A DESIGN METHODOLOGY FOR RECONFIGURABLE MILLING MACHINE TOOLS AND AN IMPLEMENTATION

by

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Abstract

Reconfigurable Machine Tools (RMTs) have been developed in response to agile manufacturing demands. The prevalent modular approach for reconfigurability, in which a machine configuration is assembled for a given part, can be a demanding task time-wise and accuracy-wise, especially for smaller-scale Reconfigurable meso-Milling Machine Tools (RmMTs).

Research in the field of Parallel Kinematic Mechanisms (PKMs) has paved the way for the design of Redundant Reconfigurable Machine Tools (RRMTs) based on such accurate mechanisms. Reconfigurability in RRMTs can be manifested through topological and geometric reconfiguration, without disassembly of the structure. The main challenge in structural design of RRMTs is selecting the optimal architecture with the required level of redundancy for the set of parts at hand. The best RRMT architecture can be selected only after decisions are taken on the design variables, and the variables that manage the redundant reconfigurability. Namely, effectively locking/unlocking dof and selecting the optimal trajectory that each joint should follow. However, to date, no comprehensive design methodology addresses the challenges that are related to design of RRMTs.
The objective of this Dissertation is, thus, to develop a design methodology for RRMTs. The methodology proposes an approach towards enhancing the performance of RMTs and attaining required machining conditions, while taking into account the possible inherent redundant reconfigurability of serial, parallel and hybrid mechanisms.

The design process is combined from two engines: synthesis and optimization. These individual tasks, which are often solved in series, are addressed here through an integrated multi-tiered optimization-based design approach. The multi-tiered optimization comprises an iterative process that transfers decisions, which are generally taken throughout the design process, to a lower or upper tier to improve the overall result. The decisions are taken on a large number of continuous/discrete parameter’s values that mutually depends upon one another in a structured manner, one that yields optimal values for a set of design variables to satisfy required process parameters.

The applicability of the proposed methodology is demonstrated through a design test case of a PKM-based Redundant RmMT (RRmMT) that can attain high stiffness while satisfying the high feed-rate requirement. The design process resulted in a new $3 \times PRPRS$ RRmMT that can be reconfigured into several different PKMs.
Acknowledgments

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Nomenclature and Acronyms

Latin Letters

\( A_i \) The position of the \( i^{th} \) curvilinear joint in the Eclipse, the UofT PKM, the Alizade PKM, and the Glozman PKM.

\( A_k \) A matrix of linkage constraints that describe the kinematic relations between the nodes on the two sides of the joints in the MSA model.

\( B \) Set of parts.

\( B_j \) Part \( j \).

\( C \) Performance metric.

\( C_i \) The position of the \( i^{th} \) prismatic joint in the Eclipse, MIT-SS-1, and the UofT PKM.

\( C_i^j \) Performance metric for the \( i^{th} \) RRMT architecture, and the \( j^{th} \) Part.

\( C_{i,k}^j \) Performance metric for architecture \( i \), configuration \( k \), and Part \( j \).

\( CO_{ij}^t \) The \( t \) configuration-part set for the RRMT that is constructed from the \( i^{th} \) and \( j^{th} \) mechanisms.

\( CON_{ik}^j \) Connectivity of the \( i^{th} \) chain of the \( k^{th} \) mechanism.

\( CP_j \) The number of points along the specified tool trajectory for the \( j^{th} \) part.

\( co_k \) The configuration for the \( k^{th} \) part.

\( d_i \) Prismatic joint’s range of travel.

\( dp_h \) The \( h^{th} \) decision variable.

\( DP \) Set of geometrical and dynamic design variables.
\( e \)  
\( x \)-axis stage range of motion.

\( f_i \)  
The degree of relative motion permitted by joint \( i \).

\( f_j \)  
The redundant constraint of joint \( j \).

\( F \)  
Vector of tool forces.

\( F_r \)  
Force acting on the tool flute in the radial direction.

\( F_t \)  
Force acting on the tool flute in the tangential direction.

\( F_N \)  
Force acting on the tool along the normal to the feed direction.

\( g(\Phi) \)  
Kinematic and dynamic constraints.

\( g_{e_n} \)  
The \( n^{th} \) generation in Step 1 of the synthesis process.

\( g_{e_m} \)  
The \( m^{th} \) generation in Step 2 of the synthesis process.

\( h \)  
The number of links in the mechanism.

\( HY \)  
Hybrid mechanism.

\( I \)  
The eye matrix.

\( J \)  
Jacobian matrix that transform from joint space velocities to task space velocities.

\( J_q \)  
Matrix of the time derivatives of the joint space variables.

\( J_x \)  
Matrix of the time derivatives of the task space variables

\( \bar{\kappa} \)  
Stiffness matrix of the active joints.

\( K \)  
Stiffness matrix at the tool-tip.

\( K_T \)  
The link stiffness in the global frame (MSA model).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{xx}, K_{yy}, K_{zz}$</td>
<td>Stiffness along Cartesian $x$, $y$ and $z$ axes for a force applied along the same axis.</td>
</tr>
<tr>
<td>$\bar{K}<em>{xx}, \bar{K}</em>{yy}, \bar{K}_{zz}$</td>
<td>Mean stiffness along Cartesian $x$, $y$ and $z$ axes for a force applied along the same axis.</td>
</tr>
<tr>
<td>$l_j$</td>
<td>Index of the checked point along the specified tool trajectory, which is related to part $j$.</td>
</tr>
<tr>
<td>$L$</td>
<td>Link length.</td>
</tr>
<tr>
<td>$L_{con}$</td>
<td>Number of configurations of an RRMT.</td>
</tr>
<tr>
<td>$(L_x, L_y, L_z)$</td>
<td>A unit vector to define the specified tool orientation.</td>
</tr>
<tr>
<td>$m$</td>
<td>The number of cooperative mechanisms (i.e. two for RRMTs).</td>
</tr>
<tr>
<td>$me_i$</td>
<td>The $i^{th}$ mechanism.</td>
</tr>
<tr>
<td>$M$</td>
<td>Mobility of a mechanism.</td>
</tr>
<tr>
<td>$M_i$</td>
<td>The $i^{th}$ actuator.</td>
</tr>
<tr>
<td>$M_{req}$</td>
<td>The minimal mobility which is required from the RRMT.</td>
</tr>
<tr>
<td>$M_{arc}$</td>
<td>Number of RRMT architectures in the database.</td>
</tr>
<tr>
<td>$M_{i,k}$</td>
<td>Mobility of chain $k$ of mechanism $i$.</td>
</tr>
<tr>
<td>$M_{WH}$</td>
<td>Mobility of the workpiece-holder.</td>
</tr>
<tr>
<td>$M_{TH}$</td>
<td>Mobility of the tool-holder.</td>
</tr>
<tr>
<td>$\mathbb{N}$</td>
<td>Natural numbers domain.</td>
</tr>
<tr>
<td>$N_{prt}$</td>
<td>Number of parts in the given set for which the RMT is designed.</td>
</tr>
<tr>
<td>${O}$</td>
<td>Cartesian coordinate system.</td>
</tr>
<tr>
<td>${O_1}$</td>
<td>The frame that is attached to the workpiece base.</td>
</tr>
</tbody>
</table>
\{O_2\}  The frame that is attached to the tool-holder base.

$p$  The number of joints in the mechanism.

$per_i$  Performance of the $i^{th}$ tier.

$P$  Prismatic joint.

$P_i$  Position of the $i^{th}$ platform joints.

$P_i^0$  Position of the $i^{th}$ platform joints in home configuration.

$Pr_c$  Performance value of the optimal RRMT.

$Pr_{-c_i}$  Performance value of architecture $i$.

$Pr_{-c_i}^j$  Performance value of configuration $k$, of RRMT $i$, with the $j^{th}$ part.

$^{l_j}Pr_{-c_i}^j$  Performance value of configuration $k$, of RRMT $R_{i,k}$, at point $l_j$, along the tool trajectory required for fabricating the $j^{th}$ part.

$^{l_j}pQ_{l,k}^j$  Posture for configuration $k$, of RRMT $R_{i,k}$, at point $l_j$, along the tool trajectory required for fabricating the $j^{th}$ part.

$PA$  Parallel mechanism.

$q$  Joint space parameters.

$\dot{q}$  Joint space velocities.

$q_{WH}$  Joint-space parameters of the workpiece-holder.

$q_{TH}$  Joint-space parameters of the tool-holder.

$\delta q$  Vector of joint space errors.

$Q$  The vector $Q = [\theta_1 \ \theta_2 \ \theta_3 \ d_1 \ d_2 \ d_3]^T$ denotes the generalized coordinates of the active joints.

$r$  The number of redundant dof.
$r_{R_i}$ The redundancy of the $i^{th}$ RRMT.

$\mathbb{R}$ Real numbers domain.

$R$ Revolute joint.

$R$ Set of RRMT architectures.

$\begin{bmatrix} 0_2 \end{bmatrix} \overline{R}$ Transformation matrix from frame $\{O_1\}$ to frame $\{O_2\}$.

$\begin{bmatrix} T \end{bmatrix} \overline{R}$ Transformation matrix from frame $\{O\}$ to frame $\{T\}$.

$R_b$ Base radius of the PKM.

$R_p$ Platform radius of the PKM.

$R_i$ The $i^{th}$ architecture.

$R_{i,k}$ Configuration $k$ of architecture $i$.

$s_i$ The $i^{th}$ set-up variable of the mechanisms' set-up.

$s_u$ A vector of set-up variables.

$S$ Spherical joint.

$SC$ Set of building blocks.

$SE$ Serial mechanism.

$SU$ Space of set-up variables.

$t$ Geometric variable.

$t_{R_i}$ The joints parameter at the $i^{th}$ discrete point along the specified tool trajectory.

$t_{str_i}$ The task space parameter at the $i^{th}$ discrete point along a specified tool trajectory.
\( T \)  
The pose of the tool with respect to the Cartesian coordinate system.

\( \tilde{T} \)  
The trajectory of the tool relative to the workpiece frame \( \{W\} \).

\([T_x \ T_y \ T_z]^T\)  
Cartesian coordinates of the origin of the tool-holder platform frame.

\([T_\alpha \ T_\beta \ T_\gamma]^T\)  
Orientation of the tool-holder platform given as (z-y-z) Euler angles.

\( \{T\} \)  
Coordinate system of the moving platform of the tool-holder.

\( TH \)  
Tool-holder mechanism.

\( TR \)  
Joint space trajectories.

\( TSTR \)  
Given set of task space trajectories that are associated with the given set of parts.

\( \{W\} \)  
Coordinate system attached to the moving platform of the workpiece-holder.

\( W \)  
The pose of the workpiece-holder moving frame \( \{W\} \) with respect to the Cartesian frame \( \{O\} \).

\([W_x \ W_y \ W_z]^T\)  
Cartesian coordinates of the origin of the moving frame of the workpiece-holder.

\([W_\alpha \ W_\beta \ W_\gamma]^T\)  
Orientation of the workpiece-holder platform given as (z-y-z) Euler angles.

\( WH \)  
Workpiece-holder mechanism.

\( X \)  
A symbol for a joint that can be prismatic or revolute.

\( x_f \)  
\( x \)-coordinate of the discrete points along the tool path given in workpiece holder frame.

\([x_f \ y_f \ z_f]^T\)  
Coordinate of the discrete points along specified tool path given in the workpiece frame.

\( x \)  
Task space parameters.
\( \hat{x} \) Task space velocities.

\( \delta x \) Vector of task space errors.

\( \Delta X \) Vector of the displacements of the nodes dof in the MSA model.

**Greek Letters**

\[ [\alpha \ \beta \ \gamma] \] Orientation along the specified tool path relative to the workpiece frame, given as z-y-z Euler rotation angles.

\( \gamma \) The rotation angle of the platform about its normal.

\( \theta_i \) The position of the \( i^{th} \) curvilinear base joints of the Eclipse, the UofT PKM, the Alizade PKM, and the Glozman PKM.

\( \theta_A, \theta_C \) Drive command to the rotational actuators in the serial mechanism.

\( \lambda \) The mechanism space (3 for planar and 6 for spatial).

\( \lambda_M \) Lagrange multiplier.

\( \tau \) 6-dof vector of the forces at the joints.

\( \varphi_i \) Revolute joints’ range of travel.

\( \Phi \) The decision variables in the optimization process of the RRMT.

\( \chi \) The rotation angle of the first revolute joint in the 3×PRPRS RRMT.

\( \Omega \) The matrix in the MSA model, which is constructed from two unit vectors, \( \tilde{m} \) and \( \tilde{k} \), that defines the frame for the revolute joint.

**Acronyms**

**CAD** Computer-aided design.

**CN** Condition Number.
dof  degrees of freedom.

3D  3-Dimensional.

FEA  Finite Element Analysis.

PKM  Parallel Kinematic Mechanism.

RMT  Reconfigurable Machine Tool.

RRMT  Redundant Reconfigurable Machine Tool.

RmMT  Reconfigurable meso-Milling Machine Tool.

RRmMT  Redundant Reconfigurable meso-Milling Machine Tool.

RPM  Rounds Per Minute.

mMT  meso-Milling Machine Tool

MSA  Matrix Structural Analysis.

Symbol

\{\mathcal{L}\}  The space of parallel kinematic mechanisms.

\{\mathcal{H}\}  The space of serial mechanism.

(\cdot)  Optional redundancy or optional mechanism in RRMT architecture.

\|\!(\cdot)\!\|  The two-norm of (\cdot).

X  Active joint.

(\cdot)^{-1}  Inverse matrix of (\cdot).

\chi^L  Lockable joint.

(\cdot)^*  The optimal value for the design variable (\cdot).
\[ |(\cdot)| \quad \text{Determinant of } (\cdot). \]

\[ \Delta(\cdot) \quad \text{A small change in } (\cdot). \]
Chapter 1 Introduction and Literature Review

The concept of Reconfigurable Machine Tools (RMTs) was developed in response to agile manufacturing demands. This design paradigm allows one to change a machine’s configuration based on the task it is intended to perform [1]. Configuration is defined herein as the topology of the machine dedicated to machining a specific part. A common approach to reconfigurability during early designs of RMTs was to change the machine configuration by exchanging sub-components [2]. Thus, supporting design methodologies focused on modular design, e.g., constructing a library of modules, and choosing the right modules for the parts.

Modularity, however, has its drawbacks, for example, reconfiguration requires disassembling/assembling of modules and recalibration. This can be a demanding task time-wise and accuracy-wise, especially for smaller-scale Reconfigurable meso-Milling Machine Tools (RmMTs). Namely, the time necessary to reconfigure an RmMT, due to the smaller size of its modules and the required tool accuracy for machining the meso-scale parts, may make them impractical to use.

meso-milling commonly refers to the machining of millimeter-size parts with features in the range of tens or hundreds of micrometers. meso-parts are characterized by their accuracy, feature tolerance/object size, which is smaller than \(10^{-3}\), and surface roughness of better than 100 nanometers [3]. The growth in demand for 3D sculptured meso-scale parts [4], and the unique machining requirements of such parts, call for new methods to support the design of corresponding RmMTs.

Redundant Reconfigurable Machine Tools (RRMTs) attain reconfigurability through the locking of selective degrees-of-freedom (dof). Namely, it is suggested that redundancy in mobility could be used to optimally reconfigure an RRMT prior to its use for a given part (or, multiple parts). For example, the 8-dof RRMT, shown in Figure 1.1(a), has a 6-dof Parallel Kinematic Mechanism (PKM) based tool-holder, and 2-dof serial based workpiece-holder. The PKM has three actuators that move along the base rail, and three actuators that move vertically, and the workpiece-holder is constructed from two linear stages. The 8-dof RRMT can be reconfigured to a 5-dof RRMT, shown in Fig. 1.1(b), through the locking of the base joints.
1.1 Thesis Statement

It is evident from the literature, that there is a need to develop a comprehensive design methodology for RRMTs. Such a methodology, must address the following challenges:

i. Consider wide range of RRMT architectures that are based on serial/parallel/hybrid mechanisms.

ii. Synthesize RRMTs that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations, and PKMs that differ in topology.

iii. Enhance the performance of a given RRMT through redundant reconfigurability without the disassembly of the structure.

iv. Manage the variety of decision making that involve with an RRMT design.

v. Determine the required level of redundancy of an RRMT.

vi. Address the unique design requirements of Redundant Reconfigurable meso-Milling Machine Tools (RRmMTs), such as the machine size and the machining conditions.
Existing methods are unable to address these challenges concurrently, with a single solution method. Thus, the overall objective of the proposed research, therefore, is *to develop a novel generic design methodology for RMTs with possible redundancy whose performance may be enhanced through reconfigurability. The developed methodology will be verified through the design and build of a prototype 5-axis RRmMT.*

1.2 Literature Review

In the following, a review is provided on architectural design efforts of machine tool pertaining to PKM-based architectures and reconfigurable design. In addition, a review of the literature on meso-milling machine tools, and meso-milling operation is also provided, as these topics are relevant to the implementation of the methodology proposed in this thesis.

1.2.1 Machine-Tool Architectures

Several prominent design efforts for machine tools, based on PKMs, and reconfigurable structures, have been proposed in the last two decades. This section reviews the design approaches with respect to the evolution of machine tools architecture shown in Fig. 1.2: 1994 - first PKM-based machine tool, [5]; 2000 - serial mMTs [6]; 2000 – PKM-based machine tool architectures designed to enhance performance [7]; 2002 – hybrid-based machine tools [8]; 2006 – PKM-based modular RMTs [9]; 2010 - redundant RMTs [10].

Although serial-based machine tools have the advantages of large workspace/footprint ratio and relatively uniform performance over the workspace, their accuracy may be limited by cumulative (stacked) joints errors. Thus, in response, PKM-based machine tools, such as the Stewart Platform based Variax, have emerged [11]. Table 1.1 compares PKM and serial mechanisms. The characteristics of PKMs include high stiffness-to-mass ratio, higher accuracy, and higher acceleration compared to those of serial mechanisms [12]. However, PKMs have drawbacks such as singularities, non-uniform distribution of mechanical stiffness over the workspace, and small workspace coverage.
These drawbacks have led to subsequent design efforts, in which hybrid machine tools have been suggested, namely, combinations of PKM with serial mechanisms [8, 14]. One example is the 5-axis MIT-SS-1 [9] with a 3-dof PKM-based tool-holder, and a 2-dof stacked serial workpiece-holder.

An alternative approach to enhance performance has been the use of kinematic redundancy. The term redundancy, as described above, is used herein to describe a mechanism with more than the minimal number of actuators required for the task, e.g., a 6-dof PKM, used as a tool-holder, for 5-axis machining. Redundant designs can provide increased dexterity, fewer singular
configurations, higher stiffness, faster machining time, and higher velocity. For example, a conventional size machine tool with redundant hybrid design that incorporates a 6-dof hexapod tool-holder, and a 2-dof serial rotating stage workpiece-holder was proposed in [15]. The three redundant dof are used in a heuristic-based trajectory optimization to minimize the Jacobian condition number. Similarly, high-stiffness trajectory planning for a multi-arm robot, was obtained using a combination of null-space search and min-max optimization [16].

Another type of design effort to enhance performance has been the use of reconfigurability, Fig. 1.3. RMTs can be optimally reconfigured to achieve the requirements of a representative set of parts at hand [17]. Such reconfigurable systems have been proposed in the past, with the most notable work being performed at the University of Michigan, Ann Arbour. A survey on the development of RMTs, such as a multi-spindle machine and reconfigurable bicept system, is presented in [18].

PKMs are used as a special class of RMTs as they possess the required flexibility and convertibility to adapt to changes, and their modular structure allows to customize them based on the task at hand. For example, comprehensive research into RMTs led to the design of a family of reconfigurable mechanisms with several identical actuated legs that are distributed around the platform and a passive leg, which is connected to the center of the platform [19-20].
The mobility of these mechanisms can be modified by locking dof of the passive leg, thus, obtaining new configurations with 3-, 4- and 5-dof, Fig. 1.4.

![Fig. 1.4](image1)

Figure 1.4. A class of reconfigurable PKMs with changing number of identical chains and one passive chain: (a) 3-dof, (b) 4-dof, (c) 5-dof [19].

In [9], a Tripod-based RMT is constructed from repetitive sub-components, as depicted in Fig. 1.5.

![Fig. 1.5](image2)

Figure 1.5. A Tripod based RMT [9].

In a more recent example [21], a reconfigurable redundant hybrid manipulator that can reach up to 7-dof is presented. This architecture consists of a 3-dof serial mechanism that is mounted on a modular PKM platform, Fig. 1.6. The PKM can be reconfigured to obtain 4-dof mobility, or 3-dof with redundant actuation. Adding the redundant dof in Configuration III and Configuration IV is done to improve the robot’s performance near singularities and to
compensate on mechanical backlashes. In [22], a stationary spindle is positioned above a 6-dof PKM-based worktable for machining complex-geometry parts.

Figure 1.6. The Cheopy PKM: Conf. I – 3-dof (3 translational motions), Conf. II – 4-dof (3 translations, one rotation), Conf. III – 3-dof controlled by 4 actuators (3 translational motions), Conf. IV – 3-dof controlled by 4 actuators (3 translational motions) [21].

Another example of using PKM in reconfigurable machines can be found in [23], where a 2-dof serial rotating head is mounted on a 3-dof PKM platform to construct the hybrid TriVariant machine tool. Reconfigurability is obtained by connecting the TriVariant in different orientations, thus, allowing flexibility in machining different parts, Fig. 1.7(a). Researchers present in [24] a 3-dof PKM-based RMT that can be reconfigured through a reconfigurable driving mechanism. The mechanism adapts the distance between the moving platform and the base to adjust the actuators vector for different tasks, Fig. 1.7(b). The 6-dof ReSI-Bot, Fig 1.7(c), is a recent RMT design that can enhance performance related criteria, such as workspace, stiffness and dexterity through geometrical reconfiguration [25].
An adaptive PKM-based machine, which uses repositioning of joints and an adjustable platform, is presented in [10]. The 9-dof system is composed from a hexapod whose links are connected to the base through three actuators. This system is reconfigured off-line, by adjusting the links’ base radius location, and on-line using the redundant dof to optimize the joint trajectories against a task. The Isoglidien-TaRb [26] is a family of reconfigurable fully-isotropic PKM-based RMTs that can be reconfigured by locking up actuators situated on the fixed base.

In [27], a 6-dof RRMT, which is based on a 3×PRRS topology is presented. Reconfiguration into lower mobility configurations is carried out by coupling of joints in adjacent chains. A 5-SPU based reconfigurable PKM is depicted in [28]. The PKM is reconfigured by displacing its attachments on the base radially.

Parallel mechanisms can also reconfigure into assembly and working mode configurations [29]. Configurations associated with the solutions to the direct kinematic model are denoted as assembly modes, and configurations that are associated with the solutions to the inverse kinematic model are denoted as working modes. In order to reconfigure between assembly/working modes, the RRMT has to go through a singular configuration. In [30], a procedure for reconfiguration in order to increase the workspace coverage of a PKM is presented. The procedure is demonstrated through reconfiguration of the Triglider.

Some of the abovementioned approaches to the design of traditional-size machine tools have also been applied to the design of meso-Milling Machine Tools (mMTs), where the motivation
has been to decrease energy consumption and footprint. The term meso-milling commonly refers to the machining of millimeter-size parts with features in the range of tens and hundreds of micrometers. Recently, demand has grown for 3D sculpted meso-scaled parts that must be milled using 5-axis machines. Example industries include biomedical (implants, such as heart stents and bone screws), consumer electronics (dies for embossing miniature letters and patterns), IT (micro moulds for fiber optic ferrules, moulds for lenses/mirrors used in optical switches), as well as aerospace and automotive, (e.g., [4, 31]).

mMTs use the same general technology as conventional size machine tools, with a few differences, such as the small size of meso-machines, the miniature tools/tool-holder, and the high spindle speed that is required (40K-500K rpm) [32-33]. mMTs have been developed, mostly, in the form of 3- and 5-axis fixed-topology machines. The University of Illinois has developed both 3- and 5-axis serial mMTs, shown in Fig. 1.8(a), with 2 μm accuracy, 20 nm resolution, a 40 mm stage-travel and 180° (B axis) and 360° (C axis) rotation [34]. Seoul National University 5-axis serial-based mMT, which is shown in Fig. 1.8(b), achieves a 50 nm resolution, 20-30 mm stage-travel, and 360° rotation (A and C axes) [35]. The latest example, shown in Fig. 1.8(c), is the 5-axis gantry based mMT developed by Brunel University (commercialized by UPM), with 0.1 μm accuracy, 5 nm resolution, and stage-travel of 225/230/165 mm for the x/y/z axes, respectively [36].

![Figure 1.8](image.jpg)

Figure 1.8. Academia mMTs: (a) University of Illinois 5-axis mMT [34], (b) Seoul National University serial 5-axis mMT [35], (c) Brunel University 5-axis mMT [37].
In industry, manufacturers have been using cutting-edge technologies, providing enhanced precision, highly specialized machines. The Robonano-0iB by FANUC, Inc., shown in Fig. 1.9(a), is a good example of a high-end performance 5-axis mMT [38]. With performances of 0.1 $\mu$m accuracy and 1 nm resolution, this expensive ($1M) mMT is intended for customers with a high manufacturing volume, primarily in the high-precision optics industry. Another notable commercialization effort, is that of Microlution Inc. (model 5100-5100-S), shown in Fig. 1.9(b), which offers 0.1 $\mu$m accuracy, 50 nm resolution, and a 63×63×63 mm workspace [39]. Atometric Inc. offers an mMT (G4-ULTRA CNC MicroMachining Center) with up to 5 axes, 0.3 $\mu$m accuracy, 100 nm resolution, and 101×101×76 mm workspace [40]. HASS mMT (Office-Mills-OM-1A), shown in Fig. 1.9(c), offers 5 $\mu$m accuracy, 15.5 nm resolution, and 203/305 stage travel for the $x$ and $y/z$ axes, respectively [41], and Kern downsized machining center (Kern Micro), which is shown in Fig. 1.9(d), features a 2 $\mu$m accuracy, and a 100 nm resolution [42].

![Figure 1.9](image_url)

**Figure 1.9.** Commercial mMTs: (a) Robonano-0iB by FANUC [39], (b) 5100-S by Microlution [40], (c) Office-Mills-OM-01 by Hass [41], (d) KERN-Micro by KERN [42].

As noted above, most existing mMTs are based on serial fixed-topology architectures. An exception is the 3-dof arch-type mMT presented in [43]. The mMT, shown in Fig. 1.10, consists of two sets of arched rails that are perpendicularly mounted into the main frame using a bearing. The rails have a common rotational center and are able to provide 3-dof rotational motion that controls the tool orientation, using two actuators at the base.
Current mMTs can cover a 6-dof workspace only by stacking stages. As a result, the errors from different stages are additive, and in order to achieve high accuracy, these machines use high-end performance components. PKM/hybrid/redundant RRmMT have not been reported in the literature, apart from the ones developed in our laboratory [44-46], and the potential of achieving high-end performance with such designs have not been investigated. By implementing the new design methodology for an RRmMT design, herein, it is assumed that improved cost effective performance (increased workspace and stiffness) can be attained.

