Second place</td>
<td>C</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>C</td>
</tr>
<tr>
<td>Third place</td>
<td>O</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>O</td>
<td>O</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

Figure 4.13: Study 7 interpolation and adjoint correction plots
Figure 4.14: Study 7 interpolation and adjoint correction error plots.
Study 8

In this study, the number of nodes is reduced by shrinking the far-field by half and increasing the spacings around the leading edge and trailing edges as seen in Table 4.29. Furthermore, the NACA0012 geometry created by ICEM-CFD for this study used more points to interpolate the aerofoil surface resulting in a smoother aerofoil surface very similar to Figure 2.4 in Section 2.1. Excellent order results were obtained for $C_D$ while bad orders were obtained for $C_L$ for the O mesh, which had a non-monotone sequence, and the H mesh, which had an overshoot estimated order. The adjoint correction method yielded almost perfect results with almost exact corrections to the next finer mesh with the exception of medium to fine correction of $C_L$ on the O mesh as in Table 4.31. This is shown through the data in Tables 4.30 and 4.31 and Figures 4.15 and 4.16. This suggests that the curvature smoothness of the mesh affects the order of mesh convergence; this is because in ICEM-CFD the aerofoil surface curve dictates the curvature smoothness of the mesh which is again illustrated in Figure 2.4 in Section 2.1. Because the $C_L$ orders are off, this study suggests that smaller spacing is required for better order estimates on the H mesh.

Note on Richardson Method: It is worth mentioning here that the capability of the Richardson method itself should be investigated in future studies, especially its specific non-linear behaviour and its sensitivity to the data. For example in this study if the $C_L$ computed on the H fine mesh was 0.00002 higher i.e. $C_L = 0.20982$, the estimated order for $C_L$ convergence would be 1.59 instead of 3.39 in Table 4.30.

Table 4.29: Study 8 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>62°</td>
<td>10%</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>20</td>
<td>1</td>
<td>78°</td>
<td>373,421</td>
</tr>
<tr>
<td>O</td>
<td>62°</td>
<td>10%</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>20</td>
<td>1</td>
<td>78°</td>
<td>307,115</td>
</tr>
<tr>
<td>H</td>
<td>N/A</td>
<td>N/A</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>20</td>
<td>1</td>
<td>N/A</td>
<td>308,662</td>
</tr>
</tbody>
</table>
### Table 4.30: Study 8 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>Coarse</td>
<td>$0.20513 \times 10^{-3}$</td>
<td>$0.12165 \times 10^{-3}$</td>
<td>2.2021</td>
<td>$0.20906$</td>
<td>$0.20950$</td>
<td>2.0518</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.13994 \times 10^{-3}$</td>
<td>$0.12659 \times 10^{-3}$</td>
<td></td>
<td>$0.20939$</td>
<td>$0.20947$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.12569 \times 10^{-3}$</td>
<td>$0.12659 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>O</strong></td>
<td>Coarse</td>
<td>$0.24552 \times 10^{-3}$</td>
<td>$0.12691 \times 10^{-3}$</td>
<td>2.5811</td>
<td>$0.20936$</td>
<td>$0.20934$</td>
<td>-0.59580</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.14690 \times 10^{-3}$</td>
<td>$0.13031 \times 10^{-3}$</td>
<td></td>
<td>$0.20934$</td>
<td>$0.20931$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.13031 \times 10^{-3}$</td>
<td>$0.12691 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>H</strong></td>
<td>Coarse</td>
<td>$0.27017 \times 10^{-3}$</td>
<td>$0.11288 \times 10^{-3}$</td>
<td>2.3640</td>
<td>$0.20810$</td>
<td>$0.20980$</td>
<td>3.3898</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.14378 \times 10^{-3}$</td>
<td>$0.11288 \times 10^{-3}$</td>
<td></td>
<td>$0.20965$</td>
<td>$0.20982$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.11902 \times 10^{-3}$</td>
<td>$0.11288 \times 10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.31: Study 8 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
<td>Coarse → Medium</td>
<td>$0.25270 \times 10^{-3}$</td>
<td>$0.14110 \times 10^{-3}$</td>
<td>0.98</td>
<td>$0.20900$</td>
<td>$0.20938$</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>$0.15180 \times 10^{-3}$</td>
<td>$0.12606 \times 10^{-3}$</td>
<td>0.97</td>
<td>$0.20940$</td>
<td>$0.20946$</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>O</strong></td>
<td>Coarse → Medium</td>
<td>$0.29100 \times 10^{-3}$</td>
<td>$0.15671 \times 10^{-3}$</td>
<td>0.90</td>
<td>$0.20930$</td>
<td>$0.20934$</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>$0.29100 \times 10^{-3}$</td>
<td>$0.15671 \times 10^{-3}$</td>
<td>0.85</td>
<td>$0.20932$</td>
<td>$0.20932$</td>
<td>0.61</td>
</tr>
<tr>
<td><strong>H</strong></td>
<td>Coarse → Medium</td>
<td>$0.32060 \times 10^{-3}$</td>
<td>$0.14463 \times 10^{-3}$</td>
<td>0.99</td>
<td>$0.20890$</td>
<td>$0.20962$</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>$0.15610 \times 10^{-3}$</td>
<td>$0.11943 \times 10^{-3}$</td>
<td>0.98</td>
<td>$0.20960$</td>
<td>$0.20979$</td>
<td>0.93</td>
</tr>
</tbody>
</table>

### Table 4.32: Study 8 ranking table

<table>
<thead>
<tr>
<th>Ranking</th>
<th>$E_{C_D}$</th>
<th>$O_{C_D}$</th>
<th>$A_{C_D}$</th>
<th>$E_{C_L}$</th>
<th>$O_{C_L}$</th>
<th>$A_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First place</strong></td>
<td>O</td>
<td>C</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>O</td>
</tr>
<tr>
<td><strong>Second place</strong></td>
<td>C</td>
<td>H</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>H</td>
</tr>
<tr>
<td><strong>Third place</strong></td>
<td>H</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>O</td>
<td>C</td>
</tr>
</tbody>
</table>
Figure 4.15: Study 8 interpolation and adjoint correction plots
4.2. 2D Mesh Convergence Studies

Figure 4.16: Study 8 interpolation and adjoint correction error plots
4.2. 2D Mesh Convergence Studies

Study 9 (AMBER2D)

This study is conducted using C meshes used in [20] and serves to give an insight to the difference between ICEM CFD generated meshes and AMBER2D [26] generated meshes. The main difference between these C meshes and the ones used in the previous studies is the smoothness of the mesh variation in the streamwise direction and through block interfaces i.e. the spacings do not jump nor shrink suddenly. These meshes yielded excellent results of accuracy, order, and adjoint error correction as in Tables 4.35 and 4.35. These meshes have smooth mesh curvature, smooth spacing variation in the streamwise direction, and almost circular cell distribution around the leading edge. What is quite interesting in this study is that the number of nodes is low, and the far-field is small (10× chord length). However, this is consistent with the repeated adjoint correction results from previous studies where the correction becomes worse as the mesh resolution increases.

Table 4.33: Study 9 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>φ</th>
<th>L_r</th>
<th>L_s</th>
<th>T_s</th>
<th>O_s</th>
<th>F_d</th>
<th>F_s</th>
<th>θ</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90°</td>
<td>N/A</td>
<td>2.5×10^{-4}h</td>
<td>2.5×10^{-4}h</td>
<td>2.5×10^{-4}</td>
<td>10</td>
<td>2</td>
<td>90°</td>
<td>190,512</td>
</tr>
</tbody>
</table>

Table 4.34: Study 9 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>C_D</th>
<th>C'_D</th>
<th>P_{C_D}</th>
<th>C_L</th>
<th>C'_L</th>
<th>P_{C_L}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.30582×10^{-3}</td>
<td>0.20515</td>
<td>0.26239×10^{3}</td>
<td>0.25129×10^{-3}</td>
<td>0.20648</td>
<td>0.20694</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.25363×10^{-3}</td>
<td>0.20647</td>
<td>0.25129×10^{-3}</td>
<td>0.20680</td>
<td>0.20694</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.25363×10^{-3}</td>
<td>0.20680</td>
<td>0.25129×10^{-3}</td>
<td>0.20694</td>
<td>2.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.35: Study 9 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>C_D</th>
<th>C'_D</th>
<th>Δ</th>
<th>C_L</th>
<th>C'_L</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.27923×10^{-3}</td>
<td>0.26234×10^{-3}</td>
<td>1.00</td>
<td>0.20511</td>
<td>0.20647</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.25592×10^{-3}</td>
<td>0.25410×10^{-3}</td>
<td>0.95</td>
<td>0.20647</td>
<td>0.20680</td>
<td>0.94</td>
</tr>
</tbody>
</table>
4.2. 2D Mesh Convergence Studies

Figure 4.17: Study 9 interpolation and adjoint correction plots

Figure 4.18: Study 9 interpolation and adjoint correction error plots
4.2.1 Discussion

After obtaining all the data required and getting them organized in the format described above, a best topology was determined for each study, so in a sense we are performing a horizontal comparison in each study between different topologies to determine the best one. Then given that one topology wins in most of the studies, the studies themselves are compared vertically for a specific topology, and with careful studying of the parameters and the shape of the mesh, conclusions were drawn about what kind of meshes provide best results in terms of accuracy, producing correct order of convergence and being suitable for applying the adjoint error correction method.

To have a broader view on the results in Section 4.2, the scores of each topology are compiled as follows. We allocate the score of 1.0 for first place, 0.5 for second place, 1/3 for third place, and 0 if not applicable. For each of the measures, $E_F$, $O_F$, and $A_F$ we add the scores of each topology through the nine studies. Table 4.48 shows the obtained overall scores to the first decimal place.

