DYNAMICS OF A UNITARY FERMI GAS

by

Alma Beth Bardon

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Physics
University of Toronto

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Abstract

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Alma Beth Bardon
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This thesis investigates the dynamics of a degenerate Fermi gas in the unitary limit of strong interactions, in particular, the dynamics of a transversely magnetized unitary Fermi gas in an inhomogeneous magnetic field. Short-range interactions between ultracold fermionic potassium atoms are tuned to their quantum mechanical limit by employing a Feshbach resonance. Spin-echo measurements of magnetization dynamics show that demagnetization is caused by diffusive spin transport with a diffusion constant that saturates at low temperatures to the conjectured quantum-mechanical lower bound $\sim \hbar/m$, where $m$ is the particle mass. A prediction for the increase in temperature due to demagnetization based on universal thermodynamics is compared to measured momentum distributions. The development of pair correlations is observed by time-resolved radiofrequency spectroscopy of Tan’s contact parameter.
Preface

My graduate work culminated in this publication:


This thesis serves as an expanded and supplemented version of the paper, with more background, experimental details, and supporting measurements. On that note, I would like to acknowledge my co-authors. S. Beattie, C. Luciuk, and S. Trotzky shared the responsibilities of data acquisition and analysis with me, as well as continued improvement and troubleshooting of the experiment. W. Cairncross developed our lab’s understanding of universal thermodynamics. D. Fine and N. S. Cheng contributed to improving the capabilities and reliability of the experiment prior to scientific fruition. G. J. A. Edge constructed the $^{40}$K dispensers. E. Taylor and S. Zhang greatly contributed to our understanding from a theoretical point of view. J. H. Thywissen was the project leader with all that entails. All authors made contributions to the writing of the paper, and therefore also to the thoughts and language that are throughout this thesis. Some parts of this thesis are adapted from the paper, including the presentation of the final results in Chapters 4 and 6. Not among the authors, but essential to the experiment’s success was D. Jervis, who fabricated the chip, as well as a number of students and postdocs who built the machine over the years.

In the early years of my Ph.D., I worked on a boson double well experiment with two senior graduate students, L. J. LeBlanc and M. H. T. Extavour, and a postdoc J. McKeever. As the
work is thoroughly discussed in their theses, I have attached a copy of the resulting paper as an appendix. My contributions to this work were assisting in the development of experimental sequences, and collecting data.

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Chapter 1

Introduction

The dynamics of a system of strongly interacting fermions is a problem relevant to diverse systems such as high temperature superconductors, neutron stars, and quark-gluon plasmas. Short-range interactions reach their quantum-mechanical limit when the scattering length that characterizes inter-particle collisions diverges. The absence of a small parameter in this unitary regime renders a quantitative and sometimes even a qualitative theoretical description challenging. Unitary interactions can be realized in a controlled laboratory system using ultracold fermionic alkali atoms tuned to a Feshbach resonance [1, 2]. Experiments with ultracold atoms have already greatly contributed to the understanding of equilibrium properties of unitary gases, notably measurements of universal thermodynamic functions [3, 4, 5] and the transition to superfluidity at low temperature [5, 6].

Recent studies of unitary non-equilibrium dynamics are beginning to shed light on the nature of transport in this extreme form of matter [7, 8, 9, 10]. For example, observations of anisotropic expansion of a unitary Fermi gas as well as the damping rate of its breathing mode indicated an unprecedentedly low viscosity [8]. Simple kinetic arguments predict limits on transport coefficients, such as the diffusivity, viscosity or conductivity, in unitary Fermi gases. In a classical ideal gas [11], these coefficients can be calculated from the mean thermal speed and the mean free path of the particles that make up the system (Table 1.1). Quantum degeneracy of a Fermi gas places a lower bound on a particle’s speed of \( \frac{\hbar k_F}{m} \), where \( \hbar k_F \) is the Fermi
<table>
<thead>
<tr>
<th>Effect</th>
<th>Particle property</th>
<th>Law</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion</td>
<td>Number</td>
<td>$\vec{j}_n = -D\nabla n$</td>
<td>$D = \frac{1}{3}v\ell$</td>
</tr>
<tr>
<td>Viscosity</td>
<td>Transverse momentum</td>
<td>$F_x/A = \frac{j_x}{\rho} = -\eta \partial_x v_x$</td>
<td>$\eta = \frac{1}{3}\rho v\ell$</td>
</tr>
<tr>
<td>Conductivity</td>
<td>Charge</td>
<td>$\vec{j}_q = \sigma\vec{E}$</td>
<td>$\sigma = nq^2\ell/(mv)$</td>
</tr>
</tbody>
</table>

Symbols: $n =$ number of particles per unit volume, $\vec{p} =$ momentum, $v =$ mean thermal speed, $\rho =$ mass per unit volume, $\ell =$ mean free path, $q =$ electric charge, $F_x/A =$ shear force per unit area, $m =$ mass of particle

Table 1.1: Phenomenological transport laws and kinetic theory predictions of transport coefficients [12].

momentum, and $m$ is the particle mass. For unitary interactions, the collisional cross-section scales as the square of the thermal de Broglie wavelength, which leads to a minimum mean free path at low temperature: the interparticle spacing, $\sim 1/k_F$. This line of argument leads to a lower bound of diffusivity of $D \gtrsim \hbar/m$. There have been two recent measurements of non-equilibrium dynamics in resonantly-interacting atomic Fermi gases that revealed spin diffusion. The first experiment separated two spin components in an elongated three-dimensional harmonic trap with opposite momentum kicks before turning on unitary interactions. The confining potential forced the clouds of opposite spin to collide. Interactions were strong enough to cause the dilute gases to bounce off each other several times before they slowly relaxed into one another. In this last step, the experiment revealed a minimum trap-averaged spin diffusivity of $6.3(3)\hbar/m$ [9]. A spin echo experiment measuring the demagnetization of a two-dimensional gas with resonant interactions found a surprisingly low spin diffusion constant of $0.0063(8)\hbar/m$ [10]. In this case, spin diffusion was driven by a gradient in the magnetization direction rather than its magnitude, so it is uncertain whether the difference in diffusivities by three orders of magnitude is due to the dimensionality or the mode of spin diffusion.

Here, we study the demagnetization of a three-dimensional transversely polarized gas at an
interaction resonance. The experiment starts with every atom in an equal coherent superposition of two resonantly interacting states. This initial state is non-interacting because the Pauli exclusion principle prohibits s-wave scattering between identical fermions, and higher partial wave scattering is suppressed at low temperatures. Over time, in the presence of a magnetic field gradient, the system loses its magnetization and turns into a mixture with unitary interactions. There are three aspects of the dynamics that we follow: 1) the loss of magnetization itself, 2) the rethermalization of the gas as a second Fermi sea and strong interactions develop, and 3) the growth of interactions as the initially non-interacting gas transforms into a unitary mixture.

**Outline**

This thesis presents a complete study of the demagnetization dynamics of a transversely-polarized three-dimensional Fermi gas at an interaction resonance

- **Chapter 2** describes the experimental apparatus. As this machine has been discussed in previous theses [13, 14], this chapter focusses on newly implemented capabilities and important calibrations.

- **Chapter 3** discusses interactions in atomic gases. Feshbach resonances, and their use in quantum simulation are described with data from our lab to support the discussion.

- **Chapter 4** presents spin echo measurements of demagnetization dynamics. A spin spiral model allows us to extract the transverse spin diffusivity as a function of temperature.

- **Chapter 5** contains a prediction for the increase in temperature due to demagnetization based on the experimentally determined equation of state for a unitary Fermi gas [5]. The prediction is compared to measured momentum distributions.

- **Chapter 6** studies the development of interactions as magnetization is lost with time-resolved radiofrequency spectroscopy of Tan’s contact parameter [15].
Chapter 2

Apparatus

The chip experiment is a versatile machine which can produce quantum degenerate gases of both $^{87}\text{Rb}$, a boson, and $^{40}\text{K}$, a fermion. Prior to the work described in this thesis, this apparatus was used to study bosons in radio-frequency (rf) dressed potentials (See Refs. [13, 14] and Appendix A). This chapter first provides an overview of the experimental apparatus, which is more thoroughly described in previous theses [13, 14], then details the more recent changes of the apparatus which are directly relevant to the experiments presented in the later chapters.

2.1 Overview

Broadly speaking, each experimental cycle can be broken into three steps: preparing the gas, performing an experiment, and measuring the result. The experimental stage consists of cold potassium atoms which are optically trapped below a “chip”. Wires on the chip serve as antennas for radio frequency manipulation of the atomic state. Direct current through the wires in combination with externally applied fields control the magnetic field, which in turn controls the atomic interaction. The full apparatus is introduced by following an experimental cycle.
2.1.1 Typical experimental sequence

In the following I present a summary of the typical stages in our experimental cycle. A standard cycle takes about 30 seconds. All equipment is controlled by an ADwin sequencer with a minimum time step of 10 µs.

**MOT:** Atoms are collected in a dual magneto-optical trap (MOT) stage. For each element, six beams (three counter-propagating pairs) intersect at the centre of a quadrupole magnetic field generated by current through a pair of coils in anti-Helmholtz configuration and a single coil with its symmetry axis perpendicular to those (the “transfer” coil). The $^{87}$Rb and $^{40}$K light is $\approx 30$ MHz red-detuned from the $|F = 2, m_F = 2\rangle$ to $|F' = 3, m_{F'} = 3\rangle$ and $|F = 9/2, m_F = 9/2\rangle$ to $|F' = 11/2, m_{F'} = 11/2\rangle$ cycling transitions of the D-2 lines respectively. The $^{87}$Rb repump light is tuned to the $F = 1$ to $F' = 2$ D-2 transition and combined with trapping light prior to amplification. The repump light for potassium ($F = 7/2$ to $F' = 9/2$ D$_2$ transition) is amplified separately, and added to only four of the six beams (two counter propagating pairs). These four intersecting repump beams each have a 1 cm diameter hole in the middle which creates a volume without repump in which cold $^{40}$K accumulates and no longer scatters any photons. Excited state collisions with $^{87}$Rb are then suppressed. The vapour pressure of both $^{87}$Rb and $^{40}$K is temporarily increased during the MOT by pulsing atom dispensers and employing arrays of 405 nm purple LEDs to release atoms from the wall of the glass cell via LIAD (light induced atomic desorption). At the end of the MOT stage, there is a quick optical molasses step, where the magnetic quadrupole field is turned off but the $^{87}$Rb beams are on. This step reaches sub-Doppler temperatures for $^{87}$Rb.

**Optical pumping:** In preparation for magnetic trapping, each atomic species is optically pumped into the low-field seeking stretched state of the ground state, $|F = 9/2, m_F = 9/2\rangle$ for $^{40}$K and $|F = 2, m_F = 2\rangle$ for $^{87}$Rb. The optical pumping beams are composed of $\sigma^+$ light which is resonant with the $F = 2$ to $F' = 2$ D-2 transition and $F = 9/2$ to $F' = 9/2$ D-2 transition for $^{87}$Rb and $^{40}$K respectively, as well as some repump light.

**Magnetic trapping:** The atoms are trapped in a quadrupole field then transferred vertically
5 cm to 200 µm below the atom chip by changing the current in the transfer coil. The chip trap is made from the magnetic field produced by current through a “z”-shaped wire on the chip (Fig. 2.1, blue wire) combined with a homogeneous external magnetic field produced from two pairs of coils, the $x$-bias and $z$-bias coils. The combination of the $z$-bias field with the field from the $z$-wire creates a local minimum in magnetic field $\sim 200\mu m$ below the chip. The $x$-bias field prevents that minimum from being zero. This field configuration traps atoms in low field seeking states, while maintaining a quantizing field.

Rf evaporation: We use a “U”-shaped wire (Fig. 2.1, light green wire) to deliver radiofrequency (rf) magnetic field which selectively removes high-momentum $^{87}\text{Rb}$ atoms from the trap. The remaining $^{87}\text{Rb}$ atoms collisionally rethermalize with a lower average energy. Collisions between $^{40}\text{K}$ and $^{87}\text{Rb}$ atoms sympathetically cool the fermionic potassium atoms in the process. It is not possible to evaporatively cool spin-polarized $^{40}\text{K}$ directly because the Pauli exclusion principle prohibits s-wave collisions between identical fermions. An advantage of the chip trap is that high gradients, and therefore tight trapping frequencies, are readily achievable. This allows rapid rethermalization through high rates leading to a relatively fast evaporation stage ($\approx 6$ s).

Transfer to optical trap: The optical trap consists of two laser beams which are crossed at their focus. While the atoms are still being evaporated on the chip, the “crossing beam” is slowly turned on to assist in mode-matching between the chip trap and the optical trap and to increase density. Additionally, the chip trap is weakened near the end of rf evaporation to prepare for mode-matching with the “trapping beam”. After rf evaporation, the intensity of both beams is ramped up, then the chip trap is ramped off, and the atoms are optically trapped.

State transfer: The remaining $^{87}\text{Rb}$ is transferred from $|F = 2, m_F = 2\rangle$ to its absolute ground state $|F = 1, m_F = 1\rangle$ with one step of microwave adiabatic rapid passage (ARP) applied with the microwave antenna (Fig. 2.1, dark purple looped wire). $^{40}\text{K}$ is transferred from $|F = 9/2, m_F = 9/2\rangle$ to its absolute ground state $|F = 9/2, m_F = -9/2\rangle$ with rf ARP through all of the $m_F$ states in $F = 9/2$. Once both species are in their ground states,
spin-changing collisions are energetically prohibited, which greatly enhances the lifetime of the mixture.

**Optical evaporation:** In the final stage of cooling, the hottest atoms are removed by weakening the optical trap until they fall out. This stage preferentially removes $^{87}\text{Rb}$ because the heavier atom has greater gravitational sag in the trap. At the end of cooling, residual $^{87}\text{Rb}$ atoms are removed with a resonant light pulse, leaving a spin-polarized cloud of 10000-70000 potassium atoms at temperatures as low as $\approx 0.15T_F$.

**Feshbach field:** We then ramp on a magnetic field near the Feshbach resonance at about 202 G. The field is produced by current through the MOT coils in Helmholtz configuration.

**Experiment:** A multitude of options are available at this stage, generally including state manipulation to a superposition or a mixture, then probing the system with spectroscopy, nuclear magnetic resonance (NMR) techniques, or thermometry.

**Field jump and imaging:** Our final measurement is an optical absorption image. Typically, we jump the field from the Feshbach resonance to 209G, where there are no interactions to affect the measurement. A combination of rf state manipulation with an option of a Stern-Gerlach pulse to separate spin states allows us to image various states as desired.

### 2.1.2 The chip

The atom chip was fabricated at the University of Toronto by D. Jervis [16]. Its design and implementation are thoroughly discussed the the PhD thesis of M. H. T. Extavour [13]. Figure 2.1 is a reproduction from that thesis, and shows the wires colour-coded for simplicity of discussion. Throughout the experimental cycle we use each wire at least once, as summarized in Table 2.1.
Figure 2.1: (a) Top view of the chip. The grey shading indicates one interconnected area which is grounded. The brightly coloured wires are summarized in Table 2.1. The black dots are through-holes for mounting the chip upside-down to a copper stack. (b) A zoomed view of the central portion of the chip. The $x$-direction wire lengths are indicated in micrometers. The diagram is not to scale. This image was originally produced in Ref. [13], which includes detailed dimensions of each wire.

<table>
<thead>
<tr>
<th>Chip wire</th>
<th>Colour</th>
<th>Use(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z-wire</td>
<td>blue</td>
<td>magnetic trapping, Stern Gerlach pulse</td>
</tr>
<tr>
<td>Rf-antenna</td>
<td>light green</td>
<td>$^{87}\text{Rb}$ rf evaporation, $^{40}\text{K}$ state preparation</td>
</tr>
<tr>
<td>Microwave antenna</td>
<td>dark purple</td>
<td>$^{87}\text{Rb}$ state preparation 6.8 GHz</td>
</tr>
<tr>
<td>U-wires</td>
<td>red</td>
<td>magnetic field gradient control</td>
</tr>
<tr>
<td>Bar wires</td>
<td>dark green</td>
<td>coherent state manipulation</td>
</tr>
<tr>
<td>Dimple wires</td>
<td>yellow</td>
<td>heating between cycles for experimental stability</td>
</tr>
</tbody>
</table>

Table 2.1: Summary of chip wires.
2.2 Crossed dipole trap

Our final trap is a crossed optical dipole trap (ODT). The basis of the trapping potential is the “ac Stark shift”, where the energy of states in the presence of far-detuned light is shifted by an amount proportional to the light intensity [17]. When the light is red-detuned from the transition, the energy of the ground state is lowered, and potential wells will form in light intensity maxima. The potential experienced by an atom in an intensity field $I(r)$ is

$$U(r) = -\frac{3\pi c^2}{2\omega_0} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) I(r), \quad (2.1)$$

where $c$ is the speed of light, $\omega_0$ is the resonant frequency, $\omega$ is the frequency of the light, and $\Gamma$ is the linewidth of the transition [17]. A focussed gaussian laser beam has a maximum intensity at its focus of $I_{\text{max}} = \frac{2P}{\pi w_0^2}$, where $P$ is the power, and $w_0$ is the waist. This gives a trap depth of

$$U_0 = -\frac{3\pi c^2}{2\omega_0} \left( \frac{\Gamma}{\omega_0 - \omega} + \frac{\Gamma}{\omega_0 + \omega} \right) \frac{2P}{\pi w_0^2}. \quad (2.2)$$

The spatial dependence of the potential in the radial direction of a focussed gaussian beam is

$$U(r) = U_0 e^{-2r^2/\omega_o^2}, \quad (2.3)$$

where $r$ denotes the radial coordinate. In a harmonic approximation of $U(r)$, the trapping frequency in the radial direction for an atom of mass $m$ is

$$\omega_r = \sqrt{\frac{4U_0}{mw_0^2}}. \quad (2.4)$$

The axial trapping frequency of a single beam is much weaker (dependent on the Raleigh range rather than the waist), which necessitates the use of a “crossing” beam to achieve low aspect ratio traps.

