SELECTING AND IMPLEMENTING RICH MATHEMATICS TASKS
IN THE MIDDLE SCHOOL

by

Lori Jane Henhaffer

A thesis submitted in the conformity with the requirements
For the degree of Master of Arts
Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education
University of Toronto

© Copyright by Lori Jane Henhaffer 2014
Selecting and Implementing Rich Mathematics Tasks in the Middle School

Lori Jane Henhaffer
Master of Arts, 2014

Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education, University of Toronto

Abstract

This collective case study of two middle school mathematics teachers examines how teachers select and implement rich mathematics tasks. The research revealed that these two teachers tend to select rich mathematics tasks from a variety of sources that meet individual needs, conform to assessments and align to characteristics of rich mathematics tasks. These two teachers perceive rich mathematics task to be open-ended, cross-curricular, student-centered, to have multiple steps, to encourage a variety of solutions and solution strategies and to support high-levels of cognitive thinking processes.

Factors identified to support rich mathematics task implementation include connecting procedures with mathematical concepts, modeling student-made solutions, enforcing scaffolding strategies and pressing for justification. Factors identified to inhibit rich mathematics task implementation include teachers overlooking opportunities to press for conceptual understanding, student behavior issues, appropriateness of the task, teachers comfort with mathematics and time.
Acknowledgements

The list of each individual I must thank for providing me with guidance and support throughout my academic and professional career is too extraordinary to mention in this short acknowledgement. However, I am so grateful for every one of you who has entered my life and made an impact on my future.

First, I would like to thank my wonderful advisor, Doug McDougall, who has inspired me and who introduced me to the participants described in this thesis. Your thoughtfulness has not only allowed me to contribute to current literature in mathematics education, but has inspired me to continue forward with my education. I am blessed to have you as my advisor and to have experienced your insights and shared knowledge.

Second, I would like to thank Jim Hewitt for acting as my second reader. I cannot thank you enough for taking the time to better this thesis and your input is truly valued.

I must thank my participants, Denise, Anthony and Gamma school personnel for your encouragement and kind words. Your dedication to your work will forever impact those who meet you, including me. I can now call many of you friends and I look forward to the wonderful differences you will make in education.

Although I have enjoyed all that the University of Toronto has offered, I cannot forget my roots back home in New Jersey. My friends celebrate my achievements and give me the encouragement to challenge myself, for they believe that my hard work and dedication will make me the best person I can be. Thank you for all of your continued support.

Ryan, you are my rock. This journey together has come with challenges, but our respect for one another trumps all else. Thank you for spending countless nights editing
my papers and for giving me sustained encouragement when I needed it most. Each step I take in life is for us and our friendship and love has no boundaries.

Finally, my family is the number one reason I am delighted to be standing where I am today. Mom, my passion and dedication to be the best that I can be is a direct reflection of your character. You are my hero and I know you would be the most proud if you were still with us today. Everything I do is for you and because of you. Dad, your generosity is contagious and I look up to the strength you have to move past difficult times and enjoy the little moments in life. You are my best friend and my whole world. Countless times I have been supported by my three brothers and it is endearing to know we will always be there for one another. Thank you to my Aunt Joy who has loved me and listened to me in the best and worst of times. As for the rest of my endearing family, thank you for supporting my decision to move to Canada. Each of you holds a special place in my heart.
## Table of Contents

### Chapter One: Introduction

1.1 Introduction ..... 1  
1.2 Research Content ..... 1  
1.3 Research Questions ..... 5  
1.4 Significance of Study ..... 5  
1.5 Background of the Researcher ..... 8  
1.6 Plan of the Thesis ..... 11

### Chapter Two: Literature Review

2.1 Introduction ..... 13  
2.2 The Reform Movement ..... 13  
2.3 The Ten Dimensions of Mathematics Education ..... 17  
2.4 Cognitive Demand of Tasks ..... 21  
2.5 Mathematics Tasks  
   2.5.1 Rich Mathematics Tasks ..... 26  
   2.5.2 Task Selection ..... 30  
   2.5.3 Task Implementation ..... 33  
2.6 Teacher Beliefs ..... 38  
2.7 Teacher Collaboration ..... 39  
2.8 Summary ..... 42

### Chapter Three: Methodology

3.1 Introduction ..... 43  
3.2 Research Context ..... 43  
3.3 Participants  
   3.3.1 Grade 6 Teacher ..... 44  
   3.3.2 Grade 8 Teacher ..... 45
Chapter Four: Findings

4.1 Introduction

4.2 Case One: Denise
   4.2.1 Characteristics of Rich Tasks
      4.2.1.1 Meeting Individual Needs
         4.2.1.1.1 Engaging Tasks
         4.2.1.1.2 Differentiation
         4.2.1.1.3 Practical Use
      4.2.1.2 Structure of Rich Tasks
      4.2.1.3 Assessments
   4.2.2 Source of Tasks
   4.2.3 Implementation
      4.2.3.1 Supporting Factors
      4.2.3.2 Inhibiting Factors

4.3 Case Two: Anthony
   4.3.1 Characteristics of Rich Tasks
      4.3.1.1 Meeting Individual Needs
         4.3.1.1.1 Engaging Tasks
         4.3.1.1.2 Practical Use
      4.3.1.2 Structure of Rich Tasks
      4.3.1.3 Assessments
   4.3.2 Source of Tasks
   4.3.3 Implementation
      4.3.3.1 Supporting Factors
      4.3.3.2 Inhibiting Factors

Chapter Five: Interpretation and Discussion of Findings

5.1 Introduction

5.2 Research Questions

5.3 Discussion of Each Research Question
   5.3.1 How do teachers select rich mathematics tasks for classroom use?
5.3.1.1 What characteristics do teachers look for in rich mathematics tasks? 79
  5.3.1.1.1 Individual Student Needs 79
  5.3.1.1.2 Characteristics of Rich Mathematics Tasks 80
5.3.1.2 What sources do teachers use in the selection of rich mathematics tasks? 82
5.3.2 During implementation, what factors support or inhibit the integrity of rich mathematics tasks? 84
  5.3.2.1 Supporting Factors 84
  5.3.2.2 Inhibiting Factors 86

5.4 Major Findings 89

5.5 Implications for Future Research 91
Chapter One: Introduction

1.1 Introduction

The purpose of this thesis is to explore and describe how middle school teachers select and implement rich mathematics tasks in their classroom. Reform initiatives are encouraging teachers to improve student achievement through the use of rich discussions and worthwhile tasks that link procedures with conceptual understanding (Curriculum and Evaluation Standards for School Mathematics, NCTM, 1989). Significant teacher support is needed if teachers are to change their culture and beliefs about teaching to better align with reform approaches (Zaslavsky & Leiken, 2004). One reason I chose this topic is because of the value it may bring to current and prospective teachers who desire to employ rich mathematics tasks in their classroom. This chapter outlines why this study is significant to existing research, the research questions this study sets out to answer and how my personal experience has led me on the journey of exploring rich tasks.

1.2 Research Content

Mathematics is commonly viewed as a set of facts and procedures that can only be mastered through memorization and the practicing of procedures. Could students in U.S. classrooms describe in detail the connection between what they engaged in during mathematics and the world in which they lived? Multiple domestic and international assessments revealed that students in U.S. classrooms were learning less mathematics than they could (Hiebert & Stigler, 1997; 2004; Silver & Kenney, 2000).

First published in 1989, the National Council of Teachers of Mathematics (NCTM) introduced the *Curriculum and Evaluation Standards for School Mathematics* describing new research on how students learn mathematics. In an effort to address low performance
scores on international assessments, the NCTM published another document in 2000 called *Principles and Standards for School Mathematics* (NCTM). Together these documents acted as guides for changes that needed to happen in order to improve mathematics education. According to NCTM documents, traditional teaching approaches were not addressing the development of "students' capacity to think and reason mathematically" (Stein, Grover, & Henningsen, 1996). Traditional classrooms could be viewed as engaging students in the procedures needed to complete mathematics tasks and the discipline of mathematics was viewed as a structured system (Battista, 1994, p. 463). Although there is no set of criteria that defines a reform classroom, the NCTM identified characteristics that should be present in math education reform classrooms. Many of these characteristics, such as teaching mathematics via real-world open-ended problems, rely heavily on the "doing of mathematics" where students understand and justify the choices they make when solving mathematics in a variety of ways.

In accordance with the NCTM, the Ontario Ministry of Education (OME) has adapted reform policies to support the learning of mathematics. The curriculum guide for K-8 mathematics supports classroom tasks that promote: 1) problem solving, 2) reasoning and justification, 3) reflection, 4) multiple solutions and solution strategies, 5) using procedures with concept understanding, and 6) communication (OME, 2005). Teachers are responsible for fostering creativity through engaging their students in mathematics tasks that are challenging and connected to the outside world in a meaningful way. In doing so, student achievement gains have the potential to improve and the desire for life-long learning may be cultivated.

In an effort to better assess the international achievement gap in mathematics, the
Third International Mathematics and Science Study (TIMSS, 1999) measured student achievement in and attitudes towards mathematics and science of grade eight students around the world (McDougall, 2004, p. 2). The video component in this study took samplings from teachers in grade eight classrooms across the globe and made comparisons of a variety of teaching dimensions, “including the ways classrooms are organized in the different countries, the kinds of mathematics problems presented to students, and the ways problems are worked on during classroom lessons” (Hiebert & Stigler, 2004, p. 11). Over half of the time Japanese students were found to be engaged in tasks that teachers specifically designed to address conceptual understanding, while no lessons recorded in the U.S. classrooms engaged students in the same manner. The grade eight United States students spent significantly more time practicing skills compared to their peers in Germany and Japan (Stigler & Hiebert, 1997). The reform recommendations proposed by NCTM were consistent with Japanese lessons indicating that students often spent time finding alternative solution strategies and solving challenging tasks that linked procedure with understanding. Schools in the U.S. already provide enough time for practicing skills and it may be beneficial to direct more attention toward designing and implementing tasks that involve drawing mathematical relationships between concepts.

The reform movement aims to further develop students’ thinking capacities (Stein et al., 1996). The National Council of Teachers of Mathematics (NCTM) describes mathematics reform initiatives to be “characterized by an increased emphasis on thinking, reasoning and problem solving” (Stein & Lane, 1996, p. 50). In mathematics, there are teaching approaches that have been recognized to improve students’ mathematical
thinking, yet some teachers continue to exclusively implement traditional approaches. As 

stated by Stigler and Hiebert (1997), making changes in the system does not ensure that 

these changes will be carried out by teachers. One highly regarded theory of learning is 

the idea that students who do not have the basic skills and knowledge of a subject, more 

prominent in economically disadvantaged communities, will easily become frustrated 

with challenging problems that ask students to reach beyond procedures to better 

understanding concepts (Stein & Lane, 1996). However, a national project based on 

researching reform practices in the U.S., known as QUASAR (Quantitative 

Understanding: Amplifying Student Achievement and Reasoning), provided insight into 

the learning gains demonstrated by students from economically disadvantaged 

communities. These middle school students, many of whom have never mastered basic 

skills, showed positive gains in achievement when engaged in a curriculum that 

supported reform initiatives (Stein & Lane, 1996).

The feeling of being inadequate is often a response given by teachers who choose not 

to implement the reform initiatives in their classroom. Teachers may find it difficult to 

challenge their students if they are not entirely sure of the relationships among the 

mathematical concepts they teach. Researchers and teachers agree that providing 

opportunities for students to feel challenged can only be achieved when the teacher has a 

comprehensive and extensive understanding of the subject (Zaslavsky & Leiken, 2004). 

Also, the role of the teacher is to make pedagogical choices based on the examination of 

student input and classwork. However, Sullivan, Clarke and Clarke (2013) found that 

making these choices based on inferences “is more difficult than it appears” (p. 62).

In addition, teachers may teach in the same manner in which they learned, because
they “learn to teach by growing up in a culture, watching their teachers teach, and then adapting these methods for their practice” (Hiebert & Stigler, 2004, p. 13). Changing teaching practice to better exemplify reform approaches would mean changing the existing structures within teachers' beliefs. In order to do so, teachers will need to be provided with the opportunity to practice implementing reform approaches to teaching and engage in collaborative activities to continuously enhance their learning and confidence.

1.3 Research Questions

This qualitative case study explores how one grade six teacher and one grade eight teacher select and implement rich mathematics tasks in their middle school classrooms. This thesis is based on the following questions:

1. How do teachers select rich mathematics tasks for classroom instruction?
   a. What characteristics do teachers look for in rich mathematics tasks?
   b. What sources do teachers use in the selection of rich mathematics tasks?

2. During implementation, what factors support or inhibit the integrity of rich mathematics tasks?

1.4 Significance of the Study

The underlying goal of NCTM's proposed reforms is to "enhance students' understanding of mathematics and to help them become better mathematical doers and thinkers" (Henningsen & Stein, 1997, p. 524). However, holding teachers accountable for such achievement with no clear path describing how to improve their instruction may not be effective. Direct research must be conducted in order to determine the exact processes that cultivate mathematics learning in the classroom. Although data shows that the deep
culture of teaching must change to make way for reform strategies, Stigler and Hiebert (1997) remind us that teachers make a choice on how they choose to teach even before they set foot in a classroom.

Students' primary experience with mathematics in classrooms is directly related to the mathematics tasks in which they engage (Henningsen & Stein, 1997; Schoenfeld, 1994). Anthony and Walshaw (2009) argue that “in the mathematics classroom, it is through tasks, more than in any other way, that opportunities to learn are made available to the students” (p. 96). Mathematics tasks have been described as bridges between students and the learning of mathematics. To completely understand mathematics means having the ability to set up problems without being told how to do so, make adjustments and recognize invalid responses, invent multiple strategies to reach conclusions, and make justifications that demonstrate connections among concepts (Stein et al., 1996). Instruction that guides students to engage in mathematics tasks that elicit this type of understanding proved to have the greatest gains on performance assessments measuring mathematical thinking and reasoning (Stein & Lane, 1996). The Ontario Ministry of Education (2005) urges teachers to "use rich problems" where students take on an investigative approach to develop a solid understanding of mathematics. Sullivan et al. (2013) argues that tasks that develop fluency are “well represented in every school mathematics text” (p. 8-9), so teachers should seek to develop conceptual understanding. The task choices that teachers make, including the initial task selection, the alterations made to the task, and the implementation of the task in their classroom, are a direct reflection of how they afford students the opportunity to engage in meaningful mathematics where procedures are directly related to conceptual understanding.
The types of rich mathematics tasks that best guide students in the conceptual learning of mathematics and foster high-level thinking have been investigated in a number of environments. Yet, little focus has been directed towards instructional characteristics that enhance or diminish the tasks' intended high-level cognitive-demands during implementation (Henningsen & Stein, 1997). Studies show that intended cognitive-demands of rich mathematics tasks are typically reduced during the implementation stage, transforming rich tasks designed to elicit mathematical meaning into procedurally focused tasks with a high interest in task correctness (Stein et al., 1996). In fact, Tzur (2008) argues that there is a significant deviation among the ways task developers intended the task to be used and teachers’ use of the task in the classroom.

Much of the early evidence from studies connecting reform oriented practices to student achievement gains in mathematics were conducted at the elementary level (Stein & Lane, 1996). Nonetheless, recently there has been a spike in similar studies being conducted in the middle grades all suggesting that rich mathematics tasks that are compatible with reform initiatives have the ability to improve student achievement. More exploration is needed to better understand the relationship between teacher, student and task (Ross et al., 2002; Stein & Lane, 1996; Stein et al., 1996). An understanding of these relationships is pivotal in developing classroom environments and teaching practices that are consistent with reform initiatives and ultimately in improving student achievement.

The significance of this study is that it contributes to the current knowledge regarding how teachers incorporate rich mathematics tasks into their classroom. This study explores factors that uphold or reduce the intended cognitive-demands of rich tasks. In addition to studying how the integrity of rich mathematics tasks are influences by teachers' decisions
during implementation, this study supports literature pertaining to how teachers initially make task selections. It is a goal of this study to orient itself in the eyes of the teacher to better understand challenges teachers experience while they strive to implement reform initiatives and how they may be supported in executing reform approaches before and during instruction.

1.5 Background of the Researcher

As described by Creswell (2013), all writing is positioned in the eyes of the researcher. In an effort to situate myself within the research, I acknowledge my own beliefs and biases as I describe the journey that has led to my involvement in this particular qualitative case study. I also discuss my own beliefs about the learning of mathematics and describe how they may influence my interpretation of the results presented in this study.

Influenced by my mother who worked as a realtor and often brought home blueprints, and my father who drew diagrams for his carpentry business, I began drawing layouts of homes before the age of 5. At first, they started as front-facing dollhouse-like sketches and eventually evolved into elaborate multi-dimensional blueprints created on the popular computer program referred to as CAD (Computer Aided Design). There was little doubt in my mind that studying architecture would be in my future and that mathematics would serve as a useful tool.

My love for the discipline of mathematics did not begin until I reached high school. For the first time, I found that my teachers and peers acknowledged my ability to grasp concepts quickly and recognize pattern with little effort. My work ethic and eagerness to understand "why" concepts worked the way they did quickly moved me to the top of my
mathematics class. I began to see how the knowledge of angles and side lengths would be useful in the world of architecture. I was immediately motiva ted by rich tasks that were related to the world beyond the classroom, yet rarely did I experience these tasks in the mathematics classroom.

