CONTRIBUTIONS TO THE DEVELOPMENT OF A REAL-TIME ULTRASOUND SIMULATOR

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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Abstract

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Realistic, real-time ultrasound simulation is important for medical education and research. This thesis continues to develop the Fast and Mechanistic Ultrasound Simulator through understanding computational artifacts, constructing metrics for reliable simulation, and improving acoustic modelling. Computational artifacts in simulated Doppler spectra were found to originate from various types of discretization, such as in sampling frequency and subsequent phase mismatches, providing insight into mitigation strategies. A study of simulation and input data parameters has determined a practical estimate of greater than 6 point scatterers/mm$^3$ within the acoustic sample volume as sufficient to support consistent and realistic Doppler simulation. Finally, frequency dependent attenuation was incorporated to model dispersive effects in tissue. The resulting simulated B-mode images are comparable to those of the Field II ultrasound simulation package, with only a modest performance penalty over the existing simulator.
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To my dear Hannah, who walks along with me and loves me much, thank you for listening.

May the days come soon where simplicity bears fruit in kindness and justice.
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Chapter 1

Introduction

Simulation can be broadly described as an imitation of some real object, situation, or process, created for the purpose of exploration, forecasting, experimentation, or teaching. It has found extensive use in these areas due to its flexibility, repeatability, and abstraction from real life. Indeed, many occupations rely upon simulation to test and teach skills in settings where ideal responses are fairly well known, without the risks associated with the genuine situation. For instance, aviation and astronautics frequently make use of simulation to test equipment and train pilots [1, 2] under a variety of routine and high-stress situations such as those that might occur once in a lifetime.

In medicine, carefully rendered simulation can likewise help provide the training needed for the skills required for diagnosis and treatment. It can do so by providing a realistic testbed to examine the causes and effects of successful and failed procedures, thereby honing intuition to avoid error in practical situations. A simulated environment also helps minimize the risk of further injury or complication for existing patients, nor violates patient autonomy, thereby maintaining ethical standards of both care and education [3]. Simulation can appear in a variety of forms designed to provide realistic feedback through visual, audio and tactile cues, such as physical props or mannequins of varying complexity, situational actors, or computational models of reality [4].

Furthermore, the intersection of increasingly advanced technology and need for skilled medical professionals has led to more interest in finding ways to augment and improve medical education. As such, many specializations including endoscopy, obstetrics, cardiology and radiology have made use of simulations as part of their curricula [5]. Consequently, simulation has demonstrated that it can provide beneficial training, such as in preparation for laparoscopy and colonoscopic procedures [6, 7], which have been shown to improve the success of such maneuvers. Similarly, in the field of diagnostic ultrasound, the training of radiologists and sonographers can be significantly enhanced by allowing rare cases to be presented to the trainee without requiring the presence of a patient.
1.1 Diagnostic Ultrasound

Diagnostic ultrasound maintains an established position within medical pedagogy as an effective imaging modality, due to its relatively good spatio-temporal resolution, non-invasive and nearly risk free operation when used within diagnostic limits, portability, and low cost compared to other imaging techniques [8]. Though typically performed in separate appointments and analyzed by radiology specialists, ultrasound has also found increasing use in point-of-care treatment situations, allowing non-specialist physicians to quickly examine patients on demand and make decisions based on their observations [9].

In principle, ultrasound imaging functions by transmitting acoustic waves in the ultrasonic range into the human body. As these waves encounter inhomogeneities in the body, such as cysts or growths, they are reflected back and received by the ultrasonic probe. The resulting changes in amplitude, timing and shape of the initial waveform provides information about the material properties of the structures inside the body. This behaviour is has been well studied in human tissue, where sound generally moves on average at 1540m/s, while in contrast, the speed of sound in air is approximately 330m/s [10].

![Figure 1.1: A basic phenomenon underlying ultrasound imaging is the nature of echoes or reflections due to inhomogeneous scatterers. The incident wavefront (red) impinges on a target, which reflects a weaker wavefront (blue), resulting in a delayed version of the incident wave at a later time, \( t_1 + t_2 \).](image)

Usually, a fairly large array of transmitting and receiving elements are employed to produce a stronger transmitted and received signal. Since distance and amplitude information is stored in the returning wavefronts, it is often important to align the signals
acquired from each element, relative to some focal point. This is accomplished through a sequence of delays, resulting in clearer contrast between background noise and the presence of an object, which in turn can be interpreted as information for medical diagnosis.

![Diagram](image)

Figure 1.2: The basic acquisition flow for an array of elements operating as a phased array is illustrated. Signals transmitted and then received by multiple array elements are inherently not aligned after reflection from some scatterer. In order to generate a strong and coherent response, a set of delays $\tau_i$ are set for each array element, based on the desired focal point. The resulting summation from each element channel improves signal contrast.

1.1.1 B-mode Ultrasound

Ultrasound imaging can be employed in many ways, but there are two ubiquitous modes of operation that will be the emphasis of this thesis. The first is B-mode or B-scan ultrasound imaging, and is used to image two-dimensional slices into the body, often to analyze sections of the heart, liver or other organs, or for prenatal assessments. Different variations capture still images or movies, or are overlaid with additional modes of imaging to further enhance diagnostic assessments.

This mode creates images through successive capturing of individual scan lines (known also as A-lines) which are spatially adjacent to one another, producing a two-dimensional depth scan into some region of the body. Figure 1.3 depicts the process. As the transducer array is shifted across the scene outlined in green, individual A-lines are captured and the envelope of each is determined. Typically this is followed by a logarithmic conversion to display within some selected power range. This is repeated for a designated number of lines (usually between 50 - 200 lines), and the resulting collection of envelopes is used to form a two-dimensional image of the interior. Depending on the parameters of the transducer, and the use of more than one focal zone, the lateral and axial resolutions of
the received signals may be improved to increase the quality of a B-mode image.

![Figure 1.3](image.png)

Figure 1.3: A simple sketch of the B-mode acquisition process is shown here. By laterally moving the transducer, a series of scan lines or A-lines (radio frequency signals) are recorded. Taking the power or magnitude of the envelope of these signals, a 2D image can be formed.

1.1.2 Doppler Ultrasound

The second major modality of interest in this thesis is Doppler ultrasound, which extracts information about the flow velocity of blood based on its relationship to the frequency content of the reflected acoustic signal. This technique can be employed using either continuous wave excitation or pulsed transmissions. The former approach converts fluctuations in the carrier frequency due to relative motion into velocity data, while the latter uses multiple echoes to determine velocity by recording changes in position over several time instants. For various reasons, most modern Doppler ultrasound systems operate on a pulsed wave basis rather than using the Doppler effect as it arises from continuous waves. However, the actual phenomenon in use has mathematical similarities to the Doppler effect, leading to the somewhat misleading name adoption of “Doppler ultrasound” [11]. In this thesis, references to Doppler ultrasound will imply pulsed wave flow techniques, unless otherwise stated. In practice, this type of assessment has high temporal resolution and has shown itself to be crucial for observing and advising procedures related to the vascular system, such as the inspection of cardiac functions, the examination for atherosclerosis, and the post-analysis of aneurysms and other forms of brain haemorrhage [12, 13].
Doppler ultrasound systems generally operate by transmitting and receiving a rapid succession of long duration acoustic pulses at a particular region (e.g. subregion within a blood vessel) for some given pulse repetition frequency (PRF) and isonation angle. In essence, this is a form of motion tracking based on rapid sampling over a defined time period. As targets such as red blood cells move through the region of interest, their relative motion induces change in each reflected pulse.

The overall ensemble of returned waves is then subject to further processing to determine changes of phase or frequency over the measurement period. Sampling from a particular depth or time within this collection of signals, a new signal is extracted and transformed using a short-time Fourier transform (or a similar mathematical tool) to visualize the changing frequency content. By examining these frequency changes, corresponding shifts in velocity can be calculated according to the Doppler formula: 

$$\Delta v = \frac{\Delta f_c}{2f_c \cos \theta}$$

for the measuring period, which are then used to inform medical diagnoses.

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**Figure 1.4:** An illustration of Doppler signal acquisition. Multiple pulses are transmitted at some PRF and angle $\theta$ into the vessel of interest. Many fast-time radio frequency signals are recorded, from which a slow-time signal is subsampled at a certain depth or time, based on the sample volume (in yellow) and sample depth (red ‘x’). Conversion to the frequency domain via a sliding short-time-Fourier transform describes the changing frequency content over the measurement period, providing flow velocity information.
1.1.3 Attenuation

In the realm of ultrasound imaging there are a few important concepts and phenomena to account for when operating or designing related tools or procedures. The first of these is attenuation, a broad category of effects that greatly influence wave phenomena, whether they be electromagnetic, acoustic or other in nature.

Attenuation is the decay of wave amplitude or energy as it travels in a medium, due to the material nature of the space in which it propagates. It generally arises from a combination of scattering due to inhomogeneities and absorption due to the conversion of wave energy into heat, light or other forms. This is simply described by an additive relation, \( \alpha = \alpha_s + \alpha_a \), corresponding to the scattering and absorption. For simple simulations or monochromatic signals this is often well encapsulated by an exponential decay with distance from the emitting sources, much like would be expected by a free space emission. However, a wider understanding of attenuation necessarily includes the effects of dispersion and frequency dependent effects.

In this case, it should be evident that for wideband acoustic signals, often pulsed transmissions, there is significant frequency information in the wave. Since human tissue is not in general a dispersionless medium, the effect of signal decay with distance is not felt evenly across all frequencies. This results in a distortion of the pulse shape as it travels, including loss of detail and information. This type of phenomenon is most readily felt in B-mode imaging, which generally uses a wideband transmit pulse (e.g. 100% fractional bandwidth) and where the region of interest encompasses a wide spatial extent; it is expected that acoustic waves broadcast throughout an entire region will experience non-uniform changes, thereby affecting the final image. However, in Doppler ultrasound the transmit signal generally has a narrow bandwidth (typically less than 10%) so that the effects of dispersion can be ignored. Correct calculation of this phenomenon is clearly necessary for a realistic simulation, and will be explained in greater detail in Chapter 4.

1.1.4 Acoustic Fields and Sample Volumes

The ability for ultrasound machines to generate useful data or images of some region of interest depends heavily upon the nature of the acoustic field generated by the imaging array or imaging transducer in transmission and reception. Arrays of transmitting and receiving elements necessarily produce an acoustic field with power profiles dependent on the size of the array, weighting or apodization of elements, and the position focal points of the lateral and elevation dimensions that result in a particular angle of the field.

In addition to the main structure of the acoustic field (otherwise known as the main “beam”), diffraction causes wave energy from the array to generate sidelobes, which are
Figure 1.5: Reproduced, with permission from Cobbold [14]. (a) and (b) demonstrate the effect of distance on a transmitted signal experiencing attenuation without dispersion compared to a signal that also experiences the effects of dispersion. The latter not only weakens in amplitude, but loses shape as the dispersive effects distort the signal.

weaker but still significantly powerful portions of the acoustic field, which may illuminate undesired regions of the medium. It is possible to mitigate this effect through careful weighting or apodization of the elements of the transducer array, and it is generally desirable to have as narrow as possible of an acoustic field to improve resolution and reduce noise. There is also an additional category of artifacts called grating lobes, which are replicas of the mainbeam occurring at intervals defined by the spacing of transducer array elements. Like sidelobes, these regions of the acoustic field can interfere with imaging by focusing energy into undesired regions of the medium. However, grating lobes can be eliminated if the spacing between transducer elements is less than $\lambda/2$.

Overall, depending on the parameters of the transducer and positioning of this field, targets of interest may or may not generate sufficiently powerful return signals, and undesired regions may receive more signal energy than intended, causing some loss of clarity in the resulting image.

This leads to the notion of sample volumes (SV), which may be understood as a defined region of high signal power within an acoustic field. Steinman et al. have analyzed the properties of a sample volume in detail [15], noting relationships to the geometry, positioning and operation of the transducer. As shown below in Figure 1.7, an acoustic field varies in power or intensity over its domain, where there is usually one area of highest power near the focal point. This region can be considered the sample volume and its size is usually demarcated by some cutoff level (e.g. -20dB corresponds to contributions within $\sim 1\%$ of peak power). Transducer geometry is a major factor in SV shape: for example, a teardrop shaped volume is applicable to circular arrays, while a linear array produces a highly directed and asymmetric sample volume. Furthermore, the size and shape of the sample volume change with the excitation duration of the transducer and
the range gating applied by the system. More excitation cycles result in elongated sample volumes, which are sometimes useful to gather information from a bigger region.

The concept of a sample volume is an important distinction because its shape and position, along with possible interference from side and grating lobes greatly affect the ability of a radiologist or ultrasound technician to observe the properties of structures and anomalies inside the body.

1.1.5 Spectral Broadening

A final accompanying phenomena related to sample volumes which affects Doppler ultrasound is the presence of spectral broadening. This phenomenon is concerned with the widening of the range of significant frequency content in an acoustic signal due to a number of factors, either relating to the geometry of the transducer or to the behaviour of the flow being observed [16].
Figure 1.7: Reproduced, with permission from Aguilar. Lateral (a) and elevation (b) views of the acoustic field down to -20dB produced by a 65x9 element linear transducer array. The scale on (a) indicates power levels inside the field in dB. The effect of lateral focusing is clear; the field as viewed laterally is very narrow, while the elevation view is significantly broader.

The cause of changes in the Doppler spectrum of a signal attributed to the geometry of the transducer are denoted “Intrinsic Spectral Broadening” (ISB), and are seen due to the finite width and extent of the acoustic beam. Meanwhile, the second main source of variations in the Doppler spectrum originates from the behaviour of the particle or scatterer itself, known as “Extrinsic Spectral Broadening”. From these sources a number of possible scenarios unfold (illustrated in Figure 1.8)

- The inhomogeneous strength of the acoustic field means that a constant speed scatterer passing through a sample volume of finite width will naturally produce variations in the returned signal amplitude, leading to a spectrum of frequency contributions, termed *transit-time broadening*.

- As a red blood cell passes through the sample volume within the beam, the relative angle with respect to the transducer surface changes. This change in angle necessarily produces a spread of frequency, widening the initially narrowband signal. This is known as *geometric broadening*.

- During a sampling period (e.g. between two pulses, when the interrogating wave reflects off the scatterer), the scatterer may accelerate and change velocity, resulting in *non-stationary broadening*.

- Lastly, throughout the entire sampling volume, the scatterer’s velocity may change
entirely, which is considered velocity gradient broadening.

![Figure 1.8: Depictions of different sources of spectral broadening. The examples in (a) and (b) are due to intrinsic properties of the ultrasound imaging system, while (c) and (d) are results from the nature of blood flow.](image)

In general, the longer a scatterer remains within the sample volume, its presence is recorded with more information in the interrogating signal, providing a better estimate of its true velocity. This is merely a spatial analogue of the uncertainty principle, and is an inherent property to consider in temporal imaging.

The presence of spectral broadening can both provide useful information about blood flow characteristics, as well as obfuscate them. For instance, the distribution of velocities is naturally a source of spectral broadening, and should be reported, however, broadening can also make it difficult to determine singular quantities, such as the peak flow velocity. As alluded to in the previous subsection, issues with spectral broadening are subject to the properties of the transducer, the parameters of the excitation signal, and also the angle at which the measurements are recorded.

## 1.2 Ultrasound Simulation

Like many other areas of medical practice, ultrasound imaging is nearly a work of art, since the ability to both acquire useful information and make meaningful interpretation requires substantial skill and intuition. Historically, training has followed an apprenticeship model of “learning by doing”, which often results in high quality education, but which is understandably time consuming. The large demand for ultrasound as a diagnostic measure, particularly with increased interest for focused ultrasonography [9] necessitates additional forms of educating students.

In order to effectively train radiologists and technicians in the use of ultrasound, simulation is often employed to achieve suitably real and immersive conditions. Similar to other areas of medical simulation, a number of different approaches may be used. Some common methods of ultrasound simulation and education are:

- Real human subjects,
• Interpolated feedback,
• Physical phantoms (e.g. agar gel models of liver, carotid arteries, etc.),
• and Computational models

Before outlining the main forms of simulation, it is important to note the use of real patients in training technicians and radiologists. Strictly speaking, this is generally not a simulation method, as actors cannot simulate interior anatomical conditions; however, all simulation approaches are to be compared to the use of diagnostic ultrasound on humans, being the subject of application of ultrasound imaging. Consenting patients can be used to give medical residents a genuine scenario while accompanying supervising practitioners can help to monitor and give feedback on the performance of the examination. Ultimately, the end goal of this type of education is to train residents well enough that their own examinations are more than sufficient to assess patients, isolate anomalies, and determine the sources of problems. Unfortunately, opportunities with live patients are usually limited in availability and variety, and not all cases have the flexibility needed to support training practice.

1.2.1 Image Interpolated Simulation

In lieu of an actual patient on which to practice the use of ultrasound imaging, some simulators use prerecorded images of previous ultrasound examinations as performed by an accomplished technician. In this case, an extensive dataset is captured at many key points throughout a scan, and mapped to the corresponding placement of the ultrasound probe at that time. Then the simulation system supplies a mock ultrasound probe, controls, and mannequin to a student, who can direct the examination based on their own decisions. As the mock probe is moved, the system interpolates readings and images based on the pre-recorded volumetric data to present feedback to the learner. MedSim currently carries such a product called Ultrasim® [17], which is capable of B-mode, Colour and Spectral Doppler imaging.

However, interpolation accuracy is limited and presents useful data only as long as the student does not stray far from the original acquisition positions. This limits exploration on the student’s part, and may also provide a misleading estimate on examination proficiency. Furthermore, such simulations are also generally unable to support spontaneous change in parameters (e.g. ultrasound frequency or focus depth), which add more constraints and restrict the quality of training a student can receive.
1.2.2 Physical Phantoms

Another option is to design high fidelity physical models, otherwise known as phantoms, which can incorporate tissue properties and fluid behaviour accurately. These are usually well calibrated models using some sort of gel or partially organic material to effectively copy tissue and blood vessel properties, and may incorporate blood-mimicking fluids with tuned pumping systems to apply vascular behaviour, such as demonstrated by Poepping et al. [18]. Using these type of models, students or researchers are at liberty to use a real ultrasound system with complete control over its parameters, and practice understanding the resulting images. However, these models are costly and time-consuming to construct, and naturally are limited in scope - they must be designed for particular situations or parts of the body, and thus are not suitable for training a wide variety of scenarios, unless the teaching institution has the time, money and storage facility to build phantoms for all such cases.

1.2.3 Computational Simulation

The final category and main focus of this thesis will be the use of computational models or computer based simulations. This class of simulation approaches rely upon quantitative calculations based on empirical data combined with mathematical models or completely mechanistic frameworks to determine the relative or absolute numeric behaviour of acoustic fields in some defined environment. From this physical simulation of reality, data can be recorded and processed as it would appear in an actual ultrasound machine, leading to realistic images and readings. A brief survey of some existing simulators is given here.

Many simulators operate based on determining the spatial impulse response (SIR) of the transducer array with respect to the field points of interest. To calculate such responses, often the transducer array is represented by a collection of smaller continuous elements. A well-known ultrasound simulator of this particular design is the Field II [19] simulation package, which is capable of a high degree of control of key aspects such as transducer geometry, excitation parameters, and environmental settings, making it a versatile platform to simulate various ultrasound interactions and imaging techniques. Other simulators designed in this manner include FOCUS, by McGough et al. based on earlier work [20] and designed for fast and accurate calculation of the near-field acoustic pressure, Ultrasim (no relation to the Ultrasim @designed by Medsim) [21] and the DREAM toolbox [22], which is based on discrete representation of the impulse response.

Some simulators represent the transducer as a collection of points rather than geometric entities like rectangles or triangles. Previous work has produced strategies such as the Distributed Point Source Method (DPSM) [23] or the Complex Equivalent Source
Method (CESM) [24], and more recently the Fast-and-Mechanistic Ultrasound Simulation (FAMUS) approach [25], which was developed to address the balance between speed and accuracy of simulation.

Alternatively, some approaches use empirical data from experiments combined with mathematical models to achieve particular imaging objectives in order to produce the final ultrasound image. These are considered to be semi-empirical approaches, and have the advantage of simplifying the required calculations, but at the cost of general simulation of the acoustic field or notable artifacts, which also limits their accuracy and flexibility of use in other applications. Examples of this type of simulation include FUSK, a fast frequency domain (or k-space) ultrasound B-mode simulator based on a point spread function convolution method demonstrated by Hergum et al. [26], Mo and Cobbold’s work on Doppler signal simulation through the use of stochastic Gaussian processes [27] and a more comprehensive setup provided by Khoshniat et al. which coupled a prescribed sample volume with a precomputed computational fluid dynamics (CFD) velocity field, in order to simulate Doppler ultrasound under more realistic blood flow conditions [28].

