Frequency-Shifted Interferometry for Fiber-Optic Sensing

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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This thesis studies frequency-shifted interferometry (FSI), a useful and versatile technique for fiber-optic sensing. I first present FSI theory by describing practical FSI configurations and discussing the parameters that affect system performance. Then, I demonstrate the capabilities of FSI in fiber-optic sensor multiplexing and high sensitivity chemical analysis.

We implemented a cryogenic liquid level sensing system in which an array of 3 fiber Bragg grating (FBG) based sensors was interrogated by FSI. Despite sensors’ spectral overlap, FSI is able to separate sensor signals according to their spatial locations and to measure their spectra, from which whether a sensor is in liquid or air can be unambiguously determined.

I showed that a broadband source paired with a fast tunable filter can be used in FSI systems as the light source. An array of 9 spectrally overlapping FBGs was successfully measured by such a system, indicating the potential of system cost reduction as well as measurement speed improvement.

I invented the the FSI-CRD technique, a highly sensitive FSI-based fiber cavity ring-down (CRD) method capable of deducing minuscule loss change in a fiber cavity from the intensity decay rate of continuous-wave light circulating in the cavity. As a proof-of-principle experiment, I successfully measured the fiber bend loss introduced in the fiber cavity with FSI-CRD, which was found to be 0.172 dB/m at a bend radius of 12.5 mm. We then applied FSI-CRD to evanescent-field sensing. We incorporated fiber tapers as the sensor head in the system and measured the concentration of 1-octyne solutions. A minimum detectable 1-octyne concentration of 0.29% was achieved with measurement sensitivity of 0.0094 dB/% 1-octyne. The same system also accurately detected the concentration change of sodium chloride (NaCl) and glucose solutions. Refractive index sensitivity of 1 dB/RIU with a measurement error of $1 \times 10^{-4}$ dB was attained for NaCl solutions.
Finally, I proposed a theoretical model to study the polarization effects in FSI systems. Preliminary results show that the model can already explain the experimental observations. It not only provides insight into how to improve system performance but also suggests potential new applications of the technique.
Acknowledgements

First and foremost, I would like to sincerely thank my supervisor Prof. Li Qian for her trust, support, and patience throughout the course of the thesis. This work would not have been possible without her sagacious guidance.

I am grateful to Prof. Peter R. Herman and Prof. Mohammad Mojahedi for kindly serving on my Ph.D. supervisory committee as well as the Ph.D. examination committee. Their suggestions were very helpful to the development of the thesis. I would also like to thank Prof. J. Stewart Aitchison and my external examiner, Prof. Jianping Yao from University of Ottawa, for joining the final oral examination committee of my Ph.D. thesis.

Many people have contributed to this thesis. I would like to acknowledge the input from Prof. Peter W. E. Smith and Prof. Joyce K. S. Poon at this project’s early stage. I thank the members – both former and current – of our research group Michael A. Galle, Rojina Ghasemi, Jason C. Ng, Dr. Bing Qi, Christopher A. Sapiano, Peyman Sarrafi, Lijun Zhang, Yiwei Zhang, Eric Y. Zhu, and Dr. Wen Zhu for their assistance and fruitful discussions. Special thanks go to Dr. Bing Qi, Yiwei Zhang, and visiting professor Dr. Ciming Zhou from the National Engineering Laboratory for Fiber Optic Sensing Technology at Wuhan University of Technology for their direct involvement in the project. The administrative support from the Photonics Group administrative assistants Belinda Li, Linda Liu, Raj Balkaran, and Beverly Chu are also highly appreciated.

I would like to extend my gratitude to Prof. Kevin P. Chen, Dr. Tong Chen, Dr. Di Xu, and Qingqing Wang from University of Pittsburgh for the successful collaboration on the cryogenic liquid level sensing project. I am grateful to Prof. Datong Song from the Department of Chemistry for helpful discussions on the measurement of chemical solutions. I would also thank engineering technologist George Kretschmann from the Scanning Electron Microscope (SEM) Laboratory at the Department of Geology for providing me with training and suggestions on the use of their SEM facility.

I would thank Prof. James S. Wallace, Robert Abader, and Mark Tadrous from the Department of Mechanical and Industrial Engineering for our enjoyable collaboration on the biofuel sensing project.

I am glad to know my fellow graduate students who have made my graduate studies more interesting, especial those who have helped me during my time as the President of the Optical Society of America Student Chapter. I would like to thank Dr. Bhavin J. Bijlani, J. Niklas Caspers, Kyle H. Y. Cheng, Arnab Dewanjee, James Dou, Dr. Luis A. Fernandes, Jason R. Grenier, Moez Haque, Dr. Emanuel Istrate, Arash Joushaghani, Dongpeng Kang, Pisek Kultavewuti, Kenneth Li, Zhongfa Liao, Steven A. Rutledge, Iliya Sigal, Xiao Sun, Zhiyuan Tang, Alex Wong, Herman M.-K. Wong, and Liang Yuan. This list is by no means complete.

I owe a debt of gratitude to Yanbing Ma, from whom I have learned the invaluables over the past eight years.

Finally, I would like to thank my parents for their selfless love, unconditional support, and
understanding. No words can express my deepest gratitude to them. This work is dedicated to my parents.

Fei Ye
Toronto, Ontario
September, 2013
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List of Acronyms

2D  two-dimensional
AI  aluminum
AM  amplitude modulation
AOM  acousto-optic modulator
ASE  amplified spontaneous emission
BD  balanced detector
BNC  Bayonet-Neill-Concelman
Co  cobalt
CRD  cavity ring-down
CRDS  cavity ring-down spectroscopy
CW  continuous-wave
CW-CRDS  continuous-wave cavity ring-down spectroscopy
DAQ  data acquisition board
DFT  discrete Fourier transform
EDFA  Erbium-doped fiber amplifier
EMI  electromagnetic interference
FBG  fiber Bragg grating
FDM  frequency-division multiplexing
FFT  fast Fourier transform
**FMCW** frequency-modulated continuous-wave

**FMS** frequency-modulation spectroscopy

**FP** Fabry-Perot

**FSI** frequency-shifted interferometry

**FSI-CRD** FSI-based cavity ring-down

**FT** Fourier transform

**FWHM** full width at half-maximum

**H₂O** water

**H₂SiF₆** hexafluorosilicic acid

**HAF** high-attenuation fiber

**HAFBG** high-attenuation fiber Bragg grating

**HF** hydrofluoric acid

**ICAS** intracavity absorption spectroscopy

**ISO** isolator

**IUPAC** International Union of Pure and Applied Chemistry

**LPG** long-period grating

**NaCl** sodium chloride

**OFM** optical fiber microwire

**OFN** optical fiber nanowire

**OPL** optical path length

**OSA** optical spectrum analyzer

**PC** polarization controller

**PD** photodiode

**PM** power meter

**PS-CRD** phase-shift cavity ring-down
RDC  ring-down cavity
RF  radio frequency
RI  refractive index

SA-FSI  single-arm FSI
SCRD  spatial cavity ring-down
SDM  spatial-division multiplexing
SEM  scanning electron microscopy

SiF₄  silicon tetrafluoride
SiO₂  silica
SMF  single-mode fiber

SNR  signal-to-noise ratio

TDM  time-division multiplexing
TF  tunable filter
TLS  tunable laser source

VOA  variable optical attenuator

WMS  wavelength-modulation spectroscopy
Chapter 1

Introduction

1.1 Fiber-optic sensing

1.1.1 An overview

More than a decade into the 21st century, our society has fully embraced the information age. Modern telecommunication technologies enable information to be transferred from one place to another anywhere on the earth within a matter of seconds. The ubiquitous employment of the Internet especially, has fundamentally transformed the way people communicate with each other, unleashing countless new opportunities that influence everyone and in almost every aspect of our daily lives: online news, online shopping, online banking, online education, online social networks, and the list goes on. More and more mobile devices, such as the iPhone or BlackBerry® series, are now connected to the Internet, which facilitates the access to information even further. Behind the scenes, an enormous amount of Internet data is exchanged via a sophisticated network of fiber optics. The information is encoded into optical signals, transmitted through long dielectric fibers at high speed, and decoded at the receiving ends. The practice of sending data through glass fibers originated from a pioneer’s vision almost half a century ago, when a 32-year-old Charles K. Kao proposed in 1966 that “a fibre of glassy material constructed in a cladded structure ... represents a possible practical optical waveguide with important potential as a new form of communication medium” [1]. For his groundbreaking contributions, Kao was awarded the Nobel Prize in Physics in 2009 [2]. Following Kao’s footsteps, scientists and engineers have devoted extensive research in the past few decades to fiber-optic communications, and this paved way to the revolution in communications today. One can proclaim that fiber optics is a cornerstone of the modern information age, and it shall continue to shape our future.

The advances of fiber-optic communications not only weave the world together, their products have benefited other fields as well. Useful tools including high quality optical sources, detectors, amplifiers, modulators, and so on are now commercially available for new applications [3,4]. The laser diodes used in fiber-optic communications, for example, can also be found
in their simpler forms in gadgets such as CD or DVD players, laser printers, or barcode readers. The fiber-optic sensing industry emerged in recent years is a notable derivative of fiber-optic communications [5–11].

A fiber-optic sensor is a device that utilizes light guided by optical fibers to measure certain physical or chemical parameters (e.g., pressure, index of refraction, chemical concentration etc.). If a fiber-optic sensor’s sensing process takes place within the fiber, it is referred to as an intrinsic fiber-optic sensor. On the other hand, if the sensing occurs outside of the fiber, it is called extrinsic. There are also hybrid fiber-optic sensors whose sensing mechanism may not necessarily be optical, whereas light is delivered through the fiber to power the sensor (e.g., an electrical sensor) or to simply carry the resultant data. The distinction between the definitions of extrinsic and hybrid fiber-optic sensors are sometimes not pellucid [5].

Fiber-optic sensors offer many advantages compared with traditional electrical sensors [5–12]. First, light inherently possesses many properties that can be manipulated and measured: intensity, wavelength (or frequency), polarization, phase, coherence, and so forth. This suggests that if one can relate a measurand change to any of above light property change and measure it, an optical sensor can be made. Therefore, considerable flexibility is possible in the design of fiber-optic sensors. Made of dielectric materials, fiber-optic sensors are immune to electromagnetic interference (EMI). For instance, they can be applied reliably in power distribution systems where traditional electrical sensors require laborious electrical insulation due to the high voltages involved [13]. Optical fibers are non-conducting, stable, and chemically inert; they are durable in many harsh environments (e.g., corrosive, high-temperature). Furthermore, fiber-optic sensors can be spark-free and thus are suitable for environmental monitoring applications in oil and mining industry where fire hazards could lead to disastrous consequences. After years of improvements, optical fibers currently have very low loss. The attenuation of a standard single-mode fiber can be as low as 0.17 dB/km [14], which allows us to place the sensor head and the analyzer kilometers apart without amplifications. This is highly desirable if in situ measurements must be carried out over a wide area or in a dangerous environment. Because of fiber’s flexibility and lightness, fiber-optic sensors can be very compact as well. A fiber-optic current sensor with a sensor head simply consisting of a fiber loop can replace a conventional transducer that may weigh up to 2 tons [15]. Small-sized gyroscopes based on fiber optics have already been used on Boeing commercial airplanes [16].

The cost reduction of fiber-optic components has been continuously driven by the telecommunications industry. In the 1980s, single-mode fibers were rare and they cost $5-10 per meter, but they were only 10 cents per meter in the 1990s and had been widely deployed. It is projected that the cost of single-mode fibers will be less than 5 cents per meter by 2020 [5]. As another example, the laser diodes in CD players cost only $3 each today, compared with $3000 in the 1980s when they were still prototypes. Their price may drop to below $1 in 10 years [5]. Consequently, the tools for fiber-optic sensing shall become cheaper and cheaper in price, but their performances and functionality shall improve as a result of the technical maturation.
Besides the fiber-optic sensing examples given above, many large-scale fiber-optic sensing systems have already been employed in the field. With current trend, we can foresee that the fiber-optic sensing industry, propelled by technical superiority and cost reduction, will flourish in the near future and take up a significant share in a sensing market dominated by conventional technologies.

### 1.1.2 Conventional fiber-optic sensor multiplexing

The multiplexing capability is a unique feature of fiber-optic sensors [5, 6, 17]. Multiple point sensors can conveniently be linked by various fiber-optic components to form a sensor network that share optical sources or detectors (see Fig. 1.1). Such an arrangement is particularly advantageous when one needs to monitor a measurand at multiple locations, or to monitor multiple measurands at the same time. The cost of a large-scale multiplexed fiber-optic sensing system can be lower due to a reduced number of key components (e.g., sources or detection units) as well as simplified cabling and packaging [5, 17]. In practice, fiber-optic sensors can be distributed along a single fiber link to form a serial array as illustrated in Fig. 1.1(a) [18]. Another topology is the parallel configuration in Fig. 1.1(b), where the sensors are placed in different channels and share the sensing light allocated by a fiber coupler [19]. Alternatively, with a few fiber couplers for each sensor, an architecture that is akin to a ladder can be achieved [Fig. 1.1(c)] [6, 17]. Variations or combinations of above topologies are also feasible [17].

Once a fiber-optic sensor network is built, special multiplexing schemes are used to interrogate the sensors with the goal of obtaining and distinguishing the measurement result from each individual sensor. Commonly used fiber-optic sensor multiplexing techniques include spatial-division multiplexing (SDM) [20], time-division multiplexing (TDM) [17, 18, 21], wavelength-division multiplexing (WDM) [17, 18, 22], and frequency-division multiplexing (FDM) (see Fig. 1.2) [5, 17]. The sensors in a SDM system share the light source, but each sensor requires its own detection unit [Fig. 1.2(a)]. In TDM, pulsed sensing light is injected into the sensor network. Different time delays are introduced between sensors so that the signal from each sensor arrives at the detection unit at a different instance of time. The signals from different sensors can therefore be separated in the time domain [Fig. 1.2(b)]. In WDM, a specific wavelength window is assigned to an individual sensor to operate within. The signals from different sensors can thus be identified from the wavelength range they belong to [Fig. 1.2(c)]. There are more than one way to implement FDM [17]. In the frequency-modulated continuous-wave (FMCW) version of FDM [23], the source is modulated periodically by a radio frequency (RF) signal, and the beat frequencies generated from the mixing of the returning sensor signals and the modulation reference are measured. Different time delays between the sensors result in a unique beat frequency for each sensor that can be used to separate sensor signals. (All optical FMCW can also be implemented, in which the optical frequency is periodically modulated [24].)

These conventional fiber-optic sensor multiplexing schemes have their respective limitations. In SDM, although the light source can be shared, as each sensor requires its own detection
Variations or combinations of above topologies can also be employed. Note the sensors in (c) are of transmission-type. Figures (a) and (b) assume reflection-type sensors which reflect the resultant data back to the detector. If they are transmission-type sensors, a detector should be placed at the end of the array in (a), and each sensor in (b) needs one detector after it.

unit, the system cost can still be high. In TDM, expensive pulsed source and fast detection electronics are required, which may also incur higher cost. For a WDM system, the sensor number is limited by the ratio of available source bandwidth to the sensor wavelength window size. A sensor’s dynamic range is limited by the size of the wavelength range assigned to it. FDM methods such as FMCW involve coherent detection, which means modulation on the source and a reference signal are required to produce beat frequencies. An ideal fiber-optic
Fig. 1.2. Commonly used fiber-optic sensor multiplexing schemes. (a) spatial-division multiplexing; (b) time-division multiplexing; (c) wavelength-division multiplexing; (d) frequency-division multiplexing.
sensor multiplexing technique should be capable of measuring and separating the signals from the sensors in the network with simple setup and shared low-cost components (e.g., continuous-wave sources and slow detectors).

1.2 Motivation behind the thesis

As fiber-optic sensing is an emerging industry, tools that enable fiber-optic sensor multiplexing or high sensitivity measurement are going to be in high demand. The goal of this thesis is to explore and expand the capabilities of a fiber-optic technique called frequency-shifted interferometry (FSI) in fiber-optic sensing, specifically in low-cost fiber-optic sensor multiplexing and high-sensitivity sensing applications. Another aim is to investigate the factors that affect the performance of an FSI-based fiber-optic sensing system to determine the limitations of the technique and discover potential directions for improvements or new frontiers. This thesis is a continuation of my master’s thesis, in which some preliminary work on FSI was carried out (FBG interrogation and gas sensing).

1.3 Major contributions of the thesis

One major contribution of the thesis is in the area of fiber-optic sensor multiplexing. FSI has many advantages over conventional fiber-optic sensing techniques. Unlike TDM, FSI uses continuous-wave (CW) light source and a slow detector, and thus pulsed light source or fast electronics are avoided. Also, it allows the sensors to operate in the same wavelength range, which is not possible for WDM. The sensor number in a channel can be increased. In contrast to the FMCW technique, FSI does not rely on source modulation or any reference signal. We applied the technique of FSI in the interrogation of an array of fiber-optic liquid level sensors. The experimental setup of our own design allows us to measure the responses of sensors while enabling active sensor tuning. This system has nicely overcome the difficulties encountered by conventional refractive-index-based fiber-optic liquid level sensing schemes, and it is particularly useful for cryogenic applications. We also showed that an FSI system using a broadband source and a tunable filter can also be employed to multiplex fiber-optic sensors. We implemented a fully automated sensing system which uses much shorter measurement time and low-cost equipment, proving that FSI is a practical fiber-optic sensing technique.

Another important contribution of the thesis is the invention of the fiber cavity ring-down (CRD) technique based on frequency-shifted interferometry (FSI-CRD). Conventional CRD techniques demand that an optical pulse (∼10 ns duration) be generated in the cavity first. The rapid transient event is then captured by fast detection electronics (∼100 MS/s sampling rate) so as to obtain the pulse intensity decay rate in the time domain. In FSI-CRD, on the other hand, CW light is launched into the optical cavity, and a slow detector is used to measure the interference signal (with a sampling rate less than 100 kS/s) between lightwaves exiting
the cavity after the same number of round trips. The light intensity decay rate is extracted from the Fourier transform of the interference signal, instead of from the direct time-domain measurement. Therefore, FSI-CRD is a novel CRD technique whose principle is completely different from conventional ones. We formulated the theory of FSI-CRD and designed the preliminary tests. We successfully carried out proof-of-concept experiments to measure fiber bend loss. As one step forward, we applied the technique to evanescent-field sensing. The challenge of this application lies in finding an experimental setup and a sensing element that can be employed reliably in such a highly sensitive environment. Through trial and error, we investigated different experimental configurations and arrived at one setup that is suitable for our purpose. We also studied the fabrication method of the sensing element (fiber tapers) by chemical etching. Fiber tapers of similar performances can be produced repeatedly. We used an FSI-CRD system incorporating a fiber taper to measure chemical solution absorption, refractive index change, or a combination of both. Our chemical absorption experiments show lower detection limit than those obtained by conventional fiber-based cavity ring-down techniques reported in literature. Our refractive index measurements also offers low measurement error that is comparable to those reported by other groups.

My work has resulted in 6 publications in prestigious journals (5 from this thesis work) and multiple publications in conferences:

**Refereed journal publications**


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1F. Ye et al. (2008) is from my master’s work. Only the conference publications from this thesis work are included.
Refereed conference publications


1.4 Organization of the thesis

This thesis is organized as follows. Chapter 2 introduces the theory of FSI. Several realizations of FSI are described, including the configurations in the forms of a Sagnac interferometer, a linear Sagnac interferometer, and a single-arm interferometer. Important parameters that dictate the performance of an FSI system (spatial resolution, spatial sensing range, dispersion, and system crosstalk) are also analyzed. In Chapter 3, a cryogenic liquid level sensing system based on FSI is presented. A review on conventional fiber-optic liquid level sensing methods is given. The design and operation principle of the liquid level sensors used in our experiments are explained. Successful interrogation of such a serial sensor array by FSI is an excellent example that demonstrates the capability of FSI as a fiber-optic sensor multiplexing tool. Chapter 4 shows that FSI is indeed a practical fiber-optic multiplexing scheme that promises low system cost. A serial array of fiber Bragg grating sensors can be interrogated in a short measurement time with an FSI system using a cheap broadband source paired with a fast tunable filter. Chapter 5 is dedicated to the highly sensitive FSI-CRD technique. The principle of FSI-CRD is presented and compared with that of conventional CRD methods. Experimental results of FSI-CRD in fiber bend loss measurements and evanescent-field sensing are also shown. Chapter 6 studies the effects of polarization in FSI-based systems. Finally, in Chapter 7, we summarize the original contributions of this study on FSI and provide the future prospects of the FSI technique.
Chapter 2

Theory of Frequency-Shifted Interferometry

In frequency-shifted interferometry (FSI), continuous-wave (CW) light at optical frequency $\nu_0$, together with its frequency-shifted copy at $\nu_0 + f$, is launched into a common optical path. After exiting the common path, the original lightwave frequency is shifted to $\nu_0 + f$ as well so that it interferes with its frequency-shifted copy, resulting in a sinusoidal interference signal $I$. By recording $I$ while sweeping $f$, one can deduce the optical path length as well as the interfering light intensity from the sinusoid oscillation rate and amplitude, respectively. If the lightwave and its frequency-shifted copy enter multiple paths of different lengths, each path contributes a unique sinusoid component to $I$ that can be separated by Fourier transform. CW lasers or low-coherent broadband sources can be employed in FSI, and no fast electronics are needed for the measurement, which promises a cost-effective way of performing fiber-optic sensing [25–32]. This chapter describes the principle of FSI, and presents a few FSI configurations as well as various parameters that characterize the performance of an FSI system.

2.1 Frequency-shifted fiber Sagnac interferometer

FSI was first constructed with a Sagnac interferometer configuration [25, 33–35]. Consider the Sagnac loop formed by a 50/50 fiber directional coupler in Fig. 2.1(a). When light enters the fiber loop from port 1 of the fiber coupler, it forms two counter-propagating lightwaves in the loop. Because of their interference at the coupler when these two lightwaves recombine, all the light power exits the loop from input port 1 (polarization effects are neglected here). However, if an optical frequency shifter is introduced asymmetrically into the loop as shown in Fig. 2.1(b), there may be power coming out from both fiber coupler ports, depending on the phase difference between the two counter-propagating lightwaves. The phase difference arises due to the asymmetry in the loop—the two fibers between the frequency shifter and the fiber coupler are of different lengths. In Fig. 2.1(b), the light traveling in the counterclockwise
Chapter 2. Theory of Frequency-Shifted Interferometry

Fig. 2.1. Sagnac interferometer configuration. (a) a regular Sagnac interferometer; (b) a frequency-shifted Sagnac interferometer.

direction passes $l_0$ first at its original optical frequency $\nu$, acquires a frequency shift $f$, then goes through $l_1$ at a frequency $\nu + f$. On the other hand, the clockwise-propagating light passes $l_1$ at frequency $\nu$, but at $\nu + f$ when going through $l_0$. Let us assume the input electric field at port 1 of the coupler to be $E_{\text{in}} = E_0 \cdot \exp(2\pi \nu t + \phi_0)$, where $E_0$ is the input field amplitude, $\nu$ is the optical frequency, and $\phi_0$ is some initial phase of the field. The electric fields of the two counter-propagating lightwaves when they meet at port 1 can be written as

$$E_{\text{cw1}} = (\sqrt{\gamma})^2 E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi(\nu + f)t + \frac{2\pi n_1 \nu}{c} + \frac{2\pi n_0 (\nu + f)}{c} + \phi_0 + \frac{\pi}{2} \right] \right\}$$  \hspace{1cm} (2.1a)

$$E_{\text{ccw1}} = (\sqrt{\gamma})^2 E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi(\nu + f)t + \frac{2\pi n_0 \nu}{c} + \frac{2\pi n_1 (\nu + f)}{c} + \phi_0 + \frac{\pi}{2} \right] \right\}$$  \hspace{1cm} (2.1b)

while the fields at port 2 are

$$E_{\text{cw2}} = (\sqrt{\gamma})^2 E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi(\nu + f)t + \frac{2\pi n_1 \nu}{c} + \frac{2\pi n_0 (\nu + f)}{c} + \phi_0 \right] \right\}$$  \hspace{1cm} (2.2a)

$$E_{\text{ccw2}} = (\sqrt{\gamma})^2 E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi(\nu + f)t + \frac{2\pi n_0 \nu}{c} + \frac{2\pi n_1 (\nu + f)}{c} + \phi_0 + \pi \right] \right\}$$  \hspace{1cm} (2.2b)
where $\gamma = 0.5$ is the coupling ratio of the fiber coupler, $n$ is the effective refractive index of the fiber mode, and $c$ is the speed of light in vacuum. Note that $(\sqrt{\gamma})^2$ is used in above two sets of equations to emphasize that each field component passes the direction coupler twice before the interference. The $\pi/2$ terms in the exponentials of (2.1) are contributed by $E_{ccw1}$’s crossing the fiber coupler once, while the $\pi$ term in (2.2b) is a result of $E_{ccw2}$’s crossing the coupler twice.

It is easy to show that the intensities at the two ports are (see Appendix A)

\begin{align}
I_1 &\propto 2\gamma^2 E_0^2 + 2\gamma^2 E_0^2 \cos \left[ 2\pi \frac{n(l_1 - l_0)}{c} f \right] \\
I_2 &\propto 2\gamma^2 E_0^2 - 2\gamma^2 E_0^2 \cos \left[ 2\pi \frac{n(l_1 - l_0)}{c} f \right]
\end{align}

As can be seen, both (2.3a) and (2.3b) have a common DC component $2\gamma^2 E_0^2$, and a sinusoidal term. Furthermore, the two sinusoidal terms are out of phase by $\pi$ (opposite sign). The differential intensity between port 1 and port 2 is therefore

\begin{align}
\Delta I = I_1 - I_2 &\propto 2\gamma^2 E_0^2 \cos \left[ 2\pi \frac{n(l_1 - l_0)}{c} f \right] \\
&\propto 2\gamma^2 E_0^2 \cos \left[ 2\pi \cdot F \cdot f \right] \\
&\propto 2\gamma^2 E_0^2 \cos \Delta \phi
\end{align}

where $F = n(l_1 - l_0)/c$, and $\Delta \phi = 2\pi \cdot F \cdot f$ is the phase difference between the two interfering

\footnote{Here we assume that the fibers are single-mode.}
field components. It is clear from (2.4) that we may consider $\Delta I$ as a function of $f$ (see Fig. 2.2). Note that although $F$ has the unit of time, it in fact describes the oscillation frequency of the sinusoid as a function of $f$. This suggests that we may obtain the information about fiber mode refractive index or fiber length by sweeping $f$ and measuring $\Delta I$, since the oscillation frequency $F$ of $\Delta I$ is proportional to $n(l_1 - l_0)$. Given $n$, one may deduce the length difference between $l_0$ and $l_1$. Similarly, one may measure the optical length difference as a function of input light wavelength, from which chromatic dispersion can be calculated [25].

