ESSAYS ON INTERMEDIATED CORRUPTION, FINANCIAL FRICTIONS AND ECONOMIC DEVELOPMENT

by

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Abstract

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Distortions that affect macroeconomic outcomes are an important avenue through which we can explain differences in cross country output and productivity. In this thesis I concentrate on two types of distortions, political economy and informational distortions. In Chapter one, I build a model of intermediated corruption where interactions between government bureaucrats and those who bribe them are mediated by a third party. I show that intermediation has significant effects on the incidence of corruption and the prices entrepreneurs pay for permits. When corruption is particularly acute, measures that increase the frequency with which government bureaucrats are audited often have the undesirable result of increasing the prevalence of corruption because of intermediation. In Chapter two I explore the link between corruption and inequality by building a model in which tax collectors are corrupt. I find that as inequality increases, the frequency of corrupt transactions increases as well. I also find that where corruption is more severe, because wealthier individuals tend to pay lower taxes, inequality is higher. I perform a few quantitative experiments to better understand this linkage. Chapter three explores distortions that are caused by adverse selection in markets with search frictions. I find that when participants are concerned about the information they reveal through their interactions in the market, the distortions to liquidity are deeper and that equilibrium selection is significantly affected. I also find that markets with reputational concerns are more sensitive to outside shocks.
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Chapter 1

Intermediated Corruption

Abstract:
This chapter uses a search theoretic framework with history dependence to model the role of intermediaries in corruption and examine the effects of policy on the level of intermediated corruption, the price of licenses and welfare. Intermediaries with longer histories earn higher premiums. The frequency of corrupt transactions is inversely related to income levels. As the government increases the fraction of profits that it legally extracts from entrepreneurs in the form of compliance and/or red tape costs, intermediation intensifies, as entrepreneurs are reluctant to obtain licenses through legal means. As a result corruption becomes endemic, particularly under regimes where business costs are high, and measures to combat corruption are ineffective as they merely transfer value to intermediaries. The quantitative exercise shows that increasing the frequency with which governments audit bureaucrat entrepreneur matches reduces welfare, and increases the equilibrium price entrepreneurs pay to obtain permits.
1.1 Introduction

The issue of bribery is ubiquitous around the world. The World Bank Enterprise Survey reports that nearly 30% of firms world-wide expect to pay bribes to public officials to ‘get things done’. Researchers at the World Bank estimate that the size of the bribery market is at least $1 trillion per year (Rose-Ackerman, 2004), which amounts to nearly 3% of the world’s GDP. Bribes paid by private citizens in search of a government good or service quite often reach corrupt bureaucrats by way of intermediaries, who facilitate this exchange of governmental services in return for a fee.\footnote{For example, Bertrand et al. (2007) find that all individuals who bribed a government official to obtain a driver’s license did so through an intermediary.} The use of intermediaries arises as both private citizens and officials attempt to shield themselves from anti-corruption measures and audits.\footnote{Here corruption is defined as in Shleifer and Vishny (1993) i.e., the sale by government officials of government property for private gain. Licensing is clearly such a case, where an official earns illegal profits from the sale of government property.} By employing a third party, officials can make the process of uncovering corrupt activities increasingly difficult, because no direct contact between a briber and bribee can be established. Furthermore, given the natural difficulty of enforcing contracts in illicit markets, corrupt bureaucrats have to rely heavily on those intermediaries who have proven themselves trustworthy in previous transactions, so as to minimize the risk of being cheated out of their share of the proceeds. As an illustration of this ‘service’ provided by intermediaries, consider a recent report on Russia on The Economist magazine, which cites a local businessman as saying: ‘You go to the local administration to get permission for something and they send you to a private firm that will sort out the paperwork for you, which happens to be owned by their relatives’.\footnote{The Economist, November 29th 2008: A special report on Russia, p10.}

The prevalence of intermediated corruption raises several questions. What determines the number of active intermediaries in corruption? How do intermediaries affect the transaction price of a license inclusive of bribes? How do policies such as corruption monitoring affect the size of intermediation, price and welfare? To address these questions, I first construct a search-theoretic model with history dependence to generate intermediated corruption as an equilibrium outcome. An interesting implication of the model is that where business costs associated with obtaining government permits are sufficiently high, auditing government bureaucrats more diligently increases the level and payoffs to intermediation. From this point of view corruption tends to be obstinate, by redistributing income to individuals who are neither bureaucrats nor direct end users of government services, it creates a service industry that is very difficult to uproot.
The choice of search is a natural environment for the study of intermediated corruption. Purchasing a service or a license in the illegal market involves substantial effort devoted to searching for the appropriate party. Because of its illicit nature, the corruption ‘market’ is unable to reduce these frictions through the public domain. Moreover, the natural lack of contract enforcement adds a layer of complexity to the interactions between intermediaries and bureaucrats. In this model an intermediary takes possession of a license from a bureaucrat after negotiating a price, and delivers the agreed upon price only after having sold the license to an entrepreneur.\footnote{This is what Rubinstein and Wolinsky (1987) call \textit{consignment}. See below for a detailed explanation.} In this manner the model allows for intermediaries to renege on an agreement with a bureaucrat and keep the proceeds of a sale. The choice to take this course of action depends on the value of having a longer history as an honest intermediary. Because of search frictions, longer histories deliver more value, but if the value of building history is sufficiently low for entrant intermediaries, they will cheat. If this is the case, bureaucrats will insure themselves against breach by demanding prohibitively high prices, thus making intermediation impossible in equilibrium.

Naturally, the decision of an individual to become an intermediary depends on his outside option, i.e. the wage he can obtain by choosing to spend his time working instead. Higher wages imply a higher continuation value needed for the individual to become an intermediary, and, keeping licensing costs constant, higher wages reduce the size of the corruption ‘market’ as measured by the number of active intermediaries. Through this channel, the model can explain the well known negative relationship between corruption and income per capita, i.e., as income per capita increases, intermediation wanes, which makes corrupt activities less feasible and less frequent. If however, the value of building a history of honest transactions is sufficiently high to induce entry into intermediation, then the model suggests that intermediaries with longer histories can extract a higher premium (the difference between the price they are paid for the license and the one they pay for it) because in any negotiations they must be compensated for being honest middlemen.

I then conduct quantitative exercises to study comparative statics associated with changes in the policy parameters of interest. First, I find that when the costs of obtaining permits legally are relatively high, increasing the frequency with which the government audits bureaucrats leads to an increase in the level of intermediation. The first effect of an increase in the frequency of auditing is to reduce the probability that an intermediary gets to engage in the exchange of permits in the corruption ‘market’. However, given
that the costs of procuring licenses legally are relatively high, entrepreneurs are reluctant
to do so and are therefore willing to give up more value to obtain them illegally. This in
turn increases the premium that intermediaries extract from entrepreneurs, thus making
intermediation more lucrative and inducing more entry. Second, this exercise suggests
that reducing the costs that governments extract from entrepreneurs, which include both
compliance and red-tape costs, is a substantially more efficient way of rooting out cor-
ruption than simply increasing the frequency of audits. The quantitative section also
investigates equilibrium selection as a function of the two policy parameters discussed
above. When the costs of obtaining permits legally are high, equilibria in which inter-
mediaries are active become more likely since entrepreneurs are willing to forgo a larger
share of the surplus in order to avoid paying these costs. This in turn increases the value
to intermediation, which induces honesty from intermediaries. On the other hand, as
licensing costs decrease, intermediated equilibria become less likely.

This chapter contributes to the existing literature on corruption by studying inter-
mediation and its effects on prices and welfare. Bertrand et al. (2007) find that the
driving ability of those that obtained their licenses through corrupt means was substan-
tially lower than the driving ability of those who did so through legal channels. This fact
gives credence to the modeling perspective of this chapter. That is, the compliance/red
tape costs are bypassed when licenses are purchased through corrupt means. Shi and
Temzelides (2004) also take a search based approach to the study of corruption, and
find that bribery arises because an official’s trading decisions are immaterial in their
consumption outcomes and they only bear a small fraction of the cost of production.
Bribery then induces bureaucrats to accept lower quality goods which may increase their
production. In this chapter however, corruption arises purely because bureaucrats hold a
 monopoly on licensing and search frictions exist only in the corruption ‘market’. Shleifer
and Vishny (1993) suggest that increasing the size of the bureaucracy may reduce the
size of bribes as competition between bureaucrats increases. One of the key results of this
chapter is that in the presence of intermediaries, increasing the size of the bureaucracy
may actually intensify the frequency of corrupt activities by increasing the size of the
intermediation market and has no effect on the average price entrepreneurs pay to obtain
permits. In fact, the only party to benefit from an increase in the size of the bureaucracy
are the intermediaries, as they can extract higher prices for their services given that they
are more likely to make a connection with a bureaucrat.

From an intermediation point of view, the seminal work was done by Rubinstein and
Wolinsky (1987, RW henceforth) and this chapter will follow closely their framework.
There are however, some departures from RW. First, this chapter only considers an en-
environment with *consignment*, where intermediaries deliver the agreed upon price only after having sold the good to an entrepreneur. This market structure allows a richer role for intermediaries than just buyers/sellers. With consignment, an intermediary’s role is closer to that of a financial institution, and trust in them is built through multiple matches. Second, this chapter allows for breaches of contract in the corruption market, where an intermediary may decide not to pay back the agreed upon price to the bureaucrat. This is a very natural extension, since, given the illegality of corruption, breaches of contract would be very difficult to prosecute. Third, I allow for history building, where intermediaries that have gone longer without breach have longer histories. This gives rise to an implicit trade-off that individuals make. If an intermediary does not repay the bureaucrat, he runs the risk of losing accumulated history, while being honest is costly.

One implicit assumption I make is that any auditing body has a limited scope of search for illegal activity. Even more strongly, I assume that the cost of uncovering illegal activity when intermediaries are involved is prohibitively high. This is not an extreme assumption. For the reasons mentioned above, proving malfeasance in cases where intermediaries are involved is difficult. However, the results of the chapter are not qualitatively affected by this assumption, all that is needed for the results to continue to hold is the assumption that the probability of uncovering a corrupt transaction is lower when intermediation is present.

In this setting the role of government is limited to a collection of rules and regulations that are utilitarian, and bureaucrats are the individuals charged with enforcing them. The issue of corrupt auditors and complete corruption is not addressed here. The starting point is that the potential for corruption exists, which enables me to focus on issues of intermediation and the division of the proceeds of corruption between individuals.

### 1.2 A Model of Intermediated Corruption

#### 1.2.1 The Model Environment

Consider a setting with a continuum of risk-neutral agents of mass one. Time is continuous. There are three types of agents, bureaucrats with time-invariant mass $B$, entrepreneurs with constant mass $E$, and workers with mass $1 - B - E$. All agents discount the future at a common discount rate $\rho$. There is an arrival rate of death (exit) $\lambda$ and, in each period $t$, an exiting agent is replaced with a newborn.\(^5\) Denote by $r = \rho + \lambda$ the effective discount rate. Entrepreneurs are endowed with a project that yields a lifetime

---

\(^5\)all rates are Poisson arrival rates.
discounted value of $A$. Both bureaucrats and workers value the license at zero. In order to operate the project, an entrepreneur must obtain a license from a bureaucrat.

This transaction can happen in two possible environments. In the public market, the entrepreneur and bureaucrat are always matched, and with probability $\alpha$ the match is audited. When the bureaucrat is audited by the government, he must give the license to the entrepreneur without charging a fee, because he is under direct observation. On the other hand, if the bureaucrat is not directly audited, he has the opportunity to sell the license in the corruption market and earn a fee.\(^6\) There is a cost to completing the transaction in the public market, which is denoted by a constant $b < A$. The constant $b$ represents a composite of red tape costs and investment costs that must be paid if the entrepreneur is to be equipped with the license in the public market. The red tape costs can be attributed to bureaucratic congestion, while investment costs are license requirements that the entrepreneur has to fulfill if he obtains the license in the public market, but are foregone when the bureaucrat issues the license in private. More often than not, skirting the official channels is done not only to avoid costly bureaucratic procedures, but also to avoid compliance with the rules that they require.

If the entrepreneur does not meet a bureaucrat in the public market she enters the corruption market where she searches for an agent that holds a license. Bureaucrats that do not issue the license in the public market also enter the corruption market and search for an agent who is in need of a license. The flow arrival rate of a match between an entrepreneur and a bureaucrat in the corruption market is $\mu$. Once a bureaucrat issues a license in either market, he is immediately endowed with another one that he can issue at will, while an entrepreneur who has obtained a license exits the market and is immediately replaced by another.

To further clarify the timing structure of the market, suppose a bureaucrat is holding a license. With probability $\alpha$, the bureaucrat is forced to give away the license through audit, in which case another license is issued to him and the process restarts. With probability $1 - \alpha$ instead he is given a chance to enter the corruption market and sell the license at a price. If the sale is to an intermediary, the bureaucrat is immediately endowed with a new license, but is still owed the proceeds of the previous sale. The figure below is a visual depiction of the timeline for the bureaucrat. If an entrepreneur receives a project and enters the market, she will always attempt to obtain the license legally first, and then try the corruption market. Therefore, the effective rate with which an entrepreneur purchases the license directly from a bureaucrat on the corruption market

\(^6\)Any such match that is unaudited subsequently breaks down.
Chapter 1. Intermediated Corruption

Figure 1.1: Timeline for the bureaucrat

is \((1 - \alpha)\mu\). The timeline for the entrepreneur is depicted below.

Figure 1.2: Timeline for the entrepreneur

Workers earn the constant wage \(w\) but can also choose to enter the corruption market and become intermediaries that obtain a license from the bureaucrat and sell it to entrepreneurs.\(^7\) However, workers are resource constrained and cannot pay for the license up front. In a match between a bureaucrat and an intermediary, the two negotiate a price and, if there is agreement, the bureaucrat issues the license to the intermediary, but the agreed upon price is not immediately paid. Instead, the intermediary pays the bureaucrat only after he has actually sold the license (see RW).

Denote an intermediary that is in search of a license as an \(n\)-type and one that is in search of an entrepreneur as an \(m\)-type. The rate of a match between an \(m\)-type intermediary and an entrepreneur is \(\gamma\). Once an \(m\)-type intermediary has sold the license

\(^7\)A worker can either be an intermediary or a worker, not both.
to an entrepreneur, he has the choice to either pay back the agreed upon price to the bureaucrat, or cheat and keep all of the proceeds of the sale to himself. There are no informational asymmetries here, and the decision of the intermediary is a binary one; pay or not. There is however, a cost to cheating. Denote by $h \in \mathbb{N}$ the history of an active intermediary. A history marks the number of consecutive successful transactions an intermediary has completed without cheating since the last time he cheated. If an intermediary with any history cheats in a period, the intermediary’s history is reset to zero.\footnote{An extension of the model where this assumption is relaxed is presented in the appendix. See also the discussion section.} A transaction includes both the purchase of a license as well as its sale to the entrepreneur. Histories are public information. However, only histories are observed.

To further clarify the definition of histories, consider a simple example. Suppose an arbitrary intermediary has completed a series of transactions and we mark his cheat/not cheat decision in a binary manner where 1 marks cheating. Then a full history of the agent could be written as follows: \{...0,1,0,1,0,0,0,0,0,1,0,0,0,0\}. In the above definition of histories, this arbitrary agent has a history of four. The implicit assumption here is one of partial memory loss, or partial record keeping, where the only part of an agent’s history that can be observed is the number of zeros after the last occurring cheat. The assumption is made for tractability by reducing an individual’s history from a sequence to an integer.

The timeline for an intermediary is depicted below. Note that the decision of whether to cheat or not determines the future history of an intermediary; if, after selling the license to an entrepreneur, an intermediary decides to cheat, he becomes a type $n$ intermediary with history 0. If not, then his history is $h + 1$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{timeline_intermediary.png}
\caption{Timeline for the intermediary}
\end{figure}
Chapter 1. Intermediated Corruption

Denote by \( n(h) \) and \( m(h) \) the mass of \( n \) and \( m \)-type intermediaries of history \( h \) and by \( Q(h) \) the probability that a representative intermediary of history \( h \) will cheat. Also denote the rate with which an intermediary of history \( h \) meets a bureaucrat by \( \pi(h) \), where \( \pi(0) = \frac{B}{n(0)+B} \) and \( \pi(h) = \tau \forall h > 0 \). I assume that the matching function for agents with history zero is such that the total number of matches between bureaucrats and intermediaries is \( M(n(h), B) = \frac{Bn(h)}{n(h)+B} \). Therefore the only history for which the distribution is instrumental in determining the rate of a match is history zero. For all other histories the rate with which an intermediary meets a bureaucrat is constant. This structure allows for these rates to weakly increase in histories, which results in intermediaries with a higher level of experience to be better at matching with bureaucrats. This captures what I think to be a salient and realistic feature of the corruption market, and one that can be empirically observed in the lobbying industry.\(^9\) The assumption that the differences are only present in the first step of acquiring a history is made to keep the model tractable, although none of the results are sensitive to this particular assumption.\(^10\)

Denote by \( \eta(h) \) the rate with which a bureaucrat meets an intermediary of history \( h \).\(^11\)

Let \( s \) and \( e \) represent the bureaucrat and the entrepreneur, respectively. Denote by \( V_i \) agent \( i \)'s expected value of being unmatched in the corruption market, by \( Z_{ij} \) the total value created by a match between agents \( i \) and \( j \), and by \( P_{ij} \) the price paid by agent \( j \) to \( i \) in the event of an agreement, where \( i, j \in \{ s, m, n, e \} \). The matches where a good is potentially exchanged are those between a bureaucrat and entrepreneur \((se)\), a bureaucrat and an \( n \)-type intermediary \((sn)\) and between an \( m \)-type intermediary and an entrepreneur \((me)\). At any random point in time a bureaucrat can be in one of two states. Either he is holding a license, in which case his value of being unmatched in the market is \( V_s \), or he is holding a license and is waiting for payment from an intermediary, in which case his value of being unmatched in the market is \( R_s(h) + V_s \), where \( R_s(\cdot) \) represents the residual value of a consignment sale. Note that \( R_s \) depends on \( h \) since the equilibrium probability of cheating will depend on histories and so will the expected payment to the bureaucrat.

The total values created by each match are as follows:

\[
Z_{sc} = A + V_s
\]

\(^9\)See for example Draca et. al. 2010.
\(^10\)Search frictions here would guarantee an increasing rate of match with respect to history for an intermediary searching for a bureaucrat.
\(^11\)Given the matching function \( \eta(0) = \frac{n(0)}{n(0)+B} \) and \( \eta(h) = \tau \forall h > 0 \).
\[
Z_{sn}(h) = V_m(h) + R_s(h) + V_s
\]
\[
Z_{me}(h) = A + (1 - Q(h)) (V_n(h + 1) - P_{sn}(h)) + Q(h)V_n(0)
\]

The last identity comes from the fact that when an entrepreneur and \(m\)-type intermediary meet, there is the value of the project to the entrepreneur \(A\), and the expected value of the intermediary given some probability of cheating \(Q(h)\). In this case, if the intermediary decides not to cheat, his history increases to \(h + 1\), but he has to pay the agreed upon price to the bureaucrat \(P_{sn}(h)\). If instead he decides to cheat he is endowed with a zero history but keeps the price \(P_{sn}(h)\).

The value functions of individuals are as follows:

\[
rV_n(h) = \pi(h) [V_m(h) - V_n(h)]
\]
\[
rV_m(h) = \gamma \left[ P_{me}(h) + (1 - Q(h))[V_n(h + 1) - P_{sn}(h)] + Q(h)V_n(0) - V_m(h) \right]
\]
\[
rV_e = (1 - \alpha) \left( \gamma \sum_h (A - P_{me}(h) - V_e) \frac{m(h)}{M} + \mu [A - P_{se} - V_e] \right) + \alpha [A - b - V_e]
\]
\[
rV_s(h) = (1 - \alpha) \left\{ \mu P_{se} + h \eta(h)R_s(h) \right\}
\]
\[
rR_s(h) = \gamma \left\{ (1 - Q(h))P_{sn}(h) - R_s(h) \right\}
\]

Equation (3) is the recursive equation for the entrepreneur where \(M \equiv \sum_h m(h)\). Note that the rate with which an entrepreneur meets an intermediary of history \(h\) is \(\gamma \frac{m(h)}{M}\), and the total expected surplus value of the match to the entrepreneur is \(A - P_{me}(h) - V_e\).

Here bargaining is modeled as Nash bargaining with equal weights.\(^{12}\) In any negotiation between two agents, the surplus that goes to each agent is simply half of the total surplus created by the match. Consider, for example, (6) below.

\[
P_{me}(h) + (1 - Q)(V_n(h + 1) - P_{sn}(h)) + QV_n(0) - V_m(h) =
\]
\[
= \frac{1}{2} [Z_{me}(h) - V_m(h) - V_n(h)]
\]

The left hand side of the equation is equal to the surplus going to the intermediary with the license, which is comprised of the price paid to the intermediary of type \(m\) by the entrepreneur \(P_{me}\) plus the expected value of the intermediary after the sale based on

\(^{12}\)The assumption of equal weights is relaxed in the quantitative section. In section 4.3 more general values of the bargaining parameter are explored.
the cheating decision, minus the loss of $V_m$ given that the intermediary is no longer an $m$-type. The right hand side represents the net surplus of the match. Equations (7) and (8) are similarly derived and represent the bargaining outcomes of the ($ns$) and ($se$) matches respectively.

$$V_m(h) - V_n(h) = \frac{1}{2}[Z_{sn}(h) - V_n(h) - V_s] \tag{7}$$

$$A - V_e - P_{se} = \frac{1}{2}[Z_{se} - V_e - V_s] \tag{8}$$

1.2.2 The Intermediary’s Decision

After an intermediary of history $h$ has sold a license to an entrepreneur, he makes the decision on whether to cheat, i.e., whether to pay the bureaucrat the price agreed upon. If the continuation payoff of not cheating is higher than the payoff from cheating, then the probability of cheating is zero. If the continuation payoff of not cheating is lower, then the intermediary cheats with probability one and if the two payoffs are equal, the intermediary is indifferent between cheating or not. Denote by $q(h)$ the probability with which an intermediary chooses to cheat.\(^{13}\) Then the above decision can be summarized as follows:\(^{14}\)

$$q(h) \begin{cases} 
= 0 & \text{if } V_n(h + 1) - P_{sn}(h) - V_n(0) > 0 \\
= 1 & \text{if } V_n(h + 1) - P_{sn}(h) - V_n(0) < 0 \\
\in [0,1] & \text{if } V_n(h + 1) - P_{sn}(h) - V_n(0) = 0 
\end{cases} \tag{9}$$

This equation shows that an intermediary is less likely to cheat if the future value of a higher history is higher and the current price is lower.

1.2.3 Distributions

In equilibrium, an agent of history $h - 1$ can move to a history $h$ only through completing a transaction and not cheating. Only $m$-type agents of history $h - 1$ can become $n$-types of history $h$ since only an $m$-type agent has a good to exchange. An $n$-type agent of history $h$ ($h > 0$) moves out of that history either through death (with rate $\lambda$) or through meeting

\(^{13}\)Here I chose lower case letters to represent an agent’s decision and upper case letters to represent all other agents’ decision. This is in line with a Nash equilibrium notation and a symmetric equilibrium will require $q(h) = Q(h)$ for all $h$.

