THREE ESSAYS ON FIRM ORGANIZATION AND TRADE

by

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Abstract

Three Essays on Firm Organization and Trade

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This thesis examines how firms organize production and its relationship to international trade.

In Chapter 1 I examine the assignment of workers to layers and firms. I use an administrative dataset of French workers to study the organization of manufacturing firms. First, I test whether higher ability workers are employed in the higher layers of firms. Second, I test whether there is positive assortative matching between workers in the different layers of firms. Third, I test whether abler managers supervise more workers. Finally, I test whether abler workers allow their managers to increase their span of control. I emphasize four results. First, abler workers are employed in the higher layers of firms. Second, I find evidence of positive assortative matching between workers in the different layers of firms.
Third, I find evidence that abler managers supervise less workers. Finally, I also find weak
evidence that abler workers allow their managers to increase their span of control.

In chapter 2 I investigate the effect market toughness has on the organization of firms.
To understand this relationship I develop a monopolistically competitive model with en-
dogenous markups and heterogeneous firms. As in Garicano (2000) and Caliendo and
Rossi-Hansberg (2012), production requires physical labour and knowledge. In the model,
market size affects markups which then affects how firms organize production and their
productivity. I also analyze how the distribution of organizations varies across markets
of different sizes. The model predicts that in larger markets firms will have more layers,
consistent with the data.

In chapter 3 I examine the impact of international trade on the sorting of heteroge-
neous agents into sectors and occupations, and within sectors the matching of agents into
teams, as well as the impact of trade on the distribution of earnings. To understand this
relationship I extend Garicano and Rossi-Hansberg (2004) to two goods and two countries.
I emphasize three results. First in an open economy, factor price equalization does not
hold. Second, in a given sector, the returns to managers and production workers change in
the same direction. And third whether international trade increases or decreases income
inequality depends on the knowledge of the country.
Dedication

To my parents and sister.
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## 3 Teams, Trade and Comparative Advantage

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Chapter 1

Sorting Within and Across French Production Hierarchies
1.1 Introduction

Ever since Coase (1937) economists have known that one of the most important problems a firm faces is how to organize inputs efficiently. However, classical economic models often abstract from firms’ organizational decisions. A firm is like a black box, whereby inputs are directly mapped into a final good. However, an understanding of how firms organize is essential, because firms determine the allocation of productive resources in the economy.\footnote{As noted by Rosen (1982): ‘The firm cannot be analyzed in isolation from other production units in the economy. Rather, each person must be placed in his proper niche, and the marriage of personnel to positions and to firms must be addressed directly.’}

One important organizational decision of firms is what types and how many workers they should hire, as well as what tasks should be assigned to which workers.

Despite much theoretical interest, very little is known empirically about how workers sort together in firms.\footnote{For example, several studies have used models of firm organization to investigate earnings inequality (Garicano and Rossi-Hansberg, 2006), offshoring (Antras, Garicano and Rossi-Hansberg, 2006, 2008) and knowledge diffusion (Dasgupta, 2012).} Several researchers have investigated whether good workers are employed in productive firms. While most empirical studies are concerned with how workers match with firms, far fewer studies have examined the different tasks workers perform, and whether better workers are employed with better workers in the other positions of a firm.

This paper fills the gap by examining how workers sort together in firms. My empirical strategy relies on the idea that firms can be thought of as hierarchical teams, composed of layers that perform different tasks. The lowest layer of a firm, for example, contains workers who focus on production, while higher layers contain individuals that perform managerial tasks. With this in mind, I examine how workers sort into teams and layers within each team. More precisely, within a team I first test whether higher ability workers are employed in higher layers. Second, across teams, I test whether there is positive assortative matching, in which the ability of an individual in one layer is positively correlated with the ability of a worker in another layer. Third, I investigate whether this sorting pattern is caused by higher ability workers allowing their managers to increase their span of control and employ larger teams, as suggested by Garicano and Rossi-Hansberg (2006).

I use an administrative dataset of French workers, the Declarations Annuelles des Données Sociales (DADS) to test these predictions. I begin by classifying employees as residing...
in the different organizational layers of firms. With my dataset, I observe four distinct layers, production and administrative workers, supervisors, senior managers, and owners and CEOs, by using occupational codes. The concept of a layer that I use is from the management hierarchy theory of the firm that was introduced by Garicano (1999) and used empirically by Caliendo, Monte and Rossi-Hansberg (2012). In theory a layer corresponds to a set employees who earn similar wages, are of similar ability and perform tasks at a similar level of authority. Since firms are hierarchical teams, layers have the added property that, within a firm, higher layers contain fewer workers who are of greater ability.

For every firm in the dataset, I calculate the total number of layers in the firm, and the size of each layer, in terms of labor hours worked. With my dataset I can observe four different types of organizations, one-layer firms, two-layer firms, three-layer firms, and four-layer firms. I show that this classification of employees into layers is meaningful and consistent with the concept of a layer discussed above.

Then for the years 1993 to 2004, I use the panel dimension of my dataset to obtain estimates of workers’ ability. I estimate a Mincerian wage regression with individual fixed effects, as in Abowd, Kramarz, and Margolis (1999). I use the individual fixed effects from my regression as my measure of worker ability.

Using these measures of the size and number of layers of firms, along with measures of worker ability in these layers, I test my main predictions. First, I conclude that higher ability workers are employed in the higher layers of firms. For example for four-layer firms, I find that an individual with a one hundred percent increase in his ability will on average reside 0.511 layers higher. Second, I find evidence of positive assortative matching between workers in the different layers of firms. For example, in four-layer firms, a one hundred percent increase in the average ability of workers in layer one is associated with a 0.320 increase in the average ability of workers in layer two. Third, I find evidence that abler managers supervise less workers. For example, in three-layer firms a one unit increase in the average ability of managers in layer three is associated with a 23.1 percent decrease in their span of control. Finally, I find only weak evidence that higher ability workers allow their managers to increase their span of control. For example, in four-layer firms, a one unit increase in the average ability of workers in layer three is associated with a 33.4 percent increase in their span of control.
In the last part of the paper, I address robustness of my results by assessing several potential threats to my empirical strategy. First, one concern with the empirical analysis is that my worker fixed effects are inconsistent. Because the worker fixed effects are incidental parameters from a wage regression, they can only be measured consistently as the number of years an individual is observed in the panel grows large. To resolve this issue, I conduct my analysis on a restricted sample of worker fixed effects for workers that I observe for at least 10 periods. Second, as discussed in Andrews, Gill, Schank, Upward (2008) my measures of workers’ ability may be misestimated and any positive correlation between the individual fixed effects is the result of a positive correlation between the estimated error of the individual fixed effects. To address this issue, I conduct my empirical analysis outside of the sample, for the year 2008, and only on the set of workers who have moved to a firm that they have never been employed in before. Taking both potential threats into account, I continue to find that higher ability workers are employed in the higher layers of firms, that the ability of individuals in one layer of a firm is positively correlated with the ability of workers in another layer, and only weak evidence that higher ability workers allow their managers to increase their span of control, and evidence that higher ability managers supervise less workers.

This paper is related to the broad literature on the theory of the firm allowing for management hierarchies. With the aim of explaining the distributions of firm size and earnings in the economy, a long-standing literature has examined how productive factors are allocated to managers with different abilities (for example Lucas (1977) and Rosen (1982)). To motivate my empirical strategy, I use a model by Garicano and Rossi-Hansberg (2006) in which agents with different cognitive abilities sort into occupations, layers and teams. Regardless of the distribution of knowledge in the economy, the equilibrium displays skill stratification, in the sense that agents with similar levels of cognitive ability sort into the same occupations and layers across firms. Agents with the least amount of knowledge become production workers, while agents with high levels of ability sort into managerial layers which correspond to higher layers of firms. The equilibrium also displays positive assortative matching, in the sense that higher ability managers organize into firms with higher ability subordinates. The mechanism behind this result is the following: in a given layer, agents of greater ability can solve a greater proportion of problems, and thus render
their subordinates more productive. In turn, because they can solve a greater proportion of problems alone, higher ability subordinates require less of their superiors’ time. This frees up the latter’s time and allows managers to supervise more workers.

This paper is most closely related to Garicano and Hubbard (2005) who examine positive assortative matching between partners and associates in law firms in Texas. Using data on lawyers’ school of education and firm of employment, they find that associates are more likely to work at the same firm as partners who went to a similarly ranked school, consistent with positive assortative matching. The nature of their data, however, does not permit them to obtain a measure of workers’ ability that varies across individuals who graduated from the same school. In addition their analysis is limited to two-layer firms: partners and associates. My dataset and classification strategy allow me to make progress on this issue, since I can identify up to four layers in firms. Finally, another distinction is that I examine the mechanism that is causing this sorting pattern: higher ability managers supervising larger teams.

More generally, this paper is also related to a large empirical literature examining sorting in labor markets. In particular, this literature is concerned with whether productive workers are matched with productive firms (see for example Abowd, Creecy, and Kramarz (2002), Abowd, Kramarz, Lengermann and Perez-Duarte (2004), Martins (2008), Andrews, Gill, and Upward (2006)). My paper examines a related but different question: how workers sort with other workers in layers and firms.

In the sorting literature my paper is most closely related to Lopes de Melo (2013), who uses matched employer-employee data from Brazil to examine whether workers of similar ability sort into the same firms. While Lopes de Melo focuses exclusively on whether workers of similar ability sort into the same firm, I also examine how workers sort into the different layers of firms, and the mechanism that is causing this pattern. In addition I make progress on an econometric issue associated with testing whether higher ability workers sort into the same firms. Specifically, a positive correlation between workers’ ability may be due to standard estimation error. I address this issue by examining whether my findings hold in the year 2008, for individuals who have moved to a firm that they have never been employed

\[^3\] Iranzo, Schivardi and Tosetti (2008) investigate whether production and non-production workers are complements or substitutes. Although in their analysis, managers are contained in their non-production worker classification, they do not focus on the relationship between managers and production workers.
The paper proceeds as follows: Section 2 presents a brief description of the management hierarchy theory of a firm and its predictions. Section 3 introduces the data. Section 4 discusses the estimation strategy. Section 5 presents the descriptive statistics and summary results, Section 6 tests the model’s predictions, and Section 7 presents robustness checks. Finally, section 8 concludes.

1.2 Model

In this section I briefly present the management hierarchy model from Garicano and Rossi-Hansberg (2004, 2006) and discuss its main conclusions. To fix ideas, I first present the model where teams have only two layers, and then describe the general setting in which teams have any number of layers. For a complete exposition and for all proofs, I refer the reader to Garicano and Rossi-Hansberg (2004, 2006) and Antras, Garicano and Rossi-Hansberg (2008).

1.2.1 Setup of the Model: Two-Layer Firms

Production requires labor and knowledge. A continuum of agents have one unit of time and are heterogeneous in their level of knowledge. For agents who spend their time in production, to produce a unit of output they need to solve the problem they confront. More specifically, agents draw one problem per unit of time spent in production. If they can solve the problem then a unit of output is produced, otherwise output is zero.

Problems are differentiated by their difficulty. The difficulty of a problem is drawn from a known distribution, and depending on his knowledge, an individual may or may not be able to solve the problem. Assume that problems are drawn from a uniform distribution with support [0, 1]. An individual with knowledge $z$ can solve all problems in the interval [0, $z$]. That is, $z$ is the fraction of problems an individual with knowledge $z$ can solve. The output of such an individual from working alone is therefore:

\[ y(z) = z. \]  

(1.1)

I abstract from the decision to acquire knowledge. For a model where agents acquire knowledge see Garicano and Rossi-Hansberg (2006).
Production, however, can occur in teams. Teams are composed of production workers, who spend their one unit of time drawing problems and attempting to solve them, and a manager, who does not draw problems but instead spends all of his time solving problems that his production workers cannot solve. In other words, production workers specialize in routine tasks (i.e. production), while managers specialize in nonroutine tasks (i.e. problem solving).

For managers to receive problems, communication is possible between managers and production workers within a team. When a production worker encounters a problem that he can solve alone, he produces a unit of output. Otherwise, he asks his manager who in turn spends a fraction of his time communicating with his worker. If the manager knows the solution to the problem, then he conveys the solution to his worker who immediately produces a unit of output. Otherwise the problem remains unsolved and nothing is produced. Let $h$ denote the time cost per problem that a manager incurs communicating with his production worker. For a manager with knowledge $z_m$, working in a team composed of $n$ production workers with knowledge $z_p$, his time constraint is the following:

$$nh[1 - z_p] = 1,$$

(1.2)

where $h[1 - z_p]$ is the total cost per unit of time that a manager incurs while working with a production worker with knowledge $z_p$. The communication technology therefore limits the amount of interactions managers can have with their workers, and this in turn determines the number of workers a manager can supervise, $n(z_p)$. Since a manager deals with all problems that his production workers cannot solve, and he can only solve a fraction $z_m$ of them, the output of such a team is the following:

$$y(z_m, z_p) = n(z_p)z_m.$$

(1.3)

The production framework has several important features. First, regardless of their occupation, workers are not perfect substitutes. Second, there is a mechanism through which managers can increase the value of their knowledge, by sharing it with their workers. That is, in this framework matching is many to one. Third, production is asymmetrically sensitive to skill. Since they can leverage their knowledge over many workers, managers are more...
important for production. And fourth, managers and production workers are complements. The mechanism behind this result is the following. Managers of greater knowledge can solve a greater proportion of problems, and thus render workers more productive. Holding the knowledge of production workers constant at $z_p$, the output of a team is increasing with the knowledge of the manager. In turn, more knowledgeable production workers increase the productivity of their manager. Since all individuals have one unit of time, managers are constrained in the number of workers that they can supervise. Because they can solve a greater proportion of problems on their own, more knowledgeable production workers spend less time communicating with managers. This frees up their managers’ time and allows them to supervise more workers. Holding the knowledge of a manager fixed at $z_m$, the output of a team is increasing in the knowledge of production workers.

In equilibrium the economy will exhibit the following properties. First there is positive assortative matching between managers and production workers. This follows from the fact that managers and production workers are complements. Second, individuals with greater knowledge will sort into managerial occupations, while individuals with lesser knowledge will become production workers. This follows from the fact that production is asymmetrically sensitive to skill. Finally, managers of greater knowledge will supervise more production workers and employ larger teams. This follows from positive assortative matching and the manager’s time constraint. The following proposition summarizes the results.

**Proposition 1.1** The equilibrium assignment of individuals to occupations and teams has the following properties:

- Individuals of greater knowledge sort into managerial occupations, while individuals with lesser knowledge become production workers
- There is positive assortative matching between managers and production workers
- Managers with greater knowledge supervise larger teams.

In Figures 1, 2 and 3, I present a numerical example where the distribution of knowledge and problems is uniform with support $[0, 1]$, and the manager’s cost of communicating with his production workers is 0.6 units of time. In this setting, in equilibrium there is only one type of organizations present: two-layer firms. In my example all individuals with knowledge below 0.723 are production workers, and all individuals above are managers. Since
managers increase the productivity of their workers, and production workers allow managers to supervise more workers, there is positive assortative matching. Figure 1 plots the assignment of production workers to their managers. The assignment function is increasing, knowledgeable managers supervise knowledgeable production workers. In this parametrization of the economy, workers with 0 knowledge form teams with managers with knowledge 0.723, while workers with knowledge 0.723 are assigned to managers with knowledge 1. In addition, since managers supervise many workers the mass of managers is smaller than the mass of production workers. Given my assumption of a uniform distribution of talent, the mass of managers is 0.723 and the mass of production workers is 0.277.

Figure 2 plots the number of production workers a manager supervises, his span of control, as a function of his knowledge. In equilibrium managers’ supervision increases nonlinearly with their knowledge. This result follows from the assignment of agents to firms, and the production technology. The least productive manager supervises 1.68 workers, while the most productive manager supervises 6.01. Finally, figure 3 plots managers’ span of control as a function of their workers’ knowledge. Because managers of greater knowledge work with more knowledgeable production workers, the function is also increasing.

\[ \int_{0.0723}^{1} n(z)g(z)dz = 0.723 \]

That is, the total demand of production workers by managers has to equal their supply.
1.2.2 Setup of the Model: Firms with More than Two Layers

When teams with more than one layer of managers can form, production has a similar structure. Communication is still possible between managers and production workers, however a production worker now has more managers available to help him solve problems. When a production worker encounters a problem that he cannot solve, he first asks his immediate manager who in turn spends a fraction of his time communicating with his worker. If the manager knows the solution to the problem, then he conveys the solution to his worker who immediately produces a unit of output. Otherwise the production worker asks the manager two layers above. If that manager knows the solution to the problem, he explains the solution to his production worker, who produces a unit output. If the manager does not know the solution, the production worker asks the manager three layers above. This process continues, until the problem is solved, or the production worker has seen a manager in every layer of the firm, at which point the problem remains unsolved.

Consider a team composed of three layers, a manager in layer three with knowledge $z_m^3$, $n_m^2$ managers in layer two with knowledge $z_m^2$ and $n_p^1$ production workers in layer one.
with knowledge $z_1^p$. Let $h$ the time cost per problem that a manager incurs communicating with a production worker and assume that this cost is the same across all managers. Since managers in layer two can solve $z_2^m$ fraction of the problems, the time constraint of a manager in layer three is the following:

$$n_p^1 h [1 - z_2^m] = 1,$$

(1.4)

where $h [1 - z_2^m]$ is the total cost per unit of time that the manager incurs while working in a team composed of managers in layer two with knowledge $z_2^m$. Similarly, the number of managers in layer two is determined by the time constraint:

$$n_p^1 h [1 - z_p] = n_m^2,$$

(1.5)

where $h [1 - z_p]$ is the total cost per unit of time that a manager in layer two incurs while working with a production worker with knowledge $z_1^p$. Again, the communication technology limits the amount of interactions managers can have with their subordinates, and this in turn determines the number of workers a manager can supervise, $n_p^1$, and the number of managers in layer two, $n_m^2$. Since the manager in layer three deals with all problems that his production workers and his managers in layer two cannot solve, and he can only solve a
fraction $z_m^3$ of them, the output of such a team is the following:

$$y(z_m^3, z_m^2, z_p^1) = n_p^1 z_m^3.$$ (1.6)

The production framework has the same features as in the previous setting. First, regardless of their occupation, individuals are not perfect substitutes. Second, there is a mechanism through which managers can increase the value of their knowledge, by sharing it with their workers. Third, production is asymmetrically sensitive to skill. And fourth, managers in layer two and managers in layer three are complements, and production workers and managers are complements. Therefore in equilibrium the economy will exhibit the same properties as in an economy with only two layer firms. The following proposition summarizes these results:

**Proposition 1.2** With three layer firms, the equilibrium assignment of individuals to occupations and teams has the following properties:

- **Individuals with the greatest knowledge sort into managerial occupations in layer three, individuals with intermediate knowledge sort into managerial occupations in layer two, while individuals with least knowledge become production workers**
- **There is positive assortative matching between managers and production workers, and between managers in the different layers of a firm**
- **For all layers, managers with greater knowledge supervise more individuals in the layers below.**

The results in proposition 1.2 are generalizable to an economy where firms can have any number of layers (see Garicano and Rossi-Hansberg (2006) for details). Furthermore, an important point to note is that any model with a production function that exhibits similar interactions between managers and production workers will, in a general equilibrium, yield similar results (Garicano and Hubbard, 2013). More specifically, as long as high skill agents raise the productivity of their subordinates, and better individuals require less supervision, then in equilibrium, high skill individuals will form firms with more and better subordinates in the layers below.
1.2.3 Discussion: Taking the Model to the Data

In this section thus far, I have presented the intuition of the model, and its predictions. The sections further below are concerned with estimating a measure of worker’s ability, testing whether there is positive assortative matching between workers in the different layers of firm, and testing the mechanism that is driving this sorting pattern: able workers require less supervision, and thus allow their superiors to supervise more of them. Given my measure of ability, testing for positive assortative matching is straightforward. As done in the literature that investigates matching between workers and firms, to test whether there is positive assortative matching between workers in the different layers of firms, I estimate a correlation between the average ability of workers in the different layers of firms. To test whether abler managers supervise more subordinates, I base myself on the equations that describe managers’ time constraint. For firms with three layers, the relevant equations are:

\[ n_1^p h [1 - z_p] = n_2^m, \]  
\[ n_1^p h [1 - z_2^m] = 1. \]

These equations have the following features. Both are a function of the number of production workers, and both depend on the knowledge of the individuals in the layer below. After rearranging the equations, and taking natural logs, one obtains the following expressions:

\[ \ln \frac{n_1^p}{n_2^m} = \ln h - \ln [1 - z_p], \]  
\[ \ln \frac{n_1^p}{1} = \ln h - \ln [1 - z_2^m]. \]

Because abler workers require less supervision, the size of a layer 1 relative to a given layer is positively correlated with the ability of workers in the layer below.

These equations can be generalized to a firm with \( L \) layers. Furthermore as managers occupy all other layers except layer 1, I drop the subscripts \( p \) and \( m \) from the equations, and rewrite the equations above in a more general form. Namely:
\[
\ln \frac{n^1}{n^{l+1}} = \ln h - \ln[1 - z^l].
\] (1.11)

where \(n^1\) denote the size of layer 1, \(n^{l+1}\) denotes the size of layer \(l + 1\) and \(z^l\) denotes the ability of agents in layer \(l\).

For a given firm, I define the span of control of workers in layer \(l\) as the ratio of the size of layer 1 to the size of layer \(l\), i.e. \(\text{span}^l = \frac{n^1}{n^l}\). According to the model, therefore, managers’ span of control should be increasing with their subordinates ability. Moreover in the model, agents’ ability is constrained to be between 0 and 1. As I obtain estimates of agents’ ability that are outside of this range, I approximate the equation (1.11) with the following equation:

\[
\ln \frac{n^1}{n^{l+1}} = \gamma_0 + \gamma_1 \hat{z}^l + u,
\] (1.12)

where \(\hat{z}^l\) represents my measure of the ability of agents in layer \(l\). If the mechanism described by the model, that determines how agents sort together into firms, holds in the data then the estimated coefficient \(\gamma_1\) should be positive and significant.

Furthermore, since there is positive assortative matching, there is a one-to-one correspondence between the ability of individuals in the different layers of a firm. Therefore, equation (1.11) can also be rewritten as:

\[
\ln \frac{n^1}{n^{l+1}} = \ln h - \ln[1 - f(z^{l+1})].
\] (1.13)

where \(f()\) is a function that maps the ability of workers in layer \(l\) to layer \(l + 1\); i.e. \(z^l = f(z^{l+1})\). I approximate equation (1.13) with the following equation:

\[
\ln \frac{n^1_p}{n^{l+1}_m} = \gamma_0 + \gamma_1 \hat{z}_{l+1} + u,
\] (1.14)

where \(\hat{z}_{l+1}\) represents my measure of the ability of agents in layer \(l + 1\). If the mechanism described by the model holds in the data then the estimated coefficient \(\gamma_1\) should be positive and significant.
1.3 Data

1.3.1 Data Description

The data are extracted from the Déclarations Annuelles des Données Sociales (DADS), which are provided and maintained by the French National Statistical Institute for Statistics and Economic Studies (INSEE). The DADS are matched employer-employee datasets and are constructed from administrative records that must be completed by all employers in France. A report must be filled by each establishment for every one of its employees, so there is a unique record for each employee-establishment-year combination. The DADS contains two datasets: a panel of workers born in October and that runs from 1976 to 2008, and from 1993 to 2008, exhaustive cross-sections of all workers in mainland France.\(^6\)

In both the panel and cross-section datasets, for each observation, there is information on employees’ characteristics, such as age, gender, and occupation, basic information on the establishment, such as location, industry and the parent firm, and basic firm level information, such as the firm’s industry. For each observation there is also information on annual earnings, denominated in 2007 euros, number of days worked, and number of hours worked.\(^7\)

As discussed further below, I use the panel dataset to obtain measures of workers’ ability. From the panel dataset, I remove incomplete observations and industries for which there are coding problems. For computational tractability, I restrict the sample to the years 1993 to 2004, and to all full-time workers who are born in October in an even numbered year, are between the ages 18 and 65 and work in mainland France. In a given year, an individual may hold multiple jobs. In case of multiple jobs, for a given year I keep the worker’s employment with the highest salary. For the years 1993 to 2004, there are 4,999,728 observations, corresponding to 753,092 workers in 399,676 firms. Appendix A and Abowd, Kramarz and Margolis (1999) provide further details on the data and information on how wages are determined in France.

From the exhaustive cross-section, I use the year 2004, and remove incomplete obser-

---

\(^6\)Until 1993 the DADS only contained information on individuals born in October in an even numbered year. From 1993 onwards, the DADS contains information on all individuals born in October.

\(^7\)Information on the total number of hours worked is only available after 1993.
vations and industries for which there are coding problems. I use the information in the cross-section to measure the total number of layers in firms and the size of each layer, in terms of total hours worked. Appendix A and Caliendo, Monte and Rossi-Hansberg (2012) provide further details on the exhaustive cross-section data.

1.3.2 Classification of Layers

To organize the data, I classify employees as residing in the different organizational layers of firms. The concept of a layer that I use is from the management hierarchy theory of the firm by Garicano (1999). In theory a layer corresponds to a set employees who earn similar wages, are of similar ability and perform tasks at a similar level of authority (Caliendo, Monte and Rossi-Hansberg, 2012). The concept of a layer that I use is therefore independent of the actual occupations of employees, such as whether they are lawyers, engineers or computer programmers. Instead it depends on their knowledge, productive ability, and their relative position in the organizational hierarchy of firms. In addition, since firms are hierarchical, layers have the added property that within a firm higher layers contain less workers who are of greater ability.

To map this concept of a layer to the data, I adopt the strategy put forth by Caliendo, Monte and Rossi-Hansberg (2012), and use one-digit occupational codes, which range from 2 to 6, to classify employees into layers. In total, I can classify employees into four distinct layers. Layer 1 corresponds to qualified and non-qualified administrative workers and blue-collar workers. It contains all workers with occupational codes 5 and 6, respectively. Layer 2 is composed of supervisors and individuals with a higher level of responsibility than ordinary workers, and contains all workers with an occupational code 4. Layer 3 is composed of senior directors and top management staff and contains all workers with an occupational code 3. Layer 4 corresponds to owners who receive a wage and CEOs. It contains all workers with occupational code 2.

I consider a firm to have a layer if there is at least one employee in the exhaustive cross-section employed there. In all, in the dataset there are four types of firms present: one-layer

---

8Since the panel of the DADS is only a 5 percent sample of the population, it is not suitable to properly measure the total number of layers and the size of each layer in a firm.
9The occupational codes range from 1 to 6. I have removed all firms operating in the agricultural and fishing industries, which correspond to occupational code 1.
Chapter 1: Sorting Within and Across French Production Hierarchies

Table 1.1: Descriptive Statistics from the Exhaustive Cross-Section Dataset

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Number of Firms</th>
<th>Number of employees</th>
<th>Average Number of hours</th>
<th>Median Number of employees</th>
<th>Median Number of hours</th>
<th>Standard Deviation Number of employees</th>
<th>Standard Deviation Number of hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160,904</td>
<td>3.45</td>
<td>3,382.28</td>
<td>2</td>
<td>2,028</td>
<td>4.73</td>
<td>4,980.97</td>
</tr>
<tr>
<td>2</td>
<td>74,676</td>
<td>8.36</td>
<td>9,437.35</td>
<td>5</td>
<td>6,084</td>
<td>10.80</td>
<td>11,971.35</td>
</tr>
<tr>
<td>3</td>
<td>52,949</td>
<td>23.90</td>
<td>31,701.27</td>
<td>11</td>
<td>13,342</td>
<td>53.59</td>
<td>80,918.29</td>
</tr>
<tr>
<td>4</td>
<td>14,434</td>
<td>59.92</td>
<td>82,872.88</td>
<td>33</td>
<td>47,106</td>
<td>96.12</td>
<td>119,415.10</td>
</tr>
</tbody>
</table>

Notes: Descriptive statistics from the exhaustive cross-section of the DADS for the year 2004.

Table 1.2: Descriptive Statistics from the Matched Dataset

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Number of Firms</th>
<th>Number of employees</th>
<th>Average Number of hours</th>
<th>Median Number of employees</th>
<th>Median Number of hours</th>
<th>Standard Deviation Number of employees</th>
<th>Standard Deviation Number of hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,160</td>
<td>11.00</td>
<td>11,844.12</td>
<td>8</td>
<td>8,233.50</td>
<td>23.14</td>
<td>25,319.03</td>
</tr>
<tr>
<td>2</td>
<td>3,322</td>
<td>19.88</td>
<td>25,989.84</td>
<td>14</td>
<td>18,173.50</td>
<td>22.11</td>
<td>28,176.84</td>
</tr>
<tr>
<td>3</td>
<td>7,860</td>
<td>67.14</td>
<td>101,683.20</td>
<td>37</td>
<td>53,741</td>
<td>115.92</td>
<td>183,040.00</td>
</tr>
<tr>
<td>4</td>
<td>5,450</td>
<td>83.61</td>
<td>128,090.50</td>
<td>53</td>
<td>81,350</td>
<td>99.54</td>
<td>155,456.00</td>
</tr>
</tbody>
</table>

Notes: Descriptive statistics from the the matched sample dataset for the year 2004.

firms, two-layer firms, three-layer firms and four-layer firms. Further below, I show that this classification of employees into layers is meaningful and consistent with the concept of a layer discussed above.

1.3.3 Merging Datasets

For the year 2004 I merge the information from the panel and exhaustive cross-section datasets together. Unlike the exhaustive cross-section, since the panel data is based on a 5 percent sample of the French population, it contains information on a sample of all firms operating in mainland France. Approximately 1 percent of firms in the panel dataset are not matched. I keep only firms that operate in the manufacturing sector, and remove any firms that operate in more than one industry and location. In total the matched dataset contains 23,916 firms that operate in 17 industries, of which 2,160 are one-layer firms, 3,322 are two-layer firms, 7,860 are three-layer firms and 5,450 are four-layer firms.

Tables 1.1 and 1.2 contains descriptive statistics of firms in the exhaustive cross-section dataset and the matched dataset for the year 2004, respectively. As is evident from the average and the median number of workers and the average and the median number of
hours worked in a firm, the matched dataset contains larger firms.\textsuperscript{10} For both measures of firm size, number of employees and the number of hours, the standard deviation in the matched sample is also greater than in the population. The sample is therefore biased towards larger firms, and is not representative of the entire population of firms. In light of this finding, the conclusions in this paper may not be valid for the entire population of workers and firms within France.

1.4 Estimating Ability

To test the predictions of the model, I need a measure of worker’s ability. I use the empirical approach of Abowd, Kramarz and Margolis (1999) which has been developed further by Card, Kline and Heining (2013) to obtain measures of workers’ ability from the data. I model log hourly wages, $w_{it}$, of worker $i$ in time $t$, as a linear function of a time-invariant worker component $\theta_i$, a time-invariant firm-layer component $\psi_{J(i,l,t)}$, time varying worker characteristics $x_{it}$, and a mean-zero error term $\epsilon_{it}$. The equation to be estimated is:

$$\ln w_{it} = x_{it}\beta + \theta_i + \psi_{J(i,l,t)} + \epsilon_{it}.$$  \hspace{1cm} (1.15)

The term $\theta_i$ captures the portable part of a worker’s wages that remain with him as he moves across firms, or layers within firms. The variation of this term reflects a worker’s productivity, bargaining ability and labor market discrimination. In the subsequent analysis, I use $\theta_i$ as my measure of workers’ ability. The terms $x_{it}$ captures how workers’ earnings evolve with changes in their observable attributes, such as labor market experience. In my estimation, I use age as a proxy for experience.\textsuperscript{11} Although in theory, workers only form firms with other workers, I include in equation (1.15) firm-layer fixed effects, $\psi_{J(i,l,t)}$, which are meant to identify firm attributes that affect every worker’s earnings in a given layer in a firm equally, such as compensation policies, bargaining strength in the labor market, and productivity. Alternatively, since not all workers are employed in the same firm throughout their career, one can interpret the firm-layer fixed effects as partially accounting for any

\textsuperscript{10}As explained further below, one reason for this result is that I obtain my measures of workers’ ability from the largest connected set of workers and firms.

\textsuperscript{11}Since in equation (1.15) age cannot be separately identified from worker and time fixed effects, I exclude any time trends from the analysis. Indeed one can show that age can be written as a linear combination of the time and worker fixed effects.
permanent influences past employees may have on the current organization, or any influences that affect individuals’ earnings in a given layer of a firm that are the result of workers in the other layers.

To identify all of the econometric parameters in equation (1.15), I assume, as in Abowd, Kramarz and Margolis (1999), that the error term \( \epsilon_{it} \) is strictly exogenous. Under this assumption, the parameter \( \beta \) can be consistently estimated as the number of workers, \( N \), the number of firm-layers, \( J \), and the number of years, \( T \), increases. The parameters \( \theta_i \) and \( \psi_{J(i,t,l)} \) can only be separately identified by workers who switch employers, or layers within employers in the panel. In the dataset, there are in total of 1,156,816 worker displacements. Since \( \theta_i \) is an incidental parameter, consistent estimates for it can only be obtained as the number of years a worker is observed grows large. Table 1 in the Appendix A presents the distribution of the number of years a worker in observed in the panel. Over 50 percent of workers are observed for 6 years or more. Similarly, \( \psi_{J(i,t,l)} \) can only be consistently estimated if the number of workers in a layer in a firm, or the number of years grows large. Table 2 in the Appendix A presents the distribution of the number of workers observed in a layer in a firm in a given year, as well as the number of years firms’ layers are observed in the panel. The average number of workers in a layer in a firm is 2.67, and over 50 percent of firms’ layers are observed for 2 years or more.

To estimate equation (1.15) I focus on the largest connected group, that is the largest group of layers within firms that, over the years, have had at least one employee in common with another layer in the same or a different organization. In the panel, the largest connected group contains 753,092 workers and 569,198 layers within firms. To estimate equation (1.15), I use the algorithm put forth by Guimaraes (2007), which builds on the algorithm of Abowd, Creecy and Kramarz (2003).

1.5 Results & Descriptive Statistics

Table 1.3 summarizes the estimation results from regression (1.15). To summarize my findings I report the standard deviation of log hourly wages, of the worker and firm-layer effects, as well as of the time-varying observables. I also report the root mean squared

\footnote{The output of the algorithm provides a non-unique set of solutions for the worker and firm fixed effects. To make the effects unique, the algorithm sets the average of the firm fixed effects to zero.}
### Table 1.3: Summary Statistics

<table>
<thead>
<tr>
<th>Sample Year</th>
<th>1993 – 2004</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Worker and Firm-Layer Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Number of Worker Effects</td>
<td>753,092</td>
</tr>
<tr>
<td>Number of Firm-Layer Effects</td>
<td>569,198</td>
</tr>
<tr>
<td><strong>Summary of Parameter Estimates</strong></td>
<td></td>
</tr>
<tr>
<td>St. Dev. of Wages</td>
<td>0.4417</td>
</tr>
<tr>
<td>St. Dev. of Worker Effects</td>
<td>0.3940</td>
</tr>
<tr>
<td>St. Dev. of Firm Effects</td>
<td>0.2508</td>
</tr>
<tr>
<td>RMSE of AKM Residual</td>
<td>0.1717</td>
</tr>
<tr>
<td>Adjusted R-Squared</td>
<td>0.8489</td>
</tr>
<tr>
<td><strong>Correlations</strong></td>
<td></td>
</tr>
<tr>
<td>Wages &amp; Worker Effects</td>
<td>0.2509</td>
</tr>
<tr>
<td>Wages &amp; Firm Effects</td>
<td>0.5073</td>
</tr>
<tr>
<td>Worker Effects &amp; Firm Effects</td>
<td>−0.1636</td>
</tr>
<tr>
<td><strong>Comparison with the Match Effects Model</strong></td>
<td></td>
</tr>
<tr>
<td>RMSE of Match Model</td>
<td>0.1490</td>
</tr>
<tr>
<td>Adjusted R-Squared</td>
<td>0.8862</td>
</tr>
<tr>
<td><strong>ADDENDUM</strong></td>
<td></td>
</tr>
<tr>
<td>Sample Size</td>
<td>4,999,728</td>
</tr>
</tbody>
</table>

**Notes:** Results from OLS estimation of equation (1.15). $X\beta$ includes age and age squared interacted with gender. The match model includes $X\beta$ the region effects and a separate dummy for each worker-firm pair, corresponding to a job.
error (RMSE) of the residuals and the adjusted R-squared of the estimation, both of which take into consideration the number of variables estimated in the model. One important point to note is that the standard deviation of the worker effects is less than the standard deviation of wages. In the model because workers of different abilities are more productive from working in firms rather than alone, individuals’ wages are amplified relative to their ability, and hence the standard deviation of wages is greater than the standard deviation of abilities, consistent with the data.

In Table 1.3, I also report correlations. The correlation between the worker and firm-layer fixed effects is $-0.1636$. This finding is similar to the empirical literature that investigates how workers sort into firms. As many researchers report there is a negative correlation between worker and firm fixed effects, estimated from a log-linear wage equation. In my analysis, I abstract from this correlation, since I am concerned with how employees in each layer of a firm match, rather than how workers match with firms. Furthermore, the correlation between the individual fixed effects and log-hourly wages is 0.2509. Therefore, individuals of higher ability earn more.

Table 1.3 also contains the adjusted R-squared and RMSE of a model with unrestricted match effects, that is a separate dummy for each worker-firm-layer job spell. If match effects are an important determinant of workers’ wages, then a model with worker-firm-layer match effects should provide a markedly better fit to the data. The match effects model has an adjusted R-squared of 0.8862 and a RMSE of 0.1490, while the adjusted R-squared from the estimation of equation (1.15) is 0.8489 and the RMSE is 0.1717. The match effects model, therefore, fits the data slightly better than a specification with separate worker and firm-layer effects. Although this indicates that a match component is present in wages, the improvement in fit is modest.

As in Card, Kline and Heining (2013), I further examine the importance of a match component to wages. In particular, I examine the wage dynamics of all individuals who changed firms, or layers within firms, in the years 1993 to 2004 with at least two consecutive years in the new and old position. I classify the origin and destination positions by the quartile of the estimated firm-layer effects and calculate the average hourly wages of agents in each cell two years before and after the move. I report the results in Table 1.4. If the error term in equation (1.15) is exogenous, Card, Kline and Heining (2013) argue that the
Table 1.4: Mean Log Wages by Transitions and Years

<table>
<thead>
<tr>
<th>Origin-Destination Quartile</th>
<th>Number of Observations</th>
<th>Two Years Before</th>
<th>One Year Before</th>
<th>One Year After</th>
<th>Two Years After</th>
<th>Change from Two Years Before and After</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 to 1</td>
<td>25,775</td>
<td>2.03</td>
<td>2.11</td>
<td>2.14</td>
<td>2.22</td>
<td>0.19</td>
</tr>
<tr>
<td>1 to 2</td>
<td>15,654</td>
<td>1.89</td>
<td>1.99</td>
<td>2.19</td>
<td>2.24</td>
<td>0.35</td>
</tr>
<tr>
<td>1 to 3</td>
<td>11,759</td>
<td>1.79</td>
<td>1.91</td>
<td>2.28</td>
<td>2.31</td>
<td>0.52</td>
</tr>
<tr>
<td>1 to 4</td>
<td>3,410</td>
<td>1.85</td>
<td>1.97</td>
<td>2.64</td>
<td>2.58</td>
<td>0.73</td>
</tr>
<tr>
<td>2 to 1</td>
<td>16,427</td>
<td>2.09</td>
<td>2.20</td>
<td>2.03</td>
<td>2.12</td>
<td>0.03</td>
</tr>
<tr>
<td>2 to 2</td>
<td>51,732</td>
<td>2.07</td>
<td>2.14</td>
<td>2.16</td>
<td>2.21</td>
<td>0.14</td>
</tr>
<tr>
<td>2 to 3</td>
<td>44,670</td>
<td>2.07</td>
<td>2.16</td>
<td>2.28</td>
<td>2.32</td>
<td>0.25</td>
</tr>
<tr>
<td>2 to 4</td>
<td>10,489</td>
<td>2.08</td>
<td>2.19</td>
<td>2.55</td>
<td>2.55</td>
<td>0.47</td>
</tr>
<tr>
<td>3 to 1</td>
<td>11,468</td>
<td>2.25</td>
<td>2.37</td>
<td>1.98</td>
<td>2.11</td>
<td>−0.14</td>
</tr>
<tr>
<td>3 to 2</td>
<td>40,717</td>
<td>2.17</td>
<td>2.25</td>
<td>2.14</td>
<td>2.21</td>
<td>0.04</td>
</tr>
<tr>
<td>3 to 3</td>
<td>109,545</td>
<td>2.29</td>
<td>2.35</td>
<td>2.38</td>
<td>2.41</td>
<td>0.12</td>
</tr>
<tr>
<td>3 to 4</td>
<td>42,056</td>
<td>2.42</td>
<td>2.50</td>
<td>2.66</td>
<td>2.69</td>
<td>0.27</td>
</tr>
<tr>
<td>4 to 1</td>
<td>3,445</td>
<td>2.65</td>
<td>2.79</td>
<td>2.05</td>
<td>2.24</td>
<td>−0.41</td>
</tr>
<tr>
<td>4 to 2</td>
<td>8,550</td>
<td>2.42</td>
<td>2.56</td>
<td>2.16</td>
<td>2.25</td>
<td>−0.17</td>
</tr>
<tr>
<td>4 to 3</td>
<td>30,478</td>
<td>2.51</td>
<td>2.60</td>
<td>2.46</td>
<td>2.50</td>
<td>−0.01</td>
</tr>
<tr>
<td>4 to 4</td>
<td>72,529</td>
<td>2.73</td>
<td>2.81</td>
<td>2.85</td>
<td>2.87</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Notes: Descriptive statistics of job transitions from the estimation of equation (1.15).

following three conditions should hold in the data. First changes in the wages of individuals who transition from one quartile to the other should be relatively symmetric. For example the wage gain of individuals who transition from first quartile to the second quartile should be similar to the wage loss of individuals who transition from the second quartile to the first. Second individuals who move to new firms, or layer within firms, within the same quartile should not experience a wage gain. And third if the error term in equation (1.15) is exogenous, the increase in wages of workers who transition to different quartiles should also be monotonically increasing with the distance of the quartiles. These conditions hold in Table 1.4. For visual aide Figure 1.4 panel (a) illustrates the wage profiles of workers in the first and fourth quartiles. The gains or losses of individuals who transition to quartiles is monotonically increasing with the distance between the quartiles, and the gains or losses are relatively symmetric. Panel (b) illustrates the wage profiles of workers that remain within the same quartile. These profiles are relatively flat. Therefore, at a minimum, the model in equation (1.15) is a relatively decent first approximation to wages.

Table 1.5 reports summary statistics of firms in the matched sample. In the table firms are grouped by the total number of layers they have. The average firm in the sample has
(a) Movers from 1st and 4th Quartile

(b) Movers within the Same Quartile

Figure 1.4: Wages of Movers
Table 1.5: Description of Manufacturing Firms by Total Number of Layers

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Average Number of Employees</th>
<th>Average Number of Hours</th>
<th>Average Wage Ability</th>
<th>Median Wage Ability</th>
<th>Standard Deviation Wage Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st layer</td>
<td>11.00</td>
<td>11,844.12</td>
<td>2.13</td>
<td>0.433</td>
<td>2.10</td>
</tr>
<tr>
<td>2nd layer</td>
<td>2.71</td>
<td>4,115.16</td>
<td>2.40</td>
<td>0.428</td>
<td>2.38</td>
</tr>
<tr>
<td>THREE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st layer</td>
<td>49.16</td>
<td>71,431.91</td>
<td>2.33</td>
<td>0.352</td>
<td>2.27</td>
</tr>
<tr>
<td>2nd layer</td>
<td>11.72</td>
<td>19,900.35</td>
<td>2.61</td>
<td>0.473</td>
<td>2.57</td>
</tr>
<tr>
<td>3rd layer</td>
<td>6.25</td>
<td>10,350.94</td>
<td>2.96</td>
<td>0.580</td>
<td>2.96</td>
</tr>
<tr>
<td>FOUR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st layer</td>
<td>58.57</td>
<td>85,769.69</td>
<td>2.36</td>
<td>0.359</td>
<td>2.29</td>
</tr>
<tr>
<td>2nd layer</td>
<td>14.90</td>
<td>25,643.13</td>
<td>2.64</td>
<td>0.501</td>
<td>2.61</td>
</tr>
<tr>
<td>3rd layer</td>
<td>8.91</td>
<td>14,515.65</td>
<td>3.00</td>
<td>0.574</td>
<td>2.98</td>
</tr>
<tr>
<td>4th layer</td>
<td>1.22</td>
<td>2,165.00</td>
<td>3.27</td>
<td>0.772</td>
<td>3.39</td>
</tr>
</tbody>
</table>

Notes: Descriptive statistics of manufacturing firms that are in both the exhaustive cross-section and panel datasets of the DADS, for the year 2004. These statistics are reported separately for firms with different number of layers. Column 1 refers to the layer within a firm. Columns 2 report the average number of employees in a given layer, while column 3 reports the average number of hours worked by employees in a given layer. These measures are obtained from the exhaustive cross-section of the DADS. Columns 4, 6 and 7 report the average log hourly wages, median log hourly hourly wages and standard deviation of log hourly wages within a layer. Columns 5, 7 and 9 report the average ability, median ability and standard deviation of ability of workers in a given layer. Measures of wages and ability values are obtained from the panel dataset of the DADS. Ability is estimated from equation (1.15). An organizational structure that is consistent with a hierarchy. In the average firm lower layers are larger than the layers above. For example for the average four-layer firm, layer 1 has 58.57 employees, layer 2 has 14.90 employees, layer 3 has 8.91 employees and layer 4 has 1.22 employees. The findings are similar if one measures the size of layers by the number of hours worked. In addition average wages and ability are greater in the upper layers of a firm. Returning again to the average four-layer firm, the log-hourly wages of workers in layer 4 are 3.27, the log-hourly wages of workers in layer 3 are 3.00, and the log-hourly wages of workers in layers 2 and 1 are 2.64 and 2.36 respectively. Therefore there is a clear ranking in wages. This pattern is similar for ability. In a firm with four layers, the average ability of workers in layer 4 is 0.772, the average ability of workers in layer 3 is 0.574, and the average ability of workers in layers 2 and 1 are 0.501 and 0.359 respectively. The classification of workers into layers, therefore has economic meaning. The evidence is consistent with the view that firms are hierarchies, in the sense that higher layers of a firm are smaller and contain workers of greater ability.
1.6 Tests

1.6.1 Testing Skill Stratification

I first verify that abler individuals occupy the upper layers of organizations. Testing whether this sorting pattern holds in my sample is important because it confirms that workers in a layer are abler than their subordinates. If this is not the case, then the model’s prediction of positive assortative matching and the mechanism that determines how individuals sort into organizations is clearly false.\textsuperscript{13} For individual $i$ employed in firm $j(i)$, I therefore estimate the following equation:

$$\text{layer}_{j(i)} = \mu_0 + \mu_1 \text{ability}_i + X_{j(i)} + u_{j(i)},$$

where $\text{layer}_{j(i)}$ is the layer in firm $j$ with a total number of $L$ layers that worker $i$ occupies, $\text{ability}_i$ is the estimated ability of worker $i$, and $X_{j(i)}$ are industry and location controls. I estimate equation (1.16), across firms with the same total number of layers, separately. In equation (1.16) I am interested in how agents sort into layers. The coefficient of interest is $\mu_1$. If abler individuals occupy the upper layers of organizations, then $\mu_1$ will be positive and significant.