In a large southern town in New Jersey, I started my senior year of high school with all the intent in the world to apply to architecture programs for college. My course elective that semester was a CAD course in which we designed objects like trebuchets and created our dream bedroom. My involvement with mathematics resulted in the production of objects and ideas, and I deemed mathematics as important concepts that would help me understand my world. Also, it was the first time I opened my mind to other occupations that would foster my love for creativity and mathematics, such as engineering. One university that I had my heart set on attending did not offer an architecture program was the University of Delaware. When I found out I had been accepted to the University of Delaware, I had a difficult decision to make. I needed to say goodbye to my dream of becoming an architect and begin imagining my life as an engineer, or I needed to say goodbye to the University of Delaware and all that it had to offer. I said goodbye to architecture.

I was asked to volunteer at the University of Delaware's Academic Enrichment Center where students could ask other students for assistance with their class work and studying habits. I signed on as a mathematics tutor and my schedule quickly began to fill up. I soon decided to tutor students in groups as this would be more efficient for both parties involved. Students enjoyed how I would use rich tasks to connect the mathematical ideas to the real-world and this helped them see mathematics as a useful
tool for understanding the way in which the world worked. Having a passion to share everything I could was only half of the reason I enjoyed this tutoring position. The part I valued the most was hearing, "I finally get it". After a month of tutoring, I went to visit my academic advisor to see if changing my major to mathematics education would be feasible. It turned out that all of the classes I had taken for engineering would count for classes needed for the mathematics education program. And just like that, my future was headed towards teaching mathematics.

Teaching eighth grade mathematics for three years at a public school in New Jersey was the most rewarding experience. Students would often describe how they had never enjoyed mathematics the way they were enjoying it in my classroom. Parents would write to me emails sharing the thrill they were experiencing as their child frequently talked about mathematics at the dinner table. Students who had never viewed themselves as accomplished mathematics students were now reaching out to me for advice on how to become a mathematics teacher. As I reflected on my teaching approaches, I believed they were below average compared to what a reform classroom should look like.

My students recognized my spirit towards and passion for mathematics, and they knew I believed in their ability to do mathematics despite what they had been told in the past. I worked hard at building positive emotions towards mathematics by introducing every lesson with real world examples and allowing the students opportunities to explore the concepts. Unfortunately, feeling the pressure for my students to perform well on state assessments, I began to return to traditional ways of teaching and many of my lessons became static and procedurally focused. The culture of the school valued traditional approaches to teaching and I did not feel the support necessary to continue with the
implementation of rich mathematics tasks.

Attending the University of Toronto for my master degree and being taken under the wings by Doug McDougall was a rewarding experience. Doug McDougall's thorough understanding of how to identify needed improvements in teaching and his knowledge of rich mathematics tasks served as a solid foundation for this study. In addition, I am the sole researcher and will be analyzing the findings from this study as a one person team. It is important that I have acknowledged my own biases and set them aside in order to provide credible findings when analyzing the collected data. Understanding how teachers strive to implement reform initiatives may provide the needed information to support current and prospective teachers in using rich mathematics tasks in their classroom.

1.6 Plan of the Thesis

This thesis consists of five chapters depicting the journey this study has endured. Chapter One introduces the study, including a rationale for its existence, and sets up what the researcher hopes to better understand.

Current research findings will be explored in Chapter Two, beginning with a comprehensive review of reform approaches to teaching. This study will align with the Ten Dimensions of Mathematics Education (McDougall, 2004) theoretical framework that identifies areas of improvement needed in education. In addition, the cognitive-demands elicited by particular tasks will be described and used to better understand the factors that enhance or diminish the richness of tasks during implementation. In addition, how this study views rich tasks will be identified, including rich task findings from existing studies. Finally, the role of the teacher and the positive impact collaboration can have on teaching is discussed, along with support strategies for
teachers working towards building reform-oriented classrooms.

Chapter Three depicts the methods and qualitative approaches used to set-up and carry-out this study. This study is a subset of a larger multi-year research study on School Improvement in Mathematics (McDougall, 2009) and a summary of how this study will contribute to the larger project will be discussed. Lastly, I explain the choices made for how data was collected, why particular analysis methods were employed, and the ways in which ethics were considered.

Analysis findings are described in Chapter Four according to the themes that emerged from the collected data. The collective case study involving two participating teachers is illustrated depicting their selection and use of rich mathematics tasks in their classroom.

Chapter Five attempts to answer all proposed research questions defined in Chapter One based on the major findings from this study. Implications for further research conclude this study.
Chapter Two: Literature Review

2.1 Introduction

Every researcher must educate themselves on the current research and knowledge in their area of study. As Arlene Fink (2005) mentions, “high-quality literature reviews base their findings on evidence” (p. 3). Using my research questions, I accessed the University of Toronto’s digital library database called Education Resources Information Center (ERIC) and typed “math* task” into the search bar. This allowed me access to multiple research studies conducted and various articles written on mathematics tasks. I narrowed the list down by selecting to view only publications after 1995 in order to find the most recent articles and studies.

After reading many articles and studies, I ventured to their references and took note of any articles the authors viewed that would benefit my study. After reading twenty or so research studies and articles found in the references sections, I began to see a pattern in the authors listed in references such as Mary K. Stein and NCTM. I studied the work completed by these authors and began to feel as though I had a strong foundation in rich mathematics tasks. Then, I journeyed into other areas that the readings had suggested in the readings that were closely tied to rich mathematics tasks such as the reform movement, cognitive demand, teacher’s beliefs and collaboration.

2.2 The Reform Movement

Some traditional classrooms may be viewed as teacher-directed, driven by tasks that strengthen procedural accuracy, and allow for single representations. Many students who attend public schools in North America may have a first-hand experience learning by way of traditional approaches. While research was conducted analyzing how students best
learned mathematics, the NCTM decided to make changes that would alter teaching approaches in North American schools to better align with student learning. In 1989, they published the *Curriculum and Evaluation Standards for School Mathematics* that would guide teachers to employ teaching approaches proven to be effective in leading to increased student achievement.

Teachers and education programs saw little evidence to move towards the reform practices until the *Trends in International Mathematics and Science Study* (TIMSS) was conducted. This study described that United States student achievement was below many of their peers from other countries despite an increase in funding (Finn, 1990, p. 600). TIMSS was a national study conducted to measure students' achievements and attitudes towards mathematics and science. One method of data collection for this study was the use of videos to capture teaching instruction from around the world. Hiebert and Stigler (1997; 2004) analyzed and compared the teaching approaches exercised by grade eight teachers in Japan, Germany and the United States, finding evidence that may serve as proof for the relationship between reform initiatives and the improvement in teaching and learning mathematics.

Fifty-four percent of Japanese lessons included tasks that placed a heavy emphasis on conceptual understanding, while conceptually focused lessons in the United States were viewed only seventeen percent of the time. Teachers in the United States often "stepped in and did the work for the students or ignored the conceptual aspects of the problem when discussing it" (Hiebert & Stigler, 2004). In doing so, many United States teachers may shield students from finding solutions to challenging tasks, and may ultimately leave a deficit in the learning of mathematics. Along with the documents published by the
NCTM, the Mathematics Association of America (1991) and the National Research Council (1989) highlight the importance of developing students' mathematical knowledge that is deeply connected to "doing mathematics" where procedures and conceptual understanding go hand in hand (Stein et al., 1996, p. 456).

There is no doubt that adapting reform principles is highly encouraged, but what would a reform classroom look like? According to Ross et al. (2002), reform classrooms should be student-directed where the teacher finds themselves as another learner in the classroom. Students should become mathematically literate, building skills in conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (McDougall, 2004). Students should be able to represent solutions in multiple ways, justify their solution, and explain how the procedures tie into the learned concept. Giving students the opportunity to investigate mathematical concepts through problem solving must be matched with scaffolding that carefully leads students towards the underlying principles, without telling students how to get there or for what they should be looking (Mathematics: The Ontario Curriculum, Grades 1-8, 2005). Textbooks should be used as a tool, but not to be followed as a curriculum guide.

Stein and Lane (1996) found that student achievement gains where students needed to reason about their conclusions and provide reflective thoughts are higher in classrooms that employ reform teaching approaches than in traditional classrooms. Evidence that reform practices contribute to student achievement in economically disadvantaged communities is also prevalent in emerging findings (Silver & Stein, 1996). Some studies report that reform strategies do not lead to significant improvement in student achievement, but no studies report that reform strategies led to lower student achievement.
(Ross et al., 2002).

Calling a reform movement into action does not ensure that reform strategies will be adapted into each and every classroom. Cuban (1988) would describe the reform movement as a second-order change. Second-order changes seek to change the existing structures of schools. Reform initiatives call for second-order changes because teachers are asked to lead student-directed classroom, giving up authority and changing the role of the teacher. Second-order changes are often the most difficult to enact and take the most time to catch on in the education system. Teachers want to experience significant gains in student achievement when using reform strategies before they make the leap to alter their beliefs and roles in the education world.

In addition to the slow adaption of reform practices due to the history of second-order changes, many studies show there is a fine line between encouraging mathematical reasoning and having students feel lost and unmotivated to continue with a rich task (Ball, 1993; Ross, Haimes, & Hogaboam-Gray, 1996; Ross et al., 2002). Students are not the only participants within the classroom who may feel overwhelmed by rich tasks. Also, teachers must feel confident in their ability to understand underlying concepts in order to lead students in the "doing" of mathematics. However, some teachers believe that their practices do support reform initiatives when in fact most of their approaches would be described as traditional (Ross et al., 2002).

In order to transform teaching to accommodate reform initiatives, Hiebert and Stigler (2004) suggest shifting priorities during planning time to studying and reflecting upon teaching approaches. Teachers are likely to improve on their teaching practices if they are continuously studying and reflecting on their classroom strategies.
Although reform practices have the opportunity to increase student achievement and motivate students to see mathematics as useful, there are a number of teachers who continue teaching in the traditional manner. Some teachers have chosen to teach via reform approaches and do not see themselves returning to their traditional roots. Other teachers choose traditional approaches to teaching because they believe they work best. There may be another subset of teachers who want to teach using reform strategies, but do not yet have to support and encouragement to do so efficiently and effectively.

2.3 The Ten Dimensions of Mathematics Education

In developing a framework for a successful mathematics program that incorporates reform initiatives, McDougall (2004) introduced the Ten Dimensions of Mathematics Framework. Each dimension that culminates to form the Ten Dimensions of Mathematics Education is meant to help administrators and teachers identify challenging areas within their school, pedagogy and curriculum, and provide suggestions for improvement. The components in this framework are in no way discrete and may overlap dependent upon the goals of the school.

Dimension One in the Ten Dimensions of Mathematics Education framework is Scope and Planning. The teacher identifies the outcomes wanted within each curriculum strand in order to build on key ideas and plan for strand integration. When planning a mathematics program, recognizing what mathematical processes would best support the integration of each key idea and lesson is necessary, even during the construction of assessments. This strand also expects the teacher to use a variety of resources to make the biggest impact when covering what is expected of them in the curriculum guide.

Meeting Individual Needs is Dimension Two in the framework. Based on student
needs and abilities, teachers must use multiple approaches to teaching in an effort to reach the whole child. Preparing for students learn at different rates and in different ways, scaffolding, varying tools and time, and using open-ended tasks are a few ways that a teacher can differentiate their instruction.

Dimension Three focuses on the Learning Environment. Setting classroom procedures and using multiple grouping strategies are some ways that teachers can meet the needs of their students. In addition, providing for opportunities where students can make choices and receive constructive feedback may enable students to feel valued.

Student Tasks is Dimension Four and is the dimension that is addressed in this study. Teachers are expected to plan for and use rich mathematics tasks as the basis of their instruction. Procedural tasks should still be introduced to students, but paired with conceptual understanding. These rich tasks should invite students to construct meaning through multiple forms of representations, involve high-order thinking, and encourage communication among students. Educators are encouraged to alter or construct tasks that are in context for their students.

Dimension Five is the Construction of Knowledge. This dimension describes a constructivist approach to teaching, exemplifying active student engagement in order to develop mathematical thinking. The teacher builds on students' prior knowledge and uses questioning techniques to move students forward in their mathematical thinking.

Communicating with Parents is Dimension Six. Open communication should be established between school and home to discuss performance and the future of the mathematics program. From Web pages to classroom newsletters, there are a variety of ways and reasons that teachers and parents should use communication resources.
Dimension Seven is Manipulatives and Technology. One way to develop conceptual understanding and supply links between concrete and abstract thinking is to incorporate a variety of manipulatives in order to cultivate student learning. Technology integration is required by the Ontario Ministry of Education (2005) and it can provide ways to explore mathematical ideas efficiently and effectively.

Student's Mathematical Communication is Dimension Eight. This dimension focuses on encouraging teachers to assign group work in order to allow students to communicate their thinking to others using oral and written methods of communication. Students should be encouraged to use mathematical vocabulary and diagrams to reason and justify their understanding of concepts.

Dimension Nine is Assessments and this dimension encourages teachers to use a variety of assessment strategies for learning, as learning and of learning, to report on student achievement. The assessments should align with curriculum objectives and expectations, and should take into account the needs of students, even when using the self-assessment strategy. The criteria being measured via assessments should be disclosed to the students and should also provide teachers with opportunities to reflect on their own teaching.

Teacher's Attitude and Comfort with Mathematics is the last dimension, but not of less importance than the other dimensions. Dimension Ten describes teachers as having a positive view of and passion for mathematics. Naturally, teachers who are comfortable with mathematics often choose to make connections among concepts and show how mathematics is useful outside the classroom walls.
As mentioned, Dimension Four, Student Tasks, is the focus in this study. Dimension Four discusses the balance of instructional tasks needed in reform-oriented classroom. Rich mathematics tasks are a subset of tasks and this framework suggests students should be exposed to rich tasks on a regular basis. In this dimension, teachers should aim for a Level Four use of rich mathematics tasks based on the Ten Dimensions rubric (McDougall, 2004). A Level Four would indicate that teachers use rich tasks with all of their students and as the foundation for their mathematics curriculum. A Level Three would indicate that a teacher often assigns rich tasks to students, but it does not guide instruction. Assigning rich tasks every so often and/or assigning the rich tasks to a select group of students would indicate a Level Two use of rich tasks. A Level One use of rich mathematics tasks means rich tasks are rarely presented in the classroom and often presented only after "regular" work is finished.

The Ten Dimensions of Mathematics Education framework embodies reform-based principles of mathematics. It guides educational personnel in identifying areas of concern within a mathematics program and suggests strategies for improvement. It is easier for a teacher to make changes and improve an area of weakness if they can easily identify the strand that needs improvement. Although teachers may find that their struggles exist in multiple dimensions, it is encouraged to focus on one dimension at a time for best results regarding the improvement of their instruction. The Attitudes and Practices for Teaching Math Survey (McDougall, 2004) can be used as a 20-question self-assessment guide for teachers to identify weak dimensions within their own teaching instruction.

Using Dimension Four within the Ten Dimensions of Mathematics Education as a framework for improving teaching will ensure that my study focuses on the processes and
choices teachers make as they select and implement rich mathematics tasks in their classroom.

2.4 Cognitive Demand of Tasks

As reform initiatives sweep North America, researchers have continued to study the precise factors that play a role in raising student achievement. Stein and Lane (1996) provide evidence that there exists a positive correlation between task implementation that focuses on the high-level cognitive demands of the instructional task and student learning gains. In providing students with the opportunity to engage in tasks that elicit high-level cognitive demands, teachers should be aware of what tasks constitute as high- and low-level cognitive demands.

One interpretation of cognitive demands "refer(s) to the kind of thinking processes entailed in solving the task" (Stein et al., 1996, p. 461). Cognitive demands are often diffused into many categories based thinking processes, ranging from memorization to the "doing of mathematics". Researchers from the QUASAR project analyzed hundreds of tasks used in classrooms and created a list of criteria (further referred to as the LCD, Levels of Cognitive Demand) that could be used to "rate" the mathematics task based on the thinking it demands of students (Arbaugh & Brown, 2005). The LCD criterion is separated into four categories, two lower-level demands and two higher-level demands. The two lower-level cognitive demands engage students in the task of memorization and the use of procedures without connection to meaning. The two higher-level cognitive demands involve students using procedures with connection to meaning and in the "doing of mathematics". Whether intended to access low-levels or high-levels of cognitive demand, mathematics tasks serve as the foundation for eliciting these thinking processes.
Similarly, Stein and colleagues (Stein & Lane, 1996; Henningsen & Stein, 1997) propose six levels of cognitive demands in which students engage during mathematics instruction; two high-levels, three low-levels and one level described as non-mathematical. The high-level cognitive demands are split into two categories based on thinking processes. The higher of the two levels provided opportunities for students to "do mathematics" and construct their own solutions to tasks without having a solution strategy or an algorithm to follow. The second level of high cognitive demands encourages students to engage in thinking processes that use procedures with connections to meaning, concepts, and understanding. Students may have a suggested pathway to follow through the problem, but the pathway is broad, allowing for opportunities to explore elsewhere.

Also, there is a close connection to the underlying conceptual ideas. Three levels of thinking processes in the mathematics classroom can be considered as low cognitive demand. These levels describe thinking processes that use procedures without connection to concepts, memorization and unsystematic and/or nonproductive exploration. Little cognitive effort is used in solving tasks that are framed in any of these three low-level thinking processes. The last level of cognitive demand recognized in mathematics classrooms is labeled non-mathematical, because either the task does not call for any mathematical activity or the students are off-task.

Awareness of cognitive demands of mathematics tasks help teachers prepare lessons that will engage students in thinking processes that may lead to gains in student achievement. In turn, performance gains are relatively small when tasks were set-up to elicit thinking that conforms to lower-levels of cognitive demand. Stein and Lane (1996)
"indicate that the greatest student gains on the mathematics performance assessment were related to the use of instructional tasks that engaged students in high levels of cognitive processing", particularly with tasks that involved the "doing of mathematics" (p. 74).

