Robust Control of Active Ankle Torque during Quiet Stance: a Servomechanism Problem

By
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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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University of Toronto

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Abstract

Treatment and restoration of motor abilities to individuals with spinal cord injury (SCI) is possible with neuro-rehabilitative systems for unsupported upright stance, also known as quiet stance. The aim of this research project is to develop a controller that will regulate active ankle torque during electrical stimulation for quiet stance. This study uses a novel technique of applying an optimal robust servomechanism controller to bring the human’s centre of mass (COM) to a specified reference position independent of external disturbances. The robust controller obtained was able to track quiet stance within 1 second, and highly attenuate both constant and Gaussian noise disturbances intensity. The robust controller presented can be considered as a step forward in neuro-rehabilitative systems for quiet stance, as previous controllers have not considered dealing with the servomechanism control problem in the regulation of ankle torque.
Acknowledgements

I would like to thank my supervisors, Professor Edward Joseph Davison and Professor Milos Popovic for their mentorship and support.

My foremost thanks goes to my father, for his endless passion and teaching of seeking light, my mother, for her modest support, my sister, for her lovely caring eyes, and my brother, for his humble character.

I would like to thank my critiques, for helping to grow my capacity to love.

love, whom without, I would be no one

Dedication

This thesis is dedicated to 1980-1988 Iranian war veterans who unwillingly had to partake in a war, and received immobility in return.
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<td>Autonomic Nervous System</td>
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<tr>
<td>ASIA</td>
<td>American Spinal Injury Association</td>
</tr>
<tr>
<td>BCI</td>
<td>Brain Computer Interfaces</td>
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<tr>
<td>CSF</td>
<td>Cerebral Spinal Fluid</td>
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<tr>
<td>CNS</td>
<td>Central Nervous System</td>
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<tr>
<td>COM</td>
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<td>Functional Electrical Stimulation</td>
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<tr>
<td>ICU</td>
<td>Intensive Care Unit</td>
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<td>PD</td>
<td>Proportional-derivative</td>
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<tr>
<td>PID</td>
<td>Proportional-integral-derivative</td>
</tr>
<tr>
<td>PNS</td>
<td>Peripheral Nervous System</td>
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<tr>
<td>SCI</td>
<td>Spinal Cord Injury, Spinal Cord Injured</td>
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<tr>
<td>SNS</td>
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1. Introduction

1.1 Motivation and Objective

Spinal cord injury (SCI) patients have partial ability or complete inability to maintain balance during quiet stance due to deficits in neuromuscular function. Restoring standing and enabling stable stance has a great impact on improving the quality of life and independence of SCI patients [1]. Restoring standing to spinal cord injured individual can also bring a number of secondary benefits for the patient such as minimization of osteoporosis, prevention of contractures in lower limbs, reduction of spasticity, stimulation of circulation, and renal function improvements [2]. Standing is an important element of locomotion, which enables one to get around the house and community. Standing is also very relevant for accomplishing everyday activities such as enabling transfers, talking to others at their eye level, or picking up objects from a shelf; having the ability to stand also provides psychological and psychosocial benefits, vocational benefits, as well as recreational benefits. This is why we have decided to work on the topic of quiet standing.

Motor system neuroprostheses employing functional electrical stimulation (FES) can provide a means to improve the overall health and mobility of individuals with paralysis resulting from spinal cord injury (SCI). The purpose of this thesis is to develop a robust controller for a closed loop neuroprostheses system regulating active ankle torque during quiet stance, in order to facilitate stable standing for spinal cord injured individuals.

By developing a closed loop model for controlling the active ankle torque, one can synthesize the artificial control of quiet standing in SCI individuals with the help of functional electrical stimulation (FES). FES systems are rehabilitative devices that enable muscle contractions by means of short low energy pulses. However, continuous FES results in progressive fatigue of muscles that may result in knee buckling, and hence a need for a mechanism to re-establish knee extension. Rapid muscle fatigue while standing can cause loss of balance, and the possibility of introducing new traumas [3]. A faster, better performing controller implies stabilization occurs during a shorter time period. This in turn allows the FES systems to artificially produce and regulate quiet standing using shorter
stimulation periods, which then exert muscles contraction less frequently resulting in delayed and/or reduced fatigue.

To further justify the need and motivation for the research described herein, we will first examine the physiological background of nervous systems and the spinal cord, and explain how the human body applies a feedback control mechanism in regulating and maintaining upright stance. An overview of SCI, types of SCI, consequences of SCI, SCI costs as well as emergency treatment and rehabilitation services necessary for the individuals after SCI are also presented, showing the great need for improved systems helping SCI patients in upright stance. Assistive technologies and FES assisted systems for restoring standing, as well as the limitations and benefits of using FES assisted closed loop systems for regulating upright stance for SCI individuals will also be discussed.

1.2 Thesis Organization

This thesis is composed of 8 chapters. Chapter 2 provides the physiological background of quiet stance. Chapter 3 is a literature review of current assistive technologies for standing, FES, FES’ viability in regulating upright stance, and past and present FES-assisted closed loop strategies for regulating upright stance. Chapter 4 presents the research question and hypotheses. Chapter 5 provides the preliminary of control theory to solve the problem at hand. Chapter 6 provides the theoretical method to obtain the closed loop plant model and presents the controller. Chapter 7 provides the results of our controller and compares it with a proportional derivative (PD) control system proposed by Masani et al. [4] and a proportional integral derivative (PID) control system proposed by Same et al [5] for FES-assisted quiet standing. In Chapter 7 our controller’s robustness is also verified. Chapter 8 revisits the test cases of Chapter 7 for a revised controller which is tuned to be better suited for FES assisted systems. Conclusions are presented in Chapter 9.
2. Physiological Background

2.1 The Nervous System

The nervous system is composed of the central nervous system (CNS) and the peripheral nervous system (PNS). CNS is the primary control center for the body and is composed of the brain and spinal cord. PNS consists of a network of nerves that connects the rest of the body to the CNS. The PNS is made up of sensory and motor divisions. Sensory division neurons conduct action potentials toward the CNS and motor division neurons conduct action potentials away from the CNS. The PNS includes the Somatic Nervous System (SNS). The somatosensory system designates senses other than hearing, taste, vision, balance and smell. Somatosensory receptors are distributed all over the body rather than concentrated at specific locations. Each of the different types of receptors responds to many different kinds of stimuli. The somatosensory system includes at least 4 distinct modalities and many submodalities. These include the senses pertaining to the skin such as touch (mechanoreception), temperature (thermoreception) modality), pressure, and vibration; proprioception or sensing of body position, kinesthesis, and body movement; as well as pain (nociception), itch, and tickle [6] [7]. The sensory receptors cover the skin and epithelia, skeletal muscles, bones and joints, internal organs, and the cardiovascular system. Figure 1 provides an overview of sensory inputs and motor outputs coming to and from the central nervous system, respectively.

![Figure 1 The Nervous System](image-url)
2.2 Spinal Cord

The brain and the spinal cord make up the central nervous system (CNS). The spinal cord together with the brain controls the muscles of the limbs and the trunk, as well as the functions of internal body organs.

From the brain, the spinal cord descends down the middle of the back and is surrounded and protected by the bony vertebral column. The spinal cord is surrounded by a clear fluid called Cerebral Spinal Fluid (CSF), that acts as a cushion to protect the delicate nerve tissues against damage from impact against the inside of the vertebrae. CSF also serves a vital function in cerebral autoregulation of cerebral blood flow. Thirty one pairs of spinal nerves originate in the spinal cord including 8 cervical, 12 thoracic, 5 lumbar, 5 sacral, and 1 coccygeal. The role of spinal cord nerves are as follows [9]:

- Cervical Nerves "C": nerves in the neck supply movement and feeling to the arms, neck and upper trunk. They also control breathing.
- Thoracic Nerves "T": nerves in the upper back supply the trunk and abdomen.
- Lumbar Nerves "L" and Sacral Nerves "S": nerves in the lower back supply the legs, the bladder, and bowl.

Figure 2 provides information on the sections of the spine and how they relate to human body parts.
2.3 Muscle Movement

For the muscle to be effectively controlled, the CNS needs information about the force the muscles are generating, the length of the muscle and the position and velocity of the joint that the muscle is actuating. The receptors inside the muscles and between the muscles and tendons are a system that provides proprioceptive feedback. Two muscle receptors important for motor control are the muscle spindles and the Golgi tendon organ afferents, which convey information on muscle contraction and stretch [10]. Spinal Reflexes are responsible for adjusting movements in response to changes in sensory feedback. The muscle, joint and skin afferents are activated in a distinct way during a movement, giving multisensory signals regarding displacement of individual joints, pressure on particular regions of the skin, or stretch of certain muscles to the motor system. When perturbations occur as the movement proceeds, the sensory signal communicating with the nervous system will evoke a reflex
response that changes the motor output [11]. Aside from receptors, feedback about movements is provided by movements of the head through the vestibular system in the inner ear, visual feedback, and feedback about pressure changes through the support surfaces and joints of the body as shown in Figure 3.

**Figure 3 Human Movement Control [12]**

The muscles of the human body can be categorized into a number of groups. Figure 4 presents front (anterior) and back (posterior) lower limb muscle groups. The Quadriceps is a group of four muscles found at the front of the thigh. The role of these muscles is to extend the leg from a bent position. The Hamstring muscles are found to the rear of the leg and contribute to the hip flexion and are used to flex the knee in the act of pulling the heel towards the buttocks. The Glutes muscles facilitate hip extension as well as lifting the leg to the side in an action called hip abduction. Gastrocnemius and Soleus muscles play a role in extending the foot at the ankle. The Tibialis Anterior muscle is primarily responsible for
moving the foot and ankle towards the head (dorsiflexion). The core muscles align
the spine, ribs, and pelvis of a person to resist a specific force, whether static or dynamic.
Core muscles allow posture maintenance, bending, twisting, and providing a stable spine for
various activities involving movement of the extremities that are required in daily life and for
most activities [13]. The core stabilization system has been divided into three distinct
subsystems: the passive subsystem, the active muscle subsystem, and the neural subsystem.
The passive subsystem consisting of the spinal ligaments and the facet articulations between
adjacent vertebrae allows the lumbar spine to support load about 10kg and less than body
mass. The active muscle subsystem itself is divided into global group and local groups. The
global group acts to increase intra-abdominal pressure while the local group control
intersegmental motion between adjacent vertebrae. The neural subsystem monitors and
adjusts muscle forces based on feedback provided by muscle spindles, Golgi tendon organs
and spinal ligaments [14].

Figure 4 Posterior and Anterior Human Lower Limb Muscle Bundles [15]
2.4 Spinal Cord Injury (SCI)

Any damage to the spinal cord that blocks communication between the brain and body causes SCI. SCI can be caused by traumatic injuries, which stretch or compress the neurological tissue of spinal cord, or non-traumatic injuries that result from developing diseases and processes such as cancer, infections, and disk degeneration of the spine. Although sensitive to the severity of the damage, SCI normally causes disconnection and dysfunction of human functions, below the level of injury.

The American Spinal Injury Association (ASIA) impairment scale is a method to examine and categorize the type of SCI incurred to an individual. Figure 5 lists the main categories of the impairment scale.
Box 1 | The ASIA Impairment Scale

Classification of spinal cord injury (SCI) severity using the American Spinal Injury Association (ASIA) Impairment Scale. The main categories of the Impairment Scale are as follows:

- **A** (complete): No motor or sensory function is preserved in the sacral segments S4–S5.
- **B** (incomplete): Sensory but not motor function is preserved below the neurological level and includes the sacral segments S4–S5.
- **C** (incomplete): Motor function is preserved below the neurological level, and more than a half of key muscles below the neurological level have a muscle grade of <3.
- **D** (incomplete): Motor function is preserved below the neurological level, and at least a half of key muscles below the neurological level have a muscle grade of ≥3.
- **E** (normal): Motor and sensory functions are normal.

Extent of injury after damage to specific spinal segments is illustrated in the figure (see American Spinal Injury Association in Online links box for the complete standard neurological classification of SCI).

Figure 5 Main Categories of ASIA Impairment Scale [16]
2.5 Types of SCI

SCI can be categorized into the following types [17]:

- Traumatic injury is a result of physical trauma to the spinal cord. They are mainly caused by motor vehicle crashes, sports injuries, falls, or other incidents.
- Acquired diseases such as viral or bacterial infections (including polio), or diseases that can cause tumours or cysts on the spine. Some diseases such as multiple sclerosis can also deteriorate such that cause lesions on the spinal cord and may result in paralysis.
- Congenital disorders, such as spinal bifida, where the spinal cord is deformed or exposed at birth.

The American Board of Physical Medicine and Rehabilitation Examination Outline for Spinal Cord Injury Medicine groups fractures, dislocations and contusions of the vertebral column into traumatic SCI and a health condition, such as disease, infection, or a tumour damages the spinal cord into non-traumatic SCI. Non-traumatic SCI is when damage is done to the spinal cord by means other than an external physical force [18].

2.6 Consequences of SCI

The consequences of SCI can be categorized into the following [19]:

- Tetraplegia (or quadriplegia): Where there is impairment of function in the arms as well as in the trunk, legs and pelvic.
- Paraplegia: Where there is impairment of the trunk, legs and pelvic, but with arm function not being impaired.

A complete or incomplete lesion of spinal cord that occurs during SCI alters whether there is partial or total motor and/or sensory deficit [18]. A complete SCI is characterized by the complete absence of sensory and motor function in the lowest sacral segments (S4–5). In an
incomplete injury there is partial preservation of sensory and/or motor function at S4–5. This is termed sacral sparing [20].

Following a SCI, the individual may experience the following medical issues [21]

- Hypercalcemia: As a result of increased bone resorption as a consequence of immobilization
- Autonomic dysreflexia: Sudden and unanticipated blood pressure elevations of 20–40 mmHg above baseline due to pain or irritation below the level of injury.
- Hyperhidrosis: Sympathetic over activity of the cephalad portion of the spinal cord immediately below the zone of injury
- Temperature regulation abnormalities
- Pain
- Cardiovascular disorders is a result of sedentary lifestyles
- Bladder management
- Bowl management
- Spasticity: Involves with slowing of stepping and of voluntary limb movements, exaggerated tendon reflexes and muscle hypertonia defined as a velocity-dependent resistance of a muscle to stretching (Lance, 1980) [22]
- Psychological issues are introduced as a result of abrupt, severe, and generally permanent change in body function

### 2.7 SCI Costs

SCI, whether traumatic or non-traumatic, often results in some level of motor and/or sensory deficit below the level of lesion. SCI is devastating and it is a lifelong medical condition. It frequently produces mobility and sensory deficits that render SCI individuals dependent on others for the rest of their lives. Hence, besides causing massive burden and reduced quality of life both for the patient and his/her family, SCI also results in massive financial costs that both the patient and his/her family as well as the health care system need to shoulder. The lifelong financial care requirements for a SCI individual can range from $1.6 to $3 million,
depending on the level of paralysis. Regardless of age or gender, individuals with high tetraplegia (C1-C4) have the highest average yearly cost, followed by low tetraplegia (C5-C8), and paraplegia. More than 60% of people living with a SCI are unemployed. The estimated annual cost of traumatic injuries (which include traumatic brain injury and other CNS injuries not limited to SCI) in Canada is approximately $3.6 billion of which $1.8 billion being direct health care costs [23]. Compared to the general abled-bodied population, Canadians with traumatic SCI are re-hospitalized 2.6 times more often, require contact with a physician 2.7 times more often, and require home care services 30 times more time amount [24].

