What's the Matter?: Educators' Understandings of Teacher Content Knowledge in Primary Mathematics

By

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Abstract

A significant number of Ontario teachers lack a background in mathematics; how this lack impacts student learning in Ontario is unclear. Prior studies indicate that high teacher self-efficacy developed through mathematics pedagogical content knowledge (MPCK) improves student learning. Therefore, this thesis examines how primary school math teachers and administrators conceptualize the value of mathematical content knowledge for teaching. Towards this goal, I have used a case-study model, interviewing three teachers and a principal from one Ontario school, along with an instructor in initial teacher math education. From the existing literature on MPCK, I assessed how participants were aware of their MPCK and the ways they attempted to improve their math teaching. A qualitative method was used to acquire contextualized data that is often absent from large scale psychological and survey-based studies. The findings indicate that teachers' understandings of the value of content knowledge in math are linked to their fundamental conceptualizations of the role of a teacher. However, the participants' understandings of the nature of math and the nature of teaching were inconsistent. This inconsistency points to a systemic misunderstanding of the role of a teacher. Moving forward, results from teachers' perspectives provide a valuable addition to a framework for larger scale MPCK research in the Canadian context.
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Chapter 1: INTRODUCTION

Introduction to the Research Study

The role of the teacher, in a philosophical sense, often goes unremarked upon. Debates on the education system focus on curriculum, pedagogy, content and students’ lives outside the classroom. Our understandings of a "good teacher" are notoriously nebulous. The "good teacher" exists within a multitude of processes and systems over which he or she has no real agency (curriculum, pedagogy, content, students’ lives outside the classroom). This nebulousness of good teaching is what drives most education research. Policy that shifts from focus on pedagogy to focus on content often obliterates the central actors in education: the teachers. The question of who the only adult in the classroom is, and why they are in the room at all, merits deep consideration. Our understandings of what it means to teach, taps into ancient epistemological questions of what it means to know something or even if we can ever know anything at all. To teach, must one know?

The practice of the teaching of math provides a lens through which to explore the broader question concerning the role of a teacher and teacher knowledge. Upon graduation, Ontario students enter an emerging knowledge-based economy where a solid understanding of math will improve their access to jobs and provide the tools to comprehend an increasingly technological society. The current math curriculum in Ontario focuses on problem-solving to develop mathematical thinking skills. Elementary school teachers are tasked to build a "solid conceptual foundation" (Ontario Ministry of Education, 2005) for students. Yet, many teachers themselves do not hold a "solid conceptual foundation" of math.

As the pendulum of education theory swings in North America, the focus shifts between absolute teacher content knowledge, and absolute knowledge of pedagogical technique.
Currently, in the context of initial teacher education at OISE in the University of Toronto, the focus is on pedagogy and instructional technique. Initial Teacher Education programs teach instructional strategy (Ontario Ministry of Education, 2004) with very minimal subject content. It is assumed that content relevant to elementary school curriculum material has already been taught in undergraduate programs. The majority of elementary Teacher Candidates have previously earned a Bachelor of Arts degree, yet receive qualification to teach most subjects. In recent years, more new teachers had an educational background in English, History, Science and Geography than in Mathematics (Ontario College of Teachers, 2008; 2009; 2010; 2011). Therefore, a significant number of Ontario elementary teachers lack an educational background in math, yet are still certified to teach the subject; in response, teachers typically develop coping strategies for bridging gaps in their own understanding of math. In this project, I explore 1) ways in which math educators see content knowledge and teaching success as interrelated, 2) how teachers bridge gaps in their own understanding, and 3) their general perspectives on the role of the teacher in the larger educational institution.

Significance

The purpose of this study is to identify and analyze a possible gap between the putative requirements of effective math pedagogy and the math content knowledge possessed by math teachers. The perpetual improvement of the Ontario curriculum will be irrelevant if policies do not consider individual subjectivities and needs of the diverse teaching population. Many studies have examined the results of standardized tests that purportedly assess students' content knowledge. Yet, there is relatively little attention paid to teacher knowledge in the elementary grades. Therefore, this study focuses on the elementary school teacher with the goal of clarifying
their subjectivities and perspectives on their own content knowledge. How teachers understand their own efficacy in the math classroom in relation to their content knowledge has direct implications for further teacher training, further research and policy development.

This research not only examines the role content knowledge plays in the minds of math educators, but importantly is a step toward developing larger issues in public education. The insight into teacher knowledge provided below prompts the reader to consider the question: "Are there any absolute criteria, such as content knowledge, by which we can judge teachers?"

**Research Questions**

The goal of this project was to examine how elementary math teachers perceive their knowledge of math as affecting their implementation of the math curriculum. Towards this goal, I analyzed teachers' understandings of their own mathematical practice to answer the following questions:

1. In what ways do math teachers conceive of personal success in their math classroom?
2. Do math teachers perceive a relationship between their math content knowledge and their ability to teach math?
3. What are the relationships between math teachers' understandings of the nature of math and their understandings of the nature of teaching?

**Background of the Researcher**

I come to this study with invested personal interest, as I will soon be qualified to teach math, without the tools I currently believe are necessary to my success as a teacher of math. This research project emerges from my desire to understand how my lack of content knowledge in
math could influence my future students’ math education. I grew up in the Toronto public school system. I never saw myself as explicitly ‘bad at math,’ yet it was never my favourite subject. My formal math training ended in grade twelve, when I took the course *Advanced Functions*. As a future teacher of math, I seek to understand how I can improve upon my knowledge and improve my self-efficacy in this subject, to best support my future students.

My previous research experiences are grounded in the field of Anthropology. I have taken linguistic anthropology courses wherein I learned transcription conventions and the influence of questioning techniques on participant responses. Most significantly, I participated in a research project where I compiled an ethnography of a university café, participating in ethnographic fieldwork and semi-structured and casual interviews, using the framework of subcultural theory. This work allowed me to identify my own positionality as a researcher. Although the current project is not an anthropological study, I will use the tools of language analysis borrowed from anthropology, to examine the interview data.

Overview

Chapter 1 includes the introduction and purpose of the study, the research questions, as well as the motivation for pursuing the topic of teacher math content knowledge. Chapter 2 contains a review of the existing literature on teacher self-efficacy and Pedagogical Content Knowledge. Chapter 3 provides the methods and procedure that were used in this study, including information about the participants and data collection instruments. In Chapter 4, I summarize the interview data, compiling participants’ responses into significant themes. Finally, in Chapter 5 I provide a discussion of the significance of the findings and highlight opportunities for future study in the subject. References and a list of appendices follow at the end.
Chapter 2: LITERATURE REVIEW

I expand on two existing threads of education research. The first thread is that of psychology research on self-efficacy as defined by Bandura (1977), particularly the role of teachers’ sense of self-efficacy within the educational institution. The second thread is that of research on teacher content knowledge, and how significantly different types of knowledge affect student success. The foundational thinker on teacher content knowledge is Lee Shulman, who proposed the unique framework of Pedagogical Content Knowledge (PCK) (1986). Woven together, these bodies of research guide the research questions concerning teachers’ self-conceptions, specifically teachers’ self-conceptions of their own math classroom efficacy. Teachers’ self-conception is measured subjectively through their perceived math PCK (MPCK). This research also provides a framework for analysis of the interview data.

Self-Efficacy Defined

Emerging from the field of cognitive theory, the concept of self-efficacy can be simply constructed as, “the individual’s perceived expectancy of obtaining valued outcomes through personal effort” (Fuller, Wood, Rapoport, & Dornbusch, 1982, p. 7). Here, I further this definition, examining if “personal effort” could be linked to PCK and to what end teachers feel inclined toward "personal effort" in math. Fuller et al. (1982) examine self-efficacy on the three levels: school wide, teacher and student. This research project focuses on the level of teacher self-efficacy, with obvious extensions to the levels of student and school self-efficacy. On the benefits of positive self-efficacy in students, Schunk and Richardson (2011) reported that "self-efficacy can affect effort, persistence, and learning. Students who feel efficacious about learning
expend greater effort to success, persist longer on difficult tasks, and learn more” (p. 16). Clearly, these outcomes reveal the positive value of student self-efficacy.

**Teacher Mindset and Teacher Self-Efficacy**

Research on teacher mindset and self-efficacy demonstrates that teachers, like students, benefit greatly from a sense of self-efficacy. According to Ashton and Webb (1986), "teacher self-efficacy was a significant predictor of student achievement" (cited in Schunk and Richardson, 2011). Self-efficacy boosted student achievement in math beyond successfully completing rote calculation: "teachers with higher self-efficacy are more likely to design challenging activities, help students succeed, and persist with students having problems" (Schunk and Richardson, 2011, p. 24). The benefits of teacher self-efficacy promote deeper, conceptual teaching that emerges through their implementation of “challenging activities” in the math classroom (Schunk and Richardson, 2011, p. 24).

Intertwined with the concept of self-efficacy is Dweck's (2006) research on mindset and the outcomes correlated to holding a particular mindset. Suh, Graham, Ferrarone, Kopeinig and Bertholet (2011) present Dweck's two formulations of mindset in simple terms: "a growth mindset holds that your basic qualities are things that you can cultivate through your efforts, whereas a fixed mindset holds that your qualities are 'carved in stone'" (p. 169). In the analysis, I examine potential relationships between self-efficacy and mindset in the math classroom.