Our trap is made from two perpendicular beams of 1064 nm light, both focussed at the location of the magnetic chip trap. ODT 1 propagates along the $x$-direction and is the tighter “trapping” beam with a gaussian waist of 21 $\mu$m. ODT 2 propagates along the $z$-direction and has a waist of 80 $\mu$m. It is the weaker crossing beam, as it mostly contributes to the trapping
Figure 2.2: Crossed dipole trap and simplified chip wire geometry. ODT 1 and ODT 2 have beam waists of 21 µm and 80 µm respectively. At typical parameters, the crossed trap has a mean trapping frequency of $\bar{\omega}/2\pi = 486(15)$ Hz, and an aspect ratio of 4:1:1.

frequency but not so much to the depth. The trapping frequencies and trap depth can be varied by adjusting the power in each beam. The waist of each beam was determined by varying the power in each beam and measuring the appropriate trapping frequency, then relating them through Equations 2.2 and 2.4. Direct imaging of the beam waists would be simpler, however our imaging optics are aligned for 767 nm-780 nm light, not 1064 nm. The beams’ orientation with respect to the chip is depicted in Figure 2.2. After we have completed cooling the atoms, we typically choose the powers such that the mean frequency is $\bar{\omega}/2\pi = 486(15)$ Hz, and the aspect ratio of the trap is 4:1:1.

2.2.1 Dipole trap optics

The light for the dipole traps originates from a single 10W fibre laser. It is split into two arms, then the power of each beam is controlled by acousto-optic modulators (AOMs), which also shift the frequency of each beam by 80 MHz in opposite directions. The light is transported close to the chamber by two high power optical fibres. Prior to entering their focusing lenses,
each beam hits a piezo-controlled mirror, which allows us to easily and reproducibly move the beams around for final touches when aligning the crossed dipole trap to the magnetic chip trap, or, more importantly, to move the trapped atoms around to directly measure gradients in magnetic field or rf power (Sections 2.7.2 and 2.5.2). ODT1 and ODT2 travel through the glass cell on the same paths as our radial and axial imaging probes, as shown in Figure 2.3. The trap and probe light is combined and separated with dichroic mirrors. These dichroics are not perfect in that they allow a little dipole trap light to make it to the cameras, which greatly simplifies dipole trap alignment.

### 2.3 Temperature control: Spin polarized DFG

The hottest atoms are removed by weakening the optical trap and allowing them to fall out. This final stage of cooling preferentially evaporates $^{87}\text{Rb}$ atoms, now due to the greater gravitational sag of the heavier atom. This puts a limit on the lowest temperatures we can achieve, because at the very end of evaporation, the differential sag separates the two species and they are no longer in thermal contact. A high trapping frequency with low trap depth, as it is for small beam waists, helps mitigate this problem. The temperature of potassium is determined by the temperature of $^{87}\text{Rb}$ and can therefore be controlled by adjusting the final trap depth of evaporation. At the end of cooling, the trap is ramped back up to the desired trap depth, then residual $^{87}\text{Rb}$ atoms are removed with a resonant light pulse, leaving a spin polarized degenerate Fermi gas. This evaporation stage takes 1.5-3s. Figure 2.4 demonstrates the control over temperature that this method allows.
Figure 2.3: Optics setup surrounding the glass cell (blue box). Diagram not to scale.
Figure 2.4: Temperature control of a spin polarized degenerate Fermi gas. $^{40}\text{K}$ is sympathetically cooled by $^{87}\text{Rb}$. The temperature is controlled by adjusting the final depth of the crossed dipole trap. The temperature is measured by fitting a Fermi momentum distribution to the atoms' density distribution after expansion during time of flight, as described in References [18] and [19], and in Section 5.2.
2.4 From a boson machine to a fermion dream...

When I made the transition from being an underling working on the boson double well project to being the senior graduate student in charge of the chip lab, my first order of business was to make cold fermions again. The experiment had successfully produced cold fermions before [20], however, they were slowly abandoned as they were difficult to work with and the boson work was successful. Upon reviving potassium, things were much the same; low and unreliable atom number, long cycle times. The following summarizes our attempted strategies to improve atom number, stability, and temperature control.

**Increased power:** Originally, all $^{87}$Rb and $^{40}$K light for the MOT was combined in one tapered amplifier (TA) which yielded a maximum of $\sim 600$ mW of combined power after spatial filtering. We assembled two additional TAs, one for $^{40}$K trapping light and the other for $^{40}$K repump light, which increased our power output to 500 mW of $^{87}$Rb trap and repump combined, 550 mW of $^{40}$K trap and 200 mW of $^{40}$K repump. Separating the light had two additional benefits: the ability to provide individual feedback to stabilize each power, and the possibility of a dark spot MOT. **Verdict:** very useful.

**Dark spot MOT:** With the help of T. Pfau, we implemented a dark spot MOT to reduce loss due to light-assisted collisions between $^{40}$K and $^{87}$Rb [21]. The centre of the MOT has a region with (ideally) no $^{40}$K repump light, so that the cooled $^{40}$K atoms will fall into a dark state. For long MOT times, we have observed the final $^{40}$K MOT number to be up to 50% higher with the dark spot, however the effect on atom number after evaporation is not as large. The dark spot is not necessary to achieve our highest atom numbers, however it does seem to improve stability. **Verdict:** useful, probably mostly for stability.

**Pulsed dispensers:** Instead of running the dispensers in the morning, and relying only on LIAD for MOT vapour pressure for the rest to the day, we started running the dispensers in pulsed mode. During the MOT the current though the dispensers is high so they briefly dispense atoms. For the rest of the cycle the current is off. Between cycles, the currents take an intermediate value, so that they stay warm but not dispensing. This reduces daily drifts in
atom number. **Verdict:** very useful.

**Microwave evaporation of $^{87}\text{Rb}$:** The rf used to evaporate $^{87}\text{Rb}$ on the chip can remove very hot $^{40}\text{K}$ atoms. We tried switching to microwave evaporation of $^{87}\text{Rb}$ using the $|F = 2, m_F = 2\rangle$ to $|F = 1, m_F = 1\rangle$ transition, with the intention of keeping all $^{40}\text{K}$ atoms. Unfortunately, we also kept the tiny fraction of $|F = 2, m_F = 1\rangle$ $^{87}\text{Rb}$ atoms that made it into the chip trap. If we evaporated away all the $|2, 2\rangle$ atoms, we were left with a little BEC of $|2, 1\rangle$. In order to make sympathetic cooling of $^{40}\text{K}$ with microwave evaporation work, we had to stop half way through, remove the contaminant $|2, 1\rangle$ atoms, then continue. It worked to cool potassium, but showed no improvement in atom number over rf evaporation, and was a big hassle. **Verdict:** not useful.

**Evaporation in decompressed hybrid trap:** Before the final stages of evaporation on the chip, we decompress the magnetic trap while increasing the confinement on the weakest axis by ramping up the crossing beam. This aids in mode-matching for transfer to the dipole trap while avoiding three body loss by decompressing the trap. **Verdict:** slightly useful.

**Sympathetic cooling in the dipole trap** The original strategy to cool potassium in the dipole trap was to put potassium in a mixture of two spin states, and evaporate them directly. There are two downsides to this: 1) directly evaporating potassium means reducing the final atom number, and 2) for our desired experiment we need spin polarized potassium, so we would have to somehow eliminate one of the spin states after evaporation. We switched to sympathetic cooling with $^{87}\text{Rb}$ in the dipole trap. We first attempted this with $^{87}\text{Rb}$ in $|F = 2, m_F = 2\rangle$, before we had a microwave source that would allow us to change the hyperfine state of $^{87}\text{Rb}$. The lifetime in the trap was insufficient to get potassium to degeneracy. As soon as we put $^{87}\text{Rb}$ in $|F = 1, m_F = 1\rangle$ and $^{40}\text{K}$ in $|F = 9/2, m_F = -9/2\rangle$ (both ground states), sympathetic cooling worked beautifully. **Verdict:** great.

**Graham’s dispensers:** The aforementioned improvements to the experiments had helped our atom number, however over a couple of frustrating months the atom number declined at an increasing pace until one day, it was no longer workable. This was the great dispenser disaster
of 2011. The enriched $^{40}\text{K}$ dispensers from Alvatec had died in a particularly damaging way; they spewed a black substance over the inside of the glass cell (Figure 2.5). As a result, we no longer had a potassium source, and the cell was too opaque to work with rubidium, so we performed a vacuum change to replace the dispensers and clean the glass cell. This time, the potassium dispensers were made in house. G. Edge made the new enriched $^{40}\text{K}$ dispensers by following the chemistry from Reference [22] and modelling the structure of the dispenser after Reference [23]. Once the experiment was reassembled with Graham’s dispensers, the atoms were plentiful, cold, and stable. **Verdict:** indispensable.
2.5 Radiofrequency state manipulation

We use rf state manipulation for two main purposes central to our scientific goals, coherent state manipulation and spectroscopy. Coherent state manipulation requires Rabi frequencies $\Omega_R$ that are much faster than our decoherence time as well as homogenous Rabi frequencies across the trapped cloud. To achieve a good spectroscopic signal, the pulse area $\Omega_R t$ must be large. We perform time-resolved spectroscopic measurements which puts a limit on the pulse time $t$ of about 100 $\mu$s, so the signal also benefits from high Rabi rate. The atom chip is a great asset to achieve high $\Omega_R$ due to the proximity of the atoms to the antenna, however a homogeneous Rabi frequency is difficult for the same reason.

The states of interest in $^{40}$K are adiabatically connected to $|F = 9/2, m_F = -9/2\rangle$, $|F = 9/2, m_F = -7/2\rangle$ and $|F = 9/2, m_F = -5/2\rangle$ at low field. For simplicity, these states will be referred to as $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively. In Chapters 4, 5, and 6 the specific states are no longer important, so I will rather label them by their role in the experiment as $|\downarrow\rangle$, $|\uparrow\rangle$ and $|p\rangle$ respectively. At magnetic fields near the Feshbach resonance the relevant rf transitions are around 40 – 50 MHz.

2.5.1 Coherent state manipulation

The first step to mastering coherent state manipulation was seeing Rabi oscillations. Figure 2.6 shows one of our first successful Rabi oscillations. Here we tune the rf frequency to be resonant to the transition, then apply a pulse of variable length before measuring the population in the two states. The pulse drives an oscillation in population between the two states at the Rabi frequency $\Omega_R$. Once we know the Rabi frequency, we can, for example, choose the pulse time to transfer all atoms from one state to another (a $\pi$ pulse), or to put all atoms in a superposition of the two states (a $\pi/2$ pulse). Chapter 4 will expand on how we use nuclear magnetic resonance techniques to extract physical parameters from the system.
Figure 2.6: Rabi flopping at 202.1 G. The atoms start in $|a\rangle$ then oscillate to $|b\rangle$ and back while a resonant rf pulse is applied. The Rabi frequency of this oscillation is 7.2(4) kHz, however our maximum Rabi frequency is $\sim 150$ kHz.

### 2.5.2 Homogeneous Rabi frequency

The bar wires (Fig. 2.1, dark green wires) are connected in series to deliver rf to the atoms. We chose the bar wires for this purpose for two reasons. First, they are the furthest wires from the atoms which makes the Rabi frequency the most homogeneous. Second, the field they produce has the appropriate polarization for the transition. The phase delay between the wires is adjusted with a length of wire external to the vacuum system to get the field from each wire to add constructively in the middle, where the atoms are. This produces the largest Rabi frequency, and minimizes its gradient at the atoms. We measure the Rabi frequency simply by observing Rabi flopping, as in Figure 2.6. The gradient in Rabi frequency is measured by moving the atoms with piezo controlled mirrors on the dipole trap beams and repeating measurements of the Rabi frequency for various positions. Figure 2.7 shows the gradient measurement for different wire configurations. The best wire configuration gave a Rabi frequency gradient of $0.018 \%/\mu m$, and is used for the rest of the measurements in this thesis.
Figure 2.7: Minimizing the gradient in Rabi frequency. The diagram shows the field in the ideal configuration, when the currents through each wire are in phase with each other. The gradient in Rabi frequency was measured by moving the crossing beam from its standard position \((x = 40 \mu m)\), then repeating measurements of the Rabi frequency. The measurements were performed under various wire configurations with the same rf source output. Using only wire PE, the Rabi frequency at the atom’s standard position was 15.3 kHz, with a gradient of 0.1 %/µm. Similarly, TL only gave 11.9 kHz−0.091 %/µm. Wire TL connected to EP was 18.3 kHz+0.23 %/µm. Wire TL connected to PE was 13.4 kHz+0.12 %/µm. The highest Rabi frequency with the lowest gradient was achieved by adding a 1 m long cable between TL and PE. This gave a Rabi frequency of 41.5 kHz+0.018 %/µm.
Figure 2.8: Rf signal generation and control chain. Open ports of all switches are connected to 50 Ω terminators.

### 2.5.3 Rf signal generation and control

We switch between three separate sources for coherent rf state manipulation and spectroscopy. The rf source and control chain is depicted in Figure 2.8.

**Micromatic:** The signal is generated through direct digital synthesis (DDS) [24]. The amplitude output is constant so the square pulses are generated through switches. The amplitude of the square pulse can be controlled with the voltage variable attenuator.

**Phaseomatic:** This DDS source has the additional capability of an external control of the gain, which is used for generating shaped pulses. It also provides a phase-shifted output with variable gain that we do not use [24].

**Keithley:** The Keithley 3390 is an arbitrary waveform generator which has an external analog input to control the phase of the signal. The phase can be switched in 60 µs. This feature is used for measuring Ramsey fringes.

Square pulses are controlled by a signal from the ADwin (10 µs or longer) or from an SRS signal generator (shorter than 10 µs) which activates a switch. Shaped pulses are made with a shaped signal from an SRS fed into the Phaseomatic gain. The timing between pulses is controlled by the ADwin.
2.5.4 Pulse shaping

The shape of an rf spectrum is the convolution of the lineshape from the ensemble of atoms and the Fourier transform of the rf pulse. For short pulses, the spectrum is dominated by the pulse’s Fourier transform. While a square pulse is simple to produce, its Fourier transform is a sinc function which features substantial sidebands in the wings of the spectrum (see Figure 2.9a). The spectroscopy performed in Chapter 6 relies on sensitive detection of small signal in the wings. It is therefore essential to remove these sidebands. This is achieved with a Blackman pulse, shown Figure 2.9b. The trade-off for flat wings is a wider central peak, which would make a Blackman pulse worse than a square pulse for precise frequency measurements.

The Blackman window is given by

\[ w(t) = a_0 - a_1 \cos \left( \frac{2\pi t}{\tau} \right) + a_2 \cos \left( \frac{4\pi t}{\tau} \right) \]  

(2.5)

where \( a_0 = 0.42659 \), \( a_1 = 0.49656 \), \( a_2 = 0.076849 \), and \( \tau \) is the total pulse length. The pulse has zero amplitude outside of \( 0 \leq t \leq \tau \).

Figure 2.10 shows several single-particle spectra from our first pulse-shaping attempts. The signal is the number of atoms transferred from a cloud of atom in the \( |b\rangle \) state only to the \( |c\rangle \) state. We successfully removed the sidebands in the spectrum with pulse shaping.
Figure 2.9: Pulse shapes and their Fourier transforms. a) 20 $\mu$s square pulse. FWHM = 60 kHz. Significant sidebands are present in the Fourier transform. b) 20 $\mu$s Blackman pulse. FWHM = 113 kHz. The structure in the wings of the Fourier transform is gone.
Figure 2.10: Effect of pulse shaping on the single particle spectrum. a) square b) cosine c) Blackman.


2.6 Tuning interactions: magnetic field control

We control the interactions between atoms by employing a Feshbach resonance (See Chapter 3). This requires the magnetic field to be stable and tuneable around \( B_z = 202.1 \) G. We control the field and its gradients with three tools: the Feshbach coils for the main field and its stability, the jump coils for fast switching of the field, and wires on the chip for gradient control.

2.6.1 The Feshbach field

The Feshbach field is produced by \( \approx 30 \) A of actively stabilized current through coils in Helmholtz configuration. The coils were originally designed primarily for the MOT and magnetic trapping stages of the experiment. As such, they are not centred on the atoms in the chip trap, but rather 2.5 cm below. This creates a gradient of \( \partial_y B_z \approx 10 \text{ G/cm} \) at the atoms, which can be compensated with current through chip wires if desired (Section 2.7). The stabilizing circuit was initially built by L. J. LeBlanc [14] then modified to its current form by N. Cheng and S. Trotzky [25].

Field calibration

The field is calibrated with rf spectroscopy, typically by measuring the \(|a\rangle\) to \(|b\rangle\) transition frequency. The magnetic field shifts the energy of this transition through the Zeeman effect. We convert the measured frequency to magnetic field through the Breit-Rabi formula [26]. The linewidth gives a conservative estimate of the stability of the magnetic field throughout the rf pulse. For instance, at \( B_z = 202.10 \) G, the stability of the field is reflected in a 1 kHz-wide spectral line centred at 44.817 MHz, which means the field is stable to at least two parts in \( 10^5 \) (Figure 2.11). The field can be stabilized with this precision from 177 G to 213 G.
Figure 2.11: Demonstration of field stability. This line was taken with a weak 20 ms long pulse. The full width half max of the Lorentzian width is 1.0(1) kHz, which corresponds to a field stability of 4 mG at 202 G. The transferred fraction does not reach 50% because the pulse area was smaller than $\pi$. The negative offset is the result of poor background subtraction in the images.
2.6.2 Jumping the field

It is difficult to change the current in the Feshbach coils quickly because of their large inductance. Typically it takes about 5 ms to change the field by 10 G if using only the Feshbach stabilization circuit. This is insufficient if you wish to abruptly change the interaction strength, for example, to turn off interaction effects during imaging. We use an additional set of coils (the $z$-bias coils) mounted on the outside of the Feshbach coils for this purpose. The “jump” circuit was built by N. Cheng [25] and works by suddenly discharging a high-voltage capacitor through the coils. The initial voltage is high to change the current quickly, then reduced to the steady state value to provide a constant offset field. For the purpose of turning off the interactions, the resultant field need not be as stable as when interactions are resonant because the dependence of the scattering length on the field is relatively flat near the zero crossing of the resonance. It takes the jump circuit approximately 10 $\mu$s to turn on the offset current required to move the field from 202 G to 209 G (zero interactions). Figure 2.12 demonstrates the current jump in the $z$-bias coils and the induced current in the Feshbach coils upon triggering the jump circuit.

2.7 Magnetic field gradient control

Gradients in magnetic field will cause atoms on one side of the cloud to evolve differently than atoms on the other. In Chapter 4 we use control of the gradient to test our model of spin diffusion.