2.2 Linear frequency-shifted Sagnac interferometer with multiple reflections

FSI can also adopt a linear Sagnac interferometer configuration, in which case the optical frequency shifter is placed asymmetrically in the fiber loop with part of the loop formed by a linear segment (see Fig. 2.3). Suppose an input light at frequency $\nu$ is sent into the interferometer from port 1 of the 50/50 fiber directional coupler $C_1$, and there are multiple reflection sites ($R_1, R_2, R_3, ..., R_N$) along the fiber at the output coupler $C_2$ as shown in Fig. 2.3. Each reflection site $R_i$ ($i = 1, 2, 3, ..., N$) introduces four reflected components at $C_1$ because of the different paths taken by the light. One component travels through the upper fiber section between $C_1$ and $C_2$ twice and is at frequency $\nu$, while another component is at $\nu + 2f$ since it goes through the frequency shifter in the lower fiber section between the two couplers twice. The remaining two components both pass the frequency shifter exactly once and thus interfere in the coupler $C_1$ at frequency $\nu + f$. These two interfering components are equivalent to the counter-propagating lightwaves in a frequency-shifted Sagnac interferometer discussed in the previous section. Essentially, each reflection site helps to form a frequency-shifted Sagnac interferometer in which the two fiber sections connected to the frequency shifter have lengths $l_0$ and $l_1 + 2L_i + l_2$, respectively. Here, $L_i$ is the distance between the $i$th reflection site and the coupler $C_2$, while $l_0, l_1,$ and $l_2$ are the fiber lengths shown in Fig. 2.3. The differential interference signal produced by the $\nu + f$ components is a sinusoid

$$\Delta I_i \propto A_i \cdot \cos \left[ 2\pi \cdot \frac{n(l_1 + 2L_i + l_2 - l_0)}{c} \cdot f \right]$$

where $A_i$ is some amplitude proportional to the reflectivity of the reflection site. Note that the components at $\nu$ and $\nu + 2f$ in fact produce DC signals at the two $C_1$ output ports, which are effectively canceled out after the differential measurement. The intermixing of the components at $\nu$, $\nu + f$ and $\nu + 2f$ generates beat frequencies at $f$ and $2f$. If a slow detector is used to measure $\Delta I$, these beat signals can be averaged out to zero (e.g., if $f \sim 100$ MHz and $\Delta I$ is measured by a 10 MHz detector). For $2L_i \gg l_1 + l_2 - l_0$, we have $l_1 + 2L_i + l_2 - l_0 \approx 2L_i$. The total differential interference signal contributed by all the reflection sites is a summation
Fig. 2.3. A linear frequency-shifted Sagnac interferometer with multiple reflection sites. $C_1$ and $C_2$: 50/50 fiber directional couplers; $R_i$: the $i$th reflector.

Fig. 2.4. Performing Fourier transform on the differential interference signal obtained from a linear frequency-shifted Sagnac interferometer with multiple reflection sites provides spatial and reflectivity information of the reflection sites. (a) the differential interference signal $\Delta I$; (b) the Fourier spectrum of $\Delta I$ in (a).

of sinusoids [28]

$$\Delta I \propto \sum_{i=1}^{N} A_i \cdot \cos \left( \frac{2\pi}{c} \frac{2nL_i}{c} f \right)$$

$$\propto \sum_{i=1}^{N} A_i \cdot \cos(2\pi \cdot F_i \cdot f)$$

(2.5)

where $F_i = \frac{2nL_i}{c}$. As one can see, $L_i$ is unique for each reflection site. Consequently, if we consider each sinusoid as a function of $f$ as before, $\Delta I$ is a summation of sinusoids at different frequencies $F_i$ [an example of $\Delta I$ is shown in Fig. 2.4(a)]. After Fourier transform (FT) on the interference signal $\Delta I$, the sinusoidal component contributed by each reflection site shall appear as a peak in the Fourier spectrum [see Fig. 2.4(b)]. The amplitude and the location of a peak in the Fourier spectrum is proportional to the interfering light intensity and the distance $L_i$ of the reflection site, respectively. Therefore, we may obtain the reflection spectrum for each and every reflection site, if we measure $\Delta I$ (as a function of $f$) and compute its FT over a range
of wavelengths [27–29]. An FSI system in this linear frequency-shifted Sagnac interferometer configuration is able to interrogate an array of reflection-type fiber-optic sensors [27–29].

### 2.3 Single-arm frequency-shifted interferometer with multiple reflections

Relying on sideband interference, an FSI system with only a single interferometer arm can also be built [32]. Compared with a linear frequency-shifted Sagnac interferometer system which has two interferometer arms, a single-arm FSI (SA-FSI) configuration is simpler and more compact. Fig. 2.5 shows a typical setup of a SA-FSI system that consists of a circulator and a modulator (e.g., an intensity modulator or a phase modulator). Given an input light field at optical frequency $\nu$ and amplitude $E_0$, the modulator can be used to produce sideband signals. When an intensity modulator is used, the output electric field from the modulator can be written as

$$E(t) = [1 + M_a \cos(2\pi ft)] \cdot E_0 \cdot e^{2\pi i\nu t}$$

$$= E_0 \cdot \exp(i2\pi \nu t) + \frac{M_a}{2} \cdot E_0 \cdot \exp[i2\pi(\nu + f)t] + \frac{M_a}{2} \cdot E_0 \cdot \exp[i2\pi(\nu - f)t]$$

where $M_a$ is the modulation amplitude and $f$ is the modulation frequency. The last two terms in above equation correspond to the sidebands at $\nu \pm f$ under such amplitude modulation (AM). When a phase modulator is used, on the other hand, an infinite number of sidebands are produced, and the output field is

$$E(t) = E_0 \cdot \exp[i(2\pi \nu t + M_p \sin(2\pi ft))]$$

$$= E_0 \cdot \exp(i2\pi \nu t) \sum_{m=-\infty}^{\infty} J_m(M_p) \exp(i2\pi mt)$$

where $M_p$ is the modulation index, $J_m(M_p)$ is the Bessel function of the first kind. With proper modulation parameters, however, the higher order sidebands in (2.6b) can be suppressed. Now consider the light components at $\nu + f$ (first order sideband) exiting the output port 3 of the

![Fig. 2.5. A single-arm frequency-shifted interferometer. $R_i$ is the $i$th reflection site.](image)
circulator. Each reflection site \( R_i \) contributes two components at \( \nu + f \): one corresponds to the sideband signal produced when input light passes the modulator for the first time and is then reflected by the reflection site, while the other is the sideband signal produced when the central band signal passes the modulator for a second time after the reflection at \( R_i \). These two components are equivalent to the two interfering components at \( \nu + f \) in a linear frequency-shifted Sagnac interferometer system discussed before. Therefore, the output intensity \( I_{SA} \) is

\[
I_{SA} \propto I_{DC} + \sum_{i=1}^{N} R_i \cos \left( 2\pi \cdot \frac{2nL_i}{c} \cdot f \right)
\]

where \( I_{DC} \) is the DC component caused by the central band signal. Similarly, a Fourier transform performed on \( I_{SA} \) which is recorded when \( f \) is swept linearly can provide us with spatial information of the reflection sites and their reflectivity information. Note that there is also interference between sidebands at \( \nu - f \). A bandpass filter can be used to select the interference signal at \( \nu - f \) or \( \nu + f \).

### 2.4 FSI system performance parameters and considerations

#### 2.4.1 Spatial resolution and spatial sensing range

The spatial resolution in an FSI sensing system refers to the minimum resolvable separation between two reflection sites. In the case of fiber-optic sensor multiplexing by using a linear frequency-shifted Sagnac interferometer configuration or a SA-FSI system, this parameter determines how closely sensors (such as fiber Bragg grating sensors) can be distributed along the fiber link. As can be seen in (2.5), the spatial resolution \( \delta L \) is proportional to the resolution of the sinusoidal components’ oscillation frequency \( F \) after the FT:

\[
\delta L = \frac{c}{2n} \cdot \delta F
\]

According to the theory of discrete Fourier transform [36], the resolution of \( F_i \) is given by \( \delta F = 1/\Delta f \), where \( \Delta f \) is the frequency sweep range of the frequency shifter. Therefore, the spatial resolution \( \delta L \) is inversely proportional to \( \Delta f \)

\[
\delta L = \frac{c}{2n\Delta f}
\]

Note this is not necessarily the minimum sensor physical separation, since the fiber between two sensors can be wound.

The spatial sensing range is the maximum distance between the system and the furthest sensor at which reliable measurement can still be made. As \( F_i = 2nL_i/c \), the further the reflection site (larger \( L_i \)), the higher the frequency \( F_i \). By Nyquist theorem, to sample \( \Delta I_i \) that oscillates at \( F_i \), we must use fine \( f \) steps such that the sampling rate is at least twice as high as
Given a frequency sweep step size of $f_{\text{step}}$, the maximum $F$ can be sampled without aliasing is $F_{\text{max}} = 1/(2f_{\text{step}})$. Therefore, the spatial sensing range $L_{\text{max}}$ is determined by the frequency shift step $f_{\text{step}}$ of the optical frequency shifter:

$$L_{\text{max}} = \frac{c}{4nf_{\text{step}}} \tag{2.10}$$

Of course, when the sensing system is very lossy, signal-to-noise ratio (SNR) requirement may also limit the spatial sensing range.

One can see that both spatial resolution and spatial sensing range are determined by the parameters of the frequency sweep. For example, if $\Delta f = 20$ MHz and $f_{\text{step}} = 0.04$ MHz, we have $\delta L \sim 5$ m and $L_{\text{max}} \sim 1293$ m. More generally, the following relation holds

$$\frac{L_{\text{max}}}{\delta L} \propto \frac{\Delta f}{f_{\text{step}}} \tag{2.11}$$

It suggests that to attain a longer spatial sensing range $L_{\text{max}}$, one must make the ratio of $\Delta f$ to $f_{\text{step}}$ large. In order to obtain finer spatial resolution $\delta L$, this ratio must be kept small.

### 2.4.2 Spectral resolution and spectral range

The spectral resolution embodies the smallest spectral feature that can be resolved by a sensing system. The spectral range is the widest wavelength interval over which spectral measurement can be made. In an FSI system, these two parameters are both determined by the optical source used. If a tunable laser is employed, the spectral resolution and spectral range are determined by the wavelength resolution and the wavelength range of the laser, respectively. When a broadband source and a tunable filter is used, the spectral resolution is given by the spectral width of the filter, and the spectral range is limited by the bandwidth of the broadband source. For example, an FSI system with an Agilent 81642A laser provides a spectral resolution of 0.1 pm and a spectral range of 130 nm (from 1510 to 1640 nm).

### 2.4.3 Dispersion effects

So far, the derivation of the differential intensity signal in an FSI system is based on narrow-bandwidth input light without the consideration of fiber dispersion. Now suppose the input light is centered at $\lambda_0$ with a bandwidth of $\Delta \lambda$. The differential interference signal $\Delta I$ becomes an integral over all the wavelength components. Under the assumption of a uniform spectrum, $\Delta I$ in a linear frequency-shifted Sagnac interferometer configuration can be written as

$$\Delta I \propto \int_{\lambda_0 - \Delta \lambda}^{\lambda_0 + \Delta \lambda} I_0 \cos \left[ 2\pi \cdot \frac{2n(\lambda)L}{c} \cdot f \right] d\lambda \tag{2.12}$$

where $I_0$ is the interference signal intensity, $n(\lambda)$ is the refractive index of the spectral component at $\lambda$ due to dispersion, $L$ is the distance between the reflector and the coupler C₂ (see Fig. 2.3).
After a glance at (2.12), one can immediately recognize that it is a summation over a continuous range of frequency components $F(\lambda) = 2n(\lambda)L/c$. In the Fourier spectrum, these components form a broadened peak around the component of the center wavelength $F(\lambda_0)$. An expression can be found for the optical path length $2n(\lambda)L$.

The optical path lengths (OPLs) for different spectral components can be written with respect to that at $\lambda_0$:

$$2n(\lambda)L = 2n_0L + \Delta \tau c$$ (2.13)

where $2n_0L$ is the OPL of the component at $\lambda_0$. $\Delta \tau$ is the spread in time, which is the product of the group delay dispersion $\tau_\lambda$ and the wavelength spread $\delta \lambda$ [37]

$$\Delta \tau = \tau_\lambda \cdot \delta \lambda$$ (2.14)

where $\delta \lambda = \lambda - \lambda_0$. Note that the group velocity dispersion parameter $D$ is related to the group delay dispersion by [37]

$$\tau_\lambda = DL_0$$ (2.15)

where $L_0$ is the length of propagation. In the case of a linear frequency-shifted Sagnac interferometer configuration, $L_0 = 2L$, thus (2.13) can be written as

$$2n(\lambda)L = 2n_0L + \tau_\lambda \cdot \delta \lambda \cdot c = 2n_0L + D2L(\lambda - \lambda_0)c$$ (2.16)

The integral in (2.12) becomes

$$\int_{\lambda_0 + \frac{\Delta \lambda}{2}}^{\lambda_0 + \frac{\Delta \lambda}{2}} I_0 \cos \left[ 2\pi \cdot \frac{2n(\lambda)L}{c} \cdot f \right] d\lambda$$

$$= I_0 \int_{\lambda_0 - \frac{\Delta \lambda}{2}}^{\lambda_0 + \frac{\Delta \lambda}{2}} \cos \left[ 2\pi \cdot \frac{2n_0L + D2L(\lambda - \lambda_0)c}{c} \cdot f \right] d\lambda$$

$$= \Delta \lambda I_0 \cos \left( 2\pi \cdot \frac{2n_0L}{c} \cdot f \right) \cdot \text{sinc}(2\pi DL\Delta \lambda f)$$ (2.17)

Therefore, the differential interference signal that includes fiber dispersion is

$$\Delta I \propto \Delta \lambda I_0 \cos \left( 2\pi \cdot \frac{2n_0L}{c} f \right) \cdot \text{sinc}(2\pi \cdot DL\Delta \lambda \cdot f)$$ (2.18)

The cosine term $I_0 \cos[2\pi(2n_0L/c)f]$ in (2.18) is the usual interference signal contributed by the component at $\lambda_0$, and the dispersion effect due to wide bandwidth $\Delta \lambda$ is accounted for by the sinc function whose oscillation is much slower than the cosine term. According to the convolution theorem [36,38], the Fourier transform of $\Delta I$ is the convolution between the Fourier transforms of a cosine and a sinc function.
Fig. 2.6. Comparison between the normalized Fourier peaks with and without dispersion effects. The thick black dashed curve is the Fourier peak contributed by the component at $\lambda_0$ without dispersion, while the red curve is the Fourier peak calculated from (2.18). The Fourier transform was carried out with a Hann window and a fast Fourier transform size of $2^{20}$.

We can estimate the dispersion effects in FSI with a practical example. The group velocity dispersion parameter $D$ for a standard SMF-28 fiber at 1550 nm is about 16.2 ps/(nm·km). Suppose the source bandwidth $\Delta \lambda$ is 35 nm, $L$ is 1 km, and the frequency shifter is an acousto-optic modulator (AOM) which is swept from 90 to 110 MHz at steps of 0.04 MHz. In Fig. 2.6, the normalized Fourier peak of the component at $\lambda_0$ is compared with that calculated from (2.18). As can be seen, the Fourier peak obtained with dispersion effects included (thin red line) matches almost perfectly with that contributed by a single wavelength component (thick black dashed line), indicating negligible dispersion effects. This is not a surprising result if one evaluates the OPL difference induced by wavelength-dependent refractive index. As at around 1550 nm, the group delay dispersion $\tau_\lambda$ is on the order of 16.2 ps/(nm·km) times 2 km, namely, $\sim 32.4$ ps/nm, the OPL difference between other wavelength components and the component at $\lambda_0$ is on the order of only 0.17 m, well below the width of the Fourier peak ($\sim 10$ m).

2.4.4 System crosstalk

In a sensing or communications system, crosstalk takes place when the signal of one channel affects that of the other. Three types of crosstalk may influence the performance of an FSI system: spectral shadowing effect, discrete Fourier transform resolution, and unwanted reflections among reflection sites.
Spectral shadowing effect

Spectral shadowing effects occur when multiple reflectors with overlapping reflection wavelengths are arranged in a serial array. As light needs to pass the upstream reflectors before reaching the downstream ones, the input light for the downstream reflectors bears the spectral characteristics, or the “shadows”, of the upstream ones (see Fig. 2.7). Spectral shadowing effects, which is a form of crosstalk, cause distortions in the measured spectra. They are not unique to FSI, but are commonplace in conventional fiber-optic sensing systems [18]. However, in an FSI sensing system, such crosstalk can be effectively removed [28].

The key to FSI’s capability of correcting spectral shadows is that FSI can resolve the reflection spectra for every individual reflector. The spectral shadow seen by the $i$th reflector can be removed by using the spectrum of the $(i-1)$th reflector. The interference signal amplitude $A_i$ of the $i$th reflector ($i \geq 2$) can be written as

$$A_i = \left( \prod_{j=1}^{i-1} T_j \right)^2 R_i$$

(2.19)

where $T_j = 1 - R_j$ is the transmittance of the $j$th reflector and $R_i$ is the reflectivity of the $i$th reflector. The spectrum for the $i$th reflector measured directly thus carries the transmittance information $T_j$ of the previous $i-1$ reflectors. It is easy to show that the reflectivity $R_i$ is

$$R_i \propto \frac{A_i}{A_{i-1}} \cdot \frac{R_{i-1}}{T_{i-1}^2}$$

$$\propto \frac{A_i}{A_{i-1}} \cdot \frac{R_{i-1}}{(1 - R_{i-1})^2}$$

(2.20)

As there is no spectral shadow for the 1st reflector, the spectrum measured experimentally for

![Fig. 2.7. Spectral shadowing effect. The $(i-1)$th sensor casts a shadow on the $i$th sensor. As a result, the reflection from $R_i$ also carries the spectral information of $R_{i-1}$.](image)
this reflector is its true reflection spectrum, namely, $R_1$ is known. The actual reflection spectra of the subsequent reflectors can be obtained sequentially by applying (2.20) [28].

**Discrete Fourier transform crosstalk**

In practice, the interference signal $\Delta I$ acquired in an FSI sensing system is a set of finite data, and it is processed through discrete Fourier transform (DFT). Due to the finite length of the data set, the peaks in the Fourier spectrum have finite widths [36, 39]. In DFT, crosstalk takes place when two frequency components are close to each other so that their Fourier peaks become overlapped in the Fourier spectrum. This is undesirable since the amplitude of one peak may be distorted by that of the other, and vice versa. According to the theory of discrete Fourier transform, increasing the sweep range of $f$ or applying a window function (e.g., a Hann

![Fig. 2.8. The effects of windowing in DFT. (a) simulated interference signal under a rectangular window; (b) simulated interference signal under a Hann window; (c) FFT of the signal in (a); (d) FFT of the signal in (b).](image-url)
window) in the DFT process can reduce such crosstalk [36, 39]. The effects of a finite data set in the DFT are illustrated in Fig. 2.8. In Fig. 2.8(a) and (b), one can see the interference signal under a rectangular and a Hann window, respectively. Comparing their corresponding DFT in Fig. 2.8(c) and (d), we can see that a Hann window can effectively suppress the side lobes of a Fourier peak.

Unwanted reflections among reflection sites

When a number of reflection sites (or sensors) are connected in series (see Fig. 2.3), undesirable multiple reflections or interference between lightwaves from different reflection sites (or sensors) may occur. This constitutes another source of crosstalk in an FSI system.

Fig. 2.9(a) illustrates the case of multiple reflections in which both interfering lightwaves follow the same routes, but are reflected back and forth between the reflection sites. The interference signal produced by the lightwaves that experience multiple reflections introduces additional peaks in the Fourier spectrum, which may distort the peaks of the desired interference signal amplitude. For instance, suppose the sensors are distributed evenly along the fiber link. The interference signal between the lightwaves reflected between R_{i-1} and R_i adds an additional amplitude to the Fourier peak of R_{i+1}. The effect of multiple reflections can be seen in Fig. 6 of [28]. Choosing sensors with low reflectivity may effectively reduce the crosstalk caused by multiple reflections.

If the coherence length of the light source is longer than twice the separation between two reflection sites, R_{i-1} and R_i, undesired interference occurs between the lightwaves reflected by R_{i-1} and by R_i. These interference signals can be shown to have the form of A_{i-1}A_i \cos \Delta \phi, where \Delta \phi is again the phase difference between the two interfering field components, and A_{i-1} and A_i are the amplitudes of the interfering fields. Table 2.1 lists the possible contributions given by this unwanted interference for a linear frequency-shifted Sagnac interferometer system.

![Fig. 2.9. The effects of unwanted reflections. (a) multiple reflections between reflection sites (or sensors); (b) interference between lightwaves coming from different reflection sites (or sensors).](image)
Table 2.1. Contributions of interference caused by unwanted reflections.

<table>
<thead>
<tr>
<th>Path of the interfering field components</th>
<th>( \Delta \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>upper arm ( \rightarrow R_{i-1} ) ( \rightarrow ) lower arm and lower arm ( \rightarrow R_i ) ( \rightarrow ) upper arm</td>
<td>( 2\pi n(2L_i + l_1 + l_2 - l_e) f/c + 2\pi n(2L_i - 2L_{i-1}) \nu_0/c )</td>
</tr>
<tr>
<td>upper arm ( \rightarrow R_i ) ( \rightarrow ) lower arm and lower arm ( \rightarrow R_{i-1} ) ( \rightarrow ) upper arm</td>
<td>( 2\pi n(2L_{i-1} + l_1 + l_2 - l_0) f/c + 2\pi n(2L_{i-1} - 2L_i) \nu_0/c )</td>
</tr>
<tr>
<td>upper arm ( \rightarrow R_{i-1} ) ( \rightarrow ) lower arm and upper arm ( \rightarrow R_i ) ( \rightarrow ) lower arm</td>
<td>( 2\pi n(2L_i - 2L_{i-1}) \nu_0/c )</td>
</tr>
<tr>
<td>lower arm ( \rightarrow R_{i-1} ) ( \rightarrow ) upper arm and lower arm ( \rightarrow R_i ) ( \rightarrow ) upper arm</td>
<td>( 2\pi n(2L_i - 2L_{i-1}) f/c + 2\pi n(2L_i - 2L_{i-1}) \nu_0/c )</td>
</tr>
</tbody>
</table>

(see Fig. 2.3) [28]. The left column of the table shows the paths of the field components and the right column lists the phase differences corresponding to the entries on the left. The interfering terms in the first two rows of the table shall appear in the Fourier spectrum at \( F_{i-1} \) and \( F_i \). The contribution from the third row is a DC component, and the contribution of the fourth row is near \( F = 0 \) for very close \( R_{i-1} \) and \( R_i \). Therefore, the terms in the first two rows add to the crosstalk. If a low-coherent light source (such as a broadband source) is used, the crosstalk caused by unwanted interference can be minimized.

### 2.5 Summary

This chapter has introduced the theory of frequency-shifted interferometry (FSI). By allowing two frequency-shifted lightwaves originated from the same source to interfere after they travel through a common path, provided that one light acquires the optical frequency shift before entering the path while the other is shifted after exiting it, we can extract the optical length of the path as well as the interfering light intensity from the interference signal. Several implementations of FSI systems have been introduced, including frequency-shifted Sagnac, linear frequency-shifted Sagnac, and single-arm interferometer configurations. With a proper configuration, an FSI system can be used to measure fiber length and dispersion [25], to locate multiple
weak reflections along a single fiber link [26], or to multiplex fiber-optic sensors [27–29]. Key parameters (spatial resolution, spatial sensing range, spectral resolution, spectral range, dispersion effects, and crosstalk) that dictate the performance of an FSI system are discussed. The advantage of FSI as a sensing system is that a CW source (even a broadband source) and a slow detector can be used. FSI allows the sensors to overlap spectrally, and after straightforward correction, sensors’ spectra can be measured accurately. As a demonstration of the fiber-optic sensor multiplexing capability of FSI, the following chapter presents a novel cryogenic liquid level sensing system based on FSI.
Chapter 3

A Cryogenic Liquid Level Sensing System Interrogated by Frequency-Shifted Interferometry

Liquid level sensing has important applications in fields such as industrial processing, quality control, and fuel storage and transportation. This chapter describes a fiber-optic liquid level sensing system based on active fiber Bragg grating (FBG) sensors multiplexed by the frequency-shifted interferometry (FSI) [29]. The operation principle of this novel liquid level sensing system is completely different from those of its conventional counterparts, and it proves to be particularly useful in cryogenic environments.

3.1 Conventional fiber-optic liquid level sensing techniques

Many techniques have been developed for the purpose of liquid level sensing. It can be achieved by mechanical [40], electrical [41], acoustic [42] or ultrasonic [43] methods. Although electrical liquid level sensors are widely applied in practice, their deployment in explosive or corrosive environments poses a threat to safety and system reliability. Furthermore, when multiple sensors are needed, electrical sensing systems entail extensive wiring, cabling, and feedthroughs. This inconveniently increases the system complexity, cost, as well as the potential of failure. In contrast, fiber-optic liquid level sensing techniques can overcome these difficulties. As mentioned in Chapter 1, fiber-optic sensors are not only immune to electromagnetic interference, but also spark-free and in general chemically inert. They can be installed safely in oil tanks or containers for most chemicals. Thanks to fiber-optic sensors’ multiplexing capability, the complexity of a fiber-optic liquid level sensing system can be reduced.

Various fiber-optic liquid level sensing techniques have been developed. The liquid level can be determined, for instance, from the optical absorption caused by the liquid, by shining a light beam from the bottom of the container through the stored liquid and measuring the
transmitted light intensity— the stronger the absorption the higher the liquid level [44]. Instead of transmission, reflection can also be used to gauge the liquid level. The light beam can be directed onto the liquid surface at an angle so that when the liquid level changes, the reflected light exhibits lateral displacement at the receiver, leading to altered light coupling efficiency, and therefore, the reflected light intensity detected becomes liquid level dependent [45, 46]. Some method relies on the liquid buoyant force, in which the strain of a bending cantilever beam attached to a buoy is measured by a fiber-optic strain sensor and is converted into the liquid level information [47]. Alternatively, the pressure measurements by a fiber-optic pressure sensor at the bottom of the liquid tank can be related to the liquid level [48]. However, most fiber-optic liquid level sensing techniques are based on the refractive index (RI) difference between the liquid and the gas [49–56].

Some fiber-optic liquid sensing schemes depends on the ambient-RI-dependent light propagation properties in an optical waveguide [49–52]. For instance, a long-period grating (LPG) can be used to determine the liquid level over its length continuously [49]. As the effective refractive indices of the cladding modes excited by the LPG are dependent on the surrounding RI, the resonance wavelength of a LPG is blueshifted when the grating is fully immersed in the liquid [minimum A in Fig. 3.1(a)] compared to that when the LPG is in the air [minimum B in Fig. 3.1(a)]. If the LPG is partially immersed in the liquid, it can be considered as two separate gratings whose lengths are related to the liquid level. As a result, two transmission minima can be found in the LPG transmission spectrum, one of which corresponds to the grating feature in the liquid [minimum A in Fig. 3.1(b)] while the other is contributed by the grating section in the air [minimum B in Fig. 3.1(b)]. The relative changes of the minimum transmission values between A and B in the spectra can thus be calibrated into the liquid level. Similarly, an etched section of fiber Bragg grating (FBG) partially immersed in the liquid can be treated as two
gratings, since the resonance wavelength of an FBG is also a function of the surrounding RI. One is able to deduce the liquid level information from the relative amplitude changes at two resonance wavelengths [51] in the transmission (or reflection) spectrum [Fig. 3.1(b)]. Note here that the resonance wavelength of an FBG is redshifted when it is immersed in the liquid. Last but not least, the loss change through an optical waveguide induced by the ambient RI variation can also be utilized to find liquid level. Methods employing devices such as side-polished fiber sections [50] or a piece of D-shaped fiber [52] have been demonstrated. They can usually provide one-dimensional liquid level information.

