ATTITUDE AND ORBIT CONTROL OF SMALL SATELLITES FOR AUTONOMOUS TERRESTRIAL TARGET TRACKING

by

Najmus Ibrahim

A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Graduate Department of Institute for Aerospace Studies
University of Toronto

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Abstract

Attitude and Orbit Control of Small Satellites for Autonomous Terrestrial Target Tracking

Najmus Ibrahim
Master of Applied Science
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University of Toronto
2013

Terrestrial target tracking using low Earth orbit satellites provides essential daily services and vital scientific data. In this thesis, the Attitude and Orbit Control System of such a terrestrial tracking satellite, Nanosatellite for Earth Monitoring and Observation Aerosol Monitor, is presented in detail. The satellite is a new generation Earth observation mission with the objective of detecting global atmospheric aerosol content through sub-degree pointing. The design is presented from initial hardware selection and budget development to operation definition and mission operation. The efficacy of performing precise autonomous Earth-pointing on a small satellite platform is validated through high fidelity simulations involving satellite and environmental dynamics, test-characterized hardware models and flight software-in-the-loop. The results provide practical target tracking methodologies which in the past have been publicly inaccessible to the author’s best knowledge and which can be now be applied to a broad range of precise Earth-pointing satellites.
Acknowledgements

First and foremost, I would like to thank my supervisor Dr. Robert Zee for giving me the opportunity to study at the Space Flight Laboratory. I am truly grateful for being able to collaborate amongst such talented staff and work on such innovative missions at the forefront of space technology. This work would not have been possible without the aid and support of my colleagues and family. I am indebted to Karan Sarda and Dr. Simon Grocott for their enduring mentorship throughout the duration of my degree. I would also like to thank Niels Roth for his openness to explore all facets of attitude and orbit control throughout our countless discussions.

This work is dedicated to my loving parents who sowed the seeds of curiosity and no matter what the challenge have always encouraged me to pursue my life’s endeavours. This achievement is as much mine as it is yours.

“Few will have the greatness to bend history itself; but each of us can work to change a small portion of events, and in the total of all those acts will be written the history of this generation.”

−Robert F. Kennedy
(1925-1968)

Najmus Ibrahim
September 2013
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# Nomenclature

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<tr>
<td>AASV</td>
<td>Astronomical Almanac Sun Vector</td>
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<td>ACS</td>
<td>Attitude Control Subsystem</td>
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<td>ADS</td>
<td>Attitude Determination Subsystem</td>
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<td>ADC</td>
<td>Attitude Determination and Control</td>
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<td>AEKF</td>
<td>Attitude Extended Kalman Filter</td>
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<td>AOCS</td>
<td>Attitude and Orbit Control System</td>
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<td>AISSat-1</td>
<td>Automatic Identification System Satellite 1</td>
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<tr>
<td>ASRP</td>
<td>Australian Space Research Program</td>
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<tr>
<td>AT</td>
<td>Atomic Time</td>
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<tr>
<td>AU</td>
<td>Astronomical Unit</td>
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<tr>
<td>CANOE</td>
<td>Canadian Advanced Nanospace Operating Environment</td>
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<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>CIRA-72</td>
<td>Committee on Space Research International Reference Atmosphere 1972</td>
</tr>
<tr>
<td>COG</td>
<td>Center of Gravity</td>
</tr>
<tr>
<td>COM</td>
<td>Center of Mass</td>
</tr>
<tr>
<td>COTS</td>
<td>Commercial-off-the-Shelf</td>
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<td>DAR</td>
<td>Display Aspect Ratio</td>
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<td>ECEF</td>
<td>Earth-Centered-Earth-Fixed</td>
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<tr>
<td>ECI</td>
<td>Earth-Centered-Inertial</td>
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<td>EGM96</td>
<td>Earth Gravitational Model 1996</td>
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<td>EGM2008</td>
<td>Earth Gravitational Model 2008</td>
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<td>ENZ</td>
<td>East-North-Zenith</td>
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<td>Extended Kalman Filter</td>
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<td>FK5</td>
<td>Fifth Fundamental Katalog</td>
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<td>FLOPS</td>
<td>Floating Point Operations</td>
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<td>GAST</td>
<td>Greenwich Apparent Sidereal Time</td>
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<td>GCRF</td>
<td>Geocentric Celestial Reference Frame</td>
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<td>GEO</td>
<td>Geostationary</td>
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<tr>
<td>GMST</td>
<td>Greenwich Mean Sidereal Time</td>
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<tr>
<td>GNB</td>
<td>Generic Nanosatellite Bus</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>GSD</td>
<td>Ground Sampling Distance</td>
</tr>
<tr>
<td>HPOP</td>
<td>High Precision Orbit Propagator</td>
</tr>
<tr>
<td>IAU</td>
<td>International Astronomical Union</td>
</tr>
<tr>
<td>ICR</td>
<td>In-Cross-Range</td>
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<tr>
<td>IERS</td>
<td>International Earth Rotation Services</td>
</tr>
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<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
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<tr>
<td>ISRO</td>
<td>Indian Space Research Organization</td>
</tr>
<tr>
<td>ITRF</td>
<td>International Terrestrial Reference Frame</td>
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<tr>
<td>J2000</td>
<td>12:00 January 1, 2000 (Terrestrial Time)</td>
</tr>
<tr>
<td>JD</td>
<td>Julian Day</td>
</tr>
<tr>
<td>JGM-3</td>
<td>Joint Gravity Model 3</td>
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Acronyms (cont.)

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<tr>
<th>Acronym</th>
<th>Description</th>
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<tr>
<td><strong>LEO</strong></td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td><strong>LTDN</strong></td>
<td>Local Time of Descending Node</td>
</tr>
<tr>
<td><strong>MITA</strong></td>
<td>Mini-Satellite Italiano a Tecnologia Avanzata</td>
</tr>
<tr>
<td><strong>MEMS</strong></td>
<td>Microelectromechanical Systems</td>
</tr>
<tr>
<td><strong>N/A</strong></td>
<td>Not Available</td>
</tr>
<tr>
<td><strong>NEMO-AM</strong></td>
<td>Nanosatellite for Earth Monitoring and Observation - Aerosol Monitor</td>
</tr>
<tr>
<td><strong>OASYS</strong></td>
<td>Onboard Attitude System Software</td>
</tr>
<tr>
<td><strong>OBC</strong></td>
<td>Onboard Computer</td>
</tr>
<tr>
<td><strong>OEKF</strong></td>
<td>Orbit Extended Kalman Filter</td>
</tr>
<tr>
<td><strong>PEF</strong></td>
<td>Pseudo Earth-Fixed</td>
</tr>
<tr>
<td><strong>PID</strong></td>
<td>Proportional-Integral-Derivative</td>
</tr>
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<td><strong>PRISMA</strong></td>
<td>Precursore Iperspettrale della Missione Applicativa</td>
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<tr>
<td><strong>PROBA-1</strong></td>
<td>Project for On-Board Autonomy 1</td>
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<tr>
<td><strong>RK4</strong></td>
<td>Fourth Order Runge-Kutta</td>
</tr>
<tr>
<td><strong>RMS</strong></td>
<td>Root Mean Squared</td>
</tr>
<tr>
<td><strong>RSS</strong></td>
<td>Root Sum Squared</td>
</tr>
<tr>
<td><strong>SAIL</strong></td>
<td>Space Avionics and Instrumentation Laboratory</td>
</tr>
<tr>
<td><strong>SFL</strong></td>
<td>Space Flight Laboratory</td>
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<tr>
<td><strong>SGP4</strong></td>
<td>Simplified General Perturbations</td>
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<tr>
<td><strong>STK</strong></td>
<td>Satellite Tool Kit</td>
</tr>
<tr>
<td><strong>S3S</strong></td>
<td>Sinclair-SFL-SAIL</td>
</tr>
<tr>
<td><strong>TAI</strong></td>
<td>Temps Atomique International</td>
</tr>
<tr>
<td><strong>TEME</strong></td>
<td>True Equator Mean Equinox</td>
</tr>
<tr>
<td><strong>TT</strong></td>
<td>Terrestrial Time</td>
</tr>
<tr>
<td><strong>TGPS</strong></td>
<td>GPS Time</td>
</tr>
<tr>
<td><strong>TLE</strong></td>
<td>Two Line Element</td>
</tr>
<tr>
<td><strong>TOD</strong></td>
<td>True of Date</td>
</tr>
<tr>
<td><strong>UTIAS</strong></td>
<td>University of Toronto Institute for Aerospace Studies</td>
</tr>
<tr>
<td><strong>UT</strong></td>
<td>Universal Time</td>
</tr>
<tr>
<td><strong>USD</strong></td>
<td>United States Dollar</td>
</tr>
<tr>
<td><strong>UTC</strong></td>
<td>Coordinated Universal Time</td>
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<tr>
<td><strong>WGS84</strong></td>
<td>World Geodetic System 1984</td>
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Nomenclature (Cont.)

General Nomenclature

\(0_{n \times n}\) zero matrix of dimension \(n \times n\)
\(1_{n \times n}\) identity matrix of dimension \(n \times n\)
\(\dot{(\cdot)}\) temporal derivative of a vector observed in inertial frame \(\mathcal{F}_i\)
\(\dot{\circ}\) temporal derivative of a vector observed in a non-inertial frame
\(\cdot\) quantity propagated forward in time onboard satellite or nominally desired quantity
\(\cdot\) quantity on unperturbed target tracking trajectory
\(\cdot\) quantity effected by the satellite to track the unperturbed target tracking trajectory
\(\cdot\) basis column vector with unity norm or quantity obtained through an estimator
\(\tilde{\cdot}\) perturbed quantity
\(\delta\) prefix indicating error term
\(\mathcal{N}\) Gaussian distribution
\(\mathcal{F}_A\) reference frame \(A\) in three dimensions
\(\mathcal{F}_a\) column vectrix whose elements are the basis vectors of \(\mathcal{F}_A\)
\(C_{ab}\) general rotation matrix from \(\mathcal{F}_b\) to \(\mathcal{F}_a\); \(C_{ab} \triangleq \mathcal{F}_a \cdot \mathcal{F}_b^T\)
\(v\) lower-case normal-face font representing real scalars, \(v \triangleq \|\mathcal{V}\|\) or \(\|v\|\)
\(\mathcal{V}\) lower-case bold-face accented font representing vector quantity
\(\mathcal{V}\) lower-case bold-face font representing real column vectors
\(V\) upper-case bold-face font representing real matrices

Subscripts

\((\cdot)_\circ\) relating to the spacecraft \(\mathcal{B}\)
\((\cdot)_\odot\) relating to the Earth \(\mathcal{E}\)
\((\cdot)_{\odot}\) relating to the Sun
\((\cdot)_a\) spacecraft body-fixed alignment frame
\((\cdot)_b\) satellite body-fixed frame
\((\cdot)_c\) relating to the center of mass of satellite \(\mathcal{B}\) or satellite body-fixed constrained frame
\((\cdot)_e\) rotating Earth-Centered-Earth-Fixed frame or error rotation term
\((\cdot)_i\) non-rotating Earth-Centered-Inertial frame
\((\cdot)_k\) the \(k\)th value of a quantity or the value at timestep \(k\)
\((\cdot)_\ell\) topocentric South-East-Zenith frame
\((\cdot)_o\) local orbit fixed frame or vector from origin of Earth-Centered-Inertial frame
\((\cdot)_a\) body-fixed sensor frame or subsatellite point
\((\cdot)_\tau\) True-Equator-Mean-Equinox frame
\((\cdot)_w\) relating to reaction wheel \(\mathcal{W}\)
## Greek Symbols

<table>
<thead>
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<th>Symbol</th>
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<tr>
<td>$\alpha_{ai}$</td>
<td>angular acceleration of $F_a$ with respect to $F_i$, used for target tracking [rad/s$^2$]</td>
</tr>
<tr>
<td>$\alpha_{bi}$</td>
<td>angular acceleration of $F_b$ with respect to $F_i$ [rad/s$^2$]</td>
</tr>
<tr>
<td>$\alpha_{ca}$</td>
<td>angular acceleration of $F_c$ with respect to $F_a$, used for target tracking [rad/s$^2$]</td>
</tr>
<tr>
<td>$\alpha_{ci}$</td>
<td>angular acceleration of $F_c$ with respect to $F_i$, used for target tracking [rad/s$^2$]</td>
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<tr>
<td>$\alpha_{\text{ca}}$</td>
<td>components of $\alpha_{ca}$ expressed in $F_c$ [rad/s$^2$]</td>
</tr>
<tr>
<td>$\alpha_{\text{ai}}$</td>
<td>components of $\alpha_{ai}$ expressed in $F_i$ [rad/s$^2$]</td>
</tr>
<tr>
<td>$\alpha_{\ell}$</td>
<td>topocentric/observation right ascension angle [$^\circ$]</td>
</tr>
<tr>
<td>$\alpha_{\rho}$</td>
<td>camera projection right ascension angle [$^\circ$]</td>
</tr>
<tr>
<td>$\beta_k$</td>
<td>shorthand form of $\kappa_{i,k}x_{i,k+1}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>nadir angle; $\gamma \triangleq \arccos \left( \frac{-\hat{r} \cdot \kappa}{|\hat{r}| |\kappa|} \right)$</td>
</tr>
<tr>
<td>$\delta\gamma$</td>
<td>nadir angle error [$^\circ$]</td>
</tr>
<tr>
<td>$\delta\omega_\parallel$</td>
<td>angular velocity tracking error parallel to the imager boresight [rad/s]</td>
</tr>
<tr>
<td>$\delta\omega_\perp$</td>
<td>angular velocity tracking error perpendicular to the imager boresight [rad/s]</td>
</tr>
<tr>
<td>$\delta\theta$</td>
<td>stacked angular error in Roll, Pitch, Yaw; $\delta\theta \triangleq [\delta\theta_1 ; \delta\theta_2 ; \delta\theta_3]^T$</td>
</tr>
<tr>
<td>$\delta\theta_1$, $\delta\theta_2$, $\delta\theta_3$</td>
<td>angular error in Roll, Pitch, Yaw, respectively [rad]</td>
</tr>
<tr>
<td>$\delta\ell$</td>
<td>topocentric/observation declination angle [$^\circ$]</td>
</tr>
<tr>
<td>$\delta\rho$</td>
<td>camera projection declination angle [$^\circ$]</td>
</tr>
<tr>
<td>$\delta s$</td>
<td>mapping error [km]</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>grazing angle between $\kappa$ and local horizontal of target [rad]</td>
</tr>
<tr>
<td>$\epsilon_d$</td>
<td>coefficient of diffuse reflection</td>
</tr>
<tr>
<td>$\epsilon_s$</td>
<td>coefficient of specular reflection</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>first three Euler parameters $[\varepsilon_1 ; \varepsilon_2 ; \varepsilon_3]^T$</td>
</tr>
<tr>
<td>$\varepsilon_e$</td>
<td>$\varepsilon$ portion of quaternion derived from rotation error $C_e$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>controller damping ratio</td>
</tr>
<tr>
<td>$\eta$</td>
<td>fourth Euler parameter</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>Greenwich Apparent Sidereal Time angle [$^\circ$]</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>Equation of the Equinoxes [$^\circ$]</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Greenwich Mean Sidereal Time angle [$^\circ$]</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>angle between $n$ and $\vec{n}_{\odot}$ [$^\circ$]</td>
</tr>
<tr>
<td>$\psi$</td>
<td>target azimuth angle [rad]</td>
</tr>
<tr>
<td>$\psi_E$</td>
<td>target East azimuth angle from $\hat{\ell}_2$ to path from $\mathbf{r}_s$ to $\mathbf{x}$ on geoid [rad]</td>
</tr>
<tr>
<td>$\kappa_s$</td>
<td>satellite-to-target vector; $\kappa_s \triangleq \mathbf{x} - \mathbf{r}_s$ [m]</td>
</tr>
<tr>
<td>$\kappa_e$</td>
<td>components of $\kappa_s$ expressed in $F_e$ [m]</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>components of $\kappa_s$ expressed in $F_i$ [m]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>magnitude of $\kappa_s$ in a geocentric frame [m]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>terrestrial longitude [$^\circ$]</td>
</tr>
</tbody>
</table>
Nomenclature (Cont.)

Greek Symbols (cont.)

λı terrestrial target longitude [%]

µ expected value (mean) of parameter

µ⊕ gravitational parameter of Earth $\mathcal{E}$; $\mu \triangleq Gm_{\oplus} = 3.986 \times 10^{14}$ N m$^2$/kg

ν absolute velocity of terrestrial position; $\nu \triangleq \dot{x}$ [m/s]

ν⊕ tangential velocity of Earth’s equator; $\nu_{\oplus} \approx 466$ m/s

ζ Earth central angle; $\zeta \triangleq \arccos \left( \frac{\mathbf{r} \cdot \mathbf{x}}{\|\mathbf{r}\| \|\mathbf{x}\|} \right)$ [rad]

ρa density of Earth’s upper atmosphere [kg/m$^3$]

ϱ body-fixed vector from $w$ to an infinitesimal mass element of wheel $W$ [m]

ϱ magnitude of $\mathbf{\varrho}$ expressed in $F_b$ [m]

$\hat{\rho}_k$ $k^{th}$ basis vector of reference frame $F_p$

$p$ standard deviation of parameter

$\Upsilon$ Vernal Equinox direction at J2000

φc terrestrial geocentric latitude [%]

φd terrestrial geodetic latitude [%]

φt terrestrial target geodetic latitude [%]

Φ state transition matrix

φ Euler-axis rotation angle [rad]

$\varphi_{ai}$ Euler-axis rotation angle from $F_i$ to $F_a$ or for facilitating $\omega_{ai}$ [rad]

$\varphi_{ca}$ Euler-axis rotation angle for facilitating $\omega_{ca}$ [rad]

$\omega_w$ vector of stacked absolute axial angular velocities $\omega_k$ of individual wheels $W$ [rad/s]

$\omega_{ai}$ angular velocity of $F_a$ with respect to $F_i$, used for target tracking [rad/s]

$\omega_{bi}$ angular velocity of $F_b$ with respect to $F_i$ [rad/s]

$\omega_{ca}$ angular velocity of $F_c$ with respect to $F_i$, used for target tracking [rad/s]

$\omega_{ci}$ angular velocity of $F_c$ with respect to $F_i$ [rad/s]

$\omega_{ei}$ angular velocity of $F_e$ with respect to $F_i$ [rad/s]

$\omega_{wb}$ relative angular velocity of wheel $W$ with respect to spacecraft $\mathcal{R}$ [rad/s]

$\omega_{wi}$ absolute angular velocity of wheel $W$ with respect to $F_i$ [rad/s]

$\Delta \omega$ shorthand notation of $\omega_{bi}$ observed and expressed in $F_b$ [rad/s]

$\omega$ absolute controller angular velocity error observed and expressed in $F_b$ [rad/s]

$\omega_{bi}$ components of $\omega_{bi}$ expressed in $F_b$ [rad/s]

$\omega_{ai}$ components of $\omega_{ai}$ expressed in $F_i$ [rad/s]

$\omega_{ca}$ components of $\omega_{ca}$ expressed in $F_c$ [rad/s]

$\omega_{d}$ desired $\omega_{bi}$ observed and expressed in $F_b$ [rad/s]

$\omega_{wi}$ components of $\omega_{wi}$ in $F_b$

$\omega_{\oplus}$ accepted constant rotational velocity of the Earth; $\omega_{\oplus} \approx 7.292115 \times 10^{-5}$ rad/s

$\omega_n$ magnitude of $\omega_{wb}$ expressed in $F_b$

$\omega_k$ absolute axial angular velocity $\omega_a$ of $k^{th}$ wheel $W$ or components of $\omega_{bi}$ in $F_b$ [rad/s]

$\omega_n$ controller natural frequency [rad/s]
### Roman Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>body-fixed unit vector for Euler-axis rotation or wheel (W) symmetry axis</td>
</tr>
<tr>
<td>(a_{ai})</td>
<td>components of Euler axis expressed in (F_i), for transforming (F_i) to (F_a) or facilitating (\omega_{ai})</td>
</tr>
<tr>
<td>(a_{ca})</td>
<td>components of Euler axis expressed in (F_a) for facilitating (\dot{\omega}_{ca})</td>
</tr>
<tr>
<td>(a_k)</td>
<td>components of the (k)th wheel (W) symmetry axis expressed in (F_b)</td>
</tr>
<tr>
<td>(A)</td>
<td>generic area variable or cross-sectional area incident to oncoming atmosphere ([\text{m}^2])</td>
</tr>
<tr>
<td>(A_k)</td>
<td>(k)th surface area of satellite ([\text{m}^2])</td>
</tr>
<tr>
<td>(AU)</td>
<td>(1AU = 149598000) km</td>
</tr>
<tr>
<td>(b)</td>
<td>satellite body-fixed vector locating (O_b) from (m) or total local magnetic field ([\text{nT}])</td>
</tr>
<tr>
<td>(b_k)</td>
<td>(k)th basis vector of reference frame (F_b)</td>
</tr>
<tr>
<td>(b_k)</td>
<td>components of local magnetic field (b) expressed in (F_b) ([\text{nT}])</td>
</tr>
<tr>
<td>(b_k)</td>
<td>magnitude of satellite body-fixed vector locating (k)th (O_b) from (O_b) ([\text{m}])</td>
</tr>
<tr>
<td>(b_k)</td>
<td>semi-minor axis of Earth; (b_\oplus \equiv 6356.752) km</td>
</tr>
<tr>
<td>(c)</td>
<td>first moment of inertia of satellite (B) ([\text{kg m}])</td>
</tr>
<tr>
<td>(c_\phi)</td>
<td>first moment of inertia of spacecraft (R) ([\text{kg m}])</td>
</tr>
<tr>
<td>(g_k)</td>
<td>vector from (O_b) to center of pressure of a single force (see Section 3.7) ([\text{m}])</td>
</tr>
<tr>
<td>(c_w)</td>
<td>geodetic datum for evaluation of (WGS84) location ([\text{km}])</td>
</tr>
<tr>
<td>(C_{nm})</td>
<td>geopotential coefficient of Earth’s mass distribution</td>
</tr>
<tr>
<td>(C_D)</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>(d)</td>
<td>desired direction unit vector used for attitude alignment</td>
</tr>
<tr>
<td>(d_k)</td>
<td>components of (d) expressed in (F_a)</td>
</tr>
<tr>
<td>(d_k)</td>
<td>components of (d) expressed in (F_e)</td>
</tr>
<tr>
<td>(d_k)</td>
<td>components of (d) expressed in (F_i)</td>
</tr>
<tr>
<td>(d)</td>
<td>controller derivative gain scaling factor</td>
</tr>
<tr>
<td>(df_{d})</td>
<td>differential aerodynamic force on (dA) ([\text{N}])</td>
</tr>
<tr>
<td>(df_{s})</td>
<td>differential solar radiation force on (dA) ([\text{N}])</td>
</tr>
<tr>
<td>(dm)</td>
<td>infinitesimal mass element of specified body ([\text{kg}])</td>
</tr>
<tr>
<td>(dm_\oplus)</td>
<td>infinitesimal mass element of Earth (\varepsilon) ([\text{kg}])</td>
</tr>
<tr>
<td>(dA)</td>
<td>infinitesimal surface area element of satellite ([\text{m}^2])</td>
</tr>
<tr>
<td>(e)</td>
<td>eccentricity of orbit</td>
</tr>
<tr>
<td>(e_\oplus)</td>
<td>eccentricity of the Earth; (e_\oplus \equiv 0.081819221456)</td>
</tr>
<tr>
<td>(f)</td>
<td>total external force on arbitrary body composed of (n) point masses; (f \equiv \sum_{k=1}^{n} f_k) ([\text{N}])</td>
</tr>
<tr>
<td>(f_a)</td>
<td>aerodynamic force exerted on spacecraft (R) ([\text{N}])</td>
</tr>
<tr>
<td>(f_k)</td>
<td>external force on (k)th point mass or surface area element of satellite (B) ([\text{N}])</td>
</tr>
<tr>
<td>(f_s)</td>
<td>solar radiation force exerted on spacecraft (R) ([\text{N}])</td>
</tr>
<tr>
<td>(f)</td>
<td>nonlinear continuous dynamics model for (\dot{x}) or components of (f) expressed in (F_b) ([\text{N}])</td>
</tr>
<tr>
<td>(f)</td>
<td>Lagrange coefficient</td>
</tr>
<tr>
<td>(f_\oplus)</td>
<td>flattening of the Earth; (f_\oplus \equiv \frac{1}{298.257})</td>
</tr>
<tr>
<td>(F_X)</td>
<td>partial derivative of (f) with respect to state</td>
</tr>
<tr>
<td>(g)</td>
<td>total external torque on spacecraft (R); (g \equiv g_\ell) ([\text{m}])</td>
</tr>
</tbody>
</table>
Nomenclature (Cont.)

Roman Symbols (cont.)

- $g_w$: torque induced on wheel $W$ by spacecraft $R$ [m]
- $g_c$: total control torque exerted on spacecraft $R$ [m]
- $g_d$: total disturbance torque exerted on spacecraft $R$ [m]
- $g_m$: magnetic disturbance or magnetic control torque exerted on spacecraft $R$ [m]
- $g_f$: components of $g_c$ expressed in $F_b$ [m]
- $g_d$: components of $g_d$ expressed in $F_b$ [m]
- $g_g$: components of gravity gradient torque expressed in $F_b$ [m]
- $g_m$: components of $g_m$ expressed in $F_b$ [m]
- $g_s$: components of solar radiation pressure torque expressed in $F_b$ [m]
- $g_w$: column of axial wheel torque $g_k$ induced on individual wheel $W$ about $w$ in $F_b$ [m]
- $g$: nonlinear observation model
- $g$: Lagrange coefficient
- $g_k$: axial torque induced on wheel $W$ [m]
- $g_k$: axial torque induced on $k^{th}$ wheel $W$ [m]
- $G_e$, $G_w$: partial derivatives of observation model with respect to state and noise variable
- $G_r$, $G_v$: partial derivatives of orbital acceleration with respect to position and velocity
- $G$: gravitational constant; $G \doteq \frac{6.673 \times 10^{-11}}{m^3/(kg \cdot s^2)}$
- $\Delta h_w$: excess absolute axial wheel momentum expressed in $F_b$; $\Delta h_w \doteq h_w - \bar{h}_w$ [N m s]
- $\bar{h}$: absolute angular momentum of spacecraft $R$ about $O_b$ [N m s]
- $\bar{h}_c$: absolute angular momentum of spacecraft $R$ about $O_b$ [N m s]
- $\bar{h}_w$: absolute angular momentum of single wheel $W$ about $O_b$ or body axial angular momentum of multiple wheels about $O_b$ [N m s]
- $h$: nonlinear motion model or components of $\bar{h}$ expressed in $F_b$ [N m s]
- $h_w$: components of $\bar{h}_w$ expressed in $F_b$ [N m s]
- $h_w$: nominally desired wheel axial angular momentum vector expressed in $F_b$ [N m s]
- $h$: spacecraft altitude [km]
- $h_a$: inertial angular momentum of wheel $W$ [N m s]
- $h_k$: inertial angular momentum of $k^{th}$ wheel $W$ [N m s]
- $h_t$: terrestrial target altitude [km]
- $h_t$: terrestrial altitude [km]
- $H_e$, $H_v$: partial derivatives of motion model with respect to state and noise variable
- $i_k$: $k^{th}$ basis vector of reference frame $F_i$
- $i_o$: in-track viewing angle [°]
- $I_3$: second moment of inertia dyadic of satellite $R$ taken about $\Theta_b$ [kg m²]
- $I_{11}$: second moment of inertia dyadic of wheel $W$ taken about $\Theta_w$ [kg m²]
- $I_0$: second moment of inertia matrix of $I$ in $F_b$ [kg m²]
- $I_a$: wheel axial-inertia matrix of identical orthogonal wheels in $F_b$; $I_a \doteq I_a 1$ [kg m²]
- $I_w$: second moment of inertia matrix of $I_w$ in $F_b$ [kg m²]
- $I_k$: component of $I_w$ about symmetry axis [kg m²]
Nomenclature (Cont.)

Roman Symbols (cont.)

\( I_t \) component of \( I_w \) about arbitrary transverse axis \([\text{kg m}^2]\)
\( J \) second moment of inertia dyadic of \( B \) about body-fixed point other than \( O_b \) \([\text{kg m}^2]\)
\( J_b \) second moment of inertia dyadic of \( B \) about body-fixed point other than \( O_b \) \([\text{kg m}^2]\)
\( J_w \) second moment of inertia dyadic of \( W \) about body-fixed point other than \( O_b \) \([\text{kg m}^2]\)
\( J \) second moment of inertia matrix of \( J \) in \( F_B \) \([\text{kg m}^2]\)
\( K \) Kalman gain
\( K_b \) derivative gain matrix for B-Dot controller
\( K_d \) derivative gain matrix for Quaternion Regulator
\( K_i \) integral gain matrix for Quaternion Regulator
\( K_m \) derivative gain matrix for Momentum Management controller
\( K_p \) proportional gain matrix for Quaternion Regulator
\( k \) index variable or proportional gain scaling factor for Quaternion Regulator
\( m \) magnetic moment of the satellite \([\text{A m}^2]\)
\( m_b \) components of \( m \) expressed in \( F_B \) \([\text{A m}^2]\)
\( m \) shorthand form of \( m_b \) \([\text{kg}]\) or Legendre polynomial order
\( m_c \) mass of spacecraft \( R \) \([\text{kg}]\)
\( m_\odot \) mass of Earth \( \odot \) \([\text{kg}]\)
\( m_s \) mass of satellite \( R \) \([\text{kg}]\)
\( m_w \) mass of reaction wheel \( W \) \([\text{kg}]\)
\( M \) facilitatory matrix in Extended Kalman filter
\( n \) outward vector normal to a specified surface
\( n \) system order or Legendre polynomial degree
\( \hat{e}_k \) \( k \)th basis vector of reference frame \( F_o \)
\( o_o \) off-track viewing angle \([\text{°}]\)
\( O_i \) center of mass of Earth and origin of \( F_i \)
\( O_b \) origin of \( F_B \)
\( \hat{p} \) body-fixed unit vector extending outwards from payload boresight
\( \hat{p} \) absolute linear momentum of satellite \( R \) \([\text{kg m/s}]\)
\( \hat{p}_s \) absolute linear momentum of spacecraft \( R \) \([\text{kg m/s}]\)
\( \hat{p}_w \) absolute linear momentum of wheel \( W \) \([\text{kg m/s}]\)
\( p \) body-fixed frame components of \( \hat{p} \) or \( F_B \) components of \( p \) \([\text{kg m/s}]\)
\( P \) covariance matrix of state or shorthand form of \(\mathbf{s}^T \hat{p} \times \hat{p}^T\)
\( P_\odot \) solar radiation pressure at 1AU; \( P_\odot \approx 4.56 \times 10^{-6} \text{N/m}^2\)
\( q \) Euler parameter (quaternion) set \([e n]^T\)
\( q_e \) Euler parameter (quaternion) set representing \( C_e \)
\( q_d \) Euler parameter (quaternion) set representing \( C_d \)
\( q_l \) Euler parameter (quaternion) set representing \( C_l \)
\( Q \) covariance matrix of noise variable \( w \)
\( P_{nm} \) Legendre polynomial of degree \( n \) and order \( m \)
\( r \) shorthand form of \( r \) \([\text{km}]\)
\( r_\odot \) absolute position of an infinitesimal mass element of spacecraft \( R \) \([\text{km}]\)
\( r_\odot \) vector from \( O_i \) towards the Sun \([\text{km}]\)
Nomenclature (Cont.)

Roman Symbols (cont.)

- \( \mathbf{r}_c \): absolute position of \( \mathbf{b} \) with respect to a geocentric frame [km]
- \( \mathbf{r}_o \): vector from \( O_i \) to \( O_b \) or initial absolute position in Lagrange coefficient algorithm [km]
- \( \mathbf{r}_s \): subsatellite point on geoid [km]
- \( \mathbf{r}_w \): absolute position of an infinitesimal mass element of wheel \( \mathcal{W} \) [km]
- \( \mathbf{r}_c \): components of \( \mathbf{r}_c \) expressed in \( \mathcal{F}_b \) [km]
- \( \mathbf{r}_t \): components of \( \mathbf{r}_c \) expressed in \( \mathcal{F}_r \) [km]
- \( \mathbf{r}_e \): components of \( \mathbf{r}_c \) expressed in \( \mathcal{F}_e \) [km]
- \( \mathbf{r}_i \): components of \( \mathbf{r}_c \) expressed in \( \mathcal{F}_i \) [km]
- \( \mathbf{r} \): magnitude of \( \mathbf{r}_c \) [km]
- \( \mathbf{R} \): covariance matrix of noise variable \( \mathbf{v} \)
- \( \mathbf{s} \): body-fixed unit vector chosen to be aligned with \( \mathbf{d} \)
- \( \mathbf{s}_e \): Earth-fixed vector from \( O_b \) to a satellite surface, to the Sun
- \( \mathbf{s}_k \): \( k \)-th basis vector of reference frame \( \mathcal{F}_s \)
- \( \mathbf{s}_s \): body-fixed vector from \( O_b \) to an infinitesimal mass element of spacecraft \( \mathcal{R} \) [m]
- \( \mathbf{s}_w \): body-fixed vector from \( O_b \) to an infinitesimal mass element of wheel \( \mathcal{W} \) [m]
- \( \mathbf{s} \): components of \( \mathbf{s} \) expressed in a body-fixed frame
- \( \mathbf{s}_b \): components of \( \mathbf{s} \) expressed in a body-fixed frame
- \( S_{nm} \): geopotential coefficient of Earth’s mass distribution
- \( S_E \): geodetic datum for evaluation of \( \text{WGS84} \) location [km]
- \( \Delta t_c \): control timestep [s]
- \( \Delta t_e \): imager exposure time [s]
- \( \Delta t_k \): timestep difference in Extended Kalman Filter; \( \Delta t_k \equiv t_k - t_{k-1} \) [s]
- \( t \): \( UT1 \) time [s]
- \( t_s \): controller settling time [s]
- \( U \): gravity potential of Earth \( \mathcal{E} \) [MJ/kg]
- \( \mathbf{v} \): arbitrary vector or absolute velocity of \( \mathbf{b} \) with respect to a geocentric frame; \( \mathbf{v} \equiv \dot{\mathbf{r}}_c \) [m/s]
- \( \mathbf{v}_o \): absolute velocity of \( \mathbf{r}_o \); \( \mathbf{v}_o \equiv \dot{\mathbf{r}}_o \) [m/s]
- \( \mathbf{v}_r \): relative velocity of \( \mathbf{b} \) as observed in \( \mathcal{F}_e \) [m/s]
- \( \mathbf{\hat{v}}_r \): unit vector of \( \mathbf{v}_r \); \( \mathbf{\hat{v}}_r \equiv \frac{\mathbf{v}_r}{||\mathbf{v}_r||} \)
- \( \mathbf{v} \): noise variable of nonlinear motion model
- \( \mathbf{v}_i \): components of \( \mathbf{v} \) expressed in \( \mathcal{F}_i \) [m/s]
- \( \mathbf{v}_o \): components of \( \mathbf{v}_o \) expressed in \( \mathcal{F}_o \) [m/s]
- \( \mathbf{w} \): noise variable of nonlinear observation model
- \( w \): pixel-width of \( \	ext{CCD} \) [pixels]
- \( \mathbf{x} \): absolute terrestrial position with respect to a geocentric frame [km]
- \( x \): state variable in Extended Kalman Filter
- \( x_p, y_p \): \( \text{WGS84} \) components of \( \mathbf{x} \) expressed in \( \mathcal{F}_e \) [km]
- \( \mathbf{y} \): measurement or orbital state vector observed/expressed in \( \mathcal{F}_e \) for geopotential integration
Nomenclature (Cont.)

