Exploring the Impact of Changes in Ontario’s Senior Mathematics Curriculum on Student Success in STEM Programs

By

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Abstract

In 2007, the Ontario Ministry of Education revised the senior mathematics curriculum in ways that altered the scope and depth of the calculus component. The courses *Advanced Functions & Introductory Calculus* and *Geometry & Discrete Mathematics* were removed from the curriculum and replaced with two new courses: *Advanced Functions* (MHF4U) and *Calculus & Vectors* (MCV4U). The purpose of this research is to begin exploring the impact of these changes on student success in post-secondary STEM programs. Data was collected via semi-structured interviews with 5 participants: three secondary school mathematics teachers from the Greater Toronto Area, and two calculus professors from a major Ontario university. Findings suggest that educators believe that students experience major difficulties in university calculus as a result of decreased exposure to pure calculus; issues with basic numeracy and algebra, predating high school; inexperience with advanced concepts; missing concepts within the curriculum; and shifts in cultural and systemic perspectives. Both secondary and post-secondary educators in this study suggest that students who enter STEM programs now encounter greater difficulty with first year calculus courses relative to previous students. The implications of the concerns brought forth by the educators interviewed are wide-ranging, suggesting that Ontario students are potentially entering university unprepared for the rigor of the mathematics that they will encounter.

*Keywords*: Calculus; Curriculum; Ontario; Student Success; University; Mathematics
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Chapter 1: INTRODUCTION

Background

Public education is an identifying feature of standard Canadian life, and as a nation, we have long prided ourselves on providing equal, accessible, and quality opportunities to our students to be competitive on a global scale. With regards to education, Canada is unique in that each province adheres to its own curriculum. As a result of varied provincial education curricula, Canadian students are entering higher education with differing skill sets and competencies in specific subjects.

Ontario, Canada’s most densely populated province, reviews curriculum standards on a cyclical basis and have recently made some dramatic changes to the high school math curriculum (Ministry of Education, 2007). Specifically, the up-to-date curriculum eliminated two math courses – Advanced Functions and Introductory Calculus and Geometry and Discrete Mathematics – to create a combined course called Calculus and Vectors (MCV4U) and a new course called Advanced Functions (MHF4U). This controversial move was made as a result of many political, academic, and social pressures placed on the Ministry of Education to re-examine the math curriculum (Ministry of Education, 2006). The resulting course (MCV4U), however, has come at a cost where students are spending less time learning fundamentals of calculus and are deprived of critical components of calculus and discrete mathematics foundations that were previously learned by students under the former curriculum.
Purpose of the Study

There has been significant research carried out in the United States on the importance of calculus in high school curriculum and its effect on student success in higher-level education (Trusty & Niles, 2003; Tyson, 2011; Sadler & Tai, 2007; Ferrini-Mundy & Gaudard, 1992; Spresser, 1981; Burton, 1989). However, the Canadian literature is severely lacking in this respect. In fact, there has been little research conducted following the curriculum reform in Ontario of the mathematics curriculum examining whether it has proved successful or not. In my own experience, I have witnessed my peers struggle in university-level calculus and physics courses after having taken the MCV4U course now offered in Ontario high schools and have consistently asked myself whether there was a common flaw in the high school math education that these students were receiving. Thus, the purpose of this study is to explore the changes made in the Ontario mathematics curriculum and determine whether the 2007 revision’s expectations are aligned with the university curriculum in this province.

Research Question

The guiding question of this study is: *in what ways has the current Ontario mathematics curriculum affected the transitional success of high school students entering university-level STEM\(^1\) programs?* Sub-questions that stem from this are:

- How has the newly reformed 2007 Ontario curriculum (MCV4U, MHF4U) affected student success in first year calculus courses and science/engineering programs from the perspectives of teachers and professors?

\(^1\) STEM: Science, Technology, Engineering, & Math
• Does the 2007 curriculum adequately prepare students for programs in science and engineering that utilize calculus?

• What difficulties do higher-level educators (i.e. University course instructors) encounter when teaching first year calculus courses and why?

• What are strategies and recommendations to remedy the declining mathematics performance in Ontario students entering STEM programs?

Background of Researcher

As a Master’s of Teaching (MT) student, I feel I have come to OISE with a very specific purpose. While I aspire to teach mathematics and science at the secondary level, I also have a keen interest in research in curriculum policy, administration, and changes, and the impact these branches have on student success. My goal is to develop the tools I need at OISE to bring meaning to curriculum standards when I am in the field and to eventually pursue a career in the academic research of curriculum reform. At the core of my belief system is that any student, anywhere, given ample opportunity, has the ability to understand the fundamentals of all subjects. Specifically, I believe that subjects that are considered to be difficult and esoteric can be taught in such a way that the basic tenets can be made useful and understandable. To expand upon this notion, it is my opinion that secondary students under the current Ontario mathematics curriculum are not being taught the full set of tools they need to excel in higher-level engineering, mathematics, and science programs.

Having gone through the Ontario school system from kindergarten to grade 12, and concurrently experiencing the university-bound stream and International
Baccalaureate (IB) program in my senior years, I had the opportunity to compare the pace, content and utility of these curricula. I completed IB mathematics and biology but chose to do my humanities and chemistry courses at the university-bound (4U) level. Being a part of the IB and 4U programs exposed me to a diverse peer group and as a result, I saw that when comparing IB mathematics and the Ontario curriculum-based courses, there was a large discrepancy in both the content and the manner in which it was being taught. The IB program focused on higher-order thinking and thus we often derived proofs for equations and theories. The 4U program was more knowledge-based and if proofs were shown, it was not expected that a student understood them (i.e. it was considered ancillary background knowledge). The most significant difference I saw was that we were taught integral calculus and applications of differentiation while these concepts are either glossed over or do not exist in the 2007 Ontario mathematics curriculum. This discrepancy was emphasized to me when I entered my first year of university and took the introductory calculus course required for all science and engineering students at the University of Ottawa. The fundamentals of integral and differential calculus were briefly introduced before higher-order thinking and difficult problem solving questions were given to us. The expectation seemed to be that we had all seen it before and thus we should be able to learn the basic material at a more rapid pace. I recall seeing my colleagues who had gone through the Ontario curriculum struggle, primarily because they had not encountered this concept before. Their struggles were reflected in the high failure rate of this first year calculus class, and this difficulty is something I encountered when speaking to colleagues from other universities as well.
I have been lucky enough to get the opportunities to learn mathematics and science in such a way that I developed an interest in both subjects and was able to excel. That is not to say that mathematics or science have always come easily to me. I had some negative experiences in my middle school years that made both subjects feel like something I would always have to work extremely hard at without any outside assistance. In high school, I had some excellent teachers for these subjects and I believe, as a direct result, was inspired to continue in further studies focusing on these subjects. I never stopped working hard to excel, but I was given resources and was challenged in my high school classes to get into the habit of questioning – a habit that many do not have the opportunity to learn. Through this, I have learned the importance of understanding the mechanics and purpose of theories, equations, etc. and do not take them for face value. These tools have brought me to where I am today and have provoked me to question the curriculum, why it is shaped the way it is, and the consequences of its framework.

Overview

This study is divided into five main chapters (I – V). In Chapter I, an introduction of the topic, the researcher, purpose and main research question are presented. Chapter II provides a comprehensive literature review on the existing research surrounding this topic. Chapter III outlines the methodology used to conduct this study. Results from the researcher’s fieldwork are presented in Chapter IV and finally, in Chapter V, the findings of this study are critically examined and reflected upon in the discussion.
Chapter 2: LITERATURE REVIEW

Introduction

The existing body of literature surrounding student success, mathematics, and student transition into university is wide-ranging, though largely focused on American studies. Student success will be defined, as this term is used frequently throughout this study. As this study focuses on curriculum alignment between the Ontario grade 12 mathematics curriculum and university-level mathematics and science, historical context for this particular mathematics curriculum is outlined in this literature review, including the transition from the Ontario Academic Credit (OAC) program to the present four-year secondary school format. Additionally, an overview of studies focusing on the impact of taking calculus on student success in engineering and applied science programs is presented.

Student Success

Student success is a broadly defined term that can encompass rigid, metric results, such as grades or pass/fail rates, and holistic, wide-ranging experiences that propel students forward. In Ontario’s “Building Pathways to Success” document (2003), a specific definition for student success is not given; however, the need to expand our societal expectations beyond grades and attendance at post-secondary institutions is recognized. A more expansive definition, guided by a report on student achievement by British Columbia’s Ministry of Education, includes an acknowledgment that the combination of “performance standards, level of satisfaction, competitiveness and growth” (Adamuti-Trache, Bluman, & Tiedje, 2013, p.2906) all contribute towards a
level of student success. Similarly, the Higher Education Quality Council of Ontario has most recently noted that an expanded definition of student success at post-secondary institutions that focuses not only on “academic achievement, retention, and engagement” (2011, p.1), but “student learning outcomes” during university and beyond (p.17) is becoming more prevalent and valued at the university level. While each of these definitions offer a specific framework to consider student success, Adamuti-Trache et al. (2013) offer a more generalized statement on student success as being, “the most successful students will be those who are best able to mobilize what they have learned in high school in order to succeed academically at university” (p.2907). Thus, though student success is most often defined through measurable data (Mullin, 2012), for the purposes of this study, student success will be framed in a context more akin to Adamuti-Trache et al.’s (2013) definition, with a particular focus on the transitional success of Ontario students entering university-level STEM programs.

