Optimization of a Wastewater Treatment Plant Expansion with Flexible Expansion Time

by

Biyun Zhang

A thesis submitted in conformity with the requirements for the degree of Master of Applied Science

Department of Chemical Engineering and Applied Chemistry
University of Toronto

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Abstract

The municipal water and wastewater sector is considered to be one of the most capital intensive industrial sectors. Optimization methods that reduce both capital and operating costs can be of great benefit to this sector. Models using real options approaches to determine the optimal size of a wastewater plant expansion can provide insights that traditional Net Present Value approaches cannot. Previous studies on optimal plant expansion utilizing the real options framework only considered a predetermined expansion time. In this study, we expand on previous work and also utilize the Monte-Carlo simulation technique to consider a wastewater plant expansion in a high growth municipality. In the enhanced model, the date of the decision for the expansion is not a predetermined fixed variable, leading to Bermudan-like Asian option formulation. The results show that a wastewater plant which faces uncertain future demand can benefit from time-flexible modular expansion.
Acknowledgement

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1 Introduction

Public infrastructure is an important aspect of modern society. Infrastructure projects demand expensive initial investments and have long-term cash flow implications over the life of the project. It is very common to find North American infrastructure projects that cost hundreds of millions of dollars (Government of Alberta, 2014) (United States Department of Agriculture 2014). In Canada, the total funding for public infrastructure has steadily increased over the past two decades – in 2013, the amount invested in public infrastructure reached nearly $30 billion (Infrastructure Canada 2014). However, according to the Federation of Canadian Municipalities, despite a committed increase to a record-high $70 billion in Canadian infrastructure spending for the next 10 years, it is estimated that there is still a gap of at least $200 billion required for repairing existing infrastructure and funding new ones (Morgan 2014). As one of the most capital-intensive industrial sectors in North America, the municipal water and wastewater sector shares the difficulties commonly faced by traditional public infrastructure projects as its need for capital expenditures are substantial. According to the Federation of Canadian Municipalities, in 2007, a total of $31 billion was needed for refurbishment of existing systems and $57 billion was needed for replacing existing systems and constructing new ones. The Conference Board of Canada reported that the average capital expenditures in municipal water and wastewater have reached $1.5 billion annually from 1998-2006, with the capital expenditures in 2006 being close to $2.4 billion for the whole country (PPP CANADA 2013). Although some advanced water treatment technologies have been developed and put into use in municipal systems, capital and research expenditures are being held back in the water sector due to restricted margins and regulated pricing leading to sub-optimal returns (Global Cleantech Center, 2013). Hence, better asset investment strategies in the wastewater sector can significantly improve capital expenditure
efficiency and allow more capital to be invested in the research and development of better treatment technologies. In addition to its capital requirements, infrastructure projects can take years to build and decades to run – the current average age of core infrastructure projects in Canada is more than thirteen years (Infrastructure Canada 2014). Cash flows related to such costly, long life-span, infrastructure projects often possess features such as deferments and staged investments. For such projects, however, common valuation methods associated with the decision making process may not properly account for the optionality component associated with future decision making, i.e., the optionality to build only when it is needed, which may potentially lead to sub-optimal capital investment strategies (Trigeorgis 1996). Therefore, finding an effective method of optimizing capital investment for infrastructure projects with embedded options may greatly reduce the capital requirements.

1.1 Objective

The objective of this thesis is to determine the optimal sizing of a wastewater treatment plant for a municipality which also has the option to expand the plant over its project life. We develop two methods, the numerical method and the simulation method, to approach the problem and demonstrate that the two methods generate consistent results. We draw engineering economic analysis and business conclusions from these results.

1.2 Recommendations and Key Contributions

We expanded the work of Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) to develop a numerical approach to determine the optimal sizing of a dual expansion for a wastewater treatment plant. The results suggest that staged and deferred investment strategy would create savings by as much as 31% compared to traditional investment strategies. We also
implemented the Monte Carlo simulation to develop a simulation method that is much more easily understood by practitioners. The results of both models match consistently, hence validating the simulation approach.

1.3 Thesis Outline
The outline of the thesis is as follows: A literature review on the topic is presented in Chapter 2. Chapter 3 will discuss the components, setup and processes of the two methods used to model the wastewater treatment plant optimization problem. With this framework established, Chapter 4 will present results specific to a set of parameters using both methods, the results are compared and discussed. Finally, we summarize, conclude, and present avenues for future work in Chapter 5.
2 Literature Review

In order to develop frameworks that can be applied to value a project with embedded options, this chapter will provide a review of topics relevant to project valuation for Infrastructure projects, especially in the wastewater treatment sector. First, we discuss the commonly used discounted cash flow (DCF)/net present value (NPV) valuation method and its shortcomings in Section 2.1. Then we introduce the real options (RO) method and its advantage in Section 2.2. In Sections 2.3 and 2.4, we will describe previous studies that utilized the RO framework in the infrastructure sector and wastewater treatment sector. General methods using the RO framework are discussed in Section 2.5. We introduce the Monte-Carlo approach and its previous applications in the industry in Section 2.6 before summarizing this chapter in Section 2.7.

2.1 Standard Valuation Methods for Infrastructure Projects

In industry, managers are often faced with the task of deciding on the capacity of a given facility before its construction. Traditionally, when decision makers come to making investment decisions, the DCF/NPV method is commonly used to value potential investments. The project’s net present value is obtained by discounting its future cash flows at some required rate of return that is assumed by the decision makers. If the net present value of all cash flows related to the project is positive, then the project is deemed profitable and the decision makers should proceed with the project (Fraser, et al. 2009). The DCF/NPV valuation model is widely used for the valuation of capital investments for public infrastructure projects, including wastewater treatment plant projects. Static models developed by Gillot, et al. (Gillot, et al. 1999) used a DCF analysis to optimize for the wastewater treatment plant size. Ferrer et al. (Ferrer, et al. 2008) presented an Excel-based tool for simulating and optimizing WWTPs using the NPV method.
Iqbal and Guria (Iqbal and Guria 2009) developed a multi-objective optimization model using a genetic algorithm technique for the optimal design of wastewater treatment plant process from a cost perspective, also using the NPV method. However, as stated by Trigeorgis (Trigeorgis, 1996), the NPV method has major drawbacks due to the limited number of possible scenarios that are usually considered. Important factors, such as management’s ability to make decisions in the future given potential new insights / information are not easily accounted for in the NPV model. This ability to react to future information can significantly reduce the risk of a project. Furthermore, the NPV method does not allow for a rigorous way to account for discount factors, especially in the case of embedded optionality. Hence, we aim to develop models that are able to value projects with embedded options.