Table 1.6 reports the regression results. Each entry in the table reports the estimated coefficient of $\mu_1$. Because of the large number of indicator variables in the regressions, I estimate equation (1.16) using OLS. Rows 1 to 3 contain the results for firms with 2, 3 and 4 layers in their organization, respectively. The first column in Table 1.6 indicates the total number of layers in firms, the second column contains the sample size of the regressions, and the third to seventh columns report the estimated value of the coefficient, $\mu_1$.

The regressions in column (3) report how agents sort into layers across firms, industries and locations. In all three regressions the coefficients are positive and significant at the one percent level. The column indicates that an individual with a one hundred percent increase in his ability and employed in a two-layer firm will on average reside 0.053 layers higher,\textsuperscript{13} The argument made in the model is that organizations exist to optimally utilize the knowledge of their workers. By shielding knowledgeable agents from easy tasks, a hierarchy allows abler individuals to focus on solving more complex or harder problems, while lower ability individuals focus on easier or commoner problems. This implies that within a firm, higher ability agents occupy the upper layers of organizations.
Table 1.6: Regression Results for Skill Stratification

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Sample Size</th>
<th>Model (3)</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>4,432</td>
<td>0.053***</td>
<td>0.044***</td>
<td>0.047***</td>
<td>0.043***</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>THREE</td>
<td>19,841</td>
<td>0.369***</td>
<td>0.347***</td>
<td>0.349***</td>
<td>0.336***</td>
<td>0.411***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>FOUR</td>
<td>16,003</td>
<td>0.469***</td>
<td>0.446***</td>
<td>0.438***</td>
<td>0.428***</td>
<td>0.511***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.020)</td>
</tr>
</tbody>
</table>

Industry FE | No | Yes | No | Yes | No |
Area FE     | No | No  | Yes| Yes | No |
Firm FE     | No | No  | No | No  | Yes|

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors (clustered at the firm level) in parentheses. OLS regressions for equation (1.16). Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\mu_1$ from equation (1.16). The dependent variable is the layer that a worker occupies. The right-hand side variable is the ability of the worker. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France.

While if he is employed in an organization with three-layers he will on average reside 0.369 layers higher and if he is employed in a four-layer firm he will on average reside 0.469 layers higher.

Even within industries and locations, Table 1.6 reports that higher ability agents occupy the upper layers of firms. To examine how agents sort into layers within industries, in column (4) I include industry fixed effects. The coefficients remain positive and significant at the one percent level. In column (5) I include location fixed effects so as to examine how agents sort into layers within locations. The coefficients in column (5) remain positive and significant. And finally, in column (6) I include both industry and location fixed effects. The findings indicate that an individual with a one hundred percent increase in his ability and employed in a two-layer firm will on average reside 0.043 layers higher, if he is employed in a three-layer firm will on average reside 0.336 layers higher, and if he is employed in an organization with four layers he will on average reside 0.428 layers higher. Therefore even within industries and within locations abler individuals occupy the upper layers of organizations.

The regressions in column (7) contains firm fixed effects and indicate how agents sort into layers within firms. In two out of three regressions, the coefficient $\mu_1$ is positive and significant at the one percent level. For four-layer firms, the result indicates that an individual with a one hundred percent increase in his ability will on average reside 0.511 layers higher. For organizations with three layers an agent with one hundred percent increase in
his ability will on average reside 0.411 layers higher. Within three and four layer firms, therefore, abler individuals are employed in the upper layers of organizations.

In column (7), the value of $\mu_1$ for two-layer firms is 0.041 and is not significant at the ten percent level. This would suggest that within two-layer firms, higher ability agents are not sorting into the upper layers of organizations. However, in light of the fact that the coefficient of $\mu_1$ in column (7) is similar in magnitude to column (6), and that in the average two-layer firm there are 1.3 observations in the dataset, these findings are inconclusive.

To summarize the findings from this section, from the evidence presented in Table 1.6, one can conclude that abler agents occupy the higher layers of organizations. In other words, agents in higher layers of firms are of higher ability than agents in layers below. I can now examine whether there is positive assortative matching between agents in the different layers of firms, and whether the mechanism that determines the sorting pattern is as suggested by the model.

### 1.6.2 Testing for Sorting

I now test for positive assortative matching.\footnote{Appendix B contains additional tests for positive assortative matching. Appendix B tests whether better workers sort into organizations with better co-workers. I adopt the approach of Lopes de Melo (2013), and investigate whether a worker’s fixed effect is positively correlated with that of his co-workers. I conduct this analysis across several dimensions and find evidence in favor of positive assortative matching.} I proceed in two steps. First, for every layer in a firm, I define a representative measure of a layer’s ability. Second, I present my estimation strategy along with the results.

According to the knowledge-based hierarchical theory of firms, layers consists of workers who are of similar ability. A representative measure of a layer’s ability, therefore, is an average of the ability of workers that are employed there. Because not all workers devote the same amount of hours to a firm, I use a weighted average ability of workers as my representative measure of a layer’s ability, where the weights are determined by the number of hours.\footnote{I use a weighted average to account for the fact that some workers may be employed for the full year in a firm. In such a case, these workers cannot have the same impact on a firm, as workers who have been employed for the entire year.} More specifically, let $N^l_j$ be the number of individuals in layer $l$ at firm $j$, $H^l_{j(i)}$ be the number of hours performed by individual $i$ in layer $l$ in firm $j$ and $H^l_j$ be the total number of hours in layer $l$ at firm $j$. The measure of the ability of layer $l$ at firm $j$ that I use, is the following:

$$\text{measure} = \frac{1}{H^l_j} \sum_{i} w_i H^l_{j(i)}$$
Table 1.7: Regression Results for Sorting Tests

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer 1</th>
<th>layer l-g</th>
<th>Sample Size</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
<th>Model (10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>2</td>
<td>1</td>
<td>142</td>
<td>0.103</td>
<td>0.116</td>
<td>0.005</td>
<td>0.145</td>
<td>0.110</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.095)</td>
<td>(0.098)</td>
<td>(0.178)</td>
<td>(0.185)</td>
<td>(0.095)</td>
<td>(0.196)</td>
</tr>
<tr>
<td>THREE</td>
<td>3</td>
<td>2</td>
<td>457</td>
<td>0.240***</td>
<td>0.240***</td>
<td>0.223**</td>
<td>0.224**</td>
<td>0.241***</td>
<td>0.227**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.068)</td>
<td>(0.072)</td>
<td>(0.089)</td>
<td>(0.094)</td>
<td>(0.068)</td>
<td>(0.094)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>662</td>
<td>0.301***</td>
<td>0.314***</td>
<td>0.277**</td>
<td>0.295***</td>
<td>0.302***</td>
<td>0.290***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.065)</td>
<td>(0.066)</td>
<td>(0.080)</td>
<td>(0.086)</td>
<td>(0.064)</td>
<td>(0.084)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>1385</td>
<td>0.233***</td>
<td>0.217***</td>
<td>0.249***</td>
<td>0.235***</td>
<td>0.235***</td>
<td>0.236***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.034)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>FOUR</td>
<td>4</td>
<td>3</td>
<td>15</td>
<td>0.636*</td>
<td>0.692*</td>
<td></td>
<td></td>
<td>0.692</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.332)</td>
<td>(0.268)</td>
<td></td>
<td></td>
<td>(0.406)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>2</td>
<td>22</td>
<td>−0.180</td>
<td>−0.158</td>
<td></td>
<td>−0.117</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.225)</td>
<td>(0.308)</td>
<td></td>
<td>(0.212)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>37</td>
<td>0.324</td>
<td>0.354</td>
<td></td>
<td>0.408</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.243)</td>
<td>(0.259)</td>
<td></td>
<td>(0.271)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>452</td>
<td>0.195***</td>
<td>0.178***</td>
<td>0.215**</td>
<td>0.169</td>
<td>0.194***</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.063)</td>
<td>(0.067)</td>
<td>(0.090)</td>
<td>(0.103)</td>
<td>(0.063)</td>
<td>(0.105)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>687</td>
<td>0.077</td>
<td>0.072</td>
<td>0.115</td>
<td>0.117</td>
<td>0.075</td>
<td>0.118</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.056)</td>
<td>(0.058)</td>
<td>(0.070)</td>
<td>(0.073)</td>
<td>(0.056)</td>
<td>(0.073)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>1249</td>
<td>0.341***</td>
<td>0.332***</td>
<td>0.332***</td>
<td>0.322***</td>
<td>0.340***</td>
<td>0.320***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.046)</td>
<td>(0.047)</td>
<td>(0.058)</td>
<td>(0.060)</td>
<td>(0.047)</td>
<td>(0.060)</td>
</tr>
</tbody>
</table>

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.18). Each cell displays the estimate of a separate regression for firms with the same total number of layers and across two layers of firms. The table only reports the value of the coefficient $\alpha_1$ from equation (1.18). The dependent variable is the estimated weighted average ability of workers in layer $l$. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l-g$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976. Entries are omitted because the sample size was too small.

\[
ability^l_j = \sum_{i=1}^{N^l_j} \frac{H^l_{ji}}{H^l_j} \cdot ability^l_i, \quad (1.17)
\]

where the summation is taken over all individuals in layer $l$ at firm $j$.\(^\text{16}\)

For firms with the same organizational structure, or the same total number of layers, positive assortative matching implies that the best workers team up with the best workers in other layers of firms. For example, when comparing two firms with the same total number of layers, say 2, the firm with the best production workers in layer 1 also employs the best

\(^{16}\)Note that this construction is only possible for layers where there is at least one employee in the panel dataset of the DADS. As the panel is only a five percent sample of the French population, for many firms $ability^l_j$ remains undefined.
managers in layers 2. In other words, there should be a positive correlation between the ability of workers in the different layers of firms. To test for positive assortative matching, I therefore estimate the following equation:

\[ ability^l_j = \alpha_0 + \alpha_1 ability^{l-g}_j + X_j \beta + u_j, \]

(1.18)

where \( ability^l_j \) is the estimated weighted average ability of all workers in firm \( j \) who are in layer \( l \), and \( ability^{l-g}_j \) is the estimated weighted average ability of all workers in firm \( j \) who are in layer \( l - g \), for \( g = 1, ..., l - 1, l \). The firm controls \( X_j \) are firm observable variables such as firm age, an indicator for whether the firm already existed in the first year I have information, 1976, as well as indicator variables for industry and location. I include industry and location fixed effects because the assignment of workers to layers and firms may be different across industries and locations. I estimate equation (1.18), for firms with the same organizational structure and for the different values of \( l \) and \( g \). In equation (1.18) I am interested in how \( ability^l_j \) varies with \( ability^{l-g}_j \) across firms with the same total number of layers. If there is a positive assortative matching, then the coefficient \( \alpha_1 \) will be positive and significant.

Table 1.7 reports the results. Each entry in the table illustrates the estimated coefficient of \( \alpha_1 \) between two layers. The first column indicates the total number of layers in firms. The second column indicates the layer for which weighted average ability is the left-hand-side variable in equation (1.18), and the third column indicates the layer for which weighted average ability is the right-hand-side variable in equation (1.18). The fourth column reports the sample size of the regressions, while the fifth to tenth columns report estimated values of the coefficient. In Table 1.7 all the standard errors are White-heteroskedasticity consistent standard errors.

Table 1.7 reports that the sample size varies across regressions, even for firms with the same total number of layers. For example, in regressions with four-layer firms the sample can be as small as 15 observations or as large as 1249 observations. For firms with a given number of layers, the sample size increases when I estimate equation (1.18) with lower layers. The reasons are twofold. First given the nature of the data, I do not observe workers in all layers of firms. And second, I am more likely to observe a worker in the lower layer of an organization.
Almost all of the coefficients reported in Table 1.7 have a positive sign. There are three notable exceptions. In row six the coefficients between the ability of workers in layers four and layers two in four-layer firms are negative but not significant. In all three cases, however, the reported coefficients are imprecise. Because the sample size is small relative to the number of control variables there is not much independent variation in the data. This lack of independent variation in the data may also account for the reported negative coefficients.

Column (5) contains no controls and tests how agents sort together into firms across industries and locations. In these regressions, the majority of the estimated values of $\alpha_1$ are positive and significant, indicating that across industries and locations there is positive assortative matching. For example, in organizations with four layers, a one unit increase in the average ability of workers in layer one is associated with a 0.341 average increase in the average ability of workers in layer two.

Even within industries and locations, there is evidence of positive assortative matching. Column (6) contains industry fixed effects and examines how agents sort together within industries, while column (7) contains location fixed effects and examines how agents sort together within locations. In most cases the coefficients are positive and significant, suggesting that even within industries or within locations, the best workers team up with the best workers in other layers of firms. Column (8) reports regression results with both industry and location fixed effects. The findings indicate that there is positive assortative matching. For example, in organizations with four layers, a one hundred percent increase in the average ability of workers in layer one is associated with 0.322 average increase in the average ability of workers in layer two. In standardized units, this implies that a one standard deviation increase in the average ability of agents in layer one corresponds to a 0.301 standard deviation increase in the average ability of agents in layer two.

Column (9) in Table 1.7 reports results of regressions with firm observables as controls. Almost all of the coefficients are positive and half are significant. Finally column (10) reports results with the full set of controls, industry, location and firm observables. The coefficients remain positive and significant. For example, in organizations with three layers, a one hundred percent increase in the average ability of workers in layer one is associated with 0.236 average increase in the average ability of workers in layer two, and with a 0.290
average increase in the ability of workers in layer three. To obtain a sense of the strength of this relationship, a one standard deviation increase in the average ability of agents in layer two, corresponds to a 0.224 and 0.259 standard deviation increase in the average ability of agents in layers two and three, respectively.

One observation from Table 1.7 is that the magnitudes of $\alpha_1$ are small. A small magnitude, however, is not necessarily inconsistent with the theory, since the assignment of agents into teams depends on the parameters of the model, and in particular on the distribution of abilities in the economy.\(^{17}\)

Furthermore, there are several rows in Table 1.7 where although the coefficients are positive, they are never significant. For example, in organizations with four layers, there appears to be no relationship between the average ability of agents in layers three and one. The same result hold for organizations with two layers as well. This suggests that there is no sorting between agents in these layers, however the fact that the coefficients are always positive indicates that there is a relationship in the data, albeit not strong.\(^{18}\)

To summarize the results, out of the possible 51 estimated coefficients, 28 are positive and significant at the five percent level, 2 are positive and significant at the ten percent level, 18 are positive but not significant, and 3 are negative and not significant. Therefore, apart for two-layer firms, these results provide evidence that there is positive assortative matching between workers in different layers of firms.

### 1.6.3 Testing the mechanism

Until now, I have found evidence that workers in a layer are of higher ability than their subordinates in the layers below, and that there is positive assortative matching between the workers in the different layers of firms. I now proceed to test the model’s mechanism behind this sorting pattern. I proceed in two steps. First I test whether agents’ span of control increases with their own ability. And second, I test whether agents’ span of control increases with their subordinates’ ability.

Let $HR^l_j$ be the total number of hours worked by employees in layer $l$ at firm $j$. I define

\(^{17}\)The small magnitudes for $\alpha_1$ are not problematic. If one were to assume a continuum of agents, as in Antras, Garicano and Rossi-Hansberg (2006), then the mass of managers will be smaller than the mass of production workers. In this case, the matching function would have a slope that is less than 1.

\(^{18}\)For two layer firms, this is consistent with the findings in table 1.6, which report that there is little evidence of sorting between agents and layers in firms.
the span of control of workers in layer $l$ as:

$$span^l_j = \frac{HR^1_j}{HR^l_j}.$$  \hfill (1.19)

In other words, my measure of the span of control of workers in layer $l$ is the ratio of the total number of hours in layer 1, to the number of hours in layer $l$.\textsuperscript{19} The argument is that all workers in layer $l$ supervise $N^l_j$ individuals in layer 1, and these individuals spend a total of $HR^1_j$ hours at the firm. Dividing by the total number of hours worked by employees in layer $l$, $HR^l_j$, one obtains the number of hours a worker in layer $l$ is expected to devote to supervising individuals in layer 1. This definition of span of control is closely related to the firms’ maximization constraint in the model, discussed in the previous section, and it has the advantage of being invariant to the number of hours in the highest layer of the organization.\textsuperscript{20}

In the model, the mechanism that is causing agents to sort together into firms is the following. Managers benefit from working with abler production workers because they take up less of their time, which allows managers to supervise more of them. Also, because abler managers can solve a greater number of problems, they increase the productivity of their workers. Therefore, in equilibrium, abler managers will be working with abler production workers and managers’ span of control will be increasing with their ability. To test the mechanism of the model, therefore, I estimate the following equation:

$$\ln span^l_j = \gamma_0 + \gamma_1 ability^l_j + X_j \beta + u_j,$$ \hfill (1.20)

where $ability^l_j$ is the estimated weighted average ability of all workers in firm $j$ who are in layer $l$, and $span^l_j$ is defined above. The controls $X_j$ are firm age, whether the firm was present in 1976, and indicator variables for industry and location. I estimate equation (1.20) for firms with the same number of layers and for different values of $l$, separately. In equation (1.20) I am interested in how the span of control of agents in layer $l$ varies with their ability. If abler subordinates render their superiors more productive by allowing them to supervise more workers, then $\gamma_1$ should be positive and significant.

\textsuperscript{19}Since I cannot observe reporting relationships within organizations, this is the only measure available. I obtain $HR^l_j$ from the exhaustive cross-section of the DADS.

\textsuperscript{20}In the Garicano & Rossi-Hansberg (2004, 2006) all workers have one unit of time available. Also, the number of workers in the top layer of a firm is normalized to one.
Table 1.8: Testing Mechanism - Managers’ Ability

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer</th>
<th>Sample Size</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>601</td>
<td>(-0.243)</td>
<td>(-0.161)</td>
<td>(-0.213)</td>
<td>(-0.109)</td>
<td>(-0.249)</td>
<td>(-0.115)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.158)</td>
<td>(0.137)</td>
<td>(0.232)</td>
<td>(0.197)</td>
<td>(0.160)</td>
<td>(0.195)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2412</td>
<td>(-0.175^{**})</td>
<td>(-0.084)</td>
<td>(-0.081)</td>
<td>(-0.037)</td>
<td>(-0.159^{**})</td>
<td>(-0.027)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.036)</td>
<td>(0.072)</td>
<td>(0.083)</td>
<td>(0.076)</td>
<td>(0.078)</td>
<td>(0.076)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1183</td>
<td>(-0.231^{**})</td>
<td>(-0.116)</td>
<td>(-0.067)</td>
<td>(-0.040)</td>
<td>(-0.273^{**})</td>
<td>(-0.067)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.111)</td>
<td>(0.097)</td>
<td>(0.121)</td>
<td>(0.110)</td>
<td>(0.108)</td>
<td>(0.109)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1918</td>
<td>(-0.120^{*})</td>
<td>(-0.068)</td>
<td>(-0.062)</td>
<td>(-0.055)</td>
<td>(-0.117^{*})</td>
<td>(-0.056)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.066)</td>
<td>(0.061)</td>
<td>(0.070)</td>
<td>(0.063)</td>
<td>(0.066)</td>
<td>(0.063)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1042</td>
<td>(-0.139)</td>
<td>(0.001)</td>
<td>(-0.216)</td>
<td>(-0.109)</td>
<td>(-0.116)</td>
<td>(-0.092)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.127)</td>
<td>(0.106)</td>
<td>(0.133)</td>
<td>(0.120)</td>
<td>(0.125)</td>
<td>(0.120)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>80</td>
<td>(0.094)</td>
<td>(-0.001)</td>
<td>(0.311)</td>
<td>(-0.021)</td>
<td>(-0.074)</td>
<td>(0.486)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.295)</td>
<td>(0.339)</td>
<td>(0.457)</td>
<td>(0.971)</td>
<td>(0.296)</td>
<td>(1.274)</td>
</tr>
</tbody>
</table>

Industry FE: No Yes No Yes No Yes
Area FE: No No Yes Yes No Yes
Firm Controls: No No No Yes Yes

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.20). Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ from equation (1.20). The dependent variable is the estimated span of control of agents in layer $l$. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.
Table 1.8 reports the regression results. The table has a similar structure to Table 1.7. Each entry in the table reports an estimated value of the coefficient $\gamma_1$. The first column reports the total number of layers in firms. The second column reports the layer for which weighted average ability is the right-hand-side variable in equation (1.20), the third column reports the sample size of the regressions, and the fourth to ninth columns report estimated values of $\gamma_1$. In Table 1.8 all the standard errors are White-heteroskedasticity consistent standard errors.

First note that the reported sample sizes in Table 1.8 vary across regressions. For example, in regressions with four-layer firms the sample size can be as small 80 observations or as large as 1,918 observations. The sample size also increases for regressions examining the mechanism in the lower layers of firms. As explained previously, this is not surprising given nature of the dataset and, because firms are hierarchies, in the dataset there are less employees in the higher layers of firms. Also the reported sample sizes are different from Table 1.7, since to estimate equation (1.20) only the ability of one employee in a layer has to be recorded in the dataset.

In Table 1.8 the vast majority of the estimates of $\gamma_1$ are negative, however only a handful of them are significant. The only consistent exception is the regressions of equation (1.20) reported in the last row, for agents in layer four in four-layer firms. In this case, the coefficients alternate sign, however they are never significant, and so they do not lead to a firm conclusion.

Column (4) contains no controls and examines how workers’ ability varies with their span of control across industries and locations. The results indicate that workers’ span of control is decreasing with their ability. For example in three-layer firms a one unit increase in the average ability of workers in layer two is on average associated with a 17.5 percent decrease in their span of control.

Within industries and locations, the relationship remains negative but not significant. Column (5) examines how workers’ span of control varies with their ability within industries, while column (6) examines the relationship within locations. In both models the evidence suggests that there may be a negative relationship between agents’ ability and their span of control, however this is not conclusive. Column (7) examines the relationship within industries and locations. The results remain the same: although the coefficients are negative,
they are not significant.

Column (8) in Table 1.8 reports results with firm observables as controls. In column (8) two coefficients are negative and significant at the five percent level. In organizations with three layers, a one unit increase the average ability of workers in layer three is associated with a 27.3 decrease in their span of control, while a one unit increase the average ability of workers in layer two is associated with a 15.9 decrease in their span of control. Finally column (9) reports results with the full set of controls, industry, location and firm observables. The coefficients remain negative but not significant.

To summarize the results reported in Table 1.8, out of the 36 estimated coefficients, 4 are negative and significant at the five percent level, 2 are negative and significant at the ten percent level, 26 are negative but not significant, and 4 are positive but not significant. In light of these findings, the evidence suggests that although there is positive assortative matching between workers in the different layers of firms, the mechanism that is driving this sorting pattern is not present in the data. Indeed, the findings suggest that the opposite may be taking place. Abler managers form teams with abler production workers, however these workers take up more of the managers’ time.

I now test whether agents’ span of control increases with their subordinates’ ability. I estimate the following equation:

\[
\ln \text{span}_{lj} = \gamma_0 + \gamma_1 \text{ability}_{l-1} + X_j \beta + u_j, \tag{1.21}
\]

where \( \text{ability}_{l-1} \) is the estimated weighted average ability of all workers in firm \( j \) who are in layer \( l-1 \), and \( \text{span}_{lj} \) is defined above. The controls \( X_j \) are the same as in equation (1.20). As before I estimate equation (1.21) for firms with the same number of layers and for different values of \( l \), separately. In equation (1.21) the variable of interest is \( \gamma_1 \). If the mechanism behind the sorting of agents into organizations is correct, and abler subordinates allow their superiors to supervise more workers, then \( \gamma_1 \) should be positive and significant.

Table 1.9 presents the results from regression (1.21). The table has a similar structure to Table 1.8. As Table 1.9 reports in most of the regressions \( \gamma_1 \) is negative. The findings are more conclusive than the results reported in Table 1.8. For example, in three-layer firms, the relationship between the average ability of workers in layer 2 and the span of control of workers in layer 3 is negative and significant throughout the table. For workers in layer
### Table 1.9: Testing Mechanism - Subordinates’ Ability

<table>
<thead>
<tr>
<th>Total Layers</th>
<th>Number of Layers</th>
<th>Sample Size</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>1</td>
<td>2863</td>
<td>−0.070</td>
<td>−0.098**</td>
<td>−0.031</td>
<td>−0.068</td>
<td>−0.055</td>
<td>−0.061</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.053)</td>
<td>(0.051)</td>
<td>(0.057)</td>
<td>(0.054)</td>
<td>(0.053)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>THREE</td>
<td>1</td>
<td>6430</td>
<td>−0.077**</td>
<td>−0.035</td>
<td>−0.061</td>
<td>−0.032</td>
<td>−0.070*</td>
<td>−0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.038)</td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>2413</td>
<td>−0.268**</td>
<td>−0.169**</td>
<td>−0.206**</td>
<td>−0.158*</td>
<td>−0.253***</td>
<td>−0.150*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.082)</td>
<td>(0.076)</td>
<td>(0.085)</td>
<td>(0.081)</td>
<td>(0.081)</td>
<td>(0.081)</td>
</tr>
<tr>
<td>FOUR</td>
<td>1</td>
<td>4494</td>
<td>−0.143***</td>
<td>−0.105**</td>
<td>−0.115***</td>
<td>−0.088**</td>
<td>−0.140***</td>
<td>−0.091**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.045)</td>
<td>(0.043)</td>
<td>(0.045)</td>
<td>(0.043)</td>
<td>(0.045)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>1918</td>
<td>−0.161*</td>
<td>−0.089</td>
<td>−0.084</td>
<td>−0.067</td>
<td>−0.158**</td>
<td>−0.067</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.082)</td>
<td>(0.074)</td>
<td>(0.086)</td>
<td>(0.080)</td>
<td>(0.082)</td>
<td>(0.080)</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>1042</td>
<td>0.198</td>
<td>0.273</td>
<td>0.249</td>
<td>0.316**</td>
<td>0.218*</td>
<td>0.334***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.131)</td>
<td>(0.131)</td>
<td>(0.153)</td>
<td>(0.148)</td>
<td>(0.129)</td>
<td>(0.147)</td>
</tr>
</tbody>
</table>

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.21). Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ from equation (1.21). The dependent variable is the estimated span of control of agents in layer $l$. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l-1$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.

In four-layer firms, however, there is evidence in favor of the mechanism suggested by the model: higher ability subordinates allow the superiors to increase their span of control. Because for the other regressions the findings are similar to Table 1.8, I only discuss the results reported in the last row of Table 1.9.

Column (7) examines how workers’ ability varies with their superiors span of control within industries and locations. For four-layer firms the coefficient is positive and significant at the five percent level. A unit increase in the average ability of workers in layer three is associated with a 31.6 percent increase in the span of control of workers in layer four. Further column (8) reports results with firm observables as controls and column (9) reports the results for the full set of controls. In column (8) the coefficient of $\gamma_1$ is positive and significant at the ten percent level. In column (9) it is positive and significant at the five percent level and indicates that a one unit increase in the average ability of workers in layer three is associated with a 33.4 percent increase in the span of control of workers in layer four. In light of these results, for workers in layer four in organizations with four layers,
there is some evidence to suggest that the mechanism is present in the data.

Therefore, the evidence is mixed. There is limited evidence in favor of the mechanism described by the model, and evidence to suggest that the opposite is taking place: abler managers form teams with abler production workers, however, these production workers take up more of the managers’ time, which limits the amount of agents managers can supervise.

1.6.4 Additional Results

In this section, I examine the data more closely. I proceed in two steps. First, I examine whether the findings in the previous section are the same across firms with different sizes. Second, I examine whether the relationship holds for mono-establishment firms, that is firms that consist of only one plant. In this section I only report estimation results for equations (1.20) and (1.21) with the full set of controls.

It may be the case that large firms are using different production technologies, or that reporting relationships within large organizations are different. To account for this, within an industry I classify firms into quartiles by size, where I calculate firms’ size from the number of workers in the organization. Tables 1.10 and 1.11 report the results.
Table 1.11: Testing Mechanism - Subordinates’ Ability

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Layer l-1</th>
<th>Sample Size</th>
<th>First Quartile</th>
<th>Second Quartile</th>
<th>Third Quartile</th>
<th>Fourth Quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TWO</td>
<td>1</td>
<td>2863</td>
<td>−0.341***</td>
<td>−0.006</td>
<td>0.589***</td>
<td>1.216***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.072)</td>
<td>(0.077)</td>
<td>(0.123)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>THREE</td>
<td>1</td>
<td>6430</td>
<td>−0.374***</td>
<td>0.131**</td>
<td>0.042</td>
<td>−0.063</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.077)</td>
<td>(0.055)</td>
<td>(0.053)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>2413</td>
<td>−0.867***</td>
<td>−0.107</td>
<td>0.136</td>
<td>−0.122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.158)</td>
<td>(0.126)</td>
<td>(0.124)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>FOUR</td>
<td>1</td>
<td>4494</td>
<td>−0.189</td>
<td>−0.013</td>
<td>−0.057*</td>
<td>−0.183***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.172)</td>
<td>(0.083)</td>
<td>(0.058)</td>
<td>(0.059)</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>1918</td>
<td>−0.052</td>
<td>−0.267</td>
<td>0.106</td>
<td>−0.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.320)</td>
<td>(0.174)</td>
<td>(0.102)</td>
<td>(0.092)</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>1042</td>
<td>−1.750***</td>
<td>−1.510***</td>
<td>−0.376**</td>
<td>1.001***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.518)</td>
<td>(0.274)</td>
<td>(0.167)</td>
<td>(0.160)</td>
</tr>
</tbody>
</table>

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.21). Each row displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ interacted with the firm size quartile, for regressions with the full set of controls. The dependent variable is the span of control of workers in layer $l-1$. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l-1$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.

From Tables 1.10 and 1.11 a weak pattern emerges. Even though the majority of the coefficients are negative, in higher quartiles, the magnitude of $\gamma_1$ decreases and even becomes positive. For example, in Table 1.11 for organizations with four layers, in the first quartile, a one unit increase in the average ability of workers in layer three is associated with a 175.0 percent average decrease in the span of control of workers in layer four, in the second quartile it is associated with a 151.0 percent average decrease, and in the third quartile the corresponding value is a 37.0 percent decrease. In the fourth quartile a one unit increase in the average ability of workers in layer three, however, is associated with a 100.1 percent average increase in the span of control of workers in layer four. This pattern does not hold uniformly throughout table. Although it less robust it is also present in Table 1.8. For example in organizations with three layers, in the first quartile, a unit a one unit increase in the average ability of workers in layer two is associated with a 56.0 percent average decrease in their span of control, while in the fourth quartile the corresponding value is a 29.6 percent increase. Therefore, there is some weak evidence that indicates that the mechanism described by the model holds in large organizations, while for small firms the evidence suggests that abler agents supervise less workers.
Table 1.12: Testing Mechanism - Mono-Establishment Firms

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer</th>
<th>Sample Size</th>
<th>Model (4)</th>
<th>Total Number of Layers</th>
<th>layer</th>
<th>Sample Size</th>
<th>Model (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>2</td>
<td>591</td>
<td>−0.135 (0.200)</td>
<td>TWO</td>
<td>1</td>
<td>2723</td>
<td>−0.072 (0.057)</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>2309</td>
<td>−0.046 (0.077)</td>
<td>THREE</td>
<td>1</td>
<td>6140</td>
<td>−0.033 (0.037)</td>
</tr>
<tr>
<td>THREE</td>
<td>3</td>
<td>1120</td>
<td>−0.075 (0.114)</td>
<td>THREE</td>
<td>2</td>
<td>2309</td>
<td>−0.161** (0.082)</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>1783</td>
<td>−0.061 (0.066)</td>
<td>FOUR</td>
<td>1</td>
<td>4203</td>
<td>−0.100* (0.045)</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>952</td>
<td>−0.102 (0.126)</td>
<td>FOUR</td>
<td>2</td>
<td>1783</td>
<td>−0.058 (0.084)</td>
</tr>
<tr>
<td>FOUR</td>
<td>4</td>
<td>82</td>
<td>0.486 (1.274)</td>
<td>FOUR</td>
<td>3</td>
<td>952</td>
<td>0.270* (0.148)</td>
</tr>
</tbody>
</table>

| Industry FE           | Yes   | Industry FE | Yes   |
| Area FE               | Yes   | Area FE     | Yes   |
| Firm Controls         | Yes   | Firm Controls| Yes   |

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equations (1.20) and (1.21). Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ from equation (1.21) with the full set of controls. The dependent variable is the estimated span of control of agents in layer $l$. The right-hand side variable in the first column is the estimated weighted average ability of workers in a lower layer, $l$. The right-hand side variable in the first column is the estimated weighted average ability of workers in a lower layer, $l − 1$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.
In addition, it may be the case that reporting relationships are only specific to a physical location. In particular if a firm is operating multiple plants, their organization may be different than what is suggested by the theory. To account for this, in Table 1.12 I report regression results for mono-establishment organizations. The results are similar to those reported in the previous section. Apart for the span of control of agents in layer four, the estimated values of $\gamma_1$ are negative. Therefore, the conclusion remains the same. There is limited evidence in favor of the mechanism described by the model, and evidence to suggest that the agents’ span of control is decreasing with ability.

1.7 Robustness Checks

1.7.1 Inconsistent Estimates

There are three threats to my estimates of workers’ ability. All stem from my estimation of worker fixed effects. First, because the worker fixed effects are incidental parameters from regression (1.15), consistent estimates for them can only be obtained as the number of years an individual is observed in the panel grows large. Since for the years 1993 to 2004, the average worker is observed for 6 years, not all of the estimates of the time-invariant component, $\theta_i$, identify a consistent measure of a worker’s ability. Although the panel is short, to get a sense of how important is this issue, I conduct my analysis on workers that I observe for at least 10 periods. Tables 1.13, 1.14, 1.15 and 1.16 present the regression results for this restricted sample.

Table 1.13 reports the tests for skill stratification. As in the previous table, higher ability agents occupy the upper layers of organizations. In addition in column (7), even within two-layer firms this relationship is now significant and indicates that an individual with a one hundred percent increase in his ability will on average reside 0.254 layer higher.

Table 1.14 reports the regression results that test for positive assortative matching. First, note that in comparison to Table 1.14 not all of the reported estimates have a positive sign. In two-layer firms, within industries and locations, a one unit increase in the weighted average ability of agents in layer one is associated with a 2.018 decrease in the weighted average ability of agents in layer two. Although this would suggest that there is negative

---

21 This removes 925 observations from the dataset.
Table 1.13: Regression Results for Skill Stratification

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Sample Size</th>
<th>Model (3)</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>1,746</td>
<td>0.118***</td>
<td>0.093***</td>
<td>0.110***</td>
<td>0.092***</td>
<td>0.254***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.027)</td>
<td>(0.026)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>THREE</td>
<td>10,374</td>
<td>0.634***</td>
<td>0.604***</td>
<td>0.612***</td>
<td>0.596***</td>
<td>0.805***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>FOUR</td>
<td>8,601</td>
<td>0.706***</td>
<td>0.685***</td>
<td>0.688***</td>
<td>0.677***</td>
<td>0.838***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.027)</td>
<td>(0.027)</td>
<td>(0.035)</td>
</tr>
</tbody>
</table>

Industry FE No Yes No Yes No
Area FE No No Yes Yes No
Firm FE No No No Yes Yes

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors (clustered at the firm level) in parentheses.

OLS regression results of equation (1.16) for workers with at least 10 years in the dataset. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient µ₁ from equation (1.16). The dependent variable is the layer that a worker occupies. The right-hand side variable is the ability of the worker. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France.

assortative matching between agents in layers one and two, the sample is small relative to the number of controls. In addition, in columns (5) and (9) the reported relationship is positive and significant at the five percent level. Therefore, these findings do not lead to firm conclusion.

Second in organizations with four layers, in contrast to the results reported in Table 1.7 there is now weak evidence in favor of positive assortative matching between workers in layers three and one. As reported in column (9), for example, a unit increase in the weighted average ability of workers in layer one corresponds to a 0.135 increase in the average ability of workers in layer three.

Third, note that some the estimated magnitudes of the coefficients are larger. For example in column (10), in three-layer firms a unit increase in the average ability of workers in layer one is associated with a 0.370 increase in the average ability of workers in layer two. In standardized units, a one standard deviation in the average ability of agents in layer one corresponds to a 0.341 standard deviation increase in the ability of agents in layers two, which is greater than the 0.224 standard deviation increase reported from Table 1.7.

For the restricted sample, Table 1.15 tests whether abler workers have a greater span of control. In general the results are similar to those reported in Table 1.8. However, in three-layer organizations, there is now convincing evidence that higher ability agents in layer
### Table 1.14: Regression Results for Sorting Tests

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer l</th>
<th>layer l-g</th>
<th>Sample Size</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
<th>Model (10)</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.354**</td>
<td>0.326</td>
<td>0.127</td>
<td>-2.018**</td>
<td>0.348**</td>
<td>-1.670</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.136)</td>
<td>(0.208)</td>
<td>(1.653)</td>
<td>(0.365)</td>
<td>(0.139)</td>
<td>(1.628)</td>
</tr>
<tr>
<td>TWO</td>
<td>2</td>
<td>1</td>
<td>38</td>
<td>0.455***</td>
<td>0.449***</td>
<td>0.502***</td>
<td>0.514***</td>
<td>0.460***</td>
<td>0.511***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.075)</td>
<td>(0.068)</td>
<td>(0.123)</td>
<td>(0.121)</td>
<td>(0.074)</td>
<td>(0.124)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>232</td>
<td>0.419***</td>
<td>0.427***</td>
<td>0.281***</td>
<td>0.274**</td>
<td>0.411***</td>
<td>0.262**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.086)</td>
<td>(0.089)</td>
<td>(0.121)</td>
<td>(0.130)</td>
<td>(0.087)</td>
<td>(0.120)</td>
</tr>
<tr>
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<td>3</td>
<td>1</td>
<td>380</td>
<td>0.382***</td>
<td>0.373***</td>
<td>0.387***</td>
<td>0.372***</td>
<td>0.381***</td>
<td>0.370***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.047)</td>
<td>(0.047)</td>
<td>(0.060)</td>
<td>(0.060)</td>
<td>(0.047)</td>
<td>(0.061)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td>9</td>
<td>0.674</td>
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<td>0.574</td>
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<td>0.574</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td>(0.469)</td>
<td>(0.469)</td>
<td>(0.469)</td>
<td>(0.469)</td>
<td>(0.469)</td>
<td>(0.469)</td>
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<tr>
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<td>2</td>
<td>14</td>
<td>0.112</td>
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<td></td>
<td>(0.183)</td>
<td>(0.969)</td>
<td>(0.969)</td>
<td>(0.969)</td>
<td>(0.969)</td>
<td>(0.969)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>19</td>
<td>0.226</td>
<td>0.554</td>
<td>0.242</td>
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<td>0.242</td>
<td>0.242</td>
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<tr>
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<td></td>
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<td>(0.334)</td>
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<td>(0.409)</td>
<td>(0.409)</td>
<td>(0.409)</td>
</tr>
<tr>
<td></td>
<td>3</td>
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<td>225</td>
<td>0.348***</td>
<td>0.358***</td>
<td>0.423***</td>
<td>0.391**</td>
<td>0.336***</td>
<td>0.369***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.092)</td>
<td>(0.105)</td>
<td>(0.158)</td>
<td>(0.171)</td>
<td>(0.093)</td>
<td>(0.169)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>379</td>
<td>0.138***</td>
<td>0.137***</td>
<td>0.114</td>
<td>0.117</td>
<td>0.135**</td>
<td>0.127</td>
</tr>
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<td>(0.065)</td>
<td>(0.114)</td>
<td>(0.119)</td>
<td>(0.068)</td>
<td>(0.121)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>679</td>
<td>0.357***</td>
<td>0.361***</td>
<td>0.318**</td>
<td>0.329**</td>
<td>0.356***</td>
<td>0.326**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.085)</td>
<td>(0.086)</td>
<td>(0.135)</td>
<td>(0.141)</td>
<td>(0.084)</td>
<td>(0.142)</td>
</tr>
</tbody>
</table>

Industry FE: No
Area FE: No
Firm Controls: No

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.18) for workers with at least 10 years. Each cell displays the estimate of a separate regression for firms with the same total number of layers and across two layers of firms. The table only reports the value of the coefficient $\alpha_1$ from equation (1.18). The dependent variable is the estimated weighted average ability of workers in layer $l$. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l - g$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976. Entries are omitted because the sample size was too small.
## Table 1.15: Testing Mechanism - Managers’ Ability

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Sample Size</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>2</td>
<td>263</td>
<td>-0.137</td>
<td>-0.024</td>
<td>-0.020</td>
<td>0.257</td>
<td>-0.206</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.242)</td>
<td>(0.189)</td>
<td>(0.398)</td>
<td>(0.403)</td>
<td>(0.252)</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>1471</td>
<td>-0.128*</td>
<td>-0.089</td>
<td>0.000</td>
<td>0.006</td>
<td>-0.115</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.090)</td>
<td>(0.083)</td>
<td>(0.099)</td>
<td>(0.090)</td>
<td>(0.090)</td>
</tr>
<tr>
<td>THREE</td>
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<td>775</td>
<td>-0.359**</td>
<td>-0.321***</td>
<td>-0.305*</td>
<td>-0.385**</td>
<td>-0.385***</td>
</tr>
<tr>
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<td></td>
<td></td>
<td>(0.140)</td>
<td>(0.125)</td>
<td>(0.155)</td>
<td>(0.152)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>1198</td>
<td>-0.031</td>
<td>-0.016</td>
<td>-0.018</td>
<td>-0.019</td>
<td>-0.035</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.080)</td>
<td>(0.073)</td>
<td>(0.097)</td>
<td>(0.086)</td>
<td>(0.080)</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>700</td>
<td>-0.165</td>
<td>-0.037</td>
<td>-0.185</td>
<td>-0.096</td>
<td>-0.143</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.144)</td>
<td>(0.118)</td>
<td>(0.168)</td>
<td>(0.149)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>FOUR</td>
<td>4</td>
<td>54</td>
<td>0.089</td>
<td>-0.159</td>
<td>0.905</td>
<td>-0.518</td>
<td>-0.119</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.381)</td>
<td>(0.563)</td>
<td>(1.010)</td>
<td>(3.235)</td>
<td>(0.376)</td>
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</tbody>
</table>

Industry FE: No, Yes
Area FE: No, No, Yes, Yes
Firm Controls: No, No, No, Yes

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.20) for workers observed for at least 10 years. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient γ₁ from equation (1.20). The dependent variable is the estimated span of control of agents in layer l. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, l. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the coefficient of the estimated firm fixed effects from regression (1.15), the age of the firm, and whether the firm was present in the first year of the panel, 1976.
Table 1.16: Testing Mechanism - Subordinates’ Ability

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer l-1</th>
<th>Sample Size</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>1</td>
<td>1225</td>
<td>-0.022</td>
<td>-0.046</td>
<td>0.049</td>
<td>0.026</td>
<td>-0.018</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.080)</td>
<td>(0.076)</td>
<td>(0.102)</td>
<td>(0.093)</td>
<td>(0.080)</td>
<td>(0.093)</td>
</tr>
<tr>
<td>THREE</td>
<td>1</td>
<td>3919</td>
<td>-0.029</td>
<td>0.025</td>
<td>0.003</td>
<td>0.041</td>
<td>-0.030</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.047)</td>
<td>(0.046)</td>
<td>(0.048)</td>
<td>(0.047)</td>
<td>(0.047)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>1471</td>
<td>-0.207**</td>
<td>-0.158</td>
<td>-0.120</td>
<td>-0.100</td>
<td>-0.199**</td>
<td>-0.096</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.100)</td>
<td>(0.096)</td>
<td>(0.105)</td>
<td>(0.103)</td>
<td>(0.101)</td>
<td>(0.103)</td>
</tr>
<tr>
<td>FOUR</td>
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<td>3014</td>
<td>-0.121**</td>
<td>-0.075</td>
<td>-0.113**</td>
<td>-0.082</td>
<td>-0.122**</td>
<td>-0.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.052)</td>
<td>(0.050)</td>
<td>(0.055)</td>
<td>(0.053)</td>
<td>(0.052)</td>
<td>(0.053)</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>1198</td>
<td>-0.050</td>
<td>-0.012</td>
<td>0.043</td>
<td>0.040</td>
<td>-0.050</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.100)</td>
<td>(0.093)</td>
<td>(0.117)</td>
<td>(0.110)</td>
<td>(0.099)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>700</td>
<td>-0.061</td>
<td>-0.023</td>
<td>-0.008</td>
<td>0.025</td>
<td>-0.034</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.151)</td>
<td>(0.136)</td>
<td>(0.168)</td>
<td>(0.161)</td>
<td>(0.150)</td>
<td>(0.163)</td>
</tr>
</tbody>
</table>

Industry FE | No | Yes | No | Yes | No | Yes  
Area FE     | No | No  | Yes | Yes | No | Yes  
Firm Controls | No | No  | No  | Yes | No | Yes  

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.21) for workers observed for at least 10 years. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ from equation (1.21). The dependent variable is the estimated span of control of agents in layer l. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l-1$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the coefficient of the estimated firm fixed effects from regression (1.15), the age of the firm, and whether the firm was present in the first year of the panel, 1976.

three supervise less workers. As reported in column (9) a one unit increase in the average ability of individuals in layer three is associated with a 40.7 decrease in their span of control.

Table 1.16 reports estimates of $\gamma_1$ from equation (1.21). Although the general conclusions are similar to Table 1.9, there are two differences. First, there is no longer any evidence to suggest that abler workers in layer three allow agents in layer four to increase their span of control. And second, excluding the last row, there are now 9 estimated values of $\gamma_1$ that are positive but not significant.

Therefore, the conclusions remain the same. Although there is evidence that the best workers team up with the best workers in other layers of firms, there is limited evidence in favor of the mechanism described by the model, and evidence to suggest that the opposite is taking place. In particular that abler managers form teams with abler production workers, however, these production workers take up more of the managers’ time, thereby limiting the amount of agents managers’ can supervise.
1.7.2 Estimation Error

A positive correlation between the individual fixed effects may be the result of using standard econometric techniques. As discussed in Abowd, Kramarz, Lengermann, and Perez-Duarte (2004) and Andrews, Gill, Schank, Upward (2008), in equation (1.15) there is a negative correlation between the worker and firm-layer effects caused from standard estimation error. When the firm-layer fixed effects in equation (1.15) are on average underestimated, the individual fixed effects will be overestimated, and when the firm-layer fixed effects are on average overestimated, the individual fixed effects will be underestimated. Because in the panel workers transition between layers within firms, this implies that my regressions may suffer from non-classical measurement error, biasing results.