Miscellaneous Symbols

\( \delta t_c \)  
onboard clock inaccuracy with respect to \( TGPS \) \( [s] \)

\( \ell \)  
pixel-length of \( CCD \) \( [pixels] \)

\( \ell_k \)  
\( k \)th basis vector of reference frame \( \mathcal{F}_t \)

\( \mathcal{B} \)  
satellite composed of \( \mathcal{R} \) and single or multiple \( \mathcal{W} \)

\( \mathcal{E} \)  
Earth body

\( \mathcal{R} \)  
spacecraft structure

\( \mathcal{W} \)  
reaction wheel

\( A_i \)  
ground area captured by imager \( [km^2] \)

\( \mathcal{E} \)  
target-tracking exclusion surface

\( \mathcal{E}_e \)  
East-aligned target-tracking exclusion surface with seasonal variation

\( \mathcal{E}_p \)  
Cross-Track-aligned target-tracking exclusion surface with seasonal variation

\( \mathcal{E}_s \)  
pushbroom-aligned target-tracking exclusion surface with seasonal variation

\( \mathcal{E}_s \)  
complete target-tracking exclusion surface; \( \mathcal{E}_s \triangleq \mathcal{E}_p + \mathcal{E}_e + \mathcal{E}_\kappa \)

\( G \)  
ground sampling distance of imager \( [m/pixel] \)

\( \mathcal{H} \)  
Hessian of cost function \( \mathcal{J} \)

\( \mathcal{J} \)  
cost function to be minimized the angular distance between \( \rightarrow s \) and \( \rightarrow d \)

\( O(\varphi^n) \)  
terms of order \( \varphi^n \) and higher

\( \mathcal{P} \)  
pixel density of imager \( [pixels^2] \)

\( \mathcal{P}_T \)  
total pixel smearing; \( \mathcal{P}_T \approx \mathcal{P}_\perp + \mathcal{P}_\parallel \) \( [pixels] \)

\( \mathcal{P}_\perp \)  
pixel smearing perpendicular to imager boresight \( [pixels] \)

\( \mathcal{P}_\parallel \)  
pixel smearing about imager boresight \( [pixels] \)

\( \mathcal{S}_e \)  
East-aligned target-tracking mounting surface with seasonal variation

\( \mathcal{S}_p \)  
Cross-Track-aligned target-tracking mounting surface with seasonal variation

\( \mathcal{S}_s \)  
pushbroom-aligned target-tracking mounting surface with seasonal variation

\( \mathcal{S}_s \)  
complete target-tracking mounting surface; \( \mathcal{S}_s \triangleq \mathcal{S}_p + \mathcal{S}_e + \mathcal{S}_\kappa \)

\( \Delta \lambda_t \)  
longitude displacement of target from satellite ground-track \( [^\circ] \)

\( \Delta t \)  
elapsed time since calculation of full \( FK5 \) transformation \( [s] \)

\( \Delta t_e \)  
imager time of exposure \( [s] \)

\( \Delta AT \)  
time offset between \( TAI \) and \( UTC \); \( \Delta AT \triangleq TAI - UTC \)

\( \Delta UT1 \)  
time offset between \( UT1 \) and \( UTC \); \( \Delta UT1 \triangleq UT1 - UTC \)

\( \mathcal{G}_a \)  
geometric center \( \mathcal{B} \) without appendages

\( \mathcal{G}_b \)  
center of mass of \( \mathcal{B} \)

\( \mathcal{G}_c \)  
center of mass of \( \mathcal{E} \)

\( \mathcal{G}_w \)  
center of mass of \( \mathcal{W} \)

\( \mathcal{G}_n \)  
center of mass of \( \mathcal{W} \)
Nomenclature (Cont.)

Reference Frames

\( \mathcal{F}_\tau \) True Equator Mean Equinox frame; see definition in Section 3.4.7

\( \mathcal{F}_a \) satellite body-fixed \textbf{Alignment} reference frame; see definition in Section 3.4.5

\( \mathcal{F}_b \) satellite body-fixed reference frame; \( \mathcal{F}_b \triangleq \begin{bmatrix} \hat{b}_1 & \hat{b}_2 & \hat{b}_3 \end{bmatrix}^T \)

\( \mathcal{F}_c \) satellite body-fixed \textbf{Constrained} reference frame; see definition in Section 3.4.5

\( \mathcal{F}_i \) Earth-Centered-Inertial reference frame; \( \mathcal{F}_i \triangleq \begin{bmatrix} \hat{i}_1 & \hat{i}_2 & \hat{i}_3 \end{bmatrix}^T \)

\( \mathcal{F}_\ell \) Topocentric reference frame; \( \mathcal{F}_\ell \triangleq \begin{bmatrix} \hat{\ell}_1 & \hat{\ell}_2 & \hat{\ell}_3 \end{bmatrix}^T \)

\( \mathcal{F}_\rho \) Camera Projection reference frame; \( \mathcal{F}_\rho \triangleq \begin{bmatrix} \hat{\rho}_1 & \hat{\rho}_2 & \hat{\rho}_3 \end{bmatrix}^T \)

\( \mathcal{F}_o \) Orbit reference frame; \( \mathcal{F}_o \triangleq \begin{bmatrix} \hat{o}_1 & \hat{o}_2 & \hat{o}_3 \end{bmatrix}^T \)

\( \mathcal{F}_e \) Earth-Centered-Earth-Fixed reference frame; \( \mathcal{F}_e \triangleq \begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \end{bmatrix}^T \)

\( \mathcal{F}_s \) generic body-fixed \textbf{Sensor} reference frame; \( \mathcal{F}_s \triangleq \begin{bmatrix} \hat{s}_1 & \hat{s}_2 & \hat{s}_3 \end{bmatrix}^T \)

Common Rotation Matrices

\( C_{yx} \) general rotation matrix from frame \( \mathcal{F}_x \) to frame \( \mathcal{F}_y \); \( C_{yx} \triangleq \mathcal{F}_y \cdot \mathcal{F}_x^T \)

\( C_{e} \) \textbf{FK5} rotation matrix describing transformation from \( \acute{ECEF} \) to \( \acute{ECI} \)

\( C_{\theta} \) sidereal time rotation matrix in \( \text{FK5} \) transformation

\( C_{\phi} \) polar motion rotation matrix in \( \text{FK5} \) transformation

\( C_{\Omega} \) precession rotation matrix in \( \text{FK5} \) transformation

\( C_{\Psi} \) nutation rotation matrix in \( \text{FK5} \) transformation

\( C_{c} \) rotation error; \( C_{c} \triangleq C_{t} C_{d}^T \)

\( C_{k} \) principal rotation about \( k \text{th} \) basis vector of a particular reference frame

\( C_{t} \) rotation matrix representing actual attitude or attitude estimate from \( \text{EKF} \)

Math

\( \triangleq \) defined as

\( \approx \) approximated as

\( \approx \) approximately equal to

\( \equiv \) equivalent to

\( \sim \) equal to

\( \rightarrow \) leads to

\( \| \cdot \| \) two-norm of vector or column vector

\( (\cdot) \times \) cross-product operator of column vectors

\( (\cdot)^T \) transpose of a matrix or column vector

\( (\cdot)^{-1} \) inverse operator of matrix or quaternion

\( (\cdot)^{\otimes} \) quaternion left hand compound operator for quaternion multiplication

\[
\mathbf{v} \times \triangleq \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}; \quad q^{-1} \triangleq \begin{bmatrix} -\varepsilon \\ \eta \end{bmatrix}; \quad q^\otimes \triangleq \begin{bmatrix} \eta 1 + \varepsilon^\times & \varepsilon \\ -\varepsilon^T & \eta \end{bmatrix}
\]
Chapter 1

Introduction

Terrestrial-pointing satellite missions provide essential daily services and crucial scientific data to both the private and public sectors. These satellites can be categorized into a variety of classes including communication, technology demonstration and Earth observation. The first communication satellite, Project SCORE, was launched in 1958 and since then communication satellites have formed an integral part of human society providing common day-to-day services. Out of these applications, long distance telephony is historically the most important. To facilitate this, typically, Geostationary (GEO) and Low Earth Orbit (LEO) communication satellites are employed to forward calls to satellite teleports which are connected to the world’s Public Switched Telephone Network. These satellites help provide phone services to remote locations and large regions without landline telecommunications. Geosynchronous satellites are also advantageous in providing television and radio services. In North America, Fixed Satellite Services are used to distribute cable television, whereas Direct Broadcast Satellites are used to transmit direct-to-home satellite television services. Communication satellites are also employed in remote regions where availability of Internet and broadband services are desired.

Earth observation missions include a wide range of satellites whose primary objectives are to study the Earth and the human environment. Unlike traditional aerial observation systems such as unmanned aerial vehicles, space based observation systems are able to provide global scale information efficiently. The RapidEye mission is a commercial multi-spectral imagery mission which was launched by the German Company RapidEye AG in 2008. The mission includes a constellation of 5 minisatellites which are all in the same orbital plane at an altitude of 630 km. Each satellite has a mass of approximately 150 kg, an imager swath width of 80 km, a ground sampling distance (GSD) of 5 m and a ±25° cross-track viewing capability with a pointing accuracy of 12'' (3σ). The constellation is able to effect a revisit cycle of 5.5 days at nadir and 1 day at cross-track for North American and European regions [1] [2]. The Advanced Satellite with New System Architecture for Observation (ASNARO) spacecraft is a Japanese high resolution Earth imaging satellite built by NEC Corporation and is anticipated to launch in 2013. The entire system is expected to have a mass of 450 kg and is projected to have a total development cost of $74.7 million. The imager onboard ASNARO has a 0.5 m GSD
Chapter 1. Introduction

from an orbital altitude of 510 km. The satellite is expected to have a pointing accuracy of 3′ (3σ), a maximum slew rate of 4°/s and be able to point ±45° with respect to nadir in both the along-track and cross-track directions [3]. The Precursore Iperspettrale della Missione Applicativa (PRISMA) mission is an Italian Earth observation mission being developed by the Italian Space Agency and is expected to launch in 2014. It involves a single small satellite of approximately 500 kg which images the Earth using hyperspectral and panchromatic cameras from a sun synchronous altitude of 700 km. The hyperspectral and panchromatic cameras have a GSD of 30 m and 5 m, respectively. The satellite is expected to have a 7 day revisit time of the European region, capitalizing on its 15° cross-track look capability. Overall, the satellite is anticipated to be able to point with an accuracy of 4.4′ (3σ) [4] [5]. A particular Earth observation mission of interest in this thesis will be the Nanosatellite for Earth Monitoring and Observation - Aerosol Monitor (NEMO-AM) mission being developed by the Space Flight Laboratory (SFL). NEMO-AM is a science-based mission with the objective of operating a 40.9 m GSD (from 650 km) dual polarization, multi-band imager to measure the atmospheric aerosol content over specific geographical locations. It is anticipated to be completed in early 2014 and operate from an altitude of 650 km, while performing sub-degree Earth pointing on a 15 kg bus.

Although the majority of these missions for communication and Earth observation have been effected by commercial large-scale satellites, there is scope for micro and nanosatellites to achieve similar, if not the same, success. In general, these commercial satellites are relatively large, have long development cycles and typically cost upwards of tens, if not hundreds, of millions of dollars. High performance small satellites have demonstrated that they are able to contend and operate in a competitive market by effecting cheaper and faster manufacturing periods than their large-scale commercial counterparts. This has been facilitated by capitalizing on advanced miniaturized commercial-off-the-shelf (COTS) hardware which have only emerged in the last decade. The task of performing terrestrial pointing on any satellite platform is highly dependent on the Attitude and Orbit Control System (AOCS) of the satellite. The objective of the AOCS is to determine the absolute location of the satellite’s target, estimate the state of the satellite (i.e. attitude, angular velocity, position and velocity) through sensory inputs, calculate the desired state to be achieved and then command actuators to achieve that state, such that mission and pointing requirements are met. To carry out rapid target acquisition and pointing, all of these tasks must be performed autonomously. This thesis presents the detailed AOCS design of NEMO-AM developed by the author which advances the capabilities of Earth-pointing small satellites under 20 kg.
1.1 Earth Target Tracking Review

The primary objective of Earth-pointing satellites is to point their payload, be it a high gain antenna, telescope or an imager, at a ground target and subsequently orient itself about the payload to achieve a specific attitude. Although satellite technology to perform Earth-pointing has been around since the early 1960s, there are only a limited number of existing works in academic literature. All of these works demonstrate only the initial pointing objective using idealistic analytical scenarios. Tangible examples of satellites which have successfully demonstrated Earth-pointing are many and overwhelmingly include commercial large-scale satellites or satellites which favour custom-built hardware over COTS parts and have masses over 80 kg. The majority of the on-orbit results for these satellites are proprietary and those that have been made accessible in recent years are limited to Earth-pointing only in the nadir direction. Nevertheless, the aforementioned analytical works and on-orbit results of successful commercial Earth-pointing satellites are elucidated in this subsection.

The works in academic literature are first reviewed. Goeree et al. [6] [7] at the University of Arizona developed a geometric tracking controller which included a quaternion feedback component for tracking attitude, a feedback damping component to track angular velocities and a feed-forward component to counteract gyroscopic coupling torques. Simulations of their UASat satellite, which housed a non-orthogonal set of four reaction wheels, showed a ground tracking pointing accuracy of 18′′. The authors considered osculating orbits and highly simplistic astro-dynamical models to approximate the motion of the satellite and its target. The satellite was also assumed to have perfect attitude and orbital knowledge and only wheel-specific hardware models were effected to demonstrate wheel zero-crossing avoidance.

Steyn et al. [8] at the University of Surrey also presented a quaternion-based Proportional-Integral-Derivative (PID) feedback controller and a generalized method to track terrestrial targets for nominally Earth pointing satellites [9]. Their implemented controller was based off the work of Wie et al. [10] which effected open loop decoupling control torque and performed inertially referenced instantaneous Euler-axis rotations. This controller boasts flight heritage on the Apollo Skylab and on NASA’s Space Transportation System program. Similar to Goeree et al., the authors also used simplistic astrodynamical models and assumed perfect attitude and orbital knowledge while employing an orthogonal set of three reaction wheels which exhibited no wheel zero-crossing characteristics. Simulations of a theoretical micro-satellite with an AOCS running at 1 Hz showed double digit arcsecond-level pointing accuracy under these idealized conditions.

Kondo et al. [11] at the Keio University and the National Space Development Agency of Japan developed a purely kinematically-derived pointing methodology for Earth observation satellites flying high fidelity determination sensors. The formulation does not consider the angular accelerations required for pointing and also leaves rotation about the payload unconstrained, thereby limiting the applicability of the methodology to realistic missions. Numerical simulations where attitude and orbital knowledge were provided by a star tracker and Global
Positioning System (GPS) model demonstrated a 4" pointing accuracy when the AOCS was operating at 2 Hz.

In all of these academic works, unrealistic simplifications in the hardware, and in the satellite and environmental dynamics are employed to demonstrate the notion of Earth-pointing. Onboard state estimation is omitted and off-line simulations which do not include flight software or hardware-in-the-loop are used to validate the control strategies. Hardware performance modeling, if considered, is only attempted for reaction wheels. Although, target-tracking control methods are discussed in detail, none of the works demonstrate an intelligent way to constrain the satellite attitude about the payload for realistic Earth-pointing objectives.

Next, on-orbit results for successful Earth-pointing satellites are reviewed. The Mini-Satellite Italiano a Tecnologia Avanzata (MITA) was an Earth observation small-satellite mission whose primary objective was to design, develop, implement and demonstrate the usability of a generic small-satellite platform for a wide variety of future Earth missions. The satellite was launched and injected into a nominal circular orbit with an inclination of 87.3° and an altitude of 450 km on 15 July 2000 from Plesetsk (Russia). The bus has a mass of 170 kg and an average power of 85 W at the end of life. The AOCS was comprised of two monoaxial horizon sensors, 5 coarse sun sensors and a three-axis magnetometer. The nominal Earth-pointing attitude was nadir pointing with an accuracy between 0.1° and 1.0° (1σ), depending on the combination of attitude sensors employed. The mission ended on 15 August 2001 when MITA re-entered the Earth’s atmosphere due to its low initial orbit.[12] [13].

CloudSat is cooperative research mission between the National Aeronautics and Space Administration and the Canadian Space Agency, designed to study the characteristics of clouds and their effects on climate and weather. CloudSat was launched on 28 April 2006 from Vandenberg Air Force Base in the United States and injected into a sun-synchronous orbit with an altitude of 705 km and inclination of 98.2°. The spacecraft has a mass of 995 kg and an average power draw of 700 W. The AOCS consisted of 14 coarse sun sensors for solar array control, a single redundant star tracker, 2 GPS receivers, 3 two-axis magnetometers, 3 dual-winding torque rods and 4 reaction wheels. CloudSat was initially administered as part of the A-Train formation flying constellation where it would trail its leading constellation satellite, CALIPSO, in the same orbit by approximately 100 s and point its payload onto the same groundtrack. The on-orbit pointing accuracy was observed to be 0.07° (3σ). Currently, CloudSat is not part of the A-Train constellation, but remains functional and is expected to be funded and operated through 2013.[14]

The Project for On-Board Autonomy 1 (PROBA-1) is a technology demonstration satellite developed by the European Space Agency as part of its MicroSat program. The spacecraft’s objective was to conduct technological experiments for future small spacecraft missions. PROBA-1 was launched 22 October 2001 from Antrix (India) into a sun-synchronous orbit with an altitude of 615 km and an inclination of 97.9°. The satellite has a mass of 94 kg, a peak power production of 90 W and an AOCS comprised of dual star trackers, a redundant three-axis mag-
netometer, a single GPS receiver, 4 magnetorquers and 4 reaction wheels. The AOCS had two main autonomous Earth-pointing modes. The first mode required PROBA-1 to acquire 5 images by scanning its spectrometer over a 20 km Earth target and sweeping across the target at 5 separate imaging angles. The second mode required the satellite to continuously point its payload towards an Earth-fixed target and image the target by slewing up to an angular speed of 0.8 °/s. On-orbit results demonstrated that by effecting an AOCS cycle at 10 Hz, PROBA-1 was able to achieve an attitude determination accuracy better than 125″ and a pointing accuracy better than 360″. After a decade in orbit, the satellite recently received a software update to its star tracker in May 2012 and continues to collect valuable Earth observation data [15].

1.2 Thesis Overview

In this thesis, a comprehensive AOCS design for NEMO-AM is presented, with a particular emphasis on the Earth-pointing portion of the mission. An overview of the satellite is given in Chapter 2. Afterwards, the fundamentals of orbital and attitude dynamics are reviewed in Chapter 3. After sufficiently introducing the relevant astrodynamics and motion equations, the generalized architecture of the AOCS onboard NEMO-AM, and the simulation framework used to validate the AOCS designs is outlined in Chapter 5. Next, target tracking considerations for NEMO-AM are presented and the rotational trajectory which facilitates tracking is derived in detail in Chapter 4. Lastly, the AOCS design of NEMO-AM is validated using simulations with test-based hardware models and flight-software-in-the-loop in Chapter 6. This validation method is not only imperative to mitigate risk and affirm performance in the flight design before launch, but also provides a foundation for practical target tracking which can be extended to various types of Earth-pointing satellite missions –something which has not been publicly existent in the past to the author’s best knowledge.
Chapter 2

NEMO-AM Overview

The Nanosatellite for Earth Monitoring and Observation Aerosol Monitor (NEMO-AM) satellite is a next-generation microsatellite being designed and built by the Space Flight Laboratory (SFL) at the University of Toronto Institute for Aerospace Studies (UTIAS), and being funded by the Indian Space Research Organization (ISRO). The satellite is developed around the Generic Nanosatellite Bus (GNB) concept, where a multipurpose and adaptable satellite bus is designed to work with a wide range of payloads with little or no modification. Consequently, much of the hardware on NEMO-AM is inherited from the GNB and likewise boasts flight heritage. This concept not only allows for shorter design cycles, but allows those periods to be focused more on mission specific goals. The primary mission objective of NEMO-AM is to detect the atmospheric aerosol content over specific geographical regions. The satellite is anticipated to have a local time of descending node (LTDN) between 09:00 and 11:00 and an orbital altitude between 600 km and 800 km. The satellite is shown in Figure 2.1 and although it is based on the GNB, it is designed with several modifications to allow it to achieve its mission and imaging objectives.

Figure 2.1: NEMO-AM body-fixed frame.
The satellite bus envelopes a volume of $20\text{ cm} \times 20\text{ cm} \times 40\text{ cm}$ and is expected to have a maximum mass of 15 kg\cite{16} with a power throughput capability of 80 W\cite{17}. A large solar panel of $60\text{ cm} \times 58\text{ cm}$ is attached on the $+X$ face for power generation. \textit{NEMO-AM} will fly the standard set of Attitude and Orbit Control System (\textit{AOCS}) components found on the \textit{GNB}, a \textit{GPS} receiver, communication antennas (S-band for uplink and downlink), onboard computers for task management, a power distribution network including batteries and solar cells, and a multi-spectral imager to capture aerosol concentration. The attitude hardware suite is composed of a single star tracker, six sun sensors, one magnetometer, three magnetorquers and three reaction wheels. Each geometric face will have two sun sensors, while each geometric body axis will have one magnetorquer and one reaction wheel positioned along its direction. The magnetometer is positioned underneath the $-X$ panel and the star tracker is canted 38$^\circ$ from the $-X$ direction, towards $+Y$, in the $XY$ plane. A detailed description of each sensor is presented in Section 5. A diagram of the placement of the satellite’s external hardware, emphasizing the attitude components, is presented in Figure 2.2.

![Figure 2.2: Placement of attitude hardware on \textit{NEMO-AM}.](image)

The optical instrument is designed to detect the light reflected by atmospheric aerosols at visual and short-wave infrared bands in three bands and two polarizations. The instrument has a 25 ms exposure and is anticipated to have a ground sampling distance (\textit{GSD}) of 40.9 m from an altitude of 650 km at fine-resolution mode, down to 160 m ground resolution at coarse-resolution mode. The imager field of view (\textit{FOV}) is projected from the $XZ$ body-fixed plane, along the boresight ($-Y$) direction. The observations will be repeated during an Imaging Campaign at different viewing angles with respect to the ground target, allowing image capture at different scattering angles. This target tracking is done by autonomously pointing the imager continuously at the target of interest and subsequently rotating about the boresight to align consecutive images as the satellite travels in its orbit. The maneuver is shown in Figure 2.3.
Figure 2.3: *NEMO-AM* target-tracking maneuver during an Imaging Campaign.

The *AOCS* requirements for *NEMO-AM* \[18\] center on the target-tracking maneuver. Preliminary requirements from *ISRO* \[19\] were developed further in detail by the author and new requirements were created to explicitly outline the operational *AOCS* capabilities of the satellite. All footprint coverage requirements and related values are with respect to the instance when the satellite is performing target tracking and pointing nadir, from an altitude of 650 km, at a target on its ground-track. The original design called for imaging maneuvers where the \(-Z\) camera axis was used to constrain the satellite’s attitude by aligning it with a chosen local heading related to the target. *NEMO-AM* was required to consistently image an area equivalent to 129 km × 8.6 km with an image jitter of 10% of a pixel or less during an Imaging Campaign. Based on this attitude, the \(+X\) panel would primarily be orthogonal to the orbital plane during its imaging maneuver. The original design also desired the imager charged-couple device (CCD) to have a pixel density of 1277 pixels × 4008 pixels for one polarization and 2554 pixels × 4008 pixels for two polarizations. This corresponded to an imager footprint of 48.6 km × 152 km for one polarization. During detailed design however, it was observed that at certain viewing angles, the desired target tracking attitude would allow sunlight to interfere with the imager \[20\] and the satellite would need to be expanded to accommodate the manufactured imager prisms. Consequently, an alternate tracking attitude was chosen instead, where the \(+X\) solar panel is used to provide stray light shielding for the camera and the \(+X\) axis is used to align consecutive images in the XZ image plane. In order to also maximize ground swath of the images, the CCD was rotated and restructured so that each polarization would be composed of 848 pixels × 2332 pixels and have an equivalent half-angle FOV of 1.53° × 4.19°. In this tracking attitude, *NEMO-AM* has three alignment options referred to as Pushbroom, Cross-Track alignment and East-alignment. In Pushbroom alignment, the payload is pointed towards the immediate subsatellite point while the \(+X\) axis is aligned with the orbit normal. In Cross-Track alignment and East-alignment, *NEMO-AM* points its camera at the desired target and rotates about the boresight so that the \(+X\) axis is aligned in the imager plane with the satellite cross-track direction (see Section 3.4) or the local East heading of the target, respectively. These three alignment options are summarized in Table 2.1. Cross-Track alignment will
result in the same image as pushbroom alignment when *NEMO-AM* is directly over a target on the satellite groundtrack. In this document, East-alignment attitude is primarily considered, since it closely resembles the Cross-Track alignment for ground-track targets and is more demanding than the pushbroom configuration. A comparison of the previous and revised tracking attitudes is shown in Figure 2.4 and the alignment options are shown in Figure 2.5.

![Figure 2.4: *NEMO-AM* target tracking attitude during an Imaging Campaign.](image)

<table>
<thead>
<tr>
<th>Alignment Name</th>
<th>Alignment Attitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-Track-Aligned</td>
<td>$-Y$ pointed at target. $+X$ aligned with orbit normal projection in imaging plane.</td>
</tr>
<tr>
<td>East-Aligned</td>
<td>$-Y$ pointed at target. $+X$ aligned with local East projection in imaging plane.</td>
</tr>
<tr>
<td>Pushbroom</td>
<td>$-Y$ pointed at nadir. $+X$ aligned with orbit normal.</td>
</tr>
</tbody>
</table>
By revising the target tracking attitude and \textit{CCD} structure, revisions to the imager footprint requirement were also needed. The original footprint requirement of 129 km $\times$ 8.6 km was scaled using the ratio between the original and new display aspect ratios of a single polarization (i.e. 1277 \textit{pixels} $\times$ 4008 \textit{pixels} vs. 848 \textit{pixels} $\times$ 2332 \textit{pixels}). Thus, the settled footprint requirement calls for consistent imaging of an area equivalent to 75.1 km $\times$ 5.7 km when the satellite is pointing nadir from an altitude of 650 km. At this specific viewing angle, each pixel corresponds to 40.9 m or equivalently 13.0\arcmin/	extit{pixel}. Thus, the allowable error in pointing which still permits a swath of 75.1 km $\times$ 5.7 km to be imaged is equivalent to 0.89\arcdeg. Likewise, the 2\sigma tolerance of this quantity is defined as the pointing requirement for \textit{NEMO-AM}. The restructuring of the imager footprint parameters is shown in Figure 2.6.

Figure 2.6: \textit{NEMO-AM} original and revised footprint requirements for nadir pointing from 650 km. CCD footprints outlined in thick black line, image footprints outlined in thin black line and corresponding continuous imaging requirements outlined in dash-dotted black line.

Supplemental requirements for imaging off-track targets and selecting viewing angles were also formulated for practical operational scenarios. The analysis behind these requirements are detailed in Section 6.
Chapter 3

Orbital and Attitude Dynamics

In this chapter, the relevant coordinate frames and time systems for this thesis are established, and the spacecraft orbital and attitude motion equations are presented.

3.1 Notation

In this work, vectrix notation adopted from Hughes [21] is used exclusively to manipulate vectors in multiple reference frames. A reference frame is represented by a column vector containing the orthonormal and dextral basis vectors of that frame and is referred to as a vectrix. The vectrix for reference frame $\mathcal{F}_i$ is denoted by $\mathbf{F}_i = \begin{bmatrix} \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 \end{bmatrix}^T$ and a vector $\mathbf{v}$ expressed in $\mathcal{F}_i$, with components $\mathbf{v}_i = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$, is denoted as $\mathbf{v} = \mathbf{F}_i^T \mathbf{v}_i$. Errors in state variables are expressed by affixing a prefix $\delta$ to the variable. The error corrupted state, which is a sum of the true state ($\mathbf{v}_i$) and its error ($\delta \mathbf{v}_i$), is expressed using an overscript on the state variable:

$$\tilde{\mathbf{v}}_i = \mathbf{v}_i + \delta \mathbf{v}_i \quad (3.1)$$

Algebraic vector operations are admissible only if all the vector components are expressed in the same frame. The dot and cross product operations in $\mathcal{F}_i$ vectrix notation are respectively:

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}_i^T \mathbf{F}_i \cdot \mathbf{F}_i^T \mathbf{b}_i = \mathbf{a}_i^T \mathbf{b}_i; \quad \mathbf{a} \times \mathbf{b} = \mathbf{F}_i^T (\mathbf{a}_i \times \mathbf{b}_i) \quad (3.2)$$

The time derivative of a vector $\mathbf{v}$ is distinguished by an overdot $\dot{\mathbf{v}}$ if it is observed in an inertial frame or by an overcircle $\overset{\circ}{\mathbf{v}}$ if it is observed in a specified rotating frame. The time derivative of a vector’s components has only one meaning and is expressed using an overdot $\dot{\mathbf{v}}$. Using this notation, the relationship between the temporal derivative of $\mathbf{v}$ observed in a non-rotating inertial frame $\mathcal{F}_i$ and decomposed into that observed in a rotating frame $\mathcal{F}_b$ is:

$$\dot{\mathbf{v}} = \overset{\circ}{\mathbf{v}} + \omega_{bi} \times \mathbf{v} \quad (3.3)$$

$$\mathbf{F}_i^T (\dot{\mathbf{v}}_i) = \mathbf{F}_b^T (\dot{\mathbf{v}}_b + \omega_{b_i} \mathbf{v}_b) \quad (3.4)$$
where the term $\dot{v}_i$ on the left hand side is the time derivative of $v$ observed and expressed in $\mathcal{F}_i$ and the term $\dot{v}_b$ on the right hand side is the time derivative of $v$ observed and expressed in $\mathcal{F}_b$. The variable $\omega_{bi}$ is the angular velocity of $\mathcal{F}_b$ relative to $\mathcal{F}_i$, expressed in $\mathcal{F}_b$. In this work, the convention will be to express all angular velocity components in the frame corresponding to their first subscript. When working with multiple coordinate systems, inevitably the components of a vector must be transformed from one frame to another. This transformation is typically done through a direction cosine matrix $C$. The rotation matrix from frame $\mathcal{F}_i$ to $\mathcal{F}_b$ is represented by $C_{bi}$. Its kinematics are described by:

$$\dot{C}_{bi} = -\omega_{bi}^\times C_{bi} = C_{bi} \omega_{ib}^\times$$ \hspace{1cm} (3.5)

Principal rotations about the basis 1-, 2- and 3-axis are represented by $C_1$, $C_2$ and $C_3$ respectively. From a controls perspective, the rotation error $C_e$ is defined as the rotation from a desired attitude $C_d$ to the actual (true) satellite attitude $C_t$. From an estimation perspective, $C_t$ is the estimated attitude and $C_d$ is the desired estimate of attitude (i.e. true attitude) of the satellite. This rotational error can be expressed as a 3-2-1 (Yaw-Pitch-Roll) Euler rotation. The rotation angles $\theta_k$ are affixed with a $\delta$ since $C_e$ is an error term:

$$C_t = C_e C_d \hspace{1cm} (3.6)$$

$$C_e = C_1(\delta\theta_1) C_2(\delta\theta_2) C_3(\delta\theta_3)$$

The angle error column vector is defined as $\delta\theta = \begin{bmatrix} \delta\theta_1 & \delta\theta_2 & \delta\theta_3 \end{bmatrix}^T$. Direction cosine matrices may also be expressed as Euler-axis rotations about a unit-norm axis $\mathbf{a}$ and a rotation angle $\varphi$:

$$C(\mathbf{a}, \varphi) = \cos \varphi \mathbf{1} + (1 - \cos \varphi) \mathbf{a} \mathbf{a}^T - \sin \varphi \mathbf{a}^\times \hspace{1cm} (3.7)$$

The rotation axis $\mathbf{a}$ can be expressed in either the initial or final frame of the transformation since $C(\mathbf{a}, \varphi) \mathbf{a} = \mathbf{a}$. If the temporal derivatives of the Euler-axis and rotation angle of a transformation between two frames are known, the relative angular velocity between the two frames can be expressed as:

$$\omega = \dot{\varphi} \mathbf{a} - (1 - \cos \varphi) \mathbf{a}^\times \dot{\mathbf{a}} + \sin \varphi \dot{\mathbf{a}} \hspace{1cm} (3.8)$$

Another representation of a finite rotation $C$ is the set of four parameters $\varepsilon$ and $\eta$, which use the same rotation axis $\mathbf{a}$ and angle $\varphi$ as an Euler-axis rotation:

$$q(\mathbf{a}, \varphi) = \begin{bmatrix} \varepsilon^T & \eta \end{bmatrix}^T = \begin{bmatrix} \mathbf{a}^T \sin \frac{\varphi}{2} & \cos \frac{\varphi}{2} \end{bmatrix}^T \hspace{1cm} (3.9)$$

Collectively, these quantities are known as Euler parameters (quaternions) and they possess the unit-norm property $q^T q = 1$. Two consecutive rotations is equivalent to two rotation matrices being multiplied in sequence. This operation can be achieved with quaternions by defining
the quaternion left-hand operator \((\cdot)\otimes\). The quaternion equivalent of two consecutive rotations \(C_2C_1\) is:

\[
q = q_2 \otimes q_1
\]

\[
q_\otimes = \begin{bmatrix}
\eta_1 - \epsilon^\times & \epsilon \\
-\epsilon^T & \eta
\end{bmatrix}
\]

(3.10)

The quaternion kinematics are described by:

\[
\begin{bmatrix}
\dot{\epsilon} \\
\dot{\eta}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
-\omega^\times & \omega \\
-\omega^T & 0
\end{bmatrix} \begin{bmatrix}
\epsilon \\
\eta
\end{bmatrix}
\]

(3.11)

A more in-depth description of quaternions can be found in Kuipers [22].

### 3.2 Geodesy

To perform terrestrial target tracking, a general set of geodetic datums are required to define the shape of the planet. As geodetic models of the Earth have evolved with the aid of satellites, they have shown that the Earth’s shape is roughly an oblate spheroid (ellipsoid) which revolves about its minor axis. The 2004 revised World Geodetic System 1984 (WGS84) ellipsoidal reference model derived from the 1996 Earth Gravitational Model (EGM96), specifies the semimajor axis and the semiminor axis to be equal to the equatorial and polar radius, respectively, and is widely used to approximate the Earth’s geoid for semi-precise calculations. The model origin is located at the Earth’s center of mass and, the mean equatorial radius \(R_\oplus\) and the semiminor axis \(b_\oplus\) are used to derive the planet’s flattening \(f_\oplus\) and eccentricity \(e_\oplus\) [23][24]:

\[
R_\oplus \approx 6378.137 \text{ km}; \quad b_\oplus \approx 6356.752 \text{ km}; \quad f_\oplus \approx 1 \frac{298.257}{298.257}; \quad e_\oplus^2 \approx 6.694 \times 10^{-3}
\]

(3.12)

The ellipsoidal representation approximates the Earth’s flattened shape to have a fractional decrease in the radius of the poles equal to \(f_\oplus R_\oplus \approx 21\) km. Terrestrial longitude \(\lambda\) and terrestrial latitude \(\phi\) are used to specify locations on the Earth. Longitude is the East-West angular displacement measured positive East of the prime meridian. Latitude is defined as the North-South angular displacement measured positive due North of the equatorial plane. Although the ellipsoidal model poses no difficulties when explicitly defining longitude, it does so with defining latitude. Geocentric latitude \(\phi_c\) is defined as the angle between the equatorial plane and a vector from the Earth’s center to the location of interest, whereas geodetic latitude \(\phi_d\), which is commonly used in maps, is the angle between the equatorial plane and the normal to the surface of the Earth model. Figure 3.1 illustrates these parameters.
The height of a terrestrial location $\mathbf{x}$, along the normal to the reference ellipsoid surface, is denoted as $h_\oplus$. The Cartesian components of $\mathbf{x}$ in an Earth-Centered-Earth-Fixed frame (see Section 3.4) can be expressed as a function of the location’s terrestrial parameters and WGS84 geodetic datums \[24\]:

$$
\mathbf{x}_e = 
\begin{bmatrix}
(C_\oplus + h_\oplus) \cos(\phi_d) \cos(\lambda) \\
(C_\oplus + h_\oplus) \cos(\phi_d) \sin(\lambda) \\
(S_\oplus + h_\oplus) \sin(\phi_d)
\end{bmatrix}
$$

(3.13)

where the geodetic datums $C_\oplus$ and $S_\oplus$ are defined as:

$$
C_\oplus \triangleq \frac{R_\oplus}{\sqrt{1 - e_\oplus^2 \sin^2 \phi_d}}
$$

(3.14)

$$
S_\oplus \triangleq \frac{R_\oplus (1 - e_\oplus^2)}{\sqrt{1 - e_\oplus^2 \sin^2 \phi_d}}
$$

(3.15)
3.3 Time Systems

In this section, the time systems used to facilitate target tracking are presented:  

- **Sidereal Time**: Based on the consecutive passing of the stars over a specific meridian. It is expressed as the hour angle between the Vernal Equinox $\Upsilon$ and a particular meridian of interest. Since the Equinox is moving with time, Mean and Apparent Sidereal Time can be distinguished at a given epoch by measuring the angle between the specified meridian and the mean and true Vernal Equinox, respectively, in the true equatorial plane. The Greenwich Mean Sidereal Time ($GMST$, $\theta_m$), and Greenwich Apparent Sidereal Time ($GAST$, $\theta_a$), can be defined as the angular distances measured positive East from the mean and apparent $\Upsilon$, respectively, to the Greenwich Meridian.

