Design, Analysis, and Prototyping of A 3×PPRS Parallel Kinematic Mechanism for meso-Milling

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Department of Mechanical and Industrial Engineering
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Abstract

Parallel Kinematic Mechanisms (PKMs) are well suited for high-accuracy applications such as meso-milling. However, drawbacks such as limited platform tilting angle and high configuration dependency of stiffness often limit their usage. In this Thesis, a new six degree-of-freedom (dof) PKM architecture based on a 3×PPRS topology is proposed, in order to address these problems.

The new PKM is presented, and its inverse kinematics and Jacobian matrix are derived. The kinematic relations are incorporated into MATLAB to calculate the workspace of the PKM. The stiffness of the new PKM is obtained using Finite Element Analysis (FEA), and configuration dependency of stiffness is investigated. The proposed new mechanism is compared with three similar existing 6-dof PKMs, and it is shown that the new PKM exhibits higher stiffness. Lastly, three meso-Milling Machine Tool prototypes were designed and built. In particular, Prototype III is based on the new mechanism.
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Nomenclature and Acronyms

Nomenclature

Latin Letters

\[ A_i = [A_{ix} \ A_{iy} \ A_{iz}]^T \]

Position of curvilinear base joint

\[ C_i = [C_{ix} \ C_{iy} \ C_{iz}]^T \]

Position of radial prismatic joint

\[ d_i \]

Radial prismatic joint travel

\[ d_{1i} \]

Tangential prismatic joint travel or RmMT

\[ d_{2i} \]

Radial prismatic joint travel for RmMT

\{E\}

Mobile platform frame at center of the platform

\[ H^{-1} \]

Transformation matrix from \( J^{-1}_E \) to \( J^{-1}_k \)

\[ J^{-1} \]

Inverse Jacobian Matrix

\[ J^{-1}_E \]

Inverse Jacobian matrix of Euler angles

\[ J^{-1}_k \]

Kinematic inverse Jacobian Matrix

\[ \Delta J^{-1}_k \]

Determinant of the kinematic inverse Jacobian Matrix

\[ L \]

Link length

\[ M_1 \]

First actuator

\[ M_2 \]

Second actuator

\{O\}

Global frame at center of the circular rail

\[ P_i = [P_{ix} \ P_{iy} \ P_{iz}]^T \]

Position of spherical joints

\[ Q = [\theta_1 \ \theta_2 \ \theta_3 \ d_1 \ d_2 \ d_3]^T \]

Joint-space coordinates

\[ \dot{Q} \]

Velocity in joint-space coordinates
\( R_b \) \hspace{1cm} \text{Radius of the circular rail}

\( R_p \) \hspace{1cm} \text{Radius of the mobile platform}

\( R_T \) \hspace{1cm} \text{Rotation matrix in } H^{-1}

\( X_p = [x_p \ y_p \ z_p \ \alpha \ \beta \ \gamma]^T \) \hspace{1cm} \text{Task-space coordinates}

\( \dot{X}_p \) \hspace{1cm} \text{Velocity in task-space coordinates}

\textbf{Greek Letters}

\[ [\alpha \ \beta \ \gamma] \] \hspace{1cm} \text{Euler angles}

\( \theta_i \) \hspace{1cm} \text{Curvilinear joint travel}

\( \Delta \theta_i \) \hspace{1cm} \text{Angle between two curvilinear joints}

\( \varphi_i \) \hspace{1cm} \text{Revolute joint travel}

\textbf{Acronyms}

3D \hspace{1cm} \text{3-Dimensional}

CAD \hspace{1cm} \text{Computer-aided design}

dof \hspace{1cm} \text{Degree-of-freedom}

FEA \hspace{1cm} \text{Finite element analysis}

FRF \hspace{1cm} \text{Frequency response function}

IKP \hspace{1cm} \text{Inverse kinematic problem}

mMT \hspace{1cm} \text{meso-Milling Machine Tool}

P \hspace{1cm} \text{Prismatic joint}
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>PCP</td>
<td>Platform center point</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-integral-derivative</td>
</tr>
<tr>
<td>PKM</td>
<td>Parallel Kinematic Mechanism</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse-width modulation</td>
</tr>
<tr>
<td>RmMT</td>
<td>Reconfigurable meso-Milling Machine Tool</td>
</tr>
<tr>
<td>R</td>
<td>Revolute joint</td>
</tr>
<tr>
<td>S</td>
<td>Spherical joint</td>
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</table>
Chapter 1  Introduction and Literature Review

1.1 Introduction

The term meso-Milling commonly refers to the machining of parts within the dimensional range of 0.5 to 5 mm, feature sizes of less than 0.1 mm, and tolerances of less than ±1µm. In recent years, miniaturized 3D sculpted parts with complex features, producible on such machines, are increasingly in demand in various industries, such as biomedical, aerospace, consumer electronics, and defense [1].

Compared to conventional size machine tools, smaller-scale meso-Milling Machine Tools (mMTs) offer several advantages, such as smaller footprint, less energy consumption, and less cost [2]. Currently, there are several mMTs that can mill parts in the range of millimeters or less. For instance, the MesoMill developed by MIT Precision Engineering Research Group [3] can machine parts in the range 6 to 25mm. The overall size of this machine is 750×400×300 mm.

Current research efforts are mostly focused on the design of mMTs based on serial kinematic mechanisms (e.g., [4], [5], [6], [7]). For serial kinematic mechanisms, the feed stages are stacked one upon another, which means the overall stiffness of the machine at the tooltip is determined by the least stiff structural component, and that positioning errors are accumulated [8].

In response, Parallel Kinematic Mechanisms (PKMs) have attracted considerable attention from both researchers and manufacturers over the past two decades. A PKM is a closed-loop kinematic chain mechanism whose end-effector is linked to the base by independent kinematic chains [8]. PKMs are well suited for high-accuracy applications, such as motion simulators [9], coordinate measuring machines [10], surgery robots [11], micro-positioning systems [12] and machine tools [13].

The main advantage of PKMs is their increased positioning accuracy compared to serial kinematic mechanisms, due to their high overall stiffness. This characteristic of PKMs can be explained by the fact that the forces in the mechanisms are mainly acted along the kinematic
chains, which have very high stiffness in compression and tension. Also, the forces are distributed among the kinematic chains, as they are connected in parallel, which avoid error build-up [14]. Moreover, PKMs can be designed to have their actuators mounted at the base, which lowers the moving mass and allow for high operational speed [15].

However, there are several challenges in the design of 5-axis mMTs using PKMs, such as relatively small workspace coverage [16] and limited end-effector/tool platform tilting angle [17]. For example, while following a continuous path on a 3D surface, the platform tilting angle of most PKMs cannot reach 90°, which is a fundamental requirement for full 5-axis milling operations. Furthermore, PKMs that can achieve 90° platform tilting angle often do not satisfy the required stiffness for milling operations [18]. Lastly, the performance of a PKM is highly pose dependent, namely, there is a large variation of performance within its workspace [14].

To address these problems, several approaches have been proposed. One approach is to design hybrid mechanisms, which is the combination of serial and parallel mechanisms [19]. However, it is not a first priority design option in this research, since it would have lower stiffness than its PKM counterparts. Another approach is to incorporate redundant degrees-of-freedom (dof) into the mechanism, namely, using more actuators than the required platform dof [20] [21]. The added redundancy can allow for more machine configurations, which can be utilized to improve machine performance (i.e., platform tilting angle) through optimization. Lastly, the third approach is to design novel PKM architectures with enhanced performance.

1.2 Overview of PKM Based 5-axis Machine Tool Designs

Several PKM based machine-tool concepts have been realized in the past. However, there is no existing PKM-based mMT design. Therefore, this literature review will provide an overview of conventional size PKM-based machine tool designs, and brief discussion of their performance criteria.

In the context of machine-tool applications, most known prototypes of commercial PKMs can be categorized into two families: (i) PKMs with fixed base joints and length varying struts, and (ii) PKMs with moving base joints and fixed length struts [22]. It can be noted that redundancies and
hybrids are extensively employed in the realized designs. It is also to be noted that these designs generally have a positioning accuracy of 3µm or worse [14].

The first family comprises machine-tool designs which are derived from the Gough-Stewart platform architecture. The Gough-Stewart platform has six prismatic actuators, and the actuators are connected to the base and the platform, usually through universal joints. One notable weakness of the Gough-Stewart platform is its low platform tilting angle [23].

This family of PKM-based machine tools includes the Hexapod 380 by CMW, as shown in Figure 1.1 [24]. It is a 5-axis milling machine tool for machining large parts used in nuclear and aerospace industry. It has a classical Stewart-platform architecture, and can machine parts with a positioning accuracy of 8µm. It has a platform tilting angle of ±30°.

![Figure 1.1: The Hexapod 380 by CMW.](image)

One way to improve platform tilting angle is to employ a hybrid mechanism into the machine design. For example, the Tricenter by Deckel Maho [25] is a 5-axis hybrid mechanism milling machine, as shown in Figure 1.2. This design is specially focused on tool dexterity. The machine has three actuators arranged in parallel to control x, y, and z motions of the central platform, which carries a two-axis serial milling heads. This hybrid mechanism milling machine is able to achieve ±60° platform tilting in x-axis, and ±90° in z-axis.
In another attempt to overcome limitation in platform tilt, the company Metrom developed a Pentapod based machine [26], as shown in Figure 1.3, which has five extensible struts carrying the platform. The kinematics has been configured so that in one tool axis, the platform is able to achieve 90° tilting angle. However, in order to achieve full 5-axis machining, a redundant rotary table needs to be incorporated in serial to the Pentapod, which makes it a hybrid mechanism.

Similar PKM based machine tools in this family include the VARIAX-Hexacenter by Gidding & Lewis, the TORNADO 2000 by Hexel, and the MIKROMAT 6X (Mikromat/IWU) [22].
The second family of PKMs have been investigated more recently: This category includes the Ecospeed [14] by DS-Technology, as shown in Figure 1.4. It is specifically designed for high speed machining of structural aircraft components from aluminum alloy plate and billet type materials. The machine is developed as a hybrid system, its rotary and z-axes are fully parallel, and are carried by the serial x- and y-stages. The machine has a platform tilting angle of ±40°, and a stiffness of 55N/μm, 50N/μm and 250/μm in x, y, and z direction, respectively.