\(^{14}\)In the notation $q(h)$, I have suppressed the dependence on the aggregate state which should appear through $V_n(0)$. The dependence will be made explicit when analyzing the equilibrium.
a bureaucrat and becoming an \( m \)-type (with rate \( \pi(h) \)). An \( m \)-type agent exits the group if he meets an entrepreneur (with rate \( \gamma \)) or through death. The only entry in the group of \( m \)-type agents is through \( n \)-type agents of history \( h \) who meet a bureaucrat. Thus, the dynamics of the measures of the \( m \)-type and \( n \)-type individuals are:

\[
\dot{n}(h) = \gamma m(h - 1)(1 - Q(h)) - (\lambda + \pi(h))n(h) \quad \forall h > 0 \tag{10}
\]

\[
\dot{m}(h) = \pi(h)n(h) - (\gamma + \lambda)m(h) \quad \forall h \tag{11}
\]

The equation of motion for the mass of \( n \)-type agents of history zero is represented by (12) below. The mass of \( n \)-type agents who enter history zero is the total mass of agents that cheat, represented by the first term in the equation and the mass of new entrants, represented by \( \delta \). Note that the first term is the sum of all agents that meet entrepreneurs and decide to cheat with probability \( Q(h) \). As in (10), exit is decided by death or meeting a bureaucrat. In a stationary equilibrium, the inflow into history \( h \) must equal outflow, which implies that the rates of change for each history and type will equal zero. Thus,

\[
\dot{n}(0) = \gamma h Q(h)m(h) - n(0)(\lambda + \pi(0)) + \delta. \tag{12}
\]

### 1.3 Equilibrium

**Definition:** A stationary symmetric equilibrium consists of a triplet \( \{q(h), Q(h), n(h)\}_{h=0}^{\infty} \), value functions, prices and distributions such that given \( \{Q(h)\}_{h=0}^{\infty} \), (i) – (vii) hold:

(i). The value functions satisfy (1) – (8).

(ii). \( q(h) \) satisfies (9) for all \( h \).

(iii). \( \dot{n}(h) = \dot{m}(h) = 0 \forall h. \)

(iv). \( V_i \geq 0 \forall i, R_s \geq 0. \)

(v). \( P_{ij} \geq 0 \forall \{ij\}. \)

(vi). \( V_n(0) = w. \)

Moreover \( Q(h) \) satisfies:

(vii). \( q(h) = Q(h) \forall h. \)

The equilibrium concept that is being applied here is that of a stationary Nash equilibrium, where an intermediary takes others’ decisions \( (Q(h)) \) as given and chooses the probability of cheating according to the best response function in (9). I concentrate on symmetric equilibria to reduce the set of applicable equilibria and simplify the analysis. Conditions (iv) and (v) are necessary for participation in the market by all agents. Condition (vi) is the free entry condition, where the value of an entrant is equal to his
outside option \(w\). In equilibrium this condition determines the number of new entrants \(\delta\).

Plugging in for equilibrium conditions and solving we get the following equations:

\[
V_n(h) = \frac{\gamma \pi(h)}{\psi(h)} (X + (1 - Q)V_n(h + 1) + QV_n(0))
\]

\[
n(h) = \frac{\gamma (1 - Q(h))}{\lambda + \pi(h)} m(h - 1) \quad \forall h > 0
\]

\[
m(h) = \frac{\pi(h)}{\gamma + \lambda} n(h)
\]

Where \(\psi(h) \equiv (2r + \gamma) (r + \pi(h)) + r(r + \gamma), \ X = \frac{x + (1 - \alpha) \gamma \sum P_{me(h)} m(h)}{\phi}, \) and \(\phi \equiv r + (1 - \alpha) \gamma + \alpha \mu / 2 + \alpha\). Note that the effective discount rate \((\gamma \pi(h)/\psi(h))\) is weakly increasing in histories. For all \(h > 0\) we have a constant discount rate where \(\pi(h) = \tau\). Equations (10) and (11) imply the following equilibrium distribution equations:

\[
n(h) = n^*(0)^h \prod_{i=1}^h g(i) \forall h > 0
\]

where \(g(h) = \frac{\gamma (1 - Q(h)) \pi(h - 1)}{(\lambda + \pi(h))(\gamma + \lambda)}\) and

\[
n^*(0) = \frac{1}{\lambda + \pi(0)} (\gamma_h Q(h)m(h) + \delta)
\]

Substituting for \(\pi(h)\) we get the following equations:

\[
n(h) = n^*(0) \left(\frac{\gamma}{\gamma + \lambda}\right)^h \left(\frac{\tau_1}{\lambda + \tau_1}\right)^{h-1} \left(\frac{\pi(0)}{\lambda + \pi(0)}\right)^h (1 - Q(i))
\]

\[
m(h) = n^*(0) \left(\frac{\gamma}{\gamma + \lambda}\right)^h \left(\frac{\tau_1}{\lambda + \tau_1}\right)^h \left(\frac{\pi(0)}{\lambda + \gamma}\right)^h (1 - Q(i))
\]

Note that both (18) and (19) above imply that \(\lim_{h \to \infty} n(h) = \lim_{h \to \infty} m(h) = 0\).

Given the Nash structure of the equilibrium described above, a result where \(Q(h) = 1\) \(\forall h\) and where intermediaries are not active is always possible. This is the self-fulfilling nature of such an outcome. In this equilibrium, the bargaining between the bureaucrat and intermediary breaks down and no good exchanges hands because \(\lim_{Q(h) \to 1} P_{sn(h)} = \infty\) since the bureaucrat must fully insure against the probability of cheating. As this probability approaches one, the only way the bureaucrat can insure against this occurrence is
by requesting a prohibitively large price, which results in the breakdown of the match.\footnote{See Appendix A for a detailed exposition of an equilibrium without intermediaries.}

In any equilibrium an entrepreneur has two settings in which to purchase the license, in the public setting, where she will have to pay the red tape cost $b$, or the corruption market, where the red tape costs are not paid. In both cases a license is procured (discounted by the rate of time preference $r$). Therefore the ‘savings’ produced by this match are $b\alpha + rA$. The amount $b\alpha + rA$ is an essential component of the bargaining process and therefore in any negotiation it will represent the size of the ‘pie’ to be shared between agents. Let $x \equiv b\alpha + rA$.

### 1.3.1 Equilibrium with Intermediaries

An equilibrium with intermediaries is one in which $n(h) > 0$ for some $h \in \mathbb{N}$, which is equivalent to the probability of cheating at history zero being less than one (see proof of lemma below). Given the large set of possible equilibria, it is necessary to derive some equilibrium properties so as to give some structure to equilibria with intermediaries. Furthermore, while an equilibrium without intermediaries always exists, this may not be the case for equilibria in which intermediaries are active. In fact, as we will see in section 4.1, there are constraints on parameters that determine the existence of such an equilibrium.

Rewriting (9) the best response function becomes:

$$ q(h) = \begin{cases} 
  0 & \text{if } D(Q(h), h) > 0 \\
  1 & \text{if } D(Q(h), h) < 0 \\
  \in [0, 1] & \text{if } D(Q(h), h) = 0 
\end{cases} $$

(23)

where

$$ D(Q(h), h) = \frac{1}{1 - Q(h)} \left( \frac{(2r + \gamma)(r + \pi(h))}{\gamma \pi(h)} V_n(h) - X - w \right) $$

is the difference function defined by (9). This function depends on $Q(h)$ both directly and indirectly through the value function $V_n(h)$. The following result reduces the set of possible equilibria by ruling out instances where an equilibrium with intermediaries exists, and the probability of cheating is one for some history $h > 0$.

**Lemma 1.** For any equilibrium with intermediaries, $q(h) = Q(h) < 1 \Rightarrow q(h + 1) = Q(h + 1) < 1$.

**Proof.** See appendix. \(\blacksquare\)
The above result implies that if an equilibrium with intermediaries exists, i.e. \( Q(0) < 1 \), then it will never be optimal for an intermediary to cheat with probability one. This is an intuitive result. If an entrant intermediary does not find it optimal to cheat with probability one, then, given that the probability of meeting a bureaucrat at histories higher than zero is higher, it must be that it is never optimal to cheat with probability one. Furthermore, the above result suggests that the only equilibrium in which an intermediary uses pure strategy to decide whether to cheat is one in which \( Q(h) = 0 \ \forall h \). In the following section we proceed with the description of this equilibrium and the conditions for its existence.

### 1.3.2 Pure Strategy Equilibrium with Intermediaries

In order for such an equilibrium to exist, it must be that it is optimal for all agents to choose \( q(h) = 0 \) for all \( h \), given a sequence \( \{Q(h)\}_{h=0}^{\infty} \) such that \( Q(h) = 0 \) for all \( h \). However, first we must describe the entry distribution. Since no intermediaries cheat, the free entry condition, \( V_n(0) = w \), uniquely determines a value \( \psi^*(0) \) (see (17) above) which in turn uniquely determines a probability of a match at history zero (\( \pi^*(0) \)). In equilibrium, this probability pins down the mass of new entrants that satisfies the free entry conditions where \( n^*(0) = \delta^* \) and \( n^*(0) \) is such that \( \pi^*(0) = \frac{B}{B + n^*(0)} \).

The nature of the equilibrium is such that intermediaries with histories higher than zero are identical and payoffs for these intermediaries are constant. The only real distinction is between intermediaries with history zero and those with positive histories. Using (1) – (8) we find the following result:

**Proposition 2.** In any equilibrium with intermediaries the following hold:

(i). \( \frac{r}{\tau} < \frac{\gamma}{r + \gamma} \)

(ii). \( \tau > \pi(0) \)

(iii). \( P_{sn}(h) < P_{sn}(0) \)

(iv). \( P_{me}(h) - P_{sn}(h) > P_{me}(0) - P_{sn}(0) \)

**Proof.** See appendix. ■

The first condition relates the two search periods of an intermediary. If the probability of meeting a bureaucrat (\( \pi(h) = \tau \)) is too low, for such an equilibrium to exist it must be that the probability of meeting an entrepreneur is very high to compensate. If an intermediary must wait for a long time before he meets a bureaucrat, then, for this equilibrium to be optimal for him, the probability of selling the license thus procured

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16 Uniqueness and existence are ensured by the continuity of both the \( \pi \) and \( \psi \) functions.
must be high. This logic applies in the reverse as well; if the probability of meeting an entrepreneur is very low, then the probability of meeting a bureaucrat must be very high to compensate. The second condition is intuitive, intermediaries with longer histories must have a higher rate of match with bureaucrats, otherwise there would be no incentive to build such histories.

The third condition relates the fact that in this equilibrium intermediaries with positive history pay a lower *consignment* price even though no intermediaries cheat. This result is due to the fact that an intermediary of positive history has a higher value of being unmatched because his probability of finding a bureaucrat is higher, so in any negotiation between a bureaucrat and an intermediary, the bureaucrat must compensate for this higher value. This result would suggest that, in the corruption market, experience matters, and that intermediaries with longer histories can command higher premiums for their services. The fourth result in the above proposition is the expression of that intuition. The difference between the sale and purchase prices (i.e. the premium) for more experienced intermediaries is higher.

Another parameter of interest is the size of the corruption market as measured by the mass of active intermediaries in equilibrium. Changes in the outside option or the size of the payoff from corruption affect the mass of intermediaries at each history.

**Lemma 3.** If \( \frac{w}{x} \) increases then \( n(h) + m(h) \) falls for all \( h \).

*Proof.* See appendix. ■

The above lemma describes a salient feature of the corruption market. As workers become richer (or the proceeds from corruption fall) the total number of workers that choose to become intermediaries \( \sum_h (n(h) + m(h)) \) falls. This straightforward result follows from the free entry condition. Since the outside option of workers increases, the value of a new entrant must increase to reflect the change. This implies that the probability that a new entrant meets a bureaucrat in the corruption market must in turn increase. This can only be achieved if the total mass of entrants falls. Since there are now fewer entrants, there must be fewer intermediaries at each history.

If one considers differences in countries’ per capita incomes, controlling for the size of corruption proceeds or effectiveness of government audits, we can see that lower income countries will have a tendency to have more pervasive and endemic corruption. The size of the corruption market is inversely related to average income.

---

\(^{17}\)This condition would be endogenous were we to allow the distribution in each history \( h > 0 \) to affect the matching rate. This however does not add any new insight to the model; the first history building step is illustrative of all others that would follow for higher histories.
It is clear from the discussion of equilibrium above that any increase in the size of the bureaucracy will only result in a larger mass of entrants in the market, but, *ceteris paribus*, the price that entrepreneurs pay will remain unchanged. This result is not an artifact of the constant rate of matching between agents with positive histories. Consider what happens to the market of history zero; when the size of the bureaucracy increases, the probability of a match for an entrant intermediary increases as well, which induces more entry, up to the point where the rate of a match remains unaffected. For workers, the equilibrium with intermediaries is clearly Pareto superior, given that a fraction of them earn a higher lifetime value, and the rest remain workers with a wage of \( w \).

In terms of efficiency, the rate at which a license is issued to an entrepreneur increases to \( \alpha + (1 - \alpha)(\mu + \gamma) \), and the number of projects approved during an arbitrary time period \( t \) increases. Depending on the reasoning behind licensing, this fact can have various implications for welfare. Note that the increase in the number of licenses issued is solely due to an increase in the number of licenses issued in the corruption market, where presumably the necessary quality controls are bypassed (remember the entrepreneur does not pay the red tape cost \( b \) in this market). Thus, the projects approved in the corruption market may result in poor quality of implementation, which can cause substantial economic losses in the long run.

### 1.4 Quantitative Evaluation and Policy Analysis

The purpose of this section is to illustrate equilibrium selection as a function of the policy parameters, to quantify some of the model’s comparative static predictions and to analyze the model’s sensitivity to some of the exogenous parameters. The main policy parameters of interest here are the audit probability \( \alpha \) and the legal costs of obtaining permits \( b \).

In fact, the following exercises will compare responses of key variables to both these parameters, with the goal of giving some insight as to what would constitute effective policy in reducing the negative effects (if any) of the size of the corruption market, especially with an eye towards entrepreneurial value.\(^{18}\) In the quantitative section I will allow for two departures from the theoretical model outlined above: First, bargaining power will be allowed to vary. Second, the matching function for matches between bureaucrats and intermediaries of history 0 will be slightly amended to \( M(n(0), B) = \frac{\text{Ren}(0)}{\sigma n(0) + B} \) where \( \sigma \) represents the intensity of search on the part of intermediaries. I will

\(^{18}\)Since the space of policy analysis is two-dimensional, a theoretical treatment of the comparative static results is cumbersome, as the welfare equation (24) below can easily attest to. The numerical analysis is just as illustrative and sheds light on all of the responses of the pertinent variables.
then use these two parameters to perform a sensitivity analysis on the key results.\textsuperscript{19}

1.4.1 Equilibrium Selection

The comparative statics and sensitivity analysis in the following sections are only relevant to the pure strategy equilibrium with intermediaries. However, as the theoretical section suggests, an equilibrium where intermediaries are not used is always possible. Considering the fact that the policy parameters $\alpha$ and $b$ are a main point of focus for the quantitative analysis, it would be appropriate at this point to discuss how they affect equilibrium selection. Figure 1.4 below depicts the combinations of $\alpha$ and $b$ over which an equilibrium with intermediaries becomes feasible. It is important to note that Figure 1.4 depicts combinations of $\alpha$ and $b$ where intermediation is feasible, which does not imply that it the only outcome of the model. The section in the graph above labeled "no intermediation" depicts combinations of $\alpha$ and $b$ that do not incentivize intermediation and thus will always induce $q(h) = Q(h) = 1$ for all histories. The section labeled "active intermediaries" depicts combinations of $\alpha$ and $b$ where both types of pure strategy as well as mixed strategy equilibria coexist given the Nash equilibrium concept employed here.

\textsuperscript{19}See appendix for sources of parameter values. The project value $A$ is normalized to one and $b$ is in percentage terms.
When both $b$ and $\alpha$ are relatively low, the “threat point” of intermediaries in their bargaining with entrepreneurs is low as well. Therefore the “savings” provided by the corruption market are small, as is the share of these savings that can be appropriated by intermediaries. This is because when business costs are low, entrepreneurs are less reluctant to purchase the license legally and therefore less willing to part with a larger share of the surplus. For larger values of $b$, the probability of audit $\alpha$ that induces intermediation fall significantly. This is due to the fact that as $b$ increases entrepreneurs will have to part with a larger share of the project if they purchase the license legally, therefore even when the probability of such an occurrence is relatively low, they are willing to give up a larger share of their profits to prevent it from happening, which increases the returns to intermediation and therefore induces entry.

One can think of the combinations of $\alpha$ and $b$ in the graph above as the expected loss to the entrepreneur in case the license has to be purchased legally. Entering the corruption market is a way of shielding oneself from such a loss. If this expected loss is sufficiently large, the entrepreneur will be willing to compensate an intermediary in possession of a license. Where intermediation is feasible, this compensation surpasses the threshold value an intermediary needs to remain honest. In short, when both $\alpha$ and $b$ are very high, there is a lot of value to being an intermediary, and so intermediaries find it advantageous not to cheat. It is also worth noting here that the relationship of $\alpha$ and $b$ along the boundary of feasibility is non-linear. This is due to the fact that at high values of $b$, intermediation becomes more frequent, and therefore a significant proportion of the licenses for sale in the market are in the hands of intermediaries. Implicitly this means more negotiating power on the side of an intermediary. In this way, the fall in $\alpha$ that is needed to keep the intermediary indifferent between cheating and being honest at high levels of $b$ is substantially smaller.

### 1.4.2 Policy Effects

I define welfare as the weighted sum of the discounted value of agents in the economy. I analyze the response of average prices for licenses, the mass of intermediaries, individual payoffs and welfare to the policy parameter $\alpha$, the probability that the government performs an audit, as well as the parameter $b$, the fraction of the project’s value that the government legally extracts from entrepreneurs. Equation (24) is a mathematical representation of this notion of welfare. The key observation is that not only do $\alpha$ and $b$ influence the values of each agent; they also affect the number of intermediaries that are active in the corruption market, thus influencing the welfare of potential intermediaries.
(workers).

\[ W(\alpha, b) = BV_s(\alpha, b) + EV_e(\alpha, b) + \sum_h n(h, \alpha, b)V_n(h, \alpha, b) + \sum_h m(h, \alpha, b)V_m(h, \alpha, b) + \]
\[ + \left( 1 - E - B - \sum_h \{ n(h, \alpha, b) + m(h, \alpha, b) \} \right) w \]

(24)

The parameters \( \alpha \) and \( b \), can be thought of as two possible mechanisms of reducing the incidence of corrupt transactions. The parameter \( \alpha \) represents the probability that a license will be sold in the illicit market, and \( b \) represents the costs an entrepreneur must incur for the license if it is obtained legally. The effects are summarized in Figure 1.5 and Table 3.

The first graph in Figure 1.5 relates the number of intermediaries as a percentage of the total population to both parameters.\(^{20}\) The first observation is that intermediation is strictly increasing in \( b \), as the red tape costs fall, the payoff to intermediation is strictly decreasing. The more striking feature of the figure however, is the positive effect that the probability of an audit has on the mass of intermediaries. When the costs of licensing legally (\( b \)) are large, the payoff to intermediation is equally large. As the probability of an audit (\( \alpha \)) increases, so does the expected loss that an entrepreneur faces, since she is much more likely to have to purchase the license legally. This in turn induces a larger share of the surplus to be transferred to intermediaries, which increases the value of intermediation and therefore the fraction of the populations that practices it. In fact, the above result suggests that increasing the frequency of audits merely transfers corrupt activities and the resulting revenue into the hands of intermediaries since the mass of bureaucrats remains unchanged. It is worth reiterating that this result only holds when compliance costs are particularly onerous, and suggests that reducing these costs may be a more effective way of eliminating corruption. In this case, reducing the costs of doing business by 50% results in a fall of 8.5% in intermediation, while increasing \( \alpha \) by the same magnitude only exacerbates the problem (see Table 3 below).

The second graph in Figure 1.5 clearly shows that entrepreneurs are made worse off with a higher frequency of audits when intermediaries are active. This is entirely due to the fact that the costs of procuring the license through the legal process are prohibitive (\( b \) is relatively large), and forcing the entrepreneurs to incur these costs through audit is welfare reducing. Inherently this results in a seemingly perverse vicious cycle of cor-

\(^{20}\)In figure 1.5, \( \alpha \) is increasing on the \( x \) axis while \( b \) is decreasing. For example 0.3 on the \( x \)-axis implies a 30% increase in \( \alpha \) or a 30% fall in \( b \) from benchmark values.
Corruption; in countries that have the highest red tape costs, where fighting corruption is critical, doing so worsens the outcome for entrepreneurs and presumably for the larger economy as projects become less profitable. Table 3 gives a quantitative picture of these effects, a reduction of $b$ is clearly far more preferable from an entrepreneur’s point of view than an increase in the frequency of audits.

A fall in $b$ reduces the welfare of potential intermediaries as the surplus to be negotiated over falls, which reduces the prices intermediaries can fetch for their services. Overall, any such discussion of welfare is limited by the partial equilibrium nature of this analysis. It is worth noting that this model omits questions of externalities and the reasons for issuing licenses, which in most cases are used to correct perceived market failures. However, one must also remember that corrupt governments often increase the amount of red tape and licensing to increase corruption proceeds that they collect, so it is not always clear that license issuance is intended to correct for these failures.
pay is falling. As Table 3 shows, welfare increases with a reduction in $b$ and falls with an increase in $\alpha$, because entrepreneurs are strictly better off in the first instance.

The last graph in Figure 1.5 explores the relationship between prices and the policy parameters $b$ and $\alpha$. As in the analysis above, increasing $\alpha$ when $b$ is relatively high actually increases the average price an entrepreneur pays in an intermediated equilibrium, because when costs are relatively high, intermediaries have higher threat points with higher $\alpha$ and can extract higher proportions of the surplus in a match with the entrepreneur. Reducing $b$ on the other hand is far more effective (see Table 3) as it greatly reduces the price both bureaucrats and intermediaries command in the corruption market.

The welfare impact of changes in the policy parameters in Figure 1.5 above is the result of the interplay between the welfare effects on entrepreneurs and intermediaries and depends on the fraction of the population that is engaged in either activity. Bureaucrat’s welfare is decreasing in both parameters (see Figure 1.6) and is a small proportion of the total. However, as Figure 1.6 below shows, the value to intermediation increases significantly with an increase in $\alpha$ at high levels of $b$, where the reasons for this increase are laid out in the discussion above. When the proportions of entrepreneurs and intermediaries in the population are similar, total welfare falls with an increase in $\alpha$. This is due to the fact that the fall in entrepreneurial welfare is quite significant, while the rise in intermediary welfare is moderated by the increase in the extensive margin of intermediation. Note that when intermediation surpasses entrepreneurship as an activity, total welfare increases in $\alpha$, as intermediaries become the dominant group.

Figure 1.6: Bureaucrat and intermediary welfare
1.4.3 Sensitivity Analysis

Section 1.4.1 discusses the sensitivity of equilibrium selection under different combinations of the policy parameters $\alpha$ and $b$. In this section, I will explore how sensitive the results of the policy effects exercise in section 1.4.2 are to changes in two exogenously given parameters of the model; the bargaining power parameter $\theta$, as well as the search intensity parameter $\sigma$. This choice reflects the fact that both parameters are more likely to vary in cross country comparisons. The bargaining power parameter could be thought of as a second dimension to the level of corruption in a country. Where corruption is pervasive, bureaucrats are less likely to be reported when engaged in corrupt activities and are therefore more likely to set the terms in any negotiation, thus having higher negotiating power. On the other hand, the search intensity parameter captures the ease with which a match with a bureaucrat can be achieved by an intermediary. When $\sigma$ is relatively low, bureaucrats are more accessible by intermediaries and so access to licenses is less cumbersome.