1.2.2 RMT Design Methodologies

A number of research works on modular-design approaches for RMTs have been reported in the literature (e.g., [47]). In [2], a systematic methodology for designing RMTs was presented. The inputs to the methodology are functional requirements extracted from process plans of the machined parts. These inputs are used to generate a set of configurations with the required tool motions. The methodology includes: translation of the machining process plan for a part into a motion matrix, identification and representation of modules in a matrix form, synthesis of configuration by multiplying integrated module matrices, and comparing it with the task motion matrix.

In [17], a generic design methodology that relates planning of the part set-up with the machine tool configuration, for the synthesis of self-reconfigurable machine tools (through
spindle re-orientation), is presented. Using part machinable features, which are translated to extended hybrid graphs, the machine-tool configuration is defined.

A method for synthesizing a minimal yet sufficient set of building blocks necessary to form an RMT for machining a family of parts is described in [48]. Using axiomatic design principles, the set of building blocks is determined by associating the identified machinable geometric features to the structural modules derived from conventional machine tools. The set of building blocks are, then, used to construct the configurations of the RMT, and the number of common modules among the original set is maximized through re-selection.

In [49], a framework for the development of RMTs that aids in identification of modules is presented. The framework includes the following steps: identification of the parameters that characterize the machine reconfigurability, and generation of preliminary geometric schematics, which allow constructing a machine configuration from building blocks using heuristics.

A concurrent optimal design of reconfigurable serial robots, which is also applicable for modular RMT was also reported [9, 50]. In this approach, a task-oriented optimization methodology determines the best configuration by combining both module types and dimensions into one problem formulation.

In [51], an RMT design approach that consists of two steps is presented: (i) synthesis of commutative designs, and (ii) selection of the best alternative. The RMT design and its module warehouse are optimized for building an RMT, by solving five single-objective problems: configurability, cost, and tool accuracy along the three Cartesian axes, $x$, $y$ and $z$.

Researches in [52], presented a synthesis method for variable topology parallel mechanisms. Axiomatic design based integrated framework, which was developed to assist the design of generic modular reconfigurable platform, is presented in [37]. The framework incorporates a theoretical model that covers various relevant micro manufacturing processes and machine tool elements, a design support system that includes a user-friendly interface, and an engine for a design evaluation.

A design methodology for PKMs, with steps to define basic elements (joints), number of actuators, connections of links and direction of joint movement with relation to the $z$-axis, is proposed in [53]. Another example of PKM design methodology is described in [54]. With the support of the design environment, machine structures are synthesized, and virtual machine
simulations are performed for validation and off-line programming. A two-stage design methodology for reconfigurable modular PKMs is depicted in [55]. The first stage selects the structure, and the second stage selects the geometrical design parameters that determine the optimal configuration for a given task.

A method that allows on-the-fly reconfiguration of PKMs is presented in [56]. The reconfiguration to lower mobility configuration is performed by disassembling chains of the PKM. A 6-dof PKM that is reconfigured by disassembly of a chain to a 5-dof PKM is shown in Fig. 1.11. In [57], a two-level method for geometric reconfiguration of a PKM-based RMTs is presented. In the higher level the particle swarm optimization algorithm is applied, and in the lower level the simplex linear programming, to determine the location of the base joints along linear rails.

![Figure 1.11. Topological reconfiguration of a 3×RRRR based-PKM [56].](image)

In [58], the most comprehensive structural synthesis methodology for PKM, which incorporates redundant configurations is presented. A set of links are created according to design objectives (link connectivity, type of joints, degree of redundancy, etc.). These links are then combined using evolutionary morphology (recombination, mutation, migration and selection), which is aimed at generating the complete set of possible solutions for the set of design objectives.

Some methods for the design of multi-modes of operation PKMs have been proposed. The modes of operation are defined as tool motion, which is limited to a specific pattern, such as rotational, or translational, motions. Reconfiguration from one mode of operation to the other requires the mechanism to pass through singular postures. In [59], a synthesis method for multi-
mode PKM is described. The synthesis is performed by: synthesis of one mode, two modes, and multi-modes chains. The chains are assembled to synthesize a PKM with multiple modes of operation. An example for a mechanism capable of mode assembly reconfiguration is the $3\times RRRRR$ based-PKM shown in Fig. 1.12.

![Diagram](image)

Figure 1.12. The $3\times RRRRR$ modes of operation [59].

A design method for PKMs that can topologically reconfigure into lower mobility configurations by switching from active constraint joint to passive locked joint is presented in [60]. Using a 6-dof PKM structure, lower mobility modes are attained without removing sub-components, through locking/unlocking joints. Workspace and kinematic analysis are performed to identify the capabilities of each of the PKM configurations. It is shown that this method can be utilized in limited mobility parallel robots, with different isomorphic configurations, for automatic reconfiguration.

Table 1.2 gives an overview of existing RMT design methodologies. As can be noted, existing design methodologies for RMTs, do not generally support RRMTs. The closest to such an approach are methods which design mechanisms with kinematic redundancy [61]. For example, researchers in [62] described a method for redundant machine tools that can increase machining productivity by decreasing the non-cutting tool movements, through a four successive optimization steps of process parameters.

The literature review reveals that no comprehensive design methodology optimizes the RRMT architecture and the process parameters, for a given set of parts. Namely, determine the required topology, level of redundancy and the distribution of dof between the workpiece- and tool-holders of a non-modular redundant reconfigurable machine. Hence, in order to support RRMTs design and make full use of their reconfigurability, a new design methodology that considers a wide range of machine design topologies, such as serial, parallel and hybrid is
proposed herein. The methodology can design RRMTs that achieve reconfigurability via redundancy, without the need to assemble/disassemble modules.

Additionally, current synthesis methods for Redundant Reconfigurable PKMs (RR-PKM), those mentioned in the literature review and others, do not consider conditions for combining a set of PKMs into a single architecture. RR-PKMs, which can be used as tool/workpiece holder of an RRMT, should support different types of reconfiguration processes to increase their reconfigurability. They should be able to reconfigure into assembly/working mode configurations and into sub-PKMs that differ in topology. Furthermore, although the functionality of joints (i.e., determining the active, passive or lockable joints, which are passive joints that can be locked through a dedicated mechanism) plays an important role in reconfigurable PKMs, past synthesis methods have, typically, overlooked this aspect. Thus, the proposed RRMT synthesis process should consider both topology and joint functionality in order to synthesize RR-PKMs.

Table 1.2. Overview of existing design methodologies for RMTs.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Description</th>
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<tbody>
<tr>
<td>Allan, et al, (2013) [60]</td>
<td>Parallel-based RMT reconfiguration method (lower mobility configurations) for ad-hoc task.</td>
</tr>
<tr>
<td>Sun, et al, (2010) [37]</td>
<td>Axiomatic design approach for the design of serial modular RmMTs.</td>
</tr>
</tbody>
</table>
1.2.3 meso-Milling

Demand for meso-parts in different fields, such as medicine, biotechnology, optics, electronics, and defense is constantly increasing. Example applications include medical implants, bio-chips, and wireless devices [63]. Parts/features are considered as meso-scale when at least two of their dimensions are in the range of 10-1000 micrometer. Generally, meso-features also require accuracy of a few microns and surface roughness of 100 nanometers [3].

High strength meso-scale parts are required in various applications: Fig. 1.13(a) - a propeller made of aluminum with a diameter of 800 µm; Fig. 1.13(b) - micro-scale mechanical components, called microbarbs, which are used for attachment of tissues using sharp piercing edges. The microbarbs are manufactured from several materials including polymer polymethyl methacrylate (PMMA) and 304 stainless steel, using a two step end-milling cutting operations. High aspect ratio parts, such as the microbarbs, are hard to machine, due to their tendency to break during cutting; Fig. 1.13(c) - an injection mold insert for manufacturing micro channels on a biochip, which are manufacture using end milling. The mold is manufactured from age-hardened steel with hardness of approximately 40 HRC, where the width and height of the channels is 100 µm; Fig. 1.13(d) - a mold of micro rim, which is made of SAE H13 steel, and the plastic molded part; Fig. 1.13(e) - a 110 copper-based heat sink. The pin fin width and the gap size between fins is 200 µm, and it is manufactured using end-milling operation.

meso-milling machining conditions are driven by the minimal chip thickness effect. The cutting mechanism in meso-milling is combined of variable portions of shearing to ploughing that are controlled by the chip thickness. Ploughing of the workpiece, which occurs when the uncut chip thickness is smaller than the minimum chip thickness, results in increase in cutting forces, chatter, deterioration of the surface roughness, and tool wear [4, 64]. Similar effect exists in meso/micro-drilling [65].

The chip thickness is controlled by the spindle speed, the number of tool flutes, and the feed-rate. Performance metrics based on forces that develop during machining, are related to the machine-tool capabilities. Thus, the machining conditions of the meso-cutting operations should be incorporated into the design process of the meso-machine tool to attain the required accuracy.
[66]. Obviously, keeping the chip thickness in the required size, larger than the minimal chip thickness, is a key to reducing the forces that are created during meso-milling.

Figure 1.13. meso-parts: (a) meso-propeller [35], (b) microbarbs [67]. (c) injection mold insert for manufacturing micro channels on a biochip [68], (d) rim mold in SAE H13 of 55 HRC (left) and the molded plastic part (right) [69], (e) heat sink (left) and the pin fin geometry (right) [70].

The spindle speed required for obtaining surface speed of 100 m/min with tool diameter smaller than 1 mm surpass 500,000 rpm [71]. Commercial spindles can reach up to 200,000 rpm. However, work is on going to develop spindle technologies and design for the high RPM required in meso-milling [72]. Current actuators technologies which are suited for mMTs cannot produce the dynamic performances that are required in meso-milling, i.e., actuators that are used in small size accurate robots are piezo-based, and the speed that they can develop, while providing sufficient thrust force, is 6-10 m/min [73]. In PKMs the transmission between the joint and tool can be 10:1. Meaning, for the tool to move in 3 m/min the actuator speed should be ~30 m/min. However, since PKMs present high stiffness compared to serial mechanisms, a PKM-based RRmMT that can attain high stiffness while satisfying the required meso-milling machining conditions can be the next step in the evolution of mMT design.
Current design methodologies in the literature do not support redundant RmMTs, in addition, no PKM-based RRmMT has been reported in the literature. Therefore, the viability of the proposed novel design methodology is demonstrated herein via the design test case of a new PKM-based RRmMT that can attain the high feed-rates required in meso-milling.

1.3 Thesis Objectives

Agile manufacturing requires the ability to quickly adapt to changes while maintaining the advantage of high quality products. Research in the field of mechanisms for machine tool in general, and specifically PKMs present an opportunity in terms of design of machine tools that are capable of adjusting their structure according to task requirements. Developments in design of PKM in combination with reconfigurability approach lead to the design of redundant reconfigurable PKM-based machine tools. In the redundant reconfigurability approach, the machine tool structure is adjusted without the need to disassemble sub-components. This can be an advantage in meso-machine tools, where the sub-components that are assembled are small and accurate calibration is required. The demand to attain high performance in machine tools is often related to machining accuracy and the stiffness of the structure. For this purpose, PKM and hybrid mechanisms that offer many advantages over conventional serial design are considered.

A design methodology for RRMTs is proposed against such a background in this research. The methodology should have the ability to compare different architectures based on desired performance that are extracted from the machining process of a given set-of-parts. This requires to analyze serial/PKM/hybrid mechanisms while taking advantage of redundant reconfigurability to ameliorate performance. Thus, the objective of this research is to establish a generic design methodology framework that can determine the topology and level of redundancy of an RRMT, which is designed for a given set of parts.

In addition, to the best of the authors’ knowledge, current design methodologies in the literature do not support the design of RRmMTs. Thus, another objective is to demonstrate the proposed novel design methodology via the design test case of a new RRmMT.
1.3.1 Design-Methodology Framework

The new design methodology should include the following two engines: (1) synthesis of RRMT architectures, and, (2) optimization of the RRMTs design. The proposed methodology must consider and integrate the information from the machining process domain into the design domain, in which decisions are made on the topology, and kinematic and static design parameters of the RRMT. For example, designing an RRMT that can machine a set of parts in minimal time depends on how fast the tool can move. The tool movement, in turn, depends on the actuators characteristics, e.g., speed and acceleration, as well as on the distance that the joints need to travel. The distances that the joints travel are associated with redundancy that leads to issues such as managing of redundancy, i.e., the decision regarding which dof to lock and which dof to activate effectively, and task allocation and trajectory resolution, i.e., determining the optimal trajectory that each joint follows.

In terms of design optimization, the complexity of the engineering design problem of RRMTs can be seen in the diversity of decision making, where a mixture of a large number of continuous/discrete parameters' values that mutually depend on one another, needs to be selected. Hence, one challenge in developing the design methodology is reducing the disturbance caused by parameters.

Other challenges in developing the proposed design methodology are related to the wide range of architectures compared, and the ability to conceptually determine the way redundant reconfigurability of an architecture can be utilized for reconfiguration without disassembling sub-components.

To demonstrate the applicability of the proposed methodology it is implemented for the design test case of a new RRmMT. The challenge in implementing the design methodology is developing software for design optimization and performance analysis of architectures. The optimization requires full motion simulations and dedicated performance evaluation modules. The design relies on the results from simulations developed within the methodology framework, and the challenge is to translate the virtual simulations results into a physical machine. Namely, incorporate engineering considerations, including interfaces between components,
manufacturing and assembly, which are not part of the simulations that yielded the optimal RRmMT.

The prototype RRmMT is built for a set of meso-scale parts. An important application of meso-milling is 3D molds, which require a 5-axis machining, namely, all the tool motions in space (besides the roll about the tool axis) are vital. Therefore, the design focuses on 5-dof RRmMT architectures. meso-parts require a relative accuracy of a feature tolerance/object size to be smaller than $10^{-3}$, and surface roughness of 100 nanometers [3]. Thus, the objective is to select the stiffest architecture from a given set of RRmMTs while attaining the machining conditions that are required in meso-milling.

1.4 Contributions and Thesis Overview

1.4.1 Contributions

This research has generated a number of contributions to the body of knowledge of machine design, to the RMT research field, and to meso-milling:

1) A novel comprehensive design methodology for RRMTs that can optimally utilize redundant reconfigurability to enhance performance:

   — A multi-tiered optimization-based design approach that has been applied to the conceptual design of RRMTs, namely, optimization that combines a large mixture of variables from different domains.

   — A synthesis process for RRMTs that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations, and full mobility sub-PKMs was developed.

   — Design of PKM-based RRmMTs for the unique machining conditions of meso-milling.

   As will be presented in Chapter 4, the methodology has been applied to design a stiff RRmMT architecture that can attain the high feed-rates required in meso-milling.
2) A novel $3\times PRPRS$-based RRmMT that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations, and sub-PKMs, such as the Eclipse [7], the Alizade PKM [74] and the UofT PKM [75].

1.4.2 Thesis Overview

In Chapter 2 the problem formulation of the synthesis process and the optimization process of RRMTs, and the main design challenges, such as kinematic modeling of lower mobility sub-configurations, applying kinematic/dynamic constraints on compared RRMTs, and analyzing the performance of RRMT variants are depicted.

Chapter 3 presents the proposed multi-tiered optimization-based design methodology. In addition, algorithmic procedure for synthesis of RRMTs, a procedure for determining the constraints on the travel-range of joints, and a procedure for optimization of joint trajectories are detailed. In order to demonstrate the design process, a simulated example of a PKM-based RRMT is illustrated using static stiffness as the primary objective function.

Chapter 4 depicts the design test case of a new RRmMT. A set of meso-scale parts is presented, and its effect on the design process is discussed. The synthesis process, and the set of architectures, which are compared throughout the design process are presented. Stiffness results of the compared RRmMTs are obtained through simulations, and the outline for a new RRmMT prototype that was designed via the proposed methodology are also presented.

Chapter 5 summaries the conclusions and contributions made in this work. Finally, recommendations for future work, which are generated from this research, are provided.

Appendix A presents the inverse-kinematic models of the compared architectures, which include the Eclipse PKM-based RRMT [7], the UofT PKM based RRMT developed in our laboratory [76], the MIT-SS-1 hybrid-based RRMT [8], and a reference $PPPRR$ serial-based RRMT.

Appendix B depicts the Matrix Structural Analysis (MSA) method for evaluating the static stiffness of the compared RRMTs.
Chapter 2  Problem Definition

RRMT design is comprised of two main sub-problems: (i) synthesis of RRMTs, and (ii) optimization of the design. This thesis addresses these two sub-problems, as well as the applicability of the proposed methodology to the design of RRmMTs. As mentioned in Chapter 1, different methodologies for designing RMTs have been proposed [77], however, to the best of my knowledge no comprehensive design methodology for RRMTs have been reported in the literature. Therefore, in order to design RRMTs, a systematic methodology, which includes complimentary design steps and principles, should be developed. To ameliorate RRMTs performance, the methodology should make full use of redundant reconfigurability, and combine information from the machining process into the design process. Since mechanisms differ in their characteristics, the design process should consider wide range of redundant architectures based on serial, parallel, and hybrid topologies and determine the level of redundancy that leads to optimal performance. The proposed methodology should also design RRmMTs that present a challenge due to the machine size and the conditions of meso-milling operation.

This chapter presents in detail the abovementioned sub-problems, and addresses the challenges in developing a design methodology. Throughout the following sections, it is important to note that although redundant reconfigurability is feasible, research on the practical design aspects of redundant mechanisms, such as lockable joints, spherical joints that have a large travel range and efficient control algorithms still continues. In addition, current spindle and actuator technologies do not satisfy the high rpm and feed-rates that are required in meso-milling. As technology advances, we can expect sub-component capabilities to improve, making redundant reconfigurable solutions more practical.
2.1 Formulation of RRMT Design Problem

The first step in developing the proposed design methodology is to formulate the problem in a mathematical framework. The performance of a machine tool can be characterized, for example, by the success of a machining task in achieving the desired accuracy within a given time constraint. An objective function, \( Pr_c \), which is a measure of success in achieving machining objectives, depends primarily on the machine performance.

The performance comparison criteria can include accuracy, stiffness, acceleration, speed, vibration, etc., which are characterized herein by a metric, \( C \). Other criteria, which relate to the machine design, such as reconfigurability and machine cost [51], can be also utilized in \( C \). This metric, in turn, can be defined in terms of variables, \( \Phi \), such as the machine architecture, and kinematic and dynamic characteristics of the building blocks, i.e., dimensions, weight, and inertia.

The actual evaluation of this metric depends on the objectives and the task assignments for each of the design specifications of the parts, such as geometrical features and material. Accordingly, the performance metric for the \( i^{th} \) RRMT architecture, and the \( j^{th} \) part is expressed as a function of \( \Phi \):

\[
C_i^j = f_i^j(\Phi).
\]  
(2.1)

Above, \( Pr_c \) is subject to the constraints, \( g(\Phi) \leq 0 \), defined by the kinematic and dynamic restrictions, such as joint ranges of motion and maximal velocity of stages. The constraints are determined according to sub-component (module) specifications in order to keep the design within the achievable technology limits. Another type of constraint, such as the machine cost or the on-line computation time for optimizing joints trajectories, which is expected to increase with every dof that will be incorporated, functions as a counterbalance to the level of redundancy.

The problem of designing an optimal \( n \)-dof RRMT architecture for the manufacturing of \( N_{prt} \) representative parts can be formulated as follows: for a given set of RRMTs \( R \), and a set of parts \( B \), solve the following optimization problem:
Given: \[ R_i, \; i \in [1..M_{arc}] ; \]
\[ B_j, \; j \in [1..N_{prt}] ; \]

Maximize: \[ Pr_c = f \left( \{ C_i^1, C_i^2, ..., C_i^{N_{prt}} \} \right) ; \] (2.2a)

Subject to: \[ g(\Phi) \leq 0, \] (2.2b)

where \( M_{arc} \) is the number of commutative RRMTs.

The process optimizes the RRMT architecture and other design variables. The problem can be solved as a multilevel optimization, i.e., first optimizing the RRMT architecture and then the building block dimensions. A different way would be to combine all the variables into one optimization problem with a single weighted objective function [50]. The latter would result in an optimization problem of combined discrete/continuous variables.

In order to realize the full potential of RRMTs, reconfigurability should be utilized to enhance the RRMT’s performance. The problem of selecting an RRMT configuration \( R_{i,k} \), for machining a given part \( B_j \), can be formulated as:

Given: \[ B_j, \; j \in [1..N_{prt}] ; \]
\[ R_{i,k}, \; k \in [1..L_{con}] ; i \in [1..M_{arc}] ; \]

Maximize: \[ Pr_{c_i^j,k} = f(C_i^j_{i,k}) , \] (2.3a)

Subject to: \[ g(\Phi) \leq 0 , \] (2.3b)

where \( k \) is the index for the RRMT configuration.

The problem of selecting an RRMT posture \( l_j^i P Q_{i,k}^j \), is formulated as follows: for a given configuration \( R_{i,k} \), and a point \( l_j \), along a specified tool trajectory for fabricating the \( j^{th} \) part, solve the following optimization problem:
Given: \[ B_j; \quad j \in [1..N_{prt}]; \]
\[ R_{i,k}, k \in [1..L_{con}]; i \in [1..M_{arc}]; \]

Maximize: \[ ^iPr_{c_{i,k}} = f \left( C \left( ^iPO_{i,k} \right) \right); \quad l_j = 1 \text{ to } CP_j; \] (2.4a)

Subject to: \[ g(\Phi) \leq 0, \] (2.4b)

where \( CP_j \) is the number of points along the \( j^{th} \) tool trajectory. The performance index of the RRMT depends on the configuration selected for each part, which in turn is a function of the optimized configuration’s postures at discrete checked points along the tool trajectory. Thus, the performance index value, \( Pr_{c_{i,1}} \), of the \( i^{th} \) architecture is denoted as:

\[
Pr_{c_{i,1}} = f \left( \left\{ ^1Pr_{c_{i,1}}, \ldots, ^{CP_1}Pr_{c_{i,1,L_{con}}} \right\}, \right) \]
\[
\left( \begin{array}{c}
\left\{ ^1Pr_{c_{i,2}}, \ldots, ^{CP_2}Pr_{c_{i,2,L_{con}}} \right\} \\
\vdots \\
\left\{ ^1Pr_{c_{i,N_{prt},1}}, \ldots, ^{CP_{N_{prt}}}Pr_{c_{i,N_{prt},L_{con}}} \right\}
\end{array} \right) \right)
\]
and the overall performance index of an RRMT as:

\[
Pr_{c} = f \left( Pr_{c_{i,1}}, \ldots, Pr_{c_{M_{arc}}} \right). \] (2.6)

In modular RMTs, each configuration is constructed individually for a given part. A number of established methods have been proposed for configuration synthesis [2, 17, 77-78]. These methods ensure kinematic viability, i.e., the ability of a configuration to perform the motions required to manufacture the part at hand. Synthesis of an RRMT, for a set of parts, should incorporate the complete set of dof for the complete set of parts.

In order to support a design process that compares alternatives and selects an RRMT, the synthesis process should create a database of RRMTs, \( R \), which serve as the search space for the optimal machine tool architecture. To conduct a search in as efficient a manner as possible, it is desired that the search space include RRMTs that can potentially machine the given set of parts. Furthermore, the synthesis process of every commutative RRMT should terminate when the architecture possesses the optimal level of redundancy. The condition for termination of the synthesis process of an RRMT can be based on performance criterion, such as accuracy, or machining time with respect to the parts at hand. The performance value, in turn, depends on the
design and process parameters that have to be determined as well. Performance of an RRMT is a function of the performance of the configurations and postures that have been selected by the optimization processes, as depicted in Eqs. (2.3)-(2.4). Therefore, the performance index defined in Eq. (2.6) can be used to determine if the termination condition is met.

The synthesis problem of the RRMT can be specified as follows: for a given set of parts, $B$, and a set of building blocks, $SC$, solve the following optimization problem:

$$\text{Given:} \quad B ; SC$$

$$R_i \equiv TH \cup WH; \ TH,WH \in \{\mathcal{L} \cup \mathcal{H}\};$$

$$\text{Maximize:} \quad Pr_{ci} = f(C(R_i, r)); \quad (2.7a)$$

$$\text{Subject to:} \quad g(\Phi) \leq 0, \quad (2.7b)$$

$$M_{TH} \cup M_{WH} \geq M_{req}, \quad (2.7c)$$

where $R_i$ is the $i^{th}$ architecture, $TH$ denotes the tool-holder mechanism; $WH$ denotes the workpiece-holder mechanism; $\mathcal{L}$ is the space of PKMs; $\mathcal{H}$ is the space of serial mechanisms; $r$ is the number of redundant dof; $M_{TH}$ is the tool-holder mobility; $M_{WH}$ is the workpiece-holder mobility; $M_{req}$ is the mobility required to machine the given set of parts.

### 2.2 Main RRMT Design Challenges

The challenges that should be addressed, in order to develop a methodology that can design serial/parallel/hybrid-based RRMTs, are presented in this section. Following the problem formulation above, the challenges are specified in the context of optimization-based design of an RRMT against a given set of parts.

**Redundant Reconfigurability**

The design of a variety of novel PKMs in the past three decades have presented researchers with an opportunity to develop modular and/or RR-PKMs. RR-PKMs can achieve redundant reconfigurability in three forms:
Case 1: Topological reconfiguration: locking/unlocking joints to achieve a lower/higher mobility mechanism. For example, by unlocking the revolute base joints of the 2×RR-based RRMT, shown in Fig. 2.1(a), the RRMT can be topologically reconfigured into the 2×RRR configuration, Fig. 2.1(b).

Case 2: Geometric reconfiguration: adjusting the size/orientation of links and joints without rearranging the chain topology. For example, by reorienting the links that are connected to the base, the 2×RR-based RRMT, shown in Fig. 2.1(a), can be geometrically reconfigured into the configuration shown in Fig. 2.1(c).

Case 3: Topological and geometric reconfiguration: combining Cases 1 and 2 to reconfigure into assembly/working mode configurations and into different sub-PKMs. For example, unlocking the base joints and adjusting the orientation of the second link in each chain, the 2×RR-based RRMT shown in Fig. 2.1(a) can be reconfigured into the working mode shown in Fig. 2.1(d).