Finally, it is important to discuss the way in which biological media can be represented in digital form as input for computational simulation. In many cases, small spherical scatterers are used to describe soft biological media in ultrasound, which for computational purposes may be resolved to a point. This is partly because individual cells in tissue and blood are generally much smaller than the interrogating wavelength, and do not reflect ultrasonic waves strongly. For example, the maximum dimension of a red blood cell (RBC) is approximately 8µm [29], close to 40 times shorter than $\lambda = 0.308$mm, for propagation of $f_c = 5$MHz and $c_0 = 1540$m/s. However, despite the fact that blood cells are weak scatterers, under normal hematocrit (RBC density) of $\sim 45\%$ one can expect to see $5 \times 10^6$ cells per cubic millimetre, which is dense enough that individual cells are no longer positionally independent of each other, and exhibit some correlation. Brody [30] and Atkinson and Berry [31] recognized the partially coherent nature of cells clustering or travelling in ensembles, providing local density fluctuations that give rise to constructive and destructive interference that can be detected by ultrasound transducers. Subsequently, the voxel approach of Mo and Cobbold [32] permits groups of red blood cells to be described by a single scatterer with an equivalent backscattering coefficient, where again we may use a point given that such groups are still smaller than a typical ultrasound transmission wavelength.
1.3 Objectives

Given the importance of and demand for ultrasonography, there is a need for immersive training programmes that are realistic in both response time and physical representation in order to fully develop the skills of future sonographers. While computer based simulations are highly attractive in terms of combining realism with flexibility, they like the aforementioned simulation methods bear an associated set of benefits and drawbacks.

One of the main difficulties of using computer based simulations, especially for those which are purely mechanistic, is the ability to perform calculations in real time. While the clear advantage of computational simulation is its high degree of numerical accuracy and thus its potential high fidelity to physics, the cost in time and number of calculations needed to precisely represent the acoustic field of the transducer and its subsequent interaction with a realistic target is generally very large, and only increases in complexity when coupled with other data driven models e.g. CFD based blood flow in vessels. Without a way to overcome this impasse, mechanistic simulation remains unsuitable for training purposes. Therefore the balance between realism and speed is one critical aspect which requires special attention.

The FAMUS simulator developed by Aguilar et al. [25] has so far been relatively successful in this regard, by demonstrating an approach which operates in near real time speed without much loss of accuracy for the acoustic (mid- and) far field, compared to Field II. However, the need for even more efficient simulation processes remain, as it is possible to couple the simulator with more detailed anatomical models or empirical CFD models of blood flow in intricate vessels, each of which involve a large number of scatterers or velocity trajectories and greatly increase the computational burden [33]. Furthermore, there are additional physical phenomena yet to be accounted for, such as dispersive effects, shear waves, and varying sound speeds due to inhomogeneous media, which also contribute to the computational workload of the simulation. Finally, there also some outstanding pieces of ultrasound imaging functionality yet to be implemented.

The purpose of the work described in this thesis is to build upon the advances demonstrated by the FAMUS framework, and strive toward achieving real-time, high accuracy performance of the ultrasound simulator. Specifically, it is proposed to:

1. Evaluate the tradeoffs between speed and realism by exploring the effects of various input data characteristics and choices of system parameters on Doppler simulation,

2. and proceed with development and implementation of frequency dependent attenuation as it arises in human body tissue.

By addressing these needs, it is hoped that the resulting improvements can provide both
realistic functionality and real-time performance, which together can create a cohesive learning environment.

1.4 Organization

As we pursue the aforementioned goals, the work described in the remainder of this thesis is organized as follows:

In Chapter 2, additional technical background concerning various computational based simulation methods is reviewed, along with the fundamental operating principles of FAMUS, the key framework on which this thesis is based.

Chapter 3 will provide discussion centred on the assessment of simulation parameters and important nuances encountered in the simulation of ultrasound physics, particularly in Doppler ultrasound. Several experiments are reported to describe these subtleties, and their effects on how simulation is performed are discussed. In particular, a major exploration culminates in the understanding of the range of detail (or number of point scatterers) required for consistent and largely realistic feedback.

Next, Chapter 4 details the development of frequency dependent attenuation, which is a necessary part of the physical model required to capture the behaviour of ultrasound in tissue. Essential issues relating to dispersion and attenuation are discussed, along with their tests. Finally, two approaches to implementation are illustrated, along with final simulation results and performance measures comparing both methods.

Finally, Chapter 5 summarizes the overall work and presents some recommended directions to pursue in future development. A major point of interest is the implementation of dynamic focusing in reception, and necessary development objectives are considered, including basic tests and discussion of key functionality.
Chapter 2

Background

In order to continue the development of simulation methodology and strike a balance between realism and real time performance, it is important to understand the theory beneath existing methods of ultrasound field simulation, in both the time and frequency domains. It is especially important to understand the major assumptions behind these approaches, and the resulting advantages and issues associated with each. Furthermore, careful design of these simulators makes them effective not only on the basis of computational efficiency and accuracy, but also in terms of their ease of use by an operator. The following sections describe some influential work and technical predecessors to FAMUS, as well as its underlying framework.

2.1 Spatial Impulse Response based Simulators

2.1.1 Time Domain Simulation

For general ultrasound simulation, it is necessary to make some assumptions about factors such as environment parameters, transducer properties and excitation behaviour in order to simplify the overall simulation. To this end, it is assumed that one can model acoustic behaviour through a linear system of transfer functions. A relatively straightforward approach is then to calculate the time domain spatial impulse response of the transmitting and receiving transducers as they interact with designated targets in the field, and via convolution with the excitation waveform, determine the final ultrasound image.

Being a versatile and popular ultrasound simulator, it is helpful to understand how this approach is used in the Field II simulation package. The method begins by assuming that the transducer surface geometry can be represented by a collection of rectangular or triangular elements, i.e. by decomposing large or complex forms of the aperture into simpler shapes. By making use of Huygens' principle, one can determine the response of
the entire aperture by collecting the contributions from each small element. By assuming that reciprocity conditions apply to the geometry being considered, the transmit–receive response can be found by first calculating the transmit impulse response.

Under infinite rigid-baffle conditions, and the use of linear acoustics in a homogeneous medium, the Rayleigh integral enables the transmit velocity potential spatial impulse response at an observation point \( \mathbf{r} \) to be expressed as

\[
h(\mathbf{r}, t) = \int_S \frac{\delta(t - |\mathbf{r} - \mathbf{r}'|/c)}{2\pi|\mathbf{r} - \mathbf{r}'|} dS,
\]

in which \(|\mathbf{r} - \mathbf{r}'|\) denotes the distance of the observation point to a point \( \mathbf{r}' \) on the transducer surface \( S \), and \( c \) is the speed of sound in the propagation medium. As expressed by the above integral, each of the element responses are summed together, resulting in a highly accurate representation of the impulse response of the entire transducer surface [34] for any arbitrary shape.

Assuming that targets in the imaging domain are represented by collections of discrete points in three dimensions, Field II calculates contributions to \( h(\mathbf{r}, t) \) via the intersection of spherical waves on an individual element originating from each point. Naturally, the phenomena at hand is best described in three dimensions; however, Field II simplifies process of finding intersections between planar elements and spherical waves by projecting down to the plane of the transducer element, and finding the intersection between a circular wave and transducer element. One approximation is made with respect to the far field. With increasing axial depth, the distance from some distant field point to positions on the same element is roughly the same. It is then possible to approximate the intersecting arc with a line segment, which is generally quicker to calculate. The error of this approximation is steadily reduced when using smaller elements, as a line better fits an arc for short distances.

In similar fashion, Ultrasim as developed by Holm also operates on the basis of decomposing the transducer geometry and numerically solving the Rayleigh integrals [21]. Meanwhile, the work of McGough et al. on FOCUS uses a different integral approach than the standard Rayleigh integral, in order to optimally calculate near field responses. Noting that the Rayleigh integral approach has slow convergence very near or at the transducer surface due to the numerical singularity introduced by the \( 1/R \) term, they developed the fast-near field method (FNM) [20] for the near field region which proved to converge more quickly and with less error when compared to results calculated using the traditional Rayleigh integral or Field II. Finally in a discrete perspective, Piwakowski and Delannoy’s work on the DREAM toolbox also represents the transducer in terms of smaller elements, but instead uses samples (termed “discrete representation”) [22] from
the centre of each element combined based on a small temporal window to construct a time averaged spatial response function for all times in the range of the spatial impulse response.

### 2.1.2 Frequency Domain Simulation

In the cases described above, the methods for locating the spatial impulse response all take place in the time domain. While these approaches usually produce accurate representations of \( h(t) \), one often has to accept the fact that closed form time domain solutions for various geometries often possess discontinuities, meaning that time domain simulation methods need to use high sampling rates in order to stably resolve these features. Alternatively, one can pursue frequency domain techniques to determine an equivalent representation of the SIR, and either perform all interactions in the frequency domain, or use an inverse Fourier transform to obtain the spatial impulse response. Li and Zagzebski and Rao et al. [35, 36] previously worked on frequency domain models of both 1D linear and 2D array transducers composed of rectangular and square elements.

The general form of this approach describes the pressure field from an arbitrary transducer surface vibrating at an angular frequency \( \omega \) based on individual elements of width \( a \) and height \( b \) as

\[
p_i(r, \omega) = -\frac{ipkcu(\omega)}{4\pi}A_0(r, \omega), \quad A_0(r, \omega) = \int_{-b/2}^{+b/2} \int_{-a/2}^{+a/2} e^{ik|r-r'|} \frac{dx'dy'}{|r-r'|},
\]  

where \( u(\omega) \) is the velocity of the surface, \( \rho \) is the density of the medium, \( c \) the speed of sound, and \( k = \omega/c \) is the wavenumber. To simplify the calculations and improve computation speed, they approximated the \( |r-r'| \) term assuming the element height approximately 1cm and the width close to half a wavelength. While it would be ideal to use something more precise like the Fresnel approximation, in order to allow beamsteering over a wide angle (in which case the lateral and axial dimensions of the focal point are comparable) the authors employed a less restrictive expression that it is essentially the Fraunhofer approximation.

The resulting method performed well over a wide range of angles with low relative error to the exact solution. In terms of computational speed, working through the frequency domain permitted much faster calculation compared to high resolution time domain methods. Interestingly, the resulting method was able to produce fairly precise point spread functions down to 10mm depth, despite the use of approximations generally suited for the far field. Lastly, the nature of the algorithm is such that later features for dispersion and dynamic focusing are easy to implement.
2.2 Point Source Simulation of Acoustic Fields

The previously mentioned simulation methods determine the spatial impulse response of a transducer through subdivision into many smaller geometric elements. This is well-suited to finding the response of an arbitrarily shaped transducer, and is usually able to closely match (to within sampling resolution) the piecewise continuous responses that are often the result of exact solutions to polygonal surfaces. However, as alluded to by several authors, this can be a time consuming process. An alternative starting point is to represent the transducer by a discrete collection of point sources, from which the overall array response is closely approximated. This is motivated by the fact that the expressions of individual point sources are themselves simpler to calculate, and that with a sufficiently large number of point sources, the overall response should still be highly accurate, provided the arrangement of sources is such that the boundary conditions of the original source radiation problem are satisfied. A few examples of this type of approach are reviewed here.

The Distributed Point Source Method (DPSM) developed by Ahmad et al. [23] proceeds to describe planar transducer surfaces as collections of point sources from which the pressure field is obtained through the integration of spherical waves via a discretized form:

\[ p(r) = \sum B_m \frac{\exp(ikfr_m)}{4\pi r_m} \Delta S_m, \quad A_m = \frac{B_m \Delta S_m}{4\pi}, \quad (2.3) \]

where \( r_m \) is the distance from the \( m \)th source to the point \( r \). To perform this summation, knowledge of the strengths or weights \( A_m \) of each point source are required, and can be obtained using a matrix equation relating wave velocity, source strength \( A_m \) and position of each point source. In order to satisfy a unique solution when non-normal surface velocities are non-zero, the method also considers the use of triplet sources, which expands the original point source into a set of three points arranged at the vertices of a centred equilateral triangle randomly oriented on the transducer plane. Ahmad et al. then simulated the model in a homogeneous fluid and also studied the case where two transducer fields interacted with each other, producing relatively accurate fields. However, this simulation method is only capable of steady state excitation, and only for the transmitted acoustic field.

A second example is the Complex Equivalent Source Method (CESM), reported by Ochmann and Piscoya [24]. This technique extends beyond describing a radiating surface with point sources positioned in real 3D space by replacing such sources augmented with positions in complex 3D space, e.g. \( y = (x_s, y_s, z_s) \rightarrow y_e = (x_s - ja, y_s - jb, z_s - jc) = y - j\beta \). Making some adjustments to satisfy boundary conditions at the surface, their modified system was capable of producing a radiated field which is approximately the
same as one for a system built with real sources. A numerical example was provided for a

1
2
3
4
5
circular piston assuming an infinite rigid baffle, showing relatively low error with respect
to the exact solution, and most importantly, that a magnitude fewer number of complex

sources compared to real sources are required for a similar degree of accuracy, leading
to rapid computation times. However, like DPSM, this calculation method was only
designed for transmission and demonstrates some instances of false radiation along the
transducer plane. Furthermore, the method is potentially problematic for high frequency
excitation, as it possesses a condition number that increases with frequency.

Finally, the last example described here is a precursor to the FAMUS framework. Given
the importance of analyzing flow in blood vessels, several authors have attempted
to perform realistic multi-physics simulation. For instance, Khoshniat et al. (2005) [28]
succeeded in coupling CFD velocity fields with Doppler ultrasound simulation through
the use of prescribed sample volumes and precomputed flow data to produce real-time
playback of Doppler spectra. However, this was accomplished using semi-empirical sim-
ulation, which maintains accuracy only under particular transducer geometry, focus, and
steering angles. On the other hand, earlier work by Oung and Forsberg (1999) [37] also
combined CFD information with acoustic field simulation to generate Doppler spectra,
but provided mechanistic accuracy at the cost of real-time speed. Such a divide has been
difficult to span, and the potential of complex multi-physics simulation is unfortunately
matched by its computational overhead. Thus, Aguilar et al. proposed a simulation tech-
nique involving point source representations of the transducer [38] in order to progress
toward a platform capable of both mechanistic simulation and real time performance.

Like the work of Ochmann and later Ahmad, this approach relies on a point source
representation of the transducer surface. However, unlike those methods, which propose
more complicated representations using complex-valued spatial positions or triplet point
sources, the methodology proposed by Aguilar et al. only requires a multitude of ordinary
monopoles. The initial equation for a rigid planar baffle is given by the Rayleigh integral,
shown below, which can be replaced by a discrete summation:

\[
p(r : \omega) = \frac{j\omega \rho v_{no} e^{j\omega t}}{2\pi} \int \int_{\infty} \xi_0(x, y) e^{-j\omega R/c} \frac{dS_0}{R}, \quad \text{and} \quad (2.4)
\]

\[
p(r : \omega) = \frac{j\omega \rho v_{no} e^{j\omega t}}{2\pi} \sum_x \sum_y \xi_0(x, y) e^{-j\omega R/c} e^{-\alpha R} \frac{\Delta x \Delta y}{R}, \quad (2.5)
\]

where the second expression also incorporates an attenuation term \( e^{-\alpha R} \), for a single
frequency, an appropriate setting in Doppler ultrasound. Effectively, the discrete sum-
mation implies that an infinite number of monopoles arranged on the surface will be able
to recreate the acoustic field at any given position \( r \).
Figure 2.1: For some arbitrary surface $S_0$, $N$ point sources are uniformly distributed so that each is associated with an equal $\Delta A$. For sufficiently large $N$, the collective contribution from each source at the position $r_s$ is nearly identical to the exact response from $S_0$.

Rigorously speaking, the use of a rigid baffle assumption requires that the normal $v_{no}$ be zero everywhere outside the surface. But in practice, because this point source approach is oriented toward real time simulation and ‘realistic enough’ calculation, it is more concerned with overall performance and accuracy near the focal zone and into the far field - much more relevant for Doppler ultrasound - meaning that while a dense arrangement of monopoles on the transducer surface $S_0$ may not exactly satisfy this requirement, the acoustic field in the region of interest is generally well matched.

Using the method given in (2.5) and distributing the monopoles so that each is associated with an equal portion of $S_0$, Aguilar et al. demonstrated that it is possible to determine the total intensity of the transmitted field, given continuous wave excitation. This was validated by comparisons of the generated field to an analytical solution of a circular piston and a numerical calculation of a linear phased array using in Field II, which is commonly taken as a “gold standard” for numeric ultrasound simulation. As expected, for regions central to the field and around the focal zone, good agreement was achieved with both the exact and numeric standards.

2.3 Acoustic Field Comparison to Rectangular Elements

At this point it is important to consider some of the differences between SIR methods obtained by transducer representations using planar elements versus point sources. One might expect that the use of point sources, rather than triangles or rectangles, is likely to produce greater error in the computed impulse response, especially in the near field. However, since computational efficiency is a crucially important aspect of simulation, it is instructive to compare the field response of both types of transducer array components,
particularly as they radiate into the far field.

For this exploration the radiated field of a single point source and square element are compared. The focus is upon single elements because although in practice arrays of elements are used for diagnostic purposes, understanding the behaviour of the building blocks of such arrays is important. To make the analysis simpler, we are mainly concerned with the differences in behaviour in a narrow window in front of an individual element, rather than wide angle behaviour from $[-\pi/2, \pi/2]$. Furthermore, we will only be concerned with the lateral variation across the field of view, as the effects of lateral and elevation dimensions are mostly independent, and in part because this thesis focuses on linear 1D arrays.

The equation for the pressure response of a rectangular element is shown below, transformed into the frequency domain to provide more straightforward manipulation. The final time domain solution can then be determined through an inverse Fourier transform.

\[
p(x_0, y_0, z : \omega) \approx \frac{j\omega \rho_0 \nu_0}{2\pi R} e^{-jk\left[z + \frac{x_0^2 + y_0^2}{2z}\right]} \int \int \text{rect}(\frac{x_1}{W})\text{rect}(\frac{y_1}{H})e^{j\frac{k}{2}(x_0x_1 + y_0y_1)} e^{-j\frac{k}{2}(x_1^2 + y_1^2)} dx_1 dy_1,
\]

which represent the Fraunhofer and Fresnel approximations, respectively. To be completely precise throughout all depths, it would be better to compare to an exact solution or numerical integration of the Rayleigh integral. However, as alluded to previously, far field behaviour will be of greatest interest to us, so for simplicity the Fraunhofer and Fresnel approximations are compared instead. In general, the behaviour of the pressure phasor is governed by the leading term containing the decaying exponential and inverse relationship to distance, directionally weighted by what is known as a \textit{directivity function} $D(\mathbf{r})$ or $D(\theta, \phi)$, indicated by the terms containing the sinc or rect functions in the expressions above. Meanwhile, for a monopole source, the pressure response is mainly

\[
p(x_0, y_0, z : \omega) = \frac{j\omega \rho_0 \nu_0}{2\pi R} e^{-jk\left[z + \frac{x_0^2 + y_0^2}{2z}\right]} W H \text{sinc}(\frac{Wx_0}{\lambda z})\text{sinc}(\frac{Hy_0}{\lambda z}), \quad \text{and} \quad (2.7)
\]
\[
p(x_0, y_0, z : \omega) = \frac{j\omega \rho_0 \nu_0}{2\pi R} e^{-jk\left[z + \frac{x_0^2 + y_0^2}{2z}\right]} \mathfrak{F} \left[ \text{rect}(\frac{x_1}{W})\text{rect}(\frac{y_1}{H})e^{-j\frac{k}{2}(x_1^2 + y_1^2)} \right], \quad (2.8)
\]
Figure 2.2: Layout of comparison between point source and square element of \( \Delta A = 0.04\text{mm}^2 \). Focus of analysis is in the far-field region, in order to satisfy the condition on distance \( R \) between the element centre and observation point.

determined by the decay in amplitude with distance, as it is by definition omnidirectional

\[
p(x_0, y_0, z : \omega) = \frac{j\omega \rho_0 v_0}{2\pi R} e^{-jk\left[z + \left(x_0^2 + y_0^2\right)/2\lambda\right]}.
\]  

(2.9)

Some common environmental parameters \((c = 1540\text{m/s}, f = 5\text{MHz}, \lambda = 0.308\text{mm})\) are selected, and we also choose the square element size to be of 0.2mm width and height, smaller than the wavelength of a transmitted wave. Thus the criterion for validation of the Fraunhofer approximation (and by extension, also the Fresnel approximation) is that \( R >> \pi (x_1^2 + y_1^2)_{\text{max}}/\lambda = 0.21\text{mm}, \) where \( R \) is the Cartesian distance from the observation point in the field to the element centre. Assuming a factor of 10 is sufficient, its accuracy should be sufficient for ranges beyond 2mm. It should be noted that essentially this is a comparison between a sinc function and a purely spherical radiating source. While there are other possible geometries for which analytical solutions exist (e.g. circular piston), use of rectangular elements is largely appropriate as many ultrasound applications employ linear arrays.

Figure 2.3 shows the normalized pressure phasor magnitude as it varies laterally in \( x \) for several axial depths \( z \) for a point source (triangular markers) and the square element for the Fraunhofer and Fresnel approximations (solid and dashed lines), assuming both source types are centred at the origin. It is quite apparent that near the axis \( |x| \leq 1\text{mm}, \) the responses are essentially the same. Moving laterally away from the origin, the magnitude of responses diverge from each other, with > 10% difference approximately 6mm from the axis, when measured at a depth of 20mm. However, it should be noted that at that position, the observation point moves to about \( \theta = \tan^{-1}(\frac{3}{10}) \approx 17^\circ \) from the axis, which might not be considered a narrow window.

It can be observed that this difference rapidly drops off as the measurement point
moves into the far field, where planar elements appear increasingly similar to that of a point source. This brings up one of the main assumptions used for point source representation: that in far field usage, the expected radiation pattern is similar between finite area and point sources. For instance, at a depth of $z = 80$mm, the deviation between the two radiators now drops below 1%, even at a wide angle from the source. Furthermore, one would expect the collective response for an array of geometric or point source elements (for sufficiently many elements) to be similar throughout the extent of the array, only diverging at the edges where diffraction may behave differently due to the different assumptions. Therefore, while there are certainly differences that arise through the use of point source elements, the deviations are not significant for far-field analysis and medical simulations, which should prove to be helpful for achieving real-time performance.

