
by

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A thesis submitted in conformity with the requirements for the degree of Masters of Applied Science
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Abstract


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The harnessing of solar energy during the operation of an unmanned aerial vehicle (UAV) provides a potential solution to combat the energy constraints. This thesis examines the practicality of a mini solar-assisted UAV and provides experimental validation in regards to energy maximization through solar-electric power management and flight path optimization. A solar-assisted UAV is constructed and shows an increase in flight time. In addition, through the application of power management techniques, an increase in net energy for the solar-assisted UAV is observed. A simulation environment is also developed providing a model for the UAV and estimations for the energy collected and consumed during flight. The simulation results are consistent with real-time measurements during flight tests. Finally, an energy-optimal flight path increasing the net energy is obtained and successfully demonstrated during flight tests.
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Nomenclature

\((\cdot)^*\)  
Optimal state, costate, control vector or matrix

\(\alpha_x\)  
Angle of attack

\(\alpha_{equ}\)  
Right ascension angle

\(\beta\)  
Sideslip angle

\(\delta_{equ}\)  
Declination angle

\(\ell\)  
Characteristic length \(\text{m}\)

\(\ell_{obs}\)  
Latitude of the observer

\(\epsilon\)  
Angle between \(\vec{x}_B\) and the thrust vector

\(\epsilon_{ecl}\)  
Obliquity of the ecliptic

\(\eta_{prop}\)  
Propeller efficiency \(\%\)

\(\eta_{sol}\)  
Solar cell efficiency \(\%\)

\(\gamma\)  
Pitch angle, rotation around \(\vec{y}_B\)

\(\lambda_{ecl}\)  
Ecliptic longitude

\(\mathcal{L}\)  
Lagrangian representing running cost

\(\nu\)  
Kinematic viscosity \(\text{m/s}^2\)

\(\omega_{local}\)  
Local hour angle

\(\phi\)  
Roll angle, rotation around \(\vec{x}_B\)

\(\psi\)  
Yaw angle, rotation around \(\vec{z}_B\)
\( \theta \)  
Solar incidence angle

\( \ell_o(\mathbf{r}(t_f)) \)  
Terminal cost

\( \lambda \)  
Costate vector or matrix, implied time dependence

\( C_1 \)  
Rotation matrix around the \( x \)-axis causing a roll rotation

\( C_2 \)  
Rotation matrix around the \( y \)-axis causing a pitch rotation

\( C_3 \)  
Rotation matrix around the \( z \)-axis causing a yaw rotation

\( C_{BE} \)  
Rotation matrix from \( \mathcal{F}_E \) to \( \mathcal{F}_B \)

\( C_{EB} \)  
Rotation matrix from \( \mathcal{F}_B \) to \( \mathcal{F}_E \)

\( f \)  
System of differential equations describing equations of motion

\( H \)  
Hamiltonian function

\( u \)  
Control vector or matrix, implied time dependence

\( x \)  
State vector or matrix, implied time dependence

\( \omega \)  
Angular velocity vector in the fixed-body frame of reference \(^{\circ}/s\)

\( \mathbf{a}_I \)  
Acceleration vector in the inertial frame of reference \( \text{m}/\text{s}^2 \)

\( \mathbf{F}_{\text{aero},W} \)  
Aerodynamic force vector in the wind frame of reference \( \text{N} \)

\( \mathbf{F}_B \)  
Force vector in the body-fixed frame of reference \( \text{N} \)

\( \mathbf{F}_{G,I} \)  
Gravitational force vector in the inertial frame of reference \( \text{N} \)

\( \mathbf{F}_I \)  
Force vector in the inertial frame of reference \( \text{N} \)

\( \mathbf{F}_{\text{thrust},B} \)  
Thrust force vector in the body-fixed frame of reference \( \text{N} \)

\( \mathbf{v}_B \)  
Velocity vector in the body-fixed frame of reference \( \text{m}/\text{s} \)

\( \mathbf{v}_W \)  
Velocity vector in the wind frame of reference \( \text{m}/\text{s} \)

\( \mathbf{v}_I \)  
Velocity vector in the inertial frame of reference \( \text{m}/\text{s} \)

\( \mathbf{w}_B \)  
Wind velocity vector in the fixed-body frame of reference \( \text{m}/\text{s} \)

\( \mathbf{w}_I \)  
Wind velocity vector in the inertial frame of reference \( \text{m}/\text{s} \)
$F_B$ Matrix of unit vectors in the body-fixed frame of reference

$F_E$ Matrix of unit vectors in the ESF frame of reference

$F_I$ Matrix of unit vectors in the inertial frame of reference

$F_W$ Matrix of unit vectors in the wind frame of reference

$\zeta$ Solar zenith angle

$\zeta_{cor}$ Corrected solar zenith angle

$a$ Solar azimuth angle

$au$ Astronomical unit km

$b$ Span m

$c$ Chord m

$C_D$ Coefficient of drag

$L$ Coefficient of lift

$C_{D_i}$ Coefficient of induced drag

$C_{D_o}$ Coefficient of drag at zero-lift

$D$ Drag force N

$d$ Distance flown m

$e$ Solar elevation angle °

$e_0$ Oswald efficiency factor

$E_{col}$ Energy collected J

$E_{con}$ Energy consumed J

$E_{net}$ Net energy J

$g$ Standard acceleration due to gravity m/$s^2$

$g_{ano}$ Mean anomaly °

$J$ Cost function, implied time dependence
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_{aug}$</td>
<td>Augmented cost function</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Lift force</td>
<td>N</td>
</tr>
<tr>
<td>$L_{lon}$</td>
<td>Mean longitude</td>
<td>°</td>
</tr>
<tr>
<td>$L_{obs}$</td>
<td>Longitude of the observer</td>
<td>°</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the UAV</td>
<td>kg</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of Julian Days since J2000</td>
<td></td>
</tr>
<tr>
<td>$P_R$</td>
<td>Power required</td>
<td>W</td>
</tr>
<tr>
<td>$P_{col}$</td>
<td>Power collected</td>
<td>W</td>
</tr>
<tr>
<td>$P_{con}$</td>
<td>Power consumed</td>
<td>W</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>Nominal power</td>
<td>W</td>
</tr>
<tr>
<td>$P_{sd}$</td>
<td>Solar spectral density</td>
<td>W/m²</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of the Earth</td>
<td>km</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>Planform area</td>
<td>m²</td>
</tr>
<tr>
<td>$S_{sol}$</td>
<td>Planform area covered by solar cells</td>
<td>m²</td>
</tr>
<tr>
<td>$T$</td>
<td>Thrust force</td>
<td>N</td>
</tr>
<tr>
<td>$t$</td>
<td>Time flown</td>
<td>s</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Final time</td>
<td>s</td>
</tr>
<tr>
<td>$t_o$</td>
<td>Initial time</td>
<td>s</td>
</tr>
<tr>
<td>$T_{R}$</td>
<td>Thrust required</td>
<td>N</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity of the UAV</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_{\infty}$</td>
<td>Freestream velocity</td>
<td>m/s²</td>
</tr>
<tr>
<td>$V_{max}$</td>
<td>Maximum velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$V_{\text{stall}}$</td>
<td>Stall velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight of the UAV</td>
<td>N</td>
</tr>
<tr>
<td>$Y$</td>
<td>Side force</td>
<td>N</td>
</tr>
<tr>
<td>$(\hat{x}_B, \hat{y}_B, \hat{z}_B)$</td>
<td>Unit vectors in the body-fixed frame</td>
<td></td>
</tr>
<tr>
<td>$(\hat{x}_E, \hat{y}_E, \hat{z}_E)$</td>
<td>Unit vectors in the ESF frame</td>
<td></td>
</tr>
<tr>
<td>$(\hat{x}_W, \hat{y}_W, \hat{z}_W)$</td>
<td>Unit vectors in the wind frame</td>
<td></td>
</tr>
<tr>
<td>$(s_{B_1}, s_{B_2}, s_{B_3})$</td>
<td>Components of $\vec{s}_B$</td>
<td></td>
</tr>
<tr>
<td>a-si</td>
<td>Amorphous silicon</td>
<td></td>
</tr>
<tr>
<td>AM</td>
<td>Air mass</td>
<td></td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society of Testing and Materials</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Capacitor</td>
<td>F</td>
</tr>
<tr>
<td>CIGS</td>
<td>Copper indium gallium selenide</td>
<td></td>
</tr>
<tr>
<td>CR1</td>
<td>Output diode</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Duty cycle</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Day number in date</td>
<td>m</td>
</tr>
<tr>
<td>EPP</td>
<td>Expanded polypropylene</td>
<td></td>
</tr>
<tr>
<td>ESF</td>
<td>Earth-surface-fixed (frame of reference)</td>
<td></td>
</tr>
<tr>
<td>FSC</td>
<td>Flight System and Controls Lab at the University of Toronto Institute for Aerospace Studies</td>
<td></td>
</tr>
<tr>
<td>GMST</td>
<td>Greenwich Mean Sidreal Time</td>
<td>hr</td>
</tr>
<tr>
<td>hr</td>
<td>Hour in UTC</td>
<td></td>
</tr>
<tr>
<td>$i_a$</td>
<td>Current flowing through node a</td>
<td>A</td>
</tr>
<tr>
<td>$i_c$</td>
<td>Current flowing out of node c</td>
<td>A</td>
</tr>
<tr>
<td>$I_{mp}$</td>
<td>Current at nominal power, $P_{\text{max}}$</td>
<td>A</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
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</tr>
<tr>
<td>$I_{sc}$</td>
<td>Short circuit current</td>
<td>A</td>
</tr>
<tr>
<td>I-V</td>
<td>Current-voltage</td>
<td></td>
</tr>
<tr>
<td>J2000</td>
<td>Reference date from January 1, 2000 at 12:00 terrestrial time or 11:58:55.816 UTC</td>
<td></td>
</tr>
<tr>
<td>jd</td>
<td>Julian Day</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Inductor</td>
<td>H</td>
</tr>
<tr>
<td>LMST</td>
<td>Local Mean Sidereal Time</td>
<td>°</td>
</tr>
<tr>
<td>m</td>
<td>Numerical month in the Gregorian calendar</td>
<td></td>
</tr>
<tr>
<td>min</td>
<td>Minutes in UTC</td>
<td></td>
</tr>
<tr>
<td>MOSFET</td>
<td>Metal–oxide–semiconductor field-effect transistor</td>
<td></td>
</tr>
<tr>
<td>MPPT</td>
<td>Maximum point power tracking</td>
<td></td>
</tr>
<tr>
<td>PSA</td>
<td>Plataforma Solar de Almería (algorithm)</td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td>MOSFET</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>Output resistance, load resistance</td>
<td>Ω</td>
</tr>
<tr>
<td>$R_{C}$</td>
<td>Equivalent resistance connected to the capacitor C</td>
<td>Ω</td>
</tr>
<tr>
<td>$R_{L}$</td>
<td>Equivalent resistance connected to the inductor L</td>
<td>Ω</td>
</tr>
<tr>
<td>sec</td>
<td>Seconds in UTC</td>
<td></td>
</tr>
<tr>
<td>SMPS</td>
<td>Switched mode power supply</td>
<td></td>
</tr>
<tr>
<td>$T_s$</td>
<td>Switching frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned aerial vehicle</td>
<td></td>
</tr>
<tr>
<td>UTC</td>
<td>Coordinated Universal Time</td>
<td></td>
</tr>
<tr>
<td>$V_I$</td>
<td>Input voltage</td>
<td>V</td>
</tr>
<tr>
<td>$V_O$</td>
<td>Output voltage</td>
<td>V</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>$V_{mp}$</td>
<td>Voltage at nominal power, $P_{max}$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{oc}$</td>
<td>Open circuit voltage</td>
<td>V</td>
</tr>
<tr>
<td>$y$</td>
<td>Year</td>
<td></td>
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</table>
Chapter 1

Introduction

1.1 Motivation

Over the past decade, there has been a resurgence of interest in unmanned aerial vehicles (UAVs) due to technological advances in aeronautics and astronautics. As sensors, motors and other control instruments become more precise, smaller and lighter, the number of applications for UAVs continues to expand. From applications including military reconnaissance drones to mimicking biologically inspired flight techniques and formations, it is no surprise that UAV research has been embraced by both industry and academia. Since UAVs provide a low risk, affordable and repeatable flight environment when compared with manned aerial vehicles, UAVs may also serve as an initial platform for new technologies, which may then be applied to commercial applications.

The operation of UAVs is often limited by on-board energy, typically from fuel or electrical sources. This holds particularly true for endurance missions such as reconnaissance or surveillance. As such, there is an increasing need to explore renewable and sustainable energy sources in aviation. Due to the elevated altitudes aerial vehicles operate at, one natural resource that is abundant and can be harnessed is solar energy.

Mini UAVs are garnering increased interest due to their size, cost and manoeuvrability that allow them to be used in situations not feasible for larger size UAVs [1]. However, solar-powered or solar-assisted mini UAVs present different constraints and unique challenges when compared to the heavily researched solar High Altitude Long Endurance (HALE) UAVs [2, 3]. While integrating solar energy to HALE UAVs has been shown to be positive, whether the same benefits are present when extending to the mini UAV classification is uncertain.
Chapter 1. Introduction

1.2 Literature Review

1.2.1 Solar UAVs

There has been increased interest in applying photovoltaic (PV) modules to UAVs in both the academic community and industry. The first recorded instance of a solar-powered aircraft was Sunrise I in November 1974, designed by R.J. Boucher from Astro Flight, Inc, funded by the Advanced Research Projects Agency (ARPA) [4]. With a wingspan of 9.75 m and weight of around 8.6 kg, Sunrise I flew at an altitude of 100 m for 20 minutes [5]. Since then, with improvements in aircraft designs, materials and PV modules, solar UAVs have been able to fly through the night. This allows for sustainable, perpetual flight. The first recorded UAV to fly through the night is AC Propulsion’s SoLong UAV with a wingspan of 4.75 m and weight of 12.8 kg and used a combination of lithium-ion batteries and Sunpower A300 solar cells [6]. The SoLong UAV was able to fly for 48 continuous hours. Today, QinetiQ’s Zephyr with a wingspan of 22.5 m and weight of just over 50 kg has been able to fly at altitudes over 20 km for 14 days [7]. A list of solar powered aircrafts flown to date and their specifications are neatly compiled in Noth’s PhD thesis [5].