![Figure 1.4: The Ecospeed by DS-Technology.](image)

The Eclipse, shown in Figure 1.5, deserves special mention [17]. It is a redundant 5-axis machine with three kinematic chains, and has a 3×PPRS topology. The Eclipse stands out from all the existing PKM based machine tools, as it is the only fully parallel 5-axis machine tool which can achieve 90° platform tilting angle. In fact, a first effort in our laboratory in designing a new mMT was based on downsizing the Eclipse mechanism [18]. Although new approaches were needed due to its insufficient stiffness, the Eclipse does provide an excellent reference for this research.

Other PKM based machine tools that belong to the second family include the HexaM by Toyota, the Urane SX by Renault Automation, and the Quickstep by Krause & Mauser [22].
1.3 Reference Parallel Kinematic Mechanisms

Along with the Eclipse mechanism, two other existing 6-dof PKMs were found to be able to reach 90° platform tilting angle, while following a continuous path along a 3D surface. They are shown in Figure 1.6: the Alizade mechanism [27], with a 3×PPRS topology, and the Glozman mechanism, with a 3×PRRS topology [28]. These PKMs all consist of three kinematic chains with a curvilinear base joint. In this thesis, they are referred to as the Reference PKMs.

Figure 1.6: Reference PKMs: (a) the Alizade PKM, (b) the Eclipse (c) the Glozman PKM.
It is to be noted, that there are other three 6-dof PKMs which are similar to the Reference PKMs, as shown in Figure 1.7: the Behi mechanism with a 3×PPRS topology [29], the Tahmasebi mechanism with a 3×PPUU topology [30], and the Ben-horin mechanism with a 3×PPRS topology [31]. The Behi mechanism is similar to the Alizade mechanism, and its base joint can be sliding on either a triangular or a rectangular rail. For Tahmasebi and Ben-horin mechanisms, there is an x-y motion stage in each of the base joints, and the base joints can be fixed at different locations on a plane.

It is shown that these mechanisms can achieve 90° platform tilting angle. However, since each of the base joints in these PKMs only has linear motion with a limited range of travel, their workspace coverage is substantially smaller than the Reference PKMs when examined using similar geometrical design parameters and are, therefore, excluded from the Reference PKMs list.

Figure 1.7: Similar mechanisms to Reference PKMs: (a) the Behi mechanism, (b) the Tahmasebi mechanism, (c) the Ben-horin mechanism.

1.4 Thesis Objective

From the literature review, it is seen that there currently exists no PKM-based mMT design, so an overview of conventional size parallel machine tools was provided. Investigating conventional size machine tools, it was found that existing PKM-based machine tools suffer from two major drawbacks: (i) limited range of platform tilting angle (e.g., the Hexapod), and (ii) low structural stiffness, which leads to inaccuracy at the tooltip (e.g., the Eclipse).
Therefore, the objective of this thesis is to develop a new Parallel Kinematic Mechanism, which is suitable for 5-axis meso-Milling Machine Tool design. The PKM needs to satisfy the following requirements:

1. Ability to reach at least 90° platform tilting angle, while following a continuous path on a 3D surface.
2. Among all the possible PKMs which meet the first requirement, the new PKM needs to attain the highest stiffness.

1.5 Organization of Thesis

The remainder of the thesis is organized as follows:

Chapter 2 presents the architecture and kinematic modelling of the new PKM. In particular, the inverse kinematics and the Jacobian matrices are derived.

Chapter 3 is divided into three sections. Section 3.1 analyzes the new PKM’s workspace and platform tilting angle. The workspace is defined with respect to the PKM’s platform tilting angle, and the effect of geometrical design parameters on the platform tilting angle is investigated. Section 3.1 concludes with a brief discussion on singularity within the new PKM’s workspace. Section 3.2 discusses the new PKM’s stiffness, as well as its configuration dependency. Lastly, Section 3.3 presents a comparative analysis of the new PKM and the Reference PKMs.

Chapter 4 describes three built RmMT prototype, as well as their respective mechanical designs. The built reconfigurable mMT prototypes are redundant mechanisms and have 9-dof, and are utilized as test beds for integrating subcomponents and conducting physical experiments.
Chapter 2  Design

In this Chapter, the architecture of the proposed new PKM is presented. Its inverse kinematic model, which describes the relationship between the position and orientation (pose) of the mobile platform and the positions of the actuated joints, is derived. Also, the inverse kinematic Jacobin matrix is obtained. These kinematic relations will be used to investigate workspace and platform tilting angle in Chapter 3.

2.1 Architecture

The new PKM, which is shown in Figure 2.1, has a $3\times$PPRS topology (the underline indicates the actuated joints). It has three identical kinematic chains that connect the mobile platform to the stationary circular rail. Each chain has two actuators, which are drawn in red. $M_1$ denotes the first actuator, which moves along the circular rail. The second actuator, denoted as $M_2$, is mounted on top of $M_1$, and it moves in radial direction with respect to the circular rail. A fixed-length link is connected to the second actuator, $M_2$, through a revolute joint, and the mobile platform is connected to the link through a spherical joint.

![Figure 2.1: The New PKM architecture.](image-url)
2.2 Kinematic Modeling

2.2.1 Notation

Figure 2.2 shows the schematic representation of the new PKM. The mechanism consists of a circular rail with radius $R_b$, and the global coordinate frame $\{O\}$ is positioned at its center point. Three actuated curvilinear joints are attached on top of the circular rail, and the position of the curvilinear joint on the $i^{th}$ ($i=1$ to 3) kinematic chain is denoted as $A_i = [A_{ix} \ A_{iy} \ A_{iz}]^T$. Then, three actuated prismatic joints which travel in radial direction with respect to the circular rail are mounted on top of the curvilinear joints, and they are located at $C_i = [C_{ix} \ C_{iy} \ C_{iz}]^T$. Next, three links of fixed length, $L$, connect the prismatic joints and the mobile platform through passive revolute and spherical joints, respectively. The revolute joints have an angular travel of $\phi_i$. The spherical joints are located $120^\circ$ apart on a mobile platform with radius $R_p$, and their positions are denoted as $P_i = [P_{ix} \ P_{iy} \ P_{iz}]^T$.

![Figure 2.2: Kinematic notation for the new PKM.](image)

The joint-space coordinates of the active joints are defined by the vector $Q = [\theta_1 \ \theta_2 \ \theta_3 \ d_1 \ d_2 \ d_3]^T$, where $\theta_i$ represents the curvilinear joint travel, and $d_i$ represents the prismatic joint travel, respectively. The mobile frame, $\{E\}$, is attached at the center of the
platform, and its z-axis is normal to the platform. In this Thesis, the pose of the mobile platform is described by its center point position, and three Euler angles representing the orientation of the mobile frame, \( \{E\} \), with respect to the base frame, \( \{O\} \). The minimum number of independent coordinates describing the pose of a 6-dof mobile platform is six, which is denoted here as \( X_p = [x_p \ y_p \ z_p \ \alpha \ \beta \ \gamma]^T \).

### 2.2.2 Inverse Kinematics

The inverse kinematics problem (IKP) of a PKM describes the mapping from the pose of its mobile platform, \( X_p \), to the joint-space coordinates, \( Q \). IKP is relatively straightforward to solve, as the actuated joint-space variables can be expressed explicitly in terms of task-space variables [8]. The purpose of solving IKP is to use the relationships in subsequent analyses. One way of using inverse kinematics relations is to verify whether a given coordinate point can be reached by the PKM, and by checking every point in a predefined volume (e.g., the volume of a sphere), the PKM’s workspace size and platform tilting angle can be derived. Also, inverse kinematics can be employed to determine the system inverse Jacobian, which can be used in singularity calculation.

To solve the IKP of the new PKM, first, the positions of the spherical joints \( P_i \) \( (i = 1 \ to \ 3) \) with respect to the mobile frame, \( \{E\} \), are calculated:

\[
P^E_1 = [R_p \ 0 \ 0]^T, \tag{2.1}
\]

\[
P^E_2 = [-R_p/2 \quad \sqrt{3}R_p/2 \quad 0]^T, \quad \text{and} \tag{2.2}
\]

\[
P^E_3 = [-R_p/2 \quad -\sqrt{3}R_p/2 \quad 0]^T. \tag{2.3}
\]

Then, the position of each spherical joint, with respect to global frame, \( \{O\} \), is defined by

\[
P_i = [x_p \ y_p \ z_p]^T + RP_i^E, \tag{2.4}
\]
where \( P_i \) is the position of the \( i^{th} \) spherical joints given in task-space coordinates. \( R \) is the rotation matrix of the mobile platform with respect to global frame. From here, the joint-space coordinates of the PKM in terms of position of the spherical joints, \( P_i \), can be expressed as:

\[
\theta_i = \tan^{-1}(P_{iy}/P_{ix})
\]

\[
\varphi_i = \sin^{-1}(P_{iz}/L), \text{ and}
\]

\[
d_i = \sqrt{L^2 - P_{iz}^2} + \sqrt{(P_{ix}^2 + P_{iy}^2)} - R_b.
\]

Given Equations (2.1) to (2.7), the positions of the actuated curvilinear and prismatic joints can be obtained as:

\[
A_i = [R_b \cos \theta_i \quad R_b \sin \theta_i \quad 0]^T, \text{ and}
\]

\[
C_i = [d_i \cos \theta_i \quad d_i \sin \theta_i \quad 0]^T.
\]

### 2.2.3 Inverse Jacobian Matrix

The inverse Jacobian matrix is defined here as the matrix relating the velocity of the mobile platform to the velocities of the actuated joints:

\[
\dot{Q} = J^{-1} \dot{X}_p.
\]

There are two types of inverse Jacobian matrix, depending on how \( \dot{X}_p \) is defined. The first type is the kinematic inverse Jacobian matrix \( (J_k^{-1}) \), which relates the mobile platform’s linear and angular velocities to the actuated joints’ velocities. The second type is the inverse Jacobian matrix of Euler angles \( (J_E^{-1}) \), which relates the mobile platform’s linear velocity and the variations of the Euler angles to the actuated joints velocities [16]. Since \( \dot{X}_p \) is defined with respect to its Euler angles variations, the inverse Jacobian matrix of Euler angle can be calculated as:
Moreover, the kinematic inverse Jacobian matrix is needed for singularity analysis, and can be obtained from the inverse Jacobian matrix of Euler angles through a simple matrix transformation, [8]:

\[
J_k^{-1} = J_E^{-1} H^{-1},
\]

(2.12)

where

\[
H = \begin{bmatrix}
I_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & R_T
\end{bmatrix}, \text{ and}
\]

(2.13)

\[
R_T = \begin{bmatrix}
0 & -\sin \alpha & \cos \alpha \sin \beta \\
0 & \cos \alpha & \sin \alpha \sin \beta \\
1 & 0 & \cos \beta
\end{bmatrix}.
\]

(2.14)
Chapter 3 Analysis

3.1 Workspace and Platform Tilting Angle

Obtaining large workspace and high platform tilting angle is a challenging issue in the design of PKMs, as they generally have small workspace relative to their footprints, and also low platform tilt [17]. The three main types of constraints that limit the workspace and platform tilting angle of a PKM are (i) the actuator limits, (ii) the passive-joint limits, and (iii) link interferences.