There are three matches where bargaining is relevant. For the sake of brevity, bargaining power is collapsed to a single parameter $\theta$ as described in the table below. The first column lists the matching agents (bureaucrats ($S$), entrepreneurs ($E$) and intermediaries ($I$)) and the second the bargaining power of the corresponding agent.

<table>
<thead>
<tr>
<th>Match</th>
<th>Bargaining Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S, E$</td>
<td>$(1 - \theta), \theta$</td>
</tr>
<tr>
<td>$S, I$</td>
<td>$(1 - \theta), \theta$</td>
</tr>
<tr>
<td>$E, I$</td>
<td>$\theta, (1 - \theta)$</td>
</tr>
</tbody>
</table>

Figure 1.7 below reproduces Figure 1.5 at different levels of the bargaining power parameter.

Note that when $\theta$ increases, the entrepreneur is better off in both of her matches and therefore entrepreneurial value increases in $\theta$. For the same reasons, as $\theta$ increases, the average price of a license paid by the entrepreneur falls as well, while maintaining the magnitude of its first order response to $\alpha$. On the other hand, the behaviour of the number of intermediaries as a response to $\theta$ is more subtle. There are two effects in operation here. First, an increase in $\theta$ reduces an intermediary’s bargaining power when he sells the license to an entrepreneur, but increases it when said license is purchased from the bureaucrat. The first effect decreases intermediation value while the second increases it. There is, however, a third effect. When $\theta$ increases, the bureaucrat’s bargaining power is

\[In the interest of space I have omitted the response of these variables to a fall in $b$. The variables respond in a similar manner.
reduced in all of his matches, which reduces his negotiating position even further when matched with an intermediary. This then increases the value to being an intermediary even further, thus inducing more entry. It is also interesting to note that an increase in the bargaining power parameter $\theta$, does not only shift aggregate welfare up due to an increase in entrepreneurial value, but it also seems to reduce the first order effect $\alpha$ has on welfare.

![Graphs showing responses to bargaining power](image)

Figure 1.7: Responses to bargaining power

Figure 1.8 below depicts the response of the level of intermediation and total welfare to changes in the search intensity parameter $\sigma$. As expected, when search intensity is low, bureaucrats are more accessible, which induces entry into intermediation. Since entrepreneurial welfare is not very sensitive to search intensity, a reduction in $\sigma$ causes an increase in total welfare due to the fact that the value to intermediation has increased as illustrated by the increase in the number of intermediaries.

### 1.5 Discussion and Conclusion

This chapter constructs a basic model of corruption with intermediation. I find that an equilibrium without intermediaries is always present given the self-fulfilling nature of individuals' beliefs. In a pure strategy equilibrium intermediaries of longer histories earn higher premiums. Changes in wages relative to the potential proceeds from corruption affect both the size of the corruption market as measured by the mass of active intermediaries, as well as payoffs to intermediaries. An increase in the probability of an audit reduces welfare for bureaucrats, while its effects on entrepreneurs and intermedi-
Figure 1.8: Responses to search intensity

aries depend on the size of the compliance/red tape costs. When these are relatively high, increasing the frequency of audits (fighting corruption) increases intermediaries’ welfare and reduces that of entrepreneurs. In essence, the reduction in entrepreneurial welfare is a result of their aversion to obtaining permits legally given the high costs, which induces them to accept a higher ask price from intermediaries and bureaucrats. In effect, increasing auditing frequency acts as a revenue redistribution mechanism from bureaucrats and entrepreneurs to intermediaries.

From a modeling perspective, one feature of note is the assumption that a cheating intermediary immediately loses all of his history. This is not a necessary condition for the existence of the equilibrium above, although it makes the analysis more tractable. However, it is possible to admit a partial loss of history so that a cheating intermediary has a history of \( h + 1 \) with some positive probability. This extension is dealt with in detail in the appendix. Naturally, this form of punishment is more general and could be applied to the model without great complication. However, in any such extension of the model, the pure strategy equilibrium with intermediaries is identical to the one described in detail in the chapter. The differences between the two versions of the model are seen in the variety of mixed strategy equilibria not analyzed here.

The choice of consignment as a modeling tool is deliberate here. Anecdotal evidence suggests that intermediaries meet with licensees privately. In that event, and given the fact that contract enforcement is difficult in illicit markets, intermediaries have ample opportunity to cheat bureaucrats out of corruption revenue. This chapter then addresses the question as to why bureaucrats would use intermediaries in the first place. The goal,
as stated in the introduction, is to generate intermediation as an equilibrium outcome. The cheating probability in the model can be thought of as the fraction of corruption revenue that an intermediary decides to keep for himself by reneging on the agreement with the bureaucrat. The goal here is to allow the intermediaries an avenue to do so and understand the mechanics that would result in an intermediated equilibrium even when cheating is a possibility.

In this environment the accumulation of histories resembles, to some extent, reputational concerns. However, intermediaries are ex-ante identical and there is no uncertainty about player types. Therefore the use of the term reputation would be non-standard and confounding in this chapter. In a model with histories, as defined here, and heterogeneous intermediaries, the interaction between the two would provide an enhanced understanding of the mechanisms at work in intermediated corruption. Naturally the outcome would depend on the source of heterogeneity of potential intermediaries, but if agent types were to be partially revealed in multiple matches, the accumulation of histories on the part of intermediaries could potentially interact with learning about agents’ types. In that case features of a presumptive common prior distribution would be essential in determining equilibrium outcomes. Understanding these mechanics would provide ample opportunity for future research.

From a more general political economy perspective, it is important to stress that corruption is not an isolated phenomenon that involves only the government official and the end-user of government service, but rather a market in itself that takes time and resources to operate. This market is innovative and becomes embedded in the economy and is difficult to uproot. In this environment corruption is endemic because it involves a larger number of actors than just bureaucrats and end users of government services. Viewed from this angle, corruption is a structure that, through its redistributive power, thrives by giving a larger number of agents a real stake in its survival. Thus, corruption becomes an implicit political bribe. In a corrupt country that holds free elections, government officials who have a record of being corrupt may not be thrown out of office because a substantial fraction of the population may have a real economic interest in the status-quo. In this way corruption makes society myopic by overemphasizing the short term gains and ignoring long term losses.

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23 On this point see Bobonis et. al. (2010).
1.6 Appendix

Appendix A: Equilibrium without intermediaries

Let us consider the equilibrium without intermediaries. Given the Nash structure of the equilibrium and given a sequence \( Q \) such that \( Q(h) = 1 \forall h \), the optimal sequence \( q \) that satisfies (6) is such that \( q(h) = 1 \forall h \). Therefore the sequence \( Q(h) = 1 \forall h \) is a Nash equilibrium. This establishes the following lemma:

**Lemma 4.** An equilibrium without intermediaries always exists.

In fact, there are multiple equilibria in which intermediaries are not active, where an equilibrium without intermediaries is one in which \( n(h) = m(h) = 0 \), \( \forall h \). However, there is one feature that these equilibria have in common, \( Q(0) = 1 \). If it is optimal for entrants to cheat, and since histories evolve one step at a time, the mass of intermediaries of history 0 will be zero, resulting in a degenerate distribution of zero mass for each history. Furthermore, any sequence of cheating probabilities \( \{Q(h)\}_{h=1}^{\infty} \) combined with \( Q(0) = 1 \), is an equilibrium without intermediaries.

The price and value functions in this equilibrium can be solved as:

\[
V_e = \frac{\alpha (A - b) + A(1 - \alpha)\mu/2}{r + (1 - \alpha)\mu/2 + \alpha} \tag{20}
\]
\[
P_{se} = \frac{1}{2} \frac{b\alpha + rA}{r + (1 - \alpha)\mu/2 + \alpha} \tag{21}
\]
\[
V_s = \frac{1}{2r} (1 - \alpha)\mu \frac{b\alpha + rA}{r + (1 - \alpha)\mu/2 + \alpha} \tag{22}
\]

The price paid by the entrepreneur (21) reflects the outside option that the entrepreneur has if she refuses to accept the offer at hand. In an equilibrium without intermediaries; an entrepreneur has only two settings in which to purchase the license, in the public setting, where she will have to pay the red tape cost \( b \) with probability \( \alpha \), or the corruption market, where the red tape costs are not paid. In both cases a license is procured (discounted by the rate of time preference \( r \)). Therefore the ‘savings’ produced by this match are \( x \). The effective discount rate for an entrepreneur looking for a license in an equilibrium without intermediaries is \( \theta = r + (1 - \alpha)\mu/2 + \alpha \) and so the total ‘savings’ produced by this match are \( \frac{b\alpha + rA}{r + (1 - \alpha)\mu/2 + \alpha} \). Note that if \( \theta < 1 \) then the ‘savings’ are magnified given that an actual discount would be applied to any future successful bargaining and when \( \theta > 1 \), the price paid by an entrepreneur is a fraction of the total ’savings’ produced by the match since the the effective discount is larger than one.
The arrival rate of matches where a license exchanges hands produced in any time interval $\Delta$ in this equilibrium is $((\alpha + (1 - \alpha)\mu)\Delta$. This is the average rate at which a new project is approved. With a rate of $\alpha$, the entrepreneur is able to get the license publicly, and given that she has not done so, she will obtain the license with rate $(1 - \alpha)\mu$ in the corruption market.

**Appendix B: Proofs**

**Lemma 1**

Proof. Suppose that $Q(h + 1) = 1$. Then we know that $\lim_{Q(h)\to1} P_{sn}(h) = \infty$, which implies that the bargaining between the bureaucrat and the entrepreneur breaks down and there is no agreement. Therefore, the value of an $n$–type intermediary of history $h + 1$ is at most $w$. Using (13) and noting that $Q(h) < 1$ by assumption, we have that $V_n(h) = \frac{\gamma\pi(h)}{(2r + \gamma)(r + \pi(h)) + r(r + \gamma)}(X + w)$. Substituting this in the equation for $D(\cdot, \cdot)$ we have:

$$D(Q(h), h) = \frac{1}{1 - Q(h)} \left( \frac{(2r + \gamma)(r + \pi(h))}{(2r + \gamma)(r + \pi(h)) + r(r + \gamma)} - 1 \right) (X + w) < 0$$

which is a contradiction. ■

**Proposition 2**

i. Proof. For $D(0, h) > 0$ we must have that $V_n(h + 1) - P_{sn}(h) - w > 0$. Plugging in for $P_{sn}(h)$ we get $V_n(h) \left( 1 - \frac{r(r + \gamma)}{\gamma\pi(h)} \right) - w > 0 \Rightarrow \frac{r}{\pi(h)} < \frac{\gamma}{r + \gamma}$. ■

ii. Proof. We find the conditions for $D(0, 0) > 0$. We have that: $D(0, 0) = \frac{2r (r + \pi(0)) + \gamma r}{\gamma\pi(0)} w - X$. The free entry condition is:

$$w = \frac{\gamma\pi(0)}{\psi(0)} (X + V_n(h)) = \frac{\gamma\pi(0)}{\psi(0)} \frac{\psi(h)}{\psi(h) - \gamma\pi(h)} X \Rightarrow$$

$$X = w \frac{\psi(h) - \gamma\pi(h)}{\psi(h) - \gamma\pi(0)}$$

Replacing in the equation for $D(0, 0)$ we have:

$$w \left( \frac{2r (r + \pi(0)) + \gamma r}{\gamma\pi(0)} - \frac{\psi(h) - \gamma\pi(h)}{\psi(h) - \gamma\pi(0)} \right) > 0$$
so we need $2r (r + \pi(0)) + \gamma r - \left(1 - \frac{\gamma \pi(h)}{\psi(h)}\right) \psi(0) > 0 \Rightarrow \frac{\pi(h)}{\psi(h)} > \frac{\pi(0)}{\psi(0)} \Rightarrow \tau > \pi(0)$. 

Proof. Note that $P_{sn}(h) = \frac{r(r + \gamma)}{\gamma \pi(h)} V_n(h)$ and $P_{sn}(0) = \frac{r(r + \gamma)}{\gamma \pi(0)} - w$. Therefore $\frac{P_{sn}(0)}{P_{sn}(h)} = \frac{\pi(h)}{\pi(0)} \frac{w}{V_n(h)}$. The free entry condition implies that: $w = \frac{\gamma \pi(0)}{\psi(0)} \left(\frac{\psi(h)}{\gamma \pi(h)} V_n(h)\right)$. Plugging this condition into the equation for $\frac{P_{sn}(0)}{P_{sn}(h)}$ we get: $\frac{P_{sn}(0)}{P_{sn}(h)} = \frac{\psi(h)}{\psi(0)} > 1$. 

iv.

Proof. First we note that:

$$2P_{me}(h) = A - V_n(h + 1) + V_m(h) + P_{sn}(h) - V \forall h$$

$$\Rightarrow 2(P_{me}(h) - P_{me}(0)) = (V_m(h) + P_{sn}(h)) - (V_m(0) + P_{sn}(0))$$

$$\Rightarrow (P_{me}(h) - P_{sn}(h)) - (P_{me}(0) - P_{sn}(0)) = \frac{1}{2} [V_m(h) - V_m(0)] + \frac{1}{2} [P_{sn}(0) - P_{sn}(h)] > 0$$

where the last inequality results from the first part and the fact that $V_m(h) > V_m(0)$. 

**Lemma 3**

The proof for this lemma involves relatively tedious algebra, below I have highlighted the major steps needed to complete the proof.

Proof. First consider the equations for both $P_{me}(h)$ for $h > 0$ and $P_{me}(0)$ given below:

$$2P_{me}(0) = \frac{2X}{\psi(0)} \Omega(\pi(0))$$

where $\Omega(\pi(0)) = ((r + \pi(0))(r(1 - c_1) + \gamma) + r(\gamma))$

$$c_1 = \gamma \pi(h) \frac{\psi(h) - \gamma \pi(h)}{\gamma \pi(h)} < 1$$

$$X = \left(\frac{x + (1 - \alpha)\gamma \sum_h P_{me}(h) \frac{m(h)}{M}}{\phi}\right) = \frac{X}{\phi} + \frac{(1-\alpha)\gamma}{2\phi} \left(2P_{me}(0) \frac{m(0)}{M} + 2P_{me}(h) \left(1 - \frac{m(0)}{M}\right)\right)$$

$$\phi = r + (1 - \alpha)\gamma + (1 - \alpha)\mu/2 + \alpha$$

$$2P_{me}(h) = c_2 X$$
where \( 1 < c_2 = \left( \psi - 2 \gamma \pi + \gamma (r + \pi) + r \psi \right) \psi (h) - \gamma \pi (h) \right) < 2 \).

From the free entry condition:

\[
X = \frac{\psi (0) \psi (h) - \gamma \pi (h)}{\psi (h)} w = \frac{\psi (0)}{\gamma \pi (0)} c_3 w
\]

where \( c_3 = \frac{\psi (h) - \gamma \pi (h)}{\psi (h)} \).

Plugging in for \( X \) and rearranging we get:

\[
\frac{x}{w} = \frac{\psi (0)}{\gamma \pi (0)} \left( c_3 \phi + \frac{c_2 c_3}{2} (1 - \alpha) \gamma \frac{m (0)}{M} - \frac{c_2 c_3}{2} (1 - \alpha) \gamma \right) - (1 - \alpha) \gamma c_3 \Omega (\pi (0)) \frac{m (0)}{M}
\]

Note that \( \frac{\partial}{\partial \pi (0)} \left( \frac{\psi (0)}{\gamma \pi (0)} \right) = \frac{\gamma \pi (0) (2 r + \gamma) - \psi (0) \gamma}{(\gamma \pi (0))^2} = - \frac{r \gamma (r + \gamma) + r \gamma (2 r + \gamma)}{(\gamma \pi (0))^2} < 0 \)

Also \( \frac{\partial}{\partial \pi (0)} \left( \frac{\Omega (\pi (0))}{\gamma \pi (0)} \right) = - \frac{r \gamma (r + \gamma) + r \gamma (2 r + \gamma)}{(\gamma \pi (0))^2} < 0 \)

Denote by:

\[
f (\pi (0)) = \frac{\psi (0)}{\gamma \pi (0)} \left( c_3 \phi + \frac{c_2 c_3}{2} (1 - \alpha) \gamma \frac{m (0)}{M} - \frac{c_2 c_3}{2} (1 - \alpha) \gamma \right) - (1 - \alpha) \gamma c_3 \Omega (\pi (0)) \frac{m (0)}{\gamma \pi (0)} M
\]

\[
\frac{\partial f (\pi (0))}{\partial \pi (0)} = \frac{r \gamma (r + \gamma) + r \gamma (2 r + \gamma)}{(\gamma \pi (0))^2} \times \left( 1 - \alpha \right) \gamma c_3 \frac{m (0)}{M} - c_3 \phi - \frac{c_2 c_3}{2} (1 - \alpha) \gamma \frac{m (0)}{M} + \frac{c_2 c_3}{2} (1 - \alpha) \gamma \right)
\]

If we can show that the above expression is negative then an increase in \( w/x \) should result in an increase in \( \pi (0) \) which reduces \( n (0) \) and as a result the fraction of all agents of both type for any history \( h \).

Rearranging the expression inside the brackets we get

\[
c_3 \left( (1 - \alpha) \gamma \frac{m (0)}{M} - \phi - \frac{c_2}{2} (1 - \alpha) \gamma \frac{m (0)}{M} + c_2 (1 - \alpha) \gamma \right)
\]

\[
c_3 \left( - \phi + \frac{c_2}{2} (1 - \alpha) \gamma \left[ 1 + \frac{m (0)}{M} \left( \frac{2}{c_2} - 1 \right) \right] \right)
\]

and given that \( c_2 < 2 \) the second term inside the bracket is positive. Since \( \phi = \)
r + (1 − α)γ + (1 − α)μ/2 + α a sufficient condition for the completion of the proof would be that

\[ \frac{c_2}{2} \left[ 1 + \frac{m(0)}{M} \left( \frac{2}{c_2} - 1 \right) \right] < 1 \]

where the last inequality is a result of \( c_2 < 2 \).

Appendix C: An Extension

Suppose that we relax the assumption of complete loss of history in case an intermediary cheats. Instead, suppose that the intermediary gets to maintain her history with some probability \( p \), and loses it with probability \( 1 - p \). One can think of \( p \) as a less onerous instrument of punishment meted out by the bureaucrats when an intermediary decides to cheat. In this case, when an intermediary cheats with probability \( q \) the total surplus created by an intermediary-entrepreneur match is:

\[ Z_{me}(h) = A + (1 - q(h)) [V_n(h + 1) - P_{sn}(h)] + q(h) [pV_n(h + 1) + (1 - p)V_n(0)] \]

which corresponds to the original case when \( p = 0 \) (i.e. the intermediary always loses all of her history). Equation (2) above then becomes:

\[ rV_m(h) = \gamma \begin{cases} \frac{P_{me}(h) + (1 - Q(h)) [V_n(h + 1) - P_{sn}(h)]}{+Q(h) [pV_n(h + 1) + (1 - p)V_n(0)] - V_m(h)} \end{cases} \]

All other equations remain unaltered. Rearranging we get:

\[ V_n(h) = \frac{\gamma \pi(h)}{\psi(h)} [X + (1 - q(h) + q(h)p) V_n(h + 1) + q(h)(1 - p)V_n(0)] \]

which corresponds to (13) when \( p = 0 \). The intermediary’s decision can be written as:

\[ q(h) \begin{cases} = 0 & \text{if } V_n(h + 1) - P_{sn}(h) > pV_n(h + 1) + (1 - p)V_n(0) \\ = 1 & \text{if } V_n(h + 1) - P_{sn}(h) < pV_n(h + 1) + (1 - p)V_n(0) \\ \in [0, 1] & \text{if } V_n(h + 1) - P_{sn}(h) = pV_n(h + 1) + (1 - p)V_n(0) \end{cases} \]

which corresponds to (9) when \( p = 0 \). Given (4) and (5) the bureaucrat prefers to induce an intermediary to cheat with probability 0. Let us explore the best response function
of the bureaucrat $p(q(h), Q)$, where both the intermediary and the bureaucrat take the sequence $Q = \{Q(h)\}^\infty_{h=0}$ as given when choosing $p$ and $q$. Define by $\hat{p}(q(h), Q)$ such that $V_n(h + 1) - P_{sn}(h) = \hat{p}(q(h), Q)V_n(h + 1) + (1 - \hat{p}(q(h), Q))V_n(0)$. Since the RHS of this expression is increasing in $p$ then for $p < \hat{p}$, $V_n(h + 1) - P_{sn}(h) > pV_n(h + 1) + (1 - p)V_n(0)$, where $\hat{p}(q(h), Q)$ is given by:

$$1 - \hat{p}(q(h), Q) = \frac{(V_n(h + 1) + X) \frac{r(r + \gamma)}{(1 - Q(h))\gamma\pi(h)}}{(V_n(h + 1) - V_n(0)) \left[ 1 + q\frac{r(r + \gamma)}{(1 - Q(h))\psi(h)} \right]} \quad (\hat{p})$$

Note that if $Q(h) = 1$, the bargaining between the bureaucrat and intermediary breaks down ($P_{sn}(h) = \infty$) which implies that $q(h) = 1 \forall p$. In this case intermediaries are not active. In short, the equilibrium without intermediation is still present in this specification of the model. It is also worth pointing out that the existence of $\hat{p}$ is predicated on the existence of an equilibrium where intermediation is possible, i.e. $q < 1$. The RHS of the equation for $\hat{p}$ above is positive if conditions $i$ and $ii$ in proposition 1 above are satisfied. Furthermore, suppose that the RHS of this expression is $> 1$. Consider the intermediary’s decision, which is determined by the sign of:

$$(1 - p)V_n(h + 1) - \frac{r(r + \gamma)}{(1 - Q(h))\gamma\pi(h)}V_n(h) - (1 - p)w$$

Given that this expression is decreasing in $p$, let’s examine the above expression when $p = 0$.

$$V_n(h + 1) - \frac{r(r + \gamma)}{(1 - Q(h))\gamma\pi(h)} [X + (1 - q(h))V_n(h + 1) + q(h)V_n(0)] - w$$

After rearranging this expression it is clear that if the RHS in the equation for $\hat{p}$ is indeed $> 1$, then the above expression is negative so that in equilibrium $q(h) = Q(h) = 1$ and intermediaries are not active. In this sense $\hat{p}$ is only defined when intermediation is possible; more specifically when the conditions of proposition 1 are satisfied.

The threshold value $\hat{p}$ defines the best response function of the bureaucrat given $q(h)$ and $Q$. The best response function for the intermediary is $q(h) = 0$ if $p < \hat{p}$, $q(h) = 1$ if $p > \hat{p}$ and $q(h) \in [0, 1]$ if $p = \hat{p}$. Therefore, in this extension of the model, the equilibria in which intermediaries are active are ones in which $p \leq \hat{p}$ and the conditions of proposition 1 are satisfied. Note however that in the pure strategy equilibrium where intermediaries are active, $q(h) = 0$, which implies that the equilibria in the two versions of the model are identical. This is apparent in the equation for $V_m$ above when $Q(h) = 0$. In this
light, the assumption of complete loss of history is a simplifying assumption that does not change the outcome of the pure strategy equilibrium with intermediaries, which is the equilibrium under scrutiny in the chapter. Naturally, the parameter $p$ will affect mixed strategy equilibrium values.

