DESIGN OF HUYGENS’ METASURFACES FOR REFRACTION AND FOCUSING

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science Graduate Department of Electrical and Computer Engineering University of Toronto

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Abstract

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The systematic design of unit cells for Huygens’ metasurfaces is presented here. The Huygens’ metasurface implements collocated electromagnetic boundary conditions enforcing the electric and magnetic surface currents of the equivalence principle. This provides the flexibility necessary for efficient interfacial beamforming applications. Furthermore, the boundary condition nature permits achieving this in an electromagnetically thin profile. This concept and its design procedure are verified through application to refraction and Gaussian-beam to Gaussian-beam focusing. The Huygens’ metasurfaces presented herein are printed on two bonded boards instead of many stacked, interspaced layers, and can be manufactured using standard PCB fabrication techniques. This simplifies the fabrication, and allows the design to be scaled to higher frequencies. These two bonded boards implement a single, collocated, sub-wavelength array of electric and magnetic dipoles. Furthermore, in contrast to traditional frequency-selective surfaces and transmitarrays, which are on the order of a wavelength thick, these designs are only $\lambda/10$ thick.
Rom. 11:36 “Because out from Him and through Him and to Him are all things.
To Him be the glory forever. Amen.”
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Chapter 1

Introduction

This thesis concerns a metasurface implementing a discontinuous field boundary condition at an interface. Enforcing this boundary condition allows the engineering of the reflected and transmitted field distributions across the interface, for a given incident field. Volumetric metamaterials mimic naturally occurring transparent media, gradually manipulating the phase of the transmitted field in that media, to achieve similar goals. In contrast to this, a metasurface introduces an abrupt field discontinuity at an interface. These cases are illustrated in Fig. 1.1 for the application of plane-wave refraction.

![Figure 1.1: Bulk and interfacial plane-wave refraction. (a) Refraction in bulk media, such as in a volumetric metamaterial, and (b) refraction at an interface, such as in a metasurface.](image)

Fig. 1.1a shows plane-wave refraction in a volumetric metamaterial, which can also be explained in terms of Huygens’ principle. This states that each point along a wavefront can be thought of as a source of secondary spherical wavelets that spread out at the speed of light. The tangential surface to all these wavelets gives the new wavefront. The destination media of differing refractive index modifies the phase velocity, causing the tangential surfaces to the wavelets to refract. The direction of the optical path depends on the immediate medium in a continuous manner and is thus gradual. In contrast to this, in the metasurface in Fig. 1.1b, the phase of each point of the planar wavefront incoming from the left is advanced at a spatial point. This is done by inserting a phase according to a roughly linear spatial profile, thus refracting the wave, and this is instantaneous at the interface.
1.1 Theoretical Background to the Huygens’ Metasurface

The same phenomena can be examined from Maxwell’s equations, which in differential form can be expressed as:

\[
\nabla \times E = -M_i - \frac{dB}{dt}, \quad (1.1)
\]
\[
\nabla \times H = J_i + J_c + \frac{dD}{dt}, \quad (1.2)
\]
\[
\nabla \cdot D = \rho_e, \quad (1.3)
\]
\[
\nabla \cdot B = \rho_m. \quad (1.4)
\]

where \(E\) is the electric field intensity, \(H\) is the magnetic field intensity, \(D\) is the electric flux density, and \(B\) is the magnetic flux density. \(J\) and \(M\) are the electric and magnetic volume current densities, and the subscript \(i\) indicates an impressed source, while \(c\) indicates a conduction current. \(\rho_e\) and \(\rho_m\) are the electric and magnetic volume charge densities. Furthermore, \(J_c\) is expressed as:

\[
J_c = \sigma E, \quad (1.5)
\]

where \(\sigma\) is the conductivity. From (1.1) to (1.4), the electromagnetic (EM) boundary conditions are derived as:

\[
\hat{n} \times (E_2 - E_1) = -M_s, \quad (1.6)
\]
\[
\hat{n} \times (H_2 - H_1) = J_s, \quad (1.7)
\]
\[
\hat{n} \cdot (D_2 - D_1) = \rho_{es}, \quad (1.8)
\]
and

\[
\hat{n} \cdot (B_2 - B_1) = \rho_{ms}. \quad (1.9)
\]

where \(\hat{n}\) is the unit normal to the boundary interface and points into region 2, \(J_s\) and \(M_s\) are the electric and magnetic surface current densities, and \(\rho_{es}\) and \(\rho_{ms}\) are the electric and magnetic surface charge densities. The surface current and charge densities are non-zero only for a perfect conductor.

Consider the plane wave at oblique incidence to a dielectric boundary in Fig. 1.2 below. Here \(\mathbf{k}_{\text{inc}}, \mathbf{k}_{\text{refr}}\)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{plane_wave_oblique_incidence.png}
\caption{Plane wave at oblique incidence to the boundary of a bulk media.}
\end{figure}
and \( \mathbf{k}_{\text{refr}} \) are the incident, reflected, and refracted wavevectors, respectively. In addition, \( \varepsilon \) and \( \mu \) are the permittivities and permeabilities in their respective media. Taking (1.6) to (1.7) for the case of a plane wave incident upon the boundary between two bulk media, the Fresnel reflection and transmission coefficients can be obtained for both the transverse electric (TE) and transverse magnetic (TM) modes [1]. The result of the analysis gives:

\[
\begin{align*}
\Gamma_{\text{TE}} &= \frac{\eta_2 \sec \theta_t - \eta_1 \sec \theta_i}{\eta_2 \sec \theta_t + \eta_1 \sec \theta_i}, \\
\Gamma_{\text{TM}} &= \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}, \\
T_{\text{TE}} &= 1 + \Gamma_{\text{TE}}, \\
T_{\text{TM}} &= (1 + \Gamma_{\text{TM}}) \frac{\cos \theta_t}{\cos \theta_i}.
\end{align*}
\]

Here \( \eta \) refers to the intrinsic impedance of the medium, \( \theta_t \) and \( \theta_i \) to the transmitted and incidence angles, \( \Gamma \) to the reflection coefficient, and \( T \) to the transmission coefficient. Other modes are linear combinations of these modes. Note the dependence of (1.10) to (1.13) on the bulk constitutive parameters by means of \( \eta \). This points to the dependence on the immediate media in a continuous manner. Thus in a volumetric metamaterial, by engineering \( \varepsilon \) and \( \mu \), it is possible to introduce field discontinuities to refract a plane wave in any arbitrary fashion as desired. This can be seen from (1.8) to (1.9), and the constitutive relationship between \( \mathbf{B} \) and \( \mathbf{H} \), as well as that between \( \mathbf{D} \) and \( \mathbf{E} \). Conductive bulk media are not considered because though these support surface current and charge densities, the large penetration losses and reflections up to about optical frequencies make these impractical for transmit applications.

Now we move on to consider the case of the Huygens’ metasurface, depicted in Fig. 1.3. In the absence of bulk media, we can no longer rely on the constitutive parameters to introduce the field discontinuities needed to change the trajectories of our plane waves. Instead from (1.6) to (1.9), we note that if we can engineer either surface current or charge densities, we can again introduce field discontinuities to change the direction of \( \mathbf{E} \) and \( \mathbf{H} \), and thus refract or reflect a plane wave in a desired fashion. The surface current densities produce discontinuous tangential fields and the surface charge densities produce discontinuous normal fields. Here we consider only the surface current densities, so that the surface charge densities are considered to be zero. This is essentially Schelkunoff’s surface equivalence principle, which is an extension of Love’s equivalence principle, in turn developed from Huygens’ principle. The difference here is that the surface currents are interpreted as quantities that can be engineered to achieve the desired effect. Thus, we can also interpret this as a problem of engineering Huygens’ sources along an interface [2]. Noting that the fields in (1.6) to (1.9) are not restricted to plane waves, this analysis can be extended to arbitrary field distributions across the interface.
This problem can also be considered from the viewpoint of surface impedances. To obtain these quantities, the surface current densities are viewed as coming from physical polarization currents generated from an array of polarizable particles. These particles are characterized on the macroscopic level by both electric and magnetic surface polarizabilities \( \alpha_{es} \) and \( \alpha_{ms} \), respectively, that account for mutual coupling [3]. These are related to the surface currents in [4] through:

\[
J_s = j\omega \alpha_{es} E_{t,av}\big|_s, \tag{1.14}
\]

and

\[
M_s = j\omega \alpha_{ms} H_{t,av}\big|_s. \tag{1.15}
\]

Here \( E_{t,av}\big|_s \) and \( H_{t,av}\big|_s \) are the average tangential electric and magnetic fields at the surface boundary. Furthermore, we go on to define the respective electric and magnetic surface impedances \( \bar{Z}_e \) and \( \bar{Z}_m \):

\[
\bar{Z}_e = (j\omega \alpha_{es})^{-1} = E_{t,av}\big|_s / J_s, \tag{1.16}
\]

and

\[
\bar{Z}_m = j\omega \alpha_{ms} = M_s / H_{t,av}\big|_s. \tag{1.17}
\]

Note that here \( \bar{Z}_e \) and \( \bar{Z}_m \) are dyadics, corresponding to the anisotropies necessary to relate the polarizations of the input and output fields. However, in this work, we consider only the simplified case of scalar impedances.

Combining (1.6) and (1.7) with (1.16) and (1.18), and dropping the tensor notation on the latter pair, we obtain the following relationships:

\[
\hat{n} \times \frac{E_2 + E_1}{2} = Z_e \hat{n} \times (H_2 - H_1) \tag{1.20}
\]

and

\[
\hat{n} \times \frac{H_2 + H_1}{2} = -(1/Z_m) \hat{n} \times (E_2 - E_1). \tag{1.21}
\]

This says that the impedances \( Z_e \) and \( Z_m \) introduce field discontinuities that enforce arbitrary field distributions across an interface. This is akin to implementing the surface currents of the equivalence principle. Therefore regardless of whether the problem is viewed as engineering surface impedances or surface current densities, the result is the same. Furthermore, the equations (1.20) and (1.21) provide a complete description of the problem in Fig. 1.3, subject to several conditions [5]. First, the EM fields are macroscopic, and thus the distances considered from the metasurface must also be macroscopic for this description to be valid. In addition to this, when implementing \( Z_e \) and \( Z_m \) the metasurface unit cells must be sub-wavelength enough that homogenization applies. This brings to mind the effective media treatment of volumetric metamaterials, and so we classify the structures here as metasurfaces. Finally, the unit cells must be sparse enough with respect to one another so that quadrupole and higher-order contributions can be neglected.

The description in (1.20) and (1.21), but using instead the perspective of polarizabilities, has been named the “Generalized Sheet Transition Conditions” (GSTCs) [5]. The full GSTCs include the normal components of the fields however, so this is a simplification. Though the perspective of polarizabilities is recent, this description in (1.20) and (1.21) is not new. The interaction of macroscopic EM fields with a metasurface in a homogeneous medium can be modeled by an impedance boundary condition and its magnetic dual [6].
The impedance boundary condition is expressed as

\[
\hat{n} \times (E_2 - E_1) = 0, \tag{1.22}
\]
\[
\hat{n} \times (H_2 - H_1) = J_s, \tag{1.23}
\]

and

\[
\hat{n} \times E_1 = Z_e J_s. \tag{1.24}
\]

Taking the magnetic dual, the admittance boundary condition which results is

\[
\hat{n} \times (E_2 - E_1) = -M_s, \tag{1.25}
\]
\[
\hat{n} \times (H_2 - H_1) = 0, \tag{1.26}
\]

and

\[
\hat{n} \times H_1 = (1/Z_m)M_s. \tag{1.27}
\]

Fig. 1.3 can also be used to illustrate (1.22) to (1.27). Provided that the impedances and the admittances are collocated and do not couple, these boundary conditions can be combined to obtain the more compact description in (1.20) and (1.21). These impedances can be related to the canonical two-port network parameters through a lattice circuit model \cite{7}. In addition to this, infinite array analysis, a locally periodic approximation, and transmission-line theory \cite{8, 9} are used to physically design the unit cells implementing the impedance profiles.

From \cite{4}, we note that electric and magnetic sheet current densities, and thus $Z_e$ and $Z_m$, can be interpreted as arrays of electric and magnetic dipoles. The infinitesimal magnetic dipole is equivalent to an infinitesimal electric loop. Thus, the surface impedances located along the same boundary, enforcing the equivalence principle, can be engineered through sub-wavelength arrays of collocated electric dipoles and loops. These electric dipoles and loops are implemented in this work as reactive sheets. Pairs of orthogonal collocated electric and magnetic dipoles, corresponding to the orthogonal electric and magnetic currents, are essentially Huygens’ sources. This leads to the naming of these metasurfaces as "Huygens’ Metasurfaces".

Due to both the equivalence principle and the uniqueness theorem, $\mathbf{Z}_e$ and $\mathbf{Z}_m$ constitute a transfer function between a predetermined excitation and the desired response. This includes field polarizations. Note the dyadic notation on the impedances here. The equivalence theorem allows the use of just the tangential fields to completely specify the surface currents on the boundary that enforce the transfer function. The uniqueness theorem guarantees that for given excitation fields on one side supporting those engineered surface currents, the transmitted and reflected fields will be recovered. In this case, the tangential fields at all boundaries are known and sufficient to uniquely specify the fields in both regions across the interface. Practically however, we are limited not only by the applications that can be conceived, but also by the impedances that can be synthesized. Ideally, the impedances are passive, lossless, and moderate in value. Often though these are active, lossy, and extreme. Thus, further research involves both the synthesis of improved unit cells, as well as methods for approximating the required transfer functions with more realizable ones. In this work, we focus and implement this ideal case to a good approximation for the application of 1D and 2D plane-wave refraction, as well as 1D Gaussian-beam to Gaussian-beam focusing.
1.2 Review of Metamaterials, Other Metasurface Descriptions, and Related Structures

This review is not meant to be exhaustive, but to provide a general overview. Engineered reflection and refraction are not new ideas, with lenses, prisms, and mirrors being in use for thousands of years. In 1968 Veselago first theorized the existence and implications of a negative-index metamaterial (NIM) [10], but this remained merely of theoretical interest since it was thought that media with negative \( \mu \) did not exist. It was not until recently that practical structures were devised to achieve negative \( \varepsilon \) [11] and \( \mu \) [12]. These were composed of thin arrays of wires and split-ring resonators (SRRs), respectively. These were subsequently combined [13] to fabricate the first volumetric NIM. Since then, metamaterials have begun again to be of practical interest, though modern metamaterials research is not restricted to just engineering a negative index.

Due to this interest in metamaterials, there has been much research done in this emerging field. Much further research into the wire and SRR media has been done, for example in [14]. Here the propagation of EM waves is observed below the cutoff frequency of the fundamental mode in a waveguide, in accordance with theoretical predictions, at the same time demonstrating the negative \( \mu \) properties of an SRR. Media with only one of \( \varepsilon \) and \( \mu \) negative are opaque to EM waves, the waves being evanescent. It is common to represent a waveguide below the fundamental cutoff mode as a plasma, since both can be represented with a negative \( \varepsilon \) and have similar frequency characteristics. Because of this, it is only when the frequency bands of the waveguide and SRR occur simultaneously for negative \( \varepsilon \) and \( \mu \) respectively, that propagation can occur where it previously did not in the separate structures. This demonstrates that the SRR has negative \( \mu \) and provides verification of NIM theory. In [15], the frequency dependence of the magnetic resonance of the SRR on its geometrical properties is investigated. Since the wire media can also be modeled as a plasma, these have negative \( \varepsilon \) below their plasma frequency. It is thus primarily the narrowband magnetic resonance of the SRR that limits the operating frequency bandwidth and location of a NIM. Thus in the interest of applications at different frequency bands, the geometrical dependence of the magnetic resonance of the SRR is explored. The same work also distinguishes between frequency resonances due to electric, magnetic, and periodic responses of the SRR. The SRR is capable of an electric response due to a dipole-like charge distribution, whereas the gap is responsible for the magnetic response. Thus, removing the gap in the SRR and comparing the response to that of a regular SRR shows an electric response when their resonances coincide, and a magnetic response when a resonance appears only in the response of the regular SRR.