A number of papers have been published in regards to the design methodology for solar powered aircraft. In Morrisey’s Master’s thesis [8], a Design Structure Matrix optimization method is used as a multidisciplinary tool to justify existing high altitude, long endurance, high aspect ratio UAVs such as NASA’s Helios [9]. Roberts et al. [10], Boldock et al. [11], and Phillips [12] provide additional methods of optimizing aircraft configuration once aerodynamic, structural, propulsive, and energetic factors have been determined. However, there is a lack of study and experimentation on solar powered aircraft in the mini size category. One noteworthy implementation towards this category is the MikroSol, NanoSol and PicoSol designed by Dr. Sieghard Dienlin [13]. While not much information is available on these aircraft, the smallest of the three, the PicoSol, had a wingspan of 0.99 m and weight of 0.127 kg. The earlier iteration, the NanoSol was reported to deliver 8.64 W of solar power. A study on the design of micro solar-powered aircraft was done by Wilson et al. [14]. In this paper, Wilson et al. conclude that at the small and micro scale, it seems likely that solar power can only be used an auxiliary power source and not the primary, sole power source. The viability of solar-assisted flight for a mini UAV will be explored in this thesis.
1.2.2 Power Management

The management of power for hybrid systems is a problem that is receiving greater attention and more research needs to be done as it is becoming more desirable to have multiple power sources in vehicles. Lee et al. [15] used a power management system utilizing both passive and active power control logic to handle power issues that an UAV using PV arrays, fuel cells and batteries may encounter. During regular operation, only fuel and solar power were utilized; power from the battery was only used during cycles of peak power, such as take-off and transient flight. Active control was applied to land the UAV in scenarios where either the fuel or solar cell is failing or the battery charge is running low. This resulted in a more efficient power distribution and better system safety. Chen et al. [16] maximized endurance by creating a hybrid energy storage system that managed the recharging of the fuel cell and battery with solar power. Shiau et al. [17] outlined and experimentally verified the design of a power management system consisting of modules for maximum power point tracking (MPPT), battery charging and power conversion. This thesis seeks to integrate power management techniques in the planning of an energy-optimal flight path for a solar-assisted UAV and design a platform that would allow the investigation of issues when managing a solar and electrical system.

1.2.3 Energy-Optimal Path Planning

By far the most popular approach to solve the energy-optimal path planning problem is to maximize the terminal battery energy, or equivalently to minimize the exhausted energy, by comparing changes in energy. Terminal battery energy refers to the remaining energy stored in the batteries at the end of the flight. Often, the contributions and losses of different energy sources are treated separately as unique modules. Examples of these modules could include the energy consumed in flight operation, the energy lost in the avionics and electronics, or the energy generated by solar power. The solution for a time-energy optimal path for solar powered UAV was studied extensively by Klesh [18]. Klesh separated the energy consumed and collected to maximize the total energy of the aircraft. The sensitivity and difficulty in solving the resulting nonlinear optimization problem have been addressed by multiple methods, including re-parametrization of the equations of motions to quaternions [19] and using a grid map to perform grid search [20].

Alternative approaches towards finding an energy-optimal path include minimizing thrust [21, 22], minimizing time for vehicles of constant thrust [23, 24] and maximizing energy change with respect to distance [25]. Research using probabilistic graph and mapping methods has also been used to find energy optimal paths by maximizing the
likelihood to find thermal energy sources \cite{26}, applying spatial cost-to-go maps \cite{27} and optimizing a designed performance index in a directed graph \cite{28}. All these methods have been used towards the search of an energy-optimal path for a generic vehicle such as a ground rover, quad-rotor or underwater vehicle. Given the studies that will be performed on power management, the method of maximizing the terminal battery energy is chosen due to the sensors used and ability to log energy data.

\section*{1.3 Research Objectives}

The literature survey above has shown success in using PV arrays to significantly extend the endurance of large scale UAVs, capable of flying at high altitudes, toward achieving perpetual flight. To this end, the design, wing configuration and geometry are optimized in an effort to minimize induced drag and to maximize the surface area available for mounting solar panels respectively. For the same aircraft configuration and flight path, a reduction in induced drag leads to lower overall fuel consumption. However, there is a gap in the research towards smaller size, mini UAVs.

The goal of this thesis is to provide experimental validation for solar-electric power management and flight path optimization towards maximizing the solar energy contribution of a solar-assisted mini UAV. Four research objectives are designed to achieve this goal. The first is the demonstration of the worthiness of installing PV arrays on a mini UAV. The second is the reduction of energy lost in the mini solar-assisted UAV through solar-electric power management. The third is the development and verification of a solar-assisted UAV model through simulation and experiments. The fourth is the design and validation of an energy optimal flight path through simulation and flight tests.

\section*{1.4 Thesis Outline}

This thesis is divided into five chapters. The first chapter provides the motivation behind this thesis and provides a brief literature survey of relevant work in the field of solar UAVs, power management and energy-optimal path planning. Chapters 2 to 5 separately address the four research objectives previously mentioned. Chapter 2 provides a high level design of the UAV and the viability of retrofitting an existing mini UAV with solar panels. The experimental set-up including justifications for the selection of aircraft configuration, solar panels, power converters and sensors is also presented. Chapter 3 demonstrates using a test bench the energy characteristics of the UAV system. Studies on impedance matching were also performed on the test bench resulting in a reduction
in energy lost. Chapter 4 develops the solar UAV model in two and three dimensions, the environment model as well as validating these models through flight tests. Chapter 5 justifies the need for path planning, explores the energy-optimal path planning problem and validates these paths through flight tests. Finally, Chapter 6 provides a summary of the project and discusses possible extensions for future research.
Chapter 2

Viability of a Solar-Assisted Mini UAV

This chapter addresses the first research objective of demonstrating the worth of installing PV arrays on a mini UAV. A high level, conceptual design of a solar-assisted mini UAV is presented. The conceptual design reveals that current, market available PV arrays can generate a significant portion of the energy required to operate a mini UAV. Next, the selection of the UAV configuration, photovoltaic cell and power conversion method used for the final design is outlined. Finally, a test bench of the solar-assisted mini flying wing UAV supported the objective by achieving a longer operation time compared to a scenario without PV arrays.

2.1 High Level Design

The problem of developing and validating an energy-optimal path planning algorithm through a solar-assisted mini UAV described in Chapter 1 can be functionally and physically decomposed into several sub-problems. Functionally, this involves the demonstration of the viability of a solar-assisted mini UAV, the development of an UAV model that incorporates the collection of solar energy, and the application of the aforementioned model to a path planning algorithm to determine an energy-optimal path. These problems are addressed in Chapter 2, Chapter 4, and Chapter 5, respectively. Physically, the solar-assisted mini UAV is comprised of the aircraft, the autopilot, the photovoltaic panels, the power conversion module, and the required sensors and instruments.

In order to demonstrate the viability of using solar energy for aircraft flight, the amount of solar power collected is compared with an approximation for the power required. The aircraft parameters used for this approximation are those of the mini UAV
developed by the Flight System and Controls (FSC) Lab at the University of Toronto Institute for Aerospace Studies and are summarized in Table 2.1. From [29], a crude approximation for power required for level, unaccelerated flight is

\[
P_R = \frac{T_R V_\infty}{\eta_{\text{prop}}} = \frac{1}{\eta_{\text{prop}}} \frac{W}{C_L/C_D} V_\infty = \frac{1}{\eta_{\text{prop}}} \frac{mg}{C_L/C_D} V_\infty, \tag{2.1}
\]

where \(P_R\) is the power required, \(T_R\) is the thrust required, \(V_\infty\) is the freestream velocity for the level, unaccelerated flight, \(\eta_{\text{prop}}\) is the propeller efficiency, \(W\) is the aircraft weight, \(C_L\) is the coefficient of lift, \(C_D\) is the coefficient of drag, \(m\) is the aircraft mass, and \(g\) is the standard acceleration due to gravity.

The coefficients of lift, \(C_L\), and drag, \(C_D\), can be obtained through polar graphs given the Reynolds Number and airfoil profile. The Reynolds Number is defined as

\[
\text{Re} = \frac{V_\infty \ell}{\nu}, \tag{2.2}
\]

where \(\ell\) is the characteristic length, and \(\nu\) is the kinematic viscosity.

Assuming a freestream velocity, \(V_\infty\), of 15 m/s and a temperature of 20 °C, the resulting Re is approximately 500 000 as calculated by (2.2). Furthermore, assuming the airfoil profile follows that of the Martin Hepperle MH60 airfoil and is constant spanwise, the pressure distribution over the airfoil is computed using XFOIL. XFOIL is an open source program that is capable of performing subsonic analysis of airfoils, obtaining aerodynamic parameters, such as the pressure distribution, given the geometry and angle of attack. The drag polar and curves for the resulting lift and drag coefficients at varying angles of attack are given by Figures 2.1a, 2.1b, and 2.1c respectively. The power required curve at \(\alpha = 0^\circ\) in Figure 2.1d is obtained by extrapolating \(C_L\) and \(C_D\) at \(\alpha = 0^\circ\) and substituting into (2.1) to provide a preliminary estimate of the power required for level, unaccelerated flight at \(\alpha = 0^\circ\) at various velocities. At the selected freestream velocity of 15 m/s, a crude approximation for the power required for level, unaccelerated flight of the FSC UAV, with parameters in Table 2.1, using (2.1) is 176 W.

The power obtained from photovoltaic cells can be expressed as

\[
P_{\text{col}} = \eta_{\text{sol}} P_{sd} S_{sol} \cos \theta, \tag{2.3}
\]
where $P_{col}$ is the power collected from the solar cells, $\eta_{sol}$ is the solar cell efficiency, $P_{sd}$ is the solar spectral density, $S_{sol}$ is the wing area covered by solar cells, and $\theta$ is the solar incidence angle with derivation in Section 4.2. Figure 2.2 shows the estimated power obtained from the solar cells at various solar cell efficiencies with the following assumptions:

1. The solar spectral density is taken as the average value for 12:00 noon in August 2011 in Toronto, Canada [30] ($P_{sd} = 900 \text{ W/m}^2$)
2. No loss in power conversion
3. Sun directly above solar cells ($\theta = 0^\circ$)
4. Solar cells cover the entire FSC UAV ($S_{sol} = S$)
As of 2014, the National Renewable Energy Laboratory of the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy records solar cell efficiencies from 13.4% in thin-film technologies to 44.7% in multijunction cells as seen in Figure 2.3 [31]. This means that for the current application of using solar cells with the FSC UAV, the potential power collected could be up to 77 W and 257 W for leading edge thin-films and multijunction cells respectively. The approximations for power collected are obtained using (2.3) with the four assumptions. However, readily available solar cells have a lower efficiency, such as the 21.5% Sunpower A300 monocrystalline panels used on satellites and solar cars and the 10.5% Ascent Solar copper indium gallium selenide (CIGS) thin-films for commercial applications. These provide a more realistic estimation of 124 W for the monocrystalline panels and 60 W for the CIGS thin-films. Taking the best cell efficiencies in 2014, multijunction cells with an efficiency of 44.7% provide all the energy needed for level, unaccelerated flight of the FSC UAV and would easily result in the ability for perpetual solar flight. Contrastingly, while the commercial Ascent Solar cells provide substantially less energy, they still provide 34.09% of the power required. With the added benefit of energy-based path planning, this preliminary comparison between the energy required and energy collected show an undeniable contribution resulting from the solar cells.
Chapter 2. Viability of a Solar-Assisted Mini UAV

2.2 Experimental Set-up

2.2.1 Overview

The UAV used for experimental flight tests is the University of Toronto Flight Systems and Controls (FSC) UAV shown in Fig. 2.4, a flying wing UAV in the mini class with main aircraft parameters summarized in Table 2.1. The FSC UAV is retrofitted with CIGS thin-film photovoltaic cells that are 9% efficient and connected in parallel to a Genasun GV-5 converter, a 94% to 99.85% efficient maximum point power tracking (MPPT) buck-boost converter with a tracker efficiency of over 99%. In addition, the FSC UAV houses a Kestral™ Flight Systems autopilot that allows for autonomous flight and way-point navigation.

2.2.2 UAV Configuration

The selection of the flying wing as the UAV configuration was driven by functionality and usability. Functionally, the flying wing typically has no horizontal tail or definitive fuselage. Structurally, the removal of the tail and dedicated fuselage means less weight is needed when compared to a conventional aircraft with similar lift characteristics. The reduction in weight results in a reduction of energy required. The reduction of this weight
Figure 2.4: Picture of the Flight Systems and Controls Zagi UAV used as the experimental platform for this thesis

Table 2.1: Parameters of FSC solar-assisted mini UAV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ (m)</td>
<td>1.52</td>
</tr>
<tr>
<td>AR (m)</td>
<td>4.6</td>
</tr>
<tr>
<td>$S$ ($m^2$)</td>
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</tr>
<tr>
<td>$c$ (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>$V_{\text{stall}}$ (m/s)</td>
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</tr>
<tr>
<td>$V_{\text{max}}$ (m/s)</td>
<td>27.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ (kg)</td>
<td>2.25</td>
</tr>
<tr>
<td>$e_0$</td>
<td>0.931</td>
</tr>
<tr>
<td>$C_{D_w}$</td>
<td>0.013</td>
</tr>
<tr>
<td>$\eta_{\text{prop}}$ (%)</td>
<td>10</td>
</tr>
<tr>
<td>$S_{\text{sol}}$ ($m^2$)</td>
<td>0.0894</td>
</tr>
<tr>
<td>$\eta_{\text{sol}}$ (%)</td>
<td>9</td>
</tr>
</tbody>
</table>

also results in lower wind loading. A lower wind loading leads to lower bending moments and hence less structural support will be needed. Aerodynamically, the lift to drag ratio is increased. The elimination of the non-lift producing surfaces decreases the wetted aspect ratio, which reduces drag. In addition, since the entire body now acts as a lift producing surface, the planform area and corresponding lift increases.

