DEVELOPMENT OF A FIBRE BRAGG GRATING SENSOR FOR ROCK DEFORMATION MONITORING

by

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A thesis submitted in conformity with the requirements for the degree of Masters of Applied Science
Graduate Department of Civil Engineering
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Abstract

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This thesis examines the theoretical and experimental performance of a fibre Bragg grating sensor for static and dynamic strain measurement in hard rock. A literature review focuses on the fundamental theory of fibre Bragg gratings, various physical demodulation schemes used to interrogate Bragg sensors with an emphasis on charge coupled device spectrometry, as well as the aliasing behaviour of Bragg gratings as sensors. A coupled numerical-analytical analysis is conducted on various sensor configurations designed for borehole deployment in order to establish the response of the strain sensors under various strain conditions. The findings of an experimental investigation of two sensors subjected to uniaxial strain within a grout and a rock specimen are presented. The experimental data confirm the feasibility of using the proposed sensor as a part of an integrated optical strain sensing network.
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Chapter 1

Introduction

It is well known that excavations, and mass changes in or on the surface of the earth induce changes in the deformation fields that exist in the earth’s crust. Such changes need to be well understood when tunneling or mining underground to ensure the safety and operability of the underground structure. Miscalculation in the short and long term mechanical behaviour of underground structures can result in failure of the structure in a variety of stable and unstable mechanisms (Salamon, 1970). Furthermore, evidence has shown that large scale geo-engineering works are correlated with induced and triggered fault rupture, which can threaten both the geo-engineered structure itself or nearby populations (Klose, 2013; González et al., 2012).

The acquisition of information in geo-engineering is typically associated with a high degree of uncertainty. The constitutive behaviour of the various materials in question, properties of discontinuities ranging from fractures to faults, geometric boundaries, and structural geology are all associated with errors in their measurement. Furthermore, the assumption of uniformity in the far-field stresses commonly adopted in most numerical simulations appears to be erroneous (McKinnon, 2006). The result of such uncertainties produces difficulties when trying to model and simulate the performance of underground structures accurately. For instance, it is exceedingly difficult, or impossible, to predict the incidence of induced remote seismicity nearby mining operations due to the inherently chaotic nature of mining induced stresses (or displacements) on regional fault behaviour (McKinnon, 2006).

Current practice in the design and operation of mines consists of detailed numerical modeling to predict the response of the rock for different mineral extraction procedures. Inherent to any comprehensive numerical modeling scheme is a description of the material properties (i.e. constitutive model), a geometric and mechanical description of discontinuities and the mechanical laws that govern them (i.e. Mohr-Coulumb, stick-slip, rate and state laws, etc.), and a description of the pre-excavation, in-situ stress state. The proper selection of material and geometric models that correctly represent the system is an inherently difficult process. This is due to the variability of the material itself and the impracticability of taking sufficient measurements of all the parameters required. As a result, assumptions are often made when sufficient data cannot be obtained, or when more accurate and comprehensive investigation methods are deemed too expensive for the project in question.

In order to test the assumptions made in the modeling phase, observation and measurement of the response of the rock mass is necessary during the excavation procedure and life of an underground structure. To this end, a series of tools and techniques have been developed. In mining, at a minimum,
convergence of the excavation is typically monitored to ensure that the boundaries of the excavation are deforming as predicted. Increasingly sophisticated monitoring schemes include the use of pressure cells installed in grouted boreholes, stress pads, vibrating wire extensometers, as well as rock overcoring and undercoring to determine the current state or change in the state of stresses and strains. Depending on how these methods are employed, good temporal and spatial resolution of measurements can be difficult to obtain or prohibitively expensive.

There are, at the moment, methods of interpreting the effects of these evolving strain fields on the surrounding rock mass with high temporal and spatial resolution; that is, through the use of seismic and microseismic monitoring arrays distributed locally (or regionally) around the geo-engineering project in question. Depending on the level of sophistication of the sensing network, it is possible to deduce the moment magnitude, the moment tensor, the hypocenter, and the stress drop at the fault after seismic slip. Inversion schemes have been used to deduce the in-situ principal stress field from the focal mechanisms of clusters of microseismic events. Readings from these events have shown that larger magnitude events can occur at a great distance from mining operations, and are not correlated with excavation processes (blasting and stope advances) (Snelling et al., 2013). This is in sharp contrast to rock mass damage near excavation surfaces and induced seismicity which are closely correlated and mechanically related (Napier and Malan, 1997; Maxwell and Young, 1996).

The use of instrumentation near the excavation, as well as the use of seismic sensing arrays can only give a limited amount of information concerning deformations occurring around a mine or other types of geo-engineering projects. Seismic information derived from arrays can only provide band-limited information of seismic deformation and thus does not include aseismic deformation processes such as creep and slow rupture. Das and Zoback (2011), and Peng and Gomberg (2010) have suggested that these aseismic processes play a significant role in the total deformation energy release of faults and are not at all constrained to subduction zones, as previously thought.

Given the current monitoring practices that are economically feasible, there remains a requirement for monitoring tools that can measure changing ground conditions with high spatial and temporal resolution, while taking measurements that are both accurate and extremely precise. Of particular interest are tools capable of measuring the evolving strain fields around excavations in hard rock environments. Assuming that an accurate, as well as a wide temporal and spatial data set can be obtained by such monitoring techniques, engineers, seismologists, and geoscientists could gain deeper insights into phenomena such as remote triggered and induced seismicity, mine stiffness changes, rock bursts and other geomechanical instabilities, among many other phenomena.

Without specifying any single motivation for the development of such sensors, the following thesis will attempt to elucidate some of the basic principles behind fibre Bragg grating sensors and their possible applications to rock deformation monitoring. First, a brief literature review will be provided explaining the mechano-optical principles of Bragg sensing as well as other optical sensing mechanisms and how they have been applied to sensing deformations in rock and in other structures. Secondly, based on the existing designs in the literature that have proven successful, the design of an optimal sensor casing will be presented with a coupled mechano-optical analysis of the sensor performance, taking into account the possibility of transverse strains as a contributor to sensor drift. Following the design and analysis of the sensor and casing, a laboratory validation study will be performed to show the possibility of using the sensor as a means to measure strains within a rock mass. Finally, the limitations of the proposed sensing system design will be discussed with proposed methods that might improve these limitations in
the future.
Chapter 2

Introduction to Fibre Optics and Fibre Bragg Grating Sensing

In this chapter, the basic principles of fibre optics and optical sensing will be presented, with emphasis on the Bragg sensing method. The fundamental physical properties governing fibre optics and Bragg gratings will be explored, and the methodology by which Bragg gratings can be employed as sensors of both temperature and strain will be elucidated. The prominent commercial and experimental methods used to acquire the optical signal information produced by Bragg gratings will be compared. Limitations on the Bragg grating sensing method such as acquisition and aliasing of dynamic strains, accuracy, and resolution of the sensing technique will be discussed.

2.1 Basic Principles of Fibre Optic Cables

The use of fibre optic sensors for monitoring is derived from decades of development in the field of fibre optics. The fibre optic cable has the unique advantage of being capable of carrying light waves through the fibre over great distances without significant loss of energy or distortion in the signal (Kao and Hockham, 1966). Through modification of the fibre, the light traveling through the fibre can be made sensitive to the external environment (Measures, 2001). These two key features of the optical fibre make them an ideal candidate for use as both a sensing element as well as the transmitter for the optical sensing signals.

The fibre optic cable consists of a cylindrical fused silica core covered with a silica cladding of slightly lower refractive index. The presence of such a cladding ensures that light of a specific wavelength bandwidth is totally internally reflected in the core of the silica fibre, allowing light to travel tens of kilometres without significant loss of power. The core of standard single mode fibre is generally around 5–10 µm in diameter with a cladding diameter of approximately 125 µm. Following production, fibres are typically coated with acrylate or polyimide in order to prevent moisture and mechanical degradation. An optical fibre schematic is presented in Figure 2.1.

For applications concerning telecommunications and sensing, single mode fibres are preferred whereby the core of the fibre is kept very small (approximately 5 or 10 µm) to prevent temporal spreading of the light wave thus permitting plane wave propagation and a wide transmission bandwidth. Multimode optical fibres consist of larger cores (up to 100 µm) and are capable of transmitting light at much higher
The dependence of fibres being single mode or multimode is dependent on the solution of the electromagnetic wave equation for fibres of given cores and cladding geometries and their respective refractive indices. For single mode optical fibres, wavelengths falling below a characteristic wavelength corresponding to the first optical mode are quickly dissipated. Additionally, some fibre optical cables have been designed to preserve a given polarization or to polarize light. This is typically accomplished through a cross sectional prestrain causing birefringence within the fibre core or by using elliptically shaped cores. For most conventional sensing applications, single mode, non-polarizing optical fibres are typically used.

Attenuation of light along the length of an optical fibre is a strong function of the wavelength with local minima of 1300 nm and 1550 nm. The process of Rayleigh scattering dominates attenuation at shorter wavelengths while infrared absorption dominates at higher wavelengths. Rayleigh elastic scattering arises from optical inhomogeneities within the fibre resulting from the manufacturing process. In modern fibres, losses near the theoretical limit of 0.2 dB/km are obtained in so called “ultra low-loss single mode optical fibres” at wavelengths centred around 1550 nm.

Attenuation also occurs as a result of micro- and macrobending of the fibre. When the fibre is bent, the power of the light wave contained in the outer cladding (that is, furthest away from the centre of curvature) is radiated. This is due to the fact that the lightwave can no longer remain perpendicular to the fibre axis without exceeding the speed of light in the medium. Significant attenuation can occur if the fibre is bent past a given radius which may cause a loss of the signal. When installation of fibre optic cables is being considered for monitoring purposes, efforts should be made to prevent excessive bending of the cables to prevent attenuation.

2.2 Principles of Bragg gratings

A fibre optic Bragg grating is defined as a periodic modulation of the core’s refractive index (Figure 2.2). This periodic modulation is accomplished by exposing the core to a high intensity interference pattern of ultraviolet light. A standard method by which fibre Bragg gratings were typically produced is termed the “Transverse Holographic Technique,” in which two coherent beams of ultraviolet light are directed at a section of the fibre (Meltz et al., 1989). In this method, the grating spatial wavelength corresponds to the angle between the two interfering beams, allowing the spatial period of the Bragg grating to be substantially different from the wavelengths of the interference beam. This method has been superseded by the phase mask technique (Hill et al., 1993). In this case, the interference patterns of ultraviolet light are produced by a phase mask, which is a periodic surface relief structure of thin silica glass through
which intense beams of ultraviolet light are passed. The phase mask causes diffraction of the incident light in two directions and can therefore create the interference patterns required to write the gratings. The phase mask technique simplifies the FBG writing due to the reduced stability requirements, easy alignment of the fibres, and the lower coherence permitted of the laser beam. It is also possible to write gratings on several parallel fibres simultaneously using this method.

The simplest type of Bragg grating consists of a uniform periodic modulation of the core refractive index. In order to suppress the side lobes that appear in uniform gratings (2.4), the index modulation is often appodized in FBG sensors. This assists in more accurate detection of the peak reflected wavelength. Chirped, phase-shifted, and superstructure gratings also exist, but they will not be discussed here. The following equation can be used to describe the perturbation to the index of refraction in the fibre

\[
\delta n_{\text{eff}}(z) = \overline{\delta n_{\text{eff}}}(z) \left[ 1 + s \cos \left( \frac{2\pi}{\Lambda} z + \phi(z) \right) \right]
\]

(2.1)

where \(\Lambda\) is the nominal grating period, \(\overline{\delta n_{\text{eff}}}(z)\) is the spatially averaged index change over one grating period, \(s\) is the fringe visibility of the index change, and \(\phi(z)\) is the grating chirp.

A fibre optic Bragg grating has the important characteristic that a narrow-band back-reflection of light is generated when the so-called phase match condition is satisfied. This reflection results from the
constructive interference from the forward propagating wave and the waves scattered by the periodic grating. A detailed description of this process is given in Othonos and Kalli (1999). In the presence of a Bragg grating, a broadband light source can be transmitted through the grating while a narrow band of light, sufficiently near the Bragg wavelength, $\lambda_B$, is reflected. The Bragg wavelength is given by the following equation

$$\lambda_B = 2n_{\text{eff}}\Lambda$$

(2.2)

where $n_{\text{eff}}$ is the mean core index of refraction and $\Lambda$ is the grating period of index perturbation.

### 2.2.1 Coupled mode theory for Bragg gratings

In order to determine the reflected spectrum of a fibre Bragg grating, coupled mode theory can be used. The derivation of the reflectivity, $R$, which is the proportion of light at a given wavelength that is reflected by the grating, provided by Erdogan (1997), is shown here, which is identical to that of Othonos and Kalli (1999) for a uniform Bragg grating.

We consider the simplified differential equations of a reflection mode with amplitude $A(z)$ where $z$ is the longitudinal fibre axis into a counter-propagating mode of amplitude $B(z)$. It is assumed that these equations retain only terms that involve amplitudes of the mode of interest, and neglect terms that have a rapidly oscillating $z$ dependence (when compared to the full form of the coupled mode equations). The resulting, simplified, coupled mode equations can be written

$$\frac{dR}{dz} = i\hat{\zeta}R(z) + i\kappa S(z)$$

$$\frac{dS}{dz} = -i\hat{\zeta}S(z) - i\kappa^* R(z)$$

(2.3)

where the amplitudes $R$ and $S$ are

$$R(z) = A(z) \exp(i\delta z - \phi/2), \quad S(z) = B(z) \exp(i\delta z + \phi/2)$$

(2.4)

The term $\hat{\zeta}$ is know as the dc self-coupling coefficient

$$\hat{\zeta} = \delta_d + \zeta - \frac{1}{2} \frac{d\phi}{dz}$$

(2.5)

and $\delta_d$ is the detuning coefficient which is independent of the axis, $z$,

$$\delta_d = \beta - \frac{\pi}{\lambda} = 2\pi n_{\text{eff}} \left(\frac{1}{\lambda} - \frac{1}{\lambda_d}\right)$$

(2.6)

the wavelength $\lambda_d$ is known as the design peak reflection wavelength for an infinitesimally small index of refraction change (i.e. $\delta n_{\text{eff}} \to 0$). For a single-mode Bragg grating, we have the simplified relations

$$\zeta = \frac{2\pi}{\lambda} \delta n_{\text{eff}}$$

(2.7)

$$\kappa = \kappa^* = \frac{\pi}{\lambda} \delta n_{\text{eff}}$$

(2.8)

We are interested solely in the case of a uniform (i.e. unchirped) grating, thus $\delta n_{\text{eff}}$ is a constant since
\( d\phi/dz = 0 \), therefore \( \kappa, \zeta, \) and \( \hat{\zeta} \) are all constants. The coupled-mode equations 2.3 can therefore be reduced to a coupled first-order differential equation with constant coefficients.

For a uniform grating of length \( L \) (from \( z = -L/2 \) to \( z = L/2 \)) with a forward propagating wave incident from \( z = -\infty \), and no backward propagating wave for \( z \leq L/2 \), the following boundary conditions can be prescribed:

\[
R(-L/2) = 1, \quad S(L/2) = 0 \tag{2.9}
\]

The reflected amplitude, \( \rho \), at \( z = -L/2 \) is given by

\[
\rho = \frac{S(-L/2)}{R(-L/2)} = \frac{-\kappa \sinh \left( L\sqrt{\kappa^2 - \zeta^2} \right)}{\zeta \sinh \left( L\sqrt{\kappa^2 - \zeta^2} \right) + i\sqrt{\kappa^2 - \zeta^2} \cosh \left( L\sqrt{\kappa^2 - \zeta^2} \right)} \tag{2.10}
\]

Finally, the reflectivity, \( R \), is given by

\[
R = |\rho|^2 = \frac{\sinh^2 \left( L\sqrt{\kappa^2 - \zeta^2} \right)}{\cosh^2 \left( L\sqrt{\kappa^2 - \zeta^2} \right) - \left( \zeta^2/\kappa^2 \right) } \tag{2.11}
\]

Reflection spectra for two similar Bragg gratings are shown in Figure 2.4.
The maximum reflectivity for a Bragg grating is given by

\[ R_{\text{max}} = \tanh^2(\kappa L) \]  

which occurs when \( \zeta = 0 \), equivalent to a wavelength of

\[ \lambda_{\text{max}} = \left( 1 + \frac{\delta n_{\text{eff}}}{n_{\text{eff}}} \right) \lambda_d. \]  

### 2.2.2 Fibre Bragg gratings as sensors

Any change of the mean index of refraction, \( n_0 \), or of the grating period, \( \Lambda \), due to changes in temperature or strains will result in a change of the Bragg wavelength, \( \lambda_B \). The fibre Bragg grating therefore constitutes an intrinsic sensor, in other words, the sensing element is contained directly within the signal transmitting medium (in this case, the optical fibre). In order to sense changes in strain and temperature, either the reflected or transmitted spectra must be detected and analyzed to establish the change in the Bragg wavelength.