It is usually difficult to obtain the complete six-dimensional workspace of a PKM, thus, different subsets of the complete workspace are usually employed for different purposes. For instance, the constant-orientation workspace is defined as all the possible positions that can be reached by the PKM with a given platform orientation [15]; the orientation workspace is defined as all the possible orientations that can be reached by the PKM at a particular position [32]; and, the dexterous workspace is defined as all the possible pose of the mobile platform that can be reached by the PKM at all platform orientations [33].

3.1.1 Definition

In this Thesis, the workspace of the PKM is defined as the maximum hemisphere size where all the positions within its volume are reachable by the platform center point (PCP), with the platform being normal to the hemispherical surface of a workpiece.

One important thing to be mentioned is that the platform needs to tilt from 0 to 90° along the hemisphere in order to meet the above workspace definition. Thus, herein, the maximum platform tilting angle is the performance criterion to be discussed, where it determines the size of the workspace. Moreover, since the platform can tilt more than 90° in some PKM configurations, the platform tilting angle is investigated over a sphere, instead of a hemisphere, where the angle is measured from the z-axis, as shown in Figure 3.1. It can also be noted here that a PKM configuration is defined as a specific PKM pose.
3.1.2 Method

In order to determine the tilting angle of the PKM, a discretization method can be employed for its simplicity [15]. Herein, a sufficiently large spherical region in Cartesian space is discretized into a finite number of points. Then, for each point within the region, the inverse kinematic problem is solved, and the kinematic constraints are imposed, which checks the PCP’s ability to reach the point. The platform tilting angle can be obtained for every reachable point, where each point is represented by its position and Euler Angles.

3.1.3 Geometrical Study

The objective for developing a new PKM architecture in this Thesis is to design a 5-axis meso-Milling Machine Tool (mMT) based on it. Thus, in order to provide useful information for machine design, a geometrical study is conducted for the PKM. Specifically, the relationships between the platform tilting angle and the design parameters are obtained. The aim of this study is to determine the set of design parameters that would result in the maximum platform tilting angle for any sphere size.

Since discretization is computationally intensive [8], the simulations are executed on a spherical surface instead of a volume, as shown in Figure 3.2 (a). The radius of the sphere in this analysis
is chosen to be 10 mm. This choice is arbitrary, as it is observed that for any given sphere radius, similar relationships would exist between the platform tilting angle and the geometrical design parameters.

In addition, only the cross-section of the spherical surface is evaluated instead of the entire surface. For PKM architectures with curvilinear rails, when the PCP is able to reach an angle at one point on the spherical surface, it can reach the same angle all around the sphere. This behavior of symmetry can be seen from Figure 3.2 (b).

Lastly, the simulation setup ensures that the reachable points are continuous along the spherical surface. Hence, in the subsequent analysis, when the results show a reachable tilting angle of 90°, it automatically implies that the PKM can reach 0 to 90°.

![Figure 3.2](image)

**Figure 3.2**: Geometrical study (a) top half of a spherical surface, (b) cross-section of the top half a spherical surface.

### 3.1.3.1 Geometrical dimensions and constraints

There are two sets of design parameters, some are geometrical dimensions, and some are joint constraints. This study varies one or two parameters at a time while fixing others, in order to obtain the relationships between the design parameters and the maximum platform tilting angle of the PKM. Geometrical dimensions include the circular rail radius, $R_b$, the link length, $L$, the platform size, $R_p$, and the minimum curvilinear joint separation. Joint constraints include
curvilinear joints’ travel, $\theta_i$, prismatic joints’ travel, $d_i$, revolute joints’ travel, $\varphi_i$, and spherical joints angle of swing, as shown previously in Figure 2.2.

The design parameters are summarized in Table 3.1 with the default values, which permit the PKM to reach at least 90° platform tilting angle.

Table 3.1: Geometrical dimensions and constraints – default values.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>New PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular rail radius ($R_b$)</td>
<td>150 mm</td>
</tr>
<tr>
<td>Curvilinear joints’ range of travel ($\theta_i$)</td>
<td>Continuous 360°</td>
</tr>
<tr>
<td>Link length ($L$)</td>
<td>165 mm</td>
</tr>
<tr>
<td>Platform radius ($R_p$)</td>
<td>20 mm</td>
</tr>
<tr>
<td>Minimum curvilinear joint separation</td>
<td>10°</td>
</tr>
<tr>
<td>Prismatic joints’ range of travel ($d_i$)</td>
<td>0–65 mm</td>
</tr>
<tr>
<td>Revolute joints’ range of travel ($\varphi_i$)</td>
<td>5°–70°</td>
</tr>
<tr>
<td>Spherical joints’ range of travel</td>
<td>±75°</td>
</tr>
</tbody>
</table>

3.1.3.2 Curvilinear rail radius

The radius of the curvilinear rail determines the footprint of the PKM. Figure 3.3 shows the relationship between the platform tilting angle and the curvilinear rail radius. The platform tilting angle achieves 90° and beyond with a curvilinear rail radius of 150 mm, and reaches maximum at 250 mm.

![Figure 3.3: Platform tilting angle vs curvilinear rail radius.](image-url)
3.1.3.3 Link length

It is assumed herein, that all PKM links have the same length. The platform tilting angle of a PKM is sensitive to link length variation. Hence, it is important to obtain their relationship. In this study, the link lengths vary from 160 to 220 mm. It is noted that, as the length of the links changes, the distance between the spherical workpiece and the ground requires to be adjusted accordingly, in order to obtain maximum platform tilting angle for a given link length, as shown in Figure 3.4.

Figure 3.4: Spherical workpiece’s height vs link length.

Figure 3.5 displays the relationship between the spherical workpiece’s height from the ground and the link length. These results were obtained by investigating a limited number of link lengths.
Figure 3.5: Height of the spherical workpiece vs link length.

Figure 3.6 shows the relationship between the platform tilting angle and the link length. It is noted that, for the design parameters shown in Table 1, the PKM can indeed reach platform tilting angles of $90^\circ$ and beyond. The figure also indicates the existence of a possible optimal link length; $165$ mm for the PKM at hand, as defined in Table 3.1.

Figure 3.6: Platform tilting angle vs link length.

3.1.3.4 Platform size

The size of the platform can also be considered as a design parameter. The platform is generally used to carry the spindle for a machine tool. Therefore, it would be helpful to know the possible
platform sizes where the PKM can reach 90° and beyond, in order to determine the types of spindles to be incorporated.

Figure 3.7 displays the relationship between the platform size and the tilting angle. The PKM at hand can reach 90° tilting angle and beyond with a platform radius 25 to 35 mm. The curve peaks at about 30 mm.

![Figure 3.7: Platform tilting angle vs platform size.](image)

### 3.1.3.5 Link length and platform size

The above geometrical studies show that there may be optimal dimensions for both the link length and the platform size. For example, for the PKM at hand, the platform tilting angle reaches a maximum value with a link length of 165 mm. Moreover, the platform size in which the PKM reaches maximum tilt is 30 mm. Hence, the next step is to analyze the combined effect of the two design parameters. This study may be useful for future research in workspace optimization.

Figure 3.8 shows the platform tilting angle, with respect to the two design parameters, it is observed that the PKM’s platform tilting angle reaches maximum when the link lengths and the platform sizes are between 160 to 170 mm, and 25 to 35 mm, respectively.
Figure 3.8: Platform tilting angle vs platform size and link length.

### 3.1.3.6 Minimum curvilinear joint separation

The minimum curvilinear joint separation, as shown in Figure 3.9, determines the size of the curvilinear actuator. It is desirable to have as small curvilinear joint separation as possible, so that the size of the curvilinear actuator can be selected with more flexibility.

Figure 3.9: Minimum curvilinear joint separation.
Figure 3.10 shows that, for the PKM at hand, for a minimum curvilinear joint separation of more than 15°, the mechanism loses its ability to reach a platform tilting angle of 90° or beyond. It can be noted that for circular rail of 150 mm radius, a minimum of 15° separation corresponds to a curvilinear actuator size of minimum 40 mm, which is currently smaller than most available conventional actuators.

![Figure 3.10](image)

Figure 3.10: Platform tilting angle vs minimum curvilinear joint separation.

3.1.3.7 Prismatic joint travel

The relation between the prismatic joint travel and platform tilting angle is also needed for actuator selection. As can be noted from Figure 3.11, for the chosen set of default design parameters, Table 3.1, prismatic actuators with more than 36 mm range of travel are sufficient for the PKM to reach 90° or beyond platform tilting angle.

![Figure 3.11](image)

Figure 3.11: Platform tilting angle vs prismatic joint range of travel.
### 3.1.3.8 Spherical joint angle of swing

A spherical joint provides 3-dof mobility at a linkage connection point. However, a spherical joint’s angle of swing is a complicating factor in the mechanical design of a PKM, especially, when trying to achieve high platform tilting angles, Figure 3.12. Currently, commercial spherical joints have at most ±35° angle of swing [34], [35].

![Spherical joint](image)

**Figure 3.12: Spherical joint.**

From Figure 3.13, it can be noted that for the PKM at hand, a ±35° spherical joint angle of swing provides very limited tilting angle at the platform. In order to achieve 90° or beyond platform tilting angle, the spherical joint would need to have at least ±60° angle of swing.