There are also some characteristics of flying wings that make them more usable in
this specific application. Flying wings are chosen due to their high manoeuvrability when compared to conventional winged aircraft. This in turn makes the control of flying wings more forgiving than conventional aircraft, especially being able to adjust to gust in windy conditions. The flying wing also typically has a larger chord due to the payload area being in the wing. For an aircraft with a similar aspect ratio, a larger chord results in a larger planform area and thus a larger surface to mount solar panels. Flying wings are also forgiving in terms of structural strength especially if constructed out of expanded polypropylene (EPP) foam. Since many tests will be performed using the aircraft, a platform that is less susceptible to breaking and repair is ideal. Most flying wings are pushers with the motor and propeller in the back. Due to the swept nature of flying wings, the chance of damaging the propeller or motor is lower than a conventional aircraft. Similarly, since there are less protruding components on a flying wing when compared to a conventional aircraft, the flying wings are less vulnerable to damage.

2.2.3 Solar Panels

CIGS thin films provide several advantages over traditional silicon based cells that make them favourable for UAVs. First, the copper, indium, gallium, and selenide is effective over a larger range of wavelengths in the solar spectra. Figure 2.5 compares the relative spectral responses of different solar cell materials in relation to the normalized AM1.5 spectrum. The AM1.5 spectrum is a standard reference spectra in the photovoltaic industry published by the American Society of Testing and Materials (ASTM) that quantifies the terrestrial global solar irradiance distribution in W/(m²·nm) at an absolute air mass of 1.5. Global irradiance includes both the direct component, straight from the sun, and diffuse radiation, such as scattering from clouds and ground reflection. By selectively mixing the material in the photovoltaic module, CIGS are capable not only of drawing radiation from the visible light spectrum (390 nm - 700 nm), but the near-infrared spectrum (700 nm - 1400 nm) as well when compared to amorphous silicon (a-Si) thin films. The larger absorption range of CIGS cells when compared to silicon cells results in the ability to generate power for longer time periods as well as low light or cloudy conditions. This is ideal for the operation of an UAV where environment conditions are bound to change during flight. Second, thin film photovoltaic cells, in particular CIGS cells, are an attractive solution for UAVs since CIGS cells have an absorption performance similar to mono- and poly-crystalline silicon cells, as seen in Figure 2.5, without the expensive cost, rigidity, thickness and weight. Depending on the construction of the module and percentage composition of copper, indium, gallium
and selenide, different spectral responses can be obtained, as shown by the three CI(G)S modules in Figure 2.5. Lower weight photovoltaic cells would decrease drag and energy required, while the thinness and flexibility of CIGS cells would reduce the aerodynamic disturbance of adding the cells on the wing.

Figure 2.5: Normalized spectral response for different types of solar modules (including three different CI(G)S modules) [32]

The CIGS thin films selected for the FSC UAV were the Ascent Solar WaveSol WSME-0045 panels shown in Figure 2.6. The WSME-0045 panels were measured to have a maximum efficiency of 9 percent and their load characteristics shown in Figure 3.1 in Chapter 3.

2.2.4 Power Conversion

The power converter selected for the FSC UAV is the Genasun GV-5 converter and the key features are summarized in Figure 2.7. The GV-5 is a buck-boost converter with a Maximum Power Point Tracking (MPPT) controller designed to charge lithium batteries. This particular converter was selected for two reasons. First, the Genasun GV-5 is designed for solar panel applications and has a MPPT controller. The importance of the MPPT controller is explained in Chapter 3. For applications such as UAVs that are energy limited, getting as much power from the PV modules is crucial. Second, several
Chapter 2. Viability of a Solar-Assisted Mini UAV

### Rated Specifications

<table>
<thead>
<tr>
<th></th>
<th>Values [33]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Power, $P_{max}$</td>
<td>4.5 W</td>
</tr>
<tr>
<td>Voltage at $P_{max}$, $V_{mp}$</td>
<td>18.5 V</td>
</tr>
<tr>
<td>Current at $P_{max}$, $I_{mp}$</td>
<td>0.243 A</td>
</tr>
<tr>
<td>Open Circuit Voltage, $V_{oc}$</td>
<td>25.5 V</td>
</tr>
<tr>
<td>Short Circuit Current, $I_{sc}$</td>
<td>0.295 A</td>
</tr>
<tr>
<td>Mass</td>
<td>0.105 kg</td>
</tr>
<tr>
<td>Dimensions</td>
<td>0.344 m \times 0.170 m</td>
</tr>
</tbody>
</table>

Figure 2.6: Ascent Solar WaveSol WSME-0045 CIGS thin film

Other research groups have successfully used a similar converter when implementing solar panels on a conventional wing aircraft [34], [35].

### Key Features

<table>
<thead>
<tr>
<th></th>
<th>Values [36]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Voltage Range</td>
<td>0 - 27 V</td>
</tr>
<tr>
<td>Continuous Rated Load Current</td>
<td>5 A</td>
</tr>
<tr>
<td>Converter Efficiency</td>
<td>94 - 99.85 %</td>
</tr>
<tr>
<td>Tracking Efficiency</td>
<td>99 + %</td>
</tr>
<tr>
<td>Operating Consumption</td>
<td>0.150 mA</td>
</tr>
<tr>
<td>Mass</td>
<td>0.080 kg</td>
</tr>
<tr>
<td>Dimensions</td>
<td>0.110 m \times 0.560 m \times 0.250 m</td>
</tr>
</tbody>
</table>

Figure 2.7: Genasun GV-5 solar power converter

#### 2.2.5 Sensor Selection

The FSC UAV uses the Kestral™ Autopilot for autonomous flight control and waypoint navigation capabilities. The autopilot has an 8-bit 29 MHz processor that measures...
the logs the states of the aircraft. This includes altitude, airspeed, roll, pitch, heading, temperature and GPS positions. The autopilot and block diagram is shown in Figure 2.8 and 2.9. The relative location of the autopilot on the FSC UAV is labelled as A in Figure 2.10.

Figure 2.8: Kestral™ Autopilot [37]

Figure 2.9: Kestral™ block diagram [37]

Electrical measurements are measured in real-time using an Arduino set-up with current sensors and XBee-PRO ZigBee radio frequency modules. The current sensor for the two lithium polymer batters is the Allegro ACS758 rated to 50 A. A lower rated, higher resolution current sensor (Allegro ACS714) is used for the solar panels. Voltages from the batteries and solar panels were obtained using a voltage divider. All these were fed into the analog inputs of an Arduino Uno with an 8-bit ATmega328 microcontroller and relayed to the ground station in real time using the XBee-PRO ZigBee embedded radio frequency modules. The relative locations of all these components on the FSC UAV are shown in Figure 2.10. This configuration of sensors allows for current and
Figure 2.10: Location of sensors on the FSC UAV
A - Kestral™ Autopilot, B - Lithium polymer batteries, C - ACS758 current sensors, D - GV-5 power converter (underneath), E - ACS714 current sensor, F - Arduino Uno, G - Intermediary circuit with voltage dividers, H - X-Bee PRO Zigbee RF module

voltage tracking in real-time for each power source. Consequently, the effects of power management studies discussed in Chapter 3 can be measured.

2.3 Test Bench Demonstration

A test bench was designed and constructed to mimic the loads the photovoltaic modules, batteries, converters and motor would experience during flight. Whereas the final FSC MUAV uses two photovoltaic modules and one MPPT power converter, the test bench allows for configurations of up to four photovoltaic modules and four converters. A picture of the test bench with four photovoltaic modules and two converters is shown in Figure 2.11a. The AXI2820/10 motor and 10x6 propeller were controlled by an electronic speed controller (ESC), which in addition to controlling the speed of the motor, provided low voltage protection for the two lithium polymer batteries. The signal for the ESC was provided using a servo tester instead of a radio receiver so that the pulses sent
Chapter 2. Viability of a Solar-Assisted Mini UAV

(a) Constructed test bench

(b) Block diagram

Figure 2.11: Test bench and block diagram replicating FSC MUAV

to the motor could be accurately repeated. Each battery pack voltage, current and power were individually monitored in real-time using separate Eagle Tree Systems eLoggers V4. The eLoggers were an early iteration of the Arduino set-up described in Section 2.2.5. Two WSME-0045 panels were connected in parallel with each battery. Because of the limited input voltage range of the Genasun GV-5 controllers, the two PV modules were placed in parallel instead of in series. As such blocking diodes were placed in series with the positive terminal of each module, as shown in Figure 2.11b. Blocking diodes prevent batteries from dumping charge into the panels in low light conditions, like during shade conditions or at night. 1N5819 ON Semiconductor Schottky barrier rectifier diodes were used as Schottky diodes have low conduction and switching losses and are extremely fast when compared to silicon diodes. A block diagram of the test bench is shown in Figure 2.11b.

A study was performed to demonstrate the promising benefits in retrofitting the FSC mini UAV with photovoltaic modules. The study makes use of the low voltage protection in the ESC in order to compare the maximum test duration with and without solar modules. Aside from the presence of solar modules, all other conditions including panel orientation, converter, batteries and motor speed were kept constant throughout each test. The solar illumination for each test ranged from 80,000-100,000 lx.

The study compared the maximum operation time of the system. Using a cut-off voltage of 9.7 V provided from the ESC, the test bench was operated at a constant pulse width of 1.6 ms at 50 Hz, representative of the motor operating at 60% throttle. A constant pulse width signifies that the input load of the motor is constant and as a result, so too are the revolutions and speed of the motor. Figure 2.12 shows the operation time of the test bench before the battery voltage reaches the minimum of 9.7 V. The maximum
operation time of the test bench with solar modules was about 2 min longer, representing a 10% extension from the duration for the operation of the test bench without solar modules. Figure 2.13 shows the total charge used by the motor at 60% throttle. The solar modules provided on average an additional 0.2 Ah, which equates to either prolonging the operation of the test bench for the same battery capacity or reducing the battery capacity discharged for the same operation time. This difference is expected to be larger during flight tests as the throttle will not be constant throughout. The presence of regions with low throttle will result in a reduction in motor load and a larger impact from the solar modules. In addition, the higher altitude will reduce the effects of shadows from trees and buildings, resulting in a larger solar density.

![Figure 2.12: Maximum operation time at 60% throttle](image)

![Figure 2.13: Total discharge at 60% throttle](image)

### 2.4 Summary

In this chapter, a high level analysis of the viability of using photovoltaic cells in relieving the energy required to operate a mini UAV was performed. Also, the selection of UAV configuration, photovoltaic cells, power conversion technique, and sensor selection for the solar-assisted FSC UAV was outlined. Finally, results from a test bench study supported the high level analysis that CIGS thin-films could provide significant contributions to a mini UAV. While the benefit is limited for the mini scale, with the addition of power management and energy-conscious path planning, the expected benefit is much greater.
Chapter 3

Solar-Electric Power Management

In the previous chapter, the high level conceptual design and test bench studies showed the energy contributions of the PV arrays to a mini UAV. This chapter addresses the second research objective of reducing the energy lost in the solar-assisted mini flying wing UAV through solar-electric power management. Specifically, the energy lost in three areas were examined: the conversion of solar energy to electrical energy, the interactions between lithium polymer batteries, and the interaction between each PV array and converter.

3.1 Converting Solar to Electrical Power

A power converter, such as power supplies or regulators, is a circuit that converts a voltage to another. One of the main advantages of a switched mode power supply (SMPS) over a common linear regulator is that a SMPS can convert large voltages at high efficiency [38]. By arranging switches, inductors, and capacitors in different configurations, any voltage can virtually be achieved. The three fundamental SMPS topologies are the buck converter, the boost converter and the buck-boost converter. The buck converter steps down the input voltage, the boost converter steps up the input voltage and the buck-boost inverts the input and can either step up or down the magnitude of the input. For ideal converters, only the duty cycle, defined as the rate at which the MOSFET is switched on and off, is needed to control the conversion of the input voltage.

In PV applications, the I-V characteristics of the PV panels are determined by the load, irradiance and temperature. The electrical properties of four WSME-0045 are shown through the measured I-V curve in Figure 3.1. Characterization of each panel was done by connecting each panel to a variable power resistor. Experiments were performed
in an indoor lab environment of 25°C illuminated by fluorescent lights with a rated luminous efficacy of 104 lm/W and a measured illuminance of 1,000 lx. Differences in electrical properties of the CIGS panels can be attributed to the variance in the material properties and manufacturing process. In Figure 3.1, Panels 3 and 4 produce notably less power under the same irradiance as panels 1 and 2. For panel 1, the most efficient panel, the maximum power is noted at 0.382 W for fluorescent lights at 1,000 lx, which results in a maximum efficiency of 9%. This will be the efficiency used for the photovoltaic models for the remaining thesis.

