Topology Control in an Indoor Wireless Sensor Network using Realistic Propagation Models

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Graduate Department of Electrical and Computer Engineering
University of Toronto

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Abstract

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This thesis considers the problem of designing an energy-efficient communication backbone for a wireless sensor network deployed in an indoor environment. We propose and implement a Topology Control scheme to determine optimal sensor node transmission power levels that maintain a desired level of network connectivity. Power optimized singly connected network is constructed using a Minimum Spanning Tree based formulation. To achieve fault-tolerance, we employ a Matroid Intersection based approach to construct a Minimum Cost Rooted $k$-Outconnected Spanning Subgraph, basis for the computation of a Minimum Power $k$-Connected Spanning Subgraph, $k > 1$. Realistic wireless channel assumptions are made to construct the optimized network topologies. Field measurements are performed in an indoor office setup to characterize the channel parameters and hence evaluate the accuracy of various path loss models suitable for such propagation environments. Network simulations validate the necessity of exercising Topology Control to achieve network longevity.
Dedication

To my parents,
Anupama and Arun Goel
and to my sister,
Mansi Goel
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AN  Application Node
APX  Approximable
BS  Base Station
CLT  Central Limit Theorem
CTS  Clear-to-Send
E2E  End-to-End
IID  Independent and Identically Distributed
LOS  Line-of-Sight
LQI  Link Quality Indication
MAC  Media Access Control
MCkOS  Minimum Cost Rooted $k$-Outconnected Spanning Subgraph
MPkCS  Minimum Power $k$-Connected Spanning Subgraph
MPkIS  Minimum Power Rooted $k$-Inconnected Spanning Subgraph
MST  Minimum Spanning Tree
NLOS  Non Line-of-Sight
PDF  Probability Density Function
PER  Packet Error Rate
PPP  Poisson Point Process
<table>
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<tr>
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<th>Description</th>
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<tr>
<td>PRR</td>
<td>Packet Reception Rate</td>
</tr>
<tr>
<td>RA</td>
<td>Range Assignment</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>RSSI</td>
<td>Received Signal Strength Indication</td>
</tr>
<tr>
<td>RTS</td>
<td>Request-to-Send</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference plus Noise Ratio</td>
</tr>
<tr>
<td>WLAN</td>
<td>Wireless Local Area Network</td>
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Chapter 1

Introduction

1.1 Motivation

Inter-operability with existing technologies and the potential of integration with the technologies of the future such as Ambient Intelligence and Internet-of-Things have lead to an ever increasing popularity of wireless sensor networks (WSNs). A sensor network is comprised of a number of small, inch-sized sensing devices called sensor nodes that consist of multiple modules to perform sensing, data aggregation, communication as well as energy management tasks [1]. Some of the applications of this class of ad-hoc networks include environmental/habitat/structural health monitoring, assisting disaster relief operations, public safety and security and detection of harmful chemical substances in a commercial manufacturing unit or a laboratory setup. Given the magnitude of its applications, several key issues related to optimal node deployment, localization, data aggregation/fusion, network backbone construction, routing, MAC layer scheduling, fault-tolerance, scalability and security need to be addressed.

Scarce energy and capacity resources are the fundamental limitations of sensor networks. Communication tasks are the major source of energy consumption in such networks. A typical sensor node can have up to 10-fold increase in battery discharge rate when its radio transceiver is performing packet transmissions at the maximum possible power level, as compared to the case when the transceiver is inactive. Network longevity is especially important in situations when a sensor network is deployed in a remote location where replacement of batteries is either impractical or undesired due to cost constraints. The proposition of energy-efficient protocols at all layers of the communication protocol stack therefore becomes essential. In [2], for example, we propose a time series prediction model with the aim to reduce the number of communications between the sensing nodes and the base station. The framework employs support vector machines and an adaptive
noise assisted data analysis method as a preprocessing step to perform one step ahead prediction of the sensed phenomenon. Furthermore, a connected communication graph is a fundamental requirement for all network operations. Communication between an arbitrary pair of sensor nodes in the network is only possible if the underlying network backbone is fully connected.

In this work, we therefore consider the problem of constructing an energy-efficient, fully connected network backbone. This is an important network design problem as it aims to enhance the lifetime of the network by discarding sub-optimal communication links from the network topology. We further extend the problem to construct energy-efficient, fault-tolerant topologies with the aim to achieve load balancing, network reliability and hence a further increase in network lifetime. Also considering the fact that network protocols that make simplistic assumptions about the physical layer can suffer performance degradation when operated under actual environmental conditions, we consider realistic propagation models to better depict a wireless channel in an indoor environment while constructing the optimized network topologies.

1.2 Contributions

- Characterization of path loss models in an indoor environment at 2.45 GHz: We perform channel measurements to capture large-scale signal variations in an indoor office setup. Based on the measurement data, four channel models, namely the Unit Disk model, Log-normal Shadowing model, WINNER II Stochastic Channel model and the Volcano Indoor Multi-Wall model are investigated. The evaluated path loss models range from purely deterministic to semi-empirical to purely site-specific. The propagation models are first parameterized and calibrated using part of the field measurement data and then evaluated both qualitatively as well as quantitatively against the actual path loss values. Such a performance evaluation, based on real channel measurements, of several channel models appropriate for indoor propagation environments thus far had been limited. Furthermore, this empirical study facilitates a fair comparison of the large-scale signal variations predicted by different path loss models by parameterizing the models based on actual channel measurements. The characterized channel models are then employed for network $k$-Connectivity analysis based on Monte Carlo simulations to observe the dependence of network backbone construction on different channel models. The network connectivity analysis explains whether the different degrees of accuracy obtained in performing path loss predictions in an indoor environment using dif-
ferent path loss models (characterized based on the channel measurements in the indoor environment) have significant impact on the network backbone construction under varying physical layer conditions or not.

• **Network $k$-Connectivity analysis:** We analyze the impact of network parameters such as node density and transmission power levels on the average behaviour of a sensor network operated under different physical layer conditions over several spatial realizations. The Monte Carlo method is used to estimate the probability of network $k$-Connectivity for a range of transmission power levels. Confidence Intervals are employed to estimate the average minimum transmission power level of the sensor nodes deployed in an indoor environment that ensure, with high probability, the construction of a $k$-Connected network topology, $k \in \{1, 2, 3\}$. This statistical framework has a direct implication on the pre-deployment phases, such as network planning and dimensioning, of a sensor network. This analysis shows the dependence of network backbone construction satisfying different connectivity levels ($k$) on a number of physical layer models making large-scale signal variations in an indoor environment.

• **Proposition of a Topology Control scheme to construct power optimized singly connected and fault-tolerant topologies:** We consider the Topology Control problem as a constrained optimization problem with the transmission power of the sensor nodes as the optimization objective and preservation of network connectivity as the problem constraint. Specifically, we construct a $k$-Connected network topology that provides a $k + 1$ approximate solution to the optimal power assignment problem for the case when $k \in \{1, 2, 3\}$, which is otherwise APX Hard. Constructed topologies support symmetric communication links and can tolerate at least $k - 1$ node failures (in the worst case) before the network disconnects and hence are fault-tolerant when $k \in \{2, 3\}$. We claim the novelty in the employment of a simpler Matroid Intersection based formulation to construct a Minimum Cost Rooted $k$-Outconnected Spanning subgraph, a basic step in the construction of power optimized fault-tolerant topologies, hence avoiding a complicated reduction to submodular flows, traditionally solved using an integer linear programming formulation. The proposed scheme is evaluated using network simulations performed in a discrete event simulator.

Part of the contributions made in this thesis will appear in the following publication:

• Gagan Goel, Scott H. Melvin, Yves Lostanlen and Dimitrios Hatzinakos, “Connectivity Analysis of Indoor Wireless Sensor Networks using Realistic Propagation
Chapter 1. Introduction


1.3 Organization

This thesis is structured as follows:

In Chapter 2, we first review the related work on realistic radio propagation modelling, especially highlighting recent work in the context of indoor channel modelling for WSNs. We then familiarize the reader with the problem of Topology Control and cite some contributions that have been made in this line of research. Notations and definitions of the elements of graph theory that we use in the subsequent Chapters are then introduced. Lastly, we discuss the mathematical framework adopted for statistical estimation of network parameters.

Chapter 3 describes in detail our methodology used to carry out channel measurements in an indoor office setup, which also includes a description of the radio equipment used and the measurement site. Formulations of the four radio propagation models that we investigate are then covered. This is followed by an empirical estimation of the channel parameters and performance evaluation of the models.

Chapter 4 starts with the description of a Max-Flow Min-Cut based algorithm that we implement to compute the vertex connectivity of a network. Next, we construct a power optimized, singly connected network topology using a Minimum Spanning Tree formulation. The last part of the Chapter describes the construction of power optimized fault-tolerant topologies through a series of algorithms that lead to the generation of directed/undirected spanning subgraphs satisfying certain connectivity requirements.

In the first half of Chapter 5, we analyze the average behaviour of a sensor network deployed in an indoor environment under different path loss models. We study the impact of node density and the transmission power of the sensor nodes following a realistic spatial distribution on various levels of network connectivity by means of the mathematical framework introduced in Chapter 2. This Chapter then describes our Topology Control scheme proposed on the basis of various spanning subgraphs constructed in Chapter 4. A procedure to evaluate the proposed scheme using network simulations follows. This includes a description of the performance metrics as well as the routing schemes considered for performance evaluation.

In Chapter 6, we highlight the main results obtained in this work and discuss the future research directions.
Chapter 2

Background

2.1 Realistic Radio Propagation Modelling

A radio propagation model attempts to quantify the behaviour of an actual wireless channel. The propagation of electromagnetic waves (EM) in wireless networks is influenced by the characteristics of the environment in which they operate. In complex indoor and dense urban environments, large-scale and small-scale radio frequency (RF) signal variations can occur as a result of [3]:

- Multi-path reflections due to the presence of metallic structures along the propagation path. This can result in signal fading at the receiving terminal. Fading is generally modelled with a Rayleigh distribution if there is no dominating line-of-sight multi-path component, otherwise it is modelled with a Ricean distribution.

- Shadowing due to signal blockage caused by obstacles in the propagation path.

- Diffraction caused by bending of an EM wave when it strikes edges of objects with dimensions larger than the signal wavelength.

Therefore, the existence of a wireless communication link between a transmitter-receiver pair is a random phenomenon. Realistic radio propagation models, that are capable of capturing such channel randomness at the physical layer, can provide accurate evaluation of protocols when performing network simulations.

2.1.1 Motivation

Experiments were conducted in an open parking lot in [4] to demonstrate the impact of radio irregularity on several sensor network protocols. This study reports anisotropic path
losses and variance in transmission power of nodes as the key causes of radio irregularity. Experimental validation is performed by analyzing Received Signal Strength Indication (RSSI) values, packet reception ratios and the communication range of MICA2 motes equipped with Chipcon CC1000 radio transceivers at different remaining battery levels. It is shown that radio propagation shows continuous variation with incremental changes in direction, i.e. the propagation media is anisotropic. The second factor resulting in radio irregularity, variance in transmission powers, is primarily attributed to the heterogeneous properties of radio hardware due to manufacturing anomalies, different rates of battery power depletion in different environments, different work loads, etc. Incorporation of such realistic channel conditions in network simulations shows that routing and Topology Control protocols suffer significant performance degradation in the presence of radio irregularity. Specifically, location based routing protocols, such as Geographic Forwarding, suffer route discovery failures. Similarly, location based Topology Control protocols, such as Geographic Adaptive Fidelity, results in decreasing connectivity levels with an increase in the irregularity metric.

Spatio-temporal properties of various radio metrics such as RSSI, Link Quality Indication (LQI) and Packet Error Rate (PER) were experimentally analyzed in [5]. Channel measurements were performed in a harsh indoor factory setting (consisting of a number of stationary and moving obstacles and machinery) using IEEE 802.15.4-compliant Chipcon CC2420 radios. It was observed that RF signals undergo signal blockage, multi-path fading due to stationary as well as moving obstacles and also radio interference from other devices and sources in the machine shop. Such effects were dominant in the cluttered regions of the factory as the RSSI values showed significant drops when the receiver was placed in close proximity with obstacles. This study provides evidence of the dependence of the wireless link quality on the complicated layout of the factory floor plan.

Dependence of multi-path fading on the characteristics of the propagation environment is shown to exist in [6]. Again, radio hardware specifically used for low-rate wireless personal area networks was employed to conduct experiments in an indoor environment. It was observed that radio channel randomness, primarily arising due to the multi-path fading phenomenon, shows spatial variation and depends on the topology of the environment where the transmitter-receiver pair is placed. To model this effect, a fading function is proposed by the authors that maps cartesian coordinates of the transceiver to a fading coefficient which is then employed for received power computations at different locations within the propagation environment.
2.1.2 Channel Modelling for Indoor Wireless Sensor Networks

Empirical studies have been performed in [7]–[9] to characterize the radio propagation conditions and propose channel models for WSNs operating in indoor environments. The authors of [7] performed channel measurements in an indoor office building and determined the small scale variations of the RF signals. It was experimentally validated that the fading characteristics conform with a Rician distribution. Characterization and performance evaluation of path loss models performed in Chapter 3 are complementary to this work in the sense that we focus solely on large scale RF signal variations which facilitates analysis of such sensor network properties as connectivity, one of the fundamental requirements for all network operations.

A radio coverage tool was devised in [8]. It was concluded that the RF signal propagation is significantly affected by the presence of walls in an indoor office building. Propagation models that take into consideration signal attenuation due to walls conform well with actual wireless channel observations. However, the coverage tool proposed in [8] is based on RSSI values measured between sensor nodes deployed in the indoor environment, which are specific to a particular sensor radio technology and are only an approximation to actual received power levels. Also, the parameterization, based on empirical data, of the Log-normal model used for the performance evaluation was not performed.

Packet Reception Rate (PRR) was used as a metric for indoor WSN channel modelling in [9]. Transmitter and receiver locations are mapped to PRR values taking into account both the non-isotropic radiation patterns of the antennas encountered in practice as well as site-specific information about the propagation environment to model varying levels of signal attenuations through different building structures. However, only a deterministic path loss model was considered for a comparative analysis against the proposed scheme.

Figure 2.1 illustrates site-specific propagation predictions performed using the Volcano Indoor Multi-Wall model [10] in an indoor office building. Radio propagation mapping is shown for two transmitters operating at 2.45 GHz. It can be observed that signal attenuation is clearly non-isotropic and is determined by the partitions encountered in the propagation path along different directions. In Chapter 3, we validate the accuracy of Volcano Indoor Multi-Wall model in performing propagation predictions in an indoor office setup and further employ this model to perform realistic path loss computations between sensor nodes deployed in an indoor environment.
2.1.3 Propagation Models for Asymptotic Connectivity Analysis

Radio propagation models are generally referred to as connection models in the context of stochastic geometry and random graph theory. Theoretical advances have been made recently to analyze the coverage, capacity and connectivity properties of wireless networks using different connection models ranging from purely deterministic to purely random models, and the hybrids thereof. In [11], the node isolation probability was quantified in the presence of channel randomness, which leads to closed form expressions for network connectivity. It was observed that an increase in the shadow fading standard deviation of the Log-normal Shadowing connection model reduces the node isolation probability and therefore allows an increase in the level of network connectivity. Furthermore, the negative impact of Rayleigh and Rician fading channels on connectivity were conformed as the fading channels resulted in an increase in the node isolation probability (hence a decrease in the network connectivity).

The critical node density, i.e. the lower bound on the node density that ensures asymptotic network connectivity (or connectivity with high probability) for a wireless ad hoc network with Log-normal Shadowing connection model was derived in [12]. It was
shown that the critical node density reduces with an increase in the standard deviation of the connection model. The results of [11] and [12] are therefore in conformity with respect to the improvement of network connectivity in a shadow fading environment.

Characterization of the minimum network parameters for asymptotic connectivity for other connection models, such as $k$ for a $k$-Nearest Neighbor model and critical node density for Random Connection and Signal-to-Interference plus Noise Ratio (SINR) model has been highlighted in [13]. In a $k$-Nearest Neighbor model, a given node connects only to its $k$ nearest neighbors. This connection model is most suitable for a dense network. A random connection model is a generalization of the Unit Disk and Log-normal Shadowing connection models. According to this model, a connection between a pair of nodes is a random function $g(x)$ of the distance $x$ between them. $g(x)$ satisfies the properties of non-increasing monotonicity and integral boundedness [14]. In interference rich scenarios where multiple network nodes transmit at the same center frequency, the SINR model is an ideal choice as it establishes a connection between a pair of nodes only when their SINR values exceed a certain threshold required for reliable communication.

