Bio-inspired Optimal Locomotion Reconfigurability of Quadruped Rovers using Central Pattern Generators

by

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Abstract

Legged rovers are often considered as viable solutions for traversing unknown terrain. This work addresses the optimal locomotion reconfigurability of quadruped rovers, which consists of obtaining optimal locomotion modes, and transitioning between them. A 2D sagittal plane rover model is considered based on a domestic cat. Using a Genetic Algorithm, the gait, pose and control variables that minimize torque or maximize speed are found separately. The optimization approach takes into account the elimination of leg impact, while considering the entire variable spectrum. The optimal solutions are consistent with other works on gait optimization, and are similar to gaits found in quadruped animals as well. An online model-free gait planning framework is also implemented, that is based on Central Pattern Generators is implemented. It is used to generate joint and control trajectories for any arbitrarily varying speed profile, and shown to regulate locomotion transition and speed modulation, both endogenously and continuously.
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Chapter 1: Introduction

1.1 Motivation

The motivation for this work arises from a need to design reconfigurable rovers that can traverse multiple terrain. Over such terrain, legged locomotion is considered far superior to wheeled locomotion. The former is inspired by the locomotion capabilities of animals, which are well studied in literature [1]. While animals cannot reconfigure their structures to imitate other animals, they can achieve different locomotion modes by reconfiguring their gaits. This continuous transition depends on their speed and other requirements, such as energy, stability etc. Thus, animals exhibit a continuum of changing gaits as a function of their speed, which is a desirable property for legged rovers. It is essential in robotics to be able to find the optimal gaits that animals use, and transition between these gaits continuously in a manner similar to animals. More importantly, such optimizations and trajectory generation tools should exhibit some level of independence from the model of the rover itself. This would enable researchers to apply a gait optimization and planning framework to robotic structures based on any animal. Unfortunately, there is a gap in literature on such frameworks, for both walking and running gaits.

Thus, there is an opportunity for research in developing a universal gait optimization framework. By applying such an optimization model to a single quadruped rover, one would obtain the series of optimum gaits (gait profile) to be used for any given speed (speed profile). The model-free trajectory generator can generate joint and control trajectories for both, speed modulation and gait transition, in a smooth and endogenous manner. Coupled with this trajectory generator, the quadruped rover model can reconfigure its locomotion parameters to use optimal gaits as a function of speed. This continuous transition is achieved with a single input parameter alone – desired speed.

The gait optimization and planning framework developed in this thesis, can be applied to any quadruped rover structure, thereby creating opportunities for further work in finding a globally optimum quadruped rover structure.
1.2 Outline

This thesis primarily consists of two research areas. Firstly, the Gait optimization problem of a quadruped rover is addressed, to minimize joint torque at fixed speeds, and maximize speed. Secondly, a trajectory generator based on a phase oscillator model of a Central Pattern Generator is introduced to transition between multiple optimum gaits endogenously and continuously, in an online manner.

Chapter 1 presents the motivation for this work, as well as an outline of the thesis. Chapter 2 examines the background of research on quadruped rover locomotion and designs, optimization algorithms, joint trajectory design and control, biological CPGs and CPG models in robotics. It also presents the main objectives of this work.

Chapter 3 describes the gait optimization problem of a rover and the optimization framework developed in this work. Firstly, the section describes the quadruped rover model and the design variables selected for the optimization process. Secondly, it describes the rover’s kinematic and dynamic equations. Thirdly, it describes the optimization objectives for two separate cases – i) minimizing torque for a fixed speed, and ii) maximizing speed. Next, it outlines the numerous instantaneous and permanent constraints that must be satisfied by the rover during its evaluation process. Finally, it presents the Genetic Algorithm used in this work.

Chapter 4 describes the online CPG-based trajectory generator. Firstly, the three modules of the CPG are described. The PONN (Phase Oscillator Neural Network) consists of phase oscillators that serve as a type of associative memory to store patterns. The CRI (CPG-Rover Interface) converts the transition of the PONN’s oscillators between their stored patterns into meaningful transitions between various joint and control configurations of the quadruped rover. The HCU (Higher Level Control Unit) modulates the working of the PONN and CRI to effectively perform two functions in an online manner – i) speed modulation, and ii) gait modulation.

Chapter 5 details the Integrated Design and Simulation Environment that was designed in this work using MATLAB®, Simulink® and SimMechanics®. This environment was used to carry out
the gait optimization of a feline quadruped rover for various speeds, and exhibit the working of the online CPG-based trajectory generator. This section consists of the rover model used in simulation, and the environment model. It also describes the separate optimization of the CPG’s internal parameters using the Genetic Algorithm presented in Chapter 3.

Chapter 6 presents and discusses the results of this work. First, it compares the optimum gaits found from the gait optimization in Chapter 3 with the results of other optimization works as well as the gaits found in quadruped animals. Secondly, it discusses the results of using the online CPG trajectory generator for two speed profiles, to exhibit stability during both, speed and gait modulation.

Chapter 7 concludes the thesis by summarizing the main contributions of this thesis. It also discusses some directions for future work.
Chapter 2: Background

Section 2.1 describes the two approaches often taken by researchers when designing rovers based on animals, and how this work combines the two approaches. Section 2.2 describes the gait optimization problem, and why evolutionary algorithms are efficient for such problems. Section 2.3 details the simulation models used by researchers in the past as well as different methods to define joint trajectories. Section 2.4 discusses the relevance of biological CPGs in controlling rhythmic locomotion as well as the key works that have adapted these to phase oscillator neural networks for robotic applications.

2.1 Biomimicry and Bioinspiration

There are two approaches used by researchers in designing quadruped rovers based on animals. In the first approach, the rover’s kinematic, dynamic and control parameters are directly derived from the morphology of the animal in consideration. This is referred to as biomimicry, many examples of which can be found in [2]. An alternative approach, referred to as bioinspiration, obtains the robot’s parameters by using algorithms that are based on nature, such as evolutionary algorithms [3], swarm algorithms [4], etc. Although many works in the literature can aid in selecting optimal design parameters through the former approach, the latter approach is more tractable because it involves optimizing a mathematical model based on kinematics and dynamics. This work combines the two approaches by categorizing the robot parameters into four groups, based on an animal’s ability to change its parameters during locomotion: i) structure parameters (e.g., length of links), ii) gait parameters (e.g., duty factor), iii) pose parameters (e.g., joint angles), and iv) control parameters (e.g., control gains.) Animals can often achieve multiple gaits by changing the parameters in the gait, pose and control categories. However, animals do not possess the ability to change their structure parameters. The structure parameters of the robot in this thesis are based on the structure of a domestic cat [5] (biomimicry). The gait, pose and corresponding
control parameters are optimized using an evolutionary algorithm (bioinspiration), thereby combining the two approaches.

Several legged rovers have been developed that can change their kinematic, dynamic and/or control parameters to use various locomotion modes [6-12]. While some rover designs use multiple locomotion modes by varying their structure parameters [13], animals cannot do so. These structure parameters include the orientation of the joints with respect to each other, length of the links, mass of the links, etc. On the other hand, gait and pose parameters can be easily modified, and include joint angles, the stride length and duration, relative leg motion, etc. Locomotion modes that are achieved by a change in gait and pose parameters alone are called gaits, many examples of which can be found in [1].

2.2 Locomotion Optimization Problem

Optimum gait generation has been shown to be a nonlinear, multi-constrained optimization problem [14]. Evolutionary algorithms (EA) have been used by many researchers for this problem for multiple reasons. Firstly, EAs do not need the formulation relating the design variables to the design attributes [3]. This allows researchers to consider the rover as a black-box during simulations, making the optimization process modular. These relations, which correspond to the quadruped rover’s dynamics equations of locomotion, are highly-coupled and non-linear. Secondly, EAs can be easily implemented when a large number of design variables are involved, their inter-relationships aren’t necessarily known, or the state space of possible solutions is disjoint [15]. Moreover, the design variable space The design variables selected for the quadruped rover in this work, belong to a multi-dimensional and coupled space, which reduces the efficacy of standard gradient-search-based optimization techniques [16]. Thirdly, EAs do not require information such as the continuity or differentiability of the associated objective function(s), and can easily find multiple local minima/maxima [3]. Lastly, with a suitable choice of population size, mutation rate and recombination rates in the EA, the algorithm can be insensitive to the initial population selected, and has strong global search capabilities.
Evolutionary algorithms have been used by many researchers. The gait optimization problem for a dog-like quadruped rover for various walking speeds is carried out in [17]. An optimization of gaits for the AIBO robot is carried out using Genetic Algorithms (GA) with the aid of AIBO hardware [18-20]. A biped humanoid robot for climbing slopes is suggested in [21], and optimized using a GA to achieve maximum forward velocity and stability, as well as minimum energy consumption. The parameters of a hexapod rover are optimized using the GA for different locomotion speeds and periodic gaits [22]. Researchers in these works have shown in simulation or hardware implementation, that the solutions obtained using the GA are indeed stable and optimum.

2.3 Locomotion and Trajectory Design in the Sagittal Plane

The locomotion of a quadruped rover is achieved by defining the trajectories of actuated joints. There are two approaches often used. In the first approach, the parameters defining the joint trajectories are directly optimized. This method has been used by various researchers, such as Vermeulen et al., who used joint trajectories based on polynomial functions [23]. The second approach involves the design of the motion of the foot or toe, and solving for the corresponding joint trajectories by Inverse Kinematics [24]. In [22], the trajectory for the toes of a hexapod robot was selected as a cycloid during the swing phase and a uniform horizontal trajectory during stance phase. This paper uses the second approach, because it allows one to take into account the leg impact at touchdown. A smooth, continuous and periodic toe trajectory is created by defining a series of Bezier functions. Animals tend to retract their paws right before touchdown to reduce leg impact as well, and this has been shown to aid locomotion periodicity in legged robots [25-26].

To cover all aspects of legged locomotion, the movement of a quadruped rover must be analyzed along three translational directions (forward-backward thrust, upward-downward heave and sideways sway), as well as three rotational directions (roll, yaw and pitch, about the respective translational directions). However, a 2D model in the sagittal plane can reveal most aspects of quadruped locomotion without the complexities of a 3D model [27], such as stabilization of the
frontal plane rolling motion which was addressed in [28]. A 2D rigid body model was used to simulate the locomotion of a horse from patterns from a given kinematic dataset [29]. A fuzzy-logic-based control strategy was developed for creating simulated planar gallops in quadrupeds [30]. A 10-link planar simulation was used to test the feed-forward and feedback strategies for controlling transverse gallops in a horse [31]. In this paper, the main body of the quadruped rover is constrained to move in the sagittal plane, described by two translational degrees of freedom (thrust and heave) and one rotational degree of freedom about an axis perpendicular to the sagittal plane (pitch).

2.4 Central Pattern Generators in Robotics

Once the optimal gaits are found by the optimization process, the rover must be able to transition between these gaits smoothly. To understand gait transition, one must look at the control mechanism in animals used to transition gaits during locomotion. There is enough evidence to suggest that rhythmic patterns in animals are generated centrally in the nervous system without the need for an external controlling input [32]. This is associated with biological Central Pattern Generators (CPG), which refer to a network of neurons that can produce periodic patterns without requiring any external rhythmic input [33]. This endogenous behaviour is known to underlie various rhythmic movements in organisms such as breathing, chewing, and digestion, locomotion etc. and serve as basic building blocks in the neural circuits in both invertebrates and vertebrates [34]. The CPG network consists of multiple coupled oscillatory centres, and it has been proposed that CPGs have at least one unit for each degree-of-freedom in the structure [35].