Thus, it is possible to increase the versatility of a single RRMT architecture through reconfiguration into configurations that differs in topology, and/or geometric parameters, without the disassembly of the structure.
Current RRMT design methods do not concurrently consider the three redundant reconfigurability cases. Therefore, the design methodology, which is presented in this thesis, should make use of the potential of these three reconfigurability cases to optimize performance. The challenge is to design RRMTs, and effectively manage their redundant reconfigurability, i.e., the decision regarding which dof to lock and which dof to activate, into which configuration

Figure 2.1. Reconfiguration from (a) $2\times RR$ to: (b) $3\times RRR$ architecture – topological reconfiguration, (c) $2\times RR$ configuration – geometrical reconfiguration, (d) a different $2\times RR$ working mode – combined topological and geometric reconfiguration.
to reconfigure, and how to allocate the task. In addition, to the best of my knowledge a single RRMT architecture that can reconfigure into lower mobility configuration, different working/assembly mode sub-configuration, and different sub-PKM has not been presented in the literature. Therefore, another challenge would be to develop a design process that considers the conditions for combining a set of PKMs into an RRMT architecture.

Synthesis of RRMTs

In the synthesis sub-problem, architecture variants are constructed for machining a prescribed set of parts. Synthesis of RRMTs is a combinatorial problem, i.e., RRMTs are a combination of two mechanisms; a tool-holder and a workpiece-holder. In turn, each mechanism is a combination of sub-components such as links and joints. The number of potential serial machine-tool architectures is small compared to PKM-based machine tools. For example, more than 1500 different topologies were counted in [58]. The complication of the synthesis problem of RRMTs can be noted from Table 2.1, where the combinations of one or two cooperating mechanisms with possible redundancy that are constructed from serial/parallel/hybrid mechanisms, are classified into nineteen different classes. The problem is further complicated by the fact that there is no limit to the level of redundancy of mechanisms, thus, it is possible to construct infinite number of RRMTs.

Table 2.1. RRMT variants.

<table>
<thead>
<tr>
<th></th>
<th>Tool-Holder</th>
<th>Workpiece-Holder</th>
<th>Number of combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Redundant serial-based RRMT</td>
<td>rSE</td>
<td>((r)SE)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>SE</td>
<td>1</td>
</tr>
<tr>
<td>Redundant parallel-based RRMT</td>
<td>rPA</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Redundant hybrid-based RRMT</td>
<td>rHY</td>
<td>((r)PA ; (r)SE ; (r)HY)</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>(r)SE</td>
<td>(r)PA</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>rPA</td>
<td>(r)PA</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>PA</td>
<td>PA</td>
<td>1</td>
</tr>
</tbody>
</table>

SE – Serial; PA – Parallel; HY – Hybrid; r – Redundant; () - optional

In addition, the synthesis process should support an optimization process that determines the level of redundancy that would obtain the best performance with the given set of parts. Since on
the one hand, there is infinite number of architectures, and on the other hand arbitrarily constraining the search space of RRMTs may lead to a sub-optimal solution, determining the synthesis termination condition is a challenge. The constraints that delimit the synthesis process should relate to design objectives and performance requirements, thus, it should be interwoven with the performance evaluation process. For example, improvement in the performance, due to added dof, can be used as a criterion that delimits the level of redundancy of an RRMT.

Current synthesis methods of RRMTs can be divided into:

*Top-down approaches*: Constructing an RRMT with all the required dof and, then, analyzing its reconfiguration capabilities (e.g., [58]).

*Bottom-up approaches*: Constructing a configuration from a library of sub-components for each part, and, then, combining these configurations into an RRMT, (e.g., the synthesis method for PKMs that can be reconfigured into multi-modes of operation [59]).

The RRMT’s design methodology should make full use of the redundant reconfigurability, thus, the synthesis process should incorporate dof that allow the three cases of reconfiguration. Synthesis methods for RR-PKMs (that can be used as tool/workpiece holder), those mentioned in the literature review and others, do not, generally, consider conditions for combining a set of PKMs into a single architecture. The RR-PKMs should be able to reconfigure into assembly/working mode configurations and into sub-PKMs that differ in topology. Furthermore, although the functionality of joints (i.e., determining the active, passive or lockable joints) plays an important role in reconfigurable PKMs, past synthesis methods have, typically, overlooked this aspect. Thus, special attention should be given to the synthesis problem of RRMTs with a RR-PKM based tool/workpiece holder, considering the topology and joint functionality.

**Kinematic Modeling**

The kinematic model of an RRMT is required for performance analysis. Inverse-kinematics derives the set of actuated joints’ parameters for a given pose of the end-effector. To analyze the performance of an RRMT, which depend on its configurations and postures, it is required to obtain a set of joint-space parameters for the tool- and workpiece- holders, at discrete points along the specified trajectories.
For serial-based machine-tools, the inverse-kinematic relations can be formulated using the Denavit–Hartenberg method, and the Homogeneous Transformation Matrices [79]. Kinematic models of PKMs enclose the relation between the joints and the tool. The geometric approach is a relatively convenient way to form the inverse-kinematic model of PKMs [80]: a vector loop equation is written for each chain, where the coordinates of the platform joints (and the tool) are specified in terms of the joints parameters. Given the set of equations, passive joints’ parameters are eliminated. From the formed set of equations, given the task space coordinates it is possible to compute the joints parameters. In this work, the inverse-kinematic models of the mechanisms that construct the RRMTs are given, and the kinematic model of each RRMT is formulated with respect to a global coordinate system, \( \{O\} \), Fig. 2.2.

A moving frame, \( \{T\} \), is attached to the center of the platform (tool), where its \( z \)-axis is normal to the platform plane. The vector \( T = [T_x \ T_y \ T_z \ T_\alpha \ T_\beta \ T_\gamma]^T \) represents the pose of frame \( \{T\} \) with respect to the global frame \( \{O\} \), where \( [T_x \ T_y \ T_z]^T \) are the Cartesian coordinates of the origin of the platform frame, and \( [T_\alpha \ T_\beta \ T_\gamma]^T \) denote the set of (\( z \)-\( y \)-\( z \)) Euler angles. Similarly, the vector \( W = [W_x \ W_y \ W_z \ W_\alpha \ W_\beta \ W_\gamma]^T \) represents the pose of the moving frame \( \{W\} \), of the workpiece-holder with respect to the Cartesian frame \( \{O\} \).
In machining, the pose of the tool frame \( \{T\} \), relative to the workpiece frame \( \{W\} \), \( \bar{T} \), is specified. Thus, the Cartesian pose of frame \( \{T\} \) that satisfies the given tool path can be obtained as:

\[
T = \bar{T} + W.
\] (2.8)

One challenge is to model the kinematics of RRMTs that reconfigure through topological reconfiguration. Locking/unlocking of dof and reorienting/reposition of joints and chains change the topology and characteristics of an RRMT. Therefore, performance analysis of the configurations of an RRMT calls for updating the kinematic model for each configuration. Namely, kinematic constraint analyses of lower mobility sub-configurations.

**Task Allocation and Trajectory Resolution**

In general, given the tool-path, the joint-space parameters of a mechanism are obtained from the inverse-kinematic model. As mentioned above, RRMTs can be constructed from two cooperating mechanisms, each redundant for the task. Therefore, a fundamental challenge with RRMTs is to allocate the task. Namely, divide a specified tool-path, which is given relative to the workpiece, into Cartesian trajectories that the tool- and workpiece-holders have to follow. The Cartesian trajectory that each mechanism follows has to be resolved into joint-space trajectories, which can be a challenge in the case of a mechanism that is redundant for the task. For example, a 6-dof PKM is redundant for 5-axis machining.

Following, as RRMT configurations and joint trajectories are not unique for a prescribed tool-path, these parameters can be selected to optimize a cost function, which is associated with the RRMT’s performance.

A number of methods have been developed to solve the trajectory-planning problem. Motion roadmaps method, which has been applied for obstacle avoidance, is presented in [81]. A database of postures is generated, then, based on an initial point and a target point the map is queried to create obstacle free trajectory. A heuristic approach for selecting the joint-space parameters of a 7-dof hybrid mechanism is depicted in [15]. Stiffness of a two armed robot during the machining of workpiece was optimized at discrete points along the given tool-path [82].
As redundancy can be utilized through task allocation and trajectory resolution for optimizing aspects of an RRMT’s performance, configurations and postures should be determined during early design phase.

**Performance Criteria**

The design of RRMTs is optimized against the given set of parts. The optimization entails iterative performance evaluation of different design variants. Therefore, the first challenge is to effectively analyze the RRMTs performance during the design process.

As mentioned in Section 2.1, performance criteria of RRMTs can include accuracy, stiffness, acceleration, speed, and vibration, which are defined by the kinematic and dynamic characteristics of the RRMT architecture. RRMTs are often constructed from PKMs, in which generally, more than one actuator controls the tool motions and several kinematic chains move simultaneously. The complex design of PKMs also results in physical interferences between chains. Hence, it is required to generate full motion simulations that can verify the feasibility of the postures of commutative PKM-based RRMTs.

One of the objectives of the design methodology is to compare PKM-based RRMTs with their serial counterparts in the design phase, using predicted performance obtained from simulations. Therefore, attention should be given to the different characteristics of serial and parallel mechanisms, and the associated approaches for performance analysis. For example, serial mechanisms are isotropic while parallel mechanisms are anisotropic [83]. Namely, structural deflections due to external forces in PKMs are posture dependent, which can lead to significant variation of performance over the workspace [84]. This results in several challenges when determining and analyzing the performance criterion for comparison among RRMTs, such as carefully sampling the performance of postures of PKMs that are redundant for the task. In the following, examples of performance criteria that can be utilized to compare RRMTs, the analysis approach, and the associated challenges are detailed [51]:

**Workspace**: the purpose of the analysis is to determine the boundary of the mechanism’s workspace. There are several types of workspace. For example, constant orientation workspace includes the points that the PKM’s end-effector can reach with a given orientation. Another type is the orientation workspace, in which all the possible orientations that are reachable by the
tooltip at a given point are tested. Workspace analysis of serial mechanisms is relatively simple, as it is possible to translate the joint motions directly into end-effector motions. PKMs' workspace analysis is more involved and requires specific procedures.

There are two main workspace analysis approaches, geometrical and discretization [85]. In the geometrical approach, the workspace covered by each chain is analyzed individually, and the PKM workspace is generated as the intersection of its chains’ workspaces. Workspace in the geometrical approach is generally obtained using Computer-Aided Design (CAD) software [86].

In the discretization method, a specified volume is covered by a cloud of points, where each point is queried as to whether or not an acceptable end-effector (tool and workpiece) pose can be obtained. The boundary of the workspace is defined by the set of reachable points. The main advantage of this method is that it allows incorporating other constraints, aside from the geometrical. However, a problem occurs when there are voids in the workspace.

A different workspace analysis, applicable for machine-tools is the kinematic workspace, which is defined as the workspace in which the machine can cover a specified path at a programmed feed-rate [87].

The workspace, which an RRMT is designed to cover, is defined by a set of Cartesian trajectories and machining conditions for fabricating a set of parts at hand. Meaning, the ability to cover a given set of tool-paths is a fundamental requirement that any proposed RRMT has to address. This requirement is utilized for elimination of solutions that are not feasible. In the case of an RRMT, which is constructed from two redundant cooperating mechanisms, the trajectories of the mechanisms’ end-effectors, for a given tool-path defined relative to the workpiece, are not unique. This implies that the required workspace of the RRMT should be obtained through an optimization process that selects a set of Cartesian trajectories for machining the set of parts.

**Stiffness**: in serial mechanisms, stiffness is directly related to joints and structure compliance, and it is relatively simple to generate a model that captures the stiffness of the mechanism. The stiffness of PKMs is associated with kinematic singularities, in which the mechanism gains or losses the ability to control dof. Therefore, the Jacobian matrix is widely used in stiffness analysis methods of PKMs. Evaluating the time derivative of the kinematic relations of a PKM, one can obtain
\[ J_q \dot{q} = J_x \dot{x}, \]  

(2.9)

where \( J_q \) is the matrix of the time derivatives of the joint space variables, and \( J_x \) is the matrix of the time derivatives of the task space variables, and the Jacobian \( J \) is formulated as:

\[ J = J_x^{-1} J_q. \]  

(2.10)

It is possible to compute the stiffness matrix of a PKM at the tool as a function of the active joints stiffness, and the mechanism’s postures over the workspace, through the following equation [88]:

\[ K = J^{-T} \kappa J^{-1}, \]  

(2.11)

where \( \kappa \) is the stiffness matrix of the active joints and \( K \) is the stiffness matrix at the tool.

Several methods that are variations of the analytical method presented above have been proposed for the analytical modeling of static stiffness of PKMs [83, 89]. The motivation in developing these methods has been to provide quick evaluation of static stiffness in relation to the PKM postures. For example, Eq. (2.11) only captures the effect of active joints on the stiffness at the tool. The virtual joint model is an enhanced framework, in which actuators and links are replaced by equivalent springs. The links bending stiffnesses are modeled as torsional springs, which are located at virtual joints, and the actuators are modeled as torsional/linear springs [90]. This framework was applied for stiffness analysis of a family of RMTs [19].

Another method for stiffness analysis, which is not specific to PKMs, is the Finite Element Analysis (FEA). However, re-meshing different PKM’s postures and solving for stiffness through the design process is tedious, computationally expensive, and therefore inefficient for optimization-based design [91].

It order to select the best postures of a PKM, stiffness of hundreds of alternate solutions may be analyzed at each point along the Cartesian trajectories. Therefore, the challenge with stiffness analysis is to formulate a model, which is on the one hand detailed enough to capture the effect of subcomponents such as passive joint, active joints and links, and on the other hand systematic and computationally efficient so it can be used as a cost function.

**Kinetostatic performance criteria:** example of several kinetostatic-based PKMs’ performance criteria that can be applied to compare RRMTs are briefly presented here.
**Manipulability indices**: manipulability belongs to a family of kinetostatic indices that are based on the kinematic Jacobian. **Manipulability** is defined as the ability of a mechanism to change tool pose in an arbitrary direction, and it is associated with the distance of the PKM’s posture from singularities. There are several ways to quantitatively estimate manipulability: (i) minimum eigenvalue of the Jacobian matrix, (ii) Condition Number \((CN)\), and, (iii) determinant of the Jacobian matrix. The second is a direct and simple way defined as [92]:

\[
CN(J) = \|J^{-1}\| \|J\|.
\]  

(2.12)

However, the applicability of these indices is limited to Jacobian matrices that are homogeneous in terms of units [92].

**Reaction forces**: the inverse Jacobian is used to calculate the reaction forces and moments in the joints due to forces and moments acting on the tool:

\[
\tau = J^T F,
\]  

(2.13)

where \(\tau\) is the vector of joint forces and \(F\) is the vector of tool forces. This relation can be used to compute the trust force that should be applied at the joints. The inverse relation can be used to calculate the forces at the actuators due to external forces applied at the tool-tip. However, this model is applicable for equilibrium state, and it does not considerers the relation during transitions.

**Position error**: joint-space errors can be translated into the task space error as:

\[
\delta x = J \delta q,
\]  

(2.14)

where \(\delta q\) is the vector of joint-space errors, and \(\delta x\) is the vector of task-space errors. This can be an efficient way of estimating the effect of actuator position inaccuracies on the position error at the tool.

**Performance index**

The design methodology should systematically evaluate the performance metric of design candidates. Figure 2.3 depicts the hierarchy in evaluation of the performance of RRMT’s, which include the performance of postures at checked points along the Cartesian path, and the performance of configurations. The performance metric in each level should be optimized to
obtain an inclusive index of the performances from the lower level, such, that the best overall performance index of a given RRMT is obtained.

Figure 2.3. Hierarchy in RRMT performance.

**Design constraints**

*Passive joints*: poses of the tool and workpiece holders are feasible only if the joint-space parameters and machine layout are within the limitations that are imposed by the kinematic and dynamic constraints. To assure that the compared RRMTs are subjected to similar constraints, the passive joint-travel range should be determined using the same criterion/assumptions, regardless of the topology of the RRMT. For example, it is possible to use specifications of commercially available products in order to determine the constraint on the range of motion of spherical joints in simulations.

*Active joints*: the given set of parts and the desired performance, dictate the machining conditions: tool (type, diameter, and material), spindle speed, feed-rate, etc. The dynamic characteristics of the actuators and the topology of the mechanism determine whether the RRMT can generate the feed-rate for machining the set of parts. To verify, in early design stages, that a synthesized RRMT can fabricate a part at hand, information from the machining process should be integrated into the design process. This is especially important for RRmMTs in which meso-milling conditions dictate high spindle speed and high feed-rate.

*Singularities*: one of the inherent drawbacks of PKMs is kinematic singularities that are typically defined with respect to the Jacobian matrix [85]. PKM singularities are classified according to the determinant of the inverse Jacobian, \(|J^{-1}|\), into three types as follows:
Type 1 singularity (serial singularity) results in a non-zero velocity vector, $\mathbf{q}$, for which the platform does not move. Type 2 singularity (parallel singularity) leads to a non-zero motion, $\mathbf{x}$, for which the joint velocities, $\mathbf{q}$, are zero. Type 3 singularity occurs when both Types 1 and 2 singularities are present at the same time.

$$
\begin{align*}
|J^{-1}| &= \pm\infty, \quad |J_q| = 0 \quad \text{Type 1 Singularity,} \\
|J^{-1}| &= 0, \quad |J_x| = 0 \quad \text{Type 2 Singularity,} \\
|J_q| &= 0, \quad \text{and} \quad |J_x| = 0 \quad \text{Type 3 Singularity.}
\end{align*}
$$

It is important to avoid singular postures that can result in extreme high/low performance values. In PKMs, in which the Jacobian is not homogeneous in terms of units, the absolute values of the determinant of the inverse Jacobian, by itself, cannot indicate whether the posture is singular [92]. It is possible to identify singular configuration though mapping of the determinant values over the workspace, where extreme performance value of a posture relative to its neighboring postures indicate on a singularity.

### 2.3 Design Methodology Outline

A design methodology for RRMTs that addresses the abovementioned challenges has been developed. Fig. 2.4 depicts the high-level architecture of the multi-tiered optimization-based design methodology, which is proposed in this thesis. The methodology considers the interaction between the synthesis, design and process domains.

Information from the machining process is incorporated into the synthesis and design processes to obtain a set of optimal design parameters for the task. The global inputs to the proposed multi-tiered optimization process are a set of parts, and a database of sub-components. The objective is to design an RRMT that yields the best performance metric value against the given set of parts. The variables in this process are the topology, the kinematic and dynamic design variables, as well as the machine's configuration, postures, and associated joint-space trajectories for each part.
Synthesis and design of RRMT architectures are concurrently addressed by the proposed methodology through a combined optimization-based design process. Performances are evaluated multiple times for the variant sets of design and process parameters that are associated with each architecture. Thus, effective means to coordinate the optimization process are required.

A multi-tiered optimization-based design approach can utilize a hierarchical process of optimization of performance. Typically, decisions have a coupled effect on different design aspects and are, therefore, transferred into lower or upper level to improve the overall result. In addition, a tiered approach is well suited for integrating domain knowledge that are coarsely related [93].

![Figure 2.4. Proposed RRMT design methodology.](image)

The optimization process is carried out as follows: the set of parameters that were selected by the upper tiers serve as constraints to the lower tiers. Performance metric value, which requires
full tool-motion simulation, is evaluated at the lowest tier. This metric and associated set of design parameters are returned from the lower tiers to the tiers above in an iterative manner that updates the parameter values. The process terminates when a pre-defined condition is obtained.

Three procedures to support the design process are presented. The first algorithmic procedure, which is incorporated into the multi-tiered design methodology, synthesizes RRMTs and iteratively incorporates redundant dof. The objective is to support the tiered optimization-based design approach, by associating synthesis with performance evaluation in an iterative manner that can effectively limit the search space, and the level of redundancy of the synthesized RRMTs. The second procedure, which is incorporated into the lowest optimization-tier, is a three level-optimization process that determines the joints trajectories and associated postures of the mechanisms for specified tool trajectories. The third procedure translates the feed-rate, which is required for machining the set of parts at hand, into constraints on active joint-travel range. By incorporating the machining constraints into the analysis process, it is guaranteed that the commutative RRMTs can cover the kinematic workspace (i.e., reach all the points along the specified tool-path at the required feed-rate). The tiers in the optimization-based design methodology are dedicated for selecting design parameters, configurations, and postures along the specified tool trajectories.

The detail of the proposed optimization-based design methodology, and the complimentary design steps and principles, are presented in Chapter 3.
Chapter 3  Design Methodology

In RMT design, the parts, for which a machine-tool is designed, are known *a priori*. According to previous literature, RMT design methodologies construct a machine configuration for each part. An RRMT, however, should combine all the dof and possible redundancy that result in optimal performance with the given set of parts. This necessitates a combined design approach, integrating the two sub-problems outlined in Chapter 2: (i) synthesis of RRMTs, and (ii) optimization of the design. These individual tasks, which are often solved in series, are addressed here through an integrated approach, one that yields optimal values for a set of design variables to satisfy required process parameters. The purpose of this thesis is to demonstrate how an appropriate integration of the synthesis, design, and machining processes, into a multi-tiered optimization-based design methodology of RRMTs, can significantly enhance performance. The optimized design variables of the RRMT range from the architecture’s topology, to the actual machine postures along the tool trajectories that are associated with the set of parts at hand.

This chapter is divided into two sections: Section 3.1, in which the proposed design methodology is formulated, and supporting procedure of the design are explained; Section 3.2, which demonstrates the effect of the different tiers from the optimization process, through performance analysis of a PKM-based RRmMT that can be reconfigured into lower mobility sub-configurations.

3.1 Design Methodology Formulation

What RRMT design problem typically entails is being given a set of parts, design a machine tool that achieves the best performance through redundant reconfigurability. The following section will elucidate the formulation of the optimization-based design approach. Additionally, it will describe the supporting processes for synthesizing RRMTs, task allocation, and translation of machining conditions into constraints on actuators motion. All the processes are formulated in a way that can support iterative multi-tiered optimization process.
Multi-level optimization is an acceptable approach for dealing with difficulties that are often experienced in engineering problems, such as decisions on a large number of variables from different domains, and complex cost functions [95]. This approach facilitates a hierarchical structure of decision-making, as it optimizes models through a selection of optimal value of the variables in upper levels and then passes them as constraints to the lower levels for evaluation of the cost function.

Examples of tiered optimization approach can be found in [96], where it has been applied for trajectory optimization, and in [97], to support the design of a blade rotor. A two-tier design methodology for modular PKMs is depicted in [55]. The structure is selected in the first tier, and the geometrical design parameters are selected in the second. Due to the complexity of the problem, a multi-tiered optimization approach was chosen for the new RRMT design methodology.

The formulation of the multi-tiered optimization is presented next. At each tier, different decision variables are found, such that the RRMT that can achieve the best performance is derived. The advantage of the tiered approach is that it allows for iterative decision-making of design variables based on a logical order of design. For example, selecting a pair of mechanisms for the RRMT architecture in Tier 1, and determining the set-up variables for these mechanisms in Tier 2. The global inputs to the proposed multi-tiered optimization approach are: the given set of \(k\) parts, \(B\), the library of sub-components from which mechanisms are constructed, \(SC\), and the required Mobility, \(M_{req}\), between the tool and workpiece. The variables for the design process are the topology, the kinematic and dynamic design variables, as well as the configuration, postures and associated joint-space trajectories for each part.

The optimization process is carried out as follows: each tier receives as input the set of parameters that were selected by the upper tiers. The performance for the complete set of parameters, \(\Phi\), is evaluated at the lowest tier by virtual simulations that implement the tool-motion simulation of the analyzed architecture at hand. The evaluated performance metric value, \(p_S(\Phi)\), and the lower tiers’ parameter values are returned to the tiers above in an iterative manner that allows each tier to update and optimize its parameters with respect to the given
parameters of the upper tiers. The process terminates when the different architecture alternatives have been analyzed and the optimal performance metric value is determined. The set of decision variables comprises the pair of mechanisms that construct the RRMT, \( me_i \) and \( me_j \); the set-up variables of the two mechanisms, \( su \); the configuration-part set, \( CO_{ij} \); the set of design variables, \( DP \); and, the joint-space trajectories, \( TR \).

**Tier 1:**

The global inputs, namely, a library of subcomponents, \( SC \), are used for constructing mechanisms from joints and links, the given set of parts \( B \), and the required Mobility, \( M_{req} \), between the tool and workpiece, serve as input to this Tier. The objective here is to obtain, one mechanism, \( me_i \), or a pair of mechanisms, \( me_i \cup me_j \), that can be used as the RRMT architecture. Namely, a combination of tool- and workpiece- holders that would yield the best achievable performance metric, \( Pr_c \). Since there exists a finite number of mechanisms that are constructed, the problem at hand is a discrete type optimization. An additional constraint that is related to this tier is the mobility of the pair of mechanisms, \( M_{WH}(me_i) + M_{TH}(me_j) \), that must be equal to, or larger than the required mobility.

Equations (3.1a)-(3.1b) describe the optimization performed by Tier 1:

Given: \( SC ; B \),

\[
(P_1) \quad \min_{me_i \cup me_j} p_1 = f_1(me_i \cup me_j, su, CO_{ij}, DP, TR), \quad (3.1a)
\]

Subject to: \( M_{WH}(me_i) + M_{TH}(me_j) \geq M_{req}, \quad (3.1b)\)

where \( su \) solves Tier 2.

**Tier 2:**

Tier 2 determines the optimal set-up variables, \( su^* \in SU \), in which the given RRMT tool- and workpiece- holders, determined by Tier 1, are positioned, one with respect to the other. \( SU \) is the vector space of set-up variables that are optimized. Since the variables associated with the set-up can be evaluated in the continuous domain, the optimization is a multi-mode, non-linear, continuous, constrained optimization. Hence, the selected set-up is determined and, then, passed on to Tier 3 as input for the determination of the best performance metric value. The constraints
that are related to this tier are geometrical restrictions, which ensure that the two mechanisms can perform the required cooperative tasks (i.e., have an overlapping workspace).

Equations (3.2a)-(3.2b) describe the optimization performed by Tier 2:

Given: \(su = (s_1, s_2, \ldots, s_r); \ s \in \mathbb{R}\);

\[
(P_2) \quad \min_{su} p_2 = f_2(me_i \cup me_j, su, CO_{ij}^t, DP, TR),
\]

Subject to: \(g_2(me_i \cup me_j, su, CO_{ij}^t, DP, TR) \leq 0\),

where \(CO_{ij}^t\) solves Tier 3.

**Tier 3:**

For a given RRMT architecture, \(R_{ij}\), examined by Tier 2, and its corresponding set of \(m\) possible configurations through redundancy, this tier determines the best achievable performance metric, \(p_3^*\), value for the set of \(k\) parts at hand (with this architecture). This objective is achieved by determining the optimal combination of RRMT configurations and parts, \(CO_{ij}^t\), i.e., \(k\) pairs, \((c_{o1}, \ldots, c_{ok})\), of part-configuration. Since there exists a finite number of possible configurations and parts, \(t \in \{1..k^m\}\), the problem at hand is a discrete combinatorial type optimization. At each iteration of the search, a possible combination of \(k\) part-configuration pairs, \(CO_{ij}^t\) is passed on to Tier 4, for the determination of the corresponding best achievable performance metric value. Hence, in this tier, the variables are all the achievable combinations of RRMT configuration-part pairs, \(CO_{ij}^t\).