2.2 Topology Control in Wireless Sensor Networks

2.2.1 Definition

Topology Control, as defined in [15], is the technique of coordinating nodes’ decisions regarding their transmission power levels in order to generate a network backbone which aims to maintain a certain connectivity level $k$ of the network while simultaneously reducing the overall network energy consumption. Simply put, given a maximum power communication graph $G_{\text{max}} = (V, E_{\text{max}})$ (A communication graph or simply a graph $G$ is an ordered pair of a set of vertices $V$ and a set of edges $E$, i.e. $G = (V, E)$. A graph is analogous to a network topology which consists of a set of nodes and the associated set of communication links between the nodes), in which the transmitting power of the nodes is set to the maximum allowable level, and whose network connectivity $K(G_{\text{max}}) \geq k$ (a more formal definition of network connectivity is given in Section 4.2), the aim of Topology Control is to generate a spanning subgraph, $G = (V, E)$ of the maximum power communication graph, with nodes now transmitting at reduced power levels, such that $E \subseteq E_{\text{max}}$ and $K(G) = k$, i.e. $G$ contains as few edges as possible while still ensuring the desired connectivity level $k$. In other words, $G$ is a sparse representation of the maximum power communication graph that, at the same time, preserves the network connectivity.
Chapter 2. Background

A recent survey [16] however, provides a broader definition of Topology Control. It refers to Topology Control as any technique that uses a controlled network parameter to reduce energy consumption and maintain the desired network properties. The controlled network parameters include the transmission power, the operating power mode of the sensor nodes (such as sleep, idle or active) and the node role (such as cluster head or sensing node in clustering based protocols).

2.2.2 Motivation

Single hop, long distance communication of a sensed phenomenon between a source node and a destination induces higher levels of interference in the sensor network and also requires sensor nodes to perform transmissions at higher power levels. Multi hop communication, one of the distinguishing features of wireless ad hoc and sensor networks, therefore becomes necessary to reduce this interference and conserve energy. Communication using short, interference-minimal multiple hops between a source-destination pair is achieved by exercising Topology Control such that the sensor nodes communicate only to the neighboring nodes at reduced transmission power levels. With energy-efficient construction of a network backbone and hence enhancements in the network lifetime as the primary design goal, Topology Control takes a network-wide perspective, unlike power control. Power control provides a channel-wide perspective and aims to adjust the node transmission power levels on a per-packet basis. Since intra-network interference, resulting due to concurrent transmissions, is directly proportional to the power used to transmit the packets, Topology Control further increases network spatial reuse and hence the overall network capacity.

2.2.3 Related Works

Adjustment of the transmission power of nodes is formulated as a constrained optimization problem in [17], with the generation of a connected and a bi-connected network as the two constraints and the maximum power used by the nodes as the optimization objective. It considers as Topology Control the construction of a communication graph that can be directly or indirectly used by a routing mechanism. Two centralized algorithms are proposed in [17] for static networks along with two distributed heuristics that adaptively adjust node transmit powers with changing network dynamics in a mobile network. The proposed framework considers a Unit Disk model for the propagation characteristics of the environment.

The objective of Topology Control in [18] is to maximize the topological network
lifetime in a two-tiered WSN consisting of sensing nodes and application nodes (AN or cluster heads) in the first tier and ANs and a base station (BS) in the second tier. Network lifetime enhancements are achieved by optimally determining the BS location in the network by means of computational geometry. To further reduce the overall network energy consumption, inter-AN relaying is implemented. A free space path loss model defines the distance dominated power consumption model for the sensor nodes and it is assumed that the proposed scheme can be extended to include multi-path reflections, interference as well as fading and shadowing effects.

The possibility of a WSN operating with heterogenous wireless devices is considered in [19] to devise a Topology Control protocol in the presence of asymmetric links with guaranteed network connectivity. The proposed algorithm first builds a strongly connected maximum power topology locally for each node, then the minimum power vicinity topology is derived based on a shortest path algorithm. The protocol offers good scalability since information exchange between the sensor nodes is limited to a local neighborhood. Due to the lack of global information, however, the algorithm leads to sub-optimal minimum transmission power levels after execution. Only a Unit Disk path loss model is considered for path loss computations and it is assumed that the protocol works correctly with any propagation model.

Topology Control is considered as an iterative process in [20] in which the network switches between the topology construction and topology maintenance phase throughout the network lifetime. Topology construction is performed locally in a distributed manner and considers for a given node, the selection of an appropriate neighboring node that has a high reachability probability to the sink. The reachability probability metric was first used in [21] to exploit the lossy nature of wireless links in a sensor network for the design of a Topology Control protocol. The aim of Topology Control in [22] is to construct a Connected Dominating Set (CDS) by selecting a set of sensor nodes in the network as active nodes that form a singly connected CDS. The selection criterion for the nodes to form CDS is based on the remaining energy of the nodes. We consider as topology control, however, the determination of optimized transmission power levels satisfying different network connectivity requirements. A distributed Topology Control scheme based on Yao Graphs and Gabriel Graphs is proposed in [23] to generate an energy-efficient communication graph (energy-efficiency is measured using distance and power stretch factors) with low interference (measured using average and logical node degrees). The constructed topology is, however, singly connected. In this thesis, along with power-optimized singly connected network topology, we achieve fault-tolerance and construct a communication backbone that is resilient to node failures. A Topology Control scheme
based on a hierarchical network structure is proposed in [24]. The authors consider
determination of optimal cluster head and sink locations in the sensor network to achieve
enhancements in network lifetime. However, fault-tolerant design of the network topology
is not considered in [24].

2.3 Graph Theoretic Definitions

The following terminologies from [15], [25] and [26] are used in Chapter 4 for the con-
struction of power-optimized singly connected and fault-tolerant topologies:

1. Directed and undirected graph: If the edges in $E$ consist of unordered vertex-pairs,
   the graph is an undirected graph. A directed graph consists of ordered vertex-pairs
   $e = (u, v) \in E$ such that $u$ and $v$ respectively represent the tail and head of $e$.

2. Vertex Degree: For an undirected graph $G = (V, E)$, the degree of a vertex $v \in V$
   refers to the number of edges in $E$ incident to $v$. If $G$ is directed, we define the
   out-degree of $v$ as the number of edges in $E$ with $v$ as the tail and similarly the
   in-degree as the number of edges in $E$ with $v$ as the head.

3. Vertex and Edge-Disjoint Paths: Two directed or undirected paths between a
   source-destination, $(s - t)$ pair are called vertex-disjoint if they do not share any
   vertices except $s$ and $t$. Similarly, a pair of paths are edge-disjoint if they do not
   share any edges.

4. Spanning Subgraph: A graph $G' = (V', E')$ is a spanning subgraph of a di-
   rected/undirected graph $G = (V, E)$ if $V' = V$ and $E' \subseteq E$.

5. Minimum Spanning Tree: Given an edge-weighted connected graph $G = (V, E)$ (an
   edge-weighted graph is a graph in which all edges are assigned non-negative real
   numbers), a Minimum Spanning Tree is a minimally connected spanning subgraph
   $G' = (V, E')$ such that $|E'| = |V| - 1$ and the sum of the weights on the edges in
   $E'$ is minimum.

6. Minimum Cost Rooted $k$-Outconnected Spanning Subgraph (MC$k$OS): Given an
   edge-weighted directed graph $G = (V, E)$, MC$k$OS is a spanning subgraph $G' =
   (V, E')$ of $G$ such that for a certain $r_0 \in V$ as the root vertex, there are at least
   $k$ vertex-disjoint directed $(r_0 - t)$ paths in $E' \forall t \in V - \{r_0\}$ and the sum of the
   weights of the edges in $E'$ is minimum.
7. Power of a graph $P(G)$: Given an edge-weighted undirected graph $G = (V, E)$, the power of the graph $G$ is defined as:

$$P(G) = \sum_{v \in V} p(v)$$

where $p(v)$ is the power of a vertex $v$ and is defined as the maximum cost of an edge $e \in E$ incident to $v$.

8. Directed Minimum Power Rooted $k$-Inconnected Spanning Subgraph (Directed MP$k$IS): Given an edge-weighted directed graph $G = (V, E)$, Directed MP$k$IS is a spanning subgraph $G' = (V, E')$ of $G$ such that for a certain $r_0 \in V$ as the root vertex, there are at least $k$ vertex-disjoint directed $(t - r_0)$ paths in $E' \forall t \in V - \{r_0\}$ and the power of the graph $G'$, as defined above, is minimum. An undirected MP$k$IS is analogously defined for an undirected, input, edge-weighted graph $G$.

9. Restricted MP$k$IS: It is an undirected MP$k$IS such that the degree of the root vertex $r_0$ is equal/restricted to $k$.

10. Minimum Power $k$-Connected Spanning Subgraph (MP$k$CS): Given an edge-weighted undirected graph $G = (V, E)$, MP$k$CS is a spanning subgraph $G' = (V, E')$ such that there are at least $k$ vertex-disjoint $(u, v)$ paths in $E' \forall u, v \in V$ and the power of the graph $G'$ is minimum.

11. Bi-direction: A Bi-direction of an undirected graph $G = (V, E)$ is a directed graph obtained by replacing every edge $e = (u, v) \in E$ by two oppositely directed edges $(u, v)$ and $(v, u)$ but both retaining the same edge weight as $e$. Figure 2.2 shows an example of an undirected graph and its Bi-direction.

12. Underlying graph: An Underlying graph of a directed graph $G = (V, E)$ is an undirected graph obtained by replacing every directed edge $e \in E$ by an undirected edge of the same weight as $e$ and then keeping any one of the two parallel edges between a pair of vertices. Figure 2.2 also serves as an example that illustrates a directed graph (Fig. 2.2b) and its corresponding Underlying graph (Fig. 2.2a).

The following Matroid theoretic definitions from [27] are required for the construction of MC$k$OS in Section 4.4.1:

1. A Matroid $M = (E, I)$ is a pair of a finite set $E$, also called the ground set and a family of subsets of $E$, $I$, also called a family of independent sets. $I$ satisfies the following properties:
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Figure 2.2: An Undirected Graph and its Corresponding Bi-direction

- $I$ is non-empty, i.e. $I \neq \emptyset$. Also, the empty set is independent, i.e. $\emptyset \in I$
- Hereditary property: Every subset of every independent set is independent, i.e. if $I_1 \in I$ and $I_2 \subset I_1 \subset E$, then $I_2 \in I$
- Augmentation property: If $I_1, I_2 \in I$ and $|I_1| > |I_2|$, then there is an element $i$ in $I_1 - I_2$ such that $I_2 \cup \{i\} \in I$

2. Matroid Intersection: Given two Matroids $M_1 = (E, I_1)$ and $M_2 = (E, I_2)$ on the same ground set $E$, the Matroid Intersection is a set of independent sets defined as:

$$M_1 \cap M_2 = (E, I_1 \cap I_2) \quad (2.2)$$

3. Independence Oracle: Given a Matroid $M = (E, I)$, the Independence Oracle is a procedure that for a given $I' \subseteq E$, checks whether $I' \in I$.

4. A bi-set $X = (X_O, X_I)$ is a pair of subsets $X_O, X_I$ of a ground set of vertices $V$ of a graph $G = (V, E)$ such that $\phi \subseteq X_I \subseteq X_O \subseteq V$. $X_O$ and $X_I$ are respectively called the outer and inner members of $X$. 
5. A directed edge \( e = (u, v) \) is induced by a bi-set \( X = (X_O, X_I) \) if the target vertex \( v \) of \( e \) is in \( X_I \) and the source vertex \( u \) of \( e \) is in \( X_O \).

6. For a directed graph \( D = (V, A) \), \( I(X) = I_D(X) = I_A(X) \), refers to the set of edges induced by \( X \) and \( i_D(X) = i_A(X) = |I_D(X)| \), where \( |X| \) denotes the cardinality of the set \( X \).

7. A directed edge \( e = (u, v) \) enters a bi-set \( X = (X_O, X_I) \) if \( e \) enters both \( X_O \) and \( X_I \).

8. For a directed graph \( D = (V, A) \), \( g(X) = g_D(X) = g_A(X) \) refers to the number of edges entering \( X \).

9. \( g \)-boundedness: For a given root vertex \( r_0 \) and an arbitrary destination vertex \( t \in V - \{r_0\} \) pair, a set of edge-disjoint \((r_0, t)\) paths is said to be \( g \)-bounded if every vertex \( v \in V - \{r_0,t\} \) is used by atmost \( g(v) \) of these paths, where \( g \) is a function such that \( g : V \rightarrow \mathbb{Z}_+ \).

10. \( \lambda_g(r_0, t; D) \) is defined as the maximum number of \( g \)-bounded \((r_0, t)\) paths in a directed graph \( D = (V, A) \).

11. A directed graph \( D = (V, A) \) is called rooted \((k, g)\)-Connected from a root vertex \( r_0 \) to \( t \) if \( \lambda_g(r_0, t; D) \geq k \) for every \( t \in V - \{r_0\} \).

12. A directed graph \( D = (V, A) \) is called \((k, g)\)-foliage if \( D \) is minimally rooted \((k, g)\)-Connected, i.e. deletion of any edge from \( D \) results in it being no longer rooted \((k, g)\)-Connected.

13. A Matroid \( M = (E, I) \) is called a Free Matroid if every subset of the ground set is independent, i.e. \( \forall I' \subseteq E, I' \in I \).

14. Direct Sum: Given two Matroids \( M_1 = (E_1, I_1) \) and \( M_2 = (E_2, I_2) \) on two disjoint ground sets \( E_1 \) and \( E_2 \) (two sets are called disjoint if they do not contain any common elements), the Direct Sum of \( M_1 \) and \( M_2 \), denoted by \( M_1 \oplus M_2 \), is a Matroid whose ground set is the union of \( E_1 \) and \( E_2 \) and whose set of independent sets is also the union of the independent sets in \( I_1 \) and \( I_2 \).

2.4 Mathematical Preliminaries

A Poisson Point Process (PPP) [13] can realistically model the node distribution in a given deployment region. Since the minimum transmission power levels required for
the sensor nodes to achieve network connectivity are functions of the underlying network model (and also of the propagation conditions between the nodes), for several realizations of the Poisson distribution of nodes the minimum power levels would in fact be random variables with an unknown probability density function (PDF). We therefore rely on Statistical Estimation of the average minimum transmission power levels of the nodes and on the Monte Carlo Method for the estimation of the probability of network connectivity for a pre-specified transmission power level. This framework is then employed in Section 5.2 to assess the average behaviour of a sensor network in terms of different connectivity levels for a number of path loss models suitable for an indoor network operation.

2.4.1 Confidence Intervals for Average Estimation

Let $P$ be a random variable representing the minimum transmission power level of nodes and $E\{P\} = \mu$ be its unknown average that we intend to estimate. Let $P_1, P_2, \ldots, P_n$ be a sequence of independent measurements of $P$ obtained by performing $n$ realizations of the Poisson Point Process, having the same PDF as $P$. The sample mean of this sequence is $M_n = \frac{1}{n} \sum_{i=1}^{n} P_i$. The sample variance can be calculated to measure the accuracy of $M_n$ in estimating $\mu$:

$$V_n^2 = \frac{1}{n-1} \sum_{i=1}^{n} (P_i - M_n)^2 \quad (2.3)$$

If $V_n^2$ is small, then we have a high degree of confidence in $M_n$ as a good estimate of $\mu$. However, if $V_n^2$ is high, then there is uncertainty in the level of accuracy that $M_n$ provides in the estimation of $\mu$. The notion of Confidence Intervals [28] then comes into practice. Instead of a single value $M_n$, it specifies an interval of values, centered around $M_n$, that contains the desired estimate with high probability. The probability with which this interval contains $\mu$ is specified upfront in terms of a Confidence Level, $1 - \alpha$. Specifically,

$$P[M_n - \epsilon \leq \mu \leq M_n + \epsilon] = 1 - \alpha \quad (2.4)$$

where $\epsilon$ is a positive constant that depends on the sample size $n$, variance of the random variable $P$, and on the confidence level $1 - \alpha$. Determination of this interval depends on the PDF of $P$, therefore, we consider the following three cases: (1) $P$ has a Gaussian distribution with unknown mean and known variance, (2) $P$ has a Gaussian distribution with unknown mean and unknown variance, and (3) $P$ is not Gaussian and has unknown mean and unknown variance.
P is Gaussian, unknown mean, known variance

Let $S_n$ be the sum of $n$ Independent and Identically Distributed (IID) random variables $P_i$’s, with mean, $E\{P_i\} = \mu$ and variance, $Var(P_i) = \sigma^2$. Then, $Var(S_n) = Var(P_1) + Var(P_2) + \ldots + Var(P_n) = n\sigma^2$ and $E\{S_n\} = E\{P_1\} + E\{P_2\} + \ldots + E\{P_n\} = n\mu$. Therefore, $M_n$ itself is a Gaussian random variable with variance, $Var(M_n) = Var(S_n) = \frac{n\sigma^2}{n}$ and mean, $E\{M_n\} = E\{S_n\} = \mu$.