The complex CPG model can select which gait to follow internally, based on simple external input signals. In many vertebrate animals, electrical stimulation in the Mesencephalic Locomotor Region (MLR) is known to result in locomotion behaviour via the CPG. This property makes it desirable to implement gait planning and control models in robotics that mimic biological CPGs. Varying the intensity of stimulation is also known to enable the CPG to automatically induce gait transition [34]. Various types of CPG models have been used in robotics, such as connectionist
models [36, 37], vector maps [38] and systems of coupled oscillators [39, 40]. All these models include the numerical integration of a set of coupled differential equations. By adjusting the coupling between the oscillators, various gaits can be achieved. In these models, there exists a direct correlation between the size of the neural network and the morphology of the animal that the structure of the robot is based on. Such a CPG model has been used for a variety of robots. It has been used to control a hexapod robot inspired by insect locomotion [41]. CPGs have been used for controlling swimming robots [42], terrestrial snake models [43], as well as biped locomotion in humanoid robots [44]. CPG-based models have also been used to control quadruped locomotion for a given gait while modulating speed [45].

It is desirable to have a CPG model which is independent from the robot morphology or number of joints. This is because the optimization framework developed in this work can be applied to any quadruped robotic structure. The use of Denavit-Hartenberg parameters [46] to describe the rover’s structure, allows for easy modification and addition of joints in any leg. A CPG framework that is independent from the robot morphology was developed in [47], where the neural network does not consider the number of degrees of freedom in the corresponding robot. The size of the neural network only depends on the number of configurations to be cycled through by any joint. Through a mapping between the phase oscillators and the joint trajectories, a gait change can be easily induced by changing the configurations to be cycled through. As the framework is relatively recent, its adaptation to a quadruped robotic structure for both walking and running gaits is missing in literature, and requires further study.

Besides the obvious relation of artificial CPGs in robotics to their biological counter parts in controlling animal locomotion, there are other properties that make CPGs ideal for trajectory generation. Firstly, CPG models generate stable cyclic trajectories, which are robust to perturbations in the corresponding state variables [34]. This property aids in creating a smooth gait transition even when the desired speed changes abruptly. Secondly, CPG models can be used to induce speed and gait transition with very few input parameters. In this work, a single input parameter – the desired speed, is used. This helps reduce the dimensionality of the gait planning and control problem. Lastly and most importantly, the CPG model can be designed to be model-
free, and the gait planning mechanism needs no knowledge of the rover’s kinematics and dynamics. This enables the implementation of the CPG to generate joint and control trajectories for any robotic structure.

2.5 Objectives of this work

Based on the gaps in the literature described above, the objectives of this work are as follows:

1. To develop a universal evolutionary optimization model to find the optimum gaits for a quadruped robot, which eliminates leg impact at touchdown. This will be accomplished by defining smooth, continuous and periodic Bezier trajectories for the motion of the toes of each leg;

2. To minimize the joint torque or maximize forward speed using the developed optimization model, while taking into account the entire gait and pose variable spectrum;

3. To evaluate the optimal gait results by comparing the trends followed by the optimal solutions to those found in other works on gait optimization as well as in quadruped animals;

4. To design a system of CPGs to generate trajectories for a quadruped rover’s joints and control gains in an online manner. This will be accomplished by cycling through a sequence of joint configurations corresponding to the optimal joint motion. The phase oscillator patterns corresponding to these joint configurations will be stored in an associative memory of phase oscillators, and retrieved endogenously by a retrieval network;

5. To generate trajectories for a smooth speed transition at a given gait. This will be accomplished by coupling the natural frequency of the retrieval network’s pacemaker to the stride time of the rover;

6. To generate trajectories for a smooth gait transition. This will be accomplished by integrating external inputs from the CPG’s higher level control unit with the CPG-Robot Interface to modify which joint configurations to be cycled through.
7. To evaluate the performance of the online CPG-based trajectory generator for two different speed cases – i) continuous speed profile using a ramp function, and ii) discontinuous speed profile using a series of step functions.
Chapter 3: Locomotion Optimization

3.1 Quadruped Rover Model

As shown in figure 1, a quadruped robot consists of a main body $B$ and four legs, behaving similarly to a parallel manipulator. The main body has a mass $M_B$ and moment of inertia $I_B$ with respect to the attached coordinate frame $\{B\}$ at its centre of mass. It is constrained to move in the sagittal plane with respect to the inertial world frame $\{W\}$, described by a horizontal position $x$, vertical position $z$ and pitch $\psi$. The legs are divided into two groups, front (F) and hind (H), and the right (R) and left (L) legs in each group are bilaterally symmetric. Each leg is modeled as a three-degree-of-freedom (DOF) serial manipulator, consisting of three parallel revolute joints: hip$(O_0)$, knee $(O_1)$ and ankle$(O_2)$, and three links of lengths $L_1$, $L_2$ and $L_3$, respectively. For the front/hind legs, these links are the Humerus/Femur $(\overline{O_0O_1})$, Radius/Tibia $(\overline{O_1O_2})$ and

Figure 1 - Rover model in the sagittal plane
Metacarpal/Metatarsal ($O_2O_3$), respectively. The leg structure resembles that of a domestic cat, without the phalanges [5]. The total leg length $L_t$ is the sum of the length of the links:

$$L_t = L_1 + L_2 + L_3$$ (1)

The ratio of the length of each link to the total leg length is selected here based on data collected for the limbs of domestic cats [5]. Each leg is connected to the main body $B$ via the leg’s hip joint. The leg touches the ground at the toe ($O_3$). The instant when each leg’s toe hits the ground during the locomotion cycle is referred to as the leg’s touchdown. The instant when its toe is lifted off the ground is referred to as the leg’s take-off. The phase between a leg’s touchdown and take-off is called stance. The phase between a leg’s take-off and touchdown is called swing. The locomotion stride is a cycle during which all four legs complete one swing and one stance phase. The successive stride cycles in quadruped animals are more or less identical [1]. The distance travelled by ($O_B$) during the stride time $T$ in the thrust direction is referred to as the stride length $\lambda$. The velocity of the main body’s center of mass ($O_B$) in the thrust direction is assumed to be uniform ($\lambda/T$). During the swing phase of a leg, its hip joint ($O_0$) moves forward with the main body $B$ along the
trajectory \( (W_0(t))_{sw} \) with respect to the world frame \( \{W\} \). During the same phase, the leg’s toe \( (O_3) \) moves along the trajectory \( (O_3(t))_{sw} \) with respect to frame \( \{0\} \) attached to the main body at the hip (figure 2). During the stance phase, the leg’s toe maintains contact with the ground, which is assumed to be rigid to prevent the toe from penetrating it. If no slippage occurs, i.e., \( (W_3(t))_{st} = \text{constant} \), the toe behaves similar to a passive pivot joint, about which the leg rotates and carries the hip \( (O_0) \) along the trajectory \( (O_0(t))_{st} \) with respect to frame \( \{3\} \) attached to metatarsal at \( (O_3) \). As illustrated in figure 2, to avoid using the toe as a reference frame during stance, one can consider the trajectory \( (O_3(t))_{st} \), which is the same as \( (O_0(t))_{st} \) but rotated about \( (O_3) \) by \( \pi \) radians [48]. As the toe moves backward along \( (O_3(t))_{st} \), the main body moves forward along \( (O_3(t))_{st} \) as desired. Each leg can be parameterized by the joint angles \( \theta_1 \) (hip), \( \theta_2 \) (knee) and \( \theta_3 \) (ankle):

\[
q(t) = [\theta_1(t) \quad \theta_2(t) \quad \theta_3(t)]
\]  

(2)

To drive the main body in the sagittal plane, the three joints in each leg are actuated to follow specific desired joint trajectories \( q(t) \). One approach is to define desired joint trajectories \( q(t) \), and derive the toe trajectories \( (O_3(t))_{sw} \) and \( (O_3(t))_{st} \), respectively, using Forward Kinematics [24]. However, to allow for a better insight into the locomotion of the rover’s body, it is desirable to assign the toe trajectories first, and derive the desired joint trajectories using Inverse Kinematics [24]. The second method is adopted in this work, and will be discussed shortly. To derive the joint trajectories \( q(t) \) from the toe trajectories using Inverse Kinematics, the joint angles \( q_0 \) for an initial configuration of the leg must be known. Since different initial conditions yield different joint trajectories, the initial conditions should also be considered as optimization variables (three for each leg group, six in total):

\[
P_a = q_0 = [\theta_{1_{AEP}} \quad \theta_{2_{AEP}} \quad \theta_{3_{AEP}}]
\]  

(3)

For each leg, either the front-most configuration, Anterior Extreme Position (AEP), or the back-most configuration, Posterior Extreme Position (PEP), can be used as the initial configuration. The former is selected in this work (see figure 2.) The toe trajectories for stance and swing are created by describing three separate trajectories: the toe’s horizontal position \( x(t) \), its vertical
position $z(t)$, and the orientation of the last link $\psi(t)$. To describe these trajectories, the position $(x_3, z_3)$ and velocity $(\dot{x}_3, \dot{z}_3)$ of the toe, as well as the orientation of the last link ($\psi_3$) and its angular velocity ($\dot{\psi}_3$) must be known at the AEP and the PEP. Using this information, the stance trajectory from AEP to PEP can be defined. During the swing phase from PEP back to AEP, one additional toe position is required to describe the maximum step clearance. Of these positions and velocities, some are dependent on each other due to the constraints (section 3.2), whereas some are a function of optimization variables already selected (additional details are in section 2.1). The additional optimization variables are the following six independent variables for each leg group:

$$P_b = [\dot{z}_{AEP} \dot{z}_{PEP} f_c \psi_{AEP} \psi_{PEP} \psi_{net}]$$  \hspace{1cm} (4)

where $\dot{z}_{AEP}$ and $\dot{z}_{PEP}$ are the vertical velocities of the toe at AEP and PEP, respectively, $f_c$ is the maximum step clearance of the toe during the swing phase, $\psi_{AEP}$ and $\psi_{PEP}$ are the angular velocities of the last link at AEP and PEP, respectively, and $\psi_{net}$ is the net change in orientation of the last link during stance/swing phases. Using these variables, the trajectories $x(t)$, $z(t)$ and $\psi(t)$ are described using Bezier functions [49], because they exhibit smooth curves, and hence small perturbations in the parameters do not result in a large fluctuation of the trajectories. The equation of a Bezier curve of order $m$ is:

$$\delta(t) = \frac{1}{(t_f)^m} \sum_{k=0}^{m} \frac{m!}{k! \cdot (m-k)!} \cdot t^k (t_f - t)^{m-k} \cdot a_k$$ \hspace{1cm} (5)

where $t$ represents time, $t_f$ represents the time taken during swing or stance, and $\delta$ represents either $x$, $z$ or $\psi$. A Bezier curve of order $m$ has $m + 1$ unknown coefficients $a_k$ ($k = 0: m$). These unknowns are solved using the position and velocity information of the toe and last link, derived from the optimization variables mentioned above. Additional details on the orders of the Bezier functions selected, as well as solving the unknowns can be found in Section 2.1.

