Viscoelastic Stokes Flow around a Cylinder using Particle Image Velocimetry

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Department of Mechanical and Industrial Engineering
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Abstract

The velocity field around a cylinder in viscoelastic Stokes flow was measured using particle image velocimetry (PIV) and compared to a Newtonian flow field at comparable Reynolds numbers ($Re$). The Newtonian test fluid was polybutene, and two Boger fluids having nearly constant viscosities were used to isolate the effects of fluid elasticity. A slowly rotating annular channel was used to generate steady flow with $Re$ of order $10^{-6}$ to $10^{-3}$, and cylinders were immersed at the centre of the channel. To approximate unbounded flow, each cylinder had a diameter many times smaller than the channel width.

The Deborah number ($De$) was used to represent the magnitude of flow elasticity, and elastic effects were measured for $De > 0.6$. Elasticity was shown to decrease flow velocities relative to comparable Newtonian flow, particularly downstream of the cylinder, consistent with the drag enhancement due to elasticity reported by other authors.
Acknowledgments

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Chapter 1
Introduction

The flow around a cylinder is a fundamental flow in fluid mechanics. In its simplest form, the problem to be studied in this thesis is planar flow around a cylinder, shown schematically in Figure 1.1.

![Figure 1.1 Unbounded uniform flow approaching an infinitely long cylinder](image)

Depending on the flow velocity $U$, the cylinder diameter $d$, and the properties of the flowing fluid, many distinct problems can be identified, classified largely by the Reynolds number, which is the ratio of inertial to viscous forces in a flow:

$$Re = \frac{\rho Ud}{\eta}$$  \hspace{1cm} [1.1]

where $\rho$ is the fluid density and $\eta$ is the viscosity.

The flow around cylinders for $Re > 1$ has been exhaustively studied for Newtonian fluids, with clearly defined flow regimes and expressions for the associated drag coefficients. For non-
Newtonian fluids, however, fundamental questions remain. The focus of this study is Stokes flow, so that density is not a factor; little work has been done in this regime for the flow of viscoelastic fluids, which exhibit both fluid- and solid-like behaviour simultaneously.

In reality, recreating the flow depicted in Figure 1.1 in an experimental setting at $Re \ll 1$ is physically impossible. No flow can be truly unbounded, and no cylinder infinitely long. Creating a flow experimentally that approximates these conditions is difficult. For Stokes flow, approximating an unbounded flow is especially challenging, because walls placed thousands of diameters away from the cylinder can still have a profound effect on the flow (White 1945).

Because the geometry of the problem is so simple, results from studying flow around a cylinder can be applied to more complex flows. An example is flow around the flagellum of a swimming microorganism, which is the primary motivation for this work.

### 1.1 Motivation

Most microorganisms generate propulsion with the motion of external appendages. These appendages can take many different forms, from arrays of cilia on the cell surface which drive the propulsion of the cell through coordinated movements, to tails which rely on helical or wavy motion for propulsion. The relevant model here is periodic planar motion of a waving flagellum. A familiar example is the mammalian spermatozoon, but many other microorganisms rely on one or more flagella for propulsion.

Investigations by Taylor (1951) and Purcell (1977) have estimated that microorganisms swim at Reynolds numbers on the order of $10^6$ to $10^4$, firmly in the Stokes regime. This swimming occurs in a variety of biological fluids, often complex fluids having non-Newtonian rheology (Lauga & Powers 2009).

Unlike macro-scale swimmers such as fish and birds, the inertialess environment of microorganisms prevents the generation of propulsion by imparting momentum to the surrounding fluid. Instead, microorganisms must rely on viscous forces coupled with periodic movements (Lauga & Powers 2009). Figure 1.2 illustrates the physics of drag-based thrust for a microorganism equipped with a waving flagellum.
If a sufficiently small segment of the flagellum is considered, it can be modelled as a straight cylinder with a uniform cross-section. This cylindrical segment, having velocity components perpendicular and parallel to the long axis ($u_\perp$ and $u_\parallel$, respectively), will experience force components which oppose the motion of the cylinder ($F_\perp$ and $F_\parallel$). If the flow resistance perpendicular to the cylinder is greater than the resistance experienced longitudinally, the resultant force acting on the cylindrical segment will generate propulsion, and the net propulsive force will be the sum of the contributions of each differential tail segment. It follows that the swimming speed of a microorganism in a given fluid is determined by the balance of the net propulsive force and the drag of the microorganism body.

The simple analysis outlined above is greatly complicated by the biological environment. One complicating factor can be fluid elasticity, which arises naturally in many biological fluids such as the cervical mucus. Human sperm cells have been shown to increase the waving frequency of their flagellum and swim in straighter paths when the fluid is viscoelastic (Katz et al. 1978). This change in swimming strategy is indicative of complex flow effects resulting from fluid elasticity.

**Figure 1.2** Drag-based thrust derived from flagellum motion.
Adapted from Lauga & Powers (2009)
There have been several recent attempts to derive an analytical solution for the swimming speed of a microorganism in a viscoelastic fluid. By modelling the viscoelastic fluid as a linear Maxwell fluid, Fulford et al. (1998) found that a mammalian spermatozoon could swim more efficiently in a viscoelastic fluid than in a comparable Newtonian fluid having the same viscosity. Fu et al. (2007) performed a similar analysis using the upper convected Maxwell model, and found the opposite result, with decreased swimming speeds in the viscoelastic fluid. Lauga (2009) added another level of complexity, determining that the impact of viscoelasticity on swimming performance depends on the shape and beat frequency of the swimming stroke.

As for experiments, very little work has been performed. The only prior research studying viscoelastic Stokes flow was conducted using a cylinder confined in a channel, and all but two prior experiments (Shiang et al. 1997, François et al. 2008) used narrow channels, creating blockages which alter the flow. It was for this reason that recent research at the University of Toronto (Wang 2012, Shiau 2013) developed a new test facility for making measurements of drag on a cylinder in viscoelastic Stokes flow. Their results with a novel experimental test geometry will be summarized in section 2.2.3, and it will be seen that they warrant further investigation.
Chapter 2
Background

This thesis reports measurements of the velocity field around a cylinder in both Newtonian and viscoelastic Stokes flows. Rheological characterization of the test fluids was required in order to quantify flow elasticity. Shear flow and its application in rheological characterization is explored in this chapter. The effect of added polymer to a Newtonian solvent, and ways to study and characterize any elastic flow effects are also discussed. A survey of relevant research relating to Newtonian and viscoelastic flow around cylinders is included later in the chapter. Finally, the major research objectives of this work are outlined in section 2.3.

2.1 Concept Review

2.1.1 Shear Flow

Shear flow is flow driven by shear stresses applied to a fluid, typically resulting from the movement of solid boundaries. A fundamental concept in fluid mechanics, shear flow can be used to measure the flow properties of fluids. In the simplest two-dimensional case, known as simple shear, a fluid is confined between two parallel plates moving relative to one another. For this case, a linear velocity profile is obtained and the velocity gradient is given by:

\[ \dot{\gamma} = \frac{du(y)}{dy} \]  

2.1

\( \dot{\gamma} \), known as the shear rate, is constant in simple shear and is equal to the ratio of the upper plate velocity to the separation of the plates, \( \frac{U}{H} \).

Fluids experience shear stresses, \( \tau_{xy} \), as a result of an applied shear rate, \( \dot{\gamma}_{xy} \). These two quantities are related by the viscosity of the fluid, \( \eta \):

\[ \eta = \frac{\tau_{xy}}{\dot{\gamma}_{xy}} \]  

2.2

For Newtonian fluids, the viscosity is independent of shear rate while that of non-Newtonian fluids typically varies with shear rate. There are many classes of non-Newtonian fluids, but their
behaviour can be broadly categorized as shear-thinning, shear-thickening, or yield stress fluids. Shear-thinning fluids, whose shear viscosities decrease with increasing shear rate, are the most common type of non-Newtonian fluid. Blood, ketchup, paint and nail polish are all examples of shear-thinning fluids. Most polymer solutions are also shear-thinning, including the solutions under investigation in this study.

2.1.2 Extensional Flow

A fluid is subjected to both shear and extension as it flows around a cylinder. Significant extension is expected near the upstream and downstream stagnation points, leading to elongation of a fluid element and the development of extensional stresses near those regions (François et al. 2008). A schematic of such an elongation, along a single axis, is shown in Figure 2.1.

For the uniaxial extension along the x-axis illustrated above, and for a constant extensional rate, \( \dot{\varepsilon} \), the stress distribution can be written as (Barnes et al. 1989):

\[
\sigma_{xx} - \sigma_{xy} = \dot{\varepsilon} \eta_E (\dot{\varepsilon}) \tag{2.3}
\]

\[
\sigma_{xx} - \sigma_{zz} = \dot{\varepsilon} \eta_E (\dot{\varepsilon}) \tag{2.4}
\]
\[ \tau_{xy} = \tau_{xz} = \tau_{yz} = 0 \]  \hspace{1cm} [2.5]

where \( \eta_E \) is the extensional viscosity. \( \eta_E \) is equal to the extensional stress divided by the extensional rate, analogous to the viscosity in shear (Barnes et al. 1989). Extensional viscosity was not measured in this thesis, however a detailed characterization of the experimental test fluids used in this thesis, in extension, was conducted by Yip (2011).

### 2.1.3 Polymer Solutions

Polymer solutions are fluids consisting of long chain polymer molecules dissolved in a solvent. In addition to being shear-thinning, polymer solutions are viscoelastic, meaning they exhibit both viscous and elastic behaviour depending on the time scale of a given flow. The behaviour of polymer solutions is determined by three main parameters: the viscosity of the solvent, the length of the polymer molecules and the polymer concentration.

The concentration of a polymer solution is classified according to the amount of overlap of the randomly-coiled polymer chains. In a dilute solution, the concentration is low enough that no interaction between the polymers takes place. Some amount of polymer interaction, including the possibility of entanglement, occurs in a semi-dilute solution. In concentrated polymer solutions, a large degree of interaction between the polymer molecules yields very high elasticity. The polymer solutions in this thesis belong to a class of fluids known as Boger fluids, which are dilute polymer solutions with a nearly constant viscosity. Boger fluids can be used in experiments to isolate the effects of fluid elasticity; this aspect is discussed in more detail in section 2.1.4.

The shear-thinning behaviour of polymer solutions is due to the deformation of the polymer molecules in response to shear. Under no shear, the long polymer chains are loosely coiled and roughly spherical in shape. As the fluid begins to flow, the polymer coils become extended and align in the direction of the flow in response to the velocity gradient within the fluid (Figure 2.2).
As illustrated in Figure 2.2, shearing of a polymer molecule can lead to the development of normal stresses. Brownian motion in the fluid causes the tendency of a deformed polymer coil to return to its undeformed state; it’s this mechanism which drives elastic recovery in a polymer molecule. Because the normal stresses generated by the polymer are directly related to the polymer elasticity, measurement of the normal stresses in a flow can be used to characterize the elasticity of a polymer solution. This is commonly done using the first normal stress difference, $N_1$, which is defined as:

$$N_1 = \sigma_{xx} - \sigma_{yy}$$  \[2.6\]

Because the first normal stress difference results from shearing of the fluid, it’s useful to consider the relationship between $N_1$ and shear rate. Measurements of $N_1$ with increasing shear rate show the magnitude of $N_1$ is approximately proportional to $\dot{\gamma}_{xy}^2$, at least at low shear rates, leading to the definition of the first normal stress coefficient, $\Psi_1$:
\[ \Psi_1 = \frac{N_1}{\dot{\gamma}_{xy}^2} \]  

which is expected to be approximately constant.

Another method of characterizing the elasticity of a polymer solution is by examining its response under oscillatory shear. A fluid under a sinusoidally varying strain input with frequency \( \omega \) and small amplitude \( \gamma_0 \) will exhibit a phase-shifted sinusoidal stress response. For the following strain input:

\[ \gamma_{xy}(t) = \gamma_0 \sin(\omega t) \]  

the oscillatory stress output can be broken down into two components, one in-phase and one out-of-phase:

\[ \frac{\tau_{xy}(t)}{\gamma_0} = G'(\omega) \sin(\omega t) + G''(\omega) \cos(\omega t) \]  

where \( G' \) is termed the dynamic storage modulus, and \( G'' \) the dynamic loss modulus. These represent the in-phase and out-of-phase stress components. For a viscoelastic fluid, \( G' \) represents the elastic response of the fluid while \( G'' \) represents the viscous response of the fluid. Newtonian fluids do not exhibit any elastic behaviour, which means \( G' \) is always equal to zero; also, \( G'' = \eta \omega \), and the oscillatory stress response for every fluid shows a phase-lag of \( \frac{\pi}{2} \) radians.

The elasticity of a polymer solution can also be characterized by the fluid relaxation time, denoted as \( \lambda \). This is a measure of the time it takes for a polymer molecule to return to its unstretched state after flow, and therefore a measure of the fluid’s time to relax after an applied stress. The determination of a fluid relaxation time from rheological characterization will be discussed in detail in section 4.2.

Comparison of the fluid relaxation time to a characteristic flow time or rate allows for the creation of a dimensionless group which expresses elastic effects in a flow. Two of the most
commonly reported dimensionless groups are the Deborah number, $De$, and the Weissenberg number, $Wi$, defined as:

\[
De = \frac{\dot{\gamma}}{t_c} \tag{2.10}
\]

\[
Wi = \dot{\gamma} \cdot \lambda \tag{2.11}
\]

where $t_c$ is a characteristic flow time and $\dot{\gamma}$ is the shear rate.

In this study, the Deborah number will be used to quantify flow elasticity. At low Deborah numbers ($De < 1$) the polymers remain unstretched and the fluid is expected to behave as a Newtonian fluid. At high Deborah numbers ($De > 1$), the flow process happens too quickly for the polymers to relax. As a result, the polymers remain significantly stretched and strong elastic effects are produced. Thus, the onset of measurable elastic effects is expected when $De \approx 1$.

### 2.1.4 Boger Fluids

Boger fluids are dilute polymer solutions which exhibit pronounced elasticity while maintaining nearly constant viscosity. Made by adding a small amount of high molecular weight polymer to a sufficiently viscous solvent, Boger fluids allow for the measurement of elastic effects without the consideration of shear-thinning (James 2009). By comparing the behaviour of Boger fluids to that of a comparably viscous Newtonian fluid at the same Reynolds number, viscous and elastic effects can be separated in viscoelastic flows, the former being determined with the Newtonian fluid. Thus, any effects that result from flow elasticity can be isolated.

Unlike low-viscosity dilute solutions, the key advantage of Boger fluids is that their high elasticity is measurable at room temperatures. Also, Boger fluids can be optically clear, allowing for the use of optical measurement techniques such as laser Doppler anemometry or particle image velocimetry.

### 2.1.5 Oldroyd-B Constitutive Model

In order to determine the relaxation time of a fluid from rheological characterization, a constitutive model is needed which can adequately describe response of the fluid under steady and oscillatory shear. The Oldroyd-B constitutive model is commonly used for some
viscoelastic fluids. The prediction of a constant viscosity, as well as separate parameters for the solvent and the polymer contributions to viscosity, makes the Oldroyd-B model a suitable choice for the characterization of dilute solutions like Boger fluids.

