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Extraction of Nonstationary Sinusoids

by

Alireza Karimi Ziarani

A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
University of Toronto

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Extraction of Nonstationary Sinusoids

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Abstract

A novel method of extraction of nonstationary sinusoids and estimation of their parameters is presented. The proposed algorithm has a nonlinear structure which renders it fully adaptive in tracking time-variations in the parameters of the targeted sinusoid including its phase and frequency. Mathematical properties of the differential equations governing the proposed algorithm are presented and the performance of the proposed algorithm is demonstrated with the aid of computer simulations. The proposed algorithm has been digitally implemented on a digital signal processor (DSP) platform and the laboratory verification of its performance is presented. Superiority of the proposed algorithm over conventional Fourier analysis and linear adaptive methods is demonstrated by its successful application to a number of real problems. Elimination of sinusoidal disturbances under time-varying conditions is a challenging problem of current research. The proposed algorithm is employed to construct an adaptive notch filter for the rejection of quasi-periodic interferences. Its performance is exemplified by its application to the problem of elimination of power line noise potentially present on electrocardiogram and telephone cables. Application of the proposed algorithm in the estimation of low-level biomedical signals polluted by noise constitutes another example of its superior performance over conventional methods. Refinement and analysis of ultrasonic waves used in non-destructive testing (NDT) of materials is presented as another useful application of the proposed algorithm. Finally, estimation of instantaneous frequency in noise, exemplified by Doppler frequency shift estimation, is demonstrated. The proposed algorithm exhibits a high degree of immunity with regard to both external noise and internal parameter settings while offering structural simplicity crucial for real-time applications.
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Chapter 1

Introduction

Nonstationary sinusoids are sinusoidal signals having time-varying parameters. An algorithm capable of estimating the parameters of a nonstationary (i.e. time-varying) sinusoid in real-time finds applications in various branches of engineering. Parameters of interest are usually the amplitude, total and constant phase, and the frequency of a sinusoidal signal. Examples of its applications are estimation of time-varying biomedical signals [1], active noise and vibration control [2], sinusoidal disturbance rejection [3], phasor measurement [4] and frequency estimation [5, 6] in power systems.

There have been numerous attempts to design an algorithm to extract a single sinusoidal signal out of a given multi-component input signal. An ideal frequency domain analysis tool such as discrete Fourier transform (DFT) loses its effectiveness when the frequency of the input signal happens to vary with time [7]. Conventional methods of signal analysis, therefore, have long been left aside when dealing with nonstationary signals. Time-frequency domain signal processing tools are being developed to tackle such problems, an example of which is wavelet transform [1]. Adaptive notch filtering is another example of recently developed techniques. An adaptive notch filter with a very sharp notch whose center frequency adaptively tracks that of the desired component of the input signal has been the subject of an active field of research [8]. Alternative methods of sinusoidal signal extraction and analysis have been proposed. One is referred to [9] and [10] for a class of signal processing algorithms, the development of which does not follow conventional methods.
This thesis presents the development and applications of a signal processing algorithm capable of extracting and estimating the individual sinusoidal components of its input signal and tracking variations of the amplitude, phase and frequency of such sinusoids over time [11]. The proposed algorithm is a generalization of two previously developed nonlinear adaptive algorithms [12]. The unique feature of the proposed algorithm is its capability in accommodating large frequency variations. The proposed algorithm is found to have numerous applications in diverse areas of engineering ranging from biomedical engineering [13, 14] to the non-destructive testing of materials [15, 16].

1.1 Review of the Existing Methods

A wide variety of signal processing methods have been employed for the extraction of sinusoidal signals of time-varying nature, or the estimation of their parameters. Each of the existing methods presents strengths and weaknesses when applied to the nonstationary problem of sinusoidal signal detection. The range of available methods of analysis and synthesis of quasi-periodic signals encompasses conventional techniques such as Fourier-based algorithms and phase-locked loops as well as recent, not so well-established signal processing methods. A brief review of some of the research activities in this area is presented in this section and the advantages and shortcomings of each family of methods are highlighted.

1.1.1 Conventional Methods

Fourier-based and adaptive filtering techniques present the most common families of signal processing algorithms used for the analysis of nonstationary signals. Alternatively, phase-lock techniques have long been employed for the extraction, or equivalently synthesis, of sinusoidal signals having time-varying phase. The majority of the conventional methods can be classified as falling within the following families of signal processing methods:
Fourier-based Methods

Fourier analysis provides the means of finding the best projection of a signal onto a space spanned by a number of fixed frequency sinusoids. In other words, given a number of distinct frequencies of fixed difference (i.e. equally spaced along the horizontal axis in the frequency domain representation), Fourier analysis provides a measure of the share of the input signal at each such frequency and thereby presents a frequency domain representation of the input signal. It is known that Fourier analysis exhibits serious shortcomings when applied to nonstationary signals because of its inherent fixed frequency assumption [7]. A description of nonstationary signals in the frequency domain using a simple Fourier transform would provide misleading results [17]. The adaptation of Fourier analysis to nonstationary scenarios presents a subclass of conventional methods of analysis of nonstationary signals. Short-time Fourier transform (STFT) is an example of this kind [1]. The adaptation in this case consists of partitioning the time-varying signal into a multiplicity of segments within each of which the signal can be assumed almost stationary. Therefore, STFT presents an approximate time-frequency domain representation of slowly time-varying signals.

There are other signal processing tools developed based on Fourier analysis for various specific applications. Some examples for specific applications in power engineering can be found in [4, 5]. It goes without saying that all such methods are of an approximating nature and presuppose only slow variations in the characteristics of the sinusoidal signal under study. Considering the inefficiency of the Fourier-based methods, alternative methods of signal analysis have been developed over the last few decades [17, 18].

Time-frequency Domain Methods

Time-frequency domain methods of signal processing are usually window-based. Given nonstationary signals as the input, the output of these methods are generally huge amounts of data yet to be analyzed. Their computational demand is very high and further post-processing and information extraction are usually required [17]. They provide solely the means of analysis of the signals. Where needed, any individual component of the input signal is yet to be synthesized, possibly by the use of a phase-locked loop (PLL).
As noted above, short-time Fourier transform is an example of time-frequency domain signal analysis methods. STFT is inherently window-based [17]. Within each window, STFT provides the best approximation of the limited-length windowed signal in a space spanned by a fixed set of sinusoids as is typical with Fourier-based methods. In other words, within each window slot, sinusoidal basis functions are used to span a space onto which the projection is made. Generalization of STFT to allow for basis functions other than sinusoidal signals amounts to wavelet transform which is another example of time-frequency methods of signal analysis [1].

Adaptive Filters

The defining characteristic of an adaptive filter is its ability to operate satisfactorily, according to a criterion of performance acceptable to the user, in an unknown and possibly time-varying environment [18]. Therefore, adaptive filtering presents a possible solution to the problem of estimation of nonstationary sinusoids [19]. Application of the adaptive filtering theory to this area has been extensively studied over the last several years [20]. Various adaptive schemes have been proposed each presenting a compromise between efficiency in terms of speed/accuracy and practicality [21].

Standard linear adaptive filters exhibit poor performance when applied to the intrinsically nonlinear problem of detection of sinusoids. Often, they require additional information about the statistical characteristics of the input signal. An example of the required additional information is an external reference signal [22]. Nonlinear algorithms such as Kalman filters are shown to have better performance when applied to nonlinear, nonstationary problems [18]. However, they suffer from complexity of the structure and sensitivity with regard to setting parameters and initial conditions.

Notch Filters

A sharp notch filter is capable of separating and extracting a desired sinusoidal component of a given signal if the frequency of the signal remains constant. However, when the problem
is nonstationary, the frequency may also vary with time. The sharper the notch filter is designed, the more inoperative it becomes if any change in the frequency of the input signal occurs. Obviously, turning the notch filter into a band stop filter by widening its rejection band, and thereby accommodating frequency variations, does not offer any better solution. The ideal solution is an adaptive notch filter which is capable of tracking the frequency of the input signal and accordingly adjusting its center notch frequency.

Various architectures have been proposed for the construction of adaptive notch filters (see [8, 23], for example). However, there is no unique approach to the design of adaptive notch filters. As a specific example, lattice-based structures have attracted attention during the last few years [9].

**Phase-locked Loops**

Unlike most signal processing algorithms which aim at the estimation of the parameters of signals, phase-locked loop (PLL) aims at the extraction of a specified component of its input signal and tracking its phase variations over time. PLL is a fundamental concept widely used in diverse areas of electrical engineering. The main idea of phase lock is the ability to generate a sinusoidal signal the phase of which coherently follows that of the main component of the input signal. PLLs have been the subject of research for several decades. An enormous amount of theoretical and practical information is now available on this rather mature subject. However, the design of efficient PLL structures is still considered an active subject of research [24]. The basic architecture of PLLs, mainly used in analog form, can be used in the design of software-based PLLs which essentially present signal processing algorithms capable of extraction of sinusoidal signals [25].

PLLs have limited frequency lock-in range within which the variations of the frequency of the desired signal are tolerated [24]. Sophisticated PLL architectures employing higher order loop filters can accommodate larger frequency variations at the expense of increasing the complexity and decreasing the robustness of the overall structure [25]. Even so, PLLs provide only phase/frequency-adjusted sinusoidal signals. Extraction of nonstationary sinusoids,
properly so called, implies the adjustment in the amplitude of the synthesized sinusoidal signal as well. This feature is not common to PLLs.

1.1.2 Recent Advances

As is clear from the discussion of conventional methods of signal analysis/synthesis applied to the problem of extraction of quasi-periodic signals, this subject is an active area of research. In recent years, the focus of the research has been on the development of frequency-tracking algorithms. Two important families of recent methods are presented in this section.

The Method of Regalia

The particular family of adaptive notch filters briefly introduced here has its roots in the time-domain method of Regalia [9]. Recently, Hsu et al. presented a modified version of the original algorithm and proved its mathematical properties [10]. The structure of the algorithm essentially consists of a second order notch filter equipped with an adaptation mechanism. The overall dynamics of the algorithm is highly nonlinear. It offers a method of extraction of time-varying sinusoids in noise [26] and the estimation of their frequency.

The structure of the adaptive nonlinear algorithm of Regalia does not fit within any of the well-established conventional methods of signal analysis and synthesis. The significance of this method is its nonlinear approach to the nonlinear problem of estimation of frequency. This method is further discussed in subsequent chapters and is used for comparison with the proposed method of this thesis.

A New Family of Nonlinear Adaptive Notch Filter Algorithms

The search for a new nonlinear adaptive notch filter algorithm was initiated a few years ago by Ghartemani. The nonlinear algorithm described by Ghartemani and Iravani [27] presents a new approach in the design of methods of extraction of sinusoidal signals in which no model for the input signal is assumed. The algorithm tries to find a pre-specified sinusoid within its input signal while tracking variations of the parameters of such a sinusoid over time. The algorithm directly estimates amplitude and phase of the input signal and instantaneously
generates the desired sinusoidal signal as its output. Simplicity of structure, high noise immunity and robustness have rendered this algorithm attractive for real-time applications [27]. This algorithm, however, does not allow for large frequency variations. Moreover, it does not directly provide an estimate of the instantaneous frequency.

A second member of this family of algorithms is presented by Ghatremani and Ziarani in [12] and provides an estimate of the instantaneous frequency of the desired component of its input signal. However, its satisfactory performance is observed only under small off-nominal frequency variations. To address this shortcoming, the proposed algorithm of this thesis presents a third member of this family of algorithms which is a generalization of the first two mentioned above.

1.2 Objective

The overwhelming desire to develop an adaptive notch filter algorithm with the aim of accommodating large variations in the frequency of the sinusoid of interest is the main driving force behind the work presented in this thesis. Thus the main objective of this thesis is the development of a method of extraction of potentially time-varying sinusoidal signals and estimation of their parameters, namely the values of their amplitude, constant and total phase, and frequency. Development of such an algorithm consists of the steps of formulation, mathematical proof of functionality and implementation.

A second objective of the thesis is the demonstration of the usefulness of the proposed algorithm through its successful application to various engineering problems in which the extraction of sinusoidal signals, or the estimation of their parameters, is desired.

1.3 Methodology

The majority of the available methods of analysis of quasi-periodic signals provide the means of estimation of parameters of sinusoids. Where needed, the sinusoidal signal itself is yet
to be synthesized by the use of phase-lock schemes, for instance. The proposed approach, however, aims directly at the extraction of the sinusoidal signal itself. A sinusoidal signal is a nonlinear function of its phase: therefore, its extraction is inherently a nonlinear problem. The methodology in the development of the proposed algorithm is the same as that employed in its precursors, namely the two algorithms suggested in [12] which are among the few methods of extraction of sinusoids treating the problem in a nonlinear way. In the proposed algorithm, the model for the targeted sinusoid is improved to include all three parameters of a sine wave distinctly. This allows for the accommodation of large variations in the parameters of the targeted sinusoid including its frequency.

1.4 Original Contributions

Original contributions of this thesis can be summarized as follows:

- Improvement/generalization of two previously developed methods of extraction of sinusoids to accommodate large frequency variations,

- Proof of mathematical properties of the proposed algorithm,

- Implementation of the proposed algorithm in software,

- Demonstration of the engineering usefulness and the advantages of the proposed algorithm by way of presenting its applications in various practical scenarios. In doing so, the following applications are developed:
  - A novel universal narrow-band interference eliminator,
  - A novel method of refinement of pulsed sine wave signals.

As well, applicability of the proposed algorithm to the general problem of frequency estimation is explored by the presentation of a preliminary study of Doppler frequency shift estimation. A thorough treatment of this problem is not attempted and is suggested as a future research direction.
1.5 Outline of the Thesis

Considering the scope of this thesis covering the development of a fundamental signal processing algorithm on the one hand and its application to various engineering problems on the other hand, the organization of the thesis is divided into two parts.

Part I of the thesis presents the development, mathematical properties and digital implementation of the core algorithm. Chapter 2 outlines the formulation of the proposed algorithm in a parallel analogy with the standard Fourier analysis. Mathematical properties of the proposed algorithm guaranteeing its desired performance are presented in chapter 3. Implementation of the algorithm in software, both within standard simulation environments and on a dedicated processor, is the subject of chapter 4.

Part II of the thesis presents examples of the application of the proposed algorithm to problems from diverse areas of engineering, ranging from biomedical to communications engineering. The main application of the proposed algorithm presented in this thesis is an adaptive notch filter useful for the elimination of power line interference. Discussions on the design of such a filter together with the results of its application to two important cases of refinement of biopotential and telephone line signals are presented in chapter 5. Chapter 6 presents a typical signal detection problem in which estimation of a faint signal polluted by various types of noise is desired. Another example of the application of the proposed algorithm in the extraction of sinusoids in noise is presented in chapter 7. Preliminary application of the proposed algorithm in the estimation of instantaneous frequency of sinusoids is presented in chapter 8.

Finally, concluding remarks including an outline of recommended future research directions and the main features of the proposed algorithm are presented in chapter 9.
Part I:

Algorithm

Part I presents the development, mathematical properties and digital implementation of the proposed core algorithm. Chapter 2 outlines the formulation of the proposed algorithm in a parallel analogy with the standard Fourier analysis. Mathematical properties of the proposed algorithm guaranteeing its desired performance are presented in chapter 3. Implementation of the algorithm in software, both within standard simulation environments and on a dedicated processor, is the subject of chapter 4.
Chapter 2

Formulation of the Algorithm

Formulation of the proposed algorithm for the extraction of nonstationary sinusoids is presented in this chapter. Development of the algorithm is presented after a brief review of one of the most widely used signal processing tools, namely Fourier analysis. The proposed algorithm is presented as a nonlinear adaptive generalization of the standard Fourier analysis which itself is linear and non-adaptive.

2.1 Fourier Analysis Revisited

One of the reasons for the tremendous growth in the use of digital signal processing methods over the last few decades is the development of discrete Fourier transform (DFT) or its computationally efficient alternative, fast Fourier transform (FFT). The idea of DFT is closely related to Fourier series representation of periodic signals. In essence, DFT of a set of sampled data provides Fourier series coefficients of a periodic signal, a single period of which is given by the given set of sampled data. In this section, the inherent assumptions behind the Fourier series analysis and its limitations are reviewed. The proposed algorithm emerges as a generalization of Fourier analysis and in an attempt to eliminate assumptions intrinsic to Fourier analysis which limit its applicability to a wide range of time-varying applications.

Given a periodic signal \( u(t) \) with period \( T_0 \), the best approximation of the signal, \( \hat{u}(t) \),
in a space spanned by a number of sinusoids \( \sin(i\omega_0 t + \delta_i), \ i = 0 \cdots N - 1 \), each having a frequency of a multiple integer of the fundamental frequency \( \omega_0 = \frac{2\pi}{T_0} \) (in rad/s) and a constant phase \( \delta_i \) (in rad), written as [28]

\[
\hat{u}(t) = \sum_{i=0}^{N-1} A_i \sin(i\omega_0 t + \delta_i) = a_0 + \sum_{i=1}^{N-1} a_i \cos(i\omega_0 t) + b_i \sin(i\omega_0 t)
\]

is achieved by the minimization of the least squares error \( ||e||^2 \)

\[
||e||^2 = \frac{1}{T_0} \int_{T_0}^{T_0} [u(t) - \hat{u}(t)]^2 dt. \tag{2.1}
\]

In general, when the approximation of \( u(t) \) is in the form of linear combination of the basis vectors, the approximating coefficients are given by the inner products of \( u(t) \) and the basis vectors [28]. Therefore,

\[
\begin{align*}
a_0 &= \frac{1}{T_0} \int_{T_0} u(t) dt, \\
a_i &= \frac{1}{T_0} \int_{T_0} u(t) \cos(i\omega_0 t) dt, \quad i = 1 \cdots N - 1, \\
b_i &= \frac{1}{T_0} \int_{T_0} u(t) \sin(i\omega_0 t) dt, \quad i = 1 \cdots N - 1.
\end{align*}
\]

One can easily verify that the least squares error to be minimized can be redefined as \( e(t)^2 = [u(t) - \hat{u}(t)]^2 \), i.e. as the instantaneous value of error, and still obtain the same expressions for the Fourier coefficients \( a_0, a_i, b_i, \ i = 1 \cdots N - 1 \) although in this case the defined error would not be mathematically well-defined.