The relative phase of a leg $\phi$ is the fraction of the stride time $T$ after which its touchdown occurs after the touchdown of an arbitrarily selected reference leg. With the front left leg serving as a reference, three relative phase parameters are defined for other legs (figure 3a). A symmetric gait refers to one in which the two legs of each leg group have a relative phase of 50% with respect
to each other. Most of the gaits for walking and low-speed running in animals are symmetric [1]. Hence, two of the three relative phases are selected to be 50%, leaving one relative phase variable $\phi_{FH} \in [0, 1]$ (figure 3b). Figures 3c-e show three common gaits, namely pace, trot and amble, respectively. The duty factor $\beta \in (0, 1)$ is the ratio of the time that a leg is in the stance phase to the stride time. Most animals have equal duty factors for the legs of each leg group [1], and the same assumption is made in here. To know the initial pitch angle of the rover during the simulation, there must exist an instant $T^*$ when at least one leg of each leg group must be in the stance phase (see section 2.1 for more detail). However, this is not guaranteed for symmetric gaits with duty factors $\beta < 0.25$, unless $0 \leq \phi_{FH} \leq \beta$ or $1 - \beta \leq \phi_{FH} \leq 1$, which are rare circumstances. Therefore, $\beta$ is selected to be at least 0.25 in the optimization. Together, $\beta$ and $\phi_{FH}$ can completely parameterize symmetric gaits in quadruped rovers, and are used as variables in the optimization (two in total):

$$\mathbf{G} = \begin{bmatrix} \beta \\ \phi_{FH} \end{bmatrix}$$  \hspace{1cm} (6)

The duration of the stance and swing phases is $\beta T$ and $(1 - \beta)T$, respectively. The velocity of $(O_B)$ can be represented by $(\lambda/T)$, which is assumed to be uniform. The net distance $\kappa$ travelled forward by the hip $(O_0)$ of each leg during its swing phase is

$$\kappa = (1 - \beta)T \cdot \left(\frac{\lambda}{T}\right) = (1 - \beta)\lambda$$  \hspace{1cm} (7)

Figure 3– Common relative leg phases in quadruped animals (a) general gait, (b) symmetric gait, (c) pace, (d) trot, and (e) lateral sequence walk. The reference leg is 0.
Since \((O_B)\) moves \(\lambda\) meters ahead in one cycle, each hip joint must move the same distance forward in one cycle as well. Hence, the distance \(\kappa\) that the leg’s hip \((O_0)\) moves forward with respect to its toe \((O_3)\) during the stance phase can be defined as:

\[
\kappa = \lambda - \bar{\kappa} = \beta \lambda
\]

To ensure that each leg returns to the same AEP after each cycle, the leg’s toe \((O_3)\) must move forward horizontally with respect to its hip \((O_0)\) by \(\kappa\) during the swing phase. For this reason \(\kappa\) is referred to as the step length (figure 2). Instead of using \(\kappa\) as an optimization variable, \(\lambda = \kappa / \beta\) is used:

\[
P_c = \lambda
\]

One further assumption is made to simplify the design. In most cat species, the duty factors for the front and hind legs are quite similar at all speeds [5]. Hence, they are represented by a single variable for all legs. Since the duty factors for the four legs are assumed the same, their step lengths should also be the same, which makes it unnecessary to have longer hind legs. Although the total leg length for the front and hind legs are the same, link length ratios were assigned based on the cat-limb data [5].

In addition, one can argue that the suitable control parameters for the joints of a quadruped rover also directly depend on the rover’s gait and pose variables. To ensure that the optimization is not limited by a fixed set of control parameters for all gaits, the control gains are included as design variables, as well. Researchers have successfully used PD gains for each joint for a quadruped rover [28]. Similarly in this work, the controls for each leg consist of three PD controllers for the hip, knee and ankle joints, with proportional and derivative gains \((K_{P,1}, K_{D,1})\), \((K_{P,2}, K_{D,2})\) and \((K_{P,3}, K_{D,3})\), respectively:

\[
K = [K_{P,1}, K_{D,1}, K_{P,2}, K_{D,2}, K_{P,3}, K_{D,3}]
\]

The Pose \((P)\), Gait \((G)\) and Control \((K)\) optimization variables \(\varphi\) are listed in table 1:

\[
\varphi = [P^F_a, P^H_a, P^F_b, P^H_b, P_c, G, K^F, K^H]
\]
The horizontal and vertical position of the toes \( (x_3, z_3) \), and the orientation \( \psi_3 \) of the metacarpal/metatarsal link, with respect to the hip frame \( \{0\} \), is related to the joint angles \( q \) of the leg by the following trigonometric equations:

\[
x_3 = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3) \tag{12}
\]

\[
z_3 = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3) \tag{13}
\]

\[
\psi_3 = \theta_1 + \theta_2 + \theta_3 \tag{14}
\]
When the leg is at the AEP, \( q \) is equal to the initial joint angles \( q_0 \) as in equation (3). Substituting \( q_0 \) in the above equations gives the position of the toe \( (x_{3,AEP}, z_{3,AEP}) \) and the orientation \( \psi_{AEP} \) of the metatarsal/metacarpal links at the AEP. From here, the position of the toe \( (x_{3,PEP}, z_{3,PEP}) \) and the orientation of the corresponding link \( \psi_{3,PEP} \) at the PEP is given by:

\[
x_{3,PEP} = x_{3,AEP} - \beta \lambda, \quad z_{3,PEP} = z_{3,AEP}, \quad \psi_{3,PEP} = \psi_{3,AEP} - \psi_{net}
\]  

(15)

For convenience, the subscript 3 is omitted, since the toe alone is being referred to. The vertical velocity of the toe and the angular velocity of the last link are known directly from the optimization variables. At any instant, if the toe is stationary with respect to the world frame \( \{W\} \) and the rover’s main body has a velocity \( V^* \) in the thrust direction with respect to \( \{W\} \), then the toe is moving at \(-V^*\) with respect to the main body (and the attached hip frame). During stance and leading up to the take-off instant, the toe’s velocity with respect to \( \{W\} \) is zero. Hence, the horizontal velocity of the toe at PEP is set to \(-V_B\), where \( V_B \) is the desired thrust velocity of the main body. At the AEP (touchdown instant), the velocity of the toe suddenly drops to zero, which leads to leg impact at touchdown. To eliminate the impact effects, the horizontal velocity of the toe at AEP is also set to \(-V_B\). Finally, one additional toe position must be known to describe the maximum step clearance during swing. At this position, the vertical velocity of the toe becomes zero, as the toe starts approaching the ground again. It is assumed that this maximum step clearance occurs halfway between \( x_{PEP} \) and \( x_{AEP} \). Therefore, this additional position is

\[
(x_m, z_m) = (x_{PEP} + (\beta \lambda/2), f_c)
\]

(16)

At this point, the following positions and velocities are known for each leg group:

\[
(x_{PEP}, z_{PEP}, \psi_{PEP}), (x_{AEP}, z_{AEP}, \psi_{AEP})
\]

(17)

Table 2 describes the boundary conditions referring to the known positions and velocities in equation (17), and summarizes how they are derived. Based on the given number of boundary
conditions, the order of the Bezier curve is selected, as shown in table 2. For a 3rd-order Bezier trajectory \( \delta(t)^{m=3} \), four unknowns must be solved for \((a_0, a_1, a_2 \text{ and } a_3)\):

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Boundary Condition</th>
<th>Variable</th>
<th>Relation to optimization variables</th>
<th>Bezier function order</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x(t)_{st})</td>
<td>(x</td>
<td>_{t=0})</td>
<td>(x_{AEP})</td>
<td>From (\theta_{AEP}), equation (12)</td>
</tr>
<tr>
<td>(x</td>
<td><em>{t=t</em>{st}})</td>
<td>(x_{PEP})</td>
<td>From (\theta_{AEP}, \lambda, \beta), equations (12) and (15)</td>
<td></td>
</tr>
<tr>
<td>(dx/dt</td>
<td>_{t=0})</td>
<td>(\dot{x}_{AEP})</td>
<td>(V_B), equation (33)</td>
<td></td>
</tr>
<tr>
<td>(dx/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{x}_{PEP})</td>
<td>(V_B), equation (33)</td>
<td></td>
</tr>
<tr>
<td>(x(t)_{sw})</td>
<td>(x</td>
<td><em>{t=t</em>{st}})</td>
<td>(x_{PEP})</td>
<td>Described above</td>
</tr>
<tr>
<td>(x</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(x_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(dx/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{x}_{PEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(dx/dt</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(\dot{x}_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(\psi(t)_{st})</td>
<td>(\psi</td>
<td>_{t=0})</td>
<td>(\psi_{AEP})</td>
<td>From (\theta_{AEP}), equation (14)</td>
</tr>
<tr>
<td>(\psi</td>
<td><em>{t=t</em>{st}})</td>
<td>(\psi_{PEP})</td>
<td>From (\theta_{AEP}, \psi_{net}) equations (14) and (15)</td>
<td></td>
</tr>
<tr>
<td>(d\psi/dt</td>
<td>_{t=0})</td>
<td>(\dot{\psi}_{AEP})</td>
<td>(\dot{\psi}_{AEP})</td>
<td></td>
</tr>
<tr>
<td>(d\psi/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{\psi}_{PEP})</td>
<td>(\dot{\psi}_{PEP})</td>
<td></td>
</tr>
<tr>
<td>(\psi(t)_{sw})</td>
<td>(\psi</td>
<td><em>{t=t</em>{st}})</td>
<td>(\psi_{PEP})</td>
<td>Described above</td>
</tr>
<tr>
<td>(\psi</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(\psi_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(d\psi/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{\psi}_{PEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(d\psi/dt</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(\dot{\psi}_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(z(t)_{st})</td>
<td>(z</td>
<td>_{t=0})</td>
<td>(z_{AEP})</td>
<td>From (\theta_{AEP}), equation (13)</td>
</tr>
<tr>
<td>(z</td>
<td><em>{t=t</em>{st}})</td>
<td>(z_{PEP})</td>
<td>From (\theta_{AEP}), equations (13) and (15)</td>
<td></td>
</tr>
<tr>
<td>(dz/dt</td>
<td>_{t=0})</td>
<td>(\dot{z}_{AEP})</td>
<td>(\dot{z}_{AEP})</td>
<td></td>
</tr>
<tr>
<td>(dz/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{z}_{PEP})</td>
<td>(\dot{z}_{PEP})</td>
<td></td>
</tr>
<tr>
<td>(z(t)_{sw})</td>
<td>(z</td>
<td><em>{t=t</em>{st}})</td>
<td>(z_{PEP})</td>
<td>Described above</td>
</tr>
<tr>
<td>(z</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(z_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(dz/dt</td>
<td><em>{t=t</em>{st}})</td>
<td>(\dot{z}_{PEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(dz/dt</td>
<td><em>{t=t</em>{st}+t_{sw}})</td>
<td>(\dot{z}_{AEP})</td>
<td>Described above</td>
<td></td>
</tr>
<tr>
<td>(z</td>
<td><em>{t=t</em>{st}+(t_{sw}/2)})</td>
<td>(z_m)</td>
<td>From (\theta_{AEP}, f_c) equation (13, 16)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 – Boundary conditions and order for the Bezier functions
\[
\delta(t) = a_0 \frac{(t_f - t)^3}{t_f^3} + 3a_1 \frac{t(t_f - t)^2}{t_f^3} + 3a_2 \frac{t^2(t_f - t)}{t_f^3} + a_3 \frac{t^3}{t_f^3}
\]  

(18)

For a 4th order Bezier trajectory \(\delta(t)^{m=4}\), five unknowns must be solved for \((a_0, a_1, a_2, a_3\) and \(a_4)\):

\[
\delta(t) = a_0 \frac{(t_f - t)^4}{t_f^4} + 4a_1 \frac{t(t_f - t)^3}{t_f^4} + 6a_2 \frac{t^2(t_f - t)^2}{t_f^4} + 4a_3 \frac{t^3(t_f - t)}{t_f^4} + a_4 \frac{t^4}{t_f^4}
\]  

(19)

The unknown Bezier coefficients are solved by evaluating the function \(\delta(t)\) and its derivative \(\dot{\delta}(t)\) at the values shown in Table 2. Once the Bezier coefficients are solved, equations (18) and (19) define \(x(t)\), \(z(t)\) and \(\psi(t)\) for the toe trajectories \((0O_3(t))_{sw}\) and \((0O_3(t))_{st}\). Note that the Bezier trajectory \(x(t)_{st}\) results in a linear function. Knowing the toe trajectories, the joint trajectories \(q(t)\) can be obtained by solving equations (12)-(14).

