THE DEVELOPMENT OF A SENSOR TO DETERMINE VELOCITY IN LIQUID ALUMINUM

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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Abstract

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This thesis presents two sensor designs to determine velocity of liquid aluminum. These sensors are referred to as Sensor1 and Sensor2 where the principle of operation is based on forced convective heat transfer. Sensor1 is constructed of one material while Sensor2 is comprised of two materials, the geometry of both is that of a circular cylinder. The immersion of the sensor into the flow can be classified as a circular cylinder in liquid aluminum crossflow. A calculation of the localized Nusselt number acting on the surface of the sensor produces an estimate of the magnitude and direction of velocity. The localized Nusselt number estimate is provided by more than one thermocouple temperature measurement from within the sensor. The Sensor1 and Sensor2 concepts were constructed and tested in flowing liquid aluminum and numerical models were developed to study the heat transfer between the liquid and sensor.

A peculiarity of this liquid metal system is that aluminum may solidify and subsequently melt onto the surface of the sensors; the solid aluminum is referred to as shell. The growth of the shell depends on various conditions, one examined in this work is the thermal resistance at the Sensor1-aluminum interface. An enthalpy numerical model is used to show that the thermal resistance at the Sensor1-aluminum interface has an important effect on the solidification and melting times of shell. The numerical model is validated with experimental measurements of localized shell solidification and melting times.

To estimate the velocity of liquid aluminum the solidification of shell must be suppressed. By incorporating two materials of drastically different thermal properties, various regions of Sensor2 may be treated as lumped systems, this mathematical treatment provides fast estimates
of the localized Nusselt number, hence the velocity can be determined in real-time. The modified Nusselt number estimated by Sensor2 is a sum of the thermal resistance from the thermal boundary layer and thermal resistance at the Sensor2-aluminum interface. By numerically modeling the flow of liquid aluminum over the Sensor2 design and by comparing these results to experimental data, the thermal resistance at the Sensor2-aluminum interface is estimated. A technique to determine the magnitude and direction of velocity from the local Nusselt number of the Sensor2 design is presented.
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E.5 Applied Cosine Distribution Heat Transfer Coefficient Function

E.6 Temperature Response and Estimated Heat Transfer Coefficient for a Cosine Function Heat Transfer Coefficient
Nomenclature

\( \Delta H \)  latent heat content
\( \Delta r \)  resistance layer thickness
\( \Delta \)  difference
\( A \)  area
\( A \)  mushy zone damping parameter
\( b \)  constant
\( b \)  time constant
\( C \)  constant temperature distribution
\( C \)  damping constant
\( c \)  specific heat
\( D \)  diameter
\( f \)  fraction
\( H \)  enthalpy
\( h \)  heat transfer coefficient
\( I \)  number of nodes in r direction
\( J \)  Bessel function of the first kind
\( J \)  number of nodes in \( \Theta \) direction
\( k \)  thermal conductivity
\( L \)  latent heat
\( L \)  number of thermocouples
\( n \)  normal direction
\( P \)  pressure
\( R \)  thermal resistance
\( r \)  radial coordinate
\( R^2 \)  coefficient of determination
\( RPM \)  revolution per minute
\( S \)  interface position
\( S \)  source term
$SPH$  liquid superheat
$T$    temperature
$t$    time
$U$    uncertainty
$u$    x-component of velocity
$V$    volume
$v$    y-component of velocity
$x$    x-coordinate
$Y$    measured temperature
$y$    value of data set
$y$    y-coordinate
$\text{Bi}$ Biot Number $= (h V)/(k_s A_s)$
$\text{Fo}$ Fourier number $= (\alpha t A_s^2)/V^2$
$\text{Nu}$ Nusselt number $= (h D)/k$
$\text{Pr}$ Prandtl number $= (\mu c_p)/k$
$\text{Re}$ Reynolds number $= (\rho u D)/\mu$

**Subscripts**

$cyl$  cylinder  
$eff$  effective  
$f$    finish  
$i$    initial  
$in$   inner radius  
$int$  interface  
$l$    liquid  
$l$    local  
$l$    thermocouple sensor number  
$m$    melting  
$m$    time index  
$out$  outer radius  
$r$    future time index  
$ref$  reference  
$s$    solid  
$s$    surface  
$sen$  thermocouple sensor  
$\text{Al}$  aluminum  
$la$   local average
S2    Sensor2

**Greek Symbols**

α    thermal diffusivity  
β    heat transfer coefficient parameter estimate  
ε    noise component  
∞    free stream  
λ    eigenvalue  
μ    dynamic viscosity  
ω    angular velocity  
ρ    density  
Θ    polar coordinate

**Superscripts**

*  non-dimensional value  
^  estimate  
→  vector  
i  index  
n  iteration index  
T  transpose  
-  average
Chapter 1

Introduction

1.1 Scope

During the late 1800’s the Hall-Heroult process was patented which pioneered commercial aluminum production. Aluminum is processed as a liquid, iron or steel production began long before this time, where steel is also processed as a liquid. Over the centuries engineers have worked to improve these processes; one means to improve a process is to study its transport variables. Measurement of the transport variables allow process events to be characterized, from this understanding an improved process may be designed. However, at the time of this research, there was a lack of sensors available to measure transport phenomena events in high temperature liquid metals. A strategic report [1] written in the year 2011 - produced by The Minerals, Metals, and Materials Society (TMS), titled: Linking Transformational Materials and Processing for an Energy Efficient and Low Carbon Economy - stated that there was a need in materials processing industries for molten metal sensors. The TMS report claimed that the introduction of such sensors would increase process efficiency up to 15%.

In this setting, high temperature is defined as greater than 933 K (660 °C). Pure aluminum undergoes solid-to-liquid phase transformation at this temperature, thus aluminum is classified as a high temperature liquid metal. Liquid steel is processed in excess of 1873 K (1600 °C), it
is another fluid classified as a high temperature liquid metal. Aluminum and steel are the two largest produced metals worldwide, where the processing steps are conducive for monitoring fluid velocity, yet there was no technique in wide scale deployment to measure the velocity of a high temperature liquid metal.

One may consider that the detection of velocity in a fluid is a standard measurement. This statement is true for air and water, which are common fluids. Hot-wire anemometry and laser-doppler velocimetry are but two techniques to probe the common fluid. Reviews of techniques to measure the velocity of a common fluid may be found in Goldstein [2] and Tavoularis [3]. A liquid metal is not a common fluid. High temperature liquid metals are corrosive and opaque which preclude the use of hot-wire and laser-doppler anemometry to probe liquid metal velocity.

With respect to primary aluminum production, measuring velocity in an aluminum reduction cell is important as the metal pad velocity affects the dissolution of alumina feed, the heat transfer within the cell and the current efficiency [4, 5]. The local intensity of the metal flow in an aluminum reduction cell also affects the relative ledge protection (amount of solidified aluminum) on the sidewall of the reactor [4]. The ledge protection helps to prevent the erosion of the refractory lining. Some operating variables of the aluminum reduction cell are the magnetic field strength and the current distribution within the cell [4]. An improved understanding of the relationship between the magnetic field strength and the current distribution in relation to the aluminum velocity may lead to cell design changes which reduce wall erosion and improve the overall operation of the cell. Measuring the localized velocity in an aluminum reduction cell will equip engineers with a means to characterize the cell performance, this in turn will improve day to day operating practices of existing cells and help design new improved aluminum reduction cells.

During secondary aluminum production, measuring the velocity of liquid aluminum helps control the production process in the following manner: aluminum scrap is typically melted in a reverberatory furnace, after melting the aluminum flows from the furnace through launders (a
series of channels) into converters for refining the alloy composition, then to a casting machine. In general, data relating to the production process is taken at the beginning and the end of the process. This data relates to the mass of aluminum scrap charged into the furnace and mass of aluminum that is solidified upon casting [6]. Having a measure of the velocity of liquid aluminum flowing in the launders will facilitate the calculation of aluminum mass flux and will aid operators in knowing precisely the quantity of aluminum that was drained from the furnace.

For steel production: many of the defects affecting steel quality are associated with the flow structure in the upper part of the mould of a continuous caster [7]. Asymmetric flows in the mould can lead to undesirable oscillations of the meniscus, where these fluctuations can trap mould slag which form as inclusions in the cast steel. Insufficient flow of steel at the surface of the mould causes the meniscus to freeze and this in turn causes surface defects in the cast steel [7]. Much research is conducted on optimizing the mould flow pattern to achieve stable casting conditions which mitigate low steel quality. Due to the lack of direct measurement methods for high temperature liquid metals the effectiveness of these optimized flow patterns in the mould cannot be monitored in liquid steel. Monitoring the velocity of steel in the mould of a continuous caster is desirable in that it will identify the flow structure and aid to predict quality of continuously cast steel.

Lastly, with respect to liquid metal alloy refining operations: advances have been made in understanding fluid flow phenomena and mixing in ladle metallurgical operations through mathematical modeling [8, 9]. There has also been interest in recirculating flows in molten metal systems as these flows are prevalent in the melt of induction furnaces [10], where these induction furnaces are used to create specialty metal alloys. The ability to experimentally study these high temperature flows and verify the mathematical models of these high temperature systems is limited because of the lack of high temperature liquid metal velocity sensors.

To address the need for liquid metal sensors, this research is concerned with the development of a high temperature liquid metal velocity sensor, the experimental fluid in focus is
1.1.1 Liquid Metal Velocity Measurement

Many techniques have been explored to detect liquid metal velocity, each employing a different measuring principle. Reviews of these techniques include those of Argyropoulos [11], Iguchi et al. [12], and Eckert et al. [13]. The terminology “contact” or “non-contact” is used to describe the measurement technique, where a contact method requires the sensor and fluid to physically interact. Generally the sensor should be placed at the location where one desires a velocity measurement. It is important to distinguish whether the technique makes contact with the liquid because at times, engineers are interested in probing the velocity so that the instrument does not disturb the existing flow pattern or contaminate the bath with unwanted elements.

At the time of this work, there were four active liquid metal velocity techniques in development, known to the author. They include the Ultrasonic Doppler Velocimetry technique, Contactless Inductive Flow Tomography, the Lorentz Force Flowmeter, and the Nail Dipping Technique.

Ultrasonic Doppler Velocimetry (UDV)

The UDV method is a contact method; however, the contact of the fluid and sensor need not occur at the location where the velocity measurement is desired. The measuring principle as outlined by Eckert et al. [13] is based on the pulsed echo technique. An ultrasound pulse is emitted from a transducer into the liquid metal and travels along a measuring line, the reflected wave is received by the same transducer and the velocity is obtained from a correlation analysis of several ultrasound pulses.

Eckert et al. [13] adopted the UDV method and tailored it to measure low and moderate temperature liquid metal flows. The UDV technique was deployed to measure the flow in liquid sodium at temperatures up to 473 K (200 °C) [14]. To extend the technique to high temperature
liquid metals Eckert et al. [15] coupled the transducer to a wave guide, as the transducers of that time had a usable temperature limit of 473 K (200 °C) and could not be placed in direct contact with a liquid metal at greater than 473 K (200 °C). The wave guide acted to decouple the transducer from the negative thermal and chemical effects of the higher temperature melt. UDV measurements with the wave guide were employed to measure lead-bismuth flows up to 573 K (300 °C) and copper-tin flows up to 893 K (620 °C) [15]. The UDV sensor was used to measure velocities in a scaled model of the mould of a continuous steel caster [16]. The liquid in the scaled experimental casting apparatus was a low melting point alloy.

An advantage of the UDV technique is the ability to deliver the velocity profile along the measuring line in real-time. The magnitude of velocity is readily available from the ultrasonic data, however a drawback of this technique is that more than one UDV sensor is required to measure the direction of velocity. For a two-dimensional flow-field, the measuring lines of two UDV sensors must intersect to derive the direction vector. This technique has not been tested in a high temperature liquid metal.

**Contactless Inductive Flow Tomography (CIFT)**

The CIFT technique is a non-contact method. In this technique, an external magnetic field is applied to the conducting liquid metal, the flow induced perturbations to the magnetic field are measured and the entire velocity field of the liquid is reconstructed. The reconstructed velocity field is estimated by an inverse algorithm. Details of the inverse algorithms developed to reconstruct the velocity field can be found in the work of Stefani and Gerbeth [17, 18, 19] and Yin et al. [20].

The technique itself has been employed to measure the velocity in the mould of a scaled model steel continuous caster, where the liquid metal was the gallium-indium-tin eutectic alloy. The specifics of the experiment can be found in Wondrak et al. [21] and Timmel et al. [22, 16].

An advantage of the CIFT technique is its capability of providing an estimate of the localized magnitude and direction of velocity within the entire domain of the flow system on which
the magnetic field acts. Perhaps a disadvantage of the technique is that it requires the solution of an inverse algorithm to estimate the flow-field. The inverse algorithm is typically mapped onto a discretized computational domain of the fluid flow vessel. With the state of computational hardware at the time, these types of inverse algorithms typically could not generate an estimate of the velocity field in real-time. The CIFT technique has not been tested in a high temperature liquid metal system.

Lorentz Force Flowmeter (LFF)

The LFF is a non-contact method. The liquid metal flow is exposed to a magnetic field, the measured force acting on the magnetic field generating system is correlated to the liquid metal velocity. The theory of the flowmeter is detailed by Thess et al. [23]. The LFF technique has been tested using both low and high temperature liquid metals, the gallium-indium-tin eutectic alloy and aluminum, as explained by Kolesnikov et al. [6].

An advantage of the LFF sensor is that it was tested for a high temperature liquid metal flowing in a duct; mean magnitude of velocity information is extracted from the technique in real-time. A disadvantage of the LFF system described by Kolesnikov et al. [6] is that it only measures channel flows, where the bulk direction of velocity is known. Thess et al. [23] show that it is theoretically possible to obtain localized velocity information from the LFF technique, however such a system is yet to be developed. The LFF measurement system as presented in Kolesnikov et al. [6] is not applicable to the aluminum electrolytic cell nor the mould of a steel continuous caster as engineers require localized velocity information to characterize these high temperature liquid metal flow systems.

Nail Dipping Technique (NDT)

The NDT is a contact method. It is a phase change (solidification) velocity measurement technique. A stainless steel rod (the nail) is dipped into liquid steel, for a period of 3-5 seconds. Liquid steel solidifies onto the nail during this time. Measurements of the solidified steel height
around the rod can be correlated to the steel velocity. Examples of measurements taken in the mould of a steel continuous caster include those of Liu et al. [24], Cho et al. [25], Ji et al. [26], Cukierski and Thomas [7], and McDavid and Thomas [27], where the liquid metal in question was liquid steel.

An advantage of the NDT is that it was tested in a high temperature liquid metal. The technique can deduce both magnitude and direction of velocity. Perhaps the method is not in the true sense real-time as manual measurements conducted by an operator of the solidified steel lump height must be taken around the stainless steel nail to infer velocity. Ji et al. [26] state in their work “the diameter of the lump was always changing in each solidified nail which created the error for the flow velocity.” Perhaps a thermal resistance at the interface of the stainless steel nail and the liquid steel affect the diameter of the lump that solidifies onto the nail. As will be shown in this thesis, the thermal resistance affects the volume of liquid metal that solidifies (lump diameter) onto a circular metallic cylinder. The correlations developed for the NDT to determine the velocity should be calibrated for thermal resistance.

1.2 Theoretical Considerations

The sensors developed in this thesis operate on forced convective heat transfer principles. In subsection 1.2.1 an overview of the principle of operation of the sensors is provided. Depending on the initial temperature condition of the sensor, the thermal resistance at the sensor and aluminum interface, and the convective conditions of the bath, liquid aluminum may solidify and subsequently melt onto the surface of the sensor. The solidification and subsequent melting can be classified as a bidirectional moving boundary problem, in subsection 1.2.2 the solidification and melting phenomena is explained with respect to this work.
1.2.1 Principles of Operation of the Sensors

The sensors which are developed in this work infer the magnitude and direction of velocity in a high temperature liquid metal by the interaction of thermal energy between the flow-field and the sensor. This thermal energy exchange requires the sensor to be in contact with liquid aluminum and at the location the velocity reading is desired. Energy is extracted by the introduction of a solid medium into the liquid; this solid does not undergo solid-to-liquid phase change. The geometry of the probe is a circular cylinder, as such the flow around the solid geometry is that of a cylinder in crossflow and knowledge of convection of a cylinder in crossflow can be exploited in the development of the sensor. These thermal phenomena are widely studied in the transport phenomena literature for common fluids. Common fluids have Prandtl numbers greater than $\text{Pr} \geq 0.7$. The Prandtl number of a liquid metal is $\text{Pr} \leq 0.1$; for instance, liquid aluminum has a Prandtl number of $\text{Pr}=0.013$ [28], liquid steel $\text{Pr}=0.1$ [28], liquid silicon $\text{Pr}=0.007$ [28] and liquid sodium $\text{Pr}=0.009$ [29].

Two forms of sensors are developed in this work, these probe designs are referred to as Sensor1 and Sensor2; schematics of the sensors are depicted in Figure 1.1. Both Sensor1 and Sensor2 can be classified as a circular cylinder. The difference between the two is the material used to construct the sensors. Sensor1 is composed of a uniform material throughout the cylinder. In Sensor2, the material is varied so that the thermal properties are adjusted at select locations around the sensor. The rationale for this material property adjustment is to simplify the data reduction technique which is utilized to infer the magnitude of velocity.

Some Theory on how the Sensors Determine Velocity

Zdravkovich [30] has extensively studied the circular cylinder in crossflow. The flow of the liquid around the circle is briefly described: as the flow approaches the leading edge of the cylinder it stagnates, the stagnation point is the location on the cylinder where the fluid velocity is zero and defined as $\Theta = 0^\circ$ in polar coordinate around the cylinder. The flow accelerates over the front of the cylinder. The boundary layers also develop over the front surface of the
Table 1.1: Flow Regimes around a Smooth Circular Cylinder after Mutlu Sumer and Fredsoe [31]

<table>
<thead>
<tr>
<th>Re</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re &lt; 5</td>
<td>No separation, creeping flow</td>
</tr>
<tr>
<td>5 &lt; Re &lt; 40</td>
<td>A fixed pair of symmetric vortices</td>
</tr>
<tr>
<td>40 &lt; Re &lt; 200</td>
<td>Laminar vortex street</td>
</tr>
<tr>
<td>200 &lt; Re &lt; 300</td>
<td>Transition to turbulence in the wake</td>
</tr>
<tr>
<td>300 &lt; Re &lt; 3 \times 10^5</td>
<td>Subcritical regime, laminar boundary layer, wake completely turbulent</td>
</tr>
</tbody>
</table>

Re: Reynolds number

cylinder; depending on the Reynolds number, the boundary layer separates from the surface. A reverse flow and vortex are formed as the boundary layer separates. The wake behind the cylinder is generated by the vortices which are swept downstream. Depending on the Reynolds number, the flow pattern near to and behind the cylinder can vary significantly, as summarized in Table 1.1.

The heat transfer coefficient distribution around the surface of a circular cylinder in cross-flow is representative of that in Figure 1.2. This figure plots the local Nusselt number ($\text{Nu}_l$) around a circular cylinder when the fluid is sodium ($\text{Pr}=0.009$), the experimental data displayed
in Figure 1.2 is generated by Ishiguro et al. [32]. The local Nusselt number is displayed for Reynolds number of \( \text{Re}=865, 1665, \) and \( 16900 \), which fall under the subcritical flow regime [31]. The development of the thermal boundary layer around the cylinder results in the form of the Nusselt function displayed in Figure 1.2. The function is higher at the forward stagnation point and lower at the diametric location. If one were to view this function in terms of energy transfer, more energy is transferred into the circular cylinder at the forward stagnation point than any other location around the cylinder. This understanding guides the sensor design. To infer the direction of velocity, locate the stagnation point of the flow, that is, the hottest point on the sensor. To infer the magnitude of velocity, obtain an estimate of the Nusselt number and compare the value to a Nusselt versus velocity correlation.

Typical experiments which are conducted for this configuration in common fluids are generated for steady state conditions; a long period of time elapses between the time the cylinder is immersed into the liquid to the point in time where the Nusselt function is captured. It is standard practice to heat the cylinder with some form of electrical resistance element during experiments in common fluids [33]. Liquid aluminum is corrosive, thus the materials which are used to construct the sensor should remain intact for the duration of the velocity measurement. The sensor should be simple and rugged to withstand the harsh chemical and thermal conditions of high temperature liquid aluminum. To mitigate the negative corrosive and mass transfer effects which may occur between the sensor material and liquid aluminum, the probe is plunged into the liquid for short duration of time. Plunging a cold sensor into the hot liquid averts the need to either joule heat or cool via gas the sensor material. The thermal energy of the liquid is utilized to heat the sensor and gain temperature recordings to infer the velocity.

In terms of manufacture, a thermal capacitive circular cylinder of uniform material is simple to construct, Sensor1 (Figure 1.1a) is of this design. To infer the Nusselt number of the Sensor1 design, one may use an inverse heat transfer algorithm to estimate the heat transfer coefficient. These inverse algorithms take time to compute an estimate, thus there is a delay between the point in time which the reading is taken and the velocity is computed. The sequen-
tial function specification algorithm developed by Beck et al. [34] is implemented to estimate
the heat transfer coefficient of a Sensor1 design; the computation of the heat transfer coefficient
estimate on a non-parallel code can take up to three days! It is not practical to wait this long for
a velocity measurement. Hence Sensor2 (Figure 1.1b) is proposed to make the mathematical
treatment of the heat transfer coefficient estimate tractable. The data reduction technique is
simplified by introducing discrete thermal capacitive lumps into the circular cylinder, where
energy flow outside of the lumps is negligible, these lumps are treated mathematically by the
lumped system analysis of Equations (1.1) and (1.2).

\[ T_{S2}^* = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp(-b t) \]  \hspace{1cm} (1.1)

\[ b = \frac{h A_s}{\rho V c_p} \]  \hspace{1cm} (1.2)

In Equations (1.1) and (1.2), \( T_i \) represents the initial temperature of Sensor2, \( T_\infty \) represents
the free stream temperature, \( b \) represents the time constant of the lump, \( t \) is the time, \( h \) the
heat transfer coefficient, \( A_s \) the surface area of the lump, \( \rho \) the density of the lump, \( c_p \) the
specific heat, and \( V \) the volume of the lump. Equation (1.1) can be modified to introduce two
non-dimensional variables, this is represented by Equation (1.3).

\[ T_{S2}^* = \frac{T(t) - T_\infty}{T_i - T_\infty} = \exp(-\text{Bi} \text{ Fo}) \]  \hspace{1cm} (1.3)

\[ \text{Bi} = \frac{h V}{k_s A_s} \]  \hspace{1cm} (1.4)

\[ \text{Fo} = \frac{\alpha t A_s^2}{V^2} \]  \hspace{1cm} (1.5)

In Equation (1.3) the non-dimensional Biot (Bi) and Fourier (Fo) numbers are introduced. The
Biot number is defined by Equation (1.4) where \( k_s \) is the thermal conductivity of the (lump)
solid material. The Biot number may be thought of as a ratio of the resistance to conduction within the solid over the resistance to convection across the fluid boundary layer [35]. When the Biot number is less than $\text{Bi} \leq 0.1$ it is assumed that the temperature within the solid is uniform. The Fourier number is defined by Equation (1.5), where $\alpha$ is the thermal diffusivity of the lump, the Fourier number is employed as a dimensionless time variable for presenting recorded temperature data of Sensor2.

The subsequent section describes the bidirectional moving boundary problem to provide context to the solidification and melting problem.

### 1.2.2 The Bidirectional Moving Boundary Problem

A peculiarity which arises in the liquid aluminum system is that aluminum may freeze onto the sensor, depending on the magnitude of heat flux drawn from the bath into the sensor. The solidified aluminum which adheres to the sensor is referred to as a shell in this thesis. The growth of a shell and subsequent melting of the shell is regulated by the balance of energy supplied to and withdrawn from the sensor-aluminum interface. To depict this phenomena,
the sequence of thermal events are described which occur when Sensor1 (a circular cylinder comprised of uniform material) is plunged into flowing liquid aluminum.

Initially the sensor is cold, at a temperature well below the liquid metal. At the instant in time when the cylinder comes in contact with liquid aluminum, energy will exchange between the cylinder and the liquid. When the magnitude of the heat flux drawn by the cylinder is larger than the magnitude associated with the convective heat flux, aluminum will solidify onto the cylinder. As the sensor material absorbs energy from the surroundings, its temperature increases with time. This in turn, will reduce the magnitude of heat flux drawn by the cylinder with time. Assuming that the heat flux from the liquid phase remains constant, there will be a point in time where the heat flux drawn by the sensor is less than the heat flux supplied to the sensor, under such a condition the aluminum shell on the sensor is melting. The aluminum shell will continue melting until a point in time where no shell exists on the sensor. This moving boundary problem may be classified as bidirectional, where the solid-liquid aluminum interface advances into the liquid and then recedes towards the solid cylinder.

To explain this phenomenon further, the Stefan condition is stated, which is an interface boundary condition applied to the solid-liquid aluminum interface. This condition is a result of the conservation of energy, the form may be viewed as a balance of energy flux.

\[ \rho_s L \frac{dS(\Theta, t)}{dt} = k_s \frac{\partial T(\Theta, t)}{\partial n} - h(\Theta, t)(SPH) \]  

In Equation (1.6), \( \rho_s \) represents the density of the solid phase (i.e. aluminum), \( L \) the latent heat of solidification, \( S \) the interface position, \( t \) time, \( k_s \) the thermal conductivity of the solid phase, \( T \) the temperature, \( n \) the normal direction to the interface in the solid, \( h \) the heat transfer coefficient, \( SPH \) the superheat of the liquid, and \( \Theta \) the angular coordinate around the cylinder.

In the case of a circular cylinder, it is assumed that the behaviour of the convective heat transfer coefficient is like that of Figure 1.2. The heat transfer coefficient will be greater at the forward stagnation point \( \Theta = 0^\circ \) and lower at \( \Theta = 180^\circ \) [32]. The data presented in
Figure 1.2 was acquired in liquid sodium with a Prandtl number of \( Pr = 0.009 \), this Prandtl number is approximately similar to that of liquid aluminum (\( Pr = 0.013 \)). Let us assume that this heat transfer coefficient distribution holds around the cylinder when the shell has grown. This coefficient distribution will vary the magnitude of heat flux being supplied at the solid-liquid aluminum interface. In consequence, the solid-liquid interface velocity will be a function of position around the surface of the cylinder. Aluminum will solidify and melt at different rates around the surface of the cylinder. Assuming that the form of the heat transfer coefficient function presented in Figure 1.2 holds at the aluminum solid-liquid interface, the shell morphology is expected to resemble that presented in Figure 1.3. Figure 1.3a displays the system at the initial state, the cylinder is schematically depicted within flowing liquid aluminum. At some point in time after the initial state a shell will grow and then melt around the cylinder. While the shell is receding the melting is expected to be faster at locations closer to the stagnation point and longer at the diametric location. This corresponds to there being less shell closer to the stagnation point and more shell at the diametric location, this is shown in Figure 1.3b. As time proceeds, the volume of shell on the cylinder will decrease, however the melting pattern is expected to hold, melting faster closer to the stagnation point (Figure 1.3c). As the melting at the stagnation point is fastest, the shell at the rear part of the cylinder will be the last to melt, shown in Figure 1.3d.

In order to further understand the solidification of the aluminum shell on the cylinder, another phenomenon must be taken into account which occurs at the sensor-aluminum interface. This phenomenon is the thermal resistance which modifies the temperature at the sensor-aluminum interface. The thermal resistance at the sensor-aluminum interface affects the ability of the sensor to freeze aluminum. To explain the effect of this thermal resistance on the shell solidification and melting, the relationship of Equation (1.7) is utilized.

\[
\frac{k_{cyl}}{\partial T(\Theta, t)} \frac{\partial}{\partial n} = \frac{1}{R} \Delta T_{\text{int}} = k_{Al} \frac{\partial T(\Theta, t)}{\partial n} \tag{1.7}
\]
Figure 1.3: Schematic of Shell Solidification and Melting on a Sensor1 Design
The $R$ term can be viewed as a thermal resistance with units of m$^2$·K·W$^{-1}$. The thermal resistance causes the temperature in the aluminum phase to be greater at the sensor-aluminum interface when compared to the temperature in the aluminum phase under perfect thermal contact conditions. This increase in temperature in the aluminum phase at the sensor-aluminum interface causes a decrease in the temperature gradient in the aluminum phase. The decrease in temperature gradient correlates to a decrease in heat flux in the aluminum phase. The thermal resistance restricts the exchange of energy at the sensor-aluminum interface. Any value of thermal resistance acts to decrease the heat flux which is drawn by the sensor when compared to a perfect thermal contact condition. Higher thermal resistance values will freeze less aluminum and in contrast lower thermal resistances will freeze more aluminum.

The thermal resistance acts to suppress the rate at which energy is drawn by the sensor and as a result, under conditions of high convective heat flux, shell may not form on the sensor. The convective heat transfer coefficient is a function of position around the cylinder. For certain flow conditions, shell may not grow on certain parts of the cylinder while it grows on other parts of the cylinder. From the convective heat transfer coefficient distribution exhibited in Figure 1.2, it is predicted that under a restrictive shell growth condition, shell will not grow on the front of the sensor ($0^\circ \leq \Theta \leq 90^\circ$ and $270^\circ \leq \Theta \leq 360^\circ$) and it will grow on the rear of the sensor ($90^\circ \leq \Theta \leq 270^\circ$).

As the solidification and melting of the liquid metal is a byproduct of the sensor making contact with the liquid system, experimental evidence of this solidification and melting behaviour is necessary to guide the sensor design.

### 1.3 Thesis Objectives

The primary objective of this thesis is to develop a contact sensor to determine velocity of liquid aluminum. The sensor design is of a circular cylinder geometry so that knowledge of forced convection heat transfer can be exploited in the principle of operation of the sensor. To
infer the direction of velocity obtain an estimate of the local Nusselt number and compare it to a Nusselt versus position function. To infer the magnitude of velocity obtain an estimate of the average Nusselt number and compare the value to a Nusselt versus velocity correlation. Embedding more than one thermocouple into a solid material that does not undergo solid-to-liquid phase change will facilitate an estimate of the local Nusselt number. To withstand the harsh chemical and thermal conditions of liquid aluminum the sensor construction should be simple and rugged. Two sensor designs are constructed and tested in this thesis, denoted Sensor1 and Sensor2, where Sensor2 is superior to Sensor1 in that the estimate of velocity occurs in real-time. The development process involved both experiments and numerical modeling, as one can imagine, the process was iterative and incremental as the experimental work occurred in liquid aluminum.

To accomplish this primary objective of developing a high temperature liquid metal velocity sensor there are several secondary objectives:

- Design and construct a thermal capacitive circular cylinder sensor of uniform material denoted as Sensor1, where Sensor1 has more than one embedded thermocouple to measure the temperature distribution within the cylinder. Test Sensor1 in flowing liquid aluminum.

- Obtain experimental data pertaining to the solidification and melting of aluminum onto Sensor1 designs constructed of steel and copper in flowing liquid aluminum at different flow velocities and superheat.

- Model numerically using a commercial CFD code, ANSYS Fluent [36] the flow of liquid aluminum around the Sensor1 geometry to predict the local solidification and melting times of the shell. Use this data to obtain an estimate of the thermal resistance at the Sensor1-aluminum interface for steel and copper Sensor1 materials.

- Design and construct a circular cylinder sensor of more than one material denoted as Sensor2, where Sensor2 has more than one high thermal conductivity lump where a ther-
mocouple is embedded into each lump. The data reduction technique of these thermocouples will provide fast estimates of the local Nusselt number. Test Sensor2 in flowing liquid aluminum.

- Model numerically in ANSYS Fluent the steady convective heat transfer of liquid aluminum around Sensor2 to generate the local Nusselt number curves for Sensor2 at various Reynolds numbers.

- Model numerically in ANSYS Fluent the transient conjugate heat transfer from liquid aluminum to Sensor2 to provide an estimate of the values of the thermal resistance at the interface of each thermal capacitive lump of Sensor2.

- Devise a technique to determine the magnitude and direction of velocity from Sensor2.

- Apply the technique to determine the magnitude and direction of velocity of the Sensor2 design to the liquid steel system, by modeling numerically in ANSYS Fluent the response of Sensor2 in flowing liquid steel.

The remaining chapters of this thesis are organized as follows: Chapter 2 provides a review of the literature relevant to this thesis. Chapter 3 describes the experimental apparatus and construction of Sensor1 and Sensor2. Chapter 4 describes the numerical models employed to predict the solidification and melting of aluminum onto Sensor1 and the rate of heat transfer to Sensor2. Chapter 5 presents results of experiments and numerical modeling of Sensor1. Chapter 6 presents the results of experiments and numerical modeling of Sensor2. A technique to determine the direction and magnitude of velocity of liquid aluminum using Sensor2 is described in chapter 6. Chapter 7 concludes this thesis.
Chapter 2

Literature Review

To develop the sensors both experiments and numerical modeling were performed to understand this thermal-fluid phenomena. Three research themes revolve around the objective of creating an invasive heat transfer sensor to determine the velocity of liquid aluminum. The first involves the solidification and melting represented as a moving boundary problem. Those studies which experimentally track the solid-liquid phase change interface are presented in section 2.1. Numerical convection diffusion phase change literature relevant to the Sensor1 design are presented in section 2.2. The second theme involves an interfacial thermal resistance at the sensor-aluminum interface. A review of the relevant literature on the thermal resistance of liquid metals on various substrate materials is provided in section 2.3. As the sensors are essentially circular cylinders in liquid aluminum crossflow, the final theme involves forced convection over a circular cylinder. A review of the experimental, analytical, and numerical literature of forced convection to cylinders in crossflow is provided in section 2.4.
2.1 Experimentally Monitoring the Solid-Liquid Phase Change Interface

As the contact of a cold metallic substrate with a hot liquid metal may result in the solidification of the liquid metal, an objective of this thesis is to investigate this phenomena by collecting data pertaining to the local solidification and melting times of shell around Sensor1 while Sensor1 is within flowing liquid aluminum. To this end, experimentally a technique to monitor the solid-liquid phase change interface is required. This section provides a review of various methods employed to monitor this solid-liquid interface.

Studies which experimentally track the solid-liquid phase change interface in various material systems under convection are discussed, where the technique used to delineate the phase front, the material system, and the type of convection, is listed. Hao and Tao [37, 38] experimented in a water system, where they tracked the solid-liquid interface under forced convection during the melting of ice spheres. They observed the melting front of dyed ice spheres using a digital video camera. The evolution of the phase front was measured using this visual technique. Studies which experimentally observed the phase front in types of wax, include those of Assis et al. [39, 40], Khodadadi and Zhang [41], Tan et al. [42], and Jones et al. [43]. In these wax systems the solid-liquid interface was measured visually by a camera as the solid phase of the wax was opaque while the liquid phase was transparent. Free-convection drives the flow-field of the aforementioned literature on the wax system.

Among materials which are classified as metals, several studies reported on natural convection in enclosures. Wolff and Viskanta [44] studied the solid-liquid interface morphology during the solidification of tin in a rectangular cavity. To probe the interface position, an L-shaped glass rod was immersed into the liquid tin and made contact with the solid-liquid interface. The interface was measured at discrete points during the solidification process, the morphology was reported in two-dimensions. Gau and Viskanta [45] reported the solid-liquid interface position during the melting of gallium in an enclosure. They employed what they
called “the pour out method” [45] to trace the solid-liquid interface at select times during the melting process and reported the shape of the interface in two-dimensions. In another study, Gau and Viskanta [46] employed both the pour out method and the glass rod probing method to measure the shape of the solid-liquid interface during the phase change of Wood’s metal (i.e. eutectic composition of wt%: Bi 50%, Pb 26.7%, Sn 13.3%, and Cd 10%). They [46] reported the advancement of the interface referenced along the height (one-dimension) of the enclosure. Szekely and Chhabra [47] reported the shape of the solid-liquid interface in two-dimensions during the solidification of lead in a cavity. Thermocouples were situated throughout the cavity to infer the interface position. Campbell and Koster [48] reported the solid-liquid interface during the melting of gallium in an enclosure. They used a radiographic technique which was non-intrusive to delineate the interface in two-dimensions.

Visual techniques to track the solid-liquid interface are readily employed in material systems where one phase is opaque and the other phase is transparent. Solid metal and liquid metal are opaque; the techniques which are commonly used in a metal system to explore the interface, come in contact with the interface. One may argue that visual (non-intrusive) observation of the phase front history is better than physically probing the interface, as the physical probe is intrusive, it may alter the phase front movement. The solid-liquid phase change interface is probed in this work by using methods that are physically intrusive like the research in the past, as to implement a non-intrusive method of tracking the solid-liquid interface in a high temperature liquid metal environment would be experimentally challenging and time consuming.