Now, if the frequency (hence the total phase) of signal \( u(t) \) happens to drift from \( f_0 = \frac{1}{T_0} \), Fourier coefficients obtained by the above minimization process still provide the best approximation that can be given in terms of the bases \( \sin(i\omega_0 t + \delta_i), \ i = 0 \cdots N - 1 \): to use a geometrical interpretation, \( \hat{u}(t) \) thus obtained is the orthogonal projection of the vector \( u(t) \) onto the space spanned by the basis vectors \( \sin(i\omega_0 t + \delta_i), \ i = 0 \cdots N - 1 \). However, the space onto which the projection is made is fixed and is not necessarily the best \( N \)-dimensional space for representation of the signal whose frequency is no longer known. Herein lies the main shortcoming of Fourier analysis; there is an inherent fixed frequency assumption involved in Fourier analysis.
Granted the frequency of the signal is fixed and known. Fourier analysis provides coefficients $a_i$ and $b_i$ for each basis sinusoid $A_i \sin(\omega_i t + \delta_i) = a_i \cos(\omega_i t) + b_i \sin(\omega_i t)$ which in turn provide estimates for the amplitude $A_i$ and constant phase $\delta_i$ of the sinusoidal components. Where needed, the sinusoidal component $A_i \sin(\omega_i t + \delta_i)$ itself is yet to be somehow synthesized, a task which is not trivial and often involves the use of a synchronization scheme such as a phase-locked loop (PLL). This presents another inherent shortcoming of the Fourier analysis. It does not offer means of extraction of sinusoids; it just provides means of estimation of amplitude and constant phase of individual sinusoids.

As regards the shortcoming of Fourier analysis due to its fixed frequency assumption, the ideal method is the one which finds not only the best approximation in a given space, but also the best space onto which the projection is to be made. This means that the ideal method would be inherently adaptive and the estimation process would be a closed loop process. Figure 2.1 illustrates this concept graphically. In general, a single sinusoidal signal $A \sin(\omega t + \delta)$ has three variables and, therefore, lies in a three-dimensional space. The additive noise polluting such a sinusoidal signal lies in a theoretically infinite-dimensional space. Real graphical representation of the polluted sinusoidal signal and its projection is, therefore, impossible given the physical limitation of the three-dimensional space. For the sake of conceptual presentation of the idea, in Figure 2.1 it is assumed that the input signal lies

Figure 2.1: Graphical representation of the concept of adaptive adjustment of the plane of projection.
in a three-dimensional space (represented by a non-planar curve) and the extracted sinusoid lies in a two-dimensional plane. Fourier analysis, although it provides the best projection in a fixed plane, is inefficient and the ideal method is the one that adaptively adjusts the plane of projection for the best results.

As regards its shortcoming in the lack of means of extraction of sinusoids, and considering that \( f(t) = A \sin(\omega t + \delta) \) is a nonlinear function of \( \omega \) and \( \phi(t) = \omega t + \delta \), it is easy to see that the problem of extraction of sinusoids is essentially a nonlinear one. Therefore, it is expected that a generalization of Fourier analysis to obviate both of the two aforementioned shortcomings would be a nonlinear adaptive algorithm. Such an algorithm is no longer an analysis method; since it estimates the parameters of the sinusoids and extracts (synthesizes) the sinusoidal components themselves, it is an analysis/synthesis method.

Derivation of the proposed algorithm outlined in the following section is more or less parallel to the way Fourier analysis was developed. Since the approximation is no longer a linear combination of basis vectors, the estimation cannot be obtained in a closed form such as an inner product; rather, a direct method of minimization has to be employed. To this end, gradient descent method is employed to provide a means of estimating parameters. Extraction of a single sinusoidal component is presented in the following section. Later in this chapter, the algorithm is extended to the case of extraction of a number of sinusoidal components. Such an extension would then be a nonlinear adaptive generalization of Fourier analysis.

### 2.2 Formulation of the Proposed Algorithm

Let \( u(t) \) represent a voltage or current signal. This function is usually continuous and almost periodic. A sinusoidal component of this function, \( y(t) = A \sin(\omega t + \delta) \), is of interest where \( A \) is the amplitude, \( \omega \) is the frequency (in rad/s), \( \delta \) is the constant phase and \( \phi(t) = \omega t + \delta \) represents the total phase of this component. Ideally, parameters \( A \), \( \omega \), and \( \delta \) are fixed quantities; but, in practice, this assumption does not hold true. In a typical situation, \( u(t) \)
has a general form of

\[ u(t) = \sum_{i=0}^{\infty} A_i \sin(\omega_i t + \delta_i) + n(t) \]  

(2.2)

in which \( n(t) \) denotes the superimposed disturbance/noise. In practice, all the parameters in (2.2) may undergo variations with time.

In the proposed algorithm, the objective is to extract a more or less specified component of \( u(t) \). Let \( \mathcal{M} \) be a manifold containing all pure sinusoidal signals defined as

\[ \mathcal{M} = \{ A(t) \sin(\omega(t) t + \delta(t)) \mid A(t) \in [A_{\text{min}}, A_{\text{max}}], \; \omega(t) \in [\omega_{\text{min}}, \omega_{\text{max}}], \; \delta(t) \in [\delta_{\text{min}}, \delta_{\text{max}}] \} \]

where \( \theta(t) = [A(t), \omega(t), \delta(t)]^T \) is the vector of parameters which belongs to the parameter space

\[ \Theta = \{ [A, \omega, \delta]^T \mid A \in [A_{\text{min}}, A_{\text{max}}], \; \omega \in [\omega_{\text{min}}, \omega_{\text{max}}] \text{ and } \delta \in [\delta_{\text{min}}, \delta_{\text{max}}] \} \]

and \( T \) denotes matrix transposition. The output is defined as the desired sinusoidal component, namely

\[ y(t, \theta(t)) = A(t) \sin(\omega(t) t + \delta(t)). \]

To extract a certain sinusoidal component of \( u(t) \), the solution has to be an orthogonal projection of \( u(t) \) onto the manifold \( \mathcal{M} \), or equivalently it has to be an optimum \( \theta \) which minimizes a distance function \( d \) between \( y(t, \theta(t)) \) and \( u(t) \), i.e.,

\[ \theta_{\text{opt}} = \arg \min_{\theta(t) \in \Theta} d[y(t, \theta(t)), u(t)]. \]

Without being concerned about the mathematical correctness of the definition of the least squares error, which strictly speaking has to map onto the set of real numbers, the instantaneous distance function \( d \) is used [29]:

\[ d^2(t, \theta(t)) = [u(t) - y(t, \theta(t))]^2 \triangleq e(t)^2. \]

Hence, the cost function is defined as \( J(\theta(t), t) \triangleq d^2(t, \theta(t)) \). Although the cost function is not quadratic, the parameter vector \( \theta \) is estimated using the gradient descent method,

\[ \frac{d\theta(t)}{dt} = -\mu \frac{\partial [J(t, \theta(t))]}{\partial \theta(t)} \]  

(2.3)
where the positive diagonal matrix $\mu$ is the algorithm regulating constant matrix. The values of the entries of this matrix control the convergence rate as well as the stability of the algorithm. Figure 2.2 illustrates the minimization process when achieved using the method of gradient descent. Given a quadratic cost function, it is clear that the algorithm employing this method converges to the minimum solution for the cost function. In more complex cases than those involving quadratic functions, gradient descent method may still achieve minimization although this is not true in general. The choice of the gradient descent method for the minimization process is intuitive and no rigorous justification can be presented at this point; its application is a reflection of an intuitive predisposition towards its common use. Global convergence of the gradient descent method is guaranteed for quadratic distance functions; otherwise, its convergence has to be directly proved.

From here on, the estimated value of parameter vector $\theta(t)$ is denoted by

$$\hat{\theta}(t) = [\hat{A}(t), \hat{\omega}(t), \hat{\delta}(t)]^T.$$

Therefore, $\hat{A}(t), \hat{\omega}(t)$ and $\hat{\delta}(t)$ represent estimated values of amplitude, frequency and con-
stant phase, respectively. The algorithm regulating constant matrix \( \mu \) is defined as

\[
\mu = \begin{bmatrix}
  m_1 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_3
\end{bmatrix}.
\]

Carrying out the mathematical manipulations indicated in (2.3), namely

\[
\frac{d\dot{\phi}(t)}{dt} = \dot{\omega}(t) + t \frac{d\dot{\omega}(t)}{dt} + \frac{d\delta(t)}{dt},
\]

(2.8)

one can write

\[
\frac{d\dot{\phi}(t)}{dt} = 2m_1 e(t) \sin(\dot{\omega}(t)t + \delta(t)),
\]

(2.9)

\[
\frac{d\dot{\omega}(t)}{dt} = 2m_2 e(t) \dot{\phi}(t) \cos(\dot{\omega}(t)t + \delta(t)),
\]

(2.10)

\[
\frac{d\delta(t)}{dt} = 2m_3 e(t) \dot{\phi}(t) \cos(\dot{\omega}(t)t + \delta(t))
\]

(2.11)

where \( e(t) \) is defined as

\[
e(t) = u(t) - \dot{\phi}(t) \sin(\dot{\omega}(t)t + \delta(t)).
\]

Since \( \phi(t) = \omega(t)t + \delta(t) \), one may write the following augmented equation:

\[
\frac{d\dot{\phi}(t)}{dt} = \dot{\omega}(t) + t \frac{d\dot{\omega}(t)}{dt} + \frac{d\delta(t)}{dt}.
\]

(2.8)

The presence of the time variable \( t \) in (2.6) and (2.8) is troublesome. In particular, the equations, the way they are, present a time-varying system; i.e. response of the system to a given input signal varies depending on when the system is initialized; a fact which is physically unjustifiable. To obviate this problem and to simplify the implementation as well as analysis of the dynamical system presented by (2.5)-(2.8), the time variable \( t \) is replaced by a constant number \( m_4 \). The resultant set of differential equations can then be written as

\[
\frac{d\dot{\phi}(t)}{dt} = 2\mu_1 e(t) \sin \dot{\phi}(t),
\]

(2.9)

\[
\frac{d\dot{\omega}(t)}{dt} = 2\mu_2 e(t) \dot{\phi}(t) \cos \dot{\phi}(t),
\]

(2.10)

\[
\frac{d\delta(t)}{dt} = \dot{\omega}(t) + \mu_3 \frac{d\dot{\omega}(t)}{dt}
\]

(2.11)
where error $e(t)$ is

$$e(t) = u(t) - \dot{A}(t) \sin \dot{\phi}(t)$$

(2.12)

and parameters $\mu_1$ and $\mu_2$ are constant numbers given by

$$\mu_1 = m_1,$$

$$\mu_2 = m_2 m_4,$$

$$\mu_3 = m_4 + \frac{m_3}{m_2 m_4}.$$ 

Equations (2.9)-(2.12) present the governing differential equations of the proposed algorithm. Development of these equations, although inspired by the concepts of the least squares error minimization and the method of gradients, does not comply with the conditions under which these concepts may be legitimately employed. This implies that the mathematical properties of the proposed algorithm such as its stability, convergence and uniqueness of solution, and its engineering usefulness have to be proved.

In chapter 3, it is shown that the dynamical system represented by the above differential equations possesses a unique asymptotically stable periodic orbit which lies in a neighborhood of the orbit associated with the desired component of the function $u(t)$. In terms of the engineering performance of the system, this indicates that the output of the system $y(t) = \dot{A}(t) \sin \dot{\phi}(t)$ approaches a sinusoidal component of input signal $u(t)$: in other words, the system is a notch filter which extracts a sinusoidal component of its input signal. Moreover, the slow variations of parameters in $u(t)$ are tolerated by the system, i.e. the filter is adaptive and the output follows the variations in the input signal.

The proposed core algorithm is a generalization of two previously developed nonlinear adaptive algorithms [12] which tackle the nonlinear problem of extraction of sinusoids in a nonlinear way. The first of the two algorithms considers an “$A \sin \phi$” model for the sinusoid of interest. Although excellent performance of such an amplitude-phase-model-based algorithm is demonstrated in applications where the frequency drift away from the nominal frequency is small [27], it does not directly provide an estimate of the instantaneous frequency. In the
second of the two algorithms, although a complete \( A \sin(\omega t + \delta) \) model for the sinusoid of interest is considered, the term \( t \frac{d\omega(t)}{dt} \) in (2.8) involving the time variable \( t \) is totally discarded. Good performance of the second algorithm is observed only under small off-nominal frequency variations [12]. Therefore, the sole advantage of the second algorithm over the first one is its capability to directly estimate the instantaneous frequency. The proposed algorithm of this thesis, however, is more general in the sense that it includes the two previously developed algorithms as special cases while accommodating large variations in the frequency of the sinusoid of interest.

Figure 2.3 shows a block diagram representation of the core algorithm. It is observed that error \( e(t) \) is the difference between input signal \( u(t) \) and output signal \( y(t) \). This implements (2.12). The top branch of the block diagram implements the differential equation related to the estimated amplitude, namely (2.9). According to (2.9), error \( e(t) \) when multiplied by the sine of \( \phi(t) \) and scaled by a constant number provides the time derivative of the estimated amplitude. These steps are graphically displayed in the top branch of Figure 2.3. The integration operation then provides an estimate of the amplitude. Likewise, the bottom branch of the block diagram implements (2.10) and (2.11). Finally, the output signal is synthesized by the product of the estimated amplitude and the sine of the estimated phase.

In Figure 2.3, the value of the initial condition of the integration operation required for computing the frequency is depicted explicitly by \( \omega_0 \). Assignment of the value of \( \omega_0 \) provides a means of more or less specifying the sinusoidal component of interest, the component that the algorithm is supposed to extract. In other words, the algorithm finds that sinusoidal component of the input signal which is closest in frequency to \( \omega_0 \), the assigned initial condition of frequency.

### 2.3 Extension

The core algorithm presented so far is intended to extract one single sinusoidal component of its input signal. To complete the analogy with Fourier analysis, a multiplicity of core units
Figure 2.3: Block diagram of the proposed algorithm. Parameters $2\mu_1$ and $2\mu_2$ in (2.9)-(2.11) are replaced by $\mu_1$ and $\mu_2$ in this figure.

Figure 2.4: Two possible ways of employing a multiplicity of core units for the decomposition of a multi-component input signal into its constituent sinusoidal components.
may be employed in parallel, or alternatively cascaded, to decompose a multi-component input signal into its constituent sinusoidal components. Figure 2.4 shows two possible combinations, both of which are useful for different applications. It is noteworthy that each core unit tries to extract a sinusoidal component whose frequency is closest to its specified initial condition. In order to avoid overlapping duties of the core units, the operating frequency of each of the units has to be within a specified range. This can be achieved by introducing range limiters within the frequency integration of Figure 2.3.

2.4 An Alternative Form

In the right-hand side of the differential equation estimating frequency, namely (2.10), the value of the estimated amplitude \( \hat{A}(t) \) may be considered to be absorbed within parameter \( \mu_2 \). Then, the equations may be rewritten in a more symmetrical form as given below.

\[
\begin{align*}
\frac{d\hat{A}(t)}{dt} &= 2\mu_1 e(t) \sin \hat{\phi}(t), \\
\frac{d\hat{\phi}(t)}{dt} &= 2\mu_2 e(t) \cos \hat{\phi}(t), \\
\frac{d\hat{\omega}(t)}{dt} &= \hat{\omega}(t) + \mu_3 \frac{d\hat{\omega}(t)}{dt}, \\
y(t) &= \hat{A}(t) \sin \hat{\phi}(t), \\
e(t) &= u(t) - y(t).
\end{align*}
\]

The above equations represent an alternative somewhat simpler form of the proposed algorithm. It is shown in chapter 3 how the mathematical properties of the proposed algorithm in its general form presented by (2.9)-(2.12) are inherited in this simplified form.

2.5 Precursors of the Proposed Method

As mentioned before, the proposed algorithm of this thesis is the third, and the most general, member of a new family of methods of extraction of nonstationary sinusoids. In the first member of this family of signal processing algorithms presented in [12], the desired output signal is assumed to be \( y(t) = A(t) \sin \phi(t) \). The derivation of the governing set of differential
equations follows the same procedure as that outlined in section 2.2 and amounts to the following dynamical system:

\[
\begin{align*}
\frac{d\hat{A}(t)}{dt} &= 2\mu_1 e(t) \sin \hat{\phi}(t), \\
\frac{d\hat{\phi}(t)}{dt} &= 2\mu_2 e(t) \hat{A}(t) \cos \hat{\phi}(t) + \omega_o, \\
e(t) &= u(t) - \hat{A}(t) \sin \hat{\phi}(t)
\end{align*}
\]

where \(\omega_o\) is the known fixed nominal frequency of the input signal. Since time-variations are assumed solely for two variables \(\hat{A}(t)\) and \(\hat{\phi}(t)\), the space in which the desired sinusoidal signal lies is two-dimensional. For the same reason, the dynamical system governing this algorithm is second order.

In the second of the two previously developed members of the family, also presented in [12], time-variations in the frequency of the desired signal is tried to be explicitly accommodated by defining the output signal as \(y(t) = \hat{A}(t) \sin(\omega(t)t + \delta(t))\). The derivation of the dynamics of this algorithm closely follows the same procedure as that outlined in section 2.2. The difference is in the way time variable \(t\) in (2.8) is treated. In this algorithm, the term \(t \frac{d\hat{\omega}(t)}{dt}\) in (2.8) involving time variable \(t\) is totally discarded. The resultant set of governing differential equations is

\[
\begin{align*}
\frac{d\hat{A}(t)}{dt} &= 2\mu_1 e(t) \sin \hat{\phi}(t), \\
\frac{d\hat{\omega}(t)}{dt} &= 2\mu_2 e(t) \hat{A}(t) \cos \hat{\phi}(t). \\
\frac{d\hat{\phi}(t)}{dt} &= \hat{\omega}(t), \\
e(t) &= u(t) - \hat{A}(t) \sin \hat{\phi}(t).
\end{align*}
\]

It is clear that this algorithm directly estimates the instantaneous frequency. However, the range of frequency variations tracked by the algorithm is narrow, just about the same as that of the first algorithm.