**Education in Ontario: A Historical Overview**

*The Ontario Education System*

The Ontario secondary school curriculum has undergone many changes in recent years, with the most notable being its shift from a 5-year, Ontario Academic Credit (OAC) program to the standard 4-year high school diploma frequently seen across North America. The OAC program had been put in place to provide university-bound students with more advanced courses to prepare them for higher education (Brady & Allingham, 2010). In a decision influenced by fiscal realities and the pressure to conform to a program in line with the rest of the continent, the Ontario government compressed the 5-
year curriculum into a 4-year program by embedding OAC content into courses throughout grades 9 – 12 and gradually phased out OAC (Brady & Allingham, 2010; Jacek, 2003). The “phasing out” of OAC (as opposed to an abrupt cancellation) led to what has become known as the “double-cohort” year where OAC students and 4-year Ontario Secondary School Diploma (OSSD) students graduated together.

The double-cohort year acts as an interesting case study for education in Ontario. Students who graduated in the 2002-2003 school year (the final year of OAC) came from both the OAC program and the now-traditional OSSD program, resulting in double the number of university-bound graduates. In spite of the year-difference in age and educational experience, OSSD graduates were shown to have performed at the same level as OAC graduates during their final year of high school (Brady & Allingham, 2010) and in first year university courses (Slavin, 2008; Tremblay, Garg, & Levin, 2007). A study examining factors affecting the student dropout rate from a first-year physics course highlighted the “anomaly” that was the double-cohort year (Slavin, 2008). Slavin found that despite the 20-year trend of declining success rates in an introductory physics course, students from the double-cohort year had a significantly lower dropout rate as compared to any other year (2008). He speculated that students in the double cohort were more competitive with one another and while some felt that OAC students had an “unfair advantage” (Jacek, 2003), the OSSD students still succeeded. Consequently, the success of the first year of the OSSD graduates presumably gave the Ontario Ministry of Education confidence that eliminating OAC would not be hugely detrimental towards Ontario students.
Ontario Calculus: A Summary of the Revised Curriculum

The elimination of the OAC year from the Ontario secondary school program forced the Ministry to take content reserved for that thirteenth year of high school and embed it into grades 9 – 12, resulting in a large scale curriculum reform. As part of this reform, the purpose and relevance of many courses previously reserved for the OAC year were re-evaluated. A course that gained significant attention from the ministry, academics, and the public was Advanced Functions and Introductory Calculus (MCB4U).

MCB4U was created post-OAC to serve as the primary calculus course offered in Ontario high schools. The content-heavy course incorporated fundamental concepts in calculus (differential calculus and often an introduction to anti-differentiation) that are widely used in engineering, mathematics, applied sciences as well as business programs. However, under the new post-OAC curriculum, the Ministry found that student enrolment and success was declining and put the whole of the grades 11 and 12 math curricula under review. In 2006, the Ministry suggested that calculus be removed from the high school curriculum, a proposal that was met with much criticism (Toronto Star, 2006; Levin, 2008; Ministry of Education, 2006). In an effort to find a solution that could be implemented by the next academic year (2007 – 2008), the Ministry put together a “Task Force”: a group of “experts” picked by the Ministry to examine the entire curriculum, speak to potential stakeholders and compile a comprehensive report with recommendations for a course of action.

The Task Force was under intense pressure from the Ministry and interest groups and working under a very limited time span, so they “began looking very early on for
compromise solutions” (Levin, 2008, p.22). Calculus was viewed as economically useful by engineering schools, technology companies, as well as the public: “Supporters of calculus argue that if science is the doorway to the new economy, then calculus is an essential key to unlocking that door” (Toronto Star, 2006). With this in mind, the Task Force sought to find a way to keep some calculus in the curriculum. The other two math courses offered at the time were Mathematics of Data Management (MDM4U) and Geometry and Discrete Mathematics (MGA4U); the former taught fundamental concepts that led to an understanding of statistics while the latter had components that are used in linear algebra, engineering, and physics disciplines. Based on enrolment rates and anecdotal evidence, the Task Force found MGA4U to be of little use to high school students on the whole and recommended eliminating the course from the Ontario curriculum. In lieu of MGA4U, they suggested that a course entitled Calculus and Vectors (MCV4U) be created that incorporated basic concepts in calculus (excluding integral calculus) and the vectors component from MGA4U (Ministry of Education, 2006). Additionally, they recommended that MCB4U become Advanced Functions (MHF4U) in response to feedback from mathematics professors stating that students lacked necessary skills to solve trigonometric problems (previously, this topic was covered in the grade 11 Functions course). These recommendations were implemented by the 2007-2008 academic year and remain in practice to this day.

In spite of these changes, however, university professors in mathematics, engineering, and sciences have noticed a marked decline in their students’ abilities to grasp basic and complex concepts in introductory courses (Slavin, 2007; Kershaw, 2010). While some professors have noted that there is an increased number of remedial math
courses being offered at universities across Canada (Kershaw, 2010), some believe that this is a “Made in Ontario” phenomenon (Slavin, 2007). Regardless, across the board there appears to be a general consensus that students are entering university with a superficial understanding of fundamental mathematics that is largely focused on memorization, and as a result, students are less interested in programs that require even a small amount of mathematics knowledge.

*Comparison of Curriculum Documents (2000 & 2007)*

A close reading of the Ministry of Ontario senior mathematics curriculum documents for 2000 and 2007 illustrates the major similarities and differences between the calculus-based courses. Because MCV4U incorporates calculus and vectors, readings of MCB4U and MGA4U were done to more thoroughly compare the course descriptions and expectations.

A more obvious difference is clear when comparing the 2007 document to the topics in MGA4U (the course removed from the curriculum). For example, in MGA4U, an entire strand is dedicated to the mathematical concept of a “proof” and problem solving (Ministry of Education, 2000); the entire 2007 document only contains the word “prove” once, and it is within the context of proving simple trigonometric identities in the grade 11 functions course and in MHF4U. Notably, though many topics are similar (the concept of a derivative, limits, etc.), the descriptive language used differs. When a topic such as the chain rule is identified in MCV4U, expectations dictate that “simple” problems are used (Ministry of Education, 2007). By comparison to MCB4U, the word “simple” is not used to describe expectations for the chain rule, or indeed, any of the
basic differentiation rules. Rather, “simple” examples are expected for topics that are
briefly introduced such as implicit differentiation (NOTE: this topic also does not exist in
the 2007 course). Additionally, though the vectors portion of MGA4U is used in
MCV4U, topics such as row reduction were not carried forward with the change, and thus
are not in the 2007-revised curriculum. Furthermore, significantly more expectations
within MCB4U are dedicated to curve sketching of derivatives and second derivatives,
relative to MCV4U (Ministry of Education, 2000; 2007). A comparison of the course
descriptions as well, demonstrates a difference in expectation. In MCB4U, students are
expected to “develop facility with the concepts and skills of differential calculus”
(Ministry of Education, 2000, p.38) as they apply to transcendental functions; the course
description for MCV4U asks that students “broaden their understanding of rates of
change to include the derivatives of polynomial, sinusoidal, exponential, rational, and
radical functions” (Ministry of Education, 2007, p.99). This reading of the two
curriculum documents demonstrates explicit and marked changes in content knowledge
and overall expectations.

Transitional Success from High School to University

High school mathematics forms the foundation on which first year science,
mathematics, and engineering courses are based. Accordingly, university professors have
long advocated for a continued emphasis on high school mathematics for university-bound students. A strong mathematics background in high school has been shown to be an indicator of success in various university programs, including the humanities, business, sciences, and engineering.
Numerous studies have been conducted that examine the effect of taking calculus and other advanced mathematics high school courses on degree attainment and course-specific success. When comparing different strands of high school mathematics in the United States (Algebra II, Trigonometry, Pre-Calculus, Calculus), researchers found that there was a “cumulative benefit” (Trusty & Niles, 2003, p.106) to taking more mathematics courses prior to entering engineering programs in university. Interestingly, even when considering background variables such as race and socioeconomic class, taking “intensive math courses had the strongest effects of all curricular areas on degree completion” (p.103) and having high school mathematics experience more than doubled the chances of completing a bachelor’s degree.