### 2.4 Fast And Mechanistic Ultrasound Simulator

Having completed a detailed survey of different approaches for calculating acoustic fields and using SIRs, and having assessed the capabilities and challenges associated with planar
and point source methods, we present an overview of the fundamental theory behind the Fast and Mechanistic Simulator in order to understand its strengths and to identify those areas in need of improvement.

2.4.1 FAMUS I

Drawing from earlier work on using point sources to generate a transmit field for Doppler ultrasound using continuous wave excitation, the first iteration of the Fast and Mechanistic Ultrasound Simulator, now referred to as FAMUS I, was developed to extend that discrete point source approach into a full-fledged simulator capable of simulating both transmit and receive acoustic fields [39]. With reference to the principle of acoustic reciprocity, the proposed improvement on past work was to generate the acoustic field in reception by reversing the roles of the monopole sources on the transducer surface with the scatterers in the field - that is, the transducer monopoles would now receive from a point in the field which itself acts as an acoustic source.

Figure 2.4: Illustration of the two stage transmit and receive process, facilitated by changing the roles of source and receiver between the transducer monopoles and the scatterer or field point of interest. The cumulative waveforms as they travel between positions and arrive at different times are shown using diamond markers (diagram) and sharp impulses (time graphs).

Considering a 1D linear array of elements, a transducer can be represented by a grid of point sources, where each array element is divided into rectangular sub-elements, and each sub-element associated with a single monopole from the grid. Similar to the validation performed using the circular piston transducer, the grid arrangement facilitates mapping each monopole to the same incremental area \( \Delta A \), avoiding bias in contribution from an individual point source. Additionally, the advantage of the regular grid is that each monopole is centred in its respective sub-element. The general setup used for transducers in FAMUS I is a grid of \( M \) by \( N \) monopoles, where \( M, N \) are odd and the indices \( m = -(M-1)/2 \ldots 0 \ldots (M-1)/2, n = -(N-1)/2 \ldots 0 \ldots (N-1)/2 \). It is important
to note that because the areas associated with each monopole extend beyond the points themselves, the spatial extent of the monopole positions is in fact smaller than the actual transducer dimensions, e.g. the transducer edge is not equivalent to the position of the outermost monopoles. The difference in assigned versus effective dimensions is more noticeable for coarsely discretized surfaces.

Using the arrangement in Figure 2.5a, the spacing between point sources in rows (elevation) is \( E/M \) and in columns (lateral) is \( L/N \), producing individual areas of \( \Delta A = EL/MN \) for each point source. If focusing is confined to the lateral plane, then the corresponding lateral and elevation focus positions are \( \mathbf{r}_L = (x_L, 0, z_L) \) and \( \mathbf{r}_E = (0, 0, z_E) \). The resulting delay to travel to a given element \( mn \) is then:

\[
\tau_{m,n} = \frac{1}{c} \left[ \sqrt{z_E^2 + \left( \frac{E(M-1)}{2M} \right)^2} - \sqrt{z_E^2 + \left( \frac{mE}{M} \right)^2} \right. \\
+ \left. \sqrt{z_L^2 + \left( \frac{L(N-1)}{2N} \right)^2} - \sqrt{z_L^2 + \left( \frac{nL}{N} - x_L \right)^2} \right],
\]  

(2.10)

and for some scatterer \( \mathbf{r}_s = (x_s, y_s, z_s) \), the time to travel from element \( mn \) is:

\[
t_{m,n}^s = \frac{r_{m,n}^s}{c} = \frac{1}{c} \sqrt{[x_s - nL/N]^2 + [y_s - mE/M]^2 + z_s^2}. \tag{2.11}
\]

With the intrinsic travel delays defined above, it is now simple to describe the transmit and receive pressure signals. For some transmit waveform \( v^T(t) \), the incident pressure from a particular monopole source \( mn \) in the array is equal to \( p_{m,n}^T = A_n v^T(t - \tau_{m,n}) \), with \( A_n \) as the transmit apodization coefficient for that source. Furthermore, due to the characterization of the transducer, we are able to describe the pressure variation in space.
using a free space radiation expression, assuming frequency independent attenuation $\alpha$, as $e^{-\alpha r}/r$. Hence the entire incident pressure on a given field point $r_s$ is:

$$p_{total}^T(t) = \sum_{m=-(M-1)/2}^{(M-1)/2} \sum_{n=-(N-1)/2}^{(N-1)/2} A_n v_T^T(t - \tau_{m,n} - t_{m,n}^s) e^{-\alpha r_{m,n}^s}/r_{m,n}^s. \quad (2.12)$$

This incident signal is then sent back to each monopole on the transducer, swapping the roles of source and receiver, in order to generate the total received signal. Delayed again by the transit time between the field point and particular monopole $mn$, the received pressure is equal to $p_{total}^R = e^{\alpha r_{m,n}^s}/r_{m,n}^s p_{total}^T(t - t_{m,n}^s)$. Thus the final received pressure from the scatterer $r_s$ is:

$$p_{total}^R(t) = \sum_{m=-(M-1)/2}^{(M-1)/2} \sum_{n=-(N-1)/2}^{(N-1)/2} B_n p_{total}^T(t - \tau_{m,n} - t_{m,n}^s) e^{-\alpha r_{m,n}^s}/r_{m,n}^s, \quad (2.13)$$

which is essentially a summation of differently weighted and delayed versions of the initial excitation waveform $v_T(t)$.

With this approach, Aguilar et al. were able to perform simulated Doppler and B-mode simulations, producing spectra and images with strong agreement with similar tests done in Field II. This algorithm is also relatively simple to calculate, being comprised entirely of summations of the initial excitation waveform adjusted by a decaying exponential term, resulting in an order of magnitude increase in speed over Field II, as shown in Table 2.1 below. Given the complexity of standard SIR approaches, the simplicity of the point source approach also provides the opportunity for parallelization in order to further improve calculation-time performance.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Spectral Doppler</th>
<th>B-mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field II [s]</td>
<td>17629</td>
<td>1694</td>
</tr>
<tr>
<td>FAMUS I, single core [s]</td>
<td>14717</td>
<td>1340</td>
</tr>
<tr>
<td>FAMUS I, four cores [s]</td>
<td>2237</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 2.1: FAMUS I Runtime Measurements, from [39]

### 2.4.2 FAMUS II

While FAMUS I was initially successful in terms of relatively realistic and very fast simulation capable of both Doppler and B-mode ultrasound, its design is somewhat limited due to its emphasis on directly manipulating the input waveforms in order to calculate the acoustic field. It is important to note that unlike previous discussion of simulation methods, FAMUS I constructs responses not based on the SIR, but through
direct summation of phased versions of the excitation wave. This inherently means that computation scales with the number of points of interest in the field (e.g. the number of scatterers needed to describe a target) and the number of samples in a given signal (dependent on the average distance to the target), which was found to be unable to reach real-time speeds. Additionally, the direct approach of calculating pressure means that incorporating dispersion and other non-linear effects is by definition very challenging.

In order to overcome some of these issues, a second version of FAMUS, called FAMUS II, was developed [40], returning to a spatial impulse response based design. Mathematically, it is easier to incorporate additional physical phenomena, such as dispersion, from a linear systems perspective. Computationally, the use of transfer functions and convolutions results in more compact manipulations, provided a linear system can be assumed. This offers the potential for faster simulation, since there is less computational work.

Geometrically, the transducer surface, monopoles and associated areas and calculation of delays are identical to that of FAMUS I (as seen in Figure 2.5). The primary change is the generation of the discrete impulse response and using it to calculate the pressure field in transmission and reception. To develop an impulse response, consider a spherical wave emitted from a monopole source $mn$ at the transducer, when excited by a surface velocity impulse. The fundamental form of the response will be that of a delta function decaying with distance and exponentially with attenuation:

$$h_{m,n}^T(r_s,t) = \delta \left[ t - \frac{r_{m,n}^s}{c} \right] e^{-\alpha \frac{r_{m,n}^s}{r_{m,n}^s}}. \quad (2.14)$$

Accounting for the wavefronts from all of the monopole sources impinging on the scatterer positioned at $r_s$, the transmit response can be described as:

$$h^T(r_s,t) = \sum_{m=-(M-1)/2}^{(M-1)/2} \sum_{n=-(N-1)/2}^{(N-1)/2} A_n \delta \left[ t - \tau_{m,n} - \frac{r_{m,n}^s}{c} \right] e^{-\alpha \frac{r_{m,n}^s}{r_{m,n}^s}}; \quad (2.15)$$

which is similar to (2.12), where each contribution is shifted according to its delay and travel times $\tau_{m,n}$ and $t_{m,n}^s$ and including the affects of lateral apodization through $A_n$. Now, to obtain the pressure waveform experienced at $r_s$, it is important to remember some fundamental relations involving pressure and velocity potential $\phi_N$:

$$p = \rho_0 \frac{\partial \phi_N}{\partial t}, \quad \text{and} \quad \phi_N(r,t) = v_E(t) * h(r,t)$$
which result in the following equivalent expressions for pressure [41]:

\[ p(r : t) = \rho_0 v_E(t) \ast \frac{\partial h(r : t)}{\partial t}, \quad \text{or} \quad (2.16) \]
\[ p(r : t) = \rho_0 h(r : t) \ast \frac{\partial v_E(t)}{\partial t}, \quad (2.17) \]

which conveniently allows the calculation of incident pressure through a convolution of the excitation waveform and the transmit impulse response. Next, to determine the received pressure, we again take into account acoustic reciprocity, reversing the roles of the scatterer at \( r_s \) and the monopoles on the transducer surface. This builds an impulse response in reception (along the return journey from scatterer to transducer), \( h^R(r_s, t) \), and via convolution the received pressure can be similarly calculated.

![Diagram](image)

Figure 2.6: The illustrated work of generating the transmit/receive impulse response based on the contributions from all \( S \) scatterers and MN monopoles. A higher density of monopoles on discretizing the transducer yields a better representation of the response.

However, based on the assumption of linear system behaviour, we can cascade multiple transfer functions to build the transmit-receive impulse response for that particular scatterer:

\[ h^{T/R}(r_s, t) = h^T(r_s, t) \ast h^R(r_s, t). \quad (2.18) \]

Furthermore, if the same transducer is used in reception, \( h^T(r_s, t) = h^R(r_s, t) \). From this, the total transmit-receive impulse response experienced by all the scatterers \( S \) in
the field (e.g. which in aggregate compose the entire region of interest) is:

\[
h_{S}^{T/R}(r, t) = \sum_{s=1}^{S} h^{T}(r_s, t) * h^{R}(r_s, t).
\]

The expression for \( h_{S}^{T/R}(r, t) \) is then easily used either (2.16) or (2.17) to determine the transmitted and received pressure at the transducer due to some complex target composed of \( S \) scatterers, which can then be used to generate ultrasound spectra or images.

![Diagram](image)

Figure 2.7: The basic flow in the FAMUS II simulation process, showing inputs and convolution stages. For simplicity, summation across sources/scatterers is implicit in \( h(t) \). In general, \( h^{T}(t) \) and \( h^{R}(t) \) are not equal, but there are many instances where they may be identical, making simulation simpler. Using this system approach, additional operations can be cascaded in between \( h_{S}^{T/R}(t) \) and the final output.

The modification to the simulation approach moves from calculating the direct waveforms to generating the SIR like other simulators. Combined with the point source representation of the transducer and the ability to determine the transmit/receive response from the transmit impulse alone (in the case of identical transducers) results in simulation which is much faster than FAMUS I by an order of magnitude and Field II by two orders of magnitude, as seen in Table 2.2. Due to the distributive nature of (2.19) the contributions of each monopole and scatterer are easily accumulated, and allows us to rapidly calculate the impulse response and subsequently calculate the pressure, given some input waveform. An additional advantage is that incorporating other effects such as dispersion can be viewed as filters or transfer functions, which can be modularly added in cascade onto the simulation process in Fig 2.7 to incorporate additional levels of physical behaviour, improving the realism of the simulation.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Spectral Doppler</th>
<th>B-mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field II [s]</td>
<td>1280.7</td>
<td>1774</td>
</tr>
<tr>
<td>FAMUS I, four cores [s]</td>
<td>42.86</td>
<td>123</td>
</tr>
<tr>
<td>FAMUS II, four cores [s]</td>
<td>7.4</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 2.2: FAMUS II Runtime Measurements, from [40]
Chapter 3

Evaluation of Simulation Processes for Speed and Realism Tradeoffs

In the previous chapters, the importance of ultrasound diagnostics and the study of blood flow was highlighted, particularly with regards to assessing vascular disease. Tools such as computational fluid dynamics are often used to assess the conditions of fluid flow in attempts to understand physiological indicators of disease [42], while on the clinical level Doppler ultrasound continues to provide radiologists with insight into patient health and disease progression. Several authors’ work on combined CFD and Doppler simulation to generate realistic Doppler ultrasound spectra (Khoshniat et al., Oung and Forsberg, and Swillens et al.) have wrestled with the complexity of integrating these two types of computational modeling and reached varying levels of success ranging from real-time operation to highly mechanistic simulation. However, it remains a difficult task as these results were obtained with compromises between realism, accuracy, and speed.

It suffices to say that although specific assumptions may make simulation more rapid, or that clever algorithms can improve computational efficiency, qualities such as accuracy, speed and realism also depend on the end use of simulation. For example, the purposes of visualization, teaching, and analysis each have different qualitative and quantitative inputs and requirements, and thus impose different conditions regarding the avoidance of and compensation for artifacts, calculation precision, or time constraints. In seeking to improve the FAMUS framework, it is then not only important to enhance its computational ability or range of functionality, but also to gain insight about the nature of ultrasound simulation itself, and the necessary conditions for ‘good’ or useful simulation. This is also essential because these requirements affect the desired capabilities of FAMUS, as it is clearly impractical to make an infinitely fast or efficient simulator.

One initial test of the abilities and performance of FAMUS I consisted of a Doppler ultrasound scan on a column of particles moving with a pulsatile Womersley flow profile,
much like what would be experienced in real arteries. Figure 3.1 displays an example Doppler spectral profile of a cardiac cycle and steady plug flow for comparison, displaying a number of physical and computational effects. For example, the presence of spectral broadening is clearly visible. In (a) the significant distribution of frequency components is due to the non-zero velocity gradient of scatterers as they follow a time-varying flow profile, while in (b) scatterers crossing a finite sample volume results in a band of frequencies, despite the fact that a single-velocity flow can be represented by a single frequency. Furthermore, acquisition issues can also be reproduced; the aliasing around $t = 0.1s$ in (a) is not a computational error, but the actual visual effect of measuring flow beyond the range of the PRF, which occurs in clinical Doppler imaging under similar circumstances. These results are encouraging because it means these phenomena are naturally captured by FAMUS. Finally, in both spectra there are some residual computational artifacts present as weak yellow banding near the main waveform, whose existence was not fully explored in earlier work [38,39].

![Figure 3.1: Current examples of Doppler spectra produced by FAMUS. Observation around the main waveform (dark bands) reveals some non-physical artifacts.](image)

In order for FAMUS to be used as a reliable simulation system, it will be important to remove or at least understand the nature of these artifacts. Also, because initial tests were fixed under certain geometric and scatterer properties, such as the number of contributing particles, volumetric density and region of analysis, there are open practical questions regarding the number of particles needed for realistic simulation, or what sort of behaviour could emerge from various configuration settings and input data. Without addressing these issues, it is unclear whether this simulation framework can be scaled to larger scenarios or how it is bound to behave in such cases, or if real-time performance
is even feasible, and with what level of realism.

The following sections outline a number of explorations undertaken to determine nuances in the process of simulation, in order to provide guidance for future use of FAMUS and work towards balancing mechanistic accuracy and speed. A series of valuable lessons regarding simulation settings and properties of input data are uncovered, culminating in key operational improvements to the use of FAMUS, particularly for Doppler ultrasound.

For subsequent experiments, Table 3.1 lists common parameter settings, and Figure 3.2 illustrates the three flow profiles used to test Doppler spectra generation throughout this chapter. These profiles are well-known and often used to evaluate flow behaviour in different channels or vessels. Plug flow is a very simple flow profile, having no variation in speed across its entire active domain, and drops abruptly to zero flow at the edges. A power law profile is expressed by

\[ v(r) = v_0 (1 - r/R)^{1/n}, \quad n \in \mathbb{R}, \]

where \( R \) is the vessel radius and behaves similarly to plug flow, with a more tapered drop to zero at the edge of the spatial domain. Finally, the Womersley-Evans model \cite{43} expresses time-varying pulsatile Womersley flow profile, commonly used to describe blood flow in vessels, as

\[
v(t, r/R) = 2v_0 \left(1 - \left(\frac{r}{R}\right)^2\right) + \sum_{m=1}^{\infty} |V_m||\psi_m|\cos(m\omega t - \phi_m + \chi_m),
\]

(3.1)

where \( R \) is the vessel radius and the first term represents steady flow. The nominal case used in experiments is designed for a 62 beat/min cardiac cycle, Womersley number \( \alpha = L(\omega \rho \mu)^{1/2} = 5.51 \) and a peak spatial velocity of \( v_{\text{max}} = 1.36\text{m/s} \).

Figure 3.2: Radially varying velocity profiles are illustrated by dashed lines for plug, power law \((n = 7)\) and Womersley pulsatile flow. For the time varying Womersley flow, the inset in the upper right shows a sample flow cycle, from which multiple profiles from different phases in the cycle are drawn. For plug and power law flow, the maximum velocity is \( v_0 \), the characteristic velocity of the steady state term in Eqn. 3.1.

Additionally, for simplicity, the terms ‘spectrogram’, ‘sonogram’, and ‘Doppler spec-
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevation Height</td>
<td>5 mm</td>
<td>Sampling Frequency</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Lateral Width</td>
<td>20 mm</td>
<td>Centre Frequency</td>
<td>5 MHz</td>
</tr>
<tr>
<td>Elements (Lat. / Ele. )</td>
<td>17 x 5</td>
<td>Speed of Sound</td>
<td>1540 m/s</td>
</tr>
<tr>
<td>Lateral Element Width</td>
<td>270 µm</td>
<td>Pulse Repetition Frequency</td>
<td>15 kHz</td>
</tr>
<tr>
<td>Kerf</td>
<td>30 µm</td>
<td>Excitation Cycles</td>
<td>9</td>
</tr>
<tr>
<td>F-number (lateral)</td>
<td>2</td>
<td>Vessel Radius</td>
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<td>F-number (elevation)</td>
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</tr>
<tr>
<td>Elevation Focus (x,y,z)</td>
<td>(0, 0, 14) mm</td>
<td>Fractional Bandwidth</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 3.1: Common parameter values for Doppler simulations.

‘tra’ may be used interchangeably throughout the chapter to refer to the frequency / time plot resulting from the short time Fourier transform of the slow-time acoustic signal derived from a specific sampling instant in a series of fast time acoustic reflections. A complete diagram of this process is shown in Figure 1.4, and a basic apparatus layout is shown in Figure 3.3 below.

![Figure 3.3: Layout of components for simulated Doppler scans. Angle and dimensions are not to scale.](image)

3.1 Effect of Velocity Profile on Noise

It is important to consider the artifacts in currently produced spectrograms, as shown in Figure 3.4 with enhanced contrast, and assess the possible factors underlying their presence. Since the current simulation framework does not yet incorporate “natural” noise as experienced in real Doppler systems, it is likely that these artifacts are of computational nature, due to the interaction of the simulation engine, operational parameters,
and input data.

![Contrast enhanced sonograms from simulated Doppler ultrasound of cardiac cycle at different sampling frequencies.](image)

Figure 3.4: Contrast enhanced sonograms from simulated Doppler ultrasound of cardiac cycle at different sampling frequencies. Yellow patches outside of main curve are numerical artifacts resulting from the choice of sampling frequency.

Careful observation of the artifacts in a typical Doppler spectrum generated using FAMUS shows that some appear to exhibit patterns of oscillation and decay similar to the main waveform of systole and diastole in the cardiac cycle, suggesting a relationship between artifacts and scatterer movements themselves. It should be noted that artifacts are most visible during diastole, perhaps indicating some phenomenon amplified for particles moving with near zero velocity. In practice, Doppler ultrasound machines are equipped with a wall filter (generally up to 300Hz), in order to screen out comparatively slow movements of blood vessel walls. However, this is not implemented into FAMUS and thus simulated Doppler spectra possess frequencies corresponding to zero velocity components. To explore this in detail, simulations of different flow speeds and profiles were performed to assess the relationship between artifacts and flow speed.

