ADAPTIVE SENSING STRATEGIES FOR OPPORTUNISTIC SPECTRUM ACCESS

by

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Graduate Department of Electrical and Computer Engineering
University of Toronto

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Abstract

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To meet the ever increasing spectrum demand, developing a mechanism for dynamic spectrum access seems inevitable. Spectrum sensing enables cognitive radios (CRs) to identify and use frequency bands (channels) that are not being used by primary users (PUs) at a particular place and time. However, sensing errors and limited sensing resources, such as sensing hardware and sensing time, introduce significant technical challenges to the development of such an ideal capability. Adaptive sensing strategies allow the sensing resources to be spent on more promising primary channels. This is achieved by exploiting past sensing outcomes of one secondary user (SU), or, as proposed in this research, multiple spatially distributed SUs. We propose adaptive sensing strategies for three different scenarios. First, we assume that a SU sequentially senses a number of primary channels to find the first available channel. We propose a two-stage spectrum detection strategy that allows the spectrum detector to quickly detect and skip through most of busy channels and spend most of its time on channels that are more likely to be idle. Second, we consider the case where multiple SUs jointly try to locate idle channels within a given sensing time, which itself is divided into a number of sensing slots. We propose a cooperative spectrum search strategy that specifies the channel to be sensed by each SU in each slot in such a way to maximize the expected number of identified idle channels. Third, we consider a primary network that operates in a synchronous time-framed fashion. We assume that the occupancy state of each primary channel over
different time frames follows a discrete-time Markov process. We propose a cooperative sensing strategy that decides which channel should be sensed by which SU in each frame. The goal is to maximize a utility function that accounts for both the number of detected idle channel-frames and the number of miss-detected busy channel-frames. We present analytical and numerical results to demonstrate the effectiveness of the proposed sensing strategies in increasing identified time-frequency spectrum opportunities and/or reducing interference with licensed systems.
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<th>Description</th>
<th>Page #</th>
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<tbody>
<tr>
<td>ARL</td>
<td>Average Run Length</td>
<td>41</td>
</tr>
<tr>
<td>ATSC</td>
<td>Advanced Television Systems Committee</td>
<td>28</td>
</tr>
<tr>
<td>BPF</td>
<td>Bandpass Filter</td>
<td>6</td>
</tr>
<tr>
<td>CCC</td>
<td>Common Control Channel</td>
<td>32</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive Radio</td>
<td>2</td>
</tr>
<tr>
<td>CRN</td>
<td>Cognitive Radio Network</td>
<td>2</td>
</tr>
<tr>
<td>EGC</td>
<td>Equal-gain Combining</td>
<td>29</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
<td>1</td>
</tr>
<tr>
<td>MAC</td>
<td>Medium Access Control</td>
<td>13</td>
</tr>
<tr>
<td>MOSS</td>
<td>Multi-operator Spectrum Server</td>
<td>32</td>
</tr>
<tr>
<td>NBP</td>
<td>National Broadband Plan</td>
<td>2</td>
</tr>
<tr>
<td>OSA</td>
<td>Opportunistic Spectrum Access</td>
<td>1</td>
</tr>
<tr>
<td>POMDP</td>
<td>Partially Observable Markov Decision Process</td>
<td>14</td>
</tr>
<tr>
<td>PU</td>
<td>Primary User</td>
<td>2</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
<td>34</td>
</tr>
<tr>
<td>ROC</td>
<td>Receiver Operating Characteristic</td>
<td>20</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal-to-Interference Ratio</td>
<td>42</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
<td>4</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary User</td>
<td>2</td>
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## List of Important Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Duration of a time frame</td>
</tr>
<tr>
<td>$c$</td>
<td>Number of primary channels</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of cooperating SUs</td>
</tr>
<tr>
<td>$P_f$</td>
<td>False-alarm probability</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Miss probability</td>
</tr>
<tr>
<td>$u$</td>
<td>Primary utilization factor</td>
</tr>
<tr>
<td>$N_{\text{search}}$</td>
<td>Average length of the search interval in Chapter 3</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of sensing slots in Chapter 4</td>
</tr>
<tr>
<td>${\delta_L}$</td>
<td>A spectrum sensing strategy in Chapter 4</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Density of primary transmitters in the network in Chapter 4</td>
</tr>
<tr>
<td>$N_I$</td>
<td>Number of detected idle channels in Chapter 4</td>
</tr>
<tr>
<td>$g_L$</td>
<td>Relative gain of the optimal sensing strategy in Chapter 4</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of time frames in Chapter 5</td>
</tr>
<tr>
<td>${\delta_k}$</td>
<td>A spectrum sensing strategy in Chapter 5</td>
</tr>
<tr>
<td>$P_{m,H}$</td>
<td>Average miss probability under channel state $H$ in Chapter 5</td>
</tr>
<tr>
<td>$P_{m,L}$</td>
<td>Average miss probability under channel state $L$ in Chapter 5</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Strategy design parameter in Chapter 5</td>
</tr>
<tr>
<td>$N_D$</td>
<td>Total number of detected idle channel-frames normalized by $nK$ in Chapter 5</td>
</tr>
<tr>
<td>$N_M$</td>
<td>Total number of miss-detected busy channels-frames normalized by $nK$ in Chapter 5</td>
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Chapter 1

Introduction

1.1 Cognitive Radio Technology

Today’s wireless systems are characterized by static spectrum allocation strategies in which each system is licensed to operate over a specific frequency band in a given geographic area. However, the spectrum may be used only over very short intervals and around a transmitter-receiver pair that are very close to each other. As a result, most frequency bands in the majority of geographical areas are not efficiently used.

While the Federal Communications Commission’s (FCC) frequency allocation chart indicates that we are running out of the usable spectrum, recent measurements reveal that the licensed spectrum is significantly underutilized [2, 7]. In [37], the authors study how to make efficient use of the spectrum freed up by digital switchover available for new uses in the UK. They scrutinize the virtues of both of allowing Opportunistic Spectrum Access (OSA) and of a dedicated license-exempt allocation. They concluded that it would be difficult to reserve anywhere near as much as 100 MHz for license-exempt use without excessively reducing the amount of spectrum available for licensed uses. Therefore, new devices should be able to make use of the licensed spectrum in an interleaved manner without causing harmful interference to licensed users. Based on these observations and
in order to meet the ever increasing spectrum demand, Cognitive Radio (CR) technology has been proposed in the literature [4, 22, 23, 34, 47, 64].

The IEEE 802.22 standard, the first wireless air interface standard based on CR technology, was published in July 2011 [1]. The IEEE Dynamic Spectrum Access Networks Standards Committee (DySPAN-SC), which replaced the IEEE P1900 Standards Committee, is in charge of standard projects related to dynamic spectrum access and CR-based networks [55]. The National Broadband Plan (NBP) is a policy document, prepared by the FCC based on the recommendations from industry and government, on how to amend spectrum policies to meet the demands of future broadband applications. This document promotes the opportunistic use of the frequency bands other than the TV white spaces. An overview of the vast array of potential applications of the CR technology in cellular networks, smart grid networks, wireless medical networks, and public safety networks is provided in [60].

CRs try to exploit frequency bands which are not being used by Primary Users (PUs) at a particular location and time. Since they are usually considered lower priority or Secondary Users (SUs), they must ensure that no harmful interference is caused to PUs. Therefore, spectrum opportunities in frequency and time should be reliably detected. A database registry, broadcasting beacon signals, and spectrum sensing are three different approaches that may be used to achieve this goal.

An important question to be answered in the area of Cognitive Radio Networks (CRNs), is how to define an available channel or a spectrum opportunity. In current wireless systems with static spectrum allocation strategies, a physical communication channel can be used by a mobile terminal for data transmission only if it is not assigned to other users in the system, e.g., other mobile terminals within the same cell in a cellular system. This notion of a spectrum opportunity lies at the heart of spectrum underutilization in legacy wireless systems. At the other extreme is the definition given by Ren et al. in [44]. According to [44], a spectrum opportunity for a particular secondary transmitter-
receiver pair is a channel over which they can successfully communicate without violating interference constraints. Therefore, a spectrum opportunity exists when the reception at the secondary receiver is successful, and the transmission by the secondary transmitter is harmless to the primary receivers. However, due to the complicated nature of the wireless medium and unpredictable propagation characteristics, such spectrum opportunities cannot always be identified in a reliable manner. This trade-off is discussed in more detail in Chapter 2.

1.1.1 Spectrum Sensing for Opportunistic Spectrum Access

The idea of spectrum sensing for OSA is to close the gap between the above two definitions for a spectrum opportunity. This is achieved by introducing some level of cognition into wireless devices which enables them to sense a particular band to discover idle frequency bands for opportunistic access. The best way to detect spectrum opportunities is to detect primary receivers within the communication range of a SU. However, this is possible only if the primary receivers cooperate by transmitting a pilot signal. Therefore, most research in the area of spectrum sensing for OSA has focused on the detection of primary transmitters [4, 9]. This approach is based on the detection of signals from the nearby primary transmitters through measurements of SUs. Matched filter detection, energy detection, and cyclostationary feature detection are three major proposed schemes for primary transmitter detection [45].

Ideally, the spectrum sensing capability should enable the SUs to detect spectrum opportunities, as defined in [44], in an error-free and delay-free manner. This corresponds to full protection of the PUs against harmful interference with the SUs and, at the same time, maximum use of the licensed spectrum by the SUs. However, as discussed in the next section, there are several technical challenges that impede the realization of such a perfect capability.
1.2 Key Technical Challenges

1.2.1 Sensing Errors

In many practical settings, there is a significant portion of the licensed spectrum that can be used by the SUs, provided that they guarantee noninterference to the primary network. However, due to shadowing and multipath fading effects, the secondary system may not be able to reliably distinguish between a white space and a very weak primary signal. The hidden terminal problem is an important issue in spectrum sensing that arises because of shadowing or fading (see Figure 1.1).

A false-alarm occurs when an idle primary channel is overlooked by a SU’s spectrum detector. Similarly, a miss occurs if a primary channel is mistakenly identified as idle by the detector. The detection sensitivity, the miss probability, and the false-alarm probability are the three metrics commonly used in the literature to measure the performance of different spectrum detection schemes. The first two are assumed to be determined by the regulatory bodies, while the third measures the secondary system’s performance in detecting spectrum opportunities. An overview of different spectrum sensing schemes proposed for CRNs, along with their strengths and weaknesses, is provided in Chapter 2.

While theoretically the detection performance can be improved at the expense of a longer sensing time, model uncertainties place fundamental bounds on the performance of signal detection in the low Signal-to-Noise Ratio (SNR) regime [51]. For example, as shown in [51], an energy detector cannot reliably detect the presence or absence of a signal if the signal power is less than the uncertainty in the noise power, no matter how many samples are taken. The SNR wall for a particular detector and an amount of uncertainty in the noise power is defined as the SNR threshold below which the detector fails to reliably detect the primary signals. As discussed in [44] and also in Section 2.1, detection errors are inevitable even in the absence of noise and multipath fading. As
A CR user cannot detect the presence of primary transmission. Harmful interference to a primary user.

Figure 1.1: The hidden terminal problem in transmitter detection schemes.

shown in [44], and an ideal spectrum detector is not sufficient to make the miss and false alarm probabilities arbitrarily small unless the ratio between the transmit powers of SUs and PUs is very small or very large.

1.2.2 Limited Sensing Resources

- **Hardware Limitations:** CRs should be able to sense a wide range of frequencies (sometimes a multi-gigahertz wide bandwidth [9]) and search for idle bands. One approach to wideband spectrum sensing is to implement a wideband filter followed by a high rate A/D converter to process the full range of the available spectrum at once. The wideband signal received by the antenna of a CR is a superposition of primary signals received over different frequency bands. Many factors such as location of the PUs as well as their operating power levels affect the signal strength in each frequency band. As a result, wideband spectrum sensing involves detection
of extremely weak signals from widely separated or severely shadowed transmitters in the presence of strong signals from nearby transmitters [9]. Consequently, stringent constraints are imposed on the linearity of the RF amplifier as well as the sampling rate and the accuracy of the A/D converter [9]. A more practical approach is to employ a tunable Bandpass Filter (BPF) to sense one channel at a time. This approach to wideband spectrum sensing results in multi-channel opportunistic spectrum access scenarios which are the focus of interest in this thesis. In these scenarios, a sensing strategy determines which channel should be sensed at a given time.

- **Sensing Overhead:** During opportunistic access to the licensed band, SUs should sense the spectrum every $T$ seconds in order to detect any reappeared primary transmissions. The choice of $T$ depends on the nature of the primary service and is set by the regulatory bodies. If a primary transmission is detected on a primary channel, the SU should immediately cease transmission on that particular channel. As shown in Figure 1.2, each time frame of duration $T$ starts with a sensing interval in which SUs stay silent and sense the licensed band. During this interval, SUs may sense a particular primary channel or search over multiple primary channels to detect spectrum opportunities. For most spectrum sensing schemes, increasing the sensing time results in a lower false-alarm probability, and hence, more spectrum opportunities can be detected by the SUs. Furthermore, a longer sensing interval allows a SU to search over more primary channels. However, a longer sensing interval leaves less time for the SUs to communicate and consequently reduces the throughput.
1.3 Motivation and Related Work

Current research in the area of spectrum sensing for CR is aimed at improving the SUs’ experience by enhancing their opportunistic use of the spectrum over both time and frequency, as depicted in Figure 1.3. A graphical overview, from the author’s perspective, of the research objective, key technical challenges, and main research directions in this area is shown in Figure 1.4. The figure also illustrates how different technical challenges are addressed by the proposed solutions in the literature. Related works in this field are mainly directed towards developing high performance spectrum detectors, introducing adaptive sensing strategies, and enhancing cooperative spectrum sensing schemes. While better spectrum detectors allow for reduced error probabilities and/or shorter sensing times, adaptive sensing strategies allow the sensing resources to be directed towards more promising primary channels. In this thesis, we focus on developing adaptive sensing strategies; more specifically, we introduce cooperative sensing strategies (Chapters 4 and 5), as shown in Figure 1.4.

1.3.1 Cooperative Spectrum Sensing

When a SU experiences severe shadowing or multipath fading, it cannot reliably detect the presence of nearby primary transmitters. To address this problem, cooperative spec-
spectrum sensing schemes are proposed in the literature. In these schemes, every SU performs its local spectrum sensing and reports the result to a common receiver, which can be one of the SUs. These results are then fused together to make the final decision about the absence or presence of PUs. As discussed in [5], such schemes are more useful when the distance between any two cooperating SUs is much smaller than the distance between the primary transmitter and any of the SUs. This is the case when the ratio between the transmit powers of SUs and PUs is very small, which is one of the extreme cases discussed in Section 2.1; consequently, the secondary system is small enough so that the distance attenuation is assumed to be the same for all the SUs.

One of the first works is that of Ghasemi and Sousa [18], who study the performance of cooperative spectrum sensing in fading environments. They consider a scenario where $n$ SUs cooperate in spectrum sensing by sharing only their final 1-bit decisions “0” (idle) or “1” (busy). They assume that all the SUs experience independent and identically distributed (i.i.d.) fading/shadowing, and all the users employ energy-detection with
Figure 1.4: Spectrum sensing for CR: a high-level overview of the research objective, key technical challenges, main research directions, and major contributions made in this research.

The same decision threshold. A SU receives decisions from \( n - 1 \) other users and fuses them together using the OR-rule or 1-out-of-\( n \) rule. Therefore, the final decision is “1” if any of the \( n \) individual decisions is “1”. It is shown in [18] that the proposed cooperative spectrum sensing scheme significantly increases the reliability of detection in these environments. From another perspective, cooperative spectrum sensing can be exploited to reduce the sensitivity requirements of each SU rather than improving the detection performance [33]. In fact, cooperation among SUs may be the only way to achieve a target system-level probability of detection in the case when each individual detector faces an SNR wall.
Assuming that the cooperating SUs employ identical energy detectors, two approaches for fusing the soft information received from cooperating users are proposed in [42] and [56]. One approach to cooperative spectrum sensing is to use a linear combination of the SUs measurements as a test statistic, as in [42]. This statistic is then compared to a threshold to decide about the occupancy of a primary channel. This test is a simple sub-optimal alternative to the more complex and optimal likelihood ratio test. However, finding the optimal combining weights is complicated as it involves solving a non-convex optimization problem. A solution to this optimization problem is found by an explicit algorithm proposed in [52]. Numerical results are provided to evaluate the effects of various system parameters on the performance of the proposed linear cooperative sensing scheme.

The authors in [56] design a linear-quadratic fusion strategy which takes into account the correlation among the nodes. They show that the proposed detector significantly outperforms the counting-rule detector, which is used in the case of independent observations. The authors in [29] extend their proposed multi-cycle detector to accommodate user collaboration. They also propose a censoring technique to reduce the number of transmissions of local test statistics and hence to reduce energy consumption.

The authors in [32] assume that each SU is equipped with a frequency selective filter and is able to sense a linear combination of multiple primary channels. Each of the several collaborating SUs reports its measurement to a fusion center, where the final decision about the occupancy of different channels is made. The final decision is obtained by applying either the matrix completion algorithm or the joint sparsity recovery algorithm to the individual measurements. The authors claim that the proposed algorithms can significantly reduce the number of samples required at each SU as well as the number of reports that should be sent from the SUs to the fusion center. A number of practical issues encountered in implementing cooperative sensing schemes are discussed in Section 2.4.
1.3.2 Adaptive Sensing Strategies

Adaptive sensing strategies decide which channel(s) should be sensed at a given time based on the past sensing outcomes. Such decisions are made in the feedback block in Figure 1.5. From the author’s perspective, the adaptive sensing strategies proposed in the literature can be divided into two main categories: channel search strategies and channel selection strategies. It is assumed that the occupancy states of the primary channels do not change during the channel search process, as it occurs within a single time frame, as shown in Figure 1.6. However, the states of the primary channels may change during the channel search process, as it occurs across multiple time frames, as shown in Figure 1.7.

![Figure 1.5: Adaptive sensing strategies.](image)

![Figure 1.6: A channel search strategy.](image)
Chapter 1. Introduction

Figure 1.7: A channel selection strategy.

Channel Search Strategies

Several channel search strategies for opportunistic spectrum access have been proposed in the literature. In a channel search process, each SU picks up a primary channel based on a search strategy to detect whether it is idle or not. If the selected channel turns out to be busy, the SU picks up another channel to sense and the search stops when an idle channel is found. However, when the spectrum utilization is medium to high, this approach results in a long search time. The authors in [31] compare the performance of three different search strategies, i.e., random search, serial search, and \( n \)-step serial search in terms of the mean time to detection of the first idle channel.

Based on a serial search strategy, the optimal sensing time for each channel is obtained in [20] such that the mean time to detection of the first idle channel is minimized. A multi-resolution spectrum sensing technique is proposed in [30] that comprises a preliminary coarse resolution sensing followed by a fine resolution sensing. In this technique, the primary channels are partitioned into several contiguous coarse sensing blocks. First, each SU randomly picks up and senses a coarse sensing block to detect if it contains an idle channel. If so, the SU serially searches over the channels within the coarse sensing block to detect the first idle channel. It is shown that the proposed sensing technique can significantly reduce the mean time to detection when the ratio of idle channels to busy channels is low. However, it is assumed that the received signal power is the same for all the busy channels and the signal power is known a priori. It is also assumed that the SUs have variable bandwidth receivers.
The authors in [41] consider a scenario where a SU senses the primary channels sequentially before it settles on a channel for secondary transmission. A sensing strategy decides when to stop sensing, and an access strategy specifies the transmit power level. The goal is to maximize a reward function that quantifies the secondary throughput as well as the energy consumed in sensing and transmission processes. It is shown that the optimal sensing strategy has a threshold structure based on the channel gain between the secondary transmitter and receiver. It is also shown that the optimal transmit power is given by a water-filling strategy [41]. A simple channel sensing order is proposed in [11], which requires no a priori knowledge about the activity of PUs. It is assumed that the primary channels are sensed according to their achievable rates. The first channel that is sensed to be idle is used for secondary transmission. It is shown, that the proposed channel search strategy is promising in improving the secondary throughput when the channel gains for different primary channels are significantly different.

**Channel Selection Strategies**

Various spectrum sensing strategies have been proposed in the literature for deciding which channel is to be sensed in the next frame. In these strategies, the channel selection process is based on exploiting the sensing outcomes obtained in the past frames to predict the states of different primary channels in the current frame. These strategies depend on a time-domain model for the PUs’ transmissions. The goal is to maximize spectrum utilization by a secondary network subject to non-interference constraints with the PUs. After identifying idle primary channels through spectrum sensing, a Medium Access Control (MAC) protocol must be adopted to determine the spectrum access scheme among the SUs [12, 57].

The spectrum sensing and access strategies proposed in [3, 10, 16, 65] focus on the utilization of time-varying spectrum opportunities. They are based on optimizing the secondary system throughput under the assumption that the occupancy of primary channels
over time follows a discrete-time Markov process. An analytical framework for opportunistic spectrum access based on the theory of a Partially Observable Markov Decision Process (POMDP) is proposed in [65], and optimal strategies for spectrum sensing and access are derived. The authors also propose a suboptimal greedy approach with lower complexity and comparable performance. A similar problem, but under the assumption of primary channels with unknown parameters, is considered in [26]. The problem of finding the optimal sensing and access strategies is transformed into a classical multi-armed bandit problem. Optimal and low complexity asymptotically optimal sensing and access strategies are derived for the case of a single SU. A game theoretic approach is taken for the case of multiple SUs and low complexity suboptimal MAC protocols are derived.