In this work, thermocouples are utilized to infer the point in time at which the solidified aluminum that forms on the surface of Sensor1 completely melts. The Sensor1 design is also extracted from the flow-field at discrete points in time during the solidification and melting process with the purpose of tracing the solidified aluminum interface around the cylinder. This work presents experimental data on the movement of the solid-liquid interface of a metal around a circular cylinder under forced convection. To the best of the author’s knowledge,
this bidirectional moving boundary problem has not been experimentally explored in a metal system moving under forced convection.

2.2 Numerical Convection-Diffusion Phase Change

Modeling the solidification and melting of aluminum onto steel and copper Sensor1 materials will provide an estimate of the thermal resistance at the Sensor1-aluminum interface. Various values of thermal resistance are imposed at the sensor-aluminum interface in the numerical model where the effect of the thermal resistance on the solidification and melting of aluminum onto a circular cylinder in forced convection is reported in chapter 5. The movement of the solid-liquid interface under forced convection is modeled using a commercial code that employs the enthalpy-porosity formulation [49] to capture the phase front. A review of some literature on numerical convection-diffusion phase change focusing on those works which track the interface under liquid metal forced convection is provided in this section.

There have been many numerical studies pertaining to the convection-diffusion phase change problem. Perhaps one of the most investigated is the melting of a material in a rectangular enclosure, where buoyancy drives the convection. An extensive list of references of phase change in this geometry may be found in the work of Hannoun et al. [50, 51]. There are fewer studies dealing with convection-diffusion phase change where the convection is forced in a metal system.

Numerical and experimental studies of forced convective melting of a metal have been performed by Melissari and Argyropoulos [52, 53], where they studied the melting of metal spheres. They [53] melted aluminum spheres of various diameters in liquid aluminum for various liquid superheat and velocity. The numerical work of Melissari and Argyropoulos [52] employed the SIMPLER algorithm [54] to model the convection-diffusion problem where the energy source terms are modified to account for phase change, using the effective heat capacity method [55] or an enthalpy method [55] depending on whether the sphere was aluminum
(isothermal phase change) or a magnesium alloy (long freezing range alloy). The numerical model was used to predict the melting time of the spheres under various velocity and liquid temperature conditions, with the objective to correlate the melting time of the sphere to the flow velocity.

Kumar and Roy [56, 57] presented a numerical study on the melting of a metal sphere in its own liquid under forced convective conditions. The metal in their study included that of copper, steel, and nickel. They employed the enthalpy-porosity approach [49] to study this convection-diffusion phase change problem and reported on the effect of superheat and velocity on the forced convective melting as it related to cladding operations.

Aluminum solidifies onto Sensor1 and subsequently melts under forced convection. The numerically predicted melting time of the solidified aluminum around the surface of Sensor1 agrees with the experimental measurements by incorporating a thermal resistance at the sensor-aluminum interface. Numerical predictions for various initial bath temperatures and liquid aluminum flow velocities were performed and these predictions are compared with experimental data performed for the Sensor1 design. Experimentally, a carbon coating is applied to the surface of both steel and copper Sensor1 designs which is one contribution to the thermal resistance. The following section provides a literature review for the thermal resistance at the sensor-aluminum interface.

### 2.3 A Thermal Resistance at the Sensor-Aluminum Interface

The focus of the forthcoming review will be on those thermal resistance studies of metal casting. Metal casting practitioners are interested in understanding the interfacial heat transfer when liquid metal comes into contact with a mould. The control of the solidification microstructure occurs from engineering the heat flow out of the casting, where control of the microstructure is useful for the mechanical integrity of a cast metal part. As such, the thermal
resistance at the casting-mould interface has gained attention in the literature.

In the casting process, liquid metal flows into the cavity of a mould. After filling the cavity, while the metal is liquid, a solid-liquid contact condition exists at the casting-mould interface. After some duration in time, heat is extracted from the liquid and a solid-solid contact forms at the casting-mould interface. Similarly, when Sensor1 is immersed into liquid aluminum it interacts with the liquid initially under a solid-liquid contact condition. Depending on the magnitude of heat flux drawn by the sensor, the liquid may solidify forming a solid-solid contact. At the sensor-aluminum interface of Sensor2 there is a solid-liquid contact condition. A thermal resistance exists at the sensor-aluminum interface of both Sensor1 and Sensor2.

The purpose here is to portray that the thermal resistance is a complex phenomena where many variables affect the heat transfer at the interface. In this work, values of thermal resistance are imposed into numerical models to provide an estimate of the magnitude of the thermal resistance at the sensor-aluminum interface. The experimental data are compared with several numerical predictions in which the thermal resistance is varied. The experimental data that coincides with a thermal resistance numerical prediction provides the estimate of the magnitude of thermal resistance.

In subsection 2.3.1 those studies which investigate the thermal resistance at the metal-mould interface are examined to identify the parameters that affect the thermal resistance. As models evolve to predict values of thermal resistance at the metal-mould interface, these models may be used in the data reduction technique presented in chapter 6 to identify the rate of heat transfer to the Sensor2 design and hence infer liquid metal velocity. Subsection 2.3.2 provides a review a comprehensive model in development to predict the thermal resistance at the metal-mould interface. As values of thermal resistance are introduced into the numerical models of this work, a review of values reported in the literature is provided in subsection 2.3.3.
2.3.1 Factors Affecting the Thermal Resistance at the Metal-Mould Interface

Griffiths [58] outlines various factors which affect the thermal resistance at the interface of the metal casting and mould. These factors include: liquid metal volume and applied pressure, liquid alloy composition and liquid alloy superheat and mould temperature, the mould material, interfacial gas composition and mould surface roughness, and the surface coating applied to the mould. In the forthcoming review, these factors are explored in relation to the sensor designs. The terminology mould, die, and chill are synonymous; these terms are used to describe the solid on which the liquid metal is cast.

Liquid Metal Volume and Applied Pressure to the Liquid Metal

There are conflicting views on the effect of the liquid metal volume on the thermal resistance. Some research shows that the liquid metal volume plays a role while others show that it does not. For instance, Akar et al. [59] studied the thermal resistance when an aluminum silicon alloy solidified against a water cooled copper chill, for different liquid metal volumes and reported no significant change in the value of the thermal resistance, when the volume of solidified metal increased. There was no external applied pressure source in the study of Akar et al. [59].

Sun et al. [60, 61] reported that the casting volume affects the thermal resistance. They calculated the thermal resistance for an aluminum alloy and magnesium alloy for different section size in a five step casting. Each step of the casting had a larger volume. They found lower thermal resistance for sections with greater volume.

If the volume of the liquid metal plays a role on the thermal resistance, one would expect that the sensor would observe different values of thermal resistance when dipped into different volume of baths. In this work, the volume of liquid aluminum which the sensor interacts is fixed at approximately 0.015 m$^3$, as this is the volume of aluminum that is charged into the experimental furnace.
Applying high pressure to the liquid metal decreases the thermal resistance. Sekhar et al. [62] investigated the thermal resistance when an aluminum silicon alloy solidified against H-13 die steel under applied pressure. They observed lowering of the thermal resistance under the application of pressure of \((1960 \times 10^5)\) Pa. Sun et al. [60, 61] applied pressures of up to \((900 \times 10^5)\) Pa to the liquid metal and also observed that an increase in pressure resulted in lowering of the thermal resistance. The pressure applied in the work of Sekhar et al. [62] is three orders of magnitude above atmospheric pressure. Although there is a pressure distribution around the sensor geometry in flowing liquid aluminum, the pressure differences around the sensors (circular cylinders) are not as great as those reported by Sekhar et al. [62] and Sun et al. [60, 61].

**Liquid Alloy Composition, Alloy Superheat, and Mould Temperature**

The alloy composition affects the thermal resistance by changing the wetting characteristics of the melt on the substrate. Bamberger et al. [63] and Muojekwu et al. [64] studied the effect of various compositions of aluminum silicon alloys on the thermal resistance. In their studies, larger silicon content in aluminum produced higher thermal resistance.

Narayan Prabhu and Ravishankar [65] modified an aluminum silicon melt by the addition of sodium; the sodium modification decreased the surface tension of the liquid and reduced the thermal resistance. Prates and Biloni [66] reported values of thermal resistance when aluminum copper alloys were cast against various substrates; the thermal resistance decreased with increasing copper content. With respect to this work, all experiments are performed in commercial purity aluminum.

A higher superheat produces lower thermal resistance. Akar et al. [59] and Muojekwu et al. [64] studied the thermal resistance when an aluminum silicon alloy solidified against a water cooled copper chill. The value of the superheat altered the magnitude of the thermal resistance. El-Mahallawy and Assar [67] measured the thermal resistance when commercial purity aluminum solidified against a copper chill for two different liquid superheat. The 115
K superheat condition produced lower thermal resistance than the 40 K superheat. Coates and Argyropoulos [68] studied the effect of liquid metal superheat on the thermal resistance when commercial purity aluminum was cast against iron alloy chills. An increase in the liquid superheat resulted in a decrease in the thermal resistance. In this work, the Sensor1 design is tested in various bath superheat; the effect of the superheat is reported with respect to the formation and subsequent melting of the shell in conjunction with a thermal resistance at the sensor-aluminum interface.

Bouchard et al. [69] performed experiments to determine the thermal resistance when copper blocks were immersed for short duration in liquid copper; during this time a copper shell solidified onto the copper block. They computed the thermal resistance for various preheated initial temperature of copper block and they found that the preheated copper blocks produced lower values of thermal resistance. Changing the initial temperature condition of the sensor material for this work acts to suppress the formation of shell by modifying the heat flux which is drawn from the liquid by the sensor. To determine the magnitude and direction of velocity the sensors are preheated to suppress the formation of solid aluminum; however, a systematic study is not performed to determine if the initial temperature of the sensor affects the thermal resistance.

**Mould Material**

It has been reported that higher thermal diffusivity materials resulted in lower values of thermal resistance. Muojekwu et al. [64] and Jayananda and Narayan Prabhu [70] investigated the thermal resistance when various chill materials were in contact with aluminum silicon melts. Chills with higher thermal diffusivity resulted in lower thermal resistance. Prates and Biloni [66] reported values of thermal resistance when aluminum copper alloys were cast against various substrates. The thermal resistance of copper was lower than steel. In this work, steel and copper Sensor1 designs are immersed into liquid aluminum and it is observed that the material has an effect on the ability to estimate the thermal resistance at the sensor-aluminum interface.
Interfacial Gas Composition and Mould Surface Roughness

Higher thermal conductivity gases produce lower thermal resistance. Argyropoulos and Carletti [71] injected helium gas at the metal mould interface while aluminum alloys were cast onto copper and iron alloy chills. The thermal resistance is lower in a helium atmosphere when compared to that of air, as helium has a higher thermal conductivity than that of air. In this work, all experiments are performed in an air atmosphere.

There are conflicting reports in the literature on the effect of surface roughness on the thermal resistance, as at times surface roughness is used synonymously with the terminology textured pattern. Bouchard et al. [72] studied the effect of various surface texture and roughness on the effect of the thermal resistance when copper blocks are dipped for short duration of time in a liquid copper alloy. They found that at the early stage of solidification rough or textured substrates produce lower thermal resistance than polished substrates. They also found evidence that the heat transfer at the solid-substrate liquid-metal interface was non-uniform on the rough or textured substrates.

Griffiths and Kayikci [73] studied the thermal resistance when an aluminum copper alloy solidified against an uncoated water cooled copper chill. Interestingly, they conclude that no consistent relationship was observed between the thermal resistance and chill surface roughness. Coates and Argyropoulos [68] and Muojekwu et al. [64] produced a relationship between the mould surface roughness and the thermal resistance. They reported that an increase in mould surface roughness increases the thermal resistance. In this work, the effect of surface roughness on the thermal resistance at the sensor-aluminum interface is not explored.

Surface Coatings

The application of a surface coating to the metal-mould interface increases the thermal resistance. Schmidt [74] calculated the thermal resistance at the die interface when casting var-
ious aluminum alloys. Three different die surfaces were investigated, a bare machined steel surface, a steel surface plasma sprayed with steel particles and a steel surface sprayed with ceramic coating. The thermal resistance was lowest for the machined (uncoated) steel surface, the plasma sprayed surface displayed intermediate thermal resistance and the ceramic coated surface displayed the highest values of thermal resistance.

Prasanna Kumar and Narayan Prabhu [75], Prates and Biloni [66], Kim [76], and Jayananda and Narayan Prabhu [70] reported the thermal resistance when alloys were cast against coated and uncoated mould materials. They found that the thermal resistance was lowest for the uncoated mould materials. Coated moulds produced larger thermal resistance values, the composition of the coating affected the magnitude of the thermal resistance.

Sekhar et al. [62] investigated the thermal resistance when an aluminum silicon alloy solidified against coated H-13 die steel. The mould coating acted to increase the thermal resistance, a thicker coating produced larger thermal resistance. The lowest thermal resistance was recorded for the uncoated die.

Hamasaid et al. [77] studied the effect of mould coating material and thickness of the coating material on the thermal resistance when aluminum alloys were cast against a die surface. The coatings they utilized were a titanium oxide coating and a graphite coating. The experimental measurements showed that the graphite coating had a lower thermal resistance when compared with the titanium oxide coating. For a given coating, an increase in the coating thickness resulted in an increase in the thermal resistance.

A carbon coating is applied to the surface of the sensors for the purpose of slowing the dissolution of the sensor material into liquid aluminum. Based on the aforementioned review, this carbon coating will act to increase the thermal resistance at the sensor-aluminum interface. A thicker coating of carbon will have a higher thermal resistance when compared to that of a thinner coating of carbon. This phenomena will be used to explain the experimental results in later chapters.
2.3.2 Solid-Liquid Thermal Contact Models

The aforementioned review of thermal resistance studies at the metal-mould interface outlines the many parameters which have been identified to affect the thermal resistance. Although, at times the data presented in the literature is contradictory, these studies demonstrate the complexity involved in understanding thermal contact interfaces. Engineers are interested in modeling thermal resistance phenomena, as a validated thermal resistance model will aid in die design. With respect to the development of a contact heat transfer sensor to determine the velocity of a high temperature liquid metal, a comprehensive model to predict the thermal resistance at the sensor-aluminum interface will aid in the data reduction technique employed to determine the velocity. This data reduction technique is presented in chapter 6.

Models have been developed to predict the thermal resistance at the metal-mould interface, where most in the literature are empirical, which fit a thermal resistance function to the experimental data which were recorded from experiments. Muojekwu et al. [64] list some of the empirical models that have been developed. To this author’s knowledge, there is one comprehensive model which predicts the interfacial heat transfer coefficient at the metal-mould interface.

Hamasaiid et al. [78] model the thermal contact resistance at the mould-casting interface based on thermal flux tube theory [79]. In this predictive model [78] the contact topography and interface characteristics are included by the solid surface roughness parameters and the mean trapped air layer at the interface. The model [78] is validated with experimental data published in the literature. Hamasaiid et al. [80] extend the application of this model to predict the thermal resistance during a high pressure die casting operation.

In this work, no attempt was made to model the thermal resistance at the sensor-aluminum interface as an experimental study of this nature to validate the model would be a great endeavor. The purpose was to demonstrate that one can predict the solidification and melting of aluminum on the surface of Sensor1 under forced convection utilizing a value of thermal resistance. In addition, the heat transfer to the Sensor2 geometry (where the formation of shell is
Table 2.1: Thermal Resistance Data Compiled by Goudie [81]

<table>
<thead>
<tr>
<th>Solid</th>
<th>Liquid</th>
<th>$R ,[m^2 \cdot K \cdot W^{-1}]$</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>Aluminum</td>
<td>$29 - 78 \times 10^{-5}$</td>
<td>1970</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>Aluminum</td>
<td>$23 - 78 \times 10^{-5}$</td>
<td>1970</td>
</tr>
<tr>
<td>Copper</td>
<td>Aluminum</td>
<td>$20 \times 10^{-5}$</td>
<td>1972</td>
</tr>
<tr>
<td>AISI1030 Steel</td>
<td>Aluminum</td>
<td>$24 \times 10^{-5}$</td>
<td>1978</td>
</tr>
<tr>
<td>Copper</td>
<td>Aluminum</td>
<td>$25 - 83 \times 10^{-5}$</td>
<td>1985</td>
</tr>
<tr>
<td>Copper</td>
<td>Aluminum</td>
<td>$8 - 50 \times 10^{-5}$</td>
<td>1991</td>
</tr>
<tr>
<td>Copper</td>
<td>Aluminum</td>
<td>$14 \pm 2.0 \times 10^{-5}$</td>
<td>1995*</td>
</tr>
<tr>
<td>Iron</td>
<td>Aluminum</td>
<td>$10.9 \pm 0.9 \times 10^{-5}$</td>
<td>1995*</td>
</tr>
</tbody>
</table>

* denotes the data from Goudie’s experiments, no coating is applied at the solid interface in all studies.

suppressed) is modeled by introducing a thermal resistance at the sensor-aluminum interface.

### 2.3.3 A Range of Thermal Resistance Values

As in this work values of thermal resistance are imposed at the sensor-aluminum interface, the work of Goudie [81] is examined, who estimated the interface thermal resistance between solid-cylinders that were immersed into liquid-metals. Goudie’s data [81] gives an appreciation for the range in magnitude of this thermal resistance. Of the aforementioned thermal resistance studies Goudie’s work is related here, as the geometry and experimental arrangement is most similar; solid cylinders of copper and iron were immersed into a stagnant bath of aluminum. In contrast to this work, a coating was not applied on Goudie’s solid cylinders. Table 2.1 lists the range of thermal resistance quoted by Goudie [81], this range of values are $(8 \times 10^{-5} \leq R \leq 8.3 \times 10^{-4}) \, m^2 \cdot K \cdot W^{-1}$, note that there is one order of magnitude range of thermal resistance. In the forthcoming numerical modeling of this transport process, values of thermal resistance are imposed at the sensor-aluminum interface. The objective is to demonstrate that the model and experimental data can be in agreement with a thermal resistance.
2.4 Forced Convection of a Circular Cylinder in Crossflow

In the following sections a review is provided of studies on forced convection of a circular cylinder in crossflow. Experimental studies (subsection 2.4.1), analytical studies (subsection 2.4.2) and numerical studies (subsection 2.4.3) are discussed. The focus of the numerical studies from the literature concern the Prandtl number and boundary conditions applied to the cylinder, as in this work a non-standard boundary condition configuration to model the heat transfer to Sensor2 is employed.

2.4.1 Experimental Studies

Zukauskas and Ziugzda [82] have published a volume which is devoted entirely to experimental techniques, analytical techniques and results thereof for the heat transfer to a cylinder in crossflow. A review of experiments performed on forced convection to circular cylinders can also be found in Zdravkovich’s [30] treatise. Zukauskas and Ziugzda, and Zdravkovich reference many experimental studies which use air as the working fluid for forced convection around a circular cylinder; the Prandtl number of air is approximately $\Pr=0.7$. A thesis on forced convection of various fluids is by Sanitjai [33], where he performed experiments on the local heat transfer to a cylinder in crossflow for the Prandtl number range of $(0.7 \leq \Pr \leq 200)$. Sanitjai’s work focused on providing a detailed analysis of the effect of the Prandtl number on the local heat transfer, as previous studies which varied the Prandtl number, focused on the average heat transfer to the cylinder. Few experimental studies have been performed on the heat transfer to a cylinder for low Prandtl number liquids in the literature. To the author’s knowledge, in the English literature there were three experimental studies. Two performed by Ishiguro and coworkers [32, 83] which investigated the heat transfer to a cylinder in crossflow in liquid sodium. The other investigation was translated to English and is by Andreevskii [84] who also used liquid sodium.
2.4.2 Analytical Studies

An analytical study of liquid metal flow across a circular cylinder is provided by Khan et al. [85], who employed an integral approach to solve the boundary layer equations to derive expressions for the average heat transfer coefficient. They solved for conditions of viscous and inviscid flow around the circular cylinder. Liquid metals have very low Prandtl number, this correlates to the thermal boundary layer being larger than the viscous boundary layer. Khan et al. [85] argued that although the viscous boundary layer is small, it cannot be disregarded in liquid metal flow solutions. Their study portrayed the effect of the viscous boundary layer on the average heat transfer coefficient from the cylinder. The average Nusselt number versus Reynolds number correlations which they derived for inviscid and viscous flow are listed as Equations (2.1)-(2.4), these correlations are utilized to validate the numerical modeling of this thesis:

**Inviscid Constant Wall Temperature [85]:**

\[ \overline{Nu} = 0.95 \text{Re}^{0.5} \text{Pr}^{0.5} \]  \hspace{1cm} (2.1)

**Inviscid Uniform Heat Flux [85]:**

\[ \overline{Nu} = 1.09 \text{Re}^{0.5} \text{Pr}^{0.5} \]  \hspace{1cm} (2.2)

**Viscous Constant Wall Temperature [85]:**

\[ \overline{Nu} = \frac{0.465}{(\text{Pr} + 0.0077)^{0.1}} \text{Re}^{0.5} \text{Pr}^{0.5} \]  \hspace{1cm} (2.3)

**Viscous Uniform Heat Flux [85]:**

\[ \overline{Nu} = \frac{0.645}{(\text{Pr} + 0.0077)^{0.04}} \text{Re}^{0.5} \text{Pr}^{0.5} \]  \hspace{1cm} (2.4)
2.4.3 Numerical Studies

In the forthcoming section, an overview is presented of the recent literature concerning the numerical simulation of forced convection over various geometry of cylinder, presenting the range of Reynolds number, Prandtl number and type of thermal boundary condition applied to the cylinder surface. Not many forced convection studies of flow around various cylinder are investigated for the Prandtl number range of \( \text{Pr} \leq 0.1 \). Prandtl numbers of \( \text{Pr} \leq 0.1 \) naturally occur in liquid metals, due to the high thermal conductivity of the liquid metal. As an aside, perhaps in the future, an engineered fluid may approach such low values of (liquid metal) Prandtl number, this class of liquid may be the nanofluid, where the thermal conductivity of the working fluid is enhanced by nano particles [86].

The material properties around the Sensor2 design are adjusted so that some regions of the probe can be modeled as adiabatic. As the literature is reviewed on forced convection to various cylinders attention is drawn to the type of boundary condition applied to the cylinder geometry, as the Sensor2 design is modeled using various non-uniform thermal boundary conditions.

Studies of Various Cylinder Geometries

Among the numerical studies of convection of circular cylinders in crossflow, a review of the pre-1996 literature can be found in Ahmad [87]. Ahmad’s review focused on those numerical studies that solve the steady flow equations for the Reynolds number range of \( (100 \leq \text{Re} \leq 500) \). The current review is focused on the post-1996 literature and the laminar flow regime.

Bharti et al. [88] numerically studied the heat transfer to a circular cylinder in crossflow for the Reynolds numbers of \( (10 \leq \text{Re} \leq 45) \) and the Prandtl numbers of \( (0.7 \leq \text{Pr} \leq 400) \). They investigated the effect of a constant wall temperature and a uniform heat flux boundary condition on the heat transfer from the surface of the circular cylinder. Publications extending the Reynolds number range to the unsteady laminar vortex shedding range include that of Patnana et al. [89] for the Reynolds numbers of \( (40 \leq \text{Re} \leq 140) \), the Prandtl numbers of \( (1 \leq \text{Pr} \leq 100) \), and various power law indices for the constant wall temperature boundary condition.
These studies concluded the average Nusselt number increases with Reynolds and/or Prandtl number. In addition, the uniform heat flux condition showed a higher value of Nusselt number compared to the constant wall temperature condition.

Dhiman et al. [90, 91] studied the forced convective heat transfer from a square cylinder in crossflow for Reynolds numbers of \((1 \leq Re \leq 45)\) and Prandtl numbers of \((0.7 \leq Pr \leq 4000)\). The thermal boundary conditions employed in [90, 91] are those of the constant wall temperature and the uniform heat flux type. From these studies it was shown that the average Nusselt number for the front surface has the highest value, the top surface intermediate, followed by the rear surface. The average Nusselt number increases monotonically with an increase in the Reynolds and/or Prandtl number and is always higher for the uniform heat flux condition than the constant wall temperature condition.

Chandra and Chhabra [92, 93] performed a numerical study of the forced convective heat transfer from a semi-circular cylinder in crossflow at Reynolds numbers of \((0.01 \leq Re \leq 39.5)\) and Prandtl numbers of \((0.7 \leq Pr \leq 100)\). They employed a constant wall temperature boundary condition. Bhinder et al. [94] numerically studied the forced convective heat transfer from a semi-circular cylinder at angle of incidence to the flow-field employing Reynolds numbers of \((80 \leq Re \leq 180)\) and various angles of incidence, the working fluid was air (Pr=0.7) with a constant wall temperature condition. The heat transfer results conform to the expected dependence on Reynolds and Prandtl numbers. The bulk of the heat transfer occurs from the front curved surface of the cylinder. The variation of the local Nusselt number is maximum at the front stagnation region, a sudden jump in the local Nusselt number variation is found at the singularity points (vertices) of the semi-circle.

Bharti et al. [95] performed a numerical study on the heat transfer from an elliptical cylinder in the steady flow regime at Reynolds numbers of \((0.01 \leq Re \leq 40)\), Prandtl Number \((1 \leq Pr \leq 100)\), for various power law indices and aspect ratio with respect to elliptical cross-section with a thermal boundary condition of constant wall temperature. Irrespective of the aspect ratios of the elliptical cylinder, the average Nusselt number increases monotonically
with an increase in the Reynolds and/or Prandtl number.

Dhiman and Shyam [96] studied the forced convective heat transfer from an equilateral triangular cylinder in the laminar unsteady flow regime at Reynolds number of (50 ≤ Re ≤ 150) and a Prandtl number of Pr=0.7 for a constant wall temperature boundary condition. Chatterjee and Mondal [97] performed a numerical study for the equilateral triangular cylinder at Reynolds numbers of (50 ≤ Re ≤ 250), at Prandtl numbers of Pr=0.7, Pr=7, and Pr=100, and various channel blockage for a constant wall temperature boundary condition. As the Reynolds number increases, the local Nusselt number also increases on any face of the equilateral triangular cylinder. The average Nusselt number increases monotonically with increasing value of Reynolds and/or Prandtl number.

Dhiman and Hassan [98] reported a numerical study of the forced convective heat transfer from a trapezoidal cylinder in the steady and unsteady flow regimes at Reynolds numbers of (1 ≤ Re ≤ 150) and a Prandtl number of Pr=0.7 for a constant wall temperature boundary condition. The square cylinder possesses larger Nusselt numbers when compared to the trapezoidal cylinder, however the heat transfer results conform to the expected dependence on Reynolds and Prandtl numbers.

The lower limit of the Prandtl number in the foregoing literature review is that of Pr=0.7. The type of thermal boundary condition utilized in the aforementioned literature is either a constant wall temperature (Dirichlet) or a uniform heat flux (Neumann) boundary condition associated around the circumference of the circle, that is between (0° ≤ Θ ≤ 360°). In the case of a non-circular geometry the value is applied uniformly around the perimeter of the cylinder. The heat transfer coefficient is modeled to the Sensor2 design by employing non-uniform or discrete thermal boundary conditions as the material properties of the Sensor2 design allow some areas to be assumed as adiabatic. The local Nusselt number distribution is obtained for the case that heat is exchanged from specific locations on the surface of the circular cylinder.
Non-Uniform/Discrete Thermal Boundary Conditions

Although non-constant or non-uniform thermal boundary conditions are not widely studied in forced convection across cylinders, they are employed in the study of natural convection in enclosures. Some literature which investigates the effect of various thermal boundary conditions imposed on enclosures are reviewed where the thermal boundary condition applied to each of the enclosure surfaces is stated.

Basak et al. [99] numerically studied the effects of non-uniform thermal boundary conditions on natural convection flows in a square cavity for the steady laminar flow regime where they investigated the effects to the system when the top boundary is zero heat flux, the side walls are specified temperature conditions and the bottom wall is where the heat flux varies sinusoidally with position. In another study, Basak et al. [100] numerically investigated the effects of various thermal boundary conditions on the natural convection in a triangular enclosure, the bottom wall of the enclosure is a zero heat flux condition, while the other two walls are a specified temperature, taking either a constant value or varying linearly along the length of the wall. Roy et al. [101] studied the natural convection in a triangular enclosure where the bottom temperature condition varies sinusoidally along the length of the wall and the remaining two walls are constant temperature conditions. Natarajan et al. [102] numerically studied the effect of natural convection in a trapezoidal enclosure. The top boundary is zero heat flux, the bottom is a constant temperature and the two side walls, which are inclined at an angle, are linear functions of temperature. In these numerical studies the type of and configuration of boundary conditions applied to the walls of the enclosure have drastic effects on the heat transfer to the fluid and consequently the flows inside the enclosures.

Discrete heat exchange locations, where the value associated with the boundary condition is discontinuous, are also employed in the study of natural convection flows in enclosures. Cheikh et al. [103] numerically studied the natural convection in a square enclosure where the bottom wall is sectioned into three, a value of uniform heat flux is imposed to a segment of the bottom wall where the remaining segments of the bottom wall are zero heat flux. The
remaining walls are sectioned in two, where either half can take a zero temperature or a zero heat flux boundary condition. Saravanan and Sivaraj [104] numerically studied the effect on natural convection in a square enclosure where the top boundary is zero heat flux and the two side walls are at a constant temperature; the bottom boundary consists of three segments, the thermal boundary condition for the middle segment is a linear temperature profile, where the two remaining segments are zero heat flux. The application of discrete heat exchange locations to the walls of the enclosure also change the heat transfer inside the enclosure by generating a flow-field that is not comparable to the standard type, where all walls are heated uniformly with the same type of boundary condition. In chapter 4 the numerical methodology to model the heat transfer to the Sensor2 design is presented where discrete and various thermal boundary conditions are employed around the circular geometry.

Modeling the Flow-Field as Inviscid

To guide the sensor development a range of magnitudes of velocity which are encountered in an industrial environment are obtained from the work of Johnson [4]. Johnson [4] performed dissolution studies in an aluminum reduction cell, with the goal of developing a correlation for the rate of dissolution of iron to the magnitude of liquid aluminum velocity in the cell. He estimated the magnitude of velocity between \((0.04 \leq u_{\infty} \leq 0.24)\) m·s\(^{-1}\) [4] in the reduction cell which correlates to Reynolds numbers of \((3400 \leq Re \leq 20400)\). The experiments performed in this work are estimated for Reynolds numbers of \((6800 \leq Re \leq 21000)\).

According to Zdravkovich [30], Mutlu Sumer and Fredsoe [31], and Williamson [105], at a Reynolds number of greater than approximately \(Re=260\) [105] the wake from the circular cylinder is turbulent and the flow structure is three-dimensional. The subcritical flow regime across a circular cylinder according to Mutlu Sumer and Fredsoe [31] spans \((300 \leq Re \leq 3 \times 10^5)\). Kravchenko and Moin [106] numerically simulate the flow over a circular cylinder in the subcritical regime at a Reynolds number of \(Re=3900\). They employ a Large Eddy Simulation (LES) model to predict the turbulence in the flow regime and compare their numerical results.
with experimental data in the literature; they find that the LES solution is in good agreement with experimental data.

LES predictions in which the large scale turbulent structures are computed and where the subgrid turbulent scales are modeled are emerging as a viable numerical technique with respect to accurately predicting the complex flow-field of flow around cylinders [107, 106]. However the LES predictions suffer from long computation times as result of numerically resolving the major portion of the turbulent scales [107, 108]. The flow-field which is imposed in the numerical predictions for Sensor2 is inviscid; this simplification is applied to reduce the computation time of the numerical calculation.

Kays and Crawford [109] and Bejan [110] state that for very low Prandtl numbers the thermal boundary layer will develop much faster than the velocity boundary layer and little error will be introduced if the velocity everywhere in the thermal boundary layer is assumed to be the free stream velocity. As stated previously Khan et al. [85] argue against this. Galante and Churchill [111] also argue against this proposition as they state the following: the application of the velocity field for potential flow for the predictions of forced convection from immersed solid objects to liquid metals is based on three premises. The first premise is that the velocity field outside the momentum boundary layer is closely approximated by potential flow; this premise is invalid in the region of separation that occurs around a circular cylinder. The second premise is that the free stream velocity is sufficiently large and the viscosity sufficiently low so that the momentum boundary layer is very thin and hence does not displace the region of potential flow significantly from the surface. Galante and Churchill [111] argue that this premise is also false, as the momentum boundary layer thickens with position along the immersed body, this thickening of the momentum boundary layer decreases the rate of heat transfer and thereby results in over prediction of the rate of heat transfer when employing a potential flow solution. The third premise is that the Prandtl number of the liquid metal is sufficiently low so that the thermal boundary layer extends far beyond the momentum boundary layer. Galante and Churchill [111] argue against this premise as the Prandtl number is low,
it is still finite which results in an over prediction of the heat transfer. They estimate that for a Prandtl number of \( Pr=0.02 \) the over estimation of the average Nusselt number is 7% when employing a potential flow solution to the momentum equation for flow over an isothermal (constant wall temperature) cylinder. At the subcritical Reynolds numbers where the experiments of this work are performed, to accurately predict the flow-field around a cylinder, a LES numerical method would need to be implemented. During this thesis the author lacked the computational hardware to perform these LES simulations, as such an approximation to the flow-field is performed by employing a potential flow solution for the Sensor2 design.
Chapter 3

Experimental Methodology

This chapter describes the experimental apparatus employed to generate liquid aluminum flow-fields in which the sensor designs were tested. The construction of Sensor1 and Sensor2 are described, and schematics of the experimental apparatus, Sensor1, and Sensor2 are presented.

A specially designed furnace was used to generate the liquid aluminum flow-fields for this study. The experimental system is portrayed in Figure 3.1, where Figure 3.1a is a schematic and Figure 3.1b is a photograph of the experimental apparatus. The experimental apparatus was comprised of the furnace which was named the Revolving Liquid Metal Tank (RLMT), an immersion apparatus which was used to insert the sensor into flowing liquid aluminum and a data acquisition system with computer to record the various signals from the experiment.

3.1 Experimental Equipment

The Revolving Liquid Metal Tank (RLMT) was an electrical resistance furnace which was capable of melting approximately 50 kg of aluminum. The crucible diameter was approximately 346 mm (13.6 inch) with a height of approximately 254 mm (10 inch). Figure 3.2a is a photograph of the RLMT crucible, charged with sectioned aluminum ingots for melting. Using Figure 3.2b as a visual guide, the construction of the furnace was as follows: focusing at the radial center of the furnace, a mild steel crucible was surrounded by electrical heating coils,
Figure 3.1: Experimental Apparatus

(a) Schematic of Experimental Apparatus

(b) Photograph of Experimental Apparatus
surrounding the heating coils was alumina insulation brick, followed by another layer of aluminosilicate fiber insulation. The underside of the steel crucible was fastened to a steel shaft which ran down the length of the furnace out of the electrically heated zone to a pulley system located at the base of the furnace structure. The pulley system was connected to an electrical motor which was used to rotate the crucible in clockwise rotation. The crucible pulley arrangement generates a liquid aluminum forced vortex flow-field. The sensors were immersed at a fixed radial position in this flow. Commercial purity aluminum was used to perform the experiments in this work; Table 3.1 lists the thermophysical properties.

To position the sensor into the RLMT, a device named the immersion apparatus was utilized. This instrument is displayed in Figure 3.1. The immersion apparatus was capable of translating the sensor in the vertical and horizontal directions in addition to having the capability to rotate the sensor about its base. The center of the sensor was placed 103 mm (4 inch) from the center of the crucible.

A National Instruments Data Acquisition (DAQ) System was used to record the various signals from an experimental run. This system comprised of an SCXI 1000 chassis, two SCXI 1125 amplifiers, and two SCXI 1320 input data modules. The DAQ was connected via a National Instruments PCI 6033E card to a 32bit Windows XP operating system computer which was used to interface with the DAQ and record the signals. Figure 3.1 portrays the DAQ and computer.