The space in which the first algorithm seeks the output is two-dimensional whereas that of the second one and that of the proposed algorithm of this thesis are three-dimensional:
however, if one wishes to categorize these three algorithms under the same formulation, one may write

\[
\frac{d\dot{A}(t)}{dt} = 2\mu_1 e(t) \sin \dot{\phi}(t),
\]
\[
\frac{d\dot{\omega}(t)}{dt} = 2\mu_2 e(t) \dot{A}(t) \cos \dot{\phi}(t),
\]
\[
\frac{d\dot{\phi}(t)}{dt} = F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)),
\]
\[
e(t) = u(t) - \dot{A}(t) \sin \dot{\phi}(t).
\]

where \( F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)) \) for each of the three algorithms is defined as

- \( F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)) = 2\mu_2 e(t) \dot{A}(t) \cos \dot{\phi}(t) + \omega_0 \) for the first algorithm.
- \( F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)) = \dot{\omega}(t) \) for the second algorithm.
- \( F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)) = 2\mu_2 e(t) \dot{A}(t) \cos \dot{\phi}(t) + \dot{\omega}(t) \) for the proposed algorithm.

A comparison of the form of function \( F(\dot{A}(t), \dot{\omega}(t), \dot{\phi}(t), e(t)) \) for the first and the third case reveals great symmetry which may be interpreted as the optimality of the design in these two cases. It is noteworthy that these two algorithms are obtained in two different spaces; in other words, this symmetry is hidden in the first inspection of the governing sets of differential equations of these two algorithms (which are not directly comparable since they do not have the same order); this is observed only after the proposed algorithm is fully developed independent of this symmetry, a fact which reinforces the apparently arbitrary formulation of the proposed algorithm. All this stated, the final judgment on the functionality, practicality and usefulness of the proposed algorithm can be made only through the mathematical proof of its performance, practical verification of the theoretical proofs and demonstration of its superiority in real applications. These issues are addressed in the following chapters.
Chapter 3

Mathematical Properties

As shown in chapter 2, a set of nonlinear non-autonomous differential equations governs the dynamics of the proposed algorithm. Since the least squares error used in the development of the proposed algorithm is not a quadratic function of the parameters to be estimated, the gradient descent method, by itself, does not guarantee the convergence and stability of this dynamical system. This chapter presents proofs of the mathematical properties of the proposed algorithm. Poincaré map theorem is employed. This theorem is reviewed in the following section. Section 3.2 summarizes the proofs of the existence, uniqueness and stability of a periodic orbit of the proposed dynamical system. The mathematically predicted behavior of the system is then demonstrated by the use of a few computer simulations at the end of this chapter.

3.1 Poincaré Map Theorem

The Poincaré map theorem is reviewed here mainly based on the exposition in [30, page 64]. Consider the following ordinary differential equation:

\[
\frac{dx}{dt} = f(x), \quad x \in \mathbb{R}^n,
\]

where \( f : U \rightarrow \mathbb{R}^n \) is \( C^r \) on some open set \( U \subset \mathbb{R}^n \). Let \( \phi(t, \cdot) \) denote the flow generated by (3.1). Suppose that (3.1) has a periodic solution of period \( T \) which is denoted by \( \phi(t, x_o) \), where \( x_o \in \mathbb{R}^n \) is any point through which this periodic solution passes. Let \( S \) be an \( n - 1 \)
Figure 3.1: Illustration of the Poincaré map theorem [31].

dimensional surface transverse to the vector field at \( x_0 \); \( S \) is referred to as a cross-section to the vector field. One can find an open set \( V \subset S \) such that the trajectories starting in \( V \) return to \( S \) in a time close to \( T \). The map that associates points in \( V \) with their points of first return to \( S \) is called \textit{Poincaré map}, and is denoted by \( P \).

**Theorem 1 (Poincaré Map)** A fixed point of \( P \) corresponds to a periodic orbit of (3.1). The stability type of the fixed point of \( P \) is the same as that of the periodic orbit of (3.1).

Figure 3.1 illustrates the concept of Poincaré map theorem. The initial point \( x_0 \) lies within sector \( S \). Under the flow of the dynamics, it arrives again in \( S \). Map \( P \) transforming \( x_0 \) to \( P x_0 \) is the Poincaré map. \( P^2 x_0 \) is the arrival point in section \( S \) after two times traversing the sector. If \( P^2 x \) happens to coincide with \( x \), a doubly periodic orbit exists. Point \( x_1 \) is an example of this type of fixed points. Finally, \( x^* \) is a simple fixed point. It returns to itself in \( S \) under the flow of the dynamics.

### 3.2 Mathematical Properties of the Algorithm

In the governing equations of the proposed algorithm, namely (2.9)-(2.12), the caret sign (\(^\wedge\)) indicates that quantities \( \hat{A}(t), \hat{\omega}(t) \) and \( \hat{\phi}(t) \) are estimated values. Since only estimated
values are used in the discussions presented in this chapter and the following chapters, the caret signs are dropped for the sake of simplicity. Therefore, from here on quantities $A(t)$, $\omega(t)$ and $\phi(t)$ represent $\dot{A}(t)$, $\dot{\omega}(t)$ and $\dot{\phi}(t)$, respectively. The equations are thus written as

\[
\begin{align*}
\frac{dA(t)}{dt} &= 2\mu_1 e(t) \sin \phi(t), \\
\frac{d\omega(t)}{dt} &= 2\mu_2 e(t) A(t) \cos \phi(t), \\
\frac{d\phi(t)}{dt} &= \omega(t) + \mu_3 \frac{d\omega(t)}{dt}, \\
e(t) &= u(t) - A(t) \sin \phi(t).
\end{align*}
\] (3.2) (3.3) (3.4) (3.5)

The expression for the error function (3.5) may be used in (3.2)-(3.4) to yield a more explicit form. The presence of sine and cosine terms in the expressions suggests framing of the equations in spherical coordinates system. If the explicit form of the governing differential equations of the algorithm is framed in the spherical coordinate system, the set of differential equations becomes

\[
\begin{align*}
\frac{d\rho(t)}{dt} &= -2\mu_1 \rho(t) \sin^2 \phi(t) + 2\mu_1 \sin \phi(t) u(t), \\
\frac{d\theta(t)}{dt} &= -\mu_2 \rho(t)^2 \sin(2\phi(t)) + 2\mu_2 \rho(t) \cos \phi(t) u(t), \\
\frac{d\phi(t)}{dt} &= \dot{\theta}(t) + \mu_3 \frac{d\theta(t)}{dt}.
\end{align*}
\] (3.6) (3.7) (3.8)

Let $u(t) = u_o(t) + \epsilon u_1(t)$ where $u_o(t) = A_o \sin \phi_o(t)$ and $\phi_o(t) = \omega_o t + \delta_o$. In other words, assume that the input signal has a sinusoidal component at frequency $\omega_o = 2\pi f_o$ and some other superimposed components. Rewrite (3.6), (3.7) and (3.8) as

\[
\begin{align*}
\frac{d\rho(t)}{dt} &= -2\mu_1 \rho(t) \sin^2 \phi(t) + 2\mu_1 A_o \sin(\omega_o t + \delta_o) \sin \phi(t) + 2\epsilon \mu_1 \sin \phi(t) u_1(t), \\
\frac{d\theta(t)}{dt} &= -\mu_2 \rho(t)^2 \sin(2\phi(t)) + 2\mu_2 A_o \sin(\omega_o t + \delta_o) \rho(t) \cos \phi(t) + 2\epsilon \mu_2 \rho(t) u_1(t), \\
\frac{d\phi(t)}{dt} &= \dot{\theta}(t) + \mu_3 \frac{d\theta(t)}{dt}.
\end{align*}
\] (3.9) (3.10) (3.11)
The following theorem deals with the existence, uniqueness and stability of a periodic orbit for this dynamical system.

**Theorem 2** Consider the dynamics represented by (3.9)-(3.11) and let $T_0 = \frac{2\pi}{\omega_0}$. This system has a locally unique, asymptotically stable $T_0$-periodic orbit which

i) for $\epsilon = 0$, coincides with $\gamma_0(t) = (A_0, \omega_0, \phi_0 = \omega_0 t + \delta_0)$, and

ii) for $\epsilon \neq 0$ lies in an $\epsilon$-neighborhood of $\gamma_0(t)$ denoted by $\gamma_\epsilon(t)$.

In engineering terms, when the input signal is a pure sinusoid, the algorithm extracts such a sinusoidal signal and provides it as the output of the system. When the input signal is a sinusoid polluted by a totality of undesired components, the output signal approaches the single sinusoidal component to which convergence is desired.

**Proof.** According to the Poincaré map theorem, the behavior of the dynamical system near its periodic orbit can be investigated using a discrete map. The fixed points of this map correspond to the periodic orbits of the original dynamics and their stability types are equivalent. A suitable section to obtain the Poincaré map is

$$S_\epsilon = \{(\rho(t), \theta(t), \phi(t)) : |\rho(t) - A_0| < \epsilon, |\theta(t) - \omega_0| < \epsilon, \phi(t) = \delta_0\}.$$  

Consider the initial point $(\rho_1, \theta_1, \delta_0) \in S_\epsilon$. After a certain time $t^*$, this point intersects $S_\epsilon$ again under the flow of the dynamics at point $(\rho_2, \theta_2, \delta_0 + 2\pi) \in S_\epsilon$. The Poincaré map is defined as follows:

$$P : S_\epsilon \rightarrow S_\epsilon$$

$$(\rho_1, \theta_1) \mapsto (\rho_2, \theta_2).$$

Suppose $\mu_1$ and $\mu_2$ are small and define $\mu_1 = \epsilon \mu_1$ and $\mu_2 = \epsilon^2 \mu_2$. With this definition, all the terms in (3.9) have a coefficient of $\epsilon$ and all the terms in (3.10) have a coefficient of $\epsilon^2$ on the trajectory connecting $(\rho_1, \theta_1)$ to $(\rho_2, \theta_2)$. The values of $\frac{d\theta(t)}{dt}$ and $\frac{d\rho(t)}{dt}$ could be written as $O(\epsilon^2)$ and $O(\epsilon)$. Therefore,

$$\rho(t) = \rho_1 + \int_0^t \frac{d\rho(\tau)}{dt}d\tau = \rho_1 + O(\epsilon) \quad \forall t \in I^* = [0, t^*],$$

(3.12)
\[
\theta(t) = \theta_1 + \int_0^t \frac{d\theta(\tau)}{d\tau} d\tau = \theta_1 + \mathcal{O}(\epsilon^2) \quad \forall t \in I^* = [0, t^*].
\]

Also, note that \( \forall t \in I^* = [0, t^*] \)

\[
\phi(t) = \delta_o + \int_0^t \frac{d\phi(\tau)}{d\tau} d\tau = \delta_o + \int_0^t \left( \phi(\tau) + \mu_3 \frac{d\theta(\tau)}{d\tau} \right) d\tau \quad \quad \quad (3.14)
\]

\[
\phi(t) = \delta_o + \theta_1 t + \mathcal{O}(\epsilon^2) + \mu_3 (\theta_1 + \mathcal{O}(\epsilon^2) - \theta_1) \quad \quad \quad (3.15)
\]

\[
\phi(t) = \delta_o + \theta_1 t + \mathcal{O}(\epsilon^2). \quad \quad \quad (3.16)
\]

Consider the last equation for \( t = t^* \). Knowing \( \phi(t^*) = 2\pi + \delta_o \), one concludes that

\[
t^* = \frac{2\pi}{\theta_1} + \mathcal{O}(\epsilon^2).
\]

The Poincaré map is computed using the following integration formulas:

\[
\rho_2 = \rho_1 + \int_0^{t^*} \frac{d\rho(t)}{dt} dt.
\]

\[
\theta_2 = \theta_1 + \int_0^{t^*} \frac{d\theta(t)}{dt} dt.
\]

Substituting from (3.12)-(3.17), one obtains

\[
\rho_2 = \rho_1 - 2\mu_1 \int_0^{t^*} [\rho_1 + \mathcal{O}(\epsilon)] \sin^2[\theta_1 t + \delta_o + \mathcal{O}(\epsilon^2)] dt + 2\mu_1 A_o \int_0^{t^*} \sin(\omega_o t + \delta_o) \sin[\theta_1 t + \delta_o + \mathcal{O}(\epsilon^2)] dt + 2\mu_1 \epsilon \int_0^{t^*} \sin[\theta_1 t + \delta_o + \mathcal{O}(\epsilon^2)] u_1(t) dt = \rho_1 - 2\mu_1 \frac{\rho_1 \pi}{\theta_1} + 2\mu_1 \frac{A_o \pi}{\theta_1} + \mathcal{O}(\epsilon^2)
\]

and

\[
\theta_2 = \theta_1 - \mu_2 \int_0^{t^*} [\rho_1 + \mathcal{O}(\epsilon)] \sin[2\theta_1 t + 2\delta_o + \mathcal{O}(\epsilon^2)] dt + 2\mu_2 A_o \int_0^{t^*} [\rho_1 + \mathcal{O}(\epsilon)] \sin(\omega_o t + \delta_o) \cos[\theta_1 t + \delta_o + \mathcal{O}(\epsilon^2)] dt + 2\mu_2 \epsilon \int_0^{t^*} [\rho_1 + \mathcal{O}(\epsilon)] \cos[\theta_1 t + \delta_o + \mathcal{O}(\epsilon^2)] u_1(t) dt = \theta_1 + \mu_2 A_o \rho_1 \frac{1 - \cos(\frac{2\pi \omega_o}{\theta_1})}{\omega_o - \theta_1} \cos(2\delta_o) - \cos(\frac{2\pi \omega_o}{\theta_1} + 2\delta_o) + \mu_2 A_o \rho_1 \frac{\cos(2\delta_o) - \cos(\frac{2\pi \omega_o}{\theta_1} + 2\delta_o)}{\omega_o + \theta_1} + \mathcal{O}(\epsilon^3).
\]
Therefore, the Poincaré map is in the form
\[
\begin{pmatrix}
\rho_2 \\
\theta_2
\end{pmatrix} = \begin{pmatrix}
\rho_1 \\
\theta_1
\end{pmatrix} + \begin{pmatrix}
2\pi\mu_1 & 0 \\
0 & A_0\mu_2
\end{pmatrix} F(\rho_1, \theta_1, \epsilon)
\]
where \( F(\rho_1, \theta_1, \epsilon) \) is defined as
\[
\begin{pmatrix}
\rho_1 - A_0 - \rho_1 + O(\epsilon) \\
\theta_1 - \omega_0 + O(\epsilon)
\end{pmatrix} + \begin{pmatrix}
\frac{\cos(2\pi\omega_0) - 1}{\theta_1 - \omega_0} + \rho_1 \frac{\cos(2L) - \cos(2\theta_0 + 2\pi\omega_0)}{\omega_0 + \theta_1} + O(\epsilon)
\end{pmatrix}.
\]
At the point \((\rho, \theta, \epsilon) = (A_0, \omega_0, 0)\), the function \( F \) vanishes: i.e. \( F(A_0, \omega_0, 0) = 0 \) and its derivative map is
\[
\begin{pmatrix}
-1/\omega_0 & 0 \\
0 & -\pi^2 A_0 (2 + \sin 2\theta_0)/\omega_0^2
\end{pmatrix}.
\]
It is observed that the derivative matrix is non-singular: therefore, the derivative map is one-to-one. Here reference is made to the implicit function theorem [32, page 8]:

**Theorem 3 (Implicit Function Theorem)** Suppose \( F : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m \) has continuous first partial derivatives and \( F(0, 0) = 0 \). If the Jacobian matrix \( \partial F(x, y)/\partial x \) is non-singular, then there exists neighborhoods \( U, V \) of \( 0 \) in \( \mathbb{R}^m, \mathbb{R}^n \), respectively, such that for each fixed \( y \in V \) the equation \( F(x, y) = 0 \) has a unique solution \( x \) in \( U \). Furthermore, this solution can be given as \( x = g(y) \), where \( g \) has continuous first derivatives and \( g(0) = 0 \).

Therefore, equation \( F(\rho_1, \theta_1, \epsilon) = 0 \) has a unique solution \((\rho_1(\epsilon), \theta_1(\epsilon))\) in an \( \epsilon \)-neighborhood of \((A_0, \omega_0)\) for which
\[
(\rho_1(0), \theta_1(0)) = (A_0, \omega_0).
\]
Note that this solution of equation \( F(\rho_1, \theta_1, \epsilon) = 0 \) corresponds to the fixed point of Poincaré map and hence to the periodic orbit of the original dynamics in (3.9), (3.10) and (3.11).

To study the stability of the periodic orbit of the system, one may consider the linearized set of Poincaré equations about \( \gamma_0(t) \)
\[
\frac{\partial P(\rho(t), \theta(t))}{\partial(\rho(t), \theta(t))} |_{\gamma_0(t)} = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} + \begin{pmatrix}
2\pi\mu_1 & 0 \\
0 & A_0\mu_2
\end{pmatrix} \frac{\partial F(\rho(t), \theta(t))}{\partial(\rho(t), \theta(t))} |_{(A_0, \omega_0, 0)}.
\]
The eigenvalues of the linearized Poincaré map which determine the stability type of this periodic orbit are computed as

\[
\begin{align*}
\lambda_1 &= 1 - \frac{2\pi \mu_1}{\omega_0} + \mathcal{O}(\epsilon^2), \\
\lambda_2 &= 1 - \left(\frac{\pi^2 A^2 \mu_2}{\omega_0^2}\right) \left(2 + \frac{\sin 2\delta_0}{\pi}\right) + \mathcal{O}(\epsilon^3).
\end{align*}
\]

(3.19)

Note that both of the above eigenvalues lie in the unit circle provided that \(\mu_1\) and \(\mu_2\) are sufficiently small. Roughly, one can see that for the values of \(0 < \mu_1 < 2f_0\) and \(0 < \mu_2 < \left(\frac{2f_0}{\lambda_0}\right)^2\) the stability is guaranteed. Therefore, the obtained unique periodic orbit is asymptotically stable.