The problem of secondary access to multiple continuous-time on-off Markovian channels under tight collision constraints is considered in [27]. It is shown that if the tolerable collision probabilities for different primary channels are below certain thresholds, the optimal access strategy is a memoryless strategy with periodic channel sensing. The proposed spectrum sensing strategy in [38] employs the $\epsilon$-greedy reinforcement method to coordinate the cooperative sensing among several SUs. Based on occupancy statistics of PUs, the proposed strategy selects the most promising primary channels and assigns the cooperating SUs to sense these channels in such a way that a desired miss probability at each channel is satisfied. Under the assumption that different SUs may see different behaviour on a given primary channel, the authors in [16] aim to maximize the expected sum-throughput of all the SUs. The throughputs resulting from different user-channel combinations are modelled as i.i.d. random processes, and the problem of learning the optimal matching of users to channels is formulated as a combinatorial multi-armed bandit problem.
1.4 Contributions and Outline

In this thesis, we assume that the spectrum available for opportunistic access is divided into a number of primary channels. Our proposed solutions can be divided into two main categories, channel search strategies and channel selection strategies. These solutions for OSA based on spectrum sensing are not only applicable to a hierarchical access scenario, i.e., where there are PUs and SUs, but also applicable to any ad hoc network scenario where there is no static spectrum allocation. In such a scenario, all the users are of the same priority. The two contributions of the thesis in the first area are:

- **A Two-stage Spectrum Detection Strategy for OSA:**

  In Chapter 3, we consider a scenario where a SU employs a tunable BPF to sense one primary channel at a time until an idle channel is found. One approach is to allocate equal sensing times to all the primary channels. However, this approach may result in a long search time if the spectrum utilization is medium to high. Since primary transmitters are located at different locations, they have different transmission powers, and they experience different fading/shadowing conditions, the signals received over different primary channels are at different power levels. Even for a single primary transmitter, e.g., a base station in a cellular network, the signals received over different channels can be at very different power levels. Based on this observation, we introduce a two-stage spectrum detector that serially searches over primary channels and focuses on those that are more likely to be idle.

  In our proposed strategy, each channel is first sensed for $T_1$ seconds. Based on the average received power in the first stage, $Y_1$, the spectrum detector decides either to proceed to the second stage, where the channel is sensed for $T_2$ more seconds, or to skip to the next channel. This process continues until an idle primary channel is found. The block diagram of a two-stage spectrum detector is shown in Figure 1.8. We show that the proposed strategy significantly reduces the average channel
search time, specially when the spectrum utilization is high.

![Block diagram of the two-stage spectrum detector introduced in Chapter 3.](image)

- **A Cooperative Channel Search Strategy for OSA:** In Chapter 4, we focus on multi-channel cooperative spectrum search within a single sensing interval, and we assume that the status of the primary channels remain unchanged during the search process. We assume that the cooperating SUs, each equipped with a tunable BPF, try to jointly detect as many idle channels as possible within a predefined sensing interval $T_S$. The sensing interval consists of $L$ sensing slots of equal duration, as shown in Figure 1.9. During each sensing slot, each SU tunes to and senses one of the primary channels and reports the outcome to other cooperating SUs. The occupancy of each channel is decided based on a fusion rule which is tailored to fit specific regulatory constraints. The main idea in this chapter is to exploit the
structure of the fusion rule to expedite the channel search process. Given the past sensing outcomes, we are interested in the optimal choice of the channels to be sensed, each by a different SU, in each sensing slot. The goal is to maximize the expected number of identified idle channels after $L$ sensing slots. We show that the proposed cooperative channel search strategy remarkably outperforms the brute-force strategy, which requires all the cooperating SUs to sense an identical channel in each slot.

![Diagram of a time frame structure with $L$ sensing slots and channels for spectrum sensing and secondary transmission.]

Figure 1.9: The structure of a time frame for the scenario considered in Chapter 4.

The contribution of the thesis in the second area is

- **A Joint Channel Selection Framework for OSA:** In Chapter 5, we consider a scenario where the primary network operates in a synchronous time-framed fashion and the occupancy of each primary channel in each time frame follows a discrete-time Markov process. There are $n$ SUs trying to identify idle channels in each frame. A frame starts with a sensing interval of fixed length in which SUs stay silent, and each of them senses the primary channel which is assigned to it by the sensing strategy. The occupancy of a primary channel is decided *individually* by each SU to produce a binary sensing outcome. We assume that the sensing outcomes obtained by all the SUs are stored in a fusion center, which can be one of the SUs.

We propose a finite-state channel model that includes the fading condition in addition to the occupancy state for each primary channel. This channel model allows
sensing outcomes obtained in different frames and under different fading conditions to be combined to provide information vectors regarding the state of each primary channel. This model is then extended to the case of multiple geographically distributed SUs. This extended model allows sensing outcomes obtained by several cooperating SUs in different frames to be fused together to furnish information vectors for each primary channel.

Based on the aforementioned channel model, we introduce cooperative sensing strategies that exploit past sensing outcomes of several cooperating SUs to choose the optimal sensing task in each frame. The goal is to maximize a utility function for the secondary network over $K$ frames, which is a function of both the expected number of detected idle channel-frames and the expected number of missed busy channel-frames. The proposed framework in Chapter 5, which introduces a design parameter, enables the system designer to devise strategies with different degrees of aggressiveness. We show that the proposed cooperative sensing strategies perform significantly better than a random selection strategy or a cooperative sensing strategy designed based on a two state On-Off channel model. This improvement is in terms of both the spectrum utilization by the SUs and the amount of interference with the PUs.

The remainder of the thesis is organized as follows. The necessary background on spectrum sensing for CR as well as a detailed overview on the related works is provided in Chapter 2. The main contributions of this thesis are presented in Chapters 3 to 5. Conclusions are given in Chapter 6.
Chapter 2

Background and Literature Review

In this chapter, we provide the necessary background on spectrum sensing for CR and an overview of various solutions proposed in the literature to tackle the technical issues discussed in Chapter 1.

2.1 What is a Spectrum Opportunity?

A definition of spectrum opportunity is given in [44]. Based on this work, a spectrum opportunity exists when the reception at the secondary receiver is successful, and the transmission at the secondary transmitter is harmless to the primary receivers. As suggested in [44], any interference constraint imposed by regulatory bodies should specify at least two parameters \( \{\eta, \zeta\} \). The first parameter \( \eta \) (the noise floor) is the maximum interference power that can be tolerated by an active primary receiver. The second parameter \( \zeta \) is the maximum allowable outage probability, i.e., the probability that the interference at an active primary receiver surpasses \( \eta \). Since opportunity detection errors are inevitable in practice, the outage probability should always be positive.

Consider a pair of SUs as shown in Figure 2.1. Neglecting the effects of shadowing and multi-path fading, a channel is an opportunity for transmission from \( A \) to \( B \) if there is no active primary receiver within distance \( r_1 \) of \( A \), and there is no active primary transmitter
within distance $R_1$ of $B$. The radius $r_1$ of the solid circle is called the interference range of the SU, which depends on the transmit power of $A$ as well as the noise floor $\eta$. The radius $R_1$ of the dashed circle is called the interference range of PUs, which depends on the transmit power of PUs as well as the interference tolerance of $B$. As can be seen from

Figure 2.1: Spectrum opportunities in space.

Figure 2.1, spectrum opportunity is a function of the transmit powers of both PUs and SUs as well as the interference constraints. Furthermore, the spectrum opportunities are not symmetric, i.e., an opportunity for transmission from $A$ to $B$ may not be an opportunity for transmission from $B$ to $A$.

The performance of a spectrum detection scheme can be described by its Receiver Operating Characteristic (ROC), which gives the probability of detection $P_d$ as a function of the probability of false-alarm $P_f$. In the absence of noise and fading, $A$ can reliably detect the existence of primary transmitters within its detection range $r_D$. This range can be adjusted by changing the detection threshold. Even with ideal detection at $A$,
there are three sources of detection errors: hidden transmitters, hidden receivers, and exposed transmitters (see Figure 2.2). Hidden transmitters are primary transmitters within distance $R_I$ of $B$, but outside the detection range $r_D$ of $A$ (node $X$ in Figure 2.2). Hidden receivers are primary receivers within the interference range $r_I$ of $A$ that their corresponding transmitters are outside the detection range $r_D$ of $A$ (node $Y$ in Figure 2.2). Exposed transmitters are primary transmitters within the detection range $r_D$ of $A$ that their corresponding receivers are outside the interference range $r_I$ of $A$ (node $Z$ in Figure 2.2).

While the first two cases result in transmission collisions between the PUs and SUs, the third case results in false-alarms. Different choices of the detection range results in different points on the ROC curve. As shown in [44], if the ratio between the transmit powers of PUs and SUs is very small or very large, the probabilities of miss and false-alarm can be made arbitrarily small and perfect spectrum detection is achieved. In the
above two cases, the separation between the communicating nodes would be very small, either in the primary or secondary network. However, even for these cases, spectrum detection does not guarantee non-interference with PUs in the presence of multipath fading, shadowing, or sensing errors.

2.2 Spectrum Detection Schemes

2.2.1 Narrowband Detection Schemes

CRs utilize spectrum detectors to distinguish between occupied and idle narrow frequency bands. If the outcome of a spectrum detection process is the null hypothesis, it means that there is no PU in a certain frequency band. The alternative hypothesis, on the other hand, indicates the existence of a PU on that particular band.

Let $x(t)$, $h$, and $v(t)$ denote the band-limited primary signal, the fading process, and the additive noise process, respectively. The binary hypothesis test for spectrum sensing for each channel is formulated as follows:

\begin{align*}
\text{Signal absent} & \quad \mathcal{H}_0 : & r(t) = v(t) & \text{for } 0 \leq t \leq T, \\
\text{Signal present} & \quad \mathcal{H}_1 : & r(t) = hx(t) + v(t), & \text{for } 0 \leq t \leq T, (2.1)
\end{align*}

where $r(t)$ is the received signal and $T$ is the sensing time. The signal is assumed to be independent of both the noise and the fading process. All processes are assumed to be stationary and ergodic unless otherwise specified [46]. The discrete-time version is obtained via sampling the received signal:

\begin{align*}
\text{Signal absent} & \quad \mathcal{H}_0 : & r[n] = v[n] & n = 1, 2, \cdots, N \\
\text{Signal present} & \quad \mathcal{H}_1 : & r[n] = hx[n] + v[n], & n = 1, 2, \cdots, N, (2.2)
\end{align*}
where \( x[n] \) denotes the samples of the primary signal, \( v[n] \) denotes the noise samples, \( r[n] \) denotes the received signal samples, and \( N \) is the number of samples. A test-statistic/threshold based detector aims to distinguish between the two hypotheses:

\[
Y(r) = Y(r[1], \cdots, r[N]) \overset{\mathcal{H}_1}{\gtrless} \overset{\mathcal{H}_0}{\lambda},
\]

(2.3)

where \( Y \) is a deterministic function and \( \lambda \) is the detector threshold. The false-alarm probability and the miss probability are given by

\[
P_f = \Pr[Y(r) \geq \lambda | \mathcal{H}_0],
\]

(2.4)

\[
P_m = \Pr[Y(r) \leq \lambda | \mathcal{H}_1].
\]

(2.5)

The form of the function \( Y \) is determined by the detection scheme. The detector threshold \( \lambda \) and the sensing time \( T \) should be chosen such that the opportunistic use of the licensed spectrum is maximized, while at the same time the regulatory constraints imposed by the primary network are satisfied. In general, there are three narrowband spectrum detection schemes: matched filter detection, energy detection and cyclostationary feature detection [6].

**Matched Filter Detection**

The matched filter is the optimal detector in stationary Gaussian noise since it maximizes the received SNR. Given a priori information of the primary signal such as the modulation type and order, the pulse shape, and the packet format, the matched filter achieves high sensing quality in a short time. However, the sensing performance significantly degrades in the case of inaccurate a priori knowledge of the primary signal [45].

**Energy Detection**

If a CR’s a priori knowledge is limited to the local noise statistics, the optimal detector is an energy detector, where the received signal in each frequency band is squared and
integrated over the sensing time [18]. The output of the integrator is then compared to a predefined threshold in order to decide whether a licensed band is occupied or not (Figure 2.3).

![Figure 2.3: Block diagram of an energy detector.](image)

Employing a sampling theory-based approach, the authors in [15] consider the problem of energy detection of an unknown band-limited signal under both AWGN and fading channels. They assume that the received BP signal can be represented as

$$ r(t) = \begin{cases} 
\mathcal{R}\{v_{LP}(t)e^{j2\pi f_c t}\}, & H_0 \\
\mathcal{R}\{[hS_{LP}(t) + v_{LP}(t)]e^{j2\pi f_c t}\}, & H_1 
\end{cases} $$

where $\mathcal{R}\{\cdot\}$ denotes the real part of a complex number, $h = \alpha e^{j\theta}$ is a slow-fading channel, $f_c$ is the carrier frequency, $S_{LP}(t) = S_c(t) + jS_s(t)$ is an equivalent low-pass representation of the unknown signal, and $v_{LP}(t) = v_c(t) + jv_s(t)$ is an equivalent low-pass AWGN process with zero mean and a flat (one-sided) power spectral density of $N_0$. Therefore, if $W$ denotes the signal bandwidth, the noise power will be equal to $N_0 W$. The signal-to-noise ratio is denoted by $\gamma = (\alpha^2 E_s)/(N_0)$, where $E_s$ is the signal energy.

It is assumed that the received signal is first filtered by an ideal BPF. Then, the output of this filter is squared and integrated over the sensing time $T$. Based on the sampling theorem for narrowband signals, the output of the integrator $Y$ under $H_1$ can be approximated by
\[ Y \triangleq \frac{1}{N_0 W} \int_0^T r^2(t) dt \simeq \frac{1}{N_0 W} \left[ \sum_{i=1}^{N/2} (\alpha_c S_{ci} - \alpha_s S_{si} + v_{ci})^2 + \sum_{i=1}^{N/2} (\alpha_c S_{si} + \alpha_s S_{ci} + v_{si})^2 \right], \tag{2.6} \]

where \( N/2 \) is the number of samples per each of the I and Q components, \( \alpha_c = \alpha \cos \theta \), \( \alpha_s = \alpha \sin \theta \), and \( S_{ci}, S_{si}, v_{ci}, \) and \( v_{si} \) represent the \( i \)th samples of the I and Q components of \( S_{LP}(t) \) and \( v_{LP}(t) \), respectively. It is shown that under \( \mathcal{H}_1 \), \( Y \) has a non-central chi-square distribution with variance \( \sigma^2 = 1 \), non-centrality parameter \( \mu = 2\gamma \), and \( N \) degrees of freedom [15]. Under \( \mathcal{H}_0 \), \( Y \) has a central chi-square distribution with \( N \) degrees of freedom. Therefore, the probability density function of \( Y \) can be written as

\[
 f_Y(y) = \begin{cases} 
 \frac{1}{2^{N/2}(N/2)} y^{N/2-1} e^{-\frac{y}{2}}, & \mathcal{H}_0 \\
 \frac{1}{2} \left( \frac{y}{2\gamma} \right)^{N/2} e^{\frac{y}{2\gamma}} I_{N/2-1} \left( \sqrt{2\gamma y} \right), & \mathcal{H}_1
\end{cases}
\]  

where \( \Gamma(\cdot) \) is the Gamma function and \( I_v(\cdot) \) is the \( v \)th order modified Bessel function of the first kind. Exact closed-form expressions for both the false-alarm probability and the miss probability based on a decision threshold \( \lambda \) are given by (see [15])

\[
 P_f = \frac{\Gamma(N/2, \lambda/2)}{\Gamma(N/2)} , \tag{2.8} \\
 P_m = Q_{N/2} \left( \sqrt{2\gamma}, \sqrt{\lambda} \right) , \tag{2.9}
\]

where \( \Gamma(\cdot, \cdot) \) is the incomplete gamma function and \( Q_{N/2}(\cdot, \cdot) \) is the generalized Marcum \( Q \)-function. The authors in [15] also derive closed-form expressions for the probability of detection over Rayleigh and Nakagami fading channels. Furthermore, they consider the improvement in detection performance when low-complexity diversity schemes such as square-law combining and square-law selection are used.

Due to its simplicity and ease of implementation, the energy detector has been widely adopted in proposed spectrum sensing methods. However, the detection performance is
highly susceptible to uncertainty in noise power. Also, energy detectors cannot differentiate primary and secondary transmissions. This capability is desirable when a SU is permitted to access a channel that is being used by another SU, but not if it is being used by a PU.

**Cyclostationary Feature Detection**

The modulated signals are characterized by sine wave carriers, pulse trains, or cyclic prefixes which results in cyclic variation of statistical properties of the received signal. This property, which is known as the cyclostationarity, can be detected by constructing the correlation function of the received signal. This feature also allows the spectrum detector to differentiate the primary signal from the noise, which is temporally white. Therefore, a cyclostationary feature detector performs well in the case of uncertainty in noise power [45].

A discrete-time zero-mean process $x[t]$ is (almost) cyclostationary, if its time-varying covariance $c_{2x}(t; \tau) = \text{E}\{x[t]x[t + \tau]\}$ can be represented in the form of a Fourier series with respect to $t$ (see [13]):

$$c_{2x}(t; \tau) = \sum_{\alpha \in A_2} C_{2x}(\alpha; \tau) e^{j\alpha t},$$

$$C_{2x}(\alpha; \tau) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} c_{2x}(t; \tau) e^{-j\alpha t},$$

(2.10)

where $C_{2x}(\alpha; \tau)$ is the cyclic covariance at cycle-frequency $\alpha$ and

$$A_2 \triangleq \{\alpha : 0 \leq \alpha < 2\pi, C_{2x}(\alpha; \tau) \neq 0\}. \quad (2.11)$$

The time-varying spectrum $s_{2x}(t; \omega) \triangleq \sum_{\tau=-\infty}^{\infty} c_{2x}(t; \tau) e^{-j\omega \tau}$ can also be represented in the form of a Fourier series:

$$s_{2x}(t; \omega) = \sum_{\alpha \in A_2} S_{2x}(\alpha; \omega) e^{j\alpha t},$$

(2.12)

$$S_{2x}(\alpha; \omega) = \sum_{\tau=-\infty}^{\infty} C_{2x}(\alpha; \tau) e^{-j\omega \tau},$$

(2.13)
where \( S_{2x}(\alpha; \omega) \) is called the cyclic-spectrum.

In [13], the authors develop tests in both time and frequency domains to check for presence of cycles in the cyclic-covariance and spectrum by searching over a set of candidate cycles. In the time domain, the objective is to detect those \( \alpha \)'s for which \( C_{2x}(\alpha; \tau) \neq 0 \), for a given set of data \( \{x[t]\}_{t=0}^{T-1} \) and for a fixed \( \tau \). The second-order cyclostationarity is present in \( x[t] \) if there exists at least one pair \( (\alpha, \tau) \) for which \( C_{2x}(\alpha; \tau) \neq 0 \). A consistent estimator of \( C_{2x}(\alpha; \tau) \neq 0 \) is given by

\[
\hat{C}_{2x}(\alpha; \tau) \triangleq \frac{1}{T} \sum_{t=0}^{T-1} x[t]x[t+\tau]e^{-j\alpha t} = C_{2x}(\alpha; \tau) + \epsilon_{2x}^{(T)}(\alpha; \tau),
\]

where \( \epsilon_{2x}^{(T)}(\alpha; \tau) \) denotes the estimation error. If we let \( \tau_1, \tau_2, \cdots, \tau_N \) be a set of lags and \( \alpha \) be a candidate cycle-frequency, we aim to solve the following hypothesis testing problem:

\[
H_0 : \alpha \notin A_2 \quad \forall \{\tau_n\}_{n=1}^{N} \quad \Rightarrow \quad \hat{C}_{2x}(\alpha; \tau) = \epsilon_{2x}^{(T)}(\alpha; \tau),
\]

\[
H_1 : \alpha \in A_2 \quad \text{for some} \{\tau_n\}_{n=1}^{N} \quad \Rightarrow \quad \hat{C}_{2x}(\alpha; \tau) = C_{2x}(\alpha; \tau) + \epsilon_{2x}^{(T)}(\alpha; \tau).
\]

Using the asymptotic normality of sample cyclic-covariance and spectrum, the authors derive constant false-alarm rate tests in both time and frequency domains to check for the presence of second-order cyclostationarity. These tests are then extended for checking the presence of cycles in the \( k \)th-order cyclic-cumulants and polyspectra. A comprehensive overview on cyclostationary and cyclostationarity detection schemes can be found in [17].

The authors in [29] extend the Neyman-Pearson type test in [13] to the case of multiple cyclic frequencies. This test is then used to detect the presence of nearby PUs in CRNs. It is shown in [49] that cyclostationary signatures may be intentionally embedded in a digital communications signal to enhance the process of signal detection and identification. The major disadvantages of cyclostationary feature detectors are their higher computational complexity and longer sensing time [6].
2.2.2 Wideband Detection Schemes

One approach to wideband spectrum sensing is to implement a wideband filter followed by a high rate A/D converter to process the full range of the available spectrum at once. This approach usually involves direct estimation of the power spectral density of the observed wideband signal. Spectrum estimation techniques can be either parametric or non-parametric. Although parametric techniques are simpler to realize, they are more prone to model uncertainties that arise due to the unreliable nature of wireless channels and spectrum access by other SUs.

In [24], a novel nonparametric and therefore robust spectrum estimation method is introduced that resolves the bias-variance dilemma presented in earlier methods and allows for a real-time estimate of the spectrum with different resolutions. A new space-time processor is also introduced that estimates the unknown directions of arrival of interfering signals. The authors extend their method to characterize the cyclostationarity property of the received signals, which can be used for signal classification. The authors confirm their analytical results through two real-life examples: wideband Advanced Television Systems Committee (ATSC) digital television signals and generic mobile radio signals.

A wideband spectrum sensing technique is proposed in [53] that employs the wavelet transform to classify non-overlapping frequency bands into black, gray, and white spaces. Based on the assumption that wireless signals are typically sparse in open-spectrum networks, the authors in [54] exploit compressive sampling to reduce the number of required samples. In [43], a wideband spectrum sensing technique is proposed that jointly detects primary signals over multiple frequency bands.

Before digital signal processing techniques can be utilized for spectrum sensing, RF amplification, mixing, and A/D conversion is performed in the wideband frontend of a CR. Therefore, wideband spectrum sensing architectures place significant constraints on the RF analogue circuits of a CR [9]. Another approach is to employ a tunable BPF to sense one frequency band at a time (narrowband sensing). We adopt this approach in
2.3 OSA through Cooperative Spectrum Sensing

When a SU experiences severe shadowing or multipath fading, it is not able to reliably detect the presence of nearby PUs. To address this issue, collaborative spectrum sensing schemes are proposed in the literature. Ghasemi and Sousa [19] studied the performance of different collaborative spectrum sensing schemes in fading/shadowing environments. In these schemes, every SU performs its local spectrum sensing and reports its measurement to a common receiver or a fusion center. These results are then mixed together to make the final decision on the availability of a primary channel.