Fostering self-efficacy in students seems to be a lofty goal. However, as a caution against creating meaningless tasks in the math class (including teacher education classes) with the sole purpose of boosting students' sense of self-efficacy, Suh et al. comment "a series of easy tasks can lead to a false sense of self-efficacy" (p. 170) which will not actually allow students to
develop into "good problem solvers [who] are flexible and resourceful" (p. 171). These concepts of developed self-efficacy overlap with Dweck's 'growth mindset', through which people believe in their ability to develop.

Wigfield et al. (1989), in a large-scale psychological study in the United States, demonstrate that students with a low sense of self-efficacy will achieve greater success learning from a teacher who holds a high sense of self-efficacy. Much research on self-efficacy (Tschannen-Moran & Hoy 2001; Wigfield et al., 1989) derives from longitudinal survey research studies in the United States. A large-scale survey research was beyond the scope of this thesis; I add instead qualitative data from the Canadian context to the discussion of teacher self-efficacy.

The Formulation of Pedagogical Content Knowledge

To examine self-efficacy in teaching, it is worthwhile to consider the many factors that can lead to heightened self-efficacy in teaching. In Bandura's (1977) foundational text on self-efficacy, he proposed that "performance accomplishments, vicarious experience, verbal persuasion and emotional arousal" (p. 143) were all sources of self-efficacy for patients having psychological treatments; their increased self-efficacy would allow them to implement appropriate coping strategies in times of stress. For this project, I specifically assess the performance accomplishments of teachers where they are able to recognize their own success in the math classroom.

Proposing a potential source of self-efficacy, Basile, Kimbrough, Koellner, & Swackhamer (2009) assert that subject content knowledge increases a teacher’s sense of self-efficacy. Yet, it is integral for this study not to solely focus on content, as content knowledge does not necessarily translate into greater teaching success, nor self-efficacy in teaching (Ball,
Hill & Bass, 2005). In a study of grade five literacy, Guo, Connor, Yang, Roehrig, Alysia, and Morrison (2012) found that “teachers' education did not significantly predict any student outcome” (p. 25). These findings underscore the nebulous nature of defining a "good teacher"; sometimes, researchers can overlook entire components of the educational system.

To Shulman (1986), in the mid 1980s, research on good teaching suffered from a lack of focus on content:

[m]y colleagues and I refer to the absence of focus on subject matter among the various research paradigms for the study of teaching as the "missing paradigm" problem. The consequences of this missing paradigm are serious, both for policy and for research (1986, p.6).

To fill this gap of lack of focus on subject matter, Shulman began to examine what types of subject matter teachers needed to know to be effective in their practice. Shulman (1986) asserted:

The teacher need not only understand that something is so; the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened or denied. Moreover, we expect the teacher to understand why a particular topic is particularly central to a discipline whereas another may be somewhat peripheral. (p. 9)

From this assertion on the value of conceptual content knowledge, Shulman (1986) went on to postulate a specific type of teacher knowledge that was separate from pure knowledge of subject matter and knowledge of pedagogical teaching strategies. This type of knowledge was synthesized from pedagogy and content, yet moved beyond both. Shulman (1986) labeled this other teacher competency as Pedagogical Content Knowledge (PCK). This specific type of knowledge enabled good teachers to understand how to teach their given subject in the context of the subject matter itself. This definition moved beyond content knowledge, “the amount and organization of knowledge per se in the mind of the teacher,” as Shulman explains, “[w]e already have a number of ways to represent content knowledge: Bloom's cognitive taxonomy, Gagné's
varieties of learning…” (1986, p. 10). This unique knowledge created a whole, PCK, which is greater than the sum of pedagogical and content knowledge in isolation.

This foundational construction of PCK initiated a large body of research that aims to establish the presence of PCK in the context of different subjects. Shulman’s definition of PCK is central to the data analysis because it provides a framework through which to explain teacher responses.

**Pedagogical Content Knowledge in Mathematics**

This section highlights several perspectives on the interrelatedness of MPCK. The context of this analysis is that only a minority of Ontario elementary teachers possess post-secondary training in math, yet, they are all expected to teach math. In Ontario, Initial Teacher Education programs, such as the Master of Teaching degree, do not teach content in subject specific classes.

A central thinker in math education is Hongxi Wu, who has participated in a significant review of math education in California. He is a former mathematician who shifted his research focus to math education, after feeling despair in the level of mathematical knowledge possessed by his first year university students. Most significantly, Wu created a textbook for pre-service math teachers called *Understanding Numbers in Elementary School Mathematics* (2011). Without explicitly referring to PCK, this text functions as a step-by-step guide for future elementary teachers to develop the appropriate math content knowledge which enables them to teach their students in a mathematically sound manner, with PCK. At the end of each chapter Wu poses content questions to the reader to test their understanding of the material; he makes them
do the math. Wu laments the lack of deep content knowledge among elementary math teachers, asserting that they often increase student confusion in math classes.

Several studies point out the deficiency of teachers who do not know enough content to effectively communicate an understanding of math to their students (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992; Ma, L., 2010; Piccolo, D., 2008; Ball et al., 2005). Borko et al. (1992) offer a case study of a pre-service teacher who desires to create math lessons that develop student understanding rather than to simply teach how to implement memorized algorithms. Yet, this pre-service teacher does not possess the content knowledge to realize her goal, as she is unable to provide a conceptual explanation of the algorithm for dividing fractions, as posed by an inquisitive student. Here, we see content knowledge as a factor that limits a teacher's self-efficacy in the math classroom.

Ma (2010) offers a cross-cultural examination of the knowledge of math teachers in the United States and in China. She emphasizes the deep content knowledge of the Chinese teachers, which leads to greater student success. These studies demonstrate that if a math teacher lacks deep content knowledge in math, the class often becomes computational with little time spent on the logical underpinnings of mathematical thinking. Central to the above research is the understanding that math content knowledge is inherently linked to pedagogical success; these above researchers all have a high math content knowledge.

K.J. Delvin, another mathematician who has turned his focus to education, comments specifically on the nature of math: "to confuse mathematical thinking with basic mastery of basic skills is akin to confusing architecture with bricklaying . . . the skills are much more easily acquired when encountered as a part of mathematical thinking." (Delvin, 2011, p. 2). To Delvin, the simplistic and separate attention paid to basic skills and mathematical thinking, often
polarized in the debate over the relative benefits of rote learning compared to a more inquiry-based approach, is at the crux of what is wrong with math in the United States (2011).

Cladwell, Karp and Williams (2011), echoing Schulman, comment on the distinction between math content knowledge and MPCK "knowing and being able to do familiar math are not enough when you are in the classroom. You also need to be able to identify and justify or refute novel claims - or better yet, assist students in thinking about these claims in guided conversations" (3). Here, they portray the teacher as an interpreter of mathematical thinking and exploration, rather than a one directional faucet. When it comes to this teacherly "profound understanding of fundamental mathematics . . . that is deep, broad and thorough" (Ma and Kessle, 2001) "there is varying opinion about level, breadth, and formality of this content" (Burrill, 2001). Attending to where teacher candidates may gain this "profound understanding" Burrill (2001) claims, unfortunately, "what teachers take from their mathematics courses does not necessarily sever them well in their classroom practice." Despite significant research, the true nature of math content needed for teaching remains unclear.

Ball (2005, 2008), a pupil of Shulman’s theories, has written a vast body of research on the possible application of PCK. She was also a practicing elementary math teacher for fifteen years. Ball’s approach is more open and questioning about the nature of math content knowledge than Wu (2012) and Ma (2010) who conclude that an integral link exists between teacher content knowledge and student success. Ball et al. (2005) reword the concept of MPCK with the term “mathematical knowledge for teaching,” which clarifies the supportive relationship between knowledge and teaching, emerging from the more abstract term “PCK.” The phrase “mathematical knowledge for teaching” clarifies that math content knowledge is in place for the teaching of math. Ball et al. (2005), in a large-scale survey-based study of elementary math
teachers, discover the “centrality of mathematical language and the need for a special kind of fluency with mathematical terms” (p. 21) as integral to the success of an elementary math teacher. Here, Ball et al. (2005) present mathematical understanding as coming through a different language entirely, one with which teachers must develop fluency.

Arguably, the most interesting part of Ball et al.’s (2005) study was the paucity of teachers with a high math content knowledge teaching in low-income, low-achieving schools. Noting this, the study asserts “[t]his was a promising finding because it suggests that improving teachers’ knowledge may be one way to stall the widening of the achievement gap as poor children move through school” (p. 44). Further, Ball et al. (2005) call for future research which could strengthen “the claim that effective teaching entails a knowledge of mathematics above and beyond what a mathematically literate adult learns in grade school, a liberal arts program, or even a career in another mathematically intensive profession such as accounting (p. 44)”; in other words, to prove the unique existence and value of MPCK.

**Synthesis of Teacher Self-Efficacy and PCK?**

This research project initially focused on PCK but was expanded given the emergent link between teacher self-efficacy, and PCK, as they are both claimed to be predictive of teacher success. Thus, the study was driven by an interest in determining whether or not and how teachers think about these two ideas, and whether they posit a correlation or causal link between the two.

There is some existing research, in the North American context, which examines the role of teacher content knowledge as related mindset and self-efficacy in the math classroom. Schunk and Richardson (2011) describe the benefit of measuring self-efficacy in the math classroom
through higher order mathematical concepts and thinking rather than through computational and procedural skills. Suh, Graham, Ferrarone, Kopeinig and Bertholet (2011) suggest that "all members of the mathematics community [, teachers and students,] must commit to developing a growth mindset" (p. 182) to ensure their success with developing mathematical confidence and skills. This study alludes to an initial resistance from the students who are used to being spoon-fed mathematical facts and processes.