With the duty factor \(\beta > 0.25\), the instant \(T^*\) is guaranteed to exist, where at least one leg from each the front and hind leg groups is in the stance phase (toe in contact with ground.) By using the joint angles \(q(t = T^*)\) in equations (12) and (13), the position of these two toes with respect to their hip frames \(\{0\}, (x_3^F, z_3^F)\) and \((x_3^H, z_3^H)\), is calculated. Then, the initial pitch \(\psi_B\) of the body is given by:

\[
\psi_B = -\sin^{-1}\left(\frac{z_3^F - z_3^H}{x_3^F - x_3^H + IGD}\right)
\]  

(20)

where IGD (Inter-Girdle Distance) is the distance between the front and hind limb hips. Only solutions in the first and fourth quadrants are selected, due to the constraints on the maximum allowable pitch (Section 3.2). Once \(\psi_B\) is known, the position of the toes with respect to the world frame \(\{W\}\) can be found by the following equation (where a pre-superscript \(W\) is added to express the vertical and horizontal positions of the toes with respect to the world frame \(\{W\}\)):

\[
\begin{bmatrix}
W_{x_3^F} \\
W_{z_3^F} \\
W_{x_3^H} \\
W_{z_3^H}
\end{bmatrix} =
\begin{bmatrix}
W_{R_0} & 0 \\
0 & W_{R_0}
\end{bmatrix}
\begin{bmatrix}
(x_3^F) \\
(z_3^F) \\
(x_3^H) \\
(z_3^H)
\end{bmatrix}
\]  

(21)
The matrix $wR_0$ represents a rotation from the hip frame $\{0\}$ to the world frame $\{W\}$. The vertical position $wz_B$ of the main body $B$ is then:

$$wz_B = \left( \frac{wz_3^F + wz_3^H}{2} \right)$$  \hspace{1cm} (23)

The horizontal position of the main body $wx_B$ is initialized to zero. Equations (12)-(23) are analytically solved and evaluated in MATLAB®, and this module is referred to as the Kinematics Trajectory Generator (KTG).

### 3.3 Dynamics Formulation:

The Inverse Dynamics (ID) of a fixed-base manipulator is well documented in the literature [50], but the development of an ID formulation for moving-base manipulators, such as biped or quadruped rovers, is an ongoing research. The dynamic equations for a six-wheeled rover are derived using Kane’s method in [51], but the work is not challenged by changing the contact points as in the case of legged rovers. The dynamic equations for a six-legged, radially-symmetric rover are derived using the Euler-Lagrange approach for a tripod statically stable gait [52]. However, in this work the gait variables describe an entire range of gaits, which may not be statically stable. The approach developed in [53] computes the ID torque in a reduced-dimensional null space of the constraints, using an orthogonal decomposition of the constraint Jacobian. Hence, contact forces need not be measured and used. Using this method in this paper, the rover’s generalized coordinates are:

$$\hat{q} = \begin{bmatrix} q^{FL} & q^{FR} & q^{HL} & q^{HR} & r_B \end{bmatrix}^T \in \mathbb{R}^{15}$$  \hspace{1cm} (24)

where $q \in \mathbb{R}^3$ represents the three joint angles for each of the four legs, and $r_B$ represents the horizontal and vertical position of the centre of mass of the main body and its orientation, with respect to the world coordinate frame $\{W\}$:

$$r_B = \begin{bmatrix} x_B & z_B & \psi_B \end{bmatrix} \in \mathbb{R}^3$$  \hspace{1cm} (25)

The ID model can be formulated using the Euler-Lagrange equations [50]:
\[ \dot{\tau} = M(\dot{\theta}) \cdot \ddot{\theta} + C(\dot{\theta}, \dot{\theta}) \cdot \dddot{\theta} + g(\dot{\theta}) - J_c(\dot{\theta})^T \cdot f \]

(26)

The generalized torque vector \( \dot{\tau} \) is:

\[ \dot{\tau} = [\tau_{FL} \tau_{FR} \tau_{HL} \tau_{HR} 0 0 0]^T \in \mathbb{R}^{15} \]

(27)

where \( \tau \) is the joint torque matrix for each of the four legs:

\[ \tau = [\tau_1 \tau_2 \tau_3]^T \in \mathbb{R}^{3} \]

(28)

The matrix \( M \in \mathbb{R}^{15 \times 15} \) represents the inertia matrix, \( C \in \mathbb{R}^{15 \times 15} \) is the matrix describing centrifugal and Coriolis effects and \( g \in \mathbb{R}^{15} \) defines the gravitational field vector. The vector \( f \in \mathbb{R}^{8} \) consists of the four horizontal \( F_x \) and four vertical \( F_z \) interaction forces between the ground and the toes:

\[ f = [F_{x_{FL}} F_{x_{FR}} F_{x_{HL}} F_{x_{HR}} F_{z_{FL}} F_{z_{FR}} F_{z_{HL}} F_{z_{HR}}]^T \]

(29)

The constraint Jacobian \( J_c \in \mathbb{R}^{8 \times 15} \) is defined as:

\[ J_c = \left[ \frac{\partial \dot{p}}{\partial \dot{\theta}} \right]_{8 \times 15} \]

(30)

where,

\[ \dot{p} = [W_{x_3}^{FL} W_{x_3}^{FR} W_{x_3}^{HL} W_{x_3}^{HR} W_{z_3}^{FL} W_{z_3}^{FR} W_{z_3}^{HL} W_{z_3}^{HR}]^T \in \mathbb{R}^{8} \]

(31)

and \( W_{x_3} \) and \( W_{z_3} \) are the vertical and horizontal position of the four toes in the world frame \( \{W\} \).

By carrying out a QR decomposition of the constraint Jacobian [53], the unconstrained and constrained dynamics can be separated, and the joint torque \( \tau \) can be solved.

### 3.4 Objective Functions

The locomotion optimization problem of a quadruped rover can be considered in two major cases: i) where the minimum energy consumption is desired at a constant ratio of the stride length \( \lambda \) to the stride time \( T \); and ii) where the maximum rover’s overall forward speed is desired (for a given torque limit). For the first case some works have used different forms of the specific resistance [54, 55], which is the ratio of the maximum available power/energy to the maximum forward velocity/distance travelled during one cycle. This work selects a sthenic criterion that minimizes the total joint torque \( \tau \) over a cycle [56]. To prevent the optimization from selecting
solutions with the shortest stride length and time as the optimum solution, the energy criterion is divided by $\lambda$:

$$ E = \frac{1}{\lambda} \int_{0}^{T} (\|\tau^{FL}\|^2 + \|\tau^{HL}\|^2) dt $$

(32)

where, $\|\tau\|^2$ is the second norm of the joint torque vector $\tau$.

For the second case, an objective function that is often used is the rover’s center of mass velocity $V$ in the thrust direction. The criterion selected here is:

$$ V_B = \frac{1}{T} \int_{0}^{T} (\dot{x}_B) dt $$

(33)

which is an average of the velocity $\dot{x}_B$ of the center of mass ($O_B$) of the rover’s main body $B$ over one cycle.

The optimization results based on the above-mentioned objectives are studied separately in this thesis. First, the minimum-torque optimization is carried out for three fixed speeds (0.1, 0.5 and 1.0 m/s). Next, a maximum-velocity optimization is carried out. The speed optimization needs another variable (stride time $T$) in addition to $\varphi$.

3.5 Constraints

There are two types of constraints. Instantaneous constraints (IC) are defined at a particular instant during the locomotion cycle. Permanent constraints (PC) must be satisfied in a specific duration or in the entire locomotion cycle.

3.5.1 Instantaneous Constraints

(a) Gait Periodicity constraints: To ensure gait periodicity, certain constraints need to be defined for the joint trajectories. Assuming that the leg motion cycle begins with the stance phase and ends with the swing phase, the final conditions at the swing phase must be the same as the initial conditions of the stance phase. For each leg, these conditions are those corresponding to the AEP position:
These conditions also ensure a smooth transition from the swing to the stance phase for each leg, and hence the entire rover.

\begin{align*}
q_{st,i} &= q_{sw,f} = q_0 \\
\dot{q}_{st,i} &= \dot{q}_{sw,f}
\end{align*}

(b) Impact constraints. As the leg transitions between stance and swing phases, sudden changes occur in the degrees of freedom and their derivatives, which can result in physical impacts on the system causing energy dissipation. Without restoring energy into the system, the system cannot repeat its periodic motion. Therefore, the transition constraints ensure that there are no sudden changes in the degrees of freedom despite the phase transition. A smooth transition between the two phases of motion is desirable for the gait trajectory; for this reason, in addition to equations (34) and (35), the initial values of the joint angles \( q(t) \) and their velocities \( \dot{q}(t) \) for the swing phase is set equal to their final values in the stance phase:

\begin{align*}
q_{st,f} &= q_{sw,i} \\
\dot{q}_{st,f} &= \dot{q}_{sw,i}
\end{align*}

In addition, the initial value of the horizontal position of the toe is also set equal to its value at the end of the swing phase. This way the robot begins its stance at the same point where it landed on. Also, the vertical position of the toe \( \mathbf{w_{3}} \) is known to be zero with respect to the world frame \( \{W\} \) during the stance phase. Hence, the toe’s vertical position at the start as well as the end of the swing phase must be zero:

\[ \mathbf{w_{3,i}} = \mathbf{w_{3,f}} = 0 \]

To smooth the changes in the rate of degrees of freedom, animals tend to retract their leg prior to touchdown. In legged locomotion, as shown by Seyfarth et al. [25] and Wisse et al. [26], leg retraction prior to touch-down helps the motion periodicity. Therefore, by reducing the absolute velocity of the toe with respect to the ground one would be able to eliminate the impact effects. The above arguments can be summarized as:

\[ \mathbf{w_{3,f}} = \mathbf{w_{3,f}} = 0 \]
(c) Numerical Computation Constraints. For certain optimization variable sets \( \varphi \), the ODE solver in SimMechanics® may not converge to any solution, or the solution may be unrealistic or impractical. To avoid such circumstances, the model computation is terminated in the optimization if the following situations occur, and the solution is discarded.

- the KTG module fails to generate joint trajectories \( q(t) \);
- the required time increment for solving the system dynamics is smaller than a threshold (e.g., \( 10^{-7} \) sec);
- the vertical position \( z_B \) of the centre of mass \( (O_B) \) of the rover’s main body \( B \) becomes negative;
- the absolute value of the pitch \( \psi_B \) of the main body \( B \) about its centre of mass \( (O_B) \) is bigger than a threshold (e.g., \( \psi_{\text{max}} = \pi/3 \) rad);
- the vertical position of any of the four toes \( w_z_3 \) is greater than the vertical position of their corresponding hip \( w_z_0 \)

3.5.2 Permanent Constraints

This group of constraints is both in the form of equalities and inequalities, which must be satisfied at all times or during a period of the locomotion cycle.

(a) Unilateral Contact Condition. This condition is placed on the vertical position of the toes during swing and stance phases. During the swing phase of a leg, its toe must be above the ground, and hence \( w_z_3 \) must be greater than zero:

\[
\left( w_z_3(t) \right)_{\text{sw}} > 0 \tag{40}
\]

During the stance phase of a leg, its toe must maintain contact with the ground, and hence, \( w_z_3 \) must be zero:

\[
\left( w_z_3(t) \right)_{\text{st}} = 0 \tag{41}
\]

(b) Non-sliding Condition. When the toe \( (O_3) \) is in contact with the ground during the stance phase, it is assumed that the friction force being applied to the toe is sufficiently large, so as to prevent
slipping. This constraint is met when the horizontal velocity of the toe of any leg in the stance phase remains zero:

\[ (\dot{w}x_3(t))_{st} = 0 \]  

and the ratio of the horizontal component to the vertical component of the ground reaction force does not exceed the surface static friction coefficient \( \mu_s \):

\[ \frac{F_x}{F_z} < \mu_s \]  

(c) **Motion Dependent Constraints.** At all times, the rover must avoid certain locomotive behaviour through the following constraints:

\[ \dot{x}_B(t) > 0 \]  
\[ (\dot{z}_B(t))_{sw,i} \geq 0 \]  

Equation (44) constrains the forward velocity of the centre of mass of the main body to remain positive, hence preventing the backward motion of the rover’s main body. Each leg must also have a positive vertical velocity at the take-off moment (initial condition of the swing), equation (45); otherwise it may penetrate into the ground.

