ESSAYS ON MONEY, BUSINESS CYCLES AND HOUSEHOLD FORMATION

by

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Abstract

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This dissertation consists of three independent essays in Macroeconomics. The first essay studies whether efficiency can be improved by introducing government-issued illiquid bonds to an economy where money is the only asset and essential. In contrast with perfectly liquid bonds, illiquid bonds can increase societal welfare in two ways: First, allocating consumption goods among heterogeneous agents more efficiently; second, stimulating consumption and output level by loosening the liquidity constraints of households. More importantly, since societal welfare is elevated persistently when the inflation rates range from a level slightly above Friedman Rule to an upper bound, this essay provides an insight into the essentiality of illiquid bonds.

The second essay provides a novel propagation mechanism of productivity shocks to explain an empirical fact: The response curve of output to a positive productivity shock reaches its peak up to eight quarters after the shock. Using a micro-founded monetary search model and focusing on agents' decisions on establishing long-term trading relationships in the goods market, I show that when a positive shock takes place in the economy, marginal agents break down previous trading relationships and explore better matching opportunities. As a result, shortly after the shock, the average productivity level of transactions increases, but the total number of transactions decreases. The calibrated model shows that the latter effect dominates, resulting a slightly decrease of aggregate output after a positive productivity shock. The search friction, together with
the monetary channel, gives rise to a delayed output response at the aggregate level.

The third essay develops a general equilibrium theory of household formation – i.e., marriage – following Coase’s theory of firm formation. Individuals in the model consume both market- and home-produced commodities, and home production is facilitated through marriage. Market frictions, including taxation, search and bargaining problems, increase marriage rates when home and market goods are substitutes. In particular, inflation, as a tax on market activity, makes household production and hence marriage more attractive, as long as singles use cash more than married individuals, which is supported by data. The prediction that inflation and other taxes affect household formation is also supported by evidence.
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The third essay in this thesis (Chapter 3) is a joint work with Kenneth Burdett, Mei Dong and Randall Wright. The views expressed in the thesis are those of the authors, and no responsibility should be attributed to the Bank of Canada, the Federal Reserve System, or any Federal Reserve Bank.
## Contents

### 1  Sustained Societal Benefit of Illiquid Bonds

1.1  Introduction ................................................................. 2
1.2  The Model ................................................................. 5  
    1.2.1  The Environment .................................................. 5
    1.2.2  The Timing of Events ........................................... 7
    1.2.3  The Economy with Perfectly Liquid Bonds .................. 8
    1.2.4  Stationary Monetary Equilibrium ................................. 10
1.3  The Economy with Directly Injected Illiquid Bonds ............... 14
    1.3.1  Essentiality of Illiquid Bonds ................................. 15
    1.3.2  Societal Welfare Comparison Between the Two Economies ... 21
1.4  Conclusion ............................................................... 23

### 2  Delayed Output Response to Productivity Shocks

2.1  Introduction ................................................................. 25
2.2  The Environment .......................................................... 29
    2.2.1  Households and Matches ....................................... 29
    2.2.2  Timing of Events ................................................ 32
    2.2.3  Quantities of Trade in the Goods Market ................... 34
    2.2.4  The Representative Household’s Decision Problem .......... 35
    2.2.5  Optimal Choices ................................................ 38
List of Tables

2.1 Calibrated parameter values ................................................. 46
3.1 Cash Holding per Adult – All Households .............................. 92
3.2 Cash Holding per Adult – Households with Bank .................... 93
3.3 Summary Statistics – 2009 FRB Boston Survey ....................... 94
3.4 Summary Statistics – Bank of Canada Survey Questionnaire ....... 95
3.5 Summary Statistics – Bank of Canada Diary Instrument ......... 96
3.6 Regression Results – Bank of Canada Survey Questionnaire .... 97
3.7 Regression Results – Bank of Canada Diary Instrument .......... 98
3.8 Summary of Macro Data Sources ............................................ 99
3.9 Panel-data GLS w/o taxes, raw data ..................................... 100
3.10 Panel-data GLS w/ OECD taxes, raw data ............................ 101
3.11 Panel-data GLS w/ Mendoza taxes, raw data ......................... 102
3.12 Panel-data GLS, Mendoza sample w/o taxes, raw data .......... 103
3.13 Panel-data GLS w/o taxes, smoothed data (HP) ..................... 104
3.14 Panel-data GLS w/ Mendoza taxes, smoothed data (HP) .......... 105
3.15 Panel-data GLS, Mendoza sample w/o taxes, smoothed data (HP) 106
3.16 Panel-data GLS, Mendoza sample w/o taxes, smoothed data (5-yr moving avg) ................................................................. 106
3.17 Individual Country - the United States, raw data ................. 107
3.18 Individual Country - the United States, smoothed data (HP) .... 107
3.19 Individual Country - Canada, raw data .......................... 108
3.20 Individual Country - Canada, smoothed data (HP) .............. 108
List of Figures

1.1 The Timing of Events ................................. 8
1.2 Societal Welfare Comparison of the Two Economies ............... 22
1.3 Welfare Gain of Illiquid Bonds .......................... 22

2.1 Timing of Events ........................................ 32
2.2 Kernel density estimation of electronics goods prices ............... 46
2.3 The transition path of \( a_s \) ................................ 48
2.4 The transition path of consumption ................................ 49
2.5 The transition path of money holding of unmatched agents .......... 50
2.6 The transition path of the value of money ........................ 51

3.1 The legend of Figure 3.2-3.4. ................................ 108
3.2 Austria, Australia, Belgium, Canada, Denmark, Finland .......... 109
3.3 France, Germany, Italy, Japan, Netherlands, New Zealand .......... 110
3.4 Norway, Spain, Sweden, Switzerland, UK, US .................. 111
Chapter 1

Sustained Societal Benefit of Illiquid Bonds
1.1 Introduction

A long standing puzzle in monetary economics is why government-issued, risk-free nominal bonds dominate fiat money in the rate of return. As a close substitute for money, short-term government bonds share the same intrinsic characteristics as money and yet earn positive nominal interest rate. Hicks (1935) clearly states that this issue is “absolutely fundamental to dynamic economics” and argues that bonds are less liquid than money and so the higher rate of return on bonds is a compensation for the illiquidity. But this argument does not resolve the puzzle – it simply replaces the difference in returns between money and bonds with the difference in the two assets’ liquidity levels. The fundamental reasons why bonds are less liquid than money are still unclear.

To uncover the reasons, researchers have taken two different approaches. The positive approach investigates directly the factors that restrict the liquidity of bonds in reality. For example, Wallace (1983) argues bonds are illiquid since government bonds are issued in large denominations and private banks are legally restricted from intermediating those denomination government debts. Andolfatto (2005) argues that, despite the fact that private banks can intermediate large denomination bonds, it is the lack of complete record keeping technology that restricts the acceptability of private-issued banknotes in transactions, hence bonds are less liquid than fiat money. Aiyagari, Wallace and Wright (1996) show that, in a monetary model with decentralized exchange (search), bonds can be traded at discount, if and only if government agents discriminate against not-yet-matured securities. However, for bonds to be completely illiquid in all these models, the legal restriction must be sufficiently severe in the sense that either the minimum denomination of bonds must be large or the government must be a non-negligible part of the exchange. Shi (2008) has eliminated this drawback of previous models and shows that even an arbitrarily small legal restriction is sufficient to make bonds illiquid in an environment where goods are traded in decentralized exchange.

However, none of the above models, except Shi (2008), analyze the normative aspect
of illiquid bonds, i.e., whether or not illiquid bonds can play an essential role in improving allocation efficiency and enhancing societal welfare level. In fact, illiquid bonds in these models are often inessential: By eliminating the legal restrictions and making bonds liquid, societal welfare does not fall and may even increase. Kocherlakota (2003) is the first to take the normative approach and emphasize the essentiality of illiquid bonds. His model is a variation of Townsend’s (1980) model of spatial separation. The main addition is that households differ in preferences and the difference is modeled by a multiplier to their utility function. He shows that, when bonds are made illiquid by the physical design, they increase social welfare relative to liquid bonds by improving intertemporal consumption sharing between heterogeneous households. However, this essential role of illiquid bonds lasts for only one period: Because the heterogeneity among households’ preferences exists only in the first period, the welfare-improving role of illiquid bonds disappears from the second period onward.

In this paper, I construct a model where illiquid bonds can improve societal welfare in the steady state. In the model economy, a fraction of households demand more consumption goods in even periods than in odd periods, while the rest of households’ demand demonstrates the opposite pattern. This setting creates heterogeneity among households in each period. Both intra-period and inter-period consumption sharing is hence desirable. Since the incomplete financial market and anonymity assumptions preclude private credit, the question remaining is whether another financial instrument, government-issued bonds for instance, can be used to facilitate consumption sharing.

If bonds are perfectly liquid, in the sense that they can be used to purchase consumption goods in goods market, then they can only perform exactly the same role as money, i.e., a medium of exchange. As a result, bonds will be traded with money at par and earn the same rate of return as fiat money. The outcome is that societal welfare cannot be improved, since stationary equilibrium allocations are identical to the case without any bonds.
If bonds are perfectly *illiquid* instead, the agents with high demand on consumption goods can sell their bonds and get fiat money to buy more consumption goods and consume immediately; the agents with relatively low demand can buy these bonds at discount and postpone their consumption to the next period, when they have high propensity to consume. In this way, all households are better off since both types can consume more when they desire more. As a result, the equilibrium allocation is a Pareto improvement upon a perfectly liquid bonds economy, and societal welfare is improved and the improvement is sustainable.

Some other papers approach the essentiality of illiquid bonds from other angles and reach the same conclusion. For example, Boel and Camera (2006) construct a monetary economy with heterogeneity in discounting and consumption risk. The bonds are illiquid in the sense that an early redemption fee is charged if agents want to convert bonds to cash before maturity. Berentsen and Waller (2011) introduce heterogeneity by assuming idiosyncratic shocks and shows that both inside bonds (credit) and outside bonds (illiquid nominal bonds) are essential. Shi (2008) imposes a legal restriction on the liquidity of bonds, which can improve societal welfare by providing partial insurance to the consumption risk coming from matching shocks. However, the paper focuses on match specific shocks and emphasizes smoothing marginal utility between matches rather than between agents.

The remainder of the paper is organized as follows. Section 1.2 introduces the environment of the model, and examines the stationary equilibrium allocations of a benchmark model: an economy in which bonds are perfectly liquid. Section 1.3 shows the welfare improving effect of illiquid bonds by comparing two sets of stationary equilibrium allocations in the economy with perfectly liquid bonds and in the economy with illiquid bonds respectively, and then offers explanations of the result. Section 1.4 concludes the paper.
1.2 The Model

1.2.1 The Environment

The model adopts the environment in Kocherlakota (2003). Time is discrete and starts at \( t = 0 \). There are a large number of households in the economy with unit measure and two types of perishable goods. Each household consists of two agents, a consumer and a producer. The producer of a household produces and sells one type of goods, and the consumer of the same household purchases and consumes the other type of goods. No household can consume its own production.

The households are characterized in 2 dimensions: (1) the types of goods they produce and consume, and (2) their tastes, i.e., how eagerly they desire consumption goods, during different periods. The households who consume type 1 goods (and produce type 2 goods) are denoted as type 1 households, and the rest are denoted as type 2 households. Type 1 and type 2 households are assumed to have equal measures. Within each type, \( s \) \((0 < s < 1)\) fraction of the households have high taste in even periods and low taste in odd periods (and are called type \( H \) households) and \((1 - s)\) fraction of them have the opposite tastes (and are called type \( L \) households). In other words, each household’s taste is assumed to interchange between high and low in any two adjacent periods and is thus identical every other period. For example, if a household’s taste is high in period \( t \), it switches to low in period \( t+1 \). In period \( t+2 \), the household’s taste switches back to high again. Similar to Townsend’s (1980) turnpike model, this setup creates an environment with heterogeneous agents who have different nominal intertemporal marginal rates of substitution.

Households live in a 3-island economy. At the beginning of each period \( t \), all households are located on island 3, where no production technology is available. The consumers in the type 1 households then go to island 1 to buy the type 1 goods and the producers in the type 1 households go to island 2 to produce the type 2 goods, while the destinations
of the producers and consumers in the type 2 households are the opposite. As a result, only type 1(2) goods are produced and transacted in island 1(2). Island 1 and island 2 are symmetric, in the sense that the economic activities taking place on island 2 are analogous to the activities on island 1. The only difference is that type 2 goods instead of type 1 goods are transacted on island 2. On those two islands, according to the strategy predetermined by their households, producers sell their products for fiat money, and consumers purchase goods from producers using their fiat money brought from island 3. After transactions, producers and consumers return to their home households on island 3. The utility of a household in any period is the utility of consumption minus the disutility of production.

Taking a type 1(2) household who has high taste in even periods and low tastes in odd periods as an example. Its preference is represented by

$$
\sum_{t=0, \ t \text{ is even}}^{\infty} \beta^t \left\{ \left( \theta \ln c_t^H - \alpha y_t^H \right) + \beta \left( \ln c_{t+1}^H - \alpha y_{t+1}^H \right) \right\}
$$

where the multiplier of the utility function in even periods, $\theta$, is the taste parameter of this household, and $\theta > 1$. This suggests that the household desires more consumption goods in even periods than in odd periods. $c_t^H$ denotes the consumption level of the household in even periods when its taste is high, while $c_{t+1}^H$ stands for the same household’s consumption choice in odd periods when its taste switches to low. Similarly, $y_t^H$ and $y_{t+1}^H$ denote the production levels of the household in even periods (when its taste is high) and odd periods (when its taste is low) respectively. The production capacity is assumed to be bounded between 0 and 1, i.e. $0 \leq y_t^H \leq 1 \ \forall t$.

I also assume $\beta > \alpha$, where $\beta$ is the discount rate and $\alpha$ is the marginal cost of production. This assumption makes welfare comparisons more straightforward by eliminating the inflationary distortion on households’ labor supply. For a type 1(2) household who has low taste in even periods and high tastes in odd periods, its taste parameter is
1 in even periods and $\theta$ in odd periods, and its production and consumption allocation are denoted with a subscript $L$.

There are two embedded assumptions in this environment, which generate the role for money. First, there is no record keeping technology and all agents are hence anonymous. This assumption rules out lending and borrowing among private agents. Secondly, the taste parameters of households are private information, which precludes the government from granting discriminatory transfers according to households’ needs. With this friction, the first best allocation is not achievable.

1.2.2 The Timing of Events

The timing of events in the economy with perfectly liquid bonds and in the economy with perfectly illiquid bonds are the same. All households start with the same amount of money holding $M_0$ at date zero. Immediately after the beginning of each period, the bonds market opens and each household is endowed with perfectly liquid bonds or perfectly illiquid bonds of the same amount. Those bonds are government-issued, one-period nominal bonds, and $B_t$ units of one-period bonds refer to the payoff of $B_t$ units of fiat money at maturity. The face value of the issuing $B_t$ is very small relative to the aggregate money stock in the economy $M_t$, which is evaluated at the beginning of period $t$. Households then buy or sell bonds at a competitive price before their agents go to island 1 or 2. After bonds market closes, agents travel to island 1 or 2 to produce and purchase consumption goods, and then they go back to their households on island 3. At the end of each period, households redeem matured one-period bonds if they hold any and receive a lump-sum transfer $L_t$ from the government. The bonds endowment $B_t$ and the lump-sum transfer $L_t$ will keep the total money supply in the economy to grow at a constant rate, $\gamma$. Hence the equation $M_{t+1} = \gamma M_t = M_t + B_t + L_t$ should always hold. Figure 1.1 shows the timing of events described above.

Notice that when the money growth rate $\gamma$ is less than 1, the government collects
Figure 1.1: The Timing of Events

(a) The no-bond economy

(b) The economy with perfectly illiquid bonds

lump-sum tax from households. In the standard monetary economy populated by representative infinitely-lived agents, the optimal monetary policy follows the Friedman rule, i.e., $\gamma = \beta$ and the nominal interest rate is equal to zero. In my model, the welfare improving role of illiquid bonds exists only when the nominal interest rate on illiquid bonds is above the Friedman Rule. If nominal interest rate equals to zero, heterogeneous agents have no mutual interest in participating in the bonds market, hence the welfare improving role of bonds disappears. There are some papers arguing that, in the environment with heterogeneity among agents, the distributional effect of monetary policy can make some agents worse off under Friedman rule. Therefore, optimal policy may involve using inflation (See Edmonds (2003), Green and Zhou (2002), and Belongia and Ireland (2002)). Those papers provide additional support to my result.

1.2.3 The Economy with Perfectly Liquid Bonds

Liquid bonds are defined as the ones that are portable and can be used as means of payment in goods market on island 1 and 2. It is straightforward to see that the real
allocation in any equilibrium with liquid bonds is the same as the one in the equilibrium with no bonds. Intuitively, if the bonds can be accepted in the goods market as universally as money, they must have the same expected rate of return and the relative price of one unit of bond in terms of money is exactly one. Bonds then become a perfect substitute of money, performing exactly the same transaction role played by money. Hence the relative price of liquid bonds and money must be identical. The equilibrium of a perfectly-liquid-bond economy is identical to the equilibrium of a cash-only economy. Because of this reason, I will characterize the equilibrium of the cash-only economy in this section.

Given a sequence of price levels \( \{ P_t \}_{t=0}^{\infty} \), the maximization problem of a household who has high taste in even periods and low taste in odd period is shown as follows:

\[
(HP) \quad \max \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \theta \ln c_t^H - \alpha y_t^H \right] + \beta^{t+1} \left[ \ln c_{t+1}^H - \alpha y_{t+1}^H \right] \right\} \]

which is subject to the following constraints:

(i) the cash-in-advance constraints at even periods and odd periods:

\[
P_t c_t^H \leq M_t^H \quad \text{and} \quad P_{t+1} c_{t+1}^H \leq M_{t+1}^H \quad (1.1)
\]

(ii) the budget constraints at even periods and odd periods:

\[
M_{t+1}^H \leq M_t^H - P_t c_t^H + P_t y_t^H + L_t \quad (1.2)
\]

\[
M_{t+2}^H \leq M_{t+1}^H - P_{t+1} c_{t+1}^H + P_{t+1} y_{t+1}^H + L_{t+1} \quad (1.3)
\]

Here, the household maximizes the discounted life-time utility by choosing consumption and production in even and odd periods: \( c_t^H, c_{t+1}^H, y_t^H \) and \( y_{t+1}^H \). Since the consumer of the household is spatially separated from the producer by assumptions, he/she is “cash constrained” in the goods market. More specifically, the cash-in-advance constraints (1.1)
prevent the consumer from accessing the credit market and buying more than what he/she can afford.

The budget constraints (1.2) and (1.3) are standard. They describe the law of motion of money of the household. The household’s money holding at the next period equals to the net money balance carried over from previous period plus the income generated by the sales of production and the lump-sum transfer from the government.

In addition, I assume that the production level is bounded between zero and one, i.e., \(0 \leq y_t^H \leq 1\) and \(0 \leq y_{t+1}^H \leq 1\). The money holdings and consumption levels in every period are restricted to be non-negative, \(M_{t+1}^H \geq 0\), \(M_{t+1}^L \geq 0\), \(c_t^H \geq 0\), \(c_{t+1}^H \geq 0\).

Denote the aggregate money supply in the economy in period \(t\) as \(M_t\), so \(M_t = sM_t^H + (1 - s)M_t^L\), where \(M_t^H\) and \(M_t^L\) are money holdings of type \(H\) and \(L\) households in the economy, respectively. I will normalize all the nominal values using \(M_t\) and denote \(m_t^H = M_t^H/M_t\), \(m_t^L = M_t^L/M_t\) and \(p_t = P_t/M_t\). The household’s optimal decisions are characterized in the appendix.

### 1.2.4 Stationary Monetary Equilibrium

A symmetric stationary monetary equilibrium in the economy with perfectly liquid bonds consists of the household’s decisions \(\{c_t^i, y_t^i, c_{t+1}^i, y_{t+1}^i, m_{t+1}^i, m_{t+2}^i\}_{t=0}^{\infty} \), \(i \in \{H, L\}\), and prices \(\{p_t\}_{t=0}^{\infty}\), such that the following conditions are satisfied: (i) Optimality: given prices and other households’ choices, \(\{c_t^i, y_t^i, c_{t+1}^i, y_{t+1}^i, m_{t+1}^i, m_{t+2}^i\}_{t=0}^{\infty} \), \(i \in \{H, L\}\), the household’s choices solve (HP); (ii) symmetry: the choices and shadow prices are the same across households of the same types \((H\) and \(L)\); (iii) goods markets clear on each island \(1\) and \(2\): \(c_t^H + c_t^L = y_t^H + y_t^L\ \forall t\); (iv) stationarity: every household has identical choices every other period: \(c_t^i = c_{t+2}^i, y_t^i = y_{t+2}^i, i \in \{H, L\} \ \forall t\). In contrast to the usual stationary conditions in the literature, condition \(iv\) describes a stationarity with a two-period length. Namely, the equilibrium allocations are identical every two periods.

The characteristics of the monetary stationary equilibrium are determined by the
government monetary policy: the inflation rate or money growth rate in this model economy. Intuitively, the inflation rate directly affects the opportunity cost of holding money and influences the households’ consumption decisions. Households reduce their money holdings in face of a higher opportunity cost of holding money. Since households cannot consume their own production and the sales of the production can only be used with one period delay, the opportunity cost of holding money also affects households’ production decisions. When inflation rate is moderate, households produce at the optimal amount. Hyperinflation will eventually shut down production activities.

As a modification of Kocherlakota(2003), the modified environment in this paper makes a large difference. In the paper, the tastes of households alternate and the heterogeneity in the aggregate level exists in every period. As a direct result, the marginal rates of substitution between agents of different types deviates from one in every period. This deviation begs the intertemporal consumption sharing between agents in every period and the societal welfare can be potentially elevated in a sustainable fashion.

Welfare analysis requires that the preference discrepancy between the two types of households cannot be too large. If \( \theta \) is too large, then the opportunity cost of holding money for type \( L \) agents becomes trivial in the sense that they are strictly better off to postpone their consumption to the next period than to consume in the current period. To prevent this trivial outcome, I restrict \( \theta \) by \( 1 < \theta < 1/\alpha \).

It is worth mentioning that \( t \) is even in all cases and \( s \), the fraction of households who have high preference, is greater than \( 1/2 \). If \( s \) is less than half, the result is slightly different. However, the difference is mainly mathematical and will not affect welfare analysis.

In the economy with perfectly liquid bonds, for a given inflation rate \( \gamma \in (\beta \theta, \frac{\theta + (1-s)}{s(\theta-1)}) \), there exists a stationary monetary equilibrium. When the inflation rate is low, i.e., when \( \gamma \in (\beta \theta, \beta/\alpha] \), the marginal cost of production at any time period is less than the discounted marginal benefit of consumption in the subsequent period for both types of
households. All the agents hence will produce at their maximum capacity, so $y_t^H = y_{t+1}^H = y_t^L = y_{t+1}^L = 1$. Money holding will be exactly the same for all the agents at all periods. In the equilibrium, the consumers will exhaust all the money balance at goods markets, so the consumption allocation is the same across households too, i.e., $c_t^H = c_{t+1}^H = c_t^L = c_{t+1}^L = 1$. In the equilibrium, households with different preferences end up with the same production and consumption level and the same money holdings in each period.

If the inflation rate increases to the range $\gamma \in \left( \frac{s\beta\theta + (1-s)\beta}{\alpha + \beta(1-s)(1-\theta)}, \frac{s\beta\theta + (1-s)\beta + s\alpha}{\alpha + \beta\theta + s\beta} \right)$, type $H$ agents’ marginal cost in even periods is higher than the discounted marginal utility of next period, and they produces at interior and $y_t^H = \frac{\beta + \alpha(1-s)(1-\gamma)}{\alpha(1-s\gamma - s)} < 1$. Foreseeing higher marginal utility in an even period, type $H$ agents’ marginal cost in odd periods is no greater than the discounted marginal utility of next periods, and so they produce at the maximum capacity $y_{t+1}^H = 1$. Similarly, type $L$ agents produce at interior in odd periods with $y_{t+1}^L = \frac{\gamma \beta (1-s) + \alpha \gamma - (1-s)\beta s + (1-s)\alpha}{\gamma[\beta(1-s) + s\alpha]}$ and at corner in even periods with $y_t^L = 1$. According, the consumption allocations are:

$$
c_t^H = \frac{\gamma \beta (1-s) + \alpha \gamma - (1-s)\beta s + (1-s)\alpha}{\gamma[\beta(1-s) + s\alpha]} \frac{1}{\alpha + \alpha s (\gamma - 1)},
$$
$$
c_{t+1}^H = \frac{\beta}{\alpha \gamma} \frac{1+s(\gamma-1)}{\beta(1-s) + \alpha s} \frac{\beta(1-s) + \alpha s}{\gamma(1-s) + s},
$$
$$
c_t^L = \frac{\beta}{\alpha \gamma} \frac{s + \gamma(1-s)}{\beta s + \alpha(1-s)} \frac{\beta s + \alpha(1-s)}{1+s(\gamma - 1)},
$$
$$
c_{t+1}^L = \frac{(\gamma - 1)\beta s + \alpha \gamma}{\gamma[\beta s + (1-s)\alpha]} \frac{\beta(1-s) + \alpha s}{\alpha(\gamma - s\gamma + s)}.
$$

Notice that the taste parameter $\theta$ does not appear in any of the equilibrium allocations. The reason is that the threshold of production decision is determined by the taste parameter of type $H$ in odd periods (type $L$ in even periods), 1, which does not show explicitly in the mathematical expression.

