Analytical Approaches to Investigating Transit Network Resilience

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Abstract

The reliability of public transit networks is of critical importance the world over. As transit demand is forecasted to grow, there exists a need to quantitatively measure the operational resilience of a transit network. Such metrics can be used to transit planners and operators to effectively mitigate the impact of service disruptions on commuters. By representing the public transit network in Toronto, Canada as a directional graph, a series of metrics based in graph theory were employed to analyze resilience. Using simulation allowed one to overcome the inherent limitations in a network science approach and introduced the concepts of ‘Importance’ and ‘Exposure’. These indices are based on the demand-weighted increased in travel cost when service disruptions occur. An analysis on the behavior of trip-makers is conducted via a sentiment analysis using Twitter. The result of the combined efforts discussed above is a framework to capture operational resilience.
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1 Introduction

The public transportation network of a major metropolitan area is a critical lifeline for joining communities and providing accessibility to work, school and social activities. A network as large and complex as a modern public transit system is not without its vulnerabilities. Disasters, either manmade or otherwise, can diminish or otherwise completely disrupt the functionality of this essential system.

Rising petroleum prices, coupled with increased populations in urban areas will result in further pressure on transit systems the world over. With this ever increasing demand, it is imperative to examine how a transit system responds to disruptions. The ability of a transit system to retain an acceptable level of performance during a disruption, and the inherent cost to users of said disruption shall define the resilience of the system.

Transit resilience is a relatively new research topic with profound impacts on both transit planning and day-to-day operations. Quantifying the resilience of a network will aid planners and operators in determining strategies to mitigate risk, and reduce the overall impact of a disruption on passengers. Furthermore, quantification of resilience allows one to maximize a constrained capital budget in order to seek the best possible improvements (in terms of improving resilience, and inherent reliability) and thus guide investment plans.

Past research on transportation network resilience has largely focused on large scale disruptions. Large scale disruptions, such as the September 11th terrorist attacks or Hurricanes Katrina and Sandy, impact many facets of daily life, including the public transit system. Although very critical to analyze, there is a growing need to study the smaller scale and more frequent disruptions that occur during routine operations. Such incidents can have profound impacts on a transit system which is operating near or even over-capacity. Ultimately, the overarching goal of this research is to provide a means to determine where targeted intervention is needed to mitigate the impacts of a disruption.
A more focused objective of this research is to develop a method for determining and quantifying the resilience of a public transportation network. This conceptual framework will be built upon the mathematical procedures and theories found in graph theory, and then expanded to consider the idiosyncrasies of human behaviour and decision making for trip making purposes. In general, a loss of performance of a network due to a service disruption will be quantified, and from this, comments on network resilience can be made. Factors which contribute to a resilient network will be discussed and analyzed.

1.1 The Resilience Catechism

The following research will seek to answer a number of questions: 1) Can network science metrics appropriately quantify transit network resilience? 2) What properties of a transit network need to be considered in order to quantify transit network resilience? 3) What are the characteristics of a transit network which make it resilient, and by extension are there critical characteristics that when absent result in a non-resilient network? 4) How can one measure the impact of a disruption on i) passengers and ii) communities, and by what metric can one use to determine the cost of a disruption?

At the onset, an attempt to examine the resilience of a network from a purely graph theory approach will be adopted. The science of complex networks allows for a unique method to determine the structural resilience of a public transit network. These structural metrics, although invaluable, have a number of limitations, most notably is that they treat all nodes and links as equivalent, a concept which is obviously not true in a public transit network that has services of varying performance levels (e.g. bus vs. metro). Therefore, it is imperative that the traditional structural metrics be updated to consider the properties of a public transit network, thus producing an analysis which more accurately measures resilience. The impact of a disruption on the users (that is to say, a user-based approach) is a key deliverable from this research. By further extending this metric to examine how regions or areas are impacted, one can shed light on the spatial impacts of a disruption.
1.2 General Approach

The methodology proposed requires detailed modelling to examine the impact of a series of disruptions on the transit system. In an attempt to answer the first proposed research question, a public transit network can be developed in a graph-based software program called NetworkX. NetworkX facilitates the creation and analysis of complex networks using the principles of graph theory. From NetworkX, a series of metrics can be obtained given a service disruption or series of disruptions. Graph theory based metrics can quantify the loss of functionality in terms of node/link removal, change in the efficiency of exchange and change in shortest path calculations. To analyze a public transit network with consideration for service attributes will require software capabilities beyond that on NetworkX.

Computational simulation of travel behaviour has seen increasing use as of late, one notable piece of software is EMME/4. A software such as this is very data-hungry, to facilitate this data need, the public transit use of the Transportation Tomorrow Survey (TTS) from 2011 will be used. Transit assignment will be conducted in EMME/4 based on the 2011 TTS data. A modified congested fare-based transit assignment model was applied in place of the generic assignment found within EMME/4. Disruptions to the transit network, in the form of service suspensions are simulated and the cost of such a disruption is determined as the change in travel time experienced by the users as they seek to find a new route to their destination. As travel demand varies both spatially and temporally, some disruptions may have larger impacts on travel time than others; therefore, to accurately predict the resilience of a network, it is imperative that the scenarios studied are exhaustive of all possible disruptions which can occur.

The loss of functionality of the network is measured as change in travel time experienced by users. This change in travel time arises as users are forced to take alternative, less attractive routes (such as busy and slow moving bus lines) or in some cases, walking large portions of their trip when no alternative exists.
1.3 Prior Research

The work contained in this thesis is an extension of several pieces of research. The initial portion regarding complex network science is built upon the ideas and principles laid out by Abigail Osei-Asamoah and Nicholas Lownes in 2014 (Osei-Asamoah & Lownes, 2014). Their work looked at the structural resilience of complex networks (with a focus on transportation networks) by calculating a number of structural indices. Such metrics are computationally simple to perform and allow for ease of comparison between networks, and thus serve as a good basis for determining resilience.

Expanding a strict Graph theory approach was necessary to both capture the realistic behaviour of passengers and to overcome the limitations inherent in a network science approach (i.e. equality in link/node weight). This thesis borrows and expands upon the concept of 'Importance' as defined and applied by Erik Jenelius and Lars-Goran Mattsson (Jenelius & Mattsson, 2012; Jenelius, Petersen, & Mattsson, 2005). Based on a study of the vulnerability of the road network of Sweden, Jenelius et al. proposed a metric for determining and rank-ordering the importance of a road link. By simulating the increase in travel time due to the disruption of a link, Jenelius et al. were able to provide a quantitative measure of the resilience of a transportation network. A more complete account of the research in network resilience can be found in the literature review, Chapter 2.

1.4 Motivation and Study Area

The public transit system is an integral part of modern urban life. For many, public transit represents the only feasible way to make such critical trips as getting to work. Both improvements and disruptions of the public transit system have far reaching impacts on the travel times of commuters.

Toronto is located on the northwestern shores of Lake Ontario in Canada. With a total population of over 2.65 million people (Statistics Canada, 2011), it is the most populous metropolitan area in Canada, and the economic heart of the province of Ontario.
Public transit within the city is provided by two separate agencies, each with a defined geographic jurisdiction. These operators are: GO Transit, which operates commuter rail services as well as inter-municipal coach buses; and the Toronto Transit Commission (TTC), which operates the underground subway lines (of which there are four) and surface-level services, including streetcar lines and buses. The TTC represents the most used transit system in Toronto, serving on average, 2.76 million passengers per day.

Toronto was recently ranked as one of the worst cities in North America for congestion, with an average travel time of 65.6 minutes in 2014 (Canadian Index of Wellbeing, 2014). Having experienced almost three decades of government neglect for transit infrastructure expansion, portions of TTC system now operate strenuously above capacity during the morning peak period.

Historically, The TTC has subscribed to a philosophy titled the “state of good repair”. The priority of which, has been the maintenance and repair of infrastructure and equipment. As of late, budget limitations have diminished the TTC’s ability to maintain this state. Aging infrastructure combined with year after year of record ridership has undermined the reliability of the TTC metro system. At the high level of demand experienced daily, even minor disruptions greatly reduce the effectiveness of the system to provide even basic functionality. With the political climate unlikely to change, there is a growing need to study the impact of the service disruptions that frequently cripple the TTC rapid transit system.

### 1.4.1 TTC Rapid Transit System

The rapid transit system of Toronto consists of four subway lines and 69 stations (Toronto Transit Commission, 2013). Figure 1.1 shows the four subway lines, streetcar lines and the GO rail lines which serve Toronto. The subway lines are as follows, Yonge-University-Spadina Line (YUS-yellow), Bloor-Danforth Line (BD-Green), Sheppard Line (Purple) and the Scarborough Rapid Transit Line (SRT-Blue). In addition, the TTC operates 11 streetcar lines, two of which operate in a dedicated right-of-way (ROW), with the remainder operating in mixed traffic.
Figure 1.1 TTC Subway Network Map. (Adapted from TTC.ca)
Figure 1.2 GO and TTC Network for the GTA. (Adapted from Metrolinx.ca)
The rapid transit system of Toronto is plagued with delays, a total of 5864 occurred in 2013 (Toronto Transit Comissions, 2013). Delays can be attributed to incidents at track level (fire, unauthorized persons), mechanical failures, unruly passengers and weather. Said delays can result in the holding of train service for several minutes, or be as serve as to result in the suspension of service on an entire line. Of the over 5000 delays, 48 of which were major delays that resulted in a service suspension of the subway system for a portion of a line.

1.5 Proposed Contribution

With limited budget, frequent delays and growing ridership, a quantifiable and defensible metric of resilience is needed. Such a metric needs to be calculated based on easily available data and presented in a manner that is accessible to transit operators and planners. This work intends to propose a metric for the resilience of a public transportation network, by way of quantifying the relative importance of transit stations contained therein. Said importance will be determined based on the demand-weighted increase in travel time experienced by transit users during the morning commuting period during various service suspension scenarios. The impact of said service suspensions will be further extended to a neighborhood level. The intent of which, is to examine a more regional measure of the resilience public transit network in Toronto. To help shed some light on the behavioral aspects of a service disruption, a Twitter sentiment analysis will be conducted. With the advent and widespread use of social media, there exists a great wealth of informal data, which previously was inaccessible, on how transit users behave during disrupted conditions. Capturing user sentiments can help shed light on aspects of transit resilience which may be difficult to model using simulation alone. Such an analysis can then serve to improve the overall framework of resilience.

The metric of resilience contained in this thesis represents a critical first step in a chain of resilience research, disruption management planning research and facets of demand-management research, of which with a focus of public transportation. Ultimately, this research will form the first link in a chain which could roughly be described as a 'Network Resilience Calculator'. The use of such a tool will provide transit planners/operators with a quantitative value of the
resilience of their network. From such a value, one can determine areas (or portions of a line) of high vulnerability. Such critical knowledge can then guide infrastructure improvement planning and disaster planning/mitigation.

1.6 Thesis Outline

To begin, a thorough literature review will be presented that examines resilience research conducted to date. The focus of this review will be research that analyzed the resilience of transportation networks. Following this, the methodological approach adapted in this work will be detailed. The relevant theory will be outlined and the formulation of proposed metrics for quantifying resilience will be presented. The results of the application of Graph theory and the EMME simulation will be discussed and analyzed. Finally, a sentiment analysis of social media, with a focus on passenger behaviour during service disruptions will be presented.
2 Literature Review

The word resilience is derived from the Latin word 'resilire' which roughly translates to 'leap back or rebound' (Latin Dictionary: Resilio, n.d.). The study of resilience (especially with respect to transportation engineering) is a relatively new field of research. In the wake of increasing climate instability and security threats, both foreign and domestic, incorporating aspects of resilience into planning is gaining traction with planners and engineers across a range of disciplines.

Resilience is a term which finds a home across a wide variety of fields. Furthermore, the word itself has a wide variety of definitions and is often synonymous in the literature with 'robustness', 'reliability', 'redundancy' and 'adaptability', among others. Resilience has been studied in a large number of fields including sociology, psychology, engineering, ecology, business, and economics. To begin this literature review a presentation of accepted definitions of resilience is presented below:

- Ecology: “A measure of persistence of systems and their ability to absorb changes and disturbances and still maintain the same relationships between populations or state variables.” (Holling, 1973).

- Economics: “‘Nurtured’ ability of an economy to recover from or adjust to the effects of adverse shocks to which it may be inherently exposed.” (Briguglio, 2006).

- Social Sciences: “The capacity of a system, community or society potentially exposed to hazards to adapt, by resisting or changing, in order to reach and maintain an acceptable level of functioning and structure.” (Huiping, 2005).

- Seismic/Earthquake engineering: "Community seismic resilience is defined as the ability of social units (e.g., organizations, communities) to mitigate hazards, contain the effects of disasters when they occur, and carry out recovery activities in ways that
minimize social disruption and mitigate the effects of future earthquakes" (Bruneau, 2003).

From the presented list, it should be apparent to the reader that there is no one classically accepted definition of resilience. Definitions of resilience can further vary within one field of study. Across the numerous definitions, one notices several common themes. These themes relate to the ability of a system to recover and persevere through disruptions/shocks. With respect to transportation engineering -to which this work pertains- resilience can take a number of definitions, including but not limited to:

- "Resilience is a characteristic that enables the system to compensate for losses and allows the system to function even when infrastructure is damaged or destroyed" (Battelle, 2007).

- "Resilience is the ability of systems to accommodate variable and unexpected conditions without catastrophic failure" (VTPI, 2010).

- "Resilience is the ability for the system to maintain its demonstrated level of service or to restore itself to that level of service in a specified timeframe" (Heaslip, Louisell, Collura, & Serulle, 2010).

- "Resilience is a characteristic that indicates system performance under unusual conditions, recovery speed, and the amount of outside assistance required for restoration to its original functional state" (Murray-Tuite, 2006).

- "[Resilience is] the ability of a transportation network to absorb disruptive events gracefully and return itself to a level of service equal or greater than the pre-disruption level of service within a reasonable time frame" (Freckleton, Heaslip, Louisell, & Collura, 2012).
• With respect to freight operations, "The ability for the transportation system to absorb the consequences of disruptions, to reduce the impact of disruptions, and to maintain freight mobility in the face of such disruptions" (Adams, Bekkem, & Bier, 2010).

2.1 Quantification of Resilience

The previous section has provided a variety of definitions on the concept of resilience across a broad spectrum of fields. Some of the aforementioned conceptual frameworks of resilience were also accompanied by methods to quantify resilience. The following section will outline and detail some methods for the quantification of resilience found in the literature.

Tamvakis and Xenidis (Tamvakis & Xenidis, 2013) conducted a comprehensive review of the quantifiable methods of resilience that exists in the literature. Based on their survey, a number of common approaches were employed, they include but are not limited to: probabilistic modelling, graph-theory applications and fuzzy inference methods.

Cox et al. (Cox, Prager, & Rose, 2011) used the London transportation system to propose a series of operational metrics for resilience. The metrics are centered on the concepts of network vulnerability, flexibility and the availability of network resources. The flexibility of the transportation system allowed the system to adjust and overcome shocks, such as the 2005 London subway and bus bombings. Cox et al. proposed the concept of the Direct Static Economic Resilience, which they defined as "the percentage of avoidance of the maximum economic disruption that a particular shock can bring about.". In their framework, resilience is increased when passengers are able to adopt mode-switching behavior when their mode of travel is compromised by a disruption.

Kevin Heaslip et al. used Fuzzy Inference to examine the resilience of a transportation network at the regional level (Heaslip, Louisell, Collura, & Serulle, 2010; Heaslip, Louisell, & Collura, 2009). As presented before in the preceding section, a definition of resilience was put forth. Heaslip's framework on resilience involves "cycles" (see Figure 1) and outputs a "network performance index" based on a Fuzzy Inference System (FIS) which captures the relationship between variables of the transportation network.
As per Figure 2.1, the cycle of resilience is composed of four distinct stages. The first stage, denoted as "normality", represents the system behaving in a regular fashion, the pre-disruption state. Upon the occurrence of a disruption, the system enters the second stage, "breakdown". During "breakdown", the system experience a loss in performance until a minimum level of performance is reached. The degree to which performance is impacted by the disturbance is referred to as "robustness" in this work. Eventually, the system enters the third phase known as "self-annealing". This third stage represents the system remaining unassisted by external sources. Finally, the system enters the "recovery" phase in which the disruption is remedied and the system returns to pre-disruption levels of operation.