![Platform tilting angle vs spherical joint angle of swing](image)

**Figure 3.13: Platform tilting angle vs spherical joint angle of swing.**
3.1.3.9 Summary

The maximum platform tilting angle of a PKM was investigated with respect to different design parameters, for a spherical workpiece of 10 mm radius. It is shown that the tilting angle is sensitive to changes in link length and platform size. Furthermore, there exists optimal solutions for the two design parameters, and this finding can be used for workspace optimization.

The default design parameters given in Table 3.1 can be adjusted to yield larger platform tilting angle. Moreover, setting the prismatic joints’ range of travel to 0 to 40 mm would be sufficient for obtaining a minimum of 90° platform tilting angle, for a spherical workspace with a radius of less than 12 mm. The modified PKM parameter values are shown in Table 3.2.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>New PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular rail radius ($R_b$)</td>
<td>150 mm</td>
</tr>
<tr>
<td>Curvilinear joints’ range of travel ($\theta_i$)</td>
<td>Continuous 360°</td>
</tr>
<tr>
<td>Link length ($L$)</td>
<td>165 mm</td>
</tr>
<tr>
<td>Platform radius ($R_p$)</td>
<td>25 mm</td>
</tr>
<tr>
<td>Minimum curvilinear joint separation</td>
<td>5°</td>
</tr>
<tr>
<td>Prismatic joint’s range of travel ($d_i$)</td>
<td>0–40 mm</td>
</tr>
<tr>
<td>Revolute joints’ range of travel ($\theta_i$)</td>
<td>5°–70°</td>
</tr>
<tr>
<td>Spherical joints’ range of travel</td>
<td>±75°</td>
</tr>
</tbody>
</table>

3.1.4 Singularities

Kinematic singularity occurs when a mechanism loses one or more dof for certain configurations, and it is related to the system inverse kinematic Jacobian matrix, $\Delta J_k^{-1}$, which was derived in Chapter 2. For PKMs, kinematic singularities are typically classified into three categories [36]:

Type I singularity, also known as serial singularity, results when the determinant of $J_k^{-1}$ goes to infinity, namely,

$$\Delta J_k^{-1} = \pm \infty$$ (3.3)
Type I singularity indicates that there exist some nonzero $\dot{Q}$ vectors that result in zero $\dot{X}_p$ vectors. Physically, it means that the mobile platform remains stationary, while the actuators are moving. Type I singularities usually occur at the boundary of the workspace, where different branches of the inverse kinematic solutions converge.

Type II singularity, also known as parallel singularity, results when the determinant of $J_k^{-1}$ is equal to zero, namely,

$$\Delta J_k^{-1} = 0$$

(3.4)

Type II singularity indicates that there exist some nonzero $\dot{X}_p$ vectors that result in zero $\dot{Q}$ vectors. Namely, the mobile platform is able to achieve motion in space, while all the actuators are completely locked.

Type III singularity occurs when both Type I and Type II singularities are present at the same time.

It is to be noted, that the inverse kinematic Jacobian matrix derived in Section 2.2.3 is not homogenous in terms of units for two reasons: (i) the PKM has two types of actuated joints, and their joint-space coordinates have different units (e.g., millimetre for prismatic joints, and degree for curvilinear joints), and (ii) the mobile platform has translational and rotational dof. Therefore, the “closeness” of a 6-dof PKM configuration to a singularity can be only determined with respect to the distribution of Jacobian determinant over the workspace. For example, a Type II singularity occurs at a configuration when the determinant of the inverse kinematic Jacobian matrix is low compared to the determinants for other configurations.

Similar to workspace and platform tilting angle analyses, singularity simulations were carried out through a discretization method. The determinant of the inverse kinematic Jacobian matrix is checked at each point on the spherical surface, which has a radius of 10 mm. If the determinant value at a particular point is relatively high or low compared to its surrounding, the point is close to being singular.
Herein, the relative high and low determinant values are chosen to be 4 and 0.25, respectively. This choice of selection is based on observations from the previously built prototypes, and existing analyses [18], [37]. The numbers do approximate the regions of singularities within the workspace of the PKM, as shown in Figure 3.14, blue points indicate singular positions.

![Figure 3.14: Singular positions within the workspace (a) perspective view, (b) top view.](image)

**3.2 Dynamic Stiffness**

The position accuracy of a PKM is directly related to its stiffness. In particular, the dynamic stiffness indicates the magnitude of the vibration response of the mechanism with respect to a time-varying load. The stiffness of a PKM is highly related to its specific configuration [14]. Hence, it is important to study the configuration-dependency of stiffness, as the result can be used to plan tool trajectory to avoid regions/directions of excessive structural vibration, and also to be used in the design of closed-loop controllers for vibration damping [38].

In this study, the minimum dynamic stiffness of the PKM developed in this Thesis was evaluated, as it relates to the maximum deformation at the PCP. The minimum dynamic stiffness occurs when the frequency of the applied force is equal to one of the natural frequencies of the PKM.
3.2.1 Configuration Dependency of Dynamic Stiffness

At a given frequency, the dynamic stiffness of a PKM can be calculated by inverting the Frequency Response Function (FRF). The Cartesian FRFs of the PKM are obtained using a commercial FEA software package, ANSYS. To study the configuration-dependency of dynamic stiffness, harmonic analysis was conducted for five different PKM configurations. Each configuration corresponds to a set of joints locations, where $\Delta \theta_i$ represents the separation between the curvilinear joints, and $d_i$ represents the travel of the prismatic joints ($i = 1$ to $3$). As can be noted in Figure 3.15, the five configurations are defined by: (i) $d_i = 25\, mm$, $\Delta \theta_i = 120^\circ$, (ii) $d_i = 0\, mm$, $\Delta \theta_i = 120^\circ$, (iii) $d_i = 50\, mm$, $\Delta \theta_i = 120^\circ$, (iv) $d_i = 25\, mm$, $\Delta \theta_i = 60^\circ$, and (v) $d_i = 25\, mm$, $\Delta \theta_i = 140^\circ$. For Configuration 5, the angles between the curvilinear joints are $140^\circ$, $140^\circ$ and $80^\circ$, respectively.

Configuration 1 can be considered as the Reference Configuration, where its prismatic-joints are at the middle of their travel range, and the curvilinear joints are equally spaced. Configurations 2 to 5 represent the relative extreme locations of the joints. By studying the five chosen PKM configurations, an approximate relationship between PKM configuration and its dynamic stiffness can be obtained. The geometrical dimensions used for this analysis were listed in Table 3.2.
Figure 3.15: PKM configurations for stiffness analysis (a) Configuration 1, (b) Configuration 2, (c) Configuration 3, (d) Configuration 4, and (e) Configuration 5.

For each harmonic analysis, a sinusoidal force of 1 N magnitude is applied to the PCP along the x, y and z-axes. For every solution, the Cartesian displacement of the PCP is captured along the direction of the applied force. The contacts between the pin and housing of revolute and
spherical joints are modeled as non-separation, and all other contacts are bonded [39]. The constant damping ratio for the PKM is set as 1%. Although the estimated damping ratio might not be a close approximation, it is nonetheless adequate for this analysis, as the absolute magnitude of stiffness values are irrelevant for comparison between different PKM configurations. Figure 3.16 shows an example of the simulation environment.

![Simulation Environment](image)

**Figure 3.16:** FEA simulation environment.

The Cartesian components of the FRFs for each PKM configuration are shown in Figure 3.17. It can be observed that the FRFs are indeed highly dependent on PKM configurations.

The minimum dynamic stiffness values along the three Cartesian axes for the five PKM configurations are summarized in Table 3.3. For ease of comparison, all stiffness values are presented as a ratio with respect to Configuration 1, which is the reference configuration. The stiffness of Configuration 1 is normalized to 1.
Figure 3.17: FRF for different PKM configurations along (a) xx, (b) yy, and (c) zz directions, respectively.
Table 3.3: Minimum dynamic stiffness for 5 different PKM configurations.

<table>
<thead>
<tr>
<th>Minimum Dynamic Stiffness</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
<th>Configuration 3</th>
<th>Configuration 4</th>
<th>Configuration 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>1</td>
<td>0.61</td>
<td>2.5</td>
<td>1.11</td>
<td>0.78</td>
</tr>
<tr>
<td>$K_{yy}$</td>
<td>1</td>
<td>0.59</td>
<td>9.34</td>
<td>0.20</td>
<td>3.59</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>1</td>
<td>1.95</td>
<td>0.28</td>
<td>0.03</td>
<td>0.42</td>
</tr>
</tbody>
</table>

3.2.2 Summary

The results given in Table 3.3 can be summarized as follows:

- Configuration 1 ($d_i = 25 \text{ mm}, \Delta \theta_i = 120^\circ$) has the intermediate values for minimum dynamic stiffness, which indicates that the choice of joints locations is appropriate for the Reference Configuration.

- Configuration 2 ($d_i = 0 \text{ mm}, \Delta \theta_i = 120^\circ$) displays the highest stiffness in the $z$-direction, and relatively low stiffness in the $x$ and $y$ directions. Low prismatic-joint travel corresponds to large $\varphi$, which implies that the links’ axes are oriented more towards the $z$-axis. Thus, the axial stiffness of the link is the main component to resist the applied forces in $z$ direction. In $x$ and $y$ directions, the links are subject more to bending, as the angle between the links and the applied force is large [39]. For links with large length-to-width ratios, the bending stiffness is lower than the axial stiffness.

- Configuration 3 ($d_i = 50 \text{ mm}, \Delta \theta_i = 120^\circ$) shows the opposite behavior compared to Configuration 2, where it has relatively high stiffness in $x$ and $y$ direction, and low stiffness in $z$ direction. The same logic from Configuration 2 can be applied here.

- Configuration 4 ($d_i = 25 \text{ mm}, \Delta \theta_i = 60^\circ$), unlike configurations where all the links of the PKM are spaced evenly, has the lowest dynamic stiffness in $y$ and $z$ directions. A possible explanation would be, as the three links are clustered together, they act somewhat like a
single cantilever beam, in which the PKM no longer retains its closed kinematic loop structure, and consequently loses the ability to resist force in all directions.

