Grade 9 Teachers Use of Technology in Linear Relations

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ABSTRACT

The purpose of this study is to examine secondary mathematics teachers’ perceptions about technology integration in teaching the grade 9 Linear Relations Unit and to investigate the impact of these perceptions and teachers’ everyday practices on the development of student tasks, construction of content knowledge, and facilitation of students’ mathematical communication within the context of the Linear Relations Unit in grade 9 mathematics.

Case studies were conducted with three mathematics teachers teaching in three urban secondary schools in Ontario. Qualitative data was collected through a series of ongoing classroom observations of the teachers. Additionally, interviews were conducted at the beginning and end of the data collection phase with each teacher.

The results from this study suggest that the teachers perceived that the integration of technology in the Linear Relations Unit assisted them to: 1) create interactive and dynamic learning environments which helped make the content meaningful to students; 2) guide their instruction and to closely monitor students’ understanding and track their progress, by providing real time feedback; 3) help struggling students move forward in their learning when they did not master the prerequisite skills required to build upon a new math concept and to help them develop math interpretative and problem solving skills; 4) differentiate instruction and address different learning styles and skills making abstract content more tangible and helping students
connect words to images and graphs; 5) teach students to verify and validate their answers and check for their correctness, as well as to avoid relying only on the visual aspect of mathematics; and 6) assist students build mathematical communication skills.

Implications of the findings for future research and suggestions to secondary mathematics teachers integrating technology, in the context of the Linear Relations Unit, are also included.
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CHAPTER ONE: INTRODUCTION

1.1 Introduction

The purpose of my study is to explore teachers’ perspectives on the role of technology and the way they are reflected in their teaching practice in a technology supported environment. Particularly, I investigated teachers’ perceptions about the integration of technology in teaching the grade 9 mathematics Linear Relations Unit and how the integration of technology impacts on three of the Ten Dimensions of Mathematics Education (McDougall, 2004): Student Tasks (Dimension 4), Constructing Knowledge (Dimension 5), and Students’ Mathematical Communication (Dimension 8).

My interest in this topic stems from my eagerness to help mathematics teachers create an interactive class environment that is structured to facilitate and support the process of learning mathematics. As the Head of a Mathematics Department in an Ontario high school, I have searched for ways to improve the quality of mathematics teaching and to enhance students’ learning experiences and their achievement in mathematics.

It is stated by the National Council of Teachers of Mathematics (NCTM, 2000) that among other engaging teaching and learning strategies and techniques, the implementation of technology in the mathematics classroom is thought to influence the teaching of mathematics, to enhance students’ learning, and to increase “students’ focus on decision making, reflection, reasoning, and problem solving” (p. 24). Along this line, I believe that the effective integration of technology has the potential to assist teachers in developing meaningful lessons, which provide opportunities for students to investigate mathematics in a variety of contexts. Also, if used efficiently, technology integration could facilitate a better understanding of abstract math concepts, help students connect among various representations, and assist them in solving
problems inspired by real life situations. However, the research regarding the integration of technology in mathematics suggests that technology in and of itself does not make any difference in students’ learning. Only when it is utilised by teachers with strong knowledge of technology, pedagogy, and mathematical content and who know how to select appropriate instructional strategies to motivate students to engage in the learning process may technology produce desirable outcomes in students’ learning and their achievement.

Based on this assumption, my study intends to examine secondary mathematics teachers’ perspectives about the role of technology in delivering the Linear Relations Unit content from the grade 9 Applied course. It also aims to identify instructional strategies integrating technology that teachers employ when developing student tasks, planning for knowledge construction, and facilitating mathematical communication in the context of the grade 9 Linear Relations Unit.

The evidence from the last four years of EQAO standardised test results indicate that, for “cognitive skills Applications and Thinking, students performed least well on the questions involving Linear Relations context” (EQAO’s Provincial Secondary School Report, 2012, p. 35). In searching for ways to find successful and effective uses of technology for the teaching and learning of mathematics, I decided to conduct this investigation in a grade 9 Applied course and to focus on the Linear Relations Unit, which is a unit that students did not perform well on in the EQAO standardised test.

This thesis outlines the context of my research, my personal background, the research questions, the conceptual framework, the existing literature relevant to teachers’ perceptions about the role of technology in mathematics education, and my research methodology. The data is presented as case studies of three mathematics teachers integrating technology in delivering the Linear Relations Unit in grade 9 mathematics. The case studies are analysed in relation to the
research questions and literature. Finally, implications of the findings and suggestions for future research are discussed.

1.2 Research Context

Research regarding the integration of technology in mathematics education suggests that the use of different technological tools significantly extends and enriches teachers’ instructional strategies and supports students’ learning in mathematics. Technology has been used to engage students in discovering the power and beauty of mathematics through explorations and investigations, multiple representations of different mathematical concepts (for example, symbolic, numerical, and graphical), and simulation of the real world. The integration of technology in mathematics lessons provides opportunities for students to conceptualise and practice abstract theories and ideas, to experience and negotiate among multiple perspectives and solutions, to examine and probe answers and conjectures, to explore questions, to generate and test hypotheses, and to construct mathematical knowledge. Technology offers possibilities to experiment and collect real data, sketch and analyse graphs, and draw valid mathematical inferences. It makes mathematics more meaningful and attractive to learners.

The importance of technology is reflected in various curriculum documents. For example, according to the Ontario Curriculum, Grade 9 and 10 Mathematics (2005):

Applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, graphing software, computer algebra systems (CAS), word-processing software, and presentation software can be used to support various methods of inquiry in mathematics. The technology also makes possible simulations of complex systems that can be useful for problem solving purposes or when field studies on a particular topic are not feasible. (p. 28)

Technology can help reduce the time spent on routine mathematical tasks, allowing students to devote more of their efforts to thinking and concept development instead of the computational aspect of the problem (The Ontario Math Curriculum, 2005).
Technology is also a complementary teaching tool that helps address students’ individual needs and different learning styles. Habre (2007) expressed that it is important to incorporate all tools available, including technology, to help students have a deeper understanding of the math concepts. Technology integration in mathematics lessons involves interaction and dialogue among students and between students and teachers. It calls for organisation and strong communication skills in order to display and present coherent mathematical ideas. Technology requires the acquisition of specific mathematical terminology and the adaptation of precise mathematical vocabulary, which are both needed to represent and communicate findings and solutions to problems. Kimmins (1995) noticed:

Precise language is essential when programming a calculator or a computer to perform a desired task. Mathematical thought has to be well-formulated and translated into the language of the calculator or computer when, for example, a computer or calculator is programmed to estimate the probability that a quadratic equation has real roots under varying conditions on the coefficients. Furthermore, mathematical communication is fostered when students use word processors and presentation software to write and present reports which include mathematical symbols, tables, and graphs. (p. 3)

According to Goldsmith and Schiffer (1997), the reform practice described by the NCTM (2000) requires reconceptualising one’s notion of teaching, learning, and mathematics. In other words, it requires a major restructuring of mathematics practice. It requires a paradigm shift in terms of beliefs and perspectives. Integrating technology as a tool that challenges the traditional approach to teaching and learning mathematics is an important part of this process of change.

Despite the concerted efforts of educators to promote technology as a medium of instruction with a great potential for fostering learning by creating real world simulations, its implementation in secondary mathematics education is lagging (Lagrange et al., 2003). While some teachers avoid using technology because they do not have adequate training and do not feel confident implementing a specific technology in their practice, others experience difficulties in adjusting their teaching strategies to settings involving technology (Monaghan, 2004). Even
teachers who are highly educated and skilled with technology do not necessarily “integrate technology on a consistent basis” (Bauer and Kenton, 2005, p. 519).

Bauer and Kenton (2005) enumerated key obstacles that might prevent the integration of technology, such as access to computer labs or technological tools, lack of appropriate software, software compatibility, Internet connectivity, technical problems, extra time required to plan technology based lessons, student skill levels, and large class size. Nevertheless, there is evidence that technology is used in the math classroom to enhance the teaching and learning process, that innovative lessons incorporate technology not only as a computational tool but also as a medium for investigations, explorations, and testing conjunctures, and that the learning outcome is improved as a result of using technology.

Thus, research on technology integration in mathematics education has indicated that technology is used in various ways to deepen students’ understandings of mathematics content. For example, the National Council of Teachers of Mathematics (NCTM, 2000) emphasised that “teachers can use simulations to give students experience with problem situations that are difficult to create without technology, or they can use data and resources from the Internet and the World Wide Web to design student tasks” (p. 25). Bransford, Brown, and Cocking (1999) suggested that technology is used to create conditions for learning that are based on the success of visualisation of concepts. These include real world contexts for learning, connections to outside experts, visualisation and analysis tools, scaffolds for problem solving, and opportunities for feedback, reflection, and revision.

Harnisch (2002b) advised that, via the Internet, technology can connect students with outside experts. They “can have access to experts in practically any field all over the globe. They can send e-mail to them and even ‘chat’ with them online” (p. 2). Harnisch and Sato (1990)
recommended integrating technology to provide scaffolds for problem solving and to provide opportunities for students to examine complex situations, to consult multiple sources and media, and to collaborate and understand the perspectives of others. As stated by the NCTM (2000), “with calculators and computers, students can examine more examples or representational forms than are feasible by hand, so they can make and explore conjectures easily” (p. 24). Alagic (2003) stated that interactive technologies like Geometer’s Sketchpad allow students to observe changes instantly, thereby providing opportunities for students to gain a more in-depth understanding of what is happening behind the scenes.

Technology can also be used to generate and collect data for conjectures. According to NCTM (2000), students can investigate characteristics of shapes using dynamic geometry software, organise and analyse large sets of data, study linear relationships and the ideas of slope and uniform change with computer representations, use simulations to study sample distributions by performing physical experiments with calculator based laboratory systems, and work with computer algebra systems that efficiently perform most of the symbolic manipulation that was the focus of traditional secondary school mathematics programs (p. 26).

Lewis (1999) discussed the importance of integrating technology to facilitate the connection of different curricular topics and interconnect subjects: “Lessons learned in one subject must be transferred to others. To take its space squarely in school curricula, technology education must establish itself not just in its own right, but crucially in relation to other subjects” (p. 49). As stated by the NCTM (2000), technology “blurs some of the artificial separations among some topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics” (p. 26).
However, although technology has great potential to impact the teaching and learning of mathematics, its integration in the classroom does not guarantee high achievement, better understandings, and learning outcome (Clark, 1983; Li, 2004). Successful and effective teaching with technology depends on many aspects, including teachers’ knowledge of technology, pedagogy, and content knowledge, quality of teaching approaches, curriculum goals, type of programs, and type of learners (Albright & Graf, 1992; Coley et al., 2000). Monagan (2004) recognised that “the whole undertaking involves a fusion of many factors” and he proposed “to find a holistic way to examine teachers’ practice” (p. 327). Ruthven and Hennessy (2002) reported that technology is most useful to enable general pedagogical aspirations, as opposed to its didactical contribution to mathematics learning.

Beliefs about the success of using technology in mathematics vary widely among teachers and impacts on teachers’ choices to adopt technology in delivering the math content. Thus, some teachers integrate technology in their lessons extensively, while others do so very rarely. Guerrero, Walker, and Dugdale (2004) classified teachers’ attitudes toward the use of technology in the mathematics classroom as “apprehensive”, whereas their students’ attitudes were “mainly enthusiastic.”

A considerable amount of research has been conducted to examining the factors that contribute to teachers’ decisions to integrate technology into their lessons. Thus, as Monaghan (2004) suggested, understanding the actual situation of the few teachers using technology is a major and complex issue. Ross, Hogaboam-Gray, and Hannay (1999) noticed that teacher confidence played a significant role in determining whether they incorporated technology in their teaching. In their opinion, the most important predictor of confidence with technology-based instruction related to teachers’ perceptions of their ability to use computers. Some teachers were
reluctant to participate in workshops and technology demonstrations in order to learn about a specific technology and its efficient implementation in the mathematics lessons because they had low computer self-efficacy. According to Wang, Ertmer, and Newby (2004), this lack of confidence contributed significantly to teachers’ dispositions toward computer use in the classroom and may be an indicator of the different levels of technology integration in the classroom.

Sugar, Crawley and Fine (2004) noted that teachers’ decisions to integrate technology into their teaching were primarily based on “teachers’ conscious reasoning about the personal consequences for doing so” (p. 211). Thus, many teachers, satisfied with their teaching outcome and their learners’ results, did not see the need and relevance of changing their teaching style and adopting new technological tools and strategies. Healy and Hoyles (2001) expressed that the use of technology might present problems for teachers who were familiarised to a certain routine of instruction. These teachers feared that technology driven lessons impacted their classroom discipline and structure. Experienced teachers (15 or more years in the classroom), who were not trained in the use of computer technologies through formal teacher education coursework, were often resistant to technology because they did not see how it fit with their content area (Plair, 2008). These teachers believed that explorations and investigations utilising technology did not translate into a better educational outcome. They raised concerns that learners would focus on the visual aspect of learning and would fail to develop rigorous formal deduction and deductive reasoning. Consequently, teachers would fail to foster critical thinking and students would fail to develop their ability to justify and proof their solutions. Other teachers believed that the use of technology in the classroom was too time consuming, required extra preparation time, and that the extra effort seemed to have little impact on students’ learning outcomes (Snyder, 2001).
In addition, there were traditional mathematics teachers who rejected the idea of incorporating educational technology in their math lessons in spite of the fact that they used technology in various forms to prepare lesson plans and worksheets for their classes, to create different assessment and evaluation documents, and to record, calculate, and analyse their students’ marks. To a certain extent, these teachers worked indirectly with educational technology and used it to facilitate and ease the teaching and learning process.

A review of the literature (Schmidt & Callahan, 1992; Drier, 2001a; Drier, 2001b) showed that some experienced teachers believed that using technology would harm students’ understanding of basic math concepts, make them dependent on technology, and not be effective as an instructional tool. Moreover, Li (2007) concluded that most teachers in her study perceived technology integration “as no more than an extra workload on both teachers and students, with little educational value for the time and effort invested” (p. 392). She also found that many teachers refused to adopt technology because they feared that technology might replace them in the classroom.

In conclusion, the research on mathematics learning with technology indicates that although technology has great potential to impact the teaching and learning of mathematics, if it is not used effectively it does not help students to better understand the math content. However, there is scarcity in the analysis of how teachers’ perceptions on the role of technology in mathematics education are reflected in their technology based lessons designed for grade 9 Applied level students, who have not been very successful in the EQAO standardized tests over the past 5 years, have attendance problems, experience some language challenges, and lack effort and engagement.
The primary goal of this study is to investigate this gap in the literature. Specifically, this thesis will explore teachers’ perceptions about the role of technology in teaching the Linear Relations Unit in a grade 9 mathematics course, analyse the adjustment of teachers’ strategies and techniques in technology based lessons, and discuss the impact of technology integration on three dimensions of mathematics education, namely student tasks, knowledge construction, and students’ communication. I will use the Ten Dimensions of Mathematics Education (McDougall, 2004) as the conceptual framework to guide the methodology and analysis of my study. This framework provides an explicit description of the essential components of a successful mathematics education program and offers initiation and guidance for instructional practices that differ from traditional practice.

The ultimate goal of this study is to inform and help mathematics teachers understand the potential benefits of using technology in teaching the Linear Relations Unit in the grade 9 mathematics course.

1.3 Significance of the Study

My research interest revolves around two main axes: teachers’ perceptions about the role of technology in mathematical education and the way their perceptions are reflected in their teaching. The decision to conduct my research with teachers teaching the grade 9 Applied course was based on the need to explore and identify instructional strategies and techniques that allow junior secondary school students of different abilities to be successful in mathematics. The Ontario Education Quality and Accountability Office (2012) stated that, over the past five years, the percentage of students taking applied mathematics who performed at or above the provincial standard increased by ten percentage points, from 34% to 44%. The increase in the number of students achieving at or above the provincial standard in mathematics is giving hope to
mathematics educators. However, there is cause for concern as 56% of grade 9 Applied students failed to meet the provincial standards and there is a sense of urgency to address this issue to prevent students from falling behind their academic peers or even international peers. This is a major concern that teachers, in collaboration with researchers in education, need to address.

As McDougall (2004) suggested, “There are many ways a teacher can facilitate an inclusive and nurturing environment” (p. 23). In my opinion, integrating technology efficiently in the teaching of mathematics offers true potential for teaching and learning, as well as for helping students to increase their achievement in mathematics. By providing opportunities for exploration and investigations, by engaging in interactive activities, and by enabling problem-based learning, the integration of technology helps students better connect with their teachers and become more engaged in the learning process. This means that students are more interested in lessons that are focussed around the use of technology, are more productive in completing school work, and participate more in classroom discussions, which in turns helps their comprehension of math content and their achievement in mathematics. Technology also offers teachers the possibility to differentiate instruction to address the needs and learning skills of their students. It also assists teachers in providing real time feedback, which informs students about their learning.

Additionally, the instant feedback may guide teachers to make important instructional decisions and to immediately address misconceptions and misunderstanding. Technology can support learning through the use of real-world problems, which supports students in making the connection between concrete contexts and abstract concepts.

As technology brings about an increase in students’ motivation and engagement and helps knowledge construction in a dynamic and interactive environment, technology integration might be one way to help deliver the content of the grade 9 Applied mathematics course in order
to deepen and enhance students learning and improve their achievement in the EQAO standardised test.

In my research, I used the Ten Dimensions of Mathematics Education as the conceptual framework for understanding best teaching practices in mathematics (McDougall, 2004). I used this framework as a guideline to help identify key elements in technology based instruction and as an organisational tool to keep track of teachers’ views on the role of technology in secondary mathematics education. The focus of my study is to investigate how the integration of technology impacts on three of the Ten Dimensions of Mathematics Education: student tasks, constructing knowledge, and students’ mathematical communication. These dimensions, directly connected to the process of learning new mathematics concepts, provide a summary of significant elements that teachers use in their instruction to help students build new understandings and communicate their thinking and mathematical arguments in more formal ways, using conventional mathematical terminology and formal vocabulary.

In this study, I investigate how teachers shape their lessons and instructional strategies when integrating technology in their teaching. The investigations were guided by the characteristics of the three selected dimensions from the Ten Dimensions of Mathematics Education framework. Out of the Ten Dimension of Mathematics Education proposed by McDougall (2004), these three essential features refer to the basic learning process. The development of student tasks communicates the nature of the learning prompted by the teacher and clarifies teachers’ expectations and their objectives in a particular educational circumstance. The construction of content knowledge constitutes the act of learning, provides information about how learning occurs (if it does occur), and informs appropriate use of student tasks and instructional strategies in that specific context. It provides avenues for teachers’ reflections about
the efficiency and effectiveness of the instructional strategies and tools used. Then, during the teaching and learning process, students communicate, debate, ask questions, and provide answers and explanations. Technology could contribute to enhance the development of student tasks, to help students comprehend and construct new knowledge and to increase students’ abilities to report their findings and understandings orally and in writing. For example, in order to learn a new concept, students must be provided with a task to explore or investigate. Then, they analyse, discuss, and negotiate new meanings and construct new content knowledge. At the end of this process, students report their comprehension of the new concepts and translate their understanding in words or mathematical symbols either orally or in writing.

![Figure 1. Dimensions of Education](image)

According to the Student Tasks Dimension, teachers design authentic problem solving tasks embedded in real life context. These tasks allow for multiple representations and multiple solutions and bridge conceptual understanding from real life to abstract representation.

According to the Constructing Knowledge Dimension, teachers use various instructional strategies to build on students’ prior knowledge, provide appropriate scaffolding, model high-level performance, and engage students through effective questioning techniques. According to the Students’ Mathematical Communication Dimension, teachers engage in practices that increase students’ oral and written communication skills (McDougall, 2004).
My study expands the understandings of the Ten Dimensions of Mathematics Education to provide evidence of effective ways to use technology in the math classroom to bring about enhanced student learning. It suggests strategies and techniques to be used in technology based lessons to improve the process of teaching and learning mathematics, specifically in relation to the development of student tasks, construction of knowledge, and facilitation of mathematics communication. The findings of my thesis add to the characteristics of the Ten Dimensions of Mathematics Education key elements specific to the student tasks, construction of content knowledge, and mathematics communication in a technology based learning environment.

Mathematics teachers could benefit from the findings of this study, as they are able to identify effective teaching strategies and techniques to be used or to be avoided in a technology driven environment. Teachers are able to get a better understanding of how to efficiently integrate technology in teaching the grade 9 Linear Relations Unit to develop rich tasks, to help students construct content knowledge, and to facilitate students’ mathematical communication. The findings of this thesis may help teachers develop an intrepid attitude in the use of technology; one that encourages them to take risks and inspires them to involve technology in the classroom environment.

1.4 Research Questions

The research questions in this study are:

1) What are mathematics teachers’ perceptions about the integration of technology in teaching the Linear Relations Unit in grade 9 mathematics?

2) How do teachers’ views on the role of technology and their everyday practice help us to better understand how they develop student tasks in the context of the Linear Relations Unit?
3) How do teachers’ views on the role of technology and their everyday practice help us to better understand the construction of content knowledge in the context of the Linear Relations Unit?

4) How do teachers’ views on the role of technology and their everyday practice help us to better understand the facilitation of students’ mathematical communication in the context of the Linear Relations Unit?

1.5 Researcher: Personal Background

I was born in Romania where I spent the first 30 years of my life. I graduated with a Bachelor of Science degree and obtained my teaching certification from Babes-Bolyai University (Romania). After graduation, I taught in the secondary school in Brasov, Transylvania (Romania) for four years.

As a child and teenager, I studied eight years of elementary school and four years of secondary school in an intense math and science program. During my schooling, the educational emphasis was on mathematics and science and the vision of the totalitarian communist regime was to prepare young students for industrial tasks and jobs. As an intermediate and senior student, I attended mathematics and science classes every day for at least one period. I enjoyed these classes although I still remember the load of homework assigned for the evenings. There were also long-term projects and assignments that required extra work on the weekends.

As part of an elite mathematics and science program in secondary school, the math and science lessons included a significant amount of problem solving designed to develop a student’s ability to think abstractly and to perform mathematical operations with numbers in an efficient manner. In spite of the fact that collaboration in the classroom was not encouraged by teachers, the high-level of thinking and abstractness of the problems assigned for homework encouraged
peers to get together and work in groups to discuss and figure out solutions to problems. Incompletion of homework was not an option and excuses were not acceptable, including: I was confused, I did not understand, and I did not know how to approach the questions.

Even though teachers accepted more than one mathematically correct way to get to the answer, precision and exactness, reported clearly and concisely in very formal vocabulary and symbols of math, were the focus of learning mathematics. Students were expected to memorise and remember basic skills and facts, as well as specific math knowledge. Calculators were forbidden, computers were at the beginning of their existence, and manipulatives were not used in the learning and teaching process.

The secondary school curricula treated Arithmetic, Algebra, Euclidean Geometry, Analytic Geometry, Statistics and Probabilities, Advanced Functions and Calculus and Vectors as distinct math subjects. Deductive reasoning received the most emphasis and rigorous proofs based on classical axioms and theorems were the norm in the mathematics courses.

Teaching mathematics through lectures was the instructional method used by teachers. The classroom was a very quiet environment where students listened to the teacher delivering the math content and then worked independently to provide solutions to various problems. There were not opportunities during the lessons for students to come together and discuss ideas and facts in order to expand their understandings and comprehend lessons more effectively. Individual work was encouraged and was something that teachers felt was essential in order to be successful in courses such as mathematics, physics, or chemistry. Teachers were very strict and they expressed outrage and put a student down for not knowing the correct answer to a particular question.
Every week, one class period was devoted to preparing students for math and science competitions. During these lessons, teachers presented high-level thinking questions and advanced theories and asked students to figure out solutions to previous local, national, and international contests. Then, students were invited to present their solutions in front of the whole class. The classroom engaged in discussions and various interpretations were introduced and demonstrated.

After graduating from university, I taught for four years at three different high schools in Brasov, Transylvania (Romania). During this period of time, I realised that students have different interests in learning and have various learning styles and needs. I also realised that direct instruction does not fit the learning style of every single student in the classroom. This was when I first realized that the educational system in Romania was missing significant components in the process of teaching and learning and that it did not encourage students to discover their abilities, nor did it help them to succeed in their academic endeavours. On the contrary, the system discouraged some students to persist in learning by destroying their self-esteem and confidence. Students who were not achieving well in mathematics and sciences were considered inferior and were looked down upon. Until the social revolution in 1989 and the very end of the communist regime, the whole educational system in Romania failed to identify the need to adjust the methods of instruction and evaluation and tailor them to reach all students effectively.

After I moved to Canada, I pursued a Bachelor of Education from the University of Ontario Institute of Technology and a Master of Education from the University of Toronto. I was so happy to discover the variety of approaches to the teaching and learning process. With a very solid mathematics background, I adapted easily to the educational system in Canada. The integration of technology in the mathematics classroom was one teaching component that
captivated my interest, especially because it makes the study of mathematics topics that were previously impractical accessible and it provides students with a rich way to gain confidence and competence in learning mathematics.

As a Head of the Mathematics Department in an Ontario secondary school, I am committed to model and promote a “hands-on” approach to learning mathematics based on the extensive use of technology and the fostering of creativity. During my eight years as a secondary school mathematics teacher in Ontario, I have implemented various teaching strategies to encourage students to participate in math lessons, to engage them in solving problems, and to make the content relevant to their everyday life. One of my interests in teaching mathematics is the adoption and integration of educational technology in the teaching and learning process. By using computers, graphing calculators, and an interactive board, I deliver engaging lessons, access various resources, and provide multiple representations to the math content. I try to captivate students’ curiosity with simulations, virtual scenarios, dynamic manipulations, problem solving situations, explorations, and investigations. These activities have created an interactive learning environment, engaged students, and fostered the learning of mathematics. New emerging technology has helped me as a math teacher to create an appropriate student-centred environment that makes mathematics fun and captivating.

As our society is progressing rapidly through the advancement of technology, my interest is to examine teachers’ beliefs about the power of technology to enhance the process of learning math and to help students become better problem solvers. Another interest is to determine how teachers plan and implement technology in their teaching practice in ways that support mathematics learning by creating innovative technology driven learning tasks that focus on
developing skills in collaboration, self-directed learning, high-level thinking, and communication.

1.6 Definitions of Terms

Technology: refers to technology used in an educational context, such as graphing calculators, a Smart Board, Mobi View tablets, clickers, computer software tools, computer algebra systems, interactive online activities (such as Gizmos), and other technology that assist in the collection, recording, organisation, and analysis of data. In the context of the grade 9 Applied level mathematics, technology encompasses a variety of interactive educational tools used to design and deliver meaningful lessons, relevant to students’ lives, to create dynamic classroom environments that motivate students to engage in the learning process, to provide students with opportunities to explore and investigate mathematical ideas and to help them make connections between real-word observations and abstract mathematical concepts, to enhance students learning experiences through the use of virtual simulations, to help deepen students connections with everyday experiences, to assist students develop problem solving skills and mathematical reasoning, to promote mathematical dialogue and help students’ develop their mathematical communication skills and expand their vocabulary.

Graphing calculator: any handheld calculator that “provides all the facilities of a scientific calculator, as well as capabilities for data analysis, linear algebra, programming, and the graphing of functions” (Penglase & Arnold, 1996, p. 59).

Smart Board: an interactive whiteboard that operates as part of a system and includes the interactive whiteboard, a computer, a projector, and whiteboard software.

Motion detector: a device that detects moving objects, particularly people.
Grade 9 Applied level students: a group of grade 9 students, who has a considerable number of students with attendance problems, experiencing some language challenges, and lacking effort and engagement.

1.7 Limitations of the Study

This study examined three secondary mathematics teachers integrating technology in teaching the Linear Relations Unit in grade 9 Applied mathematics course. Due to the small sample size, it will be difficult to make generalizations for teachers who have different characteristics than those that were studied. While large generalizations will be hard to make, this study will illuminate whether teachers, perceive technology integration in mathematics as having an impact on their instructional approach regarding three of the 10 Dimension of Mathematics in teaching the Linear Relations unit in grade 9. The three dimensions are: Student Task, Construction of Knowledge and Students’ Communication, as proposed by McDougall (2004).

It will be difficult to assert that the integration of technology in teaching Linear Relations will bring about specific classroom-based changes universally. However, the information will inform whether classroom-based changes are indeed made by those teachers who engage in integrating technology in delivering the Linear Relations Content. Additionally, participants involved in this study will be able to provide information about the aspects of teaching that are useful for supporting positive changes from integrating technology in the process of teaching and learning mathematics and those aspects that seem to impede positive change.
1.8 Plan of the Thesis

This thesis consists of five chapters. Chapter One outlines the research problem, the research context, the questions investigated, the researcher’s background, the significance of the study, and the plan of the thesis.

Chapter Two examines the literature pertaining to teachers’ perceptions on the role of technology in mathematics education and the literature related to evidence of technology based mathematics lessons’ design and implementation. The literature review summarises the Ten Dimensions of Mathematics Education framework (McDougall, 2004), which represents the lens through which the findings are organised and analysed.

Chapter Three provides detailed information about the research design, including participants, data collection, settings, procedures, and the frame analysis method. This chapter includes the rationale for the study, the protocol for observation, and the interview questions. It describes how using study participants present ethical considerations, such as the masking of identities and the protection of data.

Chapter Four presents the findings of the research study. The data is presented through the case studies of three secondary mathematics teachers integrating technology into teaching the Linear Relations Unit in grade 9 Applied mathematics course.

Chapter Five presents a discussion of the data in relation to the research questions. The findings are also linked to the literature about teachers’ perceptions on the role of technology in mathematics education and the literature related to evidence of technology based mathematics lessons’ design and implementation. Implications of these findings and suggestions for future research are discussed.
CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

Research in mathematics education has extensively explored the integration of technology in the classroom. Researchers have examined the integration of technology into the mathematics classroom with a focus on the perceptions of teachers and candidate teachers. The following review of the related literature is organised by teachers’ perceptions on the role of technology in mathematics education and evidence of technology based mathematical lessons’ design and implementation. The focus of technology integration related literature is on the development of student tasks, construction of content knowledge, and facilitation of students’ mathematical communication. The Ten Dimensions of Mathematics Education (McDougall, 2004), is the conceptual framework through which this study is organised and analysed.

2.2 Conceptual Framework

McDougall (2004) suggested that, as the understanding of what comprises best mathematical practices shifts toward a more reform oriented view, the expectations for teaching approaches and the role of the classroom teacher also change. It is widely recognised that effective mathematics instruction must incorporate more than the knowledge of number facts and basic operations, and that mathematical literacy is more than procedural fluency (McDougall, 2004). Supporting the National Council of Teachers of Mathematics’ (NCTM) Principles and Standards for School Mathematics (2000), McDougall (2004) identified five competencies necessary for mathematical literacy: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The conceptual framework guiding the analysis of this study is the Ten Dimensions of Mathematics Education (McDougall, 2004), a framework that emphasises aspects of teaching
and learning that are evident in a mathematics program. This framework provides an explicit description of the essential components of a successful mathematics education program and offers initiation and guidance for instructional practices that differ from the traditional practice (McDougall, 2004; Ross et al., 2003).

It helps teachers focus on specific aspects of their teaching that contribute to improve students’ achievement. The ten dimensions are: (1) Program Scope and Planning: Teachers take into account the strands of mathematics, their integration within units and lessons, and the connection of topics through key ideas when designing a mathematics program; (2) Meeting Individual Needs: Teachers employ a variety of lesson styles and instructional techniques to meet the needs of all students and to help them develop higher order thinking skills, such as critical thinking and problem solving; (3) Learning Environment: Teachers create appropriate learning settings and student groupings and provide constructivist feedback to enhance learning; (4) Student Tasks: Teachers design authentic problem solving tasks, embedded in real life context; tasks that allow for multiple representations and multiple solutions and tasks that bridge conceptual understanding from real life to abstract representation; (5) Constructing Knowledge: Teachers use various instructional strategies, build on students’ prior knowledge, provide appropriate scaffolding, model high level performance, and engage students through effective questioning techniques to help knowledge construction; (6) Communicating with Parents: Teachers communicate with parents about students’ achievements and about the mathematics program, and engage parent support for mathematics reform; (7) Manipulatives and Technology: Teachers employ effective use of manipulatives and technology; (8) Students’ Mathematical Communication: Teachers engage in practices that increase students’ oral and written communication skills; (9) Assessment: Teachers provide constructive feedback and advance a
transparent and authentic assessment through diagnostic, formative, and summative strategies; and (10) Teacher Attitude and Comfort with Mathematics: Teachers’ attitudes towards mathematics and their comfort with the subject matter influence students’ learning. While these dimensions may overlap at times, they provide a framework for observing the development in teaching practices.

A second lens being used to organise the data collected and to structure the findings and analysis is the list of the instructional approaches, based on specific elements of constructivist learning, which are supported by the integration of technology, provided by Boethel and Dimock (1999). They indicated that, when technology is used as a tool for learning, rather than the object of instruction or as the instructor, it can assist teachers as they strive to:

- uncover students’ prior knowledge, understanding, and beliefs;
- tap into students’ interests and provide increased motivation for learning;
- base instruction on the posing of problems;
- provide a variety of experiences, experimentation, and negotiations of meaning;
- increase the complexity of the content;
- take on the role of facilitator;
- increase the ability of students to test multiple scenarios and, thus, challenge preconceived notions or misconceptions;
- increase the authenticity of the content and context; and
- broaden the circle of social interaction to include students’ peers and experts beyond the classroom, the school, the community, and even their home country.

(2.2.1 Linear Relations Unit)

In the Linear Relations Unit in grade 9 mathematics, students are introduced to new key concepts that describe a line, such as the rate of change and y-intercept. According to The Ontario Curriculum, Grades 9 and 10 Mathematics (2005), at the end of the unit students are expected to be able to: graph lines using the rate of change and y-intercept, a table of values, and x- and y-intercepts; express a linear relation as an equation in two variables, using the rate of change and the initial value; describe the meaning of the rate of change and the initial value for a
linear relation arising from a realistic situation; and describe the effects of changing these parameters (rate of change and initial value) to the linear graph and to the linear equation (p. 41-43). In addition, the NCTM’s Principles and Standards for School Mathematics (2000) stated that “students should be able to use equations of the form \( y = mx + b \) to represent linear relationships, and they should know how the values of the slope (m) and the y-intercept (b) affect the line” (p. 226). Furthermore, the NCTM (2000) explained that knowledge of functions is a central component of algebra learning that should be emphasised throughout students’ school years beginning in the elementary grades.

In spite of these high expectations, the Education Quality and Accountability Office (EQAO) released their 2012 report for Grade 9 Assessment of Mathematics showing that grade 9 Applied students in Ontario performed poorly on linear relations (41% at or above the provincial standard in 2010, 42% in 2011, and 44% in 2012).

Many research studies have explored students’ and mathematics teachers’ knowledge of linear relations and have investigated the major conceptual problems students have in studying linear relations (Barr, 1980; Posamentier, 1998; Fennema and Franke, 1992; Choike, 2000). Researchers have shown that students develop instinctive and naïve theories, as well as preconceptions and misconceptions, about mathematics that impact their math learning. Students are attached to these theories that are constructed from real experiences and do not give them up easily.

In order to help students move away from their intuitive interpretations about mathematical ideas and to assist them to construct correct understandings of math concepts, teachers should have strong arguments and use strategic teaching approaches (Posamentier, 1998). Fennema and Franke (1992) noted that real world examples help students develop
understanding of abstract mathematics. Choike (2000) emphasised conceptual understanding that focussed on a few big ideas and encouraged multiple representations (words, tables of data, graphs, and symbols) as an “effective strategy for reaching out to students with different learning styles” (p. 557). He also recommended using learning by discovery teaching strategies to involve students in guided explorations that “are more likely to keep students engaged and hence, to encourage them to construct a personal understanding of the mathematics” (p. 559).

Research has shown that students have difficulty and misconceptions associated with the concept of slope and the concept of y-intercept. Thus, Barr (1980, 1981) investigated students’ misconceptions associated with the calculation of slope. He explained that students memorise the slope formula without taking into account the understanding of slope as a rate of change. He described that, in many cases, students replace the values for the coordinates of two points into the slope formula without considering the meaning of the formula or without checking their final answers. Errors occur when students incorrectly use the formula as run over rise, when they substitute the wrong coordinates, or when they use the points in a different order in the numerator and the denominator of the slope formula. Barr (1980) concluded that students struggle to see the slope as a ratio when it is represented in decimal form. He also suggested that teaching students to make connections between the meaning of the slope and the process of calculating the slope may prevent students from making mistakes when calculating slopes.

Moschkovich (1990), as well as Schoenfeld, Smith, and Arcavi (1993), discussed students’ difficulties to interpret linear functions and their graphs and to translate between different representations. They examined students thinking when graphing lines and reported that students often misplace the x- and the y-coordinate, as well as interchange the x- and y-axis. When using the slope ratio, students have difficulty seeing the slope as a ratio if “m” is an
integer and whether the slope is computed as “x over y” or “y over x”. Students also have difficulty identifying the initial value on the graph and struggle to use the slope as a ratio to plot new points required to draw the graph of the line. Schoenfeld, Smith, and Arcavi (1993) asserted that students’ low understanding of points on a line is determined by the absence of “Cartesian connection”. They concluded that all misunderstandings about how to identify the initial value, to use the slope to plot a new point on the graph or to connect the points on the graph contribute to incorrect graphing of the lines.

Stump (1997, 2001) studied both teachers’ and students’ understandings of slopes. She found that secondary school mathematics teachers have a limited understanding of the concept of slope. Their mathematical understanding of slope was dominated by geometric representations, whereas algebraic, trigonometric, and functional representations were less understood. Students, too, had difficulty understanding the slope as a rate of change, as few students used the slope of the line to measure the rate of change involving the appropriate variables in functional situations (Stump, 2001).

Kondratieva and Radu (2009) found that students enrolled in a precalculus university course recognised formulas for lines with a positive slope rather than for lines with a negative slope. They noticed that students recognised the formulas for particular lines in the following order of preference: horizontal, vertical, line with positive slope, and line with negative slope.

Hart (1981) argued that students do not easily connect graphs to linear equations. McDermott, Rosenquist, and van Zee (1987) also found that students struggle to connect graphs to the notion of rate of change.

Although many research studies have explored students’ and mathematics teachers’ knowledge of linear relations, and have investigated major conceptual problems within this unit
of study, a search of the literature revealed no information about teachers’ perceptions about the integration of technology in teaching the Linear Relations Unit to grade 9 Applied level students, who have not been successful in mathematics, have attendance problems, experience some language challenges, and lack effort/engagement. This study aimed to bring together three lenses: a group of at risk students, the Linear Relations Unit and the integration of technology, to investigate teachers’ perceptions about technology integration in the teaching and learning process and to explore the implementation of these perceptions in everyday practices.

2.2.2 Student Tasks

According to the NCTM (2000), teachers are the key factors in implementing effective lessons in a technology rich environment. They should use “technology to enhance their student’ learning opportunities by selecting or creating mathematical tasks that take advantage of what technology can do effectively and well - graphing, visualizing, and computing” (p. 26).

Ruthven et al. (2009) stated that incorporating new tools and resources into lessons requires teachers to develop “appropriate topic-related tasks to be undertaken, suitable activity formats to be used, and potential difficulties to be anticipated” (p. 281). Watson and Sullivan (2009) argued that “different classroom tasks afford different kinds of mathematical activity and students’ experiences of different kinds of mathematical learning” (p. 113). In their opinion, when teachers decide to use particular tasks, they are making choices about the nature of mathematical activity and learning that might take place, as well as about mathematics.