Sunil Babu Pant completed his Master’s thesis titled ‘Transportation Network Resiliency: A Study of Self-Annealing” on the “self-annealing” phase (Pant, 2012). Working under the supervision of Kevin Heaslip, his thesis examined the total loss in the network performance during this phase and looked to quantify it by way of numerical analysis. He found that the higher the degree of robustness in the network, the better performance obtained.
In 2011, Serulle (Serulle, Heaslip, Brady, Louisell, & Collura, 2011) and company working in conjunction with Heaslip further refined the past FIS model for regional resilience and adjusted the quantitative calculation.

Murray-Tuite of the Virginia Polytechnic State University put forth ten dimensions of resilience for a transportation system (Murray-Tuite, 2006). The ten dimensions are as follows: redundancy, diversity, efficiency, autonomous component, strength, collaboration, adaptability, mobility, safety and the ability to recovery quickly. Murray-Tuite looked at examining how resilience was impacted by traffic assignment under the System Optimal regime vs. the User Equilibrium regime. For quantifiability, only the last four of the ten dimensions of resilience were considered.

Bruneau et al. (Bruneau, 2003) put forth a quantitative framework for seismic resilience. A key finding of their research was the proposal of the "four R's" which related to infrastructure qualities. The four R's are as follows: Robustness, Redundancy, Resourcefulness, and Rapidity. Bruneau et al. is most known for the proposal of the resilience triangle, which can be found in Figure 2.2. Said triangle shows the loss in functionality of a system as a result of a shock. The depth of the triangle indicates how severe the loss in functionality is, the length denotes the duration of the loss and the slope of hypotenuse denotes the speed of recovery. According to Bruneau, the area of the triangle directly defines resilience, with larger values of area indicating a less resilient system.
The aforementioned Battelle used the terms redundancy and resilience interchangeably (Battelle, 2007). Battelle proposed that resilience is derived from the ability of a system to accommodate demand by having spare capacity. In addition, resilience is found by having alternative routes and properly coordinating demand on available capacity in an efficient manner.

The Victoria Transport Policy Institute, an independent research group has contributed several pieces of work to the field of transportation resilience (VTPI, 2010; Institute, 2010). In the first body of work, they noted that transportation systems need to be able to provide service given a wider range of operating conditions. They defined system quality as the level of service a system can offer under even the worst conditions. In their second piece of work, they looked to evaluate resilience at a variety of levels within the system. The levels they studied included the strategic planning level, the community level, the design level, the individual level and the economic level. They noted that a system with good information collection and provision, high mobility, high diversity, built-in redundancy, performance efficiency and strength constitutes a resilient system.

2.2 Resilience Indices

Within the body of work that involved quantifying resilience, a sub-group of literature exists which seeks to provide resilience indices. These indices seek to rank-order networks to compare the total resilience observed.
Li Zhang et al. developed a Measure of Resilience (MOR) for an intermodal transportation system (Zhang, Wen, & Jin, 2009). In their work, resilience is defined as the intermodal network performance after the disaster relative to the performance before the disaster. Using regression, a Performance Index (PI) was developed based on performance indicators (e.g. travel speeds, etc). The PI is a value that denotes the ratio of travel speed after the disaster to free flow speed, and is weighted for miles travelled. Resilience is thus measured based on the PI before and after the disaster.

Nagurney and Qiang put forth the idea of the relative total cost index (Nagurney & Qiang, 2009). This value would denote the robustness of a transportation network. The index is calculated as the ratio of increase in total cost of the network relative the original cost under a given capacity retention ratio. The index allows for the analysis of travel behaviour, such as comparing System-Optimal assignment with User-Equilibrium. A network is more robust when it has a lower relative total cost index.

Scott et al. developed the Network Robustness Index (NRI) to better determine the critical links in a network (Scott, Novak, Hall, & Guo, 2006). In the past, link criticality had been defined by volume to capacity ratios (v/c). Scott et al. noted that such measures do not consider the system-wide impacts of loss of particular links. The NRI considers network flows, available capacity of links and the topology of the network. NRI as a metric was used by Scott on three hypothetical networks. The results showed that investment to improve links with a high NRI value provided higher system benefits when compared with links with a higher v/c ratio. James Sullivan et al. applied the NRI to determine critical links on the road network in Vermont (Sullivan, Aultman-Hall, & Novak, 2010). This was the first application of the NRI to a real world road network. Their work found that application of the NRI to a real network was feasible and produced meaningful results. Furthermore, it was found that in a real world scenario, adding capacity in a multi-modal sense provided the most robustness.

In 2010, Adams et al. produced a metric to examine the operational resilience of road corridors in Wisconsin (Adams, Bekkem, & Bier, 2010). The metrics which were examined included: extra distance travelled and travel time for alternative routes, change in the volume of traffic
routed, and the difference in the level of service experienced. A Risk Priority Number (RPN) was assigned to each road segment which accounted for the probability and severity of a disruption on that segment. A linear function of the volume on the segment, its RPN and the economic value of the segment led to the determination of the resilience rating. Segments were then ordered based on their resilience rating to determine the critical corridors.

The Department of Homeland Security (DHS) in the United States produced a resilience index for mass-transit stations (Security, 2011). Their screen process involved sending inspection teams to transit stations to examine the existing elements and provide a rating for resilience from such disruptions as natural disasters or terrorist attacks. The screening process and subsequent resilience rating is based on historical values and ‘best-practice’ principles employed by the DHS. The index considers the risk of a disruption, the severity of the incident, and the quality of the present infrastructure. Infrastructure is examined for a wide range of properties, such as function/criticality in the system and vulnerability.

Jenelius and Mattsson developed a methodology for determining a road networks vulnerability (Jenelius & Mattsson, 2012). Working out of the Royal Institute of Technology in Stockholm, their goal was to develop a framework that could facilitate decision-making in terms of investment and operations of the road network in Sweden. The method involved determining the Importance (k) of links in a network. The Importance of a particular link is a function of the increase in weighted travel time that occurs when that link is cut, i.e. a disruption scenario has occurred. The initial model was used to determine and rank the most important road-network links in northern Sweden. Jenelius et al. further applied their framework in 2005 to a more detailed analysis of the road network in Sweden. They examined two scenarios and the associated impact on total travel time, an ‘equal opportunities perspective’ and a ‘social efficiency’ one. The latter involves examining the Importance of links without respect to the weighted demand, and the former applies the Importance metric as developed in the previous paper.

The work of Jenelius in the field of vulnerability analysis continued in conjunction with Cats (Cats & Jenelius, 2012). In this work, not only was network topology considered, but so too was
the concept of supply and demand in terms of public transit service availability. They noted that unlike a road network, a service disruption on a public transit network is dependent on how demand reacts to the reduction in supply. A dynamic transit assignment model was applied. The model considers passengers as adaptive decision makers and allows for the provision of real-time information and knowledge of timetables and local conditions. In the field of graph theory, the authors applied the concept of node betweenness centrality for the determination of resilience. However, Jenelius et al (Jenelius, Petersen, & Mattsson, 2005) noted that the traditional measure limitations. With that in mind, the authors built upon the traditional betweenness measures from both an operators and passenger’s perspective. Their new metric considered the dynamic nature of the transportation system in terms of supply and demand, the probabilistic path choice of commuters and variability in supply and demand with respect to days of the week. Using their updated betweenness centrality measure, the authors used a few measurements as proxies for the vulnerability of the transportation network. With respect to the transit service operator, the difference between the operating cost post and pre-disruption was the measure of vulnerability, whereas for passengers, the difference in total travel time for the same period was used as the measure. The unique mixture of graph theory and demand modelling (multinomial logit models were used for transit supply/demand availability as well as passenger route choice) was applied to the Stockholm transit network. Various forms of real-time information provision was studied and the resilience of the network was considered. The findings of the case study were of note for being novel. Prior studies in the field of network resilience characteristically found that the most connected nodes/links were the most critical in terms of resilience. This study found that this was not necessarily the case in the presence of real-time transit information provision. In fact, link/node importance was directly related to the amount of information provided to passengers. The take-away message for this piece of research is that it would be beneficial to study resilience in a dynamic setting.

In the context of freight operations, Miller-Hooks et al. used Monte Carlo simulation to measure resilience in a disrupted network (Miller-Hooks, Zhang, & Faturechi, 2012). The research looked into determining the optimal set of preparedness scenarios and recovery plans given a fixed budget and desired level of service.
King, 2015

The work of Chang and Nojima examined the transportation system performance after the Kobe Earthquake in Japan in 1995 (Chang & Nohima, 2001). Following in the footsteps of Miller-Hooks, Chang and Nojima examined the resilience of a network post-disruption, as opposed to analyzing during a disruption. The latter analysis technique would seem to be the most common in the aforementioned literature presented here. Chang and Nojima examined resilience in terms of network coverage and link accessibility.

Table 2.1 contains a summary of the resilience indices outlined this section.

**Table 2.1 Summary of Resilience Indices**

<table>
<thead>
<tr>
<th>Source</th>
<th>Metric</th>
<th>Index Approach and Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li Zhang et al.</td>
<td>Measure of Resilience (MOR)</td>
<td>Intermodal transportation system, with focus on freight movement</td>
</tr>
<tr>
<td>Nagurney et al.</td>
<td>Relative Total Cost Index</td>
<td>Applied to the road network to contrast assignment best-practices</td>
</tr>
<tr>
<td>Scott et al.</td>
<td>Network Robustness Index (NRI)</td>
<td>Applied to the road network, examined the relationship between volume, capacity and link criticality</td>
</tr>
<tr>
<td>Adams et al.</td>
<td>Risk Priority Number (RPN)</td>
<td>Operational resilience of road corridors, examined the cost of taking alternative routes during a disruption</td>
</tr>
<tr>
<td>Department of Homeland Security (DHS)</td>
<td>Resilience Index for Transit Stations</td>
<td>Screening process examining the elements of transit stations that contribute to resilience given natural disasters</td>
</tr>
<tr>
<td>Jenelius et al.</td>
<td>Link Importance and Exposure</td>
<td>Applied to the road network to determine the critical links by examining the cost of link disruption</td>
</tr>
</tbody>
</table>
2.3 Complex Network Theory Approaches

There is a growing body of work on transportation resilience which uses the concepts of Graph theory. As transportation systems take the form of networks, they lend themselves readily to the study of complex networks. There has been a vast amount of research in complex networks in a number of fields including sociology, physics and geography.

To begin the process of understanding how transportation resilience can be studied using the principles of Graph theory, it would be beneficial to examine the topological measures used in research to date. Lin and Ban of the Royal Institute of Technology produced an extensive list of topological measures and their application with respect to transportation systems (Lin & Ban, 2013). A number of these basic measures, such as node centrality and degree, will be applied in this work.

Derrible and Kennedy (Derrible & Kennedy, 2011) further produced a study outlining the applications of Graph theory to transit network design. The research serves as an inventory of the work done to-date using Network science and Graph theory. Similar to the work by Lin et al., a number of metrics were presented and will be used in this research.

De-Los-Santos et al. applied Graph theory to evaluate passenger robustness on a public transit network (De-Los-Santos, Laporte, Mesa, & Perea, 2012). They define robustness as the total travel time that all agents experience over the networks. They studied two scenarios for

<table>
<thead>
<tr>
<th>Miller-Hooks et al.</th>
<th>Resilience Index</th>
<th>Applied to freight operations, tested an optimal set of post-disaster scenarios and recovery mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chang et al.</td>
<td>Transportation System Performance Index</td>
<td>Applied to freight operations, examined link accessibility and network coverage</td>
</tr>
</tbody>
</table>
disruptions, one in which passengers would either wait for the disruption to clear or take another route, the other scenario involves the public transit agency supplying an alternative mode over the affected link. The models and indices they developed were tested on the Madrid commuter network. The results showed that specific improvements made to the network resulted in robustness increases. Furthermore, they were able to optimize upgrade schemes in order to maximize robustness.

Ip and Wang have conducted several research efforts regarding transportation network resilience using Graph theory (Ip & Wang, 2009; Ip & Wang, 2011). In their earlier work, they represented the transportation network as an undirected graph-as has been done in many studies before-with cities being represented by nodes and roadways represented by links. Their metric for resilience for a particular node was based on the number of independent paths from the node in question to all other nodes. The independent paths were further weighted for demand and nodes were weighted for population in the local area. A key tenet of resilience for the network was the reliability of the road segments. Thus, the resilience of a node is a function of the reliable paths which connect all other nodes. They then extended this metric to determine the network resilience by summing the weighted resilience of each node. In a subsequent study, the researchers denoted that resilience is a function of redundant resources, de-localized supplies and reliable delivery methods. Again, they represented the transportation network as an undirected graph, with cities as nodes and roads as links. They further applied the concept of a weighted resilience metric based on independent paths through nodes and computationally applied the metric to a hypothetical network with characteristic Chinese attributes (i.e. city size, population and density). Ip et al. proposed the concept of friability, which is the reduction in resilience that occurs when a particular link is removed from the network. Using friability, critical links could be determined, and this was then applied to the Chinese railway system, with a further case study examining the impact of an Earthquake in the Schuan basin.

Leu et al. used Melbourne as a case study for the network analysis of resilience (Leu, Abbass, & Curtis, 2010). Their work exclusively focused on the ground transportation system within Melbourne. The assessment framework for resilience applied was one based on three layers: the physical layer, the service layer and the cognitive layer. The research focused on measuring the
physical layer of resilience by representing the transportation network as an undirected graph. Centrality measures for nodes were calculated such as degree, betweenness and clustering. They noted that nodes with a higher betweenness than average may act as bottleneck nodes and would represent a high structural value within a network. Leu et al. further analyzed the network in Melbourne by removing nodes and determining topological integrity and the distance gap. Topological integrity is a measure of how disconnected a graph becomes after the removal of a node. In effect, the number of non-overlapping sub-graphs is counted. The probability that the removal of node k, resulted in the formation of n sub-graphs was calculated and a corresponding probability density function was generated. The analysis allowed for the determination of critical nodes within the Melbourne transportation network and the associated spatial damage (increased travel distance required, called distance gap) when a node fails. Their work highlighted a number of glaring weaknesses in the Melbourne network, such as lack of redundancy and low alternative routes.

Outside of transportation engineering, there has been far reaching applications of network science to study the resilience of various network types, including: power grids (Albert, Albert, & Nakarado, 2004; Crucitti, Latora, & Marchiori, 2004), the internet (Palmer, Siganos, Faloutsos, & Gibbons, 2001; Magoni, 2003), supply chain management and communications (Zhao, Kumar, Harrison, & Yen, 2011; Moreno, Pastor-Satorras, Vazquez, & Vespignani, 2003). A common feature amongst all of these pieces of research is the analysis of nodes and links in terms of node properties. Osei-Asamoah et al. built upon this body of work concerning randomized and targeted node failures within a complex graph and applied it to a transportation network. Osei-Asamoah applied two performance metrics; global efficiency and relative size of the giant component, both of which are Graph theory techniques. Disruptions were examined in three networks; the Connecticut Highway network, the Indiana Rail Road Network and a biological network created via a slime mold. The research found that the higher the redundancy in the network, the higher the resilience. Furthermore, there is a strong positive correlation between resilience and the average degree for nodes and the clustering of nodes.

Park and Kang acknowledged that traditional topological metrics contained the implicit assumption that all links in a network were of equal importance (Park & Kang, 2011). They
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proposed a method of determining the connectivity of nodes within a multimodal transit network. Connectivity was chosen as a metric as the authors felt that it was a good proxy for the efficiency and resilience of the transit network. Their metric involved determining the ‘inbound connecting power’ and the ‘outbound connecting power’, both of which were a function of the capacity of the transit lines serving the node, the speed of the lines and the length of the line. The connecting power of a node is then the average of the inbound and outbound connecting power. The work is further extended to consider transfer stations, at which passengers may exhibit mode-switching behaviour. Considering that walking distance is a factor in mode switching, Park and Kang considered that walking distance will factor in to the connectivity index of the station. Thus, at a multimodal transfer station, the connectivity index was the weighted average of connecting power given walking distance acceptance rates. The research shed light on the important point that in a transit network, generic Graph theory measures may prove to be too simple and fail to capture the intricate nature of transit passenger behavior.

Mishra et al. (Mishra, Welch, Torrens, Zhu, & Knapp, 2014) built upon the work of Park and Kang by adding in the concepts of (1) connective power of a line, (2) connecting power of a transfer center and (3) further adding the level of development in the surrounding area and the number of transit lines to the connecting power at the node level. The authors developed a GIS tool to apply the connectivity index to the Baltimore-Washington region.