<table>
<thead>
<tr>
<th>Topology</th>
<th>$E_{CD}$</th>
<th>$O_{CD}$</th>
<th>$A_{CD}$</th>
<th>$E_{CL}$</th>
<th>$O_{CL}$</th>
<th>$A_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4.0</td>
<td>2.3</td>
<td>4.3</td>
<td>3.8</td>
<td>3.8</td>
<td>4.2</td>
</tr>
<tr>
<td>C</td>
<td>3.2</td>
<td>4.8</td>
<td>2.3</td>
<td>2.5</td>
<td>3.3</td>
<td>3.7</td>
</tr>
<tr>
<td>O</td>
<td>3.8</td>
<td>3.8</td>
<td>4.3</td>
<td>2.8</td>
<td>2.0</td>
<td>3.2</td>
</tr>
</tbody>
</table>

The H mesh receives the first place for drag accuracy followed closely by the O mesh, while the C mesh scores last place. The H mesh however falls to last place for drag convergence order, with first and second places taken by the C and O meshes respectively. The H and O meshes tie for first place when it came to drag adjoint correction followed by the C mesh. As for the lift coefficient results, the H mesh scored first place in accuracy, order, and adjoint correction. The C and O meshes scored second and last places respectively in order and adjoint correction while last and second places respectively in lift accuracy. H has the best overall behaviour the lift coefficient, while the scores of accuracy, order, and adjoint correction for the drag coefficient are not consistent i.e. no topology wins in all categories.

When it comes to order of convergence, much more concrete trends appear for the lift coefficient $C_L$ i.e. it is much easier to infer what should be the mesh parameters to yield proper $P_{CL} \approx 2$. For the drag coefficient, it is more subtle. This in turn suggests that the machine zero issue has a role to play here, since the quantity $C_D$ has a theoretical exact value
of 0.00. Moreover most of the values of $C_D$ on fine meshes were on the order of $10^{-4}$, and this raises the question of are we accurate enough. But again, this suggests that functionals of zero theoretical value and close to zero grid-converged value are poor measures of a CFD solver order of accuracy estimation.

Through detailed studying of the data, scores, and meshes, the following mesh generation guidelines were drawn in order to produce CFD meshes that result in full convergence with close to theoretical order of convergence and can be used for the adjoint error estimation method. Note that all distances are in terms of chord length, and these parameters are for the fine meshes in a mesh convergence study.

### Optimal Mesh Parameters for 2D Inviscid Flow

- $F_d \geq 30$, the far field distance has some effect. In general, when the far field is too small $\leq 20$, the estimated order is a bit higher than the theoretical order, and with it being too large $\geq 50$, the estimated order is a bit less.

- $1 \leq F_s \leq 3$

- $100,000 \leq N \leq 500,000$ for a good asymptotic range.

- $O_s \leq 10^{-4}$

- $\frac{L_r}{O_s} \approx 1$ specifically and generally an aspect ratio between 1 and 4 in a leading edge region $L_r$ of 10% i.e. spanning through 10% of the aerofoil surface curve length. In other words, spacing must be best within along minimum of 10% of the aerofoil surface, perfecting spacing through the rest of the aerofoil increases accuracy but our ultimate goal is to increase accuracy while maintaining or reducing computational cost.

- Matching of mesh spacings through block interfaces both close to the aerofoil surface and close to the far field boundary while ensuring smooth spacing variation in the streamwise direction throughout physical domain. This aspect of the mesh improves results and mesh convergence tests even if the spacings are not optimal. This is shown from the results for AMBER2D and some validation studies in Section 4.4. It is also illustrated in Figure 4.19. In order to further investigate the effect of spacings along the leading and trailing edges, a study was conducted using the H mesh, the winning mesh, were nine combinations of spacings of $L_r$ and $T_r$ were used, and the resultant convergence orders are shown in Figure 4.20.
4.2. 2D Mesh Convergence Studies

Figure 4.19: Bad spacing matching (left) and good spacing matching (right)

- Using uniform spacing $u$ along the $L_r$ is best and helps with reaching desired convergence order even when the mesh has other issues like non-matching spacings through block interfaces.

- Mesh curvature smoothness across block interfaces. This is best described via figures. Figure 4.21 shows different C-meshes contrasting good versus bad curvature. This has a direct relationship to mesh skewness. The figure illustrates going from bad surface curvature (left) to a good surface curvature (right). Mathematically this means that for mesh points $\mathbf{x} = (x, y, z)$, we have $\partial \mathbf{x}/\partial \xi$ is continuous at block interfaces.

- Using smooth surface aerofoils, i.e. more points are used in the geometry generation, is particularly important for the adjoint error correction method to work properly.

- With the exception of H-meshes, node concentrations away from the aerofoil surface are to be avoided since they lead to erroneous orders of convergence even with a number of nodes exceeding one million. The nature of H-meshes dictates having node concentrations away from the aerofoil surface. Figure 4.23 shows various members of Family 4. These concentrations lead to erroneous order estimation, obtaining an overestimated order that reached in some cases fifth order.
Figure 4.20: Leading and trailing edge spacing combination study

(a) C-Mesh with unwanted concentrations  
(b) O-Mesh with unwanted concentrations

Figure 4.22: Illustrating unwanted node concentrations

- Must have good orthogonality and low skewness of cells around the leading edge and halfway through the chord length improve accuracy and convergence order significantly. In other words, having $\theta, \phi = 90^\circ$ is optimal for C and O meshes. H meshes inherently have good orthogonality around the aerofoil. The C mesh has the best of this quality since it combines low skewness cells around both the leading and trailing edges.
4.2. 2D Mesh Convergence Studies

Figure 4.21: Meshing curvature illustration

- Hyperbolic spacing variation, or a smooth spacing variation algorithm, in the stream-wise, and preferably normal, directions is required for a proper order of accuracy convergence. Having spacing jumps in the stream or normal direction will sometimes lead to erroneous order of convergence unless the jump is smooth such as shown in some of the verification studies below.

The violation of these conditions will most likely cause an estimated order of accuracy that does not match the theoretical order. A large off-wall spacing will cause the order to decrease, and the deviation of the aspect ratio at the leading edge from unity will lead to an increase of order, which can reach double or even triple the theoretical order regardless of how large the number of nodes which is used. One last note is that it is clear from the results and correlations that there are mesh parameters and conditions which result in good performance in regards to $C_D$ and others in regards to $C_L$. Having the ones related to a specific functional yields good results with respect to it and combining all parameters yields overall good results.

2D Turbulent Optimal Mesh Parameters

As for producing good 2D meshes suitable for turbulent flow simulations, the guidelines related to mesh shape, node concentrations, orthogonality and skewness remain the same. These parameters are deduces from several turbulent flow studies of which only study 9 in Section 4.4 is included. The following parameters are different:
• $O_s \leq 10^{-6}$

• Hyperbolic stream-wise spacing is used along the $L_r$.

• Flaring is required for convergence and proper order of convergence. Flaring is shown in Figure 4.23. Many studies were conducted under turbulent conditions with mesh parameters identical or similar to Figure 4.23 but without the flaring and none of the cases converged. These studies are not included.

![Figure 4.23: Flaring in the far-field beyond trailing edge.](image-url)
4.3 3D Mesh Convergence Studies

This section comprises selected 3D mesh convergence studies conducted to give some insight and a start for a similar investigation to the 2D studies for future work. The first three studies, 10, 11, and 12, are done using the H-H topology around a rectangular NACA0012 wing with a sharp trailing edge and wing tip. Study 13 is conducted with the same topology around the ONERA-M6 wing with a sharp trailing edge and a rounded wing tip.

Study 10

In this study, reasonable leading edge and trailing edge mesh spacings are applied (with reference to deductions from the 2D studies). Then in order to investigate the effect of the wing tip spacing $W_t$, it is assigned a large value of 0.2 times the chord length as shown in Table 4.37. The fine mesh in this H-H topology family has approximately 5.6 million nodes. The coarse and medium meshes converged but the fine mesh did not converge even with tweaking multiple dissipation parameters. Therefore, no convergence order and grid converged estimates were obtained as shown in Table 4.38. Nevertheless, the available results for the coarse and medium meshes were used to obtain an adjoint correction from the coarse to the medium mesh which yielded reasonable but not excellent results; only an average $\Delta$ of 0.4 as in Table 4.39.

<table>
<thead>
<tr>
<th>Study 10 mesh parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Topology</td>
</tr>
<tr>
<td>H-H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Study 10 mesh convergence data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Topology</td>
</tr>
<tr>
<td>H-H</td>
</tr>
<tr>
<td>H-H</td>
</tr>
<tr>
<td>H-H</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Study 10 adjoint correction results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid Topology</td>
</tr>
<tr>
<td>H-H</td>
</tr>
<tr>
<td>H-H</td>
</tr>
</tbody>
</table>
Study 11

In this study, the wing tip spacing $W_p$ is reduced to $10.0 \times 10^{-4}$ of the chord length and increases hyperbolically till 50% of the wing span and decreases hyperbolically again to reach the root spacing of the same value. The leading edge and trailing edge spacings are a quarter of the wing tip spacing, as shown in Table 4.40. All meshes in the family fully converged. An order of 2.6 was obtained for $C_D$ as shown in Table 4.41 and no order for $C_L$ was obtained because the sequence of functionals is not monotone. Furthermore, Table 4.42 shows that the adjoint correction method yielded poor results. This might be due to the fact that a trilinear interpolation is used but it might be due to the relatively large spacings used at the leading and trailing edges.