The benefits of self-efficacy and a growth mindset in the math classroom have a transformative effect on student achievement in math, according to Schoenfeld (2007) students and teachers with this attitude have:

- alternative approaches when they get stuck, ways of making progress when they hit roadblocks, of being efficient with (and making use of ) what they know. They also have a certain kind of mathematical disposition-- a willingness to pit themselves against difficult mathematical challenges under the assumption that they will be able to make progress on them, and the tenacity to keep at a task when others have given up (p. 60)

This conceptualization of "mathematical disposition" appears congruent with Ball et al.'s (2005) "mathematical knowledge for teaching" (p. 15). From this body of research, emerges a potential marker of teacher efficacy, or a path toward improved teaching: successful teaching in the math classes emerges from teacher self-efficacy as a growth mindset. These theoretical understandings of teacher knowledge and self-efficacy are useful insofar as they provide a system in which to contextualize the participants' perceptions of their success in the elementary math class.
Chapter 3: METHODS

Procedure

Based on a vast selection of books and articles on teacher content knowledge and teacher self-efficacy, I selected Shulman’s concept of Pedagogical Content Knowledge (PCK) to frame this project. I established a literature review that probed many different facets of the issue of teacher content knowledge in math, with particular focus on the reality that many elementary math teachers have little math content knowledge. Tied in with the framework of PCK, I also drew on psychological research on self-efficacy. I engaged in five audio-recorded interviews with Ontario educators to better understand the climate of teachers’ MPCK. I used a qualitative method to build a nuanced understanding of educators’ perceptions of their own MPCK, and their self-concept as math teachers. After the interviews, I transcribed the recordings; the transcripts constitute the data upon which the findings were based. Further, the data were analyzed to discover emergent themes, and these themes were compared to key ideas from the literature review. The potential implications of the findings are provided in the Discussion section below. Taken together, these chapters form the core of my Masters of Teaching Research Project.

Instruments of Data Collection

I collected data on the contextualized realities of teacher practice in Ontario elementary math classes using a qualitative method. As discussed, much of the existing literature on this topic was collected through survey research. To allow participants to fully explain their conceptions of math teaching and education, I personally conducted all semi-structured face-to-face audio interviews. I began each interview by explaining the procedure of the interview to the interlocutor, to help put them at ease, as recommended by Turner (2010). Examples of interview
questions are: "What do you hope students will take away from your grade x math class?" and "What do you think the biggest constraints on your success as a math teacher are?" (Appendix A). Both of these questions are non-judgmental and open ended, this strategy elicits open answers from the participants (Turner 2010). These qualitative interviews add to a body of research that is predominantly survey based.

Participants

The study participants were colleagues in a public Ontario elementary school. I recruited three elementary teachers and the school's principal. I made initial contact with the Principal because of a previous connection, as he was my own middle school math teacher and I had communicated with him briefly at a summer program where we both worked. Besides that encounter, I have no personal relationship with him. I remembered his attentiveness to math education from my experience in his classroom and was interested in the mathematical climate he would attempt to foster as an administrator. Due to his extensive background teaching math and his role as an administrator, he brings a unique perspective to this research. Additionally, I have interviewed a university math education instructor, who is able to provide insight into the context of Initial Teacher Education. The participants had diverse cultural and mathematical backgrounds which allowed the data to open up a range of possibilities for further research in MPCK and the role of a teacher. These participants are not wholly representative of Ontario’s teaching population, but will illuminate some important realities of teaching math in Ontario.

I recruited participants through initial contact with the school's principal. The professional relationship that these educators had with each other and their similar context allows for a full, rich picture of a particular manifestation of math education community. Even within a
small sample from the same community, the vast differences in participant responses reveal the greater divergence in opinion surrounding math education that could be captured in a more comprehensive study.

**Data Collection and Analysis**

I conducted interviews in the participants' offices and classrooms. With permission, I made digital audio recordings of the interviews. During the interviews, I gave the participants a typed outline of the interview questions, so they could have a sense of the structure and length of our discussion. One interview was interrupted by school fire alarm testing, and we re-located outside the school building. While conducting the interviews, I took hand written notes to supplement the audio data. Following the interviews, I transcribed the audio-recordings. Through careful analysis, I examined the similarities and differences of the responses to the existing literature, to each other and for general themes. To transcribe, I used the software ExpressScribe and the speech recognition software, Dragon Naturally Speaking. Following this, I coded the transcript with different e-highlighters to show some *in vivo* codes. I ran the transcripts through the online program Wordle, which provides a visual representation of frequently used terms, to see the most commonly used words used in each interview. With these new structured understandings of the data, I coded the transcripts for emergent themes. I then combed through the transcripts to find quotations that are concisely representative of these themes. Themes from the transcripts became the areas of focus for the subsequent analysis.
Ethical Review Procedures

I followed the ethical review approval procedures for the Master of Teaching program. I contacted potential participants through e-mail, providing a preliminary outline of the research and the interview process. When the participants responded with availability and interest, I sent an electronic copy of the Letter of Consent (Appendix B) prior to our interview date. At the beginning of each interview, I read the letter of consent, provided the participant with a copy to keep, and retained one signed copy.

The time of each interview did not exceed one hour, as to not inconvenience the participant. From the outset, I explicitly informed the participants that participation is completely voluntary and that they are able to withdraw at any time without penalty. I guided each participant through the interview procedure by outlining the types of questions I planned to ask. Following the interview, I offered the participants the opportunity to have anything they said removed from the record. During the interviews, I took hand-written notes using a pseudonym for the participant; these notes will be kept in a locked drawer at my home. The audio-recording I make will be stored on a password protected computer and no mention of the participant’s personal information is included in the file.

Limitations

The number of participants in this study is low, as this project is conducted as a starting point into further research into MPCK. This low number of participants is also due to the Master of Teaching Research Project protocol. The low number of participants who work in the same teaching context provide data that cannot be generalized to a larger population of teachers. The results should not be taken, in their current form, to influence any policy decisions. The
experiences of the five educators interviewed do not represent a whole picture of the different instances of varied math content knowledge in Ontario classrooms. This research was conducted over a limited period of time. With a greater number of participants, conducted over a longer period of time, the results of this study would be more significant. However, this study provides rich detail about the teachers and their experiences. There will be aspects of this research that the readers may be able to generalize to their own situations. The onus is on the reader to assess which aspects of this study are relevant to their own contexts.

Additionally, I arrived at this topic from a particular position and bias. The selection of this topic was motivated by my low content knowledge in math and so I may have inadvertently revealed this situation to the participants, causing them to skew their responses to my understanding. To minimize this bias, in a future study it would be beneficial to have several researchers, each with a varied MPCK.
Chapter 4: FINDINGS

It is dawn, I leave behind the busy Toronto street, turning down quiet tree-lined avenue with a basketball net in nearly every driveway. At the end of this road is a public elementary school. Within this school, there are students, teachers, caretakers and administrators all with the common goal of educating Ontario students. Though it's early morning, teachers are already bustling around the school. The walls reveal that the primary grades have been polling each other about their favourite types of candy. As I enter, the principal walks in carrying a bag of bagels for the school's small breakfast program.

The findings presented in this section include a general description of a school’s elementary math community and its relationship with the administration in the school. I share the findings from the interviews of three primary math teachers at this school, the school's principal, and a university math education instructor who has worked as a curriculum consultant in math. The participant’s names have been changed to protect their anonymity. Taken together, these vignettes and personal accounts of teacher understandings of math content knowledge demonstrate a particular instance of curriculum translated into practice. I also present particular similarities and differences between the participants’ responses and focus on areas of interest for future research and understanding.

Participant Profiles

Alanna is a grade one teacher and a self-labeled "baby-boomer". She has taught math in grades one through eight, with more time spent in the primary years. Central to her self-concept as a math teacher is her understanding of the immense benefit she received from being taught in the "traditional way," with a focus on basic computation. Alanna is up-beat and jovial when
discussing her classroom and math. She took an extended hiatus from teaching, taking time to stay home to raise her children and to start a small manufacturing company. She says this break enabled her to stay energetic in the classroom. Alanna had great success in high school math, but did not pursue additional post-secondary math training. She ranks her math content knowledge on the higher end of a scale from beginner to expert.

The second teacher interviewed, Bev, had a full teaching career in the Caribbean and has been teaching in Canada for four years in the primary grades. She currently teaches grade one. Bev explains that since moving into the Canadian system, she has had to supplement her past learning with a greater focus on methodology and strategies to integrate critical thinking skills through "experiential learning," rather than the focus on rote, which was prevalent in her home country. Since coming to Canada, Bev earned a Master’s degree in Education Management. She cites her own struggles as a math student as greatly beneficial to her ability to relate to her students. Bev never had a true interest and love for math until she took a statistics course in university. Through the interview, Bev emphasized her lack of expert knowledge and continually discussed her ongoing personal and professional development.

Chloe teaches grade three. She has taught in Canada for four years, with previous experience teaching English in Asia. Chloe conveys an intense emotional connection to her identity as a teacher, emphasizing that she is "the happiest person in the world" because of her job in the classroom. She attributes her current understandings of math to the amazing training she received at an Ontario Teachers’ College, which provided content and methodological training. Still, she perceives her math content knowledge to be low. She had minimal success in her high school math courses, and notes that she had no idea what she was "actually" doing and
was just "plugging in formulas." Chloe is actively involved in furthering her math knowledge through personal research and professional development.

