Active Control of Separation on a Low Reynolds Number Airfoil Using Synthetic Jet Actuation

by

Mark Feero

A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Graduate Department of Aerospace Studies
University of Toronto

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Abstract

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Master of Applied Science
Graduate Department of Institute for Aerospace Studies
University of Toronto
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Wind tunnel experiments were used to study the effect of excitation amplitude and frequency on flow separation using synthetic jet actuation. A synthetic jet actuator was located near the leading edge of a NACA0025 airfoil at a chord-based Reynolds number of 100,000 and angle-of-attack of 10°. Under these flow conditions, the boundary layer separated from the suction surface and failed to reattach. Low-frequency excitation was used to target flow instabilities, while high-frequency excitation was performed at time scales an order of magnitude smaller. Low-frequency excitation at the separated shear layer frequency was found to be the most effective technique for flow reattachment and drag reduction. The results suggested that flow reattachment depended on exceeding a threshold momentum coefficient that varied with excitation frequency. Furthermore, a local minimum in drag independent of excitation frequency was achieved when the momentum coefficient corresponded to an average jet velocity that matched the freestream velocity.
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Nomenclature

\( a \) \hspace{1em} \text{Isentropic speed of sound in the synthetic jet cavity}

\( A \) \hspace{1em} \text{Area}

\( A_{\text{exit}} \) \hspace{1em} \text{Area of the wake measurement plane}

\( b \) \hspace{1em} \text{Piezoelectric diaphragm thickness}

\( c \) \hspace{1em} \text{Airfoil chord length}

\( C_{aC} \) \hspace{1em} \text{Acoustic compliance of the synthetic jet cavity}

\( C_{aD} \) \hspace{1em} \text{Acoustic compliance of the piezoelectric diaphragm}

\( C_D \) \hspace{1em} \text{Drag coefficient}

\( C_L \) \hspace{1em} \text{Lift coefficient}

\( C_p \) \hspace{1em} \text{Pressure coefficient}

\( C_\mu \) \hspace{1em} \text{Momentum coefficient}

\( d \) \hspace{1em} \text{Synthetic jet slot width}

\( D \) \hspace{1em} \text{Piezoelectric diaphragm diameter}

\( \text{DC} \) \hspace{1em} \text{Burst modulation duty cycle}

\( e \) \hspace{1em} \text{Statistical uncertainty of a given mean quantity at the 95\% confidence interval} \\
\( (\equiv 2\sigma/\sqrt{N}) \)

\( E \) \hspace{1em} \text{Young’s modulus of elasticity}

\( E_{\text{app}} \) \hspace{1em} \text{Input voltage amplitude}

\( E_{\text{fg}} \) \hspace{1em} \text{Function generator voltage}

\( E_{\text{hw}} \) \hspace{1em} \text{Hot-wire voltage}

\( E_{uu} \) \hspace{1em} \text{Power spectral density of} \ u

\( E_{vv} \) \hspace{1em} \text{Power spectral density of} \ v

\( f \) \hspace{1em} \text{Frequency}

\( f_{1,2} \) \hspace{1em} \text{Synthetic jet resonant frequencies}

\( f_s \) \hspace{1em} \text{Sampling frequency}

\( f_D \) \hspace{1em} \text{Piezoelectric diaphragm natural frequency}

\( f_H \) \hspace{1em} \text{Helmholtz frequency}

\( F^+ \) \hspace{1em} \text{Dimensionless excitation frequency} (\equiv f_e X_{sep}/U_\infty)

\( F_D \) \hspace{1em} \text{Drag force}

\( h \) \hspace{1em} \text{Synthetic jet slot height}

\( K \) \hspace{1em} \text{Second-order frequency response gain}

\( K_j \) \hspace{1em} \text{Synthetic jet formation criterion constant} (Re_j/S^2 > K_j)

\( L \) \hspace{1em} \text{Streamwise distance from the synthetic jet to the measurement plane}

\( m \) \hspace{1em} \text{Number of blocks in a phase averaged cycle}
$M$ Number of cycles used for phase averaging
$n$ Number of samples contained in each block for phase averaging ($\equiv N/m$)
$N$ Number of acquired samples
$p$ Static pressure
$Re_c$ Reynolds number based on chord length ($\equiv U_{\infty}c\rho/\mu$)
$Re_j$ Jet Reynolds number ($\equiv \bar{U}_j\rho/\mu$)
$S$ Stokes number ($\equiv \sqrt{\omega d^2 \rho/\mu}$)
$St$ Dimensionless frequency expressed as a Strouhal number ($\equiv f_c/U_{\infty}$)
$t$ Time
$t^+$ Dimensionless reattachment time scale ($\equiv tU_{\infty}/X_{sep}$)
$t'$ Normalized time variable ($\equiv (t - \tau)U_{\infty}/L$)
$T$ Synthetic jet velocity period of oscillation
$Tu$ Turbulence intensity
$u, v$ Instantaneous velocity in the $(x, y)$ directions, respectively
$u', v'$ Fluctuating velocity in the $(x, y)$ directions, respectively
$u_j$ Instantaneous synthetic jet velocity
$u'_j$ Turbulent fluctuating synthetic jet velocity
$U$ Mean streamwise ($x$-direction) velocity
$U_{\infty}$ Freestream velocity
$U_o$ Average outer velocity at the wake measurement plane
$U_c$ Vortex convection velocity
$U_{sw}$ Hot-wire calibration mean velocity
$\bar{U}_j$ Time and spatially averaged synthetic jet velocity (expulsion)
$U_{j, rms}$ RMS synthetic jet velocity (expulsion)
$U_{min}$ Minimum streamwise velocity in the airfoil wake
$V$ Synthetic jet cavity volume
$w$ Synthetic jet slot length
$W$ Wake width
$W_{vv}$ Wavelet power spectrum of $v$
$x, y, z$ Global coordinates in the streamwise, transverse and cross-stream directions, respectively
$x^*, y^*, z^*$ Synthetic jet coordinates normal to the slot exit plane, along the slot width and along the slot length, respectively
$x', y'$ Airfoil coordinates, where $x'$ is aligned with the chord line
$X_{sep}$ Length of the separated flow domain
$y_t$ Airfoil thickness distribution
\(y_1, y_2\)  
Lower and upper boundaries of the wake measurement plane, respectively

\(y_{1/2}\)  
Wake half-width location

\(\alpha\)  
Airfoil angle of attack

\(\beta\)  
Cross-wire yaw angle

\(\eta\)  
Dimensionless wavelet time variable

\(\rho\)  
Density

\(\mu\)  
Dynamic viscosity

\(\lambda\)  
Wavelength

\(\omega\)  
Synthetic jet oscillation speed

\(\omega_0\)  
Dimensionless wavelet frequency

\(\Phi\)  
Phase offset between the function generator voltage and synthetic jet velocity

\(\theta\)  
Phase angle measured relative to the function generator voltage

\(\nu\)  
Poisson’s ratio

\(\sigma\)  
Standard deviation of a given mean quantity

\(\tau\)  
Time corresponding to control initiation

\(\tau_{xx}\)  
Viscous stress in the \(xx\) direction

\(\gamma\)  
Piezoelectric diaphragm volume fraction

\(\Psi_\omega\)  
Wavelet function

\(\zeta\)  
Second-order frequency response damping ratio

**Subscripts (of variables not defined above)**

\(c\)  
Carrier frequency

\(e\)  
Excitation frequency

\(m\)  
Modulation frequency

\(ws\)  
Dominant separated wake frequency

\(wa\)  
Dominant attached wake frequency

\(sl\)  
Dominant shear layer frequency

\(l\)  
Airfoil lower surface

\(u\)  
Airfoil upper surface

\(ave\)  
Spatial average in the spanwise direction \((z^*)\) of the synthetic jet

\(j\)  
Synthetic jet quantity

\(\infty\)  
Freestream quantity

\(rms\)  
Root-mean-square
1 Introduction

1.1 Low Reynolds number airfoils

The most important characteristic of any lift-generating surface is the airfoil profile. Proper performance of the airfoil is necessary for the lifting surface to function efficiently. The relevant scaling parameter for airfoils is the chord based Reynolds number,

\[ Re_c = \frac{U_\infty c \rho}{\mu}, \]  

where \( c \) is the airfoil chord length, \( \mu \) is dynamic viscosity and \( \rho \) is the density of the fluid in which the airfoil is operating. Aerodynamic performance, as characterized by the lift-to-drag ratio, decreases at lower Reynolds number due to the relatively large impact of viscous effects. Above \( Re_c = 10^6 \), the adverse pressure gradient present on the upper surface of an airfoil is generally not strong enough to cause the flow to separate [24; 9]. This is due to the fact that the flow has already transitioned to turbulence prior to reaching the adverse pressure gradient. However, for Reynolds numbers less than \( 10^6 \), the boundary layer may still be laminar when the adverse pressure region is reached. In this case, the flow may separate without reattaching, causing a significant decrease in aerodynamic performance. This is behaviour, known as “stall”, is observed even at high Reynolds number, however it typically requires a large angle of attack. Separation without reattachment may occur on airfoils at low Reynolds number even for low angles of attack. When the boundary layer separates and transitions to turbulence before reaching the end of the chord, it will reattach and form a laminar separation bubble. Separation bubbles are characterized as short and long, with long bubbles typically spanning 20-30% of the chord at a Reynolds number of \( 10^5 \) [24]. A long separation bubble is highly detrimental to airfoil performance. The boundary layer development for a typical separation bubble is shown in Figure 1. Qualitatively speaking, when the momentum in the fluid layers near the surface is unable to overcome the forces due to the adverse pressure gradient, a region of reverse flow will develop. The location of the separation point is described mathematically as the point where the shear stress at the surface vanishes, which is equivalent to the location where the velocity gradient in the boundary layer is zero [11].

The subsequent discussion reveals that at low Reynolds number, there are three possible flow regimes for the boundary layer on the suction surface:

1. Flow attached to the airfoil surface over the entire cord length,
2. Separation of the boundary layer followed by reattachment, forming a zone of recirculation,

3. Separation of the boundary layer and failure to reattach.

Aerodynamic performance decreases from regime 1 to 3. This range of Reynolds number below $10^6$ has received considerable attention from researchers due to the fact that it corresponds to a number of applications such as human powered aircraft, micro air vehicles (e.g. UAVs), rotor blades, wind turbines and low-speed aircraft. Experimental studies have been conducted to measure aerodynamic forces [9; 25; 26] and characterize the development of boundary layers with separation bubbles [6; 29; 45]. These studies concerning the characteristics of low Reynolds number flow have been accompanied by considerable interest to mitigate flow separation and thereby improve aerodynamic performance.

### 1.2 Synthetic jet actuators

Active flow control devices differ from passive control (e.g. trip wires, surface roughness) in that they add energy to the flow and have operational characteristics that can be adjusted [10]. These devices typically fall under the category of fluidic, moving object/surface or plasma. Active control is advantageous due to the fact that actuators can be designed such that no parasitic drag is introduced and, at the bare minimum, the control can be switched from “on” to “off”. A common zero-net-mass-flux fluidic
actuator for flow control is the synthetic jet. A synthetic jet actuator (SJA) is composed of a vibrating diaphragm mounted in a sealed cavity with an orifice/slot leading to the surface where control is desired. A typical synthetic jet configuration is shown in Figure 2. Deformation of the diaphragm causes the working fluid to be alternately ingested and expelled by the cavity, thereby adding momentum (but not mass) to the flow [33]. The net momentum transferred to the flow during the expulsion phase is due to the formation of a vortex pair at the orifice/slot edge(s). As the vortices detach and move away from the orifice, the resulting velocity profile resembles that of a continuous jet. Figure 3 shows an example of the typical flow fields produced during ingestion and expulsion from Yao et al. [41]. While the flow produced by a synthetic jet is similar to that of a pulsed jet, synthetic jets offer the unique feature that no external fluid source is required. This allows the production of synthetic jet devices that are extremely compact and can be easily implemented in wind tunnel models without the need for complex piping, fluid storage, etc.