The Oldroyd-B model idealizes a dilute solution as a suspension of non-interacting, infinitely extensible, linearly elastic dumbbells in a Newtonian solvent (Prilutski et al. 1983). The constitutive equation for the model is given by (Bird et al. 1987):

\[
\tau + \dot{\lambda}_1 \tau = \eta \left( \dot{\gamma} + \dot{\lambda}_2 \dot{\gamma} \right)
\]  

[2.12]

where \( \tau = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{pmatrix} \) is the stress tensor, \( \dot{\gamma} = \begin{pmatrix} \dot{\gamma}_{xx} & \dot{\gamma}_{xy} & \dot{\gamma}_{xz} \\ \dot{\gamma}_{xy} & \dot{\gamma}_{yy} & \dot{\gamma}_{yz} \\ \dot{\gamma}_{xz} & \dot{\gamma}_{yz} & \dot{\gamma}_{zz} \end{pmatrix} \) is the rate of strain tensor, \( \eta \) is the viscosity, \( \dot{\lambda}_1 \) is the fluid relaxation time and \( \dot{\lambda}_2 \) is the fluid retardation time. \( \tau \) and \( \dot{\gamma} \) represent the upper convected time derivatives of the stress and strain rate tensors, respectively.

Alternatively, the stress tensor in Eq. 2.12 can be separated to define the individual stress contributions of the polymer and solvent:

\[
\tau = \tau_p + \tau_s
\]  

[2.13]

where \( \tau_p \) and \( \tau_s \) are the stress contributions of the polymer and the solvent, respectively.

Substitution of Eq. 2.13 into Eq. 2.12 yields:

\[
\eta = \eta_p + \eta_s
\]  

[2.14]

where \( \eta_p \) and \( \eta_s \) are the contributions to the fluid viscosity of the polymer and the solvent, respectively.

For simple shear flow, Eq. 2.12 simplifies to the following relation (Bird et al. 1987):

\[
\sigma_{xx} - \sigma_{yy} = N_1 = 2\lambda_1 \eta_p \dot{\gamma}_{xy}^2
\]  

[2.15]
which suggests a constant first normal stress coefficient, $\Psi_1$, dependent only on the fluid relaxation time and the polymer contribution to the viscosity.

For small amplitude oscillatory shear with frequency $\omega$, the Oldroyd-B model predicts the following dynamic response:

$$
\frac{G'}{\omega} = \frac{\eta_p \lambda_1 \omega}{1 + (\lambda_1 \omega)^2}
$$

[2.16]

$$
\frac{G''}{\omega} = \eta_s + \frac{\eta_p}{1 + (\lambda_1 \omega)^2}
$$

[2.17]

The quantity $\frac{G''}{\omega}$ is known as the dynamic viscosity, and represents the solvent contribution to the fluid viscosity as $\omega \rightarrow \infty$:

$$
\lim_{\omega \rightarrow \infty} \left( \frac{G''}{\omega} \right) = \eta_s
$$

[2.18]

By Eq. 2.14 the polymer contribution to the viscosity is then the difference between the viscosity and the solvent contribution.

The prediction of Eq. 2.15 of a constant first normal stress coefficient does not agree with experimental measurements, which indicate the value of $\Psi_1$ is constant only in a small range of shear rates. Therefore, in order to experimentally measure the relaxation time of a Boger fluid using the Oldroyd-B model, the value of $\Psi_1$ is determined in the range of shear rates over which it is constant, or nearly so.

Another valid approach to determine $\lambda_1$ involves taking the low frequency limit of Eq. 2.16:

$$
\lambda_1 = \lim_{\omega \rightarrow 0} \left( \frac{G'}{\omega^2 \eta_p} \right)
$$

[2.19]
Due to measurement limitations which will be detailed in section 4.2, determination of a fluid relaxation time using Eq. 2.19 wasn’t possible in this thesis, and so values of $\lambda_1$ were determined using Eq. 2.15.

All of the parameters of the Oldroyd-B model are measurable by way of steady shear and oscillatory rheological tests. The procedures for making rheological measurements using a shear rheometer are outlined in the following section.

2.1.6 Rheometry

A rheometer is a measurement device used to characterize a fluid’s response to applied forces, and can be used to measure the parameters in the Oldroyd-B model. The most common rheometer is the shear rheometer, which measures the shear response of a fluid under various types of shearing. Several geometries can be used to house the fluid; samples can be contained between two parallel plates, a plate and a cone or concentric cylinders. In each case shear flow is driven by a spindle which generates relative motion of the test geometry. By applying a constant shear rate while simultaneously measuring the torque acting on the test geometry, the viscosity of the fluid sample can be determined. Shear rheometers can also be used to measure the oscillatory stress response of a fluid sample. All rheological measurements performed in this study were carried out using a shear rheometer with a cone and plate geometry, shown schematically in Figure 2.3 below.

![Cone and plate rheometer](image)

**Figure 2.3** Cone and plate rheometer
For the cone and plate geometry, a Peltier plate acts as a fixed base and maintains the temperature of fluid above. The fluid is bounded on top by a shallow-angled cone, with cone angles typically ranging between 0.5 and 4 degrees, and rotation of the spindle drives the shear flow. A small region at the tip of the cone is usually truncated to prevent interference between the cone and the Peltier plate.

To make measurements using a shear rheometer, a fluid sample having a predetermined volume is loaded onto the Peltier plate using a syringe. The cone is then slowly lowered onto the sample until the truncation gap is reached. To ensure that the sample is evenly distributed, the cone is then rotated several times, and any excess fluid is removed using a trimming tool.

All operation of the shear rheometer is controlled by the user with the help of additional software. For measurements made in steady shear, data are collected at multiple shear rates over the range of interest determined by the user. The advantage of the cone and plate geometry is a uniform shear rate everywhere in the fluid sample. For every data point to be collected, the spindle rotates with a constant angular velocity which, for a given cone angle, will generate the shear rate specified by the user. A transducer measures the torque applied to the spindle, which is then related to the shear stress in the fluid. The shear rheometer then returns the viscosity of the fluid by applying Eq. 2.2. Changes in the viscosity over a range of shear rates can reveal shear-thinning or shear-thickening behaviour.

In addition to measuring the torque, any normal forces generated by the fluid sample during shearing (see Figure 2.2, above) are also measured, and, after accounting for the area of the cone, the shear rheometer returns the first normal stress difference. Using Eq. 2.7, values of \( N_1 \) at a known shear rate yield the first normal stress coefficient, \( \Psi_1 \), which can be used to calculate the fluid relaxation time according to the Oldroyd-B model. Determination of the fluid relaxation time from measured values of \( N_1 \) is detailed in section 4.2

For oscillatory tests, the user specifies the range of angular frequencies over which the dynamic response of the fluid is to be measured. The shear rheometer applies a sinusoidally varying strain and measures the phase-shifted torque to determine the dynamic response of the fluid. Values of \( G' \) & \( G'' \) are returned for every angular frequency specified by the user, which for a
polymer solution can be then used to evaluate the contributions of the polymer and the solvent to the viscosity by applying Eqs. 2.18 & 2.14.

2.2 Historical Background

This section provides an overview of the most relevant research on flow around cylinders in the Stokes regime, first with Newtonian fluids and then with viscoelastic fluids. Though this thesis is concerned mainly with measurements of the velocity field around a cylinder, drag remains the most physically important parameter when discussing flow around cylinders. As a result, most of the pertinent analytical and experimental results are expressed in terms of drag.

2.2.1 Prior Newtonian Research

The problem of creeping flow past a cylinder in a viscous incompressible fluid was first considered by Stokes (1851), for whom the very low Reynolds number flows are now named. By neglecting the non-linear inertia terms of the Navier-Stokes equations Stokes was successful in deriving a solution for the creeping flow around a sphere in three-dimensions. But he found no solution to the two-dimensional problem of flow around a cylinder that could satisfy both the near and far boundary conditions simultaneously. This lack of solution is known as ‘Stokes’ paradox’.

Oseen (1910) determined that the paradox was a result of the neglected inertia terms which, while insignificant near the cylinder, become dominant in the far field even for $Re \ll 1$. He suggested an approximation which relied on the linearization of the non-linear inertia terms (Currie 2012). Lamb (1911) solved the Oseen approximation for unbounded flow around a cylinder, and gave an analytical expression for the viscous drag coefficient dependent on the Reynolds number. Though accurate in the far field, Lamb’s solution suffers from the inclusion of the linearized inertia terms close to the cylinder walls.

Kaplun (1957) and Proudman & Pearson (1957) showed separately that Stokes’ paradox could be resolved by implementing a Stokes-type solution in the near field and the Oseen approximation in the far field. Using a technique known as matched asymptotic expansions, the inner and outer solutions could be overlapped to yield accurate results. Kaplun’s expression for the viscous drag coefficient as a function of the Reynolds number is:
\[ C_\eta = \frac{F_d}{4\pi \eta UL} = \frac{1}{S} - \frac{0.87}{S^3} \tag{2.20} \]

where \( F_d \) is the drag force, \( U \) is the uniform flow velocity, \( L \) is the cylinder length, \( S = \frac{1}{2} - \gamma - \ln \left( \frac{Re}{8} \right) \) and \( \gamma = 0.57721... \) is Euler’s constant.

This solution pertains to unbounded flow, but every actual flow is necessarily bounded. For the case of a cylinder confined in a channel, shown schematically in Figure 2.4, there have been several attempts to find the drag.

\[ \frac{1}{\ln \left( \frac{H}{d} \right) - 0.97157 + 1.7244 \left( \frac{d}{H} \right)^2 - 1.7302 \left( \frac{d}{H} \right)^4} \tag{2.21} \]

valid for \( H/d \geq 2 \).
Similar expressions have been derived by other authors (Takaisi 1956, Brenner 1962, Taneda 1964), with slight variations in the intervals of validity. Using any one of these expressions allows for the comparison of bounded experimental data to the unbounded analytical predictions of Lamb (1911) and Kaplun (1957), as shown in Figure 2.5.

Figure 2.5 Comparison of Kaplun’s unbounded analytical solution with the confined flow solutions of White (1945) and Faxèn (1946)

Figure 2.5 illustrates the challenge of approximating unbounded Stokes flow due to the profound effect of channel walls on the drag. At the very low Reynolds numbers of interest in this thesis, walls which are 1000 times wider than the cylinder can lead to a doubling of the drag predicted by Kaplun’s unbounded solution.

2.2.2 Prior Viscoelastic Research

Several authors have attempted to derive analytical solutions for unbounded creeping viscoelastic flow past cylinders. Using an Oseen-type solution far from the cylinder matched numerically to a viscoelastic inner flow solution, Ultmann & Denn (1971) derived analytical expressions for the drag and velocity field for the flow of a linear Maxwell-type fluid around spheres and cylinders. Though the streamlines predicted by their solution were validated experimentally using dye-streaks, their solution has been characterized as an oversimplification
by other researchers (Mena & Caswell 1974, Carew & Townsend 1991), stemming from the exclusion of a retardation time in the linear Maxwell constitutive model. Mena & Caswell (1974) derived an analytical solution valid for $Re << 1$ and $De << 1$ using a similar procedure to that of Ultmann & Denn, but modelled their fluid as an Oldroyd-B type fluid. Their results predicted a downstream shift in streamlines and a decrease in the drag coefficient with increasing $De$ compared with Lamb’s Newtonian solution (1911). After attempting to correct for wall effects, experimental results from Broadbent & Mena (1974) and Manero & Mena (1981) confirmed, respectively, the drag reduction and streamline shift predicted analytically by Mena & Caswell.

The first numerical study of unbounded viscoelastic Stokes flow around a cylinder was performed by Pilate & Crochet (1977), who found a drag reduction caused by viscoelasticity at low Reynolds numbers. These results were confirmed by Townsend (1980), and are also in agreement with the aforementioned analytical and experimental work. Due to the limitations of computational analysis at that time, solutions were limited to $De << 1$. Attempts to obtain results at higher Deborah numbers led to numerical instabilities in the solution.

In a breakthrough paper by Chilcott & Rallison (1988), a numerical technique was implemented which enabled, for the first time, simulations of unbounded viscoelastic Stokes flow around spheres and cylinders at high Deborah numbers (up to $De = 16$). The constitutive model chosen for the fluid was the FENE dumbbell, which models the polymer molecules as non-interacting dumbbells connected via a non-linear elastic element with finite extensibility. The flow domain was an annulus with inner wall radius $r$ and outer wall radius $r_b$. The no-slip boundary condition was imposed at $r$, which represented the radius of the body, and a uniform flow was imposed at $r_b$, which was 10 to 20 times greater than $r$. These boundary conditions restricted the results to that of an approximate inner solution, rather than a full flow solution. The area in between the two radii was meshed, and a mesh refinement was added in a sector of the annular domain downstream of the body.

For $De \to 0$, Chilcott & Rallison predict a drag which matches the Newtonian drag value, indicating negligible elastic effects. Contrary to earlier numerical work which was restricted to very low Deborah numbers, their results show a drag enhancement with increasing Deborah number, with the onset of elastic effects occurring around $De \approx 1$. Elastic onset for $De$ near unity
is appropriate, since polymers will not be significantly deformed for Deborah numbers less than one. Velocity profiles taken along the axis aligned with the direction of the flow and passing through the middle of the cylinder show asymmetry between the upstream and downstream velocity profiles at $De = 10$, with decreased flow velocities in the downstream wake region, consistent with the reported drag increase.

Though their results represented a breakthrough in computational rheology at the time, Chilcott & Rallison were limited by the computational capabilities of their day, leading to a relatively coarse mesh resolution and the imposition of the uniform flow boundary condition so close to the cylindrical or spherical body. Assessing the impact of these simplifications is challenging, since they did not provide any theoretical or experimental validation of their work.

Viscoelastic flow has generated a great deal of numerical and experimental work for a cylinder which is confined. The geometry of the problem is defined entirely by the channel width to cylinder diameter ratio, $H/d$ (see Figure 2.4, above). Though some research has focused on a uniform upstream flow profile driven by the movement of channel walls, the more commonly studied problem is with fixed walls and a Poiseuille-type velocity profile upstream.

The confined channel flow with $H/d = 2$ has become a benchmark problem in computational rheology. Carew & Townsend (1991) were the first to explore this problem numerically, employing the Oldroyd-B model to study the impact of elasticity on the drag for a cylinder in a Poiseuille-type flow. Their results indicate a drag reduction with increasing flow elasticity when the Deborah number is much less than unity.

The numerical work of Carew & Townsend (1991) closely matched the experiment of Dhahir & Walters (1989). Flow was driven by a peristaltic pump and conditioned to produce a Poiseuille-type profile upstream of the cylinder, and both constant viscosity Boger and shear-thinning fluids were tested. Like Carew & Townsend, Dhahir & Walters found that the presence of elasticity led to a drag reduction, though Deborah numbers were limited to be much less than unity. Working in a higher Deborah number range, Verhelst & Nieuwstadt (2004) reported a drag increase with increasing Deborah number for $0.1 < De < 4$. Their laser Doppler velocimetry (LDV) measurements showed reduced flow velocities in the wake region with increasing $De$, but also confirmed significant three-dimensional flow effects which may have impacted their results.
Few viscoelastic experimental studies have been carried out for wide channels, where $H/d > 2$. Though achieving unbounded flow is virtually impossible due to wall effects (see Figure 2.5, above), experiments carried out with remote boundaries can yield insight into the flow around an isolated cylinder. Maximizing the channel width also minimizes extension associated with the contraction flow generated by large blockages.

Experiments with $H/d = 16$ were performed by Shiang et al. (1997), who used particle image velocimetry to investigate the velocity field. They used a translating channel to generate uniform flow upstream of the cylinder, and measured decreased flow velocities with increasing Deborah number along the downstream stagnation line, for $De$ ranging from 0.6 to 3.0. Elastic effects became pronounced between $De = 0.6$ and $De = 1.2$, however no intermediate measurements were made, limiting the utility of their reported measurements in pinpointing elastic onset.

François et al. (2008) made measurements of drag and velocity in microchannels for $H/d$ values of 25 and 100, respectively. A drag enhancement and decreased downstream flow velocities due to elasticity were reported beyond some critical Deborah number, consistent with the results of Chilcott & Rallison (1988) and Shiang et al. (1997); determination of the critical Deborah number for elastic onset from their data is difficult due to their chosen method of non-dimensionalization.

