Temporal Variations in the Compliance of Gas Hydrate Formations

by

Lisa Aretha Nyala Roach

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy

Department of Physics
University of Toronto

© Copyright by Lisa Roach 2012
Temporal Variations in the Compliance of Gas Hydrate Formations

Lisa Roach
Doctor of Philosophy
Department of Physics
University of Toronto
2012

Abstract

Seafloor compliance is a non-intrusive geophysical method sensitive to the shear modulus of the sediments below the seafloor. A compliance analysis requires the computation of the frequency dependent transfer function between the vertical stress, produced at the seafloor by the ultra low frequency passive source-infra-gravity waves, and the resulting displacement, related to velocity through the frequency. The displacement of the ocean floor is dependent on the elastic structure of the sediments and the compliance function is tuned to different depths, i.e., a change in the elastic parameters at a given depth is sensed by the compliance function at a particular frequency. In a gas hydrate system, the magnitude of the stiffness is a measure of the quantity of gas hydrates present. Gas hydrates contain immense stores of greenhouse gases making them relevant to climate change science, and represent an important potential alternative source of energy. Bullseye Vent is a gas hydrate system located in an area that has been intensively studied for over 2 decades and research results suggest that this system is evolving over time.
A partnership with NEPTUNE Canada allowed for the investigation of this possible evolution. This thesis describes a compliance experiment configured for NEPTUNE Canada’s seafloor observatory and its failure. It also describes the use of 203 days of simultaneously logged pressure and velocity time-series data, measured by a Scripps differential pressure gauge, and a Guralp CMG-1T broadband seismometer on NEPTUNE Canada’s seismic station, respectively, to evaluate variations in sediment stiffness near Bullseye. The evaluation resulted in a \((-4.49 \times 10^{-3} \pm 3.52 \times 10^{-3})\) % change of the transfer function of 3rd October, 2010 and represents a 2.88% decrease in the stiffness of the sediments over the period. This thesis also outlines a new algorithm for calculating the static compliance of isotropic layered sediments.
Acknowledgments

This journey, independent of the outcome, was successful for the experiences gained, opportunities awarded and perspectives reformulated. For this I wish to say thank you to the people who were a part of it. To Prof. Nigel Edwards, for agreeing to be my supervisor. Your style of supervision made the years enjoyable. I especially liked your candour and your boundless support of and contribution to my academic growth. To Prof. Bernd Milkereit and Prof. Richard Bailey, for your constructive feedback during the committee meetings. Specifically to Prof. Milkereit for your continual show interest in my progress and Prof. Bailey for the five minute meetings that save me months of work. To Krystyna Biel for your excellent work that guaranteed smooth transitions to each academic year. To Patricia Edwards, for including me at family gatherings, which made my being away from home bearable. To Eleanor Willoughby for your guidance. To Canada Science Team Integrator, Martin Heesemann for alerting me of the data availability and making data access easy. To Prof. Claire Samson for your assessment of the thesis and providing detailed feedback that allowed for an improved final product. To Reza Mir and Catherine Robin, for the stimulating conversations, the laughter and the encouragement. To the Geophysics student body for creating a cool work environment. To my all sister friends, especially: Shanda, Twanna, Isshad, Ayana and Danine for ears and words. To Khalila for the love. To Miriam for the dream. To Sciltian for bringing the rainbow.
For you my loves,
Khalila, Miriam, Sciltian
Table of Contents

Acknowledgments.............................................................................................................. iv
List of Tables ............................................................................................................................ ix
List of Figures ........................................................................................................................... x
List of Appendices .................................................................................................................. xiv
List of Acronyms .................................................................................................................... xv
Introduction ............................................................................................................................. 1

Chapter 1 Geological Setting ............................................................................................... 6
  1.1 Introduction ....................................................................................................................... 6
  1.2 Nature of in situ gas hydrate deposits ........................................................................... 14
  1.3 Detecting gas hydrate deposits ..................................................................................... 15
  1.4 Cascadia Margin gas hydrates ..................................................................................... 17
  1.5 Summary ......................................................................................................................... 24

Chapter 2 Seafloor Compliance Theory ............................................................................... 25
  2.1 Introduction ....................................................................................................................... 25
  2.2 Infra-gravity waves .......................................................................................................... 25
    2.2.1 Background ............................................................................................................... 25
    2.2.2 Pressure spectrum at NEPTUNE Canada Site 889 .................................................. 30
  2.3 Seafloor compliance: the interaction of the infra-gravity wave with the seafloor ......... 33
  2.4 New algorithm for calculating the static compliance .................................................. 38
    2.4.1 Formulation of the new algorithm ........................................................................... 38
    2.4.2 The validation of the new 1-D algorithm ................................................................. 43
  2.5 Seafloor compliance modelling for layered structure .................................................. 47
2.5.1 A sensitivity analysis .................................................................47
2.5.2 Interpreting compliance curves using the sensitivity analysis results ....55
2.5.3 An eigenparameter analysis .........................................................56
2.6 Summary ......................................................................................63

Chapter 3 NEPTUNE Canada I - Seafloor Compliance Experiment .................65
3.1 Introduction ..................................................................................65
3.2 The NEPTUNE Canada Project ..........................................................65
3.3 The Micro-g LaCoste gPhone ...............................................................68
  3.3.1 The campaign style gPhone ..........................................................68
  3.3.2 Modifications for long-term deployment ...........................................70
3.4 The Scripps Differential Pressure Gauge ..............................................71
3.5 The pressure vessel, gimbals and bottom assembly .................................74
3.6 Preparations for NEPTUNE Canada Installation .................................77
  3.6.1 Instrument calibration .................................................................77
  3.6.2 Equipment testing after shipment to Victoria, BC ............................81
  3.6.3 Assembly of SFC equipment for deployment .................................82
  3.6.4 Testing by NEPTUNE Canada .....................................................92
3.7 Deployment of the SFC experiment ......................................................94
3.8 Data processing and results ...............................................................96
  3.8.1 Method of data processing ..........................................................96
  3.8.2 Data processing ..........................................................................97
3.9 A final word ....................................................................................108
3.10 Summary ......................................................................................108

Chapter 4 NEPTUNE Canada II – Variations in sediment stiffness .......................110
4.1 Introduction ....................................................................................110
4.2 Processing procedure and results ................................................................. 113
4.3 Analysis of data .............................................................................................. 126
  4.3.1 Investigation and validation of temporal changes ....................................... 126
  4.3.2 Interpretation of results ............................................................................ 130
    4.3.2.1 Method of determining the elastic parameters ................................. 132
    4.3.2.2 Determining the variation in hydrate concentration ....................... 133
    4.3.2.3 The error in the transfer function ................................................... 143
4.4 Summary ........................................................................................................ 145
Chapter 5 Summary, Conclusions and Recommendations ............................... 148
  5.1 Summary ...................................................................................................... 148
  5.2 Conclusions ................................................................................................. 149
  5.3 Recommendations ....................................................................................... 150
References ........................................................................................................... 152
Appendices ........................................................................................................... 165
  Appendix A: Derivation of equations 2.32, 2.33 and 2.34 ............................ 165
  Appendix B: Derivation of the least squares estimate of a straight line with errors in a pair of observables ............................................................... 168
  Appendix C: Details on the data used in the estimate of temporal variations at Bullseye ............ 172
  Appendix D: Further explanations about the method used in interpreting the observed trend ................................................................................................... 174
  Appendix E: Software (MATLAB) .................................................................. 181
  Appendix F: Profiles of density, porosity and velocity for IODP 311 U1328 .................... 183
  Appendix G: Scripps Institution of Oceanography Differential Pressure Gauge Specifications ................................................................. 185
  Appendix H: Predictive sedimentary model (TPBE) ........................................ 187
List of Tables

Table 2.1: List of vectors and parameters used throughout chapter ........................................... 26
Table 2.2: Summary of performance of new matrix method .......................................................... 45
Table 2.3: Elastic parameters and compliance of models used in sensitivity analysis .................... 48
Table 2.4: TPBE variables for determining elastic parameters of sedimentary models ............... 49
Table 2.5: Eigenparameter analysis results for layer 1 over half-space ..................................... 61
Table 2.6: Eigenparameter analysis results for layer 2 over half-space ..................................... 61
Table 2.7: Eigenparameter analysis results for buried layer 1 ..................................................... 62
Table 2.8: Eigenparameter analysis results for buried layer 2 ..................................................... 62
Table 4.1: Variables used TPBE estimate of elastic parameters of gas hydrate bearing sediments ........................................................................................................................................... 133
Table 4.2: Parameters of models used in fitting the transfer function of 3rd October, 2010 ........ 136
Table 4.3: Parameters of the best fit model ..................................................................................... 137
Table 4.4: Summary of results from hydrate concentration estimation ......................................... 139
Table 4.5: Error plot of transfer function of 3rd October, 2010 .................................................... 145
Table C.1: Days with irregularities ................................................................................................. 173
Table D.1: Relationship between change in hydrate concentration and Δλ/Δµ .......................... 175
Table D.2: Relationship between percent change in µ and transfer function ............................. 178
Table D.3: Relationship between initial hydrate concentration and the change in elastic parameters ........................................................................................................................................ 180
Table F.1: DPG Specifications ....................................................................................................... 185
## List of Figures

Figure 1.1: Structure of a clathrate ................................................................. 8
Figure 1.2: Types of gas hydrate structures ...................................................... 8
Figure 1.3: Global gas hydrate occurrence inventory ....................................... 9
Figure 1.4: Gas Hydrate phase diagram ........................................................... 11
Figure 1.5: Images of exposed gas hydrate on the seafloor ............................. 12
Figure 1.6: Images of in situ gas hydrates formations .................................... 13
Figure 1.7: Microstructure of gas hydrate formations ..................................... 14
Figure 1.8: A seismic reflection profile showing a BSR .................................. 16
Figure 1.9: Tectonic map of Cascadia margin ............................................... 18
Figure 1.10: Tectonic interpretation of the Northern Cascadia Vent field .......... 19
Figure 1.11: Seismic reflection profile of the vent field in Cascadia Margin ........ 20
Figure 1.12: An illustration of a model of Bullseye Vent ............................... 21
Figure 1.13: Images of carbonates and chemosynthetic communities at Bullseye Vent .... 22
Figure 1.14: CSEM resistivity results at Bullseye Vent .................................. 23

Figure 2.1: Power spectra of the pressure and velocity channels Structure of a clathrate 32
Figure 2.2: Mathematical set of of layered model used in the derivation of the static compliance ............................................................................................................ 40
Figure 2.3: Boundary conditions for interfaces in a layered medium ................ 40
Figure 2.4: A plot of the compliance as function of frequency of layer over half-space model ............................................................................................................ 46
Figure 2.5: Sensitivity curve at 0.03Hz for a half-space and a layer over half-space models 51
Figure 2.6: Sensitivity curves (response) for a suite of frequencies for a half-space model .......................... 52
Figure 2.7: Sensitivity curves (response) for a suite of frequencies for a layer over a half-space model ............................................................................................................ 53
Figure 2.8: Surface plots of the sensitivity response to λ andμ ........................................... 54
Figure 2.9: A plot of the compliance as a function of frequency for a buried layer .... 57
Figure 3.1: Map outlining the NEPTUNE Canada network .................................. 66
Figure 3.2: Detailed map of NEPTUNE Canada ODP Site 889 instrument locations .... 67
Figure 3.3: Components of the Micro-g Lacoste gravimeter system .......................... 69
Figure 3.4: Scripps differential pressure gauge (DPG) .......................................... 72
Figure 3.5: Schematic of the relative pressure measurement mechanism ...................... 73
Figure 3.6: Schematic diagram of the DPG pressure sensor ...................................... 74
Figure 3.7: Structural components of the SFC apparatus ........................................ 75
Figure 3.8: The gimbal ............................................................................................. 76
Figure 3.9: DPG calibration set up ........................................................................... 77
Figure 3.10: Sample time-series from DPG calibration exercise ................................. 78
Figure 3.11: Step response of the DPG ................................................................. 80
Figure 3.12: Fit to the DPG step response .................................................................. 80
Figure 3.13: Instrument testing at the Geological Survey of Canada (GSC) ............... 81
Figure 3.14: Sphere fitted with mid-ring ................................................................... 84
Figure 3.15: Gimbal fitted on mid-ring ...................................................................... 84
Figure 3.16: Wired mid-ring .................................................................................... 85
Figure 3.17: Wired power supply ............................................................................. 86
Figure 3.18: Wired RS-232 to RS-422 converter ..................................................... 86
Figure 3.19: DPG electronics ................................................................................... 88
Figure 3.20: Wired gPhone ..................................................................................... 89
Figure 3.21: Assembled sphere with DPG ................................................................ 90
Figure 3.22: Schematic diagram of the assembled SFC apparatus .......................... 91
Figure 3.23: ODP mink connector .......................................................................... 91
Figure 3.24: NEPTUNE Canada junction box ............................................................ 93
Figure 3.25: Power testing of the SFC apparatus ..................................................... 93
Figure 3.26: Waveforms from Power test ............................................................... 93
Figure 3.27: Deployment of the SFC apparatus ......................................................... 95
Figure 3.28: Sample time-series from the DPG and gravimeter ............................... 98
Figure 3.29: DPG circuit filter ................................................................................. 99
Figure 3.30: Power spectral densities for 27th November, 2009 ............................. 101
Figure 3.31: Coherence, transfer function bound and phase plots for DPG-gravimeter 27th November, 2009 .......................................................... 101

Figure 3.32: PSDs of the BSR, DPG and gravimeter for 31 January, 2010 .................. 103

Figure 3.33: DPG-gravimeter and BPR-gravimeter TFB, TFP and coherence comparison 104

Figure 3.34: Tsunami data and frequency domain analysis results ................................ 105

Figure 3.35: PSDs of the BSR, DPG and gravimeter of days with the tsunami of 27th February, 2011 ................................................................. 106

Figure 3.36: DPG-gravimeter and BPR-gravimeter TFB, TFP and coherence comparison for the days with the tsunami of 27th February, 2011 ........................................ 107

Figure 3.37: Suspect connector in SFC recovery ......................................................... 108

Figure 4.1: Güralp broadband seismometer ............................................................ 111
Figure 4.2: Seismometer package on the seafloor ..................................................... 112
Figure 4.3: Seismometer being buried ................................................................. 112
Figure 4.4: Broadband seismometer frequency response ....................................... 114
Figure 4.5: Pressure and velocity record for 14th October, 2010 ............................ 116
Figure 4.6: Bandpassed pressure and velocity data (600 seconds) ......................... 116
Figure 4.7: Pressure and velocity record for 21st October, 2010 (earthquake) .......... 117
Figure 4.8: Pressure and velocity record for 13th January, 2011 (spurious spike) ...... 117
Figure 4.9: Coherence, transfer function bounds and phase for 14 October, 2010 ...... 118
Figure 4.10: Coherence, transfer function bounds and phase for 21 October, 2010 (earthquake) ........................................................................ 119
Figure 4.11: Coherence, transfer function bounds and phase for 13 January, 2011 (large variations) .......................................................... 120
Figure 4.12: Coherence, transfer function bounds and phase for 1 March, 2011 (data spikes) ........................................................................ 120
Figure 4.13: Surface plot of the coherence for full data set .................................... 122
Figure 4.14: Surface plot of the transfer function for full data set ....................... 123
Figure 4.15: Pressure spectrogram for 3rd October, 2010 ........................................ 124
Figure 4.16: Surface plot of pressure and velocity PSD of full data set .................. 125
Figure 4.17: Cummulative histogram and probablility plots of trend data ............ 128
Figure 4.18: Linear fit to data (Trend in data) .................................................................................................................. 129
Figure 4.19: Bootstrap re-sampling and F-test statistical results ......................................................................................... 131
Figure 4.20: Comparison of the transfer function of 3rd October, 2010 to the compliance of various sedimentary models .................................................................................................................. 135
Figure 4.21: Comparison of the compliance of the best fit sedimentary model and the transfer function of 3rd October, 2010 (Day 1) and schematic of the best fitting sedimentary model .................................................................................................................. 137
Figure 4.22: Comparison of the compliance of a various models ......................................................................................... 138
Figure 4.23: Comparison of the compliance response for parameter variations in best fit model .................................................................................................................................................................. 140
Figure 4.24: Relationship between the change in shear modulus and the change in normalised compliance .................................................................................................................................................................. 142
Figure 4.25: Transfer function error plot .......................................................................................................................................................... 145

Figure D.1: The relationship between the percent change in the hydrate concentration and the Δλ/Δµ .................................................................................................................................................................. 176
Figure D.2: Effect of changing elastic parameters on the transfer function .............................................................................. 177
Figure D.3: Relationship between the percentage change in hydrate concentration and percent change in the elastic parameters ........................................................................................................................................... 179
Figure F.1: Profiles of density, porosity and velocity at IODP 311 U1328 ......................................................................................... 183
Figure F.2: Compressional and shear velocities at IODP 311 U1328 ................................................................................................. 184
Figure G.1: DPG noise spectrum from manufacturer ....................................................................................................................... 186
List of Appendices

Appendix A: Derivation of equations 2.32, 2.33 and 2.34 ................................................................. 1655

Appendix B: Derivation of the least squares estimate of a straight line with errors in a pair of observables. .......................................................................................................................... 1688

Appendix C: Details on the data used in the estimate of temporal variations at Bullseye ....... 1722

Appendix D: Further explanations about the method used in interpreting the observed trend.............................................................................................................................................. 17474

Appendix E: Software (MATLAB) ........................................................................................................ 18180

Appendix F: Profiles of density, porosity and velocity for IODP 311 U1328 ...................... 18383

Appendix G: Scripps Institution of Oceanography Differential Pressure Gauge Specifications .............................................................................................................................................. 1855

Appendix H: Predictive sedimentary model (TPBE) ................................................................. 1877
List of Acronyms and Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GHSZ</td>
<td>gas hydrate stability zone</td>
<td>ADC</td>
<td>analogue-digital converter</td>
</tr>
<tr>
<td>SFC</td>
<td>seafloor compliance</td>
<td>ROPOS</td>
<td>remote operated platform for ocean science</td>
</tr>
<tr>
<td>ODP</td>
<td>ocean drilling program</td>
<td>PSD</td>
<td>Power spectral density</td>
</tr>
<tr>
<td>IODP</td>
<td>integrated ocean drilling program</td>
<td>TFB</td>
<td>transfer function bounds</td>
</tr>
<tr>
<td>BSR</td>
<td>bottom simulating reflector</td>
<td>TFP</td>
<td>transfer function phase</td>
</tr>
<tr>
<td>LWD</td>
<td>logging while drilling</td>
<td>GHBS</td>
<td>gas hydrate bearing sediments</td>
</tr>
<tr>
<td>DPG</td>
<td>differential pressure gauge</td>
<td>IRIS</td>
<td>Incorporated Research Institution for Seismology</td>
</tr>
<tr>
<td>BPR</td>
<td>bottom pressure recorder</td>
<td>HC</td>
<td>hydrate concentration</td>
</tr>
<tr>
<td>BBS</td>
<td>broadband seismometer</td>
<td>MM</td>
<td>matrix method</td>
</tr>
<tr>
<td>TPBE</td>
<td>triple phase boundary conditions</td>
<td>KL</td>
<td>Konstantin Latychev</td>
</tr>
<tr>
<td>CSEM</td>
<td>controlled source electromagnetic</td>
<td>WC</td>
<td>Wayne Crawford</td>
</tr>
<tr>
<td>ODI</td>
<td>Ocean Design Inc.</td>
<td>HS</td>
<td>half-space</td>
</tr>
</tbody>
</table>
Introduction

Overview

Marine gas hydrates occur naturally in passive and active continental margins worldwide. Estimates of the mass of methane in the oceanic environment are in excess of $10^{16}$ kg (Kvenvolden 1988). The immense stores of this powerful greenhouse gas make methane hydrates relevant in climate change science, an important potential alternate source of energy as well as a possible geo-hazard. These wide reaching and ranging influences thus make gas hydrate research pertinent. For example, on the Cascadia margin – an active margin, near Ocean Drilling Project (ODP) Site 889, – there are four very prominent blank zones. At the largest of these, the Bullseye Vent, there have been numerous research conducted in an effort to understand the nature of gas hydrate deposits (Riedel et al. 2006a). However, to date, very little is known about how they vary over time. It is crucial to the characterization of gas hydrate deposits to understand the nature of these changes. A full understanding of these deposits requires long-term observation of the gas hydrates in situ. The goal of this thesis was to investigate temporal variations in the shear modulus of a region near Bullseye Vent through the use of the seafloor compliance method and was achieved through partnership with Neptune Canada.

What motivated this research?

With the pending energy crisis and the alarm of global climate change, methane hydrate research has become increasingly important. The year 2008 epitomised the energy crisis. Crude oil was at its highest, at $122 per barrel per barrel, in almost three decades (US Energy Information Administration). Even more staggering was the fact that in the first five months of 2008, the increase in oil was $29.07 while for the same period in 2006 and 2007, the increase in oil price was (only?) $8.37 and $12.12, respectively. Commensurate to this is the fact that worldwide consumption of oil is larger than its production so at this rate, we are unable to sustain our energy budget and the need for an alternate source is critical. Despite vast investment in exploration and production, there is a general inability to replace the oil
produced each year with fresh discoveries, or even to maintain current levels of output. These facts have propelled research in gas hydrates as it is has long been assessed as a formidable energy source. World estimates of methane trapped in gas hydrates deposits range over three orders of magnitude, specifically 3,100 to 7,600,000 trillion m$^3$ for oceanic sediments (Collett 2002). If 1m$^3$ of methane hydrate is depressurized, it produces about 0.8 m$^3$ of water and 170 m$^3$ of methane (Methane Hydrate Advisory Committee, 2002). Tapped, these methane hydrate deposits could possibly supply our energy needs for centuries. Even more appealing, methane burns more cleanly than all other fossil fuels. Cleaner energy will have a reduced impact on the Earth’s global climate. Moreover, the Earth’s climate system is susceptible to rapid injections of methane into its atmosphere. This susceptibility is because atmospheric methane absorption band is less saturated so a single molecule of additional methane has larger impact on the radiative balance than a molecule of carbon-dioxide (Archer 2007). Apart from the direct effect of the release of methane, there are positive feedback mechanisms that may be induced due to its presence.

What justified this research and the use of the SFC method?

Bullseye Vent is located on a continental margin created by the subduction of one plate under another. During subduction, the methane rich sediments of the submerging plate are pushed into the gas hydrate stability zone (GHSZ) and explain the abundance of gas hydrates offshore Western Canada (Hyndman and Davis 1992). Gas hydrate existence was inferred from the presence the bottom stimulating reflector (BSR) which is known to mark the base of the GHSZ. Though a reliable indicator of the occurrence of free gas and gas hydrate, its absence is not an indication of the non-existence of gas hydrates. Reflection seismic, the traditional method of hydrate exploration, though capable of mapping the BSR, cannot clearly define the internal structure of a deposit especially in regions of seismic-blanking. Bullseye Vent is a blank zone at which coring expeditions found gas hydrates (Riedel et al. 2006b). Additionally, sporadic venting at Bullseye Vent, confirmed by the observation of plumes and changes in volume of estimated hydrates over different surveys, are indications of temporal variations at the Bullseye Vent (Willoughby et al. 2008). The presence of gas
hydrates alters the shear properties of the sediments and the seafloor compliance method is sensitive to the variations in shear modulus with depth in a sedimentary section. Shear modulus changes if pore space brine is replaced with gas hydrate but does not when replaced with free gas. Hence compliance data is reflective of gas hydrate content. Compared to other methods, for example standard boreholes, this method is fast, inexpensive and unobtrusive to the sediments since measurements are made with instruments placed on the seafloor (Willoughby and Edwards 1997). Thus it is employed in an effort to assess the nature of this hydrate system.

What did we want to know and why?

Given temporal variations, one can determine whether gas hydrates are increasing or dissociating in situ. Thus the initial questions asked were:

i) are the deposits in a stable state?

ii) if they are changing with time, how fast is this change?

iii) what kind of change is occurring: an increase or decrease?

iv) can the changes be linked to any other event- earthquakes perhaps?

v) is there an independent pattern to these changes? Perhaps a cyclical?

These point toward the general questions of where the methane gas is being stored or more importantly, when it is being released. The answers to these questions will indicate whether methane will become an increasing problem in global climate change and how much of an energy resource will be available over time.

How did we go about doing this?

Neptune Canada was ideal for the achievement of this goal because it replaced the campaign mode of research. Previous investigations of gas hydrate deposits involved annual campaigns where an entire data set consisted of measurements made over a 12 hour period (in the case of compliance) (Willoughby 2003). Such limited data reduced the inferences that can be
made and in cases where a data set is so poor that it cannot be salvaged, the campaign becomes futile. In addition, yearly campaigns proved problematic as measurements can be disrupted due to bad weather and worst yet, equipment may be lost. Furthermore, navigation is difficult during these annual campaigns which made repeat measurements on a particular site almost impossible. These lead to slow progress in gas hydrate characterisation and limited the science that can be done. NEPTUNE Canada provided an ideal opportunity to conduct research without the difficulties of annual campaigns. It also allows for the attempt at new science, in that temporal studies on known gas hydrate deposits can be conducted. A seafloor compliance (SFC) experiment was installed at NEPTUNE Canada ODP Site 889 where Bullseye Vent is located. The instrumentation included an adapted Micro-g LaCoste gPhone interfaced with a Scripps Institution of Oceanography differential pressure gauge from which data was uploaded to the NEPTUNE Canada’s data management system. This SFC apparatus failed to provide any useful data so the analysis in this thesis was done on compliance data obtained from the seismic network that was also installed at Site 889 on the NEPTUNE Canada observatory.

**Thesis structure**

The process of investigating the listed research objectives are outlined in the first four chapters of this thesis while the final chapter summarises the research and presents new research goals based on the results presented herein. Chapter 1 discusses the nature of gas hydrates and their physical expression in a sedimentary structure. It also highlights the characteristics of Bullseye Vent revealed from previous geophysical and geochemical studies and discusses the limitation of these characteristics on the traditional gas hydrate exploration method. Chapter 2 reviews the fundamentals of seafloor compliance theory: the source the deformation – infra-gravity waves and the response of the sediment to this vertical force. It details the new approach to computing the 1D compliance of a layered structure under the assumption that the speed of the gravity waves is much less than the elastic wave speeds of the sediments – the static solution. Here, a comparison is made between the compliance computed using the full solution and this static approach. This chapter ends with an analysis of the frequency-depth relation of the compliance function and its power of resolution.
Chapter 3 details the seafloor compliance instrument which was designed, manufactured and assembled by the Marine Geophysics group at the University of Toronto. It describes the NEPTUNE Canada project and its role in facilitating the long term monitoring of gas hydrates. The chapter details the preparation, assembly and deployment of the seafloor apparatus and it discusses the failure of the experiment. Chapter 4 presents results which fulfil the research goal - the observation of a long term trend in a gas hydrate system. The chapter outlines the data processing procedures, details the process of determining validating the trend, and concludes with an analysis of the trend results.
Chapter 1
Geological Setting

1.1 Introduction

Clathrates are non-stoichiometric compounds formed by the inclusion of one molecule (the guest) into the cavity of a crystal lattice of another (the host). There are many types of host (e.g. urea, cellulose, graphite, water) and guest molecules (e.g. hydrocarbons, water, noble gases, oxygen) of which natural gas hydrates are a subset (Pellenbarg and Max 2003). Natural gas hydrates are clathrates with three-dimensional hydrogen-bonded water molecules encaging halogens, noble gases, and low molecular weight hydrocarbons (Figure 1.1). Sloan (2003) describes three predominant gas hydrate structures called, structure I (sI), structure II (sII), and structure H (sH) (Figure 1.2). The formation of each structure is complex and depends primarily on the size of the gas molecules available in the sediment to act as guests. The sI unit comprises of 8 cages; 2 small and 6 large ones. A unit of sII has 16 small and 8 large cages for a total of 24 cages. sI hydrates preferentially form in natural environments since they encage small molecules like methane which is abundant. sII units encage larger molecules like propane which tend to be found in non-natural environs, i.e. oil and gas pipelines (Sloan 2003). Typically, each structure contains a single guest but under high very pressure, more than one gas molecules can be encaged. sH are least occurring and are usually packed with hydrocarbons having longer that 4 carbon atom chains (Sloan 1998).

The formation of gas hydrate, as well as its stability, is fundamentally dependent on: (i) temperature, (ii) pressure, (iii) gas composition and saturation; and (iv) pore-water composition (Tréhu et al. 2006). Its nucleation and growth depends on the size, shape and composition of the sediment grain (Clennel et al. 1999). Wherever there exist high pressures and low temperatures, and sufficient amounts of guest and host molecules in the sediments, gas hydrate formation is spontaneous. However, in the absence of any of the above four conditions, gas hydrates are unstable. The major limiting factor in gas hydrate generation is the availability of a suitable hydrocarbon to occupy the water cages (Sloan 2008).
Figure 1.2: Gas hydrate structures. $5^{12}6^4$ indicates a water cage composed of 12 pentagonal and four hexagonal faces. The pink squares indicate the number of cage types while the blue rectangles indicate number of water molecules. Reprinted by permission from Macmillan Publishers Ltd: [Nature] (Sloan 2003), copyright (2003).
The physical conditions necessary for gas hydrate generation exist along continental margins in water depths greater than 500m at low- to mid- latitudes, as well as at high latitudes of the Arctic in 300m of water. Additionally, favourable conditions for the generation of gas hydrates are found on land in permafrost in the Polar Regions (Dillon and Max 2003). The knowledge about the occurrence and distribution of gas hydrates in the natural environment is garnered from numerous investigations by universities and international consortia. Figure 1.3 shows the extent of this distribution. From which it is clear that gas hydrates occur worldwide, in both marine and permafrost sediments (Kvenvolden and Lorenson 2010).

Figure 1.3: Gas hydrates that have been recovered from samples (open circles) and inferred (solid circles) along continental margins and within the permafrost region. Source: Kvenvolden and Lorenson (2010), courtesy of the U.S. Geological Survey.

Of the possible gas molecules which can be encaged during hydrate formation, methane is abundant. Kvenvolden (1993) suggests that methane usually comprises of more than 99% of the hydrocarbon gas mixture present in sediments, thus methane-based gas hydrates are most common. Methane generation is through three major mechanisms: the fractionation of mantle material and the biogenic and thermal alteration of organic matter present in sediments (Max
et al. 2006). Biogenic and thermogenic methane sources are distinguishable from each other through their C$_1$/C$_2$ ratio (Wellsburg and Parks 2003). Usually, the amount of organic (biogenic) carbon in the local sediment is insufficient to produce substantial concentrations of hydrates (Max et al. 2006). Coffin et al. (2003), Sassen et al. (2003, 2004), Matsumoto et al. (2004) provide evidence that the additional methane source available in the gas hydrate stability zone (GHSZ) is biogenically and thermally generated outside the GHSZ. Lu et al. (2007) suggested that the thermal methane is produced from deep heat sources and is believed to arrive in the GHSZ through migration. Buffet and Archer (2004), Wellsburg and Parks (2003) and Max et al. (2006) explain mechanisms by which biogenic methane can amass in the GHSZ.

Figure 1.4 shows a simplified phase diagram for oceanic gas hydrates, illustrating the pressure and temperature conditions governing methane hydrate stability. Left of the phase boundary, methane hydrates are stable. This methane pressure-temperature boundary varies according to the type of methane hydrate present. The thermal gradient (dashed red) line demarks the left boundary of gas hydrate phase diagram. The thermal gradients, both hydro-thermal and geo-thermal, describes the depth-temperature relationship in the water column and in the sediment below the seafloor, respectively. Thus, the area bounded by the intersecting thermal gradients and pressure-temperature phase curves, outlines the hydrate stability conditions.

Within the sediments, the lower point of intersection between the phase boundary and the geo-thermal gradient delineates the bottom/base of the GHSZ. Below the GHSZ, the temperature is too high for gas hydrates to be stable and thus free gas accumulates. This results in the base of the GHSZ being itself a phase boundary, since it separates the gas hydrate and the free gas zones. The top of the GHSZ is identified by the point of intersection between the methane pressure-temperature curve and the hydro-thermal gradient. This point can occur in the water column as depicted in Figure 1.4. However, because methane hydrates are buoyant it is most likely that they would rise until they are above the GHSZ and dissociate. Hence, effectively, the top of the GHSZ is the seafloor itself. However, gas hydrate formation is not commonly observed at or near the seafloor because: (i) methane gas migrating from depth is converted to gas hydrate in sediments further below the seafloor (Xu
and Rupell 1999); (ii) gas hydrates that escape the sediment are rapidly dissolved by the low methane-concentrated seawater (Rehder et al. 2004); and (iii) of the numerous methane reducing chemical processes that occur in the near surface (Waite et al. 2009). However, gas hydrates have been observed close to the seafloor in the Gulf of Mexico, Japan Sea and northern Cascadia margin (McGee et al. 2009; Uchida et al. 2004; Suess et al. 2001; Chapman et al. 2004).