In the presence of significant transverse deviatoric strain, the Bragg grating becomes a birefringent medium, resulting in a splitting of the main Bragg wavelength into two separate peaks (Van Steenkiste and Springer, 1997). To denote the differential wavelength, the term \( \lambda_{\text{dif}} \) is introduced. Generally, the change in the average Bragg wavelength (\( \lambda_{\text{avg}} \)) and the differential Bragg wavelength can be written as a function of the internal strains and temperatures. That is

\[
\frac{\Delta \lambda_{\text{avg}}}{\lambda_{B_0}} = f(e_i^s; \Delta T) \\
\frac{\Delta \lambda_{\text{dif}}}{\lambda_{B_0}} = f(e_i^s)
\]

where \( \Delta \lambda \) is the Bragg wavelength shift, \( e_i^s \) is the change in the sensor strains for \( i = 1, 2, ..., 6 \), and \( \Delta T \) is the change in temperature. The strains, subject to a deformation field, \( u(x_1, x_2, x_3) \), have the following definition:

\[
e_1 = e_{11} = \frac{\partial u_1}{\partial x_1} \\
e_2 = e_{33} = \frac{\partial u_3}{\partial x_3} \\
e_3 = e_{12} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \\
e_4 = \gamma_{23} = \frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3} \\
e_5 = \gamma_{13} = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3} \\
e_6 = \gamma_{12} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}
\]

(2.15)

Note that in this case, the strains \( e_i \) refer to the strains corresponding to the sensor axes, where the \( x_1 \) direction is the sensor (Bragg grating) axis.

A simplified form of the functional relation 2.14 occurs when the Bragg grating is free of transverse stresses and no temperature variations are occurring. Then the conditions are such that

\[
e_2^s = e_3^s = -\nu^s e_1^s \\
e_4^s = e_5^s = e_6^s = 0
\]

(2.16)

and the the Bragg wavelength shift becomes (Van Steenkiste and Springer, 1997):

\[
\frac{\Delta \lambda_{\text{avg}}}{\lambda_{B_0}} = \left[ 1 - \frac{n_0^2}{2} (p_{12} - \nu^s (p_{11} + p_{12})) \right] e_i^s
\]

(2.17)
where $p_{11}$ and $p_{12}$ are the corresponding elements of the photoelastic Pockel’s tensor ($P_{ij}$) for the grating and $\nu^s$ is the Poisson’s ratio for the grating.

In the case where the grating is exposed only to changes in temperature, and it is free of residual strains, the sensor strains are

$$e_1^s = e_2^s = e_3^s = \alpha^s \Delta T$$  \hspace{1cm} (2.18)

and the Bragg wavelength shift is then given by (Van Steenkiste and Springer, 1997):

$$\frac{\Delta \lambda_{\text{avg}}}{\lambda_{B_0}} = \left( \alpha^s + \frac{1}{n_0} \frac{dn_0}{dT} \right) \Delta T$$  \hspace{1cm} (2.19)

where $\alpha^s$ is the coefficient of thermal expansion of the Bragg grating and $\frac{dn_0}{dT}$ is the thermooptic coefficient of the sensor.

For sensors experiencing variations is both temperature and mechanical strain, the Bragg wavelength shift is given by the addition of equations 2.19 and 2.17:

$$\frac{\Delta \lambda_{\text{avg}}}{\lambda_{B_0}} = \left( \alpha^s + \frac{1}{n_0} \frac{dn_0}{dT} \right) \Delta T + \left[ 1 - \frac{n_2^2}{2} \left( p_{12} - \nu^s (p_{11} + p_{12}) \right) \right] (e_1^s - \alpha^s \Delta T)$$  \hspace{1cm} (2.20)

where the term $(e_1^s - \alpha^s \Delta T)$ is representative of the strains that would occur in the absence of temperature change. In applications where the sensing period is greater than any period of anticipated temperature fluctuation, temperature sensors must be placed near grating sensors that are used to determine components of the strain field. This can be accomplished either by independent temperature sensors, or by using Bragg gratings that are uncoupled from any mechanical strains and therefore only sensitive to temperature fluctuations in the surrounding environment. The data derived from the temperature sensors can then be used to correct the strain data from the FBG strain sensors.

When strains are not limited to the uniaxial direction (i.e. the sensor is embedded in a continuum undergoing an arbitrary change in the state of strain), equation 2.20 does not apply. The discussion of this more general case is deferred until Chapter 3.

Since information relating to temperature and strain variation is encoded in terms of the reflected wavelength, it is possible to write multiple FBG’s of different Bragg wavelengths along a single fibre at different locations to create a sensor array. This technique, known as wavelength division multiplexing, allows multiple sensors to be embedded on a single optical fibre. The reflected spectrum (or the transmitted spectrum) is detected in order to establish the Bragg wavelength shift at each given sensor location. The maximum number of FBGs that can be written into a single cable depends on the expected changes in temperatures and strains (to ensure that the Bragg wavelengths do not begin to overlap), the bandwidth of the light emitting system, and the sensitivity bandwidth of the detection system.

### 2.3 Standard configuration of a fibre Bragg grating sensing system

Fibre Bragg gratings, when used as sensors, fall under the category of spectrometric fibre optic sensors. That is, the detection of the optical spectrum is used to determine the value of strain or temperature (i.e. the measurand) in a medium. The Bragg grating constitutes an intrinsic sensor within the fibre, meaning the fibre acts as both the sensing element and the transmitter of the sensing signal.
The Bragg grating itself forms only a portion of the total sensing system. In addition to the Bragg grating, a light source, an optical interrogator, and an optical network are also required. In this subsection, a brief review of the critical components of the Bragg sensing system will be presented.

A broadband light source, typically of a Light Emitting Diode (LED) type is required in order to transmit a wide bandwidth of light through the sensing network. The light passes through one or several Bragg gratings within the sensing network and a part, or parts, of the generated spectrum are reflected back towards the light source. In the case where the reflected spectrum is being analyzed, the reflected spectrum is diverted by means of an optical circulator (the fibre optic equivalent of a half-silvered mirror in bulk optics) to the interrogation unit. Various methods of determining the reflected spectrum of a Bragg sensing array are possible. The most common are the use of optical spectrum analyzers (OSAs), tunable filters, and interferometers. Generally, for commercial applications of Bragg grating sensors, optical spectrum analyzers are used due to their minimal cost and portability with respect to the other options. The standard setup described is depicted in Figure 2.5.

Alternatively, tunable narrowband light sources can also be used to interrogate the FBG sensors instead of a broadband light source. In this case, a tunable light source is used to scan the system, and for each sweep the maximum wavelengths of reflected light are recorded by the interrogator.

\[\text{Figure 2.5: Schematic of a standard fibre optic sensing network}\]

In all cases, careful attention must be paid to the design of the FBG sensing system. Optical fibres are inherently brittle and delicate, and their installation in structures or in boreholes must be carefully designed to ensure long-term reliability of the sensing network. Additionally, since the FBG generally constitutes a relatively short portion of the fibre (on the order of 1 cm), it is necessary to ensure that this portion of the fibre is correctly coupled to the host material. To this end, various sensor patch designs have been suggested by numerous researchers in order to couple the FBG to the host medium on
Chapter 2. Introduction to Fibre Optics and Fibre Bragg Grating Sensing

or in which the strain and temperature is to be measured. A brief review of these designs is presented in Section 2.7.

2.4 Interrogation Methods for Fibre Bragg Grating Sensors

In order to detect changes in strain within the fibre Bragg grating, the reflected peak wavelength or peak wavelengths (in the case of wavelength-division multiplexed gratings) must be demodulated. In order to measure changes of strain of one $\mu\varepsilon$, changes in the peak wavelength on the order of one nm must be detected. The most prominent methods to detect the peak wavelength change can be broken down into the following classes: passive filtering, tunable filtering, wavelength tunable source interrogation, charge-coupled device (CCD) spectrometry, and interferometry. In this section, each method is described in brief. The physical mechanism behind each measurement method is described, and the merits and limitations of each method are discussed.

2.4.1 Passive Filtering

Passive filtering relies on optical filters that attenuate the energy of light above or below a certain wavelength. When the cutoff wavelength of the filter is close to the signal wavelength, variations in the signal wavelength result in a linear variation in the output intensity of the filter. Broadband filters have a wide cutoff wavelength width, resulting in lower resolution but higher range; whereas edge filters have a higher resolution and lower range. Passive optical filters also rely on bulk optical components which places high demands on alignment, reducing the portability of the system. Additionally, for wavelength division multiplexed FBG’s, each FBG requires its own devoted filter, increasing the complexity of the system.

An early example of a passive filter was demonstrated by Melle et al. (1992). In their experiment, the reflected spectrum of an FBG was split into two beams of equal intensity; the first beam passed through a wavelength dependent filter, and the second beam served as a reference. Both beams were detected by photodetectors, converted to a voltage signal, amplified, and sent to a divider. The intensity ratio of the reference beam to the filtered beam served to define the change in the reflected peak. This method requires one filter whose linear operating range corresponds to that of the Bragg grating being used for sensing.

In a more advanced method of passive filtering, Sano and Yoshino (2003) presented arrayed waveguide gratings (AWGs) as a means to demodulate multiple FBGs simultaneously. The AWG is an array of narrowband transmission filters which are integrated onto a planar waveguide which contains no mechanical moving components. The FBG central wavelengths are designed to lie between the passbands of two adjacent AWG channels (c.f. figure 2.6), allowing for a high degree of wavelength division multiplexing. The method was shown to be successful in the detection of ultrasonic waves by Fujisue et al. (2006). In a theoretical and numerical study of AWG peak demodulation, Buck et al. (2009) noted that the bandwidth between adjacent AWG channels linearly affects the measurement range of a given channel. Furthermore, they noted that the noise characteristics of the system (and therefore the resolution of the measurement) are highly dependent on the light source quality, and the spectral width and peak of the FBG. More importantly, since AWGs were not originally intended for spectral peak demodulation of FBG sensor arrays, commercially available AWGs optimised for sensing applications are not yet available. Buck et al. (2012) customised an AWG design for the monitoring of dynamic
Chapter 2. Introduction to Fibre Optics and Fibre Bragg Grating Sensing

2.4.2 Scanning Optical Filters

Another means to determine the the Bragg grating wavelength change is to scan the incoming spectrum using a tunable filter. Fabry-Perot, acousto-optic, and Bragg grating tunable filters have been demonstrated. The demodulated output is the convolution of the tunable spectrum with that of the grating; the output is optimised when the spectrum of the filter matches that of the grating. All methods reported have provided resolutions on the order of $\sim 1 \, \mu\varepsilon$.

In the case of the Bragg grating scanning filter, a piezoelectric transducer is used to vary the length of the optical receiving end Bragg grating which seeks to match the spectrum of the sensor grating. Detection can take place in parallel, in which case one receiver FBG and one photodetector is required for each sensor FBG used (Jackson et al., 1993). A series configuration of receiver FBG’s has also been demonstrated (Brady et al., 1994) wherein each receiver FBG is modulated at different frequencies so the use of a single photodetector and coupler can be employed, rather than the multiple required for the parallel configuration.

The tunable wavelength Fabry-Perot filter relies on a Fabry-Perot cavity which acts as a filter, whereby the mirror spacing is controlled by a piezoelectric transducer (Kersey et al., 1993). A change in the mirror distance by the transducer results in a change of the filter passband. The filter can operate in a tracking mode (for a single FBG sensor) or scanning mode (for multiple sensors). Strain measurement resolutions of $0.3 \, \mu\varepsilon$ were reported using this method.

The acousto-optic tunable filter is another variation of a scanning optical filter first applied to interrogate FBG sensors by Xu et al. (1993). This method relies on the coupling between optical and acoustic waves in anisotropic crystals where certain optical waves are polarized under specific acoustic frequencies (Harris and Wallace, 1969). Similar to the Fabry-Perot and Bragg grating scanning filters,
the acousto-optic method can be used in both a scanning and tracking mode configuration (Geiger et al., 1995; Xu et al., 1996).

### 2.4.3 Interrogation using a wavelength tunable source

Another method of interrogating FBGs consists of a wavelength tunable light source rather than a broadband source. A narrow linewidth, single frequency, wavelength swept fibre laser is required for this type of interrogation. This method determines the maxima in reflected power during each wavelength sweep of the tunable laser, thereby determining the maximum reflected wavelength of each sensing FBG.

An early application of this type of source-interrogation system was demonstrated by Ball et al. (1994). In their experiment, they constructed a single frequency erbium doped fibre laser using 2 98% Bragg gratings for the resonator which was modulated by a linear piezotranslator to adjust the resonator length. The return signal was monitored by a 3 dB coupler and a photodetector connected to a digital oscilloscope. They reported a frequency inaccuracy of approximately 2.3 pm over a wavelength range of 2.3 nm.

An erbium doped wavelength-swept fiber laser was developed by Yun et al. (1998) which had an improved wavelength range of 28 nm and a linewidth of 0.1 nm. They reported strain resolutions of less than 0.47 $\mu$ε measured at 250 Hz. Swept lasers with similar capabilities are commercially available at present.

### 2.4.4 Interrogation with charge-coupled device (CCD) spectrometry

CCD spectrometry is characterised by parallel detection of the total reflected light spectrum by an array of pixels (Ezbiri et al., 1997, 1998). This method relies on a bulk-optic dispersive element (generally a finely ruled diffraction grating) that directs the reflected light across an array of detector pixels. The resolution corresponds to the number of pixels in the CCD array and the bandwidth of the dispersion grating. Typically, pixel resolution is around 0.1 - 0.4 nm. In order to improve the resolution, digital algorithms for determining the subpixel peak are required, such as the centroid algorithm (Askins et al., 1995). Section 2.5 describes various peak detection algorithms in greater detail. Using these methods, strain resolutions on the order of 1 $\mu$ε are attainable.

Standard pixels are fabricated with Indium Gallium Arsenide which require a certain integration time where the pixels are exposed to the reflected spectrum followed by a reset time which limits the measurement frequency. Commercially available interrogators at present are capable of recording spectra at 30 kHz and greater.

Due to the parallel measurement of the reflected spectrum, wavelength division multiplexing is immediately possible without any alteration of the interrogation device. In one setup, simultaneous spatial and wavelength multiplexing was demonstrated by a series of colinear CCD arrays, each interrogating a different branch of FBG sensors simultaneously (Hu et al., 1997).

### 2.4.5 Interrogation using interferometry

A final noteworthy method of peak wavelength detection is the interferometry class. This method uses the wavelength shift of the sensor and a Mach-Zehnder or Michelson interferometer to monitor the phase change of the signal. The phase resolution of this method can be extremely high, however this method is prone to temperature induced phase drifts within the interferometer and has a limited unambiguous
range of $2\pi$ radians in phase change (Othonos and Kalli, 1999). As a result, interferometry is generally best suited to monitor dynamic strains alone with a limited range without multiplexing capabilities.

More recently, Qiao et al. (2006) developed a two-wave mixing interferometer using a photorefractive crystal which was used to demodulate the spectral shift of Bragg gratings. The methodology consists of using a photorefractive crystal to mix two coherent beams of light (one weak signal beam from the reflection of the FBG and one pump beam) to measure a phase shift. Since the crystal can modify its properties as a result of long-term, quasi-static changes in strain or temperature within the FBG, it is insensitive to long period variations in these quantities without external active compensation, as is generally required in other interferometric configurations (Kersey et al., 1992). Qiao et al. (2006) note a high pass filter behaviour with a cutoff frequency of 600 Hz, and in principle, no upper limit in the detectable frequency range arising from the two-wave mixing method (apart from the restrictions imposed by the photodetection electronics and the physical FBG). Additionally, the two-wave mixing interferometer method can be extended using the same crystal to include multiple channels, allowing for wavelength division multiplexing with cross talk limited to $-30$ dB between channels. A resolution of $0.3$ pm was reported using this method. Balogun et al. (2008) and Kirikera et al. (2011) demonstrated the applicability of this method to the detection of ultrasonic waves on aluminum plates with acquisition frequencies of up to 180 kHz.