![Figure 3.1: Measured I-V characteristics of WSME-0045 under fluorescent lighting](image)

As seen in Figure 3.1, the voltage provided by the PV modules can vary. While not reached in the experiments, this voltage can range anywhere between 0 V to the rated open circuit voltage, 25.5 V in the case of the WSME-0045. In order for the converter to operate over the entire voltage range, a buck-boost converter topology was selected. A schematic of a typical buck-boost converter is shown in Figure 3.2a. The desired output is obtained by using a drive circuit to vary the duty cycle applied to the metal-
oxide-semiconductor field-effect transistor (MOSFET) Q1. L and C, along with their equivalent resistances $R_L$ and $R_C$, make up the inductor and capacitor filter for the output respectively. CR1 is the output diode. In continuous conduction mode, the behaviour of the converter can be separated into two states: the ON state and OFF state. During the ON state, the MOSFET is on, equivalently acting like a short as seen in Figure 3.2b. Since the diode CR1 is reverse biased, current only flows through the inductor in the direction indicated by $i_a$. Assuming the input voltage $V_I$ is constant, the inductor charges linearly, while the output voltage is supplied by the capacitor C. During the OFF state, the MOSFET is off and the equivalent circuit is shown in Figure 3.2c. Since the charge in the inductor cannot instantaneously drop to 0, the inductor charges the capacitor in the direction indicated by $i_c$, opposite to $i_a$. During this state the diode CR1 becomes forward biased. The output voltage is supplied by the capacitor C which is being charged by the inductor L. The resulting phenomenon is that as the MOSFET is on for a longer period of time, the inductor gets more charged. When the converter finally reaches the OFF state, the inductor acts like a voltage source, maintaining a balance between the voltage in the output and the inductor. This behaviour is summarized in the waveforms shown in Figure 3.3a. Through circuit analysis, assuming negligible losses, the output voltage, $V_O$, can be related to the input voltage, $V_I$, by varying the duty cycle, $D$, as described by equation (3.1).

$$V_O = -V_I \times \frac{D}{1-D}$$

The converter enters a discontinuous mode when the inductor fully discharges during the OFF state. There is now a third state after the inductor current and diode current reach zero until the ON state. This results in the output voltage being dependent not only on the duty cycle and input voltage, but the inductance, L, switching frequency, $T_s$, and output resistance, R, as well. This behaviour is summarized in the waveforms shown in

Figure 3.2: States of buck-boost converter [39]
Figure 3.3b and represented, without losses, by equation (3.2).

\[
V_O = -V_1 \times \frac{D}{\sqrt{2L/RT_s}}
\]  

(a) Continuous mode \hspace{1cm} (b) Discontinuous mode

Figure 3.3: Waveforms of a buck-boost converter [39]

Losses in a converter result in reduced efficiencies and are particularly relevant for applications where power is limited, as is the case for UAVs. Five principal means of power loss include: conduction, switching, gate-drive, thermal and magnetic losses. First, conduction losses occur when current flows through the capacitor, inductor, load, diodes and MOSFETs. Conduction losses only occur when the device is on, for example when the diode is forward biased or during the MOSFET ON state. Second, switching losses occur as the switching characteristics of diodes and MOSFETs are not ideal. Switching losses are directly proportional to switching frequency. Third, gate-drive losses are associated to the charging and discharging of the gate of the MOSFET and are independent of the load. Fourth, thermal losses occur when components heat up. In particular, temperature plays a role in increasing the ON resistance of MOSFETs. Fifth, magnetic losses are associated with losses due to hysteresis, eddy current and windings surrounding the magnetic core. While the converter design is out of the scope of this thesis, the identification of potential losses show the importance of carefully selecting a converter suitable for the application
of UAVs with a high efficiency.

In addition to the buck-boost converter, a maximum power point tracking (MPPT) is integrated in the power conversion process in order to ensure the power delivered by the PV panels are at a maximum regardless of the load and irradiance. Without MPPT, the point at which the converters operate at would be driven to match the battery voltage. This is typically not the maximum operating point of the PV panels. The MPPT algorithm samples the PV cells to find the voltage at which maximum power is obtained. The MPPT then forces the converter to operate at this maximum operating point and converts the resulting output voltage to the battery voltage using a high frequency, high efficiency DC-DC converter. For this thesis, a Genasun GV-5 converter with an MPPT controller which is rated at a minimum of 94% converter efficiency, and 99% tracking efficiency. The addition of the MPPT controller is advertised to increase power obtained by 50% [36]. Key features of the Genasun GV-5 were presented in Figure 2.7 in Chapter 2.

A block diagram showing how the power converter and photovoltaic module is connected to the rest of the FSC UAV is shown in Figure 3.4.

3.2 Interaction between Lithium Polymer Batteries

In order to extract the most power from both power sources, a study was performed to examine the power lost during the transfer of power from the batteries to the FSC UAV during flight. The transfer of power is at a maximum when the impedance of the batteries match that of the load. A higher load impedance restricts current flow while a higher source impedance results in power being dissipated as heat.

To study different scenarios in a controlled environment, the load profile experienced during flight was modelled in LabVIEW and replicated using the Agilent 6051A electronic load. The LabVIEW interface and block diagram are shown in Figures 3.5 and 3.6. The following pseudocode outlines this process:

- Opening load impedance data (1)
- Reading load impedance data (2)
- Closing load impedance data (3)
- Save data into array (4)
- Set the first element to be read (11)
- Load n-th element in the array (5)
Figure 3.4: Circuit block diagram of the FSC UAV with PV array
• Send \( n \)-th element to the Agilent 6051A block (10)

• Output \( n \)-th element to desired port (9)

• Increment \( n \) (6)

The interface allows the user to designate the starting time step (13) and desired output port (14), as well as the capability to stop the simulation at any time (17). Blocks (16) is a visual display of the output impedance at a specific time step (15) being sent to the desired port.

Figure 3.5: LabVIEW block diagram to electronically replicate loads using the Agilent 6051A

During ground tests, while the two lithium polymer batteries were fully charged to the same amount before each test, the final charge of each battery after operation were different. This difference indicates the batteries were discharged at different rates during the tests, signifying an inefficient transfer of power. Using the simulation environment, the load shown in Figure 3.7 was performed on four battery pairs and the resulting discharge curves shown in Figure 3.8. The initial battery charge was the same throughout. Battery pair B and C discharged the least capacity, which signifies that the pair had the closest impedance and would be the most efficient pair. Compared with the worst pair, battery pair B and C discharged 0.04 Ah less, or a 24% reduction in energy lost. The battery pair of B and C were used for all future bench and flight tests.
Chapter 3. Solar-Electric Power Management

3.3 Interaction between PV Panels and Converters

Similar to the power lost in pairing two batteries with different impedances together, there is an optimum ratio of panels to converters for each battery that minimizes energy lost. As the WSME-0045 panels produce relatively low currents in the mA range compared to the open circuit voltage at 25.5 V, multiple panels were connected in parallel instead of in series to avoid even larger voltages. The results for one, two and four WSME-0045 panels connected in parallel to charge one lithium polymer battery through the GV-5 converter is presented in Figure 3.9. The tests were performed outdoors during the same day with approximately the same environment conditions at 100,000 lx. The ratio of four panels to one converter experimentally provided the largest total capacity charged, as expected due to having the largest number of panels. However, the one panel to one converter configuration has the largest capacity charged per panel as seen in Fig-

Figure 3.6: LabVIEW interface to electronically replicate loads using the Agilent 6051A

Figure 3.7: Respective load resistances for 10 kHz frequency increases every 1/12 min

Figure 3.8: Battery capacity discharged over 0.5 min for different battery pairs
Figure 3.10. This can be explained with the impedances of each WSME-0045 panel being slightly different, resulting in a less efficient system when trying to match the battery impedance. In order to resolve this, a scenario of four individual WSME-0045 panels connected to their own GV-5 converter was performed. However, since the converters reduced the voltages to different values, there was an even larger impedance mismatch, resulting in the least efficient configuration. A method to solve this in the future could involve designing an external feedback loop to regulate the converter voltages.

![Figure 3.9: Battery capacity charged using PV panels](image1)

![Figure 3.10: Battery capacity charged using PV panels per panel](image2)

In a per panel comparison, the two panel to one converter provides 79%, and the four panel to one converter provides 68% of the current delivered by the one panel to one converter configuration. However, with the additional size and payload needed for the additional converters, for the sake of this thesis, the two to one panel to converter ratio was selected for future experiments.

### 3.4 Summary

In this chapter, the energy lost in the conversion of solar energy to electrical energy, the interactions between lithium polymer batteries and the interactions between the PV panels and converters was reduced. By pairing batteries B and C, a 24% reduction in energy lost was measured compared to the worst battery pair of C and D. In addition, the two panels per one converter configuration showed to be the most energy and performance efficient of the configurations tested. The reductions in energy lost allow for a more optimal usage of the current solar-assisted mini flying wing UAV.
Chapter 4

Solar UAV Model

Having validated the advantage of retro-fitting a mini flying wing UAV with solar panels in Chapter 2, this chapter addresses the third research objective of developing a model for the solar-assisted mini UAV and verifies the energy collected from the photovoltaic cells and energy consumed during flight through experimental flight tests.

4.1 UAV Model

The UAV model serves to describe the motions of the UAV. In order to properly develop the equations of motions, four reference references are introduced: the inertial frame of reference, the Earth-surface-fixed (ESF) frame of reference, the body-fixed frame of reference, and the wind frame of reference.

The inertial frame of reference, $\mathcal{F}_I$, is one in which the position space is isotropic and both the position and time space are homogeneous. In $\mathcal{F}_I$, the motion is described by Newton’s first law, or Newton’s law of inertia, whereby an object remains at rest or in uniform motion in a straight line unless there is a force acting upon it. In addition, Newton’s other two laws of motion are only valid in an inertial frame.

The Earth-surface-fixed (ESF) frame of reference, $\mathcal{F}_E$, has the origin at the surface of the earth and axes $\mathbf{x}_E$, $\mathbf{y}_E$, and $\mathbf{z}_E$ pointing north, east, and downward respectively. For aircraft problems, $\mathcal{F}_E$ can be approximated by an inertial frame since the aircraft movements and rotations are significantly larger than those of the earth. As such, Newton’s laws of motions are valid in the ESF frame.

The body-fixed frame of reference, $\mathcal{F}_B$, has axes aligned with the aircraft, with the origin located in the aircraft centre of mass. In this thesis, the axes $\mathbf{x}_B$, $\mathbf{y}_B$, and $\mathbf{z}_B$ are defined positive towards the fore location, outward towards the right wing, and downwards. The roll, pitch, and yaw rotations ($\phi$, $\gamma$, $\psi$) are defined accordingly to
transform from the ESF frame to the body-fixed frame.

The wind frame of reference, $\mathcal{F}_W$, has $\mathbf{x}_W$ being aligned with the velocity vector. From the body frame of reference, the $\mathbf{y}_B$-axis is rotated by the angle of attack, $\alpha_x$, and the resulting $\mathbf{z}_W$-axis is rotated by the sideslip angle, $\beta$. $\mathbf{x}_W$, $\mathbf{y}_W$, and $\mathbf{z}_W$ are positive pointing towards the fore location, right wing and downwards.

The series of rotation matrices performed on the ESF frame of reference to obtain a coordinate system in the body-fixed frame are shown in Figure 4.1 and are defined as

\[
C_3(\psi) = \begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix},
\]

\[
C_2(\gamma) = \begin{bmatrix}
\cos \gamma & 0 & -\sin \gamma \\
0 & 1 & 0 \\
\sin \gamma & 0 & \cos \gamma
\end{bmatrix},
\]

\[
C_1(\phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix},
\]

where $C_1$ is a rotation matrix around the $x$-axis causing a roll rotation, $C_2$ is a rotation matrix around the $y$-axis causing a pitch rotation, and $C_3$ is a rotation matrix around the $z$-axis causing a yaw rotation. The resulting rotation from $\mathcal{F}_E$ to $\mathcal{F}_B$ is

\[
\mathcal{F}_B = C_{BE}\mathcal{F}_E,
\]

where $C_{BE}$ is the rotation defined as

\[
C_{BE} = C_1(\phi)C_2(\gamma)C_3(\psi)
= \begin{bmatrix}
\cos \gamma \cos \psi & \cos \gamma \sin \psi & -\sin \gamma \\
-\cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi & \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi & \sin \phi \cos \gamma \\
\sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi & -\sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi & \cos \phi \cos \gamma
\end{bmatrix}.
\]

Similarly, the resulting rotation to obtain $\mathcal{F}_E$ in terms of $\mathcal{F}_B$ is given by

\[
\mathcal{F}_E = C_{EB}\mathcal{F}_B,
\]
where $\mathbf{C}_{EB}$ is the rotation defined as

$$
\mathbf{C}_{EB} = \mathbf{C}_{\psi}^T \mathbf{C}_{\gamma}^T \mathbf{C}_{\phi}^T
$$

$$
= \begin{bmatrix}
\cos \gamma \cos \psi & -\cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi & \sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi \\
\cos \gamma \sin \psi & \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi & -\sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi \\
-\sin \gamma & \sin \phi \cos \gamma & \cos \phi \cos \gamma
\end{bmatrix}.
$$

(4.7)
The velocity of the aircraft is conveniently expressed in the body-fixed frame as

\[
\mathbf{v}_B = C_2(\alpha_x)C_3(-\beta) \mathbf{v}_W \nonumber
\]

\[
= \begin{bmatrix}
\cos \alpha_x \cos \beta & -\cos \alpha_x \sin \beta & -\sin \alpha_x \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha_x \cos \beta & -\sin \alpha_x \sin \beta & \cos \alpha_x
\end{bmatrix}
\begin{bmatrix}
V \\
0 \\
0
\end{bmatrix} 
\]

\[
= \begin{bmatrix}
V \cos \alpha_x \cos \beta \\
V \sin \beta \\
V \sin \alpha_x \cos \beta
\end{bmatrix},
\]

where \( \mathbf{v}_B \) is the velocity vector expressed in the body-fixed frame of reference, \( \mathbf{v}_W \) is the velocity vector expressed in the wind frame of reference, and \( V \) is the velocity of the UAV. The kinematic equations are typically expressed in an inertial frame of reference, so the resulting equations with a \( C_{EB} \) rotation are

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = C_{EB} \mathbf{v}_B
\]

\[
\dot{x} = V \cos \alpha_x \cos \beta \cos \gamma \cos \psi + V \sin \beta(- \cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi) \\
+ V \sin \alpha_x \cos \beta(\sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi) \\
\dot{y} = V \cos \alpha_x \cos \beta \cos \gamma \sin \psi + V \sin \beta(\cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi) \\
+ V \sin \alpha_x \cos \beta(- \sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi) \\
\dot{z} = -V \cos \alpha_x \cos \beta \sin \gamma + V \sin \beta \sin \phi \cos \gamma + V \sin \alpha_x \cos \beta \cos \phi \cos \gamma
\]

The dynamic equations are obtained through re-arranging Newton’s second law. The forces acting on the UAV consist of aerodynamic, thrust and gravitational forces. The aerodynamic force is conveniently represented in the wind frame as

\[
\mathbf{F}_{aero, W} = \begin{bmatrix}
-D \\
Y \\
-L
\end{bmatrix},
\]

where \( \mathbf{F}_{aero, W} \) is the aerodynamic force vector in the wind frame, \( D \) is the drag force, \( Y \) is the side force, and \( L \) is the lift force. The thrust force is conveniently represented in
the body-axis frame as

\[
\mathbf{F}_{\text{thrust},B} = \begin{bmatrix} T \cos \epsilon \\ 0 \\ -T \sin \epsilon \end{bmatrix},
\] (4.11)

where \( \mathbf{F}_{\text{thrust},B} \) is the thrust force vector in the body-fixed frame, \( T \) is the thrust force, and \( \epsilon \) is the angle between the \( \mathbf{x}_B \) and the thrust vector. The gravitational force is conveniently represented in the inertial frame of reference as

\[
\mathbf{F}_{G,I} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix},
\] (4.12)

where \( \mathbf{F}_{G,I} \) is the gravitational force vector in the inertial frame of reference, \( m \) is the mass of the UAV, and \( g \) is the standard acceleration due to gravity.