Kilpatrick et al. (2001) believed that teachers need to design or select appropriate student tasks to facilitate the understanding of concepts and relationships, to stimulate higher order thinking and reasoning mathematically, and to help increase students’ abilities to communicate their thoughts both orally or in written solutions. They identified four types of tasks and
described how these tasks and teachers’ actions contributed to the learning of mathematics. The four types of tasks are:

- Type 1: Involves a model, example, or explanation that elaborates or exemplifies the mathematics;
- Type 2: Situates mathematics within a contextualised practical problem to engage the students, but the motive is explicitly mathematics;
- Type 3: Involves open-ended tasks that allow students to investigate specific mathematical content; and
- Type 4: Involves interdisciplinary investigations in which it is possible to assess learning in both mathematical and nonmathematical domains. (p. 118)

Watson and Sullivan (2009) remarked that some teachers fail to make distinctions about the kinds of cognitive activity likely to be prompted by a task and may adopt attractive resources because they provide fun contexts, quiet work, material for discussion, and practical activity, without analysing the nature of learning afforded (p. 111).

Many researchers in mathematical education emphasise that students should be provided with opportunities to concentrate on cognitively complex tasks, such as explorations and investigations of math concepts and problem solving. These tasks facilitate students’ connections among relevant mathematical ideas and shape students’ perceptions of mathematics (Ruthven, 2002). They provide students with a better understanding by offering multifaceted perspectives and views of the same mathematical concepts. However, as Ruthven (2009) noticed, the teacher thinking and classroom practice associated with the use of graphing technology appear to have received little attention from researchers.

McDougall (2004) noticed that “the use of rich tasks, real-life contexts, and tasks that allow for multiple representations contributes to positive student achievement” (p. 10). In the Ten Dimensions of Mathematics Education, he described rich tasks as having the following characteristics:

- Are problem based;
- Allow for multiple possible solutions;
- Enable all students to participate at their own level;
• Allow student generated problem solving strategies;
• Involve multiple representations;
• Present math in a context that makes connections to other math topics, other math strands, other subject areas, and the real world;
• Lead students to consider important mathematical ideas; and
• Expect students to reflect on and communicate their thinking. (p. 25)

In my study, I use the above features of rich tasks to analyze the quality and appropriateness of student tasks in technology based environments and to evaluate and compare teachers’ beliefs about the role of technology integration in a grade 9 Applied mathematics course.

Robutti and Ferrara (2002) described the importance of designing technology-based mathematical tasks for building students’ capacities for mathematical thinking and reasoning. In their study, a motion sensor was used to simulate “motion trips” and set an interpretation task of a space-time graph. The authors inferred that the technology made possible transitions between static and dynamic interpretations of the distance-time graphs, leading to standardised meanings for the graphs.

Hegedus and Kaput (2003) investigated learning through planning student tasks that required SimCalc software. This software allows initial access to functions through virtual simulations of everyday situations dependent on time. They concluded that the use of technology and kinaesthetic approaches led students to study functions as a means of exploring and analysing real world and simulated behaviour.

Pratt and Davison (2003) argued that the visual and kinaesthetic features of the Interactive White Board (IWB) need to be incorporated in adequate tasks that facilitate an understanding of the conceptual aspect of the theory rather than tasks that present and emphasise only the visual aspect of the problem. In their research on the use of the Interactive White Board
with dynamic geometry software, they illustrated that the visual and kinaesthetic affordances of the IWB are insufficient to help students understand the definitions of quadrilaterals.

In addition, in a study that investigated the benefits and limitations of using pre-constructed, web based, dynamic geometry sketches, Sinclair (2003) analysed the interaction within the learning environment between the design materials, the exploration process, and students’ development of geometric thinking skills. She concluded, “task questions and sketch facility must work together to create an environment for exploration and the design of the learning tasks has the potential to support or impede the development of exploration strategies and geometric thinking skills” (p. 313).

Bostic and Pape (2010) asserted that students need to be provided with suitable tasks to develop well-connected representations of mathematical concepts and to begin to use representations other than symbols in their solution strategies. They insisted that the new graphing technology, specifically TI-Nspire Computer Algebra System (CAS) technology, facilitates these connections by providing multiple representations to be viewed in a connecting field in which relationships may be investigated and abstracted, and by lightening the cognitive load of generating representations for the learner so that more effort may be expanded to solve math problems.

According to prospective teachers’ perspectives, the activities and tasks designed with technology need to be guided by the teachers through discussions and explanations rather than sending students off with a list of instructions (Habre, 2007). The use of open-ended mathematical tasks in a technology supported environment is an excellent approach, as it encourages higher order thinking.
There is very little research on teachers’ opinions on the successes or failures of their mathematics lessons and about their reflection on the efficiency of their teaching approaches. This study investigates the link between teachers’ views on the role of technology and the way that they are mirrored in their everyday teaching practice.

In conclusion, researchers have been exploring the best way to integrate technology in mathematics to enhance students’ learning experiences. While the characteristics of rich student tasks are identified as follows: are problem based, allow for multiple solutions, involve multiple representations, allow connections to other topic and subjects, and engage critical thinking, reasoning and reflections, some elements that distinguish technology-based student tasks are as follows: allow virtual simulations and explorations, promote investigations of real world, emphasise the visual aspect of problems, and facilitate an understanding of the conceptual aspect.

The focus of my research is to investigate teachers’ beliefs about the role of technology in designing mathematical tasks and to explore the way their beliefs are applied in designing and adapting student tasks in a technology supported environment to promote the learning of mathematics in a grade 9 Applied course. This study also looks to identify new elements regarding technology integration to add to the detailed descriptions of Student Task, as presented by McDougall (2004).

2.2.3 Constructing Knowledge

Constructivist theorists characterise learning as making sense of the world by “synthesizing new experiences into what we previously come to understand” (Brooks & Brook, 1993, p. 4). In constructing ideas and connecting concepts, learners draw on a complex web of experiences, perceptions, rules, and other information and “engage in a grand dance that shapes”
understandings (Brooks & Brook, 1993, p. 4). An opportunity for learning takes place when students encounter “an object, an idea, a relationship, or a phenomenon that appears inconsistent with their existing understandings” (Brooks & Brook, 1993, p. 4). As Shapiro (1994) points out, when discrepancies do arise, a learner’s first response is to seek explanations that do not require a shift in well-established understandings, and if the discrepancies seem irrelevant, they are simply ignored.

Constructivists agree that social interaction is an important component in the learning process. Duffy and Cunningham (1996) stated that thinking “is always dialogic, connected to another, either directly as in some communicative action or indirectly via some form of semiotic mediation: signs and/or tools appropriated from the socio-cultural context” (p. 177). In addition, through dialogue, learners test and refine their ideas, share multiple perspectives, and negotiate their understandings and misconceptions. According to Vygotsky (1978), “cognitive abilities and capacities themselves are formed and constituted in part by social phenomena” (p. 109).

Brooks and Brooks (1994) pointed out that “teachers’ ability to uncover students’ conceptions is, to a large degree, a function of the questions and problems posed to students” (p. 65). Good questioning is central to constructing knowledge and learning. With effective questioning techniques, teachers can guide or direct students’ attention toward the exploration and reinvention of mathematics (Martino & Maher, 1999). Henningsen and Stein (1997) found that the provision of rich tasks was insufficient for knowledge construction to occur. Teachers should build on students’ prior knowledge, provide appropriate scaffolding, model high level performance, and engage students through effective questioning. The type of questions addressed to guide students’ thinking is very important.
According to Martino and Maher (1999), open-ended questions aimed at conceptual knowledge and problem solving strategies can contribute to the construction of more sophisticated mathematical knowledge. These types of questions, “framed in such a way that a variety of responses or approaches are possible”, encourage teachers to create well thought out questions and tasks that are inclusive not only in allowing students to approach them “by using different processes or strategies but also in allowing students at different stages of mathematical development to benefit and grow from attention to the task” (Small, 2009, p. 6). Furthermore, open-ended questions engage all students to take part in the discourse rich classroom environment, to present their solutions, to articulate their thinking processes, to interpret their findings, to listen to different approaches, to notice different perspectives, and to reflect and construct knowledge.

The goals of the Ontario Curriculum Grades 9 and 10 Mathematics (2005) are to educate students to build new mathematical knowledge through problem solving; to develop and evaluate mathematical arguments and proofs; to communicate mathematical thinking coherently and clearly to others; to recognize and apply mathematics in contexts outside of mathematics; and to create and use representations to organise, record, and communicate mathematical ideas. An, Wu, and Kulm (2004) expressed that, in order to help students understand mathematics conceptually, teachers need to develop various strategies, including techniques for technology integration, to make mathematics concepts visual, live, connected, and meaningful. Along the same line, Griffioen, Seales, and Lumpp (1999) believed that the real time, interactive component of wireless mobile classrooms and the use of interactive, immediate hands-on activities are critical issues to support technology integration. They suggested that the use of
mobile computers and wireless technology in education gives students increased learning experiences, greater convenience, more flexibility, and better collaboration.

Interactive simulations and illustrations can produce a much greater depth of understanding of a theoretical concept. When technologies are used in classroom settings, they can go far beyond chalk and talk in providing better explanations and understandings of mathematics. Clarifying the role of technology in learning, Duffy and Cunningham (1996) stated:

Technology is seen as an integral part of the cognitive activity….This view of distributed cognition significantly impacts how we think of the role of technology in education and training, the focus is not on the individual in isolation and what he or she knows, but on the activity in the environment. It is the activity – focused and contextualized – that is central... The process of construction is directed towards creating a world that makes sense to us, that is adequate for our everyday functioning. (p. 187)

Gorder (2008) found that teachers need to learn to integrate technology within the context of their classroom through practice, reflection, and sharing of teaching practices. In order to help students construct mathematical knowledge, teachers need to use appropriate virtual tools to conduct investigations and demonstrations of math theory, to gather data for conjectures, to provide multiple representations, to exemplify connections among notions and ideas, and to create problems and situations that require and stimulate higher order thinking. For example, in the grade 9 Applied course, students utilising graphing calculators or graphing software could collect real data to generate a position or a velocity graph. Then, they could walk in a straight line to replicate that graph. Through investigations, students will be able to determine the characteristics of distance-time and velocity-time graphs, to interpret the graphs in terms of actual motion, and to describe “situations that would explain the events illustrated by a given graph of a relationship between two variables” (The Ontario Curriculum, Grade 9 and 10
Barak, Lipson, and Lerman (2006) indicated that using technology for active learning kept students focused, engaged, and motivated.

In summary, the related literature suggests that teachers need to develop various instructional strategies to help students understand mathematics conceptually and to make mathematics concepts visual, live, connected, and meaningful. The learning environments that most effectively support student learning allow students to explore concepts, to pose interpretations and hypotheses, to test their ideas and apply them in other contexts, to foster dialogue, and to negotiate among multiple viewpoints, and to reflect on their learning. In addition, technology integration can make content easier to understand, as it increases students’ learning experiences through real time interactive activities inspired by real life situations, visual simulations and illustrations, investigations and demonstrations of math theories, multiple representations, easy access to information, and increased social interaction and dialogue.

My study explores teachers’ perceptions of the role of technology and their instructional practices to facilitate knowledge construction in the context of Linear Relations Unit in grade 9 mathematics. Additionally, this research aims to find new elements regarding technology integration to add to the detailed descriptions of Knowledge Construction, as presented by McDougall (2004).

### 2.2.4 Students’ Mathematical Communication

Promoting classroom talk to enhance students’ learning in mathematics in a technology supported environment represents an important aspect of the process of learning mathematics. As Chapin, O’Connor, and Anderson (2003) pointed out, “classroom dialogue may provide direct access to ideas, relationships among these ideas, strategies, procedures, facts, mathematical history” and also “supports student learning indirectly, through the building of a social
environment – a community – that encourages learning” (p. 6). The NCTM (2000) suggested that students can deepen their understanding of what they are learning and consolidate their own learning via learning mathematics by communicating their thinking through numbers, images, and words.

Heller et al. (2005) stated that teachers should provide opportunities for students to engage in different mathematical communication activities that stimulate mathematical discourse in order to optimise students’ understanding of mathematics. They noticed that the effects of graphing calculator technology increased the quality and quantity of mathematical discourse and illustrated the positive impact of these tools. Picking up on this theme, Kimmings (1995) implied that mathematical communication is fostered when students use word processors and presentation software to write and present reports that include mathematical symbols, tables, and graphs (p. 3).

McDougall et al. (2006) suggested that “grouping students to foster oral communication and providing opportunities for written communication have a positive effect on student achievement” (p. 12). The authors summarised some strategies for teachers to engage students in “practices that increase students’ communication of mathematical ideas”, such as teaching students generic skills in how to give explanations (Webb & Farrivar, 1994); teaching students how to give explanations tailored to mathematical arguments (Fuchs et al., 1997); using metacognitive (self-questioning) prompts to focus students’ attention on problem elements (Cardelle-Elawar, 1995); using question stems to stimulate high level discourse in math groups (King, 1989); monitoring comprehension by having students probe a partner’s understanding of mathematical ideas using prompt cards (Mewareck & Kramarski, 1997); and addressing
differential participation in student groups, as well as creating status equity in cooperative groups by assigning competence to all students including low-achieving students (Cohen, 1994).

In summary, the literature provides evidence of instructional strategies implemented in the classroom to help students’ abilities to communicate their understanding of mathematics in writing and verbally. It also implies that technology integration could facilitate the development of students’ communication skills. Moreover, the Ontario Curriculum for Grade 9 and 10 Mathematics (2005) specified that teachers should encourage their students “to select and use the communications technology that would best support and communicate their learning” (p. 14). In spite of this, research is needed to elucidate teachers’ perspectives on the power of using technology to stimulate communication in the mathematics class, to facilitate dialogue among students and between students and their teacher, and to encourage students to provide detailed and logical solutions and explanations in both oral and written forms when learning the grade 9 Applied math content.

My study explores teachers’ perceptions of the role of technology and their instructional practices to foster mathematical discourse, which, in turn, supports students learning of linear relations concepts in grade 9 Applied mathematics. Moreover, this research intends to discover new elements regarding technology integration to add to the detailed descriptions of Students’ Mathematical Communication, as presented by McDougall (2004).

2.3 The Role of Teachers in Learning Environments Utilising Technology

Technology has become a significant part of everyday life. It plays an important role in the context of reform-based classrooms, as it contributes “to increased student achievement when used appropriately” (McDougall, 2006). According to the Technology Principle in the Principles and Standards for School Mathematics (NCTM, 2000), “Technology is essential in teaching and
learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (p. 2). Teachers are key factors in facilitating student learning in a technology rich classroom. In their study, Hennessy, Ruthven, and Brindley (2005) reported that teachers used technology only when they believed it enhanced student learning compared with other approaches. In their opinion, teachers’ commitment to integrating technology relates to “recognizing the educational value and believing in the transformative potential of the technology” (p. 185).

Utilising technology, teachers can challenge students to solve high level thinking problems; to provide settings for students to investigate and analyse real life situations, generate hypotheses, and check alternative solutions; to engage students in conducting investigations and making connections with other concepts; and to facilitate simulations which “offer students the opportunity to confront problems and make decisions in an imaginary environment that is realistic enough to provide meaningful issues and appropriate consequences” (Knapp & Glenn, 1996, p. 29). Technology-based classrooms create appropriate mediums for increasing students’ interaction, enhancing mathematical communication skills, and improving equity in mathematics.

Technology can help keep track of students’ thinking process and understanding, help identify students’ misconceptions, help monitor students’ progress and engagement level, and help provide quick feedback. Jones, Valdez, Nowakowski, and Rasmussen (1995) emphasised that, with technology integration, teachers should change their role from information gatekeepers to facilitators, guides, and co-learners, with the student as explorer, producer, cognitive apprentice, and sometimes teacher. Boethel and Dimock (1999) noted that “Computer based technology allow teachers to shift their role from expert provider of ‘right’ answers to facilitator
or coach. But, teachers remain the pedagogical experts in the classroom and as such they are the key to obtaining successful interaction” (p. 21).

The teachers’ role in a technology enhanced instructional environment is to plan, monitor, analyse, and regulate their classroom instruction. Ruthven, Deaney, and Hennessy (2009) reported that teachers adapt their classroom practice and develop their craft knowledge by establishing a coherent resource system that effectively incorporates the software, by adapting activity formats to exploit new interactive possibilities, by extending curriculum scripts to provide for proactive structuring and responsive shaping of activity, and by reworking lesson agendas to take advantage of the new time economy.

The teacher’s role in a technology-supported teaching and learning process is to reflect upon their practice in order to improve it. Teachers’ reflection upon the use of technology in a mathematics classroom is an important component that helps not only the improvement of their classroom practice but also the conceptualisation of their role in a context where technology is being used (Habre, 2007). Teachers need to understand how their lesson plans will be incorporated in the classroom and how to make the most efficient use of a specific technology. They need to reflect on how to modify or adjust their teaching strategies and techniques to design meaningful lessons that help students construct mathematics knowledge through the use of technology.

A teacher’s role is more critical and challenging when utilising technology (Habre, 2007). Although prospective teachers played a different role when teaching with the aide of technology versus teaching in a traditional math classroom, they were not able to describe the tasks, responsibilities, or characteristics of a teacher teaching with technology (Habre, 2007). This is a significant aspect that I examine in my research.
Godwin and Sutherland (2004) emphasised the vital role that teachers play in orchestrating and structuring classroom activities in such a way as to support students to focus attention on appropriate mathematical ideas. There is a potential risk of inefficiently using technology in the math classroom if the tasks lead students to focus on the technology at the expense of the mathematics (Monaghan, 2004). Habre (2007) emphasised that mathematics, not technology, should remain the focus of instruction in mathematics classrooms (p. 3). Ruthven, Deaney, and Hennessy (2009) suggested that successful classroom use of technology, particularly the use of graphing software, depends on teachers’ abilities to induct students into using it for mathematical purposes, to provide suitably pre-structured lesson tasks, to prompt strategic use of the software, and to support mathematical interpretation of the results.

2.4 Teaching Strategies Utilising Technology

Lagrange and Monaghan (2009) advocated that teaching strategies applied in “traditional” settings can no longer be applied in a routine-like manner when technology is available. Instead, new collections of techniques are likely to be developed to reflect teachers’ views on mathematics education (Pierce & Ball, 2009). In reform-based classrooms, technology is being used in activities incorporating more challenging and demanding mathematics tasks and to solve difficult problems. Pierce and Stancey (2004) noted the example of a teacher who believed that students learn best if they use by-hand (pen and paper) skills for new procedures and then later move to using technology as the procedure increases in difficulty (p. 302).

Technology has been used to make investigative lessons more viable and captivate students’ interest (Ruthven et al., 2009). As the Ontario Curriculum for Grade 9 and 10 mathematics suggests, “technology makes possible simulations of complex systems that can be
useful for problem-solving purposes or when field studies on a particular topic are not feasible” (p. 37).

Technology has been used to promote student-centred learning and to support explorations of more open-ended tasks, in which students have control over the situation. In a technology-based environment, students investigate math concepts and solve math problems individually and in collaboration with their classmates. Borba and Scheffer (2003) analysed dynamic classroom activities, using motion detectors and graphing software, to focus on the analysis of graphs of the mathematical functions. They suggested that it is possible to deepen students’ connections with everyday experiences through environments that reveal phenomena and their modelling concurrently.

Technology has been used to engage and enable less able students to participate in meaningful explorations based on real world situations. Technology rich activities engage students to learn through discovery, through manipulating virtual objects, and to build mathematical understandings and knowledge. Evaluating the contribution of graphing software to the teaching of algebraic forms, Ruthven et al. (2009) found that, in the lower and average attaining classes, the graphing software “helped make tasks accessible to students who would have found organization and presentation challenging and would have really struggled, echoing aspects of overcoming students difficulties and building assurance” (p. 289).

Technology has been used as a tool for the visualisation and exploration of mathematical concepts. Based on the results of a survey conducted with prospective teachers, Habre (2007) stated that they considered technology to be a tool for visualisation and exploration of geometrical concepts rather than an aide in the teaching of algebra. These teacher candidates affirmed that technology could not be integrated in algebra because it has “a lot of arithmetic that
must be learned by doing it by hand” (p. 6). Perhaps their lack of teaching experience or their lack of exposure to investigative and exploratory activities and simulations in their previous mathematics experiences, made them closed to the pedagogical potential of technology in all mathematical areas.

Technology has been used to enhance multiple representations, promote alternative approaches to mathematical problems, develop students’ abilities to connect ideas and make sense of their math solutions and engage students in more realistic complex real world problem solving. Bransford (2003) created complex real world setting on videodisc or CD that students use to engage in solving real world problems. Students use data from the story to create tables and graphs to help them answer questions involving car rental costs, fuel costs, rates and times of travel. Doerr and Zangor (2000) encouraged the use of graphing calculators’ as media for developing various math strategies, such as solving equations graphically and numerically rather than analytically. Regular access to technology could help students understand links between symbolic, tabular, and graphical output by making these links explicit. Similarly, Goulding and Kyriacou (2007) suggested that students could compare strategies involving tables of values (trial and improvement), symbolic manipulation methods, and reading off solutions from graphs when solving equations. Polya (2004) indicated that students became more effective at problem solving when they were able to apply various representations to the mathematical questions. Students receiving instruction using multiple representations discovered that different representations provided access to different inferences and calculations.

Technology has been used as an aide in mathematical problem solving and probing solutions. According to Ruthven et al. (2009), teachers viewed the calculators as a valuable tool for checking sketches and manipulations, for building assurance and confidence in the accuracy
of students’ work, for creating independent learners who work with less teacher exposition and more student investigation, and for peer collaboration. Graphing calculators and mathematical software provide students’ quick access to multiple representations of mathematical concepts and opportunities to explore various mathematical situations and contexts, as well as to test their findings.

Mesa and Gomez (1996) concluded that technology should be used to carry out and check calculations and to investigate concepts, test theories, and make inferences. They presented a study focusing on student’ approaches to math problems by using graphing calculators. After analysing and categorising different strategies used by students to provide solutions to the problems, the researchers found that technology was mainly used to carry out and check calculations rather than to make conjectures and explore them.

Technology has been used as a tool for increasing the pace and efficiency of lesson delivery. Ruthven et al. (2009) noticed the importance of technology in terms of time economy. They concluded that the use of graphing software to the teaching of algebraic forms made it possible to produce extremely accurate graphs extremely quickly, so that students could move through everything at a much quicker pace, allowing a topic to be addressed in only a single lesson. Assude (2005) noticed that teachers operated within a time economy in which they sought to improve the “rate at which the physical time available for classroom activity is converted into a didactic time measured in terms of advance of knowledge” (p. 187). Assessing the use of interactive whiteboards in secondary mathematics classrooms, Miller and Glover (2006) found that teachers designed lessons utilising interactive whiteboards evolving from the initial use of the board as a medium for visualisation to a medium that supported demonstration of concepts and proof of mathematical theories.
Technology has been used as a tool to help develop good mathematical communication skills. According to Kimmins (1995), the integration of technology enhanced motivation to communicate mathematics precisely in order to perform a desired task (e.g. algebra students programmed a computer to estimate the probability that a quadratic equation has real roots under varying conditions on the coefficients). It also enhanced the “ability for students to present mathematical ideas both orally and in writing (e.g. use of word processors to write reports that include mathematical symbols, tables, and graphs and use of multimedia presentation programs to communicate mathematical ideas)” (p. 2).

Research studies in education demonstrate that the integration of technology can help improve students’ self-concept and motivation (Sivin-Kachala & Bialo, 2000) and can help improve students’ scores on standardized tests (Bain & Ross, 1999). Some barriers related to the adoption of technology into teaching are: teachers’ lack of specific technology knowledge and skills (Snoeyink & Ertmer, 2001-2002; Williams et al., 2000); teachers’ unfamiliarity with the pedagogy of using technology (Hughes, 2005); teachers’ lack of technology related classroom management knowledge and skills (Lim et al., 2003; Newhouse, 2001); and teachers’ perceptions of technology as a catalyst for change in their teaching practice, from “teacher-centred” to “student-centred” when they prefer a “teacher-centred” model (Scrimshaw (2004).

In addition, teachers’ attitude and beliefs is another barrier to the integration of technology (Hermans, Tondeur, Valcke, & Van Braak, 2006). Pressures related to high stakes testing (Fox and Henri, 2005) and the perceived tensions between using technology to enhance learning and the need to conform to the external requirements of traditional examinations, which sometimes prohibit the use of technology (Hennessy et al., 2005) constitute serious barriers for technology integration.
Along the same lines, other factors that prevent technology integration are a school culture that considers the traditional instructional approach to be better than a constructivist approach (Hart, 2004), the limited time available for undergoing the technological explorations and investigations of mathematical concepts (Pierce and Ball, 2009), prior negative experiences with technology (Crisan, 2005), teachers fear of being replaced by technology (Li, 2007), and limited resources.

2.5 Summary

The research about the integration of technology in mathematics education has shown that technology has the potential to provide innovative educational opportunities for students with interactive activities that involve simulations, animations, investigations, and visual demonstrations of mathematical theories and to engage students’ problem based learning with scenarios relevant to students’ lives and inspired by authentic, real situations. Technology could be used as a motivating tool, to encourage students to participate actively in the learning process, and as a social tool that promotes mathematical dialogue and enhances students’ interactions. Technology could be used to develop meaningful lessons, to create rich tasks for students to examine, and to contribute to help the construction of content knowledge.

However, although there is evidence that technology is integrated in the learning process of mathematics, some barriers have prevented teachers from adopting technology in their lessons. Drjvers et al. (2010) suggested that, in order to encourage teachers to adopt new teaching techniques and strategies that help them benefit from technology in everyday practice, more research is needed to investigate the alternatives to traditional teaching that emerge in the technology rich classroom.
As a response to the above recommendation, my research sheds light on the relationship between teachers’ views on teaching mathematics with technology and the way their views are reflected in their teaching strategies and techniques involved in delivering the Linear Relations Unit content in grade 9 Applied mathematics. Important consideration is given to planning and delivering the course content as a result of the impact of technology on three important dimensions of mathematics, as presented by McDougall (2004), specifically student tasks, constructing knowledge, and students’ mathematical communication.
CHAPTER THREE: METHOD

3.1 Introduction

The purpose of my study was to investigate the relationship between teachers’ views on the role of technology in mathematics education and the way that technology was implemented in their teaching practice. This chapter will present the research design, discuss the process of participants’ recruitment, and explain the data collection, analysis, and ethical considerations of this study.

3.2 Research Design

This research is a qualitative study involving a multiple case study design. Merriam (1998) asserted that a case study design is appropriate when the researcher’s goal is to gain “insights, discovery, and interpretations” of a particular phenomenon of study (p. 28). My case studies were three secondary math teachers and this is in agreement with Stake (2006), who stated:

A case might be an individual person, a group, such as a family, an institution, such as a school, an agency of some kind, such as a local authority childcare service or even a whole country’s education system. Each of these different kinds of cases can be considered to be a ‘bounded system’ in the sense that it comprises a complex of interrelated elements or characteristics that has clearly identifiable boundaries. (p. 1)

Case studies are unique due to the fact that they concentrate upon a single phenomenon or entity (the case) and seek to provide a rich description of it (Stake, 1995; Merriam, 1998). For example, in my study the aim was to come to a strong understanding and to provide in-depth information about the relationship between teachers’ perspectives on the role of technology integration and the way these perspectives are reflected in their teaching practice. According to Stake (1995), case studies are useful when “the first emphasis is on understanding the case itself” (p. 8). My case study is an intrinsic case study. Stake (1995) defined the intrinsic case study as
“the study of the particularity and complexity of a single case” (p. xi). In an intrinsic case study there is no attempt to generalise, but rather to examine in depth the intrinsic uniqueness of the individual case for its own sake.

I used interviews, observations, field notes, and a collection of documents (course outlines, lesson plans, students work, etc.) to collect my data. By using a variety of empirical data, I was able to triangulate the data collected in order to gain a deep understanding of teachers’ perspectives about the role of technology in planning and delivering the Linear Relations Unit content and of their everyday practices. Including more than one case enabled me to gain a better understanding of the teachers’ perceptions about the integration of technology in mathematics and their teaching practices and strengthens the validity of my interpretations (Merriam, 1998).

In my analysis, I followed Merriam’s (1998) recommendation to pursue two stages of data analysis (p. 15). In the first stage, I examined in isolation the relationship between each teacher’s perspective on the role of technology in mathematics education and their classroom practice and, in the second stage, I conducted a cross-case analysis to compare the three cases presented.

3.3 Selecting Participants

The purpose of my research was to gain rich and in-depth insights into the relationship between teachers’ perceptions of the role of technology integration in mathematics education and the way they are reflected in their classroom practice. As Glesne (1999) remarked, qualitative researchers tend to select each of their cases purposefully. Patton (1990) suggested that:

The logic of power of purposeful sampling lies in selecting information-rich cases for study in depth. Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research, thus the term purposeful sampling. (p. 169)
Participants in this study included three secondary school teachers teaching grade 9 Applied level mathematics in three ethnically and culturally diverse secondary schools in Ontario. I initially met two of the three participant teachers at an Ontario Association for Mathematics Education (OAME) conference and I met the third teacher through the Learning Consortium Collaborative Teacher Inquiry Project (McDougall, 2012; Egodawatte, 2012). All three teachers were giving presentations on successful mathematics lesson plans, activities, and projects that integrate technology. These teachers were selected by their peers to describe their outstanding teaching approaches and to discuss practices that engage and motivate grade 9 Applied level students (a significant number of whom have attendance problems, experience some language challenges, and lack effort/engagement) to participate and learn. They did exceptionally well in helping the grade 9 Applied students improve their EQAO scores and had high expectations and standards.

By recruiting only exemplary teachers in my study, I was trying to maximize what we can learn about technology based teaching and learning strategies implemented in the grade 9 Applied Linear Relations Unit. These teachers were intrinsically motivated to integrate technology in order to create engaging learning environments and to captivate their students’ interest. Although they had strong content, pedagogical, and technological knowledge, they were willing to identify ways to improve their teaching. They self-directed their learning and were open to learning from experts, peers, and students how to integrate technology to make mathematics lessons exciting and interesting, as well as more meaningful and accessible to their students. These expert teachers’ perceptions and practices help the understanding of how and why they approached and delivered the Linear Relations content when technology was incorporated. They could serve as role models for other peers who aspire to adopt and integrate
technology into their teaching practices, as they may present engaging technology based approaches for the secondary mathematics content in workshops and professional development sessions. Their handouts, lesson plans, and interactive activities could also be compiled into ready-to-use resources to be distributed across the province to help novice teachers guide their students into the discovery and application of the linear relations mathematical content.

By selecting only exemplary teachers I did not take into account the perceptions of teachers who do not integrate technology because they believe that, although technology has great potential to impact the teaching and learning of mathematics, its integration in the classroom does not guarantee high achievement, better understanding, and learning outcome (Li, 2004). I also did not consider recording the struggles of novice teachers in the field of technology integration, when they attempt to incorporate technology in designing and delivering grade 9 Linear Relations lessons. Furthermore, I did not look at teachers who previously incorporated technology but decided to give up this approach. The perceptions of these teachers about the role of technology in teaching the Linear Relations Unit to grade 9 Applied level students could have contributed to a better understanding of their particular challenges and could have enriched the content of my research. By knowing more about obstacles that teachers encounter when integrating technology I could have focused my attention towards finding solutions for their problems and looking for suggestions to improve their teaching practices. However, due to the limited time frame for my research and my interest in studying grade 9 teachers’ use of technology in Linear Relations rather than grade 9 teachers’ struggles with the integration of technology, I felt it important to first invite exemplary teachers to be part of my study, to present their successful stories, and to discuss their approaches to teaching with technology. These highly motivated teachers used modern teaching techniques that incorporated
technology to deliver meaningful lessons and to engage grade 9 Applied students in the process of learning mathematics. They shared their experiences with using Smart Board, clickers, and TI-Nspire graphing technology and also discussed some of the advantages and pitfalls of integrating technology in everyday teaching practice.

### 3.4 Data Collection

I conducted semi-structured interviews and observations, as well as collected relevant documents (lesson plans, handouts, course outlines, and students’ work) over the course of three months: April, May, and June 2012.

#### 3.4.1 Interviews

I designed and conducted a first round of semi-structured interviews to distinguish the school culture, to identify teachers’ backgrounds and previous experiences, to explore their perceptions regarding the role of technology in mathematics education, to establish at what level teachers collaborate with peers, colleagues, and experts, and to record the main objectives and expectations when integrating technology in the grade 9 Applied Linear Relations Unit. The intention of the first interview questions was to motivate discussions about what the teachers considered to be good practices of technology integration in the grade 9 Applied mathematics content and about how they used technology to provide rich student tasks, to help students construct knowledge, to promote mathematical discourse, and to engage and motivate students (Raymond, 1997; Skott, 2001). All interviews provided me with detailed descriptions and considerable information about teachers’ perceptions and practices, which I was able to summarize, compare, and contrast. The interviews were used to identify and establish common patterns and themes that emerged between and across observations and interviews (Warren, 2001).
After the first round of interviews, I conducted a follow-up interview after each classroom observation to provide the fullest possible understanding of the relationship between teachers’ perceptions of the role of technology and the way they were reflected in classroom instruction. During these interviews, I asked specific questions to capture teachers’ thinking and feelings about their classroom instruction, to gain insights into their explanation, and to account for their actions. After each interview I prepared “a written facsimile, with key ideas and episodes captures” (Stake, 1995, p. 66).

All interviews were audio recorded using a digital recorder and later transcribed into Microsoft Word. Transcriptions of the interviews were sent via email to the three participant teachers to be checked and approved.

### 3.4.2 Observations

At first, in my classroom observations, I watched for details describing the school environment (i.e. community settings and economic, social, and cultural milieu), human environment (i.e. teachers, students, administration, staff, and parents), and the learning environment (i.e. learning situations, teacher-student relationships, discipline, and control) (Bogdan & Biklen, 2003). I observed each teacher in the classroom for about 10 to 12 periods of 75 minutes to give the participant teachers and students enough time to adjust to my presence in the classroom and to assure consistency in my findings.

During all observations, I recorded descriptive notes of teachers, students, and classroom activities. Glesne (1999) stated that, as a researcher using memo writing, “you develop your thoughts, by getting your thoughts down as they occur, no matter how preliminary or in what form, you begin the analysis process” (p. 131). In addition, I kept a record of my own
perceptions, questions, and reactions throughout the study and I referred to them when I analyzed
the findings (Merriam, 1998).

As a nonparticipant observer in the classroom, I watched and recorded notes without
becoming involved in the teaching and learning process. By not actively participating in the
dynamics of the lesson, I made teachers and students feel more comfortable with my presence in
their classroom. As I became familiar with the settings, I narrowed my observations to specific
aspects of teaching focusing on the design of student task, construction of knowledge, and
facilitation of communication.

3.4.2.1 Student Task

In my observations I aimed to identify characteristics of student task incorporating
technology (instant feedback, dynamic and interactive simulations, and manipulations) that made
the technology-based student task different from a traditional (paper-pencil) learning task. I
sought to distinguish specific elements that teachers put in place when providing technology
based student tasks to: make the instruction more efficient and effective (e.g. saving time by
obtaining quick and precise graphs); engage students in the learning process (e.g. tasks inspired
by real-life situations, tasks selected from authentic websites, tasks relevant to student life); help
build students' capacity for mathematical thinking and reasoning (e.g. making predictions,
estimating, providing arguments); and differentiate their teaching (e.g. visualizations,
simulations, and manipulations) so as to ensure that all students in their classrooms had
opportunities to interact and learn (Robutti & Ferrera, 2002).

3.4.2.2 Constructing Knowledge

Brooks and Brooks (2004) specified that good questioning is central to constructing
knowledge and learning (p. 65). In my observations I focused on capturing the kind of
questioning techniques that teachers use in a technology driven environment to guide or direct student attention toward the exploration of mathematics. For example, I looked for evidence that answers the following questions:

How do teachers use technology to activate prior knowledge, model high-level performance, and engage students through questioning?

How are the questions that teachers use to guide students’ thinking different in a technology driven environment?

How do teachers use technology to help students actively interact with the new knowledge?

How do teachers use technology to help students practice and deepen their understanding of new knowledge?

An, Wu, and Kulm (2004) stated that teachers need to develop various strategies, including techniques for technology integration, to make mathematics concepts visual, live, connected, and meaningful. In my observations I searched for different techniques that teachers used in a technology driven environment to provide better explanations and understandings of key abstract concepts in linear relations, such as rate of change, initial value, and point of intersection of two lines. Specifically, I attempted to distinguish:

How teachers used different technological tools in the classroom to: conduct investigations and demonstrations of math theory; help students gather data for conjectures; provide multiple representations; exemplify connections among notions and ideas; create problems and situations that require and stimulate higher order thinking; and help students construct mathematical knowledge.

What teachers did to increase student learning during the class session when teaching with technology.
3.4.2.3 Facilitation of Communication

Heller et al. (2005) suggested that technology plays a positive role in increasing the quality and quantity of mathematical discourse. They recommended that teachers provide opportunities for students to engage in different mathematical communication activities, which help optimize students understanding of mathematics.

During the classroom observations I looked for innovative techniques that teachers employed when utilizing technology to engage students in practices that increased their communication of mathematical ideas, such as:

- teaching students generic skills of how to give explanations (Webb & Farrivar, 1994);
- teaching students how to give explanations tailored to mathematical arguments (Fuchs et al., 1997);
- using metacognitive (self-questioning) prompts to focus students’ attention on problem elements (Cardelle-Elawar, 1995);
- using question stems to stimulate high-level discourse in math groups (King, 1989);
- monitoring comprehension by having students probe a partner’s understanding of mathematical ideas (Mewareck & Kramarski, 1997); and
- addressing differential participation in student groups and creating status equity in cooperative groups by assigning competence to all students including low-achieving students (Cohen, 1994).

In addition, I observed how the teachers used technology to ask open-ended questions that encouraged students to take part in a discourse rich classroom environment, to present their solutions, to articulate their thinking processes, to interpret their findings, to listen to different approaches, to notice different perspectives, and to reflect and construct knowledge.

Additionally, I searched for evidence that supported the teachers’ opinions that technology helps
them guide their instruction and monitor their students’ progress.

Moreover, I tried to identify vocabulary strategies that teachers incorporated in their technology driven lessons to help students appropriately and accurately use the language of mathematics.

These observations and field notes served as another check on interpretation and another means to verify interpretations of data.

3.4.3 Document Collection

I collected copies of lesson plans, handouts, and students’ work, and studied and analysed them to gain a better understanding of the teachers’ instructional practices and the alignment to their beliefs about technology integration in mathematics education.

3.5 Data Analysis

I examined the transcriptions of the interviews, my field notes and the collection of the documents several times and recorded my thoughts each time the data was reviewed (Delamont, 2002). As Bogdan and Biklen (2003) suggested, the analysis was an ongoing process that required me to categorise, synthesise, search for patterns, and interpret the data collected. In my study, data analysis was performed simultaneously with data collection to enable me to focus and shape the study as it proceeded (Glesne, 1999).