2.8 Emergency Treatment

Emergency treatment occurs immediately following a SCI. First it is ensured that the neck and spine are immobilized to prevent further damage. Once the patient is stabilized, scans are taken to locate the exact place of injury, and neurological examinations are performed to determine the extent of motor and sensory deficit. The emergency treatment depends on the severity of injury. The patient may require admission to the intensive care unit (ICU), neurosurgical observation unit, or general ward [25]. Following emergency treatment the patient is transferred to a rehabilitation facility to receive rehabilitation services.

2.9 Rehabilitation Services

SCI rehabilitation centres would normally assist the spinal cord patients in the following ways:

- They will provide medical treatment appropriate for SCI individuals and they will coordinate the patient’s rehabilitation program.
- They will provide physical rehabilitation to help patients regain motor and sensory functions.
- They will provide skills-training program to help patients perform their daily life tasks such as getting dressed and getting around home and community.
- They will provide psychological support to help patients go through emotional and psychological difficulties they typically face following drastic life changing experience caused by SCI.
They will provide counselling sessions to help patients achieve their personal, career, and other goals that will help them achieve independence.

2.10 SCI complications for standing

SCI patients face the important complication of inability to stand upright. Standing is important for accomplishing everyday activities such as enabling transfers, talking to others at their eye level, or picking up objects from a shelf. It is also an essential element of locomotion, which enables one to get around the house and community. Thus restoring standing and enabling stable stance has a great impact on improving the quality of life and independence of SCI patients [1]. Restoring standing to spinal cord injured individual can also bring a number of secondary benefits for the patient such as minimization of osteoporosis, prevention of contractures in lower limbs, reduction of spasticity, stimulation of circulation, and renal function improvements [2], as well as psychological and psychosocial benefits, vocational benefits, and recreational benefits. Design and development of technologies that would assist SCI patients in standing upright is therefore imperative.
3. Neuroprostheses Literature Review

3.1 Assistive Technologies for Restoring Standing

Assistive technology refers to a broad range of devices and tools that are conceived and applied to improve the mobility limitations faced by persons with disabilities [26].

Literature suggests five groups of assistive technology methods for restoring standing for SCI individual: orthotics, standing frames, specialized wheelchairs, functional electrical stimulation, and hybrid systems (combinations of the above) [2]. The mentioned categories can be classified into alternative devices and augmentative devices. While devices that assist with patients’ mobility are very important and are briefly introduced in the following section, the focus of this thesis is on those technologies that support standing, such as FES.

3.1.1 Alternative Devices

In case of total loss of mobility, devices such as wheelchairs and the special vehicles could be an optimal solution for helping with the mobility of SCI patients. The continuous use of wheelchair may cause health problems such as loss of bone mass, osteoporosis, skin scores, and degradation of blood circulation and physiological functions as the person is sitting for long periods of time [27].

3.1.2 Augmentative Devices

The augmentative devices may help avoid the mentioned health problems associated with using wheelchairs and allow using the patient’s remaining locomotion capability. The augmentative devices can be categorized as follows [28]:

3.1.2.1 Mobility-training Devices during Rehabilitation

The literature suggests new movement, or improvement to an existing movement can be achieved only through repetitions [29]. Robotic mobility devices are capable of providing this task and can be categorized into three groups: treadmill-training devices, ambulatory-training devices and feet-manipulator training devices.

One of the aims of treadmill-training devices is to enhance motor recovery and walking ability of people with neurological injury. Examples of such devices include Lokomat [29],
Lokohelp [30], and LOPES [31], which provide intensive training using movement repetition.

Ambulatory training devices deliver an over ground dynamic assistance so that the patient can learn to walk with proper quiet posture, with the need of less equipment. Examples of such devices are LiteGait [32], Walkaround [33], WalkTrainer [34], and KineAssist [35].

Feet-manipulator training devices are based on holding the patient’s feet to a robotic manipulator. The manipulator is responsible for supporting and leading the patient in the walking situations. As a result of these artificial feet movements the slack muscles between the toes and the hips are forced into action again [28]. Examples devices include HapticWalker [36] and GaitTrainer [37] which manipulate the legs, according to the kinematics and the desired speed.

3.1.2.2 Self-ported Devices

Self-ported devices are classified into devices that improve the function of movable parts of the body and are carried by the user (orthoses), or to substitute a lost member (prostheses) [38].

Orthoses, grouped into active and passive, work in parallel with partially functioning body part to enhance its functionality. In active orthoses, actuators or motors provide the necessary energy to allow movement. In passive orthoses, with the help of springs and links that rely on gravity balancing principles, the user provides the necessary energy for achieving balance [28]. Literature shows RoboKnee [39], and GAIT [40] are active orthoses devices.

3.1.2.3 External Devices

Examples of external devices are crutches, canes and walkers. They provide support and increase balance stability during standing.

3.2 Functional Electrical Stimulation (FES)

The restoration of arm-free standing in people with spinal cord injury can be accomplished with the help of Functional Electrical Stimulation (FES). FES is a rehabilitative technology that applies electrical currents to peripheral nerves through electrodes. The electrodes are
placed on the skin over the nerves. Also they are placed over the motor points which are cites of stimulation that produce the strongest and most isolated contraction at the lowest level of stimulation. This current causes an electric flow between the electrodes. This in part causes ions to create current in the tissue influencing the transmembrane potential leading to action potential generation. The propagation of action potential across the nerve causes contraction of a paralyzed muscle [41].

The tension produced in electrically stimulated muscle is a function of the stimulation waveform, the frequency of stimulation, and the stimulation intensity. The stimulation intensity is a function of the total charge transferred to the muscle, which depends on the amplitude and pulse width of stimulation.

In the clinical setting, three approaches are used to apply FES: implanted electrodes in combination with an implanted electric stimulator (implanted FES system), implanted electrodes attached to an external stimulator (percutaneous FES system), and surface stimulation electrodes attached to an external stimulator (surface FES system). In some applications FES systems are also used in combination with passive or active orthoses [42].

Devices that deliver FES are a type of neural prosthesis and are substituting for a neural function which is damaged or destroyed. They can be used for therapeutic stimulation to improve muscle tone, bulk and strength, to reduce spasticity, and to relief pain [43]. FES is also used for functional stimulation to generate movements or functions, which mimic normal voluntary movements. While a range of surface electrodes, percutaneous electrodes, and implanted electrodes exist for stimulation of the nerves under the skin, most FES technologies for regulating quiet stance employ surface stimulating electrodes that are applied to the skin [44].

The advantages of surface systems are that they are non-invasive, relatively easier to implement, and are relatively inexpensive, and thus can be well utilized in therapeutic applications. However placing the electrodes in the appropriate locations, achieving isolated contractions, activating deep muscles, as well as managing various electrodes and associated wirings are some of the challenges faced when using surface FES systems [45].
3.3 Why FES is a Viable Solution for Regulating Quiet Stance?

Following the SCI, individual’s muscles below the level of lesion do not receive regular exercise, they undergo disuse atrophy and convert to muscle with a higher proportion of fast-twitch fibres than is present in healthy individual’s [46]. Consequently, the affected muscles are weak and fatigue quickly. However, disuse atrophy is often a fully or partially reversible process, since the affected muscles can be re-trained with electrically stimulated weight-bearing exercises to increase their strength and fatigue resistance [47]. This re-training process can also occur as a beneficial side effect of regular FES use.

FES can be useful in inducing joint movement by stimulating the flexor and/or extensor muscles of the joint. That is through activation of spinal reflexes during FES induced muscle contractions. Spinal reflexes are feedback loops in the CNS [48]. This spinal reflexes that originate in the spinal cord below the level of injury are often present in individuals who have SCI, although these reflexes may be significantly heightened or depressed and thus altered as a result of the injury [49]. These reflexes may be activated during FES induced muscle contractions, causing exogenous contractile signals to be sent to the paralyzed muscles in parallel with the FES control signal. The angle of the joint, or the torque about a joint, whose muscles are stimulated can be regulated by varying the tension produced in the flexor and extensor muscles of the joint. As a result, the joint angle or joint torque can be controlled by modulating the stimulation pulse duration, amplitude, or frequency [50].

3.4 Limitations of FES

FES recruits motor units in a synchronous manner, meaning it stimulates all of the motor units at the same time, instead of individual stimulation of the motor units as is done by the central nervous system. For this reason, achieving tetanic contractions with FES stimulation requires a much higher stimulation frequency. The drawback of the higher frequency stimulation is the increased rate of fatigue associated with stimulated muscle contractions as compared to contractions initiated by the central nervous system [46].

Also, FES has a tendency to recruit the fast-twitch muscle fibres before the slow-twitch muscle fibres. This order of fibre recruitment, also known as non-physiological recruitment, is the opposite of natural muscle fibre recruitment order, in which the slow-twitch fibres are
recruited first. The fast-twitch fibres fatigue more quickly than slow-twitch fibres, leading to increased rate of muscle fatigue [46]. However, regular stimulation with FES causes the composition of muscle to change some of the fast-twitch fibres to slow-twitch fibres over time. This change in composition increases the fatigue resistance of the muscle, and introduces another time-varying factor that affects the response of muscle to FES [46].

Continuous FES results in progressive fatigue of muscles. Rapid muscle fatigue while standing can cause loss of balance, and the possibility of introducing new traumas. Rapid muscle fatigue may also result in knee buckling and there would need to be a mechanism to re-establish knee extension. Increasing stimulation pulse width or stimulation amplitude of the quadriceps muscle may provide re-establishment of knee extension [51], when the muscle is not extensively fatigued.

3.5 FES Assisted Systems for Regulating Quiet Stance

A literature search shows that various dynamic controllers, finite state controllers (FSCs), and artificial neural networks (ANNs) have been used to develop controllers for FES-assisted quiet stance [50].

Bajd et al implemented one of the earliest FES systems for standing in paraplegia. This system provided bilateral, open-loop stimulation of the knee extensors, which maintained the knees extended [52]. The hips were held hyperextended passively while the ankle joints were free to move. The paraplegic person was holding on to a suitable support which acted as a postural controller. The limitation to this system was that the arms of the paralyzed individual were engaged in stabilizing activity, making standing non-functional as well as the problem of muscle fatigue, which limited the duration of achievable standing. These deficiencies motivated several research groups to work on the problem of restoring functional, i.e. “unsupported” or “arm-free” standing in SCI individuals.

Jaeger et al. conducted one of the first theoretical studies by developing the single-link inverted pendulum model, a simple mathematical model of arm-free standing [2]. The only degree of freedom was the ankle joint, assumed to be under the control of closed loop FES via stimulation of calf and dorsiflexor muscles. A simple Proportional–Derivative (PD)
controller was used in simulation studies, and showed that stabilization could be achieved at least theoretically.

Abbas and Chizeck investigated the regulation of coronal plane hip angle on neural position in paraplegic subjects using PID control strategy [53]. They evaluated the performance of PID strategy by comparing them to that of an open loop stimulation system. They were able to show a reduction of 41% in root mean square (RMS) error, reduction of 52% in steady-state error, and 22% reduction in compliance as compared to the open loop system. The challenges of the study at the time were the high level of energy expenditure using functional neuromuscular stimulator (FNS) systems and the limited ability to respond to disturbances. The study was able to show by utilizing the residual sensory-motor abilities of a paraplegic in conjunction with implementing FES controlled ankle stiffness, paraplegic standing can be achieved. Hatwell et al. investigated the design of an adaptive controller to control the movement of the leg joints [54]. The results of this study indicated that control systems that facilitate user-driven, task-dependent postures can be more effective and efficient than conventional open-loop stimulation.

Matjacic and Bajd employed a mechanical rotating frame apparatus to restore standing to thoracic-level paraplegics. They examined whether a paraplegic can balance if his legs behave approximately like an ideal lower link of the double-pendulum. This introduced solid control of lower back and trunk. Not only should the knees and hips be held in extension and the ankle stiffness be accurately controlled, but also lateral motion should be prevented to achieve sagittal plane stability. They needed to compensate for loss of balance using trunk movement combined with ankle stiffness [55]. An extension to this work above, Jaime et al. (2002) examined whether a paraplegic subject is able to maintain balance during standing by means of voluntary and reflex activity of the upper body while being supported by closed loop controlled ankle stiffness using FES [56]. FES was used to produce the desired stiffness and ensure that the stiffness is maintained at a predefined level. The knees and hips of the subject were held in extended positions by a mechanical apparatus, which restricted movement to the sagittal plane. They showed that paraplegic standing can be achieved by implementing FES-controlled ankle stiffness when the residual sensory-motor abilities of the patient are utilized. Ankle stiffness produced by FES alone however was not
adequate to achieve stable stance. The idea was to make the task of stabilizing the erect body possible for the subject to do it himself using his own sensory-motor abilities. However, the FES-controlled ankle stiffness contributed to the task of stabilizing the body.

Hunt et al. investigated control of unsupported standing by implementing a closed loop model consisting of an inner loop ankle moment controller with an outer loop position controller [57]. Standard pole-placement design approach was employed for both the inner and the outer loop controllers. The model was tested with three cases: quiet standing (keeping the reference angle constant), external disturbances (while the reference angle is tracked, various disturbances were applied such as pulling/pushing of the subject, the subject stretched his arm with or without a weight, etc.), changing reference (step-wise varying reference angle). The study’s results suggested adaptive control may not be suitable for controlling quiet stance. This is because adaptive control requires persistent excitation of the plant to detect changes in the plant characteristics. Since in a quiet standing (balancing) situation, disturbances and, thus, excitation of the plant can be very small for significant periods of time, the use of a single, time-invariant controller, which is designed to perform robustly for the expected operating conditions, is more appropriate.