Appendix D: Calibration of Parameters

I calibrate the model using data for Mexico from Gamboa-Cavazos et al. (henceforth GC) as well as a collection of data from the World Bank, the International Labour Office (ILO) and the OECD. The data on corruption payments that GC analyze comes from a survey designed and financed by ITESM (see paper for details). GC divide corruption payments into two categories: low and high graft. The survey question used to define low graft asked: “On average, what percentage of total yearly revenues of firms similar to yours is devoted to extra-official payments made to low-level public officials?” For the purposes of this chapter I will only use measures of low graft as this definition of corruption best represents the type of bribe payments that the theoretical model alludes to.

GC reports that 53% of respondents admitted to making illicit extra-legal payments during the year, which I take to represent the probability that a firm has at least one encounter during the year with an individual that holds a license (either a bureaucrat or an intermediary of type $n$). To find the magnitude of $\gamma + \mu$ I use data from the World Bank Enterprise Survey which reports the amount of time managers spend dealing with licenses and permits. Using the fact that the waiting time for a Poisson process with parameter $\gamma + \mu$ is exponential with mean $\frac{1}{\gamma + \mu}$, the waiting time and the probability of illicit payments above I calibrate the time period $\Delta \simeq 3.4$ years. Furthermore, the survey participants report that on average they pay 2% of their annual revenue in bribes to deal with licensing and permit issues. Combined with a reported average profit margin of 7% in GC, one can deduce the average price entrepreneurs pay to obtain licenses/permits as a percentage of the total discounted value of the project.\footnote{The discounted value of the project $A$ is normalized to one.} I use the above targets to calibrate particular values of $\gamma$ and $\mu$.

The parameter $b$ is calibrated using the World Bank’s Doing Business Report. Specifically, I use data on actual as well as time costs for the relevant categories in the report (starting a business, enforcing contracts, paying taxes, construction permits and registering property). All of the up-front costs are calculated as a percentage of GDP/capita. To these costs I add the appropriately discounted time costs of waiting to perform each
individual task. In essence this method of calculating $b$ takes into account both the up-front as well as red-tape (time) costs of completing business with the government.

The discount factor $\rho$ is targeted using the interest rate from the OECD database. Combined with the entrepreneur exit rate of 10% reported by Tybout and Davis et al., I calibrate the value for the discount factor $r$. The parameter $\alpha$ is targeted using the World Bank’s Governance Indicators ’control of corruption’ index. The index varies from $-2.5$ to $2.5$ with the latter indicating perfect control of corruption ($\alpha = 1$). The percentage of population that are entrepreneurs ($E$) comes from the Global Entrepreneurial Monitor report of 2002. The data on wages comes from the ILO (International Labour Office).

Finally, $B$ (size of the bureaucracy as a percentage of the population) is calculated using data from the US Census bureau and includes government employment in the relevant departments (IRS, EPA, etc.) at both the federal and state level excluding departments that are irrelevant to the model (defence, state etc.). First, it is important to note that the total size of the bureaucracy in a country is not a precise measure of what $B$ stands for in the model; the number of bureaucrats who have the authority or the influence/connections to issue permits and licenses, i.e. to deliver upon a match with either an entrepreneur or an intermediary. Second, calibrating $B$ to US data underestimates the size of the bureaucracy in Mexico as overall, the Mexican government is a larger proportion of working age population than that of the US and therefore may be underestimating the effects of the policy variables mentioned above on the size of the corruption market.

Table 1 is a summary of the sources and reported values for each parameter. Table 2
presents the calibrated values for each parameter adjusted for the time period in question.

**Table 1: Parameters and Reported Values**

<table>
<thead>
<tr>
<th>Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>WBI control of corruption index</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Interest rate (OECD)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Tybout &amp; Davis et al.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>GC (price paid for licenses)</td>
</tr>
<tr>
<td>$B$</td>
<td>US Census Bureau</td>
</tr>
<tr>
<td>$\mu$</td>
<td>% of firms claiming to have paid low graft (GC)</td>
</tr>
<tr>
<td>$b$</td>
<td>WB Doing Business Report (% of GDP/capita)</td>
</tr>
<tr>
<td>$E$</td>
<td>GEM report 2002</td>
</tr>
<tr>
<td>$w$</td>
<td>ILO (% of GDP/capita)</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>% of time managers spend on dealing w/ permits (WB ES)</td>
</tr>
</tbody>
</table>

The values of key parameters are given in **Table 2**.

**Table 2: Parameter Values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu$</th>
<th>$\gamma$</th>
<th>$r$</th>
<th>$B$</th>
<th>$E$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>3.85</td>
<td>1.03</td>
<td>0.601</td>
<td>0.0089</td>
<td>0.124</td>
<td>0.0806</td>
</tr>
</tbody>
</table>
Chapter 2

Persistent Inequality, Corruption and Factor Productivity

Abstract:
Empirical studies suggest that inequality and corruption are tightly linked, yet the mechanisms underlying this linkage are imperfectly understood. This chapter uses a model with bequests, financial frictions and corrupt tax-collectors to explain the observed relationship between corruption and inequality, as well as between corruption and productivity differences. Because of financial frictions, entrepreneurial income is determined by wealth holdings. I show that when wealth is private information, taxation is regressive under corrupt regimes. Quantitatively, up to an additional 7% of aggregate income is transferred to the top 1% of the distribution due to corruption. Where wealth distributions are highly unequal, corruption is more prevalent, creating a feedback effect between corruption and inequality. Through regressive taxation, corruption induces wealth levels to inversely affect the productivity threshold entrepreneurs face when deciding whether to operate a project. This in turn induces entry at the low end of the productivity distribution and prevents entry at the high end, which has adverse effects on aggregate TFP. If corruption and inequality are particularly severe, a larger proportion of mid-productivity types end up opting out of entrepreneurship, thus reducing the mass of operating firms in the middle of the size distribution. Financial frictions amplify these effects and go a long way to explain some of the extreme wealth and income inequality that is observed across countries.
2.1 Introduction

For a large fraction of the world’s population, interactions with the government for those who need to pay taxes and/or get a permit to operate can be difficult, especially for individuals with a limited amount of funds.\(^1\) Speed, consistency, and fairness in dealing with government officials are luxuries in most of the developing world, luxuries that are relatively expensive to obtain. Therefore, in their interactions with the state, individuals in corrupt countries do not always have equal access to government services. Entrepreneurs connected with government officials, typically as a result of some form of illicit payment, tend to see real economic returns to their ‘investment’.\(^2\) If we grant the assertion that wealth is an important factor in dealings with corrupt government officials, it then stands to reason that such interactions may play a role in exacerbating inequality and its effects.\(^3\) Figure 2.1 may, at least at first pass, lend credence to the claim that there is a positive correlation between income inequality and corruption.

![Corruption and Gini Index](image.png)

Figure 2.1: Source, WB Governance Index and UNU-WIID

The link between inequality and corruption raises a series of questions. What exactly are the mechanisms that translate inequality into corruption, and vice versa? What effects, if any, do such mechanisms have on aggregate productivity and the much talked about ‘missing middle’ in the size distribution of firms? What are the quantitative predictions on the aggregate variables involved? To answer these questions I construct an overlapping generations model with bequests, in which raising capital is hampered by collateral requirements (financial frictions). The government raises revenue through taxes on entrepreneurial profits, which affect an individual’s decision on whether to become an entrepreneur. Entrepreneurs can use bribes to lower their tax rates. When wealth is

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\(^1\)See for example the World Bank’s *Doing Business Report*.


\(^3\)For the empirical evidence see You and Khagram (2004) and Gupta et. al. (1998).
private information, wealthier entrepreneurs pay lower taxes, which results in *de-facto* regressive taxation. Because taxes act as an entry barrier, wealthier individuals face lower entry thresholds as measured by the productivity level that makes them indifferent between renting out their wealth or using it as entrepreneurial capital. An outcome of the private information model is that, under certain specifications, higher inequality implies a higher frequency of corrupt transactions. In this manner the model can explain some of the persistence in both inequality and corruption as well as provide a link between inequality and productivity outcomes. The quantitative results suggest that corruption can account for a reallocation of up to an additional 7% of aggregate income to the top 1% of the income distribution. It also suggests that the middle of the size distribution of firms is especially thin in more corrupt countries and the lower tail is significantly thicker.

The mechanisms at work can be decomposed in the following steps. First, because of financial frictions, one’s wealth holdings are an important determinant of entrepreneurial income. Second, when wealth holdings are private information, corrupt bureaucrats screen individuals by offering a menu of choices that is composed of an assigned tax rate and the bribe the official asks for it. Individuals then self select by choosing the optimal menu of bribes and tax rates. This menu offers lower tax rates to wealthier individuals because they are, on the margin, more sensitive to taxation and therefore willing to pay heftier bribes. In turn, taxation becomes regressive under corrupt regimes, which increases inequality. Third, because taxation acts as a barrier to entry, and because wealthier individuals face lower tax rates, they face lower entry thresholds, which implies that on average wealthy individuals are more likely to become entrepreneurs. In this way corruption increases inequality through both channels; regressive taxation and the choice of whether to enter entrepreneurship. Fourth, when the distribution of wealth is particularly unequal, bureaucrats have very noisy ‘information’ about where a particular individual stands on the wealth distribution. To optimize, they increase the fraction of the population from which they accept bribes. This increases the number of corrupt transactions and lowers the fees that the upper tail of the wealth distribution has to pay, implicitly allowing wealthier types to keep a larger proportion of their income not only through lower taxation but also through lower bribes. As a result inequality increases, which feeds back to corruption in a vicious loop. Fifth, because of bequests inequality is transmitted across generations, which induces persistence.

\(^4\)The choice of private information with regards to wealth seems to reflect well the facts on the ground. Greece and Italy are two recent egregious examples of hiding wealth to evade taxes.  
\(^5\)On the empirical evidence for this fact see Hunt and Laszlo (2008).
As a check on the validity of the model I study the effect of corruption on the size distribution of firms measured by both effective labour as well as productivity. In the absence of corruption, there is a unique entry threshold on productivity that is determined by market aggregates and taxes. When corruption is present, bureaucrats screen individuals on their wealth by offering contracts that reduce taxes as wealth increases. As previously stated, this presents wealthier types with lower entry (productivity) thresholds and induces them to enter where they would not have in the absence of bribery. Their entrance increases demand for labour, and if the marginal product of labour is sufficiently sensitive to capital (wealth) then this increases aggregate wages. This in effect creates two counterfactual productivity thresholds; one that is lower than the unique threshold faced by all in the absence of bribery, and a much higher threshold faced only by those who are positioned low enough on the wealth distribution to be excluded from paying bribes. Therefore, a large proportion of entrepreneurs in the middle of the productivity distribution do not find it worthwhile to enter production because of their wealth holdings, resulting in the contraction of the middle of the distribution. Bearing in mind the previously mentioned feedback effects of inequality on corruption, we see that inequality also has adverse effects on the productivity distribution of operating firms.

The financial frictions component is essential for these outcomes for two reasons; first it is a factually relevant tool that links wealth with income thus making the wealth transmission mechanism pertinent for the transmission of inequality (on this point see Banerjee and Newman (1993), Cagetti and De Nardi (2006), Buera (2008, 2009), Buera and Shin (2010), Moll (2009, 2010) for a representative sample). Second it has direct effects on the mechanism through which corruption affects inequality. As the quantitative section demonstrates, when financial frictions are particularly severe, the inequality and productivity distortions discussed above are more pronounced. This is because entry thresholds depend on taxation through financial frictions. When collateral requirements are high, profits depend heavily on wealth levels and taxation can significantly lower entry incentives. On the other hand, when collateral requirements are low, the effect of taxation is minimal since capital is less dependent on wealth holdings. In this manner financial frictions compound the negative effects on inequality and the size distribution of firms that result from a corrupt bureaucracy.

The counterfactual experiment in the quantitative section is based on the effects that corruption has on outcomes for a benchmark ‘clean’ economy. I chose Sweden for the benchmark economy as it is the second least corrupt country in the World Bank
Governance Index.\textsuperscript{6} I then increase the level of corruption and record its effect on the steady state measures of the GINI coefficient, the percentage of aggregate income that accrues to the top 1% of the distribution, and productivity. I then vary the financial frictions coefficient to quantify the importance of the channel for the above mentioned outcomes. Through the mechanisms outlined above, corruption has a significant effect on inequality, whether measured by the percentage of income going to the top percentile of the distribution, or by the GINI coefficient.

Previous attempts to explain the feedback link between corruption and inequality include Alesina and Angeletos (2005). In their specification, the government’s goal is to reduce inequality and it does so by redistributing income. If voters are sufficiently concerned with fairness, larger inequality leads to demands for more redistribution. When corruption is prevalent, redistribution ends up favouring the rich, which increases inequality and restarts the loop. It is difficult however to reconcile this account with the fact that most corrupt governments have very low revenue raising capacity (see for example Besley and Persson 2011) and are therefore unable to redistribute income that they are incapable of collecting. This chapter contends that while the channel that links fairness to inequality may be present, the distortionary effect of collecting revenue is the main culprit here. On the other hand Esteban and Ray (2006) caution us that even in cases where officials are honest, lobbying can distort the signals they receive, thus causing inefficient allocation of resources. Environments with high levels of inequality can then increase lobbying intensity and amplify inefficiencies. This chapter departs from Esteban and Ray along two significant dimensions (among many others); first, in this chapter inefficiencies arise because of the way governments extract resources from entrepreneurs, not because of the way resources are allocated. This is an important distinction, because here corruption is essential in understanding how inequality persists whereas, as Esteban and Ray clearly demonstrate, this may not be so in the allocation of resources channel. Second, lobbying as Esteban and Ray define it, is a way to distort the signals government officials get, thereby distorting information about the true state of nature. In this chapter corruption is a direct way individuals employ to change how the rules apply to them, thus resulting in their unfair application.

This chapter is related to a large body of literature along two dimensions. The first links corruption to macroeconomic outcomes, (see Mauro (1995, 1998), Alesina and Angeletos (2005), Ehrlich and Lui (1999), Murphy et. al. (1993) among others). To my knowledge, this chapter is the first to provide a mechanism through which corruption and

\textsuperscript{6}The least corrupt country is Denmark but the differences are trivial.
inequality are inextricably linked without appealing to government expenditure. It also provides an explicit mechanism that links corruption to factor productivity, a channel that has not received sufficient attention. The second strand of literature studies the effect of inequality on growth and development (see Galor and Zeira (1993), Galor and Moav (2004), Persson and Tabellini (1994), Alesina and Rodrik (1994), Galor Zeira and Vollrath (2008) for a representative sample). This chapter’s contribution is to make explicit the distortions that inequality has on the way governments raise taxes and thus exacerbating the persistence of inequality. In this dimension the chapter is more in the spirit of Alesina and Rodrik (1994), where wealth distributions can affect the way individuals interact with the government. I also provide a quantitative framework for analyzing the effects of such frictions (in this case corruption and financial frictions) and do so in a dynamic setting.

For any entrepreneur, interactions with the government are multi-dimensional. They include but are not limited to; taxation, import quotas, and sometimes even the right to bid on government funded projects. All of these interactions require approval from some government agency and are open to undue influence. This chapter reduces all interactions with the government to the single dimension of taxation in order to keep the analysis tractable. It is worth keeping in mind that such analysis will underestimate the effects of corruption in aggregate because of this fact, and that taxation is used as a convenient proxy for interactions with the government.

2.2 A model of Inequality, Corruption and Productivity

2.2.1 The Model Environment

Consider a small open overlapping generations economy populated by a continuum of agents of mass one. Each generation lives for only one period during which it works, produces and consumes. At the end of the period, a new generation of equal mass is born and the old generation dies. At time $t$ each agent is endowed with a productivity level $z_t \in [z, \bar{z}]$ which is distributed according to $F(\cdot)$. Productivity is $iid$ over time.\footnote{Productivity is individual specific and can be thought of as the ability to run the project as well as the quality of the project itself. In this case productivity is being used as a generic term to capture the intangibles of the individual production function.} At $t = 0$, the initial old are endowed with wealth distributed according to $H_0(\cdot)$ with support $[\bar{a}_0, \bar{a}_0]$. I will assume that wealth and productivity are uncorrelated. This assumption is
not crucial to the qualitative outcomes of the model as long as the two are not positively perfectly correlated. In the long run, productivity realizations directly determine wealth and income outcomes, so the model is realistic in that regard.

Each agent also owns a production technology \( f(k, l) = z k^{\alpha} l^{1-\alpha} \) where \( \alpha \in (0, 1) \), \( l \) represents labour and \( k \) capital. The agent borrows (rents) capital at the world rate \( r \) and hires labour at wage \( w \). There are however, limits to how much capital an individual can borrow. This limit is determined by the amount of collateral an individual can offer. More specifically, \( k \leq \lambda a \) where \( \lambda \geq 1 \) and the amount of collateral is an agent’s wealth endowment. The parameter \( \lambda \) is commonly referred to in the literature as the measure of financial frictions and is a convenient way to parametrize the degree of financial market development.\(^8\)

Each individual has a unit of time that they supply inelastically. During its lifetime, each generation earns the market determined wage \( w \), and, depending on whether an individual decides to become an entrepreneur, profit \( \pi(a, z) = z k^{\alpha} l^{1-\alpha} - w l - r(k - a) \). Preferences are given by \( U(c, s) = c^\gamma s^{1-\gamma} \) where \( c \) represents consumption and \( s \) the amount of wealth left as bequests to the next generation. The above utility function implies the following equations for consumption and bequests:

\[
\begin{align*}
    c(a, z) &= \gamma (y(z, a) + a) \\
    s(a, z) &= (1 - \gamma)(y(z, a) + a) \\
    U(a, z) &= \delta (y(z, a) + a)
\end{align*}
\]

where \( y(z, a) \in \{\pi(a, z) + w, w + ra\} \) and \( \delta \equiv \gamma^\gamma (1 - \gamma)^{1-\gamma} \). An individual’s profit maximization problem can be written as:

\[
\max_{k,l} \quad zk^{\alpha} l^{1-\alpha} - wl - r(k - a) \\
\text{s.t.} \quad k \leq \lambda a
\]

The optimization problem above implies the following individual labour and capital demand functions:

\[
\{k(a, z), l(a, z)\} = \begin{cases} \\
\{\lambda a, \left(\frac{2}{\alpha}\right)^{1/(1-\alpha)} z^{1/\alpha} \lambda a\} \quad \text{for } z \geq z_{\min} \\
\{0, 0\} \quad \text{for } z < z_{\min}
\end{cases}
\]

\(^8\)See for example Kehoe and Levine (1993).
where \( z_{\text{min}} = \left( \frac{r}{\eta} \right)^{\alpha} \) and \( \eta = \alpha \left( \frac{1-\alpha}{w} \right) \frac{1-\alpha}{\alpha} \). Note that the productivity threshold is the result of the entry decision, which is made on the basis of \( U^e \geq U^w \) where \( U^e = \delta (\pi(a, z) + a + w) \) and \( U^w = \delta (w + (1 + r)a) \) represent the utility of the entrepreneur and worker respectively. The above solution implies the following profits for an active entrepreneur \((z \geq z_{\text{min}})\):

\[
\pi(a, z) = [\eta z^{1/\alpha} \lambda - r(\lambda - 1)] a
\]

The entry decision can be simplified to the comparison between the rate of return to wealth from entrepreneurship, \([\eta z^{1/\alpha} \lambda - r(\lambda - 1)]\) and the rate of return to savings \(r\).

Now suppose that the entrepreneur has to pay a tax rate on profit \( \tau \in (0, 1) \), so that total after tax profit is \( \pi = (1 - \tau) \{zk^{\alpha}l^{1-\alpha} - wl - r(k - a)\} \). The capital and labour demand functions remain unaltered, however the entry threshold becomes:

\[
z_{\text{min}}(\tau) = \left( \frac{r}{\eta} \left( 1 + \frac{\tau}{\lambda(1-\tau)} \right) \right)^{\alpha}.
\]

Note that \( \frac{\partial z_{\text{min}}}{\partial \tau} > 0 \) so that taxes act as an entry deterrent to potential entrepreneurs. Also, compared to the threshold without taxation, again ignoring aggregates, taxation seems to induce less entrants at first pass. However, the wage effect makes this relationship ambiguous.\(^9\)

The effect of financial frictions here is twofold. The first effect works as in the case without taxes, through the wage equation (see below). However financial frictions exacerbate the taxation effect at the individual level, as is clear in the equation for \( z_{\text{min}} \) above. Taking the limit of \( z_{\text{min}} \) as \( \lambda \to \infty \) we see that this threshold is the same as in the case without taxation \( \left( z_{\text{min}} = \left( \frac{r}{\eta} \right)^{\alpha} \right) \). Given the nonlinearity of the expression above in both \( \lambda \) and \( \tau \), the combined effect of both taxation and financial frictions is especially pronounced at high levels of taxation and financial frictions, which is an empirical reality for much of the developing world. The intuition here is relatively simple. When financial frictions are low (\( \lambda \) is high) the return to the project is high because individuals are not constrained in the amount of capital they can put into the project. In this sense \( \lambda \) is a determinant of the rate of return on capital for entrepreneurs. In this scenario taxes matter less, because they are reducing an already high rate of return. On the other hand, when financial frictions are high (\( \lambda \) is low) the rate of return on the project is very sensitive to taxation.

\(^9\)I will forgo questions of government spending and budget constraints in this environment to focus on the issues at hand. One can imagine that government spending can be used to finance public goods that enter the utility function linearly in some capacity. As will be seen shortly corruption has significant negative effects on the amount of revenue the government can raise and the way it is raised. See Tanzi and Davoodi (1997).
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Aggregation I

The aggregate labour demand equation is:

\[ L^d = \lambda \left( \frac{(1-\alpha)}{w} \right)^{1/\alpha} E(z^{1/\alpha} \mid z > z_{\min}(\tau)) E(a)(1 - F(z_{\min}(\tau))) \]

which, together with a normalized labour supply \( L^s = 1 \) implies:

\[ w = (1 - \alpha) \left( \lambda E(a) \int_{z_{\min}(\tau)}^{\bar{z}} z^{1/\alpha} f(z) \partial z \right)^{\alpha} \]

Note that \( \frac{\partial w}{\partial z_{\min}} < 0 \); a higher entry threshold reduces labour demand at the extensive margin while leaving the intensive margin unaltered.

Each individual’s entry decision depends on the aggregate state through the wage rate \( w \). In order for aggregate outcomes to be consistent with individual decisions, it must be that each individual facing the threshold \( z_{\min} \) results in an aggregate threshold that is consistent. More precisely, the existence of such a threshold requires that the equation

\[ z_{\min} = \varphi(z_{\min}) = \left[ \frac{r}{\alpha} \left( 1 + \frac{\tau}{\lambda(1-\tau)} \right) \left( \lambda E(a) \int_{z_{\min}}^{\bar{z}} z^{1/\alpha} f(z) \partial z \right)^{1-\alpha} \right]^{\alpha} \]

has a fixed point.\(^{10}\) It is relatively straightforward to show that \( \varphi' < 0 \). Taking the limits of the function as \( z_{\min} \) approaches the boundaries, we get

\[ \lim_{z_{\min} \to \bar{z}} \varphi(z) = \left( \frac{r}{\alpha} \left( 1 + \frac{\tau}{\lambda(1-\tau)} \right) \left[ \lambda E(a) E(z^{1/\alpha}) \right]^{1-\alpha} \right)^{\alpha} > 0 \quad \text{and} \quad \lim_{z_{\min} \to z_{\min}} \varphi(z) = 0 \]

which guarantees the existence of a fixed point.