It is easy to see that with a minor modification throughout the proof of Theorem 2, the same procedure may be applied to the dynamical system presented in section 2.4. The only significant qualitative difference which affects the settings of the parameters in this case is in the expression for the second eigenvalue of the linearized system. In short, \(\mu_2 A_0\) is to be replaced by \(\mu_2\).

Figures 3.2 and 3.3 illustrate the behavior of the proposed dynamical system by way of example. Figure 3.2 shows the convergence of the algorithm in response to a unit-amplitude sinusoid. The pre-set initial frequency in Figure 3.2 is the same as the frequency of the incoming sinusoid. To show the frequency-adaptive nature of the algorithm, in another example the initial frequency of the algorithm is deliberately set to be about 50% off the frequency of the incoming sinusoid. Figure 3.3 shows the convergence of the algorithm in frequency. These two examples are precursors of the results of numerous simulations and laboratory verifications which are presented in subsequent chapters to prove the engineering usefulness of the proposed algorithm after some discussions on the implementation of the proposed algorithm in the following chapter.
Figure 3.2: Illustration of the convergence of the proposed algorithm. The input signal is a unit-amplitude sine wave. The top graph shows the convergence of the dynamics to the periodic orbit associated with the extracted sinusoid; the bottom graph shows the flow of the dynamics in the time domain.
Figure 3.3: Illustration of the frequency tracking property of the proposed algorithm. The top graph shows the convergence of the algorithm to the periodic orbit associated with the extracted sinusoid; the bottom graph shows the same phenomenon in the time domain.
Chapter 4

Implementation

The signal processing algorithm described by the dynamical system (2.9)-(2.12) has a very simple structure consisting of a few arithmetic and trigonometric operations. It can be easily implemented in software in the form of a code written in a programming language or in a schematic design environment. Implementation of the algorithm in software can be done either on a personal computer or on a dedicated processor such as a digital signal processor (DSP). Alternatively, it can be implemented in hardware using either digital or analog circuitry. This chapter presents the implementation of the proposed algorithm in software form. The following section outlines the implementation of the algorithm in Matlab™ and Matlab Simulink™ computational software. Subsequently, the implementation of the algorithm on a DSP platform is presented in section 4.2.

4.1 Implementation within the Matlab Simulink™ Environment

Implementation of the proposed algorithm entails the discretization of the differential equations describing the algorithm. The discretized form of the governing equations of the proposed algorithm can be written as

\[ A[n + 1] = A[n] + 2T_s \mu_1 e[n] \sin \phi[n]. \]  \hspace{1cm} (4.1)

\[ \omega[n + 1] = \omega[n] + 2T_s \mu_2 e[n] A[n] \cos \phi[n], \]  \hspace{1cm} (4.2)
\[
\begin{align*}
\phi[n+1] &= \phi[n] + T_s \omega[n] + 2T_s \mu_2 \mu_3 e[n] A[n] \cos \phi[n] \quad (4.3) \\
y[n] &= A[n] \sin \phi[n], \quad (4.4) \\
e[n] &= u[n] - y[n] \quad (4.5)
\end{align*}
\]

where a first order approximation for derivatives is assumed, \(T_s\) is the sampling time and \(n\) is the time step index. Conversion of (4.1)-(4.5) to a software code using a high level programming language is straightforward as is further clarified in this section.

**Implementation as a Matlab Code**

The discretized form of the equations, (4.1)-(4.5), can be readily implemented as a Matlab script code. As an illustrative example, the following is the Matlab code for the core algorithm:

\[
\begin{align*}
y(n) &= A(n) \times \sin(\phi(n)); \\
e(n) &= u(n) - y(n); \\
A(n+1) &= A(n) + 2 \times T_s \times \mu_1 \times e(n) \times \sin(\phi(n)); \\
\omega(n+1) &= \omega(n) + 2 \times T_s \times \mu_2 \times e(n) \times A(n) \times \cos(\phi(n)); \\
\phi(n+1) &= \phi(n) + T_s \times \omega(n) + 2 \times T_s \times \mu_2 \times \mu_3 \times e(n) \times A(n) \times \cos(\phi(n));
\end{align*}
\]

which may be employed within a loop.

**Implementation as a Simulink Model**

Alternatively, the algorithm may be implemented as a Simulink model. Figure 4.1 shows an example of the implementation of the algorithm within the Simulink environment. The function of each block within the Simulink model is self-evident. For example, the \( \frac{1}{s} \) block is an integrator generating quantity \( x \) by inputting its derivative \( \frac{dx}{dt} \). The initial conditions are defined within such integrators. The model directly follows (2.9)-(2.12). One can describe the role of the upper branch of Figure 4.1 as the amplitude estimation process and the role of the lower branch as the phase estimation process. It is observed that the two branches are not self-sufficient and are somewhat interdependent.
Figure 4.1: Implementation of the proposed algorithm as a Simulink model.

4.1.1 Simulations

To demonstrate the performance of the proposed algorithm, a number of simulations are presented in this section. In the first numerical experiment, a pure sinusoid with unit amplitude, frequency of \( f = 60 \) Hz and random constant phase is input to the algorithm. Part (i) of Theorem 2 of chapter 3 predicts the existence of a periodic orbit for this case. The initial conditions are chosen as \( A_0 = 0, f_o = 50, \) and \( \phi_o = 0. \) The values of \( \mu \)-parameters determine the convergence speed of the algorithm. As predicted by the mathematical analysis of chapter 3, two eigenvalues of the algorithm are related to \( \mu_1 \) and \( \mu_2. \) In section 3.2, it was shown that the values of \( \mu \)-parameters have to be chosen such that the two conditions of \( 0 < \mu_1 < 2f_o \) and \( 0 < \mu_2 < \left( \frac{2f_o}{A} \right)^2 \) are roughly satisfied. Quantity \( f_o \) is the frequency of the sinusoidal signal of interest and \( A \) is its amplitude. Given \( f_o = 50, \) the choice of \( \mu_1 = 2f_o = 100, \mu_2 = (2f_o)^2 = 10000 \) for a unit-amplitude sinusoid seems to roughly satisfy the conditions. The choice of \( \mu_3 \) is interdependent on the choice of \( \mu_2. \) Generally, one may choose the value of \( \mu_3 \) such that the product \( \mu_2\mu_3 \) becomes of the same order of magnitude as \( \mu_1 \) for a balanced speed in terms of tracking the amplitude and total phase (or frequency). This is in accordance with the way the governing equations of the algorithm present the role of parameters; i.e. \( \mu_1 \) and \( \mu_2\mu_3 \) appear to determine the convergence speed in amplitude and total phase, respectively. With these considerations, it is observed that a set of values of \( \mu_1 = 100, \mu_2 = 10000, \mu_3 = 0.02 \) provides the convergence in a few cycles. Since the flow of the dynamics can be easily visualized in a few cycles time for a sinusoid, this speed
Figure 4.2: Convergence to periodic orbit \((A = 1, \omega = 120\pi, \phi = 120\pi t + \delta_0)\).

is desirable in presenting the simulation results. Therefore, this set of parameters are used for all the simulations presented in this section.

Figure 4.2 shows the performance of the algorithm in convergence to periodic orbit \((A = 1, \omega = 120\pi, \phi = 120\pi t + \delta_0)\). For easy visualization, the flow of the dynamics is presented in a Cartesian coordinate system in which the axes are \(A\cos\phi, A\sin\phi, f\). The periodic orbit, therefore, is depicted by a circle which is set off the horizontal plane at a vertical distance equal to the value of the frequency. It is observed that the algorithm converges to the periodic orbit associated with the input sinusoid in a few cycles. The extracted signal,
Figure 4.3: Performance of the algorithm in extracting a sinusoidal signal.
Figure 4.4: Phase tracking performance of the algorithm.

its amplitude and its frequency are shown in Figure 4.3.

Phase Tracking

In order to demonstrate the performance in tracking the constant phase, the previous experiment is repeated, but this time the input sinusoid is made to have a constant phase of $\frac{\pi}{2}$. Figure 4.4 shows the performance of the algorithm in tracking the phase of the input signal.

The Role of the Initial Conditions

The algorithm is insensitive to the initial conditions. To illustrate this, the same experiment as before is carried out with a different frequency initial condition $f_0 = 70$. Figure 4.5 shows the convergence of the algorithm to the same periodic orbit as in Figure 4.2. It is noteworthy that in both cases of Figures 4.2 and 4.5, the sinusoidal component to which the algorithm converges is more or less specified by the value of the initial condition $f_0$. 
Figure 4.5: Convergence to periodic orbit \((A = 1, \omega = 120\pi, \phi = 120\pi t + \delta_0)\). this time starting from a different initial point.

**Amplitude Tracking**

Figure 4.6 shows the performance of the algorithm in tracking time variations in the amplitude. A step change of 10% in the amplitude of the input signal, which is taken to be the same as before, occurs. The new value of the amplitude is tracked within a few cycles while the phase and frequency of the signal undergo transients for a few cycles.

**Frequency Tracking**

Figure 4.7 shows the performance of the algorithm in tracking the variations in the frequency of the input signal. The input signal is the same as before except that the frequency of the input signal undergoes a step change. It can be observed that the variations are effectively tracked within a few cycles. Values of the \(\mu\)-parameters are the same as before.
Figure 4.6: Response of the algorithm to a step change (dotted line) in the amplitude of the input signal.
Figure 4.7: Response of the algorithm to a step change (dotted line) in the frequency of the input signal.
Robustness and Noise Immunity

The algorithm is found to be robust with regard to its internal structure, most importantly with regard to the adjustment of its \( \mu \)-parameters. Numerical experiments show that the performance of the algorithm is almost unaffected by parameter variations as large as 50%. The algorithm is also robust with respect to its external conditions. As an example, Figure 4.8 shows the performance of the algorithm in a noisy environment. It is the same scenario as that of Figure 4.3 with the only difference that a white noise at 20 dB below the level of the sinusoidal input signal is added to the input signal. The extracted frequency is picked as an index of the performance. It is clear that the presence of about 10% noise in the input generates an error of about 0.2% in the frequency estimation at the same speed of convergence. It is notable that for each given application, one can modify the values of the \( \mu \)-parameters to accordingly balance speed and accuracy. Alternatively, one may employ post-filtering to smooth out the estimated values. It has been observed that employing a simple low-pass filter having a time constant of 0.1 s can reduce the error to about 0.05%.

Multi-component Input Signals

As explained in section 2.3, a parallel combination of core units of the proposed algorithm may be employed to decompose an input signal having more than one sinusoidal component into its constituents. In such a parallel configuration, it is important to properly limit the range of permissible frequencies for each unit so as to avoid overlapping duties among units.

Figure 4.9 shows an example of the operation of the proposed algorithm in a parallel configuration consisting of two units. The ranges of permissible frequencies of the two units are limited by virtue of the numeric limiters implemented within the integrators computing the frequency, namely the ones that take \( \frac{d\omega}{dt} \) as the input and generate \( \omega \) as the output. For this example, one of the units is set to operate within the range of 0 Hz to 50 Hz while the permissible range of the other one is set to be 50 Hz to 100 Hz. The input signal is composed of two sinusoids of equal amplitude whose frequencies are drifting over time polluted by a zero-mean white Gaussian noise at 3 dB below the signal level. The time-variations of the frequencies have been purposely set such that within the time-interval \( t = 4s \) to \( t = 6s \) the
Figure 4.8: Illustration of the effect of the presence of noise.
Figure 4.9: Performance of two units of the core algorithm connected in parallel in tracking frequencies of an input signal having two sinusoidal components. Time-varying frequencies of the two components are \( f_1 \) and \( f_2 \). The top graph shows the extracted frequency (\( f \)) by the unit operating in the range of 50 Hz to 100 Hz and the bottom graph shows the extracted frequency (\( f \)) by the unit operating in the range of 0 Hz to 50 Hz.
frequencies of both constituting sinusoids fall within one and the same range, namely the range of 0 Hz to 50 Hz. It is observed that apart from the ambiguous period of $t = 4$ s to $t = 6$ s during which the proposed configuration is in a state of confusion, each unit tracks the sinusoidal component that falls within its range. It is noteworthy that the two signals are actually exchanged between the two units after the period of confusion.

The simulations presented in this section demonstrate good performance of the proposed algorithm. The performance of the proposed algorithm is also verified in laboratory as is further illustrated in the following section.

### 4.2 Implementation on a Digital Signal Processor (DSP)

Essentially, if a realistic signal processing algorithm is properly simulated in a simulation environment, its implementation on a dedicated processor such as a DSP using a low-level programming language is an engineering effort and is devoid of scientific value. Realism of the simulation of an algorithm means that there should be no "hidden" impractical assumption which could be veiled in a simulation environment. For example, the error due to the quantization in a mixed signal system may be hidden in a simulation environment if the input signal happens to have impractically high resolution. Another example is the presence of a time variable within the equations governing an algorithm. One can define a time vector in a Matlab script code and present some results for an algorithm involving such a time variable; however, an analog circuit, for instance, knows no independent time variable and a book-keeping scheme to generate such a variable is extremely artificial and has practical implications.

In order to verify the practicality of the core algorithm in real engineering applications, the algorithm has been implemented on a Texas Instruments\textsuperscript{TM} TMS320C6711 floating point DSP platform. This section reviews the employed hardware, its programming for implementation of the targeted algorithm, and examples of the laboratory verification of the algorithm.
4.2.1 DSP Hardware

The employed core digital signal processor is a Texas Instruments TMS320C6711 floating point DSP which has a clock speed of 150 MHz and a core supply of 1.8 V. The employed hardware platform on which the DSP is mounted comprises an on-board power supply and a number of peripherals such as the coder/decoder (CODEC) which in this case is a Texas Instruments TLC320AD535. The latter is a 16-bit 2 channel analog to digital (A/D) and digital to analog (D/A) converter having a 4.96 kHz bandwidth and a maximum sampling frequency of 11.025 kHz. It can be initialized through programming using Code Composer Studio (CCS)™ which is the integrated development environment (IDE) accompanying the TMS320C6000 family of processors. The board connects to the parallel port of the host computer on which the CCS runs. CCS is the shell program through which the DSP is controlled; it allows for the management of the application by combining all of the necessities to build the executable files and monitor the results into one program.

4.2.2 Programming

The C programming language is recognized by CCS. Thus, the code for any algorithm can be written in C. Using CCS, one is able to compile the program into its assembly code and subsequently download it into the DSP. There are a number of programming conventions which have to be observed while working with CCS. The program for implementing the proposed algorithm has several lines of C code; the main part of the code implementing the algorithm is, however, very simple and is presented here as a sample:

\[
\begin{align*}
*y &= (*A0) \times \sin(*phi0) \\
*e &= *u - *y \\
*A1 &= (*A0) + 2 \times T_s \times MUE_1 \times e \times \sin(*phi0) \\
*omega1 &= (*omega0) + 2 \times T_s \times MUE_2 \times e \times (*A0) \times \cos(*phi0) \\
*phi1 &= (*phi0) + T_s \times omega0 + 2 \times T_s \times MUE_3 \times e \times (*A0) \times \cos(*phi0)
\end{align*}
\]

Comparison between this piece of code and its Matlab equivalent reveals great similarity.
AO, omega0, and phi0 represent the values of the variables at nth iteration and likewise A1, omega1, and phi1 represent the values of the variables at (n + 1)th iteration.

4.2.3 Laboratory Verification

Laboratory verification of the performance of the proposed algorithm implemented on the DSP platform is presented here through the display of input and output signals recorded by the graphical interface of CCS [33]. Experiments, the results of which are presented here, have been conducted with settings of the parameters as \( \mu_1 = 100 \), \( \mu_2 = 1000 \), \( \mu_3 = 0.08 \). The initial pre-set frequency of the algorithm is chosen to be 400 Hz or 2513.27 rad/s.

Figures 4.10-4.12 show snapshots of the real-time performance of the algorithm. The input signal, an intentionally distorted sinusoid with a frequency of 400 Hz, and the extracted sinusoidal component are shown in Figure 4.10. Figure 4.11 shows the estimated amplitude and total phase and Figure 4.12 shows the angular frequency deviation as estimated by the algorithm. Since the input signal has the same frequency as the initial condition of the frequency integrator (both 400 Hz), the \( \Delta \omega \) estimated by the algorithm approaches a steady state error, which ideally has to be zero. The observed steady state error is, of course, fully controllable by the adjustment of parameters. In all the graphs presented in this section, the horizontal axis is the time step index. Linear interpolation is used to produce continuous graphs.

In another experiment, the input signal is chosen to be a non-symmetric square wave having a frequency offset of about \( 2\pi \Delta f = -22.8 \) rad/s from the nominal frequency of \( 2\pi f_0 = 2\pi 400 = 2513.27 \) rad/s. Figure 4.13 shows the performance of the algorithm. It is observed that the algorithm retrieves the fundamental component of the input signal and the value of its frequency.

Figure 4.14 illustrates the frequency tracking capability of the algorithm for large values of frequency deviation. The input signal is again a non-symmetric square wave having a frequency offset of about \( 2\pi \Delta f = +179 \) rad/s from the nominal frequency of \( 2\pi f_0 = 2\pi 400 = 2513.27 \) rad/s.
Figure 4.10: Illustration of the laboratory verification of the performance of the algorithm. The input signal is an intentionally distorted sinusoid. The top graph shows the input signal and the bottom graph shows the extracted sinusoid. In both graphs the horizontal axis is the sample number.
Figure 4.11: Illustration of the laboratory verification of the performance of the algorithm. The input signal is an intentionally distorted sinusoid. The top graph shows the estimated amplitude and the bottom graph shows the estimated total phase. In both graphs the horizontal axis is the sample number.
Figure 4.12: Illustration of the laboratory verification of the performance of the algorithm. The input signal is a distorted sinusoid. This graph shows the angular frequency deviation as estimated by the algorithm. The horizontal axis is the sample number.