2.4.6 A comparison of the various interrogation methods

Dyer et al. (2005) provided comparative reviews of swept lasers and OSA interrogation systems described above with regards to the short and long term stability of measurements. They note nonlinearities in the measured peak wavelength for some optical spectrum analyzers and note that periodic calibration of the instrument is necessary to ensure long-term accuracy of measurements.

In terms of measurement of both static and transient loads, the optical spectrum analyzer and the arrayed waveguide grating are the only two methods capable of acquiring reflected spectra at meaningful acquisition rates. Of these methods, only the arrayed waveguide grating has been shown to provide adequate anti-aliasing ability, although the resolution and linear spectral range per channel are inferior to the optical spectrum analyzer.

Interferometric interrogation schemes have shown to be extremely sensitive to strains (on the order of $n\varepsilon$), however, due to extreme sensitivity to environmental perturbations that might alter the optical path length, these methods are only reliable in the acquisition of dynamic strains. The more recent method, 2-wave mixing, has allowed for wavelength division multiplexing, although these devices are not yet commercially available.

Table 2.1 has been compiled based on the available literature for each noteworthy interrogation scheme. The values and data contained therein are meant to be representative since each method can be customized to some degree to suit a range of purposes.
### Table 2.1: A summary of the interrogation methods presented in section 2.4.

<table>
<thead>
<tr>
<th>Class</th>
<th>Technique</th>
<th>Description</th>
<th>WDM capable(^1)</th>
<th>Anti-aliasing</th>
<th>Range</th>
<th>Resolution</th>
<th>Acquisition rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive filtering</td>
<td>Edge Filter</td>
<td>Melle et al. (1992)</td>
<td>–</td>
<td>Yes</td>
<td>20 nm</td>
<td>100 pm</td>
<td>&gt;100 kHz</td>
</tr>
<tr>
<td></td>
<td>Arrayed waveguide grating</td>
<td>Sano and Yoshino (2003); Fujisue et al. (2006); Buck et al. (2009)</td>
<td>+</td>
<td>Yes</td>
<td>4.5 nm/ch</td>
<td>10 pm</td>
<td>20 kHz</td>
</tr>
<tr>
<td>Tunable filters</td>
<td>Bragg scanning filter</td>
<td>Jackson et al. (1993); Brady et al. (1994); Paterno et al. (2006)</td>
<td>+</td>
<td>No</td>
<td>10 nm</td>
<td>1 pm</td>
<td>85 Hz</td>
</tr>
<tr>
<td></td>
<td>Fabry-Perot filter</td>
<td>Kersey et al. (1993)</td>
<td>+</td>
<td>No</td>
<td>6–30 nm</td>
<td>0.5–3 pm</td>
<td>30 Hz</td>
</tr>
<tr>
<td></td>
<td>Acousto-optic tunable filter</td>
<td>Xu et al. (1993); Geiger et al. (1995); Xu et al. (1996)</td>
<td>++</td>
<td>No</td>
<td>&gt;100 nm</td>
<td>~0.5 pm</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Wavelength tunable source</td>
<td>Tunable laser</td>
<td>Ball et al. (1994); Yun et al. (1998)</td>
<td>++</td>
<td>No</td>
<td>80 nm</td>
<td>0.5 pm</td>
<td>1 kHz</td>
</tr>
<tr>
<td>Parallel Spectral</td>
<td>CCD spectrometry</td>
<td>Ezbi et al. (1997, 1998); Askins et al. (1995); Hu et al. (1997)</td>
<td>++</td>
<td>Partial</td>
<td>80 nm</td>
<td>0.7 pm</td>
<td>35 kHz</td>
</tr>
<tr>
<td>Acquisition</td>
<td>Interferometry(^2)</td>
<td>Kersey et al. (1992)</td>
<td>–</td>
<td>Yes</td>
<td>0.2 nm</td>
<td>0.7 fm/√Hz</td>
<td>&gt;100 kHz</td>
</tr>
<tr>
<td></td>
<td>Enhanced range interferometry</td>
<td>Rao et al. (1996)</td>
<td>–</td>
<td>Yes</td>
<td>1.8 nm</td>
<td>44 fm/√Hz</td>
<td>&gt;100 kHz</td>
</tr>
<tr>
<td></td>
<td>2-wave mixing</td>
<td>Qiao et al. (2006); Balogun et al. (2008); Kirikera et al. (2011)</td>
<td>+</td>
<td>Yes</td>
<td>40 nm</td>
<td>0.3 pm</td>
<td>180 kHz</td>
</tr>
</tbody>
</table>

\(^1\) ‘–’ indicates wavelength division multiplexing is impractical, ‘+’ indicates customisation of interrogator required to match gratings, ‘++’ indicates no customisation of the interrogator is required

\(^2\) amenable only to dynamic strain measurements due to long period drift of the interferometer phase

\(^3\) values highly dependent on one another, typical values are shown here
2.5 Digital Peak Detection Algorithms for CCD spectrometry

Several methods have been proposed to determine the peak wavelength at a subpixel resolution returned by a FBG sensor using a CCD array. A summary and a quantitative analysis on the accuracy of these methods was given by Bodendorfer et al. (2009). These methods will be discussed briefly in this section. In all cases, a single, peak, wavelength $x_B$ at a given point in time is desired in a specified region of the CCD array. $x_i$ represents the central wavelength of the $i^{th}$ pixel, and $I_i$ represents the intensity of light on the $i^{th}$ pixel. A schematic of the system is given in Figure 2.7.

\begin{align*}
\text{Reflected spectra from each FBG} & \rightarrow \text{Diffraction grating} \rightarrow \text{CCD array} \\
\text{Pixel no., } i & \rightarrow \text{Intensity, } I(x_i) \\
\text{Intensity, } I(x) & \rightarrow \text{Wavelength, } x_i \\
1520\text{nm} \ldots \ldots & 1580\text{nm} \\
\lambda_{\text{max},n} & \rightarrow \lambda_{\text{max},n+1} \\
\end{align*}

\textbf{Figure 2.7: Principle of operation of a CCD array}

\subsection*{2.5.1 Centroid detection algorithm}

The centroid detection algorithm mentioned in Askins et al. (1995) consists of determining the centroid of the incident spectrum. It has the following form:

\begin{equation}
-x_B = \frac{\sum_{i=1}^{N} x_i I_i}{\sum_{i=1}^{N} I_i}
\end{equation}

Equation 2.21 can be interpreted as the geometric centre.

\subsection*{2.5.2 Quadratic fit algorithm}

The quadratic fit algorithm consists of fitting a polynomial $I_{\text{fit}}(x_i)$ to the CCD data using a least squares method. That is, we wish to minimize the expression

\begin{equation}
\frac{1}{N} \sum_{i=1}^{N} w_i (I_{\text{fit}}(x_i) - I(x_i))^2
\end{equation}
where the \( w_i \) are weighting elements. The function \( I_{fit}(x_i) \) has the following form for a quadratic polynomial

\[
I_{fit}(x_i) = ax_i^2 + bx_i + c
\]

where \( a, b \) and \( c \) are constants.

### 2.5.3 Gaussian fit algorithm

The Gauss fit algorithm is similar to the quadratic fit, except in this case, the function \( I_{fit}(x_i) \) of equation 2.22 has the following form

\[
I_{fit}(x_i) = a_0 \cdot \exp \left( -\frac{x - \mu}{2\sigma^2} \right)
\]

where \( a_0, \mu, \) and \( \sigma \) are constants representing the amplitude, mean, and standard deviation of the Gauss curve, respectively.

### 2.5.4 Linear phase operator algorithm

The application of linear phase operator on peak detection was first proposed by Zeh et al. (2004) which was based on the work of Blais et al. (2000) to extract three-dimensional information from CCD images. The linear phase operator algorithm is a combination between a finite impulse filter (the z-transform) and a linear interpolation step. The z-transform is given by

\[
Z\{h(n)\} = H(z) = \sum_{i=1}^{N-1} h_i z^{-i}
\]

The filter output can be described by the following equation, where the coefficients \( h_i \) are the coefficients for the z-transform of the fourth order and \( I_i \) is the intensity of the light at the \( i^{th} \) pixel:

\[
y(n) = \sum_{i=0}^{N} h_i I_{n-i}
\]

and the \( h_i \) for a fourth order linear phase operator are given by

\[
h_0 = 1, \quad h_1 = 1, \quad h_2 = 0, \quad h_3 = -1, \quad h_4 = -1.
\]

The filter output defines the two values \( \zeta_{LPO}(\lambda_B) \) and \( g_{LPO}(\lambda_B) \) where \( \zeta_{LPO} \) is the intensity value on the left of the filter origin, and \( g_{LPO} \) is the value on the right. The corrected subpixel wavelength, \( \lambda_{B,sub} \), is given by the following equation

\[
\lambda_{B,sub} = \lambda_B + \frac{g_{LPO}(\lambda_B)}{g_{LPO}(\lambda_B) - \zeta_{LPO}(\lambda_B+1)} \quad \text{for} \quad I_{\lambda_B-1} > I_{\lambda_B+1}
\]

\[
\lambda_{B,sub} = \lambda_B + \frac{g_{LPO}(\lambda_B-1)}{g_{LPO}(\lambda_B-1) - \zeta_{LPO}(\lambda_B)} \quad \text{for} \quad I_{\lambda_B+1} > I_{\lambda_B-1}
\]

where \( \lambda_B \) is the center wavelength of the pixel of maximum intensity, and the terms \( I_{\lambda_B-1} \) and \( I_{\lambda_B+1} \) correspond respectively to the intensities to the pixels left and right of the pixel experiencing the maxi-
mum intensity \( I_{\lambda_B} \).

### 2.5.5 A comparison of the peak detection algorithms

Over the past two decades, all of the algorithms presented above have been used for strain sensing applications. A quantitative study on the accuracy, signal-to-noise ratio, and linearity of some of these methods has been discussed by Bodendorfer et al. (2009); Zeh et al. (2004); and Buck (2012). The performance of any of the algorithms depends on the width and shape of the reflected spectrum, the signal-to-noise ratio of the digital signal (influenced by intensity fluctuations of the broadband light source, and shot-noise of the CCD interrogator \textit{inter alia}), the acquisition rate of the spectra with respect to dynamically varying strains, and the number of pixels used in the calculation.

Specifically, Bodendorfer et al. (2009) showed deterioration (i.e. an increase in the standard deviation of a series of measurements for a stationary FBG) with the centroid algorithm by increasing the pixel window, but noted stability for the other methods. Additionally, they noted that due to the asymmetry of the reflected spectra, the same peak detection algorithms give different peak positions corresponding to a total spread of about 0.1 pixels (or 9 pm) for an FBG with a spectral width of approximately 250 pm and a CCD pixel width of approximately 90 pm which they attribute to the asymmetry of the reflected spectrum. Standard deviations of the LPO, Gauss, and quadratic algorithms were also significantly better than the centroid method.

Buck (2012) showed that both fit (Gauss and polynomial), and centroid algorithms can exhibit non-linear behaviour, especially for narrow reflected spectra. This problem is exacerbated by quickly varying strains due to the requirement of minimum integration time on the CCD pixels.

An additional consideration in the selection of a peak detection algorithm is the computational intensity. Although all algorithms presented are relatively straightforward, the analysis of highly multiplexed sensor networks acquiring data at very high acquisition rates will require substantial computational capacity. Algorithms requiring optimization (polynomial and Gaussian fits) are among the most computationally intensive, while the centroid algorithm is the least intensive.

### 2.6 Dynamic Strain Monitoring with CCD spectrometry

Van Damme et al. (2007); Buck et al. (2011) and Buck (2012) discussed the consequences of utilizing CCD spectrometers for the acquisition of dynamic strains. Specifically, they focused on the aliasing of signal frequencies higher than the Nyquist frequency of the measurement device (in this case the CCD array based interrogator). For electrical sensors, and for optical sensors whose signal can be converted to an analogue electrical signal (such as the edge filter design), aliased signal components can be removed by introducing an electric bandwidth filter before digital sampling of the analogue signal. In the case of optical devices, such as the CCD array which requires a minimum pixel integration time of the incident light, a similar voltage-based anti-aliasing filter is not possible.

#### 2.6.1 Intrinsic anti-aliasing capability of CCD spectrometry

Van Damme et al. (2007) demonstrated both theoretically and experimentally that aliased components of a signal can be reduced by increasing the duty cycle parameter, \( \delta \), which is the fraction of time during
a sampling period that the system integrates light. In other words
\[ \delta = t_i f_s \] (2.28)

where \( t_i \) is the finite integration time that the pixel integrates light, and \( f_s \) is the sampling frequency of the interrogation device. The Fourier domain representation (transfer function) of a signal of frequency \( f \) is given by the function
\[ R(f) = \frac{\delta}{f_s} \text{sinc}\left(\frac{\delta \pi f}{f_s}\right) \] (2.29)

where \( R(f) \) represents the relative response amplitude of the signal. Signal frequencies greater than the Nyquist limit \( (f_g = f_s/2) \) will be incorrectly imaged at lower frequencies, given by
\[ f_{\text{image}} = |f - \mu f_s| \] (2.30)

where \( \mu \) is an integer value. Figure 2.8 demonstrates the characteristic low-pass behaviour of a CCD array at increasing duty cycles \( (\delta) \). From this figure, it is apparent that aliasing can be optimally suppressed with higher sampling frequencies and duty cycles equal to one. However, it should be noted that there are practical limitations to a high duty cycle, since some CCD arrays require a minimum reset time, in which the pixels cannot integrate light between integration cycles.

**Figure 2.8:** Amplitude attenuation for a sampling frequency, \( f_s = 500 \text{ Hz} \) \( (f_g = 250 \text{ Hz}) \), and a reference frequency of \( \omega = 30 \text{ Hz} \). Note that higher signal frequencies are attenuated for greater duty cycles, exhibiting an intrinsically low-pass behaviour. (After Van Damme et al., 2007.)
2.6.2 Signal deskewing and detection of aliasing

Buck et al. (2011) noted that the use of increased duty time alone does not adequately suppress aliasing of high frequency signals and presented a novel method to (i) deskew signals within the first Nyquist limit, and (ii) to detect and quantify the degree of signal aliasing for signals outside the first Nyquist limit. The method consists of reconstructing the reflected spectrum time history on an ideal virtual CCD array and taking the correlation with the actual measured spectrum time history. As the degree of correlation diverges, so too does the quality of the reconstructed signal. The systematic procedure suggested by Buck et al. (2011) will be briefly summarized in this section.

It is noted that a force \( f(t) \) acting on the Bragg grating induces a change of the Bragg frequency \( \lambda_B(t) \) which produces a time dependent reflection spectrum \( s(\lambda_B(t), \lambda) \), where \( \lambda \) is the continuous wavelength domain. After exposure of the CCD pixels, an integrated spectrum is obtained \( S(\omega, t_i) \) at discrete points in time \( t_i \), where \( i \) is a natural number. Following the integrated spectrum, a time-discrete Bragg wavelength is computed, \( \tilde{\lambda}_B(t_i) \), from any of the algorithms listed in section 2.5. The computed Bragg wavelengths \( \tilde{\lambda}_B(t_i) \) can be used to reconstruct a time-continuous signal \( \tilde{\lambda}(t) \), and from this signal, a reconstructed spectrum \( \tilde{S}(\lambda, t_i) \) can be determined.

The frequency spectrum of the measured signal \( \tilde{\lambda}_B(t) \) is denoted \( F_s \) and is given by

\[
F_s(\omega) = \sum_{\mu=-\infty}^{+\infty} R(\omega - \mu \cdot 2\omega_g) \cdot F_{\text{orig}}(\omega - \mu \cdot \omega_g) \tag{2.31}
\]

where \( R \) is the same transfer function described by equation 2.29, \( \mu \) is an integer indicating aliased signal components, \( \omega_g \) is the Nyquist frequency, and \( F_{\text{orig}} \) is the frequency spectrum of the forcing function \( f(t) \), or

\[
f(t) = \int_{-\infty}^{\infty} F_{\text{orig}}(\omega)e^{i2\pi\omega t} d\omega \tag{2.32}
\]

both of which are unknown.