There is also evidence that taking high school calculus plays a factor in how successful students may be in university-level calculus, engineering, and applied sciences courses. A study conducted at a British Columbian university found that students with calculus experience performed significantly better than students without this background, even when controlling for factors such as mathematical ability and gender (Fayowski, Hyndman, & MacMillan, 2009). Interestingly, this study took place after the removal of an introductory calculus chapter from the grade 12 mathematics course in British Columbia, and thus students who did have background calculus knowledge received it from either locally developed or advanced placement calculus courses. At present, this is one of very few studies conducted in Canada in recent years on the impact of calculus experience on post-secondary calculus achievement.

There has been some evidence that calculus experience has a positive effect on achievement in mathematics-based courses such as physics, in STEM programs. In a
study comparing the effects of taking high school calculus and physics on achievement in introductory calculus and physics classes, researchers found that calculus appeared to be the best predictor of success in either course (Tyson, 2011). The author also examined migrational effects (i.e. switching out of engineering into different programs) and found that “students who did not reach pre-calculus in high school were most likely to switch” (Tyson, 2011, p.770). Additionally, a Canadian study found taking the provincial grade 12 mathematics and physics courses in high school to be a positive indicator for high achievement in first year physics at a British Columbian university (Adamuti-Trache et al., 2013). Not surprisingly, researchers have also noted that when comparing the effects of taking various science courses in high school (biology, chemistry, physics) and calculus, the only cross-disciplinary predictor of success in university science programs was mathematics (Sadler & Tai, 2007). Thus, the literature suggests that prior mathematics experience is strongly related to success in the sciences.

It has also been found that the actual quantity of high school mathematics taken (number of courses as well as time spent in the course) plays a role in the transitional success of university students. Ferrini-Mundy and Gaudard (1992) found that having a year’s worth of calculus experience versus a brief introduction to the discipline had a substantial effect on student success in Calculus I and Calculus II courses in university. Similarly, increased experience with calculus appears to lead to increased success in chemistry, calculus, and computers engineering courses (Spresser, 1981). Though these studies provided quantitative and qualitative evidence of a correlation between calculus experience and transitional success in university, none of them examined whether
specific elements of the high school calculus curriculum had a particular effect on student achievement.

Nevertheless, researchers have also continuously found that university professors are seeing decreased basic conceptual and analytical skills in entering students. One researcher noted that, “the lack of the year of high school calculus can seriously handicap the first-year university student” (Burton, 1989, p.351), because it was one of few courses that focused on building a strong foundation of analytical skills. Indeed, anecdotal evidence from calculus instructors in universities suggests that students enter their classes with a “false confidence” (Ferrini-Mundy & Gaudard, 1992, p.68) as a result of previous courses emphasizing procedural methods rather than a conceptual understanding and applications of mathematics.

Conclusion

As evidenced by the existing literature, it is clear that calculus plays an important role in the transitional success of university-bound students. Though most of the literature comes from American sources, the university calculus curriculum in the United States is similar to that of in Canadian universities. However, much research is still needed to understand the effects of specific units in the calculus curriculum on student success in university, and the 2007 mathematics curriculum reform needs to be re-examined with a critical eye. The proceeding study aims to provide a comprehensive analysis through primary research and an extensive discussion of this curriculum and its effect on the transitional success of university-bound Ontarians entering mathematics, the sciences, and engineering programs.
Chapter 3: METHODOLOGY

Overview

To begin exploring the ways in which the current Ontario senior mathematics curriculum has affected the transitional success of students entering post-secondary STEM programs, I first examined the existing literature surrounding the topics of calculus, student success, and the historical context of Ontario’s school system and curriculum reviews. Following this, I conducted five semi-structured interviews with three high school mathematics teachers based in the Greater Toronto Area, and two calculus professors from a large Ontario university. These interviews were transcribed and analyzed for common themes within each grouping (teachers; professors), and across the educators.

Participants

To gain insight on the transitional success of students, it was integral to get educators’ perspectives from the secondary and post-secondary level. Teacher participants were selected based on their experience teaching the current Calculus and Vectors course (MCV4U), while professor participants were selected based on their experience teaching first year calculus to Ontario students since the 2007 curriculum revisions. Each of the participants previously taught the interviewer or her colleagues either at the secondary or post-secondary level. All names have been replaced with pseudonyms.
Teachers

1. Lucy is a retired secondary mathematics teacher with 33 years of experience. Lucy has a Bachelors degree in music with a minor in mathematics, mathematics specialist qualifications, K – 13 qualifications, and has taught calculus under the OAC curriculum, the pre-2007 *Advanced Functions and Introductory Calculus* course, and the current *Advanced Functions* and *Calculus and Vectors* courses.

2. Eric is a current secondary mathematics teacher with 13 years of teaching experience, 10 of which have included the International Baccalaureate (IB) calculus course, post-OAC calculus, and the current *Advanced Functions* and *Calculus and Vectors* courses. Eric holds a Bachelor’s degree in Mathematics and Statistics, and a Master’s degree in Pure Mathematics.

3. Corinne is a department head of mathematics with 14 years of teaching experience in Canada and Africa. She has her Honours Bachelor of Science in mathematics with a minor in religion, a Master of Science in capacity development and extension and international development and has taught calculus at the International Baccalaureate level, A-levels, as well as Ontario’s current *Calculus and Vectors* course.

Professors

1. Dr. Roger Litchfield is an associate professor with undergraduate, Master’s and PhD degrees in mathematics from Europe. Dr. Litchfield did his post-doctoral studies in mathematics in Canada and has been teaching calculus at the undergraduate level since 2008. Dr. Litchfield has experience with and currently teaches both of the first year courses, Calculus I and II for the life sciences.
2. Dr. Anthony Olsen is an assistant professor with an undergraduate degree in physics and chemistry, a second undergraduate degree in mathematics, a Master of Science degree in chemistry, a Master of Science degree in pure mathematics, and a PhD in applied mathematics. Dr. Olsen started teaching calculus as a PhD student and has been teaching calculus and differential equations courses for the physical sciences and engineering students since 2002. Dr. Olsen has experience teaching mathematics for the social sciences, and business, and currently teaches the first year Calculus I and II for the physical sciences and engineers courses, in addition to second and third year calculus and differential equations. Dr. Olsen teaches a remedial undergraduate mathematics course modeled after Ontario’s *Calculus and Vectors* course.

**Procedure**

Interviews took place in the summer of 2013 either at the participant’s place of work, office, or home. Interviews were semi-structured and recorded using a portable recording device with the intention of them each being between 45 – 60 minutes. In actuality, interviews ranged from roughly 50 – 90 minutes. Qualitative data from both the recording and in-person reactions (ex. Emotions) provided a more holistic understanding of the participant and their relationship and experiences with the research topic.

Two sets of questions were developed (see Appendices B and C) for the professors and teachers, respectively. There was significant overlap in questions, but because educators encountered students at two different stages of calculus education, some questions had to be modified to appropriately suit the participants’ experience.
Questions were grouped into four sections: background information (1), experience and conceptual understanding (2), subject-specific compliments and concerns (3), and recommendations (4). The purpose of section 1 was to get a background understanding of the participant and their experience with calculus. Section 2 focused on understanding where and why students had success and difficulty in either secondary-level calculus or first year undergraduate calculus. The purpose of the questions in section 3 was to remind participants of the curriculum change, and to explore what the impact of the change has been on students in either secondary or post-secondary courses, and in STEM programs on the whole. Lastly, section 4 gave educators opportunities to make recommendations for students under the current curriculum, amendments to the 2007-revised mathematics curriculum, and to provide any additional comments or insights that were not specifically addressed previously.

Data Collection and Analysis

Data was transcribed verbatim using audio files transferred from a personal recording device to the interviewer’s personal computer. Interviews were saved on the computer hard drive and backed up using Google drive and an external hard drive. All data was kept private and confidential.

Data was analyzed in five stages:

• First, by creating summaries of each individual interview, elucidating initial themes based on the summary points, and colour coding them on the summary sheets
• Second, by highlighting passages within each interview according to the colour coded themes from the summaries
• Third, by looking for common themes amongst teachers or professors
• Fourth, by looking for common themes across teachers and professors
• Fifth, by grouping data into six overall themes

Themes were identified by pulling common ideas from each of the interviews, and subsequently grouping results into sub-themes under the six overall themes.

Ethical Review Procedures

All of the research participants were contacted via email and provided with consent letters detailing the purpose of the study and the interviewing protocol in advance of each interview. Virtual and paper copies were provided to the participants and it was required that they read and sign the consent letter (see Appendix A) in order for the interview to proceed. Two of the interviews took place in the participants’ private offices, two at the participants’ workplace in empty classrooms, and one at the participant’s home. Participants were made aware that the interview would be recorded with a portable recording device and were each given the option of member checking the transcript and analysis of their respective interviews, however all five participants declined.

During the formal interview process, questions that the participants had were addressed before and during the interview, as they came up. Participants were informed via the consent form and in person that to maintain anonymity and confidentiality, pseudonyms would be used. Participants were given the option to email the interviewer if they had any additional comments they wanted to make after the formal interview was
complete. Participants were told that they would be made aware of when the research process was complete and would be able to have access to the finalized research paper, and any accompanying academic papers, should they request it.