2.2 The Real Options Approach

The real options approach is an extension of financial options theory on real assets. It can be applied to value a real world project with embedded options using established financial theories (Trigeorgis 1996). For example, the opportunity to invest can be viewed as a call option, where the firm has the “flexibility” to acquire the underlying asset at some “uncertain” future point in time. In such cases, due to the flexibility of the firm and the uncertainty of the environment, a “wait-and see” strategy may have great benefits to the firm. The waiting period becomes a “learning” process to the firm as it obtains more information about its option, hence its risk associated with uncertainty decreases (Smit and Trigeorgis 2003). Many studies have been carried out that show that methods that use real options are superior to standard methods relying on NPV analysis (Trigeorgis, 1996). Esty (Esty, 1999) discussed the advantages of the RO approach compared to the NPV approach when valuing large-scale complex projects that have changing discount factors. Ford, Lander and Voyer (Ford, Lander, & Voyer, 2002) used a toll
road project example to develop a RO framework for valuing strategic flexibility in construction projects where traditional valuation methods cannot capture the value associated with the flexibility. Zhao and Tseng (Zhao & Tseng, 2003) compared the optimization results of a public parking garage project obtained using both the NPV method and the RO method. In their work, they found that the value of flexibility in the project is so significant that failure to capture the flexibility can lead to incorrect economic decisions. Garvin and Cheah (Garvin & Cheah, 2004) studied the Dulles Greenway project in the United States using both the NPV and the RO method. Their results also suggest that ignoring the value of embedded option could significantly underestimate the potential value of the whole project. Therefore, in this thesis we will apply the RO approach to evaluate the wastewater treatment plant design project.

2.3 General Methods for Solving Real Options Problems

In the RO framework, the optionality involved in the real asset acts as the underlying option. The value of such option usually follows a stochastic process that can be transformed into a partial differential equation (PDE) hence can be solved either analytically or numerically. The analytical approach is typically only successful when used to value simple, straightforward European or American options where the PDEs take on special forms (Hull, 2010). For example, the well-known Black-Scholes equation is developed to analytically value a simple European option. Geske (Geske, 1979), Johnson (Geske & Johnson, 1984) and Whaley (Whaley, 1981) used analytical valuation to value American options with known and discrete dividends. Kim (Kim, 1990) expanded the use of analytical valuation method to value American options with continuous dividends. Turner (Turner, 2010) also developed analytical solutions to value barrier options. However, the underlying options in the above studies do not possess complex features such as multiple uncertainty drivers, mean-reverting properties, etc. Deriving closed form
solutions can be extremely challenging for complex options. Numerical methods need to be applied when analytical solutions cannot be derived for the PDE.

Common numerical methods include the lattice valuation, the finite difference method (FDM) and Monte-Carlo simulation. Primarily due to its ease of implementation, the lattice method is most widely used in the finance community for option pricing (Kwok, 2008). The lattice valuation model for option pricing, first developed by Cox, Ross and Rubinstein (Cox, Ross, & Rubinstein, 1979), traces the evolution of the option’s underlying variable in discrete time. For example, each node of a pricing lattice denotes a price candidate at a given point in time. The price of the option is then calculated by finding the present value of the option values at all future nodes, taking probabilities into account. The binomial lattice model has been used extensively in modeling interest rate dynamics in studies such as Ho and Lee (Ho & Lee, 1986), Boyle (Boyle, 1988), Madan, Milne, and Shefrin (Madan, Milne, & Shefrin, 1989), He (He, 1990) and Hull and White (Hull & White, 1990). The lattice method can be applied to value projects with multiple embedded options and is an efficient valuation approach for short lattice setups, but it is not a feasible method when the optimization process involves multiple state variables, and it becomes computationally expensive when the number of nodes is increased to improve accuracy (Trigeorgis, 1991). An alternative to the lattice method is the finite difference method. It discretizes the random process by approximating the solutions of the PDE on a grid and was first implemented for pricing derivatives by Brennan and Schwartz (Brennan & Schwartz, 1977). Ramaswamy and Sundaresan (Ramaswamy & Sundaresan, 1985), Brennan, Courtadon and Subrahmanyam (Brennan, Courtadon, & Subrahmanyam, 1985) valued American options on futures contracts accounting for early exercise possibilities by means of an implicit finite-difference method. Badea (Badea, 2004) utilized the finite difference method to test the
numerical case stated in Kim’s (Kim, 1990) model and obtained consistent results. Gong, He and Meng (Gong, He, & Meng, 2006) used FDM methods to develop a time-dependent volatility multi-stage compound real options model for valuing venture capital investments. Research from Nwozo and Fadugba (Nwozo & Fadugba, 2012) has shown that the FDM has the fastest convergence speed among the lattice method, the FDM and the Monte-Carlo method. Also, the FDM is very accurate when applied to non-path dependent processes, but it can require sophisticated algorithms for solving large sparse linear systems of equations. In recent years, more realistic, multifactor price models have been developed using the RO framework to value real assets with multiple embedded options, and the higher complexity has made it difficult to utilize the lattice method and the finite difference method (Cortazar, Gravet, & Urzua, 2008). The Monte-Carlo simulation method for option pricing by (Boyle, 1977), on the other hand, is flexible in handling multi-dimensional real options problems. The basis of the Monte-Carlo simulation is the strong law of large numbers, hence the evolution of an option price over time is simulated repeatedly to determine the expected price (Hull, 2010). The Monte-Carlo simulation method can be applied to a stochastic process with complex payoff functions without much effort. It is also a much more straightforward valuation method for practitioners to use. Generally, decision makers in industry tend to shy away from models based on complex financial mathematics, yet simpler approaches based on Monte Carlo simulation are accepted (Benedetti, Bixio and Vanrolleghem 2005) (Ferrer J., et al., 2008) (Cortazar, Gravet, & Urzua, 2008).

2.4 Real Options in Infrastructure

In addition to the studies cited in Section 2.2, researchers have shown that applying RO valuation approaches can provide valuable insights for the firm in making infrastructure
investment decisions when options are present in the project. Smit (Smit, 2003) used a combination of real options theory and game theory to develop a framework that captures the value of growth options for airports. He applied the lattice valuation model with data from European Airports and concluded that larger airports are expected to further strengthen their advantages against smaller ones. By integrating RO valuations with game theory principles, he managed to make a more complete assessment of strategic growth option value. Weck, Neufville and Chaize (Weck, Neufville, & Chaize, 2004) set up a RO framework for staged design of communications satellite constellations in low earth orbits using the expected number of users and their activity level as the basis. According to their results, the RO framework could lower the life cycle costs of the systems studied in the paper by more than 20% compared to the traditional methods. Neufville, Scholtes and Wang (Neufville, Scholtes, & Wang, 2006) developed a spreadsheet tool for valuing real options using a parking garage case. Almassi, McCabe and Thompson (Almassi, McCabe, & Thompson, 2013) used a finite-difference method to develop a valuation tool that assists governments involved in Public-Private Partnership (P3) projects to design a guarantee so that the government has a toll to minimize cost and reasonably mitigate the risk. Tahon, et al. (Tahon, et al., 2013) quantified the value of managerial flexibility in the decision making process for the telecommunications sector, they applied the real options framework with different valuation methods to the investment project for the next generation fixed access network rollout. Clearly, the above studies have shown that the real options approach is suitable for valuation of infrastructure projects where options to expand, delay, or abandon are present. The use of real options methods for the valuation of these infrastructure projects has significant merit and for this reason it will be implemented in the optimization problem of this study.
2.5 Real Options in Wastewater Treatment