The RLMT was instrumented with a bath thermocouple and a contact rod (Figure 3.1a). The bath thermocouple was positioned at the center of the crucible and was used to measure the temperature of the liquid aluminum. The contact rod was employed as a current path for an
Figure 3.2: Photographs of Revolving Liquid Metal Tank (RLMT) Interior
Table 3.2: Thermophysical Properties of AISI 12L14 Steel [112]

<table>
<thead>
<tr>
<th>$\rho$ [kg·m$^{-3}$]</th>
<th>$k$ [W·m$^{-1}$·K$^{-1}$]</th>
<th>$c_p$ [J·kg$^{-1}$·K$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7870</td>
<td>51.9</td>
<td>472</td>
</tr>
</tbody>
</table>

immersion circuit. The immersion circuit was used to signify the exact immersion time of the sensor into the aluminum bath. A 5V bias was applied across the sensor and the contact rod. When the sensor was immersed into the bath, a short circuit of 0V was recorded by the DAQ. The voltage drop from 5V to 0V signifies the immersion of the sensor.

### 3.2 The Sensors

Two probe designs have been constructed and tested in this thesis. The geometry of both probes are circular cylinders. The first probe design which is referred to as Sensor1, is a circular cylinder constructed of one material. The second probe design which is referred to as Sensor2 is a circular cylinder constructed of more than one material.

#### 3.2.1 Sensor1

Sensor1 consists of two parts. Part 1 was where thermocouples were inserted. This was done via threading the type K thermocouples through a 9.5 mm (0.375 inch) diameter hole which was bored along the length of the part 127 mm (5.0 inch); the diameter of the cylinder was 38.1 mm (1.5 inch). This dimensional information is depicted in Figure 3.3a. Figure 3.3b and Figure 3.3c show the underside of Part 1 where the thermocouples were located. There are eight positions for thermocouples to be housed in the Sensor1 design. The tips of the leads of the type K thermocouples all face the outer surface of the cylinder and were 1 mm (0.039 inch) from the surface. The chromel and alumel leads of the type K thermocouple form an intrinsic thermocouple junction with the sensor substrate. These leads sit on either side of the thermocouple position cut-out of Part 1, the thermocouple cut-out locations are depicted in Fig-
Table 3.3: Thermophysical Properties of C110 Copper [112]

<table>
<thead>
<tr>
<th>( \rho ) [kg m(^{-3})]</th>
<th>( k ) [W m(^{-1}) K(^{-1})]</th>
<th>( c_p ) [J kg(^{-1}) K(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8890</td>
<td>388</td>
<td>385</td>
</tr>
</tbody>
</table>

Table 3.3c. The intrinsic thermocouple junction is known to have superior response times when compared to the welded bead junction, this information is outlined in the work of Moen [113], White [114], and Henning and Parker [115]. Each thermocouple was located 45° from one another, which is shown in Figure 3.3c. The depth of the slot which houses the thermocouple was 0.12 mm (0.005 inch) smaller than the diameter of the thermocouple sheathing. Part 2 of the Sensor1 design is depicted in Figure 3.3d, it was a piece of material 63.5 mm (2.5 inch) in length which covers the thermocouples of Part 1, shielding them from the liquid aluminum. The thermocouple shielding was pinched by 0.12 mm (0.005 inch) which help to ensure that the thermocouple sheathing does not move within the slot. The chromel and alumel leads are compressed between Part 1 and Part 2 to ensure proper thermal contact of the thermocouple leads with the sensor material. This technique has been utilized in the work of Kim [76] and Osman [116] to measure the thermal response at select locations inside copper material. Experiments were performed with a Sensor1 design constructed from AISI 12L14 steel and C110 copper, photographs of these probes are shown in Figure 3.4a and Figure 3.4b. Table 3.2 and Table 3.3 list the thermophysical properties of AISI 12L14 steel and C110 copper. The length of Sensor1 that was immersed into the liquid was 127 mm (5 inch), approximately 25.4 mm (1 inch) was exposed above the bath. A coating of carbon was applied to the surface of the sensor to prevent the dissolution of the sensor material for the duration of the transient experiment. This carbon coating was applied to a cool Sensor1 design by manually spraying an aerosol colloidal suspension of carbon and ethanol onto the surface of the Sensor1 prior to each experiment. The carbon coating acted to prevent the oxidation of the iron and copper and prevented the dissolution of iron and copper into liquid aluminum.

Type K thermocouples were strapped on the external surface of Sensor1 to infer the duration
Figure 3.3: Schematic Views of the Parts used to Construct Sensor1
of aluminum shell. The leads of chromel and alumel were exposed from the sheath and made
direct contact with the liquid metal upon immersion of Sensor1. These exposed junctions were
twisted to read temperature prior to immersion into liquid aluminum and the leads were bent
toward the cylinder to touch the surface. Four external thermocouples were situated on the
surface of the cylinder. The internal thermocouples were positioned 45° from one another.
The four exterior thermocouples were situated at 90° from one another on the outside of the
cylinder, they were positioned 63.5 mm (2.5 inch) from the base of the cylinder. Figure 3.5 is a
photograph of Sensor1 coated with carbon and fitted with exterior thermocouples. Figure 3.6 is
a schematic of the location of the thermocouples from the stagnation point as they are located
around the Sensor1 geometry. The stagnation point on the surface of the sensor is estimated by
constructing a line from the center of the RLMT crucible to the point tangent to the face that
interacts with the oncoming flow-field.

3.2.2 Sensor2

Sensor2 was constructed of more than one material. The heat transfer region upon which veloc-
ity information from liquid aluminum is derived was constructed from calcium silicate material
and copper material. The remaining material of Sensor2 was constructed from mild steel; the
mild steel was used to encase the calcium silicate and copper materials. The thermophysical
properties of calcium silicate are listed in Table 3.4. Each copper element location was isolated
by a region of low thermal conductivity calcium silicate material, the rationale of this design
was to isolate the transfer of energy to the eight type K thermocouples from only the liquid
aluminum and prevent the transfer of energy within the sensor.

Figure 1.1b is a schematic of Sensor2. It consists of a mild steel holder which was used to
encase the calcium silicate and copper elements of the probe. Eight copper elements were fitted
into the calcium silicate material at 45° from one another. Figure 3.7 are various views of the
copper element of Sensor2. Figure 3.7c is an isometric view of the element, it was a 76.2 mm
(3 inch) length of copper with a 3.175 mm (0.125 inch) diameter bore which runs 25.4 mm (1
Figure 3.4: Photographs of Sensor1 Constructed of Steel and Copper
Figure 3.5: Photograph of the Sensor1 Design Coated with Carbon and Fitted with Exterior Thermocouples

Figure 3.6: Sensor1 Thermocouple Orientation with respect to flow-field Stagnation Point
inch) down the length of the element. This information is more clearly seen in Figure 3.7b, an exposed junction type K thermocouple was inserted into the bore in the copper element. Figure 3.7a is a plan view of the copper element, it was originally a 6.35 mm (0.250 inch) by 6.35 mm (0.250 inch) square bar where a radius of the dimension of the cylinder was machined into the copper element. The curvature which was machined into the copper element ensures that when assembled, the probe has a circular cross-section. A thermocouple was inserted into the bore of the copper element and fixed in place using tin-antimony solder, the thermocouple junction was exposed and the chromel and alumel leads were not welded together prior to soldering into the hole. Prior to immersion of Sensor2 into liquid aluminum the entire sensor was heated to approximately 673 K (400 °C). At a temperature of 673 K (400 °C) the tin-antimony solder was in liquid phase. The tin-antimony solder acts to couple the thermocouple junction to that of the copper material acting to reduce the thermal lag of the thermocouple. This technique has been utilized in the work of Mucciardi [117] and Goudie [81] to measure the thermal response inside a blind hole at select locations inside circular metal cylinders. In addition, preheating the sensor to this temperature reduces the heat flux drawn from the sensor-aluminum interface and suppresses the formation of shell.

Figure 3.8 are drawings of the calcium silicate housing which thermally isolate the copper elements. Figure 3.8a is a plan view of the housing and Figure 3.8b is an isometric view, showing the positions of the eight machined grooves in the calcium silicate for the copper. Figure 3.9 displays the caps which were used to thermally isolate the flow of energy from the narrow face of the copper element. There were two, an upper cap displayed in Figure 3.9a which has eight bores which were used to pass thermocouples to the copper elements, and a lower cap portrayed in Figure 3.9b.

Figure 3.10 depicts the steel holder which encases the composite sensor. Figure 3.10a portrays the steel holder upper cap, Figure 3.10b displays the steel holder lower cap, these steel caps were used to sandwich the calcium silicate and copper parts together. Figure 3.10c is the steel holder rod which runs down the length of the probe and was used to fix in position
Table 3.4: Thermophysical Properties of Calcium Silicate [112]

<table>
<thead>
<tr>
<th>$\rho$ [kg·m$^{-3}$]</th>
<th>$k$ [W·m$^{-1}$·K$^{-1}$]</th>
<th>$c_p$ [J·kg$^{-1}$·K$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.2</td>
<td>930</td>
</tr>
</tbody>
</table>

the steel end caps. The steel holder rod also acted as the attachment point to the immersion apparatus to insert and remove the probe from liquid aluminum.

Figure 3.11 are photographs which depict the components of Sensor2 (Figure 3.11a) and depict an assembled Sensor2 (Figure 3.11b). The center of Sensor2 was placed 103 mm (4 inch) from the center of the RLMT crucible. The length of Sensor2 that was immersed into the liquid aluminum was 127 mm (5 inch), approximately 25.4 mm (1 inch) was exposed above the bath. Each copper element was coated with a layer of carbon prior to preheating the Sensor2 design to approximately 673 K (400 °C), the carbon coating was applied by spraying an aerosol colloidal suspension of carbon and ethanol onto the surface of the copper element. The carbon coating prevented the dissolution of the copper element into the liquid aluminum for the duration of the experiment. The copper elements are shown in the photographs of Figure 3.11 without the coating of carbon.
Figure 3.7: Drawings of Copper Element Component of Sensor2
Figure 3.8: Drawings of Calcium Silicate Housing Component of Sensor2
Figure 3.9: Drawings of Calcium Silicate Cap Components of Sensor2
Figure 3.10: Drawings of Steel Holder Components of Sensor2
(a) Photograph of Sensor2 Components

(b) Photograph of Assembled Sensor2

Figure 3.11: Photographs of the Sensor2 Design
Chapter 4

The Numerical Models

In this chapter the numerical methods employed to study the fluid-thermal phenomena relevant to this thesis are described. Numerical investigations are performed in ANSYS Fluent [36]. Fluent is utilized to study the solidification and melting patterns of aluminum onto the Sensor1 design with the goal to estimate the thermal resistance at the experimental Sensor1-aluminum interface. The equations employed are presented in section 4.1; the computational domain and boundary conditions of the solidification and melting problem are described in subsection 4.1.2. Fluent is also employed to study the heat transfer rates to the Sensor2 design with the purpose of generating local Nusselt number curves at various Reynolds number and to estimate the thermal resistance at the interface of each thermal capacitive lump of Sensor2. The computational domain and boundary conditions for modeling Sensor2 are presented in sections 4.2 and 4.3. The discretization schemes are listed that are chosen from the Fluent code to solve these numerical problems and grid independence tests for each numerical investigation are presented.
4.1 Modeling Solidification and Melting of Aluminum on Sensor1

Numerically modeling the flow of liquid aluminum around Sensor1 to predict the local solidification and melting times of the shell will provide an estimate of the thermal resistance at the Sensor1-aluminum interface. The thermal resistance at the Sensor1-aluminum interface may then be compared with that at the Sensor2-aluminum interface. The estimate of the thermal resistance is required in the data reduction equation to determine the velocity of the liquid metal.

In this section, the algorithm employed by Fluent to model the solidification and melting of aluminum onto the surface of Sensor1 is described. ANSYS Fluent [36] is a commercial computational fluid dynamics software which is capable of solving the momentum and energy equations to model the solidification phenomenon encountered in this work. ANSYS Fluent v14.0 is utilized for this computational study. Fluent employs the enthalpy-porosity [49] approach to track the solid-liquid interface for solidification and melting phase change problems as a fixed grid is employed for the computation. The enthalpy method accounts for the latent heat in the energy equation by assigning a nodal latent heat value to each computational cell according to the temperature of the cell [49]. Upon change of phase, the nodal latent heat content of the cell is adjusted to account for latent heat absorption or evolution, the adjustment being reflected in the energy equation as either a heat sink or source [49]. The enthalpy-porosity approach to model the phase change damps the velocity in the solid nodes by employing a porosity approach. Computational cells that are undergoing phase change are modeled as pseudo-porous media, with the porosity being a function of the enthalpy content [49].

A brief description of the equations employed in the enthalpy-porosity technique are as follows: in the liquid and mushy regions, conservation of mass is required and represented by Equation (4.1), liquid metals are Newtonian fluids, thus the conservation of momentum for a
two-dimensional Cartesian system is represented by Equation (4.2) for the x-component and Equation (4.3) for the y-component.

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{4.1}
\]

\[
\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho \vec{u} u) = \nabla \cdot (\mu \nabla u) - \frac{\partial P}{\partial x} + Au \tag{4.2}
\]

\[
\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho \vec{u} v) = \nabla \cdot (\mu \nabla v) - \frac{\partial P}{\partial y} + Av \tag{4.3}
\]

\(u, v\) represent the x and y velocity components, \(\rho\) the density, \(\mu\) the dynamic viscosity, \(P\) the pressure, \(t\) time, and \(A\) the mushy zone damping parameter. The enthalpy formulation of the energy equation is represented by Equation (4.4).

\[
\frac{\partial (\rho H)}{\partial t} + \nabla \cdot (\rho \vec{u} H) = \nabla \cdot \left( \frac{k}{c} \nabla H \right) + S_H \tag{4.4}
\]

\(H\) is the enthalpy \((H = \int_{T_{ref}}^T c \, dT + \Delta H)\), \(k\) the thermal conductivity, \(c\) the specific heat, and \(S_H\) is the energy source term. The latent heat evolution is accounted for by the source term in the energy equation, it is represented by Equation (4.5):

\[
S_H = \frac{\partial (\rho \Delta H)}{\partial t} + \nabla \cdot (\rho \vec{u} \Delta H) \tag{4.5}
\]

The latent heat content \(\Delta H\) is a function of temperature. In the case of an isothermal phase change, as in the case of this work, the divergence term on the right of equation (4.5) vanishes. The latent heat content as a function of temperature is defined by the following relationship:

\[
f(T) = \begin{cases} 
L & T > T_m \\
0 & T < T_m 
\end{cases}
\]
$T_m$ is the isothermal phase change temperature and $L$ is the latent heat of fusion. To account for the zero velocities in the solid regions a value of the damping parameter $A$ must be chosen. The damping parameter $A$ in Equation (4.6) mimics the Carman-Kozeny equation for flow in porous media [49].

\[ A = -C \frac{(1 - f_l)^2}{f_l^3 + b} \]  

(4.6)

$C$ is a constant to account for the mushy region morphology, $f_l$ is the liquid fraction of the cell ($\frac{\Delta H}{L}$), and $b$ is a constant (0.001) to avoid division by zero.

### 4.1.1 Simplifications Employed

The modeling employed for the forced convective solidification and melting is via a two-dimensional simulation. Using a cylindrical coordinate reference frame (Figure 4.1), the cross-section of the revolving liquid metal tank (RLMT) is taken in the radial and angular plane, thus the axial length of the tank is not included in the numerical calculation. The mean velocity acting on the cylinder based on the location of immersion and the RLMT setting can fall within the range of $(0.08 \leq u_\infty \leq 0.25) \text{ m·s}^{-1}$, this corresponds to Reynolds numbers within the range of $(6800 \leq Re \leq 21000)$. According to Williamson [105], at these Reynolds numbers the vortex shedding behind the cylinder is three-dimensional, the simulations presented in this work do not capture this phenomenon. Based on the analysis of Mutlu Sumer and Fredsoe [31] the wake formed by the cylinder is turbulent at these Reynolds numbers. The equations employed in the solution of this problem do not invoke turbulence modeling. The aforementioned simplifications are utilized in the solution of the momentum equations to save on the time required to compute a numerical solution to the problem. With respect to energy, a thermal resistance at the sensor-aluminum interface must be accounted for, so that the numerical predictions agree with the experimental data.
4.1.2 Calculation Domain and Boundary Conditions

This numerical study considers a two-dimensional plane of the revolving liquid metal tank (RLMT). Figure 4.1a is a schematic of the domain where the crucible wall is the outer extent, characterized in the cylindrical coordinate system by a radius of 0.173 m. The sensor is located 0.103 m from the center of the crucible. The sensor (circular cylinder) has an outer radius of 0.01905 m. Figure 4.1b schematically displays the boundary and initial conditions of the system.

**Initial Condition - Bath:** The bath temperature is initialized with a value dependent on the superheat \((SPH)\) of the liquid aluminum and represented in Figure 4.1b as \(933 + (SPH)\), on the Kelvin temperature scale. The initial velocity profile of the liquid is that of a forced vortex flow-field \(U_\Theta = \omega r\).

\[
T_{bath} = 933 + (SPH) \quad (4.7a)
\]

\[
U_\Theta = \omega r \quad (4.7b)
\]

\[
\omega = \frac{2\pi (RPM)}{60} \quad (4.7c)
\]

where \(U_\Theta \ [m\cdot s^{-1}]\) is the tangential velocity component, \(\omega \ [s^{-1}]\) is the angular velocity, \(r \ [m]\) is the radial coordinate, and \(RPM\) represents the revolutions per minute that the RLMT crucible spins in the experiment.

**Initial Condition - Sensor1:** The initial temperature condition for Sensor1 is that of 298 K. The sensor wall is impenetrable and no-slip occurs on the surface of the wall. There is an imposed thermal resistance which acts at the sensor-aluminum interface denoted by \(R \ [m^2 \cdot K \cdot W^{-1}]\). This thermal resistance is uniform around the circumference of the sensor
(a) Computational Domain

(b) Boundary Conditions

Figure 4.1: Computational Domain and Boundary Conditions of the Solidification and Melting Problem
and constant in time, it exists for the entire duration of the simulation.

\[ T_{cyl} = 298 \]  \hspace{1cm} (4.8a)

\[ R = C_R \quad \text{specified constant} \]  \hspace{1cm} (4.8b)

**Crucible Wall:** The boundary condition at the crucible wall with respect to temperature is constant at a value of \( 933 + (SPH) \) K. The wall rotates with an angular velocity \( \omega \) of the experimental rotation speed of the RLMT.

\[ T_{wall} = 933 + (SPH) \]  \hspace{1cm} (4.9a)

\[ \omega = \frac{2\pi(RPM)}{60} \]  \hspace{1cm} (4.9b)

### 4.1.3 Discretization Schemes and Property Data

Here the discretization schemes and property data are listed for the solution of this numerical problem. Unstructured quadrilateral cells are generated using the ANSYS Workbench v14.0 mesh generation software [118]. The SIMPLE scheme [54] has been employed for the pressure velocity coupling. The momentum and energy spatial discretization are second order upwind schemes [36]. The transient formulation is first order implicit [36]. The residuals of continuity, x-velocity, and y-velocity are reduced to \( 10^{-5} \) and the energy residual is reduced to \( 10^{-9} \). The maximum iterations per time step is set at 200. The mushy zone parameter \( (C) \) is set constant at \( 10^5 \). The solidification and melting of aluminum onto the Sensor1 design constructed of steel and copper are both modeled. The thermophysical properties of commercial purity aluminum, AISI 12L14 steel, and C110 copper are listed in Tables 3.1, 3.2, and 3.3.
4.1.4 Grid Independence Tests

A grid independence study was performed to ascertain a suitable mesh. Figure 4.2 displays a representative mesh explored in this work. The solution on three grid sizes are explored, each with increasing grid density ranging from Grid 1 (G1) at 26029 cells, Grid 2 (G2) at 30259 cells, and Grid 3 (G3) at 45292 cells. The region around the cylinder is refined to higher degree than the remainder of the fluid domain as shown in Figure 4.2. The refined region of cells spreads beyond the circular cylinder (Sensor1) itself to capture the solidification of aluminum which forms on the surface of the probe. Figure 4.3a plots the computational domain liquid fraction as a function of time for the case of no thermal resistance at the Sensor1-aluminum interface for a copper substrate in flowing liquid aluminum at 0.08 m·s$^{-1}$ and 60 K SPH. The computational domain liquid fraction versus time was used as a means to evaluate an appropriate grid density. Good agreement between the results of Grid 2 and Grid 3 was observed on the liquid fraction versus time curve (Figure 4.3a). Figure 4.3b shows the results
Figure 4.3: Grid Independence Tests for the Solidification and Melting of Aluminum on Sensor1: Plots of Liquid Fraction versus Time in the Computational Domain for a Copper Sensor1 in Liquid Aluminum flow conditions of 0.08 m·s\(^{-1}\) and 60 K SPH, \(\dot{R} = 0 \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)

of the liquid fraction in the domain versus time for Grid 2 when the time step was varied. Three time steps of \(\Delta t = 0.01 \text{ s}, 0.005 \text{ s} \) and 0.001 s were tested. Using the results of Figure 4.3, Grid 2 (30259 cells) and a time step of \(\Delta t = 0.005 \text{ s}\) was employed for the solidification and melting predictions in this work.

### 4.2 Modeling the Steady Convective Heat Transfer of Liquid Aluminum around Sensor2

Numerically modeling the steady convective heat transfer of liquid aluminum around Sensor2 will generate local Nusselt number curves for Sensor2 at various Reynolds numbers. These curves will be used to obtain a correlation relating the Nusselt number of Sensor2 to the Reynolds number of the flow-field and hence allow an estimate of the magnitude of velocity.

A uniform flow profile is employed to model the convective heat transfer of liquid aluminum around Sensor2, this conforms to the standard cylinder in crossflow conditions which are commonly presented in the literature. The data reduction equation presented in chapter 6
to determine direction and magnitude of velocity is derived based on this standard flow configuration. The experimental configuration deviates from the standard configuration in that the experimental velocity field is initially linear, when the sensor is immersed the flow-field is transient, while the standard configuration is of a uniform velocity field. In this work, the effect of the initial linear velocity field and the transient nature of the experimental flow on the heat transfer to Sensor2 was not studied.

A cross section of the RLMT is modeled for Sensor1 to account for the energy loss in the tank. In the solidification and melting problem of Sensor1 approximately 430 kJ of energy can exchange between Sensor1 and the liquid on average 30 seconds. The experimental furnace cannot input that amount of energy per unit time, as a result the cross section of the entire tank is required to model the temperature drop of the bath. In Sensor2 the energy exchanged is estimated at approximately 30 kJ, the furnace can input that energy per unit time to make the uniform free stream inlet temperature assumption valid for the duration of the Sensor2 experiment.

In this section the mathematical formulation of the steady heat transfer from a circular cylinder in crossflow to predict the rate of heat transfer to the fluid from Sensor2 is presented. The equations which are employed do not model turbulence and the solution is time independent. The Reynolds number range investigated is (10 \( \leq \) Re \( \leq \) 30000). Four cases of thermal boundary conditions are utilized which will be elaborated on later. The continuity, momentum, and energy equations are solved in ANSYS Fluent [36] in the absence of solidification and melting and a gravitational force. Solidification and melting is not modeled as Sensor2 is preheated to approximately 673 K (400 °C), at temperatures higher than this shell did not form on the sensor. It is assumed that the flow is steady and incompressible with negligible thermal dissipation and the thermophysical properties of the fluid to be constant. The two dimensional numerical calculation solves the continuity (4.1), x-component (4.2) and y-component (4.3) of momentum, and the energy equation (4.4). In equations (4.2) and (4.3) the velocity damping source terms \( Au \) and \( Av \) are not employed, in addition the enthalpy source term \( S_H \) in equation
Figure 4.4: A Plot of the Computational Domain for the Heat Transfer from Liquid Aluminum to Sensor2

(4.4) is not employed in this formulation.

4.2.1 Calculation Domain and Boundary Conditions

The calculation domain is schematically depicted in Figure 4.4, it is a circle, with an outer diameter of $100D$, where $D$ is the diameter of the circular cylinder (Sensor2). The cylinder lies at the center of the domain, the inlet is located at $50D$ upstream and the outlet is located at $50D$ downstream from the center of the circle.

**Inlet Boundary:** Along the perimeter of one semi-circle of the domain is the inlet boundary condition, where the free stream velocity and free stream temperature are set at:

\[
\begin{align*}
    u &= U_\infty \\
    v &= 0 \\
    T &= T_\infty
\end{align*}
\]
Outflow Boundary: On the other semi-circle an outflow [36] boundary condition is specified:

\[
\begin{align*}
\frac{\partial u}{\partial x} &= 0 \quad (4.11a) \\
\frac{\partial v}{\partial x} &= 0 \quad (4.11b) \\
\frac{\partial T}{\partial x} &= 0 \quad (4.11c)
\end{align*}
\]

Surface of the Cylinder: The surface of the cylinder is impenetrable to the fluid with a zero shear stress condition of:

\[
\tau_s = 0 \quad (4.12)
\]

where \(\tau_s\) is the shear stress on the surface of the cylinder.

The thermal boundary condition however requires some elaboration. In this work four different configurations are studied:

Cylinder CWT: The first is the Dirichlet, constant wall temperature (CWT) condition:

\[
T = T_s \quad (0^\circ \leq \Theta \leq 360^\circ) \quad (4.13)
\]

Cylinder UHF: The second is the Neumann, uniform heat flux (UHF) condition:

\[
\frac{\partial T}{\partial n_s} = -\frac{q}{k} \quad (0^\circ \leq \Theta \leq 360^\circ) \quad (4.14)
\]

Typically in experimental heat transfer studies the Neumann boundary condition is applied to the surface of the cylinder as this boundary condition is relatively easier to implement experimentally than the Dirichlet condition [33]. In mass transfer studies however the Dirichlet boundary condition is more appropriate to represent the transport process at the surface of the
cylinder. These are the two commonly encountered boundary conditions in the literature, as such in this work numerical predictions of both are performed to estimate the heat transfer to Sensor2.

The remaining two conditions refer to the discrete thermal boundary conditions on the surface of the cylinder.

**Sensor2 CWT:** The third configuration consists of both Dirichlet and Neumann conditions on the surface of the cylinder:

\[ T = T_s \quad (45^\circ \times i - 9^\circ) \leq \Theta \leq (45^\circ \times i + 9^\circ) \quad i = (1 \text{ to } 8) \] (4.15)

\[ \frac{\partial T}{\partial n_s} = 0 \quad \text{elsewhere on the boundary} \] (4.16)

**Sensor2 UHF:** The final thermal boundary configuration consists of the Neumann condition where the value is discontinuous on the surface of the cylinder:

\[ \frac{\partial T}{\partial n_s} = -\frac{q}{k} \quad (45^\circ \times i - 9^\circ) \leq \Theta \leq (45^\circ \times i + 9^\circ) \quad i = (1 \text{ to } 8) \] (4.17)

\[ \frac{\partial T}{\partial n_s} = 0 \quad \text{elsewhere on the boundary} \] (4.18)

To facilitate a discussion on the numerical results some terminology is introduced. There are four configurations of thermal boundary condition which are applied to the surface of the circular cylinder. In the discussion, the designation Cylinder is used to refer to the case where the thermal boundary condition is uniform around the circumference of the circle. The designation Sensor2 is used to describe the scenario where the thermal boundary condition is discrete around the circumference of the circle.

The boundary conditions on the surface of the circular cylinder are schematically depicted in Figure 4.5. Figure 4.5a represents the Cylinder classification, where the thermal boundary conditions are uniform. The Cylinder designation can have either a CWT or a UHF boundary
Figure 4.5: Schematic of Cylinder and Sensor2 Thermal Boundary Conditions: Steady Convective Heat Transfer of Liquid Aluminum around Sensor2

condition signifying that either a Dirichlet or a Neumann condition is applied uniformly around the circle. Figure 4.5b is a diagram of the boundary conditions of Sensor2. The elements that exchange energy are shaded to highlight the position around the circle. The energy exchange occurs at 18° arc segments around the circle. The arcs are spaced 45° from one another, from arc center to arc center. They are depicted in Figure 4.5b. These arc segments take either a CWT or UHF boundary condition. The remaining perimeter of the circle has a boundary condition of a zero heat flux. In what is to follow, Sensor2 is designated as having either a CWT or a UHF boundary condition. This implies that the CWT condition occurs at the eight 18° arc segments, zero heat flux is maintained on the remainder of the circle. Similarly, when the UHF condition for Sensor2 is referred to, it is implied that the heat flux is uniform along the eight 18° arc segments, zero heat flux is maintained on the remainder of the circle. The assumption is made that the heat transfer is negligible over the regions of calcium silicate material as there is three orders of magnitude difference in the values of thermal conductivity of the copper and calcium silicate materials. The thermal conductivity of copper is 388 W·m$^{-1}$·K$^{-1}$ while the thermal conductivity of calcium silicate is 0.2 W·m$^{-1}$·K$^{-1}$.
4.3 Modeling the Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2

Numerically modeling the transient conjugate heat transfer from liquid aluminum to Sensor2 provides an estimate of the values of thermal resistance at the interface of each thermal capacitive lump of Sensor2. These thermal resistances are required in the data reduction equation to determine the velocity of the high temperature liquid metal.

The mathematical formulation of the transient conjugate heat transfer problem for determining the temperature response of the copper sensing elements of the Sensor2 design is presented here. The modeling is similar to the steady formulation except that the mathematical equation includes a transient term and all material sub-domains are modeled. Four material sub-domains are modeled, the first being the aluminum fluid, the second being the calcium silicate solid material, the third the copper solid material, and the final sub-domain a resistance layer which acts between the solid-copper and the liquid-aluminum materials. The purpose here is to obtain an estimate of the temperature response of the copper element numerically under various values of thermal resistance to compare to the experimental data. The two dimensional numerical calculation solves the continuity (4.1), x-component (4.2) and y-component (4.3) of momentum for the fluid sub-domain, and an energy equation (4.4) for each material:

4.3.1 Calculation Domain and Boundary Conditions

The two dimensional crossflow around a circular cylinder is considered again. The calculation domain is schematically depicted in Figure 4.4. The inlet boundary conditions are represented by Equation (4.10), the outlet boundary conditions represented by Equation (4.11), and the boundary condition with respect to momentum at the surface of the cylinder is represented by Equation (4.12).

A cross-sectional schematic of the Sensor2 components is presented in Figure 4.6, which labels the material sub-domains. The boundary conditions at the representative interfaces of
Sensor2 for the conjugate analysis is presented in Figure 4.7.

**Aluminum - Resistance Layer Interface:** The aluminum-resistance interface is represented by the segment $AF$ in Figure 4.7. At the aluminum-resistance layer interface the heat flux is continuous and represented by:

$$k_{Al} \frac{\partial T}{\partial n} = k_R \frac{\partial T}{\partial n}$$  \hspace{1cm} (4.19)

**Aluminum - Calcium Silicate Interface:** The aluminum-calcium silicate interface is represented by the segments $AH$ and $FG$ in Figure 4.7. At the aluminum-calcium silicate interface the heat flux is continuous and represented by:

$$k_{Al} \frac{\partial T}{\partial n} = k_{cs} \frac{\partial T}{\partial n}$$  \hspace{1cm} (4.20)

**Resistance Layer - Copper Interface:** The resistance-copper interface is represented by the segment $BE$ in Figure 4.7. At the resistance-copper interface the heat flux is continuous and represented by:

$$k_R \frac{\partial T}{\partial n} = k_{Cu} \frac{\partial T}{\partial n}$$  \hspace{1cm} (4.21)

**Resistance Layer - Calcium Silicate Interface:** The resistance-calcium silicate interface is represented by the segments $AB$ and $FE$ in Figure 4.7. At the resistance-calcium silicate interface the heat flux is continuous and represented by:

$$k_R \frac{\partial T}{\partial n} = k_{cs} \frac{\partial T}{\partial n}$$  \hspace{1cm} (4.22)

**Copper - Calcium Silicate Interface:** The copper-calcium silicate interface is represented by the segments $BC$, $CD$, and $DE$ in Figure 4.7. At the copper-calcium silicate interface
Figure 4.6: Cross-Sectional Schematic of Sensor2 Geometry Identifying the Various Material Sub-Domains

the heat flux is continuous and represented by:

\[ k_{Cu} \frac{\partial T}{\partial n} = k_{cs} \frac{\partial T}{\partial n} \] (4.23)

**Initial Temperature Condition:** The initial temperature of the calcium silicate, copper and resistance layer are initialized at a Temperature \( T_i \) which is below that of the fluid initial temperature \( T_\infty \).

\[ T_{Cu} = T_i \] (4.24a)
\[ T_{cs} = T_i \] (4.24b)
\[ T_R = T_i \] (4.24c)
\[ T_{Al} = T_\infty \] (4.24d)

**Initial Velocity & Pressure Field:** The velocity and pressure fields are initialized by using the
Figure 4.7: Schematic of the Sensor2 Interface Conditions for the Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2
steady solution to the continuity and momentum equations of this flow problem. That is the potential flow solution to the flow over a circular cylinder.

A thermal resistance is incorporated between the solid-copper and liquid-aluminum by modifying the thermal conductivity of the region which is defined as a resistance layer in Figure 4.6. The density and heat capacity of this resistance layer is that of copper. Equation (4.25) is the relationship employed to impose a thermal resistance into the numerical model between the solid-copper and liquid-aluminum. Where \( \Delta r \) is the radial thickness of the resistance layer and \( k_{eff} \) is the effective thermal conductivity of the layer. The quotient of these quantities is the thermal resistance which is imposed at the copper-aluminum interface.

\[
R = \frac{\Delta r}{k_{eff}} \quad [\text{m}^2\cdot\text{K}\cdot\text{W}^{-1}] \tag{4.25}
\]

### 4.4 Nusselt Number

Here definitions of various types of averages employed to describe the Nusselt number of the Sensor2 design are presented. The Nusselt number is defined as:

\[
Nu_l(\Theta) = \frac{h_l(\Theta)D}{k} \tag{4.26}
\]

where \( h_l(\Theta) \) is the local heat transfer coefficient, \( D \) the diameter of the circular cylinder (Sensor2), and \( k \) the thermal conductivity of the fluid.

**\( \overline{Nu} \) - Average Nusselt Number:** The average Nusselt number is represented by the following relationship:

\[
\overline{Nu} = \frac{\overline{h_l}D}{k} = \frac{1}{2\pi} \int_0^{2\pi} Nu_l(\Theta)\,d\Theta \tag{4.27}
\]
**N̅u_{la} - Local Average Nusselt Number:** The local average Nusselt number is represented by the following relationship:

\[
N̅u_{la} = \frac{h_{la} D}{k} = \frac{1}{\Theta_2 - \Theta_1} \int_{\Theta_1}^{\Theta_2} N u_t(\Theta) d\Theta
\]  

(4.28)

The local average Nusselt number is employed to describe the heat transfer to the copper elements of the Sensor2 design. The difference \((\Theta_2 - \Theta_1)\) represents the 18° arc segments where the fluid transfers energy to the cylinder.

**N̅u_{S2} - Average Nusselt Number of Sensor2:** An average Nusselt number for Sensor2, is defined as follows:

\[
N̅u_{S2} = \frac{1}{8} \sum_{i=1}^{8} N̅u_{la_i}
\]  

(4.29)

In this work the average Nusselt number for Sensor2 is defined as the average of the sectors around the circular geometry that are classified as copper material, the region of the Sensor2 geometry which is calcium silicate acts as an adiabatic surface and is not averaged in the Nusselt number. There are eight copper elements in the Sensor2 geometry hence the summation of eight in Equation (4.29).

### 4.5 Discretization Schemes and Property Data: Heat Transfer from Liquid Aluminum to Sensor2

The discretization schemes and property data that were used to solve these numerical problems related to Sensor2 are listed here. Unstructured quadrilateral cells are generated using the ANSYS Workbench v14.0 mesh generating algorithms [118]. The SIMPLE scheme [54] has been employed for the pressure velocity coupling. The momentum and energy spatial discretization is a third order MUSCL scheme [36, 119], where a transient formulation is employed the discretization order is first with respect to time [36]. The residuals of continuity, x-velocity and
y-velocity are reduced to $10^{-8}$ and the energy residual is reduced to $10^{-10}$. The thermophysical properties of Commercial Purity Aluminum, Copper, and Calcium Silicate are listed in Table 3.1, Table 3.3, Table 3.4.

### 4.6 Grid Independence Tests: Heat Transfer from Liquid Aluminum to Sensor2

A grid independence test is performed for the steady convective heat transfer of liquid aluminum around Sensor2 problem and the transient conjugate heat transfer from liquid aluminum to Sensor2 problem. Table 4.1 displays the grid parameters for the steady mathematical formulation. Four grids are explored: G1a with 44848 elements, G2a with 67248 elements, G3a with 102448 elements, and G4a with 128048 elements. Figure 4.8 displays an enlarged view of the G1a mesh around a sector of the circular cylinder for the steady convective heat transfer of liquid aluminum around Sensor2 problem. Figure 4.9 is a set of two plots that show the Nusselt number versus angle for the four grids where the Cylinder UHF boundary condition is invoked. Figure 4.9a plots the results for a Reynolds number of Re=40 and Figure 4.9b plots the results for a Reynolds number of Re=10000. There is good agreement between the four grid sizes. Figure 4.10 is two plots that show the Nusselt number versus angle for the four grids where the Sensor2 CWT boundary conditions are invoked. Figure 4.10a plots the results for a Reynolds number of Re=40 and Figure 4.10b plots the result for a Reynolds number of Re=10000. There is good agreement for the Nusselt number between the four grids, G1a is employed in this numerical study.