I started with the research questions and used the interview transcripts to identify emerging patterns and evidence that addressed my research questions. After reading and re-reading the transcripts, I defined initial codes (e.g. student tasks, interactive activities, dynamic learning environment, benefits of technology integration, real time feedback, connecting strands, collaboration, facilitation of dialogue, classroom management, strategies when incorporating technology, strategies for teaching linear relations, misconceptions, misinterpretations, level of
engagement, conceptual understanding, communication skills, vocabulary, reasoning, visual representations, multiple representations, transfer among various representations, characteristics of teachers teaching with technology, technological knowledge, pedagogical knowledge, mathematical knowledge, advantages and disadvantages of technology integration, teacher’s role). I grouped these codes into categories or themes (problem solving strategies, effective instruction, level of engagement, interactive learning environments, checking and validating solutions, connections among mathematical ideas and strands, mathematical thinking and reasoning, preconceived notions and misconceptions, student interest, big ideas, differentiated instruction, and feedback) and I connected them to the research questions.

I compared all of my evidence (including transcripts, field notes and documents), made a matrix of categories, and placed the evidence within the categories. To analyze the data from my study, I synthesized the information I obtained from various sources “into a coherent description of what he has observed or otherwise discovered” (Fraenkel & Wallen, 2000, p. 505).

Throughout the data collection process, I wrote daily field reports to systematically examine where I was and where I needed to direct my attention (Glesne, 1999). Furthermore, I explored data systematically and asked myself questions of the data, as I tried to figure out possible explanations for the data and I reflected on possible reasons (Delamont, 2002; Glesne, 1999). I aimed to find ways to make connections that would be ultimately meaningful to both myself and the reader and to explain teachers’ perceptions about the integration of technology or how they integrate technology, as well as why they used technology to deliver the linear relations content (Glesne, 1999).
3.6 Validity and Credibility Issues

Glesne (1999) emphasised that research validity is “an issue that should be taught about during research design as well as in the midst of data collection” (p. 32). I addressed researcher bias by continuously exploring my own subjectivity. I utilised Creswell’s (1998) and Lincoln and Guba’s (1985) recommended strategies to enhance the trustworthiness of the research: prolonged engagement and persistent observations, triangulation, peer review and debriefing, negative case analysis, clarification of research bias, member checking, rich thick description, and external audit.

Prolonged engagement and persistent observations attempt to surmount possible misinterpretations. To eliminate the natural tendency of people to act differently when they are being watched, I spent long periods of time in classrooms with the teachers involved in the study, so that I was able to develop trust and learn the culture of the school. By increasing teachers’ comfort with me as a researcher in their classes, they adjusted to my presence and performed in their usual manner.

Triangulation supports a finding by showing that multiple data collection methods, multiple sources, multiple investigators, and/or multiple theoretical perspectives agree with it or at least do not contradict the data. To confirm convergent findings, I used data triangulation, as proposed by Patton (1990). Data triangulation refers to using several data sources (such as interviews, classroom observations, lesson plans, and handouts) and observing teachers at different times of the school day and in different settings, such as classrooms and the teachers’ lounge.

Peer review and debriefing allows a peer to assess and comment on the findings that emerge. To ensure consistency, I asked one of my peers to read, reflect, and provide input on my
work. Negative case analysis encourages “conscious search for negative cases and unconfirming evidence so that you can refine your working hypotheses” (Glesne, 1999, p. 32). I sought out dialogue with colleagues that I knew held different opinions. These discussions allowed me to see the topic of my research through different perspectives and to reflect upon my subjectivity.

Clarification of research bias requires “reflection upon your own subjectivity and how you will use and monitor it in your research” (Glesne, 1999, p. 32). By writing my views and observations both before and after my interviews and classroom observations, I addressed preconceived opinions, reflected upon my emotions and feelings, and better observed and noticed external facts and evidence important for this study.

Member checking involves “sharing interview transcripts, analytical thoughts, and/or drafts of the final report with research participants” to ensure the researcher is representing the participants and their ideas accurately (Glesne, 1999). I shared the results with the three participants at the end of the study to ask if they perceived the results to be accurate and plausible, and I received their approval. Rich, thick description is “writing that allows the reader to enter the research context” (Glesne, 1999, p. 32). I provided a detailed description of my research. External audit refers to “an outside person [who] examines the research process and product through ‘auditing’ the fieldnotes, research journal, analytic coding scheme, etc.” (Glesne, 1999, p. 32). My supervisor and my committee members assisted me in improving my research findings and analysis. I also sought feedback from my peers in the research settings.

3.7 Ethical Considerations

Prior to conducting the research, I obtained approval from the University of Toronto Ethics Review Office. I contacted the participants, informed them of the research project, and asked them to participate. All three participants were free to withdraw from the study at any
time. During the reporting and discussion of data, none of the participants, schools, or communities were identified (pseudonyms were used). Transcripts of all interviews and observations were sent by email to the participants involved for confirmation. I invited all participants to make comments and to provide feedback about the observations and interviews in order to ensure accuracy. None of them made any comments or required any changes. At the end of the study, each participant received a copy of the results and the analysis. It is believed that no one was harmed in this study, and that the results will contribute to a growing knowledge base around the impact of technology on three dimensions of education: Student Task, Knowledge Construction, and Students’ Communication (McDougall, 2004).
CHAPTER FOUR: FINDINGS

4.1 Introduction

In this chapter, I present the findings resulting from an exploration of three mathematics teachers who used various technological tools to teach the Linear Relations Unit to their grade 9 students. The data was gathered from a variety of sources: observations of mathematics lessons that integrated technology, field notes, documents (lesson plans, handouts, course outlines, course planning, and students’ work) and interviews. I examined teachers’ classroom activities, focussing especially on occasions marked by the integration of technology in designing student tasks, constructing knowledge, and facilitating students’ mathematical communication.

For each case, I provide a short introduction to the school, some biographical information about each participant teacher, and a description of how technology was used in designing student tasks, facilitating knowledge construction, and promoting mathematical communication in teaching the Linear Relations Unit to grade 9 students. I provide a summary of each participant teacher at the end of each case, underlining some effective uses of technology in teaching grade 9 Applied level mathematics.

This chapter is organized around the dimensions of education targeted in this research, Student Task, Knowledge Construction and Students’ Mathematical Communication (McDougall, 2004). Within the Construction of Knowledge dimension, the literature was used to identify elements already discussed about how technology can support constructivist learning environments, such as, student interests and increased motivation for learning, instruction based on the posing of problems, variety of experiences, experimentation, and negotiation of meaning, complexity of the content, preconceived notions or misconceptions, authenticity of the content and context. In the context of Linear Relations unit, this chapter presents new factors that
participant teachers consider that contribute to enhance the process of learning mathematics in a technological driven environment such as, basing instruction on big ideas, providing equal opportunity to learn, encourage participation into the lesson, providing immediate feedback.

4.2 Fermat Secondary School

Fermat Secondary School is a secondary school located in the west part of the Greater Toronto Area (GTA), in an ethnically, linguistically, and culturally diverse neighbourhood that experiences considerable economic and social issues. At the time of my research, the school was home to about 1000 students. There were math courses at the Academic, Applied, and Essential (Locally Developed) levels in grade 9 and 10, as well as at the University, College, and Workplace Preparation levels in grade 11 and 12.

The school Principal emphasised the importance of integrating technology and multimedia in everyday teaching practice in order to equip students with the skills, competencies, and tools needed to succeed in the future. The Math Department is very supportive and committed to helping students succeed. Thus, the department offered an after-school program that utilised math software called IXL for students who were looking for extra help. In addition, a tutoring program called “The counting on you” ran twice a week after school to help students complete their homework or clarify math concepts and theories. For students who liked to be challenged, there was also a math club that helped them preparing for math contests hosted by the University of Waterloo.

The 2011-2012 EQAO results in the mathematics assessment of Fermat S.S.’ grade 9 students showed that 86% of students at the Academic level and 31% of students at the Applied level achieved at or above the provincial standard, at level 3 or 4. These results show how well students have met the curriculum expectations at either Academic or Applied levels to the end of
grade 9. The results at Fermat showed an increase in the students’ achievement compared to the previous year. The EQAO results from the past three years, indicating grade 9 Applied level students’ achievement at or above the provincial standard, were: 30%, 26%, and 31% (EQAO school report website).

4.2.1 The Case of Shannon

The 2011-2012 school year was Shannon’s tenth year of teaching secondary school mathematics. She had taught for two years at another school and for eight years at Fermat Secondary School teaching “all of the University and Academic level courses and many of the Applied ones” (interview, May 15, 2012). She received her bachelor’s degree in mathematics from an east European university and earned her Bachelor of Education eight years before I met her from a large university in Ontario.

Shannon considered herself comfortable with and confident in integrating technology into her everyday teaching practice. In terms of the technological tools used in her classroom, Shannon emphasised that she had been using “pretty much everything that is around in terms of technology: Smart Boards, graphing calculators, motion detectors, GSP, all sort of things like that” (Interview, May 15, 2012).

Her personal goal for teaching was “to become a better teacher every year and get involved in new projects every year, to try something new every year. And of course, technology plays a role in that” (interview, May 15, 2012). As an involved professional, Shannon intended to “stay up-to-date with the new technology and new trends in education and applications of technology” because she viewed technology as “play[ing] a more important role every year” (Interview, May 15, 2012).
An example of a project involving technology that Shannon wanted to launch into her classes in the next school year was the IXL project. This project was designed for grade 9 students and runs on a daily basis at Fermat S.S. after school. Students who needed to improve their basic skills were instructed by their teacher to go to the computer lab and were assigned to work on particular math tasks that targeted the math concepts that needed review. Shannon explained that IXL was software designed to provide drill and practice exercises for grade 9 students who needed to improve their basic skills. For example, IXL was used “to help students upgrade their skills in terms of integers, fractions, equations, algebraic expressions, etc.” (Interview, May 15, 2012). According to Shannon, this software allowed for differentiated instruction and addressed the individual needs of each student:

Its target is meant to improve only the skills where students are not very strong; so one student could need five or six questions to master a skill while another student might need 20 or 50 questions to master a skill. Now, using this software, we can address exactly the weaknesses and the strengths of our students. (Interview, June 25, 2012)

For the following school year, Shannon hoped to start this program as an after-school program and, planned to integrate it as part of her daily lessons. She felt that the instant feedback provided by this software informed her as a teacher about her students’ progress, and helped identify students’ mathematical strengths and weaknesses. Based on this information, Shannon could design the next appropriate steps and adjust her instruction in order to meet the individual needs of each student. She explained:

So right away each night I can look at the performance of my students and I can go the next day in class and I know that a certain student needs to work on a certain skill while another student can move on to the next topic. So its target is differentiated. (Interview, June 25, 2012)

On the other hand, Shannon felt that the instant feedback helped students build confidence and positive behaviours that promote learning. The full explanations and justifications provided by the software motivated students to self-direct their learning. They
played a significant role in assisting students to acquire the basics skills required to move on to a higher level of mathematics understanding.

Another technology-related project that Shannon was planning on incorporating in the following school year was the flip classroom. She had decided to adopt this teaching approach because she had heard and read about the contribution of this approach to major improvements in terms of standardised tests. A group of teachers from Shannon’s school visited a secondary school in Detroit, Michigan that was a complete flip school. These teachers came back to Fermat secondary school with enthusiasm about incorporating this strategy into teaching in the following school year. According to Shannon, the flip classroom meant that:

The new lesson/content is not delivered traditionally in class but is delivered at home through a video. So students are supposed to go home and watch the video of the new lesson. The video could be done by the teacher or it could be a YouTube video or another video assigned by the teacher. The next day, students come to class and they focus on the applications. It allows the teacher more one-on-one time with each student and it allows more collaboration among students. (Interview, June 25, 2012)

Very enthusiastic about this approach, Shannon did not wait until the next school year to try it in a grade 9 class and she also implemented it immediately in the last part of her grade 12 Calculus and Vectors Class for the 2011-2012 school year. The student feedback was, in general, very positive and encouraging. She explained that students really liked to have the chance before the exam or before the formative evaluation to go back to these lessons, to watch the important parts again, and to review the key concepts. However, with all this success, Shannon mentioned that some of her grade 12 students needed time to adjust to this approach, as they were not accustomed to completing the pre-class activities before every math class.

Shannon believed that this strategy may work well with highly motivated students. Nevertheless, she was willing to try it with Applied level classes and to experience and learn
how to engage Applied level students in taking more responsibilities about their learning and participating more actively in the learning of mathematics.

When asked to think ahead and anticipate any potential disadvantages of the flipped class model of teaching, Shannon specified that it is extremely time consuming to create and compile the videos. She did not agree with teachers who said that “once you do the videos you have them for years to come”. In her opinion, this was not true because “every year you are a different teacher, a better teacher, and you do not do the same lessons for the rest of your career” (Interview, June 25, 2012). She believed that the lessons should be improved upon over the years or should be changed according to the needs of the students. She specified, “These videos cannot represent you for the rest of your career” (Interview, June 25, 2012).

Shannon is committed to working collaboratively with other teachers to record videos for her classes. She specified some advantages to creating videos in partnership with other teachers: it was more fun and interactive; it provided opportunities for teachers to take turns in delivering the lessons and to pretend to be either the students who ask typical questions or the teacher who provides answers and clarifications; it made the lessons more dynamic and interesting; it provided different perspectives, various solutions, and diverse teaching techniques; and, implicitly, it provided learning opportunities to reach all students, as well as opportunities for professional growth.

In addition, for the next school year, Shannon planned to use Angel, which is the e-learning platform used by her school board. The platform allows students and teachers to collaborate, to work in pairs or groups, to share ideas and solutions to math problems, to have access to homework, assignments, and other information posted online by the teacher. Moreover, Shannon is very ambitious to introduce the Smart Slate into her lessons next year. The Smart
Slate is a wireless tablet used to deliver lessons and take up questions. Shannon explained that one of the benefits of using this tablet, versus the Smart Board, was that the teacher can teach the lesson from anywhere in the class and create a student-centred learning environment: “I do not need to be at the front of the room. I can walk around and students will see the result on the Smart Board” (Interview, June 25, 2012). Another benefit was that students could show their work while writing on the tablet from their desks. One disadvantage of using the tablet, as identified by Shannon, is that learning to write on it takes time and practice:

The biggest disadvantage of a tablet is that you need a little bit of time to get used to it. It does not come naturally. And I can spend time to get used to it, but students will need some more time to get used to it. (Interview, June 25, 2012)

Shannon was very active in attending professional development workshops, courses, and conferences. She was motivated to learn and get to know new curriculum resources and strategies. She also wanted to keep up-to-date on emerging technological tools for the classroom and tried to find new teaching ideas to apply in her classrooms:

In the past years, I have been involved in a lot of Smart Board professional activities. I have been part of their annual conference. I am supposed to do a three day workshop. I am supposed to become a Smart instructor. I have been doing a lot of their webinars. I have also been doing a lot of the Texas Instrument webinars. So a lot of activities. All of these professional development opportunities help me for sure to give me ideas that I was able to take and apply in my classes. (Interview, May 15, 2012)

Nevertheless, Shannon believed that the most efficient way to accumulate technology-related knowledge was her own explorations and discoveries: “I find the most efficient way to develop my own knowledge with technology is the work and the research that I do on my own outside of the classroom setting and not necessarily doing the professional development opportunities” (Interview, May 15, 2012).

Shannon suggested that the integration of technology has changed her teaching practice. She integrated technology in her lessons on a regular basis because she found that integrating
technology in the classroom made learning more interesting for students. Technology allowed her to engage students more actively, to pose and discuss real life math problems, to make the content presentation more dynamic, and to explain concepts in ways that were more attractive than textbooks:

With Smart Board or other technology tools, I create attractive and interactive lessons, presented so much different than in the textbooks. I have students manipulate things around to reveal the impact of changes. I use real life problems with real world issues to engage students. I motivate students with dynamic presentations. For example, what I can do now using the Smart Board when I talk about a certain equation at the grade 9 level, the equation of a line, right away I can graph it using a Smart Board. I can come up with a table of values in a few seconds. I did the same thing when I was using a black board; however, now it is faster and it is definitely more interesting for the kids. Kids are more interested when the same thing is done with technology. (Interview, May 15, 2012)

Shannon argued that the integration of technology made the assessment and evaluation process different. She used clickers to assess students’ understanding at different points in the lesson in order to inform her practice and modify classroom instruction, if needed:

I believe a lot in formative evaluations. The clickers that I have in my classroom are a very useful tool to test the knowledge of the students at any given time with very little effort. Kids have a graphing calculator and a clicker on their desk. When I want to see if the kids are ready to go to the next level, I throw a small question at them and right away I figure out if they are able to move on to the next level or if I need to go back and reinforce some of the knowledge that we have discussed. (Interview, May 15, 2012)

I asked Shannon to describe the teachers who were most influential to her teaching practice and to explain how her teaching instruction was different. She informed me that a number of teachers influenced her teaching practice. She particularly admired them for possessing strong subject and pedagogical knowledge. Nevertheless, her teaching approach to incorporating technology made her different from her model teachers who taught her 20 years ago:

There were a number of teachers who influenced me. The reason they had a strong influence on me was mainly because of their strong knowledge of the subject [that] they were teaching; but also because of the way they presented their subject – they made it interesting, they made it logical, they emphasised the problem solving – and I believe that
to some extent I think I took lots of things from the teachers who influenced me. And of course I have my own way. I learned new things. I started incorporating technologies, so of course I am different from the teachers that shaped me as a student 20 years ago.

(Interview, May 15, 2012)

Shannon felt that she did not need to be in total control of the technology used in the math lessons. Looking back at her teaching style from eight years prior, she explained that she used to give out handouts with steps on how to accomplish a certain task using technology, and to provide strict directions on how to proceed in performing a specific assignment. She “needed to be the master of any kind of technology that I brought to the classroom in order to explain each feature of the software or piece of equipment I was using” (Interview, June 25, 2012). Since then her strategy had changed and, she is not afraid to reach outside of her comfort zone. She very often invited students to investigate in order to discover how to carry out a certain task by using technology. She asked more leading questions and she gave more freedom to students to explore and find better approaches and solutions. She stated:

I allow for more input from students. I am able to say that I do not know how it works and let us all figure it out together because the results we get together are better. I allow students to discover more, to find answers for themselves. Five years ago, I would have given out a handout with steps on how to accomplish a certain task using technology. Right now, I ask more questions. So what do you think? Where should I go to do that? Things like that to allow more freedom for students. (Interview, May 15, 2012)

When asked what teachers needed to know to integrate technology effectively in the mathematics class, Shannon explained that teachers should be aware that students are usually better with technology than their teachers, and that teachers should be willing and prepared to learn from their students:

First of all, I think teachers need to be brave and start using technology without thinking too much about it. It is very likely that students in your class will be able to figure out what they need to do if you give them a tool. They will definitely be fast at that, and teachers need to accept that the students they are teaching are better, most of the time, with technology than the teachers themselves. So teachers should be ready to learn from their students. I think that is the main first step. Once you overcome the pressure and you
are ready to learn from your own students things will move smoothly. (Interview, May 15, 2012)

Based on her experiences with integrating technology in teaching mathematics, Shannon concluded that students do not need detailed instructions about how to proceed in finding out the solution to a math problem. She strongly believed that students were able to find ways to engage technology to solve a math problem by discussing with their peers:

I encourage students to investigate and help each other whenever they use technology. I do not provide lots of steps. I do not spoon-feed them. I allow them to investigate, to discover and I give minimum instructions when it comes to technology because I know that with communicating with their peers they are able to figure out how to use software like Geometer’s Sketchpad. (Interview, June 25, 2012)

When asked to describe the qualities and knowledge of a teacher who integrated technology efficiently and effectively in the mathematics class, Shannon suggested that competency in terms of content knowledge and content delivery, good pedagogical skills, and desire to explore and learn how to use new technological tools are essential. She said:

First of all, and no matter what technology a teacher decides to use, I believe that a teacher should be extremely competent in terms of content knowledge. The teacher must be the expert in terms of the content delivered. That is one thing. Another thing that the teacher must know very well is pedagogy: the strategies and the skills that are needed at a certain time depending on the group of kids that the teacher is dealing with. I think that, again, is a very important role of the teacher. Now, in terms of technology, I believe that the teacher should have the desire to use technology and try new technologies, and try to understand the benefits of new technology. Be aware of the benefits, be aware of the fact that the kids we are teaching these days are definitely different from the kids that we were teaching 10 or 20 years ago; these kids have access to technology, they have access to content right away. The content is at their fingertips. They need a little bit more, they need an emphasis on application and things like that. (Interview, June 25, 2012)

4.2.2 Technology Integration at Fermat Secondary School

The majority of the math classrooms at Fermat Secondary School were equipped with Smart Boards and graphing calculators. Some classrooms had clickers. Shannon was largely responsible for convincing the school’s administration to purchase the Smart Board equipment for the entire Department of Mathematics. Most of the teachers at Fermat used technology in
their lessons and they tried to create a dynamic environment where students were excited to learn. These teachers were enthusiastic about using technology in their classes and they openly and freely shared their resources and experiences of delivering successful technology based lessons: “Peers are also important. If you happen to be around teachers that use technology and are enthusiastic about using technology, that will influence you and will create an environment that is ready to try new things” (Interview, May 15, 2012).

In Shannon’s opinion, the administration played a critical role in shaping how much and how technology was used in the classroom. At Fermat Secondary School, the administration was committed to equipping the entire school with Wi-Fi and other technology required to help students reach their potential. The Math Department, in collaboration with the administration, created opportunities to support teachers while they learned how to integrate technology in their instruction. Thus, university professors from local universities were invited to present technology-based lessons, teachers were encouraged to attend professional development workshops and seminars, and sharing ideas and experiences among teachers were sustained during regular working hours (Fieldnotes, May 28, 2012).

The technological tools mostly used in Shannon’s class were the Smart Board, TI-Nspire graphing calculators, and the Geometer’s Sketchpad software.

4.2.2.1 Graphing Calculators

Shannon encouraged students to use TI-Nspire graphing calculators on a regular basis, although she believed that they were not as user-friendly as the applications found on laptops, cell phones, and math software. She explained that the graphing calculators required “prior knowledge and memorisation of steps” and they “are not as intuitive as all the other applications that they [students] see in their everyday lives” (Interview, June 25, 2012). She used graphing
calculators in her lessons because they were available and they allowed her “to meet her objectives” in the math lessons. Specifically, her main objectives were to “provide the four models for each math problem: the graphical model, the algebraic model, the numerical model (table of values), and the description in words” and to engage students in making connections among these models when solving problems:

As I said, I believe a lot in the four models of each of the applications that we did in class. I often have students looking at the graph of a line, the table of values, and looking at the algebraic expression and, of course, the description in words and making connections among the four models. In my opinion, that is extremely important. I really believe it allows for an overall approach of mathematics. (Interview, June 25, 2012)

4.2.2.2 Geometer’s Sketchpad

Shannon reserved the computer lab weekly to give her students access to Geometer’s Sketchpad (GSP) software in order to engage them in learning visually, which, in her opinion, made mathematics meaningful and promoted deep understanding. She also used this software on a regular basis with the Smart Board to demonstrate concepts and to illustrate math ideas. “Investigating slopes” was an interactive activity using GSP that Shannon designed to help students acquire a conceptual understanding of the slope and the y-intercept, and connect these two numbers to the equation of the line. She felt that the visual representations and the immediate changes in the graph that occur as a result of changing the slope and the y-intercept helped students understand conceptually how to convert the information provided on the graph into an equation. When asked what kind of learning was prompted with this dynamic activity, Shannon replied:

What I really wanted to accomplish, I wanted students to acquire a conceptual understanding of the slope and the y-intercept as opposed to a mechanical approach…I wanted students to connect the equation of a line to the slope and the y-intercept. I gave them an equation of a line and then I asked them to calculate from the graph the slope and the y-intercept and then I made them connect these two numbers (slope and y-intercept) to the equation and make the connection there that slope is represented by a certain number and the y-intercept the same way…it was more dynamic. They [students] saw
that based on the numbers that they imputed the equations looked different ways.
(Interview, June 25, 2012)

4.2.2.3 The Importance of Technology

Shannon strongly believed that technology played an important role in the process of teaching and learning mathematics. She explained that the mathematics classroom has changed dramatically over the last 10 years as more technological tools have become available to better assist students in understanding the math content. Teachers are not the single possessors of knowledge anymore or the only ones able to deliver the math content:

The difference is given by the fact that students have access to knowledge, to content knowledge, right away at their fingertips. If they want to know how to solve an equation they can solve any. There are one million applications that will solve any equation that you can imagine. If you want to divide polynomials, again you can find an application and it can be done right away. Ten years ago the teacher was the only one able to deliver that content and all of a sudden that changed. (Interview, June 25, 2012)

In Shannon’s opinion, the use of technology helped teachers to provide thorough and comprehensive explanations of the math concepts through the possibility of including real life examples in applications and problem solving, carrying on investigations, conducting simulations, and organising and interpreting data. She insisted that visualising concepts through technology deepened students’ understandings of abstract concepts and enhanced students’ mathematical communication and vocabulary:

What I believe is the advantage of technology is bringing technology in your classroom allows for more application, more real life applications, of mathematics in real life context. For example, if we go back to the grade 9 math course, I was able, using technology, to emphasise important mathematics terminology – for example, the difference between independent and dependent variables right away. I got online and I got one real life example. We graphed it, we found its equation, we interpreted its meaning, we displayed the table of values, and there you go, we had all representations (graphical, numerical, and algebraic) at our fingertips. I did simulations with motion detectors and kids were able to see the meaning of y-intercept or the slope, to discuss the steepness of the lines, to investigate the slope for parallel or perpendicular lines, or to associate the slope to the velocity of a position-time graph. All of a sudden we can easily attach meaning to abstract concepts that used to be very dry concepts and right now we can bring the real life interpretations of these concepts into the mathematical classroom. I
was talking about how to organise data, to make predictions through interpolation and extrapolation. Again, technology helped me emphasise the vocabulary/terminology through the real life examples I was using. (Interview, June 25, 2012)

Additionally, Shannon believed that technology-supported learning mathematics with understanding, promoted the development of problem solving strategies and techniques, allowed and encouraged higher level thinking skills, and enhanced the teaching of mathematical communication skills and vocabulary:

So I truly believe that technology is way more than a tool to provide a visual representation of mathematics. In my opinion various forms of technology allow and encourage higher level thinking skills. It allows students access to solving real life situations that they would not have had access to without technology and that definitely leads to [the] developing of higher level thinking skills that are so important when teaching mathematics. Another thing that comes to my mind is that with technology we can do, we can perform, all sorts of investigations that allow students to construct, to build meaning for themselves. It allows students to discover math concepts on their own. And it allows for, I am thinking in terms of communication, it improves communication in the mathematics classroom. (Interview, June 25, 2012)

Moreover, Shannon expressed that technology enabled teachers to create a dynamic learning environment that empowered students to explore and construct a conceptual understanding of mathematics and to connect abstract concepts to real life situations:

The teacher is able to create a more dynamic learning environment for students. It allows the teacher to empower students to construct for themselves the abstract math content and attach meaning to it by connecting the math classroom to real life or real situations. (Interview, June 25, 2012)

Shannon emphasised the role of technology in promoting effective teaching and learning. She explained that technology plays an important role in the analysis of data and explained that technology was used efficiently and effectively in her Linear Relations Unit when a scatter plot of data was needed in order to display the relationship between two variables. In this context, the integration of technology saved time when it came to determine the linear regression and the correlation, and it provided more opportunities for students to focus on the interpretative nature of the math problem, to discuss the type of relationship, to discuss the overall trend, and to make
predictions by using interpolations and extrapolations. Moreover, she believed that “the main benefit using technology is that it gives up a chance to present students whenever a problem is solved with a graphical approach, or an algebraic approach, a numerical approach, and also a description in words” (Interview, May 15, 2012).

In addition, Shannon viewed technology as an important tool for identifying students’ strengths and weaknesses. This information was useful for guiding and organising instruction and for making purposeful decisions about how to differentiate instruction to meet the individual needs of each student. She explained:

Another thing that I think is important to mention here is that technology allows teachers to identify the weaknesses and the strengths of the students. And it allows [for] preparing material that is technology-related material that is specific to [the] various needs of all the students in the classroom. Technology allows for differentiation. Never before were we able to target individual needs of all the students in the classroom. Right now, using technology, we have all sorts of ways to approach all the students and the different learners that we have in our regular math classroom. (Interview, June 25, 2012)

Shannon pointed out that technology empowered students to become independent, as it allowed them to work at their own pace. This helped students build their confidence and gave them an increased sense of accomplishment and personal growth:

A good thing about technology is that it gives students a chance to work at their own pace. Some students are able to finish everything that was assigned and move at a deeper level, while other students will only work through the basic questions, but each student takes ownership of his/[her] own learning and will experience success at his/[her] own level. This gives them a sense of accomplishment and personal growth. (Interview, June 25, 2012)

Shannon suggested that a significant role of technology in the math class was in assisting teachers to deliver many different types of instruction to address students’ individual needs, learning styles, and skills levels:

Definitely technology is a huge asset when talking about differentiating instruction and different needs of the various learners in our typical classroom, their different learning styles and skills levels. Now to go back to the discussion about numerical, algebraic, and graphical representation that can be presented at the same time with absolutely no effort.
Right away you can address all sorts of learners in your classroom: the logical mathematical learner, [the] visual learner, [the] kinesthetic learner. (Interview, May 15, 2012)

Shannon felt that technology helped students connect with their teachers when teachers created learning environments and used learning tools that were relevant to their lives:

Now, what technology could do, in my opinion, is enhance learning and teaching. That is one thing. Another thing that technology could do is it can give the teacher a chance to have a common language with the kids. Our students obviously are pretty good with technology and in order to speak the same language, teachers need to learn the language of technology. (Interview, May 15, 2012)

Despite the fact that Shannon considered the presence of technology in the math class to be essential, she discussed that integrating technology too early in math education could be problematic. She suggested that this could be problematic, specifically, when students were allowed to use the calculators prior to developing a strong number sense and algebraic thinking:

For example, my experience with grade 9 students is [that] when they come to grade 9 and they have absolutely no number sense because they were given calculators in grade one or two, somewhere around that age, and they were not given the opportunity to develop a number sense, that is a really bad example of using technology. (Interview, May 15, 2012)

4.2.3 Student Tasks

Shannon designed technology rich tasks that offered different levels of challenge and were accessible to students with various skills and interests. These tasks encouraged creativity and provided opportunities for students to relate to real world situations. They challenged students to select the problem of their choice (e.g. students chose the object to picture), engaged them to collaborate and discuss mathematics with their peers, and had the potential to broaden their mathematical content knowledge.

One example was the student task entitled “Creating Pictures by using GSP”. This task required students to use GSP to create a picture of an object of their choice by using only line segments. The segments must be generated by an equation of a line with a restricted domain. The
equations used to generate the line segments should have had slopes and \( y \)-intercepts in integer or fraction forms. The picture needed to include at least two pairs of parallel and perpendicular lines, and at least three horizontal and three vertical lines (Student handout, May 29, 2012).

Shannon pointed out that the main objective of this open-ended task was “to give students a chance to model the world around them by using mathematics”, “to make students understand why math is important in their everyday lives”, “to relate abstract math concepts to real life situations”, and to give students the opportunity to make appropriate decisions when applying their mathematical knowledge. She explained:

It relates abstract math concepts to real life situations. It also gives students the chance to understand that to make decisions in terms of slope and \( y \)-intercept, giving them a chance to deeply understand these concepts. If they change the slope from positive to negative, they see the result right away in their picture. They have to make the right decision at the right time in order to generate a picture according to their plan. (Interview, June 25, 2012)

Shannon emphasised that this task created the opportunity to reinforce students’ understandings of the key concepts in the Linear Relations Unit. She asked students to use the Geometer’s Sketchpad (GSP) to create examples to illustrate their conceptual understanding of positive, negative, zero, and undefined slopes, and to justify using visual representations: “why parallel lines must have equal slopes and perpendicular lines must have negative reciprocal slopes” (Student handout, May 29, 2012).

Shannon underlined that this task engaged students in practicing their written mathematical communication skills. Thus, students were asked to explain how they arrived at their end product (e.g. specify the equation of the lines, restrictions on the domain and range), to list and provide the meaning of the slope and \( y \)-intercept for each line segment, to set the domain and the range, to establish the conditions for parallel and perpendicular lines, and to write the equations of the lines involved (Field notes, May 29, 2012).
Shannon explained that one significant advantage of using technology in this task was the immediate feedback: “If the lines are completely different from their intention, and the picture looks totally different than the object they want to draw, then obviously they [students] realize that they need to go back to adjust the equations accordingly or to change the settings of the domain or the range” (Interview, June 25, 2012). She insisted that:

> By drawing the line segments, as a result of inputting the equation of the line with restrictions for the domain and range, the students connect the visual representations to the abstract equation, attach meaning to the slope and y-intercept, and gain fluency in moving between different representations of lines. (Interview, June 25, 2012)

Shannon also mentioned that the integration of technology in this task creates an environment that engaged students who might not have otherwise been engaged in completing mathematics assignments. It also showed what students were capable of achieving (Field notes, May 29, 2012).

Shannon provided another example of a technology rich student task that she assigned as independent work to her grade 9 students. The task was called A Washing Machine Problem. In this task, students explored equations and graphs of lines in order to discover and analyse the property of parallel lines (i.e. they have the same slope). This problem was inspired by a real life situation: a new dry cleaning machine uses a cleaning liquid that evaporates or condensates such that there is a loss of 2% of the initial amount of liquid after each use. Students were asked to determine and graph the equation that described the relation of the amount of the liquid remaining and the number of washes for three different initial amounts of liquid. Then, they were asked to compare the three graphs and their equations in order to notice that the parallel lines have the same slope (Student handout, June 5, 2012).

Shannon provided students with the choice of using the table of values, the equation of the line, or the graphical representation to come to the conclusion that parallel lines have the
same slope. She also created this task to encourage students of various abilities to respond to the task to the best of their ability. For example, a less able student could complete the table of values to determine the quantity of liquid after 45 uses, while a more capable student could use the equation or could extrapolate the graph to find the same answer. The task encouraged critical thinking, specifically when the students decided that they needed to set $y = 0$, in order to determine the number of washes (x-variable) until the liquid was zero. For this part of the problem students also had the choice to answer the question by using the graphical representation, the algebraic approach (to use the equation of the line), or the numerical representation (to fill in the table of values until the amount of the liquid became zero) (Field notes, June 5, 2012).

The task involved the use of graphing calculators, which helped students check the validity of their conjectures that parallel lines have the same slope. I asked Shannon to explain why she chose to use technology for this task and she stated that technology assisted students in visualising the situation and helped them produce high precision graphs faster:

The graphing calculators assisted students to visualise the situation graphically and to see that the lines which have same slopes do not intersect because they are parallel. The graphing calculators also helped students work quicker and be efficient in producing high precision graphs. The accuracy of the graphs had a positive impact on students’ understanding that parallel lines have [the] same slope. In the final part of the project students were asked to write the equation of four different pairs of parallel lines and check graphically that they are parallel. Technology played an important role in assisting students to produce fast graphs and check their conjecture. (Interview, June 25, 2012)

4.2.4 Knowledge Construction

4.2.4.1 Challenging Preconceived Notions and Misconceptions

Shannon mentioned that, when teaching slopes as a rate of change, she frequently encountered a common misconception that could be remedied with the use of motion detectors. The misconception was that, when students compared the speed for different parts of the
distance-time graph, they believed that the negative values were smaller than the positive values, instead of considering the absolute values in their comparisons and ignoring the sign, which indicates the direction of movement. She explained:

Sometimes negative slopes make confusion in the students’ minds. The confusion is that the negative symbol has physical meaning and shows the direction of the motion. This is why sometimes students misread the information provided on the graph when they are asked to compare a negative, a positive, a zero, and an undefined slope. They are tempted to conclude that the negative slope indicates a slower average speed, when in fact the negative sign indicates the object’s direction of movement. Sometimes students also consider the negative slope to show a slower average speed than the zero speed of an object at rest, just because negative numbers are smaller than zero. (Interview, May 15, 2012)

To challenge this misconception, Shannon used motion detectors and asked students to move towards and away from the detector with the same speed. This helped students understand that the rate of change represents the average speed and indicates how fast the object is moving, and helped to reinforce that the sign shows the direction of movement:

I ask them to move with a constant speed towards the detector to make them understand that when the distance decreases in time, the graph shows a line which falls from left to right and it has a negative slope. Then, I ask students to move at the same constant speed away from the detector and the distance increases in time. On the distance-time graph, the line rises from left to right and it has a positive slope. I also ask students to stop for a while such that on the graph they see a horizontal line with a slope of zero. When students compare the average speed of their real motions and connect them to the graph produced by their motions, they understood that the average speed indicates how fast the object is moving and not the direction that the object is moving. Thus, students realize that when comparing the average speeds they should ignore the sign of the slope and should consider that the steeper the line, the faster the average speed. Technology helps very much in making this concept clear to students. (Interview, May 15, 2012)

Shannon felt that the integration of technology engaged students in active learning and provided a bridge to conceptual understanding of key ideas in mathematics:

I really believe that once you perform and you do an activity like that where you see results, math results, as a result of your actions I really believe that that provides an excellent opportunity for deep understanding of the concepts and real and true understanding of the concepts. (Interview, June 25, 2012)
4.2.4.2 Student Interests and Increased Motivation for Learning

Shannon used technology to tap into her students’ interests and to increase their motivation to learn. In one lesson that I observed, for example, students used graphing calculators to investigate the relationship between the height of a person and the person’s arm span. After students measured and recorded their heights and their arm spans in a table of values, they used the TI-Nspire graphing calculators to scatter plot the data from all the students in the class, as well as to graph and determine the equation of the line of best fit. By looking at the graph, the students discussed and interpreted the appearance of the scatter plot and they noticed that there was a positive correlation between arm span and height.

Furthermore, students interpreted the meaning of the slope and y-intercepts in situations using algebraic, tabular, graphical, and verbal descriptions of linear functions. They used real data to further their conceptualisation of the linear relation (specifically of the form y = mx), to predict by using interpolation and extrapolation, to make decisions and critical judgments, and to solve problems involving the direct variation concept. Shannon found that the use of graphing calculators in this activity allowed students to quickly plot data points on the calculators and to use their time to focus on the interpretation of the graphical displays.