- **Universal Time ($UT$)**: Based on the mean motion of the Sun relative to Earth. It is the currently accepted realization of mean solar time and its purpose is to achieve a constant average solar day length of 24 hours. Since the Earth’s rotation is not uniform, one second of Universal time is not constant and is updated regularly. There are three variations of $UT$, with $UT1$ being the most important in this work. It is used to facilitate sun vector ephemeris calculations and the $FK5$ $C_{1c}$ rotation (see Section 3.4.8.1).

- **Atomic Time ($TAI$)**: Based on the cycle count of a Cesium-133 atom in a high-frequency circuit. It is abbreviated as Temps Atomique International ($TAI$) and being able to capture relativistic effects in clocks, is the most accurate of the discussed systems.

- **Coordinated Universal Time ($UTC$)**: Based on $TAI$, it is the foundation for civil time systems and ordinary clocks. It was designed to be aligned closely with $UT1$ by an offset $\Delta UT1$ which varies between $\pm 0.9$ s.

- **Terrestrial Time ($TT$)**: A timescale based on apparent geocentric observations of celestial bodies. It is offset from $TAI$ by exactly 32.184 s and is used extensively in General Perturbation applications.

- **GPS Time ($TGPS$)**: An atomic timescale which uses an independent set of atomic clocks maintained by the United States Naval Observatory. Its origin is set at 0:00 January 6, 1980 $UTC$ and has a constant 19 sec offset from $TAI$.

The different time systems are related to each other through the following expressions:

\[ UTC = UT1 - \Delta UT1 \]
\[ TAI = UTC + \Delta AT \]
\[ TT = TAI + 32.184 \text{ s} \]
\[ TGPS = TAI - 19 \text{ s} \]
The quantities $\Delta AT$ and $\Delta UT1$, together, form a part of what are known as the Earth Observation Parameters (EOP) and are published regularly by the International Earth Rotation Services (IERS) due to the irregular rotation of the Earth. Both $\Delta UT1$ and $\Delta AT$ are accumulated differences and the convention is to fix $\Delta AT$ to an integer value in seconds and update $\Delta UT1$ continuously between $\pm0.9\text{s}$. Since July 1, 2012, $\Delta AT$ has been set to 35 s [25].

### 3.4 Coordinate Frames

In this section, the most relevant coordinate frames pertaining to Earth-based target tracking are discussed. Many of these systems are Earth-based themselves, with their origin either at the Earth’s center of mass (geocentric) or on the surface of the Earth (topocentric). Terrestrial targets are identified by the target’s altitude $h_t$ above the ellipsoid, latitude $\phi_t$ and longitude $\lambda_t$. Absolute target terrestrial position and velocity are denoted by $\mathbf{x}_t$ and $\mathbf{v}_t$, and absolute spacecraft position and velocity are denoted by $\mathbf{x}$ and $\mathbf{v}$. The spacecraft height above the Earth ellipsoid is denoted by $h$. The satellite-to-target vector $\mathbf{\kappa}$ is defined as:

$$\mathbf{\kappa} \triangleq \mathbf{x} - \mathbf{r} \quad (3.17)$$

#### 3.4.1 Earth-Centered-Inertial Frame (IAU-1976 / FK5)

The inertial frame used for satellite-based applications is typically referred to as the Earth-Centered-Inertial (ECI) frame. The ECI frame has a number of different variations, but the one chosen in this work is the International Astronomical Union 1976 (IAU-1976) variant. It is a rotational transformation from an Earth-Centered-Earth-Fixed frame to an Earth-Centered-Inertial frame (see Section 3.4.8.1) and is also referred to as the Fifth Fundamental Katalog (FK5) transformation. It is very closely aligned to the standard inertial coordinate system adopted by the IERS in 1997, known as the Geocentric Celestial Reference Frame (GCRF). The GCRF is based on the International Astronomical Union 2000 (IAU-2000) system which is independent of solar system dynamics [26] and is itself accurate to at least $0.001''$ [24]. It is not adopted in this work because its computational cost does not justify the improved accuracy over the FK5 (see Section 3.4.8.1) for the desired satellite applications. Both systems are not inertially fixed in space since the Vernal Equinox and the Earth’s equator move very slowly over time. However, by defining the Equinox and equator of a particular date, a “pseudo” inertial system can be achieved. The FK5 system is defined by extrapolating a moving mean Equinox and the Earth’s equator move very slowly over time. An illustration of this frame relative to the others in this section is shown in Figure 3.2 (a).
3.4.2 Earth-Centered-Earth-Fixed Frame (ECEF)

The Earth-Centered-Earth-Fixed (ECEF) frame is a non-inertial geocentric frame composed of the dextral and orthonormal basis vectors $\mathcal{F}_e \triangleq \left[ \hat{e}_1 \ \hat{e}_2 \ \hat{e}_3 \right]^T$ and fixed to the rotating Earth. It rotates at the same angular velocity as the Earth $\omega_{\oplus}$, which is assumed to be constant:

$$\omega_{ei} = \mathcal{F}_e^T \begin{bmatrix} 0 & 0 & \omega_{\oplus} \end{bmatrix}^T$$  \hspace{1cm} (3.18)

$$\omega_{\oplus} \approx 7.292115 \times 10^{-5} \text{ rad/s}$$  \hspace{1cm} (3.19)

The $\hat{e}_1$ vector points towards the Greenwich prime meridian along the mean equatorial plane, the $\hat{e}_2$ vector points 90° East of $\hat{e}_1$ in the same plane and the $\hat{e}_3$ vector completes the triad. Consistent with the Global Positioning System (GPS) constellation, terrestrial Cartesian positions are specified in this frame using the WGS84 realization from Eq. (3.13) \cite{23} \cite{24}. The equatorial plane in this frame is identical to that realized in the WGS84 model. Throughout this work, GPS measurements are assumed to be obtained in the ECEF frame \cite{27}. This system is equivalent to the formally recognized International Terrestrial Reference Frame (ITRF).

An illustration of this frame relative to the others in this section is shown in Figure 3.2 (a).

3.4.3 Topocentric Coordinate Frame (ENZ)

An important coordinate frame for identifying terrestrial locations is the local Topocentric Coordinate System, also referred to in this work as the East-North-Zenith (ENZ) frame. It is composed of the dextral and orthonormal basis vectors $\mathcal{F}_\ell \triangleq \left[ \hat{\ell}_1 \ \hat{\ell}_2 \ \hat{\ell}_3 \right]^T$. The frame is centered at the WGS84 realization of the location of interest and like the ECEF system, rotates with the Earth at $\omega_{\ell i} = \omega_{ei}$. For a given location, $\hat{\ell}_1$ and $\hat{\ell}_2$ are aligned with the parallel of latitude and meridian passing through that location, respectively. Likewise, $\hat{\ell}_1$ points due East and $\hat{\ell}_2$ points due North from the site. The $\hat{\ell}_3$ vector points towards the geodetic Zenith which is inclined by $\phi_d$ from the equatorial plane. For a given location, the $\hat{\ell}_1 - \hat{\ell}_2$ plane is tangential to the surface of the Earth geoid. Two angles used to locate a satellite with respect to Zenith of a site are defined in this frame, with the components of $\kappa$ expressed in $\mathcal{F}_\ell$:

$$\alpha_\ell \triangleq \arctan\left( \frac{\kappa_1}{\kappa_3} \right) ; \quad \delta_\ell \triangleq \arctan\left( \frac{-\kappa_2}{\sqrt{\kappa_1^2 + \kappa_3^2}} \right)$$  \hspace{1cm} (3.20)

The first is the topocentric right ascension angle $\alpha_\ell$, which is the angle $\kappa_3$ makes with $\hat{\ell}_3$ in the Zenith-East plane and is measured positive due East. The topocentric declination angle $\delta_\ell$ is the angle $\kappa$ makes with the Zenith-East plane and is measured positive due North. During target tracking these parameters are called the observation ascension and observation declination viewing angles, respectively. An Earth-fixed target is denoted as a groundtrack target during target tracking if $\alpha_\ell$ is equal to nil at anytime during that observation. An illustration of this frame is shown in Figure 3.2 (a) and the viewing angles are depicted in Figure 3.2 (b).
3.4.4 In-Cross-Range Orbit Frame

An intermediate orbit frame between any geocentric frame and a satellite body-fixed frame is the In-Cross-Range (ICR) frame. It is a non-inertial frame with its origin at the center of mass (COM) of the satellite \( \mathbf{O}_b \) and composed of the dextral and orthonormal basis vectors \( \mathcal{F}_o \triangleq \begin{bmatrix} \hat{\mathbf{O}}_1 & \hat{\mathbf{O}}_2 & \hat{\mathbf{O}}_3 \end{bmatrix}^T \). The frame’s orientation with respect to \( \mathcal{F}_i \) is dependent on the satellite orbit. The \( \hat{\mathbf{O}}_3 \) vector points along the radial direction \( \mathbf{r} \), the \( \hat{\mathbf{O}}_2 \) vector points towards the direction of the cross product of the satellite inertial position \( \mathbf{r} \) and the satellite inertial velocity \( \mathbf{v} \). The \( \hat{\mathbf{O}}_1 \) vector points towards the direction of motion and completes the triad. The directions \( \hat{\mathbf{O}}_1 \) and \( \hat{\mathbf{O}}_2 \) are aptly named the in-track (along-track) and cross-track (off-track) directions.

Traditionally, the directions \( \hat{\mathbf{O}}_1 \), \( -\hat{\mathbf{O}}_2 \) and \( -\hat{\mathbf{O}}_3 \) are known as Roll, Pitch and Yaw, respectively [28]. Two important angles used to locate a target with respect to a satellite ground track are defined in this frame, with the components of \( \kappa \) expressed in \( \mathcal{F}_o \):

\[
o_o \triangleq \arctan \left( \frac{\kappa_2}{-\kappa_3} \right); \quad i_o \triangleq \arctan \left( \frac{\kappa_1}{-\kappa_3} \right)
\]

(3.21)

The first is the off-track viewing angle \( o_o \), which is the angle \( \kappa \) makes with the \( \hat{\mathbf{O}}_1 - \hat{\mathbf{O}}_3 \) plane and is measured positive towards \( \hat{\mathbf{O}}_2 \). The in-track viewing angle \( i_o \) is the angle between \( \kappa \) and the \( \hat{\mathbf{O}}_2 - \hat{\mathbf{O}}_3 \) plane, and is measured positive towards \( \hat{\mathbf{O}}_1 \). The angles \( o_o \) and \( i_o \) are a measure of the angular distance from the satellite to the target in the off-track and in-track direction, respectively. In this work, ground-track targets are targets which have both \( o_o \) and \( i_o \) equal to nil at some epoch in the Imaging Campaign. That is, ground-track targets are targets where the satellite passes directly overhead at some instance of the observation period. Similarly, off-track targets designated by a particular off-track viewing angle \( o_o \) are targets which require the satellite to rotate through \( o_o \) in the off-track direction when the in-track angle \( i_o \) is nil. An illustration of this frame relative to the others in this section is shown in Figure 3.2(b) and the viewing angles are depicted in Figure 3.3.

3.4.5 Body-Fixed Frames

There are four satellite-specific body-fixed coordinate frames defined in this work. All four of these frames have their origins \( O_b \) centered at \( \mathbf{O}_b \). The most general Satellite Body-Fixed frame \( \mathcal{F}_b \) is composed of the dextral and orthonormal basis vectors \( \mathcal{F}_b \triangleq \begin{bmatrix} \hat{\mathbf{b}}_1 & \hat{\mathbf{b}}_2 & \hat{\mathbf{b}}_3 \end{bmatrix}^T \), which by convention are chosen to be individually orthogonal to the idealized satellite geometric faces. That is, \( \hat{\mathbf{b}}_1 \), \( \hat{\mathbf{b}}_2 \) and \( \hat{\mathbf{b}}_3 \) correspond to the outward normals of the +X, +Y and +Z satellite geometric faces. \( \mathcal{F}_b \) is not restricted in any orientation and its attitude with respect to \( \mathcal{F}_i \) evolves according to Euler’s equations (see Section 3.6). The Satellite Body-Fixed Alignment frame \( \mathcal{F}_a \) is identical to \( \mathcal{F}_b \), but oriented in inertial space such that the vector along the outward direction of a fixed payload’s boresight \( \hat{\mathbf{p}} \) is aligned with \( \kappa \). However, this alone leaves the spacecraft attitude and rotation about \( \hat{\mathbf{p}} \) unconstrained. The Satellite Body-Fixed Constrained
frame $\mathcal{F}_c$ is identical to $\mathcal{F}_a$, except that the rotation about $\hat{p}$ is constrained by choosing a body-fixed reference direction $\hat{s}$ and aligning it as closely as possible with a desired reference direction $\hat{d}$. For imaging-specific applications, $\hat{s}$ is typically one of the axes of the imager plane and is aligned with projections of either one of the $\mathcal{F}_o$ axes or a local ordinal heading in the ENZ frame. These constraints are discussed in Section 4.2.1. The definitions of $\mathcal{F}_a$ and $\mathcal{F}_c$ are particularly useful in deriving the satellite target tracking kinematics (see Section 4.2). Since $\hat{p}$ and $\hat{s}$ are body-fixed vectors, their components are equivalent in $\mathcal{F}_b$, $\mathcal{F}_a$ and $\mathcal{F}_c$ at all times. Moreover, $\hat{p}$, $\hat{s}$ and $\hat{d}$ are all defined as unit vectors. An illustration of the $\mathcal{F}_b$ frame relative to the others in this section is shown in Figure 3.2 (a).

Since sensory measurements are unique to the sensor taking the measurements, a generic Sensor Body-Fixed frame $\mathcal{F}_s$ is defined. It is composed of the dextral and orthonormal basis vectors $\mathcal{F}_s \equiv \begin{bmatrix} \hat{s}_1 & \hat{s}_2 & \hat{s}_3 \end{bmatrix}^T$, which are chosen to be individually orthogonal to the geometric axes of the specific sensor. The sensors of interest in this work are presented in Section 5.1.1.

### 3.4.6 Camera Projection Frame

For terrestrial imaging applications, a continuously changing Camera Projection frame is used to determine the desired orientation of the target in the image. The coordinate system is composed of the dextral and orthonormal basis vectors $\mathcal{F}_\rho \equiv \begin{bmatrix} \hat{\rho}_1 & \hat{\rho}_2 & \hat{\rho}_3 \end{bmatrix}^T$. The $\mathcal{F}_\rho$ origin is coincident with the $\mathcal{F}_\ell$ frame of the target. The $\hat{\rho}_1$ vector is aligned with the desired reference direction $\hat{d}$. The $\hat{\rho}_2$ vector is formed by the cross product $-\kappa \times \hat{\rho}_1$ and $\hat{\rho}_3$ completes the triad. The orientation of the frame is dependent on the rotation of the Earth, the satellite orbit and the desired reference direction $\hat{d}$. The definition of the frame ensures that the satellite-to-target vector $\kappa$ always lies in the $\hat{\rho}_1 - \hat{\rho}_3$ plane. Consequently, when imaging, the $\hat{\rho}_1 - \hat{\rho}_2$ plane corresponds to the surface onto which the target is projected on by the satellite imager. Likewise, two angles are defined to quantify the orientation of this projection when the imager is pointed at the target and the components of $\hat{s}$ are expressed in $\mathcal{F}_\rho$:

$$
\alpha_\rho \triangleq \arctan \left( \frac{s_1}{s_3} \right) ; \quad \delta_\rho \triangleq \arctan \left( \frac{s_2}{\sqrt{s_1^2 + s_3^2}} \right)
$$

The first is the camera projection right ascension angle $\alpha_\rho$ which is the angle $\hat{s}$ makes with $\hat{\rho}_3$ in the $\hat{\rho}_1 - \hat{\rho}_3$ plane and is measured positive towards $\hat{\rho}_1$. The camera projection declination angle $\delta_\rho$ is the angle between $\hat{s}$ and the $\hat{\rho}_1 - \hat{\rho}_3$ plane and is measured positive from that plane towards $\hat{\rho}_2$. When performing observations, typically in the image, it is desired to align a body-fixed reference vector $\hat{s}$ with a reference direction $\hat{d}$ in order to capture images which are consistently oriented in the same manner. That is, when all the images from a Campaign are overlaid, a terrestrial or inertial direction is always pointing in the same direction to provide a consistent bearing. This imaging alignment objective is desired for NEMO-AM. In general, when the satellite is pointing at the target, $\hat{s}$ will not be exactly aligned with $\hat{d}$ in inertial space.
due to the satellite’s orbital inclination and right ascension. However, achieving consistently oriented images does not require this alignment in inertial space. For consistently oriented images, it is a necessary and sufficient condition that \( \mathbf{s} \) be aligned with \( \mathbf{d} \) in the \( \hat{\rho}_1 - \hat{\rho}_2 \) imaging plane. This is equivalent to \( \mathbf{s} \) having a camera projection declination angle \( \delta_\rho \) equal to nil. The camera projection right ascension angle \( \alpha_\rho \) will typically be non-zero due to the orientation of the satellite’s orbit. Inertially, the alignment condition corresponds to pointing the imager at the target and rotating about \( \hat{p} \) to achieve an attitude which minimizes the angle between \( \mathbf{s} \) and \( \mathbf{d} \) and projects \( \mathbf{s} \) onto \( \mathbf{d} \) in the imaging plane. Any subsequent rotation about \( \hat{p} \) from this attitude will result in a non-zero \( \delta_\rho \) (along with an already non-zero \( \alpha_\rho \)) and a larger angular distance between \( \mathbf{s} \) and \( \mathbf{d} \). Without loss of generality, this scenario can be observed in Figure 3.4 where \( \hat{p} = -\hat{b}_2 \), \( \mathbf{s} \) is a vector with a non-zero \( \hat{b}_1 \) component, \( \mathbf{d} \) is the target’s local East heading and imager alignment of \( \mathbf{s} \) and \( \mathbf{d} \) can only be achieved by rotating about \( \hat{p} \) and eliminating \( \delta_\rho \).

### 3.4.7 True Equator Mean Equinox Frame

The True Equator Mean Equinox (TEME) coordinate system is the frame which the Simplified General Perturbations (SGP4) propagator is implemented in. The TEME frame is denoted by \( \mathcal{F}_r \) and is closely aligned with \( \mathcal{F}_i \), but does not account for the nutational and precessional motion of the Earth. This omission can lead to knowledge errors of up to 0.23° in the orientation of the Earth in ECI. Thus, for precise calculations, it is important to convert any celestial or terrestrial states from TEME to \( \mathcal{F}_i \) or \( \mathcal{F}_e \) components before any operations are completed. Coordinate conversion from this system is discussed in Section 3.4.8.
(a) Terrestrial parameters and coordinate frames $\mathcal{F}_i$, $\mathcal{F}_e$, $\mathcal{F}_\ell$ and $\mathcal{F}_b$.

(b) Topocentric ascension $\alpha_\ell$ and declination $\delta_\ell$ viewing angles.

Figure 3.2: Depiction of $\mathcal{F}_e$, $\mathcal{F}_\ell$ and $\mathcal{F}_\alpha$, and local target viewing angles.
Figure 3.3: Off-track viewing angle $o_o$ and in-track viewing angle $i_o$ derived from $\kappa$.

Figure 3.4: Camera Projection Coordinate frame $\mathcal{F}_p$ for $\delta_\ell = 0^\circ$, $\hat{p} = -\hat{b}_2$, $d = \hat{\ell}_2$. 
3.4.8 Coordinate Transformations

In this section, the transformations between coordinate frames used in this work are discussed.

3.4.8.1 Earth-Centered-Earth-Fixed to Earth-Centered-Inertial

A terrestrial-to-celestial transformation $C_{ie}$ is essential to map coordinates from $ECEF$ to an Earth-fixed inertial frame in order to perform target tracking. The two most recognized approaches to evaluating the transformation are the $IAU-1976/FK5$ and $IAU-2000$ ($GCRF$) reduction formulas (see Section 3.4.1). The $FK5$ transformation is chosen over the $IAU-2000$ counterpart in this work since its accuracy is sufficient for the purpose of ground tracking with sub-degree pointing accuracy and it requires 344 less floating point operations ($FLOPS$). The reduction formulas which transform position vectors expressed in the $ECEF$ to the $ECI$ frame are a time-dependent sequence of four rotations $C_\phi(t)$, $C_\theta(t)$, $C_\Omega(t)$ and $C_\Psi(t)$:

$$r_i(t) = C_{ie}(t) r_e = C_\Psi(t) C_\Omega(t) C_\theta(t) C_\phi(t) r_e$$  \hspace{1cm} (3.23)

where $t$ is the $UT1$ time of the desired $C_{ie}$ rotation. For conciseness, the time variable $t$ in the reduction formulas will be omitted throughout the rest of this work. The first transformation $C_\phi$ accounts for the polar motion of date and is colloquially known as “polar wobble”. This rotation is described by two displacements $x_p$ and $y_p$ which specify the location of the reference terrestrial pole, as agreed by international committees, with respect to the Earth’s true axis of rotation. The terrestrial pole is orthogonal to the true equator of date. The maximum amplitude in the pole displacement is approximately 10 m. Rotating through $C_\phi$ from the $ECEF$ frame results in a Pseudo Earth-Fixed ($PEF$) frame.

The second rotation $C_\theta$ is equivalent to $C_3(-\theta_a)$ and accounts for most of the time dependency of the terrestrial-to-celestial transformation. It converts the rotating frame into a non-rotating True of Date ($TOD$) frame by rotating the $PEF$ frame through $\theta_a$, the Greenwich Apparent Sidereal Time.

The third and fourth rotations, $C_\Omega$ and $C_\Psi$, account for the nutation and precession of Earth’s axes due to gravitational perturbations of the Sun and Moon, respectively. The luni-solar precession has a period of approximately 26,000 years, resulting in a precessional rate of $49.846''/yr$. The luni-solar nutation is dominated by the gravitational influence of the Moon and has a principal period of 18.6 years, which is equal to the precession of the Moon’s orbital plane. The $C_\Omega$ transformation accounts for the largest number of $FLOPS$ in the $FK5$ transformation since its evaluation is dependent on a trigonometric series of up to power $n = 106$. Whereas the $IAU-2000$ reduction requires all 106 terms, using only the first 20 terms for this work is sufficient.

For autonomous target tracking missions, it is imperative to fly an onboard terrestrial-to-celestial transformation, since the tracking trajectory to be followed by the satellites rely on this transformation to construct the inertially-referenced attitude, angular velocity and angular
acceleration that are to be regulated (see Section 4.2). In terms of computational cost, the FK5 reduction requires a total of $17n + 221$ FLOPS. The rotations are based on UT1, TAI and TT time systems and thus require the knowledge of $\Delta UT1$ and $\Delta AT$ (see Section 3.3). These quantities, along with the polar motion angles $x_p$ and $y_p$, form the majority of the EOP, and are predicted and published for a given Modified Julian Date by the IERS. To compute the FK5 transformation, the standard IAU-1980 EOP data file realizations should be used, in addition to the $\Delta AT$ offset from IERS Bulletin A [25]. Given that the polar motion angles have an amplitude of $\sim 10\text{ m}$, $x_p$ and $y_p$ can be omitted over the period of a satellite’s lifetime for the mission applications in this work. However, $\Delta UT1$ and $\Delta AT$ should be updated for every target tracking campaign. The full FK5 formulation is accurate to the IAU-2000 realization to within $0.01''$ [23]. When $C_\Omega$ is truncated to 20 terms, the rotation error $\delta \theta$ in the FK5 $C_{ie}$ reduction relative to the IAU-2000 reduction is observed to be approximately $0.62''$ over the period of a typical low altitude sun-synchronous orbit, as shown in Figure 3.5.

![Figure 3.5: FK5 rotation error relative to IAU-2000 reduction vs. Time](image)

Transforming velocities from the ECEF to the ECI frame can be accomplished by simply differentiating Eq. (3.23) and approximating the derivatives of $C_\phi$, $C_\Omega$ and $C_\Psi$ as nil due to their relative magnitudes and long time constants. The absolute velocity expressed in ECI coordinates, of a non-fixed location $\vec{r}$ expressed in ECEF components is thus:

$$v_i = C_\psi C_\Omega C_\phi \vec{r}_e + C_\psi C_\Omega C_\theta C_\phi \dot{\vec{r}}_e$$ (3.24)

$$C_\theta = -\omega_{ie}^{\times} C_\theta = \omega_{ie} \begin{bmatrix} -\sin \theta_a & -\cos \theta_a & 0 \\ \cos \theta_a & -\sin \theta_a & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\vec{r}_e$ is the location’s position expressed in $F_e$ components and the angular velocity of the TOD frame about the PEF frame is approximated as $\omega_{ie} = \begin{bmatrix} 0 & 0 & -\omega_{ie} \end{bmatrix}^T$ in TOD components.

The FK5 transformation is discussed further in detail in Montenbruck [23] and Vallado [24].
3.4.8.2 True Equator Mean Equinox to Geocentric

Although the TEME frame is closely aligned to the TOD frame, a standard definition of the frame is not well defined in literature. Conceptually, the reference frame resides in between the intermediate PEF and TOD frames which are separated by $\theta_a$. The Greenwich Apparent Sidereal Time can be decomposed into the Greenwich Mean Sidereal Time $\theta_m$ and what’s known as the Equation of the Equinoxes $\theta_e$. Thus, two routes can be taken to transform TEME coordinates. To convert SGP4 coordinates to those in a standard frame, an initial 3-axis principal rotation to the PEF or TOD frame is required using $\theta_m$ or $\theta_e$, respectively. The corresponding coordinates can then be transformed to ECEF or ECI by rotating through the standard reduction formulas in Section 3.4.8.1:

$$r_e = C^T_\phi C_3(\theta_m) r_\tau$$  \hspace{2cm} (3.25)

$$r_i = C_\Psi C_\Omega C_3(-\theta_e) r_\tau$$  \hspace{2cm} (3.26)

where $r_\tau$ is the component vector of $r$ expressed in the TEME frame. If the first route to ECEF is taken, the velocity transformation requires knowledge of the angular velocity of TEME with respect to ECEF. If however the second route to ECI is considered, since the Equation of Equinoxes is much smaller than $\theta_a$, the angular velocity of TEME with respect to ECI can be assumed to be nil. Likewise, for simplicity, the (absolute) velocity components can be computed as outlined in Eq. (3.26) with $v_\tau$ replacing $r_\tau$ and without any angular velocity corrections.

3.4.8.3 Earth-Centered-Earth-Fixed to East-North-Zenith

To describe quantities relative to a terrestrial site of interest, it is important to be able to express these quantities in the ENZ frame. Since the ENZ frame is rotating with the ECEF frame, the transformation from ECEF to ENZ is time-invariant. This transformation is expressed using two consecutive principal rotations through the site’s longitude $\lambda$ and geodetic latitude $\phi_d$:

$$r_\ell = C_1(\pi/2 - \phi_d) \ C_3(\pi/2 + \lambda) \ r_e$$  \hspace{2cm} (3.27)

where $r_\ell$ are the components of $r$ expressed in the $F_\ell$.

3.4.8.4 Earth-Centered-Inertial to Spacecraft Body

The rotation from $F_i$ to $F_b$ is calculated regularly by the Attitude and Orbit Control System (AOCS) of NEMO-AM. Onboard, this rotation is estimated by the Extended Kalman Filter (see Section 5.1.2). Analytically it can be calculated using the formulation:

$$C_{bi} \triangleq F_b \cdot F_i^T = \begin{bmatrix} \hat{b}_1 \cdot \hat{i}_1 & \hat{b}_1 \cdot \hat{i}_2 & \hat{b}_1 \cdot \hat{i}_3 \\
\hat{b}_2 \cdot \hat{i}_1 & \hat{b}_2 \cdot \hat{i}_2 & \hat{b}_2 \cdot \hat{i}_3 \\
\hat{b}_3 \cdot \hat{i}_1 & \hat{b}_3 \cdot \hat{i}_2 & \hat{b}_3 \cdot \hat{i}_3 \end{bmatrix}$$  \hspace{2cm} (3.28)
3.5 Orbital Dynamics

In this section, the orbital (translational) motion equations of a satellite about Earth will be presented. These motion equations are used to construct the satellite orbit for attitude simulation purposes (see Section 5.4). Although there exist perturbative forces (see Section 3.7) other than the Earth’s gravitational pull, the effect of these forces on the orbit are on a timescale much larger than the typical target tracking observation period and thus are not evaluated in the motion equations. The application of terrestrial ground tracking with sub-degree pointing accuracy requires orbital knowledge on the order of 1 km from LEO. Simulation-based mission validation using osculating (Keplerian) elements and SGP4 orbits is insufficient for this accuracy level. Thus, a more realistic Earth model is needed to determine satellite orbits. It is important to distinguish the notion of the center of mass (COM) from the center of gravity (COG). Whereas the center of mass is the mean location of all the mass in a body, the center of gravity is the mean location of the gravitational force acting on that body. Although, the difference in their locations is minute on a satellite, this difference results in the Gravity Gradient Torque discussed in Section 3.7.1. The motion of the satellite center of mass \( \Phi_b \) is the same as if its entire mass, composed of \( n \) point masses, was concentrated at \( \Phi_b \) and driven by the sum of all the external forces:

\[
\sum_{k=1}^{n} \mathbf{f}_k = m_b \ddot{\mathbf{r}}_c
\]

(3.29)

where \( \mathbf{f}_k \) is the external force on the \( k^{th} \) point mass, \( m_b \) is the mass of the satellite and \( \mathbf{r}_c \) is the absolute position of \( \Phi_b \). The left hand side of Eq. (3.29) is equal to the total external force \( \mathbf{f} \) calculated for the system of point masses and not that on a single particle of mass \( m_b \) located at \( \Phi_b \). Thus in general, the gravitational force will not be acting at \( \Phi_b \) but at the COG of the satellite instead. However, since the difference between the two locations is on the order of \( 10^{-9} \) m \[^{[29]}\], the total external force acting on the satellite can be accurately approximated as that acting on a single particle of mass \( m_b \) located at \( \Phi_b \). Physically, this implies that the total external force \( \mathbf{f} \) does not depend on the mass distribution of the satellite. Thus, the translational motion equations of Eq. (3.29) are only dependent on \( m_b \) and, \( \mathbf{r}_c \) and its derivatives. Since only the disturbance torques \( \mathbf{g}_d \) in the attitude motion equations (see Section 3.6.1) are dependent on \( \mathbf{r}_c \) and its derivatives, the translational motion can be solved independently of the attitude motion. This approximation is enforced in this work. For conciseness, \( \mathbf{r}_c \) is represented by \( \mathbf{r} \) and its absolute temporal derivative is denoted by \( \mathbf{v} \) for the remainder of this thesis.
3.5.1 Orbital Motion Equations

To construct a realistic Earth model, the gravity potential (geopotential) \( U \) is defined as:

\[
U = \int \frac{G}{|\vec{r} - \vec{s}_\oplus|} \, dm_\oplus
\]  

(3.30)

where \( G \) is the gravitational constant, \( \vec{r} \) is the position of the satellite center of mass \( \mathcal{S}_b \) with respect to the Earth’s center of mass \( \mathcal{S}_\oplus \), \( dm_\oplus \) is an infinitesimal mass element of the Earth \( \mathcal{E} \) and \( \vec{s}_\oplus \) is the position of \( dm_\oplus \) from \( \mathcal{S}_b \). The integral in Eq. (3.30) can be rewritten in terms of spherical harmonics by using an infinite series of Legendre polynomials [23]:

\[
U = \frac{G m_\oplus}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{R^r_n}{r^n} P_{nm}(\sin \phi_c) (C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda))
\]  

(3.31)

where \( m_\oplus \) is the mass of the Earth, \( R^r_n \) is the Earth’s mean equatorial radius, \( r \) is the magnitude of \( \vec{r} \), \( \phi_c \) is the geocentric latitude of \( \vec{r} \), \( \lambda \) is the geocentric longitude of \( \vec{r} \), \( P_{nm} \) is the associated Legendre polynomial of degree \( n \) and order \( m \) and, \( C_{nm} \) and \( S_{nm} \) are geopotential coefficients which describe the Earth’s internal mass distribution. In this work, the coefficients are obtained from the 2008 Earth Gravitational Model (EGM2008). Since the gravitational force is conservative, the gravitational acceleration is equal to the gradient of the geopotential expansion. Thus the orbital motion equations are:

\[
\ddot{\vec{r}} = \nabla U
\]  

(3.32)

By exploiting inherent recurrence relations of Legendre polynomials, the acceleration \( \ddot{\vec{r}} \) of the satellite’s center of mass \( \mathcal{S}_b \) may be directly calculated in terms of its Cartesian components [23]. The position \( \vec{r} \) and velocity \( \vec{v} \) of the satellite can be evaluated by constructing a state vector \( \vec{y} = [\vec{r}^T \, \vec{v}^T]^T \) and integrating it numerically. A classical 4th Runge-Kutta integration scheme (\( RK4 \)) combined with Richardson extrapolation is employed for this task due to both their robustness and ease of implementation in obtaining numerical solutions to ordinary differential equations. Together, the combination is a 5th order extrapolation scheme. The integration is only valid if all quantities are observed and expressed in the same reference frame. Due to the nature of the integration in Eq. (3.30), the evaluated accelerations yield absolute values expressed in \( F_e \). That is, they produce the quantity \( C_{ei} \ddot{\vec{r}}_i \). The absolute \( \vec{r} \) and \( \vec{v} \) expressed in \( F_i \) can be determined by rotating the corresponding absolute accelerations through by \( C_{ie} \) in the integration scheme. This may not be immediately desired, since the \( C_{ie} \) transformation is computationally demanding. The alternative is to first perform the integration in \( F_e \), thereby computing \( \vec{r} \) and \( \vec{v} \) as observed and expressed in ECEF. Likewise, the accelerations of \( \mathcal{S}_b \) used in the integration scheme must be of the same form:

\[
\ddot{\vec{r}}_e = C_{ei} \ddot{\vec{r}}_i - 2\omega^r_{ei} \vec{r} - \omega^v_{ei} \vec{v} \]  

(3.33)
where it is assumed that the inertial acceleration expressed in $F_e$ can be obtained without having to explicitly calculate $C_{ie}$. Afterwards, if the components of the absolute velocity $\mathbf{v}$ are to be expressed in $F_i$, then the velocity components must be augmented by $\omega \times r$ before $\mathbf{v}$ is rotated through by $C_{ie}$. The components of the absolute position $\mathbf{r}$ do not require any augmentation before applying the $ECEF$ to $ECI$ transformation.

A discrete canonical timestep of twice the control cycle timestep $\Delta t_c$ is used in the $RK4$ Richardson integration scheme for expediency when simulating the orbit in the $AOCS$ analysis. When orbit data is desired at a finer interval, the integrator is supplemented with a $5^{th}$ order Hermite interpolant, thus forming a consistent $5^{th}$ order integrator and interpolant combination. This methodology can also be used for real-time attitude control which requires orbital state information when the estimated $GPS$ state is available at discrete, but not necessarily equidistant timesteps. Altogether this framework is defined as the High Precision Orbit Propagator ($HPOP$). Orbit integration and interpolation are discussed further in detail by Montenbruck and Gill [23] [30].
3.6 Spacecraft Attitude Dynamics

In this section, the attitude motion (rotational) equations are first presented for a generalized spacecraft structure \( R \) with a single reaction wheel \( W \) fixed inside. The combined body of the spacecraft and reaction wheel \( (R + W) \) is referred to as the satellite \( B \). For attitude applications, the spacecraft can be thought of as all the components of a satellite which cannot actively induce torques on \( R \) through mechanical means. The center of mass of \( R, W \) and \( B \) are \( \mathbf{c}_o, \mathbf{c}_w \) and \( \mathbf{c}_b \), respectively. An illustration of this arrangement is shown in Figure 3.6. Structural deflections due to attitude maneuvers are negligible on NEMO-AM and likewise rigid body assumptions are enforced. All temporal derivatives of vectors with an overcircle \( \circ \) are observed in \( F_b \) and as a result of the rigid body assumptions, \( \circ \) derivatives of body-fixed vectors are nil. While the mass centers of \( R, W \) and \( B \) are important, they are generally not convenient locations to construct a dynamical analysis around since they constantly change through the design lifecycle of a satellite. Thus \( O_b \) is kept separate from \( \mathbf{c}_o, \mathbf{c}_w \) and \( \mathbf{c}_b \) in order to derive the most general motion equations and understand how they are realized before practical assumptions are enforced. A more in-depth treatment of the rotational motion equations is found in Hughes [21]. Once these general motion equations are formulated, they are extended to a more practical case of a spacecraft with three orthogonally aligned wheels.