Ideally, the spatial velocity profile across the slot exit during expulsion has a “plug” or “top-hat” profile. That is, the velocity is constant across the width of the slot. Smith and Swift [34] showed that the spatial velocity profile can deviate significantly from plug flow under certain conditions, therefore the time and spatially averaged velocity is used as the characteristic velocity scale. The average jet velocity is given by
Figure 3: Phase averaged streamlines in the plane perpendicular to a 2D synthetic jet slot measured using PIV [41].

\[
\overline{U_j} = \frac{1}{T/2} \frac{1}{A_j} \int_{A_j} \int_0^{T/2} u_j \, dt \, dA_j,
\]

(2)

where \(u_j\) is the synthetic jet velocity, \(T\) is the period of oscillation and \(A_j\) is the cross-sectional area of the slot. The jet flow can be characterized according to a jet Reynolds number, \(viz.\)

\[
Re_j = \frac{\overline{U_j} \rho}{\mu},
\]

(3)

and a Stokes number, given by

\[
S = \sqrt{\frac{\omega d^2 \rho}{\mu}},
\]

(4)

where \(d\) is the slot width/orifice diameter and \(\omega\) is the oscillation speed. Note that this definition of a Stokes number is typical for synthetic jet studies, however it is generally known as the Roshko number in other oscillatory flow applications. The effectiveness with which a synthetic jet actuator is able to transfer momentum is highly dependant on these dimensionless parameters and the slot/orifice geometry. A time-averaged synthetic jet is formed when the vortex pair formed at the slot edges (vortex ring for an axisymmetric
orifice) is able to detach and convect away. As shown by Holman et al. [22], this jet formation criterion can be expressed as $Re_j/S^2 > K_j$, where $K_j \approx 1$ and 0.16 for two-dimensional and axisymmetric synthetic jets, respectively.

1.3 Flow separation control

The use of periodic excitation applied locally at the surface to mitigate flow separation and restore the aerodynamic performance of stalled airfoils is a technique that has been applied with varying degrees of success for a number of years [16]. Periodic excitation has been accomplished using pulsed jets, acoustic excitation, surface mounted diaphragms and synthetic jet actuators, to name a few. An extensive review by Greenblatt and Wygnanski [21] concluded that the majority of investigations using periodic excitation on airfoils described an optimum dimensionless frequency within the range $2 < F^+ < 4$, where $F^+ = f_e X_{sep}/U_\infty$, $f_e$ is the excitation frequency and $X_{sep}$ is the length of the separated flow domain. This range of $F^+$ corresponds to a Strouhal number,

$$St_e = \frac{f_e c}{U_\infty},$$

that is $O(1)$.

Post-stall separated flow has two dominant features: shear layer roll-up near the leading edge and large scale vortex shedding in the wake [40]. These instabilities are coupled due to the fact that vortex shedding in the wake causes global changes in circulation. Tian et al. [35] demonstrated that the frequency of the shear layer instability is larger than that of the wake instability, and the coupling between the two instabilities is non-linear. Excitation of the separated shear layer at $St_e \approx O(1)$ has proven to be effective since the frequency associated with the separated wake instability, $St_{ws}$, is also $O(1)$. However, Amitay and Glezer [2] demonstrated flow reattachment and performance improvement on a stalled airfoil for both $St_e \approx O(1)$ and for excitation frequencies an order of magnitude larger, $St_e \approx O(10)$. For $St_e$ that is $O(1)$, large vortical structures are formed that lead to unsteady reattachment and time-periodic variations in circulation. Actuation at $St_e \approx O(10)$ is essentially time invariant relative to the time scale of the flow and leads to local modification of the apparent aerodynamic shape of the flow surface, thereby changing the pressure gradient and suppressing flow separation [3]. Similar results were demonstrated by Glezer et al. [17] on a circular cylinder experiencing boundary layer separation at $Re_D = 7.55 \times 10^4$ (where $Re_D$ is Reynolds number based on cylinder diameter). The authors showed that low-frequency excitation caused cross-stream oscillations of the separated shear layer and strong coupling to the wake,
while high-frequency excitation displaced the cross-flow and lead to a favourable pressure gradient and more stable boundary layer.

The effect of excitation amplitude for SJAs is characterized using the momentum coefficient, \( C_\mu \), defined as the ratio of the time-averaged momentum of the synthetic jet to the momentum of the freestream, \( \text{viz.} \)

\[
C_\mu = \frac{\rho_j U_{j_{\text{rms}}}^2 d}{1/2 \rho U_\infty^2 c},
\]

where \( U_{j_{\text{rms}}} \) is the root-mean-square of the jet velocity at the exit plane over the expulsion half of the cycle. Amitay et al. [3] investigated the influence of \( C_\mu \) on separation control of a circular-leading-edge NACA airfoil operation between \( 3.1 \times 10^5 < Re_c < 7.25 \times 10^5 \) using a pair of SJAs. The authors found that \( C_\mu \) on the order \( \mathcal{O}(10^{-3}) \) was required to cause the flow to reattach, leading to a significant increase in lift and decrease in pressure-drag. The authors also demonstrated that the value of \( C_\mu \) required to reattach the flow decreased with the distance of the actuator to the separation point. Tuck and Soria [37] used unsteady synthetic jet actuation with \( C_\mu = 1.4 \times 10^{-3} \) to cause steady flow reattachment on a stalled NACA 0015 airfoil at \( Re_c = 3 \times 10^4 \) and \( \alpha = 18^\circ \). This lead to an increase in lift of 45%. Recent PIV measurements on the same experimental setup by Buchmann et al. [7] were used to confirm that actuation lead to transition to turbulence immediately downstream of the actuator and increased momentum near the surface. Tian et al. [35] used both amplitude-modulated and burst-modulated synthetic jet actuation to optimize the lift-to-drag ratio \( (C_L/C_D) \) on a post-stalled NACA 0025 airfoil at \( Re_c = 10^5 \) and angle-of-attack \( \alpha = 20^\circ \). A closed-loop control strategy was able to fully reattach the flow and increase \( C_L/C_D \) by a factor of 2 for \( C_\mu \approx \mathcal{O}(10^{-5}) - \mathcal{O}(10^{-4}) \). Goodfellow et al. [19] found that a SJA was able to decrease drag by 50% on a NACA 0025 airfoil operating at \( Re_c = 10^5 \) and \( \alpha = 5^\circ \) when \( C_\mu \) reached a threshold value that was \( \mathcal{O}(10^{-3}) \). Below the threshold value, negligible gain in aerodynamic performance was achieved. These results were similar to those of Amitay et al. [3]. In the work of Greenblatt et al. [20], synthetic jet actuation was applied to the separated flow over a wall mounted hump. These authors also found that the separation region was reduced and a noticeable decrease in pressure-drag was achieved once \( C_\mu \) reached a certain value.

### 1.4 Drag measurement

The impact of active flow control on the aerodynamic performance of an airfoil is typically evaluated by measuring the lift and drag coefficients. The drag coefficient is often determined experimentally from the surface pressure distribution, a force balance or us-
ing a control volume approach. Surface pressure measurements can be used to calculate the component of drag due to pressure, however the viscous component cannot be measured. A force balance measures the total drag force but measurements become difficult to perform when the drag force is very small. Therefore, a control volume approach will be used to measure the drag coefficient in this work.

Following the methodology of Van Dam [38], the drag force can be determined by considering a fixed control volume enclosing the airfoil in steady uniform flow. Assuming that control volume surfaces are sufficiently far away from the body such that the flow is undisturbed at those locations, the drag force is given by,

\[ F_D = - \int_{A_{exit}} (p - p_\infty + \rho U^2 - \rho U_\infty^2 - \tau_{xx}) \, dA \]  

where \( A_{exit} \) is the downstream measurement plane, \( p \) is pressure, \( U \) is the mean streamwise velocity and \( \tau_{xx} \) is the viscous stress, given by [31]:

\[ \tau_{xx} = 2 \mu \frac{\partial U}{\partial x} - \rho u'^2 - 2 \rho' u' - \rho u'^2. \]  

By assuming constant density and that the Reynolds stress term (\( \rho u'^2 \)) is much larger than the viscous term (\( 2 \mu \frac{\partial U}{\partial x} \)), \( \tau_{xx} \) can be rewritten in terms of the streamwise fluctuating velocity (\( u' \)) as,

\[ \tau_{xx} \approx -\rho u'^2, \]  

which can be substituted into (7) to give:

\[ F_D = - \int_{A_{exit}} (p - p_\infty + \rho U^2 - \rho U_\infty^2 + \rho u'^2) \, dA \]  

The pressure term in (10) can be eliminated by considering the \( y \) momentum equation for shear layers,

\[ \frac{1}{\rho} \frac{\partial p}{\partial y} = - \frac{\partial v'^2}{\partial y}, \]  

which can be integrated to give \( p - p_\infty = -\rho v'^2 \), where \( v' \) is the cross-stream fluctuating velocity. Finally, by applying mass conservation, (10) becomes

\[ F_D = - \int_{A_{exit}} [\rho U (U - U_\infty) + \rho (u'^2 - v'^2)] \, dA. \]
Assuming that the average velocity at the edges of the downstream measurement plane, $U_\circ$, has recovered to the freestream value, the drag coefficient can be expressed as

$$C_D = -\frac{2}{U_\circ^2 c} \int_{y_1}^{y_2} \left[U(U-U_\circ) + (\overline{u'^2} - \overline{v'^2})\right] dy. \quad (13)$$

where $y_1$ and $y_2$ are the lower and upper bounds of the measurement plane, respectively. Often, $\overline{u'} \approx \overline{v'}$ and the fluctuating velocity term in (13) can be ignored.

1.5 Motivation

The main motivation of the work contained in this thesis is to better understand the impact of synthetic jet actuation parameters on flow separation mitigation and the restoration of aerodynamic performance. The impact of excitation frequency ($F^+$ or $St$) and amplitude ($C_\mu$) on low Reynolds number airfoils undergoing laminar boundary layer separation has received considerable attention in the research community (e.g. [3; 2; 19]), however further investigation is required to understand the effects of these parameters on drag reduction. Based on previous work by Goodfellow et al. [19], it is expected that the drag coefficient will show a strong dependance on $C_\mu$ when $St_e \approx O(10)$. The question remains as to whether similar behaviour will be found when $St_e \approx O(1)$. Furthermore, a better understanding of the flow structures in the wake of the reattached flow for different excitation frequencies is desirable since this can provide information about the nature of the reattachment and aerodynamic forces (i.e. steady or unsteady).

1.6 Objectives

The objective of this thesis is to investigate the following:

- The performance of the synthetic jet actuator in quiescent conditions as characterized using velocity measurements. The frequency response of the synthetic jet exit velocity to the input signal is to be measured in order to determine the bandwidth of the actuator where sufficient control authority can be achieved.

- The sensitivity of the baseline flow conditions to changes in angle of attack and Reynolds number as determined from the surface pressure distribution over the airfoil.

- The impact of $C_\mu$ on mean velocity profiles and frequency content in the wake
using both low and high-frequency excitation ($St_e \approx O(1)$ and $St_e \approx O(10)$, respectively).

- The relationship between $C_\mu$ and $C_D$ for low and high-frequency excitation.

- The temporal characteristics of the reattachment process for high-frequency excitation including the time scale of reattachment and the transient dynamics that occur after control is initiated.