4.2.4.3 Experiences, Experimentations, and Negotiations of Meaning

Investigations and explorations with the use of technology were paramount in Shannon’s lessons. She provided the example of a lesson in which, by using technology, she facilitated students’ understanding of the properties of parallel and perpendicular lines. Thus, by using Geometer’s Sketchpad (GSP), students formulated and tested conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models. They discussed trends for slopes and the graphs of linear equations, and negotiated the attributes of linear
equations, which indicate whether two lines are parallel or perpendicular. They decided how the physical construction of a line corresponds with the numerical attributes of parallel and perpendicular lines, and then they applied properties of linear equations to related problems involving parallel and perpendicular lines. Shannon described the “Parallel and Perpendicular Lines” lesson. She explained how she guided her students to discover the property of parallel lines to have equal slopes:

In Geometer’s Sketchpad, I start with parallel lines. I asked students to open a new sketch, to draw a line, to select it, and to measure its slope. We analysed and discussed various positions of the line and the corresponding slope values, looking specifically to find explanations for the slope of a vertical line, which is undefined, and the slope of a horizontal line, which is zero. Students draw another line and drag it to make it parallel to the first one. They noted that their slopes are equal. I also taught them how to use the Construct menu to construct with precision parallel lines. At this point, after analysing the slopes of linear equations and their graphs, the students identified the attributes of parallel lines. (Interview, May 15, 2012)

Shannon also explained how she directed her students to find the property of perpendicular lines, that the product of their slopes is negative one:

Then they continue in the same way with perpendicular lines, but this time they started drawing perpendicular lines and they calculate the slopes. This time, it wasn’t necessary to show them how to construct the perpendicular lines. They figured it out by themselves. So, using the Construct menu they made perfect perpendicular lines. Then, they find the slopes of these lines, multiplied them in the calculator, and verified the property that one slope is the negative reciprocal of the other. Together, we discussed and concluded that if you multiply the two slopes, you should get -1. Then, students were assigned some application questions to practice the new properties discovered. (Interview, May 15, 2012)

Shannon provided another example of designing a technology driven interactive lesson to help students make sense of an abstract math concept: the investigation of the point of intersection of two lines by using Gizmos. She combined the physical movement of students in the class with the simulation provided by Gizmos on the Smart Board. Thus, on one hand, she experimented and modelled with Gizmos the abstract situations of the motion of a cat and a mouse chase. On the other hand, she had students pretend to be the mouse and the cat and
moving in a straight line with a speed that matched the graphical representation Gizmos displayed on the Smart Board:

For example, again going back to grade 9 where we are supposed to investigate the point of intersection of two lines, the way I made my lesson interactive [was] I went to the Gizmo activity – the famous mouse and cat activity – where you have a cat chasing a mouse and [by] varying the speed of the cat and the mouse you can talk about the point of intersection. And at the same time, you can see the distance over time of both the mouse and the cat. And for a certain speed of both the mouse and cat, you get a point of intersection or no point of intersection. And the way I made this lesson more interactive [was] I had the students look at the Gizmo activity but at the same time I had them demo the activity. I had one student being the mouse and another one being the cat and they were trying to do the same thing as the Gizmo activity. So technology helped me make my lesson more interactive. So technology definitely could play a big role if used wisely. (Interview, May 15, 2012)

As stated by Shannon, one of the benefits of using Gizmos in this lesson was the visual representation of this abstract math concept (point of intersection of two lines) and the possibility to manipulate the variables and constants and to immediately see the effects of these changes:

By using Gizmos, students became actively involved in their learning. They can manipulate variables and constants and watch the effects of changes immediately. The visual representation is the great help. Students make easily connections to real life situations and they better understand the key concepts. Gizmos makes abstract concepts fun to learn. (Interview, May 15, 2012)

Another advantage of using Gizmos in determining the point of intersection of two lines was the possibility to differentiate instruction and to allow students to explore the math concepts at their own pace:

Students could also browse for Gizmos on their own, which helps them understand the concepts at their own pace. It helps differentiate instructions in the class, as students with all ability levels could interact with Gizmos and discover the meaning at their own pace. (Interview, May 15, 2012)

4.2.4.4 Increasing the Complexity of the Content

Shannon believed that technology enriches the complexity of the content: it not only accentuated the visual aspect of the problem but also facilitated and supported students’ conceptual understanding of mathematical ideas. Thus, Shannon illustrated how she used
computer-based motion detectors to introduce the slope as a constant rate of change of the
distance with respect to time. During the class observation, I noticed that Shannon gave students
the opportunity to explore distance-time graphs by asking them to move back and forth in front
of the motion detector at various walking rates so that a plot of their motion would match the
model graph (Field notes, May 18, 2012). Shannon’s rationale for performing these simulations
was: “Simulating one person’s motion in real time allows students to understand, make
predictions and generalisations about different motion situations” (interview, June 25, 2012). She
felt that this experiment increased students’ understanding of the slope as a rate of change, and
helped them make precise predictions and interpretations of the graphical shapes of given
motions:

I am thinking that, without the motion detector, without the use of technology, students
would look at the graph on a piece of paper and would understand the concept on the
surface. We scratch the surface of this concept. When they have to move, they have to be
actively involved, they have to perform certain actions, in order to obtain the same graph.
In my opinion, this transition from a piece of paper to the actual movement insures the
conceptual understanding of, and the enduring understanding of, this concept. (Interview,
June 25, 2012)

4.2.5 Students’ Mathematical Communication

When first asked about the role of technology in developing students’ mathematical
communication, Shannon suggested that technology did not play an important role:

Right now, we focus on communication in mathematics and when we say communication
in mathematics we say proper terminology, proper graphs, presenting mathematical
format, and things like that. I do not really think that technology plays a very important
role in developing or helping students’ mathematical communication. I am thinking in
terms of creating graphs – if students create lots of graphs by using technology that will
not necessarily mean that students will be able to create their own correct graph without
technology. Same about, for example, using the CAST Rule to solve an equation. It
definitely helps you to use the CAST Rule to figure out the steps in solving an equation
and [to] avoid typical mistakes. However, if you only do it using technology it is not very
likely that students will be able to show proper mathematical format when solving an
equation. So I do not really think that technology helps students improve their
mathematical communication. (Interview, June 15, 2012)
However, when referring to the investigative lesson involving motion detectors to produce distance-time graphs, Shannon explained that the use of this technological tool targeted the development of mathematical communication skills. She explained that students needed to provide precise mathematical instructions to their peers to make them walk in such a way to match the given graphs:

I will go back to one of my favourite concepts: the rate of change and the meaning of rate of change. The way I introduce this was using the motion detector. I really, really like this approach because it allows for a more dynamic classroom first of all. It allows for improved communicated skills because peers were supposed to give mathematical instructions to the person performing the matching activity, and in order to obtain a good graph the instructions needed to be precise and using good form. (Interview, June 15, 2012)

Additionally, during my observations, I noticed that, in the “Height versus Arm Span Activity” handout, Shannon included a communication component and invited students to use appropriate mathematical language to discuss the relationship between height and arm span. She expected students to use the mathematical terminology learned in the Linear Relations Unit, particularly to use the terms increasing, decreasing, pattern, trend, rate of change, prediction, interpolation, extrapolation, and correlation. In Shannon’s opinion, the use of graphing calculator emulators and the Smart Board provided the visual representation needed to analyse information, to explore mathematical meaning, and to practice mathematical terminology:

To encourage math communication, for example, I asked students to get the domain, the range, to talk about outliers, the trend, to identify the y-intercept and discuss its meaning, to determine the rate of change and interpret its meaning. Everything was done based on the data we collected in class and data collected over the years and added to the list of present students in class. The graphing calculator produced a very quick and accurate graph and provided the line of best fit. It was our chance to make predictions and to talk about extrapolation and interpretation on a more accurate graph. We also went back and talk about positive and negative correlation and the meaning in this context. (Interview, June 25, 2012)

Shannon also encouraged students to discuss and analyse the advantages of using the scatter plot instead of the table of values in studying the relationship between height and arm
span. She asked students, “In what ways does the scatter plot, when compared to the table of values, make it easier to see relationships between height and arm span?” Students concluded that, since the scatter plot orders the data on the basis of increasing heights, and tables of data do not necessarily do this, it was easier to see the pattern from a graphical display versus from an unordered numerical presentation of the data.

Shannon also asked students to describe the relationship between the height and arm span for points that were below or above the line of best fit. Students responded that points that were above the line represented students whose arm spans were larger than their heights and the points that were below the line represented students whose arm spans were smaller than their heights.

In this context, the use of graphing calculators helped students draw graphs quickly and enabled them to investigate the effect of the outliers on the accuracy of the line of best fit. With the use of graphing calculators, Shannon was able to facilitate mathematical discussions to explain the reliability of predictions made from the graph. She used this opportunity to caution students not to use a line of best fit to make predictions beyond the bounds of the data points, or to make predictions about a population that is different from the population from which the sample data was drawn. To make this clear to her students, Shannon imported a separate set of data for babies, produced a scatter plot, and determined the line of best fit. She showed that babies have different body proportions from children and adults, are growing very fast, and the graph of their height and arm span generates a different line of best fit (Field notes, May 24, 2012).

In the same lesson, Shannon put up an article from the Toronto Star on the Smart Board to provide an example of a nonlinear relationship. This graph, having the shape of an exponential
function, described the growth of Internet use over the years. She explained how technology facilitated students’ comparison of linear and nonlinear relationships:

The article from the Toronto Star was an example of a nonlinear relationship. It was the growth of Internet use over the years, which was obviously a nonlinear relationship. So it allows students to compare and go back and talk about linear versus nonlinear relationships, which is an expectation in grade 9. (Interview, June 25, 2012)

Shannon integrated technology to engage students in dialogue while enabling formative assessment. Thus, she explained that, as a “firm believer in formative assessment”, she used Smart Board and the interactive response system (clickers) to monitor changes in students’ understanding and to check for their comprehension during the lesson. Students’ anonymous answers to the formative assessment questions provided opportunities for debate and discussion, which in turn helped students clarify concepts and actively build new knowledge:

I am a firm believer in formative assessment. Now I do it by incorporating questions in a Smart Board file and asking them [students] to use clickers. I do something else that I was not able to do before. Right now, I ask a question, and not necessarily a multiple choice question that is mostly used in a version of summative assessment, but without expecting a certain answer. Asking students to give me an answer and then analysing the different answers that are coming from students. I am doing it in an anonymous way, and I put the answers on the board and then together we discuss and analyse different answers that came from students. Students listen to their classmates’ explanations, they argue and present their understandings, and this interaction helps them actively construct knowledge. (Interview, May 15, 2012)

In addition, Shannon incorporated technology in her lessons to provoke students to discuss with their peers in order to negotiate meaning and to achieve a good understanding of linear relationships. For example, she invited students to work in groups to explore various equations and graphs of lines with different slopes. Her intention was to use technology to expose students to a large variety of examples of linear equations and graphs. Through dialogue, these examples helped students negotiate and discover the connections between the algebraic and graphical representations of linear relations:
For example, in grade 9, when studying properties of a line, what I did was I put students in groups of three or four and I asked each student in the group to use the graphing calculator to graph a line with a given slope. Each student had a different slope. Then, I asked them to get together and look at all their graphs and discuss how the graphs will change for different values of the slope, and try to connect the algebraic model that was given to them with the graphical model that they constructed for themselves in order to realize that the higher the slope the steeper the graph of the line. Or similarly, I had some of them graphing lines with positive slopes and with negative slopes. And again have them in their groups discuss and negotiate the connection between the algebraic model, positive or negative slope, and the graphical model. The graph looks a certain way with a negative slope and looks different with a positive slope. And for a slope zero and a slope undefined. (Interview, May 15, 2012)

4.2.6 Summary

The case of Shannon demonstrated how technology enables teachers to create a dynamic learning environment that: empowers students to become independent learners who explore and construct a conceptual understanding of mathematics; promotes the development of problem solving strategies and techniques, allows and encourages higher level thinking skills; and enhances the teaching of mathematical communication skills and vocabulary.

Shannon used technology to engage students more actively in the learning process, to pose and discuss real life math-related problems, to make the content presentation more dynamic, and to explain concepts in ways that were more attractive than textbooks. She integrated technology to assess students’ understanding at different points in the lesson in order to inform her practice and to modify classroom instruction to meet the individual needs of her students.

In the context of the Linear Relations Unit, Shannon’s mathematical activities can be summarized by three predominant actions while teaching with technology: graphing, displaying information and mathematical procedures and interpreting. In terms of instructional strategies, Shannon integrated technology to relate abstract concepts in linear relations to real life situations,
to conduct investigations of properties of key concepts, to promote critical thinking, and to foster mathematical communication.

Thus, to reinforce students’ understandings of the key concepts in the Linear Relations Unit, Shannon engaged students in working on a project, “Creating Pictures by using Geometer’s Sketchpad”, to model the world around them. This task helped students relate abstract math concepts (such as rate of change, x- and y-intercepts, domain, range, parallel and perpendicular lines, and point of intersection) to real life situations, and to illustrate their understandings of positive, negative, zero, and undefined slopes. In addition, Shannon used an interactive Gizmo (Modelling Linear Systems-A Game of Cat and Mouse) activity to help students make sense of an abstract math concept, the point of intersection of two lines, and to provide students with the opportunity to manipulate variables and constants and to see the immediate effects of these changes. She believed that visual representations were a great help and assisted students to make connections between real life actions and abstract math concepts.

Additionally, Shannon designed a technology-based project, “A Washing Machine Problem”, and encouraged students to use technology (TI-Nspire graphing calculators) to explore and analyse the properties of parallel lines and to check the inferences they made. She used motion detectors to challenge students’ misconceptions that negative slopes indicate a slower average rate. By asking students to move toward and away from the detector with the same speed, she helped them understand that the rate of change represents the average speed and indicates how fast the object is moving, and that the sign indicates the direction of movement.

Shannon used technology (TI-Nspire graphing calculators) to tap into her students’ interests and to increase their motivation to learn. Thus, she asked students to record their heights and arm spans, to scatter plot the data from all the students in the class, to graph and determine
the equation of the line of best fit, and to discuss and analyse the relationship between the height of a person and the person’s arm span. Shannon found that the graphing calculators allowed students to quickly plot data points on the calculators and to use their time to focus on the interpretation of the graphical displays.

TI-Nspire graphing calculators, clickers, motion detectors, Geometer’s Sketchpad and interactive Gizmos activities were the predominant technological tools used during Shannon’s mathematical activities. Shannon used the Smart Board in every lesson and mathematical activity that I observed.

Table 1. Shannon’s Classroom Observation Log

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Technological Tools</th>
<th>Date and Time</th>
<th>Student Handout</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Investigating Slopes</td>
<td>Geometer’s Sketchpad IXL-E-homework</td>
<td>May 16th</td>
<td>Investigating Slopes.</td>
</tr>
<tr>
<td>2. Linear vs. Non-Linear</td>
<td>Geometer’s Sketchpad Smart Board</td>
<td>May 17th</td>
<td>None</td>
</tr>
<tr>
<td>3. Distance Time Graph (Day 1)</td>
<td>Graphing Calculators CBR Smart Board</td>
<td>May 18th</td>
<td>Distance-Time Graphs</td>
</tr>
<tr>
<td>4. Distance Time Graph (Day 2)</td>
<td>Interactive White Board CBR</td>
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<td>Stories with Slopes.</td>
</tr>
<tr>
<td>5. The Line of Best Fit (Day 1)</td>
<td>TI-Nspire Graphing Calculators Smart Board/Clickers</td>
<td>May 24th</td>
<td>The Line of Best Fit</td>
</tr>
<tr>
<td>6. Extrapolation and Interpolation</td>
<td>TI-Nspire Graphing Calculators Smart Board</td>
<td>May 25th</td>
<td>Extrapolations and Intrapulations</td>
</tr>
<tr>
<td>7. Scatter Plots</td>
<td>Microsoft Excel Smart Board/Gizmos</td>
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<td>The Height versus Arm Span Activity.</td>
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<tr>
<td>8. Using Linear Relations to Create Pictures</td>
<td>Geometer’s Sketchpad Smart Board</td>
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<td>Using Linear Relations GSP</td>
</tr>
<tr>
<td>9. Graphing Lines of the Form y=mx+b</td>
<td>Smart Board Graphing Calculators</td>
<td>May 31st</td>
<td>Graphing Lines of the Form y=mx+b</td>
</tr>
<tr>
<td>10. Direct and Partial Variation</td>
<td>Smart Board TI-Nspire/Clickers</td>
<td>June 4th</td>
<td>Direct and Partial Variation</td>
</tr>
<tr>
<td>11. Review Linear Relations</td>
<td>Smart Board TI-Nspire Graphing Calculators</td>
<td>June 5th</td>
<td>None</td>
</tr>
<tr>
<td>12. Linear Relations Project</td>
<td>Geometer’s Sketchpad</td>
<td>Due June 5</td>
<td>Linear Relations Project – Washing Machine.</td>
</tr>
</tbody>
</table>
4.3 Euclid Secondary School

Euclid Secondary School is an ethnically and culturally diverse school located in the Greater Toronto Area. In the 2011-2012 school years, approximately 1100 students were enrolled at Euclid S.S. The school invited students to learn what they love and to pursue different courses and programs, including the International, Business and Technology (IBT) Program. The International, Business and Technology Program (IBP) is “an innovative program that enriches and extends the traditional secondary school curriculum by incorporating 21st century skills, preparing students for the challenges of an increasingly and technological demanding world” (School Board website). The Department of Mathematics offered a variety of courses from a Locally Developed course in grade 9 to an International Baccalaureate course in grade 12.

Students had access to mathematics support on-site after school and online. A math club ran once a week after school for about an hour. This club was intended for students interested in exploring mathematics beyond the school curriculum and it also prepared students to write math contests organized every year by the University of Waterloo.

The 2011-2012 EQAO results in the mathematics assessment of Euclid S.S.’ grade 9 students indicated that 90% of students at the Academic level and 43% of students at the Applied level achieved at or above the provincial standard, level 3 or 4. These results indicate an increase in the students’ achievement compared to the preceding school year. The EQAO results from the past three years, indicating applied level students’ achievement at or above the provincial standard, were: 30%, 24%, and 43% (EQAO school report website).

4.3.1 The Case of Anna

Anna graduated from an East European university with a master’s degree in Engineering. Before moving to Canada, she worked as an engineer for 10 years. Upon her arrival to Canada,
Anna reconsidered her employment options and pursued a Bachelor of Education at a large university in Ontario and became a mathematics teacher at the intermediate and senior level. She had been teaching for 12 years in the GTA and had been the Head of a Mathematics Department for four years. Anna transferred to Euclid Secondary School at the beginning of the 2011-2012 school year and was assigned to teach three math courses (grade 9 Applied, grade 10 Academic, and grade 11 Mixed level).

In her career as a mathematics teacher, Anna has taught almost all mathematics courses at the high school level. Anna felt very confident in using technology in the mathematics classroom, including graphing calculators, Smart Boards, iPhones with free math applications, Gizmos, and Geometer’s Sketchpad (GSP). She felt on par with students in terms of using of technology, and this helped her make effective connections with her students. She strongly believed that “our generation must stay at par with progress in technology; otherwise, we will widen the gaps that already exist between us educators and our students” (Interview, February 16, 2012).

Anna integrated technology into her lessons mainly “to connect with students on their level and use technological gadgets they relate to [to] teach mathematical concepts”. She also integrated technology into her lessons because it increased the attention and involvement of students in their learning process, as well as it ignited “their curiosity in mathematical concepts” (Interview, February 16, 2012). Furthermore, she felt that the integration of technology motivated students to learn because it made lessons more interactive and relevant to students’ lives and captivated their attention. She stated:

I truly believe that something dramatic has to happen in the education system so that we will start connecting with these kids in the technological fields….And if our education will not start bringing and allowing these smart technologies into our class, I think that we will go way into two different directions with kids. Motivation is a key. So if you do
not bring to class what probably would get their attention, they will never learn what you want them to learn. So I guess it is just as simple as that. We cannot teach anymore chalk and talk – no way. (Interview, February 16, 2012)

Anna started using technology when she entered the teaching profession and was considered by colleagues “to be an expert in the use of technology” (Interview, February 16, 2012). She explained that she was lucky to start her first teaching assignment in a secondary school well-equipped with technology and was inspired by a tech-savvy mentor teacher who used many of the available technologies, including Texas Instrument graphing technology, and a number of different mathematical software, such as Fathom and Geometer’s Sketchpad (GSP).

The dedication of Anna’s mentor, his knowledge about the integration of technology in mathematics, his skilfulness to provide the adequate technological tools to engage students in the exploration and investigations of math concepts, and his ability to motivate students to learn math theories and apply them in solving problems convinced Anna to adopt, learn, and integrate technology into her math lessons. She explained:

I think that I looked up to him so much that I wanted to know as much about technology as he did because when I was sitting in his class and he was teaching Data Management with a TI92, the kids did what he told them to do and every single person was engaged. And the kids that were sitting side by side, they were not talking about the movie from last night or whatever, they were talking about the math results…And he was the first person that proved to me that if you know how to use technology you are a better teacher than anybody else around you who is not using it. (Interview, February 16, 2012)

A teacher from another school trained Anna on how to use the Smart Board. When she observed this teacher delivering several successful lessons with the Smart Board, Anna noticed the increase in students’ engagement, and the efficiency and effectiveness of teaching with this technology. This influenced Anna to start learning how to integrate the Smart Board into her lessons:

[The colleague] taught me a lot in terms of Smart Boards because she was the first one to get it. I saw the increased students’ engagement again. Soon I saw that the kids are more eager to learn and I wanted to have it in my classroom as well. You bring it in, you use it,
you use it effectively, you engage these kids, and then that is what you wanted. That is what I wanted. I saw it and I wanted to replicate the lessons like that. (Interview, February 16, 2012)

Although Anna liked to attend professional workshops to learn about incorporating technology in math lessons and enjoyed learning how to use technology from observing other teachers, she loved to discover how technological tools worked and what they could do on her own. In the first interview, she mentioned:

I like going to workshops, but I like discovering things on my own also. So you give me a graphing calculator – my more thrilling experience is when I am discovering what the calculator can do on my own. Then people tell me what buttons to push, to collect data, whatever. I do both. I go to workshops but I do my own discovery. I also like to use new technology that would come in and that you would come across. I like to first just browse on my own and see what can be done with it [technology]. (Interview, February 16, 2012)

Anna’s goal for teaching in the next five years was to increase student success and to have a 100% passing rate in every class she taught taking into consideration that all students will be attending her classes daily. She believed that this goal could be reached by “implementing more technology” and by increasing communication with her students “outside of classrooms using blogs, Angel, or My Class sites so they can revisit lessons or activities on their own” (Interview, February 16, 2012).

Anna had also signed up to be part of the “flipping school project” in the next school year. This project was a new initiative of her school board and she was willing to embrace this approach to teaching because she had heard about its outstanding success from other teachers who had used it. In addition, she wanted to “let the kids be in charge of their own learning”. Anna explained that the “flipping school” teaching strategy implied that teachers have their students complete the instruction as homework, by watching a video prepared by the teacher. Class time is used to apply and practice the concepts learned outside of the classroom, and to perform enriched tasks, projects, and group assessments (Interview, June 20, 2012).
Comparing her new teaching practices incorporating technology with her initial teaching strategies that made use of the black board, Anna remarked that students were enjoying the interactive activities and explorations, were more engaged in discussions, participated actively in the learning process, and behaved more appropriately during the lesson:

To present concepts on the black board was becoming more of the chore for me and even more of a struggle to stay focused for my students. Bringing into the classroom graphing calculators, computer algebra system, and Geometer’s Sketchpad application allowed me to engage my students in lessons, discussions, explorations, etc. The attention span increased, discipline issues are fewer than ever before, and in many cases, I saw how kids are enjoying activities. (Interview, February 16, 2012)

When asked to describe the qualities and knowledge of a teacher who integrated technology efficiently and effectively in the mathematics class, Anna suggested that strong knowledge in terms of content knowledge and pedagogy, flexibility in terms of adjusting teaching approaches and adopting new technological tools, and openness in terms of learning technological tricks from students are essential. She described:

A teacher teaching with technology is open-minded and not afraid to try new things in the lessons. She is reflective. If something does not work, she does not use it again, or she re-works the existing approach. She is self-motivated and self-driven. It is not only instructional strategies that need to be adapted to new technological tools; it is also assessment strategies that need to be changed. A teacher integrating technology needs to be confident when allowing students to use technology during assessments. She is also constantly changing and adopting to his/her students' needs. Very importantly, she is knowledgeable in terms of content knowledge and pedagogy, but not afraid to learn from her students technological tricks. (Interview, June 20, 2012)

4.3.2 Technology Integration at Euclid Secondary School

According to Anna, the majority of teachers in the Math Department at Euclid S.S. were not integrating technology in their lessons: “A lot of my current colleagues are from the old school [of thinking]. And I know that three of them never touch any technology...Unfortunately only a couple of teachers know how to use TI-Nspire graphing calculators” (Interview, February 16, 2012).
At Euclid, Anna was a role model not only for her students but also for the teachers in the Department of Mathematics. She was well-known for delivering good lessons. Students talked about her lessons and her instruction techniques incorporating technology and teachers in her department were curious to learn what she did to make her students interested and willing to learn:

My colleagues started recognising some good things that came out from my class, and I think that my own students are saying that ‘she has good lessons.’ And good means that: ‘I was awake for most of the time, I actually learned something, and I did my homework’ in many cases. And that is when your colleagues start asking what you do differently that we are hearing from the kids that you have good lessons. (Interview, February 16, 2012)

Anna’s teaching strategies inspired other teachers from the Math Department to adopt technology in their lessons:

My colleague, just by mistake, walked into a class and we were doing regression [with the TI-Nspire graphing calculators] and I have a projection of what I was doing, and so she got inspired. So when she was doing quadratics, I taught her how to do regression with quadratics. (Interview, February 16, 2012)

Anna also presented herself as an enthusiastic “technology driven individual” and “the most advanced with [the] use of technology”. She was optimistic that new teacher graduates coming into this profession would be technologically literate, thus, motivating students to be interested and learn math: “They are advanced in terms of using technology... They can bring something inspiring and something good into our profession” (Interview, June 20, 2012). Anna hoped that, as the new Head of the Math Department at Euclid S.S., she would be able to make technological tools more available for teachers to use and to provide opportunities for teachers to present and attend presentations and workshops focusing on the integration of technology in teaching mathematics. She intended to invite them to discuss and reflect on their current practices with the hope that she would be able to move them towards embedding more
technology in their teaching practice. Her desire was to motivate teachers to adjust their teaching strategies and techniques to actively engage more students in the teaching and learning process:

I was able to purchase 50 new TI-Nspire calculators. I was given special funding for that. I was able to organize, toward the end of the semester, to have two day workshops on how to use TI-Nspire, and I was able to install the software for the Smart Board. So I am just hoping that people will get inspired and will start thinking about delivering the concepts in a more interactive way. So there is a lot to be accomplished in my department. (Interview June 20, 2012)

The technological tools mostly used in Anna’s class were Interactive Whiteboard (Smart Board), TI-Nspire graphing calculators, and the Geometer’s Sketchpad software.

4.3.2.1 Interactive Whiteboard, TI-Nspire Graphing Calculators

In terms of technological tools, Anna tried to incorporate the tools that were available in the school, such as the Smart Board installed in her class, the TI-Nspire graphing calculators available every day on students’ desks, the computers in the computer labs to run Gizmos activities, and Geometer’s Sketchpad (GSP). She also encouraged students to bring their own technological gadgets, such as smartphones and iPods, into the classroom. Anna was determined to train students to become familiar with using their favourite tech devices for mathematics. She explained:

I strongly believe in the new smartphones and the new smart technology. Students could use them as calculators, to look up information on the web, to graph, and play educational games. I want my students to use their favorite tech devices tools for mathematics. These technologies captivate students’ interest in the math course, motivate and engage them in completing homework, and help them increase their confidence about their math abilities. It is something that must be more visible in all classrooms; otherwise we lose these kids in the next 10 years, completely. (Interview, February 16, 2012)

Anna criticized what she felt was the inappropriate use of PowerPoint in her school. She provided the example of some teachers who used the tool to display lesson plans:

A couple of my colleagues, they have PowerPoint [presentations]. Every lesson is on a PowerPoint, Smart Board, but it is only [an] expensive projection white screen to deliver a lesson. There is no interaction. Teachers present math ideas as bullet points with no
room for critical thinking. Monotone slides encourage passivity and disengage students. (Interview, February 16, 2012)

As a counterexample to this use of technology, Anna illustrated how she used Geometer’s Sketchpad to teach transformations of functions. She explained how this instrument enabled students to perform transformations and to see the changes in the graph instantly. In her opinion, the possibility to manipulate the graph of the function and to associate the visual representation with the equation of the transformation added dynamism to the lesson, stimulated students to take part in the learning process, enhanced critical thinking, nurtured students’ dialogue, and created opportunities for students to apply and communicate mathematics. She explained:

But when you have kids doing things with you, and you tell them that this is the moment when we all get into our Geometer’s Sketchpad, or you have your own pre-made sketches and you tell them to open them up; I was doing the transformations in parabolas so I said, ‘Pull the parabola to the right. What is happening to the equation? Pull the parabola to the left.’ So that is when you get your interaction. You make students think, debate, apply, and talk mathematics. That is when you get that teaching moment. (Interview, February 16, 2012)

When asked whether she used the Smart Board to promote student- or teacher-centred instruction, Anna explained that she utilised the Smart Board fairly efficiently and even on occasions when she stood in front of the Smart Board to deliver the lesson, she managed to move back and forth from teacher centre to student centre through questioning and dialogue techniques. She said:

Smart Board allows you to visualize and it allows you to be in front of a class and to show them what is happening. And then you have a discussion and the kids are telling me to not let the cat catch the mouse. And I say, “Okay, how do I do that?” And they tell you how to: change the slope, change the speed. This is the perfect example that when you bring something in that moves and they can interact with it. It does not have to be that the kid comes to the board and touches it. It does not have to be that way. (Interview, February 16, 2012)

In one lesson that I observed entitled “Introduction to Point of Intersection: Modelling Linear Systems”, Anna used the Smart Board, in conjunction with a Gizmo activity, to engage
grade 9 students to simulate and graph two lines that represented the cat-and-mouse chase and then to determine their point of intersection. This activity was designed to connect real world slope, y-intercept, and the intersection of lines. Through experimentation, the students figured out how to adjust the speeds of the cat and mouse in order to make their motion lines intersect at different points: at the beginning, in the middle, and at the end of their chase. Students saw the effects on the graph and on the chase immediately.

By using simulations, students learned how to change the speeds to make the lines intersect at different points on their trajectories. They also practiced the skills of reading and interpreting data on the graph. For example, Anna asked students to discuss and figure out the meaning of the x-axis (how long each animal had been running) and the y-axis (the distance of each animal from where the cat started) for each animals’ movement, and to explain the meaning of the slope of each line in this situation (the constant running speed for each animal). When given different speeds for the cat and the mouse, Anna encouraged students to think critically and to answer high level thinking questions, such as ‘Does the cat catch the mouse? (e.g. It did not if the two animals were assigned equal speeds - parallel lines)’ and ‘How far does the cat run before catching the mouse?’.

In this interesting lesson, Anna used an interactive Gizmo activity to simulate the constant movement of two animals, to connect the meaning of the slope of a line to real life situations, and to interpret the meaning of x-intercept, y-intercept, and the point of interception. Although the students answered questions from their desks and the teacher manipulated the lines on the Smart Board, the interactive nature of this simulation encouraged them to maintain a high level of motivation in learning and engaged them in creating meanings and constructing new knowledge. It also fostered the development of mathematical communication skills, as students
engaged in dialogue with their peers and the teacher in order to operate and interpret the simulations from their desks (Field notes, May 9, 2012).

4.3.2.2 The Importance of Technology

Anna felt that the integration of technology in mathematics was important because teachers could better connect with their students. From her point of view, students were proficient in using various technological devices and software, and this proficiency was something to be utilised and brought into the classroom to help with the teaching and learning process. Students could build new understandings by using familiar tools and by adopting technological instruments that they used in their everyday lives. They preferred to learn in interactive and dynamic environments, to conduct investigations and explorations, or to use computer simulations that enhanced their learning experiences. She explained:

It is the power of dynamic software that you can change things instantaneously and see the impact right away. So when you get kids engaged in a discussion of what is a point [of] intersection? And where is it? Where can you get it? I think that technology allows you to connect with the students and come up with very good ideas from real life about how to present the concept, with interesting simulations, investigations, and exploration. (Interview, June 20, 2012)

Anna argued that the visual representations of mathematical ideas made the lesson attractive and interesting, aroused students’ curiosity, engaged them in discussions, and helped them to recall information, attribute meaning, think critically, and construct mathematical knowledge. In her first interview, she described how she encouraged students to explore the Pythagorean relation by constructing different regular shapes along the sides of a right angle triangle. By using the GSP, the students measured the areas of the figures drawn along the legs and the hypotenuse, and then checked that the sum of the areas of the figures drawn on the two legs was equal to the sum of the area of the figure drawn on the hypotenuse:

When you have that questioning and discussion, and I can tell you grade 9 Applied Pythagorean theorem. Bring a technology and ask them, “What would happen if I
construct a different shape above each side of a right angle triangle? Do you think that it will work?” And that is what we did, right? We did equilateral triangles because it was easy. And with the geometrical sketchpad you do it 1, 2, 3, and it takes you five minutes and then measure the areas (1, 2, and 3). It works. How is it possible? So it is not only to present a concept, but to truly take it a step further: make them think, make it interesting. I think that is when you bring that interesting moment in a class and get that attention that you should have. Truly, honestly, I think that the – and it is my experience – that the kids will kind of connect Pythagorean to that moment when…oh we constructed squares along the sides of the triangle, or equilateral triangles, or semi-circle, or a different shape, and we measured their areas. And it worked...Students are curious. They engage in the lesson and think. They will make the connection and they will learn the concepts. I think that that is more important. (Interview, February 16, 2012)

According to Anna, a significant reason that technology was an important component in the grade 9 Applied level course is that it helped capture students’ interest and attention for a longer period of time. She maintained that the integration of technology engaged students with attractive activities and investigations relevant to their everyday life, thus providing rich and unforgettable learning experiences. This aspect kept students thinking and attempting to solve math problems and increased their perseverance for long time (Field notes, May 8, 2012).

Anna believed that, while technology helped students get fast answers to their problems, it could not verify and validate their answers or check for their correctness and meaningfulness. She mentioned that “technology is not a replacement of the knowledge” and students should be in charge of checking the correctness of the answers by using any technological tool (Interview, February 16, 2012). When using technology, students should be able to reason their solutions and justify their answers to the math problems. They should recognize that technology can be problematic and that, when used inappropriately, it could lead to incorrect answers or it might provide incomplete solutions. She continued:

Students are using a good algorithm – in terms of pushing the buttons – but the answer does not seem right. They have to have the estimation process and [be] asking themselves a question of whether or not it is possible to get an answer a certain way. It is not a 100% correct answer all of the time…because it might have another answer as well or it is incorrect due to a wrong mode. For example, sine of 45 degrees is seen as of sine of 135
degrees. So if you put in 0.707 and the calculator give[s] you a 45 degree angle. Is it true? Partially. (Interview Feb 16, 2012)

From Anna’s point of view, one large disadvantage of using technology was that students relied on it too much. She indicated that many students took the answers obtained from using technology, assumed that they were correct, and they did not attempt to make sense of them or to check if they were accurate and valid: “Even though I keep telling them [the students] that reasoning is an important part of mathematics, somehow they just believe the calculator because that is it. Heavy reliance would be a disadvantage” (Interview February 16, 2012).

Anna believed that technology helped students understand difficult math concepts in fun ways. This helped students learn and remember math ideas. For example, by simulating the movement of two animals, a mouse and a cat, and manipulating their corresponding speeds that mathematically are the slopes of the lines, students were able to understand the changes on the graph and associate them with the concept of slope (e.g. the mouse running faster meant that the line was steeper) and point of intersection. Anna explained:

This is the beauty of using technology: that you can outsmart your kids in terms of teaching something difficult. At the beginning, the focus was moving the mouse closer to the cat, make the cat run faster. As you could see from the answers, they [students] knew what had to be done. So through fun we looked at the lines and the equations and point of intersection. The idea is that they had told you that they got the concept: the mouse running faster means that the line will be steeper. I think that is when you get that spark in your teaching/learning, when you bring something that kids enjoy and you see that the learning is happening whether they realise it or not. (Interview, June 20, 2012)

4.3.3 Student Tasks

Anna’s philosophy about teaching mathematics was that “what we teach in mathematics must be useable outside of the classroom, in real life” (interview, February 16, 2012). She designed her lessons to instill the elements needed to captivate students’ interest and to engage them in the process of learning. She worked to make the content relevant to students’ everyday lives and to relate it to realistic situations. Thus, Anna planned and implemented a grade 9
Applied level student task that connected various representations of linear relations and helped bridge conceptual understandings of the basic elements of data management and characteristics of linear relations.

The students, working in groups of three or four, were given 10 Styrofoam cups with a lid. They were asked to measure the vertical height of one cup, then to measure the heights of a stack of 2 cups, a stack of 3 cups, and so on. Anna encouraged students to be extremely careful in attaining a high degree of accuracy and asked them to record their results in a table. After measuring their stacks, the students used the TI-Nspire graphing calculators to graph the points (x, y), where x was the number of cups in the stack and y was the height. Then, students answered related high order thinking questions such as: Determine the maximum number of cups that can be put in a stack under your table; If you have 10 stacks of cups, each stack about 3 feet long, about how many cups you have? (Students handout: Stacking Cups).

During my observations, I noticed that some students missed the point that each time a new cup was added, the height increased by the height of the lip. To help them remain focus on the context and obtain precise graphs, Anna used the Smart Board and the TI-Nspire emulator to invite students to think about the process and whether their numbers made sense, and to check their graphing and their measurements. She conducted a class discussion about how to write the equation for this linear relation and the geometric meaning of the slope and the y-intercept of a line in this context. Then students used the TI-Nspire to construct the graph, to determine the equation for three more different size cups, and to compare these graphs to the original cup graph (Field notes, April 19, 2012).

In this task, students had a real world situation to explore and used technology to identify that the height of the stack and the number of cups in the stack represented a linear relation.
Discussions about the geometric meaning of the slope and y-intercept of a line, as well as its contextual meaning, helped students connect the abstract representation to real life. Significant aspects of the role of technology in this lesson included the possibility: to display accurate graphs, to determine instantly the equation of the line of best fit, to create multiple representations of the linear relation (a table of values, the equation, and the graph), and to provide opportunities for students to compare different graphs in terms of their rates of change and y-intercepts. Technology made this lesson time efficient. Students did not waste time graphing the data but instead analysed and interpreted data drawn from a real life situation, made connections among various representations of linear relations, and answered related questions.

Another student task incorporating technology that Anna implemented during my observations was from the TIPS4RM and was entitled “The Sum of the Interior Angles in a Polygon”. Anna’s goal with this student-centred activity was to facilitate the investigation of the sum of the angles in a polygon by utilising the Geometer’s Sketchpad software. In addition, the intention of this lesson was to help students realize the interconnection between the concepts learned in the Linear Relations Unit and those studied in the Geometry Unit. By integrating GSP in this lesson, Anna enhanced her teaching by improving visualization, favoring the construction of knowledge through pattern discovery, and providing opportunities for high level thinking.

Students worked independently to discover the general formula for the sum of interior angles in a polygon with n sides. They felt proud and happy when they discovered the geometric formula through explorations with Geometer’s Sketchpad (GSP) (Field notes, June 1, 2012). Anna’s role in this lesson was to help the students in their investigations by answering questions that “might be about the tool a student is using, might be about the concept they are trying to learn, might be about how to correct a mistake [students] made, and so on” (Interview February
16, 2012). Anna was proficient in this geometric software and quickly figured out her students’ construction and conceptual mistakes, and addressed errors in her students’ thinking.