Maxwell’s equations are form invariant under a transformation of coordinate system, but this is not the transformation of the physical space itself. Such a transformation can be used to intuitively implement some desired functionality, such as a cloak [16–18], over the actual space. This transformation can be interpreted in the original space as a new set of material parameters, \( \varepsilon \) and \( \mu \), and lends itself well to volumetric metamaterials. In [19] this invariance was applied to an entirely different problem, the computational simplification of photonic band structures, at a time when there was interest in devices such as photonic insulators. This led to the further insight of the interpretation of desired functionality in one coordinate system as a new set of material parameters in another. Thus in conjunction with the development of volumetric metamaterials, transformation optics has emerged as a powerful design tool, and an overall view is given in [20].

Due to the challenges presented by loss, temporal dispersion, and fabrication, metasurfaces, the 2D analog to volumetric metamaterials, have attracted much interest. In many cases, the metasurface can encode the same functionality as its metamaterial counterpart. Examples of metasurfaces are the high-impedance surfaces of [21] and the holographic impedance surface of [22]. In electronic devices, a common situation is
to have an antenna over a ground plane, which can be approximated as a perfect electric conductor. For low profile reasons, it is desired that the antenna be as close to the ground plane as possible. However, mounted flush against the ground plane, from image theory it is seen that the antenna current cancels with its image, and the setup does not radiate efficiently. Furthermore, induced surface current propagation and scattering causes other artifacts in the radiation pattern such as nulls, cross-polarization, and side-lobes. The high-impedance surface implements a perfect magnetic conductor over a frequency band, and an antenna mounted flush against such a conductor has its radiation reinforced and is free from surface current propagation, making such a low-profile design feasible. In the holographic impedance surface, surface-wave propagation is taken advantage of to scatter an EM wave into a desired radiation pattern based on a given input wave and an interference pattern, much like in classical optical holography. Besides holography, more general descriptions exist for the interaction between the surface wave and an impedance modulation [23].

For a more detailed review of the literature pertaining to metasurfaces, see [24].

Though not in the field of metamaterials, another approach taken to reflect and transmit incident waves in desired fashion is that of reconfigurable reflect and transmit arrays. These structures cannot be considered as metamaterials because their unit cells are typically too large to be homogenized in an effective medium model. Instead, these have been used in the antenna community for low-profile, high-gain dynamic beamforming applications in place of phased arrays which tend to be bulky, lossy, complex, and expensive. A concise and extensive literature review of reflect arrays is found in [25], and for reconfigurable transmit arrays this can be found in [26].

1.3 Review of Metasurface Literature utilizing the Generalized Sheet Transition Conditions

The GSTCs of [5] model the interaction of macroscopic EM fields with a metasurface in a homogeneous medium. The GSTCs there are not the first, but are more general than previous attempts, and an overview of that literature is described there. Described originally in terms of macroscopic surface polarizabilities, and later on in terms of susceptibilities, these can be related to the surface currents of the equivalence principle. These surface currents can then be related to impedances as described above. However, the GSTCs here will refer strictly to the polarizability and susceptibility formulations, not the impedance formulation addressed in this work. In order for the GSTCs to represent the problem, the supporting conditions of homogenization and sparsity need to be met. Since the EM fields are macroscopic, the distances from the metasurface must be comparable to this for this description to be valid. The derivation is given in detail in [5], and also concisely summarized in [27].

The GSTCs have been applied to the reflection and transmission of plane waves using a metasurface in [28]. However, there the derivation is restricted such that the transmitted and reflected angles are the same as the incident. Total transmission and reflection are derived for plane waves of arbitrary angle of incidence at both TE and TM polarizations. These conditions may require negative electric or magnetic susceptibilities. In contrast, the Huygens’ metasurface here is able to achieve total reflection and transmission for plane waves of arbitrary incidence, reflected, and transmitted angles. That is, these do not need to be identical. Furthermore, the metasurface there is composed of homogenous unit cells. In contrast to this, the Huygens’ metasurface is in general inhomogeneous, allowing for more general applications. The phenomenon of total reflectance and transmission occurring in a metasurface for arbitrary angles of incidence is further investigated in [29]. This is found to occur due to the resonant nature of the scatterers, such that the transverse susceptibilities are larger than the normal susceptibilities. This behavior degrades with losses. However, total reflection and transmission with arbitrary incident, reflected, and transmitted angles is still
not discussed there. In [30], the results in [28] are verified experimentally with yttrium-iron-garnet particles within a waveguide, but not in free space. Here we validate experimentally the GSTCs in free space with our Huygens’ metasurface. In contrast to the ceramic magnetodielectric spheres used there, our design consists of printed elements which can be manufactured using standard PCB fabrication techniques. These printed elements are able to synthesize the full range of impedances (and thus the susceptibilities) required. These GSTCs are then generalized further in [27] to include a material interface.

In [31] the GSTCs are used for parameter extraction of a metamaterial, accounting for boundary effects. The literature pertaining to the problem of boundary effects on parameter extraction is reviewed there. The connection between surface susceptibilities pertaining to a metasurface and its description in terms of bulk effective medium parameters is explored in [32]. Forcing a bulk media description upon a metasurface, the bulk parameters can no longer be considered intrinsic. Instead they vary with the thickness of the metasurface. The description of a metasurface as an infinitely thin surface is more natural than as a thin sheet of bulk media. In this case, the susceptance (or in our case, the impedance) description afforded by the GSTCs is unique to the problem.

In [33] the result of [28] was used to design a metasurface waveguide with low material and radiation losses. The further application of a metasurface to leaky-wave antennas is mentioned but not addressed in detail there. In [34] the GSTCs are used to explore guided wave propagation along a metasurface for both TE and TM polarized incidence. The metasurfaces are found to support backward and forward waves as well as analogs to surface-plasmon polaritons as well as dielectric surface waves. Furthermore, in contrast to classical media, up to three guided waves may exist independently. Finally in [35], leaky wave propagation along a metasurface is discussed.

Further applications for the GSTCs have included extraction of susceptibilities for both mono and bi-anisotropic scatterers [36–38], shielding [39], resonators [40], tunable metasurfaces [41], and plasmonic metasurfaces [42].

1.4 Motivations to the Huygens’ Metasurface and Comparisons to Related Designs

For applications such as satellite, point-to-point communications, and radar, narrow beams and high directivity are required. Satellite communications must efficiently cover long distances involving small angular sectors. Point-to-point communications allow maximized use of the frequency spectrum where several antennas are situated close together. The resolution of radar at the same range but different bearings is limited by the half-power beamwidth, outside of which the signal does not produce a useful return. Traditional approaches to achieve this high directivity include reflectors and lenses, which are not planar, are bulky, and are difficult to scan because this must be done mechanically. On the other hand, phased arrays are planar, and can be scanned with the inclusion of phase shifters, but these have a feed network which increases quadratically with the size of the array. Not only does this increase complexity, but losses increase with both frequency and the size of the aperture. These issues motivate spatial feeding of antenna arrays, such as with reflect and transmitarrays. These have the advantage of a low profile, yet without the complicated and lossy feed network of a phased array. However, for a traditional transmitarray, to obtain 360° degrees phase control and good matching, 3-4 λ/4 spaced layers have been shown to be the optimal solution [43]. The concept of a Huygens’ metasurface, proposed in [2,3], maintains this phase control and matching, while dramatically reducing the corresponding thickness.

The Huygens’ metasurface physically implements boundary conditions described by the surface equivalence principle. That is, it engineers the electric and magnetic surface currents that are excited there. This
boundary condition nature permits the same degree of phase control and matching as in traditional transmitarrays, but in a much thinner profile. This provides the flexibility necessary for a wide range of interfacial beamforming applications, which besides the plane-wave refraction and Gaussian-beam to Gaussian-beam focusing considered here have included Gaussian-beam to Bessel-beam transformation [3], active cloaking [44], and beamforming with full polarization control [45]. This design philosophy is in contrast to the traditional transmitarray, where each resonant element is tuned to the desired phase while maintaining good matching. In the Huygens’ metasurface, the surface currents of the equivalence principle are excited on collocated impedance and admittance sheets, which are implemented with orthogonal electric and magnetic dipoles, respectively. Collocated, orthogonal pairs of electric and magnetic dipoles can correspond to Huygens’ sources, from which the name Huygens’ metasurface is derived. The characteristic cardioid pattern of a Huygens’ source explains the good matching that can be achieved, since the radiation out the back of the cardioid is small.

The optimal 3-4 \( \lambda/4 \) spaced layers solution of the traditional transmitarray is due to its exclusively electric response. Since the overall thickness is not electrically small, it cannot be considered as a single interface, nor can it physically implement the surface equivalence principle. In contrast to this, the Huygens’ metasurface has both an electric and a magnetic response. Though the design occupies three layers, the overall thickness is about \( \lambda/9.3 \). Due to this extreme sub-wavelength nature, this can be considered as one interface from an EM perspective. Furthermore, the number of layers and their spacing here are just due to the physical implementation used, and are not required by theory which is the case of traditional transmitarrays. In other words, a Huygens’ metasurface theoretically comprises a single layer of electric and magnetic dipoles. Therefore, further investigations into possible unit cell designs could reduce the number of layers and their spacing even further.

More recently, it has been claimed that a certain class of thin transmitarrays generates a magnetic response from currents circulating along electric impedance sheets [46, 47]. From symmetry arguments, the circulating currents between sheets due to the magnetic response cancel except in the outer sheets, forming a loop of current. In contrast, the circuit model for a Huygens’ source unit cell is a lattice network. This provides a rigorous circuit theory analog to a set of collocated EM boundary conditions enforcing the equivalence principle [7]. In this model, the magnetic response is clearly identified and is independent of the electric response. This has been used to accurately model the electric and magnetic properties of dipoles, loops, and Huygens’ source unit cells in chapter 2.

In [48], 1D interfacial refraction and its implications on efficiency are described, without prescribing a method of implementation. This is improved upon in [2], where the boundary conditions for collocated impedance and admittance surfaces are applied to the problem. This provides the specific impedance and admittance profiles at the interface to enforce plane-wave refraction. However, unit cell synthesis is only briefly touched upon. In chapter 2, the results of [2] are used with the lattice model in [7] to synthesize such a Huygens’ metasurface. The systematic design procedure, starting from the collocated boundary conditions, is illustrated in detail. This also includes investigations into unit cell discretization, and the actual synthesis of practical unit cells using transmission-line theory, infinite array analysis, and locally periodic approximations. Furthermore, the physical intuition into the unit cell synthesis is presented alongside the procedure. This design for 1D plane-wave refraction has been fabricated and characterized, and serves as the groundwork to identify issues pertinent to the synthesis, experimental setup, and characterization of more complex designs, such as the ones presented in chapters 3 and 4. In chapter 3, the 2D extension to 1D interfacial refraction in [2] is implemented with a Huygens’ metasurface. Experimentally, 2D refraction achieves a high measured efficiency over both the oblique incidence and frequency response characterizations. This demonstrates that the insertion loss and reflections are low in the Huygens’ metasurface. The total efficiency is also measured, and this includes the effect of side-lobes in addition to matching and insertion loss. The power contained in
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the side-lobes is observed to be low in general in the Huygens’ metasurface.

In [3], the theoretical analysis for the elimination of both side-lobes and reflections in a single Huygens’ metasurface, accounting for power conservation, results in complex impedances at the interface. Thus numerical optimization is used to approximate the required impedances with passive and lossless solutions, and this extra requirement is avoided here entirely. This is because the problem here is the modified one in [2], and slight reflections are permitted, resulting in entirely reactive impedances. Furthermore, the design here consists of printed elements on two-bonded boards instead of many stacked, interspaced layers. This simplifies the fabrication, and allows for scaling to higher frequencies without the alignment issues associated with interspaced designs.

Lastly, Kymeta Corporation is an example of a company that has taken metasurface technology successfully into commercial applications. Their metasurface-based reconfigurable transmitarray technology has been applied to address the ever-increasing demand for on-the-go wireless connectivity, and the need for the infrastructure to support it, specifically on the side of the user-terminal. While wireless connectivity is commonplace in developed populations, connectivity in settings such as commercial airlines, shipping routes, as well as remote areas has much opportunity for further development of the infrastructure. Traditional solutions such as mechanical terminals and phased arrays tend to be large, heavy, expensive, power intensive, and difficult to install, among other things. In contrast to this, Kymeta’s mTenna using metasurface technology is manufactured with existing lithographical technology, and is being applied to portable satellite terminals about the size, weight, and power consumption of a standard notebook computer; low-profile fixed-satellite terminals which are easily installed even in remote areas; aeronautical terminals small enough to be installed even on the fuselage of small aircraft; and many other applications due to its flexibility and scalability. This technology stands to provide much in terms of social benefits, such as making internet accessible to poor regions of the world, field journalism, mobile hospitals, and aid deliveries, to name a few. The company has recently reached the milestone of demonstrating a Ka-band satellite link with its technology, and has secured $12 million in investments. This example proves that the opportunities for metasurface technology to be applied are abundant, and the potential benefits on society are likewise numerous. Such possibilities in an emerging field cannot be ignored.

1.5 Thesis Outlook

To date not much attention has been given to the discussion of the design of the constituent unit cells in a Huygens’ metasurface. This process will be demonstrated here in detail for the application of 1D refraction. The design will then be further validated in simulations and experimentally. 1D refraction will serve as the groundwork for more complicated designs. From the experience gained, the design and experimental characterization techniques will be improved upon considerably. These will then be applied to 2D refraction and 1D focusing, except that the experimental validation for the latter will be left to future work. This will confirm the theoretical foundations of the Huygens’ metasurface as well as prove the utility of the concept.

1.6 Thesis Outline

The outline of the remainder of this work is summarized here. In chapter 2, the design procedure of the unit cells for a Huygens’ metasurface is presented for the application to 1D refraction. The lattice circuit model, infinite array analysis, locally periodic approximation, transmission-line theory, and the effects of unit cell discretization are all addressed there. Then simulation and experimental results are presented for the 1D refraction Huygens’ metasurface. In chapter 3, this 1D refraction theory is generalized to 2D.
and experimental results are presented. In chapter 4, the theoretical background for 1D Gaussian-beam to Gaussian-beam focusing is developed, followed by simulation results. Finally, chapter 5 concludes with a summary, the major contributions, and the future directions.

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CHAPTER 2

DESIGN OF UNIT CELLS FOR THE HUYGENS’ METASURFACE

In order to illustrate the design of unit cells for the Huygens’ metasurface in detail, the theory from chapter 1 will be applied to the case of 1D plane-wave refraction. Following this, in subsequent chapters the unit cell synthesis will not be detailed. It must be stressed that this 1D refraction case is taken as the groundwork for the later designs. That is, with the help of this design, pertinent issues to the fabrication of the Huygens’ metasurface, experimental setup, and characterization should be identified here. These are then addressed in later chapters.

2.1 Design Procedure for 1D Refraction

Here we refer back to Fig. 1.3. Taking (1.20) and (1.21), we stipulate normally incident and reflected plane waves, as well as a 30° refracted plane wave as the field distributions on both sides. The design frequency is 10.00 GHz. That is \( \mathbf{E}_1 = \mathbf{E}_{\text{inc}} + \mathbf{E}_{\text{refl}} \) and \( \mathbf{E}_2 = \mathbf{E}_{\text{refr}} \) where

\[
\mathbf{E}_{\text{inc}} = E_{\text{inc}} e^{-jk_{\text{inc}} \cdot \mathbf{r}},
\]

\[
\mathbf{E}_{\text{refl}} = E_{\text{refl}} e^{-jk_{\text{refl}} \cdot \mathbf{r}},
\]

and

\[
\mathbf{E}_{\text{refr}} = E_{\text{refr}} e^{-jk_{\text{refr}} \cdot \mathbf{r}}.
\]

Here \( E_0 \) and \( k \) are the corresponding amplitudes and wavevectors, \( \mathbf{r} \) is the radial coordinate vector, and \( j \) is the imaginary unit. The time dependence is \( e^{j\omega t} \) and this time-harmonic form will be employed throughout, therefore this notation will be dropped.