Together with the aforementioned numerical study of unbounded viscoelastic Stokes flow by Chilcott & Rallison, the PIV measurements of Shiang et al. and the microfluidics experiments of François represent the only relevant velocity field measurements for viscoelastic Stokes flow around a cylinder with remote boundaries.

### 2.2.3 Viscoelastic Stokes Flow around a Cylinder in an Annular Channel

As detailed in section 2.2.1, creating unbounded uniform Stokes flow around a cylinder experimentally is very difficult, since the necessary walls can have a profound impact on the flow (White 1945). Additionally, it is challenging to maintain velocities low enough to achieve Reynolds numbers on the order of $10^{-6}$ to $10^{-3}$.

Recent experimental work by Wang (2012) sought to collect drag measurements in viscoelastic flows of Boger fluids around cylinders in wide channels. In order to generate steady, wall-driven
flow, an annular channel geometry was studied rather than a straight-walled channel. Using an annular channel, steady flow velocities could be achieved by maintaining a constant angular velocity, something easily done with a turntable coupled to a speed-controlled motor. By minimizing the diameter of the cylinders which were immersed in the centre of the channel, Wang was able to achieve a maximum channel width to diameter ratio of 267. Drag was related to small deflections of a cantilever beam to which the cylinders were fixed. Wang’s experimental setup proved promising, but too high rotation speeds and an overall lack of resolution in his drag measurements limited his measurements to Deborah numbers well above the elastic onset reported by Chilcott & Rallison.

After refinements to Wang’s experimental setup, Shiau (2013) was able to make accurate drag measurements for Deborah numbers as low as 0.25. He also employed numerical simulations and Faxèn’s expression for the bounded viscous drag coefficient to make comparisons between his results and Kaplun’s unbounded Newtonian analytical solution. The onset of elastic effects occurred at a $De$ value between 0.5 and 0.7, beyond which increased flow elasticity was shown to increase the drag, approximately doubling the expected Newtonian value when $De \approx 4$. Some of Shiau’s experimental drag measurements for various cylinder diameters and immersed lengths are reprinted in Figure 2.6 below.
Figure 2.6 Measurements of drag on a cylinder in a rotating annular channel, from Shiau (2013). Legend values are the cylinder diameter and immersion depth, respectively. \( H/d \) was 267 for the 1.19 mm cylinder and 40 for the 3.34 mm cylinder. Corrections have been applied to account for wall and end effects. The test fluid is T1, a Boger fluid characterized in section 4.2 of this thesis.

Elastic onset is evident in Figure 2.6, with any increase in the non-dimensional drag above unity directly attributable to flow elasticity. A steeper rise in drag with \( De \) was reported by Shiau for measurements made with a second Boger fluid having a higher viscosity and longer relaxation time, though onset occurred in the same Deborah number range.

Absent any measurements of the velocity field, Shiau relied on the results of previous studies to explain the mechanism of the drag increase resulting from elasticity. Typically, the drag enhancement is attributed to a stress imbalance on the cylinder surface resulting from normal stresses developed by the deforming polymer molecules as they flow around the cylinder (Chilcott & Rallison 1988, François et al. 2008). Normal stresses can be developed by polymers in shear and extensional flows, however the relative contributions of shear and extension in generating the drag enhancement for a cylinder with remote boundaries is unclear.
2.3 Research Objectives

There remains a need for experimental measurements of the velocity field around a cylinder in a viscoelastic Stokes flow for large values of $H/d$. Knowing the velocity field around the cylinder should be valuable in explaining the elastic effects observed by Shiau (2013), while also providing basic results for this fundamental study. The geometry to be considered will be the annular geometry studied by Shiau, namely, that of a cylinder immersed in the centre of a slowly rotating annular channel.

The primary goals of this study are then:

- To develop a technique for mapping the velocity field using PIV under the same experimental conditions as Shiau.

- To measure the velocity field of one or more constant-viscosity Boger fluids, and compare those velocity fields to that of a Newtonian fluid under comparable conditions, to isolate for the effects of fluid elasticity.

- Perform measurements for $0.1 \leq De \leq 20$, with a high density of measurements around the expected onset of elastic effects ($0.5 \leq De \leq 0.7$).

- Relate the measured velocity profiles to the reported drag increase.

- Examine the mechanism by which elasticity enhances drag.
Chapter 3
Experimental Design

To make measurements of the velocity field in Newtonian and viscoelastic Stokes flows required an experimental apparatus that could maintain steady flows in the Stokes regime for Reynolds numbers as low as $O(10^{-6})$. To study flow with remote boundaries, the flow geometry had to be chosen so as to maximize values of $H/d$.

In Shiau’s thesis as in this work, an annular tank, filled with either Newtonian polybutene or one of two Boger fluids, was centred on a slowly rotating turntable. One of two gearmotors was used to drive the slow rotation of the turntable. A cylinder was immersed in the centre of the annular channel, at a depth many times greater than its diameter, in order to approximate two-dimensional flow. The flow conditions were chosen to be firmly in the Stokes regime, and experimental Deborah numbers were sought between 0.1 and 20.

For making PIV measurements, a sheet of laser light illuminated a horizontal plane and reflective spheres added to each fluid acted as tracer particles. A CCD camera mounted above the annular tank was used to capture sequences of images of the fluid flowing near the cylinder. Sequences were also collected 180° opposite of the cylinder to evaluate the ‘freestream’ conditions. The velocity field in each image sequence was determined through cross-correlation analysis.

3.1 Geometry

Several aspects of the experimental design, including the geometry of the annular tanks and the chosen test fluids, were left unchanged from Shiau’s earlier experiments, so as to facilitate easier comparison between his drag measurements and the velocity measurements reported in this thesis.

3.1.1 Annular Tanks

Three annular tanks, one for each test fluid, had been fabricated by concentrically bonding two acrylic tubes to a circular base plate. Acrylic was chosen as the tank material in order to maintain optical clarity of the tank walls, allowing for PIV measurements. Each fluid was contained in a separate tank to eliminate mixing between the fluids and to avoid the time
The dimensions of the annular tanks are presented in Figure 3.1. Each tank had inner and outer wall diameters of 177.8 and 444.5 mm, respectively, resulting in a channel width of 133.4 mm. Each tank had a wall height of 127.0 mm. During experiments, tanks were filled with fluid to within 10 mm of the wall height, for a fluid depth of approximately 120.0 mm.

**Figure 3.1** Annular tank with dimensions and linear velocity profiles
3.1.2 Cylinders

A cylinder was immersed in the centre of the annular channel. Each cylinder was mounted perpendicularly into a clear acrylic horizontal sheet, as shown in Figure 3.2, which allowed for an unobstructed view around the cylinder when viewed from above.

![Cylinder and fixture isometric view](image)

**Figure 3.2** Cylinder and fixture isometric view

As illustrated below, a C-clamp held the edge of the cylinder fixture to a square retort bosshead, which in turn was clamped to a fixed support. By changing the position of the bosshead on the support, the immersion depth of the test cylinder, $L$, could be adjusted.

![Annular tank with immersed cylinder](image)

**Figure 3.3** Annular tank with immersed cylinder
In order to approximate two-dimensional flow around a cylinder, the immersion depth of the cylinder should be many times greater than the diameter. In practice, however, increasing the depth led to too high drag forces at the highest test speeds, so that the cylinders were prone to deflection and even breaking. Initial tests were performed using optically clear acrylic cylinders; however with a Young’s modulus of 3.2 GPa they proved to be too flexible, deflecting significantly even in low speed flows. With a Young’s modulus of 71.7 GPa, fused quartz was chosen as a suitable alternative. The highly stiff, optically clear material was available in diameters as low as 1.0 mm, yielding a channel width to diameter ratio of 133. An additional cylinder having a diameter of 4.0 mm was chosen for comparison, with $H/d = 33$. In order to minimize deflection and eliminate breakage, the 1.0 mm cylinder was immersed to a depth of 40 mm ($40d$) and the 4.0 mm cylinder was immersed to a depth of 80 mm ($20d$).

All PIV measurements were made in the horizontal plane halfway along the cylinder immersion depth, a depth of 20 and 40 mm for the 1.0 and 4.0 mm cylinders, respectively, illuminated by the light sheet described in detail in section 5.1.2. Using optically clear cylinders eliminated any shadows cast by the cylinder on the inside half of the channel, allowing for measurements 360° around the cylinder.

### 3.2 Drive System

The large diameter turntable provided a stable, level base for the annular tank. The existing motor system yielded rotational speeds which were too high, generating flows above the minimum Deborah number of interest. After decoupling the existing motor system, two DC gearmotors having sufficiently low speeds were selected to drive the turntable. Each motor was used separately to rotate one of two drive rollers which were frictionally coupled to the outer edge of the turntable. The diameters of the drive rollers used were 16.8 and 31.4 mm, yielding speed reductions of 59:1 and 32:1, respectively, when coupled with the 990 mm diameter turntable. A schematic of the drive system is presented in Figure 3.4 below, and motor specifications are listed in Table 3.1.
Figure 3.4 Drive system schematic

\[ \omega_t = \omega_m \frac{d_m}{d_t} \]
Table 3.1 Specifications of the high and low speed gearmotors

<table>
<thead>
<tr>
<th></th>
<th>Low speed</th>
<th>High speed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Maximum rated speed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[rpm]</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td><strong>Voltage</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[V]</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td><strong>Torque</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[in.lbs]</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><strong>Angular velocity range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_m$ [rad/s]</td>
<td>0.20 – 0.65</td>
<td>0.75 - 3.4</td>
</tr>
</tbody>
</table>

During initial tests the motors were powered directly by a variable DC power supply. However, it was observed that at low input voltages the rotation rates were inconsistent, owing to small variations in the turning resistance of the turntable. With the voltage and current held constant by the power supply, these variations were up to 10% during testing. In order to maintain a constant rotation rate under variable loading, a linear output motor speed controller with an internal potentiometer was added between the motor and the power supply. This new setup allowed the motor to maintain a constant speed (within 1% for all tests) throughout experimentation. The rotation rate of the drive roller was measured before and after each experimental run, and the average value was taken as the experimental rotation rate. The angular velocity of the turntable (and of the annular tank) was then calculated as:

$$\omega_t = \omega_m \frac{d_m}{d_t} = \left( \frac{\# \text{ of revolutions}}{\text{time [s]}} \right) \times \frac{2\pi d_m}{60 d_t} \quad [3.1]$$

where $\omega_m$ and $\omega_t$ are the angular velocities of the motor and turntable, respectively, and where $d_m$ and $d_t$ are the diameters of the drive roller and turntable, respectively, as shown in Figure 3.4.

The Deborah number was introduced in Eq. 2.1 as the ratio of the fluid relaxation time to a characteristic flow time. The characteristic flow time used in this thesis is:
\[ t_c = \frac{d}{V_c} \]  \hspace{1cm} [3.2]

where \( d \) is the cylinder diameter and \( V_c \) is the approach velocity, taken to be that at the centre of the annular channel under the assumption of rigid body rotation:

\[ V_c = \omega_i R_c = \omega_i \frac{H + d_i}{2} \]  \hspace{1cm} [3.3]

where \( R_c \) is the radial position at the centre of the annular channel, and \( H \) & \( d_i \) are the channel width and diameter of the inner wall, respectively. The value of \( R_c \) is 155.6 mm for the tank dimensions given in section 3.1.1.

Using the definition of \( t_c \) given above, \( De \) can be written as:

\[ De = \frac{\lambda V_c}{d} \]  \hspace{1cm} [3.4]

All Deborah numbers in this thesis were calculated in this manner.

Similarly, the Reynolds number from Eq. 1.1 should be redefined, replacing the uniform flow velocity \( U \) with the more appropriate approach velocity:

\[ Re = \frac{\rho V_c d}{\eta} \]  \hspace{1cm} [3.5]

The angular velocity ranges listed in Table 3.1 yielded Reynolds numbers of order \( 10^{-6} \) to \( 10^{-3} \) and Deborah numbers ranging from approximately 0.1 to 20, for the experimental fluids described in the next chapter. Therefore, the chosen gearmotors were appropriate for this study.
Chapter 4
Test Fluids

In order to evaluate the effects of elasticity in a flow, one must compare the behaviour of an elastic fluid to that of a Newtonian fluid at a comparable Reynolds number. For this study, polybutene was used as the Newtonian fluid, and two Boger fluids previously prepared in the University of Toronto Flow Measurements Lab were used as the elastic fluids. As noted in section 2.1.4, the nearly constant viscosity of Boger fluids enables the measurement of elastic effects without consideration of shear-thinning.

Rheological characterization of each fluid was necessary in order to account for their viscous and elastic properties. A shear rheometer (TA Instruments AR2000), equipped with a 40 mm, 2° cone, was used for all rheological tests. Because of the strong temperature dependence of rheological properties, small variations in temperature can cause significant changes in fluid viscosity and elasticity. All PIV experiments were carried out at ambient room temperature, and the temperature of the fluid in the tank was measured prior to each experimental trial; fluid temperatures ranged from 20.0°C to 22.5°C. Procedures (discussed in detail in the following sections) were developed to ensure that fluid characterizations accurately reflected the experimental test conditions.

4.1 Characterization of Polybutene

The Newtonian fluid was a polybutene having a molecular weight of 910 g/mol (Brenntag Indopol H-100). The only properties of interest were its viscosity and density, measured using the AR2000 shear rheometer and a digital analytical balance (Mettler AE 160), respectively. The measured fluid density was 877.8 kg/m³, which was assumed to be constant. Viscosity measurements were performed immediately after use in the annular tank, and were carried out at the experimental temperature of the fluid, measured prior to each trial using a digital thermometer (HANNA HI-8053). Figure 4.1 shows a sample measurement of the shear response of polybutene at 20°C.
The dashed lines in Figure 4.1 show the range of expected maximum experimental shear rates, as determined from simulations performed in ANSYS Fluent. As expected for a Newtonian fluid, the viscosity was found to be constant across the measurable range of shear rates, allowing the average viscosity value to be used for all calculations.

The measured average viscosity at every experimental test temperature is shown in Figure 4.2.
For an increase in temperature of 1°C, the viscosity of the polybutene decreased by an average of 8.1%, confirming a strong temperature dependence.

### 4.2 Characterization of Boger Fluids

The two Boger fluids were previously prepared by Dr. Ronnie Yip at the University of Toronto (2011), and hereafter are referred to as T1 and T3. Each fluid was a dilute solution of a high-molecular-weight polymer polyisobutylene (PIB) in a polybutene (PB), with a small amount of kerosene used to dissolve the initially solid blocks of PIB. The composition of each fluid is detailed in Table 4.1.

**Table 4.1** Composition of Boger fluids in wt% as originally prepared by Yip (2011)

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Composition</th>
<th>Solvent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T1</strong></td>
<td>0.2% wt. PIB&lt;br&gt;$M_w = 4.7 \times 10^6$ g/mol&lt;br&gt;&lt;br&gt;92.8% wt. PB&lt;br&gt;$M_w = 635$ g/mol&lt;br&gt;&lt;br&gt;7.0% wt. kerosene</td>
<td></td>
</tr>
<tr>
<td><strong>T3</strong></td>
<td>0.2% wt. PIB&lt;br&gt;$M_w = 4.7 \times 10^6$ g/mol&lt;br&gt;&lt;br&gt;92.8% wt. PB&lt;br&gt;$M_w = 910$ g/mol&lt;br&gt;&lt;br&gt;7.0% wt. kerosene</td>
<td></td>
</tr>
</tbody>
</table>
The densities of T1 and T3, measured using the same method as that for the polybutene, were 861.4 and 876.0 kg/m³, respectively. Like the polybutene, the densities of the two Boger fluids were assumed to be constant over the range of experimental test temperatures.