(d) **Maximum torque constraints.** To ensure that the optimum gait trajectories do not require excessive torque at the hip, knee and ankle joints beyond the capacity of assigned actuators, the following constraints are defined:

\[ |\tau| < \tau_{max} = [\tau_{1,max} \tau_{2,max} \tau_{3,max}] \]  

where \( \tau \) is the joint torque vector in equation (28), and \( \tau_{max} \) is the vector of maximum allowable torque at the joints.

(e) **Gait stability.** In the quadruped legged locomotion, the stability of a gait can be realized using criteria such as zero moment point [57], eigenvalues of the poincaré return map, change of angular momentum, and the ground reference points [58]. Based on a mathematical definition of stability in [59], as long as a rover is not in the basin of fall, the motion is considered to be stable. This basin of fall is a “subset of the state space that leads to a fall, (where) a point on the robot, other than a
point on the foot (toe in this case), touches the ground.” Based on this condition, a periodic gait is stable, if and only if in each period, the rover does not fall. Together, the gait periodicity constraints and numerical computation constraints mentioned previously will ensure that the rover does not fall during each period. Once an optimum periodic gait is found during which a rover does not fall in any stride period, it can be said that the gait is stable.

3.6 Optimization Algorithm

The minimum torque and maximum speed optimizations are carried out using an EA based on a Genetic Algorithm [60, 61], which has been studied and used extensively for optimizing legged rovers [17–22]. An initial population of \( n \) rovers is created, where each rover has a randomly generated set of 33 variables \( \varphi \). After evaluating all members in the population, a rank is assigned to them based on the value of the objective function in consideration. The best \( n/2 \) ranked rovers are selected in a parent candidate pool. From this pool, 2 members are randomly selected, and the member with the better rank is selected as the first parent, which is referred to as Binary tournament

![Figure 4– Recombination and Mutation – Parents P-a and P-b undergo arithmetic recombination to give children C-a and C-b, which undergo mutation to give children mC-a and mC-b](image-url)
selection. Repeating this process \( n \) times generates the parent pool. These parents undergo arithmetic recombination to generate \( n \) children, which in turn go through mutation (figure 4). The best \( n \) rovers from the combined children and parent pools are selected to represent the new population. Details on various recombination and mutation techniques used in Genetic Algorithms can be found in [60, 61]. This iterative process is repeated numerous times, until convergence is reached. The algorithm is said to have converged, when the difference between the average value of the objective function for the best 50\%, 20\% and 10\% of the population is less than 1\% of their combined average value. The number of generations is expected to differ, based on the convergence of solutions.

The population size, recombination rate and mutation rate should be selected carefully, so as to prevent a premature convergence to a local solution. If the population size \( n \) is too small, the population at any generation cannot be diverse enough, and the optimization converges prematurely. If \( n \) is too large, the number of member evaluations and associated computations required increase. Balancing both these cases, a population size of \( n = 200 \) members is selected. A suitable recombination and mutation rate can diversify the population at each generation, thereby reducing chances of premature convergence, as well. In this work, all members undergo recombination and mutation. However, only a fraction of each member’s variables are modified. This fractional rate for recombination and mutation is selected as roughly 33\% and 10\%, respectively (see figure 4).
Chapter 4: Gait Planning using Central Pattern Generators

During the optimization of the rover’s gait, pose and control variables, the KTG was used to generate joint trajectories in an offline process, for three fixed speeds. The CPG Trajectory Generator designed in this work, can endogenously generate the trajectories in an online process. It consists of five CPG units. Four of these units generate joint trajectories for the four legs, and are identical in design. The fifth CPG unit generates a trajectory for the control gains. Each CPG unit consists of three parts – i) phase oscillator neural network (PONN), ii) CPG-rover interface (CRI), and iii) higher level control unit (HCU). The PONN consists of a retrieval network of globally coupled phase oscillators, within which, patterns are stored as phase differences between the oscillators. The phase oscillators are updated synchronously by regulating them via a pacemaker oscillator [47]. The CRI maps the trajectories of the phase oscillators to trajectories of the joints and control gains. The HCU takes a single input – desired speed, and regulates the dynamics of the PONN’s pacemaker and retrieval network accordingly. This enables the HCU to select the gait to be followed, the patterns to be retrieved within each gait, and the speed at which the patterns are to be cycled through. The roles of the HCU, PONN and CRI have been summarized in figure 5, and are explained in more detail below.

Figure 5– One CPG unit of the CPG Trajectory Generator
4.1 Phase Oscillator Neural Network (PONN)

This PONN consists of a retrieval network of phase oscillators $\phi$, a pacemaker oscillator $\psi$, and a global objective function $L$. The retrieval network is a type of dynamic associative memory [62], within which, patterns (or memories) can be stored as phase differences between the oscillators. If the oscillators are required to store $K$ patterns $\xi^k \in \mathbb{R}^K (k = 1, 2, ..., K)$:

$$
\begin{bmatrix}
\pi \\
0 \\
\vdots \\
0
\end{bmatrix}, \xi^2 =
\begin{bmatrix}
0 \\
\pi \\
\vdots \\
0
\end{bmatrix}, ..., \xi^K =
\begin{bmatrix}
0 \\
0 \\
\vdots \\
\pi
\end{bmatrix}
$$

(47)

then $K + 1$ oscillators $\phi_i(t) \in [0, \pi], (i = 1, 2 ... K + 1)$ are selected. These $K + 1$ oscillators have $K$ unique phase differences, which are represented by the vector $\xi(t) \in \mathbb{R}^K$:

$$
\xi(t) =
\begin{bmatrix}
\Delta \phi_1 \\
\Delta \phi_2 \\
\vdots \\
\Delta \phi_K
\end{bmatrix}
= 
\begin{bmatrix}
\phi_{K+1} - \phi_1 \\
\phi_{K+1} - \phi_2 \\
\vdots \\
\phi_{K+1} - \phi_K
\end{bmatrix}
$$

(48)

where $\phi_{K+1}$ is a reference oscillator. Hence, $\xi(t)$ represents the state of all the oscillators in the retrieval network at any instant. This state can be updated in two ways. In the first approach, the oscillators are updated one at a time, either in predefined sequence or randomly, and is called asynchronous updating. In the second approach, all the oscillators are updated together, and is called synchronous updating. The latter approach has better convergence and uses a regulatory oscillator for periodic selection of patterns for retrieval. An additional oscillator is selected as the pacemaker $\psi$, which serves the function of a clock for one complete cycle of the stored patterns. The pacemaker’s dynamics are modelled as:

$$
\dot{\psi} = \omega, \quad \psi \in [0, 2\pi]
$$

(49)

The pacemaker’s value is the phase angle of a point moving along the edge of a circle at a rate $\omega$. The range $[0, 2\pi]$ is broken down into $K$ intervals:

$$(0, \tau_1], (\tau_1, \tau_2], ..., (\tau_{K-1}, \tau_K], \quad (0 < \tau_1 < \tau_2 < \cdots < \tau_K = 2\pi)
$$

(50)

When the pacemaker enters the $k^{th}$ interval $(\tau_{k-1}, \tau_k]$ , the oscillator state $\xi(t)$ is endogenously driven towards $\xi^k$. Since the Pacemaker values are periodic, $\xi(t)$ periodically
follows the sequence $\xi^1, \xi^2, ..., \xi^K, \xi^1, \xi^2 ...$ etc. Hence, each pattern is sequentially retrieved for a specific interval during each cycle of the pacemaker. To modulate $\xi(t)$'s endogenous state transition, an energy function $L$ is used. It is a function of the oscillators $\phi_i$, as well as coupling parameters $f_{ij}(\psi)$:

$$L = \frac{-\mathcal{K}}{4(K + 1)} \sum_{i=1}^{K+1} \sum_{j=1}^{K+1} (\cos(\phi_i - \phi_j) + f_{ij})^2$$

(51)

where $\mathcal{K}$ indicates the strength of the coupling between the oscillators. Note that $L$ has $2^K$ minima if the coupling parameters are all zero. For any pacemaker value, it must be ensured that $L$ has only one minimum, and that this minimum coincides with the pattern selected for retrieval $\xi^k$. For this, the coupling parameters are selected as follows:

$$f_{ij}(\psi) = \alpha \sum_{k=1}^{N-1} \left( \left(1 - \frac{2}{\pi} |\xi_i^k - \xi_j^k| \right) g_k(\psi) \right)$$

(52)

$$g_k(\psi) = \Theta(\psi - \tau_k) - \Theta(\psi - \tau_{k+1})$$

(53)

where $\alpha$ controls the stability of the minimum point, and $\Theta(t)$ is the Heaviside step function at $t$. If $\alpha > 1$, then the retrieval network has zero-error in retrieving the selected pattern [47]. The dynamics of the phase oscillators $\phi_i$, are governed by the following equation:

$$\frac{d\phi_i}{dt} = \frac{\mathcal{K}}{K + 1} \sum_{j=1}^{K+1} \left( f_{ij} \sin(\phi_j - \phi_i) + \frac{1}{2} \sin 2(\phi_j - \phi_i) \right) + \Gamma \eta_i(t)$$

(54)

This speed function is constructed such that the speed of any oscillator is zero, only when the energy function is at its stable minimum or unstable maximum. Note that this function only depends on the current oscillator state $\xi(t)$ and not the desired state $\xi^k$. However, equation (54) always drives the oscillators towards the desired state. Hence, $\xi(t)$ is guaranteed to evolve towards $\xi^k$ independent its current value. When the Pacemaker enters the $k + 1^{th}$ interval, the current oscillator state $\xi(t) = \xi^k$, which was the stable minimum in the $k^{th}$ interval, becomes an unstable maximum. The white Gaussian noise function $\eta_i(t)$ of intensity $\Gamma$ drives the oscillators away from this unstable maximum state. The dynamics of the oscillators drive them towards the new stable
minimum state $\xi^{k+1}$. The output of the PONN is the trajectories of the phase oscillators $\xi(t)$ as they cycle through the poses $\xi^k$. The oscillators complete a single iteration through all the $K$ states when the Pacemaker goes through one cycle from 0 to $2\pi$.

The CPG model is integrated using a second order stochastic Runge-Kutta algorithm [63], which contains another noise parameter with strength $D$.

4.2 CPG–Rover Interface (CRI)

The CRI is responsible for mapping the phase space trajectories $\xi(t)$ into joint/control space trajectories $u(t)$. For the CPGs for the four legs, this map consists of a direct correlation between the $K$ states of the phase oscillators of the CPG, and $K$ angular states for each leg of the quadruped rover. A single angular state $q^k$ (equation 1) corresponds to a vector of the hip, knee and ankle joint angle ($q^k \in \mathbb{R}^3, k = 1, 2, \ldots K$). The optimal set of design variables $\varphi$ (table 1) obtained for each speed in the minimum-torque optimization, correspond to a single gait. As illustrated in figures 7-9, ten uniformly distributed angular states are selected during the swing and stance phases of the locomotion cycle each, for the three gaits obtained. For a single leg’s CPG, $K$ is selected to be 60, with 20 states being cycled through for each gait, and the other 40 remaining inactive (see Gait modulation, section 3.3.2). As a result, 61 phase oscillators are selected for the retrieval network. As the state of the retrieval network $\xi(t)$ cycles through the states $\xi^k$, the joint state $q(t)$ cycles through the angular states $q^k$, according to the following function:

$$q(t) = \frac{1}{\pi^2} \sum_{k=1}^{K} (\xi^k \cdot \xi(t)) q^k$$  (55)

When $\xi(t)$ coincides with any $\xi^k$, their dot product is $\pi^2$, and $q(t)$ coincides with the joint state $q^k$. For the control gains CPG, the following relation is used:

$$c(t) = \frac{1}{\pi^2} \sum_{k=1}^{K} (\xi^k \cdot \xi(t)) c^k$$  (56)
where \( c^k \) refers to the state of the 12 control gains (table 1) during each gait \( (c^k \in \mathbb{R}^{12}, k = 1, 2, \ldots K) \).