In recent decades, the U.S. has increased their funding in education and educational research, but the levels of cognitive demands that students use in the classroom are lower than measures decades ago (Finn, 1990). Teachers should select tasks that are connected to high-levels of cognitive demand thinking and implement them appropriately in their classroom. In addition, the teacher must actively work at maintaining the cognitive complexity of the task without hindering the intended cognitive demand level of the task (Henningsen & Stein, 1997). Studies show that tasks often change in cognitive levels as they transfer from the set-up phase to the implementation phase (Stein & Lane, 1996). This study will explore characteristics of implementation that support high-levels of cognitive demand throughout the implementation phase, along with classroom factors that have negative effects.

An insight into classroom factors judged to influence students' engagement with high-level thinking processes was examined in a study conducted by Henningsen and Stein (1997). This study used data from the QUASAR project and analyzed it in an effort to classify factors that either supported or inhibited students' high-level cognitive engagement. The findings suggest classroom factors that most frequently support high-level cognitive demand tasks throughout the implementation phase: 1) set expectations for students to explain one or more of their solution strategies, 2) set appropriate amounts of time to work on tasks to allow for exploration of multiple thinking processes, 3) make clear connections between the task and the big idea, and 4)
give appropriate scaffolding such as having students in the class model their thinking strategies. Some factors reported as lowering the cognitive demands from high-levels to low-levels are: 1) removal of the challenging aspect of the task often in response to a push from students, 2) shifts in acknowledging the correctness of the task versus the meaning of the task, 3) unclear expectations, and 4) inappropriate amounts of time given for task completion. These factors may help to guide this study in determining if these same characteristics exist in the reform classrooms here in Ontario and what other factors could be further explored.

Studies describe a correlation between the levels of students' cognitive demand required by rich mathematics tasks and students' understanding of mathematics (Arbaugh & Brown, 2005; Hiebert & Wearne, 1993; Stein & Lane, 1996). If student achievement gains are expected to increase, then classroom tasks should be set-up and implemented in a way where students are cognitively challenged. Students who are expected to problem solve, justify their reasoning and make appropriate evaluations of their mathematical thinking are likely to have an enhanced understanding of the discipline of mathematics.

2.5 Mathematics Tasks

Tasks serve as bridges between student learning and key concepts in both traditional and reform classrooms. For the purpose of this study, the terms mathematics task and instructional task will be used interchangeably. An instructional task can be viewed as the act of engaging in an activity in order to build or develop particular concepts and/or skills (Stein & Lane, 1996, p. 52). Similarly, the International Commission on Mathematical Instruction (ICMI Study 22, 2013) defines tasks as "the mediating tools for teaching and learning mathematics" (p. 10). Instructional tasks are designed to foster student
engagement with content. According to Hiebert and Wearne (1993), instructional tasks have a significant impact on student learning and performance gains. In keeping with the reform initiatives, this same study shows that procedurally based instructional tasks produce lower performance gains than those students working through rich tasks that elicit meaning. Exposing students to a variety of tasks in the classroom enhances the learning environment because it allows for an assortment of solution methods and provides an opportunity for students to use their knowledge in an unsystematic real world manner (Horoks & Robert, 2007, p. 281). Many researchers have aimed to construct a theory for how students learn from classroom teaching. Although no one theory has been able to encompass all aspects of teaching and learning, many researchers acknowledge that one day classroom instructional tasks will serve as the core in such theory (Hiebert & Wearne, 1993; Stein & Lane, 1996).

Mathematics tasks, in general, engage students in the act of developing mathematical skills and concepts. Teachers are expected to guide students in developing these skills and concepts using a range of mathematics tasks. Learning to think in a mathematical way is cultivated by the mathematics tasks students encounter (Stein & Smith, 1998). One interpretation of a mathematics task is defined as an activity in which students focus on key mathematical concepts (Stein et al., 1996, p. 460). Similarly, the Classroom Observation Instrument (COI) defines a mathematics task as work that focuses student thinking on one mathematical idea (Henningsen & Stein, 1997). In summary, a mathematics task should serve as a vehicle for developing students' ability to think and reason mathematically.
2.5.1 Rich Mathematics Tasks

In order to improve student achievement, classrooms must offer students the opportunity to construct meaning from the mathematics, make decisions about manipulatives and strategies for completing tasks where meaning and procedures are interconnected, and interpret and reflect upon the rationality of their solution and solution strategies (Stein et al., 1996). Providing students with challenging problems will allow students to make connections between mathematical concepts and develop a deep understanding of mathematics as a discipline (Hiebert & Stigler, 2004). NCTM (1991) recommends that teachers choose and implement mathematics tasks that are worthwhile, but what features would define a worthwhile task?

A rich mathematics task is viewed as a worthwhile task because of the high-level of cognitive demand it elicits and the close relationship between procedure and conceptual understanding it cultivates. McDougall (2004) defines a rich mathematics task as: 1) encompassing a set of attributes that are grounded in a real-world problem, 2) allowing for multiple solutions, 3) providing opportunity for engagement with many solution strategies developed by students, 4) involving a variety of representations, 5) leading students towards making connections between mathematical ideas, 6) expecting students to communicate their reasoning process, and 7) assigning reflection as a continuous process.

Justifying procedures and understanding in both written and oral forms would be an additional requirement that Stein and colleagues (1996) might include in the definition of a rich mathematics task. In addition, researchers who have focused their studies on rich instructional tasks reiterate that rich mathematics tasks are not intended to provide
students with an algorithm that can be practiced over and over (Horoks & Robert, 2007; Stein et al., 1996; Stigler & Hiebert, 1997). Instead, students should be given ample time to construct their own algorithms and solutions for solving complex tasks.

Although some teachers believe that skills and concepts must be mastered before students can engage with real-world examples, evidence suggests that rich real-world tasks do provide a foundation for the learning of mathematics and the opportunity to practice skills while being immersed in high-levels of cognitive thinking processes (Gainsburg, 2008). Rich mathematics tasks are tasks that engage learners by using high-levels of cognitive demand. These rich tasks are often referred to as high-level tasks because they demand the use of thinking processes connected with higher cognitive levels of awareness and understanding. High-level tasks are often complex, less structured, and the time needed for completing the task is longer than tasks where little connection to meaning is present. The Ontario Ministry of Education (2005) promotes using challenging tasks that expects students to use higher-levels of cognitive thinking processes in order to make connections among mathematics strands, other disciplines, and the world in which they live.

The QUASAR project indicates that teachers who choose to include rich mathematics tasks with high-levels of cognitive demand in their classrooms will improve student understanding of mathematics and their ability to reason using mathematics as evidence (Arbaugh & Brown, 2005).

TIMSS evaluated the mathematical activity being implemented in classrooms around the world. The results concluded that deductive reasoning, another characteristic that can be found in the description of a rich mathematics task, was a characteristic that best
allowed students to engage in key mathematical principals in an effort to draw logical conclusions (Stigler & Hiebert, 1997). Deductive reasoning was witnessed in sixty-two percent of Japanese middle school mathematics lessons and in no middle school mathematics lessons in the United States, meaning that Japanese students were spending significantly more time proving, justifying, and analyzing mathematics via their rich mathematics tasks. Students in the United States rarely had the opportunity to explore mathematics and discuss relationships (Hiebert & Stigler, 2004).

In order for students to improve on their mathematical thinking, teachers should provide worthwhile tasks that bridge the learning of procedures to underlying concepts. In addition, rich mathematics tasks are the tasks reform initiatives support in an effort to improve student achievement, one goal in mathematics education. Hiebert and Wearne (1997) discuss how students who engage with tasks aligned to reform initiatives feature longer explanations for the reasoning behind their solutions and demonstrate the use of higher-levels of cognitive thinking processes in the end of the year assessment responses.

Stein and Lane (1996) use instructional tasks as a means of measuring the relationship among teaching and learning in middle school classrooms. Other studies suggested that students in primary grades significantly benefited from rich task instruction (Fennema, Franke, Carpenter, & Carey, 1993; Hiebert & Carpenter, 1992). Specifically, when primary school students are provided with tasks that provide them with the opportunities to engage in sense-making, versus procedures and memorization, they develop a deeper understanding of mathematics. Similarly, middle school students who are exposed to rich mathematics tasks are associated with an increased understanding of mathematics and can provide extensive responses that demonstrate their
understanding (Stein & Lane, 1996). These rich tasks included exposure to multiple representations, various solutions strategies and a need for justification of solutions.

Stein and Lane’s (1996) research also provides evidence that economically disadvantaged students are not the only students who benefit from rich mathematics tasks. The students participating in this study show greater gains in mathematics understanding and are products of economically disadvantaged communities. Many of these students have yet to fully acquire the basic elementary skills, indicating that students of all levels and from all backgrounds can benefit from rich mathematics tasks.

Other studies aimed at discovering how tasks play a role in the relationship between teaching and learning have also reported that rich mathematics tasks provide discourse among students and teachers, contributing to a deeper understanding of the underlying mathematical concepts embedded in the task (Silver & Stein, 1996; Stein et al., 1996). Communication, along with problem-solving, affords students the mechanisms to reason through a solution in order to build understanding and construct new meaning. Communicating with others allows students to refine their solutions and to view other perspectives in order to broaden their own thinking through self-reflection. Shimizu, Kaur, Huang, and Clarke (2010) argue that mathematics tasks can be a catalyst for communication among students and for delivering mathematical ideas. More connections among mathematical representations can be conjectured through communication and a continuous process of broadening students’ understanding may evolve (Ontario Ministry of Education, The Ontario Curriculum, Grades 1-8 Mathematics, 2005).

McDougall (2004) suggests that teachers use a variety of instructional tasks, both
procedurally-based and conceptually-based, to teach mathematics in a way that will reach all students. In accordance, the mathematics curriculum should be based on rich mathematics tasks and procedurally-based tasks should be linked to understanding. With a goal of improving student achievement, rich mathematics tasks can be used to create a deep interconnected understanding of mathematics that will ultimately lead to student achievement gains. More importantly, rich tasks can open doors for students to see mathematics as one unit made of up key principles, and student should be able to use their understanding of these connections to understand the world and how it works.

2.5.2 Task Selection

Task selection is also referred to in literature as task choice and closely tied to the phrase task design. Teachers can select an already made task and adapt it to better fit the needs and interests of their students or they can create an entirely new task. Teachers are also expected to choose appropriate tasks for assessments, evaluate the effectiveness of tasks in their classroom, and determine the order of tasks to best support the learning of their students.

The International Council on Mathematics Instruction (ICMI, Study 22, 2013) compiled and summarized relevant research on task design and the difficulties faced when designing and implementing tasks. ICMI (2013) regards the role of teaching to be inclusive of "the selection, modification, design, sequencing, installation, observation and evaluation of tasks" (p. 10). With little practice in the area of choosing and using appropriate tasks, teachers often trust that curriculum textbooks will cover the necessary topics while aligning with reform initiatives (Remiliard, 1999). Despite efforts to move teachers away from using textbooks as the sole provider of the mathematics curriculum,
textbooks continue to be a mainstay in traditional classrooms. Finn (1990) encourages teachers to choose and refine tasks suggested in textbooks, but to refrain from using these resources as the "embodiment of the entire course" (p. 587).

Textbooks may be followed as a way to guide the curriculum, yet Remilliard (1999) observed that teachers often do not use the tasks presented in the textbooks in the way the authors had intended them to be used. A better predictor of the selection and use of tasks generated in classrooms is their alignment with teachers' beliefs about mathematics and their assumptions pertaining to the teaching and learning of mathematics (Putnam 1992; Remilliard 1991; 1999).

A study conducted by Remilliard (1991) provides evidence linking teachers' task selection with teachers' decisions. These decisions are made based on how the teacher views the task based on their own beliefs and how students react to the task. Teacher beliefs and student diversity are a few factors that play a part in the decisions teachers make about task selection and adaptation. Therefore, it is difficult to generalize how teachers make these decisions. On the other hand, frameworks that have some substance in describing how teachers make decisions about tasks are useful for professional development programs aiming to improve teachers' task selection, and ultimately as a means of improving student achievement.

Teachers have also stated that exposing their students to all procedures similar to those they will encounter on statewide or provincial assessments is another contributing factor to task selection (Hsu, Kysh, Ramage, & Resek, 2007). One study aimed at describing how teachers evaluate the importance of real-world tasks proposes that most teachers focus on the importance of the underlying mathematical skills the task supports
and how familiar the task will be to students (Gainsburg, 2008). Little consideration was
given to whether or not the task would support the key mathematical principles that could
emerge from the task or the authenticity of the task. In summary, Gainsburg (2008)
proposes that "teachers worry more about over-challenging their students than
under-challenging them" (p. 216). Anthony andWalshaw (2009) urge teachers to provide
tasks that are challenging for the learners. They argued that “effective tasks are those that
afford opportunities for students to investigate mathematical structure, to generalise, and
to exemplify” (p. 141).

Every teacher makes numerous decisions in a day regarding complex aspects of tasks
that are typically not grounded in theory. Without any formal practice, how are teachers
expected to choose tasks that are "worthwhile" for their students? In order to change how
teachers view and make decisions about tasks, they should be given ample opportunities
to learn about the complexity of tasks and practice selecting and implementing
appropriate tasks (Arbaugh & Brown, 2005; Remilliard, 1999; Lewis, Perry, Friedkin, &
Roth, 2005; Sullivan, 1999). In doing so, teachers are encouraged to practice classifying
tasks based on cognitive-levels of thinking processes rather than on surface
characteristics (Arbaugh & Brown, 2005). We cannot expect teachers to choose practical
tasks grounded in theory without providing opportunities for them to learn and practice
how to improve their task selection process.

Task selection is an important role given to teachers, yet many teachers have had
little instruction regarding how to choose appropriate tasks for their classroom. If tasks
bridge student learning with mathematical understanding, then teacher development
programs must provide opportunities for teachers to think purposefully about tasks they
use in their classroom and how they will meet their set instructional goals.

2.5.3 Task Implementation

Tzur (2008) suggests that teachers modify tasks during the planning stage and during the implementation stage if they view that students are not responding to the task in the manner they envisioned. Student learning can be described as a direct reflection of the thinking processes in which they are engaged, and the thinking processes students encounter is a direct reflection of the mathematics tasks used in the classroom (Stein et al., 1996). Therefore, task implementation plays a crucial role is supporting or inhibiting the learning of mathematics. One interpretation of task implementation suggests that teachers and students both play a role in controlling the manner in which students are engaged in the task (Stein et al., 1996). Although it is difficult to do, Sullivan et al. (2013) suggests during task implementation teachers should “probe students’ thinking without extensively ‘telling’, and gather a sense of how individuals and groups are responding to the task(s)” (p. 140). Despite the significant impact task implementation has on student learning, various research studies provide evidence that the richness of tasks usually declines during the implementation phase (Doyle, 1988; Henningsen & Stein, 1997; Hiebert & Stigler, 2004; Horok & Robert, 2007; Liledahl, Chernoff, & Zazkis, 2007; Silver & Stein, 1996; Stein & Lane, 1996; Stein et al., 1996). Keeping the integrity of tasks intact is difficult, yet necessary to elicit a range of thinking processes that may lead to improvement in student learning.

In this study, the integrity of a task is considered supported during the integration phase if the intended cognitive demands of the task remain consistent throughout. Likewise, the integrity of a task is considered inhibited during the implementation phase.
if the cognitive demands of the task decline from a high-level (doing mathematics and procedures connected to understanding) to a low-level (procedural thinking, memorization and non-mathematical activities). In over half of the tasks set up to engage students in high-levels of cognitive thinking processes, Stein & Lane (1996) observe a decline from the intended cognitive demand level during task implementation. Likewise, another study uses levels of cognitive thinking processes to measure task integrity and reports the high frequency for a task to decline during the implementation phase (Stein et al., 1996). In this study, every task set up to require no mathematical activity remained so during implementation 100% of the time.

Mathematics tasks set up to elicit procedures without connections to meaning remained so during implementation ninety-six percent of the time. However, tasks set up to connect procedures to meaning only remained so during implementation forty-three percent of the time, indicating that fifty-seven percent of these tasks declined in cognitive level during implementation. Likewise, tasks set up to engage students in the "doing of mathematics" only remained so during implementation forty-seven percent of the time, suggesting that the integrity of fifty-three percent of tasks is lost during implementation. Evidence from this study suggests that supporting the integrity of a task during the implementation phase is significantly challenging for teachers. However, Clarke and Roche (2009) found that teachers view the advantages to using these task to be that: 1) students were more hands on, 2) the tasks were purposeful and carried meaning for the students, 3) they increased students ability to think, 4) engaging for advanced students yet helpful for students who struggle with mathematics, and 5) allowed for students to solve the task in multiple ways.
Stein et al. (1996) observed many factors that contributed to the decline in task integrity during task implementation. The majority of tasks that declined were coined as having on average 2.5 factors contributing to that decline. Classrooms where the cognitive demands of tasks declined during implementation, sixty-four percent of the time the tasks became routinized as result of teacher and/or student pressures, falling into place as the most often contributor for inhibiting task integrity.

Rich mathematics tasks that require high-levels of cognitive demand are often ambiguous. There is not a clear procedure for solving these tasks and students often feel at a personal risk when engaging in these activities. In order to reassure themselves, students often pressure the teacher to provide more detail and direction. This pressure can transform the task set up to supporting the use of high-levels of cognitive demand into a more routinized task that does not allow for the use of high-levels of cognitive demand (Henningsen & Stein, 1997, Stein & Lane, 1996; Stein et al., 1996).