2.4 Conclusion

Resilience has been defined and applied in a number of fields including transportation. In addition, there has been some work done in the quantification of resilience. Only a select few of these indices present clear, concise and well-defined metrics for determining the resilience of a transportation network. Of note in this literature review, is the lack of research that involved the analysis of public transportation networks. The research involving public transportation networks typically examined natural disasters and one-time events. In the face of increased demand and limited infrastructure improvements, it is imperative that a metric for operational resilience is developed. This research proposes to develop such a metric and apply it to one of the busiest transit systems in North America, that of Toronto, Canada. Toronto presents a unique case for
analysis as the system is both multimodal (light-rail vehicles, bus, subway) and is served by several transit agencies (both local and regional).

The research conducted in the field of resilience thus far has largely avoided complex simulation and comprehensive modelling. In many cases, the resilience indices presented are sound with a strong underlying theoretical basis, but their application is relatively poor. This work seeks to overcome past shortcomings by applying a comprehensive network model and transit assignment procedure.

As evident from this literature review, resilience has largely been studied in the context of Graph theory. The science of complex networks is not entirely appropriate for a public transportation network. Transit systems are home to a number of different modes which vastly different properties (e.g. speed, headway, etc.). In addition, the behaviour of commuters on a public transit network is influenced in no small part by the characteristics of the transit lines they utilize. Quantifying the resilience of a transit network is therefore a more involved problem than that of the road network. This work seeks to overcome both the shortcomings of Graph theory and the idiosyncrasies of a transit network and present a clear, concise and readily calculable metric of network resilience.
3 Methodology

The following will describe the methodological approach taken in this work. To be discussed are the theoretical concepts and explanations of formulations with regards to the analysis of the resilience of the Toronto public transit network. The methodology described here will roughly be broken down into two parts; the first part describes the strict network science approach to resilience, whereas the second portion will describe the EMME simulation undertaken and the concepts of node Importance and Exposure.

As has been shown in the literature review, structural metrics of a graph serve as a strong basis for analyzing the resilience of a complex network. Therefore the application of graph theory will be undertaken on a graphical representation of the Toronto transit network. Just as the literature review presented before showed the value of a network science approach, it also highlighted the limitations of such a course of action. To overcome such limitations, the underlying theories which governed the graphical approach will be appropriately modified and applied using simulation. The benefit of which is that theoretically-sound metrics are applied in a realistic environment in which the idiosyncrasies of a transit network (i.e. the differences in transit line properties) and human behaviour when travelling are captured. The outcome of which, is a framework to calculate the resilience of a public transit network to operational disruptions.

There currently exists little analysis and research in terms of the altered behaviour and travel patterns of trip-makers during disrupted network conditions. A sentiment analysis of social media will be conducted to shed light on the relatively unexposed travel behaviour during service suspensions. The benefit of such an analysis is two-fold; as outlined before, this analysis helps planners determine just how passengers are reacting to service disruptions. Secondly, the sentiment analysis can serve as a validation for the redistributed passenger flows observed from the EMME simulation.
3.1 Graph Theory Approach

The onset of this work, seeks to answer the penultimate question of how to quantify the resilience of the Toronto public transit network. This initial phase involves a more abstracted approach to analyzing resiliency using the science of complex networks. Graph theory allows one to examine the structural resilience of graphs.

The core tenants of this initial portion are based upon the work of Osei-Asamoah et al., as discussed in the literature review portion of this paper. Additionally, the concept of a topological integrity measure is borrowed from the work of George Leu et al. and their study of the resiliency of the Melbourne ground transportation network (Leu, Abbass, & Curtis, 2010).

To begin, we shall study the impact of node disruptions (i.e. removal) on the performance of network. Node disruptions occurred under two unique strategies; a targeted stratagem and a random stratagem. In both cases, after a node is removed from the network two performance measures are calculated.

The following are the key assumptions employed for the analysis conducted in the graph theory approach. The first idea to be considered is the definition of a disruption. With regards to this network analysis, a disruption is an all-or-nothing event. In reality, a disruption may not completely remove a node from a network. The next major assumption is in regards to the targeted node removal strategy. It is assumed, as based on the Osei-Asamoah work, that the most critical nodes are removed first.

3.1.1 NetworkX Background

Used for the strictly network science based portion of this work, NetworkX is a Python language package that facilitates the generation, manipulation and analysis of complex networks (https://networkx.github.io/). NetworkX has a number of features which facilitate the analysis of graphs and networks:

- Supports a wide variety of graph types including directed and undirected and multigraphs (nodes are allowed multiple edges).
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- Ability to find subgraphs, paths and circuits.

- Ability to determine degree, shortest paths, nearest neighbor connectivity, betweenness.

- Ability to visualize and construct graphs and networks in both 2D and 3D.

NetworkX was also selected for its inherent suitability with regards to this project. The software has been used on complex real-world networks with great success. NetworkX is readily capable of handling networks of 10 million nodes and 100 million edges. As a result of being built upon Python libraries the software runs efficiently and lends itself well to large-scale calculations. The following references show the application NetworkX real-world networks: (Hagberg & Conway, 2010; Hagberg, Schult, & Swart, 2008).

It should be mentioned that other graph software packages do exist. Furthermore, they too are free and in some cases provide superior visualization, thus allowing for attractive renderings of the network in question (e.g. Gephi). NetworkX was selected for a number of reasons. The aforementioned suitability to large real-world networks combined with the fact that EMME/4 modeller is coded and operates using the Python language made selecting NetworkX the obvious choice.

3.1.2 Network Graph Creation

A graph, \((G = (N, E))\), where \(N\) represents the nodes and \(E\) represents the edges) was created for the TTC surface and metro system, with nodes \((N)\) consisting of surface transit stops and metro stations. Links \((E)\) represent the connecting roads or underground tunnels which join the stops and subway stations respectively. The graph representing the TTC network was generated in NetworkX via Python scripting. The graph generated is one of every TTC surface stop and subway station, connected by links which are served by TTC services, such as bus or streetcar.

Data for the NetworkX graph was derived from the EMME network model for the Toronto and GTHA. EMME network elements (e.g. transit segment length, speed, etc) are hardcoded in the Python language. As NetworkX is a python language plugin, code was generated in order to
export the EMME Toronto network into NetworkX for analysis. The network graph, G, is spatially equivalent to the real world TTC surface and metro network, as link properties such as distance were preserved. Spatial accuracy was necessary to facilitate the calculation of shortest paths. A noteworthy artifact of the EMME network model is that intersections are represented by one node. In reality, an intersection is often home to as many four transit stops, one for each cardinal direction, for computational ease, the EMME network represents these four stops in one node, but served by all the appropriate transit modes and lines. The generated NetworkX graph has also adopted this simplifying assumption.

The graph generated in NetworkX is classified as a multi-directional graph. This classification allowed for nodes to have multiple edges incident on them, a property required given the aggregation of multiple stops to one node. Denoting edges with directionality allows for the preservation of the realistic behaviour of transit services.

3.1.3 Analysis via NetworkX

Simple metrics that capture network structure and form lend themselves readily for the comparison of one network to another. From the onset, generic comments can be made about network resilience from parameters such as: order, size, average degree, and density. NetworkX can be used to readily calculate such parameters as was done in this work.

Average degree is a measure of the local connectivity in the network. It can be found by summing the total number of connections each node has to other nodes and dividing by the total number of nodes in the network. A higher degree indicates a node is more connected to other nodes.

Density is calculated as the ratio of actual connections between nodes to the potential number of connections between nodes. The actual number of connections is self-explanatory, as it is the number of connections that exist in the network in its base state. Potential connections is a more abstract concept and represents the idea that, in theory, all nodes could be connected to each other. The higher the density in a network, the stronger the network is from a resiliency standpoint.
The betweenness centrality (BC) of a node is a built-in feature of NetworkX. Global efficiency (GE) was unfortunately not found in the NetworkX library, as a result a script to calculate GE was written for this paper. The script was validated on a test-dummy network to ensure accuracy and reduce computational time.

3.1.4 Node Disruption Strategies

As the previous section outlined, two node removal strategies were employed: targeted and random. The targeted removal is based on the rank-ordered BC value of the nodes. That is, the nodes with a higher BC value are removed first until all nodes are removed from the network.

With regards to the random removal, nodes are removed in bunches at random until all nodes have been removed. Random removals are simulated 15 times with a rotating random seed. For both strategies, the global efficiency of the network is calculated.

3.1.5 Graph Theory Resilience Metrics

Two structural metrics were calculated for the given node disruption strategies. These metrics serve as a proxy for the resilience of the public transit network. The metrics calculated are: The betweenness centrality and the global efficiency, previously abbreviated BC and GE respectively.

3.1.6 Betweenness Centrality

The BC of a node or link is a measure of how central that element is in the network. The concept was first proposed by Linton Freeman (Freeman, 1977) and has found applications in ranking node and link importance in networks. BC is a global measure of node/link load and importance and thus is preferred over a more local measure such as connectivity.
The betweenness centrality of a node, \( v \), is defined by the following equation:

\[
BC (v) = \sum_{s \neq v \neq t} \frac{d_{st}(v)}{d_{st}} \tag{1}
\]

Where:

- \( d_{st} \) is the number of shortest paths from node \( s \) to node \( t \).
- \( d_{st}(v) \) is the number of shortest paths from node \( s \) to node \( t \) which travel through node \( v \).
- Normalized values of BC range from a minimum of 0 to a maximum of 1.

### 3.1.7 Global Efficiency

The GE of a network was proposed by Latora and Marchiori (Latora & Marchiori, 2001). GE is a measure of the exchange of information within a network. Osei-Asamoah et al. proposed that GE quantifies how flow is exchanged between nodes in a transportation network.

The global efficiency of a network is defined as:

\[
GE = \frac{1}{N(N - 1)} \sum_{s \neq t} \frac{1}{Z_{st}} \tag{2}
\]

Where:

- \( Z_{st} \) is the length of the shortest path between node \( s \) and node \( t \).
- \( N \) is the number of nodes in the network.
- The GE value obtained is normalized (min 0, max 1) by dividing by the GE of an ideal network where all node pairs are connected.
3.1.8 Topological Integrity

The concept of topological integrity, as adopted from Leu et al. involves determining the number of non-overlapping sub-graphs. Determination of the number of sub-graphs (and the set of nodes within) is easily done via NetworkX. Prior to a disruption, the graph studied here is fully connected, in the sense that one can get from any node in the network to any other node. After the removal of a node (i.e. a disruption) the graph may be split into several disconnected pieces.

A topological analysis was conducted to determine the probability that a random node removal would result in the network being sub-divided into k pieces. An associated frequency distribution was generated.

As Leu et al. noted, real world networks exist in physical space and as a result the failure of a node or nodes has spatial (and therefore temporal) implications for travellers. This concept will be explained by way of an example. Imagine in real space, that three nodes, X Y and Z, represent a network. In that network, node X is connected to Y and Y is connected to Z such that one could travel from X to Z by way of Y. In the realm of network science, given a failure of node Y, node X and Z are no longer connected. In reality, there now exists an additional cost to travel which is the additional distance needed to travel from X to Z by way of another route.

In the event of a disruption at the node level, it would be valuable to calculate how the average shortest path length is impacted. It is expected that the removal of some nodes will have minimal or no impact on the average shortest path length, as those nodes may not lie on any shortest paths (i.e. fringe nodes on the outskirts of the network). Contrast this with the fact that the removal of some nodes results in the network becoming disconnected. In the case of sub-graph disconnection, the additional length required for the average shortest paths approaches an infinite length. The output of this portion of the topological analysis is a frequency distribution of the expected increase in shortest path for trip-makers.

3.1.9 Limitations

A graph analysis can be considered an attempt at explaining and quantifying network resiliency from a network science approach. Real world networks, such as the public transit systems, are
extremely complex and contain a number of interdependencies and idiosyncrasies that a graph analysis cannot fully cover. With that in mind, to comprehensively quantify resilience for a transit network, additional work must be conducted.

Although the graph constructed geographically mapped the TTC transit network and is further weighted with respect to physical link length, it fails to capture the properties of a transit network which define it from other transportation networks. Such properties include but are not limited to: volume, link/node utilization, mode type which traverses the link or serves the node, presence of transfers, and wait time.

To define, quantify and ultimately apply a metric of resilience that is useable for both researchers and practitioners, one must include the aforementioned transit network properties and build upon existing network science metrics (such as GE or BC). The second phase of this research seeks to complete this task.

3.2 EMME Network Analysis and Node Importance

EMME (“Equilibre Multimodal, Multimodal Equilibrium.”) is a transportation modelling software package that was developed in the 1970’s at the Centre for Research on Transportation (CRT) at the University of Montreal. For over 40 years EMME has been developed into a comprehensive package for simulating transportation. Currently, it is used widely across Canada, and in particular it is the software of choice for the regions in the GTHA (including Toronto, the region of study). EMME adopts a static, zone-to-zone problem definition. It follows that there is a limited concept of capacity in EMME. A simplistic congested assignment algorithm does exist within EMME. This model involves adjusting the comfort level on transit lines, in addition to the wait time at stops as congestion increases. This congested assignment involves using the Method of Successive Averages (MSA) and as a result, transit volumes on different lines may not be unique. The weaknesses outlined above highlight a need for a more comprehensive transit assignment procedure.

In an effort to more fully and accurately quantify resilience, EMME was used in conjunction with modified principles of network science. Use of the EMME network allows for one to
overcome the inherent weaknesses of the strictly graph theory based approach. In particular, it allows for the analysis of realistic trip-making behaviour, given a service disruption. Furthermore, having an accurate and detailed transit network model enables a more realistic simulation of particular types of service disruptions, which in this work are confined to subway service suspensions.

3.2.1 Transit Assignment in EMME

Generically, transit assignment is the process of finding a path through a network for a passenger. This route choice is then aggregated over all passengers in order to determine the flows (or volumes) on transit lines for a given network. At its core, transit assignment is done to facilitate the prediction of potential ridership and travel time on transit infrastructure. Additionally, the results of a transit assignment can help guide policy such as transit fares, real-time information provision, and departure time choice (Wahba, 2008).

Early research on the transit assignment problem was conducted by Florian and Spiess in 1989. They described a transit trip as consisting of several distinct trip components, each which can have an associated cost:

- Access to a transit stop/station from an origin, via walking or driving, etc.
- Wait time for transit vehicle.
- Boarding (usually a penalty) a transit vehicle.
- Fare.
- In-vehicle travel time.
- Alighting from vehicle.
- Egress from a transit stop/station to a destination, again via many possible modes.
The basic transit assignment in EMME is based on the concept of finding optimal strategies. This is done by attempting to minimize a passenger’s total travel time between their origin and destination. Roughly, a strategy can be described as the path a passenger takes to reach their destination. A path is composed of access, waiting, boarding and time spent in a vehicle. Therefore it follows the optimal strategy is one in which the weighted sum of the aforementioned components is minimized.

The generic assignment found in EMME is not well suited to the heterogeneous nature of the public transit network found in Toronto. As described before, the TTC operates a number of different technologies to move passengers, and often several modes are taken to complete one trip. The transit assignment problem is further complicated by the presence of different regional services which operate within the Toronto area. For example, many trips originate outside of the Toronto-proper area via regional (e.g. GO) bus services or commuter rail, these passengers subsequently transfer to utilize the TTC system to complete their trip. Any transit assignment that is done in the context of Toronto must consider fares in order to properly compare potentially competing paths (for a given OD pair); this is further complicated by the fact that no fare integration system exists between the multitude of regional services and the TTC.

As mentioned before, there has been considerable growth in the population of Southern Ontario (a considerable portion of this growth has taken place in and around the Toronto area), and subsequently increased pressure on regional transit services and the TTC. The unprecedented congestion experienced today in Toronto must be reflected in any transit assignment. Capacity and overcrowding has a strong impact on both route and mode choice.

### 3.2.2 Congested, Fare-Based Transit Assignment (FBTA) Model

Kucirek et al. developed a transit assignment model for EMME which considers the idiosyncrasies of the transit assignment problem in Toronto and the surrounding area (Kucirek, Vaughan, & Miller, 2014). The calibrated model has consideration for congestion (both between vehicles on shared ROW, and passenger-related capacity constraints), and the impact of fares on route selection, among other factors usually considered in transit assignment models.

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A notable approach adopted by Kucirek et al. with regards to transit modes should be discussed further. Traditionally in transit assignment, distinction between transit services, such as commuter rail or heavy rail, has been treated and defined as separate *modes* of travel. However, in this integrated network approach to transit assignment, the authors modelled all services as alternative path options. This choice allowed for congested path choice to be described and simplified the mode choice structure.