However, the control gains do not change during each gait. Hence, \( K \) is selected to be 3 in total, with one control state per gait. As a result, 4 phase oscillators are selected in the retrieval network for this CPG. The control states \( c^1, c^2 \) and \( c^3 \) correspond to the gain values in table 3 at each speed, respectively.

The resulting joint trajectories were filtered using a moving average filter [64]. A data point at the every time step is replaced by the mean of the most recent \( m \) values. This recursive process, can eliminate high frequency noise, and behaves similarly to a low pass filter.

### 4.3 Higher–level Control Unit

The HCU has two primary tasks: i) modulate speed, and ii) modulate gait, which it accomplishes by modifying the dynamics of the pacemaker in the PONN.

#### 4.3.1 Speed Modulation

Note that the CPG has a single input parameter – the desired rover speed \( v \). The minimum-torque optimization resulted in three gaits \( \varphi_1^*, \varphi_2^* \) and \( \varphi_3^* \), corresponding to the three optimization speeds \( v_1^* = 0.1 \text{ m/s} \), \( v_2^* = 0.5 \text{ m/s} \) and \( v_3^* = 1.0 \text{ m/s} \), respectively. If the \( v \) is less than a threshold \( v_1 = 0.3 \text{ m/s} \), the gait \( \varphi_1^* \) corresponding to \( v_1^* \) is used. If \( v \) is between thresholds \( v_1 \) and \( v_2 = 0.65 \text{ m/s} \), the gait \( \varphi_2^* \) is used. Lastly, if \( v \) is greater than the threshold \( v_2 \), the gait \( \varphi_3^* \) is used. At any arbitrary speed \( v(t) \), the stride time \( T \) is calculated:

\[
T(t) = \frac{\lambda}{v(t)}
\]

where \( \lambda \) is the stride length corresponding to the selected gait. The pacemaker speed \( \omega \) is set as,

\[
\omega(t) = \frac{2\pi}{T(t)}
\]
As \( v(t) \) changes, \( \omega(t) \) changes accordingly, which modulates the rate at which the oscillators \( \xi(t) \) cycle through \( \xi^k \). This directly affects the rate at which \( q(t) \) follows the desired joint positions \( q^k \) (equation 55), thereby modulating rover speed.

**4.3.2 Gait Modulation**

The HCU modulates the gait by modifying the way in which the PONN interprets of the pacemaker’s value. Instead of using \( \psi \), the retrieval network’s state \( \xi(t) \) is driven according to \( \psi - \psi_0 \), where \( \psi_0 \) is given as:

\[
\psi_0 = 2\pi\phi
\]  

(59)

and \( \phi \) is the relative phase between the front left leg (reference leg) and the leg whose CPG is in consideration. For the front right leg, \( \phi \) is selected as 50%. For the hind right leg, \( \phi \) is selected as \( \phi^{FH} + 50\% \). The phase instants are assigned values according to the duty factor (\( \beta_1, \beta_2 \) or \( \beta_3 \)), of the corresponding gait, and multiplied by a switch function as shown in table 3. When the first gait \( \psi_1^* \) is to be selected, the phase instants \( \tau_k \) for \( k = 1, 2, \ldots, 20 \) have values in the range of the pacemaker \([0, 2\pi]\) because \( I_1(t) \) is 1. However, the phase instants for \( k = 21, 22, \ldots, 60 \) will have values outside the pacemaker’s interval \([0, 2\pi]\) because \( I_2(t) \) and \( I_3(t) \) are set to 100. Hence, the retrieval network only retrieves patterns \( \xi^1, \xi^2, \ldots, \xi^{20} \). The leg’s joints cycle through \( q^1, q^2, \ldots, q^{20} \), and the control gains match \( c^1 \). When the second gait \( \psi_2^* \) is to be used, \( I_1(t) \) and \( I_3(t) \) become 100, and the phase instants for \( k = 1, 2, \ldots, 20 \) and \( k = 41, 42, \ldots, 60 \) are outside the pacemaker interval. The retrieval network only retrieves patterns \( \xi^{21}, \xi^{22}, \ldots, \xi^{40} \), thereby cycling the leg through \( q^{21}, q^{22}, \ldots, q^{40} \) with control gains \( c^2 \). Lastly, when the third gait \( \psi_3^* \) is to be used, \( I_1(t) \) and \( I_2(t) \) are set to 100. The leg cycles through \( q^{41}, q^{42}, \ldots, q^{60} \) with control gains \( c^3 \).

Consider a gait change scenario. Just prior to the gait change, the oscillator state \( \xi(t) \) is guaranteed to converge towards a certain desired state \( \xi^k \) at a rate \( \dot{\phi} \) which only depends on the current state \( \xi(t) \). As the gait changes, the desired state abruptly changes to \( \xi^{k'} \). However, the rate
\[ \dot{\phi} \] does not change, which leads to a smooth transition to \( \xi^{k'} \). If \( \xi(t) \) is smooth and continuous, then every element of \( \xi(t) \) is smooth and continuous \((\xi(t) \in \mathbb{R}^K)\). Since the elements of \( \xi^k \) are either 0 or \( \pi \), the dot product \( \xi^k \cdot \xi(t) \) in equations 55 and 56 is also smooth and continuous. Since smoothness and continuity are maintained during summations, \( q(t) \) and \( c(t) \) is also guaranteed to be smooth and continuous. Thus, the HCU can transition gaits in a smooth and continuous manner.

### 4.4 Optimization of CPG’s internal variables

A genetic algorithm based on the one described in section 3.6, was used to obtain the optimal set of internal parameters of the CPG from section 4.1:

\[
\phi_{\text{CPG}} = [\alpha \ K \ \Gamma \ D] \tag{60}
\]

The objective function is selected to represent the root mean square of the error between the optimal joint trajectory (from the torque minimization optimization) and the joint trajectory generated by the CPG units, for each of the three speeds (0.1, 0.5 and 1.0 m/s):

\[
E_{\text{rms}} = \sum_{i=1}^{3} \left( \frac{1}{T_i} \int_{0}^{T_i} \left( \|q_{d,i}(t)^{FL} - q_i(t)^{FL}\|^2 + \|q_{d}(t)^{HL} - q_i(t)^{HL}\|^2 \right) dt \right) \tag{61}
\]
where the index $i = 1, 2, 3$ represents the optimal results at 0.1, 0.5 and 1.0 m/s respectively, $T_i$ is the stride time, $q_{d,i}(t)$ is the desired optimal joint trajectory (equation 2), and $q_i$ is the corresponding joint trajectory generated by the CPG. For each set of design variables $\varphi_{CPG}$ (equation 61), the CPG trajectory generator is used to generate the joint trajectories for one stride cycle, at each of the three speeds separately. Hence, the objective function above tries to minimize the error for all three speeds simultaneously. Note that only the trajectories of one leg from each leg group is required, due to the symmetry of the rover about the sagittal plane.
Chapter 5: Integrated Design and Simulation Environment (IDSE)

The Integrated Design and Simulation Environment (IDSE) consists of the Kinematics Trajectory Generator (K-TG), the evaluation model, and the online CPG Trajectory Generator (CPG). It was developed using MATLAB®, Simulink® and SimMechanics® software. Given a set of variables $\varphi$ for any rover during the optimization process, the KTG creates the corresponding joint trajectories $q(t)$, as described in section 2.1. The evaluation model consists of a simulation of the rover and its environment. Using $\varphi$ and $q(t)$, the evaluation model is executed and the optimization attributes are calculated (section 3.4 and 3.5). This iterative step is repeated for all rovers in a population, over multiple generations in the GA. The CPG is used to generate trajectories for the joints and control gains in an online manner, based on the optimal solutions found in the locomotion optimization. It is integrated with the evaluation model to evaluate the performance of the CPG as shown in figure 6. The EM consists of the Rover model and Environment model, as discussed below.

![Diagram](image)

Figure 6– The role of the KTG vs. the CPG-trajectory generation unit
5.1 Rover Model (RM)

As shown in figure 7, the Rover model consists of a cuboidal main body, and four legs. All parts are designed in accordance to the model outlined in section 2. The Denavit-Hartenberg (DH) parameters $a$, $\alpha$ and $d$ [46] offer a convenient notation to represent the configuration of robotic manipulators, and have been used in this work for the legs of the rover. The major advantage in using DH parameters is that the structure of the rover can be modified to resemble any other animal through a change in these parameters. The fourth DH parameter $\theta$ [46] corresponds to the joint angles $q$. The DH parameters selected for the feline rover have been summarized in table 4. The joint axis of the hip, knee and ankle joints are parallel as a result of $\alpha$ and $d$ being set to zero. These DH parameters, the mass of the entire rover (3.7 kg) and the length of the main body (IGD = 0.32

![Figure 7– Rover model in simulation, showing (a) trimetric view, (b) side view](image)

<table>
<thead>
<tr>
<th>$j$</th>
<th>$a_j^F = L_j^F$</th>
<th>$\alpha_j^F$</th>
<th>$d_j^F$</th>
<th>$a_j^H = L_j^H$</th>
<th>$\alpha_j^H$</th>
<th>$d_j^H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.142 m</td>
<td>0</td>
<td>0</td>
<td>0.129 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.132 m</td>
<td>0</td>
<td>0</td>
<td>0.119 m</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.056 m</td>
<td>0</td>
<td>0</td>
<td>0.083 m</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4 – Denavit-Hartenberg (DH) parameters for the front and hind legs
m) are based on data collected for domestic cats [5]. The links are designed as uniform cylindrical rods, with a density of aluminum (2700 kg/m³), and link masses proportional to link lengths.

5.2 Environment Model (EM)

The environment model consists of a standard gravity field and a spring-damper-based ground contact model [65]. The model uses a vertical position switch to apply appropriate forces to the toes. When the vertical position of the toe is greater than zero (no contact between the toe and the ground), no external force is applied to the toe. When there is contact, vertical reaction force ($F_z$) and horizontal frictional force ($F_x$) are applied according to the following equations:

$$F_z = k \varsigma^n + b \varsigma^p \dot{\varsigma}^q$$

(62)

$$F_x = \mu F_z$$

(63)

$$\dot{\mu} = -3 \left( \frac{|W\dot{x}_3|}{s_p} \right) \mu + 3 \left( \frac{\mu_k}{s_p} \right) W\dot{x}_3$$

(64)

where $\varsigma$ is the penetration distance of the toe into the surface, $\mu$ is an internal state of the friction model, $W\dot{x}_3$ is the horizontal velocity of the toe measured in the world frame $\{W\}$, $\mu_k$ is the coefficient of kinetic friction, and $s_p$ is the displacement of the toe before saturation of friction occurs.

5.3 Online CPG Trajectory Generator (CPG)

As a result of the optimization of the internal parameters of the CPG model in section 4.1, the four identical CPG units for the legs are implemented with $\alpha = 8$, $\mathcal{K} = 2538$, $\Gamma = 0.1$ and $D = 0.01$, and integrated with time step $\Delta t = 0.0001$ s. The CPG unit for the control gains is implemented with $\alpha = 3$, $\mathcal{K} = 500$, $\Gamma = 0.1$ and $D = 0.1$, with the same time step. The number of patterns stored $K$ is selected as 60, with 20 patterns for each gait. This consists of 10 patterns during swing and stance each. The patterns used are presented along with the results in section 6.
Chapter 6: Results and Discussions

The results of the minimum torque optimization for three speeds as well as the maximum speed optimization are discussed below in sections 6.1 and 6.2. The optimization results for the gait and pose variables are summarized in table 5, and those for control variable in table 6. The results of the online gait planning using CPGs for two speed profiles is discussed in section 6.3.