The authors consider a scenario where \( n \) SUs collaborate in spectrum sensing by sharing their sensing observations. The collaborating SUs employ identical energy detectors as described in Section 2.2.1. Let \( Y_i \) denote the output of the square-law (square-and-integrate) device belonging to SU \( i \). Among different linear soft-decision combining schemes, Equal-gain Combining (EGC) is particularly useful because it does not require any channel state information. Under EGC, the test statistic is given by the sum of the sensing observations, i.e.,

\[
Y_0 \triangleq \sum_{i=0}^{n} Y_i.
\]  

(2.16)

This statistic is compared to a threshold to distinguish between the two hypotheses \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \). As discussed in Section 2.2.1, each of the random variables \( Y_i, i = 1, \ldots, n \) has a chi-square distribution. The random variable \( Y_0 \) is the sum of \( n \) independent chi-square random variables and, therefore, is another chi-square random variable. Let us denote by \( \gamma_i \) the received primary SNR at SU \( i \), and by \( \lambda \) the detection threshold used to distinguish between the two hypotheses based on the test statistic \( Y_0 \). The probability of detection
conditioned on $\gamma_i$'s and the probability of false-alarm are given by

$$Q_d = \Pr[Y_0 > \lambda | \mathcal{H}_1, \gamma_1 = l_1, \ldots, \gamma_n = l_n] = Q_{nN/2} \left( \sqrt{N \sum_{i=0}^{n} l_i}, \sqrt{\lambda} \right)$$

(2.17)

$$Q_f = \Pr[Y_0 > \lambda | \mathcal{H}_0] = \frac{\Gamma(nN/2, \lambda/2)}{\Gamma(nN/2)}$$

(2.18)

where $Q_d$ and $Q_f$ denote the collaborative probabilities of detection and false-alarm, as opposed to their individual counterparts, denoted by $P_d$ and $P_f$, respectively, and $Q_{N/2}(\cdot, \cdot)$ is the generalized Marcum $Q$-function. Note that $Q_d$ is only a function of $\gamma_0 \equiv \sum_{i=0}^{n} \gamma_i$. Therefore the probability of detection is given by

$$Q_d = \int_{\gamma_0} Q_{nN/2} \left( \sqrt{N x}, \sqrt{\lambda} \right) f_{\gamma_0}(x) dx$$

(2.19)

where $f_{\gamma_0}(x)$ is the probability density function of the sum of the individual SNRs, $\gamma_0$. Under Rayleigh fading where the random variables $\gamma_i$s are independent exponential random variables with common mean $\bar{\gamma}$, the random variable $\gamma_0$ is Gamma distributed:

$$f_{\gamma_0}(x) = \frac{x^{n-1}e^{-x/\bar{\gamma}}}{(n-1)!\gamma^n}.$$  

(2.20)

In the above collaborative sensing scheme, it is assumed that the measurements of individual SUs are known exactly at a fusion center, which can be one of SUs. However, transmitting the measured energies with high precision can incur significant communication overhead, depending on the frequency of measurements and the number of collaborating SUs. In [19], the others consider the extreme case where each SU communicates only its 1-bit (hard) decision to the fusion center. SU $i$ compares its measured energy with a local threshold to make its 1-bit decision $u_i$:

$$u_i = \begin{cases} 
0, & \text{Decide } \mathcal{H}_0 \text{ if } y_i < \lambda_i \\
1, & \text{Decide } \mathcal{H}_1 \text{ if } y_i > \lambda_i 
\end{cases} \quad i = 1, \ldots, n.$$

Under the assumption that the individual measurement are mutually independent given the hypothesis, the Neyman-Pearson test is given by

$$\sum_{i=1}^{n} u_i \log_c \left[ \frac{P_{d_i}(1 - P_{f_i})}{(1 - P_{d_i})P_{f_i}} \right] \geq_{\mathcal{H}_i} \Lambda$$

(2.21)
where $P_d$ and $P_f$ respectively denote the individual probabilities of detection and false-alarm for SU $i$. The test statistic in this case is a weighted sum of the individual 1-bit decisions, and more weight is given to more reliable decisions. If it is assumed that the decision threshold and the probabilities of detection and false-alarm are the same for all the SUs, the test statistic reduces to the sum of $n$ individual 1-bit decisions. Therefore, the final decision on the availability of a primary channel is obtained by applying the $k$-out-of-$n$ rule to the individual decisions, i.e., the final decision is $H_1$ if at least $k$ individual decisions are $H_1$. The value of $k$ depends on the selected threshold $\Lambda$. The probabilities of detection and false-alarm for the $k$-out-of-$n$ detector are given in terms of the individual probabilities $P_d$ and $P_f$:

$$Q_d = \sum_{i=k}^{n} \binom{n}{i} P_d^i (1 - P_d)^{n-i}$$  \hspace{1cm} (2.22)

$$Q_f = \sum_{i=k}^{n} \binom{n}{i} P_f^i (1 - P_f)^{n-i}. \hspace{1cm} (2.23)$$

As observed in [19], the 1-out-of-$n$ (OR-rule) detector achieves the best performance among all the $k$-out-of-$n$ detectors for many cases of practical interest. The probabilities of detection and false-alarm for the 1-out-of-$n$ detector are given by

$$Q_d = 1 - (1 - P_d)^n$$  \hspace{1cm} (2.24)

$$Q_f = 1 - (1 - P_f)^n. \hspace{1cm} (2.25)$$

It is shown in [19] that the OR-rule detector significantly increases the reliability of detection in fading and shadowing channels. Although fusing the 1-bit decisions instead of the exact measurements results in a loss of performance, it is preferred if communication overhead is an issue.

Cooperative spectrum sensing has also been proposed as a feasible solution to the problem of interference avoidance in femtocell networks [8]. Due to uncertainty in femto base stations’ number and positions, they have to be capable of integrating themselves into existing radio access networks causing the least interference to existing systems. The
Multi-operator Spectrum Server (MOSS) is proposed in [8]. The role of a MOSS is to collect spectrum sensing information from one or more femto base stations and combine it with additional information about spectrum use obtained from each operator. The server then uses this information to make decisions about spectrum use and availability in each region. Since femto base stations are connected by a backhaul network, the sensing outcomes obtained by different stations can be communicated over wired connections.

2.4 Technical Issues in Cooperative Spectrum Sensing

Although cooperation among multiple SUs can significantly improve the reliability of spectrum sensing and increase the secondary use of the spectrum, there are many technical issues that must be addressed before such benefits can be realized. The main technical issues in cooperative spectrum sensing are divided into seven categories in [5]. According to this classification, the contributions in this thesis fall into categories sensing efficiency and wideband sensing. Here, we provide a brief overview of the other technical issues encountered in implementing cooperative sensing schemes.

In any cooperative sensing scenario, SUs report their local sensing results to a fusion center using a Common Control Channel (CCC). In the case where the SUs are not wired-connected to each other, the CCC is assumed to be implemented as a dedicated frequency band in licensed or unlicensed spectrum, or as an underlay ultra-wideband channel [9]. A MAC protocol is used by the SUs to access the CCC and report the sensing results. Like any other wireless channel, the CCC is not immune to impairments such as multipath fading and shadowing. When the cooperating SUs transmit their local sensing results using a random access scheme, collisions and retransmissions are inevitable. Due to the above factors, the effect of missing and/or delayed sensing outcomes on the performance of different cooperative sensing schemes is an important issue, and it has been considered
in several recent works. The bandwidth of the CCC is another factor that limits the level of cooperation among the SUs.

Energy consumption due to local spectrum sensing and data transmission is another important issue that should be taken into account. The increase in energy consumption due to cooperative spectrum sensing depends on many factors including the number of cooperating SUs, the frequency of spectrum sensing, and the amount of sensing information which is shared among the SUs. The issue of how multiple SUs group together and cooperate in spectrum sensing is also very important from both theoretical and practical points of view. The performance of cooperative sensing can significantly degrade due to correlated shadowing or the mobility of the SUs. To improve the robustness of the sensing results, several centralized and cluster-based SU selection schemes are proposed in the literature [5].

In any cooperative sensing scheme, a mechanism for synchronizing the sensing tasks among the cooperating SUs is needed. In such schemes, the sensing outcomes obtained by several SUs are fused together to decide about the availability of each primary channel. Consequently, the sensing tasks should be coordinated and synchronized, such that a meaningful fusion of the outcomes is possible. Another reason for synchronizing the sensing tasks among the SUs is that if they are not able to distinguish between primary and secondary transmutations, e.g., in the case of energy detection, they should all stay silent simultaneously during the sensing phase. Synchronization among the SUs as well as security attacks is another important issue which is addressed in several recent works. A comprehensive overview of the literature addressing the above practical issues is provided in [5].

As explained in Chapters 4 and 5, the proposed cooperative sensing strategies in this thesis may be adjusted to account for missing or delayed sensing outcomes. The required accuracy of synchronization among the cooperating SUs in these strategies depends on the length of the time frame $T$. The time mismatch between the SUs should be small
with respect to $T$, the choice of which depends on how fast the PUs’ activity change over time. It also depends on the Quality of Service (QoS) requirements of the PUs and how they are affected by the time it takes the SUs to detect the primary transmissions. Each sensing task can be initiated by the SU that is also responsible for fusing the sensing outcomes or can be initiated based on a local clock which is synchronized with the local clocks of the other SUs once every several frames. A number of practical issues that are faced in implementing cooperative sensing schemes and are specific to the proposed solutions in Chapters 4 and 5 are discussed at the end of each chapter.

2.5 A POMDP Framework for Channel Selection for OSA

Bursty data arrivals in many applications motivated the authors in [65] to propose a decentralized cognitive MAC protocol for opportunistic use of the spectrum in the time domain. In this protocol, SUs try to detect idle time frames across several primary channels through spectrum sensing. A channel that is detected to be idle in each frame can be used according to an access strategy. For many low-cost battery-powered wireless nodes, continuous full spectrum sensing is impossible due to hardware and energy constraints. Therefore, it is assumed that each SU (with data to transmit) can sense only a subset of all primary channels in each frame and must decide whether to transmit based on the sensing outcome. In what follows, we provide a summary of the aforementioned protocol under the assumption that the spectrum sensing is error-free. More general results can be found in [65].

The authors consider a scenario where the spectrum available for opportunistic access is divided into $c$ primary channels, each with bandwidth $B_i, (i = 1, \ldots, c)$. The primary network operates in a synchronous time-framed fashion. A channel is called $I$ (idle) in frame $t$ if it is not used for primary transmission in frame $t$; otherwise it is called
Therefore, the network state in frame $t$ is given by $(S_1(t), \ldots, S_c(t))$ where $S_i(t) \in \{I(0), B(1)\}$ is the occupancy state of Channel $i$ in frame $t$. It is assumed that the occupancy state of these $c$ primary channels, i.e., the network state, follows a discrete-time Markov process with $R = 2^c$ states, transition probabilities $\{p_{ij}\}$, $i, j \in \{1, 2, \ldots, R\}$, and an initial distribution $\pi(1) = [\pi_1(1), \pi_2(1), \ldots, \pi_R(1)]$.

In each frame, a SU chooses a sensing action, i.e., a subset $A \subset \{1, \ldots, c\}$ of channels to sense. Given that the current state of the network is $j$, the SU observes a sensing outcome $O_{j,A} \in \{0, 1\}^{|A|}$ which indicates the occupancy of each sensed channel. The occupancy or availability of a primary channel is decided based on a detection threshold that is tailored to some regulatory constraints.

The reward function $r_{j,A}(t)$ is defined as the number of bits delivered when a SU senses the channels in $A$ given that the primary network is in state $j$. It is assumed that the SU transmits on a primary channel if and only if the channel is sensed to be available. It is also assumed that the number of bits delivered over each channel is proportional to its bandwidth, and no data can be delivered over a channel that is used by the primary network. If the spectrum sensing is assumed to be error-free, the reward function is defined as

$$r_{j,A}(t) = \sum_{i \in A} S_i(t) B_i$$

where $S_i(t) \in \{0, 1\}$ is the state of Channel $c$ in frame $t$. For notational simplicity, it is assumed in the rest of [65] that the set $A$ consists of the single element $a \in \{1, \ldots, c\}$, which is the index of the channel to be sensed next.

The problem of cognitive MAC design reduces to an optimization problem where the goal is to maximize the expected total reward in $T$ frames. The optimization variable is the sensing policy, which is a sequence of functions, one for each value of $t$, that map all the past sensing outcomes to a sensing action. Let $\pi_i(t)$ denote the probability that the network is in state $i$ given all the past sensing outcomes at the beginning of frame $t$ prior to the state transition. Define $\pi(t) = [\pi_1(t), \ldots, \pi_R(t)]$, which is called the belief
vector in frame $t$. The transformation of the belief vector $\pi$ from frame $t$ to frame $t+1$ is determined by the sensing action and the corresponding outcome in frame $t$:

$$\pi(t+1) = T(\pi(t)|a,o) = [\pi_1(t+1), \ldots, \pi_R(t+1)]$$

$$\pi_j(t+1) = \frac{\sum_{i=1}^R \pi_i(t)p_{i,j}\Pr[O_{j,a} = o]}{\sum_{i=1}^R \sum_{j=1}^R \pi_i(t)p_{i,j}\Pr[O_{j,a} = o]} \quad \text{(2.27)}$$

It is shown that the belief vector $\pi(t)$ summarizes all the information required for choosing a sensing action in frame $t$ [48]. Consequently, a sensing policy is given by a sequence of functions, one for each value of $t$, that map the belief vector $\pi(t)$ to the sensing action $a(t)$:

$$\delta = \{\delta_t : \pi(t) \in [0,1]^R \rightarrow a \in \{1, \ldots, c\}\} \quad \text{(2.28)}$$

Therefore, an optimal sensing policy $\delta^*$ is a solution to the following optimization problem

$$\delta^* = \arg \max_{\delta} E_\delta \left\{ \sum_{t=1}^T r_{j(t),a(t)}|\pi(1) \right\} \quad \text{(2.29)}$$

which corresponds to the optimal control of a POMDP over a horizon of $T$ frames. Here it is assumed that the transition probabilities and the initial distribution are known a priori. Otherwise, the problem falls into the category of POMDP with unknown transition parameters [35]. Let the value function $\{V_t(\pi(t))\}$ denote the maximum expected reward that can be obtained starting from frame $t$ when the current belief vector is $\pi(t)$. The recurrence relations for an optimal sensing policy $\{\delta^*_t\}$ and the associated value functions $\{V_t\}$ are given by

$$\delta^*_t(t) = \arg \max_{a} \left\{ \sum_{i=1}^R \pi_i \sum_{j=1}^R p_{i,j} \frac{1}{\sum_{o=0}^1 \Pr[O_{j,a} = o]} \Pr[O_{j,a} = o] (oB_a + V_{t+1}(T(\pi|a,o))) \right\} \quad \text{(2.30)}$$

$$V^t(\pi) = \sum_{i=1}^R \pi_i \sum_{j=1}^R p_{i,j} \frac{1}{\sum_{o=0}^1 \Pr[O_{j,\delta^*_t(t)} = o]} \Pr[O_{j,\delta^*_t(t)} = o] (oB_{\delta^*_t(t)} + V_{t+1}(T(\pi|\delta^*_t(t),o))) \quad \text{(2.31)}$$

where $\Pr[O_{j,a} = o]$ is the probability of outcome $o$ if the network state is $j$ and action $a$ is taken. Finding an optimal solution to POMDPs is often computationally intractable;
therefore, several methods have been proposed in the literature that approximate solutions to POMDPs [28].

Under the assumption that the primary channels evolve independently, a sufficient statistic for the optimal policy is given by $\omega(t) = [\omega_1(t), \ldots, \omega_c(t)]$ where $w_i(t)$ is the probability that Channel $i$ is available at the beginning of frame $t$ conditioned on the sensing history. The dimension of $\omega$ is $c$, which is significantly lower than the dimension of $\pi$, which is $R = 2^c$. The transformation of the vector $\omega$ from frame $t$ to frame $t + 1$ is given by

$$\omega(t + 1) = T^g(\omega(t)|a, o) = [\omega_1(t + 1), \ldots, \omega_c(t + 1)]$$

$$\omega_i(t + 1) = \begin{cases} 1 & \text{if } a = i, o = 1 \\ 0 & \text{if } a = i, o = 0 \\ \omega_i(t)\beta_i + (1 - \omega_i(t))\alpha_i & \text{if } a \neq i \end{cases}$$  \hspace{1cm} (2.32)

Based on the sufficient statistic $\omega$, the authors propose a suboptimal sensing policy, known as the greedy sensing policy, that aims to maximize the expected reward in each frame. Assuming that the primary channels evolve independently, each channel follows a discrete-time Markov process with two states: 0 (unavailable) and 1 (available). Let us assume that Channel $i$ transits from state 0 to state 1 with probability $\alpha_i$ and stays in state 1 with probability $\beta_i$, as shown in Figure 2.5. Given the vector $\omega$ at the beginning of frame $t$ prior to the state transition, the greedy sensing policy is given by

$$\delta^g(t) = \arg \max_{a \in \{1, \ldots, c\}} (\omega_a(t)\beta_a + (1 - \omega_a(t))\alpha_a)B_a.$$  \hspace{1cm} (2.33)
Note that as in (2.26), the expected reward for Channel $a$ is proportional to its bandwidth $B_a$. The corresponding value functions $V^g_t(\omega)$ can be obtained recursively:

$$V^g_t(\omega) = (\omega^g_a \beta^g_a + (1 - \omega^g_a)\alpha_a)B_a + \sum_{o=0}^{2} \Pr[O = o|\omega, a^g]V^g_{t+1}(T^g(\omega|a^g, o))$$

$$= (\omega^g_a \beta^g_a + (1 - \omega^g_a)\alpha_a) + \left[\omega^g_a(1 - \beta^g_a) + (1 - \omega^g_a)(1 - \alpha_a)\right]V^g_{t+1}(T^g(\omega|a^g, 0))$$

$$+ \left[\omega^g_a \beta^g_a + (1 - \omega^g_a)\alpha_a\right]V^g_{t+1}(T^g(\omega|a^g, 1)). \quad (2.34)$$

The greedy sensing policy has significantly lower complexity compared to the optimal sensing policy. However, it completely neglects the expected future reward. The performance of the proposed optimal and greedy sensing policies are evaluated through numerical simulations. It is shown that the performance gap between the optimal and greedy sensing policies is negligible; although both significantly outperform a random selection policy. It is shown later in [3] that a greedy policy is optimal when the spectrum sensing is error-free and the state transitions are positively correlated over time.

A similar problem but under an energy constraint on the SUs is considered in [10]. In the absence of an energy constraint, a SU should always sense a primary channel, even if it has no data to transmit. However, a SU with an energy constraint may choose not to sense in order to save energy, even if it has data to transmit. When there is no energy constraint, a SU with packets to transmit should always access a primary channel that is sensed to be idle. However, it may decide to wait for more favorable channel conditions under an energy constraint.

It is shown that the optimal sensing decision is obtained by comparing the probability that the channel is available in the current frame to a threshold. This probability is conditioned on all the past sensing outcomes. The optimal access strategy also has a threshold structure based on the fading condition, i.e., the SU should access a primary channel if and only if the channel is sensed to be idle and the channel gain to its intended receiver is higher than a threshold. It is shown that the imposed energy constraint has a higher impact on sensing and access strategies when the battery nears depletion or when
the channel occupancy states are negatively correlated over time.
In this chapter, we consider a scenario where a SU senses a number of primary channels one-by-one to locate an idle channel. The SU employs an energy detector to distinguish between idle and busy channels. The interference constraint imposed by regulatory bodies is specified by two parameters: the detection sensitivity and the miss probability. Given the regulatory constraints, a shorter sensing time results in a higher false-alarm probability, i.e., a higher chance of overlooking an idle channel; nevertheless, it may be beneficial by allowing a SU to search over more primary channels. In [20], the optimal sensing time is obtained in order to minimize the average time required to find the first idle channel. However, allocating equal sensing times to all the primary channels may result in a long search time if the spectrum utilization is medium to high.

To address this problem, we use the fact that the power of the signal received by the SU on a busy primary channel can vary greatly depending on several factors. These factors include the location of the primary transmitter, its transmission power, and the physical channel condition between the primary transmitter and the SU. The detection sensitivity imposed by the regulatory bodies is cautiously chosen considering the worst
case scenario; however, the signals received by the SU on a busy primary channel can have a much higher power level.

In the spectrum detection strategy proposed in this chapter, each primary channel is first sensed for a short sensing time. If the average received power in the first stage is below a certain threshold, the channel is sensed for another longer sensing time. Otherwise, the detector proceeds to the next primary channel. Although the decision to declare a channel as idle is made only after the second stage, the first stage allows the spectrum detector to quickly skip through most of the busy channels. We show that the proposed two-stage detection strategy noticeably reduces the mean time to detection of the first idle channel when the spectrum utilization is medium to high.

Note that the problem considered in this chapter is different from the spectrum sensing problems considered in the context of sequential change detection (quickest detection) [25]. A common assumption in this context is that a SU, or multiple SUs as in [63], tries to detect a change in the activity of a PU within a single primary channel. It is also assumed that the PU is initially inactive and becomes active at an unknown time. A similar assumption is that the PU is initially active and vacates the channel at an unknown time. The goal is to detect these changes as quickly as possible subject to a constraint on the Average Run Length (ARL) of false alarm. However, in this chapter, we consider a scenario where a SU is trying to find an idle channel within multiple primary channels, and it is assumed that the occupancy states of these channels do not change during the search process.