**Limitations**

The time and scope for this research project leant itself to several limitations: first, the sample size \((n = 5)\) is relatively small, especially considering the comparative aspect of the study. While the number of participants limits the generalizability of their statements, for the purposes of this study, which is the first exploration into this particular topic, speaking to five different educators gives a very broad, holistic view of the research problem. Secondly, due in part to the time constraints of the study, only qualitative interview data was used. Being that the research problem aims to address all aspects of student success, a quantitative look into student achievement (Ex. Grades in secondary versus post-secondary settings) would give a more complete answer to this problem. In the future, conducting a mixed-methods type of study would be beneficial to addressing the multiple facets of this topic. Additionally, there was no inclusion of the students’ perspective. Incorporating the student voice and their collective experiences and perceptions of their own success in secondary and post-secondary calculus, will provide further evidence of reasons why students may or may not be achieving their fullest potential.
Chapter 4: FINDINGS

All five transcripts were analyzed and grouped based on common themes relative to each other. Six major themes surfaced:

- Frustration
- Time and experience
- Fundamental skills
- *Calculus and Vectors* shortcomings
- Cultural and systemic issues
- Recommendations

Within most themes, sub-themes emerged that illustrated the commonalities and differences across participants. It is important to note that there is significant overlap between the sub-themes and themes, and that often, ideas were not expressed in isolation.

**Frustration**

Each of the participants expressed frustration over a number of issues, whether it was in reference to the current curriculum, student attitudes, or larger, systemic pressures. Lucy strongly opposed the changes in the calculus curriculum and the manner in which it was done; she consistently used strong language (“mistake”, “useless”) to express her discontent with the current state of affairs. Similarly, Eric felt frustration, not only with the curriculum itself, but also with how to effectively implement it. Corinne expressed, perhaps, the least amount of frustration relative to the other participants, but was still generally unhappy that pieces of the former curriculum had been removed. Dr. Litchfield continuously referred to student attitudes and a general feeling of dissatisfaction with
basic competency levels. Finally, Dr. Olsen expressed an overall feeling of frustration with inaction on the part of the province on whether or not to keep calculus in the curriculum at the secondary level. Each of these sources of frustration will be delved into further in subsequent themes, however it is important to note the level of general dissatisfaction felt by each of the educators regarding this topic.

**Time and Experience**

The impact of time and experience at both a mathematical and psychological level was consistently brought up by each of the participants. The diversity of their ideas can be grouped into four sub-themes: Transition from 5-year to 4-year program, maturity, revisiting topics, and practice time.

*Transition from five-year to four-year program*

Four of the five participants had either been students under the five-year Ontario education system, and/or had previously taught under it. The impact of the transition from five to four years was felt widely by each of these educators. Dr. Olsen explained, “When, of course, the high school program went from being a five year program to a four year program, that in a sense eliminated one full year of math courses, or math experience”. This year’s worth of math experience that is now necessarily exempt from the four-year program affects students’ understanding of material on the whole, as well. When comparing students who graduated under the five-year system versus the four-year, Lucy stated, “…That extra year – also, that extra year of age, they were more mature, took it, they understood it better”. Interestingly, during the double cohort year, Dr. Olsen did not observe any noticeable difference in mathematic competencies, though he still felt
that the students from the five-year program were better able to make the overall
transition into university. The impact on the transition from secondary to post-secondary,
and all of the academic, social, and mental pressures that come with it evidently is
influenced by age. Dr. Litchfield explained:

The students that I see coming into my courses are supposed to be university
students, are supposed to be adults, are supposed to be, yes, more or less
independent learners, but often they aren’t. Whether having students come a year
earlier with supposedly the same level of knowledge is worth having them come
out as, well definitely, as less developed personalities, because they’re just
younger and they don’t quite find their way into university or other aspects of
society.

The other participants spoke to the maturity that comes with age differences and the
effect this may have on student success.

*Maturity*

Further emphasizing the impact time can have on student development and
success, some of the educators spoke of the maturity levels that come with experience.
Corrine explained, “I do understand that students and teenagers, that there’s a growth and
a development and to ask students to do something they’re not developmentally ready to
do yet could certainly hinder their progress”. At the secondary and post-secondary level,
educators are seeing the consequences of younger students taking more difficult
mathematics courses. Lucy and Dr. Olsen both described students in recent years as more
“immature” than years past, with Corinne summing it up by saying, “There are many
students that need that fifth year in high school just to mature”.


Revisiting Topics

In terms of experience, having the opportunity to revisit topics after initially learning them appears to lead to a greater overall understanding of material. Specifically in mathematics, Dr. Litchfield commented:

> Mathematics is a field where everything builds on everything else…in mathematics, you cannot learn differentiation if you don’t know quadratic functions or if you don’t know polynomials, if you don’t know square roots, it just doesn’t work…[the] more they are familiar with these kinds of concepts that come from advanced functions, the easier it will be not just in my course, but in any math course.

Each of the educators echoed these statements; when asked which topics students had the greatest amount of success in learning, Dr. Olsen, Corinne, and Lucy each explained that having seen a concept before provided deeper and more successful levels of understanding further down the road. Referencing the vectors section of *Calculus and Vectors*, Eric went so far as to say, “The physics kids have a huge advantage…they’ve done it all before” whereas “the other kids who’ve never seen vectors before are struggling”. Overall, the educators felt that experience with topics promoted greater understanding of more difficult material.

Practice Time

Within the context of the current mathematics curriculum, educators were displeased with the lack of practice that students get with newer and more difficult material. Dr. Olsen stated, “They don’t get enough practice…we did probably on the order of five or six times the number of exercises, five or six times the actual homework.”
It’s significant. It is”. Dr. Olsen’s statement is not only a commentary on students’ motivation, but also speaks to the fewer opportunities being provided to extend learning. Lucy, having taught OAC, pre-2007 and the post-2007 calculus courses asserted that students need practice time with the basic rules of differentiation before they can achieve success with more difficult problems. Additionally, she noted an inconsistency with current ministry-approved textbooks which have fewer exercises and lack solution manuals, unlike university texts. In Lucy’s words, “The old calculus book had that”, implying a marked difference in current and past expectations. These educators pointedly expressed the belief that practice is critical to success in mathematics.

**Fundamental Skills**

A major theme that came out of all five interviews was that the educators interviewed felt that students now have increasing difficulty grasping fundamental skills of mathematics at the elementary, secondary, and post-secondary levels. Each educator spoke to the critical impact this has on student success in mathematics at a higher level.

**Basic Numeracy and Algebra**

The educators interviewed expressed to the interviewer the vital importance of having a solid understanding of basic numeracy and algebra. Each educator explicitly stated that students today consistently have difficulty with these skills and consequently, struggle with more complicated concepts. Dr. Litchfield commented:

We have noticed over the years that students do not know what they should know according to the high school curriculum when they come into our classes. And, I’m not talking about they don’t know the calculus part. I’m talking they don’t
know the algebraic manipulations, they don’t know inequalities, they don’t know absolute values, these kind of things.

This sentiment was repeated by both the secondary and post-secondary educators. As mathematics possesses the unique quality of being a discipline where topics build upon themselves, students cannot move forward without having a strong, initial foundation.

Lucy made a significant connection when she said:

The basic arithmetic, let’s say, with the limits, and the manipulating for example, [in] advanced functions, a lot of the weaker students found that difficult because they couldn’t factor…they couldn’t factor because they couldn’t do their times tables and that goes way back to elementary school.

Dr. Litchfield and Dr. Olsen noticed similar trends at the post-secondary level in their first-year calculus courses, commenting that while students may be able to make the conceptual leap to differentiation and integration, “they cannot simplify an algebraic equation, an algebraic expression correctly”.

Secondary-level teachers Eric and Corinne highlighted the importance of mastering these fundamental skills for a student to be able to grasp more difficult material. Eric explained that, “If you can’t do or don’t understand basic numeracy, then I think it’s quite difficult to be able to do some of those things with variables”. Both numeracy and algebra are considered to be of “vital importance” by post-secondary educators, and Corinne expressed the need to be numerate as being “critical to everyday life”. The importance of having these skills long before more complicated, higher level mathematics is introduced was consistently emphasized by each of the educators, with all
of them recognizing that inconsistencies in basic foundational backgrounds are to the
detriment of students’ post-secondary success.

*Elementary Concerns*

Some of the educators voiced concerns over what and how foundational
mathematic concepts are being taught at the elementary level. With the push for inquiry-
based learning and problem solving at the elementary level, Eric expressed the concern
that this has “create[d] another hole in that we get kids who can’t do the basic stuff like I
mentioned before, like good numeracy skills, you know, basic algebra skills”. He
elaborated that there is a decreased emphasis on rote memory work in newer pedagogical
models, and that this acts as a roadblock for students. He remarked,

I can teach kids the basic idea of calculus in two minutes. It’s the algebra that kills
them. The devil’s in the details and I think we’re not really focusing on those
details and by we, I mean the curriculum as a whole, down a lot further [than high
school].