3.6.1 Attitude Motion Equations for Generalized Satellite

The spacecraft \( R \) has a mass of \( m_o \) and is rotating with an absolute angular velocity of \( \omega_{bi} \) about an inertial frame \( F_i \) whose origin is \( O_i \). A body-fixed frame \( F_b \) is attached to the spacecraft and its origin is located at an arbitrary point \( O_b \). The location of \( O_b \) with respect to \( F_i \) is given by \( r_o \). The absolute orbital velocity of \( F_b \) is \( v_o \). The location of an infinitesimal spacecraft mass element with respect to \( F_i \) is denoted by \( \mathbf{r}_o = \mathbf{r}_o + \mathbf{s} \). The first and second moments of inertia of \( R \) about about \( O_b \) are respectively:

\[
\mathbf{c}_o = \int_R \mathbf{s} \, dm \tag{3.34}
\]

\[
\mathbf{J}_o = \int_R (s^2 \mathbf{1} - s_s s_s) \, dm
\]

where \( \mathbf{J} \), like all other second moment of inertia parameters, is a second rank tensor referred to as a dyadic [21]. A single reaction wheel \( W \) is spinning with an absolute angular velocity \( \omega_{wi} \) about a body-fixed unit vector \( \mathbf{a} \), which is denoted as its rotor spin axis. The wheel is assumed to be symmetric about \( \mathbf{a} \) and has a mass of \( m_w \). The location of its center of mass \( \mathbf{c}_w \) with respect to \( F_b \) is given by \( \mathbf{b} \). The location of an infinitesimal wheel mass element with respect to \( F_b \) is denoted by \( \mathbf{s}_w = \mathbf{b} + \mathbf{b}_w \). The first moment of inertia of \( W \) about \( \mathbf{c}_w \) is nil:

\[
\int_{W} \mathbf{b} \, dm = \mathbf{0} \tag{3.35}
\]
Using the above result, the first and second moments of inertia of $\mathcal{W}$ about $O_b$ are:

$$c_w = \int_{\mathcal{W}} (\mathbf{b} + \mathbf{\rho}) \, dm = m_w \mathbf{b}$$

$$J_w = \int_{\mathcal{W}} (s_w^2 \mathbf{1} - s_w \mathbf{s}_w) \, dm = m_w (b^2 \mathbf{1} - b \mathbf{b}) + I_w$$

where $I_w$ is the second moment of inertia dyadic of $\mathcal{W}$ about $\mathbf{\rho}_w$. Since the wheel is symmetric, its moment of inertia can be decomposed into an axial component $I_a$ about $\mathbf{a}$ and a transverse component $I_t$ about any transverse axis:

$$I_w = \int_{\mathcal{W}} \left( s^2 \mathbf{1} - \mathbf{s} \mathbf{s} \right) \, dm = I_t \mathbf{1} + (I_a - I_t) \mathbf{a} \mathbf{a}$$

The absolute angular velocity of $\mathcal{W}$ is the sum of the relative angular velocity of $\mathcal{W}$ with respect to $\mathcal{F}_b$ and the angular velocity of $\mathcal{F}_b$ with respect to $\mathcal{F}_i$. Also, since the wheel spins about $\mathbf{a}$, the relative angular velocity can be simplified in terms of the rotor spin axis:

$$\mathbf{\omega}_{wi} = \mathbf{\omega}_{wb} + \mathbf{\omega}_{bi}$$

Combining the above, the zeroth, first and second moments of inertia of $\mathcal{B}$ about $O_b$ are:

$$m = m_\odot + m_w; \quad c = c_\odot + c_w; \quad J = J_\odot + J_w$$

Figure 3.6: Rigid satellite $\mathcal{B}$ composed of spacecraft structure $\mathcal{R}$ and reaction wheel $\mathcal{W}$. 
The absolute linear momenta of the spacecraft $\mathbf{p}_o$ and the wheel $\mathbf{p}_w$ can be found by integrating the absolute velocities of the individual mass elements:

$$\mathbf{p}_o \triangleq \int \mathbf{r}_o \cdot \dot{m} = m_o \mathbf{v}_o + \omega_{bi} \times \mathbf{c}_o \quad (3.40)$$

$$\mathbf{p}_w \triangleq \int \mathbf{r}_w \cdot \dot{m} = m_w \mathbf{v}_w + \omega_{bi} \times \mathbf{b}_w \quad (3.41)$$

The absolute linear momentum of the satellite $\mathbf{p}$ is the sum of the linear momenta of the spacecraft $\mathcal{R}$ and the wheel $\mathcal{W}$:

$$\mathbf{p} \triangleq \mathbf{p}_o + \mathbf{p}_w = m \mathbf{v}_o + \omega_{bi} \times \mathbf{c} \quad (3.42)$$

The absolute angular momentum about $O_b$ of the spacecraft $\mathbf{h}_o$ and of the wheel $\mathbf{h}_w$ can be formulated by integrating the moment of linear momentum of the individual mass elements:

$$\mathbf{h}_o \triangleq \int s_o \times \mathbf{r}_o \cdot \dot{m} = \mathbf{c}_o \times \mathbf{v}_o + \mathbf{J}_o \cdot \omega_{bi} \quad (3.43)$$

$$\mathbf{h}_w \triangleq \int s_w \times \mathbf{r}_w \cdot \dot{m} = \mathbf{b} \times \mathbf{p}_w + \mathbf{I}_w \cdot \omega_{wi} \quad (3.44)$$

Since the wheel is spinning about the rotor spin axis, an important quantity is the angular momentum of $\mathcal{W}$ about $\mathbf{a}$ projected along $\mathbf{a}$:

$$h_a \triangleq \mathbf{a} \cdot (I_w \cdot \omega_{wi}) = I_a \mathbf{a} \cdot \omega_{bi} + I_a \omega_a \quad (3.45)$$

The absolute angular momentum of the satellite $\mathbf{h}$ about $O_b$ is the sum of the absolute angular momenta of $\mathcal{R}$ and $\mathcal{W}$ about $O_b$:

$$\mathbf{h} \triangleq \mathbf{h}_o + \mathbf{h}_w = \mathbf{c} \times \mathbf{v}_o + \mathbf{J} \cdot \omega_{bi} + I_a \omega_a \mathbf{a} \quad (3.46)$$

It should be noted that the cancellation of $I_t$ in $\mathbf{h}$ arises from algebraic operation and not any form of simplification. The temporal derivative of the linear momentum of the satellite $\mathbf{p}$ is equal to the total external force $\mathbf{f}$ exerted on the satellite:

$$\dot{\mathbf{p}} = \dot{\mathbf{p}}_o + \dot{\mathbf{p}}_w = \mathbf{f} \quad (3.47)$$

The temporal derivative of the angular momentum of the $\mathcal{R}$ and $\mathcal{W}$ about $O_b$ can be found by differentiating Eqs. (3.43) and (3.44):

$$\dot{h}_o = \mathbf{v}_o \times (\mathbf{c}_o \times \omega_{bi}) + \mathbf{g}_o - \mathbf{g}_{ow} - \mathbf{b} \times f_{ow} \quad (3.48)$$

$$\dot{h}_w = \mathbf{b} \times \mathbf{p}_w + \mathbf{b} \times \dot{\mathbf{p}}_w + \mathbf{g}_{ow} \quad (3.49)$$

where $\mathbf{g}$ is the total external torque induced on the satellite and $\mathbf{g}_{ow}$ is the torque induced on
The only external torques on the satellite are assumed to be the disturbance torques detailed in Section 3.7. The component of $\mathbf{g}_{\omega w}$ along the rotor spin axis $\mathbf{a}$ is denoted by $g_a$ and is related to $h_a$ through:

$$\dot{h}_a = I_a \mathbf{a} \cdot \dot{\omega}_{bi} + I_a \dot{\omega}_a = g_a \tag{3.50}$$

The angular motion equations of the satellite can be found by taking the temporal derivative of Eq. (3.46). This quantity is precisely the sum of $\dot{h}_\omega$ and $\dot{h}_w$ and after some manipulation, the angular motion equations can be written succinctly as:

$$\dot{\mathbf{h}} + \mathbf{v}_o \times \mathbf{p} = \mathbf{g} \tag{3.51}$$

The attitude motion equations of $B$ can described completely by Eqs. (3.50) and (3.51). The attitude and angular velocities of the satellite can be solved analytically, only if all the motion equations are expressed in the same reference frame. In anticipation of employing the motion equations onboard the satellite, the momenta equations are expressed in $F_b$ and likewise become:

$$\mathbf{p} = m \mathbf{v}_o - \mathbf{c} \times \omega_{bi}$$
$$\mathbf{h} = \mathbf{c} \times \mathbf{v}_o + J \omega_{bi} + I_a \omega_a \mathbf{a}$$
$$h_a = I_o \mathbf{a}^T \omega_{bi} + I_a \omega_a$$

(3.52)

Similarly, the attitude motion equations expressed in $F_b$ are:

$$\dot{\mathbf{p}} = -\omega_{bi}^T \mathbf{p} + \mathbf{f}$$
$$\dot{\mathbf{h}} = -\omega_{bi}^T \mathbf{h} - \mathbf{v}_o \times \mathbf{p} + \mathbf{g}_d$$
$$\dot{h}_a = g_a$$

(3.53)

In the above derivation, no simplifications outside of the rigid body assumption were made. In this form, since $\mathbf{h}$ is dependent on $\mathbf{v}_o$, the linear and angular momentum equations must be solved simultaneously. One way to solve the angular velocities of the satellite is to integrate Eq. (3.53) for $\mathbf{p}$, $\mathbf{h}$ and $h_a$ and then solve for $\mathbf{v}_o$, $\omega_{bi}$ and $\omega_a$ simultaneously through the system of equations in Eq. (3.52). Once the spacecraft angular velocity $\omega_{bi}$ is calculated, it can be substituted into Eq. (3.5) or Eq. (3.11) and the satellite attitude $C_{bi}$ can be found by integrating for the desired attitude parameters.

### 3.6.2 Attitude Motion Equations for Satellite with Three Orthogonal Wheels

A more practical case for consideration is one which the satellite has three structurally identical and symmetric wheels affixed orthogonally inside the spacecraft structure. The three wheels are located at $b_1$, $b_2$ and $b_3$ from $O_b$, and rotating about the body-fixed directions $a_1$, $a_2$ and $a_3$ with angular velocities of $\omega_1$, $\omega_2$ and $\omega_3$, respectively. The first and second moments of
The inertia of $\mathcal{B}$ are:

\[
\mathbf{\zeta} = \mathbf{\zeta}_o + m_w \sum_{k=1}^{3} b_k
\]

\[
\mathbf{J} = \mathbf{J}_o + 3 \mathbf{I}_w + m_w \sum_{k=1}^{3} (b_k^2 \mathbf{1} - b_k b_k)
\]

where $k$ represents the wheel index and takes the value of 1, 2, 3. The scalar motion momenta equations in $\mathcal{F}_b$ components are then:

\[
\mathbf{p} = m \mathbf{v}_o - \mathbf{c}^\times \mathbf{\omega}_{bi}
\]

\[
\mathbf{h} = \mathbf{c}^\times \mathbf{v}_o + \mathbf{J} \mathbf{\omega}_{bi} + \mathbf{I}_a \sum_{k=1}^{3} \mathbf{a}_k
\]

\[
h_k = \mathbf{I}_a \mathbf{a}_k^T \mathbf{\omega}_{bi} + \mathbf{I}_a \mathbf{\omega}_k
\]

where $h_k$ is the absolute angular momentum of the $k^{th}$ wheel about its center of mass and along $\mathbf{a}_k$. The first two scalar attitude motion equations from Eq. (3.53) are unchanged. The scalar motion equations of the individual wheels expressed in $\mathcal{F}_b$ is simply:

\[
\dot{h}_k = g_k
\]

The absolute orbital velocity $\mathbf{v}_o$ can be decoupled from the attitude motion equations if the satellite is inertially fixed ($\mathbf{v}_o = \mathbf{0}$) or if the first moment of inertia of $\mathcal{B}$ is nil. The former is impossible for a satellite in orbit. The latter is possible if one assumes the positions of $\mathbf{\Phi}_b$ and $O_b$ are coincident. Since the COG and COM of the satellite are essentially collocated at the same point (see Section 3.5), this assumption is a fairly good one, and is enforced. Physically, this implies that the satellite’s orbital position is coincident to the position of $O_b$. Any subsequent rotation of the satellite is thus a rotation about $\mathbf{\Phi}_b$. Moreover, the dependency of the attitude motion on $\mathbf{v}_o$ is removed. The scalar attitude motion equations of $\mathcal{B}$ expressed in $\mathcal{F}_b$ are then:

\[
\mathbf{J} \dot{\mathbf{\omega}}_{bi} + \mathbf{I}_a \dot{\mathbf{\omega}}_{bi} + \mathbf{c}_k^\times (\mathbf{J} \mathbf{\omega}_{bi} + \mathbf{I}_a \mathbf{\omega}_w) = \mathbf{g}_d
\]

\[
\mathbf{I}_a \dot{\mathbf{\omega}}_{bi} + \mathbf{I}_a \dot{\mathbf{\omega}}_w = \mathbf{g}_w
\]

where $\mathbf{I}_a$ is the wheel axial-inertia matrix, $\mathbf{\omega}_w$ is a vector of the individual wheel speeds and $\mathbf{g}_w$ is a vector of the individual wheel torques about $\mathbf{\Phi}_b$. These are defined as:

\[
\mathbf{I}_a \triangleq \begin{bmatrix} I_a & 0 & 0 \\ 0 & I_a & 0 \\ 0 & 0 & I_a \end{bmatrix}; \quad \mathbf{\omega}_w \triangleq \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T; \quad \mathbf{g}_w \triangleq \begin{bmatrix} g_1 & g_2 & g_3 \end{bmatrix}^T
\]
The body axial angular momentum vector of the wheels $h_w$ is equivalent to $I_a \omega_w$. With the appropriate electronics $\omega_w$ can be manipulated, and so the control torque $g_c$ is defined as:

$$ g_c \triangleq \mathcal{F}_b^T (-I_a \dot{\omega}_w) \quad (3.60) $$

The angular velocities and attitude are solved similarly as described in Section 3.6.1. Since the dependence on the absolute orbital velocity is removed, the temporal derivative of the angular momentum can be integrated to directly calculate $\omega_{bi}$. It should be noted that although the motion equations developed in this section do not contain any terms involving $r$ or its derivatives, the total disturbance torque $g_d$ does depend on the motion of $\mathcal{O}_b$ (see Section 3.7) and so the attitude motion must be solved in conjunction with the orbital motion [31].

### 3.7 Environmental Disturbances

In this section, the most important environmental disturbances which perturb both the orbit and attitude of Low Earth Orbit (LEO) satellites are discussed. The perturbative forces are minute compared to the Earth’s gravitational force and thus are not included as part of the orbit calculation since their effect on the orbit is on a timescale much larger than a typical target tracking observation period. Typically, for LEO satellites below 400 km, aerodynamic torques are the most dominant environmental torques. Solar-pressure torques are secular for Earth orbiting satellites and are highly dependent on the spacecraft’s optical properties. The gravity gradient and magnetic torques vary at a rate of $r^{-3}$ with respect to the gravitational or magnetic center. On NEMO-AM, the magnetic torques are expected to be dominant due to the design of the onboard electronics. Since the path of $\mathcal{O}_b$ is approximated to trace the satellite’s orbital path (see Section 3.6.2), the torques induced on the satellite act to perturb its attitude about $O_b$. Consequently, the disturbance torques presented in this section are evaluated as observed and expressed in $\mathcal{F}_b$.

#### 3.7.1 Gravitational Disturbance

The heterogeneous oblate shape of the Earth and the nature of its inverse-square gravitational force ensure that the gravitational field over an orbiting satellite is not uniform. Consequently, the position of the COM and COG of the satellite do not lie in same position. Although the magnitude of their difference is small enough to overlook when considering the orbital motion equations, this cannot be done when considering the gravitational disturbance torques. The positional difference in the COM and COG induce a small, but measurable gravitational torque about $\mathcal{O}_b$. Under the assumptions that the satellite is small compared to $r$, the gravity gradient torque about $\mathcal{O}_b$ arising from a spherically homogeneous and isotropic Earth is:

$$ g_g = \frac{3\mu_\oplus}{r^5} r^5 r \quad (3.61) $$
where $\mu_\oplus$ is the gravitational parameter of the Earth and all components are expressed in $F_b$. Gravity gradient torques arising from the Moon and Sun are not considered because they are at most $2.3 \times 10^{-5}$ and $1.1 \times 10^{-5}$ times smaller than that compared to the Earth. Moreover, a spherically symmetric mass distribution of the Earth is sufficient because the effect of the Earth’s non-uniform oblateness diminishes at a rate of $r^{-5}$, whereas Eq. (3.61) diminishes at $r^{-3}$ [21].

### 3.7.2 Aerodynamic Disturbance

The Earth’s atmosphere decays exponentially with altitude. Likewise, aerodynamic disturbances represent the second largest non-magnetic disturbances acting on satellites in LEO. They are difficult to accurately model due to the limited knowledge of the physical properties and interactions of the atmosphere [23]. In this work, the exponentially decaying upper atmosphere is represented by the Committee on Space Research International Reference Atmosphere 1972 (CIRA-72) model [28]. The atmosphere is assumed to co-rotate with the Earth. The dominant atmospheric force is drag, which is classically modeled as:

$$\vec{f}_a = -\frac{1}{2} C_D \rho_a A \left( \vec{v}_r \cdot \vec{v}_r \right) \hat{\vec{v}}_r$$  \hspace{1cm} (3.62)

where $C_D$ is the dimensionless drag coefficient ($\sim 2.0$), $A$ is the cross-sectional area in the direction of the incident atmosphere, $\rho_a$ is the density of the atmosphere and $\vec{v}_r$ is the velocity of the satellite relative to the rotating atmosphere:

$$\vec{v}_r = \vec{v} - \vec{\omega}_{ei} \times \vec{r}; \hspace{1cm} \hat{\vec{v}}_r \triangleq \frac{\vec{v}_r}{||\vec{v}_r||}$$  \hspace{1cm} (3.63)

Empirical deviations from the co-rotating atmosphere assumption are on the order of 40% which result in drag force errors of less than 5% [23]. The drag force acting on the satellite orbit acts to reduce its kinetic energy and angular momentum, thereby causing the eccentricity and semi-major axis to decay. It has almost no influence on the satellite’s inclination, since $\vec{f}_a$ acts in the direction opposite to $\hat{\vec{v}}_r$. The differential drag force on a surface element $dA$ is:

$$d\vec{f}_a = -\frac{1}{2} C_D \rho_a (\vec{v}_r \cdot \vec{v}_r) \left( \vec{n} \cdot \hat{\vec{v}}_r \right) \hat{\vec{v}}_r dA$$  \hspace{1cm} (3.64)

where, $\vec{n}$ is the outward normal of $dA$. If the surface area of the satellite is decomposed into $n$ simple geometric shapes ($A_1$, $A_2$, $\cdots$, $A_n$), then the drag force $\vec{f}_k$ on the $k^{th}$ surface element is found by the integral:

$$\vec{f}_k = \int_{A_k} d\vec{f}_a$$  \hspace{1cm} (3.65)
The drag force on a planar satellite surface element of area $A$ with outward normal $\mathbf{n}$ is equivalent to $-\frac{1}{2} C_D \rho_a (\mathbf{v}_r \cdot \mathbf{v}_r) (\mathbf{n} \cdot \hat{\mathbf{v}}_r) \hat{\mathbf{v}}_r A$. The total aerodynamic torque about $\mathbf{b}$ is the sum of all the torques induced on each $A_k$:

$$g_a = \sum_{k=1}^{n} \mathbf{c}_k \times \mathbf{f}_k$$

(3.66)

where, $\mathbf{c}_k$ is the vector from $\mathbf{b}$ to the center of pressure of the $k^{th}$ surface element [28].

### 3.7.3 Solar Radiation Pressure Disturbance

Photons impinging on a satellite’s surface induce a secular solar radiation pressure. The solar pressure is dependent on the optical properties of the satellite’s surface, the attitude of the satellite and the intensity and spectral distribution of the radiation. Direct solar illumination is the dominant form of radiation for satellite’s in LEO. Unlike the aerodynamic force, the solar radiation pressure is independent of altitude and acts to change the eccentricity and longitude of the perigee [23]. The incident radiation is assumed to be either absorbed, reflected specularly, reflected diffusely or a combination of the three with no secondary reflections. The total differential radiational force on a surface with outward normal $\mathbf{n}$ is:

$$d\mathbf{f}_s = -P_\odot \int \cos \theta_s (1 - \epsilon_s) \mathbf{s}_\odot + 2 \cos \theta_s (\epsilon_s \cos \theta_s + \frac{1}{3} \epsilon_d) \mathbf{n} \, dA$$

$$\alpha_\odot + \epsilon_s + \epsilon_d = 1$$

(3.67)

where $P_\odot = 4.56 \times 10^{-6}$ N/m$^2$ is the solar radiation pressure due solely to the Sun at 1AU, $\mathbf{s}_\odot$ is the unit vector from the satellite surface $dA$ to the Sun along the line of incidence, $\theta_s$ is the angle between $\mathbf{n}$ and $\mathbf{s}_\odot$, $\alpha_\odot$ is the absorption coefficient, $\epsilon_s$ is the coefficient of specular reflection and $\epsilon_d$ is the coefficient of diffuse reflection. Similar to the aerodynamic disturbance torque in Section [3.7.2](#), if the surface area of the satellite is decomposed into $n$ simple geometric shapes ($A_1, A_2, \cdots, A_n$), then the solar force $\mathbf{f}_k$ on the $k^{th}$ surface element is found by the integral:

$$\mathbf{f}_k = \int_{A_k} d\mathbf{f}_s$$

(3.68)

The total solar radiation pressure disturbance torque $\mathbf{g}_s$ about $\mathbf{b}$ is found by evaluating the right hand side of Eq. (3.66) using Eq. (3.68). The solar force on a planar satellite surface element of area $A$ with outward normal $\mathbf{n}$ is $-P_\odot A \cos \theta_s [1 - \epsilon_s] \mathbf{s}_\odot + 2 (\epsilon_s \cos \theta_s + \frac{1}{3} \epsilon_d) \mathbf{n}$ [28].
3.7.4 Magnetic Disturbance

The interaction between the total local magnetic field $\mathbf{b}$ and the satellite residual magnetic field results in a magnetic disturbance torque. On NEMO-AM, magnetic moments $\mathbf{m}$ arising from the reaction wheels and current loops (steady state or transient) are the primary sources of this disturbance. For current loops, the magnetic moment is evaluated using:

$$\mathbf{m} = NI\mathbf{A}\mathbf{n}$$  \hspace{1cm} (3.69)

where $N$ is the number of coil loops of area $A$ with a supply current of $I$ and $\mathbf{n}$ is the unit direction orthogonal to the planar projection of $A$. The magnetic disturbance torque is:

$$\mathbf{g}_m = \mathbf{m} \times \mathbf{b}$$  \hspace{1cm} (3.70)

Since the most consistent and well defined component of $\mathbf{b}$ is the geomagnetic field, the evaluation of $\mathbf{b}$ in this work for magnetic torque simulation will omit the other contributing factors, including those arising from the ionosphere, the Earth’s crust and geomagnetic storms [32]. Nevertheless, the contributions of the onboard electronics are included where applicable (see Section 6.2).
Chapter 4

Target Tracking Considerations

In this chapter special topics essential to understanding target tracking are explained. These include definitions of Earth pointing geometry and imaging parameters, derivations of the satellite ground tracking kinematics and orbit determination for tracking.

4.1 Earth Pointing Geometry

It is important to define the geometric framework of Earth pointing such that pointing and mapping metrics can be calculated. This framework requires familiarity with the reference frames presented in Section 3.4. Unless otherwise noted, all vectors in this section have their origin at the center of mass of the satellite. The parameters in this section assume a spherical Earth geoid and are sufficient for preliminary mission geometry analysis. Nevertheless, to be more exact, in this work, the oblateness of the planet is treated as the target’s altitude about a purely spherical Earth. This approximation can account for up to $0.12^\circ$ of error in geometric computations. Oblate correction terms which account for the Earth’s non-spherical shape and irregular surface exist in literature and can be applied for even more accurate calculations [33].

During target tracking, the Earth has a specific angular radius $\rho$ as observed from the satellite. The subsatellite point $r_s$ of a satellite is the point on the geoid surface from which the vector drawn along its surface normal intersects with the current satellite position. The ground track is a collection of successive subsatellite points. The nadir angle $\gamma$ is the angular distance between nadir $-r$ and the satellite-to-target vector $r_t$. The grazing angle $\epsilon$ is the angle between $r_t$ and the local horizontal ($\hat{l}_1 - \hat{l}_2$) plane. The Earth central angle $\xi$ is the angle between the satellite $r$ and the target $x$. The target azimuth $\vartheta$ is the angle between the satellite ground track and the shortest path on the geoid from the subsatellite point to the target. An illustration of these Earth pointing geometric parameters is shown in Figure 4.1.
The mathematical relationships between the geometry parameters \( \rho, \gamma, \epsilon \) and \( \xi \) are:

\[
\sin \rho = \frac{R_\oplus}{R_\oplus + h}; \quad \sin \gamma = \cos \epsilon \sin \rho; \quad \gamma + \epsilon + \xi = \frac{\pi}{2}
\]  

where, \( R_\oplus \) is the equatorial radius of the Earth and \( h \) is the altitude above the idealized spheroid. The metrics of interest during target tracking are pointing errors \( \delta\theta \) and mapping errors \( \delta s \). Pointing errors are defined as inaccuracies pertaining to the orientation of the payload boresight with respect to the desired target. Mapping errors are calculated as the distances on the surface geoid between the target and where the payload boresight maps onto the geoid. Both of these error quantities terms can be attributed to inaccuracies in the onboard clock, the attitude estimate, the orbital and target state, and control (pointing). Errors in the attitude estimation, payload mounting and pointing can all be decomposed into target azimuth errors \( \delta \theta \) and nadir angle errors \( \delta \gamma \). Target azimuth errors are equivalent to payload rotation errors about nadir (-\( \vec{r} \)). Nadir errors are the errors in the angle between \( \vec{\kappa} \) and -\( \vec{r} \). Target tracking errors arising from \( \delta \theta \) lie along the target azimuthal direction, and are a function of \( \|\vec{\kappa}\| \) and \( \gamma \):

\[
\delta \theta = \delta \theta \sin \gamma
\]

\[
\delta s = \delta \theta \|\vec{\kappa}\| \sin \gamma
\]

Similarly, target tracking errors arising from \( \delta \gamma \) lie along the angular path between \( \vec{\kappa} \) to -\( \vec{r} \), and are a function of the satellite-to-target distance \( \|\vec{\kappa}\| \) and the grazing angle \( \epsilon \):

\[
\delta \theta = \delta \gamma
\]

\[
\delta s = \delta \gamma \frac{\|\vec{\kappa}\|}{\sin \epsilon}
\]

An estimate of the satellite orbital state \((\vec{r}, \vec{v})\) is typically obtained onboard through either
orbit propagation methods, a GPS receiver or a combination of GPS raw measurements and filtering. In this work, the target state ($\vec{x}, \nu$) is computed onboard in the ECEF frame through the WGS84 realization (see Section 3.2). The magnitude of $\nu$ is dependent on the latitude of the terrestrial position and in $\mathcal{F}_i$ takes the form:

$$\|\nu\| = \nu \odot \cos \phi_c$$

(4.6)

where $\|\nu\|$ is the inertial tangential velocity of a terrestrial location observed in $\mathcal{F}_i$, $\phi_c$ is the geocentric latitude of that location and $\nu \odot$ is the Earth’s inertial tangential velocity at the equator due to its rotation about its axis, and is approximately equal to 466 m/s. Errors arising from the state estimates can be found by subtracting the true state from the estimated state. Positional errors resulting from the orbit estimate $\hat{\vec{r}}$ and the target estimate $\hat{\vec{x}}$ are thus:

$$\delta \vec{r} = \hat{\vec{r}} - \vec{r}; \quad \delta \vec{x} = \hat{\vec{x}} - \vec{x}$$

(4.7)

The positional errors can further be expressed in the ICR basis directions (see Section 3.4.4). Target tracking errors which are a result of in-track position error $\delta r_1$, observed and expressed in $\mathcal{F}_o$, lie parallel to the ground track, and are a function of $\vartheta$, $\gamma$, the satellite-to-target distance $\|\vec{\kappa}\|$, the geocentric distance to the target $\|\vec{x}\|$ and the geocentric distance to the satellite $\|\vec{r}\|$:

$$\delta \theta = \delta r_1 \sin \left(\arccos \left(\cos \vartheta \cos \gamma \right)\right) \frac{1}{\|\vec{r}\|}$$

(4.8)

$$\delta s = \delta r_1 \cos \left(\arcsin \left(\sin \xi \sin \vartheta \right)\right) \frac{\|\vec{x}\|}{\|\vec{r}\|}$$

(4.9)

The pointing and mapping errors due to cross-track position errors $\delta r_2$, observed and expressed in $\mathcal{F}_o$, have a similar relationship, but lie orthogonal to the ground track:

$$\delta \theta = \delta r_2 \sin \left(\arccos \left(\sin \vartheta \cos \gamma \right)\right) \frac{1}{\|\vec{r}\|}$$

(4.10)

$$\delta s = \delta r_2 \cos \left(\arcsin \left(\sin \xi \cos \vartheta \right)\right) \frac{\|\vec{x}\|}{\|\vec{r}\|}$$

(4.11)

The target tracking errors arising from radial position errors $\delta r_3$, observed and expressed in $\mathcal{F}_o$, lie in the angular direction from $\vec{\kappa}$ to $\vec{r}$, and are a function of $\|\vec{\kappa}\|$, $\gamma$ and $\epsilon$:

$$\delta \theta = \delta r_3 \sin \gamma \frac{1}{\|\vec{\kappa}\|}$$

(4.12)

$$\delta s = \delta r_3 \frac{\sin \gamma}{\sin \epsilon}$$

(4.13)

Onboard clock inaccuracies $\delta t_c$ directly lead to miscalculations in the celestial-to-terrestrial rotation $\mathbf{C}_{ie}$ and result in target tracking errors parallel to the Earth’s equator. These target tracking errors are a function of the inertial target velocity $\nu$, $\|\vec{\kappa}\|$, $\epsilon$ and the azimuth $\vartheta_E$.
relative to the target’s local East:

\[
\delta \theta = \delta t_c \nu \cos \phi_c \sin \left( \arccos \left( \cos \vartheta_E \cos \epsilon \right) \right) \frac{1}{\| \vec{r} \|} \]  
(4.14)

\[
\delta s = \delta t_c \nu \cos \phi_c \]  
(4.15)

where \( \vartheta_E \), shown in Figure 4.2, is the angle between the target local East \( \hat{\ell}_1 \) and the path from the subsatellite point to the target. If the onboard clock is also used to propagate the satellite positions and velocities, the pointing and mapping errors can be further compounded due to inaccuracies in the calculated orbit and the satellite-to-target vector. Earth point geometry is discussed further in detail in Vallado [24] and Larson and Wertz [33].

![Diagram of target east azimuth definition on a spherical geoid](image)

Figure 4.2: Target East azimuth \( \vartheta_E \) definition on a spherical geoid.

The error quantities expressed in this section can be amalgamated to form pointing and mapping budgets (see Section 6.2). These budgets consolidate the possible sources of pointing and mapping errors and can preliminarily assess the ability of the Attitude and Orbit Control System (AOCS) to meet mission objectives. By correctly assessing all of the relevant sources of error, suitable attitude sensors and actuators can be procured. Afterwards, the expected performance of the AOCS design can be assessed by testing the hardware and subsequently modeling them in mission simulations. Altogether, these steps mitigate the risk in the AOCS design before launch, commissioning and operation.
4.2 Target Tracking Trajectory

In order to perform ground target tracking, the satellite must follow a specific attitude, angular velocity and angular acceleration trajectory ($\mathbf{C}_{bi}$, $\mathbf{\ddot{\omega}}_{bi}$, $\mathbf{\dddot{\alpha}}_{bi}$) during which its payload is continuously pointing to the location of interest, while maintaining a desired rotational constraint about the payload. This trajectory is depicted in Figure 4.3. The rotational constraint can facilitate auxiliary objectives such as secondary pointing, image alignment and power generation. In particular, the satellite target tracking kinematics, which define the trajectory, must be computed onboard and subsequently regulated in real time. The kinematics are dependent on the inertial trajectories of the satellite and the target of interest. Examples in literature demonstrate that the target tracking problem without rotational constraint can be resolved by simply regulating the attitudes and the angular velocities of the kinematics [6] [8] [11]. More precise target tracking can be achieved by regulating higher order temporal terms of the kinematic relations and enforcing the rotational constraint about the payload.

In this section, the target tracking kinematics will be developed by deriving the desired rotations, angular velocities and angular accelerations required to follow the tracking trajectory which facilitates the mission operations of NEMO-AM. Unlike other kinematic relations derived in literature [6] [8] [11], these derivations will not exercise simplifications in timing, astrodynamics, the orbit trajectory or the Earth geoid. The relations will be formulated such that they can be readily used onboard the ACS. The overall objective is to describe the discretized rotational motion at a future timestep $t_{k+1}$, using only the knowledge at current timestep $t_k$. The trajectory must ensure that the payload is pointed at the target and the spacecraft is oriented such that the desired imaging alignment of NEMO-AM is achieved (see Section 2).

![Figure 4.3: Target tracking trajectory viewed at discretized timesteps $t_k$ and $t_{k+1}$.](image)
Throughout this section, quantities which are on the target tracking trajectory are denoted with a breve superscript \( \breve{\cdot} \), facilitatory rotations not exactly on the trajectory are denoted with a caron superscript \( \check{\cdot} \), estimated attitude and angular velocity quantities onboard are denoted with an overhat superscript \( \hat{\cdot} \) and component quantities propagated forward in time are denoted with an overbar superscript \( \bar{\cdot} \). The “current” timestep is denoted by \( t_k \) and is defined as the epoch to where both the attitude and orbital state knowledge are propagated to and synchronized. Future timesteps are those after \( t_k \) and with no loss of generality will be denoted by \( t_{k+1} \) in this section. Attitude, angular velocity and angular acceleration quantities without superscripts describe the true motion of the satellite at a particular timestep. Unless otherwise noted, angular velocity and angular acceleration components will be expressed in the frame of their first subscript. Moreover, unit vector symbols such as the payload boresight vector \( \hat{p} \), Euler-axes \( \hat{a} \), body-fixed directions \( \hat{s} \) and desired directions \( \hat{d} \) will have the overhat \( \hat{\cdot} \) on their component quantities omitted for conciseness. Body-fixed vectors such as \( \hat{p} \) and \( \hat{s} \) are observed and expressed equivalently in \( F_b \), \( F_a \) and \( F_c \) (see Section 3.4.5), and likewise have their component and timestep subscripts omitted when expressed in any of these frames.

### 4.2.1 Desired Satellite Attitude

The target tracking attitude trajectory at \( t_k \) can be constructed using only the inertial positions of the satellite, its target and desired alignment direction \( d \) at that specific time. In this section, the current trajectory attitude \( C_{bi,k} \) will be derived and with no loss of generality, can be extended to attitudes at future times by propagating the positions from timestep \( t_k \). In order to construct the inertially-referenced attitude trajectory, the satellite body-frame \( F_b \) is first aligned with the inertial frame \( F_i \) and then two consecutive rotations are applied to first align the payload with the target and subsequently constrain the rotation about the alignment direction. Since these rotations are referenced from \( F_i \) and not from the target tracking trajectory, they are simply facilitatory rotations. The first of these rotations \( C_{ai,k} \) constructs an attitude where the payload boresight \( p \) points along \( k \). The second rotation \( C_{ca,k} \) transforms \( C_{ai,k} \) into an attitude where the payload boresight remains pointed at the desired target, but the spacecraft is rotated about \( p \) so that the imaging alignment objective of NEMO-AM is satisfied (see Section 2). This alignment objective is equivalent to minimizing the angle between a body-fixed reference direction \( s \) and a desired reference direction \( d \) (see Section 3.4.6). In this work, the desired reference direction is either the orbit-normal or is a local cardinal heading attached to the target (i.e. affixed to the rotating Earth). By premultiplying \( C_{ai,k} \) by \( C_{ca,k} \), the satellite attitude on the target tracking trajectory at timestep \( t_k \) can be formulated:

\[
\tilde{C}_{bi,k} = C_{ci,k} \quad C_{ci,k} = \tilde{C}_{ca,k} C_{ai,k}
\]

(4.16)  (4.17)
The intermediate rotation $\mathbf{C}_{ai,k}$, where only the payload is pointed at the target, is first constructed using an Euler-axis rotation (see Section 3.1) through an angle $\varphi_{ai,k}$ and about an axis $\mathbf{a}_{ai,k}$. The axis $\mathbf{a}_{ai,k}$ is computed as the cross product of $\mathbf{p}$ and the satellite-to-target vector $\mathbf{r}_k$.