Given the above setup, wealth levels evolve according to:

\[ a_{t+1} = \begin{cases} (1-\gamma)[(1-\tau)\pi(a_t, z_t) + w_t + a_t] & \text{if } z_t \geq z_{\min} \, t \\ (1-\gamma)[w_t + (1+r)a_t] & \text{else} \end{cases} \]

Corruption

Now consider a case in which, prior to making the entry decision and prior to drawing their productivity, entrepreneurs can negotiate with a bureaucrat who can lower their tax rate in exchange for a bribe.\(^{11}\) The bureaucrat is corruptible, and can be paid to assign a lower tax rate to the entrepreneur. There is a cost to the bureaucrat associated

\(^{10}\)The equation is obtained by plugging in for \( \eta(w) \) in the equation for \( z_{\min} \).

\(^{11}\)The intuition here is that entrepreneurs do not know ex-ante how good the project is but want to find out the tax rates they will be charged in order to know whether they should operate or not.
with being corrupt, denoted by $\theta(a, \tau)$, where $a$ is the wealth level of the individual paying the bribe and $\tau$ the tax rate assigned to that individual. One can think of this cost as the expected loss to the bureaucrat in the (random) event that he is caught.\footnote{I will make this more explicit in the quantitative section.} It stands to reason that, all else equal, if higher wealth individuals bribe a bureaucrat, then the loss in revenue to the government is higher, which should lead to an increase in the probability of the bureaucrat getting caught and thus in the total cost to the bureaucrat.\footnote{Note that the cost to the bureaucrat to being caught need not be particularly onerous, it is sufficient that he lose all the proceeds from corruption if caught for the strategic component of what follows to be consistent.} The same holds true for the tax rate that the bureaucrat charges, where the cost of corruption is decreasing in the tax rate charged. I assume that the bureaucrat is bounded below in the amount of tax he can charge by $\tau_l$, and above by the legally mandated, government announced rate of $\phi$ where $\tau_l < \phi$. The lower bound on the tax rate can be thought of as the lowest tax rate at which the bureaucrat is still able to be corrupt with the probability of being caught less than one. Put more succinctly; at any $\tau < \tau_l$, the bureaucrat is caught with probability one, and therefore is not willing to engage in corruption. If the bureaucrat charges the legally mandated rate of $\phi$, there is no corruption and therefore the costs are zero.\footnote{If the bureaucrat charges $\phi$, he is beyond reproach because he followed the law and cannot be accused of malfeasance.} The fall in costs due to increasing the tax rate $\tau$ is larger for wealthier types since the gain in revenues to the government is also larger. The above reasoning sets up the following assumption on the cost $\theta$.

**Assumption 1**: $\theta_a \geq 0$, $\theta_\tau \leq 0$, $\theta(\cdot, \phi) = 0$ and $\theta_\tau a < 0$.

The bureaucrat is unaware of the wealth level of each entrepreneur, but knows the distribution. Denote by $b(a)$ the bribe that the bureaucrat requests from type $a$ and $\tau(a)$ the tax rate that he assigns that type.\footnote{Since this structure sets up a screening problem I will refers to individuals as types, with type implying wealth levels.} Net revenue to the bureaucrat from each type is $R(a) = b(a) - \theta(a, \tau)$. As noted above, tax rates not only affect the net profit of the entrepreneur, they also affect the entry decision. Denote by $q(\tau(a)) = P[z > z_{\min}(\tau(a))]$ the probability that an individual operates given some tax rate $\tau(a)$. The utility of an individual that pays the bribe is:\footnote{Note that the bribe here is paid ex-ante.}

$$U = \begin{cases} 
\delta \left\{ (1 - \tau(a))\tilde{\pi}(z, z_{\min}(\tau))(a - b(a)) + w + (a - b(a)) \right\} & w/p \quad q(\tau(a)) \\
\delta \left\{ (1 + r)(a - b(a)) + w \right\} & w/p \quad 1 - q(\tau(a))
\end{cases}$$
If the individual decides not to participate, the utility is:

$$U = \begin{cases} 
\delta \{ (1 - \phi) \bar{\pi}(z, \min(\phi))a + w + a \} & \text{w/p } q(\phi) \\
\delta \{ (1 + r)a + w \} & \text{w/p } 1 - q(\phi)
\end{cases}$$

where \( \bar{\pi}(z, \min(\tau)) = \lambda \eta E(z^{1/\alpha} | z > \min(\tau(a))) - r(\lambda - 1) \) and \( \tau_l \leq \tau(a) \leq \phi \). Denote by \( G(\tau) \) the function:

$$G(\tau) = q(\tau) \{ (1 - \tau) [\lambda \eta E(z^{1/\alpha} | z > \tau(a))] - r(\lambda - 1) ] + 1 \} + (1 - q(\tau))(1 + r)$$

The dependence of the function \( G \) on \( \tau \) is essential for the results that follow, therefore it is useful to establish the following result:

**Lemma 5.** \( \frac{\partial G}{\partial \tau} < 0 \).

Proof: See Appendix.

The bureaucrat then chooses a series of contracts, where each contract is a tuple, \(< \tau(a), R(a) > \) to maximize expected revenue subject to:

$$G(\tau(a)) [a - R(a) - \theta(a, \tau(a))] \geq G(\phi)a \quad \forall a \quad \text{(IR)}$$

$$G(\tau(a)) (a - R(a) - \theta(a, \tau(a))) \geq G(\tau(a')) (a - R(a') - \theta(a', \tau(a'))) \quad \forall a, a' \quad \text{(IC)}$$

The individual rationality (IR) constraint is merely the participation constraint, where the ex-ante outside option for the entrepreneur is given by the expected utility of facing the tax rate \( \phi \) with wealth holdings \( a \). The incentive compatibility (IC) condition represents the local downward constraint. Note that the cost to the bureaucrat is based on the contract offered, not on the actual wealth level of the individual accepting the contract.

As a way of illustrating the problem’s outcome, consider a sequence of contracts \(< \tau(a), R(a) > \).\(^{18}\) The shaded area in figure 2.2 depicts the set of \( \tau \) and \( R \) that are feasible for type \( h \) given some \( \tau(a^l) \) and \( R(a^l) \) where \( a^h > a^l \). As the figure clearly shows, any contract offered to type \( h \) must lie on or below the payoff curve that goes through \(< \tau(a^l), R(a^l) > \) which implies that the tax rate \( \tau(a) \) is strictly decreasing in wealth. This sets up the following result:

**Proposition 6.** Consider a menu of contracts \(< \tau(a), R(a) > \) that maximizes the bureaucrat’s revenue. Then the following hold:

---

\(^{17}\)Here I am suppressing the dependence of \( G \) on other variables for ease of notation.

\(^{18}\)See appendix for a detailed explanation on the shape of the payoff curves.
a) \( \tau(a) \) is weakly decreasing in \( a \).

b) \( U^*(a) = G(\phi)a \).

c) the optimal menu is: \( \langle \tilde{R}(a^*), \tau_l \rangle \), \( \langle 0, \phi \rangle \) where

\[
a^* = \arg \max_a (1 - H(a)) (\kappa(\tau_l)a - \theta(a, \tau_l))
\]

and \( \tilde{R}(a) = \kappa(\tau_l)a - \theta(a, \tau_l) \).

d) an unique interior solution \( a^* \) exists iff \( \rho(a) < \frac{\kappa(\tau_l) - \theta(a, \tau_l)}{\kappa(\tau_l)a - \theta(a, \tau_l)} \)

where \( \rho(a) = \frac{h(a)}{1 - H(a)} \) is the hazard rate.

e) A mean preserving spread increases \( a^* \) if \( \rho \) is increasing in \( a \).

Proof: See appendix.

---

**A Numerical Example**

To illustrate how inequality affects the menu of choices the bureaucrat offers through the threshold \( a^* \) as well as the entry decision, consider the static case in which the wealth distribution is Pareto with parameter \( \nu \) and the cost function is \( \theta(a, \tau_l) = (\phi - \tau_l)(\tau - \tau_l)^2(\tau a^2 + \mu) \). Suppose that the productivity distribution is normal. The above problem
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has a closed form solution which is described by:

\[
\frac{1}{\nu} = \frac{\kappa(\tau_l) - 2\mu a^* (\phi - \tau_l)^2}{\kappa(\tau_l) a^* - (\phi - \tau_l)^2 (\mu a^2 + \mu)}
\]

I simulate a sequence of mean preserving wealth distributions and plot the dependence of a few key parameters on the dispersion of the wealth distribution, specifically the threshold \(a^*\), the expected productivity threshold and the change in wealth at the top percentile of the distribution. *Figure 2.3* depicts the dependence of \(a^*\) on inequality.\(^{19}\)

As the dispersion of the wealth distribution (inequality) increases, the bureaucrat has a very low quality ‘signal’ as to each individual’s location within that distribution. In order to insure himself against losing a large proportion of bribes, he sets the threshold relatively low. In this way inequality is persistent; in economies where wealth is equally distributed a very small proportion of the population faces ‘favourable’ tax rates, while the larger mass of entrepreneurs pays the same marginal tax rate, thus maintaining a fairly equal distribution of wealth.

![Figure 2.3: A depiction of the wealth threshold as a function of inequality](image)

To further elaborate on this point, *figure 2.4* below plots the relationship between the percent change in the wealth holdings of the top 1 percent of the distribution and \(1/\nu\), again as a proxy for inequality. Those at the top 1 percent of the distribution that do not operate have the same relative increase in income, so the differences depicted in *figure 2.4* are purely due to the initial level of inequality. As inequality increases, as *figure 2.3* makes clear, the wealth threshold that the bureaucrat sets for accepting bribes

\(^{19}\)Each point on the x-axis represents the inverse of the Pareto index \(\nu\).
$a^*$ falls, which then reduces the actual bribe each individual above this threshold pays to the bureaucrat $\kappa(\tau_l)a^*$. Therefore the top percentile of the distribution that actually operates not only gets to keep a larger percentage of their wealth/income because they pay lower taxes; they also have to pay less in bribes because the bureaucrat is insuring himself against setting too high a threshold.

Figure 2.4: % change in wealth at the top 1%

*Figure 2.5 plots the dependence of the expected threshold on inequality. The negative relationship between these variables reflects the fact that as inequality increases and the bureaucrat reduces the wealth threshold, the proportion of entrepreneurs that pay the low tax rate increases, thus reducing the barriers to entry for those that are allowed to bribe. A simplistic version of this argument would have it that in a highly equal society, most entrepreneurs follow the rules and therefore the average entry threshold is higher with more egalitarian distributions.*

**Aggregation II**

It is worth reiterating at this point that entrepreneurs negotiate with the bureaucrat prior to the entry decision, and take aggregates as given. However, forward looking agents have all of the information required to infer aggregate outcomes, in this case the wage rate. Note that a proportion $H(a^*)$ of the population will not pay bribes and will therefore face the legally mandated tax rate of $\phi$. These entrepreneurs enter only if $z \geq z_{\text{min}}(\phi)$, and since $\phi > \tau_l$ their entry threshold is higher than those that bribe the bureaucrat. Individual labour demand remains the same as before, but aggregate demand is given
Direction of increasing inequality (falling pareto index $\nu$)

Expected productivity

Expected productivity and inequality

Figure 2.5: Productivity threshold and inequality

by:

$$L^d = \lambda \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} \left\{ [1 - F(z_{\min}(\tau))] E[z^{1/\alpha}|z > z_{\min}(\tau)] E[a - \kappa(\tau)a^*|a > a^*] [1 - H(a^*)] + 
+E[z^{1/\alpha}|z > z_{\min}(\phi)] E[a|a < a^*] H(a^*) [1 - F(z_{\min}(\phi))] \right\}$$

where the first part of the expression is the aggregate labour demand of those who are above the threshold $a^*$ and are therefore able to lower their tax rates. After some transformation:

$$L^d = \lambda \left( \frac{1 - \alpha}{w} \right)^{1/\alpha} \left[ I(z^{1/\alpha}, z(\tau), \bar{z}) I(a - \kappa(\tau)a^*, a^*, \tilde{a}) + I(z^{1/\alpha}, z(\phi), \tilde{z}) I(a, a^*) \right]$$

where $I(y_1, y_2, y_3) = \int_{y_2}^{y_3} y_1 g(x) \partial x$ and $g(\cdot)$ is the appropriate pdf. The labour demand equation implies the following equation for wages and thresholds in the aggregate:

$$w = (1 - \alpha) [\lambda I(z^{1/\alpha}, z(\tau), \tilde{z}) I(a - \kappa(\tau)a^*, a^*, \tilde{a}) + \lambda I(z^{1/\alpha}, z(\phi), \tilde{z}) I(a, a^*)]^\alpha$$

$$z_{\min}(\tau) = \frac{r\tau}{\lambda(1 - \tau)} \left( \frac{r}{\alpha} \left[ \lambda I(z^{1/\alpha}, z_{\min}(\tau), \tilde{z}) I(a - \kappa(\tau)a^*, a^*, \tilde{a}) + +\lambda I(z^{1/\alpha}, z_{\min}(\phi), \tilde{z}) I(a_{\min}, a^*) \right] \right)^{1-\alpha}$$

$$z_{\min}(\phi) = \frac{r\phi}{\lambda(1 - \phi)} \left( \frac{r}{\alpha} \left[ \lambda I(z^{1/\alpha}, z_{\min}(\tau), \tilde{z}) I(a - \kappa(\tau)a^*, a^*, \tilde{a}) + +\lambda I(z^{1/\alpha}, z_{\min}(\phi), \tilde{z}) I(a_{\min}, a^*) \right] \right)^{1-\alpha}$$
A similar argument to the one made in the case without corruption establishes the existence of a fixed point for both \(z_{\min}(\tau_l)\) and \(z_{\min}(\phi)\). Note that these are the two productivity thresholds potential entrepreneurs face in the presence of corruption. Given that \(z_{\min}(\tau)\) is increasing in \(\tau\), those entrepreneurs who are able to take advantage of the bribing opportunity will face lower (productivity) barriers to entry.

In this environment wealth evolves according to the following equations:

\[
\begin{align*}
  a_{t+1} &= (1 - \gamma) ((1 + r)a_t + w_t) & \text{if } z_t < z_{\min}(\tau_l) \\
  a_{t+1} &= (1 - \gamma) ((1 + r)a_t + w_t) & \text{if } z_{\min}(\tau_l) < z_t < z_{\min}(\phi) \& a_t < a^*_t \\
  a_{t+1} &= (1 - \gamma) (a_t + (1 - \phi)\tilde{\pi}(z, z_{\min}(\phi))a_t + w_t) & \text{if } z_t > z_{\min}(\phi) \& a_t < a^*_t \\
  a_{t+1} &= (1 - \gamma) ((1 - \tau_l)\tilde{\pi}(z, z_{\min}(\tau_l))(a_t - \kappa a_t^*) + w_t) & \text{if } z_t > z_{\min}(\tau_l) \& a_t \geq a^*_t
\end{align*}
\]

where \(\tilde{\pi}(z, z_{\min}(\tau))\) is defined above.

Since labour supply is inelastic, every individual earns labour income \(w\). Those that are below the lowest possible threshold \(z_{\min}(\tau_l)\) will not operate regardless of their standing in the wealth distribution so they earn labour and capital income. This section is described by the first equation above. Those individuals above \(z_{\min}(\tau_l)\) but below \(z_{\min}(\phi)\) will operate if and only if they hold enough wealth to bribe the bureaucrat \((a \geq a^*)\), otherwise they face the legally mandated tax rate \(\phi\) and thus don’t find it worthwhile to become entrepreneurs. These individuals are represented by the second equation above. The third equation describes those that are not able to pay the bribe because they fall below the wealth threshold, but operate nonetheless because their productivity is sufficiently high. The fourth equation describes those that are wealthy enough to pay the bribe and therefore will always face the lowest threshold \(z_{\min}(\tau_l)\).

Distortions coming from corruption here are twofold: first when comparing two economies that differ only in their levels of corruption, the more corrupt economy has higher entry at the lower end of the productivity distribution (individuals described by line four above). Second if corruption is sufficiently high and the marginal product of labour is heavily dependent on capital, a corrupt economy will have inflated wages, which increases the higher productivity threshold \((z_{\min}(\phi))\). This results in those potential entrepreneurs that fall below the wealth threshold \(a^*_t\) to face higher entry barriers where corruption is more severe. This second effect reduces the number of mid productivity types entering entrepreneurship, and if inequality is significant, reduces the mass of entrepreneurs in the middle of the distribution.
2.3 Calibration and Counterfactual Experiments

I consider the effects of corruption on aggregates by calibrating the model to a benchmark economy with no corruption. For the benchmark economy I choose Sweden. There are three main adjustments to the model described above; first, I assume that the productivity parameter $z$ is no longer iid but evolves according to an AR(1) process with persistence parameter $\rho$ and log normal error term.\textsuperscript{20} Second, I assume that individuals are heterogeneous in labour income, and that their labour productivity is perfectly correlated with their general productivity (knowledge) parameter $z$. In the calibrated model the source of heterogeneity is still $z$, but its effect is now spread across two dimensions; the ability to run a project, which was the initial interpretation of $z$ as well as labour productivity. Wages are then quoted in units of effective labour instead of just units of time. Third, I endogenize the lower bound of the tax rate that the bureaucrat charges when he accepts bribes, denoted by the parameter $\tau_l$. The revenue maximization problem for the bureaucrat in this scenario is two dimensional, and corruption levels affect not only the wealth threshold $a^*$ but also the taxation levels that result from bribery.

After having calibrated the benchmark economy, I perform a series of counterfactual experiments. First I increase the level of corruption in the benchmark economy and observe the responses of inequality as measured by the GINI coefficient as well as the percentage of income earned by the top of the distribution. I also report the effect of corruption on the lower bound of the tax rate $\tau_l$ as well as the size distribution of firms according to both effective labour and productivity. I then increase the dispersion of the productivity distribution and observe the effect on the size distribution of firms. As a check on the quantitative significance of financial frictions, I increase financial frictions by reducing the parameter $\lambda$ and document the difference in the responses of the above variables. Finally, I decompose the effect of corruption on inequality to quantify the feedback effect inequality has on corruption as illustrated by the numerical example above.

\textsuperscript{20}This structure has implications for the informational structure of the screening problem outlined above. However, to keep things simple I will assume that there is no retention of information and each generation starts anew, with bureaucrats being born each generation without any knowledge of previous history.
### 2.3.1 Calibration

The value of the calibrated parameters and their sources are given in table 1 below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>persistence of productivity</td>
<td>0.677</td>
<td>De Nardi 2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Björklund/Jäntti 1997</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>within generation prod. variance</td>
<td>0.13</td>
<td>% of entr (GEM)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8.9%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>upper bound on tax rate</td>
<td>37.5%</td>
<td>WB corporate rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>on distributed profits</td>
</tr>
<tr>
<td>$1 - \gamma$</td>
<td>fraction of wealth bequeathed</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt; 0.51</td>
<td>De Nardi 2003</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Laitner Ohlsson 1997</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>coefficient of financial frictions</td>
<td>1.75</td>
<td>Firm leverage</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Song 2005</td>
</tr>
</tbody>
</table>

The persistence of the productivity parameter ($\rho$) is assumed to be the same in Sweden as in the US as in De Nardi (2003). The source of this assumption is Björklund and Jäntti (1997), who do not reject the hypothesis that income mobility in Sweden is the same as in the US. The original source of the value is Zimmerman (1992). There is a large literature that attempts to estimate different variants of the parameter $\rho$. Both Solon (1992) and Zimmerman (1992) for example estimate that the income correlation across generations is at least 0.4, but probably higher. A persistence of 0.677 implies a wage income correlation of 0.6 across generations in this model, which seems to be relatively close to intergenerational estimates of income correlation.

I use the percentage of the population that are entrepreneurs obtained from the Global Entrepreneurship Monitor’s database to target the variance of the productivity distribution $\sigma^2$. The value reported above for Sweden is very close to the value of 0.14 reported in De Nardi (2003). The value of $\phi$ is sourced from the World Bank and represents the profit redistribution tax rate. The parameter $\gamma$ is chosen to target the
ratio of inheritance to wealth in Sweden from Laitner and Ohlsson (1997).\footnote{The ratio of 51\% seems to be a lower bound on the parameter because of issues with data quality.} In the model the only reason for inheritance is the bequest motive, and accidental bequests are not considered. However the motivation for bequests is irrelevant to the outcome of the model; inequality persistence is present whether bequests are accidental or otherwise.

Given that the model assumes a small open economy, an assumption that describes the chosen benchmark economy well, the interest rate is not endogenously determined. To target the financial frictions parameter $\lambda$, I use firm leverage as a percentage of total assets reported in the dataset of 6000 Swedish firms reported by Song (2005). The dataset covers a variety of firms with sales above 10,000 Swedish kroner and therefore firm sizes are well represented. The average leverage ratio for Swedish firms is around 75\%, which implies a $\lambda$ of 1.75 for those firms who operate.

I assume the functional form for the corruption cost function to be $\theta(a, \tau) = (\phi - \tau)^2(v_1a + v_2)$ where $v_2$ is simply a normalization parameter. To estimate this function I suppose that the costs to the bureaucrat are the expected losses from being caught. I then normalize the World Bank Governance Institute’s control of corruption index to use it as a proxy for the probability that a bureaucrat is caught being corrupt. Denote by $\omega$ the probability of being caught, then we have

$$b(a) - \theta(a, \tau_l) = (1 - \omega)b(a)$$

where the right hand side is the expected bribe that the bureaucrat retains. I then use the identity $\frac{\theta(a, \tau_l)}{b(a)} = \omega$ where $b(a) = \kappa(\tau_l)a^*$ to target the parameters of the cost function.

### 2.3.2 Responses to Increased Corruption

Figures 2.6(a-c) below depict the effect of corruption on the changes in the GINI coefficient on wealth, $\tau_l$ and the income that goes to the top 1 percent of the distribution respectively.

For the graphs relating to the GINI coefficient and the share of income that goes to the top 1\% of the distribution, the $y$-axis depicts the differences in these measures between the corrupt economy and the benchmark one, where the level of corruption is measured in the $x$-axis. Figure 2.6 c shows that corruption can account for up to 7\% of aggregate income being reallocated to the top 1 percent of the income distribution. The non-linear relationship depicted in the first figure is also an interesting feature of the outcome, inequality is more sensitive at higher corruption levels, even if the cost
function $\theta$ is linear in wealth. The direct effect here comes from the lower bound on taxes $\tau_l$, as corruption increases and costs to being corrupt (probability of being caught) fall, bureaucrats become more brazen in their promises to lower rates and because the function is quadratic in $\Delta \equiv \phi - \tau_l$, at high corruption levels the lower bound on the tax rate falls significantly. This is made clear in figure 2.6 b, and the response of $\tau_l$ as corruption increases confirms the assertion above. As can be seen from the figure 2.6 a, the GINI coefficient follows the behaviour of the upper tail of the distribution.