2513.27 rad/s. Again, it is observed that the algorithm retrieves the fundamental component of the input signal and the value of its frequency.

Considering that the computational need of the proposed algorithm is very low, one expects that its execution would not put a high computational load on the DSP. Figure 4.15 shows the computational load on the DSP. The executed program consists of a code for generating the input sinusoidal signal, communicating the data between the DSP and the host computer, as well as the code for the algorithm. The totality of the tasks involved take about 10% of the computational power of the DSP. The most time-consuming task is the data communication between the DSP and its host computer. Unfortunately, there is no direct way of measuring the computational load of the stand-alone algorithm. As well, in the implementation of the algorithm, no effort has been made to optimize the program. As a rough guess, the code for the proposed algorithm itself is expected to take less than about one percent of the CPU time for processing a signal of 400 Hz sampled at 11.025 kHz. For a typical application involving power line signals sampled at 1 kHz for example, the algorithm is expected to take about 0.1% of the computational power of a DSP running at 150 MHz.
Figure 4.13: Illustration of the laboratory verification of the performance of the algorithm for a square wave input signal. The top left graph shows the input signal, the top right graph shows its extracted fundamental component, the bottom left graph shows its estimated amplitude, and the bottom right graph shows the frequency deviation of the extracted sinusoid from its pre-set value in rad/s. In all graphs the horizontal axis is the sample number.

4.3 Comparison of the Proposed Algorithm with Existing Methods

In this section, the proposed method of extraction of nonstationary sinusoids is compared with two of the reportedly most promising methods that are of a similar kind. Extended Kalman filtering is chosen as representing an example of the conventional methods with reported success in the literature. The method of Regalia [9, 10], which has recently attracted the attention of researchers, is another example of the successful methods reported so far. In each case, the existing method is implemented and its performance is compared with that
Figure 4.14: Illustration of the laboratory verification of the performance of the algorithm for a square wave input signal whose frequency is largely different from its nominal value. The top left graph shows the input signal, the top right graph shows its extracted fundamental component, the bottom left graph shows its estimated amplitude, and the bottom right graph shows the frequency deviation of the extracted sinusoid from its pre-set value in rad/s. In all graphs the horizontal axis is the sample number.
Figure 4.15: Computational load on the DSP. The top two graphs show the input and output signals of the algorithm and the bottom figure shows a graph of the computational load on the DSP as a function of time. In the top two graphs the horizontal axis is the sample number.
of the proposed method.

### 4.3.1 Extended Kalman Filter

Kalman filtering is a model-based method of estimation of parameters of an assumed model for the process under study. If the assumed model happens to be nonlinear, a linearization process has to be done in which case the estimation procedure is called extended Kalman filtering.

The stochastic model for the extraction of a sinusoidal signal and estimation of its parameters may be written as

\[
\begin{pmatrix}
A[n+1] \\
\phi[n+1] \\
\omega[n+1]
\end{pmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & T_s \\
0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
A[n] \\
\phi[n] \\
\omega[n]
\end{pmatrix}
+ 
\begin{pmatrix}
v[n]
\end{pmatrix},
\]

(4.6)

\[
y[n] = A[n] \sin(\phi[n]) + \eta[n]
\]

(4.7)

where \( A[k] \), \( \phi[k] \) and \( \omega[k] \) represent estimated values of the amplitude, phase and frequency of the sinusoidal signal of interest at the time step indices \( n + 1 \) and \( n \), respectively. The extracted sinusoid at the time step index \( n \) is denoted by \( y[n] \). Vector functions \( v \) and \( \eta \) are random processes representing the noise within the state equation and in the output, respectively. \( T_s \) is the sampling period.

Random processes \( v \) and \( \eta \), usually zero-mean white Gaussian processes characterized by their covariance matrices \( Q_v \) and \( Q_\eta \), model the statistical properties of the input signal. A major task in using Kalman filtering is the proper guess of these two processes which relate to the extent and type of the pollution present in the input signal.

Implementation of the extended Kalman filter with the given model follows the standard iterative procedure [34]. For the simulations presented in this section, the following covariance
matrices, adopted from [35], are used:

\[
Q_v = \begin{bmatrix}
0.0400 & 0 & 0 \\
0 & 0.0006 & 0 \\
0 & 0 & 0.0314
\end{bmatrix},
\]

\[
Q_\eta = \begin{bmatrix}
4000 & 0 \\
0 & 4000
\end{bmatrix}.
\]

The initial conditions are chosen as \(A[0] = 1.1 \times 400\), \(\phi[0] = 0\) and \(\omega[0] = 1.1 \times 2\pi 50\). The values in [35] have been determined for an optimal performance for a voltage signal of 400 V oscillating at 50 Hz.

Figures 4.16 and 4.17 show the performance of the extended Kalman filter in the extraction of a sinusoidal signal and estimation of its parameters when the input signal is noise-free. Although satisfactory results are obtained using the covariance matrices and initial conditions given in [35], the method is found extremely sensitive with respect to both covariance matrices and initial conditions.

Figure 4.18 shows the performance of the extended Kalman filter in the presence of noise. Incurred error in the estimation of frequency is picked as an index of the performance. It is observed that a noise of about 10% of the level of the input signal causes an estimation error of about 3%. Figure 4.19 shows the performance of the proposed method for the same input signal. The incurred error is less than 0.2%.

With reference to the presented numerical experiments, the advantages of the proposed method over Kalman filtering can be summarized as follows:

- The proposed method offers a much higher degree of noise immunity.

- Unlike Kalman filter, the proposed method does not assume any model for the noise; it simply targets the sinusoidal component of interest within an input signal of any model.
Figure 4.16: Illustration of the performance of the extended Kalman filter in the extraction of a sinusoidal signal from a clean signal. The top graph shows the extracted sinusoid and the bottom graph shows the incurred error in the extraction process.
Figure 4.17: Incurred error in the estimation process using extended Kalman filtering. The numerical experiment is the same as that of Figure 4.16.
Figure 4.18: Illustration of the performance of the extended Kalman filter in a noisy environment. The incurred error in the estimation of the frequency is chosen as the index of the performance. The bottom graph is a magnified portion of the top graph.
Unlike Kalman filter which is extremely sensitive with respect to model parameters, the proposed method is very robust in terms of variations in its internal as well as external conditions.

4.3.2 The Method of Regalia

The dynamical system originally proposed by Regalia [9] is essentially a second order notch filter to which an adaptation mechanism is augmented. The frequency of the extracted sinusoid is directly estimated as one of the state variables of the set of governing equations of the algorithm. Hsu et al. later modified the set of governing differential equations of Regalia and proved its mathematical properties, namely the existence of a periodic orbit and its stability [10]. This algorithm has recently gained attention for its good noise immunity [26]. In this section, the modified algorithm of Hsu et al. is chosen as an example of recently developed algorithms of its kind, and its performance is compared with that of the proposed method in terms of noise immunity.
Figure 4.20: Illustration of the performance of the method of Regalia in extracting a noise-free sinusoid.

Figure 4.21: Illustration of the performance of the method of Regalia in estimating the frequency of a noise-free sinusoid.
Given an input signal $u(t)$, the following set of nonlinear differential equations describes the dynamics of the algorithm proposed by Regalia:

\[
\frac{d^2 y(t)}{dt^2} + 2\zeta\omega(t)\frac{dy(t)}{dt} + \omega^2(t)y(t) = u(t). 
\]

(4.8)

\[
\frac{d\omega(t)}{dt} = -\gamma(u(t) - 2\zeta\omega(t)\frac{dy(t)}{dt})y(t) 
\]

(4.9)

in which $y(t)$ provides an estimate of the output signal and $\omega(t)$ provides an estimate of its frequency. Constant numbers $\zeta$ and $\gamma$ are the algorithm regulating parameters. The modified algorithm proposed by Hsu et al. is governed by the following set of differential equations:

\[
\frac{d^2 y(t)}{dt^2} + 2\zeta\omega(t)\frac{dy(t)}{dt} + \omega^2(t)y(t) = \omega^2(t)u(t). 
\]

(4.10)

\[
\frac{d\omega(t)}{dt} = -\gamma(\omega^2(t)u(t) - 2\zeta\omega(t)\frac{dy(t)}{dt})y(t). 
\]

(4.11)

An optimal set of parameters of $\zeta = 0.2$ and $\gamma = 10^8$ are used in this section to present the
performance of the modified algorithm as described by the above set of differential equations. Figure 4.20 shows the performance of the method of Regalia in extracting a noise-free sinusoid. An estimate of the frequency is directly available and is shown in Figure 4.21. In Figure 4.22, the quality of the input signal is degraded by the addition of a noise of about 10% of the level of the sinusoid. An incurred error of about 1.5% is observed. Comparison of Figure 4.22 with Figure 4.19 shows the much better performance of the proposed method in terms of noise immunity. Since parameter $\gamma$ has an exceedingly large value, the algorithm of Regalia is prone to numerical errors. In this respect, the proposed algorithm is more robust.

With reference to the discussions presented in this section, it is clear that the method of Regalia is superior to conventional methods such as Kalman filtering. However, the proposed method of this thesis is more advantageous in the sense that it offers a higher degree of noise immunity and directly provides an estimate of the amplitude and phase of the extracted sinusoid. Moreover, the proposed algorithm is more robust with respect to the internal parameter settings.

High noise immunity, little computational resources demanded by the proposed algorithm due to the simplicity of its structure, and its robustness are the main features of the proposed algorithm. These features play important roles in real-time applications such as those presented in subsequent chapters.
Part II:

Applications

Part II presents examples of the application of the proposed algorithm to problems from diverse areas of engineering, ranging from biomedical to communications engineering. The main application of the proposed algorithm presented in this thesis is an adaptive notch filter useful for the elimination of power line interference. Discussions on the design of such a filter together with the results of its application to two important cases of refinement of biopotential and telephone line signals are presented in chapter 5. Chapter 6 presents a typical signal detection problem in which estimation of a faint signal polluted by various types of noise is desired. Another example of the application of the proposed algorithm in the extraction of sinusoids in noise is presented in chapter 7. Preliminary application of the proposed algorithm in the estimation of instantaneous frequency of sinusoids is presented in chapter 8.
Chapter 5

Universal Narrow-Band EMI Filter

Power line interference coupled to signal carrying cables is particularly troublesome in medical equipment such as electrocardiograms (ECGs). Cables carrying ECG signals from the examination room to the monitoring equipment are susceptible to the electromagnetic interference (EMI) of power frequency (50 Hz or 60 Hz) by ubiquitous supply lines and plugs, so much so that sometimes the ECG signal is totally masked by this type of noise. Filtering such an EMI signal is a challenging problem given that the frequency of the nonstationary power line signal lies within the frequency range of the ECG signal. There are some other technical difficulties involved, the most important of which is the low sampling frequency at which the ECG signals are taken and low computational resources available.

The history of the attempts to mitigate the power line EMI on the ECG signals goes as far back as the ECG equipment itself. This problem was one of the first to attract the attention of the developers of the adaptive filtering theory [19]. Although classical adaptive filtering provides a partial solution to the problem, the problem is still considered open and research is continuing to find the ultimate solution [23, 36].

Pollution of ECG signals by quasi-periodic interferences presents a general problem with the medical equipment. This same problem may occur in various other areas. For example, telephone lines carrying voice or data are subject to induced EMI from power lines both in the form of differential mode and common mode interference. While elimination of the
common mode EMI is trivial, in practice some residual differential mode interference always exists. Presence of such a differential mode EMI, frequency content of which lies within the frequency spectrum of the signal of interest, degrades the quality of the communication channel. The affected signal may be voice, or it may be data in the case of telephone line-based data communications. The fact that characteristics of the interfering signal including its frequency may vary over time renders the noise suppression task difficult. Various methods of reduction of power line interference have been proposed over the years each presenting strengths and weaknesses. No unique solution to this seemingly simple problem has been proposed so far [37]. Extraction or elimination of sinusoids of nonstationary frequency buried within a given signal of arbitrary frequency composition is the subject of research in some other areas of electrical engineering. Few examples of applications of means of extraction of sinusoids of nonstationary characteristics can be found in [20, 38].

The proposed signal processing algorithm, introduced in chapter 2, is found promising in the construction of a universal EMI filter suitable for various applications in which the interference is a time-varying periodic (i.e. quasi-periodic) signal. It offers a robust structure and is shown to have a high degree of immunity with respect to the external noise. This chapter presents the structure and the performance of such an adaptive EMI filter for the elimination of narrow-band interferences. Two examples of its application to ECG signals and signals carried by telephone lines are considered.

5.1 Structure of the Proposed EMI Filter

One single unit of the core algorithm can be employed to extract the quasi-periodic interference mixed with the signal [39]. This unit can effectively follow time variations in the amplitude, phase and frequency of the interfering signal. Once it is extracted, it can be subtracted from the input signal to yield a "clean" signal.

As indicated in Figure 4.8, the error incurred in the extraction process increases with the amount of noise content of the input signal. In other words, the speed/accuracy trade-off
associated with the extraction process is enhanced when the input signal has less noise content. When the proposed algorithm is employed to construct an EMI filter for removing sinusoidal interferences, the noise is the totality of the components of the input signal other than the sinusoidal interference. In order to improve the performance of the unit, the use of a band pass filter (BPF) to filter out non-interference signal components is proposed in Figure 5.1. The role of the band pass filter is to improve the signal to noise ratio (signal here meaning the interference and noise meaning all other components) at the input of the core unit. Whatever is not removed by this BPF will be effectively removed by the core unit so as to produce a single pure quasi-sinusoid which is the interference. This interference in then subtracted from the input signal to provide a refined signal.

This band pass filter does not need to be sophisticated and can be as simple as a second order filter. The BPF characterized by its transfer function $H(f)$ causes an attenuation $|H(f)|$ and a phase delay $\angle H(f)$ which are functions of frequency. Since the core unit generates the value of the instantaneous frequency in real-time, the attenuation and phase delay are known and can be restored as depicted in Figure 5.1. As a concrete example, the band-pass filter employed as the pre-filtering tool in Figures 5.1 and 5.2 is chosen to be a second order filter characterized by the following transfer function:

$$H(s) = \frac{100s}{s^2 + 100s + \omega_n^2}.$$ 

Gain and phase characteristics of this filter are shown in Figure 5.3 in which $\omega_n$ is taken to be 100 Hz for ease of visualization.

Where the interfering signal is severely distorted, harmonics may also be present. In such cases, the desired signal is not polluted by a sinusoid, but by a number of sine waves. A more general configuration as shown in Figure 5.2 may be then employed to eliminate the fundamental and the harmonics of the EMI.
5.2 Application of the Proposed Filter to ECG Signal Refinement

An ECG signal is basically an index of the functionality of the heart. For example, a physician can detect arrhythmia by studying abnormalities in the ECG signal [40]. Since very fine features present in an ECG signal may convey important information, it is important to have the signal as clean as possible. Figure 5.4 shows a clean ECG signal recorded at Beth Israel Hospital (BIH) in Boston and made available by Massachusetts Institute of Technology (MIT-BIH) [41]. The recording was done using a battery operated ECG equipment to minimize the power line EMI although some such EMI still exists which is mostly coupled at the time of recording the signals on the tape. The frequency spectrum of this signal spans from near DC frequencies to about 100 Hz. The sampling frequency in most ECG devices is 240 Hz or 360 Hz. In this case, the equipment was operated at a sampling rate of 360 Hz. Therefore, the spectrum can theoretically include frequencies from zero to 180 Hz.

ECG signals can be easily polluted by a power line noise of relatively large amplitude. Were the frequency of the power line interference accurately at 50 Hz or 60 Hz, a sharp notch filter would be able to separate and eliminate the noise [42]. The major difficulty is that the frequency can vary about fractions of a Hertz, or even a few Hertz in some countries. The sharper the notch filter is designed, the more inoperative, or rather destructive, it be-
Figure 5.2: A configuration to eliminate a severely distorted quasi-periodic EMI.
comes if any change in the frequency of the power line occurs. Of course, turning the notch filter into a band stop filter by widening its rejection band, and thereby accommodating frequency variations, does not offer any better solution since it would undesirably distort the ECG signal itself. The frequency of the power grid is usually taken as being constant when conventional EMI filters for ECGs are designed. In such arrangements, the system is very fragile with respect to power frequency variations and can become completely inoperative. Such adaptive or non-adaptive filters, those discussed in [21] for instance, greatly suffer from this shortcoming.

One of the possible alternatives to take frequency variations into account is the use of an external reference power line signal [22]. This technique, available by the use of adaptive filters only, is reported to present serious practical difficulties and is nearly impossible to implement [21]. For this reason, other methods, usually very complex and inflexible, are constantly being proposed [23, 43].

An ideal EMI filter for ECG signal refinement should act as a sharp notch filter to eliminate only the undesirable power line interference while automatically adapting itself to variations in the frequency and the level of the noise. Of course, this adaptation must be done very
Figure 5.4: Recorded ECG signal and its frequency spectrum [41].
quickly so as to keep the signal clean all the time. It is supposed to be able to work in low information background, namely that dictated by the low sampling frequency, and must be robust with respect to variations in its internal as well as external conditions. An example of internal condition is its parameter settings. External conditions can range from the temperature of the environment in which the equipment is supposed to function to the superimposed noise/distortion on the interfering power signal.