The satellite-to-target vector is most naturally expressed in $\mathbf{F}_e$ since the target can be located using the WGS84 Earth definition (see Section 3.2) and the satellite position is also typically determined in ECEF components through GPS measurements. Since the satellite body-frame is initially aligned with $\mathbf{F}_i$, the computation of the Euler-axis $\mathbf{a}_{ai,k}$ must be performed in $\mathbf{F}_i$ components:

$$\mathbf{a}_{ai,k} = \frac{\mathbf{p}^T \mathbf{C}_{ie,k} \left( \mathbf{x}_e - \mathbf{r}_{e,k} \right)}{\left\| \mathbf{p} \right\| \left\| \mathbf{C}_{ie,k} \mathbf{r}_{e,k} \right\|}$$

(4.18)

where the timestep subscript on $\mathbf{x}_e$ is omitted because the terrestrial locations are time invariant in $\mathbf{F}_e$ and the ECEF to body rotation is decomposed into the FK5 rotation calculated at $t_k$.

The rotation angle $\varphi_{ai,k}$ is the angle between $\mathbf{p}$ and $\mathbf{r}_k$. The angle has the following form in $\mathbf{F}_i$ components:

$$\varphi_{ai,k} = \arccos \left[ \frac{\mathbf{p}^T \mathbf{C}_{ie,k} \mathbf{r}_{e,k}}{\left\| \mathbf{p} \right\| \left\| \mathbf{C}_{ie,k} \mathbf{r}_{e,k} \right\|} \right] = \arccos \left[ \frac{\mathbf{p}^T \mathbf{C}_{ie,k} \mathbf{r}_{e,k}}{\left\| \mathbf{r}_{e,k} \right\|} \right]$$

(4.19)

where the rightmost expression is arrived at by exploiting the unit norm of $\mathbf{r}_k$ and realizing that the magnitude of a vector is frame-independent. The complete $\mathbf{C}_{ai,k}$ rotation is achieved by substituting Eqs. (4.18) and (4.19) into Eq. (3.7):

$$\mathbf{C}_{ai,k} = \cos \varphi_{ai,k} \mathbf{1} + (1 - \cos \varphi_{ai,k}) \mathbf{a}_{ai,k} \mathbf{a}_{ai,k}^T - \sin \varphi_{ai,k} \mathbf{a}_{ai,k}^X$$

(4.20)

The second transformation $\mathbf{C}_{ca,k}$ is a rotation about $\mathbf{p}$ which facilitates secondary objectives while target tracking. For example, rotation about the payload can be used to subject solar panels to the Sun, thereby maximizing power generation while tracking a target. For imaging satellites like NEMO-AM, the primary payload is an imager and the rotation is used to orient the camera, so that successive images are aligned in the same manner and can easily be overlaid afterwards for analysis. This alignment is equivalent to applying a transformation on $\mathbf{C}_{ai,k}$ about $\mathbf{p}$ and through an angle $\varphi_{ca,k}$ which will minimize the angular path between $\mathbf{s}_k$ and $\mathbf{d}_k$ (see Section 3.4.6). The minimization takes the form:

$$\arg \min_{\varphi_{ca,k}} J \left( \mathbf{p}, \mathbf{s}_k, \mathbf{d}_k, \varphi_{ca,k} \right)$$

(4.21)

where $J$ is the cost function to be minimized. If the future desired direction $\mathbf{d}_k$ is a cardinal heading affixed to the target, its components in $\mathbf{F}_e$ can be computed by combining the target terrestrial parameters in the transformation outlined in Section 3.4.8.3. Alternatively, if $\mathbf{d}_k$ is the orbit-normal, it’s inertial direction can be accurately assumed to be invariant over the
Imaging Campaign and its components in $\mathcal{F}_e$ are thus:

$$d_{i,k} = \frac{r_{i,k}^x v_{i,k}}{\| r_{i,k}^x v_{i,k} \|}$$  \hspace{1cm} (4.22)

The cost function is exactly the negative of the dot product of $\mathbf{s}$ and $\mathbf{d}_k$, and when expressed in $\mathcal{F}_c$ is:

$$J \equiv -s^T [\hat{C}_{ca,k} d_{a,k}]$$

$$= -s^T \left[(\cos \varphi_{ca,k} 1 + (1 - \cos \varphi_{ca,k}) pp^T - \sin \varphi_{ca,k} p^x) d_{a,k}\right]$$  \hspace{1cm} (4.23)

where the rotation $\hat{C}_{ca,k}$ has been expressed in terms of an Euler-axis rotation about $\mathbf{p}$ and through the angle $\varphi_{ca,k}$. By taking the partial derivative of Eq. (4.23) with respect to $\varphi_{ca,k}$ and setting the result to zero, the stationary point in the cost function is found to be:

$$\varphi_{ca,k} = \arctan \left[\frac{s^p d_{a,k}}{s^T (p p^T - 1) d_{a,k}}\right]$$  \hspace{1cm} (4.24)

$$d_{a,k} = \hat{C}_{ai,k} C_{ie,k} d_{e,k}$$  \hspace{1cm} (4.25)

The stationary point in Eq. (4.24) is a local minimum and corresponds to the rotation angle about $\mathbf{p}$ which minimizes the angular distance between $\mathbf{s}$ and $\mathbf{d}_k$ only when the Hessian $\mathcal{H}$ of $J$ is positive definite. The Hessian is obtained by taking the second-order partial derivative of $J$ with respect to $\varphi_{ca,k}$ and has the form:

$$\mathcal{H} = \cos \varphi_{ca,k} \left(s^T (1 - pp^T) d_{a,k}\right) - \sin \varphi_{ca,k} \left(s^T p^x d_{a,k}\right)$$  \hspace{1cm} (4.26)

Geometrically, the expression in Eq. (4.24) is equivalent to projecting $\mathbf{d}_k$ and $\mathbf{s}$ onto the plane which is perpendicular to $\mathbf{p}$, and subsequently finding the smallest rotation angle $\varphi_{ca,k}$ which when effected, produces a local extremum of $J$. The local extremum is a minimum when the two projected vectors are aligned and is a maximum when they are separated by $\pi$. Thus if the Hessian is less than zero, then the critical point is a local maximum and the $\varphi_{ca,k}$ which minimizes the angular distance between $\mathbf{s}$ and $\mathbf{d}_k$ is found by adding $\pi$ to Eq. (4.24). If the Hessian is greater than zero then Eq. (4.24) gives the desired result.

During an Imaging Campaign, the Attitude Control System (ACS) requires the future trajectory attitude $\hat{C}_{bi,k+1}$ in order to effect its target tracking maneuver for the next control timestep. The procedure detailed above can be used to construct the target tracking trajectory attitude at any timestep, provided that the satellite position $\mathbf{r}$, target position $\mathbf{x}$ and desired direction $\mathbf{d}$ are defined at that specific time. The future positions and desired direction can be propagated by integrating the satellite motion equations outlined in Section 3.5 and by computing the FK5 rotation detailed in Section 3.4.8.1.
4.2.2 Desired Satellite Angular Velocity

The future target tracking angular velocity $\dot{\omega}_{bi,k+1}$ follows a similar derivation to that of the target tracking attitude trajectory. The important distinction between the two derivations is that whereas the tracking attitude was derived using the inertial frame as a starting reference, the tracking angular velocity is essentially the temporal derivative of instantaneous attitude and as such must be calculated with respect to the tracking attitudes at timestep $t_k$ and $t_{k+1}$. The quantity $\dot{\omega}_{bi,k+1}$ can be decomposed using the additive property of angular velocities [21]:

$$\dot{\omega}_{bi,k+1} \equiv \dot{\omega}_{ci,k}$$
$$\dot{\omega}_{ci,k} = \dot{\omega}_{ca,k} + \dot{\omega}_{ai,k} \tag{4.28}$$

The first constituent of Eq. (4.28), is the absolute angular velocity $\dot{\omega}_{ai,k}$ required to slew on the trajectory from timestep $t_k$ to $t_{k+1}$, and point the payload boresight $\mathbf{p}$ from $\mathbf{κ}_k$ to $\mathbf{κ}_{k+1}$. At each timestep on the trajectory, the starting reference attitude can be thought of as a fixed parameter. At $t_k$, the rotation which facilitates this pointing slew can be computed similarly to that in Eq. (4.20), except with $\kappa_i,k$ replacing $\mathbf{p}$ in Eq. (4.18). The Euler axis of this rotation $\mathbf{a}_{ai,k}$ is calculated as the cross product of the current satellite-to-target vector $\mathbf{κ}_{i,k}$ and the future satellite-to-target vector $\mathbf{κ}_{i,k+1}$. In anticipation of using the rotational kinematics to derive the absolute angular velocity, the Euler axis is computed using $\mathcal{F}_i$ components and has the form:

$$\mathbf{a}_{ai,k} \equiv \kappa_{i,k} \times \kappa_{i,k+1} \tag{4.29}$$

$$\kappa_{i,k} = C_{ie,k}(x_e - r_{e,k}) \tag{4.30}$$

$$\kappa_{i,k+1} = \bar{C}_{ie,k+1}(x_e - \bar{r}_{e,k+1}) \tag{4.31}$$

The angular path between $\kappa_{j,i,k}$ and $\kappa_{j,i,k+1}$ can be calculated readily as before using the dot product of the two vectors:

$$\varphi_{ai,k} \equiv \arccos \left[ \frac{\kappa_{i,k}^T \kappa_{i,k+1}}{\| \kappa_{i,k} \| \| \kappa_{i,k+1} \|} \right] \tag{4.32}$$

The angular velocity $\dot{\omega}_{ai,k}$ is obtained using the angular velocity decomposition in Eq. (3.8):

$$\dot{\omega}_{ai,k} \equiv \dot{\varphi}_{ai,k} \mathbf{a}_{ai,k} - (1 - \cos \varphi_{ai,k}) \mathbf{a}_{ai,k} \cdot \dot{\mathbf{a}}_{ai,k} + \sin \varphi_{ai,k} \dot{\mathbf{a}}_{ai,k} \tag{4.33}$$
To perform the calculation in Eq. (4.33), the absolute temporal derivative of \( a_{ai,k} \) and \( \varphi_{ai,k} \) must be computed. These are as follows:

\[
\dot{a}_{ai,k} = \left[ \frac{1}{\| \kappa_{i,k} \times \bar{\kappa}_{i,k} \|} \right] \kappa_{i,k}^T \dot{\bar{\kappa}}_{i,k} + 1 - \left[ \frac{\kappa_{i,k}^T (\kappa_{i,k} \times \bar{\kappa}_{i,k})}{\| \kappa_{i,k} \times \bar{\kappa}_{i,k} \|^3} \right] \kappa_{i,k}^T \bar{\kappa}_{i,k} + 1 \tag{4.34}
\]

\[
\dot{\varphi}_{ai,k} = \left[ \frac{\kappa_{i,k}^T}{\sin \varphi_{ai,k} \| \kappa_{i,k} \| \| \bar{\kappa}_{i,k} + 1 \|} \right] \left[ \dot{\bar{\kappa}}_{i,k} + 1 - \frac{\bar{\kappa}_{i,k} + 1 (\dot{\kappa}_{i,k}^T \bar{\kappa}_{i,k} + 1)}{\| \bar{\kappa}_{i,k} + 1 \|^2} \right] \tag{4.35}
\]

The temporal derivative of \( \bar{\kappa}_{i,k+1} \) expressed in \( F_i \) components is:

\[
\dot{\bar{\kappa}}_{i,k+1} = -\omega_{ie}^x \bar{C}_{ie,k+1} \left( x_e - \bar{r}_{e,k+1} \right) - \bar{C}_{ie,k+1} \bar{v}_{e,k+1} \tag{4.36}
\]

where the timestep subscript on the components of \( \omega_{ie} \) has been omitted because the Earth’s rotational rate is assumed to be constant.

The second constituent of Eq. (4.28), is the angular velocity \( \dot{\omega}_{ca,k} \) which facilitates the rotation about \( \mathbf{p} \) to minimize the angle between \( \mathbf{s} \) and the future desired direction \( \mathbf{d}_{k+1} \). The quantity \( \dot{\omega}_{ca,k} \) is computed again by employing the Euler rotation kinematics in Eq. (3.8):

\[
\dot{\omega}_{ca,k} \equiv \dot{\varphi}_{ca,k} \mathbf{a}_{ca,k} - (1 - \cos \varphi_{ca,k}) \mathbf{a}_{ca,k}^T \dot{\mathbf{a}}_{ca,k} + \sin \varphi_{ca,k} \mathbf{a}_{ca,k} \tag{4.37}
\]

The Euler axis \( \mathbf{a}_{ca,k} \) and angle \( \varphi_{ca,k} \) required to compute \( \dot{\omega}_{ca,k} \) is derived in the exact same manner described in Eqs. (4.21) to (4.25), except with one important distinction. If \( \dot{\omega}_{ca,k} \) is to describe the angular velocity on the trajectory, the \( F_a \) frame at timestep \( t_{k+1} \) must be referenced from the trajectory attitude at \( t_k \). As a result \( \bar{C}_{ai,k} \) in Eq. (4.25) is replaced by \( \bar{C}_{ai,k} \), which is the product of the alignment transformation from \( t_k \) to \( t_{k+1} \), \( \bar{C}_{ab,k} \), and the inertial-to-body trajectory attitude \( \bar{C}_{bi,k} \) at \( t_k \):

\[
\mathbf{d}_{a,k} = \bar{C}_{ai,k} \bar{C}_{ie,k+1} \mathbf{d}_{e,k+1} \tag{4.38}
\]

\[
\bar{C}_{ai,k} = \bar{C}_{ab,k} \bar{C}_{bi,k} \tag{4.39}
\]

where \( \bar{C}_{bi,k} \) is computed using the procedure in Section 4.2.1 and \( \bar{C}_{ab,k} \) is constructed using an Euler axis rotation through the angle in Eq. (4.32) and about an axis equal to the cross product of \( \kappa_k \) and \( \kappa_{k+1} \) expressed in \( F_b \):

\[
\mathbf{a}_{ab,k} \equiv \frac{\bar{C}_{bi,k} \kappa_{i,k}^T \bar{\kappa}_{i,k} + 1}{\| \bar{C}_{bi,k} \kappa_{i,k}^T \bar{\kappa}_{i,k} + 1 \|} \tag{4.40}
\]
The temporal derivative of the Euler axis $\mathbf{a}_{ca,k}$ and Euler angle $\varphi_{ca,k}$ are also required to calculate $\dot{\boldsymbol{\varphi}}_{ca,k}$. The value of $\dot{\mathbf{a}}_{ca,k}$ is nil, because the axis is exactly the body-fixed payload vector $\mathbf{p}$. The derivative of $\varphi_{ca,k}$ can be computed by differentiating Eq. (4.24) and using the $\mathcal{F}_a$ frame referenced from the trajectory attitude as outlined in Eq. (4.39).

$$
\dot{\varphi}_{ca,k} = \frac{\mathbf{s}^T \mathbf{p}^\times \left( \dot{\mathbf{d}}_{a,k+1} \mathbf{P} \mathbf{d}_{a,k+1} - \mathbf{d}_{a,k+1} \mathbf{P} \dot{\mathbf{d}}_{a,k+1} \right)}{\left( \mathbf{P} \mathbf{d}_{a,k+1} \right)^2 - \left( \mathbf{s}^T \mathbf{p}^\times \mathbf{d}_{a,k+1} \right)^2} \quad (4.41)
$$

$$
\dot{\mathbf{d}}_{a,k+1} = - \mathbf{C}_{ai,k} \mathbf{\omega}_{ai,k} \mathbf{C}_{ae,k+1} \mathbf{d}_{e,k+1}
- \mathbf{C}_{ai,k} \mathbf{\omega}_{ie} \mathbf{C}_{ie,k+1} \mathbf{d}_{e,k+1} + \mathbf{C}_{ai,k} \mathbf{C}_{ie,k+1} \dot{\mathbf{d}}_{e,k+1} \quad (4.42)
$$

where the matrix $\mathbf{P}$ is defined as $\mathbf{P} \triangleq \mathbf{s}^T \mathbf{p}^\times$. If the future desired direction $\mathbf{d}_{k+1}$ is a cardinal heading affixed to the target, its derivative observed in $\mathcal{F}_e$, $\dot{\mathbf{d}}_{e,k+1}$, is $\mathbf{0}$. Alternatively, if $\mathbf{d}_{k+1}$ is the orbit-normal, its derivative observed in $\mathcal{F}_i$ can be assumed to be nil and its components in $\mathcal{F}_e$ and derivatives thereof are:

$$
\dot{\mathbf{d}}_{e,k+1} = \mathbf{C}_{ei,k+1} \mathbf{d}_{e,k+1} \quad (4.43)
$$
$$
\dot{\mathbf{d}}_{e,k+1} = - \mathbf{\omega}_{ei} \mathbf{C}_{ei,k+1} \mathbf{d}_{e,k+1} \quad (4.44)
$$

Finally, in order to compute the future target tracking trajectory angular velocity $\dot{\mathbf{\omega}}_{bi,k+1}$ for the AOCS controller at time $t_k$, Eqs. (4.28), (4.33) and (4.37) must be combined and transformed to $\mathcal{F}_b$ components at that timestep.
4.2.3 Desired Satellite Angular Acceleration

The future target tracking angular acceleration $\dot{\alpha}_{bi,k+1}$ can be computed readily once the $\ddot{\omega}_{bi,k+1}$ is calculated. Since $\ddot{\omega}_{bi,k+1}$ is the rate of change of $\dot{\omega}_{bi,k+1}$, the tracking angular acceleration can be constructed by decomposing the trajectory angular acceleration and differentiating the Euler rotation kinematics in Eq. (3.8):

$$\ddot{\alpha}_{bi,k+1} \equiv \ddot{\alpha}_{ci,k} \tag{4.45}$$

$$\ddot{\alpha}_{ci,k} = \ddot{\alpha}_{ca,k} + \ddot{\alpha}_{ai,k} \tag{4.46}$$

$$\alpha \equiv \dot{\varphi} a + \left[ \dot{\varphi} - \varphi \sin \varphi a^x + \varphi \cos \varphi \right] \dot{a} + \left[ \sin \varphi - (1 - \cos \varphi) a^x \right] \ddot{a} \tag{4.47}$$

The terms in Eq. (4.46) can be computed by taking the temporal derivatives of Eqs. (4.34), (4.35) and (4.41) and substituting them appropriately into Eq. (4.47):

$$\ddot{a}_{ai,k} = \frac{\ddot{\beta}_k}{\| \beta_k \|^4} \left( 2 \beta_k^T \beta_k \dot{\beta}_k + 3 \frac{\beta_k^T \beta_k \beta_k + \dot{\beta}_k^T \beta_k \beta_k}{\| \beta_k \|^5} \right) + \frac{3}{\| \beta_k \|^5} \left( \ddot{\beta}_k \beta_k \right)$$

$$\ddot{\varphi}_{ai,k} = -\| \kappa_{i,k} \| \left( \dot{\varphi}_{ai,k} \cos \varphi_{ai,k} \| \kappa_{i,k+1} \| + \sin \varphi_{ai,k} \kappa_{i,k+1}^{T} \kappa_{i,k+1} \right) \left[ -\kappa_{i,k}^T \dot{\kappa}_{i,k+1} + \kappa_{i,k}^T \frac{\dot{\kappa}_{i,k+1} \kappa_{i,k+1}}{\| \kappa_{i,k+1} \|^2} \right]$$

$$+ \left[ \frac{\kappa_{i,k}^T \dot{\kappa}_{i,k+1} \dot{\kappa}_{i,k+1} \kappa_{i,k+1} + \kappa_{i,k}^T \frac{\dot{\kappa}_{i,k+1} \kappa_{i,k+1}}{\| \kappa_{i,k+1} \|^2} \right] \frac{\sin \varphi_{ai,k} \| \kappa_{i,k} \| \| \kappa_{i,k+1} \|^3}{\| \kappa_{i,k+1} \|^3} \tag{4.49}$$

$$\ddot{\varphi}_{ca,k} = \frac{s^T p^x (\ddot{d}_{a,k+1} P d_{a,k+1} - d_{a,k+1} P \ddot{d}_{a,k+1})}{(P \ddot{d}_{a,k+1})^2 - (s^T p^x d_{a,k+1})^2}$$

$$+ \frac{2 \dot{\varphi}_{ca,k} \dot{\kappa}_{i,k+1}}{(P \ddot{d}_{a,k+1})^2 - (s^T p^x d_{a,k+1})^2} \left( \ddot{d}_{a,k+1} + s^T p^x \dot{d}_{a,k+1} s^T p^x \right)$

$$\ddot{\varphi}_{ai,k} = \frac{2 \dot{\varphi}_{ai,k}}{(P \ddot{d}_{a,k+1})^2 - (s^T p^x d_{a,k+1})^2} \left( \ddot{d}_{a,k+1} + s^T p^x \dot{d}_{a,k+1} s^T p^x \right) \ddot{d}_{a,k+1}$$

$$\ddot{\varphi}_{ca,k} = \frac{2 \dot{\varphi}_{ca,k}}{(P \ddot{d}_{a,k+1})^2 - (s^T p^x d_{a,k+1})^2} \left( \ddot{d}_{a,k+1} + s^T p^x \dot{d}_{a,k+1} s^T p^x \right) \ddot{d}_{a,k+1}$$

$$\ddot{\varphi}_{ai,k} = \frac{2 \dot{\varphi}_{ai,k}}{(P \ddot{d}_{a,k+1})^2 - (s^T p^x d_{a,k+1})^2} \left( \ddot{d}_{a,k+1} + s^T p^x \dot{d}_{a,k+1} s^T p^x \right) \ddot{d}_{a,k+1}$$

where $\beta_k \triangleq \dot{\kappa}_{i,k+1} \kappa_{i,k}$ and all of the remaining parameters are either defined in the previous section or can be derived through straightforward temporal differentiation following Section 4.2.2.
4.3 Imaging Metrics

The optical metrics for satellite remote sensing applications are established in this section. Typically, remote sensing metrics are in terms of the accuracy in pointing the optical instrument, the imaging properties of the instrument and the imaging performance during observation. The accuracy in pointing however, is independent of the optical instrument and is directly determined by the AOCS performance. Thus, only the instrument imaging properties and imaging performance metrics are discussed in this section.

The electronic functionality of the optical instrument is outside the scope of this work. However, it is important to define the instrument properties relating to remote sensing performance. The level of detail which can be resolved by an imager is proportional to the pixel resolution of the instrument. This resolution is the product of the pixel-length $\ell$ and pixel-width $w$ of the imager’s CCD. The pixel-width of the CCD is defined to be larger than the pixel-length. For NEMO-AM, the imager is positioned along the $-Y$ face (see Section 2) and likewise, the pixel-width $w$ and pixel-length $\ell$ are 2332 pixels and 848 pixels (see Figure 2.6), respectively. For remote sensing, resolved detail is measured as the distance between pixel centers measured on the ground, as captured by the satellite. This type of detail is referred to as ground sampling distance (GSD) and is computed as:

$$G \triangleq \sqrt{\frac{A_i}{P}}; \quad P \triangleq \ell \times w$$  \hspace{1cm} (4.51)

where $A_i$ is the ground area captured by imager measured in km$^2$ and $P$ is the pixel density of the CCD. The ground area is proportional to the distance from the satellite to the location being observed. Likewise, the imager has the smallest GSD and the greatest resolving capability when the satellite is directly over its location of interest. Since longer distances and smaller grazing angles $\epsilon$ (see Section 4.1) cause greater distortion of the image over the curvature of the Earth, Eq. (4.51) is, in general, the mean GSD over the field of view of the imager. The GSD of NEMO-AM as a function of orbital altitude $h$ and viewing angle $i_o$ (see Section 3.4.3) is shown on the top of Figure 4.4. The angular distance represented by each pixel can be computed by replacing $\delta s$ in Eq. (4.4) with $G$ and solving for $\delta \gamma$. This variation in angular pixel length is presented on the bottom of Figure 4.4.

While imaging, the satellite must also meet stability criteria so that during exposure, captured data is not prone to excessive pixel smearing. For target tracking applications, pixel smearing is directly a measure of the satellite angular velocity tracking error. The angular velocity which must be tracked is solely dictated by the target tracking kinematics (see Section 4.2).
Figure 4.4: NEMO-AM GSD and pixel angular distance variation for groundtrack target.

and as such, pixel smear perpendicular to and about the imager boresight are respectively:

\[
P_{\perp} \triangleq \frac{\| \kappa \|}{G} \left( \delta \omega_{\perp} \Delta t_e \right) \quad (4.52)
\]

\[
P_{\parallel} \triangleq \frac{u}{2} \left( \delta \omega_{\parallel} \Delta t_e \right) \quad (4.53)
\]

where \( \Delta t_e \) is the imager’s time of exposure, \( \delta \omega_{\perp} \) is the angular velocity tracking error perpendicular to the imager boresight, \( \delta \omega_{\parallel} \) is the angular velocity tracking error parallel to the imager boresight. The angular velocity tracking errors are computed as:

\[
\delta \tilde{\omega} = \omega_{bi} - \tilde{\omega}_{bi} \quad (4.54)
\]

\[
\delta \omega_{\perp} = \left[ (\delta \tilde{\omega}_1)^2 + (\delta \tilde{\omega}_3)^2 \right]^{\frac{1}{2}} \quad (4.55)
\]

\[
\delta \omega_{\parallel} = \| \delta \tilde{\omega}_{2} \| \quad (4.56)
\]

The total pixel smearing \( P_T \) is a combination of both \( P_{\perp} \) and \( P_{\parallel} \) and is dependent on the relative motion between the satellite and the target, and the projection of the CCD pixels on the Earth’s surface. An approximate measure of the total pixel smearing is simply the sum of
the two constituents and is used in this work exclusively due to its conservativeness:

$$\mathcal{P}_T \approx \mathcal{P}_\perp + \mathcal{P}_\parallel$$  \hspace{1cm} (4.57)

This formulation ensures that both $\mathcal{P}_\perp$ and $\mathcal{P}_\parallel$ are always positive quantities in order to provide a worst-case scenario.

### 4.4 Extended Kalman Filter

The Extended Kalman Filter (EKF) has proven to be a well established method for satellite attitude and orbit determination [23] [28], and has been shown to produce an unconstrained optimal solution to the linear stochastic estimation problem [34]. It can be very effective for mildly nonlinear non-Gaussian systems [35]. This section shall first introduce the generic formulation of the EKF and then outline the specific EKF used onboard NEMO-AM to smooth and propagate the satellite orbital and attitude states in anticipation of regulating the attitude state onboard to a desired value. The general notion of the EKF is to recursively estimate the current system state $x_k$ at time $t_k$ by approximating its probability density function (belief) through simplifications of the Bayes Filter, given current inputs, sensor measurements and the state $x_{k-1}$ at a past time $t_{k-1}$. The Bayes filter itself is an approximation since it invokes the Markov property, which enforces the conditional probability density function of $x_k$ to be dependent only on the past state $x_{k-1}$, and conditionally independent of the sequence of events that preceded it. Although this approximation admits the estimation to be done in a recursive form, it does limit the exactness because the states in most dynamic systems are a direct function of all of the previous dynamics that came before it. The motion dynamics and measurement (observation) processes are assumed to be nonlinear and time-invariant, and in preparation for using them online, have the generic discrete form:

$$x_k = h(x_{k-1}, u_{k-1}, v_k)$$

$$y_k = g(x_k, w_k)$$  \hspace{1cm} (4.58)

where, $k$ is the timestep index, $f$ is the nonlinear motion model, $g$ is the nonlinear observation model, $u_{k-1}$ is the control input, $v_k$ is the process noise variable, $y_k$ is the exteroceptive measurement and $w_k$ is the measurement noise variable. The Bayes filter is computationally expensive to calculate and is thus approximated by linearizing the motion and observation models about the previous state estimate mean, constraining the belief to be Gaussian and
constraining the noise variables to be zero-mean Gaussian:
\[ x_k \sim N(\hat{x}_k, \hat{P}_k) \]  
(4.59)
\[ v_k \sim N(0, Q) \]  
(4.60)
\[ w_k \sim N(0, R) \]  
(4.61)

where the short-hand notation \( N(\hat{x}_k, \hat{P}_k) \) represents a Gaussian distributed variable with an expected value of \( \hat{x}_k \) and a covariance of \( \hat{P}_k \). By combining the approximations, the filter takes on a recursive predictor-corrector form. First, the past belief is propagated forward in time through the motion model and \( u_k \) to compute an a priori estimate \( \hat{x}_{k-1}^+ \) and an a priori covariance \( \hat{P}_{k-1}^+ \). The predicted estimates are then corrected using the observation model and the measurement \( y_k \) to obtain a posteriori estimate \( \hat{x}_k^+ \) and posteriori covariance \( \hat{P}_k^+ \). The prediction and correction steps are:

\[ \hat{x}_k^- = h(\hat{x}_{k-1}^+, u_{k-1}, 0) \]  
(4.62)
\[ \hat{P}_k^- = H_{x,k} \hat{P}_{k-1}^- H_{x,k}^T + H_{v,k} Q H_{v,k}^T \]  
(4.63)
\[ \hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - g(\hat{x}_k^-, 0)) \]  
(4.64)
\[ \hat{P}_k^+ = (1 - K_k G_{x,k}) \hat{P}_k^- \]  
(4.65)

where the linearizing partial derivatives are defined as:

\[ H_{x,k} = \left. \frac{\partial f(x_{k-1}, u_{k-1}, v_k)}{\partial x_{k-1}} \right|_{\hat{x}_{k-1}^+, u_{k-1}, 0} \]
\[ H_{v,k} = \left. \frac{\partial f(x_{k-1}, u_{k-1}, v_k)}{\partial v_k} \right|_{\hat{x}_{k-1}^+, u_{k-1}, 0} \]
\[ G_{x,k} = \left. \frac{\partial g(x_k, w_k)}{\partial x_k} \right|_{\hat{x}_k^-, 0} \]
\[ G_{w,k} = \left. \frac{\partial g(x_k, w_k)}{\partial w_k} \right|_{\hat{x}_k^-, 0} \]

and \( K_k \) is the Kalman gain:

\[ K_k = \hat{P}_k^- G_{x,k}^T \left( G_{x,k} \hat{P}_k^- G_{x,k}^T + G_{w,k} R G_{w,k}^T \right)^{-1} \]  
(4.67)

The role of the Kalman gain is to weight the contribution of the estimated measurement error to the predicted state estimate. The expressions for the covariances \( \hat{P}_k \) are arrived at by computing the expectation of the outer product of \( \hat{x}_k - x_k \) with itself. For complex non-linear systems that can only be evaluated through numerical integration, the outer product is arrived at by assuming a zero-order hold of the continuous dynamics, linearizing and computing the error dynamics of \( \frac{d}{dt} (\hat{x}_k - x_k) \) using the state transition matrix \( \Phi \). For additive time-invariant process noise and complex motion dynamics, the continuous motion dynamics and predicted
covariance take the form:

\[
\dot{x} = f(x, u, t) + v \\
\hat{P}_k^{-} = \Phi_{x,k} \hat{P}_{k-1} \Phi_{x,k}^T + Q
\]

where the state \(x_k\) must be propagated by integrating Eq. (4.68) from \(x_{k-1}\) and taking the previous control input \(u_{k-1}\) into account. The state transition matrix satisfies the following properties owing to the zero-order hold:

\[
\Phi_{x,k} \triangleq \Phi(t_k, t_{k-1}) \\
\frac{d}{dt} \Phi_{x,k} = F_{x,k} \Phi_{x,k} \\
F_{x,k} = \left. \frac{\partial f}{\partial x} \right|_{x_{k-1}, t_{k-1}}
\]

The state transition matrix can be solved in one of two ways. It can either be numerically integrated through Eq. (4.71) or the zero-order hold can again be exploited by keeping \(F_{x,k}\) constant from \(\Delta t_k \triangleq t_k - t_{k-1}\), and solving for \(\Phi_{x,k}\):

\[
\Phi_{x,k} = e^{A\Delta t_k} \\
e^{A\Delta t_k} \approx 1 + \Delta t_k A + \frac{\Delta t_k^2}{2!} A^2 + \frac{\Delta t_k^3}{3!} A^3 + \cdots
\]

where \(A \triangleq F_{x,k}\), and the state transition matrix is expressed as a truncated series expansion of the matrix exponential. The Kalman filter is started with an initial guess \(x_0\) and \(P_0\). The initial state covariance is sufficiently large to allow appropriate correction of the state with each new measurement. The measurement noise matrix \(R\) weights the sensitivity of each new observation and is derived from previously measured sensor errors. As more observations are processed, both the Kalman gain and state covariance approach zero. If no process noise \(Q\) is added in the filter, errors arising from rounding errors, small nonlinearities and approximations to the motion and measurement model are propagated without correction and may lead to an undesired or diverging result. As \(Q\) increases, both the predicted covariance and Kalman gain also increase, making the EKF more sensitive to new observations. The value of \(Q\) must be kept small enough though to ensure that improper measurements do not result in large state corrections, but large enough so that measurements can still correct inherent process errors. Practically, \(R\) is determined through bench testing of sensors and \(Q\) is determined by off-line simulations. On orbit, these noise matrices are further tuned to achieve the desired filtering results.

The EKF’s main drawback is its first-order linearization of the motion model about the mean state estimate and not the true state itself. This small difference can cause the estimator to be inconsistent or diverge wildly when the linearized attitude motion equations are a poor
approximation of the true satellite dynamics \cite{35}. In general, there is no guarantee that the filter will converge to an acceptable solution and almost always, performance is gauged by simply testing the filter.

\section*{4.4.1 Onboard Orbit Determination}

It is apparent from Section 4.2 that some sort of orbit state propagation is required to construct the target tracking trajectory. Flight experience has shown that even for satellites flying a GPS receiver to provide position and velocity as part of their guidance solution, up to two orders of magnitude increase in accuracy can be achieved by employing an online dynamical orbit determination algorithm \cite{36}. In particular, the Extended Kalman Filter (EKF) outlined previously has proven to be a well established method for satellite orbit determination and this section shall elucidate the specific Orbital EKF (OEKF) used onboard NEMO-AM to smooth and propagate GPS orbit trajectories for the purposes of target tracking. The noise variables and state estimate are again assumed to be Gaussian as before. Since GPS measurements are produced in the form of position and velocity as observed and expressed in the ECEF frame, these quantities represent the state to be estimated:

\[ \mathbf{x} = \begin{bmatrix} \mathbf{r}^T_e & \mathbf{v}^T_e \end{bmatrix}^T \]  

(4.75)

The observation model \( \mathbf{y} \) is dependent on the GPS solution type and takes the general form:

\[ \mathbf{y}_k = \mathbf{M}_k \mathbf{x}_k + \mathbf{w}_k \]  

(4.76)

where, \( \mathbf{M} \) is a facilitatory matrix that extracts the desired GPS measurements from \( \mathbf{x} \) and \( \mathbf{w} \) is the observation noise model. If only a valid velocity solution is available from the GPS, the solution will not be processed. This is owing to the fact that velocity measurements are inherently much poorer than position measurements \cite{23} and position is unobservable from velocity alone \cite{37}. If valid position and velocity solutions are both available, \( \mathbf{M}_k = \mathbf{1}_{6 \times 6} \). If only a valid position solution is available, \( \mathbf{M}_k \) simply has \( \mathbf{1}_{3 \times 3} \) and \( \mathbf{0}_{3 \times 3} \) along its diagonal.

The predicted state \( \hat{\mathbf{x}}_k^\prime \) is propagated by numerically integrating the orbital motion equations in Section 3.5.1 across \( \Delta t_k = t_k - t_{k-1} \). The continuous dynamics which facilitate this integration can be concisely expressed as:

\[ \dot{\mathbf{x}} = \mathbf{f}(\mathbf{r}_e, \mathbf{v}_e) + \mathbf{v} \]  

(4.77)

where the orbital dynamics only depend on the satellite position and time. By enforcing a zero-order hold across \( \Delta t_k \), the position and velocity dependency of \( \mathbf{f} \) can be eliminated and instead can be viewed as only being dependent on \( t_{k-1} \) \cite{38}. This approximation allows for the predicted covariance \( \hat{\mathbf{P}}_k^\prime \) to be computed through the matrix exponential expansion of the state
transition matrix $\Phi_{x,k}$ in Eq. (4.74). The exponent matrix takes the form:

$$
F_{x,k} = \begin{bmatrix}
0_{3 \times 3} & 1_{3 \times 3} \\
G_r & G_v
\end{bmatrix} \bigg|_{x_{k-1}}
$$  

(4.78)

$$
G_r = \frac{Gm_{\oplus}}{r_e^5} \left(3r_e r_e^T - r_e^T r_e \right) - \omega_e^x \omega_e^x
$$

(4.79)

$$
G_v = -2\omega_e^x
$$

(4.80)

where $F$ contains the partial derivatives of the continuous motion dynamics $f$. The force model has been approximated as only having contributions from the central body since the accuracy requirements for the state transition matrix is generally more relaxed than the orbit trajectory itself [23]. It should be noted that the elements in $F_{x,k}$ are appended with the partial derivatives of the coriolis and centrifugal accelerations to ensure that the covariance is predicted as observed and expressed in $F_e$.