In the second part of the lesson, Anna moved to the Smart Board and, through questioning and discussions, led students to explore and discover the linear relation between the number of sides of a polygon and the sum of its interior angles. Students used the TI-Nspire graphing calculator to input the ordered pairs \((x, y)\) where \(x\) is the number of sides of a polygon and \(y\) is the sum of its interior angles) in a table of values and to graph the relation in order to check their findings that this relation was linear. She described:

> So what I did with that lesson, I took that lesson – sum of interior angles in polygons – and I put it into a linear model. So what they had to do was organise the data in a table of values and come up with first differences. At that point (we [had] finished the Linear Unit), the first differences are the same so the relationship has to be linear. I asked them to come up with a graph and they expected to have a straight line and they did get a straight line. (Interview, June 20, 2012)

Then, Anna asked to determine the equation of the line that described the linear relationship between the number of sides and the sum of the interior angles in a polygon. Anna explained that students came up with the equation although it was not an easy task:

> So now the equation, that was a difficult part because we do not have an initial value, we do not have zero, and whatever angle. Yet kids were able to come up with the equation by observing the pattern. So if I have a triangle it is 180 [degrees], if I have a quadrilateral I can split it into two triangles and add them together. They have discovered that two less the number of sides multiplied by 180 is the sum. (Interview, June 20, 2012)

As a direct application to the newly introduced concept, the students were invited to predict, determine, and then check by measuring with GSP the sum of interior angles of a decagon. For this task, Anna used two different technological tools, the Geometer’s Sketchpad (GSP) and the TI-Nspire graphing calculators, to explore and deepen students’ understanding of the relationship between the number of sides of a polygon and the sum of its interior angles. Her
hope was to reinforce the fact that this relation is linear and to show students different ways of determining the general formula for the sum of the interior angles in a polygon.

During this lesson, the use of technology made the learning process more effective and efficient, as Anna used different strategies and tools to introduce the concept of the sum of interior angles in a polygon and students explored, discovered, and applied the requested formula using two different techniques. Anna organized the investigations in such a way to show students two different methods of generating the general formula for the sum of the interior angles in a polygon, and to help them recognise the connections among the mathematical ideas and strands: “So I wanted to make a connection between geometry and linear relations, and the algebra and the equation. I do not like teaching units in isolation because they are all interconnected somehow” (interview, June 20, 2012).

4.3.4 Knowledge Construction

4.3.4.1 Challenging Preconceived Notions and Misconceptions

Anna used technology to correct students’ misconceptions and misunderstandings of mathematical concepts. She explained that the instantaneous feedback provided by technology showed her what students were having trouble with and helped her to correct misconstrued concepts:

…when we went to a [computer] lab and I asked [students] to complete a five question assessment. After a very brief lesson, I got the answers from the assessment instantaneously. So I knew exactly whether I need[ed] to go back to the lesson or correct misconstrued concepts or just get the kid on the right track. So if you have a technology and you have a very brief assessment after the lesson you know right away whether they are learning or not. (Interview, June 20, 2012)

Nevertheless, when using technology, Anna encouraged students to apply their own knowledge in order to justify their answers mathematically and to avoid relying only on the visual-spatial aspects of mathematics. She explained that many students based their answers on
what they saw and, in some situations, the visual distortion misled them to provide the correct answer. For example, students looked at two lines and declared that they were parallel because “they look like parallel” (Interview, February 16, 2012). With the help of technology, Anna determined the slopes instantly and proved the students wrong by evidence that the slopes were not equal and that implicitly the lines were not parallel. She mentioned:

Now technology allows you to instantly measure the features, the lengths, determine the slopes and y-intercepts. I can see that these lines are parallel. Are they truly parallel or is it just a visual distortion that they look like parallel? It is not enough to tell the kids that the lines are parallel because they look like [they are] parallel. I think that you do go beyond the visual representation and determine the slope to prove that they are equal. (Interview, February 16, 2012)

Anna believed in the importance of making students realize that precision is very significant in mathematics. Technology helped her to demonstrate this concept to them. She described a situation where students insisted in believing that a quadrilateral was a rectangle. To correct this misconception, she made the length of one of the parallel sides slightly longer. After measuring the lengths of the sides with the Geometer’s Sketchpad (GSP), students realized that the sides had different lengths. Based on this information, students concluded that the figure was a quadrilateral and not a rectangle. In this occasion, Anna reinforced the concept of perpendicularity. She engaged students to determine the slopes of the two lines the students believed were perpendicular in order to check that the product of their slopes is -1, which is a necessary and sufficient condition of the perpendicularity of two lines. Because the slopes were not negative reciprocal, the students concluded that the sides were not perpendicular and the angles they formed were not exactly 90 degrees. Anna described the situation:

I made a rectangle and I made it quadrilateral and I ask[ed], ‘What is it?’ So they said it was a rectangle. Grade 9 Applied, and I asked, ‘why?’ They said, ‘Because it has four sides.’ And that is it? They said, ‘Two and two are the same length’. And I said, ‘How do you know that they are the same length?’ So just the slight difference in measurements and you can prove to them. And I think that that also comes to a rich discussion. I reinforced the perpendicularity concept: two perpendicular lines have negative reciprocal
slopes. When they determined the slopes of the two perpendicular lines they were not negative reciprocal. When they measured the angles they were not both 90 degrees. And 90 degree angle and 89 degree angle are two different angles. Almost the same, but not the same. I think that that is another concept that we are trying to get across to these kids. To be exact and ‘almost’ in mathematics makes a huge difference. (Interview February 16, 2012)

4.3.4.2 Student Interests and Increased Motivation for Learning

Anna integrated technology in order to plan activities inspired from the real world and to make them relevant to her students’ lives. She aimed to foster their curiosity and get them interested in what they were learning. Thus, she encouraged students to look around everywhere they went and to try to make connections to the math concepts learned in school:

So what I wanted to really, really accomplish with that activity is when a kid is looking around and seeing a slant line or a roof of a house, they can say, ‘That can be modelled by a linear relation’. So I wanted them to make a connection [that] whatever they learned in class exists outside of a class in real life situations (interview, June 20, 2012).

An example of an activity that tapped into students’ interest occurred in one of the linear relations lessons that I observed. By using the Smart Board and TI-Nspire emulator, Anna modelled how to create a “cat whiskers” picture by using only five lines. Then, she asked students to create pictures on their own respecting the same condition, using only five lines. Anna felt that this activity involved high order thinking skills because: ”the grade 9 Applied kid has to have a very clear understanding of slope (which is rate of change) and y-intercept in order to create a simple picture using only five different lines… and they should have to recreate the picture mathematically “(Interview, June 20, 2012).

In addition, students needed to know how to restrict the domain and range to make their pictures resemble the originals. Students were excited to perform this “work of art” and they considered it “fun and cool”. One student mentioned that he would like to be an engineer who designs a building with technology (Field notes, May 8, 2012). This activity reinforced Anna’s philosophy about teaching mathematics: “What is taught in the class must be usable outside of
the classroom otherwise it [mathematics] is a useless subject” (interview, February 16, 2012). Additionally, engaging learning through technology with this activity showed an increase in students’ interest and retention, as it tapped into their interests, creativity and curiosity.

4.3.4.3 Experiences, Experimentations, and Negotiations of Meaning

Integrating technology into teaching helped Anna to make course material easily accessible, create engaging lessons, and provide a large variety of experiences for students. Grade 9 Applied level students had the opportunity to explore real world situations in order to construct the concept of linear relations, to make connections among them, and to apply them in solving complex problem solving situations that required higher order thinking skills.

When asked to provide an example of an activity or a lesson in which she used technology to facilitate knowledge and meaning construction, Anna described a task that allowed students to investigate the properties of regression lines and correlation. Thus, she asked students to measure their arm span (wingspan) and height. She collected the measurements from her students and incorporated the data into a set compiled over time from her previous years’ students. Students predicted if there was a relationship between height and wingspan and then used this set of data and the TI-Nspire graphing calculator to check their prediction, to create a scatter plot of height and wingspan data, to draw the line of best fit, to determine the linear regression equation, and to analyse the correlation. Anna liked to use the TI-Nspire to teach correlations because its features permitted students to clearly see the relationship between the variables:

…they [students] were able to put it [the set of data] into a TI-Nspire. They had instructions on how to come up with the scatter plot and what is really, really nice about TI-Nspire is when I used [the] statistic and data feature, the scatter plot has really big dots and it is really visible so you can see whether [there] is some kind of a correlation or is it a linear or do they form a line, do they form a curve. (Interview, June 20, 2012)
Students used the extrapolation or interpolation method to predict the height for a given arm span (wingspan) or vice versa: to predict the arm span (wingspan) for a given height. They discussed the correspondence between data sets and their graphical representations, and interpreted the slope in this context. Anna had students consider what would happen to the line of best fit if athlete Michael Phelps’ height and arm span were added to the dataset. Then, without changing the best fit line on the scatter plot, she added Michael’s data point and asked students to discuss where Michael’s data point was located on the scatter plot. Even more discussion followed about whether or not the scatter plot represented the relationship between height and arm span in the general population, and whether or not Michael Phelps was like them or different.

According to Anna, this activity captivated students’ attention and interest because it was related to real life, involved real people, and required the use of technology. In this circumstance, technology assisted students to draw fast and accurate graphical representations of this set of data, made an easy manipulation of the data possible, showed immediate transformations and changes, and provided opportunities for significant discussions that helped students negotiate meaning and construct new knowledge (Field notes, May 21, 2012).

4.3.4.4 Increasing the Complexity of the Content

Anna believed that technology helped students develop the skills and knowledge necessary to understand and perform complex tasks. The dynamic nature of technology, the possibility to manipulate lines and graphs, and the ability to demonstrate concepts and mathematical ideas visually added value to the teaching and learning process. For example, Anna explained that Gizmos contained interactive lessons that allowed students to investigate the point of intersection of two lines, to connect the lines to real things - such as animals in motion (e.g.
the cat and the mouse run at the same speed, the cat and the mouse started at the same location), and, consequently, to better understand abstract concepts, such as the rate of change and initial value.

Anna explained that, by using technology, students constructed knowledge and meaning by looking at concrete real world situations. They saw how mathematics applies to real life situations and were capable of making connections and transferring their real world understandings into understanding of complex abstract situations and mathematical models. In Anna’s opinion, when they were assisted by technology, students became confident and were more willing to attempt to solve math-related applications:

This is the beauty of using technology that you can outsmart your kids in terms of teaching something difficult…So through fun we looked at the lines and the equations and [the] point of intersection. The ideas that they had told you that they got the concept: the mouse running faster means that the line will be steeper …I think that this particular activity is excellent for making connections and building the confidence in kids’ learning because they gave you very smart reasons and they may not have realised what it meant mathematically at the same time but I was hoping that that connection would come later. And also students are more willing to try complex math problems once they understand the math concept involved. (Interview, June 20, 2012)

In addition, Anna acknowledged that, by integrating technology, she could explain complicated concepts and could encourage students to perform rich tasks. She believed that teaching should not focus on drill and practice exercises, which help develop basic skills, but rather it should emphasise problem-solving and interpretation of real world situations. In Anna’s opinion, technology helped students to compute easily and to reprezent graphs precisely. This allowed students to move into the interpretative domain of mathematics and to develop critical thinking. She asserted:

Is it really necessary to teach and drill our kids with long division in the elementary schools? No, and is it really necessary to drill kids with adding and subtracting fractions with current technology? No. So instead of me focussing on the dry drills I can bring the concepts and explain the concepts and tell that, ’Here is the technological tool that helps you with draw calculations and here is a task that I would like you to complete’. Instead
of four or five days spent on drills I can spend four days on enriched tasks that are related to real life situations that have meanings instead of teaching one half plus one third over and over again. I think that, with current technology, we need to make peace with ourselves that our kids may never know long division as good as we knew it, and our kids may not be able to add two fractions as fast as we did it. But they do not have to, they have tools that can help them with these drills and expand their thinking instead.

(Interview, June 20, 2012)

Anna was planning her lessons and the units around big ideas in order to help students connect key concepts and make sense of the course content. With a simple technology-based activity, she reviewed with her students many key concepts from the Linear Relations unit in a short time. For example, by graphing a line with the use of TI-Nspire, she asked students to identify the slope and y-intercept, to write the equation of the line, and then to discuss the type of the relationship (linear or non-linear) and the type variation (direct or partial).

From the teacher perspective, Anna described a few advantages of using technology (TI-Nspire) in the teaching of the Linear Relations unit such as the possibility to identify students’ misconstrued knowledge, to encourage them to read out the information from the graph, to be able to visually convey the meaning of the slope and y-intercept and to reinforce the type of the relationship by using the first differences. During the second interview, Anna explained how and why she used the graphing calculators to connect different concepts in the Linear Relations unit:

With TI-Nspire, the student is able to graph the line and it is very simple to ask questions: show me the y-intercept, pinpoint the location of the y-intercept. So that is the first step question and the second step question would be: what are the coordinates of that point? With the technology, a kid is able to show and to read out information from a graph. Recalling the table of values it is as simple as splitting the screen: is this relationship linear? So the kid would have to look at differences in y-values and we could have that discussion: look, the y-values go up by 2, so it has to be linear. (Interview, June 20, 2012)

Ana gave another example emphasising the way she identified students’ misconstrued knowledge:

You could ask if the slope was positive or negative. And right away there is a connection. If the kid would tell me that the slope was negative and the line is sloping upwards then there is something misconstrued in the knowledge. (Interview, June 20, 2012)
Anna allowed students to uncover the meaning of new concepts by providing learning opportunities through real life experiences or simulations. She stimulated students to communicate and think logically, to reason and justify their actions and solutions, and to make meaningful connections with previously learned concepts. For instance, during my observations, Anna used TI-Nspire graphing calculators to connect various representations of a linear relation and to facilitate investigations the parallelism and perpendicularity of straight lines.

So if the equation is given, all of the information is given in the equation. But the connection that they [students] can make if looking at the graph and reading all of that information from the graph and making a connection with initial value, (which is the y-intercept) and rate of change (which is the slope) and recalling the table of values and calculating first differences, and identifying the type of variation, direct or partial: it allows students to recall the key elements of the Linear Relations Unit and transfer knowledge and skills to meet new challenges, in this case, to investigate and introduce the parallelism and perpendicularity of straight lines. (Interview, June 20, 2012)

Anna felt that the advantage of using technology in this context was that it allowed for multiple representations at the same time, specifically visual representation (the graph), numerical representation (the table of values), and the algebraic representation (the equation of the line). This enhanced students’ abilities to make connections among symbolic, graphical, and numerical representations, to see the immediate impact of changes made in the slope and y-intercept, and to develop insights for constructing the concept of parallelism and perpendicularity. Anna explained that the immediate impact of a change made the use of technology in mathematical education a powerful learning tool:

When I ask them, ’What will happen to the line if the rate of change and initial value are changed?’ we could make these changes and right away we saw what type of changed the graph underwent. This is a powerful learning experience because whatever you change you see immediate impact on that line. Either it is steeper or less steep or it starts at the higher initial value, or to notice that parallel lines have [the] same slopes and perpendicular lines have negative reciprocal slopes. I think that is the power of technology that you make changes and you see immediate impact. (Interview, June 20, 2012)
Anna made connections between different strands, units, and math concepts because she wanted to expose students to a variety of ways of perceiving the same math ideas. She informed me that she “wanted to make a connection between Geometry and Linear Relations and Algebra and the equation”. She did not like “teaching units in isolation because they are all interconnected somehow” (interview, June 20, 2012). During my observations, I witnessed her teaching the sum of interior angles in polygons by connecting the Geometry and Linear Relations Units. I described this lesson under the Student Tasks section.

4.3.5 Students’ Mathematical Communication

Anna maintained that technology was a great tool for motivating and engaging students in the learning process. She explained that students learned through interaction with technology by making changes and seeing an instant impact, by simulating real life situations, by discussing scenarios relevant to their lives, and by exploring, manipulating, discovering, interpreting, and analyzing mathematical ideas. From Anna’s point of view, all these learning experiences fostered dialogue with peers and teachers, enhanced students’ interest to reason, debate, and engage in conversation and effective questioning techniques, and motivated students to work cooperatively to explain their understandings and present multiple perspectives and viewpoints in the field of mathematics.

Anna believes that math has its own language and specific vocabulary, which were important to know well before using technology. She believed that students “need to be able to talk math in order to use technology in math” (Interview, February 16, 2012). To support this statement, she provided a few examples. She explained that, in Excel or TI-Nspire to fill the first column in a table from one to one hundred, the students needed to recognize it as a sequence and put it in as a formula; to solve a polynomial equation by using an iPhone the students needed to know what a
polynomial is; and to plot a point in the Geometer’s Sketchpad (GSP) the students needed to know how to use the construction menu. She said:

If you are using a Geometer’s Sketchpad and you do not know what it means to plot a point…you cannot do that. If you are using an iPhone and you do not know what it means to solve a triangle…then you cannot do that. So you need to understand the features of any technological mathematical tool before you can use it. I think that every single technology forces mathematical vocabulary and understanding before you can do anything with it. (Interview February 16, 2012)

Anna used the Frayer Model on the Smart Board to provide students with the opportunity to use technology to share ideas, to use mathematical vocabulary, and to improve mathematical communication skills. The Frayer Model is a visual organiser that helps students clarify keywords and concepts. It is structured in such a way to help students activate prior knowledge, make connections among math ideas, provide examples and counterexamples, and think critically to find relationships between concepts. Anna explained:

With the technology, I do the Frayer Model quite often…So what I did, we had on every page of the Smart Notebook – I designed maybe 10 pages and each page had a Frayer Model and a definition, and they had to come up with an example and a counterexample. One kid did the definition, one kid came up and did the drawing (how it looks like), another kid came up and did the counterexample, and then one kid came up and connected the new concept with the concepts already learned. So I think that…that is communication. (Interview, February 16, 2012)

When asked whether the Frayer Model worked differently on a Smart Board than with a paper and pencil approach, Anna explained that students were more engaged and took more responsibility when the Smart Board was used because they knew that they would go to the board to present their part to the whole class:

When they know that it will be their turn…they take more responsibility for that…because everyone has to come up and everyone has to do something with that. I am a coach, I am sitting on the side and making sure that everyone will come up and say something. Involvement and the fact that they know that they have to do or say something is the big thing here. (Interview, February 16, 2012)
To help students enhance their math vocabulary and learn the proper meaning of math terminology, or to convey the meaning visually, Anna used the Smart Board to do matching activities. She mentioned that one way that technology benefited students in the matching activities, which prompt the development of mathematical communication skills, was the immediate feedback. As the Smart Board software made students aware of the wrong answers, they knew right away if something was mismatched and they could discuss and correct any misunderstanding. Anna explained: “The Smart Board would also allow you to do the matching. You have objects on the left and terms on the right and you match them up. Again it is dynamic and it might give you the immediate feedback if you mismatch” (Interview, June 20, 2012).

Anna was willing to learn more about how to integrate technology to help students increase their ability to communicate their actions or their understanding of the mathematical concepts. She informed me that her students were able to perform tasks by using technology (e.g. determine the coordinates of the point of intersection of two lines by using the TI-Inspire) but they found it difficult to explain the math behind their actions or to justify their solutions. She thought that these steps required good communication skills and the use of proper mathematical terminology. In her opinion, the improvement of these skills necessitated a long commitment from all teachers and the integration of technology could play a role in this:

I truly believe that students have to have proper vocabulary to successfully accomplish any activity using any technology. So what I am finding interesting here is that I would say, ‘Kids, grab these two lines using technology...find the point of intersection’, and they would actually plot the point of intersection with the TI-Nspire and they would actually come up with coordinates of the intersection. So they understand what they have to do, but I am finding it difficult when they have to repeat what they did by using specific vocabulary that they hear from me. So, with the technology, I would say, ‘Find the point of intersection and find the coordinates of the point of intersection,’ and the answer would come two and three. What is two and three? Instead of them repeating, ‘The point of intersection has coordinates of two and three,’ they skip the whole sentence and they just say two numbers. (Interview, June 20, 2012)
4.3.6 Summary

The case of Anna demonstrates that she integrated technology into her everyday practice to make the lessons attractive and interesting, to arouse students’ curiosity, to engage students in discussions about real life situations, to encourage students to think critically and construct mathematical knowledge, and to create opportunities for students to apply and communicate mathematics. Technology integration assisted Anna to design appropriate student tasks and assignments that took her philosophy about teaching mathematics and put it in practice: “what is taught in the class must be usable outside of the classroom?” (Interview, February 16, 2012).

In the context of the Linear Relations Unit, Anna’s mathematical activities can be summarised by three predominant actions while teaching with technology: analyse, graph, and interpret. In terms of instructional strategies, the integration of technology in Anna’s lessons helped connect various representations of linear relations (graphical, tabular, algebraic, and verbal descriptions) by using data drawn from a real life situation (stacking cups); interconnected units (Data Management and Linear Relations, as well as Geometry and Linear Relations); and promoted high level thinking and allowed students to move into the interpretative domain of mathematics.

Anna created a technology-based activity, Stacking Cups to invite students to explore the linear relationship between the height of a stack of Styrofoam cups and the number of cups in the stack, as well as to help students make connections among various representations of linear relations (graphical, tabular, algebraic, and verbal descriptions). The role of technology (TI-Nspire graphing calculators) in this task was to display accurate graphs, to instantly determine the equation of the line of best fit, and to create multiple representations (a table of values, the equation of the line of best fit, and the graph) of the linear relations. In addition, this activity
connected the Linear Relations Unit and the Data Management Unit, as well as helped bridge conceptual understandings of the basic elements of data management and characteristics of linear relations (e.g. rate of change, y-intercept). Anna favoured the construction of knowledge through pattern discovery and helped identify the characteristics of linear relations between the number of sides of a polygon and the sum of its interior angles with a student-centred activity. This activity was designed to facilitate the investigation of the sum of the angles in a polygon using a second approach utilising Geometer’s Sketchpad.

Anna integrated technology (TI-Nspire graphing calculators) to captivate her students’ attention with an activity related to real life. Students had the opportunity to explore the linear relations between height and arm span, to manipulate the data, to identify transformations and changes in the graphs, to make predictions by interpolation and extrapolation, and to engage in discussions in order to negotiate meaning and construct knowledge.

TI-Nspire graphing calculators and interactive Gizmos activities were the predominant technological tools used during Anna’s mathematical activities. In every lesson and mathematical activity that I observed, Anna used the Smart Board.

### Table 2. Anna’s Classroom Observation Log

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Technological Tools</th>
<th>Date and Time</th>
<th>Student Handout</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Linear and Non-Linear Relations Investigations (Day 1)</td>
<td>TI-Nspire Calculators GSP</td>
<td>April 19&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Stacking Cups</td>
</tr>
<tr>
<td>2. Linear and Non-Linear Relations Investigations (Day 2)</td>
<td>TI-Nspire Calculators GSP</td>
<td>April 20&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Linear and Non-Linear Investigations</td>
</tr>
<tr>
<td>3. Rate of Change (PPT)</td>
<td>Smart Board/Gizmos</td>
<td>April 23&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>Rate of Change</td>
</tr>
<tr>
<td>4. Distance-Time Graphs</td>
<td>Smart Board/Gizmos</td>
<td>May 4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Distance-Time Graph</td>
</tr>
<tr>
<td>5. Direct and Partial Variation (Day 1)</td>
<td>TI-Nspire Calculators Smart Board</td>
<td>May 7&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Where is the Connection?</td>
</tr>
<tr>
<td>6. Direct and Partial Variation (Day 2)</td>
<td>TI-Nspire Calculators Smart Board</td>
<td>May 8&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Create a picture using TI-Nspire</td>
</tr>
<tr>
<td>7. Introduction to the Point of Intersection</td>
<td>Smart Board Gizmos</td>
<td>May 9&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Modelling Linear Systems</td>
</tr>
</tbody>
</table>
4.4 **Socrates Secondary School**

Socrates Secondary School is located in a low income, multicultural neighbourhood in the Greater Toronto Area and has an ethnically diverse student population. There were approximately 900 students enrolled in the school and most students were taking mathematics courses offered at all levels: specifically, Essential, Applied, and Academic courses for the junior students (grade 9 and 10), and Locally Developed, College, and University courses for the senior students (grade 11 and 12). The school has a wide range of programs in the area of arts, science, and mathematics. The 2011-2012 EQAO results in the mathematics assessment of Socrates S.S.’ grade 9 students indicated that 61% of students at the Academic level and 32% of students at the Applied level achieved at or above the provincial standard, level 3 or 4 (EQAO website). The results showed a large increase in the students’ achievement compared to the previous years. The EQAO results from the past three years, indicating grade 9 Applied level students’ achievement at or above the provincial standard, were: 9%, 16%, and 32% (EQAO school report website).
Students experiencing difficulty with math received extra help every day at lunchtime in a well-organised classroom setting, supervised by a math teacher. There was also a remedial program running twice a week after school. The Math Department encourages students to write two math contests. The Canadian National Mathematics League Contest, available to grade 9 to grade 12 students, is held once a month from October to March. The Waterloo Mathematics and Computing Contests are offered for grade 9 to 11 students in February and April and for grade 12 students in April (University of Waterloo website).

4.4.1 The Case of Lee

Lee is a math teacher who had four years teaching experience at Socrates Secondary School. He had taught different math courses including: grade 9 and 10 Locally Developed, Applied, and Academic mathematics, grade eleven University Functions, and grade 12 Data Management. He also coached teams and facilitated a couple of school clubs. Lee had an Honours Bachelor of Science degree with a minor in Mathematics. He was interested in pursuing a Master of Education. Lee decided to become a math teacher because he liked mathematics and teaching, and he enjoyed playing and coaching sports:

Working with youth, being a participant in sports – not just playing but also coaching a bit – and I like math, I like doing math. I liked the ideas of science and learning science. I encompassed all of that with teaching. After first year [of university], I decided that that was what I was going to do and I did some volunteering and I really liked it and so I kept on going. (Interview, February 23, 2012)

Lee’s personal goals for teaching in the next five years were to have a leadership position in the school and to make technology one of his focuses. Technology kept Lee organised. In his opinion, modernising teaching practices by incorporating technology would make the school attractive to high achievers. Lee believed that students “are right now pretty in tune with technology”. They use technology and relate to it, “they buy into technology pretty quickly”, “they know how it helps”, and they can learn better from it. He explained:
If we put the money and invest it in technology and professional development so that teachers can learn how to use it, then I think that kids will be better off. That is the goal that I try and push for in the school. Just arm these kids with technology because that is what they are used to and what they relate to and I think they can learn better from that. (Interview, June 20, 2012)

Lee felt comfortable and competent integrating technology in his mathematics lessons. He described himself as a quick learner, who knew how to use software and numerous classroom technological instruments and tools, such as Smart Board, Mobi tablets, clickers, graphing calculators, smartphones, and Gizmos. He was in the process of exploring GeoGebra, Clips, and an online learning environment called Moodle. He had been “trying to incorporate more and more technology every term” and asserted:

I have been using technology after my first year [of teaching]. In first year, I was just trying to figure out how to teach – I still am. Once I got the in-class stuff figured out then I started adding more and more. And now that I feel more comfortable about a few things, I can add some more from there. (Interview, February 23, 2012)

Lee learned about innovative ways of teaching with technology by attending professional development sessions, offered by the school board, and by attending conferences. Moreover, Lee was an independent learner who taught himself much of what he knew about educational technology and its integration into mathematics lessons. He explained that, when he felt something was useful, then he would “take the time to figure it out and learn the ins and outs [of it]” (Interview, February 23, 2012).

When asked to reflect back and identify a teacher mentor who played an important role in modelling technology adoption for him, Lee discussed a science teacher from Socrates Secondary School. This teacher influenced Lee’s adoption of technology integration in his teaching practices:

He is the other guy that usually does technology. He is got Gizmos in all of his classes. He showed me what the Smart Board was and together we got the money to get the clickers. So bouncing ideas off of him and getting ideas and just talking about what would and what would not work in a class really helped me out. It is just getting other
teachers on board and having technology allies, I guess, [that] really got me going and got me pushed toward using technology. And then you see how good it is and then you just keep getting better at it. So I think that it is that initial push that [science teacher] gave me that was the most influential for sure (Interview, February 23, 2012).

Lee talked about how he took the time to figure out and learn something new about educational technology and about troubleshooting technological issues:

The best way to prepare for the troubleshooting is to just do it at home or during prep. If I am just new to something, I will go over all of the different things in the program that I am using, just so that I know where the button is and what I am expecting it to do. Definitely the best way to mitigate any problems is to prepare and have a dry run before. After two or three days, where you kind of get the hang of things, I do not really practice anymore. (Interview, June 20, 2012)

Lee discussed how he preferred to explore new technology prior to implementing it in the classroom:

I am comfortable as long as I know how to troubleshoot issues when they come up. I do not want to walk up in the classroom and have problems happening at the moment without a solution. I try to make sure that I know what I am doing first before I do it in the class. (Interview, February 23, 2012)

Lee also explained that when a technical problem arose during the lesson that he could not fix, he always had a backup prepared to use immediately:

I do have the odd problem in the classroom. If I am not able to fix it, like one day the clickers were not working and I could not fix them, I have an analog version. So I quickly grabbed my cards that say A, B, C, or D. I will have a backup for that particular instance. There was one time where the Smart Board was not recognising my computer. When that happens, I used my computer as the cue because I know what I am supposed to be covering and I just do it on the board. (Interview, February 23, 2012)

Lee mentioned that he could not rely on his math colleagues to fix a technological problem or discuss technological issues. When a technological problem arose, he either tried to get online help or he called the technical support lines:

I am the help. If anybody has software problems they come to me. If I need serious help, I look online and try to get online help. With clickers, I have called their helpline, which is in the States somewhere. I have done that once. Actually, it might have been Smart that I called. Other than that…no. I have someone who I bounce ideas off of about using
clickers or how to use clickers but never any help in terms of troubleshooting anything because that is my de facto role now. (Interview, June 20, 2012)

Lee strongly believed that teachers should have an intrinsic desire to learn technology in order to integrate it efficiently and successfully into mathematics lessons. They needed to be willing to invest time to learn and improve their technological knowledge and skills:

In order to use technology effectively, it has to be seamless. It cannot take five minutes to troubleshoot – it should have happened before. There are sometimes where things do not go as planned or things happen accidentally, such as the other day [when] I pressed the wrong button and I had a personal note that went on the entire screen and it was embarrassing. But I mean other than that you need that initial investment. You need to jump into it. You cannot just say that you are going to do it now. We have two Mobis and one is just kind of sitting there because no one else is willing to take the time to sit down and learn it. I have offered and they have said that they are going to sit down and do it but if you do not have that buy in then you cannot do it. It takes a certain type of person to adopt technology or to at least see the benefit. (Interview, May 3, 2012)

Lee remarked that teachers cannot learn to integrate technology if they are not interested. His opinion was grounded in his personal experiences of trying to coach other staff members about the integration of technology into teaching. He commented:

You cannot learn it if you are not interested. And I found that out by trying to teach other staff members about technology. If you are not interested and you are not willing to immerse yourself in the technology that you want to learn then you are not going to learn it to any degree that it will be successful. (Interview, May 3, 2012)

When asked to describe his perception about the qualities and knowledge of teachers who integrate technology efficiently and effectively in the mathematics class, Lee suggested that:

Teachers integrating technology have an intrinsic desire to immerse in the technology and to explore interactive and visual technological tools and teaching approaches; they have a good understanding of the curriculum, a solid knowledge base of the content areas, are familiar with student conceptual understanding and misconceptions. They are able to see the interplay between content knowledge, pedagogical knowledge and the knowledge of technology; they pay attention to students’ interest and engage them in learning mathematics, by providing authentic situations related to their lives and by helping them make the vital connection between mathematics and the world outside of school. (Interview, June 20, 2012)
4.4.2 Technology Integration at Socrates Secondary School

At Socrates Secondary School, the use of technology is supported and encouraged by the school administration and the Head of the Mathematics Department. As an example, Lee described the attitudes of the Principal and the Head of the Mathematics Department towards buying a new technological tool to be used in the mathematics classroom:

I am lucky in that the administration and [the Head of the Mathematics Department] are open to any kind of ideas that any of us have on teaching. And so I saw the [Mobi tablet] and I kind of told people about it and what the benefits are. Something like the Smart Board takes a heavy financial investment into it and this is more cost effective, and people are like ‘okay, let us give it a shot’. Provided that someone is willing to use it – the administration was adamant that they were willing to buy it but somebody has to use it and it should not be like a big paper weight….And I am lucky that there were people above me that controlled the money who were open to the idea. (Interview, May 3, 2012)

Lee pointed out that having the support from the administration to purchase new technological equipment for his math lessons, motivated and engaged him to learn how to use it:

And so I think that is probably another reason why I wanted to learn it because I did not want to be that kind of teacher that said I wanted a toy or piece of equipment and then just kind of having it lie around and being a waste. I was interested and I really wanted to try it. (Interview, May 3, 2012)

Referring to his fellow math teachers, Lee explained that they were “very keen on learning technology and incorporating it”. He explained:

I think in this department people are very keen on learning technology and incorporating it. I guess that because I am so into it that hopefully they will adopt it quicker. I am already doing it I think. Right? So hopefully teachers come and see that it is pretty easy and that it helps out. The buy-in is quicker. (Interview, June 20, 2012)

He described the particular situation of a math teacher in his department, who was new to the field of technology integration, and who had recently accessed and started to integrate clickers and the Smart Board into her math lessons:

The buy-in is quicker. But people are excited. Like [math colleague] here. We got her a digital projector so she is already using her computer and she is using clickers. She is now the third person in this school to use clickers and that is exciting. I think it is just going to grow more and more. (Interview, June 20, 2012)
However, Lee reported that it “is a little bit tougher” to get part-time math teachers on board with technology, as they belonged to different departments and had limited time to explore integrating technology into teaching. The availability of technological tools was also a factor that prevented the teachers from incorporating technology in the classroom:

They do not want to prepare a lesson on the Smart Board and then realise that it is not in [their] room and someone else is using it. And I would feel the same way. If I did not have the Mobi with me all of the time, I would be very hesitant to invest the time in preparing something if I did not have it. So the availability of technology is a big thing. (Interview, June 20, 2012)

According to Lee, some veteran math teachers were at a point where they were satisfied with their teaching approaches and were not particularly interested in learning new teaching approaches, including the integration of technology:

There are people that have been teaching for 20, 25 years and the way they have been teaching has worked for them and including technology is difficult for them because they are not savvy. (Interview, June 20, 2012)

However, among these senior math teachers, there were a few willing to incorporate technology in their teaching. Lee explained:

But actually it is funny. I had the most senior person in our school…she is asking for help with Microsoft Word so that she can format tests and stuff. I mean there is some interest in incorporating technology, which is a small part of technology. I do not know. Maybe the focus of technology is rubbing off. I do not know. (Interview, June 20, 2012)

Lee believed that school culture, especially the attitude of the administration and the Head of the Math Department, had a significant role in shaping how much and how technology was used in the classroom. In his opinion, the Department Head should be “on board and be familiar with technology so that everyone who works under them can get access to quick help” (Interview, February 23, 2012). From his point of view, technology implementation should be initiated and sustained from the top-down to be adopted and embraced by teachers on a large scale:
There definitely has to be a top-down and say, ‘This is what we are going to do. This is how technology is going to help us. This is how we are going to help you to learn how to get better at it, and we are going to make you accountable for it as well.’ It cannot just be a couple of people here and there doing it. (Interview, February 23, 2012)

To support his opinion that technology implementation in the classroom should be determined from the top-down, Lee explained that many teachers used graphing calculators because the Ontario curriculum called for teachers to use them in instructing and assessing students. He stated, “For instance the curriculum involves graphing calculators. They put it in and people do it” (interview, February 23, 2012).

4.4.2.1 Role of Technology in Mathematics Education

Lee used a few types of technology in his grade 9 mathematics lessons, such as the Smart Board, the Mobi tablet, Gizmos, clickers, TI-84 graphing calculators and calculator based ranger (CBR), Excel, and cell phones/smartphones. At the time of my research, he had recently started learning GeoGebra, an interactive geometry software, and was planning to incorporate it in his courses for the following year. In the grade 9 Linear Relations Unit, Lee used technology to quickly manipulate the parameters of the equation of the line, to create a dynamic learning environment, and to enhance his lessons with real life examples:

In terms of when you are graphing a linear relation, it [technology] helps to be able to do it [the graph] quickly and effectively. And using the interactive whiteboard, it makes it very clear. You can manipulate the line that you have just drawn so that when you are changing the conditions you can change the rate of change and move it up and down to change the initial point. When there are changes then your line changes with it, and there is that dynamic that comes with it that you would not normally get if you did it on a blackboard. And getting examples from the Internet of linear relations, like cellphone plans, that is quick and easy. (Interview, June 20, 2012)

The technological tools mostly used in Lee’s class were, Clickers, Smart Board, TI-Nspire graphing calculators, and the Geometer’s Sketchpad software.
4.4.2.2 Clickers

Lee explained that a student response system is composed of a set of handheld transmitters, a computer, and a projector. The computer has software installed that collected the students’ responses and produced a pie chart or a bar chart showing students’ answer choices. In Lee’s opinion, this system helped teachers make instructional decisions in response to students’ answers. Thus, he could invite students to discuss the submitted answers with a partner or in a small group to decide the most appropriate solution. He could also ask students to explain their thinking process, therefore, revealing misconceptions, or he could choose to provide explanations or additional examples to resolve misunderstandings.

Lee believed that clickers were designed to support mathematics teaching by engaging students and employing active learning in the classroom, as well as by providing real time feedback for the students and for the teacher monitoring students’ understanding. Lee explained that the opportunity to gather real time feedback on students’ understanding represented one of the most significant benefits of this technology. He found that clickers were great ways to get a general feel for how a class was doing while learning was taking place. I found further evidence of this belief when he described how clickers informed his teaching:

In terms of shaping the learning, I think that clickers do the most for me because if 95% of the kids get the concept then it is great and let us go on. Or if 25% of the kids get the concept then I know I have to do more on that. So that is the main thing that I find helps me out the most – is that kind of feedback. (Interview, February 23, 2012)

I asked Lee if the structure of his lessons was different when technology was integrated. In his opinion, clickers helped teachers tailor their lessons to address the needs of students in the class. He explained that clickers allowed teachers to prepare most questions in advance but they could also alter the pace and the nature of instruction by adding or removing questions if needed.
Lee discussed how the students’ instant responses had dramatically changed the course of instruction:

When you know that the kids do not get it then you kind of just have to say, ‘Alright, I had this planned but it is not going to work now, so let us just make sure that we know how to do this and we will do everything else tomorrow’ for instance. I find that, when I use the clickers to make sure that the kids are getting it, that the structure is very fluid and you have to go with the flow. (Interview, February 23, 2012)

Lee emphasised that clickers were very useful to gauge how students understood the mathematics concepts discussed. He found that:

…some of the best lessons have happened when students are kind of able to test if they know what they are doing and then being able to see if they know or if they do not know what they are doing. (Interview, February 23, 2012)

According to Lee, the fact that “every kid gets to participate” made clickers stand out as an instructional tool. The use of this technology in the math lessons provided all students an equal opportunity to engage in meaningful learning and to answer the questions without the “fear of getting it right or wrong”. Lee asserted:

[Students] do not have to stick out in front of a crowd. A lot of kids do not want to do that, a lot of students prefer to be a passive learner for instance... But with having that [clickers] you have mass participation. So that really changes the teaching strategy. Instead of having only one or two [students] answering, you get everybody involved and everybody feels like they are part of the class, as opposed to just sitting down. (Interview, February 23, 2012)

Lee felt that, in addition to helping knowledge construction, clickers represented a great technological tool to improve classroom management. He believed that students were engaged in providing an answer to the question because of the instant feedback. Lee explained, “[Students] get the feedback and they are interested in doing the question. They are sort of held to account. Everybody is doing it so sometimes they may feel like they have to submit an answer” (interview, May 3, 2012). Students’ participation and interest in doing the questions helped them “gain confidence in that small period of time, such as, ‘Oh, I can do this, I can do the question. I
know that I am getting it right’” (interview, May 3, 2012). As a consequence of students’ involvement in the learning process, classroom discipline improved: “the less time that I have to manage unwanted behaviour, and then the more time we actually have to learn something.” (Interview, May 3, 2012)

4.4.2.3 Interactive Whiteboard, Mobi View Tablet, and Gizmos

Lee used the interactive whiteboard (Smart Board) and the Mobi tablet on a regular basis in his mathematics courses. He connected the clickers to the Mobi tablet or to the Smart Board in order to receive valuable feedback from students. Lee felt that, “In order to be effective, you [the teacher] need to know where the kids are as you are going along” (Interview, February 23, 2012). He also used the Smart Board and the Mobi tablet in connection with Gizmos, an online interactive program, to explore and illustrate abstract math concepts, to provide real life simulations, and to get instant feedback from students.