Following the result in [2], in order to obtain passive and lossless electric and magnetic impedances \( Z_e \) and \( Z_m \), the amplitudes must be uniquely specified. Otherwise, these become active or lossy at different positions along the interface. To aid in this discussion we repeat the result for the amplitudes

\[
E_{\text{refl}} = \frac{E_{\text{inc}} (\cos \theta_{\text{inc}} - \cos \theta_{\text{refr}})}{\cos \theta_{\text{inc}} + \cos \theta_{\text{refr}}},
\]

and

\[
E_{\text{refr}} = \frac{2 E_{\text{inc}} \cos \theta_{\text{refr}}}{\cos \theta_{\text{inc}} + \cos \theta_{\text{refr}}}.
\]

Here \( \theta_{\text{inc}} \) and \( \theta_{\text{refr}} \) are the incident and refracted angles, respectively. Note that the reflected field is in
general much smaller than the transmitted field. The application of the design parameters and (2.1) to (2.5) into (1.20) and (1.21) results in the electric and magnetic impedances shown in Fig. 2.1 and 2.2 respectively.

![Electric impedance profile](image)

**Figure 2.1:** Electric impedance $Z_e$ profile. This in conjunction with $Z_m$ transforms a normally incident plane wave to a $30^\circ$ refracted plane wave at 10.00 GHz. Side-lobes and reflections will be suppressed.

![Magnetic impedance profile](image)

**Figure 2.2:** Magnetic impedance $Z_m$ profile. This in conjunction with $Z_e$ transforms a normally incident plane wave to a $30^\circ$ refracted plane wave at 10.00 GHz. Side-lobes and reflections will be suppressed.

Using the lattice cell model [7] detailed in section 2.3, the uncoupled electric and magnetic impedances can be related to the impedance matrix of a standard 2-port network. There, the electric and magnetic responses correspond to the shunt and series branches of the lattice network, respectively. Converting this to the scattering matrix representation, the phase of the transmission coefficient $S_{21}$ can be compared to the expected result for plane-wave refraction according to [48, 49]. From the expected result a linear phase profile is seen. Furthermore, according to (2.4) and (2.5), for the passive and lossless Huygens’ metasurface, we expect the magnitudes of $S_{11}$ and $S_{21}$ to be nearly zero and unity, respectively.

The gradient of the linear phase profile determines the spatial periodicity of $Z_e$ and $Z_m$. Fig. 2.3 illustrates the scenario and its definitions to aid the discussion. From either source [48,49], the phase insertion gradient along the infinitesimal section of boundary is given by

$$
\sin \theta_{\text{refr}} - \sin \theta_{\text{inc}} = \frac{1}{k} \frac{d\Phi}{dl}.
$$

(2.6)
Chapter 2. Design of Unit Cells for the Huygens’ Metasurface

2.2 Effects of Unit Cell Discretization

Furthermore, $Z_e$ and $Z_m$ are implemented practically by taking the profiles in Fig. 2.1 and 2.2 and then discretizing them into unit cells. Taking the discretized versions of $Z_e$ and $Z_m$ into the lattice model, we utilize the transmission-line model (TLM) full-wave solver developed in [16]. This allows us to determine the quality of the refracted beam in terms of side-lobes and reflections for a given discretization. The merit of this method is that it can be done without first synthesizing an entire profile of unit cells. On the other hand, this does not account for the complex geometries of the unit cells. The solver utilizes the impedance values alone. However, this allows for the investigation of discretization before embarking on the laborious task of synthesis for an entire unit cell profile. More detailed simulations of the entire Huygens’ metasurface including unit cell geometries are done once an acceptable discretization level is committed to. In terms of $S_{21}$, the chosen discretization along the boundary determines the transmission phase that each Huygens’ source unit cell must provide.

Fig. 2.4 demonstrates the effects of discretization on the quality of the refracted beam. Based upon these investigations, for this design, we settle on a discretization level of $\lambda/10$ or 3 mm at 10 GHz. This means that the $Z_e$ and $Z_m$ profiles are implemented with a period of 20 unit cells. This choice results in the first side-lobe and the maximum reflection being $-26.56$ dB and $-22.81$ dB from the main-lobe maximum, respectively. The increased discretization reduces the side-lobes and reflections because of the homogenization condition underlying (1.20), (1.21).

2.3 Unit Cell Design

The Huygens’ source can be envisioned as collocated infinitesimal electric and magnetic dipoles. Provided that the unit cell is sub-wavelength, the magnetic dipole can be approximated by an electric loop. In addition to this, the orientations of the dipoles must correspond with the polarizations of the input and output fields in (1.20), (1.21). This reflects the anisotropies of the impedances required to enforce the surface currents.

Keeping the above in mind, unit cells were synthesized in the HFSS commercial full-wave EM solver to obtain the required electric and magnetic impedances. Here infinite array analysis was used. This result makes sense in view of the transmission-line description of infinite array analysis [8]. Furthermore,
Figure 2.4: Total electric far-field radiation pattern. This reveals the effects of discretization on the quality of the refracted beam. This also determines the size of the unit cells in the Huygens’ metasurface. For this design, we settle on a discretization level of $\lambda/10$ or 3 mm at 10 GHz.

it is consistent with the earlier conversion of the lattice model for the Huygens’ source into transmission-line scattering parameters. Though the Huygens’ metasurface is in fact inhomogeneous, this analysis still provides an accurate result as the geometrical variations are slight along the interface. This is demonstrated in later sections in the simulations and experimental results. Fig. 2.5a and 2.5b show an example unit cell and one period of the fabricated metasurface. The printed capacitors synthesizing the impedances are visible. The detailed geometrical parameters, and the $S$ and $Z$-parameters of the entire unit cell profile, are found in the appendix, Tables. A.1 and A.2. The Huygens’ source unit cell implements a loop with the outer conductor layers, while the dipole is contained in the conductor within. Note that the 1D refraction is in the horizontal plane, and the unit cell is compatible with the field polarizations stipulated in this problem. The input and output electric field polarizations are vertically oriented. The input and output magnetic field polarizations are in the horizontal plane. The unit cells are designed as an entirely printable metasurface, and are implemented on a Rogers RO3003 substrate [$\varepsilon_r = 3.00$, tan(δ) = 0.0013].

Figure 2.5: Unit cell and Huygens’ metasurface. (a) Example of a unit cell implementing a Huygens’ source. The collocated infinitesimal electric and magnetic dipoles are approximated by sub-wavelength electric dipoles and loops. The printed capacitors synthesizing the impedances are visible. (b) One period of the fabricated Huygens’ metasurface.

In order to implement the entire range of impedances and admittances in Fig. 2.1 and 2.2, sufficient unit cell area must be retained. Otherwise, not all the required values can be patterned in the conductor. This limits the discretization that can be achieved. The unit cell dimensions are $\lambda/10$ wide by $\lambda/5$ tall by $\lambda/10$ thick. Note that since refraction is in the horizontal plane, the relevant dimensions for discretization are the horizontal and normal directions. This provides greater surface area to synthesize a wider range of
impedance values.

Since the Huygens’ source unit cell collocates the dipole and loop, coupling between these structures is minimal. This can be demonstrated by application of the lattice circuit model of [7] and parametric sweeps of the geometries. The lattice model is reproduced here in Fig. 2.6 to aid this discussion. The two-port

\[ I_1 \quad \frac{Z_m}{2} \quad I_2 \]
\[ V_1 \quad 2Z_e \quad 2Z_e \quad V_2 \]
\[ \frac{Z_m}{2} \]

Figure 2.6: Lattice circuit model. The two-port lattice network description. The electric impedances \( Z_e \) correspond to the shunt branches, while the magnetic impedances \( Z_m \) correspond to the series branches.

The impedance matrix of the symmetric and reciprocal lattice circuit model is related to the shunt \( Z_e \) branches and the series \( Z_m \) branches through

\[ Z_e = \frac{Z_{11} + Z_{21}}{2}, \]  
\[ (2.7) \]

and

\[ Z_m = 2(Z_{11} - Z_{21}). \]  
\[ (2.8) \]

Here \( Z_{11} \) and \( Z_{21} \) belong to the impedance matrix of a standard two-port network. For the parametric sweeps, the geometries of the printed dipole and loop are swept in the unit cell including the substrate but excluding the other component. Fig. 2.7a and 2.7b show the geometries of a typical printed dipole and loop, as well as the parameters being varied.

\[ g \]
\[ w \]
\[ w \]
\[ (a) \]
\[ (b) \]

Figure 2.7: Printed element geometries. (a) The geometry of the printed dipole. The capacitor width \( (w) \) and gap spacing \( (g) \) are varied to achieve \( Z_e \). (b) The geometry of the printed loop. The capacitor width \( (w) \) is varied to achieve \( Z_m \).

Fig. 2.8a and 2.8b illustrate the parametric sweeps of \( Z_e \) and \( Z_m \) from the lattice model as a function of the capacitor gap spacing in the printed dipole. The different curves represent various capacitor widths. Fig. 2.9a and 2.9b illustrate the parametric sweeps of \( Z_e \) and \( Z_m \) as a function of the capacitor width in the printed loop. Here it is seen that for the printed dipole, there is variation in \( Z_e \) but not in \( Z_m \). This demonstrates that the printed dipole responds electrically but not magnetically. This is ideal, since this causes \( Z_e \) and \( Z_m \) in the combined Huygens’ source unit cell to be uncoupled and therefore independent. However it is seen that while the printed loop responds primarily magnetically, there is still a slight electric response.

Note that the substrate alone was simulated as well, and in this case \( Z_e = -176.47 \, \Omega \) and \( Z_m = 268.42 \, \Omega \).
Chapter 2. Design of Unit Cells for the Huygens’ Metasurface

Figure 2.8: Printed dipole responses. (a) The electric response of the dipole component of the Huygens’ source unit cell alone. \( Z_e \) is plotted against the capacitor gap size for the various capacitor widths. (b) The magnetic response of the dipole component of the Huygens’ source unit cell alone. \( Z_m \) is plotted against the capacitor gap size for the various capacitor widths.

Figure 2.9: Printed loop responses. (a) The electric response of the loop component of the Huygens’ source unit cell alone. \( Z_e \) is plotted against the capacitor width. (b) The magnetic response of the loop component of the Huygens’ source unit cell alone. \( Z_m \) is plotted against the capacitor width.

Since the printed dipole has \( Z_m \) almost identical to that of the substrate, this reinforces the notion that it responds purely electrically. \( Z_m \) of the printed dipole can be viewed as belonging to the dielectric-filled transmission-line it is immersed in. The conductor comprising the printed dipole has itself no contribution. The printed dipole has no closed-loop area for the changing magnetic flux to generate a response. In contrast to this, the printed loop, which does have a closed-loop area for the changing magnetic flux, responds magnetically in a dramatic fashion. In fact, there is a strong magnetic resonance in \( Z_m \). However, it can be seen in Fig. 2.9a that there is a weak electric response in \( Z_e \). This is because the printed loop forms a shunt element capacitively coupled to the transmission-line, albeit weakly. These electric and magnetic responses for the printed loop are similar to those found for a split-ring resonator (SRR), which this loop essentially is. This shunt element shows up in the lattice model as the electric component of the response, \( Z_e \). Thus the dynamic range of \( Z_m \) is much larger than that of \( Z_e \) in the printed loop. Furthermore, this agreement of the results with intuition demonstrates that the best model for an infinite array of dipole, loop, or Huygens’ source unit cells is the lattice network.

To a good approximation then, the combined Huygens’ source unit cell can be treated as having the magnetic response exclusively due to the printed loop. Taking the electric response to be purely due to the printed dipole provides a good starting point, but because the printed loop has an electric response similar
in dynamic range, the $Z_e$ of the resulting unit cell can deviate noticeably from the profiles in Fig. 2.1 and 2.2. In addition, another source of coupling between the printed dipole and loop comes from the interaction of currents with the patterns of the conductors themselves.

Here an HFSS optimization analysis was run to fine tune each unit cell to its required point on the impedance profiles in Fig. 2.1 and 2.2. Fig. 2.10 shows one period of the realized electric and magnetic impedances corresponding to each unit cell superimposed on their ideal profiles. The impedance profiles agree well except near the resonances, where the more extreme impedances are difficult to synthesize exactly.

![Figure 2.10: Realized and ideal impedances. One period of the realized electric and magnetic impedances corresponding to each unit cell superimposed on their ideal profiles. This implements the Huygens’ metasurface which refracts a normally incident plane wave in the 30° direction.](image)

Finally, to demonstrate that the unit cells do indeed function as spatially fed Huygens’ sources, Fig. 2.11 presents the element factor of a unit cell from the profile in Fig. 2.10 in the infinite array analysis. That is, the element factor includes the contributions of mutual coupling in a infinite array. The unit cell is illuminated with a plane wave, and the characteristic cardiod far-field pattern is seen in both the E and H planes.

### 2.4 Simulated and Experimental Results for 1D Refraction

To experimentally characterize this design, we use a near-field scanner in conjunction with an ATM 90-441-6 horn antenna and a 203.2 mm double-convex hyperbolic Rexolite ($\varepsilon_r = 2.53$, $\tan(\delta) = 0.00066$ at 10.00 GHz) lens. The output after the lens launches an approximate Gaussian beam with a measured waist size of 27.5 mm which is small enough to impinge 99.9% of the power upon the metasurface at its waist. Fig. 2.13 shows the schematic diagram of the experimental setup used, taken from previous work but modified for oblique incidence measurements [50]. Table. 2.1 shows the dimensions of the setup. Fig. 2.12 shows a photograph of the experimental setup used.

Using this system, we captured the frequency response, the response to oblique incidence, as well as the near-fields with a near-field scanner. Here we will present both the simulated and experimental results for comparison.

Fig. 2.14 presents the total electric far-field radiation patterns of the HFSS simulated results at the design frequency of 10 GHz, and the measured results at the optimal frequency which has shifted to 10.35 GHz. The simulated results include dielectric and conductor losses, surface roughness effects, and the patterning of the conductors. This simulation utilizes a maximum of 25.5 GB of random access memory (RAM), and on
Figure 2.11: Element factor of a Huygens’ source unit cell from the profile in Fig. 2.10, including the contributions of mutual coupling in the infinite array analysis. The characteristic cardiod far-field pattern is seen in both the E and H planes.

Figure 2.12: Experimental setup. Includes the near-field system, horn, lens, and Huygens’ metasurface.

Figure 2.13: Experimental setup. Includes the horn, lens, and Huygens’ metasurface.

an Intel Core i7-3820 running on 2 cores, takes about 45.3 hours to converge to within 2% relative energy error. These characteristics are typical of the frequency response and oblique incidence response simulations
Table 2.1: Dimensions of the setup used to characterize the 1D refraction Huygens’ metasurface. The units of all quantities are in mm.

<table>
<thead>
<tr>
<th>h₁</th>
<th>h₂</th>
<th>h₃</th>
<th>f₁</th>
<th>z₀</th>
<th>f₁ − z₀</th>
<th>f₂</th>
<th>d</th>
</tr>
</thead>
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<td>609.6</td>
<td>186.8</td>
<td>10.9</td>
<td>175.9</td>
<td>289.8</td>
<td>88.9</td>
</tr>
</tbody>
</table>

as well. For the measurements, due to limitations in the setup, only the response for the range of angles shown in the forward direction could be captured. The measured optimal frequency is taken to be the frequency with the minimum side-lobe level, after smoothing of the ripples in the measurement. For the simulated result, the maximum side-lobe level and reflection are seen to be −17.13 dB and −9.66 dB from the main-lobe maximum, respectively. For the measured result the maximum side-lobe level is seen to be −15.17 dB from the main-lobe maximum, while the reflections remain to be characterized experimentally. The locations of the main-lobe and side-lobes are seen to be approximately identical between the simulation and the measurement after smoothing of the measured data.

It must be stressed that the measured first side-lobe level should be taken qualitatively. Meaningful quantitative results require the presence of the ripples to be addressed. These ripples in Fig. 2.14 could be attributed to higher-order Gaussian modes exciting surface currents upon the metasurface. This is due to the poor coupling of the ATM 90-441-6 horn used to the fundamental Gaussian mode. In addition to this, the waist of the fundamental Gaussian mode in the ATM 90-441-6 horn is outside the limit of the paraxial approximation. Truncation effects at the lens cause a change in the waist position and size, which need near-field diffraction calculations to account for. In the far-field, truncation causes non-Gaussian components, which could be another source of ripples. See [51] for a detailed description of these factors. In addition to this, absorbers which perform optimally in the far-field may in fact become resonant at near-field distances. Given the strong agreement between the results, and comparison with [3], it seems plausible that the refraction performance in terms of reflections should likewise be promising. This is left for a future experimental investigation.