5.2.1 Parameter Setting

One of the issues to be considered in the design of the proposed EMI filter is the setting of its parameters $\mu_1$, $\mu_2$, and $\mu_3$. The values of parameters $\mu_1$, $\mu_2$, and $\mu_3$ determine the convergence speed versus error compromise. Specifically, parameter $\mu_1$ controls the speed of the transient response of the filter with respect to variations in the amplitude of the interfering signal. Speed is traded off for the steady state error. As long as the frequency of the input signal is close to its nominal value (e.g. 60 Hz), this trade-off does not introduce a significant constraint. As the frequency of the input signal deviates from its nominal value, the filter introduces more significant trade-off between speed and steady state error: for example, within a range of $\pm 2$ Hz variations away from the nominal frequency, the filter can be adjusted to catch up with a $100\%$ step change in the amplitude of the interference within 3 cycles (of the interference) with less than $2\%$ steady state error. To illustrate the effect of the value of parameter $\mu_1$ on the overall performance of the proposed EMI filter, transient and steady state errors in eliminating an interference of constant level of 0.1 mV are shown in Figure 5.5. Values of parameters $\mu_2$ and $\mu_3$ are kept constant in this example. It is observed that a larger value of $\mu_1$ is beneficial in terms of speed but it introduces a higher steady state error.

Parameters $\mu_2$ and $\mu_3$ mutually control the speed of the transient response of the EMI filter with respect to variations in the frequency of the interfering signal. The larger the value of parameter $\mu_2$ is chosen, the faster the convergence is achieved in tracking phase (or frequency) variations over time. The cost is higher steady state error. This is illustrated in Figure 5.6 where the power line interference has a constant level of 0.1 mV and its frequency is 10 Hz away from the frequency at which the algorithm is initially set. Adjustment of the
third tuning parameter, \( \mu_3 \), is interdependent on the adjustment of \( \mu_2 \). As a simple rule of thumb, one may pick a value for parameter \( \mu_2 \) based on the potential frequency drift in the interfering signal (hence, the desired frequency tracking speed) first, and choose the value of \( \mu_3 \) such that the product \( \mu_2 \mu_3 \) becomes of the same order of magnitude as \( \mu_1 \) for a balanced speed in terms of tracking the amplitude and total phase (or frequency).

Depending on the nature of the interfering signal and its potential range of variations, it is important to establish a desirable balance of speed and error. It has been observed that the proposed EMI filter is very insensitive with respect to moderate variations of parameters \( \mu \). This feature renders the parameter adjustment process of the filter fairly simple. For the simulations in this section, a moderate choice of \( \mu_1 = 100, \mu_2 = 10000 \) and \( \mu_3 = 0.01 \) results
in an EMI reduction of a factor of about 20 while keeping the transient time short enough. This transient time is observed to be within a few cycles of the interfering power line noise. All initial conditions of integrators are set to zero except for that of frequency which is taken to be 60 Hz.

5.2.2 Performance

This section presents a number of simulations to demonstrate the performance of the EMI filter in the elimination of the power line noise on ECG signals. Figures 5.7 and 5.8 show the performance of the filter in eliminating a power line interference of 1 mV constant level whose frequency is fixed at 60 Hz. This is an elementary simulated experiment since a simple notch filter would easily eliminate such a fixed frequency noise. In the simulation, only a
Figure 5.7: A power line interference of 1 mV level at 60 Hz is added to the ECG signal to provide a polluted input signal for testing the performance of the proposed filter.
Figure 5.8: Performance of the proposed filter in eliminating a power line interference of 1 mV level at 60 Hz.
60 Hz component is added to the clean ECG signal obtained from MIT-BIH whose presence is clear in the frequency spectrum of the input signal (Figure 5.8).

To demonstrate the ability of the filter to adaptively track the variations in the noise level, the level of the EMI in the simulation is made to change with time as shown in Figure 5.9. The interference has a time-varying level oscillating between 0 to 1 mV. The error (bottom graph of Figure 5.9) oscillates between 0 and 0.05 mV. Again, error is confined to within about $\frac{1}{20}$ of the maximum noise in the input.

Figure 5.10 shows the performance of the filter in tracking the variations in frequency of the power line noise. The filter is adjusted - by virtue of its initial conditions - to extract a 60 Hz power line noise. However, the EMI in the input is oscillating at 55 Hz. Effective tracking of the unknown input frequency can be observed.

Finally, Figures 5.11 and 5.12 show the performance of the filter when all characteristics of the EMI change with time. In Figure 5.11, the filter is adjusted to extract a power line noise of 60 Hz. However, the incoming EMI has a frequency of 65 Hz. This scenario is equal to a step change in the frequency of the interfering signal. The level of the EMI also changes with time. In Figure 5.12, the interference of the time-varying amplitude is made to undergo substantial frequency drift. It is observed that the extracted frequency closely follows the frequency variations of the interference. Again, effective elimination of the superimposed EMI is observed.
Figure 5.9: Performance of the filter in eliminating a power line interference of time-varying level (oscillating between 0 and 1 mV) at 60 Hz.
Figure 5.10: Performance of the filter in eliminating an interference of 1 mV level at 55 Hz.
Figure 5.11: Performance of the filter in eliminating a power line interference of time-varying level when the frequency undergoes a step change from 60 Hz to 65 Hz at $t = 0$ s.
Figure 5.12: Performance of the filter in eliminating a power line interference of time-varying level while the frequency is varying with time as well.
5.3 Application of the Proposed Filter to the Refinement of Telephone Line Signals

A number of different techniques are reported in the literature to tackle the problem of interference elimination on telephone cables. One possible way is to estimate characteristics of the interfering sinusoidal signal buried within a given input signal by means of known signal processing techniques such as least mean squares (LMS) method, synthesize the interference and subsequently subtract it from the input signal [44]. This method usually needs a PLL to synchronize the estimated sinusoidal signal with the interference before subtracting it from the input signal. Combined estimation and phase-lock scheme usually results in an excessively complex structure which, while demanding large computational resources, suffers from stability and robustness problems. This section presents a method of extraction and elimination of quasi-periodic signals buried within a given telephone line signal which contains components of arbitrary frequency content [45, 46].

5.3.1 Performance

Computer simulations are performed to illustrate various situations in which the present interference elimination method may be employed. The proposed method of EMI elimination does not assume any frequency composition for the information signal carried by the telephone line. The information signal may be voice in the case of audio transmission, or data in the case of modem-based communications. Both cases are treated the same way by the proposed EMI eliminator; it targets the power frequency noise and eliminates it while leaving the information signal untouched. The test signal is arbitrarily taken to be a recorded bird chirp sampled at $F_s = 8192$ Hz available as a Matlab library component. Although this test signal refers exclusively to a voice-like signal and not a data type signal, its application is instructive in the sense that it can serve as an available objective test signal. Figure 5.13 shows the performance of the EMI filter in the suppression of an interference of a fixed frequency of 60 Hz from an arbitrary input signal. The level of the interference added to the original signal is taken to be equal to that of the original signal so that the signal to noise ratio (SNR) is 0 dB in the presented simulations. Figure 5.14 shows the incurred error in
Figure 5.13: Time-domain representation of the performance of the proposed EMI eliminator in the suppression of a fixed frequency interference from an arbitrary input signal.
removing the EMI. It is basically the difference between the refined signal and the original signal not yet polluted by the interference. Frequency spectra of the original, polluted and retrieved signals are shown in Figure 5.15. Figure 5.16 shows the magnified portion of the frequency spectrum of the original and retrieved signals about the frequency of the interference. It is observed that the notch filter effectively removes the interference and retains the original signal almost untouched.

To show the adaptive nature of the present interference eliminator with respect to variations in the frequency of the interfering signal, the performance of the present method when a step change in the frequency of interference occurs is shown in Figure 5.17. As noted before, the rate of convergence in tracking time-variations such as that shown in Figure 5.17 is totally controllable by means of the adjustment of \( \mu \)-parameters. The tracking capability of the present method is advantageous in devising a universal power supply noise eliminator independent of the nominal frequency of the grid. Regardless of initial frequency setting (whether 50 Hz or 60 Hz), the filter finds the instantaneous frequency of the power line interference. Figure 5.18 shows an example in which the EMI filter expects a 60 Hz power
Figure 5.15: Frequency-domain representation of the performance of the proposed EMI eliminator in the suppression of a fixed frequency interference from an arbitrary input signal.
Figure 5.16: Magnified spectra of the original and retrieved signals around the frequency of the interfering sinusoid (60 Hz).
Figure 5.17: Response of the proposed method to a frequency step change in the interfering sinusoid. Top figure shows the estimated frequency and the bottom figure shows the incurred error in the EMI suppression.
Figure 5.18: Response of the proposed method to a large frequency offset in the interfering sinusoid. The top figure shows the estimated frequency and the bottom figure shows the retrieved signal.
Figure 5.19: Performance of the proposed method in eliminating a sinusoidal interference of time-varying amplitude. The top figure is the interference, the middle figure is the estimated value of the amplitude of the interference and the bottom figure is the incurred error in the EMI suppression.

To demonstrate the adaptive nature of the proposed method in tracking variations in the level of the interference, the performance of the present method in eliminating an interference of time-varying amplitude is shown in Figure 5.19.

As noted before, the power supply interference may be distorted by the presence of harmonics. Figure 5.20 shows an example of a highly distorted interference in which harmonics of the third and seventh order are present. In such cases, a multiplicity of EMI filters,
Figure 5.20: Interference is taken to be highly distorted by the presence of harmonics. This figure shows one cycle of the interfering signal.

Figure 5.21: Frequency domain representation of the performance of the present method in the elimination of an interference of highly distorted shape. An EMI reduction of about 40 dB is observed.
connected in parallel combination as suggested by Figure 5.2, may be used for the EMI mitigation. Figure 5.21 shows the frequency domain representation of the performance of an EMI filter consisting of three units connected in parallel. The fundamental frequency of the interference is made to slightly vary over time. An EMI reduction of about 40 dB is observed. Again, the level of the desired EMI mitigation is controllable by the adjustment of parameters and at the expense of speed. Considering that the power supply noise is usually a slowly time-varying signal, one can sacrifice speed for better interference elimination.

5.4 Main Features of the Proposed EMI Filter in Contrast with Existing Methods

Available methods of elimination of periodic interferences can be broadly categorized as 1) those involving the use of reference interference signals for the purpose of synchronization of the estimated interference with the actual interference before its subtraction from the input signal, and 2) those equipped with some internal process of synthesis.

For instance, the adaptive filters of [19, 22, 44] fall within the first category whereas the recently introduced methods of [23, 36] fall within the second category. As discussed earlier in this chapter, EMI filters involving the use of reference signals are usually unattractive in practice and for this reason have not gained popularity in industry. Alternative methods of elimination of periodic interferences which do not require reference signals suffer from a number of shortcomings such as their excessively complex structures and insufficient frequency detection capabilities. For example, the method presented in [36], although it is capable of determining whether the interference is of 50 or 60 Hz frequency, is not fully frequency adaptive and is rendered inefficient if any drift from either of the two nominal frequencies occurs in the power system.

The presentation of the performance of the proposed method of EMI elimination under various conditions demonstrates its superiority over existing methods. With reference to the
discussions presented throughout this chapter. The main features of the proposed universal narrow-band EMI filter can be summarized as

- **Effectiveness in tracking large variations in the parameters of the interfering power signal such as its level and frequency.** A given setting of parameters renders the algorithm operational for a wide range of variations in the characteristics of the interference; the algorithm is insensitive with respect to both external variations (such as the noise signature) and internal parameter settings.

- **Obviating the need for the interference reference signal.** Unlike conventional adaptive methods, the proposed EMI filter extracts the interference without the need for a power reference signal.

- **Obviating the need for synchronization.** Unlike FFT-based and some adaptive algorithms which are based on the estimation of parameters of the interference, and hence have the need for a synchronization mechanism such as a PLL, the proposed EMI filter directly extracts the interference and eliminates it, and thereby renders the need for complex synchronization schemes redundant.

- **Robustness and noise immunity.** These features render the proposed method particularly attractive for application in electrically polluted environments.

- **Simplicity of the structure.** In spite of the complexity of the mathematics that ensures the stability of the core algorithm employed in the proposed EMI filter, the structure of the filter, consisting of only a few arithmetic operations, is extremely simple.
Chapter 6

DPOAE Estimation

Distortion product otoacoustic emissions (DPOAEs) are very low level stimulated acoustic responses to two pure tones presented to the ear canal. DPOAE measurement provides an objective non-invasive measure of peripheral auditory function and is used for hearing assessment [47]. DPOAE screening is becoming a standard clinical practice to predict potential sensorineural hearing loss especially in newborns.

DPOAEs have been recognized for a number years [48, 49]. However, DPOAE measurement is considered an active area of research because of the challenging nature of the signal processing task. To address the ever-increasing demand for high performance DPOAE measurement methods, a number of signal processing algorithms have been proposed in recent years [47], [50]-[52]. With the availability of powerful computational tools such as digital signal processors, a variety of commercial medical equipment dedicated to the DPOAE signal measurement is becoming available [53]-[58].

In this type of otoacoustic test, two pure tones with frequencies $f_1$ and $f_2$ are presented to the cochlea. For best results, $f_2$ is usually chosen as $1.2f_1$. Since the ear is a nonlinear structure, a number of very low level distortion products are generated due to the intermodulation process within the cochlea. Among various distortion products, the component with frequency $f_3 = 2f_1 - f_2$ is the strongest. The level of such a DPOAE signal is taken as an index of the functionality of the ear. Estimation of such a weak signal buried under two
strong artifacts in a potentially noisy background is a challenging signal processing problem.

Conventionally, the fast Fourier transform (FFT) is used as the main signal processing tool to estimate the level of the DPOAE signals. Application of FFT to this problem has a number of shortcomings among which long measurement time is the most pronounced. Long measurement time is usually required for the acquisition of a sufficiently large amount of data which, when averaged, would reduce the overall background noise effect. Unreliability of the measurements is another problem of FFT-based methods and is a direct effect of the sensitivity of such methods to background noise. In addition to the need to increase the measurement time, the tests are usually required to be conducted in low noise environments such as a sound-proof room or other types of sound-proof enclosure.

In an attempt to devise high performance DPOAE estimation techniques, linear adaptive signal processing techniques [47, 58] and maximum-likelihood estimators [59] have been employed. Such techniques generally offer better performance in terms of measurement time which may be interpreted as higher noise immunity of adaptive techniques compared to FFT. However, the need for sound-proof examination rooms is not obviated by such techniques and high performance computational resources are needed.

This chapter presents a method of DPOAE signal measurement based on the core signal processing algorithm of chapter 2. The proposed DPOAE estimation method consists of three units of the core algorithm. The two artifacts are first extracted by two units and are subtracted from the input signal. This generates a signal with a higher relative portion of the DPOAE signal. Such a signal is then input to another core unit which estimates the level of the DPOAE signal. Superior performance of the proposed technique in terms of noise immunity and fast measurements is demonstrated with the aid of computer simulations.

In order to provide a brief background on the subject, a general review of the structure of a generic DPOAE measurement device is presented in the next section. It will be seen that the heart of such an apparatus is the signal processing subsystem which, in the final analysis,
determines the performance of the overall system. Section 6.2 presents the proposed scheme for such a signal processing module. Performance of the proposed method is demonstrated by simulation results in section 6.3.

6.1 Structure of a Generic DPOAE Measurement Device

In this section a brief overview of the structure of a typical DPOAE detection system is provided. Figure 6.1 shows the generic block diagram of a DPOAE measurement device. It consists of three main modules: the data acquisition/transducers module, the signal processing module and the display.

The data acquisition unit is the medium between the processing unit and the probe which transmits and receives acoustic signals in the audio range. Components of the compound data acquisition/transducers module are illustrated in more detail in Figure 6.2. One of the main functions of this module is to convert digital signals produced by the signal processing module to analog signals which are then conditioned and converted to audio signals. Conditioning of the signals in this case may or may not include filtering. Conversely, the
Figure 6.2: Details of the data acquisition and probe units.

Figure 6.3: Main functions of the embedded software.
Figure 6.4: Block diagram of the proposed signal processing scheme.

audio signals recorded by the probe are conditioned and converted to digital signals to be processed by the signal processing module.

The heart of the system is the signal processing module which produces the digital form of the artifacts and extracts and measures the DPOAE signal. A DSP, or if the computational/architectural demand is low even a microcontroller, can be employed as the hardware platform of this unit. Signal processing is embedded as the software in such a hardware platform. Alternatively, and provided that the complexity of the signal processing algorithms remains low, the signal processing unit may be implemented solely in hardware using programmable logic array (PLA) or field programmable gate array (FPGA) technology. In an ideal case, namely when the signal processing algorithm is not excessively complex, the hardware does not require a PC for its operation; however, interfacing to a PC is usually provisioned for data management.

The display unit is the interface between the device and the operator. It can be a simple LED/LCD and/or a small printer.
6.2 Proposed Technique

Figure 6.3 shows the main functions of the software embedded in the signal processing module. The software is essentially responsible for the generation of the artifact signals and the extraction of the DPOAE signal as well as the manipulation and management of input/output data. As already mentioned, the significance of the present work is in the introduction of a signal processing technique for the extraction and measurement of the DPOAE signals. The proposed signal processing scheme employs three core units to construct a high performance DPOAE extraction module.

The input signal is often assumed to consist of two pure sinusoids with frequencies $f_1$ and $f_2$ at a very high level (usually between 60 - 70 dB) and a very low level DPOAE signal of frequency $f_3 = 2f_1 - f_2$ at about 0 dB. It is polluted by a noise, usually considered to be at about -10 dB. The noise in fact represents the totality of all undesirable signals that may be present in the environment in which the examination is being conducted as well as unavoidable white Gaussian noise. Because of the excessive degree of pollution (artifacts and noise), one single core unit set to extract the DPOAE signal out of the input exhibits poor performance. Different arrangements were studied to construct a high performance architecture. The most successful configuration is shown in Figure 6.4. Three core units are employed. The first two core units are set to extract the artifacts. They effectively do so with very small errors. The extracted artifacts are then subtracted from the input to produce a signal, of which the DPOAE signal has a higher relative portion. The third core unit is then set to extract the DPOAE signal.