We describe the regulatory constraints and the structure of the energy detector employed by the SU in Sections 3.1 and 3.2, respectively. We elaborate on the system model and the problem considered in [20] in Section 3.3. We introduce the two-stage spectrum detection strategy in Section 3.4. Finally, we evaluate the performance of the proposed spectrum detection strategy in Section 3.5.
3.1 Regulatory Constraints

In this chapter, we adopt the system model proposed in [20], where the interference due to secondary transmissions is considered harmful if it causes the Signal-to-Interference Ratio (SIR) at a primary receiver to fall below a predefined threshold $\Gamma$. This threshold depends on many factors including the primary receiver’s robustness against interference, type of the primary service, and the characteristics of the interfering signal and should be determined by regulatory bodies. Given the threshold $\Gamma$, the interference range of a secondary transmitter is defined as the maximum distance at which the resulting interference may still be harmful to primary receivers. Let $P_P$ and $P_S$ denote the transmit powers of the PUs and SUs, respectively. The interference range of a secondary transmitter $D$ is then derived from the following relation

$$\frac{P_P R^{-\alpha}}{P_S D^{-\alpha}} = \Gamma,$$  \hspace{1cm} (3.1)

where $R$ is the maximum distance between a primary transmitter and its respective receiver, and $\alpha$ is the path loss exponent. The detection sensitivity $\gamma_{\text{min}}$ is defined as the minimum SNR at which a primary signal should still be detected by a SU. Since a SU can only detect primary transmitters (not primary receivers), it should be able to detect any active primary transmitter within a range of $D + R$. Therefore, $\gamma_{\text{min}}$ is given by

$$\gamma_{\text{min}} = \frac{P_P(D + R)^{-\alpha}}{N_0 W},$$ \hspace{1cm} (3.2)

where $N_0$ is the equivalent noise power spectral density, $W$ is the signal bandwidth, and $D$ is determined from (3.1).
3.2 Energy Detection

The binary hypothesis test for spectrum sensing at each channel is formulated as follows

\[ H_0 : \quad x(n) = v(n) \]
\[ H_1 : \quad x(n) = h s(n) + v(n), \]

(3.3)

where \( x(n) \) is the received baseband signal by the SU at time \( n \), \( s(n) \) is the primary transmitted signal, \( h \) is the channel gain between the primary transmitter and the SU, and \( v(n) \) is the additive background noise. In fact, background noise is an aggregation of various sources such as thermal noise, leakage of signals from other bands, and interference due to transmissions of PUs far away [51]. In this chapter, we assume that \( v(n) \) can be approximated by a stationary white Gaussian noise with zero mean and a known variance \( \sigma_v^2 \).

When a SU’s a priori knowledge is limited to local noise statistics, the optimal detector is an energy detector, where the received signal over each frequency band is squared and integrated over the sensing time [45]. We assume that each SU is equipped with a tunable BPF followed by an A/D converter to search one frequency channel at a time. The same BPF may also be used for signal reception when a secondary connection is already established. For each channel, the test statistic is the average received signal power during the sensing time \( T \) normalized by the noise power:

\[ Y = \frac{1}{N\sigma_v^2} \sum_{n=1}^{N} \|x(n)\|^2, \]

(3.4)

where \( N \) is the number of samples, which is assumed to be equal to the time-bandwidth product, \( TW \) (Nyquist sampling). According to the central limit theorem [40], for large values of \( N \) (e.g., \( N \geq 50 \)), the test statistic \( Y \) is approximately normally distributed with mean

\[ E[Y] = \begin{cases} 1 & H_0 \\ \gamma + 1 & H_1 \end{cases}, \]

(3.5)
and variance
\[ \text{Var}[Y] = \begin{cases} \frac{2}{N} & \mathcal{H}_0 \\ \frac{2(2\gamma+1)}{N} & \mathcal{H}_1, \end{cases} \] (3.6)

where \( \gamma = \frac{P|h|^2}{\sigma^2} \).

As mentioned earlier, to avoid harmful interference to potential PUs, SUs should be able to detect primary signals at SNR levels as low as \( \gamma_{\text{min}} \). Therefore, the corresponding false-alarm and miss probabilities are given by

\[ P_f(N) = P(Y > \lambda | \mathcal{H}_0) = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{N}}{2} (\lambda - 1) \right), \] (3.7)
\[ P_m(N) = P(Y < \lambda | \mathcal{H}_1) = 1 - \frac{1}{2} \text{erfc} \left( \frac{\sqrt{N} \lambda - (1 + \gamma_{\text{min}})}{2 \sqrt{2\gamma_{\text{min}} + 1}} \right), \] (3.8)

where \( \lambda \) is the decision threshold, and \( \text{erfc}(\cdot) \) is the complementary error function. For a fixed number of samples \( N \), a lower miss probability results in a higher false-alarm probability. The regulatory constraint on the reliability of PU detection may be expressed as

\[ P_m(N)|_{\gamma=\gamma_{\text{min}}} \leq \beta, \] (3.9)

where \( \beta \) is set by the regulator. Combining (3.7), (3.8), and (3.9), we have

\[ P_f(N) = \frac{1}{2} \text{erfc} \left( \text{erfc}^{-1}(2(1 - \beta)) \sqrt{2\gamma_{\text{min}} + 1} + \frac{\sqrt{N}}{2 \gamma_{\text{min}}} \right). \] (3.10)

### 3.3 Channel Search Time Minimization

We consider a primary communication system operating over a wideband spectrum which is divided into \( c \) frequency channels of bandwidth \( W \). At each time instant, a channel is busy with probability \( u \) and idle with probability \( 1 - u \), where \( u \) is the average utilization of the primary band. We also assume that the occupancy of each channel is independent of all other channels. When the number of primary channels \( c \) is sufficiently large (such
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that \( u^c \ll 1 \), with high probability, at least one of the primary channels is idle during the channel search.

Before a connection can be established, a SU has to sense the primary channels sequentially looking for the first idle channel. The average length of the search interval \( N_{\text{search}} \) is equal to the number of samples taken per channel multiplied by the average number of channels sensed before an idle channel is found. While a shortened sensing time per channel allows a SU to search over more primary channels per unit of time, the corresponding higher false-alarm probability results in a higher number of missed idle channels. In [20], the number of samples per channel \( N \) is optimized in order to minimize the delay incurred by the SU:

\[
\tilde{N}_{\text{search}} = \min_N \frac{N}{(1-u)(1-P_f(N))} \quad \text{s. t.} \quad P_f(N) \leq 1 - \frac{1 - \sqrt{\epsilon}}{1 - u}, \tag{3.11}
\]

where \( P_f(N) \) is given by (3.10), and the constraint in (3.11) ensures that an idle primary channel is found at least with probability \( \epsilon \) before going through all the primary channels. As we will see in the next section, this approach results in a long search time when the spectrum utilization is high.

3.4 Two-stage Spectrum Detection

For large values of \( c \), we can approximate the term on the right hand side of the constraint in (3.11) to write it as

\[
P_f(N) \leq 1 - \frac{1 - \sqrt{\epsilon}}{1 - u} \approx 1 + \frac{\log_e(\epsilon)}{C_1} = 1 + \frac{\log_e(\epsilon)}{C_1}, \tag{3.12}
\]
where $C_1 = c(1-u)$ denotes the average number of idle channels. Hence the optimization problem in (3.11) can be written as

$$\hat{N}_{\text{search}} = \min_N \frac{N}{(1-u)(1-P_f(N))} \quad \text{s. t.} \quad P_f(N) \leq 1 + \frac{\log_e(\epsilon)}{C_1}. \quad (3.13)$$

As can be seen from (3.13), the higher the utilization factor $u$, the higher the average length of the search interval. This is because, in general, a spectrum sensor has to go through more primary channels before successfully identifying an idle channel. In this section, we propose a two-stage spectrum detection strategy to adjust the focus on primary channels which are more likely to be idle.

First, each channel is sensed for a time of $N_1$ samples. Based on the average received power in the first stage $Y_1$, the spectrum sensor decides either to proceed to the second stage, where the channel is sensed for $N_2$ more samples, or to move to the next channel. This process continues until an idle primary channel is found. In fact, the first stage aims to decrease the effective utilization factor for the channels included in the second stage. The decision threshold in the second stage is set according to the regulatory constraint in (3.9).

We assume a path loss model with a path loss exponent $\alpha$. We also assume that under $\mathcal{H}_1$ there is one active primary transmitter in the proximity of the SU whose location is modelled as a uniformly distributed random variable within a circle of radius $D + R$. If a primary transmitter is located at distance $r$ from the SU, the received primary SNR, $\gamma$, at the SU is given by

$$\gamma = \gamma_{\text{min}} \left( \frac{r}{D + R} \right)^{-\alpha}. \quad (3.14)$$

Using (3.2), (3.5), (3.6), and (3.14), the probability density function of the average received power in the first stage under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are given by

$$f_{N_1}(Y_1 = y|\mathcal{H}_0) = \frac{e^{-(y-1)^2/(4/N_1)}}{\sqrt{4\pi/N_1}} \quad (3.15)$$
and
\[
\begin{align*}
f_{N_1}(Y_1 = y|\mathcal{H}_1) &= \int_0^{D+R} 2r e^{-\left(y-(\gamma_{\min}(r/(D+R))^{-\alpha}+1)\right)^2/(\textcolor{red}{(4/N_1)}(2\gamma_{\min}(r/(D+R))^{-\alpha}+1))} \sqrt{4\pi(2\gamma_{\min}r^{-\alpha}+1)/N_1} \, dr \\
&= \int_0^1 2r' e^{-\left(y-(\gamma_{\min}r'^{-\alpha}+1)\right)^2/(\textcolor{red}{(4/N_1)}(2\gamma_{\min}r'^{-\alpha}+1))} \sqrt{4\pi(2\gamma_{\min}r'^{-\alpha}+1)/N_1} \, dr' \tag{3.16}
\end{align*}
\]
respectively.

In the first stage, we choose the decision threshold \(\lambda_1\) to be equal to 1 (0 dB) so that an idle channel would proceed to the second stage with probability \(1/2\). The effective utilization factor for the channels proceeded to the second stage is given by
\[
u_1(N_1) = \frac{u F_{N_1}(1|\mathcal{H}_1)}{u F_{N_1}(1|\mathcal{H}_1) + (1 - u)(1/2)} \tag{3.17}
\]
where \(F_{N_1}(\cdot|\mathcal{H}_0)\) and \(F_{N_1}(\cdot|\mathcal{H}_1)\) are the cumulative distribution functions of the average received power in the first stage under \(\mathcal{H}_0\) and \(\mathcal{H}_1\), respectively. The decision threshold in the second stage, \(\lambda_2\), is chosen such that the regulatory constraint on the reliability of PU detection is satisfied:
\[
P_m(N_2)|_{\gamma=\gamma_{\min}} \leq \beta. \tag{3.18}
\]

**Theorem 1.** For large values of \(c\), the average length of the search interval for the proposed two-stage spectrum detection strategy is approximated by
\[
N_{\text{search}} \approx \frac{N_1}{((1-u)/2 + u F_{N_1}(1|\mathcal{H}_1))(1-u_1(N_1))(1-P_f(N_2))}
\]
\[
+ \frac{N_2}{(1-u_1(N_1))(1-P_f(N_2))} \tag{3.19}
\]
where the first and the second terms represent the aggregate time spent in the first and second stage detections, respectively.

**Proof.** See Appendix A.

Note that the second term in (3.19) is similar to the objective function in (3.13) except the fact that the utilization factor \(u\) is replaced with \(u_1(N_1)\). The number of samples
in the first and second stage, \(N_1\) and \(N_2\), have to be optimized in order to achieve the minimum length of the search interval:

\[
\hat{N}_{\text{search}} \approx \min_{N_1, N_2} \left( \frac{1}{(1 - u_1(N_1))(1 - P_1(N_2))} \left( \frac{N_1}{((1 - u)/2 + u F_{N_1}(1|H_1))} + N_2 \right) \right)
\]

s. t. \(P_1(N_2) \leq 1 + \frac{\log_e(\epsilon)}{(1/2) C_1} \).

Again, the constraint in (3.20) ensures that an idle primary channel is found at least with probability \(\epsilon\).

### 3.5 Numerical Results

In our numerical example, we set \(c = 200\), \(\alpha = 3\), \(\beta = 0.001\), and \(\epsilon = 0.01\). Figure 3.1 illustrates the average length of the search interval \(\hat{N}_{\text{search}}\) as a function of the utilization factor \(u\) for the single-stage and two-stage detection strategies for \(\gamma_{\min} = -5\) dB, \(\gamma_{\min} = -10\) dB, and \(\gamma_{\min} = -15\) dB. As can be seen from this figure, the proposed two-stage detection strategy significantly outperforms the conventional single-stage strategy when the spectrum utilization is high. However, the improvement due to the proposed two-stage strategy is more noticeable for higher values of \(\gamma_{\min}\). For a given PU transmit power, a lower value of \(\gamma_{\min}\) indicates a larger detection range, \(D + R\). For such a case, based on the system model described in Section 3.4, the received power in the first stage is more likely to be lower than the detection threshold, i.e., 1. Therefore, more primary channels proceed to the next stage, and there is no significant reduction in the effective utilization factor for the second stage without a significant increase in \(N_1\). As a result, the gain due to the two-stage strategy is less significant for lower values of \(\gamma_{\min}\).

Any uncertainty in the distribution of the primary transmitters in the network, their transmit powers, or the path loss model results in an uncertainty in the cumulative distribution function \(F_{N_1}(\cdot|H_1)\), which in turn results in suboptimal values of \(N_1\) and \(N_2\), as seen in (3.20). In this numerical example, we evaluate the sensitivity of the proposed
two-stage detection strategy in Section 3.4 with respect to the path loss exponent \( \alpha \). Figure 3.1 illustrates the average length of the search interval \( \hat{N}_{\text{search}} \) as a function of the spectrum utilization factor \( u \) for the case where \( \gamma_{\text{min}} = -10 \) dB and the true value of \( \alpha \) equals 5. We evaluate the performance of the proposed strategy for two cases: using the correct value of \( \alpha = 5 \) and using a postulated erroneous value of \( \alpha = 3 \). As can be seen from Figure 3.2, the proposed two-stage spectrum detection strategy is quite robust.
against the uncertainty in the path loss exponent.

![Figure 3.2: The average length of the search interval $\hat{N}_{\text{search}}$ vs. the utilization factor $u$ for $\gamma_{\text{min}} = -10$ dB and $\alpha = 5$.](image)

3.6 Summary and Practical Considerations

In this chapter, we proposed a two-stage spectrum detection strategy that decreases the average channel search time by allowing the spectrum detector to concentrate on frequency channels which are more likely to be vacant. We showed that the proposed detection strategy significantly outperforms the conventional single-stage strategy when the spectrum utilization is high. The proposed detection strategy may be useful in any OSA scenario where a SU is searching through several primary channels to find an idle channel. However, there are a number of practical issues to be addressed before such potential is realized.

In addition to the type of the spectrum detector and $\gamma_{\text{min}}$, the improvement due to the two-stage detection strategy depends on several factors including the spectrum utilization...
factor, the path loss model, and the spatial distribution of the primary transmitters as well as their transmit powers. In the case where such information is not available, or it is not accurate, the optimal values of $N_1$ and $N_2$ cannot be obtained. Therefore, adaptive versions of the proposed spectrum sensing strategy are of particular interest. Depending on the hardware used, it may be necessary to take into account the time it takes the BPF to switch to another channel. In such a case, the spectrum detector would be more conservative in skipping through the primary channels, and a lower false-alarm probability for each channel would be desired.
Chapter 4

A Cooperative Channel Search Strategy for OSA

In this chapter, we consider a scenario where several cooperating SUs are given $L$ sensing slots to jointly locate idle channels across multiple primary channels. It is assumed that the occupancy state of the primary channels do not change during the search process. In each slot, each SU is assigned to sense one of the channels and report the outcome to the other SUs. The occupancy of each channel is decided after the $L$ sensing slots according to a fusion rule, e.g., the OR-rule. We use the structure of the fusion rule to optimize the channel search process. We aim to find the optimal rule for choosing the channels to be sensed by each SU, in each sensing slot, such that the expected number of detected idle channels is maximized. We show that the proposed cooperative channel search strategy significantly increases the opportunistic use of spectrum when compared to the brute-force strategy, where the SUs scan the primary band concurrently.

We provide a formal definition of the wide-band cooperative spectrum search problem in Section 4.1. In Section 4.2, we model this problem in its most general form as a finite horizon Markov decision process. In Section 4.3, we consider the special case where individual SUs quantize their observations to 1-bit sensing decisions, which are then
Chapter 4. A Cooperative Channel Search Strategy for OSA

fused together according to the OR-rule. We derive closed-form solutions for the optimal sensing strategy and the associated value functions for the case of two cooperating SUs and discuss the general structure of the solution for the case of more than two cooperating SUs. In Section 4.4, we provide an example to illustrate how system parameters such as the structure of the spectrum detector at each SU, the regulatory constraints, and the characteristics of the primary network are related to the model parameters in Sections 4.2-4.4. We conclude this chapter with a summary and a discussion in Section 4.5.

4.1 Problem Formulation

We assume that the spectrum available for opportunistic access is divided into \( c \) primary channels of bandwidth \( W \). There are \( n \) cooperating SUs trying to maximize the number of detected idle channels after \( L \) sensing slots while satisfying specific regulatory constraints. Each sensing slot is of length \( \tau \). At the beginning of each stage, or sensing slot, each SU is assigned to sense one of the primary channels and report its observation to a fusion center (which can be one of the SUs). In the case of energy detection, this report can be a quantized version of the accumulated energy during that sensing slot. All these reports are stored in an observation state matrix in the fusion center. The observation state, aside from the time dependence, is given at any stage by an \( n \times c \) dimensional matrix:

\[
S = \begin{bmatrix}
  s_{11} & s_{12} & \cdots & s_{1c} \\
  \vdots & \vdots & \ddots & \vdots \\
  s_{n1} & s_{n2} & \cdots & s_{nc}
\end{bmatrix} = \begin{bmatrix}
  s_1 \\
  s_2 \\
  \vdots \\
  s_c
\end{bmatrix}
\]  

(4.1)

with

\[
s_i = [s_{i1}, \cdots, s_{in}]^T
\]

(4.2)
and

\[ s_{ij} = \begin{cases} q_{ij} & \text{if the } j\text{th channel is sensed by the } i\text{th SU} \\ \times & \text{otherwise} \end{cases} \]  \hspace{1cm} (4.3)

where \( q_{ij} \) denotes the outcome of sensing the \( j\)th channel by the \( i\)th SU. In the case of one-bit quantization, we have \( q_{ij} \in \{0 \text{ (idle)}, 1 \text{ (busy)}\} \), and therefore, \( s_{ij} \in \mathcal{M} \triangleq \{0 \text{ (idle)}, 1 \text{ (busy)}, \times \text{ (not sensed yet)}\} \) for any \( i \) and \( j \); using a two-bit quantizer, we have \( q_{ij} \in \{00, 01, 10, 11\} \), and therefore, the set \( \mathcal{M} \) becomes \( \mathcal{M} = \{00, 01, 10, 11, \times \text{ (not sensed)}\} \); and so forth. Let us denote by \( S^l \) the observation state matrix after \( l \) sensing slots. The initial observation state in the fusion center \( S^0 \) is given by a matrix of all \( \times \)'s:

\[ S^0 = \begin{bmatrix} \times & \times & \cdots & \times \\ \vdots & \vdots & \ddots & \vdots \\ \times & \times & \cdots & \times \end{bmatrix}. \]  \hspace{1cm} (4.4)

Let us say that the \( i\)th SU senses the \( j\)th channel in the first slot to produce sensing outcome \( q_{ij} \). Then, at the end of slot 1, the corresponding entry in the state observation matrix is updated from \( \times \) to \( q_{ij} \). This process continues until the end of slot \( L \).

The number of detected idle channels \( N_I \) is a function of the final observation state \( S^L \) and is given by:

\[ N_I = \sum_{i=1}^{c} I(s_i^L) \]  \hspace{1cm} (4.5)

where \( s_i^L \) is the \( i\)th column of \( S^L \), and \( I \) is an indicator function. Note that \( I(s_i^L) = 1 \) indicates that the observation vector for the \( i\)th channel \( s_i^L \) satisfies the specific regulatory constraints that limit the interference to PUs, and, therefore, can be used for secondary transmissions. The imposed constraints also determine the quantization thresholds at each individual SU. In a different scenario, these constraints may be adjusted for a coarse sensing task, which is followed by a second sensing stage. In the case of two cooperating
SUs, single bit quantization, and OR fusion rule, the function $I$ is given by

$$I(s) = \begin{cases} 
1 & \text{for } s = [0 \ 0]^T \\
0 & \text{otherwise}
\end{cases}.$$  \hfill (4.6)

In this chapter, we do not aim to improve the performance of spectrum detection for a single primary channel (e.g., in terms of miss and false-alarm probabilities). As a result, we assume that the decision rule for each primary channel, i.e., the indicator function $I$ as well as the detection thresholds, is given and independent of the sensing strategy. Our goal is to speed up the channel search process by designing a sensing strategy that allows the cooperative SUs to sense the channels that are more likely to be identified as idle. This goal is achieved by exploiting the correlation among the observations of different SUs. Here, we assume that the regulatory constraints are strict enough such that the detected idle channels can be successfully used for secondary transmissions. In other words, we ignore misses in this chapter.

### 4.2 Optimal Sensing strategy

A strategy for spectrum sensing $\delta$ is a sequence of functions $\{\delta_L(S)\}$, one for each value of $L$, that map each observation state $S$ to a sensing action of the form $A = (a_1, a_2, \cdots, a_n)$, where $a_i$ denotes the index of the channel to be sensed by the $i$th SU. The goal is to maximize the expected number of detected idle channels after $L$ sensing slots given that the initial state is $S^0$:

$$\max_{\delta} E\{N_1\} = \max_{\delta} E\{\sum_{i=1}^{c} I(s_i^L)\}.$$  \hfill (4.7)

This problem can be modeled as a finite horizon Markov decision process [14] with the following reward function:

$$R(S; A) = E\{\sum_{j \in \{a_i\}_{i=1}^{n}} I(s_j)\}. \hfill (4.8)$$
Therefore, the reward function gives the expected number of idle channels instantly identified as a result of sensing action $A$. We define the value function $V_{\delta L}(S)$ as the expected number of detected idle channels after $L$ sensing slots following strategy $\delta$ starting from observation state $S$. Let $V^*_L(S)$ denote the total expected reward associated with the optimal sensing strategy $\delta^*$. The recurrence relations (Bellman equations) for the sequences $\{V^*_L(S)\}$ and $\{\delta^*_L(S)\}$ are given by

\[
\delta^*_L(S) = \arg \max_A \left[ R(S; A) + \sum_{S'} P_A(S'; S)V^*_{L-1}(S') \right], \quad L \geq 2 \tag{4.9}
\]

\[
V^*_L(S) = R(S; \delta^*_L(S)) + \sum_{S'} P_{\delta^*_L(S)}(S'; S)V^*_{L-1}(S'), \quad L \geq 2 \tag{4.10}
\]

where $A$ denotes the set of sensing actions, $P_A(S'; S)$ is the probability that action $A$ in state $S$ will lead to state $S'$, and

\[
\delta^*_1(S) = \arg \max_A R(S; A) \tag{4.11}
\]

\[
V^*_1(S) = R(S; \delta^*_1(S)). \tag{4.12}
\]

Note that the values of the reward function and transition probabilities generally depend on several factors, including the fading/shadowing characteristics of the environment, structure of the spectrum detector at each SU, regulatory constraints, density of PUs in the network as well as their transmitted powers, and correlation in the occupancy of different channels. Finite horizon Markov decision processes in general can be solved in polynomial time either by dynamic programming or by linear programming, [39].