We know that the $Q$ function represents the probability of the tail of the PDF of a standard normal random variable. Therefore

\[ Q(z) = P \left[ \frac{M_n - \mu}{\sigma\sqrt{n}} > z \right], \quad \text{or} \]

\[ 1 - 2Q(z) = P \left[ -z \leq \frac{M_n - \mu}{\sigma\sqrt{n}} \leq z \right] \]

\[ = P \left[ M_n - \frac{z\sigma}{\sqrt{n}} \leq \mu \leq M_n + \frac{z\sigma}{\sqrt{n}} \right] \quad \text{(2.7)} \]

Defining $z_{\alpha/2}$ such that $\alpha = 2Q(z_{\alpha/2})$. Eq. 2.7 can then be represented in terms of the confidence level $\alpha$:

\[ 1 - \alpha = P \left[ M_n - \frac{z_{\alpha/2}\sigma}{\sqrt{n}} \leq \mu \leq M_n + \frac{z_{\alpha/2}\sigma}{\sqrt{n}} \right] \quad \text{(2.8)} \]

Finally, the $(1 - \alpha) \times 100\%$ confidence interval of $\mu$ is

\[ \left[ M_n - \frac{z_{\alpha/2}\sigma}{\sqrt{n}}, M_n + \frac{z_{\alpha/2}\sigma}{\sqrt{n}} \right] \quad \text{(2.9)} \]

As we can see, the confidence interval depends on the sample mean $M_n$, variance of the sample sequence $\sigma$, user-specified confidence level $1 - \alpha$, and the sample size $n$. Therefore, the higher the value of $n$, the narrower the confidence interval, and hence the higher the accuracy in the estimation of $\mu$. $z_{\alpha/2}$ can easily be computed using the MATLAB function \texttt{erf()}. For example, for a 95% confidence level, i.e. $\alpha = 0.05$, $z_{0.02}$ is 1.96

P is Gaussian, unknown mean, unknown variance

Suppose the $P_i$’s are $n$ i.i.d. random variables with unknown mean $\mu$ and unknown variance $\sigma^2$. The confidence interval for this case is found by replacing $\sigma$ in Eq. 2.9 with the sample standard deviation $V_n$ obtained in Eq. 2.3:
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\[
\left[ M_n - \frac{z V_n}{\sqrt{n}}, M_n + \frac{z V_n}{\sqrt{n}} \right] \tag{2.10}
\]

Now, the confidence level for this interval is:

\[
P \left[ M_n - \frac{z V_n}{\sqrt{n}} \leq \mu \leq M_n + \frac{z V_n}{\sqrt{n}} \right] = P \left[ -z \leq \frac{M_n - \mu}{V_n/\sqrt{n}} \leq z \right] \tag{2.11}
\]

Since \( V_n \) is not the actual standard deviation of \( P \), the random variable \( \frac{M_n - \mu}{V_n/\sqrt{n}} \) is no longer a standard normal random variable. It actually follows the Student’s t-distribution with \( n - 1 \) degrees of freedom [28]. The probability in Eq. 2.11 is given as:

\[
P \left[ M_n - \frac{z V_n}{\sqrt{n}} \leq \mu \leq M_n + \frac{z V_n}{\sqrt{n}} \right] = 1 - 2F_{n-1}(-z) \tag{2.12}
\]

where \( F_{n-1}(z) \) is the cumulative distribution function of a random variable following a Student’s t-distribution with \( n - 1 \) degrees of freedom. Let us define \( z_{\alpha/2,n-1} \) such that \( 1 - 2F_{n-1}(-z_{\alpha/2,n-1}) = 1 - \alpha \). Then Eq. 2.12 represents the \((1 - \alpha) \times 100\%\) confidence interval of \( \mu \):

\[
\left[ M_n - \frac{z_{\alpha/2,n-1} V_n}{\sqrt{n}}, M_n + \frac{z_{\alpha/2,n-1} V_n}{\sqrt{n}} \right] \tag{2.13}
\]

Again, the confidence interval in Eq. 2.13 depends on the sample mean \( M_n \), the sample size \( n \), the confidence level \( 1 - \alpha \) and also on the sample variance \( V_n^2 \). For large \( n \), the confidence interval in Eq. 2.13 approaches that in Eq. 2.9 as the PDF of the Student’s t-distribution approaches the PDF of a zero-mean, unit variance gaussian random variable for increasing values of \( n \). For a 95\% confidence level, \( z_{\alpha/2,n-1} \) is 2.086 for \( n = 20 \). \( z_{\alpha/2,n-1} \) eventually approaches 1.96 as \( n \to \infty \).

\textbf{P is not Gaussian, unknown mean, unknown variance}

So far we have assumed that \( P \) follows a Gaussian distribution. But, in general, the PDF of \( P \) may not be Gaussian. As per the Central Limit Theorem (CLT), for large \( n \), the PDF of a sum of \( n \) independent random variables approaches Gaussian. This principle is applied to the case when \( P \) has an unknown distribution with unknown \( \mu \) and \( \sigma \). The sample mean \( M_n \) would now be a gaussian random variable and considering a sequence of \( M_i \)'s constitutes a new sequence of i.i.d. random variables each having approximately a Gaussian distribution. Therefore, instead of performing \( n \) independent experiments to compute \( P_i \)'s, we need to perform \( n \times m \) independent measurements of \( P \) and divide the random variables into \( m \) batches of \( n \) random variables each. We can then employ
Eq. 2.13 to compute the Confidence Interval for the case when the distribution of \( P \) is unknown by replacing \( n \) with \( m \) and \( M_n \) with the batch mean \( B_m = \frac{1}{m} \sum_{j=1}^{m} M_j \), where \( M_j = \frac{1}{n} \sum_{i=1}^{n} P_i \). This method of estimating \( \mu \) is called the method of Batch Means [28].

### 2.4.2 Monte Carlo Method for Probability Estimation

In the previous section, we statistically estimated the average minimum transmission power level of nodes in terms of confidence intervals that specify a range of values that are highly likely to contain \( \mu \). We now consider the problem of determining the Probability of network connectivity for a given power level of sensor nodes using the Monte Carlo random sampling approach [29]. The salient feature of this method is that we can determine the minimum number of realizations, \( n \), of the Poisson Point Process that we need to perform in order to attain a user-specified confidence level \( 1 - \alpha \) and error bound \( \epsilon \) in the obtained probabilities. The strong law of large numbers can be used to estimate the probability of network connectivity:

\[
P \left[ \lim_{n \to \infty} f_A(n) = p \right] = 1 \tag{2.14}
\]

where \( f_A(n) \) is the relative frequency estimator of the probability of occurrence \( p \) of an event \( A \) [28]. In our case, \( A \) represents the given transmission power level. \( f_A(n) \) is calculated by performing a series of \( n \) Bernoulli trials, where each trial represents one realization of the Poisson Point Process and a success is achieved whenever the minimum transmission power level of the nodes for that trial is below the given power level for which we desire to obtain the probability of network connectivity. However, it is impractical to perform an infinite number of trials of node distributions and this method of probability estimation does not indicate when \( f_A(n) \) approaches \( p \). This shortcoming is addressed in [29] which derives lower bounds on the stopping criterion of the Bernoulli Trials by considering the fact that the distribution of \( f_A(n) \) is already known to be Binomial:

\[
n \geq \max \left\{ \left( \frac{z_{\alpha/2}}{2\epsilon} \right)^2, \left( \frac{z_{\alpha/2} \sqrt{2\epsilon + 0.1} + \sqrt{(\epsilon + 0.1)z_{\alpha/2}^2 + 3\epsilon}}{2\epsilon} \right)^2, \left( \frac{\sqrt{63 + z_{\alpha/2}}}{2\sqrt{\epsilon}} \right)^2 \right\} \tag{2.15}
\]

Eq. 2.15 is only a function of \( \epsilon \) and \( \alpha \). Therefore, \( n \) gives the minimum number of realizations of the Poisson Point Process such that we are \((1 - \alpha) \times 100\%\) confident that the obtained frequency estimation is within \( \epsilon \) of the actual probability of network connectivity for a given transmission power level.
Chapter 3

Characterization of Path Loss Models for an Indoor Environment

3.1 Introduction

We analyze the performance of a number of path loss models in modelling a real indoor environment. Specifically, we investigate the Unit Disk and Log-normal Shadowing models along with two propagation models suitable for such propagation environments, the Volcano Indoor Multi-Wall and WINNER II Stochastic channel models. Field measurements are performed at 2.45 GHz in an indoor office setup to first parameterize and calibrate the propagation models. Channel parameters, i.e. the path loss exponent and shadow fading standard deviation, are empirically determined using a curve-fitting approach on part of the measurement data. The parameters are further characterized based on the classification of propagation scenarios: (1) Based on the distance between a transmitter and a receiver, (2) Based on LOS/NLOS conditions between them. We then evaluate the performance of the path loss models: (1) Qualitatively by a direct comparison with actual path loss values (2) Quantitatively using three error metrics. This study provides a common performance evaluation framework to assess the accuracy of various channel models that capture large-scale RF signal variations in a complex indoor environment and further facilitates the network connectivity analysis under different channel conditions since the channel parameters for different path loss models are derived from actual field measurements.


3.2 Experimental Setup

In this section, we first provide details of the radio hardware used for the channel measurements. A detailed description of the indoor environment considered as our field measurement site is then provided, along with the methodology adopted to carry out the experiments.

3.2.1 Measurement Equipment

Radio equipment that we employed for the path loss measurements consisted of a simple Transmitter-Receiver (Tx-Rx) pair, capable of operating in the Industrial Scientific and Medical (ISM) unlicensed frequency bands. Figure 3.1 shows the block diagram of the measurement equipment. An Agilent E5061B Network Analyzer was used as a signal generator at the transmitter side, transmitting an unmodulated, single tone continuous wave carrier at 2.45 GHz. The signal generator offers a frequency range of 5 Hz to 3 GHz. An Agilent E4402B Spectrum Analyzer was used at the receiver side. It offers a frequency range of 9 kHz to 3 GHz. The output of the signal generator and the input of the spectrum analyzer were directly connected to two vertically polarized, identical monopole antennas. The specifications of the antennas are similar to those used for Wireless Local Area Network (WLAN) and WSN operations worldwide. With vertical polarization, the antennas provide a toroidal shaped radiation pattern in the horizontal plane, i.e. the plane parallel to the floor of the measurement site. Since we restricted the measurements to only a single floor of the measurement site, the radiation patterns of the antennas were essentially isotropic along the floor. Transmitting and Receiving antenna heights were both adjusted to 1.5 m above the floor level.

![Figure 3.1: Radio Equipment Block Diagram](image)

In order to easily perform channel measurements at several locations within the indoor environment, the radio equipment was mounted on movable carts.
3.2.2 Measurement Site and Procedure

We considered an indoor office setup for the path loss measurements. All channel measurements were performed on the 4th floor of the Bahen Centre for Information Technology building at the University of Toronto’s St. George Campus. An RF signal can propagate through a number of scenarios in such a propagation environment, e.g. through room-to-room, room-to-corridor, corridor-to-corridor, room-to-corridor-to-room, and so on. Broadly, we can classify the propagation scenarios as either Line-of-Sight (LOS) or Non Line-of-Sight (NLOS). Figure 3.2 shows the section of the floor of the Bahen Building [30] considered for the path loss measurements. All the locations at which measurements were performed are labelled with numeric and alphanumeric points. The building is composed of several different types of materials through which an RF signal can undergo partition losses. Thin concrete walls, thick concrete walls, wooden and metallic doors, glass walls, metallic elevators, glass windows, and drywall walls are the materials that constitute the largest portion of the building layout.

As can be seen from Fig. 3.2, the target measurement site provides all the characteristics of a complex indoor environment. Propagation models based on Radio Mapping and Ray Tracing algorithms require detailed information, in terms of the building layout and construction, of the indoor environment for which radio propagation predictions (such as path loss computations) are desired. The Volcano Indoor Multi-Wall model [10] that we investigate and compare with other path loss models predicts path losses in indoor environments using a digital building model of the target measurement site.

Before starting the measurement campaign, we calibrated the radio equipment to take into account the system losses in the link budget. The Tx and Rx antennas were observed to have non-zero antenna gains, primarily due to impedance mismatching and feed-line variations, during the radio hardware calibration process. However, the antenna gains were balanced by the non-zero transceiver losses within a reasonable degree of error tolerance. The final received power reading at every measurement location was an average of 500 power samples so as to limit the undesirable effects of fast fading, which are often caused by equipment movement or human intervention while performing the experiments. For low rate, low power wireless personal area networks, the transmission power is typically a few mWs. We therefore set the transmitting power of the signal generator to 0 dBm. A frequency span of 100 kHz at the receiver was used, consisting of 1000 points, resulting in a sweep time of 275 ms.

Path loss measurements were performed at 55 different locations inside the Bahen Building, resulting in a total of 142 readings on these sites. Table 3.1 shows a complete list of the Tx-Rx locations. Each measurement resulted from a different configuration of
Figure 3.2: Section of the Bahen Building Floor Plan with all the Measurement Points
Table 3.1: Transmitter-Receiver Pairs Used

<table>
<thead>
<tr>
<th>Tx Location</th>
<th>Rx Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>F9</td>
<td>F10×2, F11</td>
</tr>
<tr>
<td>F12</td>
<td>42</td>
</tr>
<tr>
<td>13</td>
<td>(F1-F8)×2, 1-12, 14-17, 19-28</td>
</tr>
<tr>
<td>18</td>
<td>1-17, 19-36, 37×2, 38-43</td>
</tr>
<tr>
<td>31</td>
<td>14-17, 19-28</td>
</tr>
<tr>
<td>35</td>
<td>1-17, 19-33, 37-43</td>
</tr>
</tbody>
</table>

the transmitter and the receiver locations. The majority of the experiments were performed with the receiver (transmitter) placed in one of the corridors and the transmitter (receiver) placed either in one of the corridors or in a room, as can be seen from Fig. 3.2.

3.3 Path Loss Models

We now describe the radio propagation models evaluated in this empirical study, their formulations as well as the respective channel parameters used for the path loss computations.

3.3.1 Unit Disk Model

A purely deterministic path loss model, the Unit Disk model is based on a simple extension to the Frii’s Transmission equation [31]. It is also referred to as the Log-distance model or the point graph model. In a Stochastic Geometry and Random Graph Theoretic sense [13], it is known as Gilbert’s random disk graph model. The received power follows an $\alpha$ law of the distance from the transmitter:

$$P_r = P_t G_t G_r \left( \frac{\lambda}{4\pi d} \right)^\alpha$$  \hspace{1cm} (3.1)

where $P_r, P_t, G_t, G_r, \alpha, \lambda$ and $d$ are respectively the received power (in mW), transmitted power (in mW), transmitting antenna gain, receiving antenna gain, path loss exponent, wavelength of the transmitted signal (in m), and the distance (in m) between the transmitter and the receiver. For a reduction to the Frii’s formula or the free space model, $\alpha$ can be set to 2 in Eq. 3.1. Although a Unit Disk model is a very basic radio propagation model as it does not model signal blockage due to obstacles or signal attenuation when an RF signal propagates through a partition in an indoor environment, it is generic in
the sense that $\alpha$ is generally determined empirically from field measurements. In terms of the average path loss (in dB) at a distance $d$ from the transmitter, Eq. 3.1 can be written as:

$$PL(d) = PL_{fr}(d_o) + 10 \alpha \log_{10}\left(\frac{d}{d_o}\right)$$

(3.2)

where $PL(d)$ is the desired path loss at a distance $d$, $PL_{fr}(d_o)$ is the free space path loss at a reference distance, $d_o$ from the transmitter and is calculated as $PL_{fr}(d_o) = 20\log_{10}\left(\frac{4\pi d_o}{\lambda}\right)$. Values of $\alpha$ can typically range from $1.6 - 1.8$ for an Indoor LOS propagation scenario, $4 - 6$ for Indoor NLOS scenario, and $3 - 5$ for dense urban environmental conditions [3]. The Unit Disk model therefore adds both a quantitative as well as qualitative basis to the determination of large-scale signal variations, i.e. path loss. Path loss predictions by the Unit Disk model are isotropic, i.e. received signal power at a given distance from the transmitter is same in all the directions. Although the radio channel behaviour as characterized by this model does not capture channel randomness, we consider it as one of the candidate models for comparative analysis due to its extensive use as a physical layer model for wireless network protocol design and evaluation.

### 3.3.2 Log-normal Shadowing Model

The Log-normal Shadowing model extends the average path loss values as computed by the Unit Disk model by considering a probabilistic distribution around the average. It therefore offers the advantages of mathematical tractability as well as realistic predictions of actual wireless channel conditions. The path loss (in dB) at a certain distance from the transmitter follows a Log-normal PDF with a mean given by the deterministic Unit Disk model of Eq. 3.2:

$$PL(d) = PL_{fr}(d_o) + 10 \alpha \log_{10}\left(\frac{d}{d_o}\right) + \chi_{sf}$$

(3.3)

where $\chi_{sf}$ is a shadow fading random variable that accounts for signal blockage due to obstacles in the propagation path. The PDF of $\chi_{sf}$ in dB is:

$$f_{\chi_{sf}}(x) = \frac{1}{\sqrt{2\pi}\sigma_{sf}} \exp\left(\frac{-x^2}{2\sigma_{sf}^2}\right)$$

(3.4)

where $\sigma_{sf}$ is the standard deviation of the shadow fading random variable $\chi_{sf}$. $\alpha$ and $\sigma_{sf}$, which are both determined from field measurements, together quantify a Log-normal shadowing propagation model. An increase in $\sigma_{sf}$ reduces the node isolation probability
in a wireless ad hoc network and hence results in increasing network connectivity levels [12]. We statistically determine these parameters in the next section using reverse curve fitting from part of the measurement data that is used for the parameterization of the path loss models.