Lucy experienced similar challenges, and explained that significant amounts of time are
taken up in many courses by having to review elementary-level concepts such as
fractions and multiplication tables. She suggested that, “University profs need to say hey,
are kids coming in and can’t factor, and why aren’t they? Because they can’t multiply.
Where does that go? Right back to elementary school”. Lucy and Eric separately reached
the consensus that qualified elementary teachers are needed to address issues in
understanding of fundamental mathematic skills so that students are able to approach
secondary-level mathematics with a stronger base.
Application and Advanced Problems

The educators were in agreement that students encountered significant difficulty when asked to apply skills to more difficult problems, with Corinne immediately stating that students “get lost in the process”. Dr. Litchfield and Eric made similar statements, and acknowledged that students tend to memorize steps or “recipes” rather than understand problems. Eric suggested that the inability to apply theory stems from a conceptual disconnect, and that “there could be a better job done of amalgamating the pure and applied [mathematics]” in schools. Similarly, Dr. Olsen acknowledged that students are unable to make what he considered to be “obvious connections” as a result of minimal context and exposure to topics given throughout students’ mathematics schooling. When moving from basic to more advanced or complicated problems, some educators noted that students tended to encounter larger issues. Referring to the Ontario curriculum, Dr. Olsen stated that, “The grade 12 Calculus and Vectors, with the course being as it is now, it really is just an introduction to the basics of differential calculus. That is all it is now”. Overall, the educators interviewed felt that the jump to application-based problems is a struggle for many students now.

Skills Needed

In order to gauge the academic skills required at the post-secondary level, Drs. Olsen and Litchfield were both asked what fundamental skills they expected students to have when entering university. Dr. Olsen repeatedly emphasized the critical importance of understanding transcendental functions (logarithms, exponential and trigonometric functions) particularly within the STEM fields due to the growing physical applications of these mathematical foundations. Dr. Litchfield, an expert in the biological applications
of mathematics expressed similar views, and mentioned several times the need to understand concepts taught early in high school, including inequalities, absolute value functions, and even quadratic equations. Speaking to the correlational success students have in mathematics and other STEM subjects, Dr. Litchfield asserted that he is “convinced that people who are lacking math skills at that level, like advanced functions and calculus, have a harder time in the sciences as well”. Both professors recognized the connection between success in mathematics and STEM subjects, overall, and emphasized the importance of a deep understanding of functions in order to excel in university STEM programs.

**Calculus and Vectors Shortcomings**

Educators were asked to specifically address the grade 12 *Calculus and Vectors* course currently in effect in the Ontario education system, in terms of the impact of the revisions on student success in STEM programs. Significantly, the educators expressed very negative views towards its impact, citing numerous issues including disconnected concepts, missing topics, and gaps in problem solving. The educators felt that as a consequence of these issues, students are not prepared for first-year calculus or STEM programs.

*Disconnected Concepts*

Many found the amalgamation of the two mathematic sub-disciplines (calculus and vectors) to be problematic from the beginning. Eric recalled, “I remember being quite surprised when this calculus and vectors course came out because, well, they’re completely disconnected…they seem to be two very disjoint things”. Previously, calculus
was taught in *Advanced Functions & Introductory Calculus*, while vectors was a component of the *Geometry and Discrete Mathematics* course. Lucy felt that combining the two disciplines was “totally useless”, reiterating Eric’s point that *Calculus and Vectors* now exists as “two half courses”. Corinne and Lucy acknowledged that students recognize that a shift occurs in the course when switching from the calculus portion to the vectors component, and consequently, not only is this often “the weakest part of the course” but, as Lucy asserted, “Most hated the vector aspect. Hated it. Hated it”.

Speaking again to the disconnect and lost context in the course, Lucy explained:

> In the vectors and in the current *Calculus and Vectors*, there’s no culminating, you know. The kids are given little bits of this, little bits of that, but it’s not going anywhere. And even as a teacher, I found that you know, petty, like, what am I teaching?

Across the educators interviewed, frustration was most apparent when discussing the amalgamation of calculus and vectors into a single course.

**Concepts Missing**

Educators commented on concepts they believed to be important that no longer exist in the senior mathematics curriculum. A recurring topic was the substantial removal of major proofs from the senior-level courses. Eric remarked,

> I know everyone talked about how brutal the proofs unit was, but proofs is supposed to be you know, a key tenet of mathematics. And now you can get through basically all of high school math, and you could literally do it with no proofs.
Corinne similarly felt that the pure mathematics element that came with the *Geometry & Discrete Mathematics* course and gave background to the vectors was the proofs unit, and showed concern regarding its removal. Both educators felt that proofs were an integral piece of the more abstract part of mathematics, and understood that in university, students would be expected to understand them.

At the post-secondary level, Dr. Litchfield explained the purpose of teaching proofs:

I do present proofs in the course and that’s typically where most students just don’t follow…I want to show them the proofs because I think that’s one of the, well, it’s central to mathematics, a proof. But, it’s also one of the things that distinguishes mathematics from probably any other field, and where it really enables, it gives, it enables the students if they understand the idea of the proof to use that, rather than having to memorize a whole lot of things.

Dr. Litchfield’s testament further emphasized the reasoning behind including proofs in a course, and later questioned whether the students coming into his courses understand what a proof is.

Dr. Olsen raised the issue of time, suggesting that the structure of the course leaves little room for genuine practice with both transcendental functions, and the chain rule, again noting the importance of both of these concepts at the post-secondary level.

When asked to summarize her feelings on the topic, Corinne bluntly stated that, “Too much was taken out to do our students justice as they leave us and move on to another program”. Echoing this statement at a post-secondary level, Dr. Litchfield commented that, “What students don’t learn in high school will hurt them in university,
for sure”. Overall, there was the expressed belief that a significant amount of content is missing from the current curriculum.

Problem Solving

The secondary school teachers understood that a reason cited for changing the curriculum in 2007 was to improve problem-solving abilities. In spite of this intention, Dr. Litchfield explained that when students are asked abstract questions that require problem solving and reasoning skills, they struggle greatly. Eric, again, referred back to the lack of proofs in the current curriculum, and asked, “If we want to improve their mathematical problem solving ability, why did we remove a huge chunk that requires fairly significant mathematical problem solving?” Additionally, Eric considered the push for increased opportunities to problem solve at the elementary level and noted a disconnect between concrete, mechanical problem solving in math versus the more abstract, mathematical problem solving required in secondary and post-secondary mathematics courses. In contrast to Eric, Corinne pointed out, “I think reasoning and problem solving can be addressed at any level with any curriculum. I don’t know that I believe a curriculum change impacts reasoning and problem solving. I think that’s a teaching approach”. Whether as a consequence of the curriculum change or otherwise, it was evident that the educators felt that problem solving abilities had not been improved since the 2007 revisions.

Impact in First Year Calculus

Since the implementation of the revised curriculum, both professors have noticed differences in student ability. Dr. Litchfield described the range of abilities as having a “bimodal distribution”, and understood that because he teaches calculus for the life
IMPACT OF CURRICULUM CHANGES

sciences, that students have many different backgrounds. Dr. Olsen spoke to the historic changes he has noticed in going from the five-year curriculum to the current curriculum:

The bigger change happened when the course which was you know, that grade 12 calculus which worked reasonably well, became the calculus and vectors…what I found was, that transition was the big one and the thing is, is that they’re not getting as much calculus, they’re certainly not getting as much practice, they’re not going as far, you know, the transcendental functions are then sort of stuffed in at the end and not, nowhere near enough time is spent on it.

As a result, Dr. Olsen believed that competency is “definitely decreasing” in incoming Ontario students. Additionally, both Dr. Olsen and Dr. Litchfield commented that course curricula drastically changed at the post-secondary level in order to accommodate the changing entry-level skills. These accommodations led to critical components (Ex. applications of integration) of the more advanced, integral calculus being eliminated from the second semester calculus course due to time constraints.

Statements on Success

The educators spoke strongly about the impact the curriculum change has and will continue to have on students who enter STEM programs. Corinne stated:

I think it’s an advantage to some students and a disadvantage to others. I think the students going off to math and science curriculums in post-secondary education would certainly find it a challenge, and it is unfair to them because they are not adequately prepared for what is expected of them at university…I think we haven’t adequately prepared them for what’s expected of them at university.
Lucy’s response to the question of whether the change was appropriate was a powerful, resounding “no”. This topic evoked the most passion in her responses, as she proclaimed that the curriculum change was a “total mistake” and that “the courses have been dummed up”. Eric repeated this thought by stating that he “feel[s] like we’ve dumbed a lot of stuff down”. Lucy continued, calling the *Advanced Functions* “a totally useless course” explaining that she sees it as a repeat of the current grade 11 functions course.