The experimental methodology used to accomplish these objectives will be discussed in the next section.
2 Experimental Details

2.1 Wind tunnel facility

The majority of all experiments were performed in a low-turbulence recirculating wind tunnel located in the Department of Mechanical and Industrial Engineering at the University of Toronto (Figure 4). The rectangular test section is 5 m long, 0.91 m wide and 1.22 m tall. One of the side walls and the ceiling are constructed using transparent polycarbonate to allow optical access for flow visualization/optical based measurements. The flow enters the test section after passing through seven screens and a 9:1 contraction. Turning vanes installed in the four corners of the wind tunnel aid in guiding the flow around $90^\circ$ bends. The freestream velocity in the test section is adjustable from approximately 2.6 m/s to 18 m/s and was monitored using a static pitot tube located at contraction exit. The pitot tube was connected to an Omega PX653 differential pressure transducer with a dynamic range of 0.5” H$_2$O and a measurement uncertainty that is 0.3% of full-scale. A traverse system with three translational degrees of freedom installed in the test section was used to position measurement instruments with an accuracy of roughly 0.15 mm. The traverse was controlled using a National Instruments PCI-7334/UMI-7764 Universal Motion Controller along with LabView software.
The free stream turbulence intensity, $T_u$, at the inlet of the test section (with no model installed) was measured for a range of $U_\infty$ using a hot-wire probe mounted at the centre of the test section (details concerning hot-wire anemometry are discussed in Section 2.4). The turbulence intensity is defined as

$$T_u = \frac{\sqrt{\overline{u'^2}}}{U} = \frac{u_{rms}}{U},$$

(14)

where $u_{rms}$ is the root-mean-square of the fluctuating velocity. No filtering other than the
low-pass filter of the anemometer was applied to the data before computing the turbulence intensity. As shown in Figure 5, the turbulence intensity shows only moderate variation with $U_\infty$ and has a mean value of 0.22%. The hot-wire measurements at the entrance to the test section can also be used to determine to background noise caused by the wind tunnel for varying speeds. The power spectral density of the streamwise velocity, $E_{uu}$, is shown in Figure 6 and indicates that for frequencies from 1 Hz to 10 kHz, the energy contained in the flow is less than approximately $10^{-4} \text{(m/s)}^2/\text{Hz}$. Figure 6 also confirms that there are no significant flow structures associated with the geometry of the wind tunnel or the 3-axis traverse, since such structures would likely be the cause of a spectral peak that would shift in frequency with $U_\infty$. For future measurements of velocity in the wake of the airfoil, power spectra results should be significantly larger than the frequency dependant noise level to be considered free from background noise contamination.

![Figure 5: Free stream turbulence intensity at the centre of the test section inlet (no model installed).](image-url)
2.2 Airfoil model

The NACA 0025 airfoil model has a chord length $c = 300$ mm and spans the width of the test section. The airfoil is mounted 400 mm downstream of the inlet to the test section (Figure 7a) and pivots about the quarter-chord location in order to adjust $\alpha$, the angle of attack relative to the free stream velocity. The global coordinate system $(x, y)$ is defined at the trailing edge of the airfoil with $x$ parallel to $U_\infty$. A local coordinate system for the airfoil $(x', y')$ is defined at the leading edge of the airfoil with $x'$ parallel to the airfoil chord line. The model is equipped with 65 static pressure taps distributed evenly between the upper and lower surfaces at mid-span. The locations of the pressure taps along the airfoil surface are shown in Figure 7b.
(a) Schematic of the airfoil model as mounted in the test section and definition of global coordinate system.

(b) NACA 0025 profile including pressure tap locations (indicated by ◇ symbols).

Figure 7: NACA 0025 model details.

Aerodynamic $\alpha = 0^\circ$ is identified as the rotation angle of the airfoil that corresponds to a symmetric pressure coefficient ($C_p$) distribution, where $C_p$ is defined as

$$C_p = \frac{p - p_\infty}{1/2\rho U_\infty^2}.$$  

(15)

For $Re_c \leq 10^5$, these pressure measurements were performed using a Scanivalve 64-port
pressure scanner connected to an MKS 226A Baratron pressure transducer with dynamic range of 0.2 torr and a measurement accuracy that is 0.30% of full scale. Pressure measurements at higher Reynolds number \((Re_c > 10^5)\) were performed using an MKS 223B pressure transducer with a dynamic range of 100 mbar and similar measurement accuracy.

In Figure 7a, the location of the SJA relative to the pressure taps is indicated. The pressure taps were meticulously installed in the model prior to any experiments involving synthetic jet actuation by Yarusevych [42]. Following the work of Yarusevych, the model was modified to house a SJA by Goodfellow [18]. In order to avoid damaging the pressure taps, Goodfellow elected to install the SJA to one side of the ports. For the present experiments, it was desired to move the SJA upstream in order to locate the jet as close as possible to the separation point at \(\alpha = 10^\circ\). Static pressure measurements (presented in Chapter 3) were used to determine that the flow separates at \(x'/c = 0.18\) for \(\alpha = 10^\circ\) and \(Re_c = 10^5\). The average separation point is estimated as the \(x'/c\) location corresponding to the beginning of a region of constant static pressure [9]. The existing hole in the surface of the model was extended upstream such that the synthetic jet location could be varied from \(x'/c = 0.19\) to 0.34. With the SJA at the most upstream position, the remainder of the hole was covered with an aluminum cap machined to match the curvature of the airfoil, as shown in Figure 8. The consequence of this SJA placement is that the pressure taps do not provide meaningful measurements during the active control experiments, therefore excluding the possibility of measuring the change in lift coefficient due to control.

![Figure 8: SJA and aluminum cap mounted in the NACA 0025 airfoil.](image)
2.3 Synthetic jet actuator

The synthetic jet actuator used in the present experiments was designed by and used in the work of Goodfellow et al. [19]. The synthetic jet has a perpendicular configuration, meaning that the motion of the oscillating diaphragm is perpendicular to the motion of the fluid through the slot. The slot width is \( d = 0.5 \text{ mm} \) and the slot height is \( h \approx 5 \text{ mm} \), giving \( h/d \approx 10 \). In the spanwise direction, the length of the slot is \( w = 140 \text{ mm} \), giving a large aspect ratio \( w/d = 280 \). The SJA spans \( \sim 16\% \) of the airfoil. The total volume of the cavity (as determined from the 3D model) is 16,240 mm\(^3\). As shown in Figure 9a, the structure of the SJA is composed of three parts: the top plate, middle plate and bottom plate. The middle plate contains the shape of the cavity and houses the piezoelectric actuators (Figure 9c). The bottom plate mates with the middle plate in order to rigidly clamp the piezoelectric disks. Finally, the top plate seals the cavity of the middle plate and forms the 140 mm by 0.5 mm slot. Delrin inserts and neoprene gaskets are used on either side of the piezo disks to isolate them from the aluminum parts. The entire assembly is clamped tightly together by nine M4 screws that pass through each plate. Each time the SJA is reassembled, a torque wrench is used to ensure that equal torque is applied to each screw. Along the top of the plates where screws cannot pass through (since they would pass through the neck of the slot), three M3 screws are inserted through the bottom plate and into tapped holes in the middle plate. Figure 9b shows an assembled view of the SJA.

The piezoelectric actuators used are the Thunder TH-5C model produced by Face International Corporation. The Thunder actuator is a composite unimorph ferroelectric driver that is composed of three main layers held together by two layers of adhesive. The material, geometry and approximate mechanical properties of each layer are given in Table 3 (\( D, b, E, \nu \) and \( \gamma \) are the diameter, thickness, Young’s modulus, Poisson’s ratio and volume fraction, respectively). The actuator reaches a maximum centreline deflection of 0.173 mm when the input voltage is 420 V (peak-to-peak). The input signal to the SJA is created using a Rigol DG1022 function generator and amplified by a Mide QPA3202 power amplifier with a voltage gain that is adjustable from 15 to 50. Due to unpredictable behaviour of the power amplifier (and possibly the piezoelectric actuators) at voltages near its maximum output (400 V), the actuators were not operated above 350 V.
Figure 9: Synthetic jet actuator details.

2.4 Hot-wire anemometry

The majority of the measurements performed in this thesis were accomplished using constant temperature hot-wire anemometry. A single wire probe was used in experiments where only the streamwise velocity component \( (u) \) was of interest. When both streamwise and transverse \( (v) \) velocity where desired, a cross-wire probe was used. The hot-wire probes were constructed using 5 \( \mu \)m tungsten wire plated with copper such that the sensing length was 1 mm (giving a sufficient length to diameter ratio of 200). For the cross-wire probes, the wire angles were approximately \( \pm 45^\circ \) and the spacing between prongs was 1 mm. The wires were operated at an overheat ratio of 0.6 by a Dantec 56C01 main unit with 56C17 CTA bridges.
Table 3: Thunder actuator geometry and mechanical properties.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>$D$ (mm)</th>
<th>$b$ (mm)</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substrate</td>
<td>Stainless Steel</td>
<td>32.77</td>
<td>0.152</td>
<td>200</td>
<td>0.29</td>
<td>0.44</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>PZT</td>
<td>31.75</td>
<td>0.180</td>
<td>63</td>
<td>0.31</td>
<td>0.49</td>
</tr>
<tr>
<td>Superstrate</td>
<td>Aluminum</td>
<td>30.73</td>
<td>0.0254</td>
<td>69</td>
<td>0.33</td>
<td>0.07</td>
</tr>
</tbody>
</table>

2.4.1 Hot-wire calibration

External calibration of the hot-wire probes was performed using a Dantec 55D90 calibration unit. Although *in-situ* calibration is more desirable, the calibration was performed externally for both synthetic jet characterization and wind tunnel experiments. The existing layout of the test section with respect to the model and traverse locations would present a number of difficulties for *in-situ* calibration, particularly for cross-wire probes. Furthermore, for the SJA characterization experiments, *in-situ* calibration is not possible due to the oscillatory nature of the flow.

The Dantec 55D90 is comprised of a axisymmetric contraction nozzle that forms a very low-turbulence free jet when air is supplied from a pressurized tank. The fundamental operation of the unit relies on the fact that the plenum pressure (which is monitored by the unit) is linearly proportional to the velocity of the jet. A typical calibration is as follows:

1. A maximum calibration velocity is selected.
2. The desired maximum plenum pressure is selected. The maximum possible is 10 bar. When the unit is operated, the plenum pressure will gradually decrease from the maximum value down to 2 bar. Therefore, the largest pressure ratio that is possible is 5. Since $U \sim p$, the maximum velocity ratio is also 5.
3. By measuring the pressure drop across the nozzle, the velocity is increased to the desired maximum value by adjusting a nob that controls the inner orifice size.
4. The unit outputs a voltage that is equal to the plenum pressure. When the ”scan” button is pushed, the pressure (and therefore velocity) will decrease from the maximum value to 2 bar. By simultaneously acquiring the hot-wire voltage, a calibration curve can be constructed.

For single-wire calibration, either a 4th order polynomial fit or King’s Law was used depending on the nature of the flow. In theory, the oscillatory flow produced from a SJA should result in velocities that range from the maximum value down to zero. Since
calibration at very low velocities (i.e. <1 m/s) is not possible using such an apparatus, a King’s Law calibration can be used to reliably extrapolate velocity values that fall below the calibration range. Figure 10 shows a typical hot-wire calibration using both a polynomial fit and King’s Law ($E_{hw}$ and $U_{hw}$ are hot-wire voltage and velocity, respectively).