To resolve these issues I conduct my analysis only on workers who have moved to a new employer in the year 2008. For this sample of workers, any errors caused by misestimated firm-layer fixed effects will be uncorrelated with one another and uncorrelated with the workers’ span of control. In these regressions, the coefficients will only suffer from attenuation bias, however the sign of the estimated coefficients will more properly reflect the relationships of interest. Because the sample sizes are small in these regressions, unlike in the previous sections Tables 1.17, 1.18, 1.19 and 1.20 do not report the results from all models.

For the year 2008, Table 1.17 reports how workers sort into layers and organizations. Because in the sample there are not many workers employed in the same firm, regression results that examine how workers sort into layers within organizations are omitted. Further, in all the regressions reported in Table 1.17 the coefficient $\mu_1$ is positive. For firms with three and four layers, the reported coefficients are also significant at the one percent level. Therefore in three and four layer firms, higher ability agents occupy the upper the layers of organizations. In two-layer organizations $\mu_1$ is no longer significant, however, it has a similar magnitude as the results reported in Table 1.6. This suggests that there is no sorting between agents into layers, however the fact that the coefficients are always positive indicates that there is a relationship in the data, albeit not strong.

Table 1.18 reports the results of tests for positive assortative matching. Because there are not many workers employed in the same firm, Table 1.18 only reports results for three-layer and four-layer organizations. First note that in firms with three layers there is some
### Table 1.17: Regression Results for Skill Stratification

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Sample Size</th>
<th>Model (3)</th>
<th>Model (4)</th>
<th>Model (5)</th>
<th>Model (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TWO</td>
<td>444</td>
<td>0.028</td>
<td>0.031</td>
<td>0.044</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.037)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>THREE</td>
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<td>0.229***</td>
<td>0.200***</td>
<td>0.222***</td>
<td>0.196***</td>
</tr>
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<td>(0.030)</td>
<td>(0.028)</td>
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</tr>
<tr>
<td>FOUR</td>
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<td>0.258***</td>
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<td>0.195***</td>
</tr>
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<td>(0.042)</td>
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</table>

Industry FE No Yes No Yes
Area FE No No Yes Yes
Firm FE No No No No

Notes: *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \). White-heteroskedastic standard errors (clustered at the firm level) in parentheses. OLS regression results of equation (1.16) for workers with at least 10 years in the dataset. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient \( \mu_1 \) from equation (1.16). The dependent variable is the layer that a worker occupies. The right-hand side variable is the ability of the worker. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France.

### Table 1.18: Regression Results for Sorting Tests

<table>
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<th>Total Number of Layers</th>
<th>Layer 1</th>
<th>Layer l-g</th>
<th>Sample Size</th>
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<th>Model (6)</th>
<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
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<tbody>
<tr>
<td>THREE</td>
<td>3</td>
<td>2</td>
<td>18</td>
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<td>-0.407*</td>
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<tr>
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<td></td>
<td></td>
<td>(0.173)</td>
<td>(0.170)</td>
<td>(0.133)</td>
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<td></td>
<td></td>
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<td>(0.168)</td>
<td>(0.789)</td>
<td>(2.020)</td>
<td>(0.131)</td>
</tr>
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<td>95</td>
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<td>(0.090)</td>
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<td>(0.112)</td>
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<td>(0.106)</td>
<td>(0.282)</td>
<td>(0.392)</td>
<td>(0.099)</td>
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<tr>
<td>FOUR</td>
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<td>1</td>
<td>131</td>
<td>0.193**</td>
<td>0.177*</td>
<td>0.297</td>
<td>0.316</td>
<td>0.170*</td>
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</table>

Industry FE No Yes No Yes
Area FE No No Yes Yes
Firm Controls No No No Yes

Notes: *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \). White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.18) for workers with at least 10 years. Each cell displays the estimate of a separate regression for firms with the same total number of layers and across two layers of firms. The table only reports the value of the coefficient \( \alpha_1 \) from equation (1.18). The dependent variable is the estimated weighted average ability of workers in layer 1. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, \( l - g \). Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976. Entries are omitted because the sample size was too small.
Table 1.19: Testing Mechanism - Managers’ Ability

<table>
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<tr>
<th>Total Number of Layers</th>
<th>Sample Size</th>
<th>Model (4)</th>
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<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
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<tbody>
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<td></td>
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<tr>
<td>TWO</td>
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<td>(0.766)</td>
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<td>(0.996)</td>
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<td>-0.056</td>
<td>-0.088</td>
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<tr>
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<td></td>
<td></td>
<td>(0.244)</td>
<td>(0.178)</td>
<td>(0.355)</td>
<td>(0.314)</td>
<td>(0.246)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>THREE</td>
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<tr>
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</tr>
<tr>
<td>FOUR</td>
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<th>Yes</th>
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<th>Yes</th>
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<tbody>
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<td>No</td>
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<td>No</td>
<td>Yes</td>
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<td>Firm Controls</td>
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<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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</table>

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.20) for workers in new firms. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient $\gamma_1$ from equation (1.20). The dependent variable is the estimated span of control of agents in layer 1. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, $l$. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the coefficient of the estimated firm fixed effects from regression (1.15), the age of the firm, and whether the firm was present in the first year of the panel, 1976.

evidence of negative assortative matching. The majority of the reported coefficients have a negative sign, and one coefficient is significant at the five percent level. Across industries and locations a one unit increase in the average ability of agents in layer one corresponds to a 0.130 decrease in the average ability of workers in layer two. As additional controls are added, however, this relationship remains negative but is no longer significant. Given that the size of the samples are small, however, these findings to not lead to a firm conclusion.

Second, in organizations with four layers, there is evidence to suggest that better workers are employed with better workers in the other layers of firms. The majority of the reported coefficients have a positive sign, and two are significant at the five percent level. For example, across industries and locations, a one unit increase in the average ability of agents in layer one corresponds to a 0.193 average increase in the average ability of workers in layer two. These findings are consistent with previous results.

Table 1.19 and 1.20 report results that test for the mechanism. As in the previous section, there is no longer any evidence to suggest that abler workers in layer three allow agents in layer four to increase their span of control. And second, although the majority of
Table 1.20: Testing Mechanism - Subordinates’ Ability

<table>
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<th>Sample Size</th>
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<th>Model (7)</th>
<th>Model (8)</th>
<th>Model (9)</th>
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<td>−0.102</td>
<td>−0.170*</td>
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<td></td>
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<td>(0.078)</td>
<td>(0.097)</td>
<td>(0.094)</td>
<td>(0.081)</td>
<td>(0.095)</td>
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<td>−0.554**</td>
<td>−0.478**</td>
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<td>−0.234</td>
<td>−0.545**</td>
<td>−0.221</td>
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<td>(0.269)</td>
<td>(0.218)</td>
<td>(0.371)</td>
<td>(0.383)</td>
<td>(0.272)</td>
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<td>(0.095)</td>
<td>(0.113)</td>
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<td>(0.099)</td>
<td>(0.108)</td>
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<td>(0.406)</td>
<td>(1.079)</td>
<td>(1.231)</td>
<td>(1.028)</td>
<td>(1.306)</td>
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Industry FE: No, Yes
Area FE: No, Yes
Firm Controls: No, Yes

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. White-heteroskedastic standard errors in parentheses. OLS regression results of equation (1.21) for workers in new firms. Each cell displays the estimate of a separate regression for firms with the same total number of layers. The table only reports the value of the coefficient γ₁ from equation (1.21). The dependent variable is the estimated span of control of agents in layer l. The right-hand side variable is the estimated weighted average ability of workers in a lower layer, l − 1. Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the coefficient of the estimated firm fixed effects from regression (1.15), the age of the firm, and whether the firm was present in the first year of the panel, 1976.

the coefficients are negative few are significant at the five percent level.

Therefore, the conclusions remain the same. Even though there is evidence that the best workers team up with the best workers in other layers of firms, there is limited evidence in favor of the mechanism suggested by the model, and evidence to suggest that the opposite is taking place.

1.7.3 Biased Estimates

Third, the estimated worker fixed effects may be biased. If workers in a given layer render their subordinates more productive, and if their subordinates make them more productive, then this should be reflected in wages. If this is the case, then the worker fixed effect in equation (1.15) is not only identifying the productivity of a worker, but also the impact his co-workers have on his productivity. In other words, for worker i employed at time t in firm J(i,t), the estimated individual fixed effect from equation (1.15), ˆθi, is equal to: θi + ϕJ(i,t), where ϕJ(i,t) is the impact worker i’s co-workers have on his productivity.
If there is positive sorting, then it is safe to assume that $\text{cov}(\theta_i, \phi_J(i,t))$ is positive. For another worker $h$ employed at time $t$ in the same firm as $i$, then it should be the case that $\text{cov}(\theta_h, \phi_J(i,t))$ is positive as well. Hence, this would imply that there is nonclassical measurement error in my subsequent regressions.

Moreover, if the model is an accurate description of the real world, since there is positive assortative matching between workers in the different layers of firms, for a given layer, $\phi_J(i,t)$ should be increasing with $\theta_i$. Therefore, according to the model the bias should be increasing in the ability of an individual. For regressions (1.18), (1.20) and (1.21), this would further bias the coefficient of interests, $\alpha_1$ and $\gamma_1$ in favor of finding a positive result. Therefore, one interpretation of my results is that they present an upper bound on the relationships of interest.

### 1.8 Conclusion

Understanding how workers sort together with other workers into layers and firms is crucial for understanding the organization of firms. Without knowledge of the precise nature of the interactions between workers in the different layers of firms, it is difficult to comprehend how firms organize production. Additionally, pinpointing the mechanism that is causing this sorting pattern is essential for determining why workers sort together in firms. Finally, better knowledge of how workers sort into layers and firms is important for understanding earnings’ inequality, and how firms respond to changes in their market environment.

This paper directly examined how workers sort together in firms. My empirical strategy relies on the idea that firms can be thought of as hierarchical teams, composed of layers that perform different tasks. Using French administrative data, I conclude that, within firms, higher ability workers are employed in the higher layers of firms, and across firms, there is positive assortative matching between workers in the different layers of firms. Third, I find only weak evidence for the mechanism, as suggested by Garicano and Rossi-Hansberg (2006), that is causing this sorting pattern: higher ability workers allow their managers to increase their span of control and employ larger teams. Finally, I also find evidence that higher ability managers supervise less workers.

An important question remains to be answered. The findings presented in this study in-
dicate that although there is some evidence that higher ability workers allow their managers to increase their span of control and employ larger teams, there is also evidence that the opposite is taking place. An important question therefore remains to be answered: If better workers sort into firms with other better workers, what is causing this sorting pattern?
Appendix A: Data Appendix

The Panel Dataset of the DADS

To estimate worker and firm fixed effects, I use the years 1993 to 2004 from the panel dataset of the DADS. Initially, the dataset contains 24,882,933 total observations, 5,469,362 workers and 1,614,337 firms. I remove from the dataset any workers or firms that cannot be properly identified or that have missing values. For reasons of computational tractability, I restrict the sample to all workers who are born in an even numbered year, are between the ages 18 and 65 and work in continental France. I also eliminate from the sample all individuals I observe only once in the panel and who are not full-time workers. In a given year, an individual may hold multiple jobs. In case of multiple jobs, for a given year I keep the worker’s employment with the highest salary. Finally, I also eliminate all firms in the agricultural and fishing industries and all industries in which there are some coding problems present. From this sample of workers and firms, to obtain an exact estimate of worker and firm-layer fixed effects I find the largest connected group. The largest connected group contains 4,999,728 observations, 753,092 workers, 399,676 firms and 569,198 firm-layer pairs.

For the years 1993 to 2004 Table 1 presents distribution of the number of years workers are observed in the panel dataset of the DADS.

<table>
<thead>
<tr>
<th>variable</th>
<th>mean</th>
<th>p1</th>
<th>p5</th>
<th>p10</th>
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<th>p75</th>
<th>p90</th>
<th>p95</th>
<th>p99</th>
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</thead>
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<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

For the years 1993 to 2004 Table 2 presents distribution of the number of years firms are observed in the panel dataset of the DADS as well as the distribution of the number of workers that are observed in a firm in a given year.

Table 2: Distribution of the number of years layers within firms are observed in the panel and the distribution of workers per layer in a firm

<table>
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<th>p10</th>
<th>p25</th>
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<th>p99</th>
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<td>22</td>
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</tr>
</tbody>
</table>
The Exhaustive Cross-Section of the DADS

The exhaustive cross-section of the DADS contains information on all workers who earn a positive wage in a French establishment. For a given year, the observations are at the worker-establishment level. Within a firm, a worker can have multiple jobs if he is employed in two different establishments. To clean the data, I first remove any observations that do not have a positive amount of hours, days, occupation or wage reported. I also remove any observations in which the firm and individual information is missing.

For every firm we have information on its industry of operation. I also have this information for every establishment that comprises a firm. I classify firms into industries using the industry of operation of the firm. Further I remove any industries in which there are classification errors. Since I focus on manufacturing firms only, I remove any industries that are not in manufacturing.

I construct my measure of layers using the first-digit of the CS classification codes. I consider a firm as having a layer if an employee is present in that layer. I classify firms by the number of management layers present in their organization. In other words, a firm where layer 1, layer 3 and layer 3 are present is defined as a three-layer firm. To identify the layers in a firm, I use the first digit of the CS occupational codes which range from 2 to 6. Therefore in total I can identify up to four layers. Layer 1 corresponds to qualified and non-qualified administrative workers and blue-collar workers. It contains all workers with CS occupational codes 5 and 6, respectively. I group CS occupational codes 5 and 6 together because their distribution of ability are similar. Layer 2 is composed of supervisors and individuals with higher level of responsibility than ordinary workers, and contains all workers with an occupational code 4. Layer 3 is composed of senior directors and top management staff and contains all workers with an occupational code 3. Layer 4 corresponds to owners who receive a wage and CEOs. It contains all workers with a CS occupational code 2.

For every layer, I calculate the total number of employees in a layer. If a worker appears in two different establishments but in a different layer, I treat him as two separate observations. I also calculate the total number of hours per layer. If a worker appears in two different establishments, I keep both observations to calculate the total number of hours per layer. Further, I remove any firms that record a positive number of hours for workers
Chapter 1: Sorting Within and Across French Production Hierarchies

in occupations, with codes different from 2 to 6.

Appendix B: Test for Positive Assortative Matching

In this section I test whether there is positive assortative matching between workers and their co-workers in a firm. I adopt the approach proposed by Lopes de Melo (2013). Building on the frictional matching model of Shimer and Smith (2003), Lopes de Melo (2013) shows that even though wages are not monotone with respect to firm productivity, wages will be monotone with respect to workers’ skills. Therefore, if better workers are sorting together into firms, the correlation between a worker’s fixed effect and the average fixed effect of his co-workers should be positive.

To test this prediction, I conduct three exercises. First I correlate the worker fixed effect with the average fixed effect of his co-workers for all firms in the economy, and for all firms with the same number of layers, separately. Second, I perform the same exercise, but within the layers of firms. And finally, I add additional structure and estimate the following equation:

$$
\begin{align*}
\theta_i &= \alpha_0 + \alpha_1 \bar{\theta}_{-i} + X_j \beta + u_j,
\end{align*}
$$

where $\bar{\theta}_{-i}$ is the average ability of workers $i$’s co-workers. I include as controls indicators for industry, the age of the firm, whether the firm was present in 1976, and controls for industry and location. If workers of similar ability are employed in the same firms, then $\alpha_1$ should be positive and significant. I conduct these tests only for the year 2004.

Tables 1.21 and 1.22 presents the correlation results. The first column of Table 1.21 presents the correlation for all workers in the economy and for firms with the same number of layers, while the second column contains the size of the sample used to estimate the correlation. The third column presents the correlation between the worker fixed effects and the firm fixed effects, and the fourth columns presents the number of observations used to estimate the correlation. In column one, the correlations are all positive. For all firms in the economy, the correlation between the worker fixed effect and the average ability of his co-workers is 0.352. Table 1.22 presents the same correlation but across the layers within a firm, $l$. Again
all correlations are positive.

Tables 1.23 and 1.24 report regression results. In the tables all standard errors are robust. In Tables 1.23 and 1.24 not all coefficients are positive and significant. In Table 1.23, apart for firms that organize with one layer, the coefficients are positive and significant at the one percent level. In Table 1.24, the results are mixed. For the higher layers of firms, the coefficient of $\alpha_1$ is negative and significant. For example, in a four-layer firm, for a worker in layer two, a one unit in the average ability of his co-workers is associated with -0.205 average decrease in his ability. In all, we can conclude that there is positive assortative matching between workers in the same organization, however within the same layer of an organization the evidence is mixed.
Table 1.21: Regression Results Sorting from Regression 1.22

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>corr($\theta_i; \overline{\theta_{-i}}$)</th>
<th>N</th>
<th>corr($\theta; \psi$)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.352</td>
<td>31,941</td>
<td>−0.277</td>
<td>31,941</td>
</tr>
<tr>
<td>ONE</td>
<td>0.374</td>
<td>481</td>
<td>−0.559</td>
<td>2,432</td>
</tr>
<tr>
<td>TWO</td>
<td>0.382</td>
<td>1,871</td>
<td>−0.521</td>
<td>4,432</td>
</tr>
<tr>
<td>THREE</td>
<td>0.362</td>
<td>15,773</td>
<td>−0.310</td>
<td>19,841</td>
</tr>
<tr>
<td>FOUR</td>
<td>0.335</td>
<td>13,816</td>
<td>−0.290</td>
<td>16,003</td>
</tr>
</tbody>
</table>

Notes: Correlations between the ability of workers and their co-workers.

Table 1.22: Regression Results Sorting from Regression 1.22

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>layer</th>
<th>corr($\theta_i; \overline{\theta_{-i}}$)</th>
<th>N</th>
<th>corr($\theta; \psi$)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE</td>
<td>1</td>
<td>0.374</td>
<td>481</td>
<td>−0.503</td>
<td>481</td>
</tr>
<tr>
<td>TWO</td>
<td>1</td>
<td>0.361</td>
<td>1,819</td>
<td>−0.423</td>
<td>1,819</td>
</tr>
<tr>
<td>TWO</td>
<td>2</td>
<td>0.639</td>
<td>52</td>
<td>−0.805</td>
<td>52</td>
</tr>
<tr>
<td>THREE</td>
<td>1</td>
<td>0.355</td>
<td>15,026</td>
<td>−0.287</td>
<td>15,026</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>0.310</td>
<td>648</td>
<td>−0.293</td>
<td>648</td>
</tr>
<tr>
<td>THREE</td>
<td>3</td>
<td>0.539</td>
<td>99</td>
<td>−0.585</td>
<td>99</td>
</tr>
<tr>
<td>FOUR</td>
<td>1</td>
<td>0.325</td>
<td>13,127</td>
<td>−0.282</td>
<td>13,127</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>0.300</td>
<td>550</td>
<td>−0.300</td>
<td>550</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>0.429</td>
<td>139</td>
<td>−0.533</td>
<td>139</td>
</tr>
</tbody>
</table>

Notes: Correlations between the ability of workers and their co-workers within the layers of firms.
### Table 1.23: Regression Results Sorting from Regression 1.22

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>std. err.</td>
<td>$\alpha_1$</td>
<td>std. err.</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.413***</td>
<td>0.009</td>
<td>0.412***</td>
<td>0.009</td>
<td>31,933</td>
</tr>
<tr>
<td>ONE</td>
<td>−0.149</td>
<td>0.106</td>
<td>−0.174</td>
<td>0.107</td>
<td>481</td>
</tr>
<tr>
<td>TWO</td>
<td>0.182***</td>
<td>0.031</td>
<td>0.171***</td>
<td>0.031</td>
<td>1,871</td>
</tr>
<tr>
<td>THREE</td>
<td>0.405***</td>
<td>0.013</td>
<td>0.409***</td>
<td>0.013</td>
<td>15,765</td>
</tr>
<tr>
<td>FOUR</td>
<td>0.374***</td>
<td>0.015</td>
<td>0.370***</td>
<td>0.015</td>
<td>13,816</td>
</tr>
</tbody>
</table>

Industry FE: Yes, Area FE: Yes, Firm Controls: No

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.22). The table only reports the value of the coefficient $\alpha_1$ from equation (1.22). Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.

### Table 1.24: Regression Results Sorting from Regression 1.22

<table>
<thead>
<tr>
<th>Total Number of Layers</th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>layer</td>
<td>$\alpha_1$</td>
<td>std. err.</td>
<td>$\alpha_1$</td>
<td>std. err.</td>
</tr>
<tr>
<td>ONE</td>
<td>1</td>
<td>−0.149</td>
<td>0.106</td>
<td>−0.174</td>
<td>0.107</td>
</tr>
<tr>
<td>TWO</td>
<td>1</td>
<td>0.135***</td>
<td>0.032</td>
<td>0.119***</td>
<td>0.032</td>
</tr>
<tr>
<td>TWO</td>
<td>2</td>
<td>0.093</td>
<td>0.415</td>
<td>−1.014***</td>
<td>0.013</td>
</tr>
<tr>
<td>THREE</td>
<td>1</td>
<td>0.399***</td>
<td>0.014</td>
<td>0.399***</td>
<td>0.014</td>
</tr>
<tr>
<td>THREE</td>
<td>2</td>
<td>−0.057</td>
<td>0.033</td>
<td>−0.0639</td>
<td>0.068</td>
</tr>
<tr>
<td>THREE</td>
<td>3</td>
<td>−0.090</td>
<td>0.185</td>
<td>−0.144</td>
<td>0.177</td>
</tr>
<tr>
<td>FOUR</td>
<td>1</td>
<td>0.372***</td>
<td>0.016</td>
<td>0.368***</td>
<td>0.016</td>
</tr>
<tr>
<td>FOUR</td>
<td>2</td>
<td>−0.205***</td>
<td>0.066</td>
<td>−0.240***</td>
<td>0.065</td>
</tr>
<tr>
<td>FOUR</td>
<td>3</td>
<td>−0.591***</td>
<td>0.131</td>
<td>−0.595***</td>
<td>0.130</td>
</tr>
</tbody>
</table>

Industry FE: Yes, Area FE: Yes, Firm Controls: No

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. White-heteroskedastic standard errors in parentheses. OLS regressions for equation (1.18). Industry fixed effects correspond to the 18 manufacturing industries. Area fixed effects correspond to the 341 employment areas in mainland France. Firm controls include the age of the firm, and whether the firm was present in the first year of the panel, 1976.
Chapter 2

The Impact of Market Size on Firm Organization
2.1 Introduction

A rich and lengthy literature has examined the differences of firms operating in markets of different sizes and established that firms in larger markets are on average bigger and more productive.\(^1\) For example, Syverson (2004) reports that the size and productivity distributions of ready-mixed concrete plants operating in denser markets have a greater mean.

One aspect of firms that has not received much theoretical nor empirical attention in this literature, however, is their organization. The majority of studies model production as a black box where inputs are directly mapped into units of output. Understanding how firms organize production is important however, because not only do firms affect the allocation of productive resources in the economy and the returns to factors of production, firms’ productivity may also depend on organization.\(^2\) With the exception of Garicano and Hubbard (2006), organizational structure has been absent from all studies that examine the differences of firms across locations. This paper fills the gap.

In this paper, I build and empirically test the predictions of a theoretical model that highlights how market size affects the organization of firms. I model an economy with heterogeneous firms and endogenous markups, and in which production requires labor and knowledge, as in Garicano (2000) and Caliendo and Rossi-Hansberg (2012).\(^3\)

I incorporate endogenous markups using the linear demand system introduced by Ottaviano, Tabuchi and Thisse (2002) and extended by Foster, Haltiwanger and Syverson (2008). To enter a market, entrepreneurs pay a fixed cost to develop a product. Once the product is developed, entrepreneurs receive a demand draw that determines their demand schedule. Entrepreneurs then decide whether to create a firm and on the optimal quantity to produce, the price, and the organizational form that maximizes their profits.

In the model entrepreneurs create firms which organize workers into layers of different

---

\(^1\) Melo, Graham and Noland (2009) conduct a meta-analysis from 34 studies that investigate the productivity gains from working in denser markets. They find that there are positive productivity gains to working in larger markets.

\(^2\) Garicano and Rossi-Hansberg (2014) present several areas of research where the study of the organization of firms has contributed new insights.

\(^3\) Garicano and Rossi-Hansberg (2004, 2006, 2012) present similar models of the organization of firms, however in their setting workers are ex-ante heterogeneous in their ability to solve problems. In this model workers are homogeneous.
sizes. Production workers are located in the lowest layer of the firm while managers occupy all of the other layers. At the top of the organization is the highest manager, the entrepreneur.

In this framework firms are knowledge-based hierarchies and their objective is to maximize profits which requires them to organize production efficiently. To produce a problem needs to be solved. Problems vary in their difficulty, or frequency, and to solve problems agents acquire knowledge which is costly. When communication is possible between agents in an organization, it is inefficient for all workers to be able to solve all problems. As argued by Garicano (2000) it is more efficient for agents to specialize in the tasks they perform. Managers learn how to solve the difficult or infrequent problems and workers deal with easy or common problems. In addition managers do not spend their time in production, but instead solve problems other agents cannot solve. Whenever a worker encounters a problem that he cannot solve he refers it to his manager. This allows firms to economize on the use of knowledge by allowing managers, who deal with exceptional problems, to leverage their knowledge over more problems.

The number of layers of managers in a firm is also endogenous. Each type of organization contains its own menu of costs and, as Caliendo and Rossi-Hansberg (2012) reveal, the decision to add a layer of managers can be thought of as a tradeoff between fixed and variable costs. Because managers devote their time to solving problems and agents are compensated for their knowledge and their time, adding a layer of managers is costly. At a certain level of output, however, a layer of managers also allows the firm to use workers’ knowledge more efficiently. The benefit of adding a layer of managers is that agents in the lower layers of the firm are required to acquire less knowledge. When a firm adds a layer of management, therefore, it is as if the firm is increasing its fixed costs in exchange for lower variable costs (Caliendo and Rossi-Hansberg (2012)). What determines how firms organize production, however, is the level of output they produce which is ultimately determined by the production technology, their demand, and the market’s structure.

Market size affects the toughness of competition, which affects the selection of heterogeneous producers and the organization of firms. Firms with low demand draws are forced to exit bigger markets. In addition, because markups are lower in bigger markets firms are

\[4\text{In other words, marginal costs are endogenous in the model.}\]
induced to change their organizational structure in favor of more layers. Under a condition on the distribution of demand draws, I show that the distribution of organizations in bigger markets will first order stochastically dominate the distribution of organizations in smaller markets. In bigger markets, therefore, the average firm will have more layers.

Further, I test the prediction of the model, using an administrative dataset of French workers, the Déclarations Annuelles des Données Sociales (DADS) which is an exhaustive cross-section of all workers who earn a positive wage in France. The richness of the dataset allows me to separate firms into hierarchical layers. To divide firms into layers, I adopt the strategy introduced by Caliendo, Monte and Rossi-Hansberg (2012), and classify employees into residing in the different organizational layers of firms. In the data, I can observe as many as four distinct layers within a firm. At the top of the organization are owners or CEOS who earn a positive wage, followed by senior managers, supervisors, and administrative and blue-collar workers. As many firms do not employ agents in every layer, there are four types of organizations in the data: firms with one layer, firms with two layers, firms with three layers, and four-layer firms.

To empirically test the prediction of the model, I compare the distribution of organizations across French employment areas of different density using the Mann-Whitney stochastic dominance test. My main finding is that the distribution of organizations in high density markets first order stochastically dominates the distribution of organizations in low density markets. Figures 2.1 offers preliminary evidence. Panel (a) plots the cumulative distribution of organizations in manufacturing and business services in employment areas with above-median employment density (orange bars) and in employment areas with below-median employment density (blue bars), while panel (b) plots the distributions of organizations of employment areas in the first and fourth quartiles of the density distribution. According to the prediction of the model, one should expect the cumulative distribution of organizations in high density markets to be below the cumulative distribution of organizations in low density markets. Indeed, as illustrated in Figure 2.1 this pattern is evident in the data.

One limitation with comparing the distribution of firms across locations is that in most industries the market is defined at the national or global level. To address this problem, I adopt the empirical strategy of Syverson (2004) and conduct my analysis only on firms
operating in the ready-mixed concrete industry. The ready-mixed concrete industry’s high transport costs ensure that firms mostly supply concrete to surrounding areas, and so the ready-mixed market is not a single national or global unit but a collection of local geographic markets (Syverson (2008)). In this case, I continue to find evidence that the distribution of organizations in high density markets first order stochastically dominates the distribution in low density markets.

A crucial feature of the model is that the productivity of a firm and the knowledge of its labor force are determined by the way production is organized given the level of demand for its product, and the level of competition in the market. This framework therefore provides a rich set of predictions on the characteristics of firms and their labor force, and how they differ across locations.

While the aim of this paper is to establish a relationship between the organization of firms and market size, in the last part of the paper, I simulate the model to assess how productivity and income differ across locations. I find that the distribution of productivity in the larger market has a greater mean and a lower variance. I also examine whether there is left truncation, shift, and dilation in the distributions of productivity in larger markets. Since it is difficult to separate truncation, shift, and dilation in a visual comparison of distributions, I use the quantile approach introduced by Combes, Duranton, Gobillon, Puga and Roux (2012) to compare distributions. I find that the distribution of productivity in the bigger market is right-shifted, dilated, and negatively left-truncated relative to the distribution in the smaller market. I also compare the distributions of income across locations. I find that the distribution of income in bigger markets has a greater mean and relative to the smaller market and exhibits a lower variance.

This paper is related to the broad literature on the theory of the firm emphasizing management hierarchies. The main distinction between this literature and my paper, is that I model production hierarchies in a product market with endogeneous markups. My model is most closely related to Caliendo and Rossi-Hansberg (2012) who study the effect of a bilateral trade liberalization on organization and firm productivity. The main distinction between their study and mine, however, is that they embed the production framework into a monopolistically competitive model with heterogeneous firms and CES preferences. In the closed-economy of their model, changes in the size of the market do not affect the entry
Figure 2.1: Distribution of Organizations

(a) Above Median vs. Below Median

(b) First Quartile vs. Fourth Quartile
threshold, nor the organizational structure of firms, and therefore, changes in the size of the market do not affect the distribution of organizations in the economy. In contrast, in my model, because market size affects the toughness of competition, changes in the size of the market affect both the entry threshold and organizational structure of firms, and hence the distribution of organizations.

Within the management hierarchy literature, my paper is also closely related to Garicano and Hubbard (2007), who examine how managerial leverage, the number of managers per worker, in U.S. law firms is affected by the size of the market. A first difference with Garicano and Hubbard’s work is the mechanisms influencing organizations. In their study, firms’ organizational decisions are affected by uncertainty over the level of demand. As the size of the market increases, aggregate uncertainty about demand falls, and managers’ return from leveraging their knowledge over workers rises, while in my model market size affects the toughness of competition between firms, thereby causing firms to change their organization. A second difference is that in their paper firms can have at most two layers, while I allow firms to have any number of layers. This makes comparisons between the distribution of organizations across markets and sectors possible. And a third difference is that I consider firms not only in the services sector, but also in manufacturing.

This paper is also related to an empirical literature analyzing the organization of firms. Despite the vast theoretical literature on the organization of firms, there has been limited empirical work because of the lack of available data. Data problems notwithstanding, Rajan and Wulf (2006), Guadalupe and Wulf (2012), Garicano and Hubbard (2007), and Caliendo, Monte and Rossi-Hansberg (2012) have made progress in bringing empirical light to theories on the organization of firms. With the exception of Garicano and Hubbard (2007), however, there has not been any other study that examines the relationship between market size and firm organization.

Besides extending research on the organization of firms, this paper is broadly related to other topics. One is the literature emphasizing the relationship between market size and productivity. Prominent studies include Syverson (2004), Melitz and Ottaviano (2008) and Combes, Duranton, Gobillon, Puga and Roux (2012). I contribute to this literature, by considering a model where the productivity of firms is endogenous and by comparing the associated distributions of productivity across locations. This paper’s conclusion is
consistent with the literature: firms in larger markets are on average more productive.

Another is a separate literature emphasizing the relationship between market size and wages, which arises from the model’s implication for the distributions of income across locations. In the model, because firms in larger markets have on average more layers, the distribution of income exhibits a higher mean. This finding is relevant to studies that investigate how wages differ across locations and conclude that workers earn higher wages in denser markets.\(^5\) Interestingly, however, this result does not rely on any sorting between agents and locations nor on any agglomeration forces, commonly appealed to in the literature to explain these findings.

The remaining sections of the paper are organized as follows. In Section 2.2, I construct the theoretical framework, and analyze the impact market size has on the organization of firms. In Section 2.3, I introduce the data and test the main prediction of the model. In Section 2.4, I simulate the model and examine how distributions of productivity and income differ across locations. Section 2.5 concludes. I relegate all proofs to the Appendix.

2.2 Model

2.2.1 Demand

The economy contains \(N\) homogeneous workers who supply their labor inelastically. All consumers have the same linear quadratic utility function,

\[
U^c = q^c_o + \int_\Omega \alpha_i q^c_i \, di - \frac{\gamma}{2} \int_\Omega q^2 \, di - \frac{\eta}{2} \left( \int_\Omega q^2 \, di \right)^2 ,
\]  

(2.1)

where \(q^c_o\) and \(q^c_i\) represent individuals’ consumption of the numeraire good and each variety \(i\), and \(\Omega\) indexes the number of varieties available in the economy. For each variety, demand is determined by the parameters \(\alpha_i\), \(\eta\) and \(\gamma\) which are positive. The parameter \(\gamma\) measures the degree to which product differentiation between varieties is preferred by consumers. Varieties are more substitutible as \(\gamma\) approaches zero, and in the limit, varieties are perfect substitutes and consumers only care about their consumption over all varieties \(Q^c_i = \int_\Omega q^c_i \, di\). The parameters \(\alpha_i\), which varies by products, and \(\eta\) represent the degree to which heterogeneous

\(^5\)Two studies in this literature are Glaeser and Mare (2001) and Combes, Duranton and Gobillon (2008).
varieties are preferred to the numeraire good.

Since marginal utilities for all goods are bounded, a consumer may not have a positive demand for every variety. Assume that in equilibrium there is a positive demand for the numeraire good. The inverse demand function of a consumer for variety $i$ is then given by:

$$ p_i = \alpha_i - \gamma q_i^c - \eta Q_i^c. \quad (2.2) $$

From this expression, the role of $\alpha_i$ becomes clear. In this setting, $\alpha_i$ serves as a demand shifter for the differentiated product $i$. Goods with a greater $\alpha_i$ will have greater demand.

Assume that a subset $\Omega^* \subset \Omega$ of varieties with a positive measure, are produced in equilibrium. Then the demand function of variety $i \in \Omega^*$ is:

$$ q_i = Nq_i^c = \frac{N}{\gamma} \alpha_i - \frac{N}{\gamma} p_i - \frac{N}{\gamma} \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \quad (2.3) $$

where $M$ is the mass of differentiated varieties present in the economy, and $\bar{\alpha} = \frac{1}{M} \int_{\Omega^*} \alpha_i di$ and $\bar{p} = \frac{1}{M} \int_{\Omega^*} p_i di$ are the average demand shifter and prices of the $M$ consumed varieties in $\Omega^* \subset \Omega$.

The price elasticity of demand is given by the expression $\epsilon_i = \left| \frac{\partial q}{\partial p_i} \right| = \left( \frac{p_{\text{max}}}{p_i} - 1 \right)^{-1}$, where $p_{\text{max}}$ is the maximum price that a firm can charge, and is equal to $\alpha_i - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p})$. Given this expression, a decrease in $\bar{p}$ and an increase in $M$ or $\bar{\alpha}$ induce a decrease in $p_{\text{max}}$, which leads to an increase in the elasticity of demand, $\epsilon_i$. As in Melitz and Ottaviano (2008), I characterize a higher elasticity as a tougher competitive environment. The price elasticity of demand is also monotonically decreasing with $\alpha_i$, and therefore firms producing a variety with a higher $\alpha_i$ encounter a more inelastic demand curve.

Individual’s welfare is determined from the indirect utility function:

$$ U^c = I^c + \frac{M}{2\gamma} \left[ \sigma_p^2 + \sigma_\alpha^2 - 2\text{cov}(\alpha, p) \right] + \frac{1}{2} \left( \frac{\gamma}{M} + \eta \right)^{-1} (\bar{\alpha} - \bar{p})^2 \quad (2.4) $$

where $I^c$ is the consumer’s income, $\sigma_p^2 = \frac{1}{M} \int_{i \in \Omega^*} (p_i - \bar{p})^2 di$ represents the variance of prices, $\sigma_\alpha^2 = \frac{1}{M} \int_{i \in \Omega^*} (\alpha_i - \bar{\alpha})^2 di$ represents the variance of $\alpha_i$, and $\text{cov}(\alpha, p) = \frac{1}{M} \int_{i \in \Omega^*} (\alpha_i - \bar{\alpha})(p_i - \bar{p}) di$ is the covariance between prices and the demand shifters $\alpha_i$. To guarantee a positive demand for the numeraire good, assume that $I^c > \int_{i \in \Omega^*} p_i q_i^c di$. Utility rises with a decrease in average prices $\bar{p}$, or an increase in the average demand shifters $\bar{\alpha}$. Holding everything else
constant, welfare rises with the variance of prices $\sigma_p^2$. Welfare also rises with the variance of the demand shifters $\sigma_\alpha^2$, and with an increase in the number of varieties, $M$. Welfare however decreases with an increase in the covariance between prices and the demand shifters $\text{cov}(\alpha, p)$, as consumers receive a smaller share of the surplus from demand.

2.2.2 Production: General Framework

Labor is the only input used in production and is inelastically supplied in a competitive market. Agents have one unit of time, and can work either in the homogeneous good sector, the production sector, or in the schooling sector. Because agents are identical, in equilibrium they all earn the same wage $w$, irrespective of their occupation and sector of employment.

In the homogeneous good sector, the good is produced with a constant returns to scale technology that requires only labor. In the analysis that follows, the price of the homogenous good is normalized to 1. Taken together these assumptions, imply that in equilibrium wages are equal to 1.

In the differentiated goods sector, agents start new firms. I refer to an agent that starts a new firm as an entrepreneur. To enter, an entrepreneur first pays a fixed cost $f_E$ to develop a product, which is thereafter sunk. Once the product is developed, the entrepreneur obtains a demand draw $\alpha$ from a known cumulative distribution $G(\alpha)$. This draw, $\alpha$, determines the demand schedule of the firm. Given the market environment, if the demand draw is low the entrepreneur may decide to immediately exit and not produce. Alternatively, the entrepreneur creates an organization to produce his product.

In the organization production requires labor and knowledge. Agents are divided in two main categories: production workers and managers. Production worker are located in the lowest layer of the organization, while managers occupy all of the other layers. In the highest layer of the organization is the entrepreneur, who performs the same tasks as agents in that layer.

Production workers spend their one unit of time generating a production possibility. For the production possibility to become output, a problem needs to be solved. Solving problems requires knowledge. Each production possibility is associated to a problem, $z$,

---

6I describe how agents acquire knowledge further below.
drawn from a known cumulative distribution $F(z)$ with decreasing density.\(^7\) For a given realization of $z$, a worker can solve the problem if his knowledge set, $\Gamma_W$, contains $z$. If this is the case, then the problem is solved, and the production possibility becomes $A$ units of output.\(^8\) The type of problem a worker draws, however, is unobservable. For a given realization of $z$, the only information the worker has is if $z \in \Gamma_W$.\(^9\)

In an organization with zero management layers, if the worker cannot solve the problem nothing is produced. In an organization with a positive number of management layers this is not the case. If the worker cannot solve the problem, then she asks her manager one layer above. The manager spend $h$ units of time listening to the worker’s problem, and solves her problem if her knowledge set, $\Gamma_M$, includes $z$.\(^{10}\) If the manager cannot solve the problem, then the worker sends the problem to a manager one layer higher. This process continues, until the problem is solved and $A$ units of output are produced, or the problem reaches the highest layer of the organization, that is occupied by the entrepreneur. The entrepreneur also spends $h$ units of time listening to the worker’s problem, and solves the problem if her knowledge set, $\Gamma_E$, includes $z$. Otherwise, the problem remains unsolved and nothing is produced.

Agents need knowledge to solve problems. I assume that one unit of knowledge requires $c$ units of a teacher’s time in the schooling sector. As teachers earn a wage $w$ for their unit of time, the cost of a unit of knowledge is, therefore, $wc$. Because wages are normalized to one, this is simply $c$. I assume that knowledge is not cumulative. Agents must not learn to solve the easy problems before they can solve the hard ones. To solve a problem $z$, an agent’s knowledge set needs to include $z$, but the agent does not necessarily need to be able solve all problems between 0 and $z$. In other words, if the length of an agent’s knowledge set is $t$, then the cost of acquiring that knowledge is $ct$. For the remainder of the paper, I denote the knowledge set of an agent in layer $l$ in an organization with $L$ layers as $\Gamma^l_L$.

Throughout the paper, I assume an exponential distribution of problems. That is, $F(z) = 1 - \exp^{-\lambda z}$, with $\lambda > 0$. The parameter $\lambda$ determines how common problems

\(^7\)Here the dimensionality of $z$ is 1. The cumulative distribution $F(z)$ determines the difficulty of problems in the economy.

\(^8\)The parameter $A$ represents the technology of the economy.

\(^9\)In the literature, this is referred to as the "labeling assumption". This assumption is crucial for the existence of organizations.

\(^{10}\)The manager will always spend $h$ units of time listening to the workers problem, whether she knows the solution to the problem or not.
are in production. High $\lambda$ implies that problems are less costly to solve because the distribution is more concentrated around zero. Moreover as shown in Garicano (2000), in an organization it is never optimal for the knowledge of agents in the different layers to overlap, and it is optimal for agents in the lower layers to learn to solve the easiest or commonest problems, while agents in the higher layers learn to solve rarer or exceptional problems.

Production with 0 Management Layers - A One-Layer Firm

Consider an organization with one worker only, a self-employed entrepreneur. Because she is the only worker in the firm, the entrepreneur spends her one unit of time generating production possibilities. This implies that the organization’s output is bounded by the amount of output that can be produced from solving a single problem, $A$. Because the distribution of problems is a decreasing density, the entrepreneur learns to solve the commonest problems. Her knowledge set $\Gamma_1$ is therefore $[0, z_1]$ where $z'_l$ denotes the knowledge of an agent in layer $l$ employed in an organization with $L$ layers.\footnote{I slightly change the notation of Caliendo and Rossi-Hansberg (2012) where the first layer of a firm is denoted as 1.} Given her level of knowledge, the entrepreneur is able to solve $F(z) = 1 - \exp^{-\lambda z_1}$ fraction of problems, and the expected output of the organization is $A \left[ 1 - \exp^{-\lambda z_1} \right]$.

Production with 1 Management Layer - A Two-Layer Firm

Consider an organization with 1 layer of management of a two-layer firm. At the top of the organization is the entrepreneur with a knowledge set $\Gamma_2$ with length $z_2$. The entrepreneur hires workers in layer 1 who spend their time generating production possibilities and solve the commonest problems. The workers’ knowledge set is $\Gamma_2 = [0, z_2]$. Every time that a production worker encounters a problem the she cannot solve, she sends the problem to the entrepreneur, who in turn spend $h$ units of time listening to the worker. If the entrepreneur knows the solution to the problem, then she communicates the solution to her production worker and $A$ units of output are produced.

Let $n^l_L$ denote the number of workers in layer $l$ in an organization with $L$ layers. Because the entrepreneur is the only agent that occupies the highest layer of the organization $n^2_2 = 1$. Further, the knowledge of the production workers and the time an entrepreneur spends
communicating with them limits the number of workers that an entrepreneur can supervise. The fraction of problems that production workers in layer 1 cannot solve is \(1 - F(z) = \exp^{-\lambda z_2}\), and because an entrepreneur spend \(h\) units of her time communicating with a worker, the entrepreneur’s time constraint is equal to \(hn_2^1 \exp^{-\lambda z_2^1} = n_2^2 = 1\), and so an entrepreneur can hire \(n_2^1 = \frac{\exp^{\lambda z_2^1}}{h}\) workers. The entrepreneur’s time constraint also provides an expression for her span of control, which is the number of workers in the layer below that she supervises. To be specific, \(\text{span}_2 = \frac{n_2^2}{n_2^1} = \frac{\exp^{\lambda z_2^1}}{h}\).

The total knowledge in the organization is \(\Gamma_O = \Gamma_2^1 \cup \Gamma_2^2\). Because in an organization it is inefficient for the knowledge of agents in two distinct layers to overlap, the entrepreneur has knowledge \(z_2^2\) and can solve problems in the interval \([z_1^2, z_1^2 + z_2^2]\). This implies that total knowledge of the organization is \(\Gamma_O = [0, z_1^2 + z_2^2]\). Given the knowledge in the organization, the fraction of problems that are solved is \(F(z) = 1 - \exp^{-\lambda z_1^2 - \lambda z_2^2}\), and the expected output of the organization is \(An_2^1 \left[1 - \exp^{-\lambda z_1^2 - \lambda z_2^2}\right]\).

Production with \(L\) Management Layers - A \(L + 1\) Layer Firm

The production process can now be generalized to an organization with \(L + 1\) layers.\(^{12}\) At the top of the organization is the entrepreneur. The entrepreneur hires workers in layer 1 and managers in layers \(l\) who spend their time solving problems. Production workers in layer 1 spend their time generating production possibilities and solve the commonest problems. The workers’ knowledge set is \(\Gamma_{L+1}^1 = [0, z_{L+1}^1]\). Every time that a production worker encounters a problem she cannot solve, she sends the problem to a manager one layer above, who in turn spend \(h\) units of time listening to the worker. If the manager does not solve the problem, then the worker sends the problem to a manager one layer higher. This process continues until the problem is solved and \(A\) units of output are produced, or the problem reaches the entrepreneur who also devotes \(h\) units of her time listening to the problem. At this point, if the entrepreneur knows the solution to the problem, she communicates it to her worker, otherwise the problem remains unresolved and nothing is produced.

In an organization with \(L + 1\) layers, the size of each layer is determined by three factors: the cost of communication, \(h\), the number of production possibilities generated in

\(^{12}\)As shown by Garicano (2000), this organizational structure is optimal when matching problems with agents who can solve them is costly.
the organization, $n_{L+1}^1$, and the fraction of problems agents in the layers below cannot solve. For example, the size of layer $l$ is given by $hn_L^1 \exp^{-\lambda \sum_{j=1}^{L-1} z_{j+1}^L} = n_{L+1}^l$. From the equations that characterize the size of each layer, one can derive an expression for the number of workers in the layer below that a manager supervises, the span of control of managers in layer $l$. Namely,

$$span_{L+1}^2 = \frac{n_{L+1}^1}{n_{L+1}^2} = \frac{\exp^{\lambda z_{L+1}^L}}{h} \text{ for } l = 2,$$

(2.5)

and

$$span_{L+1}^l = \frac{n_{L+1}^{l-1}}{n_{L+1}^l} = \exp^{\lambda z_{L+1}^{l-1}} \quad \forall \quad 3 \leq l \leq L + 1.$$

(2.6)

Because the distribution of problems is exponential, the span of control of each manager only depends on the knowledge of agents in the layer below.