The sum of all these forces in the body-fixed frame of reference, \( \mathbf{F}_B \) is

\[
\mathbf{F}_B = \mathbf{F}_{\text{aero},B} + \mathbf{F}_{\text{thrust},B} + \mathbf{F}_{G,B}
\]

\[
= C_2(\alpha_x)C_3(-\beta)\mathbf{F}_{\text{aero},W} + \mathbf{F}_{\text{thrust},B} + C_1(\phi)C_2(\gamma)C_3(\psi)\mathbf{F}_{G,I}
\]

\[
= \begin{bmatrix} -D \cos \alpha_x \cos \beta - Y \cos \alpha_x \sin \beta + L \sin \alpha_x \\ -D \sin \beta + Y \cos \beta \\ -D \sin \alpha_x \cos \beta - Y \sin \alpha_x \sin \beta - L \cos \alpha_x \end{bmatrix} \begin{bmatrix} T \cos \epsilon \\ 0 \\ -T \sin \epsilon \end{bmatrix} + \begin{bmatrix} -mg \sin \gamma \\ mg \sin \phi \cos \gamma \\ mg \cos \phi \cos \gamma \end{bmatrix},
\] (4.13)

The force vector in (4.13) can then be related to the UAV’s acceleration through Newton’s second law. While Newton’s second law is valid only in an inertial frame of reference, the inertial terms can be expressed in another frame, such as the fixed-body frame of reference. Newton’s second law expressed in the inertial frame is

\[
\mathbf{F}_I = m \frac{d \mathbf{a}_I}{dt}
\]

\[
= m \left( \frac{d \mathbf{v}_I}{dt} + \frac{d \mathbf{w}_I}{dt} \right),
\] (4.14)
where \( \mathbf{F}_I, \mathbf{a}_I, \mathbf{v}_I, \) and \( \mathbf{w}_I \) are the force, acceleration, velocity and wind velocity vectors in the inertial frame of reference. Newton’s second law in the non-inertial, fixed-body frame is

\[
\mathbf{F}_B = m \left( \frac{d \mathbf{v}_B}{dt} + \mathbf{\omega} \times \mathbf{v}_B + \frac{d \mathbf{w}_B}{dt} \right),
\]

(4.15)

where \( \mathbf{w}_B \) is the wind velocity vector in the body-fixed frame. The vector cross product between the angular velocity vector, \( \mathbf{\omega} \), and \( \mathbf{v}_B \) is given by

\[
\mathbf{\omega} \times \mathbf{v}_B = \begin{vmatrix} x_B & y_B & z_B \\ \dot{\phi} & \dot{\gamma} & \dot{\psi} \end{vmatrix} V \cos \alpha \cos \beta \quad V \sin \beta \quad V \sin \alpha \cos \beta \\
= \begin{bmatrix} \dot{\gamma} V \sin \alpha \cos \beta - \dot{\psi} V \sin \beta \\ \dot{\psi} V \cos \alpha \cos \beta - \dot{\phi} V \sin \alpha \cos \beta \\ \dot{\phi} V \sin \beta - \dot{\gamma} V \cos \alpha \cos \beta \end{bmatrix}.
\]

(4.16)

Combining (4.15) and (4.16) results in

\[
\mathbf{F}_B = m \left( \begin{bmatrix} \dot{V} \cos \alpha \cos \beta - V \sin \alpha \cos \beta - V \cos \alpha \sin \beta + \dot{\gamma} V \sin \alpha \cos \beta - \dot{\psi} V \sin \beta \\
\dot{V} \sin \beta + V \cos \beta + \dot{\psi} V \cos \alpha \cos \beta - \dot{\phi} V \sin \alpha \cos \beta \\
\dot{V} \sin \alpha \cos \beta + V \cos \alpha \cos \beta - V \sin \alpha \sin \beta + \dot{\phi} V \sin \beta - \dot{\gamma} V \cos \alpha \cos \beta \end{bmatrix} + \frac{d \mathbf{w}_B}{dt} \right).
\]

(4.17)
Equating (4.17) and (4.13), the dynamic equations of motion are

\[
\begin{bmatrix}
\dot{V} & \dot{\psi} & \dot{\gamma}
\end{bmatrix}
\begin{bmatrix}
V \cos \alpha_x \cos \beta - V \sin \alpha_x \cos \beta - V \cos \alpha_x \sin \beta + \dot{\gamma}V \sin \alpha_x \cos \beta - \dot{\psi}V \sin \beta \\
\dot{V} \sin \beta + V \cos \beta + \dot{\psi}V \cos \alpha_x \cos \beta - \dot{\phi}V \sin \alpha_x \cos \beta \\
\dot{V} \sin \alpha_x \cos \beta + V \cos \alpha_x \cos \beta - V \sin \alpha_x \sin \beta + \dot{\phi}V \sin \beta - \dot{\gamma}V \cos \alpha_x \cos \beta \\
+ \frac{d\omega_B}{dt}
\end{bmatrix} + \begin{bmatrix}
T \cos \epsilon - D \cos \alpha_x \cos \beta - Y \cos \alpha_x \sin \beta + L \sin \alpha_x - mg \sin \gamma \\
-D \sin \beta + Y \cos \beta + mg \sin \phi \cos \gamma \\
-T \sin \epsilon - D \sin \alpha_x \cos \beta - Y \sin \alpha_x \sin \beta - L \cos \alpha_x + mg \cos \phi \cos \gamma
\end{bmatrix}.
\]

In this thesis, several assumptions are made to reduce the complexity of (4.18):

1. The effects of wind are not considered \((\omega_I = 0, \frac{d\omega_I}{dt} = 0, \omega_B = 0, \frac{d\omega_B}{dt} = 0)\)
2. No sideslip \((\beta = 0, Y = 0)\)
3. Oncoming flow is relatively aligned with the velocity vector \((\alpha_x \approx 0)\)
4. Thrust vector aligned with velocity vector \((\epsilon = 0)\)

Assumptions 2 and 3 reduce the velocity vector to \(\mathbf{v} = [V, 0, 0]^T\). The combination of assumptions 1 to 4 simplify the kinematic equations (4.9) and dynamic equations (4.18) to

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} = 
\begin{bmatrix}
V \cos \gamma \cos \psi \\
V \cos \gamma \sin \psi \\
-V \sin \gamma
\end{bmatrix},
\]

\[
\begin{bmatrix}
\dot{V} \\
\dot{\psi} \\
\dot{\gamma}
\end{bmatrix} = 
\begin{bmatrix}
\frac{T-D}{m} - g \sin \gamma \\
\frac{g \sin \phi \cos \gamma}{V} \\
\frac{L}{mV} - \frac{g \cos \phi \cos \gamma}{V}
\end{bmatrix}.
\]

In a two-dimensional model, the UAV is assumed to fly at level flight. This additional assumption constrains \(\gamma\) and \(\dot{\gamma}\) to zero as any rotation about the pitch axis will directly result in a change in altitude. Thus, the two-dimensional model, as shown in Figure 4.2,
is given by

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{V} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
V \cos \psi \\
V \sin \psi \\
\frac{\tau - D}{m} \\
\frac{g \sin \phi}{V}
\end{bmatrix}
\]  
(4.20)

If steady flight is assumed, the two-dimensional model is further simplified to

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
V \cos \psi \\
V \sin \psi \\
\frac{g \sin \phi}{V}
\end{bmatrix}
\]  
(4.21)

4.2 Energy Approximations

4.2.1 Energy Collected by Photovoltaic Cells

The amount of energy collected by the photovoltaic cells is at a maximum when the vector between the sun’s incident rays and the cells align with the vector normal to the cells. The angle between these two vectors, referred to as the solar incidence angle, is
shown in Figure 4.3. For a given time interval \([t_o, t_f]\), the total energy collected is

\[
E_{col} = \int_{t_o}^{t_f} P_{col} \, dt,
\]

where \(P_{col}\) is the power collected, expressed by

\[
P_{col} = \eta_{sol} P_{sd} S_{sol} \cos \theta,
\]

where \(\eta_{sol}\) is the solar cell efficiency, \(P_{sd}\) is the solar spectral density, \(S_{sol}\) is the wing area covered by solar cells, and \(\theta\) is the solar incidence angle.

As mentioned above, the solar incidence angle is dependent on the position of the sun and the orientation of the aircraft. Specifically, this incidence angle \(\theta\) is related to the sun azimuth, \(a\), and elevation \(e\) through a series of rotation matrices. Figure 4.4 shows how the line-of-sight from the solar panel to the sun can be obtained from the Earth-surface-fixed (ESF) frame of reference. Given the relatively long flight intervals compared to the time scale for earth’s orbit around the sun, the position of the sun is assumed constant during flight. The unit vector \(\mathbf{s}_B\) representing the direction of the sun in body-fixed frame, \(\mathbf{F}_B\), can be related to \(\mathbf{s}_E\), the sun vector in the ESF frame,
\[ \mathbf{s}_B = C_{BE} \mathbf{s}_E \]

\[
\begin{bmatrix}
\cos \gamma \cos \psi & \cos \gamma \sin \psi & - \sin \gamma \\
- \cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi & \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi & \sin \phi \cos \gamma \\
\sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi & - \sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi & \cos \phi \cos \gamma \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos \epsilon \cos a \\
\cos \epsilon \sin a \\
\sin \epsilon \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\mathbf{s}_{B_1} \\
\mathbf{s}_{B_2} \\
\mathbf{s}_{B_3}
\end{bmatrix}
\]

where \( s_{B_1}, s_{B_2}, \) and \( s_{B_3} \) are the components of \( \mathbf{s}_B \) given by

\[
s_{B_1} = \cos \epsilon \cos a \cos \gamma \cos \psi + \cos \epsilon \sin a \cos \gamma \sin \psi - \sin \epsilon \sin \gamma
\]

\[
s_{B_2} = \cos \epsilon \cos a \left( - \cos \phi \sin \psi + \sin \phi \sin \gamma \cos \psi \right) + \cos \epsilon \sin a \left( \cos \phi \cos \psi + \sin \phi \sin \gamma \sin \psi \right) + \sin \epsilon \sin \phi \cos \gamma
\]

\[
s_{B_3} = \cos \epsilon \cos a \left( \sin \phi \sin \psi + \cos \phi \sin \gamma \cos \psi \right) + \cos \epsilon \sin a \left( - \sin \phi \cos \psi + \cos \phi \sin \gamma \sin \psi \right) + \sin \epsilon \cos \phi \cos \gamma
\]

The solar incidence angle, \( \theta(\phi, \psi) \), is defined in the body-fixed frame of reference as the angle between the line-of-sight to the sun, \( \mathbf{s}_B \), and the unit normal vector, \( \hat{z}_B \). Hence

\[
\theta(\phi, \gamma, \psi) = \arccos (\hat{s}_B \cdot \hat{z}_B)
\]

or

\[
\cos \theta = \sin \epsilon \cos \phi \cos \gamma + \cos \epsilon \sin \phi \left( \cos a \sin \psi - \sin a \cos \psi \right)
\]

\[
+ \cos \epsilon \cos \phi \sin \gamma \left( \cos a \cos \psi + \sin a \sin \psi \right)
\]

\[
= \sin \epsilon \cos \phi \cos \gamma - \cos \epsilon \sin \phi \sin (a - \psi) + \cos \epsilon \cos \phi \sin \gamma \cos (a - \psi).
\]

In the two-dimensional model in which the UAV is assumed to fly at level flight, the
solar incidence angle simplifies to

\[
\cos \theta = \cos \phi \sin e - \cos e \sin \phi (\cos a \sin \psi - \sin a \cos \psi)
\]

\[
= \cos \phi \sin e - \cos e \sin (a - \psi) \sin \phi.
\] (4.28)

### 4.2.2 Energy Consumed

The energy consumed during the flight can be expressed by the standard aerodynamic lift, drag and propulsion models. The total energy consumed by the UAV during interval \([t_o, t_f]\) is

\[
E_{con} = \int_{t_o}^{t_f} P_{con} dt,
\] (4.29)

where \(P_{con}\) is the power consumed and can be expressed by

\[
P_{con} = \frac{TV}{\eta_{prop}},
\] (4.30)

where \(T\) is the thrust, \(V\) is the velocity, and \(\eta_{prop}\) is the propeller efficiency. While \(T\) can be solved in the general case through (4.18), the thrust in three-dimensions with the
previous assumptions is simply

\[ T = m\dot{V} + D + mg\sin \gamma \quad (4.31) \]

\[ D = \frac{1}{2} \rho V^2 SC_D \quad (4.32) \]

\[ C_D = C_{D_0} + C_{D_i} \quad (4.33) \]

\[ C_{D_i} = \frac{C_L^2}{\pi AR e_0} \quad (4.34) \]

\[ C_L = \frac{2L}{\rho V^2 S} \quad (4.35) \]

\[ L = mV\dot{\gamma} + mg\cos \phi \cos \gamma, \quad (4.36) \]

where \( S \) is the wing planform area, \( C_D \) is the coefficient of drag, \( C_{D_0} \) is the coefficient of drag at zero lift representing the skin friction and form drag, \( C_{D_i} \) is the coefficient of induced drag, \( C_L \) is the coefficient of lift, \( AR \) is the aspect ratio, \( e_0 \) is the Oswald efficiency factor, \( \rho \) is the air density, \( m \) is the mass of the aircraft and \( g \) is the gravitational acceleration.