The process and measurement noise are assumed to be additive as before (see Section 5.1.2) to reduce computational complexity of the flight code. The state correction, covariance correction and Kalman gain take the same form as Eq. (4.64), Eq. (4.65) and Eq. (4.67), respectively. Flight experience has shown that consistent force models should be employed in both the propagation of the orbit trajectory and the computation of the state transition matrix [23] to achieve a converging state estimate. Thus orbit perturbations which arise from third body, aerodynamic and solar contributions, and are insignificant over the filter sample rate, were chosen to be left unmodeled in both the state propagation and covariance prediction. Only the geoid gravitational acceleration is considered in the OEKF.
Chapter 5

Attitude and Orbit Control

The Attitude and Orbit Control System (AOCS) is responsible for estimating and controlling the satellite’s pose. The control can either be passive, active or a combination of the two. To perform any sort of active control, the AOCS must estimate the satellite’s orbital state ($\mathbf{r}$, $\mathbf{v}$), attitude ($\mathbf{C}_{bi}$) and angular velocity ($\mathbf{\omega}_{bi}$), and then use actuation to achieve the desired pose. The System’s scope includes attitude sensors and actuators that are necessary to determine this pose and subsequently control it, and flight software which filters information, evaluates control laws and communicates with the other subsystems of the satellite.

This chapter describes the AOCS design of the Nanosatellite for Earth Monitoring and Observation - Aerosol Monitor (NEMO-AM). It is an active control design and is based heavily on the University of Toronto Space Flight Lab’s most recent launched satellite, AISSat-1. The basic suite of attitude sensors, in the form of fine sun sensors and magnetometer, are almost identical to those on AISSat-1. The magnetorquers and reaction wheels have been flown on previous Generic Nanosatellite Bus (GNB) missions, while the GPS and star tracker are scheduled to fly on upcoming GNB missions. The implementation of the satellite dynamics on the AOCS assumes that in addition to the satellite center of gravity and center of mass being coincident (see Section 3.6.2), $\mathbf{J}_b$ and all $\mathbf{J}_w$ are also coincident. In practice, this approximation is negligible since the wheel axial inertias $I_a$ are at least 3 orders of magnitude smaller than $\mathbf{J}_b$ and, $\mathbf{J}_b$ and all $\mathbf{J}_w$ are at most separated by 10 cm [10]. When the control torques $\mathbf{g}_c$ are applied using reaction wheels, the implemented satellite motion equations on the AOCS are approximated from Eq. (3.58) and take the form:

$$\mathbf{I} \dot{\mathbf{\omega}} + \mathbf{\omega}^x (\mathbf{I} \mathbf{\omega} + \mathbf{I}_a \mathbf{\omega}_w) = \mathbf{g}_c + \mathbf{g}_d$$

$$\mathbf{I}_a \dot{\mathbf{\omega}}_w = \mathbf{g}_w$$

(5.1)

(5.2)

where the $\cdot_{bi}$ subscript has been omitted in this chapter for conciseness and the term $\mathbf{I}_a \dot{\mathbf{\omega}}$ in $\mathbf{g}_w$ has been eliminated, since $\dot{\mathbf{\omega}}_w \gg \dot{\mathbf{\omega}}$ for typical operational scenarios. The AOCS Onboard Attitude System Software (OASYS) is also similar, both in terms of the basic Attitude Extended Kalman Filter (AEKF) and core control laws, to that on the GNB. Overall heritage was
maintained on the AOCS design, legacy designs were tuned and mission specific AOCS operations were developed by the author. These mission specific operations involved implementing new celestial ephemerides, constructing slewing trajectories for tracking, designing navigational orbit determination filters and synthesising robust controllers for pointing and imaging. These operations were further integrated with the existing software to create functional system capable of successful operation from orbit injection to nominal pointing and autonomous Earth observation.

5.1 Attitude Determination

The Attitude Determination Subsystem (ADS) includes the onboard sensors required for attitude estimation and the Attitude Extended Kalman Filter (AEKF) for attitude estimation (see Section 5.1.2). Although the orbital state is not an attitude parameter, GPS receivers are considered to be a part of the ADS since their outputs help form the target tracking trajectory during payload operation (see Section 4.2). The System’s primary objective is to ensure that the attitude determination accuracy of the satellite is better than the desired control accuracy, by at least the magnitude of the control error, so that pointing requirements can be met. An outline of the attitude determination sensors and the estimation techniques onboard NEMO-AM is presented in this section. The determination hardware suite includes a three-axis magnetometer, six fine sun sensors and a star tracker for added fidelity. The pointing budgets constructed from the hardware characteristics in this section are presented in Section 6.2.

5.1.1 Attitude Sensors

5.1.1.1 Sun Sensors

Fine sun sensors form one part of the basic attitude determination hardware and a full suite of six are present on NEMO-AM, with one oriented on each body face. These units are used to measure the sun vector $\mathbf{s}_\odot$ in the sun sensor frame $\mathcal{F}_s$ and subsequently rotate it into the satellite body frame $\mathcal{F}_b$, so that it can be combined with an onboard Astronomical Almanac Sun Vector (AASV) ephemeris to facilitate the estimation of the inertial-to-body rotation $\mathbf{C}_{bi}$. The fine sun sensor, shown in Figure 5.1, is based on a pinhole arrangement overlaid on top of a CMOS array design which outputs two one-dimensional profiles when a sun spot is projected through the pinhole.

![Figure 5.1: Fine sun sensor.](image-url)
The sun profiles are used to determine the direction of $\mathbf{s}_\odot$ through a centroiding algorithm. Ground test characterization results, shown in Figure 5.2(a), demonstrate that the sensors are accurate to 0.64° for the central 90% FOV and 1.05° for the full CMOS FOV [39]. The fidelity of the sun sensor is limited by the CMOS resolution, misalignments (biases) in the pinhole position, pinhole height and imperfections in the sensor electronics and pinhole makeup. In orbit, the attitude estimate inaccuracy is also compounded by mounting misalignments, transient errors in measurement handling between multiple sun sensors and ephemeris (AASV) model errors used to compare the sun sensor profiles. The aggregate of all these errors on AISSat-1 gives an on-orbit sun vector estimation accuracy of 2.51° (2σ) [40]. Each sensor has a useful circular coverage of 45° half angle, resulting in a 88% coverage of the satellite celestial sphere, shown in Figure 5.2(b). By measuring only the sun vector, rotational information about $\mathbf{s}_\odot$ is not available. For NEMO-AM, the measured local magnetic field is combined with $\mathbf{s}_\odot$ during nominal operations to achieve full three-axis attitude estimation.

![Sun Sensor Projections on Body Celestial Sphere](image)

(a) Ground characterization result

![FOV projections in blue.](image)

(b) $\text{FOV}$ projections in blue.

Figure 5.2: Sun sensor characterization and full $\text{FOV}$ suite projection on satellite sphere.

### 5.1.1.2 Magnetometer

Magnetometers form the second part of the basic ADS hardware. A single, three-axis, sensor is present on NEMO-AM, and is shown in a boom-mounted configuration in Figure 5.3. The magnetometer is used to measure the local magnetic field $\mathbf{b}$ for comparison with an onboard 11th generation International Geomagnetic Reference Field (IGRF-11) ephemeris. It relies on three orthogonal magneto-inductive sensors, designed to alter their inductance in the presence of magnetic fields. Although the boom provided magnetic isolation from the spacecraft bus on previous GNB missions, it was determined that the magnetometer sensor could be placed internally in NEMO-AM without any detrimental effects. The unit is sensitive to all magnetic fields, including those inherent on the satellite and created by the magnetorquers (see Section 5.3.1.2). This is alleviated by strategically locating the magnetometer away from field-inducing components and not polling the sensor during magnetorquer usage.
By combining $\mathbf{s}_\odot$ and $\mathbf{b}$, full three-axis attitude information can be evaluated and corrected for noise and erroneous measurements using an appropriate estimation scheme (see Section 5.1.2). The sensors’ accuracies are dependent on the resolution of the units, alignment calibration, readout noise and thermal noise. In orbit, the attitude estimation inaccuracy is compounded by any parasitic and generated magnetic fields, mounting misalignments and ephemeris ($IGRF-11$) model errors used to compare the field measurements. The aggregate of all these errors on $AISSat-1$ gives an on-orbit magnetic field estimation accuracy of $2.89^\circ$ (2$\sigma$) [40].

Figure 5.3: Magnetometer mounted on satellite boom. Boom is not used on $NEMO-AM$.

5.1.1.3 Star Tracker

The Sinclair Interplanetary, $UTIAS/SFL$ and Ryerson $SAIL$ ($S3S$) star tracker is a lightweight and low-power unit designed for small satellite and is used onboard $NEMO-AM$ to achieve the attitude and angular velocity estimation accuracy required during Imaging Campaigns. The star tracker has been in development for three years, but its optics, shown in Figure 5.4, have flight heritage on previous interplanetary missions. It is scheduled to be flown on two $SFL$ missions before 2014. The $CMOS$ detector has a $FOV$ of $15.5^\circ \times 20.2^\circ$. To operate, the boresight of the star tracker must maintain an exclusion angle with respect to the Sun and Earth centers of approximately $60^\circ$ and $30^\circ$, respectively, if the detector is not to be upset. Consequently, on $NEMO-AM$, the sensor is positioned $38^\circ$ from the $-X$ direction towards $+Y$, in the $XY$ plane (see Section 6.3.1). The unit houses a star catalogue of 2.1 million stars, with a nominal catalogue magnitude of 5.75, which it uses to facilitate its 2Hz cadence rate [41].

Figure 5.4: $S3S$ star tracker.
The $S3S$ star tracker will be used as the only attitude knowledge source for the $AEKF$ onboard $NEMO-AM$ during its Imaging Campaigns. The star tracker calibration and performance testing demonstrate that the $2\sigma$ sensor attitude estimation errors in Roll ($\delta\theta_1$), Pitch ($\delta\theta_2$) and Yaw($\delta\theta_3$) are $0.036^\circ$, $0.004^\circ$ and $0.004^\circ$, respectively [41]. In addition to attitude estimation, angular velocity measurements from motion blur are also envisioned to be available as outputs from the sensor. As the calibration and performance testing for this feature are currently under development, the only inputs into the $AEKF$ during target tracking will be the attitude estimates from the unit. This results in a rate estimation error which must be tolerated by the target tracking controller.

5.1.1.4 Global Positioning System Receiver

The Novatel $OEMV-1$ series $GPS$ receiver shown in Figure 5.5 is a commercial off-the-shelf unit with small form factor and low power consumption which provides single point position $\mathbf{r}$ and velocity $\mathbf{v}$ solutions on the L1 frequency ($1575.42\text{ Hz}$) in the $WGS84$ realization of the $ECEF$ frame. Although the receiver is not an attitude sensor, the orbital state knowledge it provides is used as inputs for the $IGRF-11$ ephemeris model and the target tracking trajectory construction. On $NEMO-AM$, the receiver is positioned on the $+Y$ direction of the satellite to increase its communication availability with the $GPS$ constellation. For cold starts in space with no almanac, ephemerides or initial estimate of the position and current time, typical time to first fix are on the order of 7 to 15 min [40]. Typical time to first fix for hot starts are on the order of 2 to 5 min [40]. The receiver tracking algorithm is designed to tolerate peak accelerations of 10 g, which exceeds the range of what $NEMO-AM$ will experience during target tracking. The receiver has a maximum update rate of 20 Hz and its clock filter, which outputs $GPS$ Time (see Section 3.3), has a timing accuracy of 20 ns ($\text{RMS}$), compared to a 5 s clock drift measured on $GPS$ deficient $GNB$ satellites over the span of 1 day. All time-dependent attitude functions are timestamped using the receiver’s clock filter. Without any filtering of the raw $GPS$ signal, the single point solution has a $1\sigma$ accuracy of approximately 13 m in position and 0.07 m/s in velocity [27], compared to a one week $TLE$ propagation error of 6.5 km in position and 20 m/s in velocity [42].

Figure 5.5: Novatel $OEMV-1$ series $GPS$ receiver.
5.1.2 Onboard Attitude Determination

The Extended Kalman Filter (EKF) introduced in Section 4.4 is tailored and implemented onboard NEMO-AM for the purposes of estimating the spacecraft’s inertial attitude $C_{bi}$ and inertial angular velocity $\omega_{bi}$. The Attitude EKF (AEKF) implemented onboard NEMO-AM is based on the work of Leung and Damaren [43]. Together, the inertial attitude and angular velocity represent the state $x$ of interest:

$$ x = \begin{bmatrix} \omega^T \\ q^T \end{bmatrix}^T \quad (5.3) $$

where $q$ is the quaternion counterpart of $C_{bi}$ and the $(\cdot)_{bi}$ subscript has been omitted in this subsection for conciseness. Sensor measurements can be in the form of the sun vector $\mathbf{s}_\odot$, the local magnetic field $\mathbf{b}$ or star tracker observations providing direct estimates for $C_{bi}$ and $\omega_{bi}$.

For vector measurements, the observation model is essentially a transformation of the ephemeris vectors in $F_i$ components to $F_b$ components:

$$ y = \begin{bmatrix} \mathbf{v}_b \end{bmatrix} = \begin{bmatrix} 0_{3\times3} & \mathbf{M}(\mathbf{v}_i, q) \end{bmatrix} \quad (5.4) $$

$$ \mathbf{M}(\mathbf{v}_i, q) \triangleq \begin{bmatrix} \eta \mathbf{I} + \varepsilon \times \\ -\varepsilon^T \end{bmatrix} \begin{bmatrix} \mathbf{v}_i^\times \\ \mathbf{v}_i - \mathbf{v}_i^T \end{bmatrix} \quad (5.5) $$

where $\mathbf{v}_i$ is an arbitrary vector in ECI components and for the AEKF represents either the sun vector or local magnetic field as computed using the AASV or IGRF. When star tracker measurements are processed, the observation model $g$ is simply the measured quaternion from the unit. The state $x$ is propagated by numerically integrating the attitude dynamics and quaternion kinematics from $t_{k-1}$ to $t_k$. The combined dynamics and kinematics to be integrated can be expressed as follows:

$$ \dot{x} = f(x, t) = \frac{1}{2} \begin{bmatrix} 2\mathbf{I}^{-1}(-\mathbf{\omega}^\times(\mathbf{I}\omega + \mathbf{I}_a \mathbf{\omega}_w) - \mathbf{g}_w + \mathbf{g}_c + \mathbf{g}_d) \\ \eta \mathbf{\omega} + \varepsilon \times \mathbf{\omega} \\ -\varepsilon^T \mathbf{\omega} \end{bmatrix} \quad (5.6) $$

which is derived from Eqs. (3.11), (5.1) and (5.2). By holding the control and disturbances torques constant over the time period $\Delta t_k = t_k - t_{k-1}$, the continuous motion model with process noise takes the form of Eq. (4.68) and the discretized observation model with additive sensor noise is:

$$ y_k = g(x_k) + w_k \quad (5.7) $$

where the noise variables and state estimate are assumed to be Gaussian as before. The process and measurement noise are assumed to be additive, instead of being propagated through the nonlinear models. Although this is not entirely accurate, this approximation is introduced to reduce the partial derivatives of the models with respect to the noise variables to identity,
thereby reducing the computational complexity of the flight code. However, the difference in accuracy can be recovered by suitably tuning the noise matrices $Q$ and $R$. For sun sensor and magnetic field vector measurements, the partial derivative of the observation model with respect to the state is:

$$
G_{x_k} = \begin{bmatrix} 0_{3 \times 3} & -2v_i \epsilon^T + 2(\epsilon v_i^T + \epsilon^T v_i) + 2\eta v_i^\times \epsilon v_i - \epsilon^\times v_i \end{bmatrix}
$$

(5.8)

For star tracker quaternion measurements, the expression for $G_{x_k}$ simply reduces to:

$$
G_{x_k} = \begin{bmatrix} 0_{4 \times 3} & 1_{4 \times 4} \end{bmatrix}
$$

(5.9)

The a priori estimate $\hat{x}_{k-1}$ is found by integrating Eq. (4.68) with Eq. (5.6) across $\Delta t_k$. Due to the nature of the motion model and the desire to reduce computation time, the predicted covariance $\hat{P}_{k-1}$ is computed through the matrix exponential expansion of the state transition matrix $\Phi_{x,k}$ in Eq. (4.74). The exponent matrix takes the form:

$$
F_{x,k} = \begin{bmatrix} (I\omega)^\times - \omega^\times I + (I_i\omega)^\times & 0_{3 \times 4} \\
\frac{1}{2} \begin{bmatrix} \eta & \epsilon^\times \\
-\epsilon^T & - \omega^\times \omega \end{bmatrix} & \frac{1}{2} \begin{bmatrix} -\omega^\times & \omega \\
-\omega^T & 0 \end{bmatrix} \end{bmatrix}
$$

(5.10)

The state correction and Kalman gain takes the same form as Eq. (4.64) and Eq. (4.67), respectively. However, the Joseph form of the covariance update is used to ensure that $\hat{P}_{k+}^+$ is less sensitive to roundoff errors and is symmetric and positive definite:

$$
\hat{P}_{k+}^+ = (1 - K_kG_{x,k}) \hat{P}_{k}^- (1 - K_kG_{x,k})^T + K_kR_kK_k^T
$$

(5.11)

Although the posteriori updates in the AEKF have a linear form, quaternions unfortunately do not form an additive group, since rotational quaternion sequences must be combined using the left-hand operator ($\cdot$) (see Section 3.1). The linear form of the posteriori updates inevitably violates the unit-norm constraint of quaternions. The typical remedy, as effected in this formulation of the EKF, is to brute-force normalize the updated quaternion at each time step [34][44]. This brute-force approach produces a state estimate which has the same direction as the optimal constrained estimate and thus is optimal in both a geometrical sense and a mean-square error sense [45].

When multiple sensors are used onboard the satellite, the implementation of the estimator follows a cascading procedure. Over the span of each control cycle, the state is first propagated in time to the epoch of the first sensor measurement and then updated using the AEKF. These steps are repeated for the remaining measurements, until all available measurements have been processed. Following this, the state is propagated one last time to when the attitude actuators will be commanded next. This implementation ensures that when the control laws are effected, they are done so with the most recent attitude state.
5.2 Attitude Software

The AOCS is overseen by the On-board Attitude System Software (OASYS). OASYS is responsible for making all attitude-related decisions including mode transitions, carrying out external commands, performing all the necessary computations for the ADS, evaluating the control laws for the ACS, and delivering the control commands to the satellite’s main operating system, thereby interacting with the rest of the spacecraft. The operating system, named Canadian Advanced Nanospace Operating Environment (CANOE), executes OASYS at a regular frequency of 1 Hz and is responsible for communicating directly with the attitude sensors and actuators, while relaying commands and transmitting telemetry to the other subsystems of the satellite or the satellite operator. Attitude information including attitude-related ground commands, software settings, and telemetry are exchanged between OASYS and CANOE through common data structures and arrays.

The baseline GNB software architecture for OASYS and the necessary additions for Earth-pointing missions are shown in Figure 5.6. On NEMO-AM, OASYS was expanded so that it was able to accomplish mission specific operations, namely facilitating autonomous Earth pointing for imaging applications. The baseline flight code is identical to that flown on AISSat-1, which is comprised in part by an SGP4 orbit propagator, an AASV ephemeris and an IGRF-11 magnetic-field model of the Earth. The SGP4 propagator is a subfunction of the Orbit Propagator block and calculates the satellite orbital position $\mathbf{r}$ and orbital velocity $\mathbf{v}$ based on a Two Line Element (TLE) set uploaded to the satellite. During target tracking, the Quaternion Feedback Regulator (see Section 5.3.2.3) requires high fidelity orbital state estimates and as such GPS measurements filtered through the Orbital EKF (OEKF) (see Section 4.4.1) will take the place of the $\text{SGP4}$ results. To further facilitate target tracking, the GPS measurements are fed forward in time using a High Precision Orbit Propagator (HPOP) (see Section 3.5.1). The orbital state is directly fed into the Astrodynamical Models block to calculate the expected sun vector and magnetic field ephemerides. In addition, the ADS is augmented with an IAU-1976 transformation model to provide accurate knowledge of the $C_{ie}$ rotation from ECEF to ECI. These models are used in conjunction with sensors to estimate the satellite’s attitude and angular velocities by means of the AEKF. Together, the AEKF estimates and the IAU-1976 reduction formulas allow orbital and attitude parameters to be manipulated in and across $F_e$, $F_i$ and $F_p$.

The baseline flight code also includes a Sensor Processing block which processes sun sensor and magnetometer measurements into measured ephemeris vectors which can be used by the AEKF. The AEKF block linearizes the satellite motion equations about a discrete trajectory and propagates them in time. It corrects its solution using the sensor measurements from the Sensor Processing block. Consequently the block is extended on NEMO-AM to accommodate star tracker measurements. Ultimately the AEKF outputs a sensor-corrected estimate of the satellite’s attitude and rates.
The attitude and rate estimates, orbital state and IAU-1976 knowledge are all direct inputs into the Attitude Controllers block. This block constructs an attitude, angular velocity and angular acceleration trajectory (see Section 4.2) required for pointing or tracking, compares the trajectory with the estimated satellite attitude state, and outputs actuator commands required to achieve the desired trajectory. The desired attitude state is dependent on the current mode of operation of the satellite, and is effected by three main controllers: Momentum Management, B-Dot and Three-Axis Pointing. B-Dot is used to detumble the satellite after it has been injected into orbit with non-zero tip-off body rates. In addition, the controller is used to dump momentum from the satellite when the reaction wheels or body rates have reached predetermined saturation levels. The Three-Axis Pointing controller utilizes a multi-purpose Quaternion Feedback Regulator (see Section 5.3.2.3) which tracks the trajectory required for inertial pointing or target tracking. There are multiple sub-controllers under Three-Axis pointing, each designed with a slightly different implementation in order to meet specific needs. Target tracking is one such sub-controller which uses the same Quaternion Feedback Regulator as the other Three-Axis Pointing controllers, but its implementation involves more complex controller synthesis and discrete frequency-based implementation to facilitate imaging at the required accuracy (see Section 2). The Momentum Management controller is capable of running in parallel with the Inertial Pointing controller and dumps momentum accumulated through secular disturbance torques. Once the torque commands are determined, they are sent through
the Main Thread of OASYS to CANOE, which then commands the actuators. The torque commands are also forwarded to the AEKF to propagate the satellite motion in time.

5.3 Attitude Control

The primary purpose of the Attitude Control Subsystem (ACS) is to evaluate the appropriate control torques and transmit them to the attitude actuators. These torques are used to control the attitude and body rates of the satellite in order to achieve a desired state. Onboard NEMO-AM, these actuators exist in the form of magnetic torquers (magnetorquers) and reaction wheels. Magnetorquers make use of the local magnetic field to achieve control torques which change the overall inertial angular momentum $\mathbf{h}$, whereas reaction wheels store and transfer angular momentum from different parts of the satellite, leaving $\mathbf{h}$ unchanged. For NEMO-AM, the governing requirement of the ACS is to facilitate the continuous tracking of a ground target while imaging an area equivalent to $75.1 \text{ km} \times 5.7 \text{ km}$ when satellite is directly over the target at an altitude of $650 \text{ km}$. The net pointing accuracy is primarily dictated by the exactness of the target tracking trajectory constructed onboard, and by the controller and actuators which subsequently implement the torques required to follow this trajectory. The control laws operate under the assumption that the satellite rotates about its center of mass $\mathbf{b}$ (see Section 3.6.2) and its motion equations obey Eq. (5.1). Tuning the control gains and implementing them in a discrete fashion is required to achieve optimal operation, even though the bandwidth of the system and inherent errors and saturation limits of the actuators limit the overall control performance.

5.3.1 Attitude Actuators

5.3.1.1 Reaction Wheels

Reaction wheels are the primary means of actuation for NEMO-AM, and have been developed in partnership between SFL and Sinclair Interplanetary. These momentum exchange devices act to store or exchange angular momentum between the spacecraft structure and themselves, while leaving the overall inertial angular momentum unchanged. The reaction wheels are comprised of a symmetric rotating mass (rotor), which is driven by a three-phase, poly-dipole, set of electromagnetic coils. Inside the housings, hall effect sensors are used to detect the motion of the wheel and provide feedback control for the wheels. Torques about all three body axes can be applied by placing the rotor axis along each axis of $\mathbf{F}_b$ and changing the wheel speeds from cycle to cycle. These devices however have three inherent drawbacks. First, the wheels have a set angular momentum capacity and if they are used to counter secular disturbance torques, over time they will attain unwanted gyroscopic stability until they saturate. Second, the wheels also have a maximum achievable torque which limits how readily high-speed slew maneuvers can be achieved by the satellite. Lastly, if the bias speed of the devices is sufficiently low, the friction in the wheels will cause stiction. Thus a non-zero bias speed is desired for control
authority. Stiction can also be occur when the wheel speed changes sign and crosses 0 rad/s, a behaviour denoted as zero-crossing.

The reaction wheels *NEMO-AM* employs is shown in Figure 5.7. The wheels have a rotational inertia of $8.81 \times 10^{-5}$ kg m$^2$, a momentum capacity of $60 \times 10^{-3}$ N m s, a maximum control torque of $1 \times 10^{-3}$ N m and a torque ripple error of $3 \times 10^{-5}$ N m ($2\sigma$) over a 1 s control timestep.

![Large 60 $\times$ 10$^{-3}$ N m s Wheel](image)

**Figure 5.7:** Large $60 \times 10^{-3}$ N m s Wheel.

### 5.3.1.2 Magnetorquers

Magnetic control torque $g_m$ is a form of active control which changes the overall inertial angular momentum of the satellite and is the primary means of dumping accumulated momentum in the spacecraft and wheels. The interaction between the total local magnetic field and the magnetic moment $\mathbf{m}$ onboard the satellite produces this magnetic torque which is governed by Eq. (3.70). The desired magnetic moments are generated onboard *NEMO-AM* by means of magnetorquers, shown in Figure 5.8. These torquers have no control authority in the direction of the local magnetic field. The units are current-controlled copper coils fitted inside printed circuit boards which are designed in-house at *SFL*. Their magnetic moment is directly proportional to the number of coil loops, the loop area and the supply current through the coil, and can be computed using Eq. (3.69). When utilizing the magnetometer, the magnetorquers are turned off altogether to avoid corrupting the measurements [20].

![Magnetorquer](image)

**Figure 5.8:** Magnetorquer.

As actuators, the magnetorquers are turned off during target tracking and do not influence the
pointing performance of *NEMO-AM*. They are sized such that their saturation levels are able to provide torques larger than the external disturbance torques causing momentum buildup on the wheels. The magnetorquer design on *NEMO-AM* has a magnetic moment of 0.54 A·m², with a current saturation limit of 100 mA. When saturation in any of the three torquers is detected, the flight software normalizes the commanded moment with respect to the largest component of \( \mathbf{m} \), so that directionality is maintained.

### 5.3.2 Control Laws

The control torques \( \mathbf{g}_c \) which the *ADS* commands, are designed to either stabilize the satellite attitude or perform slewing maneuvers to track specific trajectories. For *NEMO-AM*, the primary mission tasks involve continuously pointing its imager towards a ground target which is moving with the Earth, while the satellite itself is traveling in its orbit. Thus, unlike inertial pointing missions, a changing attitude and rate profile must be tracked in order to meet the mission accuracy and stability requirements (see Section 2). The values of the torques are a direct result of control laws which the satellite effects based on the desired AOCS task. This subsection presents the control laws and the subsequent control torques \( \mathbf{g}_c \), implemented onboard *NEMO-AM*.

#### 5.3.2.1 Momentum Management

In the presence of external (environmental) disturbances and no active attitude control, the spacecraft accumulates angular momentum as its angular velocity is perturbed. If reaction wheels are used for active control and stabilization, then the accumulated angular momentum is transferred to the wheels instead, as the satellite attempts to reject the disturbances. If the external disturbances are constant or secular, the momentum stored in the wheels will continue to rise, until saturation is reached and control authority is lost. To avoid this detrimental effect, the momentum from the wheels must be continuously dumped. This momentum dumping is performed through the magnetic torque expression in Eq. (3.70) and has the following form expressed and observed in \( \mathcal{F}_b \):

\[
\mathbf{g}_c = \mathbf{m}_b \times \mathbf{b}_b
\]

\[
\mathbf{m}_b = -K_m (\mathbf{b}_b \times \Delta h_w) \frac{1}{\|\mathbf{b}_b\|} \tag{5.12}
\]

where \( \mathbf{m}_b \) is the generated magnetic dipole of the magnetorquer, \( \mathbf{b}_b \) is the measured total local magnetic field, \( K_m \) is a positive-definite gain matrix, \( \Delta h_w = h_w - \overline{h}_w \) is the excess axial wheel momentum, \( h_w \) is the axial angular momentum vector of the wheels and \( \overline{h}_w \) is the nominally desired wheel axial angular momentum vector. The form of Eq. (5.12) implies that no magnetic torque can be applied in the direction of the local magnetic field \( \mathbf{b}_b \). In addition, after combining Eqs. (5.12) and (5.13), it is also apparent that when the excess
wheel momentum $\Delta h$ is in the direction of $b$, again, magnetic torque cannot be generated. This limiting attribute is highly dependent on the satellite’s orbit and attitude, and is most prominent for equatorial orbits. When effecting slew maneuvers, the magnetic controller acts to counteract the commanded motion of the satellite [46]. Consequently, on NEMO-AM, the Momentum Management controller is not run in parallel to the Quaternion Feedback Regulator controller (see Section 5.3.2.3) during mission critical (i.e. target tracking) situations, so that the pointing capability is maximized. In all other nominal mission scenarios, the controller operates in the background rejecting accumulated angular momentum.

5.3.2.2 B-Dot Control

Certain situations may arise where the angular velocities of the satellite or the angular momentum in the wheels are close to saturation. To reduce elevated body rates and wheel speeds, NEMO-AM employs a B-Dot controller which has the form of Eq. (5.12), but with the following magnetic dipole observed and expressed in $F_b$:

$$m_b = -K_b \dot{b}_b$$  \hspace{1cm} (5.14)

where $\dot{b}$ is the time rate of change of the local magnetic field observed in $F_b$ and $K_b$ is a positive-definite gain matrix. This controller minimizes the angular velocity of the satellite by exploiting the fact that the rate of change in the local magnetic field due to the spacecraft orbital motion is far less than that owing to the rotation of the satellite. In practice, the angular velocity of the satellite $\omega_{bi}$ does not necessarily converge to nil, but instead aligns with the local magnetic field $b$. This limitation can be overcome due to the varying direction and magnitude of $b$ throughout a satellite’s orbit [47]. The B-Dot controller is especially useful after the satellite has been separated from its launch vehicle and has an initial tip-off angular velocity.

5.3.2.3 Quaternion Feedback Regulator

The primary controller for operational tasks onboard NEMO-AM is a three-axis linear Quaternion Feedback Regulator used for inertial and quasi-inertial pointing, and target tracking maneuvers. Inertial and quasi-inertial pointing include sun pointing, star pointing and nadir pointing, and in general is simpler to implement since targets are fixed in inertial space or moving at relatively slow rates. Target tracking refers to attitude maneuvers which continuously point the satellite at an inertially moving target and is a much more challenging task given that the dynamics of the spacecraft and the inertially moving target must be incorporated in the controller implementation. Quaternion feedback can be implemented for both types of pointing maneuvers. This feedback is especially advantageous for guaranteed globally stable large-angle attitude maneuvers [48]. The control torque has the following form expressed and observed in $F_b$:

$$g_c = -K_p \epsilon_e - K_d \Delta \omega - K_i \int \epsilon_e \, dt + \omega^x (I \omega + I_a \dot{\omega}_w)$$ \hspace{1cm} (5.15)
where the last term in $g_c$ is the gyroscopic decoupling torque, which is composed of parameters estimated by the satellite. These parameters have their overhats ($\hat{\cdot}$) omitted for conciseness. $K_p$, $K_d$ and $K_i$ are the positive definite proportional, derivative and integral matrices, respectively. Moreover, $\Delta \omega = \omega - \omega_d$ is the angular velocity error, $\omega$ is the onboard estimated angular velocity of $F_b$ about $F_i$, $\omega_d$ is the corresponding desired angular velocity and $\varepsilon_e$ is the vector portion of the quaternion error which is derived from the attitude error $C_e$ (see Section 3.1). This implementation performs body referenced Euler-axis rotations by regulating $\varepsilon_e$, while tracking the desired angular velocities $\omega_d$ (i.e. regulating $\Delta \omega$). Operationally, the angular velocities $\omega_d$ that would be tracked during nadir pointing and target tracking are the orbital angular velocity $\omega_{oi}$ and the target tracking angular velocity $\omega_{bi}$ (see Section 4.2), respectively. Since the sun vector $\vec{s}_\odot$ essentially stays fixed in inertial space, only the quaternion error is regulated during sun tracking operations. In addition, the controller also feedfowards the satellite dynamics by employing the aforementioned decoupling gyroscopic torque. Exact cancellation of the gyroscopic coupling torque is impossible, but its impact can be lessened if the estimates of the satellite second moment of inertia $I$, the satellite angular velocities $\omega$, the wheel axial-inertia matrix $I_a$ and the wheel speeds $\omega_w$ are all sufficiently accurate. For trajectories which have a parabolic shape, the desired angular accelerations must also be fed forward if the steady state errors are to approach zero over the control timestep $\Delta t_c$.

The method of choosing gains for Euler-axis rotations can be derived by considering a simpler case where $\omega$ is regulated (i.e. $\omega_d = 0$) and the integral gain and disturbance torques $g_d$ are disregarded. Under these conditions, the satellite system dynamics can be formed by combining Eq. (5.1) and Eq. (5.15). Existing input-output and Lyapunov stability proofs demonstrate that to achieve globally stable control about the Euler-axis, there must be exact cancellation of the gyroscopic coupling torque and both $K_p$ and $K_d$ must be proportional to $I$ [10]:

$$K_p = k I; \quad K_d = d I$$  \hspace{1cm} (5.16)

Assuming exact gyroscopic torque cancellation, these control gain choices can be substituted into the aforementioned satellite system dynamics, thereby reducing the motion equations of the system to:

$$\left( \ddot{\phi} + d \dot{\phi} + k \sin \frac{\phi}{2} \right) I_a = 0$$  \hspace{1cm} (5.17)

where, the Euler-axis rotation has been enforced by assuming that the rotation is to be performed about a fixed axis $a$, such that $\omega = \dot{\phi} a$. For the purposes of choosing gains, the term $I_a$ can be neglected and the rotational dynamics can be linearized to first order $O(\phi)$, so that Eq. (5.17) reduces to:

$$\left( \ddot{\phi} + d \dot{\phi} + k \frac{\phi}{2} \right) = 0$$  \hspace{1cm} (5.18)
The expression in Eq. \(5.18\) is of the form of an unforced second order equation in \(\phi\), whose damping ratio \(\zeta\) and natural frequency \(\omega_n\) satisfy the following \[49\]:

\[
d = 2\zeta\omega_n; \quad k = 2\omega_n^2
\]  \hspace{1cm} (5.19)

Thus the proportional and derivative gain matrices can be evaluated by selecting the appropriate damping ratio and natural frequency. These parameters can be determined by choosing a desired settling time \(t_s\) in between two control cycles of OASYS. The settling time to within 2% error for Eq. \(5.18\) is \[10\]:

\[
t_s = \frac{8}{\zeta\omega_n}
\]  \hspace{1cm} (5.20)

where the expression has been modified slightly from that found in literature in order to account for the linearization of \(\sin \frac{\phi}{2}\). It is important to note that in general, exact cancellation of the gyroscopic coupling torque will not be achieved due to inaccuracies in the feed forward estimation. Nevertheless, in practice, the effect of the gyroscopic torque is still diminished by feed forwarding the appropriate satellite dynamics thereby facilitating near-Euler-axis maneuvers.

For reference, a more comprehensive proof for regulating \(\Delta\omega\) exists and can be found in literature \[47\]. In addition, in the presence of external disturbance torques \(\mathbf{g}_d\), the proper choice of integral gains are able to reject the constant torque components. Although global stability is not guaranteed with the addition of integral gain, the integral gain matrix \(\mathbf{K}_i\) can be tuned by analyzing the stabilizing condition for the satellite Euler-axis maneuver \[48\].