Figure 1.14: Methane hydrate phase diagram depicting the pressure (indicated as water depth)-temperature defined stability zone in the marine environment. Methane hydrates can exist in the region bounded by the red (dashed) and green (solid) lines (blue shaded region). The upper bound is based on the local water temperature, while the lower bound (within the sediment) is based on the water depth providing the geothermal gradient is constant. Reprinted from the Expert Panel on Gas hydrates (2008) / Council of Canadian Academies.
In practice, determining the GHSZ is complex as the system is dynamic. The ability to determine it is dependent on factors such as: the variability in pressure and temperature conditions in the ocean sediment which are affected by tectonic activity, sedimentation, changes in sea level and ocean temperature; the availability of guest molecules which is dependent on the production and oxidation of the molecules; methane solubility within the pore water of the sediment; variability in phase boundary (dependent on pore-water salinity); and fluid flow (Tréhu et al. 2006).

In the marine environment, gas hydrate occurrence is present in two modes: (i) as outcrops or surface mounds (Figure 1.5); and (ii) distributed throughout the sediment filling pore-space (Figure 1.6). In the first mode, given that the hydrates are visibly assessable, research is usually straightforward. The second mode requires remote sensing methods as one of the set of techniques for detection and characterization of gas hydrate deposits. One such method is seafloor compliance, which is the subject of this thesis.

Figure 1.5: Exposed gas hydrate. (Left) A 1 metre gas hydrate outcrop at Barkley Canyon. (Right) A 4 metre by 3 metre mound. Reprinted from Chapman et al. 2004, AGU
Figure 1.6: Types of *in situ* gas hydrate formations. (Source: unknown).
1.2 Nature of in situ gas hydrate deposits

For gas hydrates hosted within sediments, sediment lithology is of great significance to its distribution. The pattern of formation of naturally occurring gas hydrates varies according to the sediment type. Gas hydrates forms preferentially in sediments with coarse grains, where they appear disseminated and pore-filling, while in fine grained sediments they appear vein-like and/or nodule and lense-like, often in regions where the permeability is slightly increased (Tréhu et al. 2006). When gas hydrates are present, the effects on the sediment in which there are buried, depends on where in the pore space the hydrates are formed, and on how hydrates nucleate and grow. These effects can be placed into three distinct categories: cementation, load bearing and pore-filling (Figure 1.7).

![Diagram of gas hydrates formations](image)

Figure 1.7: Microstructure of gas hydrates formations in sediments. Here ‘supporting matrix grain’ is referred to as load bearing. Source: Dai et al. (2004)\(^1\).

---

\(^1\) “Reprinted from Marine and Petroleum Geology, 9, Jianchun Dai, Fred Snyder, Diana Gillespie, Adam Koesoemadinata, Nader Dutta, Exploration for gas hydrates in the deepwater, northern Gulf of Mexico: Part I. A seismic approach based on geologic model, inversion, and rock physics principles, 830-844., Copyright (2008), with permission from Elsevier.”
When gas hydrates cement the sediment, they do so by binding a grain to its neighbour at a point of contact, or they coat the grain, bonding adjacent grains together (Dvorkin and Nur 1996). This mechanism results in large increases in the sediments’ shear and bulk stiffness with small amounts of gas hydrates. With load bearing (Dvorkin et al. 1999; Helgerud et al. 1999), there is an increase in the mechanical stability of the sediments because the hydrate inclusions connect sediments grains while being part of the matrix. In the pore-filling mode, the gas hydrates grow outward on the surface of the sediment grain in into the pore space without connecting two or more grains together. Here the primary effect is to increase the pore fluid bulk stiffness and the conduction properties of the fluid (Waite et al. 2009). Thus, the presence of gas hydrates in sediments changes the sediments’ physical properties enough so that there is a contrast between the non-hydrate sediment and the hydrate filled sediment.

Gas hydrates sensing capitalizes on this hydrate-filled-non-hydrate-filled contrast in sediments. The choice of geophysical techniques used in mapping a gas hydrate deposit is based on the specific physical property enhancement as result of how gas hydrate interacts with the sediment grains. For instance, gas hydrates increases the seismic wave velocities of the sediment, a property to which seismic surveys are sensitive, while electrical methods respond to the increased resistivity of the hydrate bearing sediments and SFC is sensitive to the shear modulus of the sediment. Seismic methods are by far the most employed method for detecting gas hydrates; however, there are specific instances where this method is unable to adequately map a hydrate deposit. The next section elucidates this issue.

1.3 Detecting gas hydrate deposits

The primary indicator of the presence of gas hydrates is that of a bottom simulating reflector (BSR) in a seismic reflection survey profile (Riedel et al. 2006a). This reflector separates the high velocity-hydrate filled zone and the low velocity-gas filled region below it (Figure 1.8). The difference in the seismic velocities results in a negative acoustic impedance contrast at this ‘high velocity-low velocity’ interface which reflects seismic energy of reversed polarity
to that of the seafloor and other reflections. Thus, BSRs mark the base of the GHSZ. They are often parallel to the seafloor; cutting across bedding planes because their depth is determined primarily by the pressure-temperature conditions of the sediments (Hyndman and Dallimore 2001).

Though the BSR is the first signal of a strong impedance contrast, and is widely used in gas hydrates mapping because its presence is evidence of trapped free gas beneath the GHSZ and implies the existence of gas hydrates, it is not always present when gas hydrates are absent (Dillon and Max 2003). Moreover, gas hydrates are known to occur in regions where BSRs are neither present nor extensive (Hyndman et al. 2001). Additionally, there are regions in which BSRs occur above and below the expected GHSZ marker predicted by pressure and temperature conditions. However, these unexpected BSRs origin have been attributed to mineralogical transitions, other petroleum gases and carbonate accumulations (Max 2003).

Figure 1.8: A seismic reflection profile near ODP Site 889. Shows a strong BSR with a reflection which parallels and is opposite to that of the seafloor. Source: Hyndman and Dallimore (2001).
Further to undefined and multiple BSRs, there is the issue of blanking. Blanking is the weakening of a seismic reflection as a result the accumulation of gas hydrates (Dillon and Max et al. 2006). The strength of a reflection is proportional to the acoustic impedance which is the product of the seismic velocity and density of the sediment. Impedance is primarily affected by velocity rather than density variations. In porous sediments, the velocities of non-hydrate layers, are lower than hydrate filled layers, a difference which produces strong contrast and hence, reflection. However, given that gas hydrates preferentially form in porous layers, their presence increases the velocity of the layers relative to those layers that are hydrate-free. This results in a reduced impedance contrast between adjacent layers, the requirement for strong reflections (Max et al. 2006).

Blanking as well as the BSR may be used to identify hydrate formations, however, their presence give no concrete information about the nature of the deposit (Max et al. 2006), for e.g., the quantity of gas hydrates present. There are many regions in the world which have been identified as gas hydrate provinces and exhibit one, both, or neither. One such example is Cascadia Margin which revealed both – extensive BSRs and blanking. This region has been identified to have a pervasive presence of hydrates confirmed by a plethora of geophysical and/or geochemical surveys conducted in the region since 1985 (Riedel et al. 2002). An area of active gas and fluid venting was mapped in this region where the amplitude of the BSR was frequency dependent (Hyndman and Dallimore 2001).

1.4 Cascadia Margin gas hydrates

Cascadia margin, located on Canada’s West coast, is an accretionary complex formed by the subduction of the Juan De Fuca plate beneath the North American Plate (Figure 1.9). As the Juan De Fuca plate subducts at ~45mm/yr, sediments are scraped off the oceanic crust and folded into anticlinal ridges (Riedel et al. 2001). On the North American plate portion of the deformation front, there are regions characterized by seismic blank zones. Particularly, they are located in a 4km by 2km area near (ODP) 889/890 on a bathymetric bench in water depths of ~1400m – 1500m surrounded by two 200m topographic peaks (Figure 1.10) (Riedel 2007). Between these two highs, the 350m deep sediment trough is filled with new
sediments and show complicated deformation structures. The area where the blank zones are observed is slightly uplifted by 50m relative to the surrounding seafloor (Riedel et al. 2002). The sedimentary sequence within the first 128 meters below seafloor (mbsf) in this region is layers of fine turbidite interspersed with layers of clayey silts and silty clays. Below this sequence, sediments are compacted and cemented (Riedel et al. 2001).

Figure 1.9: Gas hydrate region of Cascadia margin. Source: Riedel et al. (2007). The grey region highlights the 130 km wide band of BSR inferred gas hydrates (Dash and Spence 2011). With kind permission from Springer Science+Business Media: Marine Geophysical Researches, 4D seismic time-lapse monitoring of an active cold vent, northern Cascadia margin, 28, 2007, 355, Michael Riedel, Figure 1a.
Figure 1.10: An interpretation of a single channel seismic data outlining the tectonics of the vent field in Northern Cascadia margin. Source: Riedel et al. (2002) / AGU

The blank zones are a part of a vent field and vary in a spatial scale from 80m to hundreds of metres (Riedel et al. 2002). Four main vents have been identified in this vent field (Figure 1.11) of which the largest, most pronounced of these is called Bullseye. This ~400m diameter blank zone has been the focus of numerous geophysical and geochemical studies over the past two decades (Willoughby et al. 2008). Riedel et al. (2006a) proposed an explanation for the Bullseye structure that combines features from models of Riedel et al. (2002); Wood et al. (2002); Zühlsdorff and Spiess (2004). They concluded that Bullseye embodies a ‘network of filamentous fractures’ (Figure 1.12) through which gas and fluids are channelled and the seismic blanking observed is as a result of the presence of gas hydrates, free gas or a combination of both – a model consistent with geophysical and geochemical observations. These observations include the occurrence of uniform heat flow measured at small, episodically venting outlets throughout the region. These sporadically venting outlets were identified in video and were seen to be surrounded by chemosynthetic communities and to have carbonate outcrops (Figure 1.12). An echosounder mapped gas plumes extending from the seafloor to 500m below the sea-surface, and water samples collected over the chemosynthetic communities, up to 200m above the seafloor, contained methane (Riedel et al. 2007).
Figure 1.11: An example of a single-channel seismic data over the vent field revealing 4 blank zones. The blank zone labelled 1 is named Bullseye. Source: Riedel et al. (2006a). The Geological Society of America bulletin by Geological Society of America Copyright 2006 Reproduced with permission of GEOLOGICAL SOCIETY OF AMERICA in the format Dissertation via Copyright Clearance Center.
Figure 1.12: A cartoon of the Riedel et al. 2006a model of Bullseye Vent. The Geological Society of America bulletin by Geological Society of America Copyright 2006 Reproduced with permission of GEOLOGICAL SOCIETY OF AMERICA in the format Dissertation via Copyright Clearance Center.
Riedel et al. (2006a) suggested that since gases can be concentrated at the near surface through the conduits, there are likely to be large masses of gas hydrates within the sediments close to the seafloor. Riedel et al. (2006b) reported that large pieces of gas hydrates were recovered within the first 50 m of sediments from piston cores while LWD measurements show large resistivities and high seismic velocities within the same region. IODP 311 site U1328 is located within the Bullseye Vent (Figure 1.9) and geophysical and geochemical data characterizes Bullseye as having: (i) lithography of fine grained silty to silty clays with an average of 80% clay content within the top 200 m of the sediments; (ii) a porosity
gradient that decreases from 70% near the seafloor to 45% at 128mbsf; (iii) a density profile that increases in depth from 0 to 128mbsf from 1.7 g/cm$^3$ to 2 g/cm$^3$; (iv) a porosity of ~50% and an average compressional velocity of ~1500 m/s within the region 128mbsf–200mbsf; (v) a 1500–1800 m/s compressional velocity within 200–300 mbsf (Expedition 311 Scientists 2006); (vi) methane gas concentrations that are equivalent to 0.7% of gas or 22% of hydrate at 92mbsf; (vii) pore volume gas hydrate saturations of ~15% – ~2% within the first 45mbsf; and (viii) a BSR at ~219m (Expedition Scientists 311 2005). Controlled source electromagnetic method (CSME) measurements, along a SW–NE profile intersecting the seafloor expression of Bullseye, reveal high resistivity anomalies with respect to the background (Figure 1.14) (Schwalenberg et al. 2005). Schwalenberg et al. 2005 concluded these higher resistivities are due to higher gas hydrate concentration within the blank zone and estimated that the amount of additional hydrate is within 25% to 50% of the available pore space.

![Figure 1.14](image-url)
These accumulated observations on gas hydrates and specifically on Bullseye Vent in the Cascadia margin have illuminated the nature of gas hydrates systems and accumulations in that region. They have shown that the blanking evident on seismic sections inhibits the use of seismic methods in estimating seismic velocities and hence, hydrate concentration, under these specific conditions. Furthermore, the episodic response and sporadic behaviour of these systems point towards long-term studies in the further characterization of gas hydrate deposits. As such, geophysical methods capable of sensing other geophysical parameters should be employed to augment the current observations. This thesis focuses on one such option – a long term seafloor compliance experiment deployed near Bullseye Vent.

1.5 Summary

Gas hydrates are non-stoichiometric compounds comprised of a hydrogen bonded water lattice which encages a hydrogen molecule of which methane hydrates are a subset. Methane hydrates are most common in the natural environment due to the larger fraction of methane available in the sediments. Gas hydrates are ubiquitous in the world’s continental margin and permafrost regions because these regions provide the pressure-temperature conditions required for hydrate stability. One such oceanic region is the Northern Cascadia margin. Here, gas hydrates have been studied for over 20 years. Specifically, a vast number of geophysical and geochemical investigations have been conducted in the vent field where seismic blanking is dominant. The large quantities of gas hydrates inferred by BSR mapping, is explained by the upward expulsion of methane-rich pore fluids. Around the vent field, especially at Bullseye Vent, the evidence of gas hydrates is considerable and supports temporal variations. The limited ability for seismic surveys to elucidate the nature of the vent field suggests that other geophysical methods over long periods should be employed to aid the characterization of the blank zones. The presence of gas hydrates alters the shear properties of the sediments thus the seafloor compliance method can be used to assess the nature of this hydrate system.
Chapter 2
Seafloor Compliance Theory

2.1 Introduction

Seafloor compliance is a geophysical method used to study the elastic structure of the ocean upper crust. Its application involves monitoring the deformation of the seafloor caused by the vertical stress applied by a subset of the long-period ocean surface gravity waves called infra-gravity waves. In this chapter, the characteristics, generation and propagation of these infra-gravity waves is discussed. An expression that describes the pressure amplitude of these long-period waves at the seafloor is derived and their interaction with the ocean bottom is outlined. The chapter continues with a definition of the seafloor compliance function and a derivation of its half-space solution as well as its variation within a layered Earth under the static assumption. The chapter concludes with an investigation of the influence of elastic parameters on the compliance function, the frequency-depth response of the compliance function to the elastic structure of the sediments, and the resolving power of the compliance function. Throughout this chapter, many vectors and parameters are used and are listed in Table 2.1 for ease of reference.

2.2 Infra-gravity waves

2.2.1 Background

Infra-gravity waves are the set of ocean surface gravity waves with long periods and wave heights less than 1cm (Webb et al. 1991). They are generated in shallow water by nonlinear-wave interactions of short period wind driven waves (1 to 25s) along coastlines (Webb and Crawford 1999). Most of the long period (greater than 30s) energy is trapped at the continental shelves as edge waves but some escape to the deep ocean by scattering from irregular shelves or coasts (Webb et al. 1991) and experience little or no attenuation on propagation (Lighthill 1978).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Constants (* represents an arbitrary subscript)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>shear velocity</td>
<td>a, b, c, d</td>
</tr>
<tr>
<td>$\beta$</td>
<td>compressional velocity</td>
<td>c</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Kronecker delta</td>
<td>e</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Dilation</td>
<td>f&lt;sub&gt;c&lt;/sub&gt;</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lamé parameter</td>
<td>g</td>
</tr>
<tr>
<td>$\mu$</td>
<td>shear modulus</td>
<td>h</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Compliance</td>
<td>k</td>
</tr>
<tr>
<td>$\overline{\xi}$</td>
<td>normalised compliance</td>
<td>p</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>u</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Stress</td>
<td>v</td>
</tr>
<tr>
<td>$\phi$</td>
<td>scalar potential of sediment</td>
<td>z</td>
</tr>
<tr>
<td>$\psi$</td>
<td>vector potential of sediment</td>
<td>A, B, C, D</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular frequency</td>
<td>H</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>bulk modulus of fluid</td>
<td>P</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>wavelength of waves</td>
<td>Q</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>scalar potential of fluid</td>
<td>R</td>
</tr>
</tbody>
</table>

Table 2.1: List of vectors and parameters used throughout this chapter.
Infra-gravity waves are pervasive throughout the world’s ocean basin. In the Pacific, the frequency of storms and tropical depressions provides strong sources of ocean waves which can be converted to infra-gravity waves and hence the infra-gravity wave environment is stable both in time and space (Webb and Crawford 1999). In uniform water depth, the infra-gravity wave directional spectrum is very broad (Webb and Cox 1984). Infra-gravity waves cause the low-frequency (<0.03Hz depending on water depth) pressure field on the ocean bottom along with tides, earthquakes and tsunamis (Filloux 1980; 1983) and their temporal and spatial stability as well as the nature of the directional spectrum make these long-period waves an ideal source for long term seafloor compliance studies.

Though the infra-gravity wave band consists of both freely travelling waves and forced waves, at any given frequency, the forced waves have smaller wavelengths than the free ones (Crawford et al. 1991). Webb et al. (1991) showed, through experiments and modelling studies, that forced waves are not a contributor to the observed seafloor pressure signal at periods greater than 300s (a period that is water depth dependent). Hence, only the freely travelling surface gravity waves need to be considered as a seafloor forcing source, which is ideal, since their behaviour is predictable – they obey the surface gravity wave dispersion relation (Apel 1987):

\[ \omega^2 = gk(\omega) \tanh(k(\omega)H) \]  

(2.1)

where \( \omega \) is the angular frequency, \( k \) is the wave number, \( g \) is the acceleration due to gravity and \( H \) the water depth. The wavelength of the waves at the surface of the ocean sets the maximum frequency of the surface gravity waves which are capable of palpitating the seafloor.

Using the infra-gravity wave as a source requires knowledge of the amplitude of its pressure signal at the seafloor. This amplitude can be related to the sea surface height through a general expression which is derived here. The initial problem to be considered is that of a double half-space where the gravity wave exists in the water over sediment. In 2D, the displacement, \( u(u, w) \), of the gravity wave is a function of time, \( t \), and the Cartesian

\[ \text{Forced waves are associated with groups of short period waves (Webb et al. 1991)} \]
components, $x$ and $z$. The displacement is the gradient of only the scalar potential\(^3\), $\Phi$, whose differential equation in Fourier domain can be written as

$$
\frac{d^2\Phi}{dz^2} - \left( k^2 - \frac{\omega^2}{\alpha_f^2} \right) \Phi = 0
$$

(2.2)

where $k$ is the wave number, $\alpha_f$ is the compressional wave velocity in the water. Solutions to the Equation 2.2 are of the form:

$$
\Phi = S \cosh (qz) + T \sinh (+qz)
$$

(2.3)

where $q = \sqrt{k^2 - \omega^2/\alpha_f^2}$ from which the vertical displacement, $w$, can be computed as

$$
w = \frac{d\Phi}{dz} = q[S \sinh(qz) + T \cosh(+qz)]
$$

(2.4)

The fact that the gravity wave is a pressure source provides a second option for computing the vertical displacement. The gravity waves increases pressure in the water column and this local pressure can be related to the sea surface displacement potential through Bernoulli’s equation for incompressible flow (Prager 1961). Written in Fourier domain, the displacement potential for a streamline directly below the sea surface is

$$
\left( -\omega^2 \Phi + \frac{1}{2} \omega^2 k^2 \Phi^2 + \frac{p_f}{\rho_f} \right)_{z=0} = 0
$$

(2.5)

where $\rho_f$ is the density of the sea water and can be reduced to

$$
(-\omega^2 \Phi + p_f / \rho_f)_{z=0} = 0
$$

(2.6)

Because, for the gravity waves, the second term in Equation 2.5 is small in comparison to the other two. Hydrostatic equilibrium on the streamline leads to a perturbation pressure, $p_f$:

$$
p_f |_{z=0} = -\rho_f gw |_{z=0}
$$

(2.7)

---

\(^3\) $\Phi(k, z, \omega)$ is the only displacement gradient considered since fluids do not support shearing.
The minus sign is included to account for the opposing directions of excess pressure and displacement (an increased sea surface height is above the sea surface (the equilibrium position) which causes an opposing displacement of the seafloor). Combining Equations 2.6 and 2.7 leads to

\[-\omega^2 \Phi |_{z=0} = g w |_{z=0} \]  \hspace{1cm} (2.8)

Treating the seawater as a uniform liquid, then Newton’s law is \( \frac{\partial^2 u}{\partial t^2} = -\frac{\nabla p_f}{\rho_f} \). Given \( \nabla \cdot \mathbf{u} = -\kappa \rho_f \) where \( \kappa \) is the bulk modulus, then

\[ p_f = -\kappa \nabla^2 \Phi \]  \hspace{1cm} (2.9)

when \( \mathbf{u} \) is written as the gradient of the scalar potential. This then leads to the wave equation of the sea water

\[ \rho_f \frac{\partial^2 \Phi}{\partial t^2} = \kappa \nabla^2 \Phi \]  \hspace{1cm} (2.10)

Combining Equations 2.9 and 2.10 with Equations 2.7 and 2.8 leads to an equation that governs the pressure in the water column:

\[ p(k, z, \omega) = -\rho_f g w(k, 0, \omega) \left[ \cosh(qz) + \frac{T}{S} \sinh(qz) \right] \]  \hspace{1cm} (2.11)

Under the assumption that the seafloor is rigid (i.e., the lower half-space) there can be no displacement caused by the gravity wave. Therefore, setting Equation 2.4 equal to zero results in \( T = -S \tanh(qz) \). By the same seafloor rigidity assumption, Equation 2.11, the pressure applied to the seafloor by the gravity wave can be written at \( z = H \) as

\[ p(k, z, \omega) = -\frac{\rho_f g \zeta(k, 0, \omega)^4}{\cosh(qH)} \]  \hspace{1cm} (2.12)

\[ ^4 \text{The derivation is that of Latychev and Edwards (2003).} \]
where $\zeta$ is the infra-gravity wave height. This expression demonstrates that gravity waves decay exponentially from the source. For infra-gravity waves, $k$ is larger than $\omega/\alpha$ thus $q$ can be approximated to $k$. The $\cosh(kH)$ term greatly attenuates the part of the gravity wave spectrum with short wavelengths (high frequencies), thus only waves with wavelengths comparable or greater than the water depth can reach the seafloor and cause variations in pressure amplitudes (Crawford et al. 1998). It turns out that the wavelengths of these waves are between half and twice the water depth, i.e. $0.5H < \Lambda < 2H$ (Crawford et al. 1991). It then follows that $k = 2\pi/nH$ for $0.5 < n < 2$ which leads to a cut-off frequency for infra-gravity pressure at the seafloor: $f_c = \sqrt{g/2\pi nH}$, for $\tanh(k(H)) \approx 1$ for $n \leq 2$. If the $H = 1250m$, then $0.025Hz < f_c < 0.05Hz$ which corresponds to waves of periods greater than 20 seconds and wavelengths greater than 623m.

The simple wavenumber-frequency relation, Equation 2.1, was also derived on the assumption of a rigid seafloor. However, Latychev (2000), taking into account the seafloor deformation, arrived at a dispersion relation: $\omega^2 = gk \sinh(kH)/(\cosh(kH) - \Gamma)$ where $\Gamma = \rho_f g \xi/\cosh(kH)$, all other symbols are defined in Table 2.1. He determined however, that for water depths of ~ 1 km and Lamé parameters on the order of $10^9$ Pa, $\Gamma$ is less than a percent. This result then justifies the use of the simplified dispersion relation to set the wavenumbers for infra-gravity wave seafloor forcings. Latychev (2000) cautioned that for applications requiring long wavelengths (up to 30 km) gravity waves, that data analysis should include the $\Gamma$ correction.

2.2.2 Pressure spectrum at NEPTUNE Canada Site 889

A pressure instrument deployed on NEPTUNE Canada seafloor observatory in the eastern Pacific Ocean at Site 889, confirm the ubiquitousness of waves with periods greater than 30s. Figure 2.1 shows both the pressure spectrum of the seafloor forcing waves at 1250 m and the earth response (which will be discussed in Section 2.3) on 3rd October, 2010 and 21st October, 2010 as a function of frequency and wavelength (related to each other through Equation 2.1). The character of the pressure spectra for 3rd October, 2010 is comparable to those of other similar experiments (Webb and Cox 1984; Trevorrow et al. 1988; Yamamoto
et al. 1989; Willoughby and Edwards 2000) deployed in water depth which produces a comparable upper frequency band cut-off. The pressure power versus frequency plot of Figure 2.1A shows significant energy in two distinct bands, the infra-gravity band (less than 0.03Hz) and the microseismic band (greater than 0.1Hz). Typically, the microseismic band, whose peak shifts between 0.1Hz and 5Hz, is always present (Web et al. 1991). Microseisms are a result of non-linear interactions, near coastlines, of surface waves with opposing propagational directions. They differ from infra-gravity waves in that they do not attenuate with depth and their frequency is twice the frequency of the waves from which they are generated, whereas infra-gravity wave’s frequency is identical to their source waves and their energy attenuate with depth. Microseisms arrive at the deep ocean by propagating as elastic waves through the ocean and the seafloor (Webb and Cox 1986).

Between 0.1Hz and 0.03Hz, the energy falls into a ‘noise notch’ and represents the background noise which is sometimes lower than the inherent noise of the DPG (Appendix G) (Webb and Cox 1984). Webb et al. (1991) suggest that the high reduction of energy in this band is due to the difficulty for lower frequency (hydrodynamic) or higher frequency (acoustic) wave energy to couple into waves that can propagate to the deep ocean. They also reported that the occasional signal in this band is from Rayleigh waves from large earthquakes (Figure 2.1C shows the resulting power spectra of 21st October, 2010 during which the Gulf of California magnitude 6.7 earthquake occurred and gives clear evidence of energy in the ‘noise notch’). Webb and Cox (1986) listed two other causes for pressure fluctuation in this band: (i) atmospheric waves and (ii) turbulent flow.

In general, at the lower end of the infra-gravity frequency band, the measurement of the recorded pressure signal is limited by the magnitude of the instrument noise. The upper limit of this is predicted by the dispersion relation (Equation 2.1) and in Figure 2.1, it is consistent. The pressure, as a function of wavelength in Figure 2.1, validates the approximation of the wavelength of the waves (0.5H < Λ < 2H) which produces a signal at the seafloor. From the pressure spectra of Figure 2.1 B and D it is evident that the pressure signal increases at and continues beyond wavelengths greater than the water depth (vertical black line). This result affirms the idea that seafloor pressure variations are mainly due to freely propagating gravity waves.
Figure 2.1: Pressure and velocity power spectra for the 3\textsuperscript{rd} October, 2010 (Row 1) and 21\textsuperscript{st} October, 2010 (Row 2) recorded at the seafloor at the NEPTUNE Canada SITE 889 by the DPG and broadband seismometer, respectively. A and B characterise the power spectra, recorded by the DPG over the period of the experiment, as a function of frequency and wavelength, respectively. C and D show the pressure and velocity spectra on the day of the Gulf of California earthquake. The vertical lines in B and D indicate the water depth of the seismometer-DPG installation (1250 m). On both days, the velocity spectra mimic the pressure spectra and the level of coherence between them will be evaluated in Chapter 4.
2.3 Seafloor compliance: the interaction of the infra-gravity wave with the seafloor

The seafloor is not strictly rigid; indeed it is displaced a little by the stress applied by the infra-gravity waves. Figure 2.1 displays the velocity spectrum recorded by the seismometer that is paired with the pressure instrument (DPG) at NEPTUNE Canada Site 889. These velocities are a measure of the displacement of the sediment in response to the pressure field on the seafloor. The ratio of the vertical component of seafloor stress, $\tau_{zz}(\omega)$, to the vertical component of the seafloor displacement, $W(\omega)$, as a function of frequency, $\omega$ defines vertical seafloor compliance, $\xi(\omega)$:

$$\xi(\omega) = \frac{W(\omega)}{\tau_{zz}(\omega)}$$  (2.13)

The value of this transfer function, as a function of frequency, of a given sedimentary model is attained by solving the seismic equation of motion. Newton’s law for an elastic solid can be written as

$$\rho \frac{\partial^2 u_r}{\partial t^2} = \frac{\partial \tau_{rs}}{\partial x_s}$$  (2.14)

where $\rho_b$ and $\tau_{rs}$\(^5\) represent the bulk density and the stress of the medium, respectively. If the medium is isotropic and linear, Hooke’s law, which relates the applied stress to the strain of the material, is given as:

$$\tau_{rs} = \lambda \varepsilon \delta_{rs} + 2\mu e_{rs}$$  (2.15)

where $\varepsilon$, the dilation, is the summation of $\partial_r u_r$, $\delta$ is the Kronecker delta, $e_{rs}$, the strain is $\frac{1}{2}(\partial_z u_r + \partial_r u_z)$. Substituting Equation 2.15 into Equation 2.14, where $\lambda$ and $\mu$ are constant, results in:

\(^5\) The Cartesian coordinate vector notation: any vector $\mathbf{x}$ has general components $x_i$ where $i=1, 2, 3$ and components $(x, y, z)$. The displacement vector $u_r$ corresponds to $(u, v, w)$. Summation is implied over repeated indices.
\( \rho_b \frac{\partial^2 u_r}{\partial t^2} = (\lambda + \mu) \frac{\partial \varepsilon}{\partial x_r} + \mu \frac{\partial^2 u_r}{\partial x_s^2} \) \hspace{1cm} (2.16)

In a water depth of 1250m, the maximum frequency of infra-gravity waves that can deform the seafloor is 0.049Hz which corresponds to phase velocities\(^6\) \( c \), between 30.6 m/s and 122.5 m/s. The Earth’s compressional velocities, \( \alpha \), and shear velocities, \( \beta \), are large in comparison to these phase velocities and allow the second temporal derivatives of Newton’s law to be neglected – the static assumption\(^7\). Thus for a finite homogenous layer

\[(\lambda + \mu) \frac{\partial \varepsilon}{\partial x_r} + \mu \frac{\partial^2 u_r}{\partial x_s^2} = 0 \] \hspace{1cm} (2.17)

Following from Equation 2.17, Newton’s law can be represented in 2-D by the following equations

\[(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial z^2} + (\lambda + \mu) \frac{\partial^2 w}{\partial x \partial z} = 0 \] \hspace{1cm} (2.18)

\[\mu \frac{\partial^2 w}{\partial x^2} + (\lambda + 2\mu) \frac{\partial^2 w}{\partial z^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial z} = 0 \] \hspace{1cm} (2.19)

Assuming that the general solutions to these equations are of the form:

\[ u(x, z) = U(z) \sin(kx) \] \hspace{1cm} (2.20)

\[ w(x, z) = W(z) \cos(kx) \] \hspace{1cm} (2.21)

Applying Equations 2.20 and 2.21 to Equations 2.18 and 2.19 results in the following coupled differential equations in U and W:

\[-k^2(\lambda + 2\mu)U + \mu \frac{d^2 U}{dz^2} - k(\lambda + \mu) \frac{dW}{dz} = 0 \] \hspace{1cm} (2.22)

\[-k^2 \mu W + (\lambda + 2\mu) \frac{d^2 W}{dz^2} + k(\lambda + \mu) \frac{dU}{dz} = 0 \] \hspace{1cm} (2.23)

\(^6\) Recall phase velocity is \( c = \omega / k \) and \( \frac{1}{2} H < \Lambda < 2H \) where \( H \) is the water depth.