Teachers have been observed to unintentionally alter the rich mathematics task by creating a more routinized solution strategy during implementation. Often teachers feel uncomfortable when their students are struggling, yet stepping in to provide assistance can easily lead to a more mechanical way of problem-solving. Channeling students thinking in a linear fashion is common among teachers and this will ensure that the integrity of the task is lost during implementation. Hiebert & Stigler (2004) observed United States teachers stepping in to complete the tasks for students when it became too challenging, immediately reducing the intended cognitive levels the task was set up to elicit.

The second most common factor observed to reduce the integrity of tasks during
implementation sixty-one percent of the time was the inappropriateness of the task for the students (Stein et al., 1996). If the task did not built upon prior knowledge or have personal value to students, then in most cases the integrity of the task would decline. The task needs to provide some clarity in order for the students to feel motivated to continue, but not give direction on exactly how they should continue (Henningsen & Stein, 1997).

Stein et al. (1996) discuss other contributing factors that lead to the decline in the integrity of tasks during implementation such teachers’ direction towards correctness of answer versus conceptual understanding (i.e., in 44% of tasks), having too much or too little of time to complete the task (i.e., in 38% of tasks), lack of accountability (i.e., in 21% of tasks) and classroom management (i.e., in 18% of tasks). These findings provide evidence that setting up tasks that require the use of high-levels of cognitive demand may not ensure that the integrity of a task will remain intact during the implementation phase.

There is less research conducted on ways teachers can support the integrity of tasks during implementation. However, it is equally important to study the factors that contribute to sustained high-levels of cognitive thinking if improving student achievement is a goal. Stein et al. (1996) states that classroom cultures need to be set up so that students feel supported and accepted when taking on new challenges in the classroom. In addition, teacher should refrain from handing out materials and manipulatives, which may exemplify the teachers’ beliefs on how the problem can be solved. Instead, provide an abundance of resources and allow the students to decide which, if any, could be used in constructing a solution.

Henningsen & Stein (1997) noted prior knowledge as a main factor supporting task integrity in eighty-two percent of lessons. Prior knowledge allows for students to make
meaning and connections among mathematical ideas and it also reduces classroom management issues. In addition to prior knowledge, scaffolding was mentioned as a factor that supports the integrity of tasks in seventy-three percent of observed lessons in Henningsen and Stein’s (1997) study and in fifty-eight percent of observed lessons in a second study conducted by Stein et al. (1996). The goal of scaffolding is to support students through meaningful questions without suggesting a better solution method, as this could lead to procedural understanding without the connection to the mathematical concepts. In addition, scaffolding should continuously probe students for connections between the mathematical ideas and the task (Henningsen & Stein, 1997).

Appropriate allotted time to complete the task was another significant factor in supporting task integrity. Studies constructed by Henningsen and Stein (1997) and Stein et al. (1996) found that seventy-seven percent and seventy-one percent, respectively, of tasks that remained at high levels judged time to be significantly important in maintaining the integrity of the task. Planning for an appropriate amount of time should be acknowledged by teachers before implementing a lesson and can be altered during the lesson to best allow sufficient time for students to explore ideas and focus on the quality of their work.

In seventy-three percent of lessons that upheld the integrity of that task, students were observed modeling their own high-level thinking processes (Henningsen & Stein, 1997). Sharing solutions strategies provides an opportunity for others to view a variety of perspectives. While sharing strategies, students in classrooms that support the integrity of tasks were often pressed to provide reasoning for their solution strategy and justification for their solution. Such was present in seventy-seven percent of Henningsen and Stein's
(1997) observed classrooms that maintained the integrity of the task. Students who can
describe their thinking and make meaning from the solutions must then have a solid
understanding of the underlying mathematical ideas.

Additionally, teachers need to feel confident with the underlying mathematics within
the task in order to liberate discussions and provide appropriate scaffolding. The role of
the teacher during task implementation is vast and extremely important. Teachers have
the power to implement tasks in a way that supports or inhibits task integrity and future
in-service programs should focus on supporting teachers’ knowledge and practice
regarding the implementation of rich mathematics tasks in the classroom.

Lastly, Liljedahl and colleagues (2007) remind the education community that
selecting and developing rich mathematics tasks is a recursive process. Each time a
teacher makes decisions about tasks, whether during the selection or implementation
process, they should also be asking how the task or their instruction can be improved in
the future. In doing so, teachers have the potential to build meaningful lessons that best
support the learning of mathematics in their classroom.

2.6 Teacher Beliefs

Changing teachers’ beliefs can result in a change in teaching habits, yet it is
described as notoriously difficult to do. However, changing teachers’ beliefs to better
align with reform initiatives is considered necessary (Orlosky & Smith, 1972). Teachers
have deep rooted beliefs about teaching from experiencing the traditional system for
many years, engaging in fruitful methodology courses in teacher’s college, and
experiencing the role of teaching for themselves. Fortunately, teachers have the power to
change, despite having experienced the traditional system as a student (Ross et al., 2004).
The way teachers choose to teach in the classroom is often a direct reflection of their teaching beliefs. For instance, if a teacher chooses tasks that involve repetitive routines most often in their class, then it is likely that they strongly believe that mathematics is best learned by practicing procedures. In order to change teaching practice, one must first look into how to change their beliefs about the teaching and learning of mathematics. The preparation and implementation of classroom instruction determines student achievement (Wong, 2004). With this in mind, analyzing links between teachers and tasks, and tasks and students, is vital to helping teachers examine their own beliefs about teaching, make choices about appropriate instruction, and evaluate their teaching for future lessons (Horoks & Robert, 2007).

Altering the beliefs teachers hold towards teaching mathematics strongly correlates to how they implement tasks in the classroom, and tasks selection and implementation ultimately have the power to lower, sustain or improve student achievement. Teachers have the authority to change the nature of tasks by using questioning tactics that stress certain aspects of the task, engaging students in the use of high-levels versus low-levels of cognitive thinking processes (Stein et al., 1996). Tasks teachers currently choose for classroom instruction represents teachers’ assumptions about pedagogy and the learning of mathematics (Remilliard, 1999) because they do. The question now is how teachers’ beliefs and assumptions about the teaching and learning of mathematics can shift to support student-centered classrooms driven by rich mathematics tasks.

2.7 Teacher Collaboration

Collaboration is an “essential element” to growth in the professional world of teaching (Ross et al., 2002, p. 14). Reduced workloads, more clear understanding of
content, appropriate pacing, emotional support and new teaching strategies are just some of the factors teachers have cited as benefits they reap from collaboration (Ross et al., 2002). Collaboration is a way that teachers can plan to learn not only from others, but from themselves (Hiebert & Stigler, 2004). Despite the overwhelming evidence that collaboration does improve teaching, resulting in student gains, it is seldom witnessed in action within schools in North America. Eloquently stated by Stigler and Hiebert (1997), the long term “problem is not how we teach now but that we have no way of getting better” (p. 20). The teaching culture in North America lacks collaborative approaches, often resulting in teaching habits that do not support reform initiatives and teachers who have little time, practice or value towards improving their instruction.

Using Interdisciplinary teams is one suggestion for creating a collaborative environment in schools that encourage teachers to discuss instructional strategies and ways to improve learning in their classroom. Schools who have implemented this structure have higher student gains than schools without interdisciplinary teams (Flowers, Mertens, & Mulhall, 2003). Schools that have demonstrated the greatest student achievement valued interdisciplinary teams and meet regularly to plan for instruction and evaluate their own teaching progress. When teachers recognize that improving their instruction can be facilitated via themselves and their colleagues, versus an unknown third party who speaks with them during one day of professional development, then the culture of teaching in their school will be on a new trajectory (Hiebert & Stigler, 2004).

Similar to interdisciplinary teams, lesson study is another suggestion for creating cultures where teachers frequently share knowledge through collaboration. A lesson study is often composed of a group of teachers who teach the same discipline, while
interdisciplinary teams are often composed of teachers from a variety of disciplines. In Japanese schools, lesson study occurs routinely “as a school-based, district-based, and national activity” (Lewis et al., 2012). The beliefs underlying lesson study is that teachers will create and implement better lessons when they collaborate with others, and this will improve student’s exposure to knowledge in order to allow for more opportunities to learn (Stigler & Hiebert, 1997). Research findings from a study conducted by Lewis et al. (2012) suggests that when supports are made to engage with lesson study, teachers “are able to improve their knowledge, to build habits and dispositions that support involvement, and to improve student learning (Perry & Lewis, 2010)” (p. 368). These findings also demonstrated that lesson study was one factor that led to improved test scores in mathematics at triple the rate of the district itself.

Teachers’ knowledge, emotions and instruction can benefit from remaining open to new ideas and questions from their colleagues. Arbaugh and Brown (2005) found that teachers who are given the time to examine their practices with fellow teachers and who are willing to engage in analyzing mathematics tasks based on cognitive thinking processes, become more conscious about choosing and implementing rich tasks in their classroom. These findings suggest that teachers are willing to look critically at their own practices, and practices of their colleagues, through appropriate professional development sessions. This could be one step towards changing teachers’ beliefs about the teaching and learning of mathematics, leading to a change in the culture of teaching.

Developing efforts to improve the frequency of collaboration in classrooms will extend the confidence in teachers who wish to use rich mathematics tasks in their classroom. Lewis et al. (2012) reminds us that “we should not expect teachers to learn
without actual practice and feedback from colleagues” (p. 373). A true profession of teaching should encompass teachers collectively improving pedagogy using collaborative efforts (Stigler & Hiebert, 1997). Similar to a domino effect, collaboration supports teachers who want to improve their teaching. Improvement in teaching has the potential to result in student achievement and can lead towards an alteration in teaching beliefs (Arbaugh & Brown, 2005).

2.8 Summary

The culture of mathematics teaching remains deeply rooted in traditional approaches to teaching. Through collaboration, teachers have the opportunity to improve their practice and alter the culture of teaching to better reflect reform initiatives. Reform classrooms have the ability to improve student achievement by offering and appropriately implementing rich mathematics tasks that are student-centered, allow for multiple solutions and solution strategies, and make explicit connections between concepts and procedures.
Chapter Three: Methodology

3.1 Introduction

This section encompasses the perspective taken and approaches used that serve as the foundation for this study, including a rationale for each decision. In addition, the participants are introduced individually and their backgrounds are described. The school setting is presented, including their school-wide improvement initiatives. Next, the data collection and analysis processes are illuminated. The data collection consists of surveys, observations and interviews, and obtaining validity is further presented. The data analysis phase used the assistance of a software program called NVivo 10 to identify themes. In addition, the major themes and sub-categories are introduced. Finally, the ethical considerations are mentioned and a description for how this study maintained ethical awareness is offered.

3.2 Research Content

I chose a qualitative approach for this study to capture the uniqueness of the environment and the participants’ perspectives (Bogdan & Biklen, 2003). As the researcher, I will serve as the key instrument in collecting data and my focus will center on making meaning from the descriptive data. Specifically, I chose to use a collective case study approach bounded in one middle school and a cross case analysis to describe the findings. The exploratory nature of this study affords me the chance to describe multiple perspectives in order to better understand the participants' views. My goal is to provide a thick description of each case to better understand how a teacher makes decisions in the classroom regarding rich mathematics task selection and implementation.

I have chosen to use a constructivist perspective where “reality is co-constructed”
(Creswell, 2013, p. 36). As a researcher, observing rich mathematics tasks in action will provide data that best exemplifies the experiences in which I aim to better understand. Becoming one with the classroom allows me to sense how students and teachers view rich mathematics tasks, behaviors that emerge as a result of task implementation and how teachers respond to students struggling with the open-ended characteristic of rich mathematics tasks. In addition, numerous interviews with the participants will allow me to provide descriptive data on the lived experiences they encountered, and I intend to analyze these experiences for meaning.

3.3 Participants

Gamma Middle School is involved in the School Improvement in Mathematics Study (McDougall, 2013) where both the grade eight math teachers from Gamma and the vice principal participate in professional development sessions at a local University. It is at one of these sessions that I was introduced to Gamma’s vice principal who was enthused by my research topic and quickly volunteered her school as the data collection site. Gamma Middle School is currently focused on improving their use of rich mathematics tasks in the teaching and learning of mathematics, so my research easily coincided with their improvement plan.

After meeting with the vice principal at Gamma Middle School and sharing the goals of this research project, I was paired with two teachers who volunteered their time and classroom for this study. Denise, a grade eight mathematics teacher, and Anthony, a fairly new teacher to their grade six team, serve as the participants for the study.

3.3.1 Grade 6 Teacher

Anthony began his studies at a University in Ontario, majoring in history. Having
little idea of where his studies would take him, he happened to stop into a meeting on campus that recruits students for teacher’s college. Teaching had never crossed his mind before, but it only took him one meeting before he found himself back in the classroom at teacher’s college. His first teaching positions were in an elementary school teaching various subjects to grade one, three and five. Although he admits that students of the elementary age have a lot of personality, he finds middle school to be a much better fit for him. After eleven years of teaching, Anthony does not mind teaching mathematics and has found enjoyment teaching the music theory class assigned to him at the beginning of this year. When asked what his goals are for his classroom, Anthony suggested that his overall goal is “to get the kids to know the curriculum” and to be able to “answer a question without having to ask me how to answer the question” (Initial Interview, February 26\textsuperscript{th}, 2014).

3.3.2 Grade 8 Teacher

Denise is described as a kind-hearted individual and is considered a star teacher in terms of implementing reform initiatives in her classroom. Having attended University in the city of Toronto, Denise was certain she wanted to work with children from an early age. After attending a local teacher’s college, Denise was offered an opportunity to teach history, language and geography to grade eight students, but always had a special passion for mathematics. After serving nine years as a grade eight mathematics teacher, Denise could not be more satisfied with her career. She enjoys teaching mathematics and she respects the attitudes and experiences her students bring to her classroom. When asked what her goals are for her classroom, Denise responded that she wants students to feel as though “learning is fun” and “they can do it” (Initial Interview, February 19\textsuperscript{th}, 2014).
Denise is also a participant of the larger study, School Improvement in Mathematics (McDougall, 2013), and has attended various professional development sessions where rich task discussion has occurred. Additionally, Denise is familiar with the Ten Dimensions of Mathematics Education framework and the Attitudes and Practices for Teaching Mathematic survey.

3.3.3 School Context

Gamma middle school is found in an urban neighborhood in Ontario educating almost 400 middle school students. There are twenty-five teachers and roughly fourteen support staff members at Gamma Middle School. With students speaking over 150 first languages, Gamma Middle School is a hub of diversity. Encompassing grades six through eight, this middle school has pride that resonates from the welcoming vibe they share with visitors. In the 2010-2011 Assessment of Reading, Writing and Mathematics, Primary and Junior Divisions, only forty-eight percent of sixth grade students at Gamma Middle School suggested that they like mathematics, suggesting that over half of the grade six students did not care for mathematics. Thirty-eight percent of female grade six students surveyed said they liked mathematics, indicating that sixty-two percent of female grade six students are indifferent to mathematics or dislike mathematics. Additionally, only fifty-four percent of students surveyed in grade six said that they check their final answer to ensure that it makes sense. Lastly, only forty-eight percent of grade six students assessed by the Education Quality and Accountability Office (EQAO) tested at or above the provincial standards in the years 2011-2013.

According to the vice principal at Gamma Middle School, they would like to improve student scores on the EQAO assessment, particularly in mathematics. Their plan
for improvement aligns with the Ten Dimensions of Mathematics Education framework and the dimension they are currently working on improving is incorporating rich mathematics tasks into their teaching.

3.4 Data Collection

Data for this study was collected from February 2014 to June 2014. An Attitudes and Practices for Teaching Mathematics survey was administered (Appendix A), along with numerous observations and interviews. The collected data was gathered at the discretion of the participants and Denise and Anthony were well aware that they could drop out at any time without repercussion. All dates for data collection were previously agreed upon based on the participants’ availability and timeline for rich task implementation.

During the first meeting with each participant, I handed them a short description of the research project, including the goals of the research and the layout of the data collection. One initial goal for this meeting was to ensure that Denise and Anthony felt comfortable with having me in their classrooms and to thank them for their time and commitment to making this a successful project.

My second goal was to administer an Attitudes and Practices for Teaching Math survey (Appendix A) that is designed to illuminate teachers’ beliefs and practices towards teaching mathematics in a reform-based manner (McDougall, 2004). This 20-question survey is a self-assessment with quantitative results that are known as soon as the survey is complete. Using this survey, teachers should have a better understanding of the dimension(s) within their practice that could use improvement. In addition, I used this survey to identify the beliefs Denise and Anthony held towards the implementation of rich mathematics tasks in their classroom. Lastly, the participating teachers signed a
consent form (Appendix D) allowing all data to be used for the purpose of this study and were aware that they could back out at any time. Both participants seemed relieved that I promised not to take up much of their valuable planning time and we agreed that our initial interviews could be administered via telephone.

The initial interview with Denise came on February 19th, 2014, during her lunch hour. The conversation had via telephone was recorded and later transcribed for data analysis purposes. This initial interview started with background questions and followed with questions pertaining to the school culture and rich tasks in the mathematics classroom (Appendix B). This same interview was repeated on February 26th, 2014, with Anthony. This conversation was also recorded and later transcribed. At the end of each phone interview, an observation date was set up according to the participants’ plans to implement rich tasks in their classroom.