When asked how technology had changed his teaching practice, Lee replied that his teaching strategies were very different with the integration of technology. In his initial teaching approach, which was predominately direct instruction, students had a passive role in the teaching and learning process. At the time of my research, his approach utilised technology, was student-centred, and promoted active student participation and engagement. Students were also empowered to take ownership of their learning:

I remember in my first year [teaching], I was doing the chalk talk. I was doing, kids were watching. Now getting the kids more involved, like using Gizmos for instance for feedback, using the clickers for feedback, having kids come up and use the Smart Board, it gets them doing more. The technology has helped in promoting kids doing and the students taking more ownership in what they are doing [as] opposed to me standing up front and delivering the information. (Interview, February 23, 2012)
Lee found that the most important benefit of using the Mobi View tablet instead of the Smart Board was his ability to move around the classroom and to facilitate learning and instruction from any point in the classroom:

Being able to move around in the classroom and be[ing] able to do that classroom management piece, while you are teaching at the same time, and not be confined or lost at a chalkboard or a Smart Board, I find that to be the number one benefit of that particular technology [Mobi]. Just being able to move around and manage a classroom and do something productive at the exact same time you can get more delivered. I think it is definitely successful and the kids enjoy it. I would be able to go on a website and seek information from somewhere that kids can see some real life benefit to it. (Interview, May 3, 2012)

Lee presented other advantages of using the Mobi View tablet or the interactive whiteboard in his lessons, such as the ability to take content from somewhere outside of the classroom and apply it spontaneously into the current lesson and the possibility of using multimedia to present the concepts effectively, respectively. He explained, “This is a powerful tool [that] raises students’ interest and enjoyment. This new dimension brought into the classroom empowers teachers to make the math content more visual and understandable and also relevant to students everyday life” (Fieldnotes, April 23, 2012).

In one of the grade 9 Applied lessons I observed, Lee taught the equation of the line, accessed an online website (www.Virginmobile.ca), and picked a cell phone plan for a BlackBerry and for a smartphone in order to discuss a situation relevant to the students’ lives. He asked students to identify how much it cost to start the plan (initial value), to determine the rate of change, and to create an equation to describe the provided plan. Students were advised to ignore the applicable taxes. Then, Lee asked students to calculate the cost of the plan for three years. Working on this real life question, students were interested and engaged, participated more actively, and paid more attention to the involved mathematics concepts (Fieldnotes, April 23, 2012).
Lee explained that one major disadvantage of the Mobi View tablet was that it was difficult to write with the tool. It required a lot of practice to be able to write with the Mobi tablet:

I found that, this is where a docu[ment] camera could really come into play, students have a hard time using the Mobi because it is writing but...it is different because you have to look up and you are writing down. So I find that when I try to give the kids the Mobi, it takes forever. That is why I am trying to get a docu[ment] camera so that they do not have to write using the Mobi. It is a skill. (Interview, May 3, 2012)

I asked Lee if he thought that learning performances increased as a direct result of using the Mobi View tablet and the clickers. He indicated that students performed better because these technological tools helped teachers closely monitor students’ understanding. Assisted by technology, teachers could adjust their teaching approaches to meet their students’ interests and needs. Additionally, teachers could add more examples to their preplanned lessons. They could explain the concepts differently to help students understand the concepts better or to clarify any misunderstandings. Teachers could also access real life examples to support their lesson presentation and to make the mathematics content more appealing to their students’ lives. Lee explained:

I think that where clickers come in is, when I throw in a question and I see that only 50 % of the kids get it right, then I know that I need to do some more questions. I can quickly use the Mobi and write down a new question. Debrief that question of ‘what should I have done/what the misconceptions are’. Now that you have learned that let us try another one. And then you just try it again and again. You get that feedback right away until you are satisfied with the amount of students that get the right answer. If I did not have that instant feedback and that total feedback – like not just from one or two students but I can get it from all the students – if I did not have that technology to find out that everybody gets it or not, then I would just move on assuming that because one or two students have answered it right, then I get that false belief that everyone gets it and I can just move on. I think that improves the whole student learning process and performance as well. (Interview, May 3, 2012)

In Lee’s class, technology was used to address different learning styles and skill levels. For example, Lee used Gizmos to promote student-centred learning. He encouraged students to
explore, investigate, and learn math ideas at their own pace and level: “In terms of Gizmos they [students] are the ones that are performing the task, they are the ones that are taking the information and understanding it in their own way as they are using the program” (Interview, February 23, 2012).

For students who learn better visually, Lee used the Smart Board or the Mobi View tablet. Both provided visual tools that helped students to manipulate and interact with the math content, to break it down into basic parts, and to deal with each part separately. The ability to touch and point to images made the abstract math content more concrete and tangible to students. He expressed:

Like a Smart Board for instance…understanding composite figures. You need to find the area of this vastly complicated shape to be able to have that basis of breaking down the shape and understanding the difference between adding and subtracting it…With that Smart Board, I think that some students, not all of them got to do it, but I am sure a lot of them enjoyed breaking up the shape so that you could see all of the different parts and know that this is the strategy that you need to use to find the area. (Interview, February 23, 2012)

The colorful visual representations helped students to connect the symbolic and visual representations of particular abstract mathematics concepts, such as the rate of change and the initial value (Fieldnotes, April 26, 2012). Lee integrated technology into his math lessons to also meet the needs of kinaesthetic learners who learn better by doing. Simulation and matching graphs was one strategy that Lee engaged to help these students to explore and understand the distance-time graphs. To complete this activity, students worked in pairs and used the calculator based ranger (CBR), graphing calculators, and the view screen on the Smart Board. Lee believed that this activity facilitated students’ conceptual understanding of the distance-time graph and provided opportunities for students to see a direct applicability of mathematics in real life. He said:
I think that using the Gizmo and using the CBRs on the graphing calculators really helps out with students’ understanding of the connection between the shape and position of the graph and the direction, speed (including stopped), and starting position of their walk. I found out that with the grades nines that, especially last semester, they all understood that part which was really surprising. And doing that activity the kids were always referring back to walking towards and walking away from the machine. It also helped students see the applicability of mathematics to realistic situations. For sure that helped. (Interview, February 23, 2012)

According to Lee, this activity promoted mathematical communication. Lee encouraged students to use adequate mathematical terminology by designing tasks that required students to read the graphs and to provide walking instructions to a partner who could not see the graph. To foster and reinforce the use of an appropriate mathematical vocabulary, Lee asked students to predict what they thought the graph would look like before each walk. In their predictions students communicated their understanding of the various parts of the graph using appropriate terminology (Fieldnotes, February 23, 2012).

I observed that, when students had alternative ways to solve mathematics problems, relying on technology made it easier to present their solutions to their peers (Fieldnotes, May 2, 2012). During the lesson entitled “Connecting Various Representations of Linear Relations”, I noticed that some students solved problems provided on the handout by using the table of values, others by graphing, and others by using the equation of the line. Eventually, they all agree to have the same final answer. Students came to the same conclusion using different approaches. Technology helped students to efficiently and accurately present their different methods of solving the problem and they saw that math questions can be solved in different many ways (Fieldnotes, May 2, 2012).

Through the authentic use of technology, Lee provided equal opportunities for students to participate into his lessons anonymously, to not be afraid to make mistakes, to collaborate, to think about the questions that arose in the class, and to provide an answer whether it was right or
wrong. He also helped students individually to clarify their misunderstandings and to clear up any confusion:

I think the anonymity helps that [students’ participation] to a great extent. Students who do not answer – the great thing about using the Mobi are that I can see who is answering and who is not, even though the students cannot see. They can see whether a number has submitted an answer or not but then students still have not caught on to linking a name to a number. So I can go around and see that so and so has not answered the question. I can go up to the student and ask if they have tried the question yet and ask them if they are confused about a certain concept. So that way, while I am addressing someone else, everyone else is still able to be productive and do some work and answer the question that they are working on. (Interview, May 3, 2012)

Lee found that technology provided opportunities to students to have a voice even when they were timid and reluctant for whatever reason to share their thinking and to contribute to the classroom discussion:

Technology is kind of an equaliser. For instance, with clickers everybody gets to answer. It does not matter whether you are quiet or the most outspoken student, you are on the same level at that point and you all get to answer in the same amount of time and you get that same kind of availability to answer a question. (Interview, February 23, 2012)

### 4.4.2.4 Moodle

Around the time of my research, Lee had been introduced to Moodle, an online learning environment, and he had started to implement it in his grade 11 University course. Lee described that this interactive online environment allowed teachers to post notes, to make announcements, to provide updates to students, to post assignments and online practice quizzes, to post tutorial videos for students to watch at home for extra help or to avoid missing an important concept when absent, and to download Gizmos activities or Clips lessons (Fieldnotes, June 20, 2012). The students “can take resources from it and post resources to it or send messages to peers and the teacher. They can discuss class topics on a forum if an engaging enough topic comes up” (interview, June 20, 2012).
The existence of this website minimises the number of excuses, such as "I forgot to bring it home" or "I lost it", and enables “a lot of kids who otherwise would have said that they did not have the notes or did not learn it to have it there” (interview, February 23, 2012). Lee also noted that Moodle provided opportunities for students to check their homework, to revise notes, and to be better prepared for the next math class.

Lee’s intention for the next school year was to expand the use of Moodle to all his courses, including the grade 9 Applied course. He considered this online environment a good way to practice EQAO multiple choice questions, in particular, because students received automated feedback. He stated:

The great thing about the quizzes on Moodle is that they can provide feedback. When someone chooses the wrong answer, you can put a line that says the answer is incorrect and then you can state the misconception/reason and why you think that they picked the answer that they did. You can provide a hint as well for the student and then they can try the question again. They are getting automated feedback but there’s still feedback nonetheless….And hopefully they can learn something from that, especially when they are getting the feedback by trying again and getting it, and maybe having two or three other questions that go on that same concept so that by the end of it they will get it. (Interview, June 20, 2012)

4.4.2.5 The Importance of Technology

Lee believed that technology helped struggling students move forward in their learning when they did not master the prerequisite skills required to build upon a new mathematics concept. He explained that, in order to advance in learning mathematics, teachers should focus on the main concepts. For example, when teaching the grade 9 Linear Relations Unit, he noted that teachers could allow students who struggled with basic skills to use calculators to perform addition and subtraction of fractions, and have students dedicate more time to the new introduced concepts. He emphasised:

Sometimes teachers get so bogged down on integers that they do not teach the big concepts. Like in grade 9, which is linear relations. I am in the camp that kids can really get used to integers as you go along, use the calculator for goodness sake right now, but
then let us actually learn lineal relations because those little details do not need to get in
the way of learning. So I think that technology really helps remove some of those barriers
that do not really need to be barriers (interview, February 23, 2012).

In addition, Lee recommended using technology to help teach efficiently in terms of time
management. To detail this idea, Lee provided an example from the grade 9 math curriculum:

In terms of graphing calculators, for instance, you can get the graph instantaneously
rather than spending time manually graphing it, which is really not the idea, but seeing
the shapes and seeing how graphs change for instance. The instantaneous aspect of
technology and the removal of tedious non-important math components, I think, really
makes the learning more focused and a little bit more direct. (Interview, February 2012)

Lee believed that technology was “most useful when you are trying to make the abstract
more tangible”. Referring to the Linear Relations Unit, he expressed that technology “helps to
animate the graphs, to show differences between lines with different rates of change or initial
values, to indicate different speeds” (June 30, 2012). He noted:

There are a lot of dynamic things that happen in math, like graphing or transforming
graphs – it moves. Using technology to animate that, I find, is something that helps me
for sure, and I am sure it helps kids who need to see that to make the kind of connections.
(Interview, February 23, 2012)

Lee perceived technology as a tool that promoted effective teaching. When discussing the
clickers used to provide instant feedback of students’ understanding during a new lesson, Lee
pointed out that these testing devices informed teachers and students whether the learning had
occurred:

In order to be effective, the teacher needs to know where the kids are as you are going
along. I think the clickers provide instant feedback. And I just read the other day that the
longer you wait for feedback the less effective it is; so if the kids right away know what
they are doing or do not know what they are doing then they are more inclined to fix what
they do not know. (Interview, February 23, 2012)

4.4.3 Student Tasks

Lee was dedicated to incorporating technology into the curriculum and to designing
learning tasks so that students viewed the Smart Board, the Mobi tablet, clickers, graphing
calculators, computers, and cell phones as learning tools. He focused on technology-enhanced learning tasks to address his students’ needs. By using the Internet, Lee found resources to differentiate instruction: to provide extra support material for struggling students and more challenging questions for high achieving students.

Lee strived to engage his students in the learning process by planning student tasks with content that was meaningful to them. Thus, he developed student tasks involving technology that were connected to the real world and that were relevant to his students’ lives. In the study of the equation of a line, Lee accessed an online website (www.Virginmobile.ca) and picked a cell phone plan for the BlackBerry and one for a smartphone. He asked students to complete a number of questions relating to the cell phone plans. Based on my observations of students’ performance on this task, “I believe that this is a powerful tool that raised students’ level of interest, engagement, and enjoyment. Students were motivated to work on this contextual content that was relevant to their area of interest, participated more actively, and paid more attention to understand and apply the equation of a line in this real life problem” (Fieldnotes, April 24, 2012).

The technology-assisted learning tasks prompted students to visualise ideas in order to help them develop a conceptual understanding. He explained:

I am talking about grade 9 Applied and Academic based on the lines and the distance-time graph. There are lots of tasks so that students can visualise the line and relate that line to the movement. I think that the visual helps with the conceptual understanding. I think that is what the big plus is with technology: is the visual aspect. (Interview, February 23, 2012)

Lee strived to ensure content accessibility to students by presenting the learning tasks in a clear and logical sequence. He used Microsoft Word and Excel to develop and print out worksheets for his students. With PowerPoint presentations, he organised, structured, and summarised the content of the lesson to make it more manageable for students. He also created
his lessons by using Smart Notebook software and the Smart Response Interactive Student Response System for the clickers. His intention for the next school year was to share these files with his grade 9 Applied level students to help them review the math content at home. Lee integrated technology when designing learning tasks because he could easily alter the tasks to make them meaningful and appropriate for his current students, and also because technology-based tasks were more appealing to students. He stated:

I like using the technology because it is a digital archive of the good stuff that you do and you can easily change it when you notice that things do not go quite as well as you want, then it is just a couple of backspaces and some typing and then it is a little bit better. I think that students are more willing to be involved when technology is included. I do not want it to sound like a novelty because that is not the role of technology. But I just find that students are anecdotal, are more eager to perform a task when they can push buttons or when they can touch technology somehow. I am not sure why that is. It is just what I have noticed. (Interview, February 23, 2012)

4.4.4 Knowledge Construction

When teaching for conceptual understanding, Lee incorporated technology to motivate students to investigate and explore math concepts and ideas, to present solutions to problems, to explain their thinking process, to analyse peers’ answers, to engage in dialogue, and to negotiate solutions to mathematical problems. He varied his teaching strategies to help students recall any prerequisites skills, to provide detailed and efficient explications, and to clarify any misconceptions. Lee used a few technological tools on a regular basis, including the student response system (clickers) in junction with the Mobi View tablet. Although Lee liked to encourage his students to show their work step-by-step when solving a math problem, he utilised the student response system with multiple choice, true and false, and short answer questions as a tool for learning to guide and inform his instruction.
4.4.4.1 Taking on the Role of Facilitator

Lee took on the role of a facilitator to guide students as they worked with learning tasks. He asserted that technology helped him facilitate learning by providing opportunities to students to look at different ways of solving a math problem, to communicate their various solutions to the math problems in an efficient manner, and to document and reason the validity of their solving methods. He expressed:

So I basically see who got the right answer or how many people got the right answer and then you open that up and say, ‘Someone who got the right answer, can you explain what you did?’ And then that happens and then you just invite someone else who might have done it a different way and someone might come up. I think if people see that ‘oh, 90% of us got it right, great’ but then acknowledge that there are multiple ways of getting it right. I think that helps with the math communication as well. And it is not always that there is only one way to do a question and if you do not do it that way then you are wrong. (Interview, May 3, 2012)

4.4.4.2 Challenging Preconceived Notions and Misconceptions

Lee helped students discover, understand, and clear up misunderstandings and misconceptions by asking questions that engaged students to provide reasons for or against their answer choices and by providing access to additional resource material. In one of the lessons I observed, Lee taught his students how to eliminate a common misconception encountered in the Linear Relations Unit, specifically the confusion between the slope of the tangent to the graph at a particular point in time and the height of the graph at that specific moment. For the graph below (Figure 2), Lee asked his grade 9 Applied students to read a provided graph and to find out who was going faster at 8:25, Maria or Marco.
Figure 2. Who is going faster at 8:25?

Among the answers, a few of the students indicated that Marco was going faster because the height of his graph was greater than the height of Maria’s at that point in time. Lee explained to the students that they were confusing the slope of the tangent to the graph at 8:25, which should have been calculated as rise/run, with the highest point on the graph at that particular moment.

To challenge students’ impulses to view the highest point on the graph as the greatest slope, and to reinforce the concept of slope, Lee used the Mobi View tablet to efficiently display a few more comparable graphs. By showing students similar examples, Lee stressed that on a distance-time graph, the height is not the significant graphical characteristic when finding the rate of change or the speed (Field notes, April 24, 2012). He explained to me that he used Gizmos and calculator based rangers in his lessons to help students “see what all of the different parameters do to the graph…they can see right away moving the sliders of the Gizmos. That was very
intuitive. And moving it up and down I think gives students some type of concrete concept of this idea” (Interview, February 23, 2012).

Lee provided students with opportunities to think abstractly about the linear graphs and to eliminate their inclination to view these graphs as the concrete picture of a situation. Often students described the graph in figure 2 as showing someone travelling up a hill, due east along the top of the hill, down the hill, due east along the road, up the hill again, and due east along the top of the second hill.

![Distance-Time Graph](image.png)

**Figure 3. Reading Distance-Time Graphs**
For the reverse question, when students were asked to graph an athlete’s trip around an oval track, they sometimes drew an oval picture of the track. To train his students to think and interpret these graphs more abstractly, Lee used Gizmos from explorelearning.com and calculator-based rangers to match students’ physical motion to the graph. Lee used technology to efficiently scatter plot data and draw the line of best fit, to have students focus on major concepts (such as analysing and interpreting data), and to make appropriate decisions for the given math problems. He said:

Excel mainly because it is so ubiquitous…every computer in the board has it and most computers in every workplace will have them or some version of it. And for whatever reason when students go home, or do some other project or maybe something else in whatever part of their life, when they have to create a graph I know that they will have access to Excel or some open source version of it. (Interview, June 20, 2012)

Lee expressed that students needed to be proficient and well-equipped with interpretative math skills rather than basic skills, such as creating graphs, because technology could efficiently and accurately produce these graphs. He asserted:

We live in a world where creating graphs is not the skill anymore. We have computers for that. The main thing is to be able to interpret the information and to be able to solve a problem with it. That is where I want them to be. It is not using a ruler and making sure the dots are in the right spot. We have computers for that now. But the computer will not tell you necessarily how to interpret it or what to do with it. The benefit of Excel is to get the graph right away and then what you do with it is the challenge. (Interview, June 20, 2012)

4.4.4.3 Experiences, Experimentations, and Negotiations of Meaning

By integrating technology, Lee fostered critical thinking and created learning opportunities for students to explore, compare, and conclude. He involved mathematical situations, selected from the Internet, with real applicability in life: “That is a big benefit of using technology, you get information quickly. That is what the Internet is all about really – it is about getting information” (Interview, June 20, 2012). During my classroom observations, Lee went online and found two phone plans to compare and discuss. Instead of simply asking students to
create one linear equation to describe each plan, he invited students to determine how much money they would have to pay on a contract of three years. His hope was to shed light on the overall cost over a certain amount of time and to promote awareness among students that “It [the phone plan] costs money and you have to make it” (interview, June 20, 2012). This invitation to reflection helped students connect their learning to real life. It also kept students aware of the cost of the phone plans and may help students with their ability to manage their budget and to take control over their own lives.

4.4.4.4 Increasing the Complexity of the Content

Lee provided opportunities for students to explore and look for similarities or patterns when representing linear relations in tables, graphs, or equations. He taught his grade 9 Applied students to create connections among these three core representations of linear relations, by engaging them in manipulating the parameters in the equation and looking for the graphical and tabular implications of these manipulations. He also appointed students to solve math problems that require the use of numerical, visual, and algebraic forms in order to emphasis the equivalent role that each representation plays (see Appendix GG - Connecting Various Representations of Linear Relations). Lee encouraged students to select the most suitable representation to solve the given problem.

In the lesson I observed, two technological tools were used to assist students in developing skills for the translation among these representation: graphing calculators and clickers. The use of graphing calculators enabled students to move quickly among graphical, tabular, and algebraic representations in order to explore the link and remark on the similarities among these representations. With the use of clickers, students worked towards developing skills for the interpretation among these three representations of linear relations. This tool also enabled
the teacher to monitor students’ understanding of the link among the tabular, algebraic, and graphical representations, and to make appropriate adjustments during the lesson to make the content relevant and meaningful.

Lee integrated units when teaching various math concepts. He designed his lessons to make explicit the links between mathematical notions themselves and to help students understand how mathematical ideas interconnect and build on each other. For example, Lee engaged students in determining and graphing the relationship (linear or nonlinear) between the change in the height or the radius and the corresponding change in the volume of a cylinder. By determining the volume for different values of the height or radius and by plotting these ordered pairs on a graph with the aid of graphing calculators, the students were able to decide the type of relationship (linear or nonlinear) between the change in the height or radius and the corresponding volume. He explained:

Just by graphing, and by trying to fit a straight line through the dots and showing that it can for the height-volume relationship or it cannot for the radius-volume relationship, so that they know that it is linear or nonlinear. That was the only extent of it for the technology part. (Interview, June 20, 2012)

For the following year’s grade 9 Applied course, Lee was planning to use an interactive Gizmo from www.explorelearning.com to examine the relationship between the change in the height and the corresponding change in the volume, and the relationship between the change in the radius and the corresponding change in the volume: “I did not know how to change the height while instantaneously showing the change in volume. But I think that there is a Gizmo that does that so next semester we can actually manipulate it and we can change the height” (interview, June 20, 2012).
4.4.5 Students’ Mathematical Communication

Lee involved technology in designing mathematical explorations to help students construct knowledge and to build their communication skills and to expand their math vocabulary. For example, in the grade 9 Applied course, Lee taught the distance-time graphs with the use of CBR motion detectors and graphing calculators. Students used the CBRs to move back and forth, at a slower or a faster pace, as instructed by their peers, such that they matched a shown graph. They collected and analysed real time data related to distance and velocity and their corresponding graphs. With these technological tools, Lee engaged students in modelling and interpreting graphical information of real life situations and prompted a rich mathematical dialogue. As students tried to match the graph of their walk with a given graph, they communicated with their peers and used precise mathematical terminology. Lee asserted:

It is really great with communication because you have got this little screen and you have got one person looking at it but you have also got one person moving. There is a lot of talk between the students like: go forward, move over, stop, and go faster. There are a lot of connections between what they see on the graph and what they are instructing their partner to do. I feel that that is probably the day where they talk the most to each other about math or about what they are supposed to be doing because without that communication between the two or three students you cannot match the graph. (Interview, June 20, 2012)

According to Lee, technology was useful in connecting graphs and physical motion, helped to increase students’ conceptual and analytical understanding of graphs, fostered teamwork, and enhanced students’ mathematical communication skills:

I feel like they really get to understand that the steeper the line the faster they go. And then whether it is going up or down, moving backwards and forwards, and even just to start where the graph is it is very important. (Interview, June 20, 2012)

Lee stated that the use of technology motivated students to be more active and participate in the lessons, to explain their solutions, to interpret their peers’ answers, and to engage in rich mathematical dialogue. Referring to the interactive response system (clickers), Lee explained
that he encouraged students to explain the reason behind their answers, to present methods of solving the problems, and to show their work step-by-step. By doing this, he explained, he helped students build mathematical communication skills and develop and expand a specific mathematical vocabulary:

To encourage that [communication] with using the clickers and to say, ‘Great, you got it right.’ Then okay, now let us go to the more important reasons, like how you got it right and open up that discussion. I think that is a good starting point. You are right, that is great, and now how did you do it? and then the discussion does not end. (Interview, May 3, 2012)

From Lee’s point of view, technology integration in the mathematics lessons “is working well in a class with a lot of English language learners” because students could use an online dictionary to read the meaning of a new math word. Also the visual aspect of technological tools helped students make connections between words and diagrams or graphs. Consequently, this enhanced students’ mathematical understanding and helped build mathematical communication. He said:

And definitely to have students see what they see on the screen on the page too also helps with creating those things [initial values, x-intercept, y-intercept] and getting them to create those links between words and pictures or diagrams. The initial value is on the graph and using the technology and having a PDF with the slide show already there, they are more likely to link words with the graphical meaning. (Interview, May 3, 2012)

Lee felt that “the discussion forums and wikis and being able to talk to each other online” fostered the development of students’ communication skills (Interview February 23, 2012).

When asked how technology helped foster mathematical communication, Lee suggested that he would like to provide opportunities for students to discuss different mathematical topics online in order to help them enhance their communication skills:

I think that with the online learning community [Moodle], it is my goal to have students open discussion threads about math. Maybe something math is used to explain. A controversial topic, for instance, so that students can use some mathematical communication to talk to one another online. (Interview, February 23, 2012)
Lee used technology, in combination with different teaching strategies, to engage students in the learning process and to enhance their understanding of the mathematics content. By using the student response system (clickers), he motivated students to attempt the questions and to provide an answer without the fear of having the wrong answer. He wanted to provide equal opportunities for his students to contribute to the class discussion and to think about solutions to the math problems provided in class: “I try to have them discuss questions. I try to be positive and get them to try at least. With these new response systems it is great that everybody has an opportunity to answer the question” (Interview, May 3, 2012).

After displaying the answers provided by his students as bar graphs, Lee called on volunteers from each type of answer to explain the thinking behind their solution. If students were not willing to share their response, Lee invited someone in the class to explain their peer’s choice of answer. If a large majority of students agreed on a correct answer, Lee moved on to the next problem. If many responses were wrong, Lee retaught the involved concept, went over the answers, and provided one or more examples and counterexamples to clear up the misconception. In this environment, technology helped construction of knowledge by enhancing constructive student-student and student-teacher dialogues. (Fieldnotes, February 24, 2012)

4.4.6 Summary

Lee incorporated technology into his math lessons to motivate students to investigate and explore math concepts and ideas, to engage them in presenting different ways of solving math problems, to encourage them to explain their thinking process, to invite students to engage in dialogue in order to negotiate solutions, and to provide real time feedback. He developed student tasks involving technology that were connected to the real world and that were relevant to his students’ lives. He believed that the use of technology facilitated students’ conceptual
understanding of math concepts and provided occasions for students to see a direct applicability of mathematics in real life.

By using technological learning tools, Lee provided equal opportunities for students to participate in lessons anonymously and without fear of making mistakes, to collaborate, to think about the questions that arose in the class, and to provide an answer regardless of whether it was right or wrong. He also believed that technology helped struggling students move forward in their learning when they did not master the prerequisite skills required to build upon a new mathematics concept.

In the context of the Linear Relations Unit, Lee’s mathematical activities can be summarized by three predominant objectives while teaching with technology: making the abstract more tangible, addressing the different needs and learning styles of students, and providing real time feedback.

In terms of instructional strategies, the integration of technology in Lee’s lessons was used to challenge preconceived notions and misconceptions, to interconnect units, and to encourage mathematical communication. For example, Lee used technology (a Mobi View tablet and the Internet) to efficiently display a few comparable graphs in order to help his students eliminate a common misconception encountered in the Linear Relations Unit, specifically the confusion between the slope of the tangent to the graph at a particular point in time and the height of the graph at that specific moment. Within this activity, he also provided his students with opportunities to think abstractly about the linear graphs and to eliminate their inclination to view graphical representations as the concrete picture of a situation. In addition, Lee prompted students to practice describing linear relations and graphs and to communicate using specific linear relations mathematical vocabulary and terminology.
Furthermore, Lee used interactive Gizmos activities from explorelearning.com, as well as calculator-based rangers to match students’ motions to the graphs. In these activities, technology was useful in connecting graphs and physical motion, helping increase students’ conceptual and analytical understanding of linear relations, and engaging students in modelling and interpreting graphical information of real life situations.

Finally, Lee integrated technology (the Internet and interactive Gizmos activities) to engage students in determining and graphing the relationship (linear and nonlinear) between the change in the height or the radius and the corresponding change in the volume of a cylinder. With these activities, Lee attempted to make explicit the links between mathematical concepts, as well as to interconnect different mathematical units, particularly the Linear Relations Unit and the Geometry Unit.

Clickers, motion detectors, and interactive Gizmos activities were the predominant technological tools used during Lee’s mathematical activities. He used the Smart Board or a Mobi View tablet in every lesson and mathematical activity that I observed.

Table 3. Lee’s Classroom Observation Log

<table>
<thead>
<tr>
<th>Lesson Title</th>
<th>Technological Tools</th>
<th>Date and Time</th>
<th>Student Handout</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Creating and Using Equations from Linear Relations.</td>
<td>MobiView Internet (Virginmobile.ca) Clickers</td>
<td>April 24th</td>
<td>Creating and Using Equations from Linear Relations</td>
</tr>
<tr>
<td>2. Modelling Linear Relations with Equations.</td>
<td>MobiView Internet Clickers</td>
<td>April 25th</td>
<td>Modelling Linear Relations with Equations (Part 1)</td>
</tr>
<tr>
<td>3. Determine Initial Value and Rates of Change from Graphs and Tables of Values.</td>
<td>MobiView Clickers</td>
<td>April 26th</td>
<td>Modelling Linear Relations with Equations (Part 2)</td>
</tr>
<tr>
<td>4. Graphing and Representing Linear Relations</td>
<td>MobiView Internet</td>
<td>April 27th</td>
<td>Graphing Linear Relations</td>
</tr>
<tr>
<td></td>
<td>Topic</td>
<td>Tool</td>
<td>Date</td>
</tr>
<tr>
<td>---</td>
<td>-----------------------------------------------------------------------</td>
<td>---------------</td>
<td>---------------</td>
</tr>
<tr>
<td>5.</td>
<td>Two Steps Linear Equations.</td>
<td>MobiView</td>
<td>April 30th</td>
</tr>
<tr>
<td>6.</td>
<td>Problems Involving Linear Equations.</td>
<td>MobiView</td>
<td>May 1&lt;sup&gt;st&lt;/sup&gt;</td>
</tr>
<tr>
<td>7.</td>
<td>Connecting Various Representations of Linear Relations</td>
<td>MobiView</td>
<td>May 2&lt;sup&gt;nd&lt;/sup&gt;</td>
</tr>
<tr>
<td>8.</td>
<td>Properties of Linear Relations</td>
<td>MobiView</td>
<td>May 3&lt;sup&gt;rd&lt;/sup&gt;</td>
</tr>
<tr>
<td>9.</td>
<td>Linear Relations –EQAO practice questions</td>
<td>MobiView</td>
<td>June 7&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
<tr>
<td>10.</td>
<td>Linear Relations –EQAO practice questions</td>
<td>MobiView</td>
<td>June 8&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
</tbody>
</table>
4.5 Summary of Findings

This study has provided opportunities to teachers to discuss and demonstrate some instructional strategies and approaches using technology in teaching the Linear Relations unit in grade 9 Applied mathematics. Three secondary math teachers integrated technology in their lessons to engage students actively in the learning process, to present and discuss real life math-related problem, to provide opportunities for students to experiment by using simulations and manipulations using various technological tools, to interconnect mathematical ideas and units, to promote critical thinking and foster students’ mathematical communication. They also used technology to assess students’ understanding at different points in order to inform their practice and adjust their teaching and to differentiate class instruction to meet different needs and learning styles of their students.

While all of these teachers recognized the importance of the integration of technology as part of the math classroom and beyond, they also mentioned some caveats teachers should consider before embracing technology. For example, Shannon explained that the introduction of technology too early in mathematics education may prevent young students from developing a clear understanding of number sense and developing coherent algebraic thinking and reasoning. Anna pointed out that heavy reliance on technology is problematic and constitutes a major disadvantage for students, especially when they do not check for their answers’ correctness and meaningfulness and rely unconditionally on the displayed solutions of their technological tools.

Table 4 presents a cross-case summary of the three math teachers, indicating their teachable subjects and years of teaching experience, their learning approaches to integrating technology in mathematics lessons, the technological tools and teaching strategies used to deliver
the Linear Relations unit in grade 9 mathematics and their perceptions about significant characteristics of teachers teaching with technology.

Table 4. A Cross-Case Summary of the Three Teachers

<table>
<thead>
<tr>
<th>Teachable Subjects</th>
<th>Shannon</th>
<th>Anna</th>
<th>Lee</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematics/Physics</td>
<td>Mathematics/Science</td>
<td>Science/Mathematics</td>
</tr>
<tr>
<td>Teaching Experience (years)</td>
<td>10</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Technological Tools used in Teaching Linear Relations</td>
<td>Smart Board, TI-Nspire, Clickers, Gizmos, IXL software, GSP</td>
<td>Smart Board, TI-Nspire, Clickers, Gizmos, IPhones, GSP</td>
<td>Smart Board, TI-84, Clickers, CBR, Gizmos, Moodle, Mobi View Tablet</td>
</tr>
<tr>
<td>Future Plans to Extend Technology</td>
<td>Flip Classroom, Angel</td>
<td>Flip Classroom, IXL</td>
<td>Moodle, GeoGebra</td>
</tr>
<tr>
<td>Learning to Integrate Technology</td>
<td>Mentor teacher, Self-learner, Workshops, Conferences</td>
<td>Mentor teacher, Self-learner, Workshops, Conferences</td>
<td>Pre-service Teaching, Training, Independent Learner, Workshops, Conferences</td>
</tr>
<tr>
<td>Technology Integration Teaching’s Strategies</td>
<td>Student-Centered Explorations, Simulations, Problem-based learning, Direct instruction, Demonstrations</td>
<td>Student Centered Investigations, Simulations, Problem-based learning, Direct instruction, Demonstrations</td>
<td>Student Centered Investigations, Simulations and Manipulations, Problem-based learning, Direct instruction, Demonstrations</td>
</tr>
<tr>
<td>Teachers’ Perception about Characteristics of Teachers Teaching with Technology</td>
<td>Extremely competent in terms of the content knowledge, Know pedagogy very well, Aware of the benefits of technology, Strong desire to learn about and try new technologies, Provide applications and problems consisting of real life situations</td>
<td>Good understanding of curricula, Solid knowledge of the content area, Familiar with students’ conceptual understanding, Interplay among content, pedagogy and technological knowledge, Provide authentic situations to be solved mathematically</td>
<td>Knowledgeable, Open minded, Willing to try new things, Reflective, Self-motivated, Self-driven, Confident when allowing students to use technology, Constantly changing and adopting to his/her students' needs, Willing to learn from students</td>
</tr>
</tbody>
</table>
CHAPTER FIVE: DISCUSSIONS AND CONCLUSIONS

5.1 Introduction

In this chapter, the findings are presented and discussed. The discussion is organised around three major points. First, the research questions posed in Chapter 1 are answered and the major findings are identified and linked with the current literature. Second, there is a discussion of the implications of these findings. Finally, suggestions for possible directions for future research are considered. The analysis and findings were guided by the following questions:

1) What are mathematics teachers’ perceptions about the integration of technology in teaching the Linear Relations Unit in grade 9 Applied level mathematics?

2) How do teachers’ views on the role of technology and their teaching practice help us to better understand the development of student tasks in the context of the Linear Relations Unit?

3) How do teachers’ views on the role of technology and their everyday practice help us to better understand the construction of content knowledge in the context of Linear Relations Unit?

4) How do teachers’ views on the role of technology and their everyday practice help us to better understand the facilitation of mathematical communication in the context of Linear Relations Unit?

5.2 Research Questions

In this section, I will answer the research questions based on the three case studies.

5.2.1 Research Question 1:

What are mathematics teachers’ perceptions about the integration of technology in teaching the Linear Relations Unit in grade 9 Applied level mathematics?
Evidence from this study reveals that the integration of technology in teaching the Linear Relations Unit in grade 9 mathematics changed teaching practice of teachers. Technology helps teachers adapt, modify, and differentiate their teaching so as to ensure that all students in their classrooms have opportunities to interact and learn. All of the three teachers in this study pointed to aspects of their practice that changed their teaching strategies and techniques as technology was integrated in their lessons. These changes in aspects of practice are grouped along the following emerging themes: Visualization, Problem solving, Differentiating instruction, Motivating students, Effective teaching, and Real time feedback.

5.2.1.1 Visualisation

Shannon changed the way she designs and delivers her lessons to make them meaningful and relevant to her students’ lives. Shannon uses technology to provide thorough and comprehensive explanations of math content, by including real life examples, engaging in investigations, conducting simulations, and organising and interpreting data. She indicated that technology enables the visualisation of mathematical models and processes, of graphs and tables of values, and illustrate the effect of changes in graphs; and that this can enrich and deepen students’ understandings of abstract concepts. Shannon presented one example of a technology-driven interactive lesson that emphasised the visual aspect of the problem but also supported students’ conceptual understanding of mathematical ideas: the investigation of the point of intersection of two lines by using Gizmos. With this online simulation, students manipulated the real life situation of the motion of a cat-and-mouse chase in order to understand and determine the abstract concept of the point of intersection of two lines.

Likewise, Anna indicated that visual representations of math ideas make lessons interesting, arouse students’ curiosity, and promote mathematical dialogue and communication.
Lee indicated that colourful visual representations help students connect the symbolic and visual representations of linear relations math concepts, such as rate of change and the initial value. In his opinion, the ability to point and touch the images makes the abstract math content more concrete and tangible to students.

These findings are consistent with Alagic’s (2003) assertion that well-designed ICT-based instructional representations of mathematical inquiry/problem solving activities provide opportunities for students to translate among various representations such as tables, graphs, and text; models, visualizations, and diagrams. Consequently, these ongoing interactions/transfers help students make connections and build their own conceptual understanding of mathematical concepts and relationships.