The relevant parameters of each Boger fluid were its viscosity and relaxation time, which were both temperature dependent. Together with a characteristic dimension, a characteristic flow velocity and a characteristic flow time, these parameters define the Reynolds number (flow regime) and the Deborah number (flow elasticity), given by Eqs. 3.5 & 3.4, respectively.

Measurements of the viscous properties of T1 and T3 at 20°C are presented in Figures 4.3 & 4.4, respectively.

**Figure 4.3** Viscous properties of T1 at 20°C. The dynamic viscosity, introduced in section 2.1.5, was measured during oscillatory shear, in contrast to the viscosity which was measured in steady shear.
As shown in Figures 4.3 & 4.4, each fluid exhibited weakly shear-thinning behaviour, with nearly constant shear viscosities in the range of expected maximum experimental shear rates. Measurements beyond $\dot{\gamma} \approx 20.0$ s$^{-1}$ for T1 and $\dot{\gamma} \approx 8.0$ s$^{-1}$ for T3 were not possible due to a shear flow instability, resulting from elasticity, beyond those respective shear rates (James 2009). For consistency, the maximum low shear rate value of viscosity was used for all calculations involving $\eta$. As predicted from Eq. 2.18, oscillatory tests showed asymptotic behaviour for the dynamic viscosity at high angular frequencies. This asymptotic value represents the solvent contribution to the viscosity, $\eta_s$. By Eq. 2.14 the polymer contribution to the viscosity, $\eta_p$, is then the difference between the viscosity and the solvent contribution.

Figures 4.5-4.8 present measurements of the first normal stress difference $N_1$ and the corresponding coefficient $\Psi_1$ for both fluids.
**Figure 4.5** First normal stress difference for T1 at 20°C

**Figure 4.6** First normal stress difference for T3 at 20°C
Figure 4.7 First normal stress coefficient for T1 at 20°C

Figure 4.8 First normal stress coefficient for T3 at 20°C
Measurements of $N_1$ show a steep rise in normal stress for $\dot{\gamma} > 1.0 \text{ s}^{-1}$. The Oldroyd-B model predicts that $N_1$ is proportional to $\dot{\gamma}^2$, leading to a constant first normal stress coefficient. In practice, Figures 4.7 & 4.8 show only a range of shear rates where $\Psi_1$ is constant because normal stresses were immeasurably low at low shear rates. For each characterization, an average value was calculated for $\Psi_1$, taken from the region where the value of neighbouring data points showed less than a 5% difference, denoted by the dashed lines. These regions mostly fell above the maximum experimental shear rate ranges shown in Figures 4.3 & 4.4.

As outlined in section 2.1.5, a valid approach for calculating the relaxation time of a dilute polymer solution involves the low-frequency limit of $\frac{G'}{\omega^2}$ (see Eq. 2.19). Values of $\frac{G'}{\omega^2}$ calculated from oscillatory tests are presented for each fluid in Figures 4.9 & 4.10.

![Graph of $\frac{G'}{\omega^2}$ for T1 at 20°C](image)

**Figure 4.9** $\frac{G'}{\omega^2}$ for T1 at 20°C
The asymptotic behaviour of $\frac{G'}{\omega^2}$, predicted by the Oldroyd-B model, was not observed experimentally, likely due to measurement limitations of the AR2000. Oscillatory measurements below $\omega = 0.01 \text{rad/s}$ proved too time consuming and unreliable.

The inability to measure the low frequency limit of $\frac{G'}{\omega^2}$ led to the calculation of a relaxation time based on the first normal stress difference. Rearranging Eq. 2.15:

$$\hat{\lambda}_{N_1} = \frac{N_1}{2\varphi^2 \eta_p} = \frac{\Psi_1}{2\eta_p} \tag{4.1}$$

where the fluid relaxation time from the Oldroyd-B model $\lambda_1$ has been renamed $\hat{\lambda}_{N_1}$ and the values of $\Psi_1$ and $\eta_p$ are calculated from rheological measurements, as outlined above. Calculating the relaxation time from steady shear measurements may be appropriate for this.
thesis because recent research by James et al. (2012) showed that the drag enhancement caused by elasticity for flows through arrays of cylinders was attributable to $N_1$.

As stated in section 2.3, this thesis is interested in studying flows in the range $0.1 \leq De \leq 20$, with particular interest in $0.5 \leq De \leq 0.7$, where the onset of elastic effects is expected (Shiau 2013). In order to determine the tank rotation speed for each experiment that would yield desired Deborah numbers, values of the relaxation time at every measured experimental fluid temperature were required. Since Boger fluid characterizations at each temperature would have been very time consuming, temperature-dependent relationships for the viscosity and relaxation time of each fluid were found by fitting power law curves to the data. Since all experiments were carried out at ambient room temperature, with fluid temperatures ranging from 20.0°C to 22.5°C, characterizations of T1 and T3 were performed every 0.5°C in that range. This yielded six data points for the viscosity and relaxation time, to which the power law curves were fitted.

The measured values $\eta$ and $\eta_p$ are presented in Figures 4.11 & 4.12 over the range of experimental test temperatures. Values of the corresponding first normal stress coefficients are presented in Figures 4.13 & 4.14.
Figure 4.11 Temperature dependence of the viscous properties of T1. The solid line is the power law curve fit.

Figure 4.12 Temperature dependence of the viscous properties of T3. The solid line is the power law curve fit.
Figure 4.13 Temperature dependence of the first normal stress coefficient of T1

Figure 4.14 Temperature dependence of the first normal stress coefficient of T3
Careful examination of Figures 4.11-4.14 reveals a stronger temperature dependence for $\Psi_1$ than for either of the viscous parameters $\eta$ or $\eta_p$. In their paper on rheological modeling of polyisobutylene solutions, Quinzani et al. (1990) determined the following temperature dependencies for these parameters:

$$\eta(T_0) = \eta(T)/a_T$$  \hspace{1cm} (4.2)

$$\eta_p(T_0) = \eta_p(T)/a_T$$  \hspace{1cm} (4.3)

$$\Psi_1(T_0) = \Psi_1(T)/a_T^2$$  \hspace{1cm} (4.4)

where $T$ & $T_0$ represent the measurement temperature and a reference temperature, respectively, and the shift factor, $a_T$, is:

$$a_T = \exp \left[ E \left( \frac{1}{T} - \frac{1}{T_0} \right) \right]$$  \hspace{1cm} (4.5)

where the coefficient $E$ represents the flow activation energy divided by the gas constant.

Clearly, from Eqs. 4.2 & 4.3, the shear and polymer viscosities should scale together with temperature. The measured values in Figure 4.11 yield an average decrease in the shear and polymer viscosities of T1 of 7.3% and 7.5%, respectively, for a temperature increase of 1°C; for T3, from Figure 4.12, the shear and polymer viscosities exhibit an average decrease of 8.4% and 8.0%, respectively, for the same 1°C temperature increase. Since the average decreases in $\eta$ and $\eta_p$ with temperature are similar for the respective fluids, their relative temperature dependence is consistent with the behaviour found by Quinzani et al.

In order to determine the temperature dependence of $\Psi_1$ from Eq. 4.4, the value of $a_T$ (and subsequently the value of $E$) can be determined from Eqs. 4.2 & 4.5, using temperature and viscosity values from Figures 4.11 & 4.12. Values of $E$ were 35.7°C and 42.9°C for T1 and T3, respectively. Eq. 4.4 then predicts average decreases in $\Psi_1$ for T1 and T3 of 13% and 15%, respectively, for a 1°C temperature increase. Measured values of $\Psi_1$ from Figures 4.13 & 4.14 yield average decreases of 17% and 13% for T1 and T3, respectively. Given the uncertainty in
\( \Psi_1 \) at any temperature, the agreement between the measured and predicted temperature dependence of the first normal stress coefficient is deemed adequate.

As for the relaxation times, Figures 4.15 & 4.16 present the values of \( \lambda_{N_1} \) for T1 and T3 over the experimental temperature range, calculated using Eq. 4.1.

**Figure 4.15** Temperature dependence of the \( N_1 \) relaxation time \( \left( \lambda_{N_1} \right) \) of T1. The solid line is the power law curve fit.
Temperature dependence of the $N_1$ relaxation time ($\lambda_{N_1}$) of T3. The solid line is the power law curve fit.

Since the only two parameters of Eq. 4.1 are $\eta_p$ & $\Psi_1$, which are both highly temperature dependent, the values of $\lambda_{N_1}$ are also subject to large variations with temperature. The higher relative difference in the temperature dependencies of $\eta_p$ & $\Psi_1$ for T1 led to an average decrease of 12% in $\lambda_{N_1}$ for a 1°C increase in temperature, while for T3 $\lambda_{N_1}$ varied by only 5.2% for every 1°C change in temperature.

Accurate characterization of the Boger fluids over the experimental temperature range was important to account for any changes in viscosity and elasticity resulting from variations in fluid temperature.
4.3 Summary of Fluid Characterizations

A summary of the parameters relevant to this thesis, determined through rheological characterization, is presented in Table 4.2.

**Table 4.2** Summary of fluid characterizations (temperature $T$ in °C for all empirical relations)

<table>
<thead>
<tr>
<th></th>
<th>Polybutene</th>
<th>T1</th>
<th>T3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fluid density</strong> $\rho$ [kg/m$^3$]</td>
<td>877.8</td>
<td>861.4</td>
<td>876.0</td>
</tr>
<tr>
<td><strong>Maximum viscosity</strong> $\eta$ [Pa.s]</td>
<td>35.9 Pa.s at 20°C</td>
<td>$1150.8 \times T^{-1.632}$</td>
<td>$11964 \times T^{-1.895}$</td>
</tr>
<tr>
<td></td>
<td>30.7 Pa.s at 21.8°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$N_1$ relaxation time</strong> $\lambda_{N_1}$ [s]</td>
<td>10299 $\times T^{-3.168}$</td>
<td>110.36 $\times T^{-1.293}$</td>
<td></td>
</tr>
</tbody>
</table>

The empirical relationships presented in Table 4.2 were used throughout this thesis when calculating Reynolds and Deborah numbers for the Boger fluids.
Particle image velocimetry (PIV) is an optical technique used to measure the instantaneous velocity field in a cross-section of a flow. In contrast to traditional flow measurement devices such as hot-wires and Pitot tubes, PIV is non-invasive; this is particularly important in the Stokes regime, where even a small disturbance in the flow can have an impact over a great distance. Unlike other optical measurement techniques such as laser Doppler velocimetry, PIV enables simultaneous measurement of flow velocities at many locations in the flow. Since its development in the late 1970’s, PIV has been widely used in engineering and science to measure velocity fields in many different flowing systems (Adrian 2005).

5.1 Overview

PIV analysis infers the velocity by tracking the motion of small, reflective seeding particles added to the fluid. A sequence of flow images is collected with a known frame separation, and one of several correlation algorithms is used to determine the displacement of the particles from one image to the next. Typical PIV measurement systems are comprised of a pulsed laser, a lens or series of lenses to spread the laser beam into a thin sheet, a charged-coupled device (CCD) camera, a frame grabber, a synchronizer, and a CPU for image processing and correlation analysis. This system is shown schematically in Figure 5.1.
The following sections include details of the selected components of the PIV system used to make experimental measurements. The calibration and correlation procedures are also discussed.

5.1.1 Seeding Particles

The selection of appropriate seeding particles is critical for accurate PIV measurements. All PIV analysis assumes the motion of the particles doesn’t deviate significantly from the motion of the fluid. In order to achieve this, the particles must be small and have a density close to that of the fluid being studied. Additionally, the particles must be highly visible in the flow when

Figure 5.1 Typical PIV imaging setup.
Adapted from Kiger (2009)
illuminated by the light sheet. This is achieved by choosing particles which are highly reflective, and large enough to scatter a significant amount of the laser light.

If the density of the particles is greater than the density of the fluid, they will settle to the bottom of the test geometry. The settling velocity should be several orders of magnitude lower than the flow velocity in order for the particle motion to be considered consistent with the fluid motion. The settling velocity of a spherical particle with diameter \( d_p \) and density \( \rho_p \) in a fluid with density \( \rho \) and viscosity \( \eta \) can be calculated by determining the velocity at which the drag force predicted by Stokes matches the gravitational force acting on the sphere. This is known as Stokes settling velocity (Blanchard 1967):

\[
V_s = \frac{d_p}{18\eta} \left( \frac{\rho_p - \rho}{g} \right)
\]

where \( V_s \) is the particle settling velocity and \( g = 9.8067 \text{ m/s}^2 \) is the gravitational acceleration. Clearly, the particle settling velocity is minimized for small diameters and for small differences between the particle and fluid densities.

Stokes’ prediction for the drag on a spherical particle is only valid when the particle Reynolds number is much less than one. The particle Reynolds number \( Re_p \) is given by:

\[
Re_p = \frac{\rho_p V_s d_p}{\eta}
\]

which is minimized for small diameters and high viscosity.

A measure of the ability of the seeding particles to faithfully follow the fluid motion is the Stokes number, \( S_k \), which is defined as the ratio the particle response time to a characteristic time in the flow (Tropea et al. 2007):

\[
S_k = \frac{t_p}{t_c}
\]

where \( t_p \) is the particle response time and \( t_c \) is the characteristic flow time, defined in Eq. 3.2.

In the Stokes regime, the particle response time is given by (Elghobashi 1994):
\[ t_p = \frac{\rho_p d_p^2}{18 \eta} \]  

[5.4]

For the particle motion to be considered consistent with the motion of the fluid, a Stokes number of less than one is required.

The seeding particles used in this study were silver-coated glass spheres (Conduct-O-Fil Potters Industries). These particles had a low density (1600 kg/m\(^3\)) and an average diameter of 13 μm, and have been shown to produce adequate light scattering for PIV measurements (Shiang et al. 1997, Tachie et al. 2003, Yip et al. 2011).

The Reynolds number, settling velocity, response time and Stokes number in each test fluid is presented in Table 5.1:

**Table 5.1** Summary of seeding particle flow characteristics in each test fluid

<table>
<thead>
<tr>
<th>Particle Reynolds number ( Re_p )</th>
<th>Particle settling velocity ( V_s ) [mm/s]</th>
<th>Particle response time ( t_p ) [s]</th>
<th>Maximum Stokes number ( S_{k,\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polybutene 1.1×10(^{-12})</td>
<td>1.8×10(^{-6})</td>
<td>4.2×10(^{-10})</td>
<td>7.1×10(^{-9})</td>
</tr>
<tr>
<td>T1       18×10(^{-12})</td>
<td>7.8×10(^{-6})</td>
<td>17×10(^{-10})</td>
<td>29×10(^{-9})</td>
</tr>
<tr>
<td>T3       0.82×10(^{-12})</td>
<td>1.6×10(^{-6})</td>
<td>3.7×10(^{-10})</td>
<td>6.2×10(^{-9})</td>
</tr>
</tbody>
</table>

The particle Reynolds numbers are all well below unity and firmly in the Stokes regime. The particle settling velocities are five orders of magnitude lower than the lowest experimental channel centreline velocity, which was 0.17 mm/s. The maximum Stokes number was calculated by comparing the particle response time to the shortest experimental characteristic flow time, and is much less than one in each fluid. These values indicate that the particles followed the flow over the entire range of experimental flow velocities.