If the signal \( \lambda_B(t) \) is restricted to the first Nyquist band \((-\omega_g < \omega < \omega_g)\), a deskewing filter can be applied to the reconstructed signal \( \tilde{\lambda}_B \) by applying a deskewing filter \( B(\omega) = 1/A(\omega) \). This gives

\[
B(\omega) \cdot F_s(\omega) = B(\omega) \cdot A(\omega) \cdot F_{\text{orig}}(\omega) = F_{\text{orig}} \cdot \tag{2.33}
\]

Therefore, if the applied load exists within the first Nyquist limit, it is possible to reconstruct the original signal \( f(t) \propto \tilde{\lambda}(t) \) by 2.32 and 2.33. The reconstructed time-continuous signal \( \tilde{\lambda}(t) \) is given by a Fourier analysis of the measured frequency spectrum and deskewing filter:

\[
\tilde{\lambda}(t) = \frac{1}{2\pi} \int_{-\omega_g}^{+\omega_g} B(\omega) \cdot F_s(\omega) \cdot e^{i\omega t} d\omega
\]

\[
= \frac{1}{2\pi} \sum_{\mu=-\infty}^{+\infty} \left[ \int_{-\omega_g}^{+\omega_g} B(\omega) \cdot A(\omega - \mu 2\omega_g) \cdot F_{\text{orig}}(\omega - \mu 2\omega_g) \cdot e^{i\omega t} d\omega \right]. \tag{2.34}
\]

The reconstructed signal \( \tilde{\lambda}(t) \) can than be used to compute the ideal (undistorted) spectral response on the CCD detector, \( \tilde{S}(\lambda, t_i) \) at discrete time intervals \( t_i \). This simulated spectrum is obtained by
computing
\[
\tilde{S}(\lambda, t_i) = \int_{t_i-\Delta t}^{t_i} s(\tilde{\lambda}_B(t), \lambda) \, dt
\]  \hspace{1cm} (2.35)
where \(\Delta t\) represents the integration time of the CCD detector at a timestep \(t_i\) and \(s(\tilde{\lambda}_B(t), \lambda)\) represents the assumed time-continuous reflected Bragg signal. The actual spectrum is given by
\[
S(\lambda, t_i) = \int_{t_i-\Delta t}^{t_i} s(\lambda_B(t), \lambda) \, dt
\]  \hspace{1cm} (2.36)
which is the raw integrated data furnished directly from the CCD pixel array.

A quantitative value suggested by Buck et al. (2011) for the agreement between \(S(\lambda, t_i)\) and \(\tilde{S}(\lambda, t_i)\) is the Pearson correlation coefficient,
\[
m(S, \tilde{S}) = \frac{\text{Cov}(S, \tilde{S})}{\sqrt{\text{Var}(S) \cdot \text{Var}(\tilde{S})}}
\]  \hspace{1cm} (2.37)
where the covariance is defined through the expectation operator \(E\),
\[
\text{Cov}(S, \tilde{S}) = E((S - E(S))(\tilde{S} - E(\tilde{S})))
\]  \hspace{1cm} (2.38)
A simple outline of the method described is provided in Figure 2.9.

Buck et al. (2011) tested their method both numerically and experimentally for loads containing a single frequency component and found that the decay of the correlation coefficient was sufficient to detect aliasing when high frequency components were acting on the sensor. Obviously, the major disadvantage of the method is the inability to remove the aliased components from the signal; the method only serves as a means to determine whether aliasing is occurring. The presence of aliased components in signal acquisition for CCD interrogation of FBG sensor networks is likely the greatest disadvantage of this method. Nevertheless, the economic viability of CCD spectrometers as well as their constantly
improving performance will likely increase their bandwidth capability in the future.

2.7 Examples of fibre optic sensing in geomechanics and geophysics

The use of fibre optics in aeronautics and civil infrastructure is prevalent, and there are many applications that have been published. Fibre optic cable, which is several hundreds of microns in diameter, can be easily integrated into fibre laminates, such as carbon fibre reinforced polymers, or fibre glass reinforced polymers (Grant et al., 2003). Additionally, various manufacturers have produced fibre Bragg grating “strain patches,” which can be immediately applied onto steel or concrete surfaces (Schlüter et al., 2010; Ramos et al., 2011; Schilder et al., 2012), easing their application for structural health monitoring of bridges, buildings, dams, and other civil infrastructure. Another variation of Bragg sensing instrumentation has been designed to be directly embedded into concrete (Fenando et al., 2003; Schallert et al., 2007; Biswas et al., 2009). In this case, the sensors are encased in flanged steel tubes that protect the fibre cable and provide a strain coupling mechanism to the concrete.

Although the use of fibre optic sensors for structural health monitoring in aeronautics and civil infrastructure is common, there are fewer examples of their application in geomechanics and geophysics. This section briefly summarizes a few instances in which fibre optics and their derivative technologies have been used successfully for geomechanical, and geophysical applications.

2.7.1 Rock deformation monitoring using fibre optic technology

In a notable example of rock deformation monitoring, Cappa et al. (2005) used various fibre optic sensor schemes to measure elastic heterogeneous displacement fields in a fractured rock mass. The first type of sensors used were of the Fabry-Perot type, in which a small gap between to colinear fibers is created in a micro-capillary forming a Fabry-Perot interferometer (Choquet et al., 2000). Changes in displacement between the two ends of the fiber correspond to wavelength changes in the reflected light spectrum due to the interferometric effect of the cavity. The instrument was anchored to opposite sides of a fault connected to a hydraulic pressurization system. A second set of optic sensors based on the Brillouin scattering principle (Niklès et al., 1997) were also installed on the surface of the same rock mass and anchored at various points around the fault. Fluid was injected into the fault system, and an independent set of vibrating wire extensometers were used to compare the Fabry-Perot displacement results. A 2% relative error was observed between the Fabry-Perot measurements and the extensometer readings. The surface Brillouin-based measurement system also recorded deformations that correlated well to the injection of water into the fault.

In a similar application of optical sensors, Moore et al. (2010) used fibre Bragg gratings to monitor the long term relative displacement of fractures at the Randa rockslide site in southern Switzerland. Borehole installed sensors were grouted to boreholes that extended to fractures that had been previously been mapped and were prone to movement. The long gauge length allowed the ends of the sensors to be grouted to opposite sides of the discontinuity. Long term monitoring of the Bragg gratings revealed several transient and long term slip events that correlated well with measurements taken from traditional vibrating wire extensometers. Importantly, the authors noted an initial error in measurements that they attributed to drying shrinkage of the grout.
Valley et al. (2012) used Brillouin based fiber optic sensing (similar technology to that of Cappa et al. (2005) described above) to monitor mining induced rock mass deformation citing a better understanding of mining induced seismicity as a primary motivation for the deployment of their sensors. They deployed four lengths of fibre cables in four separate grouted boreholes in a mine pillar of 25 metre length. Adjacent boreholes with multipoint borehole extensometers provided reference measurements for the Brillouin measurement system. Relative measurements between the two adjacent systems showed qualitative similarities, which they suggest could be improved by better compliance through the rock/grout/sensor system. Investigation of the temporal and spatial measurements of the Brillouin system showed large cyclic oscillations of considerable magnitude (up to 400 $\mu \varepsilon$) which the authors suggested were artifacts of the Brillouin measurement system.

In another study, Gage et al. (2013) presented a new technique for monitoring in situ strains and temperatures in rock masses. Their design consists of pretensioned steel segments instrumented with Bragg gratings embedded into a grout. The grout is then embedded into a rock mass for long term deformation monitoring, similar to a traditional extensometer. Initial laboratory validation of their design proved to successfully measure strains in a quantitative sense. Similarly to Valley et al. (2012), they note the importance of properly coupling the optical strain sensor to the host rock material in order to obtain an accurate measure of strain.

### 2.7.2 Geophysical data acquisition using fibre optic technology

In a special application of fibre optics, Blum et al. (2008) coupled looped fibre optic cables to borehole casings over lengths of 500 to 2100 metres at the SAFOD (San Andreas Fault Observatory at Depth) borehole near Parkfield, California. The borehole fibre served as one arm a a Mach-Zehnder interferometer, with a second, unstrained reference loop kept in quadrature. An A/D fast converter sampled the fringe patterns and recorded the phase changes. Optical path differences with resolutions on the order of picometers was obtained using this method. Short-term noise levels of the sensor corresponded to less than a tenth of a nanostrain and were successful in recording strains from nearby earthquakes and permanent earthquake induced deformations (Blum et al., 2010).

A number of other researchers have presented geophone designs based on fibre Bragg gratings mounted to cantilever beams (Todd et al., 1998; Zhu et al., 2003; Lam et al., 2010; Zhang et al., 2006). Of these designs, Lam et al. (2010) have shown successful 3-axis accelerometer measurements with bandwidths of 10 to 1000 Hz, minimum detectable accelerations of $10 \mu g_{\text{rms}}/\sqrt{\text{Hz}}$, and a dynamic range of 50 $g$ using mode locked lasers and $\pi$–phase shifted Bragg gratings.

### 2.8 Summary

In this chapter, the basic principles of fibre optic cable and Bragg gratings were explained. The simplified equations governing the Bragg wavelength shift as a result of strain and temperature were discussed, and the various mechanisms of sensing this shift were examined and compared. Of these methods, the CCD technique (which was used in the context of this thesis) was examined in greater detail. Various wavelength demodulation schemes that have appeared in the literature were summarized. Intrinsic aliasing suppression, and aliasing detection and deskewing algorithms developed by other researchers were presented since aliasing is a major disadvantage of CCD demodulation for FBG peak detection. Finally, different applications of fibre optic technology in geomechanics and geophysics were summarized.
The following list summarizes some key points of the literature review that are of particular interest to the application of FBG sensors as geodynamic strain sensors:

1. The effects of transverse strains and changes in temperature are important considerations in the implementation of fibre Bragg gratings, especially when the generally triaxial changes in strain are expected within the continuum surrounding the sensor.

2. CCD spectrometry is effective in simultaneous, parallel demodulation of wavelength multiplexed FBG gratings, however aliasing is a possible source of major error when signals of high frequency and significant amplitude are present.

3. Proper coupling of the sensor to the host material is required in order to gain accurate readings of the measurand. For the case of borehole installed sensors, this is not necessarily straightforward.

The aforementioned points have been incorporated into the design of the FBG sensing system (Chapter 3).
Chapter 3

Design and Analysis of Sensor Performance

The use of optical sensors to monitor both the static and transient changes in strain that may occur in a rock mass at any given time is a complicated process. In this chapter, the use of a carbon fibre reinforced polymer (CFRP) sensor casing design, and two steel sensor casing designs to couple the Bragg sensor to the rock mass through a grouted borehole is assessed via a numerical-analytical approach. The three designs installed in a grout matrix is modeled using the finite element method (FEM). Since there is a difference in the elastic properties of the grout material and the sensor casing, the sensor casing constitutes an elastic inclusion within the grout. Consequently, the strains experienced by the sensor casing are not necessarily equal to the far-field strains of the grout matrix and therefore the rock; which is the quantity of interest. The goal is to produce a sensor protection system that optimally transfers strains from the grout matrix to the fibre Bragg grating in the fibre direction. Conversely, it is also a goal of the casing design not to transfer transverse strains from the grout matrix to the Bragg grating. This is due to the fact that the Bragg wavelength changes, generally, both as a function of axial and transverse strains that occur within the fibre. In order to avoid the necessity of multiple sensors as well as complicated inversion schemes for the determination of multiple components of the strain tensor, a casing design that prevents or minimizes the transfer of transverse strains is necessary. Using the methods derived by Van Steenkiste and Springer (1997), the sensitivity of the sensor to all components of strain for different casing designs is assessed and an optimal sensor packaging system is selected.

3.1 Mechanical Analysis of the Sensor Casings

In this section, the numerical modeling of the strain transfer between the grout and the sensor protection system is discussed. As noted, the farfield strains associated with the displacement field of the grout are not necessarily equal to strains in the casing due to the mismatch of elastic moduli of both entities. Due to the irregular geometry and arbitrary boundary conditions that could be used to model the system, stress functions are impossible to derive. Therefore, finite element analysis (FEA) was selected to determine strain fields associated with the selected materials, geometries, loads, and imposed deformations. In the forgoing, strains within the casing system will be compared to the strains actually occurring in the far-field (grout). The rock will be omitted in this simulation, since it is assumed that the rock and
Chapter 3. Design and Analysis of Sensor Performance

grout are completely compliant; that is, strains in the rock are equal to strains in the rock mass along the borehole direction. Furthermore, analysis will be conducted to determine the influence of confining pressures on internal strains of the casing. The computed casing strains will then be used to determine the optical output of the sensor (Section 3.2).

3.1.1 Material Properties and Casing Dimensions

The proper selection of the casing material and dimensions is an important consideration. The protection system must be capable of adequately transferring strain from the host material (grout) to the sensing element (the Bragg grating). It must also be able to protect the delicate and brittle fibre optic cable from intense forces it may be subjected to during the transportation, installation and service life of the sensing element. Furthermore, the protection system must be resistant to the surrounding environment. Changes in temperature, moisture, and pressure may, in the long term, degrade the protection system and adversely affect its mechanical properties resulting in erroneous strain readings. Additionally, it is advantageous to design packaging systems which can effectively transmit strains in the axial direction of the fibre whilst effectively uncoupling the fibre from strains in the transverse directions. Such features can be achieved by embedding the sensor in materials which are transversely less stiff than the silica fibre optic cable, while providing stiff outer shells that prevent severe deformations. To this end, two types of sensor patches have been considered.

The material selected for the first design was unidirectional carbon fibre reinforced polymer (CFRP). The material consists of parallel carbon fibres bonded together by a thermosetting epoxy resin matrix. The two constituent materials are the carbon fibres and epoxy, the mechanical properties of which both contribute to the bulk mechanical properties of the composite material. Due to the parallel direction of the carbon fibres, unidirectional CFRP laminates are taken to be transversely isotropic (Altenbach et al., 2004). Typical elastic properties of the material are given in Table 3.1. A schematic of the material with the associated coordinates used for the FEA are given in Figure 3.1. The optical fibre and Bragg grating is aligned with the carbon fibres. The mechanical coupling to the host material (cement grout) in the longitudinal direction is achieved through the use of two flanges at either end of the sensor patch. The friction that is mobilized along the surface of the sensor patch also contributes minimally to the longitudinal strain transfer.

The second patch design consists of a hollow stainless steel tube with two flanges at either end to provide mechanical coupling to the host material. The interior of the hollow cylinder is filled with an epoxy resin to bond the optical fibre to the outer shell. The walls of the cylinder must be kept relatively thin to minimize the longitudinal stiffness of the casing (since the steel is substantially stiffer than the grout). However, the steel must be thick enough to provide some resistance to deformations in the transverse directions to minimize the transverse strain effects on the sensor. For this reason, two different wall thicknesses were investigated as part of the analysis. The design corresponds with that proposed by Fenando et al. (2003), and a drawing with the coordinate system used in the FEA is provided in Figure 3.2, and the material properties of the various components are provided in Table 3.2.

The sensor or sensor array will in practice be placed in a borehole and grout will be used to bond the sensors to the surrounding host rock. The grout material must also be carefully designed in order to maximize the strain transfer between the rock and the sensor. The grout should be of very high strength to maintain elastic linearity over as much of the expected loading range as possible as well as to prevent cracking around the location of the sensor. Another important design consideration is sufficient
volumetric stability of the cement grout to ensure that debonding does not occur between the grout and the borehole wall. The exact means to obtain these properties are explained in Chapter 4, but for the purposes of analysis, the typical elastic properties of a high strength grout listed in Table 3.2 were used.

### 3.1.2 Finite Element Model

In order to proceed with the finite element model, it is first necessary to make certain simplifications with regards to the geometry. Although, in the most general sense, this is a three dimensional problem, it is possible to reduce the problem to a two dimensional one provided that all loading is axisymmetric. Indeed, the necessity of providing imposed displacements in the sensor axis direction (which also corresponds to the direction of the carbon fibres) as well as the addition of a confining pressure (in the \( r \) direction) results in one plane of symmetry (the \( r - \theta \) plane) and one axis of rotational symmetry (the \( y \)-axis). It is thus possible to use the axisymmetric stress-strain stiffness formulations while taking advantage of the plane of symmetry, which will necessitate only a quarter section of the entire geometry to be modeled (c.f. Figure 3.3).

In the model, 8-noded axisymmetric second order isoparametric elements were used to model both materials (ANSYS element number 18). The interface was modeled with high order contact elements (TARGE169 for the CFRP sensor protection system and CONTA172 elements for the grout). An interface coefficient of friction of 0.3 was selected without the presence of any cohesive strength. This is a conservative assumption since the presence of cohesion would only improve the mechanical strain transfer over the interface. It should be noted that although the material and geometric properties of this problem are assumed linear, the introduction of the contact elements introduce a contact nonlinearity, and therefore the solution to the finite element equations is based on an iterative Newton-Rhapson approach. Numerical convergence is tested automatically within the ANSYS program.