![Figure 2.14: Simulated and measured total electric far-field. Simulated results at the design frequency of 10 GHz, and measured results at the optimal frequency of 10.35 GHz. The simulated maximum side-lobe level is −17.13 dB, while the measured maximum side-lobe level is −15.17 dB.](image)

Fig. 2.15 and 2.16 show the simulated and measured frequency responses for several representative frequencies, including the design and optimal frequencies. The simulated results reveal that the first side-lobe level and the reflections are best at the design frequency of 10.00 GHz, being −17.13 dB and −9.66 dB, respectively. The side-lobe levels are −14.37 dB and −12.00 dB at 10.35 GHz and 10.50 GHz, respectively. The
reflections degrade to $-4.02\,\text{dB}$ and $-1.50\,\text{dB}$ at $10.35\,\text{GHz}$ and $10.50\,\text{GHz}$, respectively. The response is narrowband in nature, which can be accounted for by the highly dispersive nature of the SRR-like loops. The main-lobe shifts slightly left as the frequency increases, as per (2.6).

The measured results have been averaged here to remove ripples. These results are thus only useful for observing qualitative trends. More refined quantitative results, and their comparisons with simulations will be left to future work. The measured results reveal that the first side-lobe level is best at the optimal frequency of $10.35\,\text{GHz}$. The main-lobe location shifts left as expected. However, the main-lobe is seen to also narrow as the frequency increases. Most likely this is because the gain of the ATM 90-441-6 increases appreciably over the $0.5\,\text{GHz}$ range. In the simulations, the characteristics of the excitation are frequency invariant, so the effect is not noticeable there.

![Figure 2.15: Simulated total electric far-field frequency response. Given for several representative frequencies. The first side-lobe level and reflections deteriorate away from the design frequency of 10.00 GHz. The main-lobe location shifts left with increasing frequency.](image)

![Figure 2.16: Measured total electric far-field frequency response. Given for several representative frequencies. This should be taken qualitatively for its trends, which can be verified to be the same as the simulations. The only major difference is an extra narrowing of the main-lobe observed here, which can be accounted for.](image)

Fig. 2.17 and 2.18 show the simulated and measured responses to oblique incidence for several representative angles of incidence. The simulations reveal that, as the angles of incidence increases, the location of the main-lobe shifts right as expected. This can be predicted from (2.6). Unexpectedly, the first side-lobe level decreases monotonically as the angle of incidence varies from $-10^\circ$, instead of $0^\circ$. The back-lobe level
relative to the main-lobe is still best at normal incidence, which we designed for.

The measured results will again be taken for their qualitative trends. The location of the main-lobe shifts right with increasing angle of incidence, as expected. Due to ripples and the averaging process, the side-lobe performance here is difficult to interpret, even qualitatively. Since the results for an angle of incidence of $-20^\circ$ and $0^\circ$ are close, it is difficult to say which is superior. It does look like the result for an angle of incidence of $0^\circ$ is better, as expected, but there is room to question this because the margin is small. Generally speaking however, the measurements still agree well with the simulations.

![Figure 2.17: Simulated total electric far-field response to oblique incidence. Given for several representative angles of incidence. The location of the main-lobe shifts right as expected. The side-lobe level is best at angle of incidence $-10^\circ$, but the reflections are best at $0^\circ$.](image1)

Finally, Fig. 2.19a and 2.19b show the simulated transmitted near-field at the design frequency of 10 GHz, and the measured transmitted near-field at the optimal frequency of 10.35 GHz. That the refraction of the incident radiation is primarily in the desired direction is clearly seen in both results.

![Figure 2.18: Measured total electric far-field response to oblique incidence. Given for several representative angles of incidence. The trends here agree generally with the simulations.](image2)
Figure 2.19: Transmitted near-fields. (a) Simulated and (b) measured near-field results after normalization, showing clearly that refraction of the incident radiation is primarily in the desired direction. In both figures the metasurface is at the right boundary, and the incident beam is from the right. The dimensions of the boundaries are $14.13\lambda \times 21.33\lambda$, and the scale applies to both plots.
Chapter 3

Huygens’ Metasurface for 2D Refraction

Transmitarrays and transmissive metasurfaces must efficiently couple incident power to the transmitted beam. Inefficiency is manifested in side-lobe levels, reflections, and insertion loss. The Huygens’ metasurface embodies the Huygens’ and equivalence principles, suppressing these side-lobe levels and reflections. This is accomplished with a single, thin layer of Huygens’ sources, which contains both an electric and a magnetic response. In this paper, 2D interfacial refraction is implemented with a scalar Huygens’ metasurface. The measured total efficiency is on average 71.06% for a 70° range, the maximum being 80.87% at θ_i=0°. Moreover, it is on average 68.64% over a fractional bandwidth of 8%, the maximum being 80.87% at 10.0 GHz. This demonstrates that the insertion loss, reflection, and side-lobe powers are low in our design. Furthermore, the questions of polarization purity, and the appropriate polarization definition for a scalar Huygens’ metasurface, are addressed. Our design contains only printed elements, and consists of two bonded boards, instead of many stacked, interspaced layers. This simplifies fabrication, and makes it scalable to millimeter-wave frequencies and beyond. The design is also λ/9.3 thick, in contrast to traditional transmitarrays which require 3-4 λ/4 spaced layers to obtain the same degree of phase control and matching.

3.1 Derivation for 2D Refraction

3.1.1 Polarization Purity and Appropriate Polarization Definitions for Huygens’ Metasurfaces

Before undertaking the analysis of 2D refraction, an appropriate polarization definition needs to be determined. The Ludwig-3 polarization definition [52] is the appropriate one to use for a Huygens’ source or an array of Huygens’ sources. This has several important implications which match physical intuition.

The Ludwig-3 definition of co and cross-polarization follows the far-field lines radiated from an ideal Huygens’ source [53, 54]. Take an ideal Huygens’ source at the origin with the electric dipole along ĝ_y, and the the magnetic dipole along ĝ_x. Based on the Ludwig-3 definition, the electric field lines of this Huygens’ source define the co-polarization. The cross-polarization is orthogonal to that and is in the direction of the magnetic field. It must be stressed that these polarization directions are in general not ĝ_x, ĝ_y, ĝ_z, ĝ_θ, or ĝ_φ.
fact, the conversion to Ludwig-3 unit vectors from Cartesian unit vectors is

\[ \hat{v} = \cos \phi \sin \phi (\cos \theta - 1) \hat{x} + (\cos \theta \sin^2 \phi + \cos^2 \phi) \hat{y} - \sin \theta \sin \phi \hat{z}, \] (3.1)

and

\[ \hat{h} = (\cos \theta \cos^2 \phi + \sin^2 \phi) \hat{x} + \cos \phi \sin \phi (\cos \theta - 1) \hat{y} - \sin \theta \cos \phi \hat{z}. \] (3.2)

Here \( \hat{v} \) follows the electric field and is the co-polarization, and \( \hat{h} \) is the cross-polarization [55]. Therefore, it is seen that exciting a Huygens’ source with the magnetic and electric dipoles oriented along the \( \hat{x} \) and \( \hat{y} \) axes, it is possible to radiate a field that has \( \hat{x} \), \( \hat{y} \), and \( \hat{z} \) components. Furthermore, an ideal Huygens’ source has perfect polarization purity in all directions when measured using the Ludwig-3 definition of co and cross-polarization [56]. Thus for an array of Huygens’ sources, the co-polarization over the entire far-field is according to that of its element factor. This inherently corresponds to the Ludwig-3 definition, and when measured in this fashion has perfect polarization purity. This is independent of the radiation pattern synthesized, be it a focused spot, a refracted beam, etc. These comments provide enough background on Ludwig-3 definitions to proceed to the theoretical analysis. Further physical insights from Ludwig-3 definitions are made there.

### 3.1.2 Theoretical Analysis Using Ludwig-3 Polarization Definition

The same argument presented in [2] for the 1D situation with impedance and admittances surfaces in (1.22) to (1.27) is followed. The overall scenario and its definitions are illustrated in Fig. 3.1 to aid this discussion. Referring to the spherical coordinate system, we take the refracted angle to be \( \theta_t \) in the \( \phi_t \) plane. Furthermore, the metasurface is in the \( xy \)-plane.

![Figure 3.1: The incident, reflected, and transmitted plane waves corresponding to this Huygens’ metasurface situated in free space.](image)

Taking (1.20) and (1.21), we stipulate incident, reflected, and refracted plane waves as the field distributions on both sides. That is \( E_1 = E_i + E_r \) and \( E_2 = E_t \) where

\[ E_i = E_0 e^{-jk_i \cdot r}, \] (3.3)

\[ E_r = E_0 e^{-jk_r \cdot r}, \] (3.4)

and

\[ E_t = E_0 e^{-jk_t \cdot r}. \] (3.5)

Here \( i, r \) and \( t \) refer to the incident, reflected, and refracted plane waves, respectively. \( E_0 \) and \( k \) are the
corresponding amplitudes and wavevectors, and \( \mathbf{r} \) is the radial coordinate vector. Furthermore, the projection of the electric field along the electric dipole, according to (3.1) must be taken, since the Huygens’ source radiates an electric field according to Ludwig-3. Similarly, the projection of the magnetic field along the magnetic dipole according to (3.2) must be taken.

Polarization loss factor would be an issue if the incident electric field polarization is not parallel to the electric dipoles, and if the incident magnetic field polarization is not parallel to the axis of the electric loops. This corresponds to the anisotropic nature of our scalar impedances. Then there is a component of the incident beam that passes through the screen unaffected. This would manifest as a side-lobe in the radiation pattern. However, since this increases the cross-polarization in the direction of that side-lobe only, and not over the entire far-field sphere, we would not consider this as a degradation of polarization purity. To address polarization loss factor, we choose our electric dipoles to be oriented along \( \hat{y} \), and our magnetic dipoles along \( \hat{x} \). Furthermore, we stipulate that (3.3) be \( \hat{y} \)-polarized and that \( \phi_i = 0^\circ \) though \( \theta_i \) may vary. Note that in this case \( \hat{y} \) is identical to \( \hat{v} \). This choice also addresses the issue of polarization purity. This is because a Huygens’ source radiates an electric field that is only \( \hat{v} \)-polarized. Define an incident plane-wave that has its electric field in the \( \hat{v} \)-direction as transverse electric (TE). Thus, we can expect the output beam to remain purely TE, regardless of the refracted angle. This holds even for our oblique incidence characterization, since \( \theta_i \) is allowed to vary.

Insight can be obtained if a similar problem is first considered before examining the scenario in Fig. 3.1. Consider the action of the metasurface in the orthogonal directions, \( \hat{x} \) and \( \hat{y} \). That is, there is a Huygens’ metasurface acting along each direction. For the incident, reflected, and transmitted plane waves belonging to the metasurfaces, we stipulate that the impedances and admittances be passive and lossless. Since the procedure is identical to [2], only the final expressions for the impedance profiles, admittance profiles, and field amplitudes are given. The impedance and admittance profiles along the \( \hat{x} \) and \( \hat{y} \) axes are

\[
X_{sxX} = -\frac{\eta}{2\cos(\theta_{tx})} \cot(\Phi/2),
\]
\[
B_{smX} = \frac{\cos(\theta_{tx})}{2\eta} \cot(\Phi/2),
\]
\[
X_{sY} = -\frac{\eta \cos(\phi_Y)}{2} \cot(\Phi/2),
\]
and
\[
B_{smY} = \frac{1}{2\eta \cos(\theta_{ty})} \cot(\Phi/2),
\]

where
\[
\Phi = (\mathbf{k}_t - \mathbf{k}_i) \cdot \mathbf{r}.
\]

The angles \( \theta_{tx} \) and \( \theta_{ty} \) are the projection angles of \( \mathbf{k}_t \) onto the \( xz \) and \( yz \)-planes, respectively, according to Fig. 3.1, and \( \eta \) is the impedance of free space. The transmitted and reflected amplitudes corresponding to these profiles along the \( \hat{x} \) and \( \hat{y} \) axes are

\[
E_{tx} = \frac{2E_i \cos \theta_i}{\cos \theta_i + \cos \theta_{tx}},
\]
\[
E_{irx} = \frac{E_i (\cos \theta_i - \cos \theta_{tx})}{\cos \theta_i + \cos \theta_{tx}},
\]
\[
E_{ty} = \frac{2E_i \cos \theta_i}{\cos \theta_i \cos \theta_{ty} + 1},
\]
and
\[
E_{iry} = \frac{E_i (\cos \theta_i \cos \theta_{ty} - 1)}{\cos \theta_i \cos \theta_{ty} + 1}.
\]
Note that (3.6), (3.7), (3.11), (3.12) correspond to [2], and imply there are no side-lobes regardless of the incident and refracted angles. Thus the result in [2] is consistent with the Ludwig-3 definition. Furthermore, note that refraction along the $xz$-plane is TE, and $X_{seX}, -B_{smX}^{-1}$ both have the TE wave impedance $\eta \sec \theta_t$ in their expressions. Similarly, refraction along the $yz$-plane is transverse magnetic (TM). Intuitively then, $X_{seY}, -B_{smY}^{-1}$ both have the TM wave impedance $\eta \cos \theta_t$ in their expressions. Finally, power conservation is verified to hold in the manner described in [2].

Fig. 3.2 and 3.3 show $X_{se}$ and $-B_{sm}^{-1}$ for the design which will be considered. Here we design for normal incidence as was the case in [57,58]. In addition to this, $\phi_t = -45.00^\circ$ and $\theta_t = 30.00^\circ$, and the metasurface lies in the $xy$-plane. The design frequency is 10.0 GHz. The spatial profiles exhibit a cotangent variation similar to the expressions in (3.6) to (3.9). However, it must be pointed out that (3.6) to (3.9) do not represent cuts along the $x$ and $y$ axes in Fig. 3.2 and 3.3. The expressions in (3.6) to (3.9) are not symmetrical along $x = y$, but the results in Fig. 3.2 and 3.3 are. Evidently, the combination of the actions of the metasurface in the orthogonal directions is not equivalent to the full problem. Due to the complexity of the full 2D analytical expressions, which are obtained in identical manner to [2], the results alone are shown in Fig. 3.2 and 3.3, and their solution is left as an exercise to the reader.

![Figure 3.2: The spatial impedance $X_{se}$ profile of the Huygens’ metasurface electric response for the case of this design (normal incidence, $\phi_t = -45.00^\circ$ and $\theta_t = 30.00^\circ$).](image)

To complete the analysis, and further validate the use of the Ludwig-3 definition, the response to incident cross-polarization must be analyzed. This is done for the full problem corresponding to Fig. 3.1. If the electric dipoles are $\hat{y}$-oriented, and the magnetic dipoles $\hat{x}$-oriented, we expect that the metasurface should not respond to an incident $\hat{h}$-polarized electric field, or a $\hat{v}$-polarized magnetic field. Following the same analysis as for the co-polarization, independent of coordinate system, we arrive at the relations

$$X_{se-crosse} = \frac{1}{2} \frac{E_x}{H_y},$$

and

$$B_{sm-crosse} = \frac{1}{2} \frac{H_y}{E_x}.$$

From (3.1) and (3.2), $X_{se-crosse}$ and $-B_{sm-crosse}^{-1}$ evaluate to $\eta/2$ and $2\eta$. This holds only in the Ludwig-3 definition. Taking these into the lattice model of [7], one finds that the input impedance looking into the unit cell for the cross-polarization is $\eta$, and is matched to free space. This is important, because this means that the $\hat{h}$-polarized electric field component of an incident plane-wave must pass through the metasurface
2.8

Figure 3.3: The spatial admittance \(-B_{\text{sm}}^{-1}\) profile of the Huygens’ metasurface magnetic response for the case of this design (normal incidence, \(\phi_t = -45.00^\circ\) and \(\theta_t = 30.00^\circ\)).

and not be affected. This matches the anisotropic behavior expected of our scalar impedances. In any other coordinate system, the input impedance is not matched to free space, which means that part of the incident cross-polarization is reflected. Thus, the Ludwig-3 definition is the appropriate definition to use to model an array of Huygens’ sources.