### 5.4 Attitude Simulation

The AOCS design of NEMO-AM was validated using SFL’s high-fidelity simulation framework Mirage. Simulations were desired to be carried out over a period of 24 hours to gauge the performance of the system. The time constants of celestial phenomena, orbital phenomena and the spacecraft attitude dynamics do not necessitate longer simulations for understanding the system’s behaviour. The framework integrates three main elements. These elements include the satellite architecture, orbital environment and spacecraft dynamics in a MATLAB Simulink setting. The legacy installment of Mirage has most notably been used to validate AISSat-1, but several significant advances were implemented by the author so that satellite AOCS performance could be validated to an accuracy sufficient to verify the compliance to mission requirements.

A high-level diagram of Mirage is presented in Figure \[5.9\]. The onboard flight code OASYS is unchanged from Figure \[5.6\] and is connected to the Plant Dynamics block which amalgamates and numerically integrates representative models of the hardware, celestial phenomena, the orbital environment and the satellite dynamics. The Plant Dynamics block serves as a truth model for the simulation framework and is elaborated below.

The outer satellite structure is decomposed into planar surface elements. In addition, the satellite architecture is modeled by implementing its mass, second moment of inertia, drag co-
efficient, parasitic dipoles and surface spectral properties, along with all of its attitude sensor and actuator characteristics. Each hardware component is tested at the unit level to gauge the inherent errors in its performance (see Sections 5.1.1 and 5.3.1) and modeled accordingly in the Actuator Models and Sensor Models blocks. The timing delays associated with transmitting sensor measurements and commanding actuators are also implemented. The orbital environment (see Section 3) is simulated by integrating the 2008 Earth Gravitational Model (EGM2008) to degree 21 and order 21 inside the Orbital Dynamics block. For simulations over the desired period of 24 hours, larger gravity models do not provide greater fidelity for the computational cost incurred. Environmental disturbances including those arising from gravitational, aerodynamic, solar and magnetic forces are considered inside the aptly named Environmental Disturbances block. Since the perturbative forces are minute compared to the Earth’s gravitational force, only disturbance torques are realized while the orbital perturbative contributions over 24 hours are left unmodeled. Long-term orbital consequences of these forces, if desired, are left for auxiliary lifetime simulations. The spacecraft motion is simulated in the Spacecraft Dynamics block by approximating the motion equations using Eq. (5.1). First, the temporal derivative of the inertial angular velocities is isolated and integrated in time to solve for $\omega$. The inertial angular velocities are then substituted into Eq. (3.11), combined with the current attitude and then integrated in time to obtain the propagated quaternion parameters $\varepsilon$ and $\eta$.

The simulations are performed on a system level by combining all three elements. Measurements from the non-idealized sensor models are fed into the flight code OASYS on which the estimation and control implementations are executed. The commanded control torques are then perturbed through the noise-corrupted actuator models and applied to the Spacecraft Plant Dynamics block. The results are exported to the standard MATLAB environment and Satellite Tool Kit (STK) for analyzing the AOCS performance. This approach is able to assesses real mission scenarios and validate flight performance using ground-characterized hardware test data, so that the expected performance is predicted and tailored to mitigate mission risk before launch, and assure the likelihood of operational success.
Mirage

Figure 5.9: Mirage simulation framework with additions for Earth-pointing highlighted in red.
Chapter 6

NEMO-AM

In this chapter, the current Attitude and Orbit Control System (AOCS) design of NEMO-AM is presented in detail. First, the operational sequence of the mission and the overall AOCS capabilities of the satellite are explored. This is followed by validating the expected operational performance, as stipulated in the mission description (see Section 2), through the high fidelity simulation framework described in Section 5.4. The overall objective is to demonstrate that the AOCS design is compliant with the stipulated requirements and, its performance is robust to the operational sequence, hardware behaviour and expected orbital environment. There is confidence in this approach since its foundation is built on lessons learned from legacy missions such as AISSat-1 which were tested in a similar fashion and ultimately successful.

6.1 Attitude Modes of Operation

NEMO-AM envisions three principal operational attitude modes: Passive (determination only), Rate Damping (detumbling and momentum desaturation) and Three-Axis Control. The latter mode has several submodes including Nadir Pointing, Sun Tracking and Target Tracking. The Target Tracking submode itself can be broken down into a High Speed Communication submode for data downlink to a groundstation and a Site Imaging submode during Imaging Campaigns. A description of these modes and their anticipated performance are presented in Table 6.1. Target tracking is envisioned to be performed autonomously, during which time the AOCS must transition itself from its current state to the Target Tracking submode. This entails switching to the appropriate sensor and actuator combination, transitioning onboard dynamic filters and computing imaging specific parameters (i.e. tracking trajectory) such that the knowledge and pointing requirements are met, ideally with margin.

The nominal timeline of the attitude modes in the mission life of NEMO-AM is presented in Figure 6.1. After launch vehicle separation, NEMO-AM will be placed in an Attitude Safe mode, where the attitude subsystem is off. Once the critical aspects of the satellite have been checked out, the AOCS will be brought online progressively to commission its parts and the system as a whole.
Table 6.1: Attitude modes of operation. Expected performance based on pointing budgets.

<table>
<thead>
<tr>
<th>Mode (Submode)</th>
<th>Type</th>
<th>Description</th>
<th>Expected Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>Quiescent</td>
<td>The Attitude system is off.</td>
<td>N/A</td>
</tr>
<tr>
<td>Passive</td>
<td>Quiescent</td>
<td>No control. Determination only. Used during initialization, commissioning and as a fall-back mode in case of hardware failure.</td>
<td>≤ 3.8° (2σ) (determination)</td>
</tr>
<tr>
<td>Rate Damping</td>
<td>Maintenance</td>
<td>Magnetic actuation to remove energy from the system and minimize body rates or wheel speeds. Intended for use after a tumble-inducing event or elevated momentum levels. A magnetometer and magnetorquers alone are necessary for this mode.</td>
<td>Minimize rates after tip off in under 6 orbits.</td>
</tr>
<tr>
<td>Three-Axis (Nadir Pointing)</td>
<td>Three-Axis Control (Quasi-Inertial)</td>
<td>Use of sun sensors (if available), magnetometer, magnetorquers, GPS and wheels to keep one face of the satellite pointed directly below, at the Earth.</td>
<td>≤ 4.8° (2σ)</td>
</tr>
<tr>
<td>Three-Axis (Sun Tracking)</td>
<td>Three-Axis Control (Quasi-Inertial) + Maintenance</td>
<td>Use of sun sensors (if available), magnetometer, magnetorquers and wheels to keep the +X face (see Section 2) pointed towards the Sun while maximizing the accessibility of the GPS constellation using the GPS receiver.</td>
<td>≤ 4.8° (2σ)</td>
</tr>
<tr>
<td>Three-Axis (High Speed Communication)</td>
<td>Target Tracking</td>
<td>Use of sun sensors (if available), magnetometer and wheels to continuously point the satellite at a chosen groundstation. This is intended to maximize downlink capability.</td>
<td>≤ 4.8° (2σ)</td>
</tr>
<tr>
<td>Three-Axis (Site Imaging)</td>
<td>Target Tracking + Inertial</td>
<td>Use of GPS, star tracker and wheels to continuously point the satellite at a chosen site. The tracking maneuver is set up by slewing to hold the beginning attitude of the target tracking trajectory before beginning the Imaging Campaign.</td>
<td>≤ 0.8° (2σ)</td>
</tr>
</tbody>
</table>

Once the functionality of the AOCS components have been satisfactorily affirmed and the EKF has been brought online, NEMO-AM will be commanded to enter a Rate Damping mode to minimize angular momentum imparted to the satellite by the launch vehicle and the separation mechanism. This will be accomplished by using a magnetometer and three magnetorquers. Following this, the spacecraft will be commanded to begin nominal operations by entering the Sun Tracking submode, where the large +X panel will be continuously facing the Sun and the +Y face will oriented to maximize the accessibility of the GPS constellation (see Section 2). In this mode, the basic suite of attitude sensors (six sun sensors, one magnetometer), three reaction
wheels and three magnetorquers will be used when the satellite is in sunlight. During eclipse, the star tracker will replace the basic sensor suite since it will not be susceptible to obscuration and the improved knowledge in the attitude state will allow for more efficient satellite maintenance. While in Sun Tracking mode, ground commands will upload the scheduled observation times for Imaging Campaigns, the terrestrial coordinates \((h_t, \phi_t, \lambda_t)\) of the associated targets during those Campaigns, the corresponding inertial angular momentum required to capture those images and subsequently, the scheduled downlink transmission periods for the images captured. At the scheduled observation epochs, the satellite will exit Sun Tracking submode and enter the Site Imaging submode to carry out an Imaging Campaign (see Section 2).

During Site Imaging, \textit{NEMO-AM} will switch to using only its star tracker for attitude knowledge as it images the Earth. The satellite will autonomously set up the imaging maneuver by holding the beginning attitude of the target tracking trajectory for a predetermined time denoted as the premaneuver period. Once line of sight has been acquired by the satellite, it will compute the tracking trajectory and subsequently track it. When the Campaign is complete, the satellite will enter the High Speed Communication submode to transmit the captured science data if it is scheduled to do so. Otherwise, it will return to the Sun Tracking submode until the scheduled transmit period. The satellite will alternate between High Speed Communication Mode and Sun Tracking mode based on the available link with the ground station and until the image data remaining on the satellite is completely transmitted. Once transmission of the data is finished, the satellite will return to the Sun Tracking submode until its next scheduled Imaging Campaign. The Nadir Pointing submode is not expected to be a recurring operational mode, but it is envisioned to facilitate commissioning and other check-out procedures. Active momentum dumping will be applied in the Sun Tracking and Nadir Pointing submodes, but will be switched off during Site Imaging to maximize pointing performance.

![Figure 6.1: Nominal timeline of NEMO-AM attitude modes. Nadir pointing mode not shown.](image)

Attitude operations are conceptually straightforward, such that the mode transitions are relatively simple. The high-level operational sequencing of attitude modes and submodes is shown in Figure 6.2. The nominal progression of modes is represented using blue arrows. Should difficulties arise during operation, then autonomous transitions will follow the red arrows and...
enter fall-back modes depending on the nature of the complication. *NEMO-AM* will enter a Rate Damping mode if the angular momentum in any wheel reaches an elevated threshold. If instead the satellite momentum reaches an elevated threshold, the spacecraft will transition to Passive mode. In addition, if there are difficulties with the attitude sensors or actuators, the satellite will also enter Passive mode until it can be checked out by a ground operator. The exact details of the problems managed by *NEMO-AM* are defined in detail inside the flight software *OASYS*.

![Nominal attitude mode transition diagram for NEMO-AM](image)

Figure 6.2: Nominal attitude mode transition diagram for *NEMO-AM*. 
6.2 Pointing Budgets

Before any modeling or simulations were carried out, attitude hardware and sensors were chosen and pointing budgets were constructed in order to assess the feasibility of meeting mission requirements. The chosen attitude hardware is presented in Section 5. The pointing budgets are presented in this section and are constructed for a ground-track equatorial target from an altitude of $h = 600$ km, since this represents the worst case imaging scenario for attitude control. Since the GPS clock error is on the order of 20 ns (RMS) \[27\] and there exist sufficiently accurate models for the Earth geoid and its rotation (see Section 3), knowledge in the target state ($\mathbf{x}, \nu$) and the Earth's rotation ($\mathbf{C}_{\text{re}}$) are assumed to be perfect when a GPS receiver is available.

Each pointing budget is separated into pointing error $\delta \theta$ and ground track error $\delta s$ resultants. In addition, each line term is grouped under either short, medium or long term errors. Short term errors have a time scale on the order of the controller bandwidth. Medium term errors behave on the order of the orbital period and cannot be filtered out by the Attitude EKF ($AEKF$) or Orbital EKF ($OEKF$). Long term errors have time scales longer than the orbital period. These errors are typically biases or changes in biases. A description of the relevant error terms is presented in Table 6.2. The constituents of each category are root-sum-squared (RSS) and afterwards, the categories are arithmetically summed together. The $2\sigma$ value of each error term is used wherever possible, thereby yielding conservative $2\sigma$-like net pointing and ground track errors.

There are two pointing budgets presented in this section, with each having a different combination of attitude sensors. The first pointing budget in Table 6.3 uses the basic suite of determination sensors (six sun sensors and one magnetometer), but with aggregate sensor accuracy values characterized from AISSat-1 on-orbit data. The second pointing budget in Table 6.4 replaces the basic determination suite with a star tracker since it is the only sensor expected to be supplied to the $AEKF$ during the Imaging Campaign. This budget is representative of the expected performance during target-tracking operations for imaging and high speed communication in eclipse. The first pointing budget in Table 6.3 is representative of high speed communication in sunlight. For all modes, where the sun sensors and magnetometer are anticipated to be used exclusively, the first pointing budget, less the peak tracking error line term, is representative of the expected performance.
Table 6.2: Description of types of error presented in pointing budgets.

<table>
<thead>
<tr>
<th>Short Term Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Controller Error</strong></td>
</tr>
<tr>
<td>Peak controller error with perfect knowledge of orbital/attitude state, and rate tracking error causing pixel smearing. Rate error is decomposed into rate estimation error leading to reduced stability performance and satellite velocity errors leading to inaccurate construction of the target tracking angular velocity trajectory $\dot{\omega}_{bi,k}$.</td>
</tr>
<tr>
<td><strong>Hardware Error</strong></td>
</tr>
<tr>
<td>Wheel torque ripple causing controller error and imager exposure causing synchronization error in image capture.</td>
</tr>
<tr>
<td><strong>Sensor Error</strong></td>
</tr>
<tr>
<td>Sun sensor/magnetometer noise, and star tracker accuracy, characterized through ground tests and leading to attitude knowledge error.</td>
</tr>
<tr>
<td><strong>Orbital State Error</strong></td>
</tr>
<tr>
<td>Clock error leading to incorrect propagation of onboard TLE, when a TLE is the primary source of orbital state knowledge. Otherwise, when a GPS is the primary source, these terms are composed of the intrinsic GPS inaccuracies and the synchronization error on the GPS onboard clock.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Medium Term Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Disturbance Torques</strong></td>
</tr>
<tr>
<td>Estimated onboard parasitic magnetic dipoles and environmental disturbances in the form of gravity gradient, aerodynamic and solar pressure torques, causing controller error.</td>
</tr>
<tr>
<td><strong>Ephemeris Error</strong></td>
</tr>
<tr>
<td>Clock error and satellite position error (intrinsic TLE/GPS inaccuracies) causing an incorrect calculation of onboard Astronomical Almanac Sun Vector (AASV) and International Geomagnetic Reference Field (IGRF) ephemerides.</td>
</tr>
<tr>
<td><strong>Orbital State Error</strong></td>
</tr>
<tr>
<td>Error in orbital state due to intrinsic TLE inaccuracies.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long Term Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Actuator Error</strong></td>
</tr>
<tr>
<td>Wheel axis misalignment causing control error.</td>
</tr>
<tr>
<td><strong>Ephemeris Error</strong></td>
</tr>
<tr>
<td>Intrinsic inaccuracies in the onboard AASV and IGRF ephemerides leading to attitude knowledge error.</td>
</tr>
<tr>
<td><strong>Sensor Error</strong></td>
</tr>
<tr>
<td>Sun sensor and magnetometer inaccuracies, and attitude sensor misalignments, leading to attitude knowledge error. Sensor accuracies are either characterizations from ground testing or recent on-orbit experience from AISSat-1.</td>
</tr>
</tbody>
</table>
### Nominal Operation Pointing Budget

Table 6.3: Pointing budget for *NEMO-AM* Site Imaging operations using basic sensor suite.

<table>
<thead>
<tr>
<th>Category</th>
<th>Error Source</th>
<th>Error Value</th>
<th>$\delta \ell = 0^\circ$</th>
<th>$\delta \ell = 30^\circ$</th>
<th>$\delta \ell = 60^\circ$</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Orbital State Error</strong></td>
<td>GPS $\delta r$</td>
<td>$\delta r = 0.075$ km</td>
<td>0.01</td>
<td>0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GPS synchronization lag</td>
<td>100 ms</td>
<td>0.07</td>
<td>0.69</td>
<td>0.06</td>
<td>0.69</td>
<td>0.04</td>
<td>0.69</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>camera image synchronization</td>
<td>25 ms</td>
<td>0.02</td>
<td>0.19</td>
<td>0.01</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>wheel zero crossing</td>
<td>1 mNm</td>
<td>0.14</td>
<td>1.45</td>
<td>0.14</td>
<td>1.91</td>
<td>0.14</td>
<td>5.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Hardware Error</strong></td>
<td>peak tracking error</td>
<td>0.30$^\circ$ at $\delta \ell = 0^\circ$</td>
<td>0.30</td>
<td>3.21</td>
<td>0.26</td>
<td>3.54</td>
<td>0.10</td>
<td>3.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.26$^\circ$ at $\delta \ell = 30^\circ$</td>
<td>0.30</td>
<td>3.21</td>
<td>0.26</td>
<td>3.54</td>
<td>0.10</td>
<td>3.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.10$^\circ$ at $\delta \ell = 60^\circ$</td>
<td>0.30</td>
<td>3.21</td>
<td>0.26</td>
<td>3.54</td>
<td>0.10</td>
<td>3.70</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>rate estimation error $\sim \delta \omega$</td>
<td>0.37$^\circ$/s</td>
<td>0.74</td>
<td>7.66</td>
<td>0.74</td>
<td>10.08</td>
<td>0.74</td>
<td>27.37</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Controller Error</strong></td>
<td>RSS of Short Term Errors</td>
<td>0.8</td>
<td>8.5</td>
<td>0.8</td>
<td>10.9</td>
<td>0.8</td>
<td>28.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Medium Term Errors</strong></td>
<td>RSS of Medium Term Errors</td>
<td>0.2</td>
<td>1.6</td>
<td>0.2</td>
<td>2.1</td>
<td>0.2</td>
<td>4.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Long Term Errors</strong></td>
<td>RSS of Long Term Errors</td>
<td>3.8</td>
<td>39.6</td>
<td>3.8</td>
<td>52.2</td>
<td>3.8</td>
<td>141.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Error</strong></td>
<td></td>
<td>4.8</td>
<td>49.7</td>
<td>4.8</td>
<td>65.1</td>
<td>4.7</td>
<td>174.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.2.2 Imaging Campaign Pointing Budget

Table 6.4: Pointing budget for NEMO-AM Site Imaging operations using star tracker only.

<table>
<thead>
<tr>
<th>Short Term Errors</th>
<th>Category</th>
<th>Error Source</th>
<th>Error Value</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
<th>$\delta \theta$</th>
<th>$\delta s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\delta r$</td>
<td></td>
<td>$\delta r$</td>
<td></td>
<td>$\delta r$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Orbital State Error</td>
<td>$\delta r$ = 0.075 km</td>
<td>0.01</td>
<td>0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GPS synchronization lag</td>
<td>100 ms</td>
<td>0.07</td>
<td>0.69</td>
<td>0.06</td>
<td>0.69</td>
<td>0.04</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hardware Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>camera image synchronization</td>
<td>25 ms</td>
<td>0.02</td>
<td>0.19</td>
<td>0.01</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>wheel zero crossing</td>
<td>1 mNm</td>
<td>0.14</td>
<td>1.45</td>
<td>0.14</td>
<td>1.91</td>
<td>0.14</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Controller Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>peak tracking error</td>
<td></td>
<td>0.30°</td>
<td>3.21</td>
<td>0.26</td>
<td>3.54</td>
<td>0.10</td>
<td>3.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rate estimation error $\approx \delta \omega$</td>
<td></td>
<td>0.13°/s</td>
<td>0.16</td>
<td>1.65</td>
<td>0.16</td>
<td>2.18</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>star tracker error</td>
<td>0.04°</td>
<td>0.04</td>
<td>0.41</td>
<td>0.04</td>
<td>0.55</td>
<td>0.04</td>
<td>1.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RSS of Short Term Errors</td>
<td></td>
<td>0.4</td>
<td>4.0</td>
<td>0.3</td>
<td>4.7</td>
<td>0.2</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Medium Term Errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Disturbance Torques</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gravity Gradient,</td>
<td></td>
<td>0.10</td>
<td>1.04</td>
<td>0.10</td>
<td>1.33</td>
<td>0.10</td>
<td>3.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Aero, Solar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>parasitic dipoles</td>
<td>0.05 A m$^2$ per axis</td>
<td>0.12</td>
<td>1.24</td>
<td>0.12</td>
<td>1.59</td>
<td>0.12</td>
<td>3.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RSS of Medium Term Errors</td>
<td></td>
<td>0.2</td>
<td>1.6</td>
<td>0.2</td>
<td>2.1</td>
<td>0.2</td>
<td>4.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long Term Errors</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sensor Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>star tracker alignment</td>
<td>0.30°</td>
<td>0.30</td>
<td>5.18</td>
<td>0.30</td>
<td>6.63</td>
<td>0.30</td>
<td>15.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Actuator Error</td>
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<td>0.01</td>
<td>0.10</td>
<td>0.01</td>
<td>0.14</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RSS of Long Term Errors</td>
<td></td>
<td>0.3</td>
<td>5.2</td>
<td>0.3</td>
<td>6.6</td>
<td>0.3</td>
<td>15.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Error</td>
<td></td>
<td>0.8</td>
<td>7.9</td>
<td>0.7</td>
<td>13.4</td>
<td>0.6</td>
<td>28.7</td>
</tr>
</tbody>
</table>
Each line term is either calculated by substituting the appropriate error values in the third column of Tables 6.3 and 6.4 into the error expressions ($\delta \theta$, $\delta s$) in Section 4.1, integrating the error directly over the control timestep $t_c$ or determined through the Mirage simulation framework in Section 5.4. Since these budgets are meant to correspond to errors specifically in pointing, inaccuracies involving angular distances are decomposed into only nadir angle errors $\delta \gamma$ and not azimuth errors $\delta \vartheta$, before computing the overall pointing error $\delta \theta$. The detailed errors in satellite orientation are left to be determined through simulation (see Section 6.4). Errors in orbital or target position correlate almost one to one with the ground track error $\delta s$. GPS position and velocity inaccuracies are extracted from on-orbit data \cite{40} and conservatively estimated to be $\|\delta r\| = 75.0$ m and $\|\delta v\| = 0.06$ m/s. Although any non-zero value of $\delta r$ will lead directly to pointing inaccuracies, the typical GPS $\delta v$ is too small to cause computations in the target tracking angular velocity (see Section 4.2.2) to lead to pointing errors greater than $0.01^\circ$ \cite{20}. Thus, $\delta v$ line terms are omitted in the Controller Error category. The GPS synchronization lag is the greatest possible error in resolving the onboard GPS clock and is estimated to be 100 ms. The camera synchronization lag is the time between a commanded image and when an image has completed its exposure. Nominally, this quantity is the average imager exposure duration of 25 ms. Wheel zero-crossings are inevitable during NEMO-AM’s imaging maneuvers and from a static analysis, are estimated to result in a $0.14^\circ$ pointing error. The peak tracking error is the satellite controller error, as simulated through Mirage, in tracking the target with perfect estimation and no feed-forward of target and satellite states. This controller error is greatest when the satellite is directly over the target ($\delta \ell = 0^\circ$), resulting in the the fastest tracking slew rate. With a basic suite of sun sensors and magnetometer, the rate estimation error can be as high as $0.37^\circ$/s ($2\sigma$). If the S3S star tracker is used instead, the rate estimation error is expected to be $0.13^\circ$/s ($2\sigma$). Errors in the rate estimate are transformed directly to pointing errors by integrating the rate inaccuracies over $t_c$. The pointing error due to the disturbing environment and estimated parasitic dipoles from hardware testing are extracted from the simulation framework (see Sections 3.7, 5.4 and 6.4). Although aggregate flight characterized sun sensor and magnetometer accuracies include both short term and long term errors, for conciseness they are grouped into the long term errors category. In both pointing budgets, hardware misalignment errors are conservatively estimated as $0.30^\circ$ to account for any oversight during integration of the satellite. The general trend in the pointing budgets is that as the declination viewing angle $\delta \ell$ increases, the mapping error $\delta s$ rises due to the increased distance from the satellite to the target (see Figure 6.9(b)). The pointing error $\delta \theta$ remains relatively constant since the angular disturbances are generally independent of the declination viewing angle $\delta \ell$. 
Table 6.3 shows that with a GPS receiver and no star tracker on NEMO-AM, the RSS of the short term pointing errors are a constant 0.8° (2σ) with varying declination viewing angle, since the dominant line term error sources are independent of δℓ. The estimated RSS of the short term ground track errors range from 8.5 km to 28.1 km and are most heavily influenced by the rate estimation error of 0.37°/s arising from the use of the sun sensors and the magnetometer. The medium term errors show the same trend in pointing and ground-track errors as compared to the short term errors. The RSS of the long term pointing errors are estimated to be 3.8° (2σ) and weighted most heavily by the on-orbit sun sensor and magnetometer aggregate errors.

These aggregate errors provide the greatest contribution to the total budget, resulting in ground track errors of 39.6 km, 52.2 km and 141.6 km for δℓ values of 0°, 30° and 60°, respectively. Misalignment errors are not included separately, since they are already accounted for in the aggregate terms. Overall, with a GPS and no star tracker on NEMO-AM, a pointing error of 4.8° (2σ) and a ground track error of 49.7 km (2σ) is anticipated during target tracking directly over the target (δℓ = 0°).

Table 6.4 demonstrates that when a star tracker is used instead for target tracking, the RSS of the short term pointing and ground track errors decrease by at least a factor of two. Unlike the on-orbit aggregate sun sensor and magnetometer errors, the star tracker accuracy is standalone and thus is listed as a short term error of 0.13°/s (2σ). The medium term errors are unchanged from the pointing budget in Table 6.3 since disturbance torques are independent of the choice of hardware. The long term errors include alignment errors of the star tracker and the reaction wheels. The RSS long term pointing errors are a constant 0.5° (2σ) with varying δℓ. This value is largely owing to the star tracker misalignment because wheel misalignments are observed as disturbance torques by the satellite which can be compensated by the controller feedback loop. The star tracker misalignment is expected to be calibrated on orbit and so the total long term errors listed in Table 6.4 are expected to be less pronounced during NEMO-AM’s actual operation. Overall, exclusively using a star tracker during target tracking alleviates the errors associated with the basic sensor suite and consequently a pointing accuracy of 0.8° (2σ) and a ground track error of 7.9 km (2σ) is expected to be achieved. The RSS errors are conservative at best since the largest contributors are the controller error and start tracker misalignment line terms. These terms do not stack up in a monotonic RSS fashion and are readily countered by controller feedback, feedforward dynamics and on-orbit calibration. Thus, although the RSS errors meet the overarching pointing accuracy requirements in Section 2 in practice, the errors will be reduced in orbit.
6.3 Earth Observation Considerations

Once the baseline AOCS design for NEMO-AM was chosen, mission planning considerations were made in order to determine what terrestrial locations could be observed, where in the satellite’s orbit images could be taken and in general, what the overall imaging capabilities were for the current design. In this section, these considerations for NEMO-AM are presented for an East-alignment attitude. The different attitude options are explicitly presented in Table 2.1 and are referred to throughout this subsection. There is no loss of generality, since East-alignment very closely resembles the Cross-Track alignment for ground-track targets and is a more demanding attitude to align to than the pushbroom configuration.

First, the geometric parameters of Earth pointing (see Section 4.1) for NEMO-AM are outlined by considering the range of possible orbital altitudes and off-track viewing angles $o_o$ (see Section 3.4.4). In particular, the relationship between the satellite altitude $h$ and the Earth angular radius $\rho$, and that between the longitudinal shift of a target from the ground track $\Delta \lambda_t$ and the off-track viewing angle $o_o$, are important in correlating satellite-specific observation parameters with Earth-specific observation parameters. These relationships are presented for an equatorial target viewed from $\delta_\ell = 0^\circ$ in Figure 6.3.

![Earth Angular Radius vs. Satellite Altitude](image1)

![Off-Track Viewing Angle vs. Target Longitude from Groundtrack](image2)

Figure 6.3: $\rho$ variation with $h$, and $o_o$ variation with equatorial $\Delta \lambda_t$ viewed from $\delta_\ell = 0^\circ$.

The Earth angular radius $\rho$ is a maximum of $66.1^\circ$ at NEMO-AM’s lowest expected altitude (600 km). Moreover, due to the proximity of the Earth, a large angular displacement in the
off-track viewing angle $o_o$ results in observing a terrestrial target that is displaced by a much smaller quantity $\Delta \lambda_t$ in the longitudinal direction from the satellite groundtrack. For example, Figure 6.3 demonstrates that if $NEMO-AM$ were to observe an equatorial target with $o_o = 57.9^\circ$ from $\delta_l = 0^\circ$, the target observed on the Earth’s surface would be approximately $10^\circ$ in the longitudinal direction from the groundtrack. The $\Delta \lambda_t$ shift in the observable target grows as the target’s latitude departs from the equator. Thus, although it is important to keep in mind the relationship between the satellite-specific ($o_o$ and $i_o$) and terrestrial-specific ($\Delta \lambda_t$ and $\delta_l$) Earth pointing parameters, the off-track $o_o$ and in-track $i_o$ viewing angles will be the primary means of identifying target locations throughout the rest of this work since they are independent of the satellite and target position, whereas the terrestrial-specific parameters are not.

### 6.3.1 Star Tracker Orientation

The pointing budgets in Section 6.2 demonstrate that a star tracker is needed to meet the imaging requirements of $NEMO-AM$. The inclusion of a star tracker demands that special care be made to ensure that when the unit is imaging the stars, it is not blinded by extraneous light sources. These sources include light emitted or reflected by the Earth, Sun, Moon and the satellite bus itself. Disruptions caused by moonlight are periodic and based on the Moon’s orbit and its phase. Given the limited literature for generic Moon-specific avoidance measures and relatively long periods between subsequent new Moons, star tracker operation in the presence of moonlight is anticipated to be a very low risk and was chosen to be resolved operationally on-orbit. Reflected light from the satellite itself is addressed by ensuring the $S3S$ is fitted with a stray-light suppressing, low-reflectance coated baffle, which extends outside of the bus and limits high incidence angles of light from entering the star tracker optics [16]. Extensive literature however, does exist on imager avoidance measures for the Earth and Sun, and likewise stray light from these two celestial bodies are addressed in this subsection.

Flight experience has shown that star trackers should maintain an exclusion angle with respect to the Sun and Earth of approximately $60^\circ$ and $30^\circ$, respectively [28]. However, due to the proximity of the Earth, its angular radius $\rho$ (see Figure 6.3(b)) must also be taken into account, thus compounding the $30^\circ$ exclusion angle with the Earth’s radius. Maintaining the exclusion angles ensures the operation of the star tracker, but also limits the geographical areas on the Earth which can be imaged. Consequently, the star tracker must be mounted in an orientation such that during imaging operations, the exclusion angle constraints are met, while the overall observable areas on Earth are maximized. By considering the range of possible sun synchronous orbits and seasonal variations, a mounting position for the star tracker is first found for ground-track targets and then extended to off-track targets. The problem space can be reduced by considering the worst case conditions for star tracker obscuration during continuous ground-tracking. In total, the star tracker obscuration over the mission lifetime is sensitive to the imager alignment, orbit altitude, orbit local time of descending node ($LTDN$), the season of observation, and the target coordinates relative to the ground track. These five
parameters are elaborated further:

- **Imager Alignment**: Rotation about the boresight of the imager is used to constrain the attitude of the satellite, and can also be used to align sequential images. However, over a number of Imaging Campaigns, aligning the imager with a wide array of arbitrary headings will require different types of rotation about the imager boresight and likely bring the Sun and Earth within the field of view of the star tracker and so realistically, only a small group of headings can be chosen for alignment. Only East-alignment, Pushbroom and Cross-Track-alignment (see Section 2) are investigated.

- **Orbit Altitude**: The Earth angular radius is inversely proportional to the orbital altitude and thus for a worst case situation, only 600 km orbits are considered. At this altitude, the Earth angular radius is 66.1°, resulting in a total Earth exclusion angle of 96.1°.

- **Orbit LTDN**: The range of LTDNs for NEMO-AM (see Section 2) ensure that the satellite position is always to the West of the noon hour angle. For a given target tracking imager alignment, since the orbit is sun synchronous, changing the LTDN will only influence the satellite’s line of sight to the Sun. Consequently, the worst case scenario is when the satellite is in a 11:00 LTDN orbit and closest to the Sun.

- **Seasonal Variation**: Choosing which season will result in the worst case situation is not as straightforward as the previous three parameters, since hypothetically the combination of the Earth’s tilt with respect to the ecliptic and the target tracking maneuver may lead to potentially unfavourable scenarios in all seasons. Thus, Vernal Equinox, Summer Solstice and Winter Solstice epochs are all considered.

- **Target Coordinates**: Off-track target choices are limited by the off-track viewing angle $o_o$ that the satellite must turn through to image them, since this maneuver may expose the star tracker to stray light. For a given off-track viewing angle $o_o$, the change in off-track target longitude $\Delta \lambda_t$ is smallest at the equator. Thus, to determine the minimum possible off-track target longitude, targets on the equatorial plane are solely considered.

By investigating the worst case conditions which arise from the five parameters, candidate orientations for the star tracker are determined. This is accomplished by computing the direction of the Sun and Earth in a satellite-centered unit celestial sphere and projecting a cone, whose axis is along the direction from the satellite to the corresponding celestial body, onto that sphere. The half-angle of the cone is equal to the corresponding exclusion angle of that body. These projections on the sphere (i.e. shaded regions) are aptly referred to as exclusion surfaces $\mathcal{E}$. The unshaded areas, deemed mounting surfaces $\mathcal{S}$, are composed of the directions in the satellite-centered celestial sphere which are candidates for star tracker boresight alignment. An example of this for a non-operational attitude is shown in Figure 6.4(a), where the body axis $b_3$ of NEMO-AM is aligned with nadir. The satellite attitude associated with Imaging Campaigns, oriented for comparison with the celestial sphere plots, is presented in Figure 6.4(b).
Figure 6.4: Example of Earth exclusion projection for non-operational attitude with the $+Z$ face facing nadir (left) and the Imaging Campaign operational attitude oriented for exclusion projection plots (right). Earth exclusion surface $E$ in shaded blue and mounting surface $S$ in unshaded area.

The exclusion projection plots presented in this subsection are constructed and overlaid for entire Imaging Campaign periods. For pushbroom-alignment scenarios, the exclusion plots correspond to the time periods when NEMO-AM is in sunlight. For all other alignment types, the plots represent Campaign periods where there is direct line of sight from the satellite to its equatorial sun-lit target.

The exclusion plots for observing East-aligned ground-track targets from an altitude of 600 km in an 11:00 LTSDN orbit with seasonal variation (Vernal Equinox, Summer Solstice and Winter Solstice) are considered first in Figure 6.5. The mounting surfaces for ground-track target observation does not change greatly with seasonal variation. Although the satellite slews almost 180° during its Imaging Campaign, the $+X$ face is most influenced by the Sun due to the orbit lying West of the noon hour angle. Consequently, the majority of the mounting directions of the star tracker in Figure 6.5 are concentrated near the $-X/+Y$ plane.

When all three exclusion plots in Figure 6.5 are superimposed, an explicit mounting surface for East-alignment with seasonal variation can be defined. This exclusion (i.e. shaded) surface $E_\epsilon$ and its corresponding mounting (i.e. unshaded) surfaces $S_\epsilon$ are shown in Figure 6.6(a). A similar mounting surface $S_p$ with seasonal variation can be produced for pushbroom alignment. This mounting surface $S_p$ is shown in Figure 6.6(b) and covers a larger area as compared to that in Figure 6.6(a) for East-aligned target tracking attitudes. Consequently, if the star tracker is positioned such that its boresight lies in $S_\epsilon$, ground-track targets can be observed in both an East-aligned and pushbroom orientation. The mounting surfaces $S_\kappa$ arising from Cross-Track-alignment are nearly identical to East-alignment, and for reference are presented in Appendix A.
Figure 6.5: Sun and Earth exclusion projections for ground-track equatorial targets with $h = 600$ km, 11:00 \textit{LTDN} and local East-alignment, with seasonal variation.

Figure 6.6: Sun and Earth exclusion projections superimposed from Figure 6.5 (left) and pushbroom-alignment for the same target with seasonal variation (right). Mounting surfaces for East-alignment $\mathcal{S}_e$ (left) and pushbroom-alignment $\mathcal{S}_p$ (right) in unshaded areas.
When the capabilities of local East, Pushbroom and Cross-Track alignment are combined, a complete mounting surface $S_s$ for seasonal variation can be produced. The mounting surface $S_s$ is defined by three bounding vertices, as shown in Figure 6.7. The coordinates in the satellite-centered unit celestial sphere are explicitly defined in Table 6.5. The bounding vertices allow the star tracker to lie directly in the $-X/ +Y$ plane with a cant angle between $+15.3^\circ$ and $+38.7^\circ$ with respect to the $-X$ direction.