5.2.1.2 Problem solving

All of the teachers felt that technology promotes the development of problem solving strategies and techniques, allowing and encouraging high-level thinking skills, and enhancing the teaching of mathematical communication skills and vocabulary. For example, Shannon explained that, in the investigative lesson involving motion detectors to produce distance-time graphs, students needed to use appropriate terminology and provide precise mathematical instruction to their peers to make them walk in such a way as to match the displayed graphs. In the “Height versus Arm Span Activity”, Shannon asked students to move into the interpretative domain of the line of best fit concept and, by using relevant vocabulary, to discuss the relationship between the height and arm span for points that lay above or below the line of best fit. She also taught them not to make predictions beyond the bounds of the data points and about a population that was different from the population that the sample data was drawn.
Anna used technology to help students visually organise the main concepts and keywords using the Frayer Model, to provide examples and counterexamples, and to think critically in order to make connections among linear relations concepts. Lee took advantage of technology and encouraged students to efficiently and accurately present and discuss alternative ways to solve problems during the lesson entitled “Connecting Various Representations of Linear Relations”. He emphasised that it is important for the development of his students’ critical thinking and communication skills to see that math questions can be solved in many different ways.

This is consistent with the findings of Ruthven (2002) who asserted that students should be provided with opportunities to concentrate on cognitively complex tasks, such as explorations and investigations of math concepts and problem solving. These tasks facilitate students’ connections among relevant mathematical ideas and shape students’ perceptions of mathematics. They provide students with a better understanding by offering multifaceted perspectives and views of the same mathematical concepts.

5.2.1.3 Differentiating instruction

Shannon, Lee, and Anna suggested that technology helps them identify students’ strengths and weaknesses, as well as organise instruction more efficiently and effectively. Technology assisted them to organise instruction and to make purposeful decisions about how to vary the nature of instruction to address students’ individual needs, learning styles, and skill levels. Following this, Lee integrated technology to meet the needs of kinaesthetic learners who learn better by doing. Students worked in pairs and used the calculator-based ranger (CBR), the graphing calculators, and the screen on the Smart Board to simulate and match the distance-time graphs. For students who learn better visually, Lee used the Mobi View tablet to break down the
math content into basic parts and to deal with each part separately. The colourful illustration
helped students connect the symbolic and visual representations of particular abstract math
concepts, such as rate of change and the initial value. Also, technology empowers students to
become independent, as it allows them to work at their own pace and level. From Shannon’s
point of view, this helped students build confidence in their abilities to do math and gave them an
increased sense of accomplishment and personal growth.

Lee mentioned that the integration of technology provides incentives to students to have a
voice, even when they are timid or reluctant for any reason to share their thinking, and to
contribute with their work to the classroom discussion. In his opinion, the integration of
technology in the mathematics classroom provides equal opportunities for students to engage in
active learning and to answer questions without fear of giving the wrong answer.

Shannon and Anna spoke about how technology helps them plan engaging lessons and
activities and create interactive and dynamic environments that allow investigations and
computer simulations of linear relations concepts. In their opinions, these environments facilitate
effective connections between students and teachers, as they share a common interest in
technology. Consequently, this results in a high level of engagement and participation in the
learning process. Moreover, Anna found that the integration of technology captivates students’
interest, increases the attention and involvement of students in their learning, and ignites their
curiosity in mathematics. She mentioned that the use of technology motivates and engages
students to complete their homework. In addition, Lee explained that the integration of
technology allows students to access math resources and notes electronically, to send messages
to peers, and to engage in math related discussions. This helps minimise excuses typically used
by students for not doing homework or not being prepared for the next math class.
These findings are consistent with the Kaput et al. (2007) conclusion that technology motivates students to engage in the learning process and supports mathematical ideas in ways that are important for conceptual understanding.

5.2.1.4 Motivating students

Anna indicated that one significant change in her teaching practice was that she incorporated technology to provide concrete examples of how mathematics could be used outside of the classroom. She believed that it is very important to motivate students to study mathematics by showing them that the subject has direct applicability in practice and the real world, and that it is not a dry and isolated subject. Thus, in her class, grade 9 students investigated the properties of regression lines and correlation by using a concrete set of data produced by measuring students' arm span and height. By using this set of data and the TI-Nspire graphing calculators, students created a scatter plot, drew the line of best fit, determined the linear regression equation, analysed the correlation, and made predictions about what would happen to the line of best fit if athlete Michael Phelps measurements were added to the dataset. According to Anna, this activity engaged students in the learning process because it was related to real life, involved real people, and required the use of technology.

Similarly, Lee explained that technology helps provide examples inspired from reality, animate graphs, simulate math concepts, and bring abstract ideas to life. In teaching the equation of a line, Lee involved mathematical situations selected from the Internet with real life applicability. Specifically, he found two cell phone plans to compare and discuss. His intention with this selection was to teach students the equation of a line using real life examples and to promote awareness among students about the high cost of cell phone plans.
Shannon used technology to provide real world context to be discussed and analysed. For example, when comparing linear and nonlinear relationships, she projected an article from a local newspaper on the Smart Board. This article presented the exponential growth of Internet use over the years as an example of a nonlinear relationship.

Anna found that the integration of technology helped students learn and remember mathematics concepts easily because they were more likely to remember concepts that they discovered through explorations, simulations, and those connected to real life. For example, by manipulating the movements of a cat and a mouse, students were able to understand the changes on the linear graphs and associate them with the abstract concept of slope. In Anna’s opinion, this simulation helped students understand the abstract concept of slope by remembering that the mouse running faster indicated that the line was steeper.

Anna and Lee felt that the integration of technology increases students’ persistence and perseverance over time. In addition to promoting interactive learning, technology also helps improve classroom management and discipline. Lee specified that, as students were conducting investigations in groups or individually, they were answering formative assessment questions, communicating their understanding, discussing and arguing their solutions, carrying out their tasks, and advancing in their learning.

5.2.1.5 Effective teaching

All of the teachers remarked that the integration of technology helps make their teaching more effective in terms of time management. They thought that the removal of non-relevant math concepts made learning more focused and direct. For example, Shannon described that technology was used efficiently in the Linear Relations Unit to scatter plot data and display graphs of the relationship between two variables and to determine the linear regression and
correlation. The possibility of obtaining graphs quickly provided more time to be dedicated to
discussions about the type of relationship and the overall trend, to making predictions, and to
focussing on the interpretative nature of the problem.

Anna, Shannon, and Lee felt that one considerable advantage of using technology in the
grade 9 Linear Relations Unit was that it allows for multiple representations of mathematical
problems, specifically the graphical, algebraic, tabular, and verbal descriptions. This increased
students’ abilities to make connections among different representations, to see the immediate
impact of changes made in the slope and y-intercept, and to develop insights for the concepts of
parallelism and perpendicularity.

These findings support Jiang & McClintock (2000) statement that, “We, as mathematics
educators, should make the best use of multiple representations, especially those enhanced by the
use of technology, encourage and help our students to apply multiple approaches to mathematical
problem solving and engage them in creative thinking” (p. 19).

Furthermore, Lee and Anna felt that technology helped students move forward in their
learning even when they did not master the prerequisite skills required to build upon new
understandings. Lee clarified that students used calculators if necessary to add or subtract
fractions as they learned linear relations. Their weak computational skills did not get in the way
of learning and did not prevent them from moving forward and constructing new knowledge and
understandings. In the same manner, Anna believed that the use of technology should encourage
problem solving and the interpretation of real world situations, and that it should not focus on
drill and practice exercises. From her point of view, technology could help students compute
easily and represent graphs precisely. Consequently, students could focus on the interpretative
domain of mathematics and the development of critical thinking rather than performing dry drills and practice exercises.

These findings are consistent with the findings of Ellington (2003) in that technology (graphing calculators) is typically used to simplify tedious computations, leading to greater focus on the pedagogical point, and it can be important in students’ development of their conceptual understanding of mathematics.

5.2.1.6 Real time feedback

Additionally, Lee felt that a significant change in his practice is related to the possibility of providing real time feedback and of monitoring closely students’ understanding and learning during the lesson. He explained that he can dramatically change the course of a lesson, alter the pace and nature of instruction, add more examples, and provide alternate solutions in order to make the math content more meaningful, to clarify any misunderstandings, and to clear up any misconceptions. He found that students are interested in doing questions when they receive instant feedback and this helps them gain confidence in their ability to do math. Consequently, the learning process and classroom management improve. In the same way, Shannon described that, for grade 9 students learning the Linear Relations Unit, the instant feedback helps them correct their own mistakes, such as adjust the equations, change the settings of the domain or range, or apply appropriate conditions for the slopes of perpendicular or parallel lines.

These findings support Wenglinksy (1998) findings that technology (software) can provide students with opportunities for practice and rapid feedback in a motivating environment.

5.2.1.7 Disadvantages

While all three teachers recognised the importance of the integration of technology as part of the math classroom and beyond, they also mentioned some disadvantages to embracing
technology. For example, Shannon explained that the introduction of technology too early in mathematics education may prevent young students from developing a clear understanding of number sense and developing coherent algebraic thinking and reasoning. Anna pointed out that heavy reliance on technology is problematic and constitutes a major disadvantage for students, especially when they do not check for their answers’ correctness and meaningfulness. From her experience, there were circumstances when students used technology inappropriately and obtained incorrect solutions, which were then not verified for accuracy or validity. Being aware of this trap, Anna encouraged her students to always reason and justify their solutions when utilising technology.

This finding supports Cavanaugh and Michelmore (2000) observation that, uncritical acceptance of the graphical image on the calculator led students to error. It is also consistent with Hershkowitz and Kieran (2001) assertion that, secondary school students rely on the graphical representations of functions and ignore verifying the algebraic properties of these functions.

Anna expressed that heavy reliance on visual representation may sometimes be detrimental to students. She explained that, in many cases, students based their answers on what they saw, ignored the analysis and justification of their findings, and were misled by the visual perception to provide incorrect answers. To argue her position, Anna discussed a class example when students looked at two lines and declared that they were parallel based on them appearing parallel. Assisted by technology (Geometer’s Sketchpad), Anna was able to instantly determine the slopes of the lines and to demonstrate to the students using technology-based evidence, that the slopes were in fact slightly different and, therefore, were not parallel.

This supports Ronau et al. (2009) finding that, although some software products are attractive and easy to use, they do not necessarily enhance students’ abilities to reflect and find
appropriate solutions to the problems. For example, calculators do mathematics for students but they do not teach students to think mathematically.

In addition, Anna also felt that there is a disconnect between the standardised assessment (EQAO), which is a paper and pencil based test, and the use of technology in the classroom. She believed that students need practice with assessments that imitate the standardised tests closely to adequately prepare for these tests. She expressed that technology does not help in this sense and also raised a concern about students in their first year of university who are not allowed to use technology (graphing calculators) to perform computations or to construct graphs. In her opinion, this inconsistency between secondary school and university should be addressed.

5.2.1.8 Summary

All three teacher participants spoke about changes in their teaching practice as a result of the impact of integrating technology in the math lessons. Thus, Shannon strived to integrate technology into her lessons to make them meaningful and relevant to her students’ lives. Anna incorporated technology to provide concrete examples of how mathematics could be used outside of the classroom in real life. Lee used technology to closely monitor his students’ understanding and learning during the lesson in order to inform and adjust his teaching, to make the math content more meaningful, to clarify any misunderstandings, and to clear up any misconceptions. In addition to these reported major changes in teaching practices, integrating technology in delivering the Linear Relations Unit has an influential effect on other significant elements of the teaching and learning process, such as the development of student task, construction of content knowledge, and facilitation of mathematical communication.
5.2.2 Research Question 2:

How do teachers’ views on the role of technology and their teaching practice help us to better understand the development of student tasks in the context of the Linear Relations Unit?

Within this study, the three math teachers designed student tasks that integrated technology to make learning more effective, to keep the student at the centre, to create a learning environment that encouraged equal participation of all students in the class, to address the needs of every student and recognise individual differences, to promote the development of critical thinking, and to reinforce the connection between assessment for learning and instruction. This supports Watson and Sullivan (2009) suggestion that, when teachers decide to use particular tasks, they are making choices about the nature of learning that might take place.

Teacher’s views about the role of technology in developing student tasks are organized along the following emerging themes: modelling the real world, interactive learning environments, promoting critical thinking, checking and validating solutions.

5.2.2.1 Modelling the real world

The teachers felt that student tasks should involve real life scenarios that are engaging and attractive for students and show the applicability of math concepts to the real world. Teachers viewed technology as playing a significant role in engaging students in modelling the world around them by using mathematical models, particularly the equation of the line of best fit in the context of Linear Relations Unit. Shannon created technology-based learning tasks inspired by real life situations to reinforce students’ understanding of the key abstract concepts (slope and y-intercept) in the Linear Relations Unit. Her tasks gave students a chance to model the world around them by using mathematics, made students find arguments about why mathematics is important in life, and provided opportunities for critical thinking, specifically
reasoning, making decisions and predictions, and checking the validity of solutions. One example of such a task was called the Washing Machine Problem. In this problem, inspired by a real life situation, students explored equations and graphs of lines in order to discover and analyse the properties of parallel lines. The role of technology in this task was to produce high precision graphs faster, to assist students in visualising the situation, and to provide immediate feedback. The task required students to use technology to check their conjecture about the slopes of parallel lines and solicited students to manipulate the graphs and engage in math related discussions, which required the use of specific math terminology and practice of mathematical communication skills.

5.2.2.2 Interactive learning environments

Moreover, the teachers adopted technology to create interactive environments and planned dynamic activities and simulations that allow students to immerse themselves in the learning process. By providing interactive student tasks, these teachers invited students to experiment, investigate, and think critically in order to apply their math knowledge to make decisions and write appropriate solutions for the given math problems. The teachers’ intentions in the Linear Relations unit were to help students understand how the manipulation of the parameters (slope and y-intercept) of a linear equation affects the resulting graph and to conceptualize specific abstract concepts, such as slope, point of intersection of two lines, and parallel and perpendicular lines. In teachers’ opinions, the manipulations and simulations involving the immediate impact of changes in the graph or attaching a visual image or representation of an abstract concept lead to an increase in students’ conceptual and analytical understandings of graphs.
These findings support Arzarello and Robutti’s (2001) observation that “students’ cognitive activity passes through a complex evolution, which starts in their bodily experience (namely, running in the corridor), goes on with the evocation of the just lived experience through gestures and words, continues connecting it with the data representation, and culminates with the use of algebraic language to write down the relationships between the quantities involved in the experiment” (p. 39). These researchers have explored students’ understanding of functions in an environment involving a calculator connected to a CBR motion detector. Students were encouraged to run in different patterns and with various speeds in order to create different graphs. The activity allowed students to test conjectures about the graph of different functions and connect various representations of functions by experimenting with their own physical movement in a technology-driven environment.

5.2.2.3 Promoting mathematical thinking

In the present study, teachers utilized technology to help students produce accurate and quick graphs for the Linear Relations unit. These graphs constituted the base for discussions about how to build connections between mathematically abstract concepts (slope and y-intercept) and their visual representations, and about how to gain fluency in moving among different representations of linear relations (algebraic, tabular, graphical, and verbal descriptions). This supports Qing Li’s (2007) finding that the integration of technology is very useful because of its efficiency, as it allows easy access to information and cutting edge research and it makes learning easier. He also found that technology could provide fast, reliable information and it could enable a more professional presentation of students’ work (p. 391).

The teachers believed that technology can make learning more effective and more interesting for students. Within this study, all three teachers implemented the same technology-
based student task to engage students in investigating the relationship between the height of a person and the length of a person’s arm span in order to determine the equation of the line of best fit. The use of TI-Nspire graphing calculators in this task permitted students to plot data points quickly and to dedicate more time to the interpretation of the graphical display. After the main part of this activity, which consisted of graphing the line of best fit and determining its equation, each teacher brought in different elements to discuss.

Shannon discussed with her students the reliability of predictions made from the graph and cautioned students not to use a line of best fit to make predictions beyond the bounds of the data points, or to make predictions about a population that is different from the sample data population. To make this point, Shannon used technology (her computer and the Smart Board) to import a separate set of data for babies, produced a scatter plot, determined the line of best fit, showed that babies have different body proportions from children and adults, and illustrated that the graph of their height and arm span generates a different line of best fit. Technology made this lesson time efficient. Saving time by obtaining a quick graph allowed for more ground to be covered in lesson time.

For the same task, Lee invited students to interpret the appearances of the scatter plot, to determine the correlation, and to interpret the meaning of slope and y-intercept. By using Smart Board, students translated easily among algebraic, tabular, graphical, and verbal descriptions of linear relations. In this context, technology was used to engage active learning, stimulate critical thinking, and develop abilities to make good decisions and draw appropriate conclusions. This finding is in agreement with Kerrigan (2002) who established the benefits of using mathematics software and websites to include promoting students’ higher order thinking skills, developing and maintaining their computational skills, introducing them to collection and analysis of data,
facilitating their algebraic thinking, and showing them the role of mathematics in a real life setting.

Anna designed a student task integrating technology (TI-Nspire), which asked students to create a picture resembling a real object using only five lines. This task involved high order thinking skills, as students needed a clear understanding of the slope and y-intercept to represent the object mathematically. Her objective with this task was to make students understand that what was taught in the math lesson had applicability in real life. My feeling while observing this lesson was that engaging learning through technology increased students’ interest and retention, as it tapped into their interests, creativity, and curiosity.

Anna planned and implemented a student task integrating technology to explore the relationship between the number of Styrofoam cups with lids in a stack and the height of the stack. She asked students to write the equation for the linear relation and to discuss the meaning of the slope and the y-intercept in this context. The TI-Nspire calculator was used to produce quick and accurate graphs and to determine instantly the equations of the line of best fit for three different size cups. Students had the opportunity to use the graphical, tabular, and algebraic representations of the three relations to compare their graphs in terms of their rate of change and y-intercept. Technology made this lesson time efficient and students dedicated most of the period to analysing and interpreting the graphs. They discussed and answered related questions and solved math problems in order to make connections among various representations (graphical, tabular, and algebraic representations) of linear relations, as well as to make connections between the visual representation and the abstract concepts of slope and y-intercept.

This supports Bostic and Pape’s (2010) assertion that students need to be provided with suitable tasks to develop well-connected representations of mathematical concepts and to begin
to use representations other than symbols in their solution strategies. They insisted that the new graphing technology, specifically TI-Nspire CAS technology, facilitates these connections by providing multiple representations to be viewed in a connecting field in which relationships may be investigated and abstracted, and by lightening the cognitive load of generating representations for the learner so that more effort may be expanded to solve math problems.

Moreover, Lee, Anna, and Shannon integrated technology to train students to check and validate their solutions and to clarify any misconceptions or misunderstanding. The teachers also believed that technology played an important role in helping students remain focused on the context and making them think about the process and whether their measurements made sense. For the Styrofoam cup activity, some students needed technology to produce precise graphs in order to realise that each time a new cup was added the height increased by the height of the lip, to check the precision of their measurements, and, consequently, to be able to identify that the relation between the number of cups in the stack and the height of the stack was linear.

**5.2.2.4 Connecting mathematical concepts and ideas**

Anna integrated technology (Geometer’s Sketchpad) into the design of a student task on the sum of interior angles in polygons with two goals in mind. One goal was to apply the linear relations knowledge to identify that the relationship between the number of sides and the sum of the interior angles in a polygon is linear and then to determine the equation of this relationship, which represents the formula for the sum of the interior angles in a polygon. The second goal was to help students recognise the connections among the mathematical ideas and strands. This task engaged students in discovering the geometric formula of the sum of the interior angles in a polygon by using knowledge from the Linear Relations Unit. This constitutes an example of how
teachers interconnect units and provide multiple perspectives on and approaches to the same mathematical concept.

This is in agreement with Ruthven (2002) finding that, teachers aimed to bridge the connections among mathematical ideas and strands and they provided opportunities for students to look at the same concept through the perspective of algebra and geometry.

In my research, Shannon adopted an approach with simulations using a Gizmos activity to determine the point of intersection of two lines. In this task, Shannon’s objective was to use animation and the visual feature of technology to manipulate variables and constants and to see the immediate effects of these changes. In Shannon’s opinion, the visual representation helped students easily make connections to real life situations, which helped them to better understand abstract math concepts.

This supports Hegedus and Kaput’s (2003) conclusions that, the use of technology and kinaesthetic approaches lead students to study functions as a means of exploring and analyzing both real world and simulated behaviour.

Shannon gave her students the opportunity to explore distance-time graphs in order to understand and make predictions and generalizations about different motion situations. She used a student task involving simulations using calculator-based ranger motion detectors and graphing calculators. Lee used a similar technology-based student task to collect and analyze real time data related to distance and velocity and to interpret and discuss their corresponding graphs. According to Lee, technology played a significant role in connecting graphs and physical motion, in helping students increase their conceptual and analytical understanding of graphs, in fostering teamwork, and in enhancing students’ mathematical communication skills.
This finding is in agreement with Robutti and Ferrara (2002) who described the importance of designing technology-based mathematical tasks for building students’ capacity for mathematical thinking and reasoning. In their study, a motion sensor was used to simulate “motion trips” and set an interpretation task of a space-time graph. The authors inferred that the technology made transitions between static and dynamic interpretations of the distance-time graphs possible, leading to standardized meanings for the graphs.

5.2.2.5 Summary

In conclusion, all three teachers in this study expressed that technology has changed the way they teach. In designing technology-based student tasks, the teachers adopted teaching practices that match the description of the fourth essential component for building a successful mathematics program, namely Student Task, in the Ten Dimensions of Mathematics Education framework (McDougall, 2004). The student tasks described during the interviews and observed during the classroom visits included most of the characteristics of Student Tasks: embedded in real life context, allows for multiple representations and multiple solutions, and bridges conceptual understanding from real life to abstract representations (McDougall, 2004).

In addition, by integrating technology, these teachers were able to design dynamic student tasks involving simulations and animations that connected abstract math problems to real world context relevant to students’ lives. This motivated students to participate more actively and pay more attention for longer periods of time. Besides engaging active learning, the integration of technology stimulated critical thinking and helped students develop abilities to make good decisions and draw appropriate conclusions. It played a significant role in connecting abstract graphs to concrete physical motions, and lead to an increase in students’ conceptual and analytical understanding of linear relations.
Another significant contribution of technology to the design of student task in the Linear Relations Unit was immediate feedback. This helped the teachers closely monitor the teaching and learning process. Consequently, assessing students’ understanding and their learning progress in real time provided valuable insights about how to deliver the content to make it meaningful and informed teachers to adjust their instruction when necessary.

Moreover, the integration of technology in planning the student tasks in the context of Linear Relations unit assisted the involved teachers to teach their students to verify their conjecture about the slopes of parallel and perpendicular lines, to check the point of intersection of two lines, and to justify the validity of students’ predictions or inferences based on the equation of the line of best fit. Technology helped raise awareness among students that visual perception may mislead them to provide incorrect answers.

Moreover, the integration of technology enabled students to solve the assigned tasks efficiently. Saving time by obtaining a quick and precise graph allowed students to focus more on the interpretative nature of the math problem, to analyze and discuss the relationship between variables, to solve by using various approaches, to transfer among different representations (graphical, algebraic, tabular, and verbal), and to reflect on the shapes of the graphs and the effects of any changes on the key parameters of the equation of the line of best fit.

This study did not intend to investigate if and how student tasks that integrate technology impact student learning. The existing research on this topic has mixed results. Thus, Isikal and Askar (2005), Olkun, Altun, and Smith (2005), and Sinclair (2004) found that using information technology in the classroom benefited students’ mathematics learning. Findings from Lagrange (1999) suggested that students did not express that technology integration (computer algebra) supported their understanding of mathematics even when they liked it. Based on these
contradictory findings, general conclusions about students’ learning of mathematics in a
technology-driven environment cannot be made.

5.2.3 Research Question 3:

How do teachers’ views on the role of technology and their everyday practice help us to
better understand the construction of content knowledge?

To answer this research question, I analysed the data collected in accordance with the
related literature regarding the potential of technology for supporting knowledge construction
(Boethel and Dimock, 1999). I organized the findings along the following emerging themes:
preconceived notions and misconceptions, fostering students’ curiosity, the role of teachers in a
technology-based environment, and social interaction.

The review of the literature indicated that technology can support constructivist learning
environments when it is used as a tool for learning to assist teachers as they attempt to uncover
students’ prior knowledge, understanding and beliefs; to tap into students’ interests and provide
increased motivation for learning; to base instruction on the posing of problems; to provide a
variety of experiences, experimentation, and negotiation of meaning; to increase the complexity
of the content; to take on the role of facilitator; to increase the ability of students to test multiple
scenarios and, thus, challenge preconceived notions and misconceptions; to increase the
authenticity of the content and context; and to broaden the circle of social interaction to include
students’ peers and experts beyond the classroom, the school, the community, and even their
home country (Boethel & Dimock, 1999).

In this study, the integration of technology in the Linear Relations unit made a variety of
teaching strategies possible to help teachers make effective connections with their students, to
address the interest and build on the previous skills of their students, and to consider the diverse
learning needs of their students. All of these techniques contribute to increase students’ desire to discover, learn, and construct mathematical knowledge. By using technology, the three math teachers in this study hoped to motivate students to approach the math content more confidently, to persist longer on the tasks, and to apply their critical thinking in making solution decisions. They designed technology-based interactive activities, selected real life examples from authentic websites accessible on the Internet, used simulations, provided colourful and dynamic illustrations and animations to make explicit the connection between visual and symbolic representations.

5.2.3.1 Preconceived notions and misconceptions

The findings from this study reveal that all three teachers used technology to challenge preconceived notions and misconceptions. For example, Shannon used motion detectors to eliminate students’ misconceptions that the negative slopes show a slower average speed than the zero or positive slopes just because negative numbers are smaller. By asking students to move forward and away from the detector with the same speed, Shannon helped students understand that the slope represents the average speed and indicates how fast the object is moving, and that the sign in front of the slope shows the direction of movement.

Anna used Geometer’s Sketchpad to show students that visual perception misled them to provide an incorrect answer when they referred to parallel lines. Thus, to prove that two lines were not parallel, although they looked parallel, Anna asked students to determine the slope by using the measurement tools of Geometer’s Sketchpad and proved that the slopes of the lines were slightly different.

Lee used the Mobi View tablet to display a few graphs in order to help students differentiate between the slope of the tangent to the graph at a particular point in time and the
height of the graph at that specific moment. Lee also used Gizmos from explorelearning.com and the calculator based rangers to match students’ physical motions to graphs in order to train his students to think and interpret graphs abstractly. With these investigations, Lee taught students how to graph an athlete’s trip around an oval track by using slopes on a distance-time graph and explained why drawing an oval picture of the track is not a good mathematical description of this situation.

### 5.2.3.2 Fostering students’ curiosity

Tapping into student interest and providing increased motivation for learning was another theme identified by Boethel and Dimock (1999) to support knowledge construction in a technology-driven environment. Every teacher in this study integrated technology to help students construct knowledge by fostering student curiosity and creativity and by getting students interested in what they were learning. For example, Shannon and Anna planned an activity making use of TI-Nspire graphing calculators to create a picture of an object by using lines and their equations and properties. With this activity, Anna reinforced the significance of the key elements of linear relations and also convinced her students that linear relations concepts are useful in real life. In the same way, Lee accessed a website and asked students to compare two cell phone plans. This task engaged students in representing linear relations derived from descriptions of realistic situations, motivated them to complete the assigned math work, and encouraged them to build and practice specific mathematical vocabulary by engaging in math related discussions.

Shannon used Geometer’s Sketchpad to explore concrete models in order to facilitate students’ understanding of the properties of parallel and perpendicular lines. She also used Gizmos to manipulate the parameters of the equation of a line and to see the effects of these
changes and how they are reflected in the equation of the line. Similarly, Anna used the TI-Nspire graphing calculators to teach correlations and to invite students to extrapolate or interpolate to make predictions and inferences about the resulting graphs. She collected the measurement of her students’ arm spans and heights, asked them to use the TI-Nspire calculators to create a scatter plot of the data and to draw the line of best fit to represent the relationship between the two variables (arm span and height), and then asked them to apply critical thinking to make predictions based on the obtained graph and reflect on the findings.

This supports Boethel and Dimack’s (1999) finding that, providing a variety of experiences, experimentations, and negotiations of meaning represents another strategy that helped the math teachers in their endeavours to improve classroom instruction and facilitate knowledge construction by integrating technology.

Increasing the complexity of the content was possible with the use of technology in all three math classes. Assisted by technology, the math teachers were able to demonstrate concepts and mathematical ideas visually, to manipulate lines and graphs, and to encourage students to perform rich tasks. For example, Anna used Gizmos to help students make connections and transfer their real world understandings into understandings of complex abstract situations and mathematical models. She allowed students to investigate the point of intersection of two lines by using the speed of two animals in motion, a cat and a mouse. This activity helped students to better understand abstract concepts, such as the rate of change, the y-intercept and the point of intersection, and to transfer their understanding into solving more abstract math problems.

These findings are consistent with Martinovic & Karadag’s (2012) findings that dynamic and interactive mathematics learning environments (DIMLE), “provide opportunities for teachers
to extend both the mathematics knowledge and understanding of their students, especially in the areas that are both dynamic in nature and otherwise difficult to understand” (p. 1).

5.2.3.3 The role of teacher in a technology-based environment

All three teachers viewed themselves as having the role of a facilitator rather than a direct transmitter of knowledge. Shannon viewed her main role as facilitating knowledge constructions. She spoke about changes in the way she teaches her classes compared to the beginning of her teaching career. She is planning more interactive lessons utilizing technology, is asking more leading questions to help students discover math concepts, and is giving more freedom to students to explore and find solutions to the math problems. She explained that she was prepared and willing to learn from her students about how to integrate technology more efficiently into her lessons because, in her opinion, students are usually better with technology than their teachers.

On the other hand, Anna and Lee viewed themselves at par with students in terms of technological knowledge. Anna mentioned that she likes discovering things on her own and she likes to investigate the abilities of different technological tools in order to incorporate them efficiently into the learning process. Lee also spoke about his high level of interest in learning about new technological tools and about progressively changing his teaching approach towards becoming more of a facilitator. During my observations in Lee’s class, I noticed that the mathematics lessons were taught primarily based on a student-centred approach by facilitating class discussions in pairs or as a large group and by sharing findings and understanding from explorations of real world examples. Even in a few circumstances when he presented the information through lecture, students were encouraged to ask questions and were invited to provide instant answers by using clickers. The immediate feedback helped teachers monitor the level of understanding in the class and adjust their teaching when necessary.
5.2.3.4 Social Interaction

While all three teachers spoke about using technology to broaden the circle of social interaction to include students’ peers and experts beyond the classroom, they were mostly in the planning phase with this aspect. Thus, twice a week after school, Shannon used IXL, software designed to provide drill and practice exercises to help her students improve their basic skills. She was also planning to incorporate Angel (a virtual classroom environment) into her lessons in the upcoming school year. Angel is a platform used by Shannon’s school board to allow students and teachers to collaborate, to work in pairs or groups, and to have access to homework and other information posted online by the teacher.

Additionally, both Anna and Shannon discussed the possibility of signing up to be part of the flip school project in the following school year. This teaching strategy requires students to complete the instruction as homework by watching a video provided by the teacher and to use class time to apply and practice the concepts learned outside of the classroom when they perform enriched tasks, projects, and group assignments in the classroom.

Lee intended to adopt Moodle, an online learning environment, in the following school year. This website allows teachers to post notes, to make announcements and provide updates to students, and to post tutorial videos for students to watch. Students can also interact and discuss with their peers. Lee’s hope with the introduction of these new websites was to provide additional help and support for students to revise notes, to collaborate with peers, to be better prepared for the next math class, and ultimately to construct linear relations knowledge and advance in their learning of mathematics.

These intentions of teachers to integrate online learning environments into their teaching resonates with Martin-Blas and Serrano-Fernandez (2009) assertion that the online learning
community, implemented in the Moodle platform, helped both students and teachers to have a virtual space where they can share knowledge through different kinds of supervised activities, chats and forums. This online environment helped students reinforce their abilities and content knowledge.

**5.2.3.5 Benefits of integrating technology in teaching Linear Relations**

The results from this study suggest that the integration of technology in the Linear Relations Unit was most beneficial: a) to assist teachers to provide the instant feedback necessary to inform and guide instruction; b) to help struggling students move forward in their learning when they did not master the prerequisite skills required to build upon a new math concept and to help them develop math interpretative and problem solving skills; c) to teach students to verify and validate their answers or check for their correctness and to avoid relying only on the visual aspect of mathematics; d) to differentiate instruction and address different learning styles and skills making abstract content more tangible; and e) to base instruction on big ideas.

**a) Immediate feedback**

There is evidence from this study that the teachers incorporated technology to engage students in activities that provided them with instant feedback used in guiding and adjusting the classroom instruction in order to help students construct knowledge in meaningful way. All three teachers used Gizmos hands-on activities with students working individually in a computer lab, as well as with whole group instruction exploring linear relations concepts and practicing them by answering related questions. The instant feedback provided opportunities for students to assess the progress that they were making and to inform them of specific areas that required further studies. The instant feedback was also used by teachers as an assessment for learning, as it informed them about the level of students’ understanding and helped them making appropriate
instruction decisions. For instance, Shannon and Lee used clickers on a regular basis in their classes to provide real time feedback, to monitor their students’ understanding, and to alter the pace and nature of instruction by adding or removing questions and examples to their lesson plans as needed. Lee expressed that this technological tool encouraged student participation in the learning process as students engaged in answering the questions without the fear of getting the answer wrong.

From Lee’s point of view, the clickers helped improve classroom management, as students became interested in doing the questions. Consequently, students’ participation and interest helped them gain confidence in their ability to do mathematics and contributed to enhance students’ abilities to construct meaning and knowledge. This finding is in agreement with Li’s (2007) discovery that technology could increase students’ confidence in their mathematics abilities.

Shannon used IXL software to identify the weaknesses and strengths of her students and to provide drill and practice exercises for those students who needed to improve their basic skills in order to move on to a higher level of mathematics understanding. Likewise, Anna used graphing calculators to ask students to draw the line of best fit, to find its equation, and to determine the general trend of the data. She circulated around the class to assess students’ work and to gauge their understanding through effective questioning techniques with the clearly stated objective to help them construct knowledge about linear relations.

b) Developing problem solving skills

Within this study, technology was used to help students, who did not master the prerequisite skills, to build upon linear relations concepts. This use of technology helped them move forward in their learning. Lee expressed that students needed to be proficient and well-
equipped with interpretative math skills rather than basic skills, such as creating graphs or drawing the line of best fit. In his opinion, technology could efficiently and accurately produce these graphs and this fact helps remove some barriers that stand in the way of learning and prevent some students from moving along in their learning.

Anna mentioned that technology could help students perform basic calculations like adding fractions or dividing numbers and, as a consequence, students do not need to master these basic skills. Instead, she suggested that students could spend their class time solving enriched tasks related to real life situations in order to learn and practice problem solving skills, expand their thinking, and advance their learning.

These findings are consistent with the conclusions of Alagic (2003) that technology applications “empower students with a level of mathematical power they cannot achieve without technology and, if used appropriately, have a great potential for stimulating higher order thinking when freed from the mechanics of calculating” (p. 391). In her opinion, “performing calculations; collecting, analyzing, and representing numeric information; creating and using models and simulations; representational scaffolding higher levels of abstraction, and solving problems with mathematical premises are just some of the possibilities for the hands-on, minds-on learning experiences fostered through today’s interactive technology applications” (p. 390).

Shannon remarked that students with access to technology have access to content right away. From her point of view, if students want to perform basic operations or to solve simple problems, such as solving a linear equation, they could find online applications to do it for them. She felt important to use technology to provide thorough and comprehensive explanations of the math concepts and to help students develop higher level thinking skills and move into the interpretative nature of mathematics, rather than to insist on performing manually basic
computational skills. This finding is consistent with the finding of Lee & McDougall (2010) that “…when mechanical operating issues are overcome, the graphing calculator provides students with a common starting point, which enables teachers to focus on discussion about mathematical concepts (p. 871).

c) Checking and validating solutions

Although all three teachers insisted that the visual aspect of technology is a good addition to the learning process, Shannon and Anna suggested that a heavy reliance on it could be detrimental to students. This is why both teachers recommended that students should be taught to verify and validate their answers, to check for their correctness, and to avoid relying only on the visual representations, which could be perceived incorrectly. In Anna’s opinion, when using technological tools, students should be taught to reason their solutions, justify their answers, and check if they were accurate and valid. They must recognise that technology could lead to incorrect answers or it might provide incomplete solutions if used inappropriately.

d) Differentiating instruction

In this study, teachers used technology to differentiate instruction. Shannon and Lee expressed that technology plays a significant role in assisting teachers to deliver different types of instruction to address students’ individual needs, learning styles, and skill levels. For example, utilising IXL, Shannon could identify students’ strengths and weaknesses and she could prepare technology related material to target the individual needs of all students in the classroom.

All three teachers agreed that discussing real life examples from websites are convenient. In addition, Shannon and Lee explained that technology gives students a chance to work at their own pace and experience success at their own level, which gives them a sense of accomplishment and personal growth.
Moreover, Shannon explained that using motion detectors to help students understand that the rate of change represents the average speed is a good approach for kinaesthetic learners.

Lee emphasised that simulations and matching graphs engaged kinaesthetic learners to explore and understand the distance-time graphs. Lee also acknowledged that Smart Board and the Mobi View tablet, which displays colourful graphs, are appropriate for visual learners. These findings support Kaput et al. (2008) assertion that technology attracts teachers because it provides the possibility to simulate, visualize, and model mathematical concepts and make them accessible to all types of learners.

e) **Basing instruction on big ideas**

All three teachers agreed that technology helped them deliver time efficient lessons tied to real world experiences. By integrating technology, these teachers designed lessons anchored in realistic context, provided authentic tasks to be analysed, solved, and interpreted, and planned lessons in a structured way under the umbrella of big ideas. Lee explained that the instantaneous aspect of technology and the removal of tedious math components makes learning more focused and direct. He demonstrated that, by getting instantaneous graphs utilising technology rather than by graphing them manually, gives more time to students to concentrate on the main ideas, such as seeing the shapes and seeing the changes in the graphs. Additionally, during my observations, Anna used Smart Board, motion detectors, TI-Nspire graphing calculators, and Geometer’s Sketchpad to provide learning opportunities through real life experiences and simulations, to help students construct knowledge through investigation and experimentations, and to assure that they make meaningful connections with previously learned concepts. She stimulated students to reason and justify their actions and solutions, to think logically, and to connect various
representations of linear relations, as well as to connect the key concepts from this unit to the Geometry Unit.

The above finding, that teachers used technology to interconnect mathematical concepts and units, supports Alaric’s (2003) conclusion that the appropriate use of technology can make it easier for teachers and students to bring together multiple representations through intermediate representations or explicating links among different representations of some mathematical concepts. It also reflects NCTM (2000) recommendation that, technology could be used to "blur some of the artificial separations among some topics in algebra, geometry, and data analysis by allowing students to use ideas from one area of mathematics to better understand another area of mathematics" (p. 26).

Shannon approached all important concepts in the Linear Relations unit by using investigative tasks and explorations of the distance-time graphs produced by the computer-based motion detectors. With this activity, she introduced the concept of slope as a constant rate of change of the distance with respect to time, discussed the meaning of the y-intercept, had students write the equation of the line of best fit, discussed the type of correlation, and helped students make generalisations, predictions, and interpretations for different graphs.