Mihelj and Munih considered the minimization of a cost function that estimated the effort of ankle joint muscles via observation of the ground reaction force position relative to ankle joint axis[3]. The control criterion included minimization of stimulated ankle muscle effort, with the purpose of prolonging unsupported standing time. This study created a control system to increase the activation of plantar flexor as a consequence moving the body toward the ankle joint axis. In this study, however, the COM was assumed to be placed in posterior direction, which is physiologically incorrect.

Masani et al. examined whether the PD controller is capable of facilitating robust balance during quiet standing, despite long neurological time delays [58]. They measured kinematic parameters of a healthy subject as well as EMGs of the ankle extensors during quiet stance and applied cross correlation analysis to investigate the relationship between kinematic parameters and rectified EMGs. The study demonstrated a feed-forward mechanism is not necessary to compensate for the long closed-loop time delay in human bipedal stance as
suggested in literature prior to this paper publication, and that the PD controller can represent the activity of able bodied subjects during quiet stance.

Kim et al investigated the optimal degrees of freedom (DOF) required to achieve arm free standing using FES [59][60]. Their findings showed using only six of the twelve degrees of freedom in the lower limbs actuated via FES allows paraplegic individuals to stand freely while ensuring stability and able-bodied kinematics during perturbed arm-free standing.

Same et al. tested if a PID control strategy is capable of effectively regulating an FES system at the ankle joints during quiet stance as well as to examine whether able-bodied individuals with compromised visual, vestibular and proprioceptive senses, would exhibit improved ankle joint control if FES was applied to contract the ankle muscles using a closed-loop PID control strategy. The study’s experimental results verified that the PID control strategy developed was able to regulate FES amplitudes applied to the ankle muscles of able-bodied subjects effectively [5]. Since those results were experimentally validated, we have used the results of that study to compare with our developed system for the case where the subject’s standing position changes.

3.6 Additional Remarks

An important consideration when investigating clinical viability of a standing neuroprosthesis is transferability of the system to a portable device. This is of particular concern in studies having derivative component in their control strategy such as the PID study carried by Same et al [5]. One of the major arguments against the use of a PID control strategy for facilitating stance is that the derivative terms causes amplification of high frequency measurement noise [61]. In his theoretical study Same et al. modelled measurement noise in their standing apparatus setting by applying small fluctuations to the reference angle, as it was simpler to implement. In this research however we plan to employ a closed loop strategy which rejects external disturbances that are separate from reference angle while ensuring that the controller strategy does not cause amplification of high frequency measurement noise and would not need an additional filtering.

The feedback controllers comprising integral action result in infinite gain at zero frequency, and hence large robustness against model uncertainty at low frequencies, such as constant
offsets and disturbances. The plant gain will naturally go to zero at high frequencies, and the loop is thus protected against noise and model uncertainty. The critical area is the crossover region, i.e. for frequencies close to the closed-loop bandwidth. Here, the model needs to be sufficiently accurate to ensure robustness of stability and performance against model error [61]. This is why analytical control design procedures based on simple linear models, rather than trial and error tuning of PID controllers is preferred.
4. Research Questions, Objectives and Hypotheses

4.1 Research Questions

• In the context of FES-assisted quite stance for individuals with SCIs, can a controller employing robust servomechanism problem track a reference position, while simultaneously reject disturbances and bring the model of human quiet stance represented as a single joint, single segment, inverted pendulum system to a standing position?

4.2 Thesis Objectives

The aim of this research is to achieve the following objectives:

• To develop a theoretical procedure to transform the dynamic loop system of quiet stance, which has physiological and biomechanical properties, into a simple discrete time state space representation form.
• To develop a controller for the derived model using servomechanism control
• To assess the stability and robustness of the proposed controller using theoretical analysis and to compare the controller’s performance with relevant control systems used in the past.

4.3 Hypotheses

The proposed control strategy will yield faster tracking response with the presence of external disturbances and noticeable improvements overall, as compared to past control strategies used to balance human’s quiet standing modeled as a single segment, single joint, inverted pendulum. The proposed control strategy allows for robustness, with the presence of perturbations in the plant model (e.g. change in subject’s mass). Performance of our developed controller will be compared with a past PD control study [4] having the same plant model in their closed loop system as well as a PID system [5].
5. Preliminary Discussion for Controller design

In this section we shall explore some of the underlying control theory which will be used in the development of a controller.

5.1 Signals and Systems

Continous time signals (e.g. signals in the human body and in nature or those used in analogue transmission) are continous functions of time. Discrete time signals however appear in virtually all modern electronic devices, and are the primary way for electrical devices to send/receive signals. In this case analogue signals can be received or transferred using sampling [62]. Continous signals can be sampled to yield discrete time signals; in this case a sample of the continous signal is taken at every sampling period $T_s$ as shown in Figure 6. For instance, a signal from the brain with a sampling of 0.01s refers to taking 100 samples of the signal per 1 second.

![Figure 6 Example of Continous Time and Discrete Time Signal from [62]](image)

5.2 State Space and Matrix Representation of Systems

Classical control theory methods are based on a single-input/single-output description of a system usually expressed as a transfer function in the frequency domain, or as an impulse
response in the time domain. Such input-output methods can quickly become complicated for larger systems composed of smaller subsystems.

The state-space description of modern control theory however provides a simple description of system dynamics for multivariable systems. In this case the system dynamics are represented as a set of coupled first-order differential equations (i.e. which consist as a set of internal variables known as state variables) together with a set of algebraic equations that combine the state variables into physical output variables [63]. The state of a dynamic system refers to a set of variables (state variables) that fully describe the system and its response to any given set of inputs.

The state-space representation of a system replaces an $n^{th}$ order differential equation with a single first order matrix differential equation. The state space representation of such a system is given by the following two equations [63]:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$
$$y(t) = Cx(t) + Du(t) \quad (2)$$

In this case the first equation is called the state equation, the second equation is called the output equation, where $u$ is a vector of all input signals, and $y$ is a vector of all output signals. $\dot{x}$ is the time derivative of the state vector $x$. For an $n^{th}$ order system (i.e. where $x$ is an $n$-dimensional column vector) with $r$ inputs and $m$ outputs, the size of each of these matrices are:

- $x$ is $n \times 1$ (n rows by 1 column) dimension vector; $x$ is called a state vector which is a function of time
- $A$ is $n \times n$ dimension matrix; $A$ is the state matrix, i.e. a constant
- $B$ is $n \times m$ dimension matrix; $B$ is the input matrix, i.e. a constant
- $u$ is $m \times 1$ dimension matrix; $u$ is the input vector, which is a function of time
- $C$ is $r \times n$ dimension matrix; $C$ is the output matrix, i.e. a constant
- $D$ is $r \times m$ dimension matrix; $D$ is a constant matrix
- $y$ is $r \times 1$ dimension matrix; $y$ is the output vector which is a function of time
The state space representation is not unique; many state space systems can be used to represent any linear physical system.

A point $x_0$ in the state space is an equilibrium point of the autonomous system $\dot{x} = Ax$ if when the state $x$ reaches $x_0$, it stays at $x_0$ for all future time [64]. That is, for a linear time invariant (LTI) system the equilibrium point is the solution of the equation:

$$Ax_0 = 0 \quad (3)$$

A property of a linear system is they have one equilibrium point at the origin, while nonlinear systems may have many equilibrium points.

Since we deal with state space systems, the eigenvalue of a matrix is an important characteristic. For a matrix $A$, the eigenvalue problem consists of finding a nonzero vector $x = [x_i]_{1 \times n}$ and scalar $\lambda$ such that they satisfy the following equation:

$$Ax = \lambda x \quad (4)$$

where $\lambda$ is the scalar eigenvalue of matrix $A$, and $x$ is the corresponding right eigenvector of $A$. A necessary and sufficient condition for the above equation to have a non-trivial solution for vector $x$ is that:

$$\det(\lambda I - A) = 0 \quad (5)$$

where $I$ is the identity matrix. The $n$ roots of the characteristic equation (5) are the $n$ eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$.

### 5.3 Stability

Stability is an important property of a system, and various definitions of stability have been made in the literature. In our research we are mainly concerned with bounded input-bounded output stability (BIBO). In simple terms, a system is said to be BIBO stable when a bounded input signal (such as signals from the brain) is sent to the system, and the system produces bounded outputs (such as a bounded range of muscle contractions and hence bounded leg movement). An example of BIBO stability of a system can be observed via the impulse response of a system as follows.
Consider the impulse response \( h(t) \) of a system (1), (2) due to an impulse on the input. Based on the impulse response inputted to a dynamic system, the system’s stability can be categorized into one of the following [65]:

- **Asymptotically stable system:** The stationary impulse response, \( h(t) \), is zero; i.e.
  \[
  \lim_{t \to \infty} h(t) = 0 \quad (6)
  \]

- **Marginally stable system:** The stationary impulse response is different from zero, but bounded:
  \[
  0 < \lim_{t \to \infty} h(t) < \infty \quad (7)
  \]

- **Unstable system:** The stationary impulse response is unbounded:
  \[
  \lim_{t \to \infty} h(t) = \infty \quad (8)
  \]

Figure 7 illustrates a system’s output behavior for different stability categories.
In the frequency domain, a similar remark can be made. The transfer function of the system \( H(s) \), which is the Laplace transform of the impulse response, can be equivalently used to check for different stability properties. The transfer function, \( H(s) \) can be written as:

\[
H(s) = \frac{y(s)}{u(s)} \tag{9}
\]

The roots of the denominator polynomial of \( H(s) \) are poles of the system. The poles of \( H(s) \) could give information regarding system’s stability:

- Asymptotically stable system: Each of the poles of the transfer function lies strictly in the left half plane (has strictly negative real part)
• Marginally stable system: One or more poles lies on the imaginary axis, and all of these poles are distinct. In addition, no poles lie in the right half plane.

• Unstable system: At least one pole lies in the right half plane (i.e. has a real part greater than zero). Or: There are multiple poles on the imaginary axis.

Eigenvalues in the state-space representation can also be used to determine whether an equilibrium point is stable or unstable. A stable equilibrium point is such that a system can be initially disturbed around its equilibrium point yet it will eventually return to its original location and remain there.

The eigenvalues of a system linearized around an equilibrium point can determine the stability behavior of a system around the equilibrium point [66], and for the system:

$$\dot{x} = Ax$$  \hspace{1cm} (10)

the system is asymptotically stable if and only if $\text{Re} [\lambda_i] < 0$ for $i \in [1, n]$.

### 5.4 Observability

Consider a linear, time invariant, discrete-time system in state space form [62]

$$x(k+1) = A_d x(k), \quad x(0) = x_0$$  \hspace{1cm} (11)

where $x_0$ is unspecified, and the output $y(k)$ is given by:

$$y(k) = C_d x(k)$$  \hspace{1cm} (12)

where $x(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}^m$, and $A_d$ and $C_d$ are constant matrices of the discrete system with appropriate dimensions. The system is said to be observable when we can determine complete information about the internal state variables at any discrete time instant with only having knowledge of the system outputs over that time period.

The following result is obtained from Kalman [62]:

Lemma 1: The system (11) is observable if and only if \(\text{rank} \begin{bmatrix} C_d & C_d A_d^{n-1} \end{bmatrix} = n\)
5.5 Controllability

Consider a linear discrete-time invariant system:

\[ x(k + 1) = A_d x(k) + B_d u(k) \quad x(0) = x_0 \tag{13} \]

The system is said to be controllable if we can transfer the system using the input \( u(k) \) from any initial state \( x(0) = x_0 \) to any desired final state \( x(k_f) = x_f \) in a finite time. The following result is obtained from Kalman [62].

Lemma 2: The system (13) is controllable if and only if \( \text{rank}(B_d A B_d \ldots A^{n-1} B_d) = n \)

5.6 Designing an Observer for a Linear System

Observers are designed to reconstruct (or to estimate) the state or a linear combination of the states of the system using the input and output signals of the system.

In particular consider the linear time invariant multivariable system (1), (2):

\[ \dot{x}(t) = Ax(t) + Bu(t) \quad x(t_0) = x_0 \]
\[ y = Cx(t) \tag{14} \]

where \( x_0 \) is unknown. An observer for the system is postulated as

\[ \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) \quad \hat{x}(t_0) = \hat{x}_0 \]
\[ \hat{y} = C\hat{x}(t) \tag{15} \]

where \( \hat{x}(t_0) = 0 \). Since the initial condition \( x_0 \) of the system is an unknown, a difference in the output of the system (14) and the observer exists defined as an error signal \( e(t) \):

\[ y(t) - \hat{y}(t) = Cx(t) - C\hat{x}(t) = Ce(t) \tag{16} \]

where \( e(t) \) is defined as

\[ e(t) = x(t) - \hat{x}(t) \tag{17} \]
An observer can now be constructed that takes into the account feedback information about the observation error as follows:

\[ \dot{x}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - \hat{y}(t)) = A\hat{x}(t) + Bu(t) + KCe(t) \quad (18) \]

where K is known as the Observer gain, which is chosen by finding k so that the matrix A-KC is asymptotically stable and “fast”. Using above equations (16), (17), (18) we obtain:

\[ \dot{e}(t) = (A - KC)e(t) \quad (19) \]

the problem of finding an observer turns into finding an observer gain K such that the feedback matrix A-KC is asymptotically stable (i.e. has all eigenvalues with negative real parts), resulting in an estimation error e(t) which decays to zero for any initial condition e(t₀).

This can be achieved if the pair (A, C) is observable, which is equivalent to having the pair (Aᵀ,Cᵀ) controllable [62] [67] [68].
6. Methods

The human quiet posture exhibits erratic motion of a complex nature described as “postural sway”. How the joints and muscles of the body are coordinated for postural stability is important in understanding how the nervous system coordinates the body position in space.

Characteristics of the complex nature of human postural sway have been extensively investigated through inverted pendulum models with sensory feedback and these models have been widely used to identify physiological mechanism that control and stabilize quiet stance.

Generally, control system design deals with the problem of making a given physical system behave according to certain desired specifications. To obtain a controller for a given physical device the following steps are taken:

- In the first step, a mathematical model of the physical system is obtained. One method to achieve a mathematical model is by applying the basic laws which the physics of the system satisfies, also known as first principles modeling. Another method is called system identification, where by carrying out experiments on the physical system, a mathematical model is obtained. Very often, a combination of first principles modeling and system identification is used to obtain a model.
- In the second step, a decision is made regarding which desirable properties of the system we want to be satisfied, i.e. the design specifications. Very often, these properties can be formulated mathematically by requiring the mathematical model to have certain qualitative or quantitative mathematical properties.
- The third step is to design a mathematical model of the controller, on the basis of the mathematical model of the physical system, and the list of design specifications. This process is called a control synthesis problem [69].

