Shear Rupture of Massive Brittle Rock under Constant Normal Stress and Stiffness Boundary Conditions

By

Robert Paul Bewick

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Department of Civil Engineering
University of Toronto

© Copyright by Robert Paul Bewick (2013)
Shear Rupture of Massive Brittle Rock under Constant Normal Stress and Stiffness Boundary Conditions

Robert Paul Bewick

Doctor of Philosophy

Department of Civil Engineering
University of Toronto

2013

Abstract

The shear rupture of massive (intact non-jointed) brittle rock in underground high stress mines occurs under a variety of different boundary conditions ranging from constant stress (no resistance to deformation) to constant stiffness (resistance to deformation). While a variety of boundary conditions exist, the shear rupture of massive rock in the brittle field is typically studied under constant stress boundary conditions. According to the theory, the fracturing processes leading to shear rupture zone creation occur at or near peak strength with a shear rupture surface created in the post-peak region of the stress-strain curve. However, there is evidence suggesting that shear rupture zone creation can occur pre-peak. Limited studies of shear rupture in brittle rock indicate pre-peak shear rupture zone creation under constant stiffness boundary conditions. This suggests that the boundary condition influences the shear rupture zone creation characteristics.

In this thesis, shear rupture zone creation in brittle rock is investigated in direct shear under constant normal stress and normal stiffness boundary conditions. It is hypothesized that the boundary condition under which a shear rupture zone is created influences its characteristics (i.e., shear rupture zone geometry, load-displacement response, shear rupture zone creation relative to the load-displacement curve, and peak and ultimate strengths). In other words, it is proposed that the characteristics of a shear rupture zone are not only a function of the rock or rock mass properties but the boundary conditions under which the rupture zone is created.
The hypothesis is tested and proven through a series of simulations using a two dimensional particle based Distinct Element Method (DEM) and its embedded grain based method. The DEM is calibrated to the characteristics of Lodève sandstone which is a low porosity brittle rock that has been ruptured in the laboratory in direct shear under constant normal stress boundary conditions. The calibrated model is then used to investigate and prove the impact of the constant normal stiffness boundary condition on the shear rupture process and characteristics. The understanding gained from these simulations is then used in the analysis and re-interpretation of rupture zone creation in two mine pillars. This is completed to show the value and practical application of the improved understanding gained from the simulations. The re-interpretation of these case histories suggests that one pillar ruptured predominately under a constant stress boundary condition while the other ruptured under a boundary condition changing from stiffness to stress control.

This body of work provides an improved understanding of the shear rupture of brittle rock under both constant normal stress and normal stiffness boundary conditions through the use of calibrated numerical simulations. By applying this understanding to two field case histories, which also support the findings from the DEM simulations, it was possible to arrive at an improved interpretation of shear rupture zone creation in pillars and to provide evidence for boundary condition effects in the field.
Acknowledgments

This thesis and the research completed to support it could not have been accomplished without the support of my wife Laura and the de-stressing quality of my children, Ethan and Tyler. All made sure I had few weekends to work on this.

I never would have traveled down this academic road if it were not for my advisors Peter K. Kaiser and William F. Bawden. For this I am thankful of them. Both have provided guidance and encouragement during the ups and downs of the successful and unsuccessful research projects.

To my committee members, Drs. Bernd Milkereit and John Hadjigeorgiou, for pushing me to know more, do more, be better, and be more specific. I now appreciate the scrutiny and the reasons for it.

Mr. Navid Bahrani provided helpful assistance and advice and was always a good sounding board for the PFC2D numerical simulations.

Dr. Adam Coulson is thanked for providing the data he used for the completion of his Ph.D. This data was used in Chapter 4 of this thesis.

Drs. Petit and Wibberley are thanked for their correspondence and help related to the rupture aspects of the Lodève sandstone. Dr. Petit is thanked for his review of the content in Chapter 2 of the thesis.

My fellow grad students at the University of Toronto Misters Hamed Ghaffari, Sebastian Goodfellow, and Bryan Tatone; they have been an excellent support system.

The Natural Science and Engineering Research Council of Canada (NSERC), the University of Toronto, Mirarco-Mining Innovation, the Centre for Excellence in Mining Innovation (CEMI), and Golder Associates Ltd. If support from these organizations was not given, none of this would have been possible. Thank you.
# Table of Contents

Acknowledgments.......................................................................................................................... iv  
Table of Contents.......................................................................................................................... v  
List of Tables .................................................................................................................................. x  
List of Figures ................................................................................................................................ xi  
List of Appendices ...................................................................................................................... xxii  
Nomenclature ............................................................................................................................. xxiii  
Chapter 1 ......................................................................................................................................... 1  
1 Introduction ................................................................................................................................ 1  
1.1 Background ............................................................................................................................ 1  
1.2 Objectives and hypothesis ................................................................................................. 14  
1.3 Terminology ...................................................................................................................... 15  
1.4 Scope and methodology .................................................................................................... 17  
1.5 Organization of thesis ....................................................................................................... 18  
Chapter 2 ....................................................................................................................................... 20  
2 DEM simulation of direct shear: rupture under constant normal stress boundary condition... 20  
2.1 Introduction ....................................................................................................................... 20  
2.2 Rock type used for investigation ....................................................................................... 24  
2.3 Introduction to adopted Distinct Element Method ............................................................ 25  
2.4 Grain structure .................................................................................................................. 29  
2.5 Simulation procedures ...................................................................................................... 34  
2.5.1 Direct shear: constant normal stress ..................................................................... 34  
2.5.2 Uniaxial, biaxial, and direct tension simulations .................................................. 35  
2.6 Calibration to Lodève sandstone characteristics ............................................................... 36  
2.6.1 Calibration results ....................................................................................................... 36
2.7 Forces, mechanisms, and fracture system development ................................................... 46
  2.7.1 Distribution of forces in synthetic specimen with two aspect ratio .................... 46
  2.7.2 Fracturing process leading to rupture and rupture mechanism .......................... 48
  2.7.3 Peak shear and internal strength ........................................................................... 61
2.8 Discussion ......................................................................................................................... 64
  2.8.1 Rupture mechanism and geometry dependence on normal stress ................. 64
  2.8.2 Kinematic classification of specimen undergoing shear deformation .......... 65
2.9 Conclusions ....................................................................................................................... 68
2.10 Summary of Chapter 2 .................................................................................................... 70

Chapter 3 ....................................................................................................................................... 71

3 DEM simulation of direct shear: rupture under constant normal stiffness boundary
  condition ................................................................................................................................... 71
  3.1 Introduction ....................................................................................................................... 71
  3.2 DEM simulation of constant normal stiffness boundary condition ...................... 72
  3.3 Distribution of forces in synthetic specimens .............................................................. 75
  3.4 Cap modulus values 10 to 100GPa .............................................................................. 79
    3.4.1 Normal-shear stress-path and shear strength envelopes ................................. 79
    3.4.2 Rupture zone creation ....................................................................................... 84
    3.4.3 Shear stress versus horizontal displacement response ..................................... 89
    3.4.4 Rupture zone creation stages linked to shear stress versus horizontal
displacement response and stress-path ................................................................. 91
    3.4.5 Shear stress oscillatory behaviour ................................................................... 95
  3.5 1GPa cap modulus .......................................................................................................... 97
    3.5.1 Normal-shear stress-path and shear strength envelopes ............................... 97
    3.5.2 Rupture zone creation ...................................................................................... 99
  3.6 Discussion ....................................................................................................................... 101
3.6.1 Rupture mechanics under constant normal stiffness (10 to 100GPa cap modulus) ............................................................................................................. 101
3.6.2 Rupture connectivity, smoothing, and stick-slip behaviour ............................................. 103
3.6.3 Ultimate and residual strength envelopes under constant normal stress and stiffness ............................................................................................................... 104
3.6.4 Rupture zone creation at 1GPa cap stiffness ........................................................................ 107
3.7 Conclusions ..................................................................................................................... 108
3.8 Summary of Chapter 3 .................................................................................................... 110
Chapter 4 ..................................................................................................................................... 111
4 Fracturing process resulting in shear rupture of two mine pillars .......................................... 111
4.1 Introduction ..................................................................................................................... 111
4.2 Overview of site conditions ............................................................................................ 112
4.2.1 Golden Giant mine and trapezoidal pillar (Case 1) ...................................................... 113
4.2.2 Williams mine and sill pillar (Case 2) ................................................................ 116
4.3 Overview of engineering geology ................................................................................... 118
4.3.1 Geology ............................................................................................................... 118
4.3.2 In situ stress ......................................................................................................... 119
4.3.3 Rock strength ...................................................................................................... 122
4.3.4 Discontinuities .................................................................................................... 124
4.3.5 Seismic velocity and anisotropy ......................................................................... 126
4.3.6 Rock mass character ........................................................................................... 126
4.4 Adopted methods for data analysis ................................................................................. 127
4.4.1 Principal component analysis (PCA) .................................................................. 127
4.4.2 Loading system stiffness (LSS) .......................................................................... 128
4.4.3 Stress-path, spalling limit, and fracture initiation stress level ............................ 133
4.5 Golden Giant pillar (Case 1) ........................................................................................... 134
4.5.1 Failure process .................................................................................................... 135
5.3.5 Near excavation boundary fracturing, tensile or shear origin? (speculative) ..... 185
5.3.6 The spring-slider model (speculative) ................................................................. 186
5.4 Limitations of the adopted approach........................................................................... 187
5.5 Future research........................................................................................................... 188
5.5.1 Grain boundary and intra-grain strength characteristics................................. 189
5.5.2 Linear Coulomb strength envelope slope ......................................................... 190
5.5.3 Direct shear testing machine for strong brittle intact rock............................. 190
5.5.4 Three-dimensional grain based method (GBM) ............................................... 190
5.5.5 Directional LSS development ............................................................................. 191
References...................................................................................................................... 192
Appendices...................................................................................................................... 208
List of Tables

Table 1 Estimated grain sizes by mineral type. ................................................................. 30
Table 2 Compiled mineral properties. .................................................................................. 33
Table 3 Calibration micro-parameters. ................................................................................... 43
Table 4 Compiled strong brittle low porosity sandstone data compared to simulations. ............ 44
Table 5 1:1 and 1.5:1 aspect ratio pre-peak fracture angles and major principal stress orientation summary................................................................. 61
Table 6 Fracture angles for a cap modulus of 30GPa............................................................... 89
Table 7 Summary of stress measurements............................................................................. 121
Table 8 Rock strength testing data summary........................................................................ 124
Table 9 Summary of discontinuity sets and their characteristics.......................................... 125
Table 10 Summary of Case 1 pillar rupture zone creation. .................................................... 145
List of Figures

Figure 1.1 Change in specimen rupture mode with increasing applied lateral confining pressure, $\sigma_3'$. (a-c) Marble specimens ruptured in triaxial compression (modified from Paterson, 1958) showing change in rupture mode from axial splitting ($\sigma_3' = 0$MPa) (in the direction of applied load) to single shear rupture zone ($\sigma_3' = 3.5$MPa) (inclined relative to the applied load) and finally to conjugate shear rupture zones ($\sigma_3' = 35$MPa). The rupture mode change is schematically illustrated in (d-f). Half arrows indicate shear direction. Scale as shown. ........... 2

Figure 1.2 Spalling around underground excavations. (a) Early stage of spalling showing tensile fractures created in the corner of an underground excavation (wire mesh grid in photograph, for scale, approximately 3inches). (b) Late stage of spalling showing the opening of tensile fractures into the void space of the excavation (measuring rule shown 6inches for scale). This is also called bulking. ([a] and [b] courtesy of P.K. Kaiser)................................................................. 4

Figure 1.3 Shear rupture zones remote from excavation boundaries. (a) Shear rupture zone as observed in a South African deep gold mine (person’s hand for scale) (from Ortlepp, 1997). (b and c) Schematics showing two shear rupture zone scenarios in pillar cores. (b) Relatively wide pillar (assume pillar height 6m). (c) Sill pillar (assume square excavations 5m x 5m). Half arrows indicate shear direction. ........................................................................................................ 5

Figure 1.4 Schematic illustrations of examples where constant normal stress or stiffness boundary conditions may dominate during shear rupture zone creation. (a) Constant normal stress conditions in slender pillars that fail by shear on steep rupture surfaces (in direction of maximum shear stress), lateral pillar deformations occur in a non-restricted manner into the excavations surrounding them (assume 6m pillar height). (b and c) Shear rupture zone creation in pillar cores where, normal to the rupture zone, dilatant deformation is resisted by the surrounding rock (i.e., a constant stiffness boundary, represented by the springs) (assume pillar height in ‘b’ 6m and square excavations in ‘c’ 5m x 5m). Half arrows indicate shear direction. ......................... 6

Figure 1.5 (a) Schematic illustration of constant stress and stiffness boundary conditions normal or perpendicular to a shear rupture zone being created which generate different stress-paths shown in (b). Shear stress is the loading stress and normal stress is the boundary applied stress which stays constant for constant normal stress and increases for constant normal stiffness
boundary conditions. Half arrows in (a) indicate shear direction and arrows in (b) indicate stress-path.

Figure 1.6 Schematic diagrams of (a) normal, (b) reverse, and (c) strike-slip discontinuity orientations with boundary conditions outlined by Archambault et al. (1992). \( \sigma_n = \) normal stress, \( C = \) constant.

Figure 1.7 Compilation of shear rupture in Westerly granite under constant applied lateral pressure of 50MPa. Top shows the axial stress versus axial displacement curve. Letters along the curve relate to stages of shear rupture creation shown at the bottom. (Bottom) Progressive images of acoustic emission locations showing shear rupture creation. Upper row is a view of the specimen along-strike of the shear rupture being created and the lower row shows the same events when the shear rupture is viewed face-on ([a] and [b] from Lockner et al., 1991).

Figure 1.8 Results for triaxial compression tests conducted in a stiff-testing machine and in a stiff sealed triaxial cell. Axial sections of quartzite specimens stopped at stages of loading showing the development of a shear rupture zone relative to the stress-strain curve. Shear rupture zone creation is pre-peak (from Jaeger and Cook, 1976, after Hallbauer et al., 1973).

Figure 1.9 Schematic illustration of shear rupture zone, shear rupture surface, and damage zone.

Figure 2.1 Shear rupture zone creation process in clay. Based on Tchalenko (1970) modified from Cho et al. (2008).

Figure 2.2 Grain structure generation. (a) Initial particle packing and contacts. (b) Void centroids (black ‘dots’). (c) Creation of polygonal network with nodes at void centroids. (d) Polygonal network. (e) Polygonal network overlaid on particle assembly (a-d modified from Potyondy, 2010).

Figure 2.3 Elements forming a grain in the GBM. (a) Example single grain showing smooth-joint contacts forming the grain boundaries and the internal parallel bonded particles (not showing the parallel bonds for clarity). (b) Schematic representation of a parallel bond. (c) Schematic representation of the behaviour of a smooth-joint. (d) Schematic representation of the behaviour of a contact without a smooth-joint. (e) Examples of broken parallel bonds and smooth-joints.
Figure 2.4 Histograms and cumulative “percent passing” curves for grains measured in ‘measurement sets 1 and 2’ using SEM images provided by Wibberley (2011). 31

Figure 2.5 Simplified GBM generated for Lodève sandstone (a) and representative SEM image provided by Wibberley (2011) (b). Some mineral grains are labeled ([a] and [b]: darkest grey – quartz [Qz]; light grey – feldspar [F]; and white – calcite [C]). Scales as shown. 32

Figure 2.6 Constant normal stress direct shear simulation schematic. 35

Figure 2.7 Strength and deformation calibration results. (a) Linear Coulomb strength envelope for laboratory data (grey circles) as reported by Petit (1988) compared to DEM simulation results (white squares). (b) DEM simulation shear stress versus horizontal displacement response in comparison to descriptions provided by Petit (1988) and Wibberley et al. (2000). 40

Figure 2.8 Comparison of fracture system angles. (a)-(b) DEM simulation rupture zone images for 25 and 90MPa normal stress magnitudes, respectively (orange – grain boundary, black – intra-grain tensile fracture). (c)-(d) Orientation of fracture systems in (a) and (b), respectively (counter clockwise from horizontal left half, clock wise from horizontal right half of rose diagram); pre-peak fracutres are tensile and post-peak frauctures are predominatly shear (i.e., micro-faults). (e)-(f) Orientation of micro-faults and tensile fractures in the Lodève sandstone specimens ruptured in direct shear as measured in SEM images by Wibberley et al. (2000) for 23 and 87MPa normal stress magnitudes, respectively. 41

Figure 2.9 Example fracture systems in (a) generated through a grain (black fractures) and along grain boundaries (orange fractures) from a number of individual tensile fractures. The fracture systems in (a) are shown in (b) with the overlaid fracture system sketch (dashed lines). 42

Figure 2.10 Compiled strong brittle low porosity sandstone triaxial data (grey circles) in comparison to DEM biaxial simulation results (white squares). Rupture images of the synthetic specimen tested at various confining stress magnitudes (a-d) linked to the results plotted in principal stress space. Images show visual coherence with typical rock testing failure mode change (i.e., transition from axial splitting [e – 0MPa confining stress, from boxed region in (a)] to shear [f – 32MPa confining stress, from boxed region in (c)] as shown by the velocity vector arrows. Compiled data from Franklin and Hoek, 1970; Santarelli and Brown, 1989; Kovari et al., 1983. 45
Figure 2.11 Force chain networks in the 1:1 and 1.5:1 aspect ratio synthetic specimen. (a) 5MPa normal stress. (b) 25MPa normal stress. (c) 90MPa normal stress. (d) From Wang and Gutierrez (2010). (e) From Zhang and Thornton (2007). (f) From Cho et al. (2008). (g) From Dyer and Milligan (1984). (h) From Allersma (2005). Light to black – low to high compressive forces except (g) which is opposite. ........................................................................................................ 48

Figure 2.12 5MPa normal stress rupture zone creation and mechanism (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch................................................................. 52

Figure 2.13 25MPa normal stress rupture zone creation and mechanism (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch................................................................. 53

Figure 2.14 90MPa normal stress rupture zone creation (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch................................................................. 54

Figure 2.15 90MPa normal stress rupture mechanism (1:1 aspect ratio). (a) Shallow angle fracture system showing synthetic sense of shear (BB in Figure 2.14). (b) Steep angle fracture system showing antithetic sense of shear (CC in Figure 2.14). (orange – GB, black IG tensile fracture). See Figure 2.14 for scale of BB and CC. ......................................................................................... 55

Figure 2.16 (a) Shear stress versus horizontal displacement and orientation of the major principal stress (measured counter clockwise from horizontal) internal to the synthetic specimen (1.5:1 aspect ratio). (b) Linear Coulomb strength envelopes for specimens with 1:1 and 1.5:1 aspect ratios..................................................................................................................... 56

Figure 2.17 5MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement (δh) curve. (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. AA indicates rupture mechanism location in Figure 2.20.... 57

Figure 2.18 25MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement (δh) curve. (orange – GB,
black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. BB indicates rupture mechanism location in Figure 2.20.

Figure 2.19 90MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement (\( \delta_h \)) curve. (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. CC indicates rupture mechanism location in Figure 2.20.

Figure 2.20 1.5:1 aspect ratio rupture mechanisms. (a) 5MPa showing tensile splitting (see Figure 2.17 for location of AA), (b) 25MPa showing en échelon tensile fracture (see Figure 2.18 for location of BB), and (c) 90MPa showing antithetic and synthetic micro-faults (see Figure 2.19 for location of CC) normal stresses. Larger arrows in (c) show the trends of displacement vectors that are difficult to view. (orange – GB, black IG tensile fracture).

Figure 2.21 Internal principal stress-path determined from the measurement circle in the center of the 1:1 aspect ratio synthetic specimen compared to the Hoek-Brown strength envelope determined from the biaxial simulations.

Figure 2.22 Horizontal displacement of the lower shear box wall to reached peak shear strength and the internal principal stress-path reaching the Hoek-Brown strength envelope.

Figure 2.23 Illustration of geometric variance of pull-apart arrays depending on amount of transtension or transpression (from Peacock and Sanderson, 1994). \( A \) is the infinitesimal displacement direction (180° plane extension to 0° for plane contraction, i.e., direction of specimen dilatant or compactive movement) and \( \omega \) the angle between the vein segments and the zone boundary (approximately the orientation of the major principal stress).

Figure 3.1 Constant normal stiffness boundary condition direct shear simulation schematic.

Figure 3.2 Force chains in the synthetic specimens with an initial applied normal stress of 5MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – \( \delta_h = 0.1594 \)mm, (b) 10GPa – \( \delta_h = 0.1647 \)mm, (c) 30GPa – \( \delta_h = 0.1538 \)mm, and (d) 100GPa – \( \delta_h = 0.1518 \)mm cap modulus values.
Figure 3.3 Force chains in the synthetic specimens with an initial applied normal stress of 25MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – $\delta_h = 0.1682$mm, (b) 10GPa – $\delta_h = 0.1681$mm, (c) 30GPa – $\delta_h = 0.1681$mm, and (d) 100GPa – $\delta_h = 0.1716$mm cap modulus values. ................................................................. 77

Figure 3.4 Force chains in the synthetic specimens with an initial applied normal stress of 40MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – $\delta_h = 0.1734$mm, (b) 10GPa – $\delta_h = 0.1679$mm, (c) 30GPa – $\delta_h = 0.1733$mm, and (d) 100GPa – $\delta_h = 0.1715$mm cap modulus values. ................................................................. 78

Figure 3.5 Constant normal stress boundary condition normal-shear stress-path, and peak linear and ultimate bi-linear Coulomb strength envelopes. Also showing the residual frictional strength envelope determined from the constant normal stiffness simulations ($\phi=28^\circ$). ......................... 80

Figure 3.6 Constant normal stiffness boundary condition normal-shear stress-path, and peak and residual linear Coulomb strength envelopes. (a) Initial applied normal stress of 5MPa. (b) Initial applied normal stress of 40MPa. ................................................................................................... 83

Figure 3.7 Horizontal displacement criterion for peak maximum shear strength in the constant normal stiffness simulations. ........................................................................................................ 84

Figure 3.8 Rupture zone creation images, initial applied normal stress of 5MPa at selected horizontal displacement magnitudes ($\delta_h$) for cap modulus values of: (a)-(f) 10GPa; (g)-(l) 30GPa; and (m)-(r) 100GPa (orange – grain boundary, black – intra-grain tensile fracture). .................. 86

Figure 3.9 Rupture zone creation images, initial applied normal stress of 40MPa at selected horizontal displacement magnitudes ($\delta_h$) for cap modulus values of: (a)-(f) 10GPa; (g)-(l) 30GPa; and (m)-(r) 100GPa (orange – grain boundary, black – intra-grain tensile fracture). .................. 87

Figure 3.10 Rupture zone images for initial applied normal stress of 5MPa: (a) Stage II en échelon tension fractures showing opening mode in (b). (c) Stage IIa fractures propagating from en échelon fracture tips showing shear along original en échelon tension fractures in (d) and tip fractures opening in (e). (f) Stage III, peak shear strength showing shear along the rupture zone in (g) and (h) (orange – grain boundary, black – intra-grain tensile fracture). ......................... 88
Figure 3.11 Shear stress versus horizontal displacement response for (a) 5MPa and (b) 40MPa initial applied normal stresses ........................................................................................................................................................................ 90

Figure 3.12 Linked mechanical response of the synthetic specimen with initial applied normal stress of 5MPa. (a) Shear stress versus horizontal displacement response. (b) Normal-shear stress-path. (c) Development of minor principal stress with horizontal displacement. (d) Principal stress-path internal to the synthetic specimen ........................................................................................................................................................................ 93

Figure 3.13 Linked mechanical response of the synthetic specimen with initial applied normal stress of 40MPa. (a) Shear stress versus horizontal displacement response. (b) Normal-shear stress-path. (c) Development of minor principal stress with horizontal displacement. (d) Principal stress-path internal to the synthetic specimen ........................................................................................................................................................................ 94

Figure 3.14 Assessment of shear stress oscillations in the shear stress ($\tau$) versus horizontal displacement ($\delta_h$) curve for 25MPa initial applied normal stress and 30GPa cap modulus. (a) Shear stress versus horizontal displacement curve. (b) Fracture rates and cumulative fracture counts for grain boundary and intra-grain fractures. (c)-(f) Rupture zone images at indicated horizontal displacements using ‘arrows’ in (a) (orange – grain boundary, black – intra-grain tensile fracture). ........................................................................................................................................................................ 96

Figure 3.15 1GPa cap modulus simulation results for 5, 25, and 40MPa initial applied normal stresses. (a) Normal-shear stress-path. (b) Shear stress versus horizontal displacement response. ........................................................................................................................................................................ 98

Figure 3.16 1GPa cap modulus rupture zone creation at selected horizontal displacement ($\delta_h$) magnitudes for initial applied normal stresses of: (a-f) 5MPa; (g-l) 25MPa; and (m-r) 40MPa (orange – grain boundary, black – intra-grain tensile fracture). ........................................................................................................................................................................ 100

Figure 3.17 Apparent cohesion intercept for constant normal stiffness simulations with initial applied normal stresses of 5, 25, and 40MPa ........................................................................................................................................................................ 102

Figure 3.18 Changing post-peak ultimate rupture zone geometries for various normal stress magnitudes under constant normal stress boundary conditions. (a) 5MPa. (b) 15MPa. (c) 25MPa. (d) 40MPa. (e) 90MPa. Showing increasing rupture zone geometry irregularity with increasing normal stress magnitude. ........................................................................................................................................................................ 106
Figure 3.19 Rotation (a) and mechanism (b) of rupture in simulations with 1GPa cap modulus. Arrows in (a) indicate displacement vector trends that cannot be viewed. (b) Close up view of inset box in (a) showing displacement vectors indicating opening of the tensile fracture or rupture. ................................. 107

Figure 4.1 Longitudinal view of the Hemlo mining camp showing Williams, Golden Giant, and David Bell mines and inset geographical location of the mining camp in Ontario, Canada. (A) Golden Giant shaft pillar region where pillar Case 1 is located. (B) Williams sill pillar region where pillar Case 2 is located (modified from Coulson, 2009). ................................................. 113

Figure 4.2 Golden Giant mine shaft and Case 1 pillar region showing mining to the end of 2003. (a) Longsection view looking north showing mine development and stoping around the shaft and Case 1 pillar. (b) View looking west showing proximity of shaft to de-stress slot and ore body. ................................................................................................................................. 115

Figure 4.3 Williams mine longsection showing mining to the end of 1999, separate mining Blocks 3 and 4 separated by a sill pillar, and mining directions. The Case 2 pillar is located in the sill pillar between Easting 9412E and 9462E and levels 9390L and 9415L. ......................... 117

Figure 4.4 Lower hemisphere equal area stereographic projection of the measured principal stresses. Poles for trend and plunge determined from stress measurements (data listed in Table 7). The mean principal stress orientations are circled. Each pole’s great circle is also shown. . 120

Figure 4.5 Summary of compressive strength testing data. (a) Uniaxial compressive strength (UCS) data showing influence of foliation on UCS. (b) Triaxial strength and UCS data fit using the methodology of Hoek and Brown (1997). ........................................................................... 123

Figure 4.6 Lower hemisphere equal area stereographic projection of mapping data showing selected clusters of poles forming discontinuity sets. Poles for dip / dip direction symbolically plotted based on location relative to the deposit (i.e., footwall [F/W], hanging wall [H/W], and ore). ................................................................................................................................. 125

Figure 4.7 Load displacement curve for LSS explanation. Stage 1 is the point when the pillar is supporting the ground between two excavations. Stage 2 is the point when the pillar loses its
supporting capacity and deformation of the surrounding excavations occurs. LSS is the slope of the resulting line connecting Stage 1 and 2 (from Wiles, 2007). .......................................................... 129

Figure 4.8 Plates used for the assessment of normal and loading dip line stiffness along a rupture zone. ........................................................................................................................................ 131

Figure 4.9 Golden Giant pillar (Case 1) geometry and sections 10480E and 10495E used for assessment. Mining shown to 2003. Yellow excavation block around the Case 1 pillar is the interpreted excavation damage zone. .......................................................................................... 134

Figure 4.10 Rupture zone initiation and propagation. (a-b) Micro-seismic source locations for 2002 and 2003, respectively. (c-f) Contour of micro-seismic density (5 events per 125m³) showing progression of rupture plane east to west from 2002-02 to 2003-03 (modified from Coulson, 2009). ........................................................................................................................... 135

Figure 4.11 Compilation of micro-seismic event rates and PCA plane data for the Case 1 pillar. (a) Micro-seismic events rates and cumulative event count over time. (b) PCA ellipsoid ratio over time. (c-f) Cross section (10480E) through PCA planes showing the development of a fracture system over time. (g-j) Equal area lower hemisphere stereographic projections of PCA plane poles (dip/dip direction). (c-f) and (g-j) Show date ranges of: 2002-04 to 2003-01; 2003-01 to 2003-03; 2003-03 to 2003-04; and 2003-04 to 2003-12. (a and c-f modified from Coulson, 2009; b and g-j data provided by Coulson, 2010). .................................................................................................................. 137

Figure 4.12 PCA plane cross section 10480E close up view showing: (a) development of two pairs of conjugate en échelon arrays; and (b) change in orientation of PCA poles in the direction of array dip line for date ranges 2003-03 to 2003-04 and 2003-04 to 2003-12, respectively. The fracture orientation change evident in (b) is suggestive of shearing along the rupture zone and breakage through the initially created en échelon array of fractures. (a-b modified from Coulson, 2009; data in stereographic projections provided by Coulson, 2010). ................................................................. 138

Figure 4.13 Case 1 average pillar stress-paths for sections 10495E and 10480E considering excavations around the pillar without and with stress induced damage. In both cases, the major principal stress is higher when excavation damage is considered. .................................................................................. 141
Figure 4.14 *Case 1* pillar LSS assessment along the rupture zone for yearly increments of mining. (a) Dip line (plate perpendicular to the rupture zone) LSS (loading system). (b) Normal (plate oriented along the rupture zone) LSS. ................................................................. 142

Figure 4.15 Williams sill pillar (*Case 2*). (a) Geometry and section used for assessment (9437E). (b) Schematic change in pillar geometry over time along 9437E (grey areas indicate interpreted excavation damage zone evolution over time). ................................................................. 147

Figure 4.16 Compilation of micro-seismic event rates and source locations for the *Case 2* pillar. (a) Micro-seismic events rates and cumulative event count over time. (b) PCA ellipsoid ratio over time. (c-g) Cross section (9430E ±12.5m) of micro-seismic source locations for dates indicated. (a-b re-plotted using data provided by Coulson, 2010. c-g modified from Coulson, 2009). .......................................................................................................................................... 149

Figure 4.17 (a-e) Equal area lower hemisphere stereographic projections of PCA plane poles (dip/dip direction) for years indicated. Poles are relatively concentrated in (a) and disburse over time eventually forming a girdle line in 2004 (e). (data provided by Coulson, 2010). .......... 150

Figure 4.18 SMART-cable responses (# 1 to 5) relative to the toe of the cable. *Cables* 1 to 5 are shown on the level plan for 9390L (stope numbers, Northings, and Eastings labeled). (data provided by Coulson, 2010)........................................................................................................ 154

Figure 4.19 Schematic representation of a pillar core rupture interacting with the damage zone surrounding an excavation (that increases in depth over time) and associated extensometer response. Numbers 1-3 for extensometer curves are: (1) near excavation spalling/bulking behaviour (excavation damage envelope 2000-2002); (2) excavation damage zone beginning to interact with rupture zone (2003); and (3) intersection of excavation damage zone with rupture surface with compression of previously slabbed/spalled rock................................................................. 155

Figure 4.20 *Case 2* average pillar stress-paths for section 9437E considering excavations around the pillar without and with stress induced damage. ................................................................. 157

Figure 4.21 Empirical pillar stability graph showing case histories of stable, transition, and failed pillars, and various empirical factor of safety of 1.0 envelopes. Two pillar geometry and normalized average pillar load paths are shown. One for the changing pillar geometry over time
(left trending path), and the second for a constant pillar geometry (vertically upwards trending path). (i) to (iv) show pillar geometry change due to grey damage zones. The FOS = 1.0 line of Lunder (1994) is shown as dashed after a W/H ratio of 2.0 because pillar strength should increase with increasing W/H ratio and there are no case history data points to anchor Lunder’s FOS=1.0 line.

Figure 4.22 Case 2 pillar LSS assessment along the rupture zone for yearly increments of mining. (a) Dip line LSS (loading system). (b) Normal LSS.

Figure 4.23 Compilation of Case 2 analyses; for explanation see text. Circled seismic events are in the region of interest for the Case 2 pillar.
List of Appendices

Appendix A Additional information related to Chapter 1
Appendix B Additional information related to Chapter 2
Appendix C Additional information related to Chapter 3
Appendix D Additional information related to Chapter 4
Appendix E DEM simulation of direct shear: grain boundary and mineral grain strength component influence on shear rupture
Appendix F PFC2D direct shear fish functions
Nomenclature

All notation and symbols are defined where they first appear in the text. For convenience, they are also listed with definition below.

AE  Acoustic Emission

$c$  Cohesion

c  Cohesion of smooth-joints in PFC2D

$\bar{c}$  Cohesion of parallel bonds in PFC2D

C  Calcite

CRF  Cemented Rock Fill

CSIRO-HI  Commonwealth Scientific and Industrial Research Organisation Hollow Inclusion stress measurement cell

$D_{10}$  10% cumulative percent passing value

$D_{50}$  50% cumulative percent passing value

$D_{90}$  90% cumulative percent passing value

$D_{\text{max}}/D_{\text{min}}$  Ratio of $D_{90}/D_{10}$

DEM  Distinct Element Method

$\delta_h$  horizontal displacement of the lower shear box wall (Wall 1)

E  Young’s Modulus

E  Easting

$E_c$  Modulus of particles in PFC2D

$E_c$  Bond modulus of parallel bonds in FPC2D

ESG  Engineering Seismology Group

F  Feldspar

FW  Footwall

GB  Grain Boundary

GBM  Grain Based Method

GSC  Geologic Survey of Canada

GSI  Geological Strength Index

HW  Hanging wall

IG  Intra-Grain

$J_a$  Joint alteration
Joint roughness
Max. horizontal stress to vertical stress ratio
Normal to shear stiffness ratio of particles in PFC2D
Normal and shear stiffness factors of smooth-joints in PFC2D
Normal to shear stiffness ratio of parallel bonds in PFC2D
Level
Bond radius multiplier of parallel bond in PFC2D
Linear Energy Release Rate
Loading System Stiffness
Material constant in the Hoek-Brown strength criterion
Nuttli magnitude as recorded by the Geological Survey of Canada
Multi Point Borehole Extensometer
Number of data points
Northing
Principal Component Analysis
Particle Flow Code
Poly-Methyl-Meth-Acrylate, Plexiglas
Friction angle
Friction angle of parallel bonds in PFC2D
Density of particles in PFC2D
Quartz
Coefficient of determination
Minimum grain radius in PFC2D GBM
Maximum to minimum grain radius ratio in PFC2D GBM
Minimum radius of particles in PFC2D
Maximum to minimum radius ratio of particles in PFC2D
Major principal stress
Intermediate principal stress
Minor principal stress
Lateral pressure applied to the sidewall of a cylindrical specimen in triaxial compression
Tensile strength of smooth-joints in PFC2D
\( \bar{\sigma}_c \) Tensile strength of parallel bonds in PFC2D

\( \sigma_{ci} \) Compressive strength as determined from triaxial data fitting

\( \sigma_m \) Mean stress at a point, \( \sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \)

\( \bar{\sigma}_m \) Average mean stress in a volume

\( \sigma_n \) Normal stress

\( \sigma_t \) Tensile strength

\( s \) Material constant in the Hoek-Brown strength criterion

SEM Scanning Electron Microscope

SMART Stretch Measurement to Assess Reinforcement Tension

\( \tau \) Shear stress

\( \mu \) Coefficient of friction (\( \mu = \tan \phi \))

\( \mu \) Coefficient of friction of particles in PFC2D

\( \mu_r \) Residual coefficient of friction of smooth-joints in PFC2D

UCS Uniaxial Compressive Strength through intact rock

UCS_f Uniaxial Compressive Strength influenced by foliation

USBM United States Bureau of Mines

\( \nu \) Poisson’s ratio

W/H Width to Height ratio for pillars

\( W_k \) Kinetic energy released during rock failure

\( W_f \) Energy consumed during rock failure

\( W_t \) Total energy released during rock failure (\( W_t = W_k + W_f \))
Chapter 1

1 Introduction

1.1 Background

Rupture of massive (intact non-jointed) brittle rock is easiest envisaged at the laboratory scale considering cylindrical shaped specimens. In this laboratory scale example there are three specimens. The first is subjected to uniaxial compression (i.e., axial applied load or deformation with no lateral confining pressure applied to the sidewalls of the specimen, $\sigma_3' = 0\text{MPa} = \text{constant}$). The next two specimens are subjected to triaxial compression (i.e., axial applied load or deformation with constant lateral confining pressure applied to the sidewalls of the specimen, $\sigma_3' > 0\text{MPa} = \text{constant}$) with $\sigma_3' = 3.5\text{MPa}$ and $35\text{MPa}$, respectively. As $\sigma_3'$ is increased from 0 to $35\text{MPa}$, fracturing processes occurring in the specimens change. As a result of the fracturing processes changing, the rupture mode of the specimens also change (Paterson, 1958; Jaeger and Cook, 1979; Velde et al., 1993; Escartín et al., 1997) as summarized in Figure 1.1 for cylindrical marble specimens (after Paterson, 1958). In uniaxial compression (zero $\sigma_3'$) (Figure 1.1a and d), the dominate fracturing process in the specimen is the creation of long tensile fractures parallel to the applied axial load which result in rupture of the specimen by axial splitting. As the confining pressure is increased (Figure 1.1b-c and e-f), growth of tensile fractures is inhibited (e.g., Hoek, 1968) and short en échelon arrays of tensile fractures form with the overall effect of changing the specimen’s rupture mode from axial splitting to shear localization. The specimen subjected to $\sigma_3' = 3.5\text{MPa}$ ruptures by the creation of a single shear rupture zone inclined across the specimen. The specimen subjected to $\sigma_3' = 35\text{MPa}$ ruptures by the creation of multiple (in this case conjugate) shear rupture zones.
Figure 1.1 Change in specimen rupture mode with increasing applied lateral confining pressure, $\sigma_3'$. (a-c) Marble specimens ruptured in triaxial compression (modified from Paterson, 1958) showing change in rupture mode from axial splitting ($\sigma_3' = 0$MPa) (in the direction of applied load) to single shear rupture zone ($\sigma_3' = 3.5$MPa) (inclined relative to the applied load) and finally to conjugate shear rupture zones ($\sigma_3' = 35$MPa). The rupture mode change is schematically illustrated in (d-f). Half arrows indicate shear direction. Scale as shown.
Similar to the laboratory example outlined above, in mining, stress driven failure processes in brittle rock can be grouped by the prevalent condition when failure occurs: (1) near excavation boundaries under low confining stress conditions with high degrees of freedom for broken rock to move (i.e., minor principal stress, $\sigma_3$, typically $<5$ to $10\%$ of the Uniaxial Compressive Strength, UCS) and little deformational constraints due to the proximity of excavation void space; and (2) away from excavation boundaries under higher confining stresses and constrained degrees of freedom (i.e., rotation, translation, and dilation).

Near excavation boundaries, under constant lateral or constant normal stress boundary conditions, brittle rock predominantly fails by extensional processes leading to the progressive formation of thin slabs of rock referred to as spalls. Constant stress boundary conditions are encountered near underground excavations because rock can deform or bulk relatively unrestricted in the direction towards the void space of the excavation opening. The spalling process occurs under surface parallel compressive loading conditions with spall formation resulting from propagating tensile fractures (e.g., Kaiser et al., 2000). These tensile fractures are suggested to initiate in compression due to local tensile stress concentrations which are generated by heterogeneities (e.g., Diederichs, 1999; Lan et al., 2010) stemming from both geometric (e.g., grain shape and size, and micro-cracks) and deformation contrasts (e.g., modulus). The extent of tensile fracture propagation depends on the magnitude of confinement (Hoek, 1968). Due to the low confining stress conditions near excavation boundaries, tensile fractures can easily propagate, creating spalls of rock (Figure 1.2). This failure process and its boundary condition prevent the creation of shear rupture zones and are important for understanding near excavation behaviour but are not the focus of this thesis.
Near wall, low confinement spalling behaviour

Figure 1.2 Spalling around underground excavations. (a) Early stage of spalling showing tensile fractures created in the corner of an underground excavation (wire mesh grid in photograph, for scale, approximately 3inches). (b) Late stage of spalling showing the opening of tensile fractures into the void space of the excavation (measuring rule shown 6inches for scale). This is also called bulking. ([a] and [b] courtesy of P.K. Kaiser).

Away from excavation boundaries (e.g., in wide pillars, in abutments, and ahead of advancing stope faces) brittle rock fails by shear rupture (e.g., Gay and Ortlepp, 1979) (Figure 1.3). The fracturing processes in brittle rock leading to shear rupture are dilatant (Brace et al., 1966; Scholz, 1968; Peng and Johnson, 1972; Hallbauer et al., 1973) due to newly created fractures opening and pre-existing and newly created discontinuities shearing on or overriding asperities. When a shear rupture zone is being created and is surrounded by rock that resists dilatant deformation, confining stress magnitudes will increase during shear rupture zone creation and subsequent shearing along the formed shear rupture surface. The related increases in confining stress and associated stress-path during fracturing and shear differs from constant stress
boundary conditions (Indraratna et al., 2005) and, in the extreme, are best represented by constant stiffness boundary conditions (Obert et al., 1976; Goodman, 1976; Johnston and Lam, 1989; Indraratna et al., 1997; McKinnon and Garrido, 1998) (Figure 1.4). An example stress-path difference under constant stress and stiffness boundary conditions normal to a shear rupture zone is illustrated schematically in Figure 1.5.

Shear rupture, higher confinement (e.g., in pillar cores)

Figure 1.3 Shear rupture zones remote from excavation boundaries. (a) Shear rupture zone as observed in a South African deep gold mine (person’s hand for scale) (from Ortlepp, 1997). (b and c) Schematics showing two shear rupture zone scenarios in pillar cores. (b) Relatively wide pillar (assume pillar height 6m). (c) Sill pillar (assume square excavations 5m x 5m). Half arrows indicate shear direction.
Figure 1.4 Schematic illustrations of examples where constant normal stress or stiffness boundary conditions may dominate during shear rupture zone creation. (a) Constant normal stress conditions in slender pillars that fail by shear on steep rupture surfaces (in direction of maximum shear stress), lateral pillar deformations occur in a non-restricted manner into the excavations surrounding them (assume 6m pillar height). (b and c) Shear rupture zone creation in pillar cores where, normal to the rupture zone, dilatant deformation is resisted by the surrounding rock (i.e., a constant stiffness boundary, represented by the springs) (assume pillar height in ‘b’ 6m and square excavations in ‘c’ 5m x 5m). Half arrows indicate shear direction.
Figure 1.5 (a) Schematic illustration of constant stress and stiffness boundary conditions normal or perpendicular to a shear rupture zone being created which generate different stress-paths shown in (b). Shear stress is the loading stress and normal stress is the boundary applied stress which stays constant for constant normal stress and increases for constant normal stiffness boundary conditions. Half arrows in (a) indicate shear direction and arrows in (b) indicate stress-path.
Archambault et al. (1992) suggested that normal stiffness boundary conditions are relevant for shear of existing discontinuities or shear rupture zones that are formed or being created of strike-slip orientation and constant normal stress boundary conditions for normal or reverse discontinuity orientations when influenced by a free surface (Figure 1.6).

Figure 1.6 Schematic diagrams of (a) normal, (b) reverse, and (c) strike-slip discontinuity orientations with boundary conditions outlined by Archambault et al. (1992). ($\sigma_n = \text{normal stress}, C = \text{constant}$).

The shear rupture of massive brittle rock in mines is becoming more recognized and less considered a rupture mode unique to South African deep gold mines where the phenomenon was well documented by Gay and Ortlepp (1979) and Ortlepp (1997). Coulson (2009) detailed two case studies of shear rupture zones being created in pillars at the Golden Giant and Williams mines in the Hemlo mining camp (located in northern Ontario, Canada). In one of the pillars, the fracturing processes leading to shear rupture zone creation was associated with or contributed to a number of seismic events (Nuttli magnitude between $M_n = 0$ and $M_n = 3.5$) which caused
damage to excavations, mining delays, and increased ground support costs. Golder (2008), Itasca (2008), MIRARCO (2008), and Yao et al. (2009) reported on the damage sustained to multiple excavations on multiple levels at North mine (located in Sudbury, Ontario Canada) after the shear rupture of a sill pillar between two excavations which generated a seismic event registering $M_n = 3.0$ (local magnitude) on the mine seismic system ($M_n = 3.8$ as per the Geologic Survey of Canada) and several seismic events $M_n > 1.5$ (local magnitude) associated with a fault network in the mine. Areas of the mine were closed until rehabilitation was completed causing mining delays and increased ground support costs. While not reported in the literature, in personnel communications with Dave Black (2012) (Kidd Creek mine geologist), Kidd Creek mine (located in northern Ontario, Canada) has experienced shear rupture zone creation causing damage on multiple levels, mining delays, and increased ground support costs. From these cases, shear rupture zones have been observed in a number of mines and have negatively impacted mine finances. They caused excavation damage (creating the need for rehabilitation of the damaged areas and thus increased costs), and mining delays (destroying deposit value and increasing costs). While the cases overviewed did not report any injury to people (i.e., mine personnel), the creation of shear rupture zones and the associated excavation damage they cause have the potential to injure people and in the extreme, cause fatalities.

Even though the boundary condition surrounding a shear rupture zone being created is potentially not that of constant stress, a majority of the deformation experiments on specimens of brittle rock have been conducted under this boundary condition and form the base of shear rupture zone understanding. Brace and Bombolakis (1963) proposed that a shear fracture could not propagate in its own plane signifying that shear rupture zones initiate and propagate by linkage of pre-existing fractures. Using the location of acoustic emission (AE) events, Lockner et al. (1991), Reches and Lockner (1994), and Thompson et al. (2009) tracked the fracturing stages from pre- to post-rupture in Westerly granite specimen in triaxial compression under constant applied lateral stress conditions ($\sigma_3' = C$). These investigations show that tensile fracturing initially occurs in a random manner in the specimen (Figure 1.7a) and is followed by the initiation of a shear rupture from the sidewall of the specimen; typically just after the peak strength is reached (Figure 1.7b). The shear rupture then propagates in the post-peak region of the load-displacement curve through a cloud of fractures newly generated by stress concentrations at shear rupture tips (Figure 1.7c to f).
Figure 1.7 Compilation of shear rupture in Westerly granite under constant applied lateral pressure of 50MPa. Top shows the axial stress versus axial displacement curve. Letters along the curve relate to stages of shear rupture creation shown at the bottom. (Bottom) Progressive images of acoustic emission locations showing shear rupture creation. Upper row is a view of the specimen along-strike of the shear rupture being created and the lower row shows the same events when the shear rupture is viewed face-on ([a] and [b] from Lockner et al., 1991).
From different tests under constant stress boundary conditions, it is found that the shear rupture tip propagates by forming en échelon micro-fractures ahead of it. Experiments on rock in torsion (e.g., Cox and Scholz, 1988a; 1988b), thin section observations of fault tip zones obtained from the field coupled with experiments using PMMA (Poly-methyl-meth-acrylate, Plexiglas) (e.g., Petit and Barquins, 1988), and experiments in direct (e.g., Morgenstern and Tchalenko, 1967; Tchalenko, 1970) and simple shear (e.g., Cloos, 1928; Riedel, 1929) using clay, all support this shear rupture creation process.

Few investigations have been completed using brittle rock deformed under non-constant stress boundary conditions. Hallbauer et al. (1973), using copper jacketed cylindrical specimens of quartzite deformed in triaxial compression, stopped tests at predetermined locations along the loading path and removed the specimens for sectioning and microscope observations. In the experiments of Hallbauer et al. (1973), the lateral stress magnitudes were not kept constant during loading (a result of the copper jacket) and increased during deformation. Hallbauer et al. found that shear rupture in the specimens initiated and began to propagate pre-peak as illustrated in Figure 1.8. Their test results provide some insight into brittle rock specimen rupture under non-constant stress boundary conditions; shear ruptures are generated pre- opposed to post-peak strength as determined from the constant lateral stress boundary conditions previously summarized.
Figure 1.8 Results for triaxial compression tests conducted in a stiff-testing machine and in a stiff sealed triaxial cell. Axial sections of quartzite specimens stopped at stages of loading showing the development of a shear rupture zone relative to the stress-strain curve. Shear rupture zone creation is pre-peak (from Jaeger and Cook, 1976, after Hallbauer et al., 1973).
A number of researchers have investigated the effect of constant normal stiffness boundary conditions in direct shear for specimens containing pre-existing discontinuities (Obert et al., 1976; Johnston and Lam, 1989; Indraratna et al., 1998; Boulon et al., 2002; Szymakowski, 2003; Jiang et al., 2004) and for specimens containing no pre-existing specimen scale discontinuities (Obert et al., 1976; Archambault et al., 1992). These investigations focused on the strength characteristics of the materials tested and have shown that under constant normal stiffness boundary conditions the associated normal-shear stress-path generally follows the strength envelope generated from tests under constant normal stress boundary conditions (as in the schematic stress-path for normal stiffness boundary conditions in Figure 1.5b). Limited research has been completed using direct shear under constant normal stiffness boundary conditions to improve the understanding of the fracturing processes leading to shear rupture zone creation relative to load-displacement response of a specimen, and a specimen’s ultimate strength and ultimate shear rupture zone geometry.

In summary, understanding fracturing processes leading to shear rupture zone creation in mines is needed to protect mine personnel from injury and death, reduce rehabilitation of mine excavations damaged by unexpected shear rupture zone creation, reduce mining delays, and reduce ground support costs. The majority of the experiments that have been used to understand the shear rupture of massive rock in the brittle field have been conducted using constant stress boundary conditions. This boundary condition may not prevail when fracturing processes leading to shear rupture zone creation are constrained (i.e., as in mine pillars, abutments, and away from excavation boundaries). A limited number of experiments have been completed on specimens under constant stiffness boundary conditions containing pre-existing specimen scale discontinuities and even fewer tests have been completed on specimens of intact brittle rock. Thus, the understanding of shear rupture zone creation is incomplete at both the laboratory and mine scales.