Table 4.40: Study 11 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$W_l$</th>
<th>$W_p$</th>
<th>$W$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>N/A</td>
<td>$2.5 \times 10^{-4}h$</td>
<td>$2.5 \times 10^{-4}h$</td>
<td>$2.5 \times 10^{-4}$</td>
<td>25</td>
<td>1</td>
<td>$10 \times 10^{-4}h$</td>
<td>$10 \times 10^{-4}h$</td>
<td>2.0</td>
<td>3,042,000</td>
</tr>
</tbody>
</table>

Table 4.41: Study 11 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_L^*$</th>
<th>$P_{CD}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>Coarse</td>
<td>0.99776$\times 10^{-2}$</td>
<td>0.6599$\times 10^{-2}$</td>
<td>2.5500</td>
<td>0.19011</td>
<td>0</td>
<td>$-\infty$</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.71736$\times 10^{-2}$</td>
<td>0.18891</td>
<td>0.6599$\times 10^{-2}$</td>
<td>0.19011</td>
<td>0</td>
<td>$-\infty$</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.66976$\times 10^{-2}$</td>
<td>0.18762</td>
<td>0.18891</td>
<td>0.19010</td>
<td>0</td>
<td>$-\infty$</td>
</tr>
</tbody>
</table>

Table 4.42: Study 11 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>Coarse → Medium</td>
<td>2.1975$\times 10^{-2}$</td>
<td>1.0046$\times 10^{-2}$</td>
<td>-0.024</td>
<td>0.18762</td>
<td>0.18810</td>
<td>-0.667</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>1.0478$\times 10^{-2}$</td>
<td>0.70019$\times 10^{-2}$</td>
<td>0.18964</td>
<td>0.18964</td>
<td>0.18997</td>
<td>0.140</td>
</tr>
</tbody>
</table>
4.3. 3D Mesh Convergence Studies

Figure 4.24: Study 11 interpolation and adjoint correction plots

Figure 4.25: Study 11 interpolation and adjoint correction error plots
Study 12

In this study, the wing tip and root spacings are further reduced to the values of leading and trailing edge spacings of $2 \times 10^{-4}$ as in Table 4.43. All meshes in the family converge. However for the fine mesh, the Newton-Krylov residual froze at about $10^{-4}$ i.e. giving non-complete convergence, $\approx 50\%$ of required convergence i.e. the residual is brought to half of the desired quantity for convergence. A good order is obtained for $C_D$, around 1.8, but a poor 0.5 order is obtained for $C_L$ as in Table 4.44. Table 4.45 shows that the adjoint correction performed well in predicting $C_D$ on the next mesh while simply failing for the $C_L$ where the corrections were small and in the wrong direction.

Table 4.43: Study 12 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$W_t$</th>
<th>$W_p$</th>
<th>$W$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>N/A</td>
<td>$2.5 \times 10^{-4}h$</td>
<td>$2.5 \times 10^{-4}h$</td>
<td>$2.0 \times 10^{-4}$</td>
<td>25</td>
<td>1</td>
<td>$2.0 \times 10^{-4}h$</td>
<td>$2.0 \times 10^{-4}h$</td>
<td>2.0</td>
<td>10,650,640</td>
</tr>
</tbody>
</table>

Table 4.44: Study 12 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>Coarse</td>
<td>$0.81432 \times 10^{-2}$</td>
<td>$0.6558 \times 10^{-2}$</td>
<td>1.7863</td>
<td>0.19124</td>
<td>0.1894</td>
<td>0.5070</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>$0.70235 \times 10^{-2}$</td>
<td>1.75970 $\times 10^{-2}$</td>
<td>0.68094 $\times 10^{-2}$</td>
<td>0.19073</td>
<td>0.19073</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>$0.66948 \times 10^{-2}$</td>
<td>0.75970 $\times 10^{-2}$</td>
<td>0.68094 $\times 10^{-2}$</td>
<td>0.19043</td>
<td>0.19073</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

Table 4.45: Study 12 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-H</td>
<td>Coarse → Medium</td>
<td>$1.0261 \times 10^{-2}$</td>
<td>$0.74892 \times 10^{-2}$</td>
<td>0.58</td>
<td>$0.19124$</td>
<td>$0.19138$</td>
<td>-0.26</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>$0.75970 \times 10^{-2}$</td>
<td>$0.68094 \times 10^{-2}$</td>
<td>0.65</td>
<td>$0.19043$</td>
<td>$0.19073$</td>
<td>-0.08</td>
</tr>
</tbody>
</table>
Figure 4.26: Study 12 interpolation and adjoint correction plots

Figure 4.27: Study 12 interpolation and adjoint correction error plots
4.3. 3D Mesh Convergence Studies

Study 13 (ONERA-M6 Wing)

Results from Study 12 suggest that the singularities caused by the sharp wing tip might be the reason for the poor results. Therefore a much more common wing is used, the ONERA-M6. Moreover, standard spacings were used based upon meshes in [41]. These spacings are two orders of magnitude larger than the NACA0012 studies but provide nice, aspect ratio wise, cells around the leading and trailing edges. The spacings are shown in Table 4.46. This family yielded excellent order for $C_D$ and a low order of 1 for $C_L$ as in Table 4.47. The adjoint correction performance is acceptable since it corrected all meshes in the proper directions with a minimum $\Delta$ of 0.2. This study has a better overall performance even though it has a relatively small number of nodes.

Table 4.46: Study 13 mesh parameters

<table>
<thead>
<tr>
<th>Topology</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$W_t$</th>
<th>$W_p$</th>
<th>$W$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>N/A</td>
<td>0.15 x 10^{-2}h</td>
<td>1.25 x 10^{-2}h</td>
<td>0.15 x 10^{-2}h</td>
<td>2.5^2 x 50</td>
<td>3</td>
<td>0.3 x 10^{-2}h</td>
<td>4.2 x 10^{-2}h</td>
<td>1.5</td>
<td>3,449,952</td>
</tr>
</tbody>
</table>

Table 4.47: Study 13 mesh convergence data and adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_L^*$</th>
<th>$P_{CD}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{CL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>Coarse</td>
<td>0.41069 x 10^{-2}</td>
<td>0.11962</td>
<td>HH</td>
<td>Medium</td>
<td>0.18416 x 10^{-2}</td>
<td>0.1 x 10^{-2}</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.12624 x 10^{-2}</td>
<td>0.12393</td>
<td>HH</td>
<td>Fine</td>
<td>0.15449 x 10^{-2}</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 4.48: Study 13 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_L^*$</th>
<th>$\Delta$</th>
<th>$C_D$</th>
<th>$C_L$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>Coarse → Medium</td>
<td>0.47451 x 10^{-2}</td>
<td>0.36603 x 10^{-2}</td>
<td>0.20</td>
<td>0.11928</td>
<td>0.12059</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.20123 x 10^{-2}</td>
<td>0.15449 x 10^{-2}</td>
<td>0.51</td>
<td>0.12243</td>
<td>0.12352</td>
<td>0.71</td>
</tr>
</tbody>
</table>

Optimum Mesh Parameters for 3D Inviscid Flow

As for producing good 3D meshes suitable for inviscid flow simulations, the guidelines related to mesh shape, orthogonality and skewness take the basis from the 2D studies. The aspect that is speculated to be different is the mesh spacings. This is due to the fact that applying optimal 2D mesh spacings into 3D results leads to very large number of nodes with high computational cost. In addition, applying minimum optimal conditions leads to a high number of nodes that forces us to use more blocks, for example 32 or 96 instead of the
conventional 16 block 3D meshes which were used in the square NACA0012. Finally, with 2D parameters, the following are added:

- \( W_t \leq 10^{-3}, \ W_p \leq 10^{-4}, \ F_d \geq 25 \)
- Extremely small spacings along the wing cause convergence failure problems.

Figure 4.28: Study 13 interpolation and adjoint correction plots

Figure 4.29: Study 13 interpolation and adjoint correction plots
4.4 Validation Studies

This section presents some studies which apply and test the validity of some or all of the conclusions inferred in the previous section. All the studies here are made using C meshes. Studies 1-7 are inviscid lifting cases with the NACA0012 aerofoil. Studies 8 and 9 are run with RAE2822 aerofoil under the inviscid scenario (study 8) and the turbulent scenario (study 9). Studies 1-7 are meant to show incremental improvement of mesh performance, in all three aspects of accuracy, convergence order, and adjoint correction by implementing the conditions determined in section 4.2.1. All the NACA0012 studies are done using a smooth interpolation geometry generated by ICEM-CFD.

Validation Study 1

In validation study 1, good mesh spacing is formed around a small leading edge of 3%, and the cells upstream of the trailing edge are given a high aspect ratio, which can be seen in Table 4.49. The results were excellent for $C_L$, having an error of $3 \times 10^{-5}$, order of 2 and $\Delta$ close to 1.0 in each mesh correction, which can be seen from Table 4.50 and Figure 4.30. On the other hand, the $C_D$ accuracy is average since the error with good meshes is below $10^{-5}$ and the resulting error is above $10^{-5}$, and the order is low. The adjoint correction performed very good but not as excellent as with $C_L$. This shows that good spacings make good parameters for $C_L$, but the mesh is still not good enough for $C_D$ even with its high orthogonality shown in Figure 4.31.