### 3.3.3 Volcano Indoor Multi-Wall Model

The Volcano Indoor Multi-Wall model takes into consideration the detailed indoor building layout and construction when calculating path losses. The Volcano model can provide propagation predictions for complex Indoor, Urban and Rural environments. It is based on the Cost231 Multi-Walls Multi-Floors radio propagation model [32]. Path loss computation using the Cost231 model is as follows:

\[
PL(d) = L_{FSL} + L_c + \sum_{i=1}^{I} k_{wi} L_{wi} + k_f \left[ \frac{k_f^{i+2} - b}{k_f^{i+1}} \right] L_f
\]  

(3.5)

where \(PL(d)\) (in dB) is the desired path loss at a certain distance \(d\) from the transmitter, \(L_{FSL}\) is the free space path loss (in dB) computed using Eq. 3.2 when \(\alpha\) is set to 2, \(L_c\) is a constant loss (in dB) that can be determined from the measurement data, \(I\) represents the number of different types of walls that exist in the indoor environment, \(k_{wi}\) is the number of walls composed of \(i\)th material that are penetrated by an RF signal along the propagation path, and \(L_{wi}\) is the signal attenuation (in dB) when a signal passes through a wall composed of \(i\)th material. Lastly, the path loss computation takes into account the floor losses using \(k_f\) to represent the number of floors between the transmitter and receiver, \(L_f\) to be the loss through one floor (in dB), and \(b\) is a correction factor for propagation through multiple floors.

Since the channel measurements were restricted to only one floor of the Bahen Building, we currently do not take into consideration the Multi-Floor components of Eq. 3.5 to compute path losses. The simplified formula for the Cost231 model without considering floor-floor propagation scenario is:

\[
PL(d) = L_{FSL} + L_c + \sum_{i=1}^{I} k_{wi} L_{wi}
\]

(3.6)

\(I\) and \(k_{wi}\) in Eq. 3.6 depend on the building layout, which can be determined from the building floor plan, such as that shown in Fig. 3.2. The parameters \(L_c\) and \(L_{wi}\) can be determined experimentally by performing a series of partition measurements at the
measurement site. We determine their values during the calibration process of the Volcano model using the tuning part of the measurement data (explained in Section 3.4.1). In this study, we assume that the target propagation environment does not consist of a large number of metallic structures and hence does not offer significant multi-path reflections. 3D ray tracing capabilities of the Volcano model are therefore not employed. Accurate ray tracing abilities of the Volcano model are however validated against real measurement data in an experimental study performed on board ships [33].

### 3.3.4 WINNER II Stochastic Channel Model

The WINNER II Stochastic channel model is a geometry based model in which propagation parameters are determined qualitatively from probability distributions. The distributions are extracted from channel measurements and are defined for angle spread, delay spread, shadow fading and cross polarization ratio. A range of propagation scenarios can be considered using its generic approach to propagation modelling, such as Indoor Office, Large Indoor Hall, Indoor-to-Outdoor, Outdoor-to-Indoor, Urban/Suburban/Rural Macrocell and Bad Urban Microcell. Different scenarios are modelled by using a different set of channel parameters from their statistical distributions. Furthermore, several channel realizations can be generated for a given scenario by summing contributions of rays with specific spatio and temporal channel parameters specific to a particular propagation scenario. The desired path loss at a certain distance $d$ from the transmitter is calculated as follows [34]:

$$PL(d) = A \log_{10}(d) + B + C \log_{10}\left(\frac{f_c}{5.0}\right) + X$$  \hspace{1cm} (3.7)

where $A$ is a fitting parameter that includes path loss exponent, $B$ is the y-intercept on the scatter plot of Path Loss v/s Distance on a Log-Log scale, $C$ is a path loss frequency dependence parameter, $f_c$ is frequency of operation in GHz, and $X$ is an environment specific term that accounts for wall attenuations in NLOS scenarios.

The channel parameters of Eq. 3.7 are characterized for LOS and NLOS Indoor Office environment based on several measurement campaigns and extensive literature survey performed under the WINNER II project [34]. The LOS scenario specific propagation parameters are as follows:

$$PL'_{LOS}(d) = 18.7 \log_{10}(d) + 46.8 + 20 \log_{10}\left(\frac{f_c}{5.0}\right)$$  \hspace{1cm} (3.8a)

$$PL_{LOS}(d) = PL'_{LOS}(d) + \chi_{LOS,sf}$$  \hspace{1cm} (3.8b)
where $\chi_{\text{LOS,sf}}$ is the shadow fading parameter for the LOS scenario with $\sigma_{\text{LOS,sf}} = 3$. The NLOS scenario specific propagation parameters are as follows:

\[
P_L'(d) = 36.8 \log_{10}(d) + 43.8 + 20 \log_{10}\left(\frac{f_c}{5.0}\right) + X \tag{3.9a}
\]

\[
P_L(d) = P_L'(d) + \chi_{\text{NLOS,sf}} \tag{3.9b}
\]

where $\chi_{\text{NLOS,sf}}$ is the shadow fading parameter for the NLOS scenario with $\sigma_{\text{NLOS,sf}} = 4$ and the parameter accounting for wall attenuations is $X = 5(n_w - 1)$, wherein we assume that the majority of the rooms in the building are made of thin concrete, i.e. are light walls. Here $n_w = \lfloor \frac{d}{10} \rfloor$ defines the number of walls between a transmitter and a receiver $d$ meters apart. We can only apply Eq. 3.8b and 3.9b when $d > 3$ m. However, when $1$ m $< d < 3$ m, the WINNER II Stochastic channel model reduces to the Unit Disk model with $\alpha = 2$. Furthermore, to account for obstacles in the propagation path, the possibility of an occurrence of a LOS link between a transmitter and a receiver has a probability which is a function of the distance $d$ (in m) between them:

\[
p_{\text{LOS}} = \begin{cases} 
1 & \text{if } d \leq 2.5 \\
1 - 0.9(1 - (1.24 - 0.61\log_{10}(d))^3)^{\frac{1}{4}} & \text{if } d > 2.5
\end{cases} \tag{3.10}
\]

A weighted path loss is then computed using Eqs. 3.8b, 3.9b and 3.10 considering the probability of the occurrence of both LOS ($p_{\text{LOS}}$) as well as NLOS ($1 - p_{\text{LOS}}$) links at an arbitrary separation $d$ between the Tx-Rx pair:

\[
P_L(d) = p_{\text{LOS}} P_{L,\text{LOS}}(d) + (1 - p_{\text{LOS}}) P_{L,\text{NLOS}}(d) \tag{3.11}
\]

The WINNER II Stochastic Channel model can be used for link/system level simulations of wireless systems that operate in the frequency range of 2 to 6 GHz and supports upto 100 MHz of RF bandwidth [34].

### 3.4 Results

#### 3.4.1 Parameter Estimation

Half of the measurement data recorded was used for the parameterization/tuning of the path loss models and the other half for the performance evaluation. As we described in Section 3.2.2, 142 path loss measurements were performed at the target measurement site.
The tuning and performance evaluation data therefore consists of 71 measurements each. We perform parameterization, i.e. determination of the channel parameters $\alpha$ and $\sigma_{sf}$ empirically from the 71 measurement points, of the Unit Disk and Log-normal Shadowing path loss models. The tuning data was also used to calibrate the environment specific parameters $L_c$ and $L_{wi}$ of the Volcano Indoor Multi-Wall model. Since the WINNER II Stochastic channel model is derived from several measurement campaigns and extensive literature survey on channel modelling under various propagation scenarios [34], the channel parameters of this model are not subject to further calibration.

A reverse curve-fitting approach is adopted for the parameterization of the Unit Disk and Log-normal models. This step involves fitting a linear regression curve on the tuning data to empirically estimate the path loss exponent. The shadow fading standard deviation is subsequently computed for the given sample size (i.e. size of the tuning data set) from the estimated path loss. Let us denote $\alpha$ by $m$, $10 \log_{10} \left( \frac{d}{d_o} \right)$ by $x$, the free space path loss $PL_{fs}(d_o)$ by $c$ and the computed path loss at distance $d$ by $\hat{y}$. Then Eq. 3.2 can be written as: $\hat{y} = mx + c$. Also denote by $y$ the path loss value from the tuning data. Then the minimum mean square estimation of $m$ is of the form: $\min \{ E \{(y - \hat{y})^2\} \}$. Since $c$ is a predetermined constant, i.e. the free space path loss at a reference distance, $d_o$ of 1 m, $m$ can be written as [28]:

$$m = \frac{E \{xy\} - cE\{x\}}{E\{x^2\}} \tag{3.12}$$

The estimated path loss exponent $m$, subsequently leads to an unbiased estimation of the shadow fading standard deviation for a given sample size $n$:

$$\sigma = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \tag{3.13}$$

The results of the linear regression curve-fitting are shown in Fig. 3.3. Propagation scenarios are classified on two basis for the quantification of channel parameters: (1) Based on the LOS/NLOS conditions between the transmitter and receiver. (2) Based on the distance between them. As expected [3] and also observed from Fig. 3.3, increasing distances between the Tx-Rx pair cause an increase in the path loss exponent. For $d \leq 10$ m, $\alpha = 2.19$ as opposed to a value of 3.30 when $d > 25$ m. The propagation characteristics of our measurement site shown in Fig. 3.2 are expected to provide these results because in a complex indoor environment, there is a higher possibility of the occurrence of a NLOS link between a Tx and a Rx when the distance $d$ between them is
Figure 3.3: Scatter Plots Showing Measurement Points on a Log-Log Scale along with the Linear Regression Estimation Curves for Different Propagation Classes.

High. NLOS scenarios are known to have higher path loss exponents [3]. The path loss exponent estimations based on propagation scenarios shows that $\alpha = 3.21$ for the NLOS measurements as opposed to a significantly lower value for the LOS measurements, which was found to be 1.77. The shadow fading standard deviations also provide similar results. $\sigma_{sf}$ is higher for higher $d$ and the NLOS scenario as compared to lower $d$ and the LOS scenario respectively. A path loss exponent of 2.92 and a standard deviation of 10.27 are obtained considering all the 71 measurements in the tuning data set.

Based on the distance classification, the number of tuning measurement points belonging to the respective categories are: 23 for $d \leq 10$ m, 25 for $10 < d \leq 25$ m, 23 for $d > 25$ m. Based on the LOS/NLOS classification, 15 points constitute the LOS measurements and 56 points constitute the NLOS measurements. Weighted and Average path loss exponents and standard deviations are computed based on these two propagation classes and are reported in Table 3.2. The path loss exponent for Distance (Average) and Distance (Weighted) are the same. The $\alpha$ for LOS/NLOS (Weighted) is close to the overall value when all measurement points are considered. Now, the shadow fading standard deviation for the Distance (Average) lies between that of the Distance (Weighted) and the LOS/NLOS (Weighted). Since, the number of NLOS measurements
Table 3.2: Channel Parameters for Different Scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\alpha)</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>2.92</td>
<td>10.27</td>
</tr>
<tr>
<td>Distance (Average)</td>
<td>2.73</td>
<td>8.91</td>
</tr>
<tr>
<td>Distance (Weighted)</td>
<td>2.73</td>
<td>8.86</td>
</tr>
<tr>
<td>LOS/NLOS (Average)</td>
<td>2.49</td>
<td>7.55</td>
</tr>
<tr>
<td>LOS/NLOS (Weighted)</td>
<td>2.90</td>
<td>8.96</td>
</tr>
</tbody>
</table>

We now explain the calibration process of the building layout specific radio propagation model that we investigate in this study: the Volcano Indoor Multi-Wall model. For every measurement point in the tuning data set, we first determine the number and type of each wall encountered along the propagation path between the Tx-Rx pair. A series of linear equations are then created using this information which are used to determine the unknown channel parameters of the model. The least square method is adopted to solve the system of linear equations. For a reference distance, \(d_o\), of 1 m and system frequency of 2.45 GHz, \(L_{FSL}\) was found to be \(40.225 + 19.055\log_{10}(d)\) in Eq. 3.6 and \(L_c = -0.89\) after the calibration process, both in dB. Also, the signal penetration losses (\(L_{wi}\) values in dB) were computed for various partition materials in the building.

### 3.4.2 Performance Evaluation of Path Loss Models

A comparative analysis of seven path loss models was performed based on the channel parameterization in Section 3.4.1. Three Unit Disk models with \(\alpha = 2\) (i.e. the free space model), \(\alpha = 2.73\) and \(\alpha = 2.92\), two Log-normal Shadowing models with \(\alpha = 2.73, \sigma = 8.91\) and \(\alpha = 2.92, \sigma = 10.27\), the calibrated Volcano Indoor Multi-Wall model and the WINNER II Stochastic Channel model were evaluated against the 71 channel measurements that constitute the performance evaluation data set. The results of the path loss estimations, along with the actual path loss measurements, are reported in Fig. 3.4. In order of increasing distance between the transmitter and receiver, we assigned a measurement number sequentially to the various channel measurements.

It can be observed from Fig. 3.4a that the Unit Disk model with \(\alpha = 2\) provides a
Figure 3.4: Predicted Path Losses and Actual Path Losses for All Measurement Points
good prediction of actual path losses at shorter distances, since the propagation will be highly LOS, as noted from Fig. 3.3. At higher distances, since the propagation scenario is primarily NLOS, the Unit Disk model with $\alpha = 2$ underestimates the actual path losses. For $\alpha = 2.73$ and 2.92, the predicted path losses are higher at shorter distances but are closer to the actual values when the distance increases. The computed path loss values by both of the Log-normal models, as shown in Fig. 3.4b, conform well with actual values at intermediate distances, as opposed to the computations at shorter and longer distances. Path loss values as predicted by the WINNER II and Volcano models are comparatively more accurate than the Unit Disk and Log-normal models. However, some outliers can be seen in Fig. 3.4b resulting from the Volcano based predictions. The digital building layout incorporated by the Volcano model to perform propagation predictions is believed to be the cause of these outliers since it is only an approximation of the actual building floor plan.

The performance of the path loss models is quantitatively analyzed based on three error metrics: Mean Error ($\mu_\epsilon$), Standard Deviation of the Error ($\sigma_\epsilon$) and the Root Mean Square Error (RMSE). Results are shown in Table 3.3. Here, Error refers to the deviation of the predicted path loss from the actual value. Also shown are the results when the calibration process, as explained in Section 3.4.1, is not employed for the Volcano Indoor Multi-Wall model.

<table>
<thead>
<tr>
<th>Models</th>
<th>Error Metric</th>
<th>$\mu_\epsilon$ (dB)</th>
<th>$\sigma_\epsilon$ (dB)</th>
<th>RMSE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Space ($\alpha = 2$)</td>
<td></td>
<td>-8.60</td>
<td>11.58</td>
<td>14.36</td>
</tr>
<tr>
<td>Unit Disk ($\alpha = 2.73$)</td>
<td></td>
<td>14.30</td>
<td>10.21</td>
<td>17.52</td>
</tr>
<tr>
<td>Unit Disk ($\alpha = 2.92$)</td>
<td></td>
<td>20.26</td>
<td>9.96</td>
<td>22.54</td>
</tr>
<tr>
<td>Log-normal ($\alpha = 2.73$, $\sigma = 8.91$)</td>
<td></td>
<td>-0.71</td>
<td>13.02</td>
<td>12.95</td>
</tr>
<tr>
<td>Log-normal ($\alpha = 2.92$, $\sigma = 10.27$)</td>
<td></td>
<td>5.25</td>
<td>15.22</td>
<td>16.00</td>
</tr>
<tr>
<td>Volcano Indoor Multi-Wall (Uncalibrated)</td>
<td></td>
<td>5.55</td>
<td>9.12</td>
<td>10.62</td>
</tr>
<tr>
<td>Volcano Indoor Multi-Wall (Calibrated)</td>
<td></td>
<td>1.65</td>
<td>8.00</td>
<td>8.12</td>
</tr>
<tr>
<td>WINNER II Stochastic</td>
<td></td>
<td>6.77</td>
<td>8.65</td>
<td>10.94</td>
</tr>
</tbody>
</table>

The Log-normal model with $\alpha = 2.73$ and $\sigma = 8.91$ has the smallest $\mu_\epsilon$ value, but has a very high $\sigma_\epsilon$. The Calibrated Volcano Indoor Multi-Wall model has the smallest value of $\sigma_\epsilon$ and RMSE values as well as very low $\mu_\epsilon$. The WINNER II Stochastic and the Uncalibrated Volcano models have the next best $\sigma_\epsilon$ and RMSE values. The $\mu_\epsilon$ of the
Uncalibrated Volcano is lower than that of the WINNER II model. From these observations, the Calibrated Volcano model provides the best estimation of the actual path losses, followed by its Uncalibrated version, and then the WINNER II Stochastic Channel model. Volcano models can be used in scenarios in which detailed building information is available to the network designer and a precise behaviour of the propagation environment is desired. However, the WINNER II model can be a good alternative in the absence of the information regarding building layout and construction, of course at the expense of slightly less accurate path loss predictions.

3.5 Summary

In this Chapter, we observed that site-specific radio propagation modelling is important to accurately capture the wireless channel randomness in complex indoor environments. Channel models that take into consideration RF signal attenuation due to the presence of walls and obstacles along the propagation path, such as Volcano Indoor Multi-Wall and WINNER II Stochastic models, provide more accurate path loss predictions as opposed to purely deterministic Unit Disk models, which are traditionally and more widely used for network protocol design and evaluation. How significantly the deviations in propagation predictions using different (characterized) path loss models impact the network backbone construction under varying levels of network connectivity will be observed in Chapter 5 when we perform Monte Carlo simulations.
Chapter 4

Minimum Power $k$-Connected Spanning Subgraphs

4.1 Introduction

A connected network topology is a fundamental requirement for any network layer protocol operation. Data routing between an arbitrary pair of nodes in the network is possible only if the topology or the underlying communication graph is fully connected. Furthermore, fault-tolerance is highly desired when a sensor network operates in a harsh environment wherein node hardware can likely undergo physical damage. Furthermore, an energy-efficient design of singly-connected and fault-tolerant sensor network topologies is essential for prolonged network lifetime, since sensor nodes, typically operating on stand-alone batteries, have very limited power resources. The aim of this Chapter is therefore to construct such power-optimized topologies.