Table 4.2 displays the grid parameters for the transient conjugate heat transfer mathematical formulation. Four grids are explored: G1b with 74802 elements, G2b with 108434 elements, G3b with 151418 elements, and G4b with 199687 elements. Figure 4.11 displays an enlarged view of the G1b mesh around one of the copper heat transfer elements, where the resistance layer and the edge of Sensor2 are displayed. Figure 4.12 is a set of plots which display the non-
Table 4.1: Grid Size for The Steady Convective Heat Transfer of Liquid Aluminum around Sensor2

<table>
<thead>
<tr>
<th>Grid</th>
<th>$N_{SP}$</th>
<th>$N_{DP}$</th>
<th>$E_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1a</td>
<td>640</td>
<td>140</td>
<td>44848</td>
</tr>
<tr>
<td>G2a</td>
<td>960</td>
<td>140</td>
<td>67248</td>
</tr>
<tr>
<td>G3a</td>
<td>1280</td>
<td>160</td>
<td>102448</td>
</tr>
<tr>
<td>G4a</td>
<td>1600</td>
<td>160</td>
<td>128048</td>
</tr>
</tbody>
</table>

$N_{SP}$: number of points on the cylinder (Sensor2) surface, $N_{DP}$: number of points along the diameter of the computational domain, $E_D$: total number of elements in the computational domain.

Figure 4.8: Enlarged View of Computational Grid for the Steady Convective Heat Transfer of Liquid Aluminum around Sensor2 Numerical Modeling Problem
Figure 4.9: Grid Independence Test for the Steady Convective Heat Transfer of Liquid Aluminum around Sensor2: A plot of the Local Nusselt Number for Liquid Sodium Flow (Cylinder UHF)

Figure 4.10: Grid Independence Test for the Steady Convective Heat Transfer of Liquid Aluminum around Sensor2: A plot of the Local Nusselt Number for Liquid Aluminum Flow (Sensor2 CWT)
Table 4.2: Grid Size for The Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2

<table>
<thead>
<tr>
<th>Grid</th>
<th>N&lt;sub&gt;SP&lt;/sub&gt;</th>
<th>N&lt;sub&gt;DP&lt;/sub&gt;</th>
<th>E&lt;sub&gt;S2&lt;/sub&gt;</th>
<th>E&lt;sub&gt;D&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1b</td>
<td>640</td>
<td>140</td>
<td>29954</td>
<td>74802</td>
</tr>
<tr>
<td>G2b</td>
<td>960</td>
<td>140</td>
<td>41186</td>
<td>108434</td>
</tr>
<tr>
<td>G3b</td>
<td>1280</td>
<td>160</td>
<td>48970</td>
<td>151418</td>
</tr>
<tr>
<td>G4b</td>
<td>1600</td>
<td>160</td>
<td>71639</td>
<td>199687</td>
</tr>
</tbody>
</table>

N<sub>SP</sub>: number of points on the cylinder surface, N<sub>DP</sub>: number of points along the diameter of the computational domain, E<sub>S2</sub>: number of computational elements in the Sensor2 geometry, E<sub>D</sub>: total number of elements in the computational domain.

dimensional temperature versus Fourier number, where Figure 4.12a plots the solution with a thermal resistance of \( R = 0 \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \), and Figure 4.12b plots the solution with a thermal resistance of \( R = 1 \times 10^{-3} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \). In these plots, the non-dimensional temperature versus non-dimensional time is displayed for a copper element aligned at 0° from the stagnation point of the flow-field. There is good agreement between the temperature response of the copper element between the grid sizes, G1b is used in this numerical study.
Figure 4.11: Enlarged View of Computational Grid around a Copper Element for the Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2

Figure 4.12: Grid Independence Tests for the Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2: A plot of the non-dimensional Temperature Response of a Copper Element (Re≈14000)
Chapter 5

Experimental Results and Numerical Modeling of Sensor1

An interesting phenomenon which may occur when a very cold Sensor1 is immersed into hot flowing liquid aluminum is that aluminum may freeze onto the sensor. The solidification and melting time of the aluminum around the Sensor1 geometry is monitored for different liquid aluminum temperatures (bath superheat) and various magnitudes of velocity, for the case of a copper and steel Sensor1 design. By numerically modeling this transport process, it is observed that the thermal resistance at the Sensor1-aluminum interface plays an important role on the contour of aluminum shell which solidifies and subsequently melts onto the surface of the cylinder. Comparing the experimental solidification and melting times of aluminum to the numerically predicted solidification and melting times an estimate of the thermal resistance is provided.

Section 5.1 depicts a typical experimental response of a steel Sensor1, a means to determine the local shell solidification and melting time by monitoring the external thermocouple reading is described in section 5.2. Section 5.3 presents cross-sectional data related to the solid aluminum that forms on a steel Sensor1 under flowing liquid aluminum. The effect of various magnitude of thermal resistance on the shell contour is investigated via numerical modeling,
where the numerical predictions are compared with the experimental shell solidification and melting time data in sections 5.4 and 5.5.

5.1 A Typical Experimental Response of Sensor1

Here a typical experimental response of the thermocouples from within the Sensor1 probe is explained. Figure 5.1 plots the typical experimental result from a steel Sensor1, outlining the fourteen channels that are recorded by the DAQ. The fourteen channels correspond to eight internal thermocouples, four external thermocouples, the bath temperature, and the immersion circuit. After starting the DAQ recording, the cylinder is positioned via the immersion apparatus over the entrance of the RLMT, which is signified by line segment AB in Figure 5.1. Just after the point in time of immersion (segment BC in Figure 5.1) the external thermocouples rise rapidly and approach the value of 933 K (660 °C), the melting temperature of aluminum. The internal thermocouples also begin an ascent to the temperature of the bath after the point in time of immersion. During the segment CD, the sensor remains in the flow. The sensor was removed from the liquid aluminum at the point in time of line segment DE. The internal thermocouple data can be used to infer the direction of velocity and is outlined in appendix A.

5.1.1 The External Thermocouple Response

The external thermocouple data is employed to gain insight into the duration of aluminum shell that forms and subsequently melts with time on the Sensor1 material. Figure 5.2 plots the external thermocouple data as a function of time. In this plot, several curves are displayed. The bath temperature is shown along with the immersion circuit signal. A line which spans across the plot indicated by segment AB drawn in Figure 5.2, at the ordinate value of 933 K (660 °C), denotes the melting temperature of aluminum. The four external thermocouple responses are shown. From the immersion circuit signal, the immersion time of the cylinder is 15.2 s. The exterior thermocouple otc(25) signal intersects with the aluminum melting temperature at 21.9
s. The duration of the aluminum shell which solidifies and subsequently melts at this location is calculated from the difference of the cylinder immersion time and the time where the otc(25) curve intersects with the aluminum melting line. In this text, this difference in time will be referred to as the shell solidification and melting time (SMT), and this SMT value is a function of location around the Sensor1 geometry. The difference in the immersion time and the point in time when the otc(25) curve intersects with the 933 K (660 °C) isoline corresponds to a SMT of approximately 6.7 s. The shell solidifies and melts at location otc(25) in approximately 6.7 s. The SMT of the thermocouples at otc(115), otc(205), and otc(295) are also deduced from Figure 5.2. The shell at location otc(115) solidifies and melts at 15.6 s, the shell at location otc(205) solidifies and melts at 17.5 s, and the shell at location otc(295) solidifies and melts at 6.7 s after immersion. From the exterior thermocouple readings the duration of the shell about the surface of the cylinder can be inferred. This shell solidification and melting time is used to investigate the melting pattern around the cylinder.
Figure 5.2: Exterior Thermocouple Data as a Function of Time for a Sensor1 design Constructed of Steel

5.2 The Experimental Shell Solidification and Melting Time as a Function of Polar Coordinate

The shell SMT estimated from the four external thermocouples about the cylinder can be deduced from the external thermocouple readings as outlined. A means to infer the shell SMT as a function of polar coordinate around the Sensor1 geometry is presented here. Figure 3.6 is a schematic of the location of the thermocouples from the stagnation point as they are located around the Sensor1 geometry. To generate data at different angular locations from the stagnation point of the cylinder, a series of experimental runs were performed where the thermocouple positioning was different with respect to the stagnation point of the flow. In consequence, data of the SMT of the shell at different angular positions about the cylinder was generated. On average three experiments were performed for each probe position.
5.2.1 The Steel Sensor1 Experimental Shell Solidification and Melting Time

The shell SMT is explored around the circular Sensor1 cross-section for different bath temperatures and aluminum flow velocities on Sensor1 constructed of steel. Figure 5.3 is a family of plots which represent the shell SMT as a function of polar coordinate about the cylinder for different flow velocities and bath superheat. Figure 5.3a depicts the results of the solidification and melting time of the shell about the cylinder for a flow velocity of 0.08 m·s$^{-1}$ and bath superheat (SPH) of 30 K SPH on a Sensor1 design constructed of steel. Notice that the SMT of the shell increases from the forward stagnation point to the diametric position. The largest SMT occurs at approximately 180° about the cylinder. The shell at the side of the cylinder which faces the oncoming flow solidifies and melts fastest. Figure 5.3b plots the shell SMT under a faster flow velocity (0.25 m·s$^{-1}$), the superheat is held at 30 K SPH. Under the faster flow velocity the SMT of the shell decreases with respect to the 0.08 m·s$^{-1}$ case as this increases the convective heat flux contribution in the Stefan condition (Equation (1.6)). The trend where the shell solidifies and melts faster at the stagnation point holds for the faster velocity case. Increasing the bath superheat to 60 K SPH is depicted in Figure 5.3c while the aluminum velocity is 0.08 m·s$^{-1}$. Increasing the superheat results in a decrease in the SMT of the shell as this increases the convective heat flux contribution in the Stefan condition. The melting pattern however, is similar to the previous two cases; the shell solidifies and melts fastest at the stagnation point and longest at the position which is diametric to the stagnation point. Figure 5.3d portrays the SMT of the shell under the bath conditions of 0.16 m·s$^{-1}$ and 60 K SPH. The trend outlined previously holds for this scenario as well.
Figure 5.3: Shell Solidification and Melting Time (SMT) as Function of Polar Coordinate for Various Flow Velocities and Bath Superheat (SPH) on a Sensor1 Design Constructed of Steel

(a) 0.08 m·s$^{-1}$ (Re≈6800), 30 K SPH, Steel Sensor1

(b) 0.25 m·s$^{-1}$ (Re≈21000), 30 K SPH, Steel Sensor1

(c) 0.08 m·s$^{-1}$ (Re≈6800), 60 K SPH, Steel Sensor1

(d) 0.16 m·s$^{-1}$ (Re≈14000), 60 K SPH, Steel Sensor1
5.2.2 The Copper Sensor1 Experimental Shell Solidification and Melting Time

Figure 5.4 is another family of plots which represent the SMT of the shell about a copper cylinder at different bath superheat and flow velocities. Figure 5.4a plots data for 0.08 m·s⁻¹ and 60 K SPH, Figure 5.4b plots data for 0.25 m·s⁻¹ and 60 K SPH, Figure 5.4c plots the shell solidification and melting time for 0.08 m·s⁻¹ and 90 K SPH, and Figure 5.4d plots data for flow conditions of 0.25 m·s⁻¹ and 90 K SPH. An increase in the bath velocity increases the heat transfer coefficient to the cylinder which in turn decreases the SMT of the shell. Changes in the values of the superheat affect the convective heat flux, where a larger value of superheat correlates to larger values of convective heat flux for a given flow velocity. Thus the data which is obtained at higher superheat have smaller SMT when compared with the data at lower bath superheat. The melting pattern is such that the shell melts faster around the stagnation point when compared to the diametric location.

Table 5.1 lists the maximum and minimum recorded local shell SMT for steel and copper Sensor1 designs. Between the two materials, data at 0.08 m·s⁻¹ and 60 K bath superheat can be compared. The maximum local shell SMT are identical at 27.6 s, the minimum local shell SMT are 1.8 s for the steel Sensor1 and 1.7 s for the copper Sensor1. Figure 5.5 plots the shell solidification and melting times as a function of polar coordinate for the steel and copper Sensor1 designs for the aforementioned flow conditions. The SMT of the shell on both the steel and copper Sensor1 designs are very similar. A cause for this similarity is the carbon coating at the surface of both sensors which adds a comparable thermal resistance at the Sensor1-aluminum interface.
Figure 5.4: Shell Solidification and Melting Time (SMT) as Function of Polar Coordinate for Various Flow Velocities and Bath Superheat (SPH) on a Sensor1 Design Constructed of Copper
Table 5.1: Maximum and Minimum Recorded Local Shell Solidification and Melting Times for Steel and Copper Sensor Designs

<table>
<thead>
<tr>
<th>Material</th>
<th>$U_\infty$ [m·s$^{-1}$]</th>
<th>SPH [K]</th>
<th>max [s]</th>
<th>min [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.08</td>
<td>30</td>
<td>63.1</td>
<td>20</td>
</tr>
<tr>
<td>Steel</td>
<td>0.25</td>
<td>30</td>
<td>43.9</td>
<td>2.2</td>
</tr>
<tr>
<td>Steel</td>
<td>0.08</td>
<td>60</td>
<td>27.6</td>
<td>1.8</td>
</tr>
<tr>
<td>Steel</td>
<td>0.16</td>
<td>60</td>
<td>21.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Copper</td>
<td>0.08</td>
<td>60</td>
<td>27.6</td>
<td>1.7</td>
</tr>
<tr>
<td>Copper</td>
<td>0.25</td>
<td>60</td>
<td>18.4</td>
<td>1.5</td>
</tr>
<tr>
<td>Copper</td>
<td>0.08</td>
<td>90</td>
<td>15.9</td>
<td>2</td>
</tr>
<tr>
<td>Copper</td>
<td>0.25</td>
<td>90</td>
<td>10.7</td>
<td>1</td>
</tr>
</tbody>
</table>

$U_\infty$: velocity, SPH: bath superheat, max: maximum shell solidification and melting time, min: minimum shell solidification and melting time.

Figure 5.5: Shell Solidification and Melting Time (SMT) as a Function of Polar Coordinate for a Steel and Copper Sensor Designs in Flowing Liquid Aluminum at 0.08 m·s$^{-1}$ (Re≈6800) and 993 K (720 °C) 60 K SPH
5.3 Cross-Sectional Experimental Data of Solidified Aluminum on the Sensor1 Geometry Constructed of Steel

This section discusses the thickness of the solid aluminum shell around Sensor1. The thickness of the solid aluminum around the Sensor1 geometry is a function of the conductive heat flux drawn from the solid-liquid interface and the convective heat flux supplied to the solid-liquid interface. Photographs of the aluminum shell on the surface of a steel cylinder are shown in Figure 5.6. Figure 5.6a displays the aluminum shell which forms after 6 s in flow conditions of 0.08 m·s$^{-1}$ and 30 K SPH. Figure 5.6b is a photograph of the shell which forms after 10 s for the aforementioned flow-field setting. The extracted cylinder with shell is used to create a cross-sectional plot of solidified aluminum on the cylinder. The shell cross-section is taken at 88.9 mm (3.5 inch) from the top of the cylinder.

Figure 5.7 shows the cross-sectional thickness of the shell on a steel cylinder for extraction times of 6 s, 10 s, 15 s, and 20 s from flowing liquid aluminum at 0.08 m·s$^{-1}$ and 30 K SPH, where this time corresponds to the amount of time that Sensor1 remains in the flow-field. The aluminum thickness is explained in relation to the experimental apparatus and the Stefan condition acting at the solid-liquid aluminum interface. Figure 5.7a displays the shell on the cylinder after 6 s. In this plot, the gray shaded circular region represents the cylinder, the radial coordinate measures the distance from the center of the cylinder, the angular coordinate denotes the location around the cylinder. Zero degrees (0$^\circ$) in these plots correspond to the stagnation point of the flow-field with the cylinder. At 6 s after immersion the shell thickness is approximately uniform about the cylinder, with an average radius of 24.7 mm. Figure 5.7b depicts the shell profile after 10 s of immersion. Two experimental curves are shown in Figure 5.7b. The line labeled experiment 1 has an average radius 25.9 mm and the line labeled experiment 2 has an average radius of 26.7 mm. From 6 s to 10 s the average radius of the shell has increased. The thickness of shell around the cylinder is relatively uniform, almost circular in both Figures 5.7a and 5.7b.
Figure 5.6: Photographs of Aluminum Shell on a Sensor1 Design Constructed of Steel without Exterior Thermocouples Extracted from Flowing Liquid Aluminum at 0.08 m·s\(^{-1}\) (Re≈6800) and 963 K (690 °C) 30 K SPH
After 15 s of immersion, the shell thickness becomes visibly non-uniform. This is portrayed in Figure 5.7c. The shells are thinner in the angular region of \((240^\circ \leq \Theta \leq 360^\circ)\). From the discussion of the Stefan condition, it is expected that the shell is thinner around the stagnation point because of the non-uniformity of the convective heat flux around the cylinder. Theoretically the shell which forms on the cylinder should be symmetrical from the discussion on the form of the heat transfer coefficient function (Figure 1.2). The thinner shell should be situated between the angular coordinates of \((300^\circ \leq \Theta \leq 60^\circ)\). The thinner shell is observed in the space between \((240^\circ \leq \Theta \leq 360^\circ)\) because of the experimental apparatus. The half of the cylinder designated \((180^\circ \leq \Theta \leq 360^\circ)\) faces the wall of the RLMT crucible. From the experimental arrangement, the velocity of liquid aluminum increases between the crucible wall and the cylinder in the region of \((270^\circ \leq \Theta \leq 360^\circ)\) around the cylinder. The proximity of the cylinder to the wall causes thinner shells around the cylinder coordinates of \((240^\circ \leq \Theta \leq 360^\circ)\). This explanation is a reason for the discrepancy between the theoretical shell thickness profile and the experimental shell thickness profile.

Upon close observation of Figure 5.7c it is noticed that the shell thickness in Experiment 4 is greater than Experiment 5 at certain regions around the probe. To compare the amount of solid aluminum on the cylinder the area of the shell is calculated in Figure 5.7c, the area for Experiment 1 is 935 mm\(^2\) ± 74 mm\(^2\) and Experiment 2 is 655 mm\(^2\) ± 66 mm\(^2\). More shell has solidified on the cylinder in Experiment 4 than in Experiment 5 as indicated in Figure 5.7c, perhaps due to a different thermal resistance from the carbon coating on the cylinder between Experiment 4 and Experiment 5. Between similar experiments the author tried to hold the aluminum temperature and velocity constant. Define the rear half of the cylinder as the angular coordinates of \((90^\circ < \Theta < 270^\circ)\), and define the front half of the cylinder as \((270^\circ \leq \Theta \leq 360^\circ)\) and \((0^\circ \leq \Theta \leq 90^\circ)\). From Figure 5.7d which portrays the shells at 20 s after immersion, it is observed that the shell thickness is smaller at the front half than the rear half of the cylinder. The experimental data pertaining to the shell SMT around the cylinder and the cross-sectional solidified aluminum thickness data around the cylinder are employed to tune
the numerical model to predict the heat transfer phenomena to the Sensor1 design.

5.4 Modeling Aluminum Solid to Liquid Phase Change on Sensor1

In the next several sections of this thesis the effect of a thermal resistance which is applied at the sensor-aluminum interface on the solidification and melting of aluminum onto Sensor1 is examined. ANSYS Fluent [36] is utilized to investigate the effect of the thermal resistance on the freezing and melting of the shell. The numerical methodology employed for these predictions was discussed in section 4.1. The thickness of solidified aluminum data around the steel sensor is employed to provide information on the magnitude of thermal resistance which acts at the sensor-aluminum interface. The shell solidification and melting time data is also employed to provide information on the magnitude of thermal resistance which acts at the sensor-aluminum interface. A constant in time and uniform in space thermal resistance around the cylinder is imposed in the numerical prediction.

5.4.1 Solid Fraction Measurements versus Time Comparison with Experimental Data

One experimental measurement which is utilized to validate the numerical prediction is the solidified shell thickness data obtained from the cross-sectional plot. The solidified mass of aluminum at these points in time are compared with numerical predictions from ANSYS Fluent. By applying a thermal resistance at the Sensor1-aluminum interface and executing several numerical predictions for various values of thermal resistance the predictions and the experimental data are in better agreement.

Figure 5.8 shows the solid fraction in the domain as a function of time. The solid fraction
Figure 5.7: Experimental Shell Thickness versus Polar Coordinate on a Sensor1 Design Constructed of Steel for Flowing Liquid Aluminum at 0.08 m·s$^{-1}$ (Re$\approx$6800) and 963 K (690 °C) 30 K SPH
is related to the liquid fraction by Equation (5.1).

\[ f_s = 1 - f_l \]  

(5.1)

Each curve corresponds to a Fluent prediction where the thermal resistance \( R \) is different in magnitude. The symbols correspond to solid fraction values derived from the experimental extractions. The maximum of the solid fraction curve decreases with increasing value of thermal resistance. This signifies that less shell solidifies onto the cylinder with increasing thermal resistance, due to the decrease in the magnitude of heat flux drawn from the liquid by the sensor. The curve corresponding to \( R = 1 \times 10^{-4} \text{ m}^2\cdot\text{K}^{-1}\cdot\text{W}^{-1} \) seems to match the bulk of the experimental data, however there are a few experimental data points which fall below this curve. Take for instance the data which was captured experimentally at 15 seconds. Good agreement with the experimental data can be observed for a thermal resistance of slightly greater than \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}^{-1}\cdot\text{W}^{-1} \) for the smaller experimental solid fraction data point. The difference between the two experiments performed at 15 s is in the carbon coating which was applied to the sensor. Larger thermal resistances will result when more carbon coating is applied to the sensor, and is an explanation for the spread in the experimental solid fraction measurements portrayed in Figure 5.8. The thermal resistance also has an effect on the local solidification and melting times of the shell.

5.5 The Numerical Model Shell Solidification and Melting Time as a Function of Polar Coordinate

This section will discuss the experimental shell solidification and melting time (SMT) values as a function of polar coordinate around Sensor1 with the predicted shell SMT obtained using Fluent for various imposed values of thermal resistance. The shell SMT refers to the duration in time at which aluminum solidifies and subsequently melts at a specific location on Sensor1.
Figure 5.8: Predicted Solid Fraction versus Time Curves from Fluent and Experimental Measurements for the Solidification and Melting of Aluminum onto a Sensor1 Design Constructed of Steel immersed in Flowing Liquid Aluminum at 0.08 m·s$^{-1}$ (Re≈6800) and 963 K (690 °C) 30 K SPH

The shell SMT around the surface of the sensor vary due to the heat flux distribution around the cylinder. Agreement with the experimental and predicted shell melting times can be obtained by introducing a constant thermal resistance at the interface of the sensor and the aluminum.

5.5.1 Copper Sensor1: Liquid Conditions 60 K SPH and 0.08 m·s$^{-1}$

To begin, a discussion will occur on the effect of the thermal resistance on the local shell SMT for the case of a copper cylinder which is immersed into a liquid aluminum flow-field of 0.08 m·s$^{-1}$ (Re≈6800) and at a liquid temperature of 993 K (60 K SPH). Figure 5.9 is a family of plots which displays the experimental shell SMT distribution and the numerical predictions for the aforementioned bath variables. The shell SMT distribution is computed for several values of thermal resistance.

Figure 5.9a displays the SMT for a constant in time and uniform in space thermal resistance of $R = 1 \times 10^{-5}$ m$^2$·K·W$^{-1}$. The square symbols indicate the experimental measurements, the
circle symbols connected by line segments, represent the predictions from Fluent. Examining
the experimental data in Figure 5.9a it is observed that the aluminum SMT are smaller at
locations closer to the stagnation point. At the diametric position from the stagnation point the
shell melts at a later point in time. The numerical prediction for the case of a thermal resistance
of $R = 1 \times 10^{-5}$ m$^2$·K·W$^{-1}$ also exhibits this trend, however, the disparity in the numerical
prediction and experimental data is evident.

Figure 5.9b shows the Fluent prediction for the case that the thermal resistance is $R = 1 \times 10^{-4}$ m$^2$·K·W$^{-1}$. The increase by an order of magnitude of the thermal resistance causes
a decrease in SMT of the shell closer to the location of 0° and an increase in melting time at a
position of 180°.

At large values of thermal resistance, at a value of $R = 1 \times 10^{-3}$ m$^2$·K·W$^{-1}$ for this
superheat and flow-field, the shell does not grow on the front half of the cylinder. Figure 5.9c
shows the results of the aforementioned numerical prediction. In the case of high thermal
resistance, for this flow-field and superheat, the growth and subsequent melting of the shell
occurs at select regions around the cylinder. From Figure 5.9c this growth and melting zone
occurs at locations (90° ≤ Θ ≤ 270°) around the cylinder. A restrictive shell growth condition
is referred to when the shell solidifies at isolated segments around the cylinder, as in Figure
5.9c.

Figure 5.9d plots the numerical prediction for the case that the thermal resistance is $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$. Better agreement is observed between the experimental and predicted
shell SMT for a thermal resistance of $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$, than the previously mentioned
values of thermal resistance. The effect of superheat on the shell SMT is also examined, the
purpose is to examine the sensitivity of the change in superheat on the local SMT. Figure 5.9e
(58 K SPH) and Figure 5.9f (55 K SPH) display this data. Decreasing the superheat of the
fluid for a fixed velocity, causes an increase in the SMT of the shells around the rear half of the
cylinder at a value of thermal resistance of $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$.

The analysis of the aforementioned flow (0.08 m·s$^{-1}$, 993 K) leads to the following observa-
Figure 5.9: Experimental Shell Solidification and Melting Time (SMT) Distribution and Numerical Predictions for a Sensor1 Design Constructed of Copper in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 993 K (720 °C) 60 K SPH

(a) 60 K SPH, $R = 1 \times 10^{-5}$ m²·K·W⁻¹

(b) 60 K SPH, $R = 1 \times 10^{-4}$ m²·K·W⁻¹

(c) 60 K SPH, $R = 1 \times 10^{-3}$ m²·K·W⁻¹

(d) 60 K SPH, $R = 2 \times 10^{-4}$ m²·K·W⁻¹

(e) 58 K SPH, $R = 2 \times 10^{-4}$ m²·K·W⁻¹

(f) 55 K SPH, $R = 2 \times 10^{-4}$ m²·K·W⁻¹
tions which are made for this particular experimental arrangement of the cylinder in revolving liquid aluminum. These observations are itemized as the following:

**Observation 1:** Comparing cases at a fixed velocity, bath temperature, and initial temperature of Sensor1, with increasing uniform thermal resistance, the local shell SMT in the vicinity of the stagnation point decreases, while the local shell SMT in the vicinity of the diametric location increases.

**Observation 2:** Comparing cases at a fixed velocity, bath temperature, and initial temperature of Sensor1, a very high value of thermal resistance suppresses the formation of shell.

**Observation 3:** Comparing cases at a fixed velocity, initial temperature of Sensor1, and thermal resistance, decreasing the superheat of the liquid, causes the local shell SMT to increase at the rear half of the cylinder ($90^\circ \leq \Theta \leq 270^\circ$).

### 5.5.2 Copper Sensor1: Liquid Conditions 60 K SPH and 0.25 m·s$^{-1}$

In this section the three observations are shown under a higher velocity. Figure 5.10 plots the numerical predictions and experimental data of a copper Sensor1 for a flow velocity of 0.25 m·s$^{-1}$ and a liquid temperature of 993 K (60 K SPH). Increasing the thermal resistance from $R = 1 \times 10^{-5}$ m$^2$·K·W$^{-1}$ (Figure 5.10a) to $R = 1 \times 10^{-4}$ m$^2$·K·W$^{-1}$ (Figure 5.10b) acts to decrease the SMT of the shell on the front half ($270^\circ < \Theta < 90^\circ$) of the cylinder and increase the SMT on the rear half ($90^\circ \leq \Theta \leq 270^\circ$) of the cylinder (Observation 1). At very high values of thermal resistance $R = 1 \times 10^{-3}$ m$^2$·K·W$^{-1}$ very little shell forms around the stagnation point of the cylinder. This is a restrictive shell growth case, noted as Observation 2. The plots of Figure 5.10e and Figure 5.10f present data for the case where the thermal resistance is constant at $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$ and the liquid superheat is 58 K SPH and 55 K SPH. Decreasing the superheat of the liquid under constant thermal resistance conditions acts to increase the SMT on the rear half of the cylinder (Observation 3). Figure 5.10d presents numerical data where the thermal resistance is $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$, for this value of
thermal resistance the numerical prediction resembles the experimental data at the front half of the cylinder.

5.5.3 Steel Sensor1: Liquid Conditions 60 K SPH and 0.08 m·s$^{-1}$

Fluent predictions are performed for a circular cylinder of steel with different values of imposed thermal resistance in flowing liquid aluminum. It is observed that the disparity of the local shell SMT of the numerical predictions and experimental data is not as great for the steel Sensor1 as in the case of the copper Sensor1. Figure 5.11 displays the shell SMT for a steel cylinder in a bath at 993 K (60 K SPH) and a velocity of 0.08 m·s$^{-1}$. As in the previous sections of this work, predictions to examine the effect of the thermal resistance on the local shell SMT are computed.

Examine Figure 5.11a which plots the numerical prediction for a thermal resistance of $R = 1 \times 10^{-5}$ m$^2$·K·W$^{-1}$. For a given angular coordinate the difference in the numerical prediction, when compared to the experimental data, is not as large as in the case of the copper cylinder. Figure 5.9a plots the results for a copper cylinder at 0.08 m·s$^{-1}$ and 60 K SPH, this data may be compared to the data presented in Figure 5.11a, as the initial flow, temperature, and thermal resistance conditions are similar. It seems that the thermal resistance does not have as large an influence in the numerical prediction of the shell SMT for the case of the steel cylinder. The thermal resistance has a greater effect on the numerical prediction of the shell SMT in the case of a copper cylinder.

This characteristic is a result of the thermal conductivities of the two materials. From Table 3.2 and Table 3.3, steel and copper have thermal conductivities of 51.9 W·m$^{-1}$·K$^{-1}$ and 388 W·m$^{-1}$·K$^{-1}$. The radius of the cylinder (0.01905 m) is employed as a scale to compare the values of the material resistance to heat transport by the relationship in equation (5.2).

$$R_{cyl} = \frac{r_{cyl}}{k_{cyl}}$$ (5.2)
Figure 5.10: Experimental Shell Solidification and Melting Time (SMT) Distribution and Numerical Predictions for a Sensor1 Design Constructed of Copper in flowing Liquid Aluminum at 0.25 m·s⁻¹ (Re≈21000) and 993 K (720 °C) 60 K SPH.
The resistance associated with the transport of energy across the steel is approximately \( R_{cyl} = 4 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \) while that associated with the copper is \( R_{cyl} = 5 \times 10^{-5} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \), approximately one order of magnitude in difference. The relative values of the internal cylinder resistance and the coating resistance matter. This is further outlined in reference [120].

The effect of various magnitudes of thermal resistance on the localized shell SMT for a Sensor1 design constructed of Steel is discussed. Figure 5.11a and Figure 5.11b plot the predictions for values of thermal resistance of \( R = 1 \times 10^{-5} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \) and \( R = 1 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \), where a modification in the local shell SMT around the cylinder is noticed (Observation 1). A restrictive shell growth condition occurs where the shell formation is suppressed on all locations except the diametric location from the stagnation point on the cylinder (Observation 2). Observation 2 occurs at a contact resistance value of \( R = 1 \times 10^{-3} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \) and is depicted in Figure 5.11c. Lastly, Observation 3, where the local SMT of the shell increases from a decrease in bath superheat, can be viewed in Figures 5.11d, 5.11e, and 5.11f.

Recall Figure 5.5 which plotted the experimental SMT as a function of polar coordinate for copper and steel Sensor1 designs in flowing liquid aluminum at 0.08 m·s\(^{-1}\) and 60 K SPH. The experimental SMT values for the steel and copper sensors were very similar. Figure 5.9d and Figure 5.11d plots the experimental data and numerical predictions for a thermal resistance of \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \) for the copper Sensor1 and steel Sensor1 at the aforementioned flow conditions. The experimental and numerical results are in good agreement for both the copper and steel Sensor1 designs with thermal resistance of \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \). The numerical predictions displayed in Figure 5.10d for the copper Sensor1 at higher velocity (0.25 m·s\(^{-1}\) and 60 K SPH) also show good agreement with the experimental data at a thermal resistance of \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \). Appendix B displays numerical predictions for a steel Sensor1 at conditions of 0.08 m·s\(^{-1}\) and 30 K SPH and conditions of 0.25 m·s\(^{-1}\) and 30 K SPH. The predictions in appendix B agree with experimental data coinciding with a thermal resistance of \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \). It seems that on average the carbon coating imparts a thermal resistance on the order of \( R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1} \) in the solidification and melting studies of
Figure 5.11: Experimental Shell Solidification and Melting Time (SMT) Distribution and Numerical Predictions for a Sensor1 Design Constructed of Steel in flowing Liquid Aluminum at $0.08 \text{ m} \cdot \text{s}^{-1}$ ($\text{Re} \approx 6800$) and 993 K ($720 \degree \text{C}$) 60 K SPH
There is a complex energy interaction between the Sensor1 material with the flow-field. Examining the Stefan condition (Equation (1.6)) and the sensor-aluminum interface energy transfer relationship (Equation (1.7)), for a given phase change material, factors which affect the freezing of the material include: the thermal resistance, the convective conditions of the bath (temperature and velocity), in addition to the initial temperature condition of the sensor material. Depending on the aforementioned conditions, shell may not form on the sensor. As in the restrictive shell growth case, shell may freeze on select regions around the circular geometry.

The solidification and melting of aluminum onto the Sensor1 geometry complicates the data reduction technique to determine the magnitude of velocity. Ideally, one would like to compare the Nusselt number derived from the Sensor1 thermocouple response to deduce both the direction and magnitude of velocity. When shell forms on the sensor it complicates the Nusselt number estimate because the shell must be accounted for. The formation of shell can be suppressed onto the sensor by reducing the initial magnitude of heat flux which is drawn from the bath by the sensor. This is accomplished by modifying the initial temperature condition of the sensor, by heating the sensor to an energy state closer to that of the bath. This will in turn reduce the initial magnitude of heat flux which is drawn from the liquid aluminum by the sensor. As long as the convective flux is larger than the conductive flux, shell will not grow on the sensor material.

To infer the heat transfer coefficient acting at the surface of a copper Sensor1 material in the absence of solidification and melting an inverse heat transfer algorithm was written. The sequential function specification (SFS) algorithm pioneered by Beck et al. [34] was implemented and results pertaining to the local Nusselt number on Sensor1 is presented in appendix C. The local Nusselt number distribution can be utilized to infer the direction and magnitude of
velocity in flowing liquid aluminum. As the inverse algorithm does not generate local Nusselt number estimates in real-time it is not a viable solution for industry to estimate the velocity of liquid aluminum.

A crude technique to determine the direction of velocity from the steel Sensor1 design, when aluminum shell solidifies and melts onto the sensor, is to find the hottest sector of the sensor. This technique may be useful to smaller aluminum production facilities where a quick and cheap method to determine the direction of velocity is required to troubleshoot operating problems relating to the direction of aluminum flow. This technique is presented in appendix A.
Chapter 6

Experimental Results and Numerical Modeling of Sensor2

This chapter discusses the experimental results and numerical modeling of Sensor2. Section 6.1 presents the experimental temperature data recorded from Sensor2 while it was immersed in flowing liquid aluminum. Results from fitting the heat transfer coefficients to the experimental temperature responses are analyzed. From the Fluent predictions of Sensor2 it is observed that the local Nusselt number distribution does not resemble the typical cylinder in crossflow local Nusselt number distribution, as the boundary conditions for the Sensor2 design are unique. These local Nusselt number predictions from Fluent are discussed with respect to the thermal boundary conditions and the flow-field in section 6.2, solutions are compared for various Reynolds number. Results from modeling the transient conjugate heat transfer from liquid aluminum to Sensor2 are presented where the experimental temperature response is compared to the modeled temperature response in section 6.3. The transient conjugate heat transfer predictions provide an estimate of the thermal resistance at each copper element of the Sensor2 design. By applying a thermal resistance at the copper-aluminum interface the experimental and modeled temperature data agree well. A technique to determine the magnitude and direction of velocity using the Sensor2 design is presented in sections 6.4 and 6.5. The error
associated with the Sensor2 data reduction technique is presented in section 6.6. The Sensor2 geometry and velocity data reduction technique is applied to liquid steel in section 6.7 by numerically modeling the temperature response of a Sensor2 geometry of different materials to withstand the liquid steel environment.