![Figure 10: Typical single-wire calibration. Solid blue line is the hot-wire voltage data, red line with ◦ markers is the calibration fit.](image)

In the work of Burattini and Antonia [8], the authors showed that simple effective angle calibration of cross-wires can lead to significant errors in the estimation of both velocity components. Furthermore, they showed that at velocities below 6 m/s, the effective angle is not constant and depends strongly on velocity magnitude and direction. Therefore, a look-up table calibration similar to Burattini and Antonia [8] was used. The Dantec calibration unit has a rotary stage built into the hot-wire probe support holder such that the angle of the probe relative to the flow (yaw angle), $\beta$, can be adjusted. The calibration procedure is similar to that of a single-wire probe, with the process being repeated for each yaw angle. The cross-wire probe was calibrated for $\beta = -36^\circ$ to $36^\circ$ in $12^\circ$ increments. A typical cross-wire calibration is shown in Figure 11 for velocities from 2 m/s to 10 m/s ($E_{hw1}$ and $E_{hw2}$ are the voltages of wire 1 and 2, respectively). The drawback of the look-up table calibration technique is that any wire voltages that fall outside of the envelope of the calibration points cannot be interpreted. Therefore, some knowledge of the flow is required a priori for proper calibration.
2.5 Synthetic jet velocity decomposition

The time varying flow at a fixed spatial location near the exit of the jet can be written as

\[ u_j(t) = \langle u_j(t) \rangle + u'_j(t), \]  

(16)

where \( \langle u_j(t) \rangle \) is the coherent velocity fluctuation due to harmonic forcing of the SJA, or phase-averaged velocity, and \( u'_j(t) \) is the random turbulent fluctuation. According to Hussain and Reynolds [23], the phase averaged velocity over \( M \) cycles is defined as

\[ \langle u_j(t) \rangle = \frac{1}{M} \sum_{i=0}^{M} u_j(t + iT), \]  

(17)

where \( T \) is the period of the oscillating flow. In order to extract \( \langle u_j(t) \rangle \) from a measured hot-wire signal, it is common practice to acquire a reference signal simultaneously. In this case, the output from the function generator is acquired along with the hot-wire voltage. The phase averaging methodology employed here will be described briefly.

- The function generator voltage is used to identify the beginning of each cycle.
- Since \( f_s \) (the sampling frequency) is not necessarily an integer multiple of \( f_e \) (the excitation frequency), the first data point in each cycle will not occur at the same
phase, as shown in Figure 12a.

- The entire data set is divided into blocks each containing one cycle and the phase position of the first point in the cycle is calculated using the function generator voltage.

- The remaining points are separated in phase by $\Delta \theta = \Delta t f_e = f_e / f_s$. Figure 12b shows several blocks of a typical measurement. The fact that the data points in each block do not coincide is beneficial because it effectively leads to measurement of the entire cycle.

- The blocks are assembled into a single vector sorted by phase, thereby creating a single cycle containing the entire data set.

- The single vector is divided into a new set of $m$ blocks each containing $n = N/m$ data points. If $m$ is large enough, the measurements contained in a single block occur at approximately the same phase. The mean value of a given block is then the phase averaged velocity at a particular phase, $\langle u_j(\theta) \rangle$.

![Function Generator](image1.png)

![Hot-wire](image2.png)

(a) Raw hot-wire and velocity signals (entire sample is 5 s long).

(b) Sample blocks.

Figure 12: Phase averaging procedure. A typical measurement for $f_e = 600$ Hz at 200 V is shown.

Phase averaged cycled were computed from $N = 10^5$ data points and $m = 200$ blocks, therefore $n/N = 0.5\%$ and the resolution in phase is $1.8^\circ$. In any calculations involving jet
velocity, it is implied that phase averaged quantities are used. The statistical uncertainty in \( \langle u_j(\theta) \rangle \) is related to the standard deviation, given by

\[
\sigma(\theta) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} [u_j(\theta, i) - \langle u_j(\theta) \rangle]^2}.
\] (18)

As shown by (18), the standard deviation depends on the scatter of the data due to turbulence and the phase averaging procedure (selection of \( m \), and therefore \( n \)). If the block size is too large (large \( n \)), \( \sigma \) will increase and become meaningless due to the invalid approximation that all the measurements contained in a block correspond to the same phase.

It should also be noted that during the ingestion portion of the cycle, the flow reverses direction and therefore the velocity is negative (relative to expulsion). However, the hot-wire rectifies this signal and the entire cycle appears to have positive velocity. Signal de-rectification was not performed as it was unnecessary for the present investigation. The expulsion portion of the cycle was identified as having the larger peak velocity.

### 2.6 Measurement uncertainty

Two major sources of error contribute to the measurement uncertainty of a given quantity: the accuracy of the measurement device(s), and the statistical convergence of averaged statistics. All estimates of uncertainty presented herein are at the 95% confidence interval.

The accuracy of a velocity measurement made using a hot-wire probe is primarily influenced by probe alignment, probe construction and the calibration procedure [46]. For the purposes of this investigation, only the error associated with calibration was considered. The uncertainty was estimated according to the methodology of Yavuzkurt [46]. The measurement uncertainty due to calibration for all mean and rms velocity components was approximately 3%. The uncertainty in \( U_\infty \) (monitored using a pitot tube) due to the propagated error of the pressure transducer, barometer and thermocouple was \( \sim 1\% \).

The statistical convergence of a mean quantity depends on the standard deviation and the number of samples, \( \text{viz} \).

\[
e = \frac{2\sigma}{\sqrt{N}}
\] (19)

where \( \sigma \) is the standard deviation of a given mean quantity, and \( N \) is the number of samples in the measurement. The statistical convergence of fluctuating velocity com-
ponents was estimated according to the bootstrap resampling algorithm suggested by Benedict and Gould [4]. The bootstrap algorithm is based on drawing randomly, with replacement, a number of “bootstrap samples” each containing \( N \) data points from the original data set. Benedict and Gould [4] showed 100 bootstrap samples to be sufficient for most turbulence quantities. Mean and fluctuating velocities were converged to within 0.1\% and 0.5\%, respectively, in all cases.

2.7 Experimental procedures

The procedures for the three main experiments in this thesis are described in this section. Details for other minor experiments are described in their respective sections.

2.7.1 Synthetic jet characterization

Measurements of the velocity at the synthetic jet exit were performed with the SJA clamped in a rigid stand. A schematic of the experimental setup is shown in Figure 13. A hot-wire probe was positioned as close as possible to the centre of the slot \((y^* = 0.25 \text{ mm})\) using a traverse with very fine resolution and manual adjustment. Measurements were performed with the probe positioned at \(x^* = 0\) and \(z^* = 15.4 \text{ mm}\), which corresponds to the centre of one of the piezoelectric actuators. The actuators were driven at seven voltage amplitudes from 50 V to 350 V with frequencies ranging from 200 Hz to 1200 Hz in 50 Hz intervals. The actuators were not driven above 1200 Hz due to electrical current limitations of the amplifier at 350 V. Detailed measurements were performed near the resonant peak in 10 Hz intervals in order to more accurately determine the resonant frequency. Data was acquired at \(f_s = 20 \text{ kHz}\) in order to ensure that an adequate number of points were captured in each cycle.

The spanwise uniformity of the jet velocity was investigated by performing measurements at nine locations along the slot length. Several voltage amplitudes and frequencies were considered in order to determine the effect of these parameters on velocity uniformity.

2.7.2 Wake measurements

The velocity in the wake of the airfoil was measured using hot-wire anemometry. Streamwise velocity profiles were obtained by traversing a single-wire probe in the \(y\)-direction with the hot-wire probe located at the centre of the SJA slot in the cross-stream (\(z\)) direction. The transverse (\(y\)) axis of the traverse spans 550 mm and at \(\alpha = 10^\circ\), the trailing edge is located 242 mm from the lower limit. The measurement resolution was
12.5 mm (0.042c). At each measurement station, \( N = 10^5 \) samples were acquired at a sampling frequency \( f_s = 5 \text{ kHz} \).

The drag coefficient was calculated according to Equation (13) by measuring \( U \) at \( x/c = 2 \). Measurements presented in Section 5 will confirm that ignoring the fluctuating term in (13) is valid for this flow. Neatby and Yarusevych [27] recently showed that wake surveys for drag estimation should be performed at \( x/c \geq 4 \), however physical constraints in the current experimental arrangement limited the most downstream measurement plane to \( x/c = 2 \). Beyond \( x/c = 2 \), the fully separated flow lead to a wake that spanned almost the entire height of the test section. Assuming that the same bias error will be present on all drag measurements made at \( x/c = 2 \), it will not affect the results of this work, which focus only on the relative change in drag.

For velocity measurements with the purpose of spectral analysis, measurements were made using a cross-wire probe at a fixed location in the wake. As shown in Figure 14, a typical wake profile of \( U \) consists of regions of constant outer velocity, \( U_o \), and a shear layer where \( U < U_o \) due to the interaction of the airfoil with the oncoming flow. The location of the wake half-width, \( y_{1/2} \), is defined according to

\[
U(y_{1/2}) = \frac{U_o + U_{\text{min}}}{2}. \tag{20}
\]

where \( U_{\text{min}} \) is the minimum velocity in the wake. Velocity spectra were measured at \( y = y_{1/2} \) with \( N = 2^{21} \) and \( f_s = 5 \text{ kHz} \). Average power spectra were computed from 256 overlapping segments containing \( 2^{13} \) points, giving a frequency resolution \( \Delta f = 0.61 \text{ Hz} \).
2.7.3 Time-frequency analysis of the reattachment process

The response of the flow to harmonic excitation from the SJA was inferred from velocity measured at a fixed location in the wake. Velocity was measured at $x/c = 1$ and a $y/c$ location corresponding to the edge of the wake (indicated in Figure 14) when the flow is separated and approximately $y_{1/2}$ when the flow reattaches.

The flow was subjected to a control signal that is initially zero and then switched to harmonic excitation at a given $C_\mu$ and $St_e$ at some time $t = \tau$. Velocity data was acquired at $f_s = 5$ kHz from $t = 0$ until $t = 3$ s with $\tau = 1$ s. This was determined to be acceptable from initial measurements. The measurement was repeated 200 times in order to compute a phase-averaged response to the control and eliminate noise. This was accomplished by controlling the Rigol function generator using LabView software.

A continuous wavelet transform was used to analyze the temporal evolution of the velocity power spectra. Wavelet analysis transforms a one-dimensional time-series into a two-dimensional time-frequency image [36]. The wavelet transform was computed using a Morlet wavelet function, viz.

$$\Psi_\omega(\eta) = \pi^{-1/4} e^{i\omega_\omega \eta} e^{-\eta^2/2},$$  \hspace{1cm} (21)

where $\eta$ is a non-dimensional time parameter and $\omega_\omega$ is non-dimensional frequency. The Morlet wavelet (Figure 15) was selected due to the similarity of its shape to the oscillations present in a turbulent flow. The Matlab algorithm provided by Torrence and Compo [36] was used to compute the wavelet power spectrum of velocity (the wavelet equivalent of
Figure 15: Real (solid) and imaginary (dashed) parts of the Morlet wavelet function with $\omega_o = 6$.

a Fourier power spectrum). Rather than phase averaging in the time-domain, the 200 measurements were used to compute phase-averaged wavelet power spectra.
3 Baseline Flow: Sensitivity to $\alpha$ and $Re_c$

The characteristics of laminar separation on low Reynolds number airfoils are highly sensitive to both angle of attack and Reynolds number [28]. Relatively small changes in either parameter may lead to drastic changes in the flow conditions (i.e. laminar separation bubble or fully separated) and performance of the airfoil. Therefore, the reproducibility of the baseline flow conditions may be highly sensitive to $\alpha$ and $Re_c$. The impact of these parameters on the flow topology was investigated by measuring the static pressure distribution over the uncontrolled airfoil surface.

The effect of $Re_c$ on the $C_p$ distribution is presented in Figure 16 (note that data at $x^*/c = 0.76$ on the upper surface was excluded due to a damaged pressure tap). The results indicate that for $5.5 \times 10^4 \leq Re_c \leq 1.25 \times 10^5$, the flow separates and fails to reattach and little change is observed in the pressure distribution. As $Re_c$ is increased to $1.5 \times 10^5$, a significant change in flow topology occurs. The suction peak increases drastically from $C_p \approx -0.5$ to $C_p \approx -1.4$. This is due to the fact that the flow is no longer fully separated and instead a short separation bubble is formed that spans approximately 15% of the airfoil chord. Therefore, in order to significantly modify the flow topology, $Re_c$ would need to be increased by $\sim 150\%$. Since is is unlikely that $U_\infty$ or fluid properties would be incorrectly measured by such a large margin, the uncontrolled flow is not particularly sensitive to Reynolds number at $Re_c = 10^5$.