We implement a 2-approximate solution based on a Minimum Spanning Tree formulation to construct a power-optimized 1-Connected network. To achieve fault-tolerance, a $k+1$-approximate solution is implemented to construct power-optimized $k$-Connected networks for the case when $k \in \{2, 3\}$. To ensure that the optimized topologies maintain the desired level of connectivity, we implement a Vertex Connectivity algorithm that makes a number of calls to a Max-Flow Min-Cut subroutine. Throughout the Chapter, sensor nodes in a network will be represented by a set of vertices, $V$, and the communication links between the nodes will be represented by a set of edges, $E$, resulting in a graph formation, $G = (V, E)$. Also, weights on the edges represent path losses between the nodes predicted by a radio propagation model. The algorithmic implementations of the optimized network topologies are performed using Boost C++ Graph Libraries [35].
4.2 Computing Vertex Connectivity of a Network

A pair of vertices \((u, v)\) is \(k\)-Connected or \(k\)-Vertex Connected if there exist at least \(k\) vertex-disjoint paths between \(u\) and \(v\). Equivalently, by Menger’s Theorem [25], \(k\)-Vertex Connectivity between \(u\) and \(v\) can also be defined as the minimum cardinality of an \(u-v\) vertex separator. This notion of “local” Vertex Connectivity, \(k(u, v)\), is extended to define the Vertex Connectivity of the graph, \(K(G)\), as follows: An undirected graph \(G = (V, E)\) is \(k\)-Vertex Connected, i.e. \(K(G) = k\), if there exist at least \(k\) vertex-disjoint paths between \((u, v)\) \(\forall \ u, v \in V\). Similarly, \(G\) is \(k\)-Edge Connected if there exist at least \(k\) edge-disjoint paths between any pair of vertices. The following relation is also proven [25]:

\[
K(G) \leq \lambda(G) \leq \delta(G) \tag{4.1}
\]

where \(\lambda(G)\) and \(\delta(G)\) are respectively the Edge Connectivity and minimum vertex degree of \(G\). Equation 4.1 implies that a \(k\)-Vertex Connected graph is at least \(k\)-Edge Connected. In the rest of this thesis, Connectivity and Vertex Connectivity will be used interchangeably, unless otherwise stated.

The majority of topology optimization schemes run a Vertex Connectivity algorithm as a subroutine to check for the condition that the generated optimized topology maintains a desired level of network connectivity. This section therefore highlights the implementation of a Vertex Connectivity algorithm [36] which will later serve as a building block for our Topology Control scheme.

The algorithm runs in two phases:

1. Construction of a directed network flow graph, \(H\), from the input, undirected graph, \(G\).

2. Computation of \(K(G)\) based on a number of calls to a Max-Flow algorithm (passing \(H\) as an input) to determine the local Vertex Connectivity.

4.2.1 Network Flow Graph Construction

We adopt the following formulation proposed by Even [37] to derive a directed network flow graph \(H = (V', E')\) from a given undirected base graph \(G = (V, E)\) whose Vertex Connectivity we desire to compute:

1. Add two vertices in \(H\), \(u', u'' \forall u \in V\).
2. Add an internal edge \( e' = (u', u'') \) in \( H \) emanating from \( u' \) and entering \( u'' \), \( \forall u', u'' \in V' \).

3. Add two edges \( e'_1 = (u'', v') \) and \( e'_2 = (v'', u') \) in \( H \) \( \forall e = (u, v) \in E \).

4. Assign unit weights (capacities) to the edges in \( E' \).

Figure 4.1 shows an example of a base graph consisting of 3 vertices and its corresponding flow graph. As can be seen, \( H \) has \( 2m + n \) edges and \( 2n \) vertices, where \( n \) : number of vertices in \( G \) (also referred to as the graph order), \( m \) : number of edges in \( G \) (also referred to as the graph size).

### 4.2.2 Computing \( K(G) \)

By definition [36], \( K(G) = \min \{ k(u, v) \mid \forall u, v \in V \} \). The local Vertex Connectivity \( k(u, v) \) for \( u, v \in G \) is determined by finding the maximum flow between \( u'' \) and \( v' \) in the flow graph \( H \) computed in Section 4.2.1. This trivial definition of \( K(G) \) would require \( n(n-1) \) calls to a Max-Flow algorithm, considering all possible combinations of vertices in \( G \). However, due to symmetry and also observing the fact that \( k(u, v) = n - 1 \), \( \forall (u, v) \in E \), it suffices to make \( \frac{n(n-1)}{2} - m \) calls. We implement Edmonds-Karp [38] algorithm to compute the maximum flow (or the local Vertex Connectivity) between a pair of vertices. For general edge costs, the algorithm has an \( O(nm^2) \) time complexity. However, for our restricted case of unit capacities on the edges, it runs in \( O\left(\frac{mn^2}{2}\right) \).

Improvements have been proposed in the literature to minimize the number of calls to the Max-Flow algorithm in order to calculate \( K(G) \). We make use of the following observations made by Esfahanian and Hakimi [36] to limit the number of calls to the Max-Flow algorithm to \( O\left(n - \delta - 1 + \frac{\delta(\delta-1)}{2}\right) \), where \( \delta \) is the degree of a minimum degree vertex in \( G \):

![Flow Graph H](image)
1. Let $C$ be a minimum vertex separator in $G$, i.e. $G - C$ renders the graph disconnected. For a minimum degree vertex $v \notin C$:

$$K(G) = \min \{k(v, u) \mid u \in V - \{v\} \text{ and } u \notin N(v)\} \quad (4.2)$$

where $N(v)$ is the neighbor set of $v$.

2. For a minimum degree vertex $v \in C$:

$$K(G) = \min \{k(p, q) \mid p, q \in N(v) \text{ and } (p, q) \notin E\} \quad (4.3)$$

Since either of the above conditions are possible, the lesser of the two quantities, i.e. lesser of Eqs. 4.2 and 4.3 determine $K(G)$. The Vertex Connectivity algorithm therefore has a time complexity of $O \left(\left(n - \delta - 1 + \frac{\delta(\delta - 1)}{2}\right)\left(mn^{\frac{d}{2}}\right)\right)$. The following are the steps to compute $K(G)$ [36]:

1. Find the minimum degree vertex $v \in V$.
2. Compute $K_1 = \min \{k(v, u) \mid u \in V - \{v\} \text{ and } u \notin N(v)\}$.
3. Compute $K_2 = \min \{k(p, q) \mid p, q \in N(v) \text{ and } (p, q) \notin E\}$.
4. Set $K(G) = \min \{K_1, K_2\}$.

For the graph shown in Fig. 4.2, for example, $K(G) = 2$ since removal of vertex “1” and vertex “4” disconnects the graph into 2 individually connected components. Note that $\lambda(G) = \delta(G) = 3$ for this case. The Vertex Connectivity for the example shown in Fig. 4.2 can be computed by hand, which is otherwise not the case for large size networks and therefore it becomes essential to implement a procedure that computes $K(G)$.

### 4.3 Constructing a Near-Power-Optimal Singly-Connected Network

In this section, we consider the problem of determining optimal transmission power level of nodes such that the generated network topology (or the communication graph) is minimally 1-Connected. By minimally 1-Connected, we mean that the network nodes transmit at power levels just enough to generate a singly-connected network topology, i.e. a communication graph with $K(G) = 1$. In a purely deterministic propagation
environment, the transmission power $P_t$ follows an $\alpha$ law of the distance $d$ between a Transmitter and a Receiver:

$$P_t \propto d^\alpha \quad (4.4)$$

where $\alpha$ is the path loss exponent. However, since indoor environments generally offer signal blockage/attenuation due to obstacles/walls along the propagation path:

$$P_t \propto (d^\alpha \chi) \quad (4.5)$$

where $\chi$ is a random variable that accounts for shadowing due to obstacles. The Range Assignment (RA) problem discussed in [15] is therefore not equivalent to the problem at hand for such propagation environments since instead of determining optimal transmission power levels, the RA problem seeks to determine optimal transmission ranges of nodes that ensure network 1-Connectivity.

In order to successfully accomplish MAC layer operations such as exchange of Request-To-Send (RTS) and Clear-To-Send (CTS) packets between a pair of communicating nodes, it is required that the generated communication graph supports symmetric connections between the nodes, hence needs to be undirected. The Optimal Power Assignment problem for a singly-connected network has therefore also been termed as the Min-Power Symmetric Connectivity problem [39]. It is a special case of the more general Min-Power $k$-Connected Spanning Subgraph (MPkCS) [26] problem and holds true when $k = 1$. MPkCS is important from a network reliability perspective because at least $k - 1$ node

Figure 4.2: A 2-Connected Graph Illustration
failures can be tolerated in the generated topology, before it becomes disconnected. We consider the design of Near-Power-Optimal Fault-Tolerant topologies in the next section.

Given an input undirected, singly-connected graph $G = (V, E)$ with weights $W : w(e)$, $e \in E$ associated with the edges in $E$, let $P(G)$ and $p(v)$ be the power of the graph $G$ and the power of the vertex $v \in V$ respectively, as defined in Section 2.3. Our aim is to construct an undirected, singly-connected graph $G' = (V, E')$, $E' \subseteq E$ such that $P(G')$ is minimized. This optimization problem is APX Hard, i.e. only approximations to the exact (optimal) solution can be determined using polynomial time algorithms. This APX-Hardness of the Optimal Power Assignment problem for a 1-Connected network is proven in [40]. Let $C_{opt}$ be the cost of an optimal solution to a problem and $C$ be the cost of an approximate solution computed by some algorithm, then we say that the algorithm offers an approximation ratio of $k$ if $C \leq kC_{opt}$. Obviously, $k \geq 1$ and it is desired that $k$ is as close to 1 as possible.

Minimum Spanning Tree (MST) based formulation provides a 2-approximate solution to the Optimal Power Assignment problem for a 1-Connected network [41]. The authors of [39] achieve an even better approximation ratio of $11/6$ by an integer linear programming formulation. The performance analysis of the proposed scheme in [39] however shows that only a slight average improvement over MST based solution is achieved (5 – 6% improvement averaged over 50 instances of the network), despite an increase in the computational complexity of the algorithm.

We therefore implement Kruskal’s Minimum Spanning Tree algorithm [42] which provides a strongly polynomial time 2-approximate solution to the Optimal Power Assignment problem, as the average case approximation ratio of this approach is better than its worst case 2-approximation, implicit from the experimental results of [39]. Algorithm 1 computes the desired set of edges $E'$. Again, $n$ and $m$ are the number of vertices and edges in $G$ respectively. The running time of the algorithm is $O(m \log m)$.

Figure 4.3 shows the implementation of Algorithm 1. A MST, $G'$, is constructed on a base graph, $G$, which has $K(G) = 1$, based on the weights on the edges in $E$. The edges of MST, $E'$, are shown in red. As can be seen, $G'$ has same level of Vertex Connectivity as $G$ ($K(G') = K(G) = 1$), at the same time it is sparser than $G$ (i.e. MST has fewer number of edges as compared to the base graph). The number of edges in a minimum spanning tree is $m = n - 1$. This can also be observed from Fig. 4.3, the number of edges in $G'$ is 11 for this case since there are 12 vertices in the graph. The configuration of vertices in Fig. 4.3 is same as in Fig. 4.2.

Now, the 2-approximate solution to the Optimal Power Assignment problem for a
Algorithm 1 Kruskal’s Minimum Spanning Tree

**Input:** $G = (V, E, W)$, $W : w(e), e \in E$

**Output:** $G' = (V, E')$, $E' \subseteq E$

Sort $E : w(e_1) \leq w(e_2) \leq \ldots \leq w(e_m)$

$E' \leftarrow \emptyset$

$i \leftarrow 0$, $j \leftarrow 0$

while $j < n - 1$

$i \leftarrow i + 1$

if $E' \cup \{e_i\}$ is acyclic then

$E' \leftarrow E' \cup \{e_i\}$

$j \leftarrow j + 1$

end if

end while

Figure 4.3: MST Construction using Kruskal’s Algorithm

singly-connected network is:

$$p(v) = \max \{w(e) : e \in E' \text{ and } e \text{ is incident to } v\} \quad (4.6)$$

For a given receiver sensitivity of a sensor node transceiver, $\beta_{th}$, we can now determine the transmission power level of nodes based on the computed power assignments such that the network is minimally 1-Connected:

$$P_{tx}(v) = p(v) + \beta_{th} \quad (4.7)$$
Considering $\beta_{th} = -80.0$ dBm, typical for an IEEE 802.15.4-compliant radio, node power assignments, $p(v)$, and the corresponding transmission power levels, $P_{tx}(v)$ (in dBm), are reported in Table 4.1 for the graph shown in Fig. 4.3.

Table 4.1: Node Power Assignment and Corresponding Transmission Power levels when $k = 1$

<table>
<thead>
<tr>
<th>$v$</th>
<th>$p(v)$ (dB)</th>
<th>$P_{tx}(v)$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>64.12</td>
<td>-15.9</td>
</tr>
<tr>
<td>1</td>
<td>49.83</td>
<td>-30.2</td>
</tr>
<tr>
<td>2</td>
<td>61.02</td>
<td>-19.0</td>
</tr>
<tr>
<td>3</td>
<td>65.63</td>
<td>-14.4</td>
</tr>
<tr>
<td>4</td>
<td>65.63</td>
<td>-14.4</td>
</tr>
<tr>
<td>5</td>
<td>61.31</td>
<td>-18.7</td>
</tr>
<tr>
<td>6</td>
<td>64.12</td>
<td>-15.9</td>
</tr>
<tr>
<td>7</td>
<td>62.56</td>
<td>-17.4</td>
</tr>
<tr>
<td>8</td>
<td>47.14</td>
<td>-32.9</td>
</tr>
<tr>
<td>9</td>
<td>63.85</td>
<td>-16.2</td>
</tr>
<tr>
<td>10</td>
<td>54.73</td>
<td>-25.3</td>
</tr>
<tr>
<td>11</td>
<td>63.85</td>
<td>-16.2</td>
</tr>
</tbody>
</table>

4.4 Constructing a Near-Power-Optimal Fault-Tolerant-Connected Network

We now extend the problem of constructing a Near-Power-Optimal Singly-Connected Network to build a power-efficient network backbone that is robust to node failures. The aim of this section is to consider the Minimum Power $k$-Connected Spanning Subgraph (MPkCS) problem for $k > 1$. Since the generation of such spanning subgraphs is harder than their singly-connected counterparts, which are APX-Hard, only approximate solutions to the MPkCS problem are known. The final network topology would therefore be Near-Power-Optimal, as is the case for the optimized topology obtained in Section 4.3 using a MST formulation when $k = 1$.

Complete battery discharge of sensor nodes, operation in hazardous environmental conditions while monitoring a physical phenomenon, physical damage to node hardware during network deployment are some of the many factors that can result in node failure,
hence disrupting the normal functioning of a sensor network. Constructing fault-tolerant topologies is therefore an important network design problem since multiple node failures can be tolerated by the network before it disconnects. This design goal can further lead to enhanced network lifetime as the network can now balance load between a source-destination pair through multiple vertex-disjoint paths. Considering such potential improvements, approximation algorithms have been proposed over the past decade with the aim to construct a $k$-Connected Spanning Subgraph ($k > 1$) of minimum power and at the same time, reducing the approximation ratio of the proposed algorithms. The authors of [43] proposed an $O(\log^4 n)$ approximate solution for MPkCS when $k$ is small and an $O(\sqrt{k})$ approximation when $k$ is large, i.e. when $k \in n - o(n)$ (a function $f(x) \in o(g(x))$ if $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$). Here, $n$ and $k$ are respectively the graph order and connectivity requirement. This was improved to $O(\log n)$ for small $k$ and $O(\log^2 n)$ when $k \in n - o(n)$ in [44]. Recently, an even better approximation of $O\left(\log k \log \frac{n}{n-k}\right)$ is obtained in [45] for any $k$.

A constant ratio approximation algorithm for MPkCS is proposed in [26] for the case when $k$ does not depend on $n$. Specifically, an approximation ratio of $k + 1$ is achieved when $k \in \{2, 3\}$. Such values of $k$ have practical relevance. Consider a scenario in which a sensor network is deployed to monitor a critical activity such as fire detection in a forest or detection of a gas leakage in a chemical plant. Even if the network can tolerate a large number of node failures, accurate detection of such critical phenomena is compromised since the sensor network now has a reduced coverage of the target deployment region. Since this restricted case has important practical implications under real-life scenarios and at the same time offers ease of implementation, we will only consider the construction of Minimum Power $k$-Connected Spanning Subgraphs for the case when $k \in \{2, 3\}$.