Decreasing element sizes were tested in sequence until a suitable convergent solution was obtained. For elements comprising the sensor protection system, a final element size of approximately 0.3 mm was used for the CFRP design, and 0.1 mm for the steel designs. Where the expected strain gradients were expected to be smaller, namely, away from the sensor casing, the element size was increased to approximately 1 mm. Figure 3.3 shows the model geometry and materials, and Figure 3.4 shows the meshing applied to the geometry.

In order to apply axial strains to the model, imposed degrees of freedom were used. Nodes on the top boundary of the model (i.e. \( y = 76 \) mm in the ANSYS model), experienced imposed deformations which were used to represent the far-field axial strains of the grout material. For numerical analyses that included confining pressure, an evenly distributed load was applied to the right side of the model (i.e. \( r = 38 \) mm). To represent the symmetry of the problem, all nodes along \( r = 0 \) were constrained in the \( y \)-direction and all nodes occupying the line \( y = 0 \) were constrained in the \( r \)-direction.

The important results of the finite element analysis are presented in Tables 3.5, 3.6, and 3.7. The strains presented are those near the embedded fibre optic Bragg grating within the CFRP or steel casing corresponding to the coordinates \((x, y) = (0,0)\). It is noted that these strains do not represent the strains within the fibre. Analytical expressions exist for these strains as a function of the substrate strains and these expressions will be used in the next section to predict the optical output of the Bragg grating.

Based on the results presented in Tables 3.5 through 3.7, we can see that the sensor design is sufficient to transfer the far field strains within the grout to the CFRP and epoxy substrates. This concludes the first step in computing the resulting optical signal resulting from farfield strain fields.
### Table 3.1: Properties of the transversely isotropic CFRP used in the numerical computations

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Young’s modulus, $E_{yy}$ (GPa)</td>
<td>130</td>
</tr>
<tr>
<td>Transverse Young’s modulus, $E_{rr} = E_{θθ}$ (GPa)</td>
<td>9.65</td>
</tr>
<tr>
<td>Longitudinal shear modulus, $G_{yθ} = G_{yr}$ (GPa)</td>
<td>5.58</td>
</tr>
<tr>
<td>Transverse shear modulus, $G_{rθ} = \frac{E_{rr}}{2(1+ν_{rθ})}$ (GPa)</td>
<td>3.21</td>
</tr>
<tr>
<td>Longitudinal major Poisson’s ratio, $ν_{yr} = ν_{yθ}$</td>
<td>0.29</td>
</tr>
<tr>
<td>Transverse Poisson’s ratio, $ν_{rθ}$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

### Table 3.2: Properties of the linear elastic isotropic grout used in the numerical computations

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, E (GPa)</td>
<td>35</td>
</tr>
<tr>
<td>Poisson’s ratio, $ν$</td>
<td>0.16</td>
</tr>
</tbody>
</table>

### Table 3.3: Properties of the linear elastic isotropic stainless steel used in the numerical computations

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, E (GPa)</td>
<td>207</td>
</tr>
<tr>
<td>Poisson’s ratio, $ν$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### Table 3.4: Properties of the linear elastic isotropic epoxy used in the numerical computations

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, E (GPa)</td>
<td>1.9</td>
</tr>
<tr>
<td>Poisson’s ratio, $ν$</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Chapter 3. Design and Analysis of Sensor Performance

Figure 3.1: Geometry and ANSYS Cartesian/cylindrical material coordinate systems of the CFRP casing. All dimensions in millimeters.

Figure 3.2: Geometry and ANSYS Cartesian/cylindrical material coordinate systems of the 2 steel casing designs. All dimensions in millimeters.
Figure 3.3: Axisymmetric model geometry and materials for the steel casing design number 1. The axis of rotational symmetry (y-axis), and the plane of symmetry \((r - \theta)\) are shown.

Figure 3.4: ANSYS mesh geometry for steel casing design number 1.
Table 3.5: Essential results of the numerical analysis for the CFRP casing

<table>
<thead>
<tr>
<th>Test</th>
<th>Imposed quantities</th>
<th>Strains at sensor location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{yy}^{\infty}$ (m$\varepsilon$)</td>
<td>$\varepsilon_{yy}^c$ (m$\varepsilon$)</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.979</td>
</tr>
<tr>
<td>2</td>
<td>1.000</td>
<td>0.965</td>
</tr>
<tr>
<td>3</td>
<td>1.000</td>
<td>0.986</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>0.986</td>
</tr>
<tr>
<td>5</td>
<td>1.000</td>
<td>0.990</td>
</tr>
<tr>
<td>6</td>
<td>1.000</td>
<td>0.998</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>1.002</td>
</tr>
<tr>
<td>8</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>11</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

†The coefficient of friction ($\mu$) for test 2 was set to 0.1

3.2 Mechanical–Optical Analysis

It has been shown in the previous section that the axial component of strain at the core of the steel and CFRP casings is roughly equal to that of the far-field axial grout strain, however, the transverse strains are not equal to the far field grout strains.

In this section, the calculation of the sensor output (Bragg wavelength shift) as a function of the internal casing strains is considered. In the most general case one would consider a fibre optic sensor embedded in a generally anisotropic material. However, the most complicated design considered is that of a unidirectional CFRP for which the elastic properties can be taken as transversely isotropic. For the analysis, the properties of the sensor (silica fibre) and the coating (polyimide) will be taken to be isotropic.

The problem can be stated as follows:

Based on the strains in the substrate material (epoxy or CFRP), what are the resulting strains in the sensor. Following the strains in the sensor, what is the expected change in the optical output of the sensor.

The above problem was considered in detail in the book “Strain and Temperature Measurement with Fiber Optic Sensors” by Van Steenkiste and Springer (1997) who used Lekhnistkii’s stress functions (Lekhnitskii, 1963) for cylindrical inclusions in anisotropic media to compute the strains. Steenkiste and Springer suggest the following approach to the problem:

1. Determine the sensor strain tensor in terms of the far field strain tensor and temperature in the surrounding substrate material.

2. Compute the changes in optical and geometric properties of the sensor given the new sensor strains at a given temperature.
3. Compute the the Bragg wavelength shift as a result of the new sensor optical and geometric properties.

The problem enumerated above is known as the forward problem; for known farfield strains the optical output is determined. In their book, Steenkiste and Springer also consider the inverse problem, in other words, for given a given sensor output, the farfield strains and temperature are determined. The inverse problem is more difficult to solve. Given the dimensionality of the problem, a minimum number of sensors is required, and each sensor must be optimally oriented in the rockmass. Following the measured changes in the Bragg shift, the rock strains can be inverted (solved uniquely), provided the minimum number of sensors is used. For rock mechanics, applying multiple sensors (e.g. a minimum of seven sensors for a general three dimensional problem), each installed in its own borehole all in a close location is prohibitively difficult. Even if one were interested in only one component of the strain tensor, all components would have to be measured if one were to consider the inverse problem. Therefore, only the forward problem is considered, and the sensitivity of each of the casing designs to transverse strains.

<table>
<thead>
<tr>
<th>Test</th>
<th>$\varepsilon_{yy}^\infty$ (m$\varepsilon$)</th>
<th>$\sigma_{rr}^\infty$ (MPa)</th>
<th>$\varepsilon_{yy}^c$ (m$\varepsilon$)</th>
<th>$\varepsilon_{rr}^c$ (m$\varepsilon$)</th>
<th>$\varepsilon_{\theta\theta}^c$ (m$\varepsilon$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0</td>
<td>1.003</td>
<td>-0.291</td>
<td>-0.291</td>
</tr>
<tr>
<td>2</td>
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<td>10</td>
<td>1.002</td>
<td>-0.062</td>
<td>-0.062</td>
</tr>
<tr>
<td>3</td>
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<td>20</td>
<td>1.001</td>
<td>0.167</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
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<td>30</td>
<td>0.999</td>
<td>0.396</td>
<td>0.396</td>
</tr>
<tr>
<td>5</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>0.229</td>
<td>0.229</td>
</tr>
<tr>
<td>7</td>
<td>0.000</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.688</td>
<td>0.688</td>
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<table>
<thead>
<tr>
<th>Test</th>
<th>$\varepsilon_{yy}^\infty$ (m$\varepsilon$)</th>
<th>$\sigma_{rr}^\infty$ (MPa)</th>
<th>$\varepsilon_{yy}^c$ (m$\varepsilon$)</th>
<th>$\varepsilon_{rr}^c$ (m$\varepsilon$)</th>
<th>$\varepsilon_{\theta\theta}^c$ (m$\varepsilon$)</th>
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<tbody>
<tr>
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<td>0.970</td>
<td>-0.253</td>
<td>-0.253</td>
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<tr>
<td>2</td>
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<td>-0.095</td>
<td>-0.095</td>
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<tr>
<td>3</td>
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<td>20</td>
<td>0.971</td>
<td>0.068</td>
<td>0.068</td>
</tr>
<tr>
<td>4</td>
<td>1.000</td>
<td>30</td>
<td>0.971</td>
<td>0.223</td>
<td>0.223</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>5</td>
<td>0.000</td>
<td>0.080</td>
<td>0.080</td>
</tr>
<tr>
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<td>30</td>
<td>0.000</td>
<td>0.477</td>
<td>0.477</td>
</tr>
</tbody>
</table>
will be evaluated.

### 3.2.1 Components of the problem

For the analytical opto-mechanical analysis of this problem, the sensor and coating is taken to be isotropic. The elastic and optical properties of the silica fibre optic sensor are given in Table 3.8. The type of coating used for both the analysis and in the experiments (Chapter 4) is polyimide, the properties of which are given in Table 3.9. In this section, the substrate materials of the fibre are the CFRP or the epoxy, and the far-field (denoted by the superscript $\infty$) refers to quantities within these substrates. The elastic properties for these materials have already been given in Tables 3.1 and 3.4.

**Table 3.8: Properties of the linear elastic isotropic sensor used in the calculations**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Young’s modulus, $E^s$ (GPa)</td>
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<tr>
<td>Poisson’s ratio, $\nu^s$</td>
<td>0.16</td>
</tr>
<tr>
<td>Shear modulus, $G^s$ (GPa)</td>
<td>31.5</td>
</tr>
<tr>
<td>Thermal expansion coefficient, $\alpha^s$, ($\times 10^{-6}$/$^\circ$C)</td>
<td>0.5</td>
</tr>
<tr>
<td>Original value of the index of refraction, $n_0$</td>
<td>1.456</td>
</tr>
<tr>
<td>Pockel constant, $p_{11}$</td>
<td>0.17</td>
</tr>
<tr>
<td>Pockel constant, $p_{12}$</td>
<td>0.36</td>
</tr>
<tr>
<td>Thermooptic Coefficient, $dn_0/dT$ ($\times 10^{-6}$/$^\circ$C)</td>
<td>1.2</td>
</tr>
<tr>
<td>Fibre radius, $r_s$ ($\mu$m)</td>
<td>120</td>
</tr>
</tbody>
</table>

**Table 3.9: Properties of the linear elastic isotropic coating (polyimide) used in the calculations**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus, $E^c$ (GPa)</td>
<td>5</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu^c$</td>
<td>0.4</td>
</tr>
<tr>
<td>Thermal expansion coefficient, $\alpha^c$ ($\times 10^{-6}$/$^\circ$C)</td>
<td>38</td>
</tr>
<tr>
<td>Coating radius, $r_c$ ($\mu$m)</td>
<td>140</td>
</tr>
</tbody>
</table>

### 3.2.2 Bragg wavelength shift

The determination of the Bragg shift in terms of applied farfield strains is a complicated procedure. Only the results and the pertinent assumptions in obtaining the Bragg shift from far-field strains will be discussed here. For a full derivation of the anisotropic stress functions and their relation to the Bragg wavelength shift, the reader is referred to Van Steenkiste and Springer (1997), and Lekhnitskii (1963).

The average shift in the Bragg wavelength as a function of strains within the Bragg grating is given by equation 3.1, where the coefficients $K^*_i$ are given by the equations listed in Table 3.10. Shear strains in the transverse sensor plane result in birefringence of the Bragg grating, resulting in a spreading of
the Bragg wavelength into two distinct peaks for sufficient shear strains. The spreading of the Bragg wavelength is given by equation 3.2.

$$\Delta \lambda_{\text{avg}} = K'_1 e'_1 + K'_h e'_h + K'_4 e'_4 + K'_5 e'_5 + K'_6 e'_6 + K'_T \Delta T$$  \tag{3.1}$$

$$\Delta \lambda_{\text{dif}} = K'_s \gamma'_{\max}.$$  \tag{3.2}$$

Similarly, the sensor output as a function of farfield strains is given by equations 3.3 and 3.4, where the coefficients $K'_i$ are given in by the equations listed in table 3.11.

$$\Delta \lambda_{\text{avg}} = K'_1 e'_{\infty} + K'_h e'_{h} + K'_4 e'_{4} + K'_5 e'_{5} + K'_6 e'_{6} + K'_T \Delta T$$  \tag{3.3}$$

$$\Delta \lambda_{\text{dif}} = K'_s \gamma'_{\max}.$$  \tag{3.4}$$

where the coefficients $K'_i$ (Table 3.11) describe the changes in the optical output based on the farfield strains. The terms $e'_{i}^{\infty}$ represent the strains in the material surrounding the sensor expressed in the $x'_1, x'_2, x'_3$ material coordinate system, and the terms $e'_i$ represent the sensor strains expressed in the material coordinate system (not the sensor coordinate system). The absence of the apostrophe denotes the sensor coordinate system. (Conveniently, for the analysis of the materials considered in this section, both of these coordinate systems are coincident.) For a displacement field $u(x_1, x_2, x_3)$ referenced in an arbitrary coordinate system, $x_1, x_2, x_3$, the strains are defined as follows

$$e_1 = e_{11} = \frac{\partial u_1}{\partial x_1} \quad e_4 = \gamma_{23} = \frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}$$

$$e_2 = e_{33} = \frac{\partial u_2}{\partial x_2} \quad e_5 = \gamma_{13} = \frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}$$

$$e_3 = e_{33} = \frac{\partial u_3}{\partial x_3} \quad e_6 = \gamma_{12} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}.$$  \tag{3.5}$$

Additionally, the average and differential sensor strains are defined as follows

$$e'_h = \frac{e'_2 + e'_3}{2} \quad e'_d = \frac{e'_2 - e'_3}{2}.$$  \tag{3.6}$$

The average and differential farfield strains are defined similarly

$$e'_{h}^{\infty} = \frac{e'_{2}^{\infty} + e'_{3}^{\infty}}{2} \quad e'_{d}^{\infty} = \frac{e'_{2}^{\infty} - e'_{3}^{\infty}}{2}.$$  \tag{3.7}$$

For the cases considered in this chapter, the global and sensor axes are taken as coincident since the Bragg sensor and carbon fibres are aligned (i.e. the axis of symmetry for transversely isotropic CFRP is coincident with the Bragg grating) which greatly simplifies the analysis. The parameter $\chi^{*, \odot}$ required
The modified elastic constants for a transversely isotropic material are given by the following equations

$$\chi^{s,\varnothing} = \frac{1}{\rho^2}$$

$$\begin{vmatrix}
0 & -2v_{\text{mod}}^\nu c \frac{G_{\text{mod}}}{G_{\text{mod}}^\nu c} & -\frac{1}{G_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} & 2v_{\text{mod}}^\nu c & 0 & 0 \\
0 & \frac{3-2v_{\text{mod}}^\nu c}{G_{\text{mod}}^\nu c} & \frac{2v_{\text{mod}}^\nu c}{G_{\text{mod}}^\nu c} & -\frac{1}{G_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} & -2 & 0 \\
0 & 0 & 4 & 6 & 2 & 0 & 0 \\
0 & 6 & 2 & 6 & -2 & -6 & 0 \\
-\rho & 0 & -2(1-v_{\text{mod}}^\nu c)^2 & -\frac{1}{G_{\text{mod}}^\nu c} & \rho & \frac{2\rho^3 v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} \\
\rho & 0 & \frac{2v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} & -\frac{1}{G_{\text{mod}}^\nu c} & -\rho & \rho^3 (3-2v_{\text{mod}}^\nu c) & -2 \frac{v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} \\
-2G_{\text{mod}} & 0 & \frac{4}{\rho^2} & \frac{6}{\rho^2} & 2 & 0 & -\frac{4}{\rho^2} & -\frac{6}{\rho^2} \\
2G_{\text{mod}} & 0 & \frac{2}{\rho^2} & \frac{6}{\rho^2} & -2 & -6\rho^2 & -\frac{2}{\rho^2} & -\frac{6}{\rho^2}
\end{vmatrix}$$