The main source of inequality in this setting is at the entrepreneurial level, and over time some entrepreneurs end up accumulating a significant amount of wealth so that inequality is being generated by the upper tail of the distribution. The differences between a corrupt and clean economy depicted in figures 2.6 a-c above are a result of the low taxes the upper tail of the distribution is paying, as well as the fact that low wealth individuals are less likely to operate due to the barriers they face. If we observe how the thresholds $z_{\text{min}}(\phi)$ and $z_{\text{min}}(\tau_l)$ evolve over time we get figure 2.7 below. The dotted red line represents $z_{\text{min}}(\phi)$, the thresholds that those below $a^*$ wealth face in becoming entrepreneurs. The solid blue line is the threshold faced by all in an economy with $\omega = 1$ (probability of being caught). The solid red line represents $z_{\text{min}}(\tau_l)$, the threshold faced by those who end up paying the bribe. From the figure one can see the patterns of entry; those individuals above the solid blue line but below the dotted red line are the
counterfactual mass of entrepreneurs that is missing in the corrupt economies, and, if some of them are unable to pay bribes because of their position in the wealth distribution, they will not enter, thus reducing the mass of firms in the middle of the productivity distribution. At the upper tail of the productivity distribution, (above the dotted red line) there is no difference between the two economies as all enter, but at the lower end, it is clear that the corrupt economy will have a thicker lower tail given that some low productivity, high wealth types will operate projects that would not have been operated in the absence of corruption. Given the above discussion on entry, it is interesting to note the differences in the size of the entrant firms as measured by labour.\footnote{Labour is measured in units of effective time.} This is motivated by the following table, reported in Tybout (1998):\footnote{The last column is sourced from the World Bank and is added to the original table.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{thresholds.png}
\caption{Productivity thresholds}
\end{figure}
Table 2: Distribution of employment shares across plant sizes

<table>
<thead>
<tr>
<th>Country</th>
<th># of workers</th>
<th>1-4</th>
<th>5-9</th>
<th>10-19</th>
<th>20-49</th>
<th>50-99</th>
<th>&gt; 99</th>
<th>Corruption</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>1.3</td>
<td>2.6</td>
<td>4.6</td>
<td>10.4</td>
<td>11.6</td>
<td>69.4</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>13.8</td>
<td>4.5</td>
<td>5.0</td>
<td>8.6</td>
<td>9.0</td>
<td>59.1</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>44.2</td>
<td>17.3</td>
<td></td>
<td>38.5</td>
<td></td>
<td></td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>S. Korea</td>
<td>12</td>
<td></td>
<td>27</td>
<td></td>
<td>61</td>
<td></td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>20</td>
<td></td>
<td>29</td>
<td></td>
<td>51</td>
<td></td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>42</td>
<td>20</td>
<td></td>
<td>38</td>
<td></td>
<td></td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>Tanzania</td>
<td>56</td>
<td>7</td>
<td></td>
<td>37</td>
<td></td>
<td></td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>Ghana</td>
<td>84</td>
<td>1</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>Kenya</td>
<td>49</td>
<td>10</td>
<td></td>
<td>41</td>
<td></td>
<td></td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>90</td>
<td>5</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>77</td>
<td>7</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>Zambia</td>
<td>83</td>
<td>1</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Honduras</td>
<td>68</td>
<td>8</td>
<td></td>
<td>24</td>
<td></td>
<td></td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>58</td>
<td>11</td>
<td></td>
<td>31</td>
<td></td>
<td></td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>Philippines</td>
<td>66</td>
<td>5</td>
<td></td>
<td>29</td>
<td></td>
<td></td>
<td>0.64</td>
<td></td>
</tr>
<tr>
<td>Nigeria</td>
<td>59</td>
<td>26</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>Jamaica</td>
<td>35</td>
<td>16</td>
<td></td>
<td>49</td>
<td></td>
<td></td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>Colombia</td>
<td>52</td>
<td>13</td>
<td></td>
<td>35</td>
<td></td>
<td></td>
<td>0.56</td>
<td></td>
</tr>
</tbody>
</table>

The table makes clear that in highly corrupt countries, the tails at both the top and the bottom of the distribution are thick at the expense of the middle. How does this model fit this empirical fact? Table 3 below is a representation of such differences. While the absolute values for each entry are not essential because they were not specific targets, it is important to note that the entry discussion above is quantitatively important when we consider relative differences. In a corrupt economy the lower end of the distribution
is significantly larger at the expense of firms of medium size.\textsuperscript{24}

TABLE 3: Labour size distribution

<table>
<thead>
<tr>
<th>Labour</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = 1$</td>
<td>28%</td>
<td>60%</td>
<td>12%</td>
</tr>
<tr>
<td>$\omega = 0.4$</td>
<td>45%</td>
<td>42%</td>
<td>13%</td>
</tr>
</tbody>
</table>

As the numerical exercise at the beginning of the chapter made clear, the effects of corruption are more significant when inequality is particularly severe. The productivity distribution for Sweden is highly concentrated around the mean and so the table above underestimates the effect of corruption on some of the economies where inequality in opportunity (productivity) is more pronounced. To quantify the effect consider the table below, where I have increased the variance of the productivity distribution to 0.3:

TABLE 4: Labour size distribution ($\sigma^2 = 0.3$)

<table>
<thead>
<tr>
<th>Labour</th>
<th>Small</th>
<th>Medium</th>
<th>Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = 1$</td>
<td>31%</td>
<td>32%</td>
<td>37%</td>
</tr>
<tr>
<td>$\omega = 0.4$</td>
<td>63%</td>
<td>13%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Whereas in the first instance medium size enterprises in the corrupt economy were at 67\% of the clean one, when we increase $\sigma$ we see that ratio shrink to nearly 40\%, a significant fall given that $\sigma$ is probably still relatively small. Also, the lower tail doubles in size with higher variance.

Financial frictions are essential for the distortionary effect of corruption on the size distribution of firms and aggregate productivity. Without financial frictions, taxation is immaterial in the individual entry decision, and, given that it works through taxation, so is corruption. In this sense, financial frictions attenuate the effect of corruption on inequality and the effect on productivity through the entry decision. In order to quantify these effects I lower the coefficient of financial frictions $\lambda$ and compare the response of the GINI coefficient, the lower bound on the tax rate and the share of income allocated to the top 1\% of the income distribution in the two environments (figures 2.6 d-f). The effect of increasing financial frictions is significant, especially for the lower bound on taxes. In this way financial frictions do not only have first order effects of their own, but they tend to exacerbate the effects of other frictions such as inequality and corruption. Without

\textsuperscript{24}Note that $\omega = 0.4$ maps into a value of 0.6 on the normalized World Bank Governance Index, which is the corruption index of a large proportion of the world’s population.
financial frictions there is very little linking income and wealth, but when the two are linked because of collateral requirements, the effect of other frictions kicks in and can account for a large fraction of income and wealth inequality as well as the disparity in the size distribution of firms.

2.3.3 Decomposing the Effects of Corruption on Inequality

The discussion in the numerical example in section 2.1.4 made the case that the screening mechanism produces feedback effects between corruption and inequality. More specifically, increases in inequality reduce the wealth threshold above which the bureaucrat is willing to accept bribes, thus increasing the frequency of corrupt transactions which further increases inequality. However, the counter factual experiments above do not distinguish between the direct effect of corruption on inequality and this indirect feedback effect. Since this mechanism is important in the process of inequality persistence, it is worthwhile to try to measure it in order to gage its quantitative properties.

In order to achieve this decomposition consider the following thought experiment. Suppose the steady state equilibrium has been computed and we calculate the equilibrium bribe, wealth threshold, \( \tau \) and \( \kappa(\tau) \). From the wealth threshold \( a^* \) it is easy to compute the equilibrium fraction of individuals that actually pay the bribe. The screening outcome predicts that this fraction is dependent on inequality itself and is the channel through which the feedback mechanism amplifies the effects of corruption. Now suppose that we fix this fraction and, instead of letting individuals self-select whether to bribe depending on their wealth levels, we assign each individual a probability that he or she will pay the bribe, where this probability is the fraction of individuals that pay the bribe in the steady state. The amount each individual bribes is fixed to the value calculated in the steady state. In this experiment the wealth threshold is irrelevant because, given it is optimal to do so, an individual’s position in the wealth distribution does not determine whether or not they pay the bribe. Therefore, the only effect of corruption on inequality is direct in this experiment. Figure 2.8 below depicts the effect of corruption on inequality as measured by both the differences in the GINI coefficient between the corrupt and the ‘clean’ benchmark economy and the differences in the amount of income that goes to the top 1% of the wealth distribution.

The solid lines in figure 2.8(a,b) represent the total effect of corruption on inequality as in figure 2.6. The dashed lines represent the outcome of the experiment outlined above. figure 2.8(c) plots the fraction of the difference in inequality that is explained by the feedback effect. At low levels of corruption, the direct effect of corruption on inequality
accounts for up to 60 percent of the total, with the difference explained by the feedback effect of inequality on corruption. At high levels of corruption however, the picture changes, with roughly 80 percent of the total effect explained by the feedback channel. This suggests that the modeling choice here is highly relevant, and that the quantitative effects would be incomplete in any model which ignores these feedback effects. Given that a lot of the inequality differences are generated by differences in wealth at the top 1% of the distribution, high levels of corruption generate larger differences in the wealth held by this group between the corrupt and benchmark economies. Higher levels of inequality then affect the frequency of corrupt transactions through the wealth threshold and, as explained in section 2.1.4, increase the persistence of inequality. In short, because higher levels of corruption produce more inequality, the persistence (feedback) effect is higher where corruption is higher.

2.4 Discussion

I have constructed a model of corruption and inequality with financial frictions in which inequality plays a significant role in the level of corruption and vice versa. Societies with higher concentrations of wealth end up being more corrupt because inequality induces
bureaucrats to charge lower bribes to the higher end of the wealth distribution, but also to allow a larger proportion of the top of the wealth distribution to face lower taxes. This can explain to some extent the observed link between inequality and corruption, and it can do so without appeal to redistribution and government programs. In this environment, when wealth is not publicly observable and bureaucrats are corrupt, it is clear that any form of redistribution is going to be difficult to implement, even the targeted kind. The model seems to fit well the empirical facts regarding the size distribution of firms. More specifically, it goes a long way in explaining the puzzle of the ‘missing middle’ in the size distribution of firms.

This model has significant implications for the provision of the public good. When corruption and inequality are severe, government revenue suffers. Any redistributive scheme that promises government services or simple wealth transfers based on projections of what governments should collect rather than what they do collect is going to come up short. In order to remedy this problem some governments turn to borrowing in the international markets, but most simply reduce the amount of public goods provided. Since government services are often targeted to alleviating poverty and providing better opportunities through education, this hurts the poor disproportionately. Through this channel the model has something to say about the persistence of poverty as well.

The model has no explicit growth components to it, but its implications for growth would be significant. For example the chapter assumes that whether a generation operates or not is irrelevant for the productivity of the next generation. This assumption could be relaxed to suppose that operating has tangible benefits besides the effects of increasing one’s income and wealth. The children of successful parents are more likely to be successful themselves. If we equate success with operating a profit making enterprise, then it is possible that by putting larger hurdles for low wealth individuals to overcome, corruption is lowering growth rates, both for capital and aggregate output. Future work in this direction could reveal some salient features of corruption and its links to growth.
2.5 Appendix

A note on payoff curves

Using the methodology of Maskin and Riley (1984), note that expected utility is given by
\[ U = G(\tau(a)) \left[ a - R(a) - \theta(a, \tau(a)) \right]. \]
Total differentiation gives
\[ dU = \frac{\partial U}{\partial \tau} + \frac{\partial U}{\partial R} \frac{\partial R}{\partial \tau} = 0 \]
to get
\[ \frac{\partial R}{\partial \tau} \bigg|_{dU=0} = -\frac{\partial U}{\partial \tau} \frac{\partial U}{\partial R} = (a - R(a) - \theta(a, \tau)) G' - \theta G. \]
Given that \( \theta(\cdot, \phi) = 0 \) we have \( \frac{\partial R}{\partial \tau} \bigg|_{\tau=\phi} < 0. \) By monotonicity of \( G \) and \( \theta \) we know that \( \frac{\partial R}{\partial \tau} \) is not always positive for all \( \tau \). Figure 2 is an illustration of a possible set of payoff curves for a given type \( a \). It could be that \( \frac{\partial R}{\partial \tau} \) is positive for all of the curves, or for none of them, in this case I have illustrated some that have a positive slope for some interval. Consider a case for a type \( a \). Take two different utility levels \( U_1 \) and \( U_2 \), where \( U_2 > U_1 \). At each tax level, \( R_1 > R_2 \), which implies that the curve for \( U_2 \) lies entirely under that for \( U_1 \) for type \( a \). Furthermore since \( R_1 > R_2 \) then we have that \( \frac{\partial R}{\partial \tau} \bigg|_{\tau=\pi_2} < \frac{\partial R}{\partial \tau} \bigg|_{\tau=\pi_1} \) for all \( \tau \), which gives the curves the shape in the figure below.

![Figure 2.9: Payoff curves for an arbitrary type \( a \). \( U_1 < U_2 < U_3 \)](image)

Denote by \( \tilde{R}(a) \) the revenue level extracted from type \( a \) such that it gives this type the outside option at \( \tau = \tau_l \) so that \( G(\tau_l) \left[ a - \tilde{R}(a) - \theta(a, \tau_l) \right] = G(\phi) a. \) Rearranging we get
\[ \tilde{R}(a) = a \left( 1 - \frac{G(\phi)}{G(\tau_l)} \right) - \theta(a, \tau_l) \]
Given the above discussion, if \( \frac{\partial \tilde{R}}{\partial \tau} \bigg|_{\tau=\tau_l} < 0 \) for some type \( a \), we have that \( \frac{\partial R}{\partial \tau} < 0 \) for all utility levels for that type, i.e. the payoff curves are strictly downward sloping. Note
that $\frac{\partial \tilde{R}}{\partial \tau} |_{\tau = \tau_l} = \left( a - \tilde{R}(a) - \theta(a, \tau_l) \right) G'(\tau_l) = \frac{G(\phi)}{G(\tau_l)} a G'(\tau_l) < 0$ where the second equality comes from the definition of $\tilde{R}$ and the inequality from lemma 5. Since this result is not dependent on $a$, it implies that payoff curves are strictly downward sloping for all types.

Consider a case with two types, $a^h$ and $a^l$ where $a^h > a^l$. The combinations of $R$ and $\tau$ that give the high type the outside option (the outside option curve) lie strictly above those that give the low type the outside option if and only if $\kappa(\tau) > \theta(a^h, \tau) - \theta(a^l, \tau)$ for all $\tau \in [\tau_l, \phi]$ where $\kappa(\tau) = 1 - \frac{G(\phi)}{G(\tau)}$. To see why, note that the curve for type $i$ is described by $R^i = \kappa(\tau)a^i - \theta(a^i, \tau)$. We need $R^h - R^l > 0$, which implies that $\kappa(\tau) > \frac{\theta(a^h, \tau) - \theta(a^l, \tau)}{a^h - a^l}$. Taking limits we get the requirement $\kappa(\tau) > \theta(a^h, \tau)$ which sets up the following assumption:

**Assumption 2**: $\kappa(\tau) > \theta(a^h, \tau)$ for all $\tau$ and $a$.

Figure 2.10 depicts the outside option curves for the lowest and highest type.

![Figure 2.10: Outside option curves for $a^h$ and $a^l$](image)

**Proof of Lemma 5**:

Proof. $G$ can be written as $G(\tau) =$

$$(1 - F(z_{\text{min}}(\tau))) \left[ \eta(1 - \tau) E[z|z > z_{\text{min}}(\tau(a))] - r \right].$$

First we find the conditional distribution of $z$. $\Pr[z < x|z > z_{\text{min}}(\tau)] = \frac{\Pr[z_{\text{min}}(\tau) < z < x]}{(1 - F(z_{\text{min}}(\tau)))} = \frac{F(x) - F(z_{\text{min}}(\tau))}{(1 - F(z_{\text{min}}(\tau)))}$ and the pdf is: $\frac{f(z)}{(1 - F(z_{\text{min}}(\tau)))}$. So that $E[z^i|z > z_{\text{min}}(\tau(a))] = \frac{1}{(1 - F(z_{\text{min}}(\tau)))} \int_{z_{\text{min}}(\tau)}^x z f(z) dz$. Rearranging the terms we get:
\[
G(\zeta_{\min}(\tau)) = \eta(1 - \tau) \int_{\zeta_{\min}(\tau)}^{\zeta} zf(z) \, dz - r + rF(\zeta_{\min}(\tau)).
\]
Taking derivatives we have
\[
\frac{\partial G}{\partial \tau} \int_{\zeta_{\min}(\tau)}^{\zeta} zf(z) \, dz = -\zeta_{\min}(\tau)f(\zeta_{\min}(\tau)) \frac{\partial \zeta_{\min}(\tau)}{\partial \tau} \text{ by the Leibniz rule.}
\]
Proof of proposition 6

Proof. a) The proof is similar to Maskin and Riley (1984).

b) Consider a sequence of wealth types where: \(a < a_2 < \ldots < \bar{a}\). Here we show that \(< \tau(a_1), R(a_1) > \sim_{a_i} \tau(a_{i-1}), R(a_{i-1}) >\). Suppose that this is not true, then we have that \(< \tau(a_1), R(a_1) > \succ_{a_i} \tau(a_{i-1}), R(a_{i-1}) >\). Since \(< \tau(a_{i-1}), R(a_{i-1}) >\) is optimal for type \(i - 1\). This then implies \(< \tau(a_{i-1}), R(a_{i-1}) > \succ_{a_{i-1}} \tau(a_k), R(a_k) >\) for all \(k \leq i - 1\) by the assumption above and the first part of the proof. This then implies;

\(< \tau(a_{i-1}), R(a_{i-1}) > \succ_{a_i} \tau(a_k), R(a_k) >\) for all \(k \leq i - 1\). Now consider a scheme that keeps the same \(\tau(a_i)\) for all \(i\) but increases \(R\) to \(\bar{R} = R + \delta\) for all \(k \geq i\). For \(\delta\) small enough, \(< \tau(a_{i-1}), R(a_{i-1}) > \prec_{a_{i-1}} \tau(a_k), \bar{R}(a_k) >\). Therefore there exists another contract that gives the bureaucrat higher wealth. This is a contradiction. Therefore:

\(< \tau(a_i), R(a_i) > \sim_{a_i} \tau(a_{i-1}), R(a_{i-1}) >\). By this line of argument, the lowest level \(a\) gets no surplus so that \(< \tau(a), R(a) > \sim_{a} < \phi, 0 >\) which gives us \(b\).

c) We need to show that the optimal menu for the bureaucrat to offer is: \(< \hat{R}(a^*), \tau_1 >, < 0, \phi >\) where \(a^* = \arg \max_a (1 - H(a)) (\kappa a - \theta(a, \tau_1))\) and \(\hat{R}(a) = \kappa a - \theta(a, \tau_1)\) is the revenue that gives type \(a\) the outside option at \(\tau = \tau_1\), and \(\kappa(\tau_1) \equiv 1 - \frac{G(\phi)}{G(\tau_1)}\). We do this in three steps:

Step 1: Show that \(\tau \in \{\tau_1, \phi\}\) in any optimal contract. Suppose that there exists an optimal menu that offers a contract \(< R(a), \tau(a) >\) where \(\tau \in (\tau_1, \phi)\) for some types \(a \in [a \bar{a}]\). Consider first the contract offered to type \(\bar{a}\). Given the fact that this is the highest type and increasing revenue does not affect the IC and IR constraints of types lower than \(\bar{a}\), \(\tau(\bar{a}) = \tau_1\). Now consider the contracts offered to the two lower types, \(a_k\) and \(a_{k-1}\). Suppose that \(\tau(a_k), \tau(a_{k-1}) \in (\tau_1, \phi)\). Note that given the result in part a, \(\tau(a_k) \in [\tau_1, \tau(a_{k-1})]\). Consider a small \(\varepsilon\) increase in \(\tau(a_k)\) such that \(\varepsilon \in \epsilon\) where \(\epsilon\) is such that \(\frac{\partial R}{\partial \tau} \bigg|_{d\tau = 0}\) is constant for all \(\varepsilon \in \epsilon\). Since the IR and IC constraints for all \(a < a_k\) are unaffected by this small change in \(\tau(a_k)\), the only revenue that is affected is that of
type $a_k$ and $\bar{a}$. Denote by $\Delta_{a_k}$ the change in revenue for type $a_k$. Given that $\frac{\partial R}{\partial \theta}|_{dU=0} < 0$ we have that $\Delta_{a} > 0$ and $\Delta_{a_k} < 0$. Now since $< R(a), \tau(a) >$ is optimal it must be that this increase in $\tau(a_k)$ results in an aggregate loss in revenue so that $|\Delta_{a} | < |\Delta_{a_k} |$. If that is the case, given that $\Delta_{a}$ is constant and that the indifference curves are strictly downwards sloping, $\exists \varepsilon \in \varepsilon$ such that $|\Delta_{a} | > |\Delta_{a_k} |$ if we reduce $\tau(a_k)$ by $\varepsilon$. Therefore $\tau(a_k) \in \{ \tau_l, \tau(a_{k-1}) \}$. If that is the case, one can make the same argument to show that $\tau(a_{k-1}) \in \{ \tau_l, \tau(a_{k-2}) \}$ and so on until the last type that is included in the contract $\hat{a}$. Since $\hat{a} \bar{a}$ will receive his outside option, it is easy then to see that $\tau(\hat{a}) \in \{ \tau_l, \phi \}$ which implies the desired result.

From now on, when we refer to a contract, it is implied that $\tau = \tau_l$ whenever $R \neq 0$.

**Step 2:** Show that for any optimal contract $R(a) \in [\bar{R}(\bar{a}) \bar{R}(\bar{a})] \forall a \in [a \bar{a}]$ where $\bar{R}(a)$ denotes the revenue that gives type $a$ the outside option at $\tau = \tau_l$. ($\bar{R}(a) = \kappa a - \theta(a, \tau_l)$). Suppose that $R(a) < \bar{R}(a)$ for some $a \bar{a}$, then setting $R(a) = \bar{R}(a)$ would still violate neither IC or IR and increase revenue so that $R(a) < \bar{R}(a)$ is not optimal.

On the other side of the segment, suppose $R(a) > \bar{R}(a)$ for some $a \bar{a}$, then $R(a) = 0$ given that it violates the IR condition for type $a$. By setting $R(a) = \bar{R}(a)$ the bureaucrat can increase revenue so that $R(a) > \bar{R}(a)$ is never optimal.