The goal of this thesis is to improve the understanding of shear rupture zone creation in intact low porosity brittle rock and in massive brittle rock masses when deformed under constant stiffness boundary conditions that are applied perpendicular or normal to a shear rupture zone being created by addressing the objectives of this thesis outlined in the next section.
1.2 Objectives and hypothesis

The primary objectives of this thesis are to investigate:

- fracturing processes and mechanisms leading to shear rupture zone creation;
- shear rupture mechanisms;
- ultimate shear rupture zone geometries;
- shear stress versus horizontal displacement (load-displacement) responses; and
- strength envelopes (peak and ultimate),

of a low porosity (<10%) intact brittle rock specimen deformed in direct shear under constant normal stress and normal stiffness boundary conditions.

The secondary objective is to use the findings of the above listed to show their value and practical application through the interpretation of shear rupture processes in massive brittle rock masses in mining environments with changing boundary conditions.

For these purposes, it is hypothesized that shear rupture zone characteristics depend on the prevalent boundary condition (i.e., constant normal stress versus constant normal stiffness) during shear rupture zone creation. These shear rupture zone characteristics include:

- the fracturing processes and mechanisms leading to shear rupture zone creation relative to the shear stress versus horizontal displacement response (load-displacement response); and thus
- the ultimate shear rupture zone geometry; and thus
- the load-displacement response; and thus
- the shear rupture’s peak and ultimate strengths.

In other words, it is to be demonstrated that the characteristics of shear rupture zones are not only a function of the rock or rock mass properties in which they are created but also depend on the boundary conditions.

When brittle rock is deformed, the boundary condition applied to the rock influences the stress-path to which the rock is subjected and thus influences fracturing processes occurring in the rock (e.g., Martin, 1993; McKinnon and Garrido, 1998). Constant normal stress and stiffness
boundary conditions generate different stress-paths (as previously shown in Figure 1.5) (Obert et al., 1976; Jiang et al., 2004; Indraratna et al., 2005) and thus produce different fracturing processes (e.g., time of occurrence during deformation, orientation, frequency, and pattern) leading to different shear rupture zone characteristics.

1.3 Terminology

In an effort to focus on the essence of this thesis, the following terminology is used:

- **Stiffness** refers to that of a system confining a volume of material undergoing a failure process (e.g., normal stiffness when shear ruptures are considered). A system is an assemblage of multiple components (different rock units, structural geologic elements, mine excavation, etc.) that contribute to the boundary conditions of the strained or straining intact rock or massive rock mass. Stiffness is the ability of the material surrounding a failure process to resist deformation. Stiffness should not be confused with Young’s modulus. For example, the stiffness of a column is a function of Young’s modulus and the geometry of the column (i.e., its area and length). Therefore, stiffness is a property of a structural system which depends on the deformability of the materials that form the system (i.e., modulus) and the system’s geometry.

- **Shear rupture zone** refers to the shear rupture surface and the surrounding fracture damage zone that is created and evolves during the applied shear deformation (Figure 1.9).

- **Shear rupture surface** refers to the connected surface or fully damaged zone that is ultimately created and contained within the shear rupture zone (Figure 1.9).

- **The process of shear rupture creation** refers to the fracturing events that occur during deformation and result in generating a shear rupture zone and shear rupture surface.

- **Fracture** is a single break (crack) in a cohesional material.

- **Fracture zone or system** is created from the coalescence of multiple fractures generating a lineation. Depending on the scale of observation, a fracture zone may appear as a fracture.
• *The mechanisms leading to shear rupture* refer to the tensile or (extensional) and shear mechanisms of single fractures and or fracture systems (which are a network of individual fractures forming a larger connected fracture) being created in the rupture zone during deformation.

• *Rupture mechanism* refers to the specimen or pillar scale failure or rupture mechanism (tensile or shear).

• *Brittle rock* refers to a rock that, under applied load or deformation, fractures with an extensional or tensile component, disrupting the rock’s continuity when creating kinematically feasible failure mechanisms.

• *Peak maximum strength* is the highest strength reached during the failure process.

• *Yield point* is the stress magnitude when the yield surface (strength envelope) is reached (yield point and peak maximum strength are identical for constant normal stress boundary conditions).

• *Ultimate strength* is the lowest strength obtained with the achieved horizontal displacement of the shear box and typically has some remnant cohesion.

• *Residual strength* is the lowest frictional strength obtained with the achieved horizontal displacement of the shear box and has no cohesion (this is often not reached during a deformation limited laboratory test).

• *Intact rock* refers to a rock specimen that does not contain any specimen scale discontinuities. Intact rock consists of mineral grains separated by grain boundaries. Both grains and grain boundaries may contain fractures or other flaws.

• *Massive rock* refers to a field scale rock mass that is free of persistent joints and thus fails predominantly through intact rock.

A complete list of the nomenclature is provided in front of this thesis.
1.4 Scope and methodology

In a broad sense, the scope of this thesis is to: (1) investigate the influence of constant normal stress and normal stiffness boundary conditions on the creation of shear rupture zones in brittle rock deformed in direct shear at the laboratory scale; and (2) to apply the knowledge gained from this investigation to assist in the re-interpretation of two pillar case histories at the Williams and Golden Giant mines, located in Northern Ontario, Canada, thus showing the value and practical application of the gained knowledge. The two cases fail by the creation of shear rupture zones in massive rock (Coulson, 2009).

For this purpose, the particle based Distinct Element Method (DEM) as implemented in the commercially available Particle Flow Code in two-dimensions (PFC2D v4.00-190) (Itasca, 2011) is used to generate and calibrate a synthetic intact rock with a polygonal grain structure (that can fracture by failure of grain boundaries and mineral grains) to the direct shear (constant normal stress) laboratory test results reported by Petit (1988) and Wibberley et al. (2000) for specimens of a brittle low porosity rock. The calibrated synthetic specimen is then used to investigate shear rupture zone creation under constant normal stress boundary conditions in direct shear for low (5-15MPa), moderate (25-40MPa) and high (60-90MPa) normal stress magnitudes. This is followed by an investigation of shear rupture zone creation under constant normal stiffness boundary conditions in direct shear for the following cap modulus values: 1, 10, 30, and 100GPa.
and initial applied normal stresses 5, 25, and 40MPa. The cap modulus is a proxy for the system stiffness. These simulations provide insight into the shear rupture zone creation process and the resulting characteristics under both boundary conditions. Details related to the selected direct shear experimental set up and adopted numerical tool, PFC2D v4.00-190, are provided in Appendix A.

The understanding gained from the numerical simulations is then applied in a semi-quantitative manner to re-interpret two pillar case histories which were found to have failed by shear rupture zone creation (Coulson, 2009). In this way, the value and practical application of the gained understanding is shown. Coulson (2009) reported that the failure processes in, and the micro-seismic behaviour of the two pillar cases differed and it is demonstrated that this can be related to differing boundary conditions. It is shown that the Williams mine pillar initially failed under a deformation constrained or stiffness boundary condition, while the Golden Giant pillar failed under a constant stress boundary condition. These two cases provide field evidence in support of the hypothesis that boundary conditions affect shear rupture zone characteristics and thus mining-induced failure processes in otherwise comparable ground conditions.

1.5 Organization of thesis

This thesis consists of five chapters and six appendices. Chapter 1 introduces the overall goal, objectives, hypothesis, scope, and methodology applied to test the hypothesis.

Investigations on an intact brittle rock deformed in direct shear under constant normal stress boundary conditions are reported in Chapter 2. This chapter provides the background on the simulated brittle rock, the DEM used and its details, the methodology applied to simulate the brittle rock in direct shear under constant normal stress, and the calibration of the DEM to the available shear rupture characteristics reported by Petit (1988) and Wibberley et al. (2000). Direct shear simulations are conducted to investigate the influence of shear box length to height aspect ratios on shear rupture zone creation, rupture mechanisms, and fracture patterns developing in the synthetic specimen being deformed. Based on these simulations, an advanced understanding is gained on rupture zone creation which is found to be dependent on the normal stress magnitude.
In Chapter 3, the methodology applied to simulate the synthetic brittle rock specimen in direct shear under constant stress is adopted and modified for constant normal stiffness tests, and the direct shear simulation results under constant normal stiffness boundary conditions are presented and discussed. The results are compared to those of the constant normal stress boundary condition reported in Chapter 2.

Two pillar case histories are investigated and re-interpreted in Chapter 4 for the purpose of showing the value and practical application of the understanding gained in Chapters 2 and 3. These cases were previously studied by Coulson (2009). The case histories are re-interpreted within the framework of boundary condition effects on rupture zone creation and behaviour and show that: (1) the Golden Giant pillar ruptures under an essentially constant stress boundary condition; and (2) the Williams mine pillar ruptures under a deformation constrained condition, a situation where the boundary condition changes as mining progresses. The understanding obtained for shear rupture zone creation in the synthetic specimens deformed in direct shear reported in Chapters 2 and 3 is used to support this re-interpretation and it is shown that a more consistent understanding of the differences in rock mass response to deformation emerges. While the number of variables in a complex mining situation always leaves room for a number of alternative interpretations, the findings presented in this thesis show a high level of consistency with field observations (i.e., extensometers, seismic event timing, etc.). It is concluded that the fracturing processes leading to shear rupture zone creation at these mines was influenced by the prevailing boundary condition during shear rupture zone creation.

Finally, a summary, conclusions, and implications of the thesis are provided in Chapter 5 together with recommendations for future research.
Chapter 2

2 DEM simulation of direct shear: rupture under constant normal stress boundary condition

2.1 Introduction

In this Chapter, the fracturing processes and mechanisms leading to shear rupture zone creation are investigated in intact (non-jointed) brittle rock deformed in direct shear under constant normal stress boundary conditions. For this purpose the following are explored:

- the fracturing processes and mechanisms associated with shear rupture zone creation in the synthetic specimen;
- the synthetic specimen’s shear stress versus horizontal displacement response;
- sequential shear rupture zone creation in the synthetic specimen relative to the shear stress versus horizontal displacement response;
- the synthetic specimen’s overall rupture mechanism;
- the ultimate shear rupture zone geometry created in the synthetic specimen; and
- the synthetic specimen’s peak strength envelope.

As a side product, the influence of specimen Length to Height (L:H) aspect ratio (considering ratios of 1:1 and 1.5:1) on shear rupture zone creation is investigated to understand other boundary condition effects.

The investigation is completed using the commercially available particle based Distinct Element Method (DEM), Particle Flow Code in Two Dimensions (PFC2D, v4.00-190) (Itasca, 2011) and its embedded Grain Based Method (GBM) (Potyondy, 2010; Itasca, 2011). A synthetic specimen is generated numerically and calibrated to the available shear rupture characteristics (strength envelope, tensile strength, post-peak shear stress versus horizontal displacement response, and fracture angles) of Lodève sandstone deformed in direct shear under constant normal stress boundary conditions as reported by Petit (1988) and Wibberley et al. (2000). The sandstone is brittle and of low porosity (<2%).
First the DEM, PFC2D, and its GBM are introduced and the methodology for the generation and calibration of the synthetic specimen is presented. Next, the fracturing processes and mechanisms leading to shear rupture zone creation and the synthetic specimen’s overall rupture mechanism are investigated with a detailed analysis of the internal fracture orientations occurring in the shear rupture zone for both aspect ratios (1:1 and 1.5:1). The results from the investigations provide insight into the fracturing processes leading to rupture zone creation, the shear rupture mechanism and rupture zone geometry dependence on normal stress, and the influence of specimen aspect ratio on shear rupture zone creation. It is also shown that the post-peak shear stress versus horizontal displacement characteristic changes as the fracturing processes leading to shear rupture zone creation change and is the result of the different synthetic specimen scale rupture mechanisms and ultimate shear rupture zone geometries.

As indicated in the introduction to this thesis in Chapter 1, it is necessary to better understand the fracturing processes leading to shear rupture zone creation in intact brittle rock under various boundary conditions in an effort to better understand and interpret mining induced shear rupture zone creation. Unfortunately, limited research has focused on shear rupture zone creation in intact brittle rock (Obert et al., 1972; Petit, 1988), intact brittle analogue materials (Lajtai, 1969; Archambault et al., 1992; Cho et al., 2008), and concrete (Sonnenberg et al., 2003; Wong et al., 2005) subjected to direct shear under constant normal stress boundary conditions. The results of the limited previous investigations suggest that the shear rupture mechanism and fracturing processes leading to rupture zone creation are normal stress dependent (Lajtai, 1969; Petit, 1988; Sonnenberg et al., 2003) and result in the creation of different rupture zone geometries. Lajtai (1969), using plaster-of-Paris, and Sonnenberg et al. (2003), using concrete, suggested that the change in rupture zone geometry with increasing normal stress magnitude was a result of the specimen failure mechanism changing from dominantly tensile to shear. Lajtai (1969) also observed that fracture angles generated in specimens depended on specimen aspect ratio with fractures forming at shallower angles to the horizontal in the direction of applied shear for brick-compared to cubic-shaped specimens. Fracture angles in the ruptured specimens of Petit (1988) increased to steeper angles about the horizontal at increasing normal stress magnitudes (Wibberley et al., 2000). No aspect ratio effects were investigated by Petit (1988) where specimens ranged in aspect ratio from 0.66:1 to 0.42:1 (i.e., tall specimen). Both Lajtai (1969)
and Petit (1988) recognized that the stress state internal to a specimen is rotating when being deformed in direct shear.

Due to the rapid fracturing processes that occur in brittle rocks when being strained, the rupture of specimens composed of clay in direct (e.g., Morgenstern and Tchalenko, 1967; Gamond, 1983) and simple shear (e.g., Cloos, 1928; Riedel, 1929; Cloos, 1955; Tchalenko, 1970) have been used as analogue materials to investigate and understand fracturing processes in intact brittle rock subjected to shear deformation. These investigations have shown that fracturing leading to clay specimen rupture occurs systematically as follows (Figure 2.1, showing the rupture process described below) assuming the plane of imposed shear is the plane of maximum shear stress oriented 45° to the major principal stress:

At Stage I, just prior to reaching peak shear stress, an en échelon system of fractures forms oriented at $\phi/2$ to the direction of shearing (where $\phi$ is the angle of internal friction of the material). These fractures are called Riedel shears (R). In some cases, a conjugate Riedel shear (R’) develops that is oriented 90°- $\phi/2$. At Stage II, after peak shear stress is reached, i.e., in the post-peak region of the shear stress versus horizontal displacement curve, the Riedel shears extend horizontally (growing from their tips). With continued shear deformation, at Stage III, the Riedel shears stop growing and a new fracture, called a P shear, develops approximately symmetrical to the Riedel shears but oriented in the opposite direction (about -10° to the shear plane). The P shears interconnect the R shears. At Stage IV, continuous horizontal shears then develop (0° to 4°) isolating lenses of material and accommodating all of the displacements associated with shearing. In the direct shear experimental set up, fractures propagate from the shear box gaps (see Morgenstern and Tchalenko, 1967, their Figure 7) and it is the microscopic fractures in these larger fractures which exhibit the formation process described above.

Shear rupture zone creation in clay is interpreted based on the assumption that the major principal stress is oriented 45° to the enforced rupture zone. According to Tchalenko (1970), the 45° major principal stress orientation can be demonstrated in the Riedel experiment by putting a thin film of water on the surface of the clay. Clay surface tension is thus eliminated and the clay fails in tension with the formation of open gashes orientated 45° about the horizontal. While the investigations in clay have helped to further the understanding of the rupture of brittle rock deformed in shear, there is still debate on the rupture zone creation process and if it is
dominantly a shear fracturing or tensile fracturing process as argued by Crider and Peacock (2004) for fault creation in the upper brittle crust.

The numerical simulations reported next allow for the internal state-of-stress to be tracked during synthetic specimen deformation which is: (1) not possible in specimens deformed in the laboratory; and (2) assumed for the interpretation of shear rupture zones in clay. In addition, fracturing (location, orientation, and type – e.g., grain boundary and intra-grain), fracture mechanisms (tensile and shear), and particle displacement vectors during shear rupture zone creation are tracked during synthetic specimen deformation. All of the above listed provide an opportunity to improve the understanding of shear rupture zone creation in brittle rock deformed in direct shear under constant normal stress boundary conditions.

Figure 2.1 Shear rupture zone creation process in clay. Based on Tchalenko (1970) modified from Cho et al. (2008).
2.2 Rock type used for investigation

Petit (1988) investigated peak shear strength and normal stress dependent rupture creation in specimens of intact Lodève sandstone, a fine to medium grain brittle low porosity (<2%) rock, deformed in direct shear under constant normal stress boundary conditions. Wibberley et al. (2000) investigated the resulting fracture systems generated by Petit (1988) using scanning electron microscope (SEM) images. This rock type was chosen for this investigation because of its brittleness and it is the only well described direct shear testing series that has been completed on an intact brittle low porosity rock in the literature allowing for various shear rupture zone characteristics to be compared to numerical simulations.

The Lodève sandstone tested by Petit (1988) is not a geological unit in itself. The sandstone specimens were taken from inter-bedded sandstone\(^1\) – shale\(^2\) – cinerite\(^3\) beds (each approximately 10 cm – 1m thick) in an approximate 300m thick sequence of Lower Permian\(^4\) lacustrine shoreline and deltaic deposits located in Southern France. The sandstone was the strongest (most cohesive) rocks not affected by Permian and later faulting in the area which allowed clays and other minerals into fractures in places (Wibberley, 2011).

The sandstone is fine to medium grain and consists of calcite (cementing component originating primarily from early precipitation of pore waters), orthoclase feldspar, albite feldspar, and quartz. The calcite cementation resulted in the low porosity of <2% (Petit, 1988). No grain size analysis or composition percentages were provided by Petit (1988) or later by Wibberley et al. (2000) and this data currently does not exist (Petit and Wibberley, 2011). The mineral composition of the Permian Lodève sandstone based on information provided by Comte et al. (1985) is as follows: 45-55% feldspar; 20% carbonates; 15-20% quartz; and 10% argillaceous material. According to Petit (1988) and Wibberley et al. (2000), the sandstone does not contain argillaceous material. The composition described by Comte et al. (1985), suggesting argillaceous content, maybe

---

\(^1\) Clastic sedimentary rock composed of grains (usually quartz, feldspar, and rock fragments) from 0.0625 to 2mm bound together by a cement of quartz, carbonate, or other minerals.

\(^2\) Very fine grained clastic rock composed of silt and clay that tends to part along bedding planes.

\(^3\) A sedimentary rock composed mostly of volcanic ash.

\(^4\) Geologic period which extends from 298.9±0.2 to 252.2±0.5 million years ago.
related to the overall geologic sandstone in the area and may not be for the specific sandstone bed sampled by Petit (1988).

Petit (1988) provides quantitative details on the tensile strength and the linear Coulomb strength envelope for normal stress magnitudes approximately 15 to 100 MPa. Wibberley et al. (2000) provide angles, measured from SEM images, of the fractures and fracture systems generated in the specimens. Descriptions and figures are reported by Petit (1988) and Wibberley et al. (2000) for the final rupture zone geometries, post-peak shear stress ($\tau$) versus horizontal displacement ($\delta_h$) response of the specimens from low to high normal stress magnitudes, the creation process of the rupture zones during deformation, and general mineralogy and grain sizes of the specimens.

2.3 Introduction to adopted Distinct Element Method

PFC2D models the movement and interaction of particles which are represented as rigid circular disks that can overlap at contacts. Boundary conditions (such as constant stress or velocity) are applied along walls which are rigid borders. Particles are not bonded to walls and shear can occur at particle-wall interfaces depending on the assigned particle micro-parameters: (1) coefficient of friction, $\mu$, (2) normal to shear stiffness ratio, $k_n/k_s$; and (3) modulus, $E_c$. The particles abide by Newton’s laws of motion, and a force-displacement law is applied to each particle-particle contact. PFC2D uses an explicit finite difference method. The calculation cycle in PFC2D is summarized as follows (Schöpfer et al., 2007): (1) contacts from known particle and wall locations are updated; (2) force-displacement law is applied to each contact to update particle-particle contact forces; and (3) law of motion to each particle is applied to update particle velocity. The calculation cycle is carried out using a time-stepping routine where at each time-step a cycle is carried out. A time-step is chosen to be so small that, during a single cycle, disturbances of any particle cannot propagate further than to the particle’s immediate surroundings.

The grain based method (GBM) in PFC2D v4.00-190 is used to generate a synthetic material that mimics deformable, breakable, polygonal grains cemented along their adjoining sides. A two-dimensional disk-packing methodology is used to generate the polygonal grain structure as follows (Figure 2.2): (1) a bonded particle model with no walls is generated where the particles are of the desired grain size and variability based on the follow parameters for each grain type:
(a) percent volume composition; (b) minimum grain radius, $\overline{R}_{\text{min}}$; (c) maximum to minimum grain radius ratio, $\overline{R}_{\text{max}}/\overline{R}_{\text{min}}$ (see Figure 2.2a, showing circular particles of a desired grain size and variability where each particle-particle contact is depicted as a line joining particle centers); (2) void centroids are identified between particles as shown in Figure 2.2b as black ‘dots’; (3) the centroids of the voids (black ‘dots’ in Figure 2.2b) are then joined with lines forming a polygonal network (Figure 2.2c-d); and (4) the GBM is generated by overlaying the polygonal network on a bonded particle model of smaller particles which fill the polygonal network (Figure 2.2e).

Figure 2.2 Grain structure generation. (a) Initial particle packing and contacts. (b) Void centroids (black ‘dots’). (c) Creation of polygonal network with nodes at void centroids. (d) Polygonal network. (e) Polygonal network overlaid on particle assembly (a-d modified from Potyondy, 2010).

A grain, as shown in Figure 2.3a, is composed of grain boundaries which are represented using smooth-joint contacts (Mas Ivars, 2010). The internal structure of a grain, bound by the smooth-joints, consists of a cemented granular material bonded together by parallel bonds (Potyondy and Cundall, 2004) (Figure 2.3a-b). Smooth-joint bond breakage is thus representative of grain boundary and parallel bond breakage of intra-grain fracturing. Internal to the grain shown in Figure 2.3a are 68 parallel bonded particles (the particles forming the grain). Smooth-joints
remove the previous limitation in PFC2D where discontinuities were simulated with unrealistically rough and bumpy contacts created by the particle based nature of the DEM. Particles that are on adjacent sides of a smooth-joint can overlap during sliding, forcing the sliding path along the smooth-joint contact as schematically shown in Figure 2.3c (opposed to riding over particles along the sliding path, as in Figure 2.3d). Parallel bonds, schematically illustrated in Figure 2.3b, allow both a force and moment to be resisted between individual particles. Parallel bonds are an improvement over the contact bond which could not resist a moment (Potyondy and Cundall, 2004). Once a smooth-joint or parallel bond breaks (in either shear or tension), the contact transitions to frictional behaviour depending on the assigned smooth-joint residual coefficient of friction or particle coefficient of friction micro-parameters. Examples of broken parallel bonds internal to a grain and smooth-joints along a grain boundary are shown in Figure 2.3e. A smooth-joint is defined by the following micro-parameters: (1) normal and shear stiffness factors, $\tilde{k}_n$ and $\tilde{k}_s$; (2) tensile strength, $\sigma_c$, and shear strength (defined by a linear strength criterion), $\tau_c = c + \sigma_n \tan \phi$ (where $c$ is cohesion, $\phi$ the friction angle of the smooth-joint, and $\sigma_n$ is normal stress); and (3) a residual coefficient of friction, $\mu_r$. A parallel bond is defined by the following micro-parameters: (1) normal to shear stiffness ratio, $\tilde{k}_n / \tilde{k}_s$; (2) bond modulus, $\tilde{E}_c$; (3) tensile strength, $\tilde{\sigma}_c$, and shear strength (defined by a linear strength criterion), $\tilde{\tau}_c = \tilde{c} + \sigma_n \tan \tilde{\phi}$ (where $\tilde{c}$ is cohesion and $\tilde{\phi}$ the angle of internal friction of the parallel bond); and (4) bond radius multiplier, $\lambda$. A bond radius multiplier of 1 completely fills the gap between two particles and if the multiplier approaches zero, the material behaves as a granular material. Bond strength standard deviation can be assigned if desired. A particle is defined by the following micro-parameters: (1) minimum radius, $R_{min}$; (2) maximum to minimum radius ratio, $R_{max}/R_{min}$; (3) normal to shear stiffness ratio, $k_n/k_s$; (4) density, $\rho$; (5) modulus, $E_c$; and (6) coefficient of friction, $\mu$. 
Figure 2.3 Elements forming a grain in the GBM. (a) Example single grain showing smooth-joint contacts forming the grain boundaries and the internal parallel bonded particles (not showing the parallel bonds for clarity). (b) Schematic representation of a parallel bond. (c) Schematic representation of the behaviour of a smooth-joint. (d) Schematic representation of the behaviour of a contact without a smooth-joint. (e) Examples of broken parallel bonds and smooth-joints.
2.4 Grain structure

All current models have limitations when simulating actual grain structures of rock and simplifications are required (Lan et al., 2010). The grain structure simulated in this thesis is not intended to exactly match that of Lodève sandstone but to account for some of its grain size geometric heterogeneity, and mineral composition and properties. The synthetic grain structure generated for the Lodève sandstone is represented by three mineral types; feldspar (50%), quartz (20%), and calcite (30%); grouping the 2 feldspar mineral types together (the composition is based on Comte et al., 1985). SEM images from Wibberley (2011) were used to estimate grain sizes for each mineral grain type by manually outlining grains with a polygon around its boarder (using software where measurements can be made on scaled images) and recording the largest and smallest dimensions for two separate populations of grain diameters as follows: (measurement set 1) from a single SEM image; and (measurement set 2) from multiple SEM images. This approach was adopted because the images could not be used for automatic picking of grain type based on pixel color and grain size estimation because grain boundaries were not sufficiently defined in the SEM images. The resulting D10, D50, and D90 values are summarized in Table 1 and the histograms for measurement set 2 (which also contains measurement set 1) are shown in Figure 2.4. The grain sizes summarized in Table 1 are small (average 50% passing grain diameter of 0.27mm, D50, Table 1, measurement set 1) compared to previous PFC2D simulations (e.g., Potyondy and Cundall, 2004 where 0.55 and 1.19mm average diameter particles were used and Hazzard et al., 2000 where 1.3 to 6mm average diameter particles were used depending on the rock type being simulated; in these studies, circular particles were used to represent grains, such that one grain was one circular particle). During initial attempts to simulate the values for measurement set 1 (Table 1) using the GBM, model generation difficulties occurred because each grain needed to be represented with a number of bonded particles resulting in an unmanageable number of particles in a model (ref. Section 2.3 for a description of the GBM composition). In an effort to maintain the grain scale heterogeneity and keep the grain size as small as reasonably possible, the minimum grain size was eventually set at five times the estimated D10 value (Table 1, ‘5xD10’ column, for measurement set 1) with the maximum (Dmax) to minimum (Dmin) grain size ratio set at the values listed in Table 1 (‘Dmax/Dmin’ column, measurement set 1). The simplified grain sizes and maximum to minimum grain ratios result in a synthetic specimen with an average synthetic grain size of 1.4mm which is
of medium grain size (Press and Seiver, 1998). The grain size is within the limits of that
described by Petit (1988), with the grain structure accounting for some of the grain size
heterogeneity (i.e., $\frac{D_{\text{max}}}{D_{\text{min}}}$ ratio), and three different mineral types. Considering that the actual
grain scale and structure cannot (yet) be simulated, this is considered to be a reasonable
simplification. Measurement set 1 was used over set 2 because of the slightly larger estimated
grain sizes and measurement set 2 not being significantly different than set 1 when the two are
compared in Table 1. An example grain distribution resulting from the parameters discussed is
shown in Figure 2.5a in comparison to a representative SEM image from Wibberley (2011)
(Figure 2.5b). Visual inspection suggests that the grain composition generated is a reasonable
representation of reality. The synthetic specimen (Figure 2.5a) shows the overall grains (~1405)
and does not show the 41,388 particles which make up the grains or the smooth-joints (which
make up the grain boundaries) or the parallel bonds between the particles in each grain.

Mineral properties to aid in the selection of micro-parameters for each mineral type (feldspar,
quartz, and calcite) in the synthetic specimens were compiled from the literature and are listed in
Table 2 from the indicated sources. Not all aspects of a mineral grain’s properties can be
accommodated by the GBM such that the material properties in Table 2 should be thought of as
overall bulk mineral mechanical properties.

### Table 1 Estimated grain sizes by mineral type.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Measurement Set</th>
<th>Population</th>
<th>$^*D_{10}$ $(\mu m)$</th>
<th>$^*D_{50}$ $(\mu m)$</th>
<th>$^*D_{90}$ $(\mu m)$</th>
<th>$^*D_{\text{max}}/D_{\text{min}}$</th>
<th>$^*5xD_{10}$ $(\mu m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feldspar</td>
<td>1</td>
<td>38</td>
<td>175</td>
<td>275</td>
<td>375</td>
<td>2.14</td>
<td>875</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(136)</td>
<td>(155)</td>
<td>(255)</td>
<td>(375)</td>
<td>(2.43)</td>
<td>(765)</td>
</tr>
<tr>
<td>Quartz</td>
<td>1</td>
<td>24</td>
<td>175</td>
<td>250</td>
<td>325</td>
<td>1.86</td>
<td>875</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(108)</td>
<td>(115)</td>
<td>(185)</td>
<td>(310)</td>
<td>(2.7)</td>
<td>(575)</td>
</tr>
<tr>
<td>Calcite</td>
<td>1</td>
<td>25</td>
<td>150</td>
<td>275</td>
<td>500</td>
<td>3.33</td>
<td>750</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(102)</td>
<td>(110)</td>
<td>(215)</td>
<td>(400)</td>
<td>(3.63)</td>
<td>(550)</td>
</tr>
</tbody>
</table>

$^*D_{10}$, 10% passing value; $^*D_{50}$, 50% passing value; $^*D_{90}$, 90% passing value; $^*D_{\text{max}}/D_{\text{min}}$, $D_{90}$ value divided by the $D_{10}$ value
Figure 2.4 Histograms and cumulative “percent passing” curves for grains measured in ‘measurement sets 1 and 2’ using SEM images provided by Wibberley (2011).
Figure 2.5 Simplified GBM generated for Lodève sandstone (a) and representative SEM image provided by Wibberley (2011) (b). Some mineral grains are labeled ([a] and [b]: darkest grey – quartz [Qz]; light grey – feldspar [F]; and white – calcite [C]). Scales as shown.
Table 2 Compiled mineral properties.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Feldspar</th>
<th>Quartz</th>
<th>Calcite</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>From</td>
<td>To</td>
<td>From</td>
<td>To</td>
</tr>
<tr>
<td>Young's modulus, $E$ (GPa)</td>
<td>73</td>
<td>89</td>
<td>72</td>
<td>98</td>
</tr>
<tr>
<td>Uniaxial compressive strength, UCS (MPa)</td>
<td>180</td>
<td>290</td>
<td>200</td>
<td>328</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>-36</td>
<td>-</td>
<td>-10</td>
<td>-34</td>
</tr>
<tr>
<td>Friction coefficient polished mineral surface</td>
<td>0.12</td>
<td>-</td>
<td>0.12</td>
<td>0.25</td>
</tr>
<tr>
<td>Friction coefficient mineral gouge</td>
<td>0.55</td>
<td>0.80</td>
<td>0.55</td>
<td>0.65</td>
</tr>
<tr>
<td>Density, g/cm$^3$</td>
<td>2.54</td>
<td>2.70</td>
<td>2.47</td>
<td>2.72</td>
</tr>
<tr>
<td>Bulk modulus, $K$ (GPa)</td>
<td>50</td>
<td>63</td>
<td>36</td>
<td>38</td>
</tr>
<tr>
<td>Shear modulus, $G$ (GPa)</td>
<td>29</td>
<td>35</td>
<td>44</td>
<td>46</td>
</tr>
</tbody>
</table>
2.5 Simulation procedures

2.5.1 Direct shear: constant normal stress

The numerical direct shear set up is a simplification of the laboratory set up used by Petit (1988) where Lodève sandstone specimens with aspect ratios of 0.66:1 and 0.42:1 were used to allow for specimen anchorage and shear stress application with a 5mm wide gap left around the specimen between the upper and lower shear box walls for observation of fracturing. In the numerical simulations, a tall specimen is not needed to ensure proper anchorage and shear stress application. The tall specimen of Petit (1988) was therefore reduced to a 1:1 aspect ratio. The shear box with constant normal stress boundary conditions created in PFC2D (shown in Figure 2.6) is composed of a 50 x 50mm square synthetic specimen bound by an upper portion of the shear box which has two separate fixed lateral walls (Wall 3 and 4, Figure 2.6) and a lower portion simulated as a single wall, of ‘U’ shape (Wall 1) which moves in the horizontal direction (Figure 2.6). The contacts between all of the walls and the synthetic specimen are frictionless. A 5mm gap between the upper lateral walls and lower wall of the shear box is introduced to match the shear box gap reported by Petit (1988) (Figure 2.6). Constant normal (vertical) stress is first applied to the synthetic specimen through the use of a servo-controlled top wall (Wall 2, Figure 2.6) through applied velocity. When the desired constant normal stress magnitude is achieved throughout the synthetic specimen, the applied velocity to the top wall is arrested and constant normal stress maintained by setting the top wall (Wall 2) to act as a ‘virtual’ servo. Shear deformation is applied to the synthetic specimen through movement of the lower wall (Wall 1) by a constant applied velocity (Figure 2.6) with rotation of the shear box restricted. The shear velocity of the lower wall was chosen to ensure both the peak shear strength and post-peak shear stress versus horizontal displacement response of the synthetic specimen were not influenced by slower shear velocities considering both low and high normal stresses applied to the synthetic specimen (which were found to have different shear velocity influences). The goal was to create quasi-static loading conditions. A shear velocity of 0.04m/s was selected which relates to a displacement of the lower wall of approximately $2.75 \times 10^{-7}$ mm per time step (see Appendix B.1 for more details). Therefore, the constant normal stress simulation set up has a near infinite shear stiffness (i.e., shear displacement is controlled) and an infinitely soft normal stiffness (i.e., constant normal stress boundary conditions).
The average shear stress is determined through the reaction forces acting along Wall 4 divided by the synthetic specimen’s length. Horizontal displacement is measured based on the movement of the lower wall (Wall 1). Based on the logic adopted by Cho et al. (2008), the principal stress magnitudes and the orientation of the major principal stress internal to the synthetic specimen are determined using the measurement circle (10mm diameter) shown in Figure 2.6 in the center of the synthetic specimen. All stress components (\(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\)) are monitored for every particle within the measurement circle.

![Figure 2.6 Constant normal stress direct shear simulation schematic.](image)

2.5.2 Uniaxial, biaxial, and direct tension simulations

The investigation focuses on direct shear but other simulations were conducted and are briefly described here. The biaxial, uniaxial, and direct tension tests were conducted on rectangular
synthetic specimens, 50mm x 25mm (height to width ratio of 2.0, same height as the synthetic shear specimen but half the width). The biaxial and uniaxial (a biaxial test with zero lateral confining stress) testing procedure described by Potyondy and Cundall (2004) was followed. The term biaxial test is used for the triaxial test in PFC2D due to the two dimensional nature of the code. Under these boundary conditions, the top and lower boundaries of the synthetic specimen are moved towards each other at a constant velocity while the stress magnitude applied to the lateral boundaries is held constant. The direct tension testing procedure moves the top and lower boundaries of the synthetic specimen away from each other at a constant velocity.

2.6 Calibration to Lodève sandstone characteristics

DEM models must be calibrated such that the assigned micro-parameters in the models produce the desired macro-properties (such as strength) and various characteristics (such as load-displacement and fracture response) of the material being simulated. Typically, DEM models are calibrated to the strength and deformation characteristics of the loading condition being investigated (e.g., Hazzard et al., 2000 calibrated to the strength and deformation characteristics of rocks under uniaxial and triaxial compression). In this thesis, calibration is being completed against the reported characteristics of the Lodève sandstone deformed in direct shear. The available information for the Lodève sandstone deformed in direct shear under constant normal stress boundary conditions was outlined previously in Section 2.2. The only mechanical characteristic information missing is the deformation modulus in direct shear. To overcome this limitation, the calibration process focused on reproducing the available linear Coulomb strength envelope, tensile strength, post-peak shear stress versus horizontal displacement response, and specimen rupture zone geometric characteristics (i.e., final rupture zone geometries and fracture angles at two normal stress magnitudes) of the Lodève sandstone. The resulting calibration was then compared to strong (uniaxial compressive strength \( \geq 100 \text{MPa} \)), brittle, low porosity (\( \leq 10\% \)), sandstone data compiled from the literature to ensure compliance with the general characteristics of brittle low porosity sandstones.

2.6.1 Calibration results

Six normal stress magnitudes were simulated 5, 15, 25, 40, 60, and 90MPa to cover the range of normal stress reported by Petit (1988). The PFC2D GBM simulation results when compared to the laboratory test results of Petit (1988) are essentially identical (constant normal stress direct
shear strength test results in normal-shear stress space, Figure 2.7a, and description of the post-
peak shear stress versus horizontal displacement response of the specimens, Figure 2.7b). The
simulated tensile strength is also almost identical to that reported by Petit (1988). These
calibration aspects are summarized as follows:

(a) *Linear Coulomb strength envelope:*

Laboratory: \( \tau = 28 + \sigma_n^{0.81} \) (Figure 2.7a) \hspace{1cm} (1)

Simulation: \( \tau = 27 + \sigma_n^{0.76} \) \( R^2=0.99 \) (Figure 2.7a) \hspace{1cm} (2)

(b) *Tensile strength:*

Laboratory: -17MPa

Simulation: -16MPa

(c) *Post-peak shear stress versus horizontal displacement response:*

In both laboratory and simulation: brittle instantaneous shear stress drop at low
normal stress magnitudes with maintained peak shear stress at the highest normal
stress magnitudes (Figure 2.7b).

The final rupture zone geometries (Figure 2.8 a-b) for normal stress magnitudes of 25 and
90MPa are also similar to the geometries described by Petit (1988) where:

a) the rupture zone geometry is dependent on normal stress (as evident in Figure 2.8a-b
where the 25 and 90MPa rupture zone geometries are different);

b) at low normal stress magnitudes the rupture zone is thin with shallower fracture angles;
and

c) at high normal stress magnitudes the rupture zone is relatively wide with steeper fracture
angles (compare Figure 2.8 a-b where the 25MPa normal stress rupture zone is visually
thinner and has shallower angles compared to the 90MPa normal stress rupture zone
geometry).
The angles of fracture systems in the two rupture zone ‘snap shots’ (Figure 2.8 a-b) were determined by sketching linear trends along them using scaled images and measuring the angle of the line relative to horizontal (counter clockwise positive 0° to 90°) (PFC2D can directly output the individual fracture angles but cannot output angles of a fracture system, i.e., angles of larger fractures composed of multiple smaller fractures). An example showing two fracture systems and the resulting sketched fractures is illustrated in Figure 2.9. The fracture systems were divided into two classes to facilitate comparison to the angles measured by Wibberley et al. (2000): (1) fracture systems forming pre-peak strength which were created in the direction of the internal major principal stress and thus pure tensile fractures; and (2) fracture systems forming post-peak strength which are composed of linked fracture arrays and due to the rotation of the internal major principal stress have started to be loaded in shear and are thus no longer pure tensile fractures. The measured angles are presented in Figure 2.8c-d as rose diagrams. The fracture system angles in the synthetic specimens (Figure 2.8 c-d) are compared to the angles of micro-faults and tensile fractures measured by Wibberley et al. (2000) (Figure 2.8 e-f). ‘Micro-fault’ is the terminology used by Wibberley et al. (2000) for fracture systems in the SEM images of the specimens which showed measurable shear displacement at the scale of the SEM image. Thus, the pre-peak fractures in the synthetic specimen are comparable to the tensile fractures and the post-peak fractures are comparable to the micro-faults measured by Wibberley et al. (2000). The fracture system angles from the simulations (Figure 2.8 c-d) show the trend of increasing angles with increasing normal stress magnitudes and generally similar overall angles to the micro-faults and tensile fractures measured by Wibberley et al. (2000) (compare rose diagrams in Figure 2.8 e to Figure 2.8c, Figure 2.8f to Figure 2.8d).

It is therefore concluded that the available aspects of Lodève sandstone’s mechanical response to direct shear deformation under constant normal stress have been captured by the calibrated PFC2D GBM synthetic specimen. The resulting micro-parameters for the model components and each grain type (which are assigned separate micro-parameters) are summarized in Table 3. Comparing the micro-parameters in to the compiled grain property data in Table 2, the simulated modulus values for the particles forming the grains compare to the range of Young’s modulus values for quartz and feldspar except for calcite which was simulated with a lower modulus value. The tensile strength of the parallel bonded particle assembly forming each mineral in the synthetic specimen is similar to the uniaxial compressive strength of the minerals. The mineral
densities are also similar but it is noted that the packing of the circular particles in the grains results in pore space such that the grains are less dense than the particles forming them. Since the particles are the elementary building blocks of the grains, the particles should be of the correct density because once the arrangement of parallel bonded particles is comminuted into individual non-bonded particles, they would be too dense individually if the overall grain density was desired. The coefficients of friction for the particles in the simulations are slightly higher than the compiled values for mineral gouge. The calibration also resulted in the generation of tensile fractures along grain boundaries and in grains. This is consistent with the SEM images of Wibberley et al. (2000) where only tensile micro-fracturing was observed with micro-faults being developed from systems of tensile fractures.

The micro-parameters in Table 3 were used to simulate synthetic specimens in uniaxial and biaxial compression and the results were compared to the compiled testing data for other strong brittle low porosity sandstones. Table 4 summarizes the compiled Hoek-Brown material constant, \( m_i \) (Hoek and Brown, 1980), uniaxial compressive strength, Young’s modulus, and modulus ratio (Young’s modulus to uniaxial compressive strength) and Figure 2.10 summarizes the compiled triaxial strength data, together with the PFC2D GBM simulation results. The micro-parameters determined from calibrating the PFC2D GBM synthetic specimen to the available Lodève sandstone specific characteristics resulted in typical properties of the compiled strong brittle low porosity sandstone data. The Lodève sandstone is a low porosity brittle sandstone, and its deformation and strength properties should be reasonably bound by the compiled sandstone data, which they are. Since the micro-parameters from the calibration result in the typical characteristics summarized in Table 4 and reproduce the available Lodève sandstone characteristics in direct shear, it is concluded that the model is calibrated.

A summary of preliminary calibration results, which were completed prior to establishing the final calibration micro-parameters, can be found in Appendix B.2. These preliminary calibration results highlight the minimal variability of the synthetic specimen’s mechanical response to deformation for different synthetic specimen realizations.
Figure 2.7 Strength and deformation calibration results. (a) Linear Coulomb strength envelope for laboratory data (grey circles) as reported by Petit (1988) compared to DEM simulation results (white squares). (b) DEM simulation shear stress versus horizontal displacement response in comparison to descriptions provided by Petit (1988) and Wibberley et al. (2000).
Figure 2.8 Comparison of fracture system angles. (a)-(b) DEM simulation rupture zone images for 25 and 90MPa normal stress magnitudes, respectively (orange – grain boundary, black – intra-grain tensile fracture). (c)-(d) Orientation of fracture systems in (a) and (b), respectively (counter clockwise from horizontal left half, clock wise from horizontal right half of rose diagram); pre-peak fractures are tensile and post-peak fractures are predominantly shear (i.e., micro-faults). (e)-(f) Orientation of micro-faults and tensile fractures in the Lodève sandstone specimens ruptured in direct shear as measured in SEM images by Wibberley et al. (2000) for 23 and 87MPa normal stress magnitudes, respectively.
Figure 2.9 Example fracture systems in (a) generated through a grain (black fractures) and along grain boundaries (orange fractures) from a number of individual tensile fractures. The fracture systems in (a) are shown in (b) with the overlaid fracture system sketch (dashed lines).
<table>
<thead>
<tr>
<th>Element</th>
<th>Parameter</th>
<th>Grain 1</th>
<th>Grain 2</th>
<th>Grain 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain size</td>
<td>Volume composition (%)</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Minimum radius, $R_{min}$ (mm)</td>
<td>0.44</td>
<td>0.38</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.86</td>
<td>3.33</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>Minimum radius of particles forming grains, $R_{min}$ (mm)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.66</td>
<td>1.66</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Particle normal to shear stiffness ratio, $k_n/k_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Density (g/cm$^3$)</td>
<td>2650</td>
<td>2710</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>Modulus, $E_c$ (MPa)</td>
<td>98000</td>
<td>30000</td>
<td>89000</td>
</tr>
<tr>
<td></td>
<td>Friction coefficient, $\mu$</td>
<td>0.83</td>
<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Modulus, $E_c$ (MPa)</td>
<td>98000</td>
<td>30000</td>
<td>89000</td>
</tr>
<tr>
<td></td>
<td>Normal to shear stiffness ratio, $K_n/K_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Tensile strength, $\sigma_c$ (MPa)</td>
<td>245</td>
<td>112</td>
<td>265</td>
</tr>
<tr>
<td></td>
<td>Cohesion, $\bar{c}$ (MPa)</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Friction angle, $\phi$ (°)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Tensile and shear strength standard deviation</td>
<td>50</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Normal stiffness factor, $k_n$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Shear stiffness factor, $k_s$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Tensile strength, $\sigma_c$ (MPa)</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>Smooth-joints</td>
<td>Cohesion, $c$ (MPa)</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Friction angle, $\phi$ (°)</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Residual friction coefficient, $\mu_r$</td>
<td>0.27</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table 4 Compiled strong brittle low porosity sandstone data compared to simulations.

<table>
<thead>
<tr>
<th>Property</th>
<th>Average, mode, or range as indicated</th>
<th>Simulation Value</th>
<th>Compiled Data Population</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Compressive Strength, UCS (MPa)</td>
<td>140 – 150 (mode)</td>
<td>146</td>
<td>53</td>
<td>Li et al. (2003); Lama and Vutukuri (1978); Blair (1955)</td>
</tr>
<tr>
<td>Young's Modulus, E (GPa)</td>
<td>44 (average)</td>
<td>46</td>
<td>42</td>
<td>Belikov (1965; 1967); Broz et al. (2006)</td>
</tr>
<tr>
<td>$m_i$</td>
<td>8 – 16 (range shown on triaxial envelope, Figure 2.10)</td>
<td>9</td>
<td>5</td>
<td>Hoek and Brown (1980) table 9</td>
</tr>
<tr>
<td>Modulus ratio (E:UCS)</td>
<td>300:1 (average)</td>
<td>315:1 (i.e., 46,000MPa/146MPa)</td>
<td>Not applicable</td>
<td>Deere (1968); Martin et al. (2003)</td>
</tr>
</tbody>
</table>
Figure 2.10 Compiled strong brittle low porosity sandstone triaxial data (grey circles) in comparison to DEM biaxial simulation results (white squares). Rupture images of the synthetic specimen tested at various confining stress magnitudes (a-d) linked to the results plotted in principal stress space. Images show visual coherence with typical rock testing failure mode change (i.e., transition from axial splitting [e – 0MPa confining stress, from boxed region in (a)] to shear [f – 32MPa confining stress, from boxed region in (c)] as shown by the velocity vector arrows. Compiled data from Franklin and Hoek, 1970; Santarelli and Brown, 1989; Kovari et al., 1983.
2.7 Forces, mechanisms, and fracture system development

First, the distribution of forces in the synthetic specimens are investigated followed by the fracturing processes leading to rupture zone creation and the rupture mechanism for various applied normal stress magnitudes and for aspect ratios of 1:1 and 1.5:1. This is followed by an investigation into the relationship between peak shear strength determined from the shear box lateral wall and peak internal strength as determined from the measurement circle in the center of the synthetic specimen.

Images of the synthetic specimen after application of normal stress are summarized in Appendix B.3 and show the development of fractures at the 60 and 90MPa normal stresses due to normal stress application alone. This relates to normal stress to compressive strength ratios of 0.41, and 0.62. Although not calibrated to, the initiation of fractures in the synthetic specimen starting around 60MPa normal stress (based on the number of fractures, initiation is considered somewhere between 40 and 60MPa, i.e., 0.27 to 0.41 normal stress to compressive strength ratio) is consistent with other brittle rocks where the crack initiation threshold has been determined to be between 0.29 to 0.40 (e.g., Martin, 1993 for Lac du Bonnet Granite).

2.7.1 Distribution of forces in synthetic specimen with two aspect ratio

The aspect ratio of the shear box used for calibration (1:1) is sufficient for deforming specimens of rock containing discontinuities (ASTM D 5607) but this aspect ratio is outside that recommended for testing of granular material in direct shear (Wang and Gutierrez, 2010); suggesting an aspect ratio of 1.5:1 to 2:1. No studies have been completed on how shear box aspect ratios influence the rupture zone characteristics in intact brittle rock.

Wang and Gutierrez (2010) concluded that when the aspect ratio of the shear box is small (i.e., H approaching L; towards at 1:1 geometry), global failure of the entire specimen occurs because the contact force chains that develop from the shear box lateral boundaries propagate further into the center of the specimen avoiding the influence of the top and bottom boundaries of the shear box. Force chains show how a particle assembly is transmitting compressive forces along arrays of contacting particles with thicker black lines indicating higher particle contact forces. Large aspect ratios (long thin shear box geometries) were found to have contact force chains that are influenced by the top and bottom boundaries, with failure of the specimen occurring via
propagating shear bands from the shear box lateral boundaries towards the center of the specimen (Wang and Gutierrez, 2010). The smallest aspect ratio investigated by Wang and Gutierrez (2010) was 1.57 and they recommend shear box aspect ratios of 1.5 to 2.0.