### 4.3 OR-rule Detector

In this section, we consider the case where OR-rule is used to fuse the individual decisions made by the $n$ cooperating SUs. Upon sensing a primary channel, each SU transmits its individual 1-bit sensing decision regarding the occupancy of that channel to a fusion center. A channel can be used by the secondary network only if it is sensed by all the
cooperating SUs and all the decisions are “0”. To simplify the analysis, we assume that $c \geq nL$, and therefore, not all the channels can be sensed by at least one SU. We also assume that the joint distributions of the measurements of the cooperating SUs are independent and identical for all the channels. In other words, there is no bias toward a certain primary channel, and there is no correlation in the occupancy of different channels. The above assumptions significantly reduce the effective size of the state space in (4.1); in fact, every permutation of the columns in $S$ results in an equivalent observation state. Although the above assumptions may not hold in real situations, e.g., when the density of PUs is different for different channels, or when the cooperating SUs experience different shadowing conditions, we observe in Section 4.4 that a remarkable gain is obtainable due to the proposed sensing strategy even when the above assumptions do not hold. In this section, we will derive closed-form solutions for the optimal sensing strategy and the associated value functions for the case of two cooperating SUs and discuss the general structure of the optimal sensing strategy for the case of more than two cooperating SUs.

4.3.1 Two Cooperating SUs

For the case of two cooperating SUs, the observation state can be simplified to an ordered pair $S = (\alpha, \beta)$, where $\alpha$ and $\beta$ denote the number of channels of type I and type II, respectively (see Figure 4.1). Type I (II) channels are those that are declared idle by the first (second) SU but not yet sensed by the second (first) SU. Note that the channels that are declared busy by at least one of the SUs are not sensed in the next sensing slots and have no effect on the future sensing decisions. A similar argument applies to the already detected idle channels, that is, the channels that are declared idle by both SUs. Since we assume that $c \geq nL$, at any stage of the search process, irrespective of the sensing strategy, there exist channels that have not been sensed by any of the SUs. Therefore, the number of these channels also has no effect on the sensing strategy.

Following the above discussion, there are five distinct sensing actions, which are illus-
Type I channel: SU_1 \begin{array}{c} 0 \\ \text{X} \end{array} \quad \text{Type II channel: } SU_1 \begin{array}{c} \text{X} \\ 0 \end{array}

Figure 4.1: Channels of type I and type II for the problem considered in Section 4.3.

trated in Figure 4.2. Note that each channel is represented by a column in the observation state. In Action 1, the first and second SUs respectively sense a Type II and Type I channel. In Action 2, the first SU chooses a channel which is not sensed by any of the SUs while the second SU senses a Type I channel. In Action 3, the first SU senses a Type II channel while the second SU chooses a channel which is not sensed by any of the SUs. The cooperating SUs sense two different unexplored channels in Action 4, while in Action 5, they both sense the same primary channel.

\begin{tabular}{|c|c|c|c|c|}
\hline
Action 1 & Action 2 & Action 3 & Action 4 & Action 5 \\
\hline
SU_1 & 0 & 0 & \text{X} & \text{X} \\
SU_2 & \text{X} & \text{X} & 0 & \text{X} \\
\hline
\end{tabular}

\quad \square \quad \text{channel to be sensed by each SU}

Figure 4.2: Different sensing actions for the problem considered in Section 4.3.

Let us assume that

\[
P(q_{1i} = q_{2i} = 0) = p_0 \\
P(q_{1i} = 0, q_{2i} = 1) = P(q_{1i} = 1, q_{2i} = 0) = \frac{p_1}{2} \\
P(q_{1i} = q_{2i} = 1) = p_2
\]

for \(1 \leq i \leq c\) \hspace{1cm} (4.13)

where \(P(\cdot)\) denotes the probability of an event, and \(p_0, p_1, \) and \(p_2\) denote the probability that a channel is declared busy by none, one, or both the cooperating SUs, respectively. See Section 4.4 for an instance of how these probabilities depend on the structure of the spectrum detector, the regulatory constraints, and the characteristics of the primary
network. Figure 4.3 illustrates the state transitions and their corresponding probabilities for the special case where the current state is $S = (1, 0)$ and Action 2 is taken. Again, note that each row corresponds to a SU and each column corresponds to a channel in the observation state. As mentioned earlier, the channels that are declared busy by at least one of the SUs and the channels that are already detected idle are eliminated from the observation state. Since we assume that $c \geq nL$, there are always channels that have not been sensed by any of the SUs. Therefore, channels eliminated from the observation state are replaced with those that are not sensed by any of the SUs. The state transition probabilities $P_A(\alpha', \beta'; \alpha, \beta)$ as a result of different sensing actions are shown in Figure 4.4.

The expected number of newly detected idle channels as a result of sensing action $A$
Figure 4.4: The state transition probabilities as a result of different sensing actions for the case of two collaborating SUs.

when the current state is \((\alpha, \beta)\) is given by

\[
R(\alpha, \beta; A) = \begin{cases} 
\frac{4p_0}{2p_0 + p_1} & \text{for } A = 1 \\
\frac{2p_0}{2p_0 + p_1} & \text{for } A = 2 \\
\frac{2p_0}{2p_0 + p_1} & \text{for } A = 3 \\
0 & \text{for } A = 4 \\
p_0 & \text{for } A = 5 
\end{cases}
\]  

(4.14)

The Bellman equations are written as

\[
V^*_L(\alpha, \beta) = \max_{A_{\alpha, \beta}} \left[ R(\alpha, \beta; A) + \sum_{\alpha', \beta'} P_A(\alpha', \beta'; \alpha, \beta) V^*_L(\alpha', \beta') \right], \text{ for } L \geq 2, \text{ and } (4.15)
\]

\[
V^*_1(\alpha, \beta) = \max_{A_{\alpha, \beta}} R(\alpha, \beta; A) \]  

(4.16)
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where $A_{\alpha,\beta}$ denotes the set of admissible actions given that the state is $(\alpha, \beta)$:

$$A_{\alpha,\beta} = \begin{cases} 
\{1, 2, 3, 4, 5\} & \text{for } \alpha, \beta > 0 \\
\{2, 4, 5\} & \text{for } \alpha > 0, \beta = 0 \\
\{3, 4, 5\} & \text{for } \alpha = 0, \beta > 0 \\
\{4, 5\} & \text{for } \alpha = \beta = 0 
\end{cases} \quad (4.17)$$

as an example, Action 2 is not admissible when there is no Type I channel.

**Theorem 2.** The solution for $V^*_L = V^*_L(0, 0)$ in the recurrence relation in (4.15) is given by

$$V^*_L = V^*_L(0, 0) = Lp_0 \left( \frac{2}{2 - \rho} - \frac{1}{(2 - \rho)^2} L \left[ 1 - (1 - \rho)^{2L} \right] \right) \quad (4.18)$$

where $\rho = \frac{p_1}{2} + p_2$. Furthermore, an optimal sensing strategy $\delta^*_L$ is expressed as

$$\delta^*_L(\alpha, \beta) = \begin{cases} 
1 & \text{for } \alpha, \beta > 0, \quad L \geq 1 \\
2 & \text{for } \alpha > 0, \beta = 0, \quad L \geq 1 \\
3 & \text{for } \alpha = 0, \beta > 0, \quad L \geq 1 \\
4 & \text{for } \alpha = \beta = 0, \quad L \geq 2 \\
5 & \text{for } \alpha = \beta = 0, \quad L = 1 
\end{cases} \quad (4.19)$$

**Proof.** See Appendix B.

As the above theorem implies, the optimal sensing strategy in (4.19) does not depend on $p_0$, $p_1$, and $p_2$; although, the associated value function does. In this strategy, each user first senses a channel which is already successfully sensed by the other SU (a type I or type II channel). Therefore, Action 1, whenever admissible, is preferable to other actions. Accordingly, Action 2 and 3 are preferable to Actions 4 and 5. When there is no type I or type II channels, the cooperating SUs should sense two different unexplored
channels except at the last stage, where Action 5 is the optimal choice. Note that if the cooperating SUs scan the primary channels synchronously, i.e., if they sense the same channel in each sensing slot, the expected number of detected idle channels is equal to $L_p$. Therefore, we can define the relative gain of the optimal sensing strategy as

$$g_L(\rho) = \frac{V^*_L(0,0)}{L_p} = \frac{2}{2-\rho} - \frac{1}{(2-\rho)^2 L}[1 - (1-\rho)^{2L}]. \quad (4.20)$$

Corollary 1. The relative gain of the optimal sensing strategy $g_L(\rho)$ is an increasing function of $\rho$.

Proof. Differentiating (4.20) with respect to $\rho$ yields

$$\frac{d}{d\rho} g_L(\rho) = \frac{16(L-1)}{L} [1 - (1-\rho)^{2L}] + 16(1-\rho) \left[1 - (1-\rho)^{2L-2}\right] \geq 0; \quad (4.21)$$

note that $0 \leq \rho \leq 1$.

Corollary 2. The relative gain of the optimal sensing strategy $g_L(\rho)$ is an increasing function of the number of sensing slots $L$.

Proof. We have

$$g_{L+1}(\rho) - g_L(\rho) = \frac{[1 - (1-\rho)^2](1-\rho)^{2L}}{(2-\rho)^2 L(L+1)} \sum_{i=0}^{L-1} \left[ \left( \frac{1}{1-\rho} \right)^{2(L-i)} - 1 \right] \geq 0, \text{ for } L \geq 1 \quad (4.22)$$

and the asymptotic gain is given by $2/(2-\rho)$.

Figure 4.5 illustrates the relative gain of the optimal sensing strategy as a function of $\rho$, for different values of $L$. As it is observed, this gain is more significant for higher values of $\rho$. Figure 4.6 illustrates the relative gain of the optimal sensing strategy as a function of $L$, for different values of $\rho$. As can be seen from Figure 4.6, this gain is an increasing function of the number of sensing slots $L$. 
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4.3.2 Three or More Cooperating SUs

The optimal spectrum sensing strategy for the case of \( n > 2 \) cooperating SUs, in general, depends on \( p_i \)'s for \( 0 \leq i \leq n \), where \( p_i \) is the probability that a channel is declared busy by \( i \) out of \( n \) SUs. As an example, consider the case where three cooperating SUs try to detect idle channels when there are two sensing slots available. The observation state transition probabilities as a result of three possible sensing actions at the first stage are shown in Figure 4.7. The optimal sensing actions at the last stage, i.e., when \( L = 1 \), are also depicted in this figure. Note that there are several observation states (sensing actions) that are similar except that the role of the cooperating SUs are swapped. Since these observation states and sensing actions respectively have the same expected value and expected reward, they can be considered as a single state or action. As in the case of two cooperating SUs, channels that are declared busy by at least one of the SUs and channels that are already declared idle are eliminated from the observation state. We
The relative gain due to optimal sensing, $g_L(\rho)$, the number of sensing slots $L$.

Figure 4.6: The relative gain of the optimal sensing strategy $g_L(\rho)$ versus the number of sensing slots $L$, for different values of $\rho$.

are interested in finding the optimal sensing action at the first stage, i.e., when $L = 2$:

$$\delta^*_2(S) = \arg \max_A [R(S; A) + \sum_{S'} P_A(S'; S_0)V_1^*(S')]$$

We have

$$R(S; A) + \sum_{S'} P_A(S'; S_0)V_1^*(S') = \begin{cases} 
(1 + \rho_1 + \rho_1^2 + \rho_1^3)p_0 & A = \text{Action 1} \\
(2 + \rho_1(\rho_1 + \rho_2))p_0 & A = \text{Action 2} \\
2p_0 & A = \text{Action 3}
\end{cases}$$

where

$$\rho_1 = \frac{1}{3}p_1 + \frac{2}{3}p_2 + p_3 \quad \text{and} \quad \rho_2 = \frac{1}{3}p_1 + \frac{1}{3}p_2.$$  

The optimal sensing action as a function of $\rho_1$ and $\rho_2$ is shown in Figure 4.8, which is obviously dependent on the values of $\rho_1$ and $\rho_2$. Note that we have $\rho_1 + \rho_2 \leq 1$. In the case of more than two cooperating SUs, the recurrence relations in (4.9)-(4.12) can be solved successively, starting with $\delta^*_1(S)$ and $V_1^*(S)$. Another approach to these problems is to formulate them as linear programming problems [14].
4.4 An Example: Energy Detection in a Rayleigh Fading Environment

We consider a scenario where two SUs cooperate in spectrum sensing by sharing their 1-bit decisions, “0” (idle) or “1” (busy). We assume that the cooperating SUs experience i.i.d. Rayleigh fading, they employ energy-detectors, and they use the same decision threshold $\lambda$. A channel is declared idle only if it is sensed by “both” SUs and the individual decisions are “0”. The problem of energy detection of an unknown band-limited signal under fading channels is considered in [15]. The received signal is first filtered by an ideal BPF. Then, the output of this filter is squared and integrated over the duration of a sensing slot $\tau$:

$$ Y \triangleq \frac{2}{N_0} \int_0^T r^2(t) \, dt $$

(4.26)

where $Y$ is the accumulated energy, $r(t)$ is the received signal, and $N_0$ is the noise power spectral density. In order to protect PUs against the adverse interference caused by the secondary network, regulatory constraints are imposed on the sensitivity of each SU, or on the sensitivity of the secondary system in the case of cooperation between several SUs. As one of such regulatory constraints, the SUs are required to reliably detect primary signals at SNR levels as low as $\bar{\gamma}_{\text{min}}$:

$$ 1 - Q_d(\bar{\gamma}_{\text{min}}) = [1 - P_d(\bar{\gamma}_{\text{min}})]^2 \leq \eta $$

(4.27)

where $P_d(\bar{\gamma}_{\text{min}})$ and $Q_d(\bar{\gamma}_{\text{min}})$ denote the individual and cooperative probabilities of detection, respectively, and $\eta$ is set by the regulator. A closed-form expression for the miss probability over Rayleigh fading channels is derived in [15]:

$$ P_d(\bar{\gamma}_{\text{min}}) = e^{-\frac{\lambda}{2}} \sum_{k=0}^{N/2-2} \frac{1}{k!} \left( \frac{\lambda}{2} \right)^k + \left( \frac{1 + \bar{\gamma}_{\text{min}}}{\bar{\gamma}_{\text{min}}} \right)^{N/2-1} $$

$$ \times \left[ e^{-\frac{\lambda}{2(1+\bar{\gamma}_{\text{min}})}} - e^{-\frac{\lambda}{2}} \sum_{k=0}^{N/2-2} \frac{1}{k!} \left( \frac{\lambda\bar{\gamma}_{\text{min}}}{2(1+\bar{\gamma}_{\text{min}})} \right)^k \right] $$

(4.28)
where \( N/2 \) is the number of samples per each of the in-phase and quadrature components of the primary signal within the duration of a sensing slot \( \tau \), and the threshold \( \lambda \) is obtained by inserting (4.28) into (4.27).

Let us assume that the active PUs that are transmitting within each frequency band are independently distributed across the network according to a two-dimensional Poisson point process with density \( \mu \) (see [62]). We also assume that secondary system is small enough so that the primary SNR variations (among the SUs) due to path loss can be neglected. Therefore, we suppose that the cooperating SUs are located near to each other in the vicinity of the origin of the two-dimensional plane. Therefore, the SNR averaged over the fading, \( \bar{\gamma} \), is the same for both SUs. If we consider only the nearest primary transmitter to the origin (see Section 4.5 for a justification of the system model used), the average SNR of the primary signal at the SUs is given by

\[
\bar{\gamma} = \frac{P}{\sigma^2 R_1^b}
\]

where \( P \) is the transmitted power of the PUs, \( \sigma^2 \) is the receiver noise power at the SUs receiver, \( R_1 \) is the distance of the nearest primary transmitter to the origin, and \( b \) is the power loss exponent. It can be shown that the probability density function of \( \bar{\gamma} \) is given by [62]

\[
f(\bar{\gamma}) = \frac{2\pi \mu P^2/b}{b \bar{\gamma}^{1+2/b}} e^{-\mu \pi \left( \frac{P}{\bar{\gamma}} \right)^{2/b}} \quad \text{for} \quad \bar{\gamma} > 0.
\]

For each SU, the probability density function of the accumulated energy \( Y \) for each primary channel conditioned on the average SNR, \( \bar{\gamma} \), is given by (see [15])

\[
f(y|\bar{\gamma}) = \int_0^\infty \frac{1}{2\bar{\gamma}} \left( \frac{y}{2\bar{\gamma}} \right)^{N/2} e^{-\left( \frac{2y+\bar{\gamma}}{2\bar{\gamma}} + \frac{\gamma}{\bar{\gamma}} \right)} I_{N/2-1}(\sqrt{2\gamma y}) d\gamma \quad \text{for} \quad y > 0
\]

where \( I_v(\cdot) \) is the \( v \)th order modified Bessel function of the first kind. Since the co-operating SUs experience independent fading, the joint probability density function of the accumulated energies by the two SUs, \( Y_1 \) and \( Y_2 \), conditioned on \( \bar{\gamma} \) can be expressed as

\[
f(y_1, y_2 | \bar{\gamma}) = f(y_1 | \bar{\gamma}) f(y_2 | \bar{\gamma}).
\]
Therefore, we have
\[ f(y_1, y_2) = \int_0^\infty f(y_1, y_2|\bar{\gamma})f(\bar{\gamma})d\bar{\gamma} \quad \text{for} \quad y_1, y_2 > 0. \] (4.33)

Let us denote respectively by \( q_1 \) and \( q_2 \) the decisions made by the first and the second SUs about the occupancy of a channel. We have

\[ P(q_1 = 0, q_2 = 0) = \int_0^\lambda \int_0^\lambda f(y_1, y_2)dy_1dy_2 = p_0 \] (4.34)

\[ P(q_1 = 0, q_2 = 1) = P(q_1 = 1, q_2 = 0) = \int_0^\lambda \int_0^\infty f(y_1, y_2)dy_1dy_2 = \frac{p_1}{2} \] (4.35)

\[ P(q_1 = 1, q_2 = 1) = \int_0^\lambda \int_0^\lambda f(y_1, y_2)dy_1dy_2 = p_2. \] (4.36)

In our numerical example, we choose \( P = 1, b = 3, \bar{\gamma}_{\text{min}} = 10 \text{ dB}, \eta = 0.1, \) and \( N = 10 \). Figure 4.9 illustrates the probabilities \( p_0, p_1, \) and \( p_2 \) as a function of the density of primary transmitters in the network \( \mu \). As expected, as the density of PUs in the network increases, \( p_0 \) and \( p_1 \) approach zero, while \( p_2 \) tends to one. Note that a higher density of PUs in the network corresponds to a more crowded spectrum and a higher value of \( \rho \). It is, in fact, more likely that the cooperating SUs end up being very close to a primary transmitter for higher values of \( \mu \). Figure 4.10 illustrates the relative gain of the optimal sensing strategy versus the density of PUs in the network for the case of \( L = 32 \). As can be observed from this figure, the higher the density of PUs in the network, the higher the gain of the optimal sensing strategy with respect to the brute-force strategy in which the cooperating SUs sense the same channel in each sensing slot.

In deriving the optimal sensing strategy given by (4.19), we assume that the joint distributions of the measurements of the cooperating SUs are identical for all the channels and symmetric between the SUs (see (4.13)); i.e., there is no bias toward a particular channel or a cooperating SU. However, this assumption may not hold in real situations, where the density of PUs may be different for different channels, or when the cooperating SUs experience different shadowing conditions. In the former case, our proposed solution
is first to sort the \( c \) channels according to increasing density of PUs, then apply the same sensing strategy as in (4.19). In the following, we observe that, even for the above two cases, a remarkable gain is obtainable due to the proposed sensing strategy in (4.19).

First, we consider the case where the density of PUs in different channels are not identical. We assume that there are 64 primary channels, and they are sorted according to their corresponding density of PUs. We let the density of PUs in the \( i \)th channel, \( \mu_i \), be given by

\[
\mu_i = \mu_0(1 - e^{-i})
\]

(4.37)

where \( \mu_0 \) is a parameter that controls the general crowdedness of the primary band. Figure 4.11 illustrates the relative gain of the sensing strategy in (4.19) versus \( \mu_0 \) for the case of \( L = 32 \). As it is observed from this figure, the obtainable gain is comparable to the case where the density of PUs is the same for all the primary channels. Furthermore, this gain is more noticeable for more crowded primary bands, i.e., higher values of \( \mu_0 \).

Finally, we consider the case where the two cooperating SUs experience different shadowing conditions. Let us assume that there is an extra signal loss due to the shadowing effect which is equal to 2 dB and 5 dB at the first and the second SUs, respectively. Let us define

\[
\begin{align*}
p_0 &= P(q_1 = q_2 = 0) \\
p_{11} &= P(q_1 = 1, q_2 = 0) \\
p_{12} &= P(q_1 = 0, q_2 = 1) \\
p_2 &= P(q_1 = q_2 = 1)
\end{align*}
\]

(4.38)

where \( p_0 \), \( p_{11} \), \( p_{12} \), and \( p_2 \) denote the probability that a channel is declared busy by none, the first SU, the second SU, and both the SUs, respectively. Figure 4.12 illustrates the probabilities \( p_0 \), \( p_{11} \), \( p_{12} \), and \( p_2 \) as a function of the density of primary transmitters in the network \( \mu \). As expected, we have \( p_{11} \geq p_{12} \) for all values of \( \mu \). The relative gain
due to the sensing strategy in (4.19) versus the density of PUs in the network for the case of $L = 32$ is shown in Figure 4.13. As can be seen from this figure, the obtainable gain is comparable to the case where the cooperating SUs experience similar shadowing conditions. Moreover, the higher the density of PUs in the network, the higher the relative gain due to the sensing strategy in (4.19).