Using the plug and power law flow profiles, a series of Doppler spectra were generated with 5000 scatterers seeded in the environment and common settings described by Table 3.1 and Figure 3.3. For each flow type, spectra were generated for three speed levels based on fractions of \( v_0 = 0.15 \text{m/s} \), the characteristic velocity of the steady state term in (3.1) describing the time-varying Womersley pulsatile profile. The goal in this analysis is simply to observe the artifacts as they relate to the speeds at which the flows are moving. A table of tests is shown in Table 3.2. Note the use of speeds significantly lower than the peak velocity of the chosen Womersley flow profile; due to our interest in the abundance of artifacts during the slowest portions of the cardiac cycle, we have elected to place more
focus on slowly moving particles.

\[ v_0 = 0.15 \text{m/s} \]

<table>
<thead>
<tr>
<th>( f_s = 100 \text{MHz} )</th>
<th>Plug</th>
<th>Power Law</th>
</tr>
</thead>
<tbody>
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<td>( 0.1v_0 )</td>
<td>( n = 7 )</td>
</tr>
<tr>
<td>( 0.2v_0 )</td>
<td>( 0.2v_0 )</td>
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</tr>
<tr>
<td>( 0.3v_0 )</td>
<td>( 0.3v_0 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Table of Velocity Artifact Tests

Several simulated spectra using FAMUS I are shown in Figure 3.5. For the three speed groupings mentioned above, corresponding peak flow velocities are 0.015, 0.03, and 0.045m/s. Given a Doppler angle of 30°, their respective Doppler spectra should have main bands around 84, 168, and 252Hz. The first column of plots in Figure 3.5 display the simulated results of plug flow at various speeds. Examining these, one can see a definite correlation between the frequency at which artifacts appear and the velocity of the flow. For instance, in subfigure (a) with plug flow at 0.015m/s, the artifact bands appear around \( \geq 800 \)Hz, approximately ten times the main band frequency. Meanwhile, in subfigures (c) and (e), these bands shift to approximately 1.6kHz and 2.4kHz, growing by the same factor as the velocity increases. The consistency of this movement likely indicates a harmonic relationship to the main band. Likewise, the second column of plots shows that a progression of artifact positioning with speed is also visible for power law flow. The main difference here is that while artifacts are positioned at roughly the same frequency values, they appear to decline in volume, particularly as the flow increases speed. Similarly, the Womersley profile shown previously (refer to Figure 3.4) has characteristic banding about multiples of the main velocity/frequency component, following or mirroring the shape of the waveform. Like power law flow, its artifacts are also more dispersed than the plug profile case over the duration of the cycle. They are particularly clear in diastole (\( t > 0.5s \)), where flow begins to slow down and stabilize into a quasi-steady state.

A possible explanation could be that the distribution of velocity components noticeably affects artifacts. While the power law and Womersley flows both have continuous changes from zero to maximum velocity, plug flow has a discontinuity at the vessel wall, \( r = R \), with a flat profile that vanishes at the edge. In effect, its velocity function is windowed in space using a rectangular function, while the other profiles roll off gradually. This induces harmonics like that which appear in Fourier series representations of square waves. Figure 3.6 (a) and (b) also show this sequential banding, exaggerated by 50MHz sampling. Of course, the discrete nature of the data makes it impossible for any profile to perfectly roll off to zero, causing banding in the other profiles. However, due to gradation, the banding that occurs is weaker.
Another factor is the presence of point scatterers in the creation of artifacts. Presently, test models only account for red blood cells. Surrounding tissue structures such as vessel walls, fat, and muscle are not included, meaning there is also a “spatial window” with respect to the existence of scatterers. Hence, the reflected signal transitioning from scatterers of high velocity to zero/low velocity may exhibit a different type of abruptness compared to a reflected signal from only a high velocity scatterer, in cases where no zero/low velocity scatterer is present in the data, such as at the vessel wall.

Evidently, the behaviour of simulated Doppler spectra in regions at abrupt velocity transitions using point representations of the flow can be associated with computational artifacts. Furthermore, the apparent numerical relationship between the frequencies associated with flow velocities and artifacts implies that they are most visible at low flow speeds; in order for artifacts to appear within the frequency range of a spectrogram, its parent flow component should be roughly \( \leq (f_{PRF}/2)/10 \). In simple simulations where truncated geometries can be expected, it is important to remember this effect on the appearance of the output. Besides ensuring temporal or spatial roll-off to reduce artifacts, perhaps the use of finite sized scatterers or the inclusion of background tissue structures as slowly moving scatterers can help avoid these problems and make the changes at the edge of the spatial-temporal domain more gradual.

However, it appears that even with gradual flow profile transitions like that of power law or the Womersley profile, there are still a large number of artifacts. Therefore, discontinuities in flow profile cannot completely explain artifacts, and additional sources of computational noise are clearly present in simulation. Several additional figures are brought to the reader’s attention for later discussion. Figure 3.6 (a) through (d) show the effect of 50MHz and 200MHz sampling rates on simulated spectra. For brevity only some simulations are shown, but the plots are representative. The range of behaviour from coarse to fine sampling demonstrates a reduction in the presence of artifacts by increasing time resolution, which brings up the question of discrete time as factor in the issue of noise. Secondly, Figure 3.6 (e) and (f) show the effect of more sources on the transducer surface. These spectra show a decrease in artifacts as the surface is more finely represented, though it is less effective than increasing the sampling rate. Clearly, there are additional factors related to discreteness in time and space that affect the appearance of artifacts, which will be discussed in the following section.
Figure 3.5: Comparison of plug and power law profile Doppler spectra simulated in FAMUS II at 100MHz using an intensity-agnostic colour scheme to display flow and artifact structures. For each flow profile three speeds are depicted to assess the properties of artifacts.
Figure 3.6: (a) to (d) show spectra of plug and power law flow, for sampling frequencies of 50MHz and 200MHz. (e) and (f) demonstrate the effect of increased transducer sources. All tests were run at $3\nu_0 = 0.045\text{m/s}$. 

(a) Plug flow, 50MHz

(b) Power law flow, 50MHz

(c) Plug flow, 200MHz

(d) Power law flow, 200MHz

(e) Plug flow, 100MHz, 33x9 elements

(f) Plug flow, 100MHz, 65x17 elements
3.2 Signal Interpolation

The preceding section addressed connections between the shape and speed of flow profiles and Doppler spectrogram artifacts, indicating that discontinuities are a notable factor. However, it was also noted that temporal and spatial discretization play significant roles in artifact multiplicity and intensity, compounding upon the issue of discontinuous flow profiles.

The spectra in Figure 3.6 provided some results with respect to changes in sampling rate and transducer discretization. As one might expect, decreases and increases to sampling frequency worsen and improve Doppler spectra. Figure 3.6 (a) and (b) compared coarse sampling (50MHz) for plug and power law flow, showing an amplification of the number and intensity of artifacts, though the plug flow spectrum was more saturated. Meanwhile, at a sampling rate of 200MHz, (c) and (d) exhibited almost no artifacts. Then in (e) and (f), it was shown that by increasing the number of source monopoles by factors of 3.5 and 13, respectively, spectra artifacts were reduced, though with less efficacy. This brings back into focus two types of discretization that greatly affect FAMUS, and the differences between Field II and FAMUS.

Because FAMUS uses discrete monopole sources to represent the transducer rather than a collection of planar elements, the transmit/receive signal (FAMUS I) and impulse responses (FAMUS II) are intrinsically sparse, compared to how Field II calculates impulse responses based on the intersection of spherical waves with geometric elements. Therefore, in FAMUS extra source monopoles improve the realization of the transmit/receive signal or impulse response by accounting for more acoustic interactions.

Figure 3.7a illustrates this difference. Field II supports piecewise continuous or at least piecewise linear responses (though discrete in a digital sense), while all of the signals produced by FAMUS are collections of impulses. This means the placement of the digital impulses which comprise the signal or impulse response is contingent on the time binning or sampling resolution of the simulation. Thus, in FAMUS, the output is fairly sensitive to the sampling frequency $f_s$ set at the start of simulation. If one examines Figure 3.4, two Doppler sonograms simulated using 50MHz and 100MHz sampling are shown with readily apparent artifacts, especially in the latter half of the cardiac cycle ($t > 0.5s$). One can observe clusters of weaker intensity that populate higher frequencies above/below the main waveform, mirroring or partially mimicking the shape of the cardiac cycle.

Moreover, this dependency on sampling frequency also applies to Field II and other discrete time ultrasound simulation techniques. Jensen notes for Field II [44] that slight changes in the phase of the (impulse) response can lead to large errors, implying that phase discretization is of greater importance than the exact value of sampled amplitudes.
Thus, sonograms produced using Field II at low sampling frequencies also experience the same type of artifacts, though the design of Field II makes it less sensitive to the granularity of the sampling resolution, and the value of $f_s$ at which these artifacts become visible and obstructive is lower than the 50MHz value shown for FAMUS.

Figure 3.7: (a) Field II produces a discrete, but finer grained impulse response due to mathematically continuous elements. FAMUS can only generate a sparse signal based on monopole positions. (b) A sparse signal in FAMUS with non-integer delay is shown with sample times as bins. The fractional delay $\Delta$ is found and used to linearly interpolate the signal.

It is then reasonable to question the sensitivity of simulation results to sampling frequency, and to wonder what value of $f_s$ is suitable to use, and under what conditions. It is easy to consider increasing the sampling frequency, which should allow for even finer depiction of emitted signals, and hence better accuracy in summation or convolution. However, this choice is difficult to justify, since:

1. Increasing the sampling frequency increases the number of samples to compute, which actively works against achieving real-time performance,

2. the nominal use of 100MHz is already an order of magnitude above the carrier frequency of typical Doppler ultrasound ($\leq 7$MHz), therefore the Nyquist limit and other related principles are satisfied,

3. arbitrary increases of sampling frequency presents at best an “educated” guess; it is tedious to find a per-case metric for selecting sampling frequency, and

4. Field II functions reliably at 100MHz, a major achievement when compared to many other time domain methods that use higher sampling rates, like those assessed by Li and Zagzebski [35]; it seems both desirable and reasonable for FAMUS to be operated at a fixed sampling rate of similar magnitude.
Likewise, it is possible to entertain the use of additional source monopoles, but that mindset finds similar objection in points 1 and 3 above, as it imposes a greater computational load and program execution time, which again works in opposition to real-time performance and presents no obvious decision metric.

A better approach would be to find some way to make results more consistent and accurate, apart from changing the sampling frequency or number of source monopoles. First, it is important to understand how transmit/receive signals or impulse responses are currently constructed. As noted in (2.11), the time from an element \( mn \) to scatterer is determined from its round trip journey as \( 2t_{s,m,n} \), which determines the time at which an impulse of weight \( e^{-\alpha/R}/R, R = 2|r_s| \) occurs. However, due to discrete time and high precision position (stored as float or double precision number), some decision must be made when the travel distance is not exactly divisible by the speed of sound in the medium \( c_0 \) multiplied by the sampling frequency \( f_s \). Currently, FAMUS uses rounding to decide in which sample \( d \) the contribution from \( mn \) ends up

\[
d_{m,n} = \text{round}(2\frac{r_s}{c_0}f_s) = \text{round}(2t_{s,m,n}f_s),
\]

which is relatively simple and does not impose a large computational overhead per scatterer. Naturally, this rounding introduces some measure of error, some of which appears in aggregate through artifacts in the Doppler sonogram shown above in Figure 3.4. This is because as contributions from each source \( mn \) accumulate at samples not necessarily aligned with the continuous travel time, the velocity/frequency components in the flow are slightly misrepresented.

One way to address this without increasing temporal resolution is to calculate how the contribution of an impulse should be distributed between two adjacent samples, or in other words to interpolate the weight values. For high accuracy one could use some high order polynomial or spline interpolation scheme, but this imposes a significant overhead which may not be worth the computation time. Thus, to balance improvement of accuracy and per scatterer computation time, a simple linear interpolation scheme was tested.

For instance, in the case of FAMUS I where a full signal is transmitted to the scatterer (see Figure 2.4) from each monopole, the following process is used to adjust the incident waveform. A similar process is used to develop impulse responses in FAMUS II. Based on the positions of the monopole and scatterer the number of samples equivalent to the travel time is determined and this value is split into its integer and fractional components, \( \lfloor 2t_{s,m,n}f_s \rfloor \) and \( \Delta = 2t_{s,m,n}f_s - \lfloor 2t_{s,m,n}f_s \rfloor \). Ordinarily, this value would be rounded, shifting the signal forward or back one whole sample. Instead, we use a floor shifted copy of the signal \( x^f[d] \) and the fractional delay \( \Delta \) to construct a redistributed variant of the
transmitted signal $x[d]$, expressed below and in Figure 3.7b:

$$x[d] = \Delta x[d - 1]^f + (1 - \Delta)x[d]^f. \quad (3.3)$$

The upgrades described above were tested using the same pulsatile Womersley flow profile as used in Aguilar et al. (2010) [38], comparing the original rounding method to align signals in FAMUS I, and the distributive linear interpolation. For each method, both 1000 and 5000 scatterers were used to fill the environment, under sampling frequencies of 50MHz and 100MHz each, resulting in eight tests total.

As one can see in Figure 3.8, the use of interpolation to distribute the contribution of non-integer sample aligned impulses significantly reduces the artifacts present in the sonogram. This improvement was also tested at $f_s = 50$MHz, which is known to introduce errors similar to real problems faced by Doppler ultrasound machines (see Figure 3.9). While this approach by no means accounts for all artifacts, it is clear that Doppler spectra that would ordinarily be almost unusable (e.g. Figure 3.9a) can be largely cleaned up in (b). This method was also used on the signals involved in FAMUS II impulse response generation, which also reduces the number of artifacts in the final Doppler spectra, but with less effectiveness (see Figure 3.10). Unlike the FAMUS I case, the presence of artifacts is not drastically changed.

It is expected that this type of improvement is especially useful for FAMUS I because the interpolation directly alters the magnitude or amplitude of the transmitted signal, which is directly used by FAMUS I in summation to produce the final fast-time and sub-sampled slow-time Doppler signals. In this case, there are no complex transformations, so what matters most is a better accounting of the amplitude at each time sample, or effectively a low average $L_1$ distance (e.g. we seek $\min 1/n \sum_n |x[n] - x'[n]|$), which is the primary effect of the distribution scheme.

Meanwhile, in FAMUS II the final signal is obtained after a series of convolutions, a more complicated process than summation that is in form more related to correlation and signal energy. Because convolutions are a sequence of shift, multiply, and sum for all time points or samples, individual deviations per sample are felt across the entire output signal, meaning that error minimization here puts an emphasis on maintaining the correct shape or proportion of the signal. To this end, distribution of a misaligned impulse can actually be more detrimental as it further changes the shape of the signal. Finally, it is also important to note that FAMUS II already produces an inherently cleaner result (compare Figures 3.8a and 3.10a), which reduces the need for this modification specifically for FAMUS II.
Figure 3.8: Using an alternate display scheme to show signal intensity as a random colour, artifacts are more readily distinguishable from the oscillatory cardiac cycle. The rounding and interpolation schemes are compared for FAMUS I simulations using $f_s = 100$MHz. The first column shows results of the rounding scheme for 1000 and 5000 scatterers, and the second shows results of interpolation scheme. Clearly, the use of even simple linear interpolation cleans up artifacts considerably, for minor additional overhead.

3.3 Spatial Masking Templates

Now that we have determined some key sources of artifacts and formulated some strategies to reduce their appearance in the output Doppler spectra, it is important to consider the overall scalability of this simulation framework. The details in Table 3.1 indicate that past tests have used a collection of 5000 scatterers distributed within a 40mm long, 4.2mm radius tube. While it is possible to generate acoustic signals from such a broad domain,
we note that Doppler ultrasound generally focuses on a small portion of an entire region, centred about the sample volume of the transmitted acoustic field.

We are interested in this for a few reasons. Firstly, since the acoustic fields in transmission and reception are by no means homogeneous, it is expected that signals emanating from high power subregions contribute more significantly to final output. Secondly, the shape and location of these high power subregions are defined by the focusing properties of the transmitting and receiving transducers, which are in practice directed at the areas of interest for the Doppler scan. This implies that the most relevant information
in a Doppler spectrogram corresponds to the physical region which intersects the sample volume. Finally, in order to achieve real-time performance on a system with finite resources, it is prudent to only compute using the scatterers most needed to mimic an actual Doppler scan. Therefore, from the perspective of efficient simulation it is important to observe the effect of selecting and using various sub-collections of scatterers in the computation of Doppler spectra. In this regard, it is crucial to remember that signals from scatterers far from the focal point (and by extension, the sample volume), require more samples and hence more computational effort. Thus, it is tempting not only to use fewer scatterers, but also to confine them to a clustered group.
Additionally, in order to make selective computation effective, we must be able to easily determine which scatterers to select, otherwise we have merely substituted the quantity of calculations (many scatterers) with complexity (how to isolate relevant scatterers). However, recalling Figure 1.7 we surmise that is probably difficult to quickly compute the intersection of the acoustic field with the distribution of scatterers in space, as this depends on parameters such as centre frequency, focal point, and excitation duration, which are usually changed throughout a Doppler examination.

Previous work avoided this problem by employing simpler sample volumes, like that of Khoshniat et al. in assuming a spherical SV, and that of Evans and McDicken in choosing a teardrop shaped SV [45]. Though it would be ideal to precisely confine scatterer selection within the acoustic sample volume, this exploration is most concerned with the broad effects of using smaller and more spatially constrained collections of scatterers in simulation. Therefore, we will likewise experiment with spatial “masks” of relatively simple geometry to approximate the sample volume, and assess the results of different choices. Incidentally, this is similar to the practice of signal gating in Doppler ultrasound, where acoustic data from outside certain time periods is discarded under the assumption that it could not have originated from within the region of interest.

![Mask Shapes Illustration](image)

**Figure 3.11:** Illustration of several small spatial masks \( r = l/2 \) positioned within the vessel of interest. The larger spatial masks and the -20dB bounding box are considerably larger, and fill most of the vessel or even beyond it (not shown). The accompanying table lists their volumes for comparison.

<table>
<thead>
<tr>
<th>Mask Shape</th>
<th>Volume ( (mm^3) ) within vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere, ( r = l/2 )</td>
<td>11.15</td>
</tr>
<tr>
<td>Sphere, ( r = 3l/2 )</td>
<td>301.12</td>
</tr>
<tr>
<td>Cylinder, ( r = l/2 )</td>
<td>16.73</td>
</tr>
<tr>
<td>Cylinder, ( r = 3l/2 )</td>
<td>150.56</td>
</tr>
<tr>
<td>Box, -20dB</td>
<td>1914.68</td>
</tr>
<tr>
<td>Ellipsoid</td>
<td>44.61</td>
</tr>
</tbody>
</table>

We will assume the lateral focal point of the transducer is placed at the centre of the sample volume, and align the spatial masks to this point. Knowing that these simulations employ a nine-cycle excitation, we find an excitation length of \( l = 9\lambda = 9c/f_0 = 2.772\text{mm} \), and speculate that most important activity will occur in an axial range of this size. Thus, to contextually parametrize masks such as spheres and cylinders, their radii are set to multiples of \( l/2 = 1.386\text{mm} \). More complete details comparing the volumes and geometries of these masks are available in Figure 3.11.

Using these mask geometries, several Doppler spectra were simulated and compared in
Figure 3.12. As noted in the caption, subfigure (e) shows a Doppler sonogram obtained using a bounding box that extends to the most distant particles that respond at -20dB ($\approx 1\%$) power. This sonogram is fairly representative of the result that should be obtained from a complete Doppler scan, without using the entire domain. However, the table in Figure 3.11 shows that it by far has the greatest volume, indicating that it may use more scatterers than necessary. Examining (a) and (c), one can see that the masks used are too small and produce incomplete spectra, evident through the absence of frequency content in the first 0.3s except for the main systolic curve. Clearly, the extent of the sphere and cylinder about the focal point is sufficient to capture the strongest components of flow which shows the characteristic velocity of the cardiac cycle, but not more distal and slower particles. Also, velocity artifacts are visible in (a), likely due to the abrupt spatial truncation caused by the spherical mask.

Meanwhile, the masks used in (b) and (d) produce more complete spectra similar to the one in (e). By extending their respective radii, the masks fill nearly the entire vessel (e.g. the largest sphere diameter is 8.3mm, just shy of the vessel diameter), guaranteeing coverage of the sample volume. However, this also means they include more than just the strongest reflecting scatterers, which adds unnecessary computation. Lastly, the ellipsoidal mask in subfigure (f) is able to produce a relatively complete sonogram despite its small volume, probably because the extent of the mask more thoroughly includes crucial scatterers.

Figure 3.12f provides us with an important result, which demonstrates that it is not necessary to compute the Doppler spectra using the entire domain, and that mostly complete spectrograms can be obtained through notably smaller volumes, provided the volume of interest is wisely selected. A mask sized to the dimensions of the acoustic field will perform better, so by shaping the ellipsoidal mask in (f) such that its semi-major axis (aligned to the elevation dimension) is longer, we acknowledge the fact that the acoustic field is wider in elevation, and enclose more of the sample volume for improved simulation.

In a parallel observation, Table 3.3 records the actual number of scatterers inside the mask volumes contributing to the spectra in Figure 3.12. It appears that out of the 5000 scatterers used to seed the entire environment, in most cases fewer than 10% of the available scatterers were used to generate the simulated Doppler spectra. Notably, for the ellipsoidal mask that appeared to produce a relatively complete sonogram, only $79.5/5000 \sim 1.6\%$ of the scatterers were considered. This further reinforces the notion that a tiny, but carefully selected region may be quite sufficient for sonogram generation.

However, as discussed earlier, using masks based on simple, regular geometry is not a very pragmatic approach, because such masks are usually too approximate to fill the
<table>
<thead>
<tr>
<th>Mask Shape</th>
<th>Average Scatterer Count T =1s, 15000 timesteps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere, $r = l/2$</td>
<td>17.3</td>
</tr>
<tr>
<td>Sphere, $r = 3l/2$</td>
<td>275.9</td>
</tr>
<tr>
<td>Cylinder, $r = l/2$</td>
<td>26.3</td>
</tr>
<tr>
<td>Cylinder, $r = 3l/2$</td>
<td>131.3</td>
</tr>
<tr>
<td>Box, -20dB</td>
<td>1682.5</td>
</tr>
<tr>
<td>Ellipsoid</td>
<td>79.5</td>
</tr>
</tbody>
</table>

Table 3.3: Time averaged number of point scatterers within each spatial mask over the 15k intervals of a simulated Doppler scan. In most cases, the number of contributing scatterers is less than 10% of the total number of scatterers in the input data.

generally irregularly shaped sample volume, resulting in the loss of important scatterers. Ideally, it is of course best to use a mask of exact shape and scale to the sample volume, but this requires either pre-existing knowledge of its shape, or additional computational effort to calculate it. Instead, we could use a subregion that closely encloses or circumscribes the sample volume. Such a subregion could be shaped and sized according to the simulation context and with less foreknowledge, e.g. a blood vessel may be described by narrow cylindrical slices of scatterers.