Each of these three primary teachers briefly mentions their relationship with the "primary team," referencing an issue that has come up with the administration regarding the use of a workbook. Discussions of the EQAO test also prompt these teachers to reference the school administration and their perceived notions of the focus on the test.

I was able to interview the school's principal, to round out my understandings of how math operates in this particular setting. The principal, Paul, describes himself as having a somewhat high math content knowledge, explaining, somewhat jokingly, that further training may only assist him in aeronautics engineering. Paul has taught math across all school divisions, from primary to adult education classes. He references his own experiences as a child playing competitive chess as motivating his passion for math. Still, he explains that math has grown on him during his teaching career. Paul is "100% certain" that his Initial Teacher Education, which he took in Ontario, did not teach him any content necessary for teaching. Through taking Additional Qualification courses, attending and presenting at conferences and through personal reading, Paul became highly confident in the math classroom. He prides himself on developing his own activities that reflect the students’ interests and learning styles.

The final participant, Emily, is not involved with the daily workings of this school, yet has a role in the larger structure of how math comes to be taught in its classrooms. Emily is a university math education instructor and a former Instructional Leader for the school's board. She has brief experience in a middle school classroom. Emily's perspective overlooks the larger system in which these teachers and principal operate. She has a high math content knowledge and has frequent interaction with teachers and teacher candidates who are fearful of math.
The attitudes and understandings of these five education professionals present a small case study of how math content knowledge comes to be understood in a particular urban community. Their individual conceptions of the relation of math content knowledge to math teaching ability are not totalizing.

Although operating within the same system, the participants have divergent views on how math should be taught, attitudes, and aptitudes which all influence their own self conceptions as math teachers. In the sections below, I present the data grouped into emergent themes. These themes were organized with the intention to attend to the various perspectives on teacher content knowledge.

Content Knowledge and Educational Backgrounds of Participants

In current popular media, we see a polarizing of the methods of 'traditional' or rote teaching at one extreme, and 'New Math' which integrates multiple problem solving strategies and collaborative work at the other extreme. Each of the study participants experienced a mostly traditional, or rote, model of math education.

Two participants, Chloe and Bev, discuss their difficulty in school math classes. Chloe commented, "I had no idea what the heck I was doing in math, so, I barely made it through high school math" and Bev echoed "I know that I wasn't good in math." Speaking to her lack of engagement, Chloe elaborated "there was just a formula, I didn't understand what I was doing, why I was doing it." However, this lack of understanding disappeared when she was presented with a different style of instruction: "I went to teacher's college, I learned why was doing it, I learned the process." Bev experienced a similar shift to mathematical clarity when she perused university math courses, "I really understood math, the reasoning, and then that is when I really
developed an interest in love for it, not in high school." Chloe and Bev mark their shift in perspective toward math when the subject was no longer about plugging in formulae; disgruntled with how she had been taught, Chloe explained "I memorized how to do it, which wasn't effective because now I don't really remember anything about what I did in high school math, whatsoever." Chloe and Bev follow a similar math trajectory that started in frustration; however, each sought out further training through which they developed a consciousness of the 'whys' of math - this deeper knowledge sparked a love of math in both of them. This understanding that came through knowledge is perhaps an example of Ball et al.'s (2005) "mathematical knowledge for teaching."

Despite a newfound love of math, Chloe and Bev still feel as if their content knowledge is around early high school. Bev's apprehension around her perceived low math content knowledge came out when I ask about her current level of understanding: "If I tell you [my content knowledge level is] 'early high school math' you'll say 'you shouldn't be a teacher!'" Here, Bev's self-efficacy in the math class is hindered by her subject knowledge. This self-conception translates into their perceptions of which grades they would be confident to teach: "right now I know that I couldn't go up to grades seven and eight math, I would not feel comfortable with the amount of knowledge that I have," she continues, "I'm not comfortable going higher than elementary despite that I've had higher education, right, I'm still comfortable elementary and understand it." Bev feels able to teach elementary math "because the content is simple."

Paul experienced similar instructional methods to Chloe and Bev: "the math was kind of mechanical, sort of assembly line method structure every single year, and no one was really passionate about numbers." Unlike Chloe and Bev, Paul never really struggled with math until
late high school. Despite his relative success, he commented "I didn't learn actually either in my elementary or secondary math education, so in [one math Additional Qualification course] I learned more that I probably did in my 15 years plus of math education." Since his formal schooling, Paul has devoted significant personal effort to developing his math content knowledge. He is comfortable teaching all levels of school math, and sees himself as having a relatively high math content knowledge; though, he often underscores that he is definitely not an expert and still makes mistakes.

Alanna's understanding of her own math education is significantly different than those of Chloe, Bev and Paul. She explained, "I had the huge advantage of being taught in the traditional way!" Through a 'focus on the basics' approach, with each primary grade focused on two operations, she emphasized "by the time we had hit grade seven, you just had this very firm foundation of the tools you needed to do problem solving." Alanna had high success in high school math, but chose to pursue Fine Arts. When I asked about her choice, she revealed her inner monologue, "Let's do something different! Let's challenge ourselves! Someone should have said 'Shut up do a math degree.'" Alanna ranks her content knowledge on the "higher end." with the caveat that it "also makes it difficult to teach because then you just get it, you know, the things that you get are hard to teach." She was comfortable teaching math at the intermediate level.

The Nature of Mathematics

In teaching, we convey our sense of truth to students, and what one teacher sees as the truth of math, can be very different from the teacher down the hall. Several participants revealed their understandings of how math, as a subject, is different from others. Emily, providing the
perspective of Initial Teacher Education programs explained "the difference with math compared to, I’ll say, literacy, is that here we aren't only teaching the pedagogy we are teaching the content, right. In order to understand the pedagogy [teacher candidates] have to understand the content, there, you can't ask good math questions unless you yourself have struggled through them." To Emily, the lack of math content knowledge compared to other subjects is stark.

Central to the perspective of Bev and Chloe is a perceived factual nature of math, which put it in contrast to more 'open-ended' subjects, such as language. When asked about her favourite subject to teach Bev explained, "I still like language, there's so much more about what they think, and the expression, than as opposed to math where there's a direct path to getting to things." Chloe, too, had a similar view of primary math, "I think math can be very concrete, so I enjoy teaching the math as the concrete." After pausing, Chloe continued, diverging from Bev's perspective, "there can be different answers to math which I like, and I enjoy that and, I like when I see my kids doing that." Chloe commented "it's difficult in grade three to teach things that are not concrete. I find elapsed time in geometry, right angles difficult . . . [my students] don't have that conceptualization." Further, she explained a conflict between the abstract nature of math and her students' developmental capacity: "it's very abstract and I'm not sure if that's the right time to be capitalizing on that for them." Bev and Chloe present internal conflicts about their desires for exploratory math in their classrooms, while also holding that the primary math curriculum is generally factual, concrete and straightforward.

To Paul, the understanding of math that informs his teaching is one which goes beyond the seemingly simple operations introduced in elementary math. He explains that a teacher should be "significantly ahead of the students." Unlike Chloe and Bev, Paul expanded his conception of math beyond what is explicitly relevant to the primary curriculum "][teachers] need
to have a clear picture of, and an understanding of, number facts, and I don't mean operational facts, I mean just simply being able to tell the kids the beauty of numbers, that 1 times 1 is 1, 11 times 11 is 121, 111 times 111 is 12321 - it's a pattern." Here, he translates computational facts into a more abstract perception of "the beauty of numbers."

Emily has a more activist stance when it comes to perceptions of math, she commented:

We need to change the face of mathematics so it won't be perceived as one of these subjects that you either get it or you don't, it's become publically acceptable to say 'I'm not good at math' but it's not the same to say 'I'm not good at reading' and I think that is the difference because the public has accepted that it's okay to not be good in math.

She, like Paul, described her understandings of the beauty of numbers and the necessity to convey this creative understanding of math a problem solving to students.

When asked what they hoped their students would remember after many years out of their classroom, all participants communicated a desire for their students to have learned an appreciation of math. Some participants added they hoped students would remember particular mathematical strategies that could better the students' lives. Bev responded that she hopes students would remember "how to add quickly subtract quickly the counting up or counting back method." She then went on to explain her particular methods for these strategies. Bev was the only participant who explicitly presented computational strategies as what she hoped her students would remember. The other participants expanded on a more general sensibility with numbers which would allow students financial success and an appreciation of math.

I questioned the participants on how their content knowledge influenced their ability to answer student questions. Bev and Alanna answered that they had not ever been stumped by a student question in their math class. Chloe remembered a student that she had in grade three who was doing high school level math - she felt the need to seek a lot of help in facilitating his
learning and expressed her gratitude that the students’ parents were very knowledgeable in math. Emily felt that because of her experience preparing teacher candidates for moments of confusion in the math classroom, she has already encountered most of the questions that usually stump teachers. Paul quickly thought of a recent example of when he had been puzzled by a student question. In answering the student, he provided his meta-cognition as well as a more factual answer:

I got a question yesterday, from a student... from the middle east, and the symbols used in their mathematics education are different from ours, so, he's just asked me "Why can't I use my way of describing the solution using the symbols I learned?" and I said to him "Well math is a universal language but there are different dialects, so if I was your teacher you could use them, because I would understand your dialect. However, you need to have that conversation with your teacher."... you always get questions from students and the easy answer is, and teachers should not be afraid to say this, "I don't actually know, but I will be happy to try to solve it with you" I don't always know, I mean I don't know what 339 times 381 is but I can give you a pretty good estimate.