\(^7\) The static assumption is not a complete solution but is sufficient for this thesis.
U and W, in turn, satisfy the following decoupled fourth order equations

\[
\frac{d^4 U}{dz^4} - 2k^2 \frac{d^2 U}{dz^2} + k^4 U = 0 \tag{2.24}
\]

\[
\frac{d^4 W}{dz^4} - 2k^2 \frac{d^2 W}{dz^2} + k^4 W = 0 \tag{2.25}
\]

which are found by manipulating Equations 2.22 and 2.23. The fourth order differential equations have general solutions of the form:

\[
U(z) = (A + Bz)e^{-kz} + (C + Dz)e^{kz} \tag{2.26}
\]

\[
W(z) = (A_1 + B_1 z)e^{-kz} + (C_1 + D_1 z)e^{kz} \tag{2.27}
\]

where A, B, C, D, A_1, B_1, C_1, and D_1 are medium dependent constants. Substituting Equations 2.26 and 2.27 into Equations 2.22 and 2.23 leads to

\[
\frac{dW}{dz} = -\frac{1}{b} [a_1 A e^{-kz} + (a_1 z - 2k)B e^{-kz} + a_1 C e^{kz} + (a_1 z + 2k)D e^{kz}] \tag{2.28}
\]

where \(a_1 = k^2 + \frac{-k^2(\lambda+2\mu)}{\mu}\) and \(b = -\frac{k(\lambda+\mu)}{\mu}\) and when integrated and differentiated gives, respectively,

\[
W(z) = \frac{1}{bk^2} \left[ a_1 k A e^{-kz} + [a_1 (kz + 1) - 2k^2]B e^{-kz} - a_1 C e^{kz} - [a_1 (kz - 1) + 2k^2]D e^{kz} \right] \tag{2.29}
\]

\[
\frac{dW^2}{dz^2} = \frac{1}{b} \left[ a_1 k A e^{-kz} + [a_1 (kz - 1) - 2k^2]B e^{-kz} - a_1 k C e^{kz} - [a_1 (kz + 1) + 2k^2]D e^{kz} \right] \tag{2.30}
\]

Replacing W in Equation 2.21 with Equation 2.29 results in the following expression for the vertical displacement as a function of \(x\) and \(z\):
Assuming a periodic pressure, \( p(x, 0) = P(0) \cos(kx) \) is applied to the surface of a half-space \((z = 0)\). This pressure is the vertical stress \( \tau_{zz}(0) \) and thus the expression \( \tau_{zz} \) derived from Hooke’s law (Equation 2.15) can be used along with Equations 2.20, 2.21, 2.26 and 2.28 (see Appendix A for details) to formulate an expression for the maximum pressure applied to the surface of the half-space:

\[
P(0) = -2\mu k A - \frac{2\mu(\lambda + 2\mu)}{\lambda + \mu} B
\]  

By using the zero tangential stress condition, \( \tau_{xz} = 0 \), it is possible to obtain a relationship between the unknown parameters \( A \) and \( B \) (see Appendix A for details) as

\[
\frac{B}{A} = -\frac{k(\lambda + \mu)}{\mu}
\]  

The compliance on the surface of the half-space (plane \( z = 0 \)) can be written as the ratio of negative normal stress (applied pressure) to the vertical displacement, \( \xi = -\frac{w(0)}{P(0)} \) which leads to the expression for the compliance of a half-space (see Appendix A for details):

\[
\xi = -\frac{(\lambda + 2\mu)}{2k\mu(\lambda + \mu)}
\]  

Multiplying Equation 2.34 by the wavenumber of the source, \( k \) (which is determined from the standard dispersion relationship) removes the dependency of the compliance function on frequency resulting in the normalised compliance of a half-space being a constant:

\[
\bar{\xi} = -\frac{(\lambda + 2\mu)}{2\mu(\lambda + \mu)}
\]
The common approach for computing the compliance of a 1-D isotropic stratified earth model subject to plane wave loading is through solving the wave equation. This is because the pressure force excites compressional \((P)\) and vertical shear \((SV)\) wave which decay with depth\(^8\). In Fourier domain, the seismic wave equation is partitioned as
\[
d_{zz}^2 \phi - [k^2 - (\omega^2 / \alpha^2)] \phi = 0 \quad \text{and} \quad d_{zz}^2 \psi - [k^2 - (\omega^2 / \beta^2)] \psi = 0
\]
where \(\phi\), the scalar potential and \(\psi\), the \(y\)-component of the vector potential are solved. The resulting \(P\) and \(SV\) waves have associated body waves velocities, \(\alpha\) and \(\beta\), respectively, with amplitudes in each layer given as
\[
\phi = c_1 e^{-r_j z} + d_1 e^{+r_j z} \quad \text{and} \quad \psi = c_2 e^{-s_j z} + d_2 e^{+s_j z}
\]
where
\[
r_j = \sqrt{k^2 - (\omega^2 / \alpha_j^2)}, \quad s_j = \sqrt{k^2 - (\omega^2 / \beta_j^2)}; \quad \alpha_j = \sqrt{(\lambda_j + 2\mu_j)/\rho_j}, \quad \text{and} \quad \beta_j = \sqrt{\mu_j/\rho_j}
\]
where \(c_i, d_1, c_2\) and \(d_2\) are constants, \(j\) represents the layer, the imaginary operator, \(i\), is included for mathematical convenience. Calculating the compliance under the static assumption implies that the \(\omega^2 / \alpha^2\) and \(\omega^2 / \beta^2\) terms tend to zero and therefore, \(r_j = s_j = k\). However, this approach fails since the compliance function is undefined (both numerator and denominator of Equation 2.13 vanish). So in order to obtain a solution to the compliance, the second order terms of \(r\) and \(s\) are included – the quasi-static solution. This quasi-static method has been used to derive the analytic normalised compliance for an isotropic uniform half-space (Equation 2.35); first by Sorrels and Gofforth (1973), as well as Latychev and Edwards (2003) and Crawford (2004).

The dependence of seafloor compliance on sedimentary structure makes the measurements from this geophysical technique suitable for exploration of the shallow oceanic sediment. Specifically, the shear modulus of the sediments can be determined by inverting the vertical compliance data (Trevorrow and Yamamoto 1991) and can be employed in constraining shear velocities. In reality, two types of compliance exist: (i) the transfer function between the vertical displacement and the vertical stress; and (ii) the transfer function between horizontal displacement and the vertical stress (horizontal compliance). However, horizontal displacements caused by infra-gravity waves are very difficult to measure due to instrument limitations and the presence of seafloor currents which swamp the horizontal signal (Crawford et al. 1991). In this thesis, compliance refers to the vertical compliance.

---

\(^8\) See Latychev and Edwards (2003) for the nature of this decay.
2.4 New algorithm for calculating the static compliance

In the literature, there is a large number of numerical recipes for calculating the elastic response of the Earth to body waves (Latychev and Edwards 2003). However, there exist very few algorithms in the geophysical literature which calculate the compliance response of the Earth to elastic waves and there is none which calculates its response under fully static conditions only for a layered medium (a 1D code). A numerical recipe for the calculation of the compliance using the static solution is outlined in Section 2.3 is presented in the following section. This algorithm adopts the method described by Sorrels and Gofforth (1973) and a matrix expression that determines the static response of a medium to a surface load is deduced. The Thomson (1950)-Haskell (1953) method is then employed to propagate these equations through a layered medium by mapping the stresses and displacements in each layer to the next. The process enables the calculation of the ratio of the vertical displacement and the vertical stress (the transfer function) as a function of frequency.

2.4.1 Formulation of the new algorithm

In section 2.3, Newton’s second law and Hooke’s law was used to obtain, in the static limit, expressions in 1D for the amplitude of the horizontal, $U(z)$ and vertical displacements $W(z)$, of each layer of a layered medium as

$$U(z) = A e^{-kz} + B z e^{-kz} + C e^{+kz} + D z e^{+kz}$$  \hspace{1cm} (2.36)

$$W(z) = A e^{-kz} + \frac{k(\lambda+\mu)z+2(\lambda+3\mu)}{k(\lambda+\mu)} B e^{-kz} - C e^{+kz} +$$

$$\frac{-k(\lambda+\mu)z+2(\lambda+3\mu)}{k(\lambda+\mu)} D e^{+kz}$$  \hspace{1cm} (2.37)

The $x$-z horizontal, $\tau_{xz}$, and the vertical, $\tau_{zz}$ stresses are given as:

$$\tau_{xz} = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$  \hspace{1cm} (2.38)

$$\tau_{zz} = (\lambda + 2\mu) \frac{\partial w}{\partial z} + \lambda \frac{\partial u}{\partial x}$$  \hspace{1cm} (2.39)
Applying Equations 2.36 and 2.37 to Equations 2.38 and 2.39 at x=0, results in

\[
\begin{align*}
\tau_{xz} &= -2\mu k Ae^{-kz} - \frac{\lambda + \mu}{2k\mu(\lambda + \mu)z + 2\mu^2} B e^{-kz} + 2\mu k Ce^{+kz} + \\
&\quad \frac{2k\mu(\lambda + \mu)z - 2\mu^2}{\lambda + \mu} D e^{+kz} \\
\tau_{zz} &= -2\mu k Ae^{-kz} - \frac{2\mu(\lambda + 2\mu) + 2\mu k(\lambda + \mu)}{\lambda + \mu} B e^{-kz} + 2\mu k Ce^{+kz} + \\
&\quad \frac{2\mu(\lambda + 2\mu) - 2\mu k(\lambda + \mu)}{\lambda + \mu} D e^{+kz}
\end{align*}
\] (2.40) (2.41)

The Thomas-Haskell method is then used to propagate the displacements and stresses through the layered half-space. The problem is set up as follows. Equations 2.36, 2.37, 2.40 and 2.41 is used to formulate the following generic matrix which must be written for each layer:

\[
\begin{bmatrix}
U \\
W \\
\tau_{xz} \\
\tau_{zz}
\end{bmatrix} = 
\begin{bmatrix}
e^{-kz} & ze^{-kz} & e^{+kz} & ze^{+kz} \\
e^{-kz} & \frac{k(\lambda + \mu)z + (\lambda + 3\mu)}{k(\lambda + \mu)} e^{-kz} & e^{+kz} & -\frac{k(\lambda + \mu)z + (\lambda + 3\mu)}{k(\lambda + \mu)} e^{+kz} \\
-2\mu k e^{-kz} & -\frac{2k\mu(\lambda + \mu)z + 2\mu^2}{(\lambda + \mu)} e^{-kz} & 2\mu k e^{+kz} & 2\mu k e^{+kz} \\
-2\mu k e^{-kz} & -\frac{2k\mu(\lambda + \mu)z + 2\mu(\lambda + 2\mu)}{(\lambda + \mu)} e^{-kz} & -2\mu k e^{+kz} & -2\mu k e^{+kz}
\end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}
\]
(2.42)

Figure 2.2 illustrates the geometry of the problem. It is considered 2-dimensional in the x-z plane so the wave-front is parallel to the y-z plane and the displacement components, parallel to the x and z axes, are independent of y. The origin of z is at the (m-1)th interface. The boundary conditions describing the stresses and displacements and other assumptions about the model are listed in the Figure 2.3.
Figure 2.2: A representation of the layered model. $R_m$, a vector of stresses and displacement of the $m^{th}$ layer; $F_m$ and $E_m$ are the upper and lower linking matrices; and $Q_m$, a vector of the unknown parameters of the layer. $R$ and $Q$ can be written for each interface while $F$ and $E$ can be written for each layer.

![Layered Model Diagram]

Boundary Conditions and Assumptions

1) For a solid-solid interface the displacements and stresses are equal. i.e.

$$w_m = w_{m-1}, \quad u_m = u_{m-1}, \quad \tau_{zz_m} = \tau_{zz_{m-1}}, \quad \tau_{xz_m} = \tau_{xz_{m-1}}$$

2) For a solid-liquid interface, displacements and vertical stresses are equal but the horizontal stresses are zero.

3) The force at the seafloor is due to gravity waves

4) A uniform half-space underlies the layered model

Figure 2.3: The boundary conditions for interfaces in a layered medium.
For the homogeneous $m^{th}$ layer say, $A_m$, $B_m$, $C_m$, $D_m$ represents the unknown constants (to be determined through satisfying the boundary conditions), $\lambda_m$ and $\mu_m$ are the Lamé constants and shear modulus, respectively and, $z_m$ the layer thickness. Let

$$Q_m = \{ A_mB_mC_mD_m \}$$  \hspace{1cm} (2.43)

$$R_m = \{ u_m w_m \tau_{xz_m} \tau_{zz_m} \}$$  \hspace{1cm} (2.44)

and $E_m$ represent the matrix which links the parameters in the $m^{th}$ layer to the upper interface, $F_m$ represent the matrix which link the parameters in the $m^{th}$ layer to the lower interface. The vector expressions of Equations 2.43 and 2.44 allow for the use of a matrix equation to describe the relationship between the parameters of the $m^{th}$ layer, $Q_m$, the displacements and stresses for the upper and lower interfaces for the $m^{th}$ layer, $R_{m-1}$ and $R_m$ respectively, and the linking matrices, $E_m$ and $F_m$ for each layer as

$$R_{m-1} = E_m Q_m$$  \hspace{1cm} (2.45)

$$R_m = F_m Q_m$$  \hspace{1cm} (2.46)

$E_m$ is obtained by setting $z$ in Equation 2.42 to 0 and $F_m$ is obtained by setting $z = z_m$ (by the geometry of the problem)

An expression which describes how the stresses and displacements of the $m^{th}$ interface are linked to those at the $(m-1)^{th}$ interface can be formulated by substituting Equation 2.45 into Equation 2.46. It is:

$$R_m = F_mE_{m-1}^{-1}R_{m-1}$$  \hspace{1cm} (2.47)

Likewise, for the $(m-1)^{th}$ layer, $R_{m-2} = E_{m-1}Q_{m-1}$ and $R_{m-1} = F_{m-1}Q_{m-1}$. Therefore, the stresses and displacements of the $(m-1)^{th}$ interface can be linked to those at the $(m-2)^{th}$ through $R_{m-1} = F_{m-1}E_{m-1}^{-1}R_{m-2}$. It follows that the stresses and displacements at the $m^{th}$ interface can be mapped directly to those at the $(m-2)^{th}$ interface by

$$R_m = F_mE_{m-1}^{-1}F_{m-1}E_{m-1}^{-1}R_{m-2}$$  \hspace{1cm} (2.48)
This provides a method for propagating the stresses and displacement at the deepest interface \((R_m)\) to the stresses and displacements at the surface \((R_0)\). Thus a general matrix relationship between \(R_m\) and \(R_0\) can be written as:

\[
R_0 = E_1 F_1^{-1} \ldots E_{m-1} F_{m-1}^{-1} E_m F_m^{-1} R_m
\]  

(2.49)

The stresses and displacements at the surface can then be linked to the parameters in the bottom most layer, the \(m^{th}\) layer, through the relationship \(R_0 = E_1 F_1^{-1} \ldots E_{m-1} F_{m-1}^{-1} E_m Q_m\). And for convenience

\[
R_0 = MQ_m
\]  

(2.50)

where \(M = E_1 F_1^{-1} \ldots E_{m-1} F_{m-1}^{-1} E_m\). The \(E\) and \(F\) matrices for each layer are formulated by setting \(z\) of the matrix in Equation 4.42, respectively, to 0 and the thickness of the layer and the elastic parameters to the specific value in the layer.

Using the matrix formulation in Equation 2.50, the compliance of a uniform half-space can be determined. The half-space is a single layer with a single interface; a fluid (sea water)-solid (sediment) boundary. Thus by the boundary conditions (Figure 2.3), \(\tau_{xz} = 0\). Also, given that there are no sources within the half-space, as \(z_{hs} \to \infty\) all the terms containing \(e^{+kz}\) vanish therefore, parameters \(B_{hs}\) and \(D_{hs}\) can be set to zero. Lastly, \(M = E_{hs}\) since the half-space has a single interface. Thus, for the half-space, Equation 2.50 can be formulated as \(R_{surface} = E_{hs} Q_{hs}\)

\[
\begin{bmatrix}
U_{surface} \\
W_{surface} \\
\tau_{zzsurface}
\end{bmatrix}
= \begin{bmatrix}
e_{11} & e_{12} & e_{13} & e_{14} & A_{hs} \\
e_{21} & e_{22} & e_{23} & e_{24} & B_{hs} \\
e_{31} & e_{32} & e_{33} & e_{34} & 0 \\
e_{41} & e_{42} & e_{43} & e_{44} & 0
\end{bmatrix}
\]  

(2.51)

Thus the compliance of the half-space is given as

\[
\xi = \frac{W_{surface}}{\tau_{zzsurface}} = \frac{e_{21} + e_{22} \left( \frac{B_{hs}}{A_{hs}} \right)}{e_{41} + e_{42} \left( \frac{B_{hs}}{A_{hs}} \right)}
\]  

(2.52)
Manipulating row 3 of Equation 2.51 allows for a relationship between the unknown parameters of the half-space to be formulated:

\[
\frac{B_{hs}}{A_{hs}} = -\frac{e_{31}}{e_{32}}
\]  

(2.53)

Letting the origin coincide with the interface means that \( z = 0 \), thus the relevant elements of Equation 2.42 for solving Equation 2.52 are:

\[
e_{21} = 1, \quad e_{22} = \frac{(\lambda_{hs} + 3\mu_{hs})}{k(\lambda_{hs} + \mu_{hs})}
\]  

(2.54)

\[
e_{31} = -2k\mu_{hs}, \quad e_{32} = -\frac{2\mu_{hs}^2}{(\lambda_{hs} + \mu_{hs})}
\]  

(2.55)

\[
e_{41} = -2k\mu_{hs}, \quad e_{42} = -\frac{2\mu_{hs}(\lambda_{hs} + 2\mu_{hs})}{(\lambda_{hs} + \mu_{hs})}
\]  

(2.56)

Substituting Equations 2.54 to 2.56 into Equation 2.52 results in the analytic solution for a half-space: \( \xi(\omega) = -(\lambda + 2\mu)/2k\mu(\lambda + \mu) \).

To solve the multilayered problem, the \( M \) matrix is set up where \( E_{m,*} \) and \( F_{m,*} \) \((* = 0, 1, 2, \ldots, m)\) for each layer are obtained by substituting \( z = -z_{m-*} \) and \( z = 0 \), respectively. The compliance is then calculated by applying the boundary conditions as applied in obtaining the analytic half-space solution.

### 2.4.2 The validation of the new 1-D algorithm

The algorithm described above was coded in MATLAB (Appendix E) and is very efficient. It calculates the compliance of a 50 layer model for 200 frequencies on a 2.33 GHz Intel Core 2 Duo CPU with 2.99 GB of RAM within a few seconds. This new code will be referred to as the Matrix Method (MM). The MM algorithm was validated through two processes. One process involved comparing the solution of the half-space produced by the MM to the analytic solution. Here, two models were used: (i) a single layer half-space, and (ii) a multi-
layered half-space. The lack of available analytic solutions for the compliance of multi-layered models dictated that the second validative process involved comparing the compliance calculated by the MM to that determined by other codes. The two independent codes available were: (i) a 1D vector propagation matrix method developed by Wayne Crawford, referred to here as WC, which computes the compliance using the full solution and (ii) a 3-D finite difference code developed by Konstantin Latychev that computes the static compliance solution and is referred to as KL. In this second process, the compliance of a layer over a half-space model was compared. The comparative results are stated as the percentage difference between the MM compliance solution and the other methods and is denoted, $e_{MM}$.

Computing the compliance response of a sedimentary model requires the Lamé parameters and the wavelengths (or frequencies) of the infra-gravity waves. Here, the range of wavelengths was set between 100m (0.12485 Hz) and 20,000m (0.0054Hz). For single layer half-space test, the compliance of a range of single-block half-spaces with $\lambda/\mu$ ratios ranging between 0.6 and 43 were calculated. The elastic parameters, the $\lambda/\mu$ ratio, and the MM and analytic compliances of one of these single-block half-spaces (HS1) are listed in Table 2.2. The MM solution is identical\(^9\) to the analytic solution for the full range of loading wave frequencies. The MM also produces an identical result to the analytic half-space solution when the half-space (with the same parameters as the single-block half-space) was divided into 50 layers, each 20m thick (Table 2.2). Upon comparing the performance of the MM to the KL and WC methods in determining the compliance of a half-space; the MM performs best. The KL code solution differs from the analytic one by 0.8% and the WC by as much as 5% (Latychev 2000). The largest differences are seen in the longer wavelength (low frequency) region.

---

\(^9\) The percentage difference between the MM and the analytic solution ranges between 0 and $5 \times 10^{-14}$ percent of the analytic solution.
<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda$ (GPa)</th>
<th>$\mu$ (GPa)</th>
<th>$\xi_a$ (GPa$^{-1}$)</th>
<th>$\xi_{MM}$ (GPa$^{-1}$)</th>
<th>$e_{MM}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single layer HS1</td>
<td>4.71</td>
<td>0.314</td>
<td>1.692</td>
<td>1.692</td>
<td>0 - 10$^{-14}$</td>
</tr>
<tr>
<td>Multi-layered HS1</td>
<td>4.71</td>
<td>0.314</td>
<td>1.692</td>
<td>1.692</td>
<td>0 - 10$^{-12}$</td>
</tr>
<tr>
<td>200m layer over HS2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer</td>
<td>7.06</td>
<td>0.899</td>
<td>0.617</td>
<td>0.617</td>
<td>N/A</td>
</tr>
<tr>
<td>HS2</td>
<td>5.56</td>
<td>0.254</td>
<td>2.055</td>
<td>2.055</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of results of the matrix method (MM) validation. Details the descriptive parameters of the 3 models – the half-space and multi-layered half-space (process one) and the layer-over-half-space (process two); the analytic (a) and MM solution of the compliance for the set of frequencies between 0.005Hz to 0.12 Hz for each layer of all three models. It also lists the percentage difference between the analytic solution and the MM solution for half-space models. Key: HS – Half-space. Values are determined using the predictive model described in Section 4.3.2.1.

Phase two of the validation process verified that the MM is capable of reproducing the expected 200m layer over half-space (HS2) characteristic compliance curve (Figure 2.4). Table 2.2 lists the elastic parameters and $\lambda/\mu$ ratio of this layer over half-space model. The analytic solution of the compliance of HS2 and of a half-space with the 200m layer values are also listed in Table 2.2. These analytic compliance solutions represent the lower (magenta) and upper (green) bound, respectively, for the layer-over-half-space solution (Figure 2.4). The magenta line is the analytic compliance solution of a half-space with the Layer values (Table 2.2) while the green line represents the analytic compliance solution for the half-space (HS2 values – Table 2.2). The differences between MM compliance solution and that produced by the KL and WC codes were negligible at frequencies above 0.03Hz. Furthermore, the solutions approach each other at the higher frequencies (>0.08Hz) and match the lower compliance bound (that of the half-space (HS2), as expected). At frequencies below 0.03Hz, the MM and the KL compliance solutions fall within 2.8% of each other, while the difference between the MM and WC method increases up to 5.7%. Moreover, the MM and KL solutions approach the analytic half-space at the lower frequencies where as the WC overshoots it. However, the MM performs best at very low frequencies because as shown in Figure 2.4, its solution is identical to upper bound.
Figure 2.4: Each ‘curve’ represents the compliance as a function of frequency for half-spaces (solid horizontal lines) and for layer over half-space (non-solid lines) using three computational methods. Table 2.2 lists elastic parameter values. Solid magenta line – the analytic half-space solution using values of HS2; solid green line – the analytic half-space solution for a half-space with values of the 200m layer; dashed dotted red line, solution for the layer-over half space model using the KL code, dashed blue line – solution for the layer-over half space model using the WC and dotted solid black line – solution for the layer-over half space model using the MM. Key: HS – half-space, Lay – layer, KL – Konstantin Latychev code, WC – Wayne Crawford code, MM – matrix method code.

The nature of the layer over a half space compliance curve is evidence of a relationship between compliance-frequency and the depth of the sedimentary structure, i.e. shallow sedimentary structure is sensed at short wavelength (high frequencies) in the compliance function while deep structure is sensed by long wavelength (low frequencies). This frequency-depth-structure relationship can be investigated more formally through a sensitivity analysis and is presented next.
2.5 Seafloor compliance modelling for layered structure

The depth-frequency relationship allows for a better interpretation of seafloor compliance measurements and pinpoints the limitations of these measurements. Thus the usefulness of seafloor compliance in determining crustal structure is tied to understanding its tuning\textsuperscript{10} behaviour. In the following section, the results from an investigation of the nature of the depth-frequency relationship, through a sensitivity analysis, is presented. Additionally, an eigenparameter analysis is performed so as to provide insight into the resolution capabilities of the compliance function.

2.5.1 A sensitivity analysis

A kernel analysis can be used to quantify the compliance function’s sensitivity to each of the elastic parameters and to characterize the ‘tuning’ of the compliance function. It may be possible to perform this analysis analytically for a half-space but for a general case it is easier to resort to numerical methods. The scheme for calculating the sensitivity functions (software in Appendix E) is as follows. The model is broken into a finite number of layers each of finite thickness based on the resolution and depth required (e.g. 500 layers each of 10m thick for a total depth of 5000mbsf). Then to determine the sensitivity to a specific parameter (i.e. $\mu$) of the layered model, first, the compliance, as a function of frequency, is calculated for the model (termed the ‘original’ compliance). Then the parameter of interest (here $\mu$), of the top (or first layer) is increased by a specific percent (for e.g., 10%), thus creating a new model for which the compliance is then calculated. The percentage difference between the compliance of the original model and the new model, with respect to the original model, is then determined as a function of frequency. The percentage difference is scaled by dividing by the thickness of the layer and the percent variation for a sensitivity to the parameter in a unit – per meter. This process is then repeated for each of the layers in turn. The result is a set

\textsuperscript{10}The verb ‘tune’ is used as a synonym for the compliance depth-frequency relationship. It refers to the fact that specific frequencies are most sensitive to specific depths (i.e. where the sensitivity kernel is maximum for each frequency).
of curves which describes the sensitivity to a parameter at a specific depth, for a given range of frequencies.

Here, these partial derivatives were computed for two simple models: a half-space and a layer over a half-space. They were divided into 500 – 10m thick layers for a total depth of 5000mbfs and the compliance function was evaluated between wavelengths of 50m to 10km. For both models, the parameters $\rho, \lambda, \mu$ were investigated by perturbing each layer by 10%. Figure 2.5A shows the sensitivity function at 0.03Hz for a half-space described by the elastic parameters of the half-space (Table 2.3) while Figure 2.5B shows the sensitivity for a 220m layer-over a half-space (values are listed in Table 2.3 as layer 1 and half-space). For both models, the compliance function is insensitive to the density. This is the expected result since the analytic solution of a half-space (Equation 2.35) has no density dependence; however, in the compliance literature there has always been a density dependence, an artefact of the predictive models used to determine the elastic parameters. The predictive model for formulating the elastic parameters in this thesis (described in Chapter 4) eliminates the artificial density dependence.

<table>
<thead>
<tr>
<th>Description</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$\lambda$ (GPa)</th>
<th>$\mu$ (GPa)</th>
<th>$\varepsilon$ (1/GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>1979</td>
<td>6.53</td>
<td>0.332</td>
<td>1.58</td>
</tr>
<tr>
<td>Layer 2</td>
<td>1954</td>
<td>9.05</td>
<td>5.57</td>
<td>0.124</td>
</tr>
<tr>
<td>Half-space</td>
<td>2320</td>
<td>8.81</td>
<td>0.536</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Table 2.3: Parameters describing the models and analytic half-space compliance solution of each layer treated as a half-space.
Table 2.4: Values of variables required in the TPBE estimate of the elastic parameters in Table 2.3. Column ‘Comparative ξ’ tells how the compliances of the layers compare to the half-space.

<table>
<thead>
<tr>
<th>Description</th>
<th>Porosity (%)</th>
<th>Clay Content (%)</th>
<th>Hydrate Conc. (%)</th>
<th>Comparative ξ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>40</td>
<td>80</td>
<td>30</td>
<td>Softer</td>
</tr>
<tr>
<td>Layer 2</td>
<td>40</td>
<td>80</td>
<td>100(^\text{11})</td>
<td>Stiffer</td>
</tr>
<tr>
<td>Half-space</td>
<td>20</td>
<td>80</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2.5 highlights the response of the compliance function to both elastic parameters. In this example, at 0.03Hz, the sensitivity to \( \lambda \) decreases monotonically with increasing depth while the sensitivity to \( \mu \) has a skewed bell shape which peaks at 260m. It is clear that the compliance function is much more sensitive to the \( \mu \) than it is to \( \lambda \). The comparative scale is dependent on the values of the elastic parameters. This is an expected result since the analytic partial derivatives\(^\text{12}\) of the compliance of a half-space (Equation 2.35) with respect to the lame parameter and the shear modulus are 

\[
\frac{\partial \xi}{\partial \lambda} = -\frac{1}{(2k(\lambda + \mu)^2)} \quad \text{and} \quad \frac{\partial \xi}{\partial \mu} = -\frac{(2\mu^2 + 2\lambda\mu + \lambda^2)}{(2k\mu^2 (\lambda + \mu)^2)}.
\]

A clearer way of demonstrating that the compliance function is particularly sensitive to the shear modulus is by evaluating Equation 2.35 under the condition that \( \mu \ll \lambda \). That is \( \bar{\xi} = (\lambda + 2\mu)/(2\mu(\lambda + \mu)) = (1 + 2\mu/\lambda)/(2\mu(1 + \mu/\lambda)) \to 1/2\mu. \)

\(^{11}\) The 100% hydrate concentration is for demonstrating the compliance response to a very stiff layer in comparison to the half-space (values are derived from the triple-phase boundary equation described in Section 4.3.2.1)

\(^{12}\) The fact that derivatives are negative is consistent with the expectation that a more rigid structure would be less compressible; exhibiting less displacement for a given force. \( \lambda \) and \( \mu \) are always positive thus there is always a decrease in the magnitude of the compliance when there is an increase in either of the elastic parameters.
Figure 2.5 also gives a sense of the comparative sensitivity between both elastic parameters: the sensitivity to $\lambda$ is greatest in the very shallow sediments (<50mbsf) though in this instance, not much greater than 12 times that of $\mu$, while beyond 50mbsf, the sensitivity to $\mu$ surpasses that of $\lambda$ on the order of $10^3$.

Figures 2.6 and 2.7 show the compliance sensitivity to $\mu$ and $\lambda$ at 5 different frequencies of the same two models described above. At the different frequencies, the character of the $\lambda$-kernel remains exponential though the ‘rate of decay’ decreases with increasing frequency. The nature of the $\mu$-kernel is not as straightforward. It too varies according to the frequencies of investigation, however, the peak sensitivity at each frequency occurs at different depths within the sediment. It is this behaviour of the $\mu$-kernel that characterizes the tuning effect of the compliance function. An investigation of the relationship between the depth and the wavelength of the source waves for the two models revealed that the compliance function peaks at depths between 0.16 and 0.20 of the wavelength of the loading waves. Figure 2.8A and Figure 2.8B are plots of the normalized sensitivity as a function of depth for the full spectrum of frequencies used in the analysis of the half-space model. They elucidate the depth-frequency relationship of the compliance function. The figures are surface plots where the depth is plotted against the wavelength and the colour variation (scaled on the colour bar) shows the magnitude of the sensitivity function.
Figure 2.5: The sensitivity kernels of a half-space model (A) and a 200 m thick layer over a half-space (B). The solid red line shows the sensitivity of the compliance function to density, dotted dashed black line is the response to the Lamé parameter and the blue dashed line to shear modulus. The ‘kink’ in (B) results because the derivatives are not continuous across boundaries.

If $\lambda$ is replaced with the bulk modulus, $K$, i.e $\lambda = K - \frac{2}{3}\mu$, in the compliance solution, the derivatives of the compliance function with respect to $K$ are similar to the derivatives with respect to $\lambda$. That is, the sensitivity of the compliance function to $K$ and $\lambda$, as a function of frequency, are within a few percent of each other.
Figure 2.6: The sensitivity response for several frequencies of waves for a half-space model. (A) The sensitivity to the shear modulus and (B) the sensitivity to the Lamé parameter for the half-space with values of Layer 1. See Table 2.3 for parameter values. Each line in both graphs represents the response at specific frequencies listed in the legend.
Figure 2.7: The sensitivity response for several frequencies of waves for a layer over half-space model. (A) The sensitivity to the shear modulus, and (B) the sensitivity to the Lamé parameter for the layer (layer 1 values) over half-space (half-space values) model for both parameters. See Table 2.3 for parameter values. Each line in both graphs represents the response at specific frequencies listed in the legend.