Currently, the application of the real options method to value projects in the wastewater sector is limited. A single stage “real option” approach that copes with future demand uncertainties for wastewater treatment plant expansion was first developed by Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2010), who modeled the growth in demand for a residential wastewater treatment as a geometric Brownian motion (GBM) that is partially correlated to an appropriate traded market index. Their model transformed expected future penalty costs for wastewater connections as an Asian option with the expansion size being the strike price. The finite difference method was used to numerically solve for the capital needed to fund the single stage expansion. Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) further developed a closed form approximation to the modular expansion problem. The modular expansion model allows a second-time expansion of the plant at a pre-defined future date. Their results showed that a modular expansion requires significantly less up-front capital investment, and the overall expected expenditures were reduced compared to the single-stage expansion model. Despite the fact that cost savings can be achieved by their model – the modular expansion model can only account for a modular expansion at a pre-fixed future date, which limits the applicability of this model in practice. To relax this constraint, a RO framework is utilized to enhance their closed form model so that the modular expansion can take place on any day, hence making the embedded option of the project a Bermudan real option. The Bermudan real option is to be valued with finite difference schemes given the assumption that the random process has a closed form solution in the Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) model. However, in order to generalize the optimization and make the process more practical to engineers, a Monte-Carlo approach is developed for the optimization.
2.6 The Monte-Carlo Approach within the RO Framework

The Monte-Carlo simulation within the RO framework can be applied to random processes that do not have closed form solutions and is illustrated as a promising approach to determine Bermudan or simple multiple-exercise real options. Boyle (Boyle, Options: A Monte Carlo Approach, 1977) first applied the Monte-Carlo simulation to value financial options. Early researches on valuing American/Bermudan options using simulation methods include: Tilley (Tilley, 1993), Barraquand and Martineau (Barraquand & Martineau, 1995), Carriere (Carriere, 1996), Broadie and Glasserman (Broadie & Glasserman, 1997), Raymar and Zwecher (Raymar & Zwecher, 1997). Their work developed simulation algorithms for estimating American options, combining the forward-looking Monte-Carlo method with backward-looking dynamic programming. Later, Longstaff and Schwartz (Longstaff & Schwartz, 2001) provided a powerful least-squares Monte-Carlo approach to approximating American option prices by simulation. Its simplicity and accuracy gained attention from others in the field and many studies have been carried out on financial option valuation using similar frameworks such as Rasmussen (Rasmussen, 2002), Clement, Lamberton and Protter (Clement, Lamberton, & Protter, 2002), Hippler (Hippler, 2008). Gamba (Gamba, 2002) expanded the work of Longstaff and Schwartz to arrive at an approach for valuing capital budgeting problems with multiple embedded real options and multiple state variables. Chiara, Garvin and Vecer (Chiara, Garvin, & Vecer, 2007) used an Australian option approach in their research to value the government’s revenue guarantee of a “build-operate-transfer” type of public-private partnership in the public infrastructure sector by the use of Monte-Carlo simulations. Boogert and Jong (Boogert & Jong, 2008) applied a Monte-Carlo method to value gas storage contracts using current and expected gas prices. There are also many studies focused on WWTP projects that utilize the Monte-Carlo method. Huo, et al. (Huo, Jiang, Seaver, Robinson, & Cox, 2006) used Monte Carlo simulation
to quantify plant performance and arrived at the conclusion that conservative design assumptions may lead to better performance. McCormick, Johson and Turner (McCormick, Johnson, & Turner, 2007) used Monte-Carlo simulations to produce probability distributions for different input variables for complex WWTP systems. Belia, et al. (Belia, et al., 2009) proposed a structure for identifying the sources of uncertainties in a wastewater treatment system, but no quantitative models were discussed. Benedetti, et al. (Benedetti, Claeys, Nopens, & Vanrolleghem, 2011) assessed the convergence of Monte-Carlo simulations of wastewater treatment models to test conditions that allow convergence with fewer simulation runs. However, most of the studies related to WWTP design projects involve hydraulic, control or biological models that put focus on effluent qualities, reactor concentrations and other chemistry-related parameters. We have not come across any studies that look at the WWTP design problem solely from a financial perspective and also allow modular expansions using the Monte-Carlo method.

2.7 Summary

To summarize, the objective of this thesis is to relax the pre-fixed date constraint in the Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) model and develop a more realistic modular expansion model for the WWTP design project. The modular expansion model will allow the second modular expansion to be carried out at any point after initial expansion. To achieve this, we first expand the Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) numerical model by utilizing a finite difference scheme and arrive at a design project with an embedded Bermudan real option. Also, a practical, and easy to understand Monte-Carlo simulation is developed for general cases where a closed form solution cannot be derived from the PDE of their model. Results obtained by using both methods are compared and found to be similar to each other.
3 Methodology

This chapter presents the models of this thesis. In Section 3.1, we present the background for the project, which is rooted in the work of Lawryshyn and Jaimungal ([Lawryshyn & Jaimungal, 2010] and [Lawryshyn & Jaimungal, 2014]). Section 3.2 describes an enhancement of the latter numerical model. The enhanced model combines the use of analytical approximation and the finite difference method (FDM) to solve for optimal timing and sizing. The Monte Carlo simulation model is presented in Section 3.3. The value of the Monte Carlo approach is that it is much more intuitive than the FDM and will likely be more easily accepted by practitioners. Furthermore, it is much easier to implement the Monte Carlo approach for more complex, but arguably more realistic, stochastic processes. Finally, the method to determine the optimal sizing of the waste water treatment plant (WWTP) project is presented in Section 3.4.

3.1 Project Background

The objective of this thesis is to determine the optimal sizing of a wastewater treatment plant for a municipality that also has the option to expand the plant over its project life. The present condition is that the municipality is experiencing high population growth and must build a new wastewater treatment plant immediately. The construction is expected to take just under 2 years and the plant to be constructed is expected to operate for the next 25 years. However, the rate of growth of wastewater connections in the municipality is volatile and uncertain. Thus, the optimal plant size cannot be determined using standard methods. An undersized plant will lead to significant overflow penalty costs while an oversized plant may show poor performance and will also require significant up front capital costs that may be difficult to recoup if potential population expansion is not realized. Hence, a staged design of the plant expansion can help the
municipality to reduce the cost of an initial over-design, yet still allow for the option to expand if significant population growth is realized.