Here, Paul synthesized his knowledge of math with his attention to diversity and equity. The responses to this question on the limits of teacher knowledge showed the participants’ varying interpretations of the merit or demerit of showing vulnerability in front of students.

Of all the participants, Chloe most greatly emphasized that her understandings of math were significantly limited "I don't even know if I would understand what they're doing at University exactly, like, if I went to university math class I probably wouldn't have a clue what the heck was going on" open to the challenge, she followed with "maybe I should go and see."

The Nature of Teaching

Tied into the participants’ beliefs about teaching math are their understandings of teaching in general. Most participants highlighted the importance of constructing opportunities
for learning that are connected to students' passions. In this section, I present participants' understandings of the role of the teacher and the nature of teaching. Here, “nature of teaching” can be defined as theoretical understandings of the fundamental purpose of teaching, and how, practically, teaching should be enacted.

Each of the participants had a largely positive portrayal of their experiences teaching. When asked about a recent positive school experience some went so far as to say: "I have to be honest with you, I love teaching, I had a great time, I didn't have one bad day in my career, I had one bad hour, once, but that was fully unrelated to teaching" and another said: "I'm the happiest person in the world, ever. I get to be in the classroom for my job, and I mean waking up every morning is the best thing in my life... so when you ask about a positive experience, every day is a positive experience." Here, the participants' understanding of their identity as a teacher is included under a general state of positivity and hopefulness.

Though the participants were clear that they felt positively about their work in schools, they all have different perspectives on what that work actually is. I asked participants to choose a metaphorical object from my suggestions of a pipe, sink or faucet, that they felt best represented the role of a teacher. Even though they worked in the same school, the Principal and three teachers each had the following different responses: a pipe, a faucet, a sponge and a faucet/pipe combination. One participant stated simply that she was the faucet, letting the information out to the thirsty people; she understood the pipes as her background. The other three all had some aspect of reciprocity tied into their metaphor, with a feedback channel of information coming from the students. Paul created an extended metaphor, as he claimed the role of the teacher was similar to a sponge:
I think you take in ideas from everyone and you kind of squeeze out whatever amount you want to the situation, but you always want to make sure that you are never ever dry, because eventually you shrivel up . . . As long as you got a water source coming to feed that faucet, and its constant and pure, that's great too. And you can fill up the sink to whatever level you want. The problem is that some of your kids can't swim. So you know it's really difficult, you need to give them life preservers, and, you know, a cup to drink from because they don't understand that you can actually drink like this [cups hands], which I could never understand. But you have different learners and I think you need to kind of squeeze out what you can to each particular student to help that learner.

Through their chosen metaphors, the study participants displayed deeply held beliefs about the nature of teaching and of knowledge transmission.

Translating these understandings of teaching into classroom practice involves integrating them with the curriculum subjects. Emily has had interactions with many teachers and teacher candidates. She offered this advice to her students: "I don't want you to just be good math teachers and to be good at teaching math I want you to like it, because if you like it, then your kids will like it." Bev held a similar perspective, "when learning takes place in an atmosphere enjoyment and relaxation it's much better than sitting all the time and filling out paper, the children are happy working together, solving problems together, with different abilities." Bev continued with the perspective that this classroom is not only more enjoyable, but that more successful, memorable learning happens. Emily, who frequently emphasized her focus on content knowledge, revealed "it's not that you have to have a math degree to teach math." To Emily, a rich knowledge of primary math is necessary. Bev noted that "for earlier grades there's not so much about content . . . I've been focusing on K to three teaching of math which in itself focuses on more and methodology." Emily explained, somewhat differently from Bev that there is significant content in the primary grades, and that is "really classroom culture that makes the teaching successful . . . anybody, whether they are new or experienced, as long as they are
willing to experience what the students do, which is to do the math, then they'll be able to [be successful]."

Commenting on how to create this positive classroom culture, Paul suggested "bringing something interesting and relevant to the kids, whether it's problem solving format, whether it's something that they could design which would better an aspect of their own lives." To Paul, developing an understanding of the student’s personal lives is crucial to the role of a teacher. Bev integrates a consideration of the education her students bring from home, she noted that "what they do at home is a lot of text book and then practice worksheets, and so on. So, I find that my skills are required to inculcate thinking an inquiry 'how did you get the answer?' not just getting the answer." To these participants, the role of the teacher is to develop students' meta-cognition and interest in the subject.

Chloe and Paul discussed problem solving as central to what they hope students take away from their math classes. Chloe stated that she hopes her students know that "they can persevere and in that they can challenge everything and always try their best to do their best that's the most important thing . . . I want them to go through life and say I can kill that question." Paul emphasised the value of looking at a problem in a variety of ways, "I'd like them to come back and say 'you challenged me, you made me think, it was a hard program but at least you encouraged me to try to find different ways of solving something.' Because that's what life is all about. I don't think I was successful with everyone, and I don't think it's possible to be successful with every single person, but my hope is that I didn't turn off people from math."

Chloe and Paul tempered their hopes for student learning as they admitted that they will not succeed in every instance.
To Alanna, the place of the teacher to create their own program for the students perpetuates a system of inequality. She is a strong advocate for a textbook and workbook based program that is homogenized across different classrooms. She asserted:

You've got teacher A, who is really creative and knows a lot about a subject and the kids get this fabulous program that she has created or he has created. And then you got teacher B who couldn't care less, she gets some crappy little blackline masters books. That's just, you know, every day they get the same thing. If they had a uniform a textbook that they had to go through than the kids all would get it, certainly get the content, it may not be delivered an exciting way but they'd still get the content. They would all be equal, but it's not equal, it's not equal at all.

To Alanna, the standardized system would reduce the differences in student experience due to inter-teacher variability.

**Perspectives on Fixed and Growth Mindsets**

Emerging from how the participants discussed teaching and learning were their conceptualizations of the limits of teaching and what sort of improvement a person can create for themselves. Here, I attended to the participants’ comments on life-long learning, student potential and the limits to success.

Chloe frequently integrated her personal learning into her answers to questions about content knowledge. Chloe mentioned several Additional Qualification courses that she has taken and several professional development activities, including Math Camp which is an intensive math focused retreat for teachers. She expressed her commitment to "life-long learning." When I asked her if she had ever been stumped in the classroom by a student's question Chloe responded:

Every once in a while something comes up, that, it's an uncomfortable . . . "why don't you know? You're supposed to know everything!" . . . Which is a perfect time to take a teachable moment to say teachers do not know everything they have
to learn, and keep learning and look for answers just like they have to keep learning their whole lives.

Chloe is open about her lack of knowledge with students, and makes an active effort to seek out new knowledge from family, colleagues, articles and books if she is unsure of something.

Paul, now very confident in math teaching asserts "I wasn't born to be a math teacher, I didn't specialize in mathematics, but it kind of grew on me as time went on, I think I became a teacher of mathematics." To continue to challenge his math knowledge Paul does math contest questions, designed for elementary and middle school students, he admits "I'm stumped on certain questions occasionally . . . but I like struggling with questions." Paul's attention to his knowledge improvement is greater than to the content he already knows.

Like Chloe and Paul, Bev frequently affirmed her desire to continue to learn about teaching. Bev has not focused her recent professional development on content, explaining "what is critical at this stage [grade one] is methodology . . . so that is where I have been trying to improve or to expand my repertoire." Bev's motivation to improve is both internal and external; she feels a personal desire to learn more and is motivated by the concerns of the school's administration (led by Paul). Bev reported that because of the school's falling EQAO scores in math, "the ministry has been looking at focusing on the teaching of math from the early grades, so I have to make myself very knowledgeable. I would say that I have quite a lot of skills, but new things are being developed and so I need to improve myself, you know, in line with current thinking." Bev sees the methods for teaching math as most crucial to improving her ability to teach math.

Chloe, Peter and Bev orient themselves toward new strategies for teaching math, including the use of manipulatives and technology. In contrast, Alanna advocates for a system similar to one she experienced. She felt confident with the math training that she had received in
her secondary schooling and has not taken any significant steps to improve upon that knowledge. Alanna does say the math is involved in her daily life, as she enjoys number puzzles, such as Sudoku. To Alanna, the progress oriented focus of math education is misguided; she emphatically said: “We have to keep having new ways of teaching basic concepts! Let’s just save the money! Let’s just do it the way we have done it! The textbooks I had were fabulous, I would love to find them! I don't know why it they are why do we have to keep spending more and more and more money on the newest and best.” Her view of her own education stands to her as highly successful "you know to be honest with you, I think my schooling was fabulous I really do I remember having teachers who loved math, math teachers were the happiest teachers in the school.” Alanna described what she believed to be a reality about the nature of students, she expressed a sadness that some students just don't have "math sense . . . they just don't get it . . . Here's the thing, some people never will and it's okay. Some people will never have red hair, and it's okay. You're going to have a complete community.” Despite her repeated expression of sadness about these students with "no math sense," Alanna was confident that they could still be successful, "he may end up being a plumber making $150,000 a year! Boohoo! How sad is that? Three weeks holidays a year." Here, Alanna differs from the other participants in viewing math sense as something potentially innate in students.