In Figure 2.8A, the dark black band marks the depth-wavelength combinations with the greatest sensitivity. The linear trend of this band supports the tuning effect i.e. as the wavelength of palpitation increases, the depth of peak sensitivity also increases. The surface plot also shows the breath and sharpness of the sensitivity function at each frequency – as the wavelength increases, the images smears as one moves away from the central trending line. This tells that as the frequency increases, the peak gets less sharp and the sensitivity function...
become less localized. This means that deep structure is determined with lower resolution than shallow sediments. Though this analysis is only presented for a half-space, the same results were attained for the layer over the half-space model with the difference being the magnitude of the sensitivity.

Figure 2.8: (A) and (B) are surface plots of the sensitivity response as a function of depth and wavelength for the shear modulus and Lamé parameter, respectively. The legend describes the values of high and low sensitivity. (The curves of (A) and (B) are normalised to highlight the depth-frequency relationship).
2.5.2 Interpreting compliance curves using the sensitivity analysis results

Having completed the sensitivity analysis, the characteristic shape of Figure 2.4 can now be discussed. The nature of the compliance curve as a function of frequency, determined by all three formulations (WC, KL and MM), can be described by the tuning effect of the compliance function established through the sensitivity analysis. In the layer-over-half-space (Table 2.2 for values of 200m Layer over half-space – HS2) curves of Figure 2.4, at the higher frequencies, the compliance solution tends towards the layer values (high frequency bound) as expected since the layer is at the surface. At low frequencies, the curve approaches the low frequency bound which is the half-space values, again consistent, since the half-space is deeper.

The steep slope in the middle, a transition zone, is also a consequence of the nature of the sensitivity-depth relationship. Since the compliance function has its peak sensitivity between 0.16 and 0.20 of the wavelength of the pressure source, as the frequency of the loading wave increases, the sensing depth decreases and the compliance function ‘sees’ different parts of the sediment. In the frequency range of the transition zone, the sensitivity function is broad (not localised), so it senses a wide region. As the sensing frequencies decrease and approach the interface, both layers are ‘seen’ and the result is a superposition of both values.

To further demonstrate the compliance sensing ability and to demonstrate the ability of the matrix method to provide predictable compliance solutions, two embedded layer models were analysed. The parameters for these models are listed in Table 2.3. The models are of a 100m thick hydrate layer buried 50m below the seafloor within a half-space. The difference in each model lay in the values of the elastic parameters. Figure 2.9 shows the compliance response of the pair of models as a function of frequency and wavelength. Figures 2.9A and 2.9C show the response where the embedded layer’s compliance value is larger than the half-space while Figures 2.9B and 2.9D show the response for an embedded layer with a compliance value smaller than the half-space. These responses are aptly explained by the

---

14 The description of additional parameters used to determine the elastic parameters of the buried layers are listed in Table 2.4.
frequency–depth relationship given the relative compliance values of each of the individual layers.

In both cases, the upper and lower bounds of the compliance function are the same because the upper layer and half-space values are identical. Thus it is expected that at the higher and lower frequency ends of the spectrum, the compliance response will be that of layer/half-space. In the middle, as the frequency increases the compliance changes from the value of the half-space to a compliance value that is a combination of the layers sensed. The compliance response peaks at the frequency that maximizes/minimizes the difference between the layers sensed and then returns, at higher frequencies, to the half-space compliance value.

2.5.3 An eigenparameter analysis

The advent of generalized inverse theory has lead to the means by which the resolving power of geophysical measurements can be quantified. Eigenparameter analysis employs singular value decomposition (SVD) in determining errors in the parameters of a given model based on its resolution and on the noise in the measured data. The above sensitivity analysis illustrated how perturbing the parameters of a model affects its compliance response. The importance of a parameter is measured by the magnitude of the change it causes in the compliance when it is changed by a small value. Eigenparameter analysis is thus useful in two ways: (i) it allows the model to independently determine which of its parameters can be resolved by the measurements and to what accuracy and (ii) it allows the procedure of model fitting to be described in a physical way and thus be more meaningful.
Figure 2.9: The compliance as a function of frequency (A) and of wavelength (B) for the model depicted in E with values of Layer 1. (C) The compliance as a function of frequency and (D) of wavelength for model F. In each case, the layer is 100 m thick and is buried 50mbsf. See Table 2.3 for parameter values.
To set up an eigenparameter analysis, the model parameters are defined as $p_j, j=1, N$ where the $p_j$'s are $\lambda$, $\mu$ and $th$ (layer thickness). The data from which the model is determined are $Y_i, i=1,M$ ($M>N$) with each datum being the compliance at a specific frequency and having a standard error $e_i$. If the parameters of the sediments are slightly varied, this variation ($dp_j$) can be related to the expected change in the data ($dY_i$), through the matrix equation: $dY = Adp$. Each coefficient $A_{ij}$ is the partial derivative, $\partial Y_i/\partial p_j$; the sensitivity of $Y_i$ to a change in $p_j$. These partial derivatives are determined numerically using the central difference method (Matthews and Fink 2003).

$dY = Adp$ can be simplified to $dY_i^* = L_{iit} dp_j^*$ where $0 \leq i \leq N$ by performing a SVD on the matrix $A$ such that $A = U x L x V^T$. The diagonal elements of $L$ are the eigenvalues of $A$, the new parameters, $p^*$, are referred to as the eigenparameters and new data, $Y^*$, the eigendata (fully detailed in Edwards et al. 1981; Edwards 1997). The result is that each eigendatum independently determines the eigenparameter and a one-to-one eigendatum-eigenparameter correspondence. Edwards et al. (1981) showed that the standard errors in the eigendata are independent and concluded that the associated standard errors in an eigenparameter are the reciprocal of its corresponding element of the diagonal of the matrix $L$.

Edwards (1997) suggests that each element of $A$ be scaled in two ways before the SVD is applied: (i) divide each $\partial Y_i/\partial p_j$ term by the error, $e_i$, which rescales the units of the elements of the original data, $Y_i$, making the standard errors unity and (ii) multiply each $\partial Y_i/\partial p_j$ term by $p_j$, so as to redefine the model parameters as $log(p_j)$. This logarithmic scaling of the model parameters renders the eigensolution capable of representing physically interpretable combinations of the original model parameter, should they exist. The scalings result in the standard error of the eigenparameter being the fractional standard error of the combinations of the original parameters. The standard error is the measure of resolution of the eigenparameter; when the value for a specific eigenparameter is greater than unity, the parameter is said to be undetermined.

Accurately estimating the parameters is as important as determining how well they are resolved. Jupp and Vozoff (1975) showed that parameter inaccuracy is inversely proportional to the resolution and is directly proportional to the data inaccuracy. 'This data inaccuracy can
be translated to parameter inaccuracy as approximate ‘standard error bounds’ which measure changes in model parameters due to perturbations in the data’ – Vozoff and Jupp (1977). The standard error bound in each model parameter is calculated by summing the product of the weights of the model parameters and the standard error for each eigenparameter. This standard error bound is only valid when the standard error in the original parameter is less than 1 (Edwards 1997).

Four hypothetical models were analysed using this eigenparameter method: Model 1, a layer over a half-space model with parameter values of Layer 1 and Half-space in Table 2.3; Model 2, a layer over a half-space whose layer parameters are Layer 2 from Table 2.3; Model 3, a 100m layer buried 50mbsf within a half-space (values of layer are of Layer 1); and Model 4, similar to Model 3 but with the values of Layer 2 in Table 2.3. The frequency range at which the compliance is calculated is between 0.05Hz and 0.01Hz. These frequencies encompass the set of gravity waves that can cause deformation of the seafloor and have a coherent signal with the seafloor response. The uncertainty in the compliance data was determined from the NEPTUNE Canada BBS-DPG installation results (outlined in Section 4.3.2.3) and ranges between of 1% and 4.5% of the data. Each of the parameters was perturbed by ± 10% for the calculation of $\partial Y_i/\partial p_j$.

The results of the eigenparameter analysis of the four models described above are displayed in Tables 2.5 through 2.8. In the tables, the eigenparameters are ranked and ordered such that the eigenparameter with the smallest standard error is listed first (the first row). Each eigenparameter is a combination of the weights of the logarithm of each model parameter. Based on the magnitude of this weight, a physical interpretation of the eigenparameter can be deduced.

Table 2.5 displays the results of Model 1. For this model, the shear modulus of the half-space is resolved by the most important eigenparameter (EP1) as its weight is almost unity. Based on the eigenvalue, the compliance method is able to determine the shear modulus to within 0.5% of its true value when the error in the compliance measurement is 6%. The error bound on the shear modulus of the half-space suggests that it is determined very well, to 0.09%. However, since the gas hydrate layer in our Bullseye Vent system is estimated above the
220mbsf BSR, this resolution is of no importance to hydrate estimation. A look at the weights of the second best eigenparameters (with standard error greater than unity), eigenparameter 2 can resolve the shear modulus-thickness product of top layer to better than ~7% of its value. The third eigenparameter consists of a combination of the shear modulus and Lamé parameter of the layer as well as the Lamé parameter of the half-space – an unuseful combination. Eigenparameter 4 produces no interpretable physical property – its weights are divided between the three parameters of the layer and even at the lowest error in the original data (1%), the standard error is 94%. Below the dashed line, the resolving power of the compliance on this model is very poor since the standard errors for each eigenparameter is greater than unity.

Table 2.6 is the complement of Table 2.5, where instead, the layer is stiffer than the half-space. Here again, the most significant eigenparameter resolves the shear modulus of the half-space with the same accuracy while the accuracy of the estimate of the parameter is less at 0.17%. However, the similarity ends there. The second eigenparameter is able to resolve the thickness of the layer to within 2.2% of its value and the estimate on this thickness (or depth to the half-space) is determined to within 4.9%.

For the buried layer models 3 and 4, the first eigenparameter is able to resolve, like in the above cases, the shear modulus of the half-space with accuracies varying between 0 and 0.18%. In both cases, the accuracy of the estimate of this shear modulus is better than 0.1%. For the buried ‘soft’ layer, Model 3, the shear modulus –thickness product is the resulting physical characteristic resolved by eigenparameter 2 while for Model 4, the ‘stiff’ layer, it’s the thickness of the layer that is resolved (to within 2.1%). For both models, the remaining important eigenparameters do not produce any discernible physical characteristics.

In summary, the four models make it clear that the compliance method is very capable of resolving the shear modulus of the half-space (deep layers) reasonably well. However, the idea of the results being valid for a set of measurements of a specific kind on a particular model, as suggested by Jupp and Vozoff (1975), is not shown to be true in this simple analysis.
Table 2.5: The results of the eigenparameter (EP) analysis for layer over a half-space (using Layer 1 and Half-space values in Table 2.3). $\mu_1$ and $\mu_2$ are the shear modulus of the layer and the half-space respectively; $\lambda_1$ and $\lambda_2$ are the Lamé parameters of the layer and half-space, respectively; and $th_1$ is the thickness of the layer. The bolded numbers in each row are the weights of the most significant parameters which contribute to the eigenparameter of that row. The two rightmost columns show the standard error in the EP when estimated with errors in the original parameters of 1% and 6%, respectively. The row in red highlights the EP whose standard error is greater than unity when the uncertainty in the original data is 6%. The dashed line marks the EPs which have a standard error greater than unity for errors in the original data of 1%. The final row indicates the error bounds (EB) on the original parameters.

<table>
<thead>
<tr>
<th>EP</th>
<th>d log($\mu_1$)</th>
<th>d log($\mu_2$)</th>
<th>d log($\lambda_1$)</th>
<th>d log($\lambda_2$)</th>
<th>d log($th_1$)</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/L$_{\mu}$</td>
</tr>
<tr>
<td>1</td>
<td>0.0012</td>
<td>0.9953</td>
<td>0.0009</td>
<td>0.0011</td>
<td>0.0014</td>
<td>0.005</td>
</tr>
<tr>
<td>2</td>
<td>0.5509</td>
<td>0.0027</td>
<td>0.0249</td>
<td>0.0086</td>
<td>0.4129</td>
<td>0.071</td>
</tr>
<tr>
<td>3</td>
<td>0.1856</td>
<td>0.0000</td>
<td>0.4817</td>
<td>0.2687</td>
<td>0.0639</td>
<td>0.856</td>
</tr>
<tr>
<td>4</td>
<td>0.2615</td>
<td>0.0000</td>
<td>0.1480</td>
<td>0.0688</td>
<td>0.5217</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>0.0007</td>
<td>0.0019</td>
<td>0.3445</td>
<td>0.6528</td>
<td>0.0000</td>
<td>5E+04</td>
</tr>
<tr>
<td>EB</td>
<td>0.2813</td>
<td>0.0009</td>
<td>0.2095</td>
<td>0.1037</td>
<td>0.5092</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.6: The results of 100 metre thick layer (2) over a half-space (see Table 2.3 for values). Description of table parameters is as in Table 2.5.

<table>
<thead>
<tr>
<th>EP</th>
<th>d log($\mu_1$)</th>
<th>d log($\mu_2$)</th>
<th>d log($\lambda_1$)</th>
<th>d log($\lambda_2$)</th>
<th>d log($th_1$)</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/L$_{\mu}$</td>
</tr>
<tr>
<td>1</td>
<td>0.0018</td>
<td>0.9739</td>
<td>0.0002</td>
<td>0.0013</td>
<td>0.0227</td>
<td>0.0050</td>
</tr>
<tr>
<td>2</td>
<td>0.0761</td>
<td>0.0244</td>
<td>0.0042</td>
<td>0.0008</td>
<td>0.8945</td>
<td>0.0221</td>
</tr>
<tr>
<td>3</td>
<td>0.5820</td>
<td>0.0005</td>
<td>0.0045</td>
<td>0.3717</td>
<td>0.0412</td>
<td>1.3147</td>
</tr>
<tr>
<td>4</td>
<td>0.3047</td>
<td>0.0008</td>
<td>0.1399</td>
<td>0.5130</td>
<td>0.0416</td>
<td>5.3203</td>
</tr>
<tr>
<td>5</td>
<td>0.0354</td>
<td>0.0003</td>
<td>0.8512</td>
<td>0.1131</td>
<td>0.0000</td>
<td>10754</td>
</tr>
<tr>
<td>EB</td>
<td>0.3980</td>
<td>0.0017</td>
<td>0.1251</td>
<td>0.5363</td>
<td>0.0492</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.7: The results of 100metre thick layer (1) buried 50mbsf within a half-space (see Table 2.3 for values). Description of table parameters is as in Table 2.5.

<table>
<thead>
<tr>
<th>EP</th>
<th>d log(µ₁)</th>
<th>d log(µ₂)</th>
<th>d log(µ₃)</th>
<th>d log(λ₁)</th>
<th>d log(λ₂)</th>
<th>d log(λ₃)</th>
<th>d log(th₁)</th>
<th>d log(th₂)</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0014</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0049</td>
<td>0.0008</td>
</tr>
<tr>
<td>2</td>
<td>0.0061</td>
<td>0.9979</td>
<td>0.0003</td>
<td>0.0073</td>
<td>0.0343</td>
<td>0.0367</td>
<td>0.0315</td>
<td>0.5173</td>
<td>0.0262</td>
</tr>
<tr>
<td>3</td>
<td>0.0078</td>
<td>0.1233</td>
<td>0.0004</td>
<td>0.0550</td>
<td>0.3503</td>
<td>0.4628</td>
<td>0.0000</td>
<td>1.3612</td>
<td>0.2269</td>
</tr>
<tr>
<td>4</td>
<td>0.1500</td>
<td>0.1748</td>
<td>0.0000</td>
<td>0.0037</td>
<td>0.0225</td>
<td>0.0181</td>
<td>0.3300</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>0.5889</td>
<td>0.0094</td>
<td>0.0001</td>
<td>0.1459</td>
<td>0.0967</td>
<td>0.0335</td>
<td>0.0102</td>
<td>6.E+02</td>
<td>1.E+02</td>
</tr>
<tr>
<td>6</td>
<td>0.2391</td>
<td>0.0189</td>
<td>0.0001</td>
<td>0.4017</td>
<td>0.2661</td>
<td>0.0482</td>
<td>0.0209</td>
<td>6.E+03</td>
<td>1.E+03</td>
</tr>
<tr>
<td>7</td>
<td>0.0070</td>
<td>0.0415</td>
<td>0.0000</td>
<td>0.0052</td>
<td>0.0104</td>
<td>0.0044</td>
<td>0.5972</td>
<td>4.E+05</td>
<td>6.E+04</td>
</tr>
<tr>
<td>8</td>
<td>0.0011</td>
<td>0.0001</td>
<td>0.0011</td>
<td>0.3813</td>
<td>0.2195</td>
<td>0.3950</td>
<td>0.0012</td>
<td>2.E+07</td>
<td>3.E+06</td>
</tr>
</tbody>
</table>

Table 2.8: The results of 100 metre thick layer (2) buried 50mbsf within a half-space (see Table 2.3 for values). Description of table parameters is as in Table 2.5.

<table>
<thead>
<tr>
<th>EP</th>
<th>d log(µ₁)</th>
<th>d log(µ₂)</th>
<th>d log(µ₃)</th>
<th>d log(λ₁)</th>
<th>d log(λ₂)</th>
<th>d log(λ₃)</th>
<th>d log(th₁)</th>
<th>d log(th₂)</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td>1/100</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.9961</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0017</td>
<td>0.0002</td>
<td>0.0018</td>
<td>0.0049</td>
</tr>
<tr>
<td>2</td>
<td>0.0020</td>
<td>0.0469</td>
<td>0.0020</td>
<td>0.0077</td>
<td>0.0013</td>
<td>0.0040</td>
<td>0.0982</td>
<td>0.8381</td>
<td>0.0740</td>
</tr>
<tr>
<td>3</td>
<td>0.0256</td>
<td>0.0301</td>
<td>0.0010</td>
<td>0.2099</td>
<td>0.0050</td>
<td>0.5743</td>
<td>0.1253</td>
<td>0.2898</td>
<td>2.2477</td>
</tr>
<tr>
<td>4</td>
<td>0.0385</td>
<td>0.1767</td>
<td>0.0002</td>
<td>0.0400</td>
<td>0.0240</td>
<td>0.1416</td>
<td>0.5694</td>
<td>0.0098</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>0.0612</td>
<td>0.2477</td>
<td>0.0004</td>
<td>0.4182</td>
<td>0.0800</td>
<td>0.1525</td>
<td>0.0119</td>
<td>0.0281</td>
<td>160</td>
</tr>
<tr>
<td>6</td>
<td>0.0123</td>
<td>0.4628</td>
<td>0.0001</td>
<td>0.2006</td>
<td>0.1174</td>
<td>0.0233</td>
<td>0.1189</td>
<td>0.6467</td>
<td>3.E+03</td>
</tr>
<tr>
<td>7</td>
<td>0.8597</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0203</td>
<td>0.0066</td>
<td>0.0008</td>
<td>0.0759</td>
<td>0.0286</td>
<td>3.E+04</td>
</tr>
<tr>
<td>8</td>
<td>0.0007</td>
<td>0.0277</td>
<td>0.0003</td>
<td>0.1034</td>
<td>0.7658</td>
<td>0.1017</td>
<td>0.0003</td>
<td>1.E+07</td>
<td>2.E+06</td>
</tr>
</tbody>
</table>

EB 0.0096 0.0119 0.0012 0.0787 0.0019 0.2152 0.0481 0.0212
2.6 Summary

Gravity waves are pervasive in Pacific basin and generate the pressure field at the seafloor at periods greater than 20s. They have remarkably small amplitudes and only a subset is capable of deforming the seafloor. The gravity waves’ amplitude and the water-depth sets the frequency spectrum of waves capable of deforming the seafloor. The highest frequency of waves of a particular amplitude that can exert pressure to the seafloor have wavelengths ($\Lambda$), $\frac{1}{2}H < \Lambda < 2H$, where $H$ is the water depth. The vertical stress from these infra-gravity waves displaces the seafloor and hence seafloor compliance is defined as the frequency dependent transfer function between this stress and the resulting seafloor displacement. This vertical compliance has a unit of Pa\(^{-1}\). Compliance is studied at frequencies below 0.03Hz because the stress field produced by gravity waves in this frequency band is predictable and can be described by a simple frequency-wavenumber relation.

For the first time, the compliance of a uniform half-space was derived purely, under the static assumption. This result lead to the development of a new algorithm for calculating the compliance of a layered medium. This method was coded in MATLAB and is very efficient. The code was validated using models with varying elastic properties. The compliance response as a function of frequency was as expected. Particularly, the static algorithm computed the compliance of a half-space with negligible errors.

The relationship between the crustal elastic properties and the compliance as a function of frequency and depth was derived by the evaluation of the partial derivatives of the compliance function with respect to each elastic parameter. The result is that the compliance is much more sensitive to changes in $\mu$ than $\lambda$ and the comparative sensitivity is related to the value of the elastic parameters. Furthermore, the compliance function is not sensitive to $\rho$. A change in the shear property at a particular depth is seen by the compliance over a limited frequency band, some linear superposition over the sensitivity function. This field of view is 0.16 to 0.20 of the wavelength of the deforming waves. At a given frequency, the transfer function is controlled by the crustal structure; it is higher under regions with smaller elastic velocities.
The power of compliance function to resolve the parameter of interest was investigated through eigenparameter analysis. This method allowed for quantifying the errors in the parameter of a set of simple models and to attain a physical interpretation of the relationship between the parameters to which the compliance function was sensitive. The parameters of the four models analysed were the shear modulus, Lamé and the thickness of the layer. The results suggest that the resolving power of the compliance function is dependent on the error on the compliance function itself. The analysis show that the compliance function is capable of resolving parameters at large depths and that there is no clear relationship between a suite of similar models and the physical parameters that can be resolved by this function.
Chapter 3
NEPTUNE Canada I - Seafloor Compliance Experiment

3.1 Introduction

As discussed in Chapter 1, very little is known about the temporal variations of gas hydrate deposits though there has been evidence to support changes on scales of hours to years. It is important for gas hydrate characterization to understand the nature of these changes. In order to examine these temporal changes, the SFC method was used. As described in Chapter 2, the starting block to determining seafloor compliance is the simultaneous collection of a pressure and vertical displacement time-series on the seafloor at the site. These measurements are made by a differential pressure gauge (DPG) and a gravimeter, respectively. Measuring the pressure applied by infra-gravity waves to the seafloor is straightforward when the Cox DPG is employed. Conversely, the measurement of vertical displacement is neither as simple nor direct. It is its analogue, the vertical acceleration, which is directly measured. The Micro-g LaCoste gPhone is used in this experiment. The advent of NEPTUNE Canada cabled seafloor observatory provided the opportunity to monitor a gas hydrate deposits for periods which are not feasible through campaign style research. This chapter gives an overview of the NEPTUNE Canada project, describes the gravimeter and the DPG, and it details the preparation for the long term deployment of the SFC experiment; the data collection, processing and results; and the recovery of the apparatus.

3.2 The NEPTUNE Canada Project

NEPTUNE Canada is the first regional-scale cabled networked ocean observatory in the world. Spread across the Juan de Fuca Plate, this ground-breaking real-time sub-sea laboratory gathers and delivers data from a wide variety of instruments. NEPTUNE Canada’s offers ocean scientists with the infrastructure to study processes and events which the traditional approach to ocean science is incapable of providing. Networking is via 800km of electro–optic cables which provide continuous power, accurate time and remote access to the
set of connected instruments. Data is transmitted at 4GB/s from the seafloor to the onshore data management and archival system at the University of Victoria in British Columbia.

Figure 3.1 shows the layout of the NEPTUNE Canada cable backbone on which instruments are distributed at six sites. These sites were selected for their ability to assist in addressing the five major research goals of NEPTUNE Canada. One of these sites is ODP 889. At Site 889 the specific research objectives are to monitor changes in hydrate distribution, depth, structure, properties and venting, particularly related to earthquakes, slope failures and regional plate motions. As such NEPTUNE Canada employed a set of geophysical instruments which includes a CSEM system, an acoustic sensor, a broadband seismometer and the SFC apparatus. Figure 3.2 shows the layout of the instruments at Site 889. All instruments are specially adapted to the high pressure and low temperatures of the ocean and designed to withstand the corrosive marine environment of the North Pacific (NEPTUNE Canada 2011). This seafloor observatory is designed to be operational for 25 to 30 years and to host any instrument which outputs data electronically (NEPTUNE Canada 2011)

Figure 3.1: Map outlining the layout and coverage of the network at the NEPTUNE Canada seafloor observatory. Data used in this thesis are from the seismic station and seafloor compliance installation at ODP 889. Reprinted with permission from NEPTUNE Canada.
Figure 3.2: A detailed map of the locations of the various instruments at NEPTUNE Canada ODP 889. The aqua box identifies the location of the BBS-DPG installation, the red box marks the SFC installation. The grey area bordered with a dotted magenta line indicates the extent of gas hydrate in the region while the grey line within this magenta region outlines the surface expression of Bullseye Vent. Reprinted with permission from NEPTUNE Canada (original source: Charles Paull)
3.3 The Micro-g LaCoste gPhone

3.3.1 The campaign style gPhone

A Micro-g LaCoste gPhone relative gravimeter was secured for the NEPTUNE Canada seafloor compliance experiment because of its ability to measure changes in gravity over a wide frequency spectrum. The gPhone, having high sensitivity and a large dynamic range, is optimized to measure the ambient accelerations in the absence of seismic activity as well as the first arrivals of an earthquake. This relative gravimeter has an amplitude response which acts as a low pass filter with a cut-off frequency of 5Hz and a flat frequency response between DC and 1 Hz (Niebauer et al. 2007). Niebauer et al. (2007) reports that the gPhone is capable of detecting small changes of the earth’s gravity field with a precision of a few parts per billion (10^{-9}) over time scales of one second.

The off-the-shelf system consists of three (3) major parts (Figure 3.3). The Meter box (referred to as the gPhone) encases the sensor and the data acquisition (DAQ) circuitry; the Electronics box houses an uninterruptible power supply, GPS and Rubidium clock Timing Module, power supplies, and output connectors; and a laptop which runs the proprietary software, gMonitor, which controls the gPhone and records the gravity data.

The gPhone’s sensor comprises of a metal Zero Length Spring (LaCoste 1934) system with low drift and an electrostatic feedback system (Laswell et al. [date unknown]). The spring’s mass position is sensed and stabilized by the feedback system’s capacitive plates. The gravity range of the meter is adjustable and its linear electronic feedback system ranges between ± 25 milliGals and ± 50 milliGals. These features allow the meter to be operated for greater than 2 years without having its range screw and spring system re-adjusted. The sensor is unaffected by variations in ambient temperature through the insulated double-oven design of the gPhone. Humidity is stabilized by having the inner oven filled with dry air. Isolation from pressure changes is by the triple chamber casing. The layers of the chamber are made of thick Aluminium and all chambers are o-ringed sealed, air tight and water resistant. The entire package is 13kg with dimensions of 31cm by 32.5cm by 25.2 cm and stands on 9cm legs.
Figure 3.3: The three main components of the Micro-g LaCoste relative gravimeter connected during the initial testing at the University of Toronto lab in preparation for the SFC experiment on the NEPTUNE Canada seafloor network. The NPort is a serial-to-Ethernet device which allows the laptop to communicate with the meter box (gPhone).
The DAQ board and the electronic feedback system of the gPhone are held in the outer oven which provides the necessary stable temperature environment. Though the meter casing is designed for controlled temperature, humidity and pressure, the system is equipped for recording the ambient environment as well as the pressure and temperature of the sensor so the data can be used for gravity data corrections by the software. The accuracy of this measurement is ensured by the inclusion of 24 bit A/D converters. Overall, the gPhone has a reported resolution of 0.01µgal and a range of 7000 milliGals (Micro-g LaCoste [Date unknown]).

3.3.2 Modifications for long-term deployment

This Micro-g LaCoste system was originally designed for campaign-style field research where neither continuous power supply nor internet connection is part of the experimental set-up. However, it was purchased for long-term deployment on the NEPTUNE Canada project and thus needed to be modified to render it suitable for its intended purpose. As such, we needed to: (i) lower the input voltage from 24V to 15V; (ii) lower the power consumption of the system since the available power supply from NEPTUNE Canada was 15V with a maximum of 75W on a 370m extension cable; (iii) completely eliminate the electronics box from the system, as it was too large for inclusion in the pressure vessel and had too a high power draw; (iv) adapt for communications via the RS-422 protocol; (v) configure the DAQ circuitry of the gPhone to record the outputs from the DPG so that the output from both instruments are recorded simultaneously. This was attained by adding a 0 – 5V input to the gPhones’s analogue-to-digital converter (ADC); and (vi) include a power output of +5V from the gPhone to the DPG. For the reconfigurations ((iv) through (vi)), the gravimeter was returned to the manufacturer (Micro-g LaCoste, Denver). The adjustments in (i) and (iii) were done by the Marine Geophysics Group at the University of Toronto on consultation with the manufacturer.

The exclusion of the electronics box and the lowering of the power consumption of the system required that the only essential components be used: a clock and a power supply. An Infinity VHK100W DC-DC converter was used to power the entire SFC apparatus: the DPG
and its circuitry, the gPhone and the RS-232 to RS-422 converter. The DC-DC converter protects input reversals, provides continuous short circuit protection, has a wide input range (10V – 36V or 18W-75W) and isolated output. The RS-232 to RS-422 conversion was handled by a Black Box ICD100A, an optically isolated converter with surge protection.

3.4 The Scripps Differential Pressure Gauge

Manufactured at the Scripps Institution of Oceanography, this off-the-shelf DPG (Figure 3.4) is based on the Cox et al. (1984) design. Cox et al. (1984) requirements for a well functioning deep-sea DPG were that it should: (i) have low noise; (ii) be stable in the ambient high pressure, and low and fluctuating temperature at the seafloor; (iii) require low power; (iv) be rugged enough to endure multiple deployments to the seafloor and; (v) be simple and affordable. These stipulations resulted in the design of a differential sensor capable of sensing pressure variations on the order of 1Pa (=0.1mm of water pressure) at water depths of up to 3000m (~3×10^7 Pa) and temperatures between -2°C and +35°C (see Appendix G for manufacture’s specifications). This DPG is capable of detecting pressure fluctuations at the seafloor generated within a wide frequency range including those produced by seismic disturbances of the seabed as well as of the low frequency end of the ocean spectrum.

Figure 3.5 shows a schematic diagram of the sensing elements of DPG (Cox et al. 1984). It consists of a flexible rubber diaphragm (R), which is exposed to the ocean and facilitates seafloor pressure detection together with the oil\textsuperscript{15}-filled pressure chamber (P); and a rigid oil-filled reference chamber (V\textsubscript{c}). The rubber diaphragm and reference chamber are connected to each other through a strain gauge transducer (T) and a capillary leak. This configuration allows the DPG’s sensor to measure the ocean pressure fluctuations on short time-scales and to equilibrate (or re-equilibrate) its internal reference pressure with the ocean pressure on long time scales. In a state of equilibrium, the pressure throughout the incompressible fluid (stippled), in both the reference chamber and pressure chamber is identical and is facilitated by the capillary leak, which allow fluid to move though both chambers at a very slow rate. In

\textsuperscript{15} Compressed silicon oil.
this state, T has a constant response. However, when R is exposed to some pressure, at the seafloor say, the pressure is registered in chamber P and a pressure difference is created between the pressure chamber and the reference chamber. This deviation is sensed by T which in turn produces a voltage proportional to the observed pressure change. At very long periods, the DPG is limited by temperature fluctuations that change the pressure within the reference chamber (Webb and Cox 1986). The capillary leak has an additional function apart from that of allowing the pressure gauge to equilibrate: it prevents large pressure difference between the reference chamber and the pressure chamber when pressure and temperature conditions slowly change. In instances where the pressure changes are rapid, the transducer is protected from destruction by pressure release valves attached to the reference chamber.

Figure 3.4: Scripps Institution of Oceanography differential pressure gauge (DPG). Its dimension is 152.5 mm diameter x 200 mm long not including the connector. (Left figure from Scripps Institution of Oceanography’s website).