<table>
<thead>
<tr>
<th>Grid Topology</th>
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<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
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<td>C</td>
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<td>3 %</td>
<td>1.5$x10^{-4}$u</td>
<td>12.5$x10^{-4}$h</td>
<td>1.5$x10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>60°</td>
<td>322,580</td>
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</tbody>
</table>

<table>
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<tr>
<th>Grid Topology</th>
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<th>$C_D$</th>
<th>$C_D$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.15249$x10^{-3}$</td>
<td>0.10307$x10^{-3}$</td>
<td>0.067719$x10^{-3}$</td>
<td>1.269</td>
<td>0.21154</td>
<td>0.21202</td>
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<tr>
<td></td>
<td>Medium</td>
<td>0.08254$x10^{-3}$</td>
<td>0.067719$x10^{-3}$</td>
<td>1.269</td>
<td>0.21154</td>
<td>0.21202</td>
<td>0.21218</td>
</tr>
</tbody>
</table>

Table 4.49: Validation Study 1 mesh parameters

Table 4.50: Validation Study 1 mesh convergence data
Table 4.51: Validation Study 1 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^{+}$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^{+}$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse → Medium</td>
<td></td>
<td>0.21288 × 10^{-3}</td>
<td>0.10797 × 10^{-3}</td>
<td>0.90</td>
<td>0.21145</td>
<td>0.21201</td>
<td>0.98</td>
</tr>
<tr>
<td>Medium → Fine</td>
<td></td>
<td>0.12289 × 10^{-3}</td>
<td>0.088097 × 10^{-3}</td>
<td>0.73</td>
<td>0.21200</td>
<td>0.21215</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Figure 4.30: Validation study 1 mesh convergence results
Figure 4.31: Validation study 1 medium mesh
Validation Study 2

In this study we increase the leading edge to 7%, while maintaining the same number of nodes, which leads to an increase in the leading edge spacing, this is shown in Table 4.52. This reduces the $C_D$ error to $10^{-5}$ and increases the convergence order to 1.5 as in Table 4.53 and Figure 4.32. Furthermore the adjoint correction improves, as shown in Table 4.54. Results for $C_L$ are very similar with a slight increase in the order. This study shows that covering more area of leading edge improves results and it outweighs the cost of increasing aspect ratio.

Table 4.52: Validation Study 2 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.53: Validation Study 2 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td></td>
<td>0.16673$\times10^{-3}$</td>
<td></td>
<td></td>
<td>0.21176</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>0.10488$\times10^{-3}$</td>
<td>0.070985$\times10^{-3}$</td>
<td>1.507</td>
<td>0.21209</td>
<td>0.21218</td>
<td>2.2464</td>
</tr>
<tr>
<td>Fine</td>
<td></td>
<td>0.08309$\times10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.54: Validation Study 2 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^*$</th>
<th>$C_D^{+}$</th>
<th>$\Delta$</th>
<th>$C_L^*$</th>
<th>$C_L^{+}$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>Coarse → Medium</td>
<td>0.20688$\times10^{-3}$</td>
<td>0.11226$\times10^{-3}$</td>
<td>0.88</td>
<td>0.21171</td>
<td>0.21207</td>
<td>0.94</td>
</tr>
<tr>
<td>Medium</td>
<td>Medium → Fine</td>
<td>0.11707$\times10^{-3}$</td>
<td>0.088036$\times10^{-3}$</td>
<td>0.77</td>
<td>0.21208</td>
<td>0.21216</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 4.32: Validation study 2 mesh convergence results
4.4. Validation Studies

Figure 4.33: Validation study 2 medium mesh
Validation Study 3

In study 3, the effect of larger than optimal mesh spacings is illustrated. The fine mesh in this study has a large number of nodes and is given a uniform, but large, normal spacing going from the wall (aerofoil) to a distance of 5% of the chord length, then the spacing increases hyperbolically towards the far-field edge, as shown in Table 4.55 and Figure 4.35. The obtained orders were low and accuracy was lower for $C_L$ and higher for $C_D$. The adjoint correction from medium to fine for the $C_D$ failed, providing quite an anomaly as shown in Table 4.57.

Table 4.55: Validation Study 3 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$P_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90$^\circ$</td>
<td>3 %</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$u$</td>
<td>$3.0 \times 10^{-4}$</td>
<td></td>
<td></td>
<td>40</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.56: Validation Study 3 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.10682$\times 10^{-3}$</td>
<td>0.074688$\times 10^{-3}$</td>
<td>1.303</td>
<td>0.21147</td>
<td>0.21121</td>
<td>1.679</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.087800$\times 10^{-3}$</td>
<td>0.074688$\times 10^{-3}$</td>
<td>1.303</td>
<td>0.21198</td>
<td>0.21121</td>
<td>1.679</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.080073$\times 10^{-3}$</td>
<td>0.074688$\times 10^{-3}$</td>
<td>1.303</td>
<td>0.21214</td>
<td>0.21121</td>
<td>1.679</td>
</tr>
</tbody>
</table>

Table 4.57: Validation Study 3 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.11220$\times 10^{-3}$</td>
<td>0.089153$\times 10^{-3}$</td>
<td>0.93</td>
<td>0.21142</td>
<td>0.21196</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.095625$\times 10^{-3}$</td>
<td>0.088325$\times 10^{-3}$</td>
<td>-0.07</td>
<td>0.21197</td>
<td>0.21212</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Figure 4.34: Validation study 3 mesh convergence results
Figure 4.35: Validation study 3 medium mesh
Validation Study 4

In this study, the spacings are returned to normal like studies 1 and 2. The normal uniform spacing is reduced to 0.5% in order to avoid spacing jumps, this is shown in Figure 4.37. Accuracy results were excellent for both functionals with lower errors, and so is the obtained orders, shown in Table 4.59. The adjoint correction performed well, except for the same anomaly at the medium to fine $C_D$ correction, and this suggests an issue between adjoint correction and having blocks of uniform spacing in the normal direction. This is shown in Table 4.60. Most likely the results are not perfect because we are still limiting the leading edge spacing region to 3% of the aerofoil surface length.

Table 4.58: Validation Study 4 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_t$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_v$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90°</td>
<td>3%</td>
<td>$1.5 \times 10^{-4}$u</td>
<td>$1.5 \times 10^{-4}$h</td>
<td>$1.5 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>60°</td>
<td>351,080</td>
</tr>
</tbody>
</table>

Table 4.59: Validation Study 4 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^+$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^+$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.096916$\times 10^{-3}$</td>
<td>0.079084$\times 10^{-3}$</td>
<td>1.791</td>
<td>0.21163</td>
<td>0.21219</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.084290$\times 10^{-3}$</td>
<td>0.079084$\times 10^{-3}$</td>
<td>1.791</td>
<td>0.21204</td>
<td>0.21215</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.080619$\times 10^{-3}$</td>
<td>0.079084$\times 10^{-3}$</td>
<td>1.791</td>
<td>0.21205</td>
<td>0.21214</td>
<td>1.91</td>
</tr>
</tbody>
</table>

Table 4.60: Validation Study 4 adjoint correction results

| Grid Topology | Grid Level | || $C_D$ | $C_D^+$ | $\Delta$ | $C_L$ | $C_L^+$ | $\Delta$ |
|---------------|------------|||-------|---------|----------|-------|---------|----------|
| C             | Coarse → Medium | || 0.093120$\times 10^{-3}$ | 0.085795$\times 10^{-3}$ | 0.88     | 0.21156 | 0.21205  | 1.02     |
|               | Medium → Fine | || 0.089294$\times 10^{-3}$ | 0.087985$\times 10^{-3}$ | -1.00    | 0.21203 | 0.21214  | 0.91     |

Figure 4.36: Validation study 4 mesh convergence results
Figure 4.37: Validation study 4 medium mesh
Validation Study 5

In this study, the leading edge is extended to 10% with good spacing along with the same normal uniform spacing of study 4, this is shown in Table 4.61. Meshes resulted in functionals with excellent orders, especially on the $C_L$ side, and reasonable accuracy, being slightly less accurate than previous studies and this is most likely because of the lower number of nodes, this is shown in Table 4.62. The adjoint correction method performed flawlessly having an average $\Delta$ of 0.98. All this even though the number of nodes is less.

Table 4.61: Validation Study 5 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>98°</td>
<td>10%</td>
<td>$1.25 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$1.0 \times 10^{-4}$</td>
<td>40</td>
<td>1</td>
<td>78°</td>
<td>289,200</td>
</tr>
</tbody>
</table>

Table 4.62: Validation Study 5 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_C_D$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.84356×$10^{-3}$</td>
<td></td>
<td>0.20767</td>
<td>0.21103</td>
<td>0.21217</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.22536×$10^{-3}$</td>
<td>0.086428×$10^{-3}$</td>
<td>2.47</td>
<td>0.21103</td>
<td>0.21188</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11234×$10^{-3}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.63: Validation Study 5 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$\Delta$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.75140×$10^{-3}$</td>
<td>0.23147×$10^{-3}$</td>
<td>0.99</td>
<td>0.20713</td>
<td>0.21092</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.20055×$10^{-3}$</td>
<td>0.11214×$10^{-3}$</td>
<td>1.00</td>
<td>0.21088</td>
<td>0.21185</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Figure 4.38: Validation study 5 mesh convergence results
Figure 4.39: Validation study 5 medium mesh
Validation Study 6

This study uses the minimum optimal spacings and nodes put together as shown in Table 4.64. Order and adjoint correction results are perfect but the accuracy is not as high because of the low number of nodes.