**Step 3:** Here we finally prove the main result.

Suppose that there exists another menu $< R(a), \tau_l >$ where $a \in A \subseteq [a \bar{a}]$ that maximizes revenue for the bureaucrat, where $R(a) \in [\bar{R}(a_{\min}) \bar{R}(a_{\max})]$ as per step 2 where $a_{\min} = \min A$ and $a_{\max} = \max A$. Also $R(a) \neq R(a^*) \forall a \in A$ where $a^*$ is as defined in the proposition. Denote by $R^i \equiv \min_{\tau_l}(R(a)) \forall a \in A$. Then by the monotonicity of $\theta$ there exists an $\bar{a}$ such that $R^i = \bar{R}(\bar{a})$. Take any $\bar{a} \geq \hat{a}$. The payoff to this type is given by

$$G(\tau_l)(\bar{a} - R(\bar{a}) - \theta(\bar{a})),$$

while the payoff given by $\bar{R}(\bar{a})$ is $G(\tau_l)(\bar{a} - \bar{R}(\bar{a}) - \theta(\bar{a}))$. Since $\bar{a} \leq \hat{a}$ then $\theta(\bar{a}) \leq \theta(\hat{a})$ and $R(\hat{a}) \geq R(\bar{a})$, which implies that $R(\hat{a})$ violates the IC condition for type $\hat{a}$. This in turn implies that the optimal contract choice for type $\hat{a}$ is $\bar{R}(\bar{a})$. Since the choice of $\hat{a}$ was arbitrary, this is true for all $a \geq \bar{a}$. In that case the revenue to the bureaucrat is: $(1 - H(\bar{a}|a_{\min} \leq a \leq a_{\max})) (\kappa \bar{a} - \theta(\bar{a}, \tau_l))$ where $a \in A$. However, since we assumed that $R(a) \neq R(a^*)$ then it is clear that the bureaucrat can increase revenue by offering $R(a^*)$ to all $a \geq a^*$ which proves the result.

**d)** Note that the FOC is: $-h(a)(\kappa(\tau_l)a - \theta(a, \tau_l)) + (1 - H(a))(\kappa(\tau_l) - \theta(a, \tau_l)) = 0$. For an interior solution to exist, given the monotonicity of $\theta$, the FOC must be positive at the lower bound $a_l$ which gives us the desired result. If this condition fails to hold then the solution is $a = a_l$.

**e)** Suppose $\rho$ is increasing in $a$. Consider a mean preserving spread with $cdf \bar{H}(a) = \frac{1}{2}$.
$H(a + \xi)$ where $\xi$ has mean zero. Denote by $\tilde{a} = \arg \max_a \left(1 - \tilde{H}(a)\right) (\kappa a - \theta(a, \tau_l))$. Suppose $\tilde{a} < \hat{a}$, then it must be that $f'(\tilde{a}) = f'(\hat{a})$ given that $f'' = 0$. Also, since $f' > 0$ then $f(\tilde{a}) < f(\hat{a})$. By the FOC, we have that $0 = f'(\tilde{a}) - \rho(\tilde{a}) f(\tilde{a}) = f'(\hat{a}) - \rho(\hat{a}) f(\hat{a}) \Rightarrow \rho(\hat{a}) f(\hat{a}) = \rho(\tilde{a}) f(\tilde{a})$ which implies that $\rho(\hat{a}) > \rho(\tilde{a})$ which is a contradiction. ■
Chapter 3

Reputation in Directed Search Markets with Adverse Selection

Abstract:
This chapter introduces reputation building in directed search with adverse selection. Seller types randomly determine the quality of the asset they hold, where both a seller’s type and asset quality are private information. When sellers are matched with a buyer and an exchange occurs, the quality of the asset that a seller holds reveals information about the seller’s fundamental type, which I refer to as reputation. Markets where sellers have a higher reputation have lower liquidity and higher prices. With reputational concerns, the downward liquidity distortions caused by adverse selection are exacerbated. Equilibrium selection is affected by the incentives sellers have to earn a higher reputation. Shocks to entry costs have larger effects when sellers can build a reputation through multiple matches with buyers.
3.1 Introduction

The recent financial crisis and its effects have focused a lot of attention on the liquidity of financial markets and the effects of market participation on aggregates. In this regard, markets with search frictions and adverse selection can, to some extent, account for distortions in market participation. However, in the literature, information revealed during the course of a match between buyers and sellers does not have any repercussions for future payoffs. The question then naturally arises: how do incentives to hide and/or reveal information about the fundamental features of a market participant affect liquidity and equilibrium selection in directed search markets with adverse selection? To answer this question I construct a directed search model in which sellers build a reputation through their actions as well as the quality of their sales.

In this setting, reputation refers to the belief market participants have about a seller’s underlying fundamental type. More specifically, this underlying type, which endures throughout a seller’s life in the market, randomly determines the quality of the asset that a seller holds. When a seller and a buyer interact and a buyer learns the quality of the asset he has just purchased, he forms a belief about the seller’s type, which I refer to as reputation. In a larger sense, one can think of reputation as a form of rating that outside observers attach to a particular seller given their observations about the quality of the asset that the seller has delivered in a successful transaction. I then enhance the usual definition of a submarket to include reputation so that when buyers post a price they also post the reputation of the sellers they are willing to buy from. In this enhanced version, search is directed not only in the price/liquidity dimension but also on the reputational one. Furthermore, the action of participating in a particular submarket reveals, albeit partially, some information about a seller’s type.

This chapter suggests that markets that attract more reputable sellers are less liquid and, because of the liquidity-price trade-off, more expensive. When a semi-pooling market prevails, sellers with higher reputation hold a higher value asset in expectation. Therefore the price paid in the semi-pooling market is relatively high. Since in the lower separating market, price and tightness are unaffected by reputation, in markets that accept sellers of high reputation, the downward liquidity distortion is higher because of the higher incentives sellers have to imitate up.

The results of the chapter indicate that in the presence of reputational concerns, the downward liquidity distortions caused by adverse selection are amplified. In the absence of reputational concerns, since agents holding a low quality asset tend to imitate up, the market tightness is distorted downward. When sellers are concerned about their
reputation, they have an extra incentive to enter submarkets that deliver a higher reputation. This extra incentive causes the downward distortion to be more severe. It must be noted here that these distortions are present only when semi-pooling equilibria can be constructed. When there is full separation, reputation does not enter the mix since it does not affect the expectations buyers have about the type of asset they will receive in any particular market.

Another interesting implication of this chapter is that timing matters for equilibrium selection; the multiplicity of equilibria means that if, in the future, sellers separate according to the type of asset they hold, semi-pooling equilibria are less likely in the present. This is because reputational concerns increase the incentive that low quality asset holding sellers have to imitate up, which, as explained above, increases the downward distortion in liquidity. Therefore, any shock that increases the equilibrium tightness in the semi-pooling market has to be larger in magnitude to cause an equilibrium shift in the case of reputational concerns. The converse is also true; if a semi-pooling equilibrium prevails for sellers when they are experienced, it is easier to maintain a semi-pooling equilibrium for new sellers.

A shock to the cost of entering the market has a higher negative effect on liquidity when reputational concerns are present. The intuition for this result is fairly clear. In the presence of reputational concerns, a seller holding a low quality asset can increase his reputation by entering the semi-pooling market. In a sense, this seller has an extra incentive to imitate up beyond the usual pricing incentive. When a shock in the cost of entry reduces the mass of buyers across submarkets, since the downward distortion in markets with reputational concerns must be larger in magnitude to account for this additional incentive, the total effect on liquidity is larger. This result suggests that markets where interactions between buyers and sellers reveal some information about the underlying type of a seller are more sensitive to shocks and can shut down easier than in the absence of reputational concerns.

This chapter builds on the directed search literature starting with Peters (1991), Montgomery (1991) and Moen (1997) as well as the more recent literature that investigates the effects of private information on liquidity such as Guerrieri et al. (2010), Guerrieri and Shimer (2011) and Chang (2012). What is novel in this chapter is the consideration of information about seller types being revealed during a successful match. Delacroix and Shi (2012) also build a directed search model in which a seller’s actions send a signal to a buyer about the quality of the good a seller holds. However, in this chapter sellers are ultimately concerned with their reputation and since it is buyers who post prices, seller’s actions only reveal ex-post information about a seller’s fundamental
type. In the presence of semi-pooling equilibria as constructed by Guerrieri and Shimer (2011) as well as Chang (2012), reputational concerns determine outcomes since for some sellers there exist scenarios in which buyers overestimate the probability that a seller is of a higher type. In markets with full separation, these scenarios do not exist since buyers know with certainty the type of good they are to receive.

3.2 Environment

Consider a directed search market with a continuum of buyers and sellers. Time is continuous. Each seller is of type $i \in \{L, H\}$ with probability $\pi$ and $1 - \pi$ respectively. When a seller first enters he draws an asset from the set $S = \{s_1, s_2\}$, where $s_1 < s_2$ and the probability that a type $i$ seller draws $s_2$ is given by $\lambda^i$, where $\lambda^L < \lambda^H$. In other words, high type sellers are more likely to draw the higher value asset. Buyers’ and sellers’ valuation of an asset is the same, but the type of asset a seller is holding and the type of the seller is private information. Each seller is endowed with only two draws from the set $S$, but must sell the first asset in order to be endowed with a second one. After having sold the second time, a seller exits the market and is immediately replaced by a new seller. There is a flow holding cost $c \in \{c_l, c_h\}$ where $c_l < c_h$ to the seller which is independent of a seller’s type. The probability that a seller draws $c_h$ is $\gamma$. Buyers pay a flow cost $k$ to enter the market. Once a buyer exits the market he is immediately replaced by another.

Agents are risk neutral, infinitely lived with a discount rate of $r$. As is standard in directed (competitive) search markets, buyers post a price $p$ and sellers direct their search towards their most preferred submarket. Each submarket is defined by the price $p$ and the tightness (the ratio of buyers to sellers) denoted by $\theta(p)$.\(^1\) In each submarket, matches are bilateral and random. The Poisson rate with which a seller matches with a buyer in submarket $(p, \theta(p))$ is $m(\theta(p))$ and the rate with which a buyer meets a seller is $q(\theta)$. By the definition of the market tightness, $m(\theta)$ is an increasing function of $\theta$. Given the usual assumptions on the matching function, we have that $m(\theta) = \theta q(\theta)$.

\(^1\)This definition will be amended to include reputation in the later parts of the chapter.
3.2.1 Complete Information

Consider the case in which buyers have complete information on the type of asset a seller is holding. Denote by \( V_j(p, \theta) \) the value of a seller holding his \( j \)th asset, where \( j \in \{1, 2\} \). Then the model follows the previous literature where buyers post a price \( p \) having rational expectations about the equilibrium level of tightness \( \theta(p) \) in each submarket. It is important to note here that submarkets are also defined by the length sellers have been in the market. So one can think of this categorization as a simple rule; when buyers post a price \( p \) in a market for new sellers, that implies that only new sellers are allowed to enter. New sellers then determine which submarket to enter by maximizing their expected profit. Given that in the benchmark case there are no informational asymmetries, a seller holding a type \( s \) asset solves the following problem:

\[
\max_{\theta, p} rV_1(s, p, \theta(p)) = s - c_j + m(\theta(p))(p - V_1(s, p, \theta(p)) + E[V_2(s', p', \theta'(p'))])
\]

\[
\max_{\theta, p} rV_2(s, p, \theta(p)) = s - c_j + m(\theta(p))(p - V_2(s, p, \theta(p)))
\]

where \( E[V_2(s', p, \theta(p))|s] \) is the expected value to the seller of receiving the second asset of random quality \( s' \).

On the other side of the market, a buyers' utility is given by:

\[
rU(p, \theta(p)) = -k + q(\theta)(\frac{s}{r} - p - U(p, \theta(p)))
\]

Given that buyers face the free entry condition, in any equilibrium all buyers must be indifferent between entering the market or not, which implies that the price tightness relationship is expressed by:

\[
p = \frac{s}{r} - \frac{k\theta(p)}{m(\theta(p))}
\]

Note that in any equilibrium, the set of prices \( P \) and market tightness \( \Theta \) must be exhaustive, in the sense that any deviation in the posting price \( p' \notin P \) is not profitable for a buyer.

Solving the maximization problem for both sellers holding the first and the second

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\(^2\)Note that in this case information about a seller’s type is irrelevant since that does not affect a buyers payoff.

\(^3\)On this see Burdett, Shi and Wright (2001).
asset we get the first best solution benchmark for the market tightness:

\[
\theta_1^{FB} = \frac{c_j + rE[V_2(s', p', \theta'(p'))]}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]

\[
\theta_2^{FB} = \frac{c_j}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]

The first best solution in both cases is increasing in the ratio of the holding costs \((c/k)\). Also note that due to perfect information, the quality of the asset does not determine market tightness. However, in markets with adverse selection this outcome cannot be sustained because sellers holding low quality assets have an incentive to pretend to be a higher type.

### 3.2.2 Markets with Incomplete Information

Given that sellers’ types determine the quality of the asset that they hold, interactions between buyers and sellers are informative for the buyers. Once a buyer takes possession of an asset, its quality is immediately apparent and if the seller of the asset was new in the market and is poised to receive another asset, the buyer forms an expectation about the type of the seller. Denote this expectation by \(\mu_1\), which represents the conditional probability that a seller is of type \(H\), and by \(\phi_1\) the conditional probability that a seller holds \(s_2\) given \(\mu_1\). The derivation of these probabilities will be discussed more in detail below. I assume that an expectation about a particular seller is public information. More specifically, when a seller that has already sold an asset in the market is searching for buyers, the market participants’ beliefs about his type are common. One can consider this type of belief to be a form of reputation, since a higher \(\mu_1\) implies that the seller has a higher likelihood of holding the higher value asset \(s_2\).

Since the length a seller has been on the market and the market’s belief about that seller type \((\mu_1)\) are common, submarkets here are defined not only by the usual parameters \(\theta\) and \(p\), but also by length in the market and reputation \(\mu_1\). So when a buyer sets up a market by posting a price, he specifies both length in the market and reputation of the type of sellers he wishes to attract. So in its entirety, a submarket is defined by four parameters \((p, \theta(p), \mu_1, j)\) where \(j \in \{0, 1\}\) represents the length that a seller has been in the market.

In this chapter I will concentrate on symmetric stationary equilibria, where sellers face a stationary set of submarkets \((p, \theta(p), \mu_1, j)\) and buyers form an expectation about the conditional distribution of sellers that they attract in any submarket. Since this is an equilibrium with adverse selection, informational asymmetries will determine the types
of sellers a particular submarket attracts which will in turn determine the expectation buyers form on the quality of the asset that will be available in each submarket. The free entry condition then determines the tightness of the submarket $\theta(p)$. Also note that the aggregate distribution of assets in the market does not affect equilibrium market tightness as in Shi (2009). This block recursive property of the equilibrium implies that submarket outcomes depend only on the distribution of the quality of the asset within a submarket rather than the aggregate.

As mentioned earlier, all buyers have a common prior about new sellers, but they form beliefs about sellers that have already sold an asset. As will become clearer in the analysis below, these beliefs are not only dependent on the quality of the asset a seller sold, but also on which submarket a seller chooses to search. Therefore, in order to make the analysis tractable, I will use backward induction and first concentrate on the outcome for sellers that have already sold an asset and have an individual reputation (market belief about their type).

### 3.2.3 Experienced Sellers

Consider the problem of a seller who has already sold an asset and holds a reputation $\mu_1$. A buyer that posts the price $p$ in a submarket open to experienced sellers with a particular reputation $\mu_1$ will in equilibrium post a price that satisfies the free entry constraint. Denote by $y$ a seller’s net valuation of the asset $s_i - c_j$. Then the seller’s IC constraint requires:

$$V_2^*(y, \mu_1) = \max \left\{ \frac{y}{r}, \max \frac{y + p(\hat{y})m(\theta(\hat{y}))}{r + m(\theta(\hat{y}))} \right\}$$  \hspace{1cm} (3.5)

A seller’s net valuation of the asset, $s_i - c_j$ has four possible combinations. Suppose that $s_2 - s_1 > c_h - c_l$ so that the net valuations are ranked as follows: $y_1 = s_1 - c_h < y_2 = s_1 - c_l < y_3 = s_2 - c_h < y_4 = s_2 - c_l$. Then a buyers valuation is non-decreasing over $y$, which implies that there exists no pooling equilibrium.\(^4\)

If however, $s_2 - s_1 < c_h - c_l$,\(^5\) the ranking changes to: $y_1 = s_1 - c_h < y_2 = s_2 - c_h < y_3 = s_1 - c_l < y_4 = s_2 - c_l$ and the buyer’s valuation is non-monotone over $y$. Denote $y_p \equiv s_p - c_h$ and $s_p = (1 - \phi_1)s_1 + \phi_1 s_2$. In this case, an equilibrium in which sellers of valuation $y_2$ and $y_3$ are pooled and $y_1$ and $y_4$ separate can be constructed as follows:

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\(^4\)See GSW and Chang.

\(^5\)In this case sellers that hold the high value asset are not necessarily the ones willing to hold on to it the longest. One can think of this as a sudden need for funds or a correction of expectations about the future dividends of an asset.
1) let $\theta^*(y_1) = \theta^*_{FB}(y_1)$

2) in the pooling equilibrium let $\theta^*(y_2) = \theta^*(y_3) = \theta^*(y_p)$, where $\theta^*(y_p)$ satisfies the IC constraint in 3.5. Note that since the buyers’ valuation over $y_p$ equals $s_p$, the free entry condition is automatically satisfied.

3) for $y_4$ set $\theta^*(y_4)$ to satisfy 3.5.

By construction, $y_p < s_2 - c_h < s_1 - c_t$, and since $\theta^*(y_p)$ satisfies the IC constraint

$$\frac{s_p - c_h + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m(\theta^*(y_p))} \geq \frac{s_p - c_h + \frac{s_2 m(\theta^*(y_4))}{r} - k\theta^*(y_4)}{r + m(\theta^*(y_4))}$$

it does so for both $y_2 = s_2 - c_h$ and $y_3 = s_1 - c_t$. Furthermore, any equilibrium that satisfies the free entry condition and the $IC$ condition for sellers must induce $\theta^*_{FB}(y_1) \geq \theta^*(y_p) \geq \theta^*(y_4)$. To see that this holds in the semi-pooling equilibrium above, consider the first inequality, $\theta^*_{FB}(y_1) \geq \theta^*(y_p)$, where the latter must satisfy:

$$\max_{\theta} \frac{s_p - c_h + \frac{s_p m(\theta)}{r} - k\theta}{r + m(\theta)}$$

s.t

$$\frac{y_1 + \frac{s_1 m(\theta^*_{FB}(y_1))}{r} - k\theta^*_{FB}(y_1)}{r + m(\theta^*_{FB}(y_1))} \geq \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m(\theta^*(y_p))}$$

If the constraint above was slack, then it is easy to see that $\theta^*(y_p) = \theta^*_{FB}(y_1)$ since the solution to $\max_{\theta} \frac{s_p - c_h + \frac{s_p m(\theta)}{r} - k\theta}{r + m(\theta)}$ is the same as the one for $\max_{\theta} \frac{s_1 - c_h + \frac{s_1 m(\theta)}{r} - k\theta}{r + m(\theta)}$.

However, $\theta^*(y_p) = \theta^*_{FB}(y_1)$ would imply that $\frac{y_1 + \frac{s_1 m(\theta^*_{FB}(y_1))}{r} - k\theta^*_{FB}(y_1)}{r + m(\theta^*_{FB}(y_1))} < \frac{y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k\theta^*(y_p)}{r + m(\theta^*(y_p))}$ since $s_p > s_1$, which would violate the constraint. Therefore the constraint must hold with equality in equilibrium. This fact together with the first order conditions of the above problem implies the desired result. The reasoning behind $\theta^*(y_p) \geq \theta^*(y_4)$ is similar.\(^6\)

The behaviour of $\theta^*(y_p)$ with respect to beliefs can be easily deduced from the $IC$ constraint above. Given that the LHS is constant, any increase in $s_p$ will result in a decrease in equilibrium tightness of the semi-pooling submarket since the RHS of the equation must be increasing in $\theta$ at the point of intersection. Since $s_p$ depends positively on $\mu_1$, we have that $\frac{\partial \theta^*(y_p)}{\partial \mu_1} < 0$. Therefore, in semi-pooling submarkets that admit sellers with lower reputation (lower $\mu_1$), it is easier to find buyers for a seller. Note that this effect of reputation on market tightness affects the upper separating market for $y_2$ as well.

\(^6\)Here I have chosen to concentrate on semi-pooling equilibria with downward distortion in tightness.
through the IC constraint. More specifically we can also write that \( \frac{\partial \theta^*(y_2)}{\partial \mu_1} < 0 \). In this manner, even in submarkets where reputation plays no role in determining expectations as to the quality of the asset, it plays a role in liquidity. In fact this characteristic would suggest that markets with very reputable sellers are less liquid and more expensive.

Note that in submarkets one and four, expectations about a seller’s type do not determine the price of the exchange because buyers know the type of asset they are about to receive. In those markets the value to the seller is simply given by \( \frac{y_i + \frac{m}{r} - k \theta^*(y_i)}{r + m (\theta^*(y_i))} \). In the pooling market however, the expected value to the buyer depends on the expectations about a seller’s type through \( \phi_1 \). More specifically we can write \( V_2^* (y, \mu_1) = \frac{y_i + \frac{m}{r} - k \theta^*(y_i)}{r + m (\theta^*(y_i))} \) and by the envelope theorem we have that \( \frac{\partial V_2^* (y, \mu_1)}{\partial \mu_1} > 0 \). Therefore, if \( s_2 - s_1 > c_h - c_l \) and a semi-pooling equilibrium cannot be sustained, there is no value to reputation given that the quality of a seller’s asset will eventually be revealed. If however \( s_2 - s_1 < c_h - c_l \) there exists a semi-pooling equilibrium as described above that induces reputational concerns. As a consequence, the manner in which beliefs are formed determines the expected future payoff of a new seller.

It is important to note that the condition \( s_2 - s_1 < c_h - c_l \) allows us to construct a semi-pooling equilibrium but does not exclude the possibility of a fully separating equilibrium in the market for experienced sellers. It is worth noting here that in a fully separating equilibrium, the value function of a seller does not depend on the reputation parameter \( \mu_1 \). This is due to the fact that in such an equilibrium, buyers expect to attract sellers holding a particular type of asset, which implies that prices are determined with certainty. More specifically, a buyer’s expectations about a seller’s type are irrelevant given that they know precisely the type of asset a seller is holding in any submarket.

### 3.2.4 New Sellers

Now consider the problem of a new seller that is in possession of an asset of quality \( s_i \). Given the non-monotonicity of a buyer’s matching value, a pooling equilibrium can potentially be constructed as above. However, belief updating determines the nature of this equilibrium, so at this point it is necessary to clarify how beliefs depend on the ratios of seller types in each of the submarkets. The belief parameter \( \mu_k^1 \) is the ex-post probability that a particular seller is of type \( H \) given the type of asset he sold as a new seller \( s_i \) and the submarket he participated in \( x_k \), where \( x_1 \) denotes the submarket \((p^{FB}, \theta_1^{FB}, \pi, 1)\), \( x_p \) the pooling submarket and by \( x_2 \) the upper separating submarket.
More formally this can be written as:

\[
\mu^k_1 = \Pr[H|s, x_k] = \frac{\Pr[s, x_k|H] \Pr[H]}{\Pr[s, x_k|H] \Pr[H] + \Pr[s, x_k|L] \Pr[L]}
\]

by Bayesian updating. Rewriting \(\Pr[s, x_k|j]\) as \(\Pr[s, x_k, j]/\Pr[j] = \Pr[s|x_k, j] \Pr[j]/\Pr[j]\) we can rearrange the above expression.

\[
\mu^k_1 = \frac{\Pr[s|x_k, H]}{\Pr[s|x_k, H] + \Pr[s|x_k, L]} = \frac{1}{1 + \frac{\Pr[s|x_k, L]}{\Pr[s|x_k, H]}} \tag{\mu_1}
\]

Note that for any type of seller, as far as actions are concerned, ex-post beliefs depend on the ratio of low types to high types in the submarket they decide to enter. That is to say that when buyers post a price \(p\), their expectations about the tightness of the market that will result in equilibrium will take into account the extra incentive that sellers have to pretend to be higher types. In the market for experienced sellers for example, the trade-off was between the tightness of the market and the price paid, while in the current scenario, low quality asset sellers have incentives beyond this trade-off given that, as shown in the subsection above, future payoffs depend positively on reputation, which is negatively affected by the ratio of low to high types in a given market.