Equations (3.3a)-(3.3b) describe the optimization performed by Tier 3:

Given: \(CO_{ij}^t = (c_{o1}, c_{o2}, \ldots, c_{ok}); k \in \{1..m\}\)

\[
(P_3) \quad \min_{su} p_3 = f_3(me_i \cup me_j, su, CO_{ij}^t, DP, TR),
\]

Subject to: \(g_3(me_i \cup me_j, su, CO_{ij}^t, DP, TR) \leq 0\),

where \(DP\) solves Tier 4.
Tier 4:

A combination of $k$ part-configuration pairs, $CO^t_{ij}$, evaluated by Tier 3, serves as the input to this Tier. The objective is to obtain the corresponding optimal (kinematic and dynamic) design parameters, $DP$, that would yield the best achievable performance metric for the combination at hand. Since the design variables can be evaluated in the continuous domain, the problem is a multi-variable, multi-mode, continuous, constrained optimization. At each iteration, a solution set, $DP$, which consists of the values of the kinematic and dynamic design parameters of the RRMT architecture is passed on to Tier 5, for the determination of the corresponding best achievable performance metric value.

Hence, in this tier, the variables are all the design parameters, $(dp_1, \ldots, dp_n) \in DP$, that can be varied. The constraints that are related to this tier are engineering specifications and other feasibility issues that restrict the variability of the (continuous) kinematic and dynamic design variables. Another set of constraints that are related to this tier are geometrical restrictions, which ensure that the two mechanisms can perform the required cooperative tasks (i.e., have an overlapping workspace).

Equations (3.4a)-(3.4b) describe the optimization performed by Tier 4:

Given: $DP = (dp_1, \ldots, dp_n) ; dp \in \mathbb{N}$

$$min_{DP} p_4 = f_4(me_i \cup me_j, su, CO^t_{ij}, DP, TR),$$

Subject to: $g_4(me_i \cup me_j, su, CO^t_{ij}, DP, TR) \leq 0$, (3.4b)

where $TR$ solves Tier 4.

Tier 5:

A combination of $k$ part-configuration pairs, evaluated by Tier 3, and a set of values for the design parameters (common to all configurations), $DP$, evaluated by Tier 4, serve as the input to this Tier. The objective is to obtain the optimal joint-space trajectories, $TR$, corresponding to the required (tool) task trajectories, $TSTR$, (one for each part) at hand, where the joints parameter at every discrete point, $d$, along the tool trajectory is denoted as $tstr_d$. If the RRMT architecture considered does not have any joint-space redundancy, then, the problem at hand is one of simply evaluating the value of the objective function without any required optimization. Otherwise, the
joint-space redundancy has to be investigated to obtain optimal joint-space trajectories. The (optimized) objective function, for example, could be maximization of the machining accuracy.

Since the values of the redundant joints can be evaluated in the continuous domain, \( \mathbb{R} \), the problem is a multi-variable (depending on the number of redundant joints), multi-mode, continuous, constrained optimization. The optimization process is further complicated due to the continuous-path nature of the task-space trajectories. This problem can be addressed by executing a full tool-motion simulation, where the performance-index is evaluated only at \( d \) discretized checked-points along the path. Hence, in this lowest-tier, the variables are the (displacement/velocity/etc.) joint-space values of the RRMT. The constraints that are related to this tier are engineering specifications and other feasibility issues that restrict the variability of the redundant joints values.

Equations (3.5a)-(3.5b) describe the optimization performed by Tier 5:

Given: \( TSTR = (TSTR_1, TSTR_2, ..., TSTR_k) \);
\[ TSTR_i = (tstr_1, tstr_2, ..., tstr_d) \]

\[ \min_{TR} p_5 = f_5(me_i \cup me_j, su, CO_{ij}^t, DP, TR), \]  \hspace{1cm} (3.5a)

\[ \text{Subject to: } g_5(me_i \cup me_j, su, CO_{ij}^t, DP, TR) \leq 0. \]  \hspace{1cm} (3.5b)

The overall performance index, \( Pr_c \), for the chosen RRMT is defined as:

\[ Pr_c = p_1(\Phi^*). \]  \hspace{1cm} (3.6)

The overall scheme of the proposed multi-tier optimization based design methodology is depicted in Fig. 3.1.
Read: Set of \( k \) parts, \( B \); \textbf{Read}: Set of building blocks, \( SC \); \textbf{Read}: Required mobility, \( M_{\text{req}} \).

For \( \text{iter}_1 = 1 \) to (number of allowed iterations for Tier 1)

\[
\begin{align*}
\text{Construct two mechanisms } & m_{e_1} \text{ and } m_{e_2} \text{ from the set of building blocks } \\
\text{If the mechanism mobility } & M_{WH}(m_{e_1}) + M_{TH}(m_{e_2}) \geq M_{\text{req}}, \\
\text{Set } & m_{e_1} \cup m_{e_2}, \text{ a combination of tool- and workpiece- holders mechanisms }
\end{align*}
\]

End

For \( \text{iter}_2 = 1 \) to (number of allowed iterations for Tier 2)

\[
\begin{align*}
\text{Set } & su \in SU, \text{ the set-up parameters between the mechanisms’ pair } \\
\text{Set } & R, \text{ topology of the RRMT architecture } \\
\text{For } & \text{iter}_3 = 1 \text{ to } k^m \text{ (the number of combinations of part-configuration pairs)}
\end{align*}
\]

\[
\begin{align*}
\text{Set } & CO_{ij}, \text{ a combination(iter}_3 \text{) of } k \text{ part-configuration pairs } \\
\text{For } & \text{iter}_4 = 1 \text{ to (number of allowed iterations for Tier 4)}
\end{align*}
\]

\[
\begin{align*}
\text{Set } & DP, \text{ a solution set of kinematic and dynamic parameters } \\
\text{For } & \text{iter}_5 = 1 \text{ to (number of allowed iterations)}
\end{align*}
\]

\[
\begin{align*}
\text{Set } & TR, \text{ a solution for the joint-space trajectories } \\
\text{Evaluate the performance, } & p_5 = f(m_{e_1} \cup m_{e_2}, su, CO_{ij}, DP, TR) \\
\text{If the best performance value is found:} & \\
\text{Go to Tier 4 (The optimal joint-space trajectories are found)}
\end{align*}
\]

End

\[
\begin{align*}
\text{End}
\end{align*}
\]

\[
\begin{align*}
\text{If the best performance value is found} & \\
\text{Go to Tier 3 (The optimal kinematic and dynamic parameters are found)}
\end{align*}
\]

End

End

\[
\begin{align*}
\text{If the best performance value is found} & \\
\text{Go to Tier 2 (The optimal combination of configuration-part pairs are found)}
\end{align*}
\]

End

End

If the best performance value is found

Go to Tier 1 (The optimal set-up of the pair of mechanisms is found)

End

End

If the best performance value is found

Exit optimization (The optimal RRMT is found)

End

End

Set the RRMT to A

Set the Performance value to Pr,c

---

Figure 3.1. Pseudo-code for the five-tier optimization-based design methodology for RRMTs.
3.1.2 RRMT Synthesis

There are several synthesis methods for non-modular RMTs. For example, in [59], the synthesis of RRMTs that can be reconfigured into different modes of operation is presented. The synthesis of PKMs that can geometrically and topologically reconfigure into lower mobility sub-configurations is depicted in [58].

This thesis focuses upon a synthesis approach that pursues three objectives: (i) to embed the synthesis in an overall design optimization process, (ii) to synthesize RRMTs that are constructed from one, or two cooperating serial/parallel/hybrid mechanisms, that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations and sub-PKMs, and (iii) to determine the RRMT’s optimal level of redundancy.

For synthesis of PKM-based RRMTs that can be reconfigured into a given set of sub-PKMs, the synthesis approach proposed in this paper focuses on the method’s ability to determine the topology, level of redundancy and joint scheme. Joint Scheme is defined herein as the description of the functionality of a sequence of joints in a given topology (e.g., lockable revolute, passive prismatic, etc.). A three-step synthesis process that integrates a bottom-up approach and a top-down approach is illustrated in Fig. 3.2. First, a set of RRMT architectures with the required mobility are synthesized. These RRMTs can topologically reconfigure into lower mobility sub-configurations. Second, PKM-based tool-holders of the RRMTs from the set are combined. The combined PKMs have identical topology, and redundant dof are incorporated into an RR-PKM that can be reconfigured into assembly/working mode configurations. Third, the joint scheme of the RR-PKM is adjusted for reconfiguration into PKMs from the set that differ in topology.

Figure 3.2. A three-step RRMT synthesis process.
Reconfigurability of RR-PKM\textsuperscript{s} depends on their joint characteristics. The objective in locking joints is to enhance performance, such as stiffness. Commercial joints that can switch from a passive to a locked state are not readily available and only preliminary work has been conducted in academia. For example, a spherical lockable joint with internal brake is presented in [98]. In [99] adaptive (lockable) revolute joints that can switch from a passive to a locked state to avoid singularities during reconfiguration are studied. In [100-101], a design that allows to change the gap between the moving and stationary parts of the joint in order to have the ability to vary the accuracy and friction. These designs are based on piezo layer that when energized increases the friction between the moving parts.

3.1.2.1 Step 1 – Synthesis of RRMT\textsuperscript{s}

The three-step synthesis is translated to an algorithmic process that is interwoven into the tiered optimization-based design. As depicted in Fig. 3.3, given a set of parts, the first generation, $g_{e_1}$, of an RRMT architecture is constructed in Tier 1 of the optimization process, from a library of building blocks, such as links and 1-dof joints.

The RRMT\textsuperscript{s} may be constructed as either one single mechanism, or a combination of two. The required mobility and level-of-redundancy drive the synthesis. Herein, mobility, $M$, is defined as the total dof to be controlled, so that every link is situated in a specific position [80]. The advantage in mobility is that it is possible to derive it from the mechanism topology.

An RRMT architecture with the required mobility, is passed on to Tier 2 to determine the relative tool-holder and workpiece-holder set-up. If the RRMT does not possess the required mobility, the RRMT is rejected. In order to determine the required level of redundancy of an RRMT, an iterative process adds dof to architectures that were synthesized in Tier 1. In each iteration, a dof is added to a parent architecture from the $n^{th}$ generation, $g_{e_n}$, and the performance metric of the offspring is obtained. The iterative process continues until the improvement in performance metric of the offspring, relative to its parent, is smaller than a predefined threshold.
3.1.2.2 Step 2 – Combining Assembly/Working Mode Configurations into a RR-PKM

PKM-based tool/workpiece holders of the RRMTs that were synthesized in Step 1 are combined, to construct a RR-PKM based tool/workpiece holder that can reconfigure into different assembly/working mode configurations. The constructed RR-PKM should allow reconfiguration into these PKMs without the disassembly of the structure. Thus, it is necessary
to determine the singularity cases of the assembly/working modes, understand their effect on the platform motion, and the methods to avoid these singularities.

Topology of RR-PKMs can be represented using 1-dof revolute \( R \) and prismatic \( P \) joints. Joints with more than one dof can be denoted as a sequence of 1-dof joints. For example, a spherical joint \( S \), can be presented as a sequence of three revolute joints whose axes of rotation intersect at a single point located at the joints center. In order to combine identical-topology PKMs that are associated with assembly/working mode configurations, the singular configuration that the RR-PKM may pass during reconfiguration should be identified. Following are examples of singularities and approaches to avoid them.

**Example 1**: In chains with more than two joints, where there exist a revolute joint along the chain, singularity occurs when the links on the two sides of the revolute joint are aligned. For example, this Type 1 singularity, which is associated with the inverse-kinematic solution, divides between two working modes of a PKM with \( RRR \) chains, Fig. 3.4(a). Replacing the passive revolute joint by a lockable joint, the PKM can switch between the working-mode configurations [99].

**Example 2**: Type 1 singularity of a \( PRR \) chain is shown in Fig. 3.4(b). Reconfiguration between the working modes that are associated with this singularity can be obtained by incorporating a lockable revolute joint between the base and the prismatic joint whose orientation needs to be adjusted. The lockable joint would be unlocked before switching, and locked after the link is reoriented.
Thus, as noted above, synthesized RR-PKMs can switch between assembly/working mode configurations utilizing the capabilities of lockable joints that can change from a locked state to a passive state. Herein, clutch based lockable joints, which can be used to switch from locked to unlocked condition [102], are denoted by the superscript \(^L\).

**Example 3:** Singularities of three and six chains mechanisms with a spherical joint that is connected to the platform have been studied [103-104]. A Type 2 singularity in a three chain PKM with a spherical joint, which is connected to the moving platform, occurs when the platform’s plane and chain’s plane (defined by the location of the spherical joint and the directions of the zero-pitch screw of the chain) are co-planar, Fig. 3.5(a). This singularity can be avoided by incorporating a redundant joint into one of the chains, so that the planes are not co-planar, Fig. 3.5(b).

**Example 4:** A Type 2 singularity in a three-chain PKM with a spherical joint connected to the moving platform may also occur when the planes intersect at the center point of a joint, or at the center of the platform, Fig 3.5(c). Similar to Example 3, this singularity can be avoided by incorporating a redundant joint, such, that the plane of the chain with the redundant joint does not intersect with the other planes at the point that resulted in a singularity, Fig 3.5(d).

---

**Figure 3.4.** Working modes of: \((a)\) RRR chain, \((b)\) PRR chain.

**Figure 3.5.** Singularity avoiding: \((a)\) singular configuration (co-planar planes), \((b)\) added redundancy to avoid the singularity, \((c)\) singular configuration (four planes coincide at the center of the platform), \((d)\) added redundancy to avoid the singularity.
**Example 5:** In some cases, reconfiguration between configurations with identical topology can be obtained by adjusting geometrical constraints. For example, the tool motions of a PKM with chains in which the revolute joint axes intersect at one common point, Fig. 3.6(a), are rotational [105]. PKMs with chains in which the revolute joint axes are parallel are limited to translational tool motions, Fig. 3.6(b). Switching between translational and rotational tool motions can be obtained by incorporating to each chain revolute lockable joints that can be used for reorienting the passive joints axes, as shown in Fig. 3.6(c).

![Figure 3.6](image)

Figure 3.6. Chain arrangement for: (a) RRR - rotational tool motions, (b) RRR - translational tool motions, (c) $RR^L^L^L^L^R^R^R^L^L^R$ - lockable joints added for reorienting the joints axes.

Per the above examples, by incorporating lockable joints, that allow switching between assembly/working modes, identical topology tool-holder based PKMs, from the set of RRMTs that were synthesized in Step 1, can be combined into RR-PKMs in Step 2.

### 3.1.2.3 Step 3 – Combining PKMs that Differ in Topology into a RR-PKM

In Step 3, a RR-PKM that can be reconfigured into different sub-PKM architectures is synthesized, and the corresponding joint scheme is determined. Reconfiguration of a RR-PKM is carried out without disassembling sub-components, and the synthesis is aimed at efficient topology in terms of the number of redundant dof. Thus, only PKMs with the following characteristics are combined:

- PKMs with a number of chains that is identical to that of the RR-PKM synthesized in Step 2.
- Chains topology (i.e., joint arrangement along the chain) that can be obtained by locking/unlocking/reorienting joints of the RR-PKM synthesized in Step 2.
— Prismatic joints that move along the same path (linear, circular, etc.) as do the prismatic joints of the RR-PKM synthesized in Step 2.

Each PKM-based tool/workpiece holder of a synthesized RRMT with mobility higher than two can be reconfigured into lower mobility sub-configurations through locking of joints. Reconfigurability into different assembly/working modes and different PKMs requires further adjusting the RRMT joint scheme, which is therefore determined as follows:

— A passive joint should be incorporated when the joint is not to be locked.
— A lockable joint should be incorporated when the joint has to be locked in one configuration but unlocked in another.
— An active joint should be incorporated when the joint needs to be locked in one configuration but active in another.

3.1.2.4 Mobility and Level-of-Redundancy of RRMTs

As mentioned above mobility and level-of-redundancy drive the synthesis of RRMTs in Step 1. Mobility, can be calculated as [80]:

\[ M = \lambda \cdot (h - p - 1) + \sum f_i - \sum f_j, \]  

(3.7)

where, \( M \) is the number of dof, \( \lambda \) is the mechanism space (3 for planar and 6 for spatial), \( h \) is the number of links, \( p \) is the number of joints, \( f_i \) is the degree of relative motion permitted by the \( i^{th} \) joint, and, \( f_j \) is redundant constraint of the \( j^{th} \) joint.

Three key controlling elements contribute to the mobility of the mechanism:

— The manner in which the links and joints are connected.
— The relative orientation of the kinematic pairs.
— Special link-length constraints imposed in the building of the mechanism that ensure its mobility.

Herein, the focus is on mechanisms whose mobility can be obtained through Eq. (3.7).

Level-of-redundancy, \( r \), is defined as the number of dof that are not required for the task. For example, the level of redundancy of a 4-dof mechanism for a 3-dof task is one. The level-of-redundancy, \( r_{R_l} \), of RRMT \( R_l \), is obtained as:
\[ r_{R_i} = \sum_{m=1}^{2} (COn_i^m - M_{req}); \]  

\[ \text{subject to: } \sum_{m=1}^{2} \min(COn_i^m) \geq M_{req}, \]  

where \( COn_i^m \) is the connectivity of the \( i^{th} \) chain of mechanism \( m \). Connectivity is defined as the number of dof between two links [106]. Chain connectivity is defined as the number of dof between the base and the platform. Equation (3.8b) specifies the condition for the RRMT to possess the required mobility for machining the given set of parts.

### 3.1.2.5 RRMT Set-Up

The set of parameters that determine the position and orientation, in which the two mechanisms that construct an RRMT are located, are selected in Tier 2. Adjusting the orientation can result in different relative tool-workpiece motions, redundancies, workspace coverage and performance. A coordinate frame is assigned at the center of the base of each mechanism. The orientation is denoted as a vector \( OR \), which corresponds to the set of (z-y-z) Euler angles, that transform from the workpiece-holder frame to the tool-holder frame. The associated transformation matrix is denoted as \( \{O_2\}_{O_1}R \), where frame \( \{O_1\} \), is attached to the workpiece base, and frame \( \{O_2\} \), is attached to the tool-holder base. In Fig. 3.7, the orientation between the mechanisms of two RRMTs is shown.

\[ \{O_2\}_{O_1}R = (-90^\circ, 90^\circ, 90^\circ) \]  

\[ \{O_2\}_{O_1}R = (180^\circ, 180^\circ, 0) \]  

Figure 3.7. Orientation between the mechanisms of RRMTs.
3.1.3 Kinematic Analysis

In order to facilitate task allocation and joint trajectory selection, a three level algorithmic optimization process is performed in Tier 5. The process, which is depicted in Fig. 3.8, is designed to support RRMTs that are constructed from one, or a combination of two serial/parallel/hybrid redundant mechanisms:

**Level 1** — in case that the RRMT is redundant, the pose of the platform of the workpiece-holder, which is defined as the 5-axis Cartesian position \([W_x \ W_y \ W_z]\) and orientation \([W_{\alpha} \ W_{\beta}]\), is determined through the optimization process in Level 1.

**Level 2** — the set of joint-space parameters of the workpiece-holder, \(q_{wp}\), for the end-effector pose determined in Level 1 serves as the input to this level. The joint-space parameters of the workpiece-holder are obtained from the inverse-kinematic model. In case that the workpiece-holder is redundant for the task, its joint-space trajectories and associated posture can be optimized.

**Level 3** — the specified tool trajectory for fabricating the given part, and the posture of the workpiece-holder mechanism from Level 2, serves as the input to this level. The position \([T_x \ T_y \ T_z]\) and orientation \([T_{\alpha} \ T_{\beta}]\) of the end-effector of the tool-holder is obtained from the Cartesian trajectory and the workpiece-holder posture. The joints coordinates for a given tool-holder, \(q_{TH}\), are obtained from the inverse-kinematic model. In case that the tool-holder is redundant for the task, its joint-space trajectories and the associated posture can be optimized. The performance of the RRMT is obtained for the selected postures. The iterative process to optimize the performance repeats from Level 1 until a termination condition is met.

The task allocation algorithmic process allows individually optimizing the joint positions of the tool- and workpiece-holders. It is noted that in case the workpiece-holder has less than the number of dof required for the task, its joint parameters are simply obtained from the inverse-kinematic model, and the workpiece-holder joint trajectory optimization in Level 2 is not required. Similarly, the joint trajectory optimization in Level 3 is not required if the tool-holder is not redundant.
3.1.4 Set of Parts Effect on the Design Process

Information from the given set of parts is integrated into different phases of design process:

*Required tool-motions*: as described in the problem definition in Chapter 2, the set of parts are translated into geometrical features and associated tool trajectories. The geometric features
and trajectories can be obtained through existing methods that were developed for RMTs [2, 48]. The trajectories are further translated into required machine characteristics (such as, mobility) that are used during the design process.

Translating the set of parts into tool-motion can be separated into two sub-activities: classifying the parts and machinable features, and transforming the information into tool-motion functions. The classification of parts can be done based on Group Technology using, for example, the Opitz system [107]. Machinable features of parts can be classified through methods such as Tool Approach Direction [17]. This method associates the machinable features with the machine-tool functions. The abovementioned methods for determining the required tool motions from the set-of-parts are applicable to both modular and redundant RMTs, and therefore this subject is not addressed in this thesis.

**Joint travel range:** in order to design a PKM-based RRMT, an algorithmic procedure for determining the constraints on the allowed travel-range of RRMT’s actuators between adjacent checked points is proposed. As depicted in Fig. 3.9, given the machining conditions, the specified tool trajectory, and the dynamic characteristics of the actuators (speed, acceleration), the allowed travel range of the RRMT’s actuators are derived.

The distances that the actuators need to travel during machining are obtained from the inverse-kinematic model. As mentioned above, in redundant mechanisms, the task-space trajectories are allocated to the joints of the workpiece- and tool-holders. Thus, by incorporating the constraint on the allowed joint-travel range into the performance analysis, it is possible to select the design variables and the architectures, configurations and postures that can obtain the required machining conditions.
3.2 Simulations Analysis

Virtual simulations were developed to support the design process of RRMTs. The simulations embody the outline of the methodology presented in Sections 3.1. The aim has been to study the characteristics and implementation of redundant reconfigurability, and to identify bottlenecks in the design process. The simulations results are divided into two sub-sections: Section 3.2.1 presents the Eclipse-based RRMT, its topological reconfigurability into lower mobility sub-configurations, and its general characteristics, which are analyzed through simulations. Stiffness based performance metric that is considered in this thesis for comparing RRMTs, is also presented. Section 3.2.2 demonstrates the effect of optimization tiers on the performance of RRmMTs. The redundancy of the RRMT is the outcome of a 6-dof PKM-based tool-holder, which is utilized for 5-axis machining tasks, as well as redundant dof that are added in series to the workpiece-holder. The effect of redundant reconfigurability on performance is demonstrated.
through: (i) reconfiguration of the 6-dof PKM-based tool-holder into lower mobility sub-configurations, and (ii) task allocation and trajectory resolution. Synthesis and design of an RRMT that is constructed from a RR-PKM based tool-holder will be presented in Chapter 4. In this Chapter the focus is the multi-tiered optimization, and a hybrid RRMT design, which is based on a PKM tool-holder and redundant dof that are added in series to the workpiece-holder.

3.2.1 Analyzed RRmMT

The Eclipse-based RRmMT was chosen for its ability to attain a relatively large workspace in a compact design. The RRmMT, which is shown in Fig. 3.10, is based on a $3\times PPRS$ topology with circular base. The first ‘$P$’ denotes an actuator moving along a circular base and the second ‘$P$’ denotes an actuator moving along the $z$-axis. Each chain contains two actuated joints. In addition, each chain includes a revolute joint and a spherical joint, which are passive. More information on the inverse-kinematic model of the Eclipse can be found in Appendix A.

![Figure 3.10. 7-dof Eclipse-based RRmMT.](image)

The Eclipse can be synthesized from a database of sub-components as explained in Section 3.1.2. Reconfigurability and level-of-redundancy are analyzed throughout the design process. Since only five uncoupled dof are required for 5-axis machining, the tool-holder rotation angle about the normal to the moving platform is an inherent redundancy of the 6-dof PKM-based
RRmMT. For a given platform frame position and pitch/yaw angles, there are infinite PKM postures and the RRmMT performance can be optimized over these postures.

The inverse-kinematic analysis of the 5-dof sub-configurations of the RRmMT, shown in Fig. 3.11, is presented next, where the notation for the kinematic analysis is shown in Fig. 3.10. The parameters values that define the position and orientation of the tool-holders platform’s frame, \{T\}, in global coordinates are:

\[
T = [T_x \ T_y \ T_z \ T_{\alpha} \ T_{\beta} \ T_{\gamma}]^T,
\]

where \([T_x \ T_y \ T_z]^T\), are the Cartesian coordinates of the origin of the platform frame, and \([T_{\alpha} \ T_{\beta} \ T_{\gamma}]^T\) denote the set of (z-y-z) Euler angles of the platform frame.

The positions of the spherical joints of the PKM, \(P_i\) (i=1 to 3), in task space coordinates can be obtained from the position and orientation of the platform frame, \{T\}, and the position of the spherical joints in home configuration, \(P_i^0\), relative to the platform frame, as:

\[
P_i = [T_x \ T_y \ T_z]^T + \frac{\partial}{\partial T} R^T P_i^0,
\]

where \(\frac{\partial}{\partial T} R\) is the rotation matrix of the platform with respect to the global frame. The detailed solution for the spherical joints coordinates is expressed as:

\[
P_{ix} = T_x + P_{ix}^0 (\cos T_{\alpha} \cos T_{\beta} \cos T_{\gamma} - \sin T_{\alpha} \sin T_{\gamma}) - P_{iy}^0 (\cos T_{\alpha} \cos T_{\beta} \sin T_{\gamma} + \sin T_{\alpha} \cos T_{\gamma}) + P_{iz}^0 (\cos T_{\alpha} \sin T_{\beta}),
\]

\[
P_{iy} = T_y + P_{ix}^0 (\sin T_{\alpha} \cos T_{\beta} \cos T_{\gamma} + \cos T_{\alpha} \sin T_{\gamma}) - P_{iy}^0 (\sin T_{\alpha} \cos T_{\beta} \sin T_{\gamma} - \cos T_{\alpha} \cos T_{\gamma}) + P_{iz}^0 (\sin T_{\alpha} \sin T_{\beta}),
\]

\[
P_{iz} = T_z - P_{ix}^0 (\sin T_{\beta} \cos T_{\gamma}) + P_{iy}^0 (\sin T_{\beta} \sin T_{\gamma}) + P_{iz}^0 (\cos T_{\beta}).
\]
A tool path defines the position and the pitch and yaw angles of the tool relative to the workpiece frame. For a given Cartesian path it is possible to optimize the roll angle of the platform of a 6-dof PKM. In Sub-Configurations 1-3, shown in Fig. 3.11, the first joint in the $j^{th}$ chain, $j \in [1..3]$ is locked. The position of the locked base joint is denoted as:

$$ A_j = \begin{bmatrix} R_b \cos \theta_j & R_b \sin \theta_j & 0 \end{bmatrix}^T ; \theta_j = const, \quad (3.12) $$

where $\theta_j$ is the angle relative to the x-axis, and it is obtained as:

$$ \begin{pmatrix} P_{jy} \\ P_{jx} \end{pmatrix} = \tan \theta_j. \quad (3.13) $$

To derive the roll angle of the platform, which is redundant for the 6-dof Sub-Configuration, Eq. (3.11a) and Eq. (3.11b) are substituted into Eq. (3.13):

$$ T_y + P_{jx}^0 \left( \sin \alpha T_\beta \cos \gamma + \cos \alpha \sin \gamma \right) = T_y \quad (3.14) $$
By substituting the following trigonometric identities into Eq. (3.14):

\[
\begin{align*}
\sin \gamma &= \frac{1 - t^2}{1 + t^2}; \\
\cos \gamma &= \frac{2t}{1 + t^2},
\end{align*}
\]

a quadratic equation in \( t \) is formulated, from which it is possible to obtain the roll angle, \( \gamma \), and associated mechanism postures.