There are four phases involved in the two-stage expansion of this WWTP project:

- Phase 1: a decision on the first module size $K_1$ and the size of land reserved for a potential second module $K_{2\text{max}}$ will be made at time $t_0$. Construction will take place for the first modular plant from time $t_0$ to time $t_1$.
- Phase 2: the operation of the first modular plant starts from time $t_1$. After time $t_1$, the municipality has the option to expand the wastewater treatment plant at any variable time $t_2$ if necessary.
- Phase 3: if necessary, a decision on the second module size $K_2$ that is no larger than $K_{2\text{max}}$ will be made at $t_2$ and construction will take place for the second modular plant from $t_2$ to $t_3$.
- Phase 4: the operation of both modular plants will take place from time $t_3$ to $T$.

Figure 1 is a graph showing different phases of the two-stage WWTP project:

![Sample path for a two-stage expansion of a WWTP project](image)

To develop the optimization framework, four major factors are considered: the first is the connection rate of the wastewater plant, $X_t$, which stands for the growth of local wastewater treatment demand at time $t$. Following Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2010) and (Lawryshyn & Jaimungal, 2014), we assume that the expansion rate follows a GBM which is correlated to a traded security (for instance, a market index). This models the fact that
the general strength of the economy is reflected both in the market index and in the growth rate of a municipality. The price of the security also follows a GBM process,

\[ dS_t = \mu_S S_t dt + \sigma_S S_t dW_t, \]  

where \( \mu_s \) and \( \sigma_s \) are constants representing the drift and volatility, respectively and \( W_t \) is a Wiener process. Similarly, the connection rate (expansion rate) is modeled as

\[ dX_t = \mu_X X_t dt + \sigma_X X_t \left( \rho dW_t^t + \sqrt{1 - \rho^2} dW^\perp_t \right), \]  

where \( \mu_x \) is the drift of \( X_t \), \( \sigma_x \) is the volatility of \( X_t \), \( W_t \) and \( W^\perp_t \) are two independent Wiener processes under the real world measure, and \( \rho \) is the correlation between \( X_t \) and \( S_t \). In the risk neutral pricing measure, we obtain the following expression for a stochastic process of \( X_t \):

\[ dX_t = \bar{r} X_t dt + \sigma_X X_t \left( \rho d\bar{W}_t + \sqrt{1 - \rho^2} d\bar{W}^\perp_t \right), \]  

where

\[ \bar{r} = \mu_x - \frac{\rho \sigma_x}{\sigma_S} (\mu_s - r), \]  

and \( \bar{W}_t \) and \( \bar{W}^\perp_t \) are two independent Wiener processes under the risk-neutral measure. Table 1 shows the numerical values of the market parameters that are used in the thesis.

Table 1. Market parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market drift, ( \mu_S )</td>
<td>10%</td>
</tr>
<tr>
<td>Market volatility, ( \sigma_S )</td>
<td>16%</td>
</tr>
<tr>
<td>Connection rate drift, ( \mu_X )</td>
<td>8%</td>
</tr>
<tr>
<td>Connection rate volatility, ( \sigma_X )</td>
<td>5%</td>
</tr>
<tr>
<td>Correlation factor, ( \rho )</td>
<td>0.5</td>
</tr>
<tr>
<td>Risk free rate, ( r )</td>
<td>2%</td>
</tr>
</tbody>
</table>

The second factor to consider is the total number of connections at time \( t \), \( N_t \), which denotes the demand of the wastewater treatment service at time \( t \). Following Lawryshyn and Jaimungal
(Lawryshyn & Jaimungal, 2010), the total number of connections, \( N_t \), can be modelled as the accumulation of demand rate \( X_t \) over time:

\[
N_t = N_0 + \int_0^t X_u du,
\]

where \( N_0 \) is the initial number of connections, and negative \( N_0 \) implies initial extra capacity.

The third factor is the general construction cost, which we model to consist of two parts: the fixed construction cost, \( \alpha \), and the variable construction cost \( \gamma \). Following Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014), we assume different cost coefficients for each stage of the expansion. The cost functions for the first expansion, \( C_1 \), and second expansion, \( C_2 \), are:

\[
C_1(K_1, K_{2 max}) = \alpha_1 + \gamma_1 K_1 + \gamma_{12} K_{2 max},
\]

\[
C_2(K_2 \leq K_{2 max}) = \alpha_2 + \gamma_2 K_2.
\]

where \( \alpha_1, \alpha_2 \) are positive coefficients associated with fixed construction cost, and \( \gamma_1, \gamma_2, \gamma_{12} \) are positive coefficients associated with variable construction cost. Furthermore, \( K_1 \) is the size of the first expansion and \( K_{2 max} \) is the maximum size available for the second modular expansion. The construction coefficients used in this paper are presented in Table 2.

**Table 2. Construction cost coefficients.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module 1 fixed cost, ( \alpha_1 )</td>
<td>$3,500,000</td>
</tr>
<tr>
<td>Module 2 fixed cost, ( \alpha_2 )</td>
<td>$525,000</td>
</tr>
<tr>
<td>Module 1 variable cost 1, ( \gamma_I )</td>
<td>$860/connections</td>
</tr>
<tr>
<td>Module 1 variable cost 2, ( \gamma_{12} )</td>
<td>$172/connections</td>
</tr>
<tr>
<td>Module 2 variable cost, ( \gamma_2 )</td>
<td>$757/connections</td>
</tr>
</tbody>
</table>

The fourth factor to consider is the penalty cost, \( PC_t \), which is the cost associated with demand overflow at time \( t \). Overflow occurs when the capacity of the wastewater treatment plant cannot cope with the service demand \( N_t \). Given the service demand, \( N_t \), and the plant capacity, \( K \), the excess demand \( XD \) at time \( t \) is expressed as:
\[ XD = \begin{cases} 0, & N_t \leq K \\ N_t - K, & N_t > K \end{cases} \quad (8) \]

As shown in Equation (8), \( XD \) takes on the form of a financial call option. Furthermore, the present value of the expected penalty cost associated with demand overflow at time \( t \), \( PC_t \), can be defined as:

\[
PC_t = PC_0 e^{-(r-r_{cpi})t} \tilde{E} \left[ \max \left( N_0 + \int_0^t X_s ds - K, 0 \right) \right], \quad (9)
\]

where \( r \) is the risk-free rate, \( r_{cpi} \) is the inflation rate, and \( PC_0 \) is the initial penalty cost rate as of time 0.

Therefore, the present value of the total expected penalty cost from time \( t \) to \( T \) is:

\[
\tilde{E} [PC_{t,T;K}^{PV}] = \int_t^T PC_0 e^{-(r-r_{cpi})u} \tilde{E} \left[ \max \left( N_0 + \int_0^u X_s ds - K, 0 \right) \right] du. \quad (10)
\]

As developed by Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014), the present value of the total expected penalty cost from time \( t \) to \( T \), as stated in Equation (10), takes on the form of an Asian option.

Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014), further developed an analytical approximation for the penalty cost function of Equation (10), which leads to the expected total cost function for a given set of \( X_0 \) and \( N_0 \):

\[
\tilde{E} [PC_{t,T;K}^{PV}] \sim \int_t^T PC_0 e^{-(r-r_{cpi})u} e^{\mu u} (X_t \Phi(d_{t,u,+}) - (K - N_t) e^{-\mu \Phi(d_{t,u,-}))} du, \quad (11)
\]

where \( \Phi \) is the normal cumulative distribution term,

\[
d_{t,T;\pm} = \ln \left( \frac{X_t / K'}{\bar{\mu}_{t,T}} \right) + \frac{1}{2} \sigma_{t,T}^2, \quad K' = K - N_t, \quad \bar{\mu}_{t,T} = \bar{\mu} = \ln \left( e^{r(T-t)} - 1 \right) - \ln \bar{r},
\]
and $\sigma_{t,T} = \sqrt{\ln \left( \frac{2}{\tau + \sigma_X^2} \right) + \ln \left( \frac{e^{(2\tau + \sigma_X^2)(T-t)}}{2\tau + \sigma_X^2} - 1 - e^{(T-t)} - 1 \right) + 2\ln \tau - 2\ln \left( e^{(T-t)} - 1 \right)}$.

Finally, in their fixed-date two-stage expansion model, Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) showed that at some time $t^*$ the optimal second stage expansion size can be determined by

$$K_2^{\text{Opt}} = \min_{K_2} \left( C_2(K_2) + \mathbb{E} \left[ \text{PC}^\text{PV}_{t^*+t_{\text{const}},T,K_1+\min(K_2,K_{2\text{max}})} \right] \right) e^{-\pi^2},$$

(12)

where $t_{\text{const}}$ is the time required for construction of the expansion. Equation (12) will be used extensively in the numerical model for determining the optimal strategy at each decision point. It should be noted that Equation (12) could also be used in the Monte-Carlo simulation to reduce computational time, in view of the fact that one of the objectives was to develop a model purely based on Monte-Carlo simulation; a technique much more readily accepted by practitioners, a different optimization algorithm was utilized. In the following subsection, the numerical model methodology is presented and in the subsequent one, the simulation model is developed.

### 3.2 An Expansion of the Lawryshyn and Jaimungal Model – the Numerical Method

To develop the two-stage expansion model, we first develop a numerical method to determine the optimal timing and sizing of the second modular plant for a given set of $K_1$ and $K_{2\text{max}}$ by expanding on the single stage model developed by Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014). The analytical approximation in Equation (11) is incorporated in the numerical methods to determine the optimal sizing for a given set of $X_0$ and $N_0$ at a
specific time. To determine the optimal time and size for the second modular plant, the optimization is performed on a grid with $X_t$ and $N_t$ being the two axes.

As discussed previously, we assume that the managers have the option to expand the plant at any time after initial construction is completed during the life of the plant and decisions are made on an annual basis only. However, since the construction time of the second expansion is just under two years, it makes no sense to expand at any time after year 23. Thus, the latest decision time is at year 23. Therefore, at $t = 23\ y$, we utilize Equation (12) to determine the optimal expansion size, $K_{2Opt,t=23}$. Next, we need to compare the cost for two scenarios at year 23 – the total cost associated with expanding the plant at year 23 versus the cost of not expanding the plant at that time. The expansion strategy cost at $t = 23\ y$ can be seen as the sum of the construction cost of second modular plant sized with $K_{2Opt,t=23}$, the penalty cost during the construction where the plant size is $K_1$, and the penalty cost during the operation of the new plant where the plant size is $K_1 + K_{2Opt,t=23}$. Hence the cost of the expansion strategy at $t = 23$ is

$$C_{t=23}^{ES}(X_t, N_t) = C_2(K_{2Opt,t=23}) + E[PC_{23,23+t_c;K_1}^PV] + E[PC_{23+t_c;K_1+K_{2Opt,t=23}}^PV],$$

(13)

where $t_c$ is the length of construction. For the no expansion scenario, the total cost is just the penalty cost incurred with plant size $K_1$:

$$C_{t=23}^{NC}(X_t, N_t) = E[PC_{23;K_1}^PV].$$

(14)

Hence the optimal cost for each point $(X_t, N_t)$ at year 23 on the grid is

$$v(t = 23, X_t, N_t) = \min(C_{t=23}^{NC}(X_t, N_t), C_{t=23}^{ES}(X_t, N_t)).$$

(15)

Since decisions are only made annually, the next decision date to be considered is one year earlier, at year 22. We need to work backwards because the optimal decision of waiting at year 22 involves knowing what the outcome at year 23 is; this is Bellman’s principal of dynamic programming (Bellman, 1957). We use the finite difference method to model the evolution of
penalty values through decision dates. The expected total cost $v_{opt}$ is described by an Asian-option, as stated by Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) – the PDE for an Asian option is:

$$\frac{dv}{dt} + \bar{r}x \frac{dv}{dx} + x \frac{dv}{dy} + \frac{1}{2} \sigma^2 x^2 \frac{d^2 v}{dx^2} = rv. \quad (16)$$

Equation (16) is also appropriate for the problem at hand with the following boundary conditions:

$$\frac{\partial v}{\partial x_{x=x_{\text{max}}}} = \text{constant}, \quad (17)$$

$$\frac{\partial v}{\partial y_{y=y_{\text{max}}}} = \text{constant}, \quad (18)$$

$$\frac{\partial v}{\partial y_{y=y_{\text{min}}}} = \text{constant}, \quad (19)$$

$$v_{0,y}^t = P C_0 e^{-(r-r_{\text{cpi}})u} \mathbb{E}[\max(N_t - K, 0)] \ast (T - t). \quad (20)$$

We apply Equation (16) at $v(t = 23, X_t, N_t)$ to arrive at $v(t = 22, X_t, N_t)$. At a given time $t^* \in [22, 23]$, $x$ denotes the connection rate $X$, $y$ denotes the total number of connections $N$, and $v$ denotes the time-discounted value of the total expected cost from year 22 to $T$.

At this point, the cost of no-expansion-at-year-22 strategy is simply the discounted value of the best strategy from year 23, hence

$$C_{t=22}^{NC}(X_t, N_t) = v(t = 22, X_t, N_t), \quad (21)$$

and the cost of the expansion strategy at year 22 is

$$C_{t=22}^{ES}(X_t, N_t) = C_2(K_2^{\text{Opt}, t=22}) + \mathbb{E}[PC_{t=22}^{PV} + C_{t=22}^{PV} + C_{K_1 + K_2^2}^{\text{Opt}, t=22}], \quad (22)$$

and the optimal cost is updated as

$$v(t = 22, X_t, N_t) = \min(C_{t=22}^{NC}(X_t, N_t), C_{t=22}^{ES}(X_t, N_t)). \quad (23)$$
The methodology is repeated until \( t = 2y \), a depiction of the numerical model is presented in Figure 2.