The variation in the $C_p$ distribution with increasing $\alpha$ is shown in Figure 17, where $Re_c$ is fixed at $10^5$. At low angles of attack, the flow is characterized by a laminar separation bubble on both the upper and lower surface that covers approximately 30% of the chord. However, it is interesting to note that for $\alpha < 4^\circ$, the suction peak on the lower surface exceeds that of the upper surface. This leads to a negative lift coefficient, as demonstrated in Figure 18. This phenomena was also observed by Mueller and Batil [26] for a NACA 663-018 airfoil at $Re_c = 1.3 \times 10^5$. The lift coefficient was calculated as,

$$C_L = \cos \alpha \int_0^1 (C_{p,l} - C_{p,u}) \frac{d\left(\frac{x'}{c}\right)}{1} + \sin \alpha \int_0^1 \left( C_{p,l} \frac{dy'_t}{dx'} - C_{p,u} \frac{dy'_{t,u}}{dx'} \right) \frac{d\left(\frac{x'}{c}\right)}{1}, \quad (22)$$

where the subscripts $l$ and $u$ correspond to the lower and upper surfaces of the airfoil, respectively, and $y'_t$ is the thickness distribution of the airfoil. This formulation of $C_L$ takes into account both the axial and normal forces acting on the airfoil. At $\alpha = 7^\circ$, the flow over the airfoil becomes highly unsteady as stall begins to occur. The airfoil is fully stalled when $\alpha > 7^\circ$ and the flow separates without reattaching, as evidenced
Figure 16: $C_p$ as a function of $Re_c$ at $\alpha = 10^\circ$ ($\circ$ and $\Box$ symbols indicate the upper and lower surfaces, respectively).

by a region of constant static pressure and the sharp decrease in $C_L$ at $\alpha \approx 7^\circ$. These results indicate that at $\alpha = 10^\circ$, the angle of attack would need to be misaligned by approximately $-3^\circ \pm 0.5^\circ$ in order to significantly modify the topology of the uncontrolled flow. While the slope of the $C_L$ curve for $\alpha \geq 8^\circ$ implies a change in the flow with $\alpha$, the establishment of $\alpha = 0^\circ$ by measuring a symmetric pressure distribution whenever the airfoil was reinstalled in the test section ensured that the variation in $\alpha$ was less than approximately $\pm 0.5^\circ$. 
Figure 17: $C_p$ as a function of $\alpha$ at $Re_c = 10^5$ ($\circ$ and $\square$ symbols indicate the upper and lower surfaces, respectively).
Figure 18: $C_L$ as a function of $\alpha$ at $Re_c = 10^5$. The dashed-line is a spline fit to the data points.
4 Synthetic Jet Characterization

Prior to the flow control experiments with the SJA installed in the airfoil, the response of the exit-plane jet velocity to sinusoidal excitation was characterized in quiescent conditions. This was done in order to determine the resonant frequencies of the SJA and to acquire $U_{j\text{rms}}$, a parameter needed for the calculation of $C_\mu$. Phase averaged velocity cycles for each case were computed from 4848 individual cycles with a resolution in phase of 1.8°. The phase averaged velocity, $\langle u_j \rangle$, was converged to within 1% over the entire cycle in all cases.

Figure 19: Synthetic jet velocity for (a)/(b) $E_{\text{app}} = 250$ V and $f_e = 450$ Hz, and (c)/(d) $E_{\text{app}} = 250$ V and $f_e = 970$ Hz. Figures (b) and (d) show $\langle u_j \rangle$ (solid line) and $|E_{fg}|$ (dashed line) normalized by their respective maxima.
Figure 19 shows two typical examples of the velocity measured at the synthetic jet exit plane ($E_{app}$ is the amplitude of the applied voltage). Figures 19a and 19c show the entire velocity signal collapsed onto a single cycle (as described in Section 2.5). The phase angle $\theta$ is measured relative to the input signal from the function generator. The results demonstrate that the synthetic jet operates in a very regular manner with low turbulence, as indicated by the tight collapse of the data. This is especially noticeable in Figure 19a where the velocity magnitude is relatively small ($< 2$ m/s). In Figures 19b and 19d, the phase-averaged jet velocity (solid line) is plotted with the absolute value of the function generator voltage (dashed line). These plots show that, as expected, there is a frequency dependant phase offset between $\langle u_j \rangle$ and $E_{fg}$. The phase offset between jet velocity and driver displacement has been demonstrated both experimentally (e.g., [15]) and numerically (e.g., [30]). The phase angle between the driver velocity and jet velocity at the slot exit is zero at low frequencies due to incompressible flow in the cavity, and increases to $90^\circ$ as the excitation frequency approaches the Helmholtz frequency and the flow becomes compressible. Note that in this case the phase offset is relative to $E_{fg}$, which likely exhibits a constant phase offset from the displacement of the piezoelectric diaphragm.
The response of $\langle U_j \rangle$ to sinusoidal excitation is shown in Figure 20a. Average jet velocities from approximately 0.4 m/s up to 18 m/s were achieved. Solid lines are curve fits of the following equation to the data,

$$\langle U_j \rangle = \frac{K f_n^2}{\sqrt{(f_n^2 - f_e^2)^2 + (2\zeta f_n f_e)^2}},$$ \hspace{1cm} (23)$$

which has the form of the amplitude-frequency response of a second order system with
gain, $K$, damping ratio, $\zeta$, and natural frequency, $f_n$ (note that a DC offset was included to account for the fact that at $f_e = 0$, $\langle U_j \rangle = 0$). Due to the small slot width ($d = 0.5$ mm), the exit velocity profile was not measured and a “top-hat” profile during expulsion was assumed. In reality, $u_j$ can deviate significantly from a top-hat profile and depends both on $h/d$ and $S$. The solution to the fully-developed oscillatory flow in a pipe/channel depends only on $S$ and for $S \gtrsim 5$, the velocity profile is relatively flat [39]. In the present investigation, $S$ ranges from 4.5 to 11. Therefore, if it can be assumed that $h/d = 10$ is large enough for the oscillatory flow to become fully developed, then $u_j(y^*, t) \approx u_j(d/2, t)$.

The frequency response of $\Phi$, the phase offset between $\langle u_j \rangle$ and $E_{fg}$, is shown in Figure 20b. Solid lines are curve fits of the phase response function for a second order system, given by,

$$\Phi = - \tan^{-1} \left( \frac{2\zeta f_n f_e}{f_n^2 - f_e^2} \right) + \Phi_o$$

(24)

where $\Phi_o$ is a constant phase offset.

Gallas et al. [14] used a lumped-element-modelling approach to describe the response of a piezoelectric-driven synthetic jet actuator to a sinusoidal excitation voltage. They showed that the transfer function relating the volume flow rate through the slot to the applied voltage has a denominator that is fourth-order in Fourier space, indicating two resonant frequencies. These frequencies are bounded by, but not equal to, the Helmholz frequency of the cavity and the natural frequency of the piezoelectric diaphragm, viz.

$$f_1 f_2 = f_D f_H,$$

(25)

where $f_1$ and $f_2$ are the resonant frequencies, $f_D$ is the natural frequency of the diaphragm and $f_H$ is the Helmholz frequency. Furthermore, when the ratio of the acoustic compliance of the diaphragm, $C_{aD}$, to the acoustic compliance of the cavity, $C_{aC}$, tends to zero, the SJA behaves as a second-order system with a natural frequency corresponding to the Helmholz frequency of the cavity. The Helmholz frequency for a cavity with a rectangular slot is given by [13],

$$f_H = \frac{1}{2\pi} \sqrt{\frac{5}{6} \frac{w d a_c^2}{V h}},$$

(26)

where $a$ is the isentropic speed of sound in the cavity and $V$ is the volume of the cavity. The acoustic compliance of the diaphragm is given by,

$$C_{aD} = \frac{\pi (D/2)^6 (1 - \nu^2)}{16 E b^3}$$

(27)
The thickness, $b$, was taken as the total thickness of the piezoelectric diaphragm and the diameter, $D$, was taken as the mean diameter. Since the diaphragm is a composite made up of several materials, $E$ and $\nu$ were estimated according to linear mixing rules [12], viz.

$$E = (E\gamma)_{ss} + (E\gamma)_{pzt} + (E\gamma)_{al}, \quad (28)$$

$$\nu = (\nu\gamma)_{ss} + (\nu\gamma)_{pzt} + (\nu\gamma)_{al}, \quad (29)$$

where the subscripts ss, pzt and al refer to the stainless steel, PZT and aluminum layers, respectively. The acoustic compliance of the cavity is defined as

$$C_{ac} = \frac{V}{\rho a^2}. \quad (30)$$

For the SJA used in these experiments $C_{ad}/C_{ac} = 0.003$, therefore a second order response of the volumetric flow rate (or jet velocity) to applied voltage is expected with a strong resonant peak near $f_H$. Figure 20a shows that at each excitation amplitude, a resonant peak is seen at $f_e \approx 970$ Hz.

Estimation of the Helmholtz frequency requires knowledge of the slot height, $h$. In a typical SJA configuration, the slot height is well defined from the rest of the cavity by an abrupt change in cross-sectional area. However, the cross-section of the SJA used in these experiments resembles the schematic shown in Figure 2 (where the slot width is emphasized), and the 0.5 mm wide slot is straight for 5 mm before connecting to the larger cavity area by a smooth curve. If $h$ is taken as 5 mm, the straight portion of the slot, $f_H = 1481$ Hz. This value differs significantly from the peak at $f_e = 970$ Hz and suggests that the effective height of the slot is $> 5$ mm. Assuming $f_1 \approx f_H$, the appropriate value of $h$ in Equation (26) is $\sim 11$ mm, which is reasonable given the dimensions of the cavity. The close agreement of Equation (23) to the mean jet velocity over the range of frequencies investigated suggests that, as expected, the system is predominantly second-order. It should be noted that the fitting parameters in Equation (23), $K$, $\zeta$ and $f_n$, are considered constants, although in reality they are likely to exhibit some dependence on frequency and amplitude due to non-linear effects [14]. While the characterization of the jet was restricted to frequencies below $f_e = 1200$ Hz, a measurement at $E_{app} = 100$ V for $f_e = 50 – 2700$ Hz performed by Goodfellow [18] confirmed that a second resonant peak with significantly lower amplitude is seen at a higher frequency ($\sim 2500$ Hz). The phase response of the jet velocity, Figure 20b, also demonstrates the second order nature of the system. For each excitation amplitude, $\Phi$ closely follows the function given by Equation (24) and a $180^\circ$ phase shift occurs as $f_e$ exceeds $f_1$. However, at frequencies less than 400 Hz there is a noticeable deviation from the trend for all $E_{app}$. This is likely due to non-
linearities present in the SJA that are not accounted for in the lumped element model. For example, $\langle u_j \rangle$ deviates noticeably from a sinusoid for $E_{app} = 250$ V and $f_e = 300$ Hz (Figure 21).

![Figure 21: $\langle u_j \rangle$ for $E_{app} = 250$ V and $f_e = 300$ Hz.](image)

Excitation at $f_e = 970$ Hz provides a broad range of control authority since $\langle U_j \rangle$ can be varied from values that are much smaller than $U_\infty$ (low amplitude excitation) to values near $4U_\infty$. In terms of momentum coefficient, Figure 22a shows that $C_\mu$ spans almost three orders of magnitude at $f_e = 970$ Hz. The jet formation criterion of Holman et al. [22] was evaluated and is shown in Figure 22b, where $K_j = 1$ is indicated by the solid line. The results suggest that near the resonant peak, a jet was formed for all voltage amplitudes except $E_{app} = 50$ V. Therefore, $f_e = 970$ Hz was selected as the optimal excitation frequency of the SJA. A low-frequency excitation strategy will be discussed in Section 6.