Comparatively, Dr. Olsen said, “What they are doing now in high school is just a fraction of what used to be done, like, thinking of the grade 12 calculus course is just a fraction of what used to be done”, agreeing with the sentiments of the secondary school teachers. Overall, educators felt that the current *Calculus and Vectors* course does not contain the elements previously present in the Ontario curriculum that prepared students for success in university.

**Cultural and Systemic Issues**

Issues stemming from changes in cultural and systemic ideals consistently emerged, though no specific questions were asked in relation to this topic. The diversity of concerns brought forth provided a deeper understanding of factors outside of the curriculum that impact student success.

**Failure Rates**

The university professors discussed the failure rates in their first year courses, as Dr. Litchfield noted that failure rates of up to roughly 35% have occurred in his classes. Dr. Olsen noticed similar trends with 30 – 40% failure rates consistently showing up in his first year courses, and asked the important question, “What’s going on?”
Interestingly, Dr. Litchfield put these rates in a socio-political context when he explained that, “If we’re aiming to keep the failure rates at the same level, then either one of two things are happening. Either all of the students are getting smarter year by year by year or we’re lowering the standards year by year by year”. The implication of his comment pointed towards larger, systemic pressures impacting societal decisions.

*Cultural Shift*

Both professors noticed changes in Western culture, including the increased prevalence of math anxiety, and general attitude shifts towards mathematics. When discussing math anxiety, Dr. Litchfield said:

Maybe that’s even more important in determining success or failure at the university level than the actual grades that students get. When somebody can have math phobia or math anxiety and still do well on tests, if they have certain techniques, but just this apprehension might prevent them from going any further once they get to university.

Reiterating previous statements, Dr. Litchfield acknowledged that the nature of the course he teaches makes for encounters with many math anxious students, and this appears to impact their abilities to continue in STEM programs. He referred back to the public school system and made the comment that students are perceptive of teacher attitudes and that students are aware of teachers with math anxiety, and that this has a negative effect on student success in mathematics.

Dr. Olsen considered the broader context of Western society, and explained that “It’s okay to say you hate math” in today’s day and age. When comparing dislike for math to a dislike for reading, Dr. Olsen explained:
As a mathematics teacher that [sic] I do recognize is that in our culture, it’s okay to hate math, right? Little Johnny says I don’t like to read, everybody’s up in arms. Everybody’s up in arms saying, no, that’s terrible, right? But, Little Johnny says I hate math and people say, that’s okay.

He continued that in other areas of the world, the importance of mathematics is recognized at a very different and fundamental level as compared to Canada.

Lastly, Dr. Litchfield touched on the impact of living in the information age where students are able to look up information rather than learn and truly know it themselves. Commenting on students’ current experiences with being able to access information at any time without having to know it, Dr. Litchfield said, “I think that has many implications for success or non-success at university because they need to get over that”. Across the educators, changes in cultural attitudes appear to have had a large impact on mathematical success.

**Student Attitudes**

The educators interviewed all felt that student attitudes have changed drastically in recent years. At the post-secondary level, Dr. Litchfield perceived that students appear to have the impression that they know everything they need to know about calculus when entering university. Dr. Olsen noticed similar attitudes, and explained that, “They come here into first year calculus believing they know everything that there is to know about calculus. This is actually an attitude I’ve encountered in the last couple of years that they think that what they’ve done in high school is really all there is”. Both professors appeared perplexed by these attitude changes, but evidently believe that it is impacting on student success.
Eric and Lucy observed other changes in student attitude, as well. Eric expressed that students tend to “give up” more easily, and that there is simply a lesser willingness to commit to problems. Lucy specifically referenced *Advanced Functions* and found “the whole attitude was lacking because they thought, oh, it’s a review, I’ll do well, I don’t have to study” and in fact, found students typically did poorly with this attitude. Thus, student attitudes are thought to be powerful influences on academic success by the educators.

*Provincial Expectations*

Through discussions about the curriculum change, Corinne and Eric both speculated on the implications and purposes of the revision. Corinne recognized that by pushing the two disciplines together, that “they’re also trying to provide a course that has an [sic], that gives more students an opportunity to get a grade 12 credit”. By contrast, Eric lamented that by having a course that is a combination of calculus and vectors, “We’re forcing the kids who need the calculus to take the vector stuff that they probably don’t need”. Lucy had similar qualms, that students who are not entering the engineering or pure mathematics streams but are required to have a calculus background for their specialty are being “forced” to take the combined course. The secondary teachers overall had more comments on provincial expectations and constraints than the post-secondary educators, presumably because they are more aware of these systemic barriers.

*Technology*

A surprising finding was the educators’ perceptions on the influence of technology on student success. The overall sentiment was that there is an overreliance on
the use of calculators that is to the detriment of students’ learning. Dr. Olsen stated that calculator use is:

Too early, too much…we’re at the point now where if you tell the students they can’t use their calculator for something, even if it is something where the calculator is completely unnecessary, or not even possibly used, really, they will have a problem because they need to have it with them. It’s their little crutch.

Dr. Olsen’s assertion indicates a psychological reliance beyond curriculum expectations on using technology. Similarly, Eric and Lucy both acknowledged the value in technology, but were both clear that it “shouldn’t be a necessity for basic numeracy”.

That this sub-theme emerged on numerous occasions, though there were no specific questions addressing technology, and was commented on by secondary and post-secondary educators alike, denotes the importance of addressing technology use as a potential factor impacting student success.

**Recommendations**

Each of the educators made numerous recommendations to the Ministry, to teachers, and to students on how best to proceed in coming years with the current curriculum and to increase chances of student success. The list is certainly much more exhaustive than that which is provided, though many recommendations were implicitly mentioned in previous sections.
Top-Down Overhaul

Currently, Ontario curriculum revision for elementary and secondary mathematics appears to happen in isolation. Lucy vehemently made the recommendation that a top-down overhaul is needed, where each grade is reviewed. Lucy insisted:

I think that the curriculums need to be looked at from the top down. Start at the university level, what do the kids need to get in, and then that’s how you arrange your grade 12 courses. And then you back track to grade 11, and grade 10, and grade 9, and right back to kindergarten. A full overhaul.

Lucy continued to express this viewpoint throughout her interview, imploring educators and policy makers to communicate and listen to people at the level up in order to adequately prepare students for the future. Corinne agreed, simply stating that changes at any level “filters down as well”. To create consistency and meaningful changes, evidently a complete K–12 curriculum analysis is needed.

Increase Practice Time and Depth

Educators overall felt that more practice, and greater depth of content is absolutely essential to student success at the post-secondary level. Dr. Olsen said, “Give them more practice. Give them more practice with the more difficult things. I look in the grade 12 book…there are not enough exercises for one thing, they’re too similar, and they’re all too easy”. Similarly, Corinne and Lucy both expressed the need to “do more” for students and give them “more opportunities” to practice more difficult material in an effort to better prepare students for expectations further down the road. Eric felt that increasing demands in terms of what students are required to do to move forward – whether that is from unit to unit, or course to course – will simulate better what post-
secondary institutions require. Lastly, Dr. Litchfield expressed that a deeper understanding of fewer things, as opposed to a “superficial” knowledge of material will set students up for greater success. More explicitly, he stated, “My recommendation would be, the things they learn in high school, they should learn really well”. Across the educators, the need to learn mathematics at a deeper level was apparent.

*Course Redistribution*

Redistributing material from different courses at the high school level was a recommendation suggested by several educators. Corinne highlighted changes that could be made in specific courses, singling out *Advanced Functions* as a course that could be “condensed to provide space for some of the calculus”. She later explained that shifting more calculus material into this course would leave room for greater depth and understanding in *Calculus and Vectors*. Lucy went a step further and suggested that *Calculus and Vectors* be split into their respective courses as per the pre-2007 curriculum so that a “meatier program” could be maintained. In contrast to the two secondary teachers, Dr. Olsen, frustrated with what he considers a state of inaction in the province, stated:

> There is one of two ways to go and we need to make a decision if we’re going to go one way or the other. And that is either calculus is done in, in high school, in which case it should be done properly, or calculus is not done in high school at all, and we do it here. And we do it the way we’re gonna do it here.

*Add In*

In previous sections, what educators felt to be lacking or missing in the current courses was discussed. Additionally, these educators made recommendations as to which
topics were most critical to student success, to the extent that they should be re-integrated into the curriculum. Corinne and Eric both advocated for the re-incorporation of proofs into the senior level courses. Additionally, Eric recalled that matrices, a topic taught previously in the *Mathematics of Data Management* and *Geometry & Discrete Mathematics* courses be reintroduced as it is both “very attainable for students in high school” and it has wide-ranging implications in STEM programs. Lucy and Dr. Olsen both believed that a simple introduction to integral calculus would be an important addition to the curriculum. Dr. Olsen remarked:

I would think that it would not be a bad idea at the end of the grade 12 calculus if the teacher were to simply say to the class, okay, we’ve spent the whole term or the whole year depending on how the school works, talking about if this is a function, what is its derivative. Now, in the last week of class, why don’t we just think about going the other way. Just to introduce the idea of anti-differentiation…because it is amazing how many students can be fairly competent with differentiation but have a lot of trouble with integration.