- Configuration 5 ($d_i = 25 \text{ mm}, \Delta \theta_i = 140^\circ$) shows the largest discrepancy in stiffness between the $x$ and $y$ directions. Using the abovementioned argument from above, when two links are close to each other, they act more like a single link. While the third link is positioned far apart, the configuration translates into a two-link mechanism. In this configuration, the force applied along the $y$-axis has a large fraction of its components acting axially to the links, which results in high axial stiffness. The force applied in the $x$ direction, on the other hand, is normal to the link axis which creates bending motion, which leads to low stiffness.

To conclude, the dynamic stiffness of the PKM is highly dependent on its configuration. In general, curvilinear joints should be evenly spaced on the circular rail in order for the PKM to acquire high overall PKM stiffness. Furthermore, the stiffness of the new PKM is sensitive to the change in $\varphi$, which corresponds to prismatic-joint travel. Thus, for applications which require high stiffness in the $x$ and $y$ directions, it is desirable to operate in a large prismatic-joint travel range. Similarly, for applications which require high stiffness in the $z$ direction, it is favorable to operate in a low prismatic joint travel range.

### 3.3 Comparative Analysis

In this Section, the proposed new PKM is compared with three reference PKMs shown in Figure 3.18. All the compared PKMs are based on three-chain topology, and each contains two actuated joints, labeled as $M_1$ and $M_2$. $M_1$ is a curvilinear joint which moves along the circular rail. The primary difference between the new PKM and the Reference PKMs lies in the motion of the $M_2$. In the new PKM, it moves in radial direction with respect to the rail.
The geometrical parameters and kinematic constraints of the compared PKMs, namely, the new PKM, the Eclipse, the Alizade PKM and the Glozman PKM are summarized in Table 3.4. The design parameter values were chosen according to the following criteria: (i) the PKM can achieve at least $90^\circ$ platform tilting angle, and (ii) the dimensions are identical for all the PKMs. For the Glozman PKM, the total length of its two links is 240 mm instead of 165 mm. This extra length is required as it is the closest to the dimension of the other PKMs’ links, to achieve the $90^\circ$ platform tilting angle. It is assumed that a small difference in link length has limited effect on the dynamic stiffness of the PKMs.
### Table 3.4: Geometrical parameters and kinematic constraints of the PKMs.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>New PKM</th>
<th>Eclipse</th>
<th>Alizade</th>
<th>Glozman PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Base radius</em> $(R_b)$</td>
<td>150 mm</td>
<td>150 mm</td>
<td>150 mm</td>
<td>150 mm</td>
</tr>
<tr>
<td><em>Platform radius</em> $(R_p)$</td>
<td>25 mm</td>
<td>25 mm</td>
<td>25 mm</td>
<td>25 mm</td>
</tr>
<tr>
<td><em>Link length</em> $(L)$</td>
<td>165 mm</td>
<td>165 mm</td>
<td>165 mm¹</td>
<td>120 mm</td>
</tr>
<tr>
<td>Curvilinear base joints’ range of travel $(θ_1)$</td>
<td>Continuous 360°</td>
<td>Continuous 360°</td>
<td>Continuous 360°</td>
<td>Continuous 360°</td>
</tr>
<tr>
<td>Prismatic joints’ range of travel $(d_i)$</td>
<td>65 mm</td>
<td>65 mm</td>
<td>65 mm</td>
<td>-</td>
</tr>
<tr>
<td>Revolute joints’ range of travel $(φ_1)$</td>
<td>5°–70°</td>
<td>5°–70°</td>
<td>5°–70°</td>
<td>5°–70°</td>
</tr>
<tr>
<td>Spherical joints’ range of travel</td>
<td>±70°</td>
<td>±70°</td>
<td>±70°</td>
<td>±70°</td>
</tr>
</tbody>
</table>

#### 3.3.1 Platform Tilting Angle

To justify the above selection of design parameters, the maximum platform tilting angle is calculated for each of the compared PKMs using the same method in Section 3.1.2. The results are given in Figure 3.19, which shows that the maximum platform tilting angle for all the PKMs do exceed 90° for a sphere radius of 1 to 5 mm. One thing to be noted is that the design parameters are not optimized, so it only shows the PKMs’ ability to reach 90° platform angle, but suggests nothing about their comparative workspace size.

![Figure 3.19: Maximum platform tilting angle for the compared PKMs.](image)

¹ The link length of the Alizade PKM is defined at the middle of the prismatic joint travel range.
3.3.2 Stiffness

The dynamic analysis of the new PKM presented in Section 3.2.1 is repeated here for all four PKMs with the dimensions specified in Table 3.4, and the results are used for comparison. Three different PKM configurations are employed in this study.

The first configuration is chosen where the $M_2$ joints for each PKM have minimum travel, as shown in Figure 3.20. In the new PKM, the Eclipse PKM and the Alizade PKM, their prismatic-joints travel 0 mm for this configuration. In the Glozman PKM, its $M_2$ joints are revolute, and the joints’ travels are represented by the angle $\phi$. In order to avoid mechanical interference, the minimum travel for the Glozman PKM is set to be $30^\circ$.

Figure 3.20: Configuration 1 (a) the new PKM, (b) the Eclipse, (c) the Alizade PKM, and, (d) the Glozman PKM.
The second configuration is chosen where the $M_2$ joints for each of the PKMs are at the middle of its maximum range of travel, which is shown in Figure 3.17. In the new PKM, the Eclipse and the Alizade PKM, the prismatic joints travel is 32.5 mm, while in the Glozman PKM, the revolute joints are at 60° travel.

The third configuration is selected where the $M_2$ joints for each of the PKMs reach maximum travel, as shown in Figure 3.21. The prismatic joints travel 65 mm for the new PKM, the Eclipse and the Alizade PKM. For Glozman PKM, the revolute joints travel is set to be 90°.

![Figure 3.21: Configuration 3 (a) the new PKM, (b) the Eclipse, (c) the Alizade PKM, and, (d) the Glozman PKM.](image)

FRFs for Configurations 1 through 3 are presented in Figure 3.22, Figure 3.24, and Figure 3.25, respectively. The FRFs’ magnitudes for the Glozman PKM are large compared to the other three PKMs in the $z$ direction for Configurations 1 and 2. Hence, two separate graphs displaying the FRFs of the new, the Eclipse and the Alizade PKM in the $z$ direction can be seen in Figure 3.23 and Figure 3.26.
Figure 3.22: Configuration 1 - FRF for all the PKMs along (a) xx, (b) yy, and (c) zz directions.

Figure 3.23: Configuration 1 - FRF for all the PKMs excluding the Glozman PKM.
Figure 3.24: Configuration 2 - FRF for all the PKMs along (a) xx, (b) yy, and (c) zz directions.
Figure 3.25: Configuration 3 - FRF for all the PKMs along (a) xx, (b) yy, and (c) zz directions.

Figure 3.26: Configuration 3 - FRF for all the PKMs excluding the Glozman PKM.
3.3.3 Summary

Table 3.5 to 3.7 summarize the minimum dynamic stiffness values along the three Cartesian axes for the compared PKMs.

<table>
<thead>
<tr>
<th>Minimum Dynamic Stiffness</th>
<th>The New PKM</th>
<th>The Eclipse PKM</th>
<th>The Alizade PKM</th>
<th>The Glozman PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>0.017</td>
<td>0.064</td>
<td>0.08</td>
<td>0.0096</td>
</tr>
<tr>
<td>$K_{xy}$</td>
<td>0.018</td>
<td>0.068</td>
<td>0.051</td>
<td>0.0099</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>9.26</td>
<td>26.67</td>
<td>2.04</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Table 3.6: PKMs’ stiffnesses for Configuration 2.

<table>
<thead>
<tr>
<th>Minimum Dynamic Stiffness</th>
<th>The New PKM</th>
<th>The Eclipse PKM</th>
<th>The Alizade PKM</th>
<th>The Glozman PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>0.052</td>
<td>0.050</td>
<td>0.028</td>
<td>0.034</td>
</tr>
<tr>
<td>$K_{xy}$</td>
<td>0.061</td>
<td>0.047</td>
<td>0.027</td>
<td>0.040</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>1.88</td>
<td>0.58</td>
<td>0.19</td>
<td>4.02</td>
</tr>
</tbody>
</table>

Table 3.7: PKMs’ stiffnesses for Configuration 3.

<table>
<thead>
<tr>
<th>Minimum Dynamic Stiffness</th>
<th>The New PKM</th>
<th>The Eclipse PKM</th>
<th>The Alizade PKM</th>
<th>The Glozman PKM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>0.078</td>
<td>0.039</td>
<td>0.016</td>
<td>0.0056</td>
</tr>
<tr>
<td>$K_{xy}$</td>
<td>0.067</td>
<td>0.037</td>
<td>0.016</td>
<td>0.0057</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>2.15</td>
<td>0.21</td>
<td>0.15</td>
<td>0.017</td>
</tr>
</tbody>
</table>
The configuration-dependency of the new PKM was studied in Section 3.2.1, and the same reasoning applies to this comparative study. The PKM’ stiffness values are highly dependent on its revolute joints’ travel, \( \varphi \), and \( \varphi \) is inversely related to prismatic joints’ travel. Thus, the PKM is least stiff in the \( x \) and \( y \) directions, and stiffest in the \( z \) direction for Configuration 1. For Configurations 2 and 3, the PKM’s stiffness in the \( x \) and \( y \) directions increases, and the stiffness in the \( z \) direction drops.

For the Eclipse, the angle \( \varphi \) remains constant, as its \( M_2 \) joints all move vertically together. Therefore, the most important factor which affects this PKM’s stiffness is the locations of \( M_2 \) on the vertical columns. Each vertical column can be regarded as a cantilever beam, hence, when \( M_2 \) is positioned at its higher end, the column is subjected to bending, which leads to a decrease in stiffness. However, at the lower end of the column, there is little bending from the cantilever beam effect. Thus, the Eclipse has relatively high stiffness compared to other mechanisms at Configuration 1. In general, the Eclipse has higher stiffness than the new PKM with low \( M_2 \) travel, and lower stiffness with high \( M_2 \) travel.