Table 4.64: Validation Study 6 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>φ</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>θ</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>75°</td>
<td>7 %</td>
<td>6.25×10^{-4}</td>
<td>1.0×10^{-4}</td>
<td>1.0×10^{-4}</td>
<td>40</td>
<td>1</td>
<td>50°</td>
<td>117,780</td>
</tr>
</tbody>
</table>

Table 4.65: Validation Study 6 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^+$</th>
<th>$P_C_D$</th>
<th>$C_L$</th>
<th>$C_L^+$</th>
<th>$P_C_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.68974×10^{-3}</td>
<td>0.21962×10^{-3}</td>
<td>0.082872×10^{-3}</td>
<td>2.17</td>
<td>0.20953</td>
<td>0.21242</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.21962×10^{-3}</td>
<td>0.11405×10^{-3}</td>
<td>0.082872×10^{-3}</td>
<td>2.17</td>
<td>0.21170</td>
<td>0.21224</td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.11405×10^{-3}</td>
<td>0.082872×10^{-3}</td>
<td>0.082872×10^{-3}</td>
<td>2.17</td>
<td>0.21178</td>
<td>0.21223</td>
</tr>
</tbody>
</table>

Table 4.66: Validation Study 6 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^+$</th>
<th>Δ</th>
<th>$C_L$</th>
<th>$C_L^+$</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.71361×10^{-3}</td>
<td>0.21398×10^{-3}</td>
<td>1.01</td>
<td>0.20946</td>
<td>0.21178</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.23199×10^{-3}</td>
<td>0.11650×10^{-3}</td>
<td>1.01</td>
<td>0.21167</td>
<td>0.21223</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Figure 4.40: Validation study 6 mesh convergence results
4.4. Validation Studies

Figure 4.41: Validation study 6 medium mesh
Validation Study 7

Study 7 was conducted to verify that the splitting the mesh into further blocks does not significantly affect the results. The meshes in this study are identical to study 6 except that the blocks around the leading edge were merged leading to 3 block mesh instead of a 5 block mesh. The change in results is negligible as seen from Tables 4.68 and 4.69.

Table 4.67: Validation Study 7 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>φ</th>
<th>Lr</th>
<th>Ls</th>
<th>Ts</th>
<th>Os</th>
<th>P2</th>
<th>P3</th>
<th>θ</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>75°</td>
<td>7%</td>
<td>6.25×10⁻⁴u</td>
<td>1.0×10⁻⁴h</td>
<td>1.0×10⁻⁴</td>
<td>40</td>
<td>1</td>
<td>50°</td>
<td>117,780</td>
</tr>
</tbody>
</table>

Table 4.68: Validation Study 7 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>C_D</th>
<th>C_D'</th>
<th>P_C_D</th>
<th>C_L</th>
<th>C_L'</th>
<th>P_C_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>C</td>
<td>0.62826×10⁻³</td>
<td>0.20897×10⁻³</td>
<td>0.20897</td>
<td>0.21126</td>
<td>0.21126</td>
<td>2.02</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>0.20666×10⁻³</td>
<td>0.082014×10⁻³</td>
<td>2.15</td>
<td>0.21126</td>
<td>0.21126</td>
<td>2.02</td>
</tr>
<tr>
<td>Fine</td>
<td></td>
<td>0.11086×10⁻³</td>
<td>0.082014×10⁻³</td>
<td>2.15</td>
<td>0.21126</td>
<td>0.21126</td>
<td>2.02</td>
</tr>
<tr>
<td>Fine</td>
<td>C</td>
<td>0.11086×10⁻³</td>
<td>0.082014×10⁻³</td>
<td>2.15</td>
<td>0.21126</td>
<td>0.21126</td>
<td>2.02</td>
</tr>
</tbody>
</table>

Table 4.69: Validation Study 7 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>C_D</th>
<th>C_D'</th>
<th>Δ</th>
<th>C_L</th>
<th>C_L'</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>Medium → Fine</td>
<td>0.66805×10⁻³</td>
<td>0.20193×10⁻³</td>
<td>1.01</td>
<td>0.20889</td>
<td>0.21135</td>
<td>1.04</td>
</tr>
<tr>
<td>Medium</td>
<td>Coarse → Medium</td>
<td>0.22414×10⁻³</td>
<td>0.11315×10⁻³</td>
<td>0.98</td>
<td>0.21124</td>
<td>0.21183</td>
<td>1.00</td>
</tr>
<tr>
<td>Fine</td>
<td>Medium → Fine</td>
<td>0.22414×10⁻³</td>
<td>0.11315×10⁻³</td>
<td>0.98</td>
<td>0.21124</td>
<td>0.21183</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 4.42: Validation study 7 mesh convergence results
Figure 4.43: Validation study 7 medium mesh
4.4. Validation Studies

Validation Study 8 (RAE2822)

In study 8, the mesh configuration used in study 4 is applied to the RAE2822 aerofoil, i.e. the identical mesh is embedded around the new aerofoil, this is shown in Figure 4.45. The configuration is embedded exactly as is, causing a slight mesh deflection shown in the bottom right subfigure in Figure 4.45. The accuracy results are the same as in study 4 along with the adjoint corrections shown in Tables 4.71 and 4.72. Nevertheless, the convergence order of $C_D$ is high. This suggests that while the optimal parameters will improve performance for any geometry, further small tweaks in the mesh are required to achieve optimal performance.

Table 4.70: Validation Study 8 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>$\phi$</th>
<th>$L_r$</th>
<th>$L_s$</th>
<th>$T_s$</th>
<th>$O_s$</th>
<th>$F_d$</th>
<th>$F_s$</th>
<th>$\theta$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90°</td>
<td>7 %</td>
<td>$1.5 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-4}$</td>
<td>40</td>
<td>4</td>
<td>60°</td>
<td>311,272</td>
</tr>
</tbody>
</table>

Table 4.71: Validation Study 8 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D$</th>
<th>$C_D^*$</th>
<th>$P_{C_D}$</th>
<th>$C_L$</th>
<th>$C_L^*$</th>
<th>$P_{C_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.54651 $\times 10^{-3}$</td>
<td>0.38000 $\times 10^{-3}$</td>
<td>3.03</td>
<td>0.50107</td>
<td>0.50319</td>
<td>0.50327</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>0.40071 $\times 10^{-3}$</td>
<td></td>
<td></td>
<td>0.50286</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td>0.38262 $\times 10^{-3}$</td>
<td></td>
<td></td>
<td>0.50286</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.72: Validation Study 8 adjoint correction results

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>$C_D^i$</th>
<th>$C_D^+$</th>
<th>$\Delta$</th>
<th>$C_L^i$</th>
<th>$C_L^+$</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse → Medium</td>
<td>0.54207 $\times 10^{-3}$</td>
<td>0.38765 $\times 10^{-3}$</td>
<td>1.09</td>
<td>0.50094</td>
<td>0.50294</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>Medium → Fine</td>
<td>0.41235 $\times 10^{-3}$</td>
<td>0.38928 $\times 10^{-3}$</td>
<td>-0.63</td>
<td>0.50282</td>
<td>0.50317</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Figure 4.44: Validation study 8 mesh convergence results
Figure 4.45: Validation study 8 medium mesh
Validation Study 9 (RAE2822-Turbulent)

Study 9 comes after a series of unpresented turbulent studies with correlations to parameters deduced in Section 4.2.1. This is a study where the spacings from 2D studies are inculcated, in addition to the convergence requirements of having a very small off-wall spacing and flaring at the trailing edge. Acceptable values of error orders were obtained. However, no adjoint error correction was conducted for this study.

**Table 4.73:** Validation Study 9 mesh parameters

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>φ</th>
<th>Lr</th>
<th>Ls</th>
<th>Ts</th>
<th>Os</th>
<th>Fd</th>
<th>Fs</th>
<th>θ</th>
<th>W</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>93°</td>
<td>10%</td>
<td>2.5×10⁻⁴u</td>
<td>2.5×10⁻⁴h</td>
<td>0.01×10⁻⁴</td>
<td>40</td>
<td>1</td>
<td>70°</td>
<td>2.5</td>
<td>175,956</td>
</tr>
</tbody>
</table>

**Table 4.74:** Validation Study 9 mesh convergence data

<table>
<thead>
<tr>
<th>Grid Topology</th>
<th>Grid Level</th>
<th>C_D</th>
<th>C^*_D</th>
<th>P_{C_D}</th>
<th>C_L</th>
<th>C^*_L</th>
<th>P_{C_L}</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Coarse</td>
<td>0.013275</td>
<td></td>
<td>0.010373</td>
<td>0.0097128</td>
<td>2.45</td>
<td>0.45999</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fine</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.46:** Validation study 9 mesh convergence results
Figure 4.47: Validation study 9 medium mesh
Chapter 5

CONCLUSIONS, CONTRIBUTIONS, AND RECOMMENDATIONS

5.1 Conclusions and Contributions

In this project, a framework for manual mesh assessment and comparison is put together and is used to compare meshes of different topologies and deduce conclusions about what constitutes a “good” mesh. The framework is composed of six parts: 1) Geometry and mesh parameters, 2) mesh-family construction, 3) obtaining CFD outputs (error, order, adjoint), 4) data presentation, 5) correlations. The geometry comprises the aerofoil or wing points and curve interpolation. The mesh is defined by the mesh parameters: the topology, the leading edge skewness factor \( \phi \), the leading edge region \( L_s \) and its spacing profile \( L_s \), the trailing edge spacing profile \( T_s \), the off-wall spacing \( O_s \), the maximum spacing in the normal and stream-wise directions of the far-field \( F_s \), the leading edge orthogonality factor \( \theta \), and in addition the root and tip spacings \( W_p \) and \( W_r \) for a wing. The mesh-family construction has three aspects. First, that all the mesh parameters are applied properly in the mesh. Second, that mesh families with different topologies have the same parameters for fair comparison. Third, that the mesh families are created in accordance with the Richardson method used which requires that a coarser mesh is created by removing every second node in each direction. The CFD outputs are four: the Newton-Krylov solutions and computed functionals, the grid converged estimate obtained from the Richardson method, the convergence order obtained from the Richardson method, and the adjoint error correction. Data presentation was a big task in this project and it entailed arranging all the data in proper tables with a sensible sequence and giving names such as \( F^+ \) to a corrected functional. Correlations are made, with the built framework, between CFD outputs and mesh parameters to see which parameters resulted in good and acceptable outputs and which resulted in erroneous ones, and how sensitive the “goodness” of a mesh is to the parameters.