The specifics of sizing are left to another exploration, perhaps including a metric for deciding on key dimensions. We are less concerned with this ‘imprecision’ because the main goal of this exercise was to observe the effect of using a significantly reduced computational volume on simulated Doppler spectra, where the use of geometric masks allowed for some degree of parametrized testing. What was crucial was to intelligently choose a significantly smaller region from which to draw simulation input, while capturing most of the particles that lie within some desired power level. In this way, we accommodate both the requirement of low computation burden, and a narrowly defined spatial region to select the strongest contributing scatterers. This leads us to subject of our next study, which is to consider the effect of increasing the number of contributing scatterers by seeding particles solely within this zone.
Figure 3.12: Doppler spectra obtained through various spatial masks. (e) results from a rectangular region sized to the furthest particles of -20dB power. (a) to (d) use spherical or cylindrical masks with radii based on the excitation length of the tests. (f) uses an ellipsoidal mask whose key parameters \(a, b, c\) are mapped to the Cartesian dimensions \(x, y, z\).
3.4 Doppler Simulation Convergence

In this section we begin to address the other end of the tradeoff between speed and realism. The preceding exploration focused on reducing the spatial extent and volume in which scatterers are distributed, while maintaining a relatively good simulation in terms of completeness. By doing so, we place a loose constraint on the number of scatterers used in simulation, which is helpful in reducing the amount of computational work. On the other hand, it is also necessary to populate a simulation with enough computational elements (i.e. scatterers) in order to generate an accurate and complete Doppler scan.

The main question we proceed to examine is this: what amount of point scatterers adequately represents blood flow? In the realm of B-mode simulation, Oosterveld et al. determined that a density of 10 scatterers per cubic millimetre is sufficient to develop realistic ultrasound speckle [46], while Crombie stated that a density of 100 scatterers per \( mm^3 \) would be a good representation of red blood cell density in humans [47]. These two values provide a workable range in which to perform B-mode simulations. Meanwhile, Swillens et al. performed Doppler simulations using a density of 10 scatterers per resolution cell, defined as the volume enclosed by the -3dB lateral and elevation transducer beam widths and the axial pulse length [33], probably a small region no larger than a few cubic millimetres. However, this is a somewhat vague reference point, and we are interested in verifying this quantity.

In order to determine a similar metric for Doppler ultrasound simulations, it is necessary to observe a broad range of simulation results and assess their corresponding density settings. It is expected that as the scatterer density increases, eventually there will be a point after which the growth in density will not notably change the appearance of the resulting spectrograms. In other words, we are looking for a point of “convergence” where the acoustic interactions as calculated using FAMUS are resolved with enough detail to produce a consistent level of quality, and by virtue of operating mechanistically based on fundamental physical principles, it is hoped such consistent results will be sufficiently “real” or accurate like that of a clinical ultrasound machine. The latter remark is necessarily contingent on the evaluation of clinical professionals, and must be addressed before employing FAMUS as a medical tool. For the present, we focus on determining the minimum input needed for consistent performance.

3.4.1 Velocity-weighted Flow Data Generation for Compact Spatial Extent

Beginning with the environment described in Table 3.1, the vessel of interest has a nominal volume of \( V = \pi r^2 L = 2216 mm^3 \). With 5000 scatterers, the scatterer density is 2.26 scatterers per cubic millimetre. To experiment with density settings one can choose to
progressively increase the number of scatterers inside the vessel from 5000 to a suitably high count, and observe the output spectra. However, this approach is extremely costly in terms of computational resources, as to perform a reasonable spread of tests (for instance, up to tenfold density), the time to simulate and required memory storage grows linearly. This is clearly problematic beyond basic testing - if FAMUS is to be used in comprehensive scenarios, such as simulations based on complete vascular models spanning many centimetres in length, it cannot be constrained by the volume of the region and the total number of scatterers.

Therefore, an alternative method of conducting the analysis is required. Rather than only increasing the total number of scatterers to adjust scatterer density, we can also limit our range of consideration to a significantly smaller spatial volume, following the intuition developed in the preceding exploration. However, this approach is associated with a new set of challenges. By sharply limiting the scope of scatterer positions, consistency of flow behaviour becomes a crucial issue, and we must ensure that flow inside the selected volume appears the same as its non-truncated counterpart. This was not an issue in the previous section, because the large tube environment provided a sufficiently continuous background of moving particles. In effect, we have adopted an Eulerian perspective, focusing on a control volume through which particles flow, thereby introducing the problem of the movement of particles across the boundary of the selected volume.

A simple approach is to replace every scatterer exiting one end of this subdomain with another one at the entrance, with the same relative cross-sectional position within the cylindrical vessel (see Figure 3.13). This is somewhat like a train of boxcars connected together on a rail, being observed throughout a tunnel. As one car traverses a mouth of the tunnel, it is “replaced” by an equivalent car on the other side, as pulled by the connecting chain. Then in subsequent data generation steps, the newly introduced particles take on the velocity at the given position $r$ within the vessel cross-section.

However, this is not fully representative of the physical properties of flow. While mass conservation (or particle-by-particle consistency) is crucial, the propagation due to velocity of the fluid must also be considered, since many flow profiles are dynamic and time-varying. While particles added to the entrance immediately adopt the velocity at time $t_i$ and position $x(r)$, this does not always ensure that the spatial gaps formed by the exiting particles are filled in the case of varying velocities. Furthermore, flow should also appear uniformly random. If new scatterers are simply reinserted into the same cross-sectional position with every passage across the subdomain, there will be underlying regularity in the flow which will appear as a repeated pattern in the output Doppler spectrum. This is where the boxcar analogy breaks down, since particles are not in general strung together like beads; replacement particles appear only once the original
one exits. By introducing randomness into the cross-sectional position of the replacement scatterers, the flow should appear with no repetition artifacts. This also avoids the need to track every individual particle that exits the vessel, and instead works on a statistical level of conserving mass. Furthermore, the use of randomized cross-sectional positioning is actually a better model of the flow behaviour.

A simple thought experiment confirms this idea. To begin, consider a spatially varying flow profile. For simplicity, we will examine parabolic flow in a straight vessel, again selecting a finite length to form a control volume. At positions along or near the vessel axis, one would expect particles to exit the tube most frequently, in contrast to particles near the wall. This is because these radial positions lie among the highest velocities, while positions near the wall correspond to near zero velocity. Assuming steady flow and conservation of mass, it is also more likely that new particles entering this control volume (e.g., for flow just prior to the entrance) will appear at radial positions close to the axis, rather than near the edges of the vessel. Additionally, for discrete time intervals, one would also expect these new particles to be propagated axially into the vessel according to the velocity profile, filling the space made by the exiting particles (also shown in Figure 3.13). Thus, the use of the velocity profile to weight the random positioning in a statistical manner is a more faithful representation of the physical processes in flow.

![Figure 3.13](image)

Figure 3.13: A “boxcar” particle replacement scheme, in (a) to (c). Hatched particles leave the vessel and are replaced on the opposing side in orange. The flaw in this scheme is evident in (b), where the replacement particles are placed exactly at the mouth of the vessel. This leaves a noticeable gap (shaded red and green zones). In (d) this is remedied by also providing a random depth into the vessel, limited by the maximum depth possible, $v(r, t_i) \Delta t$. For simplicity, cross-sectional randomness is not shown.

Using this information, we can instead insert new particles, uniformly random distributed within a space defined by the vessel cross-section and depth equivalent to the flow
velocity profile at that time instant multiplied by one time step interval: \( d = v(r, t_i) \Delta t \). This method takes into account the velocity at different radial positions to weight the random position assignment, satisfying mass conservation, flow velocity, and randomness within an arbitrary volume. The resulting modification provides a tube of considerably smaller spatial extent (see Figure 3.15) which allows more precise control of volumetric density without imposing a great computational load.

3.4.2 Line and Volumetric Density Comparison Metric

In further preparation for determining a range of appropriate density values for Doppler simulation, it is important to have a metric by which different results can be compared. To quantitatively assess the convergence of the sonograms, we would like compare the density of the scatterers to the wavelength of the transducer transmissions. It is expected that as this spacing approaches and falls below the operating wavelength, the reflected signal will not be able to discriminate between particles (i.e. the domain is now “smooth” with respect to the excitation pulse), and the sonograms should appear to converge. However, volumetric density cannot be directly compared to wavelength; instead, we develop a relationship between volumetric density and transducer wavelength to produce an equivalent scatterer line spacing or line density. A brief derivation is provided below.

Given a rectangular volume \( V \) and its dimensions \( L_x, L_y, \) and \( L_z \), it can be said that for some uniform point spacing \( \Delta s \):

\[
N_i = L_i / \Delta s \quad \text{for } i \in \{x, y, z\},
\]

which represent the number of point scatterers per dimension, such that

\[
N_{total} = N_x N_y N_z = \frac{L_x L_y L_z}{\Delta s^3} = \frac{V}{\Delta s^3}.
\]

Therefore the volumetric point density and uniform point spacing can be written as:

\[
\frac{N_{total}}{V} = \frac{1}{\Delta s^3},
\]

\[
\Delta s = \left( \frac{V}{N_{total}} \right)^{1/3}.
\]
For sufficiently resolved images, the average scatterer spacing should be on the order of the wavelength of sound in the medium, implying a particle seeding of:

\[ \Delta s \approx \lambda, \quad \text{or} \quad N_{\text{total}} \approx \frac{V}{\lambda^3} \]

around which the resulting spectra should begin to reach realistic and/or convergent appearance. This gives a rough estimate of the number of particles needed in simulation.

### 3.4.3 Doppler Spectra Comparison

Finally, we are prepared to assess the convergence requirements of a typical Doppler simulation, equipped with guidelines and a numerical measure by which to evaluate the resulting spectrograms. We proceed to run a battery of simulation tests using the compact environment described in the previous section for a multitude of densities and scatterer counts in order to get an adequate picture of the behaviour of FAMUS I (see Table 3.4), using pulsatile Womersley flow over a period of one second to mimic cardiac behaviour. For corresponding transducer and simulation settings, the reader is referred to Table 3.1 at the beginning of the chapter.

### Post-processing and Image scaling

To fairly compare sonograms of different test situations (namely, of higher scatterer densities), it was necessary to (1) normalize the slow-time Doppler signals and (2) use a unified colourmap to display the final image. The increasing density of scatterers adds to the overall power of the reflected ultrasound signal, which when unprocessed can make comparison untenable. Therefore, to preserve the relative intensity of each sonogram, the slow-time signals were normalized by energy:

\[
x_{\text{norm}}(t) = \frac{1}{\sqrt{\int \| x(t) \|^2 dt}} x(t).
\]  

(3.4)
Figure 3.15: Schematic of simulated environment. Original (light grey) and shortened (dark grey) tubes are overlaid for comparison.

<table>
<thead>
<tr>
<th>Number of Scatterers</th>
<th>Volumetric Density (scat/mm$^3$)</th>
<th>Line Density (scat/mm)</th>
<th>Number of Scatterers</th>
<th>Volumetric Density (scat/mm$^3$)</th>
<th>Line Density (scat/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.651</td>
<td>1.154</td>
<td>1000</td>
<td>6.510</td>
<td>0.536</td>
</tr>
<tr>
<td>200</td>
<td>1.302</td>
<td>0.916</td>
<td>2000</td>
<td>13.019</td>
<td>0.425</td>
</tr>
<tr>
<td>300</td>
<td>1.953</td>
<td>0.800</td>
<td>3000</td>
<td>19.529</td>
<td>0.371</td>
</tr>
<tr>
<td>400</td>
<td>2.604</td>
<td>0.727</td>
<td>4000</td>
<td>26.039</td>
<td>0.337</td>
</tr>
<tr>
<td>500</td>
<td>3.255</td>
<td>0.675</td>
<td>5000</td>
<td>32.548</td>
<td>0.313</td>
</tr>
<tr>
<td>600</td>
<td>3.906</td>
<td>0.635</td>
<td>6000</td>
<td>39.058</td>
<td>0.295</td>
</tr>
<tr>
<td>700</td>
<td>4.557</td>
<td>0.603</td>
<td>7000</td>
<td>45.568</td>
<td>0.280</td>
</tr>
<tr>
<td>800</td>
<td>5.208</td>
<td>0.577</td>
<td>8000</td>
<td>52.077</td>
<td>0.268</td>
</tr>
<tr>
<td>900</td>
<td>5.859</td>
<td>0.555</td>
<td>9000</td>
<td>58.587</td>
<td>0.257</td>
</tr>
<tr>
<td>10000</td>
<td></td>
<td></td>
<td>10000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Test cases for Doppler simulation convergence. Bold face indicates the approximate number of scatterers where convergence begins. Italicized face indicates tests in range of density used for experiments by Swillens et al.

After energy normalization, the resulting sonograms were mapped to the same 128 level colourmap, to ensure equal visual comparison.

**Visual comparison**

Upon visual inspection of the sonograms in Figures 3.16 and 3.17, it was found that at around 700 scatterers (volume density = 0.651/mm$^3$) the major features and texture of the images begin to stabilize, fully developing around a thousand scatterers (1000
scatterers: volume density = 6.51/mm$^3$). One can consider sonograms converged beyond this volume density range; observation of the higher densities shows that the texture and structure of the sonograms remains more or less the same. While prior to this density level, there are notable structural artifacts. For instance, for the test run with 100 scatterers, the first 0.3 seconds of systole demonstrate vertical striping which disappears at higher densities. A reasonable point of comparison to previous work is available in Figure 3.16b. The volumetric density in this case is 2.6 scat/mm$^3$, slightly higher than the 2.26 scat/mm$^3$ used previously. Here, a transition is visible in the texture of the image: while the vertical striping is much less visible here, other parts of the sonogram still appear somewhat blocky; compare 0.2s to 0.3s across the subfigures.

It should be noted that when converged, the sonograms do not look exactly the same – there is noticeable “movement” in the texture of the sonogram – caused by the random noise added by the new scatterers at higher densities. However, this is expected phenomenon; clinical imaging results are not noise-free and the appearance of the simulated sonograms is within realism.
Figure 3.16: Sonograms prior to and during convergence range. Note clear vertical striping artifacts in the lowest density test in (a) and slight “blockiness” in (b). The spectra in (c) and (d) show increasing consistency as scatterer volumetric density reaches a suitable level.
(a) 900 scatterers, density 5.859 scat/mm$^3$
(b) 1000 scatterers, density 6.51 scat/mm$^3$
(c) 5000 scatterers, density 32.55 scat/mm$^3$
(d) 10000 scatterers, density 65.1 scat/mm$^3$

Figure 3.17: Sufficiently converged sonograms. (a) and (b) are quite similar in their appearance, with (b) is denoted as the convergence “point”, and (c) to (d) appear texturally and structurally consistent, despite the order of magnitude increase in density.
Returning to the metrics derived earlier, from the speed of sound and selected transducer centre frequency, the wavelength of the transmissions was calculated to be 0.308mm. For a volume of 153.62mm$^3$, the corresponding scatterer count is 5258 scatterers. Note that this number is related to the inverse cubed wavelength, and thus varies dramatically with minor deviation. In practice, it is likely that we do not need to exactly seed particles to this level. For instance, allowing the spacing to lengthen by one wavelength, we arrive find a figure of 657 scatterers at 2$\lambda$, which is around the point where our simulated sonograms began to stabilize (~700 scatterers).

![Plot of estimated required scatterer count vs line spacing for $Vol = 153.62$mm$^3$. Blue and red bars indicate the calculated scatterer count for $\lambda$ and 2$\lambda$, respectively.](image)

Prior references to work by Oosterveld et al. and Crombie are reassuring, since a range of 10 to 100 scatterers per cubic millimetre easily encompasses our test cases in the range of two to ten thousand scatterers. This appears to be in line with observations, as the tests which were observed to mark the transition into convergence were located at densities ranging from 4.56 scat/mm$^3$ to 6.51 scat/mm$^3$ or 700 to 1000 scatterers in the volume, respectively. This also overlaps the upper end of density values suggested in Swillens et al., providing further confirmation of these results. In this region, major features in the sonograms stabilized around one thousand scatterers and could be considered converged beyond that point, with densities beyond this range showing consistency with each other, e.g. see comments on Figure 3.17(c) and (d). The widest line spacing corresponding to these densities is around 0.464mm, or 1.5$\lambda$, for a centre frequency of 5MHz, and $c_0 =$
1540m/s. The plot in Figure 3.18 provides the range for the number of scatterers needed to densely seed the volume in the test cases used, for a spacing in the range of \([\lambda, 2\lambda]\). Note the logarithmic scale, which indicates that the required quantity of scatterers is fairly flexible, within a certain range. Furthermore, the results appear to align with earlier developments in FAMUS. Aguilar et al., used a scatterer density of approximately 2.26 scat/mm\(^3\), roughly a quarter of the density needed for convergent sonograms. This corresponds to a seeding of less than 400 scatterers in our test environment (Figure 3.16b), a test case which is indeed blocky and somewhat incomplete and hence not fully converged. Therefore, it appears that volumetric densities around and above 6.51 scat/mm\(^3\) are acceptable for dense seeding of Doppler ultrasound simulations that provide sufficient realism without undue computational burden.

### 3.5 Summary

In this chapter we developed a number of improvements for the FAMUS framework and for simulation in general. Results from the first two sections provided an understanding of computational artifacts and helped formulate some practical ways to reduce their presence in simulation output. By exploring the effect of sampling frequency on signal generation in FAMUS, a linear interpolation method was developed to improve quality without increasing the sampling frequency. Meanwhile, by analyzing several simulated flow profiles, the source of motion based artifacts in input data was isolated.

Finally, the last two sections approached the issue of improving simulation from the perspective of balancing computation speed and realism. We first examined the effects of limiting the computational volume to a smaller size and to shapes approximating the sample volume in order to calculate relevant acoustic interactions, and secondly quantified the number of point scatterers necessary for consistent simulation results. In particular, a density of 6.51 scat/mm\(^3\) was found to allow Doppler simulation results to converge to a consistent quality. However, this value should not be taken as an absolute requirement for accuracy; acceptable scatterer density is linked to simulation input, e.g. Womersley flow in a straight tube blood vessel model. More realistic vessel geometry and complicated flow patterns may require higher resolution.

Based on recommendations from both explorations, we have a better sense of what type of computational work (i.e. scatterers drawn from high power regions) and how much computation (per cubic millimetre) would satisfy or at least balance both requirements for speed and accuracy. This guideline of shrinking the computational volume while increasing detail within it will hopefully provide manageable and scalable conditions on which to base future uses of the simulation framework.
Chapter 4

Frequency Dependent Attenuation

The next major objective of this thesis is to incorporate of frequency dependent attenuation. Here, we turn our attention from Doppler ultrasound, which is highly localized in space but sensitive to temporal changes, to the B-mode or B-scan modality, which is dependent on spatial variation in the imaged region.

In order to perform realistic B-mode ultrasound imaging, it is necessary to consider the effects due to properties of the medium being scanned. As sound waves travel through a medium from source to receiver and back again, it is expected that the waves will be affected differently based on their frequency components. Simple simulations may only consider the decay of signals due to the distance traveled from one point to another, as would be expected in a dispersionless medium, or when using a mono-chromatic signal. However, in real-world situations where finite acoustic pulses are employed, the result is a signal with wideband properties, meaning that there are multiple frequency components of interest. Such is the case in B-mode imaging, which is to be addressed here.

In order to be of viable use to real time training simulations or other explorations, the simulation framework provided by FAMUS must be capable of emulating most if not all physical phenomena regarding acoustic propagation. Hence, in this chapter we aim to accurately mimic frequency dependent attenuation effects, and calculate such properties faster than Field II.

4.1 Theory

4.1.1 Attenuation and Dispersion

We begin with an overview of the physical properties and relationships that give rise to attenuation. In general, as an acoustic wave passes through a medium, a portion of the wave energy is converted to heat, light, and other forms, a process known as
absorption. Furthermore, the presence of inhomogeneities in the medium tends to reflect wave energy, leading to losses from scattering, hence the expression: \( \alpha = \alpha_a + \alpha_s \) where the components of attenuation are due to absorption and scattering, respectively. Similarly, the phenomena of dispersion arises also due to absorption and scattering.

Initially developed for understanding electromagnetic waves, the Kramers-Kronig (K-K) relations describe the relationship between attenuation and dispersion, which for acoustic waves result in a pair of relations between phase speed \( c(\omega) \) and attenuation \( \alpha(\omega) \), dependent on frequency [14]:

\[
\alpha(\omega) = \frac{-2\omega^2}{\pi} \int_0^\infty \left[ \frac{1}{c_0(\omega')} - \frac{1}{c_0(\omega)} \right] \frac{d\omega'}{\omega'^2 - \omega^2},
\]

\[
\frac{1}{c_0(\omega)} = \frac{2}{\pi} \int_0^\infty \left[ \alpha(\omega') - \alpha(\omega) \right] \frac{d\omega'}{\omega'^2 - \omega^2}.
\]

Thus, dispersion is the variation of the phase velocity of waves propagating in an attenuating medium.