For the two-dimensional case, since steady level flight is assumed, the equations further reduce to

\[ T = D = \frac{1}{2} \rho V^2 SC_D \quad (4.37) \]

\[ C_D = C_{D_0} + C_{D_i} \quad (4.38) \]

\[ C_{D_i} = \frac{C_L^2}{\pi AR e_0} \quad (4.39) \]

\[ C_L = \frac{2L}{\rho V^2 S} \quad (4.40) \]

\[ L = \frac{mg}{\cos \phi}. \quad (4.41) \]

## 4.3 Environment Model

Estimations for the sun azimuth, \( a \), and elevation, \( e \), were made using the Plataforma Solar de Almería (PSA) algorithm developed by Blanco-Muriel et al. [40]. The PSA algorithm has inputs of time and location and outputs the sun position to an accuracy of \( 1/120^\circ \). The key derivation steps are listed below.
First, the Julian Day is calculated using time and date and is given by

\[
jd = \frac{1461}{4} \left( y + 4800 + \frac{m - 14}{12} \right) + \frac{367}{12} \left( m - 2 - 12 \left( \frac{m - 14}{12} \right) \right) - \frac{3}{4} \left( y + 4900 + \frac{m - 14}{12} \right) + d - 32075 - 0.5 + \frac{1}{24} \left( hr + \frac{\text{min}}{60} + \frac{\text{sec}}{3600} \right),
\]

(4.42)

where \( jd \) is the Julian Day, \( y \) is the year, \( m \) is the numerical month in the Gregorian calendar with January attributed 1 and December attributed 12, \( d \) is the day number in specific month, \( hr \) is the hour in Coordinated Universal Time (UTC), \( \text{min} \) is the minutes in UTC and \( \text{sec} \) is the seconds in UTC.

Second, the ecliptic coordinates of the sun are approximated from the Julian Day and the ecliptic longitude and obliquity of the ecliptic are computed. The fundamental plane of the ecliptic coordinate system is the ecliptic, which mirrors the apparent, average path the sun orbits around the earth. The approximations use reference data from January 1, 2000 at 12:00 terrestrial time or 11:58:55.816 UTC and can be found in [41]. So the time since J2000 is

\[
n = jd - 2451545.0,
\]

(4.43)

where \( n \) is the number of Julian Days since J2000, and the Julian Day for J2000 is 2451545.0. The mean longitude and mean anomaly, as outlined in [41], can be approximated linearly as

\[
L_{\text{lon}} = 280.4665 + 0.9856474n
\]

(4.44)

\[
g_{\text{ano}} = 357.5291 + 0.9856003n,
\]

(4.45)

where \( L_{\text{lon}} \) is the mean longitude, \( g_{\text{ano}} \) is the mean anomaly, with the first term for \( L_{\text{lon}} \) and \( g_{\text{ano}} \) being the mean longitude/anomaly at J2000 and the second term being the daily change in mean longitude/anomaly in degrees per day. Blanco-Muriel et al. [40] provide an improved approximation to the ecliptic longitude and obliquity of the ecliptic compared to the low precision formula in [41]

\[
\lambda_{\text{ecl}} = L_{\text{lon}} + 1.9145998 \sin g_{\text{ano}} + 0.0199928 \sin 2g_{\text{ano}} - 0.0064973 - 0.0011631 \sin \Omega 
\]

(4.46)

\[
\epsilon_{\text{ecl}} = 23.4393 - 0.0000004n + 0.0022689 \sin \Omega,
\]

(4.47)
where \( \lambda_{ecl} \) is the ecliptic longitude, \( \epsilon_{ecl} \) is the obliquity of the ecliptic and \( \Omega = 122.7791 - 0.0595566n \).

Third, the ecliptic longitude and obliquity of the ecliptic in the ecliptic coordinate system is converted to the right ascension and declination in the equatorial coordinate system. The equatorial coordinate system is similar to the geographic coordinate system in that the fundamental plane is the celestial equator, a projection of the geographic Earth’s equator, and the poles are the celestial poles, a projection of the geographic North and South poles. The right ascension and declination angles can be obtained through trigonometric relations

\[
\alpha_{equ} = \arctan (\cos \epsilon_{ecl} \tan \lambda_{ecl}),
\]

\[
\delta_{equ} = \arcsin (\sin \epsilon_{ecl} \sin \lambda_{ecl}),
\]

where \( \alpha_{equ} \) is the right ascension angle and \( \delta_{equ} \) is the declination angle.

Fourth, the right ascension and declination angle in the equatorial coordinate system is converted to the azimuth and elevation angles in the horizontal coordinate system. The horizontal coordinate system is dependent on the location of the observer. The fundamental plane is the local horizon and the poles are the zenith and nadir, two imaginary points above and below the observer respectively. The local hour angle is the angle relating the plane containing the object, in this case the sun, and the plane containing the observer. The local hour angle is given by

\[
\omega_{local} = \text{LMST} - \alpha_{equ},
\]

where \( \omega_{local} \) is the local hour angle and LMST is the Local Mean Sidereal Time given by

\[
\text{LMST} = 15\text{GMST} + L_{obs}
\]

\[
\text{GMST} = 0.6974243242 + 0.0657098283n + \text{hr} + \frac{\text{min}}{60} + \frac{\text{sec}}{3600},
\]

where GMST is the Greenwich Mean Sidereal Time and \( L_{obs} \) is the longitude of the observer. The azimuth angle, \( a \), of the sun is then given by

\[
a = \arctan \frac{-\sin \omega_{local}}{\tan \delta_{equ} \cos \ell_{obs} - \sin \ell_{obs} \cos \omega_{local}},
\]
where $\ell_{obs}$ is the latitude of the observer. The zenith angle, $\zeta$, is given by

$$
\zeta = \arccos (\cos \ell_{obs} \cos \omega_{local} \cos \delta_{equ} + \sin \delta_{equ} \sin \ell_{obs}),
$$

(4.54)

and with the parallax adjustment, the corrected zenith angle $\zeta_{cor}$ is given by

$$
\zeta_{cor} = \zeta + \frac{R}{au} \sin \zeta,
$$

(4.55)

where $R$ is the radius of the Earth and $au$ is astronomical unit representing the distance form the Earth to the Sun. The elevation angle, $e$ in degrees can then be expressed as

$$
e = 90 - \zeta_{cor}.
$$

(4.56)

### 4.4 Model Validation through Flight Tests

The models described in Sections 4.2, 4.1 and 4.3 are validated through experimental flight tests using the flying wing UAV. Table 2.1 presented in Chapter 2.2.1 provides the numerical values for the FSC UAV parameters. The aircraft parameters $AR$, $S$ and $m$ were measured on the UAV. The aerodynamic parameters $C_{D_o}$ and $\eta_{prop}$ were obtained through a MATLAB Simulink model of the current flying wing UAV developed by Li [42]. The parameters not relating to the aircraft such as $\rho$ were estimated based on the average altitude the aircraft will operate at. Velocities, $V$, positions, $x, y, z$, and rotation angles $\phi, \gamma, \psi$, were all either control variables or obtained using the Kestral™ autopilot, described in Chapter 2.2.5, for validation purposes. Similarly, the panel parameters $S_{sol}$ was measured and $\eta_{sol}$ estimated through test bench experiments in Chapter 2. The environment parameters were modelled using the PSA algorithm detailed in Section 4.3. The final parameter unaccounted for is the Oswald efficiency factor. $e_0$ was estimated using a method Shevell proposed in [43].

A set of five flight tests were used to verify the accuracy of the energy approximations compared with energy values obtained from the current sensors on the UAV. The states were obtained through the Kestral™ autopilot and the sun position modelled using the PSA algorithm. The values for energy consumed are compared in Figure 4.5 and had a root mean square error of 12%. This error is acceptable considering the assumptions of the model and provided reasonable agreement between experimental and simulated results.
Parametrization was done to estimate the value for solar density, $P_{sd}$. Seeing as $P_{sd}$ varies from one flight test to the other, the initial estimation for $P_{sd}$ was selected based on the average $P_{sd}$ per month in Toronto in 2012 [30]. The value for $P_{sd}$ was then retroactively calculated using the energy collection approximation and the measured energies during flight tests. The new $P_{sd}$ would then be used for subsequent flights during the day. Figure 4.6 show model accuracy pre-parametrization, where the value for $P_{sd}$ was estimated using [30], and post-parametrization, where the value for $P_{sd}$ was calibrated using experimental flight test data on the same day. 60% of the flight data for the one flight test set was used and averaged for the calculation of the calibrated $P_{sd}$. The remaining 40% of the flight data acted as a test set with no effect on the calibrated value of $P_{sd}$. Pre-parametrization, the approximations for energy collected was $30\% - 40\%$ higher when compared to the measured energy values during flight test experimentation. Using the calibrated value for $P_{sd}$, the training set had a root mean square error of $4\%$ for the approximated energy collected, while the test set had a mean square error of only $1\%$. The recalibrated $P_{sd}$ provided a significant improvement on the approximated values for energy collected and show that the approximations were reasonable as long as calibration was done prior to experimentation. It should also be noted that the set of
Figure 4.6: Comparing approximations for energy collected against experimental values

five flights spanned over five hours, or a full flight test day, which signified that a single calibration of $P_{sd}$ was sufficient for a fairly accurate approximation. For the remaining thesis, the $P_{sd}$ values reported are the calibrated values.

4.5 Summary

In this chapter, a model for the UAV was derived and the key assumptions used to obtain the final two- and three-dimensional model was presented. In addition, approximations for the energy collected by the photovoltaic cells and consumed during for an desired path was calculated and validated using experimental flight tests.
Chapter 5

Energy-Optimal Path Planning

In Chapter 2, the advantage of retro-fitting a mini UAV with solar panels was shown through a series of test bench studies, and Chapter 4 presented an experimentally validated model to quantify this advantage. This chapter addresses the fourth and final research objective of investigating the benefits of path planning, designing an energy-optimal flight path and validating the path through simulation and flight tests.

5.1 Justification for Energy-Optimal Path Planning

5.1.1 Ground Tests

A further evaluation of equation (4.23) shows that the only independent variable resulting in changes in the energy collected is the solar incidence angle $\theta$. Figure 5.1a and Figure 5.1b show the effect of solar incidence has on power collected using the model in Chapter 4 and ground test experiments respectively. The environment parameters used for this study are listed in Table 5.1. The trend in the simulation results show maximum power at low incidence angles, with a peak at $\theta = 0^\circ$, signifying the sun is right above the panels. As the incidence angle increases, the power collected decreases as the panels receive less solar irradiance. Finally, as incidence angles increase past $90^\circ$, the sun is now below the panels and no power is received. During experiments, power is still obtained at $\theta$ greater than $90^\circ$ due to diffuse reflection. The $\pm 2\text{ W}$ spread seen in Figure 5.1b is attributed to this phenomenon along with the resolution of the Eagle Tree Systems eLoggers. For level, steady flight, the incidence angle is not expected to change dramatically. However, even a $\pm 5^\circ$ change in incidence angle can cause a $\pm 0.5\text{ W}$ change. This increase or decrease in power drawn is magnified as the flight time increases and thus justifies a need for path planning to account for the solar incidence angle.
5.1.2 Flight Tests

Through flight tests, two approaches to path selection resulting from solar incidence angles are designed to serve as examples for energy-optimal comparisons. Two variables that affect the solar incidence angle are the bank and pitch angle. In the first approach, the solar incidence angle is varied through a change in bank angle. In the second approach, the solar incidence angle is varied through a change in pitch angle. The environmental and flight conditions are presented in Table 5.2.

Table 5.2: Flight conditions for solar incidence studies

<table>
<thead>
<tr>
<th>a (°)</th>
<th>e (°)</th>
<th>$P_{sd}$ (W/m²)</th>
<th>$S_{sol}$ (m²)</th>
<th>$\eta_{sol}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>167</td>
<td>56</td>
<td>976</td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>

The first experiment compares a straight direct path with a banked one, designated by P1 and P2 in Figure 5.2, respectively. The velocity, altitude and bank angle are held relatively constant throughout the flight by the Kestral™ autopilot and assumed constant. While the terminal battery energy determined by (5.18) is negative due to
the fact that the current solar cells being insufficient towards providing perpetual flight, path P2 results in a 0.31 kJ, or 6.65%, lower net energy than P1, albeit flying a longer and more energy consuming path. This result shows that the gain in energy collected from the photovoltaic panels can outweigh the gain in energy consumed in a path that requires more energy to fly, even at a low average bank angle difference.

Figure 5.2: Flight test results for varying roll in two-dimensional model
P1 - $\phi_o = 0^\circ$, $\bar{\theta} = 35.5^\circ$, $E_{net} = -4.66 \text{ kJ}$
P2 - $\phi_o = 8^\circ$, $\bar{\theta} = 31.5^\circ$, $E_{net} = -4.35 \text{ kJ}$

The second experiment compares two paths with identical energy required, but with mirrored pitch angles. One path is pitched favourably toward the sun, and the other pitched unfavourably. Consequently, while the pitch angles of the aircraft remain the same throughout both paths, the solar incidence angle varies. This resulted in an additional 0.74 kJ, corresponding to path P4 being 13.65% more energy efficient when compared to path P3. The two flight paths and corresponding measured net energy values are presented in Figure 5.3. This second experiment shows that even a marginal change in solar incidence angle ($\Delta \bar{\theta} = 5.7^\circ$) produces a significant measured increase in the magnitude of energy collected by the solar-assisted FSC UAV. Along with the result from the previous experiment, the results warrant an investigation on path optimiza-
tion. The energy savings seen during flight tests can be expected to increase with the implementation of an energy-optimal path as described in the next section, as well as improvements in areas such as solar panels with higher efficiencies.