Figure 6.5 shows the performance of the core algorithm in the extraction of a specified sinusoidal component of its input signal. As noted before, the specification of the desired sinusoidal component is achieved by means of appropriate setting of parameter $\omega_o = 2\pi f_0$ -the initial condition of the frequency integrator- which determines the frequency of the component of interest. The input signal is a clean sinusoid of frequency $f = 3000$ Hz with a random constant phase. All the initial conditions are taken as zero. As would be expected,
Figure 6.5: Performance of a single core unit in the extraction of a sinusoidal component and its amplitude. The dashed line waveform is the input signal $u(t)$ and the output is denoted by $y(t)$. The parameters are set as $\mu_1 = 10000$, $\mu_2 = 60000$ and $\mu_3 = 0.1$. 
the convergence is achieved within about 1 ms.

When the input signal consists of a single pure sinusoid which is to be extracted, the values of the parameters \( \mu_1 \) and \( \mu_2 \) can be chosen very large so as to increase the speed of convergence without any trade-off with accuracy. However, as the degree of the pollution in the input signal -which may be quantitatively represented by the total harmonic distortion (THD) or the signal to noise ratio (SNR)- increases, there will always be a trade-off between speed and accuracy. Figure 6.6 shows the performance of the single unit set as before for a sinusoidal input but this time polluted by a white Gaussian noise with SNR=20 dB. With previous parameter settings, a noise of 100% magnitude in the input introduces about 5% error in the estimated value of the amplitude. The same experiment is now repeated with much smaller values of parameters \( \mu_1 \) and \( \mu_2 \). Value of \( \mu_3 \) is retained the same as before for all the simulations. Figure 6.7 shows the result of this case. It is clear that a reduction of parameters by a factor of 60 results in a convergence time of about 60 times longer. For this, one gains a reduction in the incurred error of a factor of about 10.

Another factor to be considered in the assignment of the values to parameters \( \mu_1 \) and \( \mu_2 \) is the amplitude of the input signal. Recalling the rough permissible range of the values of parameters \( \mu_1 \) and \( \mu_2 \) presented in chapter 3, namely \( 0 < \mu_1 < 2f_o \) and \( 0 < \mu_2 < (\frac{2f_o}{A_o})^2 \), it is observed that the choice of \( \mu_1 \) is independent of the amplitude of the input signal whereas the choice of \( \mu_2 \) depends on its amplitude. In order to achieve the same speed of convergence when the amplitude of the input signal is reduced, one has to increase the value of the parameter \( \mu_2 \) by the square of the reduction factor. Figure 6.8 shows the performance of the algorithm when this provision is made which verifies the theoretical prediction.

### 6.3 Performance

This section presents the performance of the proposed technique by the use of computer simulations. All the initial conditions are taken to be zero. The first two units which are employed to extract the artifacts must do so with as small an error as possible to produce
Figure 6.6: Performance in the presence of noise. The parameters are set as $\mu_1 = 12000$, $\mu_2 = 60000$ and $\mu_3 = 0.1$. The bottom figure is a magnified portion of the top figure.
Figure 6.7: Performance in the presence of noise. Two of the parameters are set as $\mu_1 = 200$ and $\mu_2 = 1000$.

The bottom figure is a magnified portion of the top figure.
Figure 6.8: Performance of a single core unit in the extraction of a sinusoidal component and its amplitude when the amplitude of the input signal is reduced by factor of 10. The dashed line waveform is the input $u(t)$ and the output is denoted by $y(t)$. Two of the parameters are set as $\mu_1 = 12000$ and $\mu_2 = 60000 \times (10)^2$. 


an input signal to the third unit containing the DPOAE signal as a main component, otherwise the third unit will not be able to extract the DPOAE signal with sufficient accuracy. Therefore, the parameters are chosen relatively small as $\mu_1 = 100$, $\mu_2 = 1000$ and $\mu_3 = 0.1$. With these values, it is expected that the convergence is achieved within about 100 ms. The same setting was also found suitable for the third unit. Two "forces" oppose each other in the selection criteria for the values of the parameters of the third unit. On the one hand, since SNR is very low for the third unit (the DPOAE signal being taken as the signal and the total residual errors of the first two units as the noise), the parameters must be very small. On the other hand, since the input has a very low level, the value of $\mu_2$ must be large. These two "forces" seem to cancel each other for the case of $\mu_2$ so that the original value of 50 remains a good choice. As for the value of $\mu_1$, one expects that it should be small to achieve acceptable accuracy in the extracted signal. This is true, but since only the amplitude of the extracted signal (i.e. the level of the DPOAE signal) is of interest, one can substantially increase $\mu_1$. The accuracy in the estimated extracted signal (and not necessarily its amplitude) is thus traded off which in this case is not of importance. The estimation of the amplitude remains accurate while the convergence speed is high enough. At any rate, a rough setting of the values of parameters $\mu_1$ and $\mu_2$ is to be done. The core algorithm has been shown to be very insensitive with regard to the settings of the parameters: variations as much as 50% in the values of these parameters seem to have practically no effect on the performance. The value of the amplitude directly estimated by the third unit was used as the estimate of the level of the DPOAE signal.

The input signal is synthesized by the summation of two artifacts at frequencies $f_1 = 3000$ Hz and $f_2 = 1.2f_1 = 3600$ Hz and the DPOAE signal at $f_3 = 2f_1 - f_2 = 2400$ Hz. The amplitude of the artifacts are taken as 1 V and the DPOAE level is 0.5 mV. With these values, the level of the artifacts is set at about 66 dB higher than that of the DPOAE signal. Figure 6.9 shows the performance of the proposed technique. It is confirmed that the measurement is completed in about 100 ms.

To present the dynamic performance of the proposed algorithm, the response of the algo-
Figure 6.9: Performance of the DPOAE estimation method in the measurement of a signal of 0.5 mV amplitude.
Figure 6.10: Dynamic performance of the proposed method. The level of the DPOAE signal undergoes a step change from 0.5 mV to 1 mV at $t = 200$ ms.
A number of measurements can be made consecutively. Figure 6.11 shows such an example. In each case, the frequency of the extracted DPOAE signal is shown. As the frequency decreases, there are fewer cycles in a given time interval. For example, there are 160 cycles available in 100 ms of a signal of frequency $f = 1600$ Hz whereas in the case of a signal of frequency $f = 4000$ Hz the available number of cycles are 400 in the same time interval. This explains why the estimation of the DPOAE level is better achieved at higher frequencies. It should be noted that within the range of frequencies of interest for DPOAEs, the algorithm provides acceptable estimates in about $100 - 200$ ms.
As noted before, one of the main features of the employed core algorithm is its noise immunity. Therefore, one expects the overall DPOAE estimation scheme to be immune to background noise. Usually, otoacoustic tests are conducted in quiet examination rooms so that the level of the noise floor is low enough for good results. Under such controlled conditions, a noise floor of about 10 dB lower than the DPOAE signal \((\text{SNR}=10 \text{ dB})\) exists. It is obvious that an algorithm which is able to extract a DPOAE signal highly immersed in background noise is desirable in that it alleviates the need for special quiet examination facilities such as special sound-proof enclosures. Figure 6.12 shows the performance of the proposed algorithm in a highly noisy environment where the level of the DPOAE signal is set to be at the level of the noise floor \((\text{SNR}=0 \text{ dB})\). The incurred error is negligible. In Figure 6.13, the level of noise is elevated to be 10 times larger than that of the DPOAE signal itself. The incurred estimation error is less than \(\pm 15\%\) which for most practical applications is considered to be sufficient. The excellent noise immunity of the proposed algorithm not only obviates the need for sound-proof examination rooms, but also provides a way to reduce the level of the artifacts for more patient-friendly tests.

### 6.4 Comparison with the Existing Methods

A number of works in the area of DPOAE signal estimation involve the use of adaptive noise cancellation (ANC) in reducing the level of the noise floor [47, 50]. A reference signal, usually a recording from an auxiliary microphone placed on the ear not under examination, provides a stochastic measure of the background noise which is used to mitigate the noise recorded by the microphone placed on the ear under examination. In such techniques, the subsequent use of an estimator is required. Most commonly, FFT-based estimators are used [53-57]. It is widely known that FFT estimators pose serious shortcomings when employed in DPOAE estimation. Various other techniques, usually aimed at obviating the shortcomings of FFT such as its window-based nature and its sensitivity with regard to frequency, have been proposed. One of the recently proposed methods presented by Ma and Zhang in [59] is used in this section for comparison with the new approach presented this chapter.
Figure 6.12: Performance of the proposed method in the estimation of the DPOAE signal in a highly noisy background (SNR=0 dB). The bottom figure is a magnified portion of the top figure.
Figure 6.13: Performance of the proposed method in the estimation of the DPOAE signal in an extremely noisy background (SNR = -20 dB). The bottom figure is a magnified portion of the top figure.
The method presented in [59] is an optimal maximum-likelihood estimator for the extraction of DPOAE signals. Superior performance of the method, especially in cases where FFT exhibits leakage effect 1 is observed. The signal model is assumed to consist of the two artifacts and the DPOAE signal polluted by background noise. In [59], simulated data are used for the two artifacts and the DPOAE signal whereas the background noise is a recorded noise.

The Method of Ma and Zhang

The recorded DPOAE signal is formulated as

\[ s(n) = \sum_{i=1}^{3} A_i \cos(n \omega_i + \phi_i) + \nu(n) \]

where \( \omega_i = 2\pi f_i / f_s \) with \( f_s \) being the sampling frequency in Hz and \( \nu(n) \) is the background noise. The DPOAE level is defined as \( L = 10 \log A^2 \). Given vector representations of the DPOAE signal \( s = [s(0) s(1) \cdots s(N-1)]^T \) and the background noise \( \nu = [\nu(0) \nu(1) \cdots \nu(N-1)]^T \), the received DPOAE signal vector is

\[ s = H_a w_a + H_{dp} w_{dp} + \nu \]

where

\[ H_a = \begin{bmatrix} 1 & \cos \omega_1 & \cdots & \cos((N-1)\omega_1) \\ 0 & \sin \omega_1 & \cdots & \sin((N-1)\omega_1) \\ 1 & \cos \omega_2 & \cdots & \cos((N-1)\omega_2) \\ 0 & \sin \omega_2 & \cdots & \sin((N-1)\omega_2) \end{bmatrix}^T \]

\[ H_{dp} = \begin{bmatrix} 1 & \cos \omega_3 & \cdots & \cos((N-1)\omega_3) \\ 0 & \sin \omega_3 & \cdots & \sin((N-1)\omega_3) \end{bmatrix}^T \]

\[ w_a = [A_1 \cos \phi_1 \quad -A_1 \sin \phi_1 \quad A_2 \cos \phi_2 \quad -A_2 \sin \phi_2]^T \]

\[ w_{dp} = [A_3 \cos \phi_3 \quad -A_3 \sin \phi_3]^T \]

1 Leakage is the phenomenon of the spread of energy from a single frequency to a number of frequency locations on the frequency spectrum [17]. This effect is apparent when a frequency component present in the signal is not equal to any of the distinct frequency components forming the discrete frequency spectrum generated by FFT.
Minimization of the least squares error \( \|s - H_a w_a - H_{dp} w_{dp}\|^2 \) results in the following expression for the estimated DPOAE level:

\[
\hat{L} = (\hat{w}_{dp}(1))^2 + (\hat{w}_{dp}(2))^2 \tag{6.1}
\]

where \( \hat{w}_{dp}(1) \) and \( \hat{w}_{dp}(2) \) are the first and second elements of \( \hat{w}_{dp} \) given by

\[
\hat{w}_{dp} = w_{dp} + (H_{dp}^T P_a^O H_{dp})^{-1} H_{dp}^T P_a^O v
\]

Here \( P_a^O \) is the orthogonal projection complement matrix of \( H_a \) and is given by

\[
P_a^O = I - H_a (H_a^T H_a)^{-1} H_a^T
\]

where \( I \) denotes the identity matrix.

**Comparison**

Simulated data were used for the comparison of the Method of Ma and Zhang with the proposed method. Similar results to those presented in [59] were obtained using a simulated noise of zero-mean white Gaussian distribution as the background noise. For both cases, the numerical experiments involve two artifacts of frequencies \( f_1 = 2.454 \) kHz and \( f_2 = 3.003 \) kHz. The DPOAE signal is thus at \( f_3 = 2f_1 - f_2 = 1.905 \) kHz. The sampling frequency is chosen as \( f_s = 10.24 \) kHz. The DPOAE signal is at 0 dB level while the two artifacts are at 65 dB. The experiment was repeated several times for different levels of the noise floor. Figure 6.14 compares the performance of the two methods for varying levels of the noise floor. The index of the performance is taken to be the normalized mean squares error (MSE) incurred in the estimation process defined as

\[
\text{Normalized MSE} = \frac{\text{Mean}(|L - \hat{L}|^2)}{L^2}.
\]

In the case of the proposed method, the vector of the estimated level of the DPOAE (\( \hat{L} \)) is formed after the signal is stabilized in the time domain (after about 100 ms delay).

When the incurred error exceeds the signal level, the signal is no longer recoverable. It is observed that the Method of Ma and Zhang is quite sensitive to the level of the background noise.
Figure 6.14: Comparison of the performance of the proposed method with that of [59].

noise. In fact, as soon as the noise level exceeds the DPOAE level, the signal is totally lost and the estimation process fails. As it would be expected, the proposed method has a very high degree of noise immunity of about 20 dB more than that of Ma and Zhang. In practice, an estimation process such as that of [59] has to be equipped with an averaging procedure to reduce the level of noise. Although averaging can reduce the noise, it is time-consuming. The proposed method is particularly advantageous in that it provides the estimation of the DPOAE signals almost in real-time without the need of noise reduction facilities such as sound-proof examination rooms.

The proposed method of DPOAE estimation exemplifies an application of the core algorithm in the information extraction from audiological signals which may be adapted to other problems of similar nature. As illustrated in this chapter, the main features of this new approach are its high noise immunity and little computational resources required for its implementation which render the proposed method attractive both in terms of efficiency and the costs involved in its production.
Chapter 7

EMAT Signal Analysis

Transmission of bursts of ultrasonic waves into a medium is a well-known technique for the acquisition of useful information about the structure of the medium under study. Ultrasonic waves are often used in non-destructive evaluation (NDE) of materials. Various techniques for the generation of ultrasonic waves for NDE of metallic structures exist, among which electromagnetic acoustic transduction (EMAT) has attracted considerable attention over the years due to its favorable non-contact testing feature. However, this method suffers from serious shortcomings due to the poor quality of the received signals which are often highly polluted by noise [60, 61]. A significant amount of literature deals with various methods of improvement of the quality of received EMAT signals. Coil design considerations constitute a major research trend in this regard [62]. Electromagnetic field computation has been used to assist the design of coil geometry [63]. Also, there has been considerable interest in adapting signal processing methods for noise elimination [64, 65] and flaw identification [66, 67].

In the NDE of materials using bursts of ultrasonic energy, i.e. pulsed sine waves, it is often desirable to detect the peak of the received signal, its amplitude and its time of arrival [60]. In order to detect such features of the received noisy signal, it is necessary to improve the signal quality through the elimination of electrical noise. Time averaging of a repeated signal can reduce random electrical noise. However, the main problem with this technique is the long measurement time needed which limits its applicability to real-time NDE [61].
Narrow band filtering is used as the primary method to improve signal quality. A notch filter with a sharp notch is effective in eliminating the electrical noise, but renders the equipment sensitive to potential frequency drifts. Moreover, the output signal of such a filter has to be analyzed for detection of the peak and its arrival time. Fourier transform analysis could be used but, in this case, all the time information about the peak position will be lost [60].

This chapter presents a method of noise elimination for pulsed sine wave signals [15, 16]. The aim is to detect the arrival time of the envelope peak which is necessary in order to make accurate travel time measurements of ultrasonic echoes. The structure of the proposed method is presented and its behavior is demonstrated by the aid of computer simulations. Experimental verification of the performance of the proposed method is then exemplified by the refinement of and information extraction from real measured EMAT signals [68].

7.1 Proposed Method

The core algorithm of chapter 2 exhibits a high degree of immunity with respect to noise and can, therefore, be used as a noise reduction technique [69]. Figure 7.1 shows this property. The degree of the estimation accuracy is fully controllable by the adjustment of parameters $\mu_1$, $\mu_2$ and $\mu_3$ in a trade-off with convergence speed. For each particular application, one can choose a suitable set of parameters. In general, as the frequency of the operation increases, proportionally higher values of parameters $\mu_1$ and $\mu_2$ have to be used to retain the same convergence speed in terms of the required number of cycles for convergence. Therefore, it is reasonable to divide the values of these two parameters by the nominal frequency of the input signal when expressing their values. A typical set of parameters, used in the simulations of this chapter, is $\mu_1 = 10$, $\mu_2 = 200$ and $\mu_3 = 0.08$, where the values of $\mu_1$ and $\mu_2$ are divided by the nominal frequency of the incoming signal.

The proposed method of time-domain signal analysis consists of (a) the elimination of noise from the input signal by passing it through the core algorithm, (b) the estimation of the amplitude of the extracted sinusoid, and (c) the comparison of instantaneous amplitude with
Figure 7.1: Performance of the core algorithm in the extraction of a sinusoid and its amplitude, present in a highly noisy input signal.
Figure 7.2: Performance of the core algorithm in the extraction of a pulsed sinusoid and its amplitude, present in a noisy input signal.

a defined threshold to determine the peak. Note that (b) is accomplished without further effort since the noise elimination algorithm automatically provides a direct estimate of the amplitude. The time of arrival of the peak is conveniently detectable by time-gating of the peak detection scheme; the estimated arrival time is then to be reduced by a constant time delay.