The DPG’s pressure sensor is the GE NovaSensor NPH-8-007: a solid state low pressure silicon sensor with temperature compensation. Internally, the piezoresistive strain gauge is embedded into a Wheatstone bridge configuration (Figure 3.6). The pressure range of this sensor is 7 KPa at 1 PSI H2O, and constant current application results in linear relationship between output voltage and input pressure (GE NovaSensor 2009). The sensing transducer is connected to a signal conditioning circuit which amplifies the DPG output. The resistive bridge of the pressure sensor (Figure 3.6) is connected to the DPG’s circuit board. The output signal is measured across -Out and +Out. With no differential pressure, there is no load on the bridge and a 0V difference is seen across the -In and +In pins. However, when the ocean and reference pressure differs, the bridge becomes unbalanced and a voltage, proportional to this pressure difference is seen across -Out and +Out.
3.5 The pressure vessel, gimbals and bottom assembly

The deployment package was designed for stability and instrument protection. The gPhone, the DPG circuitry and other miscellaneous components which all require protection from the corrosive elements of the sea and the high pressure conditions at the seafloor, are housed in the pressure vessel. The enclosure is a pair of 25 kg pressure-resistant Aluminium half-spheres (Figure 3.7A). Each half-sphere has an outer diameter of ~0.75m and an inner diameter of ~0.65m. The pressure vessel is completed by a mid-ring which is sandwiched between the half-spheres. This mid-ring (Figure 3.7B) is also made of Aluminium and has inner and outer diameters identical to the half-spheres. On the sides of the mid-ring there are three holes spaced equidistantly around the diameter for the marine connections. On the mid-ring’s flat upper and lower surfaces are slots for o-rings. The pressure vessel has been employed since 1998 and thus preparation for NEPTUNE Canada required re-anodization. After the re-anodization, brackets were re-attached with Blue Goop® (an oil-based thread lubricant) coated bolts to the spheres and a layer of rubber was used to separate the surface of the brackets from that of the spheres. The brackets were of two styles, one set had circular shaped holes and the other pear shaped holes and were attached alternatively.
The DPG is affixed to the bottom assembly as it must be exposed in the ocean to function. The bottom assembly (Figure 3.7C) was named for its role of securing the assembled sphere and in providing a platform on which the apparatus can sit on the sea floor. Its inner frame is made of stainless steel and is shaped as an equilateral triangular. The outer sides of the triangular frame are lined with rectangular slabs, creating open vertices which were fitted with cylinders. These cylinders have a 10 inch diameter and height of 20 inches and have disc shaped covers. The DPG is hosted in a single cylinder – the others are left empty. On the top edge of the bottom assembly there are three vertically protruding rods at the midpoint of the equilateral triangle. These rods serve two purposes: (i) securing the sphere to the bottom assembly and (ii) clamping the half-spheres onto the mid-ring. The underside of the bottom assembly is also fitted with slabs of lead to provide additional weight to its light frame. This enables the entire apparatus to remain right-side-up when lowered to the sea floor and reduces the chance of it being toppled during deployment.

Figure 3.7: (A) the pair of half-spheres, (B) the wired mid-ring and (C) the bottom assembly.
For optimal performance, the gPhone must be levelled, i.e., where the meter is horizontal in both the Long (x) and Cross (y) directions. In this ideal orientation, the gravity output is a maximum. The gimbal was designed for the task of keeping the gPhone levelled. It consists of a central box, within which the gravimeter is housed, and a flat ring which is affixed with hinges around its middle, much like a belt (Figure 3.8). The flat ring allows the gimbal to be fastened to the sphere while the hinges allow a full 360 degree revolution of the box along the x and y axes. The gimbal was originally designed to be operated electronically, however, over time, the electronic parts deteriorated and replacements were not available. This meant that a new system would have been required. Due to time constraints, this was not possible, so the old system was modified for mechanical levelling. Mechanical levelling involved attaching weights to the bottom of the gimbal box to create a vertically restoring force on the gimbal box filled with the gPhone and the experiment’s electronics. As such, the 5lb DC-DC power supply was secured to the bottom of the gimbal box for initial coarse levelling. Then additional weights were added to the bottom of this box until the system is appropriately levelled. The measure of how appropriately levelled the system is can be determined through the proprietary software, gMonitor. The standard is that the long and cross levels should be within ±5000 units.

Figure 3.8: The gimbal. Side view (left), top view (right).
3.6 Preparations for NEPTUNE Canada Installation

3.6.1 Instrument calibration

The calibration exercise was performed at Prof. Nigel Edwards’ backyard pool and had three goals: (i) to determine if the DPG responds to changes in pressure, (ii) to establish the nature of the response to pressure variations at different frequencies, and (iii) to get a conversion value for the voltage response per change in water pressure (displacement) of the DPG. Two devices were used to fulfil the three above goals: (i) a motor powered cam capable of simulating sinusoidal motion. The cam is equipped with a DC motor to which the input voltage determines the frequency with lever of the cam rotates; and (ii) a teeter-totter used to simulate pressure changes by vertically displacing the DPG in the pool. Both instruments are shown in Figure 3.9.

Figure 3.9: The DPG immersed in the pool during the calibration exercise. The teeter-totter is the wooden object on which the cam (silver object) sits.
The first two goals were investigated simultaneously. The DPG was attached to the lever arm of the cam and lowered into the pool until suspended at about 50 cm below the water surface and about 75 cm away from the pool’s edge (Figure 3.9). This immersion position reduces the sources of noise to the measured pressure changes from the simulated DPG displacement. After allowing the DPG to equilibrate at its initial submersion position, the cam was set to vertically displace it, in turn, at each frequency in the range of available frequencies (0.01Hz to 0.1Hz). The pressure fluctuation simulation was conducted until an appropriate length time-series (at least 5 cycles) was attained. Figure 3.10 shows the DPG’s response at 0.1Hz. As expected, the response is sinusoidal with the maximum voltage recorded when the DPG was at its deepest submersion and vice-versa.

![DPG response at rotation of 0.1Hz](image)

Figure 3.10: A 85 seconds time-series of the motion of the DPG when oscillated by the cam at a frequency of 0.1Hz.
The third goal was achieved by recording the step response of the DPG. The DPG was hung from the moveable end of the teeter-totter, lowered into the pool until it is completely immersed and allowed to stabilize. It was then instantaneously displaced from its resting position\(^\text{16}\), held in its new position and allowed to re-equilibrate. This step input was repeated until an almost perfect step response was achieved. Figure 3.11 shows the response to two continuous step inputs where the DPG was first hoisted to its resting position to create the first input and lowered to create the second. The circled sections on Figure 3.11 are due to human error since the step input was created manually and it was thus impossible to prevent the teeter-totter from vibrating on its stops.

The second half of the step response in Figure 3.11 was used in determining the calibration factor of the DPG. The exponential part of the curve (Figure 3.12) was fitted with a RC circuit formula \( V_R(t) = V_0 e^{-\frac{t}{RC}} \) based on the knowledge that the DPG’s circuit included high pass filter. The fit resulted in \( \frac{1}{RC} = 0.0674 \), thus a time constant, \( \tau = 14.84 \) seconds; and \( V_0 = 3.368V \). The maximum voltage corresponds to the maximum pressure change experienced by the DPG’s response to the step input and is used to determine the calibration factor. The calibration factor is the ratio of the maximum pressure change cause by the step input to the resulting maximum voltage. The maximum pressure change experienced by the DPG during the step input is described by \( P = \rho g H \) where \( \rho \), is the density of the water of the pool, \( g \) the acceleration due to gravity and \( H \), the height the DPG was displaced during the step input. \( P = 88.1Pa \) which gives a calibration factor of \(-27.84 \pm 2.36 \) Pa/V. Given that the DPG’s output was recorded by a 24 bit ADC with a 0 – 5V voltage range, the true value by which the DPG data was scaled is \( 7.2607112 \times 10^{-6} \) Pa/ADunit.

The calibration of the gravimeter was done by the manufacturer – Micro-g LaCoste (formerly LaCoste & Romberg) and details of this procedure are outlined in their *LaCoste & Romberg Instruction manual D and G gravimeters* (LaCoste Romberg 2004). The calibration constant provided was 0.0015904 to produce an acceleration measurement in Gals.

---

\(^{16}\) To displace the DPG, the top moveable arm of the teeter-totter was stepped on.
Figure 3.11: The response of the DPG to two concurrent step inputs.

Figure 3.12: The fit to the DPG decay step response.
3.6.2 Equipment testing after shipment to Victoria, BC

The University of Toronto Marine Geophysics Group arrived in Sidney, British Columbia on August 3, 2009 in preparation for the arrival of the equipment. After a 36 hour delay in delivery of the instruments, work began. The entire preparation exercise was challenging and tedious. The single task was to assemble the seafloor compliance apparatus for deployment at NEPTUNE Canada Site 889. The first step involved verifying that both instruments were fully operational after the cross country journey. To this end, the gravimeter was gimballed, interfaced with the DPG and connected to gMonitor through the laptop and placed on a gravity table at the Geological Survey of Canada (GSC) (Figure 3.13). This simple task mushroomed into a four (4) day ordeal upon the discovery that data flow from the gPhone was intermittent (i.e. flowing for a maximum of 40 minutes at any given time) and the sole laptop loaded with the proprietary software (gMonitor) had crashed. After reloading the software on a backup laptop, fixing the loose wire and much manipulation of Microsoft Vista and NPort Administrator to set the correct ports for communicating with the instruments, the DPG was plugged into the gPhone which was then levelled in its gimbals and connected to the NPort communication device. The set up was left to log overnight.

Figure 3.13: The partially assembled SFC apparatus on the gravity table at GSC.

---

17 A gravity table provides a low noise platform for recording gravity data.
The overnight data confirmed that the instruments were functioning properly and thus the mechanical levelling of the gimbal was re-tested. The goals of this test were to verify that the gPhone can be levelled, within the specifications for best performance, by attaching additional weights to the bottom of the gimbal box. With precise levelling achieved, the second goal was to ensure that if the gimbal box (packed with the gPhone and circuitry) is displaced by angles up to 90 degrees, along both x- and y- axes, that it will return to the optimal levelling position. Both test goals were successfully achieved. The set up was then dismantled and carefully transported across to the Marine Technology Centre (MTC), the NEPTUNE Canada instrument testing base. At this facility, the task was to re-assemble the SFC experiment for testing and ultimately, deployment.

3.6.3 Assembly of SFC equipment for deployment

The final assembly began with the preparation of the spheres and the mid-ring. First, the three sets of holes on the outer surface of each of the spheres were plugged with epoxy putty (Figure 3.7) to reduce the risks of corrosion. Next, each of the three holes on the mid-ring was fitted with their allotted bulkhead connector: one dummy, one for connecting power and the third for connecting the DPG to its circuitry. Then, the o-ring surfaces of the half-spheres and mid-ring were washed with 70% by volume isopropyl rubbing alcohol, and non-abrasive, low-linting and low-extractable wipers (Kimwipes®) to remove all impurities. Careful attention was placed on the o-ring surfaces as any foreign material will lessen the integrity of the seal. The o-ring surfaces on both half-spheres and mid-ring were carefully checked for blemishes by the gentle pass of a finger. The blemishes that were discovered were painstakingly removed by rubbing with 600 grit sand paper. The cycle of washing, checking for blemishes and sandpapering continued until there was a clear certainty that no impurities remained.

The bottom half-sphere was then placed in its box, equipped with a snugly fitting padded circular cut-out that prevents the surface of the sphere from being scratched. The mid-ring is placed in close proximity to the boxed half-sphere, given a final wash and greased with Darfur®. Two perfectly sized o-ring were then selected and thoroughly washed. One was
wrapped in a sheet of Kimwipes and the other was greased and placed into its groove in the mid-ring. Inserting the o-ring into its mid-ring groove is a technical task that requires two persons because of the diameter of the sphere. One person is assigned the task of laying the o-ring and the other of tracing the path of the o-ring with a pair of fingers to assist in fitting the o-ring in its groove. When the o-ring is laid, one gently runs a finger over the slotted o-ring to ensure that the o-ring sits correctly. If at this point, the o-ring jumps out of its slot, the entire laying procedure is repeated. This task may appear trivial, but on the contrary it is a rather painstaking task which must be done perfectly since this seal presents the highest risk to leakage and damage to the instruments.

The next step was to place the o-ringed side of the mid-ring on the boxed half-sphere. The sphere’s surface was given its final washed and greased. Then the two woman team lifted the mid-ring, watchfully turned it over, and with extreme caution and precision, laid it on the half-sphere. The mid-ring was then gently shifted back and forth to ensure that is perfectly aligned and is contiguous with the bottom half-sphere. This alignment also requires the three equidistant brackets, which line the inner sides of the mid-ring, be placed midway between a pair of brackets on the half-sphere (Figure 3.14). With this completed, we then turned our attention to fitting the gimballed gPhone and wires into the half-sphere.

The gimbal was carefully lowered into the boxed half-sphere in a manner so as not to disturb the alignment of the already laid mid-ring. The ring of the gimbal was aligned such that its three holes, which are also equidistant, are fitted over the three brackets of the mid ring. Lock nuts with plastic washers (for reduction in vibration) were used to secure the gimbals on the mid-ring. The bolts of the mid-ring, through which the holes of the gimbals were fitted, were covered with plastic for the damping of vibrations should the gimbals shift on the mid-ring (Figure 3.15). Wiring the equipment then followed.
The wires were arranged so as to have minimal effect on the operation of the gimbals, i.e. allowing room for its rotation. There were several sequential connections made which terminated at the designated power plug on the mid-ring (Figure 3.16). The following is a description of all the connections made working backwards from the power plug, a 7 pin Impulse LMPBH/LMPIL 7#16 male wet-mateable plug with 12” Teflon leads of 18 AWG. The power plug leads were extended and connected to two separate Molex connectors.
(Figure 3.16). These Molex connectors were used to ensure a reliable connection and to reduce the tension on the joints. Pins 1, 2, 5 and 7 of the power plug were attached to one of the Molex connectors which would be referred to as the ‘data Molex’. The power plug pins 3 and 4 were connected to the other Molex which would be referred to as the ‘power Molex’. The very long leads from the power Molex were run along the side of the gimbal ring and connected to the 15V input end of the power supply (Figure 3.17) with enough ‘give’ to allow for the full swing of the gimbals. The leads from the data Molex were then connected to the RS-422 side of the RS-232 to RS-422 converter. Pins 1, 2, 5 and 7 were connected to Tx (+) (H), Tx (-) (G), Rx (-) (K) and Rx (+) (L) respectively. The RS-232 to RS-422 converter was affixed, using industrial strength Velcro to a plastic board which was screwed onto an immovable part of the gimbal (Figure 3.18). The RS-232 to RS-422 converter was then set to the RS-422 mode with baud rate of 19200 bps which is ideal for the gPhone.

Figure 3.16: The wired mid ring with an inset of the NEPTUNE Canada 5-pin plug.
Figure 3.17: Fully wired power supply attached to the bottom of the gimbal box.

Figure 3.18: The RS-232 to RS-422 converter.
The female DB9 plug on the top surface of the gPhone is the point through which the gPhone is powered and the data is sent. Thus this pin must be connected to the power supply and the RS-232 to RS-422 converter. This connection was facilitated through a wired extension that was fitted with a male DB9 plug on one end and had the other end split into two – one set of wires connected to a female DB9 and the others to ring connectors. The wired extension was configured as follows: the male DB9 plug was wired such that pins 2, 3 and 9 had single wires and pin 8 had a pair. Then the wires from pins 2, 3 and one from pin 8 were soldered to a female DB9 such that pin 2 was connected to pin 3, pin 3 to pin 2 and pin 8 to pin 5. On the female DB9, pins 2, 3 and 5 are Rx, Tx and Signal Ground, respectively. This cross wiring meant that transmit on the gPhone becomes receive on the RS-232 to RS-422 converter and vice-versa and was necessary. The ring connectors were attached to the remaining two wires from the male DB9 – the second from pin 8 and that from pin 9. With the wired extension complete, the male DB9 end was plugged into the gPhone female DB9 plug, the female DB9 on the opposite end was plugged into the RS-232 side the RS-232 to RS-422 converter and the wires with the ring connectors were fastened to the output of the power supply. After the gPhone connections were made, the RS-232 to RS-422 converter was connected to the 24V power supply: two wires were fitted with clips on one end and ring connectors on the other. The clips were clamped to slots F (24V) and C (Power Ground) on the serial converter and the other end connected to the 24V power supply.

The final set of connections were related to the DPG. The DPG supply power board, circuit board (PG) and the filter board were placed adjacent to each other on a thin sheet of Aluminium (Figure 3.19) and attached to the top of the gPhone. Before the gPhone could be lowered into the gimbal box, a wired DB9 plug was connected to its side DB9 plug. This side male DB9 plug on the gPhone was configured to receive the DPG data so that it will be logged along with the gPhone data. This improves the simultaneity of the recorded data and makes for better interpretation of the results. The three pins wired on this male DB9 plug were pins 5, 8 and 9 which are Signal Ground, +5V and the ambient pressure (channel for the DPG data), respectively. The gPhone was carefully lowered into the box of the gimbal using a lifting strop made of malleable, light plastic. This was designed so that it can remain with the gPhone for future removal since the gPhone fits snugly in the gimbal box. The gPhone was then bolted to the gimbal box.
With the gPhone securely fastened to the gimbal box, the DPG power supply board was powered from the 24V power supply. A regulator which stepped down the 24V to 6V was added at the last minute to the DPG’s circuit after a voltage check at the output pins (the input of to the DPG) registered 24V. The power supply board then enabled the PG board. The PG board input connector was then connected to the DPG plug on the mid-ring. The leads of the female Impulse IESQ4F-BC underwater pluggable were soldered to a regular power connector which had its matching header on the PG board. Pin 1 was connected to J2-3, pin 2 to J2-4, pin 3 to J2-2 and pin 4 to J2-1 which are Out (+), In (-), Out(-) and In (+), respectively.

With all the connections made and the wires securely fastened to the gimbals (Figure 3.19), levelling of the gPhone was undertaken (to ensure that the long and cross levels fell within ± 5000 units\(^{18}\)). To the bottom of the Velcro covered gimbal box, Aluminium washers were attached and gMonitor was used to visualize the data stream from the gPhone to determine, through the values of long and cross levels, when the system was levelled.

![Figure 3.19: DPG electronics lain top an Aluminium sheet. From left to right, the power supply board of the DPG, the DPG circuit and the level shifting circuit.](image)

\(^{18}\) This increased range for levelling of the gPhone was requested from and implemented by the manufacturer, when during the preliminary instrument testing stage, it was discovered that the gPhone’s mass got jammed on its stops. So in an effort to prevent this happening during the SFC deployment the range was changed to this larger value.
Next, the upper surface of the mid-ring was washed and greased in preparation for the addition of the upper half-sphere. The second o-ring was given its final wash, greased and carefully laid (using the technique described above) in its slot in the mid-ring. Finally, the surface of the upper half-sphere was washed and greased. The half-sphere was then lifted by its brackets, flipped and lowered onto the mid-ring so that its six of brackets aligned with those of the lower half sphere. This too was slightly shifted to check the integrity of the seal. The ends of the stainless steel-tripod were then threaded through the upper three pear-shaped brackets, and the anodes were then screwed onto each of the feet of the tripod, after which the feet of the tripod were threaded through the bottom sets of pear-shaped holed brackets until the anode sat squarely on the bottom brackets. These anodes were included as an additional measure for securing the tripod to the sphere. This was essential since the tripod is the designed way of lifting the assembled sphere. Securing the tripod to the sphere was completed by attaching bolts and washers to the end of the feet of the tripod. The assembled sphere was then hoisted from its box and fitted into the bottom assembly (Figure 3.21A). The sphere was oriented such that the three pairs of circular holed brackets were aligned with the protruding rods of the bottom assembly. When the sphere was snugly fitted, bolts were added
to the top of these rods to secure the sphere to the bottoms assembly (Figure 3.21B). The final bit in the assembly of the SFC apparatus was to place the DPG in its delrin cylinder, bolt the disc to the top and connect the DPG to its mid-ring plug (Figure 3.21B). Figure 3.22 shows a schematic of the complete SFC apparatus.

On the side of the bottom assembly an Aluminium square bracket was attached at an angle to provide the best access for the Remote Operated Platform for Ocean Science (ROPOS) to make the marine connections (Figure 3.23). On this bracket, a Mink 10 pin plug which connects the SFC apparatus to the NEPTUNE Canada under-water junction box was affixed. A whip was then used to connect the SFC apparatus (via the power plug on the mid-ring) to the Mink plug on the bottom assembly.

Figure 3.21: (A) The sphere, which houses the gravimeter and the electronics, being hoisted into the bottom assembly. (B) The connected DPG and the anodes on the tripod.
Figure 3.22: Schematic diagram of the seafloor compliance experiment at NEPTUNE Canada ODP 889. Dark lines represent the power while lighter lines with arrows represent direction of data flow.

Figure 3.23: (A) The ODP mink connector and (B) the bracket which supports it.
3.6.4 Testing by NEPTUNE Canada

The assembled SFC apparatus was delivered to NEPTUNE Canada on August 17th, 2009 for testing. The NEPTUNE Canada team performed three tests: (i) a communication test; (ii) a power test; and (iii) a wet test. For these tests, three ODI cables were connected in series in semi-replication\(^{19}\) of the SFC seafloor cable structure. The entire assembled cable totalled 376 m. There were two 3m cables, one with an Impulse LPMIL-7-FS and an ODI bulkhead and the other having a 12-Way ROV and a MINK-10 end. These two cables were attached, one to each end, to a 370m cable that had two ODI 12-Way ROV plug ends.

For the communication test, the three cable extension was connected to port 1 on the mock junction box (JB) (3.24). Port 1 is the 15V, RS-422 port, which was specifically assigned to the SFC apparatus. Port 1 on the JB was then turned on via the proprietary software Ocean Works Junction Box Software (OWJBSW), and the voltage between pins 3 and 4 on the Impulse LPMIL-7-FS end of the 376m cable series was checked. With an input voltage of 15.1V confirmed, the Impulse pin end of the extension was plugged into the SFC apparatus. gMonitor was then used to confirm that the gPhone was functioning properly.

The power test followed and constituted evaluating the current draw of the SFC instrument package. Port 1 was powered down and the MINK-10 connector end of the 376m extension unplugged while the other end remained connected to the mid-ring of the SFC apparatus. The MINK-10 plug was connected to a table top power supply, an ammeter and an oscilloscope (Figure 3.25). 15V was supplied to the SFC apparatus and the resulting voltage and current waveforms were recorded (Figure 3.26). As expected, the input voltage peaked and remained constant at 15V, while the in-rush current was \(-0.34\)A.

\(^{19}\) In semi-replication because a 2 meter whip commissioned from ODI had not arrived at the time of testing.

On this bracket the Ocean Designs 1 (ODI) 12-way ROV flange mount bulkhead end of the 3 meter marine cable was affixed, the intermediary step in connecting the sphere to NEPTUNE Canada’s junction box(JB) pigtail.
Figure 3.24: Mock JB with the 376m extension cable (white cable) connected to Port 1.

Figure 3.25: Denis Hedji of NEPTUNE Canada conducting the power test.

Figure 3.26: Power test waveforms. Channel 1 reports a peak voltage of 15V while channel 2 shows a peak inrush of 0.34A.
Lastly, the wet test was done. The aim of the wet test was to ensure there were no ground faults on the pressure sphere. The power wires on the MINK-10 connector were shorted and connected to an Ohm meter which had a wire placed in the water filled test tank. The pressure sphere was then immersed into the test tank. The Ohm meter measured 20MOhms indicating that there were no ground faults. The sphere did not leak. Therefore, the pressure sphere and its contents had passed the wet test.

Having passed all the NEPTUNE Canada prescribed tests, the SFC apparatus was cleared for deployment and transported to the Esquimalt graving dock, from which it was loaded on the RV Thompson (Figure 3.27A) for the NEPTUNE Canada Instrument Installation Cruise scheduled for 20th August to 20th September, 2009.

3.7 Deployment of the SFC experiment

According to oral report from Dr. Reza Mir, the University of Toronto Marine Geophysics Group’s sole representative on Leg 2 of the NEPTUNE Canada deployment cruise on the RV Thompson, the successful installation of the SFC on the NEPTUNE Canada’s network was nothing short of miraculous. Running against the clock, due to a combination of bad weather, malfunctioning nodes and repairs to ROPOS, the deployment of the SFC apparatus occurred on 19th September, the last official day of the cruise. NEPTUNE Canada Installation blog (Owens 2009a) reported ... ‘The deployment method for this apparatus was unlike that for anything else we put into the water this cruise. It was fitted with a transponder, lifted over the water and dropped to free-fall 1259m from the surface to the seafloor.’ (Figure 3.27B). The SFC apparatus position was latitude 48.669525 degrees and longitude -126.85184 degrees. Owens (2009a) further reported that with the aid of the ROPOS, a mini-ROCLS (remotely operated cable laying system used for the deployment of short cables) was flown down to the seafloor to lay the cable and to connect the SFC apparatus to the ODP 889 instrument platform (Figure 3.27C).
Figure 3.27: (A) The SFC apparatus on deck of the RV Thompson just before deployment. (B) The moment the SFC apparatus makes its entry into the water. (C) The arm of the mini-ROCLS on its approach to the SFC apparatus for connection to NEPTUNE Canada. Source: (Owens 2009a).
3.8 Data processing and results

3.8.1 Method of data processing

In an environment where the accelerations recorded by a gravimeter on the seafloor is purely due to forces applied by the infra-gravity waves, then the vertical transfer function, \( T(\omega) \), would be the ratio of the acceleration, \( G(\omega) \), to pressure, \( P(\omega) \), as a function of frequency, \( \omega \):

\[
T(\omega) = \frac{G(\omega)}{P(\omega)}
\]

(3.1)

However, given that neither data are free of noise nor are gravity-waves the only source of seafloor deformation, the transfer function (compliance), is determined by finding the ratio of the cross correlation between both channels and the auto correlation of either the pressure or the gravity channels. This accounts for the random noise in the acceleration spectrum as well as the possibility that the accelerations in this low-frequency band is caused by non-gravity wave sources (Crawford and Webb 1999). Crawford et al. 1991 showed, through correlation studies between adjacent pressure sensors, that the pressure spectrum is less noisy than the acceleration spectrum. Adding the assumption that all the random noise is in the acceleration data allows for the calculation of the transfer function in Equation 3.1 as

\[
T(\omega) = \frac{\langle G(\omega)P^*(\omega) \rangle}{\langle P(\omega)P^*(\omega) \rangle}
\]

(3.2)

where \( G(\omega) \) and \( P(\omega) \) are the acceleration and pressure data transformed in frequency domain, \( * \) denotes their complex conjugate and \( \langle \rangle \) the ensemble average of all records used in the estimate. Equation 3.2 can be transformed into the compliance transfer function by dividing by \(-\omega^2\) since in the frequency domain, \( \text{acceleration} = -\omega^2 \times \text{displacement} \). Multiplying Equation 3.2 by the wave-number \( k \), gives the normalised compliance. Equation 3.2 is applicable to all data sets from a site, since it is only a function of the structure under the site and hence stationary in time (Webb and Crawford 1999). This method of calculating the transfer function is applied to the NEPTUNE Canada Site 889 data; the processing and results are presented in the following sections.
To assess whether the resulting compliance and hence the validity of the assumption that the measured accelerations (or its analogue, the displacement) are caused by infra-gravity waves, the coherence function is calculated. The coherence function, $\gamma^2$, accords significance on the transfer function and is defined in Bendat and Piersol (1980) as

$$
\gamma^2_{PG} = \frac{|G_{PG}(\omega)|^2}{G_{PP}(\omega)G_{GG}(\omega)}
$$

where $0 \leq \gamma^2 \leq 1$ and $G_{PG}(\omega)$ represents the cross power spectrum between the pressure and acceleration channels, $G_{PP}(\omega)$ is the pressure spectra and $G_{GG}(\omega)$ is the acceleration spectra. Munk and MacDonald (1960) state that there exists a meaningful phase relationship between the two records if the coherence is greater than $2\nu^{-\frac{1}{2}}$ for frequencies $f \pm \frac{1}{2} \Delta f$, where $\Delta f$ is the resolution of the smoothed record; and $\nu$, twice the number of harmonics used in the estimate. It follows that in the frequency range where

$$
\gamma^2 > \frac{4}{2 \times \text{number of harmonics}}
$$

the records of pressure and velocity are valid. The value computed in Equation 3.4 is referred to as the level of validity. Thus data processing involved the computation of the coherence function and its comparison with the calculated level of validity.

### 3.8.2 Data processing

On the 5th November, 2009 the SFC data were available for download from the NEPTUNE Canada website. Data were available on a daily basis during the period in which the equipment was installed and there were very few days in which the data were incomplete. Data gaps in the daily record were related to the NEPTUNE Canada system: power outages or driver machine reconfigurations. The SFC experiment was operational for a period of 6 months (November, 2009 to May, 2010) as NEPTUNE Canada personnel deemed it necessary to unplug and recover the instrument package. They indicated that their decision was based on the fact that their junction box monitoring system recorded a ground fault and
‘on the spot’ troubleshooting during the recovery cruise offered no quick fix. The rest of this chapter describes how the data was processed and the results.

First, the data from the DPG and the gravimeter were downloaded for total of ~180 days of data. Both streams of data were then scaled by their calibration factor (see section 3.6.1), plotted and closely inspected so as to note gaps and irregularities\(^{20}\). Figure 3.28 shows the recorded pressure and velocity time-series of the 27\(^{th}\) November, 2009. The pressure record (bottom panel) portrays one of the irregularities – the spurious data spike.

\[\text{Figure 3.28: Pressure and gravity time-series for 27}^{\text{th}}\text{ November 2009.}\]

\(^{20}\) Irregularities refer to sections of the data which have spurious points: inexplicably larger than expected variations (spikes in data); data gaps and days where there are known earth tremors.
Following data sorting, each of the 24 hour length records was broken into segments of 512 seconds for a total of 168 segments for the frequency domain analysis. The segments of every record which were unsuitable for the determination of the transfer function were removed and the following processing steps were performed on the remaining good segments of each record\textsuperscript{21}: (i) the mean was removed from each segment for both pressure and velocity data; ii) the demeaned segments were Hanning filtered and Fourier transformed to obtain the real and imaginary components for each record in frequency domain; and iii) in frequency domain, the pressure segments were corrected for the instrument response (Figure 3.29).

---

\textsuperscript{21} This means that there are different numbers of segments used in the computation of the transfer function of a daily record.

---
The corrected Fourier transformed records were then used to calculate the (i) power spectral densities (PSD) of both pressure and acceleration channels; (ii) coherence between the velocity and pressure channels; (iii) upper and lower bounds of the transfer function (TFB); and (iv) phase of the transfer function (TFP). The accuracy to which the transfer function can be determined is dependent on the coherence between the channels which in turn is dependent on the noise sources in the velocity and pressure data. This was the reason for removing records with earthquake signals. The accuracy of the transfer function estimate is also dependent on the number of segments used in its estimate; the larger the number of records used increases the degrees of freedom in estimating the transfer function and yields a more accurate result (Brownlow 1978). In all four calculations above, an ensemble average of the segments was done, i.e., an average over all segments was used to determine the value at each frequency for each day. The data was sampled at 1Hz, thus the maximum frequency in this Fourier domain analysis is 0.5Hz.

Figure 3.30 shows the PSD for the pressure and gravity (displacement) channels for the 27\textsuperscript{th} November, 2009. While the PSD for the gravity channel is what one expects for the Pacific Ocean (see Figure 2.1), the DPG is uncharacteristic. There are two obvious problems with the recorded pressure spectrum: (i) the 0.07Hz spike that overwhelms; and (ii) the amplitude which has a gentle slope (excluding spike) and should mimic the acceleration spectrum. These problems in the DPG data stream have also impacted the coherence, TFB and TFP. One expects that in the frequency range of interest, where gravity waves can deform the seafloor at water depths of \( \sim 1250 \) m, there would be a significant coherence. As described above the coherence function accords significance on the transfer function. Here, the level of validity\(^{22}\) is 0.01. The coherence function (Figure 3.31A) is very chaotic and the computed values are much less than unity and not much greater than the validity level. Figure 3.31B also shows that the transfer function is not well bounded since the TFB do not lay atop each other at any point. Lastly, the transfer function phase (Figure 3.31C) is also not consistent with the theoretical phase difference between displacement/pressure and acceleration. These results from the Fourier domain analysis for 27\textsuperscript{th} November, 2009 time-series are very typical, in that, every daily data set exhibits the similar characteristics.

\[^{22}\frac{4}{\nu} = \frac{4}{2 \times 168} = 0.01\]
Figure 3.30: Power spectral densities of pressure and gravity data on 27th November, 2009.

