Secondary Teachers’ Perceptions of the Role of Student Discourse and its Impact on Students’ Abilities to Solve Word Problems

By

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A research paper submitted in conformity with the requirements For the degree of Master of Teaching Department of Curriculum, Teaching and Learning Ontario Institute for Studies in Education of the University of Toronto

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Abstract

Despite a developing interest in the need for student engagement with mathematics, student voice in the classroom through mathematical discussions has been minimal while attitude towards and competency in word problems have largely been poor. Given that both areas have an inherent linguistic component, it was investigated, through the perspectives of two experienced Ontario secondary mathematics teachers, whether there could be a relationship between student discourse in a cooperative learning setting and competence with word problems. The findings suggest that student-student discourse along with student-teacher discourse enhance a comprehension of mathematical representations through a practice of meaningful mathematics and code-switching. Additionally, procedural competence in solving word problems seems to be positively correlated with a promotion of conceptual connections and a variety in problem-solving strategies through student-student discourse. The findings present implications as well as recommendations for mathematics researchers and educators to infuse discourse in classrooms as a way to improve students’ performance with word problems.

Key Words: word problems, mathematical discourse, cooperative learning, code-switching, student attitudes
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Chapter 1: INTRODUCTION

1.1 Introduction to the Research Study

Over the last two decades, with the goal to improve students’ performance in problem-solving, there has been a growth in research along with an increased demand into shifting from traditional teacher-centered instruction to student-centered learning in mathematics. Resulting from this push, two interconnected approaches have come to the forefront of mathematics research: student discourse and cooperative learning. First, discourse among students in the classroom, here referred to as student-student discourse, seems to have multiple benefits for problem-solving. For instance, Barwell (2011) states that using “mathematical language helps students gain insights into their own thinking” in addition to helping them with justification, consolidation of new concepts, and improving memory, as they "develop and express their mathematical ideas and strategies, precisely and coherently to themselves and to others," all of which are crucial skills for effective problem-solving.

The second area of interest, cooperative learning, has also gained substantial popularity in the mathematics reform literature as an essential approach to improving students’ attitudes and competency in problem-solving. Based on work by Artz and Newman (1990) and Sutton (1992), Leikin and Zaslavsky (1999) define cooperative group tasks as having four necessary conditions, all of which must be met for the task to be distinguished from general collaborative work. These conditions require: (i) the learning to take place in groups of two to six members, (ii) members to mutually and positively depend on one another and on the group as a whole, (iii) an equal opportunity to be given to
members for interaction with each other through various ways, and (iv) each member, in a certain role, to be accountable for the learning progress as well as for the work expected. Commonly promoted to new teachers, the benefits of cooperative learning seem plentiful to problem-solving. For instance, the structure of cooperative tasks allows students to both view problem-solving as a social process and to be exposed to a variety of interpretations, strategies, representations, and explanations, all while being accountable to the learning process, given students’ different roles in such group work (Artz & Newman, 1990).

Despite the developing interest, however, in both student discourse and cooperative learning as to effectuate better problem-solving, a special branch of problems in mathematics, namely word problems, has continually been a source of considerable distress for students. Used in almost every mathematics classroom from as early as grade one to the end of high school and beyond, word problems have been identified as being one of the major challenges students encounter in mathematics. Results from Educational Quality and Accountability Office (EQAO) for the 2011-2012 academic year indicated that for Ontario mathematics students, explanation of thinking when solving word problems (open response problems) was the weakest of all areas analyzed. Thus, there seems to be an urgent need to investigate factors affecting Ontario students’ performance in word problems, and more importantly, to identify steps that can be taken by students and educators as to collaboratively work towards cultivating success in this area.

1.2 Purpose of the Study

As mentioned earlier, the merits of student discourse and of cooperative learning have each become widely theorized among researchers, and promoting both practices in
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mathematics classrooms seems to have positive correlations with efficient problem-solving. However, a gap in literature currently persists regarding the relationship of student discourse, specifically within a cooperative learning context, with word problem competence. One of the two reasons for an interest in cooperative tasks is that it is hoped that with the nature of cooperative activities, an interesting mix of accountability and freedom of exploration may arise in group work. Specifically, students may feel accountable for individual effort because their unique role in a cooperative learning setting may require a particular contribution to the group’s progress. At the same time, the group as a whole, may be presented with the opportunity to share a wide variety of interpretations, representations, explanations, and strategies from different members, thereby making problem-solving a dynamic and eclectic process for students. This combination of accountability and freedom may enhance student competence with word problems. Secondly, it is hoped that perceptions of teachers whose classroom discussions are enriched with student voice, as cooperative tasks tend to facilitate, will be closely-aligned with and act as a window into student perspectives.

This study then aims to analyze two overarching student challenges associated with word problems from a discourse perspective in a cooperative learning environment. They are: (i) *comprehension of mathematical representations*, which is linked to the idea of code-switching, and (ii) *procedural competence*, a term which I use here to refer to students’ proficiency in (a) making connections to prior knowledge and (b) developing various paths to one or multiple solutions. These two challenges are both explored from two secondary Ontario mathematics teachers’ perceptions on the nature of discourse in a cooperative learning setting. It is hoped that with an analysis of student discourse in such context,
potential links to the two main challenges will be made to shed light on this largely unexplored field of study. In addition, it is hoped that a discussion about teachers’ current pedagogical practices relevant to these issues will help other educators mitigate the distress associated with word problems.

1.3 Research Questions

Although conversations with the research participants were fairly open-ended in nature, a total of nine questions had initially been planned. Each set consisting of three questions, dealt with one of the three key areas of interest: word problems, cooperative learning, and student discourse. Presented below is the central question of this study followed by the related sub-questions:

In Ontario, what are secondary mathematics teachers’ perceptions of the role of student discourse in cooperative learning tasks and its impact on students’ ability to solve word problems?

Two sub-questions guided the analysis of the data collected:

1. **In a cooperative learning context, how does discourse play a role in addressing comprehension of word problems?**

   - What is the role of student-student discourse in decoding the mathematical representations in a word problem?
   - How do teachers manage the dilemma of mediation? In other words, how do they teach the formal mathematical language without necessarily diverting attention from conceptual understanding?
2. *In a cooperative learning context, how does discourse play a role in addressing inadequate procedural competence?*

- Specifically, what role does discourse play in helping students utilize prior knowledge to develop conceptual connections?
- How may discourse be involved in helping students explore a range of problem-solving strategies?

1.4 **Background of the Researcher**

My interest in this topic arises from three aspects of my life: my background as a mathematics student in the past, my status as a pre-service teacher in the present, and my anticipated teaching career in the future. Reflecting on all the mathematics classes I have taken, or as far back as I can remember, just a few instances of open discussions come to mind. In terms of student discourse, I cannot remember any debates or arguments around mathematics, though there may have been a few. I suppose then that I have largely been a part of the traditional, teacher-centered style of learning that has persisted for decades in mathematics.

Despite not being able to recall many instances of student discourse in my classes, I do, very clearly, remember teaching myself how to solve trigonometry problems in high school. In doing so, I would use self-talk to ask myself the problem the question was inquiring. Often whispering to myself in a library, I would then respond by stating the steps needed to get to the solution. As I continued practicing this strategy, not just in mathematics, but in other subjects such as chemistry and physics as well, I began to appreciate its value in helping with clarification and retention of concepts. Consequently,
when I began practicing this approach with many of the students I tutored, I noticed that doing so helped us assess how much of what I had taught was accurately learned and remembered by my students. This sparked my interest in student discourse in problem-solving, and I wondered how it could be incorporated when teaching secondary mathematics in a classroom setting, which is what I had planned to do when I enrolled into teacher education.

As I made the transition from undergraduate studies in life sciences to my graduate program at the Ontario Institute for Studies in Education (OISE), I became exposed to a variety of collaborative and cooperative group tasks in almost all of my classes. These approaches of learning, being rather unfamiliar for me, allowed me to critically analyze and reflect on their value. The social aspect of gaining knowledge through discussions and seminars with my classmates made the learning not only engaging, but it has reshaped my perception of what it means to teach. As such, my research question developed from my desire to speak to and learn from teachers who are highly experienced in facilitating cooperative tasks in their high school mathematics classes.

Finally, as a future teacher of mathematics, it was important for me, at the start of this project, to become informed about the challenges students are facing with word problems. Interviews with educators whose teaching is heavily shaped by a student-centered philosophy was an indirect route for me to gain insight into student perspectives. As I now complete my degree in Masters of Teaching, my interest in this research topic continues to grow, and it is my hope that others in the field of education will also benefit from a discussion of this topic as follows.
1.5 Overview

This Master of Teaching Research Paper (MTRP) has been organized into five chapters along with a section for Appendices, followed by References. Chapter 1 includes an introduction to the study, its purpose and significance, the research questions that pertain to the study, as well as my personal interest to be involved with this topic. Chapter 2 contains a review of the literature on challenges associated with word problems, registers used in mathematics, as well as on discourse in cooperative tasks. Chapter 3 outlines the methodology and procedures that have been followed throughout the duration of this study. This chapter provides a description of the participants as well as the methods and instruments that were used to collect and sort through the data pertaining to this study.

Following the interviews with the two participants, the findings which emerged from the data have been outlined in Chapter 4, organized by headings and sub-headings. Lastly, Chapter 5 includes a discussion of the findings, along with correlations with the research referenced in the literature review. The chapter concludes with a list of recommendations, limitations and further steps.
Chapter 2: LITERATURE REVIEW

2.1 Theoretical Frameworks

This chapter aims to review the literature regarding discourse in mathematics classrooms in relation to word problem competence among students. In presenting and discussing the works of educators and researchers on these topics, it is hoped that a potential relationship between the two areas can be examined for further study. Before the research is presented, however, this section will attempt to briefly introduce the reader to the theoretical frameworks influencing the research process.

An influence of postpositivism guides this study, as its approach to gather knowledge in the fields of interest specified above is rather scientific in nature, but considers the possibility of additional factors that could influence the relationship being explored, such as the biases and backgrounds of myself and the research participants. Additionally, an investigation of mathematics teachers’ perceptions about infusion of discourse in cooperative tasks and its relationship with students’ competency in solving word problems is guided by existing theories on cooperative learning; use of discourse as student engagement; formal and informal mathematical registers; switching between mathematical representation and language; and a shift from teacher-centered to student-centered instruction.

Moreover, critical theory plays a role in my personal interest to research these issues. It is hoped that assumptions behind existing literature about mathematics students, teachers, and instruction itself can be analyzed and exposed. This may, in turn, facilitate fostering a mathematics classroom culture that empowers students to take part in determining the means of their own learning.
2.2 Need for Student Discourse in Mathematics Classrooms

The past twenty years of research into mathematics learning and teaching has brought into focus the emergent need for students to become actively-engaged learners, if their attitudes towards and competency in mathematics are to be improved (NCTM, 1989, 1991, 1995, 2000). As such, proponents of mathematics reform education view learning as a social endeavor, enriched with discussions, disagreements, and justifications (Bruce, 2007). However, many mathematics classrooms tend to follow the traditional, teacher-centered, knowledge-transfer approach to teaching mathematics. Interestingly, upon viewing their videotaped lessons for research purposes, many teachers have expressed surprise at the extent of the class time they spend talking—up to eighty percent for some, which, as research shows, is not uncommon in mathematics classrooms (Hiebert et al. 2004). Due to this largely-accepted norm in mathematics teaching that has persisted for decades, student voice in the classroom has long been undervalued. In response, researchers and educators have suggested and attempted to adopt changes in the structure of the classroom that shift the focus from teacher to student.

One of the key strategies for increasing student involvement in mathematics has been through encouraging verbal discussions in the classroom. The study by Nesher, Hershkovitz, and Novotna (2003) shows that students who are unable to negotiate discourse while engaging in mathematical problem-solving are unable to move forward in their learning. Indeed, McNair (2000) reports that students will exhibit a mathematical refinement of concepts if those concepts are integrated into a class discussion. Moreover, Solomon (2009) states that when students hear more content in a discussion, they learn
more. The key point here is the common use by these researchers of the word ‘discussion’ when referring to classroom discourse.

It is important to note, however, as Forman and Ansell (2001) do, that what was referred to as a classroom discussion in the past had a different meaning than its current value; in the past, a discussion entailed the teacher asking a question, which would be answered by a student, and the teacher would, in turn, affirm or correct the response. This interaction pattern would be repeated throughout lessons in the form of an IRE sequence, consisting of Initiation, a student Response, followed by teacher Evaluation (Mehan, 1979). In their discussion of development of understanding in the classroom, Edwards and Mercer (2013) discuss the frequently observed case in which it is the teacher who asks the questions, who knows the answers, and whose repeated questions imply wrong answers. The natural effect of the IRE sequence then is that students learn to become reliant on the cues provided by the teacher at the expense of practicing their own skills of questioning and meaning-making.