The aim of this research project is to develop a controller for a closed loop system regulating quiet stance. This is to be a replacement for the active torque mechanism conducted by the CNS in healthy individuals. To develop a new controller the following steps are taken:

- Determine the state space equations of the plant components
• Convert the continuous state space model to a discrete time state space model
• Determine the overall state space equation of the plant
• Develop a controller for the given plant
• Apply the discrete time controller obtained to the system
• Examine the performance of the system under different upright stance scenarios by varying the system inputs (reference input, noise input)
• Examine the robustness of the system by varying the plant parameters of the system by 10%.

6.1 On Selecting the Plant Model

Quiet stance involves continuously maintaining the body against gravity, and any external or internal disturbances. Bipedal Quiet Stance is inherently unstable. A small sway deviation from a perfect quiet position results in a torque due to gravity that accelerates the body further away from the quiet position. To maintain Quiet Stance, the destabilizing torque due to gravity must be countered by a corrective torque exerted by the feet against the support surface. A past study [70] suggested that the corrective torque is generated through the action of a feedback control system. This corrective torque, which necessarily involves a time delay due to sensory transduction, transmission, processing, and muscle contraction delays, is known as active torque. Some other studies [71] have suggested that the corrective torque is generated by muscle “tone” that acts without time delay [72] known as passive torque.

While in recent years it has been suggested that both active and passive torque play a role in regulating ankle torque, the aim of this research is to develop a controller that will regulate the active torque in a manner similar to the one generated by CNS.

In the selected system therefore, only the active torque generated via CNS is considered. The inputs to the system are the reference position of the subject’s center of mass, known as \( Y_{\text{ref}} \), as well as a noise source \( \omega(k) \). Physiologically, \( \omega(k) \) represents any environmental disturbances (external to the body) and internal disturbances caused by the body function.

Examples of environmental noise sources are uneven standing surfaces [73], added weight to the subject or sudden gentle or not so gentle contact of the subject with its environment [5].
Internal body noise sources can be the result of physiological constraints of the human body. For example, the subject closing his/her eyes while standing [4], the sway disturbance introduced due to breathing or stretching of an arm [60] of the person standing are examples of internal body noise sources.

As the above noises could be constant or could be rapidly dynamic, in our simulations we have considered both constant and rapidly varying forms of noise sources.

For example, a constant disturbance could resemble a weight added to the person while standing or a weight removed from the person [5].

A rapidly varying noise source can resemble a situation where the person is trying to achieve balance in a much more dynamic situation. A past study [58] considers a random disturbance corresponding to the summation of all internal noise inducing spontaneous body sway while another study [60] reports using a random noise source to producing a disturbance torque which is equivalent to the ankle moment generated when manipulating an object of approximately 0.2–0.3 kg with fully extended arms.

Another past study [4] reports that a Gaussian noise (with a zero mean and unit variance passed through a low pass filter) represents sway characteristics seen in experiments. This study further notes that “this noise had similar amplitude and frequency components as the one used for weak external perturbations in the study by Fitzpatrick et al.” In Fitzpatrick’s study he demonstrated that perturbations applied at the waist level through a spring (which has a rapidly varying behavior similar to a Gaussian or Random noise source) provide sway similar in magnitude and rate to the sway of normal stance.

Therefore while a constant disturbance can provide the system's step response characteristics, dynamic noise sources may better resemble some real life situations such as a person trying to stand while the wind is pushing him/her or standing on a tilting surface.

The selected plant model shown in Figure 8 from [4] is used to develop a controller for regulating active ankle torque in this thesis.
Figure 8 Plant Model Block Diagram adopted from [4]. The inputs to the controller are reference angle $\theta_{ref}(k)$ and the plant output angle $\theta(k)$ delayed by $\tau_2$. The plant inputs are controller output $u(k)$ and noise $\omega(k)$. The plant comprises the body dynamics block, and time delays block $\tau_1$, $\tau_2$, and a noise filtering block. $h$ is the distance of Center of Mass from the ankle’s axis of rotation.

A summary of system specifications from [4], shown in Figure 8, is listed in Table 1.

Table 1 Plant Model Specifications

<table>
<thead>
<tr>
<th>Parameter (further explained in the text)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{ref}$ – reference angle</td>
<td>Based on desired tracking</td>
</tr>
<tr>
<td>$\tau_1$ (“sensory” time delay)</td>
<td>0.05s</td>
</tr>
<tr>
<td>$\tau_2$ (“command” time delay)</td>
<td>0.05s</td>
</tr>
<tr>
<td>Noise</td>
<td>Gaussian noise with zero mean and unit variance, or constant noise.</td>
</tr>
<tr>
<td>$\tau_f$ (time constant of low pass filter)</td>
<td>1</td>
</tr>
<tr>
<td>$m$ (mass)</td>
<td>76kg</td>
</tr>
<tr>
<td>$I$ (moment of inertia of the body about the ankle joint excluding feet)</td>
<td>66kgm$^2$</td>
</tr>
</tbody>
</table>
The selected system [4] measured kinematic parameters of a healthy subject as well as EMGs of the ankle extensors during quiet stance, and carried cross-correlation analysis to investigate the relationship between kinematic parameters and rectified EMGs. The cross-correlations between COMdis and EMGs indicated that the body sway is closely related to muscle activities.

Also, the findings of [4] showed a postural control system during quiet stance adopts a control strategy that relies notably on velocity information. Since this system utilized the single joint single segment inverted pendulum model for the human body dynamics, the COM position and velocity were conveniently obtained.

The system used subject parameters of a typical male adult and time delays commonly used in literature. Also, the effect of disturbance was incorporated in this model which later helped in testing the robustness of our developed controller.

While past studies have reported using stimulation frequency of 35 Hz, a number of studies have suggested that muscle contraction may be enhanced through the use of electrical stimuli at higher frequencies up to 100Hz which produce synaptic recruitment of spinal motoneurons and recruit a greater number of fatigue resistant motor units through reflex activation of smaller motoneurons [60]. Similar to system [4], the sampling time of our system was therefore set to 0.01s (100 Hz).

### 6.2 Time Delay Blocks

Time delays are produced as a result of the propagation of the action potentials along the neuronal structures. The selected system categorizes time delays into the following groups:

- **Neuro-mechanical delay**: Motor command time delay representing the time loss due to the sensory-motor information process in the central nervous system and the neural
transmission from the central nervous system to the plantar flexors, marked as \( \tau_1 (0.05s) \).

- Feedback time delay representing cumulative time loss due to neural transmission from the ankle somatosensory system to the brain, marked as \( \tau_2 (0.05s) \).

Given that the time delays were 0.05s each and the simulation frequency was 100Hz, the following transfer function model was created:

\[
\tau_{tf} = \frac{e^{-s \cdot 0.05}}{s + 100} \quad (20)
\]

Using the matlab command ssdata (), the continuous time state space representation was then obtained.

\[
s = \text{tf}(\text{'s'});
\tau = 1/(s+1e2)*\exp(-s*0.05);
[a_\tau, b_\tau, c_\tau, d_\tau, Ts] = \text{ssdata}(\tau);
\]

Each time delay, namely the neuro-mechanical and feedback time delays had a continuous time state space representation of the form:

\[
a_\tau = -100 \quad b_\tau = 1 \quad c_\tau = 1 \quad d_\tau = 1
\]

6.3 Noise Transfer Function Block and Disturbance Torque

A Gaussian noise time series with zero mean and unit variance was used to model disturbance torque \( T_d \). This noise was first-order and low-pass filtered. The study suggested that this is the noise present at muscle torque generation process and it is a result of internal noise sources.

Given that the noise transfer function was:

\[
T_d = \frac{100}{s + 1} \quad (21)
\]

the continuous time state space representation of noise was obtained in Matlab as follows:

\[
[a_{Td}, b_{Td}, c_{Td}, d_{Td}] = \text{tf2ss}([0 \; 100],[1 \; 1])
\]
where $[0 \ 100]$, and $[1 \ 1]$ were the numerator and denominator noise transfer function coefficients from highest to lowest order of magnitude.

The disturbance torque $T_d$ had a continuous time state space representation of the form:

$$a_{T_d}= -1 \quad b_{T_d}= 1 \quad c_{T_d}= 100 \quad d_{T_d}= 0$$

### 6.4 Body Dynamics Block and $\theta$

In this system the body dynamics and kinematics during quiet stance was described using a single joint single segment inverted pendulum model pivoted at the ankle joints, with the center of mass (COM) of the body on average a few centimeters in front of the ankle joints. In this case, the inverted pendulum swayed back and forwards somewhat erratically around COM position. The COM (blue circle in Figure 9) deviates from the vertical line passing through ankle joint in quiet stance by an angle $\theta$ in normal quiet standing. $\theta$, also known as the sway angle, results in COM being located $y$ [cm] in from of the ankle joint. The length $y$ is alternatively called the center of mass displacement (COMdisp).
Figure 9. Body Dynamics as an Inverted Pendulum Model from [74] and [56]. $\theta$ is the sway angle, $m$ is the mass of the body, $h$ is the distance of COM from the ankle’s axis of rotation, $g$ is the gravitational acceleration, $T$ is the total ankle torque, $f$ is the ground reaction force to counter the torque $T$ created by force of gravity ($mg$) on COM. $f_v$ and $f_h$ are the vertical and horizontal component of ground reaction force respectively. $u$ is COP position. $y$ is the vertical projection of the COM position.

The body dynamics can be described as an inverted pendulum as shown in equation (22). $\theta$ is the sway angle, $\ddot{\theta}$ is the sway acceleration, $m$ is the mass of body, $I$ is the moment of inertia of the body (excluding feet), $h$ is the distance of COM from the ankle’s axis of
rotation, \( g \) is the gravitational acceleration, \( T \) is the total ankle torque, and \( \epsilon \) is the torque disturbance.

\[
I \ddot{\theta} = mgh \sin \theta + T + \epsilon \quad (22)
\]

The ankle torque should satisfy the following equation for the foot segment:

\[
T + f_v u \sim 0 \quad (23)
\]

Where \( f_v \) is the vertical component of ground reaction force and \( u \) is COP position (COPdis). If we take into account that \( f_v \sim mg \) in quiet stance, this equation shows that changes in ankle torque are immediately and linearly translated into changes of COP position. From above equations one can derive

\[
u \sim y - \frac{I}{mg} \ddot{\theta} \quad (24)
\]

where \( y \) is the COM position (COMdisp). The COPdisp and COMdisp must coincide (\( u = y \)) under the static quiet standing condition. However, due to excess ankle torque, \( u \neq y \) and the excess ankle torque generates COM acceleration (\( \ddot{\theta} \neq 0 \)). Thus reduction of excess ankle torque to facilitate alignment of the COP position and the COM projection on the standing surface represents the active mechanism for controlling quiet stance [58].

Gatev et al. who found a high correlation between COMdis and lateral gastrocnemius muscle (LG) in their study [75] suggested that the ankle torque generated by ankle extensors are almost equal to the gravity torque of the body \((mgh \sin \theta \sim T)\), and that the active mechanism stabilizes the body well.

Expressing the equation in Laplace transform gives:

\[
\text{Body Dynamics} = \frac{1}{Is^2 + mgh} = \frac{1}{66s^2 + 647.976} \quad (25)
\]

where \( s \) is the Laplace variable and the small angle approximation \( \theta \sim \sin (\theta) \) was used to simplify the equation. The continuous time state space representation of noise was obtained as follows:
Body dynamics had continuous time state space representation of the form:

\[
\begin{align*}
\text{a\_body} &= \begin{bmatrix}
0 & 2.4545 \\
4 & 0
\end{bmatrix} \\
\text{b\_body} &= \begin{bmatrix}
0.0625 \\
0
\end{bmatrix} \\
\text{c\_body} &= \begin{bmatrix}
0 & 0.0606 \\
0 & 0
\end{bmatrix} \\
\text{d\_body} &= 0
\end{align*}
\]

6.5 Overall Plant Discrete Time State Space Representation

The code in Appendix A uses the A, B, C, and D matrices, as well as sampling time (0.01s) and time delay of each plant block to find the overall plant discrete time state space representation without time delay.

The plant equation is given as:

\[
x(k+1) = A_p x(k) + B_p u(k) + E_p w(k) \quad (26)
\]

\[
y(k) = C_p x(k) \quad (27)
\]

The output of the code produces the following plant matrices:

\[
A_p = \begin{bmatrix}
0.36788 & 0.63212 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7.50061e-7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.00015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_p = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
C_p = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]


\[E_p = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0.99502];\]

\[D_p = [0 0];\]

### 6.6 Controller Design

The problem at hand is in essence a servomechanism problem in the context of control theory, and the robust control of the servomechanism problem has been addressed in the literature extensively [76][77][78][79][80][81]. A servomechanism problem is a very common problem in automatic control. According to the definition in [82], “In the servomechanism problem, it is desired to design a controller for a plant so that the outputs of the plant are independent, as much as possible, from the disturbances which may affect the system (i.e. regulation occurs), and also such that the outputs asymptotically track any specified reference input signal applied to the system (i.e. tracking occurs), subject to the requirements of maintaining closed-loop system stability.”

In this work, the goal is for the output (COM position) to achieve ‘tracking’ of a reference input (the angle at which the ankle joint must be). This needs to be done reliably such that in presence of disturbances and noise, e.g. with a push on the subject (modelled as \(\omega_k\), the system is still able to track the input. Moreover, it is essential that the system maintains stability throughout its various operation conditions. Hence regulating active ankle torque during quite standing in SCI patients will be carried out by considering the problem as a servomechanism problem.

Additionally, we aim to achieve a robust control of this servomechanism problem. The controller must be robust, meaning that the system still can track the input and reject noise, even if there are perturbations (or variations) in the assumed plant model. This means that for example, that if the mass of the SCI is in fact not exactly equal to the nominally assumed 76kg in the design of the controller, the system can still reliably both track both the input (and hence enable standing for the SCI), as well as rejection of noise and disturbances. This is quite a beneficial feature of the proposed design.
The controller used in this work originally introduced by Prof. E. J. Davison has been successfully applied here to the given three term servomechanism problem. It is shown in [76] that the controller is required to comprise a servo-compensator as well as a stabilizing device. The servocompensator, which is analogous to the Integral controller in classical control theory, plays an essential role and must be used in any servomechanism problem to assure that the controlled system is stabilizable and achieves robust control.

The tracking, regulation/noise rejection, as well as robustness of the controller is verified in the context of the system for achieving quite standing in SCI subjects.