Each educator framed these additions in the context of better preparing students for what is expected of students in STEM programs, and spoke out of concern and continued experience with struggling students at the university level.

**Summary**

Educators were asked point-blank whether they felt that students were going to be prepared for post-secondary STEM programs based on the current senior mathematics curriculum. The responses were powerful. Corinne said, “My understanding would be
that it’s, to be successful in a science program, even the technology side of it, and the engineering is absolutely dependent on being successful in math”. Eric, as well suggested that, “For a lot of those sort of middle of the road kids or kids that maybe struggled a little bit, I think that no, in some sense, we’re not doing them a great service and they’re not gonna be really well prepared”, effectively echoing previous statements. Lucy had, perhaps the most impassioned response when she said, “Are the kids prepared? I don’t think they are. Bottom line. I don’t think they are”.

Overall, educators expressed the belief that, for a wide variety of reasons, students are entering STEM programs at the university level inadequately prepared for the challenges they will encounter in mathematics courses.
Chapter 5: DISCUSSION

The purpose of this study was to begin exploring the impacts of changes to Ontario’s senior mathematics curriculum on student success at the post-secondary level. Specifically, I was asking, “In what ways has the current Ontario mathematics curriculum affected the transitional success of high school students entering university-level STEM programs?” The findings across secondary and post-secondary educators suggest that the *Calculus and Vectors* course is not adequately preparing students for success in post-secondary mathematics courses or STEM programs, and that issues in mathematical competency arise from gaps in learning far earlier than at the secondary level.

Connections to Literature

Much of the findings directly connect to the literature reviewed in Chapter 2. However, there were a number of responses that went beyond the initial scope of the research question, and thus, connections to the literature review are lacking in some areas. That the responses were so varied suggests that the implications to the research question are even broader than anticipated.

1. What difficulties do higher-level educators (i.e. University instructors) encounter when teaching first year calculus and why?

Each of the educators indicated that difficulties beyond calculus are evident with incoming students. Firstly, the issue of competency surrounding fundamental skills such as basic numeracy and algebra was at the forefront of each educator’s concerns. Indeed, it was not just the university professors who encountered students who do not fully understand the basics, but secondary teachers as well. Kershaw (2010) corroborated these
findings, in that she also found that students are by and large leaving secondary school for higher education lacking the necessary skills in algebra and numeracy. Recent PISA results (Brochu et al., 2013) have also indicated that mathematics knowledge is declining over the years, as seen by Canada and Ontario’s consistent drop in world rankings for mathematics scores. Since mathematics is a subject where the understanding of advanced material relies heavily on prior, foundational knowledge, students lacking older, basic skills will struggle to move forward with their mathematics education.

Students’ level of maturity was also called into question by the educators. Similar to Ferrini-Mundy and Gaudard (1992), both Drs. Litchfield and Olsen have observed that students are entering their first year calculus courses with the attitude that either they had a complete knowledge of calculus, or boasted overconfidently at their abilities. At the secondary level, educators found students to be immature and unwilling to commit to higher-order thinking problems. These attitudes pose challenges for educators and students alike, in that progression towards advanced mathematics will be stifled if students are not willing to open themselves to understand new material.

Students also have difficulty going from basic to advanced material in both differential and integral calculus. The assertion that students need to see certain material (i.e. integral calculus, applications of derivatives) was made more than once in order to make conceptual leaps. It is worth noting that many of the topics that are covered in first-year university calculus were previously addressed in the pre-2007 mathematics curriculum. As material becomes more advanced, it is to students’ benefit to see material in high school and then again at the university-level to truly understand the complexities
associated with a particular topic, and evidently, this opportunity is not being given to Ontario students under the current curriculum.

2. How has the reformed 2007 Ontario curriculum (MCV4U, MHF4U) affected student success in first year calculus courses and science/engineering programs from the perspectives of teachers and professors?

The current Calculus and Vectors course has considerable shortcomings. There is a significant amount of material missing from the actual curriculum that was previously present, and this is failing our students. The perception is that students are entering post-secondary education with lower competencies than previous students, an observation that is noted in the literature, as well (Slavin, 2007; Kershaw, 2010). University professors have attempted to modify curriculum to accommodate these changes, but even so, this is at the cost of them losing material or having to teach the content at a speed that students are not yet ready for.

There is also agreement that current students have less mathematics experience than previous students. Dr. Olsen did not notice a major decline in student abilities in mathematics during the double-cohort year when comparing students under the OAC curriculum relative to the four-year OSSD program; his observation has been documented by other researchers as well (Slavin, 2008; Tremblay et al., 2007). The year following the 2007 curriculum revisions, however, produced the more notable change. The consensus amongst the educators is that students are lacking in mathematic experience both at the basic level, as well as at the more difficult, calculus-based level. It is consistently seen that less experience leads to greater difficulty, and accordingly, the greater the amount of experience, the higher the likelihood of success. Adamuti-Trache et
al.’s 2013 study similarly found that students with calculus experience performed significantly better in first year calculus courses relative to their inexperienced colleagues. These findings suggest that the inexperience with calculus is to the detriment of Ontario students, and that greater difficulties and challenges will be experienced at the university-level, as a result.

Interestingly, while the educators advocated for more practice with more difficult applications, the 2007 curriculum specifically stipulates that in some cases, “simple” examples are to be used (Ministry of Education, 2007). Additionally, while the Task Force stated that changes in the curriculum were made to promote more problem solving (Ministry of Education, 2006), several of the educators felt that students coming to university do not have the problem solving skills they need, and, as was pointed out by the educators again, an entire course dedicated to problem solving (Geometry and Discrete Mathematics) was removed from the curriculum altogether, seemingly contradicting the objectives of the curriculum change. Again, the inexperience with difficult problems and with abstract, more mathematical type of problem solving (the proofs, for example) has proven to be problematic for students entering university for STEM programs and who are required to take university-level mathematics.

There is no current scholarly literature in Ontario that assesses this curriculum and its implications, rendering it difficult to draw comparisons. Thus, these findings suggest that larger-scale studies are needed to fully examine the impact of this curriculum change on student success.
3. **Does the 2007 curriculum adequately prepare students for science and engineering programs that utilize calculus?**

   It was found that across the board, the educators interviewed do not believe that students are adequately prepared for post-secondary STEM programs and/or calculus courses. While each of them did express the belief that holes in the 2007 curriculum are to blame for some of the issues students encounter, they also believe that problems extending back to elementary school (Ex. Solid understanding of basic numeracy and algebra) hurt students at every level of STEM-based education. Indeed, mathematics is seen as a predictor for success in STEM (Sadler & Tai, 2007; Tyson, 2011), so it can be assumed that a superficial mathematics background will cause significant challenges for students in university-level STEM programs, and therefore impact students’ abilities to achieve success in their respective programs.

   Additionally, cultural and systemic issues were considered. At the cultural level, it was acknowledged that mathematics simply is not valued to the same extent in Canada as it is in other parts of the world, and that in Western society, to hate mathematics is deemed “okay”. This is problematic on numerous levels, as this attitude resides not only with students, but also with parents and even teachers. Students are aware of these attitudes and what is valued at a societal level. Thus, the devaluation of mathematics can and will act as a major roadblock towards student success and commitment in math-based programs.

   A surprising finding was that educators felt that there is an overreliance on technology, such as calculators, by students. This was a topic I did not anticipate coming up in interviews, but it was evident that these educators felt strongly that students are
relying on technology to carry them through school in lieu of actually knowing things.

Issues arising from this relate back to the problems associated with knowledge of fundamental skills; students will struggle with more advanced problems if they do not know basic concepts such as multiplication or algebraic manipulation.

4. What are strategies and recommendations to remedy the declining mathematics performance in Ontario students entering STEM programs?

There were several recommendations made to the Ministry, students, and teachers regarding this curriculum, and student success in STEM programs. These included:

- K–12 full overhaul, where curriculum is analyzed from the top-down. It was suggested that secondary reviewers understand what universities expect and should create grade 12 courses that prepare students for that, then repeat the process for every grade all the way down to Kindergarten.

- Revamping of the grade 12 curriculum to include a pure calculus course, a course akin to the Geometry and Discrete Mathematics course, and the current Data Management and Probability course

- Increased practice with topics (basic and advanced problems)

- Stronger knowledge in basic numeracy and algebra

- Qualified teachers at all levels (elementary through secondary)

At a broader level, the educators’ concern with cultural attitudes towards mathematics and the impact of these attitudes suggests that within our society, there needs to be a large-scale shift towards a society that places higher value on mathematics if we expect future generations to succeed in mathematics-based disciplines.
Implications

The implications from this study are concerning, in that the evidence from these few educators suggests that students graduating from Ontario high schools are being done a disservice and are not being set up for success in higher-level education. This is cause for alarm, as students certainly possess the potential to succeed, but there may be political measures (i.e. curriculum standards) in place that are hindering students’ ability to achieve to the highest degree.