2.7.1 Gradient production

We control the magnetic field gradients $\partial_y B_z$ and $\partial_z B_z$ by adjusting the sum $\Sigma I$ and the difference $\Delta I$ of small currents through the pair of “U”-wires on the chip. The vector fields produced by $\Sigma I$ and $\Delta I$ are depicted in Figures 2.13a and 2.14a respectively. When these fields are added to the much larger applied external field $B_z$, only the $z$-component of the chip
Figure 2.12: Jump circuit response. a) and b) show the field from the $z$-bias coils and the Feshbach coils after triggering the jump circuit. The plotted fields are the signal from current sensors on each coil scaled with a previously measured static field calibration for each coil. Any other transient field sources, such as induced eddy currents, are not captured by this measurement. c) and d) show the combined field response on different time scales.
Figure 2.13: Controlling $\partial_y B_z$. a) The vector field of gradient wires only with $\Sigma I = 50$ mA and $\Delta I = 0$ mA. b) The variation in the $z$-component of the magnetic field when the vector field in a) is added to a constant offset field of $B = B_z = 202 G$ demonstrating $\partial_y B_z = 8.8 G/cm$ and $\partial_z B_z = 0.0 G/cm$. The curvatures are $\partial^2_y B_z = 80 G/cm^2$ and $\partial^2_z B_z = 0.0 G/cm^2$.

wire field contributes significantly to the total field. The resultant gradients are depicted in Figures 2.13b and 2.14b. Typically we choose $\Sigma I$ to eliminate the gradient $\partial_y B_z$ that comes from the external field (the off-centre coils mentioned in Section 2.6.1). We then adjust $\Delta I$ to achieve the desired gradient. We have no control over $\partial_x B_z$.

2.7.2 Gradient calibration

We directly measure the gradients by repeating rf-spectroscopy measurements of the field (as in Section 2.6.1) on a cloud which is translated by piezo-actuated mirrors on the trapping beams. The cloud position is measured using two orthogonal imaging systems, whose magnification is calibrated by a dropped $^{87}$Rb cloud in a magnetic-field-insensitive state, $|F = 1, m_F = 0 \rangle$. The effective pixel size for the axial and radial imaging is found to be $2.90 \pm 0.04 \mu m$ and $3.76 \pm 0.17 \mu m$ respectively. The uncertainty in pixel size results in a systematic gradient
Figure 2.14: Controlling $\partial_z B_z$. a) The vector field of gradient wires only with $\Sigma I = 0$ mA and $\Delta I = 50$ mA. b) The variation in the $z$-component of the magnetic field when the vector field in a) is added to a constant offset field of $\textbf{B} = B_z = 202G$ demonstrating $\partial_y B_z = 0.0$ G/cm and $\partial_z B_z = 16$ G/cm. The curvatures are $\partial^2_y B_z = 3.0$ G/cm$^2$ and $\partial^2_z B_z = 1600$ G/cm$^2$.

uncertainty which would scale gradients in the $x$-direction by $\pm 1.4\%$ and in the $y$- and $z$-directions by $\pm 4.5\%$.

Figures 2.15 and 2.16 summarize the gradient control and calibration while adjusting the sum $\Sigma V$ and difference $\Delta V$ of the voltages applied across the wires. The voltages are measured across $10 \, \Omega$ resistors, so $0.1 \, V \rightarrow 10 \, mA$ of current.

In Figure 2.15 we varied the sum to find the value that cancels the $y$-gradient. When $\Sigma V \neq 0$, the field from the wires adds an offset (not just a gradient) to the field so the external field is adjusted to keep the total field near 202.1G. Figure 2.15a demonstrates our gradient measurement technique. Figure 2.15b shows that the gradient varies linearly with $\Sigma V$. When $\Sigma V = 0$, the $y$-gradient from the external field is revealed as $10.4$ G/cm. $\Sigma V = 0.43 \, V$ zeroes the $y$-gradient.

In Figure 2.16, $\Sigma V$ is fixed to cancel the $y$-gradient, and $\Delta V$ is varied. Figure 2.16a show the direct measurement of the $z$-gradient which is summarized in Figure 2.16b. Figure 2.16c
Figure 2.15: Gradient calibration: controlling the $y$-gradient by adjusting the sum of the voltages across the two wires. The voltage is measured across a 10 $\Omega$ resistor, so $1 \text{ V} \rightarrow 10 \text{ mA}$ of current. a) Direct measurements of the magnetic field (via rf spectroscopy) as a function of the $y$ coordinate for various voltage sums. b) Dependence of the $y$-gradient of the magnetic field on the voltage sum, using data from a).

show the the dependence of the $x$- and $y$-gradients on $\Delta V$. Note that the scale is much smaller in these plots. For large $\Delta V$ the total gradient is dominated by the $z$-gradient, as expected. The minimum achievable total gradient of 1.6 G/cm stems primarily from the residual gradient in the $x$-direction, which we have no independent control over. An $x$-gradient of this magnitude could be produced by a displacement between the Feshbach coils and the trap of 40 mm in the $x$-direction.
Figure 2.16: Control of the $z$-gradient by adjusting the difference of voltages after setting the $y$-gradient to zero by fixing the sum. a) Magnetic field as a function of the $z$ coordinate for various voltage differences. b) Dependence of the $z$-gradient on the voltage difference, using data from a). c) Dependence of the $x$- and $y$-gradients on the voltage difference. d) Dependence of the magnitude of the total gradient on the voltage difference as obtained from the calibration.
Chapter 2. Apparatus

2.8 Imaging

2.8.1 Imaging optics

We have two orthogonal imaging systems, “radial” and “axial” imaging. Both cameras are looking directly into the two dipole beam paths, as shown earlier in Figure 2.3. Absorption imaging with the radial camera integrates the cloud’s optical density along the long axis of the trap, providing an image of the $y - z$ density distribution after time of flight. Similarly, the axial camera provides a $y - x$ image.

We typically perform optical absorption imaging of $^{40}$K on the $|F = 9/2, m_F = -9/2\rangle$ to $|F = 11/2, m_F = -11/2\rangle$ transition after jumping the field from the Feshbach resonance to 209G, where there are no interactions to affect the measurement. A combination of rf state manipulation with an option of a Stern-Gerlach pulse to separate spin states allows us to image various states as desired. Some data presented in this thesis (most of Chapter 3) were taken before we had the ability to image at high field, so the field was ramped back down to $\approx 5$ G where multiple states can be imaged at once without rf manipulation.

Low field probes

At low field, we image $^{87}$Rb on the $5^2S_{1/2} |F = 2, m_F = 2\rangle$ to $5^2P_{3/2} |F = 3, m_F = 3\rangle$ cycling transition. If we wish to image atoms in $|F = 1, m_F = 1\rangle$, we need to include some repump light on the $5^2S_{1/2} |F = 1\rangle$ to $5^2P_{3/2} |F = 2\rangle$ transition. This combined probe light is also used after the final evaporation stage to remove the remaining $^{87}$Rb atoms from the trap.

During the cycle, we move $^{40}$K atoms from $|F = 9/2, m_F = +9/2\rangle$ to $|F = 9/2, m_F = -9/2\rangle$ ($|a\rangle$) through all the intermediate $m_F$ states. To image the extreme states, we use the $4^2S_{1/2} |F = 9/2, m_F = \pm 9/2\rangle$ to $4^2P_{3/2} |F = 11/2, m_F = \pm 11/2\rangle$ cycling transitions. We switch between those transitions simply by reversing the quantizing magnetic field to flip the light’s effective polarization from $\sigma^+$ to $\sigma^-$ without touching any optics, then make small adjustments to the frequency with an AOM. Following a Stern Gerlach pulse, intermediate $m_F$
states are visible (if populated), as the low magnetic field does not separate the resonances for adjacent Zeeman states by more than the laser linewidth.

**High field probes**

At high field, we use $^{40}$K atoms in states $|a\rangle$, $|b\rangle$, and $|c\rangle$ only. Optical transitions from these states are separated by more than the linewidth of our laser, so they cannot be imaged at the same time. We image using the $|a\rangle$ to $|F = 11/2, m_F = -11/2\rangle$ cycling transition. It is possible to directly image atom in states $|b\rangle$ or $|c\rangle$, however there are no cycling transitions available, so the signal is weak. This makes it preferable to transfer the atoms to $|a\rangle$ prior to the imaging pulse.

In high field imaging, the Feshbach field provides the quantizing field for the atoms so it is only possible to get the highest efficiency imaging using the axial camera. Since this is our most important probe, we take great care in avoiding fringes in the probe structure by imaging the shadow of a razor blade onto the chip. This allows the probe to pass cleanly below the edge of the chip and therefore avoids fringes in the image.

### 2.8.2 State-selective imaging

Our imaging scheme allows us to simultaneously count the populations of atoms in states $|c\rangle$ and $|a\rangle$, leaving atoms in state $|b\rangle$ invisible. This is achieved with a Stern-Gerlach pulse to separate the trapped spin states, rf state manipulation during time-of-flight in a gradient, and finally, resonant absorption imaging on the $|a\rangle$ to $|F = 11/2, m_F = -11/2\rangle$ cycling transition (Figure 2.17). If we instead wish to count the atoms in $|b\rangle$ and $|a\rangle$, we precede the process with a mapping of $|b\rangle \leftrightarrow |c\rangle$ via an adiabatic rapid passage. Imaging occurs after jumping the magnetic field to 209 G, the zero crossing of the s-wave scattering resonance, in order to minimize interaction effects during time of flight.
Figure 2.17: Schematic of state selective imaging. A mixture of atoms in states $|c\rangle$, $|b\rangle$, and $|a\rangle$ (red, green and blue clouds, respectively) is released from the trap. A pulse of current through the “z”-wire generates a high gradient which provides a differential momentum kick to the atoms (Stern Gerlach pulse). The red (green, blue) line indicates the trajectory that $|c\rangle$ ($|b\rangle, |a\rangle$) atoms will travel. Before the atoms have time to separate, a $\pi$ pulse swaps the states of atoms in $|c\rangle$ and $|b\rangle$. The atoms fall and separate. A weak vertical gradient $\partial_y B_z$ differentially shifts the transition frequencies of the three clouds, which allows us to apply a $\pi$-pulse from state $|b\rangle$ to $|a\rangle$ to atoms on the red path (originally state $|c\rangle$) without affecting atoms on the blue path ($|a\rangle$). Finally, the blue clouds are imaged with resonant absorption imaging on the $|a\rangle$ to $|F = 11/2, m_F = -11/2\rangle$ cycling transition.
Chapter 3

Resonant Interactions

Feshbach resonances are an important tool in the field of ultracold atoms as they allow us to tune the interactions between atoms to be strongly attractive, strongly repulsive, non-existent, or anywhere in between. The experiments in this thesis employ a Feshbach resonance in $^{40}$K to achieve “unitary” interactions. In this limit, the scattering cross-section reaches the maximum value allowed by quantum mechanics, roughly the square of the thermal de Broglie wavelength. This chapter describes resonant interactions in atomic Fermi gases, then evaluates how well we achieve the unitary condition.

3.1 Interactions in ultracold atomic gases

Low-energy scattering is most often described by the method of partial waves (see Ref.[27] for example). The low energy condition is applicable when the de Broglie wavelength corresponding to the relative momentum of the particles is larger than range of the potential that characterizes the interaction. The following discussion outlines this method.

Consider two distinguishable atoms with relative momentum $\hbar k$ interacting via the potential $V(R)$, where $R$ is the vector between the positions of each atom. At large separation $V(R) \to 0$, and the energy of the pair is $E = \hbar^2 k^2 / 2m_r$, where $m_r$ is the reduced mass. The incident plane wave that describes these two atoms can be expanded as a sum of spherical
harmonics $Y^m_{\ell}$, which are written in terms of the relative angular momentum of the atoms, $\ell$. If the interaction potential is spherically symmetric, $V(R) = V(r)$, angular momentum will be conserved, and only the radial component of the wavefunction will be affected by the interaction. The radial wavefunction $\psi_{\ell}(R) = \phi_{\ell}(R)/R$ obeys

$$\frac{-\hbar^2}{2m_r} \frac{d^2 \phi_{\ell}(R)}{dR^2} + V_\ell(R)\phi_{\ell}(R) = E\phi_{\ell}(R).$$

This is simply a one-dimensional Schrödinger equation where the effective potential includes a centrifugal barrier,

$$V_\ell(R) = V(R) + \frac{\hbar^2 \ell (\ell + 1)}{2m_r R^2}. \tag{3.2}$$

Atomic interactions are typically viewed as having strong repulsion at close distances and an attractive Van der Waals tail $V(R) \to -C_6/R^6$, for large $R$. The characteristic length of this potential is $r_0 = (m_r C_6 / \hbar^2)^{1/4}$. Including the centrifugal barrier from Equation 3.2, one can define the energy threshold for scattering from a Van der Waals potential

$$E^{th}_\ell = -\frac{C_6}{r_{max}^6} + \frac{\hbar^2 \ell (\ell + 1)}{2m_r r_{max}^2}, \tag{3.3}$$

where $r_{max} = \left( \frac{6}{\ell(\ell+1)} \right)^{1/4} r_0$ is the radius of an effective barrier for partial wave scattering. Thus when $\ell = 0$ (“s-wave” scattering), there is no energetic barrier for collisions. In ultracold atomic gases, the $p$-wave ($\ell = 1$) barrier is much higher than the temperature, which renders $p$-wave (and higher $\ell$) collisions energetically inaccessible. For example, in our system $^{40}$K is cooled to 300 nK - 1200 nK, and the $p$-wave threshold in $^{40}$K is $E^{th}_1 \approx k_B \cdot 100 \mu K$ [28]. Reference [29] contains a direct measurement of the suppression of $p$-wave scattering below this threshold.

Equation 3.1 yields a finite number of bound solutions ($E < 0$) and a continuous spectrum of scattering solutions ($E > 0$). At large $R$ the scattering solutions are the sum of the incident plane wave with momentum $\hbar k$ and a scattered wave composed of the partial waves. The effect of $V(R)$ is captured for each partial wave with one parameter, the phase of the outgoing wave $\eta_\ell(k)$. Here, we only consider s-wave scattering. In this limit, it is useful to define the
scattering length $a$, 
\[
a = - \lim_{k \to 0} \frac{\tan \eta_0(k)}{k}, \tag{3.4}
\]
which can have any value, $-\infty < a < +\infty$. The $s$-wave scattering cross section depends on both the scattering length and the relative momentum of the colliding atoms, 
\[
\sigma = \frac{4\pi a^2}{1 + k^2 a^2}. \tag{3.5}
\]

When the system is weakly interacting, $ka \ll 1$, Equation 3.1 is independent of the energy of the colliding atoms, $\sigma = 4\pi a^2$. If the $s$-wave scattering length diverges, the cross-section reaches the unitarity limit $\sigma = 4\pi/k^2$.

The above discussion is for distinguishable particles; however, if the atoms in question are identical fermions, the Pauli exclusion principle prohibits $s$-wave scattering. This is a consequence of the anti-symmetry of the wavefunction with respect to exchange of identical fermions. Thus, while a mixture of ultracold fermionic atoms in two (or more) internal states will interact via $s$-wave collisions, a similar fully polarized gas will have no interactions even if the atoms are prepared in a superposition of internal states that do have a large scattering length.

### 3.2 What is a Feshbach resonance?

A Feshbach resonance occurs when the energy of two colliding atoms becomes equal to the energy of a bound state of the two atoms [30]. If the magnetic moment of the unbound pair differs from the bound pair by $\delta \mu$, the resonance condition can be achieved by tuning an external magnetic field $B$ to adjust the relative energy of the two states, as illustrated in Figures 3.1a and b. As the energy of the bound state is tuned from below that of the unbound state to above it, the $s$-wave collisional phase $\eta_0$ changes by $\pi$. As a result, the scattering length diverges at the resonance (Figure 3.1c). Tuning the magnetic field to either side of the resonance produces either repulsive ($a > 0$) or attractive ($a < 0$) interactions. The $s$-wave scattering length as a
function of the magnetic field $B$ is often parameterized as [30]

$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_0}\right)$$

(3.6)

where $a_{bg}$ is the background scattering length and $\Delta$ is the width of the resonance. When $B = B_0 + \Delta$, $a = 0$.

On the repulsive side of the resonance, the bound state is lower in energy than the unbound pair. For small binding energies, atoms can associate into molecules through collisions making a mixture of free atoms unstable to decay. It is possible to coherently associate these weakly bound Feshbach molecules by adiabatically ramping the field from above the resonance to below it [31]. The binding energy of a Feshbach molecule is

$$E_b = \frac{\hbar^2}{2m_r a}$$

(3.7)

when the scattering length is large ($a \gg r_0$). Once Feshbach molecules are present, they may relax through collisions to more deeply bound states releasing kinetic energy [32, 33]. This loss process poses challenges to experiments on the repulsive side of the resonance, complicating the interpretation of results [34].

### 3.3 Feshbach resonance as a tool for quantum simulation

The discussion now moves from interactions between two atoms to the behaviour of a many-body system, namely a degenerate Fermi gas, that interacts via two-body collisions. In this scenario, collisions are restricted to fermions near the Fermi surface which have a characteristic momentum $\hbar k_F$. The Feshbach resonance allows us to create wildly different materials with one experiment simply by changing the external magnetic field, and therefore the scattering length (Figure 3.2) [2]. When $a = 0$, the system is an ideal Fermi gas. The case of weakly attractive interactions ($1/k_F a \to -\infty$) is described by Bardeen-Cooper-Schrieffer (BCS) theory. Here, a superfluid of Cooper pairs will form below a critical temperature [35]. On the other side of the resonance ($1/k_F a \to +\infty$), the ground state consists of bound molecules.
Figure 3.1: Two channel model of a Feshbach resonance. a) Interatomic potentials for the bound (red) and unbound (blue) states. b) Resonance condition at magnetic field $B_0$. c) Divergence of the $s$-wave scattering length at a Feshbach resonance given by Equation 3.6. The inset illustrates the binding energy of the molecular state on the repulsive side of the resonance.
Figure 3.2: Quantum simulation at Feshbach resonances. The behaviour of a Fermi gas can be manipulated by tuning interactions. To produce rich phase diagrams, experiments may also vary temperature, polarization, dimensionality, or add a lattice. The experiments in Chapters 4, 5, and 6 all take place exclusively in the unitary regime.

Each molecule is made of two fermions and therefore is a boson. These are weakly interacting bosons that will form a Bose Einstein condensate (BEC) [36]. The crossover between these well understood limits \(1/|k_F a| \lesssim 1\) is difficult to describe theoretically as there is no small parameter for perturbation theories. Superfluidity is maintained throughout the crossover [37], even though the nature of the pairing varies: BCS pairs are larger than the typical interparticle spacing given by the density and the molecule size is smaller. Evidence of ferromagnetism has not yet been observed unambiguously in the upper branch of the repulsive side of the resonance because rapid atom loss to molecules hinders equilibrium experiments with repulsively interacting free atoms\(^1\) [34].