In contrast to its historical denotation, the exigency for discussions in today’s mathematics classroom refers to having an environment where students can ask each other questions, answer them, and provide feedback, while the teacher’s role in such discussions is mainly to structure their talk (Solomon, 2009). As such, to make sense of mathematics in today’s classrooms, students require more opportunities to express their thoughts in their own words, not just to their teachers but to their classmates as well. Thus, in place of a recitation sequence between the teacher and the student, open discussions among students, which cultivate mutuality in perspective, may help to improve student performance in word problems.
2.3 The Problems with Word Problems

The National Council of Teachers of Mathematics has targeted problem-solving, communication, and comprehension as critical areas for improvement (NCTM, 2000). Specifically, word problems, according to Wirtz and Kahn (1982), require the use of reflective thought, trial and error, evaluation, and decision-making in addition to other high-level cognitive skills, attitudes, and behaviors. All of these demands, combined, can lead to an insurmountable frustration for many students. The NCTM Principles and Standards notes that students are not just required to have a conceptual and procedural understanding of the problem, but they also need to be able to select and apply appropriate mathematical representations, such as expressions, equations, illustrations, graphs, and models. Moreover, they need to use reasoning to decide upon an effective problem-solving strategy, and then give a justifiable statement as to their decision, while being able to communicate their findings to their classmates and teacher (Sarama & Clement, 2008).

Compounding these demands of efficiently solving word problems are factors that make them inherently difficult for students, which may be related to both the nature of the problem as well as to the way they are taught. First of all, word problems tend to use mathematical terminology, a lot of which is rarely explicitly taught in the class. In fact, research by Blessman and Myszczak (2001) shows that one of the four main causes of confusion with word problems is, in fact, the vocabulary. Fletcher and Santoli (2003) suggest that if students are not reading textbooks, then the resources available to learn such terms are quite limited. Even the textbooks themselves are often written at a higher reading level than what is customarily practiced for that grade level (Brennan and Dunlap, 1985). Amen (2006) adds to this, noting that rarely are there books available for children
that use the vocabulary of mathematics. Consequently, frustration over an inability to understand the mathematical language results in a poor student outlook on word problems and their performance therein (Sarama & Clement, 2008).

Another factor complicating the situation when solving word problems is the emphasis students have on rules and repetitions. Test results from the National Assessment of Educational Progress, as reported by Kroll and Miller (1993), demonstrated an acceptable mastery of computation skill by a group of students, yet many of these same students were unable to apply these skills to word problems. Cummins (1991) supports this, stating that many children have the required arithmetic skills, but their inexperience with the linguistic forms used in the problems impedes them from making connections between the problem and their preexisting mathematical knowledge.

Moreover, students often attempt to solve word problems in a rush to find an answer without fully understanding the problem (Roti, Trahey & Zerafa, 2000). They tend to read quickly, looking only for cluewords and numerical data without assessing the contextual information or applying their real-world knowledge (Sowder & Harel, 1998; Szetela, 1993). A key example of a premature application of arithmetic operations to a mathematical situation, irrespective of its context, is the tendency of many students to ‘solve’ the following simplified version of a question inspired by the novelist Gustave Flaubert: “On a boat, there are 20 sheep and 6 goats. How old is the captain?” The finding that many children respond to such questions simply by adding the number of sheep and goats speaks to their lack of appreciation for the language in the problem.

Then, it is precisely to develop both comprehension as well as comfort with the use of mathematical terminology that seems to be a vital goal to overcome challenges faced
with solving word problems. Specifically, given that much of the research suggests that the issues students are facing in mathematics are due to language-based misconceptions, it has been recommended that educators explicitly teach mathematical terminology to students (Blessman and Myszczak, 2001). Amen (2006) states that math should no longer be simply about numbers; the words and vocabulary must be taught as well, thus meeting the goals set forth by the National Council of Teachers of Mathematics (NCTM, 2000). On this note, Maikos-Diegnan (2000) states that math teachers have to teach students how to interpret vocabulary and comprehend the mathematical language, thus being reading teachers in addition to being mathematics teachers.

2.4 Dilemma of Mediation

Given that two of the major issues prevalent in today's secondary mathematics classrooms, as discussed earlier, are insufficient student discourse in class and poor competency in solving word problems, the suggestions from research presented above for each issue yet present a new dilemma for educators: on one hand, teachers promoting open classroom discourse may encourage students to share their thoughts and reasoning orally, thereby empowering student voice in the classroom. On the other hand, the requirement to explicitly correct imprecise or inaccurate terminology in students' speech for the sake of improving language comprehension in word problems may undermine the attention needed for the mathematical substance. Adler (1998) refers to this dilemma of what to leave implicit and what to make explicit as the dilemma of mediation.

Lave and Wenger (1991) similarly discuss this idea, which they refer to as the dilemma of transparency. They argue, as reported by Adler (1998), that access to a practice
relates to the dual visibility and invisibility of its resources—to their transparency. These resources, such as language in the mathematics classroom, need to be both visible so that it can be noticed and used, and at the same time, invisible so that attention can be directed to the subject matter, namely the mathematical concept.

They further state that learning, or mastery, in a community of practice involves learning to talk. Thus, learning mathematics entails speaking mathematically. Supporting the NCTM’s Communication Standard in the Principles of Standards for School Mathematics, which states that students should be able to “use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000, p. 268), Montague, Krawec and Sweeney (2008) note that the ability to use mathematics vocabulary appropriately, articulate mathematical concepts, and understand mathematical terms is an indicator of content mastery. Yet, in their study of 320 low- and average-performing middle school students, majority of the student responses analyzed indicated a lack of clarity and cohesion and an overall deficiency in basic math vocabulary. Such discouraging results are what force many teachers to then consider an explicit instruction of mathematical terminology, even if it interrupts students in their speech, to make ‘visible’ a correct or incorrect usage of a mathematical term.

At the same time, many teachers also feel that conceptualization by students should be left more implicit with room for creativity, thereby encouraging their intuitions and informal expressions of mathematical ideas (Adler, 1998, pp. 30, 32). In fact, Zazkis (2000), in teaching mathematics to pre-service teachers, found that many were using informal language in their classroom discussions, and a focus on the language over the content—on form over the substance—had the potential to obscure rather than support mathematical
practice. Moreover, she reports on the frustration of some of her students having to learn terminology in a mathematics classroom while she, as a teacher on the other hand, was reluctant to repeat students’ informal language while communicating with them.

In 1978, Halliday first discussed the concept of the mathematical register, which he defined as “the meanings that belong to the language of mathematics...and that a language must express if it is used for mathematical purposes” (p. 195). According to Pimm (1987), who further developed this idea, “registers have to do with the social usage of particular words and expressions, ways of talking but also ways of meaning” (pp. 108-9). The switching between the everyday or informal and the formal register that Zazkis discusses can be described as code-switching, a term which Adler (1998) originally used to refer to alternating between two languages in a single speech act. However, Zazkis’ use of the term refers to the alternation of two registers in a monolingual mathematics classroom, with the formal register predominantly practiced by the teacher, and the informal mainly by students.

In using the formal mathematical register to discuss the concept of divisibility, for example, a definition for a relationship where there is no remainder when A is divided by B may be of the form, “A is divisible by B.” In comparison, this may be commonly described as, “A can be divided by B” in the informal register (Zazkis, 2000). Due to the differences being subtle in the expressions, ‘being divisible by’ and ‘being divided by,’ student misconceptions can often develop as a result. For instance, students may believe that when a number can be divided by another, such as A by B or when “B goes into A,” there will be no remainder, which, obviously is not always true. Managing both registers in the classroom then may certainly become a challenge for many mathematics teachers, but as
the next section discusses, an acceptance of both registers, rather than an exclusive focus on transition from one to the other, certainly seems beneficial.

2.5 A Balance of Registers

Given the developing interest on an explicit instruction of mathematical terminology, teachers have been recommended to encourage their students to shift from using informal expressions of mathematical thinking to communication with a greater use of standard mathematical language. In his 2013 report, Richard Barwell discusses how the Ontario curriculum reflects this ideology with the expectation in Grade 4 that students be able to “communicate mathematical thinking [...] using everyday language [and] a basic mathematical vocabulary,” while by Grade 8, students are expected to use “mathematical vocabulary and a variety of appropriate representations” (OME, 2005, p.65). Barwell points to the replacement of words “everyday” and “basic” by “mathematical” in these expectations of vocabulary, as students move to higher grades. In contrast to what the curriculum implies, he argues that there exists an important relationship between both the formal and the informal registers of mathematical language in classrooms, and that this relationship is not uni-directional.

Many classrooms exhibit a gap between the natural language and the mathematical symbolism with which students interact (Morgan, 2004). However, Barwell, in citing Bakhtin’s theory of language, remarks that there is a continual tension between a centripetal force towards uniformity, or the unitary language of mathematics, and “a centrifugal force towards heteroglossia”—the variety that exists in language—referring to the diverse forms of expressions students may use (p. 75). With this tension, he aims to
highlight the dialogic perspective on formal and informal language in mathematics. He suggests that both registers are important, and the role of the informal language in class is not simply to function as a scaffold to nurture a more formal language development; instead, “there is a dialogue between the two” (p. 79).

The suggestion that the use of natural language in mathematics can be helpful has come to take on that meaning through a process of translation from one symbolic system to another in areas such as algebra. Steinbring et al. (1998) suggest that natural language can have a useful role in the development of algebraic problem-solving. They propose that a “syncopate style of operating symbolically while sustaining the associated reasoning in natural language can help students to develop meaning for the algebraic manipulation” (p. 257). They further claim that in classes where students are encouraged to use natural language in communication, students of “average verbal ability” are successful in algebraic problem-solving, whereas this result, in control classes, is only observed with students who demonstrate a “high verbal performance.” (p. 257).

In addition, arguing from a cultural perspective of language, Walkerdine (1988) challenges the notion that students’ natural language can be separated from their conceptualization of mathematics. She notes that the use of relational terms, such as more and less in math problems is closely tied to “regimes of meaning” that are produced in cultural practices or sets of discourses in which children are inserted, and which they bring with them into the classroom (p. 32). This line of thinking is built upon Vygotsky’s notion (1987) of a unified process of concept formation: “the development of spontaneous and scientific concepts are closely connected processes that continually influence each other. These two types of concepts...are not separated from one another by an impenetrable
wall...They interact continually” (p. 177). These ideas are, in fact, supported by research that highlights the importance of allowing students to use the natural, informal language in a mathematics classroom as they are exposed to and explicitly taught the formal terminology.

Winsor (2007), for instance, used journal writing in his mathematics classroom to foster student engagement and communication. Over one year, he noticed that students’ use of mathematical terms in their writing had begun to increase. Important to note is the fact that the students in his class, who were ELL learners, were “allowed to write in the language they felt most comfortable with, but they were [also] required to write the mathematical terms in English” (p. 375). Though code-switching in his classroom involved the students’ native language and English, similar results can be applied to code-switching among registers, if students are encouraged to make connections between their natural register and the mathematical. Such was a finding from research by Stromberg and Ramanathan (1996) who provided their students with notes that were in a two-column format, with the first column containing notes from the teacher, and the second being available for students to write their thoughts in their own words. In comparing student scores on final class projects from the previous year, these researchers noticed improvements with this strategy, thus stressing the significance of connecting student conceptualization and natural register of mathematics with the formal, in lieu of a replacement.

The National Council of Teachers of Mathematics has recognized the gap between the formal and informal registers in mathematics, and Standard 8, in response, addresses this issue: there “is sometimes a mismatch between the ordinary language and
mathematical language,” and “students need to build a bridge between their uses of language within and outside the mathematics classroom” (NCTM, 1999). On this issue of a need to bridge the gap, Forman and Larreamendy-Joerns (1998) speak about the teacher's role as one of mediator, striving to help students bridge the everyday and formal worlds. Furthermore, in the words of Adler (1998), it is not a matter of whether or not to code-switch between the formal and informal mathematical registers or to transition from one to another, but rather when, how, and for what purpose should this code-switching be done.

2.6 Linking Discourse in Cooperative Tasks to Problem-Solving

Having discussed, in an earlier section, the emerging need for verbal discussions in mathematics classrooms to improve students' problem-solving skills, this part of the review will focus particularly on student discourse that takes place in cooperative learning tasks. The reason for choosing to examine student-student discourse in cooperative activities as opposed to student-teacher verbal interactions is because the former, unlike the latter, is particularly conducive to open discussions. Such discussions, in contrast to the conservative dialogic nature of student-teacher discourse, could be beneficial in improving students' comprehension in solving word problems. This section, then, analyzes the research conducted on the relationship between student discourse and general problem-solving in an effort to further investigate the impact of such discourse on students' ability to solve word problems, in particular.