(3.8)

where \(D\) is given by

$$\begin{vmatrix}
\frac{1}{G_{\text{mod}}^\nu c} & 2v_{\text{mod}}^\nu c & -\frac{1}{G_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} & 2v_{\text{mod}}^\nu c & 0 & 0 \\
\frac{3-2v_{\text{mod}}^\nu c}{G_{\text{mod}}^\nu c} & \frac{2v_{\text{mod}}^\nu c}{G_{\text{mod}}^\nu c} & -\frac{1}{G_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} & -2 & 0 & 0 \\
-2 & 0 & 4 & 6 & 2 & 0 & 0 \\
2 & 6 & 2 & 6 & -2 & -6 & 0 \\
-\rho & 0 & -2(1-v_{\text{mod}}^\nu c)^2 & -\frac{1}{G_{\text{mod}}^\nu c} & \rho & \frac{2\rho^3 v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} & \frac{1}{G_{\text{mod}}^\nu c} \\
\rho & 0 & \frac{2v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} & -\frac{1}{G_{\text{mod}}^\nu c} & -\rho & \rho^3 (3-2v_{\text{mod}}^\nu c) & -2 \frac{v_{\text{mod}}^\nu c}{C_{\text{mod}}^\nu c} \\
-2G_{\text{mod}} & 0 & \frac{4}{\rho^2} & \frac{6}{\rho^2} & 2 & 0 & -\frac{4}{\rho^2} & -\frac{6}{\rho^2} \\
2G_{\text{mod}} & 0 & \frac{2}{\rho^2} & \frac{6}{\rho^2} & -2 & -6\rho^2 & -\frac{2}{\rho^2} & -\frac{6}{\rho^2}
\end{vmatrix}$$

(3.9)

The modified elastic constants for a transversely isotropic material are given by the following equations

$$E_{\text{mod}} = \frac{E_{yy} + 2E_{yy}v_{zy} + \frac{E_{zz}^2}{E_{zz}}\nu_{yz}^2}{(1 + \nu_{yz})^2}$$

$$\nu_{\text{mod}} = \frac{\nu_{zy} + \frac{E_{yy}}{E_{zz}}\nu_{yz}^2}{1 + \nu_{yz}}$$

$$G_{\text{mod}} = \frac{E_{\text{mod}}}{2(1 + \nu_{\text{mod}})} = G_{yz} = \frac{E_{yy}}{2(1 + \nu_{zz})}$$

$$C_{\text{mod}} = \frac{\nu_{\text{mod}}E_{\text{mod}}}{(1 + \nu_{\text{mod}})(1 - 2\nu_{\text{mod}})}$$

(3.10)

where the absence of a superscript refers to the substrate material. Since, in this chapter, the only material considered which is transversely isotropic is the CFRP, equations 3.10 are used only for this material. For the epoxy substrate, as well as for the sensor material, the modified elastic constants are
as follows

\[ E_{\text{mod}} = E \]
\[ \nu_{\text{mod}} = \nu \]
\[ G_{\text{mod}} = \frac{E}{2(1 + \nu)} \]
\[ C_{\text{mod}} = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} = C \]  

where once again the absence of a superscript refers to the substrate material, and the addition of the \( s \) superscript refers to the corresponding elastic parameters for the sensor, and \( c \) for the coating. The parameter \( \rho \) expresses the ratio of the outer radius of the coating to the outer radius of the sensor, or,

\[ \rho = \frac{r_c}{r_s} . \]  

Also required for the computations of \( K_i^\infty \) is the constant \( \tilde{\alpha} \) defined by

\[ \tilde{\alpha} = \alpha_{yy} + \nu_{yx} \alpha_{xx} . \]  

The equations 3.3 and 3.4 were implemented in a MATLAB routine (Appendix B) in order to determine the sensor response based on arbitrary farfield strains.

**Table 3.10:** Definition of the coefficients \( K_i^s \) which define changes in optical output as a function of sensor strains

<table>
<thead>
<tr>
<th>( K_i^s )</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1^s )</td>
<td>( 1 - \frac{n_0^2}{2} p_{12} + \sin^2 \phi_0 )</td>
</tr>
<tr>
<td>( K_2^s )</td>
<td>( -\frac{n_0^2}{2} (p_{11} + p_{12}) - \sin^2 \phi_0 )</td>
</tr>
<tr>
<td>( K_3^s )</td>
<td>( -\sin^2 \phi_0 \cos(2\Omega) )</td>
</tr>
<tr>
<td>( K_4^s )</td>
<td>( -\sin^2 \phi_0 \sin \Omega \cos \Omega )</td>
</tr>
<tr>
<td>( K_5^s )</td>
<td>( -\sin \phi_0 \cos \phi_0 \sin \Omega )</td>
</tr>
<tr>
<td>( K_6^s )</td>
<td>( -\sin \phi_0 \cos \phi_0 \cos \Omega )</td>
</tr>
<tr>
<td>( K_T^s )</td>
<td>( \frac{n_0^4}{2} \left[ \frac{2}{n_0^2} \frac{dn_0}{dT} + (p_{11} + p_{12}) \alpha^s \right] )</td>
</tr>
<tr>
<td>( K_s^s )</td>
<td>( -\frac{n_0^2}{4} (p_{11} - p_{12}) )</td>
</tr>
</tbody>
</table>

The angles \( \phi_0 \) and \( \Omega \) are non-zero only when the grating planes are tilted with respect to the normal of the longitudinal sensor axis. In this analysis, the grating planes are assumed normal to the longitudinal sensor axis, therefore \( \phi_0 \) and \( \Omega \) are both zero.
Table 3.11: Definition of the constants $K_i$ which define changes in optical output as a function of farfield strains

\[
K'_1 = K'_s + \frac{2C^e K'_k}{\nu \rho} \left[ \frac{G_{mod} \rho^2 (1-\nu^2) \nu_y - (G_{mod}-G^c)(1-\rho^2)\nu_y - \nu^e \alpha_y}{\nu_y} \right] + \frac{\nu}{\nu_y} \left[ \frac{2(G^e - G_{mod}) + \rho^2 (2G_{mod} + C^c)}{2G^e + C^c} \right]
\]

\[
K'_h = \frac{4C_{mod} C^c \rho^2 (1-\nu_{mod}) (1-\nu^e) K'_k}{\nu \rho_y} \left[ \frac{C_{mod} - C^c}{\nu_y} + \rho^2 \left( 2G_{mod} + C^c \right) \left( 2G^e + C^c \right) \right]
\]

\[
K'_d = -K'_s \frac{\chi^{t\cdot\circ}}{G^e}
\]

\[
K'_4 = -K'_s \frac{\chi^{t\cdot\circ}}{G^e}
\]

\[
K'_5 = \frac{4G_{xy} G^e \rho^2 K'_s}{(G_{xy}-G^c)(G^e-G^c) + (G_{xy}+G^c)(G^e+G^c) \rho^2}
\]

\[
K'_6 = \frac{4G_{xy} G^e \rho^2 K'_s}{(G_{xy}-G^c)(G^e-G^c) + (G_{xy}+G^c)(G^e+G^c) \rho^2}
\]

\[
K'_T = K'_s + \frac{2C^e K'_k}{\nu \rho} \left[ \frac{(G_{mod}-G^c)(1-\rho^2)(1+\nu^e) \nu_y - C_{mod} \rho^2 (1-\nu^e) \nu_y + \nu^e \alpha_y}{\nu_y} \right] + \frac{\nu}{\nu_y} \left[ \frac{2(G^e - G_{mod}) + \rho^2 (2G_{mod} + C^c)}{2G^e + C^c} \right] \left( 1+\nu^e \right) \alpha_y
\]

\[
K'_s = -K'_s \frac{\chi^{t\cdot\circ}}{G^e}
\]
3.2.3 Results of the Mechanical–Optical Analysis

Using the methodology presented above, the farfield substrate strains computed in the finite element analysis were used to compute sensor drift as a function of lateral confinement ($\sigma_{rr,grout}^\infty$) applied on the grout using equation 3.1. The analytical method presented in Section 3.2.2 was implemented in MATLAB and the finite element results were used as input. The resulting output is displayed in Figures 3.5 and 3.6.

![Graph showing sensor drift as a function of confining stress](image)

**Figure 3.5:** Sensor drift as a function of transverse confinement applied to the grout. The original sensor wavelength is 1500 nm and the farfield imposed strain is $\varepsilon_{yy,grout}^\infty = 0$.

As the results indicate, sensor drift varies as a linear function of confining stress. The effects of confining stress are very pronounced on the CFRP casing system, whereas they are not as pronounced on the steel casing designs. The stainless steel design number 1 (thin tube) more accurately records the far-field axial strain, however it is slightly more sensitive to transverse strain than design number 2 (thick tube). As a result, a design closely corresponding to steel design number 1 was selected for fabrication.

3.3 Summary

In this chapter, several sensor casing configurations were investigated using a coupled numerical–analytical approach. The ability of the casing to adequately transfer strains from a grout to the internal casing substrate material was assessed numerically. Using the numerical results, the substrate strains were used to determine the optical response of the sensor which constitutes a small elastic inclusion within the substrate. The effect of transverse strains was characterized and was found to be minimal for the steel tube and flange design, but relatively large for the CFRP design.
Figure 3.6: Sensor drift as a function of transverse confinement applied to the grout. The original sensor wavelength is 1500 nm and the farfield imposed strain is $\varepsilon_{yy,grout} = -1000 \, \mu\varepsilon$. The ideal sensor indicates one that measures the farfield grout strains perfectly and is not sensitive to transverse strain.
Chapter 4

Experimentation

In order to investigate and validate the response of the FBG based sensor to measure strain in rock, a series of laboratory experiments was conducted. First, a suitable cement-based grout was developed and tested. Secondly, a set of two flanged, stainless steel tube sensor casings was fabricated based on the design developed in Chapter 3. FBG strain sensors were embedded in the sensor casings using an epoxy resin. The first of these gratings was embedded in a cylindrical grout specimen and subjected to uniaxial loading. Two linear variation differential transformers (LVDTs) were used to measure the displacement of the specimen and obtain an independent measurement of strain. The second casing was embedded in a Laurentian granite cylindrical specimen with a hollow core using the developed grout. This specimen was subjected to a similar set of uniaxial compressive loadings and optically derived strains were compared to independent strain measurements obtained by the LVDTs.

The objectives of the experimentation were manifold:

1. An analysis of the relationship between the applied strain in the grout and the measured strain in the optical FBG strain sensor.
2. An analysis of the relationship between the applied strain in the rock and measured strain in the optical FBG strain sensor.
3. Analysis of the resolution and noise characteristics of the optical measurement system.
4. Determination of the minimum and maximum quantity of strain (dynamic range) that can be detected by the optical sensing system, and the factors determining this range.
5. An quantitative analysis of the accuracy of the optical FBG sensor.

4.1 Grout Design and Testing

The function of the grout is to couple the optical strain sensor to the rock mass surrounding it. It is therefore the mechanical element that transfers strains from the rock mass to the sensor. Changes in strain oriented along the longitudinal borehole axis result from a traction acting from the rough rock surface onto the grout. Sufficient contact is therefore necessary between the grout and the rock mass to carry the axial strains from the rock mass to the sensor. In order to achieve the contact condition, volumetric stability of the grout during the curing period must be ensured. In addition to volumetric
Table 4.1: Two alternative grout mix designs

<table>
<thead>
<tr>
<th>Content</th>
<th>Design 1</th>
<th>Design 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Use (GU) Cement</td>
<td>1000 g</td>
<td>920 g</td>
</tr>
<tr>
<td>Fly Ash</td>
<td>0 g</td>
<td>80 g</td>
</tr>
<tr>
<td>Water</td>
<td>280 g</td>
<td>280 g</td>
</tr>
<tr>
<td>Gellenium 7700\textsuperscript{a}</td>
<td>10 mL</td>
<td>10 mL</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Polycarboxylate based super-plasticizer

stability, elastic linearity and high strength are desirable properties since a change in grout stiffness or the onset of cracking in the vicinity of the sensor will result in a nonlinear response of the optical sensor. The final consideration is the mobility of the grout. A low-viscosity grout is desired, since, ultimately, it will be injected into small diameter, long boreholes.

Grouts with similar properties to those mentioned above have been developed in the structural concrete post-tensioning industry for bonding of post-tensioning strands to the surrounding structural concrete (Ganz and Vildaer, 2002). Two mix designs based on those suggested by Ganz and Vildaer (2002) were developed (Table 4.1).

The two grout designs were mixed according to ASTM standard C305 (ASTM, 2012b) and cast into 50 mm cubes. The cubes were left to cure at room temperature in a humid environment for 7 and 28 days at which time uniaxial compression tests were conducted according to ASTM standard C109 (ASTM, 2012a). The strengths recorded are given in Table 4.2. One sample was used for each test.

Table 4.2: Grout strengths per ASTM standard C109 (single sample per test)

<table>
<thead>
<tr>
<th>Design</th>
<th>7-day strength</th>
<th>28-day strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design 1 (GU)</td>
<td>78.4 MPa</td>
<td>95.7 MPa</td>
</tr>
<tr>
<td>Design 2 (Fly ash)</td>
<td>71.7 MPa</td>
<td>106 MPa</td>
</tr>
</tbody>
</table>

Shrinkage tests were not conducted for these mix designs since the water to cement ratio of 0.28 is sufficient to preclude significant shrinkage (Ganz and Vildaer, 2002). The elastic modulus was determined from similar grout mixes in the sensor tests described in Section 4.4 on the full sized grout sample.

4.2 Design and Fabrication of the Sensor Casings

The sensor casing design is based on Design 1 of Section 3.1.1, which consists of a stainless steel tube with two flanges originally proposed by Fenando et al. (2003). The optical sensor is bonded to the interior of the casing with an epoxy resin which fills the entire tube cavity. The as-built drawing of the sensor casing is given in Figure 4.1. Due to the difficulty in machining a monolithic stainless steel casing, the flanges were machined separately and bonded to the stainless steel stock tubing using a silver solder joint. This was a significant deviation from the originally intended design since the solder joints are not as stiff nor as strong as a monolithic steel casing.

Both fibre Bragg gratings were written using the phase mask method with a sinc power profile to produce a sinc-like appodisation of the refractive index modulation. The appodisation suppresses the
Figure 4.1: A longitudinal section of the FBG sensor casing in the as-built configuration. All dimensions are in millimeters unless otherwise specified.
Table 4.3: Properties of the FBG sensors

<table>
<thead>
<tr>
<th>Property</th>
<th>FBG 1</th>
<th>FBG 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center wavelength, $\lambda_c$</td>
<td>1550.220 nm</td>
<td>1550.380 nm</td>
</tr>
<tr>
<td>Spectral width, $\Delta\lambda$, at -3 dB</td>
<td>0.590 nm</td>
<td>0.640 nm</td>
</tr>
<tr>
<td>Reflectivity</td>
<td>&gt; 90 %</td>
<td>&gt; 90 %</td>
</tr>
<tr>
<td>Grating length</td>
<td>5 mm</td>
<td>5 mm</td>
</tr>
<tr>
<td>Strain gauge factor</td>
<td>1.2 pm/$\mu\varepsilon$</td>
<td>1.2 pm/$\mu\varepsilon$</td>
</tr>
</tbody>
</table>

side lobes of the reflected FBG spectrum increasing the accuracy of the peak wavelength demodulation. After the FBGs were written, the fibres were recoated in polyimide which bonds well to epoxy for optimal strain transfer. The properties of FBG 1 and 2 are given in Table 4.3.

After fabrication of the sensor casings, two fibre Bragg gratings were bonded to the casings by injecting the casings with a low viscosity epoxy resin (EpoTek 301-1). The epoxy was left to cure at 23°C Celsius for 24 hours, as specified by the manufacturer. 900 micron loose plastic tubing was attached to the ends of the sensor casings to protect the fibers. One end of each sensor was spliced to an angle polished connector (APC) for future connection to the spectral interrogation device.