Figure 3.31: (A) The coherence, (B) transfer function bounds, (C) transfer function phase of the 27th November, 2009 records.
Based on the above results, the conclusion was that the DPG was not performing as expected. However, since the acceleration spectrum appeared reasonable, an attempt was made to compute the compliance by substituting the DPG pressure with the pressure from a bottom pressure recorder (BPR) located 319.7m away from the SFC installation. Figure 3.32 shows the PSD of the DPG, the BPR and the gravimeter for 31\textsuperscript{st} January, 2010. The pressure spectrum of the BPR has an incredibly high amplitude in the microseismic band and a very flat response in the infra-gravity waveband (marked with two vertical lines in Figure 3.32). The coherence, TFB and TFP were calculated in a BPR pressure-gravity analysis. Figure 3.33 shows the results of this analysis. While the coherence, TFB and TFP results in this case are much better, the highly coherent portions of the data occur outside the band of interest for an infra-gravity wave analysis. Figure 3.33 also compares the DPG pressure-gravity results to the BPR pressure-gravity and serves to highlight: (i) the poor response of the DPG; and (ii) the band in which the BPR pressure-gravity results are highly coherent. This BPR-gravimeter analysis was very preliminary and the initial results were significant enough to require in-depth analysis. However, an investigation was beyond the scope of the thesis.

Lastly, the decision was made to use the (mis)fortune of a tsunami occurrence to calculate the compliance – a sort of last straw. On the 27\textsuperscript{th} February, 2009 magnitude 8.8 earthquake occurred offshore Chile and generated a tsunami that arrived at the SFC installation at 22:46 GTM and were recorded on both the gPhone and the BPR (Figure 3.34). Figure 3.35 show the PSD for the BRP, DPG and the gravimeter while Figure 3.36 compares the DPG – gravimeter and BPR-gravimeter frequency domain results. Again, another instance where the DPG did not respond as expected. These initial tsunami results points to another direction for further investigation.
Figure 3.32: PSD of the (A) BPR , (B) DPG, and (C) gravimeter for 31 January, 2010. The vertical dashed dotted lines demark the maximum frequency band in which gravity waves can apply pressure at the seafloor in water depth of 1250 m.
Figure 3.33: (A) The coherence, transfer function phase and transfer function bounds for gravity-DPG pairing and (B) for gravity-BPR pairing. The vertical black dash dotted lines mark the frequency band of interest (0.01 Hz to 0.05Hz) while the horizontal dashed line in the coherence plots is the level of validity.
Figure 3.34: Time-series of the tsunami generated by the earthquake on 27\textsuperscript{th} February, 2010. The data is band-pass filtered between 2 seconds and 2.7 hours.
Figure 3.35: PSD of the (A) DPG, (B) BPR and (C) gravimeter for 27–28th February, 2010. The vertical dashed dotted lines demark the maximum frequency band in which gravity waves can apply pressure at the seafloor in water depth of 1250 m.
Figure 3.36: The coherence, transfer function bounds and phase for (A) the gravity-DPG combination and (B) the gravity-BPR combination.
3.9 A final word

Upon the return of the SFC apparatus to the NEPTUNE Canada base in Victoria, British Columbia, Dr. Willoughby, the University of Toronto marine geophysics group’s lead scientist on the project, travelled to inspect the recovered instrument and supervise the diagnostics. A series of test provided no obvious reason for the reported ground fault. The only suspect was a leaking outer connector (Figure 3.37). Regarding the poor operation of the DPG, we have been able to ascertain that the system completely shuts down at 8.6V i.e., the gPhone stops recording data. During the time the instrument was deployed, the gPhone produced data, so the reduced power to the system may explain the poor DPG data quality. In an effort to fully investigate the performance of the DPG, the marine geophysics group requested the return of the SFC apparatus.

![Image of oil filled mink connector leaking](figure3_37.png)

Figure 3.37: The oil filled mink connector appeared to be leaking upon inspection of the recovered SFC apparatus. Yellow arrow is ~15cm

3.10 Summary

NEPTUNE Canada, a seafloor observatory offshore British Columbia, Canada has provided the necessary infrastructure for the long-term monitoring of gas hydrate deposits. This infrastructure includes a 800km backbone cable capable of providing power and remote
access to a host of instruments. The observatory consists of six sites spread across the Juan De Fuca plate chosen to support five research themes. One of such themes is the investigation of vent processes in and temporal changes of gas hydrates systems and is facilitated by the installation of node ODP Site 889 located in the Cascadia vent field. On this node a number of experiments have been installed including a SFC experiment, which was set up specifically to investigate the temporal nature of shear properties in a region near Bullseye Vent.

The SFC experiment consists of a Scripps DPG and a Micro-g LaCoste relative gravimeter. The DPG is capable of measuring fluctuation of pressure of 1Pa at the seafloor, generated by infra-gravity waves, in a background of MPa. The gravimeter, which measures the associated acceleration of the seafloor due to the deformation by the infra-gravity waves, is capable of measuring and recording accelerations of 1microgal (which at 1Hz represents a displacement of 0.2533nm; \(a = -\omega^2 u_z\)). The gPhone is housed in a gimbal designed to maintain a vertical alignment. The apparatus is fully assembled in a spherical Aluminium pressure vessel which is supported by a bottom assembly, a deployment package designed for stability and instrument protection. The instruments and their circuitry were modified to allow for power and communication connection to NEPTUNE Canada’s nodes as well as to output data as one entity.

The calibration of the DPG resulted in a scaling factor of \(7.260712 \times 10^{-6}\) Pa/ADunit while the gPhone’s value was 0.0015904. The assembled apparatus passed the communication and power test performed by NEPTUNE Canada and was deployed at latitude 48.669525 degrees and longitude -126.85184 degrees at ~1250 m. The seafloor pressure and acceleration was measured continuously over a six month period before the SFC package was recovered. During this time, pressure signals from earthquakes and tsunamis and the resulting response of the earth were recorded. Preliminary analysis of the data lead to the conclusion that the DPG malfunctioned and thus the SFC data set could not be used to fulfil the goals of this thesis.
Chapter 4
NEPTUNE Canada II – Variations in sediment stiffness

4.1 Introduction

The failure of the SFC experiment dictated that other data be secured in an effort to fulfil the aim of the thesis. At Site 889, there exists a broadband seismometer (BBS) installation which is part of the NEPTUNE Canada’s seismograph network. The saving grace of this network is that (i) the seismometers not only records deformations from earthquakes but is also capable of recording deformations produced by long-period infra-gravity waves and (ii) each installation includes a DPG. Therefore, the data from this BBS-DPG pairing were used in the evaluation of the Bullseye Vent gas hydrate system.

The NEPTUNE Canada ocean bottom seismometer (OBS) system was designed by Güralp Systems and comprises of a CMG-5T triaxial broadband feedback strong motion (±2g) accelerometer; a CMG-1T triaxial broadband (360s – 50Hz) feedback seismometer (Figure 4.1) with three independent sensors; a 24-bit, seven channel digitizer, one for each of the sensors; an enhanced acquisition and communications module; a microprocessor-controlled orientation and levelling system with ±18 degrees of tilt compensation; and a high precision and stability real time clock with real time drift 0.5 seconds/year at constant temperature (NEPTUNE Canada 2011a). The system also includes a 20-bit channel for interfacing the DPG (NEPTUNE Canada 2011b). The entire system is housed in a cast titanium sphere, 54cm tall and 41cm diameter, capable of withstanding pressures in water depth of 3000m.

The OBS package was tested by the NEPTUNE Canada’s science team and was deemed suitable for deployment. It was deployed in September, 2009 at latitude 48°40.2322' and longitude -126°50.9260' at a depth of 1256 m (NEPTUNE Canada [Date unknown]). The seismometer package (Figure 4.2), mounted in its insulated double half-sphere pressure vessel, was surficially buried within a 53 cm diameter, 61cm deep cylindrical PVC caisson, filled with glass beads (Figure 4.3) (Owens 2009b). The DPG was placed 20m away from the seismometer on a small frame on the seafloor. The data from both instruments were
simultaneously logged but the pressure data were archived at NEPTUNE Canada Data Management and Archival System (DMAS) while the OBS data was archived at IRIS. To date, the OBS data are controlled and limited because they are embargoed by the US Navy and only the low frequency data (1Hz) are released a few months after its divergence. However, when released, obtaining the 1Hz seismometer data was direct. On the other hand, obtaining the DPG data was not. The fact that the DPG was not calibrated before its deployment required special permission be sought for its use. As such, acquiring the DPG data meant requesting them from NEPTUNE Canada who made them accessible in March, 2011 (see Appendix C for details).

Figure 4.1: Güralp broadband seismometer and strong motion accelerometer within their cast titanium sphere. Source: NEPTUNE Canada (2011).
Figure 4.2: Deployment package for the ODP 889 seismometer. The caisson (light blue) and the cased seismometer (sphere in middle) rests on a foam pad. Blue canvas bags filled with glass beads used for burying encased seismometer. Source: NEPTUNE Canada (2011).

Figure 4.3: Seismometer being buried with glass beads. Source: NEPTUNE Canada (2011)
The approach to calculating and assessing the validity of transfer function, described in Section 3.8.1, is applied to the BBS-DPG pairing. Here however, the transfer function in Equation 3.1 is described in terms of velocities instead of accelerations. This is because the seismometer records the response of the Earth to the low-frequency deformation sources as a velocity. The calculated BBS-DPG transfer function can be scaled by $-\omega$ for the true expression of vertical compliance – ratio of displacement to force applied. This application of pairing the seismometer and DPG data is becoming common practice in marine seismology because the vertical displacement component produced by infra-gravity waves are large enough to obscure very small signals from earthquakes thus limiting observations of Rayleigh waves and in shallow water, reduce the ability to detect long period deformations (Webb and Crawford 1999). Thus the measured vertical compliance function is combined with pressure records to correct the vertical seismic records, increasing the signal to noise ratio in the low frequency band (Webb and Crawford 2010; Frontera et al. 2010; Dahm et al. 2006; Crawford and Webb 2000).

### 4.2 Processing procedure and results

The processing procedure was identical to that applied to the gravimeter – DPG data (Section 3.8). First the data was sorted\(^{23}\) (see Appendix C for details). The Fourier domain analysis involved separating each 24-hour velocity and pressure segment into records of 512 seconds, removing the mean from, and Hanning filtering each 512 second record, then computing the coherence function, the transfer function bounds and phase by performing an ensemble average of power and cross spectra of the Fourier transformed records\(^{24}\). Again, the Fourier analysis was performed using only records and/or portions of records of pressure and velocity, chosen to avoid instrument noise, gaps and seismic events. The pressure segments

\(^{23}\) Sorting involved ensuring that both channels had equal time stamps; noting the gaps in the data; noting of segments of records with spurious points; and noting of segments with known earth tremors.

\(^{24}\) See section 3.8 for details on the Fourier domain analysis and the calculation of the coherence, transfer function bounds and transfer function phase.
were corrected for the assumed\textsuperscript{25} instrument response (Figure 3.29). The velocity segments needed not be corrected since the instrument response is flat within the frequency band of interest (Figure 4.4). The set of velocity and corrected pressure segments were then used in the computation of the transfer function.

![Figure 4.4: The CMG 1T broad band seismometer frequency response. Source: IRIS. The black dotted lines mark the frequency range of investigation. Amplitude in red and phase in blue. Source: Bub Hutt, IRIS](image)

Data spanning the period 1\textsuperscript{st} October, 2010 to 16\textsuperscript{th} May, 2011, were obtained. Of 232 days in the period, 228 days of data were available. Figure 4.5 shows a sample of the pressure and velocity time-series that has no visible irregularities. These 24-hour records are of the 14\textsuperscript{th} October, 2010. Figure 4.6 shows the first 600 seconds of the time-series in Figure 4.5, after the data from both channels were band-passed between 0.01Hz and 0.05Hz. This filtering allows for a cursory comparison of the two channels within the infra-gravity wave frequency

\textsuperscript{25} No information was available from NC about the internal circuitry of the DPG. However, since our own SFC experiment (Chapter 4) uses the identical instrument from the same company, I applied the filtered response from that instrument.
band. From this plot, a correlation between the pressure and velocity signals is visible, though there is an observable phase difference\textsuperscript{26} between them. To demonstrate the type of data sections that were removed before the computation of the transfer function, Figures 4.7 and 4.8 were included. On 21\textsuperscript{st} October, 2010, the Gulf of California magnitude 6.7 earthquake occurred and was recorded by both instruments (Figure 4.7) while on 1\textsuperscript{st} March, 2011 there was a portion on both channels with very large values of pressure and velocity attributed to instrument (/electronic) noise (Figure 4.8).

Sorting resulted in a total of 222 days being useable in the assessment of the targeted gas hydrate system. The transfer function bounds and phase and the coherence function of 4 days of the 222 day data set have been included to demonstrate the data set characteristics and to substantiate the procedure chosen to evaluate the long-term temporal behaviour of the Bullseye Vent. They are: 14\textsuperscript{th} October, 2010, a day with no visible anomalies; 21\textsuperscript{st} October, 2010, a day with a recorded earthquake; 13\textsuperscript{th} January, 2010, another day without any visible anomalies; and 1\textsuperscript{st} March, 2011, a day with spikes.

As discussed in Chapter 3, the transfer function bounds constrain the compliance function. The upper bound is determined by calculating the ratio of velocity power spectrum and the cross power spectrum between both channels, whereas, the lower bound is the ratio of the cross power spectrum and power spectrum of the pressure channel. The transfer function is well constrained when the upper and lower bound are identical, i.e., when plotted together on identical axes, they coincide. Figures 4.9 to 4.12 (A) show two distinct frequency bands in which the compliance is well constrained. In Figure 4.10 the ranges are ~0.007 Hz to ~0.03 Hz and ~0.1Hz to ~0.25 Hz. The lower frequency range is that which, at 1250 m, gravity waves can deform the seafloor.

\textsuperscript{26}Further analysis, discussed in section 4.3 confirms that this phase difference is ~90 degrees which is consistent with deformation under wave loading.
Figure 4.5: Pressure and velocity record for 14th October, 2010 in counts from NEPTUNE Canada 889.

Figure 4.6: The first 600s of the velocity and pressure record of 14th October, 2010 bandpassed to retain the frequencies in the gravity wave band (40s and 200s).
Figure 4.7: Gulf of California earthquake, magnitude 6.7 recorded by seismic station.

Figure 4.8: Time-series of 1st March, 2011 with a spurious section on both instruments.
Figure 4.9: Frequency domain analysis results for 14th October, 2010. (A) Transfer function bounds as a function of frequency. The upper and lower bounds on the transfer function between velocity and pressure bounds are quite close in the gravity wave band, diverge at the cut-off frequency and approach each other again in the microseismic band. (B) Coherence as a function of frequency (dashed black line marks the 0.01 significance level) between the pressure and gravity data is quite high in the gravity and microseismic bands, but non-valid in-between. (C) Transfer function phase as a function of frequency and is relatively constant in the well bounded regions. The pair of vertical black lines bounds the region 0.01Hz – 0.03 Hz.
Figure 4.10: Same general description as in Figure 4.9. (A) The transfer function bounds, coherence and transfer function phase of the full data set without the removal of the earthquake portions of the data. (B) The results when the portion of the data containing the earthquake was removed.
Figure 4.11: Description as in Figure 4.9. The average variation in the pressure and velocity data was larger than the average of the data set.

Figure 4.12: Description as in Figure 4.9. The frequency analysis was done on 56 records – the number of records remaining after the large valued data portion (Figure 4.8) was removed.
Figures 4.9 to Figure 4.12 show the coherence as a function of frequency. As previously stated in Section 3.8.1, the coherence function places a significance level on the transfer function: if greater than $4/v$ where $v$ is the number of records used in the estimate, the transfer function is valid. Here, $4/v = 0.01$ (marked by the black dashed lined) and within the frequency range 0.01Hz to 0.03Hz the coherence is greater than 0.8, considerably greater than the significance level, in all the figures. The frequency band in which these ‘double humps’ occur are coincident with the region of the transfer function bounds that atop each other. The consistency of the transfer function bound and coherence results within the frequency band 0.01Hz to 0.03Hz affirm the high quality data constraining the shear modulus at Site 889. Figure 4.13 shows that the coherence varies between 0.4 and 0.9 over the period thus, the coherence within the 0.01Hz and 0.03Hz frequency band is substantially greater than the significance level over the period. There are several factors that affect the coherence function including sea states (Wang et al. 2010) and the possibility that the transfer function is dependent on propagation direction of the source waves (Webb and Crawford 1999).

Figures 4.9 to 4.12 (C) show the transfer functions phase, which as expected, is flat and has an average value of -90 degrees in the frequency band in which the transfer function is well constrained. This average transfer function phase is consistent with the theoretical phase difference between displacement/pressure and velocity, a phase relationship observed throughout the entire data set.

Figure 4.13 shows the variation in the transfer function in the infra-gravity wave band and it is this change that will be examined in evaluation of the gas hydrate system’s long-term temporal changes. In the analysis that follows, the un-scaled pressure and velocity data were used so as to ensure that the results were not influenced by the magnitude disparity between each channel. This was because, unfortunately, the DPG was not calibrated before its deployment. This approach is acceptably valid since the aim is to determine the relative magnitude over time, i.e. the trend and not an exact value of the transfer function.
Figure 4.13: A surface plot of the daily coherence for the 222 days data set. It highlights the frequency band in which the coherence is high. For the entire data set, the double hump is visible with a wider variation in values in the gravity wave band (lower red region) than in the microseismic band (upper red region). However, in the gravity waveband, the coherence is at least 0.6 – much greater than the level of significance (0.01). Colour bar indicates the coherence.
Figure 4.14: The variation in the values of the transfer function over the 228 days period. The largest variations are in the frequency range 0.035Hz and ~0.08Hz. Colour bar gives a relative measure of the transfer function.
Lastly, a word on the nature of the power spectral densities of the pressure and velocity channels. Figure 4.15 shows a spectrogram of the pressure data for the 3\textsuperscript{rd} October, 2010. The variation in the power spectrum over a 24-hour period is not large and thus using each segment in the frequency analysis is justified. Figure 4.16 A and B show the PSD for the entire 222 days data set for pressure and velocity, respectively. Both figures show that, over the period, the pressure and velocity spectrum for each day is similar to each other and each channel is comparable. Thus, the full data set can be used in a trend analysis. The energy in the microseism band is not used in determination of seafloor compliance since a compliance calculation requires a determinable stress field, and a simple wavenumber frequency relationship for microseisms is difficult to establish. The analysis, therefore, is concentrated within the low frequency hump of the data where the stress field is well described.

Figure 4.15: Calculated power spectral densities for each 512 seconds segment of the 24 hour record of 3\textsuperscript{rd} October, 2010.
Figure 4.16: The power spectral densities of both channel for period spanning 1\textsuperscript{st} October, 2010 to 16\textsuperscript{th} May, 2011. (A) PSD for the pressure channel and (B) PSD for the velocity channel. The largest variation in amplitude is within the micro-seismic band and the variations are coincident in both channels.
4.3 Analysis of data

4.3.1 Investigation and validation of temporal changes

The NEPTUNE Canada seismometer installation is at water depth 1256m which, through the wave dispersion equation, sets the maximum frequency of infra-gravity waves capable of deforming the seafloor between 0.02Hz to 0.049Hz. The frequency domain analysis in the previous section indicates that the compliance function is well constrained and the phase relationship between the pressure and velocity channels is valid in the 0.01Hz – 0.03Hz range. Therefore, an investigation of the temporal changes was conducted within this frequency band of the transfer function. Detecting changes in variables is a common practice in the atmospheric sciences since environmental changes and trends impact the quality of life. Atmospheric literature describes statistical methods for detecting and estimating trends in long-term data sets (Hess et al 2001; Weatherhead et al. 1998; Gardiner et al. 2008). Gardiner et al. 2008 describes a straight forward approach to trend analysis which is adopted in this thesis. The method is to fit the best fitting straight line, in the least squared sense, to the data where the gradient of the line is the trend. The statistical significance of the trend is determined and bootstrap re-sampling is used to place an uncertainty on the gradient because it does not require any assumptions about the statistical distribution of the data.

The data points for determining the trend were derived by computing the mean of the transfer function within 0.01Hz to 0.03Hz frequency range for each day of the data set. This produced a total of 222 single points representing each day of the data set, to which the best straight line was fitted. A series of steps were performed in the determination of this best fitting straight line. The first step was to identify the outliers (points that deviate from normality) so that the least squares approach can be safely used. Creating a cumulative histogram and probability plot of residuals of the least squared estimate (LSE) of the data points is an easy way of spotting the deviating points in a data set. Figure 4.17A and 4.17B show the results of the normality test. The cumulative histogram gives a measure of the non-normality of the data set when compared to the theoretical curve of a normal distribution which has the same mean and standard deviation as the data. The probability plot also highlights the deviating points by showing the level of consistency with the ideal relationship represented by the dashed blue line (expected residual=actual residual). The outliers are clearly identifiable
both figures. Figures 4.17C and 4.18D show the cumulative histogram and probability plot, respectively, of the data set with the obvious outliers removed. With these outliers removed, more of the data fit the ideal relationships. Without the outliers, there remained 203 days of data spanning 1st October, 2010 to 16th May, 2011. From this subset of data, the LSE of the best fitting straight line yields an estimate of the trend of -4.49 x 10^{-3} % of 3rd October, 2010 (Day 1\(^{27}\)) of the transfer function. Figure 4.18 shows the fit to the data.

The trend was determined with data having a less non-normal distribution of errors, thus, standard statistical tools can be used to evaluate its significance and estimate an error in fitting it. The general principles of a statistical test require (i) a hypothesis, (ii) a calculable statistic, \( S_{\text{calc}} \) and (iii) a significance level aimed at determining the probability of rejecting the hypothesis. If \( S_{\text{calc}} > S_{\text{crit}}^{28} \), at the chosen significance level, the hypothesis is rejected (Bailey 2006). Following Bailey (2006), the significance of the estimate of the gradient term of the simple linear regression, \( y = bx + c \), was determined. The hypothesis tested was \( H_0: b = 0 \), i.e. is there no true or convincing relationship between the average compliance within the frequency band and the period? If the hypothesis is true, then the test statistic

\[
F = \frac{(SSE_{\text{without}} - SSE_{\text{with}})/q}{SSE_{\text{with}}/(n - m)} \quad (4.1)
\]

is expected to have a \( F_{q,(n-m)} \) distribution if the errors are normal. The test statistic was computed for a fit where the \( b \) model parameter is removed. Here, \( SSE \) is the sum of the squares errors; the subscript \( \text{with} \) refers to the model with the \( b \) term included; \( \text{without} \) refers to the model with the \( b \) term removed; \( q = 1 \), the number of parameters removed, \( n = 203 \) is the number of independent variables, and \( m = 2 \) is the number of \( \text{with} \)-model parameters. The partial F-statistic calculated for the data using Equation 4.1 resulted in \( F_{\text{calc}} = 6.10 \). The hypothesis was tested in two ways: a permutation test and a parametric test.

\(^{27}\) This is considered Day 1 because the actual first day of the data set, 1st October, 2010 has large errors.

\(^{28}\) \( S_{\text{crit}} \) is determined from a statistical table for the known distribution of the statistic calculated from the data.
Figure 4.17: (A) and (C) are the cumulative histograms of the data (red circles) overlain by the cumulative normal distribution having the same mean and standard deviation as the data (blue dashed line). (B) and (D) are the probability plot of the data (red circles) compared to the theoretical result for a Normal distribution (blue dashed line). (A) and (B) are the results using the full data set (222 data points) and (C) and (D) are the data set with outliers removed (203 data points).
Figure 4.18: Best fit straight line to the transfer function for the period 1st October, 2010 to 16th May, 2011. Black dots represent the mean of the transfer function in the frequency band 0.01Hz – 0.03 Hz. The dashed blue line is the best LSE fit. Error bars are estimated by bootstrapping the residuals of the original data (Appendix B) and represent the errors on the transfer function estimate. The errors range from 0.4% – 4 %

The permutation test was done by recalculating the $F$-statistic for a large number of permutations of the $y$ (percent transfer function change) with the $x$ (time) data with a tolerance of 0.001 to determine $F_{crit_perm}$. Figure 4.19A shows the result of the permutation test – the resulting histogram matches the theoretical $F_{1,201}$ distribution. A cumulative histogram of the partial $F$ values (Figure 4.19B) was plotted from which $F_{crit_perm} = 3.89$ at a 5% significance level. Therefore, $F_{crit_perm} < F_{calc}$, thus the hypothesis is rejected. The parametric test gives $F_{crit_para}=3.84$ which is also smaller than $F_{calc}$ at a critical value of 5%. Hence both statistical tests prove that the trend is significant: i.e. the correlation parameter, $b$ (trend) is significant at a 5% significance level. $F_{calc} = F_{crit_perm}$ at significance level 1.42 % and $F_{calc} = F_{crit_para} 1.43%.$
An estimate of the 95% confidence limit was done on \( b \) by bootstrap re-sampling with replacement, the errors of the best fit line. Figure 4.19C shows a histogram of the \( b \) values overlaying a normal distribution of identical mean and standard deviation of the data. The bootstrap error in \( b \) is 0.0035 with upper and lower limits of \(-8.06 \times 10^{-3}\) and \(-9.72 \times 10^{-4}\), respectively. A standard \( t \)-test estimated the error bounds as \(-0.0036; -8.07 \times 10^{-3}\) (upper) and \(-9.05 \times 10^{-4}\) (lower). With the trend validated and bounded its interpretation followed.

### 4.3.2 Interpretation of results

The least squares trend estimate suggests a 2.88% decrease in the transfer function over the 228 days. This transfer function decrease is linearly proportional to a decrease in the compliance of the region. Chapter 2 explained how a compliance measurement, as a function of frequency, informs about the elastic profile of the sediment and that it is particularly sensitive to the shear modulus (\( \mu \)). An increase in the shear modulus means an increase in stiffness of the sediment which causes a decrease in the compliance response. Chapter 1 established the presence of gas hydrates around the Bullseye region and described how they increase the shear modulus of the sediments in which they are found. The OBS package was deployed in a region where no \textit{in situ} information is available. As such, the interpretation of the trend relied on parameters derived from research conducted at Bullseye Vent (Section 1.4) since the OBS package is installed ~390m away (Figure 3.2) and lies the in region where gas hydrates are inferred (Figure 1.9). Therefore, the observed decrease in the transfer function was attributed to an increase in the hydrate concentration. The question to be answered is ‘\textit{How big an increase in hydrate concentration?}’ An answer to this question required first, obtaining an elastic parameter model whose transfer function (or compliance) matched the measured transfer function of Day 1 (3\textsuperscript{rd} October, 2010) of the data set, then, determining a new elastic model (by varying the initial one) whose mean compliance differs from that of the initial model by the observed change (-2.88%). The final step was to estimate the change in hydrate concentration that would result from the changed elastic parameters.
Figure 4.19: (A) Histogram of F values for dropping the gradient in the straight line fit for all possible permutations, overlain by the theoretical $F_{1,203}$ distribution (red solid line). (B) Cumulative histogram with cumulative normal distribution having the same mean and standard deviation as the data overlain. (C) Histogram of the bootstrap results overlain by the normal distribution. (D) The cumulative histogram of the bootstrap results with the normal distribution overlain.
4.3.2.1 Method of determining the elastic parameters

To compute the compliance of a model, its shear modulus (μ) and Lamé parameter (λ) is required but determining them is not straightforward. Willoughby and Edwards (1997), in their search for a method for calculating μ and λ of gas filled sediments found that: (i) the resulting compliance value was very method dependent, and (ii) the weighted mean velocity equation of Lee et al. (1996) combined with the Castagna et al. (1985) relation, gave the most reasonable result. However, Helgerud et al. (2000) pointed out that the Lee et al. (1996) method of estimating the compressional velocities of gas hydrate bearing sediments, based on combinations of Wood (1941) and Wyllie (1956) equations, is problematic because there is no systematic way of choosing parameters to describe the gas hydrate bearing rocks. As such, the Lee and Waite (2008) model was employed in this analysis. The major reason for choice was the fact that this method allows for the independent computation of the shear modulus and the Lamé parameter instead of their determination from the velocities.

Lee and Waite (2008) developed a three-phase Biot- (Biot 1956) type equation (TPBE) to calculate independently, the shear and compressional velocities of gas hydrate bearing sediments (GHBS). They assume that the gas hydrate, sediment and pore fluid form three frameworks which are homogenous and interwoven and thus represents the load-bearing stream of gas hydrate occurrence. The model includes the energy contributions due to contact between the sediment grains and gas hydrate, and the frictional losses due to the interaction between the same. Lee (2007) verified that this model can be used to calculate fast velocities of GHBS at low frequencies because the attenuation at these frequencies can be ignored. Under the condition that attenuation is ignored, the compressional and shear velocities at low frequencies can be expressed as $V_p = \sqrt{\sum_{i,j}^3 R_{ij}/\rho_b}$ and $V_s = \sqrt{\sum_{i,j}^3 \mu_{ij}/\rho_b}$ where the bulk density, $\rho_b$ and the elements $R_{ij}$ and $\mu_{ij}$ are given in Appendix H.

This predictive model requires many variables some of which are site specific. Table 4.1 lists some of the variables used in the calculations and their source. Porosity was determined from fitting a straight line to the site U1328 porosity data (Appendix F) and the gradient was determined to be -0.0002 per meter with a 0.55 porosity at the seafloor. The values of clay content (Appendix F) were also guided by the IODP 311 site U1328 results.
Table 4.1: Variables used in the TPBE estimate of the elastic parameters for gas hydrate bearing sediments. $K$ represents bulk modulus, $\mu$ the shear modulus, $\rho$ the density, $\eta$ the pore fluid viscosity, $\alpha$ the consolidation factor and $\varepsilon$ a fitting parameter. Subscripts’ $c$, $q$, $w$, and $h$ represent clay, quartz, water and hydrate respectively. The values denoted 1 are from Lee and Waite (2008) the others are taken from Lee (2007) (gathered from other sources).

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_q$</td>
<td>38 GPa</td>
<td>$\mu_q$</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>= 2650 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$K_c$</td>
<td>20.9 GPa</td>
<td>$\mu_c$</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>= 2580 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$K_w$</td>
<td>6.41 GPa</td>
<td>$\mu_w$</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>= 910 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$K_h$</td>
<td>2.29 GPa</td>
<td>$\mu_h$</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>= 1000 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>$\eta_S$</td>
<td>= 0.0018 Pa.s</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>= 40</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>= 0.12</td>
<td></td>
</tr>
</tbody>
</table>

4.3.2.2 Determining the variation in hydrate concentration

Given that the data used in this evaluation are obtained from a single instrument package in a stationary position, is it appropriate to fit a 1-D elastic model to the transfer function of Day 1. To find the best fit 1D model, I began with the simplest elastic model - a half-space, then sequentially graduated the model until the best fit to the transfer function of 31st October, 2010 was attained. This process involved building on the half-space by increasing the layers of the model and varying their elastic parameters and thickness until the best-fit was attained. The choice of parameters, thickness and number of layers were guided by applicable IOPD 311 drilling results (page 20), as they provide in situ information about the region, and the frequency-depth response of the compliance function. Figure 4.20 compares the compliance of the transfer function of Day 1 to a subset of the models tried. Table 4.2 details the values of the elastic parameters of each layer as well as the layer’s thickness, hydrate concentration,

---

29 The consolidation factor accounts for the amount of sediment stiffening due to consolidation and is site specific (Lee 2007).
30 Encapsulates the effectiveness of the porosity reduction in increasing the moduli of the sediment due to the increase in hydrate in the sediments. $\varepsilon$ is empirical and varies between 0 and 1 (Lee and Waite 2008).
31 ‘Best fit’ refers to that which resulted from all models tried in the forward modeling process of which 7 are shown here.
clay concentration, clay concentration and porosity for every model of the fitted models presented in Figure 4.20.