On March 6th, 2014, I met with Denise to observe her classroom as she introduced a rich mathematics task. During this observation, Denise introduced an artist named Sol LeWitt and gave instructions for students to create their own Sol Lewitt art on graph paper. In doing so, they would need to incorporate specific aspects into their painting such as a regular pentagon with a perimeter of 15cm and 3/12 of the graph should have dashed vertical lines of any primary color. A scaffolding sheet was handed out to all students so that they could plan for their design before the drawing and painting phase.

After the lesson was implemented and observations were taken, I met with Denise for a semi-structured post-observation interview. Some of the post-observation interview questions were: 1) How did you choose to use this task to use in your classroom, 2) What makes this task "rich" or not, 3) Did the task work well for the students and why or why
not, and 4) Did you need to provide more or less support for the students than you expected. I would begin by asking Denise how this rich task was selected and then ask her for her thoughts on how well it was implemented. Finally, I asked specific questions aligned to the decisions she made during implementation to better understand the choices she made. Although the semi-structured post-observations were not recorded, thick descriptive notes were taken.

Over the course of four months, I was invited back into Denise’s classroom four more times to observe a variety of rich mathematics tasks ranging from the Pythagorean Theorem to graphing scatterplots. Each observation was followed with a short semi-structured interview that focused on better understanding how she selected the rich mathematics tasks and why she chose to implement it each particular manner.

On March 31st, 2014, I was invited into Anthony’s classroom to observe a task he described as a rich mathematics task. After reviewing their homework on Order of Operations, the students were assigned to mixed ability groups to complete a rich mathematics task. This task asked students to design their own Order of Operations problem for a game show. Once approved by the teacher, the students could draw a picture at the top of the page to display a prize that the contestant would win if solved appropriately. After observing this rich mathematics task lesson, Anthony and I met for a post-observation semi-structured meeting where we discussed the origin of the task and the choices he made during the implementation phase. Over the course of the next three months, I returned to Anthony’s classroom two more times to make observations during the implementation of the rich mathematics tasks and engage in post-observation interviews. The second rich mathematics tasks I observed provided an opportunity for
students to demonstrate their learned knowledge regarding fractions and the purchasing of pizza.

Finally, I conducted post-observation interviews with each participant over the phone after I reread through all of my collected data (Appendix C). During these interviews, I asked questions based on inquiries I had made during the reread of the collected data and I thanked the participants for their time and support. These interviews were later transcribed for data analysis purposes.

In order to establish validity in this cross-analysis case study, I used a method called triangulation. Triangulation uses three sources as a means to accurately reflect the true perspectives of the participants. The survey, observations and interviews were used as the three sources for triangulation and proved to deepen my understanding of each participant’s views, attitudes, beliefs and practices regarding the teaching and learning of mathematics.

One of my goals during this project was to remain non-invasive during classroom observations, yet provide collaborative assistance at the teacher’s request. Overall, the participants and the administrative staff agreed that my research did not take up too much of their planning time and described the process to be easy-going.

3.5 Data Analysis

This research study places an emphasis on “collecting descriptive data in natural settings, uses inductive thinking, and emphasizes the subjects’ point of view” (Bogdan & Biklen, 2007, p. 274). Therefore, a qualitative approach with an analysis strategy that includes inductive reasoning was the best fit for this study. All recorded phone interviews were transcribed and looked over to guarantee accuracy among the transcription and the
actual recordings. An open-coding method was established where “the researcher can then look for similarities and differences among the cases” (Creswell, 2013, p. 200). In accordance with a constructivist approach, “the goal of [this research study] is to rely heavily on the participants’ views of the situation” (Creswell, 2013, p. 24).

The software program NVivo10 was used to assist with the coding process. This research software program is designed to provide organized analysis of collected data and support insights for possible connections in the data. Within the program, key ideas elicited from the data were established and labeled into nodes. All collected data was dispersed into each categorical node for organizational purposes. Within each categorical node, sub-categories were created to better organize the data for analysis purposes. Once this was complete, the categories would serve as the principal themes.

The categories that emerged from this analysis process are: 1) characteristics of task selection, 2) sources for task selection, and 3) task implementation. Within the characteristics categories, three sub-categories emerged: 1) meeting individual needs, 2) structure of rich tasks, and 3) assessments. Although no sub-categories emerged from the category sources for task selection, a variety of sources were discussed. Lastly, 1) supporting factors and 2) inhibiting factors were two emerging sub-categories within the task implementation category. Although these categories emerged from the collected data, the influence of the knowledge of exiting literature cannot be overlooked.

3.6 Ethical Considerations

Ethical considerations were taken into account prior to the start of the study. This study is a subset of a larger study in which the ethical review process was completed as a part of McDougall’s (2009) larger research project. In addition, this study is considered
low-risk as it mainly deals with adult participants in an area that is not considered sensitive.

Denise and Anthony first made verbal agreements to participate and then signed formal consent letters at the onset of the project (Appendix D). There were aware that neither their name, nor the school’s name would be used in the study. Instead, pseudonyms were constructed and used throughout the data collection and analysis phases. Although written in the formal consent letters, I also orally reiterated that at any time they were able to terminate the contract and rescind all collected data with no consequences. In addition, all data was confidential and at all times was stored on a secure network inside a locked computer.

It was mentioned to participants that, at all times, they were allowed to review collected data and findings to ensure accuracy and reliability. As reciprocity, I gave each participating teacher a fifty-dollar gift card and the vice principal a fifteen-dollar gift card as a token of my appreciation.
Chapter Four: Findings

4.1 Introduction

Two cases are presented, one depicting Denise’s perspective and one depicting Anthony’s perspective of rich mathematics tasks during the selection and the implementation phases.

4.2 Case One

Denise describes her teaching practice as “dedicated” and “motivated not only to help my students, but also to get better at what I am” (Initial Interview, February 19th, 2014). She defines herself as a mathematics teacher and has no problem stating, “I love math” (Initial Interview, February 19th, 2014). Denise uses rich mathematics tasks in her classroom approximately once a week or every other week, and these tasks tend to last a few class periods.

Based on our discussion of the Attitudes and Practices for Teaching Math survey (McDougall, 2004) result, Denise strongly agrees that she puts less emphasis on getting the right answer and more on the process followed. In addition, she suggests that she frequently introduces open-ended tasks in her classroom and provides students with an opportunity to solve these tasks in multiple ways.

Three themes emerged from the data analysis phase: attributes of rich mathematics tasks, sources used for the selection of rich mathematics tasks and factors that arise during the implementation phase.

4.2.1 Characteristics of Rich Tasks

Denise was asked about her views pertaining to rich mathematics tasks and why one task was considered richer than another throughout the data collection process. My goal is
to establish a solid perspective of what characteristics define a rich task through the eyes of Denise. The characteristics mentioned describe three underlying attributes that Denise looks for in rich mathematics task: the task should meet individual needs, exemplify a range of characteristics, and align with assessments.

4.2.1.1 Meeting Individual Needs

There are a variety of attributes Denise identifies during our discussion, with providing rich mathematics tasks that meet student needs being high on her priority list. In order to meet the needs of her students, Denise plans for rich mathematics tasks that are engaging, provide for differentiated instruction and align with real-world applications.

4.2.1.1.1 Engaging Tasks

Denise often mentions that she works at making mathematics enjoyable for students and she chooses tasks that “go with what [they are] teaching and that will engage the kids” (Initial Interview, February 19th, 2014). She loves mathematics and wants to provide occasions where students can share that positive feeling towards mathematics. I observed Denise laughing with the students and describing mathematics as “cool”.

When choosing a rich mathematics task, Denise says she tries “to picture myself in the role of the students. Will this be boring for me, or this will be fun? Providing inspiration for my students” is very important (Initial Interview, February 19th, 2014). How students react to the task and feel about the task is as important as the task itself. She describes the use of rich mathematics task to be “more fun” when the “experiences [are not] too far apart” (Final Interview, May 23rd, 2014). She hopes the use of rich mathematics tasks in no longer a single event, but instead “becomes their learning” as a whole (Final Interview, May 23rd, 2014). Denise is committed to improving the learning
of her students through engaging tasks that students enjoy.

4.2.1.2 Differentiation

Almost every rich mathematics task that Denise introduces into her classroom incorporates differentiation. For instance, the first task I observed starts off as a small scale art drawing where the students are directed to color half of the grid one color, a fourth of the grid another color, and so on. Her students have a choice on how big they wanted the grid, the colors they wanted to use, and the overall design. Once the students complete this task, Denise introduces another rich mathematics task with the same objective, but this time incorporating more difficult fractions and mathematical figures of particular lengths. Denise says:

I basically gave them instructions. So, a quarter of the grid has to be a particular color, three eighths of it has to be another color, and you have to include a parallelogram and a rectangle. They're using their knowledge of fractions to create art pieces. So, at the same time, I'm able to accommodate the IEP students, because instead of making it difficult fractions, I can use simpler fractions. And then I introduced this one, which includes a bigger grid, as well as more complicated fractions. (Initial Interview, February 19th, 2014)

Giving students choice is one strategy Denise uses to differentiate student learning. Often she will give students a final answer and say, “you write the question” (Initial Interview, February 19th, 2014). For instance, she gave the students the final answer of 1/3 and asked her students to come up with a question or questions that would result in an answer of 1/3. She describes this strategy to be a helpful in differentiating instruction because “you have the simpler responses, and then you have the more complicated ones that use the knowledge of simplifying fractions” (Initial Interview, February 19th, 2014).

4.2.1.3 Practical Use

Many of the rich mathematics problems Denise shared with her students are derived
by aligning the mathematical concepts in that task with practical real-world uses. Denise finds that assigning real-world value to the mathematics helps her students apply the mathematics to the outside world. A variety of rich mathematics tasks allow for students to “see the importance of the math” (Post-Observation Interview, April 3rd, 2014). In some traditional tasks, the students are not given the opportunity to “realize the uses for it” (Post-Observation Interview, April 3rd, 2014).

4.2.1.2 Structure of Rich Tasks

The structure of mathematics tasks is often defined by the characteristics of the mathematics task. Over the course of this study, Denise describes a variety of rich mathematics task characteristics. One characteristic that Denise looks for when selecting a rich mathematics task is the opportunity for students to use high-cognitive thinking processes. As she describes, rich mathematics tasks elicit “thinking and it is better than [her] telling them how to do it all of the time” (Post-Observation Interview, April 3rd, 2014). Denise believes her immediate assistance may lower the cognitive demand of the rich mathematics task, so she aims to refrain from sharing with the students a viable solution until they have had some time to grapple with the concepts on their own or in pairs. From Denise’s perspective, allowing the students to derive their own solutions upholds the intended high-cognitive demand of the task.

Denise mentions that, when students return from the summer, many have forgotten a great deal of the mathematics they were expected to learn in the previous year. When asked if she believes that the use of rich mathematics tasks in the previous grades would elicit a higher retention of material, Denise says:

Definitely, because I find [that the problems they use in the previous grades] are very rote based. [Students] tend to memorize the material, instead of understanding it.
Memorizing can only last so long. If you understand it, then the application becomes a lot easier. (Final Interview, May 23rd, 2014)

Denise believes that rich mathematics tasks are intended for students to understand the underlying mathematical concepts, while many non-rich tasks are directed towards knowledge questions where memorization can be employed. In addition, Denise believes that rich mathematics tasks offer students the opportunity to remember mathematical concepts for a longer period of time than would be expected of tasks that depend solely on memorization. This is because the students have a deep understanding of how the mathematical concepts are connected and applied.

Another central characteristic that makes up the structure of rich mathematics tasks mentioned by Denise is the student-centered component of the task. Denise prefers to use rich mathematics tasks in her classroom and she mentions that she is:

Not the type of teacher who sits at the front and lectures [the students] on how to solve the problem. I may show them a method, but then I let them play around with it, come up with their own solutions and share them. Sometimes it makes sense, and sometimes it does not make sense. But, regardless we talk about it. (Initial Interview, February 19th, 2014)

Denise finds herself using more student-centered tasks in her classroom than teacher-directed learning because she feels as though her students must make their own connections with the mathematics before she imparts her solution methods on them. Denise mentions, “student-led is one characteristic” of rich mathematics tasks (February 19th, 2014) because “with rich tasks, the learning is in their hands. You are helping them, but you are not really telling them. You are just asking them the right questions to get the meaning out of it” (Final Interview, May 23rd, 2014). It is apparent that Denise wants each of her students to build their own knowledge of the mathematics and Denise mentions in her post-observation interview that rich mathematics tasks afford students the
time for “exploring and having them come up with information based on their own knowledge, instead of me giving them all of the questions and answers” (March 24th, 2014). Her classroom exemplifies a community of learners where the students can learn from Denise and Denise can learn from her students.

A third characteristic Denise often looks for in the structure of rich mathematics tasks is a cross-curricular component. Although she admits that every rich mathematics task cannot be cross-curricular, she still “like[s] the idea of [rich mathematics tasks] being cross-curricular” when possible (Initial Interview, February 19th, 2014). A common subject that Denise integrates into her grade eight mathematics classroom is art. For instance, when Denise describes a rich mathematics tasks the students are currently working on, she says, “we are currently doing fractions and we are integrating it with art” (Initial Interview, February 19th, 2014). Denise’s integration of art is not a modest as draw a picture when you are finished with the mathematics. It involves a more complex component and often uses specific art vocabulary such as “use secondary colors to represent 3/8 of the grid”. Denise acknowledges that not every characteristic she mentions should be in every rich mathematics task.

From Denise’s perspective, rich mathematics tasks should be “open-ended” to allow for “different output as they say” (Initial Interview, February 19th, 2014). She believes open-ended tasks allow for a variety of solution strategies and sharing these strategies may be one way to improve the mathematical literacy of her students. Denise believes rich mathematics tasks that include an open-ended component allow for students to engage with the mathematics at a deeper level. There is no one algorithm that can be regurgitated, so students must build on their existing knowledge to create new knowledge.
This process may allow for a deeper understanding of the mathematics.

### 4.2.1.3 Assessments

Rich mathematics tasks can also be used for assessments for learning, of learning and as learning. Denise says she knew how to incorporate the Sol LeWitt task into her classroom because they:

Did a pre-test. We independently did this, and I gave them simple instructions. It was a hundred centimeter grid paper and I instructed that a quarter of this should be red, one fifth of this should be blue” and so on. (Initial Interview, February 19\textsuperscript{th}, 2014)

She uses this rich assessment for learning task as a diagnostic to identify what students know and to show her where to begin her lessons. Denise says, “my job is to observe and assess”, once again highlighting that her students are at the center of their learning and that she acts as the chaperon to their learning.

### 4.2.2 Source of Tasks

Denise is known in her education community for exceptional achievement in fulfilling reform initiatives. Her use of rich mathematics tasks feels like a “natural” role within her duties as a teacher. When asked how she selects which rich mathematics tasks to implement in her classroom, she said she does “a lot of research” (Initial Interview, February 19\textsuperscript{th}, 2014). For instance, when Denise discusses the Sol LeWitt task, she mentions that her “inspiration came from online. I first Googled math and art lessons and then I found an elementary level Sol LeWitt activity. I adapted it to my grade eight class” (Post-Observation Interview, March 6\textsuperscript{th}, 2014). Denise uses the Internet, specifically the Google search engine, as her main source for choosing rich mathematics tasks. Usually the rich mathematics tasks she uncovers must be altered to meet the needs of her students and to conform to Ontario’s grade eight curriculum. She will often “look up questions
online that go with the mathematics [they are] doing and then change the names to teachers’ names” (Post-Observation Interview, April 3rd, 2014).

Denise is a member of mathematics education communities such as “The Centre for Education in Mathematics and Computing” (Post-Observation Interview, April 3rd, 2014). She receives an email once a week from this particular education group with a problem of the week. She explains these problems of the week share similar characteristics with rich mathematics tasks, so she often adapts these tasks to better fit with her classroom and students’ needs.

Lastly, a source from which Denise commonly selects rich mathematics tasks is mathematics workshops and professional development sessions. She has attended several this school year and finds the tasks typically shared are rich and she can easily adapt them for use in her classroom. Also, the workshops and professional development sessions afford Denise the opportunity to collaborate with other mathematics teachers to alter or design rich mathematics tasks, and she finds this to be “really helpful” (Initial Interview, February 29th, 2014).

4.2.3 Implementation

Implementing rich mathematics tasks in an effective way where students are engaged in the mathematics and are creating knowledge through inquiry is a target Denise aims for each time she carries out the task. Through observations and interviews, factors are identified that support and inhibit Denise’s attempt to implement the rich mathematics tasks in her grade eight classroom.

4.2.3.1 Supporting Factors

Denise looks for rich mathematics tasks that are open ended, allowing for multiple
solution strategies and outputs. Once students have developed their own rational strategy for progressing through a problem, Denise often asks students to share their methods. During an observation, Denise was recorded saying, “you will notice there are no answers or solutions. That is because on Monday, you are going to do a gallery walk and be able to look at [other’s strategies] and comment on them” (April 3rd, 2014). According to Denise, providing opportunities for students to share their solution strategies and make comments is viewed as a vital element to their growth in learning. She wants her students to feel proud of their thinking and to acknowledge that everyone thinks in their own unique way that works best for them. During another observation (April 3rd, 2014), with permission from the student, Denise presented an example of a word problem a student created where the using knowledge of the Pythagorean Theorem. With help from her students, Denise points out the key components within the word problem that signify that the Pythagorean Theorem can be applied.