FBTA also allowed trip-makers to walk along network links as opposed to generic EMME assignment which required trip-makers to walk only on centroid connectors (links that connect zone centroids to the surface network) or transfer links (links that represent the transfer of direction in subway transfer stations). Allowing trip-makers to walk on network links is necessary to accurately model fare-based assignment. Furthermore, it better models trip-maker behaviour during service disruptions, where walking may be required to access alternative attractive transit lines.

Congestion was implemented via FBTA by way of the Bureau of Public Roads (BPR) congestion function. The BPR formulation has the form of $\alpha(Volume/Capacity)^\beta$, where $\alpha$ is the perception factor for congestion and $\beta$ is an exponent parameter. The BPR represents volume-delay impedance, this congestion-cost value is added to the generalized cost function found in EMME. As a BPR function becomes very large as the volume reaches or exceeds capacity, it represents a good fit for modelling transit congestion (in which vehicles have a finite capacity for passengers). The value obtained from the BPR function is converted into a weighted time, and is then added to the generalized cost function. Trip-makers then are assigned to transit lines in the network with a consideration for capacity limitations inherent to transit services.

Readers interested in parameter estimation and definition should consult Kucirek et al.

### 3.2.3 EMME Network Model for Toronto

The network model used in this work produced by the Travel Modelling Group (TMG) at the University of Toronto, representing the entire GTHA in the year 2012. The model is further
subdivided into time-period networks: AM, Midday, PM, Evening and Overnight. Each time period except for overnight constituting approximately three hours. Furthermore, each time period network is unique in the sense that it contains all the transit services and lines which operate at that time of the day, and their appropriate frequencies, scheduled departures, etc.

Surface routes, such as bus and streetcar, used an average line speed based on the length of route and average scheduled run time. The speed is assumed to be uniform along the line. Speeds on GO train and subway routes were applied individually to each segment based on the segment length and stop-to-stop travel times. GO bus routes used segment speeds based on the average line speed (as was done for surface routes) these average speeds were then further pro-rated based on link speed limits. This was done because GO buses travel on highways and a uniform speed would not be realistic.

3.2.4 Transit Demand Matrix

To model the impact of disruptions on the TTC network, the typical transit demand was required. This demand matrix (as EMME is zone to zone-based assignment) was generated from the 2011 Transportation Tomorrow Survey (TTS). This survey is conducted every 5 years and is a phone-based survey of approximately 5% of the households located in the GTHA. The TTS data describes the modes of both access and egress to and from transit. Furthermore, TTS data describes the transit routes utilized.

As this paper is an analysis of the resilience of the TTC network, the TTS transit demand matrix was limited to only those trips that begin or end inside the City of Toronto (all origins or destinations with a zone number less than 1000, as per the TTS numbering convention). These are the trips that would be impacted in the event of a service disruption. Persons travelling outside of Toronto, yet still in the GTHA are captured in the TTS demand matrix, but are unlikely to use TTC services, hence the need to curtail the demand matrix.

3.2.5 Concept of Importance

As outlined in the literature review, Jenelius et al. define the Importance of node $k$ as the aggregated travel cost increase, $c_{ij}^k - c_{ij}^0$, over all OD pairs which occurs when $k$ is disrupted.
Travel demand, $x_{ij}$, is used as a weight to account for the volume and severity of a disruption being proportional to the volume which passes through a given node.

$$\text{Importance} (k) = \frac{\sum_i \sum_{j \neq i} x_{ij} (c_{ij}^k - c_{ij}^0)}{\sum_i \sum_{j \neq i} x_{ij}}$$

Where:

- $x_{ij}$ is the demand from origin node $i$ to destination node $j$.
- $c_{ij}^k$ is the cost of travel from node $i$ to node $j$ given node $k$ is disrupted, $c_{ij}^0$ is the base case.

In the case where all weights, $x_{ij}$ are equal, all origins and destinations are equally important. If the transit demand is used as a weight, then the impact of a node disruption is a function of the travel demand between node pairs. Essentially, this is a measure of the ability of the transit network to provide an efficient means of transportation.

The preceding metric for node importance has a foundation in graph theory, but has been built on by considering both demand and the concept of utility, or in this case, disutility (increase in travel time). Using travel time as part of any metric for network resilience has a number of benefits. For starters, it is a readily quantifiable value. Furthermore, it is a concept which is easily understood by planners, operators and system users at all levels.

Jenelius et al. applied the concept of importance to the road network in Sweden using a demand modelling software called SAMPERS which uses a very rudimentary algorithm for calculating travel times (based on Dijkstra's shortest path algorithm). Furthermore, an underlying assumption of their work is that the travel time through the network was independent of volume, hence there was no consideration of congestion or capacity.

This work will closely adopt the concept of Importance for a node but with a few critical changes that have resulted in a novel and beneficial approach to calculating the resilience of a transit
network. For starters, the Importance of TTC subway stations will be calculated, as opposed to the importance of roadway links done by Jenelius et al. A more critical difference between their approach and the one adopted here lies within the calculation of travel times. The FBTA applied in this work considers congestion, which has a large impact on route choice and therefore travel times. Furthermore, the FBTA used here has trip-makers considering a wide variety of mode attributes when selecting route choice, as in the case of a public transit network where all transit modes are not viewed the same by a given trip-maker. The travel times used for the calculation of transit station Importance are far from the rudimentary capacity unconstrained shortest-path travel times used by Jenelius et al. This research has calculated the Importance of all 69 stations on the TTC subway system by simulating a service disruption at the station in question, in accordance with the definition of Importance.

3.2.6 Simulation: Theoretical Service Disruptions

Given the static nature of EMME, only a subset of disruptions to the TTC network could be simulated. In contrast to the graph theory approach described before, only TTC subway stations were targeted or disruption in EMME. The choice to target only the subway network was made in light of the computational requirements to run even a single EMME simulation. In NetworkX, any node, be it a subway station or surface line stop, could be targeted for removal with the effects computationally quick to complete. Targeting TTC subway stations exclusively allowed for a more detailed and complete analysis. Furthermore, the subway system represents a critical lynchpin in the Toronto transit network, on which, disruptions are commonplace and the impacts severe. The static properties of EMME require that any disruption simulated be maintained for the duration of the simulation, or in this case, assignment period. As a result, dynamic disruptions, or small-scale operational disruptions such as delayed trains could not be simulated. Therefore, in accordance with the definition of Importance, and given the confines of the simulation program of choice, a service disruption is described as follows: The subway lines which serve the station to be disrupted are turned back at the upstream and downstream stations preceding and following the station respectively. In the case of terminal stations, such as Finch, service simply terminates at the station immediately before it.
Figure 3.1 Service Disruptions in EMME
Figure 3.1 Service Disruptions in EMME

Figure 3.1 shows the TTC subway lines from the EMME model. Shown here is a sample hypothetical disruption at Lawrence station on the YUS line. The service suspension at Lawrence results in trains turning back at York Mills to the north and Eglinton to the south. Surface routes that serve Lawrence station, either as part of their trip itinerary or as a terminus, are unaffected. Trip-makers are still able to board or alight at the impacted station via surface routes. They are, however, unable to board or alight from a subway as no service will pass through the disrupted station.

### 3.2.7 Simulation: Realistic Service Disruptions

The theoretical service disruptions described above allow for the Importance of each station to be calculated in accordance with the strictest definition of the concept. As Importance was originally defined for a road system, the disruption of one link (or section of roadway) made sense. Upon examining of the operating procedures employed by the TTC, the disruption of one station in isolation is not realistic. To apply the concept of Importance to a transit system, it is imperative to simulate realistic disruptions, just as was done in the original Swedish roadway work. Allowing trains to turn back from the disrupted station is, in some cases, not possible and by extension an unrealistic assumption.

In reality, the TTC subway system contains a finite number of crossover tracks. Crossovers allow a train to reverse direction, a key element at terminal stations. The disruptions simulated earlier are, in some cases, impossible. This is because portions of lines lack a crossover. Thus, in some cases it would be impossible for just one station to be disrupted. This can be illustrated by way of an example as shown in Figure 3.2:
Crossovers allow trains to turn around at St George and Broadview stations for this portion of the BD line. Therefore, a disruption at Sherbourne would result in a loss of subway service from St George to Broadview. This is a stark contrast to the previous disruption simulation, where service remains in operation up till Bloor-Yonge (from the west) and Castle Frank (from the east), for the same disruption at Sherbourne.

The presence of crossovers has profound impacts on the calculation of node importance. The implication here is that “groups” of stations have the same importance when realistic subway operations are considered. In the aforementioned example, a disruption at Bay would be equivalent to a disruption at Castle Frank, again, because the same portion of the BD line must be shut down, due to track architecture. The location of track crossovers is listed below, separated by line:
Realistic disruption scenarios were determined based on the listed crossover location, and an analysis of TTC ‘best-practices’ during service disruptions. The Importance—denoted as ‘Actual Importance’—of each of the 69 stations was calculated based on realistic operations.
3.2.8 Concept of Exposure

Jenelius et al. developed a metric built upon Importance called Exposure. Exposure is defined by the following equation:

\[
Exposure (m) = \frac{\sum_k \sum_{i \in V} \sum_{j \neq i} x_{ij} (c_{ij}^k - c_{ij}^0)}{L \sum_{i \in V} \sum_{j \neq i} x_{ij}}
\]  

(4)

Where:

- \(x_{ij}\) is the demand from origin node \(i\) to destination node \(j\).
- \(c_{ij}^k\) is the cost of travel from node \(i\) to node \(j\) given node \(k\) is disrupted, \(c_{ij}^0\) is the base case.
- \(L\) is the number total number of possible disruption scenarios.
- \(V\) is the set of origin nodes within the Planning District (PD), \(m\).

The exposure of municipality or region \(m\) is the increase in travel time aggregated over all origin nodes \(i\) in the municipality and all destination nodes \(j\), in the entire network. Thus, given a random station failure somewhere on the TTC subway system, the expected increase in travel cost is given by the formula above. \(L\) is number of possible disruption scenarios that could occur in the network and \(V\) is the subset of origin nodes located within the municipality \(m\).

Given the above formulation, the expected Exposure of the 16 PDs which represent Toronto (Figure 3.3) will be calculated. The disruption scenarios to be considered, \(L\), are those of the realistic service suspensions described in the preceding section.
From equation (4), a worst-case scenario can be determined. This event is service suspension which causes the maximum increase in demand-weighted travel cost for a particular municipality. Knowledge of how particular portions of the City are impacted by various service suspensions represents invaluable information for both quantifying resilience, and for guiding recovery efforts given constrained resources.

3.3 Conclusion of Methodology

The methodological approach described above and adopted in this work is two-fold. Initially the concepts of Graph Theory are to be applied to the Toronto transit network. Structural metrics and indicators of topological integrity will be calculated to determine the resilience of the network to node-level disruptions. In order to have a more complete framework for the operational resilience of a transit network, simulation will be employed. Service suspensions will be modelled in an EMME network of the Toronto transit system. The concepts of Importance and Exposure will be calculated to determine the overall cost of a disruption on trip-makers. Results of both the simulation and Graph theory approach are presented in the following chapter.
4 Results

The following will present and discuss the results of the methodological approach as described in the preceding section. The NetworkX based analysis rooted in the theory of complex networks will be presented initially, followed by the EMME approach which considers the public transit network of Toronto, as it exists in reality.

4.1 Graph Theory Results

Table 4.1 below shows the structural network properties of the TTC metro and surface network. Such values allow for networks to be compared with one another and further serve as the basis for structural network measurements.

Table 4.1 TTC Network - Structural Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>TTC Metro and Surface Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order (Nodes)</td>
<td>4308</td>
</tr>
<tr>
<td>Size (Links)</td>
<td>18634</td>
</tr>
<tr>
<td>Average Degree</td>
<td>8.65</td>
</tr>
<tr>
<td>Global Efficiency</td>
<td>0.26</td>
</tr>
<tr>
<td>Density</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Table 4.1 shows the first set of metrics determined for the graph of the TTC transit system. These simple metrics capture network structure and form and lend themselves readily for comparison between networks. Degree is a measure of the local connectivity in the network. It can be found by summing the number of edges incident to a node. A higher degree indicates a node is more connected to other nodes. The average degree denotes the overall connectivity of nodes in the network. The high value of average degree as shown in Table 4.1 denotes that nodes are quite connected to one another. It should be noted that this likely a product of the
simplification of intersections from four nodes (one for each cardinal direction), to simply one node.

Density is calculated as the ratio of actual connections between nodes to the potential number of connections between nodes. The actual number of connections is the number of connections that exists in the network, as is. Potential connections is a more abstract concept and represents the idea that all nodes could be connected to each other. The higher the density in a network, the stronger the network in terms of maintaining functionality during node disruption. By extension, a dense network would be more resilient than a less dense network.

4.1.1 Betweenness Centrality of Nodes and Links

The behavioural change of the TTC network in response to node disruption was determined based on the two node removal strategies: random removal and targeted removal. The global efficiency denotes how well flow is able to traverse the network compared to that of an ideal network. Benchmarks of 25%, 50% and 75% loss in network performance (when compared with the undisrupted network) are determined.

As the targeted removal strategy is based on the BC value, it was necessary to determine the BC value of every node in the network a priori. Figure 4.1 shows the distribution of the BC values for all nodes and links in the graph:
Figure 4.1 Histograms of Node/Edge Betweenness Centrality
The plot of node BC values reveals that some nodes are in fact not traversed on any shortest paths. The finding seems counter-intuitive but inspection of the following figure readily explains this:

![Theoretical Node Betweenness Centrality Map](image)

**Figure 4.2 Theoretical Node Betweenness Centrality Map. (Adapted from ActiveNetworks.net)**

In the above figure, nodes with the red hue would represent nodes with a BC value of 0 whereas the cyan and blue nodes would have much higher values, as they are highly connected within the graph. As evident from Figure 4.21, very few nodes are central in the TTC transit network. In fact, the vast majority have a low BC value and therefore are not part of very many shortest paths.

Nodes which had a high BC value included but were not limited to: Finch station (bus terminal), Wilson Avenue and Keele Street, Victoria Park Avenue and Lawrence Avenue, Islington station. Eglinton station.

The edge BC value plot serves as a check to ensure network integrity has been maintained. In theory, all links should be traversed at least once on a shortest path. The reasoning follows that in an undisrupted network, one should be able to reach any node from an initial node starting location. Therefore if the plot had revealed any links had a BC value of 0, the network would be found faulty. Given that this did not occur, the network integrity is confirmed. Indirectly,
conducting the BC analysis on edges as serves as a means to validate the shortest-path algorithm applied.

4.1.2 Node Disruption Strategies and Results

The performance of the network under a targeted removal strategy represents a worst case scenario. Based on the aforementioned BC values, the nodes of the network are rank-ordered and removed sequentially. A node with a higher BC value is more central to a network and thus is more critical, hence its removal would have a more drastic impact on network performance than the removal of a node which is less central.

Figure 4.3 shows the network performance response to both targeted and random node removal.

![Figure 4.3 Network Performance Response to Random and Targetted Node Removal](image.png)
Figure 4.3 shows a great deal of information about the network response to node removal. The average GE from the 15 simulations at each node removal level is shown on the plot. In general, both targeted and random node disruptions results in decreased network performance. Under both strategies, the GE drops by approximately 50% after about 15% of the nodes are disrupted for the targeted scenario and after 20% for the random removal regime. The GE reaches a minimum of zero under both regimes but this occurs after only 85% node removal for the targeted case, as opposed to closer to 95% node removal for the random case.

In the case of random disruptions, the loss in GE is quite steep until about 30% of the nodes are removed, after which, the rate of decline in GE is markedly less. It should be noted that under both scenarios, the GE of the network drops considerably after the removal of a relatively small percent of the total nodes. This rapid loss of efficiency in transfer between nodes is testament to the lack of redundancy built into the network, and by extension, the lack of resilience. A more robust network would exhibit a much shallower curve for GE loss when plotted against node removal.

From this plot, a critical threshold for network performance given loss can be determined and applied by planners and operators to improve resilience and conduct analysis for disaster-management. Figure 4.4 shows the probability and cumulative density functions that were generated from the random removal of a node and the subsequent testing of the number of connected components.