**Benefits of Using Multiple Modalities for Instruction**

Four out of five participants highlighted the necessity of explaining mathematical concepts in different ways to access different learners. Much of the professional development taken by these participants was with the aim of expanding their arsenal of strategies to explain concepts to struggling students. Attending to the relationship between deep mathematical
understanding and the ability to explain concepts in different ways, Paul commented, "our grades one, two, and three teachers are struggling [with math]. Grade one to three math is not exactly really difficult math, but if you don't know how to present certain concepts in a multitude of different ways for kids" you will not be able to help them. Here, Paul is referencing the other participants in this study, and highlighting his understanding of primary math as relatively simple. He continued, noting a difficulty for teachers who were taught using a singular modality: "How are you going to learn [to explain concepts in different ways] if you didn't learn that in school yourself, you were taught one way." Emily picks up on the same concept explaining that "teachers will also tend to teach the way that they were taught and in most cases, it was rote."

**Necessity of Focus on the 'Basics'**

Comments about the *basics or foundation* of math were ubiquitous, yet the true nature of what this basis is in the mind of each of the participants seems fluid. And building on a fluid base is difficult!

To Alanna, as she repeatedly mentioned, her idea of the 'basics of math' is best learned through rote practice. She said "I think you have to have rote, you do have to have rote. [Math] is like language, we teach our children to read by pointing at each word until they get it right, well for some reason in math you're not allowed." Bev commented, too, on the absence of rote in the Ontario curriculum, saying her students were slow with multiplication tables "because [in Canada] you don't do the rote learning." Bev, in the process of explaining the solution to a word problem involving subtraction said "it's going back to the basics with addition and subtraction, getting the students to see the word problems as basic computation" yet, she moves beyond the
"basics" by then asking her students "how did you get the answer?" to find out "what is their thinking, so, that's what math teaching in the current times necessitates." Unlike Bev, Alanna does not express an interest in methods beyond rote for the primary grades.

Emily is in total disagreement with Alanna; she explained that teachers must "share that knowledge, so not me saying 'well this is how you do it' because that rote teaching won't help anybody. It's not going to help the teacher candidates and it won't help their students . . ." She affirmed the positive value of coming up with multiple solutions on developing a deep understanding of mathematical processes. Paul did not include rote computation in his understanding of the 'basics' rather he cited examples, such as estimating distances, which developed students' understanding of how numbers and math concepts exist in the real world.

Chloe's responses were congruent with the opinions of Emily and Peter, as she greatly valued giving her students opportunities to explore.

**Constraints and Challenges**

When asked about the constraints on their success, none of the participants immediately focused on their personal content knowledge. Instead, they commented on lack of knowledge about good teaching methods and a host of administrative and practical issues that seemed to be beyond their personal control.

For Chloe, the greatest constraints relate to not always knowing the "best" technique to introduce a problem and the grade three EQAO assessments. Commenting on the lack of methodological support, Chloe said "sometimes I think we [teachers] need a little bit more, we need some direction, they're telling us to move away from textbooks, but they're not necessarily teaching teachers how to move away from textbook." Chloe also feels pressure to cram teaching
in before the EQAO test in May, which conflicts with her usual approach which is centered around problem solving: "it's not anymore all about problem solving . . . it's like getting through the material . . . I don't think it's an effective way to learn . . . it's two different worlds . . . and there's no time necessarily to explore." When asked about the value of the test, Chloe said "I do think there are pockets of learning in there, but I do think a lot of it is producing for the test. And they say that you're not supposed to do that, but if you don't teach kids how to answer that question how are they going to be ready?"

Another problem that Chloe sees is the school's math program is the teacher's struggle with the content "[teachers] don't know what the content is, they're so used to having a textbook that they become dependent, and that's how I started." Chloe sees that she has overcome this constraint of content knowledge by taking courses and seeking advice from colleagues.

For Alanna, "the textbook program" is the biggest constraint. The organization of the program, in her opinion, causes confusion for the students “I feel so sorry for my children because we get a month of this strand, and then a month of that strand and we jump, and we jump, and we jump, and we have to keep moving, and moving, and moving, and they aren't consolidating the information." To Alanna, the short time in which she has to cover all the strands in the math curriculum is a big problem for student learning. She also lamented that there is no standardized workbook for the primary grades, she said "we spend half our time researching, finding books, photocopying them, stapling them, and it's all so random." To Alanna, the background research should be standardized and mandated, and should not fall to the teacher.

Bev also felt her biggest constraint in the math class was a lack of resources. She, unlike Alanna, asserted that there were always many books around, but said "there's never ever a full set
of resources [manipulatives] in the classroom. Unfortunately, she found that many teachable moments passed by because the appropriate manipulatives were not at hand, "if you can have well-stocked resources in the classroom, that allows children to explore on their own, and caters to teachers . . . you know when something happens and you have to you have the right resources to help the children at that point understand the concept." Like Chloe, Bev also commented on other teachers who might not have the required math content knowledge "when your teaching yourself at the same time as your teaching the kids is very difficult."

Paul, no longer teaching full time in a classroom, thought back to his prior experiences. He assumed that some teachers might say they were constrained by large classes; however to that he responded "I had large classes, that's not a constraint in math. . . . You're dealing with more, but more is sometimes better." Instead, Paul said skipped classes, due to school interruptions or student absences, were a real constraint. He saw parents pulling students out of school on a regular basis to be a big problem because "if you miss many building blocks than eventually you are going to find a huge hole."

Math Anxiety

Briefly mentioned by several participants was the idea of ‘math anxiety.’ Each participant was familiar with this term, and construed the concept in somewhat different ways; some saw the anxiety as a socially constructed fear, whereas others understood it as due to a lack of confidence with the content. Emily noted "a lot of the teacher candidates not only don't have the math content or background, but also have high anxiety about math." To Alanna, the concept of 'math anxiety' is a contemporary phenomenon:

Nobody had math anxiety when I went to school . . . just love of math, absolute love of math and this whole fear of math with "math anxiety" like
who gives that to them? I don't know. Think about it . . . "bodyweight" you know, who gives them the "bodyweight issue" anxiety? It's not really different - it's all fear and anxiety. I don't know, yeah.

Alanna went on to suggest that math anxiety emerged from the students’ deep awareness of their level or rank in the classroom. Chloe notes "they're already lost and they're in grade 3 and I'm like how did this happen?" Yet, unlike Alanna, who had high success as a math student, Chloe relates "I could see myself in them and this is my chance now to make sure that doesn't happen.

The Positive Value of Teacher 'Struggle' with Content

Integrated with the participants ideas about their own content knowledge was their understanding of the nature of 'struggle' with math. Some participants had recognized their own personal struggle and all noted their students' 'struggle' with math.

To Paul, it is beneficial for teachers to struggle with math "as long as they did something about that personal struggle." When it comes to students, Paul said "unfortunately I don't think there are too many people out there who really have that expertise" to know how to teach students who have been presented a math concept in many different ways, yet are still struggling. He points to a gap in experience of teachers with a high content knowledge: "because typically when you have higher level math courses offered [to teachers], they teach [teachers] who understand math already, which is really not helpful to kids who are struggling with math at all because some of those people probably never had struggles with math." Paul sees the benefit of a teacher's ability to relate to their students through a struggle with math.

Emily, too, points to the positive value of teacher's being able to empathize with their students:

The more you push the teacher candidates . . . to actually do the math, the anxiety starts to drop because now they understand the struggle that the kids
go through and they’re also learning the strategies themselves and their content knowledge starts to increase and then their confidence in teaching math increases . . . you know that struggle is really important for them to go through.

To Emily, struggle is not an inherently negative process which can lead to enhanced confidence.

Bev and Chloe are teachers who see their younger selves in their struggling students. Bev said "I'm more sympathetic to those who are struggling." Chloe revealed "I didn't do very well in math, I didn't understand math and I struggled with math all the way through . . . I don't want my students to have that, so my job is to teach them the fundamentals so that they can succeed." To Bev and Chloe, their own struggle motivates them to present math in many ways to reach all of their students.

Alanna did not feel that she has had any personal struggle with math, but does highlight that not having experienced difficulty with something, often makes it difficult to teach those who do experience difficulty.

**Professional Development**

When it came to professional development and initial teacher education, each participant focused on a particular issue. There were varied opinions of the value of PD, the financial cost and the availability of appropriate and helpful resources.

Paul lamented that math questions are often limited to the more easily navigable numbers, pointing out that text books rarely deal with fractions of sevenths or ninths; he expressed frustration that most teachers do not know why this is so, and do not seek to learn. To Paul, the onus of Professional Development is mostly placed on the teacher, and it seems that many teachers are not committed to PD in math. Paul explained that he thinks there should be a
content test for math teachers, "Why would I want to put someone into a mathematics classroom when I know for a fact that they don't understand mathematics? I think we need to have a better system in place." Speaking to his own teacher education, Paul explains "I did not gain any content knowledge in my teacher training, I will say that with 100% certainty." His ITE was "more theoretical" and helped him develop "methods." For Paul, PD in math is a priority, he attends conferences, does a lot of professional reading is part of the Association of Curriculum Development. He also commented "I like to deliver PD sessions . . . I always learn more than I deliver in a lesson from my audience so it's actually the questions they come up with that really make you think." Paul perceives that he has participated in extremely beneficial PD, and hopes that that could be the norm for all teachers.