Central to the $k + 1$ approximate solution to MPkCS [26] is an algorithm that computes a Minimum Cost Rooted $k$-Outconnected Spanning Subgraph (MCKOS). We compute the MCKOS using a recent, Matroid Intersection based approach proposed in [27]. MCKOS results in a Directed Minimum Power Rooted $k$-Inconnected Spanning Subgraph (Directed MPkIS) based on a cost transformation introduced in [46]. An undirected MPkIS is then obtained from the Directed MPkIS which is the final step in the computation of the MPkCS for $k \in \{2, 3\}$. A series of algorithms leading to the construction of such spanning subgraphs are now explained.
4.4.1 Matroid Intersection for Minimum Cost Rooted k-Outconnected Spanning Subgraph

The MCkOS problem is traditionally solved using a complicated reduction to submodular flows [47], typically solved using an integer linear programming formulation. However, a recent proposition in [27] avoids the consideration of submodular flows and the problem is solved using a comparatively simpler Matroid Intersection approach.

Since our aim is to consider vertex-disjointedness as opposed to edge-disjointedness, the function $g$ introduced in Section 2.3 is now $g: V \rightarrow 1$, i.e. we consider only those set of edge-disjoint $(r_0, t)$ paths for which every $v \in V - \{r_0, t\}$ is used by at most one of these paths.

For an input directed, weighted graph $S = (V, A)$ with edge weights $W : w(e), e \in A$ from which we desire to construct a MCkOS $S' = (V, A')$, $A' \subseteq A$, let $r_0$ be the root vertex, $A_0$ the set of all outgoing edges from $r_0$, $A^* = A - A_0$, $V^* = V - r_0$ and $S^* = (V^*, A^*)$. Also denote by $b^#(X)$, a bi-set function (a bi-set function is a function defined on a bi-set $X = \{X_O, X_I\}$) such that $b^#(X) = k(|V| - 1) + \mu(X)$, where $\mu(X) = |X_O - X_I|$. Construct two Matroids, $M^#$ on groundset $A^*$ with independent sets $I_{M^#}$, i.e. $M^# = (A^*, I_{M^#})$, and $M_2$ on groundset $A$ with independent sets $I_{M_2}$, i.e. $M_2 = (A, I_{M_2})$, with the following independence oracles [27]:

1. Independence Oracle for $M^#$: A subset $F \subseteq A^*$ is independent in $M^#$, i.e. $F \in I_{M^#}$, iff $i_F(X) \leq b^#(X)$ for every bi-set $X = (X_O, X_I)$ such that $\emptyset \subset X_I \subseteq X_O \subseteq V^*$

2. Independence Oracle for $M_2$: A subset $I \subseteq A$ is independent in $M_2$, i.e. $I \in I_{M_2}$, iff $\varrho_I(v) \leq k$ for every $v \in V^*$ and $\varrho_I(r_0) = 0$

Now compute the Matroid $M_1$ on ground set $A$ with independent sets $I_{M_1}$ as $M_1 = (M^# \oplus M_2)$, i.e. $M_1 = (A, I_{M_1})$ is the Direct Sum of $M^#$ and $M_2$, where $M_2$ is the Free Matroid on $A_0$ with independent sets $I_{M_2}$, i.e. $M_2 = (A_0, I_{M_2})$. The following important observation in [27] results in the construction of the MCkOS: Computation of the MCkOS or the Minimum Cost Rooted $(k, g)$-Connected Spanning Subgraph when $g$ is as defined above, is equivalent to the computation of a Minimum Cost $(k, g)$-foliage. The Minimum Cost $(k, g)$-foliage is computed as $S' = \min\{M_1 \cap M_2\}$ with $|A'| = k(|V| - 1)$.

We further make the following observations to optimize the construction of the MCkOS:

1. Remove all incoming edges to $r_0$ before computing the Matroids as they do not constitute the final solution of the MCkOS.
2. While computing $M_{\text{free}}$, modify the ground set $A_0$: Consider only those subsets of $A_0$ that have cardinality $= k$.

3. While computing $M_2$, modify the ground set $A$: Consider only those subsets of $A$ that have cardinality $= k(|V| - 1)$.

4. While computing $M^#$, modify the ground set $A^*$: Remove all outgoing edges from $r_0$ and consider only those subsets of $A^*$ that have cardinality $= k(|V| - 1) - k$ so that the direct sum of $M^#$ and $M_{\text{free}}$ has cardinality $= k(|V| - 1)$. Also, observing the fact that the final solution to the MCKOS is an intersection of Matroids $M_1$ and $M_2$, we further consider only those subsets of the ground set of $M^#$ that are subsets of the independent sets of $M_2$.

Our algorithmic formulation of the MCKOS using the Matroid Intersection approach is given in Algorithm 2.

Figure 4.4 shows the implementation of Algorithm 2 to construct a MCKOS, $S'$, from an input directed, weighted graph, $S$, for $k = 2$. Here, vertex “5” is considered as the root vertex. Weights on the edges of $S$ are also labelled. The edges of the MCKOS are shown in red. It is interesting to note from Fig. 4.4 that the in-degree of every vertex $v \in V - \{r_0\}$ is $k$ in the MCKOS $S'$.

Figure 4.4: MCKOS Construction using Matroid Intersection for $k = 2$ and vertex “5” as Root Vertex
Algorithm 2 Construct MCkOS

Input: $S = (V, A, W); W : w(e), e \in A$; root vertex $r_0$; connectivity requirement $k$

Output: $S' = (V, A'), A' \subseteq A$

Remove all in-edges of $r_0$

$I_{M_{free}} \leftarrow \emptyset, I_{M_2} \leftarrow \emptyset, I_{M#} \leftarrow \emptyset, I_{M_1} \leftarrow \emptyset$

Construct $M_{free} = (A_0, I_{M_{free}})$

for all $K$ such that $K \subseteq A_0$

if $|K| = k$

$I_{M_{free}} \leftarrow I_{M_{free}} \cup K$

end if

end for

Construct $M_2 = (A, I_{M_2})$

for all $I$ such that $I \subseteq A$

if $|I| = k(|V| - 1)$

if $g_I(v) \leq k$ for every $v \in V^*$ and $g_I(r_0) = 0$

$I_{M_2} \leftarrow I_{M_2} \cup I$

end if

end if

end for

Construct $M# = (A^*, I_{M#})$

Remove all out-edges of $r_0$

for all $F$ such that $F \subseteq A^*$

if $|F| = k(|V| - 1) - k$

if $F \subseteq I_{M_2}$

if $i_F(X) \leq b#(X)$ for every bi-set $X = (X_O, X_I)$ such that $\emptyset \subset X_I \subseteq X_O \subseteq V^*$

$I_{M#} \leftarrow I_{M#} \cup F$

end if

end if

end if

end for

Construct $M_1 = (A, I_{M_1})$

$I_{M_1} \leftarrow I_{M_{free}} \oplus I_{M#}$

Construct $S'$

$[S' = \min\{I_{M_1} \cap I_{M_2}\}$

4.4.2 From Minimum Cost to Minimum Power Rooted k-Inconnected Spanning Subgraph

Given an input directed, weighted graph $D = (V, B)$ with weights on the edges in $B$, i.e. $W : w(e), e \in B$, the Minimum Cost Rooted $k$-Outconnected Spanning Subgraph, $S' = (V, A')$, computed in Section 4.4.1, results in a Directed Minimum Power Rooted $k$-Inconnected Spanning Subgraph (Directed MPkIS), $J = (V, B')$, $B' \subseteq B$, when $W$ is
appropriately transformed. We make use of the weight transformation provided in [46] to compute $B'$. The correctness of this transformation such that a MC$k$OS results in a Directed MP$k$IS is proven in [46]. Algorithm 3 gives the construction of $J$ from $S'$ and $D$.

Algorithm 3 Construct Directed MP$k$IS

Input: $D = (V, B, W); W : w(e), e \in B$; root vertex $r_0$; connectivity requirement $k$

Output: $J = (V, B'), B' \subseteq B$

$B' \leftarrow \emptyset$

Apply Weight Transformation

$power \leftarrow 0$

for all $v \in V$
do

$\text{temp} \leftarrow \emptyset$

for all $e \in B$ such that $e = (v, u)$ for any $u \in V$
do

$\text{temp} \leftarrow \text{temp} \cup \{e\}$

end for

Sort $\text{temp} : w(e_1) \leq w(e_2) \leq \ldots \leq w(e_l); \{e_1, e_2, \ldots, e_l\} \in \text{temp}$

$power \leftarrow w(e_k)$

for all $e \in B$ such that $e = (v, u)$ for any $u \in V$
do

$w(e) \leftarrow \max\{(w(e) - power), 0\}$

end for

end for

Reverse all edges in $B$

for all $e = (u, v) \in B$
do

$e \leftarrow (v, u)$

end for

Construct MC$k$OS with Input: $S = D$; Transformed Weights $W$ and Output: $S' = (V, A')$

Reverse all edges in $A'$

for all $e = (u, v) \in A'$
do

$B' \leftarrow B' \cup (v, u)$

end for

4.4.3 Minimum Power 2 and 3-Connected Spanning Subgraphs

Given an input undirected, weighted base graph, $G = (V, E)$ with weights on the edges in $E$, $W : w(e), e \in E$, our aim is to construct a MP$k$CS, $G' = (V, E')$, $E' \subseteq E$, such that $G'$ preserves the connectivity property, $k$, of $G$, at the same time results in a power-optimized, sparse topology containing as minimum number of edges in $E'$ as possible. We implement the algorithm proposed in [26] to achieve a $k + 1$ approximate solution to the MP$k$CS problem when $k \in \{2, 3\}$. The solution makes a number of calls to the
Directed MPkIS algorithm, Algorithm 3, considering every vertex of $G$ as a root vertex.

The following steps lead to the construction of MPkCS [26]:

1. Construct a Bi-direction $D$ of $G$.

2. Compute a Directed MPkIS, $J$ of $D$, using Algorithm 3.

3. The underlying graph $H_v$ of $J$ is the undirected MPkIS for a given node $v \in V$ as a root vertex.

4. Repeat steps 1 – 3 considering every $v \in V$ as the root vertex.

5. Assign to MPkCS, one of the $n H_v$’s computed of minimum power, where $n$ is the graph order.

The original solution proposed in [26] requires Restricted MPkIS to be built for every $v \in V$ from the undirected MPkIS. This requires $O(l^k)$ calls to Algorithm 3 to construct the Restricted MPkIS, here $l$ is the number of edges incident to a given $v \in V$ considered as $r_0$. However, during the construction of the MCKOS using Matroid Intersection, we have already restricted the out-degree of the root vertex to $k$ so that the resultant undirected MPkIS is itself a Restricted MPkIS: Recall that while computing $M_{free}$, only those subsets of $A_0$ were considered to form $I_{M_{free}}$ that had cardinality $= k$. This restriction on the out-degree of the root vertex imposed during the MCKOS construction leads to only a single call to Algorithm 3, as opposed to $O(l^k)$ calls as given in [26] to construct the Restricted MPkIS. The algorithmic formulation to compute the MPkCS is given in Algorithm 4.

Algorithm 4 Construct MPkCS for $k \in \{2, 3\}$

**Input:** $G = (V, E, W); W : w(e), e \in E$; connectivity requirement $k$

**Output:** $G' = (V, E'), E' \subseteq E$

for all $v \in V$ do

    Construct a Bi-direction $D$ of $G$

    Construct Directed MPkIS with Input: $D = (V, B)$, $r_0 = v$, $k$ and Output: $J = (V, B')$

    Construct Underlying Graph $H_v$ of $J$

end for

$E' = \min_{v \in V}\{H_v\}$

Figure 4.5 shows the implementation of Algorithm 4 for the case when $k = 2$. A MPkCS, $G'$ is constructed on the base graph $G$ based on the weights on the edges in $E$. 
It can be seen that $K(G') = K(G) = 2$. The edges of MPkCS, $E'$ are shown in red. A comparatively sparser graph which discards two redundant edges from the base graph is obtained without compromising the connectivity level of the network.

![Figure 4.5: MPkCS Construction using Algorithm 4 when $k = 2$](image)

Table 4.2: Node Power Assignment and Corresponding Transmission Power levels when $k = 2$

<table>
<thead>
<tr>
<th>$v$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(v)$ (dB)</td>
<td>63.99</td>
<td>61.2</td>
<td>60.0</td>
<td>60.7</td>
<td>63.99</td>
<td>60.7</td>
<td>62.83</td>
<td>62.83</td>
</tr>
<tr>
<td>$P_{tx}(v)$ (dB)</td>
<td>-16.0</td>
<td>-18.8</td>
<td>-20.0</td>
<td>-19.3</td>
<td>-16.0</td>
<td>-19.3</td>
<td>-17.2</td>
<td>-17.2</td>
</tr>
</tbody>
</table>

The $k + 1$ approximate solution to the Optimal Power Assignment problem can now be computed for a fault-tolerant network. Power assignment of the nodes, $p(v)$, based on Eq. 4.6 and the corresponding transmission power levels, $P_{tx}(v)$ (in dBm), based on Eq. 4.7 are reported in Table 4.2 for the graph shown in Fig. 4.5.

4.5 Summary

This Chapter focused on the construction of a number of spanning subgraphs that provide constant ratio approximation guarantees to the optimal power assignment problem in a $k$-Connected network topology. The approximation ratio of the constructed topologies
is \( k + 1 \) for \( k \in \{1, 2, 3\} \). Our implementation of the algorithm to construct power optimized fault-tolerant topologies offers reduced running time by a factor of \( O(l^k) \) as compared to the default one, where \( l \) and \( k \) are respectively the degree of root vertex and connectivity requirement, \( k \in \{2, 3\} \). We also implemented a Max-Flow Min-Cut based vertex connectivity algorithm to keep track of the level of connectivity that the constructed topologies provide.
Chapter 5

Topology Control in an Indoor Wireless Sensor Network

5.1 Introduction

In this Chapter, we first analyze the average behaviour of a sensor network deployed in an indoor environment under different path loss models whose parameters are derived from the channel measurements performed in Chapter 3. Effect of varying transmission power levels of sensor nodes and node densities is observed on network $k$-Connectivity, $k \in \{1, 2, 3\}$. We then devise a simple Topology Control scheme, and subsequently evaluate it through network simulations, based on the power optimized singly connected and fault-tolerant topologies constructed in the previous Chapter. A realistic node power consumption model is employed during network simulations to account for energy consumption in a sensor node during network operation. End-to-End Delay and Node Remaining Energy are used as performance metrics to evaluate the topologies based on different connectivity levels, $k$ and routing schemes (Multi-Path and Energy-Aware routing). Furthermore, Network Lifetime is computed for three different transmission power settings of the nodes obtained with and without the proposed Topology Control scheme.

5.2 Average Network Behaviour over Several Spatial Realizations

Network backbone construction is directly impacted by the physical layer model. In order to understand this dependence, we perform a network connectivity analysis for four path loss models: Unit Disk with $\alpha = 2$, Log-normal Shadowing with $\alpha = 2.73$
and $\sigma = 8.91$, WINNER II Stochastic channel model and the Calibrated Volcano Indoor Multi-Wall model. Monte Carlo simulations, as introduced in Section 2.4.2, are employed to determine the probability of various levels of network connectivity for a range of transmission power levels, for a pre-specified confidence level and error tolerance in the obtained results. For 95% confidence with a 0.1 absolute error in the probabilities, i.e. setting $\alpha = 0.05$ and $\epsilon = 0.1$ in Eq. 2.15, we obtain $n \geq 245$. We therefore perform 250 realizations of the random node distribution. Also, to determine the average minimum transmission power level of nodes, we employ the notion of Confidence Intervals explained in Section 2.4.1. The obtained minimum transmission power levels for each of the 250 realizations are divided into 10 batches of 25 samples each. Then the method of batch means explained in Section 2.4.1 is used to compute the Confidence Intervals. A total of 101 transmission power levels are considered, ranging from 0 dBm to $-25$ dBm in decrements of 0.25 dBm.

Figure 5.1 shows a section of the digital building model of Bahen Center for Information Technology [10] that determines our deployment region for the random distribution of nodes. The deployment area considered is 690.9 m$^2$. Color codes of various partitions in the digital building model are also shown. Here, Doors are composed of wood and the rest of the partitions, i.e. External, Thick, Medium and Thin are composed of concrete, mentioned in decreasing levels of wall thickness. Glass partitions were also present which are not shown in Figure 5.1 but were considered during the propagation predictions. It should be noted that this digital building layout is only an approximation to the actual building construction.

### 5.2.1 A Poisson Point Process Based Network Model

A Point Process is a central concept of stochastic geometry [13] and is defined as the set of points (wireless nodes) in space whose spatial coordinates follow a probabilistic distribution. A Poisson Point Process (PPP) is generally considered to model node distribution in wireless networks as it realistically captures spatial patterns of various network entities such as distribution of base stations and mobile users in a cellular network and that of nodes in wireless ad hoc and sensor networks observed in real-life scenarios [48].

For simplicity, we will assume that the node density is constant throughout the deployment region shown in Fig. 5.1. This is the most basic PPP and is referred to as the Homogenous PPP. Let $A$ denote the area of the deployment region (in $m^2$) and $\lambda'$ be the constant node density (in nodes/$m^2$). Also let $\lambda = \lambda' A$. We implement a Homogeneous PPP based network model in a 2-Dimensional euclidian space as follows:
Chapter 5. Topology Control in an Indoor Wireless Sensor Network

Figure 5.1: Bahen Digital Building Model

- Generate the number of nodes, $n$, to be deployed at the target site from a random variable, $N$, following a Poisson distribution:

$$P(N = n) = \frac{\lambda^n}{n!} \exp^{-\lambda}$$  \hspace{1cm} (5.1)

- The $x$ and $y$ coordinates of the $n$ nodes are then independently and identically generated from a uniform distribution with $x \in [x_{\text{min}}, x_{\text{max}}]$ and $y \in [y_{\text{min}}, y_{\text{max}}]$, where $x_{\text{min}}$, $x_{\text{max}}$, $y_{\text{min}}$ and $y_{\text{max}}$ are the end coordinates of the deployment region shown in Fig. 5.1.