Table 5 – The optimal gait and pose variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Torque</th>
<th>Maximum Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10 m/s</td>
<td>0.50 m/s</td>
</tr>
<tr>
<td>Stride time, $T$ (s)</td>
<td>1.30</td>
<td>0.46</td>
</tr>
<tr>
<td>Stride length, $\lambda$ (m)</td>
<td>0.13</td>
<td>0.23</td>
</tr>
<tr>
<td>Front hip angle, $\theta_1^{FPEP}$ (deg)</td>
<td>$-120.87$</td>
<td>$-109.38$</td>
</tr>
<tr>
<td>Front knee angle, $\theta_2^{FPEP}$ (deg)</td>
<td>48.29</td>
<td>52.91</td>
</tr>
<tr>
<td>Front ankle angle, $\theta_3^{FPEP}$ (deg)</td>
<td>35.75</td>
<td>34.22</td>
</tr>
<tr>
<td>Hind hip angle, $\theta_1^{HPEP}$ (deg)</td>
<td>$-66.02$</td>
<td>$-47.29$</td>
</tr>
<tr>
<td>Hind knee angle, $\theta_2^{HPEP}$ (deg)</td>
<td>$-48.55$</td>
<td>$-66.71$</td>
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<tr>
<td>Hind ankle angle, $\theta_3^{HPEP}$ (deg)</td>
<td>73.23</td>
<td>63.20</td>
</tr>
<tr>
<td>Front toe touch-off speed, $z_\beta^{FPEP}$ (m/s)</td>
<td>0.00</td>
<td>$-0.03$</td>
</tr>
<tr>
<td>Front step clearance, $f_\zeta^F$ (m)</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>AEP metacarpal angular speed, $\psi_\zeta^{FPEP}$ (deg/s)</td>
<td>4.79</td>
<td>124.04</td>
</tr>
<tr>
<td>PEP metacarpal angular speed, $\psi_\zeta^{FPEP}$ (deg/s)</td>
<td>1.23</td>
<td>97.11</td>
</tr>
<tr>
<td>Metacarpal net orientation change, $\psi_{net}^F$ (deg)</td>
<td>36.31</td>
<td>30.82</td>
</tr>
<tr>
<td>Hind toe take-off speed, $z_\beta^{FPEP}$ (m/s)</td>
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<td>$-0.01$</td>
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<tr>
<td>Hind toe touchdown speed, $z_\beta^{HPEP}$ (m/s)</td>
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<td>$-0.05$</td>
</tr>
<tr>
<td>Hind step clearance, $f_\zeta^H$ (m)</td>
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<td>0.03</td>
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<td>$-96.31$</td>
<td>89.13</td>
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<tr>
<td>PEP metatarsal angular speed, $\psi_\zeta^{HPEP}$ (deg/s)</td>
<td>18.07</td>
<td>$-57.01$</td>
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<td>Metatarsal net orientation change, $\psi_{net}^H$ (deg)</td>
<td>15.89</td>
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<tr>
<td>Duty Factor, $\beta$</td>
<td>0.78</td>
<td>0.50</td>
</tr>
<tr>
<td>Relative Phase, $\phi^{FH}$</td>
<td>0.63</td>
<td>0.64</td>
</tr>
</tbody>
</table>
6.1 Results of Locomotion Optimization

6.1.1 Minimum Torque Optimization

For brevity, we discuss the results for one typical case, after which the remaining results are provided. Figure 8 shows the trajectories of the actuated joints in the front legs for 0.5 m/s. The corresponding angular speeds are shown in figure 9. The results for the hind legs at 0.5 m/s, as well

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Torque</th>
<th>Maximum Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.10 m/s</td>
<td>0.50 m/s</td>
</tr>
<tr>
<td>Front hip proportional gain, $K_{P,1}^F$</td>
<td>262.59</td>
<td>617.78</td>
</tr>
<tr>
<td>Front hip derivative gain, $K_{D,1}^F$</td>
<td>9.46</td>
<td>3.92</td>
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<tr>
<td>Front knee proportional gain, $K_{P,2}^F$</td>
<td>709.38</td>
<td>207.30</td>
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</tr>
<tr>
<td>Front ankle proportional gain, $K_{P,3}^F$</td>
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<td>730.51</td>
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<tr>
<td>Front ankle derivative gain, $K_{D,3}^F$</td>
<td>1.84</td>
<td>6.61</td>
</tr>
<tr>
<td>Hind hip proportional gain, $K_{P,1}^H$</td>
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<td>299.24</td>
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<td>Hind hip derivative gain, $K_{D,1}^H$</td>
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<td>Hind knee proportional gain, $K_{P,2}^H$</td>
<td>379.38</td>
<td>232.55</td>
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<td>Hind knee derivative gain, $K_{D,2}^H$</td>
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<tr>
<td>Hind ankle proportional gain, $K_{P,3}^H$</td>
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<td>517.24</td>
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<tr>
<td>Hind ankle derivative gain, $K_{D,3}^H$</td>
<td>5.33</td>
<td>7.26</td>
</tr>
</tbody>
</table>

Figure 8– Joint trajectories for the front legs – (a) hip joint, (b) knee joint, and (c) ankle joint, during one cycle at 0.50 m/s
as those for both leg groups at other speeds are similar. The stride length \( \lambda \) for 0.5 m/s was obtained as 0.23 m, which results in a stride time of 0.46 s. The stance and swing phase durations are equal, because the duty factor \( \beta \) was obtained as 0.5. This is characteristic of a transition between walking.

![Angular speeds for the front legs](image1)

**Figure 9**– Angular speeds for the front legs – (a) hip joint, (b) knee joint, and (c) ankle joint, during one cycle at 0.50 m/s

![Motion of the legs](image2)

**Figure 10**– Motion of the (a) hind and (b) front legs during one cycle, 0.50 m/s

![Relative motion of legs](image3)

**Figure 11**– Relative motion of (a) front and (b) hind legs during multiple cycles at 0.5 m/s. The bold lines represent AEP and PEP leg positions as in figure 10.
and running. Figure 10 depicts the leg motion during the swing and stance phases, corresponding to the joint trajectories in figure 8. Figure 11 shows the relative motion between any leg in the front leg group, and the corresponding diagonally opposite leg in the hind leg group. When the hind leg is at the PEP, the front leg is neither at the AEP nor at the PEP due to a relative phase of $\phi^{FH} 64\%$. The hip and knee angles decrease, and the ankle angle increases at the start of the swing phase. This behaviour is necessary to increase the absolute velocity of the toe gradually, which is stationary during stance. This also prevents the toe from hitting the ground at take-off. As the swing phase continues, the hip angle increases to move the leg forward. The knee angle increases during the first half of the swing phase, to retract the leg and achieve the required step clearance. The knee angle decreases during the second half of the swing phase to extend the leg and prepare for touchdown. As the hip and knee joint motions extend the leg at the end of the swing phase, the ankle angle increases, so as to reduce the horizontal velocity of the toe gradually to zero. This gradual decrease in the toe’s horizontal velocity at the end of the swing phase is important to eliminate the leg impact at touchdown. At the beginning of the stance phase, the decrease and increase in the knee and ankle angle, respectively, cause the leg to absorb any remaining impact at touchdown. For most of the stance phase, the knee angle increases, and the hip and ankle angles decrease, to take the leg back to the PEP configuration. At the end of stance phase, an increasing ankle angle and decreasing hip and knee angles prepare the leg to take-off with a positive hip velocity. The net change in the orientation of the metacarpal (table 5) is a decrease by nearly $31^\circ$ with respect to the rover’s main body, but the ankle angle seems to have decreased by only about $10^\circ$ in figure 8c. This is because the combined motion of the hip and knee joints will further decrease the orientation of the radius link with respect to the main body.

The torque applied at the joints in the front legs is shown in figure 12. At touchdown, there is no sudden change in the applied torque, because the rate of change of the angular speeds just before and after touchdown is the same. This continuity in applied torque means that the impact at touchdown is nearly eliminated. This is a consequence of defining smooth, continuous and periodic trajectories for the toe. Note that the applied torque for all three joints oscillates around zero during the swing phase. This means that the optimal Bezier toe trajectory results in nearly passive joint
motion, requiring minimal adjustments via applied torque. However, just before touchdown, the hip torque increases rapidly and then decreases. The ankle torque rapidly decreases and then increases. This torque profile is required to prepare the leg for touchdown, by bringing the hip angular speed to zero, and decelerating the ankle joint against its passive motion.

The convergence of the torque objective function can be seen in figure 13 for thrust speed of 0.5 m/s. As the optimization converges, the red, blue and black curves depicting the average value of the objective function for the best 50%, 20% and 10% of the population, respectively, all approach a steady value of minimum total joint torque.

The joint trajectories, joint angular speeds, joint torque and motion of the legs for 0.10 m/s and 1.00 m/s have been shown in figures 14 to 27 below. The behaviour of the front and hind legs in this case is similar to the case discussed above at 0.50 m/s.
Figure 14 – Joint angles for front legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s

Figure 15– Joint angle speeds for front legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s

Figure 16– Joint angles for hind legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s

Figure 17– Joint angle speeds for front legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s
Figure 18– Joint torques for front legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s

Figure 19– Joint torques for hind legs – (a) hip (b) knee and (c) ankle, at 0.10 m/s

Figure 20– Leg motion for (a) hind legs, and (b) front legs at 0.10 m/s
Figure 21 – Joint angles for front legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s

Figure 22 – Joint angle speeds for front legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s

Figure 23 – Joint angles for hind legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s

Figure 24 – Joint angle speeds for hind legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s
Figure 25 – Joint torques for front legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s

Figure 26 – Joint torques for hind legs – (a) hip (b) knee and (c) ankle, at 1.00 m/s

Figure 27 – Leg motion for (a) hind legs, and (b) front legs at 1.00 m/s
6.1.2 Maximum Speed Optimization

The stride length and stride time were obtained as 0.43 m and 0.11 s, respectively, which results in a maximum speed of 3.68 m/s (table 5). The duty factor $\beta$ at this speed was obtained as 0.43. The swing phase duration is longer than that of the stance phase, which is a characteristic of running gaits. The trajectories of the actuated joints of the front legs are shown in figure 28. The corresponding angular speeds are shown in figure 29. The leg motion for the front and hind legs during swing and stance is shown in figures 30 and 31. The hip, knee and ankle joints follow the same behaviour during swing and stance phases as in the case at 0.5 m/s. The ankle joint, however, follows a different curve. Note from figure 28c that the ankle angle increases from AEP (roughly 33°) to PEP (roughly 40°). However, the net change in orientation for the metacarpal (table 5) is a decrease by nearly 27° with respect to the rover’s main body. The combined motion of the hip and knee joints decrease the orientation of the radius link with respect to the main body.

![Figure 28](image1.png)

Figure 28– Joint trajectories for the front legs – (a) hip joint, (b) knee joint, and (c) ankle joint, during one cycle at 3.68 m/s

![Figure 29](image2.png)

Figure 29– Angular speeds for the front legs – (a) hip joint, (b) knee joint, and (c) ankle joint, during one cycle at 3.68 m/s
from AEP to PEP, allowing the decrease in the metacarpal orientation despite the increase in the ankle angle.

The torque applied at the joints in the front legs is shown in figure 32. Similar to the case at 0.5 m/s, there is no sudden change in the applied torque at touchdown, which means that the impact

![Figure 30](image-url)  
**Figure 30** – Motion of the (a) hind and (b) front legs during one cycle at 3.68 m/s

![Figure 31](image-url)  
**Figure 31** – Relative motion of (a) front and (b) hind legs for multiple cycles at 3.68 m/s.

![Figure 32](image-url)  
**Figure 32** – Torque applied at actuated joints in the front legs– (a) hip joint, (b) knee joint, and (c) ankle joint, during one cycle at 3.68 m/s

![Figure 33](image-url)  
**Figure 33** – Convergence of speed-based objective at 3.68 m/s
at touchdown is nearly eliminated. The applied torque for all three joints oscillates around zero during the swing phase, which means that the optimal Bezier toe trajectory results in nearly passive joint motion. The absolute maximum value of the torque is higher than the case at 0.5 m/s. Since the stride time at this speed is much shorter than that at 0.5 m/s, the increase in the magnitude of torque is expected. The results for the hind legs are similar to the above, and they are not shown in this thesis for brevity.