It is important to note that the nodes with the highest BC, which when removed cause the largest relative drop in GE, are not in fact the most central nodes from a demand perspective. The BC value of a node is a function of shortest paths, and therefore is a function of the geometric properties of the network. The result is that in a strict graph approach, such as this, the most central nodes may not in fact be the nodes with the highest utilization (i.e. demand, passenger volumes, transfers, etc.).
Figure 4.4 Frequency Distribution for Topological Integrity

The results depicted in Figure 4.4 are surprisingly positive in terms of analyzing the resilience of the network. Based on the frequency distribution, it is apparent that the removal of a random node, is not likely to break the network into two or more disconnected sub-graphs. Figure 4.4 also shows us that the maximum number of disconnected sub-graphs which can occur due to a node failure is 3. It should be noted though, that it is far likely that given a graph disconnection event has occurred, that the number of disconnected components is only two pieces. An example of the formation of a disconnected sub-graph is the 91A branch of the 91 Woodbine bus. This branch travels up O’Connor Drive from Woodbine station and into Parkview Hills via Parkview Hills Crescent, loops through Parkview Hills and returns back onto O’Connor Drive following the same route. Parkview Hills is secluded area of Toronto with only one entrance and exit. A disruption of the node at the entrance/exit to Parkview Hills will result in the formation of a disconnected sub-graph.

This type of analysis can be used as a proxy for the resilience of the network, as it is a good indicator for a severe network breakdown. An event which causes the network to become
severed represents a total failure in the ability of the transit system to operate. Users who desired to travel from an origin in one of the disconnected sub-graphs to a destination in another sub-graph can no longer complete their trip using the transit network. The benefit of such an analysis is that it allows one to observe possible network vulnerabilities, i.e. which nodes in particular, when disrupted, result in the formation of disconnected sub-graphs. The limitation of these charts is that it the analysis considers the physical network infrastructure, and only that. In reality, the formation of disconnected sub-graphs cannot occur at all (in the strictest sense), given that in the event that transit services cannot reach a destination, trip-makers will complete their trips via other modes, such as walking. Hence, the value in realistically simulating trip-maker behaviour (via EMME) is becoming apparent.

Figure 4.5 shows the PDF for the average shortest path length increase under the random node removal regime.

![Figure 4.5 PDF for Average Shortest Path Length Increase](image_url)
As apparent from Figure 4.5, the relative frequency of zero additional meters added indicates there is a high probability that the removal of a node has no impact on the average shortest path length. The additional length added to a shortest path is rounded to the nearest 100 meters for simplicity in reporting. Ignoring the cases where the network becomes disconnected, the maximum additional distance to the shortest path is approximately 1 km. Smaller distances of 200 to 300 meters are more common and can likely be attributed to the grid layout of the Toronto area and the typical TTC stop spacing for surface routes (which compose the majority of nodes in the graph).

The more resilient a network is, the more likely it would be that a node removal would have a limited impact on the average shortest path length, as redundancy (by way of services offered) would be in place. Such redundancy could be found in closely spaced parallel lines.

4.2 Analysis of the EMME Network

Visually, the 2011 AM Transit Demand Matrix is shown below in Figure 4.6. For clarity, the origin and destination demand patterns are presented separately. The vast majority of transit travel in the AM period is destined for PD 1, which roughly comprises downtown Toronto, the central-business district of the GTA.
Figure 4.6 2011 AM Origin and Demand Distribution
As apparent from the figure, most transit demand is destined for zones situated along the subway lines of the TTC. This is an artifact of the densification along transit lines and the urban form of Toronto. Other notable areas of demand in the AM period include York University, shown in the figure above, and midtown Toronto, located roughly at Yonge and Eglinton. Midtown (PD 4) is home to a concentration of office towers and other trip attractors.

The figure shows transit trip origins to have a less extreme spatial distribution. Transit trip origins are spread over a wider area than destination demand. However, a notable amount of trips do begin in PD 1, as PD 1 is home to a high density of high-rise residential buildings. The distributed nature of trip origins raises an interesting issue for expansion of transit services and in the realm of transit network resilience.

The AM period demand (including trips that begin or end within one of the 16 PDs of Toronto between 6:00 and 9:00 AM) totals 465,426 trips. The vast majority of trips in the GTHA begin or end within Toronto proper. A major assumption of this work is that demand is inelastic in nature. Although not strictly true, it is a fair assumption of the AM period, given that most trips at this time are work-based trips.
4.3 Theoretical Importance Values

The Importance of the 69 subway stations is shown below in Table 4.2:

Table 4.2 Station Theoretical Importance Values

| Station Name / Theoretical Importance (Trip Perceived Congested Minutes / Trip) |
|-------------------------------|---------------|----------------|----------------|----------------|---------------|----------------|----------------|----------------|---------------|----------------|
| FINCH                         | 4.11          | KIPLING        | 2.81           | KENNY          | 15.58         | LAWRENCE WEST | 15.67         | ELLESMEY       | 12.42         | MIDDENDORF     | 10.77         |
| NORTH YORK CENTRE             | 8.33          | ISLETON        | 4.76           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| SHEPPARD-YONGE                | 42.55         | ROYAL YORK     | 8.79           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| YORK MILLS                    | 49.13         | OLD MILL       | 12.3           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| LAWRENCE                      | 22.51         | JANE           | 3.28           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| EGLINTON                      | 166.66        | RUNNYMEDE      | 9.41           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| DAVISVILLE                    | 30.15         | HIGH PARK      | 7.29           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ST CLAIR                      | 33.57         | KEELE          | 8.05           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| SUMMERHILL                    | 13.28         | DUNDAS WEST    | 8.75           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ROSEDALE                      | 14.83         | LANSDOWNE      | 8.33           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| YONGE-BLOOR                   | 12.1          | DUFFERIN       | 10.43          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| WELLESLEY                     | 3.54          | OSSINGTON      | 10             | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| COLLEGE                       | 2.67          | CHRISTIE       | 13.25          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| DUNDAS                        | 2.08          | BATHURST       | 13.12          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| QUEEN                         | 1.9           | SPADINA        | 7.87           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| KING                          | 2.05          | ST GEORGE      | 3.73           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| UNION                         | 2.55          | ST GEORGE      | 3.73           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ST ANDREW                     | 1.41          | ST ANDREW      | 12.16          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| OSGOODE                       | 2.39          | SHHERBOURNE    | 25.75          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ST PATRICK                    | 2.57          | CASTLE FRANK   | 15.63          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| QUEENS PARK                   | 3.65          | BROADVIEW      | 15.09          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| MUSEUM                        | 3.85          | CHESTER        | 12.42          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ST GEORGE                     | 4.46          | PAPE           | 8.92           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| SPADINA                       | 5.11          | DONLANDS       | 3.28           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| DUPONT                        | 6.44          | GREENWOOD      | 10.68          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| ST CLAIR WEST                 | 11.31         | COXWELL        | 13.59          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| EGLINTON WEST                 | 8.63          | WOODBINE       | 12.26          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| GLENCAIRN                     | 8.4           | MAIN STREET    | 23.95          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| LAWRENCE WEST                 | 8.03          | VICTORIA PARK  | 27.1           | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| YORKDALE                      | 5.78          | WARDEN         | 13.56          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| WILSON                        | 5.03          | KENNEDY        | 14.63          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |
| DOWNSVIEW                     | 3.89          | JANE           | 3.28          | JANE           | 3.28          | SCARBOROUGH    | 3.3            | TOWN CENTRE    | 5.63          | MCCOWAN        | 5.63          |

Graphically, Theoretical Importance can be mapped as shown below:
Figure 4.7 Theoretical Node Importance Map
The results of the application of the strict definition of Importance are surprising. The station which when removed, has the largest impact on commute times is Lawrence station, on the YUS line. A pattern is observed showing that the northern portion of the YUS line is critical to network integrity. Unexpectedly, none of the interchange stations ranked very high in terms of Importance. Intuitively, one would expect given passenger throughput and their centrality with respect to network topology, the interchange stations (i.e. Yonge-Bloor and St George) would rank as quite critical, the results of this indicated that this is not the case.

Given the demand both within, and to PD 1, it was expected that the lower half of the YUS line, colloquially referred to as the ‘U’ would have high Importance values. In fact, the stations along the ‘U’ represent some of the least important stations in the network.

Possible explanations for these results include: efficient trip-maker re-routing, the abstracted nature of removing only one station (ignoring track architecture) and the presence of network robustness. These explanations will be expanded upon in the following section, when the realistic service disruptions are modelled and the Actual Importance values are determined.

### 4.4 Actual Importance Values

As discussed earlier, the presence of track crossovers create a series of possible disruption scenarios, which exhaustively allow for the determination of the Importance (denoted in this regard, as Actual Importance) of all 69 subway stations. The disruption scenarios are shown in Table 4.3, organized by line:
Table 4.3 Service Disruption Scenarios

<table>
<thead>
<tr>
<th>TTC Subway Line</th>
<th>Portion of Track Disrupted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yonge-University Spadina Line (YUS)</td>
<td>Union to Yonge-Bloor  Downsvieview to Lawrence West</td>
</tr>
<tr>
<td></td>
<td>St George to Yonge-Bloor  Finch to Sheppard-Yonge</td>
</tr>
<tr>
<td></td>
<td>Lawrence West to St Clair West  Eglinton to Union</td>
</tr>
<tr>
<td></td>
<td>York Mills to Eglinton  Eglinton to Yonge-Bloor</td>
</tr>
<tr>
<td></td>
<td>Lawrence West to St George  York Mills to Yonge-Bloor</td>
</tr>
<tr>
<td></td>
<td>St Clair West to Union  St Clair West to St George</td>
</tr>
<tr>
<td></td>
<td>Sheppard-Yonge to Eglinton  Downsvieview to St Clair West</td>
</tr>
<tr>
<td>Bloor-Danforth Line (BD)</td>
<td>Keele to Ossington  St George to Broadview</td>
</tr>
<tr>
<td></td>
<td>Kipling to Keele  Broadview to Warden</td>
</tr>
<tr>
<td></td>
<td>Woodbine to Kennedy  Jane to St George</td>
</tr>
<tr>
<td></td>
<td>Chester to Woodbine  St George to Union</td>
</tr>
<tr>
<td></td>
<td>Kipling to Jane  Christie to St George</td>
</tr>
<tr>
<td></td>
<td>Keele to Christie  Jane to Ossington</td>
</tr>
<tr>
<td></td>
<td>St George to Chester</td>
</tr>
<tr>
<td>Sheppard Line</td>
<td>Bayview to Sheppard-Yonge  Sheppard-Yonge (Non-YUS portion)</td>
</tr>
<tr>
<td></td>
<td>Complete Shutdown of the Sheppard Line</td>
</tr>
<tr>
<td>Scarborough Rapid Transit Line (SRT)</td>
<td>Complete Shutdown of the SRT</td>
</tr>
<tr>
<td>Terminal Stations</td>
<td>Finch  Downsview</td>
</tr>
<tr>
<td></td>
<td>Kennedy  Kipling</td>
</tr>
</tbody>
</table>
As evident from Table 4.3, a disruption can be so severe it results in the suspension of service for an entire line. Mathematically, the presence of crossovers on the track result in groups of stations having the same Importance. Table 4.4 below shows the Actual Importance values:

**Table 4.4 Station Actual Importance Values**

<table>
<thead>
<tr>
<th>Station Name</th>
<th>Actual Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>FINCH</td>
<td>4.11</td>
</tr>
<tr>
<td>NORTH YORK CENTRE</td>
<td>8.33</td>
</tr>
<tr>
<td>SHEPPARD-YONGE</td>
<td>43.13</td>
</tr>
<tr>
<td>YORK MILLS</td>
<td>72.44</td>
</tr>
<tr>
<td>LAWRENCE</td>
<td>221.51</td>
</tr>
<tr>
<td>EGLINTON</td>
<td>131.21</td>
</tr>
<tr>
<td>DAVISVILLE</td>
<td>101.22</td>
</tr>
<tr>
<td>ST CLAIR</td>
<td>101.22</td>
</tr>
<tr>
<td>SUMMERHILL</td>
<td>101.22</td>
</tr>
<tr>
<td>ROSEDALE</td>
<td>101.22</td>
</tr>
<tr>
<td>YONGE-BLOOR</td>
<td>36.36</td>
</tr>
<tr>
<td>WELLESLEY</td>
<td>65.11</td>
</tr>
<tr>
<td>COLLEGE</td>
<td>65.11</td>
</tr>
<tr>
<td>DUNDAS</td>
<td>65.11</td>
</tr>
<tr>
<td>QUEEN</td>
<td>65.11</td>
</tr>
<tr>
<td>KING</td>
<td>65.11</td>
</tr>
<tr>
<td>UNION</td>
<td>24.03</td>
</tr>
<tr>
<td>ST ANDREW</td>
<td>14.17</td>
</tr>
<tr>
<td>OSGOODE</td>
<td>14.17</td>
</tr>
<tr>
<td>ST PATRICK</td>
<td>14.17</td>
</tr>
<tr>
<td>QUEENS PARK</td>
<td>14.17</td>
</tr>
<tr>
<td>MUSEUM</td>
<td>14.17</td>
</tr>
<tr>
<td>ST GEORGE</td>
<td>32.02</td>
</tr>
<tr>
<td>SPADINA</td>
<td>6.5</td>
</tr>
<tr>
<td>DUPONT</td>
<td>6.5</td>
</tr>
<tr>
<td>ST CLAIR WEST</td>
<td>10.53</td>
</tr>
<tr>
<td>EGLINTON WEST</td>
<td>3.39</td>
</tr>
<tr>
<td>GLENCAIRN</td>
<td>3.39</td>
</tr>
<tr>
<td>LAWRENCE WEST</td>
<td>8.78</td>
</tr>
<tr>
<td>YORKDALE</td>
<td>5.06</td>
</tr>
<tr>
<td>WILSON</td>
<td>5.95</td>
</tr>
<tr>
<td>DOWNSVIEW</td>
<td>3.83</td>
</tr>
<tr>
<td>KIPLING</td>
<td>2.81</td>
</tr>
<tr>
<td>ISLINGTON</td>
<td>16.83</td>
</tr>
<tr>
<td>ROYAL YORK</td>
<td>16.83</td>
</tr>
<tr>
<td>OLD MILL</td>
<td>16.83</td>
</tr>
<tr>
<td>JANE</td>
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<td>RUNNYMEDE</td>
<td>14.57</td>
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<td>HIGH PARK</td>
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<tr>
<td>KEELE</td>
<td>33.01</td>
</tr>
<tr>
<td>DUNDAS WEST</td>
<td>17.55</td>
</tr>
<tr>
<td>LANDSDOWNE</td>
<td>17.55</td>
</tr>
<tr>
<td>DUFFERIN</td>
<td>17.55</td>
</tr>
<tr>
<td>BATHURST</td>
<td>14.18</td>
</tr>
<tr>
<td>SPADINA</td>
<td>14.10</td>
</tr>
<tr>
<td>ST GEORGE</td>
<td>21.71</td>
</tr>
<tr>
<td>BAY</td>
<td>23.98</td>
</tr>
<tr>
<td>YONGE-BLOOR</td>
<td>23.36</td>
</tr>
<tr>
<td>SHERBOURNE</td>
<td>23.36</td>
</tr>
<tr>
<td>CASTLE FRANK</td>
<td>23.36</td>
</tr>
<tr>
<td>BROADVIEW</td>
<td>35.62</td>
</tr>
<tr>
<td>CHESTER</td>
<td>32.34</td>
</tr>
<tr>
<td>PAPE</td>
<td>32.39</td>
</tr>
<tr>
<td>DONLANDS</td>
<td>32.39</td>
</tr>
<tr>
<td>GREENWOOD</td>
<td>32.39</td>
</tr>
<tr>
<td>COXWELL</td>
<td>32.39</td>
</tr>
<tr>
<td>WOODbine</td>
<td>36.18</td>
</tr>
<tr>
<td>MAIN STREET</td>
<td>23.08</td>
</tr>
<tr>
<td>VICTORIA PARK</td>
<td>23.08</td>
</tr>
<tr>
<td>VARDEN</td>
<td>23.08</td>
</tr>
<tr>
<td>LAWRENCE EAST</td>
<td>12.42</td>
</tr>
<tr>
<td>ELLESMERE</td>
<td>12.42</td>
</tr>
<tr>
<td>MIDLAND</td>
<td>12.42</td>
</tr>
<tr>
<td>SCARBOROUGH</td>
<td>12.42</td>
</tr>
<tr>
<td>TOWN CENTRE</td>
<td>12.42</td>
</tr>
<tr>
<td>MCCOWAN</td>
<td>12.42</td>
</tr>
<tr>
<td>BAYVIEW</td>
<td>15.55</td>
</tr>
<tr>
<td>DONSARTON</td>
<td>24.00</td>
</tr>
<tr>
<td>LESLIE</td>
<td>24.08</td>
</tr>
<tr>
<td>DON MILLS</td>
<td>24.08</td>
</tr>
</tbody>
</table>

In Figure 4.8, the Actual Importance values of the TTC subway stations can be seen visually. The cost of a trip in a typical transit assignment model is a unit of time/trip. Utilizing the FBTA model, which considered both fares and congestion results in a modified cost. To denote that
congestion is experienced by trip-makers and subsequently converted to a value of time, the unit of cost is modified to include the prefix ‘perceived congested’. The values obtained by analyzing the realistic disruption scenarios stand in stark contrast to the results of the abstracted single station disruptions. The fundamental cause of this disparity is the additional stations which must be by-passed given track architecture. Of particular note is the group of stations between Union and Yonge-Bloor on the YUS line. Track architecture is such that if any station on this portion is disrupted, service must be disrupted from Union to Yonge-Bloor (see Table 4.3). As discussed earlier, most trips in the AM period end in PD 1, which is served fundamentally by the YUS line. Examining the results of the Actual Importance, we see that the realistic disruption scenarios produce results in accordance with expectations, that is, higher demand-weighted increases in travel time when service on the Yonge portion of the YUS is suspended. Actual Importance values for this portion of the line are 65.1 minutes. Theoretical Importance values were approximately 2.0 minutes.