Figure 6.7: Sun and Earth exclusion projections superimposed for ground-track equatorial targets with $h = 600$ km, 11AM LTDN with local East, pushbroom and Cross-Track alignment during Vernal Equinox, Summer Solstice and Winter Solstice. Green line represents $XY$ plane. Mounting surface $S_s$ in unshaded area.

Table 6.5: Bounding vertices for Figure 6.7

<table>
<thead>
<tr>
<th>Vertex</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>Angle from $-X$ in $X/Y$ Plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>-0.95</td>
<td>0.26</td>
<td>-0.02</td>
<td>$15.31^\circ$</td>
</tr>
<tr>
<td>$v_2$</td>
<td>-0.78</td>
<td>0.60</td>
<td>0.10</td>
<td>$37.61^\circ$</td>
</tr>
<tr>
<td>$v_3$</td>
<td>-0.76</td>
<td>0.61</td>
<td>-0.17</td>
<td>$38.71^\circ$</td>
</tr>
</tbody>
</table>

Positioning the star tracker anywhere in the $S_s$ mounting surface allows ground-track targets to be imaged throughout the year with the aid of a star tracker. The actual alignment of the star tracker boresight in $S_s$ will determine what types of off-track targets can imaged. Thus, in order to determine a suitable star tracker position, different off-track exclusion areas are superimposed on Figure 6.7. If the satellite is to track an off-track target by rotating through by an off-track viewing angle of $o_o$, intuitively one would expect that the Sun and Earth exclusion surfaces will also rotate through by $o_o$. These shifts in exclusion areas are most pronounced at low values of $i_o$ and least pronounced for high values of $i_o$, since for a given off-track target, the off-track viewing angle $o_o$ is largest when the satellite is closest to the target. Imaging a target to the West of the ground track (negative values of $o_o$) will expose more of the $-X$ satellite face to sunlight, thereby causing the Earth to retreat from $S_s$ and the Sun to advance and mask $S_s$. 
The surface $S_s$ is fully masked by the Sun when an off-track target of $o_o = -26^\circ$ is observed.

When targets to the East of the ground track (positive values of $o_o$) are imaged, intuitively, one would expect that the converse would occur. That is, the Sun would retreat and the Earth would advance and mask $S_s$. However, since the Earth exclusion angle is greater than $90^\circ$, there is considerable overlap between the Earth exclusion areas at large in-track viewing angles for ground-track targets and the Earth exclusion areas at small in-track viewing angles for off-track targets. Thus, for a given absolute off-track viewing angle $o_o$, imaging a target West of the ground track through $-o_o$ would have more of $S_s$ masked by the Sun as compared to imaging a target East of the ground track through $+o_o$ and having the Earth mask $S_s$. This phenomena is shown for positive and negative off-track viewing angles of $10^\circ$ in Figure 6.8. Practically, this implies that there is more off-track imaging capability to the East of the ground track. The mounting surface $S_s$ is fully masked by the Earth when an off-track target of $o_o = +33^\circ$ is observed. This asymmetric off-track imaging phenomena is irrespective of season since it is dependent on the proximity of the Earth.

![Figure 6.8: Sun and Earth exclusion projections for off track equatorial targets with $h = 600$ km, 11:00 LTDN, $o_o = \pm 10^\circ$ and local East alignment during Vernal Equinox.](image)

It is possible to choose a boresight direction on the mounting surface in Figure 6.7 which allows NEMO-AM to observe off-track targets with both positive and negative values of the off-track viewing angle $o_o$. However, due to the aforementioned asymmetric off-track observing capabilities, it is more practical to identify candidate targets while maximizing the off-track capabilities by choosing to image off-track targets solely to the East of the ground track through a positive $o_o$ angle. To accomplish this, the star tracker boresight direction is chosen to be positioned on the body unit celestial sphere, such that it is aligned as closely as possible to the $+Y$ direction on $S_s$ as possible. This corresponds to the star tracker boresight being canted $38^\circ$ from the $-X$ direction, towards $+Y$, in the $X/Y$ plane. In doing so, the maximum satellite off-track viewing angle $o_o$ possible for NEMO-AM is approximately $+30^\circ$. This off-track capability is altitude dependent and will be reevaluated once NEMO-AM is in orbit.
6.3.2 Viewing Angle Selection

When imaging line of sight is first established by NEMO-AM, the distance from the satellite to the target causes the imager footprint to project into deep space. Images taken from these viewing angles would not provide valuable science data. Consequently, viewing angles which allow useful imaging, in the form of land-only capture, must be identified to establish when pointing requirements are to be enforced. Since NEMO-AM is expected to be in a sun-synchronous orbit, the location of the Earth with respect to the satellite is independent of the target choice. The target latitude is inconsequential since the 21 km bulge at the equator is small as compared to the satellite-to-target distances. The allowable off-track target viewing angle \( o_o \) is outlined in Section 6.3.1 for orbits at 600 km and is enforced in this section. Higher satellite orbits allow for greater imager projection and thereby greater deep space spillover, but will not be considered since the off-track pointing constraints are dictated by the star tracker operation which takes precedence over any footprint spillover. Lower altitudes result in larger viewing angle boundaries for which pointing requirements must be met and thus represent a more conservative attitude scenario. Thus, the analyses in this section are limited to investigating scenarios tracking equatorial targets which have an off-track angle \( o_o \) as large as \(+30^\circ\) from an altitude of 600 km. Throughout the span of an Imaging Campaign for the aforementioned scenario, the declination viewing angle \( \delta_\ell \) varies from \( \pm 77^\circ \), while the range varies between 600 km to 2750 km, as shown in Figure 6.9.

![Figure 6.9](image)

Figure 6.9: Variation of \( \delta_\ell \) and range \( r \) over Imaging Campaign of ground-track equatorial target from an altitude of 600 km.

During the Imaging Campaign, as the target first comes into the satellite’s line of sight, a portion of imager swath will be composed of the Earth and another portion will be exposed to space as shown Figure 6.10. As the Campaign continues, the relative coverage of the imager by the Earth and space will be a function of the imager FOV, the range \( r \) and the local target viewing angles \( \alpha_\ell \) and \( \delta_\ell \). The minimum orbital altitude of 600 km represents the case for the smallest imager footprint and the least obscuration of the imager by space. Since the S3S star tracker limits the satellite off-track viewing capabilities (see Section 6.3.1), imager characteristics with respect to
\( \delta \ell \) are considered for East-aligned tracking of equatorial off-track targets with viewing angle \( o_0 \) values of 0° and +30° from \( h = 600 \) km. The corresponding plots are shown in Figure 6.11 and Figure 6.12. Almost identical plots arise for Cross-Track alignment and are presented in Appendix B.

Figure 6.10: Relative imager swath coverage during Imaging Campaign at high \( \delta \ell \).

![Image](image1.png)

Figure 6.11: Imager characteristics for East-aligned tracking with \( o_{0,min} = 0^\circ \), \( h = 600 \) km.

![Image](image2.png)

Figure 6.12: Imager characteristics for East-aligned tracking with \( o_{0,max} = +30^\circ \), \( h = 600 \) km.
As observed, the minimum GSD and footprint area grow with increasing off-track viewing angle $o_o$. For ground-track targets, the minimum GSD and footprint at an altitude of 600 km are 37.8 m/pixel and 2820 km$^2$, respectively. However, when tracking off-track targets at the maximum viewing angle of $o_o = +30^\circ$, the imager projection spreads out over a much larger ground surface area and the minimum GSD and footprint increase to 59.5 m/pixel and 4588 km$^2$, respectively. It is evident that when targets are imaged from an altitude of 600 km, limiting the maximum range of declination viewing angle $\delta_t$ to $\pm 53^\circ$ will result in all of the imager FOV to be taken up by the Earth. At this stage of design, NEMO-AM is expected to meet pointing requirements for the aforementioned viewing angle range. At higher orbits, this range will decrease due to the larger projection of the imager and the smaller off-track constraints resulting in smaller Earth exclusion angles (see Section 6.3.1). Since imaging deep space is not detrimental, higher altitude viewing angle boundaries are not mission critical and will be reestablished, as driven by the science objectives, once NEMO-AM is launched.

6.4 Mission Simulations

Once the satellite architecture, orbital environment and celestial mechanics were modeled, the flight code to be used onboard the satellite was written. The flight code and simulation environment were integrated as described in Section 5.4 and iteratively tested to tune the software and simulate the expected satellite performance. This section presents the most important results of those simulations. The simulation results presented are performed chronologically for the different phases of the mission beginning on 08:00:00 15 Feb, 2012 UTC. NEMO-AM is initially injected in a 11:00 LTDN sun-synchronous orbit and its anticipated performance is shown in sequence for detumbling, sun pointing and target tracking during an Imaging Campaign.

The satellite moment of inertia matrix is derived from a solid model of the entire bus and is defined as:

$$\mathbf{I} = \begin{bmatrix} 0.29 & 0.01 & -0.09 \\ 0.01 & 0.55 & -0.01 \\ -0.09 & -0.01 & 0.40 \end{bmatrix} \text{kg m}^2$$  \hspace{1cm} (6.1)

6.4.1 Rate Damping

The first simulation of interest is detumbling of the satellite during Rate Damping mode. The satellite is assumed to have a parasitic dipole of 0.05 A m$^2$ in each body axis arising from the reaction wheels which have empirically been determined as the dominant form of magnetic disturbance on the satellite. The initial tip-off rate is set to 10$^\circ$/s and distributed equally among all three body axes. Both the parasitic dipole and the tip-off rate are conservatively chosen based on flight experience from AISSat-1, which uses a similar variant of the reaction wheels and a similar SFL separation system determined to induce a tip-off rate of 6$^\circ$/s [40]. Since detumbling is envisioned only for the period after separation from the launch vehicle and
in cases of elevated momentum, the performance of this mode is not critical as long as the rates can be reduced to levels at which three-axis control can be reinstated. The absolute angular momentum \( \mathbf{h}_i \) of the satellite is presented in Figure [6.13] for a portion of the rate damping period. It is observed that the initial angular momentum of 0.026 N m s decreases steadily as the satellite travels in its sun synchronous orbit and the greatest decrease in \( \mathbf{h}_i \) is observed to be near the poles of the Earth where the change in the local magnetic field is largest.

Satellite Angular Momentum vs. Distance From Equatorial Plane

![Satellite Angular Momentum vs. Distance From Equatorial Plane](image)

Figure 6.13: Rate damping angular momentum change with distance from equatorial plane.

The oscillation of the components of angular velocity in the body-fixed frame \( \mathcal{F}_b \) is shown in Figure [6.14] and is a direct result of conservation of absolute angular momentum between periods of magnetic momentum shedding. The overall results show that the initial tip-off rate is reduced in approximately 5 orbits when running the magnetorquers at 40% of their full strength. The final angular velocity is not nil since the B-Dot controller operates to align the instantaneous satellite magnetic dipole with the local magnetic field. Likewise, the magnitude of the residual angular velocity after rate damping is between 0.1°/s and 0.2°/s, which corresponds roughly to the rate of change of the Earth’s magnetic field relative to the orbit. This rate damping performance matches those seen onboard AlISSat-1.

![Spacecraft Angular Velocity in \( \mathcal{F}_b \) vs. Simulation Time](image)

Figure 6.14: Rate damping simulation with magnetometer and magnetorquers.
6.4.2 Sun Tracking

The second simulation of interest is Sun tracking. The Sun Tracking submode is expected to be the nominal (Three-Axis) mode since outside of the imaging objectives, the primary responsibilities of the AOCS system are power generation and GPS telemetry collection for orbit prediction. The Sun tracking attitude aligns the +X panel along \( \mathbf{s}_\oplus \) while constraining the +Y direction along orbit normal. Sun tracking in this manner maximizes both power generation and access to the GPS constellation for ground orbit determination using the GPS receiver (see Section 2). The slow rate of change, as observed in inertial space, in both the Sun vector and the orbit normal, make Sun tracking a quasi-inertial pointing maneuver. The Sun direction \( \mathbf{s}_\oplus \) can be set constant for a period of approximately four days while incurring a pointing error of approximately 4°. A pointing error of this magnitude during Sun Tracking does not have any considerable power or GPS data gathering consequences for NEMO-AM [20]. For simplicity in design, these approximations are adopted and the corresponding inertial pointing performance is presented in this subsection. Likewise, these results are representative of other inertially held attitudes whose primary objective is to reject disturbances.

Although the Three-Axis Inertial Pointing modes are not mission critical, the simulations of these modes are useful in gauging the validity of the pointing budgets in Section 6.2 and subsequently determining the anticipated on-orbit performance. The Sun tracking performance using a basic suite of sun sensors and magnetometer is presented in Figure 6.15 and the counterpart star tracker-only performance is shown in Figure 6.16. In both scenarios, the satellite has already been brought online, the rates have been reduced to acceptable levels and the EKF has been allowed to converge with the appropriate sensors. The satellite is maneuvered using the reaction wheels into the Sun tracking attitude and held. Momentum dumping is being applied at all times, which represents a conservative case for the attitude controller. The 2σ pointing accuracy using the basic suite of sensors and actuators is anticipated to be 1.93°. This type of Sun tracking is expected to be employed when the satellite is in sunlight and the star tracker is occluded by the Earth. When the satellite is in the Earth’s shadow, the AEKF is transitioned to using only star tracker measurements. Likewise, the pointing accuracy is significantly improved and has a 2σ value of 0.19°. Although there isn’t a need for Sun tracking in eclipse, transitioning to the star tracker ensures that the attitude state estimation does not diverge and the satellite does not exit the shadow of the Earth in a degraded state before Sun tracking for power generation or, more importantly, Target Tracking.

The on-orbit aggregate coarse sensor errors in Table 6.3 were determined through post-analysis and are conservative at best since high-fidelity sensors are not present on AISSat-1. Likewise, the simulations use sensor accuracies that are determined from ground testing to avoid being overly conservative. Thus, the on-orbit aggregate coarse sensor errors constitute in the difference between the 2σ pointing performance and those tabulated in Table 6.3 and Table 6.4. The aggregate error terms include timing errors, misalignment errors, ephemeris errors and wheel zero-crossings. Timing errors are anticipated to be eliminated in the final
satellite design because the sequence of AOCS hardware events during every control cycle will be explicitly mapped before the operating system software (CANOE) is finalized [20]. Moreover, with the presence of a star tracker, misalignment errors are expected to be calibrated on-orbit, thereby also reducing the risk of this error contribution. The IGRF errors are left unmodeled due to their unpredictable nature and the lack of documented measurements of the many magnetic field contributions (see Section 3.7.4). Nevertheless, the sensor covariances in the AEKF are tailored so that filter is more sensitive to sun sensor measurements. The wheel zero-crossing errors can be avoided on-orbit by keeping the wheels at a bias speed and setting the inertial angular momentum $\mathbf{h}$ with the magnetorquers so that the speeds do not approach zero even with the expected variation in attitude. Thus, although not all of the overly-conservative error terms are incorporated, the simulations from a practical point of view are realistic.

If an Imaging Campaign is scheduled, the satellite will exit the Sun Tracking submode to slew to maintain the initial attitude on the target tracking trajectory. This transition epoch is denoted as the premaneuver epoch. The Site Imaging simulation following the premaneuver epoch is presented in Section 6.4.4.

![Euler Angle Tracking Error vs. Simulation Time](#)

Figure 6.15: Sun tracking performance with sun sensors and magnetometer.

![Euler Angle Tracking Error vs. Simulation Time](#)

Figure 6.16: Sun tracking performance with star tracker only.
6.4.3 Target Tracking Kinematic Estimation

Once the premaneuver epoch for the Imaging Campaign is reached, the satellite slews to align itself with the initial attitude on the target tracking trajectory (see Section 4.2). The premaneuver period, defined as the time period between the premaneuver epoch and the first line of sight to the target, is used to transition from sun pointing smoothly and ensure that the satellite is ready to begin the Imaging Campaign. The premaneuver period is also used to acquire a stable GPS solution such that target tracking trajectory can be constructed onboard. To reduce computation, the FK5 transformation in the trajectory is approximated by omitting polar motion and holding the nutation and precession components constant over the Campaign period. Since the GPS receiver is positioned exactly opposite the imager (see Section 2), the initial target tracking attitude provides coverage to the GPS constellation by keeping the receiver from being occluded by the Earth. GPS almanac information, which assists in the acquisition of the constellation satellites during receiver power-up, is updated on the receiver at an infrequent rate \[50\] and consequently the premaneuver period onboard is set to 15 min—the maximum time to first fix for cold starts \[40\].

After the premaneuver period has begun and a valid GPS solution is acquired, the OEKF is initialized (see Section 4.4.1). Since the AOCS control cycle runs at 1 Hz, the prediction step is computed using only a low fidelity \(2 \times 2\) gravity force model. Since, other types of disturbance forces (see Section 3.7) are insignificant over the prediction period, they are consequently omitted. Consistency in the filter is maintained by employing a lower fidelity central gravity model for the state transition matrix. If GPS acquisition is lost in the middle of the observation, the filter runs open-loop by propagating only the prediction gravity model.

In the ideal case, Imaging Campaign simulations would be done with a GPS signal simulator integrated in-the-loop with the Mirage framework (see Section 5.4). However, in practice, it is quite complex to integrate a GPS signal simulator or realistically simulate its behaviour with the GPS constellation. Thus, an alternate approach to the Imaging Campaign simulations is taken, where the OEKF performance is characterized for the worst-case scenario of complete GPS loss after initialization. This characterization is then used as the benchmark for simulations on Mirage, thereby decoupling the simulation dependency on realistic GPS solutions.

Before presenting the results for the benchmark scenario, it is important to understand the behaviour of the OEKF when realistic signals are provided by a GPS signal simulator. The hypothetical scenario of target tracking with perfect control (i.e. perfectly tracking the target) is first simulated to understand this behaviour. The OEKF estimates starting at approximately 7 minutes before the target tracking epoch are shown in Figures 6.17 and 6.18. These plots are decomposed in the orbit frame in Appendix C. A non-zero value of the process noise \(Q\) is used to ensure that both the position and velocity estimates have some sensitivity to the GPS measurements. Although it may seem that the position estimates are far too sensitive to the measurements, the GPS position solution is inherently biased by the satellites it is tracking \[51\]. Two different sets of satellites being tracked will have two different biases in their GPS
solutions. Thus without any bias compensation, the OEKF position estimates will have offsets. This phenomenon is apparent in Figure 6.17 where 4 different offsets are observed due to 4 different sets of satellites being tracked in the first 700 s. For NEMO-AM’s mission objectives, these biases are small enough to be considered insignificant and likewise not compensated. The position estimates are however consistent (i.e. bounded by the estimated covariances). It is important to note that position errors are greatest in the radial direction since there are more GPS constellation satellites in the in-track and cross-track directions to resolve the in-track and cross-track components. The tuned sensitivity of the OEKF to new measurements can be justified by observing the position estimate at 720 s where a new set of satellites starts to be tracked but the estimate does not immediately follow the GPS measurement. The velocity estimates are observed to be both unbiased and consistent. The absolute raw GPS measurement errors are 12.22 m (2\(\sigma\)) in position and 13.58 cm/s (2\(\sigma\)) in velocity. The OEKF position and velocity errors are 12.17 m (2\(\sigma\)) and 6.22 cm/s (2\(\sigma\)), respectively.

![Figure 6.17: OEKF position error in ECEF for full GPS solution. Discontinuities represent transitions between two sets of GPS satellites (each with their own biases) being tracked.](image1)

![Figure 6.18: OEKF velocity error as observed and expressed in ECEF for full GPS solution.](image2)

In the case where the OEKF is in progress and the GPS solution is invalid or unavailable, the filter will continue to propagate with the onboard 2 \times 2 gravity model. Although the size of the model was chosen to balance the computational complexity with the small filter step, the open-loop propagation position and velocity errors are on the order of tens of meters and single digit centimeters per second, respectively. This performance can be observed in Figures 6.19 and 6.20 where GPS acquisition is lost at the target tracking epoch of 420 s.
Chapter 6. NEMO-AM

Figure 6.19: Propagated position errors for open-loop OEKF propagation. GPS acquisition intentionally dropped at 420 s mark.

Figure 6.20: Propagated velocity errors for open-loop OEKF propagation.

The corresponding target tracking trajectory construction (see Section 4.2) is robust enough to be relatively unaffected by the accumulated open-loop errors when compared to the closed-loop performance (see Appendix C). The trajectory construction errors are presented below in Figures 6.21, 6.22, and 6.23. The open-loop and closed-loop comparison is tabulated for conciseness in Table 6.6. Although the attitude trajectory appears to diverge in a consistent manner, the attitude errors are directly attributed to the FK5 nutation and precession approximations. The maximum trajectory error is seen near the 800 s mark, where the satellite flies directly overhead the target and the various error terms accumulate to form their peak.
Whereas the open-loop position errors are observed to increase in a quadratic fashion, the open-loop velocity errors do not increase at the same rate over the Imaging Campaign period because the ratio of the orbital accelerations to the orbital velocities are significantly smaller than the
ratio of the orbital velocities to the orbital positions. Nevertheless, even with complete open-loop propagation, the target tracking attitude and angular velocity errors are on the same order of magnitude as compared to closed-loop performance. The open-loop angular acceleration trajectory errors are an order of magnitude larger than the closed-loop performance since the sensitivity of the trajectory terms (i.e. $\dot{C}_{bi}$, $\dot{\omega}_{bi}$ and $\dot{\alpha}_{bi}$) to its constituents (satellite position and velocity) is proportional to the order of the time derivative of the kinematic relations. That is, if one were to evaluate the angular jerk open-loop, an even higher trajectory error would be observed. Since the minimum torques the reaction wheels are able to provide are on the order of $1 \times 10^{-5}$ N m, the target tracking trajectory construction using the OEKF is in practice identical for closed-loop and open-loop scenarios as long as a valid GPS solution is available to initialize the filter. If an initial GPS measurement is not available, the Imaging Campaign is aborted in order to avoid erroneous observations and any unforeseen dynamic effects from attempting to track a poorly formed trajectory.

Table 6.6: OEKF propagation errors and corresponding target trajectory construction errors.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Complete Closed-Loop</th>
<th>Complete Open-Loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. $</td>
<td></td>
<td>\delta r_i</td>
</tr>
<tr>
<td>Max. $</td>
<td></td>
<td>\delta v_i</td>
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<tr>
<td>Max. $</td>
<td></td>
<td>\delta \dot{\theta}</td>
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<tr>
<td>Max. $</td>
<td></td>
<td>\delta \dot{\omega}_{bi}</td>
</tr>
<tr>
<td>Max. $</td>
<td></td>
<td>\delta \dot{\alpha}_{bi}</td>
</tr>
</tbody>
</table>
6.4.4 Target Tracking for Imaging

The target tracking simulation was conducted for a northern Indian ground-track target located at a latitude of 35.3° N and a longitude of 40.9° W, beginning at the epoch 15 Feb 2012 11:00:00.00 UTC for an 11:00 LTDN sun synchronous orbit from 650 km. The simulation begins with the satellite in Sun Tracking mode, already having target coordinates and observation times uploaded onto its flight software. At 11:17:30.00 UTC, the satellite slews to align itself with the starting attitude of the imaging maneuver. The satellite then begins to track the ground location from 11:32:30.00 UTC to 11:45:49.00 UTC performing Cross-Track alignment. Magnetic actuation in general has less control authority compared to reaction wheel actuation. Likewise, during the Imaging Campaign, the magnetorquers are turned off to maximize tracking capability. The S3S star tracker is the only attitude sensor providing measurements to the AEKF, since the sun sensor and magnetometer measurement errors are two orders of magnitude larger and would only corrupt the star tracker solution. At the beginning of the target tracking epoch, a valid GPS solution is used to initialize the OEKF, however subsequent measurements are forced to be corrupted such that the OEKF runs open-loop. This simulation represents the poorest possible navigation solution and decouples the simulation framework from supplying realistic GPS solutions (see Section 6.4.3). Overall, the simulation is a worst case scenario in terms of estimation (smallest sun exclusion angle), navigation (worst-possible target tracking trajectory construction) and control (greatest possible angular velocity trajectory profile). The hardware error profiles use their 2σ values (see Section 5) to further compliment this scenario.

The first plot of interest is the target declination viewing angle $\delta_\ell$. When there is line of sight from the satellite to the target, the satellite has a declination viewing angle $\delta_\ell$ which varies between $\pm 77^\circ$ over a 12 minute span. However imaging is only expected to be done over a viewing angle range of $\pm 53^\circ$, which corresponds to approximately a 3 minute span as shown in Figure 6.24. The viewing angle changes most rapidly when the satellite flies overhead the target ($\delta_\ell = 0^\circ$), reaching a maximum absolute angular velocity trajectory value of 0.73°/s. Consequently, this epoch results in the most demanding portion of the target tracking maneuver.

![Figure 6.24: Declination viewing angle during Imaging Campaign. Expected imaging period bounded in red.](image-url)
The second plot of interest is of the disturbance torques imparted on the satellite. The largest disturbance torques are magnetic torques, arising from parasitic dipoles onboard the satellite. These dipoles are approximated to be $0.05 \text{ A m}^2$ on all three body axes and are modeled after \textit{AISSat-1} values \cite{40}. Due to the relative magnitude of these dipoles compared to the $20 \mu T$ ambient geomagnetic field, magnetic torques dominate all other forms of disturbances and are anticipated to be on the order of $10^{-6} \text{ N m}$. The next largest disturbance torques are secular gravity gradient ($10^{-7} \text{ N m}$), solar radiation pressure ($10^{-9} \text{ N m}$) and aerodynamic torques ($10^{-9} \text{ N m}$). A plot of the total disturbance torques on the satellite during the Imaging Campaign is shown in Figure 6.25.

![Figure 6.25: Disturbance torques during Imaging Campaign.](image)

Although the \textit{S3S} measurements only provide attitude measurements for the satellite, these attitude measurements are filtered to estimate both the inertial attitude and the inertial angular velocity as the satellite performs its maneuver. The attitude estimation error during the Campaign is presented in Figure 6.26. The $2\sigma$ attitude accuracy in Roll, Pitch and Yaw are $0.05^\circ$, $0.01^\circ$ and $0.04^\circ$ respectively. Although these errors may not seem to directly correspond to the error profiles of the star tracker (see Section 5.4), there is direct correlation to the pointing budget RSS magnitude (see Section 6.2) because the $2\sigma$ errors are mapped from the sensor frame, canted $38^\circ$ from the $-X$ panel, to the body frame of the satellite. Star tracker mounting misalignments are left unmodeled since star tracker misalignments on the satellite are known to result in biases in the onboard estimate and the subsequent imaging \cite{20}. Consequently, calibration is expected to be performed on-orbit to eliminate the errors in the attitude and body rate estimates of the satellite.

The attitude state estimation is valid, in part, if the star tracker is unobstructed by stray light during the Imaging Campaign. The Earth and Sun exclusion angles with respect to the star tracker boresight are plotted in Figure 6.27 for this period. As a result of the positioning of the star tracker, the Earth and Sun exclusion angles have approximately $3^\circ$ and $13^\circ$ of margin, respectively, when imaging the ground-track target in the simulation. When imaging off-track targets, less margin is expected to be available in the Earth exclusion angle and more margin is expected to be available in the Sun exclusion angle.
Once the imaging maneuver has begun, the wheels are commanded to apply the required torques to the satellite to track the ground location. Momentum is exchanged between the wheels and the spacecraft as it performs its target tracking maneuver. A plot of the wheel speeds is shown in Figure 6.28. The wheel speeds are kept well below their saturation limit of 1400 rad/s during this period, even without any active momentum dumping. Due to the nature of the target tracking maneuver, the wheels on the Y and Z axes must experience a zero-crossing to conserve the inertial angular momentum of the spacecraft. The wheel zero-crossings are observed at $\delta_\ell = 30^\circ$ and $\delta_\ell = -50^\circ$ for this target tracking scenario. These zero-crossings will inherently cause a disturbance in the target tracking performance due to the friction in the bearings and poor speed measurement in the internal hall effect sensors. The disturbance is readily observed in the Euler angle tracking errors and angular velocity tracking errors in Figures 6.29 and 6.30. Rotation error about the X, Y and Z axes represent pointing error in the latitudinal direction, alignment error of the camera axes and pointing error in the longitudinal direction, respectively. Nominally, the spacecraft is lagging the target in the longitudinal direction ($Z$ axis in $\mathcal{F}_b$). Before the first wheel zero-crossing, the satellite is pointing $0.2^\circ$ behind the target. Based on the wheel model used, once the zero-crossing occurs, the Z wheel accelerates to 0 rad/s and the spacecraft leads the target by $0.3^\circ$. The alignment error
also lags the East heading of the target by a maximum of 0.2° until the second wheel crossing when it sharply rises. Similar behaviour is observed in the angular velocity tracking errors in the Y and Z axes. The wheel zero-crossing errors are modeled conservatively as ongoing tests are still being performed on flight hardware. NEMO-AM will likely not experience the abrupt 0.5° change in tracking error observed in Figure 6.29 but these simulation results are shown to demonstrate that the satellite is able to tolerate and recover from such elevated disturbances in terms of pointing. The 2σ tracking Euler angle errors in the X, Y and Z axes are 0.06°, 0.20° and 0.32°, respectively. Although this meets the 0.89° pointing requirement presented in Section 2, the feed forward components of the three-axis controller will be tuned in the future to reduce the lag in the tracking performance. The corresponding 2σ tracking angular velocity errors in the X, Y and Z axes are 0.02°/s, 0.03°/s and 0.04°/s, respectively. The simulation framework does not include timing and mounting errors, that constitute in the main difference between the simulated 2σ pointing error and the 0.8° (2σ) predicted error in Table 6.4. Timing errors are anticipated to be eliminated in the final satellite as explained in Section 6.4.2. These onboard timing errors are expected to be negligible once the timing sequence is finalized before launch. Likewise, the mounting errors, as described in Section 6.2. are expected to be calibrated in flight and subsequently diminished.

The pointing accuracy of the spacecraft directly reflects the footprint coverage accuracy of NEMO-AM. The footprint coverage requirement desires an area of 75.1 km × 5.7 km to be continuously imaged during the Imaging Campaign (see Section 2). This requirement is most demanding when the spacecraft is directly over the satellite and the footprint of the imager is the smallest. The pointing accuracy directly over the target is 0.36° and the total 2σ pointing accuracy is 0.38° for the Imaging Campaign. Consequently, the 75.1 km × 5.7 km footprint required to be imaged is captured in full during the overhead flyby as shown in Figure 6.31.
Figure 6.29: Euler angle tracking error during Imaging Campaign.

Figure 6.30: Tracking angular velocity error during Imaging Campaign.

Figure 6.31: Footprint coverage during Imaging Campaign. Imager field of view represented in green, satellite ground track represented in blue dot trail and footprint requirement represented in red.
The stability pointing requirement of 10% pixel smear over the Imaging Campaign (see Section 2) is evaluated by computing the pixel blur using Eq. (4.57) over the timespan of an exposure (25 ms). Figure 6.32 shows the pixel blur during the simulated Imaging Campaign. Nominally, the smearing of an image, outside of the zero-crossings, is kept to a value 0.1 pixels. However, at the wheel zero-crossing, the angular velocity error can result in a pixel blur of greater than 1.1 pixels. Although this does not comply with the desired requirement, there is margin in the design due to the conservative modeling of the reaction wheels. Nevertheless, the maximum pixel blur will occur during a zero-crossing period in the Imaging Campaign. To mitigate this effect, flight operation will schedule wheel crossings deliberately, such that they occur outside of the desired imager exposure periods in order to ensure that the images to be taken, meet the stability requirements. Ongoing efforts are currently underway to eliminate the zero-crossing phenomena altogether such that the entire Imaging Campaign is compliant with the requirements.

Figure 6.32: Pixel blur during Imaging Campaign.
Chapter 7

Conclusions

In this chapter, the various works completed in this thesis are summarized. The future AOCS work to be completed on NEMO-AM is also outlined. These remaining tasks shall bring the attitude subsystem from a state of preliminary design into one which is ready to be integrated, flown and operated onboard the satellite upon launch.

7.1 Summary

In this thesis the AOCS design of NEMO-AM was presented in detail starting with the necessary theory required to perform Earth observation, the hardware choices for the system and finally the operational functionality of the satellite. An overview of the pertinent celestial systems and attitude dynamics was first outlined to provide the foundation for target tracking analysis. These topics were combined to derive the Earth pointing geometry and, the target tracking kinematics which is, in part, one of the major contributions of this thesis. Although existing work demonstrates how the essential parameters for simple pointing can be computed, gross approximations are made in their formulation which significantly limits any high accuracy application thereof. In addition, in all of these works, constraints on the pointing attitude are not considered. This approach inhibits the practicality of target tracking since any application which has a specified orientation during pointing cannot be achieved. To the best of the author’s knowledge, this thesis is the first published work which details the target tracking problem with no loss of generality. To facilitate the tracking, an orbital EKF was proposed that corrects and propagates the orbital parameters and the target tracking trajectory to construct a robust navigation solution. Even during complete GPS loss after initialization, the filter and trajectory construction algorithm are able to compute an attitude and angular velocity reference trajectory which are sufficiently accurate to be used to perform sub-degree tracking. Once the target tracking theory was rigorously developed, the practical side of NEMO-AM’s AOCS was explored. This involved outlining the attitude hardware (sensors and actuators) onboard the satellite, the attitude software OASYS, including the attitude EKF used to estimate the inertial orientation and angular velocity, and the onboard controllers used during each phase of
the mission. Overall heritage in the software and hardware-to-software interfacing was maintained, legacy designs were tuned and substantial, but necessary, additions necessary for mission specific AOCS operations were developed by the author. Next, the operational functionality and operational sequence of NEMO-AM were explored and system parameters were chosen in order to achieve the desired in-flight experience. Lastly, the theory and practice were brought together in a MATLAB Simulink setting to culminate in validating the AOCS design by combining the flight software OASYS with the simulated satellite architecture, orbital environment and spacecraft dynamics. Mission simulations were completed for the entire operational sequence of the satellite including detumbling, sun pointing and target tracking scenarios. The simulations demonstrated that the requirements outlined in Section 2 were met with margin outside of the zero-crossing situations. Altogether, this thesis provides a foundation for practical target tracking which can be extended to various types of Earth-pointing satellite missions—something that has not been published in the past, to the author’s best knowledge.

7.2 Future Work

Although the preliminary AOCS design has been completed for NEMO-AM, there still remains work to be done in integrating the satellite flight code and hardware functionality. The two most important of these integration tasks is the synchronization of the attitude hardware operation with the onboard Extended Kalman filters and the synchronization of the imager with the sensory telemetry. These tasks will facilitate in synchronizing the hardware functionality, estimators and controllers, and in ensuring that when an image is taken there is sufficient information to map those images to locations on Earth.

As the satellite is being constructed, further simulations will need to be carried out to ensure that any updates to the satellite architecture are represented in the validation procedure. Once the complete integration is finished, final tests will be preformed on the individual hardware components and the combined system to root out any deficiencies. Although the future work does not seem great, practically it is a lengthy procedure where the AOCS is iteratively modified with the entire satellite such that the Attitude System meets the desired performance and also complies with the constraints of the other subsystems. Finally, the tests are again reiterated through environmental campaigns to ensure that the satellite is operable in a space environment before launch and operation.
Bibliography


Appendix A

Supplementary Exclusion Plots

Figure A.1: Sun and Earth exclusion projections for Cross-Track alignment of ground-track target with $h = 600\text{ km}$, 11AM LTDN and seasonal variation. Exclusion surface $\mathcal{E}_\kappa$ in shaded area and mounting surface $\mathcal{S}_\kappa$ in unshaded area.
Appendix B

Supplementary Viewing Angle Results

Figure B.1: Imager characteristics for East-aligned tracking with $o_{\text{min}} = 0^\circ$, $h = 600$ km for Cross-Track alignment.

Figure B.2: Imager characteristics for East-aligned tracking with $o_{\text{max}} = +30^\circ$, $h = 600$ km for Cross-Track alignment.
Appendix C

Supplementary $OEKF$ Results

Figure C.1: In-track $OEKF$ position errors for closed-loop propagation.

Figure C.2: Cross-track $OEKF$ position errors for closed-loop propagation.
Figure C.3: Radial OEFK position errors for closed-loop propagation.

Figure C.4: In-track OEFK velocity errors for closed-loop propagation.

Figure C.5: Cross-track OEFK velocity errors for closed-loop propagation.
Appendix C. Supplementary OEKF Results

Figure C.6: Radial OEKF velocity errors for closed-loop propagation.

Figure C.7: Target tracking attitude trajectory error for closed-loop propagation.

Figure C.8: Target tracking angular velocity trajectory error for closed-loop propagation.
Figure C.9: Target tracking angular acceleration trajectory error for closed-loop propagation.