**Sub-Configurations 4-6 (vertical joint locked)**

The second prismatic joint parameter of the \( i^{th} \) chain, \( i = [1..3] \), is defined as:

\[
C_{iz} = \sqrt{L^2 - (Ci_x - PI_x)^2 - (Ci_y - PI_y)^2 + P_{iz}},
\]

(3.16)

where \( R_b \) is the base radius. \( C_{ix} \) and \( C_{iy} \) can be expressed as a function of the \( i^{th} \) prismatic joint coordinates \( P_{ix}, P_{iy} \), and the base radius \( R_b \):

\[
C_{ix}^2 + C_{iy}^2 = R_b^2,
\]

(3.17a)

\[
\frac{P_{iy}}{P_{ix}} = \frac{C_{iy}}{C_{ix}}.
\]

(3.17b)

Substituting Eqs. (3.17a)-(3.17b) into Eq. (3.16), it is possible to express \( C_{iz} \) as a function of the link length, the base radius, and the prismatic joint coordinates:

\[
C_{iz} = \sqrt{L^2 - R_b^2 + 2R_b \sqrt{P_{ix}^2 + P_{iy}^2 - P_{ix}^2 - P_{iy}^2 + P_{iz}}},
\]

(3.18)

In Sub-Configurations 4-6, shown in Fig. 3.11, the second prismatic joint in the \( j^{th} \) chain is locked. The geometric constraint: \( C_{iz} = \text{const}, \ j \in [1..3] \). Substituting the set of Eqs. (3.11a)-Eq. (3.11c) and Eq. (3.15) into Eq. (3.18) of the \( j^{th} \) chain, an eight order polynomial equation is formed from which \( t \) and the associated roll angle, can be obtained. Only solutions that do not have an imaginary part are considered, and the feasibility of every solution has to be tested.
7-dof RRmMT

A second redundant dof may exist when a 1-dof x-axis stage is incorporated to the workpiece-holder during the iterative synthesis process, which adds dof to the RRmMT. The 7-dof RRmMT has two redundant dof for the 5-axis machining task.

In the 7-dof RRmMT (Eclipse-based tool-holder and an x-axis stage workpiece-holder), shown in Fig. 3.10, the position of the tool-holders’s platform frame is adjusted with respect to the position of the workpiece-holder’s frame. Since the workpiece stage can move along the x-axis, the position of the platform frame is redefined to incorporate the x-coordinate, $W_x$, of the platform frame, $\{W\}$. The updated parameters’ values that define the position and orientation of the tool-holder platform’s frame, $\{T\}$, in global coordinates are denoted as:

$$
T = [W_x + \bar{T}_x \quad \bar{T}_y \quad \bar{T}_z \quad \bar{T}_\alpha \quad \bar{T}_\beta \quad \bar{T}_\gamma]^T, (3.19)
$$

where $\bar{T}$ is the tool trajectory relative to the workpiece holder’s frame.

3.2.2 Evaluating the Performance of the RRMTs

Examples of analysis results, obtained through simulations, of metrics that are associated with PKM-based RRmMT performance are presented in this section. As mentioned in Section 2.1.3, comparing mechanisms, especially PKMs, has been traditionally hampered by the variability of their performance over the workspace. Therefore, the performance index and the approach for comparing design alternatives are presented in this section.

It is usually acceptable that for a 5-axis serial-based machine tools the 6th dof (roll) is redundant, and it is therefore not considered in the workspace analysis. However, for PKMs, changing the roll angle results in a different machine posture without affecting the pose, which is required for 5-axis machining. The redundant dof can be utilized for optimizing a cost function associated with the RRmMT performance. The range of postures is especially large for PKMs that have base joints the can continuously travel along a circular rail, such as the Eclipse.

Workspace: The workspace analysis should capture the ability of the RRmMT to machine the parts. The workspace chosen for the analysis of the Eclipse based-RRmMT is a hemisphere, where the tool is required to move and tilt from 0° to 90° while remaining tangent to the hemispherical surface, Fig. 3.12.
The discretization approach was chosen for the analysis [85], where the hemisphere surface is discretized into a cloud of points. Workspace coverage of an RRmMT is a function of the constraints that are imposed on the structure. The main geometrical parameters of the RRmMT, and the constraints on joint motions are summarized in Table 3.1.

The travel of the joints is measured from the home position, at which the workpiece and global frames coincide, the z-axes of the platform frame and the global frame are collinear, and their x and y axes align. The joint variables at home position are summarized in Table 3.2.

### Table 3.1. Geometrical parameters of the RRmMT.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base radius ((R_b))</td>
<td>162 mm</td>
</tr>
<tr>
<td>Platform radius ((R_p))</td>
<td>18 mm</td>
</tr>
<tr>
<td>Curvilinear base joints’ range of travel ((\theta_l))</td>
<td>Continuous 360°</td>
</tr>
<tr>
<td>Prismatic joint’s range of travel ((d_i))</td>
<td>65 mm</td>
</tr>
<tr>
<td>Revolute joints’ range of travel ((\varphi_i))</td>
<td>5°–70°</td>
</tr>
<tr>
<td>Spherical joints’ range of travel</td>
<td>±70°</td>
</tr>
<tr>
<td>x-axis stage range of motion ((e))</td>
<td>-5 +5 mm</td>
</tr>
<tr>
<td>Link length ((L))</td>
<td>216 mm</td>
</tr>
</tbody>
</table>

### Table 3.2. Joint parameters at home position.

<table>
<thead>
<tr>
<th>(\theta_1)</th>
<th>(\theta_2)</th>
<th>(\theta_3)</th>
<th>(d_1)</th>
<th>(d_2)</th>
<th>(d_3)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>120°</td>
<td>240°</td>
<td>5 mm</td>
<td>5 mm</td>
<td>5 mm</td>
<td>0 mm</td>
</tr>
</tbody>
</table>
In order to demonstrate the effect of joint constraints, the coverage of the hemispherical workspace with different base joint-travel range of the 6-dof Eclipse-based RRmMT has been analyzed. As can be noted from Fig. 3.13, limiting the joint-travel range, to less than ±120°, results in partial coverage of the workspace.

![Graph showing workspace coverage](image)

**Figure 3.13.** Hemispherical workspace coverage for different base radii.

Comparing among mechanisms based on their coverage of workspace without optimizing their design parameters can lead to biased conclusions. For example, PKMs manipulability is sensitive to the dimensions of sub-components, such as links length and base radius. This can affect the ability of the mechanism to cover the workspace, Fig. 3.14. Since the coverage of workspace is a fundamental requirement for RRMTs, it will be considered herein as a constraint on the design. Namely, RRMTs that cannot cover the specified workspace are excluded from the design process.

Table 3.3 summarizes the dimensions of the main sub-components of the Eclipse-based PKM that lead to maximal coverage of workspace. The dimensions are normalized with respect to the hemisphere radius.
Table 3.3. Normalized machine dimensions (to hemisphere radius).

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Base radius</th>
<th>Height</th>
<th>Link length</th>
<th>Workspace covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eclipse</td>
<td>27</td>
<td>2.7</td>
<td>3.6</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Task-space errors**: performance of PKMs are posture dependent. Discretization of the workspace allows combining performance based constraints into the workspace analysis. For RRmMTs the desired workspace is defined by the set of parts at hand. By analyzing the ability of the tool to reach points along the associated tool-workpiece trajectories, and the performance along the trajectories, it is possible to specifically optimize the design for the parts. An example of the distribution of the volumetric tool position error, $\Delta x$, which results from a base joint error, $\Delta q$, of the Eclipse, is shown in Fig. 3.14. Note that the maximal tool position error is four times smaller than the position error of the base actuator.

![Figure 3.14. Eclipse based RRmMT - distribution of normalized volumetric tool position error presented as (a) cloud of points, and (b) surface.](image)

**Static Stiffness**: the foremost performance criterion of RRmMTs is accuracy that is directly related to stiffness. Stiffness of PKMs, in turn, is highly related to the mechanism's topology, configuration and postures [11]. Thus, for RRmMTs, it is important to identify PKMs that can maintain high stiffness along specified trajectories.

In this thesis, static stiffness of the RRmMTs at the center point of the moving platform is obtained using the MSA method [108]. The structural components that are considered in the modeling are actuators, passive joints, and links, Fig. 3.15. Details on the modeling can be found in Appendix B.
Figure 3.15. Schematics of the stiffness model of RRMTs.

In Fig. 3.16, static stiffness results of the 6-dof RRmMT, obtained from MSA and FEA, are compared. The stiffness at the platform center point is attained from the displacement along the $y$-axis for a 1N force applied along the same axis. The links of the mechanism were modeled as tubes where their inner and outer diameters are 14 and 19 mm, respectively. The material selected for the structure of the PKMs is steel, AISI 1018. The length of the PKM’s links, which are susceptible to bending, is 100 mm. The static stiffness of the actuators is taken to be 3 N/$\mu$m, based on the stiffness of commercial ultrasonic piezo actuator [33].

As can be noted, the MSA model captures the stiffness trends, and the average difference between the results obtained from the two models is about 0.8 N/$\mu$m. The bending stiffness of such a link, made of steel, is 11.4 N/$\mu$m. Hence, primarily the actuators dictate the stiffness of the RRmMT.
In order to overcome the challenge of variation of performance over the workspace, a statistical framework for the analysis of the stiffness is presented:

1. Determine the stiffness along $x$, $y$, and $z$ axes at a large number of discretized check points along specified trajectories. The number of discrete points (sample size) is set to be large enough to compare design alternatives using statistical characteristics. The redundant dof are used for maximizing the stiffness along the $y$-axis.

2. Determine the arithmetic mean and standard deviation of the stiffness values calculated in Step (i).

3. Compare architectures stiffness using statistical characteristics. For example, the architectures performance can be compared using two-tailed, null-hypothesis analysis that can determine whether one RRmMT has better properties than those of the other, beyond a desired significance level [76].

The Eclipse stiffness distribution over a hemisphere was derived using an MSA model at 317 points. The pose of the PKM, that has 6-dof, was optimized to attain the highest stiffness at each point. As can be noted from Fig. 3.17, the variation of the stiffness over the workspace is high, and comparison among architectures can be derived using statistical tools.
Figure 3.17. Static stiffness distribution over a 4 millimeter radius hemispherical workspace: (a) $K_{xx}$, (b) $K_{yy}$, (c) $K_{zz}$.

The mean and variance of the stiffness distributions are summarized in Table 3.4, and as mentioned these criterion can be used to compare among RRMT architectures.

Table 3.4. The mean and variance of the stiffness distribution of the Eclipse.

<table>
<thead>
<tr>
<th></th>
<th>$K_{xx}$</th>
<th>$K_{yy}$</th>
<th>$K_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.450</td>
<td>3.986</td>
<td>7.360</td>
</tr>
<tr>
<td>Variance</td>
<td>0.237</td>
<td>0.100</td>
<td>0.004</td>
</tr>
</tbody>
</table>
3.2.3 Design Methodology Simulations

The following example presents the design optimization process of the Eclipse-based RRmMT, and the effect of the different optimization tiers on the stiffness of the RRmMT. Two approaches for amending the performance of the RRmMT are considered here: (i) topological reconfiguration into lower mobility sub-configurations, and (ii) increasing the level of redundancy. To better illustrate the design process, the tiered-optimization based design methodology for this test case, is depicted in Fig. 3.18.
Figure 3.18. Multi-tiered optimization based RRmMT design process.
3.2.3.1 Machined Part

The stiffness of the RRmMT is analyzed with respect to Part 1. Figure 3.19 depicts the 200 $\mu$m circular slot feature of Part 1, which is made of stainless steel. During machining, the tool is normal to the slot curve. The workpiece is located along the Cartesian $x$-axis, which is the feed direction. The cutting force acting on the tool tooth has three components [109]. In order to better demonstrate the effect of the different optimization tiers on the design, only the stiffness along the normal to the feed direction, $F_N$, is considered.

![Figure 3.19. Isometric view of the machining of the feature of Part 1.](image)

To determine whether an RRmMT architecture can achieve the high feed-rate requirements for machining the meso-scale part in the design stage, the allowed actuator travel ranges between adjacent points are determined. The joints travel-range is obtained from the machining conditions of the meso-part, which are designed to assure the chip-load is higher than the minimal chip thickness.

Table 3.5 summarizes the machining conditions, and the allowed joints travel range corresponding to: points that are 0.1 mm apart, and actuators speed that ranges between 6-10 m/min. The cutting speed is based on those recommended in [65, 71] for hard-metal tools, and the machining conditions are designed to assure that the chip-load is higher than the minimal chip thickness.

<table>
<thead>
<tr>
<th>Material</th>
<th>Tool diameter [$\mu$m]</th>
<th>Cutting speed [m/min]</th>
<th>Chip load [$\mu$m/flute]</th>
<th>Spindle speed [RPM]</th>
<th>Allowed actuator travel range [mm]</th>
<th>Actuator speed [m/min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>stainless steel</td>
<td>200</td>
<td>60</td>
<td>1.2</td>
<td>95500</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>
3.2.3.2 Tier 5 - Task Allocation

In order to demonstrate the impact of task allocation on static stiffness, simulation results for the 6-dof and 7-dof RRmMTs with Part 1 are analyzed, Fig. 3.20. The stiffness of the 7-dof RRmMT is higher than the stiffness of the 6-dof RRmMT for $x<8$ mm, and lower for $x>8$ mm. The simulation results for the stiffness along the y axis can be explained with respect to the Cartesian joints trajectories along the specified tool path, which are shown in Fig. 3.21, and the stiffness map of the 6-dof tool-holder along the y-axis in Fig. 3.22.

Joint-space trajectories are optimized according to the procedure specified in Section 3.1.4. For Sub-Configuration 7 of the 6-dof RRmMT, as the workpiece-holder is static, only Level 3, which optimizes the joint-space trajectories of the tool-holder, is carried out. For Sub-Configuration 7 of the 7-dof RRmMT the workpiece-holder is constructed from one actuator, therefore, the workpiece-holder Cartesian position is optimized in Level 1, and the joint-space trajectories of the tool-holder are optimized in Level 3.

The stiffness results along the specified tool-path, $X_f = [x_f \ y_f \ z_f \ \alpha \ \beta \ 0]^T$, which is given relative to the workpiece frame, can be explained with respect to: the Cartesian coordinates of the workpiece-holder, $W$, the Cartesian coordinates of the tool-holder moving platform, $T$, and the stiffness map of the 6-dof tool-holder given relative to the Cartesian coordinates. Since the specified trajectory is a circular segment located on the x-z plane, the points along the trajectory are identified with respect to their x coordinates.

Let us analyze the stiffness of Sub-Configuration 7 of the 7-dof RRmMT, which is associated with Part 1. The specified tool path x-coordinate starts at $x_f = -15$ mm. The PKM-based tool-holder has a stiff posture at the Cartesian x-axis coordinate $T_x = -10$ mm, Fig. 3.22. Therefore, the x-axis stage moves the workpiece to $W_x = 5$ mm (marked as ‘A’), as can be noted from Fig. 3.21. Between $x_f = -5$ mm, and $x_f = 8$ mm the x-axis stage keeps its position at the end of its travel range (marked as ‘B’). The result is a decreased in the stiffness of the tool-holder, as the mechanism moves away from its stiffest posture in order to follow the given tool-path. Between $x_f = 8$ mm and $x_f = 15$ mm the x-axis stage moves towards $W_x = 5$ mm (marked as ‘C’), and the tool-holder moves toward a stiff posture at $T_x = 20$ mm.
Figure 3.20. Static stiffness of Sub-Configuration 7 of the 6-dof and 7-dof RRmMTs.

Stiffness is locally optimized at every discrete point along the path, and this is the reason that Sub-Configuration 7 of the 6-dof RRmMT is stiffer than Sub-Configuration 7 of the 7-dof RRmMT for $x_f > 8$ mm. No consideration is given to what happens away from the current posture, which leads to local sub-optimal stiffness of Sub-Configuration 7 of the 7-dof RRmMT.

Figure 3.21. 7-dof RRmMT – task allocation of Sub-Configuration 7 relative to Part 1.
It is noted that although the tool-holder is redundant, due to constraints on its joints’ motions it has a limited ability to change to a stiffer posture between adjacent points, which also contributes to the local optimization. However, since the performance criterion that is optimized is the arithmetic mean stiffness along the trajectory, the 7-dof RRmMT is better than the 6-dof RRmMT.

Utilizing the machine-tool’s redundant reconfigurability in an on-line mode can enhance performance, and allow the RRmMT to avoid singular postures [102]. The potential of such reconfigurability can be seen in Fig. 3.20, where the 7-dof RRmMT is stiffer along the first part of the Cartesian trajectory, and the 6-dof RRmMT is stiffer along the second part.

![6-dof RRmMT – stiffness map of Sub-Configuration 7 with respect to the given part.](image)

Figure 3.22. 6-dof RRmMT – stiffness map of Sub-Configuration 7 with respect to the given part.

It is noted that the postures of both the 6-dof and 7-dof RRmMTs are optimized at every checked point utilizing the inherent redundancy [102]. Thus, switching between configurations should be coordinated.

The static stiffness of PKMs is known to be related to their kinematic Jacobian [85]. For a better insight on the joint trajectories that are selected by the optimizer, the stiffness results of the PKM-based tool-holder are presented with respect to the Jacobian along a linear tool trajectory. Fig. 3.23(a) shows the static stiffness, \( K_{yy} \), of Sub-Configuration 7 of the 6-dof RRmMT, along a linear trajectory, \( x_f = (-30 \text{ to } 30) \text{ mm} \). Fig. 3.23(b) shows the associated inverse Jacobian determinant values, \( |J^{-1}| \).
Figure 3.23. The performance along the a Cartesian trajectory: (a) $K_{yy}$ stiffness (b) $|J^{-1}|$, (c) $|K|$. 

The correlation between the two graphs is apparent. Extreme $\Delta J^{-1}$ values are correlated with extreme stiffness values. For $x_f = 20$ mm, a singularity that results in higher $K_{yy}$ values also results in extreme $|J^{-1}|$ values. The determinant values, $|J^{-1}|$, increase between $x_f = 15$ mm to $x_f = 8$ mm, and this is aligned with increase in stiffness. Towards $x_f = 30$ mm and $x_f = -30$ mm the determinant values of the inverse Jacobian decreases, in what appears to be another singularity.

A difference between the trends of the stiffness and $|J^{-1}|$ values is observed between $x_f = -8$ and $x_f = 0$ (and symmetrically between $x_f = 0$ and $x_f = 8$). In this segment, $|J^{-1}|$ values increase while the stiffness decreases. The difference can be explained by the fact that $|J^{-1}|$ captures the effects of all dof, while $K_{yy}$ relates to stiffness along one axis only. This claim is supported by graphing of the determinant stiffness matrix values, $|K|$, along the trajectory, which is shown in Fig. 3.23(c). Specifically, it can be noted from Fig. 3.23(a) and Fig. 3.23(c), that between $x_f = -8$
mm and $x_f = 0$ mm, the values of $|J^{-1}|$ and $|K|$ correlate, and both of them increase. The similarity between the stiffness and $|J^{-1}|$ values is significant mainly because the Jacobian is not used to derive the static stiffness, yet the results indicate that the stiffness model captures the trends associated with the postures of the RRmMT.

### 3.2.3.3 Tier 4 - Design Parameters

In order to demonstrate the effect of design parameters on performance, the link length of the 6-dof RRmMT is analyzed with respect to the static stiffness. A set of link lengths that allows Sub-Configuration 7 of the 6-dof RRmMT to reach the points along the Cartesian trajectory is selected, and the mean stiffness, $\bar{K}_{yy}$, is analyzed. The results shown in Fig. 3.24, indicate that the stiffness along the $y$-axis of the RRmMT are inversely proportional to the link length.

![Figure 3.24. Mean stiffness of Sub-Configuration 7 of the 7-dof and 6-dof RRmMTs.](image)

The optimization is performed in Matlab software, using the *fmincon* function. The search engine is based on interior-point algorithm, which is well suited for continuous, nonlinear, constrained optimization. To allow the function to recover from non-coverage of the given toolpath, and to increase the chances of finding the global optimum, the optimization is repeated with different initial sets of parameters. An example of the convergence of the optimization search is shown in Fig. 3.25, where the link length is presented as a function of the number of search iterations. It is noted that the relation between the link length and stiffness presented in
Fig. 3.25 is near-linear. However, this is not always the relation between geometrical parameters and performance.

![Graph](image)

Figure 3.25. Convergence of Tier 4 optimization.

### 3.2.3.4 Tier 3 – Configurations

Tier 3 selects the set of configurations for the set of parts. Reconfiguring into a lower mobility sub-configuration limits the ability of the tool to cover workspace. The hemispherical workspace coverage of the sub-configurations of the 6-dof Eclipse-based RRmMT with a locked joint, is shown in Fig. 3.26. The workspace coverage of the lower mobility Sub-Configurations is limited, since the RRmMT cannot rotate about the tool axis and change to a posture that allows the tool to reach more points.

The tool-trajectories of the 7-dof RRmMT sub-configurations are shown in Fig. 3.26. It can be noted that for Sub-Configuration 2, Sub-Configuration 3, and Sub-Configuration 7 the changes in the tool trajectory are more significant. This can explain the ability of these sub-configurations to reach the points along the trajectory with the 6-dof RRmMT. In Sub-Configuration 7, the ability of the tool to adjust its position results in higher stiffness.
3.2.3.5 Tier 2 – Mechanisms’ Set-Up

In order to demonstrate the effect of the mechanisms’ set-up, two cases are analyzed: (Case a) the stage moves along the $x$-axis, (Case b) the stage moves along the $y$-axis. In both cases, the stiffness is optimized to achieve the highest stiffness along the $y$-axis. Fig. 3.27 shows the stiffness results of Sub-Configuration 7 of the 6-dof RRmMT.
As mentioned in Section 3.2, the linear stage is compliant only along its direction of motion. As can be noted from Fig. 3.27, when the workpiece-holder direction of motion is along the y-axis, (Case b), the stage stiffness governs the overall performance index value of the RRmMT, and its stiffness is significantly smaller. The area between the two graphs represents the range of stiffness values that can be obtained by adjusting the workpiece orientation.

3.2.3.6 Tier 1 – Architecture

The effect of the architecture on performance is analyzed in this sub-section. The focus is the level of redundancy of the RRmMT, specifically redundancy that is added in series to the workpiece-holder.

In order to further analyze the effect of reconfigurability and level of redundancy on performance, the stiffness of an 8-dof RRmMT is analyzed. In Step 1 in the synthesis process, which is depicted in Section 3.1.2, dof are iteratively added to the RRmMT. In each iteration additional linear stage is incorporated into the RRmMT. Since the design objective pursued in this example is to enhance the stiffness of the RRmMT along the y-axis, the actuators, which are compliant along their direction of motion, are located such that their direction of motion is along...
the $x$-axis. In Fig. 3.28, the stiffnesses along the given tool-path of the sub-configurations of the 8-dof RRmMT are shown.

![Figure 3.28. Stiffness distribution of the sub-configurations of the 8-dof RRmMT with Part 1.](image)

Simulations results indicate that the improvement in mean stiffness of the sub-configurations of the 8-dof RRmMT is less than a 1% predefined threshold. The improvement in mean stiffness is negligible since the 7-dof RRmMT was able to follow a trajectory, which is close to the stiffest trajectory that the RRmMT can obtain.

The tool trajectories associated with the 8-dof RRmMT are shown in Fig. 3.29. Via combining two $x$-axis stages, it is possible for the workpiece-holder to further adjust the position of the workpiece during machining. However, the tool-holder has to be able to adjust accordingly.
Actuators can perform as a locking mechanism. Locking in this case is the result of friction between the moving parts of the actuator. Since actuators are designed to create motion, and locking is a secondary design objective, their holding force characteristics are generally inferior to the locking characteristics of a clutch type joints, which are designed to prevent motion between two sub-components [98]. However, to the best of my knowledge, lockable joints are not commercially available.

Table 3.6 summarizes the mean stiffness results of the sub-configurations of the 6-dof, 7-dof and 8-dof RRmMTs. The stiffness results of the Eclipse-based RRmMTs are compared with those of an existing mMT. An example for a serial 5-axis mMT prototype is presented in [21]. The stiffnesses of the mMT along the x, y, and z axes are 0.84 N/µm, 0.85 N/µm, and 0.89 N/µm, respectively. The stiffness of the mMTs is determined by the compliance of the actuators/bearings that are directly translated to stiffness at the tool. Thus, considering the stiffness of the actuators (in this work), the stiffness of the serial mMT along the y-axis would be 3 N/µm. Stiffness results for the PKM-based RRmMT obtained from simulations range from...
2 to 8 N/\mu m. Thus, the stiffness of the analyzed PKM-based RRmMT, is potentially twice as high.

Table 3.6. Mean stiffness of the RRmMT sub-configurations.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Sub-Configuration 1 Base joint #1 locked</th>
<th>Sub-Configuration 2 Base joint #2 locked</th>
<th>Sub-Configuration 3 Base joint #3 locked</th>
<th>Sub-Configurations 4-6 Second prismatic joint locked</th>
<th>Sub-Configuration 7 Joints are unlocked</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-dof RRmMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{yy}$ [N/\mu m]</td>
<td>-</td>
<td>4.81</td>
<td>4.81</td>
<td>-</td>
<td>5.37</td>
</tr>
<tr>
<td>7-dof RRmMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{yy}$ [N/\mu m]</td>
<td>5.00</td>
<td>4.84</td>
<td>4.85</td>
<td>5.1</td>
<td>5.52</td>
</tr>
<tr>
<td>8-dof RRmMT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_{yy}$ [N/\mu m]</td>
<td>5.37</td>
<td>5.1</td>
<td>5.08</td>
<td>5.30</td>
<td>5.54</td>
</tr>
</tbody>
</table>
Chapter 4 RRmMT Design Test Case

In order to demonstrate the effectiveness of the proposed design methodology, herein, a detailed design test case of a new RRmMT is presented.