### 4.1 Jet velocity spanwise uniformity

The variation in $\langle U_j \rangle$ along the span of the slot ($z^*$-direction) is shown in Figure 23a. The jet velocity was normalized by the spanwise mean, $\langle U_j \rangle_{ave}$, in order to compare the two cases that were considered: $f_e = 400$ Hz and $f_e = 950$ Hz at $E_{app} = 250$ V. These actuation parameters were selected to investigate the effect of $f_e$ on the spanwise uniformity of $\langle U_j \rangle$. Every other point is located at the centreline of piezoelectric actuator (i.e. $z^*/w = 0.11, 0.37, 0.63$ and $0.89$). The results demonstrate that, as expected, there is a decrease in velocity in the regions between the actuators and at the slot edges. Excitation
frequency seems to have no effect on the spanwise uniformity, as evidenced by the collapse of the two data sets. Furthermore, the magnitude of velocity appears to have no effect on the spanwise variation in $\langle U_j \rangle$ since for $f_e = 400$ Hz and $f_e = 950$ Hz, $\langle U_j \rangle_{ave} = 0.75$ m/s and 6.90 m/s, respectively. However, Figure 23a also shows a reduction in $\langle U_j \rangle$ around the actuator at $z^*/w = 0.63$. This discrepancy was originally thought to be due to poor performance of the piezoelectric actuator at $z^*/w = 0.63$. Therefore, the “faulty” actuator was removed and the SJA was reassembled with a new actuator. The two SJA configurations are shown in Figure 24. The remaining actuators from Configuration 1 were replaced in different locations in order to eliminate the actuators themselves as the source of the velocity deficit.

The new SJA configuration was tested for four cases under different operating parameters, given in Table 4. These operating parameters were selected such that $\langle U_j \rangle_{ave} \approx 1$ m/s for Case 1 and 2, and $\langle U_j \rangle_{ave} \approx 5$ m/s for Case 3 and 4. This allowed proper investigation of the effects of excitation frequency and velocity magnitude on $\langle U_j \rangle(z^*)$. The results in Figure 23b clearly demonstrate that for all four cases, there is excellent agreement in the spanwise velocity profile. These measurements confirm that velocity magnitude and excitation frequency do not impact the spanwise uniformity of $\langle U_j \rangle$. More importantly, it is obvious from these results that the velocity deficit near $z^*/w = 0.63$ was not the result of Piezo 3 (Figure 24) or the other actuators, since a similar deficit is apparent in Figure 23b.

In an attempt to further explore the deficit in $\langle U_j \rangle(z^*)$, it was hypothesized that a
Figure 23: $\langle \bar{U}_j \rangle$ as a function of $z^*/w$. $\langle \bar{U}_j \rangle$ is normalized by the spanwise mean, $\langle \bar{U}_j \rangle_{ave}$, to highlight the relative variation in velocity. Dashed lines indicate ±10%.

Table 4: SJA operating parameters for spanwise $\langle \bar{U}_j \rangle$ measurements.

<table>
<thead>
<tr>
<th>Case</th>
<th>$E_{app}$ (V)</th>
<th>$f_e$ (Hz)</th>
<th>$\langle \bar{U}<em>j \rangle</em>{ave}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>850</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>1150</td>
<td>1.05</td>
</tr>
<tr>
<td>3</td>
<td>275</td>
<td>850</td>
<td>5.22</td>
</tr>
<tr>
<td>4</td>
<td>275</td>
<td>1150</td>
<td>4.96</td>
</tr>
</tbody>
</table>

difference in phase between the piezoelectric actuators could be the cause. The actuators are all driven by the same function generator/amplifier, therefore in theory they should be operating in phase. The variation in $\Phi$ with $z^*/w$ is shown in Figure 25 for each SJA configuration. The spanwise mean, $\Phi_{ave}$, was removed to compare the different cases with drastically different values of $\Phi_{ave}$. The results show that within approximately ±5°, $\langle \bar{U}_j \rangle$ is in phase for all $z^*$. There are several outliers near ±10°, however there is no consistent trend in $\Phi$ near $z^*/w = 0.63$ for any of the cases considered.

The preceding results indicate that the operating parameters of the SJA and the piezoelectric diaphragms themselves are not the cause of the localized deficit in $\langle \bar{U}_j \rangle$. The only remaining explanation is that the geometry of the device is the cause. Several options are possible: non-uniform slot width, variations in the geometry of the cavity, or non-uniform clamping of the piezoelectric disks. Non-uniform clamping of the disks is unlikely due to the fact that a torque-wrench was used to ensure that all bolts clamping...
the SJA together were given the same torque. The most obvious cause is non-uniformity of the slot width. The slot is nominally 0.5 mm wide and is therefore subject to potentially significant variations due to machining tolerances. In order to verify the uniformity of the slot width, an optical procedure was devised to measure $d(z^*)$. A DSLR camera equipped with a macro lens was mounted on a tripod and the SJA was clamped in a rigid stand. A section of measuring tape with 1 mm increments was fixed parallel with the top edge of the slot. The camera was aligned such that the lens was as parallel as possible with the face of the slot. A photo of the setup is shown in Figure 26. By traversing the SJA with the camera fixed in place, photos were taken of the slot. These photos are shown in Figure 27.

Figure 24: Piezoelectric actuator configurations. Configuration 2 was tested after the velocity deficit near $z^*/w = 0.63$ was observed.
Figure 25: Φ as a function of $z^*/w$. The spanwise mean, $Φ_{ave}$, is removed in order to compare the deviation from the mean.

Figure 26: Picture of the setup used to optically measure the width of the SJA slot.

The photos of the slot were imported into Matlab and converted to greyscale in order to determine the spanwise variation of $d$. Using the measuring tape for scale, the resolution of the images was determined to be 0.0148 mm/pixel. The slot was then identified as any pixels that fell below a threshold intensity level. The threshold was selected such that $d_{ave} \approx 0.5$ mm. The results for each of the four images are shown
in Figure 27 (note that two of the pictures were cropped since only a portion of these images were required).

![Graphs showing synthetic jet slot width, d, as measured using photos from a DSLR camera.](image)

Figure 27: Synthetic jet slot width, d, as measured using photos from a DSLR camera. The photos used for the image analysis are included below the corresponding plot of d versus z*/w.

The results for each of the four images were combined and the profile of d(z*) was smoothed using a 100 point (~1% of the data set) moving average, as shown in Figure 28. The black and red dashed lines indicate the standard deviation σ_d = 0.02 mm and 2σ_d = 0.04, respectively. Assuming that the local jet velocity is inversely proportional to the slot width, an increase in d at the location of the velocity deficit would suggest that the variation in slot width is the cause. The slot width appears to be uniform within approximately ±10%, but more importantly there is no trend in d near z*/w = 0.63. It is likely that this variation in d is a contributing factor to the spanwise velocity, however the relationship between slot width and jet velocity is not straightforward due to the complex dynamics of the synthetic jet.
In summary, the localized velocity deficit along the span of the synthetic jet slot was not caused by the following parameters: velocity magnitude, excitation frequency, phase offset or the slot cross-sectional area. Other contributing factors may include geometry imperfections and acoustic interactions inside the cavity, however this could not be verified with the available equipment. In the design of future synthetic jet actuators, it would be useful to use numerical simulations with a compressible flow solver to predict the spanwise jet velocity profile. This would also provide a means of quantifying the sensitivity of the jet velocity to local variations in the slot width.
Flow control using harmonic excitation was achieved by operating the synthetic jet at the resonant frequency of the jet, $f_e = 970$ Hz, which corresponds to a Strouhal number $St_e = 58$. Since $St_e \approx O(10)$, the time scale of actuation was an order of magnitude lower than the characteristic time scale of the flow over the airfoil. Therefore, the excitation is at “high frequency” relative to the flow.

Mean streamwise velocity ($U$) profiles in the wake of the airfoil at $x/c = 2$ for increasing $C_\mu$ are shown in Figure 29. The streamwise velocity was normalized by the average outer velocity, $U_o$. The average outer velocity is defined as

$$U_o = \frac{U_u + U_l}{2},$$

where $U_u$ and $U_l$ are the velocities at the upper and lower edges of the measurement plane, respectively. $C_\mu$ was varied from $2.10 \times 10^{-4}$ to $4.62 \times 10^{-2}$ by increasing the applied voltage from $E_{app} = 50$ V to $350$ V for harmonic excitation at $St_e = 58$. In the uncontrolled case, the wake extends from approximately $y/c = -0.25$ to $0.7$, giving a wake width of $0.95c$. As expected, the cross-stream location of the minimum velocity is above the trailing edge of the airfoil since the boundary layer on the suction surface separates near the leading edge and fails to reattach. The minimum velocity $U_{min} = 0.85U_o$ occurs $0.28c$ above the trailing edge. It should also be noted that the velocity near $y/c = 1$ (above the suction surface of the airfoil), $U_u$, is larger than the velocity near $y/c = -0.8$, $U_l$, by approximately 4%. This result is expected and is a consequence of the circulation generated by the airfoil. Further downstream, it would be expected that $U_u = U_l = U_\infty$.

For $C_\mu = 2.10 \times 10^{-4}$ and $1.52 \times 10^{-3}$, the flow remains separated and the mean velocity profile is relatively unchanged. However, when $C_\mu = 4.16 \times 10^{-3}$, the wake is deflected down such that $U_{min}$ occurs at $y/c \approx 0$ and the wake width decreases to $0.57c$. The minimum velocity also increases slightly to $U_{min} = 0.88U_o$. These changes are indicative of a flow that is at least partially reattached. For $C_\mu = 8.15 \times 10^{-3}$, $U_{min}$ remains located at $y/c \approx 0$ and the wake width increases to approximately $0.83c$. This suggests that the flow is still attached, however the control is slightly less effective than it was at $C_\mu = 4.16 \times 10^{-3}$. For $C_\mu = 1.35 \times 10^{-2}$ and $2.51 \times 10^{-2}$, no significant change in the $U$ is observed. At the largest momentum coefficient considered, $C_\mu = 4.62 \times 10^{-2}$, the wake width decreases once again to $\sim 0.54c$.

In order to confirm that the changes in $U$ when $C_\mu \geq 4.16 \times 10^{-3}$ corresponded to reattached flow, flow visualization was performed using a smoke-wire technique. A thin 0.076 mm diameter stainless steel wire was used to provide adequate smoke density.
while limiting the disturbance introduced to the flow. The Reynolds number based on wire diameter was approximately 25 and presented no appreciable disturbance to the flow since the Reynolds number was below the value of 47 required for laminar vortex shedding of a cylinder. The wire was installed along the centreline of the SJA $\sim 0.5c$ upstream of the airfoil leading edge and passed through the ceiling and floor of the test section. Oil applied to the wire was inductively heated using a variable transformer, which produced clear streaklines in the flow. The flow was illuminated using a remote flash focused into a thin slit that was positioned downstream of the airfoil model. Figure 30 shows the flow visualization images for each excitation amplitude. The results demonstrate that for $C_\mu \leq 1.52 \times 10^{-3}$, the flow remains separated and large vortical structures are apparent in the wake. As $C_\mu$ is increased above $1.52 \times 10^{-3}$, the flow becomes attached to the
suction surface of the airfoil and a noticeably narrower wake is observed. These images confirm that the changes $U$ described above correspond to reattached flow.