Many RI-based fiber-optic liquid level sensing systems exploit the total internal reflection (TIR) at the boundary between the sensor and the surrounding [53–56]. A probe can, for example, be the tips of two fibers that are fused together [see Fig. 3.2(a)] [53,55], or it can be a prism connected to two fibers [see Fig. 3.2(b)] [54,56]. In both cases, one fiber serves as the lead-in fiber which couples light into the sensing region (the fused fiber tips or the prism) where the light reaches the boundary between glass and the surrounding medium. If the surrounding medium is air, due to the high RI contrast, the TIR condition can be met so that a majority of the optical power can be coupled into the second lead-out fiber and be detected [Fig. 3.2(a) and (b)]. On the other hand, if the ambient medium is liquid, the RI difference is so small that the TIR condition no longer holds. As such, a large portion of light leaks into the surrounding, resulting in significant optical loss measurable at the lead-out fiber [Fig. 3.2(c) and (d)]. The sensors in these schemes often have a step-like response to the ambient medium phase change. Therefore, whether a specific sensor is exposed to the air or immersed in the liquid can be decided from the transmission loss of that sensor. An array of this type of quasi-distributed sensors is needed to monitor a wide range of liquid level [55,56].

These conventional fiber-optic liquid level sensing techniques have their limitations, especially in cryogenic environments or for aerospace applications. The technique based on differential absorption [44] must monitor an absorbing liquid and employ a light source that covers the absorption wavelength (not applicable for important fuel such as liquid hydrogen). This technique cannot be deployed reliably when there is liquid surface vibrations, surface tilt, bubbles, or floating debris. The reliability of the schemes using liquid surface reflections [45,46] are also susceptible to liquid surface tilt, in addition to the influences of lens aberrations, or poor collimating/focusing optics alignment. Both of above two methods may encounter further difficulties when measuring cryogenic liquid because of the scattering by the mist suspended above the liquid. In micro-g or high-g environments, liquid level sensing systems relying on the liquid buoyant force [47] or pressure [48] are not able to function properly. Even RI-based sensing schemes [49–56] have serious issues at cryogenic temperatures, as the RI difference between the air and the liquid is small in such environments [55]. Therefore, the sensitivity of these schemes are compromised. The methods in [49,51] are also temperature sensitive, because the spectral features of a LPG or FBG are temperature dependent. In a micro-g environment, the liquid fuel distribution in the fuel tank could be rather complex, which prevents the successful appli-
Fig. 3.2. Commonly used fiber-optic liquid sensor probes: (a) fused fiber tips; (b) prism; (c) fused fiber tips in liquid; (d) prism in liquid.

cation of [44–52]. Ideally, we seek a fiber-optic sensing system that is capable of unambiguously distinguishing liquid and gas at multiple sensing points so as to provide two-dimensional or three-dimensional liquid distribution information in a cryogenic environment. The fiber-optic liquid level sensors used in our experiment work on a completely different principle than those
as mentioned above. They can be multiplexed and nicely solve the problems conventional liquid level sensing systems have in cryogenic environments.

### 3.2 Liquid level sensor design and operation principle

The fiber-optic liquid level sensors used in our experiments are aluminum-coated high-attenuation fiber Bragg gratings (Al-coated HAFBGs) developed by our collaborator Professor K. P. Chen’s group at the University of Pittsburgh. Fiber-optic sensors are normally passive. These HAFBG sensors, however, can be actively tuned—they can be heated by the light propagating in the fiber [57,58]. This allows us to avail the thermal property differences between air and liquid as the sensing mechanism.

As their name suggests, Al-coated HAFBGs are based on fiber Bragg gratings. A fiber Bragg gratings (FBG) is a piece of optical fiber along which a periodic refractive index perturbation is

![FBG structure](image)

**Fig. 3.3.** FBG structure. It is essentially a piece of optical fiber with periodic refractive index modulation in the core along its length.

![Operation of an FBG](image)