The core algorithm performs very well when the sinusoidal component of the input signal is amplitude modulated. The algorithm basically looks for a sine wave; in its absence, i.e. when its amplitude is zero, it returns zero for the estimated amplitude and generates a zero-amplitude signal. Figure 7.2 illustrates this point. Notice that the output signal follows the
Figure 7.3: Illustration of the convergence delay of the core algorithm in detecting the peak of the input signal. The line style (solid line for the input, dotted line for the output and dash-dotted line for the amplitude) is used consistently in all the figures of this chapter.
Figure 7.4: Block diagram of the enhanced algorithm for noise elimination and peak detection of pulsed sinusoids.

The sinusoidal component of the input signal with a delay which is due to the convergence time of the algorithm. This effect is more clearly illustrated in Figure 7.3 where the algorithm is excited by a short-time (half cycle of a sine wave) signal. Observe that the detected place of the peak is delayed. This time delay is a complex function of the parameters $\mu_1$, $\mu_2$, $\mu_3$ and the values of initial conditions: fortunately, it is a relatively flat function of frequency: for each parameter setting this delay is (for all practical purposes) a constant number. The value of this delay, most conveniently measurable by simulation as done in Figure 7.3, can be used to correct the arrival time of the peak: it is sufficient to just subtract this number from the arrival time detected by the algorithm.

As explained in section 5.1, in order to further enhance the noise immunity of the algorithm, the use of a simple second order band pass filter at the input of the core algorithm is proposed. If the nominal frequency of the input signal is $f_o = \frac{\omega_0}{2\pi}$, the transfer function of the band pass filter is given by

$$F(s) = \frac{100}{s^2 + 100s + \omega_0^2}$$

This filter improves the signal to noise ratio (SNR) of the input signal of the core algorithm. However, it also introduces undesired attenuation and phase delay, especially if any drift from the nominal frequency occurs, which may happen as a result of equipment aging and other reasons. Due to the capability of the algorithm to estimate all the parameters of the extracted sinusoid, both these effects can be compensated for as demonstrated in Figure 7.4.
7.2 Experimental Verification

The experimental data \(^1\) are used to demonstrate the performance of the proposed method in signal refinement and analysis. Figure 7.5 shows the experimental setup for the EMAT system. The modulator generating high current/voltage signals to feed the EMAT transmitter produces a smoothly curved pulsed sinusoidal signal at about 1.8 MHz. The received EMAT signal is amplified and sent to a digital oscilloscope to measure the voltage across the receiver coil. Digitization is done at a high sampling frequency (102.4 MHz) to preserve

\(^1\) The experiments have been conducted at the Ultrasonic Nondestructive Evaluation Laboratory of the Department of Mechanical and Industrial Engineering of the University of Toronto under the supervision of Professor A.N. Sinclair.
Figure 7.6: Performance of the proposed method in noise elimination and peak detection of a highly noisy EMAT signal. The top graph shows the input signal. The refined EMAT signal and its amplitude are shown in the bottom graph.

signal integrity.

Figure 7.6 shows the performance of the proposed method for a set of 1024 points of recorded data. The received sine wave pack is reflected back from the bottom of a metallic cube under examination. In order to produce a worst-case scenario, no attempt was made to obtain good signal quality while recording the EMAT signals. The SNR of the input signal is estimated to be $-10$ dB. It is observed that the proposed method is fully capable of removing the electrical noise. However, the ripples detected during the periods of absence of the pulsed sine wave are somewhat undesirable. They are not electrical noise as will be later justified by reference to the frequency spectra; such ripples may be due to the pulse generator, with
Figure 7.7: Frequency spectra of the noisy input and clean output signals.

a possible contribution of ultrasonic echoes from small flaws in the test specimen. Figure 7.7 compares the frequency spectrum of the input signal to that of its refined variant. It is clear that the algorithm does not affect the frequency content of the desired signal and acts on the electrical noise only. Therefore, it is justified that what is passed through the signal refiner is in fact of sinusoidal shape at the EMAT operating frequency, whatever its interpretation may be.

In an attempt to provide a "correct" version of the received EMAT signal, the signal received by the EMAT receiver has been averaged 2048 times by a digital oscilloscope. The resultant averaged signal has 1024 data points. This averaged signal was observed to be still a bit noisy. It was then used as the input signal to the proposed algorithm. Figure 7.8 shows
further refinement achieved by the proposed algorithm. It goes without saying that the
time of arrival as detected by the algorithm in both cases of Figures 7.6 and 7.8 has to be
shortened by the amount of the convergence time-delay, which was numerically determined
to be about one cycle for this setting of parameters.

7.3 Comparison with Conventional Methods

As mentioned earlier in this chapter, the most commonly used conventional method of EMAT
signal refinement is signal averaging. Figure 7.9 shows the effect of signal averaging on the
SNR improvement. The raw measured data is averaged 128 and 2048 times to yield less
noisy signals. The signal quality enhancement is clear. The averaging is done using a digital
oscilloscope in the laboratory. Using the available equipment it is observed that averaging
the signal 2048 times takes about ten minutes to be completed. Such a long delay renders
the processing of the EMAT signals off-line which is undesirable for real-time industrial ap­
plications. Once the signal is refined using averaging, further post-processing is necessary
for the extraction of the value and the arrival time of the peak of the refined signal. This
may require a peak detection scheme.

Results of the employment of the proposed method of EMAT signal analysis, as presented
in Figures 7.6 and 7.8, clearly demonstrate the superiority of the proposed method. Signal
refinement is done simply by passing the signal through the proposed filter once, while peak
detection is achieved without further post-processing.

The refinement of EMAT signals presented in this chapter clearly illustrates the high noise
immunity feature of the core algorithm. It also extends the scope of the application of the
proposed algorithm to include problems involving amplitude-modulated sinusoids. Pulsed
sinusoids, for instance, may be encountered in various NDE problems of which EMAT signals
present an example.
Figure 7.8: Performance of the proposed method in further noise elimination and peak detection of an EMAT signal which has been averaged 2048 times. The top graph shows the input signal and the output signal and its detected amplitude. The frequency spectra of the input and output signals are shown in the bottom graph.
Figure 7.9: Illustration of the effect of averaging on the signal quality enhancement. The top graph shows the raw measured data. The middle graph shows the recorded signal which is averaged 128 times using a digital oscilloscope. The bottom graph shows the recorded signal averaged 2048 times.
Chapter 8

Doppler Shift Estimation

Real-time estimation of frequency of quasi-periodic signals is an important signal processing problem which arises in diverse areas of engineering such as radar, communications and power systems [70]. A more particular area is the estimation of Doppler frequency shift which is encountered both in aerospace and biomedical engineering. Considering that the core algorithm is capable of estimating the instantaneous frequency of sinusoids, the problem of frequency estimation is conceived as one of the potential areas of application of the proposed algorithm. This chapter presents preliminary results of the application of the proposed algorithm to the problem of frequency estimation in general and Doppler shift estimation in particular. An exhaustive treatment of all of the aspects of the problem of estimation of instantaneous frequency, which itself encompasses a wide range of applications, needs far greater research effort and is beyond the scope of this work.

8.1 Frequency Estimation

The problem of frequency estimation is particularly complicated when the sinusoidal signal of interest is polluted by white or colored noise. Various methods have been proposed, each presenting a compromise between the efficiency in terms of speed/accuracy and the computational demand. Frequency-domain algorithms adapted to time-frequency analysis of nonstationary signals are usually window-based and computationally demanding which render them inefficient for real-time applications [71, 72]. This section presents some pre-
Figure 8.1: Incurred estimation error as a function of the signal to noise ratio (SNR) of the input signal.

Preliminary results of the application of the proposed algorithm to the problem of estimation of instantaneous frequency of nonstationary signals [73].

A set of parameter settings of $\mu_1 = 100$, $\mu_2 = 10000$, $\mu_3 = 0.02$ is chosen to illustrate the application of the core algorithm to the estimation of frequency. As noted before, the speed/accuracy trade-off is fully controllable by the adjustment of parameters $\mu_1$, $\mu_2$ and $\mu_3$. For any given application, one can modify the values of these parameters to balance speed and accuracy. Incurred error is also a function of the extent of noise pollution imposed on the input signal. The cleaner the input signal is, the more accurate an estimation is achieved. Figure 8.1 is an illustration of this relationship. Figure 8.2 shows the adaptive nature of the algorithm in tracking time-variations. The time-constant of the overall system, which itself is a function of parameters $\mu_1$, $\mu_2$ and $\mu_3$, determines the inertia in tracking sharp time-variations. Depending on the application, one may re-adjust the time-constant of the
Figure 8.2: Performance of the algorithm in tracking frequency variations of a sinusoidal input signal. The frequency of the input signal is arbitrarily drifting about a nominal value.
system for optimum performance.

8.2 Doppler Frequency Estimation

Estimation of Doppler frequency shift is crucial in diverse areas of technology such as aerospace, communications and biomedical engineering. The reflected frequency shifted signal is usually buried under heavy noise of unknown composition which is often modeled by a white Gaussian distribution [74]. The challenge is usually in the fast and accurate estimation of Doppler shift when the signal is heavily polluted by noise. A variety of methods have been proposed over the years for the estimation of frequency in noise. Adaptive algorithms constitute a major trend in this area [75]. Most of the adaptive methods fall within the category of linear algorithms (see [76], for instance). Given that real-time employment of the estimation algorithms is often desired, computational efficiency is considered to be an important issue [77]. This section presents preliminary results of the frequency estimation aspect of the new algorithm in the case of Doppler frequency shift [78].

In most Doppler-based velocity measurement devices - whether electromagnetic or ultrasonic - the received signal is mixed with the transmitted signal to yield a noisy sinusoidal signal whose frequency is proportional to the velocity of the moving target. Therefore, the signal processing task of noise elimination and frequency estimation may be described by the attempt to estimate the instantaneous value of frequency \( f(t) = \frac{\omega(t)}{2\pi} \) of a signal \( u(t) \):

\[
u(t) = A(t) \sin(\omega(t)t + \phi) + n(t)
\]

where \( A(t) \) and \( \phi \) are amplitude and constant phase, respectively; \( n(t) \) is the totality of noise and interference which, in general, is of unknown composition.

A simple band pass filter having the following transfer function:

\[
H(s) = \frac{100s}{s^2 + 100s + \omega_0^2}
\]

may be used at the input of the core algorithm to enhance the signal to noise ratio (SNR) of the input signal to the algorithm. \( \omega_0 \) is a fixed frequency chosen roughly to be close to the
Figure 8.3: Performance of the proposed algorithm in tracking time-variations in the frequency of a noise-free sinusoidal signal. Constant phase of the input signal is a random number between 0 and 2π. Magnified portions of this figure are shown in Figure 8.4.

Frequency of the incoming sinusoid. This filter introduces a phase shift and an attenuation which are known functions of frequency. Since the core algorithm instantaneously estimates frequency as well as amplitude and phase, both the attenuation and the phase shift caused by the input notch filter are easily compensated.

Figures 8.3 and 8.4 show the performance of the proposed method in tracking time-variations in the frequency (or rather the total phase) of a noise-free sinusoidal signal. Speed may be conveniently described in terms of number of cycles needed for accurate estimation of frequency. Therefore, all the quantities, including frequency of which the number of cycles is
Figure 8.4: Magnified portions of Figure 8.3. The top two graphs show the tracking capability of the proposed method in detecting the phase of the input signal and the bottom two graphs show performance of the method in following frequency variations.

A representative index, are normalized without loss of information.

Figure 8.5 shows the performance of the proposed method in a highly noisy environment where SNR is about 0 dB. The input signal is highly polluted by an arbitrarily generated noise (deliberately distorted Gaussian distribution). With the given settings of parameters, the frequency estimation error is about 0.1% of the extracted frequency.

Doppler frequency shift estimation presents a general class of signal processing problems. Specific applications of the general idea of the frequency estimation using the core algorithm
Figure 8.5: Illustration of Doppler frequency detection in a highly noisy environment. Top graph shows the extracted Doppler signal and the bottom figure shows its frequency.

presented in this chapter are blood flow measurement in biomedical engineering and target velocity estimation in radar engineering. These two applications are briefly introduced in the following chapter.
Chapter 9

Conclusions

A core algorithm was developed with the intention to overcome the inherent shortcomings of the standard signal processing tools such as DFT and linear adaptive filtering. To demonstrate the power of the new core algorithm, its successful application to a number of problems of current research interests was presented.

Limited convergence speed of the new algorithm, however, poses problems in the processing of short-duration signals. This is an identified shortcoming of the algorithm and is discussed in more detail in this chapter.

Considering that the algorithm presented in this thesis is a fundamental signal processing tool, the range of its applications is not limited to those discussed in the preceding chapters and covers a wide subclass of time-frequency signal analysis/synthesis problems. Some examples of the areas of potential application of the algorithm are introduced in this chapter outlining the direction of proposed future research. Finally, a summary of the main features of the core algorithm is presented.

9.1 An Identified Limitation

Suppression of quasi-periodic interferences is demonstrated in detail in chapter 5. In the examples considered in chapter 5, the length of the signal is practically unlimited. Therefore,
no difficulty is encountered in estimating and eliminating the interference; in such cases, the algorithm has enough time (a few cycles, for example) to identify the interference. However, occasionally signals with limited length may be affected by unidentified interferences of short duration. In such cases, the algorithm loses its effectiveness in eliminating the noise due to the fact that it lacks enough information for proper identification of the noise. An example of this kind is considered here. Electromagnetic interference of quasi-periodic nature has been observed to have affected lightning recordings from the CN Tower recorded by the Lighting Laboratory of the University of Toronto [79]. The algorithm is used to eliminate the interference. Figure 9.1 shows the performance of the core algorithm in the elimination of a clearly visible interference coupled to one of the lightning recordings. The fact that
the characteristics of the interference on lightning recordings including its amplitude and frequency vary substantially within a few cycles, compounded by the presence of a large impulse, renders the signal processing task of the interference elimination difficult. It is observed that the algorithm, although it follows such variations to a certain extent, does not yield sufficiently satisfactory results.

9.2 Proposed Future Research

A summary of some research areas identified as potential areas of application of the core algorithm is provided in this section. Essentially, time-frequency signal analysis/synthesis problems are potential applications of the core algorithm or its extension as discussed in section 2.3 of chapter 2. Examples of such problems are introduced below.

- **Active noise cancellation.** Sound and vibration due to mechanically vibrating components produce significant amount of noise polluting the environments in which human operators function. Examples of such scenarios are vibration noise generated by aircraft engines and magnetic resonance imaging (MRI) devices. Active noise cancellation (ANC) is used wherever passive alternatives lose their effectiveness. The size of the passive noise suppressor is proportional to the wavelength of the sound wave: at low frequencies, passive noise cancelers are impractically bulky. Therefore, cancellation of the vibrations due to the low frequency electrical oscillations such as winding vibrations in MRI, and those generated by mechanical rotation (in aircraft or automobile engines, for instance) is usually achieved by ANC. Active noise cancellation is the technique of the generation of a sinusoidal sound wave which is meant to cancel out the vibration within a limited space enclosing the ears of the human operator. The first step in ANC is the estimation of the sinusoidal disturbance and the synthesisization of its out-of-phase counterpart. Research efforts are reported in the literature aiming at the identification of such sinusoidal disturbances [2]. The algorithm is expected to have application in this area.

- **Sinusoidal disturbance rejection in control systems.** Sinusoidal disturbances of uncertain frequency have significant effect on the overall behavior of control systems [3].
Identification of such disturbances using the new algorithm presents another potential area of its application.

- **Blood velocity measurement.** Ultrasonic blood flow measurement is one of the standard clinical methods of vascular disease diagnosis. The effect of the velocity of blood is registered as the Doppler frequency shift occurring in the ultrasonic waves used in ultrasonography. Conventionally, DFT-based methods are used for the estimation of Doppler frequency. Development of more efficient signal processing methods forms part of the current research in this area [80, 81]. Preliminary study presented in chapter 8 indicates potential applicability of the proposed method to this area.

- **Target velocity measurement in aerospace engineering.** Doppler effect is perhaps most recognized in radar and aerospace engineering. Clutter plays the role of the noise and the problem of target velocity measurement is essentially a problem of frequency estimation in noise [74, 75]. As predicted in chapter 8, this is considered as one of the areas of application of the core algorithm.

- **Carrier recovery in communication systems.** Efficient recovery of the frequency of the carrier in modulation-based communication systems has been an active area of research for many years [76]. The conventional approach of offset frequency estimation entails the use of a PLL to synthesize the carrier needed for demodulation process. Considering the capability of the new algorithm in directly extracting a sinusoidal signal (which is the carrier, in this case), the area of carrier recovery presents a potential area of its application.

- **Power frequency measurement.** The operating frequency of the stand-alone systems may undergo variations of up to few Hertz [23]. In such cases and in the starting phase of generators when the frequency is not yet stabilized, the estimation of frequency is important [5, 6]. The algorithm is expected to have application in this area.

- **Electromagnetic interference source identification.** A major research trend in electromagnetic compatibility (EMC) engineering is the application of signal processing
algorithms in interference characterization, source identification and EMI mitigation. Characterization of interferences is useful in the way it helps in the identification of potential sources within or outside a system, tracing noise and interference and suppression or mitigation of the interference [82]. The new algorithm, by the fact that it can decompose a signal into its constituent components while tracking its time variations, is expected to have application in the area of interference analysis.

9.3 Main Features of the Core Algorithm

In conclusion, the main features of the core algorithm are summarized as:

- **Capability of analysis of signals of nonstationary nature.** The algorithm is essentially a time-frequency signal processing tool which assumes no model for the input signal.

- **Capability of extraction (synthesis) of the targeted signal.** The core algorithm directly extracts the targeted sinusoid, and thereby obviates the need for complex synchronization schemes.

- **Robustness and noise immunity.** High degree of robustness is observed with respect to both internal (e.g. parameter settings) and external (e.g. presence of noise) conditions.

- **Simplicity of the structure.** The computational demand is very low, comparable with that of FFT.

These features render the new algorithm suitable for real-time industrial applications where the computational resources are limited and a high degree of robustness with respect to both internal and external conditions is desired.
References