If the inflation rate increases even more to the range $\gamma \in \left( \frac{s\beta\theta + (1-s)\beta}{\alpha + \beta(1-s)(1-\theta)}, \frac{(1-s)\beta \theta + s\beta}{\alpha + \beta s(1-\theta)} \right)$,
type $H$ agents find that the marginal cost is higher than the discounted marginal benefit of consumption in both even and odd periods, so they produce at interior in all periods, $y^H_t = \frac{\beta/\alpha - (\gamma - 1)(1 - s)}{1 + s(\gamma - 1)} < 1$ and $y^H_{t+1} = \frac{\beta}{\alpha \gamma} [\theta(\gamma - s\gamma + s) + (1 - s)] < 1$. Recall that $s$, the fraction of households who have high preference, is greater than $1/2$ in even periods by assumption. In even periods, the agents who have high consumption desire (type $H$ agents) outnumber the agents who have relatively less consumption desire (type $L$ agents). However, the situations in odd periods are exactly the opposite. Type $L$ producers hence face more demand in even periods than type $H$ producers do in odd periods. As a result, type $L$ producers produce at corner in even periods $y^L_t = 1$, even though type $H$ producers produce at interior in odd periods with $y^L_{t+1} = \frac{\beta}{\alpha \gamma} [s\theta(1 - \gamma) + (1 - s + s\gamma)] < 1$. The corresponding consumption allocations are shown as follows:

$$c^H_t = \frac{s\beta/\alpha + (1 - s)}{1 + s(\gamma - 1)} \frac{1}{s + (1 - s)/\theta},$$
$$c^H_{t+1} = \frac{\beta}{\alpha \gamma} \frac{1 + s(\gamma - 1)}{s\beta/\alpha + (1 - s)\beta} \frac{s\beta\theta + (1 - s)\beta}{\alpha \gamma},$$
$$c^L_t = \frac{s\beta/\alpha + (1 - s)}{1 + s(\gamma - 1)} \frac{1}{s\theta + (1 - s)},$$
$$c^L_{t+1} = \frac{s\beta(\gamma - 1) + \alpha \gamma}{\alpha \gamma} \frac{1}{\alpha \beta/\alpha + (1 - s)} \frac{s\beta\theta + (1 - s)\beta}{\alpha \gamma}.$$

If the inflation rate is very high in the range $\gamma \in \left(\frac{(1 - s)\beta\theta + s\theta(1 - s)}{\alpha + \beta s(1 - \theta)}, \frac{s\theta + (1 - s)}{s(\theta - 1)}\right]$, all the producers produce at interior:

$$y^H_t = \frac{\beta}{\alpha \gamma} [(\gamma - \gamma s + s) + \theta(1 - s)(1 - \gamma)] < 1,$$
$$y^H_{t+1} = \frac{\beta}{\alpha \gamma} [\theta(\gamma - \gamma s + s) + (1 - s)(1 - \gamma)] < 1,$$
$$y^L_t = \frac{\beta}{\alpha \gamma} [\theta(\gamma s + 1 - s) + s(1 - \gamma)] < 1,$$
$$y^L_{t+1} = \frac{\beta}{\alpha \gamma} [(\gamma s + 1 - s) + s\theta(1 - \gamma)] < 1.$$
And the consumption allocations are:

\[
\begin{align*}
    c_t^H &= \frac{\beta \theta}{\alpha \gamma} \left( s + (1 - s)\theta \right) \\
    c_t^{H+1} &= \frac{\beta}{\alpha \gamma} \left( \theta s^\alpha (1 - s) + (1 - s)\theta \right) \\
    c_t^L &= \frac{\beta \theta}{\alpha \gamma} \left( s + (1 - s)\theta \right) \\
    c_t^{L+1} &= \frac{\beta}{\alpha \gamma} \left( \theta s^\alpha (1 - s) + (1 - s)\theta \right).
\end{align*}
\]

Since production fully plays the role of intertemporal smoothing, the marginal rate of substitution of consumption is 1. Although the consumption allocations satisfy the criteria of Pareto efficiency, the societal welfare is actually lower than the previous cases, as the total production is much less than the cases with lower inflation rates.

When the inflation rate is very high, i.e., \( \gamma \geq \frac{s\theta + (1 - s)}{s(\theta - 1)} \), the opportunity cost of holding money is too high and fiat money is useless. Eventually, producers are better off shutting down the production and no economic activities take place. This case is not of particular interest.

### 1.3 The Economy with Directly Injected Illiquid Bonds

In contrast to liquid bonds, illiquid bonds are the ones that are not portable to the goods markets. The lack of portability however does not prevent agents from buying or selling bonds at a competitive price in the bonds market. In fact, those transactions do not necessarily involve physical exchange of assets among different entities. The government simply keeps the record of households’ bond holding. Households redeem matured bonds after their identities are successfully verified by the government. In this sense, the illiquid bonds is very similar to registered bonds in reality.
1.3.1 Essentiality of Illiquid Bonds

Let \( \{q_t\}_{t=0}^{\infty} \) be the sequence of relative prices of bonds in terms of money, and let \( \{P^b_t\}_{t=0}^{\infty} \) be the sequence of price levels as in the previous section. The maximization problem of a household who has high taste in even periods and low taste in odd period (type \( H \) household) is shown as follows

\[
\max \sum_{t=0, t \text{ is even}}^{\infty} \beta^t [\theta \ln c^b_H - \alpha y^b_H] + \beta^{t+1} [\ln c_{t+1}^b - \alpha y_{t+1}^b]
\]

subject to the following constraints: (i) the bonds market constraints in even and odd periods:

\[
M^H_t + q_t B^H_t \leq M^b_H + B_t q_t, \quad (1.4)
\]
\[
M^H_{t+1} + q_{t+1} B^H_{t+1} \leq M^b_H + B_{t+1} q_{t+1}. \quad (1.5)
\]

(ii) the cash in advance constraints in even and odd periods:

\[
P_t c^b_{t+1} \leq M^H_{t+1} \quad \text{and} \quad P_{t+1} c_{t+1}^b \leq M^H_{t+1}. \quad (1.6)
\]

(iii) the budget constraints in even and odd periods:

\[
M^b_H \leq M^H_t - P^b_t c^b_t + P^b_t y^b_t + B^b_t + L_t, \quad (1.7)
\]
\[
M^b_{t+2} \leq M^H_{t+1} - P^b_{t+1} c_{t+1}^b + P^b_{t+1} y_{t+1}^b + B^H_{t+1} + L_{t+1}. \quad (1.8)
\]

Here the superscript \( b \) shown on all control and state variables is used to distinguish the allocations in this illiquid-bond economy from the previous ones in the economy with perfectly liquid bonds. Constraints (1.4) and (1.5) describe how households allocate their
assets between fiat money and the one-period illiquid bonds. Similar to the economy with perfectly liquid bonds, the constraints in (1.6) are cash-in-advance constraints, while (1.7) and (1.8) are budget constraints.

Similar to the economy with perfectly liquid bonds, I also assume that the production is bounded between 0 and 1, i.e., \(0 \leq y_t^{bH} \leq 1\) and \(0 \leq y_{t+1}^{bH} \leq 1\), and that the consumption and money holdings are non-negative, i.e., \(c_t^{bH} \geq 0\), \(c_{t+1}^{bH} \geq 0\), \(M_t^{bH} \geq 0\), \(M_{t+1}^{bH} \geq 0\), \(M_t^{H'} \geq 0\), and \(M_{t+1}^{H'} \geq 0\). The household’s optimal decisions are characterized in the appendix.

Similar to the perfectly-liquid-bond economy case, I normalize all the nominal values using the total money stock in period \(t\), \(M_t^b\). More specifically, \(m_t^{bH} = M_t^{bH}/M_t^b\), \(m_t^{bL} = M_t^{bL}/M_t^b\), \(p_t^b = P_t^b/M_t^b\), and \(b_t = B_t/M_t^b\). Households are endowed with illiquid bonds \(B_t\) (\(B_t > 0\) and is small) at the beginning of each period and receive a lump-sum money transfer \(L_t\) from the government at the end of each period. Since the total money stock grows at a constant rate \(\gamma\), the condition \(l_t + b_t = (\gamma - 1)m_t\) should be satisfied.

In addition to the equilibrium conditions for the economy with perfectly liquid bonds, a stationary monetary equilibrium of this illiquid-bond economy should also satisfy the following conditions: \(i\) the bonds market clears at every period \(t\), i.e., \(s b_t^{bH} + (1-s) b_t^{bL} = b_t\ \forall t;\) and \(ii\) the price of illiquid bonds is stationary in every other periods: \(q_t = q_{t+2} \forall t\).

The previous section has shown that perfectly liquid bonds cannot be beneficial in the environment with heterogeneity. However, government-issued illiquid bonds, although the amount is very small relative to the total money stock, can provide intertemporal consumption sharing between agents of different consumption needs, and hence increase societal welfare. I will show how equilibrium allocation in this illiquid-bond economy is affected by the inflation rate, compare with the allocation in the previous economy, and then analyze the welfare implication.

When inflation is low \(\gamma \in (\gamma_1, \gamma_2)\), the marginal cost of production is less than the
discounted marginal benefit of consumption in the next period.\footnote{1}{\(\gamma_1\) is the positive root of }\(1 - \frac{b}{s} - \frac{s}{s+q_{t+1}}b = \frac{1}{\gamma} \frac{1 - \frac{b}{s} - \frac{s}{s+q_{t+1}}b}{1 + \frac{\beta}{\alpha} \frac{q_{t+1}}{q_{t+1} + b}}\), \(\gamma_2 = \frac{\beta + \alpha b}{\alpha - q_{t+1} \alpha (1-s)/s}\), where \(q_t\) and \(q_{t+1}\) are the bonds prices in even and odd periods, respectively. \(\gamma_1 < \gamma \leq \gamma_2\) guarantees the equilibrium production allocation is a corner solution for all households.

All producers produce at their maximum capacity 1, i.e., \(y_t^{bH} = y_{t+1}^{bH} = y_t^{bL} = y_{t+1}^{bL} = 1\). At period \(t\) (\(t\) is even), the households whose taste parameter is \(\theta\) (type \(H\)) desire more consumption goods than the rest of households (type \(L\)). To loosen the cash constraint, type \(H\) households sell the illiquid bonds in the bonds market and get more fiat money to buy consumption goods. At the same time, type \(L\) households buy bonds and postpone part of their consumption to the next period. Both type \(H\) and type \(L\) households adjust their asset holdings through the bonds market, and this adjustment could be mutually beneficial if the bonds price is less than 1.

\textbf{Lemma 1} When inflation varies from \(\gamma_1\) to \(\gamma_4\), all households will participate in the bonds market in the equilibrium\footnote{2}{\(\gamma_4\) solves }\(\gamma = \frac{\alpha \beta_{t+1}}{\alpha \beta_{t+1} + p_{t+1} s / p_{t}}\), where \(p_t\) and \(p_{t+1}\) are consumption goods prices in even and odd period, respectively.\footnote{} the competitive price of illiquid bonds is less than 1.

\textbf{Proof.} (by contradiction): If the bonds price equals to 1, the type \(L\) households who buy bonds would forgo part of current consumption without any compensation, which is not incentive compatible. In any stationary equilibrium, the bonds price therefore has to be less than 1.

The equilibrium consumption allocation is:

\begin{align*}
    c_t^{bH} &= 1 + \frac{1 - s}{\gamma} b + q_t b, \\
    c_{t+1}^{bH} &= 1 - \frac{b}{\gamma} - \frac{1 - s}{s} q_{t+1} b, \\
    c_t^{bL} &= 1 - \frac{b}{\gamma} - \frac{s}{1 - s} q_t b, \\
    c_{t+1}^{bL} &= 1 + \frac{s}{\gamma (1 - s)} b + q_{t+1} b.
\end{align*}
The price of bonds in even and odd periods $q_t$ and $q_{t+1}$ can be pinned down by the following system of equations:

\[
q_t = \frac{\beta \theta}{\gamma} \left( 1 - \frac{b}{\gamma} - \frac{s}{(1-s)} q_t b \right) + q_{t+1} b + \frac{q_t b}{\gamma (1-q+t+1)}
\]

\[
q_{t+1} = \frac{\beta \theta}{\gamma} \left( 1 - \frac{b}{\gamma} - \frac{1-s}{s} q_{t+1} b \right) + q_t b + \frac{q_{t+1} b}{\gamma (1-q+1)}
\]

Compared with the equilibrium allocation in the economy with perfectly liquid bonds, in each period, the households who have higher consumption desire can consume more and the ones who have lower desire can consume less, i.e., $c_t^{bH} > c_t^H$, $c_t^{bL} < c_t^L$, $c_{t+1}^{bH} < c_{t+1}^H$, and $c_{t+1}^{bL} > c_{t+1}^L$. Although the marginal rate of substitution of consumption still deviates from the efficiency level, the gap between them is diminished. So one can call this allocation a Pareto improvement upon the one in the economy with perfectly liquid bonds and hence societal welfare is improved by increasing allocation efficiency. Given that the total output level is the same in the two economies and the aggregate consumption level is unchanged, it can be concluded this improvement is structural.

When the inflation rate is located between $\gamma_2$ and $\gamma_3$, similar to the economy with perfectly liquid bonds$^3$, type $H$ agents produce at the interior in even periods and at the corner in odd periods, i.e., $y_t^{bH} = \frac{1/p_t - 1+s}{s} < 1$ and $y_t^{bH} = 1$, while type $L$ agents produce at the corner in even periods and at the interior in odd periods, i.e., $y_t^{bL} = 1$ and $y_{t+1}^{bL} = \frac{1/p_{t+1} - s}{1-s} < 1$, where $p_t^b$ and $p_{t+1}^b$ solve the following system of equations:

\[
\frac{\beta}{\alpha \gamma} p_t^b = \frac{1}{\gamma} \left( \frac{1-s}{s} - \frac{1-s}{s} p_t^b + \gamma - b \right) - \frac{1-s}{s} \frac{\beta^2 \theta}{\alpha \gamma} 1 - \frac{\beta(1-s)}{\alpha \gamma} p_{t+1}^b
\]

\[
\frac{\beta}{\alpha \gamma} p_{t+1}^b = \frac{1}{\gamma} \left( \frac{s}{1-s} - \frac{s}{1-s} p_{t+1}^b + \gamma - b \right) - \frac{s}{1-s} \frac{\beta^2 \theta}{\alpha \gamma} \frac{b(1-s)p_{t+1}^b}{1-s \alpha \gamma} 1 - \frac{\beta s}{\alpha \gamma} p_t^b
\]

$^3_{\gamma_3}$ solves $\frac{1}{x} = \frac{\alpha p_t^c}{\beta p_t^{cH}}$, where $p_t$ and $p_{t+1}$ are consumption goods prices in even and odd periods, respectively. When $\gamma_2 < \gamma \leq \gamma_3$, households produce at the interior when their taste is high and at the corner when their taste is low.
The important message is that, in odd periods, type $H$ agents produce the same amount in the two economies; however, in even periods, they produce more in the economy with illiquid bonds, i.e., $y_{bH}^{t+1} > y_{H}^t$ and $y_{bH}^t = y_{H}^t = 1$. What happens is that, after type $H$ households sell their illiquid bonds in even periods, their shadow value of money increases. The producers are therefore willing to produce more to exchange more money. In odd periods, however, the marginal cost is still less than the discounted marginal benefit, so the producers still produce at the corner, i.e., $y_{bH}^t = y_{H}^t = 1$. Similarly, in even periods, type $L$ agents produce the same in the two economies, but in odd periods, they produce more in the economy with illiquid bonds, i.e., $y_{bL}^{t+1} > y_{L}^t$ and $y_{bL}^t = y_{L}^t = 1$. As a result, the total output level in the economy with illiquid bonds is higher than in the economy with perfectly liquid bonds. The illiquid bonds have an real effect on the economy at aggregate level.

The equilibrium consumption allocation is:

$$c_{t}^{bH} = \frac{1 - \beta(1 - s)\frac{p_{t+1}^b}{s p_{t}^b}}{\alpha \gamma},$$

$$c_{t+1}^{bH} = \frac{\beta}{\alpha \gamma} \frac{p_{t}^b}{p_{t+1}^b},$$

$$c_{t}^{bL} = \frac{\beta}{\alpha \gamma} \frac{p_{t+1}^b}{p_{t}^b},$$

$$c_{t+1}^{bL} = \frac{1 - \frac{\beta s}{\alpha \gamma} p_{t}^b}{(1 - s)p_{t+1}^b}.$$

Once again, it can be shown that $c_{t}^{bH} > c_{t}^{b}, c_{t+1}^{bL} < c_{t}^{L}, c_{t+1}^{bH} < c_{t+1}^{H},$ and $c_{t+1}^{bL} > c_{t+1}^{L}$. Namely the households who desire more can consume more, and the ones who desire less will postpone their consumption, hence the allocation efficiency can be improved in the economy with illiquid bonds.
When inflation increases to between $\gamma_3$ and $\gamma_4$, the equilibrium allocation is:

\[
\begin{align*}
    c^{bH}_t &= \frac{\beta p^b_t}{\alpha \gamma p^b_t}, \\
    c^{bH}_{t+1} &= \frac{\beta p^b_{t+1}}{\alpha \gamma p^b_t}, \\
    c^{bL}_t &= \frac{\beta p^b_t}{\alpha \gamma p^b_t}, \\
    c^{bL}_{t+1} &= \frac{\frac{1}{\gamma} \left[ p^b_t + \frac{1}{1-s} b + (\gamma - 1 - b) \right] + \frac{\beta}{\gamma} p^b_{t+1}}{p^b_{t+1}}
\end{align*}
\]

where $p^b_t = \frac{1 + \gamma - 1 + b}{s \beta (1 - s) + \frac{1 + \gamma - 1 + b}{\alpha \gamma p^b_t}}$ and $p^b_{t+1} = \frac{\alpha \gamma}{s \beta (1 - s) + \frac{1 + \gamma - 1 + b}{\alpha \gamma}}$. The intuition of the welfare implication is very similar to the previous case, the societal welfare is enhanced since the allocation is more efficient and aggregate production is higher in the economy with illiquid bonds than the economy with perfectly liquid bonds.

When inflation is high, i.e., $\gamma \in (\gamma_4, \gamma_5)$, both types of households produce at the interior\(^5\), the equilibrium allocation is identical to the one in the economy with perfectly liquid bonds. This additional asset, illiquid bonds, cannot facilitate the intertemporal consumption sharing, the welfare improving role of illiquid bonds disappears.

As a brief summary, illiquid bonds can improve the societal welfare. This improvement are achieved through two channels: when inflation is moderate, the consumption allocation is more efficient; when the inflation is not too low, there is a real effect on aggregate consumption in addition to the allocation efficiency.

---

\(^4\gamma_4\) solves $\frac{1}{\gamma} = \frac{\alpha \gamma}{s \beta (1 - s) + \frac{1 + \gamma - 1 + b}{\alpha \gamma p^b_t}}$, where $p^b_t$ and $p^b_{t+1}$ are consumption goods prices in even and odd periods, respectively. When the inflation rate $\gamma$ is between $\gamma_3$ and $\gamma_4$, only type $L$ households produce at the corner in even periods. All other households produce at the interior.

\(^5\)at the threshold $\gamma_5$, all producers stop producing since the opportunity cost of holding fiat money is too high due to hyperinflation.
1.3.2 Societal Welfare Comparison Between the Two Economies

The previous section has shown that illiquid bonds can improve societal welfare from the theoretical perspective. This section is devoted to evaluating the societal welfare of the two economies numerically and explicitly. Two different methods are used: One is to measure the societal welfare by aggregating the discounted life time utility of all households in the economy, the other is to measure the welfare cost (% of consumption) of perfectly liquid bonds, i.e., the percentage of consumption that the households are willing to give up in order to switch from an economy with perfectly liquid bonds to an economy with illiquid bonds.

Let $w^H$ and $w^L$ be the weights on the utilities of type $H$ and type $L$ households, respectively. I calculate the weighted average of steady state utilities of all household types as follows:

$$W^i = \sum_{t=0, t \text{ is even}}^{\infty} \beta^t \begin{cases} 
    w^H \left[ \theta c^H_t - \alpha y^H_t + \beta (u(c^H_{t+1}) - \alpha y^H_{t+1}) \right] \\
    + w^L \left[ u(c^L_t) - \alpha y^L_t + \beta (\theta u(c^L_{t+1}) - \alpha y^L_{t+1}) \right] 
  \end{cases}$$

where $i = \text{liquid, illiquid}$

where $i = \text{liquid}$ denotes the economy with perfectly liquid bonds, and $i = \text{illiquid}$ denotes the economy with illiquid bonds. Different sets of parameters will only affect the welfare level of the two economies, but not the welfare ranking, as long as some restrictions, such as $\beta > \alpha$ and $1 < \theta < 1/\alpha$ are satisfied. In this sense, my result on welfare implication is robust. I pick $\beta = 0.98$, $\alpha = 0.9$, $s = 0.6$, and $\theta = 1.02$ to calculate the welfare level of the two economies, and then plot the societal welfare against the inflation rate in Figure 1.2.

As demonstrated in Figure 1.2, the societal welfare is a continuous function of the inflation rate, and it is monotonically decreasing in both economies. The societal welfare in the economy with illiquid bonds is consistently higher, and the welfare improvement
is more significant when inflation is relatively high. As explained in the previous section, when inflation is low, only allocation efficiency is improved; when inflation is relatively higher, not only allocation efficiency but also the real output has increased.

As shown in Figure 1.2, the absolute value of welfare improvement is very small. The small magnitude of improvement is resulted by the restrictions on the maximum capacity of production. To overcome this limitation, I calculate the percentage of consumption that the households are willing to give up in order to switch from an economy with perfectly liquid bonds to an economy with illiquid bonds. As shown in Figure 1.3, this relative measure gives a clearer picture on the magnitude of welfare improvement.

Similar to Figure 1.2, Figure 1.3 shows that there is welfare gain when inflation is
moderate, this gain is peaked when inflation is relatively higher and phase out when inflation is too high.

1.4 Conclusion

This paper studies the essentiality of illiquid nominal bonds in an environment with heterogeneous households. These households’ tastes alternate between periods, unlike representative household frameworks. Introducing one-period illiquid bonds into the economy enables different types of households to engage in intertemporal trade of consumption continuously and hence improves the efficiency of equilibrium allocation. In addition, the existence of stationary monetary equilibrium shows that the improvement is sustainable, which adds robustness to the essentiality of the illiquid bonds.

The structure of the model shares the same spirit as Townsend (1980) and reflects the fact that the endowments and tastes of agents are not synchronized. An overlapping generation model clearly exhibits this feature as well. The most important information conveyed in this paper is that a stationary monetary equilibrium exists in an economy marked by heterogeneous households and that illiquid bonds are essential. Therefore, monetary policy (the inflation rate for instance) can affect the efficiency and real variables in a sustainable way.
Chapter 2

Delayed Output Response to Productivity Shocks in a Monetary Search Model
2.1 Introduction

Empirical evidence suggests that technology diffuses slowly\(^1\) in the economy and aggregate output responds to productivity shocks with a delay. For example, Alexopoulos and Cohen (2009) use the number of new technology titles published in the United States between 1909 and 1949 as an indicator of technological change and find that GNP per capita is positively correlated with the lagged indicator during this period. Furthermore, the corresponding impulse response function shows that GNP attains its peak response roughly two years (eight quarters) after a positive productivity shock. Standard real business cycle models typically are not capable of replicating these empirical findings, since those models have weak propagation mechanisms and usually imply that output will reach the peak response immediately after a productivity shock.\(^2\) In this sense, RBC theory has not been completely satisfactory in predicting the short-run response of aggregate variables to productivity shocks. To understand the role of productivity shocks in business fluctuations, it is important for a model to have a strong propagation mechanism, i.e., a channel through which shocks are transited into dynamic changes in aggregate variables.

In this paper, I focus on the micro-foundation of the goods market behavior, and propose a novel mechanism in which a productivity shock affects the aggregate output level slightly upon impact but is then propagated over time. Intuitively, in the goods market, economic agents cannot always find a trading partner. In other words, “searching” and “matching” are costly. Moreover, the trading surplus of a successful match depends on the match-specific productivity level when the match is established. Naturally, if the productivity level is high, agents optimally form a bilateral informal trading relationship and trade repeatedly in the future, in order to benefit from the high productivity level and shield matching risk. If the productivity level is low, agents are better off to search

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\(^1\)Rogers (1995) discusses many examples of slow diffusion of technology advancement.  
\(^2\)For example, Cooley and Prescott (1995).
for another trading partner in the next period. In the stationary equilibrium, there exists a unique productivity threshold level. Agents form a trading relationship if and only if the productivity level is at least as high as the threshold, unless the match is destroyed exogenously.

When a positive productivity shock hits the economy, the agents’ optimal decisions change accordingly. Marginal agents, precisely those matched ones whose productivity levels are close to the original threshold, will leave the previous trading partner and look for new matching opportunities which potentially provide higher expected utility. In the paper, I examine agents’ optimal choices after a first order stochastic dominance change of the productivity distribution. I find that, immediately after the shock, a fraction of previously matched agents optimally break the existing trading relationship. Consequently, the total number of matched agents decreases (extensive margin effect). However, the average productivity level increases (intensive margin effect) because of the positive shock.

In addition to search frictions, fiat money in this model provides a novel channel to propagate productivity shocks. In the event of a positive productivity shock, since the value of money reflects the discounted future benefit of the shock, the same amount of money worth much more to agents. As a result, some agents who eventually will break up with their partners optimally postpone the process. This theory is coherent with an overshooting transition path of the value of money and a gradual transition path of the optimal threshold in the numerical exercise of the model. Without this monetary propagation channel, the transition process will be much shorter, which is undesirable and inconsistent with empirical evidence.

To evaluate the quantitative significance of the propagation mechanism, I calibrate the model using a monthly price data set of 36 consumer electronics products offered for sale at a price comparison site Shopper.com for the period from November 1999 to May 2001. Then I compute the dynamic responses of the equilibrium to a permanent
Chapter 2. Delayed Output Response to Productivity Shocks

and positive shock to productivity. I find that the propagation mechanism is able to
generate significant delays in the peak response of aggregate output. In the stationary
equilibrium, there is an optimal productivity threshold level above which agents form
long-term trading relationships. Although this critical threshold jumps immediately af-
ter the shock, it undershoots its new steady state level. Since this discontinuous dynamic
response implies match quality has to be significantly higher in order to qualify for a rela-
tionship, the total number of matches decreases. This extensive margin effect is so strong
that aggregate output actually drops slightly immediately after the positive shock, even
though the average match quality increases. Over time, the threshold converges to the
new steady state level smoothly when more and more new relationships are established.
As the productivity shock propagates into the economy, a higher steady state aggregate
output level can be attained.

Monetary search models provide an amenable environment for formulating the pro-
posed mechanism. These models lay out a natural setting to examine trading behaviors
in the goods market and resource allocations in economic environments with significant
trading frictions (see, for example, Kiyotaki and Wright (1993), Shi (1997), Lagos and
Wright (2005) and Faig (2005)). For instance, in pure random matching models, agents
are anonymous and a double coincidence of wants does not exist. Agents meet randomly,
bargain over the terms of trade and transact with each other with the facilitation of fiat
money. However, such models forsake the possibility of long-term relationships in favor
of simplicity\(^3\). In this paper, I incorporate this important aspect and demonstrate that
the observed delayed output response can be better explained through this new feature
of the model.

\(^3\)Goods market relationships are very common in the real economy. In retail markets, consumers form
loyalties to stores and brands instead of searching in pure randomness every period. A 1994 survey by
the Food Marketing Institute reports that only 24 to 27 percent of shoppers changed their supermarkets
of choice. Merchants and producers invest heavily in attracting customers and making them loyal.
According to Hall (2008), spending on advertising in the US was $264 billion or $2300 per household in
2006.
As one of the first monetary search models calibrated to investigate the quantitative implications, Shi (1998) also uses search frictions to examine a monetary propagation mechanism, and produces a hump-shaped response curve of output to shocks. There are three main differences between this paper and the one by Shi (1998). First, Shi’s model examines search frictions in both goods market and labor market, which are necessary for the propagation mechanism. Second, the intensive margin of search friction reinforces the interaction between the two frictional markets and gives rise to the hump-shaped response curve. I shut down both labor market and search intensity channels, and my framework is mainly devoted to the trading relationship in the goods market. Third, Shi’s paper focuses on the propagation mechanism of monetary shocks, while my model evaluates the output response to productivity shocks.

The short-run output response to productivity shocks has been studied empirically and theoretically. Macroeconomists have explored different ways to strengthen the propagation mechanism. For example, sticky price models seem successful in producing the observed impact of a productivity improvement on output. Kimball (1998) presents a dynamic general equilibrium model and claims that the decline in inputs and fixed-investment in plants induced by a technological improvement can even cause output to decline. Basu (1998) and Basu and Kimball (2004) calibrate DGE models and reproduce numerical results that explain the data well. Without assuming nominal rigidity in prices and wages, Francis and Ramey (2003) model habit formation and standard q-theory adjustment cost while Vigfusson (2004) proposes a flexible-price model relying on habit persistence and time-to-plan technology. The existing explanations are plausible, but these approaches rely on certain ad hoc assumptions and none of the papers investigate the goods market behavior from micro-foundations. Without assuming any price rigidities, the current paper outperforms neoclassical macro models and explains the empirical fact quite well by simply incorporating search frictions into a flexible-price regime.