### 5.1.2 Light Source

In order to adequately capture high speed flows while limiting blurring, typical PIV systems employ a double pulsed Nd:YAG laser (Adrian 2005). The laser pulses, acting in the same manner as a camera flash, compensate for inadequate shutter speed by temporarily freezing the
motion of the particles. This method requires high powered lasers as well as synchronization of the pulse separation and camera exposure. For low speed flows, capturing an adequately sharp image is possible relying solely on the camera shutter, using a continuous wave (CW) laser as the light source. The advantage of this method is simplicity, with the frame separation determined solely by the frame rate of the camera. No additional synchronization between the laser and the camera is required and lower power lasers can be used to good effect.

Indeed, the flow velocities in this study were low enough to avoid the need for pulsed lasers. Two CW Nd:YAG lasers, having output powers of 100 and 65 mW respectively, were used to illuminate the flow; two lasers were necessary for sufficient illumination of the seeding particles. The beam diameters of the lasers were 1.0 and 2.0 mm for the 100 and 65 mW lasers, respectively. Each beam passed through a borosilicate glass cylinder, acting as a lens to spread each laser beam into a thin sheet having the same thickness as their respective beam diameter. These sheets were overlapped in the region of interest of the flow, creating a light sheet having a total thickness of 2.0 mm (Figure 5.2). Overlapping the light sheets was necessary not just to increase illumination, but also to limit shadows cast by the immersed cylinder in the inside half of the channel. These shadows resulted from refraction as the laser sheets passed through the cylinder, even though optically clear fused quartz cylinders were used.

Both lasers were mounted on a laboratory jack which allowed for precise adjustment of the depth of the illuminated flow plane, which was half of the cylinder immersion depth for all experiments.
5.1.3 Image Acquisition

For this study, a 1/3 inch CCD camera (Pulnix RM-6740) equipped with a F2.5 macro zoom lens (Kowa LMZ45T3) was used to capture all flow images. The output images had an aspect ratio of 4:3, with a resolution of 640 horizontal pixels by 480 vertical pixels. A PCI-bus frame grabber (Euresys Grablink Value) captured the image stream output by the CCD sensor at a constant 200 frames per second. In order to alter the frame rate of an image sequence, a recording script was written for the image acquisition software Norpix StreamPix 4 which would
skip a set number of frames between each captured image. The frame separation $\Delta t$ between successive images could then be calculated as:

$$\Delta t = \frac{1}{\text{frame rate}}$$

[5.5]

Images were initially recorded to the random access memory of a personal computer before being transferred to the hard drive for storage. This was done to limit the chance of frames being dropped during the recording sequence.

Though the chosen lens captured highly detailed images with easily resolvable seeding particles, the relatively long focal length of 108 mm limited the angle of view and thus the maximum flow area that could be captured in a single image. Therefore, in order to capture the full width of the annular channel and the upstream and downstream regions in the flow, it was necessary to capture images at multiple locations, termed viewing windows. In all, three viewing windows were necessary to capture the width of the channel, and an additional four viewing windows, two each upstream and downstream of the cylinder, were used to capture the ‘wake’ regions. Figure 5.3 shows the layout of the seven viewing windows at which image sequences were captured.

**Figure 5.3** Layout of viewing windows A. Close-up of cylinder location. B. Full set of collected viewing windows, including 180° cross-channel measurements.
The edges of the viewing windows were overlapped in order to ensure complete coverage of the flow area of interest. Average velocity values were calculated in the overlapped regions. In order to evaluate the extent of the impact of the cylinder on the flow in the annular channel, measurements were also made across the annular channel 180 degrees opposite (upstream or downstream) of the cylinder.

As a way to reliably position the camera directly above each viewing window, the camera was mounted to a milling table which was fixed above the annular tank to a steel support frame. The milling table allowed for two-dimensional positioning of the camera to within 0.0254 mm (0.001”).

5.1.4 Viewing Window Size Calibration

In order to obtain absolute rather than relative PIV measurements of the velocity field, it was necessary to know the scale of the flow image, typically expressed in units of length per pixel. If a flow image contains resolvable geometric features, the scale of the image can be easily
calculated by comparing the size of the feature in the image (in pixels) to its known length. For this study, geometric features such as the edges of the cylinder or the channel walls were not adequately resolvable due to blurring of the flow images in these regions, as a result of the meniscus on the free surface. Furthermore, the upstream and downstream viewing windows contained no geometric features at all, being far from the cylinder and channel walls.

The absolute size of each viewing window was calibrated prior to capturing a flow sequence using a submersible scale which was inserted into the annular channel prior to each set of experiments. By adjusting the zoom and focus of the camera, the size of the camera image could be adjusted to match the size of a printed rectangular region of known dimension. The submersible scale is shown in Figure 5.5 below.

**Figure 5.5** Submersible scale. A. Isometric view of scale assembly. B. Printed scale markings. C. Camera image after having been properly sized by adjusting the zoom and focus to match the rectangular region of the printed scale. The scale of the image is 80 μm/pixel.
The assembly consisted of a clear acrylic window, the scale itself, four threaded rods and some additional hardware. The acrylic window would rest on the upper edges of the walls of the annular tank, and the rest of the assembly would hang submerged in the test fluid. The scale was made out of opaque plastic with markings which were printed onto a transparency and then bonded to the scale with marine epoxy. Both the scale and the acrylic window had radial slots machined into them so that the scale could be inserted even if a test cylinder was already mounted in the channel centre. Each viewing window had dimensions of 51.2×38.4 mm, which given the output resolution of the camera (640×480 pixels) yielded images with a scale of 80 μm/pixel, maximizing the resolvable detail of each image while being just large enough to span the annular channel with only three viewing windows.

By changing the height of the nuts which held the scale fixed along the threaded rods, the immersion depth of the scale could be adjusted so that each viewing window was sized at the same depth as the light sheet.

**Figure 5.6** Submersible scale in annular tank
5.1.5 Cross-Correlation

In order to determine the velocity field in a flowing fluid from two successive flow images, the images must first be broken down into rectangular sub-regions known as interrogation areas. When an interrogation area is small relative to the scale of the flow, it is assumed that the seeding particles in the area will move with the same velocity. The cross-correlation algorithm assesses the movement of the seeding particles between successive frames, and assigns a global resultant velocity vector in each interrogation area. For this study, the PIV analysis software VISIFLOW (AEA Technology) was used for the analysis of all PIV data. The cross-correlation algorithm employed by VISIFLOW is outlined below.

Images acquired through digital photography are by their nature discretised; each pixel represents a discrete position in the image and is associated with an intensity value. For this study, images were captured in 8-bit greyscale yielding 256\((2^8)\) unique pixel intensity values. Light scattered by a seeding particle in the flow will generate a high pixel intensity value, whereas regions in an image with no seeding particles will have a correspondingly low pixel intensity value. The first step in the cross-correlation algorithm is to determine the intensity value of each pixel in every interrogation area. Representing the pixel intensity values in the same interrogation area for two images separated in time by the frame separation \(\Delta t\) as \(f(m,n)_t\) and \(g(m,n)_{t+\Delta t}\), where \(m\) and \(n\) are positional coordinates, the discretised cross-correlation function can be written as (Dabiri 2006):

\[
C(r,s) = \frac{1}{M * N} \sum_{m=1}^{M} \sum_{n=1}^{N} \left[ f(m,n) - \bar{f} \right] \left[ g(m+r,n+s) - \bar{g} \right]
\]

where \(M\) and \(N\) are the number of horizontal and vertical pixels in the interrogation area, \(C(m,n)\) represents the discretised cross-correlation function which is being evaluated at the location \((m,n)\) and \(\bar{f} \) & \(\bar{g} \) represent the mean pixel intensities in the interrogation areas of \(f\) and \(g\), respectively.

The location of the peak correlation value yields the magnitude and direction of the displacement of the seeding particles in each interrogation area, which is combined with the known frame separation \(\Delta t\) to yield a velocity vector in each interrogation area. In practice, evaluating the
cross-correlation function only at the pixel locations can lead to positional uncertainty of the
correlation peak of up to ±1/2 pixel, which will yield unacceptably high errors in the calculated
velocity values. Fitting the cross-correlation function with a three-point Gaussian curve fit
achieves sub-pixel positional accuracy of the correlation peak and is standard practice in PIV
analysis (Willert & Gharib 1991).

It can be very computationally intensive to evaluate the sums in Eq. 5.6 over all the elements in
each interrogation area for every pixel location. As a result, most PIV analysis software
(including VISIFLOW) makes use of fast Fourier transforms (FFTs) to greatly speed up the
calculation (Willert & Gharib 1991). The pixel intensity functions $f(m,n)$ and $g(m,n)$ are
transformed in the spatial frequency domain, and a complex conjugate multiplication of the
corresponding Fourier coefficients is carried out. The cross-correlation function at every
location in the interrogation area can then be evaluated by taking the inverse Fourier transform of
the product of the aforementioned complex conjugate multiplication.
Figure 5.7 Overview of the cross-correlation algorithm

Pixel intensity and cross-correlation plots reprinted from Willert & Gharib (1991)
5.1.6 Determination of PIV Parameters

A sequence of 100 images was collected at every viewing window. The size of the images was calibrated using the procedure outlined in section 5.1.4, yielding images with a scale of 80 microns per pixel. Each image was divided into square interrogation areas of 32\times32 pixels, which would have generated velocity vectors spaced every 2.56 mm in the flow. Since the most relevant length scale in the flow is the cylinder diameter, and the diameters of the test cylinders were 1.0 and 4.0 mm, it was desirable to decrease the space between each vector in order to properly resolve smaller flow features. This was achieved by introducing a 75\% overlap of the interrogation areas, yielding vectors which were spaced 0.64 mm apart. Further decreasing the size of the interrogation areas would have produced a higher resolution vector field, however due to limitations of the camera resolution decreasing the size of the interrogation areas below 32 pixels produced less accurate flow correlations.

Seeding particles were added to each fluid in small amounts until each interrogation area had between 10 and 15 particles. Using simulated data, Keane & Adrian (1993) found that, with a seeding density greater than 10 particles per area, there is a 98\% probability of detecting a valid correlation peak. Though in theory a higher seeding density will produce more accurate results, with the current lasers and camera it was found that beyond 15 particles per area it became difficult to collect images with individually resolvable particles. A density of 10 particles per interrogation area required adding 0.15 grams of seeding particles per litre of fluid. The particles were mixed into the fluid over the course of several days using a slow mixer (Cole-Parmer Stir Pak) until no particle streaks were visible.

In order to accurately measure a velocity field using PIV, the frame separation between sequential images must be small enough so that the particles remain in the same interrogation area between frames, but large enough so that the particle displacement produces a strong correlation peak. Due to the relatively large annular channel width, velocity values across the channel varied greatly. In order to compensate for this effect, the frame rate of each collected image sequence was optimized for different viewing window positions across the channel. Experiments with simulated PIV data (Keane & Adrian 1993) identified a maximum in-plane displacement of 25\% of the side length of an interrogation area as optimal for cross-correlation analysis. As a result, the frame rate for every image sequence was chosen so that a particle
traveling at the maximum velocity expected in that particular viewing window, as determined by rigid body rotation, would travel $\frac{1}{4}$ of the length of an interrogation area. This criterion yields the following frame separation:

$$\Delta t = \frac{\frac{1}{4} \Delta X}{\omega_t R_{\text{max}}},$$

[5.7]

where $\Delta X$ is the side length of an interrogation area (2.56 mm for the 32×32 pixel interrogation areas described above), $\omega_t$ is the angular velocity of the annular channel and $R_{\text{max}}$ is the maximum radial distance from the centre of the annular tank in the viewing window under consideration. The frame rate for each viewing window was then calculated using Eq. 5.5.

In addition to managing the in-plane displacement of the seeding particles, it was important that any out-of-plane particle displacement be minimal, since PIV assumes measurements are made in a single, two-dimensional plane. Again, simulated data from Keane & Adrian (1993) predict accurate results as long as the out-of-plane particle displacement, $\Delta Z$, is less than one quarter of the width of the light sheet, which was 2.0 mm in this study. Numerical simulations performed in ANSYS Fluent, described in detail in section 6.1, confirmed that for the frame separation derived in Eq. 5.7 the maximum out-of-plane displacement for each cylinder diameter was two orders of magnitude smaller than the width of the light sheet, indicating sufficiently two-dimensional flow for accurate cross-correlation analysis.

### 5.2 Data Analysis

Prior to performing cross-correlation analysis on each image sequence, the contrast and sharpness of each image was enhanced using the image processing software ImageJ. This was found to improve correlation quality. Cross-correlation analysis of each 100-image sequence yielded 99 individual velocity fields, which were combined to produce a mean velocity field for each viewing window. Combining and averaging the individual vector fields was possible since all experiments were performed under steady-state conditions, and helped to minimize the effect of erroneous correlation peaks.

The mean velocity field data for each viewing window were imported into the CFD visualization software Tecplot 360, where smoothing of the $u$- and $v$-velocity components was performed in
order to minimize discontinuities in the data. Data collected at all seven viewing windows (collected at the cylinder location in the annular channel) or three viewing windows (collected at the 180° position) were then exported to a C++ program written for this thesis.

By default, data analysed in VISIFLOW are returned on a Cartesian grid with the origin at the bottom left corner of the flow image. In order to combine the data from each viewing window into a continuous velocity field, the first operation of the C++ program was to apply a coordinate shift to each viewing window so that their relative positions were consistent with their measured locations. A global coordinate shift was then applied, moving the origin of the combined dataset to the centre of the annular channel, which allowed for a cylindrical coordinate system to be easily implemented when practical. After performing these coordinate transformations, the velocity values in the overlapping regions of neighbouring viewing windows were averaged.

Several other operations were performed using the developed C++ code. First, a fourth order central difference approximation was developed to calculate shear and extensional rates at each node. Second, each dataset was non-dimensionalized, using several input parameters such as the cylinder diameter, channel width, characteristic flow time and approach velocity.

Velocity profiles were extracted from each dataset at three separate locations: across the annular channel at the cylinder location, circumferentially along the channel centreline immediately upstream & downstream of the cylinder and across the annular channel. The circumferential profiles were important for determining the velocity in the upstream and downstream ‘wake’ regions. The cross-channel profiles collected 180° opposite of the cylinder were used to determine whether the centreline velocity was affected by the immersed cylinder, and if so, to what extent was the approach velocity impacted. Bilinear interpolation was employed in order to extract velocity values in between nodes.

Due to unavoidable scatter in the experimental data, shear and extensional rate values derived from numerical differentiation were prone to much irregularity. In practice, the shear and extensional rate data presented in the following chapter were calculated by differentiating smooth functions which were fitted to the experimentally determined velocity profiles using MATLAB.
5.3 Validation and Error Analysis

The flow induced in an annular tank rotating at constant angular velocity will yield a linear velocity profile consistent with rigid body rotation:

\[ V(R) = \omega t R \]  

[5.8]

where \( R \) is the radial position from the centre of the annular tank and \( V(R) \) is the flow velocity evaluated at \( R \). This flow, with no immersed cylinder, was used to validate the experimental data collection procedure outlined above. Data were collected at various speeds at each of the three cross-channel viewing windows, and cross-channel velocity profiles were extracted. Sample velocity profiles measured at high and low rotation speeds are presented in Figure 5.8. The radial position across the annular channel has been normalized by the channel width as follows:

\[
\text{Non-dimensional channel width} = \frac{R - \left( \frac{d_i}{2} \right)}{H} 
\]

[5.9]

For simplicity, all cross-channel velocity profiles in this thesis are presented in this manner.
Figure 5.8 Comparison of experimentally measured velocity profiles to the expected linear velocity profile determined by rigid body rotation. \( \omega_t \) for the high and low speed measurements was \( 1.7 \times 10^{-2} \) and \( 4.2 \times 10^{-3} \) rad/s, respectively. Velocities have been non-dimensionalized by the approach velocity, \( V_c \), defined in Eq. 3.3.