Fit 1 (Figure 4.20A), excludes the uniform half-space as an appropriate fit to the transfer function and established that there is more character to the sediments at investigated site. The next logical choice was to try layer over half-space models. Fit 2 is an example of such and its compliance response, though starting to exhibit some variation as a function of frequency, does not capture the nature of the experimentally derived transfer function. Fits 3 and 4 then followed under the assumption that perhaps, more structure (by adding layers) will change the character of the compliance response. This turned out not to be necessarily true since Figure 4.20B illustrates that Fits 3 and 4 have the same characteristic shape of Fit 2, with the exception that at the higher frequencies (shallower sediments), the compliance value is smaller. Up to this point, the modelling was heavily guided by the IODP 311 Site U1328 results which describes to an extent, the sediments around Bullseye Vent (detailed in Chapter 1). This evidence of adding structure at different depths reflects the results of the sensitivity study; that adding layers with different elastic parameters at particular depths will affect the compliance response within a specific range of frequencies. Thus, the tactic was changed to manipulating the base models (Fit 2 and 3) using the sensitivity results. Fit 5, 6 and 7 are the result of this manipulation. Model 5 was tweaked to Model 6 based on its compliance response in the high frequency region of the sediment (between 0mbsf and 270mbsf). However, since the sensitivity function broadens (becomes less localized) as the sensing frequency decreases, it means that, though the aim is to better fit the shallower sediments of the best fit model (among the models tried), the elastic properties of the deep sediments must be considered given that they influence the compliance of the shallow sediments. The forward modelling process resulted in a four (4) layers over a half-space model (Fit 7) being the model of best fit to the transfer function of 3rd October, 2010. Figure 4.21A shows the transfer function with errors and Figure 4.21B is a schematic of best fit model. Table 4.3 lists the elastic parameters of the best fit model along with the seismic velocities and Poisson’s ratio for completeness.
Figure 4.20: Comparing the transfer function of Day 1 of the data set to the scaled compliance response as a function of frequency of seven different models. Table 4.2 lists the values of the parameter which describes each model. Key: TF – transfer function.
Table 4.2: Parameters describing the seven (7) models used in forward modelling of the transfer function of Day 1 (3rd October, 2010) in the data series. Key: Clay Con. – clay concentration, Th – layer thickness, Hydrate conc. – Hydrate concentration, HS – half-space.

<table>
<thead>
<tr>
<th>Fit/Model</th>
<th>Layer</th>
<th>Th (m)</th>
<th>Clay Con. (%)</th>
<th>Porosity (%)</th>
<th>Hydrate Conc. (%)</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>$\lambda$ (GPa)</th>
<th>$\mu$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HS</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>220</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>45</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>175</td>
<td>80</td>
<td>54</td>
<td>5</td>
<td>1757</td>
<td>4.32</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>10</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>160</td>
<td>80</td>
<td>54</td>
<td>5</td>
<td>1757</td>
<td>4.32</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>50</td>
<td>80</td>
<td>40</td>
<td>5</td>
<td>1988</td>
<td>5.55</td>
<td>0.223</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>45</td>
<td>80</td>
<td>60</td>
<td>95</td>
<td>1609</td>
<td>9.53</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>175</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>780</td>
<td>80</td>
<td>43</td>
<td>0</td>
<td>1940</td>
<td>5.44</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>220</td>
<td>80</td>
<td>60</td>
<td>20</td>
<td>1649</td>
<td>4.38</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>780</td>
<td>80</td>
<td>43</td>
<td>0</td>
<td>1940</td>
<td>5.44</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>100</td>
<td>80</td>
<td>60</td>
<td>22</td>
<td>1647</td>
<td>4.55</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>250</td>
<td>80</td>
<td>54</td>
<td>5</td>
<td>1757</td>
<td>4.32</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>100</td>
<td>80</td>
<td>40</td>
<td>5</td>
<td>1988</td>
<td>5.55</td>
<td>0.223</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>520</td>
<td>80</td>
<td>43</td>
<td>0</td>
<td>1940</td>
<td>5.44</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>HS</td>
<td>80</td>
<td>23</td>
<td>0</td>
<td>2271</td>
<td>8.97</td>
<td>0.452</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.21: (A) The transfer function of 3rd October, 2010 (black stars with error bars) compared to the scaled compliance of model (Fit) 7 (red solid line) as a function of frequency. (B) A schematic of model (Fit) 7.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Thickness (m)</th>
<th>$V_p$(m/s)</th>
<th>$V_s$(m/s)</th>
<th>$V_p/V_s$</th>
<th>Poisson's Ratio</th>
<th>$\varepsilon$ (1/GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>1720.7</td>
<td>314.9</td>
<td>5.464</td>
<td>0.483</td>
<td>3.17</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>1614.3</td>
<td>273.3</td>
<td>5.907</td>
<td>0.485</td>
<td>3.92</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>1736.4</td>
<td>335.2</td>
<td>5.1807</td>
<td>0.481</td>
<td>2.32</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>1730.4</td>
<td>307.6</td>
<td>5.62578</td>
<td>0.484</td>
<td>2.81</td>
</tr>
<tr>
<td>Half-Space</td>
<td>--</td>
<td>2085.4</td>
<td>446.2</td>
<td>4.674</td>
<td>0.476</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters of the best fit model.
A comparison of the transfer function of Day 1 to the compliance of the models required the transfer function of Day 1 be scaled. This scaling applied was so that the value of the experimental transfer function, at the lowest frequency (0.01Hz), matched the compliance of the specific model at that frequency. In effect, the end result is a character study instead of a true compliance response evaluation. This limitation is due to the fact that a calibration factor was not available for the DPG. Therefore, the transfer functions determined from the experiment cannot be converted to a true compliance response and thus, cannot be used to constrain the shear modulus of the sediments. However, the transfer function values over the 228 days can be used to answer the question of how much hydrate is required to cause the observed 2.88% decrease over the period. This approach is valid since the change in the transfer function is equivalent to the change in the compliance. For completeness, Figure 4.22 is included. It displays the true compliance response for a subset of the models (Fits 1 through 7 –Table 4.2) created during the process of finding the one of best fit.

Figure 4.22: The compliance as a function of frequency for the 7 models described in Table 4.2.
Having obtained a best fitting model, the next questions asked were: (i) what change in $\lambda$ and $\mu$ of this best fit model will produce the observed 2.88% decrease in compliance; and (ii) how much must the hydrate concentration change in order to account for the change in the elastic parameters? In response, the modelled $\lambda$ and $\mu$ of layer 1 and 2 were, in turn, increased incrementally by a percent until the mean of the transfer function in the 0.01Hz to 0.03Hz band showed the 2.88% decrease. These layers were varied because they are within the region of gas-hydrate stability. The incremental change of $\lambda$ and $\mu$ was done while keeping ratio of the percent change in $\lambda$ and the percent change in $\mu$ constant at 0.32 (see Appendix D for details on this approach). This process resulted in a percentage increase in $\lambda$ of 44.97% and $\mu$ of 162.37% in order to produce the 2.88% decrease in compliance when layer 1 properties were varied. For layer 2, $\lambda$ and $\mu$ were 6.38% and 19.60%, respectively. Table 4.4 provides a summary.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Initial HC</th>
<th>New HC</th>
<th>$\Delta$ HC (%)</th>
<th>$\Delta\lambda$ (%)</th>
<th>$\Delta\mu$ (%)</th>
<th>$\Delta\lambda / \Delta\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>63.9</td>
<td>41.90</td>
<td>44.97</td>
<td>162.37</td>
<td>0.28</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>14.0</td>
<td>9.00</td>
<td>6.38</td>
<td>19.60</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 4.4: Results from the estimation of the hydrate concentration. The elastic parameters of Layer 1 and Layer 2 were changed, in turn, in the best fit model (model 7) so that difference in the compliance of the changed model 7 and original model 7 matched the observed change in the transfer function. A discussion of the $\Delta\lambda / \Delta\mu$ ratio is in Appendix D. Key: HC – hydrate concentration.

Figure 4.23 shows the compliance functions which result from changing the Lamé parameter and the shear modulus of the top layer only, of layer 2 only and of both layers 1 and 2 simultaneously. Several observations are worth noting: (i) the compliance response in all three cases increases monotonically with frequency; (ii) the greatest change in the compliance response is at the highest frequencies (shallow sediment); (iii) the parameter changes in layer 1 result in the smallest compliance variation while layer 2 has the greatest and the paired layers fall between both; and (iv) the change in the compliance response is one
of a decrease. These four observations are as expected. The thinness of layer 1 predicts that its parameters must vary widely to compensate for the observed change in the experimental transfer function compared to that of layer 2 which is two and a half times layer 1’s thickness. The fact that the layers are the two uppermost ones, the variation observed must occur in the higher frequency range as outlined in the seafloor compliance theory. The slight variations in the lower frequency compliance are also supported by results of the compliance function sensitivity response (Section 2.5.1). Finally, the high frequency variation in this elastic parameter modelling exercise is supported by the observed variation in the transfer function over the 222 days of the entire data spectrum (Figure 4.14). Figure 4.14 supports varying the compliance of the uppermost layers since the daily variations in the experimental transfer function is seen in the 0.1 Hz to 0.03Hz frequency band.

Figure 4.23: The compliance response of the best fit model compared to the compliance responses of the original best fit model having variations in layer 1, layer 2 and layer 1 and 2 paired, respectively that matches the observed change in the transfer function over the period.
Further application of the TPBE resulted in an increase of 41.9% in the modelled 100m upper layer hydrate concentration, from 22% to 63.9%, explaining the percentage increase in $\lambda$ and $\mu$. Whilst for the second layer, a 9% increase in the hydrate concentration, from 5% to 14%, explains the increase in its $\lambda$ and $\mu$. Table 4.4 summarises these results. The belief is that this large increase, significantly greater than that of a half-space with properties identical to the modelled upper layer, is as a result of the sensitivity of the compliance function. The compliance function within the 0.01 – 0.03Hz band ‘looks’ much deeper than and senses very little of the upper layer, so in order for the upper hydrate layer impact on the compliance be seen, the change in its hydrate content has to be large.

Figure 4.24 shows how a percentage fractional change in the shear modulus of a specific model affects the percentage fractional change in its normalised compliance. Figure 4.24A shows the results of a set of 9 half-space models. The initial parameters of each modelled half-space reflect initial hydrate concentrations which vary from 0% to 80%, values that determine the initial shear modulus. For each of the 9 half-spaces, the initial hydrate concentration is increased to a maximum of 250%. The relationship between the fractional change in shear modulus and fractional change in the normalised compliance is non-linear. For changes of less than 100% in the shear modulus, the set of curves lay atop each other, indicating that the percentage change in the compliance is independent of the initial hydrate concentrations. For percentage changes in the shear modulus greater than 100%, there is a slight but visible separation of the curves.

A similar relationship results (Figure 2.24B) when the initial hydrate concentrations of the topmost 100m layer of the ‘best fit model’ (page 137) are varied. Here, the initial shear moduli (hydrate concentrations) of the layer were also set between 0% and 80% and each were in turn, increased to 250%. This result confirms that the fractional change in the transfer function is independent of the initial shear modulus. Additionally, for smaller changes in the shear modulus, the 9 curves are coincident. The difference between this model and the half-space is the magnitude of the responding change in the normalised compliance. Thus, the change in the transfer function is independent of the initial shear modulus but dependent on the model. Hence, in the unlikely event that the hydrate occupies the whole of the visible half-space, a much smaller change in the transfer function than observed is required.
Figure 4.24: The relationship between the change in shear modulus and the change in absolute normalised compliance of (A) a half-space and (B) the topmost layer of the best fit model for various initial hydrate concentrations. Each curve represents a specific initial hydrate concentration. An increase in $\mu$ decreases the compliance.
4.3.2.3 The error in the transfer function

The error on transfer function can be determined by fitting the best straight line through the power spectra of pressure and velocity time-series plotted on the x-y plane (Willoughby 2003; Everett and Hyndman 1967). Specifically, the transfer function error is the error of this best fitting slope. The method for calculating this best straight line and the error on its fit, was formulated following the general solutions for the least squares estimate of the best straight line detailed in York (1966). While York (1966) solution is in terms of the line’s slope and y-intercept where the set of x and y values each have errors which vary by individual measurement, here the solution must have a y-intercept being equal to zero because of the relationship between pressure and velocity – if there is no pressure applied to the seafloor, there will be no response of the earth. Appendix B outlines the formulation for the best fitting straight line with y-intercept = 0 where errors are present in both data sets.

The calculation of the best fitting slope is by Equation 4.13:

\[
b = \frac{\sum_i Z_i Y_i \beta_i}{\sum_i Z_i X_i \beta_i}
\]  

(4.1)

\(b\) must be solved iteratively with a suitable initial guess being the result of a regression of \(y\) on \(x\). The weights \(\omega(X_i)\) and \(\omega(Y_i)\) at each point, necessary for \(Z_i\) and \(\beta_i\) (Appendix B), is a measure of the errors in each of the \(x\) and \(y\) values. \(\omega(X_i) = \frac{1}{\sigma^2(X_i)}\) and \(\omega(Y_i) = \frac{1}{\sigma^2(Y_i)}\) where \(\sigma(X_i)\) and \(\sigma(Y_i)\) are the errors in the \(i^{th}\) \(x\) and \(y\) coordinates. \(\sigma(X_i)\) and \(\sigma(Y_i)\) were chosen to be the standard deviation in the power spectra of the pressure and velocity time-series respectively at a specific frequency \(f_i\). Thus \(\sigma(X_i) = \sqrt{G_{PP}(f_i)/2}\) and \(\sigma(Y_i) = \sqrt{G_{VV}(f_i)/2}\) (Bailey and Harrison 1999).

Applying the above process, an estimate of the errors in the transfer function for 3rd October, 2010 was made. The full 24 hour time-series was sectioned into records of length 512 seconds and the power spectrum for each record for each channel was calculated. This resulted in 168 frequency domain records each with 256 frequencies for each channel and the error on the best fit was determined for each frequency between 0.01 Hz and 0.03Hz. This was done by plotting, on an x-y axis, the 168 points for a specific frequency, say 0.02Hz and...
fitting the best slope. This represents a 1 degree of freedom estimate. The magnitude of the error on slope is dependent on the size of the error (the standard deviation) on the amplitude spectra at each frequency. Figure 4.24 is an example of the best fitting slope with a standard deviation estimate with 6 degrees of freedom (averaging 6 records to calculate the power at each frequency) at 0.02Hz. The error for the transfer function at 0.02Hz, determined through bootstrapping the residuals, is 1.08% of the slope of best fit. Table 4.5 lists the results of the error on the slope (i.e. the error on the transfer function) at each frequency when the standard deviation is estimated with 6 degrees of freedom. The error bars that appear on the transfer function of 3rd October, 2010 is a reflection Table 4.5 values.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>0.010</th>
<th>0.012</th>
<th>0.014</th>
<th>0.016</th>
<th>0.018</th>
<th>0.020</th>
<th>0.021</th>
<th>0.023</th>
<th>0.025</th>
<th>0.027</th>
<th>0.029</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error (%)</td>
<td>1.77</td>
<td>1.16</td>
<td>1.55</td>
<td>1.10</td>
<td>1.09</td>
<td>1.08</td>
<td>1.18</td>
<td>1.34</td>
<td>2.41</td>
<td>3.38</td>
<td>4.13</td>
</tr>
</tbody>
</table>

Table 4.5: The percentage errors as a function of frequency in the transfer function of 3rd October, 2010 for a 1% error on the pressure and velocity data. The percentage error is determined by bootstrapping the residuals for a fit of the Fourier transformed records at 6 degrees of freedom. Error on slope is 1.08% of the transfer function.
Figure 4.25: The red circles are the 0.02Hz amplitude of pressure and velocity of 3\textsuperscript{rd} October, 2010, derived in frequency domain through averaging of the Fourier transformed records. The black dashed line is the best fitting straight line determined using a modified York (1966) method (see Appendix B for details).

4.4 Summary

A partnership with NEPTUNE Canada has allowed for the investigation of the evolution of the gas hydrate concentration at a region near Bullseye Vent. 222 days of simultaneously logged time-series of pressure and velocity data, measured by the Scripps Institution of Oceanography differential pressure gauge, and the G"uralp CMG-1T broadband seismometer, respectively, were obtained. These instruments are part of the NEPTUNE Canada’s seismic network and are deployed ~390m in the North Eastern direction of Bullseye Vent, at SITE 889. The data set spans the time period of 1\textsuperscript{st} October, 2010 to 16\textsuperscript{th} May, 2011. In Fourier domain, the power-spectra of and cross-spectra between the pressure and velocity records for each of the 203 days were found by separating each 24-hour velocity and pressure segment
into records of 512 seconds. The coherence function and the transfer function bounds and phase was computed by performing an ensemble average of power and cross spectrum of the 168 Fourier transformed records for each of the 203 days. For each day of the period, the coherence function highlighted two distinct bands where the transfer function is well bounded. The lower frequency band is 0.01Hz – 0.03Hz, where gravity waves can deform the seafloor at water depths of 1256 m. Within this frequency band, the well bounded transfer function shows a linear decrease of \((-4.49 \times 10^{-3} \pm 3.52 \times 10^{-3}\)% of the transfer function of Day 1 (3rd October, 2010); a trend that is statistically significant at a 5% significance level and has errors within 0.4 and 4.0%. This decrease per day of the trend represents a -2.88 % change in the transfer function over the 228 day period. The 2.88% decrease represents a decrease in the compliance, consistent with an increase in the hydrate concentration in the sediment under the assumption that the area of study is within the gas hydrate province. Using Lee and Waite (2008) formulation for modelling hydrate filled sediments, the relationship between the observed decrease in the transfer function, and the percentage increase in the gas hydrate concentration of the sediments, was determined by fitting an elastic model to the experimental transfer function of Day 1 of the data series and varying the elastic parameters of the four layer over half-space two uppermost gas hydrate layers. The result was that a change of 162.37% in the shear modulus of the topmost layer accounts for the -2.88% decrease in the transfer function and reflects a ~42% change in the hydrate concentration. A ~20% change in the second layer’s shear modulus results in a 9% change in the hydrate concentration that matches the observed decrease in the transfer function.

A scheme for determining the error on the transfer function was formulated through the modification of York (1966) method for calculating the best least squares fit to x and y data that both contain errors. The mathematical formulation is outlined in Appendix B while a working algorithm coded in MATLAB can be found in Appendix E. The transfer function of 3rd October, 2010 errors ranged from 1.08% to 4.13%. 
Chapter 5
Summary, Conclusions and Recommendations

5.1 Summary

The actualisation of NEPTUNE Canada seafloor observatory provided the infrastructure to facilitate the investigation of temporal variations in the gas hydrate system near Bullseye Vent located landward of the subduction zone at the Northern Cascadia margin. Various experiments have been conducted at and around Bullseye and some results suggest that this gas hydrate system is perhaps evolving over time. The aim of the seafloor compliance experiment was to investigate this possible evolution. The non-intrusive method used for the exploration of Bullseye, seafloor compliance, involves the determination of the transfer function between pressure caused by the low-frequency passive source, infra-gravity waves, which continuously causes the deflection of the ocean floor, and the response to this palpitation, the acceleration. This frequency dependent transfer function is sensitive to the stiffness of the sediments below the seafloor, where the long wavelengths ‘see’ the deeper sediments and the shorter wavelengths, the shallower sediment. In a gas hydrate system the stiffening of the sediment is controlled by the presence of gas hydrates, and the seafloor compliance method is more sensitive to the shear modulus than any other of the physical properties of the sediment.

The failure of the experiment resulted in the use velocity and pressure measurements from a broadband seismometer and a differential pressure gauge, respectively to investigate the change in shear modulus of a region ~ 390m in the north eastern direction of Bullseye. In frequency domain, the transfer function bounds and phase, and coherence were computed on a total of 222 days from 1st October, 2010 to 16th May, 2011. The results dictated that the investigation of the temporal variations on the gas hydrate region be performed on the data in the high coherence frequency band 0.01Hz to 0.03Hz. The average of the transfer function in this well-bounded frequency range was computed for each day and a least squares fit of the data showed a linear decrease of \((-4.49 \times 10^{-3} \pm 3.52 \times 10^{-3})\%\) of the transfer function of Day 1 (3rd October, 2010). This linear decrease is significant at a 5% significance level and
represents a -2.88% change in the transfer function of Bullseye over the 222 days of the record’s span. This observed decrease is equivalent to an increase in the stiffness of the sediment and was related to a change in hydrate concentration. The frequency band in which this variation in the stiffness was observed corresponds to peak sensitivity of the compliance function at depths greater than 225mbsf. However, the sediments between 0mbsf and 225mbsf are also sensed. The largest variations in this stiffness occurred in the higher frequencies of the range, which correspond to the shallower sediments. A 4-layer over half-space model produced the best fit to the transfer function of Day 1 of the data set. Varying $\lambda$ and $\mu$ of this model’s uppermost layer by $\sim$45% and $\sim$163% resulted in a change in hydrate concentration of $\sim$42% to match the observed decrease in the transfer function. However, a $\sim$6% change in $\lambda$ and a $\sim$20% in $\mu$ of the second layer of the best fit model resulted instead in a 9% change in the hydrate concentration.

A 1D solution of the compliance using the static equations was outlined and an algorithm described. The matrix method algorithm is very efficient and is able to produce the solution to a half-space exactly. It also performs outstandingly at the end frequencies producing the exact value of the compliance for the deepest sediment (at the low frequency end) and the topmost layer (at the high frequency end) for frequencies between 5.4mHz and 124.9mHz. This coded algorithm was employed in this thesis.

Lastly, the thesis outlined a method for determining the errors bounds on the transfer function which accounts for the fact that seafloor compliance measurement has errors in the $x$ (pressure) and $y$ (velocity) values. The least squares approach, modified from York 1966, simultaneously minimizes the residuals in both coordinates.

5.2 Conclusions

The result of the thesis supports the previous evidence that suggests temporal changes in a gas hydrate deposit – a valuable first result. However, the statistically valid trend, which indicate a 2.88% increase in stiffness, was obtained for an area within the cold vent region where gas hydrates has only been inferred and no in situ measurement have been made.
Interpreting this increase in shear modulus within the sediments requires additional geophysical parameters to constrain a starting sedimentary model from which a best fit model to the transfer function of Day 1 may be obtained. From this best fit model the change in hydrate concentration can be determined. An attempt to interpret this increase was made in this thesis and resulted in a 4 layer over half space model of best fit. However, the constraints of this model were from parameters measured and/or derived from IODP U1328 drill hole data from the sediments at the Bullseye Vent, which is ~390m away from the site of investigation. A better model is only achievable through the use of other site specific parameters.

In this study, the geophysical setting severely limits seafloor compliance exploration of the uppermost region of the sediment, which is within the gas hydrate stability zone. However, the method remains a powerful tool that can elucidate the nature of the sediment within the depth range sensed. Combined with the new algorithm, which increases the accuracy at which the compliance of a layered structure can be determined at low frequencies, the seafloor compliance method can bound the hydrate volume and give a good estimate of how this bounded volume changes. However, with an uncalibrated differential pressure gauge, no conclusions can be drawn about the size (or volume) of the change in gas hydrate in the region studied.

### 5.3 Recommendations

The way this PhD project unfolded and the conclusions drawn allow me to make recommendations for future research, some of which would augment that presented. The following are recommendations based on the results of the thesis as well as the problems encountered over the 4 year period:

1) Using the data set above:
   - Calibration of the differential pressure gauge through comparison with the bottom pressure recorder.
• Using the calibrated pressure and velocity data, an estimate of the change in the volume of hydrate over the period can be done.

• A correlation between other events, for example, earthquakes, venting processes and temperature changes at the particular site and the observed changes in stiffness (gas hydrate content).

• Robust processing of the data in a re-evaluation of the trend and thus a robust estimate of the change in stiffness over the period. Robust processing would include estimation of the trend at individual frequencies and systematic approach determining and removal of outliers.

• Investigation of the scatter around the observed trend seen in Figure 4.18

2) Additional suggestions:

• An investigation into whether there is a pattern to the temporal variations.

• Correlations between the compliance seen at Site 889 and other sites on the network, once the DPGs are replaced.

• A study of the propagation of the infra-gravity wave and its effect on the coherence between the pressure applied by gravity waves and the response to the force applied.

• An investigation of the factors affecting coherence of the seafloor compliance e.g. sea-states. This can then clarify whether the variations, seen in shear modulus in the upper frequency limit of a compliance analysis, can be attributed to shear modulus changes.

• Comparing the changes in shear properties at various points within and outside the central Bullseye region through a network of compliance experiments.

• Other geophysical measurements to: (i) constrain the compliance input parameters to improve a model estimate, (ii) provide comparative data for estimated model, and an improved interpretation of the compliance results.
References


Bailey R (2006) PHY2603H Inverse Theory Course Notes


Everett JE, Hyndman RD (1967) Geomagnetic variations and electrical conductivity structure in south-western Australia. Phys Earth Planet Inter 1:24-34


Lee MW (2007) Velocities and attenuations of gas hydrate-bearing sediments 5264


NEPTUNE Canada (Unknown) About NEPTUNE Canada. In: NEPTUNE Canada. 


Owens D (2009b) Now there are three. In: Micro-g LaCoste. 

Owens D (2009a) Two Toronto Touchdowns. In: 


Seismological Society of America 100:1770-1778

Max MD (ed) Natural Gas Hydrate in Oceanic and Permafrost Environments. Springer -
Verlag

compliance imaging of marine gas hydrate deposits and cold vent structures. Journal of
Geophysical Research-Solid Earth 113:B07107

Willoughby EC, Edwards RN (2000) Shear velocities in Cascadia from seafloor compliance

deposits using seafloor compliance methods. Geophysical Journal International 131:751-
766

compliance method: experimental methods and results., |c[2003]

Wood AB (1941) A textbook of sound: being an account of the physics of vibrations with
special reference to recent theoretical and technical developments. Macmillan|c[1941],
New York|bMacmillan|c[1941]

of methane hydrates in marine sediments owing to phase-boundary roughness. Nature
420:656-660

Wyllie MRJ, Gregory AR, Gardner LW (1956) Elastic wave velocities in heterogeneous and
porous media. Geophysics 21:41-70

Xu WY, Ruppel C (1999) Predicting the occurrence, distribution, and evolution of methane
gas hydrate in porous marine sediments. Journal of Geophysical Research-Solid Earth
104:5081-5095


Appendices

Appendix A: Derivation of equations 2.32, 2.33 and 2.34

*Obtaining the expression for* $P(0)$ (*Equation 2.32 in Chapter 2)*

The normal stress in the $z$-plane comes directly from Equation 2.15 where $r=s=z$. Hence,

\[
\begin{align*}
\tau_{rs} &= \lambda \delta_{rs} + 2\mu \varepsilon_{rs} \\
\tau_{zz} &= \lambda \delta_{zz} + 2\mu \varepsilon_{zz} \\
\tau_{zz} &= \lambda \left( \frac{\partial u}{\partial x} \right) + 2\mu \left[ \frac{1}{2} \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right) \right] \\
\tau_{33} &= \lambda \left( \frac{\partial u}{\partial x} \right) + (\lambda + 2\mu) \frac{\partial w}{\partial z}
\end{align*}
\]

(A.1)

Using the general solutions for the displacements $u$ (Equation 2.20) and $w$ (Equation 2.21) as well as the amplitude $U(z)$ (Equation 2.26) and $\frac{dw}{dz}$ (Equation 2.28), gives

\[
\begin{align*}
\tau_{zz} &= \lambda \left( k \cos(kx) \right) U(z) + (\lambda + 2\mu) \left( \cos(kx) \frac{dW}{dz} \right) \\
\tau_{zz} &= \left[ \lambda k U(z) + (\lambda + 2\mu) \frac{dW}{dz} \right] \cos(kx) \\
\tau_{zz} &= \left[ -2\mu k A e^{-kz} - \frac{2\mu(\lambda + 2\mu) + 2k\mu(\lambda + \mu)z}{\lambda + \mu} B e^{-kz} \right. \\
&\quad \left. - 2\mu k C e^{+kz} + \frac{2\mu(\lambda + 2\mu) - 2k\mu(\lambda + \mu)z}{\lambda + \mu} D e^{+kz} \right] \cos(kx)
\end{align*}
\]

(A.2)

This normal stress, $\tau_{zz}$, is the vertical pressure applied $p(x,z)$. Let the pressure at the seafloor ($z=0$) be $p(x,0) = P(0) \cos(kx)$. For the half-space, as $z$ tends to infinity, the positive exponentials, $e^{+kz}$ terms must be zero since there are no sources at the bottom of the half-space, so $C$ and $D$ are in-turn zero. Thus,

\[
P(0) = -2\mu k A - \frac{2\mu(\lambda + 2\mu)}{\lambda + \mu} B
\]

(A.3)
**Obtaining an expression for B/A (Equation 2.33) using the x-z horizontal stress, \( \tau_{xz} \)**

The normal stress in the x-z plane comes directly from Equation 2.15 where \( r \neq s \). Hence,

\[
\tau_{xz} = \lambda \delta_{xz} + 2\mu e_{xz}
\]

\[
\tau_{xz} = \lambda \varepsilon(0) + 2\mu e_{xz}
\]

(A.4)

Using Equations 2.20, 2.21, 2.26 and 2.29 to substitute for the terms in Equation A.4 and the no shear/tangential stress boundary condition at \( z=0 \) means \( \tau_{rs} = 0 \) lead to

\[
\frac{dU}{dz} - kW = 0
\]

\[
\begin{align*}
-2\mu k A e^{-kz} &- \frac{2\mu k(\lambda + \mu)z + 2\mu^2}{\lambda + \mu} Be^{-kz} + 2\mu k C e^{-kz} \\
+ \frac{2\mu k(\lambda + \mu)z - 2\mu^2}{\lambda + \mu} De^{-kz} & = 0
\end{align*}
\]

(A.5)

Again, for the half-space, as \( z \to \infty \), \( e^{+kz} \) terms are set to zero. So at \( z=0 \)

\[
-2\mu k A = \frac{2\mu^2}{\lambda + \mu} B
\]

\[
\frac{B}{A} = -\frac{k(\lambda + \mu)}{\mu}
\]

(A.6)
Deriving the seafloor compliance of a half-space (Equation 2.34)

The compliance is of the half-space is defined as $\xi = \frac{W(0)}{\tau_{zz}(0)}$. Using the results of Equations A.2 ($P(0)$) and A.6 (B/A), and Equation 2.37 ($W(z)$ at $z = 0$) under the condition that $C$ and $D$ are zero (see above), leads to

$$\xi = \frac{W(0)}{P(0)}$$

$$\xi = \frac{A + \frac{(\lambda + 3\mu)}{k(\lambda + \mu)}B}{-2\mu kA - \frac{2\mu(\lambda + 2\mu)}{\lambda + \mu}B}$$

$$\xi = \frac{k(\lambda + \mu) + (\lambda + 3\mu)}{k(\lambda + \mu)} \left[ -\frac{k(\lambda + \mu)}{\mu} \right]$$

$$\xi = \frac{-\frac{(\lambda + 2\mu)}{\mu}}{[2k(\lambda + \mu)]}$$

$$\xi = -\frac{\lambda + 2\mu}{2\mu k(\lambda + \mu)}$$
Appendix B: Derivation of the least squares estimate of a straight line with errors in a pair of observables.

One method for estimating the errors on the compliance function is by using the error obtained in fitting a straight-line to a velocity against pressure plot as the estimate. The velocity and pressure values used in the plot are determined from their amplitude spectra. The number of points used to determine the best fitting straight-line (in the least squares sense) is based on the number of data records used in the Fourier analysis. In this thesis, every daily 24-hour time-series of pressure and velocity is treated as a data record. Each record is then broken into 512 seconds segments\(^{32}\) which results in 168 segments on which the Fourier analysis can be performed. The 168 segments can then be Fourier transformed and amplitude spectra for each computed. The amplitude, at a single frequency for the velocity and pressure, is plotted and the best straight-line determined. The error in the best fit estimate becomes the error in the compliance function at that specific frequency. The size of the compliance error is dependent on the number of segments used in determining the amplitude spectrum. It is well known the method of averaging in computing the amplitude spectrum reduces the errors in the estimate. Therefore, a balance must be struck between the number of data points used in the fit of the best straight-line and the number of segments used in the estimate of these data points. For e.g., if the amplitude spectra is computed by averaging 6 Fourier transformed segments, then 28 points can be used in the velocity-pressure plot. The best fit straight-line the best fit line must be of the form \( y = bx \). This is because for SFC, the response of the Earth is dependent on the existence and magnitude of the pressure applied, thus, when the pressure is zero, so is the response. Hence the \( y \)-intercept must be zero.

Traditionally, in fitting the best straight line, the regression of \( y \) on \( x \) is widely used where all the errors are attributed to the \( y \)-coordinate. In the SFC case, both the \( x \) (pressure) and \( y \) (velocity) values have errors, thus a method of best fit simultaneously minimizes the

\(^{32}\) 512 seconds length records are choosen because it provides the type of resolution needed for the compliance analysis in this thesis. That is, the trend analysis is performed on the average of the transfer function between 0.01 Hz and 0.03 Hz.
residuals in both coordinates. York (1966) outlines the general solution for the Least Squares Estimator (LSE) of the best straight line of the form \( y = bx + a \) where both \( x \) and \( y \) observables are subject to errors, which vary between points. Here, York (1966) formulation is modified to one that describes the LSE fit of the best straight line of the form \( y = bx \) where, again, both \( x \) and \( y \) observables are subject to errors.