4.5 Summary and Practical Considerations

In this chapter, we proposed an optimal sensing framework in order to solve the problem of wide-band cooperative spectrum search under a latency constraint. During each sensing slot, each SU tunes its receiver to one of the primary channels and reports its observation to a fusion center, where all these reports are stored in an observation state matrix. A spectrum sensing strategy maps each observation state to a sensing action that determines the indices of the channels to be sensed by different SUs. An optimal sensing strategy is one that maximizes the expected number of detected idle channels after $L$ sensing slots. We modelled this problem as a finite horizon Markov decision process and derived closed-form solutions for the case of two cooperating SUs and OR fusion rule. Through our numerical examples, we observed that the proposed spectrum sensing strategy significantly outperforms the brute-force strategy when the density of PUs in the network is high.

In the example considered in Section 4.4, we used a very simple system model to illustrate the effectiveness of our proposed cooperative channel search strategy and how it compares to the brute-force strategy. We considered only the nearest primary transmitter to the SUs to simplify the subsequent derivations. In fact, a more complicated system model, including multiple SUs and PUs, would be useful if our goal was to evaluate the performance of a cooperative sensing scheme with OR fusion rule in detecting the availability of a single primary channel. However, we assume that the decision rule for
accessing a primary channel, i.e., single bit quantization and OR fusion rule, is given, and the goal is to optimize the sensing actions in different sensing slots. As mentioned in Section 1.3.2, our focus in this thesis is on the design of the feedback block in Figure 1.5, while the access decisions are made in the feedforward block. We expect to obtain similar results for a more intricate system model because the relative gain of the proposed channel search strategy is only a function of $\rho = \frac{p_1}{2} + p_2$. This gain is always greater than one unless for the case where $p_1 = p_2 = 0$, i.e., where the primary channels are sensed to be idle by both SUs with probability one.

As a possible application of the proposed cooperative channel search strategy, we may consider wireless sensor nodes operating as SUs in a licensed spectrum. In battery constrained applications such as wireless sensor networks, the amount of energy consumed in the sensing and transmission phases is crucial. In the OSA scenario considered in this chapter, the number of idle channels found depends on the number of sensing slots $L$, which can be significantly larger than the number of cooperating SUs. Therefore, a small number of cooperating nodes are able to detect spectrum opportunities which can be used by a large number of sensor nodes. Consequently, the sensing task may be divided among different SUs to protect the sensor nodes from battery depletion. The frequency of spectrum sensing by the sensor nodes is inversely proportional to the value of $T$. Therefore, the power constraint for the sensor nodes is less significant for larger values of $T$, i.e., when the occupancy of the primary channels vary slowly. In this Chapter, we focused on the case where the number of cooperating SUs is small and they share only their 1-bit sensing decisions, “0” or “1”. Therefore, we assumed that the amount of energy consumed in the sensing and transmission phases satisfies the budget constraints. These constraints are more restrictive in the case of the brute-force strategy, where the SUs scan the primary band concurrently.

The proposed cooperative channel search strategy in this chapter may be adjusted to account for missing or delayed sensing outcomes, which are associated with an imperfect
control channel. Each missing sensing outcome can be considered as a $\times$ in the observation state matrix in (4.1). The delayed outcomes do not expire until the end of the search process, as it is assumed that the state of the primary channels remain unchanged. They can be used to update the entries of the observation state matrix whenever they are received. However, the proposed strategy is no longer optimal from a dynamic programming point of view. A number of other practical considerations encountered in implementing cooperative sensing schemes are discussed in Section 2.4.
Figure 4.7: The observation state transition probabilities as a result of three possible sensing actions for the case of three cooperating SUs.
Figure 4.8: The optimal sensing action for different values of $\rho_1$ and $\rho_2$ for the problem considered in Figure 4.7.

Figure 4.9: The probabilities $p_0$, $p_1$, and $p_2$ versus the density of primary transmitters in the network, $\mu$, for $P = 1$, $b = 3$, $\bar{\gamma}_{\text{min}} = 10$ dB, $\eta = 0.1$, and $N = 10$. 
Chapter 4. A Cooperative Channel Search Strategy for OSA

The density of primary transmitters in the network $\mu$.

Figure 4.10: The relative gain due to the sensing strategy in (4.19) versus the density of primary transmitters in the network, $\mu$, for $L = 32$, $P = 1$, $b = 3$, $\bar{\gamma}_{\text{min}} = 10$ dB, $\eta = 0.1$, and $N = 10$.

Figure 4.11: The relative gain due to the sensing strategy in (4.19) versus $\mu_0$, for $L = 32$, $P = 1$, $b = 3$, $\bar{\gamma}_{\text{min}} = 10$ dB, $\eta = 0.1$, and $N = 10$ for case where the density of PUs in different channels are not identical.
The density of primary transmitters in the network $\mu$.

Figure 4.12: The probabilities $p_0$, $p_{11}$, $p_{12}$, and $p_2$ versus the density of primary transmitters in the network, where there is an extra signal loss due to the shadowing effect which is equal to 2 dB and 5 dB at the first and the second SUs, respectively.
The density of primary transmitters in the network $\mu$.

Figure 4.13: The relative gain due to the sensing strategy in (4.19) versus the density of primary transmitters in the network, where there is an extra signal loss due to the shadowing effect which is equal to 2 dB and 5 dB at the first and the second SUs, respectively.
Chapter 5

A Framework for Joint Channel Selection in OSA

In this chapter, we consider a scenario where the primary network operates in a synchronous time-framed fashion and the occupancy of each primary channel in each frame follows a discrete-time Markov process. There are \( n \) SUs trying to identify idle channels in each frame. A time frame starts with a sensing interval of fixed length in which SUs stay silent, and each of them senses the primary channel which is assigned to it by the sensing strategy. The occupancy of a primary channel is decided individually (see Section 6.1 for a discussion on this assumption) by each SU to produce a binary sensing outcome. We assume that the sensing outcomes obtained by all the SUs are stored in a fusion center, which can be one of the SUs. We introduce cooperative sensing strategies that exploit past sensing outcomes of several SUs to choose the optimal sensing task in each frame. The goal is to maximize the opportunistic use of the spectrum across time and frequency.

These strategies are based on a finite-state channel model that embraces the fading condition as well as the occupancy state for each primary channel. This channel model allows the sensing outcomes obtained by multiple cooperating SUs in different time frames
and under different fading conditions to be fused together to furnish information vectors for different primary channels. The importance of the proposed channel model is twofold. First, it allows for more accurate fusion of asynchronous sensing outcomes obtained by multiple geographically distributed SUs. Second, it reduces the number of misses, as it allows the SUs to avoid primary channels with unfavorable fading conditions. The proposed framework in Section 5.2, which introduces a design parameter, enables the system designer to devise strategies with different degrees of aggressiveness. We show that a cooperative sensing strategy when based on the channel model introduced in Section 5.1 performs noticeably better compared to the case where the two-state channel model in [65] is used.

The remainder of this chapter is organized as follows. We propose the cooperative sensing strategy framework in Section 5.1, where we present our system model including the proposed finite-state channel model. Optimal and sub-optimal solutions for the problem of joint channel selection in an opportunistic spectrum access scenario are derived in Section 5.2. We provide simulation results in Section 5.3, where we evaluate the effect of various system parameters on the performance of the proposed solutions in Section 5.2. Finally, we provide a summary and a discussion of the results in Section 5.4.

5.1 System Model

5.1.1 The Network Model

Consider a scenario where the spectrum available for opportunistic access is divided into $c$ primary channels of equal bandwidth. The primary network operates in a synchronous time-framed fashion and the occupancy of each primary channel in each frame follows a discrete-time Markov process. There are $n$ SUs trying to detect idle channels in each frame. A time frame starts with a sensing interval of fixed length in which SUs stay silent, and each of them senses the primary channel which is assigned to it by a sensing
strategy (see Figure 5.1).

Figure 5.1: The structure of a time frame.

Figure 5.2 illustrates an example of such a cooperative multi-channel sensing scenario where there are three primary channels and two cooperative SUs. In this figure, the positions of symbols “1” and “2” specify the channels sensed by the first and the second SU, respectively, in each frame. The goal is to maximize the opportunistic spectrum use by the secondary network over $K$ frames. The optimization variable is the sensing strategy that maps the sensing history into a sensing task, which determines which channel should be sensed by which SU in each frame.

Figure 5.2: A cooperative multi-channel spectrum sensing scenario.

5.1.2 Channel Selection and Spectrum Sensing

At the beginning of each frame, a sensing action of the form $a = (a_1, \ldots, a_n)$ is taken, where $a_i \in \{1, \ldots, c\}$ denotes the index of the channel to be sensed by the $i$th SU. In the scenario where each SU corresponds to a femto base station, as proposed in [8], multiple
SUs may be able to sense and potentially access the same primary channel in one frame, i.e., \( a_i = a_{i'} \) for some \( i \neq i' \). However, it can also be assumed that no two SUs are allowed to sense the same channel in the same frame to avoid collision. A collision occurs if multiple SUs sense the same channel to be idle and therefore access the channel.

Associated with each sensing action \((a_1, \ldots, a_n)\) is a sensing outcome \(o = (o_1, \ldots, o_n)\), where \(o_i\) denotes the 1-bit sensing decision made by the \(i\)th SU regarding the occupancy of Channel \(a_i\). These binary decisions are made individually by the SUs based on a detection threshold imposed by some regulatory constraints. The sensing actions and outcomes belong to the action space \(A = \{1, \ldots, c\}^n\) and outcome space \(O = \{0, 1\}^n\), respectively. We assume that the sensing actions taken by the SUs in different frames as well as the corresponding outcomes are stored in a fusion center (which can be one of the SUs). In a femto base station scenario, this information can be transmitted over wired connections. In what follows, we denote by \(A = (A_1, \ldots, A_n)\) a sensing action and by \(O = (O_1, \ldots, O_n)\) a sensing outcome as random variables, while \(a = (a_1, \ldots, a_n)\) and \(o = (o_1, \ldots, o_n)\) denote the realizations of \(A\) and \(O\), respectively.

### 5.1.3 Cooperative Sensing Strategies

Let \(A(k) = (A_1(k), \ldots, A_n(k))\) and \(O(k) = (O_1(k), \ldots, O_n(k))\) respectively represent the sensing action determined by the sensing strategy and the associated outcome in frame \(k\). The problem of joint channel selection by the cooperating SUs boils down to an optimization problem where the goal is to maximize a utility function of the secondary network, over both time and frequency. The optimization variable is the sensing strategy, i.e., a sequence of functions \(\{\delta_k : \mathcal{A}^{k-1} \times \mathcal{O}^{k-1} \rightarrow \mathcal{A}\}\), one for each value of \(k\), that map the sensing history \(h(k)\) into a sensing action of the form \(a = (a_1, \ldots, a_n)\). As mentioned in the previous section, the sensing strategy may or may not assign multiple SUs to sense the same primary channel depending on the underlying assumptions about the interference among the secondary transmissions.
Let the binary random variable $\phi_{i,k}$ specify whether an idle channel is correctly identified by the $i$th SU in frame $k$ (a detected spectrum opportunity). Similarly, $\psi_{i,k}$ specifies whether a busy channel is mistakenly identified as idle by the $i$th SU in frame $k$ (a miss). We have

$$\begin{cases} (1, 0) & O_i(k) = 0, \text{ and Channel } A_i(k) \text{ is idle in frame } k \\ (0, 1) & O_i(k) = 0, \text{ and Channel } A_i(k) \text{ is busy in frame } k \\ (0, 0) & O_i(k) = 1 \end{cases}$$

Define the matrices $\Phi = [\phi_{i,k}]$, and $\Psi = [\psi_{i,k}]$, $i \in \{1, \ldots, n\}$, $k \in \{1, \ldots, K\}$. An optimal cooperative sensing strategy can be written in this general form:

$$\{\delta^*_k\} = \arg \max_{\{\delta_k\}} E\{f(\Phi, \Psi)\}$$

where the function $f$ quantifies the opportunistic spectrum use by the secondary network, $E\{\cdot\}$ denotes statistical expectation, and $E\{f(\Phi, \Psi)\}$ is the total expected reward.

We denote respectively by $N_D$ and $N_M$ the number of detected idle channel-frames and the number of miss-detected busy channels-frames, both normalized by the number of cooperating SUs $n$ and the number of frames $K$, i.e.,

$$N_D = \frac{1}{nK} \sum_{k=1}^{K} \sum_{i=1}^{n} \phi_{i,k} \quad (5.3)$$

$$N_M = \frac{1}{nK} \sum_{k=1}^{K} \sum_{i=1}^{n} \psi_{i,k} \quad (5.4)$$

In this work, we choose the function $f$ to be a weighted subtraction of $N_M$ from $N_D$, and therefore,

$$E\{f(\Phi, \Psi)\} = \eta \bar{N}_D - (1 - \eta) \bar{N}_M \quad (5.5)$$

where $\bar{N}_D = E\{N_D\}$ and $\bar{N}_M = E\{N_M\}$. The design parameter $\eta$ is a real number between 0 and 1. A higher or a lower value of $\eta$ results in a more aggressive or a more conservative sensing strategy, respectively. The case of $\eta = 1$ corresponds to the problem
of maximizing the expected number of detected idle channel-frames; the case of \( \eta = 0 \) corresponds to the problem of minimizing the expected number of miss-detected busy channel-frames.

Before the sensing history can be mapped into a sensing action by the sensing strategy, it must be converted into a posteriori probabilities for the current states of the primary channels. In deriving the sensing and access strategies in [65], the sensing outcomes obtained over different frames are fused together assuming that they are independent observations given the occupancy states of the primary channels. However, these observations can be significantly correlated due to the underlying fading condition. In the next section, we introduce a finite-state channel model that captures the fading condition as well as the occupancy state for each primary channel. Compared to the two state (On-Off) channel model used in [65], the proposed channel model in this work allows for a more accurate fusion of the sensing outcomes obtained through sensing a primary channel over multiple frames.

The multiuser extension of the proposed channel model is particularly useful when multiple spatially distributed SUs share their sensing outcomes. The sensing outcomes received from other users enhance the ability of a SU to estimate the fading condition as well as the occupancy state for each primary channel. A byproduct of this extra information is the ability of the system designer to devise sensing strategies with different levels of aggressiveness. As we will see in Section 5.3, the sensing strategy design parameter, i.e., \( \eta \) in (5.5), can be adjusted to make SUs more aggressive or conservative in their access to the primary band. This adjustment is made at the sensing strategy level, in contrast to the threshold adjustment that can be made at each individual SU at the detection level.
5.1.4 The Proposed Channel Model

When a primary channel is busy, the performance of the spectrum detector at each SU greatly depends on the physical channel condition between the SU and the active primary transmitter in that channel. Several fading channel models are used to describe random variations of wireless channels over time. Finite-state channel models are of particular interest because they allow for characterizing the memory of the fading process in a tractable manner [59]. In these models, the range of the received SNR is partitioned into a finite number of intervals, each representing a state, and Markov transitions between different states are assumed. Different finite-state Markov channel models are proposed and studied for various continuous fading channels such as the Rayleigh fading channel [50, 58].

Based on a similar idea, we propose a finite-state channel model that captures the fading condition in addition to the occupancy state for each primary channel. This channel model allows all past sensing outcomes obtained by geographically distributed SUs over different frames to be summarized in information vectors, one for each primary channel. Consider a scenario where the possible range of the received SNR $\gamma$ at each SU in a busy primary channel is partitioned into two intervals: $\gamma \geq \gamma_0$ (high) and $\gamma \leq \gamma_0$ (low). Based on this assumption, the state of a primary channel from a SU’s perspective in each frame is modeled as a random variable with three different outcomes: $I$ (idle), $H$ (high), and $L$ (low). Note that the two states $H$ and $L$ constitute super-state $B$ (busy), as shown in Figure 5.3. The proposed channel model can be extended to the case of $n > 1$ cooperating SUs. The state of a primary channel from the perspective of two SUs in each frame is modeled in a similar way as a random variable with a sample space

$$S = \{I(0), HH(1), HL(2), LH(3), LL(4)\}. \quad (5.6)$$

Again, $I$ indicates that the primary channel is idle; the other four states constitute super-state busy $B$, as illustrated in Figure 5.4. The state $HH$ denotes the state where the
channel is busy and the received (primary) SNR is higher than $\gamma_0$ at both the cooperating SUs. Similarly, $HL$ ($LH$) denotes the state where the received SNR is higher (lower) than $\gamma_0$ at the first SU and lower (higher) than $\gamma_0$ at the second SU, respectively. Finally, $LL$ denotes the state where the channel is busy and the received SNR is lower than $\gamma_0$ at both the cooperating SUs. By selecting the appropriate probabilities of transition between different states, the proposed channel model in Figure 5.4 can be adjusted to account for any correlation between the fading states of the two SUs. As an example, the stationary probabilities of states $HL$ and $LH$ can be made arbitrarily small when the fading states are severely correlated. For the general case of $n$ cooperating SUs, the number of states in super-state $B$ is given by $R = 2^n$. To simplify the notation in later derivations, we index the states in $S$ from 0 to $R$ (as in (5.6)). Both notations are used interchangeably throughout this chapter.

We denote by $S_i(k)$ the state of Channel $i$ from the perspective of the $n$ cooperating SUs in frame $k$. The stochastic process $\{S_i(k)\}, k \in \{1, 2, \ldots, K\}$, is assumed to be a finite-state Markov chain with state space $S$, an $(R + 1) \times (R + 1)$ transition probability matrix $P = [p_{jj'}], j, j' \in \{0, 1, \ldots, R\}$, and initial distribution...
\[ \pi_1(1) = [\pi_{i,0}(1), \pi_{i,1}(1), \ldots, \pi_{i,R}(1)]. \]

Furthermore, the state of different primary channels are assumed to evolve independently of each other. Since there are \( c \) primary channels available for opportunist access, the state of the primary network from the perspective of the SUs in frame \( k \) is given by a \( c \)-tuple \((S_1(k), \ldots, S_c(k)) \in S^c\). The above channel model takes into account the correlation among the sensing outcomes obtained in different time frames and/or by multiple spatially distributed SUs, as opposed to the two-state channel model used in [65].

The probabilistic relationship between \( O(k) \) and \((S_1(k), \ldots, S_c(k))\) is determined by the false-alarm and miss probabilities under different channel conditions. Here we assume that the cooperating SUs employ identical spectrum detectors with a common decision threshold. We denote by \( P_f \) the false-alarm probability, and by \( P_{m,H} \) and \( P_{m,L} \) the miss probabilities under channel states \( H \) and \( L \), respectively. The probabilistic relationship between \((O_1, \ldots, O_n)\) and \((S_1, \ldots, S_c)\) when sensing action \( a = (a_1, \ldots, a_n) \) is taken is
given by

\[
\Pr[O_i = 0 | A_i = a_i, S_{a_i} = s] = \begin{cases} 
P_f, & s = 0 \\
1 - P_m, & s = X_1 \ldots X_{i-1} H X_{i+1} \ldots X_n \\
1 - P_m, & s = X_1 \ldots X_{i-1} L X_{i+1} \ldots X_n 
\end{cases} \tag{5.7}
\]

for any \( i \in \{1, \ldots, n\} \) and \( a_i \in \{1, \ldots, c\} \), where \( X_{i'} \in \{L, H\} \) for \( i' \in \{1, \ldots, n\} \setminus \{i\} \). We also have \( \Pr[O_i = 1 | A_i = a_i, S_{a_i} = s] = 1 - \Pr[O_i = 0 | A_i = a_i, S_{a_i} = s] \). The data available for choosing a sensing action in frame \( k \) is given by the sensing history, i.e., \( h(k) = (a(1), \ldots, a(k-1), o(1), \ldots, o(k-1)) \in \mathcal{A}^{k-1} \times \mathcal{O}^{k-1} \), where \( a(l) \) and \( o(l) \) denote the realizations of \( \mathcal{A}(l) \) and \( \mathcal{O}(l) \), respectively.

Note that the miss probabilities, \( P_{m,H} \) and \( P_{m,L} \), as well as the transition probability matrix \([p_{ij}]\) depend on the choice of the SNR threshold \( \gamma_0 \), which is used to define channel states \( H \) and \( L \). In this work, we assume that the SNR threshold is given, and the false-alarm miss probabilities as well as the state transition probabilities are known. These parameters can be obtained using signal strength measurements before deployment of the secondary network. When such measurements are not available, reinforcement learning algorithms can be used to obtain an estimate of these parameters [35, 36].

### 5.2 Optimal and Suboptimal Solutions

Based on the system model introduced in Section 5.1 and the total expected reward given by (5.3)-(5.5), we propose optimal and sub-optimal cooperative sensing strategies in Sections 5.2.1-5.2.3. The benefits and drawbacks of each sensing strategy are also discussed. We evaluate and compare the performance of different sensing strategies through extensive numerical simulations in Section 5.3.
\[ \pi_{i,j}(k+1) = \Pr[S_i(k+1) = j|\mathbf{h}(k), \mathbf{a}, \mathbf{o}] \]

\[
\begin{cases}
\sum_{j'=0}^R p_{j'} \pi_{i,j'}(k) \prod_{l \in \mathcal{N}_i} \Pr[O_l = o_l|A_l = i, S_i = j'] & \text{if } a_l = i \text{ for } l \in \mathcal{N}_i \\
\sum_{j'=1}^R p_{j'} \pi_{i,j'}(k) & \text{if } i \notin \{a_l\}_{l=1}^n
\end{cases}
\]  

5.2.1 An Optimal Sensing Strategy

An optimal sensing strategy \( \{\delta^*_k\} \) is a solution to (5.2)-(5.5) and corresponds to the optimal control of a POMDP over a horizon of \( K \) frames. Let \( \pi_{i,j}(k) = \Pr[S_i(k) = j|\mathbf{h}(k)] \), i.e., the probability that Channel \( i \) is in state \( j \) given all the sensing actions and outcomes before frame \( k \). Define the matrix \( \Pi(k) = [\pi_{i,j}(k)], \; i \in \{1, \ldots, n\}, \; j \in \{0, 1, \ldots, R\} \). We call \( \Pi(k) \) the information matrix regarding the state of all the primary channels from the perspective of the SUs in frame \( k \). The transformation of the information matrix \( \Pi(k) \) from frame \( k \) to frame \( k+1 \) is determined by the sensing action and the corresponding outcome in frame \( k \):

\[
\Pi(k+1) = \mathcal{T}(\Pi(k)|\mathbf{A}(k) = \mathbf{a}, \mathbf{O}(k) = \mathbf{o}) \tag{5.8}
\]

where the \( (i, j) \)th element of the matrix \( \Pi(k+1) \) is given (5.9) at the top of page 87.