The convergence of the speed objective can be seen in figure 33. Unlike the torque-based objective function, the speed objective function does not converge gradually. The solution almost converges at generations 19, 33 and 35, but the optimization finds better solutions, and continues. At generation 37, the average value of the best 50%, 20% and 10% of the solutions were within 1% of each other, terminating the optimization consequently.

### 6.1.3 Optimal Gait Transition

As seen in table 4, the duty factor $\beta$ of the optimal solutions decreases with increasing speed, which has been observed in nature as well [66-67]. At 0.1 m/s, the duration of stance phase is longer than the swing phase, which is a characteristic of walking gaits [29]. At 0.5 m/s, the durations of swing and stance phases are equal, which is a transition point between walking and running. At 1.00 and 3.68 m/s, the duration of swing phase is longer than the stance phase, which is a characteristic of running gaits [29]. In a work that quantified the kinematics of domestic cats [67], it was observed that the duration of the stance phase decreases considerably as the speed increases from low to moderate speeds. This is accompanied by a relatively constant swing phase duration. The same behaviour is observed in the optimal solutions as illustrated in figure 34. As the speed increases further to 3.68 m/s, a decrease in both durations of swing and stance phase is observed.

The relative phase $\phi^{FH}$ at 0.10 and 0.50 m/s is constant at nearly 63%. This gait is between a pace (50%) and an amble (75%). At 1.00 m/s, $\phi^{FH}$ is 82%, which is between a trot (100%) and an amble. This suggests that the trot gait is preferred at higher speeds than a pace gait, as illustrated
As the speed increases to 3.68 m/s, $\phi^{FH}$ is at 79%, which is an amble. At such high speeds, most quadruped animals use asymmetric galloping gaits [5, 68]. However, since the scope of this work is limited to symmetric gaits, the optimization selects a four-step amble gait, which has been used by elephants for running [29].

### 6.1.4 AEP Joint Angles and Step Clearance

As the speed increases, the stride length $\lambda$ increases as well (table 5). To accommodate this increase, the joint angles of the hip, knee and ankle must cover a greater range. As a result, the AEP configurations of the front and hind legs move further ahead with increasing speed, as illustrated in figure 36. The leg configuration shown with a dotted line indicates the AEP joint angles from the literature at 0.8 m/s [5], which are close to the optimal solutions. Note that the front metacarpal angles are different between the optimum solutions and the solution from the literature.
This is because the metacarpal/metatarsal links in the rover play the role of the phalanges in an animal, which were omitted in the rover. The joint angles of the fourth joint in the front legs of domestic cats [5] are similar to the ankle angle in the solutions obtained.

As the speed increases, the maximum step clearance of the front and hind legs remains relatively constant, as illustrated in figure 37. This suggests that the step clearance is independent of speed. This phenomenon was observed in [22] as well, where the step clearance remained relatively constant in the entire range of speeds studied (0.00 – 5.00 m/s).
The vertical hip velocity is the opposite direction of the vertical velocity of the corresponding toe. At the beginning of stance phase (AEP configuration) the vertical hip velocity is observed to be positive for all thrust speeds. This velocity remains relatively constant for the front legs. For the hind legs, the vertical hip velocity increases with increasing thrust speed. The vertical hip velocity at the end of stance phase (PEP configuration) follows a similar trend, for both the front and hind legs, as illustrated in figure 38.

At 0.1 m/s, the duty factor corresponds to a walking gait. In this case, there exists a duration in the locomotion cycle when both legs in the same group are in stance together (double-stance). Since the simulation is executed in the 2-D sagittal plane, the rover’s main body does not have a rolling degree of freedom. Hence, if the height of the hip joints changes during the double-stance period, one of the legs will not touch the ground, which violates constraint (41). As figure 38 depicts, the optimal solution has a vertical velocity of nearly zero for both the hind and front legs at both the AEP and PEP configurations. This leads to a horizontal trajectory for all the hip joints, thereby satisfying constraint (41).

At 0.5 m/s, the hip joints of the front and hind legs have non-zero vertical hip velocity (figure 38), even though the gait is a transition between walking and running. As $\beta$ is 50% in this case, the
double-stance duration is reduced to an instant where the left and right legs in any leg group are at the AEP and PEP, respectively, or vice versa. Since the height of the hip at PEP and AEP is the same by equation (15), constraint (41) is still satisfied with non-zero vertical velocities for the hip joint. At 1.00 m/s and 3.68 m/s, the gaits are running gaits as the duty factors are below 0.5. Hence, the double-stance duration does not exist and constraint (41) is satisfied.

6.2 Results of Gait Planning using CPGs

The gaits of the quadruped rover with joint and control trajectories generated by the CPG, were analysed for two speed profiles, which have been shown in figure 39. These have been discussed below. Table 7 shows the range of speed values corresponding to the three gaits found in the minimum torque optimization. The 60 joint angle configurations $q^k$ (section 4.2) for the front and hind legs correspond to the configurations shown in figures 10, 20 and 27. The control states $c^k$ correspond to those in table 6.

![Figure 39](image)

Figure 39 – The two speed cases (a) continuous speed profile, and (b) discontinuous speed profile

<table>
<thead>
<tr>
<th>Range of speeds (m/s)</th>
<th>Speed of optimal gait (m/s)</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 – 0.35</td>
<td>0.10</td>
<td>Gait 1</td>
</tr>
<tr>
<td>0.35 – 0.65</td>
<td>0.50</td>
<td>Gait 2</td>
</tr>
<tr>
<td>0.65 – 1.00</td>
<td>1.00</td>
<td>Gait 3</td>
</tr>
</tbody>
</table>

Table 7 – The range of speeds for the optimal gaits (from minimum torque optimization)
6.2.1 Continuous Speed Profile

In this speed profile (figure 39a), the speed linearly increases from 0.10 to 0.90 m/s, and then decreases linearly back to 0.10 m/s. The joint trajectories generated by the CPG for the front leg joints have been shown in figure 40. The range of speeds have been separated by vertical dashed lines, and the corresponding gaits to be used in these regions have also been marked.

(a) Speed modulation - Consider the region between 0 to 2.8 seconds, where the joint trajectories correspond to gait 1. Note that as the speed increases linearly from 0.10 m/s to 0.35 m/s, the stride time decreases. A similar trend is noticed in the remaining speed ranges, for both increasing and decreasing speed. Hence, the CPG can modulate speed at a fixed gait within each region.
(b) Gait modulation - The speed reaches 0.35 m/s at 2.8 seconds, which results in a gait change. Even though the corresponding pose and gait variables change abruptly, the joint trajectories change smoothly towards the optimal joint trajectories for gait 2. At 6.2 seconds, when the speed reaches 0.65 m/s, a similar smooth change is observed. The motion of the rover’s front and hind legs just before and after 6.2 seconds has been shown in figures 41 and 42, which confirms this smooth gait transition. The leg configuration shown in red corresponds to that at 6.2 seconds.

The proportional and derivative control gain trajectories have been shown in figures 43 and 44, respectively. Note that the control gains do not change for a fixed gait. At the gait transition points, the control gains change smoothly to their new values (table 6, section 6.1). Hence, for a continuously changing speed profile, the CPG can modulate speed transition at fixed gaits, as well as gait transition. The rover continues to demonstrate stability according to the definition in section 3.5.2e.
Figure 43 – Proportional gain trajectories generated by the CPG

Figure 44 – Derivative gain trajectories generated by the CPG
6.2.2 Discontinuous Speed Profile

In this speed profile (figure 39b), the speed changes discontinuously. Note that the unlike the continuously case, the speed remains constant between abrupt changes. This reflects in the joint trajectories in the form of a constant stride time at each gait. The joint trajectories generated by the CPG for the front leg’s joints have been shown in figure 45. Once again, the range of speeds have been separated by vertical dashed lines, and the corresponding gaits to be used in these regions have also been marked.

The speed changes abruptly every 3 seconds. Note that the joint trajectories from 0 to 3 seconds, have the same shape as those from 6 to 9 seconds, as well as 15 to 18 seconds. All three correspond to gait 1, but have different stride times due to different speeds.

At gain transition instants, the joint trajectories transition smoothly. However, as the desired velocity of the rover changes abruptly, the rover’s motion cannot be predicted here. Figure 46 shows the motion of the rover’s front and hind legs when it accelerates rapidly from 0.10 to 0.60

![Figure 45 – Joint trajectories generated by the CPG for a discontinuous changing speed profile](image-url)
60 m/s. Up until the transition point, the rover’s motion is synchronous to its expected behaviour at 0.10 m/s. Right after the transition point, the height of the hind leg’s hip decreases, leading to a positive pitch velocity of the rover’s main body. This leads to the front legs exhibiting a hopping motion. After this, the rover settles to its expected behaviour at 0.60 m/s, and continues to demonstrate stability.

The proportional and derivative control gain trajectories generated by the CPG for this speed profile are shown in figures 47 and 48, respectively. Once again, the control gains change smoothly to their new values at the gait transition points.
Figure 47– Proportional gain trajectories generated by the CPG

Figure 48– Derivative gain trajectories generated by the CPG
Chapter 7: Conclusions and Future Work

In this work, a model-free locomotion optimization and planning framework has been presented, which was implemented on a rover model based on a domestic cat. The rover’s gait, pose and control variables were optimized within an Integrated Design and Simulation Environment. The single-objective, multi-constraint optimization problem was carried out, using a Genetic Algorithm, for minimizing a torque-based energy objective and maximizing speed.

The optimum gait was found to vary from a gait between an amble and pace at low velocities (0.10 and 0.50 m/s) to a gait between an amble and trot at high velocities (1.00 and 3.68 m/s). With increasing speed, the stride lengths of the optimal solutions increased, the duty factor decreased, the stance phase duration decreased with a relatively constant swing phase duration, the step clearance remained relatively constant, and the vertical hip velocities increased. The results are consistent with those of other works on gait optimization, as well as the gaits found in quadruped animals.

An artificial CPG trajectory generator was also implemented to generate joint and control trajectories in an online process. The CPG was used to generate these trajectories for both a continuous and a discontinuous speed profile. The trajectories generated in this manner showed that the CPG can regulate both speed modulation and gait transition.

Satisfying all the objectives outlined in section 2.5, this work has demonstrated that the locomotion optimization process based on Genetic Algorithms, and the gait planning framework based on Central Pattern Generators, can be used to find the optimal locomotion modes of a feline rover, as well as transition between them endogenously for an arbitrarily varying speed profile.

An interesting future direction could involve using the optimization framework to optimize locomotion parameters for asymmetric gaits, such as the gallop or canter, which can enable the rover to attain much higher speeds. Such a step would require decoupling of design parameters for the legs of the same leg group. Future work can also analyze and optimize gaits based on a 3D model including the roll, yaw and sway degrees of freedom in the simulation (out of the sagittal plane), which would require an active control strategy to stabilize the frontal plane perturbations.
Performing similar optimizations for structures based on other animals can yield meaningful links between the locomotion behaviour of animals across various taxa and the performance of quadruped rovers.

The CPG framework can be further improved by introducing an overlying mechanism that decides the number of joint configurations to be cycled through. As a result, the CPG does not have to consider the joint configurations belonging to gaits that it does not need to traverse. When a gait change is induced as a result of the speed input, the CPG can consider the joint configurations of the two gaits enveloping the gait transition. After a successful gait transition, the CPG would switch back to considering the joint configurations of the new gait alone. This can reduce the simulation time and computations required, and potentially make the CPG a real-time as well as online trajectory generator.

Lastly, by inverting equation 55, one can find the current state of the CPG’s oscillators as a function of the joint angles recorded by sensory feedback. This gives the CPG information regarding the error between the actual state and feedback state of its oscillators. The dynamics of the CPG’s oscillators can be modified to reduce this error, thereby turning the CPG into both a trajectory generator and controller. Of course, more studies are needed for the stability and performance analysis of such a control system.
References


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