Alizade PKM is less stiff than the new PKM, except in the \( x \) and \( y \) directions for Configuration 1, which is the least stiff configuration for the new PKM. There are two main factors which affect the dynamic stiffness of the Alizade PKM. The first one is its extensible link stiffness, which is related to the link length and the prismatic joint stiffness. The other factor is the revolute joints’ travel \( \varphi \). In the study, it can be seen that the link stiffness is the dominating factor. For example, when the angle \( \varphi \) is large for high \( M_2 \) travel, and if \( \varphi \) is the dominating factor, the stiffness in \( z \) direction should improve as \( M_2 \) travel increases. However, the study shows the opposite, which indicates that the increasing link length outweighs the effect of increasing in \( \varphi \).

The stiffness of the new PKM is also higher than that of the Glozman PKM, except in the \( z \) direction for Configuration 2. The main problem with the Glozman PKM is that, its first links act as cantilever beams, which affect the stiffness along all axes, depending on the orientation of these links. However, in Configuration 2, the second links are almost collinear with the first links, which reduce the cantilever beam effect of the first links, resulting in a high stiffness configuration for the Glozman PKM.
To summarize, the new PKM does exhibit higher dynamic stiffness than all the Reference PKMs do for Configurations 2 and 3. The only exception is the Glozman PKM, which is stiffer in the $z$ direction for Configuration 2. Furthermore, the least $x$, $y$ direction stiffnesses of the new PKM occurs in Configuration 1, and the case is the opposite for the Eclipse and Alizade PKM. Hence, the new PKM displays comparatively lower stiffness in Configuration 1. Lastly, the new PKM displays the highest stiffness value in the $x$ and $y$ directions, while the Eclipse has the highest stiffness value in the $z$ direction.
Chapter 4  Prototype

In the design of machine tools, it is not sufficient to only study the mechanisms employed, as there are more issues which need to be addressed. For instance, mechanism study in Chapter 3 did not consider the weight of the kinematic chains, which is a crucial factor in the design of meso-Milling Machine Tools (mMT). Moreover, interference is more prominent for the actual machine design, due to links having some thickness. Lastly, cost is another issue to be considered, and plays a major role in decision making. Therefore, prototyping is required to better understand the design issues as a whole.

As discussed in Chapter 3, PKMs exhibit several drawbacks for machine design, including singularities and high configuration-dependency of dynamic stiffness. Hence, in this Thesis, a redundant reconfigurability approach was chosen for the development of the PKM based mMT. Namely, kinematic redundancies were introduced by adding more degrees-of-freedom (dof) than those required by the tasks at hand, and reconfigurability was achieved through locking/unlocking of the redundant dof. This approach allows the use of redundancy to address the drawbacks of PKM-based machine tools, e.g., eliminating singularities [38], [40].

Three 9-dof redundant Reconfigurable meso-Milling Machine Tool (RmMT) prototypes were designed and built within the framework of this Thesis. The first and the second designs are based on downsizing an earlier design, namely, the Eclipse [17], and the third prototype is based on a new PKM architecture. All three RmMT designs are based on PKMs with a 3xPPPRS topology.

In this Chapter, the mechanical designs of the prototypes are described in detail, namely, presenting the architectures and the choice of selection for mechanical components. Moreover, the advantages and disadvantages for each prototype are discussed.
4.1 Prototype I

The mechanical design for Prototype I, shown in Figure 4.1, is based on the Eclipse mechanism. The Eclipse is in a family of PKMs with curvilinear base joints, and can achieve 90° tilting angle for a considerable large workspace [17]. It was investigated in detail and compared with several alternatives, including the Hexapod, and PKMs with a triangular base, by Mr. Hay Azulay [41]. The conclusion was that the Eclipse mechanism has comparatively a large achievable workspace and compact size, which makes it suitable for mMT design.

![Figure 4.1: CAD model of Prototype I.](image)

The 9-dof Prototype I consists of three chains and a circular base plate. Each chain contains a slider, which moves on the curvilinear guide. A linear actuator is mounted on the slider, and moves in tangential direction with respect to the curvilinear guide, and is referred to herein as the *tangential actuator*. On top of that, another vertically moving linear actuator is connected to the tangential actuator through an adaptor, and is referred to herein as the *vertical actuator*. Each
chain also includes an inextensible link which connects the vertical actuator to the mobile platform through revolute and spherical joints, respectively.

4.1.1 Mechanical Design

4.1.1.1 Base

The base, as shown in Figure 4.2, was machined from a single block of steel, in order to ensure its accuracy. The primary function of the base is to provide a platform to install the curvilinear guides. It is noted that, on top the base surface, a shoulder was machined with high accuracy, in order to provide accurate positioning for mounting the curvilinear guide. Moreover, the structure was designed to be heavy for vibration damping. The size of the base is 150 mm in radius and 137 mm in height. Lastly, there are considerable amount of empty space inside the base, as well as having windows on the side wall. These features were designed for easy and quick access to the workspace of the machine.

![Figure 4.2: Base design of Prototype I.](image)

4.1.1.2 Links

The link length is an important design parameter, as it is related to the platform tilting angle. However, a complete tilting angle analysis was not conducted for the prototype design, and a
simpler way of measuring tiling angle was achieved using CAD. The procedure can be found in Appendix A.

In this design, the length of the link was chosen to be 223 mm, and a maximum platform tilting angle of 43° could be achieved. It is noted that the spherical joint’s angle of swing imposed major limitation in platform rotation, and with conventional spherical joints of around ±30° angle of swing, the prototype is not able to reach 90° platform tilting.

The links were designed to be hollow tubes with a circular cross-section, which had an outer diameter of 19 mm and an inner diameter of 14.2 mm, as shown in Figure 4.3. The choice of this specific geometry was made for two reasons, (i) weight restrictions, and (ii) machine stiffness [39]. Hence, the hollow tube design was a compromise between reducing the weight and maintaining the stiffness of the links.

![Figure 4.3: Link design of Prototype I.](image)

**4.1.1.3 Mobile Platform**

Using the procedure shown in Appendix A, it was determined that for Prototype I, the platform tilting angle is inversely proportional to the size of the mobile platform. Hence, in this design, a small mobile platform of 45 mm radius was employed, as shown in Figure 4.4. This choice of dimension provides high platform tilting angle, while avoiding any interference between the mechanical components.
4.1.1.4 Adaptor

The primary function of the adaptors is to connect the two linear actuators. Similar to the design for links, the two major design considerations were stiffness and weight, and there was a tradeoff between the two. In the design for stiffness, bulky and heavy structures are preferred, and the adaptors should be made thick. However, there is a weight restriction, and the adaptors are required to be designed as light as possible. Moreover, the height of the adaptors is required to be at least the length of the vertical actuator plus half of its range of travel, in order for the actuators to move in full range. With the above design constraints in mind, the eventual dimensions of the adaptor were 75×75×150 mm, with a thickness of 5 mm, as shown in Figure 4.5.
4.1.2 Weight

Weight was not considered in neither of the workspace nor the stiffness analysis. However, it does have a significant effect on the design of the RmMT. In this design, the weight constraint is mostly imposed by the linear actuators, which provides a maximum actuation force of 30 N.

Table 4.1 shows the load of the mechanical components for the tangential actuators to carry. It can be seen that with the choice of mechanical components, the tangential actuator already needs to carry 2.8 kg, which corresponds to a force of about 27.5 N.

Weight issues resulted in significant limitations in the design of Prototype I. For instance, the links could not be made thicker to obtain higher stiffness. Hence, while this problem can be resolved by developing actuators which can provide more force, a simpler approach was to reduce the weight of the mechanical components.

<table>
<thead>
<tr>
<th>Components</th>
<th>Weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Linear Actuator Base</td>
<td>275</td>
</tr>
<tr>
<td>Vertical Linear Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td>Vertical Linear Actuator Motor</td>
<td>73</td>
</tr>
<tr>
<td>Adaptor</td>
<td>776</td>
</tr>
<tr>
<td>Link</td>
<td>482</td>
</tr>
<tr>
<td>Revolute Joint Assembly</td>
<td>378</td>
</tr>
<tr>
<td>Mobile Platform</td>
<td>58</td>
</tr>
<tr>
<td>Spherical Bearing</td>
<td>36</td>
</tr>
<tr>
<td>Stage-Housing Adaptor</td>
<td>126</td>
</tr>
<tr>
<td>Tangential Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td><strong>Total Weight</strong></td>
<td><strong>2808</strong></td>
</tr>
</tbody>
</table>

4.1.3 Stiffness

The dynamic stiffness of Prototype I was investigated by Mr. Masih Mahmoodi [18]. It was shown that the stiffness of this design was quite low. In order to identify the weakest link within
the structure, the amplitudes at different points on the machine were captured. It was noted that at around 160 Hz, the FRF at the revolute joints, which is mounted at the top of the adaptors, contributes approximately 64% of the overall deformation at the Platform Center Point (PCP). It is noted that the adaptors act like cantilever beams. Hence, the cantilever beam effect of the adaptors needed to be addressed. One solution was to make them bulky. However, this would lead to more weight. Another alternative was to reduce the height of the adaptors, which formed the basis for the design of Prototype II.

### 4.1.4 Prototype Built

The built prototype is shown in Figure 4.6. It can be noted that actuating all three kinematic chains was not a requirement for this Thesis. **Thus, in all the built prototypes, only one chain was actuated, while the others were kept passive for cost saving.** Moreover, each kinematic chain was attached on a segment of the full curvilinear guide, and consequently, full curvilinear range of travel could not be achieved.

The passive chains could be adjusted manually to simulate the motion of the actuated chain. The prototype did not demonstrate the full capability of the RmMT, but it did provide a test bed to investigate interactions between mechanical links, to evaluate actuator performance, and to obtain valuable experience for machine design, specifically, the design for tolerance and assembly. Engineering drawings for Prototype I can be found in Appendix B.
4.2 Prototype II

RmMT Prototype II, shown in Figure 4.7, was also designed based on the Eclipse mechanism. The main difference between the prototypes is the arrangement of the actuators’ positions. In Prototype II, for each kinematic chain, the adaptor is connected directly to the slider. Then, the tangential actuator is mounted on the adaptor, and the vertical actuator is attached to the tangential actuator. The focus for the design was to reduce the height of the adaptor, in order to obtain higher stiffness and less weight.
4.2.1 Mechanical Design

4.2.1.1 Base

Prototype II employed the same base as Prototype I, mostly to save design and fabrication costs. It can be noted that with this arrangement of the actuators, due to the vertical actuator occupying large amount of space within the base, the workspace of this prototype is severely limited, as interferences between the links are more noticeable.