\(^1\)There could still be hope for a ferromagnetic signal in dynamic experiments!
3.3.1 The unitary regime

This thesis focusses on the unitary regime \( (1/|k_Fa| \ll 1) \). At unitarity, the scattering length diverges, and therefore becomes irrelevant. The scattering cross-section saturates to, \( \sigma = 4\pi/k^2 \). For a degenerate Fermi gas, this becomes \( \sigma = 4\pi/k_F^2 \). In this limit, there is no remnant of any parameter or length scale that originated from the microscopic interaction. Thus, unitary Fermi gases have universal properties that are valid for any unitary Fermi gas, regardless of the origin of the interaction. Universal parameters extracted from cold atoms experiments, such as ours with \(^{40}\)K, will be applicable not only to all other atomic species in this limit, but also, for example, to the crust of neutron stars at twenty-five orders of magnitude higher density [38, 39].

3.4 Feshbach resonances in \(^{40}\)K

There are many Feshbach resonances in \(^{40}\)K which are nicely summarized in Reference [28]. For all of Chapters 4, 5, and 6 we use the \( s \)-wave resonance between the states \(|a\rangle\) and \(|b\rangle\) at 202.1 G, but we have also observed the \(|b\rangle\) \( p \)-wave resonance at 198 G and the \(|b\rangle\) and \(|c\rangle\) \( s \)-wave resonance at 174 G through loss.

3.4.1 Our observations of \( s \)- and \( p \)-wave resonances

We have used the loss of atoms to molecules and the associated trap loss to observe several Feshbach resonances in \(^{40}\)K. Figure 3.3 shows loss at the \( s \)-wave resonance 202 G in a mixture of atoms in \(|b\rangle\) and \(|a\rangle\). That loss feature is absent if only one species is present. There is a \(|b\rangle\) \( p \)-wave resonance near 198 G which reveals itself as loss whenever atoms in \(|b\rangle\) are present. Figure 3.4 shows loss at the \( s \)-wave resonance between \(|c\rangle\) and \(|b\rangle\) at 174.3G.

The peak in the loss rate near a Feshbach has been observed to be shifted slightly to the repulsive side of the resonance [34]. A two step loss mechanism, where atoms first form Feshbach molecules then decay to deeply bound molecules, can explain this shift [33]. This
Figure 3.3: Observation of $s$- and $p$-wave Feshbach resonances by loss during a 25ms hold time at each field value. Total atom number is imaged after sweeping to low field. a) Cloud contains only $|a\rangle$ atoms. No loss feature observed. b) Only $|b\rangle$ atoms present. $p$-wave loss feature is observed. c) Mixture of $|b\rangle$ and $|a\rangle$. Loss observed at the $p$-wave and the $s$-wave resonances. Grey bars are the calculated fields of the $|b\rangle$ $p$-wave, 198.4 G and 198.9 G, and the $|a\rangle$ and $|b\rangle$ $s$-wave resonances, 202.1 G from Reference [28]. The number drop from left to right in b) and c) is due to the crossing of resonances during the slow ramp down of the field.
Figure 3.4: Observation of $|b\rangle$ and $|c\rangle$ Feshbach resonance at 174.3 G [28]. The system is prepared as a mixture of atoms in $|a\rangle$ and $|c\rangle$. At the desired field, atoms in $|a\rangle$ are swept to $|b\rangle$ in 5 ms with rf adiabatic rapid passage, held for 20 ms, then swept back to $|a\rangle$ to end the hold time. The field is then lowered for low field imaging. The black and grey points are coarse and fine sampling of the feature with the same procedure.

shift in the peak loss rate means that the loss feature is not accurate in determining the centre of the resonance, $B_0$, but it provides an estimate.

An aside: Loss as a motivation for dynamic approach

It is clear from Figures 3.3 and 3.4 that one cannot neglect loss when working near a Feshbach resonance. We have two strategies for minimizing the effects of loss: 1) choose the experimental timescale to be faster than the loss timescale. 2) keep the atoms spin-polarized as long as possible to suppress $s$-wave collisions (thus increasing the loss timescale). Both strategies are accomplished by preparing the atoms in a superposition of the resonantly interacting states, and performing measurements on the timescale of the loss of coherence. Figure 3.5 shows the atom number loss curve after preparing a superposition of Feshbach states. Atom loss during the coherence time is small.
Figure 3.5: Weak loss of atom number during the coherence time of superposition. Atoms are prepared in a superposition of $|a\rangle$ and $|b\rangle$ at 202.1 $G$. After a hold time, the number remaining in $|a\rangle$ is measured. The superposition decoheres with a $1/e$ coherence time of 2.3 ms (marked in grey, and studied further in Chapter 4).

### 3.4.2 The $|a\rangle$ and $|b\rangle$ s-wave resonance in $^{40}$K

The parameters describing the $|a\rangle$ and $|b\rangle$ s-wave resonance in $^{40}$K have been both calculated and determined experimentally with only a small variation in the values. The calculations presented in Reference [28] agree well with 26 different measured Feshbach resonances in $^{40}$K, so I will use its calculated values for the $|a\rangle$ and $|b\rangle$ for the remainder of this thesis. These values are: $B_0 = 202.1\,G$, $\Delta = 6.9\,G$, and $a_{bg} = 167\,a_0$. The divergence of the scattering length is plotted in Figure 3.6a. The binding energy given by Equation 3.7 is plotted in Figure 3.6b (black line). A finite-range correction to the binding energy becomes relevant when the scattering length is no longer much larger than the range of the Van der Waals potential, $r_0 \approx 60a_0$ [40]. The correction,

$$E_b = \frac{\hbar^2}{m(a - r_0)^2},$$  \hspace{1cm} (3.8)
Figure 3.6: a) and b) Feshbach resonance. a) Divergence of the $s$-wave scattering length given by Equation 3.6, where $B_0 = 202.1\, G$, $\Delta = 6.9\, G$, and $a_{bg} = 167a_0$ [28]. b) Binding energy of Feshbach molecules given by Equations 3.7 (black) and 3.8 (red).
3.4.3 Rf spectroscopy of molecules

One way of making Feshbach molecules is to start with a mixture of the two species well above the resonance, then ramp the field down below the resonance. This adiabatically converts the atoms to Feshbach molecules whose binding energy is given by Equation 3.7 or 3.8. It is possible to break these molecules by flipping an atomic spin with an rf photon that has enough energy both for the atomic spin flip as well as the binding energy, as in Figure 3.7a. Rf dissociation spectroscopy has been used to directly probe Feshbach molecules [41].

Figure 3.7c shows an observation of molecules from our lab. The atoms were prepared in a mixture of $|a\rangle$ and $|b\rangle$ at 204 G. The field was ramped to 200.0 G (as determined by prior single particle spectroscopy on the $|a\rangle \rightarrow |b\rangle$ transition) in 5 ms to associate molecules. This was followed by a hold time of 2 ms to let the field settle to the accuracy determined in Section 2.6.1. An 80 $\mu$s Blackman pulse with a frequency detuning $\delta$ from the $|b\rangle \rightarrow |c\rangle$ transition probed the system. We measure the transfer rate by counting the relative number of atoms in the $|c\rangle$ state.

The peak near $\delta = 0$ is the spectrum of the unbound atoms. The rise in spectral weight at 210 kHz indicates the binding energy of the molecules. Above 210 kHz, the molecules are being dissociated and given additional kinetic energy. Equation 3.7 underestimates the binding energy of these molecules by $\approx 30$ kHz. The binding energy including the finite-range correction in Equation 3.8 is indicated by the solid red line at $\delta = 210$ kHz in Figure 3.7c. This is naively excellent agreement between the experiment and theory, however, this analysis neglects the spectral composition of the pulse. The Fourier transform of the 80 $\mu$s Blackman pulse has a full width at half maximum of $\approx 28$ kHz (refer to Figure 2.9 for comparison). This means that the onset of molecular signal in the rf spectrum will be at a detuning $\approx 14$ kHz below the actual binding energy, so the measured spectrum suggests a binding energy closer to $\delta = 224$ kHz. This discrepancy could arise for two reasons. Either, the molecules are relatively deeply bound, so the finite-range correction is insufficient (see Reference [40] for a calculation that increases this correction slightly), or the value of $B_0 = 202.10$ G is incorrect. If it was
instead $B_0 = 202.20$ G, as measured in Reference [42], the calculated binding energy would be 227 kHz (dashed red line).

The atomic peak in Figure 3.7c is shifted to $\delta = -5.7(7)$ kHz (see Figure 3.7c inset). This is likely due to the clock shift - a density dependent shift arising due to interactions between atoms (Figure 3.7b). The mean field energy shift of an atom in state $|b\rangle$ due to the whole cloud of atoms in state $|a\rangle$ is $\frac{4\pi\hbar^2}{m} a_{ab} n_a$ when the scattering length between them, $a_{ab}$ is small, and $n_a$ is the density of atoms in state $|a\rangle$. Interactions between the final state $|c\rangle$ and $|a\rangle$ would shift the transition frequency by $\frac{4\pi\hbar^2}{m} a_{ac} n_a$, where $a_{ac}$ is the scattering length between $|c\rangle$ and $|a\rangle$. The total shift in the transition frequency is

$$\Delta \omega = \frac{4\pi\hbar}{m} \frac{a_{ac}}{a_{ab}} (a_{ac} - a_{ab}) n_a \quad (3.9)$$

when $1/k_F > a_{ab} > r_0$ [18]. For the system in Figure 3.7c, $1/k_F = 1373a_0$, $a_{ab} = 716a_0$ and $r_0 = 60a_0$, so Equation 3.9 applies. The density of atoms in $|a\rangle$ is $\approx 4 \times 10^{13}$ cm$^{-3}$ prior to molecule formation, however, the density will be lower after atoms are converted to molecules. The result of Equation 3.9 for two estimates of density are plotted in green in Figure 3.7c. Both estimates assume $\approx 80\%$ of atoms are converted to molecules that subsequently don’t interact with atoms in either the $|c\rangle$ or $|b\rangle$ state. The low density estimate assumes no redistribution of density after molecules are formed. The high density estimate assumes the remaining atoms equilibrate to form their own smaller Fermi surface.

The clock shift in a unitary Fermi gas cannot depend on the divergent scattering length, $a_{ab} \to \infty$, but rather is determined by final state interactions. In the case of $^6$Li near a Feshbach resonance, the final state is also strongly interacting, so the clock shift is vanishingly small [43]. At unitarity, there is no clear molecular signature in the rf spectrum because the binding energy of the molecules is zero. Instead, the strong interactions reveal themselves as a characteristic high frequency tail in the rf spectrum, which is the subject of Chapter 6.
Figure 3.7: Rf spectroscopy of molecules at 200.000(4) G. a) The energy required to break a molecule by flipping a spin to a third state is increased from the bare frequency by the binding energy. b) Repulsive interactions between atoms produce a mean field clock shift. c) Fraction of atoms transferred to $|c\rangle$ from $|b\rangle$ versus the detuning of the applied 80µs Blackman rf pulse from the single particle resonance frequency. The red lines indicate the binding energies as calculated using a finite-range correction, with $B_0 = 202.1$ G (solid) and $B_0 = 202.2$ G (dashed). The inset is zoomed in on the single particle peak. The green lines are predictions of the clock shift for a higher (solid) and lower (dashed) estimate of the density of $|a\rangle$ after molecule formation.
3.5 How unitary are we?

All data presented in the remaining chapters of this thesis was taken at 202.10 G, the resonance position as determined by References [28] and [44]. A recent measurement using rf spectroscopy of the molecular binding energy concluded that the Feshbach resonance is centred at $202.20(2)$ G with a width of $7.1(2)$ G [42]. If we take the difference in the location of the resonance by 0.1 G as a systematic uncertainty, the largest value of $1/k_Fa$ in our experiment would be 0.1. This places our measurements solidly in the unitary regime ($1/|k_Fa| \ll 1$).
Chapter 4

Spin diffusion

In this chapter, we study the demagnetization of a transversely spin-polarized Fermi gas, where the two spin states involved are tuned to be resonantly interacting. An applied magnetic field gradient induces a gradient in magnetization, which in turn, drives spin diffusion. Through a simple spin diffusion model, the timescale of magnetization loss is related to the transverse spin diffusion coefficient. At low temperatures, the diffusion constant saturates near the conjectured quantum-mechanical lower bound $\frac{\hbar}{m}$, where $m$ is the particle mass.

4.1 Particle diffusion

Diffusion is the transport of particles that opposes concentration gradients, and results in mixing. Fick’s law of diffusion postulates that the diffusion current $\vec{J}$ is proportional to the negative of the concentration gradient,

$$\vec{J}(\vec{r}, t) = -D \vec{\nabla} n(\vec{r}, t),$$

(4.1)

where $n$ is the concentration, or equivalently, the density of the particles in question. The proportionality constant $D$ is called the diffusion coefficient or the diffusivity and has units of [length$^2$/time]. If particle number is conserved, the change in concentration with time is

$$\frac{\partial n(\vec{r}, t)}{\partial t} = -\vec{\nabla} \cdot \vec{J}(\vec{r}, t) = D \nabla^2 n(\vec{r}, t).$$

(4.2)
While in Fick’s law, $D$ is a purely phenomenological bulk quantity, it is possible to estimate $D$ from a microscopic picture for a classical gas. Intuitively, large velocities will increase transport of particles and collisions will impede transport. The diffusion coefficient in a gas scales as the product of the root-mean-squared (RMS) velocity times the mean free path between collisions,

$$D \sim v \ell.$$  
(4.3)

The RMS velocity is determined by the temperature $T$ (and increases as $\sqrt{T}$ for non-degenerate gases). The mean free path is inversely proportional to the density and the collisional cross section $\ell = (n\sigma)^{-1}$.

### 4.2 Spin diffusion in degenerate Fermi gases

Spin diffusion is the transport phenomenon that relaxes magnetic inhomogeneities in a many-body system. In the following, we use arrows to denote spatial vectors and bold font for vectors in spin space. We define the magnetization $\mathbf{M}(\vec{r})$ as the local spin density, which has a maximum magnitude given by the local density $n(\vec{r})/2$, (following Leggett [52]). By extending Equation 4.2 to a vectored quantity, one might expect the diffusive loss of magnetization $\mathbf{M}$ to behave as

$$\frac{\partial \mathbf{M}(\vec{r},t)}{\partial t} = D_s \nabla^2 \mathbf{M}(\vec{r},t),$$  
(4.4)

where each component of spin experiences Fickian diffusion. However, if we write the local magnetization vector as $\mathbf{M} = M\hat{e}$, the gradient in magnetization $\hat{\nabla} \mathbf{M}$ has two components,

$$\hat{\nabla} \mathbf{M} = \hat{e} \hat{\nabla} M + M \hat{\nabla} \hat{e},$$  
(4.5)

both of which drive diffusive spin currents. A gradient in magnitude drives *longitudinal* spin diffusion,

$$\vec{J}_\parallel = -D_s\|\hat{e} \hat{\nabla} \mathbf{M}.$$  
(4.6)
**Chapter 4. Spin diffusion**

Transverse spin diffusion is driven by a gradient in direction,

$$\vec{J}_\perp = -D_s^\perp \mathcal{M} \vec{e}. \quad (4.7)$$

Diffusive loss of magnetization is

$$\frac{\partial \mathcal{M}}{\partial t} = -\vec{\nabla} \cdot \vec{J}, \quad (4.8)$$

where the total spin current $\vec{J}$ includes both transverse and longitudinal spin currents, $\vec{J} = \vec{J}_\parallel + \vec{J}_\perp$. If diffusion is isotropic, $D_s^\perp = D_s^\parallel = D_s$, then Equation 4.8 will reduce to Equation 4.4, however that is not always the case.

The distinction between longitudinal and transverse spin diffusion is important only at temperatures well below the Fermi temperature because Pauli blocking changes the phase space for the collisions that determine diffusivity. [45, 46, 47]. Figure 4.1a depicts longitudinal spin diffusion in a degenerate Fermi system. The Fermi spheres at $x$ and $x + dx$ are slightly different sizes. For an up spin to move from $x$ to $x + dx$, it must originate from the narrow region of the Fermi sea that is the difference between the two up spheres. To equilibrate at its new position, it must then scatter with a down spin, with both spins ending up in available states, and conserve energy and momentum in the process. This means that scattering occurs only in a small layer around the Fermi sphere. At non-zero temperatures, the Fermi surfaces are smeared out, easing the conditions that restrict scattering. As temperature is reduced so that the system becomes more deeply degenerate, Pauli blocking suppresses collisions, which increases the mean free path between collisions. Thus the longitudinal diffusion constant $D_s^\parallel$ is expected to increase at very low temperatures.

The picture is different for transverse spin diffusion (Figure 4.1b). The Fermi spheres are the same size at $x$ and $x + dx$ however the direction of quantization changes. Consider first a spin from below the smaller Fermi surface after it has moved from $x$ to $x + dx$. Conservation of momentum and energy will require that the final states of a collision remain inside the small Fermi surface, and therefore the collision will have no effect on the magnetization. Effectively, low momentum spins remain in equilibrium as they move from $x$ to $x + dx$, so the
do not contribute to transport. This leaves spins in the region between the two Fermi spheres available for scattering. A spin from this region, when moved from $x$ to $x + dx$ will be out of equilibrium and able to scatter, even at low temperature. This picture also demonstrates a magnetization dependence of $D^\perp_s$. Large magnetization means that the region between Fermi spheres where collision occur is large, so transport will be suppressed for larger versus smaller magnetizations.

The differentiation of $D^\perp_s$ and $D^{\parallel}_s$ in a strongly interacting Fermi system below the so-called “anisotropy temperature” has been observed in liquid helium-3 [48, 49]. For a three
dimensional unitary gas, the anisotropy temperature is predicted to be $\sim T = 0.14 T_F$ [50].

4.3 The Leggett-Rice Effect

In 1968 Leggett and Rice predicted an additional mode of spin transport due to a spin-rotation effect [51, 52]. The spin current precesses around the local magnetization, which is referred to as the “molecular field”. Including spin diffusion and this effect, the spin current is given by

$$\mathbf{J} = -D_s \hat{e} \nabla M - D_s \hat{M} \nabla \hat{e} - \mathbf{J} \times \mu_{LR} \mathbf{M},$$

(4.9)

where $\mu_{LR}$ is the Leggett-Rice parameter. Clearly, if the spin current is along the direction of magnetization, $\mathbf{J} = \mathbf{J}_||$, there can be no precession and the spin rotation term will be zero. However, if $\mu_{LR} \neq 0$, transverse spin currents will be affected.