The force chains in 1:1 and 1.5:1 synthetic specimens for normal stress magnitudes of 5, 25, and 90 MPa are shown in Figure 2.11a-c for the indicated horizontal displacement ($\delta_h$) and are alongside the 1.57:1 aspect ratio specimen of Wang and Gutierrez (2010) (Figure 2.11d). The force chain networks in Figure 2.11a-c show that they develop toward the center of the synthetic specimen, are minimally influenced by the top and bottom shear box boundaries, concentrate in the lower left and upper right corners of the synthetic specimen, and are inclined at increasing angles about the horizontal due to the applied normal stress magnitude increasing. The force chain distribution described is similar to that of Wang and Gutierrez (2010) (Figure 2.11d). The force chains are also comparable to the DEM results of Zhang and Thornton (2007) (Figure 2.11e) and Cho et al. (2008) (Figure 2.11f) and laboratory experiments for crushed glass (Dyer and Milligan, 1984) (Figure 2.11g) and glass particles (Allersma, 2005) (Figure 2.11h). In summary, there are minimal differences between the force chain networks which develop in the 1:1 and 1.5:1 synthetic specimens and they are both comparable to other DEM and granular material assemblies. Based on the force chains, the 1:1 shear box aspect ratio is suitable for simulation.
2.7.2 Fracturing process leading to rupture and rupture mechanism

Under constant normal stress, rupture zone creation is dependent on normal stress ($\sigma_n$) and occurs in the post-peak region of the shear stress versus horizontal displacement curve. The
results are grouped into normal stress ranges where similar fracturing processes and rupture mechanisms occur: low (5 to 15MPa); moderate (25 to 40MPa); and high (60 to 90MPa). The 5, 25, and 90MPa normal stress simulations are representative of the three groups and are used here to illustrate the fracturing process leading to rupture zone creation and the resulting rupture mechanism in each group.

In each representative simulation, fracture development is assessed in a number of stages relative to the shear stress versus (applied) horizontal displacement curve. In the DEM simulations, parallel bond breakage between two particles and bond breakage along a smooth-joint contact are recorded as well as their respective mechanism (tensile versus shear). This allows both intra-grain (parallel bond) and grain boundary (smooth-joint) fracturing and mechanism to be assessed as well as fracture development in the deforming synthetic specimens. At each stage, intra-grain and grain boundary fracture development is recorded (number, location, mechanism, and orientation) allowing for a description of the process leading to rupture zone creation. Fracture orientation is for the fracture between two particles or along a smooth-joint contact. Since fracture systems form from a number of individual fractures in the synthetic specimen, systems of fractures were sketched to determine fracture trends at each stage by exporting the PFC2D image of the fractures and importing them into a computer tool to allow for drawing and measurement of lines along fracture systems (as shown in Figure 2.9). Orientations of the sketched fractures are plotted on rose diagrams and summarized statistically (average and standard deviation) where positive angles are counter clockwise and negative angles clockwise from the horizontal. The shear box geometry influences the stress conditions near the lateral walls. Therefore, the fractures that develop near the shear box lateral walls are not considered in the assessment of fracture development and orientation.

The representative simulations are graphically displayed in the same format in Figure 2.12 to Figure 2.14 which show from top to bottom: (1) shear box with rupture zone geometry (orange – grain boundary and black intra-grain tensile fracture) and area indicated where detailed fracture system angles are assessed (AA) (top left); (2) chart with the shear stress versus horizontal displacement curve, orientation of major principal stress ($\sigma_1$, measured counter clockwise from horizontal), grain boundary (GB) and intra-grain (IG) fracture rates and cumulative fracture counts (top right); (3) rupture zone detail at indicted horizontal displacement and internal $\sigma_1$ orientation (BB and CC indicate the locations of where particle displacement or velocity vectors
are viewed); (4) sketches of the rupture zone detail; (5) rose diagrams of the fracture system angles based on sketched detail; and (6) mechanism of fracture and or rupture plotted using particle displacement and or velocity vectors.

Particle velocity and displacement vectors explicitly show the movement of the particles and thus rupture mechanism. They are used to classify the fractures forming in the synthetic specimens as follows: (1) tensile fracture – a fracture or system of fractures formed when particle velocity and or displacement vectors indicate opening; and (2) micro-fault – a fracture or system of fractures formed when particle velocity and or displacement vectors indicate a shear condition. The orientation of the major principal stress as measured internal to the synthetic specimen using the measurement circle described in Section 2.5.1 is also used to interpret the fracture systems.

### 2.7.2.1 1:1 aspect ratio

At low normal stresses (see Figure 2.12), grain boundary (GB) fracturing occurs prior to peak shear strength. The GB fractures are oriented on average 15° (standard deviation of 15) and are predominately in line with the internal orientation of the major principal stress (13° to 25°) when being created. At or just after peak, rupture occurs across the synthetic specimen with a small incrementation of horizontal displacement. The rupture is tensile as predominantly indicted by particle velocity vectors. The major principal stress is also oriented parallel to the rupture at creation, 13°. The rupture zone is thin, relatively continuous, and composed of both grain boundary and intra-grain tensile fractures (orange and black fractures, respectively). With continued horizontal displacement, fracturing in the synthetic specimen leads to increased rupture zone connectivity, a transition from a tensile rupture to one that shows shear displacement (as indicated by particle velocity vectors on the lower half of the rupture), with increasing occurrence of intra-grain (IG) fractures as evident from the fracture rates and cumulative fracture counts. A fracture related to the shear box boundary is evident but does not influence the results. The boundary fracture occurs at low normal stress magnitudes because tensile stresses and low compressive force chains develop in the area where the fracture initiates. This is not a unique stress condition to the simulations and is also evident in the force-chains of Wang and Gutierrez (2010) (Figure 2.11d) where the light areas are zones of low compressive force chains and zones with tensile stress potential.
At moderate normal stresses (see Figure 2.13), rupture zone creation occurs in a progressive manner. Initially, an en échelon tensile fracture array develops (as indicated by particle displacement vectors showing opening) composed of both GB and IG fractures. IG fractures provide the linkage between GB fractures. Up to peak shear strength, the fracture angles average 22° (standard deviation 29) and are in-line with the internal major principal stress (25° to 35°) when being created. With continued horizontal displacement, the array links from the propagation of fractures from the tips of precursory fractures in the array and other shallow angle fracture systems. During the process, the en échelon tensile fractures transition into micro-faults (displacement vectors showing shear). Relative to rupture at low normal stresses (Figure 2.12), the rupture zone is created in a more progressive manner, is thicker, and contains steeper fracture angles. There is also increasing occurrence of IG fractures (black fractures, also evident from fracture rate and cumulative fracture counts). Fractures related to the shear box boundaries are not as event. Fractures propagating from the shear box gaps are evident but fracturing in the center of the synthetic specimen occurs first.

At high normal stresses (see Figure 2.14), fractures propagate from the shear box gaps at the first peak as indicated by (l) (when the internal principal stress is oriented 37°). Fracturing in the center of the synthetic specimen eventually occurs well after the first peak. Initially, the fractures are oriented 36° on average (standard deviation 19). With continued horizontal displacement, steep and shallow fracture systems develop. They are composed initially of en échelon tensile individual fractures that then coalesce with particle displacement vectors indicating shear (Figure 2.15). The shallow angle fracture systems have a synthetic shear sense and are related to the principal displacement of the lower shear box wall while the steep angle fracture systems have antithetic shear sense. Antithetic refers to a sense of shear opposite while synthetic refers to a sense of shear in the same direction as that applied to the synthetic specimen. The fracture systems eventually form a linked rupture zone across the synthetic specimen. Relative to the rupture zone that is created under moderate normal stresses (Figure 2.13), the high normal stress rupture zone develops in an even more progressive manner, has initial fractures which propagate from the shear box gaps, is thicker, and contains even steeper fracture angles. There is increasing occurrence of IG fractures (black fractures, also evident from fracture rate and cumulative fracture counts). Fractures related to the shear box boundaries are not event. A summary of rupture zone images for all normal stress magnitudes simulated is found in Appendix B.3.
Figure 2.12 5MPa normal stress rupture zone creation and mechanism (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch.
Figure 2.13 25MPa normal stress rupture zone creation and mechanism (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch.
Figure 2.14 90MPa normal stress rupture zone creation (1:1 aspect ratio). (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch.
2.7.2.2 1.5:1 aspect ratio

The synthetic specimens are 75 x 50 mm (L x H) and are composed of 62,082 particles. Normal stress magnitudes of 5, 25, 40, and 90 MPa were simulated. The shear stress versus horizontal displacement curves and linear Coulomb strength envelope are shown in Figure 2.16. Comparing Figure 2.16a to Figure 2.7b, additional horizontal displacement is needed to reach peak shear strength in the 1.5:1 aspect ratio specimen compared to the 1:1. The linear Coulomb strength envelopes for the 1:1 and 1.5:1 aspect ratios, however, are essentially identical (Figure 2.16b).

The fracturing process leading to rupture zone creation is summarized in Figure 2.17 to Figure 2.19 for the 5, 25, and 90 MPa normal stress magnitudes, respectively. The rupture zone creation images relative to the shear stress versus horizontal displacement curve are indicated in Figure 2.16a. The figures show the same fracturing processes leading to rupture zone geometry, and rupture mechanism dependence on normal stress (Figure 2.20) as found for the 1:1 aspect ratio.
synthetic specimen. Table 5 summarizes the pre-peak shear strength fracture angles and orientation of the major principal stress for the 1:1 and 1.5:1 aspect ratio synthetic specimen to illustrate that they are nearly identical. One noted difference is that the rupture zone is less influenced by the shear box gaps. Aside from this, the rupture creation process and rupture mechanisms occurring in the synthetic specimens are essentially the same.

Figure 2.16 (a) Shear stress versus horizontal displacement and orientation of the major principal stress (measured counter clockwise from horizontal) internal to the synthetic specimen (1.5:1 aspect ratio). (b) Linear Coulomb strength envelopes for specimens with 1:1 and 1.5:1 aspect ratios.
Figure 2.17 5MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement ($\delta_h$) curve. (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. AA indicates rupture mechanism location in Figure 2.20.
Figure 2.18 25MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement ($\delta_h$) curve. (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. BB indicates rupture mechanism location in Figure 2.20.
Figure 2.19 90MPa normal stress rupture zone creation (1.5:1 aspect ratio). See Figure 2.16 for rupture locations along the shear stress versus horizontal displacement ($\delta_h$) curve. (orange – GB, black IG tensile fracture). In the fracture system sketches, black are new and grey are precursory fractures from the previous sketch. CC indicates rupture mechanism location in Figure 2.20.
Figure 2.20 1.5:1 aspect ratio rupture mechanisms. (a) 5MPa showing tensile splitting (see Figure 2.17 for location of AA), (b) 25MPa showing en échelon tensile fracture (see Figure 2.18 for location of BB), and (c) 90MPa showing antithetic and synthetic micro-faults (see Figure 2.19 for location of CC) normal stresses. Larger arrows in (c) show the trends of displacement vectors that are difficult to view. (orange – GB, black IG tensile fracture).
Table 5: 1:1 and 1.5:1 aspect ratio pre-peak fracture angles and major principal stress orientation summary.

<table>
<thead>
<tr>
<th>Normal stress (MPa)</th>
<th>1:1 Aspect Ratio</th>
<th>1.5:1 Aspect Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-peak fracture orientation (°) (average/standard deviation)</td>
<td>( \sigma_1 ) orientation (°) (just prior to peak strength)</td>
</tr>
<tr>
<td>5</td>
<td>15/15</td>
<td>13</td>
</tr>
<tr>
<td>25</td>
<td>22/29</td>
<td>25</td>
</tr>
<tr>
<td>90</td>
<td>36/19</td>
<td>34</td>
</tr>
</tbody>
</table>

2.7.3 Peak shear and internal strength

In Section 2.6.1 it was found for biaxial tests using the calibration parameters summarized in Table 3 that the peak strength envelope fit using the procedure of Hoek and Brown (1997) results in the following Hoek-Brown strength envelope parameters:

- \( m_i = 9 \)
- \( s = 1 \)
- Compressive strength = 158MPa

The internal principal stress-path determined from the measurement circle in the center of the synthetic specimen is shown in Figure 2.21 with the Hoek-Brown strength envelope overlaid as determined from the biaxial simulations using the parameters listed above. The internal principal stress-path peak strength points are almost identical to the envelope determined from the biaxial simulations. The principal stress-paths show that at increasing normal stresses, the path becomes more compressive. The change in internal stress magnitude supports the observed change in rupture mechanism from tensile to dominately shear at higher normal stresses and is one of the dominate reasons for the mechanism change.
Figure 2.21 Internal principal stress-path determined from the measurement circle in the center of the 1:1 aspect ratio synthetic specimen compared to the Hoek-Brown strength envelope determined from the biaxial simulations.

The relationship between the internal principal stress-path and the peak shear strength of the synthetic specimen with aspect ratio of 1:1 was further investigated. Horizontal displacement to reach peak shear strength and the internal Hoek-Brown strength envelope were plotted together in Figure 2.22. Based on Figure 2.22 it is found that for normal stress magnitudes of 5 to 60MPa, peak shear strength (the point when the normal-shear stress-path reaches the linear Coulomb strength envelope) is coincident with the internal principal stress-path reaching the Hoek-Brown strength envelope. At 90MPa normal stress, there is a delay between when the internal strength of the synthetic specimen is reached and when peak shear strength occurs.
Figure 2.22 Horizontal displacement of the lower shear box wall to reached peak shear strength and the internal principal stress-path reaching the Hoek-Brown strength envelope.
2.8 Discussion

2.8.1 Rupture mechanism and geometry dependence on normal stress

As normal stress is increased under constant normal stress boundary conditions, the fracturing process leading to rupture zone creation and the related rupture mechanism changes as follows:

- At low normal stresses (5-15MPa), the synthetic specimen ruptures via a tensile splitting mechanism. The rupture occurs across the entire synthetic specimen at or just after peak shear strength with a small increase in horizontal displacement.

- At moderate normal stresses (25-40MPa), the synthetic specimen ruptures predominantly via a shear mechanism. The rupture occurs progressively, post-peak shear strength. Initially, an en échelon tensile fracture array develops. The array is then progressively connected creating a shear rupture across the synthetic specimen.

- At high normal stresses (60-90MPa), the synthetic specimen ruptures almost entirely via a shear mechanism. The rupture occurs progressively well after the first peak strength is reached. Initially, fractures propagate from the shear box gaps prior to peak shear strength. This is followed by the center of the synthetic specimen fracturing via an array of steep antithetic en échelon micro-faults which link via synthetic (shallow angle) micro-faults which are related to the principal direction of applied displacement. The antithetic and synthetic micro-faults are formed post-peak shear strength.

The simulations show that the rupture mechanism change is a result of the internal orientation of the major principal stress which tends to shallower angles at low and steeper angles at higher normal stresses and the internal principal stress-path which transitions from dominantly tensile to compressive at the scale measured with the internal measurement circle (i.e., 10mm diameter). This is reflected in a clear change in deformation field with increasing vertical deformation restraint (i.e., applied normal stress), resulting in different internal stress rotations, stress states, and rupture modes. The change in fracturing process leading to rupture zone creation and the resulting mechanism generates different rupture zone geometries with respective shear stress versus horizontal displacement (load-displacement) response changing from brittle (loss of shear stress post-peak) to overall ductile (maintained shear stress-post peak). This demonstrates, as expected, that the load-displacement response of a shear rupture zone (one of its characteristics)
is not only a function of the properties of the rock in which the rupture zone is created but its creation process and mechanism leading to a rupture zone geometry. Importantly, from a field observation perspective, rupture zones created in otherwise identical rock will look different depending on the state-of-stress at the time of rupture zone creation. In other words, in mining, where the state-of-stress changes as extraction ratio increases, the fracturing process leading to rupture zone creation will differ; mostly changing from a more distributed to localized and from a ductile to a more brittle behaviour.

The tensile splitting mode of rupture at low normal stresses which transitions to a more progressive, ductile, rupture creation process at higher normal stresses resulting in a shear rupture mechanism at the specimen scale is supported by shear box laboratory experiments conducted by Lajtai (1969) using plaster-of-Paris, Sonnenberg et al. (2003) and Wong et al. (2005) using concrete, and by the rupture change reported by Petit (1988) in specimens of Lodève sandstone.

It should be noted that while the highest normal stress simulations resulted in the propagation of fractures from the shear box gaps, this was also the fracture process reported by Morgenstern and Tchalenko (1967) for kaolin clay deformed in direct shear; the results of which were later used by Tchalenko (1970) as one of the data sets to support the notion that fracture networks and fracturing process in the shear box and Riedel shear experiment (Riedel, 1929; Cloos, 1928) using clay are similar to those generated from earthquakes at the crustal scale.

2.8.2 Kinematic classification of specimen undergoing shear deformation

Rupture zones can be classified by the relative movement of the shear box walls on the basis of kinematics (Fossen, 2011). Shear box deformation is commonly associated with simple shear which occurs when the state-of-stress internal to the specimen being deformed is oriented 45° relative to the rupture zone with the specimen undergoing no volume change. Although, most brittle materials either dilate or compact when ruptured depending on the magnitude of the boundary stresses confining them and are thus not truly simple shear.

Dilatant shear rupture is classified as extensional (pure splitting) to transtensional (meaning dilational sub-simple shear). Compactive shear ruptures are classified as transpressional
(meaning compactional sub-simple shear). The changing kinematics from extension to transpression is evident in the geometries of pull-apart arrays which are controlled by the amount of extension, transtension or transpression that occurs (Peacock and Sanderson, 1995; Ramsey and Huber, 1987, based on McCoss, 1987) as reproduced in Figure 2.23. Extensional pull-apart en échelon arrays are shallow and have a splitting rupture mechanism with the pull-apart arrays becoming steeper as the state-of-stress increases about the horizontal and kinematics transition from transtension to simple shear to transpression, and finally to compaction (contraction).

The simulation results show that deformation of the synthetic specimen in direct shear based on the normal stress magnitudes simulated does not reach simple-shear. The state-of-stress rotates depending on the applied normal stress such that the specimens are subject to extension at low and transtension at moderate to high normal stresses. Upward movement of the upper shear box wall (Wall 2, Figure 2.6) occurs at all normal stress magnitudes simulated indicating dilatant behaviour (see Appendix B.4).
Figure 2.23 Illustration of geometric variance of pull-apart arrays depending on amount of transtension or transpression (from Peacock and Sanderson, 1994). $A$ is the infinitesimal displacement direction (180° plane extension to 0° for plane contraction, i.e., direction of specimen dilatant or compactive movement) and $\omega$ the angle between the vein segments and the zone boundary (approximately the orientation of the major principal stress).
2.9 Conclusions

The understanding of shear rupture zone creation in intact low porosity brittle rock when deformed in direct shear under constant normal stress boundary conditions has been improved. This Chapter reported on the following for various normal stresses:

- The mechanism and process of shear rupture zone creation;
- Shear stress versus horizontal displacement response of the synthetic specimen;
- Shear rupture zone creation relative to the shear stress versus horizontal displacement response of the synthetic specimen;
- The shear rupture mechanisms;
- Ultimate rupture zone geometries; and
- Peak strength envelope.

This Chapter focused on understanding shear rupture zone creation in intact brittle rock deformed in direct shear under constant normal stress boundary conditions. Numerical simulations were completed using a synthetic specimen that was calibrated to the strength (shear and tensile), post-peak shear stress versus horizontal displacement response, and fracture angle characteristics of Lodève sandstone, a brittle low porosity rock, when deformed in direct shear.

It is found that:

- at low normal stresses (5-15MPa), the deformational behaviour of the synthetic specimen can be classified kinematically as extensional. The synthetic specimen ruptures in a predominantly tensile splitting mode; a process that occurs at or just after peak shear strength with a small increment of horizontal displacement.

- At higher normal stresses (25-90MPa), the deformational behaviour of the synthetic specimen is classified kinematically as transtensional. The synthetic specimen ruptures in a shear mode progressively. First, an array of en échelon tensile fractures develops followed by linkage of the array across the synthetic specimen leading to a shear rupture with related damage zone.

Rotation of the internal major principal stress in the synthetic specimen tends to higher angles about the horizontal and the internal stress field becomes increasingly compressive at higher normal stresses. This causes fracturing at higher angles about the horizontal and limits fracture
length with increasing applied normal stresses. The shallower orientations of the major principal stress and a predominately tensile stress field at low normal stresses facilitate splitting while the higher major principal stress orientations and predominately compressive stress field at higher normal stresses do not allow for splitting of the synthetic specimen. As a result, a progressive fracturing process is required to generate a rupture across the synthetic specimen.

Two results of the changing rupture mechanism and process as normal stress magnitudes are increased is: (1) the generation of different rupture geometries; and (2) different post-peak shear stress versus horizontal displacement responses. The primary cause of the change in shear stress versus horizontal displacement response of the synthetic specimen is interpreted to be the result of changes in the rupture zone geometry which is determined by the rupture mechanism. At low normal stresses, the rupture surface created is thin and nearly planar; thus not able to sustain high shear stresses. At higher normal stresses, the rupture surfaces are thicker, more discontinuous, and less planar; thus able to sustain high shear stresses.

Specimens with aspect ratios of 1:1 and 1.5:1 when deformed in direct shear exhibit similar fracturing processes, mechanisms, fracture angles, and linear Coulomb strength envelopes. Thus, an aspect ratio of 1:1 is suitable for investigating intact brittle rock rupture in direct shear.

The practical implications of the findings presented in this chapter are:

- In otherwise identical rock mass conditions, the geometric structure of a rupture zone created in shear differs depending on the normal stress state and the amount of deformation at the time of creation. As a consequence, differences in rupture zone geometry can serve as an indicator of the rupture’s post-peak load-displacement behaviour (e.g., more brittle if near planar).
- Rupture zones created at low normal stress display more post-peak brittle load-displacement behaviour and thus should be more prone to violent failure (sudden slip).
2.10 Summary of Chapter 2

A particle based distinct element method (DEM) and its grain based method (GBM) were used to generate and simulate a synthetic specimen calibrated to the rupture characteristics of an intact (non-jointed) brittle rock deformed in direct shear. The synthetic specimen was then used to investigate shear rupture zone creation under constant normal stress boundary conditions. The fracturing processes leading to shear rupture zone creation and the rupture mechanism were found to be normal stress dependent. The normal stress dependent change was found to be due to the orientation of the major principal stress and local stress concentrations internal to the synthetic specimens being deformed. The normal stress dependent rupture creation process resulted in a change to the rupture zone’s geometry and shear stress versus horizontal displacement response, and thus ultimate strength.

In the next chapter, Chapter 3, shear rupture zone creation in intact low porosity brittle rock when deformed in direct shear under constant normal stiffness boundary conditions is investigated and the results compared to those of constant normal stress reported in this Chapter.
Chapter 3

3 DEM simulation of direct shear: rupture under constant normal stiffness boundary condition

3.1 Introduction

Shear rupture zone characteristics (normal-shear stress-path, peak and residual strength envelopes, rupture zone creation and mechanism, and shear stress versus horizontal displacement response) are investigated in this Chapter for intact (non-jointed) brittle rock deformed in direct shear under constant normal stiffness boundary conditions. Understanding the role of stiffness boundary conditions on the rupture creation process is essential to achieve the goal of this thesis; to improve the understanding of shear rupture zone creation in brittle rock.

The hypothesis presented in this thesis, that shear rupture zone characteristics depend on the prevalent boundary condition (i.e., constant normal stress versus constant normal stiffness) during shear rupture zone creation, is proven in this Chapter by a comparison of the constant normal stiffness simulation test results with those presented in Chapter 2 for constant normal stress. Similar to Chapter 2, the following aspects are investigated:

- fracturing processes and mechanisms leading to shear rupture zone creation;
- shear rupture mechanisms;
- ultimate shear rupture zone geometries;
- strength envelopes (peak and ultimate); and
- shear stress versus horizontal displacement (load-displacement) response,

of an intact brittle rock specimen deformed in direct shear.

The investigation is completed using the commercially available particle based Distinct Element Method (DEM), Particle Flow Code in Two Dimensions (PFC2D v4.00-190) (Itasca, 2011) and its embedded Grain Based Method (GBM) (Potyondy, 2010; Itasca, 2011) which was previously described in Chapter 2. The synthetic specimen, calibrated to the rupture characteristics (peak shear strength envelope, tensile strength, post-peak shear stress versus horizontal displacement response, and fracture angles generated) of Lodève sandstone deformed in direct shear under
constant normal stress boundary conditions reported by Petit (1988) and Wibberley et al. (2000), is used. Calibration was also previously described in Chapter 2 Section 2.6. The sandstone is a brittle low porosity (<2%) rock. By numerically reproducing, in direct shear, the strength, deformation, and fracturing characteristics of a brittle rock, and simulating fracture nucleation, propagation, and evolution in a grain structure that allows both grain boundary and intra-grain fracturing, insight can be gained on the synthetic specimen’s fracturing in relation to the shear stress versus horizontal displacement response and strength under different boundary conditions.

First the DEM simulation set up for investigating normal stiffness boundary conditions is outlined and it is indicated that four cap modulus values are investigated: 1; 10; 30; and 100GPa. Each cap modulus value is simulated with initial applied normal stresses of 5, 25, and 40MPa. The 5 and 40MPa initial applied normal stress results are described in detail and the 25MPa initial applied normal stress results are summarized in Appendix C.3. Force chains developing in the synthetic specimens are then presented and compared to those occurring in the synthetic specimens subjected to constant normal stress boundary conditions. The Chapter is then divided into shear rupture zone creation under 10 to 100GPa and 1GPa cap modulus values which are found to have different rupture zone characteristics. In each division, the shear rupture zone characteristics are overviewed in comparison to the results for shear rupture under constant normal stress boundary conditions which were detailed in Chapter 2. The results provide insight into the mechanics of fracturing under constant normal stiffness boundary conditions as well as stick-slip behaviour initiation. Stick-slip is an instability process which is repetitive where periods of no shear motion are followed by a sudden shear displacement with an associated shear stress drop (Scholz, 2002).

While not reported in detail, for completeness, the orientation of the major principal stress internal to the synthetic specimens deformed under constant normal stiffness boundary conditions are presented in Appendix C.1.

### 3.2 DEM simulation of constant normal stiffness boundary condition

The constant normal stiffness simulation set up is based on the constant normal stress boundary condition simulation detailed in Chapter 2 Section 2.5. The synthetic specimen is created the
same way as in Chapter 2. The constant normal stiffness simulation differs from the constant normal stress simulation as follows (Figure 3.1):

(1) A cap of material is located on top of the synthetic specimen and is composed of a parallel bonded particle assembly. This cap is not breakable. Its deformability is controlled by the assigned modulus (a proxy for stiffness) of the particles and the parallel bonds between them, and the cap’s geometry (50mm length and 40mm height). The cap is not bonded to the synthetic specimen and the contact between the synthetic specimen and cap is frictionless. The following four cap particle and parallel bond modulus values are investigated: 1GPa, 10GPa, 30GPa, and 100GPa.

(2) Normal stress is applied to the synthetic specimen using an applied velocity to the top wall of the cap (Wall 2 Figure 3.1). Initial applied normal stress magnitudes of 5MPa, 25MPa, and 40MPa for each assigned cap modulus are simulated. Once the desired normal stress is achieved (defined as the ‘initial applied normal stress’) in the synthetic specimens, the applied velocity is stopped and the top wall locked. Therefore, normal stress develops during shearing as a function of the cap modulus. That is, as elastic vertical deformation, dilatant fracturing, or shear induced dilation occurs in the synthetic specimen, the movement is resisted by the cap providing feedback normal stress. The cap, therefore, simulates the influence of a material’s deformability, such as a rock or a rock mass as well as deformational controls influenced by a mine (e.g., geometry of excavations), surrounding a shear rupture zone being created. The values simulated do not represent actual values of rock mass modulus or mine stiffness. As in Chapter 2 Section 2.5.1, the shear stiffness is nearly infinite (horizontal displacement is controlled) but here in Chapter 3, the normal stiffness is variable and represented by the cap with modulus values of 1, 10, 30, and 100GPa. Re-call in Chapter 2 that the boundary condition was constant normal stress and thus in that Chapter the synthetic specimen was under a normal boundary condition that was infinitely soft (i.e., no increases in normal stress occurred during dilatant fracturing in the synthetic specimen).

Synthetic specimen monitoring is completed using the same approach adopted in Chapter 2. Shear stress is determined through the reaction forces acting along Wall 4 divided by the synthetic specimen’s length. Normal stress is determined through the reaction forces acting along Wall 2 divided by the synthetic specimen’s length. Horizontal displacement is measured based
on the movement of the lower wall (Wall 1). Principal stress magnitudes and the orientation of
the major principal stress internal to the synthetic specimen are determined using the
measurement circle (10mm diameter) shown in Figure 3.1 in the center of the synthetic specimen
based on the logic adopted by Cho et al. (2008) where each stress component ($\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$) is
monitored for every particle within the measurement circle.

Figure 3.1 Constant normal stiffness boundary condition direct shear simulation schematic.
3.3 Distribution of forces in synthetic specimens

The force chains in the synthetic specimens for initial applied normal stresses of 5, 25, and 40MPa and cap modulus values of 1, 10, 30, and 100GPa are shown in Figure 3.2 to Figure 3.4. The force chain networks in Figure 3.2 to Figure 3.4 show that they develop into the center of the synthetic specimens, have minimal influence by the top and bottom shear box boundaries, concentrate in the lower left and upper right corners of the synthetic specimens, and are inclined at increasing angles about the horizontal at increasing cap modulus values as summarized in Appendix C.2. Overall, the force chains in the constant normal stiffness synthetic specimens are consistent with those in the constant normal stress synthetic specimens reported in Chapter 2 Section 2.7.1. The cap of material added to simulate the constant normal stiffness boundary condition does not influence the force chains developing in the synthetic specimens.
Figure 3.2 Force chains in the synthetic specimens with an initial applied normal stress of 5MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – $\delta_h = 0.1594$mm, (b) 10GPa – $\delta_h = 0.1647$mm, (c) 30GPa – $\delta_h = 0.1538$mm, and (d) 100GPa – $\delta_h = 0.1518$mm cap modulus values.
Figure 3.3 Force chains in the synthetic specimens with an initial applied normal stress of 25MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – $\delta_h = 0.1682\text{mm}$, (b) 10GPa – $\delta_h = 0.1681\text{mm}$, (c) 30GPa – $\delta_h = 0.1681\text{mm}$, and (d) 100GPa – $\delta_h = 0.1716\text{mm}$ cap modulus values.
Figure 3.4 Force chains in the synthetic specimens with an initial applied normal stress of 40MPa. Line in the center indicates overall force chain orientation. (a) 1GPa – $\delta_h = 0.1734$mm, (b) 10GPa – $\delta_h = 0.1679$mm, (c) 30GPa – $\delta_h = 0.1733$mm, and (d) 100GPa – $\delta_h = 0.1715$mm cap modulus values.
3.4 Cap modulus values 10 to 100GPa

3.4.1 Normal-shear stress-path and shear strength envelopes

Under constant normal stress, the normal-shear stress-path is vertical (Figure 3.5) as it progresses to the strength envelope (defined by a linear Coulomb strength envelope of \( \tau = 0.76\sigma_n+27 \), \( R^2 \) 0.99, \( n = 6 \), equation 2, Chapter 2 Section 2.6.1). Once the envelope is reached, the peak shear strength is reached and shear stress is lost with the stress-path progressing to a bi-linear ultimate shear strength. A residual strength is not reached in these simulations for the amount of horizontal displacement applied to the synthetic specimens.

Between 5 and 40MPa, the ultimate shear strength envelope is defined by:

\[ \tau = 1.63\sigma_n \]  \hspace{1cm} (3)

and between 40 and 90MPa by:

\[ \tau = 0.71\sigma_n+22 \]  \hspace{1cm} (4)
Figure 3.5 Constant normal stress boundary condition normal-shear stress-path, and peak linear and ultimate bi-linear Coulomb strength envelopes. Also showing the residual frictional strength envelope determined from the constant normal stiffness simulations ($\phi=28^\circ$).

Under constant normal stiffness, the normal-shear stress-path is coupled (Figure 3.6a-b showing the results for initial applied normal stress magnitudes of 5 and 40MPa, respectively) and depends on cap modulus with higher normal stresses developing more rapidly for higher magnitude cap modulus values.

Each suite of results for the different initial applied normal stresses (i.e., 5, 25, and 40MPa) has its own unique strength envelope.

For an initial applied normal stress of 5MPa (Figure 3.6a):
\[ \tau = 0.91\sigma_n + 29.8, \quad R^2 0.99 \]  

(5)

For an initial applied normal stress of 25MPa (see Appendix C.3 for these results):

\[ \tau = 0.95\sigma_n + 23, \quad R^2 0.99 \]  

(6)

For an initial applied normal stress of 40MPa (Figure 3.6b):

\[ \tau = 0.96\sigma_n + 16, \quad R^2 0.99 \]  

(7)

Notice that the coefficient of friction is increasing and cohesion is decreasing. This will be discussed later.

Once the strength envelope is reached in each suite of results for the different initial applied normal stresses, the stress-path follows the linear Coulomb strength envelope. This is due to the dilation related increase in normal stress during deformation. Shear resistance increases proportional to the normal stress as the stress-path in the normal-shear stress space follows the linear Coulomb envelope. Eventually, the stress-path deviates, i.e., drops below the linear Coulomb envelope when the peak maximum shear strength is reached, and approaches the residual strength defined by a friction angle of 28° (coefficient of friction, \(\mu=0.52\)) (Figure 3.6a-b).

The constant normal stiffness simulations result in peak linear Coulomb strength envelopes with higher friction coefficients and overall, lower cohesive strengths compared to the constant normal stress simulations (i.e., friction coefficients of 0.91, 0.95, and 0.96 rather than 0.76, and cohesive strengths of 29.8, 23, and 16MPa compared to 27MPa). The constant normal stiffness friction coefficients correspond to friction plus dilation angles of 42.3°, 43.5°, and 43.8° rather than 37° for the constant normal stress simulations).

The peak shear strength for a given normal stress cannot be predicted by the linear Coulomb strength envelope under constant normal stiffness boundary conditions because the stress-path is dependent on the normal stiffness as illustrated by Figure 3.6. Under this boundary condition, the linear Coulomb strength envelope represents a yield point when the stress-path reaches it. For the simulations in this thesis, it was found that the point where the stress-path reaches failure (peak
maximum strength) can be predicted by a constant horizontal displacement criterion of approximately 0.35mm as shown in Figure 3.7.

The ultimate shear strength envelope from the constant normal stiffness simulations is lower, approaching a residual friction angle of $28^\circ$. In other words, much more deformation would be required to reach the residual strength in the constant normal stress simulations; only the post-peak ultimate strength was obtained.

In summary, the effect of the constant normal stiffness boundary condition is to resist a rock’s desire to dilate with a resultant increase in coupled normal stress. A second effect of this resistance is that the dilation characteristics of the rock during the creation of the rupture zone is changed, as will be explored later, and as a consequence, additional strength can be mobilized (resulting in a higher peak maximum strength).
Figure 3.6 Constant normal stiffness boundary condition normal-shear stress-path, and peak and residual linear Coulomb strength envelopes. (a) Initial applied normal stress of 5MPa. (b) Initial applied normal stress of 40MPa.
Figure 3.7 Horizontal displacement criterion for peak maximum shear strength in the constant normal stiffness simulations.

3.4.2 Rupture zone creation

PFC2D GBM rupture zone images were captured at selected horizontal displacements and used to describe the fracturing process leading to rupture zone creation for the 5MPa (Figure 3.8) and 40MPa (Figure 3.9) initial applied normal stresses. Particle displacement vectors, as per Chapter 2, were used to determine rupture mechanism. The images for the 25MPa initial applied normal stress simulations are summarized in Appendix C.3.

The fracturing processes leading to rupture zone creation occurs consistently in four stages (I to IV) and are independent of the initial applied normal stress for all simulations with cap modulus values between 10 and 100GPa. The stages leading to rupture zone creation are:

- **Stage I**, characterized by the occurrence of grain boundary tensile fractures (orange fractures in Figure 3.8a, g, m and Figure 3.9a, g, m) which are oriented in the direction of the internal major principal stress (Table 6, summary of fracture system angles at Stage I
for a cap modulus of 30GPa and initial applied normal stresses of 5, 25, and 40MPa; see Appendix C.4 for the rupture zone images, fracture sketches, and rose diagrams).

- **Stage II**, characterized by the development of an array of en échelon tensile fractures (composed of both grain boundary and intra-grain tensile fractures, orange and black fractures respectively, Figure 3.8b, h, n; Figure 3.9b, h, n). Figure 3.10a shows the displacement vectors indicating predominant opening along the en échelon fractures which then begin to grow fractures from their tips forming ‘sigmoidal’ shaped arrays (Figure 3.8c, i, o and Figure 3.9c, i, o, *Stage IIa*). The initially tensile en échelon fractures have changing kinematics as stresses rotate (i.e., tensile opening to shear) with tip fractures indicating that they are of opening mode (Figure 3.10b). From *Stage II* to *Stage III*, the en échelon array becomes progressively more connected (Figure 3.8d, j, p and Figure 3.9d, j, p).

- **Stage III**, characterized by peak maximum shear strength and the immersgence of a connected and irregular rupture zone (Figure 3.8e, k, q and Figure 3.9e, k, q) with both the top and bottom of the rupture surface indicating opposite sense of shear (Figure 3.10c). From *Stage III* to *IV*, the non-continuous rupture zone evolves and becomes less irregular and progresses into an almost fully continuous rupture surface across the synthetic specimen dominated by intra-granular tensile fractures (black fractures in Figure 3.8f, l, r and Figure 3.9f, l, r).

- **Stage IV**, characterized by a continuous rupture surface across the synthetic specimen with fractures having increasing angles at increasing cap modulus values (see Figure 3.8f, l, r and Figure 3.9f, l, r labeled increasing fracture angles).
Figure 3.8 Rupture zone creation images, initial applied normal stress of 5MPa at selected horizontal displacement magnitudes ($\delta_h$) for cap modulus values of: (a)-(f) 10GPa; (g)-(l) 30GPa; and (m)-(r) 100GPa (orange – grain boundary, black – intra-grain tensile fracture).
<table>
<thead>
<tr>
<th>Cap Modulus</th>
<th>Cap Modulus</th>
<th>Cap Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>10GPa</td>
<td>30GPa</td>
<td>100GPa</td>
</tr>
<tr>
<td>a)</td>
<td>g)</td>
<td>m)</td>
</tr>
<tr>
<td>$\delta_h = 0.15\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage I, GB fracturing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>h)</td>
<td>n)</td>
</tr>
<tr>
<td>$\delta_h = 0.20\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage II, en échelon fracture array</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>i)</td>
<td>o)</td>
</tr>
<tr>
<td>$\delta_h = 0.25\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage IIa, sigmoidal development</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>j)</td>
<td>p)</td>
</tr>
<tr>
<td>$\delta_h = 0.30\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Progressive linkage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>k)</td>
<td>q)</td>
</tr>
<tr>
<td>$\delta_h = 0.35\text{mm}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stage III, peak shear strength</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td>l)</td>
<td>r)</td>
</tr>
<tr>
<td>$\delta_h = 0.50\text{mm}$</td>
<td></td>
<td>Progressive smoothing, has not reached Stage IV</td>
</tr>
<tr>
<td>Increasing angle of fractures</td>
<td>$\delta_h = 0.36\text{mm}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.9 Rupture zone creation images, initial applied normal stress of 40MPa at selected horizontal displacement magnitudes ($\delta_h$) for cap modulus values of: (a)-(f) 10GPa; (g)-(l) 30GPa; and (m)-(r) 100GPa (orange – grain boundary, black – intra-grain tensile fracture).
Figure 3.10 Rupture zone images for initial applied normal stress of 5MPa: (a) *Stage II* en échelon tension fractures showing opening mode in (b). (c) *Stage IIa* fractures propagating from en échelon fracture tips showing shear along original en échelon tension fractures in (d) and tip fractures opening in (e). (f) *Stage III*, peak shear strength showing shear along the rupture zone in (g) and (h) (orange – grain boundary, black – intra-grain tensile fracture).
### Table 6 Fracture angles for a cap modulus of 30GPa

<table>
<thead>
<tr>
<th>Initial applied normal stress (MPa)</th>
<th>Stage I fracture orientation (°) (average/standard deviation)</th>
<th>σ₁ orientation just prior to yield point (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>26/12</td>
<td>26</td>
</tr>
<tr>
<td>25</td>
<td>32/18</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>32/36</td>
<td>33</td>
</tr>
</tbody>
</table>

#### 3.4.3 Shear stress versus horizontal displacement response

Under constant normal stiffness (Figure 3.11a, initial applied normal stress of 5MPa, and Figure 3.11b, initial applied normal stress of 40MPa), the shear stress versus horizontal displacement response of the synthetic specimen is initially elastic (up to approximately 0.17mm to 0.18mm of horizontal displacement) and then becomes inelastic (suggestive of strain hardening) with shear stress oscillations prior to reaching the peak maximum shear strength (Figure 3.11a-b as labeled) for cap modulus values of 10 and 30GPa. The post-peak response is always brittle with a ‘stepped’ or ‘staircase’ character. Under constant normal stress, the shear stress versus horizontal displacement response progresses from an idealized elastic-perfectly-brittle to an idealized elastic-perfectly-plastic response as the applied normal stress is increased from low (5MPa) to high (90MPa), respectively (Figure 2.7b showing the plot of shear stress versus horizontal displacement for constant normal stress boundary conditions and the change in shear stress versus horizontal displacement response schematically represented).
Figure 3.11 Shear stress versus horizontal displacement response for (a) 5MPa and (b) 40MPa initial applied normal stresses.
3.4.4 Rupture zone creation stages linked to shear stress versus horizontal displacement response and stress-path

The rupture zone creation stages outlined in Section 3.4.2 are described with reference to the shear stress versus horizontal displacement curves, normal-shear stress-path, and the major-minor principal stress-path that was tracked internal to the synthetic specimen using the measurement circle described in Section 3.2. The following references Figure 3.12 and Figure 3.13 for initial applied normal stress magnitudes of 5 and 40MPa, respectively. These two figures show labeled shear stress-versus horizontal displacement curves (Figure 3.12a and Figure 3.13a), normal-shear stress-paths (Figure 3.12b and Figure 3.13b), the development of minor principal stress internal to the synthetic specimen versus horizontal displacement (Figure 3.12c and Figure 3.13c), and the major-minor internal principal stress-path (Figure 3.12d and Figure 3.13d). Only Figure 3.12c-d and Figure 3.13c-d have not been previously presented. The linked shear stress versus horizontal displacement and stress-path compilation figure for the 25MPa initial applied normal stress simulations is summarized in Appendix C.3.

Stage I, grain boundary tensile fracturing (Figure 3.8a, g, m and Figure 3.9a, g, m) is characterized by relatively linear-elastic shear stress versus horizontal displacement response (Figure 3.12a and Figure 3.13a) occurring up to a horizontal displacement of approximately 0.17mm to 0.18mm. Stage II, development of an array of en échelon tensile fractures (Figure 3.8b, h, n and Figure 3.9b, h, n) is characterized by inelastic shear stress versus horizontal displacement response (Figure 3.12a and Figure 3.13a) and occurs concurrent with when the internal tensile strength of the synthetic specimen is reached (Figure 3.12d and Figure 3.13d, which show the developed principal stress-path internal to the synthetic specimen and it nearing the Hoek-Brown strength criterion, Hoek and Brown, 1997, for the synthetic specimen in the tensile region of the minor principal stress axis). Recall that the Hoek-Brown envelope was determined with biaxial simulations and discussed in Chapter 2 Section 2.7.3. Peak tensile stress development relative to horizontal displacement is shown in Figure 3.12c and Figure 3.13c as labeled. The internal tensile strength of the material being reach is also near or concurrent to when the linear Coulomb strength envelope is reached (yield point) in normal-shear stress space (Figure 3.12b and Figure 3.13b, which has horizontal arrows indicating the shear stress magnitude for when the linear Coulomb strength envelope is reached that can be linked to the horizontal arrows in Figure 3.12a and Figure 3.13a on the shear stress versus horizontal
displacement curves). From Stage II to Stage III (Figure 3.8 and Figure 3.9, 0.17-0.18mm to 0.35mm of horizontal displacement), the normal-shear stress-path follows the linear Coulomb strength envelope (Figure 3.12b and Figure 3.13b), with the en échelon array becoming progressively more connected. During this period, a rupture zone is created pre-peak maximum shear strength with continued inelastic deformation and increasing shear stress magnitudes that have shear stress oscillatory behaviour (Figure 3.12a and Figure 3.13a as labeled for the 10 and 30GPa cap modulus values). Stage III (Figure 3.8e, k, q and Figure 3.9e, k, q) is characterized by peak maximum shear strength and the immergence of a connected and irregular rupture zone (created pre-peak maximum shear strength) (Figure 3.12a and Figure 3.13a). From Stage III to IV (Figure 3.8f, l, r and Figure 3.9f, l, r) the non-continuous rupture zone evolves and becomes less irregular and discontinuous with an almost fully continuous rupture surface being created (Figure 3.8f, l, r and Figure 3.9f, l, r) with a strength state changing from one of cohesion \(c\) and friction \(\phi\) to one based mostly on a frictional strength of 28° (Figure 3.12b and Figure 3.13b).
Figure 3.12 Linked mechanical response of the synthetic specimen with initial applied normal stress of 5MPa. (a) Shear stress versus horizontal displacement response. (b) Normal-shear stress-path. (c) Development of minor principal stress with horizontal displacement. (d) Principal stress-path internal to the synthetic specimen.
Figure 3.13 Linked mechanical response of the synthetic specimen with initial applied normal stress of 40MPa. (a) Shear stress versus horizontal displacement response. (b) Normal-shear stress-path. (c) Development of minor principal stress with horizontal displacement. (d) Principal stress-path internal to the synthetic specimen.
3.4.5 Shear stress oscillatory behaviour

A close up view of the shear stress versus horizontal displacement curve for the simulation with a cap modulus of 30GPa and an initial applied normal stress of 25MPa is shown in Figure 3.14a alongside the PFC2D GBM rupture images (orange – grain boundary, black – intra-grain tensile fracture) at selected horizontal displacements (Figure 3.14c-f, where the locations of the images are referenced along the shear stress versus horizontal displacement curve using arrows in Figure 3.14a). The grain boundary and intra-grain fracturing occurring in this range of horizontal displacement are presented in Figure 3.14b as fracture rates and cumulative fracture counts separately for grain boundary and intra-grain fractures as well as combined (total). The shear stress versus horizontal displacement curve (Figure 3.14a) shows three shear stress oscillations with relatively instantaneous shear stress drop as indicated in Figure 3.14a (I, II, and III). The fracturing events in these three zones show increasing fracture rates from the peak shear stress of the oscillation to the post-peak points of the shear stress oscillations with both grain boundary and intra-grain fracturing occurring simultaneously (as indicated in Figure 3.14b). The cumulative fracture count curves show steps at each of the oscillations. Locations of some of the newly created fractures are indicated by the grey circled areas of the rupture zone images (Figure 3.14c-f) and indicate (along with the stepped cumulative fracture count curves) a progressive evolution of a discontinuous rupture zone or network towards a more connected and eventually continuous state with fracturing occurring in the less fractured areas. During the shear stress ‘re-charging’ phases (increasing shear stress magnitudes in the oscillations), less fracturing occurs. In some instances there are periods without fracturing during this strengthening as indicated by the term ‘quiescence’ in Figure 3.14b. Quiescence is used here to describe a period of horizontal displacement without fracturing.
Figure 3.14 Assessment of shear stress oscillations in the shear stress (τ) versus horizontal displacement (δ_h) curve for 25MPa initial applied normal stress and 30GPa cap modulus. (a) Shear stress versus horizontal displacement curve. (b) Fracture rates and cumulative fracture counts for grain boundary and intra-grain fractures. (c)-(f) Rupture zone images at indicated horizontal displacements using ‘arrows’ in (a) (orange – grain boundary, black – intra-grain tensile fracture).
3.5 1GPa cap modulus

3.5.1 Normal-shear stress-path and shear strength envelopes

For the 1GPa cap modulus simulations, peak shear strength is defined by a linear Coulomb strength envelope of (Figure 3.15a):

\[ \tau = 0.64\sigma_n + 34 \]  

Equation 8 is close to that defined for the constant normal stress simulations (equation 2, Chapter 2 Section 2.6.1).

Prior to peak shear strength, for the 25 and 40MPa initial applied normal stresses, there is an initial yield threshold that is defined by a constant horizontal displacement criterion (Figure 3.15a-b). When a horizontal displacement of approximately 0.18mm is reached, there is an initial shear stress drop (yield) followed by increased shear stress with continued horizontal displacement. For the 5MPa initial applied normal stress, once the Coulomb strength envelope is reached, failure occurs. The circles in Figure 3.15a indicate when the 25 and 40MPa initial applied normal stress normal-shear stress-paths near the horizontal displacement criterion indicated in Figure 3.15b and have shear stress drops which are more clearly shown in Figure 3.15c on the shear stress versus horizontal displacement curves (labeled). The stress drops are followed by shear stress increases until the linear Coulomb strength envelope for the 1GPa cap modulus constant normal stiffness simulation is reached (Figure 3.15a) at which point the envelope is briefly followed for a period of horizontal displacement prior to the loss of shear strength. The normal-shear stress-path then tends towards a strength envelope defined by a friction angle of 28°.

It follows that yield initiation and thus rupture process initiation in low stiffness environments is similar to that described in the constant normal stress environment. Thus, little difference should be expected when rupture occurs at low stiffness environments or constant stress environments because yield initiation and progression over peak occurs at a comparable strength threshold and there are relatively small increases in normal stress during fracture creation in the synthetic specimens at low normal stiffnesses.
Figure 3.15 1GPa cap modulus simulation results for 5, 25, and 40MPa initial applied normal stresses. (a) Normal-shear stress-path. (b) Shear stress versus horizontal displacement response.
3.5.2 Rupture zone creation

For the 1GP cap modulus simulations, the fracturing processes leading to rupture zone creation for increasing initial applied normal stress magnitudes (i.e., 5MPa to 40MPa, Figure 3.16) transition from one which occurs at a small increment of horizontal displacement across the synthetic specimen at a horizontal displacement magnitude of approximately 0.17mm (Figure 3.16b, 5MPa initial applied normal stress) to one which initiates at the right shear box gap and propagates progressively across the synthetic specimen to the left (Figure 3.16g-l and Figure 3.16m-r, 25 and 40MPa initial applied normal stress magnitudes, respectively) with continued horizontal displacement of the lower shear box wall.