**Fig. 3.4.** Operation of an FBG. It reflects the light at a resonance wavelength $\lambda_B$. 

```
introduced in the fiber core (see Fig. 3.3) [18,59–61]. When broadband light is injected into an FBG, the grating reflects the light at a resonance wavelength, the so-called Bragg wavelength \( \lambda_B \), that is related to the period \( \Lambda \) of the refractive index modulation [18,59–61]

\[
\lambda_B = 2n\Lambda \quad (3.1)
\]

where \( n \) is the refractive index of the core (Fig. 3.4). The Bragg wavelength is sensitive to both strain and temperature. The response of an FBG to strain or temperature change with the other parameter kept constant is [18]

\[
\frac{1}{\lambda_B} \frac{\delta \lambda_B}{\delta \epsilon} = 0.78 \times 10^{-6} \mu \epsilon^{-1} \quad (3.2a)
\]

\[
\frac{1}{\lambda_B} \frac{\delta \lambda_B}{\delta T} = 6.67 \times 10^{-6} \degree C^{-1} \quad (3.2b)
\]

where \( \delta \lambda_B/\delta \epsilon \) and \( \delta \lambda_B/\delta T \) are the derivatives of \( \lambda_B \) with respect to strain \( \epsilon \) and temperature \( T \), respectively. When the strain or temperature on an FBG increases (or decreases), the Bragg wavelength is shifted towards a longer (or shorter) wavelength. Because of their responsivity to these two parameters and their excellent multiplexing capability, FBGs are widely used as strain or temperature sensors [5–8,12,62].

High attenuation fibers (HAFs) are frequently used as plug type fixed attenuators in fiber-optic communication systems to eliminate end-of-line reflections, to limit the receiver input power, or to reduce undesirable nonlinear effects [63,64]. To achieve high attenuation, the core

![Fig. 3.5. Design of the aluminum-coated high attenuation fiber Bragg grating [29]. SMF: single-mode fiber; HAF: high-attenuation fiber; FBG: fiber Bragg grating; Al: aluminum.](image)
Fig. 3.6. Scanning electron microscopy images of an Al-coated HAFBG sample [29]. (a) a cross section image at the coated HAF part; (b) a zoomed-in look of the Al coating.
of HAFs are often doped with cobalt (Co). It has been demonstrated that when intense pump light is launched into an HAF section, the HAF is heated as a result of non-radiative optical absorption, which can serve as a means of tuning FBGs [57,58]. Such heating can be precisely controlled by selecting appropriate HAF dopant concentration, HAF length, and input pump power. HAFs with various dopant concentrations are available, offering attenuation from 0.002 to 20 dB/cm [57].

A schematic of the Al-coated HAFBG design is shown in Fig. 3.5 [29]. Each sensor consists of a 1-cm section of bare HAF spliced between two standard single-mode fibers (SMFs). The spliced fibers were treated with the hydrogen loading process at 2000 psi at room temperature for seven days. A 5-mm FBG was then inscribed into the HAF section using the phase mask technique [59–61]. A KrF 248 nm excimer laser was used and the HAF section was exposed to 200-mJ 3-ns pulses for 30000 cycles. The fiber was baked for 12 hours at 120 °C to relieve stress and to release the hydrogen. The two spliced points were reinforced by UV-curable adhesives. Finally, the HAF section was sputter coated with a Al film of 150 nm on a rotation stage. Scanning electron microscopy (SEM) images of an Al-coated HAFBG can be found in Fig. 3.6. A cross section cut at the Al-coated HAF fiber part is shown in Fig. 3.6(a). A zoomed-in look of the Al coating on the sample is presented in Fig. 3.6(b). When pump light (e.g., the output from an erbium-doped fiber amplifier) is sent to the sensor, the HAF section absorbs the optical power and heats up the sensor from the fiber core. This local heating has two consequences. First, it causes FBG refractive index change. Second, when the heat diffuses to the Al coating, the coating expands, leading to strain increase on the FBG. Therefore, a heated Al-coated FBG exhibits Bragg wavelength redshift due to both temperature and strain increase. As liquid has much higher specific heat capacity and thermal convection rate than gas, temperature increase is more difficult to attain for a sensor heated in the liquid, that is to say, its Bragg wavelength

![Intensity vs. wavelength graph](image_url)

*Fig. 3.7.* Sensing principle of Al-coated HAFBGs. A sensor heated in air shows significantly larger Bragg wavelength shift in the reflection spectrum than a sensor heated in liquid (blue: unheated sensor spectrum; green: heated sensor in liquid; red: heated sensor in air).
Chapter 3. A Cryogenic Liquid Level Sensing System Interrogated by FSI

shift is significantly smaller. The distinct spectral response differences exhibited by heated sensors can be considered as indicators that unambiguously distinguish the sensors in air from those in liquid (Fig. 3.7).

The Al-coated HAFBG design has many advantages. It represents an all-optical solution to active tuning of FBG sensors. Conventional ways of tuning FBGs usually involve electrical or mechanical parts. Techniques use piezoelectric coatings or electric heating [65–69] contain electrical interconnects at the sensors, whereas mechanical tuning methods entail some other actuators (probably also electrical in practice) to achieve the strain adjustment [70,71]. Not only does this complicate the packaging and wiring of the system, but it also voids many advantages of fiber-optic sensors such as being spark free and immune to electromagnetic interference. Compared with previously demonstrated in-fiber heating methods [72–75], the sensors based on HAFs offers single-end access to the sensing system, high heating light coupling efficiency, and more uniform heating profiles (minimized sensor spectral chirp). Furthermore, the heating level for each sensor can be precisely tailored by choosing appropriate HAF dopant concentration and length.

3.3 Cryogenic liquid level sensing experiments

The liquid level sensing experimental setup is shown in Fig. 3.8. It consists of three parts: a frequency-shifted interferometer (FSI), a heating source, and an Al-coated HAFBG sensor array. The sensing light from the FSI and the pump light from the heating source were coupled into the sensor array by a 50/50 fiber directional coupler C₀. The FSI configuration adopted is the linear frequency-shifted Sagnac interferometer form (see Fig. 2.3). The interferometer was constructed by connecting two 50/50 wideband fiber couplers C₁ and C₂. The frequency shifter was an acousto-optic modulator (AOM, Brimrose AMM-100-20-25-1550-2FP). A tunable laser (TLS, Santec TSL-210) was used as the light source, which has a wavelength resolution of 10 pm. The differential interference signal was measured by a balanced detector (BD, New Focus Model 2117). The output of the BD was recorded by a data acquisition board (NI PCI-6251). The heating source was an Erbium-doped fiber amplifier (EDFA). The pump power was monitored by a power meter (PM) connected to the 4th output port of C₀. There were three Al-coated HAFBGs G₁,G₂, and G₃ in the array. These HAFBGs have reflectivity of ~4%. Sensor G₁ was fabricated in an HAF section with 0.2 dB/cm loss, while G₂ and G₃ were inscribed in HAFs with 0.5 dB/cm loss. Assuming the splice loss between the SMF and HAF to be 0.1 dB, one could estimate the total optical power loss for the sensors, which is ~10% for G₁ and ~15% for the other two. The higher loss in G₂ and G₃ ensures sufficient in-fiber heating compared to that of G₁. Bare SMFs with standard acrylate jackets were used to link the sensors. Two isolators (ISOs), one at the end of the sensor array and the other before the PM, were added to avoid potential damage to the equipment (e.g., the AOM) caused by the high-power reflections from the fiber ends.
As the configuration of the FSI is a linear frequency-shifted Sagnac interferometer, the interference signal is described by (2.5)

$$\Delta I \propto \sum_{i=1}^{3} A_i(\lambda) \cdot \cos \left(2\pi \frac{2nL_i}{c} f\right)$$

(3.3)

where $A_i(\lambda)$ is proportional to the reflection intensity of the $i$th sensor at wavelength $\lambda$, $n$ is the effective refractive index of the fiber mode, $L_i$ is the distance between the $i$th sensor and $C_2$, $c$ is the speed of light in vacuum, and $f$ is the amount of frequency shift introduced by the AOM. Note that we can set $A_1 = R_1$, while $A_2$ and $A_3$ can be written as

$$A_i(\lambda) = \prod_{j}^{i-1} T_j^2(\lambda) \cdot R_i(\lambda) \quad (i = 2, 3)$$

(3.4)
where $T_j(\lambda)$ and $R_i(\lambda)$ are the transmittance of the $j$th sensor and the reflectance of the $i$th sensor, respectively.

The Al-coated HAFBG sensor array was lowered into a liquid nitrogen ($N_2$) dewar Fig. 3.9. Liquid nitrogen is ideal for the testing of this cryogenic liquid level sensing system as it is a commonly used cryogenic fluid whose boiling point is 77 K. The sensors were taped onto a long aluminum bar. The fibers linking the sensors were wound so that the physical separation between two adjacent sensors was just 2 or 3 cm. The order of the sensors from the top to the bottom is $G_1$, $G_2$, and $G_3$.

During the FSI measurement, the tunable laser output power was set to 4 mW, and the EDFA heating power was $\sim$ 1 W. The laser wavelength scan step was set to 10 pm. At each wavelength, the AOM was swept from 90 to 110 MHz at steps of 0.04 MHz. The time duration at each step was 1 ms. The data acquisition was synchronized with the AOM sweep. A LabVIEW program was developed to control the instruments and process the data.

Fig. 3.10 shows the responses of Al-coated HAFBG sensors to the in-fiber heating when
all of them were immersed in the liquid nitrogen. Without in-fiber heating, the sensor Bragg wavelengths were 1544.25, 1543.06, and 1543.01 nm for G₁, G₂, and G₃, respectively (dashed lines in Fig. 3.10). As the full width at half-maximum (FWHM) of the sensor reflection peaks was \(\sim 0.14\) nm, G₂ and G₃ have significantly overlapped reflection spectra due to the close proximity of their Bragg wavelengths (only 0.05 nm apart). FSI measurement can clearly resolve the individual spectra of G₂ and G₃ in spite of the spectral overlap. After the heating pump light was switched on, the sensors exhibited very small Bragg wavelength shifts (solid lines in Fig. 3.10). They were 0.04, 0.06 and 0.05 nm, for G₁, G₂, and G₃, respectively, corresponding to less than \(\sim 43\)% of the sensor FWHM width.

The Al-coated HAFBG sensors were raised out of the liquid nitrogen one by one, and reflection spectra of the heated sensors were measured by FSI (Fig. 3.11). Fig. 3.11(a) shows the sensor spectra when only G₁ was lifted to the air while the other two remained in the liquid nitrogen. It can clearly be seen that the Bragg wavelength of G₁ was appreciably higher, shifted by 0.50 nm to 1544.79 nm, corresponding to a shift about 3.5 times greater than the reflection peak FWHM width. Likewise, when G₂ was pulled out of the liquid, it also exhibited a large Bragg wavelength displacement [Fig. 3.11(b)]. The new G₂ Bragg wavelength was at 1543.91 nm, 0.79 nm larger than that when the sensor was in the liquid nitrogen without heating. Finally, when all the sensors were heated in the air, large Bragg wavelength shift was observed for every sensor [Fig. 3.11(c)]. G₃ in this case was also shifted by 0.79 nm. Therefore, the distinct sensor response difference between a sensor heated in air and a sensor heated in liquid nitrogen.
Fig. 3.11. Sensor responses as the Al-coated HAFBG sensor array was pulled out of the liquid nitrogen. (a) only $G_1$ in the air; (b) $G_1$ and $G_2$ in the air; (c) all sensors in the air. Dashed lines are sensor spectra without heating in liquid nitrogen and solid lines are heated sensor spectra.
can clearly indicate the phase of the fluid surrounding the sensor.

One should note that if the sensor is heated at the interface between air and liquid, the sensor essentially becomes two sections of gratings. The sensor section in the air experience more heating and expansion than that submerged in the liquid. The reflection spectrum of the sensor may broaden or even split. Fig. 3.12 illustrates a sensor reflection spectrum obtained when the sensor was heated at the interface. The reflection peak at $\sim 1544.58$ nm was contributed by the sensor section in the air, whereas the peak at $\sim 1544.28$ nm was the result of the heated sensor section in the liquid nitrogen. Such reflection peak split is particularly undesirable for a sensor array interrogated by the WDM technique, since it causes confusion in the demodulation process if the split of the reflection peak is large. However, as FSI is capable of separating the sensor signals from the spatial domain, it can easily recognize that the two split peaks are in fact originated from the same sensor. No complication is incurred in the analysis. Moreover, as FBGs are also temperature sensitive, the wide temperature change each sensor may face in cryogenic applications demands that each sensor should have very large dynamic range. For example, in the case of liquid nitrogen level measurement, the temperature can range from below 77 K to near room temperature (298 K), corresponding to a potential Bragg wavelength shift of over 2 nm for a FBG at 1550 nm. Such a large sensor dynamic range imposes severe challenges for WDM systems if a large number of sensors is required.
3.4 System parameters

Liquid level resolution

The liquid level resolution of our sensing system is dependent on how densely Al-coated HAFBG sensors are distributed in a sensor array. Although by (2.9), the spatial resolution of FSI is constrained by the AOM sweep range $\Delta f$ ($\sim 5$ m in the experiments), due to their flexibility, long pieces of fibers can be wound so that the actual physical separations between adjacent sensors are very small (about 2 to 3 cm in our experiments). Ultimately, the liquid level resolution can be on the order of the HAFBG length ($\sim 1$ cm).

Measurement time

The system measurement time $T_{\text{meas}}$ is a function of several parameters. Suppose the sensor spectra are measured by scanning the sensing light wavelength at steps of $\lambda_{\text{step}}$ over a range $\Delta \lambda$, and at each wavelength step, the AOM is swept for a range of $\Delta f$ with steps of $f_{\text{step}}$.\footnote{A different data acquisition approach is presented in Chapter 4.} The time required at each wavelength step is $\tau_{\text{step}} \cdot \Delta f / f_{\text{step}} + T_{\text{proc}}$, where $\tau_{\text{step}}$ is the step time interval at each AOM frequency step and $T_{\text{proc}}$ is the data processing time. The total measurement time is therefore

$$T_{\text{meas}} = \left( \tau_{\text{step}} \cdot \frac{\Delta f}{f_{\text{step}}} + T_{\text{proc}} \right) \cdot \frac{\Delta \lambda}{\lambda_{\text{step}}}$$

(3.5)

The optimal $T_{\text{meas}}$ can be chosen based on the system requirements. AOM sweep parameters $\Delta f$ and $f_{\text{step}}$ determines the FSI spatial resolution and spatial sensing range, respectively [see (2.9) and (2.10)]. The spectrum wavelength range and resolution are decided by the wavelength scan parameters $\Delta \lambda$ and $\lambda_{\text{step}}$. In our experiments, $\Delta f = 20$ MHz and $f_{\text{step}} = 0.04$ MHz, while $\Delta \lambda = 3$ nm and $\lambda_{\text{step}} = 0.01$ nm. The AOM step time interval is limited to $\tau_{\text{step}} = 1$ ms. The processing time on our slow computer is less than 0.5 s. Therefore, $T_{\text{meas}} \approx 5$ min. However, we could have used $\Delta f = 15$ MHz, $f_{\text{step}} = 0.1$ MHz, $\Delta \lambda = 3$ nm, and $\lambda_{\text{step}} = 0.03$ nm so that the measurement time is just 1 min, since given the sensor spacing and total fiber length, there is no need to use such high spatial resolution and spatial sensing range. The measurement time can be drastically reduced if the AOM driver hardware is improved and dedicated hardware or simpler software is used in the data acquisition and processing.

Maximum number of sensors

The maximum number of Al-coated HAFBG sensors our FSI system can accommodate is determined by two factors. First, the power budget of the sensing and heating light must be considered. Given pump light power, the dopant concentration of the HAF length can be selected to ensure that each sensor receive the same level of heating. The reflectivity of the
HAFBGs can also be calculated given the loss in the system. Second, the sensor density in an array is limited by the FSI spatial resolution while the location of the last sensor must not exceed the spatial sensing range.

3.5 Summary

From above experimental results, we demonstrate that FSI can be used to interrogate an array of active Al-coated HAFBG sensors for liquid level sensing at a cryogenic environment. Al-coated HAFBG sensors with in-fiber heating solve the difficulties encountered by conventional RI-based fiber-optic liquid level sensors at low temperatures. The distinct response difference between a sensor heated in the air and a sensor heated in the liquid is exploited as the sensing principle. The FSI multiplexing scheme enables sensors to overlap in wavelength so that more sensors can be incorporated into the system. The cost of such an FSI-based fiber-optic liquid level sensing system can be low, as only a CW source and a slow detector is required.
Chapter 4

Rapid Fiber Bragg Grating Sensor Multiplexing by Frequency-Shifted Interferometry

4.1 Frequency-shifted interferometry as a fiber-optic sensor multiplexing scheme towards commercial applications

As demonstrated in Chapter 3, the frequency-shifted interferometry (FSI) technique is able to multiplex an array of fiber-optic sensors based on fiber Bragg gratings (FBGs). In fact, commercial applications of FBG sensors are already commonplace in a wide range of industries including civil structures, oil and gas, power, aerospace, and transportation. Examples can be found from companies such as Micron Optics, Inc. or WUTOS. In our previous lab demonstration, we have shown that FSI can easily interrogate 10 FBG sensors in an array (number limited by the available FBGs in the lab) [27]. This sensor number is already comparable to that in a commercial wavelength-division multiplexing (WDM) system channel. Therefore, as a fiber-optic sensor multiplexing scheme, FSI has great commercial value. In earlier FSI experiments [27–29], the light sources used were tunable lasers. Although they provide fine spectral resolution and high spectral power, their wavelength tuning speed is rather slow and their cost is relatively high (more than $10000). The system measurement time is further limited by the sweep speed of the frequency shifter, an acousto-optic modulator (AOM). To push FSI towards commercial applications, system measurement time and cost are two factors that must be taken into account.

To relax the requirements of system equipment, a broadband source paired with a tunable filter (TF) can be employed in lieu of a tunable laser as the light source in FSI systems. The tunable optical filter technology is a mature one on which many commercial WDM systems are based (see products from Micron Optics or WUTOS). One can select a spectral band from the output of the broadband source with the help of a TF and use it as the sensing
light. One great advantage of TFs is that they can be swept rapidly. The fiber Fabry-Perot (FP) TFs manufactured by Micron Optics, for example, can cover the entire C-band at a scan frequency as high as 2.5 kHz (Micron Optics FFP-TF). That is to say, the time scale for a TF to sweep through a 3-nm wavelength range (the range scanned in the cryogenic liquid level sensing experiments in Chapter 3) or even wider is on the order of 0.4 ms, which is shorter than the current AOM sweep step time interval (∼ 1 ms). In [27–29], the interference signal is obtained at each wavelength by sweeping the AOM frequency \( f \), and this process is repeated at every wavelength step. Now, without making any changes to the AOM driver hardware, we can shorten the measurement time significantly by sweeping the AOM only once, and at each frequency step \( f \) sweeping the TF to obtain the interference data over all the wavelengths. A \( 2 \times 2 \) data set can be obtained at the end of the measurement (see Fig. 4.1). Finally, one can analyze the reflection information by performing Fourier transforms on the interference signal as a function of \( f \) at a specific wavelength (a row in Fig. 4.1). Since a full AOM sweep over 20 MHz at 0.04 MHz steps needs 0.5 s, we can in principle acquire all the interference signal data in half a second as opposed to several minutes.

4.2 FBG sensor array interrogation by an FSI system utilizing a broadband source and a tunable filter

4.2.1 Characterization of the tunable filter

The TF chosen for the experiments (FFP-TF) was manufactured by Micron Optics, Inc. It is a fiber FP filter whose transmission band is tuned by adjusting the FP cavity length with
appropriate voltage applied to piezoelectric materials. A broadband amplified spontaneous emission (ASE) source (AFC BBS 1550A-TS), a power meter, an optical spectrum analyzer (OSA, Ando AQ6317B), and a multi-wavelength meter (Agilent 86120B) were employed to characterize the TF. The transmitted light from the ASE source through the TF was first measured with the power meter, and its power was found to be 20 $\mu$W at around 1545 nm. A transmission spectrum of the TF is shown in Fig. 4.2(a). As the light directly from the filter was too low in power to be used as sensing light, a two-stage Erbium-doped fiber amplifier (EDFA, PriTel, Inc. LNHP FA-30) was employed to amplify TF output light. Fig. 4.2(b) shows the amplified filter output when the preamplification current and the second stage pump current

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**Fig. 4.2.** Transmission spectrum of the tunable filter. (a) tunable filter transmission spectrum without amplification; (b) tunable filter transmission spectrum amplified by an EDFA.
were set to 200 mA and 410 mA, respectively. The amplified filter output power was 8.863 mW. As can be seen from Fig. 4.2, the full width at half maximum width of the filter transmission peak is $\sim 0.05$ nm.

The TF response to the applied voltage was investigated next. The DC voltage was generated by a National Instruments (NI) PCI-6251 data acquisition (DAQ) board through a BNC-2110 BNC adapter. It was swept at steps of 0.02 V between 0 to 2 V in a triangular fashion, and the center wavelength of the TF transmission peak was measured by the multi-wavelength meter [Fig. 4.3(a)]. The TF response to the applied voltage shows excellent linearity.
and repeatability, as can be seen from Fig. 4.3(b). In the leading edges of the 3 voltage triangular waveform cycles [voltages steps 0–100, 200–300, and 400–500 in Fig. 4.3(a)], the TF center wavelengths measured match nicely (blue, green, and steel blue dots). In Fig. 4.3(b), a small level of TF hysteresis was illustrated by the deviation of TF center wavelengths within voltage steps 100–200 (red dots) from those within steps 0–100 (blue dots). A linear fit to the averaged TF center wavelength data (in the leading edges of the triangular waveforms) gives a slope of 5.074 nm/V [dashed black line in Fig. 4.3(b)]. The TF has a relatively wide free spectral range (FSR). The next transmission peak of the TF does not appear at $\sim 1530.8\ \text{nm}$ until the applied voltage is $\sim 8\ \text{V}$. It was also observed that the TF center wavelength may drift over a long period of time, which is probably caused by the temperature variations.

### 4.2.2 Experimental setup

The experimental setup is shown in Fig. 4.4. The FSI configuration is a linear Sagnac interferometer (cf. Fig. 2.3). The light source consists of a broadband ASE source (AFC BBS 1550A-TS), the TF characterized above (Micron Optics), and an Erbium-doped fiber amplifier (EDFA). An AOM (Brimrose AMM-100-20-25-1550-2FP) was used as the optical frequency shifter. $C_1$ and $C_2$ are two 50/50 fiber directional couplers. The interference signal was measured by a balanced detector (BD, New Focus Model 2117). The output from BD was received

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**Fig. 4.4.** Experimental setup for rapid fiber Bragg grating sensor multiplexing. ASE: amplified spontaneous emission source; TF: tunable filter; EDFA: Erbium-doped fiber amplifier; CIR: optical circulator; BD: balanced detector; $C_1$ and $C_2$: 50/50 fiber directional couplers; PC: polarization controller; AOM: acousto-optic modulator; $G_i$: $i$th FBG sensor.
by a DAQ board (NI PCI-6251) with the help of a BNC adapter (NI BNC-2110). The sensor array is formed by FBGs of 4\% reflectivity centered around 1548.6 nm [27]. The first 4 FBGs were fixed on translation stages so that their applied strain could be adjusted.

4.2.3 System synchronization

The challenge of this experiment is to synchronize the AOM sweep, TF tuning, and data acquisition. The AOM driver can be synchronized in 4 different ways: Sync per Scan, Sync per Step, Trigger per Scan, or Trigger per Step. Under Sync per Scan or Trigger per Scan mode, the AOM driver must output or receive a sync pulse at the beginning of a full frequency sweep. As the time for the AOM to complete a full sweep is relatively long (more than 0.5 s) and the DAQ does not necessarily have an identical clock as the AOM driver, Sync per Scan and Trigger per Scan options were not considered for our experiments. LabVIEW programs were developed to coordinate various instruments. In principle, one can use the synchronization signal from the AOM driver to trigger the data acquisition. Under Sync per Step mode, the AOM driver supplies an output trigger pulse at the beginning of every frequency step. However, unexpected computer processing delays occur during the measurement which cause the data acquisition to be out of sync. The interference data at certain frequency step $f$ is skipped in this case. Therefore, Trigger per Step mode was used in which an external trigger pulse must be supplied to the AOM driver. A LabVIEW program was written so that the computer generates

![Fig. 4.5. Synchronous voltage generation and data acquisition. For testing purpose, the voltage generated (to be used to drive the TF) is recorded by the BNC adapter port used for the interference signal measurement. The maximum time jitter occurred in these 200 recorded voltage traces was less than 1 ms. The time jitter is normally below 0.6 ms.](image)
synchronization signals that trigger both the AOM sweeping and data acquisition. Although this method limits the AOM sweep speed to about 0.1 s per step (constrained by the response time of the computer), it guarantees that the AOM stepping is in sync with TF tuning and data acquisition. The synchronization signals, which were 5-ms 4-V rectangular pulses, were generated from the AO0 port of the BNC adapter by the LabVIEW program. They were fed to the Trigger port of the AOM driver and back to the BNC adapter’s AO EXT REF port. The BNC adapter’s AO1 and AI0 ports were used to produce the TF applied voltage and record the interference signal, respectively.

To test the jitter of the system, the AO1 port was connected directly to the AI0 port, and synchronized voltage generation and measurement were conducted. The voltages generated were half-cycle 10-Hz triangular waveforms with a 2-V amplitude, while the data sampling rate was 50 kS/s. Fig. 4.5 shows 200 recorded voltage traces. The maximum time jitter was found to be below 1 ms (see the time difference between the leftmost single trace with the rightmost trace). The jitter from pulse to pulse more frequently lies within 0.3 ms about 0.025 s (see Fig. 4.6). Given fixed jitter in time, the wavelength error is determined by the voltage sweep rate. Since the voltage sweep rate was 0.08 V/ms and the TF response to voltage is 5.074 nm/V, this time jitter corresponds to a wavelength error of 0.121 nm. This jitter can be further corrected as will be discussed in the next section.

4.2.4 Experimental results

FBG sensor array interrogation

The FBG multiplexing capability of the system in Fig. 4.4 was tested with the synchronous LabVIEW program. The AOM driver was set to Trigger per Step mode. It was swept from 90
to 110 MHz at steps of 0.04 MHz. Fig. 4.7 shows the LabVIEW generated TF driving voltage (a half-cycle 2 Hz triangular waveform with an amplitude of 800 mV) [see Fig. 4.7(a)] and the corresponding measured interference signal under a sampling rate of 10 kS/s [Fig. 4.7(b)] from a preliminary test. Note that a 3.5-ms delay was intentionally introduced to the TF driving voltage waveform so that the interference signal during the leading edge of the voltage waveform can be acquired completely. As can be seen from Fig. 4.7(b), the measured interference signal has prominent amplitude between 0.07 and 0.1 s as well as between 0.15 and 0.18 s, since the TF center wavelength covered the reflection wavelengths of the FBG array twice. Furthermore, the interference signal after ∼ 0.125 s is a mirror image of the signal before 0.125 s. This can be explained by the fact that the TF reached its maximum wavelength at 0.125 s and started to sweep through the same wavelength range backwards, and at the same time, the interference
Fig. 4.8. FBG sensor array reflection spectra before and after the strain adjustments. (a) Normalized grating spectra before strain adjustments; (b) normalized grating spectra after the strain on the first and second FBGs was reduced while the strain on the fourth FBG was increased.

signals at the same wavelength have the same values. Because of this symmetry property, one can correct time jitter by searching the symmetry point of the interference signal and utilize it as the reference to the maximum wavelength. Also, the useful interference data within the FBG reflection wavelengths can be averaged.
To see whether this FSI system is able to discern the Bragg wavelength shifts of individual FBGs in the array, spatially-resolved reflection spectra of the sensors were measured before and after strain adjustments were made on the first, second, and fourth sensors (G1, G2, and G4, respectively). The TF applied voltage was half-cycle 10-Hz triangular waveforms with amplitude of 0.2 V on top of a DC offset of 0.4 V. A DAQ sampling rate of 50 kS/s was used. Fig. 4.8(a) shows the FBG spectra before the strain adjustments. The FBGs to be adjusted are highlighted in blue (G1), green (G2), and red (G4) in the figure. Despite the spectral overlap, FSI can resolve the reflection spectrum of every individual FBG. As can be seen in Fig. 4.8(a), the Bragg wavelengths of G3 to G9 are centered around 1548.4 nm, while those of G1 and G2 are near 1548.7 nm. Next, the strain on G1 and G2 were reduced, whereas that of G4 was increased. In response to such strain change, according to (3.2a), the Bragg wavelengths of G1 and G2 should shift towards shorter wavelengths, but the Bragg wavelength of G4 should move to a longer wavelength. The FBG reflection spectra after the strain adjustments are presented in Fig. 4.8(b). As expected, the reflection peaks of G1 (blue) and G2 (green) are now at shorter wavelengths (near 1548.4 nm), and G4’s peak (red) is at a longer wavelength (near 1548.8 nm). Note that the rest FBGs (G3 and G5 to G9) do not display Bragg wavelength shifts. As the bandwidth of TF is relatively wide and the reflection intensity is significantly weaker for the last a few FBGs at the end of the array, crosstalk is more appreciable in their spectra after normalization (compare G9 spectra around 1548.4 nm in 4.8(a) and (b)). The time it takes to acquire the entire interference data set is 25 s with the experimental parameters used. Note that the wavelength scan range of the TF was ~1.02 nm, and the wavelength step size corresponding to the data sampling rate was ~0.8 pm. If a tunable laser is used to scan over this wavelength range at the same wavelength step size, the data acquisition time with our current equipment is ~638 s, about 25 times longer.

Strain sensitivity test