As mentioned in Section 1.4, the fluctuating velocity term in (13) can be ignored when $u' \approx v'$ over the entire measurement plane. Figure 31 shows $\overline{u'}$ and $\overline{v'}$ measured at $x/c = 2$ for (a) the baseline case with separated flow, and (b) $C_\mu = 4.16 \times 10^{-3}$ and (c) $C_\mu = 2.51 \times 10^{-2}$ where the flow is attached. Since $\overline{u'}$ and $\overline{v'}$ are approximately equal over the span of the measurement plane in each case, the fluctuating term in (13) can be neglected. Therefore, $C_D$ is calculated using only the momentum term, viz.

$$C_D = -\frac{2}{U_o^2 c} \int_{y_1}^{y_2} U(U - U_o) \, dy.$$  \hfill (32)

The difference between $C_D$ calculated using (13) and (32) is 3% for $C_\mu = 0$ (Figure 31a) and 4% for $C_\mu = 2.51 \times 10^{-2}$ (Figure 31c).
Figure 30: Smoke wire flow visualization in the near wake of the airfoil.
The impact of excitation at $St_e = 58$ on the airfoil drag coefficient, $C_D$, for increasing $C_\mu$ is shown in Figure 32. $C_D$ is normalized by $C_{D_0}$, the drag coefficient of the unexcited case ($C_\mu = 0$), to highlight the relative change in drag. Figure 32 displays little change in $C_D$ for $C_\mu < 4.16 \times 10^{-3}$, since the flow remains separated. As $C_\mu$ is increased to $4.16 \times 10^{-3}$ and the flow reattaches, a significant decrease in $C_D$ of $\sim 45\%$ is observed. Further increasing $C_\mu$ to the largest value considered, $4.62 \times 10^{-2}$, causes an additional change of $\sim 10\%$, although it initially increases for intermediate values of $C_\mu$. This modest change in $C_D$ for higher $C_\mu$ is due to the fact that the flow has already been attached and suggests that the effectiveness of excitation at high $St$ on drag reduction depends primarily on exceeding a threshold value of $C_\mu$. Included in Figure 32 are the
results of Goodfellow et al. [19] at $\alpha = 5^\circ$ on the same airfoil for $Re_c = 10^5$. These results show a similar trend and also demonstrate that the threshold $C_\mu$ is approximately equal for $\alpha = 5^\circ$ and $10^\circ$, however a larger decrease in drag is possible at $5^\circ$. This is likely due to the fact that a more severe adverse pressure gradient is present on the suction surface of the airfoil at $\alpha = 10^\circ$.

Figure 32: $C_D$ as a function of $C_\mu$ for ($\circ$) $\alpha = 10^\circ$ and ($\square$) $\alpha = 5^\circ$ with $St_e = 58$. Data for $\alpha = 5^\circ$ are from Goodfellow et al. [19]. Error bars represent the relative measurement uncertainty.

5.1 Spectral Analysis

In addition to mean streamwise velocity profiles, spectral analysis of the velocity in the airfoil wake provides useful information regarding the temporal nature of the flow, i.e., steadily or unsteadily reattached. The flow visualization in Figure 30 shows evidence of smaller vortical structures in the wake when the flow is reattached. It was expected that these structures would create a noticeable peak in the power spectra of velocity. However, initial spectra computed from measurements at $x/c = 2$ showed no evidence of a spectral peak. Since it is possible that the vortices lost sufficient coherence to be detected at $x/c = 2$, spectral measurements were performed at several $x/c$ locations closer to the trailing edge for $C_\mu = 4.62 \times 10^{-2}$. At each streamwise location, the mean velocity profile was initially measured in order to determine $y_{1/2}$. The $y_{1/2}$ value was determined according to Equation (20) by applying a cubic spline fit to the data points.
The streamwise velocity profiles are shown in Figure 33 (successive $x/c$ profiles are offset by $0.1U$ for clarity). It was also postulated that for the narrower wake associated with the reattached flow, the spectral content of velocity may be sensitive to the transverse measurement location and this may have contributed to the lack of a spectral peak in the initial measurements.

Figure 33: Mean streamwise velocity at ($\circ$) $x/c = 0.75$, ($\square$) $x/c = 1$, ($\Diamond$) $x/c = 1.5$ and ($\triangledown$) $x/c = 2$ for $C_\mu = 4.62 \times 10^{-2}$. Successive profiles are offset by $0.1U$ for clarity.

The power spectra of $u$ and $v$ at $x/c = 0.75$, 1, 1.5 and 2 for $C_\mu = 4.62 \times 10^{-2}$ are shown in Figure 34 (successive spectra are stepped by an order of magnitude for clarity). Note that the power spectra are presented normalized by the variance of velocity. These results demonstrate a well defined spectral peak at $St = 4.86$ in $E_{vv}$ that was nearly indistinguishable at $x/c = 2$ but grows in magnitude with decreasing $x/c$, which confirms the loss of coherence as the vortical structures move downstream. Figure 34 also demonstrates that the peak at $St = 4.86$ is more apparent in the spectra of $v$ than $u$. At $x/c = 1$, the peak is difficult to identify in $E_{uu}$ and is not visible for $x/c \geq 1.5$. For the remainder of this work, only the power spectra of $v$ are considered.
Figure 34: PSD of $u$ and $v$ measured at $y_{1/2}$ at $x/c = 0.75, 1, 1.5$ and $2$ for $C_\mu = 4.62 \times 10^{-2}$ and high-frequency excitation at $St_e = 58$. Successive spectra are stepped by an order of magnitude for clarity.

The sensitivity of the power spectral density of $v$ to the $y$ measurement location was investigated by performing measurements at $x/c = 1$ from $y/c = -0.22$ to $-0.015$ in increments of $0.042c$, as shown in Figure 35. This range spanned the portion of the shear...
layer below the minimum velocity location. Included in Figure 35 is the profile of $U$ with the measurement locations indicated. Below the shear layer at $y/c = -0.22$, the power spectra is relatively flat before beginning to decay at $St \approx 2$ and only a sharp peak associated with the excitation frequency is observed. As $y/c$ is increased to $-0.18$, the decay is delayed slightly and there are no distinguishable peaks other than the now attenuated peak at $St_e$. A peak begins to emerge at a Strouhal number of $St = 4.86$ as the measurement location enters the shear layer at $y/c = -0.14$. This peak grows in amplitude as $y/c$ is increased and is easily identified at $y/c = -0.098$ and $-0.057$. The peak becomes attenuated and is almost indistinguishable as the measurement location nears the minimum velocity point. These measurements confirm that $y_{1/2}$ is the appropriate location for analyzing the spectral content of the velocity in the wake of the airfoil. The frequency characteristics of the uncontrolled and controlled airfoil wake were investigated further by performing measurements at $x/c = 1$ and the appropriate $y_{1/2}$ locations. Reliable cross-wire measurements at $x/c = 0.75$ were not possible when the flow was separated ($C_\mu \leq 1.52 \times 10^{-3}$) due to large flow angularity.

Figure 35: PSD of $v$ at $x/c = 1$ for $C_\mu = 4.62 \times 10^{-2}$ and high-frequency excitation at $St_e = 58$. Measurement locations are $y/c = -0.22, -0.18, -0.14, -0.098, -0.057$ and $-0.015$. The inset velocity profile shows the transverse measurement locations corresponding to the power spectra of matching colour.
The power spectra of the cross-stream velocity at $x/c = 1$ are shown in Figure 36. When the flow is fully separated, a broad peak associated with vortex shedding in the wake is centred at $St_{ws} = 0.84$. This value agrees with the typical separated wake frequency $St_{ws} \approx \mathcal{O}(1)$ that is reported in the literature (e.g. [2; 44]). The large scale vortex shedding associated with $St_{ws}$ can be seen in the smoke wire images in Figures 30(a), (b) and (c). As the flow becomes reattached at $C_{\mu} = 4.16 \times 10^{-3}$, the broad peak at $St_{wa}$ is flattened and the decay in the spectra is delayed. From classical scaling arguments, the dominant frequency associated with the wake, $St_{w}$, should scale as $St_{w} \sim U_{\infty}/W$ where $W$ is the width of the wake [35]. Therefore, as the flow over the airfoil is reattached and the wake becomes narrower, the dominant frequency of the wake is expected to increase. An example of this behaviour was demonstrated by Yarusevych et al. [44], where the flow over a NACA 0025 airfoil at $\alpha = 5^\circ$ was “naturally” reattached by increasing $Re_c$. Figure 36 shows that as $C_{\mu}$ is increased beyond the threshold value, a distinct peak in the spectra associated with the attached flow begins to emerge at $St_{wa} = 4.86$ when $C_{\mu}$ reaches $1.80 \times 10^{-2}$. Evidence of these structures with reduced spatial scale can be seen in Figures 30(g) and (h). If the wake structures can be approximated as sinusoidal waves, the frequency of the wave is related to the wavelength by

$$f = \frac{U_c}{\lambda},$$

(33)

where $U_c$ is the convection speed and $\lambda$ is wavelength. Assuming that $U_c$ remains constant, the ratio of the frequencies (or $St_{w}$) of the attached and separated flows can be approximated as $St_{wa}/St_{ws} = \lambda_{ws}/\lambda_{wa}$. As shown in Figure 29, the streamwise velocity only increases slightly in magnitude when the flow is reattached, therefore the assumption of constant $U_c$ is valid. Included in Figures 30(a) and (h) are estimates of $\lambda_{ws} \text{ and } \lambda_{wa}$ from the flow visualization images, giving $\lambda_{ws}/\lambda_{wa} = 2.6$. The estimate of $\lambda_{ws}/\lambda_{wa} = 2.6$ compares reasonably well with $St_{wa}/St_{ws} = 5.8$ determined from the power spectra.

### 5.2 Time-frequency analysis of flow reattachment

Temporal characteristics of the reattachment process were investigated by measuring the frequency content of the wake subjected to a control signal that is initially zero and then switched on at $C_{\mu} = 4.62 \times 10^{-2}$ at some time $t = \tau$. An example of this control signal is shown in Figure 37.

The velocity was measured at $x/c = 1$ and $y/c = -0.07$, a location that corresponds to the edge of the shear layer when the flow is separated and the half-width of the wake when the flow is attached. Figure 38 shows the phase-averaged magnitude of the
wavelet power spectrum for the cross-stream velocity component, $W_{vv}$. Phase-averaged
wavelet spectra were computed from 200 independent measurements. The wavelet power
spectrum is plotted against a normalized time variable,

$$ t' = \frac{(t - \tau)U_\infty}{L}, \quad (34) $$

where $L$ is the streamwise distance from the synthetic jet to the measurement location.
This normalization was selected to highlight the convective time scale of the flow. Included in the plot is a dashed line showing the cone of influence (COI), below which edge
effects due to the finite size of the time series become significant. As expected, a distinct
peak at $St_{ws}$ is observed prior to $t' = 0$. After control is switched on, a noticeable change
in the wake is observed at $t' = 1.3$ by a momentary attenuation of frequencies from $St = 3$ to 6. This is followed by a sharp increase in energy at $St_{ws}$ for $t' = 1.6$ before the vortex shedding associated with the separated flow is completely suppressed. The flow reaches a steady state after an increase in $W_{vv}$ for frequencies from $St \approx 2$ to 4.2 for $t' \approx 3.2$ to 6. Assuming that the time required for information to reach the hot-wire from the actuator is $L/U_\infty$, then any temporal variations in $W_{vv}$ beyond $t' = 1$ are due to the reattachment process. This gives a reattachment time scale of $5L/U_\infty$. In applications involving flow separation, a non-dimensional time variable, $t^+$, is conventionally defined using $U_\infty$ and $X_{sep}$. The time scale of reattachment in terms of $t^+$ is 10.9. This compares well with the investigations of Siauw et al. [32] and Amitay and Glezer [1] who found $t^+ = 9.5$ and $t^+ = 10$, respectively. The spectral peak associated with the reattached flow at $St_{wa} = 4.86$ is not observed in Figure 38, which is likely due to the resolution in frequency ($St$). The transient behaviour of the peak at $St_{ws}$ has been observed in other investigations and is caused by the starting vortex that occurs during the reattachment process as a result of the step change in circulation when control is initiated [2; 32]. The results shown in Figure 38 also confirm that steady reattachment occurs when high frequency excitation is employed.
Figure 38: Wavelet power spectrum of $v$ at $x/c = 1$ for excitation with $St_e = 58$ and $C_\mu = 4.62 \times 10^{-2}$. 
6 Aerodynamic Control: Low Frequency

6.1 Burst modulated excitation

Low frequency excitation was used to target the two characteristic frequencies associated with the separated flow: $St_{ws}$ and $St_{sl}$, the separated shear layer frequency. Yarusevych et al. [43] found $St_{sl} \approx 9.9$ for a NACA 0025 airfoil at $\alpha = 10^\circ$ and $Re_c = 10^5$. Figure 22a demonstrates that below $f_e = 200$ Hz ($St_e = 12$), $C_\mu$ is relatively small and varies little with voltage amplitude, placing these frequencies outside the effective bandwidth of the SJA. To target low frequencies that are $St \approx O(1)$, burst modulation of the high-frequency excitation at $St_e = 58$ was used. A burst modulated waveform is composed of a sine wave at a carrier frequency $f_c$ ($St_c$) that is modulated by a square wave at $f_m$ ($St_m$) with duty cycle DC, where $f_m < f_c$. The square wave varies between 0 and 1 such that the sine wave occurs in “bursts”. Figure 39 shows an example of a burst modulated harmonic control signal. This signal was created using a built-in feature in the Rigol function generator.