Considerable research has shown that when students collaborate and express their ideas aloud to each other, improvements in their thinking and reasoning processes are
observed (Rudnitsky et. al., 1995; Manouchehri, 1999; Cobb, 1992; Borasi et al, 1999; Fraivillig, 1999). Roti et al. (2000) state that the implementation of cooperative groups encourages active student participation and produces higher student achievement, specifically in mathematical problem-solving. What set cooperative tasks in a mathematics classroom apart from any collaborative activity are the expectations that students work together to collectively come up with a solution to a given problem, each taking on a unique role, and that the involvement of the teacher in student discussions be ideally minimal (Leikin & Zaslavsky, 1999). In such tasks then, rich student discourse can ensue. When discourse is encouraged and analyzed, it could have a great potential for both the students in building their problem-solving skills, and for the teacher in adjusting his/her instruction, having used the discussion as a tool for formative assessment.

Some of the common themes that emerge when analyzing successful strategies for solving word problems include comprehension of the question, understanding math vocabulary, and translating key words to mathematical representations (Sarama & Clement, 2008). These three themes are closely related to being efficient in the mathematical language, and in this study, they have been categorized under (i) comprehension of mathematical representations, to which I refer as one of the of two overarching student issues with word problems. Here, ‘representations’ is used as a general term to refer to mathematical terminology, in addition to symbols, numbers, notations, abbreviations, and conventions.

Lave and Wenger (1991) state that a participatory-inquiry approach provides students with an opportunity to learn to talk mathematics. In turn, Kotsopoulos (2007) reiterates Cummins’ (1984) point that contextualized problems that require students to
talk about what they are learning as they are learning it can allow meaning and language to be developed simultaneously. Moreover, Roti et al. (2000) showed that the cooperative group setting was particularly conducive to discourse, and most of their students started to share ideas and problem-solving strategies using mathematical language after cooperative work was incorporated. All these findings then show that student discourse in a cooperative context can facilitate acquisition of meaning in general problem-solving. Thus, it may be possible that such discourse could help overcome the challenge of comprehension of mathematical representations specifically with word problems.

Additionally, Roti et al. (2000) note that to be successful in solving word problems, (a) “students need to utilize their prior knowledge” to make connections with the problem and, (b) “to explore a range of problem-solving strategies” (p. 4). I have grouped these two requirements under the category of (ii) procedural competence, which is the second overarching issue of word problems to be explored in this study.

In the following discussion, to analyze the properties of student discourse in cooperative tasks, I look at the ways in which the how, what, and who of student engagement in discourse may improve word problem competence. First, in presenting how students use discourse, a focus on the register used in their participation with other class members will be analyzed. I discuss the potential relationship—which this study aims to investigate through interviews—between code-switching in discourse and an improved ability in word problem comprehension. Next, in discussing what students talk about, I present the view that students’ utilization of prior knowledge, which happens through discourse in cooperative tasks, is evidence of development of connections, and this connection-making may translate into applying learned concepts when solving word
problems. Finally, by discussing who students talk to in mathematical discourse, I highlight how the need to explore various problem-solving strategies to develop student competence in word problems may be met when members, having different roles in a cooperative group, collaborate with each other.

First, to analyze how students use discourse, I revisit the concept of code-switching: the practice of alternating between different mathematical representations, either within the same register or between the informal and the formal mathematical register. Because cooperative tasks promote discourse among members, and especially one where students are free to use an informal, natural register in discussing formal mathematical representations, they gain practice using both registers. Kotsopoulos (2007) states that “at first, the language may not be precise, but as students continue to work together and talk with one another and the teacher, the underlying meanings of the words evolve” (p. 304). This is in line with the research presented earlier; using the informal in conjunction with the formal register in a mathematics classroom can enhance student learning.

Here, I add that code-switching between registers, as cooperative work can allow, may aid students in the translating process between words and mathematical representations. Specifically, a word problem requires students not only to make sense of the words and translate those into mathematical representations, but also to comprehend and conversely express mathematical processes into words, relevant to the context of the problem. According to Fuentes (1998), students should be provided with a lot of experience in reading word problems and translating their meaning into numbers and symbols and vice versa. Goldman (1989) also discusses the necessity of this skill: “Techniques for developing the representation—a process sometimes referred to as
translation—are perhaps more important than execution of the appropriate arithmetic operations" (p. 48). Thus, when students are allowed to use their natural register to discuss mathematical concepts, thereby connecting their understandings with mathematical terminology and symbols, they may become competent in switching from words to representations, thereby developing a comprehension of mathematical representations.

Secondly, Schoenberger and Liming (2001) identify “lack of prior knowledge of mathematical concepts” as being one of the factors that make word problems challenging for students. However, when students work together in a cooperative activity, the nature of the discourse can be highly beneficial to problem-solving because of a balance that must exist between accountability in work and liberty in speech. Cooperative tasks, unlike collaborative activities in general, require each member of the group to be accountable in the process of problem-solving. This is usually done, as mentioned earlier, through assigning different roles to members, which helps keep discussions focused on the given mathematical problem. At the same time, with little involvement from the teacher, and with the nature of cooperative work being student-centered, the discourse allows students to share their personal thoughts and conceptualizations freely. Brantlinger (2014), in analyzing the works of Freire (1971) and Pruyn (1999), suggests that when students are positioned subjectively, as they are in cooperative tasks, not only do “they come to understand themselves as competent and informed political actors,” but they are also able to “co-determine the ends and means of their own education” (p. 201). He cites Skovsmose (1985) in saying that “as part of this subjective positioning, students should be provided opportunities to construct their own understandings of disciplinary subject matter and to take a critical perspective on this knowledge and how it is used” (p. 201).
Provided that cooperative discourse aims to, indeed, allow students to construct their own understandings of mathematics, it should become easy for students to make connections to their existing knowledge. Action research by Solomon (2009) with her own students allowed her to notice precisely this. She stated, “The more my students talk about the content, the more they seem to grasp the information being presented.” Others (Roti et al., 2000) heard comments from their students such as, “This problem reminds me of one that we solved before” with increasing frequency after students were allowed to express their understandings in their own words (p. 43). Likewise, Amen (2006) writes, “This oral discussion was also used to remind students of problems they had solved in the past” (p. 9). Thus, with the nature of cooperative work, what students discuss in discourse—namely, the problem provided in relation to their pre-existing understandings—may foster a development of connections with prior knowledge, which is the first of the two essential requirements in developing procedural competence with word problems (Roti et al., 2000).

Thirdly, when students collaborate as members in a cooperative task, each should have a unique perspective to offer, given that each works in a different role. Thus, in cooperative tasks, considering that it is other students with whom students engage in discourse, makes such discourse rich and eclectic. First of all, as Siegel, Borasi and Fonzi (1998) suggest, in classrooms where knowledge is regarded as a social construction, language takes on the role to negotiate meaning rather than being used merely to convey techniques. With each member interpreting the provided information from their unique role, multiple meanings and representations may come to be offered in discourse. This can allow students to explore various problem-solving strategies—the second important requirement for students to develop procedural competence with word problems (Roti et
al., 2000). Mercer (1995) states that the process of word problem-solving is not simply about access to the language of learning, but also about access to the language of mathematics and to new ways of using it. Thus, discourse itself may not be sufficient, but individual interpretations offered by each member, thus stressing the importance of discourse specifically in cooperative tasks, can teach students to consider various alternatives in the process of problem-solving.

Finally, in analyzing discourse among students rather than between teacher and student, it seems that discussions in cooperative activities allow more opportunities for students to practice justification of their ideas. In cooperative groups, students may feel more comfortable expressing their thoughts and reasoning as there may be a shift in focus—from using proper mathematical terminology (in explanations and questions) to investigating and critiquing mathematical concepts. Zack (1999) noted that it is important for students “to continue to explore in ways with which they feel attuned, and to resist being forced to accept procedures without understanding, ‘just because’ they are in textbooks or part of a canon” (p. 140). She further discusses the relevance of this practice of justification and questioning in regard to proof. She says that “a number of mathematicians and mathematics educators have protested against the blind acceptance of formal conclusions (Weyl, 1932, note 7), and have urged that preference be given to explanatory proofs, communicated in language which others can understand” (p. 141). Thus, with the opportunity to practice important skills such as justifying reasoning and critiquing actively, discourse in cooperative tasks may prove to be highly beneficial for students in solving word problems.
Chapter 3: METHODOLOGY

3.1 Procedure

In order to investigate a potential relationship between student-student discourse in a cooperative learning setting and two of the major issues associated with word problems—comprehension of mathematical representations and procedural competency—a qualitative research process was conducted. The study also aimed to expose teaching strategies that facilitate or exacerbate secondary mathematics students’ competence with word problems.

A qualitative approach to this study was best suited since the data to be collected from the research participants was to be analyzed from the perspective of a mathematics student and pre-service teacher, taking into account and addressing, rather than necessarily excluding, any biases. Moreover, it was important for the value of this study—aimed to educate fellow teachers and researchers in the field of mathematics—to hear the perspectives of secondary teachers regarding their practices and challenges through their own voice.

A literature review was first conducted to examine the research related to word problems, cooperative learning, and student discourse in mathematics classrooms. The participants sought were required to meet three criteria in order to take part in this study: (i) be experienced in teaching Ontario secondary mathematics for at least five years, preferably in the GTA (Greater Toronto Area), (ii) be currently teaching, or have previously taught within the last five years, and (iii) have a strong cooperative learning approach to teaching mathematics in their classrooms.
Two such teachers were approached via email with a request to take part in this study through semi-structured interviews to investigate whether word problems, with their linguistic component, can be made easier if students are encouraged to discuss mathematical concepts and representations in class with each other. Upon their interest, each participant was sent a letter of consent for the interview process, and the interviews were arranged for times convenient for both participants. At the time of the interviews, the letter of consent was explained once again and it was subsequently signed by each participant before the interviews began. Participants were also reminded that they had a right to refrain from answering any questions with which they did not feel comfortable, and if they wished to either have a part of their response omitted or personally to be completely withdrawn from the study before its completion, all data obtained from their participation would be discarded.

3.2 Participants

Before the interview questions were asked, participants were requested to describe their academic backgrounds as well as to provide an overview of their professional teaching experiences. The research findings were categorized and analyzed with respect to the literature reviewed.

The participants for this study, for whom pseudonyms have been used here to protect their identities, were two secondary mathematics teachers, teaching Grades 9 to 12. Sydney, teaching Grades 9, 10, and 12 at her current school, had been teaching mathematics exclusively for nineteen years. She graduated from a concurrent education program in 1994. Sydney then taught in a public school in United States for four years, and
has been teaching at her current school in Toronto for almost twelve years. Currently, she also works in the role of chair at the school’s math department.

The second participant, Olivia, studied mathematics at university, and then also completed a graduate degree in mathematics before teaching part-time as an instructor in two of Toronto’s colleges. Then, for seventeen years, consecutively, she taught Grades 9-12 Math in schools under the Toronto District School Board (TDSB) as well as the Peel District School Board (PDSB). Olivia has also worked as a department head, and has most recently taught two courses on Mathematics curriculum in a teacher education program.

3.3 Instruments of Data Collection

Face-to-face, semi-structured interviews with the research participants were conducted, being guided by a set of questions reviewed by the research supervisor. Responses were to be as detailed as desired by the participants, often leading to discussions not strictly corresponding to the interview questions. This was encouraged, however, as it provided additional relevant background of the participants’ views and teaching practices. Follow-up questions were also asked throughout the interviews to clarify and acquire relevant details.

According to Turner (2010), a standardized open-ended interview, such as those conducted in this project, “allows the participants to contribute as much detailed information as they desire and it also allows the researcher to ask probing questions as a means of follow-up” (p. 2). Such a balanced approach was important to follow in this study because it was believed that a discussion guided by questions pertaining to the topics of
interest that also allows the interviewees a certain liberty to shape the course of the conversation helps reveal perceptions in a more contextualized manner.

Some of the interview questions asked were: What have you noticed about students’ performance in solving word problems? What would be the extent of peer discussions that you would consider ideal in cooperative tasks? What are some of the differences you have observed between student-student discourse in a cooperative task and student-teacher discourse in an open classroom dialogue? (See Appendix B: Interview Questions for a full list of interview questions).

### 3.4 Data Collection and Analysis

To ensure accuracy in audibility during the transcribing process, the interviews were audio-recorded using a cell phone as well as a tablet, and personal written notes accompanying the responses were taken. The interviews lasted around 75 minutes, for which transcription was personally done using *The FTW Transcriber*. The data was subsequently analyzed and coded for emerging themes.