4.3 Analysis of the Optical Spectrum Analyzer Data

The optical spectrum analyzer used in the experiments (I-MON E-USB 512 by Ibsen Photonics) consists of a diffraction grating that splits the incoming beam onto a CCD array of 512 InGeAr pixels that range from approximately 1509 nm to 1595 nm, and the average pixel resolution ranges from approximately 136 to 203 pm. The pixel wavelength is calibrated by the manufacturer using a 5th order polynomial. The pixels have a minimum exposure time of 0.006 ms and a minimum reset time of 1.050 ms corresponding to a maximum measurement frequency of 947 Hz and a Nyquist frequency of 473.5 Hz.

In order to obtain subpixel resolution of the reflected light spectrum, an appropriate algorithm must be implemented in order to determine the peak reflection wavelengths. Several different algorithms have been suggested by various researchers (c.f. Section 2.5), and for the purposes of this study, the Gaussian algorithm (Section 2.5.3) and the centroid algorithm (Section 2.5.1) have been implemented in MATLAB to fit the measured spectral data from the CCD based optical interrogation unit (Appendix A). The wavelength data acquired from these two algorithms based on a rapid scan (2000 spectra at 20 Hz) of the IMON unit is plotted in Figure 4.2. Figure 4.3 shows the histograms of both the centroid and Gaussian algorithms, both of which contain a Gauss-shaped distribution. It should be noted that these algorithms differ in their wavelength reading by an average of 1.79 pm. This result is consistent with those obtained by Bodendorfer et al. (2009) and can be explained by asymmetry in the reflection spectrum of the FBG as a result of minor birefringence in the grating writing process (or in the case of these gratings embedded in epoxy; uneven curing of the epoxy resin). From the histograms, it can be seen that the precision of any given measurement is on the order of ~1 pm. However, if temperature and strain variation periods in the sensor are much greater than the measurement period, the wavelength resolution can be improved by a factor of $\sqrt{N}$, where $N$ is the number of measurements, since the noise is random and Gaussian (Vaseghi, 2008), which can greatly improve the strain resolution.
Figure 4.2: Deviation of computed wavelengths by the Gauss and Centroid algorithms, $\mu_{Gauss} = 1550.236766$ nm and $\mu_{Centroid} = 1550.238556$ nm

Figure 4.3: Comparison of histograms and Gaussian distributions for the Gaussian and centroid peak detection algorithms with their respective standard deviations ($\sigma$)
Figure 4.4: A periodogram showing the power spectral density of the stationary sensor. The data was processed using the IMON built-in Gaussian demodulation algorithm and collected at 952.4 Hz with an exposure time of 0.006 milliseconds with 200,000 samples. A constant noise spectral density at all wavelengths indicates a white noise process.

Figure 4.5: Autocorrelation function of the stationary sensor for 200,000 samples.
Figure 4.4 shows the power spectral density for the wavelength data of the Gaussian algorithm. This diagram shows a white noise profile (equal power) at frequencies up to the Nyquist frequency. Figure 4.5 shows the autocorrelation function of the stationary process, indicating an uncorrelated random process. This is an important feature when the isolation of transient, periodic signals is desired from the sensor time series by power spectrum analysis. For instance, an auto-regressive-moving-average (ARMA) model (Vaseghi, 2008) can be applied to such systems to increase the resolution and decrease the variance of the power series and isolate transient signals.

4.4 Uniaxial Compressive Tests on Grout and Rock Samples

In order to assess the accuracy of the sensors, instrumented uniaxial compressive tests were conducted. Two specimen were constructed; the first specimen consisted of an FBG sensor (FBG sensor 1) embedded in a grout cylinder (grout design 1), the second specimen consisted of an FBG sensor (FBG sensor 2) embedded in a Laurentian granite sample.

4.4.1 Sample preparation

The grout sample was constructed by fixing the sensor casing in the center of a plastic cylindrical mold (4-inch diameter by 8-inches long) with thin steel wire. A hole was cut into the grout cylindrical mold half an inch below the cylinder face so the fibre could exit the specimen. The mold interior was coated in a thin layer of oil based lubricant and filled with grout (mix design 1, Section 4.1, Table 4.1), and left to cure for one day in a humid environment at room temperature. After 24 hours, the sample was demolded and left to cure for an additional six days (for a total of 7 days) in a humid environment at room temperature. After seven days, the sample ends were finished using a diamond tipped end polisher. A drawing of the sample is provided in Figure 4.6.

The granite sample was formed using a 4-inch diameter diamond tipped rotary drill and a second concentric bore was made with a 1-inch diameter diamond tipped rotary drill. A final, 1-cm drill hole was made radially and the sample was cut to a nominal 8.5-inch (unpolished) length, one end of which corresponded to the 1-cm bore axis in order to allow one fibre end to exit the sample. The sample ends were finished using a diamond tipped end polisher. The sample was immersed in water for 12 hours before being grouted. The sensor was held in place using thin steel wire from each end of the specimen and the interior bore was filled with grout (mix design 1, Section 4.1, Table 4.1). The two ends of the grout were terminated 2-mm before the rock surface at the bottom, and 5-mm from the rock surface at the top. This was done in order to ensure the loading was applied directly to the rock and that strain was transferred via shear action at the interface of the rock and grout (as would be the case in an actual borehole). The sample was left to cure at seven days in a humid environment at room temperature. A drawing of the sample is provided in Figure 4.7.

4.4.2 Experimental procedure and results

The tests were conducted in a displacement controlled MTS loading frame. Two 6-inch LVDTs were mounted using a brace to the sides of both the grout and the rock specimen. The machine displacement was set at one micron per second in both loading and unloading of the specimen. Load was measured by a load and a pressure cell within the MTS frame. A maximum load was specified for each test,
Figure 4.6: Longitudinal section of the cylindrical grout sample. All dimensions in mm.

Figure 4.7: Longitudinal section of the cylindrical rock sample. All dimensions in mm.
and each specimen was subjected to a series of loading cycles with increasing maximum load for each test. The MTS data acquisition speed was set at 2 Hz and the optical interrogation unit was set to a data acquisition speed of 1 Hz. The interrogator was set to a pixel exposure time of 0.006 ms, and recorded the full spectrum for each time interval. The spectral information was post-processed using both the Gaussian and the centroid algorithms described in Section 2.5 using 11 consecutive pixels centered around the pixel of maximum counts. The optical strain data was then compared to the axial strains derived by the LVDT average. The results furnished by the series of tests performed on both specimens are presented in Figures 4.8, 4.9, and 4.10.

4.4.3 Experimental analysis: grout testing

From Figure 4.8, it can be seen that the strains derived from the LVDT and the FBG gauge are nearly equal for the grout specimen. In order to examine the difference between the measurements in higher resolution, Figure 4.11 examines the measured difference for the loading portions of tests G1, G2, and G3. Unloading portions were omitted, since it was observed that the LVDT did not register consistent strains with respect to load and optical strain for this portion, likely due to a small degree of slip during the load halting and reversal.

The maximum absolute error, the average absolute error, and the average relative error are given in Table 4.4 for all tests on the grout specimen, where the LVDT average strain is assumed to be the true reference strain.

| Test Index | $\max(|E_{abs}|)$ | mean($|E_{abs}|$) | mean($|E_{rel}|$) |
|------------|-----------------|-----------------|----------------|
| G1         | 29.18 $\mu\varepsilon$ | 16.04 $\mu\varepsilon$ | 4.19 % |
| G2         | 30.01 $\mu\varepsilon$ | 19.38 $\mu\varepsilon$ | 3.41 % |
| G3         | 23.57 $\mu\varepsilon$ | 12.88 $\mu\varepsilon$ | 2.51 % |

From Table 4.4 and Figure 4.11, it can be seen that the absolute accuracy of the optical strain gauge is approximately $\pm 30 \mu\varepsilon$ over the entire range of loading. Furthermore, the maximum range of the sensor in compression (0 to $-1925 \mu\varepsilon$) in the grout test was limited by the strength of the grout, as seen by the sudden fracture and loss of load bearing capacity of the specimen in test G3. This total range can likely be improved by longer curing of the grout, and a stronger mix design. Additionally, when the grout is embedded in stiffer rocks, the confining pressure acting on the grout by the rock as a result of dilation in compression will induce a confining pressure on the grout, increasing the strength. In the most ideal scenario, the range of the sensor would be governed by the linear range of the optical sensor itself, as opposed to the components that comprise the strain transfer mechanism (grout, steel casing, flange connection, etc.).

The premature failure of the grout at approximately 32 MPa, which is in sharp contrast to the 7-day 50 mm-cube strength of 78 MPa, may be attributed to the different sample geometries, and the presence of a large inclusion (the sensor casing) in the sensor tests. The shorter aspect ratio of the grout cubes provides a greater confining effect, thereby increasing the effective strength of the sample. As the load increases in the cube test, the constrained dilation of the sample progresses (Poisson’s effect) which produces a confinement that slows the progression and coalescence of fractures. In the sensor tests,
Figure 4.8: Results of the uniaxial compression experiments on the grout specimen (tested at 18-day strength)
Figure 4.9: Results of the uniaxial compression experiments on the Laurentian granite specimen
Test R4: Strain Histories

Test R5: Strain Histories

Figure 4.10: Results of the uniaxial compression high–strain experiments on the Laurentian granite specimen. Test R5 uses a corrected machine displacement reading since the LVDT were removed for protection in case the sample failed catastrophically.
where both a larger sample and elastic inclusion are present, the confining effect is not as dominant. Furthermore, the presence of the steel sensor casing provides an ideal location for the nucleation of a fracture.

### 4.4.4 Experimental analysis: rock testing

The output of the Bragg grating strain sensor embedded in the rock specimen is governed by a more complex behaviour than the all-grout sample. Figure 4.12 examines the difference between the optical sensor and LVDT derived strain as a function of LVDT derived strain (which is assumed to be the true, reference strain).

From Figure 4.12 as well as from Figures 4.9 and 4.10, it can be seen that the response of the sensor can be divided into 3 separate categories:

1. An initial linear-to-nonlinear portion from approximately 0-400 $\mu \varepsilon$ for all tests R1 through R5. In this phase, the rock and grout dilate due to the Poisson’s effect and this begins to produce a confining effect on the rock mass. An increase in normal stress across the material interface (rock-grout) is accompanied by an increase in the shear stress.

2. After the initial linear-to-nonlinear phase, a linear portion is observed from approximately 400-1050 $\mu \varepsilon$. Relative displacement is still occurring between the rock-grout interface since the difference between both sensor readings is still increasing during this portion.

3. After 1050 $\mu \varepsilon$, loss of interfacial strength with increasing load can be inferred from the sensor responses. The strain measured by the LVDT continues to increase with increasing load, however
Figure 4.12: Difference between the optical and LVDT derived strain as a function of LVDT strain for tests R1, R2, and R3 (Rock tests).

The optical sensor, embedded in the grout, registers progressively decreasing strains. Upon release of the load, elastic rebound of the rock sample results in a tensile force being transferred to the grout and the fibre sensor (Figure 4.9, Test R5).

The reason for the difference between phases 1 and 2 in the optical sensor response can likely be attributed to an evolving contact condition between the two materials. As the rock dilates during phase 1, the effective area that comes into contact between the two materials increases. As this process slows, a more simple linear friction relation between the two materials during phase 2 dominates the behaviour until failure. The reduction of strain in the grout during phase 3 is almost certainly due to failure of the interface and not the grout specifically, since the strain at failure is far lower than those observed in the uniaxial compressive tests on the grout sample alone.

The observed behaviour of the optical sensor in the uniaxial compressive tests on rock are not ideal. For a practical, field-deployable sensing system, perfect axial strain transfer (1:1) should be achieved between the rock mass and the optical sensor. In the test described here, the inner core was taken with a diamond-tipped drill bit, producing very smooth walls. In practice, percussion drilling, which produces rougher bore walls, is ideal since this will assist in the strain transfer mechanism at the interface. Additionally, expansive agents can be added to the grout mix to improve contact between the rock and the cured grout. However, expansive agents should be used with caution as they can adversely affect the strength and stiffness of cured grouts.
4.5 Summary

In this section, a series of experiments was presented demonstrating the response of the optical sensor. It was found that the sensor output using the I-MON interrogation system with LED broadband illumination produced a random, white, Gaussian noise process resulting in a wavelength resolution of less than 1 pm or 1.2 $\mu\varepsilon$. A test on a grout sample with an embedded strain gauge showed favourable accuracy ($\pm 30 \mu\varepsilon$) with respect to a reference reading over a range of 1925 $\mu\varepsilon$ in compression. In several tests on a Laurentian granite sample with an embedded optical sensor, it was found that the interface between the rock and grout limited the accurate acquisition of strains, and ultimately limited the total sensing range.
Chapter 5

Conclusion

The investigations contained in the present work have attempted to highlight some of the main features of fibre Bragg gratings and their possible application to quasi-distributed broadband strain sensing in rock mechanics, which poses a particular set of challenges for successful adaptation of the technology.

5.1 Summary

The literature review summarized the key concepts of FBG strain sensing and some of the main interrogation methods currently available for FBG demodulation with a particular focus on CCD demodulation since this was the technique of choice in the current study. Aliasing suppression and long-term measurement stability were found to be the greatest drawbacks of the CCD method. More generally, FBG gratings were found to be susceptible to drift caused by transverse strain.

Using a finite element analysis followed by an analytical approach to model possible sensor systems, the transverse strain sensitivity and the accuracy of various sensor systems was assessed. It was found that a flanged stainless steel tube filled with epoxy resin to encase the FBG sensor was accurate and could limit the effects caused by transverse stresses.

An experimental validation of the sensor system followed using an embedded FBG sensor within a grout sample, and an FBG sensor embedded in a Laurentian granite specimen. Testing on the grout specimen revealed strain resolutions of less than one $\mu\varepsilon$, and accuracy of $\pm$ 30 $\mu\varepsilon$ over a total range of nearly 2000 $\mu\varepsilon$ in compression. Tests on the rock specimen were not as successful due to the nature of the contact between the grout and rock.

5.2 Future work

A significant amount of research remains to be conducted before a reliable sensing system based on Bragg gratings can be deployed in hard rock environments for the acquisition of broadband strains. During the course of the present work, many possible improvements became apparent which could form the basis for later development. This section will attempt to summarize some of these possible improvements.

- An experimental investigation into the effects of transverse strain on sensor drift, and additional mitigation measures. These measures could include strain relief features within the epoxy that
houses the sensing fibre such as removing a small ring of epoxy surrounding the sensor to inhibit transverse strain effects within the steel casing.

- Additional investigation into the compliance between the rock, grout, and FBG sensor. Investigations into the borehole wall roughness, grout properties, and the relation between these two constituents is required for better understanding and confidence in the accuracy and the maximum range of the sensor.

- An experimental investigation into the detection and acquisition of seismically simulated strains. The use of piezoelectric transducers and a specially designed loading frame could be constructed to simulate seismic strains applied to grout and rock samples with an embedded FBG gauge. The amplitude and frequency limitations of both the optical gauge and the demodulation technique being used could possibly be assessed experimentally in this manner.

- Ruggedisation of the sensors into a field deployable unit. This would include modifying the casing ends (e.g. threading) for simple fibre splicing and connection to a rugged sensing cable for borehole deployment with colinear FBG temperature sensors to correct long-period strain readings for temperature drift.