In the following model, we assume that $\mu_{LR} = 0$, then examine the validity of the assumption in Section 4.8.1.

4.4 Spin spiral model

We measure the transverse demagnetization dynamics of a three-dimensional Fermi gas that is initially fully spin-polarized. Each atom is prepared in an equal superposition of two resonantly interacting internal states, labeled $|\downarrow\rangle$ and $|\uparrow\rangle$ (states $|a\rangle$ and $|b\rangle$ in Chapter 3), which corresponds to a gas with full transverse magnetization $M_y = 1$, where we define $M(\mathbf{r}) = 2M(\mathbf{r})/n(\mathbf{r})$ in anticipation of the experimental observable (Fig. 4.2). The states we use block any local mechanism for spin relaxation, unlike the scenario typical in liquids or solids. However, the differential magnetic moment $\Delta \mu$ between the internal states allows a magnetic field gradient $B' = \partial B_z/\partial z$ to twist the magnetization across the cloud into a spiral pattern, leading to a gradient in transverse magnetization. This gradient drives diffusive spin transport that erodes the coherence irreversibly. In contrast, for a weakly interacting Fermi gas, collisionless spin waves lead to reversible dynamics [53].
Figure 4.2: Spin spiral model. A $\pi/2$ pulse at $t = 0$ initializes the system with a homogeneous magnetization ($M_y = 1$ in the rotating frame) perpendicular to the magnetic field, which is along $z$. A spin spiral develops because of a magnetic field gradient, and drives diffusive spin currents. The upper and lower sequences show evolution without and with a $\pi$ pulse, respectively.

The evolution of transverse magnetization $M_\perp = M_x + iM_y$ in an inhomogeneous magnetic field is modelled with

$$\frac{\partial M_\perp(\vec{r}, t)}{\partial t} = -i\Omega_0(\vec{r})M_\perp(\vec{r}, t) + D_\perp^\perp \nabla^2 M_\perp(\vec{r}, t), \quad (4.10)$$

where the first term on the right hand side is the precession around the local external field, $B(\vec{r})$. This model assumes that $D_\perp^\perp$ is constant throughout the dynamics as well as across the trap, and that there are no spin-rotation effects\(^1\). If we write the solution in the form

$$M_\perp(\vec{r}, t) = A(t) \exp[-i\phi(\vec{r}, t)], \quad (4.11)$$

the phase $\phi$ will encompass the precession after preparing the system at time $t = t_0$,

$$\phi(\vec{r}, t) = \phi(\vec{r}, t_0) + \Omega_0(\vec{r})(t - t_0). \quad (4.12)$$

\(^1\)All of these assumptions are likely wrong. Nonetheless, the model captures trends surprisingly well (Figure 4.8). $D_\perp^\perp$ should depend on magnetization and temperature, both of which change throughout the dynamics. Also, the local reduced temperature varies across the trap. Disagreement between the data and this model are discussed in Section 4.8
In the rotating frame of the external field at $z = 0$, the effect of the magnetic field gradient will appear in the phase as

$$\phi(\vec{r}, t) = \phi(\vec{r}, t_0) + \alpha z t,$$  \hspace{1cm} (4.13)

where $\alpha = \Delta \mu B' / \hbar$ [54]. Substituting Equation 4.13 into 4.11 gives

$$M_\perp(\vec{r}, t) = M_0(\vec{r}, t_0) A(t) \exp[-i\alpha z t],$$  \hspace{1cm} (4.14)

where $M_0(\vec{r}, t_0) = \exp[-\phi(\vec{r}, t_0)]$.

In our experiment, the initial magnetization is homogeneous and fully polarized along the $y$-direction in spin space, $M_\perp(\vec{r}, 0) = i$. The differential equation now leads to the following equation for $A(t)$

$$\frac{\partial A(t)}{\partial t} = -D_\perp \alpha^2 t^2 A(t),$$  \hspace{1cm} (4.15)

which is easily solved. The magnetization at time $t$ will adopt a spiral pattern with a reduced amplitude,

$$M_\perp(\vec{r}, t) = i \exp[-i\alpha z t - D_\perp \alpha^2 t^3 / 3].$$  \hspace{1cm} (4.16)

The diffusivity and the magnetic field gradient determine the time scale of demagnetization. Because there is no spatial gradient in the magnitude of magnetization, these dynamics do not probe $D_\|$.  

We cannot directly measure the spin spiral, as our observable is a cloud averaged measurement of the populations of atoms in $|\uparrow\rangle$ and $|\downarrow\rangle$. Instead, the effect of spin diffusion on magnetization is measured using the spin-echo technique described in Fig. 4.2. The spin-refocusing $\pi$ pulse at $t_\pi$ swaps the population of the states $|\uparrow\rangle$ and $|\downarrow\rangle$, which causes the spin spiral to start untwisting. This partial rephasing reduces the rate of diffusion. At the echo time $t = 2t_\pi$, the spins realign. The final cloud-averaged $|M_\perp|$ is indicated by the contrast in $|\uparrow\rangle$ and $|\downarrow\rangle$ atom number after a final $\pi/2$ pulse with variable phase.

The phase of the magnetization immediately before the $\pi$ pulse is $\phi(\vec{r}, t_\pi^-) = \pi / 2 - \alpha z t_\pi$, and will be mapped to $\phi(\vec{r}, t_\pi^+) = \pi / 2 + \alpha z t_\pi$ after the pulse, which mirrors the spin spiral,

$$M_\perp(\vec{r}, t_\pi^+) = i \exp[+i\alpha z t_\pi - D_\perp \alpha^2 t_\pi^3 / 3].$$  \hspace{1cm} (4.17)
To continue the dynamics, we again substitute Equation 4.14 into 4.10, however this time 
\( M_0(\vec{r}, t_0) = \exp[2i\alpha z t_\pi] \) to ensure the spatial dependence of the magnetization at \( t = t_\pi \) matches Equation 4.17. The resulting time dependence for \( A \) is

\[
\frac{\partial A(t)}{\partial t} = -D_\perp s \alpha^2 (t - 2t_\pi)^2 A(t),
\]

which is solved by \( A(t > t_\pi) = A_0 \exp[-D_\perp s \alpha^2 (t - 2t_\pi)^3/3] \), where \( A_0 \) is chosen to match the magnetization amplitude at \( t = t_\pi \). The evolution of the magnetization after the \( \pi \) pulse is then

\[
M_\perp(\vec{r}, t > t_\pi) = -i \exp[-i\alpha z (t - 2t_\pi)] \exp[-D_\perp s \alpha^2 (2t_\pi^3 + (t - 2t_\pi)^3)/3].
\]

At the echo time \( t = 2t_\pi \), the magnetization is uniform, but reduced from its initial amplitude,

\[
M_\perp(\vec{r}, t = 2t_\pi) = -i \exp[-D_\perp s \alpha^2 t^3/12].
\]

It is at this stage when we probe the magnetization.

### 4.5 Measuring magnetization

Magnetization decay is measured using a \( \pi/2 - \pi - \pi/2 \) sequence (Figure 4.3a). The first pulse initializes the system with transverse magnetization. The second pulse at time \( t_\pi \) will refocus the spins at a time \( 2t_\pi \). The first and second pulses have the same phase, but the final pulse has a variable relative phase \( \varphi \). Varying \( \varphi \) reveals the magnitude of the trap-averaged transverse magnetization, \( |M_\perp| \), in the visibility of the oscillation in relative population, as in Figure 4.3b. Each measurement of \( |M_\perp| \) at a single hold time consists of 12 shots. We are sensitive to the relative frequency between the drive \( \omega \) and the atomic resonance \( \omega_0 \), with a precision of roughly \( 1/t \), where \( t \) is the hold time. For \( t \gtrsim 1 \) ms, we find that our field stability (roughly \( 1 \) kHz, or a few parts in \( 10^5 \)) is insufficient to preserve a reproducible relative phase throughout the sequence, resulting in a randomized phase for long hold times. Figure 4.4 shows an example of a fast decay of magnetization, where the visibility of the fringes is not yet affected by limited clock coherence. Figure 4.5 presents a similar measurement, but for a longer decay time,
Figure 4.3: Measuring magnetization. a) $\pi/2 - \pi - \pi/2$ pulse sequence to measure magnetization. The first pulse initializes the system. The second pulse is a spin refocussing pulse. The third pulse of variable phase interrogates the system, revealing the trap-averaged transverse magnetization. b) Absorption images performed according to the method in Section 2.8.2. After the final $\pi/2$ pulse with variable phase, the population in $|\uparrow\rangle$ and $|\downarrow\rangle$ are separated and counted. $|M_{\perp}|$ is reflected in the amplitude of this oscillation.

where there is still finite magnetization when there is no clock coherence left; for example, at $t = 3.02$ ms there is a large measured range of final populations, but no discernible fringe.

The most reliable way to extract the magnetization from our data in the presence of phase noise was determined with the help of some simulated fringes. The phase noise was modelled by adding a gaussian distribution of phases to each of our twelve sampled phases, $\{-180, -150, -120, -90, -60, -30, 0, 30, 60, 90, 120, 150\}$, then sampling a sine wave. The visibility of the fake fringes was analysed with three techniques: 1) fitting each fringe with a sine wave and taking its amplitude, 2) the standard deviation of the points normalized to the standard deviation of a sampled fringe with no phase noise, and 3) using only the maximum and minimum points. The results of the simulation are presented in Figure 4.6. The fit provides the worst results for large phase noise. The standard deviation method provides results that scatter above and below the actual visibility, with an average that tends to be about 4% lower than the actual visibility.
Figure 4.4: Demagnetization measurement that is unaffected by limited clock coherence. Data points lie near the fitted sinusoids for all hold times. Gradient $= 12.3 \pm 0.8$ G/cm
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Figure 4.5: Demagnetization measurement that suffers from limited clock coherence. Data points are scattered far from the fitted sinusoids for intermediate hold times, i.e. longer than the clock coherence time, but shorter than the demagnetization time. Gradient = $4.6 \pm 0.5$ G/cm
at high phase noise. The extrema method is less noisy than the standard deviation method. It never over estimates the visibility however it occasionally significantly underestimates it. On average its 5% below the actual visibility at high phase noise.

![Figure 4.6: Simulated fringes. Colours represent different data processing methods: Grey = visibility from fit. Blue = standard deviation of points. Red = (max-min)/(max+min)](image)

The data from Figures 4.4 and 4.5 is processed with the three techniques in Figure 4.7. To avoid an underestimation of the magnetization coherence time, we use the standard deviation of $N_T/N_{tot}$ as a measure of $|M_\perp|$ for all subsequent data in this thesis.

### 4.6 Measuring $D_{s\perp}$

We observe that demagnetization occurs in several milliseconds and that the timescale decreases with increasing gradient (Figure 4.7). Fitting $|M_\perp(t)|$ with an exponential decay function $\exp[-(t/\tau_M)^\eta]$, we find a range of $2.5 \lesssim \eta \lesssim 4.0$ without any systematic trend with gradient, compatible with $\eta = 3$ in Eq. 4.20. Constraining $\eta = 3$, we extract $\tau_M$ across a wide range of gradients (Figure 4.8), and fit it to find that the $(B')^{-2/3}$ scaling of Eq. 4.20 holds even for the case of the trap-averaged magnetization. At an initial temperature $(T/T_F)_i = 0.25(3)$, where $T_F$ is the Fermi temperature of the spin-polarized gas, a single-parameter fit of $\tau_M = (D_{s\perp}^\alpha^2/12)^{-1/3}$ to the data yields $D_{s\perp} = (1.08 \pm 0.09^{+0.17}_{-0.13}) \hbar/m$, where
Figure 4.7: Magnetization decay at two different gradients a) $= 12.3 \pm 0.8 \text{ G/cm}$ and b) $= 4.6 \pm 0.5 \text{ G/cm}$. Colours represent different data processing methods: Grey = visibility from fit. Blue = standard deviation of points. Red = $(\text{max-min})/(\text{max+min})$. The fitting function is a cubic decay, $\exp[-(t/\tau_M)^3]$.

The uncertainties are the statistical error from the fit and the systematic error from the gradient calibration, respectively. This is a direct measurement of the time- and trap-averaged diffusivity that does not rely on any calibration other than that of the gradient (Section 2.7.2).

Demagnetization transforms the system of $N$ particles in a single spin state to a mixture of two spin states, each with $N/2$ particles. The final Fermi energy of the trapped system $E_{F,f}$ therefore is reduced by a factor of $2^{1/3}$ compared to the initial $E_{F,i}$. Furthermore, demagnetization releases attractive interaction energy. Together these effects increase temperature (see Chapter 5 for a calculation of the magnitude of this effect), so that each measurement of $D_s^\perp$ has to be understood as a time average over a range in temperatures. The intrinsic heating together with the initial polarization of the cloud ensures that the gas remains in the normal phase throughout the evolution [37].
Figure 4.8: Magnetization decay times $\tau_M$ at various gradients. Horizontal and vertical error bars reflect the systematic error in the gradient calibration and the fit error respectively. The solid line is a single-parameter fit of equation (4.20) to the data which yields $D_s^\perp = (1.08 \pm 0.09 \pm 0.17 - 0.13) \hbar / m$. The shaded area denotes the combined statistical and systematic uncertainty.

### 4.7 The observed lower bound to diffusivity

Spin diffusion constants are expected to be bounded from below at unitarity for the following reasoning. Quantum degeneracy imposes a minimum typical velocity in a gas of $\hbar k_F / m$, (where $\hbar k_F$ is the Fermi momentum). The mean free path is limited by the interparticle spacing $\sim 1 / k_F$, which translates into a quantum lower bound of roughly $\hbar / m$ [9, 55, 56]. A measurement of $D_s^\parallel$ in a three-dimensional unitary Fermi gas yielded a minimum trap-averaged value of $6.3(3) \hbar / m$ [9]. However, $D_s^\perp$ as low as $0.0063(8) \hbar / m$ was recently observed in a strongly interacting two-dimensional Fermi gas [10]. This thousand-fold range in transport coefficients remains unexplained by theory.

In Figure 4.9, we choose a constant gradient and vary $(T / T_F)_i$. As above, we interpret the observed magnetization decay time $\tau_M$ as a measure of $D_s^\perp$. We find magnetization to decay faster at higher initial temperatures, corresponding to larger values of $D_s^\perp$. Diffusivity is larger in hotter clouds, as both the typical velocity and the mean free path increase with temperature.
Figure 4.9: Measured $D_s^\perp$ for various initial temperatures at a single gradient $B' = 18(1)\, \text{G/cm}$. To achieve temperature control, the data was collected with $N_{\text{Total}} = 1 \ldots 7 \times 10^4$. The filled circle marks the diffusivity obtained from the data in Figure 4.8 at $(T/T_F)_i = 0.25(3)$ and the shaded area covers $D_s^\perp < \hbar/m$.

Remarkably, we find $D_s^\perp$ to be bounded from below by $\simeq \hbar/m$ and that this bound is saturated in the vicinity of $(T/T_F)_i = 0.3$.

### 4.8 Deviation from the simple model

Careful examination of the demagnetization dynamics at our lowest initial temperatures suggests an acceleration of demagnetization at later times (Figure 4.10). Figure 4.11 summarizes free exponent fits $\exp[-(t/\tau_M)^\eta]$ to all data sets. $\eta$ shows no significant trend with gradient but a clear dependance on temperature. An apparently time-dependent $D_s^\perp$ could be due to its polarization dependence, as is predicted below the anisotropy temperature, where $D_s^\perp$ differs from $D_s^\parallel$ [46, 57]. It might also arise from spin-rotation effects [52, 58]. However, we find the deviations from Eq. 4.20 to be small, and we are unable to distinguish between these possibilities and other systematics.
Figure 4.10: Sample magnetization decay for the a) hottest \((T/T_F)_i = 0.8(1)\) and b) coldest \((T/T_F)_i = 0.15(1)\) clouds. The fitting function is the exponential decay function \(\exp[-(t/\tau_M)^3]\). The agreement is slightly worse at low temperature. The effect is small but the trend in a free exponent fit is more convincing (Figure 4.11b).

### 4.8.1 Fitting Leggett Rice theory to the data

While we cannot determine the source of the deviation at low temperature, we can use our data to place an upper bound on the value of the Leggett Rice parameter \(\mu_{LR}\) for a unitary Fermi gas. The combined effect of diffusion and the Leggett Rice effect on transverse magnetization is presented in Equation 30 of Reference [52]:

\[
\frac{\partial M_\perp(\vec{r},t)}{\partial t} = -i\Omega_0(\vec{r})M_\perp(\vec{r},t) + \frac{D_0(1 - i\mu_{LR}M_z)}{1 + \mu_{LR}^2 M_z^2(t)} \nabla^2 M_\perp(\vec{r},t). \tag{4.21}
\]

Reference [52] uses a similar method of solving this equation as in Section 4.4. The analogous equation to Equation 4.15 is

\[
(1 + \mu_{LR}^2 M_z^2) \ln \{A(t)/A(t_0)\} - \frac{\mu_{LR}^2}{2} \{A(t_0)^2 - A(t)^2\} = -D_0 \int_{t_0}^{t} (\nabla \phi(\vec{r},t'))^2 dt' \tag{4.22}
\]

In the case of our experiment, \(M_z = 0\), and \(A(t_0 = 0) = 0\). The linear magnetic field gradient creates a linear phase gradient that increases with time, until the \(\pi\) pulse reverses the phase dynamics. If the magnetization is probed at \(t = 2t_\pi\), equation 4.22 becomes

\[
\ln \{A(t)\} - \frac{\mu_{LR}^2}{2} \{1 - A(t)^2\} = -D_0 \left[ \int_{t_\pi}^{t} (\alpha t')^2 dt' + \int_{t_\pi}^{t=2t_\pi} (\alpha t_\pi - \alpha t')^2 dt' \right]. \tag{4.23}
\]
Figure 4.11: Free exponent fit to magnetization decay. Each magnetization decay for a) different gradients and b) different temperature was fit using the function $A \exp\left[-(t/\tau_M)^\eta\right] + C$ (four free parameters). The exponent $\eta$ shows no clear trend with gradient, but becomes larger as temperature is reduced.