The proposed design methodology, which is depicted in Chapter 3, involves five tiers and several intermediate steps, and it should be able to construct and compare RRmMTs. The contribution of the different optimization tiers to performance was demonstrated through the analysis of the Eclipse-based RRmMT that can be reconfigured into lower mobility sub-configurations. In the following, the comprehensive design process of an RRmMT that can be reconfigured into lower mobility sub-configuration, assembly/working mode configurations and sub-PKMs that differ in topology is presented.

The design process includes the following steps: (i) present a representative set of meso-scale parts, and the associated design requirements, (ii) synthesize RRmMTs, (iii) optimize the performance of the RRmMTs and select the best one. Specifically, the process is realized to demonstrate synthesis, design and analysis of serial, hybrid and parallel-based RRmMTs that are capable of the three cases of redundant reconfigurability. In order to design an RRmMT that can be reconfigured into assembly/working mode configurations and sub-PKMs that differ in topology, a novel RR-PKM based tool-holder is designed. In addition, the test case is also aimed at presenting the applicability of the methodology to the design of PKM-based RRmMTs.

This chapter is arranged as follows: Section 4.1 describes the synthesis (Step 1), design and analysis of RRmMTs that can be reconfigured into lower mobility sub-configurations. Section 4.2 presents the synthesis (Step 2), design and analysis of a RR-PKM based tool-holder that can be reconfigured into assembly/working mode configurations. Section 4.3 presents the synthesis (Step 3), design and analysis of a RR-PKM based tool-holder that can be reconfigured into sub-PKMs that differ in topology. Section 4.4 presents the $3\times PRPRS$ based RRmMT, which is the outcome from the design process. Section 4.5 shortly summarizes the design test case and the main conclusions.
4.1 Synthesis (Step 1) and Design of RRmMTs that can be Reconfigured into Lower Mobility Sub-Configurations

The methodology depicted in Chapter 3 is implemented for the design of a new RRmMT. The focus of this section is RRmMTs that are capable of topological and geometric reconfiguration. As such, performance of lower mobility sub-configurations that are obtained through locking of joints, are analyzed. Additionally, the level of redundancy of the RRmMTs is optimized, where the redundancy is utilized to enhance performance, and to obtain required machining conditions.

4.1.1 Selected meso-Scale Parts

In order to determine the representative set of parts that will be used in the design process, cutting mechanics of meso-scale parts was reviewed. Since the tool radii in meso-milling is of the same magnitude of order as the removed chip size, meso-cutting is sensitive to size effects such as the minimum chip thickness [69]. Ploughing of the workpiece, which occurs when the uncut chip thickness is smaller than the minimum chip thickness, results in an increase in cutting forces, chatter, deterioration of the surface roughness, and tool wear [4]. Understanding of the mechanics of meso-cutting and the minimum chip thickness effect are critical for selection of appropriate machining conditions and therefore should be integrated into the design process of dedicated meso-milling machine tools.

In [66], a methodology that relates the machining operation and the design of modular mMT was proposed. In a similar way, the RRmMT design test case combines parameters from the meso-cutting machining process into the design process. These requirements include spindle speeds that are higher than 500,000 rpm for cutting speed of more than 100 m/min. The cutting speed should be even higher than what is required in conventional milling in order to obtain surface roughness of better than 100 nanometers.

The RRmMT design test case is illustrated, herein, for three meso-scale parts. The parts represent microfluidic devices that are designed to control small droplets of blood, drugs, or other types of fluids. Operation of these devices requires, in turn, meso-scale channels, holes, and valves to manipulate the fluids, Fig. 4.1 [110].
Figure 4.1. Microfluidic channels with a 12 \( \mu m \) width and 125 \( \mu m \) holes [110].

Part 1, with a circular feature limited to a segment between tool angles 30\(^\circ\) to 90\(^\circ\) was presented in Section 3.2.3.1. The machined feature on Part 2 is a 5 mm linear slot, with a 150 \( \mu m \) width, Fig. 4.2(a). The part is made of stainless steel, and it is machined using end-milling similar to Part 1.

![Diagram of Part 2](image)

Figure 4.2. Machining feature of Part 2, (a) isometric view, (b) top view.

Part 3 is also made of stainless steel and the machine feature, shown in Fig. 4.3, is a 150 \( \mu m \) hole, tilted 15\(^\circ\) about the \( x \)-axis.
The main requirements from the RRmMT are summarized in Table 4.1. The position accuracy of the tool is derived from the high accuracy required in meso-milling, i.e., typical feature tolerance/object-size in the range $10^{-5}$-$10^{-3}$ [111]. Thus, the tolerance of a feature with a dimension of 100 $\mu$m should be smaller than 0.1 $\mu$m. The machine stiffness requirement is set to keep the displacement at the tool smaller than the required positional accuracy. The contribution of the structure to the tool position error is typically less than 10% of the overall error sources [85]. Applying the machining conditions that are recommended for meso-milling would result in machining forces that are smaller than 1N. Therefore, for a required positional accuracy that is smaller than 0.1 $\mu$m, the stiffness along the axes of the RRmMT should be higher than 180 N/$\mu$m. The motivation for designing mMTs has generally been to decrease energy consumption and footprint. In the design test case, the size of the machine tool is constrained by the base size (footprint) and the link length. Due to the small size of the RRmMT, energy consumption would be significantly smaller than that of conventional-size machine tools that are used for machining meso-scale parts.

As mentioned in Chapter 2, the required workspace is a function of the given set of parts. To machine 3D meso-scale parts, and to obtain the large negative rake angle associated with meso-milling [64], the RRmMT architectures should be also capable of high tilt angle [76]. A design that allows optimizing the location in which the workpiece is positioned with respect to the tool can increase the machine flexibility and efficiency (for example, by machining an array of meso-scale parts). Thus, a 30 mm radius hemispherical surface is defined as a reference workspace.
Table 4.1. RRmMT requirements.

<table>
<thead>
<tr>
<th>Performance criterion</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positional accuracy</td>
<td>&lt; 0.1 (\mu)m</td>
</tr>
<tr>
<td>Machine stiffness</td>
<td>&gt; 100 N/(\mu)m</td>
</tr>
<tr>
<td>Hemispherical workspace (radius)</td>
<td>30 mm</td>
</tr>
</tbody>
</table>

4.1.2 Synthesis Process (Step 1) of the Compared RRmMTs

Architectures of RRmMTs are synthesized from a given library of building blocks, which includes links and 1-dof joints. In an intermediate process, chains are synthesized from the library of building blocks [58-59], where 'RRR' indicates three consecutive revolute joints that can be substituted by a spherical joint. Following, the iterative synthesis process combines chains to construct RRmMTs.

For 5-axis machining of 3D parts, the required mobility is five. Since in PKMs drive command to actuators often results in coupled tool motion, 6-dof PKM-based RRmMTs are also synthesized. To synthesize RRmMT from one mechanism, the connectivity of the chains should be equal to, or higher than the specified mobility. Examples of topologies of chains that can be used to construct one mechanism-based RRmMTs are listed in Table 4.2.
Table 4.2. Topologies of RRmMTs constructed from one mechanism.

<table>
<thead>
<tr>
<th>Chains</th>
<th>Connectivity</th>
<th>Mechanism</th>
<th>Mobility</th>
<th>Level of redundancy</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPPRR</td>
<td>5</td>
<td>PPPRR</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>PPRRP</td>
<td>5</td>
<td>PPRRP</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>PRRPP</td>
<td>5</td>
<td>PRRPP</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>RRPPP</td>
<td>5</td>
<td>RRPPP</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>RPPPR</td>
<td>5</td>
<td>RPPPR</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>RPRRP</td>
<td>5</td>
<td>RPRRP</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
<tr>
<td>PRPPR</td>
<td>5</td>
<td>PRPPR</td>
<td>5</td>
<td>0</td>
<td>Serial</td>
</tr>
</tbody>
</table>

In RRmMTs that are constructed from two mechanisms, the combined mobility (of the tool- and workpiece-holders) should be equal to the required mobility. Therefore, chains with connectivity, which is smaller than five, are synthesized in the design test case. Examples of such chains and associated mechanisms that can be constructed are detailed in Table 4.3.

Table 4.4 lists several RRmMTs that can be synthesized from the mechanisms and chains that appear in Table 4.2 and Table 4.3. The RRmMTs are synthesized from a serial mechanism that has a mobility of five, from a parallel mechanism that has a mobility of five or six, or, from two cooperating mechanisms that have a combined mobility of five or six.
Table 4.3. Chains for lower mobility RRmMTs.

<table>
<thead>
<tr>
<th>Chains</th>
<th>Mechanisms</th>
<th>Topology</th>
<th>Connectivity</th>
<th>Mobility</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topology</td>
<td>CON</td>
<td>Topology</td>
<td>M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>1</td>
<td>$P$</td>
<td>1</td>
<td>Serial</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td></td>
<td>$R$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PP$</td>
<td>2</td>
<td>$PP$</td>
<td>2</td>
<td>Serial</td>
<td></td>
</tr>
<tr>
<td>$PR$</td>
<td></td>
<td>$PR$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RR$</td>
<td></td>
<td>$RR$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PPP$</td>
<td>3</td>
<td>$PPP$</td>
<td>3</td>
<td>Serial</td>
<td></td>
</tr>
<tr>
<td>$PRR$</td>
<td></td>
<td>$PRR$</td>
<td></td>
<td>PKM (planner)</td>
<td></td>
</tr>
<tr>
<td>$PRP$</td>
<td></td>
<td>$PRP$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>$RRP$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$PRRP$</td>
<td>4</td>
<td>$PRRP$</td>
<td>4</td>
<td>Serial</td>
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</tr>
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<td>$RRPP$</td>
<td></td>
<td>$RRPP$</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>$RPRP$</td>
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<td>$RPRP$</td>
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<tr>
<td>$RPPR$</td>
<td></td>
<td>$RPPR$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PPR$</td>
<td>4</td>
<td>$2\times PPR$</td>
<td>2</td>
<td>PKM (planner)</td>
<td></td>
</tr>
<tr>
<td>$PRP$</td>
<td></td>
<td>$2\times PRP$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RRP$</td>
<td></td>
<td>$2\times RRP$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PRPR$</td>
<td></td>
<td>$2\times RPPR$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RPPR$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4. RRMTs that are based on two mechanisms.

<table>
<thead>
<tr>
<th>Topology Mechanism 1</th>
<th>Topology Mechanism 2</th>
<th>Mobility CON</th>
<th>Level of redundancy $r$</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$PRRP, RRPP, RPRP, RPPR$</td>
<td></td>
<td></td>
<td>Combination of two serial mechanisms</td>
</tr>
<tr>
<td>$R$</td>
<td>$PRRP, RRPP, RPRP, RPPR$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$PP$</td>
<td>$RRR$</td>
<td>PPR, PRP, RPP</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>$RR$</td>
<td>$PPP$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$RP, PR$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$3\times RPR, 3\times RRR, 3\times PPR$</td>
<td></td>
<td>5</td>
<td>0</td>
<td>Hybrid Parallel+Serial</td>
</tr>
<tr>
<td>$3\times RPR, 3\times RRR, 3\times PPR$</td>
<td></td>
<td>6</td>
<td>1</td>
<td>Hybrid Parallel+Parallel</td>
</tr>
</tbody>
</table>
An illustrative example of the iterative synthesis process, which is performed in Step 1, is shown in Fig. 4.4. A library of building blocks that include links and joints is shown at the center. The first circle in the figure represents chains that are constructed from the building blocks. The second circle represents the 1\textsuperscript{st} generation of RRmMTs that have the required mobility for machining the given set of parts. The third circle represents the 2\textsuperscript{nd} generation of RRmMTs that have one redundant dof. Thus, an RRmMT in the n\textsuperscript{th} generation is constructed from an RRmMT that was synthesized in the (n-1) generation, and a redundant dof that is added in series to its workpiece-holder.
Figure 4.4. Synthesis process – Step 1.
Four architectures, representing the different types of RRmMTs that can be synthesized in Step 1, are compared. The Eclipse [7] and the UofT PKM [76] that are based on a $3 \times \text{PPRS}$ topology are shown in Fig. 4.5(a) and Fig. 4.5(b), respectively. The first ‘$P$’ denotes an actuator moving along a circular guide and the second ‘$P$’ denotes an actuator moving along the $z$-axis in the Eclipse, and in radial direction to the circular guide in the UofT PKM. ‘$R$’ indicates a passive revolute joint that connects the two links and ‘$S$’ indicates a spherical passive joint that connects the chain to the platform. The 6-dof Eclipse and UofT PKMs are redundant for 5-axis machining.

Figure 4.5. Compared RRmMTs: (a) the Eclipse, (b) the UofT PKM, (c) the MIT-SS-1, (d) $\text{PPPRR}$ serial mechanism.

A hybrid RRmMT, which is based on the MIT-SS-1 [8], is shown in Fig. 4.5(c). The RRmMT is constructed from a $3 \times \text{PPRR}$ PKM tool-holder and a $PR$ serial workpiece-holder. The tool-holder moves the tool along the $x$- and $z$-axes, and rotates about the $y$-axis. In this thesis the MIT-SS-1 workpiece-holder can move along the $y$-axis and rotate about the $z$-axis (in [8] it rotates about the $x$-axis). The serial mechanism shown in Fig. 4.5(d) is based on a $\text{PPPRR}$ tool-holder,
where ‘PPP’ denotes three actuators that move along the Cartesian axes, \(x\), \(y\), \(z\), and \(RR\) denotes two actuators that rotate about the \(z\)- and \(x\)-axes, respectively. The details of the inverse-kinematic models of these RRmMTs are given in Appendix A.

In every generation, in the iterative synthesis process in Step 1, a redundant dof is added to the RRmMTs. The redundant dof is added in this thesis in series, such, that it results in the synthesis of a hybrid mechanism, which, as depicted in Chapter 3, may be able to enhance stiffness while attaining the high feed-rate required in meso-milling.

### 4.1.2.1 Kinematics Constraint Analysis

Theoretically, 5-dof sub-configurations of a 6-dof RRmMT can machine 3D parts. The 6-dof PKM-based RRmMTs can reconfigure into 5-dof sub-configurations through locking of an active joint. Kinematic constraint analysis of the Eclipse-based RRmMT is depicted in Section 3.2.1. In this sub-section constraint analysis of: (i) the UofT PKM-based RRmMT, and (ii) the 6-dof MIT-SS-1 based RRmMT (synthesized in the second iteration in Step 1), are presented. Topological reconfiguration of the serial RRmMT is not analyzed since locking a dof in the 6-dof serial mechanism that was constructed in the 2\(^{nd}\) generation, would result in a sub-configuration, which is identical to the 5-dof serial-based RRmMT that was constructed in the 1\(^{st}\) generation.

**The UofT PKM-based RRmMT**

The UofT PKM-based RRmMT is included in the design test case due to its potential to attain a high tool tilt-angle and relatively large workspace [75]. The notation for the inverse-kinematic model is shown in Fig. 4.6.
Figure 4.6. 6-dof UofT PKM-based RRmMT.

The 5-dof sub-configurations of the UofT PKM-based RRmMT are shown in Fig. 4.7. In Sub-Configurations 1 to 3, one of the base chains is locked. In Sub-Configurations 4 to 6, the second joint in one of the chains is locked, and in Sub-Configuration 7 all the joints are unlocked.

Sub-Configurations 1-3 (base joint locked)

The positions of the base joints of the UofT PKM and the base joints of the Eclipse can be obtained from the same set of equations. Therefore, Eqs. (3.9) to (3.15) are used for evaluating the base joints parameters of Sub-Configurations 1-3 of the UofT PKM.

The second prismatic joint parameter in the $i^{th}$ chain are obtained from:

$$L^2 = (C_{ix} - P_{ix})^2 + (C_{iy} - P_{iy})^2 + P_{iz}^2.$$  (4.1)

$C_{ix}$ and $C_{iy}$ can be expressed as a function of $P_{ix}$, $P_{iy}$ and the base joint parameter $d_i$ as:

$$C_{ix}^2 + C_{iy}^2 = d_i^2,$$  (4.2a)

$$\frac{P_{iy}}{P_{ix}} = \frac{C_{iy}}{C_{ix}}.$$  (4.2b)

Substituting Eqs. (4.2a)-(4.2b) into Eq. (4.1) gives:

$$0 = -d_i^2 + 2d_i\sqrt{P_{ix}^2 + P_{iy}^2 + L^2 - P_{ix}^2 - P_{iy}^2 - P_{iz}^2}.$$  (4.3)
Two solutions for $d_i$ can be obtained from this quadratic equation, and the feasible solution is selected based on the locked joint position along the circular guide.

Figure 4.7. Sub-Configuration of the UofT PKM-based RRmMT.
**Sub-Configurations 4-6 (Second joint locked)**

In Sub-Configurations 4-6, which are shown in Fig. 4.7, the second prismatic joint in the \( j^{th} \) chain is locked. The parameter of the locked joint is denoted as \( d_j = \text{const}, \ j \in [1..3] \). Substituting the set of Eq. (3.11a) - (3.11c) and the trigonometric identities from Eq. (3.15), into Eq. (4.3) gives an eight order polynomial equation from which \( t \) and the associated roll angle can be obtained.

**MIT-SS-1 based RRmMT**

Kinematic constraint analysis of the 6-dof MIT-SS-1 based RRmMT is presented next. Locking a joint in the 5-dof MIT-SS-1, which is synthesized in the 1\(^{st}\) generation, would result in a 4-dof RRmMT that cannot machine 3D parts. However, it is possible to lock a joint in the 6-dof MIT-SS-1 based RRmMT, which is synthesized in the 2\(^{nd}\) generation and has a redundant dof. Thus, constraint analysis is derived for 5-dof sub-configurations in which a joint in the \( 3\times PRR \) planar PKM-based tool-holder is locked. The notation for the inverse-kinematic model of the 6-dof MIT-SS-1 based RRmMT is shown in Fig. 4.8.

![Figure 4.8. 6-dof MIT-SS-1 based RRmMT with a redundant x-axis stage.](image)
In Sub-Configurations 1-3, which are shown in Fig. 4.9, the prismatic joint in the $j^{th}$ chain is locked. The parameter of the locked joint is denoted as $C_{jz} = \text{const}, \ j \in [1..3]$.

![Sub-Configurations of the MIT-SS-1 based RRmMT.](image)

The equation that relates the platform joint location $P_j$ and the locked joint coordinates in the $j^{th}$ chain is formulated as:

$$C_{jx} = P_{jz} - \sqrt{L^2 - (P_{jx} - C_{jx})^2}.$$ \hspace{1cm} (4.4)

The redundant $x$-axis stage, which is added to the workpiece-holder, moves the workpiece to a location that allows the 2-dof tool-holder (with a locked actuator) to obtain the required pose. Given the tool path $[x_f \ y_f \ z_f \ \alpha \ \beta \ 0]^T$ with respect to the frame of the workpiece-
holder, where $\alpha$ and $\beta$ are the z-y Euler angles, the Cartesian $x$ and $z$ coordinates of the tool can be denoted as:

$$T_x = (x_f + d_6) \cos(-\alpha) - y_f \sin(-\alpha),$$

(4.5a)

$$T_z = z_f,$$

(4.5b)

where $d_6$ is the displacement of the 6th actuator, which is an $x$-axis stage added in the 2nd generation of the synthesis process (Step 1).

Substituting Eq. (4.5a) into Eq. (A.12a), and Eq. (4.5b) into Eq. (A.12b), the platform joint coordinates in Sub-Configuration 1, and Sub-Configuration 3, can be expressed as:

$$P_{jx} = (x_f + d_6) \cos(-\alpha) - y_f \sin(-\alpha) - r \cos \beta,$$

(4.6a)

$$P_{jz} = z_f - r \sin \beta.$$

(4.6b)

Substituting Eqs. (4.6a)-(4.6b) into Eq. (4.4), $d_6$ can be obtained as:

$$d_6 = \sqrt{[l^2 - (C_{jx} - z_f - r \sin \beta)^2 + C_{jx} + y_f \sin(-\alpha) + r \cos \beta \cos(-\alpha)]^2} - x_f.$$  

(4.7)

Given $d_6$, the coordinates of the platform joints in the first and third chains can be derived from Eqs. (4.6a) - (4.6b). The other joints' coordinates can be evaluated from Eqs. (A.12) - (A.15) that are given in Appendix A.

In Sub-Configuration 2, which is shown in Fig. 4.9, the actuator in the second chain is locked. The coordinates of the platform joint in the second chain can be attained as:

$$P_{2x} = (x_f + d_6) \cos(-\alpha) - y_f \sin(-\alpha) + \sqrt{3}r \sin \beta,$$

(4.8a)

$$P_{2z} = z_f + \sqrt{3}r \cos \beta.$$

(4.8b)

Substituting Eqs. (4.8a)-(4.8b) into Eq. (4.4), $d_6$ can be expressed as:

$$d_6 = \sqrt{[l^2 - (C_{jz} - z_f + \sqrt{3}r \cos \beta)^2 + C_{jz} - y_f \sin(-\alpha) + \sqrt{3}r \sin \beta \cos(-\alpha)]^2} - x_f.$$  

(4.9)

Given $d_6$, the coordinates of the platform joints in the second chain's can be derived from Eqs. (4.8a) - (4.8b), and the other joints coordinates can be evaluated from Eqs. (A.12) - (A.15).
4.1.3 Simulations Results

The design process utilizes the redundant reconfigurability to enhance the performance of synthesized RRmMT architectures. This sub-section presents stiffness results of the four RRmMTs that have been obtained from simulations. The PKM/hybrid based RRmMTs are compared to a serial mMTs [35]. The number of sub-configurations, which are analyzed in each RRmMT, depends on the topology, as depicted in the kinematics constraint analysis.

The main geometrical parameters and constraints on joint motions are summarized in Table 4.5. The parameters are selected according to the following guidelines: (i) the RRmMTs have to reach all the points along the specified trajectories, (ii) based on the size of current mMTs the base size is constraint to 300±30 mm [34, 38, 112], (iii) based on preliminary simulations of the Eclipse, the combination of the length of the two links in each chain is constraint to 250-350 mm, (iv) the travel-range of joints and actuators are based on commercially available products, or designs that were proposed in academia.

Design variables are optimized in Tier 4, and the constraints applied on the geometrical dimensions dictate that the size of the RRmMTs is similar to the size of current mMTs [34]. The link length has been optimized through the design process and the results are listed in the table.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>The UofT PKM</th>
<th>Eclipse PKM</th>
<th>MIT-SS-1</th>
<th>Serial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base radius ( R_b )</td>
<td>162 mm</td>
<td>162 mm</td>
<td>145 mm</td>
<td>-</td>
</tr>
<tr>
<td>Platform radius ( R_p )</td>
<td>18 mm</td>
<td>18 mm</td>
<td>18 mm</td>
<td>-</td>
</tr>
<tr>
<td>Link length ( | P_i - C_i | )</td>
<td>180 mm</td>
<td>216 mm</td>
<td>161 mm</td>
<td>-</td>
</tr>
<tr>
<td>Curvilinear base joints’ range of travel ( \theta_i )</td>
<td>Continuous 360°</td>
<td>Continuous 360°</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Prismatic joint’s range of travel ( d_i - R_b )</td>
<td>65 mm</td>
<td>65 mm</td>
<td>65 mm</td>
<td>65 mm</td>
</tr>
<tr>
<td>Revolute joints’ range of travel ( \varphi_i )</td>
<td>5°–70°</td>
<td>5°–70°</td>
<td>±70°</td>
<td>±180°</td>
</tr>
<tr>
<td>Spherical joints’ range of travel</td>
<td>±70°</td>
<td>±70°</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>x-axis stage (workpiece-holder)</td>
<td>±5 mm</td>
<td>±5 mm</td>
<td>±5 mm</td>
<td>±5 mm</td>
</tr>
</tbody>
</table>
The travel of the joints of the RRmMT are measured relative to the *home position* posture:

i. For the Eclipse, the base joint angle of the $i^{th}$ chain about the $z$-axis is denoted by $\theta_i$, and the travel of the second joint along the $z$-axis is denoted by $d_i$.

ii. For the UofT PKM, the base joint angle of the $i^{th}$ chain about the $z$-axis is denoted by $\theta_i$, and the radial distance of the second joint from the center of the base is denoted by $d_i$.

iii. For the MIT-SS-1, $d_1,d_2,d_3$ denote the travel of the joints along the $z$-axis of the prismatic joints of the 3×PRP based RRmMT, $d_4$ denote the linear $y$-axis stage parameter, and $\theta_1$ the rotating stage angle about the $z$-axis of the PR workpiece-holder.

iv. For the serial mechanism, $d_1,d_2,d_3$ denote the $x$, $y$, and $z$ coordinates of the linear stages, and $\theta_1$ and $\theta_3$ the angles of the rotating stages about the $z$- and $y$-axes.

The joint-space parameters of the compared RRmMT architectures at *home position* are summarized in Table 4.6.

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
</tr>
</thead>
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<td>Eclipse</td>
<td>0º</td>
<td>120º</td>
<td>240º</td>
<td>5 mm</td>
<td>5 mm</td>
<td>5 mm</td>
<td>-</td>
</tr>
<tr>
<td>UofT PKM</td>
<td>0º</td>
<td>120º</td>
<td>240º</td>
<td>5 mm</td>
<td>5 mm</td>
<td>5 mm</td>
<td>-</td>
</tr>
<tr>
<td>MIT-SS-1</td>
<td>0º</td>
<td>-</td>
<td>-</td>
<td>60 mm</td>
<td>60 mm</td>
<td>75 mm</td>
<td>0 mm</td>
</tr>
<tr>
<td>Serial mechanism</td>
<td>0º</td>
<td>-</td>
<td>0º</td>
<td>0 mm</td>
<td>0 mm</td>
<td>0 mm</td>
<td>-</td>
</tr>
</tbody>
</table>

### 4.1.4 Hemispherical Workspace Coverage

The comparison of workspace coverage gives an indication to the capability of the RRmMTs and their sub-configurations to machine meso-scale parts. The hemispherical workspace coverage of the sub-configurations of the 6-dof Eclipse-based RRmMT with a locked joint, are shown in Fig. 4.10. The workspace coverage of the lower mobility sub-configurations is limited, since the RRmMT cannot rotate about the tool axis and change to postures that allow reaching more points.
The solution for the system of inverse kinematic equations of the RRmMTs with a locked joint is not unique. For example, two solutions can be obtained for the inverse kinematic problem of Sub-Configurations 1-3. The two solutions represent postures in which the inverse kinematic problem (indicated by a red circle) is located in opposite $x$ and $y$ coordinates: where the locked base joint locations (red circle) are: Fig. 4.11(a) ($-x, -y$), and Fig. 4.11(b) ($x, y$). Since the base joint is locked at a predefined location, only one solution for $t$ is feasible.
The hemispherical workspace segments that are covered by the lower mobility sub-configuration of the 6-dof RRmMT that is based on the UofT PKM are shown in Fig. 4.12.