### 6.1 The Experimental Result of Sensor2

This section presents the recorded temperature data from Sensor2 while immersed in flowing liquid aluminum. Results from the data reduction technique employed to estimate values of the heat transfer coefficient of each copper element of the Sensor2 design are discussed. Using these estimated heat transfer coefficients, a plot is constructed of the experimental local average Nusselt number versus polar coordinate for the Sensor2 geometry.

Figure 6.1a displays the recorded temperature data from Sensor2 under liquid aluminum flow conditions of 0.16 m·s\(^{-1}\) (Re≈14000) and a bath temperature of 993 K (720 °C), which corresponds to a bath superheat (SPH) of 60 K SPH. There are eight temperature profiles displayed in Figure 6.1a, each corresponding to the thermal response of a copper element at a different angle from the stagnation point of the flow-field. The angles listed in Figure 6.1a refer to the location from the forward stagnation point, 0° represents the forward stagnation point of the flow-field. The temperature at 0° is the greatest, the temperatures at the copper elements of 135°, 180°, and 225° are among the lowest. The non-dimensional temperature response of Sensor2 is displayed in Figure 6.1b, the temperature is transformed to a non-dimensional value according to Equation (1.1), where \( T^*_{S2} \) represents the non-dimensional temperature. The Fourier number (Fo) represents the non-dimensional time, defined by Equation (1.5). In the non-dimensional transformation of temperature, the hottest element is located at the bottom of the plot and the coldest element at the top of the plot.

Figure 6.2 consists of eight plots, representing the non-dimensional temperature response of the eight copper elements. In each plot there are two curves, the first curve is the experimental
Figure 6.1: Recorded Temperature Response of Eight Copper Elements of the Sensor2 Design at 0.16 m·s$^{-1}$ (Re$\approx$14000) and 993 K (720°C) 60 K SPH, Pr=0.013

Table 6.1: Copper Element Coefficient Data for the Sensor2 Design at 0.16 m·s$^{-1}$ (Re$\approx$14000) and 993 K (720°C) 60 K SPH, Pr=0.013

<table>
<thead>
<tr>
<th>$\Theta$ [°]</th>
<th>$\overline{\text{Nu}}_{la}$</th>
<th>Bi</th>
<th>$R^2$</th>
<th>h [W·m$^{-2}$·K$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.42</td>
<td>0.056</td>
<td>0.964</td>
<td>3444</td>
</tr>
<tr>
<td>45</td>
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<td>0.981</td>
<td>2525</td>
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<td>90</td>
<td>0.79</td>
<td>0.031</td>
<td>0.988</td>
<td>1927</td>
</tr>
<tr>
<td>135</td>
<td>0.67</td>
<td>0.026</td>
<td>0.997</td>
<td>1628</td>
</tr>
<tr>
<td>180</td>
<td>0.63</td>
<td>0.025</td>
<td>0.998</td>
<td>1529</td>
</tr>
<tr>
<td>225</td>
<td>0.64</td>
<td>0.025</td>
<td>0.996</td>
<td>1551</td>
</tr>
<tr>
<td>270</td>
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<td>0.992</td>
<td>1982</td>
</tr>
<tr>
<td>315</td>
<td>1.33</td>
<td>0.052</td>
<td>0.938</td>
<td>3219</td>
</tr>
</tbody>
</table>

$\Theta$: polar coordinate from stagnation point, $\overline{\text{Nu}}_{la}$: the local average Nusselt number, Bi: Biot number, $R^2$: coefficient of determination, h: heat transfer coefficient
non-dimensional temperature response and the second curve is the fit of the lumped Equation (1.1) to the experimental data. A value of time constant denoted as coefficient \( b \), listed in Equation (1.2), is fit to the experimental temperature data. The heat transfer coefficient acting on the surface of the copper element is inferred from the relationship listed in Equation (1.2). The equations employed to calculate the time constant \( (b) \) are listed in appendix F. Examine Figure 6.2a which plots the non-dimensional temperature response of the copper element at the stagnation point of the flow. In addition to the two non-dimensional temperature curves, the fit value of the time constant \( (b) \), and the estimate of the local average Nusselt number (\( \overline{Nu_{la}} \)), is displayed within the sub-figure. A coefficient of determination \((R^2)\) is also displayed in the plot and is defined by Equation (6.1). The coefficient of determination is closer to the value of 1 for those copper elements located at \((90^\circ < \Theta < 270^\circ)\) around the probe, these locations have lower time constants, which correlate to lower heat transfer coefficients. Table 6.1 lists the Nusselt number, Biot number, and heat transfer coefficient of each copper element. The Biot numbers of the copper elements located between \((90^\circ < \Theta < 270^\circ)\) around the probe are closer to \( \text{Bi} = 0 \) which corresponds to the infinite heat propagation assumption of the lumped capacitance analysis [35].

\[
R^2 = 1 - \frac{\sum_i (y_i - \overline{y})^2}{\sum_i (y_i - \hat{y}_i)^2} \tag{6.1}
\]

Figure 6.3 plots the estimated heat transfer coefficient values as a function of position from the stagnation point of the flow-field. The Nusselt number values displayed in Figure 6.3 are those derived from the heat transfer coefficient fit. The intention of this thesis is to utilize these local Nusselt numbers to determine the magnitude and direction of velocity in liquid aluminum.

Figure 6.4 plots the local average Nusselt number of Sensor2 as a function of polar coordinate where the Sensor2 copper elements are coated with carbon in flowing liquid aluminum at a Reynolds number of \( \text{Re} \approx 14000 \), depicting the results of two experiments. Notice that the local average Nusselt number data for Sensor2 displayed in Figure 6.4 follows a decreasing trend from \((0^\circ \leq \Theta \leq 180^\circ)\) followed by an increasing trend from \((180^\circ < \Theta \leq 315^\circ)\). This decreasing trend in the Nusselt number to \(180^\circ\) from the stagnation point followed by the in-
Figure 6.2: Sensor2 Temperature data in flowing Liquid Aluminum at 0.16 m·s⁻¹ (Re≈14000) and 993 K (720 °C) 60 K SPH, Pr=0.013
Figure 6.2: Continued Sensor2 Temperature data in flowing Liquid Aluminum at 0.16 m·s⁻¹ (Re≈14000) and 993 K (720 °C) 60 K SPH, Pr=0.013
Figure 6.3: Estimated Local Average Nusselt Number of Sensor2 as a function of Polar Coordinate from stagnation point in Flowing Liquid Aluminum at 0.16 m·s⁻¹ (Re≈14000) and 993 K (720 °C) 60 K SPH, Pr=0.013 increasing trend after 180° resembles the Nusselt number function captured by Ishiguro et al. [32] in Figure 1.2.

6.2 Results from Modeling the Steady Convective Heat Transfer of Liquid Aluminum around Sensor2

The data reduction technique proposed to determine the magnitude and direction of velocity of a high temperature liquid metal will depend on the Nusselt number versus position functions derived in this section. As outlined in the literature review, it is becoming quite common to numerically predict the heat transfer to cylinders in flow, the intent in this work is to couple the correlations from the predictions of the Sensor2 geometry with a value of thermal resistance (which in the future may be obtained from a thermal resistance model) to determine the velocity of a high temperature liquid metal. The heat transfer to Sensor2 is modeled using a steady mathematical formulation as outlined in section 4.2. The local Nusselt number around
Figure 6.4: Estimated Local Average Nusselt Number of Sensor2 as a function of Polar Coordinate from stagnation point in Flowing Liquid Aluminum at 0.16 m·s$^{-1}$ (Re$\approx$14000) and 993 K (720 °C) 60 K SPH (Experiment 1 and 2 Comparison), Pr=0.013

the Sensor2 geometry is computed for liquid aluminum flow at various Reynolds number for both the Sensor2 CWT and Sensor2 UHF boundary conditions. It will be shown that the function of local average Nusselt number changes at larger Reynolds numbers and this behaviour is explained using the potential flow-field solution. From the local Nusselt number distribution, the average Nusselt number for various Reynolds numbers is computed. These values will be utilized to derive a Nusselt versus velocity correlation to infer the magnitude of velocity. By modifying the average Nusselt number predicted from Fluent with a value of thermal resistance, this modified Nusselt number data yields good agreement with the experimental average Nusselt number data.
Table 6.2: Comparison of Average Nusselt Number Values between Numerical Predictions and Literature for a Cylinder Uniform Heat Flux (UHF) Boundary Condition for liquid sodium flow, Pr=0.009

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.51</td>
<td>0.33 (55)</td>
<td>0.23 (123)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.12</td>
<td>1.03 (9)</td>
<td>0.72 (56)</td>
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<td></td>
</tr>
<tr>
<td>865</td>
<td>3.41</td>
<td>3.04 (12)</td>
<td>2.12 (61)</td>
<td>2.86 (19)</td>
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<tr>
<td>1000</td>
<td>3.67</td>
<td>3.27 (12)</td>
<td>2.28 (61)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1665</td>
<td>4.77</td>
<td>4.23 (13)</td>
<td>2.94 (62)</td>
<td>3.91 (22)</td>
<td></td>
</tr>
<tr>
<td>9800</td>
<td>4.93</td>
<td></td>
<td></td>
<td>4.93 (138)</td>
<td></td>
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<tr>
<td>10000</td>
<td>11.73</td>
<td>10.34 (13)</td>
<td>7.21 (63)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10200</td>
<td></td>
<td>5.29 (122)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16900</td>
<td>15.14</td>
<td>13.44 (13)</td>
<td>9.37 (62)</td>
<td>10.73 (41)</td>
<td></td>
</tr>
</tbody>
</table>

The values in parentheses represent how much percentage (%) the Fluent predictions are in excess.

6.2.1 An Assessment of the Inviscid flow-field Overestimate of Heat Transfer

Galante and Churchill [111] and Khan et al. [85] argue that predicting the heat transfer to a cylinder in liquid metal flow using an inviscid solution to the flow-field will overestimate the heat transfer to the cylinder. The Fluent-predicted average Nusselt number defined by Equation (4.27) in liquid sodium flow is compared with data in the literature published by Khan et al. [85], Ishiguro et al. [32], and Andreevskii [84] to assess the overestimate to heat transfer.

Figure 6.5 plots the average Nusselt number versus Reynolds number for liquid sodium flow (Pr=0.009) displaying data for the uniform heat flux (Cylinder UHF) and constant wall temperature (Cylinder CWT) boundary condition. Table 6.2 and Table 6.3 lists values of the average Nusselt number for various Reynolds number, where Table 6.2 lists data for the Cylinder UHF boundary condition and Table 6.3 lists data for the Cylinder CWT boundary condition. The following observations are made from Figure 6.5 and the data presented in Table 6.2 and Table 6.3.

The lower limit of the experimental data of Ishiguro et al. [32] is Re=865. The analy-
Figure 6.5: Nusselt Number versus Reynolds Number for Liquid Sodium Flow over a Circular Cylinder (Cylinder CWT and Cylinder UHF), Pr=0.009

Table 6.3: Comparison of Average Nusselt Number Values between Numerical Predictions and Literature for a Cylinder Constant Wall Temperature (CWT) Boundary Condition for liquid sodium flow, Pr=0.009

<table>
<thead>
<tr>
<th>Re</th>
<th>Fluent</th>
<th>Inviscid [85]</th>
<th>Viscous CWT Khan</th>
<th>Viscous UHF Khan</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.51</td>
<td>0.28 (78)</td>
<td>0.21 (141)</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1.06</td>
<td>0.90 (17)</td>
<td>0.66 (59)</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>3.05</td>
<td>2.85 (7)</td>
<td>2.10 (45)</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>9.52</td>
<td>9.01 (6)</td>
<td>6.64 (43)</td>
<td></td>
</tr>
</tbody>
</table>

the values in parentheses represent how much percentage (%) the Fluent predictions are in excess
sis of the overestimate to the heat transfer is provided for Reynolds numbers of \( Re \geq 865 \) as this also corresponds to the experimental Reynolds number range of this thesis. At Reynolds numbers of \( Re=865, 1000, 1665, 10000 \) and 16900 the Cylinder UHF Fluent predictions are approximately 12% to 13% greater than the inviscid UHF analytical solutions of Khan et al. [85]. At \( Re=1000 \) and \( Re=10000 \) the Fluent predictions are approximately 7% and 6% greater than the inviscid CWT analytical solution of Khan et al. [85]. The Fluent Cylinder UHF predictions are approximately 62% greater than the viscous UHF analytical solution of Khan et al. [85] at Reynolds numbers greater than \( Re \geq 865 \). The Fluent CWT predictions are approximately 43% to 45% greater than the viscous CWT analytical solutions. The Fluent prediction at \( Re=10000 \) is used to compare with the experimental data of Andreevskii [84] at Reynolds numbers of \( Re=9800 \) and \( Re=10200 \). The Fluent prediction is approximately 138% and 122% greater than Andreevskii’s data. The Fluent predictions are approximately 19% greater than the data of Ishiguro et al. [32] at a Reynolds number of \( Re=865 \), approximately 22% greater at a Reynolds number of \( Re=1665 \), and approximately 41% greater at a Reynolds number of \( Re=16900 \). From the experimental data and the viscous correlation, the range at which the Fluent predictions overestimate the Nusselt number can be as low as 19% and as high as 138%. More experiments are needed in the literature for liquid metal flows to cylinders to better characterize the overestimate.

6.2.2 The Local Nusselt Number versus Position around the Sensor2 Geometry

The results of local Nusselt number versus angle around the Sensor2 geometry in flowing liquid aluminum for various Reynolds numbers are presented in this section. Figure 6.6 is a family of plots depicting the local Nusselt number of Sensor2. The local Nusselt number is displayed for the Sensor2 CWT boundary condition at Reynolds numbers of \( Re=10 \) (Figure 6.6a), \( Re=100 \) (Figure 6.6c), \( Re=1000 \) (Figure 6.6e), and \( Re=10000 \) (Figure 6.6g). The local Nusselt number is also shown for the Sensor2 UHF boundary condition at Reynolds number of \( Re=10 \) (Figure
Re=100 (Figure 6.6d), Re=1000 (Figure 6.6f), and Re=10000 (Figure 6.6h). The Nusselt number behaviour changes with the Reynolds number, as with higher Reynolds numbers the thickness of the thermal boundary layer decreases resulting in an increase in the Nusselt number.

The thinning of the thermal boundary layer may be observed in the non-dimensional temperature contour plots of the flow around Sensor2. The set of plots are depicted in Figure 6.7, where the temperature field is displayed at Reynolds numbers of Re=10 (Figure 6.7a), Re=100 (Figure 6.7b), Re=1000 (Figure 6.7c), and Re=10000 (Figure 6.7d). Examining Figure 6.7a which plots the temperature field at a Reynolds number of Re=10, it is observed that the variation in the temperature along a line normal from the forward stagnation point, spans a distance greater than 4.5 cylinder diameters into the fluid stream. At a Reynolds number of Re=100 (Figure 6.7b), the temperature variation at a line normal to the forward stagnation point, decreases to less than approximately 2.5 cylinder diameters into the fluid when compared to the temperature variation at Re=10. At a Reynolds number of Re=1000 (Figure 6.7c) the temperature variation thins to less than 0.5 cylinder diameter and at a Reynolds number of Re=10000 (Figure 6.7d) the temperature variation thins to less than 0.3 cylinder diameter into the fluid stream at a line normal to the forward stagnation point.

The local Nusselt number around Sensor2 at a Reynolds number of Re=10 is analyzed. Figures 6.6a and 6.6b plot the Nusselt number versus angle around Sensor2 for the CWT and the UHF boundary conditions. The value of the Nusselt number is zero at the sectors on Sensor2 where no heat exchange occurs. The region of \(36^\circ \leq \Theta \leq 54^\circ\) at a Reynolds number of Re=10 for the Sensor2 CWT boundary condition displayed in Figure 6.6a is enlarged and displayed in Figure 6.8. Figure 6.8 is employed to explain the local Nusselt number. Just before the 36° location labeled as A in Figure 6.8 is where cooler fluid interacts with the heat transfer element. Because the fluid is cold the normal temperature gradient is large at point B labeled on Figure 6.8, this large temperature gradient is related to the large Nusselt number at the 36° location. With increasing position from 36°, between \(36^\circ < \Theta < 45^\circ\), the temperature of
Figure 6.6: Local Nusselt Number of Sensor2 versus Polar Coordinate from stagnation point for Flowing Liquid Aluminum at various Reynolds numbers and Boundary Conditions, Pr=0.013
Figure 6.6: Continued Local Nusselt Number of Sensor2 versus Polar Coordinate from stagnation point for Flowing Liquid Aluminum at various Reynolds numbers and Boundary Conditions, Pr=0.013
Figure 6.7: Contour Plots of the Sensor2 non-dimensional Temperature Field at various Reynolds Numbers for a Constant Wall Temperature (CWT) Boundary Condition in Flowing Liquid Aluminum, Pr=0.013, $Y^*=y/D$, $X^*=x/D$, $T^*=(T-T_\infty)/(T_s-T_\infty)$
Figure 6.8: Local Nusselt Number of Sensor2 at Re=10 for Sensor2 CWT Boundary Condition, Pr=0.013

the fluid increases causing a decrease in the normal temperature gradient, which results in a decrease in the Nusselt number, this is labeled as the curve $BC$ on Figure 6.8. At small values of Reynolds number the conduction effects of the fluid are greater than the convective effects, thus an increase in the Nusselt number is observed after the midpoint of the heat transfer element, labeled $C$ on Figure 6.8. This increase in Nusselt number is attributed to the conduction of energy away from the downstream ($54^\circ$) location of the heat transfer element, labeled as $E$ on Figure 6.8. The conduction of energy away from the $54^\circ$ location causes the fluid in this region to cool, thus increasing the normal temperature gradient at this location and resulting in an increasing Nusselt number (the curve labeled $CD$ on Figure 6.8).

Figure 6.6c and 6.6d plot the local Nusselt number versus angle for the Sensor2 CWT and Sensor2 UHF boundary conditions at a Reynolds number of Re=100. At a Reynolds number of Re=100 a greater influence of the convective effects is observed on the heat transfer as there is a larger degree of asymmetry in the Nusselt number curve during the segment of ($36^\circ \leq \Theta \leq 54^\circ$).

Figure 6.6e and 6.6f plot the local Nusselt number versus angle for the Sensor2 CWT and Sensor2 UHF boundary conditions at a Reynolds number of Re=1000. Figure 6.6g and 6.6h
plot the local Nusselt number versus angle for the Sensor2 CWT and Sensor2 UHF boundary conditions at a Reynolds number of Re=10000. At a Reynolds number of Re=1000 the asymmetry in the local Nusselt number between the angular coordinate of $(36^\circ \leq \Theta \leq 54^\circ)$ is more prevalent when compared to the Re=100 plot (Figure 6.6c), as the convective effects increase between the Reynolds number of Re=100 and Re=1000. At a Reynolds number of Re=10000 it may be said that the convective effects of heat transfer dominate as there is no increase in the value of the Nusselt number between the angular coordinate of $(36^\circ \leq \Theta \leq 54^\circ)$.

Attention is drawn to the maximum value of the Nusselt number between the the coordinates of $(351^\circ \leq \Theta \leq 9^\circ)$ and $(36^\circ \leq \Theta \leq 54^\circ)$ at a Reynolds number of Re=10000 (Figure 6.6g and 6.6h). The local heat transfer between the region of $(36^\circ \leq \Theta \leq 54^\circ)$ is greater than that of the region $(351^\circ \leq \Theta \leq 9^\circ)$. This result can be explained by examining the non-dimensional temperature profile at the surface of Sensor2 and the magnitude of velocity around Sensor2. The magnitude of velocity $(u^*_\Theta)$ is larger over the segment $(36^\circ \leq \Theta \leq 54^\circ)$ than over the segment $(351^\circ \leq \Theta \leq 9^\circ)$, the non-dimensional velocity profile around Sensor2 is displayed in Figure 6.9a. Figure 6.9b plots the non-dimensional temperature at the surface of the Sensor2 geometry as a function of position from the forward stagnation point. In Figure 6.9b two curves are depicted, one for the temperature at a Reynolds number of Re=10 and the other for the temperature at a Reynolds number of Re=10000. Notice that the temperature between the first non-heated segment of the circle approaches that of the free stream temperature, acting to reset the oncoming fluid temperature at the segment $(36^\circ \leq \Theta \leq 54^\circ)$ to the free stream temperature. At larger Reynolds number the temperature of the fluid between the sector $(9^\circ < \Theta < 36^\circ)$ approaches the free stream temperature to a greater degree than at smaller Reynolds numbers. This reset of the fluid temperature towards that of the free stream temperature in conjunction with the higher fluid velocity at the segment $(36^\circ \leq \Theta \leq 54^\circ)$ causes a larger maximum in the Nusselt number when comparing the Nusselt number values between the segments $(351^\circ \leq \Theta \leq 9^\circ)$ and $(36^\circ \leq \Theta \leq 54^\circ)$ at Reynolds numbers of Re$\geq$1000.
6.2.3 The Local Average Nusselt Number versus Position around the Sensor2 Geometry

In this section the location of the maximum Nusselt number around the Sensor2 geometry is further explored by computing a local average Nusselt number. Figure 6.10 is a family of curves which displays the local average Nusselt number ($\overline{Nu}_a$ Equation (4.28)) versus position from the forward stagnation point. The numerical predictions for both the Sensor2 CWT and Sensor2 UHF boundary conditions are portrayed for Reynolds numbers of Re=10 (Figure 6.10a), Re=100 (Figure 6.10b), Re=1000 (Figure 6.10c), and Re=10000 (Figure 6.10d). The function of the local average Nusselt number versus polar coordinate from the stagnation point changes somewhere between the Reynolds numbers of Re=100 and Re=1000. This change in the form of function has an effect on the technique which is employed to determine the direction of velocity. This change in the form of the local average Nusselt number implies that a separate data set for local average Nusselt number versus position must be utilized depending on the Reynolds number of the flow-field. At this point, attention is drawn to the value of the local average Nusselt number at the forward stagnation point of the flow ($0^\circ$) in the family
of curves of Figure 6.10. At Reynolds numbers of Re=10 (Figure 6.10a) and Re=100 (Figure 6.10b), the local average Nusselt number is maximum at the forward stagnation point. At Reynolds number of Re=1000 (Figure 6.10c) and Re=10000 (Figure 6.10d), the local average Nusselt number is maximum at ±45° from the stagnation point.

The Reynolds numbers of the Sensor2 experiments are estimated as Re≈9000, Re≈14000 and Re≈19000. Recall it was estimated that the Reynolds numbers of flows in an aluminum reduction cell range 3400 ≤ Re ≤ 20400. The local average Nusselt number is displayed for the Reynolds numbers of 9000, 10000, 14000, 19000, 30000 in Figure 6.10e. At Reynolds number of greater than Re≈9000 the convective effects dominate the heat transfer to the Sensor2 geometry, as a result the normalized local average Nusselt Number is portrayed in Figure 6.10f. In Figure 6.10f the local average Nusselt number is normalized using the square root of the Reynolds number of the prediction (\(\overline{\text{Nu}_{ia}}/\text{Re}^{0.5}\)), both the Sensor2 CWT and Sensor2 UHF boundary conditions are displayed in Figure 6.10f. The normalized local average Nusselt number will be employed to determine the direction of velocity.

6.2.4 The Average Nusselt Number versus Reynolds Number for Sensor2

In this section the average Nusselt number as a function of Reynolds number from the numerical predictions is compared with the experimental data of Sensor2. Figure 6.11 plots the average Nusselt number (\(\overline{\text{Nu}_{s2}}\)) of Sensor2, where the average Nusselt number is computed from Equation (4.29) by including only the sectors of the Sensor2 geometry where heat transfer occurs with the fluid. The average Nusselt number for both the Sensor2 CWT and the Sensor2 UHF boundary conditions are displayed in Figure 6.11 along with the analytical solutions of Khan et al. [85], where Khan et al. solve for the Cylinder CWT and the Cylinder UHF slip and no-slip boundary conditions. The Nusselt number values of the Sensor2 thermal boundary conditions are greater than the correlations of Khan et al. The value of the Nusselt number of the Sensor2 boundary condition is greater than that of the Cylinder boundary condition as there is less heating of the fluid around Sensor2 thus increasing the heat transfer coefficient at com-
Figure 6.10: Local Average Nusselt Number of Sensor2 in Flowing Liquid Aluminum at Various Reynolds number for the Constant Wall Temperature (Sensor2 CWT) and Uniform Heat Flux (Sensor2 UHF) Boundary Conditions, Pr=0.013
Figure 6.11: Nusselt Number versus Reynolds Number for the Sensor2 Design in Liquid Aluminum Flow, Pr=0.013

parable locations for the Sensor2 and Cylinder mathematical problems. This results in a larger Nusselt number for the Sensor2 boundary conditions than the Cylinder boundary conditions, as the average Nusselt number is obtained from only those sectors of Sensor2 that exchange energy with the surroundings.

6.2.5 Modifying the Average Nusselt Number to include a Thermal Resistance for Sensor2

The effect of adding a thermal resistance to the Nusselt number values obtained from the numerical predictions is examined here. Good agreement between the experimental data and the numerical predictions is observed when a modified Nusselt number is introduced to incorporate a value of thermal resistance. Figure 6.12 plots the average Nusselt number of Sensor2 versus the Reynolds number. In this plot, experimentally derived Nusselt numbers from Sensor2 are depicted which were calculated from averaging the estimated heat transfer coefficients obtained from Equations (1.1) and (1.2). The curve labeled $R = 0$ in Figure 6.12 is the average...
Nusselt number predictions from Fluent. There is approximately one order of magnitude difference in the value of average Nusselt number ($\bar{\text{Nu}}_{S2}$) of the numerical predictions (the curve labeled $R = 0$) and the experimental data. Experimentally, the copper elements of Sensor2 are sprayed with carbon to prevent the dissolution of the solid-copper into the liquid-aluminum. This carbon coating acts as a thermal resistance between the solid-copper and liquid-aluminum materials. In chapter 5 of this thesis the effect of thermal resistance on the solidification and melting of aluminum shells onto Sensor1 was investigated. Like Sensor2 a layer of carbon was coated to the surface of Sensor1. To incorporate the thermal resistance from the layer of sprayed carbon, a thermal resistance term is added to the common Nusselt number relationship as represented by Equation (6.2).

$$\bar{\text{Nu}}_{S2R} = \frac{1}{\nu_{S2} + R D}$$

(6.2)

The modified Nusselt number ($\bar{\text{Nu}}_{S2R}$) curves are computed, where a value of resistance is added to the predicted heat transfer coefficient obtained from Fluent. Figure 6.12 depicts the modified Nusselt number ($\bar{\text{Nu}}_{S2R}$) for resistance values of $R = 2 \times 10^{-4}, 3 \times 10^{-4}, 4 \times 10^{-4},$ and $1 \times 10^{-3}$ m$^2$·K·W$^{-1}$. The experimental data points agree with the thermal resistance values of $R = 2 \times 10^{-4}, 3 \times 10^{-4},$ and $4 \times 10^{-4}$ m$^2$·K·W$^{-1}$. Recall that the solidification and melting times of the aluminum shells on steel and copper Sensor1 designs agreed with a thermal resistance of $R = 2 \times 10^{-4}$ m$^2$·K·W$^{-1}$.

6.3 Results from Modeling the Transient Conjugate Heat Transfer from Liquid Aluminum to Sensor2

Fluent was utilized to estimate the temperature response of each copper element by applying a thermal resistance at the copper-aluminum interface. Matching the numerical temperature response to the experimental temperature response an estimate of the thermal resistance of each
Figure 6.12: Modified Nusselt Number versus Reynolds Number for the Sensor2 Design in Liquid Aluminum Flow with Experimental Data, Pr=0.013
copper element was obtained. Figure 6.1b plots the experimental non-dimensional temperature data when Sensor2 is immersed into flowing liquid aluminum at a Reynolds number of \( \text{Re} \approx 14000 \). Fluent is used to predict the temperature response of each copper element where a thermal resistance between the solid-copper and liquid-aluminum interface is incorporated by modifying the thermal conductivity of the region which is defined as resistance layer in Figure 4.6. Equation (4.25) is the relationship employed to impose a thermal resistance at the solid-copper and liquid-aluminum interface.

From the Fluent predictions, it is observed that experimentally each copper element has a different thermal resistance. Figure 6.13 plots the experimental non-dimensional temperature with those computed by Fluent for various thermal resistance between the solid-copper and liquid-aluminum. The response of the copper element aligned at the stagnation point of the flow (0°) is shown in Figure 6.13a, within this plot are several curves, the bold curve represents the experimental temperature response of the copper element aligned at 0° and is labeled \((t_c)\), the remaining curves represent Fluent temperature predictions for the copper element where values of thermal resistance are applied at the copper-aluminum interface. The labels accompanying the Fluent predictions represent the values of thermal resistance. From Figure 6.13a at a Fourier number of \( \text{Fo} < 12 \), it is observed that the prediction of temperature employing a resistance of \( R_0 = 2.5 \times 10^{-4} \text{m}^2\cdot\text{K}\cdot\text{W}^{-1} \) agrees with the experimental temperature. Table 6.4 lists the estimated thermal resistance for each copper element. The thermal resistance at the copper elements fall within the range \( 2.5 \times 10^{-4} \) to \( 6.5 \times 10^{-4} \text{m}^2\cdot\text{K}\cdot\text{W}^{-1} \). The thermal resistance at each copper element is not constant between copper segments. This is plausible as at the time of this work a means to control the thickness of the carbon coating between copper elements was lacking. The thickness of the carbon coating is related to the thermal resistance across the copper-aluminum interface as discussed in section 2.3. These thermal resistance values fall below the range of values recorded by Goudie [81] (Table 2.1), this is plausible as in Goudie’s work there was no coating applied to the cylinders. As mentioned, the coating acts to decrease the heat transfer, more than a bare substrate.
Figure 6.13: Fluent Predictions of Copper Element Temperature Response at Various Thermal Resistance Compared with Experimental Data from flowing Liquid Aluminum at 0.16 m·s⁻¹ (Re≈14000) and 993 K (720 °C) 60 K SPH, Pr=0.013
Figure 6.13: Continued Fluent Predictions of Copper Element Temperature Response at Various Thermal Resistance Compared with Experimental Data from flowing Liquid Aluminum at 0.16 m·s⁻¹ (Re≈14000) and 993 K (720 °C) 60 K SPH, Pr=0.013
### Table 6.4: Estimated Thermal Resistance at Copper Element

<table>
<thead>
<tr>
<th>Θ [°]</th>
<th>R [m²·K·W⁻¹]</th>
<th>Fo &lt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.50e-04</td>
<td>12</td>
</tr>
<tr>
<td>45</td>
<td>4.00e-04</td>
<td>22</td>
</tr>
<tr>
<td>90</td>
<td>5.50e-04</td>
<td>20</td>
</tr>
<tr>
<td>135</td>
<td>6.00e-04</td>
<td>23</td>
</tr>
<tr>
<td>180</td>
<td>6.25e-04</td>
<td>21</td>
</tr>
<tr>
<td>225</td>
<td>6.50e-04</td>
<td>33</td>
</tr>
<tr>
<td>270</td>
<td>5.00e-04</td>
<td>18</td>
</tr>
<tr>
<td>315</td>
<td>2.75e-04</td>
<td>9</td>
</tr>
</tbody>
</table>

Θ: polar coordinate from stagnation point, R: thermal resistance, Fo < the upper limit of Fourier number for which numerical and experimental curves coincide.

#### 6.3.1 Modifying the Local Average Nusselt Number to include a Thermal Resistance for Sensor2

The effect of adding a thermal resistance to the computed local average Nusselt number from Fluent is examined in this section. From the transient conjugate heat transfer predictions it was inferred that the thermal resistance varies for each copper element. Figure 6.14a plots the predicted local average Nusselt number functions for the Sensor2 CWT and Sensor2 UHF boundary conditions at a Reynolds number of Re ≈ 14000. The experimentally determined local average Nusselt number function at this Reynolds number is displayed in Figure 6.3. There is disparity in the form of the local average Nusselt function between Figure 6.14a and Figure 6.3. By adding a thermal resistance to the values of the local average Nusselt number, the numerically generated data will agree with the experimental data. The local average Nusselt number values predicted by Fluent are modified by adding the values of the local thermal resistance ascertained from Figure 6.13 and listed in Table 6.4 for each copper heat transfer element. The following relationship is employed:

\[
\overline{\text{Nu}_{\text{la}}R} = \frac{1}{\text{Fo} + R} \frac{D}{k}
\]

(6.3)
Table 6.5 lists the polar coordinates (Θ) of the copper elements with respect to the forward stagnation point of the flow-field along with the predicted local average Nusselt numbers (\(\overline{\text{Nu}_{\text{la}}\text{UHF}}\)) for the Sensor2 UHF and Sensor2 CWT boundary conditions. Another column in Table 6.5 lists the values of thermal resistance (R) ascertained from Figure 6.13. The value of thermal resistance is added to the predicted local average Nusselt numbers via Equation (6.3) to obtain a local average Nusselt number modified with thermal resistance (\(\overline{\text{Nu}_{\text{laR}}\text{UHF}}\)), the last column in Table 6.5 lists the experimentally determined local average Nusselt number derived using Equation (1.1) and Equation (1.2). Figure 6.14b plots the local average Nusselt number modified with thermal resistance (\(\overline{\text{Nu}_{\text{laR}}\text{UHF}}\)) employing the predicted Sensor2 UHF and predicted Sensor2 CWT local average Nusselt number values as the base heat transfer coefficient. The experimentally derived local average Nusselt numbers are also displayed on Figure 6.14b. The local average Nusselt number modified with a thermal resistance is in very good agreement with the experimental data. To determine the direction and magnitude of velocity using the Sensor2 geometry the thermal resistance which is imparted by the coating to protect the copper must be known.
(a) Local Average Nusselt Number obtained by Fluent Predictions for Sensor2 in Liquid Aluminum Flow $Re \approx 14000$, $Pr=0.013$

(b) Local Average Nusselt Number incorporating the Thermal Resistance for Sensor2 derived from Conjugate Heat Transfer Predictions $Re \approx 14000$, $Pr=0.013$

Figure 6.14: Effect of a Thermal Resistance Value on the Local Average Nusselt Number of Sensor2
6.4 Determining the Magnitude of Velocity

To determine the magnitude of velocity the estimated local average Nusselt number of Sensor2 ($\overline{Nu}_{la,R}$), which includes a value of thermal resistance obtained from Equations (1.1) and (1.2) for each copper element, is used with the known value of thermal resistance for each copper element to solve for the local average Nusselt number without resistance. Equation (6.4) is utilized to correct the local average Nusselt number value from the thermal resistance.

$$\overline{Nu}_{la} = \frac{D}{k} \cdot \left( \frac{1}{\frac{1}{\overline{Nu}_{la,R}} - R} \right)$$

(6.4)

Where $D$ is the diameter of Sensor2, $k$ is the thermal conductivity of the fluid, and $R$ is the value of thermal resistance imparted by the carbon coating acting at that particular copper element. The local average Nusselt number without resistance is then employed to obtain an estimate of the average Nusselt number of Sensor2 ($\overline{Nu}_{S2}$ Equation (4.29)). The estimate of the average Nusselt number of Sensor2 ($\overline{Nu}_{S2}$) can be used to estimate the Reynolds number of the flow-field. Employing the definition of the Reynolds number, the density and dynamic viscosity of liquid aluminum, and the diameter of Sensor2, the magnitude of velocity is determined from Equation (6.5).

$$u_\infty = \frac{\mu \cdot Re}{\rho \cdot D}$$

(6.5)

Equation (6.6) and Equation (6.7) represent the fit to the average Nusselt number of Sensor2 obtained from modeling the steady convective heat transfer of liquid aluminum around Sensor2, where Equation (6.6) is a result of the fit to the Sensor2 CWT boundary condition and Equation (6.7) is a result of the fit to the Sensor2 UHF boundary condition. Figure 6.11 displays the data points from the Fluent predictions of the average Nusselt number of Sensor2 versus Reynolds number, the data fit is generated for the average Nusselt number values of Sensor2 in the Reynolds number range of ($1000 \leq Re \leq 30000$). The coefficient of determination ($R^2$) for the Sensor2 CWT condition is $R^2 = 0.9999$ and the Sensor2 UHF condition is
$R^2 = 0.9999$.