My work is also related to two other strands of literature. One strand of literature in-
vestigates long-term relationships but focuses on labor market behavior and performance. For example, Andolfatto (1996) integrates labor market search into an otherwise standard business cycle model and studies its implication for aggregate fluctuations; Burdett, Imai and Wright (2004) investigate agents’ decisions on searching for new partners or forming long-term relationships but targets job-to-job transitions and endogenous instability in the labor market. The other strand of literature explores propagation mechanism of productivity and other shocks in the aggregate economy, e.g., Wang and Shi (2006) and Fout and Francis (2008). However, the focal point of my paper is the short-run output response to productivity shocks.

The paper proceeds as follows. In section 2.2, I describe the environment, lay out the households’ decision problems and derive the stationary symmetric monetary equilibrium. In Section 2.3, I calibrate the model and show the transition path of the economy after a permanent productivity shock. The last section concludes.

2.2 The Environment

2.2.1 Households and Matches

Time is discrete. There are a large number of households with many types in the economy. For each type, the number of households is large and normalized to one. These households desire a particular good, called the households’ consumption good, which is produced by a different household type. All goods are perishable at the end of each period and all households have the same discount factor \( \beta \in (0, 1) \).

Each household consists of a large number of infinitely lived buyers and sellers. A seller can produce and sell goods, while a buyer purchases consumption goods for the household. For simplicity, the measures of buyers and sellers are both normalized to 1. The buyers and sellers in a household share consumption each period and regard the household’s
utility as the common objective\(^4\). I pick an arbitrary household as the representative household within each type and use lower-case variables to denote its decisions. Other households’ decisions (aggregate variables) are denoted by the corresponding capital-case letters.

The representative household obtains utility \(u(q)\) from \(q\) consumption goods, where \(u'(q) > 0\), \(u''(q) < 0\), \(u'(0) = \infty\), and \(u'(\infty) = 0\). When a buyer and a seller form a match and decide to transact, the seller incurs utility cost \(a\phi(q)\) from producing \(q\) units of output where the cost function satisfies \(\phi'(q) > 0\), \(\phi''(q) > 0\). When the multiplier \(a\) in front of the cost function is low, the cost of production is low and the productivity of the match is high. Hence \(1/a\) is the index of the productivity level and I assume it is match-specific. \(a\) is continuously distributed with a cumulative distribution function \(F(\cdot)\) on a compact set \([a, \bar{a}]\).

At the beginning of a period, an agent (a buyer or a seller) searches for a partner randomly in the goods market. With probability \(\lambda^b\), a buyer matches with a seller from another household. Once matched, the pair moves to a specific location. The location is private information to the pair and is repeatedly accessible only by them. The pair then arbitrarily draw a cost parameter \(a\) from the distribution \(F(\cdot)\) and trade with each other. In each trade, the buyer has all the bargaining power and makes a take-it-or-leave-it offer. This assumption simplifies the determination of the trading quantities. When the pair exit the goods market, the trading location can be destroyed exogenously with probability \(\delta\). If that happens, the match is also destroyed and the pair can not trade with each other again. They have to search randomly in the next period, just like many other agents who did not find a match in the current period.

Every period after the goods market closes, the agents in a household can be divided into two groups: “matched” and “unmatched”. “Unmatched” agents are those who do

\(^4\)This assumption maintains tractability by making the distribution of money holdings across households degenerate despite the presence of random matching. (see Shi, 1997). For an alternative way to make the distribution of asset holdings degenerate, see Lagos and Wright (2005) and Faig (2005).
not have a trading partner either because they have not yet found a match or because their match has been destroyed. "Matched" agents are those who have found a match and survived the goods market. Not all matched agents stay in long-term relationships though. The household makes the decision for them depending on their match quality and the distribution of matched agents over their cost parameters. More specifically, the household chooses a cost parameter threshold $a_s \in [a, \bar{a}]$. If the matched agents’ cost parameters are above $a_s$, i.e., the match quality is not good enough, they have to search randomly for new partners, acting exactly like "unmatched" agents. If, on the other hand, the matched agents’ cost parameters are no greater than $a_s$, the agents will form long-term trading relationships and trade with their partners again next period. I will categorize the latter group as "in-relationship" agents, who constitute a subset of "matched" agents. Since the purpose of staying in the relationship is to take advantage of the high-quality match, in-relationship pairs do not draw productivity parameters again in their future trades.

Compared to other monetary models with random search, allowing agents to form long-term relationships is a unique feature of the current model. This setup provides a novel channel to propagate unexpected productivity shocks into the aggregate economy. When the aggregate economic conditions change unexpectedly, the household’s decision about the cost parameter threshold is affected. A direct effect is that, immediately after shock, some old relationships are broken in the goods market. To establish new relationships, agents have to search again to find good matches so that, in the short run, the observed aggregate economy may not respond to the shock. The detailed mechanism will be explained in section 2.3 when I study the dynamic transition of the economy after a permanent positive productivity shock.

Unlike many monetary models, information about the trading history of each household is not completely private since agents from different households can form long-term relationships and trade repeatedly. In this regard, credit arrangements may be a natural
extension of trading relationships\textsuperscript{5} and modelling credit in the current paper is an interesting perspective to explore. However, credit is not the focus of this paper and I assume credit is not available to avoid unnecessarily complicating of my analysis.

Without credit and a double coincidence of wants, every trade requires a medium of exchange. Fiat money, issued by the government and storable without a cost, fulfills that role. It is intrinsically worthless, i.e., it does not yield direct utility or facilitate production, but it is essential to facilitate all transactions.

\subsection{2.2.2 Timing of Events}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Timing_of_Events.png}
\caption{Timing of Events.}
\end{figure}

Pick an arbitrary period $t$ and suppress the time index. Figure 2.1 depicts the timing of events for the representative household in a period. At the beginning of each period, the household collects fiat money from its sellers and buyers and evaluates the money holding $m$. The household also evaluates the distribution of matched buyers and sellers over their productivity parameters, $G_b(a)$ and $G_s(a)$, where the subscript $b$ stands for buyers and $s$ sellers. For a particular $\bar{a}$, $G_b(\bar{a})$ is the measure of matched buyers whose cost parameter is no greater than $\bar{a} \in [a, \bar{a}]$. Based on the evaluation, the household then chooses the cost parameter threshold $a_s$. If a matched agent is in a low-quality (high-cost, low-productivity) match, i.e., $a > a_s$, he/she will search again in the goods market just

\textsuperscript{5}Corbae and Ritter (2004) extend the standard indivisible goods search model of Kiyotaki and Wright (1993) to study credit by allowing long-term relationships between matched agents.
like an unmatched agent. On the other hand, if a matched agent is in a high-quality
(low-cost, high-productivity) match, i.e., $a \leq a_s$, he/she will trade again with the same
partner.

Before the goods market opens, the household distributes the money balances to the
buyers according to their categories (in relationship or not) and match qualities. Each
matched agent who is in relationship, i.e., whose cost parameter is no greater than $a_s$, will
be given $x_{bm}(a)$ units of fiat money. Money is allocated across members of the household
based on match quality, creating a continuous distribution of money holding among
in-relationship buyers. After allocating money to in-relationship buyers, the household
evenly distributes the rest of the money balances among the not-in-relationship buyers so
that each one gets an allocation of $x_{bn}$ units of money. Non-in-relationship buyers then
search randomly when the goods market opens. Sellers hold zero money balance since
they only engage in production and sale.

When the goods market opens, all the buyers and sellers go to the market. The
in-relationship buyers and sellers trade with their previous partners, exchanging $q_m(a)$
consumption goods for $x_m(a)$ units of fiat money. The subscript $m$ denotes repeated
trade between "matched" agents. The rest of the agents search for new partners. If
an agent finds a match, the newly matched pair draw a productivity parameter $a$ and
trade. The amount of fiat money these buyers will pay, $x_n(a)$, and the quantity of goods
these sellers will produce, $q_n(a)$, are determined by Nash bargaining. The subscript $n$
stands for trading between "newly matched" agents. After these market activities, a
fixed fraction $\delta$ of matches are destroyed exogenously as agents exit the goods market.
The goods market is then closed and remains closed for the rest of the period.

The household then pools the consumption goods obtained by the buyers and con-
sumes. Immediately before the end of current period, each household receives a lump
sum transfer from the monetary authority. Let $\gamma$ be the gross rate of money growth so
that $M_{t+1} = \gamma M$. $\gamma$ is a positive number which is no less than $\beta$, the discount rate of
Chapter 2. Delayed Output Response to Productivity Shocks

2.2.3 Quantities of Trade in the Goods Market

Like Lucas (1990), I normalize nominal money holdings and nominal prices by the aggregate money holding per household, $M$. Let $m$ be the representative household’s money holding after normalization. Let $V(m, g_b(a)) : R_+ \times R_+^2 \to R$ be the household’s value function. Let $\omega^m$ be the shadow value of next period’s money holding, that is

$$\omega^m = \frac{\beta}{\gamma} \frac{\partial V(m_{t+1}, g_{b_{t+1}})}{\partial m_{t+1}},$$

where the subscript +1 stands for $t + 1$. The future value of fiat money is discounted by both the money growth rate $\gamma$ and the discount rate $\beta$, since $m_{t+1}$ is normalized by next period’s aggregate money stock $M_{t+1}$. Other households’ shadow value of money is denoted by the upper-case letter $\Omega^m$.

As mentioned before, in each trade, the buyer makes a take-it-or-leave-it offer to induce the seller to trade. Because the seller’s surplus in the trade is equal to $\left[\Omega^m x - a \phi(q)\right]$, the offer $(x, q)$ pushes the seller’s surplus to zero and must satisfy

$$x_m(a) = \frac{a \phi(q_m(a))}{\Omega^m} \quad (a \leq a_s) \quad (2.1)$$

for trades between in-relationship agents and

$$x_n(a) = \frac{a \phi(q_n(a))}{\Omega^m} \quad \forall a \quad (2.2)$$

for trades between newly matched agents.

In addition, since credit is not available and agents cannot go back to the household to get more money, the buyer cannot offer more money than he/she brings to the goods
market. The resulting liquidity constraints can be expressed as:

\[ x_{bm}(a) \geq x_m(a) \quad (2.3) \]

and

\[ x_{bn} \geq x_n(a) \quad (2.4) \]

for trades between in-relationship agents and newly matched agents, respectively.

Since in-relationship agents face no uncertainty regarding the match quality and the terms of trade when they enter the goods market, the household can allocate the exact amount of money they will need to extract the maximum surplus from the trade. Therefore, the matched agents spend all the fiat money they bring to the goods market and their liquidity constraints always bind. This result can be summarized in the following lemma:

**Lemma 2** The liquidity constraints of buyers who are in relationships and who trade repeatedly with previous partners are always binding, i.e., \( x_{bm}(a) = x_m(a) \forall a \geq a_s \).

### 2.2.4 The Representative Household’s Decision Problem

In each period, the representative household chooses the threshold of the cost parameter, \( a_s \), the money holdings of in-relationship and not-in-relationship buyers, \((x_{bm}(a), x_m)\), the quantities of trade of in-relationship and newly matched agents, \((x_m(a), q_m(a))\), \((x_n(a), q_n(a))\) to solve

\[
V(m, g_b) = \max \left\{ u(c) - \int_{a_s}^{A_s} a\phi(Q_m(a)) g_s(a) \, da \\
- [1 - G_s(A_s)] \lambda \int_{a_s}^{\sigma} a\phi(Q_n(a)) dF(a) + \beta V(m_{+1}, g_{b_{+1}}) \right\}. \quad (2.5)
\]

subject to a set of constraints. \( g_b(a) \) is the measure of matched buyers whose cost parameter is \( a \). Hence \( G_b(a) = \int_{a_s}^{a} g_b(a) \, da \) is the measure of matched buyers whose cost parameter is no greater than \( a \) and \( \left(1 - \int_{a_s}^{A_s} g_b(a) \, da\right) \) is the measure of buyers who search
randomly for partners. The subscript $b$ denotes a buyer. Similarly, $g_s(a)$ and $G_s(a)$ are the corresponding measures for sellers. $g_b$ in the value function is a set composed of the measures of matched buyers at each productivity level, i.e., $g_b = \{g_b(a) | a \in [a, \bar{a}]\}$. The upper-case letters in the maximand, $A_s, Q_m(a)$ and $Q_n(a)$ denotes the choices of other households.

The representative household’s consumption $c$ can be expressed as

$$c = \int_{a_s}^{a} q_m(a) g_b(a) \, da + \left( 1 - \int_{a_s}^{a} g_b(a) \, da \right) \lambda^b \int_{a_s}^{a} q_n(a) \, dF(a).$$

(2.6)

Consumption consists of two parts. The first part is the consumption bought by in-relationship agents, which can be calculated by integrating the quantity of goods traded at each productivity level, $q_m(a) g_b(a)$, over the interval $[a_s, a]$. Among the $\left(1 - \int_{a_s}^{a} g_b(a) \, da\right)$ agents who search randomly in the goods market, only a fraction $\lambda^b$ of them can successfully find a match. The expected trading quantity of each match is $\int_{a_s}^{a} q_n(a) \, dF(a)$ so the second part of equation (2.6) captures the total quantity purchased in the newly formed matches.

The constraints of the household’s problem are given by (2.3), (2.4) and the following:

(i) the law of motion of money holdings:

$$\gamma m_{t+1} = m - \left\{ \int_{a_s}^{a} x_m(a) g_b(a) \, da + \left( 1 - \int_{a_s}^{a} g_b(a) \, da \right) \lambda^b \int_{a_s}^{a} x_n(a) \, dF(a) \right\}$$

$$+ \left\{ \int_{a_s}^{A_s} X_m(a) g_s(a) \, da + \left( 1 - \int_{a_s}^{A_s} g_s(a) \, da \right) \lambda^s \int_{a_s}^{A_s} X_n(a) \, dF(a) \right\} + (2, \gamma).$$

(ii) the law of motion of the measure of matched buyers at each productivity level:

$$g_{b_s+1}(a) = \begin{cases} 
\left\{ g_b(a) + \left( 1 - \int_{a_s}^{a} g_b(a) \, da \right) \lambda^b f(a) \right\} (1 - \delta) & \text{if } a \leq a_s, \\
\left( 1 - \int_{a_s}^{a} g_b(a) \, da \right) \lambda^b f(a) (1 - \delta) & \text{if } a > a_s;
\end{cases}$$

(2.8)
(iii) the household’s budget constraint:

\[
m \geq \int_{\underline{a}}^{a_s} x_{bm} (a) g_b (a) \, da + \left( 1 - \int_{\underline{a}}^{a_s} g_b (a) \, da \right) x_{bn}. \tag{2.9}
\]

Equation (2.7) describes how the household’s money holdings evolve. As demonstrated in Figure 2.1, at the beginning of each period, the household distributes \( m \) to all buyers. The amount of money spent by in-relationship buyers can be calculated by integrating the amount of money spent at each productivity level, \( x_m (a) g_b (a) \), over the interval \([\underline{a}, a_s]\). Among the \( \left( 1 - \int_{\underline{a}}^{a_s} g_b (a) \, da \right) \) buyers who search randomly, a fraction \( \lambda^b \) of them find a match and the average amount of money spent in each trade is \( \int_{\underline{a}}^{a_s} x_n (a) \, dF (a) \). Therefore, the terms in the first set of braces capture the total money spent by buyers (cash outflow). Similarly, the expressions in the second set of braces are the money earned (cash inflow) by the in-relationship sellers and newly matched sellers. Notice that the decision variables in the latter braces are capital-case characters. This is because sellers do not have any bargaining power in their matches and the decisions are made by the buyers from other households. The last item on the right hand side of the equation is the normalized lump-sum transfer. In summary, the household’s money holding in the next period equals current period money \( m \) adjusted by the net cash flow from goods market activities and lump-sum monetary transfer. Since \( m_{t+1} \) is normalized by the next period money holding, it is multiplied by the gross money growth rate \( \gamma \).

Equation (2.8) describes how the distribution of matched agents evolves. Consider a particular cost parameter \( \tilde{a} \) and the associated measure of matched buyers \( g_b (\tilde{a}) \). If \( \tilde{a} \leq a_s \), the threshold level picked by the household, those buyers stay in their relationships with \( (1 - \delta) g_b (\tilde{a}) \) of them surviving the goods market to enter the next period as matched agents. At the same time, among the \( \left( 1 - \int_{\underline{a}}^{a_s} g_b (a) \, da \right) \) buyers who search randomly, a fraction \( \lambda^b \) of them find a match and draw a cost parameter of exactly \( \tilde{a} \) with probability \( f (\tilde{a}) \). So, after goods market, \( \left( 1 - \int_{\underline{a}}^{a_s} g_b (a) \, da \right) \lambda^b f (\tilde{a}) (1 - \delta) \) buyers enter the following
period as matched agents with cost parameter $\hat{a}$. This movement is described in the first row of equation (2.8). If, on the other hand, $\hat{a} > a_s$, none of the $g_b(\hat{a})$ buyers stay in relationships. The measure of matched $\hat{a}$ agents next period is independent of the current period. Instead, it only depends on the newly formed matches in the goods market, as shown in the second row of equation (2.8).

The budget constraint (iii) simply means that the money brought by buyers to the goods market cannot exceed the total money balance. It also shows how fiat money is allocated among different buyers. The money is first given to the in-relationship buyers who have formed high-quality matches ($a \leq a_s$) in previous periods. The rest of the money is then evenly distributed to the agents who are not in relationships and thus have to search randomly in the goods market.

### 2.2.5 Optimal Choices

Let $\mu_m(a)$ be the multiplier of the liquidity constraint (2.3), and $\mu_n(a)$ be that of (2.4). Let $\rho$ be the shadow price of (2.9). To simplify the formulas, multiply $\mu_n(a)$ by the number of buyers who search randomly in the goods market, $(1 - \int_a^{a_s} g_b(a) da) \lambda^b$, before incorporating the constraint into the Lagrangian. The optimal choices of $q_m(a)$ and $q_n(a)$ in turn satisfy the following conditions:

\[
\begin{align*}
    u'(c) &= \left[\omega^m + \mu_m(a)\right] \frac{a\phi'(q_m(a))}{\Omega^m}, \\
    u'(c) &= \left[\omega^m + \mu_n(a)\right] \frac{a\phi'(q_n(a))}{\Omega^m}.
\end{align*}
\]

In (2.10), the amount of money that a buyer must give to a seller with cost parameter $a$ in order to obtain one unit of the consumption good is $a\phi'(q_m(a)) / \Omega^m$. The cost of each unit of fiat money is equal to its future value, $\omega^m$, plus the shadow price of the liquidity constraint, $\mu_m(a)$. Thus, (2.10) requires that the cost to a buyer equal the marginal utility of consumption. (2.11) is similar to (2.10), but it applies to optimal decisions.
in new matches since the liquidity constraints can be different between in-relationship trades and newly matched trades.

The optimal choices of $x_{bn} (a)$ and $x_{bn}$ satisfy the following conditions, respectively:

$$\rho = \mu_m (a), \quad (2.12)$$

$$\rho = \lambda^b \int_{\underline{a}}^{\overline{a}} \mu_n (a) \, dF (a). \quad (2.13)$$

(2.12) simply says that the shadow price of money should equal to the shadow price of the liquidity constraints in all in-relationship trades (2.13), on the other hand, says that the shadow price of money should also equal the expected average shadow price of liquidity constraints of all newly formed matches.

The optimal choice of $a_s$ is characterized by:

$$0 = u' (c) \left( q_m (a_s) - \lambda^b \int_{\underline{a}}^{\overline{a}} q_n (a) \, dF (a) \right)$$

$$- \omega^m \left( x_m (a_s) - \lambda^b \int_{\underline{a}}^{\overline{a}} x_n (a) \, dF (a) \right)$$

$$- \rho \left[ x_{bm} (a_s) - x_{bn} \right]$$

$$+ \beta (1 - \delta) \omega^g (a_s)$$

$$- \beta \lambda^b (1 - \delta) \int_{\underline{a}}^{\overline{a}} \omega^g (a) \, f (a) \, da. \quad (2.14)$$

For a matched buyer with cost parameter $a_s$, the quantity of consumption goods he/she buys is $q_m (a_s)$ if the relationship is maintained, and $\lambda^b \int_{\underline{a}}^{\overline{a}} q_n (a) \, dF (a)$ otherwise. The difference between the two quantities multiplied by the marginal utility of consumption $u' (c)$ is the marginal utility gain (loss if the value is negative) of staying in the relationship. At the same time, the buyer spends $x_m (a_s)$ units of money if he/she stays in the relationship and $\lambda^b \int_{\underline{a}}^{\overline{a}} x_n (a) \, dF (a)$ otherwise. The difference between the amount of money multiplied by the marginal value of money $\omega^m$ is the marginal utility
loss (gain if the value is positive) of staying in the relationship. We also have to consider the liquidity service of money. The buyer carries \( x_{bm} (a_s) \) units of money to the goods market if the relationship is maintained and \( x_{bn} \) otherwise. The difference between the amount of money carried multiplied by the shadow price of the money constraint is the marginal utility lost (gained if the value is positive) through the liquidity service change. All these marginal utility gains and losses are described respectively by the first three lines of the right-hand side of equation (2.14).

If the buyer stays in a relationship and survives the goods market, the discounted future marginal gain of staying in that relationship is \( \beta (1 - \delta) \omega^g (a_s) \). At the same time, since this buyer does not search randomly in the goods market, he/she loses the opportunity to form a new match with a different cost parameter. This opportunity loss is captured by the discounted future marginal gain of all possible matches that survive the goods market \( \beta (1 - \delta) \lambda^b \int_a^\pi \omega^g (a) f (a) \, da \). These two trade-offs are described by the last two lines of the right-hand side of equation (2.14).

The total marginal gain (loss) should be exactly zero when \( a = a_s \), i.e., when the buyer’s cost parameter is at the threshold level, the buyer is indifferent between staying in the relationship and leaving it. The optimal choice of \( a_s \) is hence characterized by equation (2.14).

For \( m \) and \( g_b (a) \), the envelope conditions are as follows:

\[
\frac{\gamma}{\beta} \omega^m_{-1} = \omega^m + \rho, \tag{2.16}
\]
Chapter 2. Delayed Output Response to Productivity Shocks

\[ \omega^g_{-1} (a) = \begin{cases} 
  u'(c) \left[ q_m (a_0) - \lambda^b \int_a^\pi q_n (a) dF (a) \right] \\
  -\omega^m \left[ x_m (a_0) - \lambda^b \int_a^\pi x_n (a) dF (a) \right] \\
  -\beta (1 - \delta) \lambda^b \int_a^\pi \omega^g (a) f (a) da \\
  +\beta (1 - \delta) \omega^g (a_0) \\
  -\rho [x_{bm} (a_0) - x_{bn}] \\
  0 
\end{cases} \quad \text{if } a_0 \leq a_s, \tag{2.17} \]

where \( \omega^g_{-1} (a_0) = \frac{\partial V (m, g_b)}{\partial g_b (a_0)} \) and \( = \frac{\partial V (m_{+1}, g_{b+1})}{\partial g_{b+1} (a_0)} \).

The envelope condition for money (2.16) requires that the current value of money to equal the future value of money plus the expected liquidity services generated by money. The current value of money is given by the left-hand side of (2.16), where \( \omega^m_{-1} \) is multiplied by \( \gamma/\beta \) because \( \omega^m_{-1} \) is defined as the current value of money discounted to one period earlier. The right-hand side of (2.16) consists of the (discounted) future value of money, \( \omega^m \), and the expected liquidity services generated by money in the goods market, \( \rho \).

The envelope condition for the measure of matched agents is given by (2.17). Since \( V (\cdot) \) is a functional defined over a class of left continuous real valued functions \( g_b (a) \), the usual definition of a derivative does not apply here. Instead, I use the economic rendition of the variational derivative formulated by Volterra\(^6\) to solve the envelope condition. The detailed steps are included in the appendix.

The envelope condition for \( g_b (a) \), (2.17), captures the value of staying in the relationships. Note that it is very similar to the optimality condition for \( a_s \) (2.14). In particular, when \( a = a_s \), the envelope condition for \( g (a_s) \) is identical to the first order condition

\(^6\)For the detailed definition of Volterra’s derivative, see Wan (1970).
for \(a_s\). This is not surprising since the marginal value of staying in the relationship is captured by the trade-offs described in (2.14).

Recall that all buyers who are not in a relationship hold the same amount of money while searching in the goods market. If a buyer finds a match, depending on the cost parameter drawn by that match, he/she might not spend all the money held. If the match quality is high, the buyer wants to buy as much consumption good as possible and the liquidity constraint is binding. Using equations (2.2) and (2.4), we can derive the quantity traded as follows:

\[
q_n(a) = \left[ \frac{x_{bn} \Omega^m}{a} \right]^{\frac{1}{\xi}}.
\]  (2.18)

If the match quality is low, it might be more profitable for the buyer not to spend all his/her money, in which case the liquidity constraint is not binding, i.e., \(\mu_n(a) = 0\) if \(a > a_s\). According to the optimality condition for \(q_n(a)\), (2.11), we obtain

\[
q_n(a) = \left[ \frac{c^{-\sigma} \Omega^m}{\omega^m a \xi} \right]^{\frac{1}{\xi+1}}.
\]  (2.19)

The cut-off value of the cost parameter at which a buyer’s liquidity constraint is just binding is \(a^*\). \(a^*\) can be obtained by combining the above two equations, (2.18) and (2.19), and is defined in the following lemma.

**Lemma 3** The liquidity constraint in newly matched trade may not be binding. The threshold level of the cost parameter \(a^*\) for which the constraint is just binding is given by

\[
a^* = \Omega \left[ \frac{c^{-\sigma}}{\omega^m \xi} \right]^{\xi} [x_{bn}]^{1-\xi}.
\]
2.2.6 Symmetric Stationary Monetary Equilibrium

Definition 1 A symmetric stationary monetary equilibrium consists of the representative household’s choices, \((a_s, x_{bm}(a), x_{bm}, x_{m}(a), q_{m}(a), x_{n}(a), q_{n}(a), a^*)\), the value function \(V(\cdot)\), the shadow values of assets \((\omega^m, \omega^g(a))\) \(a \in [\underline{a}, \overline{a}]\), and other households’ choices such that the following requirements are met: (i) Optimality: given other households’ choices, the representative household’s choices solve the household’s problem and the value function satisfies (2.5); (ii) symmetry: the choices and shadow prices are the same across all households; (iii) positive and finite values of fiat money: \(0 < \omega_m^m < \infty\); (iv) non-negative and finite values of the marginal value of staying in the relationship \(0 \leq \omega^g_{-1}(a) < \infty \forall a \in [\underline{a}, \overline{a}]\); (v) stationarity: all real variables and the values \((\omega_m^m, \omega_g(a))\) are constant.

Proposition 1 The optimal choice of \(a_s\), the cost parameter threshold under which the agents stay in long-term relationships is interior.