$\lambda$ denotes the average number of nodes deployed at the target site. We consider the following values of $\lambda$ for the connectivity analysis: 8, 10, 12, 14, 16. The Volcano Indoor Multi-Wall model does consider wall/partition thickness to compute total signal attenuation through the material, but the digital building model shown in Fig. 5.1 uses infinitesimal thick walls/partitions. This results in a step increase in the path loss when the signal penetrates through the partitions and the possibility of a node being placed in the wall due to random spatial distribution is negligible.
5.2.2 Effect of Node Transmission Power Level on Network Connectivity

Figures 5.2–5.4 show a qualitative analysis of the average network behaviour for varying levels of network connectivity. Probability estimations are obtained for network $k$-Connectivity, $k \in \{1, 2, 3\}$, by means of 250 Monte Carlo simulations for the four path loss models. For a given path loss model, 101 communication graphs are obtained (as 101 transmission power levels are considered) for a given realization of the random node distribution with all nodes transmitting homogeneously at the same minimum transmission power level. Receiver sensitivity, $\beta_{th}$ is set to $-80.0$ dBm. Graph connectivity is computed using the algorithm implemented in Section 4.2.

As can be observed, the minimum power level ensuring a fixed probability of network $k$-Connectivity, $k \in \{1, 2, 3\}$, is different for different wireless channel conditions. From Fig. 5.2, for a 90% probability of the network to be 1-Connected, considering Volcano model based predictions as a benchmark for comparison, a 7.51 times increase in the power level based on Unit Disk model is required to more realistically model the minimum transmission power level for this simplistic path loss model. In order to compute
this factor, dBm values are first converted to their respective mW equivalents for different path loss models for a 0.9 probability of network 1-Connectivity. Similarly, a 2.76 times increase for Log-normal model and a 0.51 times decrease in the power level for WINNER II model is required. The respective scaling factors for 2-Connectivity case are: 13.96 times increase for Unit Disk, 3 times increase for Log-normal, 0.4 times decrease for WINNER II model, and for 3-Connectivity case are: 28.85 and 3.46 times increase for Unit Disk and Log-normal respectively, assuming 1 mW is the desired power setting for Volcano model, although it is an approximation since the required power level exceeds the considered range of 0 to $-25$ dBm, as can be seen from Fig. 5.4. With this range of transmission power levels, we cannot obtain a power level for WINNER II resulting in 90% probability of the network to be 3-Connected. The results from Section 3.4.2 and this network $k$-Connectivity analysis, $k \in \{1, 2, 3\}$ for different path loss models shows that the Unit Disk and Log-normal Shadowing models not only perform inaccurate propagation predictions in an indoor environment but the dependence of network backbone construction on these physical layer models also results in significant deviations from the benchmark. That is, the path loss prediction errors of Unit Disk and Log-normal Shadowing models does translate into large errors in transmission power estimates for dif-
Figure 5.4: 3-Connectivity Probabilities for Unit Disk, Log-normal, WINNER II and Volcano at $\lambda = 14$

The effect of node density on network connectivity is observed in Figs. 5.5–5.6. As observed from Figs. 5.5–5.6, for all path loss models, an increase in the network connectivity level requires an increase in the minimum transmission power level to ensure the same probability of network connectivity. On the other hand, an increase in node density increases the probability of network $k$-Connectivity, at a fixed transmission power level. For example, for the Log-normal model, at a $-10$ dBm power setting, network 1-Connectivity probabilities are 0.8, 0.9 and 0.94 respectively for $\lambda = 12, 14$ and 16.

Since the objective of a Topology Control protocol is to reduce the energy consumption of the network without compromising network connectivity, the confidence intervals reported in Table 5.1 further strengthen the necessity of exercising Topology Control in an indoor environment. It can be observed that even in the worst case scenario when the connectivity requirement is set to maximum and the node density is set to minimum, i.e. when $k = 3$ and $\lambda = 8$, for all channel models, the mean minimum transmission power
Figure 5.5: $k$-Connectivity Probabilities for $k \in \{1, 2, 3\}$ for Unit Disk Model and Log-normal Shadowing Model at Varying Node Densities
Figure 5.6: $k$-Connectivity Probabilities for $k \in \{1, 2, 3\}$ for WINNER II Stochastic Channel Model and Volcano Indoor Multi-Wall Model at Varying Node Densities
level is lower than the maximum allowable and commonly used 0 dBm power setting. Furthermore, an increasing $k$ with a constant $\lambda$ and a decreasing $\lambda$ with a constant $k$ lead to higher ranges of transmission powers and vice versa. The difference in the upper power limit of worst case scenario and the lower power limit of the best case scenario (when $k = 1$ and $\lambda = 16$) for Unit Disk, Log-normal, WINNER II and Volcano are 6.35 dBm, 13.95 dBm, 12.78 dBm and 12.22 dBm respectively. This is primarily due to the varying levels of randomness involved in the computation of path loss values using different channel models, with the lowest for Unit Disk (and hence least spread out confidence intervals) and the highest for Log-normal (and hence most spread out confidence intervals).

Table 5.1: 95% Confidence Intervals (in dBm) for Mean Minimum Transmission Power Levels under varying node densities ($\lambda$) and connectivity requirements ($k$)

<table>
<thead>
<tr>
<th>$k, \lambda$</th>
<th>Unit Disk</th>
<th>Log-normal</th>
<th>WINNER II</th>
<th>Volcano</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1, \lambda = 8$</td>
<td>$[-18.02, -17.70]$</td>
<td>$[-14.21, -13.74]$</td>
<td>$[-7.43, -6.37]$</td>
<td>$[-10.22, -9.37]$</td>
</tr>
<tr>
<td>$k = 1, \lambda = 10$</td>
<td>$[-18.48, -18.33]$</td>
<td>$[-16.00, -15.03]$</td>
<td>$[-8.55, -7.33]$</td>
<td>$[-11.24, -10.74]$</td>
</tr>
<tr>
<td>$k = 1, \lambda = 12$</td>
<td>$[-19.43, -19.02]$</td>
<td>$[-17.14, -16.43]$</td>
<td>$[-11.35, -8.76]$</td>
<td>$[-12.95, -10.79]$</td>
</tr>
<tr>
<td>$k = 1, \lambda = 14$</td>
<td>$[-19.94, -19.58]$</td>
<td>$[-18.20, -18.05]$</td>
<td>$[-12.54, -9.77]$</td>
<td>$[-14.05, -12.54]$</td>
</tr>
<tr>
<td>$k = 2, \lambda = 8$</td>
<td>$[-15.63, -15.44]$</td>
<td>$[-9.50, -8.18]$</td>
<td>$[-2.95, -2.15]$</td>
<td>$[-5.43, -4.64]$</td>
</tr>
<tr>
<td>$k = 2, \lambda = 10$</td>
<td>$[-16.49, -16.34]$</td>
<td>$[-11.68, -10.57]$</td>
<td>$[-3.78, -3.14]$</td>
<td>$[-6.97, -6.27]$</td>
</tr>
<tr>
<td>$k = 2, \lambda = 12$</td>
<td>$[-17.34, -17.15]$</td>
<td>$[-12.79, -11.98]$</td>
<td>$[-5.46, -4.60]$</td>
<td>$[-8.64, -6.44]$</td>
</tr>
<tr>
<td>$k = 3, \lambda = 8$</td>
<td>$[-13.98, -13.73]$</td>
<td>$[-5.91, -4.90]$</td>
<td>$[-0.60, -0.33]$</td>
<td>$[-2.73, -1.88]$</td>
</tr>
<tr>
<td>$k = 3, \lambda = 10$</td>
<td>$[-15.15, -14.87]$</td>
<td>$[-7.76, -7.37]$</td>
<td>$[-1.38, -0.94]$</td>
<td>$[-3.53, -3.21]$</td>
</tr>
<tr>
<td>$k = 3, \lambda = 12$</td>
<td>$[-15.87, -15.62]$</td>
<td>$[-9.81, -8.68]$</td>
<td>$[-2.55, -1.95]$</td>
<td>$[-5.17, -3.61]$</td>
</tr>
<tr>
<td>$k = 3, \lambda = 14$</td>
<td>$[-16.67, -16.45]$</td>
<td>$[-11.72, -10.62]$</td>
<td>$[-4.06, -3.31]$</td>
<td>$[-6.95, -5.45]$</td>
</tr>
<tr>
<td>$k = 3, \lambda = 16$</td>
<td>$[-17.02, -16.85]$</td>
<td>$[-12.21, -11.85]$</td>
<td>$[-4.90, -3.95]$</td>
<td>$[-7.41, -6.30]$</td>
</tr>
</tbody>
</table>
5.3 A Simple Topology Control Scheme

The majority of WSN applications require communication of the sensed phenomenon from all sensing nodes to a particular sink. The sink node subsequently communicates the information to a base station for further processing and analysis. We therefore assume such an application scenario and consider the “All-to-One” network traffic flow.

In order to satisfy such a data flow requirement, we need a slight modification to Algorithm 4 in Section 4.4.3 to construct the power optimized fault-tolerant topologies: Instead of considering every vertex \( v \in V \) as the root vertex \( r_0 \) to construct the Underlying Graph \( H_v \), we now consider only the sink node \( s \), to which every other node in the network communicates the sensed data to, as \( r_0 \). For the \( k = 1 \) case, however, the original MST based formulation of Section 4.3 still holds true to construct the optimized singly connected topology. In other words, only an undirected MPkIS for \( k \in \{2, 3\} \) with \( s \) as the root vertex needs to be constructed for the All-to-One network traffic scenario. Furthermore, the resultant undirected MPkIS should not be a restricted MPkIS with the degree of \( s \) restricted to \( k \). This is due to the possibility that more than \( k \) nodes in the network can have only a single direct connection to \( s \), in which case, enforcing this restriction will lead to network disconnection as it requires only \( k \) nodes to be connected to the sink. Optimizations 2 and 4 in Section 4.4.1 to compute \( M_{free} \) and \( M^\# \) are therefore modified as follows:

- While computing \( M_{free} \), do not modify the ground set \( A_0 \): All possible subsets of \( A_0 \) are in \( I_{M_{free}} \).

- While computing \( M^\# \), with all other optimizations intact, consider only those subsets of \( A^* \) that have \( k(|V| - 1) - \text{root out edges} \leq \text{cardinality} \leq k(|V| - 1) - k \), where \( \text{root out edges} \) are the number of out-edges of \( s \) in the Bi-direction \( D \) of \( G \).

The All-to-One application scenario further requires computing the vertex connectivity with respect to only the sink vertex \( s \), rather than computing the vertex connectivity of the entire graph. Let \( K'(G) \) represent the vertex connectivity with respect to \( s \). The algorithm implemented in Section 4.2 therefore needs to be modified as follows:

- Set \( K'(G) = \min \{ k(s, u) \mid u \in V - \{s\} \text{ and } u \notin N(s) \} \), where the notations have their usual meanings as given in Section 4.2.2.

The following steps illustrate Topology Control in an indoor wireless sensor network with pre-specified connectivity requirement \( k \):
1. Network Setup: Deploy the nodes in the indoor environment as shown in Fig. 5.1. Compute the path losses between all possible node pairs using the Calibrated Volcano Indoor Multi-Wall model.

2. Topology Construction:

   (a) Let $G_{P_{tx}} = (V, E_{P_{tx}})$ denote the communication graph obtained when all nodes transmit homogeneously at the same power level $P_{tx}$. Set $P_{tx} = P_{max} = 0$ dBm. An undirected edge $e = (u, v)$ is added to $E_{P_{tx}}$ at transmission power $P_{tx}$ iff $P_{r(v)}(u)$ and $P_{r(u)}(v)$ are both $\geq \beta_{th}$, where $\beta_{th}$ is the receiver sensitivity and $P_{r(v)}(u)$ is the received signal strength at vertex $u$ due to $v$ and $P_{r(u)}(v)$ is the received signal strength at vertex $v$ due to $u$.

   (b) Continue decrementing $P_{tx}$ by 0.25 dBm unless $K(G_{P_{tx}})' \geq k$ and $K(G_{P_{tx}-0.25})' \leq k - 1$

3. Topology Optimization: If $k = 1$, call Algorithm 1 with $G_{P_{tx}}$ as the input graph $G$. Else if $k \in \{2, 3\}$, call the modified Algorithm 4, again with $G_{P_{tx}}$ as the input graph $G$. Finally set the optimized transmission power level of nodes using Eqs. 4.6 and 4.7.

Unlike the Topology Construction phase, the Topology Optimization phase results in heterogeneous transmission power levels. Let $P_{tx}$ denote the node transmission power levels after the Topology Construction phase computed using Eq. 4.7 and $P_{tx, opt}$ denote the corresponding values after the Topology Optimization phase.

5.4 Results

Figure 5.7 shows the power optimized singly connected and fault-tolerant topologies (edges in red constitute the final solution) in a network of 8 nodes deployed in the indoor environment shown in Fig. 5.1. The topologies are obtained using the Topology Control scheme described in the previous section, considering an All-to-One network traffic flow with $\beta_{th} = -80.0$ dBm. Node “0” is the sink vertex to which all other nodes intend to send the network traffic. Path loss values shown on the edges are computed using the calibrated Volcano Indoor Multi-Wall model. Table 5.2 reports for different connectivity levels $k$, the power of the topologies computed using Eq. 2.1 for the base graph (with edges in black included) after Topology Construction phase and the power-optimized graphs obtained after the Topology Optimization phase. As can be observed, reduction in $P(G)$ is obtained when the Topology Optimization phase is implemented for all values
Figure 5.7: Power Optimized All-to-One $k$-Connected Network Topologies with Node 0 as the Sink
of $k$. This means that the sensor nodes will have lower (heterogeneous) transmission power levels which will result in network lifetime enhancements as compared to the (homogeneous) transmission power levels at which the node operates after the Topology Construction phase. This reduction in $P(G)$ is due to the removal of energy-inefficient edges from the network topologies which had higher edge weights (i.e. higher path loss values) without compromising network connectivity. In Section 5.4.3, we will see the effect of these different Base and Optimized graph powers on network lifetime. Network simulations using Opnet modeler [49] have been performed considering the generated topologies as the underlying networks/routing graphs to perform data routing from the 7 source nodes (nodes 1−7) to the sink (node 0).

<table>
<thead>
<tr>
<th>$k$</th>
<th>Base Graph</th>
<th>Optimized Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>528.79</td>
<td>515.54</td>
</tr>
<tr>
<td>2</td>
<td>583.54</td>
<td>559.88</td>
</tr>
<tr>
<td>3</td>
<td>597.62</td>
<td>592.99</td>
</tr>
</tbody>
</table>

### 5.4.1 Node Power Consumption Model and Simulation Setup

A typical wireless sensor node consists of a number of hardware components designed to accomplish several network related activities, such as a radio transceiver module, sensing module and a processing module. In order to account for the reduction in energy resources of a sensor node during network operation, we consider the following basic yet realistic node power consumption model in the network simulations:

$$P_{\text{Total}} = P_R + P_T + P_{\text{Gen}} + P_{\text{Idle}}$$  \hspace{1cm} (5.2)

where $P_R$ and $P_T$ are respectively the power consumption by the radio transceiver during packet reception and transmission, $P_{\text{Gen}}$ is the power consumption by the processing module while generating packets and $P_{\text{Idle}}$ is the power consumption when the node is idle. For a CC2420 radio transceiver, commonly used for sensor network applications, operating at 2.4 GHz, we consider the following values of $P_R$ and $P_T$ reported in [50]:

$$P_R = P_{R_0} = 59.1 \text{ mW}$$

$$P_T = P_{T_0} + \frac{P_{tx}}{\eta}$$
where $P_{r_0} = 26.5$ mW, $P_{tx}$ (in mW) is the distance-dependent node transmission power level and is the same as that mentioned in Section 5.3. Obviously, $P_{tx} = P_{txopt}$ for the power optimized topologies. $\eta$ is the drain efficiency of the radio transmitter and is directly proportional to $P_{tx}$. $\eta$ as a function of $P_{tx}$ is again reported in [50] and we approximate it as:

$$\eta = 0.028P_{tx} + 0.0104$$

$P_{Idle}$ is set to $60 \mu$W, as given in the Crossbow’s MICAz datasheet, considering a 3 V voltage supply. For $P_{Gen}$, we consider an arbitrary value of 3 mW.

All network simulations are performed in Opnet Modeler [49]. We assume that the sensor nodes are capable of simultaneously receiving packets from multiple sources. For the topologies shown in Fig. 5.7, we consider that all the sensor node devices, except the sink node, i.e. all source nodes are powered by 2 AA batteries, each with a voltage rating of 1.5 V and a current output of 1500 mAh. Therefore, the starting energy reserve for each such node is 16200 Joules. For the sink node, we assume that it operates with unlimited energy resources. Remaining simulation parameters are listed in Table 5.3.