The total knowledge in an organization with $L+1$ layers, is the sum of agents’ knowledge in every layer and is given by $\Gamma_O = \Gamma_{L+1}^1 \cup \Gamma_{L+1}^2 \cup \Gamma_{L+1}^3 \cup \ldots \cup \Gamma_{L+1}^L \cup \Gamma_{L+1}^{L+1}$. Total expected output is determined by the number of production possibilities generated, the fraction of problems that are solved in the organization, and the units of output that are generated when a problem is solved. The total expected output of an organization with $L + 1$ layers is therefore equal to $An_{L+1}^1 \left[ 1 - \exp^{-\lambda \sum_{j=1}^{L+1} z_{L+1}^j} \right]$.

### 2.2.3 Production: Firm’s Problem

Given its demand realization, the problem of the firm is to choose the optimal quantity to supply the market, the number of layers, as well as the knowledge and number of agents in each layer. The subsections that follow, describe the problem of a firm producing in the differentiated goods sector. As in Caliendo and Rossi-Hansberg (2012), I describe the problem of a firm in two-stages. First, for a given level of quantity, I describe the cost-efficient way to organize production. The solution to the cost-minimization problem determines the cost function of the firm. As the organizational problem of the firm is analyzed in Caliendo and Rossi-Hansberg (2012), in this section I simply summarize the results that are relevant to the subsequent analysis. For further details and proofs, I refer the reader to their paper. Second, given a demand draw $\alpha$ and the solution to the organizational problem, I solve for
the optimal quantity an entrepreneur supplies to the market.

Cost Minimization

Consider a firm producing a quantity $q$. Because a firm can organize its production with any number of layers, it chooses the organizational form that has the lowest cost. The cost function of a firm is therefore given by

$$C(q, w) = \min_{L \geq 1} \{C_L(q, w)\}$$ (2.7)

where $C(q, w)$ is the minimum cost of producing $q$ units of the differentiated variety, and $C_L(q, w)$ is the minimum cost of producing $q$ units with an organization of $L$ layers. Since wages are determined by the homogeneous good sector and are equal to 1, they will not vary in the equilibrium of the model, and I remove $w$ from the subsequent notation.

Agents are compensated for their one unit of time and the knowledge they acquire. For a given number of layers $L$ and a given quantity $q$, the entrepreneur decides the number of employees to hire in each layer, $n^l_L$, and the knowledge of the employees at a given layer, $z^l_L$, with the objective to minimize costs. The cost minimization problem of an organization with $L \geq 1$ layers is, therefore, the following:

$$C_L(q) = \min_{\{n^l_L, z^l_L\} \geq 0} \sum_{l=1}^{L} n^l_L[z^l_L + 1]$$ (2.8)

subject to

$$A \left[ 1 - \exp^{-\lambda Z_{L-1}^L} \right] n^1_L \geq q,$$ (2.9)

$$n^l_L = n^1_L h \exp^{-\lambda Z_{l-1}^L} \text{ for } L \geq 1 \geq 2,$$ (2.10)

$$n^L_L = 1,$$ (2.11)

where $Z_{L-1}^L = \sum_{l=1}^{L} z^l_L$ is the cumulative knowledge at layer $l$. As production workers are the only agents in the organization to draw production possibilities, there are in total $n^1_L$ problems in the firm. The first constraint indicates that the total output produced by the firm has to be at least $q$ units of output. The second constraint determines the size of each layer $l$ while the last constraint ensures that the entrepreneur supplies all of his time to the
A firm that optimally selects the number of layers, will never choose to have intermediate managers with zero knowledge. This is because managers with zero knowledge cannot solve any problems and do not contribute to the production output of the firm. However, these managers increase production costs, since the firm must compensate them for the one unit of time that they have supplied. In other words, holding the amount of quantity produced constant, a firm with $L+1$ layers in which intermediate managers have zero knowledge will have higher costs than an organization with $L$ layers where the intermediate managers are removed and all other layers have the same amount knowledge.

Further, a firm can reduce its output in two ways: by decreasing the knowledge of workers and problem solvers at every layer or by decreasing the number of layers. If the cost of communication are greater than the cost of acquiring knowledge, a firm would rather decrease the number of layers instead of setting the knowledge of its employees to zero. A restriction on the parameters of the model ensures that this is the case, and so all agents in a firm have positive knowledge. Namely, assume the following:

**Assumption 2.1** The parameters $c$, $\lambda$, and $h$ satisfy the following condition:

$$\frac{c}{\lambda} \leq \frac{h}{1-h}. \quad (2.12)$$

Assumption 2.1 ensures that the cost of communication are large relative to the cost of acquiring knowledge, so thus a firm would prefer to decrease the number of layers before it sets the knowledge of its employees to zero.\(^{14}\)

For a given number of layers, $L$, the marginal cost function of a firm is equal to:

$$MC_L(q) = \frac{ch}{\lambda A} \exp^{\lambda z}. \quad (2.13)$$

The cost function of the firm with one layer is given by:

$$C_1(q) = w(cz_1^1 + 1), \quad (2.14)$$

---

\(^{13}\)As explained in Caliendo and Rossi-Hansberg (2012) the last constraint is not just a normalization. It ensures that organizations cannot be replicated at the minimum efficient scale.

\(^{14}\)This assumption guarantees that the Lagrange multipliers on the non-negative knowledge constraints are zero.
where $z_1^L$ is the solution to the cost minimization problem. The cost function of the firm with $L = 2$ layers is given by:

$$C_2(q) = \frac{c}{\lambda} \left( \frac{h}{A} \exp^{\lambda z_2^L} q + \left( 1 - \frac{\exp^{\lambda z_2^L}}{h} \right) + \lambda z_2^L + \frac{\lambda}{c} \right), \quad (2.15)$$

where $z_2^L$ and $z_1^L$ are the solutions to the cost minimization problem, and cost function of the firm with $L \geq 3$ layers is given by:

$$C_L(q) = \frac{c}{\lambda} \left( \frac{h}{A} \exp^{\lambda z_L^L} q + \left( 1 - \exp^{\lambda z_L^{L-1}} \right) + \lambda z_L^L + \frac{\lambda}{c} \right), \quad (2.16)$$

where $z_L^L$ and $z_L^{L-1}$ are the solutions to the cost minimization problem. Note, that the marginal cost of the firm is increasing in the cost of acquiring knowledge relative to the knowledge requirements in production, $\xi_L$, the costs of communication $h$, and the knowledge of the entrepreneur, $z_L^L$. Intuitively, as the cost of acquiring knowledge is greater or the more difficult it is for employees to communicate with one another, then the higher will be cost of producing an additional unit of output. The marginal cost however is decreasing in the level of the production technology, $A$. Furthermore the average cost function of a firm with $L$ layers is $AV_L(q) = \frac{C_L(q)}{q}$.

The cost functions for all different $L$ have a number of characteristics in common. First, marginal costs are positive for all $L$ and $q$. In addition, the marginal cost of a firm is increasing in output, $q$. This implies that the knowledge of agents at all layers, $z_L^L$, is increasing in $q$, which implies that the size of each layer, $n_L^L$, and the span of control of managers, $\frac{n_L^L}{n_L^{L+1}}$, increase as well with $q$. Intuitively given the number of layers, an increase production requires more knowledge in the organization. Some of this additional knowledge is acquired by the entrepreneur, while the remainder is acquired by agents in the other layers of the organization. As agents in a layer acquire more knowledge, their managers can supervise more of them, which implies that the chosen organizational structure becomes larger, and the cost of producing an extra unit of output increases.

Second, because marginal costs are positive for all $L$ and $q$, the cost function in strictly increasing. In addition, the cost function is homogeneous of degree one in $w$, while conditional factor demands are homogeneous of degree zero in $w$. Third, the average cost curves are convex in $q$, attain the minimum $q_L^*$ when they intersect the marginal cost curves, and
converge to infinity when $q \to 0$ or when $q \to \infty$.\footnote{For a proof of these statements, see Proposition 1 in Caliendo and Rossi-Hansberg (2012).} Intuitively, when production is small increases in output lead to a less than linear increase in total costs and reduce average costs until the minimum efficiency scale (MES) is reached. Beyond the MES, however, an increase in output increases the average costs of the firm, because the firm needs to provide too much knowledge to agents in the lower layers of the organization.\footnote{Therefore, average costs are not monotonic function of quantity.}

Across layers the cost functions have the following properties. First, the minimum average cost is decreasing with the number of layers, and second, the level of output that attains the minimum average cost is increasing with the number of layers. These two characteristics are important, because together they imply that the regions around the MES will be part of the lower envelope of the cost functions that determine the global average cost curve when the number of layers is optimized.

Furthermore, at the level of output where a firm adds a layer of management, marginal costs decrease discontinuously.\footnote{This implies that marginal costs are not a monotonic function of quantity.} This implies that the knowledge of agents at all layers, $z_L^l$, and the span of control of managers, $\frac{n_L^l}{n_L^{l+1}}$, decrease discontinuously as well, while the number of employees at each layer, $n_L^l$, increases discontinuously. Intuitively, by allowing managers with the ability to solve harder problems to leverage their knowledge, a firm with more layers is able to economize on the knowledge of its existing employees. In other words, by adding a new layer of management, a firm is paying higher “fixed” costs, and, if its output is large enough, it can produce at lower average costs.

**Profit Maximization**

Consider the profit maximization problem of an entrepreneur with demand draw $\alpha$ and cost function $C(q)$, described in the previous section.\footnote{Recall that $C(q) = \min_L \{C_L(q)\}$} Given his demand, the entrepreneur chooses the optimal quantity of the differentiated variety to supply to the market and its price. In making his decision, the entrepreneur implicitly decides on the number of layers of the organization. Also, the entrepreneur takes the average demand shifter and prices as given (this is the monopolistic competition assumption). Given his draw $\alpha$, the entrepreneur’s maximization problem is:
\[ \pi(\alpha) = \max_{q \geq 0} p(q(\alpha))q(\alpha) - C(q(\alpha)). \] (2.17)

Note that the firms’ maximization problem can alternatively be written as:

\[ \pi(\alpha) = \max_{L} \{\pi_L(\alpha)\}, \] (2.18)

where

\[ \pi_L(\alpha) = \max_{q_L \geq 0} p(q_L(\alpha))q_L(\alpha) - C_L(q_L(\alpha)). \] (2.19)

where \( \pi_L(\alpha) \) are the profits of a firm producing with \( L \) layers. Due to changes in the number of layers, \( C(q(\alpha)) \) is not a strictly convex function. Given a demand draw \( \alpha \), to solve the firm’s maximization problem, one first determines the optimal profits for a given number of layers, and then compares these local optimal profits to find the global maximum of the profit function. For a given number of layers, the local optimal profit function of a firm is strictly concave in \( q \), \( \pi(0) = -1 \) and \( \lim_{q \to \infty} \pi(q) = -\infty \) and therefore the profit maximization problem of the firm is well-defined. In this section, I first describe the local solution to the firm’s maximization problem and then I return to the global solution.

Holding the number of layers, \( L \), constant consider the optimization problem of a firm with demand draw \( \alpha \), as described in equation (2.19). From the first order condition, the optimal quantity supplied is:

\[ q_L(\alpha) = \frac{N}{\gamma} \left\{ p_L(\alpha) - MC_L(q_L(\alpha)) \right\}. \] (2.20)

where the subscript \( L \) indicates that the number of layers is held constant at \( L \). From equations (2.3) and (2.20), one obtains an expression for the optimal quantity supplied:

\[ q_L(\alpha) = \frac{N}{2\gamma} \left\{ \alpha - \frac{M}{\gamma + M}(\bar{\alpha} - \bar{p}) - MC_L(q_L(\alpha)) \right\}. \] (2.21)
Given the number of layers in the firm, $L$, the optimal quantity supplied is determined from equation (2.21). The left-hand side is increasing with respect to quantity, while the right-hand side is decreasing. Therefore given the number of layers, for every demand draw $\alpha$, there exists a unique level of quantity, $q_L(\alpha)$, such that equation (2.21) holds with equality. Note that because the firm’s marginal cost depend on the quantity produced, a closed form solution is unavailable. The equations characterizing optimal prices, absolute markups over marginal costs ($p_L(\alpha) - MC_L(q_L(\alpha))$), absolute markups over average costs ($p_L(\alpha) - AC_L(q_L(\alpha))$), revenues and profits in terms of parameters are the following:

\[
p_L(\alpha) = \frac{1}{2} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) + MC_L(q_L(\alpha)) \right\}, \quad (2.22)
\]

\[
\mu^MC_L(\alpha) = \frac{1}{2} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) - MC_L(q_L(\alpha)) \right\}, \quad (2.23)
\]

\[
\mu^AC_L(\alpha) = \frac{1}{2} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) + MC_L(q_L(\alpha)) \right\} - \frac{C_L(q_L(\alpha))}{q_L(\alpha)}, \quad (2.24)
\]

\[
r_L(\alpha) = \frac{N}{4\gamma} \left\{ \left( \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) \right)^2 - MC_L(q_L(\alpha))^2 \right\}, \quad (2.25)
\]

\[
\pi_L(\alpha) = \frac{N}{4\gamma} \left\{ \left( \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) \right)^2 - MC_L(q(\alpha))^2 \right\} - C_L(q(\alpha)). \quad (2.26)
\]

Holding the number of layers constant, firms with different demand draws, $\alpha$, supply different quantities of the differentiated variety and set different prices. From equation (2.21) and the fact that for a given number of layers marginal costs are increasing with quantity, one can show that $q_L(\alpha)$ is an increasing function of $\alpha$. Prices are increasing with marginal costs, and hence prices increase with $\alpha$. Since quantities and prices are increasing with $\alpha$, revenues are also an increasing function of $\alpha$.

Firms with different demand draws, $\alpha$, also set different markups over marginal costs. Equation (2.23) indicates that markups are decreasing with marginal costs. Holding the
number of layers constant, an increase in \( \alpha \) has two opposing effects on a firm’s markups. First, the firm’s demand curve shifts to the right increasing \( \mu^MC_L(\alpha) \) for any given level of quantity produced, and second the firm produces a greater quantity of the differentiated variety causing marginal costs to increase and markups to decrease. The first effect however dominates, and markups are increasing with \( \alpha \).

Firms with different demand draws, \( \alpha \), also set different markups over average costs, \( \mu^AC_L(\alpha) \). Holding the number of layers constant, an increase in \( \alpha \) has two effects on a firm’s markups over average costs. First, the firm’s demand curve shifts to the right increasing \( \mu^AC_L(\alpha) \) for any given level of quantity produced, and second the firm produces a greater quantity of the differentiated variety causing average costs to increase or decrease. When average costs decrease \( \mu^AC_L(\alpha) \) increases with \( \alpha \). When average costs increase however, the effect on a firm’s demand schedule dominates, and markups are increasing with \( \alpha \). These results are summarized in the following proposition:

**Proposition 2.2** Within layers, \( q_L(\alpha), p_L(\alpha), \mu^MC_L(\alpha), \mu^AC_L(\alpha) \) and \( r_L(\alpha) \), are increasing with respect to \( \alpha \). Moreover, \( \pi_L(\alpha) \) is continuous, strictly increasing with respect to \( \alpha \) and strictly concave in \( q \).

**Proof.** see appendix.

Profits are continuous and increasing with respect to \( \alpha \).\(^{19}\) Moreover, as the slope of the profit function is increasing with the number of layers, there always exists a firm that is indifferent between producing with \( L \) or with \( L+1 \) layers. Consider such an \( \alpha \), say \( \tilde{\alpha} \). The pricing rule is to set marginal revenue equal to marginal cost. Since for a given quantity, an organization with \( L \) layers has a greater marginal cost than an organization with \( L+1 \) layers, \( q_L(\tilde{\alpha}) \) cannot be the optimal quantity supplied by an organization with \( L+1 \) layers. It follows that \( q_L(\tilde{\alpha}) > q_{L+1}(\tilde{\alpha}) \), and at the optimal quantities \( MC_L(q_L(\tilde{\alpha})) > MC_{L+1}(q_{L+1}(\tilde{\alpha})) \). Therefore, when the firm increases the number of layers, marginal costs decrease discontinuously and the optimal quantity supplied by the firm increases discontinuously as well. In contrast, prices decrease discontinuously when the firm increases the number of layers.\(^{20}\) Although prices decrease discontinuously, the increase quantities outweighs the change in prices, and revenues increase discontinuously as well. When the firm increases the number

\(^{19}\) Another expression for profits is: \( \pi_L(\alpha) = \frac{2}{N} q_L(\alpha)^2 + q_L(\alpha)MC_L(q_L(\alpha)) - C_L(q_L(\alpha)) \).

\(^{20}\) In all prices are therefore not monotone with respect to \( \alpha \)
of layers, the decrease in the marginal costs outweighs the decrease in prices, and markups increase discontinuously.

For a given $\alpha$, let $q(\alpha)$, $p(\alpha)$, $\mu^{MC}(\alpha)$, $\mu^{AC}(\alpha)$, $r(\alpha)$ correspond to the global solution to the firm’s maximization problem.\footnote{That is, $q(\alpha) = \arg \max_{\alpha} \pi(\alpha)$.} Given his demand draw $\alpha$, the entrepreneur maximizes profits and decides the number of layers of the organization. As $\alpha$ increases, the demand schedule changes and the entrepreneur re-optimizes, finding in some cases profit maximizing to restructure the organization and increase the number of layers. When the entrepreneur does not change the number of layers, $q(\alpha)$, $p(\alpha)$, $\mu^{MC}(\alpha)$, $\mu^{AC}(\alpha)$, $r(\alpha)$ have the properties of their corresponding local solutions. When the entrepreneur increases the number of layers, marginal costs decrease discontinuously, which affects the global solutions to the firm’s maximization problem. The profit function $\pi(\alpha)$ is the upper envelope of the local profit functions $\pi_{L}(\alpha)$. It is continuous and strictly increasing in $\alpha$. These results are summarized in the following proposition:

**Proposition 2.3** Within layers, $q(\alpha)$, $p(\alpha)$, $\mu^{MC}(\alpha)$, $\mu^{AC}(\alpha)$, and $r(\alpha)$, are increasing with respect to $\alpha$. When a firm increases the number of layers $q(\alpha)$, $\mu^{MC}(\alpha)$, $\mu^{AC}(\alpha)$, and $r(\alpha)$ increase discontinuously, while $p(\alpha)$ decreases discontinuously. Moreover, $\pi(\alpha)$ is continuous and strictly increasing with respect to $\alpha$.

**Proof.** see appendix. $\blacksquare$

### 2.2.4 Equilibrium

Prior to entering the market entrepreneurs pay a fixed cost $f_{E}$, in units of labor, to obtain a demand draw $\alpha$ from a known distribution $G(\alpha)$. This cost can be thought of as research and development costs that entrepreneurs incur before entering the market. The support of the distribution of $\alpha$ is $[\alpha_{M}, \infty]$. Under the assumption that $\alpha_{M}$ is small enough, there will always be demand draws that imply negative profits from operations, and so exit rates will always be positive.\footnote{In the simulations below, I assume that the distribution of demand draws is Pareto and equal to $G(\alpha) = 1 - \alpha^{k}$.} Therefore, given the mass of firms operating in the market, $M$, a demand draw $\alpha_{D}$ exists such that all entrepreneurs with draw $\alpha < \alpha_{D}$ choose not to
produce. For the marginal firm that is indifferent between entering the market and staying out, its profit is equal to zero. In other words, the zero profit condition is

$$\pi(\alpha_D, M) = 0. \quad (2.27)$$

For the entrepreneur earning zero profit, equation (2.27) also implies that the price of a unit supplied is equal to average costs. Further, the optimal quantity entrepreneur with demand draw $\alpha_D$ produces is the solution to the equation:

$$q_D = \frac{N}{2\gamma} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M} (\bar{x} - \bar{p}) - MC(q_D) \right\}, \quad (2.28)$$

which provides an expression for the term $\frac{\eta M}{\gamma + \eta M} (\bar{x} - \bar{p})$:

$$\frac{\eta M}{\gamma + \eta M} (\bar{x} - \bar{p}) = \alpha_D - MC(q_D) - \frac{2\gamma}{N} q_D. \quad (2.29)$$

Substituting (2.29) back into (2.26) yields an expression for the operating profits of a firm, conditional on successful entry, as a function of the parameters, the endogenously determined demand and quantity cutoffs, and the entrepreneur’s demand draw:

$$\pi(\alpha, \alpha_D, q_D) = \left[ \frac{2\gamma}{N} q_D + MC(q_D) - \alpha_D + \alpha - \frac{\gamma}{N} q(\alpha) \right] q(\alpha) - C(q(\alpha)). \quad (2.30)$$

Because the equilibrium involves solving for $\alpha_D$ and $q_D$, profits are also denoted as a function of the demand and quantity cutoffs. For the entrepreneur with demand draw $\alpha_D$, equation (2.27) is equal to:

$$\pi(q_D) = \frac{\gamma}{N} q_D^2 + MC(q_D)q_D - C(q_D). \quad (2.31)$$
A point to note is equation (2.31) is independent of $\alpha_D$. Therefore the solution to equation (2.31) is independent of the endogenously determined demand cutoff and the mass of firms operating in the differentiated goods sector.

The expected profits of entry, $V^e$, common to all producers, is equal to the expected profits before entrepreneurs know their actual demand minus the sunk entry cost. If profits were negative for all realizations of $\alpha$, no entrepreneur will enter the industry. As long as some firms produce, the expected profit of firms will be pushed down to zero by the unrestricted free entry of new firms. Accounting for entry costs, unrestricted free entry implies that expected firm profits, $V^e$, are zero. This yields the equilibrium free entry condition:

$$
\int_{\alpha_D} \pi(\alpha, \alpha_D, q_D) dG(\alpha) = f_E. \tag{2.32}
$$

where $\alpha_D$ is the demand draw from the marginal firm that is indifferent between entering and exiting the market and $q_D$ is quantity produced by the marginal firm. Because of equation (2.29), profits of a firm with demand draw $\alpha$ are a function of $\alpha_D$ and $q_D$. Given the solution to equation (2.31), equation (2.32) is only a function of the demand cutoff $\alpha_D$.

Finally, the mass of firms operating in equilibrium is equal to the mass of entrants times the probability of successful entry. Therefore, the mass of entrants is given by $M_E = M/(1 - G(\alpha_D))$.

The equilibrium is a set of values, $q_D$, $\alpha_D$, and $M$, that solve equations (2.29), (2.31) and (2.32), given the parameters of the model and the distribution of demand draws $G(\alpha)$.\footnote{By rewriting the model using equation (2.29), I am able to solve the model and obtain results without having to make any assumptions on the distribution of demand draws $G(\alpha)$.} I prove that:

**Proposition 2.4** If $\eta > \eta_\alpha$, then there exists a unique equilibrium.

**Proof.** see appendix. ■

Note that in equilibrium labor markets also clear. Labor is used for several purposes, as workers in the homogeneous sectors, as workers and managers in the differentiated good sector, as teachers, and to design new products. Let $H$ be the mass of workers in the
homogeneous good sector. As the total mass of agents in the economy is given by $N$, the labor market clearing condition is given by:

$$H + \frac{M}{1 - G(\alpha_D)} \left\{ f_E + \int_{\alpha_D} C(q(\alpha))dG(\alpha) \right\} = N. \quad (2.33)$$

The equilibrium is based on the assumption that consumers have a positive demand for the homogeneous good. Since consumers derive all of their income from their labor, I assume that consumers spend less than their unit of income on differentiated varieties. The assumption that $\eta > \eta$ ensures that this is the case. Namely, it guarantees that:

$$N - \frac{M}{1 - G(\alpha_D)} \left\{ f_E + \int_{\alpha_D} C(q(\alpha))dG(\alpha) \right\} > 0. \quad (2.34)$$

### 2.2.5 Comparative Statics with respect to Market Size

The comparative statics of interest is how a change in the size of the market, $N$, affects the decisions of firms. From the implicit function theorem it follows that:

$$\frac{\partial \alpha_D}{\partial N} = -\frac{\partial V^e / \partial N}{\partial V^e / \partial \alpha_D}. \quad (2.35)$$

I show in the appendix that the numerator of expression (2.35) is positive, while the denominator is negative. Hence an increase in the size of the market leads to an increase in the demand cutoff $\alpha_D$. Intuitively, when $N$ increases there will be two effects. First there will be the direct effect: holding the number of entrants fixed, when the size of the market increases, firms will increase their sales and profits. Secondly there will be an indirect effect: increased profits for entrants implies that potential profits increase as well, raising the expected value of entry. To preserve the equilibrium condition that $V^e = 0$, the term $\frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p})$ must rise, and under a restriction on the parameter $\eta$, $M$ must rise in order to lower expected profits.\(^{25}\) In equilibrium this lowers the demand for firms’ sales and raises

\(^{24}\)Note that in the differentiated good sector only the term $\eta M$ enter the equations.

\(^{25}\)Otherwise it may be the case that $(\bar{\alpha} - \bar{p})$ rises and $M$ decreases with $N$.\]
the bar for successful entry into the market, increasing the demand cutoff. Therefore, bigger markets induce tougher selection.\footnote{Note that in bigger markets, the term \( \frac{nM}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \) increases, inducing a decrease in \( p_{m, \alpha x} \) and an increase in the elasticity of demand, \( \epsilon_i \).} This result is summarized in the proposition below:

**Proposition 2.5** An increase in \( N \) induces an increase in \( q_D, \alpha_D \) and \( \frac{nM}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \). In addition, if \( \bar{\eta} > \eta \), then an increase in \( N \) induces an increase in \( M \).

**Proof.** see appendix. ■

To ensure that the mass of firms operating in equilibrium increases with the size of the market, I assume that the parameter \( \eta \) is bounded above by \( \bar{\eta} \).\footnote{I make this assumption to ensure that \( M \) will increase, however this assumption may be unnecessary. See the appendix for further details. Further in the appendix, I also show that there will always exist an \( \eta \) in the set \([\eta, \bar{\eta}]\).} Moreover, whether the number of varieties increases or decreases does not affect the remaining results.

Further, from equation (2.31), it follows that an increase in \( N \) also induces an increase in the quantity produced by the marginal firm, \( q_D \). One important point of note is that although the quantity produced by the marginal firm, \( q_D \) increases with \( N \), this does not imply that the firm with demand draw \( \alpha_D \) will increase its quantity in larger markets. At first glance, these statements may seem contradictory, however as I have formulated the general equilibrium of the model, I am solving for \( q_D \) and \( \alpha_D \) independently of one another. One way to think about this, is that there is a mapping between the set of demand draws, \( \alpha \)'s and the set of quantities, \( q \)'s, that depends on \( N \) and on the other parameters of the model, and I am solving for the lower bounds of both sets. How each individual producer is affected by a change in \( N \), however, depends on their demand schedule. As I discuss below, the response to bigger markets is heterogeneous across firms.

Having established the impact of an increase in market size on the aggregate parameters of the model, \( \alpha_D \) and \( M \), I now turn to individual producers. To demonstrate the results I turn to numerical simulations of the model. Figure 2.2 presents numerical simulations of two economies, one with \( N = 500 \) and the other with \( N = 1000 \), and where demand is drawn from a Pareto distribution with coefficient \( k = 3.95 \) and with support \([1, \infty]\), so \( G(\alpha) = 1 - \alpha^{-3.95} \). Panel (a) illustrates the quantities produced by firms. Firms with different demand draws respond differently to larger markets. An increase in the size of the market has two opposing effects on firms’ demand schedules. First, an increase in \( N \)
rotates firms’ demand curves outwards along the quantity-axis, increasing demand. Second, an increase in market size increases the term $\frac{nM}{\gamma+nM}(\bar{\tau} - \bar{p})$, which causes a downward shift in firms’ demand curves lowering demand. For firms with sufficiently high demand draws the first effect dominates and they are induced to produce more. For firms with low demand draws the second effect dominates inducing them to supply less quantity to the market.

Panels (b) and (c) illustrate the markups charged by firms. Firms in bigger markets charge lower markups. Because the term $\frac{nM}{\gamma+nM}(\bar{\tau} - \bar{p})$ increases with $N$, firms’ demand curves become more elastic. Consequently, for any quantity chosen, both firms’ markups over marginal costs and their markups over average costs decrease with $N$. This is true even for firms that change the number of layers in their organization. In addition the impact of lower markups implies that profit per unit decreases, and therefore to attain any given level profits firms will have to produce more output.\textsuperscript{28}

Panel (d) illustrates the prices charged by firms. Firms in larger markets charge lower prices. There are two factors that determine how the prices charged by firms change in larger markets. First, firms in bigger markets produce different quantities of output which alters their marginal costs. Firms that do not change their organizational structure and produce more output increase their marginal costs, while firms that produce less output and do not change their organization decrease their marginal costs. For firms that change their organization the opposite takes places. Since prices are a markup over marginal costs, holding markups constant an increase in marginal costs induces firms to charge higher prices, while a decrease in marginal costs induces firms to charge lower prices. Second in larger markets demand is more elastic and so markups over marginal costs decrease as demonstrated in panel (b). Overall the change in markups dominates any changes in marginal costs and prices decrease with $N$.

Panel (f) illustrates the profits earned by firms. Firms’ profits may decrease or increase with respect to the size of the market. There are two factors that determine how profits change in larger markets. First as demonstrated in panel (c), firms in larger markets charge lower markups over average costs which causes their profits to decrease. Second as illustrated in panel (a), the quantity produced by firms changes. Holding markups constant, an increase

\textsuperscript{28} To better understand this statement, consider the mapping between quantities and profits. In such a case, for any given level profit, in a bigger market because profits per unit produced are lower, a firm will have to produce more output.
in the quantity produced induces profits to increase. Therefore firms that reduce their quantity, profits decrease with \( N \), because both effects work in the direction to lower profits. In contrast firms that increase their quantity the effects work in the opposite direction and their profits may increase in larger markets. The reason is that although they charge a lower markup over average costs, firms that sufficiently increase their quantity, the second effect dominates, and their profits increase. For sufficiently large firms this is the case.

Withdrawing the Pareto assumption, the change in profits can more generally be described from firms’ demand schedules.

For a firm with demand draw \( \alpha \), its change in profits with respect to \( N \) is equal to:

\[
\frac{\partial \pi(\alpha)}{\partial N} = q(\alpha) \left[ -\frac{\partial \eta_M}{\partial N} \eta_M (\bar{\alpha} - \bar{p}) + \frac{\gamma}{N^2} q(\alpha) \right].
\]

The first term in parentheses in equation (2.36) is the result of a downward shift in firms’ demand caused by larger markets, and which leads to lower profits, while the second term, is the result of an outward rotation of firms’ demand schedules which increases profits. From equation (2.36), it follows that for firms producing a quantity greater than

\[
\left[ \frac{\eta_M}{\gamma + \eta_M} (\bar{\alpha} - \bar{p}) \right] \frac{N^2}{\gamma},
\]

the benefit from an outward rotation outweighs the loss from a downward shift in demand, and profits increase. In addition, because organizations with a greater number of layers produce a greater quantity, it follows from equation (2.36) that in larger markets profits rise faster for organizations producing with more layers.

Consider an entrepreneur with a demand draw \( \beta_{L,L+1} \) that is indifferent between two organizations. This entrepreneur’s profits satisfy the condition: \( \pi_L(\beta_{L,L+1}) = \pi_{L+1}(\beta_{L,L+1}) \). Because in bigger markets profits of organizations with more layers rise more rapidly, it follows that the distance between \( \beta_{L,L+1} \) and the marginal firm \( \alpha_D \), decreases with \( N \). Intuitively, bigger markets induce firms to lower their markups. At the same time firms have the option to change their organization. By adding a new layer of management, firms are paying higher “fixed” costs, and, if their output is large enough, they can produce at lower average costs, which allows them to increase their markups and their profits. Therefore, in bigger markets firms will they have an incentive to increase their number of layers so as to economize on their average costs. This result is summarized below:

\[29\] This analysis does not rely on the distribution of demand draws \( G(\alpha) \) to be Pareto.

\[30\] In other words, the slope of the profit function is steeper in larger market for firms producing with more layers.
Figure 2.2: The Impact of Market Size on Individual Producers. Blue (N = 500) Orange (N = 1000).
Proposition 2.6 The distance between the marginal entrepreneur with demand draw, $\alpha_D$, and the entrepreneur that is indifferent between two organizational forms, $\beta_{L,L+1}$, is decreasing with respect to $N$. In addition, the change in the distance between $\alpha_D$ and $\beta_{L,L+1}$, decreases with $L$.

Proof. see appendix. ■

An important point to note is that Proposition 2.6 does not imply that $\beta_{L,L+1}$ decreases with $N$.\textsuperscript{31} When $N$ increases, there are two effects that bring about the result in Proposition 2.6.\textsuperscript{32} First bigger markets induce tougher selection and the demand cutoff $\alpha_D$ increases. Second because firms’ demand schedules change with the size of the market, firms’ profits from producing with different organizations change, and the demand draw at which an entrepreneur is indifferent between two organizational forms changes as well. In the proof to Proposition 2.6, I provide a lower and an upper bound to the change in $\beta_{L,L+1}$ with respect to $N$. Both the lower and upper bounds may be positive or negative, and depend on the parameters of the model.

I now examine how the distribution of organizations changes with $N$. Let $\Lambda_N$ represent the discrete distribution of organizations in the economy. Relative to the marginal entrepreneur, Proposition 2.6 indicates that in bigger markets the demand draw at which an entrepreneur is indifferent between two organizational forms decreases. Alone, however, this result does not provide a ranking of distribution of organizations across locations. Under that assumption that the distribution of demand draws, $G(\alpha)$, has a non-decreasing hazard rate, I can rank that the distribution of layers across markets of different sizes.\textsuperscript{33} The following proposition contains the main theoretical result:

Proposition 2.7 Suppose $N' > N$. If the distribution of demand draws $G(\alpha)$ has a non-decreasing hazard rate, then the distribution of layers $\Lambda_{N'}$ first order stochastically dominates $\Lambda_N$.

Proof. see appendix. ■

\textsuperscript{31}Proposition 2.6 does not rely on the distribution of demand draws $G(\alpha)$ to be Pareto.

\textsuperscript{32}In addition, in the proof to Proposition 2.6 I have also shown that the quantity produced by entrepreneur $\beta_{L,L+1}$ with $L$ layers, $q_L(\beta_{L,L+1})$ decreases with $N$, while the quantity produced with $L + 1$ layers, $q_{L+1}(\beta_{L,L+1})$ increases with $N$.

\textsuperscript{33}The Pareto distribution, regularly used in the heterogeneous firm literature, always has a non-decreasing hazard rate.
Proposition 2.7 contains the main theoretical result of the paper. In the following section I empirically test the validity of this proposition, by comparing the distribution of organizations across locations within mainland France. In the final part of the paper I return to the model and conduct simulations to assess how the distributions of productivity and income differ across locations.

2.3 Empirical Analysis

2.3.1 Data Description

The first dataset used for this study is from mandatory reports collected by the French National Statistical Institute (INSEE) for the year 2002, the Déclarations Annuelles des Données Sociales (DADS). The DADS is an exhaustive cross-section of all workers who earn a positive wage in France. INSEE collects administrative information from all employers and self-employed workers within France. A report must be filled by each establishment for every one of their employees so there is a unique record for every establishment-employee pair. For every employee within an establishment there is information on the total number of hours worked, annual salary, and occupation. For every establishment there is also information on the region and industries in which it operates, as well as the parent firm of the establishment. For consistency with the model, I aggregate the data to the level of the firm. Hence there is a unique record for every firm-employee pair.

The second dataset used in this study are from customs declarations submitted to French Customs. At the eight-digit product level, for almost every French firm the customs data contains information on the annual shipments to each country, so for every exporter there is a unique record at the firm-country-product pair.\textsuperscript{34} From this dataset, for every firm I retain information on the total value of shipments, the number of products, and the number of countries exported to. In my analysis I augment the DADS dataset with export level controls because as shown in Caliendo, Monte and Rossi-Hansberg (2012) firms’ exporting decisions also affect their organizational structure.

For my analysis the geographical regions of interest are the 341 employment areas (zones

\textsuperscript{34}The almost is due to reporting thresholds for compulsory declarations. If a firms total exports to a country not in the European Union below 1000 euros or 1000 kg, then it does not need to complete a declaration report. For more information on the customs dataset see Crozet, Head, and Mayer (2012).
d’emploi) within mainland France. These areas correspond to a geographical space in which most of the inhabitants reside and work within the area, and in which firms can find most of the labor to meet their employment needs. Most employment areas correspond to a city and its surrounding area, or to a metropolitan area. Briant, Combes, and Lafoucade (2010) and Combes, Duranton and Gobillon (2008) provide further information on geographical areas in continental France.

### 2.3.2 Classification of Layers

In contrast to the number of theoretical papers, there has been limited empirical work examining the organizational structure of firms because of the difficulty in obtaining data. The richness of the DADS, however, allows me observe the hierarchy of firms and to separate firms into layers. In the model a layer corresponds to a set employees who earn similar wages, have similar knowledge and perform tasks at a similar level of authority (Caliendo, Monte and Rossi-Hansberg (2012)). This concept of a layer, therefore, is independent of the actual occupations of employees, such as whether they are lawyers or computer programmers. Instead it depends on their wages, knowledge, and position in a firm’s hierarchy.

To map this concept to the data and to separate firms into layers, I implement the empirical approach introduced by Caliendo, Monte and Rossi-Hansberg (2012). With this method, I can divide a firm into as many as four separate layers. I use one-digit occupational codes, ranging from 2 to 6, to classify employees into layers. Layer 1, the lowest layer in firms, corresponds to qualified and non-qualified administrative workers and blue-collar workers. It contains all workers with occupational codes 5 and 6, respectively. Layer 2 is composed of supervisors and individuals with a higher level of responsibility than ordinary workers, and contains all workers with an occupational code 4. Layer 3 is composed of senior directors and top management staff and contains all workers with an occupational code 3. Layer 4 corresponds to owners who receive a wage and CEOs. It contains all workers with occupational code 2. Caliendo, Monte and Rossi-Hansberg (2012) provide evidence that this classification is meaningful and consistent with the theoretical concept of layer.

For every firm I then count the total number of layers. A firm has a layer if there is at

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35In the data, occupational codes range from 1 to 6. Because I remove all firms operating in the agricultural and fishing industries, workers with occupational code 1 are removed from the data.
least one agent with the corresponding occupational code employed in the firm. Of course, not all firms have an employee in every layer. As many firms do not employ agents in every layer, there are four types of organizational structures in the data: firms with one layer, firms with two layers, firms with three layers, and firms with four layers.

Because the model is concerned with single product firms operating in a market, I retain all firms with establishments that operate in the same employment area and industry, in all manufacturing sectors and in business services. For my baseline estimates I take a strict interpretation of the model and retain all firms with employees in adjacent layers starting from layer 1. I also remove firms with one employee or less. For the year 2002, I end up with data on 157,444 firms. Of the 157,444 firms in the data, 16,987 have four layers, 41,646 have three layers, 41,496 have two layers, and 57,405 are one-layer firms.

2.3.3 Measure of Market Size: Demand Density

The exogenous variable in the model is the size of the market, \( N \). I measure this empirically using the density of an employment area in the year 1999, which is defined as the total population residing in an employment area divided by an employment area’s surface measured in kilometers squared. Population and land areas are provided by INSEE.

2.3.4 Descriptive Statistics

Table 2.1 presents summary statistics of firms operating in all of France and across employment areas. The average number of layers in a firm is 1.113, the average number of workers in a firm is 16.20 and the average hourly wage is 11.83. Table 2.1 also groups employment areas by their density and reports summary statistics across the different quartiles of the density distribution. In denser markets firms tend to employ more workers and pay higher hourly wages. For example, the average number of workers in firms located in employment areas that are in the first quartile of the density distribution is 15.27, the average number of workers in firms located in employment areas that are in the second quartile is 15.86, while in the third and fourth quartiles the average number of workers is 16.37, and 16.43.

36The industrial classification that I use is the NES 114 French classification provided by INSEE.
37Many researchers related density to the size of a market, such as Ciccone and Hall (1996), Campbell and Hopenhayn (2005), Sato, Tabuchi, and Yamamoto (2012) and Combes, Duranton, Gobillon, Puga and Roux (2012).
In addition, firms located in denser markets tend to have more organizational layers. In the first quartile of the density distribution, the average number of layers is 0.846, in the second quartile it is 0.932, while in the third and fourth quartiles it is 1.052, and 1.271. Therefore, from Table 2.1 firms in denser markets are on average bigger, pay higher wages, and have more layers in their organization.

Table 2.2 lumps employment areas together based on their employment density and compares the distribution of organizations in employment areas across the different quartiles of the density distribution. From Table 2.2 it is apparent that in denser markets the distribution of organizations has a greater percentage of firms with a greater number of layers. Comparing the distribution of organizations from the first and fourth quartile, for example, it is apparent that in the fourth quartile there is a larger percentage of firms with three and four layers in their organization than in the first quartile, while the first quartile has a larger percentage of two-layer and one-layer firms. These results suggest that as the density of an employment area increases firms have a tendency to increase the number of layers in their organization, consistent with the model’s prediction from Section 2.2.
2.3.5 Empirical Results

To summarize the discussion from Section 2.2 the model implies that in bigger markets the distribution of organizations should first order stochastically dominate the distribution of organizations in smaller markets. To test this implication I proceed in four steps. First, at the firm level, I regress the total number of layers in firms on measures of density. Second, for every market I compute the percentage of firms with one, two, three and four layers, and regress this measure on measures of density of the market. Third, I group employment areas based on their employment density and compare the distribution of organizations in employment areas with above-median density and below-median employment density, as well as the distribution of organizations of employment areas in the first and fourth quartiles of the density distribution. To compare these distributions, I apply the Mann-Whitney stochastic dominance test. And fourth, as a robustness check, I carry out the same analysis on firms operating in the ready-mixed concrete industry.

Firm Level Regressions

I first examine how market size affects the organization of firms. Because in most industries the market is determined at the national or global level, the goal of this section is to show that local markets affect the organization of firms. Namely, for firm $i$ located in employment area $a$, I estimate the following equation:

$$org_{i(a)} = \alpha + \gamma \log density_a + X_i \beta + Z_a \zeta + \epsilon_{i(a)},$$

(2.37)

where $org_{i(a)}$ is the total number of layers in firm $i$ located in employment area $a$, $density_a$ is the density of employment area $a$ in the year 1999, measured as total population divided by area in $km^2$, and $X_i$ are additional firm controls, such as the size of the firm, the industry the firm operates in, and whether the firm is an exporter, and $Z_a$ are employment area controls, such as market potential and average hourly wage excluding the return to skills in an employment area. In the analysis, I include market potential, defined as the density of surrounding markets as a control, because the demand from surrounding markets may have
an impact on the organization of firms.\textsuperscript{38} I also include the average hourly wage excluding skills as a control because in the model wages are normalized to one across markets, and so markets affect the organization of firms through demand only.\textsuperscript{39} Essentially, this variable controls for any differences in the costs to hiring workers across locations that are unrelated to skills. I exclude skills from wages because in the model wages are endogenous. Finally, I also include export level controls because as shown in Caliendo, Monte and Rossi-Hansberg (2012) exporters tend to have more layers in their organization.

Table 2.3 reports the results. Column 1 presents the results of a simple specification with local density as a control. In column 2, I cluster standard errors at the employment area level and include industry fixed effects. The regressions indicate that within industries, a one unit increase in the density of an employment area is associated with an average increase of 4.8 percent layers in firms. In columns 3 to 5, I estimate equation (2.37) with market potential and average hourly wage excluding the returns to skills as controls. In all three columns local density continues to have a positive impact on the organization of firms, however, when market potential is introduced as a control, the magnitude of local density is reduced by half.

In column 6, I control for the size of firms and the coefficient on local density remains positive and significant. Although in the model market size affects the quantity produced by firms, and there is a one to one mapping between quantity and the size of a firm, and the size of a firm and its organization, in column 6 local density continues to have a positive influence on the organization of firms. In part this may be due to differences in the relationship between size and the organization of firms across industries, or because firms are producing vertically differentiated products. As reported further below, in the

\textsuperscript{38}Market potential of an employment area $a$ is equal to:

\[
marketpotential_a = \sum \frac{density_{a'}}{distance_{a,a'}}.
\]

\textsuperscript{39}To obtain an estimate of average hourly wages excluding skills I do the following. First using the panel of all workers born in October, for the years 2002 to 2007 I estimate the following equation:

\[
\ln wage_{it} = \alpha_0 + \alpha_1 age_i + \alpha_2 age_i^2 \cdot gender_i + \alpha_3 age_i^3 \cdot gender_i + \alpha_4 age_i^4 \cdot gender_i + ind_j \cdot occ_i + time_t + \epsilon_{it}
\]

where $wage_{it}$ is the hourly wage of worker $i$ in year $t$, $gender_i$ is the gender of worker $i$, $ind_j$ are industry fixed effect, $occ_i$ are occupation fixed effects and $time_t$ are time fixed effects. Second, from this equation I retain the residual and calculate the mean residuals across years and employment areas.
ready-mixed concrete industry this is not the case. Once firms’ size is added as a control, local density does not affect the number of layers in firms.

In column 7, I control for the exporting status of a firm, while column 8 includes controls for the number of countries a firm exports to and the average value of exports per country. In both cases, the impact of local density on the organization of firms remains positive and significant. Finally, in column 9 I include all controls. Local density continues to have a positive effect on the number of layers in firms. The result indicates that a unit increase in density corresponds to a 2.2 percent increase the number of layers in firms. Therefore, local density is an important determinant of the organization of firms. As predicted by the model, firms in denser markets tend to have more layers.

**Employment Area Regressions**

Having established the size of the local market is an important determinant of the organization of firms, I now analyze the distribution of organizations across locations. Because their interpretation is intuitive and straightforward, I begin with a simple linear regression models on the data. Namely, for employment area $a$, I estimate the following equation:

$$p^L_a = \alpha + \gamma \log \text{density}_a + X_a \beta + \epsilon_a,$$

where $p^L_a$ is the percentage of firms in employment area $a$ producing with $L$ layers, $\text{density}_a$ is the local density of employment area $a$ in the year 1999, measured as total population divided by area. According to the model presented above, the coefficient $\gamma$ should be negative for lower values of $L$ and positive for higher values.

Table 2.4 reports the results. Each entry in the table reports results from a separate regression. Column 1 reports results when the dependent variable is the percentage of firms producing with one layer, while Columns 2, 3 and 4 report regressions results when the dependent variable is the proportion of firms producing with two, three, and four layers respectively. In Table 2.4 all standard errors are robust.