2.3 Quantitative Analysis

2.3.1 Calibration and Parameterizations

This subsection describes the choice of functional forms and benchmark parameters for the model. The model is calibrated to quarterly US data. The rate of time preference is set to be \(\beta^{-1} = 1.01\) in order to match a 1% quarterly real interest rate. I adopt the following widely used functional form for utility:

\[ U(c, l) = u(c) - v(l), \]
where \( c \) is consumption and \( l \) is labor input. The utility of consumption is given by a CRRA function,

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \]

where \( \sigma \), the coefficient of risk aversion, is set to the standard value of 2. The disutility of production, which is derived from the production function and the disutility of labor is taken to be:

\[ v(l) = v_0 \frac{l^{1+\kappa}}{1+\kappa}. \]

I assume that the production exhibits decreasing returns to scale, in particular,

\[ F(l) = f_0^\alpha, \quad 0 < \alpha < 1. \]

The labor input required to produce \( q \) goods can be found by inverting the above production function. That is \( l = (q/f_0)^{1/\alpha} \). By the duality of the production and cost functions, the cost of production in terms of the disutility of labor can be written as

\[ v(l) = \frac{v_0}{(1+\kappa)(f_0)^{1+\kappa/\alpha}} q^{(1+\kappa)/\alpha}. \]

Defining \( \phi_0 = \frac{v_0}{(1+\kappa)(f_0)^{1+\kappa/\alpha}} \) and \( \xi = (1+\kappa)/\alpha \), the cost function can be written as \( c(q) = \phi_0 q^\xi \).

According to recent work by Rogerson and Wallenius (2007), the intertemporal elasticity of substitution in aggregate labor supply is high so I set \( \kappa = 0.33 \). \( \alpha \), the share of labor income in output, is .64 according to Shi and Wang (2006) and Christiano (1988). Therefore, \( \xi = (1+\kappa)/\alpha = 2.078 \). I will explain how to pin down \( \phi_0 \) later in the calibration of the distribution function of cost parameters.

\( \delta \), the exogenous separation rate of matched agents when exiting the goods market, is set to be 25/4. This is in line with 25% annual customer turnover rate in Hall (2008).
Assuming an urn-ball matching function, the matching rate is

\[ \lambda^b = \frac{M (B, S)}{B} = \frac{S [1 - \exp(-B/S)]}{B}, \]

according to Berentsen, Menzio and Wright (2008). Since the number of buyers and sellers are the same in any period, i.e., \( B/S = 1 \), it follows that \( \lambda^b = 0.632 \).

Suppose the distribution of productivity is captured by a truncated exponential distribution, \( F(a) = \frac{1 - e^{-z(a - \overline{a})}}{1 - e^{-z(a - \overline{a})}} \), where \( \overline{a} \) and \( a \) are the upper and lower bounds of the cost parameters and \( z \) captures the curvature of the distribution function. One advantage of using this particular distribution function is that we can clearly analyze a change in productivity that positively affects all the new matches and produces a productivity distribution that first order stochastically dominates its predecessor. When a positive shock hits the economy, \( z \) increases to \( z' \) and \( F(a|z') > F(a|z) \) \( \forall a \). Since \( a \) is the cost parameter, this means that the probability of drawing better quality match is higher for all agents who search randomly.

In the model, the match quality varies across different matches as does the trading price of consumption goods. As a result, there is a price distribution in the economy. Calibrating this distribution to a macroeconomic level data set, I can pin down the two parameters \( z \) and \( \phi_0 \). After normalizing \( \overline{a} \) to 1, I can also calibrate the lower bound of cost parameter \( a \).

The price data consists of monthly price observations for 36 consumer electronics products offered for sale at the price comparison site Shopper.com for the period November 1999 to May 2001. The details of how to gather and compile the data set can be found in Baye, Morgan and Scholten (2003). Following Lach (2002), I estimate the kernel density function of the log price of electronics products pooled over stores and months. Log prices are expressed as deviations from the month’s average and the log price average is adjusted to zero in every month. In this way, all the variation in the densities is within
Chapter 2. Delayed Output Response to Productivity Shocks

a month. Using the same method, I then calculate the kernel density function of the prices generated by my model. The targeted parameters can be calibrated by minimizing the distance between the two functions. Figure 2.2 shows the kernel estimate of the price density from the data.

![Price Densities](image)

Figure 2.2: Kernel density estimation of electronics goods prices.

The last parameter that needs to be calibrated is the quarterly inflation rate $\gamma$. Since the price distribution data covers the period November 1999 to May 2001, the choice of $\gamma$ should be consistent with this time frame. The average annual inflation rate during this period was 3.26% so $\gamma$ is set to be 0.8%. As a brief summary, the calibrated values of the parameters are shown in Table 2.1:

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\xi$</th>
<th>$\sigma$</th>
<th>$\delta$</th>
<th>$\lambda^b$</th>
<th>$a$</th>
<th>$\bar{a}$</th>
<th>$z$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.990</td>
<td>2.078</td>
<td>2</td>
<td>0.0625</td>
<td>0.632</td>
<td>0.626</td>
<td>1</td>
<td>0.668</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Table 2.1: Calibrated parameter values
2.3.2 Dynamic Responses to a Permanent Productivity Shock

The main purpose of this paper is to provide a new propagation mechanism through which productivity shocks diffuse slowly into the aggregate economy and to explain why output does not peak immediately following a positive shock of this sort. The complexity of the model makes it very difficult to obtain an analytical solution for the theoretical response so, in this subsection, I parameterize the model using the calibration described above and examine how the model economy responds to a positive, one time permanent shock to the distribution of productivity(cost) parameters.

To conduct the numerical experiment, the model needs to be discretized. More specifically, I first discretize the support of the cost parameter \([a, \bar{a}]\). I then discretize the corresponding distribution over the support. The discretization creates mass points at each cost parameter \(a\). The finer the grid on the support, the more accurate the simulation result. Increasing the number of grid points, however, also increases the computation time dramatically. So, for this exercise, I choose \(N = 100\) points. All optimality conditions, (2.10), (2.11), (2.18), (2.19), (2.16) and (2.17), should be satisfied at each mass point \(a\). Notice that the right-hand side of the first order condition for \(a_s\) (2.14) may not be exactly equal to zero, since the true threshold level \(a_s\) might lie between 2 mass points. In such case, I choose the \(a_s\) that yields the smallest deviation from zero.

Now, consider a first order stochastic dominance change of the productivity, i.e., \(z\) increases in the distribution function \(F(a) = \frac{1-e^{-z(a-a)}}{1-e^{-z(a-ar{a})}}\). I first calculate the old and new steady state allocations and then use numerical methods to simulate the transition path of these two sets of allocations. When the shock hits, the control variables, particularly the value of money and the value of staying in relationships change instantly. I use a shooting algorithm to guess the initial response and the whole transition path of the shadow value of money \(\{\omega_m^t\}_{t=0}^T\) and the value of staying in the relationship at every cost parameter level \(a\), \(\{\omega^g_t(a)\}_{t=0}^T\ \forall a\). I set \(T\) to be a relatively large number, for instance...
20, so that I can solve the model over the next 20 quarters\(^7\). Using the dynamic system, (2.16) and (2.17), I can then solve for the optimal path of \(\omega^m\) and \(\omega^g\) \((a)\) numerically.

The Transition Path of the Threshold Cost Parameter \(a_s\)

Figure 2.3: The transition path of \(a_s\).

Figure 2.3 shows the transition path of \(a_s\), the threshold cost parameter chosen by the household. When the shock happens, all new matches will benefit from the advancement in productivity. Moreover, it is optimal for marginal agents to break up their current relationships and search for new and better partners. The vertical axis denotes the percentage of agents who are in relationships. \(a_s\) drops immediately from the old steady state (the red star) after the shock then gradually converges to the new steady state (the black pentagon). The new steady state is lower than the old one, meaning that the

\(^7\)While there are advantages to using a higher value of \(T\), the main disadvantage is that it geometrically increases the computation difficulty.
household is more selective about which agents it keeps in relationships. The reason is
that, after the shock, it is more likely to find a higher quality match, even under random
search. The blue line plots the transition path. The path is not completely smooth due
to the discretization of the model.

The Transition Path of Aggregate Output/Consumption

![Transition path of consumption](image)

Figure 2.4: The transition path of consumption.

Figure 2.4 illustrates the transition path of consumption. After the shock, the number
of matches drops immediately since some previously in-relationship agents optimally
break up. This in turn affects the total number of consumption goods in the economy.
Recall that the household consumes all the consumption goods so aggregate output level
in this set up is the same as the total consumption/production. As shown in Figure
2.4, the consumption/aggregate output level falls by less than 1% when the shock hits.
However, as more and more new relationships are established, aggregate consumption
increases gradually towards the new steady state.

The Transition Path of the Value of Money and Money Holdings

When previously matched agents break up, the amount of money originally allocated to them is redistributed evenly across all the agents who search randomly. As a result, the money holdings of unmatched agents jumps up immediately after shock, as shown in Figure 2.5. At the same time, the fact that there are more money-holding agents searching randomly combined with the fact that these agents are more likely to find high-quality matches increases the chances of encountering a binding liquidity constraints. In response, the value of money also jumps up immediately after shock and, as shown in Figure 2.6, the jump amounts to roughly 4%. Over time, more and more long-term relationships are formed and both the value of money and the money holdings of unmatched agents converges to their new steady state values.
According to the numerical results, we can conclude that when a positive productivity shock hits the economy, marginal agents will break up with their old partners and search for better trading opportunities in the goods market. In the short run, although a lot of agents actively search for new partners, the number of matches decreases since it takes time for new matches to form. Without assuming price stickiness or other types of shocks, for example a demand shock, the search friction alone induces a gap between breaking old relationships and establishing new ones and this gap results in an output drop in the short run. Over time though, as more and more new relationships are established, the economy gradually converges to the new higher steady state.
2.4 Conclusion

To explain why aggregate output takes several periods to peak following a positive productivity shock, this paper takes a new approach and models trading relationships in an environment where productivity levels are heterogeneous and agents search randomly in a frictional goods market. In this framework, when the match quality is high enough, agents optimally form long-term relationships and trade with each other repeatedly instead of looking for new partners each period. When an exogenous positive productivity shock hits, however, there are marginal agents who are better off breaking up their old relationships in order to explore better opportunities in the goods market. As a result, shortly after the shock, the number of transactions decreases (extensive margin effect) even though the average productivity of the economy increases (intensive margin effect). When the negative extensive margin effect dominates, aggregate output may even drop in the short run. The dynamic response curves shown in the numerical exercise are consistent with the theory.

The search friction is the key to generating the delayed output response here. Standard business cycle models, which happen to overlook this important friction, yield a very strong positive contemporaneous correlation between productivity shocks and aggregate output, a result clearly at odds with the empirical evidence. Adding search frictions into the model and allowing agents to form long-term relationships creates a mechanism whereby matches are preserved to shield against matching risk. Following technological progress, active search is the only way to adopt the better technology, but the search friction prevents technology advances from diffusing immediately. Search frictions are thus the driving force behind households’ decision regarding the break up of old relationships and the establishment of new ones. The mechanism does not rely on any other ad hoc assumptions, such as price rigidity and habit formation, so the results can be generated in a fully flexible price model.

It should be noted that money is assumed to be the only asset in the model by assump-
tion. An interesting extension, however, would be modeling credit trade between agents in long-term relationships. Although incorporating credit into the model is unlikely to change the model’s predictions about how output responds to productivity shocks, it raises an in-depth question about the coexistence of two different assets. Whether or not credit will improve the model’s allocation is not immediately obvious.

Another worthwhile extension would be the modeling of stochastic productivity shocks and stochastic monetary shocks since a stochastic framework would allow us to conduct a fuller comparison of the data and the theory and further test the explanatory power of the model.
Chapter 3

Marriage, Markets and Money: A Coasian Theory of Household Formation
For centuries marriages, births, and other family behavior have been known to respond to fluctuations in aggregate output and prices. Gary Becker (1988).

3.1 Introduction

This paper is about marriage – i.e., household formation. In order to understand the institution of a household, it helps to contemplate how economists think about other institutions, such as firms. In a classic paper, Coase (1937) asks why the economy has some activity organized within business firms, as opposed to independent self-employed individuals, who contract with one another as needs arise. Production could in principle be carried on without organizations like firms, with all activity orchestrated by the market. Why do firms emerge? Coase says this happens when an entrepreneur begins to hire people, forming a team under the entrepreneur’s direction, and considers conditions where this dominates contracting out individual tasks.

If markets are efficient, it should not be preferable to hire people into a firm rather than contracting for individual goods and services as needed. Coase argues, however, that there are a number of transactions costs involved in using the market, including information costs of the type that search theorists analyze, as well as bargaining costs:

The main reason why it is profitable to establish a firm would seem to be that there is a cost of using the price mechanism. The most obvious cost of ‘organizing’ production through the price mechanism is that of discovering what the relevant prices are. This cost may be reduced but it will not be eliminated by the emergence of specialists who will sell this information. The costs of negotiating and concluding a separate contract for each exchange transaction which takes place on a market must also be taken into account.

In addition, he emphasizes the effects of various policies:

Another factor that should be noted is that exchange transactions on a market and the same transactions organized within a firm are often treated differently by Governments or other bodies with regulatory powers. If we consider the operation of a sales tax, it is clear that it is a tax on market transactions.
and not on the same transactions organized within the firm. Now since these are alternative methods of ‘organization’ – by the price mechanism or by the entrepreneur – such a regulation would bring into existence firms which otherwise would have no raison d’être. ... Similarly, quota schemes, and methods of price control which imply that there is rationing, and which do not apply to firms producing such services for themselves, by allowing advantages to those who organize within the firm and not through the market, necessarily encourage the growth of firms.

Firms thus arise to help avoid costs and inconveniences associated with markets. There are limits to what can be produced internally, perhaps, due to decreasing returns, so markets still have a role. But firms’ very existence testifies to the fact that markets are not frictionless, and to the idea that organizing certain activities within such institutions can help to ameliorate search, bargaining, taxation and other costs. To give some concrete examples, an entrepreneur may sometimes need legal, accounting, secretarial or other services, all of which are available on the market. One can always try to find independent contractors to perform these duties – but this involves transactions costs. When these costs are sufficiently high, it becomes worthwhile to bring some of the activity in house, by which we mean setting up one’s own legal team, accounting department or secretarial pool. As more and more activity is performed in house, we have the genesis of the organization known as a firm.¹

We think this approach can help us understand the emergence of other organizations, as Coase (1992) himself suggests. Here we study households, or families. A narrow reading of Coase might suggest the theory does not apply to families, because he said it was important for a firm to have an “employee and employer” relationship resembling a

¹Coase was aware of alternative candidates for a theory of the firm, including specialization, risk allocation, and the notion that entrepreneurs have better knowledge or judgement, but dismissed them because in principle these can all be handled by the market: “What has to be explained is why one integrating force (the entrepreneur) should be substituted for another integrating force (the price mechanism).” Following Coase, Alchian and Demsetz (1972) argued that firms emerge because team production is more efficient than individuals working at arm’s length, through the market, but success depends on managing opportunistic behavior. Monitoring is necessary, and is more effective if the monitor is residual claimant (see also Williamson 1981 and references therein). We do not incorporate intrafamily monitoring explicitly in this paper, although one could.
“slave and master” relationship: workers are not independent contractors paid to deliver specified products, but are subject to direction and control by the firm. Of course, this could be said of some families, and it does not apply to all firms, e.g., cooperatives or partnerships. We think households can be profitably analyzed using Coasian logic, even if their internal operations better resemble happy families or partnerships than “slave-master” relationships. As with legal, accounting or secretarial services for firms, many goods and services for individuals can be provided by the market or the household, including cooking, cleaning and child care. Even companionship and sex can be obtained at home or on the market. When the costs of using markets are high, individuals, like firms, are more inclined to bring activity in house, especially when market and home commodities are good substitutes, and when home production is enhanced by forming a household that operates more or less as a team.

We are not proposing that a transitory blip in sales taxes on any given day will trigger a stampede to the altar, but if individuals find themselves in a long-term situation where the cost of using the market is higher, they will be more inclined to set up households and engage to a greater extent in home rather than market activity. Of course, love may have something to do with it, too, but it is by no means a radical idea to suggest that economic considerations impinge on marriage and other family behavior, as Becker emphasizes in the epigraph, and as is well recognized by popular media.\(^2\) Especially when one considers frictions, in the sense that it takes time and other resources to find an acceptable, let alone ideal, partner, it is not trivial to get married and start a family. Rational individuals do not search forever, but use reservation strategies, stopping when they find someone with whom the benefits of partnership formation outweigh the benefits.

\(^2\)Based on census data, USA Today (5/5//11, p. 3) concludes “Just as growing affluence let many Americans live with fewer people, the recession, high unemployment and the housing bust now are forcing some people to double up.” The next day they ran a similar story on having children. Family considerations clearly are affected by business cycles, and probably still more affected by longer-term changes – e.g., secular declines in marriage rates may be related to transactions costs decreasing with the advance of technology (as they say, it is easier to shop online than in line)
of continued search.

To study how transaction costs affect reservation strategies we develop a formal search-based model of marriage, by which we mean household formation, since we do not have a lot to say here about whether this involves certification by a church, city hall or captain at sea, on the one hand, or what they used to call living in sin, on the other. We use a general equilibrium framework, by which we mean individuals engage in more than simply looking for partners, as in most previous economic analysis of marriage (see below for references). In addition to searching for and eventually settling down at least temporarily with partners, individuals here also participate in markets where consumption goods, labor and assets are traded. It is important to have retail goods markets to capture the idea that some demands may be satisfied either by the market or the household. It is also useful to have asset markets, for reasons that will become clear, and to have labor markets, since this actually simplifies the analysis on multiple dimensions. Some of our goods markets have explicit frictions, including taxation, search and bargaining. One extension also has frictions in the payment system, which makes money essential, and allows us to study the effect of inflation as a tax on market activity.

The monetary version of model is interesting for the following reason. First, we would argue that being single is cash intensive, since goods and services like meals, cleaning etc. that can be provided by the home or market are more likely purchased on the market by singles. These items are not always purchased using currency, of course, but they are certainly purchased this way more often than their home-produced substitutes, since, by definition, household production is not even traded, let alone traded using cash (with exceptions, like paying children to do their chores). Moreover, dating – going to bars, taking taxis, leaving tips etc. – is clearly more cash intensive than hanging around the house – watching TV, having family dinners, etc. – and it seems reasonable to think that singles engage more in dating-like activity. All of this suggests that being single is on the whole cash intensive (also with exceptions, like paying the nanny). We show
this is supported by micro data. Given this, theory predicts that inflation leads to more marriage. Given that, we can go to the macro data using not only information on sales and income taxes, but also inflation, on which we have many more countries and longer time series. We examine marriage rates in a panel of countries to see how they correlate with inflation, as well as other taxes, and factors like unemployment and output.

In terms of the literature, we are not the first to notice a similarity between households and firms. Long ago, Becker (1973) proposed “marriage can be considered a two-person firm with either member being the ‘entrepreneur’ who ‘hires’ the other,” and search theorists often use their equations almost interchangeably to discuss marriage or employment, as discussed in the survey by Burdett and Coles (1999). But it is novel to rigorously apply Coasian logic to marriage in a dynamic general equilibrium model with explicit frictions. Of course some of the ideas can be found elsewhere. Pollack (1985) surveys what he calls the transactions cost approach to family behavior.\textsuperscript{3} Again, the difference is that we use dynamic general equilibrium theory with explicit frictions. Our approach is also related to much work on home production; see surveys by Greenwood et al. (1995) and Gronau (1997) (or, for a more up-to-date list of citations, see Aruoba et al. 2011). See also work in the Rupert (2008) volume, the paper by Siow (2008), and references therein, for research that is related.

The paper is organized as follows. Section 2 describes the environment. Section 3 presents baseline results on how marriage is affected by frictions – i.e., search, bargaining.

\textsuperscript{3} Relatedly, in gender studies Jacobsen (2007, 64-66) also emphasizes transactions costs can be reduced through living with others: “many household production activities are time-consuming to contract for separately. In order to duplicate the activities of one household member performing non-market activities, it may be necessary to hire a maid, cook, butler, plumber, and others. There are often substantial monetary costs involved as well, such as the plumber who charges a fixed amount per service call as well as an hourly rate. Search costs are included in this category.” She also understands the point of Coase we mentioned in footnote 1, when she says “The ability to specialize and thereby increase per capita output available to household members is the factor most cited by economists in considering the economic rationale for household formation ... However, it is not obvious... that it is necessary for persons to live together in order to reap the benefits from specialization and trade. This model is also applied to trade between countries, but does not imply that countries should also merge their legal and social systems and operate as one nation.”
and taxation. Section 4 discusses extensions. Section 5 presents the empirical analysis, using both micro data on cash usage, and macro data relating marriage to a list of aggregate variables. Section 6 concludes.

3.2 Environment

Time is discrete and continues forever. There are two types of individuals, men and women, each with measure 1/2, so the total population has measure 1. Except for their labels, men and woman are treated symmetrically. They all discount across periods at rate $\beta \in (0, 1)$. There are two types of firms, producers and retailers, owned by individuals. The measure of production firms is irrelevant, due to constant returns, while the measure of retail firms is $n$. In each period, agents interact in three distinct markets: (1) a frictionless market, in the spirit of Arrow-Debreu, where they trade assets, labor and some goods; (2) a market where they trade other goods, incorporating frictions, in the spirit of Kiyotaki-Wright; and (3) a market where single individuals search for marriage partners, in the spirit of Burdett-Coles. To help keep track of the different markets, we refer to them as AD, KW and BC.\(^4\)

Denote the value function of an individual in each of the markets by $V_1$, $V_2$ and $V_3$, with subscripts describing in the order in which they convene. In AD, we assume for ease of exposition that a good $x$ can be produced one-for-one using labor $l$ (it is an easy extension to go beyond one good and a linear technology). Good $x$, which we choose as numeraire, can be purchased from producers by individuals for consumption, or by retail firms for conversion into a different good $y$ to be sold in the KW market. Generally, if retailers make an investment in AD of $k$ units of $x$, and sell $y \leq k$ units in KW, they can

\(^4\)Although we label our frictional goods and marriage markets KW and BC, we do not mean to neglect other contributions, any more than we mean to slight other work in GE theory by calling our frictionless market AD. Work on frictional goods markets is surveyed by Williamson-Wright (2010) and Nosal-Rocheteau (2011). For marriage markets, see Becker (1991) and references therein, and more recently Shi (2001), Mortensen (1988), Burdett-Coles (1997), Burdett-Wright (1998), Eeckhout (1999), Shimer-Smith (2000), Burdett et al. (2004), Atakan (2006) and Smith (2007).
convert unsold inventories \( k - y \) into \( \rho(k - y) \) units of \( x \) in the next AD market. Hence, the opportunity cost of selling \( y \) in KW is

\[
c(y, k) = \rho(k) - \rho(k - y).
\] (3.1)

Single individuals participate in the BC market,\(^5\) where \( \lambda \) is the probability of meeting a potential partner, each period (it is a trivial extension to have singles access BC with probability less than 1 each period). If they meet no one, they continue directly to the next AD market. But if a man and woman meet, they mutually decide whether to enter into a partnership that we call marriage. Marriages break up at a rate of \( \delta \), which is exogenous for now. What makes this market interesting is that not all partnerships are created equal: when a man and woman meet, generally, they draw a payoff pair \((z, z')\) describing the flow utilities each would receive if they were to enter into a relationship. Here we focus the scenario where \( z = z' \) with probability 1, which means they agree on how much they get from each other, so that we can ignore bargaining in this part of the model. Draws of \( z \) across meetings and time are i.i.d. and the CDF is \( F(z) \). In equilibrium, individuals choose a reservation value \( R \), such that they are willing to enter into marriage when \( z \geq R \).

Here \( z \) can reflect home production, including not only the drudgery (or the joy) of cooking, cleaning etc., but also the joy of sex and companionship (or the drudgery, as the case may be). The idea is that individuals may be able to engage in some home production on their own, but can potentially do more, or do better, in a partnership with a high \( z \). This captures the notion that household production is facilitated by household formation. It stands in for a more detailed description of household activities, which

\(^5\)We assume single agents meet with each other randomly in the BC market. The time consuming matching process can also be modeled in a directed search framework, where agents are able to direct their search to the most attractive alternatives. In an environment with heterogeneous agents on both sides of the frictional market, Shi (2001) shows that the matching pattern is not always positive assortative and non-positive assignments can be decentralized efficiently under some circumstances. For this reason, a directed search set-up of the current model is potentially interesting.
generally involve decisions about the allocation of time and capital, as discussed in the work mentioned in the Introduction. Following that approach, one might write for a married couple

$$z = \max_{l_h, l'_h, k_h} \{ \chi(l_h, l'_h, k_h, \xi) - l_h - l'_h \}, \quad (3.2)$$

where $l_h$ and $l'_h$ are the spouses’ hours of home work, $k_h$ is their joint home capital, including the house, appliances etc., and $\chi$ is a home production function with $\xi$ a component specific to the partnership. In this specification, randomness in $\xi$ across matches generates randomness in $z$.

Rather than going into post-marriage decisions concerning home work and other inputs, in order to focus on the prior decision to get married, in the first place, we take $z$ as an exogenous random variable. Obviously, whether or not this is innocuous depends on the application, and one can go into more detail. But the essential ingredient for our purposes is simply that people differ in how much they are attracted to each other, how well they work together, etc., as captured by $z$. In Burdett and Coles (1997), $z$ is called pizzazz, which might be related to love, and we will have more to say about that below. For now, $z$ remains the same across periods, except that at rate $\delta$ the relationship breaks up for good. In terms of notation, any individual has martial status indexed by $z \in [\tilde{z}, \bar{z}] \cup \{s\}$, where $z = s$ means they are single, and otherwise $z$ gives the quality of their relationship.

In sum, there are four types of commodities: labor $l$ and good $x$ are traded in AD; a different good $y$ is traded in KW; and there is a non-traded home-produced BC good $z$. Within-period utility is $U = U(x, y, z) - l$, which is linear in $l$, to simplify the analysis. To focus attention on interaction between home production and commodities acquired in frictional markets, $y$ and $z$, we let $U(x, y, z) = U(x) + u(y, z)$.$^6$ Whether $y$ and $z$

---

$^6$We could eliminate $x$ entirely; as it does not complicate the analysis, we include it to show not all consumption need be acquired in frictional markets. One could also eliminate $y$, use $u(x, z)$, and impose a tax on $x$, but this would not allow one to discuss search, bargaining and payment frictions.
are substitutes or complements depends on whether $u_{yz} < 0$ or $u_{yz} > 0$. To determine whether marriage *per se* is a substitute for markets, however, one cannot just take a derivative, since matrimony involves a discrete change in state from $z = s$ to $z \in [\bar{z}, \underline{z}]$. As benchmark, we assume

$$u(y, s) = \varepsilon_0 v(y) \text{ and } u(y, z) = \varepsilon_1 v(y) + z \forall z \neq s,$$

(3.3)

where $v$ satisfies the standard assumptions plus $v(0) = 0$, and we relegate more general results to Appendix C.1. In (3.3) marriage affects one’s payoff in two ways: it changes the marginal utility of $y$ when $\varepsilon_0 \neq \varepsilon_1$ and it gives a flow utility $z$, over and above what one gets while single.\(^7\) The key feature is that, when $\varepsilon_0 > \varepsilon_1$, getting married reduces the marginal utility of $y$, meaning that market commodities and marriage *per se* are substitutes.

### 3.3 Benchmark

We begin with the case where the retail market has bilateral trade, involving search and bargaining, but no credit frictions. The plan is to describe activities in the AD, KW and BC markets, then define equilibrium, and then characterize its properties.