Table 5.3: Network Simulation Parameters

<table>
<thead>
<tr>
<th>Packet Size</th>
<th>1 Kb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Rate</td>
<td>32 Kbps</td>
</tr>
<tr>
<td>Transmission Rate</td>
<td>3, 4, 5 packets/s</td>
</tr>
<tr>
<td>Slot Duration</td>
<td>0.03125 s</td>
</tr>
<tr>
<td>Simulation Time</td>
<td>24 Hours</td>
</tr>
</tbody>
</table>

Slot Duration refers to the time slot after which a node extracts a packet from its queue (if the queue is not empty) and sends it to a neighbor node listed in its routing table. Transmission Rate means the rate at which every source node in the network generates packets to be communicated to the sink node. Packet Size specified in Table 5.3 includes both the payload and the header information. The packet header contains such information as the node id of the source, node id of the destination and time stamp at which packet is generated by the source node.

### 5.4.2 Performance Metrics and Routing Protocols Considered

For performance evaluation of the topologies, we consider and hence define the following performance metrics:
End-to-End Delay

End-to-End Delay (in seconds) is defined as the difference between the time stamp at which a packet arrives at the queue of the sink node and the time stamp at which that packet is queued at the source node for transmission. This metric is updated on a per-packet basis at the sink node.

Node Remaining Energy

Remaining energy of a source node at time $t$ is computed as the difference between the node energy reserve at the beginning of network operation and the total energy consumption until the $t$th time instant of network operation. Energy consumption is computed using the node power consumption model and the simulation parameters described in Section 5.4.1. This metric is updated per second during the network simulation time.

Network Lifetime

We define Network Lifetime as the time it takes $k$ source node(s) ($k \in \{1, 2, 3\}$) to fail (in other words, to completely exhaust their energy resources) in a $k$-Connected network topology. It should however be noted that failure of $k$ nodes in a $k$-Connected network does not necessarily disconnect the network. For example, for the All-to-One 2-Connected network topology shown in Fig. 5.7b, removal of nodes 6 and 7 still keeps the network connected.

To route packets from the source nodes to sink, we consider the following routing schemes:

Multi-Path Routing

Multi-Path routing (or $k$-Paths routing) will be valid only when $k \in \{2, 3\}$ since when $k = 1$, only a single path exists from every source node in the network to the sink. Furthermore, the power optimized singly connected network topology using the MST formulation shown in Fig. 5.7a directly provides the routing paths from the source nodes to the sink.

Now, $k$-Paths routing is performed by selecting the $k$ node-disjoint paths from every source node in the network to the sink such that the sum of the edge weights along the $k$ paths is minimum. The $k$ paths are equally utilized while transmitting packets in order to achieve load balancing.
Energy-Aware Routing

This routing scheme is again valid only for the $k \in \{2, 3\}$ case. We implement Energy-Aware routing as follows:

- All source nodes directly transmit to the sink if a direct communication link exists (i.e. an edge in the optimized network topologies shown in Fig. 5.7 exists) between a given source node and the sink.

- For source nodes which can only communicate using multiple hops to the sink, the $k$ node-disjoint paths are chosen probabilistically. In order to compute the probabilities of the $k$ paths to route packets from the source nodes to the sink, the average (mean) remaining energy per intermediate node of the $k$th path is computed (intermediate nodes are the nodes except the source node and the sink in the $k$th path), let us denote it by $E_m$, where $m \in [1, k]$. Then the probability $p_m$ with which the $m$th of the $k$ paths is chosen to transmit packets is computed as follows:

$$p_m = \frac{E_m}{\sum_{i=1}^{k} E_i}$$

For $k$-Paths routing, $p_m = \frac{1}{2}$ for $k = 2$ and $p_m = \frac{1}{3}$ for $k = 3$ as all $k$ paths are equally utilized to route packets from the source nodes to the sink, as stated earlier. This holds true even if the source nodes have a direct link to the sink.

5.4.3 Performance Evaluation

Table 5.4 shows the overall (for all source nodes) and node 1, 2 and 7 mean End-to-End Delays, mean E2E delays, (in seconds) for all connectivity levels, routing schemes and packet transmission rates (Tx Rate in packets/sec). Along any given row, E2E increases from left to right, i.e. it increases with an increase in the Tx Rate. It can be seen that an increase in the connectivity level generally reduces the E2E delays except for the case when $k$ changes from 1 to 2 with Tx Rates = 3, 4 packets/sec for node 1. This is because for the singly connected All-to-One network topology, node 1 always transmits its packets directly to the sink. But, when $k$ changes to 2, Multi-Path routing causes an increase in the E2E delay. Multi-Path routing is however essential in order to reduce the load on the link between node 1 and the sink node 0 as it acts as a bottleneck when the connectivity level is lowest ($k = 1$) and Tx Rate is highest (5 packets/sec), leading to a mean E2E delay of 3753.65 sec, shown in Table 5.4b. Nodes furthest away from the sink in a singly connected network topology, e.g. node 7 in Fig. 5.7a corresponding to Table 5.4d, will
have the best E2E delay improvements as compared to other source nodes when network connectivity increases. This is because they now have shorter, multiple node-disjoint paths to route packets as opposed to the only available comparatively longer path in a singly connected topology.

Table 5.4: Mean End-to-End Delay (in seconds)

(a) Overall

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>Tx Rate = 3</th>
<th>Tx Rate = 4</th>
<th>Tx Rate = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>0.2463</td>
<td>0.3259</td>
<td>3754.78</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>0.1985</td>
<td>0.2104</td>
<td>0.2299</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>0.1560</td>
<td>0.1625</td>
<td>0.1714</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>0.1564</td>
<td>0.1625</td>
<td>0.1706</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>0.1126</td>
<td>0.1155</td>
<td>0.1190</td>
</tr>
</tbody>
</table>

(b) Node 1

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>Tx Rate = 3</th>
<th>Tx Rate = 4</th>
<th>Tx Rate = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>0.0741</td>
<td>0.1329</td>
<td>3753.65</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>0.1893</td>
<td>0.2016</td>
<td>0.2211</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>0.0571</td>
<td>0.0617</td>
<td>0.0677</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>0.1195</td>
<td>0.1249</td>
<td>0.1324</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>0.0531</td>
<td>0.0556</td>
<td>0.0584</td>
</tr>
</tbody>
</table>

(c) Node 2

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>Tx Rate = 3</th>
<th>Tx Rate = 4</th>
<th>Tx Rate = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>0.3202</td>
<td>0.4026</td>
<td>3756.02</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>0.1911</td>
<td>0.2039</td>
<td>0.2242</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>0.0573</td>
<td>0.0623</td>
<td>0.0691</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>0.1846</td>
<td>0.1915</td>
<td>0.2005</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>0.0531</td>
<td>0.0557</td>
<td>0.0587</td>
</tr>
</tbody>
</table>

(d) Node 7

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>Tx Rate = 3</th>
<th>Tx Rate = 4</th>
<th>Tx Rate = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>0.3875</td>
<td>0.4727</td>
<td>3751.34</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>0.1881</td>
<td>0.1982</td>
<td>0.2145</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>0.1837</td>
<td>0.1903</td>
<td>0.2000</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>0.1828</td>
<td>0.1889</td>
<td>0.1964</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>0.1784</td>
<td>0.1815</td>
<td>0.1855</td>
</tr>
</tbody>
</table>
Load balancing when $k \in \{2, 3\}$ (which implicitly means Multi-Path routing) is evident from Tables 5.4a–5.4d for the highest Tx Rate as a significant drop in the E2E delay is observed when $k$ changes from 1 to 2. Improvements in E2E delay with higher connectivity levels are prominent when packet transmission rate of the source nodes is high. On the other hand, Energy-Aware routing, as defined in the previous section, further result in improvements in E2E delays over the Multi-Path routing scheme.

We now analyze the variation in energy consumption of 3 source nodes for 24 hours of network operation when operated under varying network connectivity levels, routing schemes and node transmission rates. Figures 5.8–5.10 shows the results. Here, MP refers to Multi-Path routing and EA refers to Energy-Aware routing, which, as stated earlier, are not valid when $k = 1$. We illustrate the effect of different transmission rates (Tx Rate = 3, 5 packets/sec) on node energy consumption only for one of the source nodes i.e. Node 1 since similar effects hold true for the rest of the source nodes as well. From Fig. 5.8, it can be seen that an increase in the Tx Rate from 3 to 5 packets/sec for a given $k$ and routing mechanism depletes the energy reserve of Node 1 faster.

![Figure 5.8: Remaining Energy of Node 1 after 24 Hours of Network Operation](image)

Nodes closest to the sink will have highest positive impact on remaining energy when $k$ changes from 1 to a higher level due to load balancing. This can be observed from Fig. 5.8. Remaining energy of Node 1 increases when $k$ changes from 1 to 2. A slight
Figure 5.9: Remaining Energy of Node 2 after 24 Hours of Network Operation

Figure 5.10: Remaining Energy of Node 7 after 24 Hours of Network Operation
improvement is further observed in a 3-Connected network, as compared to a 2-Connected network. Nodes 2 and 7, which are furthest away from the sink in the singly connected network, however show opposite trends when $k$ increases from 1 to 2 shown in Figs. 5.9 and 5.10 respectively. This is because when $k = 2$ they start routing packets from the neighboring nodes, which is otherwise not the case when $k = 1$. On the other hand, Energy-Aware routing results in better energy reserves of the source nodes, as compared to the Multi-Path routing scheme. This improvement enhances when Tx Rate increases, refer to Fig. 5.8.

Table 5.5 lists the network lifetime, as defined in Section 5.4.2, when the network operates with and without Topology Control. $P_{tx}$ and $P_{tx_{opt}}$ respectively represent node transmission power levels obtained after the execution of Topology Construction and Topology Optimization phase described in Section 5.3. $P_{tx_{max}}$ represent the power level resulting in a maximum power communication graph in which all source nodes homogeneously transmit at the maximum allowable power levels. We assume $P_{tx_{max}} = 0$ dBm. Network simulations are still performed with the same optimized communication graphs (i.e. same communication links) shown in Fig. 5.7. Enhancements in network lifetime are evident from Table 5.5 when Topology Control is exercised. Network lifetime enhancements are also obtained when $k$ increases or when Energy-Aware routing is employed instead of Multi-Path routing.

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>$P_{tx_{opt}}$</th>
<th>$P_{tx}$</th>
<th>$P_{tx_{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>86.0</td>
<td>80.42</td>
<td>66.10</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>140.33</td>
<td>133.82</td>
<td>122.73</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>174.10</td>
<td>156.39</td>
<td>143.16</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>191.54</td>
<td>182.57</td>
<td>171.74</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>261.61</td>
<td>250.53</td>
<td>234.75</td>
</tr>
</tbody>
</table>

Percentage improvement in the network lifetime with respect to the Maximum Power Communication Graph is shown in Table 5.6, corresponding to Table 5.5.

Figure 5.11 shows the energy consumption of all source nodes in the network when $k = 3$, considering Energy-Aware routing and a Tx Rate of 3 packets/sec. As can be observed, nodes closest to the sink for the topology shown in Fig. 5.7c, i.e. Nodes 1, 2 and 3 have the highest rate of depletion in energy reserve and nodes furthest away from sink, i.e. Nodes 6 and 7 have the lowest.
Table 5.6: % Increase in Network Lifetime w.r.t. Maximum Power Communication Graph

<table>
<thead>
<tr>
<th>Connectivity and Routing</th>
<th>$P_{tx_{opt}}$</th>
<th>$P_{tx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>30.11</td>
<td>21.66</td>
</tr>
<tr>
<td>$k = 2$, Multi-Path</td>
<td>14.34</td>
<td>9.04</td>
</tr>
<tr>
<td>$k = 2$, Energy-Aware</td>
<td>21.61</td>
<td>9.24</td>
</tr>
<tr>
<td>$k = 3$, Multi-Path</td>
<td>11.53</td>
<td>6.31</td>
</tr>
<tr>
<td>$k = 3$, Energy-Aware</td>
<td>11.45</td>
<td>6.72</td>
</tr>
</tbody>
</table>

Figure 5.11: Remaining Energy of All Sensor Nodes for Complete Network Lifetime Operation at $k = 3$, Energy Aware Routing, Tx Rate = 3 packets/sec

The network lifetime in this case is 941798 seconds (or 261.61 hours) computed according to the network lifetime metric defined in Section 5.4.2, i.e. the network aborts operation as soon as $k = 3$ nodes completely exhaust their energy resources. In this case the three nodes to fail due to complete battery discharge are Nodes 2, 3 and 1, mentioned in the order in which they fail. The change in the energy depletion rate of some source nodes when other nodes start to fail is evident from Fig. 5.12.
5.5 Summary

Significant energy savings are possible in a sensor network deployed in an indoor office setup by allowing sensor nodes to operate at lower transmission power settings without compromising network connectivity. This is evident from our analysis of average network behaviour over spatial realizations. We also validated the necessity of exercising Topology Control to achieve network longevity by means of network simulations. Also, results indicate that higher network connectivity levels in a sensor network, with an obvious advantage of fault-tolerance, lead to better load balancing and hence further enhancements in network lifetime and lesser End-to-End delays at higher packet transmissions rates.
Chapter 6

Conclusions and Future Work

Accurate modelling of a radio channel is necessary for the analysis and design of practical wireless sensor network protocols. Furthermore, proposition of energy-efficient and fault-tolerant network topologies is essential for enhancing network lifetime and making the network resilient to node failures. In this work, we proposed and implemented a Topology Control scheme for a sensor network deployed in an indoor environment by making realistic assumptions about the physical layer model.

We first validated that radio propagation models that make simplistic assumptions about the actual channel conditions do not accurately capture the radio channel randomness that is inherent in complex propagation environments. We characterized channel parameters for different propagation scenarios based on field measurements performed in an indoor office setup. We observed that path loss exponents of the Unit Disk model and shadow fading standard deviations of the Log-normal Shadowing model increase with increasing distances from the transmitter. These parameters are lower for LOS scenarios as compared to NLOS scenarios. Based on the characterized channel parameters, we then take the measurement data as a benchmark for performance evaluation to investigate the degree of accuracy of the path loss models. As opposed to Unit Disk and Log-normal Shadowing models, Calibrated as well as Uncalibrated Volcano Indoor Multi-Wall model and the WINNER II Stochastic channel model provided more accurate predictions of actual path loss measurements with their respective RMSE values of 8.12, 10.62 and 10.94 dB.

We then constructed power optimized singly connected and fault-tolerant topologies that serve as a building block for the proposed Topology Control scheme. We considered a Minimum Spanning Tree based formulation to construct minimum power singly connected network topology providing a 2 approximate solution to the optimum power assignment problem for a singly connected network. A number of directed/undirected
spanning subgraphs were constructed that satisfy different connectivity criterion to finally build power optimized fault-tolerant topologies. Specifically, we employed a Matroid Intersection based approach to construct Minimum Cost Rooted $k$-Outconnected Spanning Subgraph which serves as the primary step in the construction of a $k + 1$ approximate solution to the Minimum Power $k$-Connected Spanning Subgraph problem (or the power optimized fault-tolerant topologies) for the case when $k \in \{2, 3\}$.

Using the Monte Carlo method, we statistically estimated the probability of network $k$-Connectivity, $k \in \{1, 2, 3\}$ for a range of node transmission power levels. We employed Confidence Intervals to estimate the average minimum transmission power level of nodes that ensure, with high confidence, the construction of a $k$-Connected network topology. Such a network connectivity analysis for a specified propagation environment can facilitate the pre-deployment phase of a sensor network, e.g. network planning and dimensioning.

By means of network simulations performed in a discrete event simulator, the proposed Topology Control scheme was evaluated for the All-to-One network traffic scenario. We observed that a singly connected topology can cause bottlenecks in the network at high transmission rates and hence can result in very high End-to-End delays. Such bottlenecks however do not occur when there are multiple node-disjoint paths between the source and sink nodes, i.e. when the topology is 2 or 3-Connected, and hence results in load balancing with significantly lower End-to-End delays. We achieved a 1.63x increase and a 2.23x increase in Network Lifetime with a 2 and a 3-Connected network topology w.r.t singly connected network. Also, we obtained 30.11%, 14.34% and 11.53% increase in Network Lifetime w.r.t the maximum power communication graph for $k = 1, 2$ and 3 respectively, thus demonstrating the necessity of exercising Topology Control to achieve network longevity.

Extensions and improvements to the current work are possible both from a Channel Modelling as well as a Topology Control point of view. Field measurements can be extended to multiple floors and even multiple indoor environments to generalize the characterization of path loss models. SINR model can be used as the physical layer model to account for intra-network interference and inter-network interference arising from other wireless networks simultaneously operating in the same frequency bands. Network longevity achieved using power optimized topologies is traded off with the additional amount of communication and computational overhead added on the sensor nodes (which are generally equipped with limited processing capabilities) with their implementation. Practical validity of the proposed scheme using an experimental test bed therefore needs to be done. One of the limitations of the algorithm to construct Minimum Cost
Rooted $k$-Outconnected Spanning Subgraph is its applicability to base graphs with limited number of edges. Integration with graph partitioning algorithms can help address this issue. Making the power optimized fault-tolerant topologies scalable to large size networks is therefore an important future perspective. We also envisage integration of such energy-efficient wireless sensor networks with future wireless network technologies such as smart cities and internet-of-things.
Bibliography


[40] A. Clementi, P. Penna, and R. Silvestri, “Hardness results for the power range assignment problem in packet radio networks,” in Randomization, Approximation, and