![Burst modulated sine wave](image.png)

Figure 39: Burst modulated sine wave with $f_c/f_m = 10$ and $DC = 50\%$.

Figure 40 shows typical power spectra of the SJA exit-plane jet velocity for burst modulated excitation with (a) $f_c/f_m = 60$ and (b) $f_c/f_m = 54.5$. These two cases were considered in order to determine whether $f_c$ not being an integer multiple of $f_m$ significantly effects the power spectra of $u_j$. Distinct peaks in the spectra are seen at both $f_m$ and $2f_c$, along with the harmonics of $f_m$ ($2f_m, 3f_m$, etc.). Note that due to rectification of the hot-wire signal in the time-periodic reversing flow of the jet, the
frequency of the carrier signal appears to be $2f_c$ rather than $f_c$ (this was also observed for harmonic excitation of the SJA). The non-linear interaction of the square and sinusoidal waves is apparent by the peaks observed at $2f_c \pm f_m$, $2f_c \pm 2f_m$, etc. As shown in Figure 40, $f_c$ being an integer multiple of $f_m$ has little effect on the spectra. Increasing DC from $\sim 2\%$ to $\sim 50\%$ increases the power contained at higher frequencies, but $E_{uu}$ near $f_m$ remains relatively unchanged. These results clearly demonstrate that despite $f_c \gg f_m$, there is significant energy contained at the modulation frequency. This makes burst modulation a viable technique for targeting frequencies below the bandwidth of the SJA.

![Graph](image)

(a) $f_c = 600$ Hz, $f_m = 10$ Hz, and $E_{app} = 200$ V.

![Graph](image)

(b) $f_c = 600$ Hz, $f_m = 11$ Hz, and $E_{app} = 200$ V

Figure 40: PSD of $u_j$ measured at the jet exit plane using burst modulated excitation.
6.2 Wake measurements

The effect of low frequency excitation on the mean streamwise velocity in the wake is shown in Figures 41 and 42. The carrier frequency and duty cycle were fixed at $St_c = 58$ and DC=50%. Similar to high frequency excitation, excitation at $St_m = St_{ws} = 0.84$ and $St_m = St_{sl} = 9.9$ lead to reattached flow with a narrower wake that was shifted down towards $y/c = 0$ and had a larger value of $U_{\text{min}}$ when $C_\mu$ exceeded a threshold value. For $St_m = 0.84$, the flow reattaches at $C_\mu = 1.52 \times 10^{-3}$, which is 63% less than the threshold $C_\mu$ for harmonic excitation at $St_c = 58$. The smallest wake width was achieved at $C_\mu = 1.52 \times 10^{-3}$. Increasing $C_\mu$ to $4.16 \times 10^{-3}$ caused the wake to widen slightly, however as $C_\mu$ continued to increase there was relatively little change in the shape of the velocity profile. As $St_m$ was increased to 9.9, the flow reattached when $C_\mu = 2.10 \times 10^{-4}$, the lowest momentum coefficient considered and an order of magnitude smaller than what was required for high frequency harmonic excitation. Compared to the larger values of $C_\mu$, the wake profiles at $C_\mu = 2.10 \times 10^{-4}$ and $1.52 \times 10^{-3}$ are slightly wider. No significant change is observed in the mean velocity profiles for $C_\mu \geq 4.16 \times 10^{-3}$.

For both $St_m = 0.84$ and $St_m = 9.9$, flow reattachment causes the minimum velocity to increase to the same value that was noted for high frequency excitation, $U_{\text{min}} \approx 0.88 U_o$.

Power spectra of the cross-stream velocity in the wake of the airfoil for burst modulation at $St_m = 0.84$ and $St_m = 9.9$ are shown in Figures 43 and 44, respectively. The modulation and carrier frequencies are indicated using arrows placed along the $St$ axis. As shown in Figure 43, the vortex shedding at $St_{ws}$ persists for $C_\mu \geq 4.16 \times 10^{-3}$, which suggests that the flow reattachment is unsteady when $St_m = 0.84$. Furthermore, the spectral peak at $St_wa$ that was evident for high frequency excitation is not seen. The narrowing of the initially broad peak at $St_{ws}$ demonstrates that excitation at $St_m = 0.84$ organizes the separated wake instability. This unsteady reattachment and organized vortex shedding in the wake was noted by Amitay and Glezer [1] and leads to time-periodic changes in circulation, and therefore unsteady aerodynamic forces. The authors also showed that excitation at $St \approx \mathcal{O}(1)$ causes the formation of large vortical structures that convect downstream near the airfoil surface. As $C_\mu$ increases, the peak at $St_m$ increases in magnitude and harmonics up to $4St_m$ emerge. The frequency content of the wake is drastically different for excitation at $St_m = 9.9$ (Figure 44). No evidence of coherent structures at $St_m$ is observed in the wake, and the same peak associated with attached flow for high frequency excitation, $St_wa = 4.86$, begins to emerge and grow in magnitude with increasing $C_\mu$. This suggests that the flow is steadily reattached. It is also interesting to note that the peak at $St_wa$ begins to appear at $C_\mu = 8.15 \times 10^{-3}$, compared with $C_\mu = 2.51 \times 10^{-2}$ for high-frequency excitation (Figure 36). This may be due to
more efficient organization of the wake when forcing is applied at $St_m = St_{sl}$. The power spectra suggest that modulation at $St_{sl}$ appears to be more effective in suppressing the large-scale vortex shedding in the wake associated with the separated flow.

![Graphs showing mean streamwise velocity at $x/c = 2$ for varying $C\mu$](image)

Figure 41: Mean streamwise velocity at $x/c = 2$ for $St_m = 0.84$ and varying $C\mu$. 

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Figure 42: Mean streamwise velocity at $x/c = 2$ for $St_m = 9.9$ and varying $C_\mu$.

Figure 45a compares the variation in $C_D$ with $C_\mu$ for the three excitation frequencies considered in this work. For $St_m = 0.84$, there is an initial increase in $C_D$ due to the organization of the vortex shedding at $St_{ws}$. As $C_\mu$ is increased to $1.52\times10^{-3}$, the threshold momentum coefficient required for reattachment is reached and $C_D$ is decreased by $\sim30\%$. A similar decrease in $C_D$ is observed for $St_m=9.9$, however it is achieved for the lowest $C_\mu$ considered, $2.10\times10^{-4}$. At $C_\mu=4.16\times10^{-3}$, all three excitation strategies reach a minimum value of $C_D$ and continuing to increase $C_\mu$ has a relatively small effect.
Figure 43: PSD of $v$ at $x/c = 1$ for increasing $C_\mu$ and burst modulated excitation at $St_m = 0.84$. Excitation frequencies are indicated on the abscissa.

Harmonic excitation at $St_e=58$ performs marginally better at the largest value of $C_\mu$, however the energy required is double that of burst modulation with $DC=50\%$. The results in Figure 45a demonstrate that appreciable drag reduction can be achieved for a smaller threshold $C_\mu$ when burst modulation is employed at either $St_m = St_{ws}$ or $St_m = St_{sl}$. However, the results also suggest that the value of $C_\mu$ required for maximum drag reduction is independent of $St_m$. A possible explanation for the common minimum in $C_D$ for all excitation strategies may be inferred from the ratio of the average jet velocity to the freestream velocity, as shown in Figure 45b. While the threshold $C_\mu$ values for drag reduction occur at arbitrary values of $\langle U_j \rangle / U_\infty$, the minimum drag occurs at approximately $\langle U_j \rangle / U_\infty = 1$. Increasing $\langle U_j \rangle$ beyond $U_\infty$ initially causes a small increase in $C_D$. These results imply that while flow reattachment depends on primarily on meeting a threshold value of $C_\mu$, the ratio $\langle U_j \rangle / U_\infty$ could play an important role.
towards reaching maximum drag reduction. This concept could be investigated further by performing similar measurements at a constant angle of attack and varying Reynolds number. These experiments would be useful in determining whether $C_\mu$ or $\langle \overline{U_j} \rangle / U_\infty$, typically referred to as the blowing ratio, is the more important parameter for post-stall flow reattachment.
Figure 45: $C_D$ as a function of (a) $C_\mu$ and (b) $\langle \tilde{U}_j \rangle / U_\infty$ for $St_e=58$, $St_m=0.84$ and $St_m=9.9$. 
7 Conclusions

Synthetic jet actuation was used to mitigate flow separation and improve the aerodynamic performance of a stalled NACA 0025 airfoil operating at $\alpha = 10^\circ$ and $Re_c = 10^5$. Two actuation strategies were considered: high-frequency and low-frequency excitation. Low-frequency excitation was used to excite the natural instabilities present in the fully separated flow, viz. the local shear layer instability ($St_{sl}$) and global wake instability ($St_{ws}$). For each actuation strategy, the effect of $C_\mu$ on the wake velocity and drag coefficient was investigated.

Prior to performing flow control experiments, the exit-plane velocity of the synthetic jet was characterized in quiescent conditions. The results demonstrated that the SJA behaved approximately as a second-order system with a resonant peak at $f_e = 970$ Hz ($St_e = 58$). Time averaged jet velocities up to $\sim 18$ m/s were obtained at this frequency and $C_\mu$ varied from $2.10 \times 10^{-4}$ to $4.62 \times 10^{-2}$ with increasing voltage amplitude. The spanwise variation in $U_j$ was measured under several operating conditions and in each case, a consistent spanwise velocity profile was observed. A velocity deficit was found over a section of the jet span, however the results demonstrated that it was not caused by actuation parameters, the piezoelectric diaphragms or the slot geometry.

High frequency excitation was performed at $St_e = 58$, the lowest resonant frequency of the SJA. As $C_\mu$ was increased to $4.16 \times 10^{-3}$, a threshold value of $C_\mu$ was reached that caused the wake to narrow and shift downwards towards the trailing edge. These changes corresponded to steady reattachment of the boundary layer to the airfoil surface. This reattachment was accompanied by a decrease in $C_D$ of $\sim 45\%$. The peak in power spectra of the cross-stream velocity at $St_{ws} = 0.84$ was suppressed for the reattached flow and at large values of $C_\mu$, a new peak associated with smaller-scale vortex shedding in the wake at $St_{wa} = 4.86$ emerged. Prior to suppression of the large scale shedding in the wake at $St_{ws}$, a momentary increase in vortex strength occurs that is likely due to a step change in circulation.

Excitation at the frequency of the wake instability ($St_m = 0.84$) was found to be slightly more effective than high $St_e$ excitation, however the large scale vortex shedding at $St_{ws}$ is no longer suppressed and becomes more organized. At $St_m = 9.9$, the flow was reattached for $C_\mu = 2.10 \times 10^{-4}$, an order of magnitude less than the threshold $C_\mu$ for high frequency excitation. The large-scale vortex shedding in the wake was suppressed for $St_m = 9.9$. For both $St_m = 0.84$ and 9.9, $C_D$ was decreased by $\sim 30\%$ once the threshold $C_\mu$ was achieved. Interestingly, both high-frequency and low-frequency excitation caused a local minimum in $C_D$ at $C_\mu = 4.16 \times 10^{-3}$ where $\langle U_j \rangle / U_\infty \approx 1$. The duty cycle was
fixed at 50% for this work, however the effect of this parameter on flow reattachment is also of interest since the energy required by the SJA scales with DC.
References