The rupture zone progressively develops right to left in the synthetic specimen at larger horizontal displacement magnitudes for increasing initial applied normal stresses (compare Figure 3.16j and Figure 3.16p for 25 and 40MPa initial applied normal stress magnitudes which are for the same horizontal displacement of 0.30mm but show different rupture lengths across the synthetic specimen). This is discussed in more detail later.
Figure 3.16 1GPa cap modulus rupture zone creation at selected horizontal displacement ($\delta_h$) magnitudes for initial applied normal stresses of: (a-f) 5MPa; (g-l) 25MPa; and (m-r) 40MPa (orange – grain boundary, black – intra-grain tensile fracture).
3.6 Discussion

3.6.1 Rupture mechanics under constant normal stiffness (10 to 100 GPa cap modulus)

The mechanical response of brittle rock deformed in direct shear under the boundary condition of constant normal stiffness is different from that of constant normal stress. Once the linear Coulomb strength envelope is reached, unlike the condition for constant normal stress, the peak shear strength is not yet achieved but a yield point is reached followed by a strain or displacement hardening effect. The strength envelope is followed by the normal-shear stress-path as deformation accumulates. During this deformation period, a discontinuous rupture zone is created in the synthetic specimens prior to reaching the peak maximum shear strength. The linear Coulomb strength envelope therefore does not represent a conventional peak strength threshold under constant normal stiffness boundary conditions. It describes when yield initiates and then represents a fracturing threshold with a horizontal displacement threshold controlling the final or peak maximum shear strength. Therefore, the coupling between the normal and shear stress is an important mechanism during rupture zone creation under constant normal stiffness boundary conditions.

One of the critical differences between the boundary conditions is the creation of a post-peak strength rupture zone under constant normal stress and a pre-peak maximum strength rupture zone under constant normal stiffness. The pre-peak rupture zone creation in the constant normal stiffness simulations is generated as a result of the dilatant fracturing and shearing in the synthetic specimens interacting with the cap material of assigned modulus. As dilation occurs in the synthetic specimen, the opening is resisted by the cap material increasing normal stress magnitudes. Thus, an increase in shear stress is possible to be sustained by the synthetic specimen, i.e., an increase in normal stress results in an increase in shear strength as per the linear Coulomb strength criterion:

$$\tau = \sigma_n \tan \phi + c$$

(9)

However, the simulations also show that the stiffness constraint leads to an increase in the frictional term ($\phi$) by adding a dilation component ($i$) such that equation 9 can be re-written in the form of equation 10 or equation 11:
\[
\tau = \sigma_n \tan(\phi + i) + c \tag{10}
\]

or as

\[
\tau = \sigma_n \tan(\phi) + c_{(\text{function of } \sigma_n)} \tag{11}
\]

Equation 10 suggests that there is a dilation term independent of the normal stiffness and represents conventional thinking, where as equation 11 suggests that the friction term is unaffected but that the cohesion term is affected by the stiffness related change in stress-path. This latter interpretation is new and was tested by determining the apparent cohesion intercepts along the stress-paths of the constant normal stiffness simulations by back projecting a line represented by a friction angle of 28° (i.e., the residual strength of the constant normal stiffness simulations representing the frictional strength of the synthetic material). This is illustrated by Figure 3.17 which shows the apparent cohesion for each constant normal stiffness simulation determined from the back projections. The rate of increase in apparent cohesion in Figure 3.17 begins to level off at higher cap stiffness magnitudes showing that for the simulations presented in this thesis, an infinitely increasing apparent cohesion is not possible. Theoretically, apparent cohesion should level off to a constant apparent cohesion magnitude at some point.

![Figure 3.17 Apparent cohesion intercept for constant normal stiffness simulations with initial applied normal stresses of 5, 25, and 40MPa.](image)
The post-peak rupture zone creation under constant normal stress can now also be better understood. It occurs because dilatant fracturing in the synthetic specimens does not generate increases in normal stress and therefore, no increases in shear strength during rupture zone creation. Thus, shear strength is lost during rupture zone creation. However, in the constant normal stiffness simulations, past the yield point (when the stress-path reaches the Coulomb strength envelope), dilation leads to strain hardening with a strain dependent cohesion intercept leading to a steepening of the strength envelope. This steepening is evident from the apparent increase in friction or friction plus dilation. Increasing dilation with increasing normal stress is however contrary to common understanding in that materials are less dilatant at higher confinement or normal stress magnitudes. Thus, it is concluded that the strain hardening effect is due to increasing cohesion (as shown in Figure 3.17) as previously discussed above. This can be understood as a process whereby more asperities, rock bridges, and intact rock fractures (intragrain fractures in the simulations) have to occur to cause failure. This cohesion loss process is explored further in the next section.

3.6.2 Rupture connectivity, smoothing, and stick-slip behaviour

Stick-slip behaviour of a surface in brittle rock with a sliding contact (under dry and room temperature conditions) results from the abrupt brittle fracture of locked asperities (Byerlee, 1970). Asperity influence on stick-slip behaviour is also evident from the descriptions of rupture surfaces undergoing stick-slip behaviour in Brace and Byerlee (1966) which generated a thin powder of material and thus must have involved fracture of asperities. According to Byerlee (1970), the force needed to overcome and fracture the asperities holding the system in a stable state is the shear force. If sliding occurs in this way along a brittle surface then the shear force will increase when the surfaces become locked and decrease when fractured (Byerlee, 1970). These initial findings of asperity control on stick-slip initiation have been corroborated by the more recent works of Lei et al. (2003) and Thompson et al. (2009) where it was found that stick-slip events in a fractured specimen loaded in triaxial compression initiated from geometric heterogeneities (asperities). In all cases, stick-slip instability was preceded by fracture of intact material in locations of geometric heterogeneity with the stick-slip event occurring because the loading system was not capable to respond fast enough to the rapid fracturing and resulting displacements. In the case of Thompson et al. (2009), the initiation points for the stick-slip were in the ‘locked’ regions along the rupture surface with minimal micro-fracture damage.
While stick-slip behaviour related to energy release depends on the loading system stiffness (i.e., overall system response, Scholz, 2002) the initiation of the stick-slip instability based on the above described occurs due to a cohesion loss process.

The shear oscillatory character of the shear stress versus horizontal displacement curve under constant normal stiffness boundary conditions described in Section 3.4.5 is the result of local fracturing along the discontinuous rupture zone. The fracturing events appear similar to those prior to stick-slip instability, e.g., as in the cases of Byerlee (1970) and Thompson et al. (2009). The simulation results show that the oscillatory shear stress drops under constant normal stiffness boundary conditions, which create the ability for shear stress ‘re-charging’ during rupture creation, are directly related to the rupture-smoothing process; that is, the transition of a discontinuous ‘locked’ cohesional rupture zone into a less geometrically heterogeneous, continuous zone with eventually non-cohesive frictional behaviour. The results are in general agreement with other studies (e.g., King, 1986; King and Nabelek, 1985) where discontinuous rupture zones are recognized as a primary factor for controlling the initiation and termination of seismic events which highlights the cohesional nature of rupture ‘locking’ and the control of cohesion loss on the initiation of stick-slip behaviour.

A practical implication of the cohesion loss process and associated shear stress oscillations is that the strain hardening effect outlined in the previous section makes rupture zones stronger with increasing deformation. Therefore, the rupture zones have the potential to store more energy which is released as higher magnitude events. This interpretation is relevant for stick-slip behaviour because cohesion loss is not a gradual process but sudden leading to the shear stress oscillations each time a ‘locked’ area is starting to fail. Normal stiffness, therefore, allows asperities to ‘lock’ creating the potential for energy release.

### 3.6.3 Ultimate and residual strength envelopes under constant normal stress and stiffness

Under constant normal stress, the post-peak strength is defined by an ultimate strength envelope with bi-linear shape. The shape is similar to the one reported by Patton (1966) for discontinuities with surfaces composed of saw tooth asperities. Patton found that once normal stress magnitudes were sufficient to suppress shear ride over the saw tooth asperities, the ultimate Coulomb strength envelope changed from one characterized by a friction angle ($\phi$) plus dilation
component \((i, \text{ equal to the saw tooth asperity angle})\) to one based on an apparent cohesion \((c')\) and friction angle \((\phi)\). This model was developed from constant normal stress shear tests and thus is validated by the simulations presented here.

Under constant normal stiffness, it is possible to accumulate much more damage (fracturing) after the yield point until the peak maximum strength is reached. Hence, the rupture zone is much more strained than in the constant normal stress simulations and therefore closer to the deformation needed to reach the residual strength. As a consequence, once the peak maximum strength is reached, there is a rapid and more severe strength drop such that the post-peak strength envelope is defined by a residual friction angle of approximately 28°. Therefore, two of the primary reasons for the ultimate versus residual post-peak strength state are: (1) the amount of horizontal displacement achieved in the numerical simulations which is greater in the normal stiffness compared to normal stress boundary condition; and (2) the stress-path which results in the synthetic specimen fracturing predominantly pre-peak maximum strength under constant normal stiffness compared to post-peak under constant normal stress boundary conditions.

An additional reason for the bi-linear ultimate strength state under constant normal stress boundary conditions is the change in rupture zone creation mechanism leading to the different rupture zone geometries post-peak which are not evident in the constant normal stiffness simulation results. Under constant normal stress boundary conditions, at 5MPa and 15MPa normal stress magnitudes, the rupture zones are created due to tensile splitting of the synthetic specimen. The rupture zones are thin and relatively planar (Figure 3.18a-b). Starting at 25MPa normal stress, rupture zones are created progressively via an en échelon fracturing process. As a result, the rupture zones are thicker, more irregular, and discontinuous (Figure 3.18c-e). The thin planar rupture zones at 5 and 15MPa normal stresses generate conditions that facilitate shear displacement with a mechanism that is dilatant thus friction angle \((\phi)\) plus dilation component \((i)\) behaviour. The irregular rupture zones generated at higher normal stress magnitudes \((\geq 25\text{MPa})\) act like the saw tooth asperities in Patton’s discontinuities increasing the apparent cohesive strength component of the synthetic specimen.
Figure 3.18 Changing post-peak ultimate rupture zone geometries for various normal stress magnitudes under constant normal stress boundary conditions. (a) 5MPa. (b) 15MPa. (c) 25MPa. (d) 40MPa. (e) 90MPa. Showing increasing rupture zone geometry irregularity with increasing normal stress magnitude.
3.6.4 Rupture zone creation at 1GPa cap stiffness

Rupture is initiated from the right shear box gap in the simulations with 1GPa cap modulus. The rupture mode is tensile (Figure 3.19b). The non-symmetric rupture zone creation is due to rotation of the shear box caused by the low modulus cap material (Figure 3.19a). The rotation in the simulations occurred even though the shear box walls were not allowed to rotate. This rotation is also evident in the plane-strain finite element simulations completed by Kutter (1971) where it caused normal stresses to be lower towards one side of the numerical simulation enhancing tensile fracture initiation at that location. Rupture propagation occurs at increasing horizontal displacement magnitudes because of the higher normal stresses initially applied and then generated in the simulations which suppresses instantaneous tensile fracture propagation and the ability of the synthetic specimen to dilate.

Figure 3.19 Rotation (a) and mechanism (b) of rupture in simulations with 1GPa cap modulus. Arrows in (a) indicate displacement vector trends that cannot be viewed. (b) Close up view of inset box in (a) showing displacement vectors indicating opening of the tensile fracture or rupture.
3.7 Conclusions

Constant normal stiffness direct shear DEM simulations were conducted using synthetic specimens calibrated to the rupture characteristics of a brittle low porosity sandstone deformed in direct shear under constant normal stress boundary conditions. The results of the simulations were compared to those for constant normal stress. It is found that shear rupture zone characteristics in the synthetic specimens are different when subjected to constant normal stiffness and normal stress boundary conditions. The two boundary conditions generate different fracturing processes leading to rupture zone creation and thus different ultimate rupture zone geometries as well as different shear stress versus horizontal displacement responses due to the coupling between the normal and shear stresses (stress-path) under constant normal stiffness boundary conditions.

The hypothesis proposed in this thesis in Chapter 1 (i.e., shear rupture characteristics depend on the prevalent boundary condition [constant normal stress versus constant normal stiffness] during shear rupture zone creation) is proven by the simulation results. The understanding of shear rupture in intact brittle rock under constant normal stiffness boundary conditions has been enhanced.

Under constant normal stiffness boundary conditions:

- pre-maximum shear strength (pre-peak), a discontinuous rupture zone forms;
- the shear stress versus horizontal displacement response is always brittle (i.e., has near instantaneous large shear stress drops) during the transition from peak maximum strength to a linear residual frictional shear strength;
- specimen rupture occurs via a shear mechanism with rupture zone creation occurring in a consistent process regardless of initial applied normal stress magnitude;
- shear stress oscillatory behaviour is evident in the shear stress versus horizontal displacement curves indicative of the stress drops prior to stick-slip instability;
- the shear stress oscillatory behaviour is caused by cohesion loss processes (creation of new fractures which cause the cumulative fracture curves to become stepped);
- the strain hardening effect of the boundary condition leads to an increase in apparent cohesion opposed to friction plus dilation angle;
• the linear Coulomb strength envelope is not a conventional failure criterion, it is a threshold at which rupture creation (yield) is initiated;
• the yield threshold (when the stress-path reaches the Coulomb strength envelope) is defined by a constant horizontal displacement threshold of 0.17 to 0.18mm; and
• peak maximum shear strength is defined by a constant horizontal displacement threshold of approximately 0.35mm.
3.8 Summary of Chapter 3

A grain based distinct element method was used to simulate the fracturing processes leading to shear rupture zone creation in an intact (non-jointed) brittle rock specimen deformed in direct shear under constant normal stiffness boundary conditions. The investigation revealed that constant normal stiffness boundary conditions generate different fracturing processes leading to shear rupture, ultimate rupture zone geometries, ultimate strength, shear stress versus horizontal displacement (load-displacement) response, and rupture zone creation relative to the load-displacement response compared to the constant normal stress boundary condition. Under constant normal stiffness boundary conditions, rupture zone creation relative to the load-displacement curve occurred pre-peak shear strength. This is found to be the result of a feedback normal stress generation process which couples increases in normal stress, due to the desire of the rupture zone to dilate with related increases in shear stress, producing a complex normal-shear stress-path that reaches and then follows the strength envelope. Shear stress oscillations in the load-displacement curves, suggestive of the initiation of stick-slip instability, occurred during fracture formation while the normal-shear stress-path followed the strength envelope. The simulation results improved the understanding of the fracturing processes, shear stress versus horizontal displacement response, and initiation of stick-slip behaviour of shear ruptures that are being created surrounded by rock where constant normal stiffness conditions prevail over constant normal stress.

In Chapter 4, the understanding gained from the simulation results presented and discussed in both Chapters 2 and 3 is used to aid in the re-interpretation of two mine pillar case histories within the framework of boundary condition influence on rupture zone creation and behaviour (i.e., micro-seismic and deformation). Having proved the hypothesis proposed in this thesis based on the calibrated numerical simulations in Chapter 2 and 3, the goal of the re-interpretation in Chapter 4 is to show the value and practical application of boundary condition influence on rupture zone creation and behaviour (i.e., not to simulate the rupture of the pillars).
Chapter 4

4 Fracturing process resulting in shear rupture of two mine pillars

4.1 Introduction

The investigations presented in Chapters 2 and 3 focused on numerical simulations using synthetic specimens subjected to direct shear deformation inducing the creation of shear rupture zones at the laboratory scale under constant normal stress and normal stiffness boundary conditions. In these Chapters, it was found that the boundary conditions influenced the shear rupture zone characteristics (rupture zone creation relative to the shear stress versus horizontal displacement response, rupture mechanism, synthetic specimen shear stress versus horizontal displacement response, peak and ultimate/residual strength, and ultimate rupture zone geometry).

In this Chapter, the failure process of two mine pillars previously studied by Coulson (2009) are re-interpreted within the framework of boundary condition effects on shear rupture zone creation and behaviour (micro-seismic and deformation). The understanding gained from the previous investigations in Chapters 2 and 3 is used for the re-interpretation of these larger field scale cases for the purpose of showing value and practical application. Hence, the findings presented in this chapter are to some extent speculative and semi-quantitative. However, as will be illustrated, there is ample evidence to show that the differences in the behaviour of the two cases can be attributed to differences in stress and stiffness boundary conditions.

The first pillar, *Case 1*, is of trapezoidal shape, is isolated from active mining, and located in the Golden Giant mine at a depth below surface of approximately 720m. This pillar fractured by the creation of two pairs of conjugate rupture zones which developed over the period of a few months (Coulson, 2009). The rupture process is reflected in micro-seismic patterns. No large magnitude seismic events or rockbursts were generated during fracturing and rupture (i.e., all events were of Nuttli magnitude<sup>5</sup>\( Mn < 0 \)). The second pillar, *Case 2*, is a footwall sill pillar at

---

<sup>5</sup> Nuttli magnitude as recorded by the Geologic Survey of Canada.
the Williams mine located at a depth below surface of approximately 915m. This pillar fractures by the creation of a single rupture zone over a four year time period (Coulson, 2009). The fracturing leading to the rupture of the pillar was associated with micro-seismicity but several seismic events (magnitude range from Mn = 0.5 to 3.5) and a rockburst (Mn = 2.7) were generated by or associated with the rupture process.

Both pillars are hypothesized (as by Coulson, 2009) to fail by fracturing processes leading to shear rupture zone creation. They are observed to have different behaviour and fracture process characteristics. Here it is additionally hypothesized that the first pillar (Case 1) ruptures post-peak strength under constant stress boundary conditions while the second pillar (Case 2), has a rupture process that initiates and develops pre-peak strength with a fully connected rupture surface forming rapidly in the post-peak region under a stiffness boundary condition changing to stress. Evidence supporting these hypotheses and re-interpretations are presented in this Chapter thus showing value and practical application of the understanding gained from Chapters 2 and 3.

### 4.2 Overview of site conditions

The Williams, Golden Giant, and David Bell mines form the Hemlo mining camp located 1000km north of Toronto, Ontario, Canada near the shoreline of Lake Superior (Figure 4.1). All three mines are extracting the same north-north-east (20° clockwise from geographic North) moderate to steeply dipping (60° to 75°), 3 to 40m thick, tabular, gold deposit primarily using sub-level stoping with backfill (this mining method is summarized in Appendix D.1). Historically, cemented rockfill and sandfill were used in the primary and secondary stopes, respectively. All mines are now using cemented paste backfill. The original daily production rates from the three mines were approximately 6000, 2500, and 1500 tonnes per day, respectively. The local mine coordinate system is oriented approximately 20° clockwise from North and is used as the reference coordinate system in this Chapter. The reference datum for ground surface at Williams mine is 10322m and 5321.5m for the Golden Giant mine.
Figure 4.1 Longitudinal view of the Hemlo mining camp showing Williams, Golden Giant, and David Bell mines and inset geographical location of the mining camp in Ontario, Canada. (A) Golden Giant shaft pillar region where pillar Case 1 is located. (B) Williams sill pillar region where pillar Case 2 is located (modified from Coulson, 2009).

4.2.1 Golden Giant mine and trapezoidal pillar (Case 1)

The Golden Giant shaft was sunk in the hanging wall and crossed the ore bearing formations into the footwall of the deposit to within 9m of the ore zone and 30m south of the David Bell mine around the 4600L (Figure 4.2). As mining progressed in both the Golden Giant and David Bell mines, stresses increased in the shaft pillar area around the 4600L. As a result, spalling occurred in the west and east walls of the shaft and on lateral development above and on the 4600L, respectively. Strainbursting also occurred on the 4600L in October of 1998. Subsequent to the strainbursting, production constraints necessitated mining the ore in the shaft pillar. To accomplish this and to alleviate the high stress conditions in the shaft pillar as inferred from observations of rock mass behaviour (i.e., spalling and rockbursting), a de-stress slot was designed and constructed (Coulson, 1998; McMullan et al., 2004).
The de-stress slot initially consisted of three stopes 14m wide and 12m thick (ore thickness) oriented along the ore body excavated to within 9m of the shaft (Coulson, 2009) (Figure 4.2a). The initial slot was then expanded in thickness and vertically using a pyramidal sequence into the hanging wall country rock (non-mineralized) an additional hanging wall to footwall thickness of 4m at a 66° angle followed by a short vertical section. The slot was backfilled using paste and expanding 0.25” polystyrene balls creating a low modulus material to control the closure of the slot. Construction of the de-stress slot commenced in April 2002 and was completed in September 2005. The de-stress slot project included improvements to the micro-seismic monitoring system in the region.

Between 02/2003 and 12/2003 (mm/yy) an isolated pillar (Case 1 pillar) located in the Golden Giant mine above mined stopes at the David Bell mine, east of the de-stress slot between the 4620L and the 4600L (Figure 4.2a) failed. This isolated pillar did not generate any seismic events $M_n \geq 0$ as it was deformed to failure (Coulson, 2009).
Figure 4.2 Golden Giant mine shaft and Case 1 pillar region showing mining to the end of 2003. (a) Longsection view looking north showing mine development and stoping around the shaft and Case 1 pillar. (b) View looking west showing proximity of shaft to de-stress slot and ore body.
In 1989, a 32 channel micro-seismic system was installed by Queen’s University (Kingston, Ontario Canada) specifically to monitor the shaft pillar area. This system was upgraded in February 2002 to an eighty channel Engineering Seismology Group (ESG) Hyperion system prior to mining the de-stress slot. The system was modified up to February 21, 2003 at which point it consisted of 4 triaxial (1Hz to 5kHz) and 26 uniaxial (50Hz to 5kHz) accelerometers (sensor spacing was approximately 20 to 50m and 60 to 140m for the uniaxial and triaxial sensors, respectively). Within the shaft pillar area and around the Case 1 pillar, the mean source location error is 4m (mode 3.5m) ranging from 1 to 8m (Coulson, 2009).

4.2.2 Williams mine and sill pillar (Case 2)

The Williams deposit was mined from two horizons (Block 3 and Block 4) (Figure 4.3), both of which were planned to be extracted using a pyramidal primary-secondary mining sequence (25m sub-level spacing with a 20m stope width). This sequence was constructed in Block 3. Block 4’s extraction commenced using the pyramidal approach but ground control issues arose and a fully symmetrical pyramid sequence was not achievable, resulting in a half chevron sequence. A series of stope failures in the increasingly highly stressed pillar between mining Blocks 3 and 4 resulted in the creation of a 50m high sill pillar (located between the 9450L, 871m below surface, and the 9390L, 931m below surface, LeBlanc and Murdoch, 2000). This was eventually mined using a pillarless retreat method from east to west due to anticipated ground control difficulty (as forecast from numerical stress models indicating high stress conditions) (Coulson, 2009).
Figure 4.3 Williams mine longsection showing mining to the end of 1999, separate mining Blocks 3 and 4 separated by a sill pillar, and mining directions. The Case 2 pillar is located in the sill pillar between Easting 9412E and 9462E and levels 9390L and 9415L.

Even with the sequence and approach adopted, Block 4 developed instabilities in the sill region, where the Case 2 pillar is located. A zone of high stress was pushed ahead of the pillarless sequence leading to the following sequence of activities and events:

- 1996 – when the stope undercuts were developed to the 40m ore thickness, stress induced and structurally controlled caving occurred at the extreme west and east ends of the sill pillar (LeBlanc and Murdoch, 2000);
- 1999 – damage to the footwall access drifts occurred as a result of a Mn = 3.0 seismic event;
- 2000 to May 2003 – 10 seismic events greater than Mn = 0 occurred and were interpreted at the time as being related to a potential faulted foliation discontinuity near the footwall
access drifts. A footwall cross cut pillar strainburst between stopes 26 and 25 was recorded as a Mn = 2.7 seismic event; and

- September 2003 – a Mn = 3.5 seismic event occurred approximately 30m away in the hanging wall and was also suspected at the time to be related to a faulted foliation discontinuity.

After the Mn = 3.5 event in September of 2003, the sill was largely aseismic and mining continued without difficulty. As evident from the above outlined ground instability history, the majority of the difficulties for mining in the sill volume were located in the region around and in the footwall access drifts.

The micro-seismic system was initially installed in September of 1999 after the Mn = 3.0 seismic event caused damage to the footwall infrastructure (Bawden et al., 2000). This initial system consisted of an eight channel portable ESG system using only uniaxial accelerometers (50Hz to 5kHz frequency range). This system was changed on September 22, 2000 to an eighty channel ESG Hyperion system with 48 uniaxial and 4 triaxial accelerometers (1Hz to 5kHz range). Within the sill pillar, the mean source location error is 5m (mode 4m) ranging from 0.5m to 12m (Coulson, 2009).

### 4.3 Overview of engineering geology

#### 4.3.1 Geology

The resource being mined is a lode gold deposit (Schnieders and Smyk, 1991) locally hosted in the Lake Superior Shear Zone (Hemlo Shear Zone of Lin, 2001) (Thompson, 2006) which is a sinistral transpressional zone (Lin, 2001). Lode deposits have ore that fills fissures in the rock formation. The area has undergone four generations of ductile deformation and has been folded multiple times (Lin, 2001). Regionally, the deposit and shear zone are located in the Heron Bay-Hemlo greenstone belt of the Wawa subprovince (Superior province, Ontario) (Lin, 2001). This belt contains sedimentary, felsic, intermediate, volcanic, and intrusive rocks (Lin, 2001; Muir, 2003).

The mine geology is simplified into hanging wall, ore zone, and footwall rock units (Coulson, 2009). The hanging wall of the deposit is composed of metasedimentary rocks. The ore zone is divided into two altered mineralized units which are extensions of the footwall rocks: (a)
feldspathic sericitic ore; and (b) baritic feldspathic ore. The ore zone ranges in thickness from 3 to >40m (averaging approximately 25m). The footwall (Moose Lake Porphyry, Lin, 2001) is composed of: (a) altered muscovite schist; (b) quartz crystal tuffs and feldspathic schists; and (c) quartz eye porphyry. The footwall rocks are described by Lin (2001) as quartz ± feldspar porphyry with the contact between the hanging wall metasediment and the footwall being hard to determine. Both units have gradual contacts. As evident from the division of footwall rocks and ore units into sericitized classes, the porphyry has been sericitized and metamorphosed into muscovite schist with some relict quartz phenocrysts (quartz-eye porphyry).

All of the units are characterized by foliation (described as schistosity at the mine) which has the same orientation as the ore body (approximately 60° to 75° dip towards mine north) as well as felsic porphyry intrusives and mafic sills which are parallel and perpendicular to the deposit. These intrusives are 0.1 to 0.5m thick and are spaced approximately 0.2 to 2m.

Late stage intrusive dykes (diabase and lamprophyre) and faults are also present in the deposit region (Coulson, 2009). The diabase dykes are large, widely spaced (>1000m) discontinuities (5 to 50m thick persisting >1000m) which cross cut the deposit at high angles. The lamprophyre dykes are smaller than the diabase dykes (0.2 to 1m thick persisting approximately >10m) and occur in swarms or clusters. The faults in the mine are limited in number and are identified from slickensides and/or gouge material. These faults are associated with foliation and dyke contacts. The foliation faults are thin (1 to 3cm) and have been traced between 10 to 30m (Kazakidis, 1990).

4.3.2 In situ stress

Stress measurements have been completed using the United States Bureau of Mines (USBM) and CSIRO-HI cells (Golder 1985; 1988a; 1988b). Both are overcoring stress measurement methods. The measurements that were successful and not identified to be influenced by excavations near the measurement locations are shown on a lower hemisphere equal area stereographic projection in Figure 4.4 and summarized in Table 7. The major principal stress is oriented approximately perpendicular to the deposit strike (trend range 346° to 033°), the intermediate principal stress is oriented approximately parallel to the deposit (trend range 057° to 096°), and the minor principal stress is moderately dipping to near vertical (plunge range 57° to 80°). The ratio of the maximum horizontal to vertical stress ($k_o$) is on average 1.7 (ranging from 1.3 to 2.0) and the ratio of the
minimum horizontal to the vertical stress is on average 1.3 (ranging from 1.2 to 1.3). A number of back analyses using numerical stress models have been completed at the mines and the state-of-stress which correlates with mine observations has a $k_o$ of 2.0 with principal stress orientations as summarized in Table 7 ‘as used in mine back analyses’ (Bawden et al. 2000; McMullan et al. 2004; Bawden and Jones 2005). This state-of-stress is used for analysis purposes.

Figure 4.4 Lower hemisphere equal area stereographic projection of the measured principal stresses. Poles for trend and plunge determined from stress measurements (data listed in Table 7). The mean principal stress orientations are circled. Each pole’s great circle is also shown.
Table 7 Summary of stress measurements.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Trend (°)</th>
<th>Plunge (°)</th>
<th>Magnitude (MPa)</th>
<th>Depth (m)</th>
<th>Max. Horizontal to Vertical Stress Ratio</th>
<th>Min. Horizontal to Vertical Stress Ratio</th>
<th>Type</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ₁</td>
<td>004</td>
<td>02</td>
<td>12</td>
<td>300</td>
<td>2.0</td>
<td>-</td>
<td>USBM and CSIRO-HI Cell</td>
<td>Golder (1985)</td>
</tr>
<tr>
<td>σ₂</td>
<td>096</td>
<td>40</td>
<td>8</td>
<td>300</td>
<td>-</td>
<td>1.3</td>
<td>Golder (1985)</td>
<td></td>
</tr>
<tr>
<td>σ₃</td>
<td>271</td>
<td>50</td>
<td>6</td>
<td>300</td>
<td>-</td>
<td>-</td>
<td>Golder (1985)</td>
<td></td>
</tr>
<tr>
<td>σ₁</td>
<td>034</td>
<td>05</td>
<td>35</td>
<td>720</td>
<td>1.9</td>
<td>-</td>
<td>USBM and CSIRO-HI Cell</td>
<td>Golder (1985)</td>
</tr>
<tr>
<td>σ₂</td>
<td>304</td>
<td>07</td>
<td>24</td>
<td>720</td>
<td>-</td>
<td>1.3</td>
<td>Golder (1985)</td>
<td></td>
</tr>
<tr>
<td>σ₃</td>
<td>160</td>
<td>80</td>
<td>19</td>
<td>720</td>
<td>-</td>
<td>-</td>
<td>Golder (1985)</td>
<td></td>
</tr>
<tr>
<td>σ₁</td>
<td>274</td>
<td>60</td>
<td>40</td>
<td>1100</td>
<td>1.5</td>
<td>-</td>
<td>USBM</td>
<td>Golder (1988b)</td>
</tr>
<tr>
<td>σ₂</td>
<td>057</td>
<td>24</td>
<td>34</td>
<td>1100</td>
<td>-</td>
<td>1.3</td>
<td>Golder (1988b)</td>
<td></td>
</tr>
<tr>
<td>σ₃</td>
<td>334</td>
<td>15</td>
<td>27</td>
<td>1100</td>
<td>-</td>
<td>-</td>
<td>Golder (1988b)</td>
<td></td>
</tr>
<tr>
<td>σ₁</td>
<td>346</td>
<td>24</td>
<td>36</td>
<td>1100</td>
<td>1.2</td>
<td>-</td>
<td>USBM</td>
<td>Golder (1988b)</td>
</tr>
<tr>
<td>σ₂</td>
<td>067</td>
<td>19</td>
<td>34</td>
<td>1100</td>
<td>-</td>
<td>1.2</td>
<td>Golder (1988b)</td>
<td></td>
</tr>
<tr>
<td>σ₃</td>
<td>302</td>
<td>57</td>
<td>29</td>
<td>1100</td>
<td>-</td>
<td>-</td>
<td>Golder (1988b)</td>
<td></td>
</tr>
<tr>
<td>σ₁</td>
<td>358</td>
<td>10</td>
<td>0.0437MPa/m</td>
<td>-</td>
<td>2.0</td>
<td>-</td>
<td>Used in mine back analyses</td>
<td>Coulson (2009)</td>
</tr>
<tr>
<td>σ₂</td>
<td>93</td>
<td>28</td>
<td>0.0299MPa/m</td>
<td>-</td>
<td>-</td>
<td>1.4</td>
<td>Coulson (2009)</td>
<td></td>
</tr>
<tr>
<td>σ₃</td>
<td>250</td>
<td>60</td>
<td>0.0214MPa/m</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Coulson (2009)</td>
<td></td>
</tr>
</tbody>
</table>
4.3.3 Rock strength

Uniaxial compressive (Golder, 1986; University of Toronto, 1988), Brazilian indirect tensile (Noranda Technology Centre, 1988), and triaxial (Queen’s University, 1994) strength testing has been completed and are summarized in Table 8 and Figure 4.5. The testing is not grouped by rock type because comparison of test results indicated similar uniaxial compressive strengths, Young’s modulus, and Poisson’s ratio mean and ranges. The indirect tensile strength appears to depend on rock type (Table 8). Not all of the rock units have been tested and the triaxial test specimens were not described geologically so it is not known what rock unit they belong to.

The triaxial dataset was fit using the methodology of Hoek and Brown (1997) through all of the triaxial and UCS data where failure occurred through intact rock (Table 8 and Figure 4.5b). The estimated intact rock strength Hoek-Brown envelope has a material constant $m_i$ (Hoek and Brown, 1980) of 19 with compressive strength, $\sigma_{ci}$, 178MPa. The compressive strengths determined from the intercept of the Hoek and Brown strength envelope on the major principal stress axis (y-axis, Figure 4.5b) are similar to that determined from the uniaxial compressive strength (UCS) data alone (e.g., the footwall rocks have an average UCS of 175MPa).

Foliation is dominate in the rock mass as described next in Section 4.3.4. Specimens that failed or were influenced by the foliation have a mean compressive strength of approximately 100MPa compared to an average of 175MPa for specimens not or minimally influenced by foliation (as indicated in Figure 4.5a and summarized in Table 8).
Figure 4.5 Summary of compressive strength testing data. (a) Uniaxial compressive strength (UCS) data showing influence of foliation on UCS. (b) Triaxial strength and UCS data fit using the methodology of Hoek and Brown (1997).
Table 8 Rock strength testing data summary.

<table>
<thead>
<tr>
<th>Property</th>
<th>Average</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Unit</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Compressive (Intact), UCS</td>
<td>175</td>
<td>80</td>
<td>280</td>
<td>MPa</td>
<td>35</td>
</tr>
<tr>
<td>Uniaxial Compressive (Foliation), UCSf</td>
<td>100</td>
<td>60</td>
<td>210</td>
<td>MPa</td>
<td>11</td>
</tr>
<tr>
<td>Young’s Modulus, E</td>
<td>55</td>
<td>30</td>
<td>93</td>
<td>GPa</td>
<td>35</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu$</td>
<td>0.28</td>
<td>0.12</td>
<td>0.40</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>Indirect Tension (HW), $\sigma_t$</td>
<td>-8</td>
<td>-7</td>
<td>-10</td>
<td>MPa</td>
<td>5</td>
</tr>
<tr>
<td>Indirect Tension (Ore), $\sigma_t$</td>
<td>-13</td>
<td>-7</td>
<td>-17</td>
<td>MPa</td>
<td>5</td>
</tr>
<tr>
<td>Indirect Tension (FW), $\sigma_t$</td>
<td>-20</td>
<td>-16</td>
<td>-24</td>
<td>MPa</td>
<td>3</td>
</tr>
</tbody>
</table>

Triaxial (fit through triaxial and UCS data $\sigma_{ci}$ (MPa) $m_i$

178 19 52

4.3.4 Discontinuities

Mapping data collected from the Williams (Bronkhurst et al., 1993) and Golden Giant mines (Kazakidis, 1990) in dip and dip direction is presented as poles on a lower hemisphere equal area stereographic projection in Figure 4.6 showing 4 discontinuity clusters as summarized in Table 9. Two are major sets (occurring systematically in the mine): Set 1, a steeply dipping (to the north) foliation set (schistosity, which are open along the foliation planes and thus considered a discontinuity set for rock mass characterization purposes); and Set 2, a sub-vertical joint set. Set 3 (sub-horizontal joint set) is prevalent in the upper mining levels and becomes less evident with depth and is a minor set at the Case 1 and Case 2 pillar depths (Coulson, 2009). Set 4 (moderately dipping, to the east and west, conjugate set) is a minor set (occurring less frequently but more evident in the deeper levels of the mine). Set 4 was noted by Coulson (2009) as being random in the Case 1 and Case 2 pillar areas. Sets 2 and 3 tend to terminate on Set 1 (Coulson, 2009). Discontinuity surface characteristics as well as spacing and persistence estimates are summarized in Table 9 and are typically rough, planar to undulating, and unaltered but can range to smooth and slightly altered with no sets being fully persistent.
Figure 4.6 Lower hemisphere equal area stereographic projection of mapping data showing selected clusters of poles forming discontinuity sets. Poles for dip / dip direction symbolically plotted based on location relative to the deposit (i.e., footwall [F/W], hanging wall [H/W], and ore).

Table 9 Summary of discontinuity sets and their characteristics.

<table>
<thead>
<tr>
<th>Set ID</th>
<th>Type</th>
<th>Class</th>
<th>Dip (°)</th>
<th>Dip Direction (°)</th>
<th>Jr</th>
<th>Ja</th>
<th>Spacing (m)</th>
<th>Persistence (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Foliation</td>
<td>Major</td>
<td>70</td>
<td>011</td>
<td>1.5</td>
<td></td>
<td>0.75-2</td>
<td>0.1-1</td>
</tr>
<tr>
<td>2</td>
<td>Joint</td>
<td>Major</td>
<td>85</td>
<td>093</td>
<td>1.5</td>
<td></td>
<td>0.75-1</td>
<td>0.5-2</td>
</tr>
<tr>
<td>3</td>
<td>Joint</td>
<td>Minor</td>
<td>14</td>
<td>195</td>
<td>1.5-3</td>
<td>1-2</td>
<td>0.5-3</td>
<td>1-5</td>
</tr>
<tr>
<td>4</td>
<td>Joint</td>
<td>Random</td>
<td>41-55</td>
<td>090-267</td>
<td>1-3</td>
<td></td>
<td>0.75-1</td>
<td>2-10</td>
</tr>
</tbody>
</table>

Where Jr is joint roughness and Ja joint alteration after Barton et al. (1974); Barton (2002)
4.3.5 Seismic velocity and anisotropy

A velocity survey was completed at the Golden Giant mine (Kazakidis, 1990) parallel and perpendicular to the foliation. P-wave velocity differences were noted (mean velocity between 6063 to 6057m/s with a 5% greater velocity when parallel to the foliation).

From a rock strength anisotropy perspective (Section 4.3.3) it is expected that there would be some deformation anisotropy in the rock mass (also indicated by the p-wave velocity anisotropy). It has been shown by Tonon (2004) that in transversely isotropic materials, stress-paths determined from elastic analyses are essentially the same for isotropic and transversely isotropic materials.

4.3.6 Rock mass character

The rock mass characteristics for the hanging wall, footwall, and ore zone rock units are similar (Couslon, 2009) and are described in unison as follows for the pillar cases:

- The rock mass is composed of Strong (50 to 100MPa when loaded such that failure along the foliation is facilitated) to Very Strong (100 to 250MPa when failure predominates through intact material) rocks and contains two dominate discontinuity sets (Set 1 – Steeply dipping Foliation and Set 2 – Sub-vertical jointing), a minor sub-horizontal joint set (Set 3), and a random moderately dipping conjugate joint set (Set 4). The surface characteristics of the discontinuity sets are typically rough, planar to undulating, and unaltered but can range to slightly altered. The rock mass is predominately massive and can range to blocky when the 2 major sets and 1 minor set are present (GSI ranging from 57 to 100 but typically >75). Foliation is the dominate rock mass characteristic. Other than the noted strength anisotropy, the foliation does not appear to influence the deformation characteristics of the rock and is considered isotropic with respect to deformation properties.

GSI (Geological strength index, e.g., Hoek, 1999; Marinos et al., 2005) is a rock mass characterization system developed for reliable rock mechanics input data (e.g., rock mass properties for numerical models). The GSI chart, indicating the GSI range for the Williams and Golden Giant mine, is provided for reference in Appendix D.2.
4.3.6.1 *Estimated rock mass strength*

The parameters required for rock mass strength estimation were determined based on the rock mass character and laboratory testing described in the previous sections. The following summarizes the estimated rock mass strength envelope input parameters:

- Mean envelope: GSI of 75, UCS of 178MPa, and $m_i$ of 19.

4.4 **Adopted methods for data analysis**

Four methods are used in later sections of this Chapter to assist in the data interpretation which are described at this time. The first is the Principal Component Analysis (PCA) method (e.g., Urbancic et al., 1993; Saccorotti et al., 2002) which was used by Coulson (2009) to extract planar trends from clouds of micro-seismic data related to the two pillar cases. The second is Loading System Stiffness (LSS) (Wiles, 2002; 2007) which is used in this Chapter to estimate directional changes in stiffness along the shear rupture zones being created using a three dimension elastic boundary element stress modeling tool. The third is the spalling limit (Kaiser et al., 2000) which is used to assess changes in failure mode for a given stress-path. The fourth is the fracture initiation limit (e.g., Martin, 1993; Kaiser et al., 2000) which is used to assess when fractures could initiate in rock for a given stress-path.

4.4.1 **Principal component analysis (PCA)**

Coulson (2009) used PCA to assess planar trends in the micro-seismic data at the Golden Giant and Williams mines where it was determined that the PCA planes represent newly created fractures. The PCA results of Coulson (2009) are used and re-interpreted in this Chapter.

PCA is a statistical method that is used to determine three dimensional linear trends along the principal axes of a coordinate system from a defined cloud of scattered data points. The method uses least square regression to fit vectors along the principal coordinate axes. The resulting principal axis vectors are then used to construct ellipsoids which are volumetric or planar. When the ratio between the largest and the smallest vector of each ellipsoid is $<2.5$, the ellipsoid is volumetric and does not have a planar trend. When the ratio is $>2.5$, the ellipsoid represents a plane which has a dip and dip direction. This method has been used to determine the orientation of planes in micro-seismic data clouds (e.g., Urbancic et al., 1993; Saccorotti et al., 2002) which
are assumed to be related to either pre-existing discontinuities or newly created fractures or fracture zones.

In PCA, groups of hypocenters are first compiled by finding all of the hypocenters that are contained within a sphere of radius D centered on the reference focus. The coordinates of the hypocenters in the sphere are used to define a scatter matrix $S$ (Cooley and Lohnes, 1971):

$$ S_{lm} = \frac{1}{K} \sum_{j=1}^{K} (x_{ji} - X_{0i})(x_{jm} - X_{0m}), \quad i, m = 1, 2, 3 $$

where $j$ is the event number from 1 to $K$ and the $x$’s are the Cartesian coordinates of the hypocenter with the subscripts $(i, m)$ corresponding to longitude, latitude, and depth on taking values of 1, 2, 3, respectively. The $X_0$’s represent the arithmetic average of the three Cartesian coordinates of the hypocenters, being:

$$ X_{0m} = \frac{1}{K} \sum_{j=1}^{K} x_{jm}, \quad m = 1, 2, 3 $$

where the index $m$ has the same meaning described previously. The eigenvectors and eigenvalues of the matrix $S$ give the principal axes of an ellipsoid which best fits the cloud of hypocenters.

Coulson (2009) recognized that micro-seismic events can be related both in space and in time. Therefore, the PCA method was applied to a 50 event moving time window until all the events in a cloud were sampled and analyzed. The 50 event window size was determined by Coulson (2009) through a sensitivity analysis which looked at continuous moving windows of different size event by event. The 50 event moving window did not influence the determined planar trends in the micro-seismic data compared to smaller window sizes.

4.4.2 Loading system stiffness (LSS)

LSS is used in this Chapter to evaluate directional stiffness aligned along and perpendicular to the dip line of the rupture zones being created in the two pillar cases. LSS is calculated using the routine embedded in the three dimensional elastic numerical stress modeling tool MAP3D v58 (Wiles, 2011) which is based on the boundary element method.

An example is provided here to introduce LSS based on the example provided by Wiles (2002; 2007). A pillar is located between two excavations. At Stage 1 (Figure 4.7), the pillar is
supporting the ground in between the two excavations. In Stage 2 (Figure 4.7), the pillar loses its load carrying capacity (i.e., it fails) and can no longer support the ground. As a result, the equivalent excavation span increases and convergence occurs in both the back and floor. The response of the ground resulting from the failure of the pillar is used to estimate LSS which is the slope of the load-displacement curve (Figure 4.7).

![Figure 4.7 Load displacement curve for LSS explanation. Stage 1 is the point when the pillar is supporting the ground between two excavations. Stage 2 is the point when the pillar loses its supporting capacity and deformation of the surrounding excavations occurs. LSS is the slope of the resulting line connecting Stage 1 and 2 (from Wiles, 2007).](image)

The example illustrated by Figure 4.7 is two dimensional and in three dimensions, LSS is more complex because there are different LSS values in each direction and different values for every surface in a model. In an effort to simplify LSS in three dimensions, Wiles (2002; 2007) found that LSS is equivalent to normalizing the Local Energy Release Rate (LERR) by the square of the mean stress in a volume of material (e.g., the pillar in the example above) and thus can be simplified to this value:

$$
\frac{1}{LSS} = \frac{2\cdot LERR}{\sigma_m^2} 
$$

where $\sigma_m$ is the mean stress at a point $\sigma_m = \frac{1}{3} (\sigma_1 \sigma_2 \sigma_3)$ and $\bar{\sigma}_m$ is the average value of $\sigma_m$ in a volume.

Returning to the example provided above, the total amount of energy ($W_t$) released from the rock mass surrounding the excavations is the total area under the triangle (Figure 4.7):
\[ W_t = W_k + W_f \]  \hspace{1cm} (15)

where \( W_k \) is the released kinetic energy and \( W_f \) is the energy consumed in failing the pillar.

LERR is:

\[ LERR = \frac{W_t}{\text{volume}} \]  \hspace{1cm} (16)

where volume is that of the pillar that fails.

In MAP3D, the total energy transfer is calculated as the integral of the stresses through their deformations overall all elements. Assuming the pillar loses all load carrying capacity as in the example provided, the total energy transfer divided by the volume of the pillar is the LERR.

LSS is typically evaluated for pillars that are of rectangular to cubic shape and represent an overall volumetric LSS (e.g., Wiles, 2002; 2007). Directional LSS is assessed in this thesis using an array of 3 plates that are each 5 x 5x 1m (L, W, H) and are aligned along and perpendicular to the dip line of the rupture zones being created in the two pillar cases as interpreted based on the PCA analysis (see Figure 4.8). In this way, the dominate deformation in the calculations described above is in the direction of the largest plate span with minimal influence of the plate edges allowing for a directional LSS to be calculated.

The plates aligned along the dip line of the rupture zone thus provide an indication of the stiffness normal to the rupture zone and those aligned perpendicular to the dip line provide an indication of the stiffness in the direction of the dip line (Figure 4.8). The stiffness in the direction of the dip line is thus representative of the stiffness of the system driving shear along a rupture zone (i.e., the loading system). Therefore, while the directional stiffness methodology was developed to assess normal stiffness (plates aligned along the dip line of the rupture zone), it also provides the opportunity to assess the loading system stiffness in the direction of the rupture zone. During failure, in areas with a high loading stiffness, relatively little energy would be released where as in areas of low stiffness, a high energy release potential would be anticipated.
The following analyses were completed to test the directional stiffness concept using LSS in MAP3D:

1. *Volumetric LSS*: Assessment of a 5x5x5m cube in the center of each pillar case with and without the evolution of damage around excavations surrounding the pillar cases. This allows for a consistency check on the initial LSS values prior to mining which should be the same for both cases and how LSS changes in the pillar in a uniform manner with and without excavation damage taken into account.

2. *Trial directional LSS*: Assessment of 5x5x1m plates in the center of each pillar case with the evolution of damage around the excavations surrounding the pillar cases. The plates are aligned in three orientations with the largest faces: (1) N-S; (2) E-W; and (3)
vertically up-down. In this way, the N-S plate orientation is approximately in the
direction of the major principal stress, the E-W plate orientation is approximately in the
direction of the intermediate principal stress, and the vertical up-down plate orientation is
approximately in the direction of the minor principal stress. Therefore, prior to mining,
the N-S plate should have the lowest LSS value (softest) and the vertical up-down plate
the highest LSS value (stiffest). Also, this assessment allows for the LSS values in
different directions to be determined to see if the directional LSS assessment using plates
is viable.

For the volumetric LSS assessment, the following are found (see Appendix D.3 for supporting
figures):

- Volumetric LSS values are initially the same for both cases (Case 1 and Case 2) prior to
  mining. Indicating that the LSS methodology of Wiles (2002; 2007) consistently
  evaluates LSS.
- For the Case 1 pillar, LSS remains nearly constant in models without and with damage
  zone evolution around excavations surrounding the pillar. This indicates no change in
  stiffness due to mining.
- For the Case 2 pillar without excavation damage zone evolution, LSS values remain
  constant indicating no change in stiffness due to mining.
- For the Case 2 pillar with excavation damage zone evolution, LSS values decrease to
  lower values with a rate change starting at the end of 2001.

For the trial directional LSS assessment, the following are found (see Appendix D.3 for
supporting figures):

- For both Case 1 and Case 2 pillars, prior to mining, as was expected, the softest plate
  orientation is N-S (largest plate face in the approximate direction of the major principal
  stress) and the stiffest is vertically up-down (largest plate face in the approximate
direction of the minor principal stress).
- For both Case 1 and Case 2, LSS values are different in different directions as mining
  progresses yearly. For Case 1, the vertical plate orientation is variable during the mining
  steps due to proximity to the evolving excavation damage zone. For Case 2, starting at
  the end of 2000, LSS values are mainly constant at their respective different values.
The LSS methodology and directional LSS assessment using plates can be used to assess LSS changes and thus directional stiffness changes along rupture zones.

4.4.3 Stress-path, spalling limit, and fracture initiation stress level

Stress-paths plotted in principal stress space ($\sigma_1$ versus $\sigma_3$) for each pillar are used in this Chapter to assess potential fracture initiation, failure mode, and the nearing and or crossing of the potential rock mass strength envelope which was outlined in Section 4.3.6.1. The stress-paths are determined in each pillar case using MAP3D v58 (Wiles, 2011) and a query line down the center line of the pillar cases is used to average the major and minor principal stresses.