### 4.1.2 Tissue Response

For most applications of diagnostic ultrasound, tissue is assumed to respond linearly to acoustic excitations, because of their small energies or signal levels. Additionally, as diagnostic ultrasound operates in the megahertz range, the movement of tissue is assumed to be negligible over the scanning interval (for instance, the fastest tissue movements belong to heart valves, at approximately 1m/s [48]). Therefore, it is often assumed that tissue is a linear time invariant system, and we are subsequently interested in the frequency response or transfer function of such a system.

A common way to mathematically describe the effect of attenuation through a medium is by a simple power law expression, \( \alpha(f) = \alpha_0 f^n \) where \( n \in (1, 2) \) is typical for tissue, with \( n = 2 \) for water [49]. This gives rise to a simple formulation of the magnitude transfer function describing the propagation of an acoustic wave through a slab of tissue, for some distance or tissue thickness \( z \):

\[
|H(\omega, z)| = e^{-\alpha_0 \left( \frac{\omega}{c_0} \right)^n z}. \quad (4.1)
\]

It is important to also consider the phase of this transfer function. Typically, we expect that the phase velocity \( c(\omega) \) of transmissions in tissue does not change much with frequency, so a simple approximation would be that \( c(\omega) = c_0 \) is constant. This in turn implies a linear relationship between phase \( \Theta \) and frequency, \( n = 1 \), leading to the following transfer function:
Figure 4.1: An illustration of the transmission of some excitation $E$ and the corresponding response $R$, determined by the transfer function $H$ of a block of tissue of thickness $z$ and some attenuation value $\alpha$.

\[ \Theta(\omega, z) = \omega \tau_b z, \quad (4.2) \]
\[ H(\omega, z) = e^{-\alpha_0 |\omega| z} e^{-j\omega \tau_b z}. \quad (4.3) \]

where the “bulk propagation delay” $\tau_b = 1/c_0$, which is the response proposed by Kak and Dines [50].

Unfortunately, the impulse response corresponding to this transfer function (4.3) is non-causal, making it inconsistent with the physics. This is because although phase velocity is nearly constant, it is not completely so, and this small variation with frequency has large implications on the phase of the response. Gurumurthy and Arthur detail their encounter with this problem and proposed an alternative transfer function to resolve the issue of causality. The full derivation is described in Appendix A of [51], but the approach is briefly outlined as follows.

If some magnitude response $|H(\omega)|$ has finite energy and satisfies the Paley-Weiner condition, such a magnitude response is that of a causal function [52]. The original frequency response in (4.1) does not quite obey these requirements, but with a small modification imposing a high-frequency limit, it does. It then remains to locate the corresponding phase function, which is comprised of an all-pass component, based on the propagation delay in the medium (in other words, a linear phase expression), and a minimum-phase component provided by the Hilbert transform. The resulting expression of a suitable transfer function is:

\[ \Theta(\omega, z) = \omega \tau z - \frac{\omega \alpha_0 z}{\pi^2} \ln(\omega), \quad (4.4) \]
\[ H(\omega, z) = e^{-\alpha_0 |\omega| z} e^{-j\omega \tau z} e^{-j\frac{\omega \alpha_0 z}{\pi^2} \ln(\omega)}. \quad (4.5) \]
where \( \tau = \tau_b + \frac{\alpha_0}{\pi^2} \tau_m \), and the minimum phase delay factor \( \tau_m = 20 \). This expression is satisfactory for our needs, as it is both accurate in magnitude to an arbitrarily large frequency, and admits a causal response.

Figure 4.2: A comparison of the time domain impulse responses rendered by inverse transforming the corresponding attenuation transfer functions in (4.3) and (4.5), respectively. The waveforms are measured at depth of 2cm, for \( \alpha_0 = 0.5\text{dB/MHz-cm} \), where the speed of sound is 1540m/s.

The two transfer functions, inverse transformed into the time domain, are compared in Figure 4.2. The dotted line indicates the propagation delay of 1.29\( \mu \text{s} \), or the time it takes for sound to travel 2cm given \( c_0 = 1540\text{m/s} \) in the medium. For the non-causal response, we obtain a symmetric response, implying that we receive information before the signal has properly begun, which is clearly not a viable option in real conditions. Meanwhile, one can see that for the causal response the bulk of the signal energy of the causal response occurs after this arrival time. Also note the asymmetry in the causal response, and the lowered peak amplitude, compared to its non-causal cousin, a result of the dispersive effects in this medium.
4.2 Practical Considerations

Besides general theory regarding attenuation and dispersion, we should also note the circumstances under which these phenomena occur. Ideally speaking, when the contribution of a transmitted and backscattered wave is received by some transducer array, the path lengths from each differential area on the transducer surface to the point of interest must be individually calculated and used to determine the appropriate attenuation values, such as in Figure 4.3a. Additionally, dispersion is generally present in the propagation of wide-band signals in soft biological tissue. However, its effect on the characteristics of the received signal can generally be assumed to be small compared to the effects of attenuation. Furthermore, in the past it has been neglected when compared to variations in the phase speed caused by the inhomogeneous nature of soft tissue. Likewise, the same assumptions have been made in what follows.

In previous implementations of FAMUS II, this approach is used to construct the transmit (and receive) impulse response of the transducer at a given point in the field. Using the distances of each point source on the transducer surface to the field point, an appropriate damping can be applied to each contribution, $e^{-\alpha_0 r}$, based on a fixed or frequency independent attenuation value $\alpha_0$, such as in Figure 4.3b. This can be achieved by merely adding a multiplicative factor to each contribution, and is a sufficient approximation when used in situations such as Doppler ultrasound. However, a more complete description of the phenomenon of attenuation accounts for all frequency dependent variations, as shown in (4.5).

Figure 4.3: Diagrams of path lengths to a field point $r$ from the transducer surface. (a) shows an array of rectangular elements (only some paths shown), while (b) shows the point source approximation used by FAMUS II, and the resulting impulse response.

In fact, frequency independent attenuation may be better described as the centre frequency attenuation, $\alpha_c$, and is really more of a subset of frequency dependent attenuation. This is because for a situation such as Doppler ultrasound, the excitation signal is
usually relatively long in time, resulting in a narrowband signal centred about the centre
frequency. Thus the value of attenuation (denoted frequency independent attenuation)
is taken to be that which corresponds to the centre frequency, \( f_c \), as it is less important
to consider the variation of attenuation over frequency within the passband.

However, while equation (4.5) is accurate, directly using it for all path lengths is
computationally intensive because it would have to be considered for each pair of source
element and field point. We would like to consider some acceptable approximations to
make the task of incorporating frequency dependent attenuation simpler, while main-
taining most of its accuracy.

Firstly, it is helpful to rewrite the attenuation transfer function in a manner which
more visibly describes the contribution of the centre frequency. This can be done by
expanding (4.5) into the following form:

\[
H(\omega, z) = e^{-\frac{\alpha_c}{2\pi} z} e^{-\alpha_0 \frac{(\omega - \omega_c)}{2\pi} z} e^{-j\omega \tau z} e^{-j\frac{\omega_0 z}{\pi^2} \ln(\omega)},
\]

or alternatively,

\[
H(\omega, z) = \exp\left(-\frac{\alpha_c}{2\pi} z\right)\exp\left(-\alpha_0 \frac{(\omega - \omega_c)}{2\pi} z\right)\exp\left(-j\omega \tau z\right)\exp\left(-j\frac{\omega_0 z}{\pi^2} \ln(\omega)\right)
\]

\[
= \exp\left(-\frac{\alpha_c}{2\pi} z\right) A(\omega, z),
\]

where \( \omega_c = 2\pi f_c \), \( \alpha_c = \alpha_0 \omega_c \), and \( A(\omega, z) \) indicates the part of the transfer function which
describes the frequency dependent attenuation effects. Thus, the frequency independent
and dependent factors are now separately termed.

A similar formulation, which was used by Jensen [53] allows us to see the frequency
independent attenuation in the context of the complete phenomenon of attenuation. By
splitting (4.5) into two constituent parts, we are free to consider frequency dependent
attenuation as a separate expression to be applied later. Thus in practice it is easy to han-
dle wideband scenarios, such as B mode imaging, by use of the additional multiplicative
expression.

As mentioned before, because the transmit / receive impulse is an aggregate of con-	ributions from each pair of element and scatterer, it would be more precise to use the
distances of each pair in the frequency dependent calculation. The time domain analogue
of this would be a non-stationary convolution, which is considerably more complicated
than previous procedures. We should note, however, that at field points reasonably dis-
tant from the transducer, the variation in the path lengths of the source elements to the
field points becomes very small, or in other words \( |\mathbf{r}_i + \mathbf{r}| \approx |\mathbf{r}_{\text{mean}}| \). Therefore, for
most practical purposes of simulation it is sufficient to use the average distance from a
scatterer to the transducer array as the determining factor in the frequency dependent
attenuation value used to modify the impulse response.

Figure 4.4: The illustration in (a) demonstrates the mean distance from transducer to field point that can be used to approximate the frequency dependent attenuation value, while (b) shows the order of errors expected by this approximation, which are less than 1dB over the typical width of a transducer. (Taken from Jensen [53], Figure 2.20)

4.3 Implementation

Having addressed fundamental theory of frequency dependent attenuation and some reasonable approximations for calculating it, application to the FAMUS framework can begin. Note that because the mathematics have been formulated in terms of a linear time-invariant system, FAMUS II is the simulator most easily adapted through the addition of transfer functions representing the desired acoustic behaviours. FAMUS I, due to its direct summation approach, does not lend itself to simple modification.

Since the mechanics of attenuation have been described in the frequency domain, it is most straightforward to perform our manipulations in that domain. The relevant calculations can be easily inserted into the existing framework after the initial step of calculating the transmit impulse response of a single scatterer, accounting for independent attenuation.

The block diagram in Figure 4.5 below illustrates the flow of operations required to calculate frequency dependent attenuation and apply it to the corresponding impulse response. Mathematically, we take advantage of the fact that convolutions are multiplications in the Fourier domain to make the process of applying the frequency dependent attenuation much more simple than the time domain equivalent of a convolution. Assuming that the transmit and receive transducers are the same, we begin with the transmit impulse response $h^T_{k,\text{indep}}(t, \mathbf{r})$ of the transducer to a specific scatterer $k$ (like that shown
in Figure 4.3b), which is also assumed to be generated with the independent attenuation shown in (4.8):

\[ h_{k,\text{indep}}^T(t, \mathbf{r}) \xrightarrow{F} H_{k,\text{indep}}^T(\omega, \mathbf{r}), \]

\[ H_{k,\text{indep}}^T(\omega, \mathbf{r}) = H_{k,\text{indep}}^R(\omega, \mathbf{r}). \]

Then applying frequency dependent attenuation:

\[ H_k^T(\omega, \mathbf{r}) = H_{k,\text{indep}}^T(\omega, \mathbf{r})A_k(\omega, \mathbf{r}), \]

or

\[ H_k^{T/R}(\omega, \mathbf{r}) = (H_{k,\text{indep}}^T(\omega, \mathbf{r}))^2A_k(\omega, 2\mathbf{r}), \]

which gives us the transmit or transmit-receive response, with frequency dependent attenuation applied, for a given scatterer. Note that for the transmit / receive (T/R) case, we have used a distance of \(2|\mathbf{r}|\) to accommodate the corresponding two-way travel distance. At first glance, it seems more appropriate to square the complete impulse response including the frequency dependent attenuation:

\[ H_k^{T/R}(\omega, \mathbf{r}) = (H_k^T(\omega, \mathbf{r}))^2 = (H_{k,\text{indep}}^T(\omega, \mathbf{r})A_k(\omega, \mathbf{r}))^2. \]

But because \(A(\omega, \mathbf{r})\) is an exponential function with a linear argument in \(\mathbf{r}\), it is mathematically equivalent to double the distance instead, which requires fewer additional multiplications and is thus faster to compute.

Accounting for all such scatterers \(k\) or field positions of interest, we obtain the following total transmit (T) or total transmit-receive (T/R) responses, accounting for frequency dependent attenuation, below:

\[ H_T^T(\omega, \mathbf{r}) = \sum_{k=0}^{M} H_{k,\text{indep}}^T(\omega, \mathbf{r})A_k(\omega, \mathbf{r}), \quad \text{and} \quad \quad (4.9) \]

\[ H_T^{T/R}(\omega, \mathbf{r}) = \sum_{k=0}^{M} (H_{k,\text{indep}}^T(\omega, \mathbf{r}))^2A_k(\omega, 2\mathbf{r}), \quad \quad (4.10) \]

which we may finally inverse Fourier transform to obtain their respective impulse responses. Also, because the FAMUS II framework only computes the individual \(k^{th}\) impulse response for times where \(h_{k,\text{indep}}^T(t, \mathbf{r}) > 0\), the function is displaced in time, that is, it is not aligned to an absolute time \(t_0\) origin, and must also be appropriately delayed in time before the total impulse response can be generated. This necessitates calculating
an appropriate phase shift vector, based on the time delay – phase shift equivalency in the Fourier domain:

\[ s(t - \tau) \xrightarrow{\mathcal{F}} S(\omega)e^{-j\omega\tau}. \]

Using the earliest arrival time of any component of the impulse response of an individual scatterer \( k \), and the earliest possible arrival time over all scatterers, we formulate a delay of \( \tau = \tau_{k,\text{min}} - \tau_{\text{total,\text{min}}} \) or \( \tau = 2(\tau_{k,\text{min}} - \tau_{\text{total,\text{min}}} \) with the second expression relevant for a two-way round trip time in transmit and receive. This allows us to build a phase vector \( P_k(\omega, r) = \exp(-j\omega\tau) \), yielding the following two equations:

\[
H^T(\omega, r) = \sum_{k=0}^{M} H_{k,\text{indep}}^T(\omega, r) A_k(\omega, r) P_k(\omega, r), \quad \text{and} \quad \tag{4.11}
\]

\[
H^{T/R}(\omega, r) = \sum_{k=0}^{M} (H_{k,\text{indep}}^T(\omega, r))^2 A_k(\omega, 2r) P_k(\omega, r). \tag{4.12}
\]

Effectively, the effects of frequency dependent attenuation can be envisioned as a separate vector, \( A_k(\omega, r) \), and the phase shift vector \( P_k(\omega, r) \) to be multiplied with the impulse response in the frequency domain for the contributions of each scatterer or field point \( k \), allowing us to build up the entire field.
4.3.1 Computational Considerations

Due to the discrete nature of the simulation and the desire for real-time performance, a number of important notes are considered below to help improve calculation efficiency.

Vector Lengths

When operating in the frequency domain using discrete signals, it is necessary to take into account the properties of the Fast Fourier Transform in order to efficiently perform manipulations. It is ideal to work with vectors of power-of-two length, in order to retain the speed benefits of the FFT algorithm. Thus, for all practical manipulations of (4.12), it is necessary to zero pad the initial vectors of $h_{k,\text{indep}}(t, r)$ to the nearest power of two before working with them in the frequency domain.

Precomputed Attenuation and Phase Information

In Figure 4.5, it should be noted that there are some key precomputation steps needed to efficiently produce accurate results. The two main quantities of interest to be calculated ahead of time are the frequency dependent attenuation and phase shift vectors, as mentioned in the section before. This avoids having to calculate the exact values on the attenuation and phase curves during runtime, which would drastically affect real-time performance. Because the bounds on the range of these values can be estimated beforehand (e.g. the maximal phase difference or time delay between signals), it is best to compute this information ahead of time.

It is important that the precomputation step of frequency domain quantities take into account the largest expected number of time samples, as the final inverse Fourier transformed impulse vectors cannot contain any more samples than were originally allocated in the frequency domain. Both flavours of simulators based on the FAMUS framework locate the earliest possible receive sample time and compare it to the latest possible receive sample time to determine the duration of the total impulse response. The illustration in Figure 4.6 below shows this clearly.
Hence, for any given simulation, we can determine this length and set the length dimension of the precomputed data to be $L = 2^\log_2(\text{total length})$, thereby accommodating both constraints of 1) the required number of samples and 2) power of two vector length. Then in accordance with the simulation domain, we compute the attenuation function $A(\omega, r)$ for all possible axial distances in steps of the sampling frequency, e.g. if the simulation is ranged at a depth of 10cm, then we require $2 \cdot 10\text{cm} \cdot f_s/c_0 \approx 13000$ axial sample points, for pulse-echo transmissions. The lowest upper bound satisfying this condition will be $2^{14} = 16384$. We use this value to size a matrix of values to be used at our convenience during simulation. Likewise, to speed up the simulation, a similar matrix is calculated for the phase vectors.

**Downsampling Compatibility**

Given the longest impulse duration, we recall the recommendation regarding vector lengths and require also that the length of the first dimension of these precomputed matrices be padded up to a power-of-two size. Besides providing the speed advantage in later FFT and IFFT operations, this requirement has the effect of making frequency domain calculations fairly easy to scale. Since we rely on multiplication of vectors (instead of time domain convolutions) in the frequency domain, it is important that all vectors...
be of the same length in order to carry out the operation, and guarantee that the same frequency components are multiplied together.

By keeping all vectors powers of two in length, and computing the largest number of samples for the stored matrix, finding a length compatible attenuation or phase vector to use in calculation of (4.12) is merely a task of indexing to the correct distance $|r|$ and downsampling by the corresponding factor of two (master length / vector length = $2^n$, $n \in \mathbb{R}$).

![Figure 4.7: Assignment of frequency domain vector sample values to a smaller vector of length $M < N$, where $N/M \mod 2 = 0$. This allows perfect alignment of frequency bins dictated by the simulation setting of $\Delta f = f_s/N$, where $N$ is the maximum sized vector length.](image)

**Conjugate Symmetry**

Lastly, for further computational benefits, it is helpful to take advantage of the conjugate symmetry of the DFT of real signals [54]. When performing an FFT (or generally, a DFT) of a purely real signal, there is guaranteed duplication of information in the resulting vector. Namely, for a general complex vector of length $N$, the following relation holds:

$$S(k) = S(N - k)^*.$$  \hspace{1cm} (4.13)

Hence for even values of $N$, we know that $S(0)$ and $S(N/2)$ are real, and given $S(1)...S(N/2 - 1)$ we know the remaining values of the sequence up to $S(N - 1)$. And likewise, if $N$ is odd, we know $S(0)$ is real, and given $S(1)...S((N - 1)/2)$ we know the rest of the sequence. Therefore, in general for the attenuation and phase vectors, we only need to calculate roughly half the number of samples of the final vector length.

### 4.3.2 Intermediate Results

Based on these details in theory and practice, application of the flow in Figure 4.5 allows us to generate some promising transmit impulse responses and corresponding radio-frequency (RF) signals, for a specified excitation signal.

The plots in Figure 4.10 compare the results of using the frequency dependent capabilities of Field II and the newly implemented module in the FAMUS II framework.
Beginning with the same independently attenuated impulse response as produced by Field II, we separately ran the FAMUS II and Field II simulators for frequency dependent attenuation, to see the output transmit impulse. We can see that in Figure 4.10a, the dashed (Field II) and solid (FAMUS II) traces appear to be consistently of the same duration and amplitude, though they are shifted in time from each other. Meanwhile, Figure 4.10b shows a comparison between the attenuated transmit impulse resulting from a direct calculation from theory in MATLAB, the FAMUS II framework with manual time shifting, and Field II (solid, dot marker, and dashed lines respectively). It is clear from this figure that the same results are consistently obtained between all simulators and methods, meaning that the approach as outlined in the Implementation section can be trusted relative to that of Field II.

4.3.3 Phase Properties

At this point there is the question of the temporal shift seen in Figure 4.10a, when comparing Field II and FAMUS II. After the final step of the inverse Fourier transform,
Figure 4.10: Transmit impulse responses for Field II and FAMUS II at z=4cm, $\alpha_0 = 0.1, 0.3$ and 0.5 dB/MHz·cm, for an unfocused emission. (a) shows that the results of both simulators are very similar, except for some time delay. In (b) results of the FAMUS II frequency dependent module are manually aligned with results from Field II, showing their near congruency.

The time signal generated out of the FAMUS II framework is delayed in time, at least when compared to the data from Field II. Consulting [55], it appears that Jensen, et al. additionally supply a minimum phase requirement on their output impulse response in application into Field II.

In systems theory [56] it is well known that for a transfer function with some magnitude response $|H(\omega)|$, there are several possible choices of phase $\Theta(\omega)$ ranging from minimum to maximum phase. Returning to equation (4.4) here and the discussion of its development, we recall that there are several components to the phase of the generally non-minimum transfer function. We are most interested in the partial expression $\omega\tau z = \omega(\tau_b + \frac{\alpha_0}{\pi\tau_m})z$, as the latter logarithmic term is required for causal behaviour. This expression shows us that as attenuation constant $[\alpha_0] = \text{dB/MHz·cm}$ increases, so linearly does the corresponding time delay, which is consistent with the results shown earlier.

While consistent selection of the attenuation transfer function would not affect the magnitude and relative position of features of a particular simulation, it is evident that the choice of phase does change the absolute timing of characteristic portions of the response signal. It is convenient to have the attenuation transfer function be minimum phase (or near as possible within numeric accuracy) for the sake of data processing. This is because minimum phase systems have their energy concentrated near the beginning of the impulse
response, i.e. the delay of the energy is minimal and \( h(t) \) satisfies: 
\[
\min \sum_{n=m}^{\infty} |h(n)|^2, \forall m \in \mathbb{Z}^+, \\
\]
otherwise known as energy compaction [57]. This in turn means that all eventful behaviour \((h(t) > 0)\) occurs as early as possible, reducing the number of non-zero, relevant digital samples in the signal vector. Thus, for the sake of simulation we are similarly interested in having the FAMUS II framework produce a minimum-phase attenuation transfer function.