Alanna has taken several Additional Qualification courses, but none in math. When I brought up AQ courses, Alanna provided a "little complaint" that "AQ courses cost roughly $1000, how does that help the teaching community by charging them to improve themselves? That is absolutely stupid, they should be free!" When discussing her initial education program, Alanna noted that it was "very rarely content, it was more pedagogical concepts." She expands "it was all theoretical, it was all philosophy . . . we didn't learn . . . any classroom management skills . . . we never did lesson plans, so what we would do is, you'd choose a subject of interest and you'd write a paper on it." Alanna celebrated the current model which integrates practical strategies into ITE programs. Unlike Paul, Alanna did not expand on any positive professional development that she had participated in, math or otherwise. Bev offered her perspective on ITE as well, agreeing with Paul:

I would think that content should form part a necessary part of [teacher candidates'] testing because not everyone has that content level. I'm speaking from sheer ignorance, but depending on what your major is in your
first degree, you may or may not have taken extensive math courses, right, so if you're going to teach at the [Junior/Intermediate] level then that should be taken into consideration.

When it comes to finding appropriate resources for content knowledge, Bev found some difficulty: "you need the content and then the methodology is always adaptable, you look at how the children are learning in the classrooms and you can always find strategies, but finding content is not always easy, but it should be." Bev asserted the importance of content knowledge, yet did not have explicit suggestions of where to find this knowledge.

**Summary of Findings**

The five interviews I conducted with math educators provided me with an extensive data set through which to analyze the posed research questions. In the above section, I categorized the data into significant themes, including participants' content knowledge and educational backgrounds, their views of the nature of math and the nature of teaching, and their ideas of positives teaching strategies in their math classrooms. The participants' comments provide a new lens through which to observe the existing research cited in the literature review. Significantly, there is a clear divide between educators with a high confidence in their math content knowledge (Paul, Alanna and Emily) and a low confidence in their math content knowledge (Bev and Chloe). However, there is no consistent behavior of those at either end of the spectrum; for example, engagement in mathematical professional development was not correlated to perceived level of knowledge. This difference in disposition toward professional development seems to emerge out of the participants' mindsets and views of the role of the teacher, rather than their confidence in their content knowledge.
Chapter 5: DISCUSSION

I entered into this research project with a perception of my own limited math content knowledge. I had particular understandings of how this perception of my limited knowledge could influence my teaching. Now, with the ability to see current educators’ perspectives alongside the existing research on PCK, self-efficacy and mindset, I have come to new understandings of the position of content knowledge in contemporary math education. In this section, the project’s research questions are explored through interpretation of the data provided by the participants. I then discuss the potential implications of this study on educational policy, practitioners and Initial Teacher Education. Further, I discuss the limits of this particular study. Finally, I discuss the rich possibilities for future study on the nature of teacher content knowledge in elementary math.

Research Questions

The project’s research questions were:

1. In what ways do teachers conceive of personal success in their math classroom?

2. Do teachers perceive a relationship between their math content knowledge and their ability to teach math?

3. In what ways do teachers' understandings of the nature of math relate to their understandings of the nature of teaching?

Having the privilege of researching in a singular context, I was able to see that even within this small community of primary educators, there is vast difference in teacher self-conception in relation to successful practice in the math classroom. These questions prompted an expanse of varied responses. Interestingly, through the process of compiling and coding the data, it became clear that participants felt most efficacious in their classrooms when they felt a true sense of agency, provided to them through institutional structures, materials and knowledge.
(1) Teacher Perceptions of Personal Success and Struggle in the Primary Mathematics Class

As optimal teacher knowledge is debated in the research on successful math teaching, educators, too, wrestle with what sorts of understandings of pedagogy and content best support their success in teaching. This negotiation of what kind of teacher knowledge to focus on in teacher development is central to the creation of the field of study of PCK (Schulman, 1967), from which follows Ball et al.’s "mathematical knowledge for teaching" (2005). The study participants, save Emily, did not have the inclination to specifically articulate a special type of knowledge that synthesizes pedagogy and content.

However, participants’ stories of success seemed to highlight PCK in action. For example, Paul recounted his experience counseling a student from the Middle East on different mathematical dialects; here, Paul used his knowledge of math to structure his understanding of how to best teach that student math. It seems as if Chloe and Bev worked toward a synthesis of their convictions about good teaching with their understandings of math through engaging in relevant math professional development; though without the terminology, they convey a sense of subject-informed teaching practices. Alanna's PCK seemed to be the most absolute, but in a completely different way than Schulman would have intended: her understanding of math as a fact-based, rote subject informs her pedagogical technique of teaching math in a rote way. Alanna had a full synthesis of content and pedagogy - yet it is one that is not valued by any of the literature advocating for teachers to have a profound understanding of math (Ma, 2010; Wu, 2011).

The lack of consistency in teacher opinion on the value of rote learning, the value of methodological training and the value of conceptual and procedural math content knowledge
underlie the lack of consistency in attitudes about teacher knowledge. With such divergent views on a successful ways to implement the primary math curriculum, even from such a small sample, we see a fundamental rift at the foundation of math education. It is necessary to further examine the influence of this rift.

Participants' perceptions of the constraints on their success were as varied as their perceptions of their successes. For some, the limiting factors in their math classes were systems that were beyond their control, such as time, curriculum, the textbook, standardized tests and lack of resources. These conceptions are all fixed within the larger educational institution and seem out of an individual teachers' immediate control. Alanna did not engage in personal development to improve her math teaching, instead, she focused on changes that the administration, led by Paul, could implement. Alanna's great academic success with rote math has perhaps fostered in her what researchers labeled a false sense of self-efficacy, that accompanies simplified tasks (Suh et al 2011). Paul noted that he sometimes engaged in administrative structures, such as timetabling, to allow for a better success in his math class. The participants who identified areas of limitation arising from their own lack of math content knowledge or former lack of math content knowledge took steps to improve themselves. These educators, including Paul, Bev and Chloe seemed to have a growth mindset (Dweck, 2007), that allows them to expand on their limitation and improve their efficacy through structures that are accessible to them.

Through self-conceptualizations of success and struggle, and through identifying the factors that lead to each, participants revealed their particular mindsets toward learning and education and learning as a whole. Alanna, who through the data I collected, appears to have a fixed mindset with regard to her own math knowledge, seemed to project this understanding onto
her students. She asserted that some students will just not be good at math. Teacher perceptions
of their own successes and struggles reveal their particular mindset toward growth and learning.
Significantly, most participants’ accounts of success in the math classroom draw on teacher
knowledge and resources compatible with Schulman's conceptualization of PCK.

(2) Understandings of Content Knowledge in Relation to Teaching

Focusing more specifically on participants' discussions of their content knowledge, rather
than their general successes and constraints, I aim to demonstrate the particular situation of
content in the myriad systems that influence teacher success. Participants did not immediately
highlight their content knowledge as the most apparent influence on their ability to teach math.
Participants saw a host of other factors as significantly more important than content on their
students' success in the primary grades, where there is a perception that the math is
uncomplicated. For example, Bev saw methodologies for teaching as the most influential in the
lower grades and took steps to improve her knowledge of pedagogical technique. When probed,
participants did acknowledge a link between their content knowledge and ability to teach.

Paul and Emily, who had the most significant mathematical content training of all
participants, held views congruent with those of Ma (2010), Wu (2011), Delvin (2011) on the
necessity of teachers to have a deep understanding of basic math and mathematical thinking.
However, because they themselves are confident in their mathematical knowledge for teaching, it
was not a personal constraint. Participants, including Emily, Paul, Bev and Chloe more readily
acknowledged lack of content knowledge as creating difficulty for teachers other than
themselves. They felt new teachers who do not have significant math content training would face
significant challenges. There was a general feeling that activities for teaching math were easier to
find than sources on content knowledge. This aligns with Initial Teacher Education programs in Ontario whose most significant focus is on teaching pedagogy and methodology, rather than content.

Those participants who elaborated on their personal struggle with school math, described a feeling of self-efficacy in math that came along with their improved understanding of the subject. Emily described this same process in her teacher candidates who "do the math." Here, struggle is constructed as a process of engagement through which students and teachers develop a deeper conceptual understanding of math. In math, it seems that anxiety can preclude one from seeing struggle as a positive process. Bandura (1977) explains "[p]eople fear and tend to avoid threatening situations they believe exceed their coping skills, whereas they get involved in activities and behave assuredly when they judge themselves capable of handling situations that would otherwise be intimidating " (pg. 141). No teachers labelled an active math anxiety within themselves, despite some feelings of low content knowledge, as they all had developed appropriate coping skills in relation to math.

(3) The Conflicting Natures of Mathematics and Teaching

The most significant finding, in relation to the development of research on PCK, is a fundamental conflict between some teachers’ views of math and their views of good teaching. Teachers' understandings of the potential of math limit what types of activities they are able to implement in their math classrooms. On one hand, some participants view primary math as concrete, simple, factual and basic; on the other hand, they view good teaching as exploratory, critical, collaborative and constructivist. The dissonance between these conceptualisations of math and teaching creates a practical difficulty for teachers who attempt to create exploratory
activities for a subject whose nature they believe to be rigid. This dissonance is elevated by the EQAO test, with its rigid focus on prescriptive answers; Chloe commented that her preferred style of exploratory learning and how she feels she needs to prepare her students' for the EQAO test seem to be "two different worlds." Even participants with significant PD in math methodology still felt an insecurity in their understanding of the content and their ability to pick optimal strategies for teaching. These primary teachers do not have those profound understandings of math identified by Ma and Kessle (2001).