In a pooling equilibrium, the expression \(\frac{\Pr[x_k, L]}{\Pr[x_k, H]}\) for each submarket is:

\[
x_1 : \frac{\pi}{(1-\pi)} \frac{(1-\lambda^L)}{(1-\lambda^H)}
\]

\[
x_p : \frac{\pi}{(1-\pi)} \frac{(1-\lambda^L)(1-\gamma) + \lambda^L\gamma}{(1-\lambda^H)(1-\gamma) + \lambda^H\gamma}
\]

\[
x_2 : \frac{\pi}{(1-\pi)} \lambda^L
\]

It is interesting to note that since \(\lambda^L < \lambda^H\) the ratio of low to high types is so that \(\frac{\Pr[x_1, L]}{\Pr[x_1, H]} > \frac{\Pr[x_p, L]}{\Pr[x_p, H]} > \frac{\Pr[x_2, L]}{\Pr[x_2, H]}\). As a way of an illustration consider \(\Pr[H|s_1, x_1]\) and \(\Pr[H|s_1, x_p]\). Since \(\Pr[s_1|x_1, j] = \Pr[s_1|x_1] = 1\) we have that

\[
\Pr[H|s_1, x_1] = \frac{1}{1 + \frac{\Pr[x_1, L]}{\Pr[x_1, H]}} = \frac{1}{1 + \frac{\pi}{(1-\pi)} \frac{(1-\lambda^L)}{(1-\lambda^H)}}
\]

\[
\Pr[H|s_1, x_p] = \frac{1}{1 + \frac{\pi}{(1-\pi)} \frac{(1-\lambda^L)(1-\gamma) + \lambda^L\gamma}{(1-\lambda^H)(1-\gamma) + \lambda^H\gamma}}
\]
which implies that in a case with no reputational concerns ($\lambda^L = \lambda^H$), updating does not change the initial prior, as expected. When, however, reputational concerns are present, $\Pr[H|s_1, x_p] > \Pr[H|s_1, x_1]$ so that $\mu_1$ is larger if a seller enters the pooling submarket, which illustrates the extra incentive a seller holding an asset of value $s_1$ has to enter it. A similar comparison can be made between $\Pr[H|s_2, x_p]$ and $\Pr[H|s_2, x_2]$.

Having described belief updating, we now consider the pooling equilibrium in submarkets with only new sellers. A pooling submarket with new sellers can be constructed in a similar manner to the pooling equilibrium above, according to the following steps:

1) let $\theta^*(y_1) = \theta^F(y_1)$

2) in the pooling equilibrium let

$$\theta^*(y_p) = \arg \max_\theta \frac{sp - ch + \frac{s_p m(\theta)}{r} - k \theta + m(\theta)E[V_2(s', p', \theta'(p'))]|\mu_p^1]}{r + m(\theta)}$$

s.t

$$y_1 + \frac{s_1 m(\theta^F(y_1))}{r} - k \theta^F(y_1) + m(\theta)E[V_2(s', p', \theta'(p'))]|\mu^1]$$

$$\geq y_1 + \frac{s_p m(\theta^*(y_p))}{r} - k \theta^*(y_p) + m(\theta)E[V_2(s', p', \theta'(p'))]|\mu^p]$$

where $\mu^i$ refers to the ex-post belief on the type of a seller currently participating in market $i$. Since $\Pr[H|s_1, x_p] > \Pr[H|s_1, x_1]$ as argued above, a seller holding $s_1$ has a higher future value if he participates in the pooling market given that $\mu^1 < \mu^p$. In short, participating in the pooling market gives a low quality asset holding seller more “cover”, which may lead buyers to overestimate the likelihood that a seller is of high type. It is also important to emphasize here that both types of sellers have this incentive to try to induce buyers into a higher belief about their type. But, since high type sellers are less likely to be holding on to a low quality asset, on average, it is mostly low quality sellers that are trying to confound buyers.

For ease of notation, denote $E[V_2(s', p', \theta'(p'))]|\mu^1]$ by $T(\mu^i)$. Because in expectation an experienced seller’s value is constant save for the decision as to which market the seller will participate, this notation is particularly convenient.

3) for $y_4$ set $\theta^*(y_4)$ to satisfy a new seller’s version of 3.5:

$$V^*_1(y, \pi) = \max\{\frac{y}{r}, \max_{y_i} \frac{y + p(y_i)\mu(y_i) + m(\theta(y_i))}{r + m(\theta(y_i))}\}$$

The argument as to why such an equilibrium exists are very similar to those made for
constructing the pooling equilibrium above. The same argument applies to showing that
the downward IC constraint binds and that in equilibrium \( \theta^*(y_1) > \theta^*(y_p) > \theta^*(y_4) \), i.e.
that there is downward distortion in the market tightness. However, as argued above, the
IC constraint in the case of new sellers contains an extra term that is action dependent,
which implies that if there is downward distortion in a pooling equilibrium, the degree
of the distortion will depend on reputational concerns. In fact, since sellers’ future value
depends on the type of submarket they will be allowed to enter (as denoted by \( \mu_1 \)),
and this reputation is determined through their interactions as new sellers, the role of
reputation here is crucial. This can be seen from the parameter \( \phi_1 \), which is defined as
\[ \Pr[s_2] \] for an individual with reputation \( \mu_1 \). We know that \( \phi_1 = (1 - \mu_1)\lambda_L + \mu_1\lambda_H \),
and that \( s_p = (1 - \phi_1)s_1 + \phi_1s_2 \), which implies that \( \frac{s_p}{\mu_1} = (\lambda_H - \lambda_L)(s_2 - s_1) \). If high
types are much more likely to receive \( s_2 \) and/or the differences in the buyer’s valuation
increases, the value of having a high reputation for a new seller increases, which induces
more distortion in the market tightness. Therefore it is important to formalize the effect
that reputational concerns have on market liquidity (\( \theta \)).

**Claim 7.** Denote by \( \tilde{\theta}(y_i) \) the equilibrium tightness in a semi-pooling equilibrium where
\( \lambda_L = \lambda_H \). Then \( \tilde{\theta}(y_i) > \theta(y_i) \) for all \( y_i \), where \( \theta(y_i) \) represents equilibrium tightness in
the case of \( \lambda_L < \lambda_H \).

**Proof.** If \( \lambda_L = \lambda_H \), as illustrated above, ex-post beliefs that a seller holding \( s_1 \) is of type
\( H \) are the same as the common prior \( 1 - \pi \). Therefore, the future value to a seller in this
case is \( T(\pi) \) regardless of which market he enters. Rewriting the condition for \( \theta^1_{FB}(y_1) \)
we have:
\[
\tilde{\theta}^1_{FB} : \frac{c_j + rT(\pi)}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]
while in the case of \( \lambda_L < \lambda_H \), this is given by:
\[
\theta^1_{FB} : \frac{c_j + rT(\mu_1)}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]
where \( \mu_1 < 1 - \pi \) and therefore \( T(\mu_1) < T(\pi) \). Given that the RHS of the above
expressions is increasing in \( \theta \) due to the concavity of \( m \), it must be that \( \tilde{\theta}^1_{FB} > \theta^1_{FB} \).
Now consider the tightness of the pooling market. With no reputational concerns, the
IC constraint implies:
\[
y_1 + \frac{s_1m(\tilde{\theta}^1_{FB})}{r} - k\tilde{\theta}^1_{FB} + m\left(\tilde{\theta}^1_{FB}\right)T(\pi) = \frac{y_1 + \frac{s_p^m(\tilde{\theta}^*(y_p))}{r} - k\tilde{\theta}^*(y_p) + m\left(\tilde{\theta}^*(y_p)\right)T(\pi)}{r + m\left(\tilde{\theta}^*(y_p)\right)}
\]
(A)
while in the case of reputational concerns we have:

\[
\frac{y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k \theta_1^{FB} + m(\theta_1^{FB}) T(\mu_1)}{r + m(\theta_1^{FB})} = \frac{y_1 + \frac{s_1 m(\theta_1^{*}(y_p))}{r} - k \theta_1^{*}(y_p) + m(\theta_1^{*}(y_p)) T(\mu_1)}{r + m(\theta_1^{*}(y_p))}
\]

(B)

where \(T(\pi) > T(\mu_1^{p}) > T(\mu_1^{1})\). Given this the RHS of A lies strictly above that of B for all \(\theta\) as depicted in the figure below. Now consider \(\Delta LHS|_{\theta_1^{FB} = \tilde{\theta}_1^{FB}} = \frac{m(\tilde{\theta}_1^{FB})}{r + m(\tilde{\theta}_1^{FB})}(T(\pi) - T(\mu_1))\), which, given the fact that \(\tilde{\theta}_1^{FB} > \theta_1^{FB}\) and that both \(\tilde{\theta}_1^{FB}\) and \(\theta_1^{FB}\) are maximands, is strictly smaller than \(\Delta LHS\). If \(\Delta RHS|_{\theta_1^{*}(y_p) = \tilde{\theta}_1^{*}(y_p)} < \Delta LHS|_{\theta_1^{FB} = \tilde{\theta}_1^{FB}} < \Delta LHS\) then \(\theta_1^{*}(y_p)\) lies strictly to the left of \(\tilde{\theta}_1^{*}(y_p)\) as per the figure below. Given that \(\theta_1^{*}(y_p)\) and \(\tilde{\theta}_1^{*}(y_p)\) are smaller than their respective first best market tightness, the RHS for both A and B is increasing in \(\theta\). Moreover, since \(T(\pi) > T(\mu_1^{p})\) the curves have the shape depicted in the figure below. Now \(\Delta RHS|_{\theta_1^{*}(y_p) = \tilde{\theta}_1^{*}(y_p)} = \frac{m(\tilde{\theta}_1^{FB})}{r + m(\tilde{\theta}_1^{FB})}(T(\pi) - T(\mu_1))\), which, given that \(T(\mu_1^{p}) > T(\mu_1^{1})\) is strictly smaller than \(\frac{m(\tilde{\theta}_1^{FB})}{r + m(\tilde{\theta}_1^{FB})}(T(\pi) - T(\mu_1)) = \Delta LHS|_{\theta_1^{FB} = \tilde{\theta}_1^{FB}}\). \(\blacksquare\)

Figure 3.1: IC Condition

Claim 1 above gets at a fundamental aspect of directed search markets where reputational concerns play a role; namely that the liquidity distortion in such markets is deeper. Although in this section we concentrate on semi-pooling markets with downward distortion, a similar argument can be made when an equilibrium with upward liquidity distortion (as in Chang) can be constructed.

However, the multiplicity of equilibria in markets for both new and experienced sellers raises questions about the relationship between timing and equilibrium selection through
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the reputation channel. As clarified above, if a fully separating equilibrium prevails in markets for experienced sellers, then there is no return to reputation, since payoffs are equal conditional on the type of asset a seller holds. If this is the case, it seems that the incentive sellers have to imitate up due to reputational concerns vanishes, which should in theory make the selection of a semi-pooling equilibrium for new sellers less likely. It also seems intuitive that the converse should hold, so that in the case when a semi-pooling equilibrium prevails in the market for experienced sellers, then it is more likely that a semi-pooling equilibrium prevails in the market for new sellers too. In this way, a seller’s continued market presence coupled with reputational concerns has deep implications for equilibrium selection. This intuition leads to the following result:

Claim 8. Denote by \( \hat{\theta}(y_p) \) and \( \hat{\theta}(y_2) \) the equilibrium tightness in markets for new sellers in the semi-pooling and fully separating cases respectively when there is full separation in markets for experienced sellers and by \( \theta(y_p) \) and \( \theta(y_2) \) when there is a semi-pooling equilibrium in the market for experienced sellers. Then the following must hold:

1. \( \hat{\theta}(y_p) - \hat{\theta}(y_2) < \theta(y_p) - \theta(y_2) \)

2. in the absence of reputational concerns \( \hat{\theta}(y_p) - \hat{\theta}(y_2) = \theta(y_p) - \theta(y_2) \).

Proof. Consider the IC constraints for both cases. When there is a semi-pooling equilibrium in markets for experienced sellers \( P \) describes the downward IC constraint when the semi-pooling equilibrium prevails in markets for new sellers while \( S \) describes the downward IC constraint when the

\[
\begin{align*}
y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m(\theta_1^{FB}) T(\mu_1^1) &= \frac{s_1 m(\theta^*(y_p))}{r} - k\theta^*(y_p) + m(\theta^*(y_p)) T(\mu_1^P) \quad (P) \\
y_1 + \frac{s_1 m(\theta_1^{FB})}{r} - k\theta_1^{FB} + m(\theta_1^{FB}) T(\mu_1^1) &= \frac{s_2 m(\theta^*(y_2))}{r} - k\theta^*(y_2) + m(\theta^*(y_2)) T(\mu_1^2) \quad (S)
\end{align*}
\]

where for now let us suppose that \( T(\mu_1^1) < T(\mu_1^P) < T(\mu_1^2) \).

On the other hand, when there is a separating equilibrium in markets for experienced sellers \( \hat{P} \) describes the downward IC constraint when the semi-pooling equilibrium prevails in markets for new sellers while \( \hat{S} \) describes the downward IC constraint when the
separating equilibrium prevails in these markets:

\[
\frac{y_1 + \frac{s_1 m(\theta_{1FB})}{r} - k_1^{FB} + m(\theta_{1FB}) T(\mu_1^1)}{r + m(\theta_{1FB})} = \frac{y_1 + \frac{s_2 m(\hat{\theta}(y_p))}{r} - k_1^{*} + m(\hat{\theta}(y_p)) T(\mu_1^2)}{r + m(\hat{\theta}(y_p))} \tag{\hat{P}}
\]

\[
\frac{y_1 + \frac{s_1 m(\theta_{1FB})}{r} - k_1^{FB} + m(\theta_{1FB}) T(\mu_1^1)}{r + m(\theta_{1FB})} = \frac{y_1 + \frac{s_2 m(\hat{\theta}(y_2))}{r} - k_1^{*} + m(\hat{\theta}(y_2)) T(\mu_1^2)}{r + m(\hat{\theta}(y_2))} \tag{\hat{S}}
\]

where \(T(\mu_1^1) = T(\mu_1^p) = T(\mu_1^2)\) since there is separation in markets for experienced sellers, and therefore the value of an experienced seller depends only on the quality of the asset he holds.

Consider equations \(\hat{P}\) and \(\hat{S}\). The fact that, both \(\hat{\theta}(y_p)\) and \(\hat{\theta}(y_2)\) are smaller than \(\theta_{1FB}\) implies that \(\frac{\partial \text{RHS}}{\partial \mu}|_{\theta=\hat{\theta}(y_2)} > 0\) and \(\frac{\partial \text{RHS}}{\partial \mu}|_{\theta=\hat{\theta}(y_2)} > 0\) (see figure above). Since the RHS of \(P\) is equal to the RHS of \(S\), and since \(s_2 > s_p = (1 - \phi_1)s_1 + \phi_1 s_2\), it must be that \(\hat{\theta}(y_2) < \hat{\theta}(y_p)\). The same argument can be used to show that \(\theta^*(y_2) < \theta^*(y_p)\). Note however that in the case of \(P\) and \(S\), \(T(\mu_1^p) < T(\mu_1^2)\) while in the case of \(\hat{P}\) and \(\hat{S}\), \(T(\mu_1^p) = T(\mu_1^2)\) which implies that \(\hat{\theta}(y_p) - \hat{\theta}(y_2) < \theta(y_p) - \theta(y_2)\). All we need to show at this point is that \(T(\mu_1^p) < T(\mu_1^2)\) when there is semi-pooling in the market for experienced sellers. This holds iff \(\mu_1^2 > \mu_1^p\). Now \(\mu_1^2\) is given by

\[
\frac{1}{\frac{x}{1 - x} \frac{\lambda_L}{\lambda_H}}
\]

while \(\mu_1^p\) is given by \(\Pr[H|s_1, x_p]\) above. In order to show the desired result it is sufficient to show that

\[
\frac{\lambda_H}{\lambda_L} > \frac{(1 - \lambda_H)(1 - \gamma) + \lambda_H \gamma}{(1 - \lambda_L)(1 - \gamma) + \lambda_L \gamma}
\]

which holds iff \(\lambda_H > \lambda_L\). Part 2 simply follows from the fact that in the absence of reputational concerns \(T(\mu_1^p) = T(\mu_1^2)\).

Given the multiplicity of equilibria in markets for both new and experienced sellers, equilibrium selection depends on the expectations of buyers as to what market tightness will result in equilibrium when a certain price is posted. Part 1 of the claim above highlights the fact that when a fully separating equilibrium occurs in the market for experienced sellers the distance between the semi-pooling and fully separating equilibria in markets for new sellers gets smaller as measured by the difference between the respective equilibrium tightness. This is due to the fact that a fully separating equilibrium for expe-
rienced sellers makes the action of choosing a submarket to participate in for new sellers irrelevant to future value. This is because the outcome for experienced sellers depends only on the quality of the asset a seller holds. On the other hand, when a semi-pooling equilibrium prevails in the market for experienced sellers, there is added incentive for new sellers to gain in reputation, which increases their incentive to pool, thus making the selection of a pooling equilibrium in the market for new sellers more likely. In fact, the claim above tells us that the deviation needed to jump from a semi-pooling to a separating equilibrium in the market for new sellers is smaller when there is full separation in markets for experienced sellers. More specifically, if there is a semi-pooling equilibrium in the market for experienced sellers, any shock that reduces the semi-pooling equilibrium tightness $\theta(y_p)$ in the market for new sellers has to be larger to induce an equilibrium switch. In this way, equilibrium selection in one submarket (experienced sellers) determines equilibrium selection for another submarket (new sellers). Part 2 of the claim simply establishes the fact that when there are no reputational concerns, any incentive that sellers have to pool vanishes, which makes equilibrium selection in the market for new sellers independent of the outcome in the market for experienced sellers.

It is important to reiterate here that the presence of reputational concerns exacerbates the incentive that sellers holding $s_1$ have to enter the pooling submarket. Therefore, intuitively, reputational concerns can amplify the effects of shocks as measured by the response of market tightness. To be more specific, consider an increase in the entry cost $k$. This reduces the probability that a seller will find a buyer in any submarket. However, because with reputational concerns sellers holding $s_1$ have an extra incentive to enter the pooling submarket, the fall in the tightness of the latter must compensate for this. This intuition is represented by the claim below:

\begin{claim}
Denote by $\theta(y_p)$ and $\tilde{\theta}(y_p)$ the equilibrium market tightness in the pooling submarket in the case of $\lambda^L < \lambda^H$ and $\lambda^L = \lambda^H$ respectively. Suppose there is a one time increase in $k$. Then $\Delta \theta(y_p) < \Delta \tilde{\theta}(y_p) < 0$.

\end{claim}

\begin{proof}
First, it is useful to show that the response of $\theta_{1}^{FB}$ is larger in magnitude than that of $\tilde{\theta}_{1}^{FB}$, where $\tilde{\theta}_{1}^{FB}$ represents the first best tightness for the case where $\lambda^L = \lambda^H$. Note that we have already shown that $\theta_{1}^{FB} < \tilde{\theta}_{1}^{FB}$. Consider the expression for $\theta_{1}^{FB}$.

\[
\theta_{1}^{FB} : \frac{c_j + rT(\mu_1)}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]

\[
\tilde{\theta}_{1}^{FB} : \frac{c_j + rT(\pi)}{k} = \frac{r + m(\theta) - \theta m'}{m'}
\]
Taking derivatives with respect to $k$ we have:

$$\frac{\partial \theta^F}{\partial k} : \frac{c_j + rT(\mu_1)}{k^2} = \frac{\partial \theta}{\partial k} \left( \frac{(r + m(\theta)) m''}{(m')^2} \right)$$

$$\hat{\theta}^F : \frac{c_j + rT(\pi)}{k^2} = \frac{\partial \theta}{\partial k} \left( \frac{(r + m(\theta)) m''}{(m')^2} \right)$$

Given that $T(\pi) > T(\mu_1)$ as shown above and the concavity of $m$ we have that $0 > \frac{\partial \hat{\theta}^F}{\partial k} > \frac{\partial \theta^F}{\partial k}$. Now consider the $IC$ constraints for both cases for the submarket for new sellers:

$$f(\theta^F, \mu_1) = y_1 + \frac{sp(y_p)}{r} - k\theta^*(y_p) + m(\theta^*(y_p)) T(\mu_1)$$

$$f(\hat{\theta}^F, \pi) = y_1 + \frac{sp(y_p)}{r} - k\hat{\theta}^*(y_p) + m(\hat{\theta}^*(y_p)) T(\pi)$$

where

$$f(x, y) = \frac{y_1 + \frac{sp(x)}{r} - kx + m(x) T(y)}{r + m(x)}$$

Due to the above result, we know that the $\Delta f(\theta^F, \mu_1) < \Delta f(\hat{\theta}^F, \mu_1) < 0$. We also know that at any $\theta$, the derivative with respect to $\theta$ for the RHS of $C$ is strictly smaller than that of $D$ (see figure above). Given that the change in the left hand side is larger in magnitude for $C$ then implies the desired result.

The intuition for a larger fall in liquidity in the pooling submarket in the case of reputational concerns was given above. What is as interesting is the fact that the effect is not isolated to the semi-pooling market tightness. The first best tightness falls more in the case with reputational concerns because the costs to waiting are lower for those sellers whose reputation is relatively low ($\mu_1 < \pi \Rightarrow T(\pi) > T(\mu_1)$). In that case, individuals are willing to wait longer to meet a buyer, causing the larger drop in liquidity. In this way, reputational incentives result in higher sensitivity of liquidity to cost shocks.

### 3.3 Conclusion

In this chapter I have constructed a model with adverse selection in markets with directed search. I find that reputational concerns deepen the impact that adverse selection has on market liquidity as measured by the ratio of buyers to sellers. Furthermore, the
results suggests that equilibrium selection is significantly affected by the reputational mechanism, with the type of equilibrium that prevails in the second period of a seller’s life determining the type of equilibrium that occurs when the seller is a new entrant. More specifically, if a fully separating equilibrium is the outcome in the later stages of a seller’s career, then a semi-pooling equilibrium is harder to maintain when the seller is new. The converse seems to also be true. Moreover, shocks that reduce market participation seem to have larger effects when sellers have reputational concerns.

The modeling choices of the chapter in terms of the types of sellers and quality of assets available in the market are intentionally limited to highlight relevant features of the results. However, any extension of the model that utilized multiple types on both dimensions could be achieved without substantial change to the main results of the chapter. Where the difficulty lies however, is in the extension of a seller’s life to more than two periods. This approach is more difficult to undertake due to the large set of informational possibilities that would result in such a scenario. Nonetheless, further work in this direction could prove fruitful and helpful in understanding market participation in this environment.
Bibliography