As a future teacher, the findings from this study indicate to me that I need to be more aware of students’ deficiencies with fundamental skills, and that I need to help them develop those skills and challenge them with problems that are more advanced than the ones they currently encounter. Additionally, where I can, I must provide opportunities for students to work with material such as proofs that can enhance their problem solving abilities. Establishing a classroom with clear expectations about the use of calculators, the importance of simply knowing things, and what the purpose of mathematics is may be a small step towards remedying some of the issues that came to light from this study, but it is a step nonetheless.

Limitations

This study benefited from taking a qualitative approach to the research question, as this is not frequently done when assessing student success in higher education. That said, only speaking to two university professors limits the generalizability of the findings, particularly because there is no standardized curriculum in university, and thus experiences and expectations may significantly differ. Additionally, because themes that
were entirely unexpected emerged from the study, literature to support these ideas was limited. On the note of scholarly literature, most of what was used was taken from American studies, where the education system is relatively different from Canada’s provincially governed education system. Thus, the comparisons made with some of the literature must be considered within the context of resources available.

**Future Directions**

This study can be expanded in numerous ways. Firstly, it would be prudent to perform a quantitative assessment of student performance going from high school to university, and comparing the results of current students to pre-2007 students. Additionally, because the study focused on success in STEM programs, speaking with professors and teachers who teach STEM subjects outside of calculus (for example, physics) would offer insight into the broader implications of this curriculum. Lastly, as per one of the recommendations, a full K – 12 curriculum analysis is needed. A close examination and comparison of the elementary and secondary curricula must occur in order to better understand issues emerging from this study.

**Conclusion**

As a beginning mathematics teacher with a STEM background, delivering a strong mathematics education with the purpose of fostering student curiosity and success is of the utmost importance to me. The findings of this study have highlighted prevalent issues within and external to the 2007 revised curriculum that affect student success, and it is critical that educators and policy-makers consider these concerns as soon as possible.
In order to set our students up to reach their potential, we must consider better curriculum alignment at the elementary, secondary, and university level. We need to maintain the integrity of mathematics education by building strong foundational skills at the beginning of a student’s mathematics education, and help students apply these skills to higher-level problems and concepts. Furthermore, as a society, we must recognize that our attitudes towards mathematics will influence students’ feelings and commitment towards the discipline and associated subjects. In today’s economy where jobs in STEM fields are at a high, we want to encourage students to harness a curiosity and love for STEM subjects, and recognize the value in mathematics for their future success.
REFERENCES


Appendices

Appendix A: Informed Letter of Consent

Date: ______________________

Dear ____________________,

I am a second year graduate student at the Ontario Institute for Studies in Education (OISE) at the University of Toronto currently enrolled as a Master’s in Teaching candidate. As part of the requirements for this program and my own personal research interests, I am studying the effect of the Ontario math curriculum on the transitional success of high school students entering math or math-related programs in university. I believe that your level of experience and knowledge on the subject will provide invaluable insights into my topic.

The primary research gathered will be used for a major research paper that is designed to give teacher candidates an opportunity to explore educational topics using qualitative research techniques. My research supervisor, who is overlooking the process, is Professor Jim Hewitt at OISE.

The interview process will take 45 – 60 minutes and be recorded using a handheld audio recording device. There is potential for a follow-up interview that will take 15 – 30 minutes but it is not required. The interview may take place at any time or location that is convenient to you.

All names and any other vulnerable information will remain confidential, only to be seen by myself and my supervisor. The contents of the interview(s) will be transcribed verbatim and used as part of this research paper with the possibility of publication. The contents may be discussed/used during informal class discussions, conferences, and/or journal articles. The raw data from the interview will be disposed of within 5 years of the interview date. You are free to decline answering any specific questions and may pull out of the interview at any time. You will be well-informed about the topic and are subject to minimal risk throughout the process.

If you agree to the conditions above, please sign the attached consent form. Please do not hesitate to contact me and/or Jim Hewitt (jhewitt@oise.utoronto.ca) if you have any further questions or concerns. Thank you for all of your assistance.

Sincerely,

Gurpreet Sahmbi
Phone: 905 808 5464
Email: gurpreet.sahmbi@mail.utoronto.ca
Consent Form

I acknowledge that the topic of this interview has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time without penalty.

I have read the letter provided to me by Gurpreet Sahmbi and agree to participate in an interview for the purposes described.

Signature: _____________________________________________

Name (printed): ____________________________________________

Date: _________________________________
APPENDIX B: Interview Questions (Professors)

SECTION 1: Background Information

1. What is your name and occupation?

2. What is your academic background?
   a. Which Universities have you attended?
   b. What did you major/minor/specialize in?

3. How long have you been teaching math?
   a. How long have you been teaching calculus?
   b. Have you ever taught Calculus (or calculus-related courses) elsewhere? If so, where?

SECTION 2: Experience and Conceptual Understanding

4. Based on your experience in the classroom, which concepts in first year calculus do you think students have the greatest amount of success understanding?

5. Why do you think students are able to readily understand these concepts?

6. Based on your experience in the classroom, which concepts in first year calculus do you think students have the least amount of success understanding?

7. Why do you think students encounter difficulties with these concepts?

SECTION 3: Subject-Specific Compliments and Concerns

In 2007, the Ontario government made large changes to the senior-level math curriculum. Specifically, the revised curriculum called for the elimination of the Discrete Geometry and Calculus courses, and the creation of the new Advanced Functions and Calculus & Vectors courses. The change-up in course offerings also resulted in the elimination of a previously taught concept, anti-differentiation, at the high school level.

8. Following these changes (i.e., post-2008), can you describe the level of mathematical competence that first year students (on the whole) bring to your course?
   a. How does this level of competence manifest itself in your course?

9. What skills do you consider to be essential for success in first-year calculus?

10. Do you consider the absence or inexperience with (self-described) imperative skills to be detrimental to student success in university STEM programs? If so, why? If not, why not?
11. Have you had to modify your course to accommodate provincial curriculum changes?
   a. If so, please describe what changes you have made.

12. In your opinion, how important is success in math to completing a STEM program?

13. Do you think, under the current curriculum, Ontario high schools are adequately preparing graduates for success in calculus and STEM programs?
   a. If not, how could students be better prepared for university-level math?

SECTION 4: Recommendations

14. Do you have any recommendations for amendments to the current high school calculus curriculum? If so, please describe.

15. Do you have any advice for students under the current curriculum who will be taking calculus/calculus-related courses who are entering post-secondary education?

16. Do you have any additional comments?
APPENDIX C: Interview Questions (Teachers)

SECTION 1: Background Information

1. What is your name and occupation?

2. What is your academic background?
   a. Which Universities have you attended?
   b. What did you major/minor/specialize in?

3. How long have you been teaching math?
   a. How long have you been teaching calculus?
   b. Have you ever taught Calculus (or calculus-related courses) elsewhere? If so, where?

SECTION 2: Experience and Conceptual Understanding

4. Based on your experience in the classroom, which concepts have students had the greatest success in understanding?

5. Why do you think students are able to readily understand these concepts? How important is it to grasp these concepts?

6. Based on your experience in the classroom, which concepts have students had the least amount of success understanding?

7. Why do you think students encounter difficulties with these concepts? How important is it to grasp these concepts?

SECTION 3: Subject-Specific Compliments and Concerns

In 2007, the Ontario government made large changes to the senior-level math curriculum. Specifically, the revised curriculum called for the elimination of the Discrete Geometry and Calculus courses, and the creation of the new Advanced Functions and Calculus & Vectors courses. The change-up in course offerings also resulted in the elimination of a previously taught concept, anti-differentiation, at the high school level.

8. Do you think these revisions (i.e. Course changes, etc.) were appropriate?
   a. Do you feel that all of the important concepts in high school calculus have been retained?

9. What specific skills do you consider to be essential for success in first year calculus?
10. Do you consider the absence or inexperience with (self-described) imperative skills to be detrimental to student success in university STEM programs? If so, why? If not, why not?

11. Do you think that there are concepts that should be reintegrated into the Ontario curriculum? If so, which ones? If not, why not?

12. Does the revised curriculum address concerns raised about the previous curriculum (Ex. “Not enough time to build reasoning and problem-solving skills”)

13. In your opinion, how important is success in math to completing a STEM program?

14. Do you think, under the current curriculum, Ontario high schools are adequately preparing graduates for success in calculus and STEM programs?
   a. If not, how could students be better prepared for university-level math?

SECTION 4: Recommendations

15. Do you have any recommendations for amendments to the current high school calculus curriculum? If so, please describe.

16. Do you have any advice for students under the current curriculum who will be taking calculus/calculus-related courses who are entering post-secondary education?

17. Do you have any additional comments?