Overall, the measured velocity profiles in Figure 5.8 show good agreement with the expected linear velocity profile. Significant deviation is seen only near the channel walls, which was a result of the meniscus there. The light that passed through the curvature of the meniscus refracted, leading to blurring in the flow images at the channel walls, rendering cross-correlation analysis ineffective in those regions. After comparison of the data presented in Figure 5.8 with their expected values, better agreement was found at high rotation speeds. By calculating the average percentage error of the flow profiles (excluding the regions near the channel walls) for several different rotation speeds, the accuracy of the measurement technique over the experimental range was determined.
Figure 5.9 Average percentage error of experimentally measured velocity profiles at varying rotation speeds

Figure 5.9 shows the average percentage error decreasing with speeds. The error was caused by difficulties in measuring low velocities, on the order of 0.1 mm/s, in the low speed range, even with the linear speed controller maintaining constant angular velocities at those speeds.
Chapter 6
Results and Discussions

6.1 Fluent Simulations

Since there are no published analytical solutions for the velocity field around a cylinder in a rotating annular channel, experimental results were compared to numerical simulations performed using ANSYS Fluent. These simulations were performed for both the 1.0 and 4.0 mm cylinders, at immersion depths matching the experimental test conditions. For all simulations, the fluid was modelled as Newtonian, having a density of 877.8 kg/m$^3$ and a viscosity of 35.9 Pa.s, matching the properties of the polybutene at 20° C. Simulations were carried out in the Stokes regime, with Reynolds numbers of 7.7×10$^{-5}$ and 3.2×10$^{-4}$ for the 1.0 and 4.0 mm cylinders, respectively.

For Newtonian Stokes flow, the non-dimensionalized velocity field is expected to be independent of the Reynolds number. This behaviour was confirmed by carrying out simulations at several different Reynolds numbers, for which no variation was found in the shape of the extracted velocity profiles. As a result, velocity profiles from just one simulation for each cylinder diameter were expected to match the experimentally measured results with polybutene.

The simulation velocity profiles for both cylinder diameters are presented in Figures 6.1 & 6.2. The circumferential velocity profiles are plotted against the angular position, $\theta$, in degrees, for a cylindrical coordinate system with its origin located concentrically with respect to the annular tank. The convention in this thesis is for the cylinder position to be located at 0° for all angular position measurements, with any angular position upstream of the cylinder expressed as a negative value. Cross-channel velocity profiles are plotted against the non-dimensional channel width, defined in section 5.3 (Eq. 5.9).

Unless otherwise stated, all velocities in this chapter have been non-dimensionalized by the approach velocity, $V_c$, defined in section 3.2 (Eq. 3.3).
**Figure 6.1** Cross-channel velocity profiles at cylinder location (0°) determined via Fluent simulation

**Figure 6.2** Circumferential velocity profiles determined via Fluent simulation
As is expected for Newtonian Stokes flow, the circumferential velocity profiles presented in Figure 6.2 exhibit symmetry upstream and downstream of the cylinder. Interestingly, the non-dimensional velocity along the channel centreline never reaches unity, achieving a steady value of 0.970 and 0.897 for the 1.0 and 4.0 mm cylinders, respectively. This indicates that the immersed cylinder affects the flow around the entire annulus, and that the approach velocity predicted from rigid body rotation is never fully recovered. These velocity deficits of 3.0 and 10.3% for the 1.0 and 4.0 mm cylinders, respectively, are investigated later in this thesis by comparing experimental velocity measurements made across the annular channel 180° from the cylinder location to the approach velocity as determined from rigid body rotation.

Fluent simulations were also useful in verifying the two-dimensionality of the experimentally generated flow at the measurement depths, as PIV measurements in the vertical plane were not possible due to distortion caused by the curvature of the tank walls. As was reported in section 5.1.6, the maximum out-of-plane displacement of the seeding particles in the measured flow plane between sequential flow images was two orders of magnitude smaller than the 2.0 mm width of the light sheet, allowing for valid cross-correlation analysis.

6.2 Newtonian Results with Polybutene

Experiments with the polybutene were carried out for both cylinder diameters over a range of angular velocities, with Reynolds numbers ranging from $10^{-6}$ to $10^{-3}$. The experimental test conditions are summarized in Table 6.1. Measurements were performed at six different Reynolds numbers for each cylinder diameter in order to confirm the $Re$ independence of the flow in the Stokes regime.
### Table 6.1 Summary of experimental test conditions with polybutene

<table>
<thead>
<tr>
<th></th>
<th>1.0 mm cylinder</th>
<th>4.0 mm cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Temperature range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T$ [°C]</td>
<td>20 - 21.8</td>
<td></td>
</tr>
<tr>
<td><strong>Viscosity range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$ [Pa.s]</td>
<td>35.9 - 30.7</td>
<td></td>
</tr>
<tr>
<td><strong>Channel width to diameter ratio</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H/d$</td>
<td>133</td>
<td>33</td>
</tr>
<tr>
<td><strong>Angular velocity range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_i$ [rad/s]</td>
<td>$2.2 \times 10^{-3} - 1.1 \times 10^{-1}$</td>
<td>$2.1 \times 10^{-3} - 1.1 \times 10^{-1}$</td>
</tr>
<tr>
<td><strong>Reynolds number range</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Re$</td>
<td>$9.4 \times 10^{-6} - 4.7 \times 10^{-4}$</td>
<td>$3.6 \times 10^{-5} - 1.9 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

6.2.1 Velocity Profiles

The measured velocity profiles across the annular channel at the cylinder location and 180° opposite of the cylinder are compared to the velocity profiles determined from Fluent simulation in Figures 6.3-6.6.
Figure 6.3 Cross-channel velocity profiles at the cylinder location for the flow of polybutene around the 1.0 mm cylinder
Figure 6.4 Cross-channel velocity profiles at the cylinder location for the flow of polybutene around the 4.0 mm cylinder
Figure 6.5 Cross-channel velocity profiles at 180°, opposite of the cylinder for the flow of polybutene around the 1.0 mm cylinder
Figure 6.6 Cross-channel velocity profiles at 180°, opposite of the cylinder for the flow of polybutene around the 4.0 mm cylinder
The cross-channel velocity profiles presented in Figures 6.3-6.6 exhibit generally good agreement with the velocity profiles predicted from Fluent simulations. Like the measured linear velocity profiles in section 5.3, accurate measurements near the channel walls are not possible due to the meniscus on the free surface. A meniscus also occurred at the free surface around the edges of the cylinder. This resulted in distortion of the flow images near the cylinder walls.

A significant departure from the Fluent profiles near the cylinder walls can be seen at high Reynolds numbers in Figures 6.3 & 6.4. This is due to disruptions on the fluid surface, which caused distortion of the flow images. Figure 6.7, below, shows the two major features of this surface disruption: a rise in the fluid surface level at the leading edge of a cylinder, and a depression behind the trailing edge. As the flow neared the cylinder, the level of the fluid surface would rise and reach a maximum at the leading edge. The fluid level remained elevated as flow continued around the cylinder, creating the lateral distortions seen in Figures 6.3 & 6.4 at high Reynolds numbers. As the flow approached the trailing edge, a depression in the fluid surface would develop. This depression persisted downstream for several diameters. The surface disruptions were more pronounced for the 4.0 mm cylinder, though their effects were seen with the 1.0 mm cylinder as well.
Figure 6.7  Surface disruptions around the 4.0 mm cylinder in polybutene with $Re = 1.9 \times 10^{-3}$

A. View from downstream of the cylinder, showing the level of the fluid sloping upwards toward the cylinder. B. Side view of cylinder. The fluid surface is raised at the leading edge of the cylinder, and a depression can be seen behind the trailing edge.
Circumferential velocity profiles are presented in Figures 6.8 & 6.9. The angular position along the channel centreline has been non-dimensionalized by the cylinder diameter as follows:

$$\text{Non-dimensional arc position} = \frac{s}{d} = \frac{R_c \theta}{d} \quad [6.1]$$

where $R_c$ is the radius of the channel centreline (155.6 mm), $\theta$ is the angular position expressed in radians and $s$ is the arc length.
Figure 6.8 Circumferential velocity profiles along the channel centreline for the flow of polybutene around the 1.0 mm cylinder.
Figure 6.9 Circumferential velocity profiles along the channel centreline for the flow of polybutene around the 4.0 mm cylinder.
Again, the velocity profiles show generally good agreement with the Fluent predicted profiles. The velocity profiles for flow around both cylinders exhibit symmetry upstream and downstream of the cylinder. Some deviation from the Fluent predicted profile is seen in Figure 6.9 downstream of the cylinder at high Reynolds numbers. This can be explained by the depression seen on the fluid surface downstream of the cylinder. Other slight discontinuities in the profiles are a result of the data being stitched together from five separate viewing windows.

6.2.2 Low-Velocity Contour Plots

Since PIV enables measurement of the velocity field in two-dimensions, data can also be presented as two-dimensional contour plots, with regions coloured by velocity and separated by lines having a constant value. By presenting contour plots of non-dimensional flow velocities below a threshold value, one can compare the shape and extent of the ‘wake’ around the cylinders in flows at various Reynolds numbers. Figure 6.10 presents low-velocity contour plots for polybutene around the 1.0 and 4.0 mm cylinders. The low-velocity region has been defined as the region having a flow velocity of less than half of the approach velocity $|V|/V_c \leq 0.5$. The data in Figure 6.10 are presented on a Cartesian grid having its origin at the centre of the cylinder, non-dimensionalized by the cylinder diameter, and the dashed curves indicate the channel centreline.
Figure 6.10 Low-velocity contours ($|V|/V_c \leq 0.5$) around a cylinder in polybutene for various Reynolds numbers.
The shape and length of the low-velocity regions shown Figure 6.10 are generally symmetrical about $y/d = 0$, as expected for Newtonian Stokes flow. The extent of the low-velocity region is consistent across a range of Reynolds numbers, extending approximately eight diameters upstream and downstream for the 1.0 mm cylinder, and three and a half diameters upstream and downstream for the 4.0 mm cylinder. For both cylinder diameters, some discrepancies can be seen in the high Reynolds number range (the two rightmost contours for each diameter), the result of the surface disruptions described in the previous section.

### 6.3 Viscoelastic Results with Boger Fluids

PIV measurements of the velocity field around the 1.0 and 4.0 mm cylinders were carried out for both Boger fluids, T1 and T3, in order to evaluate the effects of elasticity. The experimental test conditions are summarized in Table 6.2. Shiau’s (2013) measurements of drag identified that the onset of elastic effects for the same fluids occurred at $De = 0.6 \pm 0.1$. It was therefore important for the experimental range to encompass this region, to obtain values well above and below the expected onset value.
Table 6.2 Summary of experimental test conditions with Boger fluids T1 and T3

<table>
<thead>
<tr>
<th></th>
<th>T1 – 1.0 mm</th>
<th>T1 - 4.0 mm</th>
<th>T3 – 4.0 mm</th>
<th>T3 – 4.0 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature range</td>
<td></td>
<td></td>
<td>20 – 22.5</td>
<td></td>
</tr>
<tr>
<td>(T , [°C]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viscosity range</td>
<td></td>
<td>8.7 – 7.1</td>
<td></td>
<td>41.0 – 32.8</td>
</tr>
<tr>
<td>(\eta , [Pa.s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relaxation time</td>
<td>0.78 - 0.54</td>
<td></td>
<td>2.3 – 2.0</td>
<td></td>
</tr>
<tr>
<td>(\lambda_{N_1} , [s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular velocity range</td>
<td>2.6×10^{-3}</td>
<td>4.3×10^{-3}</td>
<td>1.1×10^{-3}</td>
<td>3.0×10^{-3}</td>
</tr>
<tr>
<td>(\omega_t , [rad/s]</td>
<td>9.6×10^{-2}</td>
<td>1.1×10^{-1}</td>
<td>6.1×10^{-2}</td>
<td>1.1×10^{-1}</td>
</tr>
<tr>
<td>Reynolds number range</td>
<td>4.4×10^{-5}</td>
<td>3.0×10^{-4}</td>
<td>4.4×10^{-6}</td>
<td>4.4×10^{-5}</td>
</tr>
<tr>
<td>(Re )</td>
<td>1.6×10^{-3}</td>
<td>7.6×10^{-3}</td>
<td>2.3×10^{-4}</td>
<td>1.5×10^{-3}</td>
</tr>
<tr>
<td>Deborah number range</td>
<td>0.25 – 10.28</td>
<td>0.10 – 2.92</td>
<td>0.34 – 20.08</td>
<td>0.24 – 9.01</td>
</tr>
<tr>
<td>(De )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[1] A discussion of the high temperature dependence of \(\lambda_{N_1} \) for T1 can be found in section 4.2.

6.3.1 Velocity Profiles

Like the Newtonian results, velocity profiles across the annular channel and along the channel centreline were extracted from the experimentally measured velocity fields. The profiles are presented alongside Newtonian counterparts to illustrate the effects of elasticity. Because good agreement was found between the experimentally measured Newtonian velocity profiles and the Fluent profiles, all viscoelastic results are plotted against the Fluent profiles. Any significant deviation with T1 or T3 from the Fluent profile for a given diameter, i.e. from the Newtonian result, constitutes an elastic effect.

The cross-channel velocity profiles in T1 and T3 for both cylinders are presented in Figures 6.11-6.14.
Figure 6.11 Cross-channel velocity profiles at the cylinder location for the flow of T1 around the 1.0 mm cylinder
Figure 6.12 Cross-channel velocity profiles at the cylinder location for the flow of T1 around the 4.0 mm cylinder.
Figure 6.13 Cross-channel velocity profiles at the cylinder location for the flow of T3 around the 1.0 mm cylinder
Figure 6.14 Cross-channel velocity profiles at the cylinder location for the flow of T3 around the 4.0 mm cylinder
The effects of elasticity on the flow are evident in the cross-channel velocity profiles presented in Figures 6.11-6.14. At low Deborah numbers ($De < 0.5$) the measured velocity profiles show good agreement with the Fluent profiles, indicating minimal elastic effects. For higher Deborah numbers, flow elasticity increased, and velocities across the channel are lower than the Newtonian values at an equivalent speed. Elastic effects are particularly evident on the inner half of the channel.

At high Deborah numbers the surface disruptions described in section 6.2.1 caused distortions in the flow images near the cylinders, which affected the measured velocity profiles. This effect is particularly evident in Figure 6.14 for the velocity profiles at $De = 5.00$ and $De = 9.01$. Additionally, for the higher $De$ flows of T3 with both cylinders, an air cavity developed at the trailing edge of the cylinder. This cavity created even larger disruptions on the fluid surface, and in some cases created a void in the test fluid at the measurement depth. Measurements of the length and depth of the air cavity for various Deborah numbers are presented later, in section 6.3.4.

The measured velocity profiles across the channel 180° opposite of the cylinder for T1 and T3 are presented in Figures 6.15-6.18.
Figure 6.15 Cross-channel velocity profiles 180° opposite of the cylinder location for the flow of T1 around the 1.0 mm cylinder.
Figure 6.16 Cross-channel velocity profiles 180° opposite of the cylinder location for the flow of T1 around the 4.0 mm cylinder
Figure 6.17 Cross-channel velocity profiles 180° opposite of the cylinder location for the flow of T3 around the 1.0 mm cylinder
Figure 6.18 Cross-channel velocity profiles $180^\circ$ opposite of the cylinder location for the flow of T3 around the 4.0 mm cylinder
Figures 6.15-6.18 show that velocity profiles measured 180° opposite the cylinder location were not affected by any surface disruptions (with the exception of the meniscus near the channel walls). As a result, continuous velocity profiles with little scatter were able to be extracted over all values of $De$ in the experimental test range. The four figures show that velocities across the channel were less than their Newtonian counterparts for $De > 0.6$. Velocities halfway across the channel show the centreline velocity deficits measured for Newtonian flow are increased with increasing flow elasticity, in some cases more than doubling the deficits of 3.0 and 10.3% measured for the 1.0 and 4.0 mm cylinders, respectively. The increased deficits are an indication that elasticity changed the velocity field everywhere in the tank, not just near the cylinder. The increases in the centerline velocity deficit with increasing flow elasticity beyond $De \approx 0.6$ are quantified in more detail in section 6.4.