To test the FBG strain sensitivity attainable with this FSI system, an FBG sensor was glued between two translation stages. The initial fiber length between the two glued points was 84.75 cm (l = 84.75 cm). The displacement δl was increased at steps of 10 µm, and the Bragg wavelength of the FBG was measured with the FSI system. The TF applied voltage was half-cycle 10-Hz triangular waveforms with amplitude 1.3 V and a DC offset of 0.5 V. Fig. 4.9 shows the FBG Bragg wavelength as a function of applied strain (δl/l). The data exhibit good linearity, and the FBG strain responsivity was found to be 1.11 pm/µε, about 8% lower than the theoretical value 1.21 pm/µε [calculated by using (3.2a) with λB = 1550.55 nm]. Wavelength calibration error could contribute to this inconsistency. The TF was also characterized by an OSA which shows that the TF response to applied voltage is 5.358 nm/V, leading to strain responsivity of 1.17 pm/µε, only 3% below the theoretical expectation. The strain resolution of the system is mainly determined by the bandwidth of the TF (~0.05 nm). Given the strain sensitivity of 1.11 pm/µε, it is ~22 µε.
4.3 Summary

Above experiments demonstrate that an FSI system equipped with a broadband source and a tunable filter as the light source can be applied in fiber-optic sensor multiplexing. Nine FBG sensors in an array were successfully interrogated. Strain sensitivity of 1.11 pm/µε and a strain resolution of ∼ 22 µε were achieved in our current system. In this FSI system, costly tunable laser is replaced by a broadband source paired with a TF, which leads to lower system cost. As TF can be tuned at very high rate (∼ kHz), the measurement speed of the system can be significantly improved.
Chapter 5

Cavity Ring-Down Technique Based on Frequency-Shifted Interferometry

5.1 Conventional cavity ring-down (CRD) techniques

5.1.1 Traditional absorption spectroscopy

Absorption spectroscopy, which studies the absorption of electromagnetic radiation induced by matter, is an important tool that has been widely applied to the analysis of atoms and molecules [76, 77]. In a traditional direct absorption spectroscopy experiment (see Fig. 5.1), light with initial intensity $I_0$ is shined through a sample, and the transmitted light intensity $I$ is measured. The relation between $I$ and $I_0$ is governed by the Beer-Lambert law

$$ I(\lambda) = I_0(\lambda) \cdot e^{-\alpha(\lambda) \cdot l} $$  \hspace{1cm} (5.1)

where $\lambda$ is the incident light wavelength, $\alpha(\lambda)$ is the wavelength-dependent absorption coefficient of the species present in the sample, and $l$ is the light-sample interaction length. An absorption spectrum is obtained if $I$ is recorded as a function of the incident light wavelength $\lambda$. Since $\alpha(\lambda)$ is related to the species number density $n$ and absorption cross section $\sigma(\lambda)$ by $\alpha(\lambda) = n \cdot \sigma(\lambda)$, while $\sigma(\lambda)$ is distinctive for any species, different atoms or molecules exhibit different absorption characteristics. From the measured absorption spectrum, not only can one identify the sample composition, but also quantify the concentrations of various existing species. Thanks to the developments in optics and photonics, excellent light sources such as lasers are now available to provide sensing light with high spectral power density and narrow line width, enabling spectrum acquisitions at high accuracy and spectral resolution [76].

Despite of the aforementioned advantages, to detect samples with weak absorptions, traditional absorption spectroscopic techniques usually entail the measurement of small optical attenuation on a relatively large background, and the detection sensitivity is further limited by the light source intensity fluctuations. Many techniques have been devised to improved
the detection sensitivity. As suggested by (5.1), one simple way to achieve this goal is to adopt a long interaction length \( l \) [78,79]. Alternatively, as in the case of intracavity absorption spectroscopy (ICAS), the sample is placed in a laser cavity so that the effect of intracavity absorption can be enhanced through laser oscillations [80]. Techniques such as wavelength- or frequency-modulation spectroscopy (WMS or FMS) rely on heterodyne detections of frequency-modulated light [81–83]. As the noise that falls outside of the band around the modulation frequency is eliminated by lock-in detection, high sensitivity can be achieved. Recently, the cavity ring-down spectroscopy (CRDS) has become a popular choice for many applications.

5.1.2 Theory of cavity ring-down spectroscopy (CRDS)

The cavity ring-down (CRD) technique is a highly sensitive method for measuring optical losses [84–88]. Rather than attempting to determine the magnitude of a minute intensity drop caused by the sample absorption, CRDS measures how fast the intensity decays as the light circulates in an optical cavity and interacts with the sample. In a typical CRD experiment, the optical cavity, usually referred to as the ring-down cavity (RDC), is formed by two highly reflective mirrors (reflectivity \( R \geq 0.99 \)). An optical pulse is introduced into the cavity so that it starts to bounce back and forth between the two mirrors (see Fig. 5.2). Each time when the pulse hits one mirror, a small portion of the pulse power leaks out of the cavity, which can be observed by placing a detector on one end of the cavity. Suppose there is some absorbing sample in the cavity with absorption coefficient \( \alpha \) and interaction length \( l \). In the light of (5.1), the detected intensity \( I_m \) contributed by a pulse exiting the cavity after \( m \) round trips (\( m \geq 0 \)) can be expressed as [85]

\[
I_m = I_{in} e^{-\alpha l} \cdot R^{2m} e^{-2m \cdot \alpha l} \cdot T = I_0 \cdot R^{2m} e^{-2m \cdot \alpha l} \quad (m = 0, 1, 2, \ldots)
\]  

(5.2)

where \( I_{in} \) is the intensity of the pulse before it is coupled into the cavity, \( T \) is the mirror transmittance, \( R \) is the reflectivity of the mirrors, and \( I_0 = I_{in} T^2 \cdot e^{-\alpha l} \) is the intensity of the first pulse detected outside of the cavity.\(^1\) After some simple algebraic manipulations, (5.1) can be expressed as

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\(^1\)The factor of 2 in front of \( m \) accounts for the fact that the pulse needs to make double passes in the cavity before exiting from the same mirror again.
be rewritten as

\[ I_m = I_0 \cdot e^{2m(lnR - \alpha l)} \]  

(5.3)

It is clear from (5.3) that \( I_m \) undergoes an exponential decay as the round trip number \( m \) increases. We may replace \( m \) by a continuous variable time \( t = 2mL/c \), where \( L \) is the cavity length and \( c \) is the speed of light, so that the envelope of (5.3) can be expressed as a function of time [85]

\[
I(t) = I_0 \cdot \exp \left[ \frac{ct}{L} (\ln R - \alpha l) \right]
= I_0 \cdot \exp \left[ -\frac{c(-\ln R + \alpha l)}{L} \cdot t \right]
= I_0 \cdot e^{-t/\tau} 
\]

(5.4)

where \( \tau \), the so-called CRD time, is the time it takes the light intensity to fall to \( 1/e \) of its initial value

\[
\tau = \frac{L}{c(-\ln R + \alpha l)} 
\]

(5.5)

Higher loss in the RDC produces a shorter \( \tau \). An absorption spectrum can be plotted if one evaluates \( \tau \) from the decay of \( I \) as a function of \( \lambda \), and use the relation [85]

\[
\alpha(\lambda) \frac{l}{L} = \frac{1}{c\tau(\lambda)} + \frac{\ln R}{L}
= \frac{1}{c\tau(\lambda)} - \frac{1}{c\tau_0} 
\]

(5.6)

Note that the absorption can be obtained on an absolute scale. The term \( 1/(c\tau_0) = -\ln R/L \) in (5.6) is a background that can be determined by measuring the CRD time \( \tau_0 \) for an RDC without the sample.
For more general cases, in which multiple absorbing or scattering species coexist and their distributions are nonuniform in the cavity, $\alpha_l$ in (5.5) should be replaced by $\sum_i \sigma_i \int_0^l n_i dx$, the summation of all the contributions, with $\sigma_i$ being the corresponding absorption or scattering cross section of the $i$th species and the integrations carried out along the direction of light propagation. As $R$ is very close to unity, an approximation is often made in (5.4)–(5.6) such that $\ln R \approx -(1 - R)$. The effects of broadband absorption or scattering can also be included into an effective loss factor $R_{\text{eff}}$ in place of the mirror reflectivity $R$ [84,85].

The minimum detectable absorption of CRD techniques can be obtained from (5.6), which is determined by the minimum detectable CRD decay time deviation $\Delta \tau_{\text{min}}$ from $\tau_0$ [85]

$$\left[ \frac{\alpha(\lambda) l}{L} \right]_{\text{min}} \approx \frac{\Delta \tau_{\text{min}}}{c \tau_0^2} = -\ln R_{\text{eff}} \left( \frac{\Delta \tau}{\tau_0} \right)_{\text{min}}$$

(5.7)

Here we have used $R_{\text{eff}}$ to include the broadband absorption or scattering. For a gaseous sample filling out the RDC ($l = L$), the right hand side of (5.7) gives the minimum detectable absorption coefficient directly. The superior sensitivity of CRD techniques emanates mainly from two sources. First, since CRD techniques measure the decay rate of an optical pulse rather than its absolute intensity, they are immune to source intensity fluctuations. Second, due to the high-finesse of the RDC, long light-sample interaction length up to several of kilometers can be achieved. With a simple experimental setup, a minimum detectable absorption coefficient of $10^{-8}$ cm$^{-1}$ is readily achievable (e.g., with $l = L = 20$ cm, $R_{\text{eff}} = 0.9999$, and 1% accuracy in the determination of $\tau$, $\alpha_{\text{min}} = 5 \times 10^{-8}$ cm$^{-1}$). Smaller $\alpha_{\text{min}}$ as low as $10^{-9} - 10^{-14}$ cm$^{-1}$ can be achieved with refined setup or CRD variations [89–93]. Trace gas detection limits on the order of part per billion by volume (ppbv) or lower have been achieved with CRDS [91,92,94].

### 5.1.3 Continuous-wave CRDS

Conventional CRD techniques always rely on generating an optical pulse in the RDC and measuring its decay rate in the time domain. Pulsed lasers are natural choices of light sources to excite the cavity [see Fig. 5.3(a)] [84–88]. There are CRD techniques which employ continuous-wave (CW) laser sources and are thus referred to as CW-CRDS [84–86,89–91,95,96]. However, the term is somewhat a misnomer, because “CW” is only relevant to the light source, whereas an optical pulse is still required in the RDC for an observable ring-down event. Strictly speaking, these so-called CW-CRDS schemes are “pulsed” as before. Nevertheless, compared with using a pulsed laser in CRD experiments, the main advantage of CW-CRDS is its high spectral resolution and signal-to-noise ratio, as the narrow-line-width CW light can be coupled efficiently into the RDC. To produce a pulse from a CW laser, an optical switch can be used to abruptly block the light injecting into the RDC [Fig. 5.3(b)] [89,90,92,97,98]. Alternatively, one can modulate the CW laser source [Fig. 5.3(c)]. For a CW diode laser, the injection current can be rapidly switched above and below the lasing threshold [95,99]. The output wavelength of the CW laser can also be detuned from the RDC resonance wavelength so that light can no
Fig. 5.3. Various ways of generating pulses in conventional CRD techniques. (a) using a pulsed laser; (b) using an optical switch to chop the output from a CW laser; (c) modulate a CW laser or detuning its output wavelength from the resonance wavelength of the RDC; (d) detuning the resonance frequency of the RDC with a piezoelectric translator (PZT).

longer be coupled efficiently into the RDC [96]. Similarly, quickly adjusting the RDC length can induce low coupling efficiency, enabling pulsations [Fig. 5.3(d)] [91]. Once a pulse starts to circulate in the RDC, time-domain measurement can be done to evaluate the intensity decay rate of the pulse.

Two variants of CRD techniques do not require optical pulsation. One method is referred to as the spatial cavity ring-down (SCRD) in which a CW light beam is launched into a high-finesse plane Fabry-Perot (FP) cavity at an oblique angle [100,101] (see Fig. 5.4). As the beam bounces back and forth in the cavity, it exhibits lateral displacement due to the non-normal incidence. A detector array such as a two-dimensional (2D) beam analyzer is then utilized to measure the transmitted beam profile. The spatial intensity distribution of the transmitted beam exhibits exponential decay as a function of lateral displacement. The cavity loss can be deduced from a decay distance analogous to the decay time in conventional time-domain CRD schemes. Although this technique avoids pulses and fast electronics, it is sensitive to the
input beam size, the FP alignment, and the FP tilt angle. The effect of diffraction on the transmitted light intensity distribution must also be treated. To the best of my knowledge, no actual sensing experiment has been done so far with this scheme other than the proof-of-concept demonstration of SCRD signals [100, 101].

Another scheme that does not require pulsation is the phase-shift cavity ring-down (PS-CRD) technique [102–104]. PS-CRD requires intensity modulation on a CW input light beam and lock-in detection (Fig. 5.5). It can be shown that the phase shift $\phi$ experienced by the output beam is $\phi = -\arctan(\Omega \tau)$, where $\Omega$ is the modulation angular frequency and $\tau$ is the CRD time [102, 103, 105]. By measuring $\phi$ with a lock-in amplifier, one can deduce $\tau$ and thus the loss in the RDC. As PS-CRD requires lock-in detection and measures the CRD time, it is in a sense still a time-domain CRD technique.
5.1.4 Fiber-based CRD techniques

Free-space RDCs formed by two mirrors are most frequently employed in CRDS for the measurement of gaseous samples, yet in recent years, waveguide-based RDCs have been demonstrated, opening doors for new CRD applications [85, 106–113]. Optical fibers prove particularly useful in constructing these waveguide-based RDCs as a result of their versatility. Two common RDC configurations are linear cavities and loop cavities, as shown in Fig. 5.6. The former [Fig. 5.6(a)] is analogous to a free-space RDC, in which light also bounces back and forth between two mirrors except that it is guided by an optical fiber instead of traveling in free space. The mirrors can be highly-reflective coatings on both ends of fiber facets [107, 110] or a pair of reflectors such as strong fiber Bragg gratings (FBGs) [108]. A fiber loop cavity [Fig. 5.6(b)] can be built conveniently by connecting the outputs of fiber directional couplers [95, 98, 99].

One attractive advantage of using fiber-based RDCs is that depending on what sensor head is incorporated into the RDC, the same CRD system can be used for different applications. A few types of sensor heads commonly used in a fiber RDC are illustrated in Fig. 5.7. A small gap between two well aligned fiber facets, for example, allows the light to pass through the sample so that the absorption of the analyte can be measured [Fig. 5.7(a)]. However, a majority of these sensor heads utilizes the interaction between the evanescent field and the ambient environment. A fiber taper [Fig. 5.7(b)] or a section of etched fiber [Fig. 5.7(c)] provides evanescent field

![Commonly used fiber RDCs. (a) a linear cavity; (b) a loop cavity.](image-url)
Fig. 5.7. Commonly used sensor heads in a fiber RDC [111]. The light is assumed to propagate from the left to the right, and schematic mode field distributions are also shown in the figures. (a) a gap between fiber facets; (b) a fiber taper; (c) an etched fiber; (d) a long-period grating (LPG) pair; (e) a field access block.

at the thinned region of the fiber so that if absorption or refractive index (RI) change occurs in the surrounding medium, the transmission loss of the taper or the etched fiber is altered, which can be gauged by the measured $\tau$. A long-period grating (LPG) inscribed in a single-mode fiber is capable of coupling light propagating in the core into cladding modes or coupling the light in cladding modes back to the core mode. In a LPG-pair sensor head [Fig. 5.7(d)], the first LPG excites the cladding modes and produces evanescent field, whereas the second LPG brings the light back to the core. The loss between the two LPGs is again related to ambient absorption or RI index change. Similarly, a side-polished fiber in a field access block [Fig. 5.7(e)] offers evanescent field at the polished region, which can be exploited for sensing. With various kinds of sensor heads, not only can fiber-based CRD systems be used for chemical sensing [95, 98, 103, 109] and RI sensing [97, 99], but they have also been applied in measuring temperature [114], strain [115], pressure [116, 117], or detecting biological cells [118] and so on. This again shows the versatility of fiber-optic sensors. However, the insertion loss of a sensor head used in a fiber-based CRD system must be low, otherwise it limits the sensitivity and detection limit [111].
In the next section, I am going to show how frequency-shifted interferometry (FSI) can be used in CRD measurement with fiber-based RDCs. This novel FSI-based CRD (FSI-CRD) technique does not rely on any time-domain measurement of a decaying optical pulse. Instead, FSI-CRD is able to deduce the intensity decay rate of CW light circulating in a fiber RDC from its interference signal. The requirements of optical pulsation and fast electronics imposed by conventional CRD experiments are therefore relaxed. FSI-CRD can potentially be a more cost-effective alternative to conventional CRD methods.

5.2 Principle of the FSI-based CRD (FSI-CRD) technique

The FSI-CRD technique measures the interference between the CW lightwaves exiting an RDC after the same number of round trips. It is capable of resolving the intensity of light after it exits the RDC from the Fourier spectrum of the interference signal, and therefore, we can deduce the intensity decay rate of CW light traveling in the RDC. Both a linear cavity or a loop cavity can be employed in an FSI-CRD system. This section explains how FSI can be applied in CRD measurement.

5.2.1 FSI-CRD with a linear cavity

A fiber linear RDC formed by a pair of highly reflective reflectors can be used for FSI-CRD in a linear frequency-shifted Sagnac interferometer configuration (Fig. 5.8). This setup is analogous to the one in Fig. 2.3 in that the light coming back to the input directional coupler $C_0$ after $m$ round trips in the RDC is equivalent to the light coming back from a distance of $L_0 + md$ away, where $d$ is the cavity length. Therefore, when CW light is launched into the interferometer from port 1, we can apply (2.5) to obtain the differential interference signal $\Delta I$ with small modifications

\[
\Delta I \propto \sum_{m=0}^{\infty} I_m \cdot \cos \left\{ \frac{2\pi n[l_1 + 2(L_0 + md) + l_2 - l_0]}{c} f \right\}
\]

\[
\propto \sum_{m=0}^{\infty} I_m \cdot \cos \left\{ \frac{2\pi n[L_s + m2d]}{c} f \right\}
\]

\[
\propto \sum_{m=0}^{\infty} I_m \cdot \cos \{2\pi \cdot F_m \cdot f\}
\]

(5.8)

where $L_0$ is the distance between $C_1$ and $R_1$, $L_s = 2L_0 + l_1 + l_2 - l_0$ is a length constant, $d$ is the length of the RDC (the separation between $R_1$ and $R_2$), $F_m$ is the oscillation frequency of the sinusoidal component contributed by the lightwaves that exit the RDC after $m$ round trips, and $I_m$ is proportional to their intensity. Note that $F_m$ is equally spaced by $2nd/c$, and $I_m$ is determined by the reflectivities of $R_1$ and $R_2$ as well as by the additional loss in the cavity. The latter can be written as
Fig. 5.8. An FSI-CRD system with a linear cavity. $C_0$ and $C_1$: 50/50 fiber directional couplers; $R_1$ and $R_2$: highly-reflective reflectors.

\[
I_m = \begin{cases} 
I'_0 R_1 & \text{for } m = 0, \\
I'_0 (1 - R_1)^2 R_1^{m-1} R_2^m \kappa_f^2 \kappa^2 & \text{for } m \geq 1.
\end{cases}
\] (5.9)

where $I'_0$ is the initial light intensity, $\kappa_f$ is the transmittance in the RDC that includes the fiber loss and splice loss, and $\kappa$ is the transmittance of some sensing element. If the attenuation coefficient at the sensing element is $\alpha$ and the interaction length is $l$, then $\kappa = e^{-\alpha l}$. Rearranging the terms of $I_m$ for $m \geq 1$, we have

\[
I_m = I'_0 \frac{(1 - R_1)^2}{R_1} \cdot (R_1 R_2 \kappa_f^2)^m \cdot (e^{-\alpha l})^{2m} \\
= I' \cdot \exp \left[ -m \ln (R_1 R_2 \kappa_f^2) - 2m \alpha l \right] \\
= I' \cdot \exp \left[ -m (\ln \kappa_c + 2\alpha l) \right]
\] (5.10)

where $I' = I'_0 (1 - R_1)^2 / R_1$, and $\kappa_c = R_1 R_2 \kappa_f^2$ is the round-trip transmittance of an empty cavity (without any sensing element). As can be seen from (5.10), $I_m$ decays exponentially as $m$ increases. By measuring $\Delta I$ as a function of $f$ and taking its Fourier transform, we can see a series of equally-spaced exponentially decaying peaks following a strong peak in the spectrum (Fig. 5.9). We can convert $F$ in s into distance in m by using the relation

\[
L = \frac{c}{n} \cdot F
\] (5.11)

After the conversion, we have the CRD decay transient, a series of exponentially decaying peaks, in the spatial domain. We may replace $m$ by a continuous distance variable $L_t = 2md$ which is the distance traveled by the light in the RDC so that $m = L_t / (2d)$. Equation (5.10) then becomes

\[
I_m = I' \cdot \exp \left( -\frac{\ln \kappa_c + 2\alpha l}{2d} L_t \right)
\] (5.12)
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Fig. 5.9. FFT spectrum of $\Delta I$ obtained from an FSI-CRD system with a linear RDC. The first strong peak is contributed by the reflection from the first mirror of the RDC. The subsequent peaks decay exponentially due to RDC loss and can be considered as a CRD decay transient.

Analogous to the CRD time $\tau$, we can define a $1/e$ decay distance, or the CRD distance $\Lambda$

$$\Lambda = \frac{2d}{-\ln \kappa_c + 2\alpha l}$$  \hspace{1cm} (5.13)

By fitting $\Lambda$, the attenuation information $\alpha l$ in the RDC can be found. In a sense, FSI-CRD can be considered as a spatial-domain CRD technique. Note that although the setup in Fig. 5.8 is similar to that in Fig. 2.3, we are interested in the intensity decay rate as light bounces back and forth between $R_1$ and $R_2$, rather than the reflection spectra of these two reflectors.

As an RDC requires that $R_1$ and $R_2$ have high reflectivity, the first reflection from $R_1$ is a very strong one which is the source of the first strong peak in the Fourier spectrum. For instance, suppose $R_1 = R_2 = 0.99$, the first peak is $10^4$ times higher than the second peak. Crosstalk in the discrete Fourier transform (DFT) (see Section 2.4.4) may occur between the first peak and the subsequent CRD decay transient if the resolution of the Fourier transform is not high enough, or the cavity length is very short. Fig. 5.10 shows the simulation result of DFT crosstalk in an FSI-CRD system using a linear cavity with the following parameters: $R_1 = R_2 = 0.99$, $L_s = 60$ m, $d = 40$ m, $\kappa_f = 0.99$, $\kappa = 0.98$, $\Delta f = 20$ MHz, and $f_{\text{step}} = 0.01$ MHz. The DFT was done with a Hann window and an FFT size of $2^{20}$. The blue curve in the figure is the Fourier spectrum with both the first reflection and the CRD decay transient, while the red curve depicts only the CRD decay transient without the crosstalk from the first reflection. As can be seen in Fig. 5.10(a), the first Fourier peak at 60 m is considerably higher.
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Fig. 5.10. Simulation results of DFT crosstalk in an FSI-CRD system with a linear cavity. The following parameters were used: $R_1 = R_2 = 0.99$, $L_s = 60$ m, $d = 40$ m, $\kappa_f = 0.99$, $\kappa = 0.98$, $\Delta f = 20$ MHz, and $f_{\text{step}} = 0.01$ MHz. A Hann window was used with an FFT size of $2^{20}$. (a) the strong peak caused by the first reflection located at 60 m in the Fourier spectrum; (b) a close look at the exponential decaying Fourier peaks. Blue line is the FFT result with the first reflection and red is the FFT result without the influence of the first peak. Note that the red Fourier peaks are too small to be seen in (a).

than the subsequent exponentially decaying peaks. In fact, we are unable to see the exponential decay with the vertical scale in this figure. The decaying peaks can be found in a zoomed-in image of the Fourier spectrum, which also shows that once the side lobes of the first reflection fall off considerably, the crosstalk becomes minimal, as suggested by the good overlap between the blue and red curves after $\sim 1000$ m [Fig. 5.10(b)]. Therefore, the peaks in this region can be fitted to find $\Lambda$. However, the detector dynamic range is limited. If a high signal-to-noise ratio (SNR) is required, the first strong peak may prevent the proper amplification of the actual CRD decay transient. This may limit the use of a linear cavity in FSI-CRD from a practical point of view. FSI-CRD with a fiber loop cavity, on the other hand, does not have such issue.

5.2.2 FSI-CRD with a loop cavity

A typical experimental setup for FSI-CRD with a loop cavity is shown in Fig. 5.11. In this case, the fiber loop cavity is embedded in a frequency-shifted Sagnac interferometer (see Section 2.1). The RDC is formed by connecting the proper output ports of two highly unbalanced fiber directional couplers $C_1$ and $C_2$ (e.g., 99.5/0.5 couplers). When CW light is launched into the interferometer from port 1, it is divided equally into two parts by the 50/50 fiber directional coupler $C_0$ and starts to circulate in the RDC in opposite directions. A small portion of the light exits the RDC every time when the light completes a round trip. If the coherence length of the light source is shorter than the RDC length, interference occurs at $C_0$ between the two counter-propagating lightwaves that leave the RDC after the same number of round trips. The setup is...
equivalent to an amalgamation of infinitely many frequency-shifted Sagnac interferometers with incremental sizes. These Sagnac interferometers’ sizes increase by the cavity length \( d = l_2 + l_3 \) each time due to an additional round trip made by the light. The differential interference signal \( \Delta I \) at the output ports of the interferometer is

\[
\Delta I \propto \sum_{m=0}^{\infty} I_m \cdot \cos \left\{ 2\pi n \left[ l_1 + l_2 + m(l_2 + l_3) + l_4 - l_0 \right] / c \right\} \\
\propto \sum_{m=0}^{\infty} I_m \cdot \cos \left\{ 2\pi n L_s + m d \right\} / c \\
\propto \sum_{m=0}^{\infty} I_m \cdot \cos \left\{ 2\pi F_m \cdot f \right\} 
\]

(5.14)

where \( L_s = l_1 + l_2 + l_4 - l_0 \) is a length constant, \( m \) is the round trip number \( m = 0, 1, 2, \ldots \). Note that the expression of \( L_s \) for a loop cavity is different from that of a linear cavity [30].

The sources of optical loss include the coupler loss (at \( C_1 \) and \( C_2 \)), fiber loss, splice loss, as well as the loss at any sensing element inserted in the cavity. The amplitude of the sinusoidal component \( I_m \) is

\[
I_m = I'_0 (1 - \gamma)^2 \gamma^{2m} \cdot \kappa_2 \kappa_3^m \cdot \kappa^{m+1} 
\]

(5.15)

where \( I'_0 \) is some initial light intensity, \( \gamma \) is the coupling ratio of the couplers \( C_1 \) and \( C_2 \) (e.g., \( \gamma = 0.995 \) for a 99.5/0.5 fiber coupler), \( \kappa_i \) is the transmittance of the \( i \)th fiber section, and \( \kappa \) is the transmittance of some sensing element in fiber section \( l_2 \) [30]. Assuming an attenuation coefficient \( \alpha \) and an interaction length \( l \) at the sensing element, we can write \( \kappa = e^{-\alpha l} \). Equation (5.15) can then be rewritten as

\[
I_m = I'_0 \kappa_c^m \cdot e^{-\alpha \cdot ml} = I'_0 \cdot e^{-(\alpha l - \ln \kappa_c) \cdot m} \\
= I'_0 \cdot \exp \left( -\frac{-\ln \kappa_c + \alpha l}{d} \cdot L_t \right) 
\]

(5.16)
where \( I' = I'_0(1 - \gamma)^2 \cdot \kappa_2 \cdot e^{-\alpha l} \) is some initial amplitude, \( \kappa_c = \gamma^2 \kappa_2 \kappa_3 \) is the transmittance of the empty cavity, and \( L_t = md \) is the distance traveled by the light in the RDC. \( I_m \) exhibits exponential decay as \( L_t \) increases. Note that light only makes single pass through the cavity every round trip in a loop RDC, and therefore, compared with (5.12), there are no factors of 2 in front of \( \alpha l \) and \( d \) in (5.16). The CRD distance for an FSI-CRD system with a loop RDC is

\[
\Lambda = \frac{d}{-\ln \kappa_c + \alpha l} \tag{5.17}
\]

Again, the factors of 2 in front of \( \alpha l \) and \( d \) do not present in (5.17) for a loop RDC when compared with (5.13). Following the same data processing procedure as before (by taking FFT on \( \Delta I \) and fitting the exponential decay), we can deduce the loss information in the RDC.

One great advantage of incorporating a loop RDC in an FSI-CRD system is that it does not produce the strong first reflection that occurs if a linear RDC is used. The intense light which is not coupled into the RDC simply vanishes at the unused output ports of the fiber coupler. Therefore, all the Fourier peaks in the spectrum can be used for the fitting of cavity loss, and much higher signal-to-noise ratio can be achieved.

### 5.3 FSI-CRD fiber bend loss measurements

As a proof-of-principle demonstration, I introduced bend loss in a fiber loop RDC and measured the cavity loss with the FSI-CRD technique [30]. A piece of bent fiber is intrinsically lossy. For a propagating mode to maintain the mode pattern in the bent section, the outer evanescent wavefront must travel faster to catch up with the inner one. This is not realizable when the outer evanescent tail is too far from the fiber core. That part of the field is then coupled into radiation modes and the energy is lost [37]. The simplest fiber bend loss model assumes infinite cladding outside the fiber core, which predicts that the attenuation coefficient \( \alpha \) of the bent fiber section decreases exponentially as the bend radius \( R \) increases [37, 119, 120]. In practice, however, as a result of finite fiber cladding and one or multiple coating layers, \( \alpha \) deviates from a simple exponential function of \( R \). More sophisticated models have been developed to account for these effects [121–124]. For a given bend radius \( R \), one can increase the bend loss by extending the length of the bent section. This can be attained experimentally by winding the fiber around a cylindrical mandrel.

The FSI-CRD experimental setup for measuring fiber bend loss is shown in Fig. 5.12(a) [30]. The fiber loop cavity was formed by a pair of 99.5/0.5 fiber directional couplers \( C_1 \) and \( C_2 \). It has a cavity length of \( \sim 48 \) m (Corning® SMF-28™). As bend loss is wavelength dependent [122], a CW tunable laser (Agilent 81642A) was used as the light source. Its output was launched into the 50/50 fiber coupler \( C_0 \) of the frequency-shifted Sagnac interferometer through a circulator. The frequency shifter was an acousto-optic modulator (Brimrose AMM-100-20-25-1550-2FP), and the differential interference signal from the RDC was measured by a 10-MHz balanced
detector (New Focus Model 2117). The output of the balanced detector (BD) was recorded by a data acquisition board (NI PCI-6251) and processed by a computer. Two polarization controllers (PCs) were used to optimize the interference fringe visibility. Note that the optical frequency shift introduced by the frequency shifter is on the order of 100 MHz, corresponding to sub-pm wavelength shift. Therefore, high wavelength resolution can be obtained in the measurement.
As shown in 5.12(b), Fiber bend loss was introduced by winding a section of the fiber loop cavity around a mandrel (an optical post holder of 25-mm outer diameter). The fiber in the RDC that is not on the mandrel is kept straight or at a very large bend radius to minimize additional bend loss. Furthermore, to avoid unintentional bending, the fiber in the RDC was taped down onto an optical table. Two more mandrels were erected about the main mandrel so that the three mandrels are along a straight line. One could tightly wind the fiber on the central mandrel by fixing it on the two side mandrels with scotch tapes [yellow squares in 5.12(b)]. With above setup, the cavity loss can be changed precisely by altering the number of fiber turns wound on the mandrel. The transmittance of the fiber due to bending can be written as

$$\kappa = e^{-\alpha l} = e^{-\alpha \cdot N \cdot 2\pi R}$$