#### 3.3.1 Equilibrium

For individuals in the AD market, the state variables are marital status $z$ and debt $d$ brought in from the previous period. The value function satisfies

$$V_1(d, z) = \max_{x, l} \{U(x) - l + V_2(0, z)\} \text{ st. } x = wl(1 - \eta) - d + \Delta,$$

\(^7\)What one gets while single is normalized to 0, but we allow $z < 0$, so that being with some people is worse than being alone.
where \( w = 1 \) is the real wage, given a linear technology, \( \eta \) is a labor income tax rate, and \( \Delta \) is other net income from transfers, dividends etc. Notice that individuals pay off all debt in the AD market, which is without loss of generality, given \( \mathcal{U} \) is linear in \( l \). Hence, we do not have to track any distribution across agents as a state variable, which is one simplification that comes from having labor in the model (exactly as in Lagos and Wright (2005)). Using the budget equation, we reduce the problem to

\[
V_1(d, z) = \max_x \left\{ U(x) - \frac{x + d - \Delta}{1 - \eta} + V_2(0, z) \right\}.
\]

Hence, \( x \) is determined by the FOC \( U'(x) = 1/(1 - \eta) \), and \( \partial V_1/\partial d = -1/(1 - \eta) \), independent of \((d, z)\).\(^8\)

Production firms in the AD market are trivial, since with a linear technology they are willing to demand any amount of labor and supply any amount of output at \( w = 1 \). Retail firms solve the slightly less trivial problem

\[
\max_k \left\{ -k + \frac{1}{1 + r} \Pi(k) \right\},
\]

where \( k \) is an investment of AD goods in inventories, while \( \Pi(k) \) is revenue accruing in the KW market, derived below, discounted because it is only paid over to shareholders in the next AD market. The appropriate discount factor with quasi-linear preferences is always \( 1 + r = 1/\beta \) and therefore the FOC is \( \beta \Pi(k)' = 1 \).

In the KW market, trade is bilateral, and involves individuals getting \( y \) from retailers in exchange for a debt commitment \( d \). Let \( A\alpha_0 \) be the arrival rate of a spending opportunity for a single and \( A\alpha_1 \) the arrival rate for a married individual. Thus \( A \) measures the general matching efficiency in the KW market, while \( \alpha_0 \) and \( \alpha_1 \) are specific to marital status. We usually assume \( \alpha_0 \geq \alpha_1 \), with a simple special case being the one

\(^8\)Without changing any substantive results, we can make this look more like standard GE theory by replacing \( x \) in utility with \( x \in \mathbb{R}^n \) and replacing it in the budget equation with \( px \) where \( p \in \mathbb{R}^n \).
where married individuals do not participate in KW at all, $\alpha_1 = 0$. If $\alpha_0 > \alpha_1$ then marriage and markets substitutes in terms of opportunities, just like $\varepsilon_0 > \varepsilon_1$ means they are substitutes in preferences. In any case, we have

$$V_2(0, z) = z + A\alpha_1 [\varepsilon_1 v(y_z) + V_3(d_z, z)] + (1 - A\alpha_1)V_3(0, z), \quad (3.5)$$

where $(y_z, d_z)$ denotes terms of trade between a retailer and an individual with marital status $z$, and it is understood that for singles the first term on the RHS vanishes.

For retail firms in the KW market, first, let $\sigma$ denote the fraction of single individuals. Then the probability a retailer meets a single individual is $\sigma A\alpha_0 / n$ and the probability a retailer meets a married individual is $(1 - \sigma)A\alpha_1 / n$. To see this, for the first probability, note that the total number of meetings between retailers and singles is $\sigma A\alpha_0$, then divide by the number of retailers; similarly for the second probability. This assumes all retailers participate in the KW market, which is true as long as $\prod(k) \geq 0$ (we check this below; otherwise, only a fraction participate, and arrival rates adjust to satisfy zero profit, exactly as in Pissarides 2000). Retail profit$^9$ is

$$\prod(k) = \rho(k) + \frac{\sigma A\alpha_0}{n} [(1 - \tau)d_s - c(y_s, k)] + \frac{(1 - \sigma)A\alpha_1}{n} \int_R (1 - \tau)d_z - c(y_z, k)1 - F(R)dF(z),$$

where $\tau$ is a sales tax rate levied on KW consumption $y$ but not AD consumption $x$ (merely to reduce notation).

$^9$To derive this, start with

$$\prod(k) = \left[1 - \frac{\sigma A\alpha_0}{n} - \frac{(1 - \rho)A\alpha_1}{n}\right] \rho(k) + \frac{\sigma A\alpha_0}{n} [d_s(1 - \tau) + \rho(k - y_s)] + \frac{(1 - \rho)A\alpha_1}{n} \int_R [d_z(1 - \tau) + \rho(y_z - k)] \frac{dF(z)}{1 - F(R)}.$$

The first term is revenue in AD for retailers who do not trade in KW; the second is revenue for those who trade with single individuals; and the last is revenue for those who traded with married individuals with $z \geq R$. This reduces to $\prod(k)$ using (3.1).
Moving to the BC market, for single individuals, the value function satisfies

$$V_3(d, s) = \lambda \int_R \beta V_1(d, z) dF(z) + [1 - \lambda + \lambda F(R)] \beta V_1(d, s).$$

(3.6)

In words, with probability \( \lambda \), one meets another single, and when \( z = R \) one enters the next period married, while with probability \( 1 - \lambda + \lambda F(R) \) one either meets no one or meets someone with \( z < R \) and remains single. For married individuals,

$$V_3(d, z) = \delta \beta V_1(d, s) + (1 - \delta) \beta V_1(d, z).$$

(3.7)

With probability \( \delta \) the match breaks up and one enters next period single, while with probability \( 1 - \delta \) one remains in happy matrimony. Notice \( \partial V_1/d = -1/(1 - \eta) \), independent of \( z \).

This completes the description of payoffs in the different markets. We now discuss the terms of trade in KW. In general, a generic trading mechanism maps a meeting into a pair \((y, d)\). While there are many approaches one could take, for now we use the generalized Nash bargaining solution. To implement this, first note that for an individual with marital status \( z \) the trading surplus is

$$S(z) = \varepsilon_z v(y) + V_3(d, z) - V_3(0, z) = \varepsilon_z v(y) - \frac{\beta d}{1 - \eta},$$

by virtue of \( \partial V_1/d = -1/(1 - \eta) \), where \( \varepsilon_z = \varepsilon_1 \) if \( z \in [\underline{z}, \overline{z}] \) and \( \varepsilon_z = \varepsilon_0 \) if \( z = s \). Similarly, the surplus for the retailer is

$$\hat{S}(z) = \beta(1 - \tau)d + \beta \rho(k - y) - \beta \rho(k) = \beta(1 - \tau)d - \beta c(y, k)$$

(this also depends on \( k \), but that is subsumed in the notation since \( k \) is constant across
KW meetings). The generalized Nash bargaining solution is found by solving

$$\max_{y,d} S(z)^{\theta} \hat{S}(z)^{1-\theta} \text{ st. } y \leq k. \quad (3.8)$$

Assuming the constraint $y \leq k$ does not bind, the solution satisfies the FOC

$$\beta c_y(y,k) = (1-\tau)(1-\eta)\varepsilon_z v'(y) \quad (3.9)$$

$$(1-\tau)\beta d = (1-\theta)(1-\tau)(1-\eta)\varepsilon_z v(y) + \theta \beta c(y,k). \quad (3.10)$$

For singles, with $\varepsilon_z = \varepsilon_0$, we denote the outcome by $(y_0, d_0)$; for married, with $\varepsilon_z = \varepsilon_1$, we denote it by $(y_1, d_1) \forall z \in [\underline{z}, \overline{z}]$. In either case, from (3.9), retail trade $y$ is efficient except for taxes – i.e., marginal utility equals marginal cost iff $\tau = \eta = 0$ and then (3.10) determines $d$ as a function of $\theta$\(^{10}\). Also, for future reference, the surplus for buyers conditional on $z$ can be reduced to

$$S(z) = S_z = \frac{\theta[(1-\tau)(1-\eta)\varepsilon_z v(y_z) - \beta c(y_z, k)]}{(1-\tau)(1-\eta)}. \quad (3.11)$$

It is now a simple calculation to show:

**Lemma 4** Given $\alpha_0 \geq \alpha_1$ and $\varepsilon_0 \geq \varepsilon_1$, with at least one inequality strict, we have $y_0 > y_1$, $d_0 > d_1$, $S_0 > S_1$ and $A\alpha_0 S_0 > A\alpha_1 S_1$.

Lemma 4 delivers sharp predictions, in part, because we have a labor-leisure choice and $l$ enters $U$ linearly. Without this feature the model is far less tractable, unless $U$ is linear in consumption, as assumed in some related work, but is inappropriate here since we want to know how results depend on whether marriage and markets are substitutes

\(^{10}\)The outcome depends on a buyer’s marital status as an inevitable implication of generalized Nash bargaining, although this vanishes if $\theta = 1$. It also might vanish if $z$ were private information, but that complicates things considerably. A more facile approach is to use price posting or price taking, instead of bargaining, but we want bargaining included in the Coasian frictions.
or complements. Lemma 4 pertains to the case where marriage and markets are substitutes in preferences and/or opportunities; to get the results for complements, one can simply reverse all the inequalities. We think the natural case is the one where they are substitutes, not only based on introspection, but on estimates in the literature. In this case, singles get a higher expected surplus from the retail market, since they trade more on both the extensive and intensive margins, given \( \alpha_0 > \alpha_1 \) and \( \varepsilon_0 > \varepsilon_1 \). Also, notice \((y_z, d_z) = (y_1, d_1) \quad \forall z \in [z, \bar{z}], \) given the preference specification (3.3); more generally, Appendix C.1 shows \( y_z \) is decreasing in \( z \) iff \( u_{yz}(y, z) < 0 \), which says people in better marriages buy less retail iff market and home goods are substitutes in the usual sense.

We now return to the retailer’s problem. Inserting the bargaining solution, and assuming an interior solution, the FOC \( \beta \prod'(k) = 1 \) becomes

\[
1 + r = \rho'(k) - \frac{(1 - \theta)A}{n}[\sigma \alpha_0 c_k(y_0, k) + (1 - \sigma)\alpha_1 c_k(y_1, k)],
\]

The LHS is the marginal cost of the investment \( k \) made in the previous AD market in terms of this period’s numeraire. The first term on the RHS is a standard return on investment, while the second captures the expected cost reduction from bigger \( k \) when trading in KW, multiplied by \( 1 - \theta \) because this must be shared with customers via bargaining – a typical holdup problem. To focus on marriage decisions, we can simplify retailers’ problem by assuming inventories can be stored at the rate of time preference: \( \rho(k) = (1 + r)k, \) or \( c(y, k) = (1 + r)y. \) With this specification, there is no holdup problem, any \( k \in [y_0, \infty) \) maximizes profit, and the constraint \( y \leq k \) never binds. Thus, we can assume \( k = y_0 \) and proceed ignoring \( y \leq k. \)

We now come to the heart of the model: the marriage decision. By definition of the

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11 Using different methods and different data, Rupert et al. (1995), McGrattan et al. (1997), and Aguiar-Hurst (2007) all find substitution elasticities between 1.5 and 2.0.

12 A linear storage technology is useful for the same reason a linear technology for turning \( l \) into \( x \) is useful – it lets us focus on marriage, rather than standard production and investment decisions. Notice also that \( \prod(k) \geq 0, \) so that all \( n \) retailers are happy to participate in KW.
reservation value, $V_1(d, R) = V_1(d, s)$, and because $V_1$ is linear in $d$, $R$ is independent of $d$. Using (3.4)-(3.5), we reduce $V_1(d, R) = V_1(d, s)$ to

$$A\alpha_1 S(R) + V_3(0, R) + R = A\alpha_0 S(s) + V_3(0, s). \tag{3.12}$$

Before substituting in $V_3$, we use standard methods from search theory to write

$$\int R V_1(0, z) dF(z) = [1 - F(R)] V_1(0, R) + \int R \frac{\partial V_1(0, z)}{\partial z} [1 - F(z)] dz = [1 - F(R)] V_1(0, R) + \int R \frac{(1 - F(z)) dz}{\beta(r + \delta)}$$

where the first equality uses integration by parts, and the second inserts $\partial V_1 / \partial z$, found by differentiating the value functions iteratively. Substituting this into $V_3$ and $V_3$ into (3.12), we arrive at

$$R = A\alpha_0 S_0 - A\alpha_1 S_1 + \frac{\lambda}{r + \delta} \int R [1 - F(z)] dz. \tag{3.13}$$

where $S_0$ and $S_1$ are given in (3.11), simplified here because $\rho(k) = (1 + r)k$ implies $\beta c(y, k) = y$.

To see what (3.13) means, consider the standard reservation wage equation from elementary job search theory (e.g., Rogerson et al. 2005),

$$R = b_0 - b_1 + \frac{\lambda}{r + \delta} \int R [1 - F(w)] dw, \tag{3.14}$$

where $b_0$ ($b_1$) is the value of leisure plus government transfers when unemployed (employed). As is well understood, (3.14) equates the per period value of working at the reservation wage $R$ to the net cost of working, given by difference $b_0 - b_1$, plus the opportunity cost, which is the appropriately capitalized expected return to continued search for a better wage. Similarly, (3.13) equates the value of marrying a reservation partner $R$
to the difference between the value of entering the retail market single instead of married, 
\( A\alpha_0S_0 - A\alpha_1S_1 \), plus the opportunity cost, the return to continued search for a better 
marriage partner.

Given \( R \), for a single person, the probability of marriage – never more appropriately 
called \textit{the hazard rate} – is \( H = H(R) = \lambda[1 - F(R)] \). The steady state fraction of singles 
is then

\[
\sigma = \sigma(R) = \frac{\delta}{\delta + H(R)}. 
\]  

(3.15)

A statistic on which we focus, because this is what we have in the data discussed below, 
is the number of new marriages per period, \( \phi = \phi(R) = \sigma(R)H(R) \) (to remember our 
notation, \textit{sigma} is a stock and \( \phi \) is a flow). To define equilibrium, in addition to the above 
accounting relationships, we need to take feasibility (market clearing) into account. In 
KW and BC all trades are bilateral, so feasibility is automatic, while the AD feasibility 
condition is not important for what we do – it simply determines total employment

\[
L = \sigma l(s) + (1 - \sigma) \int_R l(z)dF(z),
\]

which we do not need to analyze the other variables of interest.\(^{13}\)

Putting the relevant pieces together leads to the following definition:

\textbf{Definition 2} A \textit{(steady state) equilibrium} is a list \((R, y, z, d, s)\) such that:

\( R \) solves the reservation equation \((3.13)\); \((y, z, d)\) solves the bargaining conditions \((3.9)-(3.10)\) \( \forall z \); and

\( \sigma \) solves the steady state condition \((3.15)\).

\subsection*{3.3.2 Results}

The envelope theorem implies the RHS of \((3.13)\) is decreasing in \( R \), so there is at most one 
solution. To ensure existence of an interior equilibrium, \( R \in (z, \bar{z}) \), a sufficient condition 
is that the best possible marriage beats being single and being single beats the worst

\( 13\) Again, this follows from \( l \) entering \( U \) linearly. For the record, AD market clearing is

\[
x + nk = L + [n - \sigma A\alpha_0 - (1 - \sigma)A\alpha_1]p(k) + (1 - \sigma)A\alpha_1\rho(k - y_1) + \sigma A\alpha_0\rho(k - y_0).
\]

Also, if \( x \in \mathbb{R}^n_+ \) we can solve for it and \( p \in \mathbb{R}^n_+ \) \textit{independently} of equilibrium in KW and BC. This \textit{dichotomy} obtains because of separability between \( x \) and \((y, z)\); it is not true generally.
possible: $z + A \alpha_1 S_1 > A \alpha_0 S_0 > z + A \alpha_1 S_1$. Given this, we have:

**Proposition 2** There exists a unique equilibrium with $R \in (z, \bar{z})$.

One can easily derive several results on marriage markets that parallel standard results on labor markets, including $\partial R/\partial \lambda > 0$, $\partial R/\partial r < 0$ and $\partial R/\partial \delta < 0$. Thus, increasing the arrival rate $\lambda$, or decreasing the rate at which one discounts the future of relationships, in terms of either $r$ or $\delta$, makes people more picky when it comes to tying the knot.\footnote{It is easy to sign how $\sigma$ and $\phi$ change with $r$; it is ambiguous what happens when $\delta$ increases, since this changes both the divorce and marriage rates, or when $\lambda$ increases, since this raises the arrival rate but also increases $R$ and the effect on (market clearing) $H$ could go either way. But, as in job search, going back to Burdett (1981), one can impose restrictions to ensure certain reasonable results – e.g., $\partial \sigma/\partial \lambda < 0$ if $F(z)$ is log-concave.}

Much more novel are the effects on marriage of frictions in the retail market, our Coasian transactions costs, including parameters describing search ($A, \alpha_z$), bargaining $\theta$ and taxation ($\tau, \eta$), as well as preferences $\varepsilon_z$:

$$\frac{\partial R}{\partial A} = \frac{\alpha_0 S_0 - \alpha_1 S_1}{D}, \quad \frac{\partial R}{\partial \alpha_0} = \frac{AS_0}{D}, \quad \frac{\partial R}{\partial \alpha_1} = -\frac{AS_1}{D},$$

$$\frac{\partial R}{\partial \theta} = A(\alpha_0 - S_0 - \alpha_1 - S_1),$$

$$\frac{\partial R}{\partial \tau} = \frac{\theta A(\alpha_1 y_1 - \alpha_0 y_0)}{D(1 - \tau)^2(1 - \eta)}, \quad \frac{\partial R}{\partial \eta} = \frac{\theta A(\alpha_1 y_1 - \alpha_0 y_0)}{D(1 - \tau)(1 - \eta)^2},$$

$$\frac{\partial R}{\partial \varepsilon_0} = \frac{\theta A \alpha_0 v(y_0)}{D}, \quad \frac{\partial R}{\partial \varepsilon_1} = -\frac{\theta A \alpha_1 v(y_1)}{D},$$

where $D = 1 + H/(r + \delta) > 0$. We conclude the following:\footnote{We do not give a formal proof, since it is too easy. The results on $R$ follow immediately from Lemma 4. Then, obviously, $H$ moves in the opposite direction to $R$, and a little calculus shows $\sigma$ moves in the opposite direction to $H$ while $\phi$ moves in the same direction as $H$.}

**Proposition 3** Given $\alpha_0 \geq \alpha_1$ and $\varepsilon_0 \geq \varepsilon_1$, with at least one inequality strict, we have $\partial R/\partial A > 0$, $\partial R/\partial \alpha_0 > 0$, $\partial R/\partial \alpha_1 < 0$, $\partial R/\partial \theta > 0$, $\partial R/\partial \tau < 0$, $\partial R/\partial \eta < 0$, $\partial R/\partial \varepsilon_0 > 0$ and $\partial R/\partial \varepsilon_1 < 0$. Also, $\sigma$ moves in the same direction as $R$, while $H$ and $\phi$ move in the opposite direction, with respect to changes in these parameters.
In terms of economics, the assumption on the $\alpha_z$'s and $\varepsilon_z$'s is that marriage and markets are substitutes in terms of either preferences or opportunities. Now consider the retail search parameters. Higher $\alpha_0/\alpha_1$ increases the trading probability for singles relative to a married people, making marriage less attractive and increasing $R$. An increase in overall search efficiency $A$ increases $R$ because singles are more invested in the retail market – they consume more on the extensive margin, given $\alpha_0 > \alpha_1$, and on the intensive margin, given $\varepsilon_0 > \varepsilon_1$. This is typical Coasian logic: individuals facing greater (lower) frictions in markets are more (less) inclined to bring activity in house. Similarly, as regards bargaining and taxation, a lower $\theta$ and higher $\tau$ or $\eta$ all make individuals more inclined to marry. Again, this happens because markets and marriage are alternative ways to provide consumption, the way markets and firms are alternatives in Coase’s original thesis.

Some of the results rely on specification (3.3): for an arbitrary $u(y, z)$, Appendix C.1 shows, e.g., that we can still sign the impact of arrival rates, but not taxes. However, for any $u(y, z)$, everything in Proposition 3 holds as long as $\alpha_1$ is not too big – i.e., married people are not too involved in KW. The results do not depend on details of the retailers’ problem or the pricing mechanism. We leave as an exercise the derivation using Walrasian pricing, and instead, since we need it later, consider the Kalai’s (1977) alternative (to Nash’s) bargaining solution:

$$\max_{y, d} S(z) \text{ st } S(z) = (1 - \theta) \left[ S(z) + \hat{S}(z) \right] \text{ and } y \leq k. \quad (3.16)$$

The FOC for $y_z$ is the same as before, while the FOC for $d_z$ changes to

$$[\theta(1 - \tau)(1 - \eta) + 1 - \theta] \beta d_z = (1 - \eta)[(1 - \theta)\varepsilon_z v(y_z) + \theta \beta c(y_z, k)], \quad (3.17)$$

which is different from (3.10), except in special cases like $\tau = \eta = 0$ or $\theta = 1$. Appendix C.2 shows the qualitative effects are the same as in Proposition 3.
3.4 Extensions

We consider the following four issues: dating, love, divorce and money.

3.4.1 Dating

For this application, we reverse the order of the BC and KW markets, and interpret the latter as dating. Thus, when two singles meet in BC, \( z \) is not observed; rather, after participating the KW market as a pair, it is revealed and enjoyed in the next AD (one could also let them learn more slowly as in Jovanovic 1979). Once \( z \) is known the pair decides whether to marry. For a single in AD who did not date in the previous period, or dated but realizes \( z < R \), the problem is the same as above, but for an individual that realizes \( z \geq R \) we have

\[
V_1(d, z) = z + \max_{x,l} \{ U(x) - l + V_2(0, z) \} \text{ st } x = l(1 - \eta) - d + \Delta,
\]

since now \( z \) is enjoyed in AD iff one gets married.

The BC value functions satisfy

\[
V_2(0, s) = \lambda \mathbb{E} V_3(0, \tilde{z}) + (1 - \lambda) V_2(0, s)
\]

\[
V_2(0, z) = (1 - \delta) V_3(0, z) + \delta V_3(0, s).
\]

Notice the expectation in front of \( V_3(0, \tilde{z}) \) for those on dates, since \( \tilde{z} \) is random. Also, in KW there are now married individuals, dating individuals and singles, with value functions satisfying

\[
V_3(0, s) = A \alpha_0 [\varepsilon_0 v(y_s) + \beta V_1(d_s, s)] + (1 - A \alpha_0) \beta V_1(0, s)
\]

\[
\mathbb{E} V_3(0, \tilde{z}) = A \alpha_1 [\varepsilon_1 v(y_{\tilde{z}}) + \beta \mathbb{E} V_1(d_{\tilde{z}}, \tilde{z})] + (1 - A \alpha_1) \beta \mathbb{E} V_1(0, \tilde{z})
\]

\[
V_3(0, z) = A \alpha_1 [\varepsilon_1 v(y_z) + \beta V_1(d_z, z)] + (1 - A \alpha_1) \beta V_1(0, z).
\]
Chapter 3. Marriage, Markets and Money

Using Nash bargaining, (3.9)-(3.10) and Lemma 4 still apply, and in particular, it does not matter that \( \tilde{z} \) is not known on a date, since \( (y_z, d_z) = (y_1, d_1) \forall z \in [\tilde{z}, \overline{z}] \).

Following the procedure used to get (3.13), we have\(^{16}\)

\[
R = (1 - \lambda - \delta)(A\alpha S_0 - A\alpha_1 S_1) + \frac{\lambda}{r + \delta} \int_R [1 - F(z)] dz. \tag{3.21}
\]

All the derivatives of \( R \) with respect to parameters take the same sign as in the benchmark model. Hence, the results are robust to changing the order of markets, and adding uncertainty/learning. Although nothing especially dramatic happens here, working it out is a prerequisite for the extensions to follow.

3.4.2 Love

To this point, married individuals simply enjoy \( z \) as a "warm glow" from being with their partners. Here we follow Becker (1974) and consider love in terms caring and sharing.

Using the timing in Section 4.1, for a single that did not meet anyone in the BC market the AD problem is as the same. For a single that was on a date, but realizes \( z < R \) so there is no marriage, the AD problem is

\[
V_1(\bar{d}, z) = \max_{x,l} \{U(x) - l + V_2(0, s)\} \text{ st } x = l(1 - \eta) - \bar{d} + \Delta,
\]

where \( \bar{d} = (d + d')/2 \) averages one’s debt \( d \) and that of one’s date \( d' \) (called "going Dutch"). If they marry, however, they average their utilities and consolidate budgets

\[
V_1(\bar{d}, z) = z + \max_{x,x',l,l'} \left\{ \frac{U(x) + U(x')}{2} - \bar{l} + V_2(0, z) \right\} \text{ st } \bar{x} = \bar{l}(1 - \eta) - \bar{d} + \bar{\Delta},
\]

where \( \bar{l} = (l + l')/2, \bar{x} = (x + x')/2, \) and so on.

\(^{16}\text{We calculate } EV_1(0, \tilde{z}) \text{ as follows. If } z \geq R \text{ then } \partial V_1(0, z)/\partial z = 1/\beta(r + \delta), \text{ so } V_1(0, z) = V_1(0, R) + (z-R)/\beta(r+\delta) \text{ is linear in } z. \text{ Then after integration by parts, } EV_1(0, \tilde{z}) = V_1(0, R) + \int_R [1 - F(z)] dz / \beta(r + \delta). \)
In the KW market, for a single who is not dating, the problem is also the same as before. For a single on a date, however,

\[ EV_3(0, \tilde{z}) = (A\alpha_1)^2[\varepsilon_1 v(y_1) + \beta EV_1(d_1, \tilde{z})] + (1 - A\alpha_1)^2\beta EV_1(0, \tilde{z}) \]

\[ + A\alpha_1(1 - A\alpha_1) \left[ \frac{\varepsilon_1 v(y_1)}{2} + \beta EV_1(\frac{d_1}{2}, \tilde{z}) \right] \]

\[ + (1 - A\alpha_1)A\alpha_1 \left[ \frac{\varepsilon_1 v(y_1)}{2} + \beta EV_1(\frac{d_1}{2}, \tilde{z}) \right] \]

assuming pairs search for retailers independently (one could change that). Thus,

\[ EV_3(0, \tilde{z}) = (A\alpha_1)[\varepsilon_1 v(y_1) + \beta EV_1(d_1, \tilde{z})] + (1 - A\alpha_1)\beta EV_1(0, \tilde{z}) \quad (3.22) \]

For a married individual in the KW market, a similar calculation leads to almost the same result, the only difference being that \( z \) is known,

\[ V_3(0, z) = (A\alpha_1)[\varepsilon_1 v(y_1) + \beta V_1(d_1, z)] + (1 - A\alpha_1)\beta V_1(0, z). \quad (3.23) \]

Notice (3.22)-(3.23) are identical to (3.19)-(3.20) in the previous model, and hence the models generate the same predictions. This confirms Becker’s (1974, fn. 9) intuition that “when the degree of caring becomes sufficiently great, behavior becomes similar to that when there is no caring.” Simply put, when two individuals fully internalize each others’ well-being, it is impossible for one to do anything to make the other happier. If one increases \( l \) so the other can reduce \( l' \), say, the increase in leisure gives the latter more utility, but this is exactly offset by the loss of leisure by the former. As interesting as this may or may not be, the main implication for our purposes is that Proposition 3 holds exactly as stated.
3.4.3 Divorce

In the empirical work below we look at flows into partnerships (marriage), but not flows out (divorce). Why? First, in the baseline model, the divorce hazard \( \delta \) is exogenous, but since the stock \( \sigma \) is endogenous, so is the flow \( \delta(1 - \sigma) \). It is easy to show the following: when the frictions change so that \( R \) decreases – i.e., so that people are more inclined to marriage – one may naively think the divorce flow \( \delta(1 - \sigma) \) should go down; it actually goes up. To see why, note that in steady state the flows in and out are equal, \( \delta(1 - \sigma) = H\sigma \). So, if some change makes people flow into marriage at a higher rate, the stock \( s \) adjusts until the flow out is also higher. This is also true when we endogenize the divorce hazard \( \delta \), in a generalization of Section 3.4.1, by having married individuals learn about each other gradually, or, alternatively, change their mind about each other, over time.\(^\text{17}\)

The point is that even if the divorce hazard falls, the flow \( \delta(1 - \sigma) \) generally does not. This is relevant because it suggests paying less attention to divorce than marriage, which we want to do anyway, because we have less divorce data, and we trust it less. We trust the divorce data less because by the time two people get divorced, they may well have been estranged for a long time. Even if they are living apart, it can take years for a divorce to become official in the records. Of course, the marriage records are not 100% pure in this regard, since couples can live together before getting married, but at least in some of our micro data we can control for this by treating common law couples as married. Whatever data issues arise with marriage, they are probably worse for divorce. We are not suggesting that future work should ignore divorce – just that it helps keep the current project manageable to concentrate for now on flows into marriage.