3.1.3 The 2D Phase Insertion Gradient

In [48], 1D refraction at an interface in a homogeneous medium is described as

\[
\sin \theta_t - \sin \theta_i = \frac{1}{k} \frac{d\Phi}{dl},
\]

In [49], where the medium may differ across the interface, (3.17) can be extended to a generalized law of refraction, and a generalized law of reflection is also derived. This generalization of Snell’s law is due to the addition of the phase insertion gradient at the interface. However, this is only in one dimension. In [59], this is generalized to 2D. Specializing the result of that analysis to a metasurface in a homogeneous medium, the resulting phase insertion gradient along orthogonal directions is

\[
\frac{d\Phi}{dx} = (\sin(\theta_t) \cos(\phi_t) - \sin(\theta_i) \cos(\phi_i))k, \tag{3.18}
\]

and

\[
\frac{d\Phi}{dy} = (\sin(\theta_t) \sin(\phi_t) - \sin(\theta_i) \sin(\phi_i))k. \tag{3.19}
\]

Here \(\frac{d\Phi}{dx}\) is the phase insertion gradient along \(x\), and \(\frac{d\Phi}{dy}\) is the phase insertion gradient along \(y\), necessary for an incident plane wave along \(\phi_i, \theta_i\) to be refracted along \(\phi_t, \theta_t\). Furthermore, \(\Phi\) here is the same as that in (3.10). As noted in [2], (3.18) and (3.19) are descriptive but not prescriptive. That is, (3.18) and (3.19) describe the phase gradients necessary to obtain 2D refraction. However, these do not prescribe a method to engineer these gradients. In the implementation of [59], the response is purely electric. The anomalous refraction, which is according to the phase insertion gradient and the phase index, has a reflection on the incident side. There is also ordinary in-plane refraction, which depends only on the phase index. Furthermore, the anomalous refraction goes into the cross-polarization, and is weak compared to the ordinary refraction.

A pertinent novel aspect of our work is to implement this type of 2D refraction [59] without any strong
reflections by means of a suitably developed Huygens’ metasurface. The combination of the electric and magnetic response cancels the ordinary refraction on the transmitted side. The anomalous reflection on the incident side is also canceled. Thus, there is only a small ordinary reflection on the incident side, along with strong anomalous refraction on the transmitted side. Furthermore, the radiation remains with the co-polarized component. The total efficiency at the design point is high in the experimental section here, being 80.87%. The results in (3.18) and (3.19) will be used for theoretical comparisons to experimental measurements.

3.1.4 Design Procedure and Transmission-Line Model Simulation for 2D Refraction

The design parameters for this illustration of 2D refraction using a Huygens’ metasurface are: an operating frequency of 10.0 GHz, a normal incidence angle, a refracted angle of $\theta_t = 30.00^\circ$, and a plane of refraction of $\phi_t = -45.00^\circ$. Taking the metasurface to be in the $xy$-plane, this results in spatial periods for $X_{se}$, $B_{sm}$, and $S_{21}$ along both principle axes of 8.00 cm.

The impedances and admittances are implemented by taking the profiles in Fig. 3.2 and Fig. 3.3 and then discretizing into unit cells. Taking the discretized versions of $X_{se}$ and $B_{sm}$ into the Huygens’ source circuit model, we again utilize the transmission-line full-wave solver developed in [16]. This allows us to determine the quality of the refracted beam in terms of side-lobes and reflections for a given discretization, without first synthesizing an entire profile of the unit cells. Though this method is a simplification, it provides a good first approximation. Based upon these investigations, for this design, we take square unit cells and settle on a discretization level of $\lambda/7.5$ or 4 mm. However, note that the discretization in the $\phi_t = -45.00^\circ$ plane is actually reduced to about $\lambda/5.3$ or 5.66 mm.

Taking the plane of refraction to be $\phi_t = -45.00^\circ$ simplifies the design procedure. The impedance and phase profiles in the $xy$-plane can be generated by taking units cells along a cut of the metasurface in either the $x$ or $y$ directions. Then the remainder of the profile is obtained by staggering the rows or columns of unit cells. However, examination of Fig. 3.2 and Fig. 3.3 shows that the periods of $X_{se}$ and $B_{sm}$ along the plane of refraction are also reduced compared to those along the principle axes. Thus there are 20 unit cells per period along both principle axes, and 10 unit cells per period along the plane of refraction. The two factors of a coarser discretization, in terms of both wavelengths and unit cells per period, are expected to adversely affect the results compared to the 1D refraction case in chapter 2.

This can be demonstrated simulating the cuts along either principle axis, and along the plane of refraction. Since $\phi_t = -45.00^\circ$, both principle axes contain identical $X_{se}$ and $B_{sm}$ profiles, and both axes produce identical results. Fig. 3.4 shows the simulated far-field radiation patterns for either principle axis, and the plane of refraction, using the transmission-line solver. Along either principle axis, the first side-lobe level and the maximum reflection are $-31.61$ dB and $-23.76$ dB from the main-lobe maximum, respectively. Along the plane of refraction, the first side-lobe level and the maximum reflection are $-20.61$ dB and $-21.98$ dB from the main-lobe maximum, respectively. The effects of discretization on performance are clearly seen. Furthermore, the presence of side-lobes when the analysis following [2] predicts none is due to the use of an incident Gaussian beam instead of a uniform plane wave.

The design is implemented on Rogers RO3003 substrate [$\varepsilon_r=3.00$, $\tan\delta=0.0013$, thickness=1.524 mm, 0.5 oz. cladding], using Rogers 2929 Bondply [$\varepsilon_r=2.94$, thickness=0.0504 mm]. The 2929 Bondply is chosen for its $\varepsilon_r$ and thinness. Though 0.0381 mm was available, 0.0504 mm was chosen to properly fasten the layers. Fig. 3.5a shows the design of a typical unit cell. The detailed geometrical parameters, and the S and Z-parameters of the entire unit cell profile, are found in the appendix, Tables A.3 to A.6. The Huygens’ source unit cell implements a loop with the outer conductor layers, while the dipole is contained in the
Chapter 3. Huygens’ Metasurface for 2D Refraction

Figure 3.4: The simulated far-field radiation patterns for the Huygen’s metasurface, for either principle axis, and the plane of refraction. These are predicted using the transmission-line solver, for the chosen unit cell discretization level of $\lambda/7.5$ or $4\,\text{mm}$.

...conductor within. Thus, the total thickness of the metasurface is about $\lambda/9.3$. Note that the unit cell design is fully printed, and contains both the electric and magnetic responses. The incident electric field is polarized in the vertical direction, while the incident magnetic field is polarized in the horizontal direction. The unit cells are designed and optimized using the HFSS full-wave solver according to the discussion in chapter 2. Fig. 3.5b shows the fabricated metasurface. The metasurface dimensions are $240 \times 240 \,\text{mm}$, being 60 unit cells square.

Fig. 3.5 shows one period of the ideal impedance profiles along either principle axis which must be synthesized according to Fig. 3.2 and Fig. 3.3. From the prior discussion on the staggering of unit cells, this one cut represents the impedance profiles over the entire $xy$-plane accurately. Note here that $X_{sc}$ and $-B_{sm}^{-1}$ have been represented as the impedances $Z_e$ and $Z_m$, respectively. The realized values are extracted from HFSS using the Huygens’ source circuit model. These are superimposed on the ideal and match well, even at the resonances.

Fig. 3.7 shows one period of the ideal phase profile along both principle axes, corresponding to Fig. 3.6, and obtained using the Huygens’ source circuit model. This again matches well with the synthesized phase
profile, which is superimposed. The impedance and phase profiles along the plane of refraction contain a subset of the displayed values. Therefore these also match well.

3.2 Experimental Results for 2D Refraction

To better adhere to Gaussian optics theory, some improvements have been made to the setup in chapter 2. First, a conical horn providing an asymmetric field pattern in the E and H-planes was obtained. This provides 93% coupling efficiency to the fundamental Gaussian mode. The nominal specifications of this horn at 10.0 GHz are a 3 dB beamwidth in the E and H-planes of 16.50° and 14.00°, respectively. Furthermore, the gain at this frequency is 21 dB. The horn aperture is 132.08 mm in diameter. Characterization of this horn yields a 3 dB beamwidth in the E and H-planes of 16.01° and 13.46°, respectively. These are close to
the nominal values. The $S_{11}$ of the horn is measured to be better than $-30$ dB at 10.0 GHz.

To focus the Gaussian beam while minimizing edge-diffraction effects, a Rexolite lens is placed flush against the aperture of the conical horn. This was then characterized with a planar near-field scanning system over frequency and compared with quasi-optical predictions using ABCD matrices [51, 60]. For the design frequency of 10.0 GHz, the minimum waist position from the lens aperture and its size are predicted to be 106.1 mm and 24.3 mm, and 63.1 mm and 27.9 mm, in the E and H-planes, respectively. These are measured to be 108.5 mm and 26.4 mm, and 61.0 mm and 30.4 mm, in the E and H-planes, respectively. Furthermore, the input to the metasurface is verified to be Gaussian-like in the far-field. This could be made more rigorous with gaussicity calculations [61] which are typical in quasi-optical processing. Nevertheless, this demonstrates improvement in higher-order Gaussian modes and edge-diffraction, and confirms adherence to Gaussian beam optics theory up to the metasurface input. Fig. 3.8 shows the experimental setup.

Figure 3.8: Experimental setup for 2D refraction. Includes the planar near-field system, horn, lens, and Huygens’ metasurface.

The Huygens’ metasurface for 2D refraction is placed at the 10.0 GHz minimum phase flatness point of the E and H-planes in the horn and lens system. This is because the derivation here assumes a plane wave at normal incidence. The minimum phase flatness point is calculated over 3D space from the plane of the metasurface to the near-field probe. The algorithm for the phase flatness at each transverse plane in space between the metasurface and near-field probe is expressed as:

$$\text{Phase Flatness} = \frac{1}{w_{0E,f}} \int_{w_{0E,f}} \left| \angle E\text{-plane} - \angle_{\text{ave}}E\text{-plane} \right|$$

$$+ \frac{1}{w_{0H,f}} \int_{w_{0H,f}} \left| \angle H\text{-plane} - \angle_{\text{ave}}H\text{-plane} \right|.$$  \hspace{1cm} (3.20)

That is, the magnitude of the deviation from the average phase is averaged over the waist in the E and H-planes. This point is 111.0 mm from the lens aperture, where the waists in the E and H-planes are 26.4 mm and 36.9 mm, respectively. For the metasurface placed at this point with dimensions 240 × 240 mm, the measured field tapers to $-21.17$ dB in the H-plane and $-31.15$ dB in the E-plane at the edges. For the Rexolite lens with diameter 203.2 mm, the measured field tapers to $-20.09$ dB in the H-plane and $-43.79$ dB in the E-plane at the edges. For comparison, an aperture four times the illumination waist, which is optimal for quasi-optical processing, has an edge taper of $-34.74$ dB. Not meeting this requirement, a deviation from the expected waist size and position should be expected [51]. Fig. 3.9a and 3.9b show the normalized electric field distribution at 10 GHz at the lens and metasurface input, respectively. To deal with these spillover
effects, both absorbers and conductive sheets at the metasurface were used in the characterization. However, both had a negative effect on the far-field ripples. Given the close spacing of the lens from the metasurface, and the thickness of the absorbers, the latter are in the near-field of the lens output, causing them to become resonant. In the absence of time-domain gating, the conductive sheets contribute to multipath reflections. TRL calibrations [62] could alternatively be used to account for scattering from the environment, and de-embed the S-parameters of the metasurface from those of the setup, but this was not investigated. Besides what has been mentioned, further improvements to the setup could be used to obtain a cleaner input to the metasurface and more accurate measurements. These include corrugated horns, larger Rexolite lenses, and achromatic doublets [63].

The characterization of the quasi-optical system has demonstrated that these residual spillover levels do not critically affect performance. This can also be seen in Fig. 3.10, which presents the measured far-field electric field pattern, and is raw data. Note that the measured angle and plane of refraction are $\theta_{t,\text{obt.}} = 29.29^\circ$ and $\phi_{t,\text{obt.}} = -44.45^\circ$, respectively. These are close to their predictions $\theta_{t,\text{pred.}} = 30.00^\circ$ and $\phi_{t,\text{pred.}} = -45.00^\circ$. The azimuth, elevation, and $\phi_t$ cuts along the main-lobe maximum at the design point of 10.0 GHz and normal incidence are given, and the far-field ripples are much improved over [57, 58].

The transmitted beam is not Gaussian. This is because the screen is optimized for an incident plane wave.
wave, not an incident Gaussian beam. Furthermore, the synthesized $X_{sc}$ and $B_{sn}$ values are not exact. This results in a slight variation from the required transmitted amplitude as a function of position, corresponding to the unit cell positions in Fig. 3.6 and Fig. 3.7. The measured beamwidths are dependent on the incident beam used in the characterization. Here these are large due to the large beamwidth of the incident Gaussian beam. That is, the Gaussian beam minimum waist radius which impinges on the metasurface is small, which is not a good approximation to the uniform plane wave used in the analysis. Choosing a Gaussian beam with a larger waist radius, the beamwidths from the metasurface would be expected to narrow. However, this increases spillover.

Comparing Fig. 3.4 with Fig. 3.10, the measured cuts have more ripples than the simulated ones, due to the residual spillover, but this increase is slight. Furthermore, the same trend is seen in both figures. That is, the side-lobe levels are increased in the $\phi_t$ plane compared to the azimuth and elevation planes. This was found in simulation to be due to the decreased discretization in terms of both wavelengths and unit cells per period in the plane of refraction compared to the principle axes.

The measured side-lobe levels in Fig. 3.10 in the azimuth and elevation cuts, and the plane of refraction are -21.96 dB, -18.51 dB, and -14.61 dB, respectively. This compares with -31.61 dB and -20.61 dB, along either principle axis and the plane of refraction in Fig. 3.4, respectively. Furthermore, though the measured efficiencies will be demonstrated to be high in the following sections, the simulated efficiencies are much higher than these. The simulated total efficiency, accounting for reflections, insertion loss, and side lobes, is 99.41% along either principle axis, and 98.32% along the plane of refraction. This discrepancy is because the transmission-line solver does not account for details such as substrate and conductor losses, mutual coupling between cells, and the other parasitic effects. Thus Fig. 3.4 will not be compared further with Fig. 3.10 for quantitative data. The transmission-line solver also does not include the effects of frequency detuning on the printed elements synthesizing the impedances, necessary for accurate frequency response simulations. It also cannot describe refraction when it is not limited to a 2D plane, necessary for the oblique incidence simulations. The transmission-line simulations are useful only to show the qualitative effects of discretization, before the synthesis of the physical unit cells. These limitations have been observed in the 1D refraction problem in chapter 2. It has been seen there that HFSS accurately models the behavior of the Huygens’ metasurface. However, the full structures for the 2D refraction problem are too computationally intensive to simulate in HFSS. The memory usage and computational time in HFSS for the 1D refraction problem have been discussed in chapter 2. Therefore, in the following sections for the oblique incidence and frequency response characterizations, only measurement results will be presented.

### 3.2.1 Oblique Incidence Characterization at 10.0 GHz

Here $\phi_i$ is kept constant at $0^\circ$, while $\theta_i$ is varied in steps of $10^\circ$ for both the theoretical and measurement results. Due to the limitations of the setup, the scan range is $140^\circ$ in the azimuth and elevation planes. It should be noted that the Huygens’ metasurface has been designed for normal incidence, and thus the plane of incidence in this case can also be taken to be the plane of refraction. However, the plane of oblique incidence is not the same as the plane of refraction. The plane of oblique incidence is the $xz$-plane. In contrast, as seen in Fig. 3.11, which presents the predicted and obtained output angles in spherical coordinates, the main beam for oblique incidence comes out in many different $\phi_t$ planes. The main-lobe predictions are made from (3.18) and (3.19), and agree well with the measurements. The discrepancies should be attributed to slight alignment issues and edge-diffraction effects, which have been demonstrated in the previous section to not affect performance critically. Desiring a total efficiency greater than 50%, the lower bound of $\theta_i$ reported here is $-20^\circ$. Due to the limitations of the setup, the upper bound of $\theta_i$ is $50^\circ$, where the total efficiency also becomes poor. Total efficiency is addressed further on in this section.
Figure 3.11: Predicted and obtained output angles for the oblique incidence characterization, for $\phi_i = 0^\circ$. (a) $\phi_t, \text{pred}$, and $\phi_t, \text{obt}$, and (b) $\theta_t, \text{pred}$, and $\theta_t, \text{obt}$. Both sets of angles agree well.