The effect of elasticity on the velocity field is particularly evident in the circumferential velocity profiles, which include the upstream and downstream ‘wake’ regions. Figures 6.19-6.22 present the measured circumferential velocity profiles in T1 and T3.
Figure 6.19 Circumferential velocity profiles along the channel centreline for the flow of T1 around the 1.0 mm cylinder
Figure 6.20 Circumferential velocity profiles along the channel centreline for the flow of T1 around the 4.0 mm cylinder
Figure 6.21 Circumferential velocity profiles along the channel centreline for the flow of T3 around the 1.0 mm cylinder
Figure 6.22 Circumferential velocity profiles along the channel centreline for the flow of T3 around the 4.0 mm cylinder
For Deborah numbers below 0.5, the measured velocity profiles in Figures 6.19-6.22 exhibit symmetry upstream and downstream of the cylinder, consistent with numerical simulations and experimental Newtonian results. As $De$ is increased, the velocities are reduced along the channel centerline. This reduction in velocity is asymmetrical about the cylinder, with the most pronounced reductions seen downstream of the cylinder. The asymmetry in the upstream and downstream flows is indicative of a loss of fluid momentum, consistent with the drag enhancement reported by Shiau (2013).

6.3.2 Low-Velocity Contour Plots

Plots of the low-velocity contours around the cylinders can yield additional insight into the impact of elasticity on the flow. These plots have been created and comprise Figures 6.23-6.27. Like the Newtonian results, the low-velocity region has been defined as the region where the flow velocity is less than 50% of the approach velocity, and each contour is plotted on a Cartesian grid with its origin at the centre of the cylinder. Comparisons between the Newtonian velocity contours and the contours in T1 and T3, for the lowest Deborah numbers, are presented in Figure 6.23 for both cylinder diameters. The $De$ values are all below the expected onset value of 0.5, and so Newtonian behaviour is expected.
Figure 6.23 Low-velocity contours in Boger fluids at the lowest experimental Deborah numbers, compared with contours in polybutene. Since $De$ is below the expected onset value of 0.5 for each Boger fluid, Newtonian behaviour is expected.

The shape and length of the low-velocity regions are consistent across all three fluids, indicating essentially Newtonian behaviour with minimal or no elastic effects when $De$ is below the expected onset value of 0.5.
Low-velocity contours around both cylinders in T1 and T3 are presented in Figures 6.24-6.27. At higher Deborah numbers for the 4.0 mm cylinder in T3, the downstream wake extended well beyond the measured flow area (limited by the travel of the milling table used for positioning the CCD camera). As a result, in order to fit a closed velocity contour at each value of $De$, the low-velocity region in the high $De$ range of Figure 6.27 has been defined as the region having non-dimensional velocities less than 30% of the approach velocity.
**Fluid: T1**

\[ \text{d=1.0 mm} \]

\[
\begin{align*}
\text{Re} &= 4.5 \times 10^5, \quad \text{De} = 0.25 \\
\text{Re} &= 1.1 \times 10^4, \quad \text{De} = 0.60 \\
\text{Re} &= 1.5 \times 10^4, \quad \text{De} = 0.87 \\
\text{Re} &= 2.3 \times 10^4, \quad \text{De} = 1.30 \\
\text{Re} &= 4.6 \times 10^4, \quad \text{De} = 2.98 \\
\text{Re} &= 9.3 \times 10^4, \quad \text{De} = 6.03 \\
\text{Re} &= 1.6 \times 10^5, \quad \text{De} = 10.28
\end{align*}
\]

**Figure 6.24** Low-velocity contours \((|V|/V_c \leq 0.5)\) around the 1.0 mm cylinder in T1 for increasing Deborah numbers. Elastic effects are evident at \(De = 0.87\) and beyond.
**Fluid: T1**

\[ d = 4.0 \text{ mm} \]

Re=3.0×10^{-4}  \quad De=0.10

Re=6.9×10^{-4}  \quad De=0.25

Re=1.8×10^{-3}  \quad De=0.60

Re=2.2×10^{-3}  \quad De=0.85

Re=3.4×10^{-3}  \quad De=1.32

Re=7.6×10^{-3}  \quad De=2.92

**Figure 6.25** Low-velocity contours (|V|/Vc ≤ 0.5) around the 4.0 mm cylinder in T1 for increasing Deborah numbers. Elastic effects are evident at De = 0.60 and beyond.
Fluid: T3

d=1.0 mm

Figure 6.26 Low-velocity contours \((|V|/V_c \leq 0.5)\) around the 1.0 mm cylinder in T3 for increasing Deborah numbers. Elastic effects are evident at \(De = 0.85\) and beyond.
Fluid: T3
d=4.0 mm

Figure 6.27 Low-velocity contours around the 4.0 mm cylinder in T3 for increasing Deborah numbers. **Top:** Low $De$ contours, with the low-velocity region defined as $|V|/V_c \leq 0.5$. **Bottom:** High $De$ contours, with the low-velocity region defined as $|V|/V_c \leq 0.3$. Elastic effects are evident at $De = 0.62$ and beyond.
Since it has been established that the low $De$ contours in Figure 6.23 match the Newtonian contours reasonably well, any difference in the shape and length of the contours at higher $De$ is directly attributable to flow elasticity. It’s clear from Figures 6.24-6.27 that as $De$ increases, the length of the low-velocity region increases as well. Additionally, significant asymmetry between the upstream and downstream regions is developed. The increased asymmetry is a clear indication of the drag enhancement measured by Shiau (2013).

A notable feature of Figure 6.27 is the irregularity in the contours at the trailing edge of the cylinder at high $De$. The distortion resulted from the surface disruptions and air cavity which hampered flow measurements.

Table 6.3 presents the lowest value of $De$ at which a deviation in the length and shape of the low-velocity region is evident for each test case. Since measurements were made at discrete, limited intervals in the experimental test range, these values should be taken only as estimates of the upper bound of Deborah numbers at which elastic effects become significant. To pinpoint more precisely the onset of elastic effects would require more measurements in the low $De$ range. Such an endeavour may not be worthwhile, since the Deborah number is highly dependent on the method used to determine the fluid relaxation time, which can vary greatly for different constitutive equations and changes in the rheological characterization methods. Determination of elastic onset is also dependent on the sensitivity of the measurement method; in practice, onset is expected to be a gradual process rather than a sharp change in the flow regime.

<table>
<thead>
<tr>
<th></th>
<th>1.0 mm</th>
<th>4.0 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T1</strong></td>
<td>$De = 0.87$</td>
<td>$De = 0.60$</td>
</tr>
<tr>
<td><strong>T3</strong></td>
<td>$De = 0.85$</td>
<td>$De = 0.62$</td>
</tr>
</tbody>
</table>

That the measured values in Table 6.3 match up so closely for each cylinder diameter across both test fluids may be an indication that elastic onset is in part dependent on the experimental geometry, namely the channel width to diameter ratio, though no firm conclusions can be drawn from these four data points alone. No clear trend relating geometry and onset is evident from Shiau’s (2013) data. His experiments measured drag for cylinders ranging in diameter from 1.19
to 3.34 mm; elastic onset was observed for $0.5 \leq De \leq 0.7$, with the Deborah number defined in the same manner as in the present work (Eq. 3.4).

### 6.3.3 Deformation Rates and the Role of $N_f$

Fluid elasticity is caused by stretching polymers at high shear and extensional rates. A first attempt at the calculation of rate values from experimental data involved numerical differentiation of the measured velocity field data using a fourth-order accurate central difference approximation. Though shear and extensional rates were successfully calculated using this approach, scatter in velocity data led to large fluctuations in the deformation rates. In order to present a clearer picture of the shear and extensional rates in the flow, smooth curves were fitted to the experimental velocity profiles using MATLAB, and the resulting continuous functions were differentiated.

In their numerical work, Chilcott & Rallison (1988) suggested that regions of high extension in the immediate vicinity of the upstream and downstream stagnation points, along with high shear rates in the flow around the walls of the cylinder, would lead to polymer extension and deformation, followed by a gradual relaxation of the polymer back to its undeformed state as it flows downstream. The stresses developed as result of the polymer deformation lead to additional stresses on the cylinder surface, and any net force resulting from an upstream/downstream asymmetry in these stresses will generate additional drag (François et al. 2008).

Differentiation of the fitted curves yielded the two following strain rate components:

$$\frac{d|V_y|}{dx} = \dot{\gamma}_x \tag{6.2}$$

$$\frac{d|V_\theta|}{ds} = \dot{\varepsilon}_\theta \tag{6.3}$$

where $|V_y|$ & $|V_\theta|$ are the magnitudes of the cross-channel velocity in the $y$-direction (for the Cartesian coordinate system defined in section 6.2.2) and circumferential velocity in the $\theta$-direction (for the cylindrical coordinate system defined in section 6.1), respectively. $x$ & $s$ are
the positions across the channel and along the channel centreline, and $\dot{\gamma}_x$ & $\dot{\varepsilon}_\theta$ represent the calculated strain rate components.

In order to validate the procedure outlined above, shear and extensional rate profiles calculated from Fluent predictions were compared to the profiles derived from experimental data at low Deborah numbers. The profiles were multiplied by their respective characteristic flow times, $t_c$ (defined in Eq. 3.2), in order to facilitate the comparison between data at different rotation speeds. Sample plots are presented in Figures 6.29 & 6.28.

![Graph](image)

**Figure 6.28** Comparison of the measured circumferential extensional rates with the Fluent prediction for the 4.0 mm cylinder in T1 at $De = 0.10$. 

Figure 6.29  Comparison of the measured cross-channel shear rates with the Fluent prediction for the 4.0 mm cylinder in T1 at $De = 0.10$.

Good agreement was found between the Fluent and experimental rate profiles, validating the method of differentiating fitted curves to calculate deformation rates. Since the experimental velocity measurements near the cylinders were susceptible to errors (a result of the meniscus around the cylinder and the surface disruptions described in section 6.2.1), some discrepancies between the Fluent and experimental rate profiles near the cylinder were expected and are evident in Figures 6.28 & 6.29.

Figures 6.30-6.33 present the extensional rate profiles derived from the circumferential velocity profiles. In order to assess the magnitude of the deformation rates relative to the polymer relaxation, the extensional rates in these plots have been non-dimensionalized by the fluid relaxation times at their respective experimental temperatures.
**Figure 6.30** Circumferential extensional rate profiles for a flow of T1 around the 1.0 mm cylinder

**Figure 6.31** Circumferential extensional rate profiles for a flow of T1 around the 4.0 mm cylinder
Figure 6.32 Circumferential extensional rate profiles for a flow of T3 around the 1.0 mm cylinder

Figure 6.33 Circumferential extensional rate profiles for a flow of T3 around the 4.0 mm cylinder
Accurate measurement of the non-dimensional extensional rate at the cylinder walls was not always possible due to the surface disruptions detailed above. These disruptions were particularly impactful for the 4.0 mm cylinder (Figures 6.31 & 6.33) and at high Deborah numbers.

Though the range of the y-axis varies between the plots, the magnitude of the extensional rates at a given Deborah number is fairly consistent between the two fluids for a given diameter. For example, Figures 6.31 & 6.33 for flow with $De = 0.85$ and $d = 4.0$ mm show a maximum (negative) value of $\dot{\varepsilon}_0 \lambda_{N_1}$ of approximately 0.17 for both T1 and T3. The extensional rates were much lower for the smaller cylinder, being roughly halved in magnitude at comparable Deborah numbers.

In steady extensional flows, linear elastic constitutive models, including the Oldroyd-B model, predict that polymer coils will remain unstretched when $\dot{\varepsilon} \lambda < 0.5$ (Larson 1999). Non-dimensional extensional rates above 0.5 will lead to polymer extension as viscous drag forces overcome the contraction forces of the polymer coil, and will result in extensional stresses. Thus, non-dimensional extensional rates above 0.5 are required for significant elastic effects resulting from extensional stresses.

In Figures 6.30-6.33 the values of $\dot{\varepsilon}_0 \lambda_{N_1}$ rarely exceed 0.5, and do so only for high Deborah numbers. For experiments with $De$ between 0.5 and 1.5, where pronounced elastic effects were evident in the low-velocity contours of Figures 6.24-6.27, the maximum non-dimensional extensional rates were five to ten times lower than the value necessary for elastic effects resulting from extension. Thus, stresses generated in extension alone cannot be the cause of the drag enhancement reported by Shiau (2013), or the associated suppressed downstream wake velocities reported in this thesis.

The other mechanism which could account for drag enhancement is the development of normal stresses in shear. Figures 6.34-6.37 present the shear rate profiles derived from experimental cross-channel velocity profiles, non-dimensionalized by their respective values of $\lambda_{N_1}$.
Figure 6.34 Cross-channel shear rate profiles for the flow of T1 around the 1.0 mm cylinder

Figure 6.35 Cross-channel shear rate profiles for the flow of T1 around the 4.0 mm cylinder
Figure 6.36 Cross-channel shear rate profiles for the flow of T3 around the 1.0 mm cylinder

Figure 6.37 Cross-channel shear rate profiles for the flow of T3 around the 4.0 mm cylinder
Like the extensional rates, the shear rates at the cylinder walls can be subject to large discrepancies due to the surface disruptions detailed above, particularly for the 4.0 mm cylinder and at high Deborah numbers. Shear rate profiles for the 4.0 mm cylinder in T3 at $De = 5.00$ and 9.01 were deemed too unreliable, and have been omitted in Figure 6.37 for this reason.

Comparison of the magnitude of $\dot{\gamma}_x \lambda N_1$ in Figures 6.34-6.37 with values of $\dot{\varepsilon}_\theta \lambda N_1$ (Figures 6.30-6.33) reveals maximum shear rates are typically double the extensional rates for the same diameter and Deborah number. It is reasonable, then, to consider the role of shearing in the drag increase observed by Shiau.

As shown in Figure 2.2, shearing of a polymer molecule will lead to the development of normal stresses. The first normal stress difference, $N_1$, is the difference between the two normal stress components and was defined in Eq. 2.6. Recent experiments by James et al. (2012) showed that the drag enhancement caused by elasticity for the flow of Boger fluids through arrays of cylinders was caused by $N_1$ stresses. An analysis by James (2014) of other experiments with elastic effects attributed to extensional stresses found several other instances where $N_1$ stresses are at least as relevant as those due to extension, including the case of flow around a confined cylinder.

The action of $N_1$ stresses on a cylinder was proposed in James et al. (2012). In normal-tangential coordinates relative to a given streamline, the components of $N_1$ can be written as:

$$N_1 = \sigma_s - \sigma_n$$

[6.4]

where $\sigma_s$ and $\sigma_n$ are the streamwise and normal stress components, respectively. Figure 6.38 explores the development of a drag force resulting from $N_1$ stresses.
Figure 6.38 Drag increase from flow elasticity. **A.** Schematic of the stress imbalance resulting from the development of $N_1$ stresses in shear, adapted from James et al. (2012). The normal stress component $\sigma_n$ acts directly on the cylinder surface, while the streamwise stress component $\sigma_s$ generates a hoop stress on the cylinder walls. Asymmetry in the shear rate, being higher on the upstream side of the cylinder, before dissipating as the flow passes around the cylinder, leads to a stress imbalance and a net drag force (François et al. 2008).