where $N$ is the number of fiber turns on the mandrel. Equation (5.18) suggests that the output intensity of the fiber section decays exponentially as $N$ increases.

During the experiment, the output power of the tunable laser source (TLS) was set to 3.6 mW at 1550 nm. The coherence control mode of the TLS was switched on so that its effective

\[\text{Fig. 5.13. Typical experimental FSI-CRD transients: (a) measured when } N = 0 \text{ (b) measured when } N = 8 \text{; (c) measured when } N = 20.\]
linewidth was more than 50 MHz, providing a coherence length $l_c$ much shorter than the cavity length ($l_c < 6 \text{ m} \ll d$). There was no interference between the lightwaves exiting the RDC after different round trip numbers. The acousto-optic modulator (AOM) was swept from 90 MHz to 110 MHz at steps of 0.02 MHz with a step time interval of 1 ms. The data acquisition board (DAQ) was synchronized with the sweep of the AOM. A LabVIEW program was developed to control various instruments and perform synchronized data acquisition and processing. The cavity loss was measured as the fiber turn number $N$ was increased from 0 to 20 at steps of 2. At each $N$, the PCs were adjusted to optimize the polarization. DFT was performed on the acquired differential interference signal $\Delta I$ to obtain the FSI-CRD transients. A Hann window was applied in the fast Fourier transform (FFT) process with an FFT size of $2^{20}$. One hundred CRD decay transients were recorded, normalized, and averaged. The cavity loss was then deduced by fitting the first 20 peaks in each averaged CRD transient with an exponential function.

A few experimental FSI-CRD transients are shown in Fig. 5.13. Fig. 5.13(a), (b), and (c) were obtained when $N = 0$, 8, and 20, respectively. Clear exponential decays can be seen in these figures. Moreover, a comparison from (a) to (c) verifies that as $N$ increases, the exponential decay becomes more rapid, in agreement with the theory. The peaks at the zero distance in the figures was caused by imperfect cancellation at the BD. Equation (5.16) and (5.18) suggest that the cavity loss in log scale should have a linear relation with $N$, which is indeed the case as illustrated in Fig. 5.14. The fitted slope of the data was found to be 0.0135 dB/turn which is equivalent to attenuation of 0.172 dB/m at $R = 12.5$ mm. The average uncertainty for the
Direct measurement of the fiber bend loss was carried out to compare the results with those from FSI-CRD by using the setup in Fig. 5.15. The same TLS was used to launch 1550-nm light into a piece of SMF-28 fiber. The fiber was wound on the mandrel in the same fashion as before, and the transmitted power was measured by an optical power meter (PM) at different \( N \). The power at each \( N \) was averaged over 50 measured values (PM averaging time was 500 ms). The transmission loss was then calculated with reference to the transmitted power measured when the fiber was kept straight. Fig. 5.16 shows the normalized fiber bend loss in dB as a function \( N \). A linear fit to the data (red dashed line in Fig. 5.15) gives a slope of 0.0170 dB/turn, or 0.217 dB/m at \( R = 12.5 \) mm. However, the linearity of the direct loss measurement data was rather poor compared with that in Fig. 5.14. Another linear fit was performed in the more linear region (green line in Fig. 5.16), with the data points for \( N = 0, 2, 4 \) removed. The slope of the data in this region was 0.0141 dB/turn, or 0.180 dB/m, which is in good agreement with the FSI-CRD results.
with the FSI-CRD measurement result. As no polarization adjustment was performed in the
measurement, the PM relative uncertainty due to polarization was used as the error bars in
Fig. 5.16. The poor data linearity in Fig. 5.16 could be contributed by the polarization effects
and long-term laser power drift.

Fiber bend loss measurement using FSI-CRD successfully demonstrated that FSI-CRD is
indeed a viable alternative to the time-domain CRD techniques. With a CW light source
and a slow detector, light decay rate in an optical cavity can be measured without the need
of any optical pulses. Like many interferometric fiber-optic sensing systems, an FSI-CRD
system is also sensitive to polarization effects. Input light polarization state may evolve as
it travels through the RDC for multiple times, resulting in non-optimal interference fringe
visibility. In an FSI-CRD transient, this effect appears as periodic amplitude modulation on
the exponentially decaying Fourier peaks. However, once the polarization is optimized and RDC
is carefully packaged, this effect can be minimized (see Chapter 6). The cavity length that can
be employed in an FSI-CRD system is determined by the frequency shifter sweep parameters
and the coherence length of the light source. In above experiment, with \( f_{\text{step}} = 0.02 \text{ MHz} \) and
\( \Delta f = 20 \text{ MHz} \), a spatial resolution of \( \sim 10 \text{ m} \) and a spatial sensing range of over 5 km were
achieved. A cavity length of \( \sim 48 \text{ m} \) is much longer than the source coherence length, and it
leads to identifiable individual peaks in the FFT spectrum (see Fig. 5.13). In principle (if the
decay is not too fast), all the Fourier peaks before 5 km can be used for the exponential fit
since there is no aliasing. The next section presents the experimental results of an FSI-CRD
evanescent-field sensing system in which a fiber taper was incorporated as the sensor head.

5.4 FSI-CRD evanescent-field sensing

Although the success of fiber bend loss measurement proves that FSI-CRD is capable of de-
tecting loss change inside an RDC, no real sensor head was used in this experiment [30]. To
compare the performance of FSI-CRD technique with conventional fiber CRD techniques, a
fiber taper was inserted into the RDC as the sensor head to measure evanescent-field chemical
absorption [95,98] and refractive index change [97,99].

5.4.1 Fabrication of fiber tapers

Fiber tapers have found their wide applications in areas that require evanescent fields, strong
light confinement, and mode conversion or filtering [125,126]. Examples include mode coupling
[127–131], fiber-optic sensing [95,99,132,133], particle manipulations [134,135], and nonlinear
optics [136,137]. A biconical fiber taper is shown in Fig. 5.17. It consists of a small-diameter
uniform waist region between two conical transition regions. Due to the high refractive index
contrast between air and glass at the thin waist region, the taper waist supports multiple
propagation modes. A portion of the optical power extends beyond the physical boundary
of the taper waist and reach the surrounding medium as evanescent fields. By controlling
the profile of the transition region, mode conversion can be achieved with these tapers. High nonlinearity can also be attained with a taper at an appropriate size [126]. Fiber wires with diameters smaller than 1 \( \mu \text{m} \) is usually referred to as optical fiber nanowires (OFNs), while wires with diameters larger than 1 \( \mu \text{m} \) is often called optical fiber microwires (OFMs).

Fiber tapers are most frequently fabricated by heating and pulling a section of fiber [125,126]. To date, the flame-brushing technique and its variants [126] have been popular choices for producing high-quality fiber tapers that are thin, long, uniform, or low loss. In this technique, a small flame (e.g., sustained by burning a mixture of oxygen and butane) is scanned, or “brushed”, along a fiber while it is being pulled (see Fig. 5.18). The taper profile can be

Fig. 5.17. Structure of a biconical fiber taper.

Fig. 5.18. Fiber taper fabrication by the flame-brushing technique. A section of fiber is heated by a sweeping flame, and the taper is formed as the fiber is stretched.
Fig. 5.19. Fiber taper fabrication by HF etching. A piece of bare fiber is immersed in a HF solution drop on a Plexiglas plate. The taper waist region is formed in the fully immersed section of the fiber, while the transition regions are formed at the menisci.

precisely controlled by fine tuning the flame size, motion and the fiber pulling speed. The fiber tapers fabricated by heating and pulling have a constant ratio of local cladding radius $\rho_{cl}(z)$ to local core radius $\rho(z)$, where $z$ is the distance along the fiber taper [138].

Fiber tapers can also be made by chemical etching [99,139,140]. Hydrofluoric acid (HF) is frequently used as the etchant. Silicon tetrafluoride ($\text{SiF}_4$), hexafluorosilicic acid ($\text{H}_2\text{SiF}_6$), and water ($\text{H}_2\text{O}$) are produced in the chemical reactions between HF and silica ($\text{SiO}_2$):

\begin{align}
\text{SiO}_2 + 4\text{HF} & \rightarrow \text{SiF}_4 + 2\text{H}_2\text{O} \quad (5.19a) \\
\text{SiO}_2 + 6\text{HF} & \rightarrow \text{H}_2\text{SiF}_6 + 2\text{H}_2\text{O} \quad (5.19b)
\end{align}

where the second equation dominates at high HF concentrations [139]. When a section of bare fiber (with its protection coating layers removed) is immersed in a drop of HF solution (Fig. 5.19), the acid attacks the glass (fused silica) from all directions at the same etch rate, as glass is an amorphous material. Therefore, a uniform cylindrical thin waist region can be obtained in the fully immersed fiber section. The transition regions of the taper are formed at the exposed fiber sections near the menisci between the HF solution drop and the air. Due to the Marangoni effect, the solution at the meniscus is driven by the liquid surface tension and migrates along the fiber [140,141]. The HF concentration reduces as the solution moves further away from the meniscus, and therefore, the etch rate of the solution decreases accordingly, creating a tapered fiber profile in the etching process. Although HF is a weak acid, it is toxic and highly corrosive. One must handle HF solutions under a fume hood with appropriate personal protective equipment (splash-proof chemical goggles, plastic face shield, double-layered gloves, lab coat, chemical-resistant apron, and close-toed shoes) and with extreme care. Because of the availability of the tools in the lab for HF etching, this method was chosen to fabricate the tapers used in the FSI-CRD evanescent-field sensing experiments.
The setup for fiber taper etching is shown in Fig. 5.20. To fabricate one taper, the protection coating layers of a 6-cm long section from a piece of 1.5-m standard SMF-28 fiber is mechanically stripped and then cleaned with isopropyl alcohol before the fiber is fixed by magnets at both ends on two XY translation stages. The two stages are \( \sim 12 \) cm apart and are aligned along a straight line as illustrated in Fig. 5.20. They were fixed on a long rectangular optical breadboard. The stripped fiber section is kept in the middle between the two stages. The fiber is straightened and its tension can be adjusted by tuning the displacement of the two translation stages. A third XYZ translation stage is used to support a piece of Plexiglas which serves as the holder of the HF solution.

Before the start of etching, the XYZ translation stage is adjusted so that the Plexiglas plate is in a horizontal position, about 1 mm below the stripped fiber section (Fig. 5.21). With the help of a plastic pipette, HF solution drops are applied to the bare fiber section from the middle then towards both ends so that a symmetric large HF solution drop is formed along the fiber, leaving a section of 5-mm exposed stripped fiber on each side. The resultant fiber taper length is thus \( \sim 5 \) cm. This length ensures that the taper can be fully immersed in chemical solutions on a standard microscope slides (76 × 26 mm).
Diameter of the etched tapers

The waist diameter of a fiber taper fabricated by chemical etching is a function of the etch rate and the etch time. The etch rate depends on parameters such as etchant type, etchant concentration, target material, impurities, temperature, and so on [142–144]. Our lab is not equipped with precise temperature control. All the HF etching was carried out at room temperature (∼20°C) under a fume hood. We can control the HF concentration and the etch time. Scanning electron microscopy (SEM) was used to investigate the HF etch rate on the fiber glass at several etchant concentrations. At a given HF concentration, the etched fiber diameters were measured as a function of etch time by SEM, from which the etch rate can be deduced. For a specific etch time, 5 pieces of SMF-28 fibers were stripped, cleaned, and each was immersed in a total of ∼1 ml HF solution (20 drops from the pipette) as described previously. When the desired etch time was reached, the HF solutions were removed by using paper towels and the samples were flushed with distilled water. The same procedure was repeated for a range of etch times between 5 and 45 min. Fig. 5.22 presents the waist region SEM images of the fiber samples etched by 48% HF. As the same magnification was used from (a) to (e), these figures clearly illustrate the reduction of fiber diameter as the etching progress proceeds. Fig. 5.23(a)-(c) show the measured fiber diameter versus etch time for HF solutions at 48%, 20%, and 10%, respectively. Fig. 5.23(d) compares the etching results for the three HF concentrations, where the blue curve is the 48% HF data, the red curve is the 20% HF data, and the green curve is the 10% HF data.

There are several observations from Fig. 5.23. First, the fiber diameter decreases linearly as the etch time increases. The slopes were found to be −2.11 μm/min, −0.45 μm/min, and −0.14 μm/min in Fig. 5.23(a)-(c), respectively. They correspond to etch rates of 1.1 μm/min for 48% HF, 0.23 μm/min for 20% HF, and 0.07 μm/min 10% HF. Note that the fiber diameter reduction rate is twice as fast as the etch rate, since glass is etched from all directions. The
Fig. 5.22. SEM images of the etched fiber taper samples at the waist regions prepared by using 48% HF for various etch times. (a) 5 min; (b) 15 min; (c) 25 min; (d) 35 min; (e) 45 min.
intercepts of the three fitted lines with the y-axis were 118.8 µm for 48% HF, 128.5 µm for 20% HF, and 127.9 µm for 10% HF. As the intercept of the y-axis represents the initial diameter of a fiber, this number should be the same for all three linear fits. Another linear fit was performed on the first 3 points of 48% HF data in Fig. 5.23(d), and this fit provides a y-axis intersection that is 125 µm, closer to the intercepts of the other two concentrations. The slope of the new fit is $-2.58 \mu m/\text{min}$, corresponding to an etch rate of 1.29 µm/min. It can be seen from Fig. 5.23(d) that the 48% HF data points rise above the fitted at longer etch times, which suggests that the etch rate at this concentration decreases for long etch times with the current etching method. This might be due to the relatively rapid depletion of the acid at a high chemical reaction speed and evaporation. However, we may assume from the excellent linearity of the data points below 20 min that with our current etching setup, the etch rate of 48% HF is constant for an etch time below 20 min. Similarly, within an etch time of 45 min, the etch rates remain constants for 20% and 10% HF solutions. Another observation is that the etched fiber diameter can be controlled with higher accuracy if a lower-concentration HF solution used. This can clearly be seen from the smaller error bars (standard deviations of the etched fiber diameters) in the fiber diameter measurement for lower HF concentrations. Finally, note the
diameter of a SMF-28 fiber is $125.0 \pm 0.7 \mu m$ [14], which implies there is a percentage error in the diameter reading from the SEM image.

**Loss of the etched tapers**

The loss of the etched fiber section starts to increase when the diameter of the taper becomes small such that the evanescent field extends out of the glass and is disturbed by the surrounding medium. The loss of the taper can be monitored by measuring its transmission in real time using a tunable laser and an optical power meter. Alternatively, it can be monitored with the FSI-CRD technique, if the fiber is spliced into the RDC before etching. Fig. 5.24 shows the experimental relationships between fiber taper transmission (normalized to initial transmitted power) and the etch time at two HF concentrations. They were measured by using a tunable laser and a power meter, and the transmitted power of the etched sample was recorded every 10 s. The blue curve was obtained from a fiber sample etched by 27% HF solution. It can be seen that the transmission of the sample started to decrease at around 8500 s, followed by a drastic stall till about 10800 s, after which the transmission essentially became zero. The red curve is the transmission of a sample etched by 20% HF. It exhibits similar characteristics except that its drop in transmission did not occur till after 12000 s. It took longer for this sample to reach a diameter at which the evanescent filed is perturbed, since a diluted HF solution has a slower etch rate.

Fig. 5.24. Normalized fiber taper transmission loss as a function of etch time measured for two samples. One sample was etched by 27% HF solution (blue curve), while the other was etched by 20% HF solution (red curve).
Fig. 5.25. Surface roughness of fiber samples etched by 48% HF solution for different etch time. (a) 5 min; (b) 25 min; (c) 45 min.
Fig. 5.26. A comparison of surface roughness between fiber samples etched by different HF concentrations after an etch time of 45 min. (a) 10% HF; (b) 20% HF; (c) 48% HF.
Fig. 5.27. SEM images of the etched fiber taper. (a) taper transition region; (b) detailed look of the taper waist region.
Surface roughness is another well-known source of loss for optical waveguides [145]. Rough waveguide boundaries introduce additional scattering loss to the propagation mode. It was found that at a fixed HF concentration, the smoothness of the taper surface deteriorates as the etch time increases. Fig. 5.25 shows the SEM images of sample surfaces prepared by 48% HF under different etch time. Comparing Fig. 5.25(a) to (c), one can see that small holes start to form and become more dense as the etching proceeds. Given the same etch time, the surface smoothness is poorer for tapers prepared with higher HF concentrations, as shown in Fig. 5.26. These samples have different waist diameters, and they were etched for 45 min by 10%, 20%, and 48% HF in Fig. 5.26(a), (b), and (c), respectively. At the same diameter, tapers fabricated by high-concentration HF solutions tend to have higher loss due to rougher surfaces.

To obtain a thin and low-loss fiber taper within a short fabrication time, we can immerse a fiber section in high-concentration HF solution first to trim most of the fiber cladding, followed by finer etching in low-concentration HF solutions to ensure better controlled waist diameter and surface roughness. The tapers used in the experiments were sequentially etched by 48%, 21%, 11%, and 5% HF solutions. SEM images of a taper used in the experiments is shown in Fig. 5.27. A section of the fiber taper transition region is shown in Fig. 5.27(a) which illustrates a gradual and smooth transition. The taper waist section can be found in Fig. 5.27(b). We can see that this region shows excellent uniformity.

5.4.2 Evanescent-field sensing experimental setup

A schematic of the entire setup is presented in Fig. 5.28. It is similar to the one used for fiber bend loss measurements [Fig. 5.12(a)]. Here, an amplified spontaneous emission (ASE) source (AFC BBS 1550A-TS) was used as the light source. There are several motivations for this. First, an ASE source is cheaper than a tunable laser source. Second, its output has random polarization, and as a result, the interference signal is more stable. Moreover, it can demonstrate that in FSI-CRD, there is indeed no need for modulation on the light source. The output power of the ASE source was reduced from $\sim 20$ mW to $\sim 6.8$ mW by a variable optical attenuator (VOA). The attenuated sensing light was coupled into the frequency-shifted Sagnac interferometer incorporating the RDC. The fiber taper was located in the RDC as shown in the figure. The RDC was formed by a pair of 99.5/0.5 fiber directional couplers.

As the cavity loss is to be measured as a function of chemical solution concentration in the FSI-CRD evanescent-field sensing experiments, there are two requirements on the setup. First, the cavity loss change induced by undesirable disturbances must be kept minimal. Second, to successfully carry out the measurements on chemical solutions, one must be able to control the solution concentrations accurately. To meet above two requirements, the setup in Fig. 5.29 was adopted. Two XY translation stages were aligned and screwed onto an optical breadboard (not shown in Fig. 5.29). The taper was fixed at both ends on the translation stages by magnets (brown blocks in Fig. 5.29). A small amount of tension was exerted on the taper so that it was kept straight. It was found that changing the tension on a taper could alter its transmission loss.
Therefore, the two XY translation stages were not adjusted during the solution measurement. The RDC is carefully packaged into a paper box to minimize the undesirable disturbance. The remaining free fiber segments were also taped down, as any unexpected change of fiber bend could cause cavity loss change. A microscope slide attached to a XYZ translation stage was used as the solution holder. The orientation of the glass slide was adjusted so that it was horizontal and was parallel to the fiber taper. To minimize the probable perturbation to the fiber taper, instead of changing the sample, the test solution concentration was changed by first immersing the taper in the solvent (or low-concentration solution), followed by additions of high-concentration solutions. The solvent (or low-concentration solution) was applied to the taper with a 2-ml glass pipette. Another 1-ml pipette equipped with a pipette filler was employed to control the added solution volume at 0.1-ml precision. As in the fiber bend loss measurements, the AOM was swept from 90 to 110 MHz at steps of 0.02 MHz with a step time interval of 1 ms, which renders the system a spatial resolution and a spatial sensing range of \( \sim 10 \text{ m} \) and \( > 5 \text{ km} \), respectively. The polarization controllers were first adjusted before the measurement and left unchanged throughout the experiments.

### 5.4.3 FSI-CRD measurement of 1-octyne absorption

As our ASE source and other optical components are designed for the C-band (1535–1565 nm), to use them for evanescent-field absorption measurement, we selected 1-octyne dissolved
Fig. 5.29. Details of the setup for chemical solution addition. The fiber taper is held under tension between two XY translation stages. The amount of solution added to the taper can be accurately controlled by a pipette and a pipette filler. A microscope slide attached to a XYZ translation stage is used to hold the solution.

in decane as the test samples, since 1-octyne has an overtone absorption peak centered at 1532.5 nm [95] and decane (solvent) does not interact with the 1-octyne (solute). Another advantage of this test solution is that the RI of the two chemicals are very close to each other \((n_{\text{decane}} = 1.412 \text{ and } n_{\text{octyne}} = 1.417)\) [98]. The RI-induced cavity loss change is thus minimized. It was estimated that the maximum RI change for 1-octyne concentration variations between 0 to 5\% (or 0.34 M) is on the order of \(10^{-4}\) refractive index unit (RIU). Finally, CRD-based 1-octyne solution measurements have been reported in the literature [95,98], and we can compare the performance of FSI-CRD with conventional fiber-based CRD schemes.

To carry out the experiments, the setup in Fig. 5.29 was placed under a fume hood. A quantity of 10\% (percentage by volume, that is, volume of solute over volume of solution), or equivalently 0.679 M 1-octyne solution was prepared by dissolving 1-octyne (Sigma-Aldrich) in decane (Fisher Scientific). The solution was shaken by hand for over 20 min in order to
guarantee that 1-octyne was fully dissolved. The microscope slide was aligned carefully under the taper section. To measure the cavity loss in real time, the FSI-CRD system was set to record one cavity loss value every 5 s. At around 60 s after the start of the data acquisition, 1-ml decane was added to the microscope slide to fully immerse the taper by using the 2-ml pipette. Then, the 1-ml pipette was used to add 10% 1-octyne solution to the slide. At 150, 250, 350, 450 s, 0.1-ml 10% 1-octyne solutions were added, whereas at 560, 670, and 780s, 0.2-ml 10% solutions were added. The resulting 1-octyne concentrations range from 0 to 5%. A typical FSI-CRD transient measured at 5% 1-octyne concentration is shown in Fig. 5.30. The first 30 peaks in such a transient were fitted with an exponential to deduce the cavity loss. The particular FSI-CRD transient in Fig. 5.30 indicates a cavity loss of 0.9059 dB. When the taper was in the air, the cavity loss was 0.6449 dB. The last 10 cavity loss values before each solution addition were averaged and used as the cavity loss at the previous solution concentration. The standard deviation of them was also computed. The solution on the microscope slide was not stirred during the measurement. Time was given to the solution to homogenize before an amount of new solution was added.

The cavity loss as a function of time measured by FSI-CRD is presented in Fig. 5.31. FSI-CRD is sufficiently fast to capture the mixing process of the low- and high-concentration solutions. As soon as 10% 1-octyne solution was added, the cavity loss increased and showed a clear step. As the high-concentration solution diffused and became diluted, the cavity loss exhibited an asymptotic transient before stabilization. Excellent linearity between the cavity
loss (in dB) and 1-octyne concentration was obtained as shown in Fig. 5.32. The slope of the fitted line suggests that the measurement sensitivity is 0.0094 dB/% octyne. The standard deviations of the measured cavity loss at various 1-octyne concentrations range from $2 \times 10^{-4}$ to $6 \times 10^{-4}$ dB, and the latter was used as the size of the error bars in Fig. 5.32. The minimum detectable 1-octyne concentration implied by the noise level at $6 \times 10^{-4}$ dB is on the order of 0.06% (or 0.004 M).
To estimate the system detection limit conforming to the standard recommended by the International Union of Pure and Applied Chemistry (IUPAC) [146, 147], low-concentration 1-octyne solution sample at 0.15% (0.010 M), 2.5 times the detection limit level estimated above, were prepared and measured by FSI-CRD. Before each measurement, the old sample is removed by flushing the taper gently with the prepared solution. The standard deviation of the cavity loss was measured and was found to be $9 \times 10^{-4}$ dB, calculated over the results over 11 samples. Given the sensitivity of 0.0094 dB/% octyne deduced above, this corresponds to a detection limit of 0.29% (0.020 M) 1-octyne, calculated as 3 times the standard deviation divided by the sensitivity. Note that the cavity loss standard deviation measured this way is larger, which is probably caused by the disturbances to the taper during the process of changing solution samples. The detection limit of 0.29% 1-octyne is the lowest (to the best of our knowledge) among fiber-based CRD methods (1.05% in [95] and 0.62% [98]). Furthermore, neither [95] nor [98] followed the IUPAC standard.

The effect of ASE source bandwidth

In CRD experiments, non-exponential light decay may result if the sensing light has a wider bandwidth than the absorption feature (e.g., the absorption line width). It has been shown, however, that if the sample absorbance is much smaller than unity after multiple trips, the light decay deviation from exponential is negligible [88]. In our experiments, although the bandwidth of the ASE source exceeds the octyne absorption linewidth, the maximum single-pass loss at 5% 1-octyne contributed by the absorption is only 0.0478 dB (see Fig. 5.32), which indicates an effective attenuance of 0.011 (the term attenuance is used to reflect the fact that only the evanescent part of the guided light interacts with the absorbing medium [148]). Even for 30 round trips, the attenuance is much smaller than 1, implying a good single exponential decay in our experiment [88].

Numerical simulations were also performed to explore the effects of broad source bandwidth on the FSI-CRD transients. The width of the 1-octyne absorption band is around 20 nm [95]. In the simulations, a Lorentzian absorption line centered at 1532 nm with a width of 20 nm was assumed, and a broadband sensing light that resembles the spectrum of our ASE source was constructed [blue curve in Fig. 5.33(a)]. The effective transmission in the cavity was assumed to be 80% ($\sim 0.85$ dB). The sensing light intensity between 1510 and 1580 nm attenuated by the absorption were summed for different round trip numbers. The effective attenuance (the attenuance that includes the attenuation at all wavelengths) were deduced by fitting each calculated intensity sum by an exponential function. The simulations show that even when the peak attenuance of the absorption line is 0.234, which gives an effective attenuance of 0.106 (almost 10 times larger than the effective attenuance in our experiments), the CRD transient [blue in Fig. 5.33(b)] can still be described excellently by a single exponential decay (red in the same figure). The red curve in Fig. 5.33(a) is the single-pass sample spectrum. In Fig. 5.33(b), the calculated total light intensity change is nicely matched with a single exponential decay.
Fig. 5.33. The effect of broadband sensing light on the FSI-CRD transient. The absorption line was assumed to be a Lorentzian with 20 nm line width centered at 1532 nm, which has a peak attenuance of 0.234. (a) simulated source spectrum (blue) with single-pass sample spectrum (red); (b) the calculated total light intensity decay (blue) with fitted single exponential function (red). Note the dip at 1540 nm in the simulated source spectrum in (a) is artificially created to mimic the actual ASE source spectrum. The fitted single exponential function matches almost perfectly with the calculated intensity decay even though the effective attenuance is as large as 0.106.

Therefore, we may conclude that in our experiments, in spite of the broad source bandwidth, the FSI-CRD transients are still single exponentials.
Another issue related to the source bandwidth is the fiber dispersion. As discussed in Section 2.4.3, this effect is very small in an FSI system with a source bandwidth of 35 nm and a total (double-pass) fiber length of 2000 m. As in the 1-octyne measurements, 30 FSI-CRD transient peaks were fitted, the maximum distance light travels is below 2000 m. The numerical example in Section 2.4.3 applies. Thus, the dispersion effect in our experiments is negligible.

The effect of light source intensity noise

One advantage of conventional CRD scheme is that the measurement is independent of pulse-to-pulse intensity fluctuations. In FSI-CRD, the source intensity noise within the measurement time may add to the acquired interference signal. To see its effects on the loss measurement, one should take the intensity noise power spectrum into account. During data processing, a Fourier transform is performed on the interference signal so that the noise power is spread over a range of frequencies in the Fourier domain \( F \) (or equivalently, in the spatial domain \( L \)). Given a total noise power level, if the noise is broadband, its influence on the FSI-CRD becomes small.