\(^{17}\)See Burdett and Wright (1998) for the method; the simple idea is to generalize our model the same way Mortensen and Pissarides (1994) generalize Pissarides’ baseline model. As they point out, they do this since they want to study job destruction as well as job creation flows.
3.4.4 Money

For reasons that have to do with empirical work, here we briefly sketch a monetary version of the model.\textsuperscript{18} Since money only has a role when credit is imperfect, assume now that individuals are *anonymous* in the KW market. Hence, they can renege on debt with impunity, and a medium of exchange becomes essential. This role is played by fiat money. The money supply $M$ grows at gross rate $\pi$, which equals the gross inflation rate in stationary equilibrium. Changes in $M$ can be accomplished using lump sum transfers if $\pi > 1$ or taxes if $\pi < 1$, or alternatively, using changes in government spending on the AD commodity $x$ (the results are the same for the variables on which we focus). Also, here we abstract from love and dating, and return to the baseline timing where BC follows KW follows AD.

For an individual in AD with $m$ dollars and marital status $z$,

$$V_1(m, z) = \max_{x,l,m} \{U(x) - l + V_2(\hat{m}, z)\} \text{ st } x + \psi \hat{m} = l(1 - \eta) + \psi m + \Delta,$$

where $\psi$ is the AD price of money in terms of $x$, and the FOC $\partial V_2/\partial \hat{m} = \psi/(1 - \eta)$ implies $\hat{m}$ is independent of $m$. In the KW and BC markets, for a single,

$$V_2(m, s) = A\alpha_0[\epsilon_0 v(y_s) + V_3(m - p_s, s)] + (1 - A\alpha_0)V_3(m, s)$$

$$V_3(m, s) = \lambda \int_R \beta V_1(m, z)dF(z) + [1 - \lambda + \lambda F(R)]\beta V_1(m, s)$$

where $p_z$ denotes the dollars paid for $y_z$ in KW. The equations for a married individuals are similar. Hence, $\partial V_3(m, z)/\partial m = \beta \psi_+/(1 - \eta)$ where $\psi_+$ is the value of money in the next AD market. Bargaining in the KW market is similar to the perfect-credit model, except that we add the constraint $p \leq \hat{m}$, which always binds in equilibrium – at least, as long as the nominal interest rate, defined below, is not 0. Here we use Kalai bargaining,

\textsuperscript{18}For more details, see surveys by Willaimson-Wright (2010) and Nosal-Rocheteau (2011).
which reduces the algebra a lot compared to Nash without affecting the substantive results (see fn.19).

Kalai bargaining now yields a $y_z$ that satisfies $\beta \psi_+ \hat{m} = (1 - \eta)g(y_z)$, where

$$g(y_z) = \frac{(1 - \theta)\varepsilon_z v(y) + \theta y}{\theta(1 - \tau)(1 - \eta) + 1 - \theta} \quad (3.24)$$

The FOC for $\hat{m}$ from AD can be written

$$\frac{\psi}{1 - \eta} = A\alpha_z \varepsilon_z v'(y_z) \frac{\beta \psi_+}{g'(y_z) 1 - \eta} + (1 - A\alpha_z) \frac{\beta \psi_+}{1 - \eta},$$

using $dy_z/d\hat{m} = \beta \psi_+/(1 - \eta)g'(y_z)$. Defining the nominal rate $i$ via the Fisher equation,

$$1 + i = \pi/\beta,$$

this collapses to $i = A\alpha z \mathcal{L}(y_z)$, where

$$\mathcal{L}(y_z) = \frac{\varepsilon_z v'(y_z)}{g'(y_z)} - 1 = \theta \frac{(1 - \tau)(1 - \eta)\varepsilon_z v'(y_z) - 1}{(1 - \theta)\varepsilon_z v'(y_z) + \theta} \quad (3.25)$$

A stationary monetary equilibrium is given by a solution to $i = A\alpha z \mathcal{L}(y_z)$. It is not hard to show this exist iff $i < \bar{i} = A\alpha_0 \theta (1 - \tau)(1 - \eta)/(1 - \theta)$, and when it exists it is unique because $\mathcal{L}'(y_z) < 0$.

In terms of economic results, it is easy to check $y_0 > y_1$ and $\hat{m}_0 > \hat{m}_1$ if $\varepsilon_0 \geq \varepsilon_1$ and $\alpha_0 \geq \alpha_1$ with one inequality strict – i.e., singles buy more retail goods and hence hold more money iff marriage and markets are substitutes in terms of preferences and/or opportunities. It is also easy to check $\partial \hat{m}_z/\partial i < 0$ and $\partial y_z/\partial i < 0$ – i.e., higher nominal interest or inflation rates decrease money balances and retail trade for everyone. The generalization of (3.13) is

$$R = A\alpha_0 S_0 - A\alpha_1 S_1 - ig_0 + ig_1 + \frac{\lambda}{r + \delta} \int_R [1 - F(z)]dz,$$

where $g_z = g(y_z) = \beta \psi_+ \hat{m}_z/(1 - \eta)$. Compared to the benchmark, when individuals
Chapter 3. Marriage, Markets and Money

choose $R$ they now have to take into account the cost of carrying money, $ig_z$. In terms of frictions, the effects are qualitatively the same as Proposition 3 (see Appendix C.2). We also have a new effect, $\partial R/\partial i = (g_1 - g_0)/D$, which we highlight as follows:

**Proposition 4** A unique stationary monetary equilibrium exists iff $i < \bar{i}$. In monetary equilibrium $\partial R/\partial i < 0$ iff $g_0 > g_1$, a sufficient condition for which is $\varepsilon_0 \geq \varepsilon_1$ and $\alpha_0 \geq \alpha_1$ with at least one inequality strict. Also, $\sigma$ moves the same direction as $R$, while $H$ and $\phi$ move in the opposite direction, with respect to changes in $i$.

The key prediction is that as long as $g_0 > g_1$ – which means being single is cash intensive, which is the case as long as marriage and markets are substitutes – inflation like any other tax makes individuals more inclined to move economic activity out of the market and into the home. In this way, inflation encourages marriage. This conclusion is robust to various extensions, including integrating cash and credit models (as in Dong 2010), and using other trading mechanisms in the retail market.\(^{19}\) Also, in the basic model we can add uncertainty, learning and endogenous divorce, change the timing, or incorporate alternative notions of love. The theoretical predictions are robust. We now move to empirical analysis.

### 3.5 Evidence

We consider two types of information. First, under the maintained assumption that marriage and markets are substitutes, as some empirical work suggests (recall fn.11),

\(^{19}\)For Nash bargaining, the algebra is messier, as one can see from comparing (3.24) and (3.25) to

$$
\begin{align*}
g(y_z) &= \frac{\theta y_z \varepsilon_z v'(y_z) + (1 - \theta) \varepsilon_z v(y_z)}{(1 - \tau)(1 - \eta) \theta \varepsilon_z v'(y_z) + 1 - \theta} \\
\mathcal{L}(y_z) &= \frac{\theta[(1 - \tau)(1 - \eta) \varepsilon_z v'(y_z) - 1] \varepsilon_z v(y_z) - \varepsilon_z v'(y_z) (1 - \tau)(1 - \eta) \theta \varepsilon_z v'(y_z) + 1 - \theta + \Gamma}{(1 - \tau)(1 - \eta) \theta \varepsilon_z v'(y_z) + 1 - \theta} \\
\end{align*}
$$

where $\Gamma = \theta(1 - \theta)[(1 - \tau)(1 - \eta) \varepsilon_z v'(y_z) - y_z] \varepsilon_z v''(y_z)$. One can still show $\partial R/\partial i < 0$ iff $g_0 > g_1$, and $g_0 > g_1$ holds under the same conditions, but there is one technicality: we cannot sign $\mathcal{L}'$ with Nash bargaining, while we know $\mathcal{L}' < 0$ with Kalai bargaining. Still, the method in Wright (2010) gets around the problem, and the results go through.
theory predicts singles use markets and hence money more than otherwise similar married people. We examine micro evidence on this. Second, given singles hold more money, theory predicts inflation like other market frictions increases the propensity to marry. We examine macro data on marriage flows across countries and time, to see how they relate to inflation, taxation and other variables.\textsuperscript{20}

### 3.5.1 Micro

There is some existing work that bears on the idea that being single is cash intensive. Consider Klee’s (2008) study of how people pay, using US grocery-store scanner data. These data do not include demographic information, of course, so she compares payment patterns across census tracts. Although not primarily interested in the effects of marital status, she reports that after controlling for the number of items purchased, their values, income, age and other factors, marriage – i.e., being in a census tract with more married people – significantly decreases the probability of using cash by 0.466 and increases the probability of using credit cards by 0.249.\textsuperscript{21} Also, the probability of using cash (credit) increases (decreases) on Friday and Saturday, consistent with the idea that going out is cash intensive, and given single people go out more, this again suggests that they use more money. Klee considers other explanations for the weekend effect (e.g., people get paid on Friday), but concludes “the type of items bought on Friday and Saturday – beer and cigarettes in particular – are more likely to be purchased with cash,” suggesting to us that dating is cash intensive. Her bottom line is “census tracts with a higher percentage of married households are less likely to use cash,” consistent with the hypothesis in question.\textsuperscript{22}

\textsuperscript{20}One can interpret these exercises as “tests” under the maintained hypothesis that marriage and markets are substitutes; alternatively, one can take the model as given and interpret the data as indicating the extent to which marriage and markets are substitutes.

\textsuperscript{21}Klee has data on other payments methods, too. For the record, marriage increases the probability of using checks and debit cards by the relatively small 0.115 and 0.102.

\textsuperscript{22}We briefly mention some other related work. Liu (2008) regresses cash holdings on income, expenditure and demographics, and finds a dummy for married is significantly negative. Duca and Whitesell
Moving to our own analysis, which can be focused more directly on the question at hand, consider first the Italian Survey of Household Income and Wealth, which collects information on currency holdings and spending from several thousand individuals every two years between 1993 and 2004.\textsuperscript{23} Tables 1 and 2 present summary statistics (Tables are at the end). Table 1a reports currency holdings in each year for households with \(N = 1\) adult, which we take to be single people, as well as households with \(N = 2\) and \(N = 3\). Currency holding is on average 54\% higher for individuals in households with \(N = 1\) than \(N = 2\). Although our theory focused on \(N = 1\) or 2, we also report that adults in households with \(N = 1\) hold about double the money held by those with \(N = 3\). These numbers do not control for expenditure, however, which may be important if single and married people differ in their spending behavior for other reasons. Table 1b corrects for this by dividing currency per adult by expenditure per adult (i.e., total household expenditure over \(N\)). Currency per adult over expenditure per adult is on average around 23\% higher for people in households with \(N = 1\) than \(N = 2\), and 28\% higher for those with \(N = 1\) that \(N = 3\).

We tried a number of different ways to investigate the robustness of these findings. Table 1c divides currency per adult by cash (instead of total) expenditure per adult, which changes the numbers to 19\% and 29\%. Tables 2a-c report the same measures as Tables 1a-c restricting the sample to those individuals with bank accounts, which somewhat reduces the number of observations, but makes sense to the extent that those without bank accounts may be different in ways the theory does not take into account (e.g., perhaps these individuals are involved in illegal activity, although this is speculation). This

\textsuperscript{23}Francesco Lippi was very generous sharing this data with us and helping us understand it. For more on using this data, see Alvarez and Lippi (2009) Lippi and Secchi (2009).
changes the numbers slightly, but not a lot. We also parsed the sample by distinguishing between households with no bank, with a bank but no access to an ATM, and with a bank and access to ATM; we also divided cash by durable (instead of total) expenditure. The results were broadly similar. In terms of statistical confidence, the standard errors are sufficiently low in Table 1 that clearly these differences are highly significant. Overall this evidence is very much consistent with the underlying hypothesis: at least in Italy, being single is cash intensive.

Consider next the 2009 Survey of Consumer Payment Choice by the Federal Reserve Bank of Boston, which contains data on individual cash in the wallet and total cash (in the wallet plus elsewhere), as well as income and other demographic information.\textsuperscript{24} Tables 3 reports numbers for the following categories of marital status: single; divorced or separated; widowed; nonmarried (the sum of the first three); and married plus common law. Column 1 shows that without controlling for income or expenditure, married and divorced individuals have less cash in the wallet than single or widowed individuals. Column 2 adjusts cash in wallet by annual household income in thousands. By this measure, married have substantially less cash in the wallet than nonmarried individuals. This may be an over-correction, however, since it divides individual cash holdings by household income. Column 3 rectifies this by dividing household income by the number of adults (individuals over age 15), giving cash per person over income per person. This again indicates that married people hold less cash in wallet: by this measure, nonmarried hold around 68\% more than married people.

Columns 4-6 redo 1-3 using total cash holdings. The same pattern emerges, with Column 6 indicating that after controlling for household size and income nonmarried hold around 127\% more money than married people. Columns 7-12 in Table 3 restrict the sample by eliminating individual observations with cash in the wallet over $1,000 or total cash holding over $10,000. Again this seems reasonable to the extent that people with

\textsuperscript{24}See Schuh and Stavins (2010) for more discussion of the data.
that much money may be engaged in activities not in the model (e.g., illegal activities). In this restricted sample, Columns 9 and 12 show that nonmarried hold around 67% more money in their wallets, and around 141% more money in total, than married individuals. Hence, it is also the case in America that being single is cash intensive, and this is true not only when we consider cash on hand, but also in the proverbial cookie jar and elsewhere.

Consider finally the Bank of Canada’s 2009 Methods of Payment Survey, which also contains information on currency holdings, spending, income and demographics from 6,868 respondents in a Survey Questionnaire, plus a subsample of 3,465 individuals in a 3-day Diary Instrument providing details on all transactions. Table 4 provides statistics from the Survey Questionnaire for the same categories of marital status used with Boston Fed data. Columns 1-3 are cash holdings, cash holdings divided by annual household income in thousands and cash holdings divided by household income adjusted by the number of adults (in this case, individuals over age 18). Columns 4-6 redo 1-3 using cash spending rather than cash holding. The numbers continue to indicate that married individuals hold the least cash, although the differences are not as big. By the measure in Columns 3 and 6, nonmarried hold on average around 7% more cash than married and after controlling for household size and income, and nonmarried spend approximately 1/3 more cash. Columns 7-12 of Table 4 restrict the sample by eliminating individual observations where cash holding or weekly cash expenditure exceeds $1,000. Columns 9 and 12 indicate that, in the restricted sample, nonmarried hold around 40% more cash than married, and spend around 48% more.

Arguably the Diary Instrument is more reliable than the Survey, even if the sample is smaller. The Diary numbers are reported in Table 5. Columns 3 and 6 indicate that average cash holding is about 1/3 greater for nonmarried people, and cash spending 2/3 greater, after appropriately controlling for household income and size. Columns 9 and 12 show similar results after again eliminating observations above $1,000. Most of the

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25See Arango and Welte (2012) for more on this dataset.
relevant differences – e.g., either cash holdings or spending, normalized by household income, for married vs. nonmarried – are again highly significant in both the Survey and Diary data, without or with truncating observations over $1,000. On the whole, these data constitute strong evidence in favor of the hypothesis that being single is cash intensive in Canada, too.\textsuperscript{26}

We also report OLS regression results on the Canadian data.\textsuperscript{27} The LHS variable is cash holding (or spending) divided by household income, normalized by household size iff dummies for size are not included on the RHS. In addition to household size, RHS variables include age, education etc., plus dummies for marital status. To keep the amount of information manageable, results are only reported using two marital states: we group single, divorced, separated and widowed as nonmarried; and group common law together with married. We give results for both the Survey and Diary excluding observations with cash holding or spending above $1,000. Tables 6 and 7 use the Survey and Diary data, resp., and yield fairly similar estimates. To understand the units, the average married person in the Diary sample, after eliminating observations over $1,000, holds around $77, which yields 2.825 after dividing by individual annual income (Table 5b, Column 7). So the average married person has annual income $27,349. A coefficient on the nonmarried dummy of 1, which is close to the estimates, means that if the identical person were nonmarried, they would hold 1/10 of 1\% more in terms of annual income, which is an extra $27, or a 35\% increase.

The key result for our purposes is this: for both cash holding and spending, no matter which of the various runs one considers, the coefficient on the nonmarried dummy is positive and significant, usually at the 1\% level. Also of interest is the finding that the

\textsuperscript{26}The Bank of Canada data also have payment information that can be analyzed as in Klee (2008). A quick summary from results in Arango et al. (2011) is this: the probabilities of using cash in any transaction for single and married people are 55\% and 48\%, resp., and the probabilities of using credit cards are 20\% and 25\%, resp.

\textsuperscript{27}We ran similar regressions on the Boston Fed data, and the results were similar, but not quite as strong (the marital status dummies were significant in some but not all runs).
unemployed and less educated tend to use more cash. We emphasize, however, that the reason the nonmarried dummy is significant is not that being single is correlated with being unemployed or less educated, since the relevant coefficient is still positive and significant when unemployment and education are on the RHS. Also, males use more cash, although this is not always significant. Finally, when we do not divide cash by household size on the LHS, but instead include size dummies on the RHS, individuals from bigger households use less cash, although again this is not always significant. While there is more one can do with the data, on this issue, we are prepared to rest our case: all the evidence makes it hard not to agree that being single is cash intensive.

### 3.5.2 Macro

Elementary economics tells us that market frictions encourage people to substitute out of markets and into home production, and as long as household production is facilitated by household formation, this encourages marriage. Since being single is cash intensive, as documented above, inflation leads to more marriage (Proposition 4). The logic is even simpler for the effects of sales or income taxes (Proposition 3). These effects are more likely to be operative to the extent that marriage and markets are good substitutes, as empirical work suggests (fn.11). Given this, we examine marriage rates in a panel of countries over the last half century, taking into account the effects of inflation and taxation, as well as growth and unemployment.

Table 8 provides a summary of the data. We use number of marriages and population data to compute the marriage flow \( f \) (marriages per 1000 population), from 1950 to 2004, for all countries in the United Nations Common Database (UNCDB). There are 275 countries in total in the UNCDB – a lot more than there are at any point in time, since many come into or go out of existence over the period. Given missing observations and other problems, we have at most 152 usable countries, and often far fewer, depending on what other variables we include in the regressions. Inflation is available for many
of these countries, using either the CPI or GDP deflator, from International Financial Statistics (IFS). Our GDP deflator data covers around 150 countries starting in 1970, while our CPI series is longer but has fewer countries, so we report results for both. We have OECD consumption tax rates every four years before 1990, and every two years up to 2000. Since this leaves a lot of gaps, we also use the tax series constructed in Mendoza et al. (1997), hereafter the Mendoza taxes. A big advantage of the Mendoza data is that they include labor and capital, as well as consumption, taxes; the down side is that they are available for only 18 countries.\footnote{These countries are: US, UK, Austria, Australia, Belgium, Denmark, France, Germany, Italy, New Zealand, Netherlands, Norway, Sweden, Switzerland, Canada, Japan, Finland and Spain.} In general, while taxation may be at least as important as inflation, the advantage of inflation is that the data is readily available for many more countries and years.

Although we did not explicitly incorporate aggregate changes in output or unemployment in the formal model, it would not be hard to do so, and might be interesting in future work. And even if one does not have strong priors on the effects of unemployment or output growth, one wants to control for these factors empirically for the following reason. Whatever relation one might find between marriage, on the one hand, and inflation or taxation or any other independent variable, on the other, it could be dismissed by arguing that any independent variable like inflation or taxation is merely standing in for changes in output or unemployment that are correlated with it. Hence, we include in the empirical analysis real output growth and unemployment rates. Our output data is also from IFS. For unemployment, there are various sources, including OECD Labor Force Statistics, UN and IFS data. The OECD data are available for a small subset of countries, while the others are available for around 70 countries. We tried them all, but in the interests of space we report results only for IFS unemployment (the main conclusions were similar for the others).

Since we have a panel, we use GLS. Table 9 reports the results of a first cut at the
data, where we run marriage rates on GDP deflator and CPI inflation rates in Columns (1)-(4) and Columns (5)-(8), resp. In each case, the first Column has only inflation and inflation squared on the RHS, where the latter is included because one should clearly expect a nonlinear relationship, given that there are some observations of extremely high inflation rates, and marriage rates are bounded above. Then the second Column includes output growth, the third includes unemployment, and the fourth includes both. Table 9 does not include taxes on the RHS, which means we can use a large sample, including as many as 3,453 observations across 67 to 152 countries, depending on what is on the RHS. In this Table, there is really not much going on: the coefficients on inflation may be positive, but they are tiny, and usually not significant; output growth does very little; and the only significant result is that unemployment reduces marriage. But, as we said, this is only a first cut.

Table 10 redoes Table 9 including OECD consumption taxes on the RHS, which reduces the number of countries and observations. Still the negative effect of unemployment is there, and now output sometimes shows up positive and significant. Also, consumption taxes seem to reduce marriage. We come back to taxes later; for now, the result on which we focus is that the inflation coefficients now are positive and often highly significant. Table 11 repeats the exercise using Mendoza taxes, reducing the number of countries even more. Again we postpone discussion of fiscal variables to concentrate on inflation. In Table 11 the inflation effect is positive, and usually highly significant, at least when we use the GDP deflator. To see if the differences between Table 9 and Table 11 are due mainly to adding taxes or changing the sample, Table 12 runs the model without taxes on the Mendoza countries, which tend to be more advanced (fn.28). The results are much stronger here – in contrast to Table 9, basically everything is significant in Table 12, even though there are fewer observations. In particular, in all runs the coefficients on inflation, and usually also on inflation squared, take the expected signs and are highly significant. For the countries in the Mendoza sample, without controlling for taxes the
effect of inflation is clear; and if the results are somewhat weaker when taxes are included, in Table 11, this may be due at least in part to having fewer observations.

As mentioned above, one usually takes more seriously any given model’s predictions about some components of the data than others, and we tend to think our theory applies better over longer horizons. In studying business cycles, one often takes the objects of interest to be deviations from slowly moving trends, defined using an HP filter or some other smoothing procedure, with the idea being that some models are more useful for thinking about high- than low-frequency data. Since our model seems relatively useful for thinking about lower frequencies, the objects of interest are not the deviations but the trends themselves. Table 13 shows results using HP trends on the RHS, without taxes, so that we can use the biggest set of countries and years. The results are much stronger than those for the raw data in Table 9, and in particular, the effect of inflation is usually highly significant. Table 14 redoes Table 13 including taxes, which reduces the number of countries and observations a lot, but still inflation has a significant positive effect. Table 15 runs the model without taxes on the Mendoza countries using the smoothed data. The effects of inflation continue to be consistently positive and highly significant. And to show the exact smoothing procedure does not matter much, Table 16 redoes Table 15 using five-year moving averages instead of the HP filter, with very similar results.

Our conclusions from all this are the following. First, output growth (unemployment) tends to encourage (discourage) marriage. We find this interesting. Second, inflation performs as expected based on Proposition 4, especially when we smooth the data to focus on lower frequencies. We find this satisfying. Third, given Proposition 3, we find the effects of consumption and labor taxes puzzling, since it is unclear why taxes should decrease marriage. One answer might involve a model with multiple market goods, some of which are substitutes for home production – e.g., food – while others are complements – e.g., consumer durables. For the sake of argument, suppose the former tend to be purchased with cash and the latter on credit. Then inflation encourages home production
and marriage as agents substitute home cooking for restaurant meals, while at the same
time taxation discourages marriage since it raises the cost of durables used at home. We
do not find this story too contrived, although one needs to work out the details, and that
is left for future research. Another avenue to consider is that in some countries tax codes
have a direct impact on the cost of getting married, as discussed by Chade and Ventura
(2002) and references therein. For now, the main point of looking at the aggregate data
was to see if inflation and/or taxation have an impact on marriage. It seems that they
do.

One can also look at individual countries, although there are so many that a care-
ful analysis is beyond the scope of this project. But, as a teaser, we can show some
information that readers may find useful. Figures 1-3 provide scatter plots of marriage
and inflation for the Mendoza countries, with the right panels using raw data and the
left using data smoothed by the HP filter. Regression lines are shown in each case,
although one hardly needs these to see a clear positive relationship in most countries.
We do not want to make too much of these plots, but it would seem wrong to not show
them. Also, we are aware that the 18 plots do not constitute 18 independent pieces of
information, since many of these countries have had similar inflation experiences over
the last half century, as well as participating in more or less similar events that could
affect marriage – e.g., changes in social customs, or demographic developments, like baby
boomers coming of age at similar times. Still, the plots do convey information.

While a complete country-by-country analysis must wait, we report results for the
US and Canada in Tables 17-20. For the US, when taxes are not included, inflation
has a highly significant positive impact on marriage. Once taxes are included, however,

29The points are color-coded by decade: red – 1950s; blue – 1960s; purple – 1970s; green – 1980s; and
black – 1990s. This allows one to trace out the history of inflation and marriage from the scatter for
each country about as easily as looking at the time series.

30The exceptions include Denmark, where the relationship goes the ”wrong” way, and a few others,
where it is basically flat. Finn Kydland suggests a potentially relevant feature of Denmark is that many
people live common law, so perhaps the marriage data is not as reliable.
the effect is reduced to 0, if not negative, although again this may be partly due to sample size. But what is remarkable is that in the US the effects of taxation are very much in line with theory: the relevant coefficients are all positive and usually highly significant. Coasian logic thus works well for the US, even if fiscal seem more important than monetary considerations. For Canada, the coefficients are positive for consumption and capital taxes, but negative for labor taxes. It is easier to think inflation has the predicted effect in Canada – for smoothed data, the relevant coefficients are positive and highly significant in 5 out of the 8 runs. If inflation effects are harder to see in Canada and the US, it may be because these countries do not have so much inflation, or variation in inflation, compared to the world at large. This all seems worth additional study, but must be left for future work.