![Figure 2. Depiction of the numerical model.](image)

### 3.3 The Simulation Method

The simulation method uses the Monte Carlo process to simulate the demand growth and therefore to determine the optimal. We think it is a more straightforward approach compared to the numerical method and is easier to understand from a practical perspective. In contrast to using the analytical method to determine the expected total cost in Section 3.2, at each decision making time point – i.e., as before, \( t = 2, 3, ..., 23 \) years, we simulate individual paths of the future demand growth at different time steps on a grid of \( X_t \) and \( N_t \); i.e., at each time step \( t^* \), for each grid point \((i, j)\) we simulate \( X_t \) from \( t = t^* \) to \( t = T \), for \( N \) paths and thus we arrive at a \( N_{path} \times M_{time} \) matrix, \( X_{t^*, T} \), where \( N_{path} \) is the number of paths (per grid point) and \( M_{time} \) is the number of time steps for the simulation (e.g. at \( t = 23 \) we would be simulating until \( t = T = 25y \), so \( M_{time} = (25 - 23)/dt \)). Integrating \( X_{t^*, T} \) over the time period from \( t = t^* \) to \( T \), we will
arrive at a matrix representing the number of connections during the period of \( t^* \) to \( T \), \( N_{t^*,T}^{i,j} \), where the \( k\)-th, \( l\)-th element of this matrix is given by
\[
N_{t^*,T}^{i,j} |_{k,l} = N_{t^*,T}^{i,j} + \sum_{m=1}^{l} X_{t^*,T}^{i,j} |_{k,m} \delta t, \tag{24}
\]
where \( \delta t \) is the time step in the Monte Carlo simulation. Thus, for any plant size \( K \) the present value of the penalty cost for each simulation path is given by
\[
PC(K)_{t^*,T}^{i,j} |_{k} = \sum_{m=1}^{M_{\text{time}}} e^{-r(t \text{_{m}} - t)} \max \left( N_{t^*,T}^{i,j} |_{k,m} - K, 0 \right) \cdot PC_0 \delta t \tag{25}
\]
where \( PC(K)_{t^*,T}^{i,j} \) is a vector of length \( N_{\text{path}} \), and the expected penalty cost for any total plant size \( K \) over the period \( t^* \) to \( T \) is given by averaging the vector so that
\[
PC(K)_{t^*,T}^{i,j} = \frac{1}{N_{\text{path}}} \sum_{k=1}^{N_{\text{path}}} PC(K)_{t^*,T}^{i,j} |_{k}. \tag{26}
\]

For expansion cases, we simplify our problem slightly, by assuming a discrete number of possible modular expansion sizes are available, i.e. \( K_2 \in \{K_2^1, K_2^2, K_2^3, ..., K_2^n\} \). For each possible modular expansion size \( K_2^l \) where \( l = 1,2, ..., n \), the total cost consists of the construction cost, the penalty cost during construction with plant size \( K_1 \) and the penalty cost after construction is done with plant size \( K_1 + K_2^l \), hence,
\[
C(K_2^l)_{t^*,T}^{i,j} = C_2(K_2^l) + PC(K_1)_{t^*,T_+t_c}^{i,j} + PC(K_1 + K_2^l)_{t^*+t_c,T}^{i,j} \tag{27}
\]
where \( t_c \) is the duration of construction.

Starting at \( t = 23 \), for the case of no construction, the total cost is simply the penalty cost incurred with plant size \( K_1 \), thus, we have
\[
C_{t=23}^{NC,i,j} = PC(K_1)_{t=23,T}^{i,j} \tag{28}
\]
For the expansion strategy at \( t = 23y \), we utilize Equation (27) and calculate the expansion strategy cost as

\[
C_{t=23}^{ES;ij} = \min(C(K_2^{1,ij})_{23,T}, C(K_2^{2,ij})_{23,T}, \ldots, C(K_n^{n,ij})_{23,T}).
\]  

(29)

Thus, the value of the optimal cost at each grid point \((i, j)\) at \( t = 23y \) is calculated as

\[
v_{t=23}^{ij} = \min(C_{t=23}^{NC;ij}, C_{t=23}^{ES;ij}).
\]  

(30)

At the next expansion decision point, \( t = 22y \), we consider two scenarios: the case of no expansion and the case of expansion. In the case of no expansion, we simulate \( X_t \) from \( t = 22y \) to \( t = 23y \) to get \( X_{t=22,23}^{ij} \) and \( N_{t=22,23}^{ij} \). We apply Equation (25) to find \( PC(K)_{t=22,23}^{ij} \). Then, for each end point \( X_{t=22,23}^{ij} \mid_{k,M\text{time}} \) and \( N_{t=22,23}^{ij} \mid_{k,M\text{time}} \), we use linear interpolation to determine the value of \( v_{t=23}^{ij}(X_{t=22,23}^{ij} \mid_{k,M\text{time}}, N_{t=22,23}^{ij} \mid_{k,M\text{time}}) \). Thus, the cost for no expansion becomes

\[
C_{t=22}^{NC;ij} = \frac{1}{N_{path}} \sum_{k=1}^{N_{path}} (PC(K)_{t=22,23}^{ij} \mid_{k}) + \sum_{k=1}^{N_{path}} \left( v_{t=23}^{ij}(X_{t=22,23}^{ij} \mid_{k,M\text{time}}, N_{t=22,23}^{ij} \mid_{k,M\text{time}}) \right) e^{-r(23-22)}.
\]  

(31)

For the expansion strategy, we proceed as before, with the key distinction that we simulate \( X_t \) through the entire remaining time domain, such that we calculate \( X_{t=22,T}^{ij} \) and \( N_{t=22,T}^{ij} \) and utilize Equation (27) to determine

\[
C_{t=22}^{ES;ij} = \min(C(K_2^{1,ij})_{22,T}, C(K_2^{2,ij})_{22,T}, \ldots, C(K_n^{n,ij})_{22,T}).
\]  

(32)

Then, Equation (30) is used to determine \( v_{t=22}^{ij} \). The methodology continues in a recursive fashion to \( t = 2y \). For \( t = 0 \) to \( 2y \) we simply simulate \( X_t \), determine \( N_t \) for each path, calculate the interpolated value of \( v_{t=2} \) for each case, discount to \( t = 0 \) and calculate the average to find the final value.
3.4 Optimization of $K_1$ and $K_{2\text{max}}$

Both the Numerical and Simulation Models can be used to optimize the WWTP expansion by minimizing the cost. The total cost is given by

$$TC = C_1(K_1, K_{2\text{max}}) + O_{K_1} + C_2(K_2 \leq K_{2\text{max}}) + O_{K_1+K_2},$$

where $C_1(K_1, K_{2\text{max}}), C_2(K_2 \leq K_{2\text{max}})$ are the construction costs incurred in phase 1 and phase 3, respectively and were presented in Equations (6) and (7). $O_{K_1}$ is the sum of the operating cost from the construction of first expansion to the construction of second expansion. Hence,

$$O_{K_1} = E[P_{C_{t_0,t_1;0}}] + E[P_{C_{t_1,t_2;K_1}}],$$

and $O_{K_1+K_2}$ is the sum of the operating cost from the construction of second expansion to the end of project life,

$$O_{K_1+K_2} = E[P_{C_{t_2,t_3;K_1}}] + E[P_{C_{t_3,T;K_1+K_2}}].$$

where $t_0, t_1, t_2, t_3, T$ are the time points presented in Figure 1, and $E[P_{C_{t_0,t_1;0}}], E[P_{C_{t_1,t_2;K_1}}], E[P_{C_{t_2,t_3;K_1}}]$ and $E[P_{C_{t_3,T;K_1+K_2}}]$ are operating cost presented in Equation (10). On a grid of $K_1$ and $K_{2\text{max}}$, the point that yield the smallest $TC$ value is the optimal design of the two-stage wastewater treatment plant expansion.