There are three topologies which are compared for the 2D case, namely H, C, and O topologies. Meshes with H-H and C-H topology were constructed using ICEM-CFD for 3D comparison; however due to the incompatibility of the C-H structure on a sharp wing tip or trailing edge with the current
solver, no C-H studies are included. Most of the studies were under lifting inviscid flow conditions. Multiple studies were conducted under turbulent conditions but only one is included. The adjoint method uses bicubic interpolation for 2D studies and trilinear interpolation for 3D studies. Overall when it comes to topology, the H mesh scores first place followed by the C mesh and the O mesh comes last. When it comes to mesh parameters, the studies show that with carefully chosen mesh spacings around the leading edge, good orthogonality and skewness factors, smooth spacing variation, and a reasonable number of nodes, excellent CFD results can be obtained from the mesh in terms of accuracy of computed functionals, determined convergence order and adjoint error estimation. Nevertheless, since excellent 2D meshes are already known by experience, the main contribution here is monitoring and studying the sensitivity of the mesh performance to each of the parameters. 3D studies are included for the sake of completeness and to give an example for future work that should entail an extensive study of mesh behaviour for the 3D meshes. The inferred conclusions about what makes a “good” mesh are validated by implementing them incrementally in Section 4.4.

5.2 Recommendations

The nature of this project and research is that it is easily extendable into bigger projects, possible for masters or doctorate theses. There are many details in each part of the research done that should be more thoroughly investigated and improved if possible. First is regarding geometry generation, even though ICEM-CFD is considered to be one of the best mesh generators in terms of degrees of freedom, the user has to be very careful while creating the geometries. Therefore, a method to implement AMBER2D aerofoil surface generation into ICEM-CFD should be developed. Second is regarding mesh generation, another method that implements AMBER2D orthogonality and skewness conditions should be implemented into ICEM-CFD instead of the available smoothing options, see Appendix A. These two recommendations are to ensure that anomalies and erroneous results do not come up for a non-mesh related reason. Third, the effectiveness of the Richardson method for CFD mesh convergence order estimation should be investigated, this includes the method’s sensitivity to data change and its accuracy in estimating the grid converged solutions and convergence orders. Fourth, the results from this work should be validated and tested with other solvers. Fourth, this framework is very prone to expanding with more parameters, more topologies, more studies, and splitting the research into separate parts, one for each functional. Fifth, the adjoint error plots mentioned in Section 3.6 should be applied and used further as a guide to help with inferring conclusions and with manual optimization of meshes. And finally, an interesting but very challenging task would be to use the results from this work to try to track the sources of error in the CFD solutions theoretically from the transformed equations introduced in Chapter 3.
REFERENCES


5.2. Recommendations


5.2. Recommendations


Appendix A

CFD Mesh Generation

In this appendix I give two helpful tips for ICEM-CFD users to generate nice C and O meshes and to be used in 3D mesh generation as well.

**Tip 1:** is regarding matching mesh spacings across block boundaries. When the edge spacing around the airfoil is set and the same spacing is assigned to all parallel edges, the resulting mesh has unwanted node concentrations near non-critical areas is shown in Figure A.1. Even though these meshes perform well if the other mesh parameters are tuned, matching the spacings leads to an automatic smooth mesh curvature near the leading edge as in Section 4.2.1. To match edges, follow the steps shown in Figures A.2-A.5.

**Tip 2:** is regarding the mesh smoothing utility in ICEM-CFD which increases orthogonality and skewness qualities of a mesh. However, the Multiblock smoother is recommended instead of the overall Orthogonality smoother. The Orthogonality smoother performs well with the Laplace option. Figures A.6 and A.7 show a sample of the Orthogonality smoother effect. Using the Multiblock smoother with C and O meshes on the blocks surrounding the aerofoil leads to better results. Figures A.8, A.9, and A.10 show a sample of the Multiblock smoother. Both samples are given on a C mesh surrounding a NACA0012 aerofoil.
Figure A.1: Unwanted node concentration
Figure A.2: Blocking tab → Pre-mesh params
Figure A.3: Using the edge editing tool select the edge to set its spacing
Figure A.4: Copy parameters to selected edges
Figure A.5: Choose the selected edge to match the first (top) edge
Figure A.6: C mesh before smoothing
Figure A.7: C mesh after 20 iterations of orthogonality smoother
Figure A.8: C mesh before smoothing
Figure A.9: Selecting blocks for multiblock smoother
Figure A.10: C mesh after 20 iterations of multiblock (Sorensen, Thomas & Middlecoff method) smoother
Appendix B

CODES

Richardson Extrapolation Method Code

function gcomparison(ca,k)

| %Cd and Cl values | %
| clc |
| cd=zeros(3,3); % cd(1,:) --> C-Grid, cd(1,1) --> Coarse, cd(1,2)--> med, and so on. |
| cl=zeros(3,3); |
| n=zeros(3,3); |

| % Initialization of Mesh Ratios % |
| r1 = zeros(1,3); % Ratio of medium mesh to coarse mesh, basic case=2 |
| r2 = zeros(1,3); % Ratio of fine mesh to coarse mesh, basic case=4 |

| % C-Grid % |
| if k==1 |
|  if ca==1 |
|    cd(1,1)=0.00006939; |
|    cd(1,2)=0.00001547; |
|    cd(1,3)=0.000004237; |
|    cl(1,1)=0; |
|    cl(1,2)=0; |
|    cl(1,3)=0; |
elseif ca==2
    cd(1,1)=0.00020511;
    cd(1,2)=0.00013994;
    cd(1,3)=0.00012569;
    cl(1,1)=0.20906;
    cl(1,2)=0.20939;
    cl(1,3)=0.20947;
end
elseif k==2
    if ca==1
        cd(1,1)=0.3759;
        cd(1,2)=0.4002;
        cd(1,3)=0.4619;
        cl(1,1)=0;
        cl(1,2)=0;
        cl(1,3)=0;
    elseif ca==2
        cd(1,1)=1.6019;
        cd(1,2)=0.00009991;
        cd(1,3)=0.00008573;
        cl(1,1)=0.1698;
        cl(1,2)=0.2119;
        cl(1,3)=0.2120;
    end
end

\%
\textbf{Number of Nodes in the family of meshes} \%
\begin{verbatim}
n(1,1)=23855;  \%coarse
n(1,2)=94041;  \%medium
n(1,3)=373421; %fine
\end{verbatim}
\%
\textbf{Mesh Ratios} \%
r1(1,1) = sqrt(n(1,2)/n(1,1));
r2(1,1) = sqrt(n(1,3)/n(1,1));

\%
\textbf{O-Grid} \%
\begin{verbatim}
if k==1
    if ca==1
        cd(2,1)=0.0001485;
        cd(2,2)=0.00003036;
        cd(2,3)=0.000008098;
        cl(2,1)=0.0;
        cl(2,2)=0.0;
        cl(2,3)=0.0;
    elseif ca==2
        cd(2,1)=0.00024552;
        cd(2,2)=0.00014690;
        cd(2,3)=0.00013031;
        cl(2,1)=0.20936;
        cl(2,2)=0.20934;
    end
end
\end{verbatim}
cl(2,3)=0.20931;

end

elseif k==2
    if ca==1
        cd(2,1)=0.0001714;
        cd(2,2)=0.00005741;
        cd(2,3)=0.00002939;
        cl(2,1)=0.0;
        cl(2,2)=0.0;
        cl(2,3)=0.0;
    elseif ca==2
        cd(2,1)=0.0002658;
        cd(2,2)=0.0001912;
        cd(2,3)=0.0002318;
        cl(2,1)=0.2089;
        cl(2,2)=0.2079;
        cl(2,3)=0.2066;
    end
end

% Number of Nodes in the family of meshes %
n(2,1)=19565;
n(2,2)=77271;
n(2,3)=307115;

% Mesh Ratios %
r1(1,2) = sqrt(n(2,2)./n(2,1));
r2(1,2) = sqrt(n(2,3)./n(2,1));

% H-Grid %

if k==1
    if ca==1
        cd(3,1)=0.0001506;
        cd(3,2)=0.00002885;
        cd(3,3)=0.000007903;
        cl(3,1)=0;
        cl(3,2)=0;
        cl(3,3)=0;
    elseif ca==2
        cd(3,1)=0.00027017;
        cd(3,2)=0.00014378;
        cd(3,3)=0.00011901;
        cl(3,1)=0.20910;
        cl(3,2)=0.20965;
        cl(3,3)=0.20980;
    end
else if k==2
    if ca==1
        cd(3,1)=0.00005183;
        cd(3,2)=0.00002126;
        cd(3,3)=0.00002132;
end
\[ \text{else if ca==2} \]
\[ c_{d}(3,1) = 0.00011189; \]
\[ c_{d}(3,2) = 0.000080142; \]
\[ c_{d}(3,3) = 0.000070547; \]
\[ c_{l}(3,1) = 0.212046; \]
\[ c_{l}(3,2) = 0.212270; \]
\[ c_{l}(3,3) = 0.212345; \]
\[ \text{end} \]
\[ \text{end} \]

---

\[ \% \text{Number of Nodes in the family of meshes} \% \]
\[ n(3,1) = 19840; \]
\[ n(3,2) = 77894; \]
\[ n(3,3) = 308662; \]

\[ \% \text{Mesh Ratios} \% \]
\[ r_{1}(1,3) = \sqrt{n(3,2)/n(3,1)}; \]
\[ r_{2}(1,3) = \sqrt{n(3,3)/n(3,1)}; \]