After integration, this can be rearranged into

$$\mu_{LR}^2 \exp[-D_0 \alpha^2 t^3/6 + \mu_{LR}^2] = \mu_{LR}^2 A(t)^2 \exp[\mu_{LR}^2 A(t)^2].$$

The equation can be solved by using the Lambert $W$ function for which $z = W(z)e^{W(z)}$, therefore the amplitude should decay according to

$$A(t) = \sqrt{\frac{1}{\mu_{LR}^2} W\left(\mu_{LR}^2 \exp[-D_0 \alpha^2 t^3/6 + \mu_{LR}^2]\right)}.$$

Figure 4.12 presents the results of fitting Equation 4.25 to the data from Figure 4.9, with two additional fit parameters for the experimental imaging efficiency and offset. We find the maximal value of $|\mu_{LR}|$ is $\sim 1$ at our lowest temperatures, and $|\mu_{LR}|$ decreases as the temperature increases. The fitting function is not very sensitive for small values of $\mu_{LR}$ and therefore fails above $\sim 0.5T/T_F$. When $\mu_{LR} \neq 0$ the apparent diffusivity extracted using Equation 4.20 overestimates the diffusion constant. A lower bound to diffusivity is still observed, however $D_\perp$ saturates at $\sim 2\hbar/m$. A more sensitive measurement of $\mu_{LR}$ could be made by choosing $M_z \neq 0$, and repeating the spin echo measurement. In that case, the phase of the magnetization fringe will be sensitive to the effect.
Figure 4.12: Fit results including the Leggett Rice effect. a) Leggett Rice parameter $|\mu_{LR}|$ versus temperature. The fit fails above $\sim 0.5T/T_F$ and sets $\mu_{LR} = 0$. b) $D_{\perp}$ versus temperature. Black points come from the Leggett Rice fit. Red points are the values from Figure 4.9. When the fit fails at higher temperature, and sets $\mu_{LR} = 0$, the diffusivities agree exactly.
Chapter 5

Temperature change due to demagnetization

Our demagnetization experiment begins with a degenerate Fermi gas where each atom is prepared in an equal superposition of resonantly interacting spin states. This state is non-interacting because the Pauli exclusion principle prohibits s-wave scattering between identical fermions and higher partial wave scattering is suppressed at low temperature. Demagnetization will transform the system into a fully interacting unitary mixture. Chapter 4 investigates spin diffusion as the mechanism behind demagnetization, and the growth of interactions that result is presented in Chapter 6. This chapter discusses the effect of demagnetization on the temperature of the system.

During demagnetization, the system transforms from having a single Fermi sea with $N$ particles and Fermi energy $E_{F,i}$ to having two Fermi seas with the Fermi energy of the minority particles growing and the one for the majority particles shrinking. Eventually, both take the same value $E_{F,f}$ and the particle numbers are equal, $N_\sigma = N/2$ (Figure 5.1). Therefore $E_{F,f}$ is reduced by a factor of $2^{1/3}$ compared to $E_{F,i}$. The relaxation of the Fermi surfaces as well as the release of attractive interaction energy leads to a rise in temperature. This effect has been discussed in the context of spin echo experiments on liquid $^3$He, neglecting interactions [59].
Figure 5.1: The initial state is an ideal gas composed of $N$ atoms, where each atom is prepared in an identical superposition of two states. During demagnetization the system develops a second Fermi sea with a Fermi energy that grows from zero. In the final state, both Fermi seas are equilibrated with a Fermi energy that is reduced by a factor of $2^{1/3}$.

### 5.1 Calculating the temperature rise

The temperature rise is calculated assuming that the total energy, $E$, and total number of particles, $N$ is conserved in the isolated trapped gas. This is a valid assumption because the lifetime and the inverse heating rate is much longer than the timescale of demagnetization dynamics. First, we derive the total energy of an ideal Fermi gas for a given number and temperature. Then, using the experimentally determined equation of state for a unitary Fermi gas [5], we infer the final temperature.

#### 5.1.1 The initial state: Ideal Fermi gas

For an ideal Fermi gas at temperature $T$, the average occupation of state $\nu$, with energy $E_{\nu}$, is given by the Fermi-Dirac distribution [18],

$$
\langle n_\nu \rangle = \frac{1}{e^{(E_\nu - \mu)/k_B T} + 1}.
$$

(5.1)

The chemical potential $\mu$ is fixed by $N = \Sigma_\nu \langle n_\nu \rangle$, and the total energy is $E = \Sigma_\nu E_\nu \langle n_\nu \rangle$. 

### Initial state

- **Non-interacting**: $|\uparrow\rangle + |\downarrow\rangle$
- **Final state**: Strongly-interacting

\[ E_{F,i} = \hbar \omega (6N)^{1/3} \quad E_{F,f} = \hbar \omega (6N_\sigma)^{1/3} \]
Our atoms are confined to a harmonic trap \( V(r) = \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2) \). The energy separation between states is given by \( \hbar\omega_{x,y,z} \). When the thermal energy \( k_B T \equiv 1/\beta \) is much larger than \( \hbar\omega_{x,y,z} \), we can neglect the discrete level structure and consider only the density of particles in phase space and therefore replace summations with the corresponding integrations (Thomas-Fermi approximation)[18]. We also employ the local density approximation, where the local properties of the gas at point \( r \) are assumed to be equal to those of a homogeneous gas with chemical potential \( \mu_{\text{loc}}(r) = \mu - V(r) \). For a trapped gas the occupation of a phase space cell defined by position \( r \) and momentum \( p \) is then

\[
 f(r, p) = \frac{1}{e^{\beta(\frac{p^2}{2m} + V(r))} + 1} = \frac{1}{e^{\beta(\frac{p^2}{2m} - \mu_{\text{loc}}(r))} + 1} \tag{5.2}
\]

The density of a trapped ideal Fermi gas evaluates to

\[
 n(r) = \int \frac{d^3p}{(2\pi \hbar)^3} f(r, p) = -\frac{1}{\lambda_{dB}^3} Li_{3/2}(-e^{\beta\mu_{\text{loc}}(r)}) \tag{5.3}
\]

where \( \lambda_{dB} = \sqrt{2\pi\hbar^2/(mk_BT)} \) is the deBroglie wavelength. The functions \( Li_n \) are \( n \)th order polylogarithms, defined as

\[
 Li_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n} = \frac{1}{\Gamma(n)} \int_{0}^{\infty} \frac{x^{n-1}dx}{e^x z^{-1} + 1} \tag{5.4}
\]

where \( \Gamma(n) = (n-1)! \) is the Gamma-function for integer \( n \). These functions repeatedly appear in Fermi systems.

The total atom number in a harmonic potential with mean trap frequency \( \bar{\omega} = \sqrt{\omega_x\omega_y\omega_z} \) is given by

\[
 N(\beta, \mu) = -\left(\frac{1}{\beta\hbar\bar{\omega}}\right)^3 Li_3(-e^{\beta\mu}) \tag{5.5}
\]

The total energy is

\[
 E(\beta, \mu) = -\frac{3}{\beta} \left(\frac{1}{\beta\hbar\bar{\omega}}\right)^3 Li_4(-e^{-\beta\mu}) \tag{5.6}
\]

To calculate the total energy given the number and the temperature, one uses Equation 5.5 to determine the chemical potential, which can then be substituted in Equation 5.6.
5.1.2 The final state: Unitary mixture

The density equation of state for a homogeneous spin balanced unitary Fermi gas has been experimentally determined in References [5, 60, 61]. It is typically given in terms of

\[ n(\mu, \beta)/n_0(\mu, \beta) = g(\beta \mu) \]  

where \( n(\mu, \beta) \) is the density of the unitary mixture, and \( n_0(\mu, \beta) \) is the density of an ideal gas with the same chemical potential and temperature, given by Equation 5.3. The function \( g(\beta \mu) \) can be parameterized in three parts. At high temperature \( (\beta \mu < -1.25) \), the equation of state is well described by the viral expansion

\[ n \lambda^3 dB = e^{\beta \mu} + 2b_2 e^{2\beta \mu} + 3b_3 e^{3\beta \mu} + \ldots \]  

The viral coefficients are known up to third order, with \( b_1 = 1 \), \( b_2 = 3\sqrt{2}/8 \), and \( b_3 = -0.29095295 \) [62]. For intermediate temperatures \( (-1.23 < \beta \mu < 4) \) we interpolate the experimental results of Ref. [5]. Finally, the low temperature \( (\beta \mu > 4) \) equation of state is given by

\[ n \lambda^3 dB \rightarrow \frac{4}{3\sqrt{\pi}} \left( \frac{\beta \mu}{\xi} \right)^{3/2} \]  

where \( \xi \approx 0.376(4) \) is the Bertsch parameter [5], which relates the zero-temperature chemical potential of a unitary gas to its Fermi energy, \( \mu(T = 0) = \xi E_F \).

M. J. Ku and M. W. Zwierlein define the functions [63]

\[ f_n(\beta \mu) = \frac{1}{\Gamma(-3/2 + n)} \int_{-\infty}^{\beta \mu} \frac{f(x)}{(\beta \mu - x)^{-5/2+n}} \, dx \]  

where \( f(x) = -g(x) Li_{3/2}(-e^x) \), for a harmonically trapped gas in the local density approximation. These functions are analogous to Fermi-Dirac integrals for non-interacting gases, but incorporate the modified equation of state in \( f(x) \). The equations for total number and energy of a unitary gas are

\[ N(\beta, \mu) = \left( \frac{1}{\beta \hbar \omega} \right)^3 f_3(\beta \mu), \]
and
\[ E(\beta, \mu) = \frac{3}{\beta} \left( \frac{1}{\beta \hbar \omega} \right)^3 f_4(\beta \mu). \] (5.12)

Knowledge of the equation of state allow us to calculate the energy given the temperature and number as in the case of an ideal gas.

### 5.1.3 The temperature rise

The final temperature is calculated numerically. Equations 5.5 and 5.6 are used to find the total energy for the given number and initial temperature. Then Equations 5.11 and 5.12 are used to find a combination of \( \mu \) and \( \beta \) that satisfy energy and number conservation. The red line in Figure 5.2 shows the temperature rise that would occur only from the relaxation of Fermi surfaces of an ideal gas. The blue line is the prediction for the unitary mixture, which includes the energy released when interactions are present. These lines approach each other at high temperature, where the density is reduced and interactions play a weaker role. At high temperature the reduced temperature continues to increase even when the temperature increase from demagnetization is negligible purely from the redefinition of the Fermi temperature.

### 5.2 Measuring temperature

To measure the temperature of a gas, we turn off the trapping potential and let the cloud expand during a time of flight. If interactions are not present during the expansion, then atoms which originally occupied the phase space cell \( f(r_0, p) \) will occupy \( f(r_0 + pt/m, p) \) after a time \( t \). The density at point \( r \) and time \( t \) will be the sum of contributions from all particles with initial conditions such that \( r_0 + pt/m = r \), so we integrate over the intial phase space given by Equation 5.2,

\[ n(r, t) = \int \frac{d^3p}{(2\pi \hbar)^3} f(r - pt/m, p). \] (5.13)
Figure 5.2: Calculated temperature rise after demagnetization of an ideal gas in the non-interacting case (red) and with unitary interactions (blue). The dashed line indicates the change in reduced temperature that stems purely from the redefinition of $T_F$ (constant absolute temperature). It gives a lower limit for the change in $T/T_F$ at any temperature.

Solving this analytically for a harmonic potential reveals that ballistic expansion is a scaling transformation, where each spatial dimension is rescaled by the factor $b_i(t) = \sqrt{1 - \omega_i^2 t^2}$,

$$n(r, t) = \frac{1}{b_x(t)b_y(t)b_z(t)} n\left(\frac{x}{b_x(t)}, \frac{y}{b_y(t)}, \frac{z}{b_z(t)}, t = 0\right). \quad (5.14)$$

To extract the temperature from the density distribution, we follow the fitting method in Reference [18] which explicitly separates cloud size and shape. Noting that hot clouds ($T/T_F \gg 1$) have gaussian density distributions given by the radius $\sigma_i = \sqrt{\frac{2k_BT}{m\omega_i^2}}$, and that the characteristic radius of a degenerate Fermi gas saturates to $R_{Fi} = \sqrt{\frac{2E_F}{m\omega_i}}$ at low temperature, this method defines a fit parameter that interpolates between the two limits:

$$R_{i}^2 = \frac{2k_BT}{m\omega_i^2} f(e^{\mu_\beta}) \quad (5.15)$$

where $f(x) = \frac{1+x}{x} \ln(1-x)$.

Absorption imaging integrates along the line of sight, yielding a 2D image. The fit function
for this image is
\[
n_{2D}(x, y) = n_{2D,0} \frac{Li_2 \left( - \exp[q - \left( \frac{x^2}{R_2^2} + \frac{y^2}{R_2^2} \right) f(e^q)] \right)}{Li_2(-e^q)}.
\] (5.16)

The parameter \( q = \mu / \beta \) is determined by the shape of the cloud only, and therefore is directly related to how deeply degenerate the cloud is:
\[
\frac{T}{T_F} = \left[ -6 Li_3(-e^q) \right]^{1/3}.
\] (5.17)

Both the size of the cloud and the shape parameter contribute to the measurement of the absolute temperature,
\[
k_B T = \frac{1}{2} m \omega_i^2 \frac{R_i^2}{b_i(t)^2} \frac{1}{f(e^q)}.
\] (5.18)

Figure 5.3 shows an absorption image of a degenerate cloud, and a comparison of fitting the image with either a 2D gaussian distribution or Equation 5.16. The gaussian fit is clearly wrong for the cold cloud, and significantly overestimates the temperature. We use Equation 5.16 for all quoted temperature measurements in this thesis.

5.2.1 Measured temperature rise

Our coherent manipulation of the spin does not change the kinetic degrees of freedom. Therefore we take a measurement of the temperature of a cloud before creating a superposition as the initial temperature \((T/T_F)\). It is this temperature which is quoted in the Chapter 4.

Once the cloud becomes interacting, we cannot use the ballistic approximation during expansion. We circumvent this problem by jumping the magnetic field to the zero crossing of the Feshbach resonance where the \( s \)-wave scattering vanishes right before releasing the cloud (Section 2.6.2). Figure 5.4 shows absorption images taken in this manner throughout the demagnetization dynamics. By visual inspection, it is clear that the cloud is heating up because it is spreading out. However, there is an asymmetry of unknown origin in the density distribution for short hold times. Nonetheless, we fit these images with Equation 5.16 and plot the results in Figure 5.5.
Figure 5.3: Temperature measurement. a) Average of 18 absorption images of identically prepared spin-polarized clouds after expansion during a 5.5 ms time of flight at 209 G. b) Gaussian fit to the averaged image. c) Fermi fit to the averaged image. e) Residuals of the gaussian fit show clear structure. f) Residuals of the Fermi fit are smaller than e) and show less structure. The Fermi fit of individual images yielded a reduced temperature of $T/T_F = 0.22(1)$ and an absolute temperature of 360(20) nK and 340(20) nK as determined from the horizontal and vertical radiuses respectively. The gaussian fits estimated much higher temperatures, 640(20) nK and 610(20) nK from the horizontal and vertical directions. The errors are standard deviations from averaging over 18 images.
Figure 5.4: Raw images showing the increase in temperature during demagnetization in a magnetic field gradient of $8.2(7)$ G/cm. The absorption images show atoms in state $|\downarrow\rangle$ after a 5.5 ms expansion during time-of-flight. The atoms were prepared in a superposition then held for a variable hold time before jumping the magnetic field to 209 G, the zero crossing of the Feshbach resonance to minimize interaction effects during expansion.
Figure 5.5 combines all results in this chapter. The data points are results from fitting data in Figure 5.4 with Equation 5.16. The horizontal lines are the initial temperature as measured in Figure 5.3a and the unitary prediction for the temperature rise. The green curve is an ideal gas prediction of the temperature dynamics which takes the measured demagnetization curve from Chapter 4 and assumes a thermal equilibrium at all times. Disagreement between the data and the unitary prediction can probably be attributed to a deficiency in the temperature measurement; it is apparent that the clouds get a momentum kick that we could not trace. However, we can still see the timescale of the temperature rise. The red and blue lines are fits of \( \exp\left(-\left(t/\tau_T\right)^3\right) \) to the measured temperature increase, which yields \( \tau_T = 2.5(3) \) ms. This is similar to the magnetization decay time for the same gradient, \( \tau_M = 2.3(8) \) ms. A fit to the calculated ideal gas curve gives a timescale which is faster than the magnetization loss, \( \tau_{T_{\text{ideal}}} = 1.7 \) ms. The difference between \( \tau_T \) and \( \tau_{T_{\text{ideal}}} \) could be due to interactions having a greater effect near the end of demagnetization, or due to slow thermalization. This could be checked by repeating the measurement at different gradients to see whether \( \tau_T/\tau_M \) is constant.
Figure 5.5: Temperature rise during demagnetization. Red and blue points are from the vertical and horizontal fits to the images in Figure 5.4, using Equation 5.16. Errorbars are the standard deviation from averaging. The black lines represent the initial temperature of the spin polarized cloud measured before making a superposition, and the prediction for the final temperature of the unitary mixture. The green line is an ideal gas prediction which is based on the measured magnetization decay time and assumes thermal equilibrium throughout the dynamics.
Chapter 6

Contact Dynamics

Chapter 4 demonstrated that spin diffusion at the quantum limit was responsible for the decoherence of a superposition of resonantly interacting states. In this chapter we observe the development of interactions that arise as a result of decoherence using rf spectroscopy to measure Tan’s contact parameter, \( C \). A comparison of development of contact with the magnetization data from Chapter 4 reveals enhanced pair correlation compared to a statistical mixture of spins.

6.1 What is the contact?

Tan’s contact is a central property of a system of strongly interacting fermions. It is a measure of the number of pairs of fermions of opposite spin with small separations. Knowledge of the contact translates to knowledge of many of the system’s universal properties through a series of relations known as the Tan relations [64, 65, 66] which hold true for any state of the system: superfluid or normal fluid, few-body or many-body, ground-state or finite temperature, homogeneous or trapped gas, balanced or imbalanced spin populations [67]. Remarkably, this single parameter relates microscopic properties such as the single-particle momentum distribution to bulk properties such as the internal energy and its change with the scattering length.


6.1.1 My favourite Tan relations

The contact $C$ is an extensive quantity that is the integral over the local contact density $C(R)$. The contact density $C(r) = \langle g^2 \psi^\dagger_\uparrow(r) \psi^\dagger_\downarrow(r) \psi_\downarrow(r) \psi_\uparrow(r) \rangle$ is a local measure of the pair correlation, i.e., the number of pairs of opposite spins at short distance, where $g$ is the coupling constant and $\psi_\sigma$ is the annihilation operator with spin $\sigma$ [67]. As is clear from its definition, $C$ is also proportional to the interaction energy.