Sub-Configuration 1  Sub-Configuration 2  Sub-Configuration 3

Sub-Configurations 4-6  Sub-Configuration 7

Figure 4.12. Hemispherical workspace coverage (top view) of the sub-configurations of the 6-dof RRmMT that is based on the UofT PKM.

The hemispherical workspace segments that are covered by the sub-configurations of the 6-dof MIT-SS-1 based RRmMT (2

nd generation) are shown in Fig. 4.13. The sub-configurations cannot cover the complete hemisphere. However, their coverage is sufficient for machining the given set of parts.

Sub-Configuration 1, Sub-Configuration 2  Sub-Configuration 4

Figure 4.13. Hemispherical workspace coverage of the sub-configurations of the MIT-SS-1.
The serial-based RRmMT can cover the hemispherical workspace, as its joints range of motion allows it to reach all the points, and the structure is not limited by kinematic singularities and collision between chains.

4.1.4.1 Machining Conditions

In order to determine in the design stage, whether an RRmMT architecture can obtain the high feed-rate requirements for machining the set of meso-scale parts, the allowed actuator-travel range between adjacent checked points is determined. Table 4.7 summarizes the machining conditions, and the allowed joint-travel ranges corresponding to: adjacent checked points that are 0.1 mm apart, and actuators that are limited by a maximum speed of 10 m/min. The cutting speeds are based on those recommended in [65, 71] for hard-metal tools. The maximal travel distance of the base joint with a speed of 10 m/min between points is 4 mm. In a similar way, for Part 2 and Part 3 the maximal travel distance between checked points are 6.3 mm and 3 mm, respectively.

Table 4.7. Machining conditions and allowed actuators travel-range.

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Tool diameter [µm]</th>
<th>Cutting speed [m/min]</th>
<th>Chip load [µm/flute]</th>
<th>Spindle speed [RPM]</th>
<th>Allowed travel range [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Stainless steel</td>
<td>200</td>
<td>60</td>
<td>1.2</td>
<td>95,000</td>
<td>4</td>
</tr>
<tr>
<td>#2</td>
<td>Stainless steel</td>
<td>150</td>
<td>60</td>
<td>0.8</td>
<td>130,000</td>
<td>6.3</td>
</tr>
<tr>
<td>#3</td>
<td>Stainless steel</td>
<td>150</td>
<td>35</td>
<td>2</td>
<td>75,000</td>
<td>3</td>
</tr>
</tbody>
</table>

4.1.4.2 Stiffness Analysis

The combined stiffness of a configuration-part set is the performance index used for comparing design alternatives. Full motion simulations indicate that all the RRmMTs can reach the points along the tool trajectories that are required for machining the given set of parts. The cutting forces during machining oscillate with respect to the cutter location. However, to better demonstrate the effect of design alternatives, the architectures are compared based on their stiffness along the y-axis.
Topological reconfigurability and level of redundancy effect

The performance of lower mobility sub-configurations obtained through topological reconfiguration is demonstrated next. The sub-configurations should have the required mobility for machining the part and reach all the points along the specified tool trajectory. In order to demonstrate the effect on performance, stiffness of 5-dof sub-configurations along the specified trajectory for machining Part 1 is analyzed. The analysis of the Eclipse-based RRmMT was presented in Section 3.2.3.4. Similarly, the UofT PKM and the MIT-SS-1 based RRmMTs are analyzed next.

In redundant RRmMTs task is allocated and the joint trajectories are optimized according to the procedure depicted in Section 3.1.3. The workspace coverage of the 5-dof sub-configurations of the UofT PKM-based RRmMT is limited, and they cannot reach all the points along the trajectory associated with Part 1. A redundant dof is added to the RRmMTs in the 2nd generation in Step 1 of the synthesis. The dof is a serial stage, which is added to the workpiece-holder, and is located such that its feed direction is along the x-axis. Through allocation of the task between the tool and workpiece, the tool-holder speed can be reduced. In doing that, some combinations of the tool and workpiece speeds can attain the feed-rate required for machining the meso-scale parts. In Fig. 4.14, the stiffness distribution of the sub-configurations of the 7-dof RRmMT, which is based on the UofT PKM and an x-axis stage tool-holder are presented. However, only Sub-Configuration 2, Sub-Configuration 3 and Sub-Configuration 7 can machine Part 1.
Figure 4.14. Stiffness distribution of the sub-configurations of the 7-dof RRmMT that is based on the UofT PKM along the trajectory associated with Part 1.

The Cartesian tool-trajectories of the sub-configurations of the RRmMT are shown in Fig. 4.15. Changes in the stiffness along the trajectory are associated with changes in the Cartesian tool trajectory. For example, the point indicated by (a) in Fig. 4.14 and Fig. 4.15 mark a change in tool trajectory, which is associated with a change in the stiffness of the RRmMT.

![Sub-Configuration 2: Second base joint locked](image)

![Sub-Configuration 3: Third base joint locked](image)

![Sub-Configuration 7: joint are unlocked](image)

Figure 4.15. Tool trajectories of the 7-dof RRmMT that is based on the UofT PKM: Cartesian (green), specified (blue).

All 5-dof of the RRmMT that is based on the MIT-SS-1 are required for 5-axis machining. Thus, it is not possible to topologically reconfigure this RRmMT into lower mobility su-
configurations and fabricate the given set of parts. The stiffness distributions of the 6-dof RRmMT that is based on the MIT-SS-1 are shown in Fig. 4.16. The stiffness distribution of Sub-Configuration 1 and Sub-Configuration 3 are similar, since the $x$ coordinates of the locked joints in these sub-configurations are symmetrically located with respect to the $y$-axis. Sub-Configuration 4 has 6-dof and it is therefore redundant for the task, and the joints trajectories can be optimized to enhance the stiffness. Optimizing the joint trajectories resulted in a significant stiffness increase.

![Figure 4.16. Stiffness distribution of the MIT-SS-1 based RRmMT with part 1.](image)

In order to analyze the effect of level of redundancy, stiffness results of the four RRMT architectures that have been constructed in the first second and third generations of the synthesis process are analyzed. In Fig. 4.17, the stiffness distributions of the UofT PKM-based RRmMT with one, two, and three redundant dof are presented.
In Fig. 4.18, the stiffness distributions of the 5-dof (no redundancy), 6-dof (one redundant dof), and 7-dof (two redundant dof) MIT-SS-1 based RRmMTs are presented. The stiffness of the MIT-SS-1 along the $y$-axis is related to the sub-configurations of its PKM-based tool-holder. Adding the first redundant dof improves the stiffness of the RRmMT. However, the difference between the RRmMT with two redundant dof and the RRmMT with one redundant dof is less than 1%.

The stiffness of the serial-based RRmMT is dictated by its joints stiffness. The 3 N/µm stiffness of the $y$-axis stage is directly translated to the stiffness at the tool along the $y$-axis, and the redundant $x$-axis stages that are added to the workpiece-holder do not have an effect on the serial mechanism stiffness along the $y$-axis.
The performance index used to compare design alternatives is the combined stiffness, which is the sum of the mean stiffnesses obtained with the given set of parts. The sub-configuration part-set is selected in Tier 3 and the geometrical dimensions are optimized in Tier 4. Hence, the design parameters may change according to the selected set. For the link length, which is optimized in the design test case, the maximal difference in length is 5 mm. Since for the 6-dof PKM-based RRmMTs and the three parts, there are $343 \left(7^3\right)$ different combinations of configuration-part sets, the mean stiffness of every configuration-part is presented separately, and the performance index is obtained by combining the highest mean stiffness with each part. This allows to select the configuration-part set that attains the highest combined mean stiffness.

Every RRmMT, which is synthesized in the iterative process in Step 1, is a design alternative. For the Eclipse, the UofT PKM, the MIT-SS-1, and the 5-dof serial-based RRmMTs, the improvement in stiffness between the 2nd generation and the 3rd generation, with respect to Part 1, is less than a 1% threshold. Therefore, Table 4.8 summarizes only the stiffness of the configuration-part sets that are associated with the RRmMTs that were constructed in the 1st and 2nd generations.

Figure 4.18. Stiffness distribution of the 5-dof, 6-dof and 7-dof MIT-SS-1 based RRmMTs with Part 1.
The first generation of the Eclipse and the UofT PKM-based RRmMTs comprises of a 6-dof tool-holder, which is redundant for 5-axis machining. The first generation of the MIT-SS-1 and the serial mechanism are constructed with 5-dof.

Table 4.8. Configuration-part set performance.

<table>
<thead>
<tr>
<th>Chapter 5</th>
<th>RRmMT</th>
<th>Architecture ($R_g$)</th>
<th>Tool -holder</th>
<th>Workpiece-holder</th>
<th>Part 1</th>
<th>Part 2</th>
<th>Part 3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>Stiffness [N/µm]</td>
<td>Stiffness [N/µm]</td>
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</tr>
<tr>
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<td>Eclipse</td>
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<td>6</td>
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<td>6.4</td>
<td>18.9</td>
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<td>0.1</td>
<td>0.1</td>
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<td>MIT-SS-1</td>
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<td>x-axis stage</td>
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<td>1,3</td>
<td>Mean</td>
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<td>0.36</td>
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<td>Mean</td>
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<td>5.5</td>
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<td>15.4</td>
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<td>Var</td>
<td>0.14</td>
<td>0.15</td>
<td>0.12</td>
<td>0.41</td>
</tr>
</tbody>
</table>

As expected, higher level of redundancy of the RRmMTs increases the ability of sub-configurations to machine parts, and increases the structural stiffness along the given trajectory. The highest combined mean stiffnesses of the four RRmMT architectures are presented in Fig. 4.19. Increase in mean stiffness due to redundancy is between 5-10% for the 6-dof PKM-based RRmMTs, and 40% for the 5-dof RRmMT that is based on the MIT-SS-1. The contribution of the redundancy to the stiffness of the 6-dof PKMs is constrained by the dynamic characteristics of current actuator technology that limits postures changes along the trajectory. However, the redundancy enables the PKM-based RRmMTs to obtain the high feed-rate that is required in meso-milling.

Fundamental two-tailed, null-hypothesis analysis clearly shows that the sub-configurations of the UofT PKM, which was synthesized in Step 1, has better stiffness properties, beyond a significance level of 99%. Therefore, the 7-dof RRmMT that is based on the UofT PKM is selected for the three parts, and the configuration-part set is $CO_{2,5} = [777]$, where ‘2’ is the index for the tool-holder mechanism, and ‘5’ is the index for the workpiece-holder mechanism.

7-dof
6.1 Synthesis and Design of RRmMTs that can be Reconfigured into Assembly/Working Mode Configurations

In Step 2, the objective is to synthesize a RR-PKM based tool-holder that can be reconfigured into assembly/working mode configurations. Table 4.9 includes an example set of full-mobility PKMs that are based on three and six isomorphic-chain topologies. These PKMs can be synthesized from building blocks in Step 1.

Each of the PKM-based tool-holders can topologically reconfigure into lower mobility sub-configurations by applying the holding-force of an actuator to restrict the tool’s motions. Since chains of a mechanism may limit its ability to reconfigure, the RR-PKM is synthesized from three chain PKMs.

Table 4.9. Three and six chains PKMs.
<table>
<thead>
<tr>
<th>Alizade et al. [74]</th>
<th>Alizade PKM</th>
<th>$3\times PRPS$</th>
<th>6</th>
<th>Spatial</th>
<th>moves along a curvilinear rail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ray et al. [44]</td>
<td>UofT PKM</td>
<td>$3\times PPRS$</td>
<td>6</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Glozeman and Shoham [113]</td>
<td>Glozeman PKM</td>
<td>$3\times PPRS$</td>
<td>6</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Plitea et al. [27]</td>
<td>Recrob</td>
<td>$3\times PRRS$</td>
<td>6</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Byun and Cho [114]</td>
<td></td>
<td>$3\times PRRP$</td>
<td>6</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Chen [57]</td>
<td></td>
<td>$3\times PRPS$</td>
<td>6</td>
<td>Spatial</td>
<td>First prismatic joint moves in radial direction</td>
</tr>
<tr>
<td>Kong et al. [59]</td>
<td></td>
<td>$3\times RRR$</td>
<td>3</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Gao et al. [115]</td>
<td></td>
<td>$3\times RRR$</td>
<td>3</td>
<td>Planar</td>
<td></td>
</tr>
<tr>
<td>Gao et al. [115]</td>
<td></td>
<td>$3\times RPS$</td>
<td>3</td>
<td>Spatial</td>
<td></td>
</tr>
<tr>
<td>Yu et al. [116]</td>
<td>Hexaglide,</td>
<td>$6\times PSS$</td>
<td>6</td>
<td>Spatial</td>
<td>Configurations differ in the direction of motion of the prismatic joint</td>
</tr>
<tr>
<td></td>
<td>Linapod,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>HexaM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Muruganandam and Pugazhenti [117]</td>
<td>Hexapod</td>
<td>$6\times SPS$</td>
<td>6</td>
<td>Spatial</td>
<td></td>
</tr>
</tbody>
</table>

Four PKM architectures that can machine 3D parts due to their high tool tilt-angle are considered for the synthesis of the RR-PKM. The PKMs are based on a $3\times PXXS$ topology [76]: Fig. 4.20(a) the Eclipse [7], Fig. 4.20(b) the UofT PKM [76], Fig. 4.20(c) the Glozeman PKM [113], and, Fig. 4.20(d) the Alizade PKM [74]. The symbol ‘X’ denotes a joint that can be prismatic or revolute. The topologies differ in no more than one joint, and are, therefore, suitable for synthesizing a RR-PKM.

![PKM architectures](image)

Figure 4.20. PKM architectures: (a) the Eclipse, (b) the UofT PKM, (c) the Glozeman PKM, (d) the Alizade PKM.

Assembly/working mode configurations of a PKM have the same topology, therefore, the Eclipse and the UofT PKM, that are based on the same $3\times PPRS$ topology are combined first. In order to synthesize the two PKMs into a single RR-PKM, reconfiguration requirements are
analyzed and lockable joints are added to support the reconfiguration process. Reconfiguration from the Eclipse to the UofT PKM requires the second prismatic joint to reorient. Thus, a lockable joint is added to each chain of the $3\times PPRS$ PKM. The revolute lockable joint is incorporated before the second prismatic joint, such that the RR-PKM joint scheme becomes $3\times PR^LPRS$. The lockable revolute joints allows the first link in each chain to rotate from the radial orientation shown in Fig. 4.21(a) to a vertical orientation shown in Fig. 4.21(d), where the angle $\chi$, shown in Fig. 4.21(c), changes from $0^\circ$ to $90^\circ$.

Reconfiguration into a different working/assembly mode requires planning, so that the RR-PKM does not get into singularity. For example, during the transition from the configuration shown in Fig. 4.21(a) into the configuration shown in Fig. 4.21(d), the mechanism goes through a singular posture where the links’ axes are orthogonal to the normal of the platform. This singularity can be avoided by locking the second revolute joint during the reconfiguration process [99]. Hence, the joint scheme of the RR-PKM after Step 2 is $3\times PR^LPR^LS$.

**Figure 4.21.** RR-PKM reconfiguration: (a)-(d) from the UofT PKM to the Eclipse.

Assembly/working modes can be utilized to increase the workspace and ameliorate the performance of RRmMTs. Enhancing the dynamic characteristics of a RR-PKM through assembly mode reconfiguration was presented in [99]. In the following example, reconfiguration is used to enhance the mean stiffness of the 7-dof RRmMT that is based on the $3\times PR^LPR^LS$ tool-holder and an $x$-axis stage workpiece-holder. The mean stiffnesses $\bar{R}_{yy}$ and $\bar{R}_{zz}$ that are associated with machining of the three parts are obtained from simulations. As can be noted from Table 4.10, the mean stiffness of the Eclipse based configuration of the RRMT along the $z$-axis is...
higher than the stiffness of the UofT PKM based configuration, while the UofT PKM based configuration mean stiffness along the y-axis, $\bar{K}_{yy}$, is higher than the stiffness of the Eclipse based configuration. Thus, for higher mean stiffness along the y-axis the $3\times PR^lPR^lPS$ RR-PKM is reconfigured into the UofT PKM configuration, and for higher stiffness along the z-axis, it is reconfigured into the Eclipse configuration.

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Part 1</th>
<th>Part 2</th>
<th>Part 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>The UofT PKM 7-dof</td>
<td>$K_{zz}$ [N/µm]</td>
<td>4.45</td>
<td>3.26</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}_{yy}$ [N/µm]</td>
<td>6.12</td>
<td>6.41</td>
</tr>
<tr>
<td>The Eclipse 7-dof</td>
<td>$K_{zz}$ [N/µm]</td>
<td>8.56</td>
<td>8.63</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}_{yy}$ [N/µm]</td>
<td>5.54</td>
<td>5.72</td>
</tr>
</tbody>
</table>

6.2 Step 3 - Synthesis and Design of RRmMTs that can be Reconfigured into Sub-PKMs that Differ in Topology

In Step 3, the joint scheme of the RR-PKM based tool-holder that was synthesized in Step 2 is adjusted, such that it can be reconfigured into sub-PKMs that differ in topology.

The chains of the $3\times PRPS$ Alizade PKM are constructed from a prismatic actuator, $M_1$ that moves along a circular rail, a passive revolute joint, a second prismatic actuator, $M_2$, and a spherical joint that connects the chain to the platform. To reconfigure from the Eclipse/UofT PKM into the Alizade PKM and vice versa, both the first and second revolute joints in each chain of the $3\times PR^lPR^lS$ RR-PKM should be lockable. The RR-PKM joint scheme supports that: in the Eclipse/UofT PKM, the first revolute joint is locked and the second revolute joint is unlocked; in the Alizade PKM, the first revolute joint is unlocked and the second revolute joint is locked. The reconfiguration process from the UofT PKM to the Alizade PKM is shown in Fig. 4.22(a-e).
The second PKM that differs in topology is the $3\times PRRS$ Glozman PKM. The chains of the Glozman PKM are constructed from a prismatic actuator, $\mathcal{M}_1$ that moves along a circular rail, a passive revolute joint, an active rotating actuator, $\mathcal{M}_2$, and a spherical joint that connects the chain to the platform. The Glozman PKM topology is built-in the $3\times PR^LPR^LPR^S$ but its joint scheme is different. The Glozman PKM requires the second revolute joint to be active. Thus, replacing the second revolute lockable joint in each chain with an actuator, to get a $3\times PR^LPR^LPR^S$ topology, would allow the RR-PKM to reconfigure into the Glozman PKM. In order to complicate the example at hand, active actuators that can change from active state to passive state are assumed not to exist for small-scale machine tools. Thus, the $3\times PR^LPR^LPR^S$ RR-PKM cannot be reconfigured into the Glozman PKM, and a RR-PKM variant with a $3\times PR^LPR^LPR^S$ topology has to be synthesized. The reconfiguration process from the Eclipse to the Glozman PKM, which would be possible with active joints that changes into passive state is shown in Fig. 4.23.
Figure 4.23. Reconfiguration from (a) the Eclipse to (h) the Glozman PKM.

The configurations of the synthesized $3\times PRPRS$ RR-PKM are shown in Fig. 4.24. A joint that is locked is considered as part of the link, and omitted from the topology notation. For example, reconfiguration process of the $3\times PRPRS$ RR-PKM into the Glozman PKM requires to unlock the first revolute joint and to lock the second prismatic joint, which results in the configuration topology: $3\times PRPRS \equiv 3\times PRRS$.

Adjusting the joint scheme the $3\times PRPRS$ RR-PKM can be reconfigured into other sub-PKMs, such as the $3\times RPRS$, in which the first prismatic (base) joint in each chain of the RR-PKM is locked.

---

<table>
<thead>
<tr>
<th>Topology</th>
<th>$3\times PRPRS$</th>
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<tr>
<td>joint scheme</td>
<td>$3\times PRPRS$</td>
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<tr>
<td>Configuration</td>
<td>The Eclipse PKM</td>
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Figure 4.24. Sub-PKMs and working/assembly mode configurations of the $3\times PRPRS$ RR-PKM.
The stiffnesses of the four sub-PKMs to which the $3 \times PR^lPRS$ RRmMT can reconfigure are compared with respect to the hemispherical workspace. The procedure used in Section 3.2.2 for calculating the static stiffness of the proposed PKM is utilized here. Table 4.11 summarizes the means and variances of the static stiffness of the RRmMTs over a hemispherical surface with a 4 mm radius. A two-tailed, null-hypothesis analysis showed that the UofT PKM has better stiffness properties along the $x$ and $y$ axes than those of the other three RRmMTs, beyond a significance level of 99%, and the Eclipse is the stiffest along the $z$-axis.

Table 4.11. Mean and variance values of PKMs’ static stiffness distribution.

<table>
<thead>
<tr>
<th></th>
<th>The UofT PKM [N/µm]</th>
<th>The Eclipse PKM [N/µm]</th>
<th>The Alizade PKM [N/µm]</th>
<th>The Glozman PKM [N/µm]</th>
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</thead>
<tbody>
<tr>
<td>$K_{xx}$</td>
<td>Mean: 5.691</td>
<td>Mean: 4.450</td>
<td>Mean: 3.686</td>
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<tr>
<td></td>
<td>Variance: 0.349</td>
<td>Variance: 0.237</td>
<td>Variance: 0.028</td>
<td>Variance: 0.020</td>
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<tr>
<td>$K_{yy}$</td>
<td>Mean: 4.710</td>
<td>Mean: 3.986</td>
<td>Mean: 3.247</td>
<td>Mean: 2.038</td>
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<tr>
<td></td>
<td>Variance: 0.316</td>
<td>Variance: 0.100</td>
<td>Variance: 0.103</td>
<td>Variance: 0.053</td>
</tr>
<tr>
<td>$K_{zz}$</td>
<td>Mean: 5.690</td>
<td>Mean: 7.360</td>
<td>Mean: 3.601</td>
<td>Mean: 5.487</td>
</tr>
<tr>
<td></td>
<td>Variance: 0.228</td>
<td>Variance: 0.004</td>
<td>Variance: 0.108</td>
<td>Variance: 0.292</td>
</tr>
</tbody>
</table>

6.3 Design of the $3 \times PR^lPRS$ based RRmMT

In this section several design aspects of the $3 \times PR^lPRS$ based RRmMT are presented. As mentioned above, a revolute joint is incorporated to each chain of the $3 \times PPRS$ RR-PKM to construct a $3 \times PR^lPRS$ based RRmMT. In order to construct an RRmMT that can reconfigure into different sub-PKM the following design aspects should be addressed:

i. Reconfiguration into different sub-PKM may result in motions that are outside of the RRmMT joints travel range. In the design test case, attention should be given to the design of spherical joints that connect the chains and platform. For example, let us follow on the spherical joint orientation during reconfiguration from the UofT PKM to the Alizade PKM, and to the Eclipse PKM. The spherical joint position of the UofT PKM in home position is shown in Fig. 4.25(a). As can be noted, the joint range of motion (continuous lines) is symmetric compared to the joint orientation (dashed line).
The orientation of the spherical joint of the Alizade PKM in home configuration is shown in Fig. 4.25(b). The range of motion of the spherical joint to one side is smaller compared to the range of motion to the other side, which limits coverage of the workspace. The spherical joints position of the Eclipse after reconfiguring from the UofT PKM is shown in Fig. 4.25(c). As can be noted from the figure, the spherical joint orientation is outside of the maximal range of travel. Therefore, a lockable revolute joint that can compensate on the change in the orientation of the spherical joint, is incorporated at the end of the chain.

(a) (b) (c)

Figure 4.25. Spherical joints orientation in home position:
(a) the UofT PKM, (b) the Alizade PKM, (c) the Eclipse.

ii. The optimal link length may differ between PKMs. Thus, to attain optimal RRmMT performance the link length should be adjusted according to the PKM. For the $3\times PR^T PR^T S$ RRmMT the link length when reconfiguring into the Eclipse should be 216 mm, and when reconfiguring into the UofT PKM it should be 180 mm.

iii. In case the workpiece location with respect to the platform does not change between PKMs, the tool, which is connected to the platform of the RRmMT architectures, should point toward the same direction. For example, in the design test case the tool of the Eclipse, Alizade, and the UofT PKM points towards the base.

iv. The distance between the base and moving platform of the tool- and workpiece holders may differ between configurations, and the design of the RRmMT should consider that.

A CAD model of the $3\times PR^T PR^T S$ RRmMT that can be reconfigured into the Eclipse, the UofT PKM, and Alizade PKM is shown in Fig. 4.26.
6.3.1 Built Prototypes [75]

Two RRmMT prototypes that are based on the Eclipse and the UofT PKM were designed and built in our laboratory. The RRmMT, shown in Fig. 4.27(a) is based on the Eclipse, and the RRmMT shown in Fig. 4.27(b) is based on the UofT PKM. The RRmMTs consist of three identical chains, where each chain is attached to a stage that moves along a curvilinear guide. The curvilinear guide and stage unit chosen for the RmMT is the HCR 15A+60/150R made by THK ltd. This stage can be moved to a desired location and then locked. Two FB075 linear stages, which are actuated by HR8 ultrasonic motors, manufactured by Nanomotion, are placed on top of the curvilinear stage.
The first linear stage moves tangentially with respect to the curvilinear guide (Fig. 4.27(a)), or, radially (Fig. 4.27(b)), and the second linear stage, which is mounted on top of the first one, moves in radial direction. A revolute joint connects the linear stage with a fixed length link. The link is then connected on its other side through a Seiko Hephaist SRJ008C spherical joint, to the moving platform.

The structural components such as the base, links, and platform of the prototypes are made from AISI 1018 steel. The built prototype was used as a test bed for integrating sub-components such as spindle, sensors and actuators. For example, a simple commercial spindle was incorporated into the center of the mobile tool platform, allowing for further study of the interaction between the spindle and the RRmMT [118]. In terms of human interface, the architecture of the RRmMT allows for quick and easy access to the tool and workpiece. In addition, the mechanism is constructed such that the tool is supported from below, which allows for less interference between the spindle and the mechanism.

### 6.3.2 Reconfiguration Software Module

A software module, which is designed to support the reconfiguration of the $3 \times PPRS$ RRmMT that can reconfigure to the Eclipse and the UofT PKM, was developed. The module can analyze
and select the best configuration for a given set of parts based on a performance criterion determined by the user.