Fig. 3.12 graphically presents the trajectory of the main beam as $\theta_i$ is scanned from $-20^\circ$ to $50^\circ$. This is the 2D far-field electric field pattern in the azimuth and elevation planes, normalized to the incident beam maximum. The main beam rotates about the origin. Fig. 3.13 graphically presents the cross-polarization as $\theta_i$ is scanned from $-20^\circ$ to $50^\circ$. These are also normalized to the incident beam maximum. The peaks of the cross-polarization pattern follow the location of the incident Gaussian beam. Note that $-20^\circ$ becomes $20^\circ$ looking at the scan plane from the side opposite the conical horn antenna.

The efficiency is defined here as:

$$\text{Eff.} = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100\%.$$  \hspace{1cm} (3.21)

Here $P_{\text{output}}$ refers to the power out of the metasurface, and $P_{\text{input}}$ refers to the power into the metasurface. Power not arriving at the output from the input is due to reflections and insertion loss. The total efficiency is defined here as:

$$\text{Total Eff.} = \frac{P_{\text{main-lobe}}}{P_{\text{input}}} \times 100\%.$$  \hspace{1cm} (3.22)

Here $P_{\text{main-lobe}}$ refers to the power contained in the main-lobe only. This accounts for the power in the side-lobes in addition to the reflections and insertion loss.

Fig. 3.14 presents the measured efficiencies and total efficiencies. The efficiency is on average 82.51% for a $70^\circ$ range, the maximum being 93.02% at $\theta_i = -10^\circ$. This demonstrates that the insertion loss and reflections are low in our design. The total efficiency is on average 71.06% for the same $70^\circ$ range, the maximum being 80.87% at normal incidence. That the power contained in the side-lobes is low over most of the oblique incidence range can be observed from the difference of the efficiency curves in Fig. 3.14.

The maximum side-lobe levels cover the entire far-field scan. In addition, these are relative to the main-lobe maximum, and no smoothing is done on the raw data, in contrast to chapter 2. Fig. 3.15 demonstrates that these levels are less than $-10\,\text{dB}$ over the entire oblique incidence range.

Cross-polarization levels were never a consideration in the design, but these are reported here. These levels are due to surface currents radiating on the traces orthogonal to the vertically-polarized electric field in the unit cells such as in Fig. 3.5a. These are also the maximums over the entire far-field scan, and relative to the main-lobe maximum. Fig. 3.16 demonstrates that these levels are usually less than $-10\,\text{dB}$ over the entire oblique incidence range.
Figure 3.12: Normalized 2D far-field electric field co-polarization patterns, for $\phi_i = 0^\circ$. (a) $\theta_i = -20^\circ$, (b) $-10^\circ$, (c) $0^\circ$, (d) $10^\circ$, (e) $20^\circ$, (f) $30^\circ$, (g) $40^\circ$, (h) $50^\circ$. 
Figure 3.13: Normalized 2D far-field electric field cross-polarization patterns, for $\phi_i = 0^\circ$. (a) $\theta_i = -20^\circ$, (b) $-10^\circ$, (c) $0^\circ$, (d) $10^\circ$, (e) $20^\circ$, (f) $30^\circ$, (g) $40^\circ$, (h) $50^\circ$. 
Figure 3.14: Efficiency and total efficiency for the oblique incidence characterization. The former measures the power losses due to insertion loss and reflections. The latter includes the effect of side-lobes in addition to this.

Figure 3.15: Measured side-lobe levels for the oblique incidence characterization.

In HFSS simulations of the unit cells, the cross-polarization is negligible. In the experiments, cross-polarization arises because the incident E-field is not perfectly aligned with the axis of the electric dipoles. This causes radiation from the orthogonal traces. Furthermore, the Huygens’ sources in our metasurface are not ideal. This is apparent in Fig. 3.6, and Fig. 3.7. Therefore, based upon the Ludwig-3 definition, cross-polarization is higher than it should be. In chapter 2, cross-polarization was not measured, the design there being a proof-of-concept before moving on to this more complex design.

3.2.2 Frequency Response at Normal Incidence

The relevant comments made for the oblique incidence characterization hold here as well. The entire frequency range is captured from 9.0 to 11.0 GHz in 0.1 GHz increments. However, the frequency points with a total efficiency of less than 50% are discarded.

Fig. 3.17 presents the predicted and obtained output angles in spherical coordinates, and the predictions also use a 0.1 GHz increment. Here (3.18) and (3.19) are used for the predictions with the simplification
Chapter 3. Huygens’ Metasurface for 2D Refraction

that the phase gradients do not change with frequency. Thus, the detuning with frequency of the printed
elements synthesizing the impedances is not modeled. The loops are essentially split-ring resonators, so
this simplification is reasonable only within a narrow bandwidth of the design frequency. The obtained

Figure 3.16: Measured cross-polarization levels for the oblique incidence characterization.

main-lobe angles only qualitatively follow predictions. $\theta_{t,\text{obt}}$ decreases with frequency, and $\phi_{t,\text{obt}}$ oscillates
around $\phi_{t,\text{pred}}$. Here the difference in scale between Fig. 3.11 and Fig. 3.17 must be noted. The discrepancies
between the predicted and obtained angles here are on the same order as those in Fig. 3.11.

Fig. 3.18 presents the measured efficiencies. The efficiency is on average 84.34\% over a fractional band-
width of 8\%, the maximum being 94.70\% at 9.8 GHz. The total efficiency is on average 68.64\% over the
same fractional bandwidth of 8\%, the maximum being 80.87\% at 10.0 GHz.

Fig. 3.19 presents the side-lobe levels, which are less than $-10$ dB over the frequency response, and
Fig. 3.20 presents the cross-polarization levels, which are less than $-11$ dB over the same range.

3.2.3 Near-Field Measurements

Finally, we conclude the measurements with an isosurface plot of the near-fields. This data is obtained
from a 2D scan of the near-field, which is then Fourier transformed to obtain data in the entire 3D volume.

Figure 3.17: Predicted and obtained output angles for the frequency response. Note the scale here in contrast
to that in Fig. 3.11. (a) $\phi_{t,\text{pred}}$ and $\phi_{t,\text{obt}}$, and (b) $\theta_{t,\text{pred}}$ and $\theta_{t,\text{obt}}$. 

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Figure 3.18: Efficiency and total efficiency for the frequency response. The former measures the power losses due to insertion loss and reflections. The latter includes the effect of side-lobes in addition to this.

Figure 3.19: Measured side-lobe levels for the frequency response.

This is done for normal incidence at 10.0 GHz, and demonstrates that refraction is primarily in the desired direction. The isosurface is the 3D extension of a contour plot, showing surfaces of equal magnitude. Note that only the near-fields between the plane of the metasurface and the near-field probe can be obtained. Fig. 3.21 presents the normalized near-field electric fields.
Figure 3.20: Measured cross-polarization levels for the frequency response.

Figure 3.21: 3D visualization of the normalized near-field electric fields at 10.0 GHz and normal incidence. Refraction of the beam is clearly demonstrated.
Chapter 4

Huygens’ Metasurface for 1D Gaussian-Beam to Gaussian-Beam Focusing

Here we design a Huygens’ metasurface for 1D Gaussian-to-Gaussian beam focusing with highly suppressed reflections. Instead of utilizing (1.20), (1.21), the paraxial approximation and the definition of the transmission coefficient $S_{21}$ are relied upon. It will be demonstrated that for waist sizes, the procedure is reliable beyond the limits of where the paraxial approximation does not typically require corrections. For $0.45\lambda \leq w_0 \leq 0.9\lambda$, first-order corrections are required [51], and we demonstrate a design in the middle of this range where $w_0 = 0.723\lambda$ or 21.7 mm. Here $w_0$ is the minimum waist size. Furthermore for this design, $Z_e$ and $Z_m$ are purely imaginary, and the Huygens’ metasurface exhibits highly suppressed reflections. However, the minimum waist position cannot be stipulated in addition to its size.

For the characterization of this 1D lens, the relevant parameters for a theoretical X-band corrugated horn used at 10.00 GHz are an aperture size of 132.08 mm, and a 16.5° half-power beamwidth. This corresponds to a calculated waist of 50.2 mm at the aperture, where the 1D lens will be placed. This is to minimize truncation effects. The minimum waist is 114.6 mm into the horn where its size is 42.1 mm. Similar to the case of plane-wave refraction, the electric field polarization is taken to be perpendicular to the plane of focusing. In contrast to the ATM90-441-6 horn used earlier, the minimum waist radius within the horn is well within the paraxial approximation. Furthermore, this conical horn provides 91% coupling efficiency to the fundamental Gaussian mode [51]. Simulation results for this 1D lens are presented. Detailed experimental results will be the subject of future work.

4.1 Derivation for 1D Focusing

The field pattern at the input of the metasurface is that of the conical horn antenna. This input Gaussian beam has its minimum waist size and position, and its waist size at the metasurface, as described at the beginning of the chapter. Fixing the waist size at the metasurface, from the paraxial approximation of a Gaussian beam, the corresponding minimum waist size and position are not unique. That is, the field profile with its 50.2 mm waist impinging on the Huygens’ metasurface can correspond to another Gaussian beam, with a different minimum waist size and position. The paraxial approximation for the electric field
component of the Gaussian beam as used here is given as

$$E(x, z) = E_0 \frac{w_0}{w(z)} e^{-\frac{x^2}{w^2(z)}} e^{-jkz - jk R(z)^2 + j\xi(z)}. \quad (4.1)$$

Here $E_0$ is the amplitude of the Gaussian beam, $x$ is the transverse coordinate, $z$ is the normal coordinate, and $w(z)$, $R(z)$, and $\xi(z)$ are defined as

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad (4.2)$$

$$R(z) = z \left[1 + \left(\frac{zR}{z}\right)^2\right], \quad (4.3)$$

and

$$\xi(z) = \arctan\left(\frac{z}{z_R}\right). \quad (4.4)$$

Finally, $z_R$ is the Rayleigh range, given as

$$z_R = \frac{\pi w_0^2}{\lambda}. \quad (4.5)$$

The definition of the transmission coefficient $S_{21}$ is

$$S_{21} = \frac{E_2^+}{E_1^-} \bigg|_{E_2^- = 0}. \quad (4.6)$$

Here $E_1^-$ is the incident electric field to the input port, and $E_2^+$ is the reflected electric field to the output port. The output port is matched, therefore $E_2^-$ which is the incident electric field to the output port, is zero. This reveals that to obtain an $S_{21}$ profile with unity magnitude across the Huygens’ metasurface, the amplitudes of the input and output field profiles must be the same on both sides. The action of the Huygens’ metasurface is then to provide the phase difference between the field profiles. Furthermore, because $S_{21}$ has unity magnitude, the reflections are expected to be highly suppressed.

Thus for focusing, given the input Gaussian beam, an output Gaussian beam can be found such that the field profiles across the entire Huygens’ metasurface are the same. The minimum waist sizes and positions in the regions across metasurface need not be the same. Then $S_{21}$ has a magnitude profile of unity as just described. Furthermore, the $S_{21}$ phase profile can be found from the phases of the field profiles at the Huygens’ metasurface. This is parabolic as expected from ray-optics theory. Stipulating a desired output $w_0$, the paraxial approximation dictates the minimum waist position and maximum field amplitude.

Taking the lattice model in [7], the scattering parameters are then converted to $Z_e$ and $Z_m$, which are found to be purely imaginary. At this stage, the unit cells can be synthesized according to the procedure outlined in subsection 2.3.

### 4.2 Demonstration of Design Procedure for Focusing

Given the aperture waist of 50.2 mm, this can be focused to an $w_0$ in the range of 10.9 mm to 50.2 mm. The lower bound is obtained from the full-width half-maximum of a diffraction-limited sinc, which is 0.603 $\lambda$. The diffraction limit is below the $w_0 = 0.45 \lambda$ limit for first-order corrections to the paraxial approximation. In fact, at this point, the Gaussian beam model fails completely [51]. We demonstrate a design where $w_0 = 0.723 \lambda$ or 21.7 mm. This is in the middle of the range of where the paraxial approximation requires first-order corrections. Despite this, with respect to synthesizing a desired $w_0$, the paraxial approximation
is still an accurate tool.

Following the design procedure outlined in subsection 4.1, we specify an aperture waist of 50.2 mm and an $w_0$ of 21.7 mm. Then the paraxial approximation yields a minimum waist location of 101.1 mm from the Huygens’ metasurface. The amplitude of the Gaussian at the output is predicted to be 1.94 times that of the Gaussian at the input. From the definition of the $S_{21}$, a magnitude profile of unity is obtained. Thus $S_{11}$ and the reflections are expected to be highly suppressed. Fig. 4.1 shows the corresponding phase profile. The profile is quadratic, and the focal spot predicted from ray-optics is 96.2 mm from the metasurface.

![Figure 4.1: $S_{21}$ phase profile across the Huygens’ metasurface. The profile is quadratic.](image)

Taking the lattice model, the scattering parameters can be converted to their corresponding $Z_e$ and $Z_m$ profiles across the Huygens’ metasurface. Fig. 4.2 and 4.3 illustrate the profiles. Note that the impedances are purely imaginary. The Huygens’ metasurface is passive and lossless as desired.

![Figure 4.2: $Z_e$ profile across the Huygens’ metasurface. The impedances are purely imaginary.](image)

### 4.3 Simulation Using Transmission-Line Model

Using the transmission-line full-wave solver provides a quick verification prior to unit cell synthesis. Fig. 4.4 shows the normalized complex magnitude of the electric field. There, minor reflections are seen. However, as demonstrated in Fig. 4.5, these decay at far-field distances. There it is also seen that the first side-lobe and reflections are $-26.72$ dB and $-28.62$ dB from the main-lobe, respectively. Note also from Fig. 4.5 that...
the far-field pattern is that typical of a Gaussian beam. The $w_0$ obtained is 21.3 mm, 0.013$\lambda$ away from the desired $w_0$ of 21.7 mm. Considering just the $w_0$, this demonstrates the validity of this approach even when the paraxial approximation requires first-order corrections. However, the obtained minimum waist position is 80.8 mm, 0.67$\lambda$ away from the desired position of 101.1 mm. This is due to ignoring the first-order corrections to the paraxial approximation. We demonstrate this assertion in the following section. In addition to this, the output amplitude relative to the input is 1.4059 $\text{V}_\text{m}$ compared to the expected 1.94 $\text{V}_\text{m}$. This deviation is also due to ignoring the first-order corrections in the paraxial approximation. This also demonstrates that specifying $w_0$, the position and maximum amplitude cannot be controlled.

Figure 4.4: Normalized complex magnitude of the electric field. Obtained with the transmission-line model. The output minimum waist size obtained is 21.3 mm, 0.013$\lambda$ away from the design value. Its position is 80.8 mm from the Huygens’ metasurface. The input minimum waist size is 42.1 mm, and its position is 114.6 mm from the Huygens’ metasurface. The near-field reflections decay into the far-field.

### 4.4 Further Investigation of Design Procedure for Focusing

This method is further investigated in the manner of subsection 4.1 to 4.3 for the designed output $w_0$ in the range of 21.7 mm to 49.0 mm. That is, for the middle of the range where the paraxial approximation requires first-order corrections to about the limit of the aperture waist. Table 4.1 demonstrates the accuracy of $w_0$ obtained for this range of design values. The obtained $w_0$ values are from the transmission-line solver. It is seen that the obtained $w_0$ is always within 0.027$\lambda$ of the design value. The second row of Table 4.1 refers to the limit of the paraxial approximation where typically no corrections apply, and the last two rows to the aperture waist limit.