To illustrate the power of the contact I will summarize a few of the Tan relations. A more complete list can be found in Reference [15]. Full derivations can be found in the papers of S. Tan [64, 65, 66].

The contact appears as a prefactor to the tail of the momentum distribution $n_\sigma(k)$ for both spin species

$$n_\sigma(k) \to C/k^4.$$  \hspace{1cm} (6.1)

This equation is valid in the region $|a^{-1}| \ll k \ll |r_0^{-1}|$, where $r_0$ is the range of the interaction.

The contact determines the change in energy with respect to the interaction strength via the adiabatic relation.

$$\left( \frac{dE}{da^{-1}} \right)_s = - \frac{\hbar^2}{4\pi m} C.$$ \hspace{1cm} (6.2)

The density-density correlator at short distances depends on the contact density $C(R)$,

$$\langle n_1(R + \frac{1}{2}r) n_2(R - \frac{1}{2}r) \rangle \to \frac{1}{16\pi^2} \left( \frac{1}{r^2} - \frac{2}{ar} \right) C(R),$$ \hspace{1cm} (6.3)

where $R$ is the position in the trap, $n_\sigma(R)$ is the density distribution of spin $\sigma$ in the trap, and $r$ is the small separation between the two spin states. The integration of this equation over a sphere of radius $s$ yields the number of pairs in the sphere

$$N_{\text{pair}}(R, s) \to \frac{s^4}{4} C(R).$$ \hspace{1cm} (6.4)

The volume of the sphere is simply $V = \frac{4}{3} \pi s^3$. One would naively expect the number of pairs to scale as $V^2$ as the volume is reduced; however, Equation 6.4 predicts a $V^{4/3}$ scaling. This implies that the number of pairs in a small volume is larger than one would expect by
extrapolating from large volumes due to the strong correlations associated with large scattering lengths.

Other Tan relations describe the energy relation (sum of kinetic and interaction energies), the energy change due to a sudden change in scattering length, the Virial theorem (kinetic + interaction - potential energies), the relationship between pressure and energy in a homogeneous system, and the rate of inelastic 2-body losses.

6.2 Measuring the contact with rf spectroscopy

Prior experimental work has measured the equilibrium contact through photo-association, rf spectroscopy, momentum expansion, and Bragg spectroscopy [68, 69, 70, 71, 72]. Although contact has been shown to relate various thermodynamic and many-body properties of a short-range interacting gas, it has so far been studied only in equilibrium and with an unmagnetized gas [70, 73, 74]. We perform rf spectroscopy of the contact using the technique first demonstrated by Stewart et. al. [70], which we adapt for time-resolved measurements of contact dynamics.

We apply an rf pulse coupling the $|\uparrow\rangle$ state to an initially unoccupied internal state $|p\rangle$ that interacts only weakly with $|\uparrow\rangle$ and $|\downarrow\rangle$ (Figure 6.1). The transfer rate to $|p\rangle$ is measured as a function of the frequency detuning $\delta$ above the single-particle resonance. Large positive detunings will probe short range pair correlations because energy and momentum conservation requires that a collision occurs during the spectroscopy pulse. In a strongly interacting gas in equilibrium, the high-frequency tail of such a spectrum is known to be proportional to Tan’s contact parameter and scales as $\delta^{-3/2}$ [64, 67, 75, 74, 76, 77, 78, 70].

The rate $\Gamma$ of atom transfer from state $|\uparrow\rangle$ to the weakly interacting probe state $|p\rangle$ obeys the sum rule

$$\int \Gamma(\omega) d\omega = \frac{1}{2} \pi \Omega_R^2 N\uparrow,$$

where $\Omega_R$ is the Rabi frequency of the applied rf pulse. In the limit of a non-interacting Fermi
Figure 6.1: Rf spectroscopy of pair correlations. The frequency of the rf is detuned above the single particle transition frequency. To satisfy energy and momentum conservation, the transition can only occur if two opposite spins are close enough together to collide during the spectroscopy pulse. Thus, the transition rate for large positive detunings is a measure of short range pair correlations.

gas, this sum rule is reproduced by Fermi’s golden rule. In the presence of strong interactions, $\Gamma(\omega)$ scales as $\Omega_R^2/(8\pi\sqrt{m/\hbar}\delta^{3/2})C$ for large positive detunings $\delta = \omega - \omega_0$, where $\omega_0$ is the single-particle resonance frequency. We express the detuning in units of the Fermi energy, $\Delta = \hbar\delta/E_{F,f}$, where $E_{F,f}$ is calculated for a balanced mixture with $N_\uparrow = N_\downarrow = N/2$. We furthermore introduce the normalized transfer rate

$$\tilde{\Gamma}(\Delta) = \frac{E_{F,f}}{\hbar\pi\Omega_R^2N_\uparrow}\Gamma$$

such that $\int \tilde{\Gamma}(\Delta)d\Delta = 1/2$. Its asymptotic behaviour takes the form

$$\tilde{\Gamma}(\Delta) \to \frac{1}{2^{3/2}\pi^{2}\Delta^{3/2}}\frac{C}{k_F N},$$

so that the rescaled quantity $2^{3/2}\pi^{2}\Delta^{3/2}\tilde{\Gamma}(\Delta)$ reveals $C/(k_F N)$, the contact per particle in units of $k_F$, at $\Delta \gg 1$ [76, 77, 78, 70].

In our experiments, we measure the fraction of atoms $N_p/N_\uparrow$ transferred to the probe state by a Blackman pulse of length $\Delta t$ and with mean Rabi frequency $\Omega_R$. We choose the rf
power and pulse duration such that we probe the transition in the linear regime where \( \Gamma(\delta) = N_p(\delta) / \Delta t \). Using the asymptotic behavior stated above, we obtain the contact from the fraction of atoms transferred, measured at a single detuning \( \delta \), via

\[
\frac{C}{k_F N} = \frac{4\pi \delta^{3/2}}{\Omega_R^2 \Delta t} \sqrt{\frac{m}{\hbar}} \frac{1}{k_F N^\uparrow},
\]

(6.8)

where the Fermi wave number \( k_F \) is calculated using the measured value for \( N^\uparrow = N/2 \).

### 6.3 Dynamic measurement of contact

Prior measurements of contact have all been performed on systems in equilibrium. The desire to do a dynamic measurement prompted us to ask whether upon a sudden change in the scattering length, would the contact instantly respond, or would it take time to develop. The scattering length can be changed in two ways: suddenly changing the magnetic field near a Feshbach resonance, or maintaining the field and changing the states involved in the interaction. We chose the latter method because it can be much faster. Sudden changes in field can induce eddy currents, and require a settling time to become stable. Instead, we prepared a mixture of atoms in the two weakly interacting states \(|\downarrow\rangle\) and \(|p\rangle\) at the resonance field for \(|\uparrow\rangle\) and \(|\downarrow\rangle\). In Figure 6.2, the mixture was suddenly transformed to a unitary mixture with a 3.3 \( \mu s \) \( \pi \)-pulse to move the population from \(|p\rangle\) to \(|\uparrow\rangle\). After a variable hold time (10 \( \mu s \)-5ms), the contact was measured with a 20 \( \mu s \) rf pulse. We find a finite value of the contact for our shortest hold time and no subsequent dynamics, which implies that it had fully developed within 30 \( \mu s \). This observation is consistent with contact developing on the timescale \( \hbar/E_F \approx 8 \mu s \).

### 6.3.1 Birth of interactions

We now return to the theme of this thesis: the transformation of an ideal Fermi gas to a unitary Fermi gas through demagnetization. While our result for the transverse spin diffusivity in Chapter 4 reflects the suppression of transport by strong scattering, it does not reveal how
Figure 6.2: Instantaneous development of contact. A mixture of atoms in the two weakly interacting states $|\downarrow\rangle$ and $|p\rangle$ was prepared at 202.1G. The mixture was suddenly transformed to a unitary mixture with a 3.3 $\mu$s $\pi$-pulse on the $|p\rangle$ to $|\uparrow\rangle$ transition. After a variable hold time, 20 $\mu$s Blackman pulse detuned 125 kHz above the $|\uparrow\rangle$ to $|p\rangle$ transition frequency probed the contact. The shortest hold time was 10 $\mu$s. The sample had $N = 12000$ and $T = 0.5T_F$.

interactions emerge in the many-body system. In this complementary set of measurements, we study the microscopic transformation of the gas by following the dynamical evolution of contact that is enabled by demagnetization. The only difference in the sequences is that the magnetization probe ($\pi/2$-pulse with 12 different phases) is replaced with a spectroscopy pulse. The timing sequence is depicted in Figure 6.3 (see Figure 4.3a for comparison). The development of interactions following loss of coherence has been observed once before by rf spectroscopy of a clock shift near a Feshbach resonance [43].

Figure 6.4 shows the rf spectrum for a gas probed after a 10 $\mu$s and a 2.5 ms hold time. At short hold time, the spectrum exhibits only the single-particle peak, while after a longer hold time, the spectrum develops a high-frequency tail with the $\delta^{-3/2}$ scaling of Equation 6.7.

Figure 6.5 shows how the full rf spectrum develops with time. Whenever a high-frequency tail is observed, we find that it has a $\delta^{-3/2}$ scaling for $\delta \gtrsim 4E_{F,f}/\hbar$, indicating that pair cor-
Figure 6.3: Contact measurement pulse sequence. The system is initialized with a $\pi/2$ pulse, which prepares all atoms in an equal superposition of the two resonantly interacting states. After a time $t$, the contact is probed with a 20 $\mu$s Blackman on the $|\uparrow\rangle$ to $|p\rangle$ transition. An optional spin reversal pulse can be included at $t/2$.

Figure 6.4: Signature of the contact in the high frequency tail. Rf spectra for $t = 10$ $\mu$s (solid line) and 2.5 ms (dashed line), both without a spin-refocusing pulse. The red line in the right panel is a fit of a power-law $\propto \delta^{-3/2}$ to the high-frequency tail for $t = 2.5$ ms.

relations can be described with a contact parameter throughout the dynamics. The transfer rate on the vertical axis is scaled, $2^{3/2} \pi^2 \Delta^{3/2} \tilde{\Gamma}(\Delta)$ to reveal the contact per particle as a plateau.

The plateau value can in principle be measured with one single experimental cycle at a fixed detuning rather than taking a full spectrum; each hold time in Figure 6.5 sampled 38 different detunings. Figure 6.6 compares the contact dynamics obtained from the full spectra to that of a fixed detuning measurement with three repetitions at each hold time. The noise is less on the fixed detuning measurement and it reduces the number of experimental cycles by a factor of $\sim 13$. This single shot method is used for all subsequent contact measurements.
Figure 6.5: Time-resolved spectroscopy to measure Tan’s contact. The transfer rate \( \tilde{\Gamma} \) rescaled with \( s \delta^{3/2} \) versus hold time, where \( s \equiv \pi^2(2\hbar/E_{F,f})^{3/2} \), revealing a plateau for large positive detunings \( \delta \gtrsim 4E_{F,f}/\hbar \). For the data in this figure, \( N \approx 5 \times 10^4 \) and \( (T/T_f)_i \approx 0.2 \).
Figure 6.6: Validity of single shot measurement. Comparison of the contact values extracted from these full spectra (open circles) and those measured at a fixed detuning $\delta/2\pi = 125$ kHz (filled circles). The latter method allows single-shot study of contact dynamics, and thus reduced statistical noise as seen here. Error bars are statistical.

### 6.4 Linking contact and magnetization

Clearly, Figure 6.5 demonstrates that interactions develop when a transversely-polarized unitary Fermi gas loses its magnetization. Figure 6.7 shows that it is possible to maintain a non-interacting transversely-polarized Fermi gas by preserving its magnetization. Instead of the initial $\pi/2$ used to create the superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$, we perform adiabatic rf dressing to create the superposition. A frequency of strong rf field with Rabi frequency $\Omega_R$ is swept from a large detuning $\delta_{\uparrow\downarrow}$ to the transition frequency of $|\uparrow\rangle$ to $|\downarrow\rangle$, i.e. from $\delta_{\uparrow\downarrow} \gg \Omega_R$ to $\delta_{\uparrow\downarrow} = 0$. When $\Omega_R \gg \delta_{\uparrow\downarrow}$, the superposition is an eigenstate of the system, therefore when all atoms are in this state, there are no dynamics as long as the rf field remains. The applied magnetic field gradient $\partial B_z/\partial z$ makes the detuning dependent on the position in the trap, $\delta_{\uparrow\downarrow}(z)$. The condition to maintain uniform magnetization with this method is $\Omega_R \gg \Delta\delta_{\uparrow\downarrow}$ where $\Delta\delta_{\uparrow\downarrow}$ is the spread in detuning accessed by atoms in the trap. Once the rf field is removed, the contact develops. The same method of preserving a pure state has been used to prolong the life of persistent currents of bosons in a ring trap [79].
Figure 6.7: Supression of contact growth by rf dressing. The time evolution of contact is studied using single-shot spectroscopy as described in Figure 6.6, after holding the atoms in a dressed state with equal population of $|\uparrow\rangle$ and $|\downarrow\rangle$. The state was created by sweeping a strong rf field from below the resonance to the resonance, then holding it at the resonant frequency for 4, 15, 50, 200 or 600 ms. The rf field was then stepped off, and the contact measured after a variable hold time. The difference in the maximum value of the contact may be explained my atom loss during the dressing time; approximately half the atoms are lost during the longest dressing time. The heating rate during dressing was not measured.

To directly compare magnetization and contact, we perform a rf sequence with an spin reversal pulse just as in Chapter 4. Figure 6.8 shows that, under various protocols, the contact starts at zero and grows in time towards a maximal value of $C_{\text{max}}/(k_F N) = 1.53(4)$, where $\hbar k_F = \sqrt{2m E_{F,f}}$ is the Fermi momentum in the final state and $N$ is the total number of atoms. This is comparable to equilibrium values observed previously at $T/T_F \approx 0.35$ in Reference [73], which lies between the initial and final temperatures of this data (as discussed in Chapter 5).

At longer times ($t > 5$ ms), Figure 6.8a shows a slow reduction of contact, which is likely due to heating; however, in this work we focus on the short-time dynamics. A fit using an empirical rise function $f(t) = C_{\text{max}}(1 - \exp[-(t/\tau_C)^\eta])$ to the short-time data yields an exponent
Figure 6.8: Effect of a spin reversal pulse on the development of the contact. The time evolution of contact is studied using single-shot spectroscopy as described in Figure 6.6, at $B' = 8.2(7) \text{ G/cm}$. a) Contact versus hold time $t$ with (in green) and without (in red) a spin-refocusing $\pi$ pulse at $t/2$. Solid lines show a fit to $C_{\text{max}}(1 - \exp[-(t/\tau_C)^3])$. b) Contact versus hold time $t$ with a $\pi$ pulse applied at $t_\pi = 1 \text{ ms}$. At the echo time, the spin spiral has untwisted and no longer drives a diffusive spin current. Error bars in a) and b) show statistical error across repeated measurements.

of $\eta = 3.6(3)$ with a spin-reversal and $\eta = 2.8(2)$ without a spin-reversal, reminiscent of the magnetization loss function $\propto \exp[-(t/\tau_M)^3]$. Further connection of contact to magnetization is demonstrated by Figure 6.8b, which traces the contact during a spin-reversal sequence: the rise of $C/(k_F N)$ is slowed by the refocusing pulse and plateaus at the spin-echo time, around which transverse spin diffusion is suppressed.

Figure 6.9 compares the $1/e$ time scales of magnetization loss, $\tau_M$, and contact rise, $\tau_C$, with and without an echo. A linear relationship is found, which is surprising at first, since magnetization is a one-body vector observable and contact is a two-body scalar observable. The connection becomes apparent upon examination of the possibilities for a two particle wavefunction, which is the product of the spatial wavefunction and the spin state. The spin-triplet states are symmetric under exchange of particles, and the singlet state is antisymmetric. For
fermions, the total wave function must be antisymmetric. The contact interaction requires that two particles be in the same location, which occurs only with an even spatial wavefunction. Therefore, the spin state that generates the contact must be the antisymmetric spin-singlet state. For uncorrelated spin pairs, the probability to be in a spin-singlet state is \( \rho_{ss} = (1 - |M_\perp|^2)/4 \).

Combining this assumption with the diffusion model for magnetization predicts \( \tau_C = \tau_M/2^{1/3} \) and \( \tau_C = \tau_M/2 \) with and without an echo, respectively. The maximum singlet probability for a given magnetization is \( \rho_{ss} = 1 - |M_\perp| \), and would instead give \( \tau_C/\tau_M \) that is \( 2^{1/3} \) larger. Data in Figure 6.10 show an approximately linear relation whose slope is between these two limits.

Comparing the full range of measured values for normalized \( C \) and \( |M_\perp| \) at various times and gradients in Figure 6.10 also shows a functional form between the uncorrelated \( C \sim 1 - |M_\perp|^2 \) and the fully paired \( C \sim 1 - |M_\perp| \). E. Taylor and S. Zhang performed a calculation based on a large-\( N \) expansion [80, 81, 82] that predicts that \( C(M_\perp) \) changes between these limiting behaviors as \( T \) goes from \( 2T_c \) to \( T_c \), where \( T_c \) is the critical temperature for pair superfluidity. The calculation appears in the supplements of Reference [83].

Alternatively, an apparent reduction in \( C(M_\perp) \) might arise from a lag in the evolution of

Figure 6.9: Contact growth time, \( \tau_C \), versus demagnetization time, \( \tau_M \) with (green) and without (red) a spin-refocusing pulse), measured at various gradients \( B' \). Shaded areas interpolate between the limits of fully paired and uncorrelated spins.
Figure 6.10: Normalized contact versus magnetization obtained by relating measurements of \(|M_\perp(t)|\) and \(C(t)\) for all sampled gradients \(B'\). The solid and dashed lines shows the prediction for uncorrelated and fully paired spins respectively. A typical statistical error bar is shown on a single point.

\(C\) behind \(|M_\perp|\). However we find no statistically significant dependence on gradient, which is evidence for a quasi-equilibrium of \(C\) during demagnetization. A true steady-state transport measurement, on the other hand, would suffer from an inhomogeneous magnetization due to imbalanced chemical potentials in the trap. Our dynamic measurement avoids this problem, since spin transport is strongly suppressed on the millisecond time scale \([7, 9]\).