The user interface, which is shown in Fig. 4.28, includes:

*Architecture* panel: from this panel, the assembly/working mode or sub-PKM is selected. For example, the Eclipse and the UofT PKM configurations of the $3\times PR^LPR^LPR^S$ RRmMT.

*Configuration-Part set-up* panel: from this panel, the sub-configuration for each of the parts is selected.

*Performance criterion* panel: from this panel, the performance for optimizing the configuration-part set-up is selected (the default is the RRmMT $\bar{K}_{yy}$ mean stiffness).

*Simulation Results* panel: in this panel, the configuration-part performance is presented.

6.4 Conclusions

A comprehensive design test case of a new RRmMT, which follows the principles of the developed design methodology, has been presented in this chapter.
First, four serial/parallel/hybrid RRmMTs were synthesized. Reconfiguration of the RRmMTs into lower mobility sub-configurations, by locking through holding-force of actuators was demonstrated, and the effect of topological reconfiguration on workspace coverage and stiffness was analyzed. Second, two 3×PPRS PKM based tool-holders from Step 1, were combined into a single RR-PKM based tool-holder that can be reconfigured into different assembly/working modes. The synthesis required to identify the singular configurations, which divide between the assembly/working mode configurations, and to analyze the approach for avoiding them, i.e., by incorporating redundant lockable joints. Third, the joint scheme of the RR-PKM was adjusted to allow reconfiguration into 3×PRRS and 3×PRRS PKMs. Combining the Eclipse, Alizade and UofT PKMs, which are designed with base joints that move along circular rail, resulted in a new 3×PRPS RRmMT.

Design aspects of PKM-based RRmMTs were also discussed. Analyses indicate that PKM-based RRmMTs can obtain higher stiffness compare to serial-based RRmMTs. However, the PKM-based RRmMTs cannot attain the required feed-rate due to the transmission between the joints and the tool. Thus, redundant dof were added in series to the workpiece-holder of the PKM-based RRMTs, to construct a hybrid-based RRmMT that can attain the machining conditions for fabricating meso-scale parts. Following, throughout the optimization based design test case the level of redundancy of the hybrid-based RRmMT was optimized in order to enhance performance, to obtain required machining conditions, and to support reconfiguration.
Chapter 7 Conclusions and Recommendations

The research described in this thesis is aimed at developing a new design methodology for redundant reconfigurable machine-tools. This chapter provides a summary of conclusions and contributions made. Recommendations for future work are also detailed.

7.1 Conclusions

The main challenge in structural design of RRMTs is to select the optimal architecture for the set of parts at hand. Research in the field of PKMs presents an array of opportunities for the design of mechanisms capable of adjusting their structure according to task requirements, without the disassembly of the structure. This has paved the way for a design methodology that proposes an approach towards enhancing the performance of RMTs, while taking into account the possible inherent redundant reconfigurability of a wide range of architectures (serial/PKM/hybrid).

Two engines are combined to create the new design methodology, synthesis and optimization. Design variables have to be optimized and redundant reconfigurability has to be managed in order to determine which RRMT is the best for a given set of parts. Thus, a challenge in developing the new design methodology for RRMTs was the diversity of decision-making on a large number of continuous/discrete parameter’s values that mutually depend upon one another. This engineering problem is addressed through multi-tiered optimization that comprises an iterative process that transfers decisions to a lower or upper level to improve the overall result. Given a set of parameters/constraints, the RRMT performance is evaluated in the lowest tier. The tiered approach allows organizing decisions that must be taken throughout the design process in a structured manner.

The decisions on the topology and the level of redundancy of the RRMT are interwoven with the decisions on the design variables, and the variables that are associated with managing of the redundant reconfigurability, such as the RRMT configuration for the given part. Thus, a three-step algorithmic synthesis process that constructs RRMTs is incorporated into the multi-tiered optimization process. The synthesis is driven by the mobility and level of redundancy that are
required to fabricate a given set of parts with desired machining conditions. Reconfigurability of PKMs was specifically addressed throughout the development of the design methodology. Thus, the process synthesizes RRMTs that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations, and full mobility sub-PKMs that differ in topology.

Step 1 synthesizes a variety of RRMT architectures that can be constructed from one or two cooperating mechanisms. In this step, redundant dof are iteratively added to each RRMT in series until the optimal level of redundancy is obtained, and the synthesized RRMTs can be topologically and geometrically reconfiguration into lower mobility sub-configurations. In Step 2 PKM-based tool/workpiece-holders that were synthesized in Step 1 are combined into a RR-PKM that can be reconfigured into assembly/working mode configurations. The required flexibility is obtained by incorporating lockable joints. In Step 3 the joint scheme of the RR-PKM is adjusted to allow reconfiguration into sub-PKMs that differ in topology.

In order to make full use of the redundant reconfigurability, the configurations are selected for each task. Additionally, an algorithmic process is depicted for selection of machine postures through task allocation and trajectory resolution into joint trajectories, at discrete points along specified tool trajectories.

As mentioned above, the methodology is designed to attain the optimal RRMT for a given set of parts. Due to the small size of sub-components and the long calibration time for reconfiguration, redundant reconfigurability approach can be attractive for RRmMTs. In this context, it is shown that, generally, the combination of small tool and minimal chip thickness effect, call for high spindle speed and high feed-rate. Naturally, keeping the chip thickness within the required size is key to reducing forces during meso-milling. Thus, an algorithmic process for translating information from the meso-milling process domain into the design domain was proposed in order to attain the high feed-rates that are required in meso-milling. The machining conditions for a set of meso-scale parts are translated into constraints on the RRmMT’s active joints motions.

The applicability of this methodology is demonstrated by its ability to assist in the design of a PKM-based RRmMT that can satisfy the high feed-rate requirement, so as to avoid high-milling forces. Throughout the design process, redundant reconfigurability effect is analyzed. For example: performance of lower mobility sub-configurations that are obtained through locking of
joints, and task allocation and trajectory resolution of sub-configurations that are redundant for the task are utilized for enhancing stiffness. Furthermore, the design test case exhibits the ability of the proposed methodology to address the diverse decision-making through a comprehensive design process. Full kinematic motion simulations of six RRmMTs and a module for evaluating the RRmMTs stiffness have been developed in Matlab to support the design test case. The process resulted in the design of a new $3 \times PR^LPR^L S$ based RRmMT that can be reconfigured into sub-PKMs, such as the Eclipse [7], the Alizade PKM [74], and the UofT PKM, which was developed in our Laboratory [76].

7.2 Contributions

This research has generated a number of contributions to the body of knowledge of machine design, to the RMT research field, and to meso-milling:

1) A novel generic design methodology for RRMTs that can optimally utilize redundant reconfigurability to enhance performance:

   — A multi-tiered optimization-based design approach that has been applied to the conceptual design of RRMTs.

   — A method for synthesis of RRMTs that can be reconfigured into lower mobility sub-configurations, assembly/working mode configurations, and sub-PKMs that differ in topology.

   — Can design RRmMTs for the unique machining conditions of meso-milling.

2) A novel $3 \times PR^LPR^L S$ based RRmMT.
7.3 Recommendations for Future Work

One of the main challenges in this work has been to concurrently address the specific design issues of RRMTs and RRmMTs in an integrated approach that combines synthesis, design and machining conditions. Although the design methodology that has been devised successfully meets this challenge, there is still room for improvement in order to make the proposed methodology more comprehensive. Some suggestions for future work are provided below.

7.3.1 Automating the Design Process

Efficient design of RRMTs relies on a synthesis process, which enumerates the pertinent architectures for the given set of parts. One beneficial avenue of improvement would be automating the synthesis process. Similar to the design methodology presented here, the automated synthesis process has to be integrated with the optimization and performance evaluation to add iteratively redundant dof until the termination condition is met. In addition, the synthesis process should support the automatic processes of kinematic modeling and performance evaluation of the RRMTs.

7.3.2 Inverse Kinematic models of RRmMT Configurations

The kinematic analysis of the lower mobility configurations of the Eclipse, the UofT PKM, and the MIT-SS-1 based RRmMTs indicate that for the set of inverse-kinematic equations the solution is not necessarily unique. Therefore, there is a need for a method for selecting the desired machine posture at points along tool trajectories, which are associated with the given set of task-space parameters [85].

7.3.3 On-Line Performance Evaluation

The ability to evaluate the performance of an RRmMT on-line is crucial for enhancing its capabilities. For example, uncertainties in meso-milling process, such as tool wear, can quickly deteriorate the machine-tool performance due to increased milling forces. Thus, predicting the
forces at the next checked point along the prescribed tool trajectory would allow updating the, machining conditions and machine posture for compensating on unexpected changes.

7.3.4 Engineering Considerations of RRMTs design

Commercial dedicated hardware that can support reconfiguration is currently not available. For example, lockable joints that can switch between active, passive and locked states, and spherical joints that can cover the wide range of motion that is required for reconfiguration. Lockable joints should present higher locking force compared to the holding force obtained by actuators, and their stiffness should be higher compared to that of passive joints, to make reconfigurability through locking of dof attractive.

7.3.5 Concurrently Address the Design and Task Processes

In order to capture the information from the set of parts and to efficiently implement it into the design of a new RRMT, the relation between the part characteristics and the structural design should be further investigated.

Optimizing the design process can be improved by incorporating additional requirements based on the machining conditions. Cutting mechanics of meso-parts has been studied intensely in the last few years. This has led to identifying size effects that are unique to the machining of meso-scale. It is now known, for example, that the cutting coefficients of meso-scale parts should take into account the grain size, the tool radius, the rake angle, the tool wear, the tool material, the tool coating, lubrication, temperature, etc. Accurate evaluation of the cutting coefficients may allow to enhance RRMTs performance.

Out of these parameters, the approach angle (rake angle) would be of interest for the design of the machine tool. The rake angle determines, along with the spindle speed and feed-rate, the chip load for the process. The ability to evaluate the relation between changes in the tool rake angle, which is associated with the accuracy of the machine tool, and the configuration and posture selection of the RRMT, can improve the machine-tool design and associated performance.

An additional route for improving the design methodology is to incorporate the performance of the machine-tool into meso-milling machining simulations. For example, simultaneously
simulate the meso-milling machining operation (tool-workpiece interaction) and the RRmMT reaction to the machining process. This would allow to analyze concurrently the effect of the machining process on the architecture and the effect of the structure on the machining process.
References


G. Gogu, "Isoglidien-TaRb: a family of up to five axes reconfigurable and maximally regular parallel kinematic machines," presented at the International Conference on Smart Machining Systems, Gaithersburg, 2007.


Appendix A - Inverse Kinematic Models

This Appendix presents the inverse-kinematic models of four RRmMTs that are compared in Chapter 3 and Chapter 4.

A.1 The Eclipse

The notation for the inverse-kinematic model of the Eclipse-based RRmMT with an x-axis stage workpiece-holder is shown in Fig. A.1. The Eclipse [7] is constructed from a $3 \times PPRS$ topology that has a circular rail of radius, $R_b$, on which three curvilinear joints are mounted. The mobility of the tool-holder, $M_T$, is equal to 6.

The global coordinate system, $\{O\}$, is positioned at the center of the circular rail. The position of the curvilinear joint is denoted as $A_i = [A_{ix} \ A_{iy} \ A_{iz}]^T$, where $i$ (i=1 to 3) is the chain's index. The prismatic joint is mounted on top of the curvilinear joint, and it moves along the $z$-axis. $C_i = [C_{ix} \ C_{iy} \ C_{iz}]^T$, denotes the position of the $i^{th}$ prismatic joint. In each chain, a passive revolute joint connects the vertical prismatic joint to a fixed length link. This link is connected on its other end to the moving platform through a spherical joint. The links are denoted as $L$, the angular travel of the revolute joints are denoted as $\varphi_i$, and the coordinates of the spherical joints are denoted as $P_i = [P_{ix} \ P_{iy} \ P_{iz}]^T$. The vector $Q = [\theta_1 \ \theta_2 \ \theta_3 \ d_1 \ d_2 \ d_3]^T$ is used to denote the generalized coordinates of the active joints, where $\theta_i$ and $d_i$ represent the curvilinear joint travel, and the prismatic joint travel, respectively.
Figure A.1. Kinematic notation for the 7-dof Eclipse-based RRmMT.

Frame $\{T\}$ is attached to the center of the moving platform of the tool-holder, and the vector $T = [T_x \ T_y \ T_z \ T_\alpha \ T_\beta \ T_\gamma]^T$ defines its Cartesian position and orientation (pose), with respect to the global frame $\{O\}$. Frame $\{W\}$ is attached to the center of the workpiece-holder, and the vector $W = [W_x \ W_y \ W_z \ W_\alpha \ W_\beta \ W_\gamma]^T$ defines its Cartesian position and orientation (pose), with respect to the global frame $\{O\}$. In home configuration, the axes of the tool- and workpiece-holders, are parallel to the global frame axes. For 5-axis machining the specified tool-path with respect to the workpiece frame, which is required for machining a part, is denoted as $X = [x_f \ y_f \ z_f \ \alpha \ \beta \ 0]^T$.

The positions of the spherical joints, $P_i$ ($i=1$ to 3), in task space coordinates (inverse-kinematics) can be obtained from the position and orientation of frame $\{T\}$, and the position of the spherical joints, $^TP_i$, relative to the platform frame, as:

$$P_i = [T_x \ T_y \ T_z]^T + \overset{^T}{R}^TP_i,$$

(A.1)

where $\overset{^T}{R}$ is the rotation matrix of the platform with respect to the global frame.

The joint-space coordinates of the PKM, are expressed as:

$$\theta_i = \tan^{-1}(P_{iy} / P_{ix}),$$

(A.2)
\[ d_i = \sqrt{L^2 - \left(R_b - \sqrt{P_{ix}^2 + P_{iy}^2}\right)^2 - P_{iz}}. \] (A.3)

Given Eqs. (A.1) to (A.3), the position of the curvilinear and prismatic joints can be obtained as:

\[ A_i = [R_b \cos \theta_i \ R_b \sin \theta_i \ 0]^T, \quad (A.4a) \]
\[ C_i = [R_b \cos \theta_i \ R_b \cos \theta_i \ d_i]^T, \quad (A.4b) \]

and the \( i^{th} \) revolute joint angle can be computed as:

\[ \varphi_i = \sin^{-1}\left(\frac{d_i - P_{iz}}{L}\right). \] (A.5)

### A.2 The UofT PKM

The UofT PKM-based RRmMT, shown in Fig. A.2, differs from the Eclipse PKM, in the second joint in each chain, which moves in radial direction [76]. The mobility of the tool-holder, \( M_{TH} \), is equal to 6.

The notation for the inverse-kinematic model is similar to the Eclipse-based RRmMT notation, thus, herein only the differences between the kinematic models are discussed. The second joint variable, \( d_i \), denotes the prismatic joint travel, which moves in the radial direction relative to the circular guide. The joint-space coordinates of the PKM, in terms of the position of the spherical joints, are expressed as:

\[ \theta_i = \tan^{-1}\left(\frac{P_{iy}}{P_{ix}}\right), \quad (A.6) \]
\[ \varphi_i = \sin^{-1}\left(P_{iz} / L\right), \quad (A.7) \]
\[ d_i = \sqrt{L^2 - P_{iz}^2 + \left(P_{ix}^2 + P_{iy}^2\right)}. \quad (A.8) \]

The position of the curvilinear and prismatic joints are denoted as:

\[ A_i = [R_b \cos \theta_i \ R_b \sin \theta_i \ 0]^T, \quad (A.9a) \]
\[ C_i = [d_i \cos \theta_i \ d_i \sin \theta_i \ 0]^T. \quad (A.9b) \]
A.3 The MIT-SS-1 Hybrid Mechanism

The MIT-SS-1 is constructed from a $3 \times PRR$ PKM based tool-holder that moves along the $x$- and $z$-axes and rotates about the $y$-axis, and a $RP$ workpiece-holder that rotates about the $z$-axis and moves along the $y$-axis [8]. The notation for the MIT-SS-1 based RRmMT is shown in Fig. A.3.
As mentioned above, the 5-axis tool-path with respect to the workpiece-holder is denoted as 
\[ X_f = [x_f \ y_f \ z_f \ \alpha \ \beta \ 0]^T. \] The transformation matrix from the workpiece-holder frame to the tool-holder frame is specified as:
\[
\begin{bmatrix}
\cos W_\alpha & -\sin W_\alpha & 0 & 0 \\
\sin W_\alpha & \cos W_\alpha & 0 & W_y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad (A.10)
\]

where, \( W_\alpha = -\alpha. \)

The tool-holder motions are limited to the \( x-z \) plane, and the Cartesian coordinates and orientation that the tool has to follow are:
\[
T_x = x_f \cos(-\alpha) - y_f \sin(-\alpha), \quad (A.11a)
\]
\[
T_z = z_f, \quad (A.11b)
\]
\[
T_\beta = \beta. \quad (A.11c)
\]

The link length, \( L \), the triangular platform's side, \( 2r \), and the \( x \) coordinates, \( C_{lx} \), of the first link in the \( i^{th} \) chain are constants. For the 3dof tool-holder that moves in the \( xz \) plane, given the specified tool position \((T_x \ 0 \ T_z)\), and orientation \((0 \ T_\beta \ 0)\), plane as the vector of \( z-y-z \) Euler angles, the platform joint coordinates, the platform joints \( P_1 \) and \( P_3 \) of the 1\textsuperscript{st} and 3\textsuperscript{rd} chains can be obtained from the following set of equations:
\[
P_{lx} = T_x + r \cos T_\beta; \ i \in [1,3], \quad (A.12a)
\]
\[
P_{lz} = T_z + r \sin T_\beta; \ i \in [1,3]. \quad (A.12b)
\]

The platform joint coordinates in the 2\textsuperscript{nd} chain \( P_2 \) can be obtained from the following set of equations:
\[
P_{2x} = T_x + \sqrt{3}r \sin T_\beta. \quad (A.13a)
\]
\[
P_{2z} = T_z + \sqrt{3}r \cos T_\beta. \quad (A.13b)
\]

The coordinates of joints can be derived from the set of equations:
\[
(P_{ix} - P_{jx})^2 + (P_{ix} - P_{jz})^2 = (2r)^2; \ i \neq j \ i, j = [1..3]. \quad (A.14)
\]

The \( z \) coordinate of the actuators, \( C_{lz} \), of the 3-dof PKM are obtained as:
A.4 A 5-dof Serial Mechanism

The kinematics of the 5-dof serial mechanism, which is based on a PPPRR topology, can be derived as the transformation from the base to the tool [119]. The actuators are located one on top of the other. The first stage moves along the \( x \)-axis, the second stage, moves along the \( y \)-axis. The third stage, which is mounted on top of the second stage, moves along the \( z \)-axis, the fourth stage rotates about the \( x \)-axis, and the last stage rotates about the \( z \)-axis. The notation for the inverse-kinematic model is shown in Fig. A.4.

\[
C_{ix} = P_{ix} + \sqrt{L^2 - (P_{ix} - C_{ix})^2}. \tag{A.15}
\]

Given the tool position \((x_f \ y_f \ z_f)\) in the workpiece-holder frame, and a unit vector to define the orientation \((L_o, L_y, L_z)\), the objective is to find the machine generalized coordinates (drive command variables) to the stages: linear \((d_x, d_y, d_z)\), and rotary: \((\theta_A, \theta_C)\). The transformation matrix from the workpiece frame to the tool frame, which describes these five transformations, is formulated as:

\[
\begin{align*}
\begin{bmatrix}
\vec{T} \vec{R} \\

\end{bmatrix} & =
\begin{bmatrix}
W \vec{R} \vec{1} \vec{R} \vec{2} \vec{R} \\

\end{bmatrix} \\
\end{align*} \tag{A.16}
\]

The 1\(^{st}\) transformation is translation:

\[
\begin{equation}
W \vec{R} \vec{1} =
\begin{bmatrix}
1 & 0 & 0 & d_x \\
0 & 1 & 0 & d_y \\
0 & 0 & 1 & d_z \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \tag{A.17}
\end{equation}
\]

The 2\(^{nd}\) transformation is rotation about \( z \) by:
The 3\textsuperscript{rd} transformation is rotation about \( x \) by \( \theta_A \):

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta_A & -\sin \theta_A & 0 \\
0 & \sin \theta_A & \cos \theta_A & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} \text{(A.19)}

The transformation matrix from the base to the tool is:

\[
\begin{bmatrix}
\cos \theta_C & -\sin \theta_C & \sin \theta_A \sin \theta_C & d_x \\
\sin \theta_C & \cos \theta_C & \sin \theta_A \cos \theta_C & d_y \\
0 & \sin \theta_A & \cos \theta_A & d_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} \text{(A.21)}

which can be divided into rotational and translational components, where the translational component is:

\[
\begin{bmatrix}
x_f \\
y_f \\
z_f
\end{bmatrix} = \begin{bmatrix}
d_x \\
d_y \\
d_z
\end{bmatrix}.
\]  \hspace{1cm} \text{(A.21)}

The orientation component can be obtained from the rotational matrix, which is extracted from the transformation matrix, \( R_w^T \). The tool orientation in the tool frame is denoted as a unit vector along the \( z \)- axis:

\[
\begin{bmatrix}
L_x \\
L_y \\
L_z
\end{bmatrix} = \begin{bmatrix}
\cos \theta_C & -\cos \theta_A \cos \theta_C & \sin \theta_A \sin \theta_C \\
\sin \theta_C & \cos \theta_A \cos \theta_C & -\sin \theta_A \cos \theta_C \\
0 & \sin \theta_A & \cos \theta_A
\end{bmatrix} \begin{bmatrix}0 \\
0 \\
1
\end{bmatrix},
\]  \hspace{1cm} \text{(A.22)}

and the relation between the tool orientation and the actuators rotation angles is expressed as:

\[
\begin{bmatrix}
L_x \\
L_y \\
L_z
\end{bmatrix} = \begin{bmatrix}
\sin \theta_A \sin \theta_C \\
-\sin \theta_A \cos \theta_C \\
\cos \theta_A
\end{bmatrix}.
\]  \hspace{1cm} \text{(A.23)}

The drive commands for the rotating actuators (\( \theta_A \quad \theta_C \)), are obtained as:

\[
\theta_A = \cos^{-1}(L_z),
\]  \hspace{1cm} \text{(A.24)}

\[
\theta_C = \text{atan2} \left( \frac{L_x}{\sin \theta_A}, \frac{L_y}{\sin \theta_A} \right).
\]  \hspace{1cm} \text{(A.25)}
Appendix B - Matrix Structural Analysis Method

The following gives an overview on the procedure for evaluating the static stiffness of the compared PKMs’ stiffness, using the Matrix Structural Analysis method [108]:

i. The PKM’s links are modeled as beam elements, with each element consisting of two nodes. The elements allow for bending deformations in the two lateral directions with respect to the beam orientation as well as axial deformation along the beam. The stiffness matrix, $K^i$, of the $i^{th}$ element is represented with respect to its local frame. The matrix is assembled from four sub-matrixes:

$$
K^i = \begin{bmatrix}
K_{11}^i & K_{12}^i \\
K_{21}^i & K_{22}^i
\end{bmatrix},
$$  \hspace{1cm} (B.1)

where $K_{jk}^i$ ($j,k=1,2$) are the stiffness matrices of the first and second nodes of the $i^{th}$ element. The link stiffness matrix is transformed from the link local frame to the global frame, \{O\}, shown in Fig. B.1.

![Coordinate notation for the UofT based RRmMT static stiffness model.](image)

Figure B.1. Coordinate notation for the UofT based RRmMT static stiffness model.

ii. The transformed stiffness matrix of each mechanism chain, $K_T$, is assembled according to the boundary conditions on each link: Fig. B.2(a) - two beams with rigid connection;
Fig. B.2(b) - two beams connected by a revolute joint; Fig. B.2(c) two beams connected by a revolute joint when one of the beams is fixed to the base.

\[
K = \begin{bmatrix}
K_{11}^1 & K_{12}^1 & 0 \\
K_{21}^1 & K_{22}^1 + K_{11}^2 & K_{12}^2 \\
0 & K_{21}^2 & K_{22}^2
\end{bmatrix}; \quad K = \begin{bmatrix}
K_{11}^1 & 0 & 0 \\
K_{21}^1 & K_{22}^1 & 0 \\
0 & 0 & K_{11}^2 + K_{22}^2
\end{bmatrix}; \quad K = \begin{bmatrix}
K_{11}^1 & 0 & 0 \\
0 & K_{11}^2 & K_{12}^2 \\
0 & 0 & K_{21}^2 + K_{22}^2
\end{bmatrix}
\]

Figure B.2. Stiffness matrix assembly.

\( \text{iii.} \) Kinematic constraints for rigid bodies and revolute joints are obtained from the relation between the nodes in the two sides of the link. For example, the kinematic relations between two nodes, \( j \) and \( k \), that are connected by a revolute joint, shown in Fig. B.3 are:

\[
dx_j = dx_k \tag{B.2a}
\]

\[
\theta_j(x,z) = \theta_k(x,z) \tag{B.2b}
\]

and the kinematic relation is formulated as:

\[
\begin{bmatrix}
I_{3\times3} & 0 & I_{3\times3} & 0 \\
0 & \Omega_{3\times2} & 0 & \Omega_{3\times2}
\end{bmatrix}
\begin{bmatrix}
dx_{1\times3} \\
\theta_{1\times3}
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}, \tag{B.3}
\]

where, \( I \) is the eye matrix, and \( \Omega \) is defined as:

\[
\Omega = \begin{bmatrix}
m_x & m_y & m_z \\
k_x & k_y & k_z
\end{bmatrix}, \tag{B.4}
\]

which is constructed from two unit vectors, \( \bar{m} \) and \( \bar{k} \) that define the revolute joint.
iv. Linkage constraints are considered through matrices that describe the kinematic relations between the nodes on the two sides of the joint. These constraints are assembled into a matrix, $A_k$, according to the nodes numbers.

v. Evaluating the extremum of the total potential energy under the kinematic constraints using Lagrange multipliers leads to the following set of equations:

$$
\begin{bmatrix}
K_T & A_k^T \\
A_k & 0
\end{bmatrix}
\begin{bmatrix}
\Delta X \\
\lambda_M
\end{bmatrix}
=
\begin{bmatrix}
F \\
0
\end{bmatrix}
$$

where, $\lambda_M$ is the Lagrange multipliers vector, $F$ is the force vector applied on the nodes, and $\Delta X$ is the vector of the displacements of the nodes dof. The displacements of all the chain’s nodes can be determined from Eq. (B.5).

vi. The stiffness matrix of the mechanisms is constructed in a similar way to Step (ii), using the three chains stiffness matrices obtained in Step (v). The three PKMs compared in this paper are assembled according to the model shown in Fig. B.4, where the links are represented by Nodes 1 to 3. The platform elements are modelled as rigid connections, and the associated Nodes are 4 to 7. Subsequently, the static stiffness of the mechanism is evaluated at Node 7, which is the platform center point.
Figure B.4. Schematics of the overall stiffness model of three chains based PKMs.