4.2.1.2 Links

In Prototype II, the links were designed as hollow tubes with the same circular cross-section as Prototype I, as shown in Figure 4.8. However, due to interference and limited operating space, the link length was reduced in order to obtain comparable platform tilting angle as Prototype I. The link length for the design was 128 mm, and the mobile platform was able to achieve 40° of tilting.

Figure 4.7: CAD design of Prototype II.
4.2.1.3 Mobile Platform

The size of the mobile platform chosen for Prototype II was 45 mm. Although a smaller mobile platform was preferred for Prototype II, due to limited space, the size of the mobile platform was nonetheless kept the same as Prototype I, in order to avoid any interference between the links.

4.2.1.4 Adaptors

Due to adjusted actuators’ arrangement in Prototype II, the vertical actuator range of travel did not need to be taken into account in the design of the adaptors. Thus, it allowed for a shorter adaptor height. The height of the adaptor in Prototype II was 83 mm, which is a 45% decrease from Prototype I, as shown in Figure 4.9. With the same thickness, a shorter adaptor results in a reduction of both its cantilever effect and its weight.

Figure 4.8: Link design of Prototype II.
4.2.2 Stiffness

The dynamic stiffness of Prototype II was also investigated by Mr. Masih Mahmoodi [18], and it was shown that Prototype II has improved dynamic stiffness over Prototype I. This can be attributed to the decreased adaptor height and link length. However, despite the improvement, the dynamic stiffness of Prototype II is still nowhere near desirable, and the adaptors were again found to be the main source of accuracy. It was noted that at a frequency of 248 Hz, approximately 42% of the overall PCP displacement is due to the cantilever beam effect of the adaptors.

4.2.3 Weight

Table 4.2 shows the weights of the mechanical components for a tangential actuator to carry. In Prototype II, the weight of the mechanical components is reduced to 2.4 kg. The most significant change lies in the fact that both the adaptors and the links are shorter.
Table 4.2: Component weights for Prototype II.

<table>
<thead>
<tr>
<th>Components</th>
<th>Weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Linear Actuator Base</td>
<td>275</td>
</tr>
<tr>
<td>Vertical Linear Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td>Vertical Linear Actuator Motor</td>
<td>73</td>
</tr>
<tr>
<td>Adaptor</td>
<td>472</td>
</tr>
<tr>
<td>Link</td>
<td>198</td>
</tr>
<tr>
<td>Revolute Joint Assembly</td>
<td>351</td>
</tr>
<tr>
<td>Mobile Platform</td>
<td>58</td>
</tr>
<tr>
<td>Spherical Bearing</td>
<td>36</td>
</tr>
<tr>
<td>Stage-Housing Adaptor</td>
<td>337</td>
</tr>
<tr>
<td>Tangential Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td>Total Weight</td>
<td>2404</td>
</tr>
</tbody>
</table>

4.2.4 Prototype Built

The built Prototype II is shown in Figure 4.10. Similar to Prototype I, the passive chains can be adjusted manually to simulate motion of the actuated chains. Prototype II was used primarily to conduct experiments on configuration-dependency of stiffness [38]. Engineering drawings of Prototype II can be found in Appendix C.
4.3 Prototype III

Different alternatives were considered to compensate the cantilever beam effect from the adaptors, and none of them resulted in significant improvement. Thus, the design approach had to shift from downsizing existing designs to developing an RmMT based on a new PKM architecture, namely, by arranging the actuators in a different way, the adaptors were completely eliminated from the design. The mechanical design of Prototype III is shown in Figure 4.11.

The 9-dof RmMT consists of three identical kinematic chains, where each chain is attached to a slider that moves along a curvilinear guide rail. The slider can be moved to a desired location and, then, locked. Two linear actuators are placed on top of the slider. The first linear actuator moves tangentially with respect to the curvilinear guide, and the second linear actuator, which is mounted on top of the first one, moves in a radial direction, and is referred herein as the radial actuator. A revolute joint connects the linear actuator to a fixed length link, and the link is subsequently connected to the mobile platform through a spherical joint.
To demonstrate redundancy, Figure 4.12 shows how the RmMT can be reconfigured by locking the sliders on the curvilinear guide at different positions. For instance, this particular configuration of the RmMT allows the platform to obtain higher tilting angle than when the three sliders are spaced evenly apart.
4.3.1 Mechanical Design

4.3.1.1 Base

The structure of Prototype III is inverted compared to the previous prototypes. Thus, the mobile platform is positioned above the base. Hence, the base of the prototype was designed as a simple circular plate, with a radius of 150 mm, as shown in Figure 4.13. It has several advantages over the base design for Prototypes I and II. For instance, it is easier to machine, requires less material and time to fabricate, and it is stiffer due to its bulky structure.
4.3.1.2 Links

The links were designed as hollow tubes with the same cross-section as for Prototypes I and II, as shown in Figure 4.14. Based on CAD-based measurements, the length of the link for Prototype III was designed to be 168 mm, which is its optimal value for achieving best tilting angle. It is noted that the link length chosen here is similar to the simulation results obtained in Chapter 3. This is an interesting observation, and it may suggest that the 6-dof mechanism based simulations can be used to investigate geometrical design of the 9-dof RmMT.

4.3.1.3 Mobile Platform

The mobile platform was redesigned for Prototype III, in order to house a commercial spindle.
The platform is shown in Figure 4.15. A hole with a 10 mm radius was machined at the center of the mobile platform, and the tolerance was designed to be sliding fit with respect to the spindle. In order to lock the spindle in place, a simple locking mechanism was implemented, where a side screw was inserted into the clamp, in order to tighten the spindle onto the platform. The mobile platform had a radius of 20 mm.

![Figure 4.15: Platform design of Prototype III.](image)

4.3.2 Stiffness

From Mr. Masih Mahmoodi’s analysis, Prototype III displayed better stiffness than Prototype I and II, which coincides with the simulation results obtained from mechanism studies. This improvement of the stiffness is due to the removal of the adaptors from the design, which eliminates the cantilever beam effect.

4.3.3 Weight

It is seen from Table 4.3, that the weight of Prototype III is the lowest compared to the previous prototypes. This is expected, as the third prototype did not have the adaptor, which was one of the heaviest components in the kinematic chains. Hence, by designing the RmMT based on the new PKM, the weight problem is reduced.
Table 4.3: Component weights for Prototype III.

<table>
<thead>
<tr>
<th>Components</th>
<th>Weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Linear Actuator Base</td>
<td>275</td>
</tr>
<tr>
<td>Radial Linear Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td>Radial Linear Actuator Motor</td>
<td>73</td>
</tr>
<tr>
<td>Link</td>
<td>232</td>
</tr>
<tr>
<td>Revolute Joint Assembly</td>
<td>378</td>
</tr>
<tr>
<td>Mobile Platform</td>
<td>31</td>
</tr>
<tr>
<td>Spherical Bearing</td>
<td>36</td>
</tr>
<tr>
<td>Stage-Housing Adaptor</td>
<td>216</td>
</tr>
<tr>
<td>Tangential Actuator Stage</td>
<td>302</td>
</tr>
<tr>
<td><strong>Total Weight</strong></td>
<td><strong>1845</strong></td>
</tr>
</tbody>
</table>

4.3.4 Prototype Built

The RmMT was built into a prototype, shown in Figure 4.16. The prototype can be used as a test bed for subcomponents integration and experiments. For example, as mentioned above, a simple commercial spindle was incorporated into the mobile platform, allowing for further study of the interaction between the spindle and the RmMT. Moreover, some preliminary machining tasks were carried out using the spindle, and it was shown that continuous trajectories can be achieved on a workpiece. Lastly, the prototype is currently being used for vibration analysis.

In terms of human interface, the architecture of the RmMT permits quick and easy access to the workspace. In addition, the mechanism is constructed such that the tool is supported from below, which allows for less interference between the spindle and the mechanism. Engineering drawings of Prototype III can be found in Appendix D.
4.4 Actuators and Passive Joints

All the prototypes discussed above have 3×PPPRS topology. Hence, same actuators and passive joints were implemented into the prototypes throughout the designs.

4.4.1 Linear Actuator

The selection of the linear actuator was based on two criteria, (i) high accuracy, and (ii) compact size. In the prototype designs, the Nanomotion FB075 linear actuator, powered by HR8 ultrasonic motor, was chosen, as shown in Figure 4.17. There are 8 pieces of piezo element per actuator, which in total provide a maximum stall force of 30 N. It has an encoder resolution of 10 nm, and its range of travel is 40 mm. The actuator drives a stage with a size of 75×75×30 mm.
Figure 4.17: Nanomotion FB075 linear actuator.

Delta Tau Turbo PMAC Clipper Drive was selected as the controller for the Nanomotion FB075 linear actuators. It is a micro-controller suitable for small-scaled multi-axis machining applications, and with expanded hardware, it can drive up to 8 actuators simultaneously.

The controller algorithm has two components: the dead zone compensator and the PID controller. The gains of the controllers are adjusted with respect to the input actuator speed, and are experimentally tuned. With the Delta Tau controller, the achievable accuracy of the linear actuator at steady state is ±50 nm.

4.4.2 Curvilinear Guide Rail and Slider

Originally, the plan was to have curvilinear actuators mounted on the base, instead of passive curvilinear guides. This way, the RmMT prototypes can have all of their 9-dof actuated. Moreover, some linear motor manufacturers, for instance, the Nanomotion do make curvilinear actuators, usually by custom demand. However, custom made actuators are expensive. Thus, passive HCR 15A+60/150R curvilinear guides made by THK ltd were chosen for the prototypes, shown in Figure 4.18, as they are sufficient to provide circular motion and demonstrate reconfigurability at this stage of the design.
4.4.3 Revolute Joint Assembly

The cross-section of the revolute joint assembly is shown in Figure 4.19. It contains two conventional bearings, each with a tolerance of 8 µm. The joint assembly comprises several individual components, for ease of manufacturing and assembly. It consists of a housing, which holds the bearings. A shaft is attached to the bearings, for which the tolerance was chosen to be intermediate fit between the two, in order for quick assembly/disassembly. Finally, the shaft is connected to the link, through intermediate fit. This revolute joint assembly did not provide high accuracy (e.g., sub-micron accuracy), due to the choice of bearings and excessive connections between the parts. In order to improve accuracy, the housing can be made as a single piece, and bearings with high precision can be employed, e.g., angular contact bearing. Engineering drawings of the revolute joint assembly can be found in Appendix E.
4.4.4 Spherical Joint

The Seiko Hephaist SRJ008C spherical joint, shown in Figure 4.20, was selected in the design of the prototypes due to its extended angle of swing, and high precision. The spherical joint is one of the least stiff components in the RmMT prototype, mostly due to the thin thickness of its shaft. Thus, it is desirable to use a large spherical joint in the design. However, a large spherical joint leads to interference problem between the housing and the mobile platform. Eventually, the size of the housing was chosen to be 30 mm in radius. Furthermore, as mentioned in Chapter 3, the use of a spherical joint complicates the design of PKMs with high platform tilting angle, which suggests that the tilting angle of the PKM based RmMT, is also limited by the spherical joint.