The best fit line can be obtained by minimizing the sum of

\[
S = \sum_i \left[ \omega(X_i)(x_i - X_i)^2 + \omega(Y_i)(y_i - Y_i)^2 \right] \quad \text{(B.1)}
\]

where \( X_i \) and \( Y_i \) are the observations and \( \omega(X_i) \) and \( \omega(Y_i) \) are the weights of the observations. \( x_i \) and \( y_i \) are the adjusted values of the observations which make the sum, \( S \), a minimum and must lie on the best straight line. For \( x_i \) and \( y_i \) to lie on the best straight line

\[
y_i = bx_i \quad \text{where } i = 1, 2, \ldots, n \quad \text{(B.2)}
\]

If \( b, x_i \) and \( y_i \) minimize, then

\[
\partial S = \sum_i \left[ \omega(X_i)(x_i - X_i)\partial x_i + \omega(Y_i)(y_i - Y_i)\partial y_i \right] = 0 \quad \text{(B.3)}
\]

From Equation B.2 we can write

\[
\partial y_i = b\partial x_i + x_i\partial b = 0 \quad \text{(B.4)}
\]

Multiplying Equation (B.4) by its own undetermined multiplier, \( \lambda_i \), and equated it to Equation B.3, it follows that

\[
\sum_i \left[ (\omega(X_i)(x_i - X_i) + b\lambda_i)\partial x_i + (\omega(Y_i)(y_i - Y_i) - \lambda_i)\partial y_i \right. \\
\left. + \lambda_i x_i\partial b \right] = 0 \quad \text{(B.5)}
\]

Equation B.5 leads to
\[ \sum_i (\omega(X_i)(x_i - X_i) + b\lambda_i) = 0 \]  
\[ \Rightarrow x_i - X_i = -\frac{b\lambda_i}{\omega(X_i)} \]  
\[ \sum_i (\omega(Y_i)(y_i - Y_i) - \lambda_i) = 0 \]  
\[ \Rightarrow y_i - Y_i = \frac{\lambda_i}{\omega(Y_i)} \]

\[ \sum_i \lambda_i x_i \partial b = 0 \]  
\[ \Rightarrow \sum_i \lambda_i x_i = 0 \]

Substituting Equations B.6 and B.7 into Equation B.2 results in

\[ \lambda_i = (bX_i - Y_i)W_i \]  
\[ \text{where} \]

\[ Z_i = \frac{\omega(X_i)\omega(Y_i)}{\omega(X_i) + b^2\omega(Y_i)} \]  

From Equation B.8, we may write

\[ \sum_i \lambda_i x_i = \sum_i (b\lambda_i - Y_i)Z_i \left[ X_i - \frac{b\lambda_i}{\omega(X_i)} \right] = 0 \]  

which expands to
Equation B.12 is referred to as the ‘Least Squares Cubic’ equation and can be solved iteratively for the best fit, \( b \) by substituting an initial guess of \( b \) into the \( Z_i \) equation. The sought value of \( b \) is one of the three attained roots. However, the ‘cubic’ equation can be algebraically reduced to the ‘Least Squares Linear’ equation:

\[
\sum_{i} Z_i^2 X_i^2 = 2b^2 \sum_{i} \frac{Z_i^2 X_i Y_i}{\omega(X_i)} - b \sum_{i} Z_i X_i^2 + b \sum_{i} \frac{Z_i^2 Y_i^2}{\omega(X_i)} + \sum_{i} Z_i X_i Y_i = 0
\]  
(B.12)

\[
b = \frac{\sum_{i} Z_i Y_i \beta_i}{\sum_{i} Z_i X_i \beta_i}
\]  
(B.13)

where

\[
\beta_i = Z_i \left( \frac{X_i}{\omega(Y_i)} + \frac{b Y_i}{\omega(X_i)} \right)
\]  
(B.14)

Again, a value for \( b \) must be attained iteratively from a suitable first guess.

The \( x \)-residuals and \( y \)-residuals are calculated through Equations B.6 and B.7 and are

\[
x_i - X_i = -\frac{b}{\omega(X_i)} (b X_i - Y_i) Z_i
\]  
(B.15)

\[
y_i - Y_i = \frac{(b X_i - Y_i) Z_i}{\omega(Y_i)}
\]  
(B.16)

The weights, \( \omega(X_i) = \frac{1}{\sigma^2(X_i)} \) and \( \omega(Y_i) = \frac{1}{\sigma^2(Y_i)} \) where \( \sigma(X_i) \) and \( \sigma(Y_i) \) are errors in the \( x \)-coordinate and \( y \)-coordinate of the \( i^{th} \) point. An estimate of the errors on the slope is determined by bootstrapping the residuals.

This mathematical recipe was coded in MATLAB (function \texttt{ErrorNC.m} - see Appendix E), validated and used in the estimate of the errors in the compliance data.
Appendix C: Details on the data used in the estimate of temporal variations at Bullseye

The data used in Chapter 4 to determine the long-term behaviour of gas hydrate deposit off shore Cascadia margin were obtained directly from NEPTUNE Canada. Martin Heesemann, the research theme investigator at NEPTUNE Canada, created a ftp folder (ftp://ftp.neptunecanada.ca/pub/SeismometerData/) and loaded all the archived data from the seismometer network installed at NEPTUNE Canada. This route was taken since that the time of this analysis (June, 2011) all data were not yet archived at IRIS. The data set included DPG pressure data (LDD) and vertical velocity (LHZ) data from all three sites: Barkley Canyon, ODP 1027 and ODP 889. The data format is miniSEED. Pressure and velocity data were downloaded for all three sites but only SITE ODP 889 DPG data appeared reasonable.

Instrument details about the DPG can be found by navigating the NEPTUNE Canada’s site (http://www.neptunecanada.ca) while the meta data for the seismometers is housed at IRIS DMC MetaData Aggregator (http://www.iris.edu/mda/NV). To access the LHZ and LDD data directly from IRIS one can go to the IRIS’ time-series builder at http://www.iris.edu/ws/timeseries/builder to download data files from any channel in a range of file formats. The initial information needed to access the NC seismometer network data is as follows: Network: NV; Station: NC89 or NCBC or NC27; Location: -- ; Channel: LHZ or LDD. The long-term analysis of a hydrate deposit was performed with the data from instruments located at ODP 889. The data set used spans 1st October, 2010 to 16th May, 2011 for a total of 228 days. Of the possible 228 days, there were 6 days where no DPG data was available (Table C.1 B) and there were 25 days that had anomalous spikes and/or contained large seismic variations due to earthquakes. The anomalous sections were removed from each of the 25 days (Table C.1 A) and the remaining sections were used in the initial frequency domain analysis. A test for normality was performed on the 222 days available.

33 For further information, one should navigate IRIS’s website.

34 A test for normality requires a probability plot. ‘A probability plot is a plot of the ranked actual residuals of the data against the expected residuals for the rank of a normal distribution with a standard deviation equal to that of the data’ (Bailey 2006, Chapter 3)
The normality test reduced the useable days to 203. The 25 days which were excluded from the analysis are listed in Table C.1 B.

<table>
<thead>
<tr>
<th>A</th>
<th>Days with Spikes</th>
<th>B</th>
<th>Omitted Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>useable records (ur)</td>
<td># of ur</td>
<td>Day</td>
</tr>
<tr>
<td>01-Oct-10</td>
<td>[134:168]</td>
<td>35</td>
<td>17-Oct-10</td>
</tr>
<tr>
<td>04-Oct-10</td>
<td>[1:167]</td>
<td>167</td>
<td>20-Nov-10</td>
</tr>
<tr>
<td>05-Oct-10</td>
<td>[6:47 91:168]</td>
<td>120</td>
<td>25-Nov-10</td>
</tr>
<tr>
<td>28-Oct-10</td>
<td>[1:142]</td>
<td>142</td>
<td>02-Dec-10</td>
</tr>
<tr>
<td>24-Nov-10</td>
<td>[1:133 161:168]</td>
<td>141</td>
<td>10-Dec-10</td>
</tr>
<tr>
<td>09-Dec-10</td>
<td>[1:151 ]</td>
<td>151</td>
<td>11-Dec-10</td>
</tr>
<tr>
<td>12-Dec-10</td>
<td>[138:168]</td>
<td>31</td>
<td>25-Dec-10</td>
</tr>
<tr>
<td>15-Dec-10</td>
<td>[1:128]</td>
<td>128</td>
<td>29-Dec-10</td>
</tr>
<tr>
<td>16-Dec-10</td>
<td>[139:168]</td>
<td>30</td>
<td>31-Dec-10</td>
</tr>
<tr>
<td>17-Dec-10</td>
<td>[1:69 88:168]</td>
<td>150</td>
<td>13-Jan-11</td>
</tr>
<tr>
<td>21-Dec-10</td>
<td>[1:120 146:168]</td>
<td>143</td>
<td>18-Jan-11</td>
</tr>
<tr>
<td>01-Feb-11</td>
<td>[1:113 123:168]</td>
<td>159</td>
<td>11-Feb-11</td>
</tr>
<tr>
<td>02-Feb-11</td>
<td>[1:111]</td>
<td>111</td>
<td>12-Feb-11</td>
</tr>
<tr>
<td>03-Feb-11</td>
<td>[8:168]</td>
<td>161</td>
<td>13-Feb-11</td>
</tr>
<tr>
<td>14-Feb-11</td>
<td>[80:168]</td>
<td>89</td>
<td>09-Mar-11</td>
</tr>
<tr>
<td>01-Mar-11</td>
<td>[1:29 142:168]</td>
<td>56</td>
<td>07-Apr-11</td>
</tr>
<tr>
<td>03-Mar-11</td>
<td>[1:137 140:168]</td>
<td>166</td>
<td>19-Apr-11</td>
</tr>
<tr>
<td>05-Apr-11</td>
<td>[1:63 107:168]</td>
<td>125</td>
<td>21-Apr-11</td>
</tr>
<tr>
<td>16-May-11</td>
<td>[1:147]</td>
<td>147</td>
<td>11-May-11</td>
</tr>
</tbody>
</table>

Table C.1: (A) Lists the days with spikes/visible earth tremors and details which sections of the 24-hour record was retained for use in the trend analysis. (B) Lists the days that were deemed unusable through the normality test.
Appendix D: Further explanations about the method used in interpreting the observed trend

Having observed a decrease in the transfer function (compliance) over the period, the answer to the question: what change in hydrate content will cause the observed change in the transfer function? was sought. To do this, the steps were followed:

**Step 1: Match the transfer function of Day 1**

The relevant parameters, $\lambda$ and $\mu$, required in the determination of the compliance were calculated from the TPBE by Lee and Waite 2008. Then, by methodically varying $\lambda$ and $\mu$, a sedimentary model whose compliance response best fits the transfer function of the first day of the trend was determined.

**Step 2: Determine the relationship between relevant elastic parameters**

In order to match a change in the transfer function, the elastic parameters must vary. As indicated in Chapter 1, the presence of gas hydrates increases the shear strength of the sediments. An increase in shear strength means an increase in stiffness of the sediments or a decrease in its compliance. Therefore $\mu$ must be increased in the model in an effort to match the decrease in the observed transfer function. Given that compliance is dependent on both $\lambda$ and $\mu$ the relationship between them was investigated. The investigation involved using the TPBE (which calculates the values of $\lambda$ and $\mu$ separately) to determine the ratio of the percent change in $\lambda$ and the percent change in $\mu$ (the $\Delta\lambda\Delta\mu$ ratio) given a specific percent change in hydrate concentration. To do this, a layer (or half-space) is described by a set of elastic parameters (hydrate concentration, $\lambda$ and $\mu$). Then, the initial hydrate concentration of this layer (or half-space) is increased to for e.g. 100% and the resulting percent changes in the values of $\lambda$ and $\mu$ (Chapter 2) recorded. The $\Delta\lambda\Delta\mu$ ratio is noted for each increment of hydrate percent change. Figure D.1 shows the results of such an exercise. There are two sets of nine curves; one set represents a layer with porosity of 60% (markers with lines-shown on legend) while the other set represents a layer with a 40% porosity. Each curve in the set

---

35 The change in compliance is defined as the change over a range of frequencies (in this thesis it is 0.01 to 0.03Hz). The change, as a percent, is determined by computing the average of the transfer function within the specified frequency band for Day 1 and for the last day of the data set, and calculating the percentage difference between them with respect to Day 1.
denotes the initial hydrate concentration of the layer. For example, for an initial hydrate concentration of 20% (black line with circle) the $\Delta \lambda / \Delta \mu$ ratio decreases from 0.3431 to 0.1424 as the hydrate concentration increases from 20% to 80% (i.e. a change in the hydrate concentration of the layer of 80%). The family of curves for specific set of initial model parameter relationship between $\lambda$ and $\mu$ (the $\Delta \lambda / \Delta \mu$ ratio) is dependent on the initial hydrate concentration of the layer and the percentage increase in this concentration. Figure D.1 shows that: (i) the initial hydrate concentration determines the spread/range of the $\Delta \lambda / \Delta \mu$ ratio; and (ii) the initial parameters also influence the spread of the $\Delta \lambda / \Delta \mu$ ratio. Thus the $\lambda$-$\mu$ relationship should be considered in the process of computing the change in $\lambda$ and $\mu$ when determining the change in the transfer function. For a layer, having an initial concentration of 20%, one can start with a $\Delta \lambda / \Delta \mu = 0.32^36$ for some initial guess of an increase in hydrate concentration of 15%. This means that if $\mu$ is increased by 10% then $\lambda$ must be increased by 3.2%. The best value of $\Delta \lambda / \Delta \mu$ is determined iteratively. Using the first guess in fitting the model gives an approximate value of the change in hydrate concentration. From there, the parameter model can be run again, with the new hydrate concentration, to determine the new $\Delta \lambda / \Delta \mu$ ratio. And so forth. Table D.1 shows an example of the relationship between the change in hydrate concentration and the $\Delta \lambda / \Delta \mu$ ratio.

<table>
<thead>
<tr>
<th>$\Delta h_c$ (%)</th>
<th>$\Delta \lambda$ (%)</th>
<th>$\Delta \mu$ (%)</th>
<th>$\Delta \lambda / \Delta \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-21</td>
<td>0.76</td>
<td>2.21</td>
<td>0.34</td>
</tr>
<tr>
<td>20-22</td>
<td>1.53</td>
<td>4.47</td>
<td>0.34</td>
</tr>
<tr>
<td>20-23</td>
<td>2.31</td>
<td>6.78</td>
<td>0.34</td>
</tr>
<tr>
<td>20-23</td>
<td>3.10</td>
<td>9.14</td>
<td>0.34</td>
</tr>
<tr>
<td>20-24</td>
<td>3.90</td>
<td>11.54</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Average $\Delta \lambda / \Delta \mu$ : 0.34

<table>
<thead>
<tr>
<th>$\Delta h_c$ (%)</th>
<th>$\Delta \lambda$ (%)</th>
<th>$\Delta \mu$ (%)</th>
<th>$\Delta \lambda / \Delta \mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-25</td>
<td>3.90</td>
<td>11.54</td>
<td>0.34</td>
</tr>
<tr>
<td>20-30</td>
<td>8.09</td>
<td>24.39</td>
<td>0.33</td>
</tr>
<tr>
<td>20-35</td>
<td>12.59</td>
<td>38.77</td>
<td>0.32</td>
</tr>
<tr>
<td>20-40</td>
<td>17.44</td>
<td>55.00</td>
<td>0.32</td>
</tr>
<tr>
<td>20-45</td>
<td>22.69</td>
<td>73.44</td>
<td>0.31</td>
</tr>
<tr>
<td>20-50</td>
<td>28.39</td>
<td>94.60</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Average $\Delta \lambda / \Delta \mu$ : 0.32

Table D.1: The relationship between change in hydrate concentration and the $\Delta \lambda / \Delta \mu$ ratio.

---

36 The $\Delta \lambda / \Delta \mu$ ratio value represents the average of all $\Delta \lambda / \Delta \mu$ ratios for a range of percentage change in hydrate concentrations from 0% to 50%.
Figure D.1: The relationship between the percent change in the hydrate concentration and the ratio between the percentage change in $\lambda$ and the percentage change in $\mu$ (the $\Delta\lambda/\Delta\mu$ ratio) for various initial hydrate concentrations. The set of curves with the marked with the line-shape combination has a porosity of 0.60 whereas the curves marked with only shapes have a porosity of 0.40 (the remaining model parameters are identical).

For a small percent change in the transfer function, changing only the shear modulus produces a percent change in the hydrate concentration that differs, by less than 0.4%, from the concentration produced when both Lamé parameters are changed (Figure D.2). If however, the change in transfer function is large, changing both parameters using the above method allows for greater accuracy in estimating the hydrate concentration.
Figure D.2: Effect of changing only $\mu$ and both $\lambda$ and $\mu$ to match the observed change in transfer function. Also demonstrates the relationship between the percentage change in the transfer function and the percent change the shear modulus. Given that the $\Delta \lambda / \Delta \mu = 0.30$, then the percentage change in the Lamé parameter can be determined.

**Step 3: Determine the percentage change in $\lambda$ and $\mu$ to match the observed change in transfer function**

The values of $\lambda$ and $\mu$ of the layer are increased according to the established $\lambda - \mu$ relationship described above; from 0% until the observed change in the transfer function is attained. The best fit model’s two upper most layers are 100m and 520m thick, respectively. With the BSR estimated at ~220m (IODP 311 report) these are the two layers whose change in stiffness will be as a result of a change in hydrate concentration. As such, the values of $\lambda$ and $\mu$ were increased in each of these layers until the observed decrease in the transfer function was obtained. Figure D.2 shows how a percentage increase in both parameters (red dashed line) affects the transfer function response for the upper two layers of best fit model (Section 4.4.2.2). For example, in layer 1, for a percentage change in the compliance of -1.66% then a 90% change in $\mu$ (a 27% change in $\lambda$ if $\Delta \lambda / \Delta \mu = 0.30$) is required (Table D.2).
### Table D.2: The relationship between the percentage change in the shear modulus and the respective change in the transfer function (compliance).

| $\Delta \mu$ (%) | $|\Delta \tilde{\varepsilon}|$ (%) | $\Delta \mu$ (%) | $|\Delta \tilde{\varepsilon}|$ (%) |
|------------------|---------------------------------|------------------|---------------------------------|
| 0                | 0.00                            | 90.0             | -1.660                          |
| 5                | -0.12                           | 90.5             | -1.667                          |
| 10               | -0.24                           | 91.0             | -1.674                          |
| 15               | -0.35                           | 91.5             | -1.681                          |
| 20               | -0.46                           | 92.0             | -1.688                          |
| 25               | -0.57                           | 92.5             | -1.695                          |
| 30               | -0.67                           | 93.0             | -1.702                          |
| 35               | -0.76                           | 93.5             | -1.708                          |
| 40               | -0.86                           | 94.0             | -1.715                          |
| 45               | -0.95                           | 94.5             | -1.722                          |
| 50               | -1.04                           | 95.0             | -1.729                          |
| 55               | -1.12                           | 95.4             | -1.728                          |
| 60               | -1.21                           | 95.5             | -1.729                          |
| 65               | -1.29                           | 95.6             | -1.730                          |
| 70               | -1.37                           | 95.7             | -1.731                          |
| 75               | -1.44                           | 95.8             | -1.722                          |
| 80               | -1.52                           | 95.9             | -1.723                          |
| 85               | -1.59                           | 96.0             | -1.725                          |
| **90**           | **-1.66**                       | 96.3             | **-1.726**                      |
| 95               | -1.73                           | 96.4             | -1.727                          |
| 100              | -1.80                           | 96.5             | -1.729                          |

**Step 4: Matching resulting change in $\lambda$ and $\mu$ to the change in hydrate concentration**

The final stage is to determine the size percentage change in hydrate concentration that will result from the calculated change in the elastic parameters. Again, TPBE is used for the matching. Figure D.3 shows the way the hydrate concentration varies as the shear modulus increases and the corresponding Lamé variation for the three upmost layers in the best fitting model from Section 4.4.2.2. Here, for a 90% change in $\mu$ (a 27% change in $\lambda$) of layer 1 (initial hydrate concentration=22%, porosity = 60%), the change in hydrate concentration is
29% i.e. the new hydrate concentration is 51% (Table D.2). This means that the hydrate concentration of the first layer of the best fitting model increased by 29% when the observed decrease in the transfer function is 1.66%.

Figure D.3: Relationship between the percent change in hydrate concentration and percent change in the shear modulus (Top) and the Lamé parameter (Bottom). The curves represent the relationship when the parameters of the topmost layer (dashed blue line) are changed, the second layer (dashed red line) is changed, and both layers are changed.
<table>
<thead>
<tr>
<th>hc</th>
<th>Δhc (%)</th>
<th>Δλ (%)</th>
<th>Δμ (%)</th>
<th>Δλ/Δµ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.22</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>---</td>
</tr>
<tr>
<td>0.27</td>
<td>5</td>
<td>4.0</td>
<td>11.5</td>
<td>0.34</td>
</tr>
<tr>
<td>0.32</td>
<td>10</td>
<td>8.2</td>
<td>24.4</td>
<td>0.34</td>
</tr>
<tr>
<td>0.37</td>
<td>15</td>
<td>12.8</td>
<td>38.8</td>
<td>0.33</td>
</tr>
<tr>
<td>0.42</td>
<td>20</td>
<td>17.7</td>
<td>55.1</td>
<td>0.32</td>
</tr>
<tr>
<td>0.47</td>
<td>25</td>
<td>23.0</td>
<td>73.8</td>
<td>0.31</td>
</tr>
<tr>
<td>0.52</td>
<td>30</td>
<td>28.8</td>
<td>95.2</td>
<td>0.30</td>
</tr>
<tr>
<td>0.57</td>
<td>35</td>
<td>35.2</td>
<td>120.2</td>
<td>0.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>hc</th>
<th>Δhc (%)</th>
<th>Δλ (%)</th>
<th>Δμ (%)</th>
<th>Δλ/Δµ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.47</td>
<td>25</td>
<td>23.04</td>
<td>73.77</td>
<td>0.31</td>
</tr>
<tr>
<td>0.48</td>
<td>26</td>
<td>24.16</td>
<td>77.82</td>
<td>0.31</td>
</tr>
<tr>
<td>0.49</td>
<td>27</td>
<td>25.30</td>
<td>81.91</td>
<td>0.31</td>
</tr>
<tr>
<td>0.50</td>
<td>28</td>
<td>26.46</td>
<td>86.27</td>
<td>0.31</td>
</tr>
<tr>
<td>0.51</td>
<td>29</td>
<td>27.64</td>
<td>90.68</td>
<td>0.30</td>
</tr>
<tr>
<td>0.52</td>
<td>30</td>
<td>28.84</td>
<td>95.229</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table D.3: The relationship between the initial hydrate concentration, the change in hydrate concentration, the change in Lamé parameters and the Δλ/Δµ ratio.
Appendix E: Software (MATLAB)

Several scripts and function were written in the MATLAB language to facilitate the processing and analysis of the data presented in this thesis. All of the codes written by me are included in this section. Most of them are heavily annotated, however, those that are not will probably be clear to MATLAB users. Scripts which were also employed but were not written by me are not included due to my lack of permission to distribute but their source have identified.

Description of a select list of codes used in this thesis

Functions:
- `calcNCompMM35.m` - calculates the compliance using the matrix propagation method.
- `eigANA.m` - calculates the eigenparameters for a layered earth model.
- `ErrorNC.m` - calculates the errors on the transfer function using a modified York regression method.
- `freANA1.m` - calculates the PSD, power spectra, coherence, transfer function (and compliance) bounds, transfer function phase for a given time-series. There are options for filtering using a Butterworth filter and for changing the resolution.
- `frechetKER2_1.m` - calculates the sensitivity kernels for a layered model. The sensitivity to the elastic parameters lambda and mu as well as density is determined.
- `probplot.m` - plots the cumulative histogram and a probability plot of the errors in the fit of the data; Test of Normality.
- `trendANA2.m` - performs statistical analysis of the trend. F-testing the straight line fit, bootstrapping the errors in the fit and determining the confidence levels.
- `velTPBEvp.m` - calculates the compressional modulus of the sediment (can also return the compressional velocity if desired) using the TPBE (triple phase boundary equation by Lee and Waite, 2008).
- `velPBEvs.m` - calculates the shear modulus of the sediment (can also return the shear velocity if desired)
Scripts:

- **createmodel.m** - determines the elastic parameters of the sediments using TPBE.
- **FDanalysis.m** - uses freANA1.m for the frequency domain analysis of the NC SITE 889 seismometer and DPG data. UNFILTERED.
- **FDanalysisfil.m** - uses freANA1.m for the frequency domain analysis of the NC SITE 889 seismometer and DPG data. FILTERED.
- **FitTrend.m** - used to plot fits of trend, determine the sensitivity kernels and the eigenparameter analysis for the best fitting model.
- **loadLHZDPG.m** - plots all the daily records and determined percentage of data available for each day.
- **shearmodCal.m** - determines the relationship between elastic parameters and hydrate content.

**Description of the list of function from other sources.**

- **dtanh.m** - stable inverse of the tanh function. Wayne Crawford. Available from: [http://www.ipgp.fr/~crawford/Homepage/Software.html](http://www.ipgp.fr/~crawford/Homepage/Software.html)
- **gravd.m** - determines the wavenumbers of gravity waves. Wayne Crawford.
- **lsq.m** - calculates the least squares fit to a straight line using the MLE. R.C. Bailey, 2008. Inverse Theory course PS2 solution. University of Toronto.

All the functions and scripts can be cut and pasted in to MATLAB and be ran.

All software are available but are omitted from paper copy due to size (75 pages).
Appendix F: Profiles of density, porosity and velocity for IODP 311 U1328

Figure F.2: Sonic waveform data, $P$-wave velocity ($V_p$), and $S$-wave velocity ($V_s$) obtained with the wireline Dipole Sonic Imager tool in Hole U1328C. Key: UD = upper dipole, LD = lower dipole. $V_p$ is monopole. Data available from: http://brg.ldeo.columbia.edu/logdb/.
# Appendix G: Scripps Institution of Oceanography

## Differential Pressure Gauge Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pass Band</strong></td>
<td>2 Hz to 500 seconds</td>
</tr>
<tr>
<td><strong>Nominal Gain</strong></td>
<td>1 mV/Pa at 1 Hz</td>
</tr>
<tr>
<td><strong>Noise</strong></td>
<td>$5 \times 10^{-3} \text{ Pa}^2 / \text{ Hz at 100 seconds}$</td>
</tr>
<tr>
<td><strong>Dimensions</strong></td>
<td>152.5 mm diameter x 200 mm long, not including connector.</td>
</tr>
<tr>
<td><strong>Internal Media</strong></td>
<td>dimethylpolysiloxane (silicone oil), UV sensitive</td>
</tr>
<tr>
<td><strong>Weight in air</strong></td>
<td>5 kg</td>
</tr>
<tr>
<td><strong>Weight in seawater</strong></td>
<td>1.5 kg at 15°C</td>
</tr>
<tr>
<td><strong>Operating Temperature</strong></td>
<td>-2 to +35°C</td>
</tr>
<tr>
<td><strong>Depth Rating</strong></td>
<td>6000 m</td>
</tr>
<tr>
<td><strong>Operation Orientation</strong></td>
<td>any</td>
</tr>
<tr>
<td><strong>Hardware Options</strong></td>
<td>Metric or US</td>
</tr>
<tr>
<td><strong>Connector</strong></td>
<td>4 pin Impulse or custom</td>
</tr>
</tbody>
</table>

---

37 Reprinted from the reprinted from Scripps Institution of Oceanography Marine EM Laboratory website
Figure G.1: The power spectral density (PSD), noise spectrum and magnitude-squared coherence for two DPG data sets collected in a temperature controlled vault at IGPP. 120 minutes of data were used (reprinted from Scripps Institution of Oceanography Marine EM Laboratory website)
Appendix H: Predictive sedimentary model (TPBE)

At low frequencies can be expressed as $V_p = \sqrt{\sum_{i,j} R_{ij} / \rho_b}$ and $V_s = \sqrt{\sum_{i,j} \mu_{ij} / \rho_b}$ where the bulk density, $\rho_b = (1 - \phi)\rho_s + (1 - C_h)\phi + C_h\phi \rho_h$ and $R_{ij}$ and $\mu_{ij}$ are:

$$R_{11} = [(1 - c_1)\phi_s]^2 K_{av} + K_{sm} + 4\mu_{11}/3$$
$$R_{12} = r_{21} = (1 - c_1)\phi_s\phi_w K_{av}$$
$$R_{13} = R_{31} = (1 - c_1)(1 - c_3)\phi_s\phi_h K_{av} + 2\mu_{13}/3$$
$$R_{22} = \phi_w^2 K_{av}$$
$$R_{23} = R_{32} = (1 - c_3)\phi_h\phi_w K_{av}$$
$$R_{33} = [(1 - c_3)\phi_h]^2 K_{av} + K_{hm} + 4\mu_{33}/3$$

$$\mu_{11} = [(1 - g_1)\phi_s]^2 \mu_{av} + \mu_{sm}$$
$$\mu_{12} = \mu_{21} = \mu_{22} = \mu_{23} = \mu_{32} = 0$$
$$\mu_{13} = \mu_{31} = R_{31} = (1 - g_1)(1 - g_3)\phi_s\phi_h \mu_{av}$$
$$\mu_{33} = [(1 - g_3)\phi_h]^2 \mu_{av} + \mu_{hm}$$

$$\phi_s = 1 - \phi \quad \phi_w = (1 - C_h)\phi \quad \phi_h = C_h\phi$$

$$c_1 = \frac{K_{sm}}{\phi_s K_s} \quad c_3 = \frac{K_{hm}}{\phi_h K_h}$$
$$g_1 = \frac{\mu_{sm}}{\phi_s \mu_s} \quad g_3 = \frac{\mu_{hm}}{\phi_h \mu_h}$$

$$K_{av} = \left[ \frac{(1 - c_1)\phi_s}{K_s} + \frac{\phi_w}{K_w} + \frac{(1 - c_3)\phi_h}{K_h} \right]^{-1}$$

$$\mu_{av} = \left[ \frac{(1 - g_1)\phi_s}{\mu_s} + \frac{\phi_w}{2\omega \eta} + \frac{(1 - g_3)\phi_h}{\mu_h} \right]^{-1}$$

$K$ is the bulk modulus, the subscripts $w$, $s$, and $h$ refer to water, sediment and hydrate respectively, $sm$ and $hm$ represent the sediment and hydrate frameworks, respectively, and $\mu$
and $\rho$ have their usual meanings. $\omega=2\pi f$ where $f$ is the frequency of the source and of which the results are independent. $\phi$ is the non-framework porosity and $C_h$ is the hydrate saturation.

The bulk and shear moduli of the sediment and hydrate frameworks are derived from the following expressions:

$$K_{hm} = \frac{K_h(1 - \phi_{ah})}{1 + \alpha \phi_{ah}}$$  \hspace{1cm} (G.7)

$$K_{sm} = \frac{K_s(1 - \phi_{as})}{1 + \alpha \phi_{as}}$$

$$\mu_{hm} = \frac{\mu_h(1 - \phi_{ah})}{1 + \gamma \alpha \phi_{ah}}$$  \hspace{1cm} (G.8)

$$\mu_{sm} = \frac{\mu_s(1 - \phi_{as})}{1 + \gamma \alpha \phi_{as}}$$

$$\phi_{ah} = 1 - \phi_h \quad \phi_{as} = \phi_w + \varepsilon \phi_h \quad \gamma = \frac{1 + 2\alpha}{1 + \alpha}$$  \hspace{1cm} (G.9)

The values of the bulk modulus and shear modulus for the composite sediment, $s$, can be determined using the Hill’s (1952) averaging method (Lee and Collett 2001):

$$K_s = \frac{1}{2} \left[ \sum_{i=1}^{m} f_{Si} K_{Si} + \left( \sum_{i=1}^{m} f_{Si} K_{Si}^{-1} \right)^{-1} \right]$$  \hspace{1cm} (G.10)

$$\mu_s = \frac{1}{2} \left[ \sum_{i=1}^{m} f_{Si} \mu_{Si} + \left( \sum_{i=1}^{m} f_{Si} \mu_{Si}^{-1} \right)^{-1} \right]$$  \hspace{1cm} (G.11)

where $K_{Si}$ and $\mu_{Si}$ are the bulk and shear modulus respectively of the $i^{th}$ components having volume fractions are $f_{Si}$. $m$ is the number of components. In the sediment modelled here, there are two components-clay and quartz.