The Ontario Ministry of Education policy, through curriculum documents, and Initial Teacher Education programs have set up system that encourages teachers to move away from rote tasks and textbooks, as noted by all participants. Yet, there is no mandatory content training for teachers that allows them to develop the mathematical understanding required to develop exploratory tasks, which require a fluency in mathematical thinking. The one participant, Alanna, who strongly advocated for the use of a workbook in math to standardise the system, advocated for such because she felt that some teachers cannot be trusted to find good resources to plan engaging lessons. However, Alanna herself feels satisfied with her level of content knowledge, and her understanding of the nature of math. There is no structure within this system that motivates her to change; Alanna scoffs at the continual 'innovation' in education theory and has firmly set beliefs.

The willingness for professional development and growth is completely in the hands of the teacher. In a system that has so many external pressures, it is somewhat surprising that this concept of the role of a teacher is left open to interpretation; there is little accountability for a teacher's daily work. When discussing the role of the teacher through metaphor, all participants except for Alanna highlighted some element of reciprocity and desire to accumulate more
information. The fundamental concept of the role of the teacher is significantly different in the minds of each of the participants. Is the teacher a puppet of the education system who implements content and curriculum from a pre-set standardised system or is a teacher someone who seeks out information and synthesises it in new ways to suit their students' learning? Is a teacher's job to innovate or to transmit? The inconsistencies between the participants on these seemingly simple questions point to a larger inconsistency in the education system surrounding the true ability of teachers to influence the system in which they operate.

As elementary teachers in Ontario are most likely to be from an Arts or Social Sciences background (OCT, 2008; 2009; 2010; 2011), the gap between teacher educational background and classroom subject material comes to the fore in the math classroom. The inconsistency between teacher views of teaching and math sheds light on the systemic issue of the uncertain agency of the teacher. Some participants choose to exert their own agency in relation to their lack of content knowledge through personal learning, while others choose to relinquish agency to the education system and simply implement a textbook program.

**Personal Implications**

This research project has significant implications on my understandings of teaching and the values of content knowledge. Personally, as a beginning teacher, I see it as important to foster a growth mindset within myself. I recognize my own limited math content knowledge and will work to develop a deep conceptual understanding of the basics of math through working, and even 'struggling' through practice problems in books such as Wu's *Understanding Numbers in Elementary Mathematics* (2011). Beyond me, this project has implications for practitioners of math education, education policy and for Initial Teacher Education.
Implications for Practitioners

The primary recommendation of this study for math educators is to encourage self-reflection and an assessment of their understandings of the nature of math and teaching. In the quick pace of the school year, educators can slip into the automatic process of information transmission, without realizing their own agency over the situation. Although some educators might continue to feel anxiety toward math, the growth through struggle demonstrated by several of the participants reveals a potential model for self-efficacy in math though improved understanding. Additionally, a positive value could emerge from the development of Professional Learning Communities in schools that discuss the more abstract nature of math and strengthen teachers shared understanding of the subject.

Implications for Policy

Even this pilot-scale study reveals a rift in understanding about the role of a teacher in the larger system. Although there is mandated professional development and a promotion of conceptual math instruction, there is an obvious gap between the idealized policy and the practice. Because this problem seems to emerge from confusion about the agency of teachers, it is difficult to suggest that a solution could come through creation of a policy that would further remove the agency of teachers, or further structure their roles. This project reveals a need for a greater attention to be paid toward the role of the teacher as the central agent in implementing the curriculum.
Implications for Teacher Education

The literature and interview data suggest the positive value and self-efficacy that comes along with having a solid understanding of content knowledge. The preliminary findings suggest that content knowledge has a rich potential to improve teaching, when synthesized with pedagogical technique. Teacher education programs should develop ways to help students self-assess the limits of their own content knowledge, and develop strategies through which to help them boost this knowledge. If we are able to eventually build truly positive, safe and accepting classrooms it behooves teachers to have the content to teach in this environment with all the initial conditions for learning.

Study Limitations

As this study was limited to a small number of participants, the findings may not be generalizable to the larger population. There may be aspects of this study that readers may find useful in contextualizing their own situations. The variance in participants’ views and experiences, over a small sample was very large; whether such variance would be true of a much larger sample or the population of teachers as whole, would be worth investigating. A more significant limitation of this data is that the self-concept of teachers and their teaching is filtered solely through their own eyes. Due to the ethics protocol, I was unable to collect survey or test data on teacher content knowledge. Significant to this study would also be an inclusion of the students’ perspectives on their teachers' attitudes and understandings toward math.
Future Studies

The findings provide a substantial contribution to the continuation of earlier research on PCK. To extend this study, it would be important to gather data on teachers’ math content knowledge. Examining perceived teacher knowledge alongside testable knowledge could help us to better understand the presence of math anxiety and self-efficacy in teachers. Additionally, interviews with students which focus on their perceptions of teacher self-efficacy would help to demonstrate the true value of teacher self-efficacy, related to content knowledge on students. A larger scale study could also provide insight into how social factors, such as gender, age, ethnicity and class may play into the development of teacher self-efficacy in the math classroom.
REFERENCES


### Appendix A: Interview Questions

<table>
<thead>
<tr>
<th>Conceptual Component</th>
<th>Interview Questions</th>
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<tbody>
<tr>
<td><strong>Introductory Questions</strong></td>
<td>I’d like to talk with you about your ideas about math content knowledge, and your perspective on its importance to teachers such as yourself. We'll focus on your background and interest in math and your feelings toward teaching the subject. I’d like to start off with a few questions to find out your specific teaching context.</td>
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<td></td>
<td>• Do you consider yourself a teacher of math? For how long?</td>
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<td></td>
<td>• In what grades have you taught math?</td>
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<td></td>
<td>• Briefly describe your classroom context (types of students, neighbourhood…)</td>
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<tr>
<td><strong>General Questions</strong></td>
<td>• Can you think back to a recent positive experience in the classroom? Follow up: What made it so special?</td>
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<td></td>
<td>• Is there any subject that really stands out to you as your favorite to teach? least favorite?</td>
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<td></td>
<td>o What makes planning and instruction during those classes distinct for you?</td>
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<td></td>
<td>o Do you have an educational background in it?</td>
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<tr>
<td></td>
<td>• Thinking back to your teacher education, do you feel the classes focused on content knowledge or instructional strategy? or both?</td>
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<tr>
<td><strong>Pedagogical Content Knowledge</strong></td>
<td>• How do you feel about math? Teaching math?</td>
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<td></td>
<td>• Have you had any post-secondary math training? A Qualification courses? A</td>
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<tr>
<td></td>
<td>• Any math-related PD sessions? A</td>
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<td></td>
<td>o Briefly, can you explain something you gleaned from one of those courses that is relevant to your teaching today?</td>
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<td></td>
<td>• On a scale of 1-10, 1 being early high-school math and 10 being a math expert what would you rate yourself? why? What would you need to boost it (3 --&gt; 5)?</td>
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<td></td>
<td>• Was there anything in your teacher training that really supports your current practice of teaching math?</td>
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<td></td>
<td>• How do you go about planning a math lesson? (if not described above)</td>
</tr>
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<td></td>
<td>o To what extent do you rely on commercial resources, textbooks and teacher guides? what type? (or self-developed tasks)</td>
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</table>
Have you ever come across something that you do not fully understand in the math curriculum, what do you do?

- Have you ever been stumped in the classroom? (ex. why do two negative numbers equal a positive, dividing fractions)
- If you bumped into a former student of yours, now an adult, what do you hope they still remember from your grade x math class?
- What do you think are the biggest constraints on your success as a math teacher?
- Speaking metaphorically, is the role of the math teacher most similar to a pipe, a faucet, a sink - or something else entirely? Can you explain your choice?
- Finally, maybe as a result of these questions, to what extent do you think your own background in math has had an impact on your teaching in this subject area?
Appendix B: Letter of Consent for Interview

Date: _______________

Dear _______________,

I am a graduate student at OISE, University of Toronto, and am currently enrolled as a Master of Teaching candidate. I am studying the content knowledge of math teachers for the purposes of investigating an educational topic as a major assignment for our program. I think that your knowledge and experience will provide insights into this topic.

I am writing a report on this study as a requirement of the Master of Teaching Program. My course instructor who is providing support for the process this year is Arlo Kempf. My research supervisor is Cathy Marks Krpan. The purpose of this requirement is to allow us to become familiar with a variety of ways to do research. My data collection consists of a 45 minute interview that will be audio-recorded. I would be grateful if you would allow me to interview you at a place and time convenient to you. I can conduct the interview at your office or workplace, in a public place, or anywhere else that you might prefer.

The contents of this interview will be used for my assignment, which will include a final paper, as well as informal presentations to my classmates and/or potentially at a conference or publication. I will not use your name or anything else that might identify you in my written work, oral presentations, or publications. This information remains confidential. The only people who will have access to my assignment work will be my research supervisor and my course instructor. You are free to change your mind at any time, and to withdraw even after you have consented to participate. You may decline to answer any specific questions. I will destroy the audio-recording after the paper has been presented and/or published which may take up to five years after the data has been collected. There are no known risks or benefits to you for assisting in the project.

Please sign the attached form, if you agree to be interviewed. The second copy is for your records. Thank you very much for your help.

Yours sincerely,

Researcher: Kathleen Morris

Supervisor: Cathy Marks Krpan

Course Instructor: Arlo Kempf
Consent Form

I acknowledge that the topic of this interview has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time without penalty.

I have read the letter provided to me by Kathleen Morris and agree to participate in an interview for the purposes described.

Signature: ________________________________

Name (printed): ________________________________

Date: ________________