6.7 **Controller Properties**

Designing a robust controller for a linear, time-invariant, multivariable system (plant) so that asymptotic tracking/regulation occurs independent of input disturbances and arbitrary perturbations in the plant parameters (assuming that the plant perturbations do not destabilize the system), is the basis of robust control in the servomechanism problem.

6.8 **Controller Equation**

Let the state of the controller be represented by the variable \( z(k) \) with inputs \( y_{\text{ref}}(k) \) and \( \omega(k) \) and output \( u(k) \). \( A_c, B_c_y, B_c_yref, \) and \( C_c \) are the controller state space matrices. The controller equation is given as:

\[
\begin{align*}
z(k+1) &= A_c z(k) - B_c_yref y_{\text{ref}}(k) + B_c_y y(k) \\
u(k) &= C_c z(k)
\end{align*}
\]

The output of the code for the controller matrices are:

\[
A_c = [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 -697.6004 0 0; \\
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0; \\
-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 0 0 0 0 0 0 -64842.07951 -5601.41621 -4.59832; \\
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; \\
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;]
\]
\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -0.13297 & 0 & 0;
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -0.21289 & 0 & 0;
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -0.30867 & 0 & 0;
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.58444 & 0 & 0;
0 & 7.50061 \times 10^{-7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.47234 & 0.01000 & 7.50061 \times 10^{-7};
0 & 0.00015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11.32694 & 1.00049 & 0.00015;
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99005;
\end{bmatrix}
\]

\[
B_{c_y} = [1 ; 697.60004 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0.01288 ; 0.07093 ; 0.13297 ; 0.21289 ; 0.30867 ; 0.41556 ; 0.52815 ; 11.425060 ; 0 ];
\]

\[
B_{c_yref} = [-1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0 ];
\]

\[
C_c = [-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 0 0 0 0 0 -64842.07951 -5601.41621 -4.59832 ];
\]

\[
D_c = [0 ];
\]
6.9 Closed Loop System

The closed loop representation of the entire system (Controller + Plant) then becomes:

\[
\begin{bmatrix}
x(k + 1) \\
z(k + 1)
\end{bmatrix} =
\begin{bmatrix}
A_p & 0 \\
B_c_y * C_p & A_c
\end{bmatrix}
\begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix} +
\begin{bmatrix}
B_p \\
0
\end{bmatrix} u(k) +
\begin{bmatrix}
E_p \\
0
\end{bmatrix} w(k) +
\begin{bmatrix}
0 \\
-1 \cdot B_c_yref
\end{bmatrix} y_{ref}(k)
\]

\[u(k) = C_c * z(k)\]

\[
\begin{bmatrix}
x(k + 1) \\
z(k + 1)
\end{bmatrix} =
\begin{bmatrix}
A_p & B_p * C_c \\
B_c_y * C_p & A_c
\end{bmatrix}
\begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix} +
\begin{bmatrix}
E_p \\
0
\end{bmatrix} w(k) +
\begin{bmatrix}
0 \\
-1 \cdot B_c_yref
\end{bmatrix} y_{ref}(k)
\]

\[y_k = [C_p \ 0] \begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix}\]

\[u_k = [0 \ C_c] \begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix}\]

So given that the Controller has state space matrices: \(A_c, B_c_y, B_c_yref,\) and \(C_c\) and the plant has state space matrices: \(A_p, B_p, C_p,\) and \(E_p,\) the system’s closed loop state space representation is given by:

\[A = [A_p \ B_p \times C_c; B_c_y \times C_p \ A_c];\]

\[B=[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;1 \times B_c_yref];\]

\[C=[C_p \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 ];\]

\[D=0;\]

where \(y_k\) and \(u_k\) are the outputs of the plant and the controller, respectively. Since the order of the plant is 16 and the order of Controller is 15, the dimension of the closed loop system becomes 31.

The following Matlab commands are used to obtain the times series output of the closed loop system (\(y\_system\)) and controller output (\(u\_system\)) respectively. \texttt{ss()} and \texttt{lsim()} are utilized to find the time series of the output from the state space matrices. In the state space
representation of the closed loop system, depending on the desired signal to be observed, we utilize the corresponding state space matrices. For instance, the plant output and controller output signals are calculated using the corresponding C matrices, C1 and C2 respectively. The variable U is defined as a two column matrix, with first column being the times series of the reference angle, and the second column being the times series of the noise input. Variables controller_y and sys_y are used as intermediate variable to convert the state space matrices to a state space object in Matlab, and then the time series is found by running lsim on these variables.

\[
B_{sys} = \begin{bmatrix} B & E \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix} C_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]

\[
sys_y = \text{ss}(A, B_{sys}, C1, D, 0.01);
\]

\[
[y_{system}, t, x] = \text{lsim}(sys_y, U, t);
\]

\[
C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_c \end{bmatrix};
\]

\[
\text{Controller}_y = \text{ss}(A, B_{sys}, C2, D, 0.01);
\]

\[
[u_{system}, t, x] = \text{lsim}(\text{Controller}_y, U, t);
\]

### 6.10 Simulink Model

Our developed controller uses the same plant model as the PD based controller study by Masani et al. [4]. Based on the information provided in the study by Masani et al. [4], the PD control system was recreated in Simulink to compare our developed controller with the Masani et al. PD controller system for a variety of reference angles and noise inputs.
7. Results

7.1 Controller Results in Comparison to PD Controller

In this section the simulation results of our developed controller with the Plant model [4] described in Section 5 is shown. The closed system comprising the plant and controller has two inputs, namely the reference angle input \( y_{\text{ref}} \) (rad) and the noise \( \omega \) (rad). The output of the system in angular form (\( \theta_{\text{output}} \), also known as COM position), length form (COMdisp) as well as the controller output (u) for the following inputs are shown and examined:

- Constant Reference input, \( y_{\text{ref}} = 0.1 \) rad (~5°), \( \omega = 0 \) rad
  [Figure 10, Figure 11, Figure 12, Figure 13, Figure 14, Figure 15, Figure 16]

- Constant Noise input, \( y_{\text{ref}} = 0 \) rad, \( \omega = 0.244 \) rad (~14°) rad
  [Figure 17, Figure 18, Figure 19, Figure 20, Figure 21]

- Gaussian Noise input, \( y_{\text{ref}} = \) Gaussian Noise, \( \omega = 0 \) rad
  [Figure 22, Figure 23, Figure 24, Figure 25, Figure 26]

- Varying Reference Input, \( y_{\text{ref}} = \) Step change, \( \omega = 0 \) rad
  [Figure 27, Figure 28]

Please refer to Appendix B for our system and the PD system Simulink code to obtain results for the above input cases.
7.1.1 Constant Reference Angle

To achieve quiet standing, the COM makes an angle of about or less than 0.1 rad (~5°) with respect to the vertical. In order to test if the developed controller could bring an SCI individual’s COM position to the desired standing angle, a reference input of 0.1 rad shown in Figure 10 is inputted to the system.

![Figure 10. Constant Reference Angle of 0.1 rad Input](image)

Figure 11 depicts our system plant output angular position (COM position) results and Figure 12 depicts the PD control system COM position results which are based on our Simulink simulations of the PD system in [4]. The inputs to both systems are a reference angle of 0.1 rad (~5°) and 0 rad noise input. Our controller tracks the reference angle exactly within 0.6 seconds, while it will take approximately 6 seconds for the PD control system to reach a value that is in fact larger 0.23 rad (~14°) than the desired reference input. The exact tracking achieved with our controller can be clearly seen here as a benefit over systems with PD control as they normally cannot achieve exact tracking.
Figure 11. Plant output (COM position) using robust control strategy when a constant reference angle input (Figure 10) was inputted to the system.

Figure 12. Plant output (COM position) using PD control strategy when a constant reference angle input (Figure 10) was inputted to the system.
Figure 13. Controller torque output generated using robust control strategy when a constant reference angle (Figure 10) was inputted to the system.

Figure 14. Controller torque output generated using PD control strategy when a constant reference angle (Figure 10) was inputted to the system.
Figure 13 and Figure 14 depict our system and the PD control system controller torque output for the case of constant reference input of 0.1 rad and 0 rad noise input.

In our system the controller torque fluctuates between +/- 600 N.m until it reaches and maintains the value of -65 N.m after 0.8s. Since the position of COM with respect to the vertical is annotated with a sign, a command torque of -65 N.m corresponds to exerting a torque in an opposite direction to COM. This is because COM is located in front of the ankle joint, and the backward ankle torque is continuously applied to the body to prevent it from falling. In the PD control system simulations, the controller torque was found to fluctuate between +/- 150 N.m and reaches and maintains the value of -150 N.m in steady-state.

The maximum ankle torque that a person can generate in isometric contractions (where the highest torques can be generated) is 250-260 Nm per each leg. The torque produced by our controller is larger than the tolerable amount. By changing the gain of the controller, our system was able to exactly track the reference input and keep the torque within the tolerable range. Comparing Figure 13 (COM position output of our system) to Figure 15 (COM position output of our system with controller gain adjusted), it can be seen by changing the gain of the controller exact tracking now occurs at 0.9s instead of 0.6s. And by comparing Figure 14 (Controller torque output of our system) with Figure 16 (Controller torque output of our system with controller gain adjusted), it can be seen that tolerable ankle torque joint fluctuation can be achieved with slightly longer tracking period.
Figure 15. Plant output (COM position) using robust control strategy with controller gains adjusted when a constant reference angle input (Figure 10) was inputted to the system.

Figure 16. Controller output torque generated using robust control strategy with controller gains adjusted when a constant reference angle (Figure 10) was inputted to the system.
7.1.2 Constant Noise Disturbance

A case studied in regulating quiet stance occurs when there is a constant disturbance applied to the COM. To better examine the effect of constant disturbance, the reference angle is set to zero, and a constant noise input of 0.244 rad (~14° which is the threshold standing angle the COM can deviate from vertical [5]) is applied as shown in Figure 17.

![Figure 17. Constant Noise Input](image)

As can be seen in Figure 18, and Figure 19, it takes approximately 6s for our controller and the PD controller to stabilize the system when a constant disturbance of 0.244 rad is present. In our system the plant output ($\theta$) varies very slightly up to 6e-4 rad (0.03°) at 0.32s, and exactly tracks the input within 6s. In the PD control system the plant output ($\theta$) goes up to 0.05 rad (3°) and remains at 0.05 rad after 8s. Thus, our controller attenuates the amplitude of the constant noise input by three orders of magnitude and also exactly tracks the reference angle which is 0 rad. This is while the PD system only attenuates the noise by one order of magnitude and it also does not exactly the track the reference angle, as it reaches a non-zero steady-state of 0.05 rad.
Figure 18. Plant output (COM position) using robust control strategy when a constant noise disturbance (Figure 13) was inputted to the system.

Figure 19. Plant output (COM position) using PD control strategy when a constant noise disturbance (Figure 13) was inputted to the system.
Figure 20. Controller output torque generated using robust control strategy when a constant noise disturbance (Figure 13) was inputted to the system.

Figure 21. Controller output torque generated using PD control strategy when a constant noise disturbance (Figure 13) was inputted to the system.
As seen in Figure 20, it takes about 6s for the controller to reach the steady state torque of about -25N.m, to counter the constant noise disturbance applied to the subject, while the PD system reaches a steady state torque of -60N.m as seen in Figure 21.

### 7.1.3 Gaussian Noise Disturbance

This study examines the anterioposterior body sway patterns when there is disturbances in the form of Gaussian noise present [4]. The Gaussian noise was chosen to characterize the summation of internal and external noise during quiet stance including body sway [58].

Figure 22 depicts the COM displacement and controller torque output simulation results of a previous study [4] using a PD control strategy with zero reference input, and Gaussian noise input. The plots show COMdisp has a range of 2cm and the controller torque has a range of 40 N.m.

![Figure 22 Past Study PD Control Strategy Simulation Results for Gaussian Noise Input [4]](image)

Since the output of our system is an angle measure, one may ask how COMdisp, which is a length measure, can be derived from the system’s output, which is an angle. Due to the geometry of the ankle during standing the following relationship holds between angle and y (COMdisp) shown in Figure 9 we have:
Figure 23 depicts the behavior of Gaussian noise inputted to our system. The Gaussian Noise fluctuates +/- 0.244 rad (+/-14°) from the vertical.

\[ y = h\sin(\theta) \quad (28) \]

Figure 24 depicts the system behavior with reference angle of 0 rad and a Gaussian noise input. While the noise input amplitude fluctuates between +/- 0.244 rad (which is the threshold standing angle the COM can deviate from vertical [5]), our system output amplitude is between +/- 1.5e-4 rad. This shows once again that our controller attenuates the amplitude of noise inputs by three orders of magnitude. However the PD system output amplitude is between +/- 2.5e-3 rad, meaning that it has attenuated the +/- 0.244 rad input noise by two orders of magnitude.
Figure 24. Plant output (COM position) using robust control strategy when a Gaussian noise disturbance (Figure 23) was inputted to the system.

Figure 25 depicts our system center of mass displacement (COMdisp) for reference input of 0 rad and Gaussian noise input. The COMdisp of our system oscillates between +/- 1.5 mm, while the PD system COMdisp oscillates between +/-2.5 mm.
Figure 25. Plant output (COMdisp) using robust control strategy when a Gaussian noise disturbance (Figure 23) was inputted to the system.

Figure 26 the controller torque exerted about the ankle joint for the case of zero reference input and Gaussian noise input.
Figure 26. Controller output torque generated using robust control strategy when a constant noise when a Gaussian noise disturbance (Figure 23) was inputted to the system.
7.1.4 Reference Angle Step Change

Another case studied in regulating Quiet Stance occurs when there is a change in the standing position of the individual. A past study employing a PID control strategy [5] has examined moving the reference position of a subject standing from 5° to 9°, and then back to 5° between times 10-18s of the 40s simulation period (Red plot in Figure 27). The blue plot in Figure 27 depicts the past system results while Figure 28 depicts our developed controller results for the case of a reference angle step change. Please note that the system using PID controller had an initial offset position of 14°, while it is 0° in our system. The difference in initial position is not important in this comparison as we are focusing on the behaviors of the system when the reference input is varied between times 10-18s. As can be seen in Figure 27, while both systems track the reference input in a similar fashion, the past system’s [5] results show more oscillation up to 10° at time 10s while our system has a smaller duration and amplitude of oscillation and remains close to the reference angle.