- Design of a field deployable interrogation package. This would include both field deployable hardware and software to manage one or multiple sensing arms placed in boreholes at a mine site or other locations of interest. The hardware would include a data acquisition unit, a single or several interrogators, a broadband illumination source, and possibly optical switches for monitoring of several sensing arms with a single interrogation unit. An ideal software package would process and manage data in real time with little or no supervision. The software could include real-time data processing tools such as transient signal detection and power spectrum analysis, aliasing detection, and alarm thresholds for safety

Indeed, the various techniques used to demodulate FBG spectra are constantly improving in terms of their accuracy, bandwidth, resolution, and long term stability. Commercially available CCD demodulation equipment for FBG sensors with measurement rates approaching 30 kHz have recently become available. As the bandwidth and resolution of these devices surpasses the relevant bandwidths of interest to engineers and geoscientists, parallel demodulation of many FBG gratings without aliasing could become a possibility. As increasingly sophisticated technology becomes available, older demodulation equipment could be replaced without changing the physical sensors installed in the ground, which is a unique feature of the FBG sensing system. Furthermore, integrated sensing networks incorporating three-axis accelerometers, temperature sensors, strain sensors, and pressure sensors all functioning on FBG technology are not difficult to envision.
Appendix A

Peak Detection Implementation

This appendix contains the MATLAB code for the centroid and Guassian peak detection algorithms.

function [cent_trace,gauss_trace] = peak_detection(wavelength,power,thresh,noise_amp)

% A function that will return the peak traces for multiple FBG gratings
% over an array of 512 pixels using the centroid and gaussian fit
% algorithms
%
% INPUT:
% wavelength: A vector corresponding to the central wavelength of each of
% the 512 pixels
% power: A 2 dimensional array (i,j) corresponding to the counts
% on each pixel (i) at any timestep (j)
% thresh: Threshold, or the minimum number of counts on a given pixel
% such that the algorithm will track the peak over time
% noise_amp: The average number of counts on a pixel without being
% exposed to any light.
%
% OUTPUT:
% cent_trace: A 2 dimensional array (i,j), where peak i is computed for
% timestep j using the centroid algorithm
% gauss_trace: A 2 dimensional array (i,j), where peak i is computed for
% timestep j using the Gauss fit algorithm

%% Initialization

% Create time indexing array
length_temp = length(power(1,:));
time_vector = 1:length_temp;
% normalize power
power = power - noise_amp;

%% Peak Detection and Max Trace

no_peak = 0;

% Determine number of peaks from sweep of first timestep
i = 1;
while i <= 512
    if power(i,1) > thresh
        no_peak = no_peak+1;
        pix_peak(no_peak) = i;
        i = i+6;
    else
        i = i+1;
    end
end

% initialize max trace index array
max_trace_ind = zeros(no_peak, length_temp);

% compute max trace index array (i.e. pixel referenced)
i = 1;
for j = 1:length_temp
    peak_ind = 0;
i = 1;
    while i <= 512
        if power(i,j) > thresh
            peak_ind = peak_ind+1;
            [val,sub_ind] = max(power(i:i+6,j));
            max_trace_ind(peak_ind,j) = i+sub_ind-1;
            i = i+6;
        else
            i = i+1;
        end
    end
end

% compute max trace array (i.e. wavelength referenced)
max_trace = zeros(no_peak, length_temp);
for i = 1:no_peak
    for j = 1:length_temp
        max_trace(i,j) = wavelength(max_trace_ind(i,j));
    end
end

%% Centroid Algorithm
% number of pixels on each side of center included in the centroid calc.
N_h_centroid = 6;
% initialize centroid trace array
cent_trace = zeros(no_peak, length_temp);
% compute centroid trace array
for i = 1:no_peak
    for j = 1:length_temp
        numerator = 0;
        denominator = 0;
        for k = (max_trace_ind(i,j)-N_h_centroid) : (max_trace_ind(i,j)+N_h_centroid)
            numerator = numerator + wavelength(k)*power(k,j);
            denominator = denominator + power(k,j);
        end
        cent_trace(i,j) = numerator/denominator;
    end
end

%% Gauss Fit Algorithm
% number of pixels on each side of center included in the gauss fit calc.
N_h_gauss = 5;
% number of terms in the guass function
N_ord_gauss = 1;
% initialize gauss trace array
gauss_trace = zeros(no_peak, length_temp);

for i = 1:no_peak
    for j = 1:length_temp
        x_g = zeros(N_h_gauss*2+1,1);
        y_g = zeros(N_h_gauss*2+1,1);
        index = 1;
        for k = (max_trace_ind(i,j)-N_h_gauss) : (max_trace_ind(i,j)+N_h_gauss)
            x_g(index) = wavelength(k);
            y_g(index) = power(k,j);
            index = index + 1;
        end
        gauss_trace(i,j) = polyfit(x_g, y_g, N_ord_gauss);
    end
end
Appendix A. Peak Detection Implementation

```matlab
x_g(index,1) = wavelength(k);
y_g(index,1) = power(k,j);
index = index+1;
end

[FITTEDMODEL] = fit(x_g,y_g,['gauss',num2str(N_ord_gauss)]);
coef = coeffvalues(FITTEDMODEL);

a_gauss = coef(1,1);
b_gauss = coef(1,2);
c_gauss = coef(1,3);

gauss_trace(i,j) = b_gauss;
end
end

%% Plotting

% Plot the first spectrum obtained by the CCD spectrometer
figure(1)
plot(wavelength,power(:,1))
grid on;

% Plot the wavelength time histories for Gauss and centroid algorithms
figure(2)
plot(time_vector,gauss_trace(1,:),time_vector,cent_trace(1,:));
grid on;
```
Appendix B

Optomechanical Analysis

This appendix contains the implementation of Van Steenkiste and Springer (1997) optomechanical equations in MATLAB to compute the optical output of a Bragg grating embedded in a transversely isotropic material with respect to changes in far field strains and temperature. The elastic constants of the sensor, sensor coating, and substrate are defined internally (within) the function, and are not passed to the function as parameters.

function [Rlamda_avg, Rlamda_dif] = coated_Bragg_Sens_Output(e_inf_1,e_inf_h,e_inf_d,...
  e_inf_4,e_inf_5,e_inf_6,gamma_inf_max,dT)

% The following function is based on Steinkiste and Springer, chapter 24.
% Sensor aligned with the fibers (material and sensor axes coincident)

% INPUT:
% e_inf_1: axial strain
% e_inf_h: average strain (orth. axes, 2,3)
% e_inf_d: differential strain (orth. axes)
% e_inf_4: gamma_2,3
% e_inf_5: gamma_1,3
% e_inf_6: gamma_1,2

% OUTPUT:
% Rlamda_avg: (delta.lamda_avg)/lamda_B
% Rlamda_dif: (delta.lamda_dif)/lamda_B

%------------------------------------------------------------------------
% Cell 1: Definition of relevent constants
%------------------------------------------------------------------------
% definition of the pockels constants - Typical sensor

62
\[
p_{11} = 0.17; \\
p_{12} = 0.36; \\
p_{13} = p_{12}; \quad p_{23} = p_{12}; \\
p_{22} = p_{11}; \quad p_{33} = p_{11}; \\
p_{44} = (p_{11} - p_{12})/2; \\
\]

%%%--------------------------------------------------------------------------
%%% Elastic constants of the sensor -----------------------------------------

\[
E_s = 73.1e9; \\
ns = 0.16; \\
Gs = E_s/(2*(1+ns)); \\
Cs = (ns*E_s)/((1+ns)*(1-2*ns)); \\
alpha_s = 0.5e-6; \\
\]

%%%--------------------------------------------------------------------------
%%% Elastic constants of the coating ----------------------------------------

\[
E_c = 5e9; \\
nc = 0.4; \\
Gc = E_c/(2*(1+nc)); \\
Cc = (nc*E_c)/((1+nc)*(1-2*nc)); \\
alpha_c = 38e-6; \\
\]

%%%--------------------------------------------------------------------------
%%% Elastic properties of the transversely isotropic substrate ---------------

\[
E_{xx} = 130e9; \\
E_{yy} = 9.65e9; \\
G_{xy} = 5.58e9; \\
G_{xz} = G_{xy}; \\
G_{yz} = 3.21e9; \\
nyx = 0.29; \\
nzx = nyx; \\
nzy = 0.5; \\
alpha_{xx} = 0.022e-6; \\
alpha_{yy} = 22.5e-6; \\
alpha_{zz} = alpha_{yy}; \\
\]

%%%--------------------------------------------------------------------------
%%% Ratio of the coating to sensor diameters
dia_c = 280;
dia_s = 240;
p = dia_c/dia_s;

% modified thermal expansion coefficient

alpha_bar = alpha_yy + nyx*alpha_xx;

% Computation of G_mod, C_mod, n_mod

E_mod = (Eyy + 2*Eyy*nyz + (Eyy^2/Exx)*nyx^2)/(1+nyz)^2;
n_mod = (nyz + Eyy/Exx*nyx^2)/(1+nyz);
G_mod = E_mod/(2*(1+n_mod));
C_mod = n_mod*E_mod/((1+n_mod)*(1-2*n_mod));

% computation of D_det

D = zeros(8,8);

D(1,1) = -1 / Gs;
D(1,2) = -2 * ns / Gs;
D(1,3) = -2 * (1-nc) / Gc;
D(1,4) = -1 / Gc;
D(1,5) = 1 / Gc;
D(1,6) = 2 * nc / Gc;

D(2,1) = 1 / Gs;
D(2,2) = (3-2*ns) / Gs;
D(2,3) = 2 * nc / Gc;
D(2,4) = -1 / Gc;
D(2,5) = -1 / Gc;
D(2,6) = -(3-2*nc) / Gc;

D(3,1) = -2;
D(3,3) = 4;
D(3,4) = 6;
D(3,5) = 2;

D(4,1) = 2;
D(4,2) = 6;
\[ D(4,3) = 2; \]
\[ D(4,4) = 6; \]
\[ D(4,5) = -2; \]
\[ D(4,6) = -6; \]
\[ D(5,3) = -2 * (1-nc) / (Gc*p); \]
\[ D(5,4) = -1 / (Gc*p^3); \]
\[ D(5,5) = p / Gc; \]
\[ D(5,6) = 2 * (p-3) * nc/Gc; \]
\[ D(5,7) = 2 * (1-n_mod) / (G_mod*p); \]
\[ D(5,8) = 1 / (G_mod*p^3); \]
\[ D(6,3) = 2 * nc / (Cc*p); \]
\[ D(6,4) = -1 / (Gc*p^3); \]
\[ D(6,5) = -p / Gc; \]
\[ D(6,6) = -p^3 * (3-2*nc) / Gc; \]
\[ D(6,7) = -2 * n_mod / (C_mod*p); \]
\[ D(6,8) = 1 / (G_mod*p^3); \]
\[ D(7,3) = 4/p^2; \]
\[ D(7,4) = 6/p^4; \]
\[ D(7,5) = 2; \]
\[ D(7,6) = -4/p^2; \]
\[ D(7,7) = -6/p^4; \]
\[ D(8,3) = 2/p^2; \]
\[ D(8,4) = 6/p^4; \]
\[ D(8,5) = -2; \]
\[ D(8,6) = -6/p^2; \]
\[ D(8,7) = -2/p^2; \]
\[ D(8,8) = -6/p^4; \]
\[ D_{\text{det}} = \text{det}(D); \]

\%--------------------------------------------------------------------------
\% Computation of Xs1 ------------------------------------------------------
\%--------------------------------------------------------------------------

\[ Xs1 = \text{zeros}(8,8); \]
\[ Xs1(1,2) = -2 * ns / Gs; \]
\[ Xs1(1,3) = -2 * (1-nc) / Gc; \]
\[ Xs1(1,4) = -1 / Gc; \]
\[ Xs1(1,5) = 1 / Gc; \]
Appendix B. Optomechanical Analysis

\[ Xs1(1,6) = 2 \times nc / Gc; \]

\[ Xs1(2,2) = (3-2*ns) / Gs; \]
\[ Xs1(2,3) = 2 \times nc / Cc; \]
\[ Xs1(2,4) = -1 / Gc; \]
\[ Xs1(2,5) = -1 / Gc; \]
\[ Xs1(2,6) = -(3-2*nc) / Gc; \]

\[ Xs1(3,3) = 4; \]
\[ Xs1(3,4) = 6; \]
\[ Xs1(3,5) = 2; \]

\[ Xs1(4,2) = 6; \]
\[ Xs1(4,3) = 2; \]
\[ Xs1(4,4) = 6; \]
\[ Xs1(4,5) = -2; \]
\[ Xs1(4,6) = -6; \]

\[ Xs1(5,1) = -p; \]
\[ Xs1(5,3) = -2 \times (1-nc) / (Gc*p); \]
\[ Xs1(5,4) = -1 / (Gc*p^3); \]
\[ Xs1(5,5) = p / Gc; \]
\[ Xs1(5,6) = 2 \times p^3 \times nc / Gc; \]
\[ Xs1(5,7) = 2 \times (1-n_mod) / (G_mod*p); \]
\[ Xs1(5,8) = 1 / (G_mod*p^3); \]

\[ Xs1(6,1) = p; \]
\[ Xs1(6,3) = 2 \times nc / (Cc*p); \]
\[ Xs1(6,4) = -1 / (Gc*p^3); \]
\[ Xs1(6,5) = -p / Gc; \]
\[ Xs1(6,6) = -p^3 \times (3-2*nc) / Gc; \]
\[ Xs1(6,7) = -2 \times n_mod / (C_mod*p); \]
\[ Xs1(6,8) = 1 / (G_mod*p^3); \]

\[ Xs1(7,1) = -2 \times G_mod; \]
\[ Xs1(7,3) = 4 / p^2; \]
\[ Xs1(7,4) = 6 / p^4; \]
\[ Xs1(7,5) = 2; \]
\[ Xs1(7,6) = 0; \]
\[ Xs1(7,7) = -4 / p^4; \]
\[ Xs1(7,8) = -6 / p^4; \]

\[ Xs1(8,1) = 2 \times G_mod; \]
Xs1(8,3) = 2 / p^2;
Xs1(8,4) = 6 / p^4;
Xs1(8,5) = -2;
Xs1(8,6) = -6 * p^2;
Xs1(8,7) = -2 / p^2;
Xs1(8,8) = -6 / p^4;

Xs1_det = (1/D_det) * det(Xs1);

% Other constants

n0 = 1.456; % original value of the index of refraction
dn0dT = 1.2e-5; % thermooptic coefficient
phi0 = 0; % grating angle (degrees)
omega = 0; % 0 for phi0=0, relevant where phi0 != 0

% Computation of the Ks_i values

Ks1 = 1 - n0^2*p12/2 + sind(phi0)^2;
Ksh = -n0^2*(p11 + p12)/2 - sind(phi0)^2;
Ksd = -sind(phi0)^2*cos(2*omega);
Ks4 = -sind(phi0)^2*sind(omega)*cosd(omega);
Ks5 = -sind(phi0)*cosd(phi0)*sind(omega);
Ks6 = -sind(phi0)*cosd(phi0)*cosd(omega);
KsT = n0^2/2*(2*dn0dT/n0^3 + (p11 + 2*p12)*alpha_s);
Kss = -n0^2/4 * (p11 - p12);

% Computation of the Kp_i values

Kp1 = Ks1 + 2*Cc*Ksh * ( C_mod*p^2*(1-nc)/n_mod*nyx - ... 
(G_mod-Gc)*(1-p^2)*nc - nc*Cs*(2*(Gc-G_mod)+ ... 
p^2*(2*G_mod+Cc/nc))/(2*Cc) ) / ( nc * ( 2 * (G_mod-Gc) ...
* (Cc/nc - Cs/ns) + p^2 * (2*G_mod+Cc/nc) * (2*Gc+Cs/ns));

Kph = ( 4 * C_mod * Cc * p^2 * (1-n_mod) * (1-nc) * Ksh ) / ...
    ( n_mod * nc * ( 2 * (G_mod-Gc) * (Cc/nc-Cs/ns) + p^2 * ...
      (2*G_mod+Cc/nc)*(2*Gc+Cs/ns) ) ) ;

Kpd = -Ksd * Xs1_det / Gs;

Kp4 = -Ks4 * Xs1_det / Gs;

Kp5 = ( 4 * Gxy * Gc * p^2 * Ks5 ) / ...
    ( (Gxy-Gc) * (Gc-Gs) + (Gxy+Gc) * (Gc+Gs) * p^2 ) ;

Kp6 = ( 4 * Gxy * Gc * p^2 * Ks6 ) / ...
    ( (Gxy-Gc) * (Gc-Gs) + (Gxy+Gc) * (Gc+Gs) * p^2 ) ;

KpT = KsT + ( 2*Cc*Ksh * ( (G_mod-Gc) * (1-p^2) * (1-nc) * ...
    alpha_c - C_mod*p^2*(1-nc)*alpha_bar/n_mod + ...
    nc*Cs*(2*(Gc-G_mod)+p^2*(2*G_mod+Cc/nc))*(1+ns)*...
    alpha_s/(2*Cc/ns) ) ) / ( nc * ( 2 * (G_mod-Gc) * ... 
      (Cc/nc - Cs/ns) + p^2 * (2*G_mod+Cc/nc) * (2*Gc+Cs/ns)));

Kps = -Kss * Xs1_det / Gs;

%-----------------------------------------------------------------------
%% Computation of Rlamda_avg, Rlamda_dif -------------------------------

Rlamda_avg = Kp1*e_inf_1 + Kph*e_inf_h + Kpd*e_inf_d + Kp4*e_inf_4 + ...
              Kp5*e_inf_5 + Kp6*e_inf_6 + KpT*dT;

Rlamda_dif = Kps*gamma_inf_max;
Bibliography