---

\[ \% \% \text{Newton’s Method to find P} \% \%
\]
\[ p = \text{zeros}(2,3); \]
\[ \text{for } i = 1:2 \]
\[ \text{if } i == 1 \]
\[ p(i,1) = 7; \]
\[ p(i,2) = 8; \]
\[ p(i,3) = 6; \]
\[ \text{else if } i == 2 \]
\[ p(i,1) = 8; \]
\[ p(i,2) = 5; \]
\[ p(i,3) = 8; \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{err1 = 1;} \]
\[ \text{err2 = 1;} \]
\[ \text{for } i = 1:3 \]
\[ \text{while } \text{err1} >= 10E-14 \]
\[ R = (c_{d}(i,3) - c_{d}(i,2)) / (c_{d}(i,2) - c_{d}(i,1)); \]
\[ G = (((r_{2}(1,i) \cdot p(1,i)) - (r_{1}(1,i) \cdot p(1,i))) \cdot R) - (r_{1}(1,i) \cdot p(1,i)) + 1; \]
\[ G_{p} = ((\log(r_{2}(1,i)) \cdot (r_{2}(1,i) \cdot p(1,i)) \cdot R) - (\log(r_{1}(1,i)) \cdot (r_{1}(1,i) \cdot p(1,i)) \cdot (R+1)); \]
\[ \text{pnew} = p(1,i) - (G_{p}) / G; \]
\[ \text{err1} = \text{abs}(p(1,i) - \text{pnew}); \]
\[ p(1,i) = \text{pnew}; \]
\[ \text{end} \]
\[ \text{err1 = 1;} \]
\[ c_{d}(i) = c_{d}(i,3) + ((c_{d}(i,3) - c_{d}(i,2)) / ((r_{1}(1,i) \cdot p(1,i)) - 1)); \]
while err2 >= 10E-14  
R = (c(i,3) - c(i,2)) / (c(i,2) - c(i,1));  
G = ((r2(1,i) .* p(2,i)) - (r1(1,i) .* p(2,i))) * R - (r1(1,i) .* p(2,i)) + 1;  
Gp = (log(r2(1,i)) .* (r2(1,i) .* p(2,i))) * R - (log(r1(1,i)) .* (r1(1,i) .* p(2,i))) * (R + 1);  
pnew = p(2,i) - (G / Gp);  
err2 = abs(p(2,i) - pnew);  
p(2,i) = pnew;  
end  
err2 = 1;  
cls(i) = c(i,3) + ((c(i,3) - c(i,2)) / ((r1(1,i) .* p(2,i)) - 1));  
end

%% Setting up values for plotting %

for i = 1:3  
v1(i) = (abs(cd(i,1) - cds(i)));  
v2(i) = (abs(cd(i,2) - cds(i)));  
v3(i) = (abs(cd(i,3) - cds(i)));  
v11(i) = (abs(cl(i,1) - cls(i)));  
v22(i) = (abs(cl(i,2) - cls(i)));  
v33(i) = (abs(cl(i,3) - cls(i)));  
h1(i) = (sqrt(n(i,1)));  
h2(i) = (sqrt(n(i,2)));  
h3(i) = (sqrt(n(i,3)));  
ordercd(i) = (1/2) * (((log(v3(i)) - log(v2(i))) / (log(h3(i)) - log(h2(i)))) + ((log(v2(i)) - log(v1(i))) / (log(h2(i)) - log(h1(i))));  
ordercl(i) = (1/2) * (((log(v33(i)) - log(v22(i))) / (log(h3(i)) - log(h2(i)))) + ((log(v22(i)) - log(v11(i))) / (log(h2(i)) - log(h1(i))));  
end

%% Plotting %

ycdc = [v1(1), v2(1), v3(1)];  
yclc = [v11(1), v22(1), v33(1)];  
ycko = [v1(2), v2(2), v3(2)];  
ycdh = [v1(3), v2(3), v3(3)];  
yclh = [v11(3), v22(3), v33(3)];  
xc = [h1(1), h2(1), h3(1)];  
xo = [h1(2), h2(2), h3(2)];  
xh = [h1(3), h2(3), h3(3)];  
figure  
subplot(1,2,1)  
p1 = loglog(xc, ycdc, '-ro');  
hold on  
p2 = loglog(xo, ycko, '-bs');  
hold on
 Diablo Input Parameters File

Inviscid

```bash
&OPTIMIZER
  opt_method = 'flowsolve',
  jstrm% obj_fun = 'clcd', jstrm% restart = .false.,
  jstrm% adj_flev = 1,
```
jtstrm% grad_tol = 1.d-6, jtstrm% adj_tol = 1.d-8,  
jtstrm% max_iter = 200,  
jtstrm% max_funevl = 500,  
jtstrm% vol_cnstrnt = .true.,  
jtstrm% volume = 0.5d0,  
jtstrm% vol_pen = 10000.d0,  
jtstrm% cledg_cnstrnt = .true.,  
jtstrm% cledg_pen = 10.d0  
jtstrm% c0edg_cnstrnt = .true.,  
jtstrm% c0edg_pen = 10.d0,  
jtstrm% min_angle = 3.d0,  
jtstrm% lbfgs_m = 20

END

&HYBROPT

END

&OPPTS

op_pts% nopt = 1  
op_pts% wmpo(1) = 1.000 d0,  
op_pts% fsmachs(1) = 0.500 d0,  
op_pts% alphas(1) = 1.500 d0,  
op_pts% reno(1) = 6.000 d2,  
op_pts% wfls(1) = 1.000 d2,  
op_pts% cltars(1) = 2.500 d0,  
op_pts% wdsls(1) = 1.000 d0,  
op_pts% cdltars(1) = 0.021 d0,  
op_pts% dvalfa(1) = .true.,  
op_pts% dvmach(1) = .true.

END

&FILES

grid_file_prefix = 'grid',  
output_file_prefix = 'results',  
opt_file_prefix = 'optimize'

END

&MESH

GRID% INCR = 2,  
GRID% MOVE_TOL = 1.d-12,  
GRID% NU = 0.2 d0

END

&PLANFORM

PLAN% SPAN_DELTA = 0.d0  
PLAN% ONE_Sweep_DELTA = 0.d0,  
PLAN% LE_DELTA = 0.0 d0,  
PLAN% TE_DELTA = 0.d0,  
PLAN% DIHEDRAL_DELTA = 0.d0,  
PLAN% TWIST_DELTA = 0.d0

END

&ASOLVER

NHALO = 2,

DIABLO% ISTREAM = 1,  
DIABLO% IGROUND = 2,  
DIABLO% VISCOUS = .false.,  
DIABLO% VISCROSS = .true.,  
DIABLO% INT_ONLY = .false.
DIABLO % JAC_METH = 1,  
DIABLO % INTT_PERTURB = .false.,  
DIABLO % VISC_SRF_SAT = 1,  
DIABLO % VISC_TRUE_AVE = .true.,  
DIABLO % USE_FRSTRM = .true.,  
DIABLO % TURBULNT = .false.,  
DIABLO % DUAL_CONSIST = .true.,  
DIABLO % ORDER = 2,  
DIABLO % IDMODEL = 1,  
DIABLO % P_SWITCH = .false.,  
DIABLO % DIS2(1) = 0.0d0, DIABLO % DIS4(1) = 0.04d0,  
DIABLO % DIS2(2) = 0.0d0, DIABLO % DIS4(2) = 0.04d0,  
DIABLO % DIS2(3) = 0.0d0, DIABLO % DIS4(3) = 0.04d0,  
DIABLO % V1 = 0.025d0, DIABLO % Vn = 0.25d0,  
SAT %V1 = 0.025d0, SAT %Vn = 0.25d0,  
SAT %tau = 1.0d0, SAT %prec_tau = 1.0d0,  
SAT %inTau = 1.0d0, SAT %prec_inTau = 1.0d0,  
SAT %INV_SRF_RDC = 2.d0,  
DIABLO % STRTUP_PREC = 'schur',  
DIABLO % NEWTON_PREC = 'schur',  
DIABLO % LEV_FIL = 1, DIABLO % PREC_dlf = 6.d0,  
DIABLO % SCHUR_JTS = 5, DIABLO % SCHUR_TOL = 1d-2,  
DIABLO % REORDER = 1,  
DIABLO % NK_TIME = 7, DIABLO % DT_MIN = 0.01d0,  
DIABLO % A = 0.01d0, DIABLO % B = 1.25d0,  
DIABLO % BETA = 1.5d0,  
DIABLO % NK_STRTUP = 2, DIABLO % DROP_TOL = 1.d-3,  
DIABLO % UPDAT_FRQ = 1, DIABLO % STRTUP_TOL = 1d-2,  
DIABLO % NEWTON_TOL = 1d-5,  
DIABLO % NK_JTS = 200, DIABLO % KRYLV_MAX = 60,  
DIABLO % REL_TOL = 1.d-10, DIABLO % ABS_TOL = 1.d-12