Figure 27 Plant output (COM Position) using PID control strategy when a reference angle step change (orange line) was inputted to the system. Image from [5]
7.2 Controller Robustness

In order to test the robustness of the developed controller, we introduced perturbations to the body dynamics component of the plant. The mass of the subject is varied from 90% to 110% of the nominal $m = 76$kg. As seen in Figure 29, Figure 30, Figure 35, and Figure 36 the controller indeed tracks the constant reference input in the presence of plant perturbation, hence confirming robustness. Additionally in the presence of constant and Gaussian noise, the controller also produces a robust result as seen in Figure 31, Figure 32, Figure 33, Figure 34, Figure 37, Figure 38, Figure 39, and Figure 40.

Figure 28 Plant output (COM Position) using robust control strategy when a reference angle step change (red line) was inputted to the system.
7.2.1 +10% Change in Mass

7.2.1.1 Constant Reference Angle of 0.1 rad

![COM Position](image1)

**Figure 29. COM Position during Constant Reference Angle and +10% Change in Mass**

![Controller Torque](image2)

**Figure 30. Controller Torque during Constant Reference Angle and +10% Change in Mass**
7.2.1.2 Constant Noise Disturbance 0.244 rad

Figure 31. COM Position during Constant Noise Disturbance and +10% Change in Mass

Figure 32. Controller Torque during Constant Noise Disturbance and +10% Change in Mass
7.2.1.3 Gaussian Noise Disturbance of 0.244 rad

Figure 33. COM Position during Gaussian Noise Disturbance and +10% Change in Mass

Figure 34. Controller Torque during Gaussian Noise Disturbance and +10% Change in Mass
7.2.2 -10% Change in Mass

7.2.2.1 Constant Reference Angle

Figure 35. COM Position during Constant Reference Angle and -10% Change in Mass

Figure 36. Controller Torque during Constant Reference Angle and -10% Change in Mass
7.2.2.2 Constant Noise Disturbance

Figure 37. COM Position during Constant Noise Disturbance and -10% Change in Mass

Figure 38. Controller Torque during Constant Noise Disturbance and -10% Change in Mass
7.2.2.3 Gaussian Noise Disturbance

Figure 39. COM Position during Gaussian Noise Disturbance and -10% Change in Mass

Figure 40. Controller Torque during Gaussian Noise Disturbance and -10% Change in Mass
8. Revisited Controller

In this chapter the simulation results of a revised controller are presented. The revised controller is an additional design attempt, with controller gains adjusted for achieving less rapid variations in the sway angle and ankle torque. The following shows that such controller is indeed possible, and achieves the goals of the robust servomechanism controller, i.e. it is stable, achieves exact tracking, rejects disturbances, and is robust against perturbations in the plant model. The matrices of the revised controller are presented in Appendix C.

8.1 Revisited Controller Results in Comparison to PD Controller

As in the previous section, the closed system comprising the plant and controller has two inputs, namely the reference angle input \( y_{\text{ref}} \) (rad) and the noise \( \omega \) (rad). The output of the system in angular form (\( \theta_{\text{output}} \), also known as COM position), length form (COMdisp) as well as the controller output (\( u \)) for the following inputs are shown and examined:

- **Constant Reference input**, \( y_{\text{ref}} = 0.1 \text{ rad (~5°)} \), \( \omega = 0 \text{ rad} \)
  [Figure 41, Figure 42, Figure 43, Figure 44]

- **Constant Noise input**, \( y_{\text{ref}} = 0 \text{ rad} \), \( \omega = 0.244 \text{ rad (~14°) rad} \)
  [Figure 45, Figure 46, Figure 47, Figure 48]

- **Gaussian Noise input**, \( y_{\text{ref}} = \text{Gaussian Noise} \), \( \omega = 0 \text{ rad} \)
  [Figure 49, Figure 50, Figure 51]

- **Varying Reference Input**, \( y_{\text{ref}} = \text{Step change} \), \( \omega = 0 \text{ rad} \)
  [Figure 52, Figure 53]
8.1.1 Constant Reference Angle

To test the performance of our revised controller for quiet standing, a reference input of 0.1 rad (~5º) shown in Figure 10 is inputted to the system.

Figure 41 depicts our system plant output angular position (COM position) results and Figure 42 depicts the PD control system COM position results which are based on our Simulink simulations of the PD system in [4]. The inputs to both systems are a reference angle of 0.1 rad (~5º) and 0 rad noise input. Our controller tracks the reference angle exactly within 1 second, while it will take approximately 6 seconds for the PD control system to reach a value that is in fact larger 0.23 rad (~14º) than the desired reference input. The exact tracking achieved with our controller can be clearly seen here as a benefit over systems with PD control as they normally cannot achieve exact tracking.

Figure 41. Plant output (COM position) using robust control strategy when a constant reference angle input (Figure 10) was inputted to the system.
Figure 42. Plant output (COM position) using PD control strategy when a constant reference angle input (Figure 10) was inputted to the system.

Figure 43 and Figure 44 depict our system and the PD control system controller torque output for the case of constant reference input of 0.1 rad and 0 rad noise input.

In our system, the revised controller torque fluctuates between +/- 130 N.m until it reaches and maintains the value of -65 N.m after 1s. Since the position of COM with respect to the vertical is annotated with a sign, a command torque of -65 N.m corresponds to exerting a torque in an opposite direction to COM. This is because COM is located in front of the ankle joint, and the backward ankle torque is continuously applied to the body to prevent it from falling. In the PD control system simulations, the controller torque was found to fluctuate between +/- 150 N.m and reaches and maintains the value of -150 N.m in steady-state.

The maximum ankle torque that a person can generate in isometric contractions (where the highest torques can be generated) is 250-260 Nm per each leg. It can be seen that tolerable ankle torque joint fluctuations are achieved with slightly longer tracking period resulting with not rapid fluctuations as observed with initial controller.
Figure 43. Controller torque output generated using robust control strategy when a constant reference angle (Figure 10) was inputted to the system.

Figure 44. Controller torque output generated using PD control strategy when a constant reference angle (Figure 10) was inputted to the system.
8.1.2 Constant Noise Disturbance

To observe the effect of constant disturbance on the revised controller, the reference angle is set to zero, and a constant noise input of 0.244 rad (~14°).

As can be seen in Figure 45, and Figure 46, it takes approximately 6s for our controller and the PD controller to stabilize the system when a constant disturbance of 0.244 rad is present. In our system the plant output (θ) varies very slightly up to 0.015 rad (0.86°) at 1.11s, and exactly tracks the input within 6s. In the PD control system the plant output (θ) goes up to 0.05 rad (3°) and remains at 0.05 rad after 6s. Thus, our controller completely rejects the constant noise disturbance and tracks only the reference angle which in this case is 0. This is while the PD system does not exactly track the reference angle, as it reaches a non-zero steady-state of 0.05 rad.
Figure 45. Plant output (COM position) using robust control strategy when a constant noise disturbance (Figure 13) was inputted to the system.

Figure 46. Plant output (COM position) using PD control strategy when a constant noise disturbance (Figure 13) was inputted to the system.
Figure 47. Controller output torque generated using robust control strategy when a constant noise disturbance (Figure 13) was inputted to the system.

Figure 48. Controller output torque generated using PD control strategy when a constant noise disturbance (Figure 13) was inputted to the system.
As seen in Figure 47, it takes about 6s for the controller to reach the steady state torque of about -25N.m, to counter the constant noise disturbance applied to the subject, while the PD system reaches a steady state torque of -60N.m as seen in Figure 48.

8.1.3 Gaussian Noise Disturbance

Figure 49 depicts the system behavior with reference angle of 0 rad and a Gaussian noise input. While the noise input amplitude fluctuates between +/- 0.244 rad (which is the threshold standing angle the COM can deviate from vertical [5]), our system output amplitude is between +/- 1.5e-3 rad. For the case of Gaussian noise input our revised controller attenuates the amplitude of noise by two orders of magnitude.

![COM Position during Gaussian Disturbance](image)

**Figure 49.** Plant output (COM position) using robust control strategy when a Gaussian noise disturbance (Figure 23) was inputted to the system.

Figure 50 depicts our system center of mass displacement (COMdisp) for reference input of 0 rad and Gaussian noise input. The COMdisp of our system oscillates between +/- 1.5 cm.

Figure 51 shows the controller torque exerted about the ankle joint for the case of zero reference input and Gaussian noise input.
Figure 50. Plant output (COMdisp) using robust control strategy when a Gaussian noise disturbance (Figure 23) was inputted to the system.

Figure 51. Controller output torque generated using robust control strategy when a constant noise when a Gaussian noise disturbance (Figure 23) was inputted to the system.
8.1.4 Reference Angle Step Change

Another case studied in regulating Quiet Stance occurs when there is a change in the standing position of the individual. A past study employing a PID control strategy [5] has examined moving the reference position of a subject standing from 5° to 9°, and then back to 5° between times 10-18s of the 40s simulation period (Red plot in Figure 52). The blue plot in Figure 52 depicts the past system results while Figure 53 depicts our developed controller results for the case of a reference angle step change. Please note that the system using PID controller had an initial offset position of 14°, while it is 0° in our system. The difference in initial position is not important in this comparison as we are focusing on the behaviors of the system when the reference input is varied between times 10-18s. As can be seen in Figure 52, while both systems track the reference input in a similar fashion, the past system’s [5] results show more oscillation up to 10° at time 10s while our system has a smaller duration and amplitude of oscillation and remains close to the reference angle.

![Figure 52 Plant output (COM Position) using PID control strategy when a reference angle step change (orange line) was inputted to the system. Image from [5]](image-url)
8.2 Controller Robustness

In order to test the robustness of the developed controller, we introduced perturbations to the body dynamics component of the plant. The mass of the subject is varied from 90% to 110% of the nominal $m = 76$kg. As seen in Figure 54, Figure 55, Figure 60, and Figure 61 the controller indeed tracks the constant reference input in the presence of plant perturbation, hence confirming robustness. Additionally in the presence of constant and Gaussian noise, the controller also produces a robust result as seen in Figure 56, Figure 57, Figure 58, Figure 59, Figure 62, Figure 63, Figure 64, Figure 65.
8.2.1 +10% Change in Mass

8.2.1.1 Constant Reference Angle of 0.1 rad

Figure 54. COM Position during Constant Reference Angle and +10% Change in Mass

Figure 55. Controller Torque during Constant Reference Angle and +10% Change in Mass
8.2.1.2 Constant Noise Disturbance 0.244 rad

Figure 56. COM Position during Constant Noise Disturbance and +10% Change in Mass

Figure 57. Controller Torque during Constant Noise Disturbance and +10% Change in Mass
8.2.1.3 Gaussian Noise Disturbance of 0.244 rad

Figure 58. COM Position during Gaussian Noise Disturbance and +10% Change in Mass

Figure 59. Controller Torque during Gaussian Noise Disturbance and +10% Change in Mass
8.2.2 -10% Change in Mass

8.2.2.1 Constant Reference Angle

Figure 60. COM Position during Constant Reference Angle and -10% Change in Mass

Figure 61. Controller Torque during Constant Reference Angle and -10% Change in Mass
8.2.2.2 Constant Noise Disturbance

Figure 62. COM Position during Constant Noise Disturbance and -10% Change in Mass

Figure 63. Controller Torque during Constant Noise Disturbance and -10% Change in Mass
8.2.2.3 Gaussian Noise Disturbance

Figure 64. COM Position during Gaussian Noise Disturbance and -10% Change in Mass

Figure 65. Controller Torque during Gaussian Noise Disturbance and -10% Change in Mass
9. Discussion and Conclusions

The aim of this research project was to develop a controller for a neuro-rehabilitative closed loop system regulating active ankle torque during quiet stance, for individuals with Spinal Cord Injury (SCI). Given any desired reference input and any noise input, our controller attenuated and rejected the noise and it tracked and stabilized the system to any desired reference input, by treating the problem as a robust servomechanism control problem. The proposed control strategy holds a great potential for neurorehabilitative systems regulating quiet stance, as the controller is least affected by perturbations to plant and disturbances applied to the closed loop system, while asymptotically tracking the desired angular standing position.

9.1 Discussion

Given any desired angular standing position (reference input), our controller exactly tracks this reference input, i.e. it provides tracking. Our revised controller generates a torque that does not fluctuate to very high torque amplitudes (+/- 130 N.m as compared to +/- 600N.m of the initial controller). This was done as the maximum ankle torque that a healthy person can generate in isometric contractions (where the highest torques can be generated) is 250-260 Nm per each leg, and may even be less for SCI individuals. The controller also reaches this exact tracking within 1s, which is faster than existing systems, but also not too rapid. This tracking timeframe is achieved to be compliant for future use with FES assisted systems, and the output behavior is favorable in neurorehabilitative systems for quiet stance as rapid torque generation is not feasible with FES assisted systems.

In presence of constant disturbance (noise) to the system, our controller completely rejects the constant noise and exactly tracks the desired angular standing position, i.e. it provides regulation. For example, if a person experiences a constant disturbance such as an added weight, the controller is able to maintain the desired standing position of the individual while completely countering the disturbances. Given a Gaussian disturbance, our controller highly rejects the noise by two orders of magnitude and exactly tracks the reference input. For example, if there is a dynamic disturbance such as push of wind being applied on the
individual, the controller is able to attenuate such noise fluctuation to within 0.1° in the output, which is almost zero in the control of quiet stance. The amplitude of the applied Gaussian noise was within +/-14° which is the maximum angular position a healthy individual can deviate with respect to the vertical as seen in experiments.

Given perturbations of up to +/- 10% to the plant model, our controller is still able to exactly track the reference input, i.e. it is robust. In our results we have shown exact tracking is achieved with a change in mass of the subject by +/-10% (which also affected the moment of inertia) with respect to the nominal value of m=76kg. This means the controller is robust for a wide range of patients with different mass and moments of inertia. In addition, change in the subject mass over periods of time, such as weight gain or weight loss, does not affect the performance of the controller, and it does not require that the neuroprosthesis of the SCI individual is readjusted.

The properties that the robust servomechanism controller provides (i.e. tracking, regulation, and robustness) makes it superior to control strategies such as PD and PID mainly due the comprehensive use of the closed loop signals such as the measurable plant outputs and interconnected signals as well as utilizing controller gains. PD and PID controllers however mainly make use of the error signal input.