The potential for fracture initiation in each pillar case is assessed using the bi-linear failure envelope cut-off (Kaiser et al., 2000). This envelope is defined by a principal stress difference ($\sigma_1 - \sigma_3$) = 1/3 to 1/2 UCS. When this envelope is reached by the stress-path, fracturing is anticipated to initiate and when exceeded, fractures are expected to propagate. These processes are recorded in the form of acoustic emission or micro-seismic events at a mine.

The failure potential of each pillar case is assessed using the spalling limit defined by Kaiser et al. (2000). The spalling limit is not a single line but a range of stress conditions between thresholds defined by ratios of $\sigma_1/\sigma_3 = 10$ to 20 for intact rock and potentially lower for heterogeneous rock masses. To the left of these thresholds (low confining stress magnitudes, $\sigma_3$), preferential fracture propagation can occur in tensile or extensional failure modes. To the right of these thresholds (high confining stress levels, $\sigma_3$), tensile fracture propagation is inhibited and fractures accumulate to cause macro-scale shear failure or shear rupture.
4.5 Golden Giant pillar (Case 1)

The Golden Giant pillar (Figure 4.9) is an isolated pillar with no current active mining surrounding it. It is located near the shaft of the Golden Giant mine and above mined out stopes of the neighboring David Bell mine. The pillar is trapezoidal in shape approximately 53m east-west (length), 6 to 25m north-south (thickness), and 20m in vertical dimension (Figure 4.9c).

Figure 4.9 Golden Giant pillar (Case 1) geometry and sections 10480E and 10495E used for assessment. Mining shown to 2003. Yellow excavation block around the Case 1 pillar is the interpreted excavation damage zone.
The pillar, based on micro-seismicity, fails without the occurrence of any large magnitude seismic events (i.e., all events are of \( M_n < 0 \)). Failure is driven by mining causing progressive loading and straining of the pillar (Coulson, 2009).

In this section, first the failure process based on micro-seismic data and PCA data (plotted on lower hemisphere stereographic projections and graphically in cross sectional view) is presented. Next, an assessment of the change in PCA plane ellipsoid ratio and the principal stress-path in the pillar are presented. Directional stiffness is then assessed using the LSS method. Finally, a summary and interpretation of the rupture zone creation process are provided.

### 4.5.1 Failure process

Micro-seismic source locations and micro-seismic event density (Figure 4.10) show that seismicity initiates near the eastern end of the pillar at the start of 2002 and progresses progressively westerly across the pillar to the start of 2003. The point of initiation is where the pillar has the smallest width to height ratio when excavation damage is considered.

![Figure 4.10](image)

**Figure 4.10** Rupture zone initiation and propagation. (a-b) Micro-seismic source locations for 2002 and 2003, respectively. (c-f) Contour of micro-seismic density (5 events per 125m\(^3\)) showing progression of rupture plane east to west from 2002-02 to 2003-03 (modified from Coulson, 2009).
This progression is associated with a rapid increase in micro-seismic event rates (Figure 4.11a).

Graphical plots of the PCA derived planes are suggestive of the creation of a rupture zone where:

- **Between dates 2002-04 and 2003-01**: there is random fracturing in the pillar both visually (Figure 4.11c) and as indicated by lack of clear trend in the poles of the PCA planes plotted on an equal area lower hemisphere stereographic projection (Figure 4.11g) (concentration of poles, 10 to 12%).

- **Between dates 2003-01 and 2003-03**: the PCA poles concentrate to a defined orientation of 52°/168° (dip / dip direction) but are still dispersed (Figure 4.11h) (concentration of poles, 27-30%). The PCA planes visually define a zone with a thickness, length, and general orientation (Figure 4.11d).

- **Between dates 2003-03 and 2003-04**: there is further concentration of PCA plane poles to a defined mean orientation of 47°/161° (dip / dip direction) (Figure 4.11i) (concentration of poles, 49-55%). Visually the PCA plane data shows two pairs of conjugate en échelon arrays of fractures developing (some more evident than others) (Figure 4.11f and Figure 4.12a).

- **Between dates 2003-04 and 2003-12**: some fractures in the pairs of the en échelon arrays have changed orientation to be in the direction of the array dip lines (Figure 4.11f and Figure 4.12b). This is also evident in the PCA plane poles plotted on the stereographic projection where there is still a mean fracture orientation of 56°/164° (dip / dip direction) but a girdle line of poles has developed indicating a change in orientation of some fractures with a mean orientation of 77°/344° (dip / dip direction) (Figure 4.11j). The change in fracture orientation to be aligned with the dip line of the rupture zone is suggestive of shearing and breakage through the initially created en échelon array of fractures (Figure 4.12b).
Figure 4.11 Compilation of micro-seismic event rates and PCA plane data for the Case 1 pillar. (a) Micro-seismic events rates and cumulative event count over time. (b) PCA ellipsoid ratio over time. (c-f) Cross section (10480E) through PCA planes showing the development of a fracture system over time. (g-j) Equal area lower hemisphere stereographic projections of PCA plane poles (dip/dip direction). (c-f) and (g-j) Show date ranges of: 2002-04 to 2003-01; 2003-01 to 2003-03; 2003-03 to 2003-04; and 2003-04 to 2003-12. (a and c-f modified from Coulson, 2009; b and g-j data provided by Coulson, 2010).
Figure 4.12 PCA plane cross section 10480E close up view showing: (a) development of two pairs of conjugate en échelon arrays; and (b) change in orientation of PCA poles in the direction of array dip line for date ranges 2003-03 to 2003-04 and 2003-04 to 2003-12, respectively. The fracture orientation change evident in (b) is suggestive of shearing along the rupture zone and breakage through the initially created en échelon array of fractures. (a-b modified from Coulson, 2009; data in stereographic projections provided by Coulson, 2010).
4.5.2 PCA ellipsoid geometry

The change in PCA plane ellipsoid geometry over time is shown in Figure 4.11b. Ellipsoid geometry is the ratio of the length of the PCA plane along dip to the length as measured along strike. This ratio has been used to identify pre-peak and post-peak strength states for rock undergoing deformation (e.g., Trifu and Urbancic, 1996; Coulson, 2009). The pre-peak to post-peak transition occurs when there is a rapid increase in ellipsoid ratio.

The ratio is relatively constant at a value of 5 until approximately 03/2003 (Figure 4.11b) at which time the ratio rapidly increases to approximately 25 followed by a progressive more gradual decay back to a constant value of 5 around 07/2003.

4.5.3 Stress-path

The stress-path in the Golden Giant pillar was assessed using the three dimensional elastic boundary element code MAP3D v58 (Wiles, 2011). Coulson (2009) previously used MAP3D (as well as Examin3D, RocScience, 2009) for stress analysis of mining sequences at the Golden Giant mine. For the purpose of this thesis, the mine model built by Coulson (2009) was updated to include excavation damage around the Case 1 pillar resulting from induced stresses. The stress analysis was carried out in yearly mining steps (i.e., no mining, 2000, 2001, 2002, 2003). The input parameters used in the MAP3D model are presented in Appendix D.4 and the stopes mined in each year are shown in Appendix D.6.

The rock around the underground excavations and stopes surrounding the Case 1 pillar have been damaged from high stresses. Therefore, the stress-paths were assessed in models without and with stress induced damage around excavations surrounding the pillar taken into account. The depth of damage was estimated using the depth of yield in the calibrated two dimensional plastic finite element numerical stress models of Coulson (2009) and it was assumed that the damage zone rock failed to a cohesionless material with little to no load carrying capacity.

Two sections along the pillar were analyzed: (1) near the initiation point of micro-seismicity at the eastern end of the pillar (10495E) (Figure 4.9b); and (2) near the midpoint of the pillar (10480E) (Figure 4.9b). These two sections are of interest because the modeling completed is elastic and the state-of-stress in the center of the pillar will not be characteristic of the state-of-stress initiating the failure process at the eastern end. Thus, the eastern end stress-path provides
an indication of stress magnitudes at initiation of the failure process while the pillar center stress-path provides an indication of stress magnitudes prior to rupture zone progression across the pillar. Since the numerical modeling completed cannot simulate rupture propagation across the pillar, the stress-path in the center of the pillar along section 10480E is not anticipated to near the potential rock mass strength envelope.

Stress-paths for each section are plotted in principal stress space in Figure 4.13 for the average stress magnitudes across the core of the pillar. Both stress-paths are above the bi-linear failure envelope cut-off, to the right of the spalling limits, and highly confined ($\sigma_3 > 30\text{MPa}$). When excavation damage is taken into account, the stress-path representative of the eastern end of the pillar (10495E), where the failure process initiates, is above the potential rock mass strength envelope while the stress-path in the pillar core is well below the potential strength envelope.

The stress-path at initiation (section 10495E) indicates that failure should occur due to shear rupture. The stress-path in the center of the pillar (section 10480E) is not near the rock mass strength envelope because, as previously noted, the model is elastic. As the initiated shear rupture propagates across the pillar, as indicated by the micro-seismic source locations (Section 4.5.1), stresses will be locally increased ahead of the propagating rupture tip creating the conditions for rock mass failure to occur, i.e., the pillar failure is due to rupture zone propagation across the pillar not due to the pillar stress representative of section 10480E being near or at the strength of the rock mass.
Figure 4.13 *Case 1* average pillar stress-paths for sections 10495E and 10480E considering excavations around the pillar without and with stress induced damage. In both cases, the major principal stress is higher when excavation damage is considered.

4.5.4 Assessment of normal and dip line LSS along the rupture zone

The LSS assessment was completed along Section 10480E in the center of the pillar. Rupture zone dip and dip direction was considered as 70°/165° and the plates described in Section 4.4.2 were oriented to align with this direction. Starting in the year 2000, the normal (plate aligned
along the rupture zone) (Figure 4.14b) and dip line (plate aligned perpendicular to the rupture zone) (Figure 4.14a) LSS values along the rupture zone remain essentially constant with average values of approximately 4,000MPa and 6,000MPa, respectively, to the end of 2003.

Figure 4.14 Case 1 pillar LSS assessment along the rupture zone for yearly increments of mining. (a) Dip line (plate perpendicular to the rupture zone) LSS (loading system). (b) Normal (plate oriented along the rupture zone) LSS.
4.5.5 Summary and interpretation

Fracturing processes leading to shear rupture zone creation in brittle rock and brittle analogue materials deformed in the laboratory under constant stress conditions (e.g., triaxial compression with constant lateral confinement and direct shear with constant normal stress) have been shown to initiate at peak strength (e.g., Lajtai, 1969, Morgenstern and Tchalenko, 1967; Lockner et al., 1991); which is near coincident with maximum acoustic emission event rates (e.g., Scholz, 1968); with rupture propagation and creation occurring in the post-peak region of the load-displacement curve (e.g., Lajtai, 1969, Morgenstern and Tchalenko, 1967; Lockner et al., 1991). Peak strength also has been identified using PCA ellipsoid geometry (e.g., Trifu and Urbancic, 1996; Coulson, 2009) where the pre-peak to post-peak transition occurs when there is a rapid increase in PCA ellipsoid ratio.

Shear rupture zone creation in direct shear under constant normal stress boundary conditions reported in Chapter 2 is also consistent with the above summarized where peak strength occurs at peak fracture rates with rupture zone creation occurring post-peak strength. In Chapter 2, it is also reported that the created rupture zone geometries are dependent on the applied normal stress magnitude with the shear stress versus horizontal displacement response transitioning from completely brittle to ductile at low and high normal or confining stress magnitudes, respectively.

The fracturing process leading to rupture creation in the Case 1 pillar is summarized in Table 10 and is described as follows:

- Normal to the rupture zone and perpendicular to the dip line, LSS remains constant during rupture zone creation suggesting constant boundary conditions normal to the rupture zone and constant energy release potential (based on the perpendicular to the dip line constant LSS values). It is not known (yet) if the constant LSS values indicate a constant stiffness, constant stress, or a constant boundary condition in general because this is the first case history evaluated with this new methodology.

- Initially, 04/2002 to 01/2003, random fracturing occurs in the pillar. Consistent with the stress state in the pillar being above the bi-linear cut-off (based on the stress-path) with relatively constant micro-seismic event rates and PCA ellipsoid geometry.

- A dense trend of fractures visually appears between 01/2003 to 03/2003.
• Around 02/2003, peak micro-seismic event rates, a rapid increase in ellipsoid ratio, and a fracture zone that can be visually defined from the PCA data occurs suggesting peak strength is reached.

• From 03/2003 to 12/2003, conjugate pairs of en échelon fracture arrays form, micro-seismic event rates decay and PCA ellipsoid ratio deceases to a relatively constant low value suggesting the peak to post-peak transition of the pillar.

The fracturing process leading to shear rupture zone creation in the Case 1 pillar is consistent with the boundary conditions being constant stress (peak strength occurs at the point of maximum micro-seismic event rates concurrent with the rapid increase in PCA ellipsoid ratio with the shear rupture zone being created as the micro-seismic event rates decay and PCA ellipsoid ratios reduce, i.e., post-peak strength). The LSS values assessed along the rupture zone both normal to the rupture zone and perpendicular to the dip line remain constant. Because the other data for this case indicates constant stress boundary conditions, the LSS assessment indicates either a constant boundary condition in general, or a sufficiently low normal stiffness such that constant stress boundary conditions apply. This normal stiffness LSS value is approximately 4000MPa on average (Figure 4.14b) and will be considered as a threshold between constant stress and stiffness boundary conditions when evaluating the Case 2 pillar in the next section.

By normalizing the normal stress magnitudes to UCS, the simulation results presented in Chapter 2 can be compared to the Case 1 pillar rupture. The normal stress to UCS ratio ranges from 0.17 to 0.61 for the 25 to 90MPa normal stresses simulated in Chapter 2. In the Golden Giant Case 1 pillar, the normal stress to UCS ratio at peak strength is between 0.2 and 0.55 for the excavations surrounding the pillar without and with damage zones, respectively, and thus is comparable to the normalized ratios for rupture zone creation under moderate to high normal stresses from Chapter 2. This suggests that the Case 1 pillar should form a relatively thick rupture zone (which is does based on PCA plane locations), the load-displacement response of the pillar should have no to little stress drop post-peak and thus little to no energy release (which it also does based on micro-seismic monitoring data). The dip line constant LSS values (loading system) also support the non-changing energy release behaviour of the pillar during rupture zone creation. When LSS values driving shear along the rupture zone are approximately greater than 6000MPa on average,
little seismic energy release should be anticipated based on this case. This LSS value will be considered as a threshold between little to no energy release and energy release potential when evaluating the Case 2 pillar next.

Table 10 Summary of Case 1 pillar rupture zone creation.

<table>
<thead>
<tr>
<th>Time Period (month/year)</th>
<th>Observation from PCA planes</th>
<th>Event rate (#/day)</th>
<th>Ellipsoid Ratio</th>
<th>Max. PCA Pole Concentration</th>
<th>Stress-Path (considering excavation damage)</th>
<th>Normal stiffness</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>04/2002 to 01/2003</td>
<td>Random fracturing</td>
<td>Relatively constant event rates</td>
<td>Constant low ratios (~5)</td>
<td>10 to 12%</td>
<td>All paths to the right of the spalling limits.</td>
<td></td>
<td>Pre-peak strength</td>
</tr>
<tr>
<td>01/2003 to 03/2003</td>
<td>Defined dense trend of fractures</td>
<td>Peak rate 02/2003, 80 events, and start of rate decay</td>
<td>Increase starts on 02/2003. Peak ratio (25) 03/2003. High ratios (&gt;10) 02/2003 to 07/2003</td>
<td>27 to 30%</td>
<td>All paths to the right of the spalling limits.</td>
<td></td>
<td>Peak strength on 02/2003</td>
</tr>
<tr>
<td>03/2003 to 04/2003</td>
<td>Localized conjugate pairs of en échelon fracture systems</td>
<td>Continued event rate decay</td>
<td></td>
<td>49 to 55%</td>
<td></td>
<td></td>
<td>Post-peak</td>
</tr>
<tr>
<td>04/2003 to 12/2003</td>
<td>Localized banding with fracture orientation change</td>
<td>Relatively constant event rates</td>
<td>Constant low ratios (~5) after 06/2003</td>
<td>Girdle</td>
<td></td>
<td></td>
<td>Post-peak reaching some ultimate strength state with shear along the rupture zone</td>
</tr>
</tbody>
</table>
4.6 Williams sill pillar (Case 2)

The Williams sill pillar is in the vicinity of active mining. The pillar, based on micro-seismicity, fails with the occurrence of a number of larger magnitude seismic events \((M_n \geq 0)\) and a rockburst \((M_n = 2.7)\) as a result of mining causing progressive loading and straining of the rock. This section specifically focuses on the data located in a region between Eastings 9400E and 9450E (from stope 26 to 23) and levels 9390L and 9415L (see Figure 4.15).

A cross section through 9437E (middle of stope 24) and between levels 9415L and 9390L (Figure 4.15) is used as the primary section of analysis to investigate the failure processes and behaviour along the sill’s strike. The analysis of a limited section is applicable due to the geometry of the sill pillar and the relatively linear advance of mining from east to west. As mining progresses, a stress front ahead of the mining advance is generated, this front of high stress is systematically progressed across the sill pillar and thus every section of ground in the sill is subjected to a similar stress-path during mining progression and to similar behaviour due to similar rock mass characteristics across the sill pillar volume as outlined in Section 4.3. The region selected for analysis along the sill pillar also contains more information about the rock mass behaviour in the form of extensometer data than other areas.

First the failure process based on micro-seismic data and PCA data (plotted on stereographic projections) is presented. Next, a detailed assessment of the change in PCA plane ellipsoid ratio is completed. This is followed by assessment of extensometers and the principal stress-path in the pillar. Directional stiffness is then assessed using the LSS method followed by an empirical assessment of pillar stability based on the pillar’s changing effective width to height ratio. Finally, a summary and interpretation of rupture zone creation is provided.
4.6.1 Failure process

Micro-seismic source locations suggest the progressive development of a fracture plane in the center of the pillar (Figure 4.16c-g) with a distinct localization in 06/2003 (Figure 4.16g). There are a series of increases in event rates (#/week) creating a stepped cumulative frequency curve with the highest event rate occurring prior to a rapid decline in seismic events (Figure 4.16a).
The largest event rate increases on 06/2001, 05/2003, 09/2003 are coincident with larger seismic events or rockbursts, $M_n = 3.1$, $M_n = 2.7$, and $M_n = 3.5$, respectively, that are located within or near the *Case 2* pillar region of interest (Coulson, 2009).

Unlike the Golden Giant pillar fracture process described in Section 4.5 and shown in Figure 4.12, there are an insufficient number of PCA planes in *Case 2* to show the fracturing process leading to rupture zone creation. Although, the understanding from the observations between the PCA plane plots and poles plotted on lower hemisphere equal area stereographic projections can be used to gain additional understanding (i.e., the girdle line in the stereographic projections appears to be related to fracture orientation change suggesting shearing along a rupture zone, see Figure 4.12). The poles plotted on stereographic projections yearly from 1999 to 2004 are shown on Figure 4.17. Unlike the Golden Giant pillar case where the poles tended to become increasingly concentrated to a more defined orientation up to the interpreted peak strength followed by a girdle line developing, for this case, the poles start relatively concentrated in 1999-2000 (Figure 4.17a, pole concentration 40-45%, $38^\circ/181^\circ$, dip / dip direction) and then progressively show a scattering in 2001 (Figure 4.17b, pole concentration 18-20%, $47^\circ/166^\circ$, dip / dip direction) but with evident orientation clustering. This behaviour progresses and in 2003 a non-concentrated or disbursed girdle line emerges (Figure 4.17d) with a more defined girdle line in 2004 (Figure 4.17e). From the Golden Giant case data (Section 4.5.1), the girdle line is potentially related to fracture orientation changes along the rupture zone being created suggesting shearing along a created rupture zone.
Figure 4.16 Compilation of micro-seismic event rates and source locations for the Case 2 pillar. (a) Micro-seismic events rates and cumulative event count over time. (b) PCA ellipsoid ratio over time. (c-g) Cross section (9430E ±12.5m) of micro-seismic source locations for dates indicated. (a-b re-plotted using data provided by Coulson, 2010. c-g modified from Coulson, 2009).
Figure 4.17 (a-e) Equal area lower hemisphere stereographic projections of PCA plane poles (dip/dip direction) for years indicated. Poles are relatively concentrated in (a) and disburse over time eventually forming a girdle line in 2004 (e). (data provided by Coulson, 2010).
4.6.2 PCA ellipsoid geometry

Ellipsoid geometry is the ratio of the length of the PCA plane along dip to the length as measured along strike. This ratio has been used to identify pre-peak and post-peak strength states for rock undergoing deformation (e.g., Trifu and Urbancic, 1996; Coulson, 2009). The pre-peak to post-peak transition occurs when there is a rapid increase in the ratio.

PCA ellipsoid ratio, Figure 4.16b, is relatively constant at a value of 8 between 11/1999 and 06/2000. The ratio increases to 16 just prior to 07/2000 followed by a progressive more gradual decay back to a constant value of 5 around 03/2001.

4.6.3 Extensometer response

Stretch Measurement to Assess Reinforcement Tension, SMART-cables (Hyett et al., 1997), which are multi-point borehole extensometer type instruments, are located in the volume of interest. Deformations relative to the toe of the cables are shown on Figure 4.18 which also has a plan view of the 9390L indicating the locations of the five cables in the back of the access drift (four at crosscut intersections). Each cable is discussed separately as follows:

- **Cable 1**: indicates approximately elastic drift response up to 06-2000. After this date, near boundary deformations increase due to the development of excavation parallel fracturing (near boundary dilation/slabbing) (as per ISRM, 1981 and Cording et al., 1971 which summarize example plots of displacements and mechanisms causing the displacements). This behaviour is evident to 10-2002 with approximately 60mm of displacement tapering off rapidly to <5mm at 5m depth. The average bulking\(^6\) is nearly zero in the first meter while the rock is bulking up to 5 and 6% in the second meter. The readings for 12-2002 suggest that the depth of failure is suddenly increasing to 5m with 6% bulking between 4 and 5m. The near boundary deformations remain constant during

---

\(^6\) Rock mass bulking refers to a volumetric or unidirectional length change as the rock mass is strained during the stress-driven failure process. Bulking includes geometric volume increases resulting from a geometric non-fit of hard, strong rock blocks or slabs. Broken bulking rock can only move into an excavation, therefore, the bulking factor or percentage \((BF)\) is defined as the ratio of change in length \(d\), perpendicular to the excavation boundary over an original length, \(l\):

\[
BF = \frac{d}{l} \times 100\%
\]
the propagation of the failure zone suggesting compression of broken rock near the excavation (0-4m) to accommodate the bulking at 4-5m.

- **Cable 2**: indicates approximately elastic drift response up to 12-2001. After this date, near boundary deformations increase due to the development of excavation parallel fracturing (near boundary dilation/slabbing). This behaviour continues and increases by 05-2003 to approximately 50mm of displacement tapering off rapidly to <5mm at 4m depth. The average bulking at that time is approximately 1%. The readings for 06-2003 suggest, again, a sudden deepening of the failure zone from 4 to 10m (and possibly deeper). During this propagation, most of the bulking is concentrated at 6 to 10m with less than 1% bulking. The broken rock closer to the excavation stops bulking or even compresses (between 2 to 4m depth along the cable). The 06-2003 readings are suggestive of a sudden deep seated failure propagation related to the fracture/rupture zone.

- **Cable 3**: indicates approximately elastic drift response up to 01-2002. After this date, the displacements are deep seated (~5m) with less than 0.4% bulking at a depth of 3-7m. Again, zero or negative bulking is observed near the excavation (0-2m). This is indicative of the closure of stress induced fractures due to the interaction of the fracture/rupture zone forming at depth and the excavation damage zone.

- **Cable 4**: indicates similar response compared to Cable 2 with the 12-2002 displacements indicating near boundary stress fracturing and deeper seated displacements along discontinuities (from 5m to 8m) (in this case the creation of a fracture/rupture zone). Between 12-2002 and 01-2003 bulking of about 1% at a depth of 6 to 10m (or beyond) indicates deep seated fracture processes with associated rock compression closer to the excavation.

- **Cable 5**: indicates approximately elastic drift response prior to 2002. From 12-2002 onwards, bulking increases at a depth of 3 to 5m reaching about 2 to 3%. Between 12-2002 and 01-2003, again, the depth of failure propagates rapidly to 7-8m or beyond with a bulking of 4% at a depth of 7-8m indicating deep stress induced fracturing far from the excavation boundary. The displacements for 01-2003 are again suggestive of zero or negative bulking of the previously fractured ground. This stress induced fracturing, when related to the micro-seismic data, again suggests an interaction of the excavation damage zone with a fracture/rupture zone forming in the pillar.
Overall, there is little response along the cables to the end of 2000 suggestive of near elastic excavation response. This is followed by near boundary displacements suggestive of up to 6% bulking (as a result of stress induced slabbing/spalling) near the excavations generally mid to end of 2002. Between 06 or 12-2002 and 06-2003, the bulking zone suddenly deepens to between 4 and 10m. This behaviour change is clearly related to deep seated rock mass failure and is suspected to be the result of a fracture/rupture zone developing in the core of the pillar which in turn interacts with the excavation damage zone (previously spalled/slabbed rock). Bulking at depth imposes deformations on this stress damaged rock mass which then compresses the fractured rock (e.g., Cable 2 or 4, Figure 4.18) or stops bulking (e.g., Cable 1, Figure 4.18). Due to the fact that there is no evidence of a gradual propagation of the depth of failure or bulking, it is reasonable to assume that the bulking at depth (after 06 or 12-2002) is directly related to the creation of a shear rupture zone in the pillar core. This fracture zone then interacts with the excavation damage zone to cause the observed extensometer records. This process of rupture zone interaction with the excavation damage zone influencing the extensometer measurements is illustrated schematically in Figure 4.19.
Figure 4.18 SMART-cable responses (# 1 to 5) relative to the toe of the cable. Cables 1 to 5 are shown on the level plan for 9390L (stope numbers, Northings, and Eastings labeled). (data provided by Coulson, 2010).
Figure 4.19 Schematic representation of a pillar core rupture interacting with the damage zone surrounding an excavation (that increases in depth over time) and associated extensometer response. Numbers 1-3 for extensometer curves are: (1) near excavation spalling/bulking behaviour (excavation damage envelope 2000-2002); (2) excavation damage zone beginning to interact with rupture zone (2003); and (3) intersection of excavation damage zone with rupture surface with compression of previously slabbed/spalled rock.
4.6.4 Stress-path

The stress-path in the Williams pillar was assessed using the three-dimensional elastic boundary element code MAP3D v58 (Wiles, 2011). Coulson (2009) previously used MAP3D (as well as Examin3D, RocScience, 2009) for stress analysis of mining sequences at the Williams mine. For the purpose of this thesis, the mine model built by Coulson (2009) was updated to include: (1) the observed excavation damage around the Case 2 pillar resulting from induced stresses; and (2) the footwall crosscut drifts which were used to access the ore. The stress analysis was carried out in yearly mining steps (i.e., no mining, 2000, 2001, 2002, 2003, 2004). The input parameters used in the MAP3D model are presented in Appendix D.4 and the stopes mined in each year are shown in Appendix D.7.

Similar to the Golden Giant case, the stress-paths were assessed in models without and with a simulated evolution of stress induced damage around the excavations surrounding the pillar. The modeled evolution of excavation damage over time was determined using the extensometer data by correlating it to a deviatoric stress criterion. A normalized deviatoric criterion \((\sigma_1 - \sigma_3)/\text{UCS}\) of 0.52 was found to match the interpreted depth of stress induced fracturing as determined using Cable 4 up to 12-2002 (i.e., prior to the influence of the rupture zone being created in the pillar core). The criterion was then used to build damage zone envelopes around excavations in the region of interest for each mining step. This was completed by exporting the 0.52 normalized deviatoric contour for each year of mining (from models that did not take excavation damage into account) and importing it into the model to construct the damage zone envelopes around the excavations in the region of interest. The correlation between the deviatoric stress criterion and the interpreted depth of spalling for Cable 4 is shown in Appendix D.5.

The average pillar stress-paths without and with excavation damage evolution for section 9437E through the pillar in principal stress space are shown in Figure 4.20. When excavation damage is not taken into account, the stress-path is compressive well beyond 2003, above the bi-linear cut-off, and well below the potential strength envelope for the rock mass.

When excavation damage is considered, the stress-path shows an increase in the major \((\sigma_1)\) and minor \((\sigma_3)\) principal stress to 2001 and then progresses to lower and lower magnitudes of minor principal stress at a more or less constant major principal stress. The stress-path crosses the spalling limit \((\sigma_1/\sigma_3 = 10)\) in 2002 and the potential rock mass strength (plus spalling limit \(\sigma_1/\sigma_3\))
= 20) in 2003. Extensional fracturing would therefore be expected in or just before 2002. The calculated stress-path becomes tensile (negative values of $\sigma_3$) in 2004. This tensile stress point is an artifact of the elastic continuum model that allows tension. In reality, tension is capped by the rock mass tensile strength which is much less than the modeled tension.

Figure 4.20 Case 2 average pillar stress-paths for section 9437E considering excavations around the pillar without and with stress induced damage.

The stress-path that was determined from the model which considered excavation damage evolution clearly illustrates the failure sequence supported by seismic clustering in combination with extensometer responses (rock mass bulking). Up to the end of 2001, the pillar core is stable with a safety margin when compared to the anticipated rock mass strength. In 2002, the stress state gets close to or exceeds the spalling limit (at $\sigma_1/\sigma_3 = 10$). Thus, the damage mechanism in
the pillar core changes from confined shear (which is not reached because the major principal stress is not high enough) to processes with extensional failure.

In 2003, the stress-path reaches the failure envelope for shear failure and the combined effect of extensional damage above the initiation threshold and shear failure leads to sudden rupture zone creation with related rock mass straining. This in turn causes a deepening of the excavation damage zone and thus deep seated bulking as reflected in the extensometer readings described in the previous section.

### 4.6.5 Pillar geometry change

As previously noted in Section 4.6.4, excavation damage occurred around the excavations forming the sill pillar. The evolution of excavation damage over time results in a change in effective pillar geometry (width and height). The change in pillar width was assessed along 3 sections and pillar height assessed across the middle of the pillar (Figure 4.21i-v; grey zones represent expected damage zones, interpreted based on extensometer readings and modeling). The resulting width to height ratios, assuming that the damaged rock has no supporting or stabilizing effect, were then calculated and the three values averaged. The average stress-path in the pillar considering excavation damage was then normalized by the average uniaxial compressive strength of the rock (i.e., 178MPa). The normalized average pillar stresses and corresponding average pillar width to height ratios were then plotted on the empirical pillar stability chart (Figure 4.21) with compiled case history data from multiple sources (i.e., Hedley and Grant, 1972; Brady 1977; von Kimmelman et al., 1984; Krauland and Soder, 1987; Hudyma, 1988; Sjoberg, 1992; Lunder, 1994) and various empirical stability lines for a factor of safety of 1.0. It must be noted that the resulting pillar geometry path cannot be directly related to the data (failed, transition, and stable cases) shown on the plot in Figure 4.21 because the actual widths were used for the graph and not the effective widths. Thus, the increase in normalized average pillar stress considering no pillar geometry change over time is also plotted (vertical upwards path for W/H = 3.1 in Figure 4.21). In reality, due to some supporting effect of the damage zone, the actual path is likely in between the two but closer to the path shown with black diamonds.

When the pillar geometry is assumed to remain unchanged, as was assumed for all data points in Figure 4.20, the path is vertically upwards and does not cross any empirical factor of safety of 1.0 envelopes; suggesting that the pillar should have been stable. The path considering the Case
2 pillar with effective geometry change progresses up and to the left (Figure 4.21) crossing empirical factor of safety of 1.0 envelopes between 2001 and 2002. As of mid 2001, it approaches the zone with predominantly failed pillars (red squares). The extensometers show that pillar wall damage occurred at that point and increased to greater depths as loading continued. By 2003, the effective W/H is well into the failure zone.
Figure 4.21 Empirical pillar stability graph showing case histories of stable, transition, and failed pillars, and various empirical factor of safety of 1.0 envelopes. Two pillar geometry and normalized average pillar load paths are shown. One for the changing pillar geometry over time (left trending path), and the second for a constant pillar geometry (vertically upwards trending path). (i) to (iv) show pillar geometry change due to grey damage zones. The FOS = 1.0 line of Lunder (1994) is shown as dashed after a W/H ratio of 2.0 because pillar strength should increase with increasing W/H ratio and there are no case history data points to anchor Lunder’s FOS=1.0 line.
4.6.6 Assessment of normal and dip line LSS along the rupture zone

The LSS assessment was completed along Section 9437E. Rupture zone dip and dip direction was considered 48°/180° and the plates described in Section 4.4.2 were oriented based on this.

Starting in the year 2000, dip line LSS values (loading system) along the rupture zone decease from approximately 7,800MPa to 2,000MPa (on average) (Figure 4.22a, see Figure 4.8 for location of plates) and are on average below the average value of LSS of approximately 6000MPa for the Case 1 pillar (see Section 4.5.4). Since the Case 1 pillar did not generate any seismic events Mn ≥ 0 for the average dip line LSS of 6000MPa, the Case 2 pillar may have different energy release potential. Based on the micro-seismic data, the Case 2 pillar did release energy because a number of seismic events (Mn > 0) were generated.

The normal LSS change for each plate is described as follows (Figure 4.22b):

- Plate A (Top) increases in LSS from 2000 to 2002 at which point it rapidly declines to 2003.
- Plate B (Middle) increases in LSS from 2000 to 2001 at which point it decreases to 2003.
- Plate C (Bottom), LSS remains constant from 2000 to 2001 then declines to a constant value between 2002 and 2003.
- On average, LSS increases from 2000 to 2001 and then declines to 2003.

The LSS average normal stiffness value is greater than the average of approximately 4000MPa for the Case 1 pillar (see Section 4.5.4) to the end of 2002. This potentially indicates a different influence of normal stiffness for Case 2 and a potential change in boundary condition some time in or after 2002.
Figure 4.22 Case 2 pillar LSS assessment along the rupture zone for yearly increments of mining. (a) Dip line LSS (loading system). (b) Normal LSS.
4.6.7 Summary and interpretation

Four different interpretation scenarios are explored next in an effort to understand the pillar failure process for Case 2:

1. Failure under constant stress boundary condition;
2. Failure under constant stiffness boundary condition;
3. Failure due to changing effective pillar geometry resulting in the pillar core stress exceeding the rock mass strength; and
4. Failure due to a combination of a stiffness boundary condition and a changing pillar geometry resulting in a change in pillar core stress and pillar boundary condition to one of stress control.

Figure 4.23 summarizes the analyses and results for the Case 2 pillar previously outlined in the sections above and shows when the following occur:

- Magnitude seismic events;
- Micro-seismic event rates;
- Peak ellipsoid ratio;
- Peak strength as interpreted by Coulson (2009);
- Crossing of the fracture initiation threshold;
- Near excavation and deep seated deformations as determined from the extensometers;
- Crossing of the spalling limits;
- Crossing of the empirical pillar stability lines;
- Crossing of the estimated Hoek-Brown rock mass strength envelope; and
- Crossing of the constant normal stiffness to constant normal stress boundary threshold based on relative comparison between the Case 1 and Case 2 pillar normal stiffness LSS values.
Figure 4.23 Compilation of Case 2 analyses; for explanation see text. Circled seismic events are in the region of interest for the Case 2 pillar.
4.6.7.1 **Interpretation 1 – constant stress boundary**

Assuming that the *Case 2* pillar fails under a constant stress boundary condition, as assumed by Coulson (2009), peak strength occurs at the point of highest ellipsoid ratio between July and November of 2000 and the shear rupture zone in the pillar is created post-peak strength (Figure 4.23). While a possible interpretation, this interpretation is not consistent with when the stress-path crosses the potential strength envelope in principal stress space, the occurrence of maximum micro-seismic event rate, or the crossing of various empirical pillar stability lines. All of these occur after the occurrence of the peak ellipsoid ratio. Therefore, the pillar likely did not fail under a constant stress boundary condition.

4.6.7.2 **Interpretation 2 – constant stiffness boundary**

From Chapter 3, shear rupture zone creation in brittle rock at the laboratory scale under constant normal stiffness boundary conditions has the following characteristics:

- a discontinuous rupture zone created pre-peak, before the maximum strength is reached;
- a shear stress versus horizontal displacement response that is always brittle (i.e., has near instantaneous large shear stress drops) during the transition from peak to a frictional residual shear strength;
- shear stress oscillatory behaviour in the shear stress versus horizontal displacement curves indicative of the stress drops prior to stick-slip instability;
  - the shear stress oscillatory behaviour creates steps in the cumulative fracture count curves; and
- a yield point defined by the linear Coulomb strength envelope and a peak maximum shear strength defined by a constant horizontal displacement threshold which occurs at the peak fracturing rate.

Assuming a stiffness boundary condition, the peak strength was reached in September of 2003 at the peak micro-seismic event rate and the largest seismic event of Mn = 3.5. This point occurs at or just after the stress-path crosses the estimated Hoek-Brown strength envelope and the effective pillar geometry path crosses various empirical pillar stability lines. This point also occurs prior to the PCA poles plotted on the lower hemisphere stereographic projections indicating a girdle line which, based on *Case 1*, indicates shearing along the created rupture zone. In Chapter 3, shearing along the rupture zone was only evident once or just after peak
The largest magnitude seismic event is also consistent with the large brittle stress drops that occurred in the normal stiffness simulations in Chapter 3 once the peak maximum strength was reached. The largest magnitude seismic event is consistent with the largest stress drop during transition from peak maximum strength to residual strength.

A stiffness boundary condition is also supported based on the following:

- The increasing normal stiffness along the rupture zone (on average) between the end of 2000 and 2001 followed by the subsequent loss in rupture zone normal stiffness. Potentially indicating changing boundary conditions (Figure 4.22b).

- Greater normal stiffness values in Case 2 compared to those on average for the Case 1 pillar (which is best described by a constant stress boundary condition). The Case 2 pillar has normal stiffness values that are higher than Case 1 on average between years 2000 and the end of 2002 indicating a potential stiffness boundary condition.

- The multiple seismic events which occurred in the Case 2 region of interest prior to the interpreted peak strength from November 2000 to September 2003. During this time, steps are noted in the micro-seismic event cumulative frequency curve. This period is analogous to when the shear stress oscillatory behaviour of the load-displacement curve occurs as the stress-path follows the strength envelope in Chapter 3. In the simulations, during this time, steps are also noted in the fracture event cumulative frequency curves. Recalling that the shear oscillations are indicative of stick-slip behaviour, the multiple occurrences of seismic events and the stepped nature of the micro-seismic cumulative frequency curve in the Case 2 pillar region prior to peak maximum strength supports this interpretation. It should be noted that the seismic events did not occur in the core of the Case 2 pillar or with the extraction of stopes. The seismic events were located in the rock mass out approximately 30m around the Case 2 pillar region (except for the Mn = 2.7 strainburst which occurred in an excavation defining part of the pillar. It is interpreted that shear displacement along the rupture zone during its creation caused the seismic events due to hanging wall/footwall closure. The displacement driven mechanism of seismic events away from excavation boundaries in mines is argued by Blake (1972) to be the dominate mechanistic driver.
The apparent creation of localized fractures in the pillar core based on deep seated SMART-cable response between the end of 2002 and the mid to end of 2003 (prior to peak strength). Only under stiffness boundary conditions was pre-peak rupture zone creation observed in the numerical simulations outlined in Chapter 3.

Peak micro-seismic event rates do not coincide with peak ellipsoid ratio (speculatively interpreted to be when the internal stress-path of the pillar reaches the pillar peak strength envelope, i.e., yield point, which is then followed).

4.6.7.3 Interpretation 3 – effective pillar geometry change

Changes in effective pillar width to height ratio result in changing the state-of-stress in a pillar. As pillar width to height ratio decreases, a pillar’s core state-of-stress changes from a confined toward an unconfined state. This then changes the failure mode of the pillar from shear rupture to tensile splitting (as summarized in Chapter 1 for laboratory specimen of marble, Paterson, 1958) as the stress-path approaches and eventually crosses the spalling limit.

The stress-path in the Case 2 pillar, considering excavation damage and thus effective pillar width to height ratio change, shows a progressive lowering of confining stress with a more or less constant major principal stress. Assuming this as the primary process leading to pillar failure, the pillar stress-path crosses the spalling limits in 2002 / 2003 and nears the estimated rock mass strength envelope in 2003 (also see Figure 4.20). These points are consistent with when peak micro-seismic event rates occur in the pillar. There is also a defined localized linear trend in the micro-seismic data on 06/20/2003 (see Figure 4.16g) which is either indicative of sliding along a created shear rupture zone or tensile splitting of the pillar due to the change in effective pillar width to height ratio and the stress-path crossing the spalling limit.

Interpretation 3 is not consistent with the pre-peak generation of large seismic events (unless directly associated with stoping; which they appear not to be), the localized fracturing in the pillar core, or the early development of peak ellipsoid ratio.

4.6.7.4 Interpretation 4 – combination

As always, an alternate interpretation is one that embraces elements of all three potential contributing factors, i.e., links the previous 3 interpretations. This fourth interpretation assumes
that the initial boundary conditions surrounding the Case 2 pillar are of a stiffness condition and the peak ellipsoid ratio speculatively indicates when the internal peak strength envelope of the pillar is reached which cannot be tracked with the three dimensional elastic numerical stress modeling tool. As mining continues around the pillar, damage zones around the excavations surrounding the pillar develop. Initially, these damage zones increase the major principal stress and normal stiffness along the rupture zone being created in the pillar core. During this time, pre-peak fracture systems are created in the pillar core and result in early (pre-peak) generation of seismic events and the stepped nature of the micro-seismic event cumulative frequency curve. The damage zones continue to deepen around the excavations. This creates the ability for the pillar core fracture systems, creating a rupture zone, to interact with the excavation damage zones. This interaction is the potential cause of the Mn = 2.7 pillar wall strainburst in May of 2003. As the pillar core fracture systems dilate and interact with the excavation damage zones, they will have the effect of loading the damage zones to the point where, if the rock mass characteristics are ideal, a violent failure could occur and generate a seismic event/rockburst.

This interpretation is consistent with the SMART-cable monitoring data which shows dilation in the pillar core concurrent with the Mn = 2.7 pillar wall strainburst. At this point, the damage zones have now deepened sufficiently to generate low confining stress conditions in the pillar. This allows for longer tensile fractures to propagate and the pillar boundary condition to change from one of stiffness to one of constant stress (as also supported by the decreasing normal stiffness along the rupture zone from the start of 2002 to the end of 2003) leading to pillar rupture and a large energy release as the pillar fails undergoing a stress drop to a residual strength state.

This interpretation is most consistent with all records of rock mass behaviour, monitoring data, and analyses completed.

In summary, while the situation is rather complex and conclusive answers cannot be provided with certainty, based on the learning’s from Chapters 2 and 3 it is suggested that the Case 2 pillar failed by a transition from a stiffness to a stress boundary condition. Even if there are some remaining uncertainties about the failure conditions and mode, it is suggested that the interpretation of: interaction of an excavation damage zone with a shear rupture zone being
created in the pillar provides a sound interpretation. This interpretation is more plausible than those previously advanced.

4.7 Discussion

In this section, the differences between the analyses completed in this thesis related to the Case 1 and Case 2 pillars and that of Coulson (2009) are briefly compared and discussed.

4.7.1 Golden Giant pillar (Case 1)

The Case 1 pillar interpretation is in line with Coulson (2009). The primary differences in this thesis being that the analysis of the stress-path in the Case 1 pillar was: (1) conducted without and with the development of damage zones around the excavations surrounding the pillar; and (2) two pillar stress averaging sections were used (the first near the point of rupture zone initiation, section 10495E, and the second in the middle of the pillar, section 10480E). Coulson (2009) only analyzed the stress-path in the middle of the pillar along section 10480E without excavation damage taken into consideration.

The consequence of this is that Coulson (2009) interpreted shear rupture zone creation just above the fracture initiation limit and not near or at the anticipated peak rock mass strength. In this thesis, shear rupture zone creation occurs well above the fracture initiation limit and near the estimated peak rock mass strength envelope. The interpretation in this thesis is consistent with shear rupture zone creation theory and observations in the laboratory where shear rupture zones are created at or just after peak strength. Only at low confining stress magnitudes in the field where tensile fractures can propagate long distances does rock fail at the fracture initiation stress magnitude. The point of rupture zone initiation and consideration of excavation damage are needed in elastic modeling to interpret mining induced failure case histories.

4.7.2 Williams pillar (Case 2)

The primary differences in this thesis compared to Coulson (2009) for the analysis of the Case 2 pillar are: (1) the stress-path which was assessed without and with the development of damage zones around the excavations surrounding the pillar; (2) the inclusion of the cross cut drifts in the region of interest; (3) the assumed stiffness boundary condition; and (4) the assumed change in pillar geometry with mining. Coulson (2009) only analyzed the stress-path in the middle of the
pillar without excavation damage taken into consideration, no cross cut drifts, assumed constant stress boundary condition, and no change in pillar geometry as a potential mechanism driving the failure process.

The consequence of this is that Coulson (2009) interpreted shear rupture zone creation near the fracture initiation limit (not near or at peak strength), to the right of the Mogi line (which is defined as a linear slope of $\sigma_1/\sigma_3 = 3.4$, Mogi 1966) which would suggest plastic rock mass behaviour. Mogi’s line represents the transition from brittle fracture to ductile flow in rocks. To the right of the line, ductile flow occurs which is characterized by a stress-strain curve without any drop in strength after the yield point. To the left of the line, brittle fracture occurs which is characterized by a sudden and large drop in strength after the yield point. Furthermore, shear rupture was interpreted at a point in time that was not consistent with when the stress-path crossed the potential strength envelope in principal stress space, the occurrence of maximum micro-seismic event rate, or the crossing of various empirical pillar stability lines all of which occurred after the occurrence of the peak ellipsoid ratio. This ratio clearly cannot be used to interpret peak strength when the boundary condition is not of constant stress.

The interpretation in this thesis is consistent with shear rupture zone creation theory under constant normal stiffness transitioning to normal stress boundary conditions, and observations in the laboratory where brittle shear rupture zones are created far to the left of the Mogi line.

4.8 Conclusions

In this Chapter, two pillar case histories were analyzed and re-interpreted. It was found that the Golden Giant pillar (Case 1) failed by the creation of a shear rupture zone which initiated at the east end of the pillar (at the location of the smallest W/H ratio) and propagated westerly across it. The processed monitoring data in the form of micro-seismic event rates, PCA plane locations, PCA plane ellipsoid ratios, stress-paths, and directional stiffness were consistent with shear rupture zone creation occurring under essentially constant stress boundary conditions. Based on the constant normal stress simulations on the synthetic specimen in Chapter 2, typical rupture processes and rupture characteristics were established and used for this re-interpretation. This case example was consistent with constant stress boundary behaviour.
While mine failure cases are commonly interpreted using the assumption of constant stress boundary conditions, previously not recognized and different assumptions were required to interpret the Williams mine pillar (Case 2) case history. The Case 2 pillar failed by the creation of a shear rupture zone which progressively developed in the pillar core. The processed monitoring data in the form of micro-seismic event rates and source locations, PCA plane ellipsoid ratios, SMART-cables, stress-paths, and directional stiffness were consistent with shear rupture zone creation occurring under an initial stiffness boundary condition which transitioned to a constant stress boundary condition as the pillar geometry changed due to stress induced rock mass damage around the excavations surrounding the pillar eventually interacting with the rupture zone forming in the pillar.

Both cases are consistent with laboratory testing data where brittle fracturing processes occur to the left of the Mogi line. Under constant stress boundary conditions, shear rupture zones are initiated at or just after peak strength and are created post-peak. Under constant stiffness boundary conditions, shear rupture zones are initiated pre-peak maximum strength.

The incorporation of the majority of the mine development excavations and damage zones around the excavations forming the pillar cases as well as the consideration of two extreme boundary conditions (constant stress and stiffness) were essential for the interpretation of the two cases. In many mines, it is only assumed that constant stress boundary conditions prevail and failure is interpreted based on this assumption. This can result in incorrect interpretation of the failure case and thus incorrect learning’s and future design recommendations. Furthermore, some mine developments are often excluded from mine scale three dimensional elastic numerical stress models as is the excavation damage zone evolution. As a consequence, modeling results will not follow the actual stress-paths.

It is recommended that: (1) the assumptions forming the base of an interpretation be clearly stated and the data associated with the case used to test the assumptions. If the data does not fit the assumptions, the data needs to be checked and if found correct, then new assumptions are needed. The cases in this Chapter: (1) highlight the need to consider different boundary conditions and reasons for failure development; and (2) the need for mine infrastructure, in addition to stopes to be included in areas of interest as well as the depth of stress induced damage evolution around excavations over time. If no data is available to calibrate the depth of damage
to actual conditions around excavations, it can be estimated using a method such as that proposed by Martin and Christiansson (2009).

4.9 Summary of Chapter 4

Two pillar case histories were analyzed and re-interpreted. Each pillar was interpreted to have undergone a failure process resulting in shear rupture. In the first case, at the Golden Giant mine, the rupture zone was created in the pillar over a period of a few months and did not generate any seismic events $M_n \geq 0$ (Nuttli magnitude). In the second case, at the Williams mine, the rupture zone was created in the pillar over the period of a few years and contributed to or was the direct cause of a number of seismic events with magnitudes exceeding $M_n = 2.7$. Each case was re-evaluated using micro-seismic data, Principal Component Analyses, assessed stress-paths from three dimensional elastic numerical stress models, and where available, extensometer data and pillar geometry changes. A new directional loading system stiffness methodology was developed and used to assess stiffness changes normal to and in the direction of the dip line of the rupture zones. Each case was analyzed and re-interpreted assuming constant stress and stiffness boundary conditions. The field monitoring data was used to test the boundary condition assumptions. It was found that the first pillar case ruptured under a boundary condition that was best described by constant stress while the second pillar case ruptured under boundary conditions changing from stiffness to stress control. These case histories highlight the importance of properly testing the assumptions used to interpret field monitoring data and provide evidence for boundary conditions other than constant stress to exist in mines thus showing the value and practical application of the understanding of shear rupture zone creation under different boundary conditions gained from Chapters 2 and 3.
Chapter 5

5 Conclusions, implications, and future research

In this Chapter, the following are summarized:

- The research work carried out;
- The original conclusions made towards understanding shear rupture zone creation;
- The resulting implications based on the conclusions;
- The apparent limitations of the adopted approach; and
- Recommendations for future research.