Table 2.4 also reports results from different specifications. Model 1 regresses the percentage of firms with a given number of layer on the local density of employment area, while model 2 controls for the market potential of an employment area, model 3 controls for the average hourly wage excluding skills, and model 4 controls for both market potential and
Table 2.3: Firm-Level Regression Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>log density</td>
<td>0.077***</td>
<td>0.048***</td>
<td>0.046***</td>
<td>0.024***</td>
<td>0.022***</td>
<td>0.023***</td>
<td>0.017***</td>
<td>0.017***</td>
<td>0.022***</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>log market potential</td>
<td>0.073***</td>
<td>0.074***</td>
<td>0.034***</td>
<td>0.069***</td>
<td>0.069***</td>
<td>0.035***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.011)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log firm size</td>
<td>0.568***</td>
<td>0.534***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.003)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average hourly wage</td>
<td>5.94</td>
<td>6.94</td>
<td>3.992</td>
<td>8.150**</td>
<td>7.820**</td>
<td>4.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4.265)</td>
<td>(4.011)</td>
<td>(3.092)</td>
<td>(3.717)</td>
<td>(3.669)</td>
<td>(2.964)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exporter</td>
<td>0.920***</td>
<td>0.758***</td>
<td>0.328***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.027)</td>
<td>(0.029)</td>
<td>(0.009)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of countries</td>
<td>0.024***</td>
<td>-0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average value per country</td>
<td>0.005***</td>
<td>-0.004***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Industry FE | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
Clustered S.E. | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
Sample Size | 157,444 | 157,444 | 157,444 | 157,444 | 157,444 | 157,444 | 157,444 | 157,444 | 157,444 |

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Standard errors reported in parentheses. OLS regression results for equation (2.37). Each column displays the estimate from a separate regression. Density measures the local density of an employment area. Market potential of an employment area measures the density of surrounding employment areas, and average hourly wage is the hourly wage excluding skills. Exporter is an indicator variable for whether firms export in the year 2002, number of countries measures the number of countries a firm exports to, and average value per country measure the average value of shipments to a given country.
average hourly wages.

From Table 2.4 a pattern emerges from the data that is consistent with the model. In column 1 the reported estimates of $\gamma$ are negative and significant in all of the specifications, indicating that in denser markets there are less firms producing with only one layer. This result holds even after adding market potential and the hourly wage as controls. Column 2 reports that the percentage of firms producing with two layers may either increase or decrease with the density of a local market, however the coefficient $\gamma$ is only significant at the ten percent level when it is positive. Finally, columns 3 and 4 report that in denser markets there is a greater percentage of firms producing with three and four layers. These results hold even after adding market potential and the hourly wage as controls. Taken together, the regressions in model 4 indicate that a one unit increase in the density of an employment area is associated with a 3.3 percent decrease in the proportion of one-layer firms, a 0.2 percent increase in the proportion of two-layer firms, a 1.9 percent increase in the proportion of three-layer firms and a 0.5 percent increase in the proportion of four-layer firms.

Within each model in Table 2.4, it is important to note the estimated coefficients of $\gamma$ never change from a positive value to a negative value. Firms in denser markets are less likely to be one-layer firms and more likely to be two, three, or four-layer firms. These findings therefore suggest that the distribution of organizations in denser markets first order stochastically dominates the distribution of organizations in less dense markets.

**Ordinal Stochastic Dominance Tests**

In this section I formally test whether the distribution of organization in denser markets first order stochastically dominates the distribution in less dense markets, using the Mann-Whitney test. The underlying hypothesis of the test is that both distributions are the same, while the alternative is that one distribution has systematically larger values than the other. I conduct this test for all firms in the sample and separately for each industry. Table 2.5 reports the results.

Columns 1 and 2 in Table 2.5 compare the distribution of organizations in employment areas with above-median and below-median employment density, while Column 3 and 4 compare the distribution of organizations in employment areas in the first and fourth quar-
Table 2.4: Employment Area Regressions

<table>
<thead>
<tr>
<th>Model</th>
<th>Employment Area Level</th>
<th>Percent</th>
<th>Percent</th>
<th>Percent</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>log density</td>
<td>-0.042***</td>
<td>-0.001</td>
<td>0.025***</td>
<td>0.010***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>log density</td>
<td>-0.033***</td>
<td>0.002*</td>
<td>0.020***</td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>log market potential</td>
<td>-0.034***</td>
<td>-0.015***</td>
<td>0.021***</td>
<td>0.017***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td></td>
<td>log density</td>
<td>-0.041***</td>
<td>-0.001</td>
<td>0.025***</td>
<td>0.009***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>-1.875</td>
<td>0.195</td>
<td>1.667</td>
<td>1.208*</td>
</tr>
<tr>
<td></td>
<td>hourly wage</td>
<td>(2.028)</td>
<td>(0.946)</td>
<td>(1.112)</td>
<td>(0.713)</td>
</tr>
<tr>
<td></td>
<td>log density</td>
<td>-0.033***</td>
<td>0.002*</td>
<td>0.019***</td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td></td>
<td>log market potential</td>
<td>-0.034***</td>
<td>-0.015***</td>
<td>0.021***</td>
<td>0.017***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.006)</td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.002)</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td>-2.096</td>
<td>0.095</td>
<td>1.807</td>
<td>1.322**</td>
</tr>
<tr>
<td></td>
<td>hourly wage</td>
<td>(1.965)</td>
<td>(0.966)</td>
<td>(1.131)</td>
<td>(0.658)</td>
</tr>
<tr>
<td>Sample Size</td>
<td></td>
<td>341</td>
<td>341</td>
<td>341</td>
<td>341</td>
</tr>
</tbody>
</table>

Notes: *** \( p < 0.01 \), ** \( p < 0.05 \), * \( p < 0.1 \). Robust standard errors reported in parentheses. OLS regression results for equation (2.38). Each column displays the estimate from a separate regression. Density measures the local density of an employment area. Market potential of an employment area measures the density of surrounding employment areas and average hourly wage is the hourly wage excluding skills.
tiles of the employment areas distribution. Across all industries the results in Table 2.5 indicate that the distribution of organizations in denser areas first order stochastically dominates the distribution in less dense areas. As one would expect the findings are stronger for comparisons between employment areas in the first and fourth quartiles of the density distribution. The probability that a random draw of an organization from employment areas with below-median density is greater than a random draw of an organization from employment areas with above-median is 0.419, while the corresponding value for the comparison between the first and fourth quartiles is 0.383.

Furthermore, comparisons of the distribution of organizations within industries yield similar conclusions. There are four industries, however, that yield different results: the apparel and leather industry, the pharmaceutical, perfume and soap industry, the ships, aircraft and railroad equipment industry and the textile industry. In the latter three industries, the Mann-Whitney test cannot reject the null hypothesis that the distributions are equal, while for the apparel and leather industry, the test indicates that the opposite is taking place: the distribution of organizations is less dense areas first order stochastically dominates the distribution in denser areas. Despite this case, there is overwhelming evidence to suggest that the evidence is consistent with the model and distribution of organization in denser markets first order stochastically dominates the distribution in less dense markets.

Table 2.6 conducts the same test, but controlling for the impact market potential has on the organization of firms.\textsuperscript{40} The findings are a bit weaker than those reported Table 2.5, however the results are fairly consistent with the model. The distribution of organization in denser markets first order stochastically dominates the distribution in less dense markets.

\textbf{Robustness Check: Ready-Mixed Concrete Industry}

The important exogenous variable in the model is size of the domestic market which affects firms’ demand. One concern with the empirical analysis thus far, is that most industries produce tradeable goods, and because the model is concerned with the impact the domestic market has on the organization of firms, one can argue that in most industries since the market is defined at the national or global level, the findings are not conclusive. To account for

\textsuperscript{40}Namely, at the firm-level I estimate equation (2.37) with local density, market potential and industry fixed effects as controls. Then for every firm, I remove the estimated impact market potential has on their organization.
Table 2.5: Mann-Whitney Distributions Test

<table>
<thead>
<tr>
<th>Industry</th>
<th>$H_o$ Distributions are Equal</th>
<th>Probability</th>
<th>$H_o$ Distributions are Equal</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL Industries</td>
<td>0.000</td>
<td>0.419</td>
<td>0.000</td>
<td>0.383</td>
</tr>
<tr>
<td>Food, beverages, tobacco</td>
<td>0.000</td>
<td>0.412</td>
<td>0.000</td>
<td>0.489</td>
</tr>
<tr>
<td>Apparel, leather</td>
<td>0.000</td>
<td>0.548</td>
<td>0.004</td>
<td>0.546</td>
</tr>
<tr>
<td>Publishing, printing, recorded media</td>
<td>0.000</td>
<td>0.400</td>
<td>0.000</td>
<td>0.359</td>
</tr>
<tr>
<td>Pharmaceuticals, perfumes, soap</td>
<td>0.178</td>
<td>0.465</td>
<td>0.257</td>
<td>0.452</td>
</tr>
<tr>
<td>Domestic appliances, furniture</td>
<td>0.001</td>
<td>0.473</td>
<td>0.000</td>
<td>0.457</td>
</tr>
<tr>
<td>Motor vehicles</td>
<td>0.034</td>
<td>0.449</td>
<td>0.000</td>
<td>0.408</td>
</tr>
<tr>
<td>Ships, aircraft, railroad equipment</td>
<td>0.967</td>
<td>0.501</td>
<td>0.694</td>
<td>0.518</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.000</td>
<td>0.416</td>
<td>0.000</td>
<td>0.371</td>
</tr>
<tr>
<td>Electric and electronic equipment</td>
<td>0.000</td>
<td>0.426</td>
<td>0.000</td>
<td>0.403</td>
</tr>
<tr>
<td>Building materials, glass products</td>
<td>0.000</td>
<td>0.438</td>
<td>0.000</td>
<td>0.404</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.322</td>
<td>0.488</td>
<td>0.169</td>
<td>0.471</td>
</tr>
<tr>
<td>Wood, paper</td>
<td>0.000</td>
<td>0.449</td>
<td>0.000</td>
<td>0.416</td>
</tr>
<tr>
<td>Chemicals, rubber, plastics</td>
<td>0.000</td>
<td>0.460</td>
<td>0.000</td>
<td>0.435</td>
</tr>
<tr>
<td>Basic metals, metal products</td>
<td>0.000</td>
<td>0.434</td>
<td>0.000</td>
<td>0.414</td>
</tr>
<tr>
<td>Electric and electronic components</td>
<td>0.038</td>
<td>0.467</td>
<td>0.488</td>
<td>0.482</td>
</tr>
<tr>
<td>Consultancy, advertising, business services</td>
<td>0.000</td>
<td>0.410</td>
<td>0.000</td>
<td>0.386</td>
</tr>
</tbody>
</table>

Notes: Results of Mann-Whitney stochastic dominance test. Columns 1 and 3 report the p-values of the test. Column 2 reports the probability that a random draw of an organization from employment areas with below-median density is greater than a random draw of an organization from employment areas with above-median density. Column 4 reports the probability that a random draw of an organization from employment areas in the first quartile of the density distribution is greater than a random draw of an organization from employment areas with fourth quartile of the density distribution.
Table 2.6: Mann-Whitney Distributions Test

<table>
<thead>
<tr>
<th>Distribution of Organizations Below Med</th>
<th>Probability</th>
<th>Distribution of Organizations Above Med</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0 are Equal</td>
<td>H0 are Equal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL Industries</td>
<td>0.000</td>
<td>0.459</td>
<td>0.000</td>
</tr>
<tr>
<td>Food, beverages, tobacco</td>
<td>0.000</td>
<td>0.413</td>
<td>0.000</td>
</tr>
<tr>
<td>Apparel, leather</td>
<td>0.000</td>
<td>0.461</td>
<td>0.000</td>
</tr>
<tr>
<td>Publishing, printing, recorded media</td>
<td>0.000</td>
<td>0.450</td>
<td>0.000</td>
</tr>
<tr>
<td>Pharmaceuticals, perfumes, soap</td>
<td>0.562</td>
<td>0.516</td>
<td>0.565</td>
</tr>
<tr>
<td>Domestic appliances, furniture</td>
<td>0.646</td>
<td>0.503</td>
<td>0.089</td>
</tr>
<tr>
<td>Motor vehicles</td>
<td>0.230</td>
<td>0.478</td>
<td>0.087</td>
</tr>
<tr>
<td>Ships, aircraft, railroad equipment</td>
<td>0.880</td>
<td>0.504</td>
<td>0.901</td>
</tr>
<tr>
<td>Machinery</td>
<td>0.000</td>
<td>0.437</td>
<td>0.000</td>
</tr>
<tr>
<td>Electric and electronic equipment</td>
<td>0.000</td>
<td>0.464</td>
<td>0.010</td>
</tr>
<tr>
<td>Building materials, glass products</td>
<td>0.002</td>
<td>0.471</td>
<td>0.000</td>
</tr>
<tr>
<td>Textiles</td>
<td>0.000</td>
<td>0.445</td>
<td>0.000</td>
</tr>
<tr>
<td>Wood, paper</td>
<td>0.009</td>
<td>0.477</td>
<td>0.015</td>
</tr>
<tr>
<td>Chemicals, rubber, plastics</td>
<td>0.050</td>
<td>0.480</td>
<td>0.083</td>
</tr>
<tr>
<td>Basic metals, metal products</td>
<td>0.000</td>
<td>0.449</td>
<td>0.000</td>
</tr>
<tr>
<td>Electric and electronic components</td>
<td>0.805</td>
<td>0.504</td>
<td>0.151</td>
</tr>
<tr>
<td>Consultancy, advertising, business services</td>
<td>0.000</td>
<td>0.467</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: Results of Mann-Whitney stochastic dominance test. The results are obtained from removing the impact market potential has on the organization of firms. Columns 1 and 3 report the p-values of the test. Column 2 reports the probability that a random draw of an organization from employment areas with below-median density is greater than a random draw of an organization from employment areas with above-median density. Column 4 reports the probability that a random draw of an organization from employment areas in the first quartile of the density distribution is greater than a random draw of an organization from employment areas with fourth quartile of the density distribution.

Table 2.7: Distribution of Organizations in Ready-Mixed Concrete Industry Across Employment Areas

<table>
<thead>
<tr>
<th>Number of Firms</th>
<th>One Layers</th>
<th>Two Layers</th>
<th>Three Layers</th>
<th>Four Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>917</td>
<td>0.381</td>
<td>0.283</td>
<td>0.252</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>202</td>
<td>0.504</td>
<td>0.267</td>
<td>0.183</td>
</tr>
<tr>
<td>2nd Quartile</td>
<td>215</td>
<td>0.372</td>
<td>0.413</td>
<td>0.186</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>204</td>
<td>0.333</td>
<td>0.215</td>
<td>0.323</td>
</tr>
<tr>
<td>4th Quartile</td>
<td>296</td>
<td>0.337</td>
<td>0.246</td>
<td>0.300</td>
</tr>
</tbody>
</table>
Table 2.8: Distribution of Organizations in Ready-Mixed Concrete Industry Across Urban Areas

<table>
<thead>
<tr>
<th></th>
<th>Number of Firms</th>
<th>One Layer</th>
<th>Two Layers</th>
<th>Three Layers</th>
<th>Four Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>919</td>
<td>0.380</td>
<td>0.284</td>
<td>0.252</td>
<td>0.082</td>
</tr>
<tr>
<td>1st Quartile</td>
<td>409</td>
<td>0.447</td>
<td>0.303</td>
<td>0.232</td>
<td>0.017</td>
</tr>
<tr>
<td>2nd Quartile</td>
<td>134</td>
<td>0.335</td>
<td>0.350</td>
<td>0.216</td>
<td>0.097</td>
</tr>
<tr>
<td>3rd Quartile</td>
<td>161</td>
<td>0.329</td>
<td>0.260</td>
<td>0.285</td>
<td>0.124</td>
</tr>
<tr>
<td>4th Quartile</td>
<td>215</td>
<td>0.320</td>
<td>0.223</td>
<td>0.288</td>
<td>0.167</td>
</tr>
</tbody>
</table>

In this section, I adopt the approach of Syverson (2004) and examine the distribution of organizations in the ready-mixed concrete industry (NAF 266E). The ready-mixed concrete industry is a suitable industry to test the model’s predictions, because the industry’s high transport costs ensure that firms mostly supply concrete to surrounding areas, and so the ready-mixed market is not a single national or global unit but instead a collection of local geographic markets (Syverson (2008)).

In the ready-mixed concrete industry, the measure of the size of the domestic market is determined by the construction sector. To measure the size of the construction sector, I use the number of construction workers per market area which I obtain from the exhaustive cross-section of the DADS (NAF 252). Further, as argued by Syverson (2004), the size of the construction sector is an appropriate measure of density and is exogenous for the following reasons. First, the construction sector purchases most of the ready-mixed concrete industry’s output. Second, because ready-mixed concrete accounts for a small percentage of the construction sector’s costs, causation travels from construction demand to concrete competitiveness and not in the other direction. Therefore the measure of density in the following regressions is not only exogenous but it directly relates to the model. And third, because ready-mixed concrete is a relatively small industry, it generally has a small effect on the employment area and therefore, any employment area variables can be considered exogenous.

Further in a given year, as there are few observations in a single geographical market, I make the following changes to the dataset. First, I extend the dataset to the years 2002 to 2007. As I can observe a firm for multiple years, this has the added advantage that I can also control for attributes unique to firms. And second, I extent the analysis and also study the relationship of interest at a broader geographical level, at the urban area level.
Urban areas only cover part of mainland France and correspond to metropolitan areas. In addition, because firms operate in more than one industry, I remove firms with less than 75% of their workforce employed in the ready-mixed concrete sector.\textsuperscript{41}

For completeness, Tables 2.7 and 2.8 report the distribution of organizations across employment and urban areas. The tables suggest that in denser areas there is a greater mass of firms that produce with more layers.

Furthermore, Table 2.9 reports regression results from equation (2.37). In these regressions, there are no controls for market potential or the exporting status of firms because local density is defined by the size of the construction sector and firms do not export ready-mixed concrete. Column 1 presents the results of a simple specification with local density defined at the employment area level while column 2, includes year fixed effects. Within years, the regressions indicate that a one unit increase in the density of an employment area is leads to an average increase of 14.7 percent layers in firms. In columns 3 I control for the average hourly wage excluding skills.\textsuperscript{42} The results remains the same.

In Columns 3, 4 and 5, I estimate equation (2.37) with local density defined at the urban level. In all three regressions in denser urban areas ready-mixed concrete firms have more layers. Within years, the results indicate that a one unit increase in the density of an urban area leads to an average increase of 20.2 percent layers in firms.

Columns 6 and 7 control for the size of firms. Controlling for firms’ size, there is no relationship between local density and organization, consistent with the model. In the model firms’ cost schedules are the same across locations, and hence market size only affects the decision of firms by changing their demand schedules. In addition, in the model there is a one to one relationship between quantity and the size of firms and between firms’ size and their organization. In other words, once controls for the size of firms are included in regression (2.37) there should be no independent variation in the size of the market that affects the number of layers in firms. This is reported in columns 6 and 7.\textsuperscript{43} Finally, columns 8 and 9 examine whether this also holds within firms. Controlling for firms’ size, there is

\textsuperscript{41}There are about 40 firms that change markets in the years 2002 to 2007. As this paper is not about the location decisions of firms, I also exclude these firms from the dataset.

\textsuperscript{42}Because I focus on the ready-mixed concrete industry, and average hourly wage excluding skills is supposed to identify any additional costs associated with hiring workers across employment areas, when I calculate the average hourly wage excluding skills, I remove all workers that are employed in the ready-mixed concrete sector. This ensures that average hourly wage excluding skills is exogenous in the regressions.

\textsuperscript{43}See the proof of proposition 2.6.
Table 2.9: Ready-Mixed Concrete Results

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment Area:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log density</td>
<td>0.144**</td>
<td>0.147*</td>
<td>0.0147*</td>
<td>0.052</td>
<td>−0.103</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.076)</td>
<td>(0.078)</td>
<td>(0.048)</td>
<td>(0.400)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Urban Area:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log density</td>
<td></td>
<td></td>
<td></td>
<td>0.200***</td>
<td>0.202***</td>
<td>0.179***</td>
<td>0.040</td>
<td>0.122</td>
<td></td>
<td>0.343***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.059)</td>
<td>(0.058)</td>
<td>(0.060)</td>
<td>(0.046)</td>
<td>(0.437)</td>
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</tr>
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<td>0.695***</td>
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<td>(0.068)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.068)</td>
</tr>
<tr>
<td>average</td>
<td>−0.591</td>
<td></td>
<td></td>
<td>43.452</td>
<td>17.361</td>
<td>32.896</td>
<td>−11.929</td>
<td>−27.511</td>
<td></td>
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<tr>
<td>Year FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Area FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
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<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
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<td>No</td>
<td>No</td>
<td>Yes</td>
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<td>919</td>
<td>919</td>
<td>917</td>
<td>919</td>
<td>917</td>
<td>919</td>
</tr>
</tbody>
</table>

Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Standard errors reported in parentheses. OLS regression results for equation (2.37). Each column displays the estimate from a separate regression. Density measures the local size of the construction sector. Average hourly wage is the hourly wage excluding skills calculated excluding workers in the ready-mixed concrete industry.
no relationship between local density and organization.

The results reported in Table 2.9, are therefore consistent with the model. Local density is an important factor influencing the organization of firms. Firms in the ready-mixed concrete industry operating in denser markets have more layers. In addition, the evidence suggests that density is affecting the organizational decisions of firms through demand, the same mechanism in the model.

Table 2.9 reports regression results of equation (2.38). Each entry in the table reports results from a separate regression. Column 1 reports results when the dependent variable is the percentage of firms producing with one layer, while Columns 2, 3 and 4 report regressions results when the dependent variable is the proportion of firms producing with two, three, and four layers respectively. In Table 2.4 all standard errors are robust.

In Table 2.10 a similar pattern emerges as in Table 2.4. In denser markets there are less firms producing with one and two layers, while there are more firms producing with three and four layers. The results remain the same even after controlling for the average hourly wage excluding skills across employment and urban areas.

Finally, Table 2.11 reports stochastic dominance test of the distribution of organization across employment and urban areas. In all four cases, the results indicate that the distribution of organizations in denser areas first order stochastically dominates the distribution in less dense areas. The Mann-Whitney test rejects the hypothesis that the distributions are the same at the one percent level, and always indicates that the probability of a random draw of an organization from denser employment areas is greater than a random draw from less dense employment areas. These findings are consistent with Proposition 2.7.

### 2.4 Numerical Simulations

In this section I return to the model and present numerical simulations of two economies of different sizes, one with $N = 500$ and the other with $N = 1000$, and where demand is drawn from a Pareto distribution with coefficient $k = 3.95$ and with support $[1, \infty]$, so $G(\alpha) = 1 - \alpha^{-3.95}$.\(^{44}\) The complete set of parameters chosen for each model are listed in Table 2.12. Because the objective of this study is to investigate how market size affects

\(^{44}\)The parameter $k$ is chosen to relatively large for computational tractability.
### Table 2.10: Ready-Mixed Concrete Results

<table>
<thead>
<tr>
<th></th>
<th>Percent 1 Layers</th>
<th>Percent 2 Layers</th>
<th>Percent 3 Layers</th>
<th>Percent 4 Layers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 1: Employment Area</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log density</td>
<td>−0.014</td>
<td>−0.049**</td>
<td>0.022</td>
<td>0.042**</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.016)</td>
</tr>
<tr>
<td><strong>Model 2: Employment Area</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log density</td>
<td>−0.012</td>
<td>−0.056**</td>
<td>0.025</td>
<td>0.014**</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>average</td>
<td>−11.475</td>
<td>−28.083*</td>
<td>−12.008</td>
<td>−4.599</td>
</tr>
<tr>
<td>hourly wage</td>
<td>(19.918)</td>
<td>(14.862)</td>
<td>(15.528)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample Size</td>
<td>613</td>
<td>613</td>
<td>613</td>
<td>613</td>
</tr>
</tbody>
</table>

**Model 1: Urban Area**

|                     |                  |                  |                  |                  |
| log density         | −0.012           | −0.052           | 0.009            | 0.056**          |
|                     | (0.050)          | (0.039)          | (0.047)          | (0.025)          |

**Model 2: Urban Area**

|                     |                  |                  |                  |                  |
| log density         | −0.016           | −0.053           | 0.012            | 0.057**          |
|                     | (0.050)          | (0.039)          | (0.048)          | (0.025)          |
| average             | −23.387          | 29.654           | −11.133          | 4.866            |
| hourly wage         | (19.819)         | (15.541)         | (19.134)         | (9.183)          |
| Year FE             | Yes              | Yes              | Yes              | Yes              |
| Sample Size         | 433              | 433              | 433              | 433              |

*Notes:*** p < 0.01, ** p < 0.05, * p < 0.1. Robust standard errors reported in parentheses. OLS regression results for equation (2.38). Each column displays the estimate from a separate regression. Density measures the size of the construction sector. Average hourly wage is the hourly wage excluding skills calculated excluding workers in the ready-mixed concrete industry.*

### Table 2.11: Mann-Whitney Distributions Test - Ready-Mixed Concrete Results

<table>
<thead>
<tr>
<th></th>
<th>Probability</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H_0) Distributions are Equal</td>
<td>(H_0) Distributions are Equal</td>
<td>(H_0) Distributions are Equal</td>
</tr>
<tr>
<td>Employment Area:</td>
<td>0.000</td>
<td>0.396</td>
</tr>
<tr>
<td>Urban Area:</td>
<td>0.000</td>
<td>0.404</td>
</tr>
</tbody>
</table>

*Results of Mann-Whitney stochastic dominance test. Columns 1 and 3 report the p-values of the test. Column 2 reports the probability that a random draw of an organization from employment areas with below-median density is greater than a random draw of an organization from employment areas with above-median density. Column 4 reports the probability that a random draw of an organization from employment areas in the first quartile of the density distribution is greater than a random draw of an organization from employment areas with fourth quartile of the density distribution.*
Table 2.12: Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>A</th>
<th>h</th>
<th>λ</th>
<th>c</th>
<th>γ</th>
<th>η</th>
<th>k</th>
<th>α_m</th>
<th>f_E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>500</td>
<td>10</td>
<td>0.42</td>
<td>28</td>
<td>14</td>
<td>5</td>
<td>3</td>
<td>3.95</td>
<td>1</td>
<td>1.75</td>
</tr>
<tr>
<td>Model 2</td>
<td>1000</td>
<td>10</td>
<td>0.42</td>
<td>28</td>
<td>14</td>
<td>5</td>
<td>3</td>
<td>3.95</td>
<td>1</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Notes: Parameters used in simulations of Models 1 and 2.

Table 2.13: Equilibrium Values

<table>
<thead>
<tr>
<th></th>
<th>α_D</th>
<th>q_D</th>
<th>( \frac{n^M}{\gamma + n^M}(\bar{\alpha} - \bar{\alpha}) )</th>
<th>M</th>
<th>( \bar{\alpha} )</th>
<th>( \bar{p} )</th>
<th>( \beta_{0,1} )</th>
<th>( \beta_{1,2} )</th>
<th>( \beta_{2,3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2</td>
<td>3.294</td>
<td>7.361</td>
<td>3.031</td>
<td>3.375</td>
<td>4.453</td>
<td>0.005</td>
<td>3.379</td>
<td>3.971</td>
<td>6.804</td>
</tr>
</tbody>
</table>

Notes: Equilibrium Values from simulations of Models 1 and 2.

the organizational decision of firms, except for the size of the domestic market, \( N \), the parameters chosen in both models are identical.

Table 2.13 presents the equilibrium values from both models. An increase in market size induces tougher selection, and increases the demand draw of the marginal firm, \( \alpha_D \). In tougher markets the term \( \frac{n^M}{\gamma + n^M}(\bar{\alpha} - \bar{\alpha}) \) increases as well. These results are consistent with Proposition 2.5 from the previous section. In the simulations, an increase in the size of the market also leads to an increase in the number of varieties, \( M \). Moreover, as demonstrated in Figure 2.2 in the previous section, larger markets also affect the individual decision of firms. To summarize the discussion, the response to bigger markets is heterogeneous across firms. Profits and quantities produced may increase or decrease with the size of the market, while prices, markups over marginal costs and markups over average costs decrease in bigger markets.

Table 2.13 reports the demands draws at which entrepreneurs are indifferent between two organizational forms, \( \beta_{L,L+1} \). Larger markets affect the distribution of organizations through two channels. First, bigger markets induce tougher selection and increase the demand cutoff \( \alpha_D \). And second, because markups over marginal and average costs are lower in bigger markets, firms’ are induced to re-organized production in favor of more layers. Because the quantities produced by firms change as well, it is not always the case that the cutoffs \( \beta_{L,L+1} \) decrease in bigger markets. However, the distance between \( \alpha_D \) and \( \beta_{L,L+1} \) does decrease with \( N \). This result was proven in Proposition 2.6.

Figure 2.3 presents the cumulative distribution of organizations. The cumulative distribution of organizations of model 2 is below the distribution from model 1. Because the
Figure 2.3: Cumulative Distribution of Organizations

Pareto distribution always satisfies the non-decreasing hazard rate property, it follows from Proposition 2.7 that the distribution of organizations in model 2 first order stochastically dominates the distribution of organizations in model 1. Applying the Mann-Whitney test to the simulated data confirms the result. It rejects the hypothesis that both distributions are equal at the one percent level (the p-value of the test statistic is 0.000) and reports that the probability that a random draw of an organization from model 1 is greater than a random draw of an organization from model 2 is 0.306.

I now turn to productivity and investigate how the distribution of productivity is different across locations. I take advantage of the cost function and define firm-level productivity as the inverse of average costs. The measure of firm-level productivity that I use in my analysis is, therefore, equal to:

\[
a(q) = \frac{q}{C(q)} = \frac{1}{AV(q)}. \tag{2.39}
\]

A previous section characterized the cost function of firms. The important points from that section are the following. First, unlike most models with heterogeneous firms, in this model firms’ marginal costs are not equal to their average costs. The only exceptions are at the MES points. Second, because both marginal costs and average costs depend on the quantity
produced and on the organization of a firm, they are endogenous, and so productivity is endogenous as well. Third average cost are neither constant nor a monotonic function of quantity. This implies that firm-level productivity will also be neither constant nor a monotonic function of quantity. And fourth, the minimum average cost is decreasing with the number of layers, and the level of output that attains the minimum average cost is increasing with the number of layers. In terms of productivity this implies that firms with a greater number of layers can attain a greater productivity. In addition because the quantity produced by firms and their organization depend on the size of the market, the productivity of firms will also depend on the size of the market. Taken together all these assertions indicate that the distribution of productivity will be different across locations.

Figure 2.4 illustrates the heterogeneous responses in productivity as a result of operating in a bigger market. The productivity of firms with relatively low values of $\alpha$ decreases in larger markets, whereas firms with high demand draws have their productivity increase. Note that this is different from the findings of Caliendo and Rossi-Hansberg (2012), where in a closed-economy, an increase in market size only raises wages and the number of firms, but does not affect the quantities produced by firms, and thus their organizational structure and productivity remain the same.
Figure 2.5 presents the distribution of productivity from both economies. Although the shape of the Pareto distribution is invariant to truncation, the figure shows different shares of small, medium and high productivity firms across markets. The fraction of low productivity and medium productivity firms decreases in the larger market, while the mass of high productivity firms increases. Because markups are lower in bigger markets, firms are induced to reorganize. At the same time, because they are induced to produce more output, firms in the middle of the distribution increase their productivity in larger markets, thereby increasing the mass at the tail. In other words, a larger market makes firms in the middle of the distribution more productive because it incentivizes them to reorganize and produce with more layers.

The distribution of productivity in model 2 has a mean of 5.21 and a variance of 0.076. In comparison, in model 1 the mean of the distribution of productivity is 4.94 and its variance is 0.141. Note that these values are qualitatively consistent with empirical studies examining the distribution of productivity across locations. A conclusion emerging from these studies is that the distribution of productivity in denser markets has a higher mean and a lower variance (for example, Syverson (2004)).

To draw more meaningful comparisons between both models, I adopt the approach of
Table 2.14: Qualitative on the Distributions of Productivity

<table>
<thead>
<tr>
<th></th>
<th>( \hat{A} )</th>
<th>( \hat{D} )</th>
<th>( \hat{S} )</th>
<th>( R^2 )</th>
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</thead>
<tbody>
<tr>
<td>Shift Only</td>
<td>0.274</td>
<td>-</td>
<td>-</td>
<td>0.602</td>
</tr>
<tr>
<td>Truncation Only</td>
<td>-</td>
<td>-</td>
<td>0.303</td>
<td>0.894</td>
</tr>
<tr>
<td>Shift &amp; Truncation</td>
<td>0.105</td>
<td>-</td>
<td>0.241</td>
<td>0.933</td>
</tr>
<tr>
<td>Shift &amp; Dilation</td>
<td>0.274</td>
<td>0.756</td>
<td>-</td>
<td>0.676</td>
</tr>
<tr>
<td>Shift &amp; Dilation &amp; Truncation</td>
<td>0.437</td>
<td>0.183</td>
<td>-0.904</td>
<td>0.997</td>
</tr>
</tbody>
</table>

Notes: Comparisons of distribution of productivity from model 1 and 2. \( \hat{A} \) measures shift, \( \hat{D} \) measures dilation and \( \hat{S} \) measures left-truncation.

Combes, Duranton, Gobillon, Puga and Roux (2012) and estimate to what extent the distribution of productivity in model 2 is shifted, dilated, and left-truncated relative to the distribution of productivity in model 1.\(^{45}\) Given that any results are based on numerical simulations of the model, however, I only draw qualitative conclusions.\(^{46}\) Table 2.14 reports the results. Column (1) reports estimates for shift. In all cases, there is positive shift in the distribution of productivity in model 2 relative to model 1. Column (2) reports estimates of dilation. In both cases, there is evidence of positive dilation in the distribution of productivity in model 2 relative to model 1. Finally, column (3) reports estimates of truncation. Without taking dilation into account, the coefficient for left-truncation is positive. However, as the last row indicates, when shift, dilation, and left-truncation are all accounted for, the coefficient of left-truncation is negative. Although there is positive selection in the model, and as illustrated in Figure 2.5 the productivity of the least productive firm increases in the bigger market, because very few firms in model 2 are producing with only zero layers, the distribution of productivity in model 1 is left-truncated relative to the distribution in model 2.

I now turn to income and examine how the distribution of income varies across locations. In the model, because wages are normalized to 1, income is equal to \( cz_l^1 + 1 \) and so the distribution of income is similar to the distribution of knowledge in the economy. Panel (a) in Figure 2.6 illustrates the distribution of knowledge in the economy. The fraction of agents with intermediate levels of knowledge is bigger in the larger market, while the fraction of agents with low levels of knowledge is smaller. The effect is due to the increased

\(^{45}\)For a description of how to test for shift, dilation and left-truncation, see Combes, Duranton, Gobillon, Puga and Roux (2012) or Combes, Duranton and Gobillon (2012).

\(^{46}\)Indeed, numerical simulations are not conclusive. The proper approach is to first calibrate the model to the data and then perform these tests.
Figure 2.6: Distributions of Knowledge & Income
number of firms producing with more layers in the bigger market. Because a bigger market incentivizes firms to add layers of management, the knowledge of existing workers decreases. At the same time, because firms employ more intermediate managers, there are more agents with intermediate levels of knowledge in the economy. The second effect dominates and the mass of agents with low levels of knowledge is reduced in the bigger market.

Panel (b) presents the distribution of income. The distribution of income closely resembles the distribution of knowledge. To draw more meaningful comparisons, in Table 2.15 I report the mean and variance of the distribution of income from both models. Because these results are again based on numerical simulations, I only draw qualitative conclusions. As reported in Table 2.15 the distribution of income has a higher mean but a lower variance in the bigger market relative to the smaller market. A higher mean is consistent with empirical studies that examine how wages differ across locations. A conclusion emerging from these studies is that workers earn higher wages in denser markets. The numerical simulations suggest that the model is to be able to qualitatively account for this fact.

The simulations, however, are unable to account for the fact that wage inequality is greater in denser areas, documented in the empirical literature. As there is a very large weight given to firms with low demand draws, perhaps this may due to the value of the Pareto shape parameter $k$ chosen in the simulations.

### 2.5 Conclusion

This paper has examined how market size affects the organization of firms. To accomplish this, I embed the production framework developed by Garicano (2000) and further extended by Caliendo and Rossi-Hansberg (2012) in a monopolistically competitive model with heterogeneous firms and endogenous markups. The essence of the argument is that bigger markets lower markups and ultimately induce firms to re-organize production in favor of more layers. This implies that in denser markets the distribution of organizations will first
order stochastically dominate the distribution of organizations in less dense markets. Using French data, I find empirical support for this prediction.

Furthermore a fundamental feature of the model is that the productivity of firms and the knowledge of their labor force are determined by the way production is organized. This has implications for the distribution of productivity and income across locations. I simulate the model to assess how productivity and income differ across locations. Applying the approach developed by Combes, Duranton, Gobillon, Puga and Roux (2012), I find that the distribution of productivity in the bigger markets is right-shifted, dilated, and negatively left-truncated relative to the distribution in the smaller markets. I also compare the distributions of income across locations. I find that the distribution of income in bigger markets has a greater mean relative to the smaller market however it exhibits a lower variance.

The findings of this paper have several implications. More directly, they suggest that the distribution of productivity and income across locations depend on the organization of firms. More broadly, the paper’s findings suggest that an understanding of the organization of firms may yield additional insights into the composition of workers and firms in markets. As noted by Rosen (1982): “The firm cannot be analyzed in isolation from other production units in the economy. Rather, each person must be placed in his proper niche, and the marriage of personnel to positions and to firms must be addressed directly.”

Still there is work that remains to be done. In particular, one interesting extension would be to incorporate agglomeration forces to the model and examine how this affects the organization of firms and the distributions of productivity and income across markets of different sizes.
2.6 Appendix

2.6.1 Proof of Propositions 2.2 & 2.3

We first derive the results for quantity \( q(\alpha) \).

\[
\frac{\partial q(\alpha)}{\partial \alpha} = \frac{N}{2\gamma} \left\{ 1 - \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} \frac{\partial q(\alpha)}{\partial \alpha} \right\}
\]

Rearranging yields:

\[
\frac{\partial q(\alpha)}{\partial \alpha} = \frac{\frac{N}{2\gamma}}{1 + \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} \frac{N}{2\gamma}}.
\]

Since within layers \( \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} > 0 \), quantity is increasing with \( \alpha \). Since \( \frac{\partial q(\alpha)}{\partial q(\alpha)} = -\frac{N}{2\gamma} \), when the firm increases the number of layers, quantity will shift up discontinuously.

Now moving on the prices. Since

\[
\frac{\partial p(\alpha)}{\partial \alpha} = \frac{1}{2} \left\{ 1 + \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} \frac{\partial q(\alpha)}{\partial \alpha} \right\},
\]

and within layers, marginal costs are increasing with quantity, and quantity is increasing with \( \alpha \), \( p(\alpha) \) is increasing with respect to \( \alpha \). Furthermore since,

\[
\frac{\partial p(\alpha)}{\partial MC} = \frac{1}{2},
\]

when the firm increases the number of layers, marginal costs will decrease discontinuously, and thus prices will decrease discontinuously as well.

Now moving on to markups over marginal costs. Since

\[
\frac{\partial \mu^{MC}(\alpha)}{\partial \alpha} = \frac{1}{2} \left\{ 1 - \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} \frac{\partial q(\alpha)}{\partial \alpha} \right\},
\]

and by substituting the expression for \( \frac{\partial q(\alpha)}{\partial \alpha} \), within layers \( \mu(\alpha) \) is increasing with respect to \( \alpha \). Furthermore since,

\[
\frac{\partial \mu^{MC}(\alpha)}{\partial MC} = \frac{1}{2},
\]
when the firm increases the number of layers, marginal costs will decrease discontinuously, and thus markups over marginal costs will increase discontinuously as well.

Now moving on to markups over average costs. By definition markups over marginal costs, and markups over average costs are equal to:

$$\mu_{MC}(\alpha) = p(\alpha) - MC(q(\alpha)),$$

$$\mu_{AC}(\alpha) = p(\alpha) - AC(q(\alpha)).$$

It therefore follows that:

$$MC(q(\alpha)) - AC(q(\alpha)) = \mu_{AC}(\alpha) - \mu_{MC}(\alpha).$$

Marginal costs are increasing with quantity, while average cost curves are convex and attain their minimum when they intersect their associated marginal cost curve. It therefore follows that:

$$\frac{\partial [MC(q(\alpha)) - AC(q(\alpha))] \partial q(\alpha)}{\partial \alpha} > 0,$$

which in turn implies that:

$$\frac{\partial \mu_{AC}(\alpha)}{\partial \alpha} > \frac{\partial \mu_{MC}(\alpha)}{\partial \alpha}.$$

Since $$\frac{\partial \mu_{MC}(\alpha)}{\partial \alpha} > 0$$ it follows that $$\frac{\partial \mu_{AC}(\alpha)}{\partial \alpha} > 0.$$ Markups over average costs are also equal to: $$\mu_{AC}(\alpha) = \pi(\alpha)/q(\alpha).$$ The numerator of the derivative of this expression with respect to marginal costs is equal to:

$$q(\alpha) \frac{\partial \pi(\alpha)}{\partial MC} + \pi(\alpha) \frac{\partial q(\alpha)}{\partial MC}.$$

The first term is equal to: $$q(\alpha) \frac{\partial \pi(\alpha)}{\partial MC} = q(\alpha) \left[-\frac{N}{2\gamma}MC(q(\alpha)) - q(\alpha)\right],$$ where I have used the expression for $$\frac{\partial \pi(\alpha)}{\partial MC}$$ derived below. The second term is equal to $$\pi(\alpha) \frac{\partial q(\alpha)}{\partial MC} = -\pi(\alpha) \frac{N}{2\gamma}.$$ Using the expression for profits, $$\pi(\alpha) = \frac{N}{2\gamma} q(\alpha)^2 + q(\alpha)MC(q(\alpha)) - C(q(\alpha)),$$ and eliminating common terms implies that the numerator is equal to:
\[-\frac{q(\alpha)^2}{2} - \frac{N}{2\gamma} C(q(\alpha)),\]

which is negative. The denominator of the derivative of \(\mu^{AC}(\alpha) = \pi(\alpha)/q(\alpha)\) with respect to marginal costs is equal to \(q(\alpha)^2\) which is always positive. It therefore follows that when the firm increases the number of layers, marginal costs will decrease discontinuously, and thus markups over average costs will increase discontinuously as well.

Now moving on to revenues. Since

\[
\frac{\partial r(\alpha)}{\partial \alpha} = q(\alpha) \frac{\partial p(\alpha)}{\partial \alpha} + p(\alpha) \frac{\partial q(\alpha)}{\partial \alpha},
\]

it follows that within layers, revenues are increasing with respect to \(\alpha\). Since

\[
\frac{\partial r(\alpha)}{\partial MC} = -\frac{N}{2\gamma} MC(q(\alpha)),
\]

as the firm increases the number of layers, marginal costs will decrease discontinuously, revenues increase discontinuously as well.

Now moving on to profits. From the firms maximization problem we know that,

\[
\pi(\alpha) = \left\{ \alpha - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) - \frac{\gamma}{N} q^*(\alpha) \right\} q^*(\alpha) - C(q^*(\alpha)),
\]

where \(\ast\) denotes the optimal quantities chosen by the firm. By the envelope theorem,

\[
\frac{\partial \pi(\alpha)}{\partial \alpha} = q^*(\alpha).
\]

Thus profits are increasing with respect to \(\alpha\). Since the optimal quantity produced, \(q^*(\alpha)\) is increasing with the number of layers \(L\), it follows that the slope of the profit function \(\pi(\alpha)\) is increasing with \(L\).

Consider the firm that is indifferent between producing with layers \(L\) and \(L + 1\). Then it follows that

\[
\pi_L(\alpha) = \pi_{L+1}(\alpha).
\]
Since, within layers profits are differentiable, and continuous when the firm is indifferent between layers \( L \) and \( L + 1 \), profits are continuous with respect to \( \alpha \).

I now show that holding the number of layer fixed, profits are concave with respect to \( q \).
From equation (4), we know that
\[
\pi(\alpha) = p(\alpha)q(\alpha) - C(q(\alpha)).
\]
Substituting in for \( p(\alpha) = \alpha - \frac{2\gamma}{N}q - \frac{\eta M}{\eta M + \gamma}(\overline{\alpha} - \overline{p}) \), and taking the second order derivative with respect to \( q \) yields,
\[
\frac{\partial^2 \pi(\alpha)}{\partial^2 q} = -\frac{2\gamma}{N} - \frac{\partial MC(q(\alpha))}{\partial q},
\]
which is negative. Thus profits are concave in \( q \).