Table 4.2 demonstrates the accuracy of $f_{\text{out}}$ obtained for the range of $w_0$ design values. Here $f_{\text{out}}$ refers to the position of the minimum waist. The $f_{\text{out}}$ predictions are from (4.1) to (4.5). The obtained values are from the TLM solver. It is observed that the deviation in $f_{\text{out}}$ improves as the paraxial approximation
Figure 4.5: Far-field radiation pattern of the electric field. Obtained with the transmission-line model. The pattern is typical of a Gaussian beam in the far-field. The reflections are seen to be small in the far-field.

<table>
<thead>
<tr>
<th>$w_0$ Desired (mm)</th>
<th>$w_0$ Obtained (mm)</th>
<th>$w_0$ Deviation ($\lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.7</td>
<td>21.3</td>
<td>0.013</td>
</tr>
<tr>
<td>27.0</td>
<td>26.2</td>
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<td>0.023</td>
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</tr>
<tr>
<td>36.0</td>
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<td>0.0067</td>
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</tr>
</tbody>
</table>

Table 4.1: Desired and obtained $w_0$. The obtained $w_0$ is always within 0.027 $\lambda$ of the design value. This is from the middle of the range where the paraxial approximation requires first-order corrections to about the aperture waist.

becomes increasingly valid. The last row is a discrepancy from this trend. In section 1.1, the final comment made is that the distances from the metasurface must be macroscopic in order for the description to be valid. This is because the fields themselves are macroscopic, as is the Gaussian beam considered here, which is also a far-field distribution. Close to the metasurface, the near-fields begin to dominate the description, and the last predicted $f_{out}$ in Table 4.1 is approaching a wavelength from the metasurface. This illustrates the constraint given in section 1.1.

The same trends as for $f_{out}$ exist for the output amplitude relative to the input. In addition to this, from the transmission-line solver, the reflections are small in the far-field for all entries in the tables. The far-field patterns are also typical of Gaussian beams, as in Fig. 4.5.

### 4.5 Simulation Using HFSS

Unit cells were synthesized in HFSS using the method described in subsections 2.2 and 2.3. These are shown in Fig. 4.6, and the printed inductors and capacitors synthesizing the impedances are visible. The detailed geometrical parameters, and the S and Z-parameters of the entire unit cell profile, are found in the appendix, Tables A.7 to A.10. The unit cell dimensions, substrate, and bonding film used are all identical to
Table 4.2: Desired and obtained $f_{\text{out}}$. The deviation in $f_{\text{out}}$ improves as the paraxial approximation becomes increasingly valid. Here $f_{\text{out}}$ refers to the position of the minimum waist.

<table>
<thead>
<tr>
<th>$w_0$ (mm)</th>
<th>Desired (mm)</th>
<th>Predicted (mm)</th>
<th>Obtained (mm)</th>
<th>Deviation (λ)</th>
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</thead>
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<td>101.0</td>
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<td>109.3</td>
<td>0.47</td>
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<td>33.0</td>
<td>127.4</td>
<td>116.3</td>
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<td>36.0</td>
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<td>124.5</td>
<td>116.0</td>
<td>0.28</td>
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<td>108.8</td>
<td>0.19</td>
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<td>60.5</td>
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<td>49.0</td>
<td>36.0</td>
<td>45.7</td>
<td>0.323</td>
<td></td>
</tr>
</tbody>
</table>

those for the 2D refraction design. However, instead of working with the impedances $Z_e$ and $Z_m$, it is more straightforward to work with the transmission coefficient $S_{21}$. This is because the $S_{21}$ magnitude profile is unity, while the variation is in its phase. For passive and lossless plane-wave refraction, as described in section 2.1 and [2], small reflections must exist. In that case, the impedances $Z_e$ and $Z_m$ are more straightforward to work with. The $S_{21}$ magnitude and phase profiles, though respectively close to unity and roughly linear, are not as apparent.

![Figure 4.6: Unit cells for focusing. Unit cells synthesized in HFSS using the method described in subsections 2.2 and 2.3 for this lens design. The printed inductors and capacitors synthesizing the impedances are visible.](image)

Fig. 4.7 and 4.8 show the desired and synthesized $S_{21}$ magnitude and phase profiles. Note that in the center of the screen, where most of the incident Gaussian beam energy is concentrated, the match between the profiles is most important. This ensures the highly suppressed reflections, while providing the desired output beam properties. The magnitude matches closely across most of the screen, but most importantly in the center. The phase is seen to match closely across the entire screen.

Fig. 4.9 shows the desired and synthesized $S_{11}$ magnitude profiles. Note that in the center of the screen, where most of the incident Gaussian beam energy is concentrated, $S_{11}$ should be as close to zero as possible to minimize reflections. This is demonstrated here in our design. There are stronger reflections at several positions along the metasurface, namely ±66, 70, 74, 106 mm. From Fig. 4.2, the $Z_e$ profile at these points is extreme in value. These impedances are more difficult to synthesize, thus the sub-optimal $S_{11}$ and $S_{21}$ at these points. Examining Fig. 2.8a and 2.9a, the typical $Z_e$ response is seen to be non-resonant in nature. The combination of $Z_e$ from the dipole and loop would need to be resonant to achieve such extreme shunt impedances. Note however that in contrast, Fig. 2.9b shows the $Z_m$ response of the loop to be resonant. Thus, there is no difficulty in achieving the required $Z_m$ profile in Fig. 4.3. However, among these positions with stronger reflections, at the positions closest to the center (±66 mm), the amplitude of the Gaussian profile is 18% that at the center.
Figure 4.7: Desired and synthesized $S_{21}$ magnitude profiles. In the center of the screen, where most of the incident Gaussian beam energy is concentrated, the match between the profiles is most important.

Figure 4.8: Desired and synthesized $S_{21}$ phase profiles. In the center of the screen, where most of the incident Gaussian beam energy is concentrated, the match between the profiles is most important.

Figure 4.9: Desired and synthesized $S_{11}$ magnitude profiles. In the center of the screen, where most of the incident Gaussian beam energy is concentrated, $S_{11}$ should be as close to zero as possible to minimize reflections.

Fig. 4.10 presents the normalized complex magnitude of the total electric field profile obtained in HFSS. This simulation utilized a maximum of 25.4 GB of RAM, and took about 51.0 hours to complete for the same convergence criteria as in the 1D refraction design. The Huygens’ metasurface lies along the interface of
Chapter 4. Huygens’ Metasurface for 1D Gaussian-Beam to Gaussian-Beam Focusing

Figure 4.10: HFSS normalized complex magnitude of total electric field. The output minimum waist size obtained is 24.5 mm, and its position is 81.5 mm from the Huygens’ metasurface. The input minimum waist size is 42.1 mm, and its position is 114.6 mm from the Huygens’ metasurface.

The discontinuous electric field profiles. Note that across this interface, the field profiles are about identical, as would be expected when $S_{21}$ has unity magnitude along the boundary. The minimum waist size $w_0$ obtained is 24.5 mm, which is $0.093 \lambda$ and $0.107 \lambda$ off from the analytical and transmission-line model results. These were 21.7 mm and 21.3 mm, respectively. The minimum waist position is 81.5 mm or $0.017 \lambda$ off from the transmission-line model result. The deviation in the minimum waist position from the analytical result is much greater as discussed in section 4.3. Fig. 4.11 presents the normalized complex magnitude of

Figure 4.11: HFSS normalized complex magnitude of scattered electric field. This demonstrates clearly that most of the incident power is redirected to the output.

the scattered electric field profile in HFSS. This demonstrates clearly that most of the incident power is redirected to the output.

Fig. 4.12 shows the far-field radiation pattern of the synthesized lens. In the forward direction, the shape of the far-field pattern is similar but not quite that of a typical Gaussian beam. In the backward direction, the reflections are seen to have decayed in the far-field. The first side-lobe level and reflections are $-23.31$ dB and $-12.02$ dB from the main-lobe, respectively.

Comparing Fig. 4.4 and 4.5 with Fig. 4.10 to 4.12, there are some discrepancies between the result of the transmission-line solver and the HFSS results. Clearly, the realized scattering parameters obtained in Fig. 4.7 to 4.9 for the unit cells are not quite ideal. However, using the synthesized $S_{11}$ and $S_{21}$ in Fig. 4.7 to 4.9 in the transmission-line solver, $w_0$ and $f_{out}$ are identical to before, and the far-field pattern is typical of a Gaussian as in Fig. 4.5. Thus, the discrepancies can be attributed to the simplified analysis of the transmission-line solver compared to HFSS. For instance, the effect of interactions between the EM fields with the complex geometries of the unit cells are not accounted for.
Figure 4.12: HFSS far-field radiation pattern of synthesized lens. The pattern in the forward direction is similar but not quite that of a typical Gaussian beam. In the backward direction, the reflections are seen to have decayed in the far-field.
Chapter 5

Conclusion

The groundwork in the 1D refraction design has provided the experience based upon which improvements to the experimental setup and the characterization techniques have been made. The improvements to the original experimental setup in section 2.4 have been described in section 3.2. These can be summarized as a better quasi-optical adherence. Because of this, the measured data obtained in chapter 3 have far less ripples than those in chapter 2. The most significant result of this is that no smoothing of the data is required in chapter 3. In addition to this, the improved experimental setup captures data from the full range of oblique incidence and frequency response that is meaningful. This was defined to be a total efficiency greater than 50%. The improvements in the characterization techniques are mainly the determination of both efficiency and total efficiency. In addition to this, global side-lobe levels and cross-polarization levels are included. In sections 3.2.1 and 3.2.2 the measurement of efficiency has permitted an indirect but strong inference to the reflections and insertion loss. These have been demonstrated to be low over both oblique incidence and frequency response around the design point. The measurement of total efficiency has demonstrated that the power contained in side-lobes is low in general. Finally, the cross polarization levels are demonstrated to be in an acceptable range.

Though the experimental results have not been reported, at least in simulation, the 1D Gaussian-beam to Gaussian-beam focusing lens performs as expected. In contrast to dielectric lenses, this passive and lossless lens has highly suppressed reflections, and would find much use in the application to quasi-optical systems. The drawback of our approach, both for refraction and focusing, is the narrowband nature of our design. Typically, a dielectric lens would be more wide-band. However, this design is also planar, less costly to produce, able to be manufactured with standard PCB fabrication techniques, and also able to be scaled to higher frequencies. In contrast to both transmitarrays and dielectric lenses, our design is also λ/10 thick.

Thus, the experimental results for 2D refraction, as well as the simulations for 1D Gaussian-beam focusing, have supported the concept of the Huygens’ metasurface and its unit cell design procedure, the latter of which has been demonstrated in detail. The main contributions of this thesis are summarized below:

1. To summarize the literature pertaining to Huygens’ metasurfaces and similar research such as the GSTCs, and differentiate this work from that of traditional transmitarrays. This has been done in chapter 1.

2. To provide a detailed illustration of the systematic design procedure pertaining to Huygens’ source unit cells. This is mainly in chapter 2, and has been published in [57, 58].

3. To determine appropriate polarization definitions for scalar Huygens’ metasurfaces, as well as address the issue of polarization purity. This is in chapter 3, and has been submitted for publication in [64].
4. To prove the concept of the Huygens’ metasurface through the application to refraction and focusing.

The groundwork for this is in chapter 2. However, this is mainly in chapters 3 and 4, and has been published in [58], and submitted for publication in [64].

5.1 Future Directions

The concept of the Huygens’ metasurface has potential for much further development. In addition to the full-polarization control Huygens’ metasurfaces described recently in [45], some addition avenues for further research are addressed here.

5.1.1 Experimental Validation of the 1D Gaussian-Beam to Gaussian-Beam Focusing Lens

The 1D Gaussian-beam to Gaussian-beam focusing lens still remains to be characterized experimentally. This design has in fact been fabricated twice. However, the first board is over-etched, so the manufacturer has re-spun the design at no cost to us. For the second design, fabrication issues have also been encountered for the unit cell vias. This results in adjacent unit cells being shorted together, or open circuited within the vias themselves. The manufacturer has narrowed this problem down to their stock of Rogers 2929 Bondply. In addition to this, though the manufacturer states minimum trace width and spacing capabilities at 2 mils, it has been explained after this incident that this applies only to small designs. For larger boards, these tolerances need to be increased. However, the same manufacturer was responsible for the 2D refraction design, and no such issues were encountered or mentioned during that time.

Despite these problems, the second board yields some useful results. Fig. 5.1 demonstrates that in the H-plane, the measured minimum waist size is between the HFSS and TLM results at the design point of 10.0 GHz over almost the entire 1.2 GHz frequency range. Note that this 0.6 cm variation in the minimum waist size is just $0.2\lambda$.

![Figure 5.1: The measured H-plane minimum waist size.](image)

Fig. 5.2 demonstrates that in the H-plane, the measured minimum waist position is almost between the HFSS and TLM results over a frequency range of 10.1 to 10.6 GHz. Note that the variation in minimum waist position in this frequency band is about 0.6 cm which is again just $0.2\lambda$. However, in the E-plane, the minimum waist size and position do not follow the simulated trends. In addition to the possible fabrication
errors, in the E-plane the focusing occurs closer to the metasurface and with a smaller minimum waist size. Thus, it is more extreme than in the H-plane. Recall that the conical horn used here has an asymmetric field pattern in the E and H-planes.

The far-field patterns have been measured and are far from Gaussian-like. Thus, the total efficiency has not been determined. However, efficiency has still been measured, and is presented in Fig. 5.3. Ignoring the 9.6 GHz data point, which is far below 50% as a threshold at the frequency bounds, the average efficiency over this band is 65.77%. This is much less than the 84.43% in the 2D refraction design though the frequency range is larger here. However, given that the minimum waist size and position are both valid over just a 6% fractional bandwidth instead of the full 12% here, the value of the metric determined here is uncertain. The cross-polarization levels are also presented in Fig. 5.4, and are much degraded in comparison to those in Fig. 3.16 and 3.20.

Finally, observing the near-field profiles of the normalized complex electric field, a double-focus is observed in the H-plane in Fig. 5.5. This occurs also in the E-plane. In HFSS, taking adjacent unit cells and shorting them, and open-circuiting the unit cell vias, this double-focus has not been successfully reproduced. Furthermore, it has been demonstrated in chapter 3 that in the improved experimental setup, quasi-optical
adherence has been improved, so this double focus should not be attributed to edge-diffraction effects.

The unit cells used in this 1D focusing design are the same in nature as those used for 2D refraction. This strongly suggests that the degradation in matching, insertion loss, and cross-polarization observed here, as well as the double focus in Fig. 5.5 and the unpredictable E-plane focusing characteristics, are due mainly to the fabrication issues encountered. To facilitate fabrication requirements, future designs will scale to higher frequencies to allow for the minimal trace width and spacing tolerances.
5.1.2 Passive, Lossless and Reflectionless Transmitarrays with Suppressed Side-Lobes

The passive and lossless plane-wave refraction in [2] is capable in theory of eliminating the side-lobes entirely, though it permits a small amount of reflections. In the supplemental material to [3], it is demonstrated that if the reflections are in addition also eliminated from the problem, the impedances become complex. In Fig. 2.4, taking the problem in [2], it is seen from the TLM solver that the side-lobe levels are influenced mainly by the discretization levels. This ignores the complexities of material losses and parasitic field-unit cell geometry interactions, among other things. In fact, at $\lambda/20$, the maximum side-lobe level is less than -30 dB. The reflections are also demonstrated to be low in general, however this is not due primarily to discretization levels.

Taking a more extreme case of plane-wave refraction, such as from normal incidence to $60^\circ$, the reflections are simulated in the TLM solver to increase, even for a discretization level of $\lambda/20$. Thus, passive, lossless, and reflectionless refraction is not possible for extreme cases following the approach of [2].