To begin this process of data analysis, I listened to both interview recordings at least twice, and read the transcripts a few number of times after printing them out. I then highlighted any comments that were of interest, regardless of the specific questions they were in response to since the participants often recalled additional information during the interview after already having answered the relevant question. I then grouped the participants’ comments into categories such as *student-student discourse, challenges with word problems*, and *textbook word problems*. The comments were rearranged several times
throughout the process of analysis to find links among the data, such as comparisons between the two participants’ perceptions.

### 3.5 Ethical Review Procedures

This research study followed the Ethical Review Approval procedures for the Master of Teaching program at the Ontario Institute for Studies in Education (OISE). The participants were provided with a consent form (*A: Letter of Consent*) along with sufficient time to review it before agreeing to the terms clearly outlined therein. The form clearly stated the purpose of the research study, how the data would be collected, as well as the nature of the interviews that were required for analysis.

Specifically, participants were informed that the researcher would take notes during semi-structured, face-to-face interviews, which would be recorded for subsequent analysis. They were informed that the notes, audio recordings, as well as transcripts could be shared with them, if they so desired, and that they would not be made available to anyone else, except the research supervisor and course instructor, if needed. As mentioned earlier, the participants were clearly informed about their right to refrain from answering any question during the interview. In addition, they understood that pseudonyms would be used to protect their identities as well those of any students, colleagues, or institutions that they mention. Lastly, both participants were offered access to this research paper upon its publication.
Chapter 4: FINDINGS

This chapter aims to present the major findings from two face-to-face, semi-structured interviews conducted with this study’s participants, Sydney and Olivia. It will be organized by themes that emerged upon reviewing the recorded conversations. In discussing these themes, I will quote the participants’ responses and will refer to some of the literature cited earlier in Chapter 2.

4.1 The Four Main Challenges with Word Problems

The first set of questions that Sydney and Olivia were asked dealt with students’ performance in solving word problems. Four main challenges with competence arose during the interviews, each of which are exacerbated, according to these participants, by certain teacher practices themselves. The participants attributed students’ struggle with word problems to (i) fixed mindsets, (ii) nature & content of the word problems, (iii) negligence of context, and (iv) insufficient practice. These challenges are discussed below:

4.1.1 Fixed mindsets.

One of the biggest challenges students seem to be facing in working with word problems is low self-confidence. In describing students’ response to encountering a word problem, both Sydney and Olivia used the expressions “freak out” and “freeze” throughout the interviews. Regarding her students facing a word problem, Sydney reported that “as soon as they see it, they all freak out.” Olivia also elaborated on this issue of confidence, saying, “They just freeze because they’ve learned to freeze. Every time there’s a word problem, they’re gonna go, ‘Uh...I’m not gonna be able to do this,’ and it doesn’t matter if it’s like the second question.”
When I asked the participants to share their thoughts on the poor results in EQAO open-response questions, they both felt that persistence with word problems is seriously lacking amongst students. Olivia reported on the EQAO results, stating that “it's not kids getting things wrong; it’s kids leaving things blank.” In line with this, Sydney mentioned, “I think that what the EQAO is measuring is we have uncreative thinkers who don’t have faith in themselves to take a chance...they don’t know how to do that problem. So, they’re not gonna try it.” Olivia added to this concern, saying, “I really think that giving up, that instantaneous giving up, that ‘I look at it, I don't know how to do it, I give up’ is one of the biggest problems.”

From both the participants’ responses, it seems as though students are doubting their own ability, and as a result, are failing to make efforts in problem-solving. Olivia reported on this low self-confidence indurating into a fixed mindset over time, which may discourage students from even attempting a given problem, hence the pages left blank. She said:

That growth mindset, ‘I don't know how to do it now, but I can learn how to do it, or I could figure it out’ ...is universal, but most problematic in math: ‘I'm not good at math, which releases me from all responsibility to be good at math...So, the fact I can't do it, it's okay.' So, I really, really think that persistence with word problems may be a bigger issue.

When I asked Sydney to share why she thought students are suffering from such low confidence, and whether it is an issue of students being shy, she said, “it’s sometimes being shy, but also kids who are not shy, who’re just not confident about suggesting something in math because they could be wrong.” Later during the interview, she added to this point, saying, “I think that a lot of kids are scared of doing word problems because they think that it has to be done a certain way. They don’t know the way.”
Teacher practices.

According to the participants, it seems that years of poor performance perpetuates low self-confidence among students, and this can develop into a fixed mindset regarding competence with word problems. Thus, it is important to investigate what teachers can do to help resolve some of the challenges. A discussion on this topic with the participants brought forward some teacher practices that were responsible in worsening the situation. Sydney felt that students develop a fixed mindset because their teachers do not offer them the flexibility to explore a variety of strategies in problem-solving. She mentioned that, “I think with mathematics, confidence is key. So, some kids will naturally have a lot of confidence because they’re given that from home, and other kids will come in with not a lot of confidence. Ability might be there, but [not] the confidence. ‘Take a chance! Try this!’—they’ve never really been encouraged that way before. So then, how can they do that suddenly?”

Another interesting issue that Olivia brought forward was regarding the practice of teaching students to mainly rely on answers provided in the book as a way of self-evaluating. She stated:

One of the problems is [that] the answers are at the back of the book. So, the kids never learn the need to reflect on the reasonability of their work. They check at the back of their book. That is the most efficient way for them to decide if their answer is reasonable...And they learn to question their own work...even if their work is right, and the back of the book is wrong.

Olivia felt that this practice is partially responsible for students doubting themselves since they are rarely given the opportunity to assess and question the accuracy and validity of their own work. Thus, she added, “one of the best things you can do for students is to take away the answers. And then they actually talk again. They’ll talk to each other [and ask
each other.] ‘What did you get?’” Moreover, consistent with Sydney’s comment about students being scared of not knowing the one right way to derive the answer because they think the problem “has to be done a certain way,” Olivia mentioned that teachers are often prescribing the path for students to follow in problem-solving. This is evident in word problems, for example, that are split into parts, each of which is meant to scaffold the student’s response. Despite a well-meaning intention by teachers, Olivia feels that expecting the same linearity from all students in their responses not only limits their exploration in problem-solving, but can also be responsible for many lost opportunities for teachers to learn something new. In regards to word problems with scaffolded parts, she stated:

We’re probably wrong most of the time because either the kids don’t need it at all, or what we provide just confuses them. That is so linear. First, you do this, then you do this, then you do this, and the only way you can get here is by going through that path? What if that kid sees the problem in a different way [and] would still end up there, but doesn’t see it as passing through those three checkpoints? And they see it as going a different route.

It may be a disservice, and maybe you had an opportunity as a teacher to see a whole different way of thinking about the problem and still getting the true, right answer…but you’re forcing them to think about a problem in a particular way.

Thus, both Sydney and Olivia felt that certain teacher practices need to be changed if students’ attitude towards mathematics is to be improved. Sydney mentioned that teachers themselves have to demonstrate to students that risk-taking is valuable in mathematics, saying, “I think the teacher can model it. If that was going to be your philosophy for the year...you can model it in front of the kids. ‘Okay! Let’s take that idea!’ and the group of kids can see that the teacher is accepting. And I model it in my own teaching...”
4.1.2 Nature & content of word problems.

The second major challenge that was identified consistently throughout both interviews was that of the nature and content of the word problems that students are being assigned. The nature of many textbook problems, according to Olivia, is such that it deprives students the opportunity for critical thinking. She mentioned that “the critical thinking piece of problem-solving is extracted because everything has been predigested, and handed to you: ‘Here you go.’” The given and required information are, in fact, sometimes so clearly stated that students can simply ignore the words, undermining the purpose of a word problem altogether. She said, “What the kids do [is that] they look for the numbers, they ignore the words, and probably 85% [of the time], they’re perfectly fine doing that because the words were meaningless. They didn’t affect the outcome of how you’re supposed to solve the problem anyway.” This comment supports the research of Sowder and Harel (1998) describing students’ tendency to read a problem quickly, searching for only clue words and numerical data. Such practice, however, does not prepare students for tests such as EQAO where open-response problems are not so similar.

Referring to textbook problems, Olivia mentioned:

I didn’t really do a lot of those, I’ll be honest, and I would really urge you to look at EQAO and look at open tasks. Those are not like word problems in the textbook...Those practice questions that everybody likes to do and assign so many of, those aren’t on EQAO at all...So, EQAO is entirely word problems, and the open responses are bigger word problems with many ways of doing them.

Sydney, as well, talked about the nature of textbook word problems, saying, “I tend to stay away from rote textbook problems.” She mentioned that in doing those problems, the students are “not thinking. Because we don’t teach them to think. We
teach them to be little monkeys,” as they are required to blindly repeat an algorithm taught to them by teachers. This concern was similarly reported by Zack (1999), protesting “against the blind acceptance of formal conclusions” without critical thinking. In line with Sydney’s opinion, and supporting Cummins (1991), Olivia spoke about the emphasis that is placed on computation skills:

These aren’t problems. These are just words set around an algorithm...They want you to practice an algorithm, so they’ll set up a context that enforces you to use that algorithm. But that’s not what life is like. When you get a real problem, it’s not all set up for you that way. Like [in] a real problem, first thing you do is figure out, ‘Well, how am I gonna do this?’ ‘What do I need to know?’ ‘How am I going to get what I need to know?’ Not when it comes on a piece of paper: ‘Here’s all the information you need, and now solve it’.

Olivia further noted that such problems never ask our students to think “because all of the thinking was done behind the curtain, and it was brought to you where the thinking was no longer necessary.” Thus, it seems that such textbook practice with word problems that teachers assign to their students fails to sufficiently prepare them, not only for tests such as EQAO but also for real-life problems, where critical thinking is essential.

To then better prepare their students, both the participants stated that they use word problems whose content is relevant and engaging, unlike the ones from a book. Both Sydney and Olivia mentioned the idea of students being bored with textbook problems. Sydney stated that she believes that her students “need to be playing with math all the time...doing problem-solving questions...And not from a book. That’s boring!” Olivia also talked about the idea of playing with mathematics as she asked her Grade 11 Advanced Functions class to keep a journal in which they would record various functions they encountered: “If they saw an interesting function, they would record it in their function
book. I didn’t want them to record the ones we studied together, I wanted them to play. ‘Use your phone, your tablet, your computer. Play! Play with functions.’” The students would then question each other, as the functions would be posted around the classroom for a gallery walk. Later during the interview, Olivia re-expressed her concern with boring content, saying, “It’s crazy! Little kids cannot learn enough, fast enough, and by the time they’re 14, they’re bored with learning. How can you be bored with learning? It’s because what we’ve done to the learning. We’ve made it boring.”

Olivia further mentioned an important point about the irrelevance of the content of many word problems that students face, making them boring. “Those word problems are terrible,” she said:

> Kids look at them, and they go, ‘Are you kidding me? This is supposed to be describing the real world? My world is never going to contain this word problem,’ and that’s why kids can’t see mathematics as being relevant to their futures.

She gave an example of a time when she heard another teacher talking about real-world math, and the question consisted of Shelly finding out how many goals her boyfriend scored in a game with 87 goals. In reference to this, during the interview, Olivia said, “You know, I’m sorry...a hockey game doesn’t have 87 goals, and if Shelly needs to know how many goals her boyfriend scored...she would call him.” She added that students “are not stupid. Yeah, they’re young adults, they’re kids, but they’re not stupid, and nobody is gonna sit down and do this in order to find out how many goals her boyfriend scored. She’ll call him [to find out].”

Furthermore, it is clear, as these participants have noted, that students become interested when the content of the problem is regarding something that matters to them
and is part of their world. Sydney supported this point, saying, “If I’m going to give them a problem, I want it to be kind of interesting and engaging in and of itself. Like things like, *Find the height of the flagpole out there but you’re not allowed to leave the room.*” Similarly, Olivia reported teaching mathematics to her split Gr. 9/10 Applied class by doing a lot of projects: “I love projects. Those are real problems. Like, ‘*This is your task. You’ve got two weeks. Figure it out. Tell me what you need.*’” She added later on that the word problem questions are “so contrived in order to get a way to describe the math, you’re gonna think that nobody ever does anything like this in real life because nobody ever *does* do anything like this in real life.”