We can examine the elements of the attenuation transfer function developed in section 4.1.2. Here, we can see that the comprehensive phase of (4.5) (blue solid) appears to be mostly dominated by the linear phase contribution given by \( \frac{\alpha_0}{\pi} \tau_m z \) (red dashed), which is large with distant values of \( z \) and high values of \( \alpha_0 \). However, we can see that the dispersive component (green dashed) adds a smaller logarithmic component that alters the overall behaviour of the curve. Practically speaking, to correct for minimum phase means to ensure that over the range of frequencies of interest, \( H(\omega) = |H(\omega)|\exp(\Theta(\omega)) \) should have no net change in phase, that is, \( \Theta(0) = \Theta(f_{\text{max}}) \).

This requires us to determine the full unwrapped phase of the vectors \( A(\omega, z) \) generated in the precomputed matrix, for some arbitrary upper frequency limit \( f_s/2 \) (commonly \( f_s = 100\text{MHz} \), though simulation requirements may vary). A simple approach for doing

![Figure 4.11: The overall phase and phase components of (4.5) are shown along side the phase of (4.3), for an attenuation of 0.7dB/MHz-cm at z=8cm.](image)

mostly dominated by the linear phase contribution given by \( \frac{\alpha_0}{\pi} \tau_m z \) (red dashed), which is large with distant values of \( z \) and high values of \( \alpha_0 \). However, we can see that the dispersive component (green dashed) adds a smaller logarithmic component that alters the overall behaviour of the curve. Practically speaking, to correct for minimum phase means to ensure that over the range of frequencies of interest, \( H(\omega) = |H(\omega)|\exp(\Theta(\omega)) \) should have no net change in phase, that is, \( \Theta(0) = \Theta(f_{\text{max}}) \).

This requires us to determine the full unwrapped phase of the vectors \( A(\omega, z) \) generated in the precomputed matrix, for some arbitrary upper frequency limit \( f_s/2 \) (commonly \( f_s = 100\text{MHz} \), though simulation requirements may vary). A simple approach for doing
this is to track the principal phase function of any given $A(\omega, z)$, which is generally displayed from $[-\pi, \pi]$ and increase or decrease the overall phase value by $\pm 2\pi$ when a discontinuous jump occurs (dependent on direction of change). This is more or less equivalent to tracking the winding of the function $Arg(H(\omega))$ around the unit circle on the complex plane, as each clockwise revolution about a zero will increase phase by $2\pi$ radians, while a revolution about a pole will decrease phase by $2\pi$ radians.

![Figure 4.12: Example of phase unwrapping. Overlay of the principal and unwrapped phase dictated by the sample function $\Theta(\omega) = \omega + 0.2\sin(\omega - 1)$. Wrap around points are indicated by $\pm \pi$ (dashed lines).](image-url)
This is implemented alongside the calculation of the attenuation vectors. Brief pseudocode for this technique is shown below:

```matlab
phase = atan2(Im(signal), Re(signal));
tolerance = pi;
unwrap = phase;
counter = 0;
for i = 1:length(phase)-1
    if abs(phase(i+1) - phase(i)) > tolerance
        counter = counter + sign(phase(i) - phase(i+1));
    end
    unwrap(i+1) = phase(i+1) + counter*2*pi;
end
```

Using the greatest phase deviation at $\Theta(2\pi f_s/2)$, we can then construct a linear correction factor to maintain minimum phase while preserving the overall behaviour of the transfer function. This results in the phase plot of the attenuation transfer function and resulting impulse response shown in Figure 4.13. Here we can see that the overall net phase of the functions does not change from $[-f_s/2, f_s/2]$ where the sampling rate is taken to be 100MHz, for both the attenuation transfer function or the resulting transmit response. Within these frequency bounds the phase is free to change and oscillate, but at its limits returns to its origin (non zero for the case of the transmit response), bounding the phase function and keeping the energy delay to a minimum.

Finally, after applying this correction to the attenuation transfer function, the transmit impulse responses behave as expected. Figure 4.14 below demonstrates the impulse responses for several different locations in the field at a variety of attenuation constants. In general, the responses are comparable between simulators, though one will note some increased discrepancy at longer ranges or higher attenuation values.

### 4.4 Simulation and Results

With the initial addition of the frequency dependent attenuation module into the FAMUS II framework, we are now able to complete the full transmit-receive functionality and apply it to simulations of interest, namely B-mode image scans, which benefit greatly in realism when dispersion is properly considered.
Figure 4.13: Phase plot for the attenuation transfer function, $A(\omega, z)$ (solid) and transmit impulse response, $H(\omega, z)A(\omega, z)$ (dashed), on axis at $z=4$cm for an unfocused emission and seven attenuation constants $\alpha_0 \in [0.1, 0.7]$dB/MHz·cm. The dotted line indicates zero net change in the phase of $A(\omega, z)$. Likewise, the phase of $H(\omega, z)A(\omega, z)$ has no net change across $[-f_s/2, f_s/2]$.

### 4.4.1 Simulation Environment

Tests were conducted on an Apple MacBook Pro (OS X 10.6) equipped with a 2.3 GHz Intel i5 dual-core processor and 8GB of RAM. For computation, we employ a well-established high speed C/C++ based Fast Fourier Transform library called FFTW [58], which is integrated into the Intel Math Kernel Libraries (MKL), a series of specialized libraries designed to take advantage of the hardware and instruction sets of recent Intel-designed processors.

**Phantoms**

To run the B-mode image scans and assess the capabilities of FAMUS II, three main phantoms were generated: one consisting of three on-axis point scatterers, a simple set of two on-axis hyperechoic cysts (3mm diameter) centred at 50 and 70mm, and a more
Figure 4.14: Comparison of transmit impulse responses for Field II (solid) and FAMUS II (dashed) at seven attenuation constants, given a 3.5MHz transducer focused at 4cm along the z-axis.

A comprehensive phantom containing point scatterers and hyper- and hypo-echoic cysts of varying diameters distributed widely in space, as used by Jensen for demonstration purposes in the development of Field II. All simulations were run under the following environment:
Table 4.1: General settings for phantoms and test setup. Phantom #1 is not listed due to its simplicity.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Phantom #2</th>
<th>Phantom #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phantom dimensions (L x W x H)</td>
<td>50mm x 40mm x 10mm</td>
<td>60mm x 40mm x 10mm</td>
</tr>
<tr>
<td>Number of scatterers</td>
<td>20000</td>
<td>100000</td>
</tr>
<tr>
<td>Speed of sound</td>
<td>1540 m/s</td>
<td>1540m/s</td>
</tr>
<tr>
<td>Attenuation</td>
<td>0.5dB/MHz · cm</td>
<td>0.5dB/MHz · cm</td>
</tr>
<tr>
<td>Excitation pulse</td>
<td>2 cycles</td>
<td>2 cycles</td>
</tr>
<tr>
<td>Centre frequency</td>
<td>3.5MHz</td>
<td>3.5MHz</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>100MHz</td>
<td>100MHz</td>
</tr>
<tr>
<td>Scan lines</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>

Transducer

For all tests, a rectangular transducer of overall width and height 3cm x 5mm, operating at 3.5MHz and laterally focused at 60mm, was used to isonate the selected phantoms. A two cycle pulse (Hanning windowed) was transmitted with no apodization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements (Lateral x Elevation)</td>
<td>65 x 9</td>
</tr>
<tr>
<td>Lateral Element Width</td>
<td>440µm</td>
</tr>
<tr>
<td>Elevation Height (total)</td>
<td>5mm</td>
</tr>
<tr>
<td>Lateral Width (total)</td>
<td>31.36mm</td>
</tr>
<tr>
<td>Kerf</td>
<td>50µm</td>
</tr>
<tr>
<td>Lateral Focus</td>
<td>60mm</td>
</tr>
<tr>
<td>Elevation Focus</td>
<td>0mm</td>
</tr>
</tbody>
</table>

Table 4.2: Transducer settings

4.4.2 Results and Analysis

One of the first tests examined is the behaviour of the frequency dependent attenuation module when zero attenuation is specified. In theory, it should produce nearly identical results to the preceding FAMUS II implementation, which lacks such capabilities. A straightforward test in practice is to compare results of simulating phantom # 2, at 0dB attenuation, with Field II and both the earlier version of FAMUS II and the newly updated version. The result of this B-mode image test is more or less the same between all simulators, even for a modestly complex phantom (20k scatterers). Having passed this basic sanity check of performance at no attenuation, we can now analyze the results when non-zero attenuation is applied. Figures 4.16 and 4.17 illustrate the results of some basic simulation tests to check the overall frequency dependent attenuation module of FAMUS II against that of Field II. To make the frequency dependent effects more visible, the B-mode images were not corrected with time gain compensation (TGC). It can be seen that the results of the implementation are structurally consistent with the established behaviour of Field II.
There are, however, some noticeable differences. In the simple point scatterer test (Fig. 4.16) it should be noted that the response near the scatterers at z=43mm and z=53mm change more abruptly at the distal edge of the spreading filament and have a harsher or sharper appearance, whereas the equivalent Field II image is smoother. It is also evident that the overall energy level of the background is higher in the FAMUS results. On the other hand, the results using a larger scale test (e.g. a phantom with
many thousands of scatterers) appear to display better congruency than the small point scatterer tests, as seen in Figure 4.17. The characteristic “blurring” of the cyst features and background speckle is similar in both the Field II and the FAMUS II generated B-mode images with increasing depth, as are the background characteristics.

The other visual issue that causes problems is the result of using varying frequency vector lengths between individual scan lines of the B-mode image. In operation, the algorithm shown in Fig 4.5 determines the maximal signal length in samples for a given scan line and allocates a vector of samples for the next greatest power of two, in order to allow the FFT algorithm to operate efficiently without loss of information. However, because of the lateral movement of the transducer with respect to the scatterers changes the relative distances, the maximal length may itself vary above the next power of two, causing the allocated number of samples to increase by another factor of two. This results in noticeably different signal energy between successive scan lines, as can be seen in Figure 4.17c. This may be avoided by specifying a consistent length or number of samples which all scan lines must match, but at the cost of forcing some scan lines to have more samples than necessary, increasing runtime.

Overall, the frequency dependent attenuation implementation in FAMUS II has performed well, despite the aforementioned differences in results. The updated implementation is capable of producing zero attenuation images consistent with previous simulation results, accurately calculating the behaviour for non-zero attenuation, and generating these images with very similar quality to that of Field II. The issues mentioned in the analyses above do require improvement, which will be addressed next.
Figure 4.17: Two hyperechoic cysts (Phantom #1: 3mm diameter, at z=50mm, 70mm) were simulated in (a) and (b) by Field II and FAMUS II at $\alpha_0 = 0.5\text{dB/MHz-cm}$. In (c) incongruous FFT vector lengths across scan lines cause problems for consistent energy levels. However, attenuation effects remain correct.
4.5 Hybrid Approach

The successful implementation of frequency dependent attenuation is a major step forward in adding essential functionality to the FAMUS II framework. However, there are some major performance issues to address, along with some accuracy and display issues, as noted at the end of the last section. To remedy these issues, an alternative processing method was developed and compared to the previous approach.

4.5.1 Runtime and Computation Analysis

The initial implementation described above takes place entirely in the frequency domain. While it is true that all operations can be done this way because theory defines $A(\omega, z)$ in the frequency domain as seen in (4.6), it is computationally expensive to approach the problem this way.

On the machine used in development, FAMUS II without frequency dependent attenuation can execute on the order of a minute to simulate a small B-mode image (20000 scatterers, 50 scan-lines). However, the addition of the frequency dependent code increased this significantly (See Table 4.3). While it is expected that incorporating another layer of physics, namely the additional vector multiplications of the attenuation vector $A_k(\omega, z)$ or an extra time convolution, should increase computation time by some degree, the five-fold increase in runtime is inconsistent with real-time functionality.

<table>
<thead>
<tr>
<th>Simulator</th>
<th>Attenuation (dB/MHz·cm)</th>
<th># Scatters</th>
<th>Runtime (s)</th>
<th>Precomputation (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field II</td>
<td>0.0</td>
<td>20000</td>
<td>1063</td>
<td>n/a</td>
</tr>
<tr>
<td>Field II</td>
<td>0.5</td>
<td>20000</td>
<td>1262</td>
<td>n/a</td>
</tr>
<tr>
<td><strong>FAMUS II (time domain)</strong></td>
<td>0.0</td>
<td>20000</td>
<td><strong>65</strong></td>
<td>n/a</td>
</tr>
<tr>
<td>FAMUS II</td>
<td>0.0</td>
<td>20000</td>
<td>200 (315*)</td>
<td>37 (38*)</td>
</tr>
<tr>
<td>FAMUS II</td>
<td>0.5</td>
<td>20000</td>
<td>202 (321*)</td>
<td>38 (36*)</td>
</tr>
</tbody>
</table>

Table 4.3: Test run times for basic cyst phantom (#2). Runtime of FAMUS II without any frequency dependent attenuation code is included for reference. (*Where a fixed length of samples is used across all separate scan lines to avoid the visual error in Figure 4.17c. Note that precomputation time is unaffected.)

Examining the computational requirements of a purely frequency domain approach, we can see that the cost of high resolution multiplications can be quite expensive. Firstly, we note that the handling of complex numbers in a digital system is fundamentally more burdensome. Given a complex number $z = a + bi$, the digital representation of $z$ requires two units of numerical storage, whether they be of single or double precision. Consequently, the operations required to calculate the result of equation 4.12 per individual
sample are

\[ H_k = a + bi, \]
\[ A_k = c + di, \]
and \[ P_k = e + fi. \]

Thus:

\[ H_{total} = H_k^2 A_k P_k. \]

Intermediate terms:

\[ (1) = a^2 - b^2 + 2abi = (3) + (4)i, \]
\[ (2) = ce - df + (cf + de)i = (5) + (6)i, \]
\[ (1)(2) = (a^2 - b^2)(ce - df) - 2ab(cf + de) \]
\[ + [2ab(ce - df) + (a^2 - b^2)(cf + de)]i, \quad \text{or} \]
\[ (1)(2) = (3)(5) - (4)(6) + [(4)(5) + (3)(6)]i. \]

As can be seen, even if some intermediate terms are calculated once and reused (terms 3 - 6), the result for each sample of the \( H_{total} \) vector requires four multiplications and three sums, along with the initial overhead.

Secondly, in order to carry out the manipulations described in the Implementation section, we find that all frequency domain vectors must be of the same length, otherwise it is impossible to apply sample-by-sample sums or multiplications. Given an approach that locates the longest possible response for a given scan line, this often results in many thousands of samples (e.g. for Figure 4.17b, the longest zero-padded signal requires \( 2^{14} \) or 16834 samples). This translates into additional computational costs of

\[
\text{multiplications: } 3 \sum_k \| \text{length}(h_k) - 2^\lceil \log_2(\text{total impulse length}) \rceil \| \\
\text{sums: } \sum_k \| \text{length}(h_k) - 2^\lceil \log_2(\text{total impulse length}) \rceil \|
\]

for \( k \) scatterers.

Since the length of an individual one-way impulse \( h_k \) may be on the order of hundreds of samples, while the total impulse response is on the order of thousands of samples, this results in an order of magnitude more calculations to handle the frequency domain vector – clearly an impediment to real-time performance. Therefore, it is important that we assess the advantages of time domain manipulations to see if any gains can be determined, given that the preceding implementation of FAMUS II executes far more rapidly.
4.5.2 Time-Frequency Domain Approach

The original time domain only approach constructs the total impulse response from individually time delayed impulse responses. This is like creating a patchwork of overlapping vectors, or using piecewise contributions. In contrast, the frequency domain approach of constructing the total impulse response forces us to consider the entire signal at all times. However, since the frequency domain operations are necessary for only the description of the attenuation $A_k(\omega, z)$, an alternative procedure is to remain in the frequency domain only when applying the frequency dependent attenuation vector, before returning to the time domain. The main advantage of this approach is twofold:

1. The length of FFT vector must only accommodate the size of an individual product $H_k^2A_k$, and
2. Phase shifts can be handled as time delays.

Hence by reordering the process of simulation to follow Figure 4.18 below, we greatly simplify the computational burden.

However, one additional precomputation and frequency domain consideration is required. By individually considering the transmit-receive-attenuated impulse response for a given scatterer $k$, we must remember to provide sufficient time support for the final result. Recall that in order to perform a discrete linear convolution of two signals of lengths $M$ and $N$ without aliasing, the final signal must be of length $L \geq M + N - 1$. Similarly, in the frequency domain, the multiplication of the corresponding Fourier pairs must be at least $L$ samples long. Therefore in practice, we must set aside FFT vectors that are $L = \text{length}(h_k^T) + \text{length}(h_k^R) + \text{length}(a_k) - 2$ samples long, where $a_k(t) \leftrightarrow A_k(\omega)$.

In the previous implementation, the effects of dispersion on temporal duration were implicitly included in the FFT vectors that encompassed the entire impulse response for the scan line. That is, for echoes that occur between the earliest and latest receive times, there is sufficient sample space to broaden in time and accommodate the elongation of wave pulses. However, this does truncate those echoes occurring near the latest receive times, where the effects of attenuation would cause the time duration to extend past the maximum pre-dispersion length, causing loss of information. Hence, we also expect the signals produced by the proposed hybrid method to be more accurate.

Practically speaking, since the frequency dependent attenuation is originally calculated in the frequency domain for a limited extent (up to $f_s/2$), the time domain signal is technically infinite in duration. However, this is impractical, and not a good representation of the equivalent signal (as equation 4.5 indicates, it is the frequency domain representation which is infinite, which implies a finite time duration). Thus, to determine the approximate time duration of $a_k(t)$, we locate the point at which the signal drops to
Figure 4.18: A modified version of Figure 4.5 to lower the computational burden of operating solely in the frequency domain.

below 1% of its peak amplitude, as shown in Figure 4.18. This duration can be easily computed before simulation, and retrieved as necessary.
4.5.3 Final Results

With these additional optimizations in hand, simulation runtime is improved, and is only approximately a factor of two larger than the non-frequency dependent attenuation capable simulator. In Table 4.4 below, we compare the calculation times of Field II, FAMUS II using the frequency domain only approach, and FAMUS II using the hybrid time-frequency domain approach to perform frequency dependent attenuation in simulation.
<table>
<thead>
<tr>
<th>Simulator</th>
<th>Attenuation (dB/MHz-cm)</th>
<th># Scatterers</th>
<th>Runtime (s)</th>
<th>Precomputation (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Phantom # 2</td>
<td></td>
</tr>
<tr>
<td>FAMUS II (frequency domain)</td>
<td>0.5</td>
<td>20000</td>
<td>202 (321*)</td>
<td>38 (36*)</td>
</tr>
<tr>
<td>FAMUS II (hybrid domain)</td>
<td>0.5</td>
<td>20000</td>
<td>118</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Phantom # 3</td>
<td></td>
</tr>
<tr>
<td>Field II</td>
<td>0.0</td>
<td>100000</td>
<td>3651</td>
<td>n/a</td>
</tr>
<tr>
<td>Field II</td>
<td>0.5</td>
<td>100000</td>
<td>4210</td>
<td>n/a</td>
</tr>
<tr>
<td>FAMUS II (hybrid domain)</td>
<td>0.0</td>
<td>100000</td>
<td>510</td>
<td>13</td>
</tr>
<tr>
<td>FAMUS II (hybrid domain)</td>
<td>0.5</td>
<td>100000</td>
<td>524</td>
<td>13</td>
</tr>
</tbody>
</table>

Table 4.4: Runtimes for basic cyst phantom (#2) compared between the frequency domain only and hybrid domain approaches. Lower resource requirements of the latter improve execution and precomputation time. Also displayed are Field II and FAMUS II hybrid approach runtimes for the comprehensive phantom (#3). (*Where a fixed length of samples is used across all separate scan lines to avoid the visual error in Figure 4.17c. In practice, a ~ 3x speed up is achieved.)

We also include the results of several simulated phantoms in the figures below. The scan images in Figure 4.20 simulate the same three scatterer phantom as in Figure 4.16, but using the hybrid time-frequency domain approach to determine the result at an attenuation of 0.5dB/MHz-cm. The image in a) utilizes the minimum power of two equal to or greater than $L = \text{length}(h_T^k) + \text{length}(h_R^k) + \text{length}(a_k) - 2$, as specified earlier. Meanwhile in b), we have purposely set longer FFT vectors to provide further time domain smoothing or interpolation. The resulting B-mode scans are generally more accurate, particularly the PSF of the furthest scatterer at z=53mm, where we can see the angle or spread of the energy is closer to that of Field II. The new results also have similar energy distribution to the Field II scan, in contrast to 4.16b which has considerable background energy. Though there still exist some rough artifacts in the minimal FFT vector length case, they are less prominent than before and tend to occur further from the high power regions of the PSFs. In the case where these artifacts cause problems, it is possible to extend the FFT vector lengths, as in b). Furthermore, in significantly more populated simulations (thousands of scatterers), the effect of these artifacts is generally obscured through averaging. The one main difference between the results of the hybrid approach and Field II is the absence of shadowing after the last point scatterer at z=53mm, potentially a result of insufficient estimation of length($a_k$).

Continuing the comparison of scan images, the simulation results of phantom #2 using the hybrid approach are shown as well, in Figure 4.21. Similar to the first images shown using the frequency domain only approach to frequency dependent attenuation, the hybrid approach can produce satisfactorily comparable behaviour.