Simulating realistic disruptions further resulted in a greater importance of the transfer stations of Yonge-Bloor and St George. These transfer stations serve as the interchange points for trip-makers commuting from the east and west end of the GTA respectively. It is at these two stations where passengers must transfer from the BD line to the YUS line in order to complete their trips (again, most of which terminate in PD 1). The Theoretical Importance had greatly underestimated the impact of a disruption at either of these transfer stations, a problem which was remedied in the Actual Importance analysis.
Figure 4.8 Actual Node Importance Map
As shown in Figure 4.8, Yonge-Bloor, the busiest station on the TTC network, is in fact the fourth most important station in terms of the impact on travel times when disrupted. The criticality of these BD-YUS transfer stations is a function of not only the immense volume they serve during the AM period, but also on the amount of additional stations which must have their service suspended, in the event of a disruption at the transfer station in question. For example, a disruption at Yonge-Bloor (on the YUS portion) results in service suspension from Union station to Eglinton Station. A similar disruption at St George results in a suspension of service between Union station and St Clair West station. Geographically, both these disruptions result in the suspension of service for almost the same number of stations (1 additional station is suspended in the Yonge-Bloor case), and a relatively similar distance of travel (6.5 km for the St George case and 7.7 km for the Yonge-Bloor case). Furthermore, a disruption at either transfer station results in the hindrance of flow from the east or west to the south, i.e. PD 1. Given these facts, it would seem surprising that the Importance of Yonge-Bloor is three times that of St George (96.96 vs. 32.02), especially considering how similar they are.

The disparity in Importance highlights the necessity in weighting travel times with respect to demand. Yonge-Bloor station serves the most passengers of any other station on the TTC network. Combine that with the fact that demand is skewed towards both the eastern end of the BD line and from the Finch terminus of the YUS line and the result is the difference in Importance between the two key transfer stations.

We turn our attention to the two most critical stations, Lawrence and Eglinton. Interestingly, these stations have the highest Importance values under both the theoretical regime and the realistic disruption regime. The explanation of such a result is that in fact the theoretical approach and the realistic one are almost identical with respect to these stations. In more specific terms, the location of crossovers around Lawrence station allow for trains to turn back at York Mills to the north and Eglinton to the south. This in fact is the exact scenario which is simulated in the theoretical approach, hence the identical Importance values for both regimes. The same conditions apply to the case of a disruption at Eglinton using a realistic approach, and this is reflected in the close values of Importance.
The increase in demand-weighted travel time given a disruption at either Lawrence or Eglinton is in some cases orders of magnitude greater than other stations, a surprising result especially given the previous discussion of Yonge-Bloor which is a critical transfer hub.

Briefly, the high Importance value of Lawrence station can be explained in terms of the AM trip demand pattern, trip-maker routing, and TTC network robustness. These factors will be elaborated below.

To begin this analysis of the relatively high Importance of Lawrence, let us examine the nature of the AM demand. As evident from demand matrix presented before, there is considerable demand originating from the Finch terminus of the YUS line and heading south towards downtown Toronto. Hence, a disruption at Lawrence interrupts the travel pattern of a great many trip-makers. Contrast the importance of Lawrence with other stations on the University-Spadina portion of the YUS line. Disruptions on this western part have a similar geographical impact, but overall have less of an impact on the network. The primary reason for such a stark contrast in the results is the demand on the University-Spadina portion is far less than that of the Yonge portion of the line.

The two largest contributors to the Importance value obtained for Lawrence include factors associated with trip-maker behaviour and the extent of the TTC network’s robustness. In the event of a service suspension at Lawrence, passengers from the north of Toronto and outlying regions are unable to complete their trip on the Yonge portion of the YUS line. A strategy analysis of the trips made post-disruption, in EMME, reveal that the vast majority of trip-makers would be expected to switch over to the University-Spadina portion of the YUS line in order to travel south. Given that the YUS line as a whole is already operating at or above capacity, a loss of a portion of this line has a profound impact on travel times, and this is echoed in the Importance value of not just Lawrence, but for many of the stations on the Yonge portion of the YUS line.

Just how those trip-makers switch over to the University-Spadina portion is an important point for discussion and ties directly into the overall resilience of the TTC network. Again, inspecting
the expected behaviour of tip-makers during the disruption has rendered valuable findings. In order to transfer from the disrupted portion of the YUS line, to the operating portion to the west, the vast majority of trip-makers would board busses which run predominately east and west. Bus service is found on Finch, Wilson, Sheppard, Lawrence and Eglinton Avenues, and on all the streets mentioned, serves stations on both portions of the YUS line. Unfortunately, these bus lines operate at very high load factors during the AM period. The additional demand of passengers from the disrupted subway line would create great delays. These delays become so burdensome that trip-makers end up walking to the next station where subway service is available in order to continue their trip. Figure 4.9 shows the redistribution of flow post-disruption. For reference, the undisrupted case is also shown. The thickness of the bars represents the total volume on each link segment. The colour code used is described as follows. The differences in flow between the base case and the disrupted case are calculated; positive values (i.e. when volume was higher in the base case, or pre-disruption) are shown as red bars, and negative values (i.e. when the transit volume was higher in the disrupted scenario) are shown as green bars. For clarity, the TTC subway system and the GO rail lines are shown in purple.

As apparent from the figure, trip-makers re-route themselves to the Spadina portion of the YUS given that the service south of Lawrence is not possible. Also apparent is the manner in which they make this westbound pilgrimage. The presence of the green volume bars indicates an increase in volume on the bus services that operate on Finch, Sheppard and Wilson Avenues. A total of 16,425 new trips are made on these three bus corridors alone due to the disruption at Lawrence station, a significant increase in demand on routes which already operate close to capacity during the AM period. Trip-makers who originate just east of the disrupted portion of the YUS line are forced to decide between heading to the western portion of the subway or make a southbound trip to the BD line. Those who elect to head south take one of two options, as apparent from Figure 4.9; utilizing a number of primarily north-south bus services or board a GO train destined for Union station in PD 1 (a total of 3966 trips). Geographically, making such a bus trip would be time-consuming (some trips as long as 10 km), saying nothing of the traffic conditions present in the AM period. These trip-makers alight from bus services at a number of stations along the BD line. The approximate 4000 additional persons on the BD line represent the
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equivalent of four subway trains at capacity (TTC T1 Six-Car Subways have an operational
capacity of 1000 persons (Toronto Transit Commission, 2013)). During the AM period, there is
little slack on the BD line to absorb this additional demand, hence congestion is expected and the
trip times are long.

The preceding discussion has served to explain the causes for the relative Importance (both
Actual and Theoretical) of Lawrence station when compared to other stations on the TTC
network. The high number of passengers impacted, the vast distances of northern Toronto and
the congested alternative routes result in an average additional travel time of 221 perceived
congested minutes per trip. In an effort to produce a framework for the analysis of resilience, one
must move past the proximate causes of increased travel time and determine the ultimate factors
that led to such a result. Surely, trip-makers having to find an alternative route to their
destination is not unique to the Lawrence case. Furthermore, in a capacity-constrained network
such as Toronto’s, the volume on most lines is high during the AM period. It would be beneficial
to ask why the stations between York Mills and Yonge-Bloor are the most critical stations in the
network. Although not shown here, diagrams for redistribution of transit volumes would look
similar for these other stations to the case of Lawrence. It is proposed that the greatest indicator
for the relative Importance of a station is the lack of high-quality alternative transit services. In
this definition, 'high-quality' will refer to frequency, capacity and to a lesser extent, right-of-way.
The next major indicator of Importance for a station will be the volume on the line to which the
station belongs. Let us turn our attention to the network in an effort to garner evidence for this
hypothesis.
Figure 4.9 Disrupted Flow (Green) vs. Non-Disrupted Flow (Red): Case of Lawrence Station
Without consideration of the demand profile for the AM period, one would expect that the stations on the University-Spadina portion of the YUS would too have high Importance values. But, as discussed earlier, the skewing of demand in Toronto towards the north east is so significant that similar disruptions on the University-Spadina portion to those found on the Yonge portion of the YUS line, for example, Lawrence, have a much smaller impact on demand-weighted travel times.

By way of an example, the impact of additional available high-quality transit services will be shown, as it relates to the Importance of a station. In Figure 4.10, the flow redistribution is shown for the case of a service suspension between St George and Broadview station. As presented before, the GO rail lines and subway system is shown in purple, but now the TTC streetcar system has been added in blue. Given the nature of demand, Yonge-Bloor represents a critical transfer point for passengers coming from the east and heading southbound on the YUS line. Thus, it is expected that such a service suspension would have profound impact on the average increase in demand-weighted travel time, given that passengers could no longer reach the Yonge-Bloor station. The Importance of this group of stations is 23.98 perceived congested minutes per trip, a full order of magnitude less than that of the Lawrence case. The disparity in the Importance value can be attributed to the presence of the predominantly east-west running street car lines which serve as an additional means of accessing the downtown core of Toronto from the outlying planning districts that hug the BD line. Examination of Figure 4.10 does in fact reveal an increase in ridership on these streetcar lines by an average of 2798 trips, and a maximum increase of an additional 7836 on the 504 King Street route. This network of streetcars affords a portion of Toronto with a level of resilience that other areas-namely those outside of the downtown area-lack. The location of transit services will tie into the concept of Exposure, which is to be presented in the following section.

Contrast this network of streetcars (in addition to bus services which operate in and around the subway and streetcar network) with the lack of robustness that is found north of the BD line, in particular, PDs 4, 5, 11 and 12 (see Figure 3.3). In these PDs, other than the subway, only infrequent and intermittently spaced bus routes serve in north-south capacity. The strength of the
streetcar network, outside of its additional capacity over a bus, is that it runs predominantly parallel to the BD line. The result of which is a series of additional high-quality transit lines which can be utilized in the event of a subway service suspension.

The station spacing north of the Yonge-Bloor station on the YUS line is considerably greater than the spacing found on the BD line. This difference in distance cannot be ignored when examining disruption impacts. Disruptions on the YUS line may result in trip-makers being forced to walk considerable distance to access a functioning transit service. Such an issue is less of concern for disruptions in and around PD 1. Although a considerable number of trip-makers are impacted by a service shut down on the lower-half of the YUS, the close proximity of streetcars and destinations allows for short walks. The result of which is the relatively low level of Importance for these stations when compared to the upper-half of the YUS.
Figure 4.10 Disrupted Flow (Green) vs. Non-Disrupted Flow (Red): Case of St George to Broadview
4.5 Exposure of the Toronto Planning Districts

As postulated at the onset of this work, network resilience is strongly impacted by network topology. Station Importance calculations strongly supported this argument, the evidence of such can be found in the large difference in Importance values based on the location of the disruption relative to other transit services in the network. The concept of Exposure, as discussed in the methodology, measures the demand-weighted increase in travel time per trip that begins in a particular PD and ends anywhere else in the network. The impact of a service disruption can therefore be studied in terms of the cost to particular areas of Toronto. Based on the subway network topology and the location of high-quality transit alternatives, one expects that the fringe PDs (i.e. PDs 8, 9, 10, 11, 12, 15 and 16) would be the most exposed, the primary reasons being that these PDs have sparse trip origin demand and few good alternatives to make transit trips. Figure 4.11 shows the expected exposure given a random failure on the TTC subway system for trips originating from that PD. Expected Exposure is average increase in travel time for trip-makers originating from a specific PD over all possible disruption scenarios (as outlined in Table 4.3).

As evident from the Exposure map, on average, the most exposed PDs are 12, 5 and 16 - a finding which when examined in the light of the Importance values is not unexpected. Trip-makers travelling from PD 12 or 5 have an average increase in demand-weighted travel time of 69.7 and 62.2 Trip-Perceived Congested minutes per trip, respectively. A few service suspensions might result in immense increases in demand-weighted travel time, chiefly service disruptions impacting the stations between Yonge-Bloor and Lawrence. In fact, the top 5 most impactful disruptions involve service suspensions on the YUS line north of Bloor-Danforth line and south of Finch station. It is no surprise then to find that trip-makers who originate east of the YUS line (PD 5, 12) can be greatly impacted by the average service disruption. Trip-makers originating from PDs 11 and 4 are more advantageously positioned in the event of the failure of the YUS line. Spatially, these residents are closer to the functional University-Spadina line and thus we see these PDs are less exposed. The impact of the Don Valley, which snakes through
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Toronto, likely also contributes to the relatively greater increase in travel time for those originating from PD 4 and 12.
Figure 4.11 Expected Exposure (Trip Perceived Congested Minutes / Trip)
Surprisingly, the majority of fringe PDs (7, 8, 9, 10, 15) are relatively unexposed, with PD 16 being the exception. It was expected that the vast distances these trip-makers need to travel and the few alternative transit services would result in a high expected increase in travel-time. It turns out that the topology of the GO transit lines facilitates a high-quality alternative mode of transit for these trip-makers, and as a result, these fringe PDs remain relatively unexposed.

4.5.1 Maximum Exposure by Disruption Scenario

Each PD will be subject to a ‘worst-case scenario’, that is, the service disruption that has the greatest impact on the travel time of trip-makers who originate from within it. Analysis of ‘worst-case scenarios’ has originally been a studied in the field of Game Theory. More recently, network resilience studies involving ‘worst-case’ have looked at the impact of terrorists hoping to cause the maximum amount of damage, a core tenet of which is that said terrorist has perfect knowledge.

\[
\text{Exposure}_{\text{max}}(m) = \max \frac{\sum_{i \in V} \sum_{j \neq i} x_{ij}(c_{ij}^k - c_{ij}^0)}{\sum_{i \in V} \sum_{j \neq i} x_{ij}}
\]  

Where:

- \( x_{ij} \) is the demand from origin node \( i \) to destination node \( j \).
- \( c_{ij}^k \) is the cost of travel from node \( i \) to node \( j \) given node \( k \) is disrupted, \( c_{ij}^0 \) is the base case.
- \( V \) is the set of origin nodes within the PD, \( m \).

The maximum Exposure of each PD is mapped in Figure 4.12. The service suspension which results in the maximum Exposure for each PD is shown in Appendix A. Similar to the results found from the expected Exposure, PDs 4, 5 and 12 were the most Exposed. The maximum increase in travel time occurs for trip-makers originating from PD 12 in 5 of the 35 possible service suspension scenarios. The next highest occurrence is in PD 5, with 4 service suspension
scenarios resulting in maximum travel time increases. Both the expected Exposure and the maximum Exposure analysis has revealed that disruptions greatly impact trip-makers who originate east of the YUS line. Transit authorities could build upon such results by looking to add transit services in these areas, either permanently or in the event of a service disruption.

4.5.2 Expanded Exposure

As apparent from Equation 4, the demand used in the calculation of Exposure is based on trips originating within the PD of interest and travelling to any other destination outside of the PD. This definition was built with the trip pattern that exists during the AM period, that is one in which the vast majority of trips are headed to PD 1. This calculation, although valuable, fails to capture intra-PD trips. Furthermore, the aforementioned analysis may under predict the impact of some service disruptions on a certain PD, especially when more than one service disruption impacts the same PD (or set of PDs). The calculation for Exposure was adjusted to include both trips that originate from the PD of interest and terminate in the same PD. Thus, intra-PD trips are included. Figure 4.13 shows the expected Expanded Exposure of the PDs of Toronto. Immediately, one can see that by adjusting the bounds on trip-makers, the Exposure of PD 1 has greatly increased.