**Sensor2 CWT:** The following relationship is employed for the Sensor2 CWT prediction in liquid aluminum ($Pr=0.013$) and valid for the Reynolds number range of $(1000 \leq Re \leq 30000)$:

$$Re = \left( \frac{\bar{Nu}_{S2}}{0.2729} \right)^{\frac{1}{0.4779}} \tag{6.6}$$

**Sensor2 UHF:** The following relationship is employed for the Sensor2 UHF prediction in liquid aluminum ($Pr=0.013$) and valid for the Reynolds number range of $(1000 \leq Re \leq 30000)$:

$$Re = \left( \frac{\bar{Nu}_{S2}}{0.2899} \right)^{\frac{1}{0.4871}} \tag{6.7}$$

### 6.5 Determining the Direction of Velocity

To determine the direction of velocity, plot the normalized local average Nusselt numbers corrected for the thermal resistance at the copper element, as a function of position around the Sensor2 geometry. Assuming that the thermal resistance is known for each copper element the form of the plot will resemble that of Figure 6.15 when corrected for the thermal resistance and normalized by the Reynolds number, where Figure 6.15a displays the normalized local average Nusselt number versus angular coordinate on a cartesian plot and Figure 6.15b displays this information on a polar plot. Knowing the thermal resistance at the copper-aluminum interface and knowing the Reynolds number from the fit to the steady convective heat transfer predictions derived as Equations (6.6) and (6.7), Equation (6.8) is employed to normalize the values of local average Nusselt number ($\bar{Nu}_{la}$). The eight normalized local Nusselt number values are assembled, preserving the sequence of each copper element with respect to the polar coordinate about the circle, until they resemble that of Figure 6.15 to determine the direction of velocity. Table 6.6 list the values of the normalized local average Nusselt numbers ($\bar{Nu}_{la}/Re^{0.5}$) for the polar coordinate from the forward stagnation point, for the Sensor2 CWT and Sensor2
Table 6.6: Normalized Local Average Nusselt Number Values derived from the Sensor2 Constant Wall Temperature (Sensor2 CWT) and Uniform Heat Flux (Sensor2 UHF) Numerical Predictions for liquid aluminum flow, Pr=0.013

<table>
<thead>
<tr>
<th>$\Theta$ [$^\circ$]</th>
<th>$\overline{\text{Nu}}_\text{la}/\text{Re}^{0.5}$</th>
<th>CWT</th>
<th>UHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.19</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>0.32</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.27</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>135</td>
<td>0.16</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>0.03</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>0.16</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>0.27</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>315</td>
<td>0.32</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

UHF boundary conditions.

$$\frac{\overline{\text{Nu}}_\text{la}}{\text{Re}^{0.5}} = C$$  \quad (6.8)

Table 6.7 lists numerically predicted values of Sensor2 with the purpose of illustrating the direction finding technique. Table 6.7 lists the copper element number which is labeled sequentially around Sensor2, the local average Nusselt numbers $\overline{\text{Nu}}_{\text{la}}$ of Sensor2 where these values include a thermal resistance imparted from the carbon coating, and the normalized local average Nusselt number computed using Equation (6.8). The Sensor2 UHF predictions listed in Table 6.5 are employed to illustrate the direction finding technique of the local average Nusselt number with resistance ($\overline{\text{Nu}}_{\text{la},R}$) listed in Table 6.7. The Reynolds number of the flow is $\text{Re}=10000$ and the thermal resistance is $R = 3 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}$ at each copper element for the data presented in Table 6.7. Comparing the values of the normalized local average Nusselt numbers in Table 6.7 to those in Table 6.6 for Sensor2 UHF boundary condition, it is observed that the copper element numbered as 6 in Table 6.7 correlates to that of 0$^\circ$ in Table 6.6. Thus the copper element labeled 6 is aligned with the forward stagnation point of the flow-field, this copper element represents the direction of velocity.
Figure 6.15: Normalized Local Average Nusselt Number derived from the Sensor2 Constant Wall Temperature (Sensor2 CWT) and Uniform Heat Flux (Sensor2 UHF) Numerical Predictions for liquid aluminum flow, Pr=0.013
Table 6.7: Example of Direction Finding Local Average Nusselt Number Values for a Reynolds number of Re=10000 and Thermal Resistance of \( R = 3 \times 10^{-4} \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1} \) at each copper element using the Sensor2 Uniform Heat Flux (Sensor2 UHF) Prediction as a Heat Transfer Coefficient Base for liquid aluminum flow, Pr=0.013

<table>
<thead>
<tr>
<th>No</th>
<th>( \overline{\text{Nu}}_{\text{laR}} )</th>
<th>( \frac{\overline{\text{Nu}}_{\text{la}}}{\text{Re}^{0.5}} )</th>
<th>( \Theta [^\circ] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3042</td>
<td>0.23</td>
<td>135</td>
</tr>
<tr>
<td>2</td>
<td>1.1843</td>
<td>0.08</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>1.3042</td>
<td>0.23</td>
<td>225</td>
</tr>
<tr>
<td>4</td>
<td>1.3215</td>
<td>0.30</td>
<td>270</td>
</tr>
<tr>
<td>5</td>
<td>1.3268</td>
<td>0.34</td>
<td>315</td>
</tr>
<tr>
<td>6</td>
<td>1.2873</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1.3268</td>
<td>0.34</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>1.3215</td>
<td>0.30</td>
<td>90</td>
</tr>
</tbody>
</table>

No: the copper element number, \( \overline{\text{Nu}}_{\text{laR}} \): the local average Nusselt number recorded by Sensor2 which includes a value of thermal resistance, \( \frac{\overline{\text{Nu}}_{\text{la}}}{\text{Re}^{0.5}} \): the normalized local average Nusselt number recorded by Sensor2, \( \Theta \): polar coordinate with respect to forward stagnation point

6.6 The Error of Sensor2

The Taylor series method outlined by Coleman and Steele [121] is employed to determine the error in the proposed data reduction equation of Equation (6.4), where this equation is utilized to determine the local average Nusselt number (\( \overline{\text{Nu}}_{\text{la}} \)). The uncertainty in the relationship is estimated by taking the thermal conductivity and diameter of the Sensor2 geometry as constants, the purpose here is to show that the local average Nusselt number calculation is very sensitive to the error in the thermal resistance and error in the estimated heat transfer coefficient represented as \( \overline{\text{Nu}}_{\text{laR}} \) (the local average Nusselt number with thermal resistance). Equation (6.9) is the relationship which was derived to compute the uncertainty in the local average Nusselt number \( \overline{\text{Nu}}_{\text{la}} \). The derivation is listed in appendix G.

\[
U^2_{\text{Nu}_{\text{la}}} = \left( \frac{\overline{\text{Nu}}^2_{\text{laR}} \cdot k}{\left( \frac{k}{D} \cdot \overline{\text{Nu}}_{\text{laR}} \cdot R - 1 \right)^2} \right)^2 \cdot U^2_{\text{R}} + \left( \frac{1}{\left( \frac{k}{D} \cdot \overline{\text{Nu}}_{\text{laR}} \cdot R - 1 \right)^2} \right)^2 \cdot U^2_{\overline{\text{Nu}}_{\text{laR}}} \quad (6.9)
\]
Listed in Table 6.8 are values of the local average Nusselt number which includes a thermal resistance ($\overline{\text{Nu}}_{la,R}$), the thermal resistance ($R$), the local average Nusselt number ($\overline{\text{Nu}}_{la}$), and corresponding uncertainty values of these variables when a ratio of the uncertainty of the variable divided by the variable value (the percentage error) is assumed. Table 6.8 examines the effect of the uncertainty in the thermal resistance and uncertainty in the estimate of the local average Nusselt number with resistance on the error of the local average Nusselt number. To examine the uncertainty in the local average Nusselt number, values listed in Table 6.5 are employed. The first three rows in Table 6.8 utilize the local average Nusselt number values for the Sensor2 UHF boundary condition ($\overline{\text{Nu}}_{la\text{,UHF}}$) and the corresponding thermal resistance ($R$) value at the stagnation point ($0^\circ$). The remaining three rows in Table 6.8 utilize the ($\overline{\text{Nu}}_{la\text{,UHF}}$) and the ($R$) values at $180^\circ$ from the stagnation point listed in Table 6.5. The first row of Table 6.8 assumes an error of 20% in the $\overline{\text{Nu}}_{la,R}$ and $R$ values, this corresponds to a 412% error in the local average Nusselt number estimate ($\overline{\text{Nu}}_{la}$) at the $0^\circ$ copper element. At 10% error in the values of $\overline{\text{Nu}}_{la,R}$ and $R$ the error in the $\overline{\text{Nu}}_{la}$ is 206%. At 1% error in the values of $\overline{\text{Nu}}_{la,R}$ and $R$ the error in the $\overline{\text{Nu}}_{la}$ is 20%. The fourth row of Table 6.8 assumes an error of 20% in the $\overline{\text{Nu}}_{la,R}$ and $R$ values at the copper element of $180^\circ$ from the stagnation point, at this relative error the error in the $\overline{\text{Nu}}_{la}$ estimate is 170%. At 10% error in the values of $\overline{\text{Nu}}_{la,R}$ and $R$ the error in $\overline{\text{Nu}}_{la}$ is 85%. At 1% error in the values of the $\overline{\text{Nu}}_{la,R}$ and $R$ the error in the $\overline{\text{Nu}}_{la}$ is 8%.

For this velocity technique to be a viable industrial solution more research must be conducted to quantify the error associated with the estimate of the local average Nusselt number which includes a resistance ($\overline{\text{Nu}}_{la,R}$). Research must also be conducted to ascertain the error in the thermal resistance imparted by the coating of carbon to the copper element. As explained in the introduction of this thesis comprehensive models to assess the value of thermal resistance at the interface of a metal substrate with coating in contact with flowing liquid metal is scarce in the literature with the author aware of one by Hamasaiid et al. [78]. Perhaps more experimental research should be undertaken to better quantify the thermal resistance in liquid metal systems.
Table 6.8: Sensitivity of the Error in the estimate of the local average Nusselt number

<table>
<thead>
<tr>
<th>$\overline{Nu}_{la\text{R}}$</th>
<th>$U_{\overline{Nu}_{la\text{R}}}$</th>
<th>$U_{\overline{Nu}_{la}}$</th>
<th>$R$</th>
<th>$U_R$</th>
<th>$U_R$</th>
<th>$U_{\overline{Nu}_{la}}$</th>
<th>$U_{\overline{Nu}_{la}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.54</td>
<td>0.20</td>
<td>0.309</td>
<td>2.5e-04</td>
<td>0.20</td>
<td>5.00e-05</td>
<td>96.29</td>
<td>23.34</td>
</tr>
<tr>
<td>1.54</td>
<td>0.10</td>
<td>0.154</td>
<td>2.5e-04</td>
<td>0.10</td>
<td>2.50e-05</td>
<td>48.14</td>
<td>23.34</td>
</tr>
<tr>
<td>1.54</td>
<td>0.01</td>
<td>0.015</td>
<td>2.5e-04</td>
<td>0.01</td>
<td>2.50e-06</td>
<td>4.81</td>
<td>23.34</td>
</tr>
<tr>
<td>0.56</td>
<td>0.20</td>
<td>0.112</td>
<td>6.25e-04</td>
<td>0.20</td>
<td>1.25e-04</td>
<td>6.22</td>
<td>3.65</td>
</tr>
<tr>
<td>0.56</td>
<td>0.10</td>
<td>0.056</td>
<td>6.25e-04</td>
<td>0.10</td>
<td>6.25e-05</td>
<td>3.11</td>
<td>3.65</td>
</tr>
<tr>
<td>0.56</td>
<td>0.01</td>
<td>0.005</td>
<td>6.25e-04</td>
<td>0.01</td>
<td>6.25e-06</td>
<td>0.31</td>
<td>3.65</td>
</tr>
</tbody>
</table>

$k$: thermal conductivity of aluminum fluid ($92 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$), $D$: diameter of Sensor2 (0.0381 m), $\overline{Nu}_{la\text{R}}$: local average Nusselt number with thermal resistance included, $U_{\overline{Nu}_{la\text{R}}}$: uncertainty of local average Nusselt number with thermal resistance included, $R$: thermal resistance $\text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$, $U_R$: uncertainty of thermal resistance, $U_{\overline{Nu}_{la}}$: uncertainty of local average Nusselt number, $\overline{Nu}_{la}$: local average Nusselt number

6.7 Extending the Sensor2 Design to the Liquid Steel System

In this section it is demonstrated that the Sensor2 geometry can be employed to measure the velocity in another widely produced high temperature liquid metal, namely that of liquid steel. In this thesis experiments are not performed in liquid steel but numerical predictions of the steady convective heat transfer of liquid steel around Sensor2 and the transient conjugate heat transfer to the Sensor2 geometry are performed. The purpose is to demonstrate the applicability of the proposed magnitude and direction finding technique to the liquid steel system. Liquid steel processing temperatures are in excess of 1873 K (1600 °C), copper undergoes solid to liquid phase transition at 1358 K (1085 °C) thus copper will not be a suitable temperature capturing element for Sensor2, here tungsten is recommended which melts at 3695 K (3422 °C). In addition, the author is unaware whether the calcium silicate material will withstand the harsh liquid steel environment therefor it is proposed to change the housing material to alumina. Alumina undergoes solid to liquid phase transition at 2345 K (2072 °C) and is a commonly used refractory material in steel processing. The thermophysical properties of liquid iron are listed in Table 6.9, tungsten in Table 6.10, and alumina in Table 6.11.

The numerical methodology outlined in section 4.3 is utilized where the transient conjugate heat transfer from liquid aluminum to Sensor2 is modeled, however for this case the material
Table 6.9: Thermophysical Properties of Iron [28]

<table>
<thead>
<tr>
<th>( \rho ) [kg m(^{-3})]</th>
<th>( c_p ) [J kg(^{-1}) K(^{-1})]</th>
<th>( k ) [W m(^{-1}) K(^{-1})]</th>
<th>( \mu ) [kg m(^{-1}) s(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6977</td>
<td>824</td>
<td>36.2</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Values \( \rho, c_p, k, \mu \) displayed for a temperature of 1873 K (1600 °C).

Table 6.10: Thermophysical Properties of Tungsten [112]

<table>
<thead>
<tr>
<th>( \rho ) [kg m(^{-3})]</th>
<th>( k ) [W m(^{-1}) K(^{-1})]</th>
<th>( c_p ) [J kg(^{-1}) K(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>19250</td>
<td>175</td>
<td>135</td>
</tr>
</tbody>
</table>

Properties of the system are adjusted to represent the liquid steel system. It is assumed that the resistance layer which is imparted from the layer of carbon coating remains for the liquid steel Sensor2 design, meaning that in the physical construction of the tungsten elements they would be sprayed with carbon or another ceramic material. As such the values of thermal resistance derived from Figure 6.13 are imposed to each tungsten element aligned with the flow-field. Listed in Table 6.4 are the thermal resistance values of the element with respect to the polar coordinate from the forward stagnation point.

Figure 6.16 plots the predicted non-dimensional temperature response of the eight tungsten elements in the Sensor2 geometry with alumina housing in flowing liquid steel at \( \text{Re}=14000 \) and \( \text{Pr}=0.1 \). Like the experimental response of the liquid aluminum Sensor2 design, the tungsten element at \( 0^\circ \) is the hottest in the liquid steel and the elements located at \( 135^\circ, 225^\circ, \) and \( 180^\circ \) are among the coldest.

Figure 6.17 plots the non-dimensional temperature data fit to the transient conjugate heat transfer Fluent predictions of Sensor2 with tungsten elements and alumina housing in flowing liquid steel.

Table 6.11: Thermophysical Properties of Alumina [112]

<table>
<thead>
<tr>
<th>( \rho ) [kg m(^{-3})]</th>
<th>( k ) [W m(^{-1}) K(^{-1})]</th>
<th>( c_p ) [J kg(^{-1}) K(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3690</td>
<td>18</td>
<td>880</td>
</tr>
</tbody>
</table>
Figure 6.16: Predicted non-dimensional Temperature Response of Eight Tungsten Elements in the Sensor2 Design with Alumina Housing in flowing Liquid Steel at Re=14000, Pr=0.1

liquid steel. Visually examining the fit to the data curves in Figure 6.17 for the tungsten elements in liquid steel, it is noticed that the fit to the data curves do not coincide as well with the Fluent predictions when compared with the fit to the experimental data curves displayed in Figure 6.2 for the copper elements in liquid aluminum. Table 6.12 lists the local average Nusselt number of the tungsten element, corresponding Biot number, the coefficient of determination, and the heat transfer coefficient derived from fitting Equation (1.1) and Equation (1.2) to the predicted temperature data from Fluent for the liquid steel system. The non-dimensional temperature fit of Equations (1.1) and (1.2) for the copper elements housed in calcium silicate in flowing liquid aluminum provide better coincidence to the experimental non-dimensional temperature data when compared with the predictions of the tungsten elements housed in alumina in flowing liquid steel. The coefficient of determination values listed in Table 6.1 for the copper, calcium silicate, liquid aluminum system are closer to the value of 1 when compared with the corresponding coefficient of determination values listed in Table 6.12 for the tungsten, alumina, liquid steel system. Some Biot numbers listed in Table 6.12 for the liquid steel system are greater than Bi=0.1 which is the commonly stated criterion for the lumped capacitance method to be valid [35], perhaps a one-dimensional analytical solution to the heat conduction
Table 6.12: Tungsten Element Coefficient Data for the Sensor2 Design Fluent Prediction at Re=14000, Pr=0.1

<table>
<thead>
<tr>
<th>Θ[^°]</th>
<th>( \overline{Nu}_{ar} )</th>
<th>Bi</th>
<th>( R^2 )</th>
<th>h [W·m(^{-2})·K(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.34</td>
<td>0.115</td>
<td>0.941</td>
<td>3174</td>
</tr>
<tr>
<td>45</td>
<td>3.05</td>
<td>0.105</td>
<td>0.958</td>
<td>2903</td>
</tr>
<tr>
<td>90</td>
<td>2.83</td>
<td>0.097</td>
<td>0.961</td>
<td>2694</td>
</tr>
<tr>
<td>135</td>
<td>2.51</td>
<td>0.086</td>
<td>0.966</td>
<td>2388</td>
</tr>
<tr>
<td>180</td>
<td>1.90</td>
<td>0.065</td>
<td>0.995</td>
<td>1811</td>
</tr>
<tr>
<td>225</td>
<td>2.49</td>
<td>0.086</td>
<td>0.968</td>
<td>2373</td>
</tr>
<tr>
<td>270</td>
<td>2.86</td>
<td>0.098</td>
<td>0.961</td>
<td>2722</td>
</tr>
<tr>
<td>315</td>
<td>3.19</td>
<td>0.110</td>
<td>0.949</td>
<td>3039</td>
</tr>
</tbody>
</table>

\( \Theta \): polar coordinate from stagnation point, \( \overline{Nu}_{ar} \): the local average Nusselt number with thermal resistance, Bi: Biot number, \( R^2 \): coefficient of determination, h: heat transfer coefficient

The equation with a convective heat transfer boundary condition may produce a better fit to the temperature data.

Figure 6.18 is a plot of the estimated local average Nusselt number with thermal resistance for the Sensor2 design in flowing liquid steel. The local average Nusselt number modified with thermal resistance data points of Figure 6.18 resemble those of the experimental curves of Figure 6.4 for the liquid aluminum system. The decreasing trend in the Nusselt number from \((0^° \leq \Theta \leq 180^°)\) followed by an increasing trend from \((180^° < \Theta \leq 315^°)\) is observed in the liquid steel system like the liquid aluminum system.

Figure 6.19 plots the Fluent predictions of the average Nusselt number of Sensor2 versus Reynolds number for Prandtl numbers of Pr=0.1 representing liquid steel and Pr=0.01 representing liquid aluminum. As expected, the larger Prandtl number fluid exhibits larger average Nusselt numbers for Sensor2 than the lower Prandtl number fluid. A curve fit to the average Nusselt number of Sensor2 in the Reynolds number range of \((1000 \leq Re \leq 30000)\) is performed on the Sensor2 CWT Fluent predictions for liquid steel (Pr=0.1) the coefficient of determination \( R^2 \) is \( R^2 = 0.9999 \), and for the Sensor2 UHF predictions in liquid steel the coefficient of determination is \( R^2 = 0.9997 \). Equation (6.10) and Equation (6.11) are the Reynolds number correlations for the Sensor2 CWT and the Sensor2 UHF boundary condition.
Figure 6.17: Temperature data fit to Transient Conjugate Heat Transfer Fluent Prediction of the Sensor2 Geometry with Tungsten Elements and Alumina Housing in flowing Liquid Steel at Re=14000, Pr=0.1
Figure 6.17: Continued Temperature data fit to Transient Conjugate Heat Transfer Fluent Prediction of the Sensor2 Geometry with Tungsten Elements and Alumina Housing in flowing Liquid Steel at Re=14000, Pr=0.1
Figure 6.18: Estimated Local Average Nusselt Number with Resistance for the Sensor2 Design with Tungsten Elements and Alumina Housing in flowing Liquid Steel at Re=14000, Pr=0.1
for liquid steel.

Sensor2 CWT: The following relationship is employed for the Sensor2 CWT prediction in liquid steel (Pr=0.1) and valid for the Reynolds number range of (1000 ≤ Re ≤ 30000):

$$Re = \left( \frac{Nu_{S2}}{0.6364} \right)^{\frac{1}{0.4910}}$$  \hspace{1cm} (6.10)

Sensor2 UHF: The following relationship is employed for the Sensor2 UHF prediction in liquid steel (Pr=0.1) and valid for the Reynolds number range of (1000 ≤ Re ≤ 30000):

$$Re = \left( \frac{Nu_{S2}}{0.8006} \right)^{\frac{1}{0.4806}}$$  \hspace{1cm} (6.11)

Table 6.13 lists the polar coordinate of the tungsten element with respect to the forward stagnation point of the flow-field along with the predicted local average Nusselt numbers for the Sensor2 UHF and Sensor2 CWT boundary conditions for liquid steel. The values of the
thermal resistance and the local average Nusselt number modified with thermal resistance according to Equation 6.3 are listed, lastly there is a column listing the fit of the local average Nusselt number with resistance to the predicted temperature response. Figure 6.20 plots the local average Nusselt number data modified with thermal resistance from the Sensor2 CWT and Sensor2 UHF predictions and the fit to the conjugate heat transfer predictions for liquid steel. Notice that the fit to the conjugate heat transfer predictions does not coincide as well as the Nusselt number estimates of the liquid aluminum data of Figure 6.14b. Perhaps the one-dimensional transient solution to the conduction equation will produce a better fit and hence better coincidence with the local average Nusselt numbers modified with thermal resistance.

Figure 6.21a plots the local average Nusselt number normalized by the square root of the Reynolds number on a cartesian plot while Figure 6.21b plots this data on a polar plot. Table 6.14 lists values of the normalized local average Nusselt number for the Sensor2 CWT and Sensor2 UHF steady numerical predictions for liquid steel (Pr=0.1). As the Prandtl number of the fluid has increased so has each constant associated with the temperature registering element
Table 6.13: Local Average Nusselt Number Values modified with Thermal Resistance for flowing liquid steel Re=14000, Pr=0.1

<table>
<thead>
<tr>
<th>Θ [°]</th>
<th>$\bar{N}u_{la\text{H}}$ UHF</th>
<th>$\bar{N}u_{la\text{H}}$ CWT</th>
<th>$R \ [m^2\cdot K\cdot W^{-1}]$</th>
<th>$\bar{N}u_{laR\text{UHF}}$</th>
<th>$\bar{N}u_{laR\text{CWT}}$</th>
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$\Theta$: polar coordinate from stagnation point, $\bar{N}u_{la\text{H}}$: the local average Nusselt number, UHF: uniform heat flux, CWT: constant wall temperature, $R$: thermal resistance, $\bar{N}u_{laR\text{UHF}}$: the local average Nusselt number modified with thermal resistance, fit: fit of Equations (1.1) and (1.2) to Fluent prediction to obtain local average Nusselt number modified with thermal resistance

Figure 6.20: Effect of a Thermal Resistance Value on the Local Average Nusselt Number of Sensor2 in flowing liquid steel Re=14000, Pr=0.1
Table 6.14: Normalized Local Average Nusselt Number Values derived from the Sensor2 Con-
stant Wall Temperature (Sensor2 CWT) and Uniform Heat Flux (Sensor2 UHF) Numerical
Predictions for flowing liquid steel, Pr=0.1

<table>
<thead>
<tr>
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<th>( \frac{\overline{Nu}}{Re^{0.5}} )</th>
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<th>UHF</th>
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<tr>
<td>315</td>
<td>0.87</td>
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</table>

of the Sensor2 design. Notice that the shape or functional form of the normalized local average
Nusselt number plot resembles that of the liquid aluminum predictions. As such, as long as
suitable materials can be found to withstand the chemical attack of a high temperature liquid
metal, this technique can be employed to most high temperature liquid metal flows.
Figure 6.21: Normalized Local Average Nusselt Number derived from the Sensor2 Constant Wall Temperature (Sensor2 CWT) and Uniform Heat Flux (Sensor2 UHF) Numerical Predictions for flowing liquid steel, Pr=0.1
Chapter 7

Closure

7.1 Conclusions

This thesis addresses the need for a high temperature molten metal velocity sensor by constructing and testing sensors in flowing liquid aluminum. Two sensors are constructed, Sensor1 is a design of uniform material and of a circular cylinder geometry where eight thermocouples are situated at 45° around the polar coordinate of the circle. Sensor2, is similar to Sensor1 except that it is comprised of two materials of very different thermal properties and provides fast estimates of the local Nusselt number which is used to infer velocity.

To determine the velocity of liquid aluminum using Sensor2:

1. Estimate the local average Nusselt number from fitting the temperature response to Equations (1.1) and (1.2).

2. Compute the local average Nusselt ($\bar{Nu}_{la}$) number corrected for thermal resistance imparted by the carbon coating via Equation (6.4).

3. Use these corrected values to compute the average Nusselt number for Sensor2 ($\bar{Nu}_{s2}$).

4. Correlations of the Reynolds number as a function of the Nusselt number of Sensor2 are listed as Equations (6.6) and (6.7) for liquid aluminum. Compute the Reynolds number
from the correlation and determine the magnitude of velocity using Equation (6.5).

5. The direction of velocity is located by comparing the computed normalized local average Nusselt number \((\overline{Nu}/Re^{0.5})\) values to the constant values for liquid aluminum. The polar plot of the normalized Nusselt numbers for the liquid aluminum system is displayed in Figure 6.15b, which identifies the direction of velocity.

Conclusions resulting from the experiments and numerical modeling of the Sensor1 and Sensor2 designs are outlined below:

1. To estimate the Nusselt number, the solidification of aluminum onto the sensor must be suppressed. The solidification and melting time (SMT) of aluminum was experimentally monitored for different bath superheat and various magnitudes of velocity for the case of the Sensor1 design constructed of copper and steel. Comparing numerical predictions with experimental SMT data, it was observed that the thermal resistance at the Sensor1-aluminum interface plays and important role on the contour of the aluminum shell which solidifies and subsequently melts onto the surface of Sensor1. The numerical predictions and experimental SMT are in good agreement for a thermal resistance at the Sensor1-aluminum interface in the order of \(R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\).

2. The Sensor2 design alleviates the need for a lengthy inverse heat transfer computation of the Nusselt number, as in the case of Sensor1. Discrete lumps of high thermal conductivity copper material are embedded into a low thermal conductivity calcium silicate material of the Sensor2 design; the lumps are treated by the lumped system analysis of Equations (1.1) and (1.2), and allow fast estimates of the localized Nusselt number.

3. The maximum value of the numerically predicted local average Nusselt number versus polar coordinate change between the Reynolds numbers of Re=100 and Re=1000 for the unique Sensor2 boundary conditions. At Reynolds numbers of Re\(\leq\)100 the maximum value of local average Nusselt number occurs at the stagnation point; however at
Re \geq 1000 the maximum value occurs at \pm 45^\circ from the stagnation point. This is a result of the higher fluid velocity in combination with the temperature of the fluid approaching the free stream temperature at the element located at \pm 45^\circ from the stagnation point.

4. The modified Nusselt number estimated from Sensor2 is a sum of the resistance from the thermal boundary layer and coating applied to the sensor-aluminum interface. Good agreement is obtained when a value of thermal resistance is added to the numerically predicted fluid heat transfer coefficient. Results from modeling the transient conjugate heat transfer from liquid aluminum to Sensor2 show that values of thermal resistance imparted by the carbon coating are different at each copper element. To determine the magnitude and direction of velocity using the Sensor2 design, the value of thermal resistance imparted by the carbon coating must be known to obtain a reliable Nusselt number estimate.

Although this thesis deals with liquid aluminum, the velocity technique has application to liquid steel. Numerical predictions of the local average Nusselt numbers for Sensor2 in liquid steel were generated. From these simulations, it was shown that the procedure to determine the velocity from Sensor2 can be applied to liquid steel by changing the materials used to fabricate Sensor2. Reynolds number correlations for liquid steel were numerically derived and listed as Equations (6.10) and (6.11), the constants of these relationships are different than those of liquid aluminum. The polar plot of the normalized Nusselt numbers for liquid steel is displayed in Figure 6.21b, these constants are larger for liquid steel as the Prandtl number of steel is greater than aluminum.

7.2 Future Work

The following are recommendations to improve and extend this work:

- A better understanding of the thermal resistance at the interface of the copper and liquid aluminum imparted by the carbon coating is needed. An experimental technique to
quantify the thermal resistance imparted by the carbon coating and an error estimate of this value of thermal resistance will reduce the error of the proposed equations to obtain the magnitude and direction of velocity. Experiments into different types of coating materials and the effect on the thermal resistance may determine alternate coatings to use on the sensor which do not change with immersion into liquid aluminum.

- Research into the error in the estimate of the heat transfer coefficient, from employing the lumped capacitance equations of the copper temperature element, will identify whether a one-dimensional analytical solution to the copper energy equation will provide better heat transfer coefficient estimates.

- An investigation into whether the comprehensive thermal resistance model of Hamasaiid et al. [78] is valid for the copper elements coated with carbon of the Sensor2 design will help in testing and validating the range of application of the model of Hamasaiid et al. [78] and advance the scientific knowledge of liquid metal thermal contact phenomena.

- Galante and Churchill [111] and Khan et al. [85] argue that the potential flow solution to the heat transfer over a circular cylinder overestimates the rate of heat transfer. The subcritical flow regime of the flow over a circular cylinder was the regime in which experiments were performed in this research. This subcritical flow regime results in three dimensional vortex shedding behind the cylinder, where the wake is turbulent. Predictions of the local Nusselt number of the Sensor2 design employing the unique boundary conditions of Sensor2 and using a large eddy simulation (LES) model may produce Nusselt number estimates which are in better agreement with the real flow. This in turn will improve the Nusselt number estimates to determine the magnitude and direction of velocity.
References


Appendix A

Determining the Direction of Velocity from the Steel Sensor1 Temperature Data

A quick method to infer the direction of velocity in flowing liquid aluminum, using a steel Sensor1 design which is immersed into a low superheat fluid, is to examine the temperature differences around the cylinder. This technique may be beneficial to smaller aluminum production facilities where a cheap and quick method to determine the direction of velocity is required to troubleshoot operating problems relating to the direction of aluminum flow.

The Sensor1 design houses eight thermocouples, four temperature differences can be computed by utilizing a set of diametric thermocouples which are situated around the probe. The heat transfer coefficient distribution around the surface of the cylinder will create a temperature difference around the probe. The hottest region around the circular cylinder will be the stagnation point of the flow-field, assuming that the flow-field is that of a uniform flow over a circular cylinder. It is assumed that the heat transfer coefficient distribution like that of Figure 1.2 holds when solidified aluminum is around the circular geometry.

Computing the diametric thermocouple temperature differences will identify the semi-circle which is hotter than the other semi-circle. In a relative sense, one semi-circle is hot while the other semi-circle is cold. Identifying the hot semi-circle will provide an indication
of the stagnation point of the flow-field. This direction finding technique is explained in the following sections by first discussing the interior thermocouple response of a steel Sensor1 immersed into flowing liquid aluminum.

A.1 Interior Thermocouple Response

The interior thermocouple readings at two positions within a steel Sensor1 immersed into flowing liquid aluminum at 0.08 m·s\(^{-1}\) (Re\(\approx\)6800) and a temperature of 963 K (690 °C) 30 K SPH is discussed in this section. Figure A.1a plots the interior and exterior thermocouple response at a location of 25° from the stagnation point. Figure A.1b plots the temperature response of a thermocouple located at 205° from the stagnation point. Examining the temperature response of the thermocouple at 205° from the stagnation point (Figure A.1b), one observes an inflection point occurring on the curve, labeled as point A in Figure A.1b. This inflection point is observed occurring at the point in time when the shell melts at the outer thermocouple otc(205) which is located at 205° from the stagnation point. This inflection in the thermocouple reading can be explained by the shell acting as a resistance to energy transfer over the surface of the probe. This resistance to energy transfer is active for as long as the shell exists on the probe, once the shell melts and is removed from the surface of the Sensor1 material, the resistance to energy flow from the blanket of solid aluminum is also removed. The inflection point occurring in thermocouple tc(205) represents the melting of the shell at the location of 205° from the stagnation point and an increase in the heat flux. This increase in the heat flux causes the temperature rise of thermocouple tc(205) to increase after the time at which the shell melts.

Figure A.2 plots the temperature versus time of the thermocouples located at both the inside and the outside of the probe at locations of 25° and 205° from the stagnation point. Qualitatively it can be seen that a difference occurs between the values of temperature between the thermocouples tc(25) and tc(205). This temperature difference is a result of the heat transfer coefficient distribution around the surface of the cylinder in addition to the duration of the so-
Figure A.1: Interior and Exterior Thermocouple Response at two locations within a Sensor1 Design Constructed of Steel in flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 963 K (690 °C) 30 K SPH

lidified aluminum which forms on the surface of the cylinder. In the section to follow it will be shown how this temperature difference information can be used to infer the direction of velocity of the bath.

### A.2 A Procedure to Determine the Direction of Velocity using the Diametric Temperature Difference

This section will outline a technique to determine the direction of velocity from using the internal thermocouple measurements. In order to portray the technique, the diametric thermocouple difference must be defined. Sensor1 houses eight internal thermocouples, referring to Figure 3.6, the thermocouples are labeled with respect to the stagnation point of the flow, however this labeling can be arbitrary, for example from 0 to 7. As there are eight thermocouples, there are four pairs of thermocouples which are located 180° from one another about the sensor geometry. Using the diametric thermocouple pairs, compute the difference in temperature between a pair. The four differences are defined as diff1 through diff4. Plot the curves corresponding
Figure A.2: Interior and Exterior Thermocouple Response at 25° and 205° from the Stagnation Point of the flow-field for a Sensor 1 Design Constructed of Steel in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 963 K (690 °C) 30 K SPH

to diff1 through diff4. Figure A.3 depicts the difference curves for the flow conditions of 0.08 m·s⁻¹ and 30 K SPH for a steel Sensor1. Examine Figure A.3a which is the curve for diff1. In this plot, the immersion circuit is shown along with a line demarcating the value of zero. The points on the immersion circuit labeled A though F were outlined previously in Section 5.1. If the diff1 curve lies above the zero line, it signifies that the thermocouple to the left of the subtraction is hotter than the thermocouple to the right of the subtraction symbol. Conversely, if the diff1 curve lies below the zero line, the thermocouple to the left of the subtraction symbol is colder than the thermocouple to the right of the subtraction. From Figure A.3a location (82) is hotter than location (262). Figure A.3b plots the diff2 curve and indicates that location (127) is colder than location (307). Notice in this curve, part of the plot is above the zero line for time less than 40 s. In this scenario, the thermocouple which is identified as hot will be taken after the period in time when the shell begins to melt at the surface of the probe. From the four external thermocouple measurements the shell melts at the surface after 47.3 s. The temperature difference curve diff3, represented in Figure A.3c tells us that location (172) is colder
than location (352). Lastly, location (217) is colder than location (37) which is illustrated by the diff4 curve of Figure A.3d. Figure A.3e is a diagram which denotes the locations of the thermocouples within the probe geometry. Figure A.3e labels the thermocouples which were previously identified as hot and cold. A line is drawn across the circle to identify the semicircle which is hot and the other which is cold. As the flow-field which interacts with the cylinder in the experiments is nearly symmetric, it can be concluded that the oncoming flow interacts with the probe between thermocouple locations (352) and (37) as indicated in Figure A.3e. The hottest point of the probe lies between tc(352) and tc(37), as such this region of the probe interacts with the flow-field vector.
Figure A.3: Diametric Thermocouple Difference Data for a Sensor1 Design Constructed of Steel in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re\(\approx\)6800) and 963 K (690 °C) 30 K SPH
Appendix B

Supplementary Numerical Model Shell
Solidification and Melting Time as a Function of Polar Coordinate Data

This appendix provides supplementary data pertaining to predicted shell solidification and melting time (SMT) values obtained from Fluent for a steel Sensor1 at low superheat (SPH) 30 K SPH. The numerically predicted SMT data is plotted against experimental SMT data and the results of thermal resistance at the Sensor1-aluminum interface are discussed with respect to the three observations outlined in section 5.5.