4.1.3 Negligence of context.

Olivia identified the third challenge regarding student competence in solving word problems, which stems mainly from teachers’ language being both imprecise and inaccurate. Such language, according to her, is largely responsible for students’ tendencies to ignore the context of a word problem, with the question about the number of sheep and goat on the ship ‘in relation to’ the captain’s age being a prime example. She mentioned that students are “actually taught through experience...to ignore the context of the problem,” agreeing with the findings of Sowder and Harel (1998) and Szetela (1993) on students ignoring contextual information in problems. She recalled cautioning a teacher in a course she was instructing where the teacher had to come up with ways to prompt students to explain their thinking. One of the prompts was, “When you see ‘more than,’ what do you think of?” Referring to this, Olivia mentioned:

I get that; she’s an elementary school teacher, and the simple thing to teach is that ‘more than’ means to add. But you know, if I’m ten years older than
someone else, that someone else is ten years younger than me. So, if you teach those simple rules...and you don’t teach the relationship that the ‘more than’ is describing between two things, they [students] are always gonna make the same mistakes. They’ll see ‘more than,’ they’ll put plus, and away they go, and now they’ve got it wrong.

In addition, concepts that could have multiple meanings in various contexts require teachers to address those meanings with special care as to avoid generalizing rules. Olivia discussed this by saying that when students arrive at a solution, such as \( x=3 \), its meaning would depend on the context of the problem, but teachers, in her opinion, often use oversimplified language to teach concepts, thereby overlooking the value of context and multiple interpretations. While \( x=3 \) may be a solution, it is also an equation. She added:

[But] we solve equations. So, how do you solve \( x=3 \)? Is it the solution, or is it an equation? On a line, it’s a point, on a Cartesian grid, it’s a line, and in three-dimensional space, it’s a plane. So yeah, kids are confused. So much is in context, and we take all that context away by teaching procedures.

When I answered “part of a whole” in response to her question as to what a fraction is, she asked me, “Then, is five-thirds a fraction? Is \( \sqrt{3}/2 \) a fraction?” She elaborated, saying,

So, we’re precise, but we’re not really precise, and teachers don’t like those questions. But our muddiness, we pass on to our students, you know? We’re not clear about the differences, or the distinctions, or the true definition. And we pass that on.

She mentioned feeling that teachers expect more precision from students than they themselves actually deliver in classrooms, and the mistakes that these teachers make are really important for students to understand where they are. To illustrate this point, she pointed to the common misconception with the terms, ‘minus,’ ‘negative,’ and ‘subtract’:

It’s a common one, and kids get confused but it’s because they don’t know the difference because their teachers never presented them. To them, it’s just a
symbol that looks the same, and if the teacher routinely exchanges one for the other, then, of course, the students are going to routinely exchange one for the other...So, you can’t ignore; there’s a lot of work on helping students decode, but it’s not sufficient in mathematics because those words relate things together, and the words don’t sit by themselves. ‘More than’ doesn’t have meaning until you know what two things are being described, and where the relationship between them is.

4.1.4 Insufficient practice.

Finally, the fourth major challenge that became prominent during the discussions with both participants was that of teachers valuing procedural over inquiry and application skills, which results in students not having sufficient practice with word problems. Sydney offered two main reasons for this teacher preference:

I think that one way of making kids more confident with word problems would be doing those kind of Mensa Logic problems in class. Two reasons why teachers don’t do it: a) they’re not confident they can solve the problems themselves. And the second thing is, they’re pressed for time with all this other curriculum that I’m sure they’re like, ‘Oh, I can’t fit it in.’

Making an analogy of teaching students to better solve word problems with training young athletes to effectively play a sport, Sydney suggested that students will need a lot of practice in class in order for them to perform well on evaluations with word problems:

“You gotta play a lot of games. And then, class could be practicing the skills, playing the game, practicing the skills, playing the game. Going back and forth.” Citing back to the first reason she mentioned why teachers do not practice enough word problems with students in class, she added:

We have to approach mathematics in a balanced way where they’re learning the skills but they’re seeing those kinds of questions all the time, and I’m just not sure that the teachers at the primary level have the background to be able to do that, to be perfectly honest.
Olivia, as well, spoke about the issue of insufficient practice, as teachers spend more time teaching the procedures, running out of time to work with word problems, and in turn, assigning them for homework. In doing so, she said that teachers assume that students will be able to apply the skills on their own. She gave an example of how a lesson may be spent teaching students variations of a question with different numerical values as opposed to doing any application.

They do so few...It’s all like, ‘Here’s how you solve this one, and if you change the 3 to a -2, well now you have to do this, and if you change the -2 to 1.5, now you have to do this,’ and then the bell rings. So, how do we deal with word problems? There’s no time to do those at the end of the day, and they [students] are left to do those on their own.

Olivia then provided another example of math classes often being skill-focused:

I taught a lot of Grade 11 College-level [Math], and they're supposed to be doing Applied mathematics, and yet they never saw applications, which are word problems, right? Because teachers don't teach kids applications of mathematics; they spend more time on the skills. It's all backwards...they should be doing things with the math, not learning the math in isolation.

Moreover, on the issue of an emphasis being on skills and procedures, she invited me to reflect on the math tests I had created for students in previous practicum placements:

Picture all of the tests you wrote. All the skill-based stuff is on the first two pages, and the word problems come on the last two pages. Well, what message is that delivering to kids? That the word problems are less important...we really just wanna make sure you know these skills.

While some would agree with Olivia that the placement of word problems may suggest to students that they are not as important, others would argue that testing students on the skill-based material first is important for them to feel confident and for teachers to ensure they have taught the big ideas. In fact, when Sydney was asked, at the beginning of
the interview about how often her students work with word problems in the high school mathematics courses she had taught most recently, her response was:

10% [of the time.] You’re definitely doing more skill-based before you get to those. So, it’s not the focus of what we’re doing...I don’t think we really do. I think those things—being able to piece together that kind of information—is the pinnacle of where you wanna get them to...but it takes a while.

However, later on, she voiced a concern for insufficient practice with word problems, which fails to expose students to a variety of questions:

The other thing I wanna emphasize as well is that they’ve gotta get used to seeing all types of different questions...And so, if the kids aren’t exposed to a variety of different problems all the time, all the time, [then they will not be very successful at solving them].

Perhaps, like Sydney, other teachers may be challenged by such a contradiction between what they believe and what they actually practice.

4.2 Discourse and Meaningful Mathematics

One of the three requirements noted by Roti et al. (2000) for effective word problem-solving by students is “participating in discourse with others.” Both Sydney and Olivia mentioned strongly believing in regularly promoting student-student discourse in their classrooms. With respect to mathematics, Sydney, for instance, mentioned that her students “work on all their problems together. Nobody’s quiet. The only time they have to be quiet and do stuff by themselves is when I’m testing them. That’s it. Otherwise, I expect them to learn from each other.” When I asked Olivia how often she would want her students to be discussing mathematics with each other, she too strongly voiced the same opinion, saying, “Every day. Every day. I would really want them to be talking with each other every day.” The participants’ responses revealed two main reasons for which they
highly valued student-student discourse around mathematics, and those are discussed below:

4.2.1 Student-student discourse around real mathematics.

In discussions with Sydney and Olivia regarding mathematics and discourse, both their views revealed an interesting distinction between how mathematics is commonly practiced in classrooms and what it really is about. Supporting Nesher, Hershkovitz, and Novotna (2003), who had shown that students who were unable to take part in discourse while engaging with mathematical problem-solving were unable to progress in their learning, Sydney and Olivia stated that they promote discourse among their students because that is a practice associated with doing “real mathematics.” For instance, Olivia contrasted procedural operation questions with which students often are presented in classes to those that stimulate rich discussions, saying, “I want them talking everyday...even [for] a short problem or a short question...actual word questions, not just math questions.” She explained further to distinguish between these two:

With a standard word problem, what are they gonna talk about? ‘How do you do this?’ ‘Oh, you set this up as an equation, and you solve.’ Or, ‘You factor this out, and you solve.’ What’s there to talk about? It’s not a discussion! It’s a question and a statement, but that’s not a discussion!

A discussion is a discussion [with questions like,] ‘What does this mean? What do you think it means? I think it means this. Let’s agree on meaning first.’

She mentioned that she believes in talking about the meaning, which is the first part of problem-solving. In fact, this meaning “may not be clear, as is real life.” In addition to offering problems that are truly meaningful and that can stimulate rich discussions amongst students, Olivia felt that strategies of problem-solving as well need to feel real.
Sydney voiced the same concern, saying that even writing out answers on paper for the teacher, as opposed to having discussions with peers, can feel unrealistic to students. When I asked her why writing may be more fake than explaining in words, she said that it is because other students actually need to know the answer, and that need is not shared by the teacher. She elaborated on this, saying:

One feels real and one feels fake. There is a reason because I'm explaining it to someone who doesn’t understand, but when you write it down to give it to the teacher, who already knows the answer, there is no point. Kids are pragmatic. You gotta have a reason for doing it.

Sydney further highlighted the distinction between mathematics commonly practiced in classrooms and that done by ‘real mathematicians,’ for whom discourse is a natural aspect of work. She said that “people need to talk things out. That’s how mathematicians work.” On this note, she presented a personal account of her father and the essential role discourse played in his practice:

My father was a physicist for forty years, and he had a chalkboard in his office because his grad students and his colleagues would come in, and they’d argue about stuff...they’d talk about it; they didn't look at a little, tiny question by themselves and scratch their heads for twenty minutes.

In accordance with Kotsopoulos (2007), Olivia felt the same as Sydney in this regard, saying, “If you've got people to talk to, even better! Because to me, real work is done by groups of people talking to each other.” Moreover, in presenting her view of discourse being inherently associated with real mathematics, Sydney also stated that while practicing discourse during problem-solving, students’ focus may, in fact, shift to actual problem-solving from simply answering questions. She noted that “when you give problems, and you
present them to kids, and they’re allowed to discuss them, they don’t even realize what they’re doing. But if you put them in a box on the page...then everyone just freaks out.”

4.2.2 Student-teacher discourse.

After hearing both participants’ responses about student-student discourse in their classrooms, I wanted to ask them about the discourse that takes place between them and the students, which I refer to as student-teacher discourse. Initially, both participants mentioned that students are much more conservative in their speech with teachers than they are with each other, which as discussed in the previous section, is not the ideal form of classroom communication according to Sydney and Olivia. For instance, when I asked Sydney what the communication is like when her students are speaking to her, she said, “I would say, [they are] slightly more formal with me.” Likewise, Olivia stated that students are very shy to challenge her ideas, even when she is incorrect:

If I’ve made a mistake, eventually, a hand will go up, and it’ll be kind of an apologetic ‘Excuse me...but...’ They don’t talk to each other that way, [which is more like,] ‘What do you mean? How can that be?’ And they’ll challenge each other. Because they know they can.

It seemed then that Sydney and Olivia both preferred student-student discourse over student-teacher discourse in their mathematics classrooms. Upon further discussion, however, a key benefit of their discourse with students became evident; in speaking with the teachers, the students learned to practice code-switching. Both participants expressed the importance of managing a balance of registers, between the informal and the formal mathematical language. When I asked Olivia about the nature and content of student-student discourse, she mentioned that her students were using a variety of informal expressions in their speech: “When they’re working on something, it completely varies. A
lot of ‘thingies.’ ‘This thingie’ and ‘that thingie,’ ‘You have to move this thingie over here,’ and that sort of thing.” Moreover, she pointed to the imprecise use of mathematical terms, saying that for students, “‘Solve’ is the single word in math. Like everything is ‘solve’. ‘How do you solve this?’ And I’m looking at an expression. ‘What do you want me to solve? Give me a word for it because ‘solve’ isn’t the right word.”

A similar frustration with inaccurate terminology was expressed by Sydney, when I asked her about the registers her students use in class: “Some people say, ‘Sub.’ They'll say, ‘FOIL,’ which drives me crazy...It's not a verb! It's not even a thing!” She provided further examples of students using imprecise expressions such as, ‘Move it over there,’ or ‘Shove it over there, and just divide it away’ when they are simplifying algebraic equations, but she added, “I think that’s part of communication.” Similarly, Olivia stated that she probes for clarification in order to encourage students to reflect on their language, yet she understands that students need time to develop fluency, as she offered an analogy of the learning process:

If I happen to be walking around, and I happen to hear a ‘thingie’ or a ‘solve’ when I didn’t think ‘solve’ was the right word to use, then I’ll stop and say, ‘Well, what might it be?’ But I mean they’re just talking it out...and it’s just like writing a draft of a novel; you’re not gonna worry about your commas when you’re just trying to figure out where the character is going to go. You wanna get that down [first], and then you add to it.

Sydney, too, made an analogy of this situation, as she rubbed her belly with one hand while patting her head with the other to demonstrate the challenge students face in learning multiple ideas at once:

It’s like doing this. This is hard to do when you’re like three years of old, but I can do it right now...I accept it [imprecise language] from Grade nines
because they're just learning, and they're just trying to make it all happen. So, they can't focus on what they're saying and make it happen at the same time.