In addition to sharing students’ solution strategies, Denise often uses a scaffolding technique to ensure the task is appropriate for her students. When asked why scaffolding is important, Denise said:

I do not want to set them up for failure. So, it is baby steps. I will start them with something small and when I know that they have mastered it, based on what I see, I'll extend it with another activity. If I know that they are still struggling with it, then I will not give it to them. (Initial Interview, February 19th, 2014)

Setting students up for success is a goal that Denise chooses to carry out each day. During a later observation, Denise says, “the first thing you may want to do is plan to use this smaller grid paper to make a plan. The last thing you want is to be ninety percent done and realize you have made a mistake” (Observation, March 6th, 2014). Denise
makes a suggestion, but does not scold those who choose to go immediately to the second part in the task. She merely provides a warning sign that they can choose to follow or choose not to follow. During an activity where students are plotting classmates’ heights versus their foot size, a student asks Denise if they could start the numbering on their axis at a number other than zero. Denise responds with, “that could work and I would look at our example from yesterday” (Observation, March 24\textsuperscript{th}, 2014). The previous day, the students in Denise’s class worked on a small-scale scatter plot where the axes did not begin at zero. Denise used this previous task as a scaffold to this larger richer task, so the students were able to return to her notes from the previous day and the student’s question was answered.

In addition to task scaffolding, Denise uses questioning tactics that also scaffold the learning for her students. She would often set up questions like “I wonder if the length of the hypotenuse would double if the lengths of the two legs doubled” and “how can we find the area of this strange figure if we only know the area of figures like squares, circles and other quadrilaterals”. Denise often uses questions to place thoughts in her students’ heads and allows them to contemplate how they can answer the question without specific directions for doing so.

Another supporting factor during rich mathematics task implementation is the press for justification. In numerous occasions, Denise was witnessed pressuring her students for a rationale on why their solution made sense. Denise says, “explain your mathematical calculations” (Observation, March 6\textsuperscript{th}, 2014), "why is twelve the hypotenuse" (Observation, April 3\textsuperscript{rd}, 2014) and "why is this length only eight centimeters and not sixteen" (Observation, April 3\textsuperscript{rd}, 2014). Denise advises her students to check their
mathematical calculations and to look at the larger picture to ensure their solution contextually makes sense. Denise says she likes “having [students] prove their answers and coming up with reasons why it makes sense, instead of [her] telling them this is how you do it and apply it to these problems” (Post-Observation Interview, April 3rd, 2014). When asked why she thinks justification is important in her classroom, Denise says, “because sometimes they (her students) have low confidence. So, pushing them a little and letting them believe that they can do it is a big thing for me” (Initial Interview, February 19th, 2014). Denise links having students provide justification for their solutions to student confidence. From Denise’s perspective, if students can communicate their thinking in a logical manner, then they will feel more confident in their mathematical abilities.

Another factor that supports the implementation of rich mathematics tasks in Denise’s classroom is the link between procedural fluency and conceptual understanding. Whenever procedures are discussed, Denise takes time during class to discuss the underlying mathematics and why the procedure can be used. In one observation of a rich mathematics task involving the real-world problems that can be solved using the Pythagorean Theorem, Denise asks her students, “is it faster to walk somewhere diagonally or around (outlines the legs in the right triangle)” (April 3rd, 2014). Denise’s goal was to provide practice using the Pythagorean Theorem and also make the connection that the length of the hypotenuse is always shorter than the combined lengths of the legs. In addition, this mathematics task provided an opportunity for students to solve practical real-world tasks using a procedure. In doing so, students were afforded the chance to connect the procedure with applications beyond the classroom walls. For
example, the students were given a television size and asked to determine if it would fit inside a custom cabinet of specific dimensions.

The rich mathematics task engages students in determining solutions for authentic issues using mathematical procedures. Additionally, Denise asks students if they “notice a relationship” while using procedurally based tasks to shift the focus back towards the conceptual mathematical underpinnings (Observation, March 6th, 2014). In another observation, one student asked Denise how to find the whole when one half is eleven. Instead of Denise responding with a procedural response, Denise used the mathematical understanding of one-half and asked:

Denise: Well, what's one half mean?
Student: I'm not sure.
Denise: Okay, well is two fourths the same as one half?
Student: Yes.
Denise: Is five tenths the same as one half?
Student: Yes.
Denise: Is ten twentieths the same as one half?
Student: Yes.
Denise: So what's the same as one half with a numerator of eleven?
Student: What's equal to one half with a numerator of eleven? …Twenty-two!
(Observation, March 6th, 2014)

Denise chooses to answer this student’s questions by revisiting the fundamental idea of halves. This type of response upholds the integrity of the rich mathematics task by stressing the conceptual understanding beneath the procedures.

Before the implementation phase comes into play, Denise mentions that collaboration is one factor that she finds to support the use of rich mathematics tasks in her classroom. During Denise’s final interview, she mentions how working with the other grade eight mathematics teachers allows for the “development [of rich tasks]” and the opportunity to “follow through with it and then reflect on it” (May 23rd, 2014). Denise believes this
process of creating and reflecting on the task as a pair or as a team will improve the richness of the task before and after the implementation phase because multiple perspectives are being taken into account.

### 4.2.3.2 Inhibiting Factors

Similar to factors that support the integrity of the rich mathematics task, there are a variety of classroom factors that diminish the integrity of the rich mathematics task during the implementation phase. In Denise’s classroom, some factors that may have diminished the integrity of the rich mathematics task were student behavior and the appropriateness of the task. Denise also alludes to the idea that time and teachers’ comfort level may also be two factors that inhibit the use of rich mathematics tasks prior to implementation.

One factor observed to impact the integrity of the rich mathematics task during implementation was student behavior. Denise describes a time when she attempted to use a rich mathematics task in the computer lab with a SmartBoard:

> I am not a SmartBoard user. One time I tried to and I was thrown off from ever using it again. This was a couple of years back and I brought [my students] down to use it. I found it to be more chaotic, students arguing to get up on the SmartBoard. It was just out of control instead of more focused. (Initial Interview, February 19th, 2014)

Denise’s students were excited about the rich mathematics task and the incorporation of technology. Their excitement turned into behavior issues and the rich mathematics task Denise had planned was never enacted. In addition, it was observed that behavior issues often lead to other factors inhibiting the rich mathematics task. For instance, when Denise acknowledged that students were off-task and not engaging in the activity, she would step in an effort to refocus their thinking. However, when she stepped in to regain their
attention, she would divulge too much information alluding to strategies for solving the task and reducing the rich component of the task.

Another factor observed to inhibit the integrity of the rich mathematics task during implementation is the appropriateness of the task based on students’ prior knowledge. In Denise’s initial interview, she described how students’ “readiness” and “students’ comfort with [the material]” are factors that could diminish the intended richness of the task (February 19th, 2014). Denise describes that if the students do not have the prior knowledge needed to engage effectively in the rich mathematics task, there is little learning that will occur.

Outside of the implementation phase, Denise mentions that time and teachers’ comfort with mathematics are two factors that inhibit teachers planning to use rich mathematics tasks in the classroom:

I teach three of the grade eight maths, and there is another teacher who teaches one of the grade eight maths. I feel like it is so rushed sometimes that we do not have a chance to sit there and say, okay, this worked out and this did not work out. Mentally I will do it, and I will put little notes on it. But, to sit down and have a full blown conversation, most of the time it is impossible. (Final Interview, May 23rd, 2014)

Denise describes that she chooses to collaborate with the other grade eight mathematics teachers to provide rich mathematics tasks for their students, yet there is little time for the teachers to converse. Denise comments on why she does not use rich mathematics task everyday:

I think there is no time to really plan it accordingly. You do not just want to give [the rich mathematics task] to [the students] without being able to watch them, observe them, talk with them and assess them. Doing that every day, with all of the interruptions and with the different things going on within the school and curriculum, would just be too much. (Final Interview, May 23rd, 2014)
Denise describes that implementing rich mathematics tasks is a process and is usually implemented across a few days of instruction. For this reason, there are outside factors that need to be considered, such as field trips, school assemblies and provincial testing. Denise prefers that the students have little interruptions during the implementation of the rich mathematics task so that she has the opportunity to discuss their thinking with them and monitor their progress in an effective manner. Although Denise admits rich mathematics tasks can take “a lot of time for implementation”, she also admits that, “it is worth it” (Post-Observation Interview, April 3rd, 2014).

In addition to time inhibiting the planning for, implementation of and reflection on rich mathematics tasks, Denise describes that some teachers choose not to use these tasks in their classroom because:

They are not comfortable with the subject and they feel unprepared for any questions that may come up. We are lucky because we are specialized in this area and we are ready for any challenging questions or just questions in general. We are confident that we are thinking of the right questions to ask them. (Post-Observation Interview, April 3rd, 2014)

Denise describes how the open-ended nature of rich mathematics tasks can leave teachers feeling uneasy and unprepared. It is her perception that, if a teacher is not confident in their mathematical ability, then implementing rich mathematics tasks may be uncomfortable for the teacher as many non-routine questions may arise. At the same time, the questions that do ensue from the implementation of rich mathematics tasks are a main reason for why Denise chooses to use of these types of tasks in her own classroom.

In addition to teachers feeling uncomfortable using rich mathematics tasks because of the unrestricted questions that may emerge, Denise suggests, “teachers
are not comfortable with the idea of assessing [the rich mathematics task]” and “that is one of the biggest reasons why teachers do not use it on a daily basis” (Final Interview, May 23rd, 2014). Denise imagines that some teachers feel a burden instead of feeling encouraged to use rich mathematics tasks because they are unsure of how to assess students during the implementation phase.

4.3 Case Two: Anthony

Anthony hopes to create a sense of independence for his students in terms of having them engage in the mathematics without his continuous direction. He views being able “to simplify everything so that the kids can see it in a way that [is] easier for them” as his greatest strength as a mathematics teacher (Initial Interview, February 26th, 2014). Anthony uses rich mathematics tasks in his classroom every few months or whenever his grade six team develops one for cross-curricular use.

Based on our discussion of the results from the Attitudes and Practices for Teaching Math survey (McDougall, 2004), Anthony strongly agrees that he puts less emphasis on getting the right answer and more on the process followed. In addition, Anthony describes how frustrating it can be when the students rely on “calculators to do all of their work for them” (Initial Interview, February 26th, 2014).

Three themes emerged from the data analysis phase: attributes of rich mathematics tasks, sources used for the selection of rich mathematics tasks, and factors that arise during the implementation phase.

4.3.1 Characteristics of Rich Tasks

Throughout the data collection process, Anthony engages in numerous interviews and describes a variety of characteristics that he imagines would need to be included in a
rich mathematics task. The three characteristics Anthony looks for in a rich mathematics task are that the task meets individual needs, exemplifies a range of structural characteristics and aligns with diagnostic and EQAO assessments.

4.3.1.1 Meeting Individual Needs

When Anthony visualizes rich mathematics tasks, he envisions a task where students are engaged and can connect with the practical use for the mathematics.

4.3.1.1.1 Engaging

Anthony expects rich mathematics to energize his students and keep them focused. One rich mathematics task Anthony introduces to his grade six mathematics classrooms requests that students design their own Order of Operations word problem for a contest. Students are expected to draw a prize of their choice at the top of their worksheet for their winning contestants. After implementing this rich mathematics task, Anthony discusses how “word problem keep kids connected because they know about winning the prizes” (Post-Observation Interview, March 31st, 2014). Throughout the lesson, students were engaged in the activity because of their excitement in being able to design a winning prize.

4.3.1.1.2 Practical Use

A second characteristic Anthony believes is an attribute of rich mathematics tasks is the ability it has to connect the mathematics to practical uses in the world. For example, Anthony introduces a rich mathematics task in his classroom where students work in groups and determine how many pizzas and how many sodas they will need for a class party if one student says they could eat 1/5 of a pizza, another student thinks they can eat 8% of a pizza and so on. Then, students are expected to determine how much money they
will have left if they pay with four twenty dollar bills. Anthony says, “It is everyday life.
It is buying pizza. It is paying for things and figuring out how much is left. I look for real-world connections” when I make a rich mathematics task (Post-Observation Interview, April 28th, 2014). Anthony prefers that students can see how they as grade six students can use mathematics outside the walls of the classroom.

4.3.1.2 Structure of Rich Tasks

A rich mathematics task is structured to include a variety of characteristics that integrate to form the foundation of the task. Over the course of this study, Anthony describes a variety of characteristics that he believes a rich mathematics task may encompass, such as high-cognitive thinking processes, cross-curricular learning, open ended responses, multiple steps and alignment with assessments.

Anthony believes that a rich mathematics task should incorporate application and thinking questions and move away from asking strictly knowledge-based questions. Anthony says rich mathematics task:

Require [student] thinking and their application of their knowledge towards the problem. We try and avoid questions that are just basically knowledge questions like 'is this greater than or less than' or 'is this an even number or an odd number'. The questions where it is just memorization and you can get the answer right half of the time, even if you did not know the answer. We are focusing more on the thinking questions and the application questions. (Initial Interview, February 26th, 2014)

It is Anthony’s perception that rich mathematics tasks include questions where students must understand the mathematics to be able to answer the question. The teacher does not have to speculate if the student guessed an answer or not, because rich mathematics tasks include questions where student need to apply their knowledge and engage in high-level thinking processes. In addition, Anthony suggests that rich mathematics task include “rational thinking” (Post-Observation Interview, April 28th,
Students will need to make sense of their answer when using rich mathematics tasks and justify their thinking.

Anthony also describes rich mathematics tasks as having a “multi-curricular” component (Initial Interview, February 26th, 2014). Anthony explains, “at the top (of the worksheet) they can draw something, which includes art” (Post-Observation Interview, March 31st, 2014). From Anthony’s perspective, including a place for a drawing integrates the discipline of art into the task, adding an extra element of richness to the task. In one rich task that Anthony has implemented earlier in the school year, the students had “to choose the right graph and they had to do a presentation basing their information. It was a media literacy thing, persuading the audience to tell which restaurant was the best based on the data they got from their meals” (Initial Interview, February 26th, 2014). This rich mathematics task incorporates mathematics, media literacy and language arts. When choosing a rich mathematics task, Anthony says, “we look at what we're doing in the various classes and we see how we can make a cross-curricular task that goes well with what we're teaching in our different subjects” (Initial Interview, February 26th, 2014). Even when deciding to use a rich mathematics task, Anthony describes how the cross-curricular component is a main characteristic for determining how the rich mathematics task is developed for use.

A third characteristic that Anthony suggests exists in a rich mathematics task is the opportunity for students to engage in multiple steps in the solution process (Initial Interview, February 26th, 2014; Post-Observation Interview, April 28th, 2014). He and his fellow grade six colleagues are “trying to focus on multiple step problems, because those are the ones that require their thinking and their application of their knowledge” (Initial
Interview, February 26th, 2014). Anthony links multiple step problems with students having to use higher cognitive thinking processes, both characteristics of rich mathematics tasks.

The open-ended characteristic of rich mathematics tasks is one component Anthony believes is needed for the mathematics task to be rich. For example, during the implementation of one of Anthony’s rich mathematics tasks, he encourages his students “to come up with their own skills testing question. They can write it in their notebooks and give it to their friends to try” (Post-Observation Interview, March 31st, 2014). From Anthony’s perspective, a rich mathematics task should include open-ended opportunities for students to offer their own creativity.

4.3.1.3 Assessments

Assessments are the third overarching characteristic that Anthony looks examines when selecting a rich mathematics tasks. Anthony describes that he and the other grade six teachers:

Started doing a diagnostic before each unit. We started this year, it is called the ONAP diagnostic and it just looks at where we see the kids are struggling on the unit that we are about to do. We are going to focus on teaching those areas so that the kids can try and improve. (Initial Interview, February 26th, 2014)

Anthony believes that the rich mathematics tasks he chooses for his classroom should elicit mathematical concepts in the areas where students are struggling. In addition, Anthony teaches grade six which is “an EQAO every year [and] they want to see the thinking of the student and not just the answers” (Post-Observation Interview, March 31st, 2014). The markers of the Ontario standardized test are looking for student responses that demonstrate students’ mathematical thinking, so Anthony sees the value in using rich mathematics tasks where students are asked to demonstrate these same processes.
4.3.2 Source of Tasks

When asked from where does Anthony select rich mathematics tasks, Anthony says, “I just thought about it” using the characteristics explained above (Post-Observation Interview, March 31st, 2014). He thinks about what material he needs to cover and how the material is connected with the real-world, and from this he develops a rich mathematics task. During a post-observation interview, Anthony describes how he simply “made [the rich mathematics task] up” the night before (Post-Observation Interview, April 28th, 2014). There is no particular place Anthony selects rich mathematics task from, but instead he relies on his knowledge of pedagogy and his understanding of the characteristics that should be found in rich mathematics tasks.

4.3.3 Implementation

Selecting rich mathematics tasks is one step towards using rich mathematics tasks in the classroom, while implementing the tasks in a way that upholds the task integrity is yet another step. To discover what other factors support or inhibit the implementation of rich mathematic tasks in Anthony’s classroom, many observations were made and notes were taken.

4.3.3.1 Supporting Factors

The only factor observed in Anthony’s classroom to support the implementation of rich mathematics tasks was the press for conceptual understanding. As the students came to conclusions regarding how many pizzas they would need to order for the class party, many of the students had three and eighty-one-hundredths written down as their solution. Anthony discusses with multiple students how that solution is significant:

Anthony: How many pizzas would you need?
Student: 3 and 80/100.
Anthony: Have you ever order pizza before?
Student: Yeah.
Anthony: Can you order 3 pizzas and part of another one? Can you order only some of a pizza?
Student: No
Anthony: So how many pizzas would you need?
Student: 40... No, 4 whole pizzas.
(Observation, April 28th, 2014)

Anthony walks around pressing students to rationalize their thinking and come to an understanding of their solution. Many of the students involve themselves deeply in the mathematics that they forget the real-world connection it provides.