3.5 Methodology Summary

In this chapter, we developed two methodologies within the RO framework. First, we enhanced the work of Lawryshyn and Jaimungal (Lawryshyn & Jaimungal, 2014) to develop a modular WWTP expansion model. By utilizing the finite difference scheme, we relaxed the constraint on the pre-fixed expansion date and transformed the problem into a Bermudan real options valuation. Second, we developed a more practical Monte-Carlo simulation approach so
that the RO framework can be applied to more general random processes. Both methods will be applied to the same set of parameters and consistent results are expected.
4 Results and Discussion

In this chapter we provide some of the key results of applying the two RO valuation methods developed in Chapter 3. In particular, we provide a comparison between the numerical model and the simulation model. As stated in Table 1, the drift $\mu_X$ and volatility $\sigma_X$ of $X_t$ before risk-adjustment under the risk-neutral measure are assumed to be 10% and 16% for the optimization process, the values of $\mu_S$ and $\sigma_S$, representing the drift and volatility of the market index $S_t$, are set to be 8% and 5%, respectively. Assuming a project life of 25 years and construction time of 1.99 years for both stages, this section presents the results of an optimization performed with a correlation parameter $\rho = 0.5$, a risk free rate of 2% and an inflation rate of 0%. The construction coefficients are provided in Table 2. The initial penalty rate for overflow is assumed to be $5000 per connection over the operational capacity of the plant. For the grid in both method setups, the X axis is set from 0 to 4000 while the N axis is set from -200-$K_1$ to 40000 to capture the 99.99th percentile of the stochastic process of $X_t$ and $N_t$.

4.1 Second Expansion Results: Optimization of $K_2$ for a Single Set of $K_1$ and $K_{2\text{max}}$

In this section, we present the results on the optimization of the second modular expansion size $K_{2\text{opt}}$. Optimization results are obtained assuming the original capacity to be 200 connections, hence $N_0 = -200$; and we assume an initial demand growth rate of 81 connections per year, hence $X_0 = 81$. Also, we assume the first module size $K_1$ to be 1000 and maximum second stage expansion size $K_{2\text{max}}$ to be 5000.
4.1.1 Optimal Expansion Time, $t_{opt}$

This section presents the results on the optimal expansion time, $t_{opt}$, as we perform optimizations on a yearly basis, backward in time, from the end of project life to the beginning of the project life.

The red blocks in Figure 3 show the scenarios where the expansion strategy is not favored at $t = 23y$. It is clear that all spots on the grid yield “no expansion” option as the optimal strategy for both the numerical and the simulation method. Given the total project life of 25 years, the wastewater treatment plant will only run for 2 more years before the end of the project lifespan at year 23. However, the construction of the plant alone will take almost 2 years, leading to the fact that the second module of the plant will be put into use for only minimal amount of time after the construction is complete. Thus, it is not feasible for the decision makers to seek an expansion at year 23.

![The numerical method](image1)

![The simulation method](image2)

**Figure 3** Optimal expansion time for each method at year 23. The left figure shows the results using the numerical method while the right figure shows the results using the simulation method. This figure shows the optimal expansion time at year 21 as a function of connection rate $X_t$ and number of connections $N_t$. The numerical method and the simulation method give the same values here. Parameters used are in Table 1 and 2.

Figure 4 shows the optimal exercise boundary at $t = 22y$. As the second modular plant may run for essentially one year after construction, the expansion strategy is observed as the better strategy almost everywhere except some points in the corner of the grid for both methods. The result is consistent with expectation as the penalty resulted from overflow when either $X_t$ or $N_t$ is
large and overpowers the construction cost incurred by the second modular expansion, hence an expansion is favored in such cases. However, as the connection rate $X_t$ gets smaller, a negative connection number $N_t$ suggests that available capacity may be large enough to prevent potential demand overflow from happening, therefore, the “no expansion” strategy is favored at points where $X_t$ is small and $N_t$ is negative.

The numerical method

The simulation method

Figure 4 Optimal expansion time for each method at year 22. This figure shows the optimal expansion time at year 22 as a function of connection rate $X_t$ and number of connections $N_t$. The numerical method and the simulation method give the consistent values here. Parameters used are as in Table 1 and 2. Expanded views are provided in Figures 5 and 6 below.

Figure 5 shows a comparison of the optimal exercise boundaries at year 22 and year 21 for the numerical method while Figure 6 shows a comparison of the optimal exercise boundaries at year 22 and year 21 for the simulation method. Blue blocks in the figures represent where an immediate second expansion decision is favored, while the red blocks represent the region where a second expansion is not favored given the connection rate and numbers at that time.

For the numerical method, it is clear that the expansion region becomes bigger as time progresses backward from year 22 to year 21. In the real options context where we treat the modular expansion as the real asset, the expansion size is considered as the strike size, total demand is considered as the price of the underlying asset, and the penalty is considered as the
option value, then the time left for the plant to run for is the time to maturity of the underlying option. Given all other factors equal, the value of an option increases as the time to maturity of the option becomes longer. Hence, the numerical model produced consistent results as the expansion is favored for more points on the grid to decrease the penalty value when the time for the project to run for increased.

For the simulation method, a change in area size is also observed but has less details due to a wider step size. However, the resulting feasible area shows results consistent with the numerical method.

The numerical method

\[ t_{opt} \text{ at } t = 22 \quad \quad t_{opt} \text{ at } t = 21 \]

Figure 5 Enlarged corner area t_{22} vs t_{21} by the numerical method. This figure shows the optimal expansion time at year 22 and 21 as a function of connection rate \( X_t \) and number of connections \( N_t \). Parameters used are as in Table 1 and 2.
The simulation method

Figure 6 Enlarged corner area $t_{22}$ vs $t_{21}$ by the simulation method. This figure shows the optimal expansion time at year 22 and 21 as a function of connection rate $X_t$ and number of connections $N_t$. Parameters used are as in Table 1 and 2.

Figure 7 shows the optimal exercise boundary at $t = 2y$, that is, the earliest possible time for a second expansion, expanded views for the optimal exercise boundary shown in Figure 7 are presented in Figure 8 and Figure 9. The dark blue blocks show the region where an immediate second expansion decision is favored, the light blue blocks represent where a second expansion is optimal at year 3, while he red blocks represent the region where a second expansion is not favored.