5.1 Summary

The creation of shear rupture zones in high stress mines often causes disruptions to the mining cycle, leads to increased costs in the forms of production delays and ground support, and creates a hazard for mine personnel. Hence, factors affecting shear rupture zone creation are clearly important to understand but limited research has been focused in this area. Due to the limited research and the potential for various boundary conditions to surround a shear rupture zone being created in an underground mine setting, this thesis explored the fracturing processes leading to shear rupture zone creation in massive (intact non-jointed) low porosity brittle rock subjected to constant normal stress and normal stiffness boundary conditions. In reality, of course, temporal combinations of the two boundary conditions are to be expected.

It was hypothesized for this thesis that the boundary condition perpendicular or normal to an evolving shear rupture zone would influence its key characteristics:

- the fracturing processes and mechanisms leading to shear rupture zone creation relative to the shear stress versus horizontal displacement response (load-displacement response); and thus
- the load-displacement response; and thus
- the shear rupture zone’s peak and ultimate strengths.
Most importantly, it was hypothesized that the ultimate shear rupture zone geometry would depend on the boundary conditions during creation. In other words, it was to be demonstrated that the characteristics of shear rupture zones were not only a function of the rock mass properties but the boundary conditions under which the rupture zone was created.

The thesis addressed the following objectives to test these hypotheses and to improve the understanding of shear rupture zone creation:

- fracturing processes and mechanisms leading to shear rupture zone creation;
- shear rupture mechanisms;
- ultimate shear rupture zone geometries;
- shear stress versus horizontal displacement (load-displacement) response; and
- strength envelopes (peak and ultimate/residual).

The objectives were achieved and the hypotheses were proven by the following:

- The development of a synthetic specimen calibrated to the strength (linear Coulomb envelope and tensile strength), post-peak shear stress versus horizontal displacement response, and fracture angle characteristics of Lodève sandstone deformed in direct shear under constant normal stress boundary conditions. Lodève sandstone is a low porosity (<2%) brittle rock.
  - Including a comparison of simulated and actual peak shear strength envelopes, tensile strength, post-peak shear stress versus horizontal displacement response, and shear rupture fracture zone geometries.
- The investigation of shear rupture zone creation in the synthetic specimens in direct shear under constant normal stress boundary conditions for normal stresses ranging from 5 to 90MPa.
- The development of a numerical direct shear DEM simulation set up and its use for the investigation of shear rupture zone creation in a synthetic specimen under constant normal stiffness boundary conditions for 1 to 100GPa cap modulus values under initial applied normal stresses of 5 to 40MPa.
• A demonstration of the effects of stress versus stiffness boundary conditions on the shear rupture zone creation process and the resulting rupture zone geometry as well as the resulting strength and deformation characteristics.

• The analysis and re-interpretation of two pillar case histories at the Golden Giant mine (Case 1) and the Williams mine (Case 2) which failed due to the creation of shear rupture zones. A re-interpretation based on the thesis hypothesis, that the effect of boundary conditions affected the pillar rupture zone creation process and behaviour, was presented to demonstrate the value and practical implications of the findings of the direct shear simulations under different boundary conditions. The findings from Chapters 2 and 3 were used in the re-interpretations. The following were utilized for the analysis and re-interpretation of the two pillar cases:
  - micro-seismic event rates and locations;
  - PCA plane ellipsoid ratio and locations;
  - stress-paths in the pillars assessed using three dimensional elastic numerical stress models with approximation of excavation damage zone evolution;
  - a newly developed methodology for assessing both the normal loading system stiffness along and the driving shear loading system stiffness perpendicular to the rupture zones using three dimensional elastic numerical stress models;
  - extensometers; and
  - mining induced effective pillar geometry change due to stress induced damage around the excavations surrounding the pillars.

With this input and the learning’s from the shear simulations (Chapters 2 and 3), the two cases were re-interpreted with the following outcome:

• Case 1: pillar behaviour was interpreted as being representative of conditions expected for a constant stress boundary condition; and

• Case 2: pillar behaviour was interpreted in four ways; first assuming a constant stress boundary condition, then assuming a constant stiffness boundary condition, then assuming a changing effective pillar geometry driving the instability process, and finally a combination of the previous assumptions. It was found that the Case 2 pillar is best
representative of a rupture process transitioning from a boundary condition of stiffness to stress control.

5.2 Conclusions

The main contributions of this thesis are summarized in three areas related to improving the understanding of mining-induced shear rupture zone creation (1 and 2) and showing the value and practical application of the improved understanding (3): (1) in direct shear under constant normal stress boundary conditions; (2) in direct shear under constant normal stiffness boundary conditions; and (3) in the field under either or changing stress and stiffness boundary conditions.

When mining, both the stress and deformation boundaries are changed. This thesis demonstrates, as an important conclusion, that shear rupture zones induced by mining are not only dependent on the rock mass properties in which they are created but are controlled, to a large extent, by the boundary conditions imposed by the mining system. That this is true is not new, Salamon (1970) amongst others has long shown that the mining system affects, for example, pillar energy release behaviour and McKinnon and Garrido (1998) have shown that boundary conditions have an influence on fracutre orientation and sequence of development. What is new is that this thesis demonstrates conclusively that the evolution and final characteristics of a shear rupture zone depend on the boundary condition under which it is created.

The implication of this improved understanding of shear rupture zone creation is of greatest value when mining causes localized failure as in confined pillars, abutments, or when mining into discontinuous fault structures.

While not demonstrated in this thesis, the findings of this work also assist in better understanding differences in fault-slip (double-couple source mechanism) rockbursting created by mining. As is shown in Chapters 2 and 3, the post-peak load-displacement characteristics of near identical rock masses changed drastically depending on the boundary condition. Under constant normal stress, rupture at low normal stress was more brittle (Chapter 2), whereas, it was more brittle under higher normal stresses when under high normal stiffness (Chapter 3). Since shear rupture at low normal stresses under constant normal stress boundary conditions will release less energy compared to higher normal stresses under constant normal stiffness boundary conditions (due to
the stress drop increase), conditions of shear rupture in high normal stiffness zones are most critical for mining because they have the potential to release more energy.

The following summary lists the conclusions related to shear rupture zone creation in brittle rock, first under constant normal stress boundary conditions (findings from Chapter 2) and then under constant normal stiffness boundary conditions (findings from Chapter 3). Note that there is some overlap when the Chapter 3 conclusions are summarized because some of the conclusions for the constant normal stress boundary condition were not evident prior to Chapter 3.

From Chapter 2 (shear rupture zone creation under constant normal stress boundary conditions), it follows that:

- Massive brittle rock subjected to direct shear:
  - at low normal stresses (5-15MPa) ruptures via tensile splitting. The specimen scale rupture occurs at or just after peak shear strength at a small increment of horizontal displacement;
  - at moderate normal stresses (25-40MPa) ruptures predominately in shear and the fracturing process leading to the shear rupture in the specimen is more progressive. At or just after the peak shear strength is reached, an en échelon array of tensile fracture systems form. With increasing shear deformation, the en échelon array of tensile fractures turns into micro-faults (fractures with shear displacement). These micro-faults act as shear flaws generating tensile fractures from their tips. Eventually the array links forming a continuous shear rupture zone; and
  - at high normal stresses (60-90MPa) ruptures in the same manner as described above for moderate normal stresses but the internal fractures and the resulting fracture system arrays are created at steeper angles.

- The state-of-stress internal to a specimen being deformed in direct shear is rotating and is initially largely tensile at low and compressive at higher normal stresses. As a result, at low normal stresses, the internal major principal stress rotates to an orientation nearly in line with the direction of applied displacement creating fractures at shallow angles and the condition for fracture propagation to occur across the specimen due to a
predominately tensile stress field. At moderate and high normal stresses, the internal major principal stress rotates to higher angles that are not in line with the direction of applied displacement and the internal major principal stress becomes more compressive. Therefore, fractures cannot easily propagate across the specimen creating the necessity of a progressive fracturing process leading to shear rupture zone creation in the specimen.

- The kinematics of the specimens being deformed never reach a state of simple-shear. At low normal stresses, the specimen is kinematically classified as extensional, and at moderate and high normal stresses the specimen is kinematically classified as transtensional (i.e., dilatant sub-simple-shear).

- Specimens with aspect ratios of 1:1 and 1.5:1 have essentially the same peak strength envelopes, rupture mechanism dependence on normal stress, and fracture angles. Aspect ratios of 1:1 are suitable for investigation purposes.

The following conclusions emerge from Chapter 3 for shear rupture zone creation in brittle rock under constant normal stiffness boundary conditions:

- The characteristics of shear rupture zones created under constant normal stiffness boundary conditions differ from those under constant normal stress. Different fracturing processes lead to different rupture zone creation, shear stress versus horizontal displacement response of the specimen, and ultimate rupture zone geometries. These differences are generated because of the coupling between the normal and shear stresses (non-constant normal stress stress-paths).

- Under constant normal stiffness boundary conditions, once the linear Coulomb strength envelope is reached, peak shear strength is not yet achieved but yield is initiated in the specimens (yield point). Further strain-hardening\(^7\) occurs as the strength envelope is followed. During the hardening process, fracture formation occurs prior to reaching the peak maximum shear strength. Due to the stress-path followed, the linear Coulomb strength envelope is followed with the increase in normal stress controlled by the normal stiffness. Fracturing continues to evolve as the rock is strained until the point of failure

\(^7\) Strain-hardening is used because of its common use in the literature based on metal behaviour. In brittle rock, strain-strengthening would more accurately reflect the actual physical change in the material.
(peak maximum shear strength) is reached. The point of failure or peak maximum shear strength is reached at a near constant horizontal displacement threshold suggesting that the point of failure for a shear rupture zone is strain rather than stress controlled.

- Once the linear Coulomb strength envelope is reached (yield point) in the constant normal stiffness simulations, shear stress oscillations are evident in the shear stress versus horizontal displacement curves. The oscillations are similar to those found during stick-slip behaviour of brittle rocks in the laboratory. The shear stress oscillations found in this thesis are the result of local fracturing along the discontinuous rupture zone and are representative of rupture zone smoothing processes. The oscillations are also represented as steps in the cumulative number of fracture curves. The results indicate that each stick-slip instability event is associated with a cohesion loss increment. Steps in micro-seismic monitoring cumulative number of event curves as plotted using mine data could be used to identify rock mass volumes in a mine under different boundary conditions.

- Past the yield point (when the stress-path reaches the linear Coulomb strength envelope), dilation leads to strain hardening which is interpreted as a strain dependent increase in apparent cohesion intercept resulting from a need to break through more intact rock at increasing confinement (normal stress). This can be understood as a process whereby more asperities, rock bridges, and intact rock fractures (intra-grain fractures in the simulations) have to occur to cause failure.

- Under constant normal stiffness, the post-peak strength envelope is defined by a residual friction angle of approximately 28° due to the amount of horizontal displacement achieved in the numerical simulations and the stress-path which results in the synthetic specimen fracturing predominantly pre-peak maximum strength.

- The bi-linear ultimate strength state under constant normal stress boundary conditions is found to be a result of the change in rupture zone creation mechanism and thus the different rupture zone geometries created post-peak under different normal stress magnitudes which are not evident in the constant normal stiffness simulation results. Rupture zone characteristics are thus not only a function of the properties of the material in which they are created but are also a function of the resulting rupture zone geometry.
which under constant normal stress conditions depends on the magnitude of the applied
normal stress which acts to change the rupture mechanism occurring in the specimens.

- At the low normal stiffness cap modulus values of 1GPa, the boundary condition is found
to be one of quasi-constant normal stress.

- Under constant normal stress boundary conditions, rupture zone creation in the specimens
occurs post-peak strength. Post-peak rupture zone creation occurs because dilatant
fracturing in the specimens does not generate increases in normal stress and therefore no
increases in shear strength during rupture zone creation. Thus, shear strength is lost
during rupture zone creation under this boundary condition.

The following are the conclusions derived from the re-evaluation of shear rupture zone creation
in two mine pillars which also served to show the value and practical application of the
understanding gained from Chapters 2 and 3. It is suggested that the first pillar (Case 1, Golden
Giant mine) failed largely under a constant stress boundary condition and the second pillar (Case
2, Williams mine) failed differently because of an initially stiff boundary condition changing to
one of stress control as reported in detail in Chapter 4:

- Different rupture zones exhibiting different behaviours (e.g., micro-seismic) were created
in the two pillar cases due to mining even though the rock mass characteristics in the
locations were similar. Thus it is concluded that the boundary conditions directly
contributed to the noted differences.

- The Golden Giant pillar case (Case 1) was re-interpreted assuming a constant stress
boundary condition but using a stress-path that took into account the creation of rock
mass damage around excavations.
  - Peak strength occurred at the point of maximum micro-seismic event rates
    concurrent with the rapid increase in PCA ellipsoid ratio with the shear rupture
    zone being created as the micro-seismic event rates decayed and PCA ellipsoid
    ratios reduced, i.e., post-peak stress). The stress-path is to the right of the spalling
    limits where shear rupture zone creation in a confined rock mass is expected to
    occur.
The Loading System Stiffness (LSS) values assessed along the rupture zone both normal to the rupture zone and perpendicular to the dip line remain constant. Because the other data for this case indicates constant stress boundary conditions, the LSS assessment indicates a sufficiently low normal stiffness such that constant stress boundary conditions apply. This normal stiffness LSS value is approximately 4000 MPa on average.

For the Williams mine pillar case (Case 2), it was necessary, for interpretation purposes, to take into account changing boundary conditions during shear rupture zone creation. Thus, a new (previously not recognized) factor that of differing and changing boundary conditions was required to re-interpret the case. Particularly, the need for an initial stiff boundary condition was demonstrated. In this case:

- Peak ellipsoid ratio was speculated to indicate when the internal peak strength envelope of the pillar was reached. As mining continued around the pillar, damage zones around the excavations surrounding the pillar developed increasing the normal stiffness along the rupture zone being created in the pillar core. During this time, pre-peak fracture systems were created in the pillar core and resulted in early (pre-peak) generation of seismic events. The damage zones continued to deepen around the excavations. This created the ability for the pillar core rupture fracture systems to interact with the excavation damage zones. This interaction caused the Mn = 2.7 pillar wall strainburst in May of 2003.

- The interpretation is consistent with the SMART-cable monitoring data which showed dilation in the pillar core concurrent with the Mn = 2.7 strainburst. At this point, the damage zones had deepened sufficiently to generate low confining stress conditions in the pillar. This allowed for long tensile fractures to propagate and the pillar boundary condition to change from one of stiffness to one of constant stress (as supported by the decreasing normal stiffness along the rupture zone from the start of 2002 to the end of 2003 to a value below the threshold LSS value determined from the Case 1 pillar) leading to pillar rupture and a large energy release as the pillar failed undergoing a stress drop to a residual strength state.
• The required stiffness boundary condition assumption for the interpretation of the Case 2 pillar shows the existence of stiffness boundary conditions. Additional cases are needed to prove the impact of this boundary condition.

• A methodology was developed to assess directional stiffness using the LSS method developed by Wiles (2002; 2007) using the three dimensional elastic boundary element tool MAP3D v58. The methodology allowed for the assessment of directional stiffness in the two cases assessed. For Case 1 it was determined that a normal stiffness threshold of 4000MPa indicated a sufficiently low magnitude such that constant normal stress boundary conditions occur. This threshold is supported by the Case 2 pillar’s transition from stiffness to stress boundary control.

• Mine developments, which are not in all cases included in three dimensional mine stress analysis models, and the associated excavation damage zones, must be included for assessment of stresses. Incorrect conclusions may be drawn if the majority of the stress damaged excavations are not taken into account.

5.3 Discussion of implications

Some of the potential consequences and practical implications of the research findings are summarized next. The implications which are of a more speculative nature are indicated to define applicability boundaries.

5.3.1 Direct shear test result interpretation

Constant normal stress direct shear test result interpretation must consider a rupture mechanism change from tensile to shear around a normal stress to UCS ratio of 0.17. This was found in Chapter 2 for the simulations with constant normal stress boundary conditions. This change in rupture mode could generate curved Coulomb strength envelopes depending on specimen grain boundary strength, grain interlocking, and grain strength. This is supported by the direct shear results of Cresswell and Barton (2003) for locked sands. They tested specimens free of cementation but with interlocked fabric having a measurable cohesive strength and specimens which have trace iron oxide and clay cementation which were noticeably more cohesive and easier to sample. They interpreted that their curved Coulomb strength envelopes for the material
were due to tensile rupture at low and shear rupture at higher normal stress magnitudes. They used the results reported by Lajtai (1969) using plaster-of-Paris to support their interpretation. Both Cresswell and Barton (2003) and Lajtai (1969) had to infer the rupture mechanism change. The DEM simulations presented in this thesis explicitly show this mechanism change.

The direct shear simulation results in Chapters 2 and 3 also show that a simple shear condition does not occur in the synthetic specimen which requires a major principal stress orientation of $45^\circ$ relative to the rupture surface and no specimen volume change. Simple shear should not be assumed to dominate in specimens of intact brittle rock during rupture zone creation in direct shear.

### 5.3.2 Mine failure case history interpretation

Mining rock mechanics practitioners commonly assume constant stress boundary conditions for the interpretation of mine failure prediction and forensic analysis of case histories. Other boundary conditions need to be considered if the case history’s data does not fit the theory associated with the rupture of rock in the brittle field under constant stress boundary conditions as found in Chapter 4. The case history’s data should be used to test the assumed boundary or loading condition. If the data does not fit the assumptions, then either the data quality is poor or the assumptions are incorrect and need to be modified.

### 5.3.3 Energy release potential and its dependence on mine geometry

For energy to be released during a failure process, the stiffness of the loading system deforming a volume of rock must be softer than the post-peak slope of the load displacement curve of the failing rock mass or shear rupture zone as discussed in Chapter 4. Under constant normal stress boundary conditions, it was found in Chapter 2 that the shear stress versus horizontal displacement curves become less brittle at increasing magnitudes of normal stress. Therefore, if a fracturing process leading to shear rupture zone creation is occurring in a mine environment under constant normal stress boundary conditions, the failure process can be controlled by maintaining sufficient confining stress. Under this condition, even if in a soft loading system, relatively little energy release would occur during a failure process due to the maintained magnitudes of shear stress post-peak stress. A mining strategy that maintains confinement magnitudes in the area could be an effective control measure to ensure a stress drop does not
result allowing for a release of energy during failure. In other words, constant stress environments do not seem to be conducive to mining conditions where shear rupture zone creation would cause large magnitude seismic events.

In contrast, it was found in Chapter 3 for direct shear under constant normal stiffness boundary conditions that once the linear Coulomb strength envelope is reached, and the rock mass hardens until a full shear rupture zone is created, the brittleness increases with increasing mine stiffness. This suggests that, in otherwise comparable ground conditions, higher energy release potential is to be expected in situations where the normal stiffness during rupture zone creation is high. In other words, high normal stiffness environments should be conducive to shear rupture zone creation with large magnitude seismic event potential. Changes in stress (e.g., leading to unclamping) should therefore cause larger events in areas where shear loading occurred in a high normal stiffness environment.

The findings in Chapter 3 also suggest that shear stress oscillations primarily occur in the shear stress versus horizontal displacement curves at higher normal stiffness which also created stepped cumulative number of fracture curves, and thus are indicative of high normal stiffness environments. Such stick-slip conditions are detectable by seismic monitoring and co-located events and stepped cumulative number of micro-seismic event curves are potentially indicative of the formation of a shear rupture zone with high seismic event magnitude potential (in a high normal stiffness zone). Means to soften the normal stiffness without stress relaxation would be required to prevent a shear rupture zone from having high energy release.

Furthermore, if a shear rupture zone is being created under constant normal stiffness boundary conditions, maintaining a confinement magnitude may not control the energy release potential of the failure process because of the shear stress oscillations along the shear stress versus horizontal displacement curves. Rupture zone creation pre-peak shear strength would be associated with seismic energy release due to the oscillating nature of the shear stress versus horizontal displacement curves caused by the feedback normal stress, where if in a soft loading system, each oscillation and associated stress drop would result in seismic events pre-peak strength with even larger events post-peak as the strength state of the rupture zone transitioned to frictional.
5.3.4 Properties of shear rupture zones

For the numerical stress modeling of shear rupture zones it is necessary to establish representative properties. This thesis demonstrates that the rupture zone characteristics will differ depending on the stress or stiffness environment within which the rupture zone is created and thus the rupture zone’s properties.

Under constant normal stress boundary conditions, the final rupture zone geometries in the synthetic specimen changed from thin and relatively planar at low normal stresses to thick and irregular at higher normal stresses. This geometry change was found to be one of the drivers for the change in shear stress versus horizontal displacement response of the synthetic specimen in Chapter 3. Therefore, the geometry of a rupture zone influences a rupture zone’s characteristics and thus its properties. The implication for numerical modeling, therefore, is that there are no unique constitutive relationships for rupture zones. The rupture zone’s characteristics are not just a function of the rock mass properties in which the rupture zone is created but also of the rupture zone geometry (which is linked to the boundary condition under which it is created).

It follows that estimation of the properties of a rupture zone requires an understanding of the conditions under which the rupture zone is or was created and thus a detailed understanding of the rupture zone’s geometry.

5.3.5 Near excavation boundary fracturing, tensile or shear origin? (speculative)

Near excavation boundary fracturing (spalling/slabbing) has been shown to be tensile in origin as discussed in the introduction of this thesis in Chapter 1. Recently, Barton (2006; 2012) has argued that for intermediate strength rocks such as marble and schist, near excavation boundary fracturing processes are of shear origin. This thesis shows that tensile rupture occurs at low normal stresses (normal stress to UCS ratios <0.17) under constant normal stress boundary conditions when a specimen is subjected to shear deformation. Therefore, near boundary fracturing is potentially of a tensile mechanism even if the loading or deformation condition is in shear. Once the shear induced tensile rupture is created, shear displacement occurs hiding the initial tensile origins of the fractures making an interpretation of the origin of the fractures difficult.
5.3.6 The spring-slider model (speculative)

The spring-slider model is often used to describe the repetitive nature of earthquakes. The model consists of a block resting on a surface being pulled at a constant velocity through a spring. The contact between the block and the surface it is resting on is considered frictional. As the spring attached to the block is pulled, there will be a point when the block starts to slide. Because the spring stretches during deformation, when the block begins to slide, the spring accelerates the block depending on the stiffness of the spring. This acceleration and then subsequent deceleration, is the energy released during an earthquake (Scholz, 2002).

In Chapter 3, direct shear results under constant normal stiffness boundary conditions are reported and it was found that shear oscillations in the shear stress versus horizontal displacement curves were the result of fracturing along the discontinuous rupture zones while the normal-shear stress-path followed the linear Coulomb strength envelope. These oscillations were then discussed as being similar to stick-slip oscillations but for very different reasons (cohesion loss due to fracturing).

If the constant normal stiffness boundary condition is relevant for faulting and frictional sliding along rough faults and discontinuities in the Earth’s crust under certain conditions (i.e., strike-slip faulting, faulting processes away from the Earth’s and other free surfaces) then the standard spring slider model consisting of a smooth planar frictional contact under constant normal stress boundary conditions should be complimented with a rough contact (representative of having geometric heterogeneity along a fault) under constant normal stiffness boundary conditions such that during sliding, dilation can occur (through asperity ride over or new dilatant fracturing) generating a normal stress feedback process which is not considered or taken into account in the current slider models.

Investigations of normal stiffness in the spring slider model have been completed such as those of Dieterich and Linker (1992) considering the coupling between the shear spring and the normal spring using a smooth contact block model and the influence of variable normal stress on rock friction by Linker and Dieterich (1992). These results show that stick-slip instability is influenced by the coupling between normal and shear stress and normal stress history. Therefore, there is reason to suspect that normal stiffness boundary conditions with a rough contact which allows dilation will have an influence on stick-slip behaviour.
5.4 Limitations of the adopted approach

The known limitations of the adopted approach for the assessment of shear rupture zone creation in low porosity brittle rock are summarized as follows:

- A synthetic specimen was generated to be a simplified representation of the Lodève sandstone reported by Petit (1988) and Wibberley et al. (2000). While it was possible to capture the mineral grain composition based on Comte et al. (1985) and be within the grain size range reported by Petit (1988), it was not possible to represent the grain size of the Lodève sandstone as measured from the SEM images provided by Wibberley (2011) or the actual composition of Petit’s specific specimens, the data for which does not exist. This limitation does not impact the results of this thesis because the generated synthetic specimen with the limitations on grain size and mineral composition was calibrated to the strength, post-peak deformation, and fracture angle characteristics of the Lodève sandstone. Therefore, the synthetic specimen is representative of those characteristics even with the noted two limitations.

- In the present work, only polygonal grain shapes were used to represent the grain structure of the Lodève sandstone. Not all of the grains in the Lodève sandstone are polygonal and therefore the grain structure could be better represented in the synthetic specimen by developing a routine that takes the SEM images of the Lodève sandstone and ‘maps’ an identical smooth-joint network of the grain boundary geometry. Pixel numbers were used to try and automatically pick grains and grain boundaries from the images but the SEM images were not of sufficient quality to complete this task. Therefore, it may not be possible to develop a ‘mapping’ routine based on the information available. This aspect should be explored by future works using other rock types.

- A two-dimensional approach was used to investigate shear rupture zone creation as obtained from shear tests on rock specimens. It is valid for the adopted modeling approach because of the direct shear condition being simulated and the current inability to generate a grain network in three-dimensions. In reality, shear rupture zones are three-dimensional. The influence of out-of-plane heterogeneities therefore has not been taken into account in this thesis.
• The calibration of the synthetic specimen to the available characteristics of the Lodève sandstone resulted in only tensile fracturing of grains and grain boundaries. While the calibration is consistent with the SEM images of the rupture zones created in Lodève sandstone reported by Wibberley et al. (2000), the allowance for some shear bond breakage in the simulations may impact the results and should be tested. It is not anticipated that the allowance of shear bond breakage will impact the results to the extent that the conclusions in this thesis will change. Other PFC2D simulation studies have shown that even when shear bond strength is set equal to tensile bond strength (e.g., Diederichs, 1999; Cho et al., 2008) tensile bond breakage dominates and controls synthetic specimen rupture and behaviour.

• While only two dimensional simulations were carried out, long simulation run times resulted on a Intel Xeon 2.8GHz 6 core processor with 24Gb of RAM:
  
  o the 1:1 aspect ratio constant normal stress synthetic specimen with 41,388 particles (plus approximately 25,000 smooth-joints) took approximately 2 days to run at low normal stresses (5-15MPa) and 2 weeks to run at high normal stresses (60-90MPa);
  
  o the 1.5:1 aspect ratio constant normal stress synthetic specimen with 62,082 particles (plus approximately 40,000 smooth-joints) took approximately 8 days to run at low normal stresses (5-15MPa) and 30 days to run at high normal stresses (60-90MPa); and
  
  o the constant normal stiffness simulations took 5 to >30 days to run for low and high initially applied normal stresses, respectively.

5.5 Future research

Five areas that need additional research to further the understanding of shear rupture zone creation in brittle rock are identified as:

1. Investigation of the influence of grain boundary and intra-grain strength characteristics on shear rupture zone creation and various characteristics (e.g., peak strength, ultimate strength, shear stress versus horizontal displacement response, rupture zone geometry). This research has already been initiated and is summarized in Appendix E.
2. Influence of linear Coulomb strength envelope slope on shear rupture zone characteristics under constant normal stiffness boundary conditions.

3. The development of a direct shear testing machine to allow for the rupture of specimens of very strong (>100MPa) brittle rock under both constant normal stress and stiffness boundary conditions.

4. The development of a three dimensional Grain Based Method (GBM) in a three dimensional DEM numerical modeling tool to allow for the simulation of shear rupture zone creation in intact brittle rock to be investigated.

5. Further development of the directional LSS concept.

The above areas for future research are briefly discussed in the following sections.

5.5.1 Grain boundary and intra-grain strength characteristics

The heterogeneous nature of a rock’s micro-structure (e.g., grain boundaries, grains, grain size and shape, micro-fractures, and pore space) influences fracture initiation and development and thus rock strength and deformation behaviour. The influence of grain size on the strength and deformation properties of rock (e.g., Fredrich et al., 1990; Eberhardt et al., 1999), the dominance of grain boundary and intra-grain fracture under tensile and compressive loading conditions, respectively (Mosher et al., 1975), and grain size and shape heterogeneity which plays a key role in the development of grain scale tensile stresses creating the condition for tensile fracture initiation (Lan et al., 2010) have been investigated to understand how rock fractures at the micro-scale. Two of the primary components of a rock’s micro-structure are mineral grains and the boundaries that surround them. The influence of grain boundaries has been the subject of previous investigation. Rosengren and Jaeger (1968) heat treated a coarse grained marble which resulted in the separation of grain boundaries. Testing was completed on both non-heat and heat treated marble specimens allowing for specimens with grain boundary strength and no to little grain boundary strength to be assessed. Under triaxial compression, it was found that at low confining stress magnitudes, the strength difference between the non- and heat-treated marbles was large and with increasing confining stress, the strength of the heat treated marble approached that of the non-heat treated. The micro-structure of rock and its influence on fracturing, strength, and deformation behaviour are still far from being understood as well as the separate influence of
grain boundary and intra-granular strength components. The GBM implemented in the DEM PFC2D is ideally suited to investigate the influence of these two grain strength components. This investigation has already been initiated and is documented in Appendix E.

5.5.2 Linear Coulomb strength envelope slope

Under constant normal stiffness boundary conditions, the normal-shear stress-path reaches then follows the linear Coulomb strength envelope. This envelope limits the state-of-stress in the synthetic specimen and to some extent controls the shear stress magnitude allowed for a given normal stress. Therefore, the slope of the linear Coulomb strength envelope may have an influence on shear rupture zone creation and its characteristics.

5.5.3 Direct shear testing machine for strong brittle intact rock

There is a current need to develop a testing machine that can deform very strong brittle rock specimens in direct shear under both constant normal stress and normal stiffness boundary conditions with shear stress application controlled by a constant acoustic emission event rate allowing for the complete shear stress versus horizontal displacement response of specimens to be determined. This machine currently does not exist. The development of such a machine would allow for shear rupture zone creation in very strong intact brittle rocks to be investigated which has been limited to weak brittle analogue materials (e.g., Cho et al., 2008; Lajtai, 1969) and concrete under constant normal stress boundary conditions. Published results are even more limited for intact brittle rocks and analogue materials under constant normal stiffness boundary conditions where Obert et al. (1976) and Archambault et al. (1992) present the only available results.

5.5.4 Three-dimensional grain based method (GBM)

Shear rupture zones are three-dimensional and involve fracturing along grain boundaries and in grains. A three-dimensional numerical model which simulates a grain structure that can fracture along and in grains may provide a better understanding of the fracturing processes leading to shear rupture zone creation and the ultimate shear rupture surface and zone geometry.
5.5.5 Directional LSS development

In Chapter 4, plates were used to assess the directional Loading System Stiffness (LSS) values. The plates were found to be able to assess LSS differences in different directions. This method can be developed further using spheres. A sphere could potentially allow for a LSS tensor to be developed and thus LSS in any direction to be determined.
References


Appendices

Appendix A Additional information related to Chapter 1

A.1 Experimental set up for synthetic brittle rock tests

In the laboratory, two experimental set ups are suited for investigating the influence of boundary conditions on shear rupture zone creation in specimens of intact brittle rock: (1) triaxial compression; and (2) direct shear.

In triaxial compression, the shear rupture zone location is not known \textit{a priori} and the test is typically conducted by applying an axial load to a cylindrical specimen while the lateral pressure applied to the specimen sidewalls ($\sigma_3'$) is held constant. In this set up, the lateral boundary condition is relative to the specimen sidewalls and not the shear rupture zone being created (Figure A.1). A non-constant, stiffness controlled, lateral pressure can be applied during axial loading by ‘jacketing’ the specimen with sleeves composed of different materials with different deformation properties (e.g., copper as in the case of Hallbauer et al., 1973), by allowing the lateral pressure to increase during deformation where by the lateral pressure increase is a function of the stiffness of the hydraulic system controlling the application of force resisting specimen dilation, or by using a fast-acting servo controlled lateral pressure system programmed to simulate a ‘virtual’ spring.
Figure A.1 Triaxial testing arrangement. (a) Cut away view of triaxial cell (from Hoek and Brown, 1997). (b) Schematic illustrating how the lateral boundary pressure applied to the specimen sidewall is not normal to the shear rupture zone.

In direct shear, the shear rupture zone location and orientation is enforced and thus known. The test is typically conducted using cubic or parallelepiped shaped specimen with Length to Height (L:H) aspect ratios of 0.6:1 to 2:1. First, a normal stress is applied to the top boundary of the specimen followed by shearing of the top and/or bottom with the normal stress held constant during specimen deformation. Normal stiffness boundary conditions can be applied to the top of the specimen through springs of various stiffness or by using a fast-acting servo controlled normal pressure system programmed to simulate a ‘virtual’ spring (e.g., Jiang et al., 2004) (Figure A.2).
As indicated in Chapter 1, the goal of this thesis is to investigate the influence of boundary conditions normal to a shear rupture zone being created. Since boundary conditions cannot be applied normal to a shear rupture zone in triaxial compression and the rupture zone location is not known \textit{a priori}, the direct shear experimental set up is better suited and has been selected for this thesis.
A.2 Simulation with a two dimensional particle based distinct element method (DEM)

Numerical models are used in this thesis to reproduce laboratory test results and to extrapolate to other boundary conditions (i.e., constant stiffness) because:

- the ability to detail the fracturing processes during laboratory testing is still limited, e.g., it is difficult if not impossible to distinguish grain boundary from intra-grain fracturing using acoustic emission (AE) monitoring (e.g., Thompson et al., 2009, used a state-of-the-art AE monitoring system but could not identify AE event locations with sufficient precision and accuracy to distinguish grain boundary versus intra-grain fracturing). According to Lockner and Beeler (2002), grain boundary and intra-grain fracturing occurs during shear rupture zone creation and it is thus necessary to understand their contribution to shear rupture zone creation. Grain boundary and intra-grain fracturing can be tracked in a numerical model;

- the capacity of currently available laboratory direct shear testing machines is currently restricted to weak (compressive strength <40MPa) intact brittle rocks and cannot be used for testing very strong (compressive strength 100 to 250MPa) intact brittle rocks. This limitation is not present in a numerical framework; and

- stresses are continuously rotating in a specimen being deformed in direct shear (Lajtai, 1969; Petit, 1988) and cannot be measured readily in a laboratory specimen. Thus it is not traceable how stress rotation or orientation influences shear rupture zone creation during specimen deformation. The state-of-stress in a specimen during deformation can be tracked in a numerical model.

Numerical models:

- are used to simplify the response of complex natural systems (e.g., intact rock and rock masses) subjected to loads and deformations, and help understand the mechanisms controlling their response (Starfield and Cundall, 1988);
- do not need to be complete explicit representations of the complex systems they are being used to simulate; and

- only need to consider components which are essential for their purpose (Hoek et al., 1990; Jing, 2003).

The purpose of the numerical model in this thesis is to simulate shear rupture zone creation in intact brittle rock. Shear rupture zone creation involves fracturing and wear processes (Scholz, 2002). These processes transition an intact brittle rock material to that of a granular material (e.g., Ben-Zion and Sammis, 2003; Brodsky et al., 2011) in an evolutionary manner as deformations accumulate in the shear rupture zone. The fracturing and wear processes do not occur uniformly across the zone. Thus, shear rupture zones geometrically are irregular, have finite three dimensional extents, and can be composed of a single rupture surface or of multiple interacting shear rupture surfaces forming a segmented shear rupture zone (Twiss and Moores, 2007). Regardless of whether a single shear rupture zone or segmented shear rupture zone is considered, deformation is eventually concentrated in the center of the shear rupture and tappers off to its extent which is defined as a shear rupture’s tip line. The concentration of deformations in the center generates increased fracturing and wear at this location compared to the tip line. Intact material is typically comminuted to a granular material in these locations. An example of this at the field scale is fault gouge. A created shear rupture zone with a segmented shear rupture surface is illustrated in Figure A.3 with locations of granular material labeled. Shear rupture zone creation also involves both grain boundary and intra-grain fracturing (e.g., Wibberley et al., 2000; Lockner and Beeler, 2002). Grain boundary fracture processes dominate pre-peak stress (Mosher et al., 1975; Martin and Chandler, 1994; Tromans and Meech, 2002) and intra-grain fracturing processes dominate post-peak stress under constant stress boundary conditions.
Figure A.3 Schematic of a segmented shear rupture surface surrounded by a damage zone showing locations of previously intact material comminuted into granular material. Image of fault showing comminuted granular material on right (courtesy of Wibberley, 2011).
Since shear rupture zone creation involves the evolutionary transition of an intact material to that of a granular material, the fracturing of grain boundaries, and fracture creation inside grains, a numerical tool that takes these processes into account is considered to be an essential component of the rupture process investigation.

The current options available to simulate the grain structure of intact brittle rock have been summarized by Lan et al. (2010). Polygonal networks may be more representative of grain structures than circular shaped grains as assumed in standard particle based codes, e.g., Hazzard et al. (2000), or the triangular and square shaped elements used in various finite element codes (e.g., Tang, 1997; Valley et al., 2010). Therefore, a grain structure composed of a polygonal network is chosen for this thesis.

Only two commercially available numerical tools simulate polygonal grain structures that can fracture. Both use a Grain Based Method (GBM) and are two-dimensional: (1) UDEC-GBM (Lan et al., 2010; Itasca, 2011); and (2) PFC2D-GBM (Itasca, 2011; Potyondy, 2010). Both UDEC (Lan et al., 2010) and PFC2D (e.g., Diederichs, 1999; Hazzard et al., 2000; Potyondy and Cundall, 2004; Cho et al., 2008) have been previously used to simulate fracturing leading to the rupture of intact brittle rock. The current formation of the UDEC-GBM allows only for grain boundary fracturing and is therefore limited to investigations pre-peak stress where this fracturing type dominates. In the PFC2D-GBM, the user can control grain boundary and intra-grain fracturing. The PFC2D-GBM also allows for the process of an intact material transitioning into a granular material to be taken into account. PFC2D models the behaviour of rigid circular particles that are bonded along their contacts creating an intact synthetic specimen. When the synthetic specimen is then deformed, bonds break and individual particles or particle assemblies are comminuted which are analogous to the granular material formed during shear rupture zone creation. PFC2D and its GBM are therefore best suited for the investigation of fracturing processes leading to shear rupture zone creation both pre- and post-peak stress and has been chosen for this thesis.

An added advantage of PFC2D over other codes (e.g., ELFEN, Rockfield, 2007, which is a two dimensional hybrid finite element discrete element code) is that it simulates fracture processes leading to shear rupture zone creation explicitly with no empirical or constitutive relations used.
to define, *a priori*, fracturing or its influence on material behaviour such as deformation response to loading (Cundall, 2000).

The direct shear experimental set up is used in this thesis for reasons outlined in Appendix A.1. Two dimensional simplification of the direct shear test is possible because direct shear induces a plane strain condition in specimens being deformed (e.g., Lajtai, 1969; Potts et al., 1987; Petit, 1988) and the resulting shear rupture is perpendicular to the out-of-plane constant strain boundary. Therefore, while shear rupture zones have a third dimension in nature, the rupture zone’s creation and propagation process in direct shear is propagating in the direction of movement and without out-of-plane displacement components and can thus be considered as a two dimensional failure process similar to processes encountered during rough joint shear.

**Appendix B Additional information related to Chapter 2**

**B.1 Deformation rate applied to lower shear box wall**

Diederichs (1999) found that faster deformation rates in particle based DEMs resulted in less brittle post-peak behaviour of the axial stress versus axial strain curve for synthetic specimens loaded in biaxial compression. The influence of deformation rate (velocity) was investigated under constant normal stress boundary conditions for the 15MPa and 60MPa normal stress cases to ensure that the deformation rate influence did not change at low or high normal stress magnitudes and that the rate was sufficiently slow to assume quasi-static deformation conditions. It is important that the deformation rate is sufficiently slow to allow perturbations to be dissipated throughout the synthetic specimen faster than new loads and displacements are applied. It was found that a shearing rate slower than 0.08m/s did not influence the peak stress achieved but had some influence on the post-peak response of the synthetic specimen (Figure B.1). Deformation rate was found to have more influence at the lower normal stress assessed (15MPa) with the difference between the 0.08 and 0.01m/s being only minimal for the 60MPa normal stress assessment.

The 0.08m/s assessed deformation rate is slightly slower than the one used by Cho et al. (2008) (rate of 0.1m/s) for direct shear simulations calibrated to the response of a brittle analogue material. Cho et al. found that deformation rates lower than 0.1m/s had no effect on peak shear
stress. They did not investigate the influence of deformation rate on post-peak shear stress versus horizontal displacement response.

A deformation rate of 0.04m/s was adopted and determined to be acceptable for the reduction of runtime and the minimization of numerical noise. Relative to Diederichs’ results for his choice of deformation rate, the compromise is similar.

Figure B.1 Influence of deformation rate (velocity) under constant normal stress boundary conditions as applied to the lower shear box wall on peak shear stress and the post-peak behaviour of the shear stress versus horizontal displacement curve. Values in brackets are the approximate number of days required to run each simulation.
B.2 Preliminary calibration

A preliminary calibration was completed for five synthetic specimen realizations (specimens 1 to 5). Each synthetic specimen was generated using different seeds for the grain structure and setting of the particle bond and smooth-joint micro-parameters. Each synthetic specimen was simulated at normal stresses of 5, 15, 25, 40, 60, and 90MPa. This preliminary calibration was completed to refine the calibration and assess the sensitivity of different synthetic specimen realizations. The PFC2D micro-parameters of the preliminary calibration synthetic specimen are summarized in Table B.2. The final calibration micro-parameters are also re-summarized here in Table B.1 to allow for easy comparison between the two.

The shear stress versus horizontal displacement, orientation of the major principal stress internal to the synthetic specimen, and total cumulative fracture count versus horizontal displacement are shown in Figures B.2 to B.7. While there are some differences between the simulation results, they are essentially the same. The final rupture zone geometries and rupture zone creation process and mechanisms (not shown) are also similar. This shows that there is little variability between realizations of the synthetic specimens allowing for one realization (i.e., seed) to be studied opposed to multiple.

The preliminary calibration peak shear strengths are shown in Figure B.8. The results match the laboratory results of Petit (1988) up to a normal stress of approximately 40MPa. After this point, the simulated peak shear strength becomes lower than the laboratory data. The deviation in peak shear strength between the laboratory and preliminary calibration results corresponds approximately to the point when fractures are starting to be induced in the synthetic specimen during normal stress application. The normal stress magnitude this begins to occur at is approximately one third times the uniaxial compressive strength (UCS) of the preliminary calibration synthetic specimen. Therefore, the deflection in the strength envelope could be due to a pre-damage process due to the applied normal stress surpassing the crack initiation threshold for the synthetic specimen. Slope changes similar to this have been reported by Lajtai (1969) in his plaster-of-Paris specimen. Lajtai’s slope change also occurred at approximately 0.33UCS.

It was found that by increasing the intra-grain properties (normal bond strength and friction coefficient of the grains), strength increases occurred at higher normal stresses as compared to
lower normal stresses which were minimally influenced. An interesting conclusion is that
classified strengths are more controlled by intra-grain micro-parameters opposed to grain
boundary micro-parameters. This is supported by the results of Mosher et al. (1975) who found
that grain boundary fractures predominated in tension and intra-granular fractures predominated
in compression.
<table>
<thead>
<tr>
<th>Element</th>
<th>Parameter</th>
<th>Grain 1</th>
<th>Grain 2</th>
<th>Grain 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Quartz</td>
<td>Calcite</td>
<td>Feldspar</td>
</tr>
<tr>
<td>Grain size</td>
<td>Volume composition (%)</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Minimum radius, $R_{min}$ (mm)</td>
<td>0.44</td>
<td>0.38</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.86</td>
<td>3.33</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>Minimum radius of particles forming grains, $R_{min}$ (mm)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.66</td>
<td>1.66</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Particle normal to shear stiffness ratio, $k_n/k_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Density (g/cm$^3$)</td>
<td>2650</td>
<td>2710</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>Modulus, $E_c$ (MPa)</td>
<td>98000</td>
<td>30000</td>
<td>89000</td>
</tr>
<tr>
<td></td>
<td>Friction coefficient, $\mu$</td>
<td>0.83</td>
<td>1.05</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallel bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth-joints</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Element</td>
<td>Parameter</td>
<td>Grain 1</td>
<td>Grain 2</td>
<td>Grain 3</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>------------------------------------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td><strong>Grain size</strong></td>
<td>Volume composition (%)</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Minimum radius, $R_{min}$ (mm)</td>
<td>0.44</td>
<td>0.38</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.86</td>
<td>3.33</td>
<td>2.14</td>
</tr>
<tr>
<td><strong>Particles forming grains</strong></td>
<td>Minimum radius of particles forming grains, $R_{min}$ (mm)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Maximum to minimum radius ratio, $R_{max}/R_{min}$</td>
<td>1.66</td>
<td>1.66</td>
<td>1.66</td>
</tr>
<tr>
<td></td>
<td>Particle normal to shear stiffness ratio, $k_n/k_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Density (g/cm³)</td>
<td>2650</td>
<td>2710</td>
<td>2600</td>
</tr>
<tr>
<td></td>
<td>Modulus, $E_c$ (MPa)</td>
<td>98000</td>
<td>30000</td>
<td>89000</td>
</tr>
<tr>
<td></td>
<td>Friction coefficient, $\mu$</td>
<td>0.60</td>
<td>0.80</td>
<td>0.65</td>
</tr>
<tr>
<td><strong>Parallel bonds</strong></td>
<td>Modulus, $\bar{E}_c$ (MPa)</td>
<td>98000</td>
<td>30000</td>
<td>89000</td>
</tr>
<tr>
<td></td>
<td>Normal to shear stiffness ratio, $\bar{k}_n/\bar{k}_s$</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>Tensile strength, $\bar{\sigma}_c$ (MPa)</td>
<td>190</td>
<td>90</td>
<td>210</td>
</tr>
<tr>
<td><strong>Smooth-joints</strong></td>
<td>Cohesion, $\bar{c}$ (MPa)</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Friction angle, $\bar{\phi}$ (°)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Tensile and shear strength standard deviation</td>
<td>50</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Normal stiffness factor, $\bar{k}_n$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Shear stiffness factor, $\bar{k}_s$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>Tensile strength, $\sigma_c$ (MPa)</td>
<td>28</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Cohesion, $c$ (MPa)</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Friction angle, $\phi$ (°)</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Residual friction coefficient, $\mu_r$</td>
<td>0.27</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Figure B.2 5MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.3 15MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.4 25MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.5 40MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.6 60MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.7 90MPa normal stress preliminary calibration results. (a) Shear stress versus horizontal displacement response. (b) Orientation of the major principal stress internal to the synthetic specimen. (c) Total cumulative fracture count versus horizontal displacement.
Figure B.8 Normal stress versus peak shear stress for the laboratory results of Petit (1988), the final calibration results, and the preliminary calibration results.

B.3 Constant normal stress rupture zone image summary

Images of the synthetic specimen after application of normal stress are summarized in Figure B.9 and show when fractures are induced in the specimens after normal stress application alone.
Figure B.9 Synthetic specimen after application of normal stress. (a) 5MPa. (b) 15MPa. (c) 25MPa. (d) 40MPa. (e) 60MPa. (f) 90MPa. Orange – grain boundary, black – intra-grain tensile fracture. Fracturing due to normal stress application alone starts at 60MPa.
Rupture zone images representative of peak, post-peak, and final states are summarized in Figures B.10 and B.11.

<table>
<thead>
<tr>
<th>5MPa</th>
<th>Peak</th>
<th>Post-Peak</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)-(c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)-(f)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g)-(i)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>15MPa</th>
<th>Peak</th>
<th>Post-Peak</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>25MPa</th>
<th>Peak</th>
<th>Post-Peak</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>(g)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure B.10 Constant normal stress boundary condition shear rupture zone images for normal stress magnitudes of (a)-(c) 5MPa, (d)-(f) 15MPa, and (g)-(i) 25MPa. Orange – grain boundary, black – intra-grain tensile fracture. All specimens 50mm x 50mm.
<table>
<thead>
<tr>
<th></th>
<th>Peak</th>
<th>Post-Peak</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>40MPa</strong></td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>60MPa</strong></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>90MPa</strong></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure B.11 Constant normal stress boundary condition shear rupture zone images for normal stress magnitudes of (a)-(c) 40MPa, (d)-(f) 60MPa, and (g)-(i) 90MPa. Orange – grain boundary, black – intra-grain tensile fracture. All specimens 50mm x 50mm.
B.4 Shear box dilation

Figure B.12 shows that the synthetic specimen is dilatant for all normal stress magnitudes simulated indicating that a simple shear condition is not reached. Simple shear requires no specimen volume change to occur.