&END

Turbulent

&OPTIMIZER
  opt_method = 'flowsolve',
  jstrm% obj_fun = 'clcd', jstrm% restart = .false.,
  jstrm% adj_flev = 1,
  jstrm% grad_tol = 1.d-6, jstrm% adj_tol = 1.d-8,
  jstrm% max_iter = 200, jstrm% max_funevl = 500,
jtstrm% vol_constrt = .true.,
jtstrm% volume = 0.5d0,
jtstrm% vol_pen = 10000.d0,

jtstrm% c1edg_constrt = .true.,
jtstrm% c1edg_pen = 10.d0

jtstrm% c0edg_constrt = .true.,
jtstrm% min_angle = 3.d0,
jtstrm% c0edg_pen = 10.d0,

jtstrm% lbfgs_m = 20

\END
\HYBROPT
\END
\FOPPTS
  op_pts% nopt = 1
  op_pts% wmpo(1) = 1.000d0,
  op_pts% fsmachs(1) = 0.500d0,
  op_pts% alphas(1) = 2.000d0,
  op_pts% reno(1) = 3.000d6,
  op_pts% wfls(1) = 1.000d2,
  op_pts% cltars(1) = 2.500d0,
  op_pts% wdfs(1) = 1.000d0,
  op_pts% cdtars(1) = 0.021d0,
  op_pts% dvalfa(1) = .false.,
  op_pts% dvmach(1) = .false.
\END
\FILES
  grid_file_prefix = 'grid',
  output_file_prefix = 'results',
  opt_file_prefix = 'optimize'
\END
\MESH
  GRID% INCR = 2,
  GRID% MOVE_TOL = 1.d−12,
  GRID% NU = 0.2d0
\END
\PLANFORM
  PLAN% SPAN_DELTA = 0.d0
  PLAN% ONE_SWEEP_DELTA = 0.d0,
  PLAN% LE_DELTA = 0.d0,
  PLAN% TE_DELTA = 0.d0,
  PLAN% DIHEDRAL_DELTA = 0.d0,
  PLAN% TWIST_DELTA = 0.d0
\END
\SOLVER
  NHALO = 2,
  DIABLO% ISTREAM = 1,
  DIABLO% IGROUND = 2,
  DIABLO% VISCOUS = .true.,
  DIABLO% VISCROSS = .true.,
  DIABLO% INT_ONLY = .false.,
  DIABLO% JAC_METH = 1,
  DIABLO% INTT_PERTURB = .false.,
  DIABLO% VISC_SRF_SAT = 1,
DIABLO% VISC_TRUE_AVE = .true.,
DIABLO% USE_PRSTRM = .true.,
DIABLO% TURBINT = .true.,
DIABLO% DUALCONSIST = .true.,
DIABLO% ORDER = 2,
DIABLO% TURB_DELAY = 0,

DIABLO% IDMODEL = 1,     DIABLO% PSWITCH = .true.,
DIABLO% DIS2(1) = 2.00d0, DIABLO% DIS4(1) = 0.04d0,
DIABLO% DIS2(2) = 2.00d0, DIABLO% DIS4(2) = 0.04d0,
DIABLO% DIS2(3) = 2.00d0, DIABLO% DIS4(3) = 0.04d0,
DIABLO% VI = 0.025d0,     DIABLO% Vn = 0.25d0,

SAT%VI = 0.025d0,        SAT%Vn = 0.25d0,
SAT%tau = 1.0d0,          SAT%prec_tau = 1.0d0,
SAT%S12_tau = 1.0d0,      SAT%prec_S12_tau = 1.0d0,
SAT%inTau = 1.0d0,        SAT%prec_inTau = 1.0d0,
SAT%inS12tau = 1.0d0,     SAT%prec_inS12tau = 1.0d0,

SAT%INV_SRF_RDC = 2.0d0,

DIABLO% STRTUP_PREC = 'schur',
DIABLO% NEWTON_PREC = 'schur',

DIABLO% LEV_FIL = 3,     DIABLO% PREC_dlf = 7.0d0,
DIABLO% SCHUR_ITS = 5,   DIABLO% SCHUR_TOL = 1d-2,
DIABLO% REORDER = 1,

DIABLO% NK_TIME = 7,     DIABLO% DT_MIN = 0.001d0,
DIABLO% A = 0.001d0,     DIABLO% B = 1.25d0,
DIABLO% BETA = 1.5d0,

DIABLO% NK_STRTUP = 2,   DIABLO% DROP_TOL = 1.d-4,
DIABLO% UPDAT_FRIQ = 1,  DIABLO% STRTUP_TOL = 1d-2,
DIABLO% NEWTON_TOL = 1d-2,

DIABLO% NK_ITS = 200,    DIABLO% KRYLV_MAX = 70,

DIABLO% REL_TOL= 1.d-12, DIABLO% ABS_TOL= 1.d-12

END

Adjoint Correction Diablo Code

subroutine interpSetup()
!
!
! Purpose: calculate errors from adjoint variables
!
! Pre: 1) iresults.q is opened and the values are stored in blk(;)\%q
!       2) iresults.q is interpolated using interp_result on cresults.q
!
!
! Post: 1) Write Cl, Cd and Adjoint Error Correction Terms to screen
!

variables specification

!— local variables
integer :: ib, j, k, m, n, n2
real(kind=dp) :: adj_err_corr
real(kind=dp), allocatable :: sol2 (:), sol3 (:), sol4 (:)
character (len=128) :: adj_file_name, sol_file_name

! begin main execution

if (.not. allocated(sol)) & allocate(sol(Jmat%nloc))

!— Bicubic Interpolation
!— interpolated adjoint variables are written in iadjresults.q
!— store the values into sol (ie. Test 2 of Victor’s)
call readInterpQ(blk, nBlk, 1)
write(*,*) ’complete read iadjresults.q’

do ib = 1, nBlk
  do n = 1, nVar
    do m = 1, blk(ib)%jkmmax(3)
      do k = 1, blk(ib)%jkmmax(2)
        do j = 1, blk(ib)%jkmmax(1)
          sol((blk(ib)%indx(j,k,m)-1)*nVar + n) = blk(ib)%q(j,k,m,n)
        end do
      end do
    end do
  end do
end do

do ib = 1, nBlk
  call unscaleAdjointVars(blk(ib), sol)
end do

call readInterpQ(blk, nBlk, 2)
write(*,*) ’complete read iresults.q’

!— sound speed must be initialized prior to calling adjDotRes()
do ib = 1, totBlk
  call calcPS(blk(ib), ib)
end do

call getRHS()
write(*,*) ’complete Residual computation’

!— update ghost blocks, change processors
call exchangeQ()

write(*,*) 'complete block update'

!— copy residual to b
! call setRHSvector(.false.,Pmat%nloc,b)
! write(*,*) 'residual copied to b'

!— output AeroCoeffs of interpolated flow solution
call calcAeroCoeffs()
write(*,*) 'coefficients calculated'

!— adjoint error correction
adj_err_corr = adjDotRes()
if(myrank == 0) write(*,*) 'Adjoint Error Correction Term: ', adj_err_corr
if(myrank == 0) write(*,*) 'Cl: ', Cl
if(myrank == 0) write(*,*) 'Corrected Cl: ', Cl + adj_err_corr
if(myrank == 0) write(*,*) 'Cd: ', Cd
if(myrank == 0) write(*,*) 'Corrected Cd: ', Cd + adj_err_corr

call setRHSvector(.false.,Pmat%nloc,b)

!do ib = 1,nBlk
! blk(ib)%q_old = 0.d0
! call scaleAdjointVars(blk(ib),sol)
! call updateBlockSolution(blk(ib),1.d0,sol)
! end do
! call exchangeQ()

call setRHSvector(.false.,Pmat%nloc,b)

!— write error maps
do ib=1,size(b)
  b(ib) = sol(ib)*b(ib))
end do

do ib=1,nBlk
  blk(ib)%q_old = 0.d0
  call updateBlockSolution(blk(ib),1.d0,b)
end do

call exchangeQ()
call writeQ(blk,nBlk)
if(myrank==0) write(*,*) 'Writing Error Map'
end subroutine interpSetup

! 
Grid Coarsening Code

```fortran
program coarsen_grid

! Purpose:
! Removes every other grid line from a mesh
!
! Pre:
! 1) the input file grid.g should exist
! 2) each block must have an odd number of nodes in each direction
!
! Post:
! 1) a new grid file is produced which has been coarsened, and has
!   extension *.g.new
!
! Author: J. Hicken, February 2005
!
implicit none

integer :: nblks, ib, file_stat
integer :: j, k, m
integer, dimension(:,:), allocatable :: jbmax, kbmax, mbmax
integer, dimension(3) :: jkmmax
double precision, dimension(,:,:,:), allocatable :: x, y, z
integer, parameter :: old_unit = 10, new_unit = 20
character (len = 128) :: file_name

! begin main execution

!--- open the input file and check for problems
file_name = 'grid.g'
open (unit = old_unit, file = file_name, status = 'old', &
     form = 'unformatted', iostat = file_stat)
if (file_stat /= 0) then
    print *, 'Error in coarsen_grid :: error opening ', file_name
    stop
end if

write (*,*), 'Opened grid.g...

!--- open the new grid file and check for problems
file_name = 'grid.g.new'
open (unit = new_unit, file = file_name, status = 'unknown', &
     form = 'unformatted', iostat = file_stat)
if (file_stat /= 0) then
```


...
do  ib=1,nblks

write (*,*) 'On block = ',ib

read(old_unit) &
(((x(j,k,m), &
j=1,jbmax(ib)), k=1,kbmax(ib)), m=1,mbmax(ib)), &
(((y(j,k,m), &
j=1,jbmax(ib)), k=1,kbmax(ib)), m=1,mbmax(ib)), &
(((z(j,k,m), &
j=1,jbmax(ib)), k=1,kbmax(ib)), m=1,mbmax(ib))

write(new_unit) &
(((x(j,k,m), &
j=1,jbmax(ib),2), k=1,kbmax(ib),2),m=1,mbmax(ib),2), &
(((y(j,k,m), &
j=1,jbmax(ib),2), k=1,kbmax(ib),2),m=1,mbmax(ib),2), &
(((z(j,k,m), &
j=1,jbmax(ib),2), k=1,kbmax(ib),2),m=1,mbmax(ib),2)

end do

close(old_unit)
close(new_unit)
deallocate(x)
deallocate(y)
deallocate(z)
deallocate(jbmax)
deallocate(kbmax)
deallocate(mbmax)

end program coarsen_grid
Appendix C

MESHES

Study 1

Figure C.1: Study 1 meshes
Study 2

Figure C.2: Study 2 meshes
Study 3

Figure C.3: Study 3 meshes
Study 4

Figure C.4: Study 4 meshes
Study 5

Figure C.5: Study 5 meshes
Study 6

Figure C.6: Study 6 meshes
Study 7

Figure C.7: Study 7 meshes
Study 8

Figure C.8: Study 8 meshes
Study 9

Figure C.9: Study 9 meshes
Figure C.10: Study 15 ONERA-M6 mesh