It is important to also note that the current study is based on simulations, and does not use an actual human in the closed loop system. Our controller has been developed for a plant model of the human dynamics based on previous published results in the literature. Real-world scenarios of controlling quiet stance for SCI individuals may therefore differ from the models that are used in the present and past studies, as such models may not fully capture all kinematics and dynamics of the complex nature of human quite stance. For instance, the current plant is modelling a single joint (ankle joint), and does not consider the joints of the hip, trunk and the knee, or consider non-linearities due to fatigue and spasticity. However, the models employed are a good approximation of the real-world, and are typically used in the design and study of controlling quiet stance. Additionally, since the ankle is farthest from Central Nervous System (experiencing the longest time delay) and controls the highest mass,
one may argue that as this controller is able to control the ankle joint, it is likely that it will control other joints as well.

But more importantly, the general robust servomechanism approach proposed in this thesis to control quiet stance, enables us to achieve quiet standing (if at all possible) with little information about the plant itself, i.e. even if all the plant dynamics cannot be exactly modelled via expressions. In a clinical setting, using such robust servomechanism controller over existing controllers, such as the PD and PID controllers, could be very advantageous. PD and PID controller require continuous tuning of gain parameters by the experimenter and this could be tedious and costly for both for the clinical facility and SCI individual. With the robust servomechanism controller, the controller can self-tune its gains through an iterative process until it reaches the solution (if it exists). This can be used to safely “train” a controller for an SCI subject. In such a training, the person is placed in a safe harness and is allowed to lose balance safely several times, such that the servomechanism controller attains several measurable outputs about the behavior of the overall plant (i.e. the human subject with all its dynamics present). The controller would then self-tune and finally allow the person to stand upright. If after a number of iterations, the controller was not able to make the individual stand, this means that there are no robust servomechanism controller solutions for that individual and quiet stance cannot be achieved.

9.2 Summary and Conclusions

The overall closed loop system comprises a plant (modeling body dynamics and associated time delays) as well as the controller (mimicking the CNS). The inputs to the overall closed loop system were the reference angular position of the subject’s center of mass, known as yref (rad) as well as noise source ω(k) (rad). In the literature it has been suggested that yref is about 0.1 rad (~5°) during quiet standing while the noise depends on the environmental and the individual’s physiological constraints.

The plant model was derived from a previous study which utilized PD control strategy to analyze spontaneous anteroposterior body sway [4]. The system used subject parameters of a typical male adult with time delays commonly used in literature. Also, the effect of
disturbance was incorporated in this model. The sampling time of our system was 0.01s (100Hz). The frequency of 100Hz used by the plant model has shown to produce synaptic recruitment of spinal motor neurons and recruit a greater number of fatigue resistant motor units through reflex activation of smaller motor neurons [60].

The methods used to develop our controller were as follows: First the plant components were identified and converted from transfer function to a continuous time state space representation form. Then the plant blocks continuous state representation form was converted into a discrete time state-space representation form and an overall discrete time plant model was derived. We were then able to create a discrete time controller for the derived plant. Finally, the closed system discrete time state space representation was found.

The plant model represents the body dynamics and kinematics during quiet stance including neuro-mechanical and feedback time delays. The body dynamics during quiet stance can be described as using a single joint single segment inverted pendulum model pivoted at the ankle joints, with the center of mass of the body (COM) on average a few centimeters in front of the ankle joints. The time delays represented the time loss incurred as a result of the distance the fired neuron needs to travel from the brain to the ankle joint and vice versa.

The two inputs to the plant were the torque generated by the controller and a noise input. Physiologically the noise input represents any environmental disturbances (external to the body) and internal disturbances caused by the body function. Examples of environmental noise sources are uneven standing surface [73], added weight to the subject or sudden drop of weight from the subject [5]. Internal body noise sources can be result of physiological constraints of the human body. Examples of internal noise sources are but not limited to the subject closing his/her eyes while standing [4], and the sway disturbance introduced due to breathing or stretching of arm [60] of the person standing.

As the noises could be constant or rapidly dynamic, in our simulations we considered constant and rapidly varying forms of noise sources. For example, a constant disturbance resembled a weight added to the person while standing or a weight removed from the person.
A rapidly varying noise source resembled a situation where the person is trying to achieve balance in a much more dynamic situation. Therefore while a constant disturbance could provide the system's step response characteristics, dynamic noise sources could better resemble real life situations such as a person trying to stand while the wind is pushing him/her or standing on a tilting surface.

For the design of the controller, regulating active ankle torque during quite standing in SCI patients was treated here as a servomechanism problem. This enabled us to achieve tracking of a reference input (the angle at which the leg must be) at the output (COM position). This was done reliably such that in the presence of disturbances and noise, e.g. a push to the subject, the system was still able to track the input. Moreover the system maintained stability throughout its various operation conditions.

Additionally, we achieve robust control of this servomechanism problem, for perturbations in the plant model. This means that for example, if the mass of the SCI is not exactly equal to the nominally assumed 76kg in the design of the controller, the system still does not fail and can still reliably track the input (and hence enable standing for the SCI), as well as reject noise and disturbances. This is quite a beneficial feature of the proposed design as it provides greater flexibility in handling SCI patients.

In order to examine the performance of the developed controller and determine how well it worked, we compared the results of our controller with a previous study employing PD control strategy. The PD system was selected because both our system and the PD system shared the same plant model. This way, improvements seen in any results could be entirely attributed to the controller since all the other parameters were similar in the two systems.

In the future, our developed system could be implemented in an in-lab setting to restore quiet standing for SCI individuals with the help of FES, and this controller could then be used as a framework for nero-rehabilitative systems. The proposed control strategy in this work can be considered as a step forward in neuro-rehabilitative systems for quiet stance.
References


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Appendix A

The code below presents the method used to convert the continuous time model of plant from study [4] into discrete time. The continuous time state space representation of plant blocks are derived from their transfer functions and converted into discrete time. The overall plant state space representation is then derived.

```matlab
clc;
close all;
clear all;

%time delays
t1=0.05;
t2=0.05;

%noise transfer function parameters
noise_num=10;
noise_denom=5;

%HDG
Kp=20*180/pi;
Kd=10*180/pi;

%Body parameters
m=76*0.88;
g=9.81;
h=0.87;
l=m*h;
%body transfer function parameters
body_num=1/l;
body_denom=m*g*h/l;

h=0.01;
```

%Body dynamics
```matlab
%body = 1/(l*s^2-m*g*h);
```
x1 = 0;
x2 = 0;
u1 = 0;
u2 = 0;

x = [x1 x2];
u = [u1 u2];

a_body = [0 1; body_denom 0];
b_body = [0; body_num];
c_body = [1 0];

[body_num_check body_denom_check] = ss2tf(a_body, b_body, c_body, 0);

T = 0;
[A_body B_body C_body] = c_d_ejd(a_body, b_body, c_body, 0, h);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%
% tou_M
a_tou_M = -100;
b_tou_M = 100;
c_tou_M = 1;
T = t1;
[A_tou_M, B_tou_M, C_tou_M] = c_d_ejd(a_tou_M, b_tou_M, c_tou_M, T, h);

[tou_M_num_check tou_M_denom_check] = ss2tf(a_tou_M, b_tou_M, c_tou_M, 0);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%
% tou_F
a_tou_F = -100;
b_tou_F = 100;
c_tou_F = 1;
T = t2;
[A_tou_F, B_tou_F, C_tou_F] = c_d_ejd(a_tou_F, b_tou_F, c_tou_F, T, h);

[tou_F_num_check tou_F_denom_check] = ss2tf(a_tou_F, b_tou_F, c_tou_F, 0);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%
% Td
tou_D_rows = 1
tou_D_cols = 1
RN = random('unif', -1, 1, tou_D_rows, tou_D_cols);
a_Td = -5;
b_Td = 100;
c_Td = 1;

[num_denum] = ss2tf(a_Td, b_Td, c_Td, 0)
a_Td = -1;
b_Td = 100;
c_Td = 1;
[num denum]=ss2tf(a_Td,b_Td,c_Td,0)

T= 0;
[A_Td, B_Td, C_Td] = c_d_ejd(a_Td,b_Td,c_Td,T,h);

[Td_num_check Td_denom_check] = ss2tf(a_Td,b_Td,c_Td,0);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %
%tou_M
A1 = A_tou_M;
B1 = B_tou_M;
C1 = C_tou_M;
%tou_F
A2 = A_tou_F;
B2 = B_tou_F;
C2 = C_tou_F;
%tou_d
A3 = A_Td;
B3 = B_Td;
C3 = C_Td;
%body
A4 = A_body;
B4 = B_body;
C4 = C_body;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %

A_p=[A1 zeros(6,6) zeros(6,2) zeros(6,1);
     zeros(6,6) A2 B2*C4 zeros(6,1);
     B4*C1 zeros(2,6) A4 B4*C3;
     zeros(1,6) zeros(1,6) zeros(1,2) A3];

B_p=[B1;
     zeros(6,1);
     zeros(1,1);
     zeros(2,1)];

C_p=[zeros(1,6) C2 zeros(1,1) zeros(1,2)];

C_p_theta=[zeros(1,6) zeros(1,6) C4 0];

C_p=C_p_theta;

E_p=[zeros(6,1);
     zeros(6,1);]
zeros(2,1);
B3;

eig(A_p)

save 'ABCD' A_p B_p C_p E_p

function [A,B,C]=c_d_ejd(a,b,c,T,h)

% Given a LTI continuous system:
% dx/dt = a*x + b*u(t-T),
% y = c*x , T > 0
%
% It is desired to approximate this system to a discrete time system which has no no time delay terms:
% z(t+h) = A*z(t) + B*v(t),
% y(t) = C*z(t)
%
% where h > 0 is the sampling interval, T=h*k is the total time delay,
% (where T is given and h is given) and k = T/h = the number of sampling intervals,
% and where k is a positive integer.
%

close

k = (T/h);
k = round(k);

if k <=1,disp(' Problem exists since k = T/h < 1'),pause, end
if k <=1,disp(' Problem exists since k = T/h < 1'), end

[n,m]=size(b); [r,n]=size(c); d = zeros(r,m);

disp('Discrete open loop system with sampling interval h and no time delay')
[ad,bd]=c2d(a,b,h); cd=c;
A=ad
B=bd
C=cd

%pause

%disp('Adding discrete servo compensator to discrete system')
%if T==0
 % A=[ad bd;zeros(m,n) zeros(m,m)];
 % B=[zeros(n,m);eye(m)];
 % C=[cd zeros(r,m)];
if k >= 1
    p=(k-1)*m;
    tem1=zeros(p,n+m);
    tem2=eye(p);
    tem3=zeros(n,p);
    tem4=zeros(m,n+m+p);

    A=[ad bd tem3];
    A=[A;tem1 tem2];
    A=[A;tem4];

    B=[zeros(n,m);zeros(p,m); eye(m)];

    C=[cd zeros(r,m) zeros(r,p)];
end
%end

[N,M]=size(B); [R,N]=size(C);
D=zeros(R,M);
[N,N]=size(A);

disp(' Delay time T=')
T
disp('Corresponding sampling time interval h=')
h
disp('and order of resultant discrete plant N=')
N
%pause

disp('Comparison of continuous system with no time delay')
disp(' to sampled system with time delay')

sys_c = ss(a,b,c,d)
step(sys_c)
title('Unit step response to continuous system with no time delay')
%pause

sys_d = ss(A,B,C,D)
dstep(A,B,C,D,1)
title('Unit step response to sampled system with time delay')
xlabel('Number of samples of size h --- ignore time')
%pause
Appendix B

The section provides Matlab codes for the system parameters, controller and plant state space matrices values, closed loop system state space matrices, as well as output plots for the following trials:

- **Constant Reference input**, $y_{ref} = 0.1 \text{ rad } (~5\degree)$, $\omega = 0 \text{ rad}$
  [Figure 10, Figure 11, Figure 12, Figure 13, Figure 14, Figure 15, Figure 16]

- **Constant Noise input**, $y_{ref} = 0 \text{ rad}$, $\omega = 0.244 \text{ rad } (~14\degree) \text{ rad}$
  [Figure 17, Figure 18, Figure 19, Figure 20, Figure 21]

- **Gaussian Noise input**, $y_{ref} =$Gaussian Noise, $\omega = 0 \text{ rad}$
  [Figure 22, Figure 23, Figure 24, Figure 25, Figure 26]

- **Varying Reference Input**, $y_{ref} =$ Step change, $\omega = 0 \text{ rad}$
  [Figure 27, Figure 28]
Constant Reference input

Our System Matlab Code for Constant Reference Input

yref=1 rad, ω = 0 rad

clc;
clear all;
close all;
%
%Controller
A_c=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; -3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-12 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832];
B_c_y=[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
B_c_yref=[-1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
C_c=[-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-12 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832];
D_c=[0];
%
%Plant
A_p=[0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];

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B_p = [0; 0; 0; 0; 1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
C_p = [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0];
E_p = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
D_p = [0 0];

A = [A_p B_p*C_c; B_c_y*C_p A_c];
B = [0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;1*B_c_yref];
C = [C_p 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
D = [0 0];

E = [E_p 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
B_sys = [B;E];

sys = ss(A,B_sys,C,D,0.01);

C2 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C_c];
Controller_y = ss(A,B_sys,C2,D,0.01);

load ('ConstantReference.mat');
U(1:usize,1) = ConstantReference(2,1:usize);
t_Constantyref = ConstantReference(1,1:usize);
[y_Constantyref,t,x] = lsim(sys,U,t_Constantyref);
[U_Constantyref,t,x] = lsim(Controller_y,U,t_Constantyref);
fignum=1;
figure(fignum);
plot(t_Constantyref, U(:,1));

grid on;
set(gca,'gridlinestyle','-.');

title(' Constantyref')
xlabel('time (s)') % x-axis labels
ylabel('Constantyref (rad)') % y-axis label

saveas(figure(fignum),[pwd 'Results\Constantyrefinput.jpg']);

fignum=fignum+1;

figure(fignum);

plot(t,y_Constantyref);

grid on;
set(gca,'gridlinestyle','-.');

title(' COM Position during Constant yref')
xlabel('time (s)') % x-axis labels
ylabel('	heta_{output}(rad)') % y-axis label

saveas(figure(fignum),[pwd 'Results\Constantyref.jpg']);

fignum=fignum+1;
figure(fignum);

plot(t,U_Constantyref);

grid on;
set(gca,'gridlinestyle','-.');

title('Controller Torque during Constant yref')
xlabel('time (s)') % x-axis label
ylabel('Output U(N.m)') % y-axis label

saveas(figure(fignum),[pwd 'Results\Constantyref_u.jpg']);
% PD System parameters
clc;
clear all;
close all;

% time delays
tou1=0.05;
tou2=0.05;

% noise transfer function S coefficient
touf=1;

% HDG
Kp=20*180/pi;
Kd=10*180/pi;