Note that \( a_l = i \) for \( l \in \mathcal{N}_i \) indicates that Channel \( i \) is sensed by a number of SUs, the indices of which are contained in set \( \mathcal{N}_i \), and \( i \notin \{a_l\}_{l=1}^n \) indicates that Channel \( i \) is not sensed by any of the SUs in frame \( k \). It can be shown that the matrix \( \Pi(k) \) summarizes all the information required for choosing a sensing action in frame \( k \) [48]. The expected reward in each frame is given by (5.10) at the top of page 88.

Let \( \{\delta^*_k\} \) and \( \{V^*_k\} \) denote an optimal sensing strategy and the associated value functions. The value function \( V^*_k(\Pi) \) denotes the total expected reward starting from frame \( k \) when the current information matrix is \( \Pi \). The recurrence relations for an optimal
Chapter 5. A Framework for Joint Channel Selection in OSA

\[ r(\Pi, a) = \eta \sum_{i=1}^{n} \pi_{ai,0} \Pr[O_i = 0 | A_i = a_i, S_{ai} = 0] \]
\[ - (1 - \eta) \sum_{i=1}^{n} \sum_{j=1}^{R} \pi_{ai,j} \Pr[O_i = 0 | A_i = a_i, S_{ai} = j]. \] (5.10)

sensing strategy \{\delta_k^*\} and the associated value functions \{V_k^*\} are given by [48]

\[ \delta_k^*(\Pi) = \arg \max_a \left[ r(\Pi, a) + \sum_{o\in O} \Pr[o | \Pi, a] V_{k+1}^*[T(\Pi | a, o)] \right] \] (5.11)
\[ V_k^*(\Pi) = r(\Pi, \delta_k^*(\Pi)) + \sum_{o\in O} \Pr[o | \Pi, \delta_k^*(\Pi)] V_{k+1}^*[T(\Pi | \delta_k^*(\Pi), o)] \] (5.12)
for \(k \leq K - 1\), and

\[ \delta_K^*(\Pi) = \arg \max_a r(\Pi, a) \] (5.13)
\[ V_K^*(\Pi) = r(\Pi, \delta_K^*(\Pi)) \] (5.14)

where \(\Pr[o | \Pi, a]\) is the probability of outcome \(o\) if the current information matrix is \(\Pi\) and action \(a\) is taken. This probability is given by

\[ \Pr[o | \Pi, a] = \prod_{i \in \{a_1, \ldots, a_n\}} \left( \sum_{j=0}^{R} \pi_{i,j} \prod_{l \in N_i} \Pr[O_l = o_l | A_l = i, S_l = j] \right). \] (5.15)

Note that the above problem represents a dynamic programming problem over a continuous state space \(\mathcal{W}\), i.e., \(\Pi \in \mathcal{W}\) and

\[ \mathcal{W} = \{ [w_{i,j}] \in \mathbb{R}^{c \times (R+1)} | \forall i, j : w_{i,j} \geq 0, \forall i : \sum_{j=0}^{R} w_{i,j} = 1, \}. \] (5.16)

It is shown in [48] that the optimal value function \(V_k^*(\cdot)\) is piecewise-linear and convex in its argument \(\Pi\), and it can be represented as a finite set of vectors. The “one-pass” algorithm proposed in [48] exploits this structure to compute the optimal sensing strategy and the set of value functions for any given finite-horizon POMDP. However, solving POMDPs exactly is often computationally intractable. Several methods have been developed in the literature that approximate solutions for POMDPs [28]. Based on
these methods, we propose two suboptimal cooperative sensing strategies in the next two sub sections. The performance of the proposed finite-memory sensing strategy in Section 5.2.3 approaches that of the optimal strategy as the memory is increased.

5.2.2 The Greedy Sensing Strategy

We may resort to a greedy sensing strategy that maximizes the expected reward in each frame:

$$\delta^g_k(\Pi) = \arg \max_a r(\Pi, a)$$  \hspace{1cm} (5.17)

where $r(\Pi, a)$ is given by (5.10). The greedy sensing strategy has significantly lower complexity compared to the optimal sensing strategy given by (5.11)-(5.14). Furthermore, it takes into account all the past sensing actions and outcomes. However, it completely neglects the expected future reward. If we adopt a two state model as in [65], the reward function in (5.10) reduces to

$$r(\Pi, a) = \sum_{i=1}^{n} \left[ \eta (1 - P_t) \pi_{a_i,0} - (1 - \eta) P_m \pi_{a_i,1} \right]$$

$$= \sum_{i=1}^{n} \left[ \eta (1 - P_t) \pi_{a_i,0} - (1 - \eta) P_m (1 - \pi_{a_i,0}) \right]$$

$$= n(\eta - 1) + [1 + \eta (1 - P_t - P_m)] \sum_{i=1}^{n} \pi_{a_i,0}$$ \hspace{1cm} (5.18)

where $P_m$ is the miss probability averaged over states $H$ and $L$. The corresponding cooperative sensing strategy is given by

$$\delta^g_k(\Pi) = \arg \max_a \sum_{i=1}^{n} \pi_{a_i,0}$$ \hspace{1cm} (5.19)

which is independent of $\eta$. As shown in Section 5.3, the greedy sensing strategy based on our proposed multi-state channel model significantly outperforms its counterpart that is based on a two-state channel model. Furthermore, it can be tailored to achieve different aggressiveness-conservativeness trade-offs by adjusting the parameter $\eta$ accordingly.
To find the maximizing argument in (5.17), and therefore, the sensing task in each frame, the reward function in (5.10) should be evaluated for all the admissible sensing actions. The number of these actions is given by $c^n$ when multiple SUs are allowed to sense and potentially access the same channel, and by $\binom{c}{n}$ when no two SUs are allowed to sense the same channel. As a result, determining the sensing task in each frame can be computationally demanding even for the greedy sensing strategy when the number of primary channels $c$ and/or the number of cooperating SUs $n$ is large. To overcome this problem, another suboptimal sensing strategy is proposed in the next section.

### 5.2.3 The Finite-memory Sensing Strategies

Due to computational issue associated with the greedy sensing strategy, we propose another suboptimal solution to the problem in (5.11)-(5.14), which is obtained by truncating the memory at a certain number of the most recent sensing actions and outcomes [28]. Since they are a finite number of possible truncated memory vectors, the problem of finding the optimal finite-memory sensing strategy reduces to the optimal control of a Markov decision process with a finite-state space. The higher the memory of a strategy, the closer its performance to that of the optimal strategy.

In such a strategy, the last few sensing actions and outcomes are used as an index into a precomputed lookup table to determine the sensing task in each frame. Therefore, no run-time computation is required. Setting aside the computational issue, the finite-memory strategy has the advantage of taking into account the expected future reward, which is ignored by the greedy strategy. However, truncating the memory at a certain number of the most recent sensing actions and outcomes leads to some performance degradation compared to the greedy strategy, which takes into account all the past sensing information.
For positive integers $k$ and $m$, define $h_k^m$ as
\[
    h_k^m = \begin{cases} 
        (a(1), \ldots, a(k - 1), o(1), \ldots, o(k - 1)) & k \leq m \\
        (a(k - m), \ldots, a(k - 1), o(k - m), \ldots, o(k - 1)) & k > m
    \end{cases}.
\] (5.20)

Let $M \geq 1$ represent the maximum number of sensing actions and outcomes that are used by the sensing strategy to choose a sensing action in each frame. The data available for decision making in frame $k$ is therefore given by $h_k^M$. Now, define $T^M(\Pi, h_k^M)$ as
\[
    T^M(\Pi, h_k^M) = \begin{cases} 
        T(\Pi, a(k - 1), o(k - 1)) & \min\{k - 1, M\} = 1 \\
        T^{k-2}(T(\Pi, a(1), o(1)), h_{k-2}^k) & 3 \leq k \leq M \\
        T^{M-1}(T(\Pi, a(k - M), o(k - M)), h_{M-1}^k) & k > M \geq 2
    \end{cases}.
\] (5.21)

where $T^1 = T$, as defined in (5.8) and (5.9). Note that for $k > M$, we have $\Pi(k) = T^M(\Pi(k - M), h_k^M)$; i.e., the information matrix in frame $k$ is a function of the information matrix in frame $(k - M)$, namely, $\Pi(k - M)$, as well as the $M$ most recent actions and outcomes $h_k^M$. However, $\Pi(k - M)$ depends on all the sensing actions and outcomes before frame $k - M$, which we assume are not available to a finite-memory decision maker that bases its decisions on the $M$ most recent frames. The upper and lower bounds on the performance of infinite-horizon finite-memory strategies over all possible values of $\Pi(k - M)$ are derived in [61]. In this work we replace the information matrix $\Pi(k - M)$ with the stationary distribution $\Pi(1)$.

Based on the above assumption and given the initial distribution $\Pi(1)$, an optimal finite-memory strategy is a sequence of functions $\{\delta_k^M : \mathcal{A}^l \times \mathcal{O}^l \to \mathcal{A}, l = \min\{k - 1, M\}\}$, each defined over a finite set, that map $h_k^M$ into a sensing action $a \in \mathcal{A}$. Define
\[
    \Pi^M(k) = \begin{cases} 
        \Pi(1) & k = 1 \\
        T^M(\Pi(1), h_k^M) & k > 1
    \end{cases}.
\] (5.22)
The recurrence relations for an optimal sensing strategy \( \{ \delta^M_k \} \) and the associated value functions \( \{ V^M_k \} \) are given by

\[
\delta^M_k(h^M_k) = \text{arg max}_a \left[ r(\Pi^M(k), a) + \sum_{o \in \{0,1\}^n} \Pr[O|\Pi^M(k), a] V^M_{k+1}(h^M_{k+1}) \right] \quad (5.23)
\]

\[
V^M_k(h^M_k) = r(\Pi^M(k), \delta^M_k(h^M_k)) + \sum_{o \in \{0,1\}^n} \Pr[O|\Pi^M(k), \delta^M_k(h^M_k)] V^M_{k+1}(h^M_{k+1}) \quad (5.24)
\]

for \( k \leq K - 1 \), and

\[
\delta^M_K(h^M_K) = \text{arg max}_a r(\Pi^M(K), a) \quad (5.25)
\]

\[
V^M_K(h^M_K) = r(\Pi^M(K), \delta^M_K(h^M_K)) \quad (5.26)
\]

where \( r(\Pi^M(k), a) \) and \( \Pr[O|\Pi^M(k), a] \) are as defined in (5.10) and (5.15), respectively.

As shown in Section 5.3, performance degradation due to memory truncation is less significant in more dynamic spectrum conditions. This is because the sensing actions and outcomes before frame \( k - M \) are less important in choosing a sensing actions in frame \( k \).

### 5.3 Numerical Results

In this section, we evaluate the effects of several system parameters on the performance of the cooperative sensing strategies proposed in Section 5.2. In particular, we study the effects of the dynamics of primary activity as well as the crowdedness of the licensed band. We also study the role of the cooperative sensing strategy design parameter \( \eta \) in finding a trade-off between the aggressiveness of the SUs to access the primary channels and the necessity of limiting the interference with the PUs.

In the numerical examples, we consider a scenario where \( n = 2 \) SUs try to detect spectrum opportunities within \( c = 5 \) primary channels over \( K \) frames. We assume that the cooperating SUs are not permitted to sense and access the same channel in one frame. The occupancy of each primary channel in different frames follows a discrete-time Markov
process. Each channel transits from super-state $B$ to state $I$ with probability $\alpha$ and stays in state $I$ with probability $\beta$, as shown in Figure 5.5-a. Super-state $B$ is decomposed into $R = 4$ states based on the received primary SNR at each SU. We choose $p_{0j} = (1 - \beta)/4$, $p_{j0} = \alpha$, and $p_{jj} = 1 - \alpha$, for $j \in \{1, \ldots, 4\}$. The state transitions and their corresponding probabilities for the case of two cooperating SUs are illustrated in Figure 5.5-b. As shown in this figure, there is no transition between different states in super-state $B$. We also assume that the initial distribution for the state of each channel is equal to the stationary distribution of the underlying Markov chain.

![Diagram](image)

Figure 5.5: The state of a primary channel from the perspective of two SUs, as considered in Section 5.3.

The above assumptions imply the channel states $H$ and $L$ are equally likely when a primary channel is in super-state $B$. They also imply that the fading conditions are independent for the two cooperating SUs. The probability that a given channel is in super-state $B$ in any frame is given by the parameter $u = (1 - \beta)/(1 + \alpha - \beta)$; hereafter, we
call this parameter the primary utilization factor. Table 5.1 contains brief descriptions of all the parameters of the system model considered in this section as well as the parameters related to the cooperative sensing strategies proposed in Section 5.2.

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_f$</td>
<td>False-alarm probability</td>
</tr>
<tr>
<td>$P_{m,H}$</td>
<td>Average miss probability under channel state $H$</td>
</tr>
<tr>
<td>$P_{m,L}$</td>
<td>Average miss probability under channel state $L$</td>
</tr>
<tr>
<td>$c$</td>
<td>Number of primary channels</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of cooperating SUs</td>
</tr>
<tr>
<td>$K$</td>
<td>Number of time frames</td>
</tr>
<tr>
<td>$M$</td>
<td>Memory of the finite-memory strategy proposed in (5.23)-(5.26)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>The probability that a primary channel transits from super-state $B$ to state $I$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The probability that a primary channel stays in state $I$</td>
</tr>
<tr>
<td>$u$</td>
<td>Primary utilization factor, $u = (1 - \beta)/(1 + \alpha - \beta)$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Strategy design parameter</td>
</tr>
</tbody>
</table>

### 5.3.1 Greedy versus Finite-memory Sensing Strategies

In the first example, we choose $P_f = 0.05$, $P_{m,H} = 0.05$, $P_{m,L} = 0.5$, $K = 10$, $u = 0.5$, and $\eta = 0.5$. We evaluate the performance of the proposed greedy sensing strategy in (5.17) and that of the finite-memory sensing strategies in (5.23)-(5.26) with two different values of the memory, i.e., $M = 1$ and $M = 2$. Figure 5.6 illustrates the average number of detected idle channel-frames and the average number of miss-detected busy channel-frames, $\bar{N}_D$ and $\bar{N}_M$, versus the probability that a channel transits from super-state $B$ to state $I$, i.e., $\alpha$ for the case where $u = 0.5$. Note that both values are normalized by the number of frames $K$ and the number of cooperating SUs $n$. For the sake of comparison, we have also included the results that correspond to the case where the two SUs are randomly assigned to sense two separate channels in each frame. The corresponding
total expected rewards, i.e., $E\{f(\Phi, \Psi)\} = (\bar{N}_D - \bar{N}_M)/2$, are shown in Figures 5.7. As can be seen from these figures, the proposed sensing strategies significantly outperform the random selection strategy except for values of $\alpha$ very close to 0.5. It is also observed that the performance of the finite-memory strategy approaches that of the greedy strategy even for small values of memory $M$. Note that performance improvement due to increase in the memory of a sensing strategy is less significant for values of $\alpha$ closer to 0.5. This is because the importance of past sensing information in regard to current sensing actions diminishes more rapidly for these values of $\alpha$.

Figure 5.6: $\bar{N}_D$ and $\bar{N}_M$ versus the transition probability $\alpha$ for different sensing strategies where $u = 0.5$ and $\eta = 0.5$.

In the second example, we choose $P_f = 0.05$, $P_{m,H} = 0.05$, $P_{m,L} = 0.5$, $K = 10$, $\alpha = 0.05$, and $\eta = 0.5$. Figure 5.8 illustrates $\bar{N}_D$ and $\bar{N}_M$ versus the primary utilization factor $u$ when the finite memory and random selection strategies are used. The corresponding total expected rewards are shown in Figures 5.9. As can be observed from these figures, the finite-memory strategies significantly outperform the random selection strategy in terms of both the number of detected idle channel-frames and the number of miss-detected
Figure 5.7: The total expected reward $E\{f(\Phi, \Psi)\}$ versus the transition probability $\alpha$ for different sensing strategies where $u = 0.5$ and $\eta = 0.5$.

busy channel-frames, specially, in the medium range of $u$. The benefits of our proposed channel model, which embraces the fading condition as well as the occupancy state of each primary channel, become more clear in Section 5.3.2.

5.3.2 Aggressive versus Conservative Sensing Strategies

In the third example, we choose $P_f = 0.05$, $P_{m,H} = 0.05$, $P_{m,L} = 0.5$, $K = 40$, $\alpha = 0.05$, and $u = 0.8$. Figure 5.10 illustrates $\bar{N}_D$ and $\bar{N}_M$ versus the sensing strategy design parameter $\eta$ when the greedy sensing strategy in (5.17) is used (Strategy 1). For the sake of comparison, we have also evaluated the performance of the proposed greedy sensing strategy when the two-state (On-Off) channel model in [65] is used (Strategy 2). The corresponding total expected rewards, i.e., $E\{f(\Phi, \Psi)\} = \eta\bar{N}_D - (1 - \eta)\bar{N}_M$ are shown in Figure 5.11. As shown in Section 5.2.2 and also observed from Figures 5.10, the performance of Strategy 2, in terms of $\bar{N}_D$ and $\bar{N}_M$, does not depend on the choice of parameter $\eta$. However, Strategy 1, devised based on the channel model introduced
Figure 5.8: $\bar{N}_D$ and $\bar{N}_M$ versus the primary utilization factor $u$ for different sensing strategies where $\alpha = 0.05$ and $\eta = 0.5$.

In Section 5.1, can be adjusted to achieve different degrees of aggressiveness. This is because the proposed channel model enables the sensing strategy to exploit the knowledge about the fading state as well as the occupancy state for each primary channel. This extra information together with the design parameter $\eta$ allows the channel selection criterion to be adapted based on the reward function in (5.17). As can be seen from 5.10, both strategies achieve the same number of detected idle channel-frames when $\eta \approx 0.75$. However, the number of miss-detected busy channel-frames is significantly lower for Strategy 1. As shown in figure 5.11, the performance improvement due to the proposed channel model is more significant for lower values of $\eta$, which correspond to more conservative sensing strategies.

5.4 Summary and Practical Considerations

In this chapter, we introduced cooperative spectrum sensing strategies that leverage past sensing outcomes of several cooperating SUs to choose an optimal sensing task in
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Figure 5.9: The total expected reward $E\{ f(\Phi, \Psi) \}$ versus the primary utilization factor $u$ for different sensing strategies where $\alpha = 0.05$ and $\eta = 0.5$.

Each frame. Based on the proposed finite-state channel model, asynchronous sensing outcomes obtained by several cooperating SUs are fused together to provide a posteriori probabilities for current states of multiple primary channels. We derived optimal and sub-optimal sensing strategies that decide which channel is to be sensed by which SU in each frame. We showed that the resulting spectrum reuse by a secondary network is significantly enhanced over a random selection strategy or a cooperative sensing strategy designed based on a two state On-Off channel model. As the detection threshold in a spectrum detector balances the trade-off between false-alarm and miss probabilities, the design parameter introduced in Section 5.2 allows the system designer to devise strategies with different levels of aggressiveness.

As a possible application of the proposed cooperative sensing strategy framework, we may consider self-organized femtocell networks. In these networks, cooperative spectrum sensing can be used by the femto base stations to reduce the interference to macro users as well as the other femto users. As mentioned in Chapter 2, the role of the MOSS proposed in [8] is to collect spectrum sensing outcomes from one or more femto base
Figure 5.10: $\bar{N}_D$ and $\bar{N}_M$ versus the strategy design parameter $\eta$ for different sensing strategies where $\alpha = 0.05$ and $u = 0.8$.

stations and combine it with additional information about spectrum use obtained from multiple cellular operators. The MOSS then uses this information to make decisions about spectrum use and availability in each area. Therefore, the sensing outcomes are not the only information available to the spectrum server, as it also receives information regarding the licensed spectrum that macrocells use from the operators. However, there are several reasons why the sensing outcomes can still be beneficial.

Spectrum sensing can be useful in detecting opportunities in the downlink. As an example, consider the case where a mobile terminal is very close to its serving base station, and therefore, the base station’s transmit power corresponding to that mobile terminal is relatively low. In such a case, a low power femto user near the edge of the macrocell may be able to use the same channel in an opportunistic manner without causing harmful interference to the macro user. A similar situation occurs if the macro base stations are equipped with multiple antennas and perform transmit beamforming. In such a case, the transmit power in the downlink is directed toward the macro users in each cell. Therefore, the same channel may be used by a femto user and a macro user
Figure 5.11: The total expected reward $E\{f(\Phi, \Psi)\}$ versus the strategy design parameter $\eta$ for different sensing strategies where $\alpha = 0.05$ and $u = 0.8$. 

simultaneously depending on their relative location in the cell. Another scenario is when femto base stations are utilized in areas where there are coverage issues with the cellular networks.

As the number of states in super-state busy $B$ grows exponentially with the number of cooperating SUs $n$, the proposed framework can become computationally prohibitive for a large number of cooperating SUs. In such a case, only a small number of SUs can be selected to cooperate based a user selection scheme. Several centralized and cluster-based user selection schemes have been proposed in the literature. Such schemes are shown to be very effective in reducing the cooperation overhead, i.e., the bandwidth and energy requirements, as well as increasing the cooperative gain, i.e., the robustness of the sensing results [5].

The proposed cooperative sensing strategies in this chapter may be adjusted to account for missing or delayed sensing outcomes, which are a consequence of an imperfect control channel. One way to address this issues is to update the information matrix in (5.8) only based on the available sensing outcomes. When a sensing outcome is missing,
the belief vector for each channel is just updated based on the Markov dynamics of the underlying channel model, as if that channel is not sensed in that frame. The delayed sensing outcome can be used to re-compute the belief vector for the frame when the outcome is obtained. The updated belief vector is then used to re-compute all the belief vectors afterwards until the current frame. The two suboptimal strategies proposed in this chapter can be readily applied to the belief vectors that are obtained based on incomplete or delayed sensing information. However, since the SUs or frames corresponding to the missing or delayed outcomes are not known a priori, an optimal sensing strategy, from a dynamic programming point of view, cannot be obtained. A number of practical issues encountered in implementing cooperative sensing schemes in general are discussed in Section 2.4.
Chapter 6

Conclusion and Future Work

Current research in the area of spectrum sensing for CR is focused on improving the opportunistic use of the spectrum while avoiding harmful interference to licensed users. Adaptive spectrum sensing strategies allow the sensing resources to be directed towards more promising primary channels. These strategies are therefore capable of increasing identified time-frequency spectrum opportunities as well as reducing interference with licensed systems. In this research, we introduced three adaptive sensing strategies under three different scenarios.