**B.** Streamlines generated from experimental velocity data for the 1.0 mm cylinder in T3 at $De = 3.00$. 
Given the relative magnitude of the extensional and shear rates, and the asymmetry between the upstream and downstream flow regions, it is reasonable to conclude that the previously reported drag enhancement was caused by normal stresses developed in shear.

6.3.4 Air Cavity Characterization

In addition to the surface disruptions described in section 6.2.1, air cavities developed behind both cylinders in T3 at high Deborah numbers. For the 1.0 mm cylinder, the cavity developed only for $De = 20.0$, which was the highest value of $De$ tested over all experiments. The extent of the cavity is depicted in Figure 6.39.
While making measurements of the cavity, a slight deflection of the 1.0 mm cylinder was observed. This deflection was 1.6° at $De=10.14$ and increased to 3.6° for $De=20.01$ and would have resulted in a maximum shift in the position of the cylinder in the measurement plane of approximately 1.25 mm. This shift is significant, being greater than the size of the cylinder, and could have impacted the high $De$ measurements since the actual position of the cylinder would

**Figure 6.39** Air cavity behind the 1.0 mm cylinder. The cavity extended 3.0 diameters in depth and 1.4 diameters in length downstream.
have been shifted downstream relative to the assumed 0° position. Measured flow velocities at the trailing edge would then be lower than the actual velocities.

Figure 6.40 shows that the cavity developed behind the 4.0 mm cylinder in T3 became apparent when $De = 1.99$, and increased with $De$, covering the entire immersed length of the cylinder (80 mm, 20$d$) when $De$ was 9.04. Measurements of the depth and downstream length of the cavity are presented in Figure 6.41.
Figure 6.40 Development of the air cavity behind the 4.0 mm cylinder with increasing Deborah number.
Figure 6.41 Measurements of the depth and length of the air cavity developed behind the 4.0 mm cylinder in T3.

Unlike the 1.0 mm cylinder, no deflection of the 4.0 mm cylinder was observed at any value of $De$ or $Re$, since the larger diameter led to increased stiffness.

While making drag measurements in the same annular channel, Wang (2012) also observed cavities at the trailing edge of the cylinder. His experiments were carried out at higher speeds, with Reynolds numbers on the order of $10^3$ to $10^2$ and Deborah numbers of order $10^1$ to $10^2$. Notably, even at high speeds, Wang did not observe any cavity with his Newtonian fluid, confirming that cavity formation in the Stokes regime is caused by elasticity. In a surprising result, he found that the development of the air cavity had no effect on the drag, suggesting the drag enhancement at high Deborah numbers is dominated by stresses acting on the leading edge, and that stresses acting on the trailing edge are negligibly small.

6.4 Centreline Velocity Deficit

The velocity profiles across the channel at 180° (Figures 6.5-6.6 for the polybutene and Figures 6.15-6.18 for the Boger fluids) yield the velocity at the centre of the channel for each cylinder in
Newtonian and viscoelastic flows. Velocities are lower than the approach velocity predicted by rigid body rotation (Eq. 3.3), and the deficit between the predicted and the measured velocities increases with \(De\). Having accurate values of the centreline velocity will facilitate corrections to the drag measurements reported by Shiau (2013), presented in section 6.4.1, and values of the ‘freestream’ velocity will be used in section 6.5 to make comparisons to previously reported velocity values.

Focusing on the centre of the channel at 180°, the velocity deficits measured experimentally and computed numerically for the 1.0 and 4.0 mm cylinders, have been compiled and are presented in Figures 6.42 & 6.43.

**Figure 6.42** Compilation of centreline velocity deficits for the 1.0 mm cylinder. The values of \(Re\) and \(De\) are for the Newtonian and viscoelastic fluids, respectively.
Figure 6.43 Compilation of centreline velocity deficits for the 4.0 mm cylinder. The values of $Re$ and $De$ are for the Newtonian and viscoelastic fluids, respectively.

Though there is considerable scatter in the data, the general trend of elastic onset near $De \approx 1$ is clear. Measurements made with the 4.0 mm cylinder exhibit less scatter since, for the same fluid at comparable Deborah numbers, flow velocities are four times higher for the 4.0 mm cylinder. The centreline velocity deficit measurements, having been made 180° opposite of the immersed cylinder, were not impacted by the surface disruptions near the cylinder walls. Measurement error is then governed by the linear relation presented in Figure 5.9, with high angular velocities leading to less measurement error.

6.4.1 Corrections to Prior Drag Measurements

When conducting his drag measurements, Shiau (2013) assumed a linear velocity profile across the annular channel, governed by Eq. 5.8 for rigid body rotation, because the present measurements were not available. In order to accurately determine the effects of elasticity and facilitate valid comparisons between Shiau’s wall-corrected low $De$ results and Kaplun’s unbounded solution, the Newtonian velocity deficits should be accounted for in Shiau’s corrected data. Since he studied cylinder diameters and immersion depths which were different from those in the present study, additional measurements of the centreline velocity deficit were
required to determine the magnitude of correction needed. Additional Fluent simulations, carried out in the same manner described in section 6.1, were performed for cylinders with diameters of 1.0, 2.0 and 4.0 mm, with immersion depths varying from 10 mm to 80 mm. These ranges encompassed all of Shiau’s experimentally measured diameters and depths. The resulting centreline velocity deficits for each cylinder diameter at each immersion depth were then plotted in MATLAB, and bilinear interpolation was used to derive corrections to the centreline velocities for each combination of diameter and depth measured by Shiau.

Figure 6.44 presents the surface derived from interpolation between the numerically measured centreline velocities, expressed as a percentage of the approach velocity determined from rigid body rotation (Eq. 3.3), for various cylinder diameters at varying depths.

![Figure 6.44](image)

**Figure 6.44** Fluent predicted centreline velocities for 1.0, 2.0 and 4.0 mm cylinders at various depths with interpolated surface.

Shiau measured the drag on cylinders at small immersion depths compared to the present study, and the Fluent predictions indicate a decreasing centreline velocity deficit with decreasing immersion depth. This means that the resulting corrections to Shiau’s data fall between 1.0 and
4.0% and represent only a small shift in his results. Corrected drag results are presented in Figures 6.45 & 6.46.

**Figure 6.45** Measurements of drag on cylinders in T1 from Shiau (2013), corrected for the centreline velocity deficit. Legend values are the cylinder diameter and immersion depth, respectively. $H/d$ was 267 for 1.19 mm cylinder and 40 for the 3.34 mm cylinder. Corrections have been applied to account for wall and end effects. Uncorrected data were presented in Figure 2.6.
Figure 6.46 Measurements of drag on cylinders in T3 from Shiau (2013), corrected for the centreline velocity deficit. Legend values are the cylinder diameter and immersion depth, respectively. $H/d$ was 267 for 1.19 mm cylinder and 40 for the 3.34 mm cylinder. Corrections have been applied to account for wall and end effects.

Comparison of Shiau’s data for T1 presented in Figure 2.6 (uncorrected for the centreline velocity deficit) with the newly corrected results in Figure 6.45 shows a slight improvement in the agreement between the Newtonian equivalent drag and Shiau’s measured drag at low Deborah numbers. The corrections do not change his conclusion that onset started for $De$ between 0.5 and 0.7.

6.5 Comparison with Previously Reported Velocity Profiles

The only two previous studies which included values of the velocity field for unbounded flow or flow in wide channels were the unbounded numerical study of Chilcott & Rallison (1988) and the experimental work of Shiang et al. (1997), with $H/d = 16$. Chilcott & Rallison imposed a uniform flow condition at a circular boundary 10 diameters away. Shiang et al. also studied a uniform flow, which was created using a translating channel. Velocity data from both groups
was non-dimensionalized by the uniform flow velocity, $U$, with a value of unity indicating freestream conditions.

As demonstrated in section 6.4 of this thesis, for the annular geometry under consideration there is no single value of the ‘freestream’ velocity at a given experimental speed, due to the centreline velocity deficit which is dependent on diameter, immersion depth and Deborah number. Therefore, in order to make valid comparisons between prior velocity data and the velocities measured in this thesis, the current velocity values should not be non-dimensionalized by the approach velocity $V_c$, defined in Eq. 3.3 (which depends only on the tank rotation speed) as is done elsewhere in this thesis. Instead, a new velocity, $V_{eff}$, is defined as:

$$V_{eff} = V_c \times (100\% - \text{centreline velocity deficit})$$

which is termed the effective freestream velocity. $V_{eff}$ accounts for the deviation of the velocity field 180° opposite of the cylinder from the linear velocity profile produced by rigid body rotation, and allows for a more appropriate comparison to literature values non-dimensionalized by a uniform flow velocity.

Velocity measurements from this thesis are compared to velocity profiles published by Chilcott & Rallison in Figure 6.47. All profiles represent velocities along the primary flow axis, which was linear for Chilcott & Rallison and circumferential in this thesis, and the distance from the cylinder has been non-dimensionalized by the diameter.
Figure 6.47 Comparison of measured circumferential velocity profiles at $De \approx 10$ with the numerical results of Chilcott & Rallison (1988).
For non-Newtonian flow, Chilcott & Rallison only reported velocity profiles for $De = 10$, and so only the profiles from this thesis having a similar Deborah number were included in Figure 6.47. Because of the uniform velocity boundary condition they imposed, their reported non-dimensional velocities must reach values of unity when $x/d = \pm 10$. This leads to large differences between the experimental measurements of this study at $De = 10$, and does not accurately reflect the nature of the elastic effects, which have been shown in this study to extend much further downstream than upstream. As noted in section 2.2.2, their choice to impose the uniform velocity boundary condition so close to the cylinder was strongly influenced by the limited computing power of their day. Of course, the curvature of the annular tank could also have contributed to the differences between the two datasets.

Chilcott & Rallison expressed the relative contributions of the polymer and the solvent to the viscosity of their modelled fluid using the parameter $c$:

$$c = \frac{\eta}{\eta_s} - 1$$

[6.6]

where $\eta$ and $\eta_s$ are the viscosity and the solvent contribution to the viscosity. Their modelled fluid had $c = 0.5$ for the velocity profile included in Figure 6.47. T1 and T3 had values of $c$ equal to 0.63 and 0.64, respectively, at the experimental temperatures of the included profiles. The difference in fluid properties could have accounted for some of the discrepancy between the two datasets.

Shiau (2013) didn’t make drag measurements at $De \approx 10$, but interpolation between his lower $De$ measurements ($De \leq 4$) and measurements he performed at higher Deborah numbers ($De \geq 17$) yields an approximate range for the drag coefficient. For T1, the non-dimensional drag at $De \approx 10$ is expected to range from 1.7 to 2.0, whereas for T3 the range is 2.3 to 2.7. Chilcott & Rallison reported a non-dimensional drag of 1.75, at the low end of values predicted Shiau’s data. Thus, while Chilcott & Rallison’s velocity values were likely skewed by the uniform flow boundary condition, their results for the drag increase resulting from elasticity, relative to the expected Newtonian value, may be more reliable.
In Figure 6.48, velocity measurements from this thesis are compared to the velocity profile published by Shiang et al. at $De \approx 3$. The Boger fluid studied by Shiang et al. had properties comparable to T3, with a viscosity of 43 Pa.s and a relaxation time of 1.8 s. Their fluid had a value of $c$ equal to 0.54, indicating a lower polymer contribution to the viscosity than either T1 or T3.
Figure 6.48 Comparison of measured circumferential velocity profiles at $De \approx 3$ with the experimental results of Shiang et al. (1997). Upstream velocity values were not reported by Shiang et al.
Shiang et al. only reported downstream velocity values. Since their experiments were carried out in a comparatively narrower channel, with $H/d = 16$, comparison between their measurements and the values reported in this thesis, with $H/d = 33$ and 133, is difficult. Qualitatively, the shape of the velocity profile reported by Shiang et al. is consistent with the experimental measurements of this thesis. Furthermore, better agreement is seen between their values the current results with $H/d = 33$ rather than 133, indicating possible convergence as the channel width to diameter ratio approach a common value. Additional velocity measurements in the annular channel with $H/d = 16$ (requiring a diameter of 8.3 mm for the annular tanks of this thesis) would be needed to make a definitive assessment of the agreement between the datasets.

The curvature of the annular tank may limit the validity of comparisons to experiments carried out in straight channels.
Chapter 7
Concluding Remarks

7.1 Summary

This thesis examined the effect of elasticity on the flow field around cylinders in viscoelastic Stokes flows. Numerical and experimental results with a Newtonian fluid provided a baseline, and experimental results with two Boger fluids were used to isolate for the effects of elasticity without the complication of shear-thinning.

An annular channel, rather than a straight channel, and a low-speed gearmotor generated a known, steady flow field. Sufficiently small cylinder diameters yielded channel width to diameter ratios of 33 and 133, larger than any previously reported experimental velocity measurements, with the intention of isolating effects related only to the cylinders.

PIV was used to measure the velocity field in multiple locations around the cylinder and at 180° opposite of the cylinder. Flow image sequences at individual viewing windows were analysed separately via cross-correlation analysis and later combined to form a single dataset encompassing a large area around the cylinder. This procedure was validated by making measurements of the known velocity field in the annular tank with no cylinder in the flow.

Numerical simulations performed in ANSYS Fluent indicated a deficit in flow velocity along the channel centreline approaching a cylinder. This deficit was confirmed through results with Newtonian polybutene. Newtonian velocity profiles generated numerically and measured experimentally exhibited symmetrical upstream and downstream velocities, as expected for Stokes flow.

The effects of elasticity were measurable beyond $De = 0.85$ for the 1.0 mm cylinder and $De = 0.60$ for the 4.0 mm cylinder. Fluid elasticity decreased the velocities around the cylinder, with more pronounced decreases downstream and in the inner half of the annular channel. Increasing flow elasticity led to asymmetry between the upstream and downstream velocities, with lower downstream velocities indicative of a drag enhancement.
Differentiation of the velocity profiles yielded values of the shear and extensional rates across the annular channel and circumferentially along the channel centreline, respectively. Shear and extensional rates were calculated close to the cylinders and near the upstream and downstream stagnation points. Based on previously reported studies, the elastic flow effects were attributed to polymer extension in these regions, which resulted in increased stresses acting on the cylinder. Extensional rates were found to be too low to generate significant stresses and account for the observed elastic effects. Normal stresses, resulting from shearing of polymer molecules, are suggested as the dominant stress components leading to the drag enhancement previously reported by Shiau (2013).

Comparisons of the experimental velocity profiles to prior experimental and numerical studies showed large discrepancies. Differences between the studies (such as channel geometry, fluid rheology and the imposed boundary conditions) make drawing any conclusions from the comparisons difficult.

7.2 Conclusions

- Fluid elasticity caused lower velocities relative to Newtonian values, particularly downstream of the cylinder and in the inside half of the annular channel, for Deborah numbers above 0.6.

- Surface disruptions, and an air cavity behind the cylinder, were observed at high Reynolds and Deborah numbers, respectively, limiting the accuracy of optical measurements near the cylinder.

- The decreasing velocities and increasing asymmetry in the wake region with $De$ are consistent with the drag enhancements reported by Shiau (2013).

- The high strain rates near the stagnation points and near the cylinder surface likely resulted in high polymer deformation in those regions. The generated stress components associated with such a deformation are proposed as the cause of the reported drag enhancement.
The relative magnitudes of extensional and shear rates suggest $N_1$ stresses developed in shear flow around the upstream cylinder walls are the primary contributor to the drag enhancement.
References
Faxèn O. 1946. Forces exerted on a rigid cylinder in a viscous fluid between two parallel fixed planes


Kiger K. 2009. Introduction of Particle Image Velocimetry (slides)