### 2.6.2 Proof of Proposition 2.4

The equilibrium of the model is determined from the zero-profit condition and the free-entry condition:
\[
\pi(\alpha_D, M) = 0, \tag{2.40}
\]
\[
\int_{\alpha_D} \pi(\alpha, M)dG(\alpha) = f_E, \tag{2.41}
\]
where \( M \) denotes the mass of firms operating in equilibrium and \( \alpha_D \) is the demand draw of the entrepreneur that is indifferent between entering and exiting the market.
First I transform the equilibrium to be a function of \( q_D \) and \( \alpha_D \).
From the first order condition of the firm’s maximization problem, for a given \( \alpha \), quantity is determined by the equation:
\[
q(\alpha) = \frac{N}{2\gamma} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M}(\overline{\alpha} - \overline{p}) - MC(q(\alpha)) \right\}.
\]
For a given \( \alpha \) there exists a unique quantity \( q(\alpha) \), that is a solution to the expression of above. Rewriting this equation yields an expression of the term \( \frac{\eta M}{\gamma + \eta M}(\overline{\alpha} - \overline{p}) \) as a function
the demand draw, $\alpha$, and the optimal quantity produced, $q(\alpha)$:

$$\frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) = \alpha - MC(q(\alpha)) - \frac{2\gamma}{N} q(\alpha).$$

Substituting this expression in the profit of the firm yields:

$$\pi(\alpha, q(\alpha)) = p(\alpha)q(\alpha) - C(q(\alpha))$$
$$= \left[ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) - \frac{\gamma}{N} q(\alpha) \right] q(\alpha) - C(q(\alpha))$$
$$= \left[ \frac{2\gamma}{N} q(\alpha) + MC(q(\alpha)) - \frac{\gamma}{N} q(\alpha) \right] q(\alpha) - C(q(\alpha))$$
$$= \left[ \frac{\gamma}{N} q(\alpha) + MC(q(\alpha)) \right] q(\alpha) - C(q(\alpha))$$
$$= \frac{\gamma}{N} q(\alpha)^2 + MC(q(\alpha))q(\alpha) - C(q(\alpha)).$$

For the marginal firm, it follows that the relationship between $q_D$ and $\alpha_D$ is:

$$\frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) = \alpha_D - MC(q_D) - \frac{2\gamma}{N} q_D.$$

Substituting this expression in the profit of the marginal firm yields:

$$\pi(q_D) = p(\alpha_D)q_D - C(q_D)$$
$$= \left[ \alpha_D - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) - \frac{\gamma}{N} q_D \right] q_D - C(q_D)$$
$$= \left[ \frac{2\gamma}{N} q_D + MC(q_D) - \frac{\gamma}{N} q_D \right] q_D - C(q_D)$$
$$= \left[ \frac{\gamma}{N} q_D + MC(q_D) \right] q_D - C(q_D)$$
$$= \frac{\gamma}{N} q_D^2 + MC(q_D)q_D - C(q_D).$$

For the marginal firm, I now rewrite all the variables as a function of $q_D$ and the parameters of the model. For the marginal firm prices, markups and revenues are equal to:
\[ p_D = \frac{1}{2} \left[ \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) + MC(q_D) \right] \]
\[ = \frac{1}{2} \left[ \frac{2\gamma}{N} q_D + 2MC(q_D) \right] \]
\[ = \frac{\gamma}{N} q_D + MC(q_D). \]

\[ \mu_D = \frac{1}{2} \left[ \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC(q_D) \right] \]
\[ = \frac{1}{2} \left[ \frac{2\gamma}{N} q_D \right] \]
\[ = \frac{\gamma}{N} q_D. \]

\[ r_D = \frac{N}{4\gamma} \left[ \left( \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)^2 - MC(q_D)^2 \right] \]
\[ = \frac{N}{4\gamma} \left[ \frac{4\gamma}{N} q_D MC(q_D) + \frac{4\gamma^2}{N^2} q_D^2 \right] \]
\[ = q_D MC(q_D) + \frac{\gamma}{N} q_D^2. \]

For a firm with demand draw \( \alpha \), I rewrite quantities, prices, markups and revenues as a function of \( q_D, p_D, \mu_D \) and \( r_D \), and the parameters of the model. This yields the following expressions:

\[ q(\alpha) = \frac{1}{2} \left[ \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC(q_D) \right] \]
\[ = \frac{N}{2\gamma} \left[ \alpha - \alpha_D + \frac{2\gamma}{N} q_D + MC(q_D) - MC(q(\alpha)) \right] \]
\[ = q_D + \frac{N}{2\gamma} [\alpha - \alpha_D + MC(q_D) - MC(q(\alpha))]. \]
\[ p(\alpha) = \frac{1}{2} \left[ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) + MC(q(\alpha)) \right] \]

\[ = \frac{1}{2} \left[ \alpha - \alpha_D + \frac{2\gamma}{N}q_D + MC(q_D) + MC(q(\alpha)) \right] \]

\[ = p_D + \frac{1}{2} \left[ \alpha - \alpha_D - MC(q_D) + MC(q(\alpha)) \right]. \]

\[ \mu(\alpha) = \frac{1}{2} \left[ \alpha - \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) - MC(q(\alpha)) \right] \]

\[ = \frac{1}{2} \left[ \alpha - \alpha_D + \frac{2\gamma}{N}q_D + MC(q_D) - MC(q(\alpha)) \right] \]

\[ = \mu_D + \frac{1}{2} \left[ \alpha - \alpha_D - MC(q_D) - MC(q(\alpha)) \right]. \]

\[ r(\alpha) = p(\alpha)q(\alpha) \]

\[ = \left[ p_D + \frac{1}{2} \left[ \alpha - \alpha_D - MC(q_D) + MC(q(\alpha)) \right] \right] \left[ q_D + \frac{N}{2\gamma} \left[ \alpha - \alpha_D + MC(q_D) - MC(q(\alpha)) \right] \right] \]

\[ = r_D + q_D \frac{1}{2} \left[ \alpha - \alpha_D - MC(q_D) + MC(q(\alpha)) \right] + p_D \frac{N}{2\gamma} \left[ \alpha - \alpha_D + MC(q_D) - MC(q(\alpha)) \right] \]

\[ + \frac{N}{4\gamma} \left[ (\alpha - \alpha_D)^2 - (MC(q_D) - MC(q(\alpha)))^2 \right]. \]

Furthermore, one can show that the derivatives of these equations with respect to \( \alpha \) are as above.

The equilibrium is determined by the solution to the following three equations:

\[ \frac{\eta M}{\gamma + \eta M}(\bar{\alpha} - \bar{p}) = \alpha_D - MC(q_D) - \frac{2\gamma}{N}q_D. \quad (2.42) \]

\[ \Delta ZCP = \pi(\alpha_D, q_D) = 0, \quad (2.43) \]

\[ \Delta FE = \int_{\alpha_D} \pi(\alpha, \alpha_D, q_D)d\alpha - f_E = 0, \quad (2.44) \]

Here \( M, q_D \) and \( \alpha_D \) are variables that are to be determined. Note that the solution to
equation (2.43) depends only on \( q_D \) and the parameters of the model. Given the solution to equation (2.43), equation (2.44) is only a function of \( \alpha_D \) and the parameters of the model. Finally, once \( q_D \) and \( \alpha_D \) are both determined, \( N \) is determined equation (2.42). Therefore to prove that a solution exists, I need to show that there exists a \( q_D \) and \( \alpha_D \) such that equations (2.43) and (2.44) are satisfied.

First, I show that a solution to equation (2.43) exists. First, consider the slope of the profit function:

\[
\frac{\partial \pi(q_D)}{\partial q_D} = \frac{2\gamma}{N}q_D + q_D \frac{\partial MC(q_D)}{\partial q_D} > 0.
\]

When \( q_D \) is sufficiently large, since \( MC(q_D) > AC(q_D) \), it follows that \( \pi(q_D) = \frac{2\gamma}{N}q_D^2 + q_D MC(q_D) - C(q_D) > 0 \). Since \( \lim_{q_D \to 0} C(q_D) = w \) and \( \lim_{q_D \to 0} MC(q_D) = 0 \) it follows that \( \lim_{q_D \to 0} \pi(q_D) < 0 \). Therefore, there exists a unique \( q_D \) exists such that \( \pi(q_D) = 0 \).

Second, consider the equation (2.44). Using Leibniz’s integral rule, the slope of the free-entry condition is:

\[
\frac{\partial \Delta FE}{\partial \alpha_D} = -\pi(\alpha, \alpha_D, q_D) dG(\alpha_D) + \int_{\alpha_D} \frac{\partial \pi(\alpha, \alpha_D, q_D)}{\partial \alpha_D} dG(\alpha).
\]

The first term by definition is equal to zero. I now show that the second term is positive.

Using the expression for profits, and after eliminating common terms, it follows that:

\[
\frac{\partial \pi(\alpha, \alpha_D, q_D)}{\partial \alpha_D} = \left[ \frac{2\gamma}{N} q(\alpha) + q(\alpha) \frac{\partial MC(q(\alpha))}{\partial q(\alpha)} \right] \frac{\partial q(\alpha)}{\partial \alpha_D},
\]

where for the quantity expression it follows that

\[
\frac{\partial q(\alpha)}{\partial \alpha_D} = -\frac{\frac{N}{2\gamma}}{1 + \frac{N}{2\gamma} \frac{\partial MC(q(\alpha))}{\partial q(\alpha)}} < 0.
\]

Therefore \( \Delta FE \) is downward sloping. Further, when \( \alpha_D = \alpha_M, \Delta FE > 0 \), and in the limit, when \( \alpha_D \) approaches infinity \( \lim_{\alpha_D \to \infty} \Delta FE < 0 \). Hence, there exists a unique \( \alpha_D \) such that \( \Delta FE = 0 \).

For a given number of layers \( L \), there exists a solution to \( \pi_L(\alpha^L_D, q^L_D) = 0 \), and thus there are a discrete set of potential solutions. I now show that from this set, there is only one
combination of $\alpha^L_D, q^L_D$ that satisfies the equilibrium.

Suppose not. Consider two possible solutions $\alpha^L_D, q^L_D$ and $\alpha^{L+1}_D, q^{L+1}_D$, associated with organizations with $L$ and $L + 1$ layers respectively. Without loss of generality, assume that $\alpha^L_D < \alpha^{L+1}_D$. In this case it follows that $\pi_L(\alpha^L_D, q^L_D) = \pi_{L+1}(\alpha^{L+1}_D, q^{L+1}_D) = 0$. By Proposition 2.2 it follows that all firms with demand draws in the interval $[\alpha^L_D, \alpha^{L+1}_D]$, will earn positive profits producing with an organization with $L$ layers. By Proposition 2.2 it also follows that for the entrepreneur with demand draw $\alpha^{L+1}_D$, $\pi_L(\alpha^{L+1}_D) > \pi_{L+1}(\alpha^{L+1}_D) = 0$, and so he will earn positive profits producing with an organization with $L$ layers. Therefore $\alpha^{L+1}_D, q^{L+1}_D$ is not an equilibrium solution.

I now show that if $\eta > \bar{\eta}$ both the homogeneous and differentiated goods will be produced in equilibrium. For expositional simplicity, I define $K = \frac{\eta M}{\gamma + \eta M} (\bar{\sigma} - \bar{p})$ and $B = \int_{\alpha_D} \frac{\bar{\alpha}}{N} q(\alpha) \frac{g(\alpha)}{1 - G(\alpha_D)}$. First consider the term $\bar{\alpha} - \bar{p}$. This can be rewritten as:

$$\bar{\alpha} - \bar{p} = \int_{\alpha_D} \left[ \alpha - \left[ \alpha - K - \frac{\gamma}{N} q(\alpha) \right] \right] \frac{g(\alpha)}{1 - G(\alpha_D)}$$

$$= K + B.$$

It then follows that:

$$\frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) = \frac{\eta M}{\gamma + \eta M} [K + B]$$

$$= K.$$

And by isolating terms it follows that:

$$M = \frac{\gamma K}{\eta B}.$$  (2.45)

Next consider the equilibrium condition:

$$N - \frac{M}{1 - G(\alpha_D)} \left\{ f_E + \int_{\alpha_D} C(q(\alpha)) dG(\alpha) \right\} > 0.$$

which can simply be rewritten as $N > M\bar{r}$. By substituting in the expression for $M$ from
above, it follows that

$$\eta > \frac{\gamma K_T}{N_B}. \quad (2.46)$$

Thus if \( \eta > \eta = \frac{\gamma K_T}{N_B} \) both the homogeneous and differentiated goods will be produced in equilibrium.

### 2.6.3 Proof of Proposition 2.5

Consider an increase in \( N \). From the zero-profit equation, it follows that

$$0 = -\frac{\gamma}{N^2}q_D^2 + \frac{2\gamma}{N}q_D \frac{\partial q_D}{\partial N} + q_D \frac{\partial MC(q_D)}{\partial q_D} \frac{\partial q_D}{\partial N},$$

which after rearranging terms yields the result

$$\frac{\partial q_D}{\partial N} = \frac{\frac{\gamma}{N^2}q_D}{\frac{2\gamma}{N} + \frac{\partial MC(q_D)}{\partial q_D}}. \quad (2.47)$$

Since the denominator and numerator are both positive, it follows that

$$\frac{\partial q_D}{\partial N} > 0. \quad (2.48)$$

Consider the equation characterizing the expected profits of entry \( V^e \)

$$\int_{\alpha_D} \pi(\alpha, \alpha_D, q_D) dG(\alpha) = f_E.$$  

From this equation, it follows that:

$$\frac{\partial \alpha_D}{\partial N} = -\frac{\partial V^e/\partial N}{\partial V^e/\partial \alpha_D}. \quad (2.49)$$

In the previous section, I showed that the denominator in equation (2.49) is negative. I now show that the numerator is positive. The profit of a firm with demand draw \( \alpha \) is

$$\pi(\alpha, \alpha_D, q_D) = \left[ \frac{2\gamma}{N}q_D + MC(q_D) - \alpha_D + \alpha - \frac{\gamma}{N}q(\alpha) \right] q(\alpha) - C(q(\alpha)), \quad (2.50)$$
and using the envelope theorem yields

\[
\frac{\partial \pi(\alpha, \alpha_D, q_D)}{\partial N} = \left[ \frac{2\gamma}{N} q_D + MC(q_D) - \alpha_D + \alpha - \frac{\gamma}{N} q(\alpha) \right] q(\alpha) - C(q(\alpha))
\]

\[
= \left[ -\frac{2\gamma}{N^2} q_D + \frac{\partial MC(q_D)}{\partial N} \frac{\partial q_D}{\partial N} + \frac{2\gamma}{N} \frac{q_D}{\partial N} + \frac{\gamma}{N^2} q(\alpha) \right] q(\alpha)
\]

\[
= \left[ \frac{\gamma}{N^2} (q(\alpha) - q_D) \right] q(\alpha),
\]

where here \( \alpha_D \) is held fixed and I substituted for \( \frac{\partial q_D}{\partial N} \) using equation (2.47). Therefore since

\[
\frac{\partial V_e}{\partial N} = \int_{\alpha_D} \frac{\partial \pi(\alpha, \alpha_D, q_D)}{\partial N} dG(\alpha)
\]

and \( \frac{\partial \pi(\alpha, \alpha_D, q_D)}{\partial N} \) is positive, \( \frac{\partial V_e}{\partial N} > 0 \) and thus the numerator in equation (2.49) is positive. Therefore \( \alpha_D \) is increasing with respect to \( N \).

Rearranging the first order condition of the firm’s maximization problem gives

\[
\alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) = \frac{N}{2\gamma} q_D + MC(q_D),
\]

and taking the derivative of this expression with respect to \( N \) yields:

\[
\frac{\partial \alpha_D}{\partial N} - \frac{\eta M}{\gamma + \eta M} \frac{\partial (\bar{\alpha} - \bar{p})}{\partial N} = -\frac{\gamma}{N^2} q_D.
\]

Hence it follows that

\[
\frac{\partial \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p})}{\partial N} = \frac{\partial \alpha_D}{\partial N} + \frac{\gamma}{N^2} q_D > 0.
\]

I now show that if \( \bar{\eta} > \eta \) the mass of firms in the differentiated goods sector, \( M \), will increase with \( N \). In the proof of Proposition 2.4, I showed that \( M \) can be rewritten as:

\[
M = \frac{\gamma K}{\eta B},
\]

where \( K = \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \) and \( B = \int_{\alpha_D} \frac{\gamma}{\gamma - G(\alpha_D)} q(\alpha) \). Taking the derivative of this expression with respect to \( N \) yields:
\[
\frac{\partial M}{\partial N} = \frac{\gamma}{\eta B^2} \left[ B \frac{\partial K}{\partial N} - K \frac{\partial B}{\partial N} \right].
\]

Hence \( \frac{\partial M}{\partial N} \) is positive if and only if \( B \frac{\partial K}{\partial N} - K \frac{\partial B}{\partial N} \) is positive. I do not know the sign of \( \frac{\partial B}{\partial N} \). If it is negative then it automatically follows that \( \frac{\partial M}{\partial N} \) is positive, and I do not have to make an assumption on \( \eta \). However assume that \( \frac{\partial B}{\partial N} \) is positive. Then using the expression for \( M \) from above yields:

\[
\frac{\partial k}{\partial N} > \frac{K}{B} = \frac{M \eta}{\gamma},
\]

and by isolating \( \eta \), one obtains following inequality:

\[
\frac{\gamma \frac{\partial k}{\partial N}}{M \frac{\partial B}{\partial N}} > \eta. \tag{2.52}
\]

Hence if \( \bar{\eta} = \frac{\gamma \frac{\partial k}{\partial N}}{M \frac{\partial B}{\partial N}} > \eta \), the mass of firms in the differentiated goods sector, \( M \), will increase with \( N \).

I now show that there always exists an \( \eta \) such that \( \bar{\eta} > \eta > \underline{\eta} \). Substituting the expressions for both terms yields:

\[
\frac{\gamma K \bar{\tau}}{NB} < \frac{\gamma \frac{\partial k}{\partial N}}{M \frac{\partial B}{\partial N}}. \tag{2.53}
\]

After rearranging terms it follows that

\[
\frac{M \tau}{N} < \frac{B \frac{\partial k}{\partial N}}{K \frac{\partial B}{\partial N}}.
\]

By assumption, for \( \eta \) to be in the set \([\underline{\eta}, \bar{\eta}]\) the following two conditions must simultaneously hold:

\[
M \tau < N
\]

\[
K \frac{\partial B}{\partial N} < B \frac{\partial K}{\partial N}.
\]

where by assumption \( \frac{\partial B}{\partial N} \) is positive. Hence from these two conditions it follows that:
\[ \frac{M \bar{r}}{N} < 1 < \frac{B_{lN}^{2K}}{K \partial B_{lN}^{2}}. \]

Therefore there always exists an \( \eta \in [\eta, \bar{\eta}] \).

### 2.6.4 Proof of Proposition 2.6

Consider an entrepreneur with demand draw \( \beta_{L,L+1} \) that is indifferent between two organizational forms \( L \) and \( L + 1 \). Then it follows that his profits are the same and:

\[ \pi_L(\beta_{L,L+1}, N) = \pi_{L+1}(\beta_{L,L+1}, N). \]

In this section, I first show how \( q_L \) and \( q_{L+1} \) change with respect to \( N \). Then I examine how \( \beta_{L,L+1} \) changes with \( N \). Finally, I examine how \( \beta_{L,L+1} \) changes relative to \( \beta_{L+1,L+2} \) with respect to \( N \).

Substituting the expression for profits, implies that:

\[ \frac{\gamma}{N} q_L^2 + MC_L(q_L)q_L - C_L(q_L) = \frac{\gamma}{N} q_{L+1}^2 + MC_{L+1}(q_{L+1})q_{L+1} - C_{L+1}(q_{L+1}), \]

where \( q_L \) and \( q_{L+1} \) are the quantities produced by the entrepreneur with demand draw \( \beta_{L,L+1} \) when he is producing with \( L \) or \( L + 1 \) layers. Taking the derivative of this expression with respect to \( N \) and eliminating any common terms yields:

\[ -\frac{\gamma}{N^2} q_L^2 + 2\gamma q_L \frac{\partial q_L}{\partial N} + q_L \frac{\partial MC_L}{\partial q_L} \frac{\partial q_L}{\partial N} = -\frac{\gamma}{N^2} q_{L+1}^2 + 2\gamma q_{L+1} \frac{\partial q_{L+1}}{\partial N} + q_{L+1} \frac{\partial MC_{L+1}}{\partial q_{L+1}} \frac{\partial q_{L+1}}{\partial N}. \]

(2.54)

Since \( q_L \) is the optimal quantity supplied by the entrepreneur, it satisfies the first order condition of the firm’s maximization problem:

\[ q_L = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC_L(q_L) \right]. \]

Similarly because \( q_{L+1} \) is the optimal quantity supplied by the entrepreneur, it satisfies the equation:
\[ q_{L+1} = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) - MC_{L+1}(q_{L+1}) \right]. \]

From these two expressions it follows that:

\[ MC_L(q_L) + \frac{2\gamma}{N} q_L = MC_{L+1}(q_{L+1}) + \frac{2\gamma}{N} q_{L+1}. \]  \hspace{1cm} (2.55)

Taking the derivative of equation (2.55) with respect to \( N \) yields:

\[ \frac{\partial MC_L(q_L)}{\partial q_L} \frac{\partial q_L}{\partial N} - \frac{2\gamma}{N^2} q_L + \frac{2\gamma}{N} q_L \frac{\partial q_L}{\partial q_L} = q_{L+1} \frac{\partial MC_{L+1}(q_{L+1})}{\partial q_{L+1}} \frac{\partial q_{L+1}}{\partial N} - \frac{2\gamma}{N^2} q_{L+1} + \frac{2\gamma}{N} \frac{\partial q_{L+1}}{\partial N}. \]  \hspace{1cm} (2.56)

Multiplying equation (2.56) by \( q_{L+1} \) gives:

\[ q_{L+1} \frac{\partial MC_L(q_L)}{\partial q_L} \frac{\partial q_L}{\partial N} = \frac{2\gamma}{N^2} q_L q_{L+1} + \frac{2\gamma}{N} q_{L+1} \frac{\partial q_L}{\partial q_L} = q_{L+1} \frac{\partial MC_{L+1}(q_{L+1})}{\partial q_{L+1}} \frac{\partial q_{L+1}}{\partial N} \frac{\partial q_{L+1}}{\partial N} = \frac{2\gamma}{N^2} q_{L+1} + \frac{2\gamma}{N} \frac{\partial q_{L+1}}{\partial N}. \]

and substituting this expression into equation (2.54) it follows that the following holds:

\[ -\frac{\gamma}{N^2} q_L + \frac{2\gamma}{N} q_L \frac{\partial q_L}{\partial N} + q_L \frac{\partial MC_L}{\partial q_L} \frac{\partial q_L}{\partial N} = q_{L+1} - \frac{\partial MC_L}{\partial q_L} \frac{\partial q_L}{\partial N} - \frac{2\gamma}{N^2} q_L q_{L+1} + \frac{2\gamma}{N} q_{L+1} \frac{\partial q_L}{\partial q_L} + \frac{2\gamma}{N^2} q_{L+1} - \frac{\gamma}{N^2} q_{L+1}^2, \]  \hspace{1cm} (2.57)

which can be rearranged into:

\[ \frac{\partial q_L}{\partial N} \left[ \frac{2\gamma}{N} (q_L - q_{L+1}) + (q_L - q_{L+1}) \frac{\partial MC_L}{\partial q_L} \right] = \frac{\gamma}{N^2} (q_{L+1} - q_L)^2. \]  \hspace{1cm} (2.58)

Since \( q_L \) is less than \( q_{L+1} \) it follows that the term on the right hand-side is positive, while the expression in brackets on the left-hand side is negative. Hence from equation (2.58) it follows that:

\[ \frac{\partial q_L}{\partial N} < 0. \]  \hspace{1cm} (2.59)

Performing the same steps as above, but multiplying equation (2.56) by \( q_L \) yields:
which implies that

\[ \frac{\partial q_{L+1}}{\partial N} > 0. \] (2.61)

Hence for the two quantities \( q_L \) and \( q_{L+1} \) such that an entrepreneur is indifferent between two organizational forms, it follows that \( q_L \) is decreasing with \( N \) while \( q_{L+1} \) is increasing with respect to \( N \). Therefore when controlling for market size, larger firms will have more layers.

I now examine how \( \beta_{L,L+1} \) changes with respect to \( N \). Returning to the first order condition of the firm’s maximization problem it follows that:

\[ q_L = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) - MC_L(q_L) \right], \]

which can be rewritten as

\[ q_L = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \alpha_D + \alpha_D - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) - MC_L(q_L) \right]. \]

Taking the derivative of this expression with respect to \( N \) and isolating common terms yields:

\[ \frac{\partial q_L}{\partial N} \left[ 1 + \frac{N}{2\gamma} \frac{\partial MC_L(q_L)}{\partial q_L} \right] = \frac{q_L}{N} + \frac{N}{2\gamma} \frac{\partial (\beta_{L,L+1} - \alpha_D)}{\partial N} + \frac{N}{2\gamma} \frac{\partial \left( \alpha_D - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) \right)}{\partial N}. \] (2.62)

Since \( \frac{\partial q_L}{\partial N} \) is negative and the expression in brackets in positive, it follows that the left-hand side in equation (2.62) is negative. This implies

\[ \frac{q_L}{N} + \frac{N}{2\gamma} \frac{\partial (\beta_{L,L+1} - \alpha_D)}{\partial N} + \frac{N}{2\gamma} \frac{\partial \left( \alpha_D - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) \right)}{\partial N} < 0, \]

which can be rewritten as
\[ \frac{\partial (\beta_{L,L+1} - \alpha_D)}{\partial N} < -\frac{2\gamma}{N^2} q_L - \frac{\partial \left( \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)}{\partial N}. \]

Substituting for the expression

\[ \frac{\partial \left( \alpha_D - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)}{\partial N} = -\frac{2\gamma}{N^2} q_D + \frac{2\gamma}{N} \frac{\partial q_D}{\partial N} + \frac{\partial MC(q_D)}{\partial q_D} \frac{\partial q_D}{\partial N}, \]

where \( q_D \) is the quantity produced by the marginal firm yields

\[ \frac{\partial (\beta_{L,L+1} - \alpha_D)}{\partial N} < -\frac{2\gamma}{N^2} (q_L - q_D) - \left[ \frac{2\gamma}{N} + \frac{\partial MC(q_D)}{\partial q_D} \right] \frac{\partial q_D}{\partial N} < 0. \quad (2.63) \]

and therefore because \( \left[ \frac{2\gamma}{N} + \frac{\partial MC(q_D)}{\partial q_D} \right] \frac{\partial q_D}{\partial N} \) is positive, the distance between \( \beta_{L,L+1} \) and \( \alpha_D \) decreases with \( N \). Namely,

\[ \frac{\partial (\beta_{L,L+1} - \alpha_D)}{\partial N} < 0. \]

I now proceed to analyze how \( \beta_{L,L+1} \) changes with \( N \). Returning to the first order condition of the firm’s maximization problem it follows that:

\[ q_L = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC_L(q_L) \right]. \]

Taking the derivative of this expression with respect to \( N \) and isolating common terms yields:

\[ \frac{\partial q_L}{\partial N} \left[ 1 + \frac{N}{2\gamma} \frac{\partial MC_L(q_L)}{\partial q_L} \right] = q_L + \frac{N}{2\gamma} \frac{\partial \beta_{L,L+1}}{\partial N} - \frac{N}{2\gamma} \frac{\partial \left( \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)}{\partial N}. \]

Since the term on the left-hand side is negative, it follows that:

\[ \frac{\partial \left( \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)}{\partial N} - \frac{2\gamma}{N^2} q_L > \frac{\partial \beta_{L,L+1}}{\partial N}. \]

which provides an upper bound to \( \frac{\partial \beta_{L,L+1}}{\partial N} \). Performing the same exercise with respect to \( q_{L+1} \) implies
\[ \frac{\partial \beta_{L,L+1}}{\partial N} > \frac{\partial \left( \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \right)}{\partial N} - \frac{2\gamma}{N^2} q_{L+1}. \]

which provides a lower bound to \( \frac{\partial \beta_{L,L+1}}{\partial N} \).

Consider two entrepreneurs who are indifferent producing with two types of organizations. Denote the demand draw of the entrepreneur who is indifferent between \( L, L + 1 \) layers as \( \beta_{L,L+1} \) and the demand draw of the entrepreneur who is indifferent between \( L + 1, L + 2 \) layers as \( \beta_{L+1,L+2} \). I now examine how \( \beta_{L,L+1} \) and \( \beta_{L+1,L+2} \) change relative to each other with respect to \( N \). From the first order condition of the firm’s maximization problem it follows that:

\[ q_L(\beta_{L,L+1}) = \frac{N}{2\gamma} \left[ \beta_{L,L+1} - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC_L(q_L(\beta_{L,L+1})) \right], \]

where \( q_L(\beta_{L,L+1}) \) is the quantity supplied by the firm with demand draw \( \beta_{L,L+1} \) using an organization with \( L \) layers. This implies that

\[ \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) = \beta_{L,L+1} - MC_L(q_L(\beta_{L,L+1})) - \frac{2\gamma}{N} q_L(\beta_{L,L+1}). \]

Returning to the first order condition of the firm’s maximization problem with demand draw \( \beta_{L+1,L+2} \) and substituting in the expression for \( \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) \) from above, it follows that:

\[ q_L(\beta_{L,L+1}) + \frac{N}{2\gamma} [\beta_{L+1,L+2} - \beta_{L,L+1} + MC_L(q_L(\beta_{L,L+1})) - MC_{L+1}(q_{L+1}(\beta_{L+1,L+2}))], \]

where \( q_L(\beta_{L,L+1}) \) is the quantity supplied by the firm with demand draw \( \beta_{L+1,L+2} \) using an organization with \( L + 2 \) layers. Taking the derivative of the this expression with respect to \( N \) and isolating common terms yields:

\[ \frac{\partial q_{L+1}(\beta_{L+1,L+2})}{\partial N} \left[ \frac{2\gamma}{N} + \frac{\partial MC_{L+1}(q_{L+1}(\beta_{L+1,L+2}))}{\partial N} \right] - \frac{\partial q_{L+1}(\beta_{L,L+1})}{\partial N} \left[ \frac{2\gamma}{N} + \frac{\partial MC_{L+1}(q_{L+1}(\beta_{L,L+1}))}{\partial N} \right] = \]

\[ \frac{2\gamma}{N^2} (q_{L+1}(\beta_{L+1,L+2}) - q_{L+1}(\beta_{L,L+1})) + \frac{\partial \beta_{L+1,L+2}}{\partial N} - \frac{\partial \beta_{L,L+1}}{\partial N}. \]
Since $\frac{\partial q_{L+1}(\beta_{L+1,L+2})}{\partial N} < 0$ and $\frac{\partial q_{L+1}(\beta_{L,L+1})}{\partial N} > 0$, it follows that the term on the left-hand side is negative, and therefore:

$$\frac{-2\gamma}{N^2}(q_{L+1}(\beta_{L+1,L+2}) - q_{L+1}(\beta_{L,L+1})) > \frac{\partial \beta_{L+1,L+2}}{\partial N} - \frac{\partial \beta_{L,L+1}}{\partial N}.$$ 

This provides an upper bound to $\frac{\partial \beta_{L+1,L+2}}{\partial N} - \frac{\partial \beta_{L,L+1}}{\partial N}$. Since $q_{L+1}(\beta_{L+1,L+2}) - q_{L+1}(\beta_{L,L+1}) < 0$, it follows that

$$\frac{\partial \beta_{L+1,L+2}}{\partial N} - \frac{\partial \beta_{L,L+1}}{\partial N} < 0. \quad (2.64)$$

Therefore the distance between the demand draws $\beta_{L,L+1}$ and $\beta_{L+1,L+2}$ decreases with $N$.

I can provide a lower bound for this expression as well. Using the same argument as above, but replacing $q_{L+1}(\beta_{L+1,L+2})$ with the quantities produced by the entrepreneur with demand draw $\beta_{L+1,L+2}$ using an organization of $L + 2$ layers, $q_{L+2}(\beta_{L+1,L+2})$, and replacing $q_{L+1}(\beta_{L,L+1})$ with the quantities produced by the entrepreneur with demand draw $\beta_{L,L+1}$ using an organization of $L$ layers, $q_L(\beta_{L,L+1})$ yields:

$$\frac{\partial \beta_{L+1,L+2}}{\partial N} - \frac{\partial \beta_{L,L+1}}{\partial N} > \frac{-2\gamma}{N^2}(q_{L+2}(\beta_{L+1,L+2}) - q_L(\beta_{L,L+1})).$$

### 2.6.5 Proof of Proposition 2.7

Let $\beta_{L,L+1}$ denote the demand draw at which an entrepreneur is indifferent between organizations $L$ and $L + 1$. Let $G(\alpha)$ be the cumulative distribution of demand draws, and let $\Lambda_N$ be the cumulative distribution of layers. It then follows that the probability mass of firms producing with at most $L$ layers is:

$$\Lambda_N(L) = \frac{[G(\beta_{L,L+1}) - G(\alpha_D)]}{1 - G(\alpha_D)}. \quad (2.65)$$

Taking the derivative of this expression with respect to $N$ yields:

$$\frac{\partial \Lambda_N(L)}{\partial N} = \frac{[1 - G(\alpha_D)] \left[ g(\beta_{L,L+1}) \frac{\partial \beta_{L,L+1}}{\partial N} - g(\alpha_D) \frac{\partial \alpha_D}{\partial N} \right] + [G(\beta_{L,L+1}) - G(\alpha_D)] g(\alpha_D) \frac{\partial \alpha_D}{\partial N}}{[1 - G(\alpha_D)]^2},$$

which, after eliminating common terms and adding and subtracting by the term, $[1 - G(\alpha_D)] g(\beta_{L,L+1}) \frac{\partial \beta_{L,L+1}}{\partial N}$,
can be rewritten as:

\[
\frac{\partial \Lambda_N(L)}{\partial N} = \frac{[1 - G(\alpha_D)] g(\beta_{L,L+1}) \frac{\partial \beta_{L,L+1}}{\partial N} - [1 - G(\beta_{L,L+1})] g(\alpha_D) \frac{\partial \alpha_D}{\partial N}}{[1 - G(\alpha_D)]^2} \\
= \frac{[1 - G(\alpha_D)] g(\beta_{L,L+1}) \left[ \frac{\partial \beta_{L,L+1}}{\partial N} - \frac{\partial \alpha_D}{\partial N} \right] + [[1 - G(\alpha_D)] g(\beta_{L,L+1}) - [1 - G(\beta_{L,L+1})] g(\alpha_D)] \frac{\partial \alpha_D}{\partial N}}{[1 - G(\alpha_D)]^2}
\]

(2.66)

Because the denominator is always positive, \( \frac{\partial \Lambda_N(L)}{\partial N} \) will be negative as long as the numerator is negative. In previous sections I have shown that \( \frac{\partial \beta_{L,L+1}}{\partial N} - \frac{\partial \alpha_D}{\partial N} \) is negative, and that \( \frac{\partial \alpha_D}{\partial N} \) is positive. Hence the numerator in equation (2.66) will be negative if the following condition holds:

\[
[1 - G(\alpha_D)] g(\beta_{L,L+1}) \leq [1 - G(\beta_{L,L+1})] g(\alpha_D),
\]

which can be rewritten as

\[
\frac{g(\beta_{L,L+1})}{[1 - G(\beta_{L,L+1})]} \leq \frac{g(\alpha_D)}{[1 - G(\alpha_D)]},
\]

(2.67)

Equation (2.67) is the hazard rate of the distribution of demand draws, \( G(\alpha) \). Thus as long as \( G(\alpha) \) has a non-increasing hazard rate, it follows that the probability mass of firms producing with at most \( L \) layers, \( \Lambda_N(L) \) will be decreasing with respect to \( N \). Therefore, if \( N' > N \), it follows that the distribution of layers in economy \( N' \), \( \Lambda_{N'} \), will first order stochastically dominate the distribution of layers in economy \( N \), \( \Lambda_N \).

### 2.6.6 When a Firm Increases Number of Layers it Produces More Output

In this subsection we characterize what happens to firms when they increase the number of layers. Since profits are increasing with \( \alpha \), consider the firm that is indifferent between layers \( L \) and \( L + 1 \). For this to be the case, the following condition must hold:

\[
\pi_L(\alpha) = \pi_{L+1}(\alpha).
\]
In other words, for the firm with demand draw $\alpha$ its profits from producing with $L$ and $L+1$ layers are the same. There are three cases to consider: (i) $q_L(\alpha) = q_{L+1}(\alpha)$ (ii) $q_L(\alpha) > q_{L+1}(\alpha)$ and (iii) $q_L(\alpha) < q_{L+1}(\alpha)$.

Consider case (i). From the firm’s optimization problem at $q_L(\alpha)$, the following holds:

$MR(q_L(\alpha)) = MC_L(q_L(\alpha))$. Similarly, at $q_{L+1}(\alpha)$, the following holds: $MR(q_{L+1}(\alpha)) = MC_{L+1}(q_{L+1}(\alpha))$. Since the quantities produced are the same, the marginal revenues should be equalized as well as the marginal costs. However this latter point is a contradiction since for any $\alpha$ it is the case that $MC_{L+1}(q(\alpha)) < MC_L(q(\alpha))$.

Consider case (ii). At $q_L(\alpha)$ we know that: $MC_L(q_L(\alpha)) > MC_{L+1}(q_{L+1}(\alpha))$. Since for any given number of layers $L$, marginal costs are increasing with the quantity produced, it must be the case that $q_{L+1}(\alpha) > q_L(\alpha)$.

Therefore only case (iii) remains. Further, since it is the case that $\pi_L(\alpha) = \pi_{L+1}(\alpha)$, it follows that $p_L(\alpha) - AC_L(q_L(\alpha)) > p_{L+1}(\alpha) - AC_{L+1}(q_{L+1}(\alpha))$.

### 2.6.7 Additional Proofs

Consider the equation that determines the optimal quantity produced by a firm with demand draw $\alpha$:

$$q = \frac{N}{2\gamma} \left\{ \frac{\alpha - \eta M}{\gamma + \eta M} (\bar{\pi} - \bar{p}) - MC(q) \right\}.$$

This equation defines a mapping, $q(\alpha)$, from the set of demand draws $[\alpha_M, \infty]$ to the set of quantities $[0, \infty]$, where for all $\alpha \in [\alpha_M, \alpha_D]$, $q(\alpha) = 0$ and at $\alpha_D$, $q(\alpha_D) = q_D$. In addition, for Proposition 2.3, it follows that $q(\alpha)$ is a one-to-one mapping, and so its inverse, $\alpha(q)$, is well-defined.

With $\alpha(q)$, I can now proceed to analyze the model from the perspective of the quantities produced, $q$. That is, in these proofs, the primitive of the equations is no longer $\alpha$ but instead $q$.

First consider how $\alpha(q)$ changes with $q$. From the quantity equation, it follows that:
\[ 1 = \frac{N}{2\gamma} \left\{ \frac{\partial \alpha(q)}{\partial q} - \frac{\partial MC(q)}{\partial q} \right\}, \]

rewritten from the perspective of \( q \), which implies that:

\[ \frac{\partial \alpha(q)}{\partial q} = \frac{2\gamma}{N} + \frac{\partial MC(q)}{\partial q}. \] (2.68)

Therefore \( \frac{\partial \alpha(q)}{\partial q} \) is increasing with \( q \). In addition, within layers, one can directly relate \( \frac{\partial \alpha(q_L)}{\partial q_L} \) to \( \frac{\partial q_L(\alpha)}{\partial \alpha} \) with the following expression, derived from the optimal quantity equation:

\[ \frac{\partial q_L(\alpha)}{\partial \alpha} = \frac{1}{\frac{2\gamma}{N} + \frac{\partial MC_L(q_L(\alpha))}{\partial q_L(\alpha)}} = \frac{1}{\frac{\partial \alpha(q_L)}{\partial q_L}}. \] (2.69)

Now consider how \( \alpha(q) \) changes with respect to marginal costs. From the quantity equation it follows that:

\[ \frac{\partial \alpha(q)}{\partial MC} = 1. \] (2.70)

Therefore a quantity produced with a greater number of layers, is mapped into a lower \( \alpha \).

Now consider the price associated with a quantity, \( q \). From the optimal price equation:

\[ p(q) = \frac{1}{2} \left\{ \alpha(q) - \frac{\eta M}{\gamma + \eta M} (\alpha - \bar{p}) + MC(q) \right\}, \]

rewritten from the perspective of \( q \), and it follows that:

\[ \frac{\partial p(q)}{\partial q} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial q} + \frac{\partial MC(q)}{\partial q} \right] = \frac{\gamma}{N} + \frac{\partial MC(q)}{\partial q}. \] (2.71)

Therefore, the price charged for quantity \( q \) is increasing with \( q \).

Now consider how \( p(q) \) changes with respect to marginal costs. From the price equation it
follows that:

\[
\frac{\partial p(q)}{\partial MC} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial MC} + 1 \right]
\]

\(= 1.\) \hfill (2.72)

Therefore a quantity produced with a greater number of layers, is mapped into lower prices.

Now consider the markup over marginal costs associated with a quantity, \(q\). From the markup equation:

\[
\mu_{MC}(q) = \frac{1}{2} \left\{ \alpha(q) - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC(q) \right\},
\]

rewritten from the perspective of \(q\), and equation (2.68) it follows that:

\[
\frac{\partial \mu_{MC}(q)}{\partial q} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial q} - \frac{\partial MC(q)}{\partial q} \right]
\]

\(= \gamma \frac{N}{2} \). \hfill (2.73)

Therefore, the markups over marginal costs charged for quantity \(q\) is increasing with \(q\). Now consider how \(\mu_{MC}(q)\) changes with respect to marginal costs. From the markups equation it follows that:

\[
\frac{\partial \mu_{MC}(q)}{\partial MC} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial MC} - 1 \right]
\]

\(= 0.\) \hfill (2.74)

Therefore a quantity produced with a greater number of layers, does not change markups over marginal costs.

Now consider the revenues associated with a quantity, \(q\). From the revenue equation:

\[
r(q) = \frac{\gamma}{N} q^2 + q MC(q),
\]

rewritten from the perspective of \(q\), it follows that:
\[ \frac{\partial r(q)}{\partial q} = \frac{2\gamma}{N} q + MC(q) + q \frac{\partial MC(q)}{\partial q}. \]  \hspace{2cm} (2.75)

Therefore, the revenues earned for quantity \( q \) is increasing with \( q \). Now consider how \( r(q) \) changes with respect to marginal costs. From the revenue equation it follows that:

\[ \frac{\partial r(q)}{\partial MC} = q. \]  \hspace{2cm} (2.76)

Therefore a quantity produced with a greater number of layers, is mapped into lower revenues.

Finally, consider the profits associated with a quantity, \( q \). From the profit equation:

\[ \pi(q) = \frac{\gamma}{N} q^2 + qMC(q) - C(q), \]

rewritten from the perspective of \( q \), it follows that:

\[ \frac{\partial \pi(q)}{\partial q} = 2\frac{\gamma}{N} q + q \frac{\partial MC(q)}{\partial q}. \]  \hspace{2cm} (2.77)

Therefore, the profits earned for quantity \( q \) is increasing with \( q \). Now consider how \( \pi(q) \) changes with respect to marginal costs. From the profit equation it follows that:

\[ \frac{\partial \pi(q)}{\partial MC} = q - \frac{\partial C(q)}{\partial MC}. \]  \hspace{2cm} (2.78)

Therefore, because \( \frac{\partial C(q)}{\partial MC} = q \), a quantity produced with a greater number of layers is mapped into the same profits.

Now consider the markup over average costs associated with a quantity, \( q \). From prices it follows that:

\[ p(q) = \mu^{AC}(q) + AC(q) \]
\[ = \mu^{MC}(q) + MC(q), \]

and therefore it follows that:
\[\mu^{AC}(q) - \mu^{MC}(q) = MC(q) - AC(q).\]

Taking the derivative with respect to \(q\) yields:

\[
\frac{\partial \mu^{AC}(q)}{\partial q} - \frac{\partial \mu^{MC}(q)}{\partial q} = \frac{\partial MC(q)}{\partial q} - \frac{\partial AC(q)}{\partial q} > 0.
\]

Therefore, this implies that:

\[
\frac{\partial \mu^{AC}(q)}{\partial q} > \frac{\partial \mu^{MC}(q)}{\partial q} > 0.
\]

(2.79)

Therefore, the markups over average costs charged for quantity \(q\) is increasing with \(q\). Now consider how \(\mu^{AC}(q)\) changes with respect to marginal costs. From the markups equation it follows that:

\[
\frac{\partial \mu^{AC}(q)}{\partial MC} = \frac{\partial}{\partial MC} \left( \frac{\pi(q)}{q} \right) = 0.
\]

(2.80)

Therefore a quantity produced with a greater number of layers, does not change markups over average costs.

### 2.6.8 Additional Proofs with respect to \(N\)

Consider the equation that determines the optimal quantity produced by a firm with demand draw \(\alpha\):

\[q = \frac{N}{2\gamma} \left\{ \alpha - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC(q) \right\}.\]
This equation defines a mapping, \( q(\alpha) \), from the set of demand draws \([\alpha_M, \infty]\) to the set of quantities \([0, \infty]\), where for all \( \alpha \in [\alpha_M, \alpha_D] \), \( q(\alpha) = 0 \) and at \( \alpha_D \), \( q(\alpha_D) = q_D \). In addition, for Proposition 2.3, it follows that \( q(\alpha) \) is a one-to-one mapping, and so its inverse, \( \alpha(q) \), is well-defined.

With \( \alpha(q) \), I can now proceed to analyze the model from the perspective of the quantities produced, \( q \). That is, in these proofs, the primitive of the equations is no longer \( \alpha \) but instead \( q \).

Now consider a change in the size of the market, \( N \). From the quantity equation, it follows that:

\[
0 = q + N \left\{ \frac{\partial \alpha(q)}{\partial N} - \frac{\partial \eta M (\bar{\alpha} - \bar{p})}{\partial N} \right\},
\]

which implies that:

\[
\frac{\partial \alpha(q)}{\partial N} = \frac{\partial \eta M (\bar{\alpha} - \bar{p})}{\partial N} - \frac{2\gamma}{N^2} q.
\] (2.81)

Therefore, a given quantity \( q \), may be assigned to a bigger or a smaller \( \alpha \) when the market size increases. In addition, within layers, one can directly relate \( \frac{\partial \alpha(q_L)}{\partial N} \) to \( \frac{\partial q_L(\alpha)}{\partial N} \) with the following expression, derived from the optimal quantity equation:

\[
\frac{\partial q_L(\alpha)}{\partial N} \left[ \frac{2\gamma}{N} + \frac{\partial MC_L(q_L(\alpha))}{\partial q_L(\alpha)} \right] = \frac{2\gamma}{N^2} q - \frac{\partial \eta M \frac{\partial \alpha(q_L)}{\partial N}}{\partial N} = -\frac{\partial \alpha(q_L)}{\partial N}.
\] (2.82)

Now consider the price associated with a quantity, \( q \). From the optimal price equation:

\[
p(q) = \frac{1}{2} \left\{ \alpha(q) - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) + MC(q) \right\},
\]

rewritten from the perspective of \( q \), and equation (2.68) it follows that:
\[
\frac{\partial p(q)}{\partial N} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial N} - \frac{\eta M}{\gamma + \eta M} \frac{\partial (\bar{\alpha} - \bar{p})}{\partial N} \right]
\]
\[
= -\frac{\gamma}{N^2} q. \tag{2.83}
\]

Therefore, an increase in the market size \(N\) decreases the price charged for quantity \(q\).

Now consider the markup over marginal costs associated with a quantity, \(q\). From the markup equation:

\[
\mu^{MC}(q) = \frac{1}{2} \left\{ \alpha(q) - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) - MC(q) \right\},
\]
rewritten from the perspective of \(q\), and equation (2.81) it follows that:

\[
\frac{\partial \mu^{MC}(q)}{\partial N} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial N} - \frac{\eta M}{\gamma + \eta M} \frac{\partial (\bar{\alpha} - \bar{p})}{\partial N} \right]
\]
\[
= -\frac{\gamma}{N^2} q. \tag{2.84}
\]

Therefore, an increase in the market size \(N\) decreases the markup over marginal costs charged for quantity \(q\).

Now consider the markup over average costs associated with a quantity, \(q\). From the markup equation:

\[
\mu^{AC}(q) = \frac{1}{2} \left\{ \alpha(q) - \frac{\eta M}{\gamma + \eta M} (\bar{\alpha} - \bar{p}) + MC(q) \right\} - \frac{C(q)}{q},
\]
rewritten from the perspective of \(q\), and equation (2.68) it follows that:

\[
\frac{\partial \mu^{AC}(q)}{\partial N} = \frac{1}{2} \left[ \frac{\partial \alpha(q)}{\partial N} - \frac{\eta M}{\gamma + \eta M} \frac{\partial (\bar{\alpha} - \bar{p})}{\partial N} \right]
\]
\[
= -\frac{\gamma}{N^2} q. \tag{2.85}
\]

Therefore, an increase in the market size \(N\) decreases the markup over average costs charged
for quantity $q$.