The optimal expansion plane for the numerical method has two flat surfaces – the small flat “no expansion” area at the small-$X_t$-small-$N_t$ corner, and the large “expansion at year 2” plane. The optimal expansion plane for the simulation method is very similar to that of the numerical method, except for several spots with an optimal timing at year 3. This, however could be justified by the noise of the simulation process. As $K_{2max}$ is limited to 5000, a large $X_t$ may lead
to rapid demand growth that can’t be met even with a second expansion, hence the difference between building an expansion at year 2 and year 3 is much less significant comparing to that when $X_t$ is small. Nevertheless, a very sharp boundary can be observed between the “expansion favored” and “no expansion favored” regions for both methods. The results suggest that given the sets of assumptions proposed in the Chapter 3, the second expansion should be either carried out immediately at the earliest possible time or never be carried out over the lifetime of the project.

\[ \text{The numerical method} \]
\[ \text{The simulation method} \]

Figure 7 Optimal expansion time for each methods at year 2. This figure shows the optimal expansion time at year 2 as a function of connection rate $X_t$ and number of connections $N_t$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2. Expanded views are presented in Figure 8 and Figure 9.
Figure 8 Enlarged corner area at $t_2$, the numerical method. This figure shows the optimal expansion time at year 2 as a function of connection rate $X_t$ and number of connections $N_t$. Parameters used are as in Table 1 and 2.

Figure 9 Enlarged corner area at $t_2$ by the simulation method. This figure shows the optimal expansion time at year 2 as a function of connection rate $X_t$ and number of connections $N_t$. Parameters used are as in Table 1 and 2.
4.1.2 Optimal Total Expansion Cost, $v_{opt}$

This section presents the results on the optimal expansion cost, $v_{opt}$, as we perform optimizations on a yearly basis, backward in time, from the end of project life to the beginning of the project life.

Figure 10 shows the expected cost for the numerical method while Figure 11 shows the expected cost for the simulation method. At initial state, the cost plane is a very smooth plane except for the areas at the corner of small $X_t$ and small $N_t$ values for both methods. With the small number of connections and low growth rates, the existing capacity of the plant will be able to accommodate future demands, which results in the flat “option is out-of-the-money” area.

Initial state – Year 23

![Figure 10 Optimal expected total cost for the numerical method at year 23. This figure shows the optimal expansion cost at year 23 as a function of connection rate $X_t$ and number of connections $N_t$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2.](image)
The simulation method

Figure 11 Optimal expected total cost for the simulation method at year 23. This figure shows the optimal expansion cost at year 23 as a function of connection rate $X_t$ and number of connections $N_t$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2.

Figure 12 and Figure 13 show the optimal cost for the numerical method and the simulation method at year 2, respectively. As time progresses backwards from year 23 to year 2, the cost plane maintains the kinky-plane shape with a smaller flat area for both methods. Since the time to maturity for the option increases, the potential total penalty increases, therefore, the option is more likely to be in the money – leading to a smaller “out-of-the-money” area. Also, since the potential penalty increases, the total expected penalty increases in response – resulting in a cost plane with larger $r_{opt}$ values for each node. At year 2, the numerical method and the simulation method produced consistent results.
The numerical method

The simulation method

Figure 12 Optimal expected total cost at year 2 by the numerical method. This figure shows the optimal expansion cost at year 2 as a function of connection rate $X_t$ and number of connections $N_t$. Parameters used are as in Table 1 and 2.

Figure 13 Optimal expected total cost at year 2 by the simulation method. This figure shows the optimal expansion cost at year 2 as a function of connection rate $X_t$ and number of connections $N_t$. Parameters used are as in Table 1 and 2.
4.2 First Expansion Results: Optimization of $K_I$ and $K_{2\text{max}}$

In this section, we present the optimization results on the first modular expansion that takes place at the start of the project ($t = t_0$). Figure 14 shows a surface plot of the expected total cost for the two-stage WWTP project with respect to $K_I$ and $K_{2\text{max}}$. The values at each node are obtained using a same set of the assumptions as discussed in this chapter. It is clear that a global minimum exists as the optimal WWTP design strategy.

![Surface plot of expected total cost](image.png)

**Figure 14** Expected Total Cost with respect to $K_I$ and $K_{2\text{max}}$ at year 0. This figure shows the expected total cost at year 0 as a function of $K_I$ and $K_{2\text{max}}$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2.

Figure 15 shows the changes in expected total cost with respect to size of $K_I$, it is evident that the expected total cost decreases as $K_I$ increases from 500 to 2000 for all scenarios. The smaller first expansion modular plant capacity, $K_I$, will likely lead to higher total cost of the project before the second modular expansion is initiated as the demand has a higher chance of going over the capacity, therefore incurring a large penalty cost. As $K_I$ increases beyond 2000,
the expected total cost increases mainly due to the construction cost. The construction cost for a large $K_1$ is going to be the dominant factor in the total cost function as the demand is less likely to go over the capacity, leading to insignificant overflow penalty cost.

The minimum point is observed near $K_1 = 2000$ for all cases of $K_{2\text{max}}$, it is expected since the initial connection rate $X_0$ is 81, initial connection rate $N_0$ is -200, and the risk-neutral drift, $\bar{r}$, is 0.4%, the expected demand in 25 years will be near 2000 connections. Hence, a bigger $K_1$ will likely be oversized and smaller $K_1$ will likely be too small.

![Figure 15 Changes in expected total cost with respect to size of $K_1$. This figure shows the expected total cost at year 0 as a function of $K_1$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2.](image1)

Figure 16 shows the changes in expected total cost with respect to size of $K_{2\text{max}}$. With smaller $K_1$’s, the expected total cost decreases as $K_{2\text{max}}$ increases. This is because the penalty cost incurred by smaller capacity of the first modular plant can be recovered with a bigger $K_{2\text{max}}$. 

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Nevertheless, when $K_1$ gets bigger, the first modular plant is likely to cover all the treatment demands over the project life, making bigger $K_{2\text{max}}$ less desirable as it requires a higher construction cost compared to potential savings from a second modular plant. Hence, a nearly linear relationship between total cost and $K_{2\text{max}}$ is observed in Figure 16 for series of large $K_1$ values.

The minimum of the plot is found at $K_1 = 2000$ and $K_{2\text{max}} = 4000$ with a total capital requirement of $6.93 \times 10^6$ dollars. Therefore, the optimal expansion strategy would be to build a plant of size 2000 for immediate operation and another plant of size 4000 for future expansion.

![Figure 16](image)

*Figure 16 Changes in expected total cost with respect to size of $K_{2\text{max}}$. This figure shows the expected total cost at year 0 as a function of $K_{2\text{max}}$. The numerical method and the simulation method give the same values here. Parameters used are as in Table 1 and 2.