Service suspensions south of the BD line would shut down either half of the YUS line or, in the case of Union station, the entire lower-half of the U portion of the YUS line. Trip-makers who originate in PD 1, have much shorter trips on average, and in the event of a service suspension, often walk. The impact of this great many short trips can now be seen in the Expanded Exposure of PD 1, which is significantly higher than previously. Overall, the Expanded Exposure analysis shows that poorly serviced PDs are highly sensitive to the average service suspension, and therefore represent relative points of low resilience in the TTC network.
Figure 4.12 Maximum Exposure by PD (Trip Perceived Congested Minutes / Trip)
Figure 4.13 PD Expanded Exposure (Trip Perceived Congested Minutes / Trip)
4.6 Application of Importance and Exposure: 2013 Disruption

As discussed earlier, it is the hope that this research forms a basis for a resilience framework that transit operators or planners could use to analyze the state of their network and determine quantitatively the level of robustness currently inherent. With such a framework, one could also examine the impact of service expansions. To facilitate this, the measures of Importance and Exposure could be applied to the network on a year by year basis. An overall increase in network performance under the various service suspension regimes would indicate an increase of resiliency. As demonstrated in this thesis, such metrics are readily calculable, intuitive and capture a great number of factors that govern transit trip-maker behaviour.

On an annual basis, the TTC releases a report which documents all of the service disruptions which occur on the rail network for the calendar year. This report contains data on the nature of the disruption, where it occurred and the portion of the subway line which is impacted. As all possible service disruption scenarios can be generated in EMME, the data found in this disruption report lent itself well to analysis. Several network level metrics were calculated based on the 2013 disruption data report. These metrics include: average increase in travel time per trip, and the total delay experienced by all trip-makers.

4.6.1 Overview of 2013 Disruption Data

Given the static nature of EMME, only disruptions which resulted in a service suspension can be analyzed and modelled in this framework. The 2013 data contained 43 incidents involving a service suspension greater than 10 minutes. Disruptions were as small as one station (trains turn back at Wilson and do not serve Downsview) and as large as an entire line (Sheppard Line). The top five occurring disruptions were:

- Kipling to Jane
- St George to Yonge-Bloor
- Kipling
A disruption impacting Kipling station occurred in 11 of the 43 incidents in 2013. Of note also is the relatively high-frequency of disruptions which close the entire lower half of the YUS line. This was the second most common disruption in 2013. Given the critical nature of the YUS line, and the demand to PD 1, such a result should be alarming to TTC operators. It should be noted that during 2013, construction continued on the Union station revitalization project (Metrolinx, 2014). During this project, there were numerous construction related disruptions and thus the numbers found in the 2013 disruption data may be artificially inflated. Nonetheless, as this is a study of network behaviour during periods of compromised integrity, such incidents serve as a valuable source of education for the impact on passengers.

### 4.6.2 Findings and Discussion

Using the theoretical underpinnings of Importance, trip-makers experienced an increase of 33.9 perceived congested minutes per trip on average for all delays in 2013. Service suspensions in 2013 resulted in a total delay of 26.4 million trip-minutes, of which 11.8% can be attributed to the disruptions involving Kipling station, the station with the highest rate of service suspension. Interestingly, the fifth most common disruption, that of the Sheppard line, accounts for 14.8% of the total delay experienced during 2013. Similar disruptions on the northern portion of the YUS line are relatively infrequent in their occurrence, but account for large portions of the total delay experienced. For example, in 2013, there were two service suspensions at Lawrence station, which accounted for 11.2% of the total delay minutes of the year.

The disruption pattern exhibited here lends itself well to the study of risk assessment. On the one hand, there are frequently occurring disruptions at Kipling (to Jane) which individually are not very severe. On the other hand, there exists the situation where a service disruption results in an extreme detriment to the travel times of trip-makers, but is relatively less frequent in occurrence. Ideally, one would hope that through future service planning, infrastructure expansion and the
refinement and implementation of Disaster Risk Management (DRM), both the frequency and severity of all disruptions can be reduced.

Until such a time, this work serves to provide a means by which year over year, the performance of the network under disrupted conditions can be analyzed and compared. The adaptive nature of EMME allows for the accurate and thorough modelling of a great number of possible scenarios.

4.7 Behavior during a Service Suspension: Shortcomings and

Disruptions to a transit network, especially during periods of high demand can have wide ranging adverse impacts on passenger experience. Disruptions generate high levels of uncertainty with regard to routing and expected travel times. Research has shown that there is a large subset of behavioural responses exhibited by trip-makers during a disruption, including but not limited to: mode-choice alterations and trip abandonment. With regards to the latter, it has been noted earlier in the thesis that demand has been assumed to be inelastic in this investigation. Given the nature of AM peak period trips, this assumption can be considered sound. The majority of AM period trips are school or work trips, which for the most part are completed regardless of network conditions.

Mode choice alterations are a critical facet in the behavioural patterns witnessed during non-status quo network conditions. Alternative modes to transit given a disruption include:

- Auto
- Carpool
- Taxi
- Rideshare
- Car sharing programs (e.g. Autoshare™)
- Active-modes of travel (e.g. bicycle), and
Trip-makers in this analysis are not able to switch their mode (i.e. from transit to any other mode) except to walking. There currently exists no data from the TTS that examines the mode choice patterns given a disruption, thus modelling the switching was for the most part, impossible. Furthermore, it was the hope of this study to examine the ultimate impact of disruptions and make comments on the resilience of the TTC network. For trip-makers already on route when a disruption occurs, few of the aforementioned alternative modes are viable. For example, a trip-maker who has already left home and then encounters a disruption is unlikely to go back home to get their car. Contrast this with travellers who are aware of a disruption prior to the commencement of their tour. These individuals have significantly more options in their choice subset. Given these facts, a conservative approach was adopted, and no mode switching was allowed. The result is a more complete picture of the impact of a service disruption.

The exception to this ban of mode switching lies in the inclusion of Fare-Based Transit Assignment (FBTA). All trip-makers have the option to walk on all network links and centroid connectors of the EMME Toronto model. During a disruption, this inclusion in the mode set allows for trip-makers to walk to access alternative transit services, walk the disrupted portion of the line until they reach a point where service has resumed, or in the worst case, walk the entire length of their trip.

Essentially, the need to resort to walking (when previously this was not required) represents a total failure of the transit system during a disruption. Trip-makers forced to complete their trip by walking constitute a group of individuals who have paid the ultimate penalty in terms of travel times. It is proposed in this work, that additional walking time can be used as a proxy for the resilience of transportation network. Limitations to such a metric have been discussed above, such as the inability to include all possible alternative modes. An additional limit exists in the fact that EMME does not allow for multiclass transit assignment. As a result, trip-makers assigned to the transit network are all homogenous. Obviously, this is a simplification that is an artefact of the coding of EMME, and by no means captures the complete picture of trip-maker behaviour. For example, an elderly commuter (or any person of restricted mobility) may not be
able to walk in the event of a disruption. As a result, it is obvious that the additional average walk times presented in Table 4.5 are artificially inflated. They are determined under the pretense that in the event of transit services being unavailable, a trip-maker will walk. Nonetheless, the values shown in the table are of some value. Furthermore, the additional walking time when combined with the other metrics of this paper help form a more complete picture of the resilience of the Toronto transit network.

Table 4.5 Trip Walk Time Increase

<table>
<thead>
<tr>
<th>Average Increase in Walk Time by Disruption (Minutes/Trip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Union to Yonge-Bloor: 33.2</td>
</tr>
<tr>
<td>St George to Yonge-Bloor: 19.4</td>
</tr>
<tr>
<td>Lawrence West to St Clair West: 13.0</td>
</tr>
<tr>
<td>York Mills to Eglinton: 28.3</td>
</tr>
<tr>
<td>St Clair West to Union: 22.2</td>
</tr>
<tr>
<td>Lawrence West to St George: 22.3</td>
</tr>
<tr>
<td>Sheppard to Eglinton: 19.0</td>
</tr>
<tr>
<td>Downsview to Lawrence West: 3.0</td>
</tr>
<tr>
<td>St Clair West to St George: 23.5</td>
</tr>
<tr>
<td>York Mills to Yonge-Bloor: 50.1</td>
</tr>
<tr>
<td>Finch to Sheppard-Yonge: 14.6</td>
</tr>
</tbody>
</table>

Eglinton to Yonge-Bloor: 27.5
Christie to St George: 4.3
Table 4.5 shows the average increase in walking time per trip for each of the possible disruption scenarios. As evident from the tabled data above, some disruption scenarios result in a large increase in walking time per trip-maker. It should be noted that this is the additional time a trip-maker spends walking, on average, for a given disruption scenario. Hence these values represent a portion of the total Actual Importance value, with the exception being that these are in real minutes, as opposed to perceived minutes which is how Importance is presented. Unexpectedly, the critical disruptions from the Importance analysis are also scenarios in which a great amount of additional walking time is observed. Of note, are the cases where subway service is suspended from either Yonge-Bloor or Union, up well into PD 4. In such events, passengers walk as much as an hour in order to complete their trips. Service suspension impacting PD 4 tend to be the most severe cases, largely due to the reasons discussed in Section 4.4. The walk analysis conducted here has not shed further light on passenger behaviour expected during a disruption. The fifty-plus-minutes of additional walking time for this subset of disruptions arises largely from the real lack of alternative high-order transit operations. Trip-makers are forced to walk vast distances to access operational transit.

Also evident from the above table is the impact of service disruptions within PD 1. The majority of trips that begin in PD 1 use the subway for short trips (several stops). The remainder use the streetcar network or walk. In the event that any portion of the lower half of the YUS line is shut down, a great number of trip-makers turn to walking to complete their travel. Although these walks are short in comparison to some of the journeys required during other suspensions, their large volume produces a substantial additional average walk time. In general, we see a great additional walk time for service suspensions which lie in or along areas of higher demand. Hence, suspensions impacting travel from the north of the city or the east have a greater overall impact (given Importance is demand-weighted and additional walk time is averaged over all trips).

Overall, there is almost no correlation \((r = 0.23)\) between the geographic length of impacted service and the additional average walking time experienced by trip-makers. This is strong evidence to support the idea that additional walking time can be used as a proxy of resilience.
4.8 Summary of Results

Quantification of resilience involved the application of a methodology which stemmed largely from two fields: network science and transportation demand modelling. Application of a network science approach examined the impact of node disruptions on the GE of the TTC network. The topological integrity of the network was determined by examining the formation of disconnected sub-graphs, which represent a critical failure of the network, and by extension, the transit system. The Graph theory approach showed that GE drops rapidly after the removal of only a small subset of nodes, indicative of a lack of redundancy in the TTC transit network.

Examining the behaviour of trip-makers during a service suspensions provided insights on the resilience of the public transportation network in Toronto. The concept of Importance allowed for the rank-ordering of the TTC subway stations in terms of their relative impact on trip-maker travel times when disrupted. The comprehensive EMME network model for Toronto captured the topology of the transit network and the demand patterns that exist during day-to-day operation. The result of which, is a detailed and accurate study of the resilience of the transit network, when subjected to operational disruptions. The most critical stations, in terms of their impact on passenger travel-time when disrupted, resided on the northern portion of the YUS line. Travel through this area of Toronto is limited when using the TTC. As a result passengers have few viable alternatives when subway service is disrupted.

The metric of Exposure allowed for a more spatial measure of network resilience. The PDs within Toronto were analyzed to determine the increase in travel time for trip originating within the PD in question. This allowed for the PDs of Toronto to be compared and comments on their resilience were made. PDs which had fewer transit alternatives were the most expose and these were most often found in the north eastern part of the city.

Finally, the framework for operational resilience was applied to service disruption data from the year 2013. The result of which are a number of metrics for passenger delay, and metrics concerning the most exposed regions of Toronto.
5 Twitter Sentiment Analysis

Sentiment analysis (or opinion mining) is conducted in order to determine the attitude of a person with respect to some topic or their overall tonality. The attitude may be his or her judgment, evaluation, emotional state or the intended emotional communication. Historically, sentiment analysis has been conducted by businesses to computationally study the opinions, appraisal and emotions regarding products.

The sentiment of a message can roughly be broken down into the following three categories as outlined in the equation below:

$$Sentiment = f(Holder, Target, Polarity)$$ (6)

Where:

- The holder is the person (or group) that expresses the view.
- Target is what or whom the sentiment refers and.
- Polarity is the nature of the sentiment, typically it is either positive, negative or neutral.
- Polarity can further be described in terms of its value (strong vs. weak).

A fourth piece of data may also be included which contains information regarding the context of the sentiment, such as time, place, location, etc.

Classification of the sentiment is a growing field of research. To date, there are several methods available including: Lexicon based classification, General Inquirer Lexicon, Linguistic Inquiry and Word Count and Opinion Lexicon (Minqing & Bing, 2004; Maite, Julian, Milan, Voll, & Stede, 2011; Stone, Dunphy, Smith, & Ogilvie, 1966; Pennebaker, Booth, & Francis, 2007)
Increasingly, opinion mining is being conducted via machine learning algorithms. These treat sentiment classification as a type of topic-based categorization (Pang, Lee, & Vaithyanathan, Thumbs up? Sentiment Classification using Machine Learning Techniques, 2002).

The aforementioned methods of sentiment classification have historically been applied at the document-level (that is, written works), or for customer-based reviews on products/services. In such cases, the holder and target is usually explicit, as a result, the opinion mining can often be considered simplified.

With the advent of social media, massive amounts of information can be found online which previously did not exist. Social media services, such as Twitter, enable people to generate, share, analyze and discuss content either created or discovered. With the explosion of social media in the 21st century, sentiment analysis research and application has come to the forefront for businesses, government agencies and public health officials looking to analyze and enhance their services.

The cost of collecting data via social media is orders of magnitude less than that of traditional methods, such as surveys. Cost and the fact that collecting data in real-time is impossible via traditional methods, are strong motivators for increased research and use of sentiment analysis (Musakwa, 2014).

As mentioned before, one such social media website is Twitter (www.twitter.com). Twitter is real-time social sharing platform, where users make short text (which can include an image) based posts, colloquially referred to as 'tweets', which can then be shared to other Twitter users. Twitter is often used on mobile devices, the result of which is that Twitter data has good spatial and temporal coverage. Data mining on Twitter is especially attractive as all tweets are made public (unless explicitly set to private), tweets are time stamped and can have location data included.

Sentiment analysis on Twitter (and related social media) is a more involved process than typical sentiment analysis, such as product reviews. Tweets do not have ratings and often the target of the tweet is not explicit. It is therefore needed to examine clusters of tweets in order to gain
insight on the general feelings with regards to a specific topic. Twitter opinion mining has a number of issues associated with it. Given that tweets are limited to 140 characters, the sentiment(s) expressed are confined to the sentence level, not the document level. As a result, determining the polarity given such limited data can be troublesome. Furthermore, communication on the internet can be ambiguous (i.e. involving abbreviated words or colloquialisms), and involve informal language. Historically, sentiment analysis has been conducted on smaller data sets. Twitter poses a unique problem in that there are millions of tweets which can be examine and analyzed, thus generating a computational issue which cannot be overlooked. Recent improvements in research in the field of social media opinion mining as seen researchers using hashtags, emoticons and smileys to assist in determining the polarity of a tweet (Davidov, Tsur, & Rappoport, 2010; Go, Richa, & Huang, 2009).

5.1 Literature Review on Sentiment Analysis

A thorough survey of the research conducted in the field of sentiment analysis was conducted by Pang and Lee (Pang & Lee, 2008). The remainder of this literature review will focus on sentiment analysis and opinion data mining of social media with respect to public transportation services.

In Gauteng Province, South Africa, a new high speed train system was implemented for the 2010 world cup. Public opinions on transit given the service improvement was examined via mining of Twitter data (Musakwa, 2014). Gauteng officials felt that commuters played a key role in determining whether the implementation of a new transit service resulted in a "safer, [more] reliable, [more] dependable and smart[er] public transit system". Results from the sentiment analysis were largely positive. Negative sentiments related almost exclusively to delays and the cost of the system. In relation to transit network disruptions, the focus of this thesis, the Gauteng officials were able to study tweets relating to a series of disruptions in February and March of 2014. Gauteng operators noted that tweets during disruptions disseminate notice of the disruption to other users and “inform connected commuters to make travel changes”. The value of the tweets in cases of disruptions is that they allow other travel arrangements to be made and provide up-to-date operational information.
Gauteng officials concluded that social media increased the public participation in the transit planning process and participation in relaying information to other commuters about a number of operational issues.