Now consider the revenues associated with a quantity, $q$. From the revenue equation:

$$r(q) = \frac{\gamma}{N} q^2 + qMC(q),$$

rewritten from the perspective of $q$, it follows that:

$$\frac{\partial r(q)}{\partial N} = -\frac{\gamma}{N^2} q^2. \quad (2.86)$$

Therefore, an increase in the market size $N$ decreases the revenues earned for quantity $q$.

Finally, consider the profits associated with a quantity, $q$. From the profit equation:

$$\pi(q) = \frac{\gamma}{N} q^2 + qMC(q) - C(q),$$

rewritten from the perspective of $q$, it follows that:

$$\frac{\partial \pi(q)}{\partial N} = -\frac{\gamma}{N^2} q^2. \quad (2.87)$$

Therefore, an increase in the market size $N$ decreases the profits earned for quantity $q$. 

Chapter 3

Teams, Trade and Comparative Advantage
3.1 Introduction

It is well known that workers are heterogeneous along a variety of dimensions. Workers differ in their education, experience and cognitive abilities. These differences affect agents’ labor force decisions and outcomes. In particular, differences in agents’ ability will determine the sector they are employed in, whether they are managers or production workers, and the number and type of agents they will form teams with. These decisions will ultimately determine their earnings as well as whether they better or worse off from international trade. In the literature, several studies have examined the effects of trade in a setting where heterogeneous agents sort into different sectors, or within a sector where agents sort into an occupation and match together to form teams. However, to my knowledge, there has not been a study that examines the effects of trade in a setting where heterogeneous agents simultaneously sort into sectors and occupations, and match into teams.

In this paper, I develop a model to understand how the sorting of heterogeneous agents into sectors and occupations, and the matching of agents into teams within sectors, determines trade flows, earnings and earnings inequality. To accomplish this, I extend the model of Garicano and Rossi-Hansberg (2004, 2006) to two sectors and two countries.

In both sectors, production requires labor and knowledge. Agents are endowed with one unit of time, and to produce a unit of output, a problem needs to be solved. Problems however vary in their level of difficulty, and agents are heterogeneous in their ability to solve problems. When communication is possible between agents, it is inefficient for all agents to attempt to solve all problems. Instead, as argued by Garicano (2000), agents work in teams, composed of one manager and many production workers, and specialize. In teams, production workers use their time to generate production possibilities while managers do not, instead they focus on solving problems that production workers cannot solve. When a production worker is confronted with a problem that he cannot solve, he asks his manager who spends a fraction of his time communicating with his worker. If the manager knows the solution to the problem, then he conveys the solution to his worker who produces a unit of output. Otherwise, nothing is produced. In teams, therefore, production workers specialize in routine tasks (i.e. production), while managers specialize non-routine tasks (i.e. problem solving).
The two sectors vary along the following dimension. In one sector, communication is possible between agents and production takes place in teams, while in the other sector communication is not possible and agents are self-employed and work alone. Furthermore, since agents are heterogeneous in their ability to solve problems, they choose the sector and occupation that maximizes the earnings.

I first examine the closed economy. I show that an equilibrium of the model is characterized by two ability thresholds, where agents with ability above the higher threshold sort into managerial occupations and form teams with agents below the lower threshold. The remaining agents, with ability between both thresholds sort into the self-employment sector. The equilibrium also exhibits positive assortative matching, in the sense that higher ability managers form teams with higher ability production workers. In addition, the earnings function is convex.

I then consider an open economy with two countries with different distributions of ability. I find that the country with more able workers has a comparative advantage in the sector where goods are produced in teams, while the other country has a comparative advantage in the self-employment sector.

In the open economy, I also show that factor price equalization (FPE) does not hold. Although the production technology is the same in both countries, identical workers located in different countries do not always earn the same income. This is due to two factors. First, depending on the country that he resides in, a worker may choose to work in a different sector. Second since countries are not identical, the assignment of production workers to managers is different. Both of these factors ensure that factor price equalization will not hold in the open economy.

I also find that a transition from autarky to frictionless trade causes the earnings of managers and production workers to move in the same direction. This finding is different from the well-known Heckscher-Ohlin model, where a transition from autarky to frictionless trade causes the returns to different factors of production to move in the opposite direction. Further since countries are not identical the country with a more knowledgeable population specializes in the production of the good produced in teams, while the other country specializes in the good produced by self-employed workers. Furthermore, I find that international trade always increases income inequality in the country with the more knowledgeable
population. In the other country, whether international trade increases or decreases income inequality depends on their level of knowledge and the cost of communication between managers and workers.

This paper is related to a literature in international trade that investigates how heterogeneous agents sort into industries. Prominent research papers in this literature include Costinot (2009), Costinot and Vogel (2010) and Ohnsorge and Trefler (2012). All of these studies emphasize agents’ comparative advantage from producing in different industries, and investigate how the distribution of skills determines aggregate trade flows across countries, and the impact trade has on earnings inequality. By allowing agents to simultaneously sort into sectors and occupations, my paper adds new dimension to this literature.

This paper is also related to a literature in international trade that investigates how heterogeneous agents form teams with one another. Prominent research papers in this literature include Grossman and Maggi (1999) and Grossman, Helpman and Kircher (2013). These studies examine how different technologies determine how agents form teams, and they investigate how the distribution of skills determines aggregate trade flows as well as the impact trade has on earnings inequality. The main difference between this paper and the existing literature is that I provide a micro-founded model for the formation of teams, and allow agents’ to simultaneously decide on the sector they are employed in, whether they are managers or production workers, and the number and type of agents they will form teams with.

Also this paper is similar to Antras, Garicano and Rossi-Hansberg (2006) who extend Garicano and Rossi-Hansberg (2006) to study how offshoring, or the formation of international teams, affects the distribution of earnings in both the domestic and foreign country. Although I use the same production technology as this study, my paper is different along two dimensions. First I consider two sectors, and second I only allow for the formation of domestic teams.

The paper will proceed as follows. Section 3.2 presents the closed economy version of the model. Section 3.4 defines a closed-economy competitive equilibrium, and shows that it exists. Section 3.5 discusses how the equilibrium changes, with the model’s exogenous parameters. Section 3.6 presents the open-economy model and discusses some comparative static results. And finally, section 3.7 concludes. All proofs are relegated to the appendix.
3.2 The Model

3.2.1 Production

There are two sectors in this economy.\(^1\) In sector 1 production occurs in teams, composed of one manager and many workers, while in sector 2 all workers are self-employed. In both sectors production requires labor and knowledge. To produce a unit of output, agents draw a problem that needs to be solved. Problems vary in their difficulty and the difficulty of problems is drawn from a cumulative distribution \(F(\omega)\) with support \(\Omega = [0, 1]\).

Agents are heterogeneous in their ability to solve problems. Agents draw their knowledge, \(z\), from a cumulative distribution \(G(z)\), with support \([0, \alpha]\), where \(\alpha \leq 1\). The boundary point \(\alpha\) represents the maximum amount of knowledge an agent can possess in the economy.\(^2\) I assume that knowledge is cumulative. Agents with knowledge \(z\), are able to solve all problems in the set \(A = [0, z]\). In addition, I assume that the actual problem drawn is unobservable; that is, it is impossible to label problems. When agents are confronted with a problem, they only know if they can solve it or not.\(^3\) Thus an agent with knowledge \(z\), when confronted with an unobservable problem \(\omega\), can produce a unit of output, if and only if \(\omega \in A\).

Good 1 is produced in teams, composed of one manager and many production workers. Production workers specialize in routine tasks (i.e. production), while managers specialize in nonroutine tasks (i.e. problem solving). Workers use their unit of time to generate production possibilities while managers do not, they focus on solving problems. To produce a unit of output, workers draw a problem from the distribution, \(F(\omega)\). The type of problem a worker draws is unobservable, and if the worker can solve the problem, then a unit of output is produced. If the worker cannot solve the problem, he asks his manager who in turn spends a fraction, \(h\), units of his time communicating with his worker. If the manager knows the solution to the problem, then he conveys the solution to his worker who immediately produces a unit of output. Otherwise, nothing is produced.

\(^1\)The production framework and the closed form solutions to the equations of the model are similar to Antras, Garicano and Rossi-Hansberg (2006).
\(^2\)If \(\alpha\) is less than 1, then the most knowledgeable agent will not be able to solve every problem.
\(^3\)This assumption is crucial to the model. If problems were observable, then organizations would not arise. That is, through market transactions there would be a one-to-one matching of problems and agents who can solve them.
Managers are the residual claimants of the earnings of a team. Consider a team composed of a manager with ability $z_m$ and $n$ workers with ability $z_p$. Because workers draw problems and the manager spends his time solving problems, the output of a team is determined by the number of problems drawn, $n$, and the proportion of problems the manager can solve, $nF(z_m)$. Each problem solved yields one unit of output that can be sold at price $p_1$. In addition, every worker with ability $z_p$ earns a wage $w(z_p)$. Thus the manager’s rents are given by the team’s revenues minus its wage costs and is equal to $n [p_1 F(z_m) - w(z_p)]$.

The communication technology limits the amount of interactions managers can have with their workers, and thus determines the amount of workers a manager can supervise. Because managers have one unit of time and whenever a worker cannot solve a problem the manager spends $h$ units of time communication with him, a manager in a team with workers of ability $z_p$ can supervise at most $n [1 - F(z_p)]$ workers. A point to note is that since $h$ is less than one, the structure of the model implies that a manager can supervise more than one worker, and thus matching between managers and workers is one to many.

A manager with ability $z_m$ therefore chooses the size of his team, $n$, and the ability of his workers $z_p$, with the aim to maximize his rents subject to his time constraint. Formally, the manager’s problem is:

$$R(z_m; w) = \max_{\{z_p, n\}} n [p_1 F(z_m) - w(z_p)]$$

subject to

$$hn [1 - F(z_p)] = 1$$

(3.1)

The first-order condition to the managers’ problem (3.1) is given by:

$$w'(z_p) = \frac{[p_1 F(z_m) - w(z_p)] F'(z_p)}{1 - F(z_p)}, \quad \forall z_p \leq z_1,$$

(3.2)

where $z_1$ denotes ability of most knowledgeable worker in sector 1. More formally, as will be shown below, the equilibrium will be characterized by values $z_1$ and $z_2$ such that all agents with knowledge between $[0, z_1]$ will choose to be production workers in sector 1, all agents

---

4The production function has three important requirements for matching identified by Kremer and Maskin (1997): individuals with different abilities are imperfect substitutes, output is differentially sensitive to skill, and inputs are complements.

5If $h$ is greater than or equal to 1 then managers are better off not working in teams and being self-employed.

6Managers only hire workers of a single ability. See Antras, Garicano and Rossi-Hansberg (2006) for details.
with knowledge between \([z_1, z_2]\) will be self-employed workers in sector 2, and all agents with ability between \([z_2, \alpha]\) will choose to be managers.

Given the production framework, managers and workers are complements. Managers of greater ability can solve a greater proportion of problems, and thus render workers more productive. Similarly, more able workers increase the productivity of managers. Because they can solve a greater proportion of problems on their own, more able workers spend less time communicating with managers. This frees up their managers’ time and allows them to supervise larger teams. As a result, in sector 1 there will be positive assortative matching between managers and production workers. I summarize this result in the following proposition:

**Proposition 3.1** In sector 1, managers and production workers are complements.

**Proof.** See Appendix.

In sector 2 agents are neither managers nor do they work for managers.\(^7\) Since they are not members of a team, they cannot use the knowledge of other agents to produce and therefore their earnings depend solely on their ability. Because he can solve a fraction \(z\) of problems, a self-employed agent with ability \(z\) will produce \(F(z)\) units of good 2 which he can sell at a price \(p_2\), which I normalize to 1. Therefore his earnings from operating in sector 2 are \(S(z) = F(z)\).

To summarize, in both sectors, agents draw problems from the same distribution, \(F(\omega)\). The only difference between both sectors is the communication technology. In sector 1 goods can be produced in teams and managers and production workers are complements. The parameter \(h\) determines the degree of complementarity between managers and production workers. In contrast, in sector 2, goods can only be produced by self-employed agents. There are no complementarities in production between agents in sector 2. Indeed, one way to think about sector 2 is that the communication technology, \(h\), is greater than 1.

### 3.2.2 Consumer’s Problem

Consumers have a Cobb-Douglas utility over goods produced in both sectors, labeled \(x_1\) and \(x_2\). An agents’ income depends on the sector he is employed in and his occupation. For an

\(^7\)Self-employed agents also possess 1 unit of time.
agent with ability \( z \), his earnings from being a production worker in sector 1 are \( w(z) \), from managing a team in sector 1 are \( R(z) \), and his earnings from being a self-employed worker in sector 2 are \( S(z) \). Therefore given these occupational choices, an agent with ability \( z \) will choose the sector and occupation that maximizes his earnings. To summarize, an agent with skill level \( z \) faces the following utility maximization problem:

\[
\max_{\{x_1, x_2\}} x_1^\beta x_2^{1-\beta}
\]

\[
\text{s.t } p_1 x_1 + x_2 = E(z),
\]

where \( E(z) = \max \{w(z), S(z), R(z)\} \) represents his earnings from the occupation that provides him with the highest return. For an agent with ability \( z \), the optimal consumption bundle of \( x_1 \) and \( x_2 \), as a function of income and prices is:

\[
x_1 = \beta \frac{E(z)}{p_1},
\]

\[
x_2 = (1 - \beta) E(z).
\]

### 3.2.3 Market Clearing Conditions

There are three markets in this economy: the labor market, the market for good 1 and the market for good 2. In equilibrium all three markets must clear. I first describe the labor market clearing condition, and then proceed to the markets for the final goods.

Let \( m(z_p) \) represent the skill level of a manager that is matched to a worker with ability \( z_p \). Because managers and production workers are complements, it follows that in an equilibrium there will be positive sorting, implying that \( m(z_p) \) is increasing and invertible. In equilibrium the demand for any given worker with ability \( z_p \) must equal its supply. The labor market clearing condition thus has the following form:

\[
\int_{m(0)}^{m(z_p)} n(m^{-1}(z)) g(z) dz = \int_0^{z_p} g(z) dz, \quad \forall z_p \leq z_1
\]

where \( m^{-1}(z) \) is the ability of a worker matched to a manager with ability \( z \). The left-hand
side of equation (3.5) represents the demand for workers by managers, while the right-hand side represents the supply of workers. Since equation (3.5) holds for values of $z_p \leq z_1$, we can substitute for $n(m^{-1}(z))$ and take the derivative with respect to $z_p$ to obtain the slope of the assignment function. Namely,

$$m'(z_p) = \frac{h[1 - F(z_p)]g(z_p)}{g(m(z_p))} \quad (3.6)$$

Equation (3.6), along with the conditions that the least knowledgeable manager is matched with the worst worker (i.e. $m(0) = z_2$), and the most knowledgeable manager is teamed up with the best worker (i.e. $m(z_1) = \alpha$) determine the assignment function $m(z)$.

An important point to note is the slope of the assignment function depends on the parameter $h$. When communication costs are high, production workers’ knowledge is more important for production, and thus better workers will be assignment to better managers.

In equilibrium the aggregate demand for goods $x_1$ and $x_2$ must equal their aggregate supply. Agent $z$ wants to consume $\beta E(z)g(z)$ units of $x_1$ and $(1 - \beta)E(z)$ units of $x_2$. In sector 1, every worker with ability $z$ is assigned to a manager with ability $m(z)$ and hence can produce $F(m(z))$ units of good 1. In sector 2, every self-employed worker with ability $z$ produces $F(z)$ units of good 2. Therefore, the goods market clearing conditions for $x_1$ and $x_2$, respectively, are

$$\frac{\beta}{p_1} \int_0^\alpha E(z)g(z)dz = \int_0^{z_1} F(m(z))g(z)dz,$$

$$\int_0^\alpha (1 - \beta)E(z)g(z)dz = \int_{z_1}^{z_2} F(z)g(z)dz.$$  \quad (3.7)

Equation (3.6), along with the conditions that the least knowledgeable manager is matched with the worst worker (i.e. $m(0) = z_2$), and the most knowledgeable manager is teamed up with the best worker (i.e. $m(z_1) = \alpha$) determine the assignment function $m(z)$.

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$$\frac{\beta}{p_1} \int_0^\alpha E(z)g(z)dz = \int_0^{z_1} F(m(z))g(z)dz,$$

$$\int_0^\alpha (1 - \beta)E(z)g(z)dz = \int_{z_1}^{z_2} F(z)g(z)dz.$$  \quad (3.7)

Every agent’s budget constraint must also balance. An agent’s income is derived from the sector he is employed in and his occupation, and is determined endogenously. In the economy, therefore, aggregate income must be equal to the total value of production, as shown explicitly in equation (3.9):

$$\int_0^\alpha E(z)g(z)dz = p_1 \int_0^{z_1} m(z)g(z)dz + \int_{z_1}^{z_2} F(z)g(z)dz \quad (3.9)$$

These conditions follow from the fact that $m(z_p)$ exhibits positive sorting.
3.3 Equilibrium

The definition of a competitive equilibrium is the following:

**Definition 3.2** A competitive equilibrium consists of a wage function \( w(z) \), a rent function \( R(z) \), an assignment function \( m(z) \), a relative price \( \{p_1\} \) and a pair of thresholds \( \{z_1, z_2\} \) such that the following conditions hold:

1. Agents maximize utility
2. Managers maximize rents
3. All markets clear

Condition 1 of the definition of an equilibrium implies that every agent chooses the occupation that maximizes his income, and agents’ demand for goods 1 and 2 satisfy the first-order condition to the consumer’s maximization problem. Condition 2 implies managers’ decisions solve (3.1) and given the wage function, \( w(z) \), the most able managers are unwilling to hire agents with ability above \( z_1 \).\(^9\) Furthermore for condition 2 to be satisfied in an equilibrium two conditions must be met. First the earnings function must be continuous on the interval \([0, \alpha]\), and differentiable at all values of \( z \) except at the threshold values \( z_1 \) and \( z_2 \).\(^10\) Since each agent chooses the occupation and sector with the highest earnings and agents can always choose to become self-employed workers in sector 2, the continuity of the earnings function implies that an agent with ability \( z_1 \) must be indifferent between being a production worker and a self-employed worker, and agent \( z_2 \) must be indifferent between being a manager and a self-employed worker. In other words, the earnings of the production worker with ability \( z_1 \) must satisfy the condition \( w(z_1) = F(z_1) \), and the earnings of the manager must satisfy the condition \( R(z_2) = F(z_2) \). Second, at \( z_1 \) for managers to be unwilling to hire agents with ability above \( z_1 \), the marginal return of production workers must be less than the marginal return of self-employed workers: i.e. \( w'(z_1) < F'(z_1) \).

Condition 3 implies that the markets for goods 1 and 2 clear, the labor market clears, and

\(^9\)As shown in the appendix, this is prevented from happening as long as \( w'(z_1) < F'(z_1) \).
\(^10\)If the earnings function is not continuous at points \( z_1 \) and \( z_2 \), agents marginally below or above will wish to deviate from their occupation. Differentiability of wage and rent function is required from the manager’s optimization problem.
aggregate income equals aggregate production. In addition, labor market clearing implies that equation (3.6) describes the slope of the assignment function, and the boundary points of the assignment function satisfy the conditions: \( m(0) = z_2 \), and \( m(z_1) = \alpha \).

To simplify the analysis, assume that \( G(z) \) and \( F(z) \) are uniformly distributed over their respective domains. From (3.6) and the boundary condition \( m(0) = z_2 \), the assignment function is equal to:

\[
m(z) = z_2 + h \left( z - \frac{z^2}{2} \right), \quad \forall z \leq z_1.
\]

Substituting this expression into (3.2) and imposing the boundary condition \( w(z_1) = F(z_1) \) yields the wage equation for workers in sector 1:

\[
w(z) = p_1 z_2 - \sigma (1 - z) + 1/2 p_1 h z^2, \quad \forall z \leq z_1
\]

where \( \sigma = \frac{p_1 z_2 + 1/2 p_1 h z^2 - z_1}{1 - z_1} \).\(^{11}\) The slope of the wage function, and the marginal return to a worker’s ability are given by:

\[
w'(z) = \sigma + p_1 h z, \quad \forall z \leq z_1.
\]

The marginal return to a worker’s ability is increasing with his ability, and therefore wages are convex. This effect is captured by the quadratic expression \( 1/2 p_1 h z^2 \) which arises from the fact that workers of different ability are imperfect substitutes for one another, and is determined by the level of complementarity between managers and production workers as well as the relative price of good 1. The second component to the marginal return to ability, \( \sigma \), is determined by the demand and supply of workers in sector 1.

The rents of a manager with ability \( z \) are given by:

\[
R(z) = \frac{p_1 z - w(m^{-1}(z))}{h \left[ 1 - m^{-1}(z) \right]},
\]

and using the envelope theorem, the marginal return to a manager’s ability is given by:

\[
R'(z) = \frac{p_1}{h \left[ 1 - m^{-1}(z) \right]}.
\]

\(^{11}\)From the equilibrium condition \( R(z_2) = F(z_2) \) it follows that \( \sigma = z_2 h > 0 \).
Since the assignment function $m(z)$ is invertible and less than one, the marginal return to a manager’s ability is increasing and convex. In addition, the marginal return to a manager’s ability is equal to $p_1n(z)$. Therefore the higher is the relative price of good 1 or the greater the number of production workers in his team, the greater will be the marginal return to a manager’s ability.

The earnings of self-employed workers in sector 2 are given by:

$$S(z) = z.$$  

The slope of the earnings function, and the marginal return to a self-employed worker’s ability is 1. The marginal return to a self-employed worker’s ability is therefore increasing with his ability and earnings are linear in ability. Since agents in sector 2 do not work in teams, there are no complementarity components influencing their earnings.

Under the assumption of uniform distributions, the expressions for the supply of goods 1 and 2 are given by:

$$\int_0^{z_1} m(z)g(z)\,dz = \frac{1}{6\alpha} [6z_2z_1 - hz_1^3 + 3hz_1^2],$$

$$\int_{z_1}^{z_2} zg(z)\,dz = \frac{1}{2\alpha} [z_2^2 - z_1^2].$$

Proposition 3.3 states that an equilibrium exists under the assumption that $G(z)$ and $F(z)$ are uniformly distributed.

**Proposition 3.3** Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of $\alpha$ and $h \in [0, \overline{h}]$ there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings function is convex and the sets of managers, self-employed workers and production workers are connected.

**Proof.** See Appendix. ■

Condition (2) from the definition of an equilibrium requires that managers at $\alpha$ must be averse to hiring workers slightly above $z_1$. For very high values of $h$ this is not always the case. The manager’s problem indicates that a team’s size, output and revenues, are inversely
related to the costs of communication. For a given value of \( h \), managers can always increase their revenues by employing more knowledgeable agents at the cost of higher wages. In an equilibrium, it must be the case that the increase in labor costs is greater than the increase in revenues; otherwise, managers would not be maximizing their profits. When communication costs are high this is not the case. When \( h \) is high the knowledge of production workers is important for the team’s production and managers with ability \( \alpha \) will have an incentive to deviate from their match and employ more knowledgeable agents. By employing more knowledgeable agents, their revenues will increase by more than the additional labor costs.\(^{12}\)

Since \( h \) is the fraction of time a manager spends communicating with a worker, it cannot be greater than 1. As discussed in the appendix, condition (2) is equivalent to the restriction \( h < \frac{1}{z_1 + z_2} \). Therefore, these two restrictions imply that \( h < \min \left\{ \frac{1}{z_1 + z_2}, 1 \right\} \). Since there does not exist a closed form solution for the equilibrium values of \( z_1 \) and \( z_2 \), it is not possible to obtain an expression for \( \frac{1}{z_1 + z_2} \) as function of the parameters of the model. Hence, it is impossible to accurately determine an upper bound on \( h \). One restrictive assumption is for \( h \) to be less than the smallest value \( \frac{1}{z_1 + z_2} \) can undertake in the domain \([0, \alpha] \times [0, \alpha]\), that is \( h < \overline{h} = \min \left\{ \frac{1}{2\alpha}, 1 \right\} \). This assumption guarantees that an equilibrium exist for all values of \( h \) in the interval \([0, \overline{h}]\).\(^{13}\)

Figure 3.1 depicts earnings as a function of ability in an economy with parameters \( \beta = 0.5, h = 0.4 \) and \( \alpha = 1 \). The earnings function is continuous, and nondifferentiable at points 0.4305 and 0.8649, where an individual is indifferent between becoming a production worker and a self-employed worker, and a self-employed worker and a manager, respectively. For agents in sector 1, earnings are increasing with ability and are convex, while in sector 2 earnings increase linearly with ability. In this numerical example the least able workers earn 0.2557 while the most productive workers earn 0.4305. Further, since managers can leverage their ability across many workers they benefit more from working in teams than production workers. Their earnings increase faster than the earnings of both production and self-employed workers. The least able managers earn 0.8649, while the most productive managers earn 1.1644.

\(^{12}\)Mathematically, when \( h \) is high the condition \( w'(z_1) < F'(z_1) \) is no longer satisfied.

\(^{13}\)Simulations of the model suggest that \( h \) can be as high as 0.92.
3.4 Comparative Statics

Assume consumers value goods 1 and 2 equally (i.e. $\beta = 0.5$). This section examines the comparative statics of the model. First, it examines how a change in the distribution of worker abilities affects the equilibrium. Second, it investigates how a change in the cost of communication, $h$, affects the economy. And finally, it relates these findings to changes in earnings inequality.

The following proposition indicates how the cutoffs, $z_1$ and $z_2$ change with $\alpha$:

**Proposition 3.4** An increase in $\alpha$ has the following effect:

1. The maximum ability of a worker, and the minimum ability of a self-employed worker, $z_1$, increases

2. The minimum ability of a manager, and the maximum ability of a self-employed workers, $z_2$, increases.
Proof. See Appendix.

Increasing $\alpha$ decreases the density $\frac{1}{\alpha}$ and changes the distribution of worker abilities. Intuitively, as $\alpha$ increases, holding everything else constant, more agents will choose to become workers, increasing $z_1$. As a result, the most knowledgeable managers will now be able to manage larger teams and the aggregate output of good 1 increases. The increase in aggregate output, however, decreases the relative price of good 1, thereby reducing the earnings of the existing set of managers. This last effect forces the least knowledgeable of managers to exit the industry for self-employment, thereby increasing $z_2$.

Simulations, presented in Figure 3.2, provide additional information on how a change in the distribution of worker abilities affects the equilibrium. First, consistent with proposition 3.4 in panel (a) both ability thresholds increase with $\alpha$. Second, as illustrated in panel (b) the mass of production workers, self-employed workers and managers, increases with $\alpha$. Since the sets of managers and self-employed workers are larger, it follows that the aggregate output of goods 1 and 2 increase with $\alpha$ as well, as shown in panel (c). Third, notice in panel (d) that the relative price of good 1, $p_1$ decreases as $\alpha$ increases. Since the price of good 1 is equal to the ratio of the aggregate supplies of goods 2 and 1, it follows that aggregate output of good 1 increases by more than the aggregate output of good 2.\footnote{The fact that the relative price of good 1 is equal to the ratio of the aggregate supplies of goods 2 and 1 follow directly from equation (3') of the appendix.}

Figure 3.3 presents numerical simulations on how a change in $\alpha$ affects earnings and earnings inequality. Despite a decrease in the relative price of good 1, as shown in panel (a), earnings increase for all production workers and managers. For production workers there are two reasons for this effect. First, since the thresholds, $z_1$ and $z_2$ increase with $\alpha$, production workers are of higher ability. Second, from the production workers’ perspective they are assigned to more productive managers. Both these effects lead to higher earnings for production workers. For managers, since the threshold, $z_1$ increases with $\alpha$, they are of higher ability. In contrast, because $z_1$ increases as well, some managers are assigned to less knowledgeable production workers, which has a negative impact on their earnings. The fact that they are more knowledgeable, however, outweighs the loss from being assigned to less able workers, and their earnings increase.

Figure 3.3 panel (b) presents measures of inequality. Inequality increases with $\alpha$ in both...
Figure 3.2: Equilibrium Simulations for $h = 0.4$, $\beta = 0.5$
(a) Earnings of Best and Worst Production Workers

(b) Earnings of Best and Worst Self-Employed Workers

(c) Earnings of Best and Worst Managers

(d) Inequality

Figure 3.3: Equilibrium Simulations for $h = 0.4$, $\beta = 0.5$
sector 1, the difference in earnings between the most knowledgeable manager and
the least knowledgeable production worker increases with $\alpha$. As $\alpha$ increases, the ability of the
most knowledgeable manager increases and he is assigned to more knowledge production
workers, and so he benefits more from working in teams than the least able production
worker. In sector 2, because the mass of workers in the self-employed sector increases,
inequality increases in that sector as well.

Consider the effect of an increase in the cost of communication. The following proposition
summarizes how $z_1$ and $z_2$ change with $h$:

**Proposition 3.5**  A decrease in $h$ has the following effect:

1. The maximum ability of a worker, and the minimum ability of a self-employed worker,
   $z_1$, increases

2. The minimum ability of a manager, and the maximum ability of a self-employed work-
   ers, $z_2$, increases.

**Proof.** See Appendix.

The intuition behind proposition 3.5 is the following. As communication costs decrease,
managers devote less of their time to each worker and so they can spread their knowledge
across more workers. As a result the size of existing teams increases. Since the output of
all existing teams increases, the aggregate output of good 1 increases as well. The increase
in aggregate output depresses $p_1$, however, and reduces the rents of the existing managers,
forcing the least knowledgeable of them to exit the industry for self-employment. In turn,
the mass of self-employed workers increases, which also increases the aggregate output of
good 2, increases the relative price $p_1$ and forces the least qualified self-employed workers
to exit the industry and work in a team.

Figure 3.4 also provides information on how a decrease in communication costs affects
the equilibrium. First, as shown in panel (a) both ability thresholds increase, consistent with
proposition 3.4. Second, as illustrated in panel (b) the mass of production workers, increases
while the mass of self-employed workers and managers, decreases. Third, as indicated in
panel (c) of the figure, the aggregate output of both goods 1 and 2 increases. The output
of good 1, however, increases more than the output of good 2, and hence the relative price of good 1 decreases, as shown in panel (d).

In addition, since agents are sorting into different sectors and occupations, the assignment between managers and production workers changes as well. The corollary below indicates how a change in $h$ affects the assignment between managers and production workers.

**Corollary 3.6** Let $z_1$ be the upper bound on the initial set of workers. Let $z_2'$ refer to the lower bound on the set of managers at $h' < h$. Then for a decrease in $h$ there exists an $\eta$ such that:

1. All existing workers from $[0, \eta]$ get matched to more knowledgeable managers
2. All existing workers from $[\eta, z_1]$ get matched to less knowledgeable managers
3. All remaining managers from $[z_2', m(\eta)]$ get matched to less knowledgeable workers
4. All remaining managers from $[m(\eta), \alpha]$ get matched to more knowledgeable workers

**Proof.** See Appendix.

Figure 3.5 presents further simulation results on the impact a change in communication costs has on earnings and earnings inequality. Despite a decrease in the relative price of good 1, as shown in panel (a), earnings increase for the least knowledgeable production workers. There are two factors affecting this result. First, the least able production workers are assigned to higher ability managers, which render them more productive. Second, because $h$ is lower, these workers are less important to the team’s production depressing their earnings. The first effect dominates, however, and earnings for the least knowledgeable production workers increase. The earnings of agents at the thresholds, $z_1, z_2$ increase as well. However this does not imply that individuals with ability $z_1$ and $z_2$ are earning more with a decrease in $h$, a point I return to in the paragraph below. When communication costs decrease, earnings also increase for the most knowledgeable managers. In addition to allowing them to supervise larger teams, a decrease in $h$ also allows them to form teams with workers of greater ability. Both effects operate in the direction to increase their earnings.

In Figure 3.5 panel (d) presents measures of inequality. When communication costs decreases, inequality decreases in both sectors. In sector 1, the difference in earnings between
Figure 3.4: Equilibrium Simulations for \( \alpha = 0.8, \ \beta = 0.5 \)
(a) Earnings of Best and Worst Production Workers  
(b) Earnings of Best and Worst Self-Employed Workers  
(c) Earnings of Best and Worst Managers  
(d) Inequality

Figure 3.5: Equilibrium Simulations for $\alpha = 0.8$, $\beta = 0.5$
the most knowledgeable manager and the least knowledgeable production worker decreases with $h$. In sector 2, because the mass of workers in the self-employed sector decreases, inequality decreases in that sector as well.

Holding $\alpha$ constant at 1, Figure 3.6 compares the distribution of earnings under different communication costs. There are several comparisons worth pointing out. First, the earnings of all production workers increase as $h$ is reduced from 0.4 to 0.1, while the earnings of all self-employed workers remain the same. Second, not all remaining managers earn more when communication costs decrease. The rents earned by the most knowledgeable managers rise with a decrease in $h$, while the least knowledgeable managers have their earnings reduced in part because they are assigned to less knowledgeable production workers. Third, notice that the agents that switched from being a manager in sector 1 to being a self-employed worker in sector 2 had their earnings decline the most.

\[ z \]

This simply follows from the fact that the earnings of a self-employed worker with ability $z$ is simply $z$. 

---

Figure 3.6: Population Earnings for $\beta = 0.5$, $h = 0.4$ and $h = 0.1$, $\alpha = 1$. 

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3.5 Open Economy

In this section, I extend the model to an open-economy. Assume that there are 2 countries, labeled Home and Foreign.\(^{16}\) Both countries have populations of the same size.\(^{17}\) Assume that Home has a more knowledgeable population than Foreign. For simplicity, allow the domain of \(G(z)\) to be \([0, 1]\) and the domain of \(G^*(z)\) to be \([0, \alpha]\), for some \(\alpha < 1\).

Goods are traded in international markets however, unlike in Antras, Garicano and Rossi-Hansberg (2006), managers are able to form teams only with agents in their own country.\(^{18}\) Assume that production technologies are identical in both countries and that managers’ costs to communicating with a production worker are the same in both countries.

Under these assumptions, the decision problems encountered by agents are identical to their closed-economy counterparts. In particular, for both the domestic and foreign countries, problem (3.1) describes managers’ maximization problem, problem (3.3) describes consumers’ maximization problem and their occupational choice, and equation (3.5) describes the labor market clearing condition. In an open-economy, the goods market clearing conditions, are slightly different. As each good is traded in international markets, their prices are determined by the international demand and supply. The open-economy goods market clearing conditions for \(x_1\) and \(x_2\), therefore, are:

\[
\frac{\beta}{p_1} \left[ \int_0^\alpha E(z)g(z)dz + \int_0^1 E(z)g(z)dz \right] = \int_{z_1}^{z_1} m(z)g(z)dz + \int_{z_1}^{z_1} m(z)g(z)dz, \quad (3.10)
\]

\[
(1 - \beta) \left[ \int_0^\alpha E(z)g(z)dz + \int_0^1 E(z)g(z)dz \right] = \int_{z_1}^{z_2} F(z)g(z)dz + \int_{z_1}^{z_2} F(z)g(z)dz, \quad (3.11)
\]

where the price of good 2 has been normalized to 1. The definition of an equilibrium is similar to the definition of a closed-economy equilibrium, however, it takes into account the two countries. It is presented below:

\(^{16}\)Variable referring to the foreign country will a indexed by *

\(^{17}\)This condition ensures that trade patterns are the result of differences in the distribution of abilities between the two countries.

\(^{18}\)This condition restricts the analysis to studying how communication costs affect trade patterns between countries. This assumption is equivalent to assuming that the costs of communication for international teams are greater than one.
Definition 3.7 A competitive open-economy equilibrium consists of a set of wage functions \( \{w(z), w^*(z)\} \), a set of rent functions \( \{R(z), R^*(z)\} \), a set of assignment functions \( \{m(z), m^*(z)\} \), a relative price \( \{p_1\} \) and set of a pair of thresholds \( \{(z_1, z_2), (z_1^*, z_2^*)\} \) such that the following conditions hold:

1. Agents in both countries maximize utility
2. Managers in both countries maximize rents
3. All markets clear

The proposition below indicates that an open-economy equilibrium exists. It is very similar to its closed-economy counterpart, however, it has the additional claim that factor price equalization (FPE) does not hold. Although production technologies are identical in both countries, two agents of the same ability but who reside in different countries, will generally not have the same earnings. This is due to two factors. First, because the thresholds, \( z_1 \) and \( z_2 \) are different in both countries, these agents may not be operating in the same sector. Second, because the assignment of managers to production workers is not the same, the composition of teams will not be the same as well. In fact, a production worker in the Home country will form a team with a more knowledgeable manager than the same agent in the Foreign country.

Proposition 3.8 Let \( G(z), G^*(z) \) and \( F(z) \) be uniformly distributed over their domains. For any values of \( \alpha, h \in [0, \overline{h}] \) there exists a competitive equilibrium. Moreover, in such an equilibrium, in either country there is positive sorting, the earnings function is convex, and the sets of managers, self-employed workers and production workers are connected. In addition, factor price equalization (FPE) does not hold.

Proof. See Appendix. ■

As in the closed-economy, \( h \) has be to restricted in the interval \([0, \overline{h}]\). Since there are two economies, there are three upper bound restrictions on \( h \), \( h < \overline{h} = \min \left\{ \frac{1}{z_1^* + z_2^*}, \frac{1}{z_1 + z_2}, 1 \right\} \). The first two restrictions are the result of condition 2 of the definition of an equilibrium, and the third results from the fact that a manager, in either country, cannot spend more than 1 unit of his time communicating with his workers (i.e. \( h \leq 1 \)). Since there does not exist a closed form solution for the occupational choice variables \( \{z_1^*, z_2^*, z_1, z_2\} \), it is
impossible to obtain a closed form expression for $\overline{h}$. One assumption is for $h$ to be less than the smallest value $\frac{1}{z_1+z_2}$ and $\frac{1}{z_1+z_2}$ can undertake in their respective domains, that is $h < \overline{h} = \min \{ \frac{1}{2\alpha}, \frac{1}{2}, 1 \}$. Since $\alpha < 1$, $\frac{1}{2}$ is always smaller than $\frac{1}{2\alpha}$, $\overline{h}$ simplifies to: $\overline{h} = \min \{ \frac{1}{2}, 1 \}$.\textsuperscript{19}

In sector 1 because agents can work in teams, and managers can leverage their knowledge over their production workers, knowledge is more important for production. Further, since Home has a more knowledgeable population, it has a comparative advantage in the sector where knowledge is more valuable, sector 1. The following proposition indicates how trade affects both economies:

**Proposition 3.9** For any values of $\alpha$, $h \in [0, \overline{h}]$ the following hold:

1. The relative price of good 1 in an open economy, is between Home and Foreign’s autarkic prices

2. In Home, $z_1$ increases while $z_2$ decreases, while in Foreign the opposite takes place

3. In Home, production of good 1 increases and production of good 2 decreases, while in Foreign the opposite takes place.

**Proof.** See Appendix. ■

The previous section established that in a closed economy the relative price of good 1, $p_1$, falls as $\alpha$ rises. By assumption Home’s population is more knowledgeable, and so in autarky the relative price of good 1 is higher in the Foreign country. With the opening of trade Home’s producers of good 1 can increase their earnings by exporting to Foreign’s consumers, while Foreign’s producers of good 2 can increase their earnings by exporting to Home’s consumers. As a result, Home will export and expand its production good 1, while Foreign will export and expand its production of good 2. In equilibrium, this process ensures that the relative price of good 1 is the same in both countries.

International trade also has an effect on occupational thresholds. Since with trade it becomes more profitable for agents in Home to produce good 1, there is an influx of managers and workers into the industry. As a result, the ability of the most knowledgeable worker, $z_1$, increases and the ability of the least knowledgeable manager, $z_2$, decreases with trade.

\textsuperscript{19}Simulations of the model suggest that $h$ can be as high as 0.65
In turn, this implies that the mass of workers and managers increases in the Home country, while the mass of agents in the self-employed sector decreases with trade. In contrast, in the Foreign country the opposite takes place, the ability of the most knowledgeable worker, \( z_1^* \), decreases, and the ability of the least knowledgeable manager, \( z_2^* \), increases with trade. This implies that in the Foreign country the mass of workers and managers decreases with trade, and the mass of self-employed workers increases with trade. Furthermore, as the following corollary indicates, international trade also affects the assignment of workers and managers.

**Corollary 3.10** For the Home country, let \( z_1 \) be the upper bound on the set of workers and \( z_2 \) be the lower bound on the set of managers in the closed economy. For the Foreign country, let \( z_1^* \) be the upper bound on the set of workers and \( z_2^* \) be the lower bound on the set of managers in the open economy. Then for any \( \alpha \) less than one, a move towards frictionless trade will have the following impact on the assignment of workers to managers:

1. In the Home country all workers from \([0, z_1]\) get matched to less knowledgeable managers
2. In the Home country all managers from \([z_2, 1]\) get matched to more knowledgeable workers
3. In the Foreign country all workers from \([0, z_1^*]\) get matched to more knowledgeable managers
4. In the Foreign country all managers from \([z_2^*, \alpha]\) get matched to less knowledgeable workers

The corollary above indicates that all agents in the Home country who were managers in autarky are assigned to more knowledgeable workers. From the managers’ budget constraint, it follows that \( n(m^{-1}(z_m)) \) increases. In words, all agents in Home who were managers in autarky are assigned to more knowledgeable and larger teams. In the Foreign country the opposite takes place. All agents who were managers in autarky are assigned to less knowledgeable workers and thus manage smaller teams. Therefore, as the following corollary indicates, in the Home country, in every team that is headed by an agent who was a manager
in autarky, the average output per person increases whereas in the Foreign country in every
team that is headed by an agent who was a manager in autarky, the average output per
person decreases.\textsuperscript{20}

**Corollary 3.11** Under trade, in the Home country the productivity of all agents who were
managers in autarky increases. In the Foreign country, the productivity of all agents who
were managers in autarky decreases.

Until now, the discussion has focused on how trade leads to a reallocation of resources
within countries. However, since trade determines the relative price $p_1$, the earnings of the
factors of production are also affected. More specifically, as the proposition below indicates,
in the Home country, the earnings of all agents who become workers and managers increase,
whereas in the Foreign country, the opposite takes place. In addition, since a self-employed
worker with ability $z$ earns $z$, the earnings of all agents who were self-employed in autarky,
and remain self-employed under trade, remain the same. Therefore it follows that in sector
1 the earnings of managers and production workers move in the same direction.

**Proposition 3.12** Under trade, in the Home country the earnings of all production workers
increase, and the earnings of all managers increase. In the Foreign country the opposite takes
place. The earnings of all agents who remain self-employed are unaffected by trade.

**Proof.** See Appendix. \hfill \blacksquare

The top panel of Figure 3.7 compare the distribution of earnings in the Home country
in the closed economy and in the open economy when the Foreign country has an $\alpha$ equal
to 0.8 and 0.4, respectively. First notice that the earnings of all agents who are production
workers in the open economy increase. This holds even for agents that form teams with
less knowledgeable managers. Since for workers their earnings increase, but in the Home
country the relative price of good 1 increases as well, it is difficult to establish the welfare
implications of trade. Second notice that the earnings of all self-employed workers remain
the same.\textsuperscript{21} The previous discussion indicated that the relative price of good 1 increases

\textsuperscript{20}More specifically, in a team headed by a manager of ability $z$, the average output per person is equal
to $z \frac{n(m-1(z))}{n(m-1(z)) + 1}$.

\textsuperscript{21}This simply follows from the fact that the earnings of a self-employed worker with ability $z$ is simply
$p_2 z$ and $p_2$ is normalized to 1.
in the open economy, and so all agents who remained self-employed, are worse off. Third, notice that all managers earn more in the open economy. The rents earned by all agents who were managers in the closed economy rise, and the rents earned by all agents who became managers in the open economy are greater than their earnings from being self-employed. Similar to the case for workers, since managers’ rents increase, but the relative price of good 1 increases as well, it is difficult to establish the welfare implications of trade. I return to the welfare implications of trade shortly.

The bottom panel of Figures 3 and 4 compare the distribution of earnings in the Foreign country in the closed economy and in the open economy when the Foreign country has an \( \alpha \) equal to 0.8 and 0.4, respectively. The impact of trade on the distribution of earnings in the Foreign country are the opposite of what takes place in the Home country. First, the earnings of all agents who are production workers in the open economy decrease. Since for workers their earnings decrease, but in the Foreign country the relative price of good 1 decreases as well, it is difficult to establish the welfare implications of trade. Second the earnings of all self-employed workers remain the same. The previous discussion indicated that the relative price of good 1 decreases in the open economy, and so all agents who remained self-employed, are worse off. Third, all managers earn less in the open economy. The rents earned by all agents who are managers in the open economy decrease. Similar to the case for production workers, since managers’ rents decrease, but the relative price of good 1 decreases as well, it is difficult to establish the welfare implications of trade.

Figure 3.8 presents how trade affects the welfare of all agents in the economy in the cases when \( \alpha = 0.8 \) and \( \alpha = 0.4 \). The top panels present the welfare changes of all agents in the Home country, while the bottom panels present the change in welfare for the Foreign countries. First, notice that in the Home country, although the majority of workers are better off in the open economy, there is a set of workers that are worse off from international trade. These workers are the most knowledgeable workers in sector 1. Second, notice that in the Home country all workers who are self-employed are worse off from trade. And third, although the most knowledgeable managers are better off from trade, there is a set of managers that begins from the least knowledgeable manager who are worse off from trade.

The pattern of gains and losses are in the opposite direction for the Foreign country. In the Foreign country, all workers are worse off in the open economy and although the majority
(a) Home Country $\alpha = 0.8$ & $h = 0.4$

(b) Home Country $\alpha = 0.4$ & $h = 0.4$

(c) Foreign Country $\alpha = 0.8$ & $h = 0.4$

(d) Foreign Country $\alpha = 0.4$ & $h = 0.4$  

Figure 3.7: Earnings in Closed vs Open Economy
Figure 3.8: Change in Agents’ Welfare from International Trade

(a) Home Country $\alpha = 0.8$ & $h = 0.4$

(b) Home Country $\alpha = 0.4$ & $h = 0.4$

(c) Foreign Country $\alpha = 0.8$ & $h = 0.4$

(d) Foreign Country $\alpha = 0.4$ & $h = 0.4$
of self-employed workers are better off from trade, there is a set of self-employed workers who are worse off. These agents are a subset the individuals who switched from being a production worker in the closed economy to being self-employed in the open economy. And the majority of managers are better off from trade, however depending on the value of $\alpha$, the most knowledgeable managers in the Foreign country may be worse off. Therefore, we can conclude from this discussion that the impact of trade is not Pareto improving. I summarize this finding below:

**Summary 3.13** *In the both the Home and Foreign country, not all agents gain from trade.*

### 3.6 The Impact of Trade on Inequality

Figure 3.9 presents simulation results on the impact of trade on earnings in the Home and Foreign countries, respectively. In the Home country, as illustrated in panel (a) of Figure 6, in all cases trade leads to an increase in wage inequality in sector 1. There are three reasons for this result. First, since the relative price of good 1 increases in the open economy, the revenues from a unit of output produced increases. Second, since the ability of the least knowledgeable worker $z_1$ increases under trade, the most knowledgeable managers get to form teams with better workers. This allows them to increase the size of their teams and produce more. Third, since the ability of the least knowledgeable manager decreases in the open economy, the least knowledgeable workers get assigned to a worse manager. Therefore, the proportion of problems that can be solved by the new manager decreases, which makes the workers less productive. A second implication of trade is that in the self-employed sector, since the ability of the least knowledgeable production worker increases and the ability of the most knowledgeable self-employed worker decreases, the difference in the earnings between them decreases. Therefore in the Home country, trade leads to an increase in inequality in the comparative advantage sector, whereas it leads to a decrease in inequality in the comparative disadvantage sector. The overall impact of trade on the Home country is an increase in inequality.