A clear signal as to whether the observed effect is due to pairing would be to collect data as in Figure 6.10, but in addition, vary the temperature.
Chapter 7

Conclusion

This thesis presented three complementary measurements of the demagnetization dynamics of a unitary Fermi gas: nuclear magnetic resonance (NMR) techniques to study magnetization dynamics, time-of-flight absorption images to measure changing momentum distributions, and time-resolved rf spectroscopy to measure the development of short-range pair correlations.

The magnetization dynamics of Chapter 4 showed that transverse spin diffusion in a magnetic field gradient was responsible for the loss of magnetization. The transverse spin diffusivity of a 3D unitary Fermi gas saturated a lower bound of $3D D^\perp_s \sim \hbar/m$ for the lowest probed temperatures. A diffusion constant of $\hbar/m$ challenges a quasiparticle-based understanding of transport by implying the necessity of maximally incoherent quasiparticles. For a complete picture of spin diffusion in unitary gases, our results must be combined with previous measurements of the minimum trap-averaged 3D longitudinal diffusivity $3D D^\parallel_s = 6.3(3)\hbar/m$ [9] and 2D transverse diffusivity $2D D^\perp_s = 0.0063(8)\hbar/m$ [10]. Together they suggest that while the kinetic prediction of a lower bound is consistent with the 3D longitudinal and transverse diffusion measurements, the role of dimensionality is not understood.

In Chapter 5 we used a previous measurement of the equation of state of a spin-balanced unitary Fermi gas to predict the temperature rise in our system. As there was a flaw in our dynamic measurement of the temperature, a direct comparison of initial and final temperatures
did not yield quantitative agreement. We could however observe that the timescale to reach the final temperature was similar to that of the magnetization loss.

Chapter 6 presented the first dynamic measurement of Tan’s contact parameter. We showed upon a sudden change of interaction strength, the contact is fully developed within our shortest measurement timescale ($< 30\mu$s). Then we followed the development of interactions as magnetization is lost. A direct comparison of the dynamic contact measurements with the magnetization loss data from Chapter 4 revealed that short-range pair correlations were enhanced with respect to a statistical mixture.

Our study of unitary spin transport could be extended in several ways. A clear next step that requires no hardware change to the experiment is to study the Leggett Rice effect. A spin echo experiment similar to the one in Chapter 4 but with a different initial pulse angle will reveal the effect in the phase of the population oscillation. Once the Leggett Rice effect is understood, the magnetization dependence of transverse spin diffusion at low temperature could be probed. The addition of an optical lattice beam to our experiment would allow direct comparisons of spin transport in two and three dimensions.

To eliminate issues of trap averaging for both spin diffusion and contact measurements, we could selectively probe the centre of the cloud by using hollow core beams to pump atoms at the edge of the cloud into dark states prior to detection [72].

A temperature dependent study of the relationship between contact and magnetization is warranted. The superfluid transition temperature for a spin-balanced unitary Fermi gas is $0.167(13)\ T_F$ [5], and smaller for spin-imbalanced gases [37]. It would be interesting to see the temperature dependence of the spin-singlet population above and below the superfluid transition temperature.

It seems that strong interactions keep our gas in a quasi-equilibrium throughout the dynamics; the timescale of the temperature rise and growth of contact are closely linked to the demagnetization timescale. This could be assessed further in the following ways: 1) A measurement of the equation of state as a function of polarization would allow us to calculate the
quasi-equilibrium temperature dynamics for our system and directly compare with the measured timescale. 2) We could also perform careful equilibrium measurements of the contact as a function of temperature and magnetization to compare with the dynamic contact curve. For this, we would need to develop better control over the temperature of mixtures than we currently have. 3) We could push to faster timescales with larger magnetic field gradients to see when this “quasi-equilibrium” breaks. However, this can only go so far because atoms will be pushed out of the trap by very large gradients. 4) Dynamically verify (or not) Tan’s energy relation by comparing the contact dynamics to the time-dependent release energy of the cloud (remove the jump to the zero crossing for the temperature measurement).

In conclusion, this thesis was centred around the simple idea of putting a system of atoms in a superposition of resonant interacting states and observing the transformation of an ideal Fermi gas to a unitary mixture as coherence is lost. As a result, we have touched on bounds to the kinetic theory of transport, made use of universal thermodynamics, and observed the development of short-range pair correlations. Clearly, there is a lot to be learned from the dynamics of unitary Fermi gases.
Appendix A

Double Well

The first three years of my PhD were spent working on the BEC double well experiment. The work has been thoroughly discussed in Lindsay LeBlanc’s PhD thesis [14]. Attached is a copy of our published Letter [84].
Quantum mechanical transport is a consequence of spatial variations in phase. Superfluids behave like perfect inviscid irrotational fluids, whose velocity is the gradient of a local phase, so long as the confining potential is smooth on the scale of the healing length. Where the density is small, as it is near surfaces, quantum kinetic terms must be added to the classical hydrodynamic equations. Macroscopic quantum coherence phenomena, such as Josephson effects, emerge when superfluids are weakly linked across such a barrier region.

Josephson effects have been demonstrated with superconductors [1], liquid helium [2,3], and ultracold gases in both double-well [4,5] and multiple-well optical trapping potentials [6]. The canonical description of these experiments employs a two-mode model [7–9], in which a sinusoidal current-phase relationship emerges. Hydrodynamics of a local phase, so long as the confining potential is vanishing when $V_b$ vanishes when $V_b$ is adjusted from below to above the BEC chemical potential, $\mu$, the density in the link region decreases until it classically vanishes when $V_b = \mu$. The healing length in the link region, $\xi$, increases with $V_b$ and dictates the nature of transport through this region. Oscillatory dynamics spanning three octaves are observed as we smoothly tune $\xi$ from $0.3d$ to $2d$, where $d$ is the separation between the wells.

Examination of the dynamics of an elongated BEC in a double well is timely. Recent experiments have created squeezed and entangled states by adiabatically splitting a BEC [12–14]. The degree of squeezing inferred in the elongated case [12,13] seems to exceed what would be expected in thermal equilibrium [14], raising the possibility that out-of-equilibrium dynamics may be important. With much remaining to be explored in these systems, this work represents the first study of the dynamics in the crossover regime.

Our experiment begins as $^{87}\text{Rb}$ atoms in the $|F = 2, m_F = 2\rangle$ ground state are trapped on an atom chip and evaporatively cooled in a static magnetic potential $B_S(r)$, as described elsewhere [15]. To prevent gravitational sag and to compress the trap in the weak direction (with characteristic trap frequency $\omega_t = 2\pi \times 95$ Hz), we add an attractive optical potential with a 1064 nm beam. We dress the static potential with an oscillating radio-frequency (RF) magnetic field [16,17] radiating from two parallel wires on the atom chip [Fig. 1(a)]. In the rotating-wave approximation, the adiabatic potential created by the combination of the static chip trap, the rf dressing, and the optical force is

$$U(r) = m_F' g_F \hbar \delta(r) \sqrt{\Omega_1^2(r) + \frac{1}{2} m \omega_r^2 y^2},$$

where $m_F' = 2$ is the effective magnetic quantum number, $\delta(r) = \omega_t \mu - |\mu g_F B_S(r)/\hbar|$ is the detuning, $\Omega_1(r) = |\mu g_F B_{1g}(r)/\hbar|$ is the rf Rabi frequency, $B_{1g}(r) = |B_3(r) \times B_n(r)|/|B_3(r)|$ is the amplitude of the rf field locally perpendicular to $B_3(r)$, $\mu_0$ is the Bohr magneton, $g_F$ is the Landé $g$ factor, $\hbar$ is the reduced Planck’s constant, and $m$ is the atomic mass. By assuming the individual wells are harmonic near each minimum, calculations show that $\omega_r = 2\pi \times 425$ Hz, and $\omega_t$ varies from $2\pi \times 350$ Hz to $2\pi \times 770$ Hz as we tune from low to high barriers. For comparison between theory and experiment, we account for small corrections to Eq. (1) beyond the rotating-wave approximation [18,19].
After turning on the dressing field at a frequency \( \nu_{\text{rf}} = 2 \pi \times 765 \text{ kHz} \), where the trap is a single well, the population dynamics (and the out-of-equilibrium system is avoid self-trapping [4]). To initiate the dynamics, the power double well [Fig. 1(c)].

Population \( Z \) rises and the dressed state potential splits along \( x \) direction into two elongated traps [20]. Upon release of the potential bias, we find that the barrier \( V_b / \mu \) rises and the dressed state potential splits along \( y \) direction while compressing the sample along \( x \). Atoms are trapped \( 190 \mu \text{m} \) from the chip surface. (b) A schematic one-dimensional cut at \( t = -0.5 \text{ ms} \) through trapping potential along \( x \) (solid line) in the presence of linear bias (dashed line) and (c) balanced potential at \( t = 0 \), with \( Z_0 \neq 0 \).

After turning on the dressing field at a frequency \( \omega_\nu = 2 \pi \times 765 \text{ kHz} \), where the trap is a single well, we evaporatively cool to produce a BEC with no discernible thermal fraction. In 20 ms, we adiabatically increase \( \omega_\nu \) to a new value characterized by \( \delta_0 = \delta(r = 0) \), such that the barrier \( V_b \) rises and the dressed state potential splits along the \( x \) direction into two elongated traps [20].

Using a second 1064 nm beam weakly focused off-center in \( x \), an approximately linear potential is added across the double-well junction to bias the population towards one well [Fig. 1(b)]. By applying the bias beam before and during the splitting process, we prepare systems of atoms with a population imbalance \( Z = (N_R - N_L) / (N_R + N_L) \), where \( N_R \) (\( N_L \)) is the number of atoms in the right (left) well. The range of initial population imbalances \( Z_0 = Z(t = 0) \) we use is 0.05 to 0.10, small enough to avoid self-trapping [4]. To initiate the dynamics, the power of the bias beam is ramped off in 0.5 ms (faster than the population dynamics) and the out-of-equilibrium system is allowed to evolve for a variable time \( t \) in the symmetric double well [Fig. 1(c)].

To measure the time-dependent population \( Z(t) \), we freeze dynamics by rapidly increasing both \( B_{\text{ext}} \) and \( \omega_\nu \) to separate the wells by 70 \( \mu \text{m} \), where \( V_b / \mu \sim 10^4 \). We release the clouds from the trap and perform standard absorption imaging along \( y \) after 1.3 ms time-of-flight [Fig. 2(b)]. Analysis of these images allows us to determine \( N_R \) and \( N_L \) to a precision of \( \pm 50 \) atoms.

Upon release of the potential bias, we find that the population \( Z(t) \) oscillates about \( Z = 0 \) [Fig. 2(a)] [21]. To analyze the dynamics, we use a Fourier transform (FT) to find the dominant frequency components [Fig. 2(c)]. We repeat this measurement at many values of \( V_b / \mu \), where \( \mu \) is the Thomas-Fermi chemical potential, by varying \( \delta_0 \). For the purposes of this analysis, we ignore the decay of this signal, the \( 1/e \) time constant of which is typically two oscillation periods.

When the barrier is low, \( Z(t) \) consists displays two dominant frequency components. For higher barriers, the amplitude of the higher-frequency mode decreases until only a single frequency rises above the noise floor. The white points in Fig. 3 give these frequencies as a function of the experimental parameter \( \delta_0 \) and the calculated ratio of barrier height to chemical potential, \( V_b / \mu \). The ensembles used in Fig. 3 had total atom number \( N = 6600 \pm 400(\pm 1700) \), where the error bar is statistical (systematic).

In the low- and high-barrier limits, simple models can be used to understand the dynamics. For low barriers, the hydrodynamic equations of motion can be used to estimate the frequency of population oscillation. Assuming a harmonic population response for some \( Z_0 \), the response frequency is

\[
\omega_{\text{ID}}^3 = -\frac{2}{mNZ_0} \int_S \rho \hat{n} \cdot \nabla(U + g \rho) dS, \tag{2}
\]

where \( \rho \) is the density of the condensate at \( t = 0 \), \( S \) is the surface in the \( y-z \) plane bisecting the double well, and \( \hat{n} \) is the vector normal to this surface. Since \( \omega_{\text{ID}} \) in Fig. 3 shows good agreement with the lower frequency mode at low barriers. Since tunneling cannot contribute to hydrodynamic transport, \( \omega_{\text{ID}} \to 0 \) as \( V_b \to \mu \).

The breakdown in hydrodynamics also coincides with an increasing healing length, as shown in the inset of Fig. 3.

In the opposite limit, when tunneling dominates transport, a Josephson model [8] accurately predicts the frequency of the highest barrier points,
The structure and origin of the higher-lying dynamical mode can be studied within the simulations. If our trap were smoothly deformed to a spherical harmonic potential, the two observed modes would connect to odd-parity modes [11]: the lower mode connects to the lowest $m = 0$ mode (coming from the $\ell = 1$ mode at spherical symmetry, where the quantum numbers $\ell$ and $m$ label the angular momentum of the excitation and its projection along the axis of symmetry, $y$, respectively), while the higher mode originates from the lowest $m = 2$ mode ($\ell = 3$ at spherical symmetry) [24].

With insight from GPE simulations, the observation of a second dynamical mode, which was not seen in previous experimental work [4,5], can be explained. In a purely harmonic trap, a linear bias excites only a dipole mode [25]. By breaking harmonicity along the splitting direction, $x$, the barrier allows the linear perturbation ($\ell = 1, m = 0$, where $x$ is the azimuthal axis) to excite multiple Bogoliubov modes [26]. Numerical studies show that two additional ingredients are required to excite the higher mode. First, atom-atom interactions couple the $x$ excitation to the transverse ($y, z$) motion through the nonlinear term in the GPE. Second, the anisotropy of the trap in the $y, z$ plane mixes the $m = 0$ and $m = 2$ modes such that each of the resulting modes drives population transfer between wells.

Figure 4 shows the relative strength $R_1 = a_1 / (a_1 + a_2)$ of the lower frequency mode as a function of the barrier height. The amplitude $a_1$ ($a_2$) of the lower (higher) frequency mode is extracted from a decaying two-frequency sinusoidal fit. The modes have comparable strength, even in the linear perturbation regime, when the barrier is below the chemical potential. The small spread in the GPE amplitudes shown by the grey band indicates that the higher mode is excited independently of the initial imbalance, and is not simply due to a high-amplitude nonlinearity.

FIG. 3 (color online). Frequency components of population imbalance vs rf detuning (measured) and barrier height to chemical potential ratio (calculated). Experimental points (white circles) represent the two dominant Fourier components at each detuning; error bars represent uncertainty contributed by noise in the FT from a single time series, but do not include shot-to-shot fluctuations. The spectral weight is represented through the color map, which has been linearly smoothed between discrete values of $V_b/\mu$ and darker colors indicate greater spectral weight. All calculations use $N = 8000$ and $Z_0 = 0.075$, and a single-parameter fit of the data to the GPE curves shifts all experimental points by $\delta_{\text{shift}} = 2\pi \times 5.1$ kHz [19] to compensate for a systematic unknown in $B_3(0)$. Statistical vertical error bars are shown, while a typical horizontal statistical error bar is shown at $V_b/\mu = 0.5$. Dashed lines represent 3D GPE frequencies, the solid line the plasma oscillation frequency predicted by the Josephson model, $\omega_p$, and the dotted line the hydrodynamic approximation, $\omega_{\text{HD}}/2\pi$. White bars at $V_b/\mu \sim 0.1$ indicate the bounds of the GPE simulation corresponding to the systematic plus statistical uncertainty in atom number. Inset: ratio of healing length, $\xi$, to interwell distance, $d$, as a function of $V_b/\mu$. $\xi$ is calculated at the center of the barrier.

$$\omega_p^2 = \frac{1}{\hbar^2} \Delta E \left( \Delta E + N \frac{\partial \mu_{\text{loc}}}{\partial N} \right),$$

where $\Delta E$ is the energy difference between the symmetric and antisymmetric ground states of the double-well potential, $N$ is total atom number, and $\mu_{\text{loc}}$ is the chemical potential on one side of the well [19]. The agreement is surprisingly good even for $V_b$ just above $\mu$, beyond which the frequency decreases exponentially. To our knowledge, this constitutes the first direct observation of tunneling transport of neutral atoms through a magnetic barrier, only inferred, for instance, in Refs. [12,22].

To explain the crossover behavior and the existence of the higher-frequency mode, we turn to numerical solutions of a time-dependent three-dimensional Gross-Pitaevskii equation (GPE) [8,23], which should describe all mean-field dynamics at $T = 0$. The slope and separation of the measured frequencies are well captured by the GPE, as shown in Fig. 3, though the decay of population imbalance is not reproduced by these simulations.
The trend in $R_1$ reflects the shape of the trap. When the barrier is raised from zero, the higher mode is at first more easily excited due to an increased anharmonicity along $x$ as the trap bottom becomes flatter. By further increasing the barrier, the higher-frequency mode disappears from the population oscillation spectrum due to the vanishing excitation of transverse modes. As the wave functions in each individual well are increasingly localized to the effectively harmonic minima, the linear bias no longer excites intra-well transverse motion. Furthermore, in the linear perturbation regime, the interwell Josephson plasma oscillation, like all Bogoliubov modes, cannot itself trigger any other collective mode.

In conclusion, we have studied the quantum transport of a BEC in a double-well potential throughout the crossover from hydrodynamic to Josephson regimes. Apart from fundamental interest, knowing and controlling the nature of superfluid transport is crucial for technological applications of weak-link-based devices, such as double-slit interferometers [12,20,27–29]. The adiabatic transformation of a BEC from a single- to a double-well trapping potential has been discussed in recent experimental works [12,14,22,30,31] in the context of the Josephson model, valid at high barriers [32]. Our work demonstrates that for $V_b < \mu$, the lowest mode frequency will lie below that estimated by the Josephson model. Furthermore, the higher-lying mode we observe approaches the lowest collective mode as $\omega_{c} \ll \omega_{i}$ [19] and may be important to the dynamics of splitting in strongly anisotropic double wells [12,33]. Whether using splitting to prepare entangled states [14], or recombination [31] to perform closed-loop interferometry [67], an improved understanding of double-well dynamics provides a foundation for controlling mesoscopic superfluids.

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21. When the average value of $Z$ differs from zero, we subtract the average $\bar{Z}$ from all values of $Z_i$. In all experiments, $|\bar{Z}| < 0.05$.
24. We checked this numerically by deforming our trap into a fully harmonic axially symmetric trap, and following the mode frequencies throughout this process.
Bibliography