B.1 Steel Sensor1: Liquid Conditions 30 K SPH and 0.08 m·s$^{-1}$

Here the local shell SMT is discussed when various magnitudes of thermal resistance are applied to a steel Sensor1 design in flowing liquid aluminum of low superheat. Figure B.1 is a set of plots that shows the predictions for a steel cylinder in a bath with velocity of 0.08 m·s$^{-1}$ and a temperature of 963 K (30 K SPH). Observation 1 is viewed between Figures B.1a and B.1b with
thermal resistance values of $R = 1 \times 10^{-5} \text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$ and $R = 1 \times 10^{-4} \text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$. A computation is not solved for a very high value of thermal resistance ($R = 1 \times 10^{-3} \text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$), hence data is not presented for Observation 2. Observation 3 can be viewed in Figures B.1c, B.1d and B.1e which present predictions for a thermal resistance of $R = 2 \times 10^{-4} \text{m}^2 \cdot \text{K} \cdot \text{W}^{-1}$, with superheat values of 30 K SPH, 28 K SPH, and 25 K SPH.

### B.2 Steel Sensor1: Liquid Conditions 30 K SPH and 0.25 m·s$^{-1}$

The last set of numerical predictions examine the effect of the thermal resistance at the Sensor1-aluminum interface for the case of a steel Sensor1 design which is in liquid aluminum of low superheat and high velocity. Figure B.2 is a family of plots that displays the predictions for a steel cylinder in a bath with high velocity of 0.25 m·s$^{-1}$ and a temperature of 963 K (30 K SPH). Observation 1 is viewed in Figures B.2a and B.2b. A solution is not computed for Observation 2. Observation 3 is viewed in Figures B.2c, B.2d, and B.2e.
Figure B.1: Experimental Shell Solidification and Melting Time (SMT) Distribution and Numerical Predictions for a Sensor1 Design Constructed of Steel in flowing Liquid Aluminum at 0.08 m·s\(^{-1}\) (Re\(\approx\)6800) and 963 K (690 °C) 30 K SPH

(a) 30 K SPH, \(R = 1 \times 10^{-5} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)

(b) 30 K SPH, \(R = 1 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)

(c) 30 K SPH, \(R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)

(d) 28 K SPH, \(R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)

(e) 25 K SPH, \(R = 2 \times 10^{-4} \text{ m}^2\cdot\text{K}\cdot\text{W}^{-1}\)
(a) 30 K SPH, $R = 1 \times 10^{-5}$ m$^2$.K.W$^{-1}$

(b) 30 K SPH, $R = 1 \times 10^{-4}$ m$^2$.K.W$^{-1}$

(c) 30 K SPH, $R = 2 \times 10^{-4}$ m$^2$.K.W$^{-1}$

(d) 28 K SPH, $R = 2 \times 10^{-4}$ m$^2$.K.W$^{-1}$

(e) 25 K SPH, $R = 2 \times 10^{-4}$ m$^2$.K.W$^{-1}$

Figure B.2: Experimental Shell Solidification and Melting Time (SMT) Distribution and Numerical Predictions for a Sensor1 Design Constructed of Steel in flowing Liquid Aluminum at 0.25 m·s$^{-1}$ (Re≈21000) and 963 K (690 °C) 30 K SPH
Appendix C

An Inverse Method to Determine Velocity Using Sensor1

Knowing the local heat transfer coefficient or the local Nusselt number, the magnitude and direction of velocity can be determined. The magnitude of velocity can be extracted by averaging the local heat transfer coefficient, then extracting the magnitude of velocity from an average heat transfer coefficient (Nusselt number) versus velocity curve. The direction of velocity can be determined from the reconstruction of the temperature field within Sensor1 or by examining the local Nusselt number function. The stagnation point of the flow-field will correspond to the location of greatest heat transfer, that is the hottest region of the Sensor1 geometry. This hot region will correspond to the direction vector assuming that the functional form of the local Nusselt function resembles Figure 1.2. A means to estimate the local Nusselt number of Sensor1 is through an inverse heat transfer algorithm.

Inverse heat transfer analysis can be thought of, according to Orlande [122], as a practical tool employed in thermal engineering. A review of the subject matter can be found by Orlande [122] which lists the various regularization techniques that have matured to treat the ill-posed nature of the inverse heat transfer problem. The problem in this work is an inverse boundary problem, where an estimate of the heat transfer coefficient that acts at the surface of Sensor1
is obtained. This appendix begins with an explanation of the inverse heat transfer algorithm. Section C.2 presents results from applying the inverse algorithm to a copper Sensor 1 in flowing liquid aluminum.

C.1 Formulation of the Inverse Heat Transfer Coefficient Problem

The solution methodology of Beck’s Sequential Function Specification (SFS) algorithm [34] will be presented briefly in the subsequent sections, as this algorithm is employed to estimate the heat transfer coefficient acting on the Sensor 1 geometry for a period in time that Sensor 1 is immersed into flowing liquid aluminum. More detailed information of this algorithm can be found in the work of Beck et al. [34], Woodbury [123], and Osman [116]. The notation presented by Beck et al. [34] and Osman [116] are utilized to describe the SFS algorithm in this thesis.

The estimation of the heat transfer coefficient on the surface of the sensor using the SFS method requires the solution of the following sub-problems:

1. Solving the forward (direct) heat conduction problem

2. Calculating the sensitivity coefficients

3. Solving the system of algebraic equations for the unknown heat transfer coefficient distribution

In the following sections, each sub-problem will be elaborated upon.

C.1.1 Solution of the Forward Heat Conduction Problem

The temperature distribution inside Sensor 1 is represented mathematically by the heat conduction equation, where the material is assumed to be homogeneous and isotropic, and the
thermophysical properties of the material to be temperature independent. Equation (C.1) expresses in cylindrical coordinates the heat conduction equation:

$$\frac{\partial T (r, \Theta, t)}{\partial t} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T (r, \Theta, t)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T (r, \Theta, t)}{\partial \Theta^2} \right]$$

\((R_{in} < r < R_{out})\)

\((0 \leq \Theta \leq 2\pi)\)

\((0 < t \leq t_f)\)  \hspace{1cm} (C.1)

\(T\) represents the temperature, \(\alpha\) the thermal diffusivity, \(t\) the time, \(r\) the radial scale, and \(\Theta\) the angular scale. Equation (C.1) is subject to the conditions of Equations (C.2), (C.3), and (C.4):

**Boundary Condition - Sensor1 Inner Surface:** At the inner bored surface of the Sensor1 geometry an insulative boundary condition is imposed.

$$\left. \frac{\partial T (r, \Theta, t)}{\partial r} \right|_{r=R_{in}} = 0, \quad (0 \leq \Theta \leq 2\pi)$$

\(\hspace{1cm} (C.2)\)

**Boundary Condition - Sensor1 Outer Surface:** At the outer surface of the Sensor1 geometry is the convective heat transfer boundary condition.

$$-k \left. \frac{\partial T (r, \Theta, t)}{\partial r} \right|_{r=R_{out}} = h(\Theta, t) \left[ T (R_{out}, \Theta, t) - T_\infty (t) \right]$$

\((0 \leq \Theta \leq 2\pi)\)  \hspace{1cm} (C.3)

**Initial Condition:** A constant initial temperature for Sensor1 is assumed.

$$T (r, \Theta, 0) = C \quad \hspace{1cm} (C.4)$$
The thermal conductivity of the material is denoted by \( k \) and the heat transfer coefficient by \( h(\Theta, t) \), the initial temperature distribution of the cylinder is represented by \( C \) and is constant. The heat transfer coefficient is dependent on position and time, and is unknown. The finite control volume method outlined by Patankar [54] is used to solve Equation (C.1) with conditions (C.2), (C.3), and (C.4). Appendix D discusses the discretization schemes and lists the nodal equations employed to solve the forward heat conduction problem. Appendix D.1 explains the various procedures that are executed to test the implementation of the forward heat conduction algorithm.

In order to estimate the heat transfer coefficient distribution at the surface of Sensor1, temperature readings from within Sensor1 are provided to the SFS algorithm. Embedded 1 mm (0.039 inch) from the surface of Sensor1 are eight type K thermocouples, where the transient temperature histories are known. All thermocouples are at the same depth from the surface but at different angular positions. The angular positioning of the thermocouples within the cylinder is constant and represented by:

\[
\frac{2\pi}{L}
\]

\( L \) is the number of thermocouples; in this work the angular spacing of the thermocouples is 45\(^\circ\). The temperature at the thermocouple location can be represented by the following notation (C.5):

\[
T(r_{sen}, \Theta_l, t) = T_{l,m} \quad (l = 1, 2, \ldots, L) \quad (0 < t \leq t_f)
\]  

\( r_{sen} \) is the radial location and \( \Theta_l \) is the angular location of the thermocouple. The time interval \((0 < t \leq t_f)\) is discretely represented by time intervals \( \delta t \), this is the increment of time for which the temperature data is stored by the data acquisition system, \( m \) represents the time index, \((m = 1, 2, \ldots, M)\). The measured temperature data which is input into the inverse heat transfer coefficient code is contaminated with noise. As is customary in the inverse literature...
this temperature measurement is represented as the following (C.6):
\[ \vec{Y} = \vec{T} + \vec{\epsilon} \] (C.6)

\( \vec{Y} \) is the temperature measurement vector at the thermocouple locations, \( \vec{T} \) in the ideal sense represents the temperature vector without noise, and \( \vec{\epsilon} \) is the noise vector which adds some error to the measured reading. In the laboratory setting, one contribution to the noise vector is a result of the interaction of the thermocouple connection lines with the ambient electromagnetic radiation from the lab. This interaction causes the thermocouple temperature reading to fluctuate about a mean value when the thermocouple is measuring a body in thermal equilibrium. It is assumed that the approximating model for the heat transfer coefficient \( h(\Theta, t) \) over the region \( (0 \leq \Theta \leq 2\pi) \) involves parameters denoted by \( \beta \) [34]. The number of parameters is equal to the number of temperature measurements from within the sensor \( (L = 8 \text{ eight thermocouples}) \).

C.1.2 The Sequential Function Specification Algorithm

The solution technique employed to solve the inverse heat transfer coefficient problem is that of Beck’s sequential function specification (SFS) algorithm [34]. It stabilizes the ill-posed nature of the problem by utilizing future time information to generate an estimate of the heat transfer coefficient. The solution of the estimate of the heat transfer coefficient is determined by the least squares minimization method.

For Beck’s SFS method it is necessary to assume the form of the space and time variation of the heat transfer coefficient \( h(\Theta, t) \) [34], over the intervals \( (0 \leq \Theta \leq 2\pi) \) and \( (t_m \leq t \leq t_{m+r-1}) \) where \( r \) is the number of future time steps used at time index \( m \). The assumption adopted in this work regarding the time variation of the heat transfer coefficient, is that at future times the heat transfer coefficient is constant, which is represented by Equation (C.7).
\[ h_m(\Theta) = h_{m+1}(\Theta) = h_{m+2}(\Theta) = \ldots = h_{m+r-1}(\Theta) \] (C.7)
For the spatial approximation of the heat transfer coefficient it is assumed that the variation is linear between the eight estimates of $\beta$. The SFS method for solving the two dimensional inverse heat transfer coefficient problem is formulated as the problem of estimating sequentially over time the optimal values of the set of parameters $\beta$ which minimizes the residual of the least squares objective function [116]. Using the notation presented in Osman [116] the following relation (C.8) represents the sum of squares of the residual of temperatures at the eight thermocouple locations:

$$ s^r_m(\beta_m) = \sum_{l=1}^{L} \sum_{i=1}^{r} [Y_{l,m+i-1} - T_{l,m+i-1}(\beta_m)]^2 $$

(C.8)

$r$ is the number of future time steps, $Y_{l,m+i-1}$ represents the temperatures which are measured by the thermocouples and $T_{l,m+i-1}$ represents the calculated temperatures from the forward heat conduction algorithm for a given estimate of $\beta_m$. To minimize the above relation the gradient of the sum is taken with respect to the unknown vector $\beta_m$ and the gradient is set equal to zero for the given estimate of $\beta_m$. Using the notation of Beck [34] the gradient is the following (C.9):

$$ \tilde{Z}^T(\hat{\beta}_m) \left[ \tilde{Y} - T(\hat{\beta}_m) \right] = 0 $$

(C.9)

The abbreviation is introduced (C.10):

$$ \tilde{Z}^T(\hat{\beta}_m) = \nabla_{\beta_m} T^T(\hat{\beta}_m) $$

(C.10)

The method used to solve for the new estimate of $\beta_m$ as suggested by Beck [34] is that of the Gauss Newton linearization procedure. The method is derived by replacing $T(\beta_m)$ by the approximate first order expansion about $T$ for an estimate of $\beta_m$. The Gauss Newton linearization procedure leads to the following form (C.11):

$$ \left[ \tilde{Z}^T(\hat{\beta}_m) \tilde{Z}(\hat{\beta}_m) \right] \left( \hat{\beta}_{m+1} - \hat{\beta}_m \right) = \tilde{Z}^T(\hat{\beta}_m) \left[ \tilde{Y} - \tilde{T}(\hat{\beta}_m) \right] $$

(C.11)
The superscript $n$ represents the iteration index

$$
\hat{\beta}_{m+1}^n = \hat{\beta}_m^n + \left[ (\tilde{Z}^n)^T \tilde{Z}^n \right]^{-1} (\tilde{Z}^n)^T [\tilde{Y} - \tilde{T}^n] \tag{C.12}
$$

where

$$
\tilde{Z}^n = \tilde{Z} (\hat{\beta}_m^n) \tag{C.13}
$$

$$
\tilde{T}^n = \tilde{T} (\hat{\beta}_m^n) \tag{C.14}
$$

The matrix $Z^n$ is an $[r(L \times L)]$ sensitivity matrix and is represented by:

$$
\tilde{Z}^{[r(L \times L)]} = \begin{bmatrix}
\tilde{Z}_m & \tilde{Z}_{m+1} & \tilde{Z}_{m+i-1} & \tilde{Z}_{m+r-1}
\end{bmatrix}^T \tag{C.15}
$$

where

$$
\tilde{Z}_{m+i-1}^{L \times L} = \begin{bmatrix}
\frac{\partial T_{1,m+i-1}}{\partial \beta_{1,m}} & \cdots & \frac{\partial T_{1,m+i-1}}{\partial \beta_{L,m}} \\
\vdots & & \vdots \\
\frac{\partial T_{L-1,m+i-1}}{\partial \beta_{1,m}} & \cdots & \frac{\partial T_{L-1,m+i-1}}{\partial \beta_{L,m}}
\end{bmatrix} \tag{C.16}
$$

The elements of $\tilde{Z}_{m+i-1}^{L \times L}$ are referred to as the step function sensitivity coefficients and are denoted by:

$$
Z_{L_1,L_2} (m+i-1) = \frac{\partial T_{L_1,m+i-1}}{\partial \beta_{L_2,m}} \tag{C.17}
$$

(i = 1, 2, \ldots, r)

### C.1.3 Solution of the Sensitivity Coefficients

To solve for the sensitivity coefficients the finite difference method is used to approximate the derivative. The step function sensitivity coefficients are approximated using the forward
difference formula:

\[
Z_{L_1,L_2}(m + i - 1) = \frac{\partial T_{L_1,m+i-1}}{\partial \beta_{L_2,m}} = \frac{T_{L_1,m+i-1}(\beta_1, \ldots, \beta_{L_2}, \ldots, \beta_L) - T_{L_1,m+i-1}(\beta_1, \ldots, \beta_{L_2}, \ldots, \beta_L)}{\delta \beta_{L_2}}
\]

(C.18)

where \( \delta \beta \) is the finite difference interval, in this work the interval chosen is \( \delta \beta = 10^{-2} \).

**Solution of the Equations for the Estimates of \( \beta \)**

Equation (C.11) can be written in the following form:

\[
\tilde{C}_m^n \times \tilde{b}_m^n = \tilde{d}_m^n
\]

(C.19)

where \( \tilde{C}_m^n \) is a \([L \times L]\) matrix defined by:

\[
\tilde{C}_m^n = \sum_{i=1}^{r} \left( \tilde{Z}_{m+i-1}^n \right)^T \tilde{Z}_{m+i-1}^n
\]

(C.20)

\( \tilde{b}_m^n \) and \( \tilde{d}_m^n \) are \([L \times 1]\) vectors defined by:

\[
\tilde{b}_m^n = \tilde{\beta}_{m+1}^n - \tilde{\beta}_m^n
\]

(C.21)

\[
\tilde{d}_m^n = \sum_{i=1}^{r} \left( \tilde{Z}_{m+i-1}^n \right)^T \left[ \tilde{Y}_{m+i-1} - \tilde{T}_{m+i-1} \right]
\]

(C.22)
The matrix $C^n_m$ and vector $d^n_m$ are given in component notation as:

$$C^n_m = \begin{bmatrix}
\sum_{l=1}^L \sum_{i=1}^r \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{1,m}} \right)^2 & \cdots & \sum_{l=1}^L \sum_{i=1}^r \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{L,m}} \right)^2 \\
\sum_{l=1}^L \sum_{i=1}^r \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{1,m}} \right) \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{1,m}} \right) & \cdots & \sum_{l=1}^L \sum_{i=1}^r \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{L,m}} \right) \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{L,m}} \right)
\end{bmatrix}$$

(C.23)

$$d^n_m = \begin{bmatrix}
\sum_{l=1}^L \sum_{i=1}^r \left[ \gamma_{l,m+i-1} - T^n_{l,m+i-1} \right] \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{1,m}} \right) \\
\cdots \\
\sum_{l=1}^L \sum_{i=1}^r \left[ \gamma_{l,m+i-1} - T^n_{l,m+i-1} \right] \left( \frac{\partial T^n_{l,m+i-1}}{\partial \beta_{L,m}} \right)
\end{bmatrix}$$

(C.24)

The system of equations is solved in MATLAB [124] using the biconjugate gradients stabilized method (BICGSTAB) [124] function to ascertain the update to the estimate (the vector $b^n$). At the time index $m$, the iterative solution begins by selecting an initial value of the estimate $\beta^n$ and then calculating the corresponding temperature vector $T$ at the location of the thermocouples within Sensor1. The sensitivity coefficients are then calculated. The system of equations is then solved for $b^n$ which is used to update the estimate of $\beta^n_m$. The iterations are terminated when the following criterion is met:

$$\left\lVert \frac{\tilde{b}^n}{\tilde{\beta}^n_m} \right\rVert = \left[ \left( \frac{b^n_1}{\beta^n_{1,m}} \right)^2 + \left( \frac{b^n_2}{\beta^n_{2,m}} \right)^2 + \cdots + \left( \frac{b^n_L}{\beta^n_{L,m}} \right)^2 \right]^{\frac{1}{2}}$$

(C.25)

Once the tolerance is less than $10^{-5}$ the iterations end and the time index is advanced. A new estimate is computed by the following:

$$\hat{\beta}^{n+1}_m = \hat{\beta}^n_m + \tilde{b}^n_m$$

(C.26)

Appendix E discusses three heat transfer coefficient functions that are utilized to test the SFS algorithm implementation. These heat transfer coefficient functions include: a step function.
heat transfer coefficient function (appendix E.1), a triangular pulse heat transfer coefficient function (appendix E.2), and a cosine distribution heat transfer coefficient function (appendix E.3).

C.2 Inverse Heat Transfer Results

In this section the results of the inverse heat transfer algorithm are discussed when a copper Sensor1 is immersed into flowing liquid aluminum at 0.08 m·s$^{-1}$ (Re$\approx$6800) and a bath temperature of 1053 K (780 °C) or 120 K SPH. The copper Sensor1 is preheated to approximately 585 K (312 °C) prior to immersion into the liquid aluminum, and the sensor is also coated with carbon. From the discussion in chapter 5, under this initial condition and convective bath condition, shell does not form on the Sensor1 design.

C.2.1 The Thermocouple Lag

According to Kim [76] the abrupt change in temperature occurring at the initial moment of thermal contact between a liquid metal and a substrate cannot be fully reflected due to the finite response time of a typical welded bead junction thermocouple. This lag in the thermocouple response can negatively affect the estimates of the heat transfer coefficients, as the lag will result in a lower estimate of the heat transfer. If the thermocouple lag is large, a correction to the temperature reading should be applied to accurately reflect the temperature at the measuring junction of the thermocouple.

Kim [76] performed experiments whereby magnesium and aluminum alloy strips were cast against copper substrates; the copper substrate was instrumented with type K thermocouples of an intrinsic junction type. Osman [116] performed experiments whereby hot copper spheres were immersed into water. An intrinsic thermocouple junction was created between the thermocouple leads and the copper sphere, the leads were peened onto the copper to create the measuring junction. As Kim and Osman state, this measuring junction does not require a cor-
rection because of the rapid response.

The type K thermocouples that were embedded into the copper Sensor1 were exposed where the leads of the thermocouple form an intrinsic measuring junction between the two halves of the Sensor1 cylinder. These leads are pinched to hold the measuring junction in place. A correction is not applied to the measuring junction of the thermocouples of the Sensor1 design as the means by which these thermocouples are installed into Sensor1 does not necessitate a correction.

C.2.2 Determining the Direction of Velocity using Sensor1 Constructed of Copper coated with Carbon

In this section a means to determine the direction of velocity is presented by examining the temperature contours of Sensor1 generated by the inverse algorithm. Figure C.1 schematically shows the interaction of Sensor1 with the liquid aluminum vector field. The angular coordinate is labeled around the probe with zero representing the stagnation point of the flow-field with Sensor1. This labeling is used throughout the family of contour plots of Figure C.2.

Figure C.2 is a set of plots that shows the reconstructed temperature field within the probe for various times after it was immersed into the flowing liquid aluminum. Two types of figures are displayed in Figure C.2. The plots in the left column are those of temperature contours occurring within Sensor1. The temperature contours are obtained from the SFS algorithm estimate of the heat transfer coefficients. The right column of plots of Figure C.2 demark the point in time after immersion on a temperature versus time graph where the contour plot is generated.

The temperature contours of Figure C.2 are discussed by examining the region around the Sensor1 geometry which is hottest as this region will signify the stagnation point around the sensor and subsequently indicate the direction vector of liquid aluminum. Figure C.2a plots the temperature field inside the probe 0.1 s after it is immersed into the liquid aluminum flow-field. Examining Figure C.2b, this is the point in time when the temperature signals begin to
ascend towards the bath temperature. The probe temperature is relatively uniform after 0.1 s, the boundaries of Sensor1 are beginning to heat. At 2.4 s after immersion (Figure C.2d) the probe continues to heat up. There are two hot spots on Sensor1 which can be seen in Figure C.2c, one between the angular coordinate of \(330^\circ \leq \Theta \leq 30^\circ\) and the other from \(150^\circ \leq \Theta \leq 270^\circ\) around the sensor. At 3.4 s after immersion into the flow-field, a region which can be classified as hottest begins to emerge. This region is between \(330^\circ \leq \Theta \leq 30^\circ\) in Figure C.2e. At 4.4 s after immersion (Figure C.2g) the hottest region is more clearly seen between \(330^\circ \leq \Theta \leq 30^\circ\) around the surface of the sensor. At 5.4 s after immersion (Figure C.2i) the dark contour which signifies the hottest region within the sensor can be seen bounded between the angular coordinate of \(330^\circ \leq \Theta \leq 30^\circ\). At subsequent times, at 6.4 s (Figure C.2k), 7.4 s (Figure C.2m), 8.4 s (Figure C.2o), 9.4 s (Figure C.2q), and 9.9 s (Figure C.2s) the hottest region on the probe is bounded between \(330^\circ \leq \Theta \leq 30^\circ\). Using the premise that the hottest region on the sensor will correspond to the stagnation point of the flow-field, the flow vector can be inferred. This is diagrammatically portrayed in Figure C.3 and correlated to the experimental positioning of the probe in the flow-field as shown in Figure C.1.
Figure C.2: Temperature Field Reconstruction and Thermocouple Readings using Sequential Function Specification Algorithm for a Copper Sensor Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 1053 K (780 °C) 120 K SPH
Figure C.2: Continued Temperature Field Reconstruction and Thermocouple Readings using Sequential Function Specification Algorithm for a Copper Sensor Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 1053 K (780 °C) 120 K SPH
Figure C.2: Continued Temperature Field Reconstruction and Thermocouple Readings using Sequential Function Specification Algorithm for a Copper Sensor Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 1053 K (780 °C) 120 K SPH
Figure C.2: Continued Temperature Field Reconstruction and Thermocouple Readings using Sequential Function Specification Algorithm for a Copper Sensor1 Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s$^{-1}$ (Re≈6800) and 1053 K (780 °C) 120 K SPH

Figure C.3: Flow Vector Interaction with the Hot Temperature Region of Sensor1
C.2.3 The Nusselt number Estimate using Sensor1 Constructed of Copper coated with Carbon

The Sequential Function Specification algorithm estimates the values of the heat transfer coefficient on the surface of Sensor1 at the angular coordinate of the eight thermocouples. Linear interpolation between the estimated coefficients is applied to reconstruct the temperature field within Sensor1. This section will discuss the estimate of the local Nusselt number in relation to determining the direction of velocity. The local Nusselt number is computed from the inverse heat transfer coefficient estimate according to Equation (4.26).

Figure C.4 is a series of plots which display the Nusselt number versus time and the temperature versus time at a specific location around Sensor1. The location around the probe is denoted by the angle at which the thermocouple forms with respect to the stagnation point of the flow-field. The Nusselt number acts on the surface of the probe, while the temperature plot portrayed is from the thermocouple located at 1 mm (0.039 inch) from the surface of Sensor1. The transient Nusselt number data displayed in Figure C.4 was used to obtain the time average local Nusselt number (from \(0 \leq t \leq 10 \text{ s}\)).

The time average local Nusselt number values are plotted in Figure C.5, the inverse estimate of the local Nusselt number function can be used to determine the direction of velocity. Two experiments are displayed in Figure C.5, the estimated local Nusselt number at 340° is the greatest when compared with the remaining estimated local Nusselt number values, this maximum in local Nusselt number corresponds to the heat transfer around the stagnation point, where the heat transfer is greatest.

The data from the SFS inverse estimate of the local Nusselt number of Sensor1 presented in Figure C.5 is more irregular when compared with the local Nusselt number data of Ishiguro et al. [32] (Figure 1.2). Examining Figure C.5 the local Nusselt number data decreases from \((25^\circ \leq \Theta \leq 70^\circ)\) followed by an increase in the local Nusselt number from \((70^\circ < \Theta \leq 160^\circ)\). This decreasing and increasing pattern is observed between \((160^\circ < \Theta \leq 340^\circ)\). This irregular pattern introduces complexity in the direction finding technique. Ideally, two adjacent heat
Figure C.4: Local Nusselt Number versus Time from Inverse Estimate and Experimental Temperature versus Time Data for the Eight Thermocouple Locations of a Copper Sensor Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 1053 K (780 °C) 120 K SPH (Experiment 1)
Figure C.4: Continued Local Nusselt Number versus Time from Inverse Estimate and Experimental Temperature versus Time Data for the Eight Thermocouple Locations of a Copper Sensor Coated with Carbon in Flowing Liquid Aluminum at 0.08 m·s⁻¹ (Re≈6800) and 1053 K (780 °C) 120 K SPH (Experiment 1)
C.2.4 A Comparison of the Local Nusselt Number Curves of Sensor1 and Sensor2

In this section the local Nusselt number of Sensor1 estimated by the SFS inverse algorithm is compared with the local average Nusselt number of Sensor2 estimated by the lumped system analysis of Equations (1.1) and (1.2). Using the premise that the stagnation point of the flow-field corresponds to the region of greatest heat transfer, the purpose here is to demonstrate that it is comparatively easier to distinguish the region of greatest heat transfer using the experimental Nusselt number results of Sensor2 than the experimental Nusselt number results of Sensor1.
Figure C.5 plots the time average local Nusselt number from the inverse estimate as a function of polar coordinate for a copper Sensor1 coated with carbon in flowing liquid aluminum at a Reynolds number of Re≈6800, where the results of two experiments are displayed. Figure 6.4 plots the local average Nusselt number of Sensor2 as a function of polar coordinate where the Sensor2 copper elements are coated with carbon in flowing liquid aluminum at a Reynolds number of Re≈14000, depicting the results of two experiments. Notice that the local average Nusselt number data for Sensor2 displayed in Figure 6.4 follows a decreasing trend from (0° ≤ Θ ≤ 180°) followed by an increasing trend from (180° < Θ ≤ 315°). This decreasing trend in the Nusselt number to 180° from the stagnation point followed by the increasing trend after 180° resembles the Nusselt number function captured by Ishiguro et al. [32] in Figure 1.2.

The data from the SFS inverse estimate of the local Nusselt number of Sensor1 presented in Figure C.5 is more irregular when compared with the local average Nusselt number data of Sensor2 (Figure 6.4). Therefore it would seem that the estimated local average Nusselt number from Sensor2 is more amenable to determine the direction of velocity than the estimated local Nusselt number from Sensor1. This result, in addition to the short time required to estimate the local average Nusselt number of Sensor2, makes Sensor2 a better choice of sensor for the industrial setting.
Appendix D

The Finite Control Volume Algorithm

The finite control volume (FCV) algorithm was used to solve the forward heat conduction problem. The FCV technique discussed in Patankar [54] was utilized to spatially discretize Equation (C.1) to the computational domain outlined by the geometry of Figure D.1. Figure D.1 is a schematic of the computational domain with boundary conditions. In the discretization of the partial differential equation uniform mesh spacing was utilized. The spacing of the nodes in the domain is denoted as:

\[ \Delta r = \frac{R_{\text{out}} - R_{\text{in}}}{I - 1} \]  \hspace{1cm} (D.1)

\[ \Delta \Theta = \frac{2\pi}{J} \]  \hspace{1cm} (D.2)

where \( I \) and \( J \) represent the number of nodes located in the \( r \) direction and \( \Theta \) direction respectively. A schematic of the nodal network is shown in Figure D.2. The FCV discretization is based on the integral form of the law of conservation of energy and is represented as follows:

\[ \int \int_{CS} - (\vec{q} \cdot \vec{n}) \, dA = \int \int_{CV} \rho c \frac{\partial T}{\partial t} \, dV \]  \hspace{1cm} (D.3)

where \( \vec{q} \) is the heat flux which crosses the control surface, \( \vec{n} \) is the unit normal vector to the control surface, \( dA \) is the elemental area of the control surface, \( dV \) is the elemental volume, \( T \) the temperature of the node, \( t \) the point in time where the computation is solved, \( \rho \) represents the density and \( c \) the heat capacity of the material. The control surfaces are located midway
Figure D.1: Schematic of the Computational Domain with Boundary Conditions employed to solve the Transient Heat Conduction Equation for Sensor1

\[-k \frac{\partial T(r, \theta, t)}{\partial r} \bigg|_{r=R_{out}} = h(\theta, t) \left[ T(R_{out}, \theta, t) - T_\infty(t) \right] \]

\[\frac{\partial T(r, \theta, t)}{\partial r} \bigg|_{r=R_{in}} = 0\]

Figure D.2: Schematic of Nodal Network employed in the Forward Finite Control Volume Algorithm to solve the Transient Heat Conduction Equation for Sensor1

Figure D.2: Schematic of Nodal Network employed in the Forward Finite Control Volume Algorithm to solve the Transient Heat Conduction Equation for Sensor1
between control points. The control volume is a segmental slice in the cylindrical coordinate system as shown in Figure D.2.

The energy per unit time crossing the boundaries for an interior node \((i, j)\) can be represented as:

\[
\iint_{CS} - (q \cdot \vec{n}) \, dA = - [q_{rin} A_{rin} - q_{rout} A_{rout} + q_{\Theta in} A_{\Theta in} - q_{\Theta out} A_{\Theta out}] \tag{D.4}
\]

where

\[
q_r = -k \frac{\partial T}{\partial r} \tag{D.5}
\]

\[
q_{\Theta} = -k \frac{r \partial T}{\partial \Theta} \tag{D.6}
\]

\[
A_{rin} = r_{i+\frac{1}{2},j} \Delta \Theta \tag{D.7}
\]

\[
A_{rout} = r_{i-\frac{1}{2},j} \Delta \Theta \tag{D.8}
\]

\[
A_{\Theta in} = (r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j}) \tag{D.9}
\]

\[
A_{\Theta out} = (r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j}) \tag{D.10}
\]

using the relationships given, the heat flux balance can be represented in terms of temperature as:

\[
\iint_{CS} - (q \cdot \vec{n}) \, dA = k \frac{T_{i+1,j} - T_{i,j}}{\Delta r} r_{i+\frac{1}{2},j} \Delta \Theta - k \frac{T_{i,j} - T_{i-1,j}}{\Delta r} r_{i-\frac{1}{2},j} \Delta \Theta
\]

\[
+ \frac{k}{r_{i,j}} \frac{(T_{i,j+1} - T_{i,j})}{\Delta \Theta} \left( r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j} \right) - \frac{k}{r_{i,j}} \frac{(T_{i,j} - T_{i,j-1})}{\Delta \Theta} \left( r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j} \right) \tag{D.11}
\]

The accumulation of energy term can be approximated by the following:

\[
\iiint_{CV} \rho c \frac{\partial T}{\partial t} \, dV = \rho c \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n}}{\Delta t} \frac{1}{2} \left( r_{i+\frac{1}{2},j}^2 - r_{i-\frac{1}{2},j}^2 \right) \Delta \Theta \tag{D.12}
\]

the elemental volume is expressed as:

\[
V = \frac{1}{2} \left( r_{i+\frac{1}{2},j}^2 - r_{i-\frac{1}{2},j}^2 \right) \Delta \Theta \tag{D.13}
\]

The time derivative is evaluated by using the Alternating Direction Implicit (ADI) algorithm of Peaceman and Rachford \cite{125}. The time step for the ADI scheme is one half of the discrete
time period for which data is recorded by the data acquisition system. In this case:

$$\Delta \tau = \frac{\Delta t}{2}$$  \hspace{1cm} (D.14)

where \(\Delta t\) is 0.1 s for the inverse estimates presented in this work. The following coefficients can be used to represent the constants in the net energy balance equations of (D.11) and (D.12):

- \(a_P(i,j) = \rho c \frac{1}{\Delta r} \Delta \tau \left( r_{i+\frac{1}{2},j}^2 + r_{i-\frac{1}{2},j}^2 \right) \Delta \Theta \)  \hspace{1cm} (D.15)

- \(a_N(i,j) = k \frac{1}{\Delta r} r_{i+\frac{1}{2},j} \Delta \Theta \)  \hspace{1cm} (D.16)

- \(a_S(i,j) = k \frac{1}{\Delta r} r_{i-\frac{1}{2},j} \Delta \Theta \)  \hspace{1cm} (D.17)

- \(a_E(i,j) = k \frac{1}{\Delta \Theta} \left( r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j} \right) \)  \hspace{1cm} (D.18)

- \(a_W(i,j) = k \frac{1}{\Delta \Theta} \left( r_{i+\frac{1}{2},j} - r_{i-\frac{1}{2},j} \right) \)  \hspace{1cm} (D.19)

The two difference equations, for each half of the ADI time step scheme are as follows for node \((i,j)\):

**node \((i,j)\) r direction update:**

\[-a_S(i,j)T^{n+\frac{1}{2}}_{i-1,j} + [a_S(i,j) + a_P(i,j) + a_N(i,j)] T^{n+\frac{1}{2}}_{i-\frac{1}{2},j} - a_N(i,j)T^{n+\frac{1}{2}}_{i+1,j} = a_W(i,j)T^n_{i,j-1} + [-a_W(i,j) + a_P(i,j) - a_E(i,j)] T^n_{i-\frac{1}{2},j} + a_E(i,j)T^n_{i,j+1}\]  \hspace{1cm} (D.20)

**node \((i,j)\) \(\Theta\) direction update:**

\[-a_W(i,j)T^{n+\frac{1}{2}}_{i,j-1} + [a_W(i,j) + a_P(i,j) + a_E(i,j)] T^{n+\frac{1}{2}}_{i+\frac{1}{2},j} - a_E(i,j)T^{n+\frac{1}{2}}_{i,j+1} = a_S(i,j)T^{n+\frac{1}{2}}_{i-1,j} + [-a_S(i,j) + a_P(i,j) + a_N(i,j)] T^{n+\frac{1}{2}}_{i-\frac{1}{2},j} + a_N(i,j)T^{n+\frac{1}{2}}_{i+1,j}\]  \hspace{1cm} (D.21)

The solution of the ADI system of equations is computed by sending the matrices associated with the system to the BICGSTAB [124] function in MATLAB [124].