5.2 Path Planning

The path planning problem that will be presented is similar to the traditional two-point boundary value problem, but with the energy models presented in Chapter 4 included. The two-point boundary value problem seeks to find the path bounded by the endpoints. The addition of the energy models serve to limit the solutions of the two-point boundary value problem to those that are energy-optimal. The general path planning problem is first derived, followed by the two-dimensional and three-dimensional problems.

5.2.1 General Optimization Problem

In the general optimization problem, one seeks to find the control values, \( u(t) \), that alters the states, \( x(t) \), in order to optimize a cost function \( J \). The cost function can be typically expressed as

\[
J(t, x(t), u(t)) = \ell_o(x(t_f)) + \int_{t_o}^{t_f} \mathcal{L}(t, x(t), u(t))dt,
\]

where \( \ell_o(x(t_f)) \) is the terminal cost, \( x(t_f) \) is the state vector evaluated at terminal time \( t_f \), and \( \mathcal{L} \) is the Lagrangian representing the running cost. For dynamic problems, an augmented cost function using a vector of Lagrange multipliers is used to allow for dynamic constraints. This augmented cost function is expressed as

\[
J_{aug}(t, x, u, \lambda) = \ell_o(x(t_f)) + \int_{t_o}^{t_f} \mathcal{L}(t, x, u)dt + \int_{t_o}^{t_f} \lambda^T [f(t, x, u) - \dot{x}]dt
\]

\[
= \ell_o(x(t_f)) + \int_{t_o}^{t_f} \left[ \mathcal{L}(t, x, u) + \lambda^T f(t, x, u) + \dot{\lambda}^T x \right] dt
\]

\[
- \lambda^T x(t_f) + \lambda^T x(t_o),
\]

where \( \lambda \) is the vector of Lagrange multipliers or costates, and \( f \) is the system of differential equations describing the equations of motion. For completeness, the state, costate, and control vectors are all functions of time, so \( x = x(t), \dot{x} = \dot{x}(t), \lambda = \lambda(t), \dot{\lambda} = \dot{\lambda}(t) \) and \( u = u(t) \).
Figure 5.3: Flight test results for varying pitch in three-dimensional model
P3 - pitched away from sun, $\theta = 46.7^\circ$, $E_{\text{net}} = -5.42$ kJ
P4 - pitched towards sun, $\theta = 41.0^\circ$, $E_{\text{net}} = -4.68$ kJ
For the cost function $J_{aug}$ to be an optimum, $\delta J_{aug}$ must be zero. As such,

$$
\delta J_{aug} = \ell_o \bigg|_{x(t_f)} \delta x(t_f) + \int_{t_o}^{t_f} \left[ \mathcal{L}_x \delta x + \mathcal{L}_u \delta u + \lambda^T \left[ f_x \delta x + f_u \delta u \right] + \dot{\lambda}^T \delta x \right] dt \\
- \lambda^T(t_f) \delta x(t_f) + \lambda^T(t_o) \delta x(t_o) \\
= \left[ \ell_o \bigg|_{x(t_f)} - \lambda^T(t_f) \right] \delta x(t_f) \\
+ \int_{t_o}^{t_f} \left[ \mathcal{L}_x + \lambda^T f_x + \dot{\lambda} \right] \delta x + \left[ \mathcal{L}_u + \lambda^T f_u \right] \delta u \right] dt, 
$$

(5.4)

$$
\delta x(t_o) = 0. 
$$

(5.5)

where $(\cdot)_x = \frac{\partial}{\partial x}$ and $(\cdot)_u = \frac{\partial}{\partial u}$. Equation (5.5) simply regroups the terms in Equation (5.4) in terms of $\delta x$ and $\delta u$, as well as assuming the initial state is fixed, so that $\delta x(t_o) = 0$.

The first order necessary conditions of optimality are obtained by solving for $\delta J_{aug}$,

$$
\frac{\ell_o}{x(t_f)} - \lambda^T(t_f) = 0 \tag{5.6}
$$

$$
\mathcal{L}_x + \lambda^T f_x + \dot{\lambda} = 0 \tag{5.7}
$$

$$
\mathcal{L}_u + \lambda^T f_u = 0. \tag{5.8}
$$

If one introduces a Hamiltonian function as

$$
H(t, x, \lambda, u) = \mathcal{L}(t, x, u) + \lambda^T(t) f(t, x, u), 
$$

(5.9)

the first order necessary conditions (5.6)-5.8 can be rewritten as

$$
\lambda^T(t_f) = \ell_o \bigg|_{x(t_f)} \quad \text{Transversality condition} \tag{5.10}
$$

$$
\dot{\lambda}^T = -H_x \quad \text{Costate equations} \tag{5.11}
$$

$$
H_u = 0 \quad \text{Optimality condition.} \tag{5.12}
$$

In order to ensure the stationary point is a minimum, the Legendre-Clebsch condition must also be satisfied, i.e. the Hessian must be negative-semidefinite. This is defined as

$$
H_{uu} \leq 0. \tag{5.13}
$$

The final necessary for optimality is to ensure the solution is a global minimum. Pontryagin’s minimum principle claims that if $u^*$ is the optimal control vector, then $H$ along the optimal trajectory is a global minimum. The principle is stated without proof.
as

\[
\mathbf{H}(t, \mathbf{x}^*(t), \lambda^*(t), \mathbf{u}^*(t)) \leq \mathbf{H}(t, \mathbf{x}^*(t), \lambda^*(t), \mathbf{u}(t))
\] (5.14)

or

\[
\mathbf{u}^*(t) = \arg\min_u \mathbf{H}(t, \mathbf{x}^*(t), \lambda^*(t), \mathbf{u}(t)).
\] (5.15)

If the Hamiltonian is independent of time, i.e. \( \frac{\partial \ell_o}{\partial t} = 0 \), then two special cases arise for \( t \in [t_o, t_f] \):

- \( \mathbf{H}(\mathbf{x}^*(t), \lambda^*(t), \mathbf{u}^*(t)) = \text{constant} \) \hspace{1cm} \text{Fixed final time} (5.16)
- \( \mathbf{H}(\mathbf{x}^*(t), \lambda^*(t), \mathbf{u}^*(t)) = 0 \) \hspace{1cm} \text{Free final time} (5.17)

### 5.2.2 Two-Dimensional Problem Formulation

The optimal path planning problem in two-dimensions is to maximize the net gain in energy with respect to speed and roll angle. The goal is to find an optimal UAV path between initial state \([x_0, y_0]\) and final state \([x_f, y_f]\) within the time interval \([t_0, t_f]\) where the initial time \(x_0\) is known and the final time is constrained by \( t_f \leq T_M \), where \( T_M \) is the maximum flight duration.

Using the kinematic and energy models stated in (4.20), (4.22), and (4.29), the objective function and constraints are

\[
\max_{\phi, V} J = \min_{\phi, V} -J = -E_{\text{net}} = -[E_{\text{col}} - E_{\text{con}}] = -\int_{t_o}^{t_f} (P_{\text{col}} - P_{\text{con}}) \, dt
\] (5.18)
such that
\[
\begin{align*}
\dot{x} &= V \cos \psi \\
\dot{y} &= V \sin \psi \\
\dot{\psi} &= \frac{g \tan \phi}{V}
\end{align*}
\]
\begin{align*}
[x(t_0), y(t_0), \psi(t_o)]^\top &= [x_0, y_0, \psi_o]^\top \\
[x(t_f), y(t_f)]^\top &= [x_f, y_f]^\top \\
|\phi| &< \frac{\pi}{2},
\end{align*}
\tag{5.19}

where the last two conditions are the constraints - one for the initial position and heading and one for the final position. The control variables \(\phi\) and \(V\) are assumed constant within each interval, but not necessarily for the entire flight path.

For the objective function in (5.18), the Hamiltonian function at one instance in time, as defined in (5.9), is
\[
H(x, y, \psi, \lambda_x, \lambda_y, \lambda_\psi, \phi, V) = -P_{col} + P_{con} + \lambda_x V \cos \psi + \lambda_y V \sin \psi + \lambda_\psi \frac{g \tan \phi}{V},
\tag{5.20}
\]
where \(\lambda_x, \lambda_y, \) and \(\lambda_\psi\) are the costates for states \(x, y,\) and \(\psi\) respectively. The state and costate equations, as defined in (5.11), are given by
\[
\begin{align*}
\dot{x} &= \frac{\partial H}{\partial \lambda_x} = V \cos \psi \\
\dot{y} &= \frac{\partial H}{\partial \lambda_y} = V \sin \psi \\
\dot{\psi} &= \frac{\partial H}{\partial \lambda_\psi} = \frac{g \tan \phi}{V} \\
\dot{\lambda}_x &= -\frac{\partial H}{\partial x} = 0 \\
\dot{\lambda}_y &= -\frac{\partial H}{\partial y} = 0 \\
\dot{\lambda}_\psi &= -\frac{\partial H}{\partial \psi} = \eta_{sol} P_{sd} S_{sol} \cos \theta \cos (\alpha - \psi) \sin \phi + \lambda_x V \sin \psi - \lambda_y V \cos \psi.
\end{align*}
\tag{5.21-5.26}
The first-order optimality condition given in (5.12) is

$$\frac{\partial H}{\partial \phi} = \eta_{sol} P_{sd} S_{sol} \left( \sin \phi \sin e + \cos e \sin (a - \psi) \cos \phi \right) + \frac{4m^2 g^2 \tan \phi \sec^2 \phi}{\eta_{prop} \rho V \pi R e_0}$$

$$+ \lambda_\psi \frac{g \sec^2 \phi}{V} = 0$$

(5.27)

$$\frac{\partial H}{\partial V} = \frac{3}{2\eta_{prop}} \rho V^2 S \left( C_{D_o} + \frac{4m^2 g^2 \sec^2 \phi}{\rho^2 V^4 \pi^2 R e_0} \right) - \frac{8m^2 g^2 \sec^2 \phi}{\eta_{prop} \rho V^2 \pi R e_0} + \lambda_x \cos \psi$$

$$+ \lambda_y \sin \psi - \lambda_\psi \frac{g \tan \phi}{V^2} = 0$$

(5.28)

while the second-order Legendre-Clebsch condition given in (5.13) that must be satisfied for the optimum to be a minimum is

$$H_{uu} = \frac{\partial^2 H}{\partial (\phi, V)^2}$$

$$= \begin{bmatrix} H_{\phi\phi} & H_{\phi V} \\ H_{V\phi} & H_{VV} \end{bmatrix}$$

(5.29)

$$\leq 0$$

where

$$H_{\phi\phi} = - \eta_{sol} P_{sd} S_{sol} \left( - \cos \phi \sin e + \cos e \sin (a - \psi) \sin \phi \right)$$

$$+ \frac{12m^2 g^2 \tan^2 \phi \sec^2 \phi}{\eta_{prop} \rho V \pi R e_0} + \frac{4m^2 g^2 \sec^2 \phi}{\eta_{prop} \rho V \pi R e_0} + \lambda_\psi \frac{2g \tan \phi \sec^2 \phi}{V}$$

(5.30)

$$H_{\phi V} = H_{V\phi}$$

$$= - \frac{4m^2 g^2 \tan \phi \sec^2 \phi}{\eta_{prop} \rho V \pi R e_0} + \frac{4m^2 g^2 \sec^2 \phi}{\eta_{prop} \rho V \pi R e_0} - \lambda_\psi \frac{g \sec^2 \phi}{V^2}$$

(5.31)

$$H_{VV} = \frac{3}{\eta_{prop}} \rho V S \left( C_{D_o} + \frac{4m^2 g^2 \sec^2 \phi}{\rho^2 V^4 \pi^2 R e_0} \right) - \frac{8m^2 g^2 \sec^2 \phi}{\eta_{prop} \rho V^3 \pi R e_0} + \lambda_\psi \frac{2g \tan \phi}{V^3}.$$  (5.32)

Finally, the final constraint to ensure that the global minimum solution satisfies the free final time problem as expressed in (5.17) is

$$H(x^*(t_f), y^*(t_f), \psi^*(t_f), \lambda_x^*(t_f), \lambda_y^*(t_f), \lambda_\psi^*(t_f), \phi^*(t_f), \phi^*(V)) = 0$$

(5.33)
5.2.3 Three-Dimensional Problem Formulation

Similar to the two-dimensional problem, the optimal path planning problem in three dimensions is to maximize the net loss in energy, but this time with respect to the lift coefficient, $C_L$ and roll angle $\phi$. The goal is to find an optimal UAV path between initial state $[x_0, y_0, z_0]$ and final state $[x_f, y_f, z_f]$ within the time interval $[t_0, t_f]$ where the initial time $x_0$ is known and the final time is constrained by $t_f \leq T_M$, where $T_M$ is the maximum flight duration. The resulting objective function and constraints are

$$\max_{C_L, V} J = - \int_{t_0}^{t_f} (P_{collected} - P_{consumed}) \, dt$$

such that

$$[x(t_0), y(t_0), z(t_0)]^T = [x_0, y_0, z_0]^T$$

$$[x(t_f), y(t_f), z(t_f)]^T = [x_f, y_f, z_f]^T$$

$$|\phi| < \frac{\pi}{2}.$$  \hspace{1cm} (5.35)