Both participants revealed that they understand that when students are just beginning to grasp the mathematical concepts, insistently correcting them on the language can be disruptive to their learning. However, Sydney also explained how she corrects imprecise language when she feels it is necessary to do so:

If they're using informal language or incorrect terms like ‘FOIL,’ they will say it, I will correct it, [and] they will repeat back what I've said. They'll say it wrong again, I'll correct it, they'll repeat back, and we just go through that feedback over and over again, [but] not in one conversation.

I'll only do it if it's not distracting...Sometimes I'll let things go because that's not the hill I wanna die on right then. But other times, if I wanna make a point about the language 'cause it's actually important in the context of what we're doing, then I'll jump on it...it totally depends on the context.

Olivia responded similarly, in appreciating the value of context and of having a balance in registers, although she did state that the ultimate goal is, indeed, to transition students’ register from the informal to the formal:

You teach when there's an opportunity to teach, but you don't restrict the flow of thoughts, you know, because you insist on having well-structured language. That's crazy.

There is a place and time for requiring the correct terminology, and when that's the right place and time, then you do require it. The rest of the time, you don't interject...But there's still a formalizing that has to take place. They do need to learn...

4.2.3 Code-switching.

From a discussion around both student-student as well as student-teacher discourse, it became evident that both are important for Sydney and Olivia in developing what was earlier called code-switching. As discussed earlier in Chapter 2, code-switching in this study refers to the alternation between the two registers: the informal, natural and the
formal, mathematical. As cited earlier, Fuentes (1998) states that students should be provided with a lot of experience in reading word problems and translating their meaning into numbers and symbols, and vice versa.

Accordingly, Olivia mentioned that when students are given such extensive practice where they are allowed to discuss problems and strategies with each other, they become less reliant on their teachers. Speaking of her students, she said that when they have been talking about mathematics with each other, they transition from being dependent to being independent and interdependent learners:

They’re not looking to me for the answer anymore; they’re looking to each other and to themselves for the answer. It’s about what they feel, what they think, what they can bring to it, what they can connect with. It’s just a whole different way of learning.

This belief seems to be closely aligned with Brantlinger’s research (2014) that when students are positioned subjectively, “they come to understand themselves as competent and informed actors and co-determine the ends and means of their own education” (p. 201). Given that Olivia had pointed out that discourse amongst students is much more eclectic than it is with teachers, she agreed that student-student discourse offered a greater opportunity for students to learn meaning-making of mathematical concepts and representations, in comparison to student-teacher discourse. She added, “That’s why I think they learn so much more from each other than they’ll ever learn from me.”

In terms of code-switching, Olivia felt that students benefit from speaking to each other because they learn to recognize many mathematical representations and terms, as they engage in open discussions with each other, which are not too common with teachers. In turn, the students may not only become familiar with those representations and
language, but they may also begin to appropriately use them when working with word problems. She noted:

If they’ve done a lot of exploration, then they've learned that the words form around the math anyway. If you're talking about...a simple task like a simple linear equation, and you're talking about it, there’re all kinds of words that form around it. So, [they] recognize that those words are there. They’re sort of embedded in the coding of mathematics. You have to just let them come out...

Indeed, Sydney felt that words become much easier to come out for students if they have had an opportunity to discuss the problem with each other. Even the problems that would have initially seemed intimidating to students become engaging if they can generate student-student discourse. She said:

I find that when I put those kind of logic problems at the beginning of the class, where the kids are really engaged, they’re not scared of those problems, for some reason. That doesn't frighten them...that generates conversation, and some kid will get up and explain their solution, and some other kid will get up and explain their solution.

Interestingly, Olivia also spoke about students’ fear vanishing when the written mathematical words are demystified through open discussions. She said:

They’re always talking about math. So, the words aren’t scary anymore. They’re printed words rather than oral words. They’re talking about math all the time, so the words that go around when you’re constructing a word problem, they’ve dealt with those words. They’ve spoken them.

Speaking about her experience teaching a Grade 9 class where she incorporated tasks promoting discourse among students, Olivia said that “the teaching was less active, and more organic. The kids were there, they had the concept, [and] all we had to do was name it, and maybe put some formality around it. I did the formalizing, but they did all the conceptual learning.” Here, she touched on the value of student-teacher discourse in code-switching, as it equips students with the mathematical terminology, which they can use to
better communicate their ideas and to better comprehend problems. She mentioned further, “Once you start to make that connection for them, it’s really just giving them some vocabulary and giving them some ways of looking at something they’re already familiar with.”

4.3 Procedural Competence

4.3.1 Conceptual connections.

*Cooperative tasks promote conceptual connections.*

As stated in Chapter 2, Roti et al. (2000) note that the second main requirement for students to be able to solve word problems successfully is being able to “utilize their prior knowledge to make sense of the language in the problem.” Given that cooperative tasks make learning student-centered, they can facilitate students in constructing their own understandings of mathematics, which naturally relies on making use of their prior knowledge.

Olivia mentioned that when students are forced “to think about the problem in a particular way...it’s not doing students a service; it’s not allowing them to come up with their own methods, and our own curriculum says that students are supposed to come up with their own methods.” Thus, tasks in which students were given the freedom to inquire through mathematics, while being able to discuss with their peers, allowed them to construct their own understandings, which requires connecting to prior knowledge. The student-centered nature of the tasks was evident in the use of ‘they’ and ‘them’ in Olivia’s comment:

What I tended to do was present the students with a problem and let them go at it, and I would just walk around and listen to what they were saying, watch
what they were doing...and that allowed me to get a sense of where they were taking the problem.

She also discussed the importance of students’ questions and their preferences in allowing them to lead the curriculum, “Like honestly, they’re kids, and you’re telling them what they’re supposed to do today for the next seventy minutes, and they’re supposed to care about it? Did you ask them? No.” Later on, Olivia mentioned that her teaching came to be shaped by what students were saying,

The way I taught in the last five years or so was designed so the students were leading the curriculum. I knew what I had to cover, but I didn’t have any sense of how I was going to cover it...or what pieces I was gonna connect together, until I listened to the students and saw what connections they were making.

Thus, by making her mathematics lessons student-centered with students being given many projects, being asked to record their own questions in journals, and having mathematical discussions and arguments about each other’s work, Olivia was able to facilitate a shift where connections to previous lessons were made regularly. Supporting Roti et al. on their requirement of effective problem-solving, she added further, “That to me was the biggest problem. Students learn the stupid skills and the procedures, but they don’t know how to connect them together, or how to connect them to the problem. And without those connections, they’re lost.”

**Teacher Strategies Promote Conceptual Connections.**

While cooperative tasks, according to Olivia, certainly seemed to enhance utilization of prior student knowledge, she also mentioned a few personal pedagogical strategies that promoted connection-making for students. For instance, she refrained from textbook-use because she wanted her students to both appreciate the interconnectivity of mathematical
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concepts as well as to retain the information they had learned at the beginning of the year. Speaking about textbooks, she said, “One of the things that helps students [was that] I didn’t use textbooks because I want students to learn math concepts; I don’t want them to learn 6.1.” She added that for her, textbooks are “very linear; page after page after page, and learning [on the other hand] is not linear...kids go all kinds of different ways, they circle back...”

Another pedagogical technique she practiced was to consistently refer back, when learning new concepts, to lessons her students had already learned, either in her class or in earlier grades. For instance, she described a task related to probability and algebra that she facilitated in her classroom early in the school year, where she asked her students to post their graphs around the room for a gallery walk. She mentioned that she “kept referring to that single task for months afterwards” to avoid isolation of concepts. “We keep connecting back to that, Remember back to the beginning of the year?” she added. Even when students were learning a concept for the first time, she would want them to have a discussion based on how this new knowledge related to what they already remembered from previous grades:

Even in lessons in advanced functions, when I had to teach them how to divide a polynomial by a polynomial by long division—and that was something they had not done before—I would still make sure that there were still interesting questions that they could discuss...Something like, ‘So what did we just learn? ‘How can you apply that to factoring in grade 10?’

By modelling connection-making, Olivia taught her students to refer back to their prior knowledge, “[I] let them play with it [math], and what usually happens is that [someone will ask,] ‘Why didn’t we learn to do this in grade 10? This would’ve been easier than learning how to factor because this is so straightforward.”
Sydney, too, referred to this idea of playing with mathematics and presenting it as interconnected. One of her approaches to doing this was to have evaluations that tested a broad range of concepts. Moreover, she also stressed the idea of revisiting lessons, throughout the year, that she had taught at the beginning. She said, “A lot of my problems are always mixed in, like the tests are cumulative because I’m like, ‘Alright, we’re going to visit it again. Oh! We’re going to do factoring again. Oh! We’re gonna see all this stuff, all the time.’” Here, she presented an analogy using sports to describe the importance of playing and constantly practicing with mathematics:

Those skills have to stay fresh because if you haven’t served [a ball] for three or four days, and you go play a game, [it will not be easy]. Then the teacher will say, ‘Oh, I taught it!’ And yeah, somebody taught me how to hit a baseball once, and I’m not very good at that. Because I haven’t done that in 20 years.

Moreover, in line with Olivia’s comments, Sydney also spoke to the non-linearity of mathematics. She mentioned using various math contest questions in her classes since each question is different from the other, thereby testing a whole range of concepts: “Each problem is different. All the questions on the page...are all different. It’s not in the slope section, it’s not in the unit; it’s just all combined in one big thing, and it’s way more interesting. It’s more engaging.”

### 4.3.2 Cooperative Tasks Allow an Exploration of Problem-Solving Strategies.

The third requirement noted by Roti et al (2000) for students to have better competence in solving word problems is for them to “explore a range of problem-solving strategies.” Both Sydney and Olivia supported the idea that cooperative tasks offer students the opportunity to explore a variety of interpretations, representations, and strategies to comprehend the problems, plan solutions, and finally derive the answers. However, in
order to encourage students to contribute to and explore this variety when working in
groups, these teachers mentioned first personally demonstrating to their students that
variation in interpretation and in mathematical processes are not only acceptable, but also
valuable. Olivia, for instance, noted that if a question is open to interpretation, then
teachers should “accept both interpretations. What difference does it make,” she asked,
“‘Cause it’s harder for you to mark? Big deal. You’re a teacher, and you’re teaching, and
maybe you’ll learn something new.” She mentioned that the attitude teachers must model
should be along the lines of thinking, “Wait a minute! This could be interpreted in multiple
ways. How cool is that!”

Sydney, as well, stated that with her instructions to students, she ensures that they
understand that they have a range of choices, and they can choose the most efficient
method. She gave an example of teaching factoring: “I don’t put anything on to that like,
‘Now do it three different ways.’ I show the kids three different ways, and they choose. ‘Here
are the three different ways you can factor, and I don’t care which way you do it.’” Speaking
about students who are not confident about suggesting their ideas in class, she also
mentioned that she views it her responsibility to encourage participation and risk-taking,
saying:

[There are students] who’re just not confident about suggesting something in
math because they could be wrong, and that’s where you have to generate the
comfort. ‘It’s okay to throw an idea and have it be wrong.’ That’s okay. That’s
how we generate more ideas. That’s how discoveries happen. People try stuff
and they fail. They try stuff and they fail.

Olivia’s belief about being accepting of variety in processes was also reflected in a similar
comment:
Very few places in your life are you forced to do something one particular way, and if you don’t do it that way, no one will accept it. As long as you get it done, people are happy, especially if it was something that they didn’t know how to do. Nobody asks you, ‘How did you do this?’ unless they’re interested in learning.

Moreover, both participants felt that discussions in cooperative tasks allow students to be involved in debates and arguments with each other over differences in their problem-solving methods. This, according to Sydney, teaches students a very important lesson in class: “that math is debatable.” She elaborated with this, saying:

I think it’s very powerful for kids. Like, even in my regular class, a kid will put an answer [and] we always have different ways of solving it, and value them all. I sit down a lot of times, and even when somebody’s going totally off track, I let them go off track…I try to guide them but just so the kids realize, ‘This is real math we’re doing’ because a lot of them are just trained like, ‘This is how you divide a fraction. Please write it down…just copy it down.’

Olivia also presented an example of how her students were encouraged to learn this lesson. She gave her students a question which asked if it was mathematically possible to have a given conclusion, and the students could use a variety of methods to answer the question:

You can do it algebraically, you can do it numerically, you can do it graphically—all these different ways you can look at it. There was a question, ‘Is it possible for...’ and it was so funny because when the answers were posted, the students [debated with each other], ‘Yes, it’s true; yes, you can,’ ‘No, you can’t,’ ‘Yes, you can...’ for two days.