In Chapter 3, we considered a scenario where a SU employs a tunable BPF to sense the primary channels sequentially until the first idle channel is found. We introduced a two-stage spectrum sensing strategy that accelerates the channel search process by allowing the spectrum detector to focus on primary channels which are more likely to be idle. We showed that the proposed sensing strategy significantly reduces the average channel search time when the primary spectrum utilization is moderate to high.

In Chapter 4, we considered a scenario where collaborating SUs jointly try to detect spectrum opportunities in a wide-band spectrum within a predefined spectrum sensing interval. We assumed that each SU is equipped with a BPF and is able to sense one channel at a time. The sensing interval consists of a number of sensing slots. During
each slot, each of the cooperating SUs is assigned to sense one of the channels and report its observation to the other users. The occupancy of each channel is decided at the end of the sensing interval based on a fusion rule that is fitted to a regulatory constraint or a coarse sensing task. We aimed to maximize the expected number of identified idle channels by optimally choosing the index of the channel to be sensed by each collaborating SU at each slot. We modeled this problem in its most general form as a finite horizon Markov decision process. We derived closed-form solutions for an optimal spectrum sensing strategy and the associated value functions for the case of two cooperating SUs where individual sensing decisions are fused together according to the OR-rule. We show that the gain due to the optimal sensing strategy is noticeable when the spectrum utilization is high.

In Chapter 5, we considered the joint channel selection problem for OSA where several SUs cooperate to detect idle time frames across multiple primary channels. We proposed a finite-state channel model that allows the asynchronous sensing outcomes obtained by the geographically distributed SUs to be converted into a posteriori probabilities for the current states of the primary channels. Based on this channel model, we introduced cooperative sensing strategies that determine which channel should be sensed by each SU at any given time. We showed that the proposed sensing strategies result in an increase of the spectrum reuse by the secondary network, while the amount of interference to the primary network is significantly reduced. As the detection threshold balances the trade-off between the false-alarm and miss probabilities for detecting primary signals in a single frequency channel, the design parameter introduced in Chapter 5 allows the system designer to devise cooperative sensing strategies with different levels of aggressiveness.
6.1 Future Work

In Chapter 3, we considered a scenario where a SU employs an energy detector to sense the primary channels one-by-one to find an idle channel. A similar two-stage detection strategy may be applied to any detection scheme, and in particular to cyclostationary feature detection schemes. Cyclostationary feature detectors are robust to uncertainty in the noise power as they are able to differentiate the primary signal from the noise. Since a major drawback of cyclostationary feature detectors is their long sensing time, a two-stage cyclostationary feature detector can be very effective in reducing the average channel search time.

The proposed sensing strategy in Chapter 3 is adaptive in the sense that each channel is first sensed for a short time interval, and if the average received power in the first stage is below a certain threshold, the channel is sensed for a longer time interval. The proposed sensing strategy in Chapter 4 is adaptive in the sense that each channel is first sensed by a SU, and if the average received power by the first SU is below a certain threshold (If we assume that an energy detector is used, and there are two SUs), the channel is sensed by the other SU. In general, we can consider a cooperative channel search strategy where each SU employs a two-stage spectrum detector. Such a channel search strategy can further reduce the average channel search time and/or improve the reliability of the sensing results.

In Chapter 5, we focused on exploiting the sensing outcomes received from multiple SUs in the channel selection process. In the OSA scenario considered in this chapter, the decision to access a primary channel in a frame is made individually by each SUs based only on its own measurement in that frame. However, the benefits of the proposed channel model in Section 5.1.4, which allows for a more accurate fusion of the sensing history into belief vectors for various primary channels, would be more evident if we adopt a cooperative access strategy as well as a cooperative sensing strategy. A small variation in the belief vectors may not have a significant impact on the choice of channels.
to be sensed (as the order of the channels in the list may not change dramatically), but it may have a noticeable impact on the number of false-alarms or misses.

The joint design of the sensing and access strategies requires the joint design of the feedforward and feedback blocks in Figure 1.5, as the feedforward block is responsible for making the access decisions. While most of the literature in the area of cooperative spectrum sensing is focused on designing the feedforward block, and the feedback block is generally neglected, in Chapters 4 and 5 of this thesis, we focused on the role of adaption and the design of the feedback block. The problem of finding the jointly optimal sensing and access strategies in such a cooperative sensing scenario is left as a future work. Another possible future work is the joint design of channel search and channel selection strategies in a scenario where multiple SUs cooperate to detect spectrum opportunities across multiple frames, and each frame starts with multiple sensing slots.
Appendix A

PROOF OF THEOREM 1

Proof. Let us denote by $p_1$ and $p_2$ the probabilities that a channel is successfully passed through the first and the second stage, respectively. Since $\lambda_1$ is chosen to be 1, and from (3.15) and (3.16), we have

$$p_0 = (1 - u)/2 + u F_{N_1}(1|H_1) \quad \text{(A.1)}$$

$$p_1 = (1 - u_1(N_1))(1 - P(N_2)) + u_1(N_1)P_m(N_2) \approx (1 - u_1(N_1))(1 - P(N_2)) \quad \text{(A.2)}$$

Note that a very small value of $\beta$ in the regulatory constraint in (3.18) (e.g., $\beta = 0.001$ in our numerical example in Chapter 3) ensures that the second term in (A.2) can be neglected. Let us assume that the spectrum detector successfully locates an idle channel after going through the first stage and the second stage of spectrum sensing for $i$ and $j$ times, respectively. To find the average search time before an idle channel is found, we need to find the expected length of the search interval over all possible values of $i$ and $j$.

The expected length of the search interval is given by

$$N_{\text{search}} = p_1 p_2 (N_1 + N_2) + p_1 (1 - p_1) p_2 (2N_1 + N_2) + \cdots + p_1 (1 - p_1)^{c-1} p_2 (cN_1 + N_2)$$

$$+ (p_1^2 - p_2) p_2 \sum_{i=2}^{c} \binom{i-1}{i-2} (1 - p_1)^{i-2} (iN_1 + 2N_2)$$

$$+ p_1^2 (1 - p_2)^2 p_2 \sum_{i=3}^{c} \binom{i-1}{i-3} (1 - p_1)^{i-3} (iN_1 + 3N_2)$$
\[+ \cdots + p_1^{c-1}(1 - p_2)c^{-2}p_2 \sum_{i=c-1}^{c} \binom{i - 1}{i - c + 1}(1 - p_1)^{i-c}(iN_1 + 3N_2)\]
\[+ p_1^c(1 - p_2)c^{-1}p_2 \sum_{i=c}^{c} \binom{i - 1}{i - c}(1 - p_1)^{i-c}(iN_1 + 3N_2)\]
\[= \sum_{j=1}^{c} p_1^j(1 - p_2)^{j-1}p_2 \sum_{i=j}^{c} \binom{i - 1}{i - j}(1 - p_1)^{i-j}(iN_1 + jN_2)\]
\[= \sum_{j=1}^{c} p_1^j(1 - p_2)^{j-1}p_2 \sum_{i=j}^{c-j} \binom{k + j - 1}{k}(1 - p_1)^{k+ j}((k + j)N_1 + jN_2)\]
\[= p_1p_2 \sum_{l=0}^{c-1} p_1^l(1 - p_2)^l \sum_{k=0}^{c-l-1} \binom{k + l}{k}(1 - p_1)^{k+ l}((k + l + 1)N_1 + (l + 1)N_2)\]
\[= p_1p_2[(a_1 + a_2)N_1 + a_2N_2] \quad (A.3)\]

where

\[a_1 = \sum_{l=0}^{c-1} \sum_{k=0}^{c-l-1} (l + 1) \binom{k + l}{k}(1 - p_2)^l p_1^l(1 - p_1)^k \quad (A.4)\]
\[a_2 = \sum_{l=0}^{c-1} \sum_{k=0}^{c-l-1} k \binom{k + l}{k}(1 - p_2)^l p_1^l(1 - p_1)^k. \quad (A.5)\]

From the Taylor series expansion, we have

\[\sum_{l,k=0}^{\infty} \binom{l + k}{k} x^k y^l = \frac{1}{1 - x - y} \quad (A.6)\]
\[\sum_{l,k=0}^{\infty} \binom{l + k}{k} x^k y^l = \frac{x}{(1 - x - y)^2} \quad (A.7)\]
\[\sum_{l,k=0}^{\infty} \binom{l + k}{k} x^k y^l = \frac{y}{(1 - x - y)^2} \quad (A.8)\]

for \(|x + y| < 1\). Here, we choose \(x = 1 - p_1\) and \(y = p_1(1 - p_2)\); therefore, we have
\[|x + y| = 1 - p_1p_2 < 1.\] From (A.4)-(A.5) and (A.6)-(A.8), and for large values of \(c, a_1\)
and $a_2$ are approximated by

$$a_1 \approx \frac{1}{p_1 p_2},$$  \hspace{1cm} (A.9)

$$a_2 \approx \frac{1 - p_1}{p_1^2 p_2}. \hspace{1cm} (A.10)$$

From (A.1), (A.2), (A.3) and (A.9)-(A.10), we have

$$N_{\text{search}} \approx \frac{N_1}{p_1 p_2} + \frac{N_2}{p_2}$$

$$= \frac{N_1}{((1 - u)/2 + u F_{N_1(1|H_1)}(1 - u_1(N_1))(1 - P_f(N_2))} + \frac{N_2}{(1 - u_1(N_1))(1 - P_f(N_2))},$$

(A.11)
Appendix B

PROOF OF THEOREM 2

Proof. First we show that the optimal sensing strategy is given by (4.19). The proof is based on induction on \( L \) and proof by contradiction. First we show that (4.19) holds for \( L = 1 \) and 2. For \( L = 1 \), we have

\[
\delta_1^*(\alpha, \beta) = \arg \max_{A_{\alpha, \beta}} R(\alpha, \beta; A). \tag{B.1}
\]

The optimality of (4.19) for \( L = 1 \) follows directly from (4.14), since we have

\[
R(\alpha, \beta; 1) = \frac{4p_0}{2p_0 + p_1} \geq R(\alpha, \beta; 2) = R(\alpha, \beta; 3) = \frac{2p_0}{2p_0 + p_1} \geq R(\alpha, \beta; 5) = p_0 \geq R(\alpha, \beta; 4) = 0. \tag{B.2}
\]

Therefore, we have

\[
V_1^*(\alpha, \beta) = \begin{cases} 
\frac{4p_0}{2p_0 + p_1} & \text{for } \alpha, \beta > 0 \\
\frac{2p_0}{2p_0 + p_1} & \text{for } \alpha > 0, \beta = 0 \\
\frac{2p_0}{2p_0 + p_1} & \text{for } \alpha = 0, \beta > 0 \\
p_0 & \text{for } \alpha = \beta = 0 
\end{cases}. \tag{B.3}
\]

For \( L = 2 \), we have

\[
\delta_2^*(\alpha, \beta) = \arg \max_{A_{\alpha, \beta}} [R(\alpha, \beta; A) + \sum_{\alpha', \beta'} P_A(\alpha', \beta'; \alpha, \beta) V_1^*(\alpha', \beta')]. \tag{B.4}
\]
Appendix B. PROOF OF THEOREM 2

Consider the case where \( \alpha \geq 2 \) and \( \beta = 0 \). Then we have \( A_{\alpha,0} = \{2,4,5\} \). The right-hand side (R.H.S.) of (B.4) for various admissible actions becomes

for \( A = 2 \), R.H.S. of (B.4) = \( \frac{2p_0}{2p_0 + p_1} + (p_0 + \frac{p_1}{2})V_1^*(\alpha,0) + \frac{2p_0}{2p_0 + p_1} \) \( V_1^* (\alpha - 1,0) \) \hfill (B.5)

for \( A = 4 \), R.H.S. of (B.4) = \( p_0 + (p_0 + \frac{p_1}{2})^2 V_1^* (\alpha + 1,1) + \frac{2p_0}{2p_0 + p_1} \) \( (p_0 + \frac{p_1}{2}) (p_0 + p_2) V_1^* (\alpha + 1,0) \) \hfill (B.6)

for \( A = 5 \), R.H.S. of (B.4) = \( p_0 + V_1^* (\alpha, 0) = p_0 + \frac{2p_0}{2p_0 + p_1} \) \hfill (B.7)

Note that \( \frac{4p_0}{2p_0 + p_1} \geq \frac{2p_0}{2p_0 + p_1} \). Analogously, for \( S = (1,0) \), we have

for \( A = 2 \), R.H.S. of (B.4) = \( p_0 \left( 1 + \frac{p_1}{2} + p_2 + \frac{2}{2p_0 + p_1} \right) \) \hfill (B.8)

for \( A = 4 \), R.H.S. of (B.4) = \( p_0 + \frac{2p_0}{2p_0 + p_1} \) \hfill (B.9)

for \( A = 5 \), R.H.S. of (B.4) = \( p_0 + \frac{2p_0}{2p_0 + p_1} \) \hfill (B.10)

Hence the maximizing action for \( S = (\alpha,0) \) where \( \alpha > 0 \) is Action 2, which is also given by (4.19). Similar results are obtained for other points in the \( \alpha\beta \)-plane.

Let us now assume that the sensing strategy given by (4.19) is optimal for up to some \( L \geq 2 \) but not for \( L + 1 \). With \( L + 1 \) sensing slots left, an optimal sensing strategy can be expressed as

\[ \delta^{opt} = \{\delta_1^*, \ldots, \delta_L^*, \delta_{L+1}^{opt}\}. \] \hfill (B.11)

Since the strategy \( \delta^* = \{\delta_1^*, \ldots, \delta_{L+1}^*\} \), which is given by (4.19), is not optimal, we have

\[ \delta_{L+1}^{opt} (S) \neq \delta_{L+1}^* (S) \] \hfill (B.12)

\[ V_{\delta^{opt}_{L+1}} (S) > V_{\delta^*_{L+1}} (S) \] \hfill (B.13)

for some \( S \) in the \( \alpha\beta \)-plane. Let \( \delta' \) be a strategy of the form

\[ \delta' = \{\delta_1^*, \ldots, \delta_{L-2}^*, \delta_{L-1}^*, \delta_L^*, \delta_{L+1}^*\}. \] \hfill (B.14)
From the optimality of \{\delta_1^*, \ldots, \delta_{L-2}^*, \delta_{L-1}^*, \delta_L^*\}, we have

\[
V_{\delta^*,L+1}(S) \geq V_{\delta^*,L+1}(S).
\] (B.15)

for any \(S, \delta_{L-1}^*,\) and \(\delta_L^*\)

**Proposition 1.** For any \(S\) and \(\delta_{L+1}^{\text{opt}}\), we can choose \(\delta_{L-1}^*\) and \(\delta_L^*\) such that

\[
V_{\delta^*,L+1}(S) = V_{\delta_{L+1}^{\text{opt}}}(S)
\] (B.16)

which is in contradiction with (B.13) and (B.15). Therefore we have

\[
\delta_{L+1}^{\text{opt}} = \delta^*.
\] (B.17)

**Proof.** We show the proof for a special choice of \(S\) and \(\delta_{L+1}^{\text{opt}}\). Similar results are obtained for any arbitrary \(S\) and \(\delta_{L+1}^{\text{opt}}\). Suppose there are \(L + 1\) sensing slots left, and \(S = (1, 0)\). We have \(\delta_{L+1}^*(1, 0) = 2\). Let us assume that \(\delta_{L+1}^{\text{opt}}(1, 0) = 4 \neq 2\). We have

\[
V_{\delta_{L+1}^{\text{opt}},L+1}(1, 0) = 0 + (p_0 + \frac{p_1}{2})^2V_L^*(2, 1) + (p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)V_L^*(2, 0)
\]

\[
+ (p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)V_L^*(1, 1) + \frac{p_1}{2}^2V_L^*(1, 0)
\]

\[
= p_0(2 + \frac{p_1}{2} + 2p_2 + \frac{p_1}{2p_0 + p_1}) + [(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2) + (\frac{p_1}{2} + p_2)^2]V_{L-1}^*(0, 0)
\]

\[
+ [(p_0 + \frac{p_1}{2})^2 + 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^2]V_{L-1}^*(1, 0) + (\frac{p_1}{2} + p_2)(p_0 + \frac{p_1}{2})^2V_{L-1}^*(2, 0)
\]

\[
= p_0[3 + (\frac{p_1}{2} + p_2)^2 + (\frac{p_1}{2} + 2p_2) + [(\frac{p_1}{2} + p_2)^2 + (p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)]
\]

\[
+ 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^2]V_{L-2}^*(0, 0) + [(p_0 + \frac{p_1}{2})^2 + 5(p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)]
\]

\[
+ 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^2]V_{L-2}^*(1, 0) + [(p_0 + \frac{p_1}{2})^2 + (p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)]
\]

\[
+ (p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)^2]V_{L-2}^*(2, 0)
\] (B.18)

where we used the fact that \(\delta_i^{\text{opt}}(S) = \delta_i^*(S)\) and \(V_{\delta_i^{\text{opt}},L}(S) = V_i^*(S)\) for \(0 \leq i \leq L\). If we let

\[
\delta_i^*(S) = 4, \quad \text{for all } S
\] (B.19)
and

$$\delta'_{L-1}(S) = \begin{cases} 
3 & \text{for } S = (0,1), (1,1) \\
\delta^*_L(S) & \text{otherwise}
\end{cases}$$

We have

$$V_{\delta',L+1}^*(1,0) = \frac{2p_0}{2p_0 + p_1} + (p_0 + \frac{p_1}{2})V_{\delta',L}^*(1,0) + (\frac{p_1}{2} + p_2)V_{\delta',L}^*(0,0)$$

$$= \frac{2p_0}{2p_0 + p_1} + (\frac{p_1}{2} + p_2)^3V_{\delta',L-1}^*(0,0) + 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^2V_{\delta',L-1}^*(1,0)$$

$$+ (p_0 + \frac{p_1}{2})^2(p_1 + p_2)V_{\delta',L-1}^*(0,1) + (p_0 + \frac{p_1}{2})^2(p_1 + p_2)V_{\delta',L-1}^*(1,1)$$

$$+ (p_0 + \frac{p_1}{2})^2(p_1 + p_2)V_{\delta',L-1}^*(2,0) + (p_0 + \frac{p_1}{2})^3V_{\delta',L-1}^*(2,1)$$

$$= p_0[3 + (\frac{p_1}{2} + p_2)^2 + (\frac{p_1}{2} + p_2)^3] + [(\frac{p_1}{2} + p_2)^5 + (p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)^2]$$

$$+ 3(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^3V_{L-2}^*(0,0) + [(p_0 + \frac{p_1}{2})^2 + 5(p_0 + \frac{p_1}{2})^2(\frac{p_1}{2} + p_2)^2]$$

$$+ 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)^4V_{L-2}^*(1,0) + [(p_0 + \frac{p_1}{2})^3(\frac{p_1}{2} + p_2)]$$

$$+ (p_0 + \frac{p_1}{2})^2(p_1 + p_2)^3V_{L-2}^*(1,1) + (p_0 + \frac{p_1}{2})^3(p_1 + p_2)V_{L-2}^*(2,0)$$

$$= V_{\delta^*_L,L+1}^*(1,0) \quad (B.21)$$

where we used the fact that $\delta'_i(S) = \delta^*_i(S)$ and $V_{\delta^*_i,i}(S) = V^*_i(S)$ for $0 \leq i \leq L - 2$. \hfill \Box

Here, we aim to derive a relation for expected number of idle channels detected following the optimal sensing strategy. From (4.14), (4.19), and the state transition probabilities in Figure 4.4, we can write

$$V_{L}^*(0,0) = (p_0 + \frac{p_1}{2})^2V_{L-1}^*(1,1) + (\frac{p_1}{2} + p_2)^2V_{L-1}^*(0,0) + 2(p_0 + \frac{p_1}{2})(\frac{p_1}{2} + p_2)V_{L-1}^*(1,0) \quad (B.22)$$

$$V_{L}^*(1,0) = \frac{2p_0}{2p_0 + p_1} + (p_0 + \frac{p_1}{2})V_{L-1}^*(1,0) + (\frac{p_1}{2} + p_2)V_{L-1}^*(0,0) \quad (B.23)$$

$$V_{L-1}^*(1,1) = \frac{4p_0}{2p_0 + p_1} + V_{L-2}^*(0,0). \quad (B.24)$$
If we let $w(L) = [V^*_L(0,0) \ V^*_L(1,0)]^T$ and $\rho = \frac{p_1}{2} + p_2$, we have the following difference equation

$$w(L) = Aw(L - 1) + Bw(L - 2) + C \quad (B.25)$$

where

$$A = \begin{bmatrix} \rho^2 & 2(1 - \rho)\rho \\ \rho & 1 - \rho \end{bmatrix}, \quad B = \begin{bmatrix} (1 - \rho)^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and } C = p_0 \begin{bmatrix} 2(1 - \rho) & \frac{1}{1 - \rho} \end{bmatrix}^T \quad (B.26)$$

with the initial conditions

$$w(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T, \quad w(1) = p_0 \begin{bmatrix} 1 \\ \frac{1}{1 - \rho} \end{bmatrix}^T. \quad (B.27)$$

Taking $z$-transform on both sides of (B.25), we obtain

$$W(z) = z^{-2} \left[ I - z^{-1}A - z^{-2}B \right]^{-1} \left[ (A + z^{-1}B)w(1) + C/(1 - z^{-1}) \right] + z^{-1}w(1) \quad (B.28)$$

where $W(z)$ is the $z$-transform of $w(L)$. Taking the inverse $z$-transform and solving for $V^*_L(0,0)$, we obtain

$$V^*_L(0,0) = Lp_0 \left( \frac{2}{2 - \rho} - \frac{1}{(2 - \rho)^2 L} [1 - (1 - \rho)^2] \right). \quad (B.29)$$
Bibliography