To ensure optimality, the Hamiltonian function is

$$H(V, x, y, z, \gamma, \psi, \lambda_V, \lambda_x, \lambda_y, \lambda_z, \lambda_\gamma, \lambda_\psi, C_L, \phi) = P_{collected} - P_{consumed} + \lambda_V \dot{V} + \lambda_x \dot{x}$$

$$+ \lambda_y \dot{y} + \lambda_z \dot{z} + \lambda_\gamma \dot{\gamma} + \lambda_\psi \dot{\psi},$$

where $\lambda_V, \lambda_x, \lambda_y, \lambda_z, \lambda_\gamma,$ and $\lambda_\psi$ are the costates for states $V, x, y, z, \gamma,$ and $\psi$ respectively, while $\dot{V}, \dot{x}, \dot{y}, \dot{z}, \dot{\gamma},$ and $\dot{\psi}$ are given in (4.19). The costate equations are given by

$$\dot{\lambda}_V = \frac{3}{2 \eta_{prop}} \rho V^2 S \left( C_{D_0} + K C_L^2 \right) + \lambda_V \frac{1}{m} \rho V S \left( C_{D_0} + K C_L^2 \right) - \lambda_x \cos \gamma \cos \psi$$

$$- \lambda_y \cos \gamma \sin \psi - \lambda_z V \sin \gamma - \lambda_\gamma \left( \frac{1}{2m} \rho S C_L \cos \phi + \frac{g}{V^2} \cos \gamma \right)$$

$$- \lambda_\psi \frac{1}{2m} \rho S C_L \sin \phi \sec \gamma$$

$$\dot{\lambda}_x = 0$$

$$\dot{\lambda}_y = 0$$

$$\dot{\lambda}_z = 0$$

$$\dot{\lambda}_\gamma = \eta_{sol} P_{sd} S \left( \cos \epsilon \cos \gamma \cos \phi \cos (a - \psi) + \sin \epsilon \sin \gamma \cos \phi \right) + \lambda_V g \cos \gamma$$

$$+ \lambda_x V \sin \gamma \cos \psi - \lambda_y V \sin \gamma \sin \psi - \lambda_z V \cos \gamma - \lambda_\gamma \frac{g}{V} \sin \gamma$$

$$- \lambda_\psi \frac{1}{2m} \rho V S C_L \sin \phi \sec \gamma \tan \gamma$$

$$\hspace{1cm} (5.37)$$

$$\hspace{1cm} (5.38)$$

$$\hspace{1cm} (5.39)$$

$$\hspace{1cm} (5.40)$$

$$\hspace{1cm} (5.41)$$
\[
\dot{\lambda}_\psi = \eta_{sol} P_{sd} S ( - \cos e \sin \phi \cos (a - \psi) + \cos e \sin \gamma \cos \phi \sin (a - \psi)) \\
+ \lambda_x V \cos \gamma \sin \psi - \lambda_y V \cos \gamma \cos \psi. 
\] (5.42)

The first-order optimality and second-order Legendre-Clebsch conditions are numerically approximated during implementation. Similar to the two-dimensional problem, the final constraint of \( H^* = 0 \) must be satisfied for free final time.

### 5.3 Implementation

The optimal paths satisfying the objective functions, constraints, and corresponding optimality conditions presented in Section 5.2 are approximated numerically using the \texttt{fmincon} function in the Optimization Toolbox and the numerical ordinary differential equation solver \texttt{ode45} function in the MATLAB environment. The path is discretized into \( n \) intervals of equal duration with constant control inputs within each interval. \texttt{fmincon} performs a constrained optimization to minimize the objective function by altering the control inputs while \texttt{ode45} solves for the states given these control inputs. Table 5.3 lists the optimization parameters and conditions used for the thesis unless otherwise stated.

**Table 5.3: Optimization conditions**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \phi_o (\circ) )</th>
<th>( V_o (m/s) )</th>
<th>( t_o (s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0</td>
<td>14</td>
<td>0</td>
</tr>
<tr>
<td>([\phi_{min}, \phi_{max}] (\circ))</td>
<td>([V_{min}, V_{max}] (m/s, m/s))</td>
<td>([t_{o, min}, t_{f, min}] (s, s))</td>
<td></td>
</tr>
<tr>
<td>([-45, 45])</td>
<td>([0, 30])</td>
<td>([0, 600])</td>
<td></td>
</tr>
</tbody>
</table>

### 5.4 Impact of Solar Cell Efficiency on Optimal Path

Simulation results for an energy-optimal path using the two-dimensional model in Section 5.2.2 are presented here. The objective is to demonstrate, given ideal conditions, the difference in optimal paths for the UAV at two different extremes in solar cell efficiency, 0% and 95%. While in this study it is the solar efficiency that is changed, the net effect would be the same if the solar density, \( P_{sd} \), area covered by solar cells \( S_{sol} \) or sun positions \( a \) and \( e \) were changed. The boundary conditions and initial conditions
are listed in Table 5.4. The paths are discretized into 16 nodes and are shown, along with the time histories of the state variables, in Figure 5.4. While the path travelled by the UAV with 95 % efficient cells requires more energy than the one with 0 % efficient cells, 13.517 kJ compared to 12.013 kJ respectively, the net energy total for the former is significantly lower, −0.890 kJ compared to −12.013 kJ for the latter. These results are summarized in Table 5.5. The simulation example demonstrates that the amount of energy collected can significantly affect the optimal path taken by the UAV.

Table 5.4: Initial and boundary conditions for two-dimensional paths

|                |               |               |               |               |               |
|----------------|----------------|----------------|----------------|---------------|
| \( a (\degree) \) | \( e (\degree) \) | \( P_{sd} (W/m^2) \) | \( S_{sol} (m^2) \) | \( \eta_{prop} (%) \) | \( S (m^2) \) |
| 0              | 45             | 380            | 0.1566         | 70            | 0.1566         |
| \( V_0 (m/s) \) | \( \psi_0 (\degree) \) | \([x_0, y_0]^T (m)\) | \([x_f, y_f]^T (m)\) | \( T_M \) |
| 15             | 62             | \([0, 0]^T\) | \([700, 1300]^T\) | 300          |

Table 5.5: Simulation results of two-dimensional paths

<table>
<thead>
<tr>
<th>( \eta_{sol} (%) )</th>
<th>( t (s) )</th>
<th>( E_{col} (kJ) )</th>
<th>( E_{con} (kJ) )</th>
<th>( E_{net} (kJ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>298.8</td>
<td>0</td>
<td>12.013</td>
<td>−12.013</td>
</tr>
<tr>
<td>95 %</td>
<td>299.1</td>
<td>12.627</td>
<td>13.517</td>
<td>−0.9</td>
</tr>
</tbody>
</table>

### 5.5 Flight Test Area

Flight tests were performed at Koffler Scientific Reserve at Jokers Hill and the optimal path was designed for this specific location and the desired test date. The location, date and time were used to predict the sun position, while the boundaries and terminal constraints were strategically selected given the flight location. The flight location is shown in Figure 5.5. The scheduling of flight tests were also on days that were sunny or slightly cloudy with winds less than 10 km/h to maximize \( P_{sd} \) and reduce the effects of winds and gusts.
Figure 5.4: Simulation results of two-dimensional energy-optimal paths for 0% and 95% efficient solar cells
5.6 Optimal Flight Path

The optimal path for the objective function in (5.18) and satisfying the constraints in (5.19) specific for the flight area is shown in Figure 5.6. The environment and boundary conditions are summarized in Table 5.6 and the results listed in Table 5.7. The simulated results were obtained using the conditions set in Table 5.6 and 5.3. The Kestral autopilot on-board the mini flying wing UAV is limited to either control position or roll. Since the change in bank angles is small compared to the distances travelled, way-point navigation was prioritized. The experimental results were obtained through the on-board current and voltage sensors at the same positions and velocities determined through simulation.

Table 5.6: Environment and boundary conditions for experimental test

<table>
<thead>
<tr>
<th>$a$ (°)</th>
<th>$e$ (°)</th>
<th>$P_{sd}$ (W/m$^2$)</th>
<th>$[x_0, y_0, \psi_0]^\top$ (m)</th>
<th>$[x_f, y_f]^\top$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>152.74</td>
<td>59.6</td>
<td>499</td>
<td>$[18.25, -18.94]^\top$</td>
<td>$[174.7, 104.6]^\top$</td>
</tr>
</tbody>
</table>
Figure 5.6: Experimental test on optimal path with $\psi_0 = 40^\circ$ and $P_{sd} = 499 \text{W/m}^2$

Table 5.7: Optimal flight path

<table>
<thead>
<tr>
<th></th>
<th>$E_{col}(J)$</th>
<th>$E_{con}(kJ)$</th>
<th>$E_{net}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>53.63</td>
<td>3.196</td>
<td>-3.142</td>
</tr>
<tr>
<td>Experiment</td>
<td>33.97</td>
<td>2.462</td>
<td>-2.428</td>
</tr>
</tbody>
</table>
It is important to note that while only the optimal flight path with $\psi_0 = 40^\circ$ is shown, a sweep of initial heading angles from $0^\circ$ to $110^\circ$ was evaluated. Within this set of initial heading angles, the only initial heading to satisfy the constraints in (5.19) given the current specifications of the UAV was at $40^\circ$. In the next section, the final constraint on the range of bank angles was relaxed and the results compared.

From the results in Table 5.7, one can conclude that given the current set-up the energy collected from the solar panels is significantly lower than the energy used. The large error between the experimental results and the designed optimal path results could be attributed to the limitations in the Kestral™ autopilot. While the waypoints for the autopilot were chosen to best match the positions and velocities of the designed optimal path, there was no control on the bank angles. This is seen in the dissimilarity between the experimental and designed bank angles shown in Figure 5.7.

![Figure 5.7: Comparison between the bank angles and velocities of experimental path and designed optimal path](image)

**Figure 5.7: Comparison between the bank angles and velocities of experimental path and designed optimal path**

### 5.6.1 Relaxed Constraints

As previously mentioned, the only initial value for heading between $0^\circ$ and $110^\circ$ that provided a solution satisfying the constraints in (5.19) was $\psi_0 = 40^\circ$. However, if the constraint on the bank angle is relaxed, optimal solutions are possible and are shown in Figure 5.8. Experimental flight tests using the positions and velocities of these simulated energy-optimal paths were performed and the results summarized in Figure 5.9. As expected, as the flight path deviates from the most direct path at $\psi_0 = 40^\circ$, the energy consumed increases, however so too does the energy collected. In addition, Figure 5.9
shows that the model for the UAV and energy approximations are very accurate to the energy measured during flight. The error in the model was below 1% for average energy collected and 10% for average energy consumed.

With the current experimental set-up, the photovoltaic cells only provide 1% to 3% of the total energy required. There are several factors that have caused this to be significantly lower than the high level approximations in Chapter 2.1 and the earlier simulation results. The high level approximation assumed the best case scenario: a high \( P_{sd} \), no power conversion losses, \( \theta = 0 \) and the entire FSC UAV filled with photovoltaic cells. All four of these approximations proved to be unrealistic during experiments as the recalibrated solar density was approximately 50% lower, losses were seen due to impedance mismatches, the solar incidence angle was non-zero, and the photovoltaic cells only covered 14% of the UAV planform. Compared with the previous simulation results, the simulation assumed the planform area equalled the planform area covered by solar cells, and the propeller being much more efficient. Both of these differences resulted in a much lower energy collected and higher energy consumed measured during flight tests.
5.6.2 Verification of Energy-Optimal Flight Path

In order to show that the design energy-optimal flight path is indeed optimal, the set of experimental paths for a given initial heading is compared. Since there are fluctuations in each iteration during flight tests, the path closest to the designed path should have the lowest $E_{net}$ while the variations with same initial heading should be less energy-optimal. Figure 5.10 shows the experimental paths taken for the simulated energy-optimal path with $\psi_0 = 110^\circ$. The closest path was experimental path #2 and this is supported by having the lowest experimental $E_{net}$ in Figure 5.11. All paths aside from path #2 had a larger $E_{net}$ as they were further from the optimal path.
Figure 5.10: Simulated energy-optimal path with $\psi_0 = 110^\circ$ and corresponding experimental flight paths.

Figure 5.11: $E_{net}$ for various flight paths with $\psi_0 = 110^\circ$. 
5.7 Summary

In this chapter, the justification for energy-optimal path planning is demonstrated through experimental ground and flight tests. The energy-optimal path problem is derived in the general case and the specific problems including cost function and constraints for two- and three-dimensions are presented. A simulation study on the effect of photovoltaic cell efficiency on the optimal path showed expected behaviour at high efficiencies. The energy-optimal path for the specific flight area was obtained and the energy approximations validated through flight tests. Relaxing the constraints on the optimization allowed for solutions with different initial heading values and the simulated paths validated through flight tests. Reasons were provided to explain the low contributions of the energy collected through the photovoltaic cells relative to the energy required. Finally, the energy optimality of a designed path is justified experimentally by comparing multiple experimental flight paths with identical initial conditions against the one closest to the designed path.
Chapter 6

Conclusions

In this thesis, the overarching focus was to approach the design of a solar-assisted mini UAV through a combination of aircraft control and power management perspectives and validate each step through simulation and experiments. This goal was achieved through four research objectives. First, the potential for reduced energy consumption or equivalently prolonged endurance of a solar-assisted mini UAV was demonstrated through high level conceptual design. This estimation was further supported by comparing the energy obtained through CIGS thin-films on a test bench with similar loads to the FSC UAV. Second, the energy lost in the mini UAV system was reduced through power management. This included the selection of a buck-boost converter with a MPPT controller, and the studies on the optimal pairing of lithium polymer batteries and ratios of panel-converter ratios. Third, an accurate model of the solar-assisted UAV was verified through flight tests. A two dimensional model relating the states of the UAV to the energy collected from the solar modules was derived. This was further extended to a three dimensional model to allow for changes in altitude. Fourth, an energy-optimal flight path was designed and validated through simulation and flight tests.

Further work for this project can be done to extend each category of solar aircraft design, power management and path planning. In regards to solar aircraft design, this thesis was concerned with retrofitting an existing UAV. Possible extensions include designing the aircraft configuration to use the least amount of energy for the desired flight path and increasing the coverage and efficiency of the solar panels on the aircraft. In regards to power management, future work could involve regulating the voltages from the converters and batteries to minimize impedance mismatches. In regards to path planning, viable avenues include a more sophisticated autopilot to better match the planned path and further refining the models used. As mentioned in Chapter 4, the equations of motions are currently extensions to the unicycle model.
Bibliography


[37] Procerus Technologies, Kestrel Autopilot v2.4, 2008.