The participants collectively provided three reasons for which they observe debates in their mathematics classrooms: a) the nature of the questions given promotes rich and open discussions, b) teachers allow and encourage debates and discussions without much personal involvement or interjections, and c) students feel driven to prove their point to their peers having a different opinion. What such debates then facilitate, according to
Sydney and Olivia, is the development of students’ justification skills, which are crucial in math, particularly in solving word problems. Olivia speaks to this debating pattern she observes in class and the associated justification students learn to practice:

They get angry with each other. It’s hilarious. I know that I’m on to something if they start to argue. They’ll physically get up and move and start pointing because they’ve got to make their point against your point, and when I see kids getting up and moving to be able to argue something, I know I’ve got a really good problem...I love it when kids are arguing about it, ‘No!’ And their voices go up, but they’ll never do that to me.

Likewise, Sydney mentioned observing similar behaviors, and described her students as being actively engaged in discussions: “There’re argumentative for sure, and they enjoy discussing and arguing about why they’re right and the other guy is wrong. So they’re argumentative...” She had earlier mentioned incorporating, in her class, a lot of “logic problems where kids have to explain their work to the rest of the class.” In explaining and convincing their peers of their point, the students, according to Olivia, “learn the value of justifying [even] when they don’t really want to.” She said, “It actually convinces other people of your point. Justifying is a wonderful thing. We do that all the time.”

Lastly, when I asked Olivia if she thought student-student discourse in cooperative tasks would help students with comprehension, decoding, and interpretation of word problems, she replied:

I think so, I think so. I mean, it would have to. You cannot teach a kid how to answer every possible different contextual question you could ask. You have to give them the skills and confidence, but they’ll be able to decode it themselves. They’ll be able to extract the meaningful or important information. They’ll be able to figure out the method to use the information...

In fact, in addition to helping students, Olivia mentioned that teachers themselves can have a lot to learn if they remain open to variety from students. She said:
Maybe they’ll come up with an answer I wasn’t thinking, and then I’ll get really excited because I’m learning something new. You know a really good lesson, I walk away with something I didn’t know before, even if it’s just a different way of seeing something.

4.4 Student-Student Discourse and Persistence

In addition to the issues of comprehension of mathematical representations and procedural competency, a key issue emerged from the responses of both participants—that of persistence with word problems. Both Sydney and Olivia felt that promoting mathematical discourse amongst their students offered the advantage of teaching students that if they discuss their thinking with each other, over time, they will eventually find a solution. Olivia mentioned that the timely nature of a rich discussion enforces persistence in mathematics. She said, “The fact that you didn’t understand it when you first read it doesn’t mean you have to skip it; it means it’s gonna take you more time.” She added that it teaches students that real problems take time. Sydney also spoke of often having long discussions as part of problem-solving in her classroom. At times, she noted, she lets students go on explaining even if their reasoning is inaccurate since she still values the lessons students can learn from each other. She said:

[Sometimes,] they’re just agreeing and agreeing, and not questioning anything. They fall into these lazy patterns, and so I’ll watch a whole fifteen minutes go by with somebody giving an explanation that’s totally wrong, and nobody catches them. And then we talk about that’s actually the lesson we’ve learned. ‘Did anybody think anything else that was different?’

Therefore, by letting students spend time discussing the problem, even if their answers are incorrect, Sydney believes that she promotes persistence. Olivia, along the same lines, said that she realizes that sometimes the discussion can take longer than planned, but perhaps that is that what students need to learn about mathematics: “I want
them talking every day, and when I say talking, [I mean] at least for three minutes. You know, I have timers, and five minutes usually turn into eight or ten because they would get involved.”

Thus, both participants seemed to support that it is student-student discourse around mathematics that can teach them persistence, especially when encountering a difficult problem. Speaking of particularly challenging problems, Sydney mentioned, “When something’s really hard, they’re gonna be like, ‘Whoa...’ I think if you gave those kids that problem, and they played around with it for a while, and talked about it, they could work their way through it.” Olivia also shared the same opinion, saying:

If you allow kids to work together rather than in rows... I think they learn persistence with word problems. Like they learn that there are problems that cannot be solved to completion within a five-minute period of time. Maybe you’ve gotta struggle with it, maybe you’ve gotta look at it, talk it out, or think it through...
Chapter 5: DISCUSSION

This chapter first presents the purpose that motivated this study, as discussed in Chapter 1. This is followed by a discussion of the findings, which has been organized into six sub-categories that analyze participants’ responses related to: student-student discourse, student-teacher discourse, balance of registers, teacher as mediator, conceptual connections, and exploration of variety in problem-solving strategies. The chapter concludes with a list of recommendations compiled from participants' responses in addition to this study’s limitations and suggested areas for further research.

5.1 Overview

Despite an interest and recent efforts to shift from teacher-centered to student-centered instruction in mathematics through student discourse in classrooms, research that has emerged over the past few years has not been sufficiently comprehensive in investigating potential links between student discourse and word problem competence, especially from a cooperative learning perspective. Given that both student discourse and word problems have an inherent linguistic component, it was considered valuable to explore whether student discourse can positively impact performance in solving word problems, based on the perspectives of two Ontario secondary mathematics teachers.

5.2 Discussion of the Findings

5.2.1 Discourse facilitates comprehension of mathematical representations.

Student-student discourse.

The discussion around discourse revealed that Sydney and Olivia both strongly believe in the value of student-student communication in their classrooms to make sense of
mathematics. As Morgan's work (2004) points to a concerning gap between mathematical representations and students’ natural language, both participants felt that student-student discourse can help close this gap if students are encouraged to have rich discussions around ‘meaning,’ which often tends to happen naturally in such discourse. Students might ask each other questions such as, “What does this mean? What do you think it means,” as mentioned by Olivia.

Moreover, the concept of code-switching was supported by the participants’ responses in its role to decode mathematical terminology, symbols, and graphs such as those that Olivia's students recorded in their journals. The participants felt that if students were allowed to discuss the meaning of a term or a mathematical relationship presented in a word problem, it would help them decode this information into concepts that they could grasp. This view was directly in accordance with research by Seinbring et al. (1998) that suggested that in algebra, an approach to operate “symbolically while sustaining the associated reasoning in natural language can help students to develop meaning for algebraic manipulation” (p. 257). Even if students were using informal expressions in discussions, the participants, supporting Barwell (2013), revealed that the role of the informal language in class was not simply to facilitate a shift towards the formal register; in fact, the use of the natural language in class offered the additional benefit of promoting development of conceptual connections for students.

Interestingly, student-student discourse was often referred to, by the participants, as “debates,” “arguments,” and “discussions,” and not simply as a question-response dialogue, such as, “How do you do this? – Oh, you set this up as an equation, and you solve.” This shed light on these teachers’ views about what it means to practice “real
mathematics,” supporting Lave and Wenger (1991) that mastery in a community of practice involves learning to talk, and learning mathematics entails speaking mathematically. These findings confirm that, when students are allowed to use their natural register to discuss mathematical concepts, they may become competent in switching from representations to words, and vice versa, thereby improving comprehension of word problems.

*Student-teacher discourse.*

In addition to student-student discourse, the participants were asked to elaborate on student-teacher communication. One of the main drawbacks to the latter was evident in the participants’ regret of their students not feeling confident enough to challenge teachers’ ideas as they did with each other. In response, to foster critical thinking skills among her students, Olivia mentioned asking her students if they were sure about their responses. Often, this would prompt students to recheck their work, as if it were incorrect, because they were accustomed to the expectation of providing alternative reasoning when such questions were asked, reinstating Mehan (1979) that in traditional teacher-centered classrooms, repeated questions from teachers imply wrong answers. Interestingly, even in classrooms such as those of Sydney and Olivia who heavily incorporate cooperative tasks in mathematics learning, students may come to easily doubt themselves.

On the other hand, despite this limitation, the concept of code-switching was clearly evidenced through Olivia’s and Sydney’s recounts of student-teacher communication regarding mathematical vocabulary. Whereas student-student discourse, through code-switching, functioned mainly to decode various mathematical representations, such as graphs or inequality signs, into conceptual understanding for students, student-teacher
discourse, in turn, facilitated code-switching between registers. Here, participants reported on their communication with students as facilitating an augmentation, rather than a mere translation, in vocabulary knowledge. In other words, student-student discourse allowed students to give meaning to mathematical representations using their natural register, upon which the formal vocabulary instruction was added through student-teacher discourse.

**Balance of registers.**

Some researchers (Lave & Wenger 1991; Adler 1998) have reported on teachers facing a dilemma of explicitly teaching mathematical terminology, of which there seems to be an urgent need, while still keeping students’ focus on the content. Barwell (2013) had urged the significance of keeping a balance of the natural and the formal mathematical registers, citing Vygotsky’s notion (1987) of learning: “the development of both spontaneous and scientific concepts” are closely connected and “not separated from one another by an impenetrable wall” (p. 177). It was not, however, clear from pre-existing literature how such a balance could be established and maintained within the classroom setting. Responses to this question during the interviews presented an approach that was shared by both Sydney and Olivia.

First, both teachers expressed a minor frustration over the inaccuracies in students’ language in class, such the acronym ‘FOIL’ being used as a verb or ‘solve’ being presumed as the prompt and purpose for every question. However, both participants also emphasized that learning of the proper terminology is an effort that requires time, and so patience must be demonstrated by teachers. They felt that it was unreasonable to expect a mastery of language when students were simply beginning to grasp the concepts, comparing it to the
stage of planning a story’s plot in a rough draft. More importantly, they believed that constant correction of terminology could, in fact, disrupt the flow of learning, similar to what Adler (1998) had reported regarding her own students who would become frustrated over their teacher’s corrections during their speech.

While exhibiting such a tolerance to inaccuracies in students’ language, Sydney and Olivia also strongly believed in the need for a proper use of mathematics vocabulary, supporting Blessman and Myszczack (2001) with the finding that one of the main causes of confusion in word problems is an unfamiliarity of the vocabulary. Thus, both participants reported on being persistent with corrections when they felt it was appropriate to do so. They both referred to the idea of context, with Sydney stating that she makes corrections only if it is not distracting. At times, when it is important to point out the incorrect terminology, she does not hesitate to do so; “it totally depends on the context,” she concluded.

Thus, the way Sydney and Olivia manage a balance of registers in their classroom seems to be through facilitating both student-student discourse and student-teacher discourse, but at different times. The student-student discourse generally precedes student-teacher discourse, which has the effect of students becoming more responsible for their learning; they first construct their own understandings, and the terminology is subsequently introduced by teachers. Sydney noted that with such an approach, even when she persistently corrects inaccurate usage of mathematical language, the students, having had a chance to build a conceptual understanding through prior student-student discourse, become more focused on getting their meaning across, and not being obstructed by their teacher’s corrections. This is unlike the student frustration that Adler (1998) noticed with
her students. Olivia, referring to such an approach where teaching becomes “less active, and more organic,” noted that her students tend to, as such, develop a particular concept largely on their own, and all she has to do is “name it, and maybe put some formality around it.” She said, “I did the formalizing, but they did all the conceptual learning.”

**Teacher as mediator.**

Discussions around student-student and student-teacher discourse with Sydney and Olivia, as reported in the previous sections, revealed an interesting similarity in how these teachers view themselves in their classrooms. With an emphasis in recent literature to transition from teacher- to student-centered lessons, Sydney and Olivia exemplify how students can be empowered when teachers trade their roles of leaders to become facilitators, much like the recommendation made by Forman and Larreamendy-Joerns (1998) for teachers to act as “mediators.” As discussed earlier, Olivia mentioned instances in her own class where students doubt themselves because they still believe that the teacher will always be correct. For instance, some of her students tend to think that they are wrong simply when she asks them if they are sure about a response, illustrating the pattern noted by Edwards and Mercer (2013) where the teacher’s repeated questions imply wrong answers in a traditional mathematics classroom. Despite such instances, however, particular pedagogical practices by Sydney and Olivia were evident of handing over a certain autonomy to students themselves.

For instance, both Sydney and Olivia tend to be, based on their responses, relatively patient as student-student discussions proceed in their classrooms. Sydney stated that sometimes she lets her students “go on” even when they are incorrect. She further pointed to how this can often act as an opportunity for her to teach her students to be critically
analytical of what they are taught, thereby taking control of their own learning. Likewise, it was evident that Olivia had a student-centered philosophy with her use of ‘let’ in her responses throughout the interview; phrases such as, “let them go at it,” “let them play with it,” and “let them talk about math” were frequently used by her. In addition, she pointed to the student practice of mathematical justification in her classroom that can really flourish when she refrains from interjecting. This practice seems to be in line with Solomon (2009) who views classroom discussions where the teacher’s role in discussions is mainly to structure the talk. In fact, she mentioned enjoying listening to students debate with each other because that, to her, was an indication that students were involved and invested in the learning. Moreover, both these teachers referred to noise levels in their classrooms often rising during student-student discourse, which they felt was not necessarily an issue, but perhaps, a positive sign of engagement.