Eric Mai and Rob Hranac analyzed Twitter data-and the inherent sentiments contained- in the hopes that tweets could be used to locate incidents on the road network in California (Mai & Hranac, 2012). The idea behind the research was that department of transportation authorities have a limited ability to detect incidents on such an expansive road network. Therefore, if Twitter could be used to determine where accidents had occurred and the severity of said accident, then finite resources could be allocated efficiently.

Researchers at Purdue University proposed using Twitter as a metric for rider satisfaction for the Chicago Transit Authority (CTA) subway system (Collins, Hasan, & Ukkusuri, 2013). The CTA is a system similar in size and volume to the TTC, and thus researcher findings from this work may have applications here in Toronto. The authors noted how traditional performance metrics were designed for transit agencies, not passenger point of a view.

Similar to the TTC, the CTA must be austere in their fiscal behaviour. The CTA looked to sentiment analysis as a means of gaining feedback regarding “fare increases, service changes, and provide a means to monitor safety due to a lack of personnel.”. Results of the sentiment analysis showed that largely, opinions regarding the CTA were negative, especially those regarding service and safety. An interesting finding of the research, is the strong negative sentiments which were found after delays due to a fire and flooding. A key conclusion from this work, and one found in several other pieces of research outlined by Pang and Lee, is that users are more likely to voice negative sentiments than positive sentiments.

Bas Stottelaar used the Netherlands rail system as a basis for generating a tool to use social media in order to analyze public transit (Stottelaar, 2012). A key question he asked was whether or not it was possible to use social media to gain insights on the quality of public transit service provided. To answer said question, Bas turned to a sentiment analysis of the Twitter messages in the Netherlands. He applied heuristics to determine the location of the message and further filter
them for content (i.e. topic, mode of public transit discussed, etc). The results of the research suggest that Twitter could serve as an excellent tool for monitoring public opinion on public transportation.

As this literature review has undoubtedly shown, there is little research regarding the use of social media opinion mining with respect to public transportation, and associated passenger sentiments. The vast majority of research conducted in the field has concerned itself with corporate or government opinion mining. The little work that has been done with a focus on public transportation had a limited scope. In particular, almost no work has been done to analyze the sentiments of passengers under disrupted network conditions.

Examination of sentiments held and expressed during a network disruption can provide insight into passenger behavior under disrupted conditions. It could further be used by transportation authorities to enact disruption management techniques, and mitigate overall damage (i.e. travel time increases) over the network.

5.2 Methodological Approach

The sentiment analysis conducted in this work involves a more qualitative approach than typical opinion mining, such as those involving machine learning. Data mining was conducted and gathered from the Twitter streaming API. This allowed for real-time access of content posted on Twitter. Using the API allowed for the tracking of keywords, thus messages could be filtered for relevance to the TTC. A Python library called *Tweepy* was used to help facilitate the access and use of the Twitter API.

Keywords logged included but where not limited to:

- Toronto Transit Commission
- Toronto Transit
- TTC
In general, keywords were not case sensitive, nor were they context sensitive. The result of which is the production of tweets regarding a wide variety of subjects. Given this, the top three keywords, in terms of effectiveness for generating relevant content were: #TTC, TTC and Toronto Transit Commission. Generating 78%, 16% and 4% of relevant tweets.

In total, 546 tweets were saved and deemed relevant to this analysis. Tweets were collected within 8 hours of a major disruption resulting in a service suspension on the TTC subway network. Tweets collected to contrast, that is, those collected during 'business as usual' conditions, were collected during either the AM or PM peak. The PM peak further corresponds with the maximum volume on Twitter as a whole. The filtering conducted was done in order to obtain tweets which involved one or more of the following topics:

- Delays and associated increased travel times
- Crowding on the network
- Wait times (perceived or actual)
- Mode switching behavior/Alternative route planning
- TTC level announcements regarding disruptions and subsequent service
5.3 Twitter Analysis

The TTC maintains a strong and active presence on social media. On Twitter, specifically, they operate @TTChelps for customer service, @TTCnotices for service advisories and @bradTTC, the official account of Brad Ross, Head of communications for the TTC. During a disruption, all three accounts would be active relaying information to commuters and the public in general. Several non-official accounts exist (e.g. @TTCing) which serve in a similar capacity to their official counter-parts.

At the onset of a disruption, TTC social media accounts create postings informing commuters (and would-be commuters) of the nature of the disruption, location and portion of the line impacted. Upon the resolution of the disruption, the aforementioned social media accounts inform riders that regular service has now resumed. The social media presence of the TTC plays a critical role in informing passengers of the nature of a service disruption. These tweets regarding the occurrence of a disruption represent the first class of tweets to be presented in this work. Passenger behavior-such as route choice-is undoubtedly impacted upon the acquisition of service suspension related information. An interesting phenomenon was observed with regards to official TTC tweets. Many passengers do not follow the official TTC accounts, and therefore would likely not receive information about current disruptions. In many cases, passengers would ‘retweet’ official announcements such that the information is relayed through their social network. This altruistic behavior would have profound impacts on the level of dissemination of disruption related information.

Currently there exists very little information about passenger behavior during transit network disruptions. Conventional survey methods would be almost impossible to apply in the time frame of a typical service suspension. Furthermore, surveys done after the fact would fail to capture a lot of critical information, such as passenger perception of transit. As outlined in the literature review, social media mining provides a means for researchers to analyze passenger behavior given a network level disruption.
The second class of tweets to be discussed is the official TTC tweets that direct current commuters and would-be commuters of alternative routes during the duration of the disruption:

Brad Ross  @bradTTC  3h hours ago
If heading downtown from the east, consider the 506 Carlton streetcar from Main Street Stn as an alt to Line 2. #TTC

Chris Boddy  @TPSChrisBoddy  8h hours ago
RT @bradTTC: As we wait clear on the University portion of Line 1, Yonge is a good alt to Bloor Stn., then Line 2 to Spadina Stn. #TTC

This tweet followed a disruption on the BD line from St George or Pape station during the evening PM peak. The result of which is a diminished ability for commuters to get from PD 1, downtown, to the eastern end of Toronto and Scarborough and vice versa. Tweets such as these help alert commuters of alternative routes within the network and help to highlight some of the existing redundancy contained therein. For an experienced commuter, the existence of parallel routes (when they do in fact exist) would be common knowledge. But, for less experienced commuters, such a tweet would serve as invaluable information, and allow for the completion of a trip that would otherwise be impossible to make.

The third class of tweets to be discussed are those from commuters and regard some mode switching behavior observed during a disruption. An example is included below:

@AidanD10  2h hours ago
You owe me $3 #TTC. Plus the cost of the cab I'm taking now. @johntoryTO @TTChelps

Similar messages regarded walking, taking a bicycle or utilizing the shuttle bus service which was implemented and is typically implemented during a service suspension on the subway network. The vast majority of these type of tweets are those who switched from transit to another
mode, and within that group, most switch to walking. This class of tweets sheds some light on the real lack of redundancy in the network for many portions of the subway system. Reports of individuals having to walk-and adding as much as two additional hours-speak volumes to the level of resiliency inherent in the current network. For many passengers, the only alternative to complete their trip when subway service has been suspended is walking. It should be noted that the choice to walk to complete a disrupted trip is not only a function of the lack of redundancy in the network, other factors such as financial constraints (i.e. unable to take a cab) or lack of capacity in the improvised shuttle bus service. A possible second order effect of a tweet such as the one above, that is to say, one regarding mode switching, is the impact on other commuters. Other commuters upon hearing of the requirement to switch modes may be more inclined to follow suit, possibly a form of cognitive anchoring is at play here.

Turning our attention to the shuttle bus service put in place during a disruption, we can make some comments regarding network resilience from the insights of passengers. A plethora of messages exist from customers who have embarked upon or attempted to embark on the replacement bus service. The following messages cover both those sentiments respectively:

@meaganpau 6h hours ago
#TTC delays, all day errday. Shuttle bus = I have no idea where I am & driver doesn't know where she's going. I just wanna get home.

@bintagirl 8h hours ago
Waiting for shuttle at Bview. Nightmare. #ttc #transit #topoli

It is unreasonable to expect an ad hoc bus service to be able to provide the capacity required to move a volume of passengers typically found on a subway line. With that said, the general breakdown in quality of service provided by the replacement shuttle bus service further highlights the lack of network resilience in the Toronto transit system. The operation of a replacement bus service could be used to make further comments regarding the disaster
management strategies currently employed by the TTC, although such a discussion is outside the scope of this work.

As mentioned before, many stations do in fact connect to either streetcar lines or frequent bus service (which could be considered higher quality transit than typical surface routes). In such circumstances the results of this thesis have shown that these surface routes see increased use, and in many cases help strengthen network resilience. An example is the 511 and 509 street car lines in the west, and the 505, 504 and 506 lines from the east, which help facilitate travel to the downtown core during major disruptions on the southern portion of the YUS line.

The last class of tweets to be discussed are those which make reference to an increase in travel time as a result of a service suspension. As mentioned earlier, little data exists as to the impact of large scale disruptions. Travel time is a relatively simple metric to understand for both commuters and transit operators, yet contains a wealth of information regarding system level performance. Tweets that discuss passenger travel time given a disruption can help shed some light on the impact of a service suspension.

5.3.1 Contrasting Disrupted-state vs. ‘Business as usual’

A critical difference in Twitter use between the disrupted-state and normal operations is the total volume of tweets produced and or exchanged regarding the TTC. During periods where no disruption is taking place, there is a considerable less amount of activity on Twitter that uses the aforementioned keywords. The majority of content during normal operating conditions are press releases from official TTC accounts.

With regards to tweets from passengers, those that are generated tend to comment on the crowding of surface routes (in particular Finch and Dufferin routes). The next most common topic observed regarded schedule reliability, again of predominately surface routes. Tweets of this nature tended to include images of predicted vehicle arrival times from various apps, with an accompanying caption commenting on the inaccuracy of the prediction. In some cases, comments were made regarding increased wait times for vehicles (and therefore increased total
travel time). The increase in travel time related to these cases was on the order of minutes; contrast this to the hours added to some trips during the disruptions.

The mining conducted was unable to produce any results of mode switching behavior during normal operating conditions. Although some commuters held negative sentiments regarding crowding conditions or lack of adherence to the posted schedule, they made no indication of switching their mode to complete their trip, or for any future trips.

In general, tweets from passengers during normal conditions or disrupted conditions carried a negative sentiment. Whether these tweets represented overall passenger feelings is unknown, what is known is that passengers were more likely to make negative comments on Twitter than positive ones.

5.3.2 Sentiment Analysis Conclusions

This qualitative sentiment analysis of tweets generated during and post-network disruption provided insights on passenger behavior and highlighted the lack of resilience in the TTC metro network. The former of which is relatively unstudied, and the latter has been discussed at length and modeled in this work. As evidenced here, social media can serve as an invaluable tool to make decisions in trip planning given disrupted conditions. Monitoring and maintaining a presence on social media can help transit operators manage their system. Using a more in-depth and quantitative sentiment analysis during a disruption could help quantify the level of satisfaction passenger’s experience. Thus, transit authorities would have a metric from which they could compare disaster management strategies from a passenger perspective.

The contrast between disrupted network conditions and normal network conditions on Twitter was stark. The overall increase in negative sentiment observed during a disruption is testament to the frustration commuters feel due to the lack of redundancy in the network. The weak and vulnerable state of the TTC network often leaves commuters unable to complete trips via transit or results in unacceptable increases in travel time.
The presence of real-time disruption information to passengers allows them to optimize their trips and make informed decisions. Unfortunately in the context of a network like the one found in Toronto, alternative transit routes are scarce. Further improvements in opinion mining could involve performing longitudinal analysis which also carry a spatial aspect. That is, tweets which make comments on increases in travel time due to a disruption could be mined for their location, and thus an OD pair could be derived. Mined over enough suitable tweets, the result would represent an OD matrix of actual disrupted travel times. From such data, similar disruptions can be compared year to year and a transit agency could judge the efficacy of service improvements. In effect, a potential additional measure of resilience could be derived from Twitter opinion mining.
6 Conclusions and Future Work

Resilience has been studied in a wide range of fields, including transportation engineering. Much of the research to date has adopted a conceptual framework of resilience using qualitative and descriptive means, rather than quantitative techniques. This research study has proposed a quantitative method to analyze the operational resilience of the multi-modal public transportation network of Toronto. The methods applied here were formulated based on graph theoretic approaches taking into consideration the unique properties that characterize a public transit system. In addition, the concepts of Importance and Exposure were applied using the EMME model software package to incorporate impacts on ridership demand. This thesis has demonstrated that transit system resilience can be quantified by considering network topology, transit service attributes and trip-maker behaviour. This thesis has also demonstrated the value of simulation for the analysis of complex problems, such as network-wide service disruptions. The methodology adopted in this work has provided novel insights regarding the quantification of resilience for a public transportation network.

Although the results of this work are specific to Toronto (given OD pattern and network topology), the techniques employed and the analysis conducted (Global Efficiency, Importance and Exposure) could be applied to any sufficiently detailed transit network. Furthermore, it is proposed that such detailed analysis is invaluable for examining the impact of transit service improvement and disruption management techniques. It should be noted that the measure of resilience proposed here is one with a user perspective in mind. It is important to consider that resilience targets need to be applied with caution, and that the operator perspective should be also considered.

The take-away points from this work are as follows:

- The impact of a service disruption, and by extension the resilience of a network to operational variances, is strongly affected by the presence of parallel high-quality
transit services, ideally with exclusive Right-of-Way (ROW). Resilience is strongly influenced by the spare capacity of available infrastructure.

- Better management of existing capacity can help improve resilience. The Twitter analysis has shown that operators can take an active role in directing demand towards under-utilized routes during a service disruption. These types of practices fall under the concepts of incident management, which operators need to invest in and improve.

- The provision of information to passengers can help mitigate the impact of a service disruption. Information provision is relatively inexpensive and can greatly improve passenger opinion on public transit.

- As shown in the analysis of Exposure, the concept of resilience is both spatial and temporal in nature. The same operational disruption impacts different users in profound ways. “Have PDs” and “have not PDs” have been identified in the sense that PDs which lack redundant high-quality transit services are more exposed and by extension less resilient.

- The diversity in both demand, and the impact of a given disruption means that no simple fix can be applied in order to improve resilience for the entire network. Instead, a series of improvements are needed to tackle the complex and spatially variable concept of resilience.

6.1 Future Work

6.1.1 Dynamic Assignment

The most critical improvement needed for this work is the application of these quantitative theories to a dynamic assignment model. The static nature of EMME prevents the most accurate analysis of the potential and actual impact of operational disruptions. In EMME, prior to assignment, a trip-maker is aware of the service disruption modelled, as a result they re-route themselves accordingly. In the same stream, a disruption must also be static in nature. The suspension of service must last for the entire period of analysis, an unrealistic assumption given
how disruptions occur in reality. Relaxing the static hard limit would allow for the inclusion of transit service which is implemented in the event of a disruption, i.e. shuttle bus services.

An improvement to this model would involve probabilistic occurrence of service disruptions of varying degrees. For example, only service suspensions were examined in this work. The vast majority of operational disruptions on the TTC are delays of varying length due to a number of issues, such as Passenger Assistance Alarms. There would be considerable value in modelling the stochastic nature of the more frequent minor delays in addition to major service disruptions. From this a more complete and accurate picture of network resilience can be determined.

6.1.2 Incorporating Surface Routes and Disruption Superposition

There would be value in examining the impact of surface route disruptions in addition with those of the subway system. In particular, loss of service of a streetcar route can have profound impacts on trip-maker travel times. There exists a non-zero probability that disruptions or delays can occur in parallel. Exceptional cases may involve major service suspensions on more than one line of the TTC.

6.1.3 Improved models of Disrupted Behavior

There exists little research on the behavior of transit trip-makers during disrupted conditions. As a result, most models apply the same set of decision making algorithms to the disrupted case, as they do to status quo situations. It is without a doubt that during a disruption, trip-maker perception to different mode and route alternatives change, as well as their willingness to abandon trips or alter tours. The need to incorporate these behavioral differences is paramount to obtaining accurate and realistic results from a study of resilience.
7 Bibliography


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## Appendix A

### Maximum Exposure by Selected Disruption Scenario -

Max increase in travel cost per trip (Trip perceived congested minutes / Trip)

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