![Figure 5.6: The far-field radiation pattern for plane-wave refraction from normal incidence to $60^\circ$.](image)

It is possible to use the Huygens’ metasurface to enable passive, lossless, and reflectionless plane-wave refraction with the suppressed side-lobes. This holds even for extreme cases such as normal incidence to $\pm 60^\circ$. However, past this mark, even this solution begins to fail. This is most likely a numerical issue, however, due to the extreme range of impedances that is required. The details to this solution will be presented in future work, and at present it suffices just to disclose some results. Fig. 5.7 shows the far-field patterns for normal incidence to a refracted angle of between $-60^\circ$ and $+60^\circ$. It is observed that reflections and side-lobes remain highly suppressed throughout. Fig. 5.8 shows the power contained in all reflections as a percentage of the total power incident on the Huygens’ metasurface. It is observed that this is usually around 0.2%. Note that these results are from the TLM solver, not HFSS, and also that this is not a demonstration of reconfigurable beamforming. That is, the Huygens’ metasurfaces are static.
Figure 5.7: The far-field patterns for normal incidence to a refracted angle of between -60° and +60°.

Figure 5.8: The power contained in all reflections as a percentage of the total power incident on the Huygens' metasurface.
Appendix A

Geometries and Two-Port Parameters of Huygens’ Source Unit Cells

Tables A.1 to A.10 provide all the geometrical parameters, as well as the Z and S-parameters, related to the 1D refraction, 2D refraction and 1D focusing designs. The cell dimensions and materials are discussed in the relevant chapters. Fig. 2.5a provides the geometry of the unit cells used for 1D refraction. Fig. A.1a to A.1c provide the geometry of the unit cells used for 2D refraction and are shown here. Fig. 4.6a to 4.6c provide the geometry of the unit cells used for 1D focusing. The trace width and spacing in all cases are 10 mils, except for Fig. A.1c, where the interdigitated fingers have 5 mils, and between the vias of the unit cells in chapters 3 and 4, where the spacing is 4 mils. The via diameters are all 0.2 mm. Finally in Tables A.7 to A.10, the cell numbering begins with the center of the lens and proceeds outward.

Figure A.1: Unit cells for 2D refraction. Unit cells synthesized in HFSS using the method described in subsections 2.2 and 2.3 for this design. The printed capacitors synthesizing the impedances are visible. (a) Cell type 1, (b) cell type 2 and, (c) cell type 3.
### Table A.1: Geometries of the unit cells used in the 1D refraction design

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<th>Cell Number</th>
<th>Number of Gaps</th>
<th>Loop Width (mils)</th>
<th>Dipole Gap Width (mils)</th>
<th>Dipole Width (mils)</th>
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### Table A.2: S and Z-parameters of the unit cells used in the 1D refraction design

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<th>Cell Number</th>
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<th>$S_{21}$</th>
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<td>1442.94</td>
</tr>
<tr>
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<td>9.90E-1</td>
<td>-45.28</td>
<td>2549.84</td>
</tr>
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<td>9.89E-1</td>
<td>-14.20</td>
<td>7166.90</td>
</tr>
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<td>9.31E-1</td>
<td>-23.69</td>
<td>-3415.27</td>
</tr>
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<td>9.87E-1</td>
<td>47.39</td>
<td>-3674.32</td>
</tr>
<tr>
<td>17</td>
<td>2.15E-1</td>
<td>9.63E-1</td>
<td>99.57</td>
<td>-2761.57</td>
</tr>
<tr>
<td>18</td>
<td>5.68E-2</td>
<td>9.86E-1</td>
<td>102.59</td>
<td>-1212.71</td>
</tr>
<tr>
<td>19</td>
<td>3.13E-2</td>
<td>9.88E-1</td>
<td>152.57</td>
<td>-873.54</td>
</tr>
<tr>
<td>20</td>
<td>4.75E-2</td>
<td>9.89E-1</td>
<td>204.60</td>
<td>-630.00</td>
</tr>
</tbody>
</table>
Table A.3: Geometries of cell type 1 used in the 2D refraction design

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>Number of Gaps</th>
<th>Loop Width (mils)</th>
<th>Dipole Gap Width (mils)</th>
<th>Dipole Width (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>76.00</td>
<td>94.98</td>
<td>148.99</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>83.25</td>
<td>94.84</td>
<td>149.87</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>98.64</td>
<td>94.99</td>
<td>149.69</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>132.63</td>
<td>94.83</td>
<td>149.31</td>
</tr>
<tr>
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<td>5</td>
<td>20.17</td>
<td>32.07</td>
<td>20.52</td>
</tr>
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<td>6</td>
<td>5</td>
<td>148.36</td>
<td>31.57</td>
<td>21.23</td>
</tr>
<tr>
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<td>5</td>
<td>141.96</td>
<td>30.19</td>
<td>26.52</td>
</tr>
<tr>
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<td>149.97</td>
<td>82.18</td>
<td>30.00</td>
</tr>
<tr>
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<td>3</td>
<td>102.12</td>
<td>85.69</td>
<td>25.00</td>
</tr>
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<td>106.08</td>
<td>91.21</td>
<td>36.58</td>
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<tr>
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<td>2</td>
<td>108.81</td>
<td>93.71</td>
<td>55.12</td>
</tr>
<tr>
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<td>2</td>
<td>111.84</td>
<td>71.18</td>
<td>137.54</td>
</tr>
<tr>
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<td>2</td>
<td>112.98</td>
<td>92.59</td>
<td>96.12</td>
</tr>
<tr>
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<td>67.65</td>
<td>94.97</td>
<td>94.96</td>
</tr>
<tr>
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<td>1</td>
<td>68.84</td>
<td>90.17</td>
<td>137.25</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>70.72</td>
<td>91.05</td>
<td>147.43</td>
</tr>
<tr>
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<td>1</td>
<td>72.98</td>
<td>93.36</td>
<td>148.29</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>66.61</td>
<td>90.17</td>
<td>137.25</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>70.72</td>
<td>91.05</td>
<td>147.43</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>72.98</td>
<td>93.36</td>
<td>148.29</td>
</tr>
</tbody>
</table>

Table A.4: Geometries of cell type 2 used in the 2D refraction design

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>Loop Length (mils)</th>
<th>Dipole Gap Width (mils)</th>
<th>Dipole Width (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>80.00</td>
<td>95.00</td>
<td>150.00</td>
</tr>
</tbody>
</table>

Table A.5: Geometries of cell type 3 used in the 2D refraction design

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>Loop Length (mils)</th>
<th>Dipole Length (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>81.05</td>
<td>90.05</td>
</tr>
<tr>
<td>7</td>
<td>84.99</td>
<td>90.56</td>
</tr>
</tbody>
</table>
### Table A.6: S and Z-parameters of the unit cells used in the 2D refraction design

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>$S_{11}$</th>
<th>$S_{21}$</th>
<th>$Z_e$</th>
<th>$Z_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.79E-2∠152.52°</td>
<td>9.91E-1∠65.04°</td>
<td>286.24</td>
<td>-466.46</td>
</tr>
<tr>
<td>2</td>
<td>2.55E-1∠138.07°</td>
<td>9.60E-1∠47.41°</td>
<td>311.16</td>
<td>-221.63</td>
</tr>
<tr>
<td>3</td>
<td>3.79E-1∠118.76°</td>
<td>9.20E-1∠28.72°</td>
<td>394.00</td>
<td>-39.72</td>
</tr>
<tr>
<td>4</td>
<td>3.57E-1∠100.87°</td>
<td>9.28E-1∠10.57°</td>
<td>663.75</td>
<td>72.70</td>
</tr>
<tr>
<td>5</td>
<td>7.99E-2∠82.35°</td>
<td>9.91E-1∠11.84°</td>
<td>-2961.80</td>
<td>111.59</td>
</tr>
<tr>
<td>6</td>
<td>2.55E-1∠138.07°</td>
<td>9.60E-1∠47.41°</td>
<td>-610.85</td>
<td>302.34</td>
</tr>
<tr>
<td>7</td>
<td>3.57E-1∠118.76°</td>
<td>9.20E-1∠28.72°</td>
<td>302.34</td>
<td>-113.87</td>
</tr>
<tr>
<td>8</td>
<td>3.06E-1∠167.38°</td>
<td>9.48E-1∠103.28°</td>
<td>-106.45</td>
<td>693.86</td>
</tr>
<tr>
<td>9</td>
<td>3.29E-1∠156.95°</td>
<td>9.40E-1∠113.87°</td>
<td>-81.94</td>
<td>814.74</td>
</tr>
<tr>
<td>10</td>
<td>1.06E-1∠139.73°</td>
<td>9.87E-1∠133.54°</td>
<td>-69.33</td>
<td>1524.51</td>
</tr>
<tr>
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<td>8.21E-3∠144.31°</td>
<td>9.91E-1∠151.01°</td>
<td>-48.77</td>
<td>2916.42</td>
</tr>
<tr>
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<td>1.23E-2∠151.24°</td>
<td>9.89E-1∠169.06°</td>
<td>-17.16</td>
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</tr>
<tr>
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<td>9.85E-1∠172.41°</td>
<td>5.91</td>
<td>-7301.00</td>
</tr>
<tr>
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<td>9.87E-1∠154.77°</td>
<td>42.34</td>
<td>-3370.70</td>
</tr>
<tr>
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<td>8.87E-2∠139.35°</td>
<td>9.85E-1∠135.91°</td>
<td>66.68</td>
<td>-1642.70</td>
</tr>
<tr>
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<td>9.88E-1∠119.10°</td>
<td>110.72</td>
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</tr>
<tr>
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<td>9.89E-1∠101.29°</td>
<td>152.29</td>
<td>-903.87</td>
</tr>
<tr>
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<td>9.90E-1∠82.97°</td>
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<td>-660.92</td>
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### Table A.7: Geometries of cell type 1 used in the 1D focusing design

<table>
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<tr>
<th>Cell Number</th>
<th>Number of Gaps</th>
<th>Loop Width (mils)</th>
<th>Dipole Gap Width (mils)</th>
<th>Dipole Width (mils)</th>
</tr>
</thead>
<tbody>
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<td>77.31</td>
<td>94.94</td>
<td>149.88</td>
</tr>
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<td>77.10</td>
<td>94.85</td>
<td>149.06</td>
</tr>
<tr>
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<td>1</td>
<td>76.00</td>
<td>94.56</td>
<td>149.12</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>74.03</td>
<td>95.00</td>
<td>149.83</td>
</tr>
<tr>
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<td>1</td>
<td>73.40</td>
<td>94.85</td>
<td>139.17</td>
</tr>
<tr>
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<td>1</td>
<td>71.83</td>
<td>92.47</td>
<td>150.00</td>
</tr>
<tr>
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<td>1</td>
<td>70.45</td>
<td>91.09</td>
<td>150.00</td>
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<td>1</td>
<td>69.63</td>
<td>95.00</td>
<td>111.73</td>
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<td>67.76</td>
<td>93.92</td>
<td>109.00</td>
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<td>118.74</td>
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<td>53.08</td>
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<td>35.80</td>
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<td>1</td>
<td>148.77</td>
<td>94.83</td>
<td>149.54</td>
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<td>1</td>
<td>85.67</td>
<td>95.00</td>
<td>149.69</td>
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<td>94.98</td>
<td>138.38</td>
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<td>145.00</td>
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<td>94.92</td>
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<td>85.67</td>
<td>95.00</td>
<td>149.69</td>
</tr>
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<td>65.94</td>
<td>93.94</td>
<td>95.69</td>
</tr>
<tr>
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<td>106.60</td>
<td>87.42</td>
<td>41.39</td>
</tr>
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</table>
## Appendix A. Geometries and Two-Port Parameters of Huygens’ Source Unit Cells

### Table A.8: Geometries of cell type 2 used in the 1D focusing design

<table>
<thead>
<tr>
<th>Cell Number</th>
<th>Number of Gaps</th>
<th>Loop Width (mils)</th>
<th>Dipole Width (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
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<td>100.49</td>
<td>114.32</td>
</tr>
<tr>
<td>15</td>
<td>2</td>
<td>90.48</td>
<td>76.18</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
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</tr>
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<td>34.88</td>
<td>39.40</td>
</tr>
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<td>24</td>
<td>2</td>
<td>97.12</td>
<td>107.42</td>
</tr>
<tr>
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<td>2</td>
<td>64.15</td>
<td>44.87</td>
</tr>
</tbody>
</table>

### Table A.9: Geometries of cell type 3 used in the 1D focusing design

<table>
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<th>Cell Number</th>
<th>Loop Length (mils)</th>
<th>Dipole Gap Width (mils)</th>
<th>Dipole Width (mils)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
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<td>150.00</td>
</tr>
</tbody>
</table>

### Table A.10: S and Z-parameters of the unit cells used in the 1D focusing design

<table>
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<th>Cell Number</th>
<th>$S_{11}$</th>
<th>$S_{21}$</th>
<th>$Z_e$ (Ω)</th>
<th>$Z_m$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.16E-2∠148.16°</td>
<td>9.89E-1∠60.09°</td>
<td>297.03</td>
<td>-394.32</td>
</tr>
<tr>
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<td>8.84E-2∠150.03°</td>
<td>9.88E-1∠61.70°</td>
<td>285.83</td>
<td>-404.85</td>
</tr>
<tr>
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<td>7.22E-2∠155.00°</td>
<td>9.89E-1∠66.02°</td>
<td>267.87</td>
<td>-452.03</td>
</tr>
<tr>
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<td>7.28E-2∠17.31°</td>
<td>9.88E-1∠70.72°</td>
<td>287.02</td>
<td>-578.25</td>
</tr>
<tr>
<td>5</td>
<td>8.14E-3∠145.03°</td>
<td>9.90E-1∠79.99°</td>
<td>222.73</td>
<td>-627.69</td>
</tr>
<tr>
<td>6</td>
<td>1.68E-1∠23.29°</td>
<td>9.89E-1∠90.15°</td>
<td>191.14</td>
<td>-765.95</td>
</tr>
<tr>
<td>7</td>
<td>2.78E-2∠20.44°</td>
<td>9.88E-1∠101.16°</td>
<td>158.94</td>
<td>-944.18</td>
</tr>
<tr>
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<td>4.79E-2∠164.19°</td>
<td>9.88E-1∠114.96°</td>
<td>114.01</td>
<td>-1120.54</td>
</tr>
<tr>
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<td>9.88E-1∠130.98°</td>
<td>86.46</td>
<td>-1664.06</td>
</tr>
<tr>
<td>10</td>
<td>5.16E-2∠68.88°</td>
<td>9.87E-1∠149.28°</td>
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<td>-3045.24</td>
</tr>
<tr>
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</tr>
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</tr>
<tr>
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</tr>
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<td>9.91E-1∠117.67°</td>
<td>-120.45</td>
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</tr>
<tr>
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<td>2.52E-2∠8.46°</td>
<td>9.96E-1∠90.76°</td>
<td>-190.90</td>
<td>781.74</td>
</tr>
<tr>
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<td>1.67E-2∠41.67°</td>
<td>9.98E-1∠60.34°</td>
<td>-329.60</td>
<td>448.80</td>
</tr>
<tr>
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<td>9.76E-1∠28.77°</td>
<td>-1307.25</td>
<td>277.91</td>
</tr>
<tr>
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<td>3.28E-1∠96.95°</td>
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<td>824.39</td>
<td>87.92</td>
</tr>
<tr>
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<td>2.86E-1∠133.95°</td>
<td>9.52E-1∠43.58°</td>
<td>324.04</td>
<td>-180.07</td>
</tr>
<tr>
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<td>7.26E-2∠161.67°</td>
<td>9.88E-1∠75.45°</td>
<td>226.23</td>
<td>-538.71</td>
</tr>
<tr>
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<td>2.42E-2∠168.26°</td>
<td>9.88E-1∠119.26°</td>
<td>107.50</td>
<td>-1251.17</td>
</tr>
<tr>
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<td>1.97E-2∠142.76°</td>
<td>9.87E-1∠161.19°</td>
<td>29.73</td>
<td>-4302.38</td>
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<tr>
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<td>8.87E-3∠174.03°</td>
<td>9.90E-1∠154.49°</td>
<td>-42.28</td>
<td>3299.59</td>
</tr>
<tr>
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<td>4.24E-3∠69.51°</td>
<td>9.94E-1∠108.88°</td>
<td>-135.02</td>
<td>1057.07</td>
</tr>
<tr>
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Bibliography