Furthermore, their view of their role as a guide was also highlighted by references to their physical position with respect to students’ in the classroom. Sydney, for instance, mentioned that she often sits down while the students go to the front and write their solutions on the blackboard, thereby demonstrating a reciprocity that can exist in learning between teacher and students. Moreover, with respect to students’ location within the classroom, she mentioned that in her classes, students are not always required to be seated, as some are free to work on the whiteboard at the back of the room. Olivia, too, mentioned this idea of student mobility in class with her references to gallery walks and students getting up and pointing as they make their arguments in mathematical debates. It seems then that beyond a student-centered philosophy of learning, these teachers, through
various day-to-day actions, play a key role to assist and guide their students towards engagement and empowerment in learning mathematics.

**5.2.2 Discourse in cooperative tasks develops procedural competence.**

*Cooperative tasks promote conceptual connections.*

One of the two aspects linked to the issue of *procedural competence* in word problems is being able to make connections to prior knowledge. Although their responses did not necessarily suggest that discourse specifically among students was conducive to connection-making, Sydney and Olivia both provided examples of how student-teacher discourse is used in their classrooms to allow students to make links to what they have already learned. Moreover, working in a cooperative environment where students are free to discuss the problem with each other seemed to act as a source of reminders of previously learned concepts. On this note, Sydney agreed that one student in a group may remember something that others may have forgotten, and so, discourse in such context can help students consolidate their learning.

In addition, both participants mentioned avoiding the use of textbooks because they felt that the linearity of books expects students to move from one concept to another in a sequence, when in fact, according to Olivia, “kids go all kinds of different ways, [and] they circle back…” Another strategy that Sydney mentioned was to explicitly point out concepts that students should be able to remember from previous lessons. She mentioned using constant reminder questions because, according to her, without such cues, it can be easy to forget what has been learned, which is a similar case to not being able to serve a ball after a prolonged period without practice.
Thus, it appears that students tend to make connections to their prior knowledge as a result of some of these approaches mentioned. On making connections, Olivia, for instance, described her students’ remarks such as, ‘*Why didn’t we learn to do this in grade 10? This would’ve been easier.*’ This was a similar finding as reported by Roti et al. (2000) who noticed a more frequent use of terms such as ‘remember’ and ‘remind’ by students in discourse with each other. For example, after engaging in discourse, a student commented, “This problem reminds me of one that we solved before.” These findings also align with those of Amen (2006) who noticed an improvement in memory with more classroom discourse: “...oral discussion was also used to remind students of problems they had solved in the past” (p. 9).

*Discourse in cooperative tasks exposes students to a variety of strategies.*

The second component of *procedural competence* that was investigated in this study related to students being cognizant of various interpretations of mathematical concepts in a word problem as well as to having an ability to apply a multitude of problem-solving strategies. Given that word problems are modelled after ‘real-life’ situations to prepare them for ‘real world,’ it is important for students to see problem-solving as an application of mathematics where multiple paths, as opposed to a single method to finding solutions, exist. Zack (1999) noted that it is important for students “to continue to explore in ways with which they feel attuned, and to resist being forced to accept procedures without understanding, ‘just because’ they are in textbooks or part of a canon” (p. 140). Likewise, both Sydney’s and Olivia’s responses suggest that to be able to effectively solve word problems, students need to be both familiar with a variety of problem-solving techniques and to the idea that math is debatable. In fact, both participants mentioned several
instances of having debates and discussions in their classrooms, which present opportunities for students to share their unique perspectives, thereby benefiting the class as a whole in being equipped with various interpretations as well as problem-solving strategies.

What may encourage an exploration of variety among students in problem-solving could often be, according to the participants, direct communication by the teacher that various interpretations are acceptable, and are a part of mathematics. In addition, modelling acceptance of variety of solutions on tests could also help students learn that “math is debatable,” as stated by Sydney. A single problem, Olivia said, could be done “algebraically, numerically, [or] graphically,” so as teachers, we must be open to interpretations when accepting student work, and often, we might learn something new.

5.2.3 Summary & recommendations.

In light of the discussions presented in this report, this section compiles some of the key practices mentioned and suggested by Sydney and Olivia, in their roles as secondary mathematics teachers, to improve students’ attitudes and performance, not just in word problems, but in mathematics in general:

1. Foster Student-Student Discourse Frequently in Class

- Teachers must be patient to let students discuss mathematics with limited interruptions or corrections at this stage
- A discourse among students can help to make math more meaningful; student-student discourse, unlike a question-response sequence in student-teacher dialogue, constitutes “real mathematics”
1. Let students question each other instead of always expecting teacher explanations

2. Use Student-Teacher Discourse to Promote Connection-Making
   - Let students engage in discourse among themselves to grasp new concepts. Then, use student-teacher discourse to build upon that knowledge, connecting mathematical concepts with formal terminology
   - Probe students to make links to prior knowledge: "How is this related to what you did in Grade 10 factoring?"
   - Explicitly link new concepts to those already taught: "This is like when we did...", "Remember when we...?"

3. "Let" Students Take Control of Their Learning
   - Encourage movement within the classroom, especially by encouraging students to write on the board at the front
   - Facilitate activities that allow students to move around the classroom, such as Gallery Walks

4. Promote Opportunities for Exploration and Self-Assessment
   - Foster self-checks by removing answers, when possible
   - Avoid pre-digesting problems with all required information clearly stated
   - Avoid unnecessary scaffolding of questions

5. Make Problems Relevant by Letting Students "Play" with Mathematics
   - Embed problem-solving in activities such as journals where students record what they find interesting about a given topic
   - Allow students to use technology in problem-solving
6. **Be Mindful of Own Language**

- Avoid teaching simple rules to decode comprehension of word problems
- Embed context when defining concepts and terminology (i.e. \( x=3 \) as a solution to an equation, but a line as a graph)

7. **Remind Students that Mathematics is Debatable to Teach Persistence**

- Accept variety of interpretations whenever possible
- Model through own practices
- Provide time for debates to develop and last

### 5.4 Limitations

Various factors limit the scope of this research study. First of all, my personal bias as a researcher, a mathematics student, as well as a pre-service teacher has shaped the way I have analyzed the data offered by the participants. Although my personal background and interest were addressed in Chapter 1 of this study to inform the reader of such bias, I recognize that as a qualitative researcher with an interest and investment in the topic being explored, the results may not be objectively presented.

Secondly, despite a need to study student-student discourse and student competence in solving word problems, access to views on these topics from students themselves was not possible in this study. The perceptions of their teachers, which may be influenced by a variety of factors, such as memory and demand characteristics, were the only source of data that was collected and analyzed in the form of notes and audio recordings. It was hoped, in response to this limitation, that literature that reports
students’ views as well as their teachers’ would be a source of guidance in conjunction with the interviews conducted to report common themes and patterns.

Moreover, a small sample size, with just two participants, significantly limits the impact of this research; any implied generalizations as well as recommendations presented here must be mindful of this fact. Furthermore, there is a geographic limitation on this study, as the participants recruited were both teachers in the Greater Toronto Area. However, in spite of the above limitations, the findings of this study, based on the insights of the participants, may provide key information related to teaching practices. These practices may assist and enlighten researchers, teachers, curriculum developers, and school leaders in their work to support educators and students in their journey of teaching and learning mathematics.

5.5 Further Steps & Conclusions

With word problems aimed to model “real-life” situations and to prepare students for the application of mathematics in their daily lives, the development of problem-solving skills have been emphasized for years in high school mathematics. However, this study reveals that students’ attitude and their lack of deep understanding of mathematics vocabulary seem to be major factors negatively impacting their abilities to solve word problems.

This study has presented the views of two Ontario secondary mathematics teachers who use cooperative learning tasks in their classrooms to promote student-student discourse around mathematics. Their views on student-student discourse and the impact it has on the comprehension of mathematical representations and procedural competence
suggest that the promotion of both cooperative learning and student-discourse can assist students in solving word problems.

To further explore these areas, more studies are needed to investigate the impact of student discourse on their competence in solving word problems. This would help educators in building a deeper understanding of this teaching approach. Moreover, some teacher recommendations presented in the previous section may present additional challenges for teachers, and understanding those potential challenges is important if theoretical implications are to transform into actual classroom practices. In addition, more research on how to assist teachers in infusing math discourse and collaborative learning in their practice would provide principals and school leaders with valuable information that could be used in adjustments of curriculum and in professional development. On this note, in the words of Olivia, it may be valuable to remember, not just for students, but also for educators that as we think about problem-solving, we must remember that "problems don’t lend themselves to testing; problems lend themselves to being explored. A real problem gets explored and investigated.”
A: Letter of Consent
Informed Consent Form

**Topic of Research:** Discourse in Cooperative Tasks & Competence in Solving Word Problems

**Researcher:** Muhammad Haroon, Master of Teaching candidate, OISE, U of T

Dear __________,

This invitation letter is to request your participation in a study being conducted by Muhammad Haroon, a Master of Teaching candidate at the Ontario Institute for Studies in Education, under the supervision of Cathy Marks Krpan with Patrick Finnessy as the course instructor. The study aims to investigate teacher perceptions on student discourse in cooperative learning tasks in a mathematics classroom and its potential relationship with students’ competence in solving word problems.

Your participation in this semi-structured interview is voluntary, and you will have full right to refrain from answering any questions with which you do not feel comfortable. Your responses will be recorded using an audio-recording device, and will be subsequently transcribed for analysis. You will have full access to the audio recordings as well as any notes taken during the interview, should you request them.

If you make a request to have a part of the interview response omitted, it will not be used in the study. If you choose to withdraw from the study before its completion, all data obtained from your participation in the study will be discarded. Furthermore, if you choose not to have your data published upon the completion of the study, data obtained from your participation will be deleted and will not be used in the study, as long as the request is made prior to the publication of the findings.

Participation in the present study will require a face-to-face interview, lasting around 60 minutes at a place and time of your convenience. You may be asked to take part in a follow-up interview, lasting a maximum of thirty minutes. Further consultation may be required via email or phone.

The findings obtained from this study will be published on a University of Toronto website (“T-Space”) and will also be presented to other OISE students at a research conference. The results of this research may also be published in a scholarly publication and/or reported in a scientific presentation. In any case, the identity of all participants will remain fully confidential. There is minimal risk for you as a participant.

You may withdraw from the study at any point. If you have any questions or concerns, the researcher will be happy to address them. If you are interested in receiving feedback about
the study results, please contact Muhammad Haroon at the contact information provided below.

Please sign the attached form, if you agree to be interviewed. The second copy is for your records. Thank you very much for your help.

Sincerely,

Muhammad Haroon
m.haroon@mail.utoronto.ca
Researcher’s Phone Number: ________________

Research Supervisor: Cathy Marks Krpan
Research Supervisor’s E-mail Address: ________________
Research Supervisor’s Phone Number: ________________
I acknowledge that the topic of this interview has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time without penalty.

I have read the letter provided to me by Muhammad Haroon and agree to participate in an interview for the purposes described.

__________________________  ____________________________  _________________
Printed Name                  Signature                    Date
Appendix B: Interview Questions

Research Questions:
In Ontario, what are secondary mathematics teachers’ perceptions of the merits of student-student discourse in cooperative tasks in relation to student competence in word problems?

Sub-questions:
1. How does student-student discourse compare with student-teacher dialogue in promoting connections to prior mathematical knowledge?

2. In cooperative tasks, how can code-switching of registers possibly affect performance in solving word problems?

3. How does the exploration of a variety of strategies in a cooperative context impact students' abilities to solve word problems?

Interview Questions:
These questions were intended for participants who identify as using cooperative tasks in their mathematics classrooms.

1. How often do your students work with word problems in high school mathematics courses that you currently teach?

2. What have you noticed about students’ performance in solving word problems?

3. Do your students experience any challenges in solving word problems? If so, how have you resolved or plan to help resolve some of the major challenges students face with word problems?

4. How do you define cooperative learning?

5. What are your thoughts on using cooperative learning in a high school mathematics classrooms?

6. Do you use cooperative learning in your teaching? If so, how frequently do you use this approach? In what particular ways?

7. What would be the extent of peer discussions that you would consider ideal in cooperative tasks?

8. What have you observed about the nature and content of student discourse in the cooperative tasks you facilitate in class?

9. What are some of the differences you have observed between student-student discourse in a cooperative task and student-teacher discourse in an open classroom dialogue?
References