To experimentally test the stability of the ASE source power, the output of the ASE source was attenuated by two VOAs and was connected to one of the two photodiodes (PD) on BD. The other unused PD was blocked completely. The attenuation of the VOAs was adjusted so that the output voltage of the BD was at \( \sim 3.96 \) V, below the BD saturation voltage (\( \sim 8.5 \) V). The same data acquisition system was used to record the BD output voltage for 1 s at a sampling rate of 100 kS/s (all experimental settings). Fig. 5.34(a) shows the measured PD voltage as a function of time. The average voltage is 3.956 V with a standard deviation of 0.021 V. The average voltage and the voltage levels one standard deviation away from the average are marked by a solid red line and dotted red lines, respectively. In Fig. 5.34(b), the Fourier spectrum of the measured voltage was calculated by using the same parameters as in the experiment (with a Hann window and an FFT size of 2\(^{20}\)). The DC removed noise spectrum is shown in Fig. 5.34(c). Comparing Fig. 5.34(b) and (c), one can see the amplitude of the DC component in the spatially-resolved Fourier spectrum is \( \sim 10^5 \), more than 51 dB higher than the noise level (below 1). For 1-octyne measurement presented above, the first peak of the experimental FSI-CRD transient in Fig. 5.30 is 33 dB above the noise level. The effects of ASE source intensity noise on the FSI-CRD transients are negligible.

As a comparison, the same test was carried out with the TLS used in FSI-CRD fiber bend loss measurements. The TLS was operated under the coherence control mode. The output of the BD is shown in Fig. 5.35(a). Compared with the ASE source output, the fluctuation of the TLS power is much larger. The standard deviation of the recorded power is 0.366 V. The Fourier spectrum of the laser output power exhibits a few lines equally spaced at \( \sim 194.5 \) m, 388.5 m, 582.5 m, 776.5 m, and 971 m. These lines are contributed by the internal modulation of the laser, as they disappear if the coherence control is switched off. The strongest peak near 194.5 m is 25 dB below the DC component while the rest of the noise floor is 35 dB below [see Fig. 5.35(b) and (c)]. In the fiber bend loss measurements, the initial peak in the FSI-CRD
Fig. 5.34. ASE source intensity noise and its effect on the FSI-CRD transient. (a) ASE source noise measured by a photodiode; (b) DC component in the Fourier spectrum; (c) detailed look of the ASE source intensity noise level in the Fourier spectrum with the DC component removed. In (a), the solid red line is the average of the photodiode voltage, and the dotted red lines mark one standard deviation from the average.
Fig. 5.35. TLS source intensity noise and its effect on the FSI-CRD transient. (a) TLS noise measured by a photodiode; (b) DC component in the Fourier spectrum; (c) detailed look of the TLS intensity noise level in the Fourier spectrum with the DC component removed. In (a), the solid red line is the average of the photodiode voltage, and the dotted red lines mark one standard deviation from the average.
transient is more than 23 dB above this noise floor (see Fig. 5.13). Furthermore, the locations
of the Fourier peaks introduced by the TLS coherence control do not coincide with the FSI-
CRD transient peaks, and their amplitudes decrease rapidly. Thus, the contribution of the TLS
power fluctuation to the FSI-CRD transient is also negligible.

Note that the FSI-CRD technique is immune to long-term source power drift, as the loss
information is deduced on an absolute scale within its short measurement time. Therefore, in
our FSI-CRD experiments, the loss measurement is immune to long term source fluctuation,
and the short-term fluctuation within the measurement time is negligible.

5.4.4 FSI-CRD measurement of sodium chloride solution and glucose solution concentrations

Similar experimental procedures were followed to examine another taper’s response to sodium
chloride (NaCl) solutions and to glucose solutions. In both cases, water was used as the solvent.
The sensor’s response to NaCl solution concentration changes is mainly due to the ambient
refractive index change [99], whereas the response to glucose solution concentration changes is
a combination of RI change [149] and absorption [150].

Two NaCl solutions at 5% and 20% were prepared. The taper was first immersed in 5% NaCl
solution, and then 20% solutions were added to the taper by following the same procedure used
in 1-octyne measurements. Cavity loss was recorded as a function of time when the solution
concentration was changed. Similar step-like cavity loss increases were observed. Fig. 5.36(a)
shows the cavity loss as a function of NaCl concentration. The standard deviations of the
cavity loss for stabilized solutions were averaged, and its value was found to be $1 \times 10^{-4}$
dB. The concentration of NaCl solution can be converted into refractive index which is also
labeled on the axis [99]. Note that the cavity loss has a nonlinear relation with RI over a
wide index range. Such nonlinear behavior, contributed by the nonlinear dependence of the
mode characteristics on the surrounding medium RI, is common to evanescent-field RI sensing
techniques [99, 151, 152]. Sensor calibration is required to gauge the RI from the measured
cavity loss. For small NaCl concentrations less than 13% (NaCl weight divided by the solution
weight), a linear fit can be performed and the sensitivity is $1 \times 10^{-4}$ dB per $1 \times 10^{-4}$ RIU in
this region. The sensitivity is lower at higher NaCl concentrations. As in the NaCl solution
concentration measurements, 5% and 20% glucose solutions were prepared and tested by FSI-
CRD [Fig. 5.36(b)]. In the wavelength region provided by the ASE source, glucose absorbs [150],
and thus the cavity loss is influenced by both absorption and RI. The cavity loss change due to
glucose concentration variation can be also be measured to a high accuracy by the FSI-CRD
technique. The data in Fig. 5.36 were fitted by quadratic curves (shown in blue) which can
serve as the calibration curves for the measurement of these two solutions.
5.4.5 Further discussions on the FSI-CRD experiments

All above experiments were conducted at room temperatures. Distilled water was used to clean the tapers after the experiments. The cavity loss was found to rise, when measured in the air, after the taper was moved out of the liquid each time (on the order of 0.01 dB if the liquid is water). It is probably due to the taper wear and tear induced by the liquid surface tension. Such mechanical degradation has been well-known to the industry [126]. Special treatment is required to achieve long-term use of the tapers. Polymers such as Teflon are often employed for
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Fig. 5.37. Cavity loss change during the taper cleaning process. (a) the entire cleaning process; (b) the details of cavity loss as the taper was in and out of water.

the purpose [126, 151]. Fig. 5.37 shows the cavity loss change during the cleaning process of a taper after it was taken out of NaCl solution. Fig. 5.37(a) illustrates the entire cleaning process while (b) presents the details as the taper was moved into and out of water. Between 0 and around 100 s, the taper was immersed in NaCl solution. At 100 s, the taper was removed from the solution, and because of the residual solution on the taper, the loss increased significantly (to more than 2 dB). After the solution holder was cleaned, the taper was rinsed in water again at around 175 s which reduced the cavity loss back to around 0.6 dB. Since the solution was drastically diluted, the taper was clean when it was raised into the air again at about 220 s. The
cavity loss is lower when the taper was in the air. As can be seen from Fig. 5.37(b), the cavity loss when the taper was in the air increased slightly every time after the taper was cleaned in water. The FSI-CRD system also demonstrated good long-term stability. The standard deviation of the cavity loss was $1 \times 10^{-4}$ dB over a period of 20 min when a taper was kept in the air (see Fig. 5.38).

5.5 Summary

FSI-CRD is a novel fiber-based cavity ring-down technique. It measures the interference signal of CW light traveling in a fiber ring-down cavity, and the cavity loss information is deduced from the Fourier transform of the interference signal. Unlike conventional CRD techniques, FSI-CRD does not require the generation of optical pulses in the RDC. It also avoids fast detection units which are necessary for the measurement of optical pulse intensity decay in conventional schemes. An FSI-CRD system only needs a CW light source (such as a broadband source), an acousto-optic modulator, and a slow detector, which can potentially reduce the cost of a sensing system. As a proof-of-concept experiment, this technique was first applied to fiber bend loss measurements [30]. With a fiber taper incorporated into the RDC as the sensing element, the FSI-CRD technique was employed to measure chemical solution concentration based on absorption [31]. A detection limit of 0.29% 1-octyne dissolved in decane was attained, which is the best result so far for fiber-based CRD systems [95, 98]. The same setup was utilized for refractive index sensing [31] as well. RI sensitivity of 1 dB/RIU was obtained with...
a measurement error of $1 \times 10^{-10}$ dB. Above results can be further improved by using a refined sensing element (e.g., a thinner or longer fiber taper). The successful evanescent-field sensing demonstrations suggest that FSI-CRD is indeed a viable alternative to conventional fiber-based CRD techniques. It promises a cost-effective high-sensitivity tool for chemical analysis and biomedical applications.
Chapter 6

Modeling of Polarization Effects in Frequency-Shifted Interferometric Systems

The performance of many interferometric fiber-optic sensing systems is sensitive to polarization effects [6, 8, 153–156], and systems based on frequency-shifted interferometry (FSI) are no exception. This chapter first shows a few examples that illustrate the polarization effects observed experimentally in FSI systems, and then it introduces the Jones calculus as a useful tool for modeling polarization effects. Finally, it presents a preliminary model which can already explain some of the experimental observations. It can serve as a foundation for further theoretical work on the polarization evolutions in FSI systems, which may potentially expand the applications of our technique.

6.1 Observed polarization effects in FSI systems

The performances of many fiber-optic interferometric sensing systems are strongly influenced by the interfering light coherence and polarization [153, 154]. Consider the superposition $E_{\text{tot}}(r, t)$ of two vector electric fields $E_1(r, t)$ and $E_2(r, t)$

$$E_{\text{tot}} = E_1 + E_2$$  \hspace{1cm} (6.1)

where

$$E_1(r, t) = E_{01} \cdot \exp[i(\omega t - k_1 \cdot r + \delta_1)]$$  \hspace{1cm} (6.2a)

$$E_2(r, t) = E_{02} \cdot \exp[i(\omega t - k_2 \cdot r + \delta_2)]$$  \hspace{1cm} (6.2b)
The intensity of the $\mathbf{E}_{\text{tot}}(\mathbf{r}, t)$ is [157]

$$I = \langle \langle |\mathbf{E}_{\text{tot}}|^2 \rangle \rangle$$

$$= \langle \langle |\mathbf{E}_1|^2 \rangle \rangle + \langle \langle |\mathbf{E}_2|^2 \rangle \rangle + \langle \langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle \rangle + \langle \langle \mathbf{E}_2^* \cdot \mathbf{E}_1 \rangle \rangle$$

$$= I_1 + I_2 + I_{12}$$

(6.3)

where $\langle \langle ... \rangle \rangle$ represents time average, $I_1 = \langle \langle |\mathbf{E}_1|^2 \rangle \rangle$, $I_2 = \langle \langle |\mathbf{E}_2|^2 \rangle \rangle$, and $I_{12} = \langle \langle \mathbf{E}_1^* \cdot \mathbf{E}_2 \rangle \rangle + \langle \langle \mathbf{E}_2^* \cdot \mathbf{E}_1 \rangle \rangle$. Note $I_1$ and $I_2$ are the intensities of $\mathbf{E}_1$ and $\mathbf{E}_2$, respectively, and the interference term $I_{12}$, which causes the total intensity to deviate from the sum of the component intensities, can be written as [37]

$$I_{12} = 2 \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos(\mathbf{K} \cdot \mathbf{r} - \delta)$$

(6.4)

where $\mathbf{K} = \mathbf{k}_1 - \mathbf{k}_2$ and $\delta = \delta_1 - \delta_2$. As can be seen from (6.4), the interference fringe visibility $V = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}})$, where $I_{\text{max}}$ and $I_{\text{min}}$ are the maximum and minimum attainable

![Fig. 6.1. The effects of polarization on FSI-CRD transients (experimental data). (a) an FSI-CRD transient under optimized cavity polarization; (b) an FSI-CRD transient under non-optimal cavity polarization; (c) FSI-CRD transient under another non-optimal cavity polarization.](image)
total intensities, respectively, is dependent on the degree of mutual coherence (constant $\delta$ if mutually coherent) as well as on the polarization of the interfering fields. Polarization fading occurs when the polarization states of the interfering fields are misaligned, or in the worst case, become orthogonal [153].

The phenomenon of polarization fading can be observed in FSI systems. In our experiments, we use standard single-mode fibers (SMFs) which may possess a small degree of birefringence due to imperfection during the manufacturing process, or due to fiber bending and twisting. With the linear frequency-shifted Sagnac interferometer configuration, when polarization fading is present for the signal from a sensor, the measured sensor reflection spectrum amplitude is reduced. The polarization effect is more prominent in an FSI-CRD system as light travels multiple times through the same fiber cavity. Fig. 6.1 compares the experimental FSI-CRD transient under optimal condition with non-optimal ones. The interference signal in FSI-CRD is contributed by the light that exits a fiber ring-down cavity (RDC) after multiple trips. If the RDC alters the light polarization states in a way such that the two states remain aligned at the time when the lightwaves interfere, a fine exponential decay can be observed in the transient, from which the cavity loss can be deduced [Fig. 6.1(a)]. On the other hand, amplitude modulation on the FSI-CRD transients arises if the light polarization states are misaligned [Fig. 6.1(b) and (c)]. In the latter case, complex yet stable transients are produced, and cavity loss may be hard to extract.

The investigation of polarization effects in FSI systems serves two purposes. First, it may provide useful information on how to improve the performance of FSI systems. Specifically, I am going to show that in the case of FSI-based fiber-optic sensor multiplexing (linear Sagnac interferometer configuration), the system can be made polarization independent. I shall also show that the output polarization states of an FSI-CRD system can always be optimized. Another benefit from the study of polarization effects is the potential applications of such polarization effects. In an FSI-CRD system, for example, light travels multiple trips through the RDC, and therefore, the modification made to light polarization states by the RDC is amplified. It may be possible to extract the polarization properties of the RDC from the measured FSI-CRD transient. Novel techniques could be invented based on this principle to characterize optical devices. To describe the polarization effects theoretically, we may turn to the tool of Jones calculus.

### 6.2 Jones calculus

In a series of papers published in the 1940s and 1950s [158–165], R. C. Jones formulated a method involving $2 \times 1$ and $2 \times 2$ complex matrices to treat optical field polarization problems, which is referred to as the Jones calculus or the Jones matrix formalism. In this section, I shall review the fundamentals of the Jones matrix formalism.
6.2.1 Introduction to Jones vectors and Jones matrices

Without loss of generality, let us assume that the optical field is propagating in the $z$-direction. The $x$ and $y$ components of the electric field can be expressed as [166]

\[ E_x(z, t) = E_{0x} \cdot \exp[i(\omega t - kz + \delta_x)] \] (6.5a)

\[ E_y(z, t) = E_{0y} \cdot \exp[i(\omega t - kz + \delta_y)] \] (6.5b)

### Table 6.1. A few common Jones vectors [166].

<table>
<thead>
<tr>
<th>Light polarization states</th>
<th>Normalized Jones vectors</th>
</tr>
</thead>
</table>
| $x$-polarized                             | \[
\begin{bmatrix} 1 \\ 0 \end{bmatrix}\] |
| $y$-polarized                             | \[
\begin{bmatrix} 0 \\ 1 \end{bmatrix}\] |
| right-hand circularly polarized           | \[
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ +i \end{bmatrix}\] |
| left-hand circularly polarized            | \[
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}\] |

### Table 6.2. Jones matrices for a few optical components [166].

<table>
<thead>
<tr>
<th>Optical components</th>
<th>Jones matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>polarizer</td>
<td>$J_p = \begin{bmatrix} p_x &amp; 0 \ 0 &amp; p_y \end{bmatrix}$, where $p_{x,y} \in [0, 1]$</td>
</tr>
<tr>
<td>rotator</td>
<td>$J_{\text{ROT}}(\theta) = \begin{bmatrix} \cos \theta &amp; \sin \theta \ -\sin \theta &amp; \cos \theta \end{bmatrix}$, where $\theta$ is the rotation angle</td>
</tr>
<tr>
<td>retarder (phase shifter)</td>
<td>$J_R(\phi) = \begin{bmatrix} e^{+i\phi/2} &amp; 0 \ 0 &amp; e^{-i\phi/2} \end{bmatrix}$, where $\phi$ is the total phase retardation</td>
</tr>
</tbody>
</table>
where \( \omega \) is the field oscillation frequency, \( k \) is the wave vector, \( E_{0x} \) and \( E_{0y} \) are the amplitudes of the field components, and \( \delta_x \) and \( \delta_y \) are their corresponding phases. The Jones vector \( |E\rangle \) associated with this input electric field\(^1\) is a \( 2 \times 1 \) column vector defined as [166]

\[
|E\rangle = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{0x} \cdot e^{i\delta_x} \\ E_{0y} \cdot e^{i\delta_y} \end{bmatrix}
\]

(6.6)

where \( E_x = E_{0x} \cdot e^{i\delta_x} \) and \( E_y = E_{0y} \cdot e^{i\delta_y} \). With this definition, the intensity of the field can be written as [166]

\[
I = \langle E|E\rangle = \begin{bmatrix} E_x^* & E_y^* \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = E_x^*E_x + E_y^*E_y
\]

(6.7)

where \( \langle E \rangle = [E_x^* \ E_y^*] \) is the complex transpose of \( |E\rangle \). A few frequently encountered Jones vectors are listed in Table 6.1. General elliptically polarized light can be expressed as a superposition of either two linearly polarized components (first two rows of Table 6.1) or two circularly polarized components (last two rows of the same table).

Optical components in Jones matrix formalism are represented by \( 2 \times 2 \) complex matrices. The entries of a matrix describe how the polarization components are changed after the light goes through the optical component (see Table 6.2 for a few examples). Given an input polarization \( |s\rangle = [s_x \ s_y]^T \) and an optical component with Jones matrix \( M \) [see Fig. 6.2(a)], the output polarization \( |t\rangle = [t_x \ t_y]^T \) is (the superscript “\( T \)” denotes the matrix transpose)

\[
|t\rangle = M|s\rangle
\]

(6.8)

If light passes through a number of cascaded optical elements [Fig. 6.2(b)], the Jones matrix \( M \) representing the entire system is the product of individual component Jones matrices (\( M_1, M_2, M_3, \ldots, M_n \)). Therefore, the output polarization \( |t\rangle \) is simply

\[
|t\rangle = M|s\rangle = M_n \ M_{n-1} \cdots \ M_1|s\rangle
\]

(6.9)

Sometimes, the birefringence of an optical component is defined in a coordinate system that is rotated with respect to that of the input polarization. In this case, rotator matrices (see Table 6.2) can be used to convert the polarization states from one coordinate system to another. For example (Fig. 6.3), a rotated retarder (a phase shifter whose slow and fast axes do not coincide

\(^1\)The bra-ket notation follows that of [167].
Fig. 6.2. The Jones vector representing the light polarization changes from $|s\rangle$ to $|t\rangle$ after the light goes through an optical system. The output polarization state $|t\rangle$ is obtained by multiplying $|s\rangle$ by the Jones matrix associated with the optical system. (a) Light transmission through a single optical component; (b) light transmission through multiple cascaded optical components.

Fig. 6.3. A retarder with its $x$-axis being the fast axis, rotated by an angle $\theta$ with respect to the $x$-axis of the input Jones vector.

with the $x$ and $y$ axes which are used to define the input polarization) can be written as [166]

$$J_R(\phi, \theta) = J_{\text{ROT}}(-\theta) J_R(\phi) J_{\text{ROT}}(\theta)$$

$$= \begin{bmatrix}
\cos \frac{\phi}{2} + i \sin \frac{\phi}{2} \cos 2\theta & i \sin \frac{\phi}{2} \sin 2\theta \\
-i \sin \frac{\phi}{2} \sin 2\theta & \cos \frac{\phi}{2} - i \sin \frac{\phi}{2} \cos 2\theta
\end{bmatrix}$$
where $\phi$ is the total phase retardation between the field components and $\theta$ is the angle between the two coordinate systems (counterclockwise from the $x$-axis to the fast axis when viewed towards the $-z$ direction). In (6.10), the first rotation matrix $J_{\text{ROT}}(\theta)$ rotates the input Jones vector to the coordinate system of the rotator $J_R(\phi)$, and the second rotation matrix $J_{\text{ROT}}(-\theta)$ rotates the retarded Jones vector back to its original coordinate system. Thus, Jones vectors $|s\rangle$ and $|t\rangle$ are both defined in the same coordinate system. The eigenvectors of (6.10) are linear polarization states

$$|s_+\rangle = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix} \quad (6.11a)$$

$$|s_-\rangle = \begin{bmatrix} \sin \theta \\ -\cos \theta \end{bmatrix} \quad (6.11b)$$

with eigenvalues $e^{+i\phi/2}$ and $e^{-i\phi/2}$, respectively.

$$J_R(\phi, \theta)|s_+\rangle = e^{+i\phi/2}|s_+\rangle \quad (6.12a)$$

$$J_R(\phi, \theta)|s_-\rangle = e^{-i\phi/2}|s_-\rangle \quad (6.12b)$$

We can more generally construct the phase retarder of elliptical birefringence from an elliptical polarization state [166]

$$|s_{\text{el}+}\rangle = \begin{bmatrix} \cos \alpha \\ e^{i\delta} \sin \alpha \end{bmatrix} \quad (6.13)$$

where $\phi$ is the phase retardation, and $\delta = \delta_y - \delta_x$ is the phase difference between the two field components. Given a field (6.5), $\alpha$ is an auxiliary angle of the polarization ellipse defined by $\tan \alpha = E_{0y}/E_{0x}$. Obviously, $\tan \alpha = \sin \alpha / \cos \alpha$ in (6.13). The elliptical phase retarder associated with $|s_{\text{el}+}\rangle$ is [166]

$$J_R(\phi, \alpha, \delta) = \begin{bmatrix} \cos \alpha & -e^{-i\delta} \sin \alpha \\ e^{i\delta} \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \begin{bmatrix} \cos \alpha & e^{-i\delta} \sin \alpha \\ -e^{i\delta} \sin \alpha & \cos \alpha \end{bmatrix}$$

$$= \begin{bmatrix} e^{i\phi/2} \cos^2 \alpha + e^{-i\phi/2} \sin^2 \alpha & (e^{i\phi/2} - e^{-i\phi/2}) e^{-i\delta} \sin \alpha \cos \alpha \\ (e^{i\phi/2} - e^{-i\phi/2}) e^{i\delta} \sin \alpha \cos \alpha & e^{i\phi/2} \sin^2 \alpha + e^{-i\phi/2} \cos^2 \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos \frac{\phi}{2} + i \sin \frac{\phi}{2} \cos 2\alpha & i e^{-i\delta} \sin \frac{\phi}{2} \sin 2\alpha \\ i e^{i\delta} \sin \frac{\phi}{2} \sin 2\alpha & \cos \frac{\phi}{2} - i \sin \frac{\phi}{2} \cos 2\alpha \end{bmatrix} \quad (6.14)$$

The other eigenvector of (6.14) is

$$|s_{\text{el}-}\rangle = \begin{bmatrix} e^{-i\delta} \sin \alpha \\ -\cos \alpha \end{bmatrix} \quad (6.15)$$
\[ |s_{el+}\rangle \text{ and } |s_{el-}\rangle \text{ satisfy} \]
\[
\begin{align*}
J_R(\phi, \alpha, \delta)|s_{el+}\rangle &= e^{i\phi/2}|s_{el+}\rangle \\
J_R(\phi, \alpha, \delta)|s_{el-}\rangle &= e^{-i\phi/2}|s_{el-}\rangle
\end{align*}
\] (6.16)

Note that (6.14) reduces to a linear rotated retarder (6.10) for \( \delta = 0 \) and \( \alpha = \theta \).

### 6.2.2 Jones matrices for bidirectional systems

In many applications, light may travel through an optical component in both directions (e.g., in an FSI system). How the polarization states evolve in such a bidirectional system has been discussed in Jones’ original work [158]. In the case of reverse light propagation through an optical component with a (forward propagation) Jones matrix \( M \), the input and output polarization states, \( |s\rangle \) and \( |t\rangle \), are related by [158]

\[ |t\rangle^T = |s\rangle^T M \] (6.17a)

or equivalently,

\[ |t\rangle = M^T |s\rangle \] (6.17b)

where \( M^T \) is the Jones matrix of the optical component associated with reverse light propagation. However, as pointed out by [168, 169], (6.17) is valid when the light propagation is reversed while the optical component is fixed, which implies not only an optical component rotation of 180° about the \( y \)-axis, but also a \( \pi \) phase difference between the two \( x \)-components of the field. This is demonstrated in Fig. 6.4. Suppose the input vector \( |s\rangle \) is defined in the \( x-y-z \) coordinate system in Fig. 6.4(a). For backward propagation, in a right-handed coordinate system \( x'-y'-z' \) as defined in Fig. 6.4(b), the optical component can be considered to be rotated by 180° about the \( y \)-axis, but the sign of the \( x \)-component of \( |s\rangle \) is flipped.

Without changing the polarization basis of the input Jones vector, we can rotate the optical component about the \( y \)-axis only. The relation between the Jones matrices of the optical component for forward and backward propagation becomes [168]

\[
M = \begin{bmatrix}
m_{11} & m_{12} \\
m_{21} & m_{22}
\end{bmatrix}
\] (6.18a)

\[
M_B = \begin{bmatrix}
m_{11} & -m_{21} \\
-m_{12} & m_{22}
\end{bmatrix} = \overline{M}
\] (6.18b)

where we use a transformation from a \( N \times N \) matrix \( A \) to \( \overline{A} \) defined as (called \( n \)-transpose in [168])

\[
\overline{A}_{ij} = (-1)^{i+j}A_{ji}
\] (6.19)
Fig. 6.4. The convention used in [158] to describe bidirectional light propagation. (a) input polarization $|s\rangle$ is defined in the coordinate system $x$-$y$-$z$; (b) for backward propagation, in a right-handed coordinate system $x'$-$y'$-$z'$ whose $y'$-axis is aligned with the original $y$-axis, one sees that the optical component is rotated by $180^\circ$ about the $y$-axis with a sign change in the $x$-component of $|s\rangle$.

For Jones matrices, we only work with the case where $N = 2$. Note that negative signs are added to the anti-diagonal entries of the backward propagation Jones matrix $M_B = \overline{M}$ compared with the transpose operation in (6.17b). We can see Jones’ formalism (6.17) converge to (6.18) if we consider the $\pi$ phase difference (a change in sign) in the $x$-component of the field (assuming the optical component is rotated by $180^\circ$ about the $y$-axis)

$$F_x M^T F_x^\dagger = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} m_{11} & -m_{21} \\ -m_{12} & m_{22} \end{bmatrix} = \overline{M}$$

(6.20)

where $2 \times 2$ diagonal matrix $F_x = \text{diag}[-1,1]$ accounts for the flip of sign in the field $x$-
component. If the rotation of the optical component is assumed to be about the $x$-axis, $F_y = \text{diag}[1, -1]$ should be used in (6.20), which leads to the same result.

It can be shown that [168]

$$A_n A_{n-1} \cdots A_1 = \overline{A_1} A_2 \cdots \overline{A_n} \quad (6.21)$$

for a number of $N \times N$ matrices $A_1, A_2, A_3, \ldots, A_n$. Therefore, if there are multiple optical components, the relation between the forward and backward Jones matrices is

$$M = M_n M_{n-1} \cdots M_1 \quad (6.22a)$$

$$M_B = \overline{M} = \overline{M_n} \overline{M_{n-1}} \cdots \overline{M_1}$$

$$= \overline{M_1} \overline{M_2} \cdots \overline{M_n} \quad (6.22b)$$

---

**Fig. 6.5.** Light bidirectional propagation through a rotated retarder. (a) Forward propagation in the direction of positive $z$; (b) Backward propagation through the rotated retarder which is rotated about $y$-axis by $180^\circ$ (the original $x$ and $y$ axis of the rotated retarder is rotated to $x'$ and $y'$ position). The entry side (marked in red) of the optical component for forward propagation becomes the exit side for backward propagation.
where $M_1, M_2, M_3, \ldots, M_n$ are the Jones matrices of individual optical components. The output polarization can be calculated by using (6.9).

Fig. 6.5 illustrates bidirectional light propagation through a rotated retarder. We adopt a right-handed coordinate system and assume that light propagates in the positive $z$ direction (out of the page towards the reader). By Jones’ relation (6.17), $\mathbf{J}_R(\phi, \theta) = \mathbf{J}_R^T(\phi, \theta) = \mathbf{J}_R(\phi, \theta)$, which means that the Jones matrix for reverse propagation is the same as the one for forward propagation. However, it is apparently not true in Fig. 6.5(b), since for reverse propagation, the rotated retarder is turned by $180^\circ$ about the $y$-axis (or equivalently, the $x$-axis), and the rotated angle viewed from the coordinate system of the input light polarization basis becomes $-\theta$. Note the original $x$ and $y$ axis are rotated to $x'$ and $y'$ in Fig. 6.5(b). The Jones matrix for backward propagation is

$$
\mathbf{J}_R(\phi, \theta) = \mathbf{J}_R(\phi, -\theta)
$$

This is correctly described by (6.18). Therefore, the relation in (6.18) should be used, if one wants to work in the coordinate system defined locally [154,170–173].

### 6.2.3 Jones matrices for lossless systems

In the absence of fiber nonlinearity and polarization-dependent loss, and under the assumption that the usual loss term has been factored out, Jones matrices become unitary [167]. For such lossless systems, we may write the unitary Jones matrix $\mathbf{T}$ as [167]

$$
\mathbf{T} = e^{i\phi_0} \mathbf{U}
$$

where $\phi_0$ is the common phase experienced by both field components, and $2 \times 2$ matrix $\mathbf{U}$ is also unitary with a determinant $\det \mathbf{U} = 1$. In group theory (see Appendix B or [174–177]), $\mathbf{U}$ belongs to the so-called special unitary group SU(2) [167]. In general, $\mathbf{U}$ can be written as [175]

$$
\mathbf{U} = \begin{bmatrix}
    a & b \\
    -b^* & a^*
\end{bmatrix}
= \begin{bmatrix}
    e^{i\xi} \cos \eta & e^{i\zeta} \sin \eta \\
    -e^{-i\zeta} \sin \eta & e^{-i\xi} \cos \eta
\end{bmatrix}
$$

where complex numbers $a = e^{i\xi} \cos \eta$ and $b = e^{i\zeta} \sin \eta$ satisfy $|a|^2 + |b|^2 = 1$ ($a, b \in \mathbb{C}$), and the matrix is characterized by three real continuous parameters $\xi, \eta$, and $\zeta$ ($\xi, \eta, \zeta \in \mathbb{R}$) called
the Cayley-Klein parameters. Alternatively, $U$ can be expressed as the product of three matrices [175]

$$
U = \begin{bmatrix}
e^{i\rho} & 0 \\
0 & e^{-i\rho}
\end{bmatrix}
\begin{bmatrix}
\cos \eta & \sin \eta \\
-\sin \eta & \cos \eta
\end{bmatrix}
\begin{bmatrix}
e^{i\psi} & 0 \\
0 & e^{-i\psi}
\end{bmatrix}
$$

(6.26)

where $\rho$ and $\psi$ are two real parameters such that $\rho + \psi \equiv \xi$ and $\rho - \psi \equiv \zeta$. From Table 6.2, we can immediately recognize that (6.26) is a rotator with a rotation angle $\eta$ sandwiched between two retarders with $2\rho$ and $2\psi$ as phase shifts. Furthermore, since matrix multiplications are closed for elements of SU(2), the Jones matrix for a system formed by cascaded components with matrices $U_1, U_2, U_3, \ldots, U_n \in \text{SU}(2)$ can be characterized by only 3 parameters, either in the form of (6.25) or (6.26), independent of the component number $n$. By (6.18) and (6.22), the Jones matrix for reverse propagation is then

$$
U_B = U = \begin{bmatrix}
a & b^* \\
-b & a^*
\end{bmatrix}
= \begin{bmatrix}
e^{i\xi} \cos \eta & e^{-i\xi} \sin \eta \\
-e^{i\xi} \sin \eta & e^{-i\xi} \cos \eta
\end{bmatrix}
= \begin{bmatrix}
e^{i\psi} & 0 \\
0 & e^{-i\psi}
\end{bmatrix}
\begin{bmatrix}
\cos \eta & \sin \eta \\
-\sin \eta & \cos \eta
\end{bmatrix}
\begin{bmatrix}
e^{i\rho} & 0 \\
0 & e^{-i\rho}
\end{bmatrix}
$$

(6.27)

With (6.25)–(6.27), we can start to model the polarization effects in FSI systems.

### 6.3 Modeling polarization evolution in FSI systems

In our experiments, FSI systems are built with single-mode fibers (SMFs) or SMF-based components. It is well known that SMFs have very low polarization-dependent loss. The power level we worked with is also very low ($< 10$ mW), and thus nonlinear effects are negligible. To model polarization evolution in FSI systems, we assume that the Jones matrices of fiber sections are elements of the special linear group SU(2). In other words, we require that there is no fiber nonlinearity, no polarization-dependent loss, and that the loss term has been factored out. We also require that there are no Faraday effects and that fiber directional couplers do not alter light polarization. Under these assumptions, we attempt to explain the polarization effects observed in the experiments (Fig. 6.1), and seek conditions under which the interference fringe visibility is maximized.

---