In the Foreign country the impact of trade on earnings depends on the size of the Foreign country, $\alpha$. Inequality always increases in the comparative advantage sector. In the self-employed sector, since the ability of the least knowledgeable worker decreases and the ability
of the most knowledgeable self-employed worker increases, the difference in the earnings between them increases. However whether trade leads to an increase or decrease in overall inequality depends on the inequality in sector 1. In sector 1, although inequality always decreases with trade, its overall impact on the economy depends on the level of knowledge in the economy. When \( \alpha \) is low, because the knowledge of managers and production workers in teams is low, the assignment of production workers to managers has a small impact on their earnings, and so the change in the most knowledgeable managers’ earnings are not greatly affected by forming teams with less knowledgeable production workers. In contrast, when \( \alpha \) is high, the assignment matters and managers earnings are strongly affected from forming teams with less knowledgeable production workers, and so inequality decreases in sector 1, and in the overall economy. Finally, when the knowledge of the Foreign country approaches the knowledge of the Home country, trade has a smaller impact on the allocation of agents into sectors and teams, and so inequality in the Foreign country is less impacted by trade.

These numerical findings are summarized below:

**Summary 3.14** The impact of trade on earnings inequality is the following:

1. In Home, earnings inequality in the comparative advantage sector increases
2. In Home, earnings inequality in the comparative disadvantage sector decreases
3. In Home, earnings inequality in the economy increases
4. In Foreign, earnings inequality in the comparative advantage sector increases
5. *In Foreign, earnings inequality in the comparative disadvantage sector may increase or decrease*

6. *In Foreign, earnings inequality in the economy may increase or decrease*

From Figures 3.10 and 3.11, holding $\alpha$ constant at 0.4, one can also assess how changes in the costs of communication affect the open-economy equilibrium. First notice that in both the Foreign and Home country, as communication costs decrease the ability of the least knowledgeable manager and the most knowledgeable worker increase. The intuition is the same as in the closed-economy. As communication costs decrease, managers can spread their knowledge across more workers, and as a result team size increases. Since the output of all existing teams increases, it follows that the aggregate output of good 1 increases as well. However the increase in aggregate output depresses the relative price $p_1$, reducing the rents of the existing managers and forcing the least knowledgeable to exit the industry for self-employment. Second, notice that in both countries, the mass of production workers increases with a decline in $h$ and the mass of managers decreases with $h$. In the Home country and the mass of self-employed workers increases with a decline in $h$, while in the Foreign country, the opposite is the case, the mass of self-employed workers decreases with $h$. The intuition for this result is the following. In the Home country, as communication costs decrease, the mass of self-employed workers increases, and this leads to an increase in the relative price of good 1. In the Foreign country, this forces the least qualified self-employed workers to exit the industry and work in a team. The results are summarized below:

**Summary 3.15** *In an open-economy a decrease in $h$ has the following effect:*

1. *In the Home country the maximum ability of a worker, and the minimum ability of a self-employed worker, $z_1$, increases*

2. *In the Home country the minimum ability of a manager, and the maximum ability of a self-employed workers, $z_2$, increases.*

3. *In the Home country the mass of production workers and of self-employed workers increases, while the mass of managers decreases.*

4. *In the Foreign country the maximum ability of a worker, and the minimum ability of a self-employed worker, $z_1^*$, increases.*
Figure 3.10: Equilibrium Simulations for Home Country $\alpha = 0.4, \beta = 0.5$
Figure 3.11: Equilibrium Simulations for Foreign Country $\alpha = 0.4$, $\beta = 0.5$
5. In the Foreign country the minimum ability of a manager, and the maximum ability of a self-employed workers, \( z^*_2 \), increases.

6. In the Foreign country the mass of production workers, while the mass of self-employed workers and of managers decreases.

Furthermore from panel (f) of Figures 3.10 and 3.11, one can also assess how changes in the communication costs affect earnings in the open-economy equilibrium. First notice that as \( h \) decreases, in both countries, the earnings of the least knowledgeable worker increases. The least knowledgeable workers forms teams with the least knowledgeable managers. Because, when \( h \) decreases the least knowledgeable manager can solve a greater proportion of problems, he renders his workers more productive, which in turn increases their wage. Second, notice that in both countries, the earnings of the most knowledgeable manager increase with a decline in \( h \). Since the most knowledgeable manager forms teams with the most knowledgeable workers, and a decrease in communication costs increases \( z_1 \), the most knowledgeable manager now forms teams with more knowledgeable workers. This implies that the number of workers that he supervises increases, which in turn increases his earnings. Third, since in the Foreign country the mass of workers in the self-employed sector decreases, inequality in the self-employed sector decreases as well. In contrast, in the Home country since the mass of workers in the self-employed sector increases, inequality in the self-employed sector increases with an decrease in \( h \). And fourth, notice that in either country, the distance between the earnings of the most knowledgeable manager and the least knowledgeable worker decreases with a decline in \( h \). This is because as communication costs decrease the least knowledgeable worker benefits more from working with a more knowledgeable manager. Overall inequality, therefore, decreases in either country. The conclusions are summarized below:

**Summary 3.16** In an open-economy a decrease in \( h \) has the following effect:

1. In the Home country it decreases earnings inequality in sector 1

2. In the Home country it increases earnings inequality in sector 2

3. In the Home country, earnings inequality in the economy decreases
4. In the Foreign country it decreases earnings inequality in sector 1

5. In the Foreign country it decreases earnings inequality in sector 2

6. In the Foreign country, earnings inequality in the economy decreases

3.7 Conclusion

In conclusion, this paper presents a model where heterogeneous agents sort into industries and occupations and match into teams and investigates how trade affects earnings inequality. In order to accomplish this it extends Garicano and Rossi-Hansberg (2006) to two goods and two countries. The main findings of the paper are that under certain assumptions a competitive equilibrium exists in a closed and open economy. Moreover, in such an equilibrium there is positive sorting, the earnings functions are convex and the sets of managers, self-employed and workers are connected. In addition, in the open economy, identical workers located in different countries do not always earn the same income, the country with a more knowledgeable population specializes in the production of the good produced in teams, while the other country specializes in the good produced by self-employed workers. This study also finds that a transition from autarky to frictionless trade causes the earnings of managers and production workers to move in the same direction. Furthermore, this paper finds that in the Home country international trade increases income inequality, however whether international trade increases or decreases income inequality in Foreign country depends on the ability distribution of the country.

One limitation of the paper, is that in both the closed and open economy a decrease in communication costs lowers earnings inequality. This finding is at odds with the empirical labor literature which concludes that technology has been the cause of the observed increase in earnings inequality. Perhaps the model’s result is partly due to the fact that in one sector communication costs are so high that production does not happen in teams.

Another limitation of this paper, is that it does not allow for the cost of communication to vary across countries. Integrating this feature in the present model would provide an insight into how differences in technology influence the patterns of trade. Furthermore, in such a setting, one can also investigate what happens when the less efficient technology converges to the more advanced one. Another drawback of the present model is the assumption that the
distribution of knowledge and problems are uniform. Although this assumption simplifies the analysis, it limits the robustness of the findings. A question of interest is how the results change when the distribution of knowledge and the distribution of problems have another form.
3.8 Appendix

**Proposition 3.1:** In sector 1, managers and production workers are complements.

**Proof.**

The argument in this section is similar to the first part of the proof of Theorem 1 in Antras et. al (2006). Let \( R(z_m, z_p) \) denote the rents of a manager of ability \( z_m \) assigned workers of ability \( z_p \). The equilibrium is characterized by an assignment function \( m(z) \) such that \( z_m = m(z_p) \). Since managers are maximizing rents, it follows that

\[
\frac{\partial R(z_m, z_p)}{\partial z_p} = 0.
\]

From the expression above, we can find an expression for \( \frac{\partial z_m}{\partial z_p} \). Namely,

\[
\frac{\partial z_m}{\partial z_p} = -\frac{\partial^2 R(z_m, z_p)/\partial z_p^2}{\partial^2 R(z_m, z_p)/\partial z_p \partial z_m}.
\]

Because the manager is solving a maximization problem, the numerator is negative. Also, the denominator is positive since from

\[
\frac{\partial R(z_m, z_p)}{\partial z_m} = \frac{p_1}{h[1 - z_p]},
\]

it follows that

\[
\frac{\partial^2 R(z_m, z_p)}{\partial z_p \partial z_m} = \frac{p_1}{h[1 - z_p]^2} > 0.
\]

Therefore, the equilibrium has positive sorting. \( \blacksquare \)

**Proposition 3.3:** Let \( G(z) \) and \( F(z) \) be uniformly distributed over their domains. For any values of \( \alpha \) and \( h \in [0, \bar{h}] \) there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings function is convex and the sets of managers, self-employed workers and production workers are connected.

**Proof.**

To show that an equilibrium exists, the following conditions must be satisfied:

i. The sets of managers, self-employed and workers are connected
ii. Agents do not want to deviate from their occupational choices.

iii. There exists an equilibrium.

i.) The sets are connected.

To show that the sets of workers, self-employed and managers are connected, begin by assuming that this is not the case. Suppose that the sets of managers, self-employed and workers is disconnected. That is

\[ W = [a_1, a_2] \cup [a_4, a_5], \quad S = [a_2, a_3] \cup [a_5, a_6] \quad \text{and} \quad M = [a_3, a_4] \cup [a_6, a_7]. \]

For each interval \([a_1, a_4]\) and \([a_4, a_7]\) solve the agents’ problem under the restriction that teams can be formed only with members in the same interval. Then it follows in the interval \([a_1, a_4]\) that \(m(a_1) = a_3, m(a_2) = a_4\). Similarly in the interval \([a_4, a_7]\) it follows that \(m(a_4) = a_6, m(a_5) = a_7\).

For this setup to be an equilibrium, the earnings function has to be continuous. Thus at \(a_4\) it must be the case that \(R_{14}(a_4) = w_{47}(a_4)\).

Since demand and supply in this model are homogeneous of degree zero, the assignment function is independent of prices, and equation \(\sigma = p_2 z_2 h\) is also independent of prices, the equilibrium bundles in the economy are not affected by proportional price changes. Further, in any equilibrium either

\[ \frac{p_1}{p_2} > 1 \quad \text{or} \quad \frac{p_1}{p_2} \leq 1 \]

Consider the case \(\frac{p_1}{p_2} \leq 1\). Then normalize \(p_1\) to 1. Since \(w_{14}(a_2) = a_2 p_2\), \([F(z)p_2]' = p_2 \geq 1\) and \(R'_{14}(z) = \frac{1}{h[1 - m^{-1}(z)]}\), it follows that \(R_{14}(a_4) > a_4\). Therefore, since \(a_1 < a_2 < a_3 < a_4 < a_5 < a_6 < 1\)

\[ w'_{47}(a_4) = \frac{a_6 - w_{47}(a_4)}{1 - a_4} = \frac{a_6 - R_{14}(a_4)}{1 - a_4} < \frac{a_6 - a_4}{1 - a_4} < 1 \]

Consider the case \(\frac{p_1}{p_2} < 1\). Then normalize \(p_2\) to 1. Using a similar argument as above, it follows that \(R_{14}(a_4) > a_4\). Then the following is also true:

\[ w'_{47}(a_4) = \frac{p_1 a_6 - w_{47}(a_4)}{1 - a_4} = \frac{a_6 - R_{14}(a_4)}{1 - a_4} < \frac{p_1}{h[1 - a_2]} = R'_{14}(a_4) \]
Substituting the expression for $R_{14}(a_4)$, the inequality amounts to showing that

$$p_1 a_6 [1 - a_2] h < p_1 - w_{14}(a_2) = p_1 - p_2 a_2$$

Which follows since $p_1 a_6 h < p_1 < p_1^{1 - a_2/p_1}$.

Now consider manager $a_6$. If he were to hire $a_4 - \epsilon$ he would earn

$$\Pi(a_6, a_4 - \epsilon) = \frac{p_1 a_6 - R_{14}(a_4 - \epsilon)}{h [1 - (a_4 - \epsilon)]}$$

Since $R_{14}(a_4) = w_{47}(a_4)$ and $w_{47}'(a_4) = \frac{p_1 a_6 - w_{47}(a_4)}{1 - a_4}$

$$\lim_{\epsilon \to 0} \frac{\partial \Pi(a_6, a_4 - \epsilon)}{\partial \epsilon} = \frac{R_{14}'(a_4) - w_{47}'(a_4)}{h [1 - a_4]} > 0$$

Manager $a_6$ would increase his earnings if he hires $a_4 - \epsilon$. Therefore, he has an incentive to deviate. Thus, in an equilibrium the sets of managers, workers and self-employed must be connected.

ii.) Agents do not want to deviate from their occupational choices.

First, one must show that managers with knowledge $\alpha$ do not want to hire workers with ability greater than $z_1$. Without loss of generality, normalize $p_2$ to 1. A manager with ability $\alpha$ may decide to hire a worker with ability $z_1 + \epsilon$ at wages $F(z_1 + \epsilon)$. Then he would earn

$$\Pi(\alpha, z_1 + \epsilon) = \frac{p_1 \alpha - F(z_1 + \epsilon)}{h [1 - (z_1 + \epsilon)]}$$

And since $F(z_1) = w(z_1)$ and $w'(z_1) = \frac{p_1 \alpha - w(z_1)}{1 - z_1}$

$$\lim_{\epsilon \to 0} \frac{\partial \Pi(\alpha, z_1 + \epsilon)}{\partial \epsilon} = \frac{F'(z_1) - w'(z_1)}{h [1 - z_1]} > 0$$

as long as $F'(z_1) > w'(z_1)$. At $z_1$, this is equivalent to $\sigma + z_1 h = (z_2 + z_1) h < 1$. Points $z_1$ and $z_2$ are endogenously determined in the model and are therefore a function of $h$, however since there does not exist a closed form solution to $z_1$ and $z_2$, to ensure that an equilibrium exists we impose the condition that $h$ is less than the largest value the ratio $\frac{1}{z_1 + z_2}$ can take. In the domain $[0, \alpha] \times [0, \alpha]$, the largest value $\frac{1}{z_1 + z_2}$ can ever have is $\frac{1}{2 \alpha}$.
Thus \( h < \min\left\{ \frac{1}{2\alpha}, 1 \right\} \).

Similarly, at point \( z_2 \), it must the case that condition \( F'(z_2) < R'(z_2) \). With some manipulation, one can show that it is always satisfied.

iii.) There exists an equilibrium.

To show that an equilibrium exists, one has to show that the following system of equations has a solution.

\[
\begin{align*}
h \left[ z_1 - \frac{z_2^2}{2} \right] + z_2 &= \alpha \quad (1) \\
p_1 z_2 + 1/2p_1 h z_1^2 - p_2 z_1 &= p_2 z_2 h (1 - z_1) \quad (2) \\
\frac{\beta}{p_1} \int_0^a E(z)g(z)dz &= \int_0^{z_1} m(z)g(z)dz \quad (3) \\
\frac{1-\beta}{p_2} \int_0^a E(z)g(z)dz &= \int_{z_1}^{z_2} F(z)g(z)dz \quad (4) \\
\int_0^a E(z)g(z)dz &= p_1 \int_0^{z_1} m(z)g(z)dz + p_2 \int_{z_1}^{z_2} F(z)g(z)dz \quad (5)
\end{align*}
\]

Equation (1) is the condition \( m(z_1) = \alpha \), while equation (2) results from \( R(z_2) = p_2 F(z_2) \).

Equations (3) and (4) describe the goods market clearing conditions, and equation (5) describes the fact total income equals total expenditures.

STEP 1: Rewrite the above system of five equations and five unknowns into a system of two equations and two unknowns.

From equation (5), the term \( \int_0^a E(z)g(z)dz \) can substituted into equations (3) and (4) yielding:

\[
\begin{align*}
\frac{\beta p_2}{p_1} \int_{z_1}^{z_2} F(z)g(z)dz &= (1 - \beta) \int_0^{z_1} m(z)g(z)dz \quad (3') \\
(1 - \beta) \frac{p_2}{p_1} \int_0^{z_1} m(z)g(z)dz &= \beta \int_{z_1}^{z_2} F(z)g(z)dz \quad (4')
\end{align*}
\]

Clearly, equations (3’) and (4’) are identical to one another. Further, substituting for \( p_1 \) from (3’) into (2), integrating the expressions and rearranging the terms yields:

\[
(1 - \beta) [6z_2 z_1 - h z_1^3 + 1/2 h z_1^2] [z_2 h (1 - z_1) + z_1] = 3 \beta [z_2^2 - z_1^2] [z_2 + 1/2 h z_1^2] \quad (2’)
\]

Therefore, the system has been reduced to the following two equations in two unknowns:

\[
\begin{align*}
h \left[ z_1 - \frac{z_2^2}{2} \right] + z_2 &= \alpha \quad (1) \\
(1 - \beta) [6z_2 z_1 - h z_1^3 + 1/2 h z_1^2] [z_2 h (1 - z_1) + z_1] &= 3 \beta [z_2^2 - z_1^2] [z_2 + 1/2 h z_1^2] \quad (2')
\end{align*}
\]
The domain of interest is \([0, \alpha] \times [0, \alpha]\). Equation (1) can be rewritten in the form of \(z_2\) as a function of \(z_1\). Equation (2'), however, represents a specific isoline of a function \(y = f(z_1, z_2)\) in the \(\mathbb{R}^2\) plane. Therefore both functions have a graphical representation in a standard two-dimensional plot.

**STEP 2:** Show that at the point \(z_1 = 0\) equation (1) is above equation (2').
Evaluate equations (1) and (2') at the point \(z_1 = 0\). Equation (1) yields the following expression \(z_2 = \alpha\) while equation (2') yields the expression \(3\beta z_2^2 = 0\). The latter expression has one solution \(z_2 = 0\).

**STEP 3:** Show that at the point \(z_1 = \alpha\) equation (2') is above equation (1).
Evaluate equation (1) at the point \(z_1 = \alpha\). Equation (1) provides the solution \(z_2 = \alpha - \frac{h}{2} \alpha\) which is smaller than \(\alpha\). For equation (2'), notice than in the relevant domain \([0, \alpha] \times \mathbb{R}_{\geq 0}\), the left hand side is positive, and the right hand side will be positive if and only if the following condition holds:

\[
 z_2^2 - z_1^2 > 0
\]

Thus it follows that at \(z_1 = \alpha\) equation (2') is above equation (1).

**STEP 4:** Show that equation (1) is decreasing in the interval \([0, \alpha]\).
Totally differentiating equation (1) yields

\[
 h[1 - z_1] \, dz_1 + dz_2 = 0
\]

Therefore \(\frac{dz_2}{dz_1} = -h[1 - z_1]\) is negative.

**STEP 5:** Show that equation (2') is increasing in the interval \([0, \alpha]\).
Totally differentiating equation (2') yields

\[
 dz_2 [A] + dz_1 [B] = 0
\]

where

\[
 A = (1 - \beta)(6z_2h(1 - z_1) + z_1) + (1 - \beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1 - z_1) - 3\beta(z_2 + 1/2hz_1^2)2z_2 - 3\beta(z_2^2 - z_1^2)
\]

\[
 B = (1 - \beta)(6z_2 - 3hz_1^3 + 6hz_1)(z_2h(1 - z_1) + z_1) + (1 - \beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)(1 - hz_2) + 3\beta(z_2 + 1/2hz_1^2)2z_1 - 3\beta(z_2^2 - z_1^2)hz_1
\]
Claim 1: \( A \) is negative.

Proof:

First note that:

\[
(1 - \beta)(6z_2z_1 - h z_1^3 + 3 h z_1^2)h(1 - z_1)1/2 h z_1^2 < (1 - \beta)(6z_2z_1 - h z_1^3 + 3 h z_1^2)z_1
\]

With some manipulation, one can show that:

\[
(1 - \beta)6z_1(z_2h(1 - z_1) + z_1)(z_2 + 1/2 h z_1^2) < 6\beta(z_2 + 1/2 h z_1^2)^2 z_2
\]

Then it follows that:

\[
(1 - \beta)6z_1(z_2h(1 - z_1) + z_1)(z_2 + 1/2 h z_1^2) + (1 - \beta)(6z_2z_1 - h z_1^3 + 3 h z_1^2)h(1 - z_1)(z_2 + 1/2 h z_1^2) < \\
3\beta(z_2 + 1/2 h z_1^2)^2 z_2 + (1 - \beta)(6z_2z_1 - h z_1^3 + 1/2 h z_1^2)(z_2h(1 - z_1) + z_1)
\]

This expression is equivalent to:

\[
(1 - \beta)6z_1(z_2h(1 - z_1) + z_1) + (1 - \beta)(6z_2z_1 - h z_1^3 + 3 h z_1^2)h(1 - z_1) < \\
3\beta(z_2 + 1/2 h z_1^2)^2 z_2 + \frac{(1 - \beta)(6z_2z_1 - h z_1^3 + 1/2 h z_1^2)(z_2h(1 - z_1) + z_1)}{(z_2 + 1/2 h z_1^2)}
\]

And using the fact that equation \((2')\) holds with equality, the expression above can be rewritten as

\[
(1 - \beta)6z_1(z_2h(1 - z_1) + z_1) + (1 - \beta)(6z_2z_1 - h z_1^3 + 3 h z_1^2)h(1 - z_1) < 3\beta(z_2 + 1/2 h z_1^2)^2 z_2 + 3\beta(z_2^2 - z_1^2)
\]

Thus \( A \) is negative.

Claim 2: \( B \) is positive.

Proof:

First notice that

\[
(6z_2 - 3h z_1^2 + 6h z_1)(z_2h(1 - z_1) + z_1)(z_2 + 1/2 h z_1^2) > (6z_2z_1 - h z_1^3 + 3 h z_1^2)(z_2h(1 - z_1) + z_1)h z_1
\]

Then it follows that

\[
(1 - \beta)(6z_2 - 3h z_1^2 + 6h z_1)(z_2h(1 - z_1) + z_1) + 3\beta(z_2 + 1/2 h z_1^2)^2 z_2 + (1 - \beta)(6z_2 - h z_1^3 + 3 h z_1^2)(1 - h z_2) > \\
(1 - \beta)h z_1\frac{(6z_2z_1 - h z_1^3 + 3 h z_1^2)(z_2h(1 - z_1) + z_1)}{z_2 + 1/2 h z_1^2}
\]

Using the fact that equation \((2')\) holds with equality, the expression above can be rewritten as
Thus $B$ is positive.

Therefore, from Claims 1 and 2, we can conclude that for equation (2') \( \frac{dz_2}{dz_1} = -\frac{B}{A} > 0 \). Therefore, since equation (1) is above equation (2') when $z_1 = 0$, equation (2') is above equation (1) when $z_1 = \alpha$, and equation (1) is decreasing over the interval $[0, \alpha]$, while equation (2') is increasing over the same interval, equations (1) and (2') intersect once over the domain $[0, \alpha]$. Thus, the proposition is true.

iv.) The earnings function is convex.

It remains to show that the earnings function is convex. Since $\forall z \leq z_1, w'(z) = \sigma + h z$ the wage function is convex, and since $\forall z \in [z_1, z_2], F(z) = z$ the earnings function of self-employed workers is also convex, and since $\forall z \geq z_2, R'(z) = \frac{p}{h[1-m-1(z)]}$ and there is positive sorting, the rent function is also convex. Therefore the earnings function is convex.

v.) An equilibrium exhibits positive sorting

This result follow directly from the previous proposition.

\[ \blacksquare \]

**Proposition 3.4:** An increase in $\alpha$ has the following effect:

1. The maximum ability of a worker, and the minimum ability of a self-employed worker, $z_1$, increases

2. The minimum ability of a manager, and the maximum ability of a self-employed workers, $z_2$, increases.

**Proof.**

The proof in proposition 3.3 showed that the equilibrium is characterized by the two equations:

\[
h \left[ z_1 - \frac{z_1^2}{z_2} \right] + z_2 = \alpha \quad (1)
\]

\[
(1 - \beta) \left[ 6z_2 z_1 - h z_1^3 + 1/2hz_1^2 \right] \left[ z_2 h(1 - z_1) + z_1 \right] = 3\beta \left[ z_2^2 - z_1^2 \right] \left[ z_2 + 1/2hz_1^2 \right] \quad (2')
\]
From the equation (1) it follows that

\[
\frac{dz_1}{d\alpha} = \frac{1}{h(1 - z_1) + \frac{dz_2}{dz_1}}.
\]

Since from equation (2’) it follows that \( \frac{dz_2}{d\alpha} > 0 \), the expression above is positive. Furthermore, since \( \frac{dz_2}{d\alpha} = \frac{dz_2 \, dz_1}{d\alpha \, dz_1} \), \( \frac{dz_2}{d\alpha} \) is positive because \( \frac{dz_2}{dz_1} > 0 \) and \( \frac{dz_1}{d\alpha} > 0 \).

**Proposition 3.5:** A decrease in \( h \) has the following effect:

1. The maximum ability of a worker, and the minimum ability of a self-employed worker, \( z_1 \), increases

2. The minimum ability of a manager, and the maximum ability of a self-employed workers, \( z_2 \), increases.

**Proof.** Let \( \beta = 0.5 \). Then the equilibrium is characterized by the two equations:

\[
\begin{align*}
\frac{h}{h}\left[z_1 - \frac{z_1^2}{2h}\right] + z_2 &= \alpha \\
[6z_2z_1 - hz_1^3 + 1/2hz_1^2]\left[2z_2h(1 - z_1) + z_1\right] &= 3\left[z_2^2 - z_1^2\right]\left[z_2 + 1/2hz_1^2\right] \quad (2’)
\end{align*}
\]

Taking total derivatives with from equation (1) and (2’) and setting \( d\alpha = 0 \), yields the following expressions:

\[
\begin{align*}
[D] \, dh + [E] \, dz_1 + dz_2 &= 0 \quad (1’) \\
[B] \, dz_1 + [A] \, dz_2 + [C] \, dh &= 0 \quad (2’)
\end{align*}
\]

where,

\[
\begin{align*}
[A] &= 6z_1(z_2h(1 - z_1) + z_1) + (6z_2z_1 - hz_1^3 + 3hz_1^2)h(1 - z_1) - 6(z_2 + 1/2hz_1^2)z_2 + 3(z_2^2 - z_1^2), \\
[B] &= (6z_2 - 3hz_1^2 + 6hz_1)(z_2h(1 - z_1) + z_1) + 6(z_2 + 1/2hz_1^2)z_1 + (6z_2 - hz_1^3 + 3hz_1^2)(1 - hz_1) - 3(z_2^2 - z_1^2), \\
[C] &= (z_2h(1 - z_1) + z_1)(3z_1^2 - z_1^3) + (6z_2z_1 - hz_1^3 + 3hz_1^2)(1 - z_1)z_2 - 3/2(z_2^2 - z_1^2)z_1, \\
[D] &= z_1(1 - \frac{z_1^2}{h}) , \\
[E] &= (1 - z_1)h.
\end{align*}
\]
It is straightforward to see that $D > 0$ and $E > 0$. From the proof of proposition 3.3, we know that $A < 0$ and $B > 0$. Furthermore, with some manipulation one can also show that $C > 0$.

From equation (1') it follows that

$$\frac{dz_2}{dh} = -[D] - [E] \frac{dz_1}{dh}.$$ 

Substituting that expression into equation (2'') yields

$$\frac{dz_1}{dh} = \frac{-[C] + [A][D]}{[B] - [A][E]},$$

which is negative. Therefore, from the two expressions above it follows that $\frac{dz_2}{dh} < 0$, if and only if $[E][C] < [D][B]$. Indeed, with some manipulation one can show that this inequality is true. □

**Corollary 3.6:** Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. Let $z_1$ refer to the upper bound on the initial set of workers. Let $z_2'$ refer to the lower bound on the new set of managers. Then for a decrease in $h$ there exists an $\eta$ such that:

1. All existing workers from $[0, \eta]$ get matched to more knowledgeable managers

2. All existing workers from $[\eta, z_1]$ get matched to less knowledgeable managers

3. All remaining managers from $[z_2', m(\eta)]$ get matched to less knowledgeable workers

4. All remaining managers from $[m(\eta), \alpha]$ get matched to more knowledgeable workers

**Proof.**

Let $h' < h$. Then we know that $z_2 < z_2'$ and $z_1 < z_1'$. In order to determine how workers’ assignment is affected by a change in communication costs, we want to compare $m'(z)$ with $m(z)$ in the following manner:

$$m(z) - m'(z) = h \left( z - \frac{z_2^2}{2} \right) + z_2 - h' \left( z - \frac{z_2^2}{2} \right) - z_2'$$

$$= \left( h - h' \right) \left( z - \frac{z_2^2}{2} \right) + \left( z_2 - z_2' \right)$$

At $z = 0$, $m(z) - m'(z) = z_2 - z_2' < 0$. 
At $z = z_1$, $m(z) - m'(z) = \alpha - h' \left( z_1 - \frac{z_1^2}{2} \right) - z_2' > 0$. This follows from the fact that $z_1 < z_1'$ and at $z = z_1'$, $m'(z_1') = \alpha$, and $\frac{dm'(z)}{dz} = h'(1 - z) > 0$.

Since $h > h'$, it follows that $m(z) - m'(z)$ is increasing. That is,

$$\frac{d(m(z) - m'(z))}{dz} = (h - h')(1 - z) > 0.$$ 

By the Intermediate Value Theorem, there exists a $z = \eta$ such that $m(\eta) - m'(\eta) = 0$. Hence for all $z \in [0, \eta]$ , it follows that $m(z) - m'(z) < 0$, and workers get assigned to more knowledgeable managers when communication costs are $h'$. At $z = \eta$, it follows that $m(\eta) - m'(\eta) < 0$, and workers get assigned to managers of the same ability. And, for all $z \in (\eta, z_1]$ , it follows $m(z) - m'(z) > 0$, and workers get assigned to less knowledgeable managers when communication costs are $h'$. Therefore, statements i and ii are true.

Since the matching function is monotonic and invertible, statement iii and iv follow directly from i and ii.

**Proposition 3.8:** Let $G(z)$, $G^*(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of $\alpha$, $h \in [0, \overline{h}]$ there exists a competitive equilibrium. Moreover, in such an equilibrium, in either country there is positive sorting, the earnings function is convex, and the sets of managers, self-employed workers and production workers are connected. In addition, factor price equalization (FPE) does not hold.

**Proof.**

To show that an equilibrium exists in an open economy setting, the following conditions must be satisfied:

i. In each country, the sets of managers, self-employed and workers are connected

ii. Agents do not want to deviate from their occupational choices.

iii. There exists an equilibrium.

The proofs for conditions i and ii are identical to the closed-economy framework. Further, to show that an equilibrium exists, one has to show that the following system of equations has a solution.

$$h \left[ z_1^* - \frac{z_1^* 2}{2} \right] + z_2^* = \alpha$$ \hspace{1cm} (3.12)
\[ p_1 z_2^* + 1/2 p_1 h z_1^*^2 - p_2 z_1^* = p_2 z_2^* h (1 - z_1^*) \] (3.13)

\[ h \left[ z_1 - \frac{z_1^2}{2} \right] + z_2 = 1 \] (3.14)

\[ p_1 z_2 + 1/2 p_1 h z_1^2 - p_2 z_1 = p_2 z_2 h (1 - z_1) \] (3.15)

\[ \frac{\beta}{p_1} \left[ \int_0^\alpha E(z) g^*(z) dz + \int_0^1 E(z) g(z) dz \right] = \int_0^{z_1^*} m(z) g^*(z) dz + \int_0^{z_1} m(z) g(z) dz \] (3.16)

\[ \frac{1 - \beta}{p_2} \left[ \int_0^\alpha E(z) g^*(z) dz + \int_0^1 E(z) g(z) dz \right] = \int_{z_1^*}^{z_2^*} F(z) g^*(z) dz + \int_{z_1}^{z_2} F(z) g(z) dz \] (3.17)

\[ \int_0^\alpha E(z) g^*(z) dz = p_1 \int_0^{z_1^*} m(z) g^*(z) dz + p_2 \int_{z_1^*}^{z_2^*} F(z) g^*(z) dz \] (3.18)

\[ \int_0^1 E(z) g(z) dz = p_1 \int_0^{z_1} m(z) g(z) dz + p_2 \int_{z_1}^{z_2} F(z) g(z) dz \] (3.19)

Equations (11) and (13) describe the condition that in each country, the most knowledgeable worker is matched to the most knowledgeable manager, while equations (12) and (14) result from \( R(z_2^*) = p_2 F(z_2^*) \) and \( R(z_2) = p_2 F(z_2) \), respectively. Equations (15) and (16) describe the goods market clearing conditions, and equations (16) and (17) describe the fact total income equals total expenditures in the foreign and domestic country, respectively.

STEP 1: Rewrite the above system of eight equations and eight unknowns into a system of two equations and two unknowns.

Substituting equations (17) and (18) into (15) or (16) yields the expression
\[
\frac{\beta p_2}{p_1} \left[ \int_{z_1^*}^{z_2^*} F(z)g^*(z)dz + \int_{z_1^*}^{z_2^*} F(z)g(z)dz \right] = (1-\beta) \left[ \int_{0}^{z_1^*} m(z)g^*(z)dz + \int_{0}^{z_1^*} m(z)g(z)dz \right].
\]

Equations (11) and (13) can be rewritten in the form of \( z_2^* \) as a function of \( z_1^* \), and \( z_2 \) as a function of \( z_1 \). Furthermore, isolating \( \frac{p_2}{p_1} \) in equation (19) and substituting it into (12) and (14) yields the two equations

\[
\frac{\beta}{1-\beta} \frac{B}{A} = \frac{z_2^*(z_1^*)h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^2}, \tag{3.21}
\]

\[
\frac{\beta}{1-\beta} \frac{B}{A} = \frac{z_2(z_1)h(1-z_1) + z_1}{z_2(z_1) + 1/2hz_1^2}, \tag{3.22}
\]

where \( B = \int_{z_2(z_1)}^{z_2(z_1^*)} F(z)g^*(z)dz + \int_{z_1^*}^{z_2(z_1^*)} F(z)g(z)dz = \frac{1}{260} \left[ z_2^2(z_1^*)^2 - z_1^2 \right] + \frac{1}{2} \left[ z_2(z_1)^2 - z_1^2 \right] \)

and \( A = \int_{0}^{z_1^*} m(z)g^*(z)dz + \int_{0}^{z_1^*} m(z)g(z)dz = \frac{1}{60} \left[ 6z_2^*(z_1^*)^3 - hz_1^3 + 3h z_1^2 \right] + \frac{1}{6} \left[ 6z_2(z_1)z_1 - hz_1^3 + 3h z_1^2 \right] \)

with \( z_2^* \) and \( z_2 \) are written as functions of \( z_1^* \) and \( z_1 \). Equations (20) and (21) provide a system of 2 equations in 2 unknowns, \( z_1^* \) and \( z_1 \). The domain of interest is \([0, \alpha] \times [0, 1]\).

Equation (20) represents a specific isoline of a function \( y = f_1(z_1^*, z_1) \) in the \( \mathbb{R}^2 \) plane. Similarly, equation (21) also represents a specific isoline of a different function \( y = f_2(z_1^*, z_1) \) in the \( \mathbb{R}^2 \) plane. Therefore, equations (20) and (21) can be thought of as functions of three variables \( (z_1^*, z_1, y) \)

\[
f_1(z_1^*, z_1) = \frac{\beta}{1-\beta} \frac{B}{A} - \frac{z_2^*(z_1^*)h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^2}, \tag{20}
\]

\[
f_2(z_1^*, z_1) = \frac{\beta}{1-\beta} \frac{B}{A} - \frac{z_2(z_1)h(1-z_1) + z_1}{z_2(z_1) + 1/2hz_1^2}, \tag{21}
\]

evaluated at \( y = 0 \).

STEP 2: Show that at the point \( z_1^* = 0 \) equation (20) is above equation (21).

Let \( z_1^* = 0 \). Then \( z_2^* = \alpha \), and from equation (20) it follows that \( \frac{\beta}{1-\beta} \frac{B}{A} = h \). Substituting
this into (21) yields:

\[ f_1(0, z_1) = h - \frac{z_2(z_1)h(1 - z_1) + z_1}{z_2(z_1) + 1/2hz_1^2}. \]

Since \( h < \frac{1}{z_1 + z_2(z_1)} \), and \( \frac{1}{z_1 + z_2(z_1)}(1/2hz_1 + z_2(z_1)) < 1 \) it follows that

\[ 1/2h^2z_1 + hz_2(z_1) < 1. \]

As a result,

\[ f_1(0, z_1) = h - \frac{z_2(z_1)h(1 - z_1) + z_1}{z_2(z_1) + 1/2hz_1^2} < 0, \]

which implies that equation (20) is above equation (21) when \( z_1^* = 0 \).

STEP 3: Show that at the point \( z_1 = 0 \) equation (21) is above equation (20).

Let \( z_1 = 0 \). Then \( z_2 = 1 \), and from equation (21) it follows that \( \frac{\beta}{1-\beta} \frac{B}{A} = h \). Substituting this into (20) yields:

\[ f_1(z_1^*, 0) = h - \frac{z_2^*(z_1^*)h(1 - z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^{*2}}. \]

Since \( h < \frac{1}{z_1^* + z_2^*(z_1^*)} \), and \( \frac{1}{z_1^* + z_2^*(z_1^*)}(1/2hz_1^* + z_2^*(z_1^*)) < 1 \) it follows that

\[ 1/2h^2z_1^* + hz_2^*(z_1^*) < 1. \]

As a result,

\[ f_1(z_1^*, 0) = h - \frac{z_2^*(z_1^*)h(1 - z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^{*2}} < 0, \]

which implies that equation (21) is above equation (20) when \( z_1 = 0 \).

From steps 3 and 4, and the fact that equations (20) and (21) are continuous, it follows that there exists at least one point where the equations intersect. Therefore, there exists at least one equilibrium.

STEP 4: Show that the equilibrium is unique.
From equation (20) it follows that

$$\frac{dz_1}{dz_1^*} = -\frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) - \frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) \frac{\partial}{\partial z_1^*} \left( \frac{z_2^*(z_1^*) h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right).$$

(3.23)

From equation (21) it follows that

$$\frac{dz_1}{dz_1^*} = -\frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) \frac{\partial}{\partial z_1^*} \left( \frac{z_2(z_1) h(1-z_1) + z_1}{z_2^*(z_1) + 1/2h z_1} \right).$$

(3.24)

When equations (20) and (21) intersect, the expression $\frac{\beta}{1-\beta} \frac{B}{A}$ has the same value in both equations. As a result, its partial derivatives with respect to $z_1^*$ and $z_1$ will also be equal in equations (20) and (21).

Claim 1: $\frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) < 0$ and $\frac{\partial}{\partial z_1} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) < 0$.

Proof:

The partial derivative of $\frac{\beta}{1-\beta} \frac{B}{A}$ with respect to $z_1^*$ is equal to:

$$\frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) = \frac{\beta}{1-\beta} \left[ \frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) \right].$$

From equation (11) it follows that $\frac{dz_2(z_1^*)}{dz_1^*} = -h(1 - z_1^*) < 0$, and since $A > 0$ the first term in the fraction above is negative. Since $B > 0$, and substituting $\frac{dz_2(z_1^*)}{dz_1^*} = -h(1 - z_1^*)$ into $\left[ 6 \frac{dz_2(z_1^*)}{dz_1^*} z_1^* + 6z_2^*(z_1^*) - 3h z_1^* + 6h z_1^* \right] A$ and canceling similar terms, we obtain the expression $\left[ 3h z_1^* + 6z_2^*(z_1^*) \right] B$, which is positive. Therefore, $\frac{\partial}{\partial z_1^*} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) < 0$. A similar argument shows that $\frac{\partial}{\partial z_1} \left( \frac{\beta}{1-\beta} \frac{B}{A} \right) < 0$. Hence the claim is true.

Claim 2: $\frac{\partial}{\partial z_1^*} \left( \frac{z_2^*(z_1^*) h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right) > 0$ and $\frac{\partial}{\partial z_1} \left( \frac{z_2(z_1) h(1-z_1) + z_1}{z_2^*(z_1) + 1/2h z_1} \right) > 0$.

Proof:

The partial derivative of $\frac{\partial}{\partial z_1^*} \left( \frac{z_2^*(z_1^*) h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right)$ with respect to $z_1^*$ is equal to:

$$\frac{\partial}{\partial z_1^*} \left( \frac{z_2^*(z_1^*) h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right) = \left[ \frac{dz_2(z_1^*) h(1-z_1^*)}{dz_1^*} \right] \left[ \frac{z_2^*(z_1^*) + 1/2h z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right] - \left[ \frac{dz_2(z_1^*) h(1-z_1^*)}{dz_1^*} \right] \left[ \frac{z_2^*(z_1^*) + 1/2h z_1^*}{z_2^*(z_1^*) + 1/2h z_1^*} \right].$$

First note that since $h < \frac{1}{z_2 + z_1}$, it follows that $z_2^* z_1^* > 2h z_2^* z_1^*$. Second, note that $z_2^* z_1^* > z_1^2$, and that $h(1 - z_1^*) z_1^* > h^2 z_2^* z_1^*(1 - z_1^*)$. Therefore, it follows from these three inequalities that:
Thus, \( \frac{\partial}{\partial z_1^*} \left( \frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^*} \right) > 0 \). A similar argument shows that \( \frac{\partial}{\partial z_2^*} \left( \frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^*} \right) > 0 \). Hence the claim is true.

From the two claims above, along with expressions (22) and (23), we can conclude that when equations (20) and (21) intersect in the domain \([0, \alpha].X.[0, 1]\), equation (20) is always steeper than equation (21). Therefore, these equations intersect only once, and so the equilibrium is unique.

The steps required to show that the earnings functions are convex, and the equilibrium exhibits positive sorting in Home and Foreign, are the same as in the closed economy. Therefore, it remains to show that factor price equalization does not hold. The earnings of workers with ability \( z_1^* = 0 \) and \( z_2 = 0 \) are:

\[
\begin{align*}
    w^*(0) &= p_1 z_2^* - \sigma = (p_1 + h) z_2^* w(0) = p_1 z_2 - \sigma = (p_1 + h) z_2 \\
    w^*(0) &= p_1 z_2^* - \sigma = (p_1 + h) z_2^* w(0) = p_1 z_2 - \sigma = (p_1 + h) z_2
\end{align*}
\]

Since \( z_2 \) is not equal to \( z_2^* \), it follows that their earnings are not equal. Hence, factor price equalization does not hold.

\[ \blacksquare \]

\textbf{Proposition 3.9:} For any values of \( \alpha, h \in [0, \bar{h}] \) the following hold:

1. The relative price of good 1 in an open economy, is between Home and Foreign’s autarkic prices

2. In Home, \( z_1 \) increases while \( z_2 \) decreases, while in Foreign the opposite takes place

3. In Home, production of good 1 increases and production of good 2 decreases, while in Foreign the opposite takes place.

\textbf{Proof.}

That the relative price of good 1 is between Home and Foreign’s autarkic prices follows directly from the market clearing conditions for goods 1 and 2 in the both the open and closed economies. To show how the thresholds \( z_1 \) and \( z_2 \) are affected by international trade, I consider how these thresholds adjust from a change in the price of \( p_1 \). Given \( p_1, z_1 \) and \( z_2 \) are determined by the following equations:
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\[ h \left[ z_1 - \frac{z_1^2}{2} \right] + z_2 = \alpha, \quad (3.25) \]

\[ p_1 z_2 - z_2 h(1 - z_1) + 1/2 p_1 h z_1^2 = p_2 z_1. \quad (3.26) \]

Taking the total derivative of equation (refopenproof1) and isolating terms gives:

\[ \frac{\partial z_2}{\partial p_1} = -h[1 - z_1] \frac{\partial z_1}{\partial p_1}, \]

which is negative and therefore there is a negative relationship between \( \frac{\partial z_2}{\partial p_1} \) and \( \frac{\partial z_1}{\partial p_1} \).

Taking the total derivative of equation (refopenproof2) and isolating terms gives:

\[ [z_2 + 0.5 * h * z_1^2] + [2 - h(1 - z_1)] \frac{\partial z_2}{\partial p_1} = [1 - h z_2 - h] \frac{\partial z_1}{\partial p_1}. \]

Consider the case that \( \frac{\partial z_1}{\partial p_1} < 0 \). From above it follows that \( \frac{\partial z_2}{\partial p_1} > 0 \) which implies that expression on the left-hand-side is positive. Hence because the term \([1 - h z_2 - h]\) is positive, the inequality cannot hold, and so there is a contradiction. Therefore it must be the case that \( \frac{\partial z_1}{\partial p_1} > 0 \). The claims in the proof now follow. \( \blacksquare \)

**Proposition 3.12:** Under trade, in the Home country the earnings of all production workers increase, and the earnings of all managers increase. In the Foreign country the opposite takes place. The earnings of all agents who remain self-employed are unaffected by trade.

**Proof.**

Consider the wages of agents with ability 0. Their earnings are equal to

\[ w(0) = [p_1 - h] z_2. \]

Taking the derivative of this expression with respect to \( p_1 \) yields:

\[ \frac{\partial w(0)}{\partial p_1} = z_2 + [p_1 - h] \frac{\partial z_2}{\partial p_1}. \]

Substituting in the expression for \( \frac{\partial z_2}{\partial p_1} \) and using the fact that since \( w(0) < z_2, \ p_1 - h < 1, \) it follows that \( \frac{\partial w(0)}{\partial p_1} > 0 \). Further since only at point \( z_1 \) is it the case the \( w(z) = z \) and for all other \( z < z_1, \ w(z) > z \) it follows that the wages of all production workers increase with trade.
Now consider the rents of the least knowledgeable manager $z_2$ in the new open economy. In both the open and close economy this agent earned $z_2$. Therefore, since by construction of the equilibrium $R(z) > z$ for all $z > z_2$ the earnings of agents who were previously unemployed increase with trade. Now consider the least knowledgeable manager $z'_2$ in the closed economy. Because he is assigned to better workers, the slope of the rents function will be greater. Therefore, for this agent $z'_2$ the rents function will have a higher level, and a steeper slope under trade. It therefore follows that the earnings of all agents who are managers increases with trade. ■
Bibliography