3.6 Conclusion

Coasian theory recognizes that markets and other ways of organizing economic activity coexist, and the choice to use one or the other depends on costs and benefits. The use of the market entails frictions, including search and bargaining costs, taxation and inflation. When these are big, it is desirable to bring certain activities in house by substituting out of market and into household production. Given the latter is facilitated by household formation, this increases marriage. We formalized this in a dynamic general equilibrium model, where agents trade goods, labor and assets, plus search for partners. Although the setup has a lot of detail, it simplifies nicely, and delivers very clean predictions. We think the model stands on its own as a contribution to the economic theory of marriage. But we also provided some empirical work, where we found a positive relationship between inflation and marriage in the macro data, which makes sense in light of the micro evidence that being single is cash intensive. The effects of taxation are less clear, in general, although they are there for the US. More can be done. We wanted here mainly to
illustrate that one can build dynamic general equilibrium models with frictional marriage and goods markets, and to suggest this may be interesting, not only in theory, but also empirically.
Table 3.1: Cash Holding per Adult – All Households

(1) Cash Holding per Adult – All Households

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<tbody>
<tr>
<td>N = 1</td>
<td>292.580</td>
<td>297.610</td>
<td>317.030</td>
<td>326.060</td>
<td>312.930</td>
<td>317.780</td>
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<td>1837</td>
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<td>201.711</td>
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<tr>
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<td>1207</td>
<td>1384</td>
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<td>1.641</td>
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<td>1.545</td>
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(2) Cash Holding per Adult ÷ Total Expend. per Adult – All Households

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<tbody>
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<td>obs</td>
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<tr>
<td>obs</td>
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<td>3195</td>
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<tr>
<td>obs</td>
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<td>1207</td>
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(3) Cash Holding per Adult ÷ Cash Expend. per Adult – All Households

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</tr>
</thead>
<tbody>
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<td>(0.440)</td>
<td>(0.381)</td>
<td>(0.574)</td>
<td>(0.456)</td>
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<td>obs</td>
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<tr>
<td>obs</td>
<td>(3.140)</td>
<td>(3.270)</td>
<td>(3.630)</td>
<td>(3.530)</td>
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<td>1.156</td>
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<td>1.367</td>
<td>1.253</td>
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<td>1.293</td>
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Notes: Data source – Italian Survey of Household Income and Wealth. N = j for j = \{1, 2, 3\} is the number of adults in a household. Standard errors in parentheses.
Table 3.2: Cash Holding per Adult – Households with Bank

### (Table 3.2a) Cash Holding per Adult – Households with Bank

<table>
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<td>(7.700)</td>
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<td>(6.522)</td>
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<tr>
<td>obs</td>
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<td>901</td>
<td>874</td>
<td>1112</td>
<td>1343</td>
<td>1420</td>
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<tr>
<td>N = 2</td>
<td>194.700</td>
<td>224.560</td>
<td>202.900</td>
<td>188.690</td>
<td>176.910</td>
<td>181.590</td>
<td>194.551</td>
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<tr>
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<td>(2.971)</td>
<td>(3.352)</td>
<td>(3.782)</td>
<td>(3.036)</td>
<td>(2.918)</td>
<td>(2.896)</td>
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<td>1089</td>
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<tr>
<td>N = 3</td>
<td>149.280</td>
<td>163.250</td>
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<td>139.990</td>
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<td>145.720</td>
<td>148.393</td>
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<td>(3.515)</td>
<td>(3.954)</td>
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<td>(3.676)</td>
<td>(4.091)</td>
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<td>1071</td>
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### (Table 3.2b) Cash Holding per Adult ÷ Total Expend. per Adult – Households with Bank

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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.280)</td>
<td>(0.230)</td>
<td>(0.293)</td>
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<td>(0.214)</td>
<td>(0.195)</td>
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<tr>
<td>obs</td>
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<td>901</td>
<td>874</td>
<td>1112</td>
<td>1343</td>
<td>1420</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.132)</td>
<td>(0.142)</td>
<td>(0.164)</td>
<td>(0.134)</td>
<td>(0.133)</td>
<td>(0.137)</td>
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<td>7.657</td>
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<td>(0.230)</td>
<td>(0.189)</td>
<td>(0.208)</td>
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<tr>
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### (Table 3.2c) Cash Holding per Adult ÷ Cash Expend. per Adult – Households with Bank

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<td>15.988</td>
<td>16.653</td>
<td>16.716</td>
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<td>(0.410)</td>
<td>(0.361)</td>
<td>(0.502)</td>
<td>(0.416)</td>
<td>(0.344)</td>
<td>(0.359)</td>
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<td>874</td>
<td>1111</td>
<td>1343</td>
<td>1420</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.255)</td>
<td>(0.215)</td>
<td>(0.294)</td>
<td>(0.246)</td>
<td>(0.296)</td>
<td>(0.301)</td>
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<td>2759</td>
<td>2864</td>
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</tr>
<tr>
<td></td>
<td>(0.336)</td>
<td>(0.291)</td>
<td>(0.386)</td>
<td>(0.311)</td>
<td>(0.332)</td>
<td>(0.415)</td>
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<td>1071</td>
<td>1216</td>
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Notes: Data source – Italian Survey of Household Income and Wealth. \( N = j \) for \( j = \{1, 2, 3\} \) is the number of adults in a household. Standard errors in parentheses.
Table 3.3: Summary Statistics – 2009 FRB Boston Survey

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<tr>
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<td>83.151</td>
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</tr>
<tr>
<td></td>
<td>(12.848)</td>
<td>(0.562)</td>
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<tr>
<td>divorced or separated</td>
<td>63.896</td>
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</tr>
<tr>
<td></td>
<td>(8.240)</td>
<td>(0.517)</td>
</tr>
<tr>
<td>widowed</td>
<td>85.740</td>
<td>2.610</td>
</tr>
<tr>
<td></td>
<td>(10.527)</td>
<td>(0.405)</td>
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<tr>
<td>nonmarried</td>
<td>76.909</td>
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<tr>
<td></td>
<td>(7.665)</td>
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<tr>
<td>married or common law</td>
<td>64.340</td>
<td>1.802</td>
</tr>
<tr>
<td></td>
<td>(3.258)</td>
<td>(0.548)</td>
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<tr>
<td>observations</td>
<td>2132</td>
<td>2125</td>
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Notes:
2. (1) & (7) cash in wallet (in USD); (2) & (8) cash in wallet over household income (in 1k USD); (3) & (9) cash in wallet over household income adjusted for the household size (adults 15 years old or older); (4) & (10) total cash holding (cash in wallet and cash held elsewhere in USD)); (5) & (11) total cash holding divided by household income; (6) & (12) total cash holding divided by household income after adjusting for the household size.
3. Nonmarried includes single, divorced, separated and widowed.
4. Standard errors are in parentheses.
### Table 3.4: Summary Statistics – Bank of Canada Survey Questionnaire

<table>
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<th>Cash in wallet</th>
<th>Cash spending</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(4) (5) (6)</td>
</tr>
<tr>
<td><strong>single</strong></td>
<td>69.699 (5.501)</td>
<td>1.848 (0.132)</td>
</tr>
<tr>
<td></td>
<td>3.621 (0.382)</td>
<td>93.502 (9.029)</td>
</tr>
<tr>
<td></td>
<td>2.149 (0.153)</td>
<td>4.455 (0.601)</td>
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<tr>
<td><strong>divorced or separated</strong></td>
<td>73.007 (6.799)</td>
<td>2.737 (0.451)</td>
</tr>
<tr>
<td></td>
<td>3.984 (0.707)</td>
<td>75.627 (4.682)</td>
</tr>
<tr>
<td></td>
<td>2.627 (0.239)</td>
<td>3.895 (0.521)</td>
</tr>
<tr>
<td><strong>widowed</strong></td>
<td>95.745 (15.004)</td>
<td>2.996 (0.378)</td>
</tr>
<tr>
<td></td>
<td>4.301 (0.740)</td>
<td>104.259 (17.573)</td>
</tr>
<tr>
<td></td>
<td>2.982 (0.396)</td>
<td>4.289 (0.829)</td>
</tr>
<tr>
<td><strong>nonmarried</strong></td>
<td>72.513 (4.207)</td>
<td>2.177 (0.155)</td>
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<td>3.764 (0.322)</td>
<td>89.291 (6.138)</td>
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<td>2.343 (0.125)</td>
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<td>83.229 (11.908)</td>
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<td>3.514 (1.012)</td>
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<td>1.440 (0.121)</td>
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<tr>
<td><strong>obs</strong></td>
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<td>5038 4995</td>
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### Table 3.4b: Summary Statistics – Bank of Canada Survey Questionnaire

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<td>(7) (8) (9)</td>
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<td><strong>single</strong></td>
<td>65.821 (4.192)</td>
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<td>3.373 (0.318)</td>
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<td>2.050 (0.140)</td>
<td>4.408 (0.599)</td>
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<tr>
<td><strong>divorced or separated</strong></td>
<td>69.204 (5.827)</td>
<td>2.364 (0.262)</td>
</tr>
<tr>
<td></td>
<td>3.446 (0.448)</td>
<td>75.627 (4.682)</td>
</tr>
<tr>
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<td>2.627 (0.239)</td>
<td>3.895 (0.521)</td>
</tr>
<tr>
<td><strong>widowed</strong></td>
<td>95.745 (15.004)</td>
<td>2.996 (0.378)</td>
</tr>
<tr>
<td></td>
<td>4.301 (0.740)</td>
<td>104.259 (17.573)</td>
</tr>
<tr>
<td></td>
<td>2.982 (0.396)</td>
<td>4.289 (0.829)</td>
</tr>
<tr>
<td><strong>nonmarried</strong></td>
<td>68.939 (3.377)</td>
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<td></td>
<td>1.292 (0.095)</td>
<td>2.873 (0.277)</td>
</tr>
<tr>
<td><strong>obs</strong></td>
<td>6170 6170</td>
<td>5026 4995</td>
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</tbody>
</table>

**Notes:**
1. Data source – the 2009 Methods of Payment Survey by the Bank of Canada.
2. (1) & (7) cash in wallet (in CAD); (2) & (8) cash in wallet over household income (in 1k CAD); (3) & (9) cash in wallet over household income adjusted for household size (adults 18 years old or older); (4) & (10) weekly cash spending (in CAD); (5) & (11) weekly cash spending over household income; (6) & (12) weekly cash spending over household income adjusted for household size.
3. Nonmarried includes single, divorced, separated and widowed.
Table 3.5: Summary Statistics – Bank of Canada Diary Instrument

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<th></th>
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</thead>
<tbody>
<tr>
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<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>single</td>
<td>68.978</td>
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<td>3.509</td>
<td>96.129</td>
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<td>(3.456)</td>
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<td>(0.566)</td>
<td>(0.963)</td>
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<td>(7.832)</td>
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<td>(9.438)</td>
<td>(0.460)</td>
<td>(1.058)</td>
</tr>
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<td>2.940</td>
<td>111.855</td>
<td>2.026</td>
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<tr>
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<td>(6.802)</td>
<td>(0.086)</td>
<td>(0.194)</td>
<td>(6.126)</td>
<td>(0.144)</td>
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Notes:
1. Data source – the 2009 Methods of Payment Survey by the Bank of Canada
2. (1) & (7) cash in wallet (in CAD); (2) & (8) cash in wallet over household income (in 1k CAD); (3) & (9) cash in wallet over household income adjusted for household size (adults 18 years old or older); (4) & (10) weekly cash spending (in CAD); (5) & (11) weekly cash spending over household income; (6) & (12) weekly cash spending over household income adjusted for household size.
3. Nonmarried includes single, divorced, separated and widowed.
Table 3.6: Regression Results – Bank of Canada Survey Questionnaire

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<th>(6) CS,r</th>
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Notes:
1. Data source – the 2009 Methods of Payment Survey by the Bank of Canada; sample restricted to cash holding or spending less than $1000.
2. Nonmarried includes single, divorced, separated and widowed.
3. Dependent variable in (1)-(4): cash in wallet (CW) or cash spending (CS) over household income after adjusting for household size.
4. Dependent variable in (5)-(6): cash in wallet (CW,r) or cash spending (CS,r) over household income without adjusting for household size.
5. Base group: married, female, employed, college and above, age 18 – 25, household size= 1 if dummy included in regression.
6. Standard errors in parentheses; p-values: *** p < 0.01, ** p < 0.05, * p < 0.1.
Table 3.7: Regression Results – Bank of Canada Diary Instrument

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<th>(5) CW_r</th>
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<td>0.752***</td>
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Notes:
1. Data source – the 2009 Methods of Payment Survey by the Bank of Canada; sample restricted to cash holding or spending less than $1000.
2. Nonmarried includes single, divorced, separated and widowed.
3. Dependent variable in (1)-(4): cash in wallet (CW) or cash spending (CS) over household income after adjusting for household size.
4. Dependent variable in (5)-(6): cash in wallet (CW_r) or cash spending (CS_r) over household income without adjusting for household size.
5. Base group: married, female, employed, college and above, age 18 – 25, household size= 1 if dummy included in regression.
6. Standard errors in parentheses; p-values: *** p < 0.01, ** p < 0.05, * p < 0.1.
## Table 3.8: Summary of Macro Data Sources

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### Table 3.9: Panel-data GLS w/o taxes, raw data

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<td>-0.000***</td>
<td>-0.000***</td>
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<td>-0.085***</td>
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Notes: Definitions of variables: $\pi_D$ – inflation measured by GDP deflator; $\pi_C$ – inflation measured by CPI; $\gamma$ – real output growth rate; $u$ – unemployment rate; and $NC$ – number of countries. Standard errors in parentheses; p-values: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. 
Table 3.10: Panel-data GLS w/ OECD taxes, raw data

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Notes: See notes for Table 3.9, and $\tau$ – consumption tax rates from the OECD statistics.
Table 3.11: Panel-data GLS w/ Mendoza taxes, raw data

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Notes: See notes for Table 3.9, and $\tau_c$ – consumption tax rates, $\tau_h$ – labor income tax rates, $\tau_k$ – capital income tax rates; all tax rates are from Mendoza et al. (1997).
### Table 3.12: Panel-data GLS, Mendoza sample w/o taxes, raw data

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Notes: See notes for Table 3.9.
Table 3.13: Panel-data GLS w/o taxes, smoothed data (HP)

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Notes: See notes for Table 3.9.
Table 3.14: Panel-data GLS w/ Medoza taxes, smoothed data (HP)

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Notes: See notes for Table 3.11.
### Table 3.15: Panel-data GLS, Mendoza sample w/o taxes, smoothed data (HP)

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<tr>
<td>(\pi_D)</td>
<td>0.125*** (0.020)</td>
<td>0.177*** (0.021)</td>
<td>0.126*** (0.022)</td>
<td>0.174*** (0.025)</td>
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<tr>
<td>(\pi_D^2)</td>
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<td>-0.004*** (0.001)</td>
<td>-0.004*** (0.002)</td>
<td>-0.005*** (0.002)</td>
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<tr>
<td>(\pi_C)</td>
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<td></td>
<td>0.264*** (0.040)</td>
<td>0.224*** (0.038)</td>
<td>0.394*** (0.047)</td>
<td>0.390*** (0.047)</td>
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<td>(\pi_C^2)</td>
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<td></td>
<td></td>
<td>-0.013*** (0.003)</td>
<td>-0.010*** (0.002)</td>
<td>-0.018*** (0.003)</td>
<td>-0.018*** (0.003)</td>
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<td>(\gamma)</td>
<td>7.570*** (1.420)</td>
<td>6.614*** (1.616)</td>
<td>22.684*** (2.195)</td>
<td>2.267 (3.724)</td>
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<tr>
<td>(u)</td>
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<td></td>
<td>-0.054*** (0.016)</td>
<td>-0.057*** (0.015)</td>
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Notes: See notes for Table 3.9.

### Table 3.16: Panel-data GLS, Mendoza sample w/o taxes, smoothed data (5-yr moving avg)

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<td>(\pi_D)</td>
<td>0.139*** (0.011)</td>
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<td>-0.005*** (0.001)</td>
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<td>-0.016*** (0.003)</td>
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Notes: See notes for Table 3.9.
Table 3.17: Individual Country - the United States, raw data

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<td>$\pi_C$</td>
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Notes: See notes for Table 3.11.

Table 3.18: Individual Country - the United States, smoothed data (HP)

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Notes: See notes for Table 3.11.
### Table 3.19: Individual Country - Canada, raw data

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<tbody>
<tr>
<td>$\pi_C$</td>
<td>0.102 (0.183)</td>
<td>0.124 (0.174)</td>
<td>0.772*** (0.195)</td>
<td>0.810*** (0.185)</td>
<td>0.493* (0.260)</td>
<td>0.275 (0.212)</td>
<td>0.116 (0.154)</td>
<td>0.004 (0.128)</td>
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<td>$\pi_C^2$</td>
<td>0.004 (0.124)</td>
<td>0.004 (0.015)</td>
<td>-0.042** (0.015)</td>
<td>-0.042*** (0.014)</td>
<td>-0.027 (0.017)</td>
<td>-0.011 (0.014)</td>
<td>-0.007 (0.010)</td>
<td>0.003 (0.009)</td>
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<tr>
<td>$\gamma$</td>
<td>13.326** (5.389)</td>
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<td>9.028*** (2.859)</td>
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Notes: See notes for Table 3.11.

### Table 3.20: Individual Country - Canada, smoothed data (HP)

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<td>$\pi_C$</td>
<td>0.409 (0.266)</td>
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<td>1.258*** (0.184)</td>
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<td>0.861*** (0.143)</td>
<td>0.694*** (0.167)</td>
<td>0.732*** (0.116)</td>
<td>0.042 (0.118)</td>
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<td>-0.083*** (0.015)</td>
<td>-0.054*** (0.012)</td>
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<td>$\gamma$</td>
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<td>5.720* (3.278)</td>
<td>14.787*** (2.186)</td>
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Notes: See notes for Table 3.11.

---

**Figure 3.1:** The legend of Figure 3.2-3.4.
Figure 3.2: Austria, Australia, Belgium, Canada, Denmark, Finland
Figure 3.3: France, Germany, Italy, Japan, Netherlands, New Zealand
Figure 3.4: Norway, Spain, Sweden, Switzerland, UK, US
Appendix A

Appendix to Chapter 1

A.1 Optimality Conditions of the Economy with Perfectly Liquid Bonds

In the economy with perfectly liquid bonds, a household who has high taste in even periods choose how much to consume and to produce, and how much money to be carried over to the next period. Let $\mu^H_t$ and $\mu^H_{t+1}$ be the multipliers of the cash-in-advance constraints in (1.1). Let $\lambda^H_t$ and $\lambda^H_{t+1}$ be the multipliers of the budget constraints (1.2)
and (1.3). The optimality conditions are:

\[
\begin{align*}
c^H_t &: \frac{\beta^t \theta}{c^H_t p_t} = \mu^H_t + \lambda^H_t \\
c^H_{t+1} &: \frac{\beta^{t+1}}{c^H_{t+1} p_{t+1}} = \mu^H_{t+1} + \lambda^H_{t+1} \\
M^H_{t+1} &: \gamma \lambda^H_t = \mu^H_{t+1} + \lambda^H_{t+1} \\
M^H_{t+2} &: \gamma \lambda^H_{t+1} = \mu^H_{t+2} + \lambda^H_{t+2}
\end{align*}
\]

(A.1)\(\quad\) \(\quad\) \(\quad\) \(\quad\)

(A.2)\(\quad\) \(\quad\) \(\quad\) \(\quad\)

(A.3)\(\quad\) \(\quad\) \(\quad\) \(\quad\)

(A.4)

\[
\begin{align*}
y^H_t &: -\alpha \beta^t + \lambda^H_t p_t \begin{cases} 
> 0, & \text{if } y^H_t = 1 \\
= 0, & \text{if } y^H_t \in [0, 1] \\
< 0, & \text{if } y^H_t = 0
\end{cases} \\
y^H_{t+1} &: -\alpha \beta^{t+1} + \lambda^H_{t+1} p_{t+1} \begin{cases} 
> 0, & \text{if } y^H_{t+1} = 1 \\
= 0, & \text{if } y^H_{t+1} \in [0, 1] \\
< 0, & \text{if } y^H_{t+1} = 0
\end{cases}
\end{align*}
\]

(A.5)\(\quad\)

(A.6)

Although not shown explicitly, low type households’ optimality condition are very similar; their optimal choices are denoted using a superscript \(L\) and their taste parameter 1 in even periods and \(\theta\) in odd periods.

Consider the case in which all producers produce at their maximum capacity, i.e., \(y_i^t = y^i_{t+1} = 1(i = H, L)\). Since all households produce at the corner, the marginal cost of production should be no greater than the discounted marginal benefit. the inflation rate hence has to satisfy \(\gamma \leq \beta / \alpha\) by conditions (A.5) and (A.6). In addition, since we focus on monetary equilibrium, the shadow value of money \(\mu_i^t\) and \(\mu_{t+1}^i\) \((i = H, L)\) should be strictly positive. As a result, the inflation rate is restricted to be above \(\beta \theta\). So when \(\gamma \in (\beta \theta, \beta / \alpha)\), \((\theta < 1 / \alpha)\), the equilibrium allocation is \(y_i^t = y^i_{t+1} = 1\) and \(c_i^t = c^i_{t+1} = 1\) \((i = H, L)\). Using similar logic, the results shown in the paper can be deducted.
A.2 Optimality Conditions of the Economy with Illiquid Bonds

In the economy with illiquid bonds, I normalize all the nominal values at time $t$ using the total money stock at the same period $M^b_t$. More specifically, $m^{bH}_t = M^{bH}_t / M^b_t$, $m^{bL}_t = M^{bL}_t / M^b_t$, $p^b_t = P^b_t / M^b_t$, and $b_t = B_t / M^b_t$.

Besides the multipliers of cash-in-advance constraints (1.6) and budget constraints (1.7) and (1.8), let $\psi^H_t$ and $\psi^H_{t+1}$ be the multipliers of the bonds market constraints (1.4) and (1.5), and let the variables with superscript $bH$ or $bL$ denote the type $H$ or $L$ type households’ allocations. The optimality conditions are:

\[
\begin{align*}
&c^{bH}_t : \quad \frac{\beta t}{c^{bH}_t p^b_t} = \mu^{bH}_t + \lambda^{bH}_t \quad \text{(A.7)} \\
&c^{H}_{t+1} : \quad \frac{\beta t+1}{c^{H}_{t+1} p^b_{t+1}} = \mu^{bH}_{t+1} + \lambda^{bH}_{t+1} \quad \text{(A.8)} \\
&M^{H'}_t : \quad -\psi^H_t + \mu^{bH}_t + \lambda^{bH}_t = 0 \quad \text{(A.9)} \\
&M^{H'}_{t+1} : \quad -\psi^H_{t+1} + \mu^{bH}_{t+1} + \lambda^{bH}_{t+1} = 0 \quad \text{(A.10)} \\
&M^{bH}_{t+1} : \quad -\gamma \lambda^{bH}_t + \psi^H_{t+1} = 0 \quad \text{(A.11)} \\
&M^{bH}_{t+2} : \quad -\gamma \lambda^{bH}_{t+1} + \psi^H_{t+2} = 0 \quad \text{(A.12)} \\
&b^{H}_t : \quad -q_t \psi^H_t + \lambda^{bH}_t \leq 0 \text{ and } b^{H}_t \geq 0 \text{ with c.s.} \quad \text{(A.13)} \\
&b^{H}_{t+1} : \quad -q_{t+1} \psi^H_{t+1} + \lambda^{bH}_{t+1} \leq 0 \text{ and } b^{H}_{t+1} \geq 0 \text{ with c.s.} \quad \text{(A.14)} \\
y^{bH}_t : \quad -\alpha \beta t + \lambda^{bH}_t p^b_t \begin{cases} > 0, \text{ if } y^{bH}_t = 1 \\ = 0, \text{ if } y^{bH}_t \in [0, 1] \\ < 0, \text{ if } y^{bH}_t = 0 \end{cases} \quad \text{(A.15)} \\
y^{bH}_{t+1} : \quad -\alpha \beta t+1 + \lambda^{bH}_{t+1} p^b_{t+1} \begin{cases} > 0, \text{ if } y^{bH}_{t+1} = 1 \\ = 0, \text{ if } y^{bH}_{t+1} \in [0, 1] \\ < 0, \text{ if } y^{bH}_{t+1} = 0 \end{cases} \quad \text{(A.16)}
\end{align*}
\]
Again, consider the case in which all producers produce at their maximum capacity, i.e., \( y^i_t = y^i_{t+1} = 1 (i = H, L) \). To obtain more consumption goods in even periods when their taste parameter is high, type \( H \) households sell their illiquid bonds for fiat money to finance their purchasing. As a result, type \( H \) households’ bond market constraints are not binding, i.e., \( b^H_t = 0 \) and \( b^L_{t+1} = 0 (t = 2k, k = 0, 1, 2...) \) by (A.13) and (A.14). Using this result, we can solve equilibrium consumption allocation as a function of the bond prices \( q_t \) and \( q_{t+1} \). The closed-form solution of the bonds prices can be solved by applying the market clearing conditions. Similar to the economy with perfectly liquid bonds, we can determine the equilibrium allocations at different levels of the inflations rate.
Appendix B

Appendix to Chapter 2

B.1 The Envelope Condition for the Measure of Matched Agents

Proof. Case 1: When \( a > a_s \), the change in the measure of agents who are not in a relationship has no effect on the value of staying in a relationship at any value of the cost parameter \( a \), so \( \frac{\partial V(m,g_b)}{\partial g_b(a)} = 0 \) if \( a > a_s \).