Figure B.12 Vertical displacement of the upper shear box wall (Wall 2) at the various normal stresses simulated.
Appendix C Additional information related to Chapter 3

C.1 Major principal stress orientation internal to the synthetic specimens

Figure C.1 Orientation of the major principal stress internal to the synthetic specimens (counter clockwise from horizontal). (a) 5MPa initial applied normal stress. (b) 25MPa initial applied normal stress. (c) 40MPa initial applied normal stress.
C.2 Force chain orientation in center of synthetic specimens

Figure C.2 Orientation of force chains in center of the synthetic specimens showing increasing angle at increasing applied normal stress and cap modulus value.
C.3 Constant normal stiffness: 25MPa initial applied normal stress simulation results

Figure C.3 Constant normal stiffness boundary condition rupture zone creation images with initial applied normal stress of 25MPa at selected horizontal displacement magnitudes for cap modulus values of: (a)-(f) 10GPa; (g)-(l) 30GPa; and (m)-(r) 100GPa (orange – grain boundary, black – intra-grain tensile fracture).
Figure C.4 Linked mechanical response of the synthetic specimen under constant normal stiffness boundary conditions with initial applied normal stress of 25MPa. (a) Shear stress versus horizontal displacement response. (b) Normal-shear stress-path. (c) Development of minor principal stress with horizontal displacement. (d) Principal stress-path internal to the synthetic specimen.
C.4 Constant normal stiffness: 5, 25, and 40MPa initial applied normal stress and 30GPa cap modulus fracture angles

Figure C.5 5MPa initial applied normal stress and 30GPa cap modulus fracture angle analysis. See Figure C.6 for locations of rupture zone images along the shear stress versus horizontal displacement curve. (orange – grain boundary, black – intra-grain tensile fracture). In the fracture sketches, black – new and grey precursory fracturing from the previous rupture image.
Figure C.6 5MPa initial applied normal stress and 30GPa cap modulus shear stress versus horizontal displacement curve indicating locations of rupture zone images in Figure C.5. Also showing the internal principal stress orientation (counter clockwise from horizontal), grain boundary, intra-grain, and total fracture rates and cumulative fracture counts.
Figure C.7 25MPa initial applied normal stress and 30GPa cap modulus fracture angle analysis. See Figure C.8 for locations of rupture zone images along the shear stress versus horizontal displacement curve. (orange – grain boundary, black – intra-grain tensile fracture). In the fracture sketches, black – new and grey precursory fracturing from the previous rupture image.
Figure C.8 25MPa initial applied normal stress and 30GPa cap modulus shear stress versus horizontal displacement curve indicating locations of rupture zone images in Figure C.7. Also showing the internal principal stress orientation (counter clockwise from horizontal), grain boundary, intra-grain, and total fracture rates and cumulative fracture counts.
Figure C.9 40MPa initial applied normal stress and 30GPa cap modulus fracture angle analysis. See Figure C.10 for locations of rupture zone images along the shear stress versus horizontal displacement curve. (orange – grain boundary, black – intra-grain tensile fracture). In the fracture sketches, black – new and grey precursory fracturing from the previous rupture image.
Figure C.10 40MPa initial applied normal stress and 30GPa cap modulus shear stress versus horizontal displacement curve indicating locations of rupture zone images in Figure C.9. Also showing the internal principal stress orientation (counter clockwise from horizontal), grain boundary, intra-grain, and total fracture rates and cumulative fracture counts.
Appendix D Additional information related to Chapter 4

Appendix D.1 Description of sub-level stoping with backfill

Sub-level stoping is ideally applied to moderate (>60°) to steeply dipping (>75°) tabular ore deposits. An undercut and overcut are developed at a vertical interval which depends on the stope height determined during the design stage. Cross cuts are then excavated at a spacing which depends on the stope width also determined during the design stage. At the Williams and Golden Giant mines, primary-secondary stoping is used. In this method, primary stopes are mined first and then filled. Secondary stopes are then mined. In the Williams/Golden Giant mines, the stopes are sequenced to maintain a triangular or chevron shape. This is accomplished by mining vertically with a lead stope, then outward along the slope or rill of the triangle/cheveron toward its base (Bronkhorst and Brouwer, 2001). A longsectional view of the primary-secondary stoping at Williams mine is schematically shown in Figure D.1.

Figure D.1 Schematic longsection of primary-secondary stoping at Williams mine (from Bronkhorst and Brouwer, 2001).
Appendix D.2 Geological strength index (GSI) chart

The GSI chart from Hoek and Marinos (2000) indicating the range of values for the Golden Giant and Williams mine rock masses is shown in Figure D.2.

![GSI chart](image)

Figure D.2 Geological strength index (GSI) chart with range of values for the Golden Giant and Williams mine rock masses.
Figure D.3 Volumetric LSS assessment using a cube in the Center of: (a) *Case 1*; and (b) *Case 2* pillars at the Golden Giant and Williams mines, respectively.
Figure D.4 Directional LSS assessment using plates in the Center of: (a) Case 1; and (b) Case 2 pillars at the Golden Giant and Williams mines, respectively.

**Perpendicular**: plate face oriented E-W, approx. in direction of intermediate principal stress.

**Horizontal**: plate face oriented up-down, approx. in direction of minor principal stress.

**Parallel**: plate face oriented N-S, approx. in direction of major principal stress.
Appendix D.4 MAP3D v58 input parameters

Pre-mining stress state:
- $\Delta \sigma_a$ variation -0.0437 MPa/m
- $\Delta \sigma_b$ variation -0.0299 MPa/m
- $\Delta \sigma_c$ variation -0.0214 MPa/m
- $\Delta \sigma_a$ tend 358°
- $\Delta \sigma_a$ plunge -10°
- $\Delta \sigma_c$ trend 0°

Control parameters:
- Maximum # of time steps (NLD) 10000
- Maximum # of iterations (NIT) 10000
- Stress tolerance (STOL) 0.1
- Relaxation parameter (RPAR) 1.2
- Element length (AL) 4
- Grid spacing (AG) 2
- Grid discretize (DOL) 2
- Element discretize (DON) 1
- Matrix lumping (DOC) 2
- Element lumping (DOE) 4
- Grid lumping (DOG) 4
- Aspect ratio (DOR) 5
Appendix D.5 Depth of excavation damage correlation

The correlation between the depth of interpreted stress induced damage based on SMART-Cable 4 and a normalized deviatoric stress criterion is shown in Figure D.5.

Figure D.5 Depth of interpreted stress induced damage based on SMART-Cable 4 compared to the deviatoric criterion of $(\sigma_1 - \sigma_3/178) = 0.52$. 
Appendix D.6 Golden Giant mine MAP3D stope sequence

The yearly stope sequence used in the MAP3D models of the Golden Giant mine are shown in Figures D.6 to D.7.

Figure D.6 Golden Giant stopes mined to 12-2000 and 12-2001.
Figure D.7 Golden Giant stopes mined 12-2002 and 12-2003.
Appendix D.7 Williams mine MAP3D stope sequence

The yearly stope sequence used in the MAP3D models of the Williams mine are shown in Figures D.8 to D.12.

- **Step 1:** No mining

- **Step 2:** Footwall drifting

- **Step 3:** Mining to 12-2000

Figure D.8 Williams stopes mined 12-2000.
Figure D.9 Williams stopes mined 12-2001.
Figure D.10 Williams stopes mined 12-2002.
Figure D.11 Williams stopes mined 12-2003.
Figure D.12 Williams stopes mined 12-2004.
Appendix E DEM simulation of direct shear: grain boundary and mineral grain strength component influence on shear rupture

Abstract

The influence of mineral grain and grain boundary strength is investigated in intact (non-jointed) brittle rock specimens subjected to direct shear using a particle based Distinct Element Method (DEM) and its embedded Grain Based Method (GBM). The adopted numerical approach allows for the independent control of grain boundary and mineral grain strength. The investigation reveals that in direct shear the normal stress ($\sigma_n$) applied to a rock specimen relative to its uniaxial compressive strength (UCS) determines the resulting rupture mechanism, the ultimate rupture zone geometry and thus its shear stress versus horizontal displacement response. This allowed for the development of a rupture matrix based on this controlling parameter. Mineral grain strength reductions result in the lowering of the apparent cohesion intercept of the linear Coulomb strength envelope, while grain boundary strength reductions change the linear Coulomb strength envelope to a bi-linear or curved shape. The impact of grain boundary strength is only relevant at low normal stress (or $\sigma_n$/UCS ratios <0.17) where tensile and dilatant rupture mechanisms dominate. Once shear rupture begins to be the dominate rupture mechanism in a brittle rock, the influence of weakened grain boundaries is minimized.
**E1.0 Introduction**

An intact brittle rock, assuming no presence of fluids or gases, is predominately composed of mineral grains separated by grain boundaries; the former may or may not contain cracks or other flaws. The overall peak strength of an intact brittle rock is therefore composed of three strength components derived from: (1) grain boundary strengths; (2) mineral grain strengths; and (3) grain ‘interlock’. There is evidence suggesting that these three strength components affect the overall peak strength of an intact brittle rock specimen differently depending on the confinement magnitude and therefore the fracturing processes leading to rupture zone creation and behaviour (such as load-displacement response). The purpose of this thesis section is to investigate grain boundary and mineral grain strength components on shear rupture zone creation and strength through the use of a synthetic rock model with a grain structure.

Cresswell and Barton (2003) tested specimens of locked sand free of cementation but with an interlocked fabric having a small measurable cohesion intercept and specimens which had trace iron oxide and clay cementation which were noticeably more cohesive and easier to sample. Based on the direct shear testing results reported by them, the change in rupture mechanism in direct shear at increasing normal stresses could generate curved or bi-linear peak Coulomb strength envelopes (i.e., similar in shape to the strength envelopes for saw-tooth discontinuities reported by Patton, 1966). At low normal stresses, the strength difference between the non-cemented and cemented specimens was large and tended towards the peak strength envelope of the cemented specimens at higher normal stresses (Figure E.1). The strength envelope shape change was interpreted by Cresswell and Barton (2003) to be due to tensile rupture at low and shear rupture at the higher normal stress magnitudes tested. The interpretation was supported by the results of Lajtai (1969) where the rupture surfaces generated in intact specimens of plaster-of-Paris deformed in direct shear at various normal stresses were used to infer the dominate specimen rupture mechanism which also transitioned from tensile to shear at increasing normal stresses.
Figure E.1 Peak strength envelopes determined from direct shear testing plotted in normal-shear stress space for cemented and non-cemented locked sand showing the bi-linear character of the envelopes due to dominate failure mechanism and how the non-cemented sand’s peak strength envelope tends to the peak strength envelope for the cemented sand at higher normal stresses (re-plotted and re-interpreted based on data from Cresswell and Barton, 2003).

The triaxial testing results of Rosengren and Jaeger (1968) (Figure E.2) also provide insight into the influence of grain boundary strength and grain interlocking and the shape of the peak strength envelope. Rosengren and Jaeger heat treated a coarse grained marble which resulted in the separation of grain boundaries and the retention of a highly interlocked grain structure of low porosity (<4%). Testing was completed on both the non-heat and heat treated marble specimens allowing for specimens with grain boundary strength and no to little grain boundary strength to be assessed in the highly interlocked structure. Under triaxial compression, it was found that at
low confining stress magnitudes, the strength difference between the non- and heat-treated marbles was large and with increasing confining stresses, the strength of the heat treated marble approached but never reached that of the non-heat treated (i.e., intact) (Figure E.2). Confining stress in triaxial compression acts to suppress rock specimen dilation transitioning the failure mechanism from dominantly tensile to shear rupture and therefore may be one of the factors contributing to the strength increases at higher confining stresses as was shown by Bahrani et al. (2011). The results of Rosengren and Jaeger (1968) for non- and heat-treated interlocked marble specimens show the same impact to the peak strength envelope determined from the direct shear testing results of Cresswell and Barton (2003) for locked sands where the influence of weak grain boundaries in a material with an interlocking grain structure is minimized at higher normal or confining stresses once the failure mode the specimen changes from dominantly tensile to shear.

Figure E.2 Peak strength data points determined from triaxial testing plotted in principal stress space for non-heat treated and heat-treated marble specimens showing how the heat-treated specimen strength tends towards the non-heat treated at higher confining pressures once the failure mechanism changes from dominantly tensile to shear (data from Rosengren and Jaeger, 1968).
The influence of grain size heterogeneity and thus the interlocked nature of hard brittle rocks in uniaxial compression was investigated by Lan et al. (2010) and later by Bewick et al. (2012). The results of Lan et al. and Bewick et al. show the impact of mineral grain strength on failure and are thus relevant to summarize. Lan et al. (2010) compared calibrated synthetic brittle rock specimens of heterogeneous grain size distributions to synthetic specimens with the same assigned model parameters but a homogeneous grain size distribution. They concluded that grain size heterogeneity had a dominate effect on inducing large tensile stress localizations at relatively low strains in the synthetic rock and thus impacted tensile crack initiation and the peak strength of a specimen. Bewick et al. (2012) found that the results of Lan et al. were bias because they used a regular Voronoi tessellation (i.e., honeycomb geometry) as their homogeneous model grain structure and a single loading direction. The honeycomb geometry is anisotropic and the direction of loading highly impacts model behaviour as shown schematically in Figure E.3. The single loading direction evaluated by Lan et al. (2010) is such that little tensile stress is generated along grain boundaries (i.e., Figure E.3b and E.3e). This is significant because the grains in their models were not breakable and the failure behaviour is highly dependent on the ability of grain boundaries to break. Bewick et al. (2012) investigated the impact of the two anisotropic honeycomb grain structures on tensile stress generation in an elastic framework and then the impact of breakable grains for DEM synthetic brittle rocks composed of both heterogeneous and homogeneous grain structures and concluded that the synthetic rocks with homogeneous grain size distribution with non-breakable grains resulted in higher strengths but the homogeneous grain size distributions with breakable grains resulted in similar strengths to the synthetic specimens with heterogeneous grain size distributions. Therefore, peak strength is not highly dependent on the heterogeneity induced by grain geometry; it is dependent on mineral grain strength. These results thus suggest that in interlocked specimens with low dilation potential (i.e., the honeycomb grain structure shown in Figure E.3b and E.3e), the mineral grain strength is the dominate active strength component and if mineral grain strength could be reduced in identical specimens, the effect would be to lower the compressive strength of the material.
Figure E.3: (a-b) Two extreme honeycomb grain structures where (a) is highly tensile stress generating and (b) is not. The structure shown in (b) was used by Lan et al. (2010). (c-d) Show the honeycomb structure in (a) and its similarity to the trellis structure in (d) which is a highly extensional structure when deformed in the direction shown. This structure limits the need for failure to occur through mineral grains. (e) Honeycomb grain structure in (b) showing its non-tensile stress generating potential requiring failure to occur through mineral grains. (d) re-drawn from Trollope (1969).
There are clearly interplays between the influence of grain boundary and mineral grain strength in grain structures with interlocking that appear to be dependent on the dilation potential and thus the dominate failure mechanism of a material. Dusseault and Morgenstern (1978), for Athabasca Oil Sands, suggested that the mechanism responsible for the change in the influence of grain boundary and mineral grain strength on the peak strength envelope of a material with high grain interlocking is the suppression of dilatant specimen behaviour which forces failure to occur through the locked mineral grain structure opposed to allowing the grains to more freely dilate and move around one another. This is the same mechanism found by Patton (1966) to influence the active strength components associated with the shear deformation of saw-tooth discontinuity surfaces (Figure E.4a). Patton found that at low normal stress magnitudes, it was possible for the saw-tooth asperities or override each other. The strength at these low normal stresses was one composed of friction ($\phi$) plus dilation angle ($i$). Once shear ride could no longer occur over the asperities, due to normal stress magnitudes sufficient to suppress the ability of the discontinuity to dilate, failure was forced through the asperities transitioning the strength at these higher normal stresses to one composed of friction ($\phi$) plus apparent cohesion ($c'$) as schematically illustrated in Figure E.4a. It follows that for intact rock with an interlocked grain structure, at low normal stresses, dilation and dominantly extensional failure processes allow for breakage of grain boundaries and shear ride over and around grains and thus strength deriving mainly from friction ($\phi$) plus dilation angle ($i$) (as shown schematically in Figure E.4b) and at higher normal stresses once dilation to some extent is suppressed and dominantly shear failure processes dominate, failure is forced through the mineral grains (as shown schematically in Figure E.4c) with strength deriving mainly from friction ($\phi$) plus apparent cohesion ($c'$).

Therefore, reductions in grain boundary strength should have a larger impact on overall peak strength when a specimen’s behaviour is dilatant / extensional and be minimized once dilation is suppressed and shear rupture occurs (as shown in Figure E.5b). Likewise, reductions in mineral grain strength should impact apparent cohesion such that lower mineral grain strengths should reduce the apparent cohesive strength (as shown in Figure E.5a). While there is evidence to suggest the impact of dilation suppression and thus failure mechanism on the influence of grain boundary and mineral grain strength on the overall peak strength of an intact brittle rock, it is not clear how grain boundary and mineral grain strengths separately will influence the strength of an intact rock or the resulting failure geometries, and load-displacement response when deformed.
In this section of the thesis, the shear rupture process is investigated and the influence of mineral grain and grain boundary strength, on the peak and ultimate strength envelopes, rupture zone geometries, and shear stress versus horizontal displacement responses of an intact brittle rock composed of a highly interlocked grain structure deformed in direct shear using a calibrated particle based Distinct Element Method (DEM) and its embedded Grain Based Method (GBM). The GBM implemented in the DEM allows for both the fracturing of grain boundaries and mineral grains and is thus ideally suited to investigate the influence of these two rock specimen strength components. Intact (non-jointed) brittle rock when deformed in direct shear ruptures differently depending on the applied normal stress (Lajtai, 1969; Petit, 1988; and as shown in Chapter 2 of the thesis) with the rupture mechanism transitioning from tensile splitting to shear as normal stress magnitudes increase. Therefore, grain boundary and mineral grain strengths are anticipated to have different contributions to the overall peak strength of the material depending on the dominate failure mechanism and potentially impact the resulting failure geometry and shear stress versus horizontal displacement response. Based on the findings, a rupture characteristic classification is introduced that permits the differentiation of shear rupture structures and related strength characteristics based on the ratio of applied normal stress to a specimen’s uniaxial compressive strength (i.e., $\sigma_n / \text{UCS}$ ratio). A detailed description of the simulation methodology, synthetic specimen generation, calibration, and boundary conditions are presented in Chapter 2 of the thesis.
Figure E.4 (a) Patton’s Coulomb strength envelopes in normal-shear stress space showing bi-linear shape and strength components. (1) Indicates dilatant or extensional failure along the saw-tooth discontinuity such that the asperities ride over each other (friction angle, $\phi$, and dilation angle, $i$, Coulomb strength components). (2) Indicates a dilation suppression condition forcing failure to occur through the saw-tooth asperities (friction angle, $\phi$, and apparent cohesion, $c'$, Coulomb strength parameters). (b) Schematic regular honeycomb grain structure of an idealized intact rock showing shear ride over asperities and tensile opening along grain boundaries. (c) Zoomed in view of grey shaded area in (b) showing hypothetical failure for (i) shear ride over asperities with dilation and (ii) no shear ride over asperities with failure through mineral grains.
Figure E.5 (a) The lowering of apparent cohesion \( (c') \) due to the influence of reductions in mineral grain strength. (b) The change in friction \( (\phi) \) and dilation angle \( (i) \) due to the influence of reductions in grain boundary strength.
E1.1 Background

The particle based DEM as implemented in the commercially available Particle Flow Code in Two Dimensions (PFC2D v4.0-190) (Itasca, 2011) and its embedded grain based method (GBM) (Potyondy, 2010) are used to simulate the creation of rupture zones in numerically generated synthetic specimens (deformed in direct shear) that have been calibrated to the shear strength, tensile strength, post-peak shear stress versus horizontal displacement response, and fracture angles of Lodève sandstone ruptured in direct shear under constant normal stress boundary conditions (Petit, 1988; Wibberley et al., 2000). The methodology, synthetic specimen generation, calibration, and the boundary conditions used in this investigation are described in detail in Chapter 2 of the thesis. The direct shear synthetic specimen (50mm x 50mm) is a simplified representation of Lodève sandstone and is composed of 41,388 particles and approximately 1405 mineral grains (composition of 50% feldspar, 30% calcite, and 20% quartz) with an overall average grain size of 1.4mm. The synthetic specimens are sheared at six normal stresses of 5, 15, 25, 40, 60, and 90MPa. Separate strengths are assigned to the grain boundaries and each mineral grain type allowing for the separate influence of grain boundary and mineral grain strength to be assessed.

Mineral grain and grain boundary tensile bond strengths are incrementally reduced in the synthetic specimen from the baseline calibrated values (0% strength reduction) to investigate the influence of strong versus weak mineral grains (assessment of mineral grain strength at 50 and 75% strength reductions from the calibrated values) and strong versus weak grain boundaries (assessment of grain boundary strength at 25 and 50% strength reductions from the calibrated values); for their influence on the synthetic specimen’s rupture zone creation process, shear stress versus horizontal displacement response, and peak and ultimate shear strength envelopes when deformed in direct shear. Each of the strength reductions is investigated in direct shear at the six previously mentioned normal stresses generating five suites of direct shear simulation results (including the calibration results with the baseline 0% strength reduction). The calibration of the synthetic specimen to the available characteristics of the Lodève sandstone resulted in individual bond breakage in the synthetic specimen in tension alone. Therefore, only the influence of tensile bond strength is considered in this investigation.
The grain boundary and mineral grain strength reductions and resulting tensile bond strengths assessed in the synthetic specimens are summarized in Table E.1 alongside the bond strengths resulting from the calibration to the Lodève sandstone rupture characteristics (i.e., 0% strength reduction, see Chapter 2 Table 3). The synthetic specimens with the various bond strength reductions (Table 1) were first simulated in uniaxial compression and direct tension (synthetic specimen size 50mm x 25mm, length to height ratio of 2.0) to determine each of the strength reductions suite’s respective uniaxial compressive strength (UCS), Young’s modulus (E), and direct tensile strength ($\sigma_t$) (Table E.2). The compression and tension testing results summarized in Table 2 show that the grain boundary strength reductions minimally influence the UCS and have a large impact on the direct tensile strength of the synthetic specimen. Conversely, mineral grain strength reductions minimally influence the direct tensile strength and have a large impact on the UCS of the synthetic specimen. For all the bond strength reductions, the synthetic specimen’s Young’s modulus is essentially not influenced.

The intact rock strength-modulus classification chart for intact sedimentary rock developed by Deere (1969) (Figure E.6) was used to classify the resulting strength and modulus characteristics of the synthetic specimens with their respective bond strength reductions into strength classes (i.e., high, medium, and low strength). The calibration (0% strength reduction), and grain boundary strength reductions (25 and 50%) all relate to high strength sandstone (Table E.2; Figure E.6). The mineral grain strength reductions relate to medium strength sandstone (50% reduction) and low strength sandstone (75% reduction) (Table E.2; Figure E.6). The strength classes identified above (high, medium, and low strength sandstone) for the mineral grain bond strength reductions are used for reference in the following sections.
Figure E.6: Classification of intact sedimentary rocks based on laboratory uniaxial compressive strength (UCS) and Young’s modulus (E) showing locations of the UCS and E for the various synthetic specimens with mineral grain tensile bond strength reductions of 75%, 50%, and 0% and grain boundary tensile bond strength reductions (all high strength sandstone) (from Martin et al., 2003; based on Deere, 1969).
Table E.1: Mineral grain and grain boundary strength reductions and resulting tensile bond strengths.

<table>
<thead>
<tr>
<th>Grain Type</th>
<th>Mineral Grain Tensile Bond Strength (MPa)</th>
<th>Grain Boundary Tensile Bond Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strength Reduction</td>
<td>0%</td>
</tr>
<tr>
<td>Quartz</td>
<td>245</td>
<td>123</td>
</tr>
<tr>
<td>Calcite</td>
<td>112</td>
<td>56</td>
</tr>
<tr>
<td>Feldspar</td>
<td>265</td>
<td>133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grain Type</th>
<th>Standard Deviation</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Calcite</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Feldspar</td>
<td>50</td>
<td>25</td>
</tr>
</tbody>
</table>

0% strength reduction is for the bond strengths resulting from the calibration to the Lodève sandstone rupture characteristics are described in Chapter 2 of the thesis.
Table E.2: Mineral grain and grain boundary strength reductions and resulting synthetic specimen strength, modulus, and strength class.

<table>
<thead>
<tr>
<th>Grain Boundary Bond Strength Reduction</th>
<th>Uniaxial Compressive Strength, UCS, (MPa)</th>
<th>Young's Modulus, $E$, (GPa)</th>
<th>Direct Tensile Strength, $\sigma_t$, (MPa)</th>
<th>E/UCS</th>
<th>Deere (1969) Strength Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>146</td>
<td>46</td>
<td>-16</td>
<td>317</td>
<td>High strength sandstone</td>
</tr>
<tr>
<td>25%</td>
<td>154</td>
<td>45</td>
<td>-12</td>
<td>292</td>
<td>High strength sandstone</td>
</tr>
<tr>
<td>50%</td>
<td>131</td>
<td>45</td>
<td>-8</td>
<td>344</td>
<td>High strength sandstone</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mineral Grain Bond Strength Reduction</th>
<th>Uniaxial Compressive Strength (MPa)</th>
<th>Young's Modulus, $E$, (GPa)</th>
<th>Direct Tensile Strength, $\sigma_t$, (MPa)</th>
<th>E/UCS</th>
<th>Deere (1969) Strength Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>146</td>
<td>46</td>
<td>-16</td>
<td>317</td>
<td>High strength sandstone</td>
</tr>
<tr>
<td>50%</td>
<td>93</td>
<td>46</td>
<td>-16</td>
<td>495</td>
<td>Medium strength sandstone</td>
</tr>
<tr>
<td>75%</td>
<td>54</td>
<td>46</td>
<td>-12</td>
<td>852</td>
<td>Low strength sandstone</td>
</tr>
</tbody>
</table>
E2.0 Synthetic specimen rupture test results and interpretation

The testing of the synthetic specimens were conducted to investigate two separate influence factors, mineral grain and grain boundary strength, on the peak and ultimate strength envelopes, shear stress versus horizontal displacement (load-displacement) response, rupture zone creation relative to the load-displacement response, and ultimate rupture zone geometry. First the influence of mineral grain strength is presented followed by grain boundary strength. The rupture zone geometry is described with reference to the following terminology:

- **Shear rupture zone**, the shear rupture surface and the surrounding damage zone that is created and evolves during the applied shear deformation (Figure E.7).

- **Shear rupture surface**, the connected surface or fully damaged zone that is ultimately created and contained within the shear rupture zone (Figure E.7).

Figure E.7 Schematic illustration of shear rupture zone, shear rupture surface, and damage zone.

E2.1 Influence of mineral grain tensile bond strength

E2.1.1 Rupture zone creation and shear stress versus horizontal displacement response

Each synthetic specimen suite with mineral grain strength reductions (i.e., 0, 50, and 75%; high, medium, and low strength sandstone, respectively) has a different UCS (as summarized in Table E.2) as previously outlined in Section E.1.1. Therefore, to compare the sequential rupture zone geometry developments and shear stress versus horizontal displacement (load-displacement) responses of the respective synthetic specimen mineral grain strength reduction suites, it is
necessary to normalize the applied normal stress magnitudes to the UCS for each suite of results. The normalized values are summarized in Figure E.8 which shows load-displacement curves for each suite of mineral grain strength reductions and their respective $\sigma_n/\text{UCS}$ ratios. In Chapter 2 it was found that the rupture mechanism of the synthetic specimen changed from tensile splitting to en échelon tensile fracture array development at $0.17$ ($\sigma_n/\text{UCS}$) and then to shear en échelon fracture array development at $>0.41$ ($\sigma_n/\text{UCS}$). Thus, the results are initially grouped into $\sigma_n/\text{UCS}$ ratios of $<0.17$, $0.17 \leq x \leq 0.41$, and $>0.41$ where common rupture mechanisms are anticipated to occur. The $\sigma_n/\text{UCS}$ ratios investigated in detail are shaded in grey in Table E.3 and correspond to $\sigma_n/\text{UCS}$ ratios: $\leq 0.10$ (anticipated to rupture via tensile splitting); $0.27-0.28$ (anticipated to rupture via en échelon tensile fracture array development); and $0.62-0.74$ (anticipated to rupture via en échelon shear fracture array development). Each group is described separately. The load-displacement curves shown in Figure E.8 indicate the locations of the rupture zone images (i.e., hollow circles) shown in Figures E.9 to E.11 which illustrate the rupture zone geometries at the various locations along the load-displacement curves (Figure E.8) and rupture mechanism using particle velocity or displacement vectors. Particle velocity and displacement vectors explicitly show the movement of the particles in the synthetic specimens and thus the rupture mechanism.
Table E.3 Normal stress to UCS ratio and dilation angle for each mineral grain tensile bond strength reduction.

<table>
<thead>
<tr>
<th>Normal stress (MPa)</th>
<th>5</th>
<th>15</th>
<th>25</th>
<th>40</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>146</td>
<td>0.03</td>
<td>0.10</td>
<td>0.17</td>
<td>0.27</td>
<td>0.41</td>
</tr>
<tr>
<td>50%</td>
<td>93</td>
<td>0.05</td>
<td>0.16</td>
<td>0.27</td>
<td>0.43</td>
<td>0.65</td>
</tr>
<tr>
<td>75%</td>
<td>54</td>
<td>0.09</td>
<td>0.28</td>
<td>0.46</td>
<td>0.74</td>
<td>1.11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dilation angle (°)</th>
<th>0%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>146</td>
<td>64</td>
<td>57</td>
</tr>
<tr>
<td>50%</td>
<td>93</td>
<td>50</td>
<td>44</td>
</tr>
<tr>
<td>75%</td>
<td>54</td>
<td>52</td>
<td>35</td>
</tr>
</tbody>
</table>

Grey shading indicates selected $\sigma_n$/UCS ratios for detailed analysis.
Figure E.8 Shear stress versus horizontal displacement response of the synthetic specimens for the different strength classes. (a) High strength sandstone (0% mineral grain bond strength reduction). (b) Medium strength sandstone (50% mineral grain bond strength reduction). (c) Low strength sandstone (75% mineral grain bond strength reduction). Circles along the curves indicate the locations of the rupture zone images shown in Figures E.9-E.11. Grey circles, squares, and triangles indicate the groups of results assessed together. Top right shows example idealized load-displacement responses.
Figure E.9 $\sigma_n$/UCS ratios ≤0.10. Rupture zone images correspond to the locations of the shear stress versus horizontal displacement curves (load-displacement) in Figure E.8 (grey circles) at increasing horizontal displacements ($\delta_h$) left to right. Rupture mechanism (tensile splitting) is shown at the bottom most row using particle displacement vectors for: AA medium strength sandstone; and BB low strength sandstone. Rupture mechanism for the high strength sandstone was previously shown in Chapter 2. Fractures in the rupture zone images, orange – grain boundary and black-mineral grain tensile fractures.
Figure E.10 $\sigma_n$/UCS ratios 0.27-0.28. Rupture zone images correspond to the locations of the shear stress versus horizontal displacement curves (load-displacement) in Figure E.8 (grey squares) at increasing horizontal displacements ($\delta_h$) left to right. Rupture mechanism (en échelon tensile fracture array development) is shown at the bottom most row using particle displacement vectors for: AA medium strength sandstone; and BB low strength sandstone. Rupture mechanism for the high strength sandstone was previously shown in Chapter 2. Fractures in the rupture zone images, orange – grain boundary and black-mineral grain tensile fractures.
Figure E.11 $\sigma_n$/UCS ratios 0.62-0.74. Rupture zone images correspond to the locations of the shear stress versus horizontal displacement curves (load-displacement) in Figure E.8 (grey triangles) at increasing horizontal displacements ($\delta_h$) left to right. Rupture mechanism (shear fracture array development) is shown at the bottom most row using particle displacement vectors for: AA medium strength sandstone. Rupture mechanism for the high strength sandstone was previously shown in Chapter 2. Fractures in the rupture zone images, orange – grain boundary and black-mineral grain tensile fractures.
At $\sigma_n$/UCS ratios $\leq 0.10$ (Figure E.9), rupture zone creation occurs at or just after peak shear strength via tensile splitting (Figure E.9 AA and BB) and the load-displacement curves have large stress drops post-peak strength and can be idealized as elastic-brittle (Figure E.8 grey circles). As the mineral grain strength is reduced, the rupture zones become more dominated by intra-grain tensile fractures (black fractures) with less occurrence of grain boundary tensile fractures (orange fractures). The damage zones surrounding the rupture surface are also greatly reduced at lower mineral grain strengths.

At $\sigma_n$/UCS ratios $0.27-0.28$ (Figure E.10), rupture zone creation occurs at or just after peak shear strength progressively. First, an array of en échelon tensile fractures develops (Figure E.10 AA and BB). These fractures then turn into shear flaws as the state-of-stress rotates in the synthetic specimen (see Chapter 2). The shear flaws propagate fractures from their tips creating a connected rupture zone. The load-displacement curves have less of a stress drop post-peak strength and the curves resemble a strain-weakening response (Figure E.8 grey squares). The damage zone surrounding the rupture surface, again, is greatly reduced at lower mineral grain strengths.

At $\sigma_n$/UCS ratios $0.62-0.74$ (Figure E.11), fractures propagate from the shear box gaps at the first peak as indicated by (I) and the load-displacement curves have no to little shear stress drops and can be idealized as elastic-plastic (Figure E.8 grey triangles). Fracturing in the center of the synthetic specimen eventually occurs well after the first peak at ‘I’. With continued horizontal displacement, steep and shallow fracture systems develop. No dominate mode of formation is evident for these fracture systems. They are initially composed of en échelon individual tensile fractures that then coalesce with particle displacement vectors indicating shear (Figure E.11 AA). The shallow angle fracture systems (or more appropriately micro-faults because they show shear displacement) have a synthetic shear sense (associated with the principal movement of the shear box) while the steep angle fracture systems (or more appropriately micro-faults because they show shear displacement) have antithetic shear sense. Antithetic refers to a sense of shear opposite while synthetic refers to a sense of shear in the same direction as that applied. The fracture systems eventually form a linked rupture zone across the synthetic specimen. The damage zone surrounding the rupture surface, again, is greatly reduced at lower mineral grain strengths.
It was initially assumed that the rupture zone creation mechanisms would be the same for the outlined groups of $\sigma_n$/UCS assessed. Based on the above presented results, the $\sigma_n$/UCS ratio has a clear influence on rupture zone creation and mechanism as well as the resulting load-displacement response of the synthetic specimens. One of the dominate effects of increasing normal stress is the suppression of shear box dilation or the vertical displacement of the top (upper) shear box wall. The effect of normal stress on vertical displacement of the upper shear box wall in the simulations with the various mineral grain strength reductions is presented next.

**E2.1.2 Shear rupture dilation**

The vertical displacement of the upper shear box wall (Wall 2, Figure E.12d), i.e., a reflection of the collective dilation in the shear rupture zone, is plotted versus horizontal displacement in Figure E.12a-c. Figure E.12a-c shows the change in dilation angle (the slope of the horizontal displacement versus vertical displacement curve) which becomes near zero at $\sigma_n$/UCS ratios between 0.97 and 1.11 (i.e., a transition from dilatant to non-dilatant synthetic specimen vertical displacement behaviour). The dilation angles are summarized in Table 3 below the $\sigma_n$/UCS ratios previously summarized and are also shaded grey to indicate the $\sigma_n$/UCS ratios investigated in detail. The tabulated summary of dilation angle shows that each $\sigma_n$/UCS ratio grouping: $\leq$0.1; 0.27 to 0.28; and 0.62 to 0.74; has similar dilation angles and thus vertical displacement responses.
Figure E.12: Vertical displacement of the upper shear box wall (Wall 2) versus horizontal displacement of the lower shear box wall (Wall 1). (a) 0% mineral grain strength reduction. (b) 50% mineral grain strength reduction. (c) 75% mineral grain strength reduction. (d) Schematic representation of the direct shear simulation showing wall numbering and boundary conditions.
E2.1.3 Peak and ultimate shear strength

The influence of mineral grain strength is illustrated in Figure E.13 which shows peak (Figure E.13a) and ultimate (Figure E.13b) linear and bi-linear Coulomb strength envelopes fit using linear regression of the data points for each mineral grain strength reduction. First, the peak strength envelopes are discussed followed by the ultimate.

As the mineral grain strength is reduced from the calibrated baseline (0% strength reduction) synthetic specimen (squares on Figure E.13a), the peak Coulomb strength envelope shifts downwards to lower apparent cohesive strength intercepts (i.e., 27, 21, and 13MPa for 0, 50, and 75% mineral grain strength reductions, respectively) and progressively becomes bi-linear (Figure E.13a). The medium (50% reduction) and low (75% reduction) strength sandstone’s peak strength envelopes develop into a bi-linear shape due to the application of 90MPa and 60MPa normal stress magnitudes, respectively. These normal stresses are near the UCS, i.e., 93 and 54MPa, of the medium and low strength sandstone classes, respectively. The bi-linear Coulomb envelope is caused by the vertical displacement characteristic of the synthetic specimen changing from dilatant to non-dilatant behaviour as evident from the vertical displacements in Figure E.12b-c for the $\sigma_n$/UCS ratios of 0.97 and 1.11, respectively. The high strength sandstone has positive vertical displacements of the top shear box wall (Figure E.12a) and does not develop the bi-linear shape.

The ultimate shear strengths for each sandstone strength class are shown in Figure E.13b. All have a bi-linear character similar to Patton (1966) with transition normal stress around 25MPa for the high and medium strength sandstones and a transition normal stress around 15 to 20MPa for the low strength sandstone. These bi-linear transition points are associated with the rupture mechanism change from dominantly tensile splitting to shear rupture processes. Patton (1966), using specimens deformed in direct shear containing discontinuities with surfaces composed of saw-tooth asperities, found that once normal stress magnitudes were sufficient to suppress shear ride over the saw-tooth asperities the ultimate Coulomb strength envelope changed from one characterized by a friction angle ($\phi$) plus dilation component ($i$, equal to the saw-tooth asperity angle) to one based on an apparent cohesion ($c'$) and friction angle ($\phi$) as overviewed in detail in Section E1.0. The bi-linear transition normal stress magnitudes are generally the point when the rupture mechanism of the synthetic specimens changes from tensile splitting to en échelon
tensile fracture array development (i.e., shear rupture process). The splitting mechanism creates a relatively continuous and planar rupture surface while the tensile and eventually shear en échelon fracture mechanisms create relatively discontinuous and non-planar rupture surfaces across the synthetic specimens. The continuous planar rupture surface and dilatant synthetic specimen behaviour at normal stresses < 25MPa for the high and medium strength sandstones and <20MPa for the low strength sandstone allows for shear ride over surface asperities along the created rupture surface. Thus the linear Coulomb strength envelope fit using linear regression to the data points at normal stresses <25MPa have a friction angle ($\phi$) plus dilation angle ($i$) strength (i.e., progressing from high to low strength sandstone $\phi + i$ angles of: 59°; 54°; and 50° or friction coefficients, $\mu$, of 1.64, 1.39, and 1.19). The discontinuous and non-planar rupture surface with less dilatant (and eventually slightly compactive) behaviour at normal stresses >20MPa forces failure through the isolated lenses of rock composed of mineral grains that have formed along the irregular rupture zone. This thus increases the apparent cohesive strength ($c'$) of the synthetic specimen which is essentially the same at $c' = 22$MPa for the high and medium strength sandstones and slightly lower at $c' = 16$MPa for the low strength sandstone where there is less interlock strength associated with the mineral grain structure due to the weaker grains.

While the apparent cohesive strength is essentially the same for the linear Coulomb strength envelope fit through the data points at normal stresses >25MPa (i.e., $c' = 22$MPa, Figure 10b), the ultimate strength envelopes progressing from high strength to low strength sandstone (0% to 75% reduced strengths) at normal stresses >25MPa have lower and decreasing friction angles and thus friction coefficients ($\phi = 36^\circ$ to $17^\circ$ or $\mu = 0.72$ to 0.31) even though all of the synthetic specimens have the same assigned frictional parameters (see Table 3 in Chapter 2). This strength component change is also noted for the dilatant frictional strengths at normal stresses <25MPa (i.e., decreasing $\phi + i$ angles of 59°, 54°, 50°). This change in ultimate strength at decreasing mineral grain strengths is interpreted to be due to the influence of grain interlocking whereby the low strength sandstone class allows for easier fracturing of the grains due to the reduced mineral grain strengths and thus less mineral grain interlocking compared to the medium and high strength sandstones.

In summary, the peak Coulomb strength envelopes have a bi-linear shape when specimen behaviour changes from dilatant to non-dilatant; the influence of reducing the mineral grain strength is to lower the apparent cohesive strength component, shifting the peak Coulomb
strength envelope lower while only slightly influencing the peak friction angle; the bi-linear ultimate strength envelopes are due to a failure mechanism change from dilatant tensile splitting at low to shear rupture mechanisms at higher normal stresses creating more discontinuous rupture zones forcing failure to occur through lenses of rock composed of mineral grains. The friction angle change for the ultimate strengths at normal stresses $>25\text{MPa}$ and the changing $\phi+i$ angles at normal stresses $<25\text{MPa}$ are also due to the weakened grains which do not allow for grain interlocking. These observations illustrate processes that justify the use of non-linear failure criteria for such rock.
Figure E.13 Peak (a) and ultimate (b) Coulomb strength envelopes for the different mineral grain strength reductions.
284

**E2.1.3 Rupture matrix**

In Section E3.1.1, the load-displacement response, rupture zone geometry, and rupture mechanism were investigated and it was shown that these characteristics depended on the $\sigma_n$/UCS ratio. In Section E3.1.2, the vertical displacement character of the synthetic specimen was investigated and it was shown that this characteristic also depended on the $\sigma_n$/UCS ratio. Table E.4 summarizes all of the investigated normal stresses for each mineral grain tensile bond strength reduction and shows that as the $\sigma_n$/UCS ratio increases: (1) the rupture mechanism changes from tensile splitting to shear; (2) the dilation angle decreases; and (3) the load-displacement response transitions from one with a large stress drop post-peak strength (elastic-brittle) to one with little to no stress drop (elastic-plastic). Most importantly, Table E.4 shows that the rupture of the synthetic specimens, which are representative of intact brittle rocks, can be classified based on the $\sigma_n$/UCS ratio as identified in the grey shaded areas of Table E.4. This classification is shown graphically in Figure E.14 where at each $\sigma_n$/UCS ratio the ultimate rupture zone geometry is plotted along with its idealized load-displacement response. The lines (which should be viewed as fuzzy and not definite divisions) dividing the rupture zones in Figure 11 are based on the grey shaded zones in Table 4. The $\sigma_n$/UCS ratio classes, introduced in Section E2.1.1 are summarized as follows (Figure E.14):

- $\sigma_n$/UCS between 0.03 and 0.16, defined here as Class T (Tensile): the resulting rupture is a relatively thin rupture zone which develops instantaneously across the specimen. Its load-displacement response can be idealized as elastic-brittle.

- $\sigma_n$/UCS between 0.17 and 0.28, defined here as Class ET (en échelon tensile array): the resulting rupture is a relatively wide zone of linked en échelon fractures. The en échelon fractures develop first at or near peak strength. These fractures then link between peak and ultimate strength. The load-displacement response can be idealized as strain-weakening with a less rapid stress drop post-peak compared to Class T.

- $\sigma_n$/UCS between 0.41 – 0.46, defined here as Class M (mixed mode or transition between Class T and ET): the resulting rupture is a relatively wide zone again consisting of a linked en échelon fracture system with fractures beginning to propagate from the shear
Due to the mixed fracturing processes, the failure processes occurring at these \( \sigma_n/\text{UCS} \) ratios have been classified as mixed mode and also represent the idealized load-displacement behaviour transitioning from slightly strain-weakening to elastic-plastic.

- \( \sigma_n/\text{UCS} \) between 0.62 – 1.11, defined here as Class S (Shear): the resulting rupture is a wide zone which develops via synthetic and antithetic micro-faults. The load-displacement response can be idealized as an elastic-plastic response with no to little post-peak stress drop.

The developed chart in Figure E.14 is limited to a \( \sigma_n/\text{UCS} \) ratio of 1.11 because after this ratio the synthetic specimen becomes compactive and the rupture zone geometry is visually difficult to distinguish.

Table E.4 and Figure E.14 were generalized and transferred into a rupture matrix (Fig. E.15). In this matrix, normal stress forms the vertical axis and uniaxial compressive strength the horizontal. Lines (which should be viewed as fuzzy and not definite divisions) of constant \( \sigma_n/\text{UCS} \) ratio are indicated and divide the matrix into the identified classifications based on the results outlined above as summarized in Table E.4 and Figure E.14. In an attempt to confirm the general application of this matrix, the transition from tensile splitting to the shear rupture of concrete (\( \sigma_n/\text{UCS} \) 0.15-0.16; Sonnenberg et al., 2003; Wong et al., 2005), and the \( \sigma_n/\text{UCS} \) ratios for identifiable failure mechanisms from Lajtai (1969) (i.e., en échelon tensile fracturing leading to rupture in plaster-of-Paris; \( \sigma_n/\text{UCS} \) 0.15 and 0.35) were compiled. The laboratory results for these materials support the general applicability of the matrix although it is noted that the simulations were conducted on synthetic specimen with a lower bound UCS of 54MPa, thus limiting the matrix in Figure E.15 to approximately 50MPa (as shown in Figure E.15). The results for concrete and plaster-of-Paris, stated above, are outside the range of the matrix (i.e., UCS of the concrete was 41MPa; UCS of plaster-of-Paris was 4MPa). Even though these results are outside the developed matrix range of simulated conditions, they fit to the defined classifications.
Table E.4: Summary of mineral grain tensile bond strength reduction influence on rupture zone characteristic.

<table>
<thead>
<tr>
<th>Normal Stress (MPa)</th>
<th>Sandstone Strength Class</th>
<th>Intra-Grain Strength Reduction (%)</th>
<th>$\sigma_n/\text{UCS}$</th>
<th>Dilation Angle (°)</th>
<th>Rupture Mechanism</th>
<th>Idealized Load-Displacement Response</th>
<th>Assigned Overall Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>High (UCS=146MPa)</td>
<td>0</td>
<td>0.03</td>
<td>64</td>
<td>Tensile splitting</td>
<td>Elastic-Brittle</td>
<td>T</td>
</tr>
<tr>
<td>5</td>
<td>Medium (UCS=93MPa)</td>
<td>50</td>
<td>0.05</td>
<td>50</td>
<td>Tensile splitting</td>
<td>Elastic-Brittle</td>
<td>T</td>
</tr>
<tr>
<td>5</td>
<td>Low (UCS=54MPa)</td>
<td>75</td>
<td>0.09</td>
<td>52</td>
<td>Tensile splitting</td>
<td>Elastic-Brittle</td>
<td>T</td>
</tr>
<tr>
<td>15</td>
<td>High</td>
<td>0</td>
<td>0.10</td>
<td>57</td>
<td>Tensile splitting</td>
<td>Elastic-Brittle</td>
<td>T</td>
</tr>
<tr>
<td>15</td>
<td>Medium</td>
<td>50</td>
<td>0.16</td>
<td>44</td>
<td>Tensile splitting</td>
<td>Elastic-Brittle</td>
<td>T</td>
</tr>
<tr>
<td>25</td>
<td>High</td>
<td>0</td>
<td>0.17</td>
<td>50</td>
<td>En échelon tensile array</td>
<td>Strain-weakening</td>
<td>ET</td>
</tr>
<tr>
<td>25</td>
<td>Medium</td>
<td>50</td>
<td>0.27</td>
<td>39</td>
<td>En échelon tensile array</td>
<td>Strain- weakening</td>
<td>ET</td>
</tr>
<tr>
<td>15</td>
<td>Low</td>
<td>75</td>
<td>0.28</td>
<td>35</td>
<td>En échelon tensile array</td>
<td>Strain- weakening</td>
<td>ET</td>
</tr>
<tr>
<td>40</td>
<td>High</td>
<td>0</td>
<td>0.27</td>
<td>46</td>
<td>En échelon tensile array</td>
<td>Strain- weakening</td>
<td>ET</td>
</tr>
<tr>
<td>60</td>
<td>High</td>
<td>0</td>
<td>0.41</td>
<td>30</td>
<td>Mixed tensile/shear array</td>
<td>Strain- weakening</td>
<td>M</td>
</tr>
<tr>
<td>40</td>
<td>Medium</td>
<td>50</td>
<td>0.43</td>
<td>27</td>
<td>Mixed tensile/shear array</td>
<td>Strain- weakening</td>
<td>M</td>
</tr>
<tr>
<td>25</td>
<td>Low</td>
<td>75</td>
<td>0.46</td>
<td>32</td>
<td>Mixed tensile/shear array</td>
<td>Strain- weakening</td>
<td>M</td>
</tr>
<tr>
<td>90</td>
<td>High</td>
<td>0</td>
<td>0.62</td>
<td>25</td>
<td>Dominantly shear</td>
<td>Elastic-Plastic</td>
<td>S</td>
</tr>
<tr>
<td>60</td>
<td>Medium</td>
<td>50</td>
<td>0.65</td>
<td>18</td>
<td>Dominantly shear</td>
<td>Elastic-Plastic</td>
<td>S</td>
</tr>
<tr>
<td>40</td>
<td>Low</td>
<td>75</td>
<td>0.74</td>
<td>24</td>
<td>Dominantly shear</td>
<td>Elastic-Plastic</td>
<td>S</td>
</tr>
<tr>
<td>90</td>
<td>Medium</td>
<td>50</td>
<td>0.97</td>
<td>4</td>
<td>Dominantly shear</td>
<td>Elastic-Plastic</td>
<td>S</td>
</tr>
<tr>
<td>60</td>
<td>Low</td>
<td>75</td>
<td>1.11</td>
<td>4</td>
<td>Dominantly shear</td>
<td>Elastic-Plastic</td>
<td>S</td>
</tr>
<tr>
<td>90</td>
<td>Low</td>
<td>75</td>
<td>1.67</td>
<td>-2</td>
<td>Not Applicable</td>
<td>Elastic-Plastic</td>
<td>Not Applicable</td>
</tr>
</tbody>
</table>
Figure E.14 Division of rupture in the synthetic specimens based on the \( \sigma_n/\text{UCS} \) ratio considering common rupture mechanisms, rupture zone geometries, and idealized shear stress versus horizontal displacement responses. Fractures in the rupture zone images, orange – grain boundary and black-mineral grain tensile fractures.
Figure E.15 Generalized rupture zone matrix based on Figure E.14 and Table E.4. Fractures in the rupture zone images, orange – grain boundary and black-mineral grain tensile fractures.
E2.2 Influence of grain boundary tensile bond strength

E2.2.1 Rupture zone creation and shear stress versus horizontal displacement response

The detailed rupture zone creation process is identical to that described in Chapter 2 of the thesis. The ultimate rupture zone geometries are shown in Figure E.16 along with the shear stress versus horizontal displacement (load-displacement) responses of the synthetic specimens for grain boundary strength reductions of 25 and 50%. Overall, the rupture zone geometries have similar trends with the noted difference of increasing occurrence of grain boundary fractures (orange fractures in Figure E.16, compare the 25 and 50% grain boundary strength reduction ultimate rupture zone geometries). The load-displacement responses are similar for normal stresses >25MPa while the peak strength and stress drop post peak strength at normal stresses <25MPa, as evident in the load-displacement curves, are influenced by the grain boundary strength reduction resulting in the lowering of peak strength and the post-peak response of the load-displacement curves becoming less brittle.
Figure E.16 Influence of grain boundary (GB) strength reduction on the shear stress versus horizontal displacement response and ultimate rupture zone geometries (at the indicated locations along the load-displacement curves). Fractures in the rupture zone images, orange – grain boundary and black – mineral grain tensile fractures.
E2.2.1 Peak and ultimate shear strength

As the grain boundary tensile bond strength is reduced, for normal stress magnitudes <40MPa, the peak linear Coulomb strength envelope becomes bi-linear and shifts lower as shown in Figure E.17a which is a plot of the peak strength results for synthetic specimens with reduced grain boundary strengths fit bi-linearly using linear regression. At 5MPa normal stress, the largest peak strength decreases occur compared to the baseline 0% strength reduction results. The peak strength decrease associated with the weakening of grain boundaries progressively diminishes at higher normal stress magnitudes. Minimal strength drops occur once a normal stress magnitude of 40MPa is reached and this trend continues to the 90MPa normal stress case.