4.3.3.2 Inhibiting Factors

There are a few factors in Anthony’s classroom that may contribute to the decline in the richness of the mathematics task during the implementation phase. These factors include supporting procedural understanding without connection to the conceptual understanding, overlooking cognitive opportunities, behavior issues, and time. Although Anthony has positive intentions, there are a few instances where the richness of the mathematics task may have been inhibited.

One factor that may have inhibited the intended richness of the mathematics task was Anthony supporting procedural understanding without the connection to the conceptual understanding. Anthony says:

Let us look at something (writes 84 / 6 + 2 x 5 on the board). Mena, you have to do the division and multiplication first. If you do it eighty-four divided by six plus two times five, you will get the wrong answer. Even the calculator will give you the wrong answer. (Observation, March 31st, 2014)

There is no mention of why division and multiplication needed to be completed first in left to right fashion, or why there is a specific order to these operations. In another observation, Anthony responds to a student who is asking about adding fractions by
saying, “you only need to add the numerators. The denominator will stay 100” (Observation, April 28th, 2014). The student proceeds to follow his direction, yet may not have any conceptual understanding of why the denominators remain the same. The purpose of this rich mathematics task was for students to understand adding fractions, so this response may have inhibited the richness of this task by focusing on the procedural aspect more than the conceptual understanding.

Overlooking opportunities to discuss the underlying concepts in mathematics is another factor observed to potentially reduce the integrity of the rich mathematics task. During an observation, Anthony engages in a conversation with a student regarding the number of slices in each pizza box:

   Student: How many pizza slices are in one box?
   Anthony: It doesn't ask about pizza slices.
   Student: Well, we could break the pizzas into 10 slices instead of 100 to make it more realistic.
   Anthony: You could do that, but you don't need to make it that complicated.
   (Observation, April 28th, 2014)

This student notices the unrealistic nature of assuming each pizza box contains 100 slices and presents a way to make the problem more realistic. However, without recognizing, Anthony makes the decision that what the student is doing it too difficult and suggests accepting the impractical component within the problem. In another observation, Anthony and his students are discussing factors of forty-five and thirty-six:

   Anthony: What else works for 45 and 36?
   Student: What about zero?
   Anthony: No, 0 is not a factor.
   Student: Why?
   (Observation, March 31st, 2014)

The concept of zero can be difficult for students to comprehend, yet it is overlooked during the implementation of the rich mathematics task. Anthony does not answer the
student when he asks why zero is not a factor of forty-five and thirty-six, and the student may not understand either the concept of zero or the concept of factors.

Another contributing factor that may inhibit the rich component to a mathematics task is behavioral issues. Anthony believes that rich mathematics tasks should be introduced to students after they have learned the underlying mathematical concepts for extended practice and not as a means of learning the concepts. Anthony discusses the beginning of teaching a mathematical concept when he says, “one group argued and did not get along. Whether it was a rich task or not, it would not have made a difference” in their behavior (Post-Observation Interview, April 28th, 2014). Anthony views behavior issues as being the same whether or not a rich mathematics task is being implemented, but does mention, “as long as [the students are] with peer they like, then they do not mind” rich mathematics tasks (Post-Observation Interview, April 28th, 2014). This suggests that Anthony believes pairing or grouping the students with other students is one way to avoid behavior issues during implementation. Also, Anthony mentions, “one student (describes where he sits in the classroom) will not do anything no matter what, so it would not matter for him if the task was rich or not” (Post-Observation Interview, April 28th, 2014). Anthony perceives student behavior to be a distinct and separate entity from the task being used in the classroom.

The last factor observed in Anthony’s classroom that may inhibit the integrity of the rich mathematics task is the class time needed to implement the task. Anthony perceives time to be the most difficult factor when it comes to using rich mathematics tasks in his classroom. He says, “with the EQAO four weeks away, we just do not have time to work on one problem for one and a half classes” (Post-Observation Interview, March 31st,
Anthony views rich mathematics tasks as time consuming and believes they do not allow for students to experience enough of the mathematical concepts and procedure in the limited time they have before standardized testing. Anthony says, “It is hard to [use rich tasks] every day. Sometimes you need to sit them down and have them learn through memorization. [Using rich tasks] is a couple of times a month thing. Maybe once a month” (Post-Observation Interview, April 28th, 2014). This suggests that Anthony views teacher-directed drills as a quick method for teaching the mathematics presented in the grade six curriculum and believes rich mathematics tasks should be used when the teacher has some extra time in their planning.

During one observation, Anthony asked if I could stay to observe another mathematics class because the previous class “had an assembly, so the class was cut short. [He] was not able to do the extended activity, but [he would] have more time with the next class” (Post-Observation Interview, March 31st, 2014). The extended activity was the portion of the lesson Anthony considered rich because it asks students to compile all that they had just learned. Again, this may suggest that Anthony believes rich mathematics tasks should be an extension activity once the mathematical concepts have been learned. When asked if Anthony learned anything from the professional development sessions the school prepared on rich mathematics tasks, Anthony says, “to say that any teacher did not already know that tasks should be challenging and should have multiple solutions would not be true. There is just no time for them” (Post-Observation Interview, March 31st, 2014). From Anthony’s perspective, he believes in the value that rich mathematics tasks may bring to the classroom, but indicates that
time is the number one reason for his decision to only use rich mathematics tasks every so often in his grade six classroom.
Chapter Five: Interpretation and Discussion of Findings

5.1 Introduction

First, this chapter revisits the research questions first posed in Chapter One. Next, each research question is delineated and the findings from the analysis are presented regarding how teachers select and implement rich mathematics tasks. Then, the major research findings from this study are identified. Finally, a suggestion is made concerning how this study may benefit the larger study in which it is derived and implications for future research are discussed.

5.2 The Research Questions

The following research questions set the foundation for this multiple case study:

1. How do teachers select rich mathematics tasks for classroom instruction?
   a. What characteristics do teachers look for in rich mathematics tasks?
   b. What sources do teachers use in the selection of rich mathematics tasks?

2. During implementation, what factors support or inhibit the integrity of rich mathematics tasks?

These research questions are further examined in the following sections and findings will be presented from both cases in this study.

5.3 Discussion of Each Research Question

5.3.1 How do teachers select rich mathematics tasks for classroom instruction?

Teachers select rich mathematics tasks based on their perception of task characteristics and from multiple sources.
5.3.1.1 What characteristics do teachers look for in rich mathematics tasks?

Denise and Anthony look for rich mathematics tasks that meet the needs of individual students. In order to meet the needs of their students, they select tasks that are engaging, provide opportunities for differentiation and are relatable for students beyond the classroom. They also prefer for the rich mathematics tasks to have the following characteristics: elicits high-levels of cognitive thinking processes, possesses a cross-curricular component, is open ended to allow for the formation of multiple solution strategies, remains student-centered and allows for multiple steps. Lastly, Denise and Anthony perceive assessments as contributing factors in their task selection decisions.

5.3.1.1.1 Individual Student Needs

In this study, Denise and Anthony value many attributes of rich mathematics tasks. Anthony and Walshaw (2009) highlight the significance of considering the needs of diverse learners. For example, both teachers select mathematics tasks that meet individual student needs. In particular, they select tasks they believe their students will enjoy solving. They admit that these rich mathematics tasks may change from year to year based on the changing interests of students, but should always provide for engaging experiences. According to Hodge, Visnovska, Zhao, and Cobb (2007), issues that engage students the most have a personal or societal relevance. Both teachers look for the task to provide a personal or societal relevance. Also, Anthony and Denise demonstrate that teachers want their students to appreciate mathematics. This notion aligns with the findings from the Slavit et al. (2009) study where rich mathematics tasks did not add an engagement component for all students, yet there were “noticeable increases in the amount of depth of
discussion…and increased enthusiasm” (p. 547-548).

Denise also uses these rich mathematics tasks to differentiate instruction in her classroom so that all students feel their own level of success in mathematics. This suggests that rich mathematics tasks allow for each student to shine individually and feel confident as a learner of mathematics. The nature of rich mathematics tasks often encourages students to find a collection of solutions using many strategies, thus meeting the learning styles of a variety of learners. This finding adds to current literature because teachers may now view rich mathematics tasks as vehicles for providing differentiated instruction.

The rich mathematics tasks that Anthony and Denise select exemplify attributes of mathematics in the real-world. Sullivan et al. (2013) argues that real-world practical tasks “‘come alive’ for students through showing them a purpose for what they are studying, and making mathematics more engaging for them” (p. 39). Deciding how many pizzas are needed among a group of individuals and determining if a new television will fit into a cabinet allows students to relate ‘school mathematics’ to the world beyond the classroom. Many students can relate to buying pizza and searching for a new television, so mathematics is perceived as a useful tool, even in the life of a middle school student.

5.3.1.1.2 Characteristics of Rich Mathematics Tasks

Shimizu et al. (2010) argue that during the selection of mathematics tasks, teachers think of attributes such as “content, complexity, degree of openness, form and representation” (p. 4). Similarly, Denise and Anthony use attributes as a guide in the selection of rich mathematics tasks. Many characteristics that Denise and Anthony describe to be a component of rich mathematics tasks align with McDougall (2004)
definition of rich mathematics tasks (p. 25). For example, Denise and Anthony perceive rich mathematics tasks to be open-ended, to encourage multiple solution strategies, to promote student-centered environments and to demand the use of multiple steps in the solution process. Neither Anthony nor Denise could remember having been exposed to the definition of a rich mathematics task, yet this finding suggests that, even without a formal definition in mind, teachers do view rich mathematics task characteristics in similar fashions.

In addition to these characteristics, Anthony and Denise also perceive rich mathematics tasks to elicit high-levels of cognitive thinking processes. They prefer that their students do not memorize the material, yet understand the mathematics. This finding is consistent with the findings in Stein et al. (1996), suggesting teachers attempt to formulate and implement tasks that connect procedures to mathematical concepts or present novel tasks where sophisticated mathematical thinking is facilitated. Also, Anthony teaches grade six, which is a year when the students write large scale mathematics tests (EQAO). He believes that students will score better on the EQAO if they engage with tasks that elicit high-levels of cognitive thinking processes, because thinking is a contributing factor within the marking scheme. With this in mind, Anthony chooses to implement rich mathematics tasks as a practice for the EQAO where open-ended thinking questions are used. This suggests that rich mathematics tasks can be used to prepare students for standardized tests.

Denise and Anthony select cross-curricular learning as an important characteristic of rich mathematics tasks. They view the ‘richness’ in the task as allowing students to not only see the mathematical concepts elicited by the task, but also make connections to
other disciplines. They encourage their students to view mathematics as a useful tool across learning. This finding is consistent with McDougall’s (2004) definition of rich mathematics tasks where there is integration among mathematics strands and other subject areas.

A finding that may supplement current research is the parallel teachers draw among the mathematical concepts to be learned through the use of rich mathematics tasks and assessments. Both Denise and Anthony use a variety of diagnostic assessments in their classrooms. The results of these diagnostic assessments are a driving factor in selecting rich mathematics tasks for classroom use. The diagnostic assessments describe which areas of mathematical concepts students feel least confident and teachers select rich mathematics tasks that align with these areas to encourage improvement in understanding.

An interesting observation in this study is that Denise and Anthony find the disciplines of mathematics and art to easily integrate with one another. Denise uses art often in mathematics because she finds that there are shared similarities among the two disciplines. However, unlike Anthony, Denise believes that the integration of mathematics with the discipline of art extends beyond drawing a picture when the task is complete. She expects her students to know artistic terminology and introduces the names and biographies of artists in her mathematics classroom. This finding suggests teachers see a parallel among mathematics and art and creating lessons to integrate the two disciplines is simpler than integrating mathematics with other disciplines. This also suggests that teachers’ view the integration of mathematics among other disciplines in divergent ways.
5.3.1.2 **What sources do teachers use in the selection of rich mathematics tasks?**

Although textbooks are widely used in the selection of mathematics tasks, neither Anthony nor Denise describes textbooks as sources they would use to select rich mathematics tasks. Instead, Denise uses a blend of rich mathematics tasks located on the internet and developed in collaborative settings.

The Internet is a massive portal for selecting rich mathematics tasks. Denise uses the Internet to search for tasks because of the convenience it offers. However, Denise reiterates that searching for these tasks can be quite time consuming due to the number of tasks she must sift through and the adaptations she must make in order for the rich task to well serve her students and the Ontario grade eight curriculum. Sullivan et al. (2009) mentions that “while the availability and accessibility of interesting teaching ideas is steadily increasing, the challenge of converting those ideas to student learning is substantial as ever” (p. 85). If a virtual portal existed where Ontario teachers could post, share and reflect on rich mathematics tasks that align with curriculum expectations, then this may one way to curtail the time needed for teachers to find and adapt rich mathematics tasks for their classroom. This finding suggests that teachers may need a more convenient and less time-consuming method for selecting rich mathematics tasks and adapting them to meet the needs of students and the Ontario course expectations.

Sullivan et al. (2013) suggest that “there is evidence that teacher beliefs influence their choice of tasks as well as the order in which they use particular task types” (p. 141). Denise believes that implementing reform initiatives promotes student achievement and provides for more engaging lessons. It is also her belief that she must continuously reflect
on her practices and improve her teaching strategies. Through collaborative means, such as attending workshops and professional development sessions, Denise is energized to continually build on her knowledge of rich mathematics tasks and works with others to develop and improve these tasks. Borko (2004) says “strong professional learning communities can foster teacher learning and instructional improvement” (p. 6). In addition, Denise chooses to remain connected with new and improved rich mathematics tasks through online emails from educational communities. This finding suggests that teachers who are proponents of reform initiatives may use collaborative settings as ways to improve upon their knowledge of rich mathematics tasks and, naturally, are more exposed to rich mathematics tasks than those who are not proponents of reform envision.

Anthony only chooses to participate in workshops or professional development sessions that are mandatory. Although he also aims to improve the mathematical understanding for each of his students, Anthony believes that many of the traditional approaches to teaching are equally as effective as reform strategies. When selecting rich mathematics tasks, Anthony creates these tasks by merging his views of rich task characteristics with his knowledge of pedagogy and content pedagogy. This finding is consistent with Gainsburg (2008) where the majority of teachers select real-world tasks from personal experience or make them up in the heads. However, Zaslavsky (1995) reminds the education community that teachers “will always have more to learn, even with respect to what they think they already know” (p. 19).

5.3.2 During implementation, what factors support or inhibit the integrity of rich mathematics tasks?

5.3.2.1 Supporting Factors
Based on McDougall’s (2004) rubric for Dimension Four in the Ten Dimensions Framework, Denise’s implementation of rich mathematics tasks might be placed at a Level Three, indicating that she regularly selects and implements rich mathematics tasks that demands students “to make and justify choices, to integrate ideas, to plan strategies, and to explain their thinking” (Sullivan et al., 2013, p. 9). Rich mathematics tasks are often used in Denise’s classroom as a means of learning mathematical concepts.

Anthony’s use of rich mathematics tasks might be placed in the Level Two column of the rubric indicating that, every so often, he will select and implement rich mathematics tasks in his classroom, but mainly after the mathematical concepts have been introduced. Anthony uses rich mathematics tasks as an extension of learning instead of the basis for learning. This finding compliments Battista’s (1994) conclusion that teachers’ beliefs are the key to successfully implementing the reform movement in mathematics education. This finding also suggests teachers’ beliefs regarding rich mathematics tasks plays a role in how frequently they choose to use rich mathematics tasks and the learning purpose of the task.

Anthony and Denise both aim to connect procedures with conceptual understanding during task implementation in an effort to support the ‘rich’ component within the task. According to Sullivan et al. (2013), student should “work on tasks which require them, for example, to make and justify choices, to integrate ideas, to plan strategies, and to explain their thinking” (p. 9). Neither Denise nor Anthony approve of teaching one method for solving a problem and having the students practice the solution steps repeatedly without an underlying explanation for why these procedures work. As Denise and Anthony continuously remind students of the connection among their procedures and
the underlying mathematics, the task remains rich in that students are using high-levels of cognitive thinking processes. This study proposes during the implementation of rich mathematics tasks, the task has the potential to remain rich when the teacher connects procedures with mathematical concepts.

Although connecting procedures to conceptual understanding was the only support mechanism witnessed in Anthony’s classroom, Denise also encourages multiple solutions strategies where students can model their solutions, uses scaffolding, presses students for explanations of chosen procedures and presses for justification for solution strategies. Stein, Engle, Smith, and Hughes (2008), emphasize the importance of “selecting particular students to present their mathematical responses” (p. 321). Denise prefers her students answer the questions posed by the rich mathematics task and encourages them to think rationally about why their solution strategy is valid. She rarely accepts a strategy without asking her students why their solution is justifiable.

Additionally, the use of scaffolding is one way Denise encourages higher-levels of cognitive thinking processes without divulging too much information regarding solutions or solution strategies. This finding supports Stein et al. (1996) where students who are introduced to rich mathematics tasks designed to encourage higher-level thinking processes generally use multiple solution strategies and justify their answers. Additionally, this finding also supports Stein et al. (1996) who suggest scaffolding that does not take away from the overall challenge of the rich mathematics task can uphold the intended integrity of the task.