5.2.3.6 Summary

In summary, the results of this study reveal that, when integrating technology into teaching the Linear Relations Unit, teachers used various instructional strategies, built on students’ prior knowledge, provided appropriate scaffolding, and engaged students through effective questioning techniques to help knowledge construction. In addition, the integration of technology assisted teachers to provide real time feedback to inform students about their progress and also informed teachers about their students’ levels of understanding. This aspect
gave opportunities to teachers to make appropriate instruction adjustments to help students progress in the learning process. Technology was used to help struggling students to move forward in their learning even when they did not master the prerequisite skills required to build upon linear relations concepts.

Furthermore, the integration of technology supported students to check and validate their answers, especially when visual representations, which could be perceived inaccurately, were involved. Moreover, by integrating technology, teachers made learning more focused and direct, as instruction was based on big ideas. The instantaneous aspect of technology permitted students to draw quick and precise graphs and to concentrate on the main ideas, such as seeing the shapes and seeing the changes in the graphs. The removal of tedious math components, such as spending time drawing graphs or computing manually, helped teachers devote more time to developing mathematical understanding, reasoning, and interpretations of linear relations math concepts.

Finally, the integration of technology in teaching the Linear Relations Unit assisted teachers to differentiate instruction to address students’ individual needs, learning styles, and skill levels. Thus, technology gave students as a chance to work at their own pace and experience success at their own level, which gave them a sense of accomplishment and personal growth.

This study did not investigate the impact of technology integration on students’ conceptual understanding. Some researchers, such as Adam (1997) and O’Callaghan (1998), found that the implementation of instructional technology (graphing calculators and computer algebra systems) had a positive effect on students’ conceptual understanding. Others, such as Goos et al. (2003) and Hativa (1988), found that similar instructional technologies had no effect. In Hativa’s study, a computer-based curriculum confused the student more than it helped. While
my qualitative investigation did not provide a definitive answer to whether technology integration improves students’ conceptual understanding and facilitates construction of content knowledge, the thick descriptions illustrate why technology may be effective in certain situations, as well as showing some limitations.”

5.2.4 Research Question 4:

How do teachers’ views on the role of technology and their everyday practice help us to better understand the facilitation of mathematical communication?

As stated in the Revised Ontario Curriculum Grade 9 and 10 Mathematics (2005), “Communication is an essential process in learning mathematics. Through communication students are able to reflect upon and to clarify ideas, relationships, and mathematical arguments” (p. 16). In this study, teachers did not perceive technology as playing a significant role in facilitating the development of students’ mathematical communication. All three teachers expressed the desire to learn more about how to integrate technology to help students increase their ability to communicate their actions and their understanding of the mathematical concepts orally and in writing.

However, to a certain extent, they used technology to enhance their students’ mathematical communication. For instance, Shannon and Lee explained that motion detectors, used to produce distance-time graphs, targeted the development of mathematical communication skills in addition to contributing to help increase students’ conceptual and analytical understanding of graphs. Students needed to provide precise mathematical instruction, such as go forward, move over, stop, and go faster, to their peers to make them walk in such a way to match a shown graph.
5.2.4.1 Developing mathematical vocabulary

During classroom observations, I noticed that Shannon included communication components in the activities incorporating technology. For example, in the “Height versus Arm Span Activity”, she encouraged students to use the mathematical terminology learned in the Linear Relations Unit (domain, range, increasing, decreasing, linear, nonlinear, rate of change, x-intercept, y-intercept, prediction, outliers, trend, interpolation, extrapolation, and correlation) to analyse and describe the graphs obtained with the TI-Nspire graphing calculators.

Lee stated that the integration of technology motivated students to participate more actively in the lesson, to explain their solutions, to analyse and discuss their peers’ answers, and to engage in rich mathematical dialogue. In Lee’s opinion, the visual aspect of technology and the display of graphs, images, and equations engage students in expressing mathematical ideas by using words in oral communication and symbols and numbers in written communications.

This study shows that the integration of technology, such as the use of clickers and the Smart Board, provided all students equal opportunity to engage in meaningful learning and to practice written and oral communication. Simulations using motion detectors and Gizmos activities encouraged student interaction and stimulated them to learn and practice terminology and vocabulary specific to linear relations, which is necessary to communicate precisely with peers to provide appropriate instructions. Furthermore, technology provided students the possibility to move with fluency among various representations of linear relations, such as graphical, tabular, and algebraic, and consequently to practice communicating their mathematical thinking coherently and to refine their mathematical communication skills.
5.2.4.2 Presenting solutions

Lee also described that, using the clickers, he encouraged communication by asking students to clarify and explain the reasons behind their answers, to present their solutions to their peers, to think about alternate ways of solving the problem, and to show their work step-by-step. Also, he expressed that the integration of technology helped students make connections between words (for example, rate of change, y-intercept, equation of the line of best fit) and diagrams or graphs, which in turn, enhanced students’ mathematical understanding and helped build mathematical communication skills. Lee also felt that wikis and forum discussions foster the development of communication skills by providing opportunities for students to discuss different mathematical topics by using appropriate terminology and specific vocabulary.

Furthermore, Anna remarked that students’ communication skills are poor and required improvement. She indicated that her students were able to perform math tasks utilising technology (e.g. determine the coordinates of the point of intersection of two lines by using TI-Nspire calculator) but they found it difficult to explain the math behind their actions or to justify their solutions. She believed that technology might help students communicate their understandings or findings and she was willing to learn more about how to help students develop mathematical communication skills by integrating technology.

5.2.4.3 Organizing concepts

Anna used the Frayer Model on the Smart Board to provide students with the opportunity to organise the key elements and concepts, to make connections among mathematical ideas, to provide examples and counterexamples, to share ideas, to practice specific mathematical vocabulary, and to improve mathematical skills. In her opinion, the integration of technology (Smart Board in this particular case) encouraged students to take more responsibility, as they
were asked to go to the board to present their assigned tasks to the whole class. She also used the Smart Board to reinforce definitions of key concepts from the Linear Relations Unit and to solve problems by using matching activities. She indicated that the immediate feedback provided by the Smart Board software prompted the development of mathematical communication skills.

**5.2.4.4 Summary**

In summary, the three math teachers used technology on a small scale to prompt the development of mathematical terminology and vocabulary and to foster students’ communication skills in the Linear Relations Unit. They believed that simulations with motion detectors target the development of students’ communication skills, as students need to provide precise mathematical instructions to their peers to match a shown graph.

As proposed in the Ten Dimensions of Mathematics Education framework, the findings of this research are in agreement with the eighth essential component of a successful mathematics education program: Students’ Mathematics Communication (McDougall, 2004). This component specifies that teachers should engage in practices that increase students’ oral and written communication skills.

The three math teachers used various opportunities to encourage students to communicate verbally and in writing by appropriately using the vocabulary and math symbols specific to linear relations. They provided opportunities for students to collaborate in order to create Frayer diagrams, which help organize the key elements and concepts of linear relations, compare and contrast their characteristics, provide examples and counterexamples, make connections, and practice mathematical vocabulary and symbols. Teachers also created activities that engaged students in math related discussions and interpretations, which made use of specific linear relations terminology. They provided feedback to students on their use of conventions, symbols,
and specific terminology and provided real life problems, selected from the Internet to be analysed and discussed. Teachers provided opportunities for students to work in groups to investigate and explore linear relations concepts, to solve problems, to discuss their findings and solutions, and to practice oral and written communication skills. These teachers created supportive and respectful learning environments that made their students comfortable to present solutions and ask questions.

This study did not focus on students’ perceptions of the role of technology in fostering the development of mathematical communication skills but instead deeply examined the instructional strategies that math teachers take when teaching the Linear Relations Unit using technology.

5.3 Major Findings

Based on the data collected from this study, there were three major findings, related to teachers’ perceptions about the integration of technology in teaching the Linear Relations Unit to grade 9 Applied level students, who have not been successful, have attendance problems, experience some language challenges, and lack effort and engagement. The major findings are summarised as follows:

1) Teachers identified that integrating technology assisted them to create interactive and dynamic learning environments, which, in turn, helped make the content more meaningful to students. The visual and dynamic aspects of technology helped students to develop better conceptual understandings. In the context of the Linear Relations Unit, students saw the immediate effects of changing the key parameters when they manipulated the line to have a different slope. The teachers also regarded obtaining quick and easy examples of linear relations from the Internet (such as information
about cell phone plans) as another significant aspect of technology-specific learning. In addition, simulations engaged students in active learning and helped them to visually comprehend abstract math concepts.

2) Teachers emphasised that, by providing real time feedback, technology supported and guided their instruction and assisted them to closely monitor student understanding and to track their progress.

If 90% of students got the correct solution the teacher could acknowledge the multiple ways of arriving at the right answer and invite different students to present their different solutions to the same problem. With the realisation that there is more than one way to solve a question correctly, this learning aspect could encourage students to take on a problem with more confidence. Thus, in this context, technology assists teachers in encouraging students to explain their solutions and their thinking processes, as well as to model for their peers how to approach and solve a particular math problem.

If only 50% of the students get a question right, then the teacher learns that s/he needs to do additional questions with the class to clarify the concept. By getting immediate feedback, the teacher can modify his/her original lesson plan, ask him/herself questions to modify his/her teaching (“What could I have done differently? What misconceptions are the students showing?”), and add new questions and explanations until s/he is satisfied with the number of students with the correct answer. In this context, technology helps to improve both the whole student learning process and performance.
3) Teachers shared that technology helped to improve classroom management and the level of students’ engagement. Teachers expressed that one of the greatest benefits of using technology in the classroom was their ability to move around the classroom to monitor student understanding and progress while at the same time, delivering the math content and managing the classroom effectively. When technology was integrated, the math content was more visual, understandable, and relevant to students’ everyday lives. Teachers visited websites to gather information from outside of the classroom and applied it spontaneously into their lessons. They used multimedia to present math concepts effectively and to bring outside information that was relevant and beneficial to students’ daily lives. Teachers also instantly identified students who had not answered the questions and asked them if they were confused about a particular concept. In this instance, while teachers were addressing one student, the others were able to continue answering the questions they were working on.

5.4 Implications of Findings for Future Research

The present study reveals that, to a certain extent, the participant math teachers integrated technology to help students develop and practice mathematical communication skills. However, the teachers felt that they did not have enough technological and pedagogical knowledge to integrate technology in order to help students develop mathematical communication skills and practice mathematical vocabulary and terminology. These skills, vocabulary, and terminology are necessary to investigate and discover new mathematical understandings, to comprehend a new mathematical topic, to explain the thinking process, to present solutions to the math problems, and to dialogue with peers and teachers. Research has been conducted to identify strategies to foster mathematical communications skills.
There is scarcity of information about how to integrate technology in secondary mathematics education in order to foster the development of students’ mathematical communication skills. Therefore, future research might look at investigating secondary teachers’ practices of integrating technology to enhance students’ mathematical communication skills. Some research questions could be: How does the integration of technology in secondary mathematics education contribute to enhance students’ mathematical communication skills? How can educators maximise the development of mathematical communication skills and expand students’ mathematical vocabulary in a technology driven environment?

While this study looks into teachers’ perceptions about the role of technology in the development of student tasks in the context of the grade 9 Linear Relations Unit, it was not intended to investigate if and how student tasks integrating technology impact student learning. The existing research on this topic has mixed results. A number of researchers found that using information technology in the classroom benefits students’ learning in mathematics (Isikal & Askar, 2005; Olkun, Altun, & Smith, 2005; Sinclair, 2004). Findings from Lagrange (1999) suggested that students did not express that technology integration (computer algebra) supported their understanding of mathematics even when they liked it. Based on these contradictory findings, general conclusions about students’ learning of mathematics in a technology-driven environment cannot be made. Future research should focus on the students’ perceptions about if and how student tasks that integrate technology impact their learning. A research question could be: How do students perceive student tasks that integrate technology to impact their learning?

The present study explores secondary mathematics teachers’ perceptions of the role of technology and investigates technology based strategies used to help students construct new content knowledge in the context of the Linear Relations Unit in grade 9 mathematics. However,
This study did not investigate the impact of technology integration on students’ conceptual understanding. Further research could focus on students’ perceptions about the role of technology in facilitating the construction of content knowledge in the context of the Linear Relations Unit. A research question could be: In the context of the Linear Relations Unit, what are students’ perceptions about the role of technology in the construction of content knowledge?

This study investigates teachers’ perceptions about the role of technology in enhancing students’ mathematical communication skills and examines the instructional strategies that math teachers take when teaching the Linear Relations Unit with technology. However, it did not focus on students’ perception of the role of technology in fostering the development of students’ mathematical communication skills. Therefore, new research could be conducted to find out about students’ perceptions about the role of technology in enhancing their communication skills and expanding their mathematical vocabulary. A new research question could be: What are students’ perceptions about the role of technology in helping them better communicate mathematically?

Moreover, this study is limited to exploring teachers’ perceptions about the role of technology and the way their perceptions are reflected in everyday teaching practices in the Linear Relations Unit in grade 9 mathematics. Nevertheless, investigations could be conducted in any grade and for any mathematics unit to reveal best teaching practices in a technology driven environment at different grade levels in order to enhance students’ understanding of mathematics in each particular context.
5.5 Suggestions to Mathematics Teachers Integrating Technology

From this study, a few recommendations can be made to teachers integrating technology or thinking about integrating technology in the Linear Relations Unit. The following ideas would be helpful and necessary for teachers:

- Be prepared to learn from your students about how to integrate technology efficiently. Give them freedom to investigate how to use technology to solve math problems.

- Use technology to provide real-time feedback. It will provide information to your students about their learning progresses and it will inform you about your students’ levels of understanding.

- Teach students to check and validate solutions provided by technology and to avoid relying blindly on answers generated by technology.

- Help struggling students move forward in their learning, even when they did not master a prerequisite skill, by allowing them to use technology to perform basic computations and to draw precise and accurate graphs.

- Integrate technology to convey meaning to abstract math concepts and to help students connect words to images and graphs.

5.5.1 Other suggestions based on my experience after conducting the study

- Attend professional development workshops and seminars, as well as explore technological tools on your own to investigate their potential and discover unrevealed features.

- Integrate technology to differentiate instruction, to meet the individual needs of all students, and to support their different learning styles and skills.

- Incorporate technology to create interactive and dynamic learning environments inspired by real-life situations selected from websites, to provide opportunities for students to
investigate and explore by using simulations, to organise and interpret data by using multiple representations (graphical, tabular, algebraic, and verbal descriptions), and to connect symbolic and visual representations of abstract math concepts (such as rate of change, initial value, and point of intersection of two lines), all of which make the content meaningful to students.

• Provide students with the possibility to move among various representations of linear relations (graphical, tabular, algebraic, and verbal descriptions), to practice communicating their mathematical thinking coherently, and to refine their mathematical communication skills.

• Communicate with other teachers who are enthusiastic about integrating technology in mathematics (and other subjects). They will influence you positively, encourage you, and assist you when you need support.

• Integrate technology to provide equal opportunities for all students to engage in meaningful learning and to practice written and oral communication.
REFERENCES


Appendix A.  Stories with Slopes.

Stories with Slope

One sunny June morning, Leslie decided to go for a jog. Use the graph below to write a creative story about the events that occurred during her jog. Include as much mathematical terminology as possible.

When Leslie returned from her jog at noon, she went to the fridge and took out a bottle of water. The graph to the right represents the volume of water in the bottle over time. Write a story about the volume in the bottle. Include as much mathematical terminology as possible.

At 8:00 p.m., Amir turned the water on to fill his cylindrical tub.

- The water in the tub rose at a rate of 4 cm per minute.
- When the water reached a height of 36 cm, he pulled out the plug.
- The water drained at a rate of 6 cm per minute.

Draw a graph showing the height of the water in the tub.

What time was it when the tub was drained?
Tell Me A Story and Lines

A communication level will be assigned based on the correct use of mathematical symbols, labels, and conventions.

A(2) 1. The graph describes Rami's walk with a motion detector. Tell the story that describes this graph. Use distances and times in your story.

2. A story is described in each question. Sketch the graph that describes the story in the screen provided.

A(2) Begin 5 m from the wall. Walk toward from the wall for 5 seconds. Stop for 5 seconds. Run back to your starting position. Stop.

A (2) Begin at the wall. Walk very slowly away from the wall for 3 seconds. Increase your speed for 3 seconds. Stop for 3 seconds. Walk very slowly toward the wall for 3 seconds. Run back to the wall. Stop.

A(3) 3. Jen tried her new snowboard at the One Plank Only Resort. The graph shows her first run. Tell the story that describes Jen's first run.

...
Stories with Slope (continued)

At the beginning of September, Rick weighed 90 kilograms. He decided to start an exercise plan and lose some weight. The graph below represents Rick's weight over the next 40 weeks. Use the graph to write a story about Rick's weight loss. Include as much mathematical terminology as possible.

![Graph showing weight over time]

The Ace Taxi Company charges a flat fee of $2.50 plus $0.50 per kilometre. Draw a graph to show the cost of a cab ride between 0 and 8 km long.

![Graph showing cost vs distance]
Appendix B. The Height versus Arm Span Activity

Height versus Arm Span

What is the relationship between the height of a person and the length of the person’s arm span?

Measure the height and arm span of students.
1. Sketch a graph predicting the relationship between the height of the person and the person’s arm span:

2. Data Collection

<table>
<thead>
<tr>
<th>Arm Span (cm)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Make a scatter plot using a graphing calculator. Sketch below.

4. Use first differences to estimate a rate of change.
5. Estimate the y-intercept (starting point.)

6. Find a trend line for the data using the estimated rate and y-intercept.

7. Graph your trend line over the scatter plot and adjust the parameters y-intercept and rate of change, if necessary, for a better fit.

8. What are the units of slope for the trend line?

9. What is the meaning of the y-intercept in the trend line?

10. Use the trend line to determine how tall a person is with an arm span of 137 cm. Write an equation and solve in at least three ways.

11. Use your trend line to determine what arm span a 214 cm tall person would have? Write an equation and solve in at least four ways.

12. Make a general statement about the relationship between the height of a person and the person’s arm span.

13. Which is the independent variable and which is the dependent variable in this problem situation?
Appendix C. Using Linear Relations GSP Assignment

LINEAR RELATIONS GSP ASSIGNMENT

Due Date: ________

Overview
- A separate Geometer's Sketchpad (GSP) document (file) is required for each question. Each file must be named in the following way: Question#_YourName.gsp. (e.g. Question2_AnisahMostafa.gsp, Question3_SalmanHaider.gsp).
- A single folder called "GSP Assignment" must be used to contain all GSP files.
- When the assignment is completed, both an electronic copy and a paper copy must be submitted.

The electronic copy is submitted by copying the folder "GSP Assignment" to
1: in YourTeacherLastName_GSP Assignment YourName, where YourTeacherLastName represents your teacher's last name and YourName represents your name.

Questions
As outlined above, create a separate GSP document for each of the following questions.

1. Create an example that illustrates why any point lying on an axis must have a c-o-ordinate that is equal to zero. Show examples of points on both axes (i.e. on both the x-axis and the y-axis).

2. Create an example that illustrates why:
- lines that lean to the left (i.e. slope downward to the right) have negative slope and a decreasing y-co-ordinate
- lines that lean to the right (i.e. slope upward to the right) have positive slope and an increasing y-co-ordinate
- horizontal lines have zero slope and a constant y-co-ordinate
- vertical lines have undefined slope and a constant x-co-ordinate

3. Create an example that illustrates why parallel lines must have equal slopes and why perpendicular lines must have negative reciprocal slopes.

4. Create an interesting picture that consists entirely of line segments. For this question, you must not use the line tool to produce the line segments. Instead, you must generate each line segment by using an equation of a line with a restricted domain. (This will be demonstrated in class.)

Requirements
(a) All your slopes and y-intercepts must be in integer or fraction form.
(e.g. numbers such as ... -3, -2, -1, 0, 1, 2, 3 .... or \( \frac{1}{2}, \frac{3}{2}, \) etc)
(b) Your picture must include at least two pairs of parallel lines.
(c) Your picture must include at least two pairs of perpendicular lines.
(d) Your picture must include at least three horizontal lines.
(e) Your picture must include at least three vertical lines.

Note: To enter the equation of a vertical line you need to select "Graph," then "Plot New Function," then "Equation" and change the default option "y = f(x)" into "x = f(y)."

Helpful GSP Hints
- To define a c-o-ordinate system, select "Graph" from the menu bar, then choose "Show Grid."
- To generate a line with an equation, select "Graph" then choose "Plot a New Function."
- To create restrictions on a line, right-click on the graph of the line, select "Properties" then choose "Plot." Enter the lowest and highest values of the x-co-ordinate.
- The colour and/or thickness of a line can be changed by right-clicking on it and selecting "Thick" and/or "Colour."
- Make adjustments to your equations or restrictions to ensure that the lines connect to form your desired picture.
Appendix D. Linear Relations Project – Washing Machine

Linear Relationships Rich Task

A Washing Machine Problem

NAME: ____________________________

Due: June 5th, 2012
A new dry cleaning machine has been designed. At the end of each cleaning cycle, the dry cleaning liquid will be reduced by evaporation and condensation. Each time the machine is used there is a loss of 2% of the initial amount of liquid filled.

1. Initially the machine is filled with **1000mL** of dry cleaning liquid. How much liquid will remain after the machine has been used once, twice, ..., ten times?

<table>
<thead>
<tr>
<th>Number of washes (n)</th>
<th>Amount of liquid left mL (a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000mL</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<td>6</td>
<td></td>
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<tr>
<td>7</td>
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<td></td>
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<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
2. Develop an equation that uses \( x \) and \( y \) to determine the amount of liquid left in any number of washes.

<table>
<thead>
<tr>
<th>Number of washes ((x))</th>
<th>Amount of liquid left ((y))</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000mL</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<tr>
<td>10</td>
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</tbody>
</table>

*HINT: Look at the difference each time. Explore what you can do to the number of washes to achieve that difference.
3. Draw a graph of your equation. Find the x and y intercepts and the gradient of the line.

Use TI-Nspire to check your graph.

4. What is the quantity of liquid after 45 uses?

5. After how many washes is the amount of liquid exhausted?
A Washing Machine Problem

6a. If the original quantity of liquid was 2000mL, how would your equation change?

b. Find the quantity of liquid after 45 uses with this new equation.

c. Graph this equation by using TI-Nspire. Sketch the graph on the graph provided below.

d. After how many washes is the amount of liquid exhausted?
7a. If the original quantity of liquid was 5000 mL, how would your equation change?

b. Find the quantity of liquid after 45 uses with this new equation.

c. Graph this equation by using TI-Nspire. Sketch the graph on the graph provided below.

d. After how many washes is the amount of liquid exhausted?
8a. Make a comparison of your answers to Questions 4, 6b and 7b. What do you notice?

<table>
<thead>
<tr>
<th>Slope 1 (for 1000mL)</th>
<th>y-intercept (for 1000mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope 2 (for 2000mL)</td>
<td>y-intercept (for 2000mL)</td>
</tr>
<tr>
<td>Slope 3 (for 5000mL)</td>
<td>y-intercept (for 5000mL)</td>
</tr>
</tbody>
</table>

b. What pattern do you notice about the slopes?

c. Using a TI-Nspire graphing calculator, graph all three equations in the same window. What do you notice? Write a conjecture about the relationship between the slopes of parallel lines.

d. Describe the meaning of the slope and the y-intercept in each case.

d. Use your TI-Nspire graphing calculators to verify your conjecture. Write the equation of four different pairs of parallel lines. Then, by using the graphs determine the slopes and check to see if they are parallel. Record your results below.

<table>
<thead>
<tr>
<th>Slope 1</th>
<th>Slope 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

e. Does this confirm your conjecture? _____
Explain.

END PROJECT
Appendix E. Stacking Cups

Stacking Cups

Student Materials
1. Get a stack of Styrofoam cups.

a. Find the height of 1 cup. Find the height of a stack of 2 cups. Find the height of a stack of 3, 4, and 5 cups. Make a table of values of the number of cups versus the height of the stack. Using your table, plot points on a graph where the x-coordinate is the number of cups and the y-coordinate is the height of the stack.
b. Develop an equation for the height of x cups using your work from part a.
c. What is the slope of this line and what is its physical significance?
d. What height does the formula give when there are zero cups? What is the physical significance of the vertical intercept?
e. Determine whether a stack of 100 cups will fit under the table you are working on. Write a few sentences describing how you determined this.
f. Determine the maximum number of cups that can be put in a stack under your table. Write a few sentences describing how you determined this.
g. If you have 10 stacks of cups, each stack about 3 feet long, about how many cups do you have?

Using a centimeter ruler, carefully measure the stacks to the nearest 0.1 cm. Record the data in the table.

<table>
<thead>
<tr>
<th>Number of cups</th>
<th>Stack height</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>29</td>
</tr>
</tbody>
</table>

As data are added, the points will plot and a best fit line and the equation of that line will appear on the graph.

Now let's consider some different cup types

For each cup shown, how will the graph change compared to our original cup? Construct a graph showing all three cups.

Determine the equation of the line of best fit for all three situations.
Appendix F. Where is the Connection?

Instructions:
- Chose the positive initial value of a linear relation. Write that value as the point coordinates on the sheet provided.
- Chose the values for rise and run. Write these values and the rate of change on the sheet provided.
- On the grid paper plot your initial value.
- From the initial value, draw the Rise and Run and plot another point. Record the coordinates of that point.
- From that point draw again the Rise and Run and plot another point. Record the coordinates of that point.
- Repeat this until you have plotted 6 points. Record the coordinates of each point.
- Draw a line through all the points. Is this relation partial or direct variation?
- Organize the coordinates of all plotted points, included the initial point in the table of values.

Part B: Graphing Calculator
- Enter the table of values into the graphing calculator.
- Construct the scatter plot graph.
- State the equation of the line using the regression. Record the equation of the line on the sheet provided.
- Answer the questions.

<table>
<thead>
<tr>
<th>Initial Value</th>
<th>Rise</th>
<th>Run</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table of Values:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equation of the line: ________________________________________________________________________

Is this a partial or direct variation? Justify your answer.

What is the connection between the rate of change and the equation?

What is the connection between initial value and the equation?

Name: ____________________________
Did I really get it?

Chose the equation from the envelope. State the

Initial value: ___________ Rise: ___________ Run: ___________

Is this a partial or direct variation? Justify your answer.

______________________________

Graph the relation on the grid paper.

Did I really get it?

Chose the equation from the envelope. State the

Initial value: ___________ Rise: ___________ Run: ___________

Is this a partial or direct variation? Justify your answer.

______________________________

Graph the relation on the grid paper.

Did I really get it?

Chose the equation from the envelope. State the

Initial value: ___________ Rise: ___________ Run: ___________

Is this a partial or direct variation? Justify your answer.

______________________________

Graph the relation on the grid paper.
<table>
<thead>
<tr>
<th>Initial Value</th>
<th>Rise</th>
<th>Run</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>1</td>
<td>( y = \frac{2}{5}x - 1 )</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>( y = \frac{-2}{5}x + 1 )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>( y = \frac{3}{4}x )</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>4</td>
<td>( y = \frac{-3}{4}x )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
<td>( y = 3x - 2 )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>6</td>
<td>( y = -3x + 4 )</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
<td>7</td>
<td>( y = \frac{1}{2}x )</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
<td>8</td>
<td>( y = \frac{-1}{2}x )</td>
</tr>
<tr>
<td>-3</td>
<td>-4</td>
<td>9</td>
<td>( y = x - 3 )</td>
</tr>
</tbody>
</table>
Appendix G.  Create a picture using TI-Nspire Calculators

Instructions:
- Choose Graph mode on your Nspire.
- Enter at least 5 different linear equations (one at a time) in your entry line.
- Before you turn off your calculator, show your picture to your teacher for evaluation.

Group A

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Without doing any calculations, determine whether this relation is linear or not. Explain how you know.

Is this a direct or partial variation? Justify your answer.

Determine the formula/equation of this relation:
Appendix H. Modelling Linear Systems

Objective: Experiment with a system of two lines representing a cat-and-mouse chase. Adjust the speeds of the cat and mouse and the head start of the mouse, and immediately see the effects on the graph and on the chase. Connect real-world meaning to slope, y-intercept, and the intersection of lines.

Assessment Questions:
1. The graph below represents a chase between a cat and a mouse. What statement is true about this chase?
   - A. The cat and the mouse started at the same position.
   - B. The cat and mouse run at the same speed.
   - C. The cat runs faster than the mouse.
   - D. The mouse runs faster than the cat.

   ![Graph of cat and mouse chase]

2. The graph below represents a chase between a cat and a mouse. For how much time did the chase last before the cat caught the mouse?
   - A. about 1.5 seconds
   - B. about 2 seconds
   - C. about 6 seconds
   - D. about 18 seconds

   ![Graph of cat and mouse chase]

3. The graph below represents a chase between a cat and a mouse. How far did the cat run before catching the mouse?
   - A. 3 feet
   - B. 4 feet
   - C. 15 feet
   - D. 20 feet

   ![Graph of cat and mouse chase]
Appendix I. Exploration: Point of Intersection

May 14, 2012

Finding the Point of Intersection

Student Name:

Practice Questions:

1. Two health clubs, Super Fit and Body Plus, offer monthly memberships. The total monthly cost for each club is represented by the graphs below. Which of the following is true?
   a. Body Plus is always cheaper.
   b. Super Fit is always more expensive.
   c. Super Fit is cheaper if the number of visits is fewer than 7.
   d. Body Plus is more expensive if the number of visits is greater than 9.

2. Maria wants to rent a snowmobile for a day and considers two rental companies. The relationship between the total cost of renting from Trails-R-Us and the number of kilometres travelled is represented by the graph below.

   Off-Roads charges a flat rate of $90 for a day with unlimited kilometres.

   At how many kilometres is the total cost the same at both rental companies?
   a. 70 km
   b. 80 km
   c. 90 km
   d. 100 km

   Explain your work:
Appendix J. Interpolations and Extrapolations

Examine the scatter plots for data related to weather at different latitudes. The Gizmo includes three different data sets, one with negative correlation, one positive, and one with no correlation.

1. Which of the following scatter plots would have a trend line with a positive slope?

   - A. Scatter plot A
   - B. Scatter plot B
   - C. Scatter plot C
   - D. Scatter plot D

2. Which of the following could be the equation of the trend line of this scatter plot?

   - A. \( y = 0.7x + 14 \)
   - B. \( y = 0.7x - 14 \)
   - C. \( y = -0.7x + 14 \)
   - D. \( y = -0.7x - 14 \)
3. If you graphed the number of times you go to the movies as one variable and the total amount of money you spend on movie tickets as the other variable, what type of slope would you expect the trend line to have?
   - A. positive
   - B. negative
   - C. near zero
   - D. Cannot be determined.

4. If an equation of the trend line is \( y = 2.41x - 67.66 \), where \( x \) is degrees north latitude and \( y \) is average annual snowfall in inches, estimate the annual snowfall in Dodge City, Kansas, which has a latitude of 38 degrees north.
   - A. 23.92 inches
   - B. 32.07 inches
   - C. 108.07 inches
   - D. 159.24 inches

5. In the Gizmo, the equation for the trend line between latitude and average temperature is \( y = -1.27x + 104.28 \) (\( x \) = degrees north latitude and \( y \) = average temperature in degrees Fahrenheit). Estimate the average temperature in Omaha, Nebraska, which has a latitude of 41 degrees north.
   - A. 5.47 degrees Fahrenheit
   - B. 52.21 degrees Fahrenheit
   - C. 63.28 degrees Fahrenheit
   - D. 156.35 degrees Fahrenheit

Open response: 6. Juan draws the first three terms of a pattern as shown below. The pattern continues to grow in the same way. Complete the following table according to the pattern.

<table>
<thead>
<tr>
<th>Term number, ( n )</th>
<th>Number of dots, ( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Graph the data from the table on the grid above. Add a scale for the \( N \)-axis. Draw a line or curve of best fit for the data.
Appendix K. GSP Exploration: Finding the Sum of Interior Angles

Finding the Sum of Interior Angles—Investigation using the GSP

Geometer's Sketchpad Directions:

1. Set preferences to allow for the automatic labeling of points and for angle measures in degrees.
2. Construct quadrilateral ABCD
3. Choose a vertex of the quadrilateral and draw all diagonals from that point.
4. Record the number of triangles formed in the table (as seen below).
5. Using GSP, verify that the sum of the angles in a triangle equals 180 degrees by finding the size of each angle of the triangle and then finding the sum.
6. Find the total number of degrees of the interior angles of the polygon you are investigating.
7. Repeat steps 2 - 6 for each additional polygon listed in the table below, or until you can establish a pattern.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Sides</th>
<th>Sum of Interior Angles</th>
<th>Shape</th>
<th>Each Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>180°</td>
<td>△</td>
<td>60°</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>360°</td>
<td>□</td>
<td>90°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>540°</td>
<td>□</td>
<td>108°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>720°</td>
<td>□</td>
<td>120°</td>
</tr>
<tr>
<td>Heptagon (or Septagon)</td>
<td>7</td>
<td>900°</td>
<td>□</td>
<td>128.57...°</td>
</tr>
<tr>
<td>Octagon</td>
<td>8</td>
<td>1080°</td>
<td>□</td>
<td>135°</td>
</tr>
</tbody>
</table>

... ... ... ...

Examine your table. Can you determine a rule which can be used to predict the sum of the interior angles of any polygon? Write your answer in the row labeled "n" in the table.

<table>
<thead>
<tr>
<th>Any Polygon</th>
<th>n</th>
<th>(n-2) × 180°</th>
<th>n</th>
<th>(n-2) × 180° / n</th>
</tr>
</thead>
</table>

Using your prediction (rule), what would be the sum of the interior angles of a decagon?

Verify your answer using the GSP software:
1. Draw a decagon. Do not draw the diagonals.
2. Select and measure each angle of the polygon.
3. Using the calculator tool, compute the sum of these angles.
4. Does the sum agree or disagree with your prediction? State why it does or does not agree.
Appendix L. So Give Me the Formula!

<table>
<thead>
<tr>
<th># of Sides</th>
<th>Sum of interior angles</th>
<th>First Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is this relation Linear or Not?

Can you graph it?

What is the lowest independent value you can have?

So what is the formula?
Appendix M. Creating and Using Equations from Linear Relations

Tuesday, April 24, 2012

Clacker Question

C = x + 10
C = y + 100
C = z + 20
C = 150

equations

Determine two linear equations that could be used to calculate the total cost C of a power tool and the number of months it is used. Let x be the number of months the tool is used, and let y be the number of dollars paid for the tool. Let z be the number of years the tool is used, and let w be the number of months the tool is used.
Question:

How many would it cost to produce 75
products?

The cost, C, is expressed by the equation

\[ C = \text{cost of production} + \text{fixed costs} \]

\[ \text{Cost} = 5000 + 5x \]

Which option correctly states the total cost to produce 75 products?

- A) $5050
- B) $5060
- C) $5070
- D) $5080
Appendix N.  Consent Forms

Sample Information Letter and Assent Form to Participate in the Research (Students)

Dear ___________,

I am writing in order to request for permission to participate in educational research at your school as a part of my PhD dissertation work. I am a fifth year PhD student at the Ontario Institute for Studies in Education of the University of Toronto (OISE/UT).

In my PhD research, I have been looking into exploring teachers’ views on the role of technology and the way they are reflected in their teaching practice in a technology-supported environment. The research regarding the integration of technology in mathematics education suggests that the use of different technological tools significantly extend and enrich teachers’ instructional strategies and support students’ learning in mathematics. However, there is scarcity of the analysis of how teachers’ views on the role of technology in mathematics education are reflected in the design and implementation of their technology-based lessons. This is why there is a need for additional research in this area.

The goal of this study is to explore teachers’ belief about the role of technology in mathematics education, analyze the adjustment of teachers’ strategies and techniques in technology-based lessons, and discusses the impact of technology use on three dimensions of mathematics education, the development of student tasks, construction of content knowledge and facilitation of mathematical communication. The research would happen in the period of January of 2012 to June of 2012.
I would also like to assure you that the identity of the school, the teachers and students will be protected. I believe my research will reveal good teaching and learning strategies and techniques and efficient classroom use of technology that stimulate students motivation, increase student engagement and interaction with peers, invigorate student participation in constructive discussions and debates and consequently lead to improve their understanding of the mathematics content. Should you agree to approve my study, please sign both copies of the attached consent form and keep one for your records.

For questions about your rights as a research participant contact the Office of the Research Ethics from the University of Toronto, at 12 Queen’s Park Cres. West, McMurrich Building, 3rd Floor, Toronto, ON M5S 1S8, phone 416-946-3273, fax 416-946-5763 or email: ethicsreview@utoronto.ca

Thank you in advance for assistance and cooperation.

Thank you for your time. I am looking forward to hearing from you.

Sincerely yours,

Elena Corina Georgescu

________________________  ______________________
Researcher Signature      Date

________________________  ______________________
Participant Signature      Date

CONTACT INFORMATION:
Principal Investigator:    Supervisor:
Elena Corina Georgescu    Dr. Douglas McDougall, Associate Professor
Department of Curriculum, Teaching & Learning
Ontario Institute for Studies in Education
University of Toronto
Dear ____________,

I am writing in order to request for permission for your child ________________ to participate in educational research at his/her school as a part of my PhD dissertation work. I am a fifth year PhD student at the Ontario Institute for Studies in Education of the University of Toronto (OISE/UT).

In my PhD research, I have been looking into exploring teachers’ views on the role of technology and the way they are reflected in their teaching practice in a technology-supported environment. The research regarding the integration of technology in mathematics education suggests that the use of different technological tools significantly extend and enrich teachers’ instructional strategies and support students’ learning in mathematics. However, there is scarcity of the analysis of how teachers’ views on the role of technology in mathematics education are reflected in the design and implementation of their technology-based lessons. This is why there is a need for additional research in this area.

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Thank you for your time. I am looking forward to hearing from you.

Sincerely yours,

Elena Corina Georgescu

__________________________  __________________________
Researcher Signature Date

__________________________  __________________________
Participant Signature Date

CONTACT INFORMATION:
Principal Investigator: Elena Corina Georgescu  
Supervisor: Dr. Douglas McDougall, Associate Professor  
Department of Curriculum, Teaching & Learning  
Ontario Institute for Studies in Education, University of Toronto
Sample Information Letter and Consent Form to Participate in the Research

(Teachers)

Dear __________,

I am writing in order to request for permission to participate educational research at your school as a part of my PhD dissertation work. I am a fifth year PhD student at the Ontario Institute for Studies in Education of the University of Toronto (OISE/UT)

In my PhD research, I have been looking into exploring teachers’ views on the role of technology and the way they are reflected in their teaching practice in a technology-supported environment. The research regarding the integration of technology in mathematics education suggests that the use of different technological tools significantly extend and enrich teachers’ instructional strategies and support students’ learning in mathematics. However, there is scarcity of the analysis of how teachers’ views on the role of technology in mathematics education are reflected in the design and implementation of their technology-based lessons. This is why there is a need for additional research in this area.

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Sincerely yours,

Elena Corina Georgescu

_________________________  _________________________
Researcher Signature       Date

_________________________  _________________________
Participant Signature       Date

CONTACT INFORMATION:
Principal Investigator:    Supervisor:
Elena Corina Georgescu     Dr. Douglas McDougall, Associate Professor
                          Department of Curriculum, Teaching & Learning
                          Ontario Institute for Studies in Education,
                          University of Toronto