As normal stress magnitudes increase, the influence of the weakened grain boundaries decreases leading to a transition in strength to that of the mineral grains or intact rock. The weakened grain boundaries no longer control the shear strength once dilation is inhibited to the extent that en échelon shear fracturing processes occur in the synthetic specimens. This occurs at normal stresses approximately >40MPa.

The ultimate strengths of the synthetic specimens with 25 and 50% grain boundary strength reductions (Figure E.17b) show that grain boundary strength reductions have a minimal influence on ultimate strength.
Figure E.17 Influence of grain boundary strength reduction on synthetic specimen peak (a) and ultimate (b) Coulomb strength envelopes.
E2.3 Combined influence of mineral grain and grain boundary strength

The direct shear synthetic specimen is composed of a well interlocked grain structure with consideration for both mineral grain and ground boundary strength components. These components were found to have different influences on peak shear strength. Mineral grain strength reductions result in the lowering of the overall peak Coulomb strength envelope (cohesion intercept) independent of the applied normal stress magnitude. Grain boundary strength only influenced the peak shear strength at low normal stress magnitudes (<40MPa) changing the shape of the linear Coulomb strength envelope to bi-linear or curved. For the grain boundary strength reduction results, the changing peak Coulomb strength envelope with increasing applied normal stress was found to be due to a change in rupture mechanism from predominantly tensile at low, to shear at higher normal stresses. The different failure mechanisms exploit different strength characteristics (i.e., grain boundaries when tensile splitting dominates and mineral grains when shear dominates as also shown by the influence of the two strength components on the UCS and direct tensile strengths of the synthetic specimens, Table E.2).

These results suggest that mineral grain and grain boundary strength influence the shear strength of a rock in combination at low normal stresses or low confinement conditions where dilation is less restricted and extensional processes dominate the failure process exploiting both the lower grain boundary and mineral grain strength characteristics. At moderate to high normal stresses or confinements where dilation is inhibited changing the failure mechanism of a rock specimen to shear rupture, the influence of grain boundaries on peak shear strength is reduced because failure is forced through the grains. In this case, the mineral grain strength and thus grain interlocking ability are the primary components controlling the strength of the material. The results presented suggest that there is less influence of grain boundaries on a rock’s peak shear strength when rupture occurs via shear processes (i.e., the initial formation of arrays of en échelon fractures creating discontinuous rupture zones). This is supported by the results of Cresswell and Barton (2003) and Rosengren and Jaeger (1968) previously summarized.
3.0 Conclusions

A particle based DEM and its embedded GBM were used to investigate the influence of mineral grain and grain boundary strength on rupture zone creation, ultimate rupture zone geometries, the shear stress versus horizontal displacement response, and peak and ultimate strength envelopes of an intact brittle rock. It was found that the $\sigma_n/\text{UCS}$ ratio controlled the occurrence of rupture mechanisms in the synthetic specimens and thus the resulting rupture zone creation process and ultimate rupture zone geometry and thus the resulting shear stress versus horizontal displacement response and dilatant character of the synthetic specimens. This allowed for the development of a rupture matrix based on the $\sigma_n/\text{UCS}$ ratio that can be used to understand when different rupture mechanisms and characteristics should be expected to occur in brittle rock subjected to shear deformation. While somewhat speculative, the rupture matrix developed may provide the opportunity to classify shear rupture zones observed in rock masses but further work is required for reasons of scale effects. Although, many (e.g., Scholz, 2002; Thompson et al., 2009) now suggest the laboratory results provide a direct view into the mechanisms and processes occurring at the field scale.

The conclusions related to the separate influence of mineral grain and grain boundary strengths are also summarized as follows:

As mineral grain strength is reduced in the synthetic specimen, the influence of mineral grain strength is to:

- reduce the uniaxial compressive strength while only having a minor influence on the direct tensile strength;
- reduce or lower the cohesive strength component of the peak linear Coulomb strength envelope;
- change the normal stress magnitude when a specimen transitions from dilatant to non-dilatant behaviour; the lower the mineral grain strength the less normal stress that needs to be applied prior to the transition occurring;
• change the peak linear Coulomb strength envelope to bi-linear when non-dilatant behaviour occurs in a specimen at an applied normal stress magnitude;
• reduce the ability for grain interlocking to contribute to the strength of a rock specimen; and
• reduce the thickness of the fracture damage zone around a rupture surface that is created in a specimen.

As grain boundary strength is reduced in the synthetic specimen, the influence of grain boundary strength is to:
• reduce the direct tensile strength while only having a minor influence on the uniaxial compressive strength; and
• change the peak Coulomb strength envelope to a bi-linear shape for normal stresses where the failure mechanism is via tensile splitting (i.e., at $\sigma_n$/UCS ratios <0.17); that is, the influence is depending on applied normal stress with no to little influence of grain boundary strength reduction once shear rupture is the dominate failure mechanism occurring in the specimens.

Grain boundary strength reductions do not or inimally influence the ultimate strength, the fracturing processes leading to rupture zone creation, or the overall ultimate rupture zone geometries.

References


Appendix F PFC2D direct shear fish functions

The following are the fish functions for the constant normal stress and stiffness direct shear simulations conducted in this thesis. The functions were based on Cho (2008) but modified for use with the GBM, servo controlled normal stress, and constant normal stiffness boundary condition. Mr. N. Bahrani is thanked for his assistance in the modification of the fish functions. Cho, N., 2008. Discrete Element Modelling of Rock Pre-Peak fracturing and dilation. Ph.D. Thesis. University of Alberta.

The general steps to running the fish functions are as follows:

1. Generate a specimen to be deformed in a shear box. The specimen should be the same size as the desired shear box.
2. Run ds_shear-box.dvr to create the shear box around the specimen.
3. Run ds_norm-stress to apply the desired normal stress prior to shearing.
4. Run ds_shear-test to start the direct shear simulation.

; Filename : ds_shear-box.dvr
;=====================================================================================================================================
; Purpose : Creation of a shear box for direct shear test
;=====================================================================================================================================
;
; restore Dgr-spc.sav ;; restore specimen generated using fishtank
;
SET echo off
  call ds_meas_circle.fis ;; measurement circle installation
  call ds_shear.fis ;; direct shear setup
  call clumpgen.fis ;; clustered particle model (ClsPM) fish functions
  call clump-param.dat ;; ClsPM micro-properties
  call %fist%2d_3d\crk.fis ;; crack fish function setup
SET echo on
;
; Type of shear test:
SET cons_norm = 0 ;; if 0 -> constant normal load, if #0 constant normal stiffness
;
; Generate clustered particle model (ClsPM) or conventional bonded particle model (CBPM)
SET cluster = 0 ;; if cluster = 0 generates CBPM, if cluster = 1 generates ClsPM
;
SET shr_boxgap = 5.0e-3 ;; shear box gap
;
; platen properties
SET pl_ba_Ec = 20.0e9 pl_ba_krat = 2.5
SET pl_pb_Ec = 20.0e9 pl_pb_Rmult = 1.0 pl_pb_krat = 2.5
; for crack setup
plot add fish crk_item
;

---------------------------------------------------------------------
SET md_run_name = 'ds_'
title
shear box
;

; solve average 0.00001 maximum 0.00001 ;; lowers mean/max. unbal. force to mean/max.
contact force ratio to below 1%
;
delete mc ;; delete measurement circles previously generated in fishtank
set mc_rad_fact = 15.0 ;; measurement circle scale factor
ds_install_meas_circles ;; install measurement circles
;
create shear wall ;; generate shear box
;
ds_shearbox_plotview ;; plot shearbox
;
SET md_tag_name='sb' ;; save shear box (sb)
md_save_state
;

---------------------------------------------------------------------

; End of File ds_shear-box.dvr
Return

; Filename : ds_norm-stress.dvr
;======================================================================
; Purpose : Apply normal stress by increasing velocity of the wall with id 2
;======================================================================
;
restore ds_sb.sav
;

---------------------------------------------------------------------
SET md_run_name = 'ds_ns'
title
normal stress = 25.0MPa
;

---------------------------------------------------------------------

SET norm_req = 25.0e6 ;; applied normal stress
ini xvel 0.0 yvel 0.0 spin 0.0
SET ds_knxfac = 1.1 ;; stiffness scale factor
SET md2_thick = 1.0
ds_shear_wallstiff ;; apply stiffness properties to walls
cycle 100
crk_init ;; reset crack information and start tracking cracks
;
=====================================================================;
; Install test monitoring variables and histories
;
history reset
history nstep=100
history id=1 ds_wsyy2 ;; normal stress from wall 2
history id=2 ds_ave_ytop ;; normal stress from top meas. circles
history id=3 ds_ave_ybot ;; normal stress from bottom meas. circles
history id=4 ds_wsyy1 ;; normal stress from wall 1
history id=5 crk_num_pnf ;; tensile parallel bond cracks
history id=6 crk_num_snf ;; tensile smooth joint cracks
history id=7 crk_num ;; total number of cracks
history id=8 norm_cen ;; vertical stress in centre of box
;
=====================================================================;
;
SET ds_knxfac = 1.1
ds_shear_wallstiff
;
; parameters required for accelerating wall with id 2 for normal stress in normal stiffness shear test
; Warning: depending on the value of normal stress the following values may have to be changed
;
SET Np_vel=0.05 Np_cyc=2000 Np_stages=10 Nrun_cycle = 100
;
ds_accel_normalwall ;; accelerates upper wall (id=2) to the given velocity
ds_runnormal ;; increases stresses from upper wall until the desired normal stress is achieved in the sample
;
solve average 0.001 maximum 0.001
;
SET md_tag_name='-1MPa'
md_save_state
ds_ns_his_save ;; save normal stress history
;;
=====================================================================;
; End of File ds_norm-stress.dvr
Return
; Filename : ds_shear-test.dvr
;
=====================================================================
; Purpose : perform direct shear test
;
=====================================================================
;
restore ds_ns-1MPa.sav
;
;
=====================================================================
SET md_run_name = 'ds_1MPa'
title
normal stress = 25.0MPa
;
=====================================================================
ini xvel 0.0 yvel 0.0 spin 0.0
cycle 10
; plot add fish ds_draw_shear_sample
SET md2_thick = 1.0
;SET fishcall 0 ds_shear_wss ; stresses and displacements calc. setup
;
=====================================================================
; Install test monitoring variables and histories
;
history nstep=100
history id=1 crk_num ; microcracking
history id=2 crk_num_pnf ; parallel bond tensile cracks
history id=3 crk_num_psf ; parallel bond shear cracks
history id=20 crk_num_snf ; smooth joint tensile cracks
;
history id=4 ds_wdxx ; horizontal disp.
history id=5 sd_ave ; average shear disp. from subsequent position of four balls
history id=6 ds_wdsx ; shear stress from wall 4
;history id=7 ds1_xy ; shear stress from mc200
;history id=8 ds_sxy ; average shear stress from measurement circles spec centre
;
history id=9 ds_wsyy1 ; normal stress from wall 1 (lower 'U' wall)
history id=10 ds_wsyy2 ; normal stress from wall 2 (upper wall)
history id=11 ds_ave_ybot ; average normal load bottom meas. circles
history id=12 ds_ave_ytop ; average normal load top meas. circles
;history id=13 ds_ave_y ; normal stress by averaging top mc & bottom mc
;history id=14 ds_wsyy_ave ; average normal stress upper and lower walls
;
history id=15 ds1_ms1 ; maximum principal stress inside meas. circle 200
history id=16 ds1_ms3 ; minimum principal stress inside meas. circle 200
history id=17 ds1_ang ; direction of maximum principal stress from mc200
history id=18 ds1_x ; horizontal stress from mc200
history id=19 ds1_y ; vertical stress from mc200
history id=21 avg_stra_yy ; strain rate average top circles
history id=22 strainxx_mc8 ; strain rate in x
history id=23 strainxx_mc14 ; strain rate in x
history id=24 strainxx_mc11 ; strain rate in x
history id=25 strainyy_mc8 ; strain rate in y
history id=26 strainyy_mc14 ; strain rate in y
history id=27 strainyy_mc11 ; strain rate in y

; solve average 0.001 maximum 0.001
prop xdisp=0.0 ydisp=0.0
; ;
===================================================================== ; Specify in calling routine:
SET p_vel=0.04 p_cyc=1000 p_stages=10
SET ds_peakfac =0.6 ds_e_peakfac = 1.8 ; post peak stopping criteria
SET ds_save_state = 1 stress_record_pt = 2.0e6
SET ds_save_step = 20000 ; model is saved every so many steps
SET ds_knxfac = 1.1 run_cycle = 100
;
; servo control setup in the Constant Normal Load shear test:
SET y_servo = 1 ; 1 -> for constant normal load, 0 -> for constant normal stiffness
;
d_shear_wallstiff
init_pos ; initial position of four balls at the lower boundaries
d_accel_shearwall
d_shear_plotviews
d_runshear
;
SET md_tag_name='-result'
md_save_state
;
d_s_his_save
;;
===================================================================== ; End of File ds_0p01.dvr
Return
;
Filename : ds_shear.fis
;
===================================================================== ; Purpose : Direct shear test Simulation routines
def create_shear_wall
;
; Shear box generation using wall element
; Input : pl_thk - platen thickness for normal load
;        shr_boxgap - shear box gap
;        w1_0, w2_0, w3_0, w4_0 - current wall position
;
; Output : waddl, wadd3, wadd4 - wall pointers
;
; The values of w1_0, w2_0, w3_0, w4_0 as well as w_x/w_y for wadd1to4 are
; obtained from material genesis using fishtank for the following summation:
;
cur_wy1 = w1_0 + w_y(wadd1)
cur_wx3 = w3_0 + w_x(wadd3)
cur_wx4 = w4_0 + w_x(wadd4)
;
; first the four walls are deleted and then three walls
; with IDs 1, 3 and 4 are created
;
command
del wall 1 3 4
end_command
_x1 = cur_wx3
_y1 = -0.5 * shr_boxgap - 0.5 * pl_thk
_x2 = cur_wx3
_y2 = cur_wy1
command
  wall id=1 nodes (@_x1,@_y1) (@_x2,@_y2)
  print _x2
  print _y2
end_command
_x2 = cur_wx4
_y2 = cur_wy1
command
  wall id=1 nodes (@_x2,@_y2)
  print _x2
  print _y2
end_command
_x2 = cur_wx4
_y2 = -0.5 * shr_boxgap - 0.5 * pl_thk
command
  wall id=1 nodes (@_x2,@_y2)
  print _x2
  print _y2
end_command
_x0 = cur_wx3
_y0 = 0.5 * s_height + shr_boxgap
_x1 = cur_wx3
_y1 = 0.5 * shr_boxgap - 0.5 * pl_thk
command
  wall id=3 nodes (@_x0,@_y0) (@_x1,@_y1)
    print _x0
    print _y0
    print _x1
    print _y1
end_command
_x1 = cur_wx4
_y1 = 0.5 * shr_boxgap - 0.5 * pl_thk
_x2 = cur_wx4
_y2 = 0.5 * s_height + shr_boxgap
command
  wall id=4 nodes (@_x1,@_y1) (@_x2,@_y2)
end_command
;
  wadd1 = find_wall(1)
  wadd2 = find_wall(2)
  wadd3 = find_wall(3)
  wadd4 = find_wall(4)
;
; ds_platen is called which makes the platen
;
clustered_model
end
;
=====================================================================
def clustered_model
;
; first reset color index of all balls in the model to 0
;
  bp = ball_head
  loop while bp # null
    b_color(bp) = 0
    bp = b_next(bp)
  end_loop
;
; create clumped/clustered assembly
;
if cluster = 1
  cho_clumps
endif
;
_y0 = 0.5 * s_height - pl_thk
\_yl = 0.5 * s\_height \\
; release platen balls from clumped assembly \\
if clp\_clusters = 0 ; clumped model \\
command \\
    clump release range y=(_\_y0, _\_yl) \\
end\_command \\
endif \\
; 
if cons\_norm # 0 
    ds\_cons\_norm\_stiff 
endif 
end 

=====================================================================
def ds\_cons\_norm\_stiff 
; platen setup for normal stress application 
; Input : pl\_thk - platen thickness 
; pl\_ba\_Ec - modulus of balls inside platen 
; pl\_pb\_Ec - modulus of parallel bond for between balls inside the platen 
; pl\_pb\_Rmult - radius multiplier of the parallel bond for the balls inside the platen 
; pl\_pb\_krat - parallel bond stiffness ratio 
; pl\_ba\_krat - ball stiffness ratio 
; 
; first reset color index of all balls in the model to 0 
; 
_\_y0 = 0.5 * s\_height - pl\_thk 
_\_yl = 0.5 * s\_height 
; 
; then change the color index of platen balls to 1 
; 
bp = ball\_head 
loop while bp # null 
    if b\_y(bp) > _\_y0 
        b\_color(bp) = 1 ; orange 
    endif 
    bp = b\_next(bp) 
endloop 
; 
; assign modulus to balls which make the platen 
; 
bp = ball\_head 
loop while bp # null 
    if b\_y(bp) > _\_y0 
        pl\_ba\_kn = 2.0 * pl\_ba\_Ec * 1.0 
endloop
pl_ba_ks = pl_ba_kn / pl_ba_krat
b_kn(bp) = pl_ba_kn
b_ks(bp) = pl_ba_ks
endif
bp = b_next(bp)
endloop
;
; assign properties to parallel bonds inside platen
;
cp = contact_head
loop while cp # null
   pb = c_pb(cp) ; address of the parallel bond at this contact
   if pb # null then ; parallel bond is present
      _b1 = c_ball1(cp)
      _b2 = c_ball2(cp)
      if b_y(_b1) > _y0
         if b_y(_b2) > _y0
            pb_rad(pb) = pl_pb_Rmult
            pl_radsum = b_rad(c_ball1(cp)) + b_rad(c_ball2(cp))
            pl_pb_kn = pl_pb_Ec / pl_radsum
            pl_pb_ks = pl_pb_kn / pl_pb_krat
            pb_kn(pb) = pl_pb_kn
            pb_ks(pb) = pl_pb_ks
            pb_nstrength(pb) = 10.0e20
            pb_sstrength(pb) = 10.0e20
         endif
      endif
   endif
   cp = c_next(cp)
endloop
ds_platen_interface
end
;
====================================================================
def ds_platen_interface
;
; _y2 and _y3 are first defined to create platen-rock interface
;
et2_rlo = mg_Rmin
   _y2 = 0.5 * s_height - pl_thk - et2_rlo * 2.5
   _y3 = 0.5 * s_height - pl_thk
;
; platen rock interface properties are then assigned with interface color index of 2 which is blue
;
command
   prop col = 2 fric=0.0 ks=0.0 pb_n=1e-1 pb_s=1e-1 range y= ( @_y2 , @_y3)
end_command
def ds_shear_wallstiff
end

=====================================================================
def ds_accel_normalwall
end

=====================================================================
def ds_runnormal
    Nrun_cycle = Nrun_cycle
    loop while 1 # 0
d_shear_wss
d_mc_ns
    command
        print ds_wsy2
        print ds_ave_ytop
        print norm_cen
        print ds_ave_ybot
        endcommand
    if abs(norm_cen) > norm_req
        exit
    endif
    command
        cycle @Nrun_cycle
    endcommand
    endloop
end
;

=====================================================================  

def ds_servo
    while_stepping
        ds_get_gain
        ds_get_ss
    if y_servo = 1 ; apply servo control condition for wall 2 in CNL shear test
        ud = (gy * (wsyy + norm_req))/2
        w_yvel(wadd2) = -udy
    endif
end
;

def ds_get_ss
    xdif = w_x(wadd3) - w_x(wadd4)
ydif = w_y(wadd2) - w_y(wadd1)
new_xwidth = s_width + xdif
new_height = s_height + ydif
wsxx = 0.5 * (w_xfob(wadd4) - w_xfob(wadd2)) / (new_height * 1.0)
wsyy = 0.5 * (w_yfob(wadd1) - w_yfob(wadd2)) / (new_xwidth * 1.0)
wexx = 2.0 * xdif / (s_width + new_xwidth)
weyy = 2.0 + ydif / (s_height + new_height)
wevol = wexx + weyy
end
def ds_get_gain
    alpha = 0.5
    count = 0
    ave_stiff = 0
    cp = contact_head
    loop while cp ≠ null
        if c_ball1(cp) = wadd1
            count = count + 1
            avg_stiff = avg_stiff + c_kn(cp)
        endif
        if c_ball2(cp) = wadd1
            count = count + 1
            avg_stiff = avg_stiff + c_kn(cp)
        endif
        if c_ball1(cp) = wadd2
            count = count + 1
            avg_stiff = avg_stiff + c_kn(cp)
        endif
        if c_ball2(cp) = wadd2
            count = count + 1
            avg_stiff = avg_stiff + c_kn(cp)
        endif
        cp = c_next(cp)
    end_loop
    nycount = count / 2.0
    avg_stiff = avg_stiff / count
    gy = alpha * (s_width + 1.0) / (avg_stiff * nycount * tdel)
end

def init_pos
    ; obtain position of four balls located at the lower boundaries before the test
    ball1 = ball_near2 (0.5*s_width, - 0.5 * s_height)
    ball2 = ball_near2 (-0.5*s_width, - 0.5 * s_height)
    ball3 = ball_near2 (0.5*s_width, - 0.25 * s_height)
    ball4 = ball_near2 (-0.5*s_width, - 0.25 * s_height)
    ip_b1 = b_x(ball1)
    ip_b2 = b_x(ball2)
    ip_b3 = b_x(ball3)
    ip_b4 = b_x(ball4)
end
def shear_disp
;
; obtain position of the four balls during the test and calculate shear displacement in mm
;
sd_b1 = b_x(ball1) - ip_b1
sd_b2 = b_x(ball2) - ip_b2
sd_b3 = b_x(ball3) - ip_b3
sd_b4 = b_x(ball4) - ip_b4
sd_ave = ((sd_b1 + sd_b2 + sd_b3 + sd_b4) / 4) * 1000
end
;

def ds_shear_wss
; Compute shear stress, normal stress and horz. Disp.
; Input: waddl, wadd4 - wall pointer
; md2_thick - particle disk thickness
; if "set disk on" then md2_thick = 1
;
; horizontal displacement from wall 1, absolute value in x direction
ds_wdxx = abs(w_x(wadd1)) * 1000
;
; w_yfob(wp) is out of balance force of wall wp in y direction
; Therefore normal stress is calculated by dividing force in y direction
; acting on wall 1 by sample width times the sample thickness
;
ds_wsyy1 = abs(w_yfob(wadd1)) / (s_width * md2_thick)
ds_wsyy2 = abs(w_yfob(wadd2)) / (s_width * md2_thick)
ds_wsyy_ave = (ds_wsyy1 + ds_wsyy2) / 2
;
; shear stress is calculated from force on wall 4 (right wall) in x direction
;
ds_wdsx = abs(w_xfob(wadd4)) / (s_width * md2_thick)
;
; maximum shear stress required for post-peak stopping criterion
ds_wdsx_max = max( ds_wdsx_max, abs(ds_wdsx))
end
;

def ds_accel_shearwall
;
; input: ds_wsyy_req = -1.0e6, p_vel = 0.1, p_cyc = 1000, p_stage = 10
;
; pin wall 2 in constant stiffness shear test
if cons_norm # 0
    command
wall id=2 xvel = 0.0 yvel = 0.0
endcommand
endif
;
_delvel = p_vel / p_stages  ; 0.1/10  = 0.01
_niter = p_cyc / p_stages ; 1000/10 = 100
_vel = 0.0
;
; the following loop brings the velocity from 0.01 to 0.1
; in 10 stages each takes 100 cyc.
;
loop ap_ii (1,p_stages)
    _vel = _vel + _delvel
    _fvel = _vel
    command
        wall id=1 xvel= @_fvel
        cycle @_niter
    end_command
end_loop
end
;
=====================================================================def ds_runshear
;
; input: run_cycle = 100,
;        ds_peakfac = 0.7, stops model when post peak stress is at 0.7 * peak stress
;        ds_e_peakfac = 2.0, stops model when post peak displacement is at 2.0 * peak shear
displacement
;        stress_record_pt = 2.0e6, ds_save_step = 500
;
run_cycle = run_cycle
; ds_wdsx_max is maximum wall shear stress
; ds_wdsx_max = -1.0e-20
loop while 1 # 0 ; infinite loop
    ds_mc_ns ; normal stress from measurement circles
    ds_shear_wss ; stress components and displacements from walls
    shear_disp ; shear displacement from position of four balls
    ;
    command
        cycle @run_cycle
    end_command
    ;
    ; ds_wdxx is horizontal displacement, and ds_wdsx is shear stress
    ; define shear displacement at peak stress and call is ds_wdxx_peak
    if abs(ds_wdsx) = ds_wdsx_max then
        ds_wdxx_peak = ds_wdxx
    end_if
; post peak stopping functions
if abs(ds_wdsx) <= (ds_peakfac * ds_wdsx_max) then
if abs(ds_wdxx) > abs(ds_e_peakfac * ds_wdxx_peak) then
exit
end_if
end_if

if ds_save_state = 1 then
if ds_wdsx >= stress_record_pt then
tg_cycle = tg_cycle + run_cycle
end_if
if tg_cycle = ds_save_step then
md_run_name = string(md_run_name)
cycle_num = string(cycle)
_fname = md_run_name + string('-') + cycle_num + string('.sav')
command
save @_fname
end_command
p_fname = md_run_name + string('-') + cycle_num + string('.nom')
command
set log on
set logfile @p_fname
print ds_wdsx
ds_crk_ang_norm
set log off
end_command
p_fname = md_run_name + string('-') + cycle_num + string('.shr')
command
set log on
set logfile @p_fname
print ds_wdsx
ds_crk_ang_shr
set log off
end_command
p_fname = md_run_name + string('-') + cycle_num + string('.bmp')
command
set plot bmp
plot set back white
plot add ball y y y y y y y y y y y y y y outline off
plot add wall black
plot add fish crk_item red blue red blue magenta black
plot add fish pload_color o
plot add fish platen_color lblue
plot add fish interface_color black
plot hardcopy file @p fname
endcommand
tg_cycle = 0
end_if
end_if
;
end_loop
end
;
=====================================================================
def ds_crk_ang_norm
;
; Computes the tension induced orientation of crack based on the
; information from the "crk.fis"
; Input : _crk_x, _crk_y - position of the current crack
;          _crk_normx, _crk_normy - crack normal vector
;          _crk_rad - size of the crack
; Output : _crk_ang - orientation of the current crack
;
crkp = crk_head
loop while crkp # null
  crk_getdata
  ds_crk_x0 = _crk_x - _crk_rad * _crk_normy
  ds_crk_y0 = _crk_y + _crk_rad * _crk_normx
  ds_crk_x1 = _crk_x + _crk_rad * _crk_normy
  ds_crk_y1 = _crk_y - _crk_rad * _crk_normx
  _cdist_x = ds_crk_x1 - ds_crk_x0
  _cdist_y = ds_crk_y1 - ds_crk_y0
if _cdist_x # 0.0 then
  _crk_slope = _cdist_y / _cdist_x
  nn_ang = (180.0 / pi)*atan2(abs(_cdist_y), abs(_cdist_x))
else
  nn_ang = 90.0
end_if
  _crk_ang = nn_ang
if _crk_slope < 0 then
  _crk_ang = -_crk_ang
endif
;
if _crk_fail = 3 then ; crack by tension
  command
    print _crk_x
    print _crk_y
    print _crk_ang
  endcommand
endif
;
crp = mem(crp+crk_NEXT)
end_loop
def ds_crk_ang_shr
;
; Computes the shear induced orientation of crack based on the
; information from the "crk.fis"
; Input:  _crk_x, _crk_y - position of the current crack
;        _crk_normx, _crk_normy - crack normal vector
;        _crk_rad - size of the crack
; Output: _crk_ang - orientation of the current crack
;
  crkp = crk_head
loop while crkp # null
  crk_getdata
  ds_crk_x0 = crk_x - _crk_rad * _crk_normy
  ds_crk_y0 = crk_y + _crk_rad * _crk_normx
  ds_crk_x1 = crk_x + _crk_rad * _crk_normy
  ds_crk_y1 = crk_y - _crk_rad * _crk_normx
  _cdist_x = ds_crk_x1 - ds_crk_x0
  _cdist_y = ds_crk_y1 - ds_crk_y0
  if _cdist_x # 0.0 then
    _crk_slope = _cdist_y / _cdist_x
    nn_ang = (180.0 / pi)*atan2(abs(_cdist_y), abs(_cdist_x))
  else
    nn_ang = 90.0
  end_if
  _crk_ang = nn_ang
  if _crk_slope < 0.0 then
    _crk_ang = -_crk_ang
  end_if
  ;
  if _crk_fail = 4 then ;; crack by shear
    command
      print _crk_x
      print _crk_y
      print _crk_ang
    end_command
  end_if
  crkp = mem(crkp+crk_NEXT)
end_loop
end
;
=====================================================================  

def make_arrays
 array ds_md_pt1(dim)
 array ds_md_pt2(dim)
array ds_md_pt3(dim)
end
;
=====================================================================
def platen_color
;
plot_item
stat= set_color( 0 ) ; color for single balls
bp = ball_head
loop while bp # null
  if b_y(bp) > _y0
    ds_md_pt1(1) = b_x(bp)
    ds_md_pt1(2) = b_y(bp)
    ds_rad1 = 1.0 * b_rad(bp)
    stat = fill_circle( ds_md_pt1, ds_rad1 )
  endif
  bp = b_next(bp)
end_loop
end
;
=====================================================================
def interface_color
;
plot_item
stat= set_color( 0 ) ; color for single balls
bp = ball_head
loop while bp # null
  if b_y(bp) > _y2
    if b_y(bp) < _y3
      ds_md_pt2(1) = b_x(bp)
      ds_md_pt2(2) = b_y(bp)
      ds_rad2 = 1.0 * b_rad(bp)
      stat = fill_circle( ds_md_pt2, ds_rad2 )
    endif
    endif
  bp = b_next(bp)
end_loop
end
;
=====================================================================
def pload_color
;
plot_item
stat= set_color( 0 ) ; color for single balls
bp = ball_head
loop while bp # null
  if b_y(bp) > _y5
    if b_y(bp)
if \( b_y(bp) < _y6 \)
    \[ ds_mdPt3(1) = b_x(bp) \]
    \[ ds_mdPt3(2) = b_y(bp) \]
    \[ ds_rad3 = 1.0 \times b_rad(bp) \]
    stat = fill_circle( \( ds_mdpt3, ds_rad3 \) )
    endif
endif

bp = b_next(bp)
end_loop
end

===================================================================== 

def ds_shearbox_plotview
    if cons_norm = 0
        ds_shearbox_consload_plotview
    else
        ds_shearbox_consstiff_plotview
    endif
end

===================================================================== 

def ds_shearbox_consstiff_plotview

    ;
    ; view of the shear box
    ;
    command
    plot create ShearBox
    plot set back white
    ;plot add ball outline off y y y y y y y y y y y y y y
    ;plot add ball outline off y o blu r gre lgrea mag br lgrea lblue lc lgrea dgra lo
    ;plot add clump y y y y y y y y y y y y y y y y outline off
    ;plot add measure black id off
    plot add wall black
    plot add fish platen_color lblue
    plot add fish interface_color black
    plot add fish crk_item red blue red blue magenta black
    plot show
    endcommand
end

===================================================================== 

def ds_shearbox_consload_plotview

    ;
    ; view of the shear box
    ;
    command
    plot create ShearBox
def ds_shear_plotviews
;
; a set of useful plot-views for the direct shear test.
;
command
    plot create shear_stress_disp1
    plot set title text 'Shear and normal stress (Pa) vs. horizontal disp (mm) calculated from walls'
    plot set back white
    plot add his 6 9 10 vs 4
    plot show
endcommand
end
;

def ds_ns_his_save
command
    his write 1 2 3 4 5 6 7 8 file ds_ns_history.his
endcommand
end
;

def ds_his_save
;
; Defines history set to save
command
    his write 1 2 3 20 4 5 6 9 10 11 12 15 16 17 18 19 21 22 23 24 25 26 27 file ds_test_history.his
end_command
end
;

; End OF File ds_shear.fis
return
def delete_mc
;
; delete existing measurement circles
;
command
    delete m 1
    delete m 2
    delete m 3
endcommand
end

def ds_install_meas_circles
;
;
; The meas. circle radius is scaled by the specimen using
; "mc_rad_fact".
;
; INPUT:  s_height - sample height
;         s_width - sample width
;         mc_rad_fact - measurement circle scale factor
;
meas_rad = (s_width / mc_rad_fact)
meas_rad2 = 0.575*(s_width / mc_rad_fact)
meas_rad3 = 2*(s_width / mc_rad_fact)
;
;---------------------------------------------------------------------------
;Installs a large central measurement circle
;
meas_x = 0.0
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad3 id=200
end_command
mp200 = find_meas(200)
;
;---------------------------------------------------------------------------
;Installs measurement circles close to bottom wall
;
meas_x = 0.0e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=1
end_command
mp1 = find_meas(1)
;
meas_x = 0.6e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=2
end_command
mp2 = find_meas(2)
;
meas_x = 1.1e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=3
end_command
mp3 = find_meas(3)
;
meas_x = 1.6e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=4
end_command
mp4 = find_meas(4)
;
meas_x = -0.6e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=5
end_command
mp5 = find_meas(5)
;
meas_x = -1.1e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=6
end_command
mp6 = find_meas(6)
;
meas_x = -1.6e-2
meas_y = 0.0 - 0.5 * s_height + 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=7
end_command
mp7 = find_meas(7)
; Installs measurement circles close to top wall

meas_x = 0.0e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=8
end_command
mp8 = find_meas(8)

meas_x = 0.6e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=9
end_command
mp9 = find_meas(9)

meas_x = 1.1e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=10
end_command
mp10 = find_meas(10)

meas_x = -0.6e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=12
end_command
mp12 = find_meas(12)

meas_x = -1.1e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
  measure x=@meas_x y=@meas_y rad=@meas_rad id=13
end_command
mp13 = find_meas(13)

;
meas_x = -1.6e-2
meas_y = 0.5 * s_height - pl_thk - 1.5 * meas_rad
command
    measure x=@meas_x y=@meas_y rad=@meas_rad id=14
end_command
mp14 = find_meas(14)
;
; -----------------------------------------------------------------------------------------------
; Installs measurement circles at specimen midpoint
;
meas_x = 0.0e-2
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=15
end_command
mp15 = find_meas(15)
;
meas_x = 0.25e-2
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=16
end_command
mp16 = find_meas(16)
;
meas_x = 0.5e-2
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=17
end_command
mp17 = find_meas(17)
;
meas_x = 0.75e-2
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=18
end_command
mp18 = find_meas(18)
;
meas_x = 1.0e-2
meas_y = 0.0 - pl_thk/2
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=19
end_command
mp19 = find_meas(19)
;
meas_x = 1.25e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=20
end_command
mp20 = find_meas(20)
;
meas_x = 1.50e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=21
end_command
mp21 = find_meas(21)
;
meas_x = 1.75e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=22
end_command
mp22 = find_meas(22)
;
meas_x = 2.0e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=23
end_command
mp23 = find_meas(23)
;
meas_x = 2.25e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=24
end_command
mp24 = find_meas(24)
;
meas_x = -0.25e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=25
end_command
mp25 = find_meas(25)
;
meas_x = -0.50e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=26
end_command
mp26 = find_meas(26)
;
meas_x = -0.75e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=27
derm_command
mp27 = find_meas(27)
;
meas_x = -1.0e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=28
derm_command
mp28 = find_meas(28)
;
meas_x = -1.25e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=29
derm_command
mp29 = find_meas(29)
;
meas_x = -1.50e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=30
derm_command
mp30 = find_meas(30)
;
meas_x = -1.75e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=31
derm_command
mp31 = find_meas(31)
;
meas_x = -2.0e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=32
derm_command
mp32 = find_meas(32)
;
meas_x = -2.25e-2
meas_y = 0.0 - pl_thk/2
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=33
derm_command
mp33 = find_meas(33)
;
;------------------------------------------------------------------
; Installs second row (top) of measurement circles in specimen midpoint
;
meas_x = -0.1e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=34
end_command
mp34 = find_meas(34)
;
meas_x = 0.15e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=35
end_command
mp35 = find_meas(35)
;
meas_x = 0.4e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=36
end_command
mp36 = find_meas(36)
;
meas_x = 0.65e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=37
end_command
mp37 = find_meas(37)
;
meas_x = 0.9e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=38
end_command
mp38 = find_meas(38)
;
meas_x = 1.15e-2
meas_y = 0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=39
end_command
mp39 = find_meas(39)
;
meas_x = 1.4e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=40
end_command
mp40 = find_meas(40)
;
meas_x = 1.65e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=41
end_command
mp41 = find_meas(41)
;
meas_x = 1.9e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=42
end_command
mp42 = find_meas(42)
;
meas_x = 2.15e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=43
end_command
mp43 = find_meas(43)
;
meas_x = -0.35e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=44
end_command
mp44 = find_meas(44)
;
meas_x = -0.6e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=45
end_command
mp45 = find_meas(45)
;
meas_x = -0.85e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=46
end_command
mp46 = find_meas(46)
;
meas_x = -1.1e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=47
end_command
mp47 = find_meas(47)
;
meas_x = -1.35e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=48
end_command
mp48 = find_meas(48)
;
meas_x = -1.6e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=49
end_command
mp49 = find_meas(49)
;
meas_x = -1.85e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=50
end_command
mp50 = find_meas(50)
;
meas_x = -2.1e-2
meas_y = 0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=51
end_command
mp51 = find_meas(51)
;
;------------------------------------------------------------------
; Installs second row (bottom) of measurement circles at midpoint
;
meas_x = - 0.1e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=52
end_command
mp52 = find_meas(52)
;
meas_x = 0.15e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=53
end_command
mp53 = find_meas(53)
;
meas_x = 0.4e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=54
end_command
mp54 = find_meas(54)
;
meas_x = 0.65e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=55
end_command
mp55 = find_meas(55)
;
meas_x = 0.9e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=56
end_command
mp56 = find_meas(56)
;
meas_x = 1.15e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=57
end_command
mp57 = find_meas(57)
;
meas_x = 1.4e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=58
end_command
mp58 = find_meas(58)
;
meas_x = 1.65e-2
meas_y = -0.003
command
    measure x=@meas_x y=@meas_y rad=@meas_rad2 id=59
end_command
mp59 = find_meas(59)
;
meas_x = 1.9e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=60
end_command
mp60 = find_meas(60)
;
meas_x = 2.15e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=61
end_command
mp61 = find_meas(61)
;
meas_x = -0.35e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=62
end_command
mp62 = find_meas(62)
;
meas_x = -0.6e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=63
end_command
mp63 = find_meas(63)
;
meas_x = -0.85e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=64
end_command
mp64 = find_meas(64)
;
meas_x = -1.1e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=65
end_command
mp65 = find_meas(65)
;
meas_x = -1.35e-2
meas_y = -0.003
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=66
end_command
mp66 = find_meas(66);
meas_x = -1.6e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=67
disable end_command
mp67 = find_meas(67);
meas_x = -1.85e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=68
disable end_command
mp68 = find_meas(68);
meas_x = -2.1e-2
meas_y = -0.003
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=69
disable end_command
mp69 = find_meas(69);
-----------------------------------------------------------------------------
;Installs upper most row of measurement circles at midpoint
;
meas_x = 0.0e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=70
disable end_command
mp70 = find_meas(70);
meas_x = 0.25e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=71
disable end_command
mp71 = find_meas(71);
meas_x = 0.5e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=72
disable end_command
mp72 = find_meas(72)
;
meas_x = 0.75e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=73
end_command
mp73 = find_meas(73)
;
meas_x = 1.0e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=74
end_command
mp74 = find_meas(74)
;
meas_x = 1.25e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=75
end_command
mp75 = find_meas(75)
;
meas_x = 1.50e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=76
end_command
mp76 = find_meas(76)
;
meas_x = 1.75e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=77
end_command
mp77 = find_meas(77)
;
meas_x = 2.0e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=78
end_command
mp78 = find_meas(78)
;
meas_x = 2.25e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=79
end_command
mp79 = find_meas(79)
meas_x = -0.25e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=80
end_command
mp80 = find_meas(80)
meas_x = -0.50e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=81
end_command
mp81 = find_meas(81)
meas_x = -0.75e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=82
end_command
mp82 = find_meas(82)
meas_x = -1.0e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=83
end_command
mp83 = find_meas(83)
meas_x = -1.25e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=84
end_command
mp84 = find_meas(84)
meas_x = -1.50e-2
meas_y = 0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=85
end_command
mp85 = find_meas(85)
meas_x = -1.75e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=86 end_command
mp86 = find_meas(86)
;
meas_x = -2.0e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=87 end_command
mp87 = find_meas(87)
;
meas_x = -2.25e-2
meas_y = 0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=88 end_command
mp88 = find_meas(88)
;
;------------------------------------------------------------------------
;Installs lowest row of measurement circles at midpoint
;
meas_x = 0.0e-2
meas_y = -0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=89 end_command
mp89 = find_meas(89)
;
meas_x = 0.25e-2
meas_y = -0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=90 end_command
mp90 = find_meas(90)
;
meas_x = 0.5e-2
meas_y = -0.006
command
  measure x=@meas_x y=@meas_y rad=@meas_rad2 id=91 end_command
mp91 = find_meas(91)
;
meas_x = 0.75e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=92
d end_command
mp92 = find_meas(92)
;
meas_x = 1.0e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=93
d end_command
mp93 = find_meas(93)
;
meas_x = 1.25e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=94
d end_command
mp94 = find_meas(94)
;
meas_x = 1.50e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=95
d end_command
mp95 = find_meas(95)
;
meas_x = 1.75e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=96
d end_command
mp96 = find_meas(96)
;
meas_x = 2.0e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=97
d end_command
mp97 = find_meas(97)
;
meas_x = 2.25e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=98
d end_command
mp98 = find_meas(98)
;
meas_x = -0.25e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=99
end_command
mp99 = find_meas(99)
;
meas_x = -0.50e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=100
end_command
mp100 = find_meas(100)
;
meas_x = -0.75e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=101
end_command
mp101 = find_meas(101)
;
meas_x = -1.0e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=102
end_command
mp102 = find_meas(102)
;
meas_x = -1.25e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=103
end_command
mp103 = find_meas(103)
;
meas_x = -1.50e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=104
end_command
mp104 = find_meas(104)
;
meas_x = -1.75e-2
meas_y = -0.006
command
measure x=@meas_x y=@meas_y rad=@meas_rad2 id=105
end_command
mp105 = find_meas(105)
\[
\begin{align*}
\text{meas}_x &= -2.0\times 10^{-2} \\
\text{meas}_y &= -0.006 \\
\text{command} \\
\text{measure } x &= @\text{meas}_x \ y &= @\text{meas}_y \ rad &= @\text{meas}_\text{rad2} \ id &= 106 \\
\text{end_command} \\
\text{mp106} &= \text{find_meas}(106) \\
; \\
\text{meas}_x &= -2.25\times 10^{-2} \\
\text{meas}_y &= -0.006 \\
\text{command} \\
\text{measure } x &= @\text{meas}_x \ y &= @\text{meas}_y \ rad &= @\text{meas}_\text{rad2} \ id &= 107 \\
\text{end_command} \\
\text{mp107} &= \text{find_meas}(107) \\
\text{end} \\
\end{align*}
\]

====================================================================
======
def \text{ds\_mc\_ns}
; \normal\ stress\ calculated\ from\ measurement\ circles
; \oo = \text{measure}(\text{mp200}, 1) \\
\oo = \text{measure}(\text{mp1}, 1) \\
\oo = \text{measure}(\text{mp2}, 1) \\
\oo = \text{measure}(\text{mp3}, 1) \\
\oo = \text{measure}(\text{mp4}, 1) \\
\oo = \text{measure}(\text{mp5}, 1) \\
\oo = \text{measure}(\text{mp6}, 1) \\
\oo = \text{measure}(\text{mp7}, 1) \\
\oo = \text{measure}(\text{mp8}, 1) \\
\oo = \text{measure}(\text{mp9}, 1) \\
\oo = \text{measure}(\text{mp10}, 1) \\
\oo = \text{measure}(\text{mp11}, 1) \\
\oo = \text{measure}(\text{mp12}, 1) \\
\oo = \text{measure}(\text{mp13}, 1) \\
\oo = \text{measure}(\text{mp14}, 1) \\
; \\
\text{ds1\_y} = \text{m\_s22}(\text{mp1}) \\
\text{ds2\_y} = \text{m\_s22}(\text{mp2}) \\
\text{ds3\_y} = \text{m\_s22}(\text{mp3}) \\
\text{ds4\_y} = \text{m\_s22}(\text{mp4}) \\
\text{ds5\_y} = \text{m\_s22}(\text{mp5}) \\
\text{ds6\_y} = \text{m\_s22}(\text{mp6}) \\
\text{ds7\_y} = \text{m\_s22}(\text{mp7}) \\
\text{ds8\_y} = \text{m\_s22}(\text{mp8}) \\
\text{ds9\_y} = \text{m\_s22}(\text{mp9})
ds10_y = m_s22(mp10)
ds11_y = m_s22(mp11)
ds12_y = m_s22(mp12)
ds13_y = m_s22(mp13)
ds14_y = m_s22(mp14)
ds15_y = m_s22(mp200)

; ds_sum_y1 = ds1_y + ds2_y + ds3_y + ds4_y + ds5_y + ds6_y + ds7_y
ds_sum_y2 = ds8_y + ds9_y + ds10_y + ds11_y + ds12_y + ds13_y + ds14_y

; average normal stress obtained from top measurement circles:
ds_ave_ybot = (ds_sum_y1) / 7

; average normal stress obtained from bottom measurement circles:
ds_ave_ytop = (ds_sum_y2) / 7

; normal stress by averaging top and bottom measurement circles
ds_ave_y = (abs(ds_ave_ytop) + abs(ds_ave_ybot)) / 2

norm_cen = ds15_y
end

def avg_stra_yy
    ; strain rate calculated from top measurement circles
    oo = measure(mp8, 2)
    oo = measure(mp9, 2)
    oo = measure(mp10, 2)
    oo = measure(mp11, 2)
    oo = measure(mp12, 2)
    oo = measure(mp13, 2)
    oo = measure(mp14, 2)

    strainxx_mc8 = m_ed11(mp8)
    strainxx_mc9 = m_ed11(mp9)
    strainxx_mc10 = m_ed11(mp10)
    strainxx_mc11 = m_ed11(mp11)
    strainxx_mc12 = m_ed11(mp12)
    strainxx_mc13 = m_ed11(mp13)
    strainxx_mc14 = m_ed11(mp14)

    strainyy_mc8 = m_ed22(mp8)
    strainyy_mc9 = m_ed22(mp9)
    strainyy_mc10 = m_ed22(mp10)
    strainyy_mc11 = m_ed22(mp11)
strainyy_mc12 = m_ed22(mp12)
strainyy_mc13 = m_ed22(mp13)
strainyy_mc14 = m_ed22(mp14)

avg_stra_yy = (strainyy_mc8 + strainyy_mc9 + strainyy_mc10 + strainyy_mc11 + strainyy_mc12 + strainyy_mc13 + strainyy_mc14) / 7

end

=====================================================================
def ds1_ang
;
; Computes current stress path and orientation of maximum principal stress within a measurement circle.
;
; Input : mp1 - measurement circle pointer
;         mp2 - measurement circle pointer
;         mp3 - measurement circle pointer
;         mp4 - measurement circle pointer
;
; Output :   ds1_ms1 - maximum principal stress
;    ds1_ms3 - minimum principal stress
;    ds1_ang - direction of maximum principal stress
;
;
ds1_x = m_s11(mp200)
ds1_y = m_s22(mp200)
ds1_xy = 0.5 * (m_s12(mp200) + m_s21(mp200))
_sxx = ds1_x
_syy = ds1_y
_sxy = ds1_xy

; pr_stress computes principal stress from _sxx, _syy and _sxy for mc200
;
pr_stress
ds1_ms1 = pr_smax
ds1_ms3 = pr_smin
ds1_ang = (180.0/pi) * 0.5 * atan2 ( -2.0*_sxy, (-_sxx+_syy) )
end

=====================================================================
def pr_stress
;
; computes principal stresses
; Input : _sxx, _syy, _sxy
;
; Output : pr_smax, pr_smin
avg_str = 0.5 * (_sxx + _syy)

mohr_rad = sqrt((0.5 * (_sxx - _syy))^2 + _sxy^2)

pr_s1 = (-avg_str + mohr_rad)
pr_s3 = (-avg_str - mohr_rad)

if pr_s1 > pr_s3 then
    pr_smax = pr_s1
    pr_smin = pr_s3
else
    pr_smax = pr_s3
    pr_smin = pr_s1
end_if

; Filename : sb_geom.dat
;
; Purpose : geometry of a shear box sample in CNL and CNS for direct shear test
;
;=====================================================================
; SET Rock_s_height = 50.0e-3 ; rock sample height
; SET Rock_s_width = 25.0e-3 ; rock sample width
; SET pl_thk = 0.0 ;25.0e-3 ; non-zero value for CNS - zero value for CNL
;
;=====================================================================