Currently, commercial spherical joints can achieve ±30° angle of swing, which is lower than the required, e.g., ±60°. There are several design alternatives which result in higher angle of swing. For instance, magnetic spherical joints provide more than ±90°. However, the problem with magnetic spherical joint is their inability to carry large loads, and they also have low precision [42].

Another feasible solution is, instead of using a single spherical joint, a revolute joint and a universal joint can be connected together, and the combined joint assembly can achieve the 3-dof
rotation.

Figure 4.20: Seiko Hephaist SRJ008C spherical joint.

4.5 Machining Experiments

Machining experiments were carried out using the built Prototype III. The purpose for performing the experiments was to investigate whether a continuous tool path could be achieved on a particular workpiece. It is to be noted that only one kinematic chain was actuated, and the full capability of the RmMT prototype could not be demonstrated. The CAD model for the setup can be seen in Figure 4.21.
4.5.1 Spindle Assembly

The spindle assembly included the spindle and the end mill. In this experiment, the Combi 24 E-Type micro-motor was selected as the spindle unit. The motor can attain a rotational speed of up to 20k rpm with an output power of 25 W.

The speed of the spindle was controlled through a commercial available microcontroller, Arduino. Pulse Width Modulation (PWM) was used to control the voltage supplied to the spindle,
which subsequently controlled the spindle speed. The PWM produced a digital signal switched between on and off in the form of square wave. The on and off switch simulated the output voltage 0 (on) and 5V (off). Therefore, by changing the pulse width, the output values can be varied. In this case, spindle speed can be varied from 0 rpm to 20000 rpm by changing the pulse width from 0% to 100% [43].

The end mill used in this experiment was made of carbide and has two flutes. The shank diameter was 1/8 inch.

4.5.2 Workpiece

The spindle did not possess high output power and torque. Thus, the machining experiment could not be conducted using hard workpiece materials. In the experiment, the workpiece material selected was machinable wax. It is generally used in industry for prototyping, as it is soft, does not generate much heat, and has little chip formation.

4.5.3 Workpiece Holder Assembly

The workpiece holder assembly consisted of a vertical column and a simple workpiece clamp. With only two actuators, the height of the platform could not be adjusted. Hence, the vertical column was designed so the workpiece was just at the height of the end mill tip. In this experiment, a simple clamp was used to hold the workpiece in place, as the accuracy of the results was not important, and little force was present in machining wax.

4.5.4 Tool Path

The tool path was planned as tracing the letter “U”, which is marked by several points, as shown in Figure 4.22. At each end mill position, the corresponding actuators’ travels were recorded. Once the all the actuators’ travels were obtained with respect to the positions of the end mill, the actuator data were then input into the controller, where straight paths could be traced by moving the actuators from one set of actuators’ travels to the next.

It can be noted that, in order to plan the tool path systematically, a full kinematic model of the redundant RmMT is required. Appendix F shows the proposed procedure for formulating inverse
kinematics of the RmMT.

Figure 4.22: Tool path for the letter “U”.

4.5.5 Results

Although the RmMT prototype was restricted in motion due to incomplete actuation and joint constraints, the RmMT prototype was able to trace simple curves on a workpiece, as shown in Figure 4.23.

Figure 4.23: A machining experiment.
4.6 Summary

In this Chapter, three built RmMT prototype designs were presented, and the selections for the mechanical components as well as the actuators were discussed. The built prototypes were used as test beds to investigate problems which were not addressed in the computer simulations, for instance, weight problem and interference. Most notably, Prototype III was designed based on the new PKM. It exhibits better stiffness, and has less of a weight problem compared to Prototypes I and II. Lastly, a simple machining experiment was conducted on the third prototype.
Chapter 5  Conclusions and Future Research

5.1  Conclusions

The research described in this Thesis is aimed at developing a new parallel kinematic mechanism (PKM) for meso-Milling Machine Tool (mMT) design, with the focus on dynamic stiffness and platform tilting angle. The proposed new PKM has a six degree-of-freedom (dof) 3×PPRS topology.

The inverse kinematics of the new PKM and its inverse kinematic Jacobian matrix were derived. The workspace and platform tilting angle of the PKM were studied through a discretization method. The platform tilting angle was evaluated with respect to different design parameters, including geometrical dimensions of the components and joint constraints. From the study, it was found that there may be optimal dimensions for both the link length and the platform size for the new PKM, which suggests that workspace optimization can be conducted in the future. Singularities were also discussed, and singular positions within the workspace were identified.

The configuration-dependency of stiffness for the new PKM was studied. The PKM was evaluated at five different configurations with respect to its joints’ travel. The results show that the new PKM displays high configuration-dependency of dynamic stiffness.

Furthermore, comparative analysis was conducted, in which the new PKM was subjected to comparison with three reference PKMs that can achieve at least 90° platform tilting angle, namely, the Eclipse, the Alizade PKM and the Glozman PKM. The dynamic stiffness of the mechanisms was compared for three different configurations, and the results show that the new PKM displays better dynamic stiffness than the reference mechanisms.

It was concluded that it is not sufficient to only study the mechanism in the geometric design of machine tools. For instance, weight of the mechanism was shown to be an important design consideration. Thus, to better understand the design issues as a whole, three mMT prototypes were designed and built. Prototypes I and II were designed based on the Eclipse mechanism, and Prototype III was designed based on the new PKM.
In these designs, a redundant reconfigurability approach was chosen in an attempt to address the drawbacks of PKMs, including configuration-dependency of stiffness and singularities. From analyzing the prototypes, it was concluded that Prototype III does exhibit better performance in stiffness and weighs less.

Lastly, a machining experiment was carried out on the built Prototype III to demonstrate the motion of the Reconfigurable meso-Milling Machine Tool (RmMT) prototype. Although limited by incomplete actuation and joint constraints, a continuous path was followed on a workpiece by the end mill.

5.2 Future Work

The following are recommendations for the design of PKM-based mMTs in the future:

1. A full algorithm for workspace optimization should be developed, in order to better assist the mechanical design, e.g., selecting dimensions for mechanical components.

2. For configuration dependency of stiffness analysis, a more systematic way of evaluation should be considered, and more PKM configurations need to be evaluated, in order to obtain a full map of the dynamic stiffness within the workspace.

3. Currently, MATLAB simulations are based on the 6-dof mathematical model of the new PKM. However, the mathematical model need to be derived for the 9-dof redundant machine tools, and implemented into the simulation environment, e.g., workspace analysis.

4. In this Thesis, the new PKM exhibits several drawbacks when used for the RmMT design, most notably, singularity problems and insufficient platform tilting angle due to joint constraints. Thus, in the future design approach, these issues should be considered.
References


Appendix A – Measuring Tilting Angle with CAD

Figure A.1 shows the setup of measuring platform tilting angle. The green hemisphere represents a 10 mm radius workspace and the tool is normal to the surface of the hemisphere.

In order to measure platform tilting angle, the tool is moved along the surface of the hemisphere, while the joint constraints and interferences are checked. The configurations of the machine where no interference and joint constraint are detected are said to be reachable by the tool, and the corresponding platform’s orientations indicate the platform’s tilting angle. It can be noted that since the spherical joints impose the strictest constraint, their angle of swing is marked on the CAD model for clearer observation, as shown in Figure A.2.
It can be noted that CAD measurements only approximate the platform tilting angle, and are not systematic. Several works in the literature have proposed ways of using CAD to investigate the workspace of PKMs [44], [45], [46], and should be taken into considerations in the next steps of this research.
Appendix B – Engineering Drawings for Prototype I

Figure B.1: Engineering drawing of the base.
Figure B.2: Engineering drawing of the link.
Figure B.3: Engineering drawing of the mobile platform.
Figure B.4: Engineering drawing of the adaptor.
Appendix C – Engineering Drawings for Prototype II

Figure C.1: Engineering drawing of the link.
Figure C.2: Engineering drawing of the adaptor.
Appendix D – Engineering Drawings for Prototype III

Figure D.1: Engineering drawing of the base.
Figure D.2: Engineering drawing of the link.
Figure D.3: Engineering drawing of the mobile platform.
Appendix E – Engineering Drawings of the Revolute Joint Assembly

Figure E.1: Engineering drawing of the central housing.
Figure E.2: Engineering drawing of the side housing.
Figure E.3: Engineering drawing of the shaft.
Appendix F – Inverse Kinematics of the 9-dof Redundant RmMT

Due to added redundancy, there is no closed-form kinematic solution to the 9-dof mechanism. However, a kinematic model for performance evaluation can nevertheless be proposed.

As shown in Figure F.1, the 9-dof mechanism can be reduced to 6-dof by locking the sliders on the curvilinear guide at the desired positions, which, in this case, the travel of the slider, $\theta_i$, is known. Then, for each set of $\theta_i$, the 6-dof mechanism consists of three linear actuators which move tangentially, and three linear actuators which move radially, with respect to the curvilinear guide. Their travels are denoted by $d_{1i}$ and $d_{2i}$ respectively, which are the joint-space variables of interest. The inverse kinematic solution for the 6-dof mechanism, which is a function of $\theta_i$, can be solved, using similar procedures as in Section 2.2. Finally, the complete kinematic model of the 9-dof mechanism can be established by calculating the inverse kinematics for all the required sets of $\theta_i$.

Figure F.1: Top view – schematics of the 9-dof Redundant RmMT.