% Body parameters
I=66;
m=76;
g=9.8;
h=0.87;
**Constant Noise input**

Our System Matlab Code for Constant Noise Input

\[ \text{yref} = 0 \text{ rad}, \omega = 1 \text{ rad} \]

```matlab
clc;
clear all;
close all;

%%% Controller
A_c = [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 -697.6004 0; 0 0 1 0 0 0 0 0 0 0 0 0 0 0 -9.88639e-12 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 6.21881e-14 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0 1.23759e-12 0 0; -3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-13 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832; 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.01288 0 0; 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 -0.07093 0 0; 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 -0.13297 0 0; 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 -0.21289 0 0; 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 -0.30867 0 0; 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0.58444 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0.47234 0 0.01000 7.50061e-7; 0 0.00015 0 0 0 0 0 0 0 0 0 0 0 0 -11.32694 1.00049 0.00015; 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0.99005];

B_c_y = [1 ; 697.6004 ; 9.88639e-12 ; -3.10940e-14 ; 6.21881e-14 ; 1.23759e-12 ; 0 ; 0.01288 ; 0.07093; 0.13297 ; 0.21289 ; 0.30867 ; 0.41556 ; 0.52815 ; 11.425060 ; 0];

B_c_yref = [-1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];

C_c = [1 -3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-13 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832];

D_c = [0];

%%% Plant
A_p = [0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
$B_p = [0; 0; 0; 0; 1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0.99005]$;

$C_p = [0; 0; 0; 0; 0; 0; 0; 0; 0; 1; 0.01000; 7.50061e-7; 0.09812; 1.00049; 0.00015]$;

$E_p = [7.50061e-7; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 1.00049; 0.01000; 7.50061e-7; 0.09812; 1.00049; 0.00015; 0.99005]$;

D_p = [0 0]

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
% Closed loop system
A = [A_p B_p*C_c; B_c_y*C_p A_c]
B = [0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;1*B_c_yref]
C = [C_p 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
D = [0 0]

E = [E_p ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0.99502]
B_sys = [B, E]

sys = ss(A, B_sys, C, D, 0.01);

Controller_y = ss(A_c, B_c_y, C_c, D_c, 0.01);
Controller_yref = ss(A_c, B_c_yref, C_c, D_c, 0.01);

t = 0:0.01:(2-0.01);

fignum = 1;
% Constant Noise

U = zeros(600,2);
U(:,2)=1;
[y_con_time, t] = lsim(sys, U, t);

fignum = fignum + 1;
figure(fignum);

plot(t, y_const_noise);
grid on;
set(gca,'gridlinestyle','-.');

title('COM Position during Constant Disturbance')
xlabel('time (s)')

end
ylabel('\theta_{output}(rad)') % y-axis label

saveas(figure(fignum),[pwd '\Results\const_noise_y.jpg']);

[U_const_noise_y,t,x] = lsim(Controller_y,y_const_noise,t);
[U_const_noise_yref,t,x] = lsim(Controller_yref,U(:,1),t);

fignum=fignum+1;
figure(fignum);
plot(t, U_const_noise_y + U_const_noise_yref);
grid on;
set(gca,'gridlinestyle','-.');
title('Controller Torque during Constant Disturbance')
xlabel('time (s)') % x-axis label
ylabel('U(N.m)') % y-axis label

saveas(figure(fignum),[pwd '\Results\const_noise_u.jpg']);

PD System Simulink Model for Constant Disturbance

![Simulink Model](image)

%PD System parameters
clc;

Figure 67 PD System Recreated in Simulink
clear all;
close all;

%time delays
tou1=0.05;
tou2=0.05;

%noise transfer function S coefficient
touf=1;

%HDG
Kp=20*180/pi;
Kd=10*180/pi;

%Body parameters
I=66;
m=76;
g=9.8;
h=0.87;
Gaussian Noise input

Our System Matlab Code for Gaussian Noise Input

yref=Gaussian Noise, ω = 0 rad

clc;
clear all;
close all;

%Controller
A_c= [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
0 0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 -697.6004 0 0;
0 0 1 0 0 0 0 0 0 0 -9.88639e-12 0 0;
0 0 0 0 1 0 0 0 0 0 3.10940e-14 0 0;
0 0 0 0 0 1 0 0 0 0 -6.21881e-14 0 0;
0 0 0 0 0 0 1 0 0 0 0 -1.23759e-12 0 0;
-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-13 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832;
0 0 0 0 0 0 0.36788 0.63212 0 0 0 -0.01288 0 0;
0 0 0 0 0 0 0 0 1 0 0 0 -0.07093 0 0;
0 0 0 0 0 0 0 0 0 0 1 0 0 -0.13297 0 0;
0 0 0 0 0 0 0 0 0 0 0 1 0 -0.21289 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 1 -0.30867 0 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0.58444 0;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.47234 0.01000 7.50061e-7;
0 0.00015 0 0 0 0 0 0 0 0 0 -11.32694 1.00049 0.00015;
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.99005];

B_c_y= [1 ; 697.6004 ; 9.88639e-12 ; -3.10940e-14 ; 6.21881e-14 ; 1.23759e-12 ; 0 ; 0.01288 ; 0.07093; 0.13297 ; 0.21289 ; 0.30867 ; 0.41556 ; 0.52815 ; 11.425060 ; 0 ];

B_c_yref= [-1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];

C_c=[-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-13 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832];

D_c=[0];

%Plant
A_p= [0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ];

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B_p = [0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ];
C_p = [0 0 0 0 0 0 0 0 0 0 0 0 1 0 0];
E_p = [0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0.99502 ];
D_p = [0 0 ];

-----------------------------------------------

% Closed loop system
A = [A_p B_p*C_c; B_c_y*C_p A_c];
B = [0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;1*B_c_yref ];
C = [C_p 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ];
D = [0 0 ];

E = [E_p ; 0 ; 0 ; 0 ; 0 ; 0; 0; 0 ; 0 ; 0 ; 0; 0 ; 0 ; 0 ; 0 ];
B_sys = [B,E];

sys = ss(A,B_sys,C,D,0.01);

C2 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ];
Controller_y = ss(A,B_sys,C2,D,0.01);

-----------------------------------------------

% Gaussian Noise

load ('GaussianNoise.mat');
U(1:usize,2) = GaussianNoise(2,1:usize);
t_gaussian = GaussianNoise(1,1:usize);
[y_Gaussian_noise,t,x] = lsim(sys,U,t_gaussian);
[U_Gaussian_noise_y,t,x] = lsim(Controller_y,U,t_gaussian);
fignum=1;
figure(fignum);
plot(t_gaussian, U(:,2));

grid on;
set(gca,'gridlinestyle','-.');

title('Gaussian Noise')
xlabel('time (s)')
ylabel('Gaussian Noise (rad)')

saveas(figure(fignum), [pwd '\Results\gaussian_noise.jpg']);

fignum=fignum+1;

figure(fignum);
plot(t,y_Gaussian_noise);

grid on;
set(gca,'gridlinestyle','-.');

xtitle('COM Position during Gaussian Disturbance')
xlabel('time (s)')
ylabel('\theta_{output} (rad)')

saveas(figure(fignum), [pwd '\Results\gaussian_noise_y.jpg']);

ym=sin(y_Gaussian_noise)*-9.8*0.87;

fignum=fignum+1;
figure(fignum);

plot( t,ym );
grid on;
set(gca,'gridlinestyle','-.');

title('COM Position during Gaussian Disturbance')
xlabel('time (s)')
ylabel('COMdisp (m)')

saveas(figure(fignum), [pwd '\Results\gaussian_noise_y_m.jpg']);

fignum=fignum+1;

figure(fignum);
plot( t(1:1000), ym(1:1000,1));
grid on;
set(gca,'gridlinestyle','-.');

title('COM Position during Gaussian for first 10 Seconds')
xlabel('time (s)') % x-axis labels
ylabel('COMdisp (m)') % y-axis label

saveas(figure(fignum),[pwd "\Results\gaussian_noise_y_mZoom.jpg"]);

fignum=fignum+1;
figure(fignum);

plot(t,U_Gaussian_noise_y);
grid on;
set(gca,'gridlinestyle','-.');
title('Controller Torque during Gaussian Disturbance')
xlabel('time (s)') % x-axis label
ylabel('Output U(N.m)') % y-axis label

saveas(figure(fignum),[pwd "\Results\gaussian_noise_u.jpg"]);
**Varying Reference Input**

Our System Matlab Code for Varying Reference Input

\[ y_{\text{ref}} = \text{Step change [Figure]}, \omega = 0 \text{ rad} \]

```matlab
clc;
clear all;
close all;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%Controller
A_c= [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 -9.88639e-12 0 0; 0 0 0 1 0 0 0 0 0 3.10940e-14 0 0; 0 0 0 0 1 0 0 0 0 -6.21881e-14 0 0; 0 0 0 0 0 1 0 0 0 -1.23759e-12 0 0; -3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-12 0 0; 0 0 0 0 0 0 0.36788 0.63212 0 0 0 -0.01288 0 0; 0 0 0 0 0 0 0 0 1 0 0 0 -0.07093 0 0; 0 0 0 0 0 0 0 0 0 1 0 0 -0.13297 0 0; 0 0 0 0 0 0 0 0 0 0 1 0 -0.21289 0 0; 0 0 0 0 0 0 0 0 0 0 0 1 -0.30867 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0.58444 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.47234 0.01000 7.50061e-7; 0 0.00015 0 0 0 0 0 0 0 0 0 0 -11.32694 1.00049 0.00015; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.99005];
B_c_y= [1 ; 697.60004 ; 9.88639e-12 ; 6.21881e-14 ; 1.23759e-12 ; 0 ; 0.01288 ; 0.07093; 0.13297; 0.21289; 0.30867; 0.41556; 0.52815; 0.47234; 0.01000; 7.50061e-7];
B_c_yref= [-1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
C_c= [-3287.75100 -1.17749 -0.65864 -0.57884 -0.50533 -0.43942 -0.38476 -2.51351e-13 -4.31893e-12 -1.34393e-12 -4.12137e-12 -1.19640e-11 -5.60364e-11 -64842.07951 -5601.41621 -4.59832];
D_c=[0];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% 
%Plant
A_p= [0.36788 0.63212 0 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0; 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 ];
```
\[ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0; \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0; \\
7.50061 \times 10^{-7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.00049 & 0.10000 & 7.50061 \times 10^{-7}; \\
0.00015 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.09812 & 1.00049 & 0.00015; \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99005 \\
\end{bmatrix}; \]

\[ \begin{bmatrix}
B_p = [0; 0; 0; 0; 0; 1; 0; 0; 0; 0; 0; 0; 0; 0]; \\
C_p = [0 0 0 0 0 0 0 0 1 0 0]; \\
E_p = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0.99502]; \\
\end{bmatrix}; \]

\[ \begin{bmatrix}
B = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 1*B_c_yref]; \\
C = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]; \\
D = [0 0]; \\
\end{bmatrix}; \]

% Closed loop system
\[ \begin{bmatrix}
A = [A_p B_p*C_c; B_c_y*C_p A_c]; \\
B = [0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 1*B_c_yref]; \\
C = [C_p 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]; \\
D = [0 0]; \\
\end{bmatrix}; \]

\[ \begin{bmatrix}
E = [E_p :0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0]; \\
B_sys = [B, E]; \\
\end{bmatrix}; \]

% Simulation Trials
\[ \begin{bmatrix}
\ldots \\
\end{bmatrix}; \]

\[ \begin{bmatrix}
sys = ss(A, B_sys, C, D, 0.01); \\
C2 = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 C_c]; \\
Controller_y = ss(A, B_sys, C2, D, 0.01); \\
\end{bmatrix}; \]

\[ \begin{bmatrix}
\ldots \\
\end{bmatrix}; \]

% Step Change in Noise
\[ \begin{bmatrix}
tsize = 40; \\
usize = tsize * 100; \\
U = zeros(usize, 2); \\
\end{bmatrix}; \]

% load ('GaussianNoise.mat');
\[ \begin{bmatrix}
U(1:usize, 1) = degtorad(5); \\
U(1000:1800, 1) = degtorad(9); \\
t_Stepchange = 0:0.01:tsize - 0.01; \\
\end{bmatrix}; \]
[y_Stepchange,t,x] = lsim(sys,U,t_Stepchange);

[U_Stepchange,t,x] = lsim(Controller_y,U,t_Stepchange);

fignum=1;

figure(fignum);

plot(t_Stepchange, radtodeg(U(:,1)));

grid on;
set(gca,'gridlinestyle','-.');

title(' Step change Reference Input')
xlabel('time (s)') % x-axis labels
ylabel('y_{ref} (deg)') % y-axis label

saveas(figure(fignum),[pwd '\'Results\Stepchange_yref.jpg']);

fignum=fignum+1;

figure(fignum);

plot(t,radtodeg(y_Stepchange));

grid on;
set(gca,'gridlinestyle','-.');

title(' COM Position during Stepchange Reference Input')
xlabel('time (s)') % x-axis labels
ylabel('\theta_{output}(deg)') % y-axis label

saveas(figure(fignum),[pwd '\'Results\Stepchange_y.jpg']);

fignum=fignum+1;
figure(fignum);

plot(t,radtodeg(U_Stepchange));

grid on;
set(gca,'gridlinestyle','-.');

title('Controller Torque Stepchange Reference Input')
xlabel('time (s)') % x-axis label
ylabel('Output U(N.m)') % y-axis label

saveas(figure(fignum),[pwd '\'Results\Stepchange_u.jpg']);
Appendix C

Revised Controller Matrices

\[ A_c = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.367879 & 0.632121 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.01452 & 0 & 0 \\
0 & 0.367879 & 0.632121 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.05206 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05477 & 0 & 0 \\
0 & 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.05843 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.06035 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.937675 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.93613 & 0.010002 & 7.50E-07 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.93613 & 0.010002 & 7.50E-07 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.99005 \\
\end{bmatrix} \]
\[ B_{c_y} = \begin{bmatrix} 1 \\ 0.014517 \\ -5.42E-13 \\ 1.98E-13 \\ -4.52E-14 \\ -2.37E-29 \\ 0.052058 \\ 0.054775 \\ 0.056576 \\ 0.058434 \\ 0.06035 \\ 0.062325 \\ 0.064361 \\ 0.206752 \\ 0 \end{bmatrix} \]

\[ B_{c_yref} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]
\[
\begin{bmatrix}
C^{-1} &=&
\begin{bmatrix}
-206.559 & -0.3782 & -0.2211 & -0.20592 & -0.19138 & -0.17749 & -0.16426 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -11406.5 & -1696.28 & -2.25121
\end{bmatrix}
\end{bmatrix}
\]