For the nodes which correspond to the convective boundary condition the following modifications are made to the flux terms of the control surface: node\((I,1)\)

\[
\int_{CS} -(q \cdot n) dA = h(1) (T_{I,1} - T_\infty) r_{I,1} \Delta \Theta - k \frac{(T_{I,1} - T_{I-1,1})}{\Delta r} r_{I-\frac{1}{2},1} \Delta \Theta
+ k \frac{(T_{I,2} - T_{I,1})}{\Delta \Theta} (r_{I,1} - r_{I-\frac{1}{2},1}) - k \frac{(T_{I,1} - T_{I,J})}{\Delta \Theta} (r_{I,1} - r_{I-\frac{1}{2},1})
\]  \hspace{1cm} (D.22)
\[ a_P(I, 1) = \rho c \frac{1}{\Delta t} \frac{1}{2} \left( r_{I,1}^2 + r_{I-1,1}^2 \right) \Delta \Theta \quad (D.23) \]
\[ a_N(I, 1) = h(1)r_{I,1}\Delta \Theta \quad (D.24) \]
\[ a_S(I, 1) = k \frac{1}{\Delta r} r_{I-\frac{1}{2},1} \Delta \Theta \quad (D.25) \]
\[ a_E(I, 1) = k \frac{1}{r_{I,1}} \frac{1}{\Delta \Theta} \left( r_{I,1} - r_{I-\frac{1}{2},1} \right) \quad (D.26) \]
\[ a_W(I, 1) = k \frac{1}{r_{I,1}} \frac{1}{\Delta \Theta} \left( r_{I,1} - r_{I-\frac{1}{2},1} \right) \quad (D.27) \]

**node \((I, 1)\) r direction update:**

\[ -a_S(I, 1)T_{\Delta r}^{n+\frac{1}{2}} + [a_S(I, 1) + a_P(I, 1) + a_N(I, 1)] T_{\Delta t}^{n+\frac{1}{2}} = a_W(I, 1)T_{\Delta r}^n + [-a_W(I, 1) + a_P(I, 1) - a_E(I, 1)] T_{\Delta t}^n + a_E(I, 1)T_{\Delta t}^{n+1} + a_N(I, 1)T_{\Delta t}^\infty \quad (D.28) \]

**node \((I, 1)\) \(\Theta\) direction update:**

\[ -a_S(I, 1)T_{\Delta \Theta}^{n+\frac{1}{2}} + [a_S(I, 1) + a_P(I, 1) + a_E(I, 1)] T_{\Delta t}^{n+1} - a_E(I, 1)T_{\Delta t}^{n+1} = a_S(I, 1)T_{\Delta \Theta}^n + [-a_S(I, 1) + a_P(I, 1) + a_N(I, 1)] T_{\Delta r}^{n+\frac{1}{2}} + a_N(I, 1)T_{\Delta r}^\infty \quad (D.29) \]

**node \((I, j)\)**

\[
\iint_{CS} - (q \cdot n) dA = h(j) \left( T_{I,j} - T_{\infty} \right) r_{I,j}\Delta \Theta - k \frac{(T_{I,j} - T_{I-1,j})}{\Delta r} r_{I-\frac{1}{2},j}\Delta \Theta
+ k \frac{(T_{I,j+1} - T_{I,j})}{\Delta r} \left( r_{I,j} - r_{I-\frac{1}{2},j} \right)
- k \frac{(T_{I,j} - T_{I,j-1})}{\Delta \Theta} \left( r_{I,j} - r_{I-\frac{1}{2},j} \right) \quad (D.30)
\]

\[ a_P(I, j) = \rho c \frac{1}{\Delta t} \frac{1}{2} \left( r_{I,j}^2 + r_{I-1,j}^2 \right) \Delta \Theta \quad (D.31) \]
\[ a_N(I, j) = h(j)r_{I,j}\Delta \Theta \quad (D.32) \]
\[ a_S(I, j) = k \frac{1}{\Delta r} r_{I-\frac{1}{2},j} \Delta \Theta \quad (D.33) \]
\[ a_E(I, j) = k \frac{1}{r_{I,j}} \frac{1}{\Delta \Theta} \left( r_{I,j} - r_{I-\frac{1}{2},j} \right) \quad (D.34) \]
\[ a_W(I, j) = k \frac{1}{r_{I,j}} \frac{1}{\Delta \Theta} \left( r_{I,j} - r_{I-\frac{1}{2},j} \right) \quad (D.35) \]
node (I, j) \( r \) direction update:

\[
- a_S(I, j) T_{I-1,j}^{n+\frac{1}{2}} + [a_S(I, j) + a_P(I, j) + a_N(I, j)] T_{I,j}^{n+\frac{1}{2}} \\
= a_W(I, j) T_{I,j-1}^n + [-a_W(I, j) + a_P(I, j) - a_E(I, j)] T_{I,j}^n + a_E(I, j) T_{I,j+1}^n + a_N(I, j) T_{\infty}^n
\]  

(D.36)

node (I, j) \( \Theta \) direction update:

\[
- a_W(I, j) T_{I,j-1}^{n+1} + [a_W(I, j) + a_P(I, j) + a_E(I, j)] T_{I,j}^{n+1} - a_E(I, j) T_{I,j+1}^{n+1} \\
= a_S(I, j) T_{I-1,j}^{n+\frac{1}{2}} + [-a_S(I, j) + a_P(I, j) + a_N(I, j)] T_{I,j}^{n+\frac{1}{2}} + a_N(I, j) T_{\infty}^n
\]  

(D.37)

node (I, J)

\[
\int\int_{CS} (q \cdot n) \, dA = h(J) (T_{I,J} - T_{\infty}) r_{I,J} \Delta \Theta - k \frac{(T_{I,J} - T_{I-1,J})}{\Delta r} r_{I-\frac{1}{2},J} \Delta \Theta \\
+ \frac{k}{r_{I,J}} \frac{(T_{I-1,J} - T_{I,J})}{\Delta \Theta} \left( r_{I,J} - r_{I-\frac{1}{2},J} \right) - \frac{k}{r_{I,J}} \frac{(T_{I,J} - T_{I,J-1})}{\Delta \Theta} \left( r_{I,J} - r_{I-\frac{1}{2},J} \right)
\]  

(D.38)

\[
a_P(I, J) = \rho c \frac{1}{\Delta \tau} \frac{1}{2} \left( r_{I,J}^2 + r_{I-\frac{1}{2},J}^2 \right) \Delta \Theta \]  

(D.39)

\[
a_N(I, J) = h(J) r_{I,J} \Delta \Theta \]  

(D.40)

\[
a_S(I, J) = k \frac{1}{\Delta r} r_{I-\frac{1}{2},J} \Delta \Theta \]  

(D.41)

\[
a_E(I, J) = \frac{k}{r_{I,J}} \frac{1}{\Delta \Theta} \left( r_{I,J} - r_{I-\frac{1}{2},J} \right) \]  

(D.42)

\[
a_W(I, J) = -\frac{k}{r_{I,J}} \frac{1}{\Delta \Theta} \left( r_{I,J} - r_{I-\frac{1}{2},J} \right) \]  

(D.43)

node (I, J) \( r \) direction update:

\[
- a_S(I, J) T_{I-1,j}^{n+\frac{1}{2}} + [a_S(I, J) + a_P(I, J) + a_N(I, J)] T_{I,j}^{n+\frac{1}{2}} \\
= a_W(I, J) T_{I,j-1}^n + [-a_W(I, J) + a_P(I, J) - a_E(I, J)] T_{I,j}^n + a_E(I, J) T_{I,j+1}^n + a_N(I, J) T_{\infty}^n
\]  

(D.44)
node \((I, J)\) direction update:

\[
- a_W(I, J)T^n_{I,J-1} + [a_W(I, J) + a_P(I, J) + a_E(I, J)] T^{n+1}_{I,J} - a_E(I, J)T^n_{I,J-1} = a_S(I, J)T^{n+\frac{1}{2}}_{I,J-1} + [-a_S(I, J) + a_P(I, J) + a_N(I, J)] T^{n+\frac{1}{2}}_{I,J-1} + a_N(I, J)T^n_{\infty}
\]

\((D.45)\)

For the insulative Neumann boundary condition: node\((1, 1)\)

\[
\int \int_{CS} -(q \cdot n) dA = k \frac{(T_{2,1} - T_{1,1})}{\Delta r} r_{1+\frac{1}{2},1} \Delta \Theta
\]

\[
+ \frac{k}{r_{1,1}} \frac{(T_{1,2} - T_{1,1})}{\Delta \Theta} \left(r_{1+\frac{1}{2},1} - r_{1,1}\right) - \frac{k}{r_{1,1}} \frac{(T_{1,1} - T_{1,j})}{\Delta \Theta} \left(r_{1+\frac{1}{2},1} - r_{1,1}\right)
\]

\((D.46)\)

\[
a_P(1, 1) = \rho c \frac{1}{\Delta r} \frac{1}{2} \left(r_{1+\frac{1}{2},1}^2 + r_{1,1}^2\right) \Delta \Theta
\]

\((D.47)\)

\[
a_N(1, 1) = k \frac{1}{\Delta r} r_{1+\frac{1}{2},1} \Delta \Theta
\]

\((D.48)\)

\[
a_S(1, 1) = 0
\]

\((D.49)\)

\[
a_E(1, 1) = \frac{k}{r_{1,1}} \frac{1}{\Delta \Theta} \left(r_{1+\frac{1}{2},1} - r_{1,1}\right)
\]

\((D.50)\)

\[
a_W(1, 1) = \frac{k}{r_{1,1}} \frac{1}{\Delta \Theta} \left(r_{1+\frac{1}{2},1} - r_{1,1}\right)
\]

\((D.51)\)

node \((1, 1)\) \(r\) direction update:

\[
[a_P(1, 1) + a_N(1, 1)] T^{n+\frac{1}{2}}_{1,1} - a_N(1, 1)T^n_{2,1}
\]

\((D.52)\)

\[
= a_W(1, 1)T^n_{1,J} + [-a_W(1, 1) + a_P(1, 1) - a_E(1, 1)] T^n_{1,1} + a_E(1, 1)T^n_{1,2}
\]

node \((1, 1)\) \(\Theta\) direction update:

\[
- a_W(1, 1)T^{n+1}_{1,J} + [a_W(1, 1) + a_P(1, 1) + a_E(1, 1)] T^{n+1}_{1,1} - a_E(1, 1)T^{n+1}_{1,2}
\]

\((D.53)\)

\[
= [a_P(1, 1) + a_N(1, 1)] T^{n+\frac{1}{2}}_{1,1} + a_N(1, 1)T^{n+\frac{1}{2}}_{2,1}
\]

node \((1, j)\)

\[
\int \int_{CS} -(q \cdot n) dA = k \frac{(T_{2,j} - T_{1,j})}{\Delta r} r_{1+\frac{1}{2},j} \Delta \Theta
\]

\[
+ \frac{k}{r_{1,j}} \frac{(T_{1,j+1} - T_{1,j})}{\Delta \Theta} \left(r_{1+\frac{1}{2},j} - r_{1,j}\right) - \frac{k}{r_{1,j}} \frac{(T_{1,j} - T_{1,j-1})}{\Delta \Theta} \left(r_{1+\frac{1}{2},j} - r_{1,j}\right)
\]

\((D.54)\)
\[
ap(1, j) = \rho c \frac{1}{\Delta r} \frac{1}{2} \left( r_{1+\frac{1}{2}, j}^2 + r_{1, j}^2 \right) \Delta \Theta \quad (D.55)
\]
\[
an(1, j) = k \frac{1}{\Delta r} r_{1+\frac{1}{2}, j} \Delta \Theta \quad (D.56)
\]
\[
as(1, 1) = 0 \quad (D.57)
\]
\[
aE(1, j) = \frac{k}{r_{1, j}} \frac{1}{\Delta \Theta} \left( r_{1+\frac{1}{2}, j} - r_{1, j} \right) \quad (D.58)
\]
\[
aW(1, j) = \frac{k}{r_{1, j}} \frac{1}{\Delta \Theta} \left( r_{1+\frac{1}{2}, j} - r_{1, j} \right) \quad (D.59)
\]

**node (1, j) r direction update:**

\[
[aP(1, j) + an(1, j)] T_{1,j}^{n+\frac{1}{2}} - an(1, j) T_{2,j}^{n+\frac{1}{2}} \nonumber = aW(1, j) T_{1,j-1}^{n} + [-aW(1, j) + aP(1, j) - aE(1, j)] T_{1,j}^{n} + aE(1, j) T_{1,j+1}^{n} \quad (D.60)
\]

**node (1, j) \(\Theta\) direction update:**

\[
- aW(1, j) T_{1,j-1}^{n+1} + [aW(1, j) + aP(1, j) + aE(1, j)] T_{1,j}^{n+1} - aE(1, j) T_{1,j+1}^{n+1} \nonumber = [aP(1, j) + an(1, j)] T_{1,j}^{n+\frac{1}{2}} + an(1, j) T_{2,j}^{n+\frac{1}{2}} \quad (D.61)
\]

**node (1, J)**

\[
\int\int_{CS} -(q \cdot n) \, dA = \frac{k (T_{2,J} - T_{1,J})}{\Delta r} r_{1+\frac{1}{2}, J} \Delta \Theta 
\]
\[
+ \frac{k}{r_{1,J}} \frac{(T_{1,1} - T_{1,J})}{\Delta \Theta} \left( r_{1+\frac{1}{2}, J} - r_{1,J} \right) - \frac{k}{r_{1,J}} \frac{(T_{1,J} - T_{1,J-1})}{\Delta \Theta} \left( r_{1+\frac{1}{2}, J} - r_{1,J} \right) \quad (D.62)
\]
\[
ap(1, J) = \rho c \frac{1}{\Delta r} \frac{1}{2} \left( r_{1+\frac{1}{2}, J}^2 + r_{1, J}^2 \right) \Delta \Theta \quad (D.63)
\]
\[
an(1, J) = k \frac{1}{\Delta r} r_{1+\frac{1}{2}, J} \Delta \Theta \quad (D.64)
\]
\[
as(1, J) = 0 \quad (D.65)
\]
\[
aE(1, J) = \frac{k}{r_{1,J}} \frac{1}{\Delta \Theta} \left( r_{1+\frac{1}{2}, J} - r_{1,J} \right) \quad (D.66)
\]
\[
aW(1, J) = \frac{k}{r_{1,J}} \frac{1}{\Delta \Theta} \left( r_{1+\frac{1}{2}, J} - r_{1,J} \right) \quad (D.67)
\]
For the nodes which lie on the fictitious boundary of \( j = 1 \) and \( j = J \) their nodal descriptions are as follows: node(i,1)

\[
\left\{ \begin{array}{l}
[a_P(1, J) + a_N(1, J)]T_{1,j}^{n+\frac{1}{2}} - a_N(1, J)T_{2,j}^{n+\frac{1}{2}} \\
= a_W(1, J)T_{1,j}^n + [-a_W(1, J) + a_P(1, J) - a_E(1, J)]T_{1,j}^n + a_E(1, J)T_{1,j}^n \\
\end{array} \right.
\]  

(D.68)

for node \( (1, J) \) \( r \) direction update:

\[
- a_W(1, J)T_{1,j}^{n+1} + [a_W(1, J) + a_P(1, J) + a_E(1, J)]T_{1,j}^{n+1} - a_E(1, J)T_{1,j}^{n+1} \\
= [a_P(1, J) + a_N(1, J)]T_{1,j}^{n+\frac{1}{2}} + a_N(1, J)T_{2,j}^{n+\frac{1}{2}}
\]  

(D.69)

\[
\int\int_{CS} -(q \cdot n) \, dA = \frac{k}{\Delta r} \frac{1}{r_i + \frac{1}{2}, 1} \Delta \Theta - \frac{k}{\Delta r} \frac{(T_i, 1 - T_{i-1, 1})}{r_i - \frac{1}{2}, 1} \Delta \Theta \\
+ \frac{k}{r_i, 1} \frac{(T_i, 2 - T_{i, 1})}{\Delta \Theta} \left( r_i + \frac{1}{2}, 1 - r_i - \frac{1}{2}, 1 \right) \left( r_i + \frac{1}{2}, 1 - r_i - \frac{1}{2}, 1 \right)
\]  

(D.70)

\[
a_P(i, 1) = \rho c \frac{1}{\Delta r} \frac{1}{2} \left( r_i^2 + r_{i, 1}^2 \right) \Delta \Theta
\]  

(D.71)

\[
a_N(i, 1) = \frac{k}{\Delta r} r_i + \frac{1}{2}, 1 \Delta \Theta
\]  

(D.72)

\[
a_S(i, 1) = \frac{k}{\Delta r} r_i - \frac{1}{2}, 1 \Delta \Theta
\]  

(D.73)

\[
a_E(i, 1) = \frac{k}{r_i, 1} \frac{1}{\Delta \Theta} \left( r_i + \frac{1}{2}, 1 - r_i - \frac{1}{2}, 1 \right)
\]  

(D.74)

\[
a_W(i, 1) = \frac{k}{r_i, 1} \frac{1}{\Delta \Theta} \left( r_i + \frac{1}{2}, 1 - r_i - \frac{1}{2}, 1 \right)
\]  

(D.75)

for node \( (i, 1) \) \( r \) direction update:

\[
- a_S(i, 1)T_{i-1, 1}^{n+\frac{1}{2}} + [a_S(i, 1) + a_P(i, 1) + a_N(i, 1)]T_{i, 1}^{n+\frac{1}{2}} - a_N(i, 1)T_{i+1, 1}^{n+\frac{1}{2}} \\
= a_W(i, 1)T_{i,j}^n + [-a_W(i, 1) + a_P(i, 1) - a_E(i, 1)]T_{i,j}^n + a_E(i, 1)T_{i,j}^n
\]  

(D.76)

\[
- a_W(i, 1)T_{i,j}^{n+1} + [a_W(i, 1) + a_P(i, 1) + a_E(i, 1)]T_{i,j}^{n+1} - a_E(i, 1)T_{i,j}^{n+1} \\
= a_S(i, 1)T_{i-1, 1}^{n+\frac{1}{2}} + [-a_S(i, 1) + a_P(i, 1) + a_N(i, 1)]T_{i, 1}^{n+\frac{1}{2}} + a_N(i, 1)T_{i+1, 1}^{n+\frac{1}{2}}
\]  

(D.77)
node \((i, J)\)

\[
\int_{CS} - (q \cdot n) \, dA = k \left( \frac{T_{i+1,J} - T_{i,J}}{r_{i+\frac{1}{2},J}} - \frac{T_{i,J} - T_{i-1,J}}{r_{i-\frac{1}{2},J}} \right) \Delta \Theta
\]

\[
+ k \left( \frac{T_{i+1,J} - T_{i,J-1}}{r_{i,J}} \right) \left( r_{i+\frac{1}{2},J} - r_{i-\frac{1}{2},J} \right) \Delta \Theta
\]

(D.78)

node \((i, J)\) r direction update:

\[
-a_S(i, J) T_{i-1,J}^{n+\frac{1}{2}} + \left[ a_S(i, J) + a_P(i, J) + a_N(i, J) \right] T_{i,J}^{n+\frac{1}{2}} - a_N(i, J) T_{i+1,J}^{n+\frac{1}{2}}
\]

\[
= a_W(i, J) T_{i,J-1}^n + [-a_W(i, J) + a_P(i, J) - a_E(i, J)] T_{i,J}^n + a_E(i, J) T_{i+1}^n
\]

(D.84)

node \((i, J)\) \(\Theta\) direction update:

\[
-a_W(i, J) T_{i,J-1}^{n+\frac{1}{2}} + \left[ a_W(i, J) + a_P(i, J) + a_E(i, J) \right] T_{i,J}^{n+\frac{1}{2}} - a_E(i, J) T_{i+1,J}^{n+\frac{1}{2}}
\]

\[
= a_S(i, J) T_{i-1,J}^{n+\frac{1}{2}} + [-a_S(i, J) + a_P(i, J) + a_N(i, J)] T_{i,J}^{n+\frac{1}{2}} + a_N(i, J) T_{i+1,J}^{n+\frac{1}{2}}
\]

(D.85)

D.1 The Finite Control Volume Test Case

The solution accuracy of the finite control volume algorithm is tested by comparing results from the numerical computation with an analytical solution. The solution for the case of a cylindrical geometry with a convective heat flux boundary condition may be found in Yener and Kakac [126]. The one-dimensional distribution of energy is represented by Equation (D.86) subject to the boundary conditions of Equation (D.87), Equation (D.88), and the initial condition of Equation (D.89).

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]

(D.86)
Table D.1: Finite Control Volume Test Condition Data

<table>
<thead>
<tr>
<th>Initial Condition Data</th>
<th>Boundary Condition Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = 298 \text{ [K]} )</td>
<td>( h = 1000 \text{ [W·m}^{-2}·\text{K}^{-1}] )</td>
</tr>
<tr>
<td>( T_\infty = 1023 \text{ [K]} )</td>
<td>( R_{out} = 0.01905 \text{ [m]} )</td>
</tr>
</tbody>
</table>

**Inner Sensor1 Boundary:** At the inner bore of the Sensor1 geometry the gradient of the heat flux in the radial direction is set to zero.

\[
\frac{\partial T}{\partial r} \bigg|_{r=0} = 0 \tag{D.87}
\]

**Outer Sensor1 Boundary:** At the outer Sensor1 boundary there is the convective heat flux condition.

\[
-k \frac{\partial T}{\partial r} \bigg|_{r=R_{out}} = h \left[ T(R_{out}, t) - T_\infty \right] \tag{D.88}
\]

**Initial Condition:** The initial temperature of Sensor1 is constant and set at \( C = 298 \text{ K (25 ^\circ \text{C})} \).

\[
T(r, 0) = C \tag{D.89}
\]

The analytical solution is given by the following set of equations:

\[
\lambda k J_1(\lambda r_0) - h J_0(\lambda r_0) = 0 \tag{D.90}
\]

\[
\frac{T(r, t) - T_\infty}{T_i - T_\infty} = \frac{2}{r_0} \sum_{n=1}^{\infty} \frac{1}{\lambda_n} \frac{J_1(\lambda_n r_0) J_0(\lambda_n r)}{J_0^2(\lambda_n r_0) J_1^2(\lambda_n r_0)} \exp \left( -\alpha \lambda_n^2 t \right) \tag{D.91}
\]

In Equations (D.90) and (D.91), \( \lambda \) represents the eigenvalue, \( J_1 \) the Bessel function of the first kind to order one, and \( J_0 \) the Bessel function of the first kind to order zero. The thermophysical properties of copper listed in Table 3.3 were employed for the finite control volume test. The data listed in in Table D.1 are used to test the algorithm.

A comparison between the numerical output and the analytical solution is presented for two scenarios. Figure D.3a plots the temperature as a function of time for a point located at \( r = \)
Figure D.3: Finite Control Volume Test Case Comparison of Analytical Solution and Numerical Result

0.017145 m showing the analytical and numerical curve. The two curves have good agreement for the temporal variation. Figure D.3b plots the temperature as a function of position at 10 s. The spatial comparison of the numerical and analytical solution are in good agreement.
Appendix E

The Inverse Heat Transfer Coefficient Test Cases

This appendix will outline the numerical parameters which were used to investigate the solution accuracy of the inverse algorithm. To test the solution of the inverse algorithm, a series of numerical experiments were performed. In the numerical experiment, the temperature solution to the direct (forward) problem is computed given an applied heat transfer coefficient function. The temperature profile at simulated thermocouple locations are extracted from the forward numerical solution. Noise is added to the thermocouple temperature data. The noisy thermocouple data is input into the inverse code to test the ability of the algorithm to reconstruct the heat transfer coefficient function. Three heat transfer coefficient functions will be presented to test the solution of the inverse problem. Various degrees of noise are added to the temperature solution for each heat transfer coefficient function.

Envision a cylinder comprised of copper at an initial temperature of 298 K (25 °C) which is in a liquid at a temperature of 1023 K (750 °C). On the surface of the cylinder is an applied heat transfer coefficient which varies perhaps in space and time. The thermal response of the cylinder is dependent on the heat transfer coefficient distribution. The direct problem using the finite control volume method is used to solve for the thermal response of the cylinder. At specific
Table E.1: Inverse Heat Transfer Coefficient Test Case Data

<table>
<thead>
<tr>
<th>Initial Temperature</th>
<th>Outer Boundary Temperature</th>
<th>Domain Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 298 \text{[K]}$</td>
<td>$T_\infty = 1023 \text{[K]}$</td>
<td>$R_{in} = 0.005 \text{[m]}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_{out} = 0.019 \text{[m]}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I \text{ nodes 29 in } r$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$J \text{ nodes 32 in } \Theta$</td>
</tr>
</tbody>
</table>

locations within the geometry the temperature data is then contaminated with error (noise). This noisy data is input into the inverse sequential function specification (SFS) algorithm to reconstruct the heat transfer coefficient function.

The simulated experiment employs eight internal thermocouples to mimic the experimental arrangement; these thermocouples are 45° from one another and 1 mm from the surface of a copper cylinder. Table 3.3 lists the thermophysical property data of copper. Table E.1 lists the initial temperature and boundary temperature data of the system with input parameters to the forward numerical algorithm.

### E.1 Step Function Heat Transfer Coefficient Test Case

The first test case presents the reconstruction of a step function heat transfer coefficient. The function is depicted in Figure E.1. From time of $(0 < t \leq 2) \text{ s}$ a time index of $(0 < m \leq 20)$ a value of $100 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ was applied to the surface of the cylinder, at $2.1 \text{ s}$ (time index 21) the value of the heat transfer coefficient was raised to $1000 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$ and held until $8 \text{ s}$ (time index 80) after which time the value drops to $100 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}$. As there is symmetry in the thermal response of the cylinder with respect to the angular coordinate, one thermocouple reading is shown in Figure E.2a, in relation to the applied heat transfer coefficient function. Figure E.2a is the thermal response from the solution to the forward energy problem; the temperature response is free from noise. A noise vector with a variance of 0.0025 and mean of 0 was added to the temperature response and shown in Figure E.2b. The reconstructed heat
Figure E.1: Applied Step Heat Transfer Coefficient Function

The heat transfer coefficient is shown in Figure E.2b as circular symbols. Three future times are used in the solution to regularize the ill-posed nature of the inverse problem. From Figure E.2b it can be said that the reconstruction of the function is good for the case of a small variance in the temperature reading. Figure E.2c plots the reconstructed heat transfer coefficient function for a variance of 0.01, mean 0 and using three future times. Notice that there is more scatter in the reconstructed heat transfer coefficient function. Figure E.2d plots the heat transfer coefficient function for the scenario when a variance of 0.49 was applied to the temperature measurement which is input into the SFS algorithm. Notice the large deviations which occur in the reconstructed heat transfer coefficient from the applied function. From these numerical experiments one may infer that the temperature data with more noise will result in a heat transfer coefficient function which has larger deviations from the ‘true’ form.

## E.2 Triangular Pulse Heat Transfer Coefficient Test Case

This section presents the results from the triangular pulse heat transfer coefficient test case. The pulse varies temporally and does not change around the cylinder. The triangular heat pulse begins at 0 s (time index 0) with a value of heat transfer coefficient of 100 W·m$^{-2}$·K$^{-1}$ and increases linearly to a value of 1000 W·m$^{-2}$·K$^{-1}$ at 5 s (time index 50). The heat transfer
Figure E.2: Temperature Response and Estimated Heat Transfer Coefficient for a Step Heat Transfer Coefficient Function
Figure E.3: Applied Triangular Pulse Heat Transfer Coefficient

coefficient then decreases linearly from 5.1 s (time index 51) to a value of 100 W·m$^{-2}$·K$^{-1}$ at 10 s (time index 100). This is portrayed in Figure E.3. Figure E.4a plots the heat transfer coefficient function and corresponding temperature response 1 mm from the surface. Figure E.4b plots the results for a 0.0025 variance of noise with 0 mean added to the thermocouple data. Figure E.4c depicts the reconstructed heat transfer coefficient for a variance of 0.01 error and 0 mean added to the temperature data. Figure E.4d shows the reconstructed function for a thermocouple reading with a noise vector with variance of 0.49 and 0 mean. The greater the noise in the temperature readings that is input into the SFS algorithm, the larger the scatter in the reconstructed heat transfer coefficient function at a future time regularization parameter of $r = 3$.

E.3 Cosine Distribution Heat Transfer Coefficient Test Case

The cosine distribution heat transfer coefficient spatially and temporally varies the applied values of the coefficient. The exact form of the function is listed in equation (E.1). Where $j$ corresponds to the $\Theta$ node around the surface of the cylinder, $\Delta\Theta$ is the mesh spacing in $\Theta$, and $m$ is the time index. The function values are graphically displayed in Figure E.5. Figure E.6a plots the temperature response of five of the eight thermocouples, the polar position around the
(a) Temperature Response 1 mm below Surface with Applied Triangular Pulse Heat Transfer Coefficient Function

(b) Variance of 0.0025 applied to Temperature Signal (future time $r = 3$)

(c) Variance of 0.01 applied to Temperature Signal (future time $r = 3$)

(d) Variance of 0.49 applied to Temperature Signal (future time $r = 3$)

Figure E.4: Temperature Response and Estimated Heat Transfer Coefficient for a Triangular Pulse Heat Transfer Coefficient Function
surface of the probe is noted in parentheses. The heat transfer coefficient which was applied at the corresponding angle around the cylinder is also displayed in Figure E.6a. This data set was generated using the forward heat conduction algorithm; the temperature signal contains no noise. Figure E.6b depicts the reconstructed heat transfer coefficients when a variance of 0.0025 of noise vector and a mean of 0 was applied to the temperature readings of the thermocouples. Figure E.6c displays the estimated heat transfer coefficient when a variance of 0.01 and mean of 0 in error was applied to the temperature data. Figure E.6d shows the reconstructed heat transfer coefficients for the scenario when noise of variance of 0.49 and mean 0 was applied to the thermocouple signal. For clarity the signals at 0° and 180° around the cylinder are shown in Figure E.6d. Larger scatter in the reconstructed heat transfer coefficients is observed with greater noise in the temperature signal.

\[ h(j, m) = 175 \left[ \cos ((j - 1) \cdot \Delta \Theta + 5) \right] \exp(-0.005m) \] 

(E.1)
(a) Temperature Response 1 mm below Surface with Applied Cosine Function Heat Transfer Coefficient

(b) Variance of 0.0025 applied to Temperature Signal (future time $r = 3$)

(c) Variance of 0.01 applied to Temperature Signal (future time $r = 3$)

(d) Variance of 0.49 applied to Temperature Signal (future time $r = 3$)

Figure E.6: Temperature Response and Estimated Heat Transfer Coefficient for a Cosine Function Heat Transfer Coefficient, values in parentheses represent polar coordinate on cylinder, $h$ corresponds to the heat transfer coefficient, $tc$ corresponds to the temperature, values in parenthesis refer to polar coordinate
Appendix F

The Time Constant (b) Calculation

To compute the estimate of the time constant (b) the lumped capacitance relationship of Equation (1.1) is rearranged to solve for the parameter b. This is shown as Equation (F.1) where the sub-script m refers to the index in time at which data is stored from the data acquisition system. The time constant \( b_m \) can be computed exactly for each point in time at which temperature data is stored. The parameter \( \bar{b} \) is estimated by averaging the vector of \( b_m \) values that are obtained from Equation (F.1). The average is represented by Equation (F.2).

\[
b_m(t) = \frac{-1 \log(T_{S2}(t))}{t_m}
\]  \hspace{1cm} \text{(F.1)}

\[
\bar{b} = \frac{1}{m} \sum_{1}^{m} (b_m)
\]  \hspace{1cm} \text{(F.2)}
Appendix G

The Derivation of the Error Equation

The data reduction equation employed to estimate the velocity using Sensor2 is the following:

\[
\bar{N_u_{la}} = \frac{D}{k} \cdot \left( \frac{1}{\frac{1}{\frac{k}{\bar{N_{laR}}} - R}} \right)
\]  \hspace{1cm} (G.1)

where \( D \) is the diameter of the sensor, \( k \) the thermal conductivity of the fluid, \( \bar{N_{laR}} \) the estimated local average Nusselt number which includes the thermal resistance from the Sensor2 heat transfer element, and \( R \) is the thermal resistance. Alternatively Equation (G.1) may be written as:

\[
\bar{N_u_{la}} = C_1 \cdot ((C_2 \cdot \bar{N_{laR}})^{-1} - R)^{-1}
\]  \hspace{1cm} (G.2)

where

\[
C_1 = \frac{D}{k}
\]  \hspace{1cm} (G.3)

and

\[
C_2 = \frac{k}{D}
\]  \hspace{1cm} (G.4)
The propagation of the uncertainty in the measured variables into the uncertainty of the result is represented by Equation (G.5) after Coleman and Steele [121].

\[ U_{95} = \left( \sum_{i=1}^{J} \left( \frac{\partial r}{\partial X_i} \right)^2 \cdot U_i^2 \right)^{\frac{1}{2}} \]  

(G.5)

The uncertainty associated with the thermal resistance is:

\[ \frac{\partial \bar{Nu}_{la}}{\partial R} = \frac{\partial}{\partial R} \left( C_1 \cdot ((C_2 \cdot \bar{Nu}_{laR})^{-1} - R)^{-1} \right) \]  

(G.6)

\[ = \frac{C_1}{((C_2 \cdot \bar{Nu}_{laR})^{-1} - R)^2} \]  

(G.7)

\[ = \frac{1}{C_2^2 \bar{Nu}_{laR}^2} \cdot \frac{2 \cdot R}{C_2 \cdot \bar{Nu}_{laR}} + R^2 \]  

(G.8)

\[ = \frac{C_1}{1 - 2 \cdot R \cdot C_2 \cdot \bar{Nu}_{laR} + R^2 \cdot C_2^2 \cdot \bar{Nu}_{laR}^2} \]  

(G.9)

\[ \frac{\partial \bar{Nu}_{la}}{\partial \bar{Nu}_{laR}} = \frac{C_1 \cdot C_2 \cdot \bar{Nu}_{laR}^2}{(C_2 \cdot \bar{Nu}_{laR} \cdot R - 1)^2} \]  

(G.10)

The uncertainty associated with the estimated local average Nusselt number data fit is:

\[ \frac{\partial \bar{Nu}_{la}}{\partial \bar{Nu}_{laR}} = \frac{\partial}{\partial \bar{Nu}_{laR}} \left( C_1 \cdot ((C_2 \cdot \bar{Nu}_{laR})^{-1} - R)^{-1} \right) \]  

(G.11)

\[ = \frac{C_1 \cdot C_2}{(C_2 \cdot \bar{Nu}_{laR})^2} \cdot \left( \frac{1}{C_2 \cdot \bar{Nu}_{laR} - R} \right)^2 \]  

(G.12)

\[ = \frac{C_1 \cdot C_2}{(1 - 2 \cdot R \cdot C_2 \cdot \bar{Nu}_{laR} + R^2 \cdot C_2^2 \cdot \bar{Nu}_{laR}^2)} \]  

(G.13)

\[ \frac{\partial \bar{Nu}_{la}}{\partial \bar{Nu}_{laR}} = \frac{C_1 \cdot C_2}{(C_2 \cdot \bar{Nu}_{laR} \cdot R - 1)^2} \]  

(G.14)
which leads to the overall uncertainty in the measurement:

\[
U_{\text{Nu}_{\text{la}}}^2 = \left( \frac{\sqrt{\text{Nu}_{\text{la}}^2 \cdot \frac{k}{D}}}{\left( \frac{k}{D} \cdot \text{Nu}_{\text{la}} \cdot R \right)} \right)^2 \cdot U_R^2 + \left( \frac{1}{\left( \frac{k}{D} \cdot \text{Nu}_{\text{la}} \cdot R - 1 \right)^2} \right)^2 \cdot U_{\text{Nu}_{\text{la}}R}^2
\] (G.16)