Finally, we have also illustrated the effects of simulation on the larger phantoms that are used by Jensen to examine the effects of frequency dependent attenuation. In Figure 4.22, comparisons of the complex phantom at zero attenuation and some frequency depen-
Figure 4.20: Simulations of the same phantom as in Fig 4.16, using the hybrid approach. The Field II result is given in (c) for comparison. Overall, the new images are closer in intensity, and have fewer rough artifacts.

dent attenuation are shown. The simulators produce comparable images: the degradation of the cysts and speckle with distance is similar in both, along with the behaviour of the focal region (z=60mm) and the overall energy levels throughout the B-mode scan. Hence we can see that both Field II and FAMUS II perform similarly, with the added benefit that FAMUS II has a runtime close to an order of magnitude smaller.

The optimizations described above overall result in faster simulation, as fewer calculations are required to determine the acoustic field. Secondary benefits include a lower memory overhead, since the frequency domain FFT vectors are shorter and there is no
Figure 4.21: Test of Field II and FAMUS II using the hybrid approach for phantom #2, using 20k scatterers and attenuation of 0.5dB/MHz·cm. The hybrid approach also provides similar behaviour to Field II.

need to compute phase shift vectors, and mitigation of the length and scaling issue as shown in Figure 4.17c, since the final “assembly” of the total impulse takes place in the time domain, after proper attenuation and scaling, avoiding any signal energy mismatches between scan lines.
Figure 4.22: Simulation runs of Field II and FAMUS II for phantom #3, using 100k scatterers (3.33 scat/mm$^3$) and attenuation of 0 and 0.5dB/MHz·cm.
4.6 Summary

Previous work by Aguilar et al. on efficient and precise US simulation methods has given rise to two new simulation frameworks: FAMUS I [39] and FAMUS II [40]. Following the success of the impulse response methodology used by FAMUS II and its ability to recreate Doppler and B-mode ultrasound images given frequency independent attenuation values, it was determined that realism necessitated incorporating frequency dependent attenuation to account for its effects in tissue, most notable in wideband applications (eg. B-mode scans).

Based on models of tissue behaviour, two methods were developed to more accurately model the impulse response, through the frequency domain or in a hybrid time-frequency domain approach. Both methods take advantage of precomputed matrices of attenuation values, parameters of which are easily set at the beginning of simulation. The key to efficient application of frequency dependent attenuation lies in the analogue of frequency domain multiplication to time convolutions, allowing the system’s frequency response to be appropriately modified. As illustrated by comparison to Field II and theoretical calculations, frequency dependent attenuation is correctly applied to FAMUS II.

Inevitably, some approximations were employed: using the mean distance to the transducer to calculate $A(\omega, r)$, implicitly assuming $\alpha(f) = \alpha_0 f^n |_{n=1}$, and constant phase speed $c(\omega) = c_0$. These are reasonable because signal variations across the transducer vanish with distance, because attenuation in tissue is roughly linear with frequency in diagnostic ranges ($1-10$MHz) [49], and finally because in small diagnostic signals, effects due to attenuation dominate those due to changes in phase speed.

While capable of accurately mimicking frequency dependent attenuation effects, the frequency domain-only approach was inefficient when applied to long impulse signals, which are common for real phantoms. In response, the hybrid domain method was developed to avoid unnecessary multiplications and sums and lower the overall computational burden of constructing the system response. The resulting method maintains its frequency dependent effects, is somewhat more accurate than the frequency domain only variant, and is only a factor of two slower than the current FAMUS II implementation, making it a viable approach for real-time performance with additional functionality.

At this point we have successfully included the effects of frequency dependent attenuation in tissue. We have demonstrated that realism in ultrasound simulation can be built up through a modular approach of layering successive transfer functions to compose the entire system response, with each stage representing specific physical phenomenon. Future work can now proceed in developing other key aspects of ultrasound imaging, such as the ability to handle multiple focus regions in reception through the use of beamforming.
Chapter 5

Summary and Future Work

5.1 Summary

The contributions of this thesis have proceeded along two main tracks: refinements to existing simulation processes and development of improved simulation procedures, and the implementation of more precise physical models toward furthering realistic simulation.

The experiments in Chapter 3 relating to simulation processes were able to provide insight into the nature and source of computational artifacts in simulated Doppler spectra, namely the effect of phase mismatches in the summation of acoustic signals due to sampling resolution. It should be noted that such artifacts appear at certain multiples of the main / average flow velocity, and are especially visible for low velocities as the corresponding frequencies and harmonics appear well below the Nyquist limit of the given pulse repetition frequency. Attempts to reduce noise due to granularity in the simulation (e.g. transducer point sources, sampling intervals, or point scatterer representation of fluid and media) without increasing sampling frequency were applied to FAMUS I and II via linear interpolation of the signal components. This technique was effective in FAMUS I due to direct manipulation of the acoustic radio-frequency signal, but only provided minor improvement to the spatial impulse generation procedures in FAMUS II, owing to the sensitive nature of convolutions. While some sources of discretization are innate to the simulation methodology, better awareness and understanding of computational noise helps to design strategies to overcome or account for their effects.

The second half of the chapter focused on exploring simulation performance under computational volumes of limited spatial extent. Experiments revealed that fidelity is increasingly maintained for those volumes that conform to the shape and scale of the transmit–receive acoustic field, particularly within the sample volume. This heuristic provides an efficient bound on the number and extent of point scatterers to be considered for both accuracy and speed. These results then led to a complementary study of the
point scatterer densities capable of supporting consistent Doppler simulation. Through a sequence of tests involving pulsatile Womersley flow, it was found that a quantity of greater than 6 point scatterers per cubic millimetre would be sufficiently dense to allow a convergence in the quality of simulated Doppler spectra, which was also correlated to the wavelength of the transducer system. However, it is important to understand that this numerical value is not an absolute figure of merit but rather a representation of a qualitative and mechanistic benchmark obtained for the straight tube Womersley flow used in this thesis; depending on transducer and flow profile properties, different density values may be necessary, e.g. a tortuous and turbulent flow may require much finer resolution. Regardless, this was instrumental in providing an understanding of the minimal amount of computational work required, as consistency is a necessary precursor to realism.

The combination of these two studies provides crucial benchmarks for future simulation scenarios, demonstrating a qualitative understanding of what and where to simulate, combined with a quantitative metric on how many acoustic interactions to calculate, thereby framing bounds on suggested input data resolution.

The second major direction of this work took place in Chapter 4, centred around the implementation of frequency dependent attenuation arising from dispersion. Adding to the previously existing FAMUS II architecture, a frequency domain based approach to calculating the effect of dispersion on transmit–receive signals was developed and integrated into FAMUS II for the simulation of B-mode images. Results obtained from the attenuated transmit impulse response were comparable to analytic calculations and results produced by the Field II ultrasound simulation package. Full B-mode simulations also displayed expected attenuation behaviour, and were virtually indistinguishable to those of Field II, with the advantage of considerably lower execution times.

Further refinements were made to the initial frequency domain based method to address residual computational artifacts, signal energy consistency across the imaging domain, and execution time. The resulting technique operates in a hybrid of the time and frequency domains, with an additional two-fold reduction in run time. Both methods made use of strategic precomputation of commonly reused quantities in order to provide high precision calculations combined with a small increase in computation time, though it involves some degree of memory storage overhead dependent on the spatial domain. Overall, the result is an effective module for calculating frequency dependent attenuation, expanding the functionality of FAMUS II.
5.2 Future Work

Overall, progress has been made on the utility and reliability of FAMUS as an accurate, high speed ultrasound simulator, bringing it closer to a suitable platform for real time medical training and research. However, there are still many areas of realism and functionality in ultrasound simulation which can be explored or improved, several of which are discussed below.

5.2.1 Validation

Of primary concern is validation of simulated images and spectra with outputs as produced by commercial ultrasound equipment. While this thesis has been fruitful in verifying that its mechanistic computation of acoustic phenomena is reliable, significant work remains in checking that reality is well-modelled. Ultimately, the user base for this technology will be ultrasound technicians and radiologists who depend on “correct” images to see into the human body and make diagnoses. There are at least two main objectives involved here:

- Comparison between simulated and actual ultrasound images and spectra. This itself requires validated rendering of tissue and blood, i.e., the digital phantom must be comparable to real biological media, including movement and flow conditions.
- Confirmation from focus groups consisting of clinicians: we must take into account the experience and intuition of expert users to evaluate the readiness of this tool.

Therefore, in order for FAMUS to be useful as a medical training simulator, we must also ensure that according to the experience of already trained clinicians and by comparison with physically generated images, what this simulation framework produces is commensurate with reality.

5.2.2 Integration with Computational Fluid Dynamics Models

In addition to the necessity of validation, one of the next major steps in preparing FAMUS for use as a training or investigative tool requires evaluating and improving its performance when processing flow data derived from more realistic data sources, such as complex blood vessels modelled using CFD methods. The research reported by Oung and Forsberg and Khoshniat et al. [28, 37] used an artificial stenosis and the carotid artery, respectively, as the sites of simulation and analysis, so it is natural to consider similar regions for the next stage of development.

It will be challenging to incorporate non-standard geometry and substantially larger data sets while maintaining performance and accuracy. In particular, the suggestions
made in Chapter 3 will be important to guide selection of scatterer volumes for computation, where the explorations in Sections 3.3 and 3.4 helped develop an idea of appropriate scatterer densities and spatial extent that would be beneficial for simulation.

For simple flow scenarios, it is relatively easy to decide upon appropriately sized and shaped computational volumes that capture both flow geometry and sufficient numbers of scatterers, e.g. a cylindrical slab is characteristic of a straight tube vessel, and can be made thin to lower computation costs. However, for situations like the carotid artery, it is a complex task to subdivide the domain into small enough yet suitably comprehensive computational volumes, as the situation does not conform to any regular geometry.

It is important to develop new approaches for the organization of scatterer data in terms of efficient memory storage and rapid algorithmic access for simulation while at the same time being representative of the underlying physical environment, e.g. dividing arterial branches into small regions with enough extent to encompass flow behaviour and transducer sample volumes. This could be possibly facilitated through determining flow trajectories from CFD velocity data, which would provide a description of how fluid moves and clusters, and in turn reasonable areas of division in the spatial data. This is important because one of the most appealing advantages of FAMUS is the ability to provide a robust and rapid Doppler ultrasound simulation platform, which makes it necessary to be able to easily adapt to arbitrary geometry and flow conditions.

5.2.3 Multi-region and Dynamic Focusing

The second area of interest surrounds the approaches used to improve image quality in B-mode imaging and its cousins. The use of a single focal point in phased array imaging provides a straightforward method for clearly depicting the physical structures inside the body, but only for a small zone about this focal spot. This makes it difficult to assess a broad region for anomalies or determine disease pathology. Improved clarity can be obtained through multiple scans, but it is simpler to assign multiple focal regions in reception, which allows for the radiologist a better comparison of physiology at different depths and ultimately better diagnostic performance. Furthermore, most ultrasound systems are capable of near-continuously sweeping the focal zone with the movement of the acoustic echoes, producing what is known as dynamic receive focusing.

Evidently, providing FAMUS with this functionality would enable it to be a more effective training tool, and expand its scope of use. More information is available in Appendix A, which details some preliminary work on understanding the processes involved in the development of dynamic focusing for FAMUS.
5.2.4 Additional Ultrasound Imaging Modalities

Finally, it is prudent to continue developing other modes of ultrasound imaging, in order to expand the range of functionality that FAMUS can provide and to reflect the availability of real technologies. Several examples include:

1. Colour Flow Imaging (CFI),
2. Power Doppler,
3. Synthetic Aperture Ultrasound, and
4. Plane Wave Imaging

The first two modalities are variations of Doppler ultrasound, and are commonly present in clinical settings, making them good candidates for simulation. Meanwhile, the latter two options reflect more recently researched forms of imaging that carry potential for rapid or high resolution imaging, which could open FAMUS to additional research opportunities. Development and implementation of these modalities requires a sufficiently flexible and modular construction of the FAMUS architecture, which may require modification to suit the wide variety of modes and their physical requirements.

For instance, in Colour Flow and Power Doppler imaging, additional manipulation of a background reference is required to centre corresponding flow information on the physical structures within the human body. This type of functionality would then involve some efficient scheme for interleaving the acquisition of B-mode and Doppler information. For Synthetic Aperture or Plane Wave imaging approaches, it is necessary to have individual control over the acoustic signals transmitted and received at individual transducer elements, both during acquisition and for post processing. In computation this would require organization of data on the level of single elements, not unlike the programmatic suggestions made in Appendix A.4.
Appendix A

Multiple Region and Dynamic Focusing

As mentioned in previous chapters, B-mode scanning constructs images through the reception of sequential acoustic scan lines. To obtain sufficient contrast and clarity in each line, the elements of the transducer are usually focused at some position into the medium. The particular selection of focal point, active elements and their apodization weights defines the shape of resulting acoustic beam and sample volume, which has major influence on which depths of the medium are seen most clearly. In general, the acoustic beam in transmission and reception is not required to be the same, and the overall combination of both beams can be refined to produce the best image. The purpose of applying multiple focal zones to an acoustic beam is to improve the contrast or clarity of the returning backscattered reflection, because this increases the number of points in space where acoustic waves are aligned constructively in phase. A brief discussion of relevant concepts and implementation suggestions is provided here.

A.1 Focusing Fundamentals

Multiple focus zones can be applied in both transmission and reception, though in practice it is most common to only use multiple receive focal zones; multiple transmit focal zones require more than one transmission because it is not possible to adjust the incident waves for each focal depth once they have been emitted from the transducer surface.

On the other hand, as transmitted signals pass through a medium, a wake of backscattered energy (the reflected waves) is emitted from the disturbed tissue and other intermediary objects and returns to the receiving transducer. In order of increasing distance, it is possible to tune the delays of the receiving transducer to align the signal information arriving at each individual transducer element. Doing so based on a set of pre-determined
regions into the medium gives rise to multiple receive focus zones. Taking this idea further, *dynamic* receive focusing is achieved through constantly tracking the returning acoustic reflection, in order to maximize the use of multiple focal zones. This procedure is not perfectly continuous, and is usually implemented in ultrasound machines by increasing the focus by 1cm per $13\mu s$. This is duration needed for sound to travel 2cm at $c_0 = 1540 m/s$, or a round trip of 1cm [59]. Therefore, one can consider dynamic focusing as using very many focal zones in reception. Hypothetically, it is possible to change the delays at an even faster rate/smaller distance, but the benefits might be negligible for the effort used. By finely discretizing into 1cm long zones, relatively good coverage is provided. Overall, this is particularly advantageous for distinguishing details over a broad imaging range and depth, which is crucial for good imaging.

![Figure A.1: Disturbances caused by the emitted wave return to the transducer at increasingly longer times, $t_1...t_3$. Returning signals are aligned in increasing order based on the round trip travel time/distance into the medium.](image)

In addition to real ultrasound machines, multi-region or dynamic focusing is incorporated into many ultrasound simulators, including Field II. Past work by Nikolov [60] and Kortbek [61] produced and refined what is known as the Beamforming Toolbox (BFT) for Field II, a separate platform for augmenting the pre-existing capability for multiple zone and dynamic focusing, and extending support for other image generation approaches, such as synthetic aperture ultrasound.

An important distinction to be made between the general description of dynamic focusing in ultrasound systems and that of the more generic process of beamforming as discussed by Nikolov and Kortbek, is the notion of online and offline processing. In an online implementation, adjustments are made to the way data is processed (in this case, how time samples are delayed and summed) at the time of acquisition as acoustic echoes return to the transducer. This is the most straightforward way to incorporate this feature into B-mode ultrasound systems and often results in a lower memory intensive hardware. However, offline processing requires storing the acoustic reflection data from every channel (nominally the receive elements of the transducer) separately, for more complex delay-and-sum calculations or multiple stage image generation techniques. The latter is more general and capable of producing synthetic aperture style images as well as dynamically focused images, but is done so after data acquisition and thus requires more
memory storage. It would also be helpful to provide offline processing to FAMUS, as it greatly expands the type of processing and uses available to the simulator. However, it is simplest to first add dynamic focusing in FAMUS as an online processing stage, mimicking the behaviour of common ultrasound machines.

### A.2 Implementation Constructs

In order to support multiple focal zones, it is necessary to handle multiple sets of delay times for the transducer elements, which are associated with progressively deeper focal distances. Most ultrasound machines employ a system of active elements and apodization weights to size the effective aperture. The ratio of the effective aperture size (accounting for viewing angle and active elements) to the focal depth is called the F-number. This produces a narrow acoustic beam throughout the medium, and is necessary for maintaining fine resolution and constant relative isonation so that more distant regions do not appear darker than the near field, which would obscure comparison of tissue properties.

Essentially, for very shallow depths, only a few transducer elements will be active in contributing to the transmitted or received signal in order to avoid very high power levels. Likewise, for distant points in tissue, many transducer elements must be active in order to maintain sufficiently high levels, due to the $1/R$ decay in signal amplitude. The number of active elements increases with depth until the entire array is in use, at which point the effective beam will diverge, resulting in a loss of resolution.

![Figure A.2](image)

**Figure A.2:** For every focal point, a different set of delay times is required for all active elements. As the focal point recedes into the far field, delays are nearly uniform. Also, in order to maintain a constant beamwidth, the number of active elements must change with focal length, $F = D/A$. When all available elements are active, the acoustic beam broadens with increasing distance and loses resolution.
To obtain or design the active element and delay sequences, it is helpful to think of the concept of a focus timeline, radiating out from the centre of the transducer, which demarcates positions according to travel time for the transition of discrete focal zones, as well as the central point of focus within a given zone. By associating a depth for each of these enumerated regions, it is then possible to calculate the necessary time delays for each transducer element and the number of active elements required to maintain constant illumination or F-number. Figure A.3 illustrates the division of a domain into several focal zones, which are indicated by travel times, e.g. \([0, t_1), [t_1, t_2)... [t_4, \infty)\). For simplicity, the radial distance to any focus point is set to be in the middle of a given zone (denoted by the solid coloured circle), except for the first and final zones. Despite the structure of concentric rings defining focal zones, it is important to remember that each zone has a specific focal point \(focus_1, focus_2... focus_4\) in \(\mathbb{R}^3\), each of which is distributed along the focus timeline emanating from the reference. This timeline of focus points can be steered in the domain according to angles \((\theta, \phi)\) depending on the particular application.
A.3 Field II Functionality

Before developing a multiple zone or dynamic focus implementation for FAMUS, it is important to assess how Field II performs in this regard, since it has been used as a benchmark in previous feature development. As mentioned previously, Field II comes preprogrammed with two functions for multi-zone and dynamic focus. Using the same ultrasound cyst phantom as in Chapter 4, a simple comparison of four focal zones versus dynamic receive focusing was tested.

![Figure A.4: Comparison of dynamic receive focus and a four-zone multiple zone receive focus scheme in Field II. Visual differences are slight; dynamic focus is able to produce a more consistent background energy level throughout the image, providing better contrast to the targets.](image)

Examining Figure A.4, it is apparent that the use of four focal zones in reception has very similar results to when using dynamic focus, with few visible differences. The most obvious difference is a more consistent dark background in the dynamically focused image, which improves the contrast of the targets. In practice, this is important for distinguishing weaker or smaller targets, such as tissue abnormalities early in growth.

In terms of simulation, the use of multiple region focus through six non-overlapping focal zones of 10mm depth should be identical to using dynamic focus, assuming that Field II advances its focal depth at a rate of 10mm/13μs. This helps set a target for what type of functional results FAMUS should be able to produce.
A.4 FAMUS Architecture

Finally, some consideration for necessary computer program organization and data structures follows. While the addition of frequency dependent attenuation may be imagined as an additional transfer function stage in the overall impulse generation process (see Chapter 4) that is realized through multiplication with an additional data vector, implementing functionality for multiple focus zones is a more involved proposition.

It may help to separate the transducer entity into physical and operational aspects, since the number and position of transducer monopoles refer to the physical properties of the transducer and need only be specified once (at the start of execution), while there can easily be many focal zones desired by the user and changed during operation of the simulator. Figure A.5 shows the existing transducer data structure with delay times and apodization weights as independent attributes indexed by focal zone. However, this does not lend much support for controlling individual transducer elements which are composed of several monopoles, meaning that it is difficult to specify active elements and therefore the F-number in use. Because multiple focal zones will be handled, a more natural alternative may be to consider a focal zone as the logical unit of organization, with quantities dependent on a specific focal zone included as attributes within the unit. Critical pieces of information to be associated with each focal zone are delay times (across the range of active elements), the list of active elements, and their apodization weightings, if applicable. This is also shown in Figure A.5 as a proposed structure. In this way, focal zones and their corresponding attributes can be created and accessed as required in simulation.

Additionally, Figure A.5 illustrates a method for associating source monopoles to transducer elements, the Grid ID, given that FAMUS subdivides transducer elements into collections of point sources. This is important for providing sufficient control over the contributing monopoles to settings and operations that affect whole elements. For instance, in order to support variable aperture sizing through F-numbers, one requires the ability to address subgroups of point sources that lie within the boundaries of a given transducer element, and subsequently enable or disable them based on the state of the transducer element. This coarser grouping is also convenient for handling transducer apodization, as it makes it simpler to control the relative amplitude of several monopoles at once.
Figure A.5: Possible refinement of existing data structures in FAMUS to organize focal zones and their necessary properties: delay times, apodization weights, and active elements. Also shown is a method for mapping monopoles to transducer elements.
References


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