5.3.2.2 Inhibiting Factors
Some perceived disadvantages Clarke and Roche (2009) found when using contextualised practical problems were that: 1) the task was too long and took up too much time, 2) some students arrived at a conclusion much sooner than others, 3) students with low confidence had trouble determining how to start the task, 4) students were more interested in the answer than the process, and 5) not all real situations are relevant to the middle school curriculum. Some of these same factors were identified in this study regarding the implementation of rich mathematics tasks. Overlooking opportunities to press for cognitive understanding, student behaviors, appropriateness of the task, teachers’ comfort with mathematics and time are five factors identified to potentially inhibit the implementation of rich mathematics tasks. In addition, this study suggests that task inhibitors build upon one another, signifying that one inhibiting factors can easily lead to other inhibiting factors. This aligns with the findings from Stein et al. (1996) where more than one inhibiting factors were observed during each task implementation phase. This tornado effect makes it difficult to identify exactly which factors directly inhibit the integrity of the rich mathematics task or if a combination of these factors ultimately inhibit the tasks’ integrity.

Sullivan et al. (2013) found that teachers had difficulty maintaining the challenging aspect of the task, observing teachers to reduce the difficulty in response to student reactions. When the teachers intercept and provide too much support, it is typically to keep on schedule or to address student behavioral issues. This finding also alludes to the findings from Stein et al. (1996) that planning for appropriate amounts of time is essential to upholding the integrity of the rich mathematics task. It may also add to current research
that behavioral issues in the classroom lead teachers to step in and do the difficult work for the students, reinforcing the negative behavior.

A new finding from this study suggests overlooking opportunities to press for conceptual understanding is a factor that may limit the high-levels of cognitive thinking processes in which students’ engage. The rich mathematics tasks observed prompted students to ask ‘why’ and ‘how’ questions, yet, in some cases, they were told either that it did not matter or their question was ignored altogether. It is beyond the scope of this study to identify the rationale for teachers avoiding or disregarding these questions, but it is worthy to note that this study proposes that the implementation of these tasks could be richer if the opportunities to press for conceptual understanding were not overlooked.

Although there were only a few instances where student behavior became an issue, Anthony and Denise refocused the students’ attentions by stepping in to do the difficult part of the task for the students. This suggests that students who misbehave will receive the answers without having to do the work. Adding to current research, this finding also suggests that teachers attend to misbehavior by lessening the complexity of the task, thus inhibiting the intended integrity of the rich mathematics task.

Another factor deemed to influence the integrity of the rich mathematics task is the appropriateness of the task regarding the task alignment with students’ prior knowledge. Denise uses scaffolding, differentiation and diagnostic assessments to ensure the rich mathematics tasks aligns with students’ prior knowledge. Anthony uses diagnostic assessments and implements rich mathematics tasks after students are exposed to the mathematical concepts, so aligning these tasks to students’ prior knowledge was not observed to be an inhibiting factor. However, Denise mentions in an interview that she
perceives an inhibiting factor of rich mathematics tasks to be the appropriateness of the task with students’ prior knowledge.

This finding suggests that teacher do make an effort to align rich mathematics tasks to students’ prior knowledge and perceive that the failure to do so may result in the integrity of the task being inhibited. Similarly, teachers’ comfort with mathematics was not observed to be an inhibiting factoring during the implementation phase in either of these case studies, but it was mentioned by Denise as a potential rich mathematics task inhibitor. This aligns with the findings from Sullivan et al. (2013) where challenges “associated with a teacher’s own mathematical knowledge did not tend to be articulated in direct survey questions but were evident through discussion, observation, and response to seemingly unrelated questions” (p. 141).

In accordance with the findings from Stein et al. (1996) and Sullivan et al. (2013), time is a significant factor that inhibits the integrity of rich mathematics tasks during implementation. Both Anthony and Denise struggle to find the time to select and implement rich mathematics tasks in the classroom. This finding suggests teachers view rich mathematics task selection and implementation to be vastly time consuming. In addition, Denise believes that rich mathematics tasks can improve students’ understanding of mathematics, so the extra time it takes to select and implement these tasks is worthwhile. However, Anthony believes rich mathematics tasks take up too much time from direct instruction. These findings suggest that teachers’ beliefs regarding reform initiatives can be used to predict whether or not rich mathematics tasks will be used in a teacher’s classroom and how frequently that will be used.

5.4 Major Findings
This thesis depicts a multiple case study of two middle school mathematics teachers and describes how these teachers select rich mathematics tasks and the factors involved in supporting or inhibiting these tasks during implementation. A particular set of attributes defines rich mathematics tasks and a variety of sources are used during the task selection phase. Additionally, there are a number of factors that support and inhibit the integrity of the rich mathematics task during the implementation phase.

The following is a summary of the major findings presented in this multiple case study:

1. The teachers in this study select rich mathematics tasks that are engaging, exemplify real-world problems, provide for differentiation of instruction and align with diagnostic and standardized assessments.

2. The characteristics that the teachers in this study use to define rich mathematics tasks are open-ended, cross-curricular, have multiple steps, encourage a variety of solutions and solution strategies, promote student-centered classrooms and support high-levels of cognitive thinking processes.

3. The teachers in this study who support reform initiatives are more likely to implement rich mathematics tasks in their classrooms than teachers who support traditional teaching approaches.

4. Factors that may support the integrity of rich mathematics tasks during implementation include connecting procedures with mathematical concepts, modeling student-made solutions, enforcing scaffolding strategies and pressing students to justify their solutions.
5. Factors that may diminish the integrity of rich mathematics tasks during implementation include overlooking opportunities to press for conceptual understanding, student behavioral issues, the appropriateness of task in regards to students’ prior knowledge, teacher’s comfort with mathematics and time.

5.5 Implications for Future Research

This qualitative case study of two middle school teachers is part of a larger project referred to as the School Improvement in Mathematics Study (McDougall, 2013). This overarching multi-dimensional study may benefit from the findings presented in this collective case study since rich mathematics task is one of the Ten Dimensions, the framework used in the School Improvement in Mathematics Study. Having a thorough understanding of how teachers select rich mathematics task and the difficulties they encounter will benefit the improvement teams by designing ways to eliminate the factors teachers’ view as difficult when trying to select rich mathematics tasks. Thus, more teachers are likely to use rich mathematics tasks in their classrooms. Second, school improvements teams can use the supporting and inhibiting factors presented in this thesis to prepare their teachers for implementing these rich mathematics tasks for optimal student achievement.

The findings presented in this study contribute to current research, yet there are many questions that remain. For instance, if teachers view rich mathematics tasks as vehicles for preparing students for provincial assessments, then do the students who use rich mathematics tasks on a daily basis score higher on these assessments than students who do not use rich mathematics tasks? As teachers feel pressure for their students to achieve
success on these standardized assessments, a study that measures the effects of rich mathematics tasks on EQAO scores could prove highly valuable.

The findings in this study suggest that teachers need a time-saving method for selecting and adapting rich mathematics task for their classroom. Having a convenient way to access rich mathematics tasks that already align with the curriculum and course expectations may increase the use of rich mathematics tasks in classrooms. A suggestion would be to design a virtual space where Ontario teachers can post and find lessons according to subject, grade and course. This would allow teachers to quickly access a variety of rich mathematics tasks where minor adjustments may need to be made to align with their specific group of learners. A future study may include encouraging teachers to use this portal and provide feedback on whether or not teachers find this virtual space to benefit their use of rich mathematics tasks. In addition, Sullivan et al. (2013) found that the “most productive task adaptation or development came through teachers working in teams using the mathematical concepts or misconceptions as the starting point” (p. 29). Collaboration may provide teachers with efficient and effective tools for selecting rich mathematics tasks and supporting reflection on these tasks during the implementation phase.

One task inhibitor generally leads to other factors that inhibit the task. Future studies are encouraged to make connections among these factors to address why one factor may lead to other factors. In addition, there may be a link among factors that support rich mathematics tasks during implementation as well, so future studies may decide to look into this as well.

Researching student achievement using rich mathematics tasks is beyond the scope
of this study. However, the QUASAR project indicates students’ mathematical thinking and reasoning increases when rich mathematic tasks are set up and implemented in a way that uphold a high-level of cognitive demand. Future studies are encouraged to use student achievement gains to measure the effectiveness of rich mathematics tasks. Changing teachers’ beliefs about rich mathematics tasks could be advanced if there are a number of valid and reliable studies that connect student achievement gains to rich mathematics tasks.

Although this qualitative case study depicts the perceptions of two middle school mathematics teachers and their selection and use of rich mathematics tasks, there is little room for generalizability (Creswell, 2013). A large number of cases would need to be examined and arrive at the same conclusions before generalizations can be made about how teacher select and implement rich mathematics tasks. Therefore, researchers are encouraged to extend the purpose of this study and examine other cases of middle school mathematics teachers.
References


Appendix A:

Attitudes and Practices to Teaching Math Survey (McDougall, 2004, p. 87-88)

Instructions:
Circle the extent to which you agree with each statement, according to the A to F scale below. Then, use the charts at the top of the next page to complete the Score column for each statement.

A  Strongly Disagree   B  Disagree   C  Mildly Disagree
D  Mildly Agree   E  Agree   F  Strongly Agree

1. I like to assign math problems that can be solved in different ways.
2. I regularly have all my students work through real-life math problems that are of interest to them.
3. When students solve the same problem using different strategies, I have them share their solutions with their peers.
4. I often integrate multiple strands of mathematics within a single unit.
5. I often learn from my students during math because they come up with ingenious ways of solving problems that I have never thought of.
6. It is often not very productive for students to work together during math.
7. Every student should feel that mathematics is something he or she can do.
8. I plan for and integrate a variety of assessment strategies into most math activities and tasks.
9. I try to communicate with my students’ parents about student achievement on a regular basis as well as about the math program.
10. I encourage students to use manipulatives to communicate their mathematical ideas to me and to other students.
11. When students are working on problems, I put more emphasis on getting the correct answer rather than on the process followed.
12. Creating rubrics is a worthwhile exercise, particularly when I work with my colleagues.
13. It is just as important for students to learn probability as it is to learn multiplication.
14. I don’t necessarily answer students’ math questions, but rather ask good questions to get them thinking and let them puzzle things out for themselves.
15. I don’t assign many open-ended tasks or explorations because I feel unprepared for unpredictable results and new concepts that might arise.
16. I like my students to master basic operations before they tackle complex problems.
17. I teach students how to communicate their math ideas.
18. Using technology distracts students from learning basic skills.
19. When communicating with parents and students about student performance, I tend to focus on student weaknesses instead of strengths.
20. I often remind my students that a lot of math is not fun or interesting but it’s important to learn it anyway.
**Attitudes and Practices to Teaching Math Survey Scoring Chart**

For statements 1–5, 7–10, 12–14, and 17, score each statement using these scores:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

For statements 6, 11, 15, 16, 18, 19, and 20, score each statement using these scores:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

To complete this chart, see instructions below:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>$\div 3 = 5$</td>
</tr>
<tr>
<td>2. Meeting Individual Needs</td>
<td>2, 6, 7, 15, 16</td>
<td></td>
<td></td>
<td>$\div 5 = 5$</td>
</tr>
<tr>
<td>3. Learning Environment</td>
<td>3, 5, 6</td>
<td></td>
<td></td>
<td>$\div 3 = 3$</td>
</tr>
<tr>
<td>4. Student Tasks</td>
<td>1, 2, 11, 15, 16</td>
<td></td>
<td></td>
<td>$\div 5 = 5$</td>
</tr>
<tr>
<td>5. Constructing Knowledge</td>
<td>5, 11, 14, 15, 16</td>
<td></td>
<td></td>
<td>$\div 5 = 5$</td>
</tr>
<tr>
<td>6. Communicating With Parents</td>
<td>19, 9</td>
<td></td>
<td></td>
<td>$\div 2 = 2$</td>
</tr>
<tr>
<td>7. Manipulatives and Technology</td>
<td>10, 18</td>
<td></td>
<td></td>
<td>$\div 2 = 2$</td>
</tr>
<tr>
<td>8. Students’ Mathematical Communication</td>
<td>3, 6, 10, 17</td>
<td></td>
<td></td>
<td>$\div 4 = 4$</td>
</tr>
<tr>
<td>9. Assessment</td>
<td>8, 11, 12, 19</td>
<td></td>
<td></td>
<td>$\div 4 = 4$</td>
</tr>
<tr>
<td>10. Teacher’s Attitude and Comfort with Mathematics</td>
<td>4, 7, 13, 15, 20</td>
<td></td>
<td></td>
<td>$\div 5 = 5$</td>
</tr>
</tbody>
</table>

**Step 1** Calculate the Average Score for each dimension:
1. Record the score for each Related Statement in the third column.
2. Calculate the Sum of the Scores in the fourth column.
3. Calculate the Average Score and record it in the last column.

For example:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>$\div 3 = 5$</td>
</tr>
</tbody>
</table>

**Step 2** Calculate the Overall Score:
1. Calculate the Total Score of the sums for all 10 dimensions in the fourth column.
2. Calculate the Overall Score by dividing the Total Score by 38.

For example:

<table>
<thead>
<tr>
<th>Total Score (All 10 dimensions)</th>
<th>152</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Score (Total Score $\div$ 38)</td>
<td>4</td>
</tr>
</tbody>
</table>

**Step 3** Interpret the results:

<table>
<thead>
<tr>
<th>Average Score for Each Dimension</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average scores will range from 1 to 6. The higher the average score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking, with respect to the dimension. A low score indicates a dimension that a teacher might focus on for personal growth and professional development.</td>
<td>The overall score will range from 1 to 6. The higher the overall score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking and the more receptive that teacher will likely be to further changes in his or her practice.</td>
</tr>
</tbody>
</table>
Appendix B

Semi-Structured Initial Interview Questions

Background Synopsis:
1. What is your name?
2. How many years have you been teaching?
3. What grades and subjects do you teach?
4. How long have you been here at this school?
5. Did you teach anywhere else before and if so, what grades and subjects did you teach?
6. Which class or grade level is your favorite to teach? Why?
7. What made you become a teacher?
8. What is your educational background?

School Culture:
1. Can you describe strengths about being a teacher in this particular school?
2. What are some challenges you have faced in trying to create a culture that supports student achievement in mathematics?
3. Are there opportunities within the school for collaboration? Is so, can you explain more? If not, can you explain more?

Mathematics Classroom:
1. What goals do you have for your students regarding mathematics?
2. What would you identify as strengths within your practice?
3. What are some challenges you have faced teaching Grade 6/8 mathematics?
4. How do you select the tasks used in your math classroom?
5. What does it mean, to you, for a task to be "rich"?
6. Do you use rich tasks in your classroom? If so, can you explain more? If not, can you explain more?
Appendix C

Semi-Structured Final Interview Questions

For Denise:

1. During our initial interview, you mentioned that one of the biggest struggles for you as a grade 8 math teacher is student’s retention of the material they learned in 6 and 7. Do you believe that if richer math tasks were used in those grades, then the retention of the material would be higher? Why or why not?

2. During our after-observation discussions, you mentioned that the majority of the rich tasks you select come from the internet. Is there any other place you select these tasks from besides the internet?

3. You have also mentioned that you use rich task every so often. Does every so often mean every other day, every few days, every other week? What are some reasons why you choose *not* to use rich math tasks every day?

For Anthony:

1. In the rich math tasks I observed in your classroom, you mentioned that you made them up. Have you ever used the internet as a source for selecting a rich mathematics task?

2. Do you believe that you would use more rich tasks in your classroom if you were given them at workshops or professional development sessions? Why or why not?

3. Have you ever tried to use rich mathematics tasks to differentiate instruction? Why or why not?
Appendix D

Consent Form

Dear ________,

We are investigating the programs, policies and activities that contribute to student success in mathematics. The purpose of the project is to contribute to the knowledge base regarding school improvement in mathematics and the use of a framework to guide improvement in mathematics instruction as well as to learn about teaching and learning in Grade 8 mathematics.

The External Research Review Committee of the TDSB and the University of Toronto Ethics Office has approved this study. The school principal has approved of this study.

The project will address such issues as what makes a school successful in terms of improving student achievement in mathematics. We want to know how school administration works collaboratively with teachers to put into place both processes and programs that are effective. We want to see how the use of the Ten Dimensions framework helps with school improvement in mathematics.

We would like you to participate in this project by allowing us to conduct an interview with you. It will take about 45 minutes and it will be tape-recorded. We will conduct the interview during the school day and in your school. You will be given a summary of the interviews and observations. You will also be given an opportunity to receive a summary of the report. I will also be working with you on mathematics improvement. I also plan to work with your school mathematics improvement team to identify strategies to improve mathematics in your school. I would like to tape-record some of these meetings.

We will not use your name or anything else that might identify you in the written work, oral presentations or publications. The information remains confidential. You are free to change your mind at any time, and to withdraw even after you have consented to participate. You may decline to answer any specific questions. We will destroy the tape recording after the research has been presented and/or published which may take up to five years after the data has been collected. There are no known risks or benefits to you for assisting in the project.

Please sign the attached form, if you agree to be interviewed. The second copy is for your records. Thank you very much for your help.

Yours sincerely,

Lori Jane Henhaffer
OISE/University of Toronto
lori.henhaffer@mail.utoronto.ca

Douglas McDougall
OISE/University of Toronto
doug.mcdougall@utoronto.ca
Collaborative Inquiry Project: Grade 8 Mathematics

I acknowledge that the topic of this interview has been explained to me and that any question that I have asked has been answered to my satisfaction. I understand that I can withdraw at any time without penalty.

I have read the letter provided to me by Doug McDougall and agree to participate in an interview for the purpose described.

Signature:

Name (printed): ________________________________

Date: ___________

Lori Jane Henhaffer  Douglas McDougall
OISE/University of Toronto  OISE/University of Toronto
lori.henhaffer@mail.utoronto.ca  doug.mcdougall@utoronto.ca