1 The special unitary group SU(2) is of order 3 and has the three Pauli matrices as generators [175].

2 It can be easily shown that $J_{\text{ROT}}(\theta), J_{\text{R}}(\phi) \in \text{SU}(2)$. Similarly, $J_{\text{R}}(\phi, \theta) \in \text{SU}(2)$.

3 The matrix product between two elements of SU(2) is also an element of SU(2). See Appendix B for more details.
6.3.1 Frequency-shifted Sagnac interferometer

First, let us consider the case of frequency-shifted Sagnac interferometer configuration (Fig. 6.6). The Jones matrices for clockwise propagation and counterclockwise propagation, \( M_c \) and \( M_a \), respectively, have the same form as (6.25)–(6.27)

\[
M_c = \begin{bmatrix}
e^{i\zeta} \cos \eta & e^{i\zeta} \sin \eta \\
-e^{-i\zeta} \sin \eta & e^{-i\zeta} \cos \eta
\end{bmatrix}
\]

(6.28a)

\[
M_a = \begin{bmatrix}
e^{i\zeta} \cos \eta & e^{-i\zeta} \sin \eta \\
-e^{i\zeta} \sin \eta & e^{-i\zeta} \cos \eta
\end{bmatrix} = M_c
\]

(6.28b)

Obviously, if \( M_c \) is the identity matrix, the output polarization states of the clockwise and counterclockwise light are equal, and the interference fringe visibility is optimized (\(|t_c\rangle = |t_a\rangle\), where the two polarization states are defined in their corresponding local coordinates \(x'-y'-z'\)). More generally, to optimize the interference fringe visibility, we require \( M_c |s\rangle = M_a |s\rangle \). This implies \( e^{i\zeta} \sin \eta = e^{-i\zeta} \sin \eta \), that is, \( \zeta = k\pi \), where \( k \) is an integer \( (k \in \mathbb{Z}) \).

Fig. 6.6. Input and output polarization of a frequency-shifted Sagnac interferometer. The coordinate system for the input polarization state \(|s\rangle\) is \(x-y-z\), while that for the output polarization states \(|t_c\rangle\) and \(|t_a\rangle\) is \(x'-y'-z'\).
6.3.2 Linear frequency-shifted Sagnac interferometer

A linear frequency-shifted Sagnac interferometer with multiple reflection sites is a good example of a cascaded optical system (Fig. 6.7). Light from a reflection site \( R_i \) needs to go through all the \( i \) fiber sections bidirectionally. Let us define the forward direction as from left to right for the fiber segments linking the reflection sites and as clockwise for the two fiber segments between the fiber directional couplers (see Fig. 6.7). Following the paths of the light, we may write the clockwise and counterclockwise Jones matrices, \( M_{ci} \) and \( M_{ai} \), for the \( i \)th reflector as

\[
M_{ci} = U_B \left( U_1 U_2 \cdots U_{i-1} U_i \right) \left( U_i U_{i-1} \cdots U_2 U_1 \right) U_A \\
= U_B U_{\text{lin}_i} U_A \tag{6.29a}
\]

\[
M_{ai} = U_A \left( U_1 U_2 \cdots U_{i-1} U_i \right) \left( U_i U_{i-1} \cdots U_2 U_1 \right) U_B \\
= U_A U_{\text{lin}_i} U_B \tag{6.29b}
\]

where \( U_{\text{lin}_i} = \left( U_1 U_2 \cdots U_{i-1} U_i \right) \left( U_i U_{i-1} \cdots U_2 U_1 \right) \). But by (6.22), we have

\[
M_{ai} = M_{ci} = U_B U_{\text{lin}_i} U_A \\
= U_A U_{\text{lin}_i} U_B \tag{6.29c}
\]

Fig. 6.7. Input and output polarization of a linear frequency-shifted Sagnac interferometer. The coordinate system for the input polarization state \(| s \rangle \) is \( x-y-z \), while that for the output polarization states \(| t_{ai} \rangle \) and \(| t_{ci} \rangle \) from the \( i \)th reflector \( R_i \) is the local coordinate system \( x'-y'-z' \).
After comparing (6.29b) and (6.29c), we may conclude that
\[ U_{\text{lin}i} = U_{\text{lin}i} \]  
(6.30)

Note that each of the individual Jones matrices \( U_A, U_B, \) and \( U_i \) has the form of (6.25). As \( U_A, U_B, \) and \( U_i \) are in SU(2), their multiplications are closed so that \( U_{\text{lin}i}, M_{ci}, \) and \( M_{ai} \) are also elements of SU(2). Therefore, \( U_{\text{lin}i} \) has the form of (6.25), and to satisfy (6.30), the \((1,2)\) entry of \( U_{\text{lin}i} \) must be a real number, that is, \( [U_{\text{lin}i}]_{12} \in \mathbb{R} \). Similarly, to optimize the interference fringe visibility for a random input polarization \( |s\rangle \), we require \( M_{ci}|s\rangle = |t_{ci}\rangle = M_{ai}|s\rangle \), or
\[ M_{ci} = \begin{bmatrix} a_i & b_i \\ -b_i^* & a_i^* \end{bmatrix} = M_{ai} \]  
(6.31)

which implies \( b_i = b_i^* \), that is, \( b_i \in \mathbb{R} \). Without loss of generality, we may assume that
\[ U_{\text{lin}i} = \begin{bmatrix} e^{i\xi_0} \cos \eta_0 & \sin \eta_0 \\ -\sin \eta_0 & e^{-i\xi_0} \cos \eta_0 \end{bmatrix} \]  
(6.32)

and that
\[ U_A = \begin{bmatrix} e^{i\xi_A} \cos \eta_A & e^{i\zeta_A} \sin \eta_A \\ -e^{-i\xi_A} \sin \eta_A & e^{-i\zeta_A} \cos \eta_A \end{bmatrix}, \quad U_B = \begin{bmatrix} e^{i\xi_B} \cos \eta_B & e^{i\zeta_B} \sin \eta_B \\ -e^{-i\xi_B} \sin \eta_B & e^{-i\zeta_B} \cos \eta_B \end{bmatrix} \]  
(6.33)

Then we may find an expression for \( M_{ci} \) or \( M_{ai} \), which are characterized by 8 parameters \((\xi_0, \eta_0, \xi_A, \zeta_A, \eta_A, \xi_B, \zeta_B, \) and \( \eta_B \)). The imaginary part of the \((1,2)\) entry of \( M_{ci} \) is
\[ \text{Im}[M_{ci}]_{12} = \sin(\xi_B + \xi_0 + \zeta_A) \cos \eta_B \cos \eta_0 \sin \eta_A \]
\[ + \sin(\xi_B - \xi_A) \cos \eta_B \sin \eta_0 \cos \eta_A \]
\[ - \sin(\zeta_B + \zeta_A) \sin \eta_B \sin \eta_0 \sin \eta_A \]
\[ + \sin(\zeta_B - \xi_0 - \xi_A) \sin \eta_B \cos \eta_0 \cos \eta_A \]  
(6.34)

To optimize the output polarization states from the \( i \)th reflection site, we need to set \( \text{Im}[M_{ci}]_{12} = 0 \). Suppose we seek a condition that optimizes the polarization for light from all the reflection sites, (6.34) must be zero for arbitrary \( \xi_0 \) and \( \eta_0 \). Assume we can control the Jones matrices \( U_A \) and \( U_B \) (these two fiber sections can be placed in the control center of the sensing system), we need to choose \( \xi_A, \zeta_A, \eta_A, \xi_B, \zeta_B, \) and \( \eta_B \) in a way such that all the 4 terms in (6.34) vanish:
\[
\begin{align*}
\cos \eta_B \sin \eta_A &= 0 \\
\sin(\xi_B - \xi_A) \cos \eta_B \cos \eta_A &= 0 \\
\sin(\zeta_B + \zeta_A) \sin \eta_B \sin \eta_A &= 0 \\
\sin \eta_B \cos \eta_A &= 0
\end{align*}
\]  
(6.35)
Two sets of conditions that satisfy (6.35) are

\[
\begin{align*}
\cos \eta_A &= 0 \\
\cos \eta_B &= 0 \\
\sin(\zeta_B + \zeta_A) &= 0
\end{align*}
\]  

(6.36)

and

\[
\begin{align*}
\sin \eta_A &= 0 \\
\sin \eta_B &= 0 \\
\sin(\xi_B - \xi_A) &= 0
\end{align*}
\]  

(6.37)

For example, (6.36) can be satisfied when we choose \( \eta_A = \eta_B = \pi/2 \) and set \( \zeta_A = \pi/2 \) and \( \zeta_B = -\pi/2 \). Thus \( U_A \) and \( U_B \) can be two half-wave plates rotated by \( \pm \pi/4 \) [see (6.10)]. Alternatively, if we have \( \eta_A = \eta_B = 0 \) while \( \xi_A = \xi_B = \pi/4 \), condition (6.37) is met. In this case, \( U_A \) and \( U_B \) are simply two aligned quarter-wave plates. Once optimized, the interference fringe visibility is independent of the input polarization state.

6.3.3 Fiber loop cavity embedded in a frequency-shifted Sagnac interferometer (an FSI-CRD system)

Now let us consider the polarization effects in FSI-CRD systems (Fig. 6.8). Again, we define the forward propagation as in the clockwise sense. Let the fiber loop cavity, such as a ring-down cavity (RDC), be formed by two fiber segments represented by Jones matrices \( U_L \) and \( U_S \). The

Fig. 6.8. Input and output polarization of an FSI-CRD system. The input polarization state \( |s\rangle \) is defined in the coordinate system \( x-y-z \). The light exiting the ring-down cavity after \( m \) round trips contribute two output polarization states \( |t_{cm}\rangle \) and \( |t_{am}\rangle \) in clockwise and counterclockwise directions, respectively, which are defined in their local coordinates \( x'-y'-z' \).
corresponding Jones matrices experienced by light after \( m \) round trips (\( m = 0, 1, 2, \ldots \)) in the RDC is

\[
M_{cm} = U_B(U_LU_S)^m U_LU_A
= U_B U_{RDCm} U_A
\]

(6.38a)

for clockwise propagation, where \( U_{RDCm} = (U_LU_S)^m U_L \) is the Jones matrix describing the polarization evolution as light circulates in the RDC, and

\[
M_{am} = U_A(U_LU_S)^m U_LU_B
= M_{cm} = U_B U_{RDCm} U_A
= U_A U_{RDCm} U_B
\]

(6.38b)

for counterclockwise propagation. By using (6.33) and writing

\[
U_{RDCm} = \begin{bmatrix}
e^{i\xi_m} \cos \eta_m & e^{i\zeta_m} \sin \eta_m \\
e^{-i\xi_m} \sin \eta_m & e^{-i\zeta_m} \cos \eta_m
\end{bmatrix}
\]

(6.39)

we can find the entries of \( M_{cm} \)

\[
[M_{cm}]_{11} = e^{i(\xi_B + \xi_m + \xi_A)} \cos \eta_B \cos \eta_m \cos \eta_A
- e^{i(\xi_B + \zeta_m - \zeta_A)} \cos \eta_B \sin \eta_m \sin \eta_A
- e^{i(\zeta_B - \zeta_m + \xi_A)} \sin \eta_B \sin \eta_m \cos \eta_A
- e^{i(\zeta_B - \zeta_m - \zeta_A)} \sin \eta_B \cos \eta_m \sin \eta_A
\]

(6.40a)

\[
[M_{cm}]_{12} = e^{i(\xi_B + \xi_m + \xi_A)} \cos \eta_B \sin \eta_A
+ e^{i(\xi_B + \zeta_m - \zeta_A)} \cos \eta_B \sin \eta_m \cos \eta_A
- e^{i(\zeta_B - \zeta_m + \xi_A)} \sin \eta_B \sin \eta_m \sin \eta_A
+ e^{i(\zeta_B - \zeta_m - \zeta_A)} \sin \eta_B \cos \eta_m \cos \eta_A
\]

(6.40b)

and \( [M_{cm}]_{22} = [M_{cm}]_{11}^* \) while \( [M_{cm}]_{21} = -[M_{cm}]_{12}^* \). To optimize the output polarization states \( (M_{cm}|s) = (M_{am}|s) \), one requires \( \text{Im}[M_{cm}]_{12} = 0 \), where

\[
\text{Im}[M_{cm}]_{12} = \sin(\xi_B + \xi_m + \zeta_A) \cos \eta_B \cos \eta_m \sin \eta_A
+ \sin(\xi_B + \zeta_m - \zeta_A) \cos \eta_B \sin \eta_m \cos \eta_A
- \sin(\zeta_B - \zeta_m + \xi_A) \sin \eta_B \sin \eta_m \sin \eta_A
+ \sin(\zeta_B - \xi_m - \zeta_A) \sin \eta_B \cos \eta_m \cos \eta_A
\]

(6.41)

It can be seen from (6.41), to make \( \text{Im}[M_{cm}]_{12} = 0 \), some restrictions must be imposed
on $U_{RDCm}$. As each of the matrices $U_A$, $U_B$, $U_L$, and $U_S$ is characterized by 3 parameters, to describe the system, one needs to deal with 12 independent parameters, which is rather involved. We now show even that is the case, the output polarization states of an FSI-CRD system with a fiber loop RDC can always be optimized by two polarization controllers—one before one RDC input (PC$_1$) and the other inside the RDC (PC$_2$), as shown in Fig. 6.9.

The simplest way to optimize output polarization states is to make the Jones matrix experienced by light into an identity matrix. We can recognize that if a forward unitary Jones matrix $U$ is equal to the identity matrix $I$, the backward propagation matrix is $U = U^{-1}$, since the anti-diagonal entries of the matrix are zeros. For $m = 0$, the Jones matrix for clockwise propagation of the system is

$$M_{c0} = U_B U_L U_A$$

$$= \begin{bmatrix} e^{i\xi_B \cos \eta_B} & e^{i\xi_B \sin \eta_B} \\ -e^{-i\eta_B} \sin \eta_B & e^{-i\eta_B} \cos \eta_B \end{bmatrix} \begin{bmatrix} e^{i\xi_L \cos \eta_L} & e^{i\xi_L \sin \eta_L} \\ -e^{-i\xi_L} \sin \eta_L & e^{-i\xi_L} \cos \eta_L \end{bmatrix} U_A$$

$$= \begin{bmatrix} e^{i\xi_{BL} \cos \eta_{BL}} & e^{i\xi_{BL} \sin \eta_{BL}} \\ -e^{-i\xi_{BL}} \sin \eta_{BL} & e^{-i\xi_{BL}} \cos \eta_{BL} \end{bmatrix} U_A$$

(6.42)

As $U_B$ and $U_L$ are all elements of SU(2), their product $U_{BL}$ is also in SU(2), which can then be expressed by parameters $\xi_{BL}$, $\zeta_{BL}$ and $\eta_{BL}$, as shown in the last equation. Another important property of SU(2) is that its elements are invertible. Therefore, we can have $M_{c0} = I$ by adjusting PC$_1$ so that $U_A = (U_B U_L)^{-1}$. In this case, $\xi_A = -\xi_{BL}$, $\zeta_A = \zeta_{BL}$, and $\eta_A = -\eta_{BL}$. Optimized $U_A$ can be written as

$$U_{Aopt} = \begin{bmatrix} e^{-i\xi_{BL} \cos \eta_{BL}} & -e^{i\zeta_{BL}} \sin \eta_{BL} \\ e^{-i\xi_{BL}} \sin \eta_{BL} & e^{i\zeta_{BL} \cos \eta_{BL}} \end{bmatrix}$$

(6.43)
Similarly, for $m \geq 1$, we can tune PC$_2$ to optimize $U_S$ so that the $U_{\text{Sopt}} = U_L^{-1}$, where

$$U_{\text{Sopt}} = \begin{bmatrix} e^{-i\xi_L} \cos \eta_L & e^{i\zeta_L} \sin \eta_L \\ e^{-i\zeta_L} \sin \eta_L & e^{i\xi_L} \cos \eta_L \end{bmatrix} \tag{6.44}$$

When (6.44) is met, the polarization state of light remains the same after each round trip in the RDC for both directions of propagation, since $U_L U_{\text{Sopt}} = I$. If both (6.43) and (6.44) are satisfied, the clockwise propagation Jones matrix becomes the identity matrix

$$M_{cm} = U_B(U_L U_{\text{Sopt}})^m U_L U_{\text{Aopt}}$$

$$= U_B(U_L U_L^{-1})^m U_L U_{\text{Aopt}} = U_B U_L (U_B U_L)^{-1}$$

$$= I \tag{6.45}$$

One can achieve (6.45) experimentally by first adjusting PC$_1$ to optimize the fringe visibility of the first peak in the FSI-CRD transient, and then adjusting PC$_2$ to optimize the subsequent peaks. As the system for each round trip is described by an identity matrix, the fringe visibility is optimized for any input polarization states. For unpolarized light, the input polarization can be decomposed into uncorrelated $x$- and $y$-components. As a Sagnac interferometer is self-referenced, the interference occurs between light and a coherent portion of itself, which still can be considered as polarized. Therefore, above arguments can also be applied to describe the polarization evolution of unpolarized broadband light in an FSI-CRD system.

Simulations were run based on this model to reconstruct the polarization effects in an FSI-CRD system. In the unoptimized case, the output polarization state becomes input-polarization-dependent. For example, the FSI-CRD transients in Fig. 6.10(a) and (b) were obtained for a system with arbitrary input polarization states $|s_1\rangle = [\cos(\pi/3) \ e^{i1.5\pi}\sin(\pi/3)]^T$

| Table 6.3. Jones matrix parameters used in the simulations of Fig. 6.10. |
|-----------------|-----------------|
| $\xi_A$         | $0.28\pi$       |
| $\xi_A$         | $0.15\pi$       |
| $\eta_A$        | $0.66\pi$       |
| $\xi_B$         | $\pi/3.5$       |
| $\eta_B$        | $\pi/3.5$       |
| $\xi_L$         | $\pi/3.5$       |
| $\eta_L$        | $\pi/1.3$       |
| $\xi_S$         | $0.7\pi$        |
| $\eta_S$        | $0.23\pi$       |
| for (a) and (b) | for (c) and (d) |
| $0.15\pi$       | $0.25\pi$       |
| $0.75\pi$       | $0.55\pi$       |
| $0.75\pi$       | $0.25\pi$       |
Fig. 6.10. Simulated FSI-CRD transients with non-optimized system polarization. The parameters of the Jones matrices used in the simulations are shown in Table 6.3. (a) and (b) were obtained with input polarization states $|s_1\rangle = [\cos(\pi/0.3) \ e^{i\pi/4} \cdot \sin(\pi/0.3)]^T$ and $|s_2\rangle = [\cos(1.65\pi) \ e^{i\pi/4} \cdot \sin(1.65\pi)]^T$ in one system polarization configuration. (c) and (d) are the FSI-CRD transients produced by these two polarization states in another system.

and $|s_2\rangle = [\cos(1.65\pi) \ e^{i\pi/4} \cdot \sin(1.65\pi)]^T$, respectively. In a different system, these two input states produce the FSI-CRD transients shown in Fig. 6.10(c) and (d). The parameters of the Jones matrices used in the simulations are listed in Table 6.3. For the same system, the FSI-CRD transients produced by $|s_1\rangle$ and $|s_2\rangle$ are different. As can be seen, the transients in Fig. 6.10(a) and (c) are very similar to the experimentally observed transients in shown Fig. 6.1. Note that in Fig. 6.1(b) or (c), the transient is a superposition of contributions from various polarization states due to the broadband unpolarized ASE input light. Given matrices $U_B$ and $U_L$, we can optimize the system polarization. If we select the system used to obtain Fig. 6.10(a) and (b), we can find $U_{Sopt}$ by choosing $\xi_S = -\pi/3.5$, $\zeta_S = \pi/8.3$, $\xi_S = -\pi/3.5$, and compute the inverse of $U_B U_L$ for $U_{Aopt}$. Fig. 6.11(a) and Fig. 6.11(b) show the FSI-CRD transients after optimization computed for input states $|s_1\rangle$ and $|s_2\rangle$, respectively. The transients show nice
Fig. 6.11. FSI-CRD transients for optimized system polarization. (a) and (b) are the transients produced by $|s_1\rangle$ and $|s_2\rangle$ when the first system in Fig. 6.10 is optimized. (c) and (d) are the optimized transients for the second system in Fig. 6.10.

exponential decay, as expected. Similarly, the second system used to obtain Fig. 6.10(c) and (d) can be optimized. The optimized FSI-CRD transients for the same two input polarization states are shown in Fig. 6.11(c) and (d).

6.4 Summary

This chapter explores the polarization effects in FSI systems. I propose a preliminary model based on Jones calculus which suggests that the output polarization states can be optimized for all reflection sites in the case of FSI-based multipoint sensing. I also show that in an FSI-CRD system, the output polarization states can always be optimized with two polarization controllers. As light travels multiple times in the cavity of an FSI-CRD system, the birefringence of the cavity is amplified [172,173]. Information on polarization evolution in the cavity can potentially be extracted from the FSI-CRD transients (e.g., Fig. 6.1). The preliminary model proposed can be used as a foundation for further theoretical and experimental investigation of polarization effects, which may lead to new methods of characterizing the birefringence of optical devices.
Chapter 7

Conclusions and Future Work

7.1 Summary of results

As a spin-off from the development of modern telecommunication, fiber-optic sensing emerged into an important new frontier of the sensing industry. Not only have fiber-optic sensors and sensing techniques displayed many advantages over conventional electrical sensors (e.g., low loss, immunity to electromagnetic interference, compactness etc.), but they also can perform tasks that incapacitate conventional sensors (e.g., quasi-distributed or distributed sensing). The focus of this thesis work, frequency-shifted interferometry (FSI), is a versatile and promising addition to the realm of fiber-optic sensing. Through this thesis work, I maturated the theory of FSI, and successfully demonstrated that FSI can be applied to fiber-optic sensor multiplexing and high-sensitivity sensing applications.

As a fiber-optic sensor multiplexing scheme, FSI enables one to separate the sensor signals from their spatial location and measure their reflection spectra, with the help of a continuous-wave (CW) light source, a slow detector, and a frequency shifter (e.g., an acousto-optic modulator). It offers several advantages over conventional fiber-optic sensing multiplexing schemes, including time-division multiplexing (TDM), wavelength-division multiplexing (WDM), and the frequency-modulated continuous-wave (FMCW) technique (a form of frequency-division multiplexing, or FDM). Dissimilar from TDM, which requires expensive pulsed source and fast electronics to distinguish sensor signals in time, FSI only uses CW light source and slow detectors. Also, FSI allows the sensors to overlap spectrally, in contrast to WDM which demands that each sensor should operate in its unique wavelength window. This relaxes the requirements on the sensors and can potentially increase the maximum sensor number in the sensing system. Unlike FMCW \[23,24\], FSI does not need a reference signal since it is based on a Sagnac interferometer which is a reference to itself. Therefore, FSI may lead to a cost-effective fiber-optic sensing system that accommodates a large number of sensors and has a compact setup.

We implemented a liquid level sensing system based on FSI. The fiber-optic sensors employed were fiber Bragg grating (FBG) sensors inscribed in high attenuation fibers (HAFs) coated by aluminum (Al). These Al-coated high-attenuation fiber Bragg grating (HAFBG) sensors can
be heated by guided light in the fiber, and they show distinct spectral responses to heating when they are immersed in different media (appreciable Bragg wavelength shift in air but small shift in liquid). We constructed an Al-coated HAFBG array consisting of 3 sensors which have similar Bragg wavelengths. We were able to provide heating to the sensors and interrogate all of them with FSI despite the fact that the sensors spectrally overlap with each other, since sensor signals were separated from their spatial location in the sensor array. The system unambiguously distinguished the sensors in air from those in liquid nitrogen. This liquid level sensing system has nicely solved the low sensitivity issue encountered by conventional refractive-index-based fiber-optic liquid level sensing schemes at cryogenic temperatures.

As a step towards practical applications, I demonstrated that in lieu of a tunable laser source (TLS), a broadband source with a tunable filter can be incorporated into an FSI system for fiber-optic sensor multiplexing. As the measurement speed of our current TLS-based FSI system is mainly limited by the frequency sweep speed of the acousto-optic modulator (AOM) and by the tuning speed of the sensing light wavelength, using a fast tunable filter may greatly reduce the wavelength tuning time. I built a fully automated FSI system for the interrogation of an array of 9 FBG sensors. Although the FBGs have similar Bragg wavelengths, our system was able to clearly identify the Bragg wavelength shifts of individual sensors. The strain sensitivity of an FBG measured by the system was found to be 1.17 pm/µε, in good agreement with the theoretical value (1.21 pm/µε). A strain resolution of ∼ 22 µε was achieved, determined by the bandwidth of the tunable filter used. Without any changes to the AOM driver, the data acquisition time was 25 s, which could potentially be reduced to 0.5 s if dedicated hardware were used. The cost of such a system can be reduced and the measurement speed can be drastically improved.

An important contribution of this thesis is my invention of the FSI-CRD technique, a fiber cavity ring-down (CRD) method based on the principle of FSI, which can be employed as a sensitive analytical tool for chemical or biomedical applications. In this scheme, the intensity decay rate of CW light circulating in an optical cavity is deduced from the Fourier spectrum of the light interference signal. The cavity loss information can be derived from the light intensity decay rate. If a sensor head with measurand-dependent transmission loss is incorporated into the cavity, the optical cavity loss can be used to gauge the measurand. I developed the theory of FSI-CRD and conducted experiments in two stages. First, as a proof-of-concept experiment, I measured fiber bend loss introduced in the fiber cavity with FSI-CRD, which was found to be 0.172 dB/m at a bend radius of 12.5 mm. Then, we chose fiber tapers as the sensing head in our FSI-CRD system for evanescent-field sensing. We found a repeatable recipe for fabricating fiber tapers by chemical etching. We developed a reliable experimental setup for measuring minuscule taper loss change induced by chemical absorption or refractive index (RI) change. We successfully measured the concentration of 1-octyne dissolved in decane based on its absorption. A minimum detectable 1-octyne concentration of 0.29% was achieved with measurement sensitivity of 0.0094 dB/% 1-octyne. This detection limit is the
best among those reported in literature for similar measurement conducted by conventional CRD techniques \[95, 98\]. We also used the same system to measure solution RI changes and a combination of RI and absorption changes. The concentration change of sodium chloride (NaCl) and glucose solutions could be accurately detected by our FSI-CRD system. RI sensitivity of 1 dB/RIU with a measurement error of \(1 \times 10^{-4}\) dB was attained for NaCl solutions.

To understand polarization effects in FSI systems, I formulated a theoretical model utilizing the Jones calculus. Based on this preliminary model, the experimentally observed polarization phenomena in FSI systems (especially in the FSI-CRD experiments) can be explained. I showed that for FSI-based fiber-optic sensor multiplexing systems (linear frequency-shifted Sagnac interferometer configuration), the interference fringe visibility can be optimized for every sensor simultaneously. I also showed that in FSI-CRD, it is always possible to optimize the interference fringe visibility by two polarization controllers so that the FSI-CRD decay transient can provide useful cavity loss information.

Thus far, the project on FSI has resulted in 8 publications in peer-reviewed leading journals in the field of optics and photonics. I am the first author of 5 publications, and among them, 4 are contributed by this thesis work (listed in Chapter 1). A solid theoretical and experimental foundation has been laid by this thesis work for future advances of the FSI technique.

### 7.2 Future work

There are a few directions in which the FSI project can proceed: prototyping, improvement of system performance, and expansion of applications.

It is clear that FSI can serve as a practical fiber-optic sensor multiplexing scheme \[27–29\]. The number of sensors interrogated by an FSI system has already reached 10, comparable to that in a commercial WDM sensing system channel, limited only by the sensors available in our lab \[27\]. We have come to a stage where a prototype of an FSI-based fiber-optic sensing system can be built. Instead of fine but expensive lab equipment, less costly hardware with more specialized functions can be used for the purpose of system control, data acquisition and processing. The experiments in Chapter 4 is an excellent start for such development.

There is still much room for the improvement of FSI system performance. For example, one major limitation of our current FSI system is its poor spatial resolution. The AOM currently in our lab has a frequency sweep range of only 20 MHz, which provides a spatial resolution above 5 m, which is not that impressive for field applications (the spatial resolution in WDM is only limited by the sensor length, and sub-meter spatial resolution can be achieved in TDM \[21\]). A different frequency shifter with more robust sweeping functionalities or alternative modulation schemes can be explored to achieve finer spatial resolution and longer sensing range. If the frequency can be swept at a higher speed, the measurement time of the system can also be reduced. The single-arm FSI system with a phase modulator as the frequency shifter (1 GHz frequency sweep range) has already reached a spatial resolution of only 0.1 m \[32\]. It can be
projected that further refinement of spatial resolution can be attained in the near future. The coherence length of the light source may also limit the minimum separation between adjacent sensors in a single fiber link, as crosstalk may occur when light from one sensor interferes with that from another sensor. This issue can be alleviated if a low-coherence source is employed (such as the broadband source in Chapter 4). FSI measurement speed is another area that can be significantly improved. Our current measurement speed (on the order of minutes) is mainly restrained by the AOM sweep speed and the sensing light wavelength scan speed. Some of the WDM systems reported have already reached read out frequencies at $\sim 10$ kHz [22,178].

Last but not least, with the invention of the FSI-CRD techniques, a wide variety of applications are awaiting FSI. On one hand, more sensitive measurement can be done by FSI-CRD instead of conventional CRD methods. With different sensor heads, FSI-CRD can be utilized to measure temperature, pressure, strain, or to detect biomolecules, chemical gases and so on. On the other hand, as FSI is an interferometric technique and light travels through the cavity for multiple times, the cavity birefringence is amplified. From the FSI-CRD decay transient, one may obtain information on how the light polarization changes after each pass through the cavity. To have a deeper understanding of the polarization effects, we need to design and perform experiments to test the validity of the proposed preliminary FSI polarization model. We may be able to exploit the polarization phenomena in FSI-CRD and devise a scheme to measure light polarization change or cavity birefringence. It can turn to a novel method of characterizing the birefringence of optical devices. The technique may also be deployed for electric current sensing applications.

In conclusion, FSI is a useful, practical, and versatile tool which has an auspicious future in many fields of fiber-optic sensing, including fiber-optic ranging or sensor multiplexing, and high sensitivity analysis. We can foresee that with further studies on the technique, commercial products based on the principle of FSI shall appear in the market, benefiting the development of our society.
Appendix A

Derivation of Interference Signals in a Frequency-Shifted Sagnac Interferometer

To derive the interference signal at the detectors in a frequency-shifted Sagnac interferometer, let us start with equations (2.1) and (2.2). At port 1 the total field is

\[ E_1 = E_{cw1} + E_{ccw1} \]

\[ = \gamma E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi (\nu + f)t + \frac{2\pi nl_1\nu}{c} + \frac{2\pi nl_0(\nu + f)}{c} + \phi_0 + \frac{\pi}{2} \right] \right\} \]

\[ + \gamma E_0 \cdot \exp \left\{ i \cdot \left[ 2\pi (\nu + f)t + \frac{2\pi nl_0\nu}{c} + \frac{2\pi nl_1(\nu + f)}{c} + \phi_0 + \frac{\pi}{2} \right] \right\} \quad (A.1) \]

The intensity is

\[ I_1 \propto |E_1|^2 = (E_{cw1} + E_{ccw1})(E_{cw1} + E_{ccw1})^* \]

\[ = E_{cw1}E_{cw1}^* + E_{ccw1}E_{ccw1}^* + E_{cw1}E_{ccw1}^* + E_{ccw1}E_{cw1}^* \]

\[ = \gamma^2 E_0^2 + \gamma^2 E_0^2 \exp \left[ i \cdot 2\pi \frac{n(l_0 - l_1)}{c} f \right] \]

\[ + \gamma^2 E_0^2 \exp \left[ i \cdot 2\pi \frac{n(l_1 - l_0)}{c} f \right] \]

\[ = 2\gamma^2 E_0^2 + 2\gamma^2 E_0^2 \cos \left[ 2\pi \frac{n(l_1 - l_0)}{c} f \right] \quad (A.2) \]

which is precisely (2.3a). Following the same steps, one can show that

\[ I_2 \propto |E_2|^2 = 2\gamma^2 E_0^2 - 2\gamma^2 E_0^2 \cos \left[ 2\pi \frac{n(l_1 - l_0)}{c} f \right] \quad (A.3) \]
Appendix B

Group Theory

This appendix only lists the concepts of group theory that are related to the applications in Chapter 6. Interested readers may consult other references on the subject [174–177] for in-depth discussions.

B.1 Some definitions

Definition of groups

Given a set $G$ and a rule of multiplication such that $G$ is closed under this multiplication (i.e., the product of two elements in $G$ also belongs to $G$), we may call $G$ with its multiplication rule a group if the following axioms are satisfied [174]:

- The multiplication is associative:
  For any elements $x, y, z \in G$, $(xy)z = x(yz)$ holds.

- There is an identity element in $G$:
  For every $x \in G$, there exists $e \in G$ such that $xe = x = ex$.

- There is an inverse in $G$ for every element of $G$:
  For every element $x \in G$, there exists $x^{-1}$ such that $x^{-1}x = e = xx^{-1}$.

Note that the identity element $e$ of a group are unique. The inverse of an element in the group is also unique. If a subset $H \subset G$ forms a group itself under the same multiplication of $G$, then $H$ is called a subgroup of $G$.

B.2 Matrix groups

The set of invertible $n \times n$ matrices form the $n \times n$ general linear group [176]:

$$GL_n(\mathbb{K}) = \{ A \in M_n(\mathbb{K}) \mid \det A \neq 0 \}$$  (B.1)
where $M_n(\mathbb{K})$ is the set of $n \times n$ matrices whose entries are in a commutative field $\mathbb{K}$. For our purpose, we only need to work on general linear groups with real numbers or complex numbers as matrix entries ($\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$), which are referred to as real or complex general linear groups, $GL_n(\mathbb{R})$ or $GL_n(\mathbb{C})$, respectively. $GL_n(\mathbb{R})$ has a subgroup, the $n \times n$ special orthogonal group

$$ SO(n) = \{ A \in GL_n(\mathbb{R}) \mid A^T A = I \text{ and } \det A = 1 \} $$

(B.2)

Similarly, the $n \times n$ special unitary group defined by

$$ SU(n) = \{ A \in GL_n(\mathbb{C}) \mid A^\dagger A = I \text{ and } \det A = 1 \} $$

(B.3)

is a subgroup of $GL_n(\mathbb{C})$. The $2 \times 2$ unitary Jones matrices $U$ with unity determinants defined in (6.24) are elements of $SU(2)$. We can also describe polarization evolution in Stokes space [167]. In this case, one deals with elements of $SO(3)$ which are $3 \times 3$ unitary matrices $R$. $U$ can be converted into $R$ or vice versa with the help of the pauli spin vector [167]. The formulations in Jones calculus and in Stokes space are equivalent.
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