Case 2: when \( a \leq a_s \):

Since \( g_b(a) \) is not a continuous function of \( a \), the usual derivative is meaningless. Let \( h(a) \) be a variation of the function \( g_b(a) \) defined on the domain \([a, \pi] \). Assume \( h(a) \neq 0 \) in a neighborhood of \( a_0 \) \( (a_0 < a_s) \) and \( h(a) = 0 \) otherwise. \( \max |h(a)| = \eta \), \( h(a) \) never takes opposite signs and the diameter of the neighborhood is \( \Delta \). Using the definition of a Volterra derivative

\[
\frac{\partial V(m,g_b)}{\partial g_b(a)} = \lim_{\Delta \to 0} \lim_{\eta \to 0} \frac{\mathcal{F}(\tilde{g}_b) - \mathcal{F}(g_b)}{\int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h(a) \, da}
\]

where \( \tilde{g}_b(a) = g_b(a) + h(a) \) and \( \mathcal{F} \) is the objective function of the households maximization problem.
For simplicity, assume \( h(a) = \eta \forall a \in (a_0 - \Delta/2, a_0 + \Delta/2) \), so \( \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h(a) \, da = \eta \Delta \)

\[
\frac{\partial V(m, g_b)}{\partial g_b(a)} = \lim_{\eta \to 0} \frac{u(\tilde{c}) + \beta V(\tilde{m}_{+1}, \tilde{g}_b_{+1}) + \int_\alpha^{a_\varepsilon} \mu_m(a) \left( x_{bn}(a) - \frac{m(a) - m(b)}{\Omega} \right) \frac{g(a) \, da}{\eta} + \rho \left[ m - \int_\alpha^{a_\varepsilon} x_{bn}(a) \, da - \left( 1 - \int_\alpha^{a_\varepsilon} \tilde{g}_b(a) \, da \right) x_{bn} \right] + \int_\alpha^{\pi} \left[ \left( 1 - \int_\alpha^{a_\varepsilon} \tilde{g}_b(a) \, da \right) \lambda^b \mu_n(a) \left( x_{bn} - \frac{m(a) - m(b)}{\Omega} \right) \right] \frac{dF(a)}{\eta \Delta} = \lim_{\eta \to 0} \frac{u(c) - \beta V(m_{+1}, g_{b+1}) - \int_\alpha^{a_\varepsilon} \mu_m(a) \left( x_{bn}(a) - \frac{m(a) - m(b)}{\Omega} \right) g(a) \, da - \rho \left[ m - \int_\alpha^{a_\varepsilon} x_{bn}(a) \, da - \left( 1 - \int_\alpha^{a_\varepsilon} g_b(a) \, da \right) x_{bn} \right] - \int_\alpha^{\pi} \left( 1 - \int_\alpha^{a_\varepsilon} g_b(a) \, da \right) \lambda^b \mu_n(a) \left( x_{bn} - \frac{m(a) - m(b)}{\Omega} \right) \right] \frac{dF(a)}{\eta \Delta}}
\]

\[
\tilde{c} = \int_\alpha^{a_\varepsilon} q_m(a) \tilde{g}_b(a) \, da + \left( 1 - \int_\alpha^{a_\varepsilon} \tilde{g}_b(a) \, da \right) \lambda^b \int_\alpha^{\pi} q_n(a) \, dF(a)
\]

\[
= \int_\alpha^{a_0 - \Delta/2} q_m(a) g_b(a) \, da + \int_\alpha^{a_0 + \Delta/2} q_m(a) [g_b(a) + h(a)] \, da + \int_\alpha^{a_\varepsilon} q_m(a) g_b(a) \, da
\]

\[
+ \left( 1 - \int_\alpha^{a_0 - \Delta/2} g_b(a) \, da - \int_\alpha^{a_0 + \Delta/2} [g_b(a) + h(a)] \, da - \int_\alpha^{a_\varepsilon} g_b(a) \, da \right) \lambda^b \int_\alpha^{\pi} q_n(a) \, dF(a)
\]

with

\[
\tilde{c} = c + \eta \Delta \left[ q_m(a_1) - \lambda^b \int_\alpha^{\pi} q_n(a) \, dF(a) \right]
\]

by the intermediate value theorem of integration, where \( a_1 \) is some number located in the neighborhood of \( a_0 \) and

\[
\lim_{\Delta \to 0} \frac{u(\tilde{c}) - u(c)}{\eta \Delta} = \lim_{\eta \to 0} \frac{u(c + \eta \Delta \left[ q_m(a_1) - \lambda^b \int_\alpha^{\pi} q_n(a) \, dF(a) \right]) - u(c)}{\eta \Delta}
\]

\[
= u'(c) \left[ q_m(a_1) - \lambda^b \int_\alpha^{\pi} q_n(a) \, dF(a) \right]
\]
Appendix B. Appendix to Chapter 2

\[ \gamma \tilde{m}_{+1} = m - \left\{ \begin{array}{c}
\int_{a_0}^{a_0 + \Delta/2} x_m (a) g_b (a) da + \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} x_m (a) [g_b (a) + h (a)] da \\
\phi_a^{a_s} x_m (a) g_b (a) da + \left( 1 - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} g_b (a) da - \int_{a_0 + \Delta/2}^{a_0 + \Delta/2} [g_b (a) + h (a)] da \\
- \int_{a_0 + \Delta/2}^{a_s} g_b (a) da \right) \lambda^b \int_a^\pi x_n (a) dF (a) \\
\int_{a}^{A_s} X_m (a) dG_s (a) + \left[ 1 - G_s (A_s) \right] \lambda^s \int_a^\pi X_u (a) dF (a) \end{array} \right\} + \tau_{+1} \]

\[ \gamma \tilde{m}_{+1} = \gamma m_{+1} - \left[ \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} x_m (a) h (a) da - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h (a) da \lambda^b \int_a^\pi x_n (a) dF (a) \right] \]

\[ \gamma m_{+1} - \eta \Delta \left[ x_m (a_2) - \lambda^b \int_a^\pi x_n (a) dF (a) \right] \]

by the intermediate value theorem of integration, where \( a_2 \) is some number located in the neighborhood of \( a_0 \).

Similarly, if \( a_i \in (a_0 - \Delta/2, a_0 + \Delta/2) \)

\[ \tilde{g}_{b_{+1}} (a_i) = \left\{ \tilde{g}_b (a_i) + \left( 1 - \int_{\tilde{a}}^{a_s} \tilde{g}_b (a) da \right) \lambda^b f (a_i) \right\} (1 - \delta) \]

\[ = [g_b (a_i) + h (a_i)] (1 - \delta) + \left( 1 - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} g_b (a) da - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} [g_b (a) + h (a)] da \right) \lambda^b f (a_i) (1 - \delta) \]

\[ \tilde{g}_{b_{+1}} (a_i) = g_{b_{+1}} (a_i) + \left[ h (a) - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h (a) da \lambda^b f (a) \right] (1 - \delta) \]

\[ = g_{b_{+1}} (a_i) + [\eta - \eta \Delta \lambda^b f (a_i)] (1 - \delta) \]
If \( a_i \notin (a_0 - \Delta/2, a_0 + \Delta/2) \)

\[
\tilde{g}_{b+1}(a_i) = g_{b+1}(a_i) - \left[ \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h(a) \, da \right] \lambda^b f(a_i)(1 - \delta),
\]

\[
\frac{\partial g_{b+1}(a_i)}{\partial g_b(a_0)} = \lim_{\Delta \to 0, \eta \to 0} \frac{\tilde{g}_{b+1}(a_i) - g_{b+1}(a_i)}{\int_{a_0 - \Delta/2}^{a_0 + \Delta/2} h(a) \, da} = \begin{cases} 
\lim_{\Delta \to 0, \eta \to 0} \frac{[\eta - \eta \Delta \lambda^b f(a_i)](1 - \delta)}{\eta \Delta} & \text{if } a_i \in (a_0 - \Delta/2, a_0 + \Delta/2) \\
\lim_{\Delta \to 0, \eta \to 0} \frac{-\eta \Delta \lambda^b f(a_i)(1 - \delta)}{\eta \Delta} & \text{if } a_i \notin (a_0 - \Delta/2, a_0 + \Delta/2).
\end{cases}
\]

To calculate the partial exchange of \( V \) w.r.t \( g_{b+1} \) and \( m+1 \), respectively:

\[
\lim_{\Delta \to 0, \eta \to 0} \frac{\beta V(\tilde{m}_{+1}, \tilde{g}_{b+1}) - \beta V(m+1, g_{b+1})}{\eta \Delta}
\]

The partial derivative w.r.t \( m+1 \) is

\[
\lim_{\Delta \to 0, \eta \to 0} \frac{\beta V(\tilde{m}_{+1}, g_{b+1}) - \beta V(m+1, g_{b+1})}{\eta \Delta} = \lim_{\Delta \to 0, \eta \to 0} \frac{\beta V(m+1 - \eta \Delta^{1/\gamma} \left[ x_m(a_2) + \lambda^b \int_{\frac{\pi}{2}}^{\pi} x_n(a) \, dF(a) \right], \tilde{g}_{b+1}) - \beta V(m+1, \tilde{g}_{b+1})}{\eta \Delta} = -\frac{\beta}{\gamma} V_1(m+1, g_{b+1}) \left[ x_m(a_2) - \lambda^b \int_{\frac{\pi}{2}}^{\pi} x_n(a) \, dF(a) \right].
\]
The partial derivative w.r.t $g_{b+1}$ is

$$
\lim_{\Delta \to 0, \eta \to 0} \frac{\beta V (m_{+1}, \tilde{g}_{b+1}) - \beta V (m_{+1}, g_{b+1})}{\eta \Delta}
$$

$$
\lim_{\Delta \to 0, \eta \to 0} \frac{\beta V (m_{+1}, \tilde{g}_{b+1}(a_1), \tilde{g}_{b+1}(a_2), \ldots) - \beta V (m_{+1}, g_{b+1} (a_1), g_{b+1} (a_2), \ldots)}{\eta \Delta}
$$

$$
= \int_{a}^{a+\Delta/2} \left[ \lim_{\Delta \to 0, \eta \to 0} \frac{\beta V (m_{+1}, \tilde{g}_{b+1}(a_i)) - \beta V (m_{+1}, g_{b+1}(a_i))}{\eta \Delta} \right] da_i
$$

$$
+ \int_{a-\Delta/2}^{a} \left[ \frac{\beta \partial V (m_{+1}, \tilde{g}_{b+1}(a_i))}{\partial g_{b+1}(a_i)} \right] da_i
$$

$$
+ \int_{a-\Delta/2}^{a+\Delta/2} \left[ \lim_{\Delta \to 0, \eta \to 0} \frac{V (m_{+1}, \tilde{g}_{b+1}(a_i)) - V (m_{+1}, g_{b+1}(a_i) + [\eta - \eta \Delta \lambda^b f (a_i)] (1 - \delta))}{\eta \Delta} \right] da_i,
$$

and the last term is

$$
\int_{a-\Delta/2}^{a+\Delta/2} \left[ \lim_{\Delta \to 0, \eta \to 0} \frac{\beta \partial V (m_{+1}, \tilde{g}_{b+1})(a_t)}{\partial g_{b+1}(a_t)} \right] \frac{[\eta - \eta \Delta \lambda^b f (a_t)] (1 - \delta)}{\eta \Delta} da_t
$$

$$
= \lim_{\Delta \to 0, \eta \to 0} \frac{\beta \partial V (m_{+1}, \tilde{g}_{b+1})(a_t)}{\partial g_{b+1}(a_t)} \frac{\eta \Delta - \eta \Delta^2 \lambda^b f (a_t)}{\eta \Delta} (1 - \delta)
$$

$$
= \beta \partial V (m_{+1}, \tilde{g}_{b+1})(a_t) (1 - \delta).$$

where $a_t \in (a_0 - \Delta/2, a_0 + \Delta/2)$,
All together:

\[
\lim_{\Delta \to 0, \eta \to 0} \frac{\beta V (m_{+1}, \tilde{g}_{b,1}) - \beta V (m_{+1}, g_{b,1})}{\eta \Delta}
= -\frac{\beta V_1 (m_{+1}, g_{b,1})}{\gamma} \left[ x_m (a_2) - \lambda^b \int_a^\pi x_n (a) dF (a) \right] \\
+ \beta (1 - \delta) \frac{\partial V (m_{+1}, \tilde{g}_{b,1})}{\partial g_{b,1} (a)} \\
- \int_a^{a - \Delta/2} \left[ \frac{\beta \partial V (m_{+1}, \tilde{g}_{b,1})}{\partial g_{b,1} (a_i)} \right] \lambda^b f (a_i) (1 - \delta) da_i \\
- \int_{a + \Delta/2}^a \left[ \frac{\beta \partial V (m_{+1}, \tilde{g}_{b,1})}{\partial g_{b,1} (a_i)} \right] \lambda^b f (a_i) (1 - \delta) da_i
\]

\[
\int_a^{a_3} x_{bm} (a) \tilde{g}_b (a) da = \int_a^{a_0 - \Delta/2} x_{bm} (a) g_b (a) da + \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} x_{bm} (a) [g_b (a) + h (a)] da \\
+ \int_{a_0 + \Delta/2}^{a_3} x_{bm} (a) g_b (a) da \\
= \int_a^{a_3} x_{bm} (a) g_b (a) da + \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} x_{bm} (a) h (a) da \\
= \int_a^{a_3} x_{bm} (a) g_b (a) da + \eta \Delta x_{bm} (a_4)
\]

by the intermediate value theorem of integration, where \(a_3\) is some number located in the neighborhood of \(a_0\)

\[
\left(1 - \int_a^{a_3} \tilde{g}_b (a) da\right) x_{bm} = \left(1 - \int_a^{a_0 - \Delta/2} g_b (a) da - \int_{a_0 - \Delta/2}^{a_0 + \Delta/2} [g_b (a) + h (a)] da \right) x_{bm} \\
- \int_{a_0 + \Delta/2}^{a_3} g_b (a) da \\
= \left(1 - \int_a^{a_3} g_b (a) da\right) x_{bm} - \eta \Delta x_{bm}
\]

\[
\left\{ \begin{align*}
\rho & \left[ m - \int_a^{a_3} x_{bm} (a) \tilde{g}_b (a) da - \left(1 - \int_a^{a_3} \tilde{g}_b (a) da\right) x_{bm}\right] \\
- \rho & \left[ m - \int_a^{a_3} x_{bm} (a) g_b (a) da - \left(1 - \int_a^{a_3} g_b (a) da\right) x_{bm}\right]
\end{align*} \right\}
\]
\[
= \lim_{\Delta \to 0, \eta \to 0} \frac{\rho \left[ - \int_{a}^{a_s} x_{bm} (a) \tilde{g}_b (a) \, da - \left( 1 - \int_{a}^{a_s} \tilde{g}_b (a) \, da \right) x_{bn} \right] - \rho \left[ - \int_{a}^{a_s} x_{bm} (a) g_b (a) \, da - \left( 1 - \int_{a}^{a_s} g_b (a) \, da \right) x_{bn} \right]}{\eta \Delta}
\]

\[
= \lim_{\Delta \to 0, \eta \to 0} \frac{-\rho \left[ \int_{a}^{a_s} x_{bm} (a) \tilde{g}_b (a) \, da - \int_{a}^{a_s} x_{bm} (a) g_b (a) \, da \right] - \rho \left[ \left( 1 - \int_{a}^{a_s} \tilde{g}_b (a) \, da \right) x_{bn} - \left( 1 - \int_{a}^{a_s} g_b (a) \, da \right) x_{bn} \right]}{\eta \Delta}
\]

\[
= \lim_{\Delta \to 0, \eta \to 0} \frac{-\rho \eta \Delta x_{bm} (a_4) + \rho \eta \Delta x_{bn}}{\eta \Delta} = -\rho \left[ x_{bm} (a_4) - x_{bn} \right]
\]

by the intermediate value theorem of integration, where \( a_4 \) is some number located in the neighborhood of \( a_0 \)

\[
\int_{a}^{a_s} \mu_m (a) \left[ x_{bm} (a) - \frac{a \phi (q_m (a))}{\Omega} \right] \tilde{g}_b (a) \, da - \int_{a}^{a_s} \mu_m (a) \left[ x_{bm} (a) - \frac{a \phi (q_m (a))}{\Omega} \right] g_b (a) \, da
\]

\[
= \eta \Delta \int_{a}^{a_s} \mu_m (a) \left[ x_{bm} (a) - \frac{a \phi (q_m (a))}{\Omega} \right] \, da
\]

\[
\int_{a}^{a_s} \left[ \left( 1 - \int_{a}^{a_s} \tilde{g}_b (a) \, da \right) \lambda^b \right] \mu_n (a) \left[ x_{bn} - \frac{a \phi (q_n (a))}{\Omega} \right] \, dF (a)
\]

\[
- \int_{a}^{a_s} \left[ \left( 1 - \int_{a}^{a_s} g_b (a) \, da \right) \lambda^b \right] \mu_n (a) \left[ x_{bn} - \frac{a \phi (q_n (a))}{\Omega} \right] \, dF (a)
\]

\[
= - \int_{a}^{a_s} \lambda^b \eta \Delta \mu_n (a) \left[ x_{bn} - \frac{a \phi (q_n (a))}{\Omega} \right] \, dF (a)
\]
Going back to the envelope condition of \( g_b (a) \)

\[
\frac{\partial V (m, g_b)}{\partial g_b (a)} = \lim_{\Delta \to 0, \eta \to 0} \left\{ \begin{array}{l}
\frac{u(c) + \beta V (\tilde{m} + 1, \tilde{g}_b) + \int_a^{a_m} \mu_m (a) \left[ x_m (a) - \frac{\alpha (q_m (a))}{\Omega} \right] \tilde{g} (a) da}{\eta \Delta} \\
+ \rho \left[ m - \int_a^{a_m} x_m (a) \tilde{g}_b (a) da - \left( 1 - \int_a^{a_m} \tilde{g}_b (a) da \right) x_{bn} \right] \\
+ \int_a^{a_m} \left[ \left( 1 - \int_a^{a_m} \tilde{g}_b (a) da \right) \lambda \right] \mu_n (a) \left[ x_{bn} - \frac{\alpha (q_n (a))}{\Omega} \right] dF (a) \\
- u(c) - \beta V (m + 1, g_b + 1) - \int_a^{a_m} \mu_m (a) \left[ x_m (a) - \frac{\alpha (q_m (a))}{\Omega} \right] g (a) da \\
- \rho \left[ m - \int_a^{a_m} x_m (a) g_b (a) da - \left( 1 - \int_a^{a_m} g_b (a) da \right) x_{bn} \right] \\
- \int_a^{a_m} \left[ \left( 1 - \int_a^{a_m} g_b (a) da \right) \lambda \right] \mu_n (a) \left[ x_{bn} - \frac{\alpha (q_n (a))}{\Omega} \right] dF (a) \end{array} \right\}
\]

\[
\frac{\partial V (m, g_b)}{\partial g_b (a)} = u'(c) \left[ q_m (a_1) - \lambda \int_a^\pi q_n (a) dF (a) \right] \\
- \frac{\beta}{\gamma} V_1 (m + 1, g_b + 1) \left[ x_m (a_2) - \lambda \int_a^\pi x_n (a) dF (a) \right] \\
+ \frac{\beta (1 - \delta) \partial V (m + 1, \tilde{g}_b)}{\partial g_b (a_5)} \\
- \int_a^{a_m} \left[ \beta \partial V (m + 1, \tilde{g}_b) \right] * \left[ \lambda f (a_i) (1 - \delta) \right] da_i \\
- \int_a^{a_m} \left[ \beta \partial V (m + 1, \tilde{g}_b) \right] * \left[ \lambda f (a_i) (1 - \delta) \right] da_i \\
- \rho [x_{bn} (a_4) - x_{bn}] \]

when \( \Delta \to 0 \) and \( \eta \to 0 \), \( a_1, a_2, a_3, a_4 \to a \), it can be obtained that

\[
\frac{\partial V (m, g_b)}{\partial g_b (a)} = u'(c) \left[ q_m (a) - \lambda \int_a^\pi q_n (a) dF (a) \right] \\
- \omega^m \left[ x_m (a) - \lambda \int_a^\pi x_n (a) dF (a) \right] \\
+ \frac{\beta (1 - \delta) \partial V (m + 1, g_b + 1)}{\partial g_b + 1 (a)} \\
- \beta (1 - \delta) \int_a^{a_m} \frac{\beta \partial V (m + 1, g_b + 1)}{\partial g_b + 1 (a_i)} * f (a_i) da_i \\
- \rho [x_{bn} (a_4) - x_{bn}] \]
Appendix C

Appendix to Chapter 3

C.1 Generalization of the Benchmark Case

Here we consider a general utility function $u(y, z)$, rather than (3.3). In the KW market, the value functions and trading surpluses satisfy

$$V_2(0, s) = A\alpha_0[u(y_s, s) + V_3(d_s, s)] + (1 - A\alpha_0)[u(0, s) + V_3(0, s)]$$

$$V_2(0, z) = A\alpha_1[u(y_z, z) + V_3(d_z, z)] + (1 - A\alpha_1)[u(0, z) + V_3(0, z)]$$

$$S(z) = u(y, z) - u(0, z) - \frac{\beta d}{1 - \eta}$$

$$\hat{S}(z) = \beta(1 - \tau)d - \beta c(y, k).$$

Nash bargaining (Kalai is similar) in the baseline economy implies

$$\beta c_y(y, k) = (1 - \tau)(1 - \eta)u_y(y, z)$$

$$(1 - \tau)\beta d = (1 - \theta)(1 - \tau)(1 - \eta)[u(y, z) - u(0, z)] + \theta \beta c(y, k).$$

From this it is easy to verify $\partial y / \partial z \simeq u_{yz}(y, z)$, as claimed in the text.
Next, to derive the reservation equation, use $V_1(0, R) = V_1(0, s)$ to get

$$A\alpha_1 S(R) + V_3(0, R) + u(0, R) = A\alpha_0 S(s) + V_3(0, s) + u(0, s).$$

As always, we have

$$V_3(0, R) = \delta \beta V_1(0, s) + (1 - \delta) \beta V_1(0, R),$$

$$V_3(0, s) = [1 - \lambda + \lambda F(R)] \beta V_1(0, s) + \lambda \beta \int_R V_1(0, z) dF(z),$$

where in this case integration by parts yields

$$\int_R V_1(0, z) dF(z) = [1 - F(R)] V_1(0, R) + \int_R \frac{A\alpha_1 S'(z) + u_z(0, z)}{\beta(\tau + \delta)} [1 - F(z)] dz.$$

Combining these expressions, we get the generalized version of (3.13)

$$u(0, R) + A\alpha_1 S(R) = u(0, s) + A\alpha_0 S(s) + \frac{\lambda}{r + \delta} \int_R [u_z(0, z) + A\alpha_1 S'(z)] [1 - F(z)] dz,$$

where

$$S(z) = \frac{\theta \{ (1 - \tau)(1 - \eta) [u(y_z, z) - u(0, z)] - \beta c(y_z, k) \}}{(1 - \tau)(1 - \eta)}.$$

Generalizing Proposition 2, a sufficient condition for equilibrium with $R \in (\bar{z}, \bar{z})$ is now

$$u(0, \bar{z}) + A\alpha_1 S(\bar{z}) > u(0, s) + A\alpha_0 S(s) > u(0, \check{z}) + A\alpha_1 S(\check{z}).$$

In terms of Proposition 3, we have

$$\frac{\partial R}{\partial \alpha_0} = \frac{A S(s)}{D[u_z(0, R) + A\alpha_1 S'(R)]},$$

$$\frac{\partial R}{\partial \alpha_1} = \frac{-A S(R) + A \alpha_1 \int_R S'(z)[1 - F(z)] dz}{D[u_z(0, R) + A\alpha_1 S'(R)]}.$$

Inserting $S'(z)$, the term multiplying $D > 0$ in the denominator simplifies to $u_z(0, R) + A\alpha_1 S'(R) = (1 - A\alpha_1 \theta) u_z(0, R) + A\alpha_1 \theta u_z(y_R, R) > 0$. Hence, $\partial R/\partial \alpha_0 > 0$. As for
\[ \frac{\partial R}{\partial \alpha_1}, \text{since } S'(z) \simeq u_{yz}(y, z), \text{it is negative at least when } y \text{ and } z \text{ are substitutes. We also have} \]

\[
\begin{align*}
\frac{\partial R}{\partial A} &= \frac{\alpha_0 S(s) - \alpha_1 S(R) + \frac{\lambda \alpha_1}{r+\delta} \int_R S'(z)[1 - F(z)] dz }{D[u_z(0, R) + A\alpha_1 S'(R)]} \\
\frac{\partial R}{\partial \theta} &= \frac{A[\alpha_0 S(s) - \alpha_1 S(R)] + \frac{\lambda \alpha_1}{r+\delta} \int_R S'(z)[1 - F(z)] dz }{D[u_z(0, R) + A\alpha_1 S'(R)]\theta} \\
\frac{\partial R}{\partial \tau} &= \frac{\theta \beta A[\alpha_1 c(y_R) - \alpha_0 c(y_s)] + \frac{\theta \lambda \alpha_1}{r+\delta} \int_R u_{yz}(y, z) \frac{\partial y_z}{\partial \tau} [1 - F(z)] dz }{D[u_z(0, R) + A\alpha_1 S'(R)](1 - \tau)^2(1 - \eta)} \\
\frac{\partial R}{\partial \eta} &= \frac{\theta \beta A[\alpha_1 c(y_R) - \alpha_0 c(y_s)] + \frac{\theta \lambda \alpha_1}{r+\delta} \int_R u_{yz}(y, z) \frac{\partial y_z}{\partial \eta} [1 - F(z)] dz }{D[u_z(0, R) + A\alpha_1 S'(R)](1 - \tau)(1 - \eta)^2},
\end{align*}
\]

where in the last two expressions

\[
\begin{align*}
\frac{\partial y_z}{\partial \tau} &= \frac{-(1 - \eta)u_y(y_z, z)}{\beta c_{yy}(y_z, k) - (1 - \tau)(1 - \eta)u_{yy}(y_z, z)} < 0 \\
\frac{\partial y_z}{\partial \eta} &= \frac{-(1 - \tau)u_y(y_z, z)}{\beta c_{yy}(y_z, k) - (1 - \tau)(1 - \eta)u_{yy}(y_z, z)} < 0,
\end{align*}
\]

which means the integrands have the opposite sign of \( u_{yz} \).

Even given the sign of \( u_{yz} \), we cannot sign these expressions, in general, and can construct numerical examples with \( \frac{\partial R}{\partial \tau} > 0 \) or \( \frac{\partial R}{\partial \tau} < 0 \). The specification in (3.3) has \( u_{yz} = 0 \), so that changes in \( z \) do not affect the marginal utility of \( y \), although marriage \textit{per se} does when \( \varepsilon_0 \neq \varepsilon_1 \). In this case, all of the derivatives can be signed, as in Proposition 3. Also, as mentioned in the text, when \( \alpha_1 \to 0 \) all results in Proposition 3 again hold.

For completeness we derive \( \frac{\partial R}{\partial i} \) for a general \( u(y, z) \) in the monetary economy. Using Kalai bargaining, \( R \) satisfies

\[
\begin{align*}
u(0, R) + A\alpha_1 S(R) &= A\alpha_0 S(s) + u(0, s) + i g(y_R, R) - i g(y_s, s) \\
&+ \frac{\lambda}{r+\delta} \int_R [A\alpha_1 S'(z) - ig_z(y_z, z) + u_z(0, z)][1 - F(z)] dz.
\end{align*}
\]
where
\[ g(y, z) \equiv \frac{(1 - \theta)[u(y, z) - u(0, z)] + \theta \beta c(y, k)}{\theta(1 - \tau)(1 - \eta) + 1 - \theta}, \]
and \( y \) solves (3.25). One can derive
\[
\frac{\partial R}{\partial i} = \frac{g(y_R, R) - g(y_s, s) + \frac{\lambda}{r + \delta} \int_R \left\{ [A\alpha_1 u_{yz} - (i + A\alpha_1)g_{yz}] \frac{dy_z}{dt} - g_z \right\} [1 - F(z)] dz}{D[A\alpha_1 S'(R) - ig_z(y_R, R) + u_z(0, R)]}
\]
The sign of \( \partial R/\partial i \) is ambiguous, in general, but using the specification (3.3) or letting \( \alpha_1 \to 0 \), we get Proposition 4.
C.2 Qualitative Results of Kalai Bargaining Solution

For the perfect credit baseline model with Kalai bargaining, the effects of \((A, \alpha_z)\) are the same, while:

\[
\begin{align*}
\frac{\partial R}{\partial \theta} &= \frac{A(\alpha_0 S_0 - \alpha_1 S_1)}{D\theta[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \varepsilon_0} &= \frac{\theta(1-\tau)(1-\eta)A\alpha_0 v(y_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \varepsilon_1} &= -\frac{\theta(1-\tau)(1-\eta)A\alpha_1 v(y_1)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \tau} &= \frac{\theta\beta A(\alpha_1 d_1 - \alpha_0 d_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \eta} &= \frac{(1-\tau)\theta A(\alpha_0 d_0 - \alpha_0 g_0) + i\theta(g_1 - g_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]}.
\end{align*}
\]

For the monetary model with Kalai bargaining, again the effects of \((A, \alpha_z)\) are the same, and:

\[
\begin{align*}
\frac{\partial R}{\partial \theta} &= \frac{A(\alpha_0 S_0 - \alpha_1 S_1) + i(S_0 - S_1)}{D\theta[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \varepsilon_0} &= \frac{[A\alpha_0 \theta(1-\tau)(1-\eta) - i(1-\theta)]v(y_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \varepsilon_1} &= -\frac{[A\alpha_1 \theta(1-\tau)(1-\eta) - i(1-\theta)]v(y_1)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \tau} &= \frac{(1-\eta)\theta A(\alpha_1 g_1 - \alpha_0 g_0) + i\theta(g_1 - g_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]} \\
\frac{\partial R}{\partial \eta} &= \frac{(1-\tau)\theta A(\alpha_1 g_1 - \alpha_0 g_0) + i\theta(g_1 - g_0)}{D[\theta(1-\tau)(1-\eta) + 1 - \theta]}.
\end{align*}
\]

The signs of all these are as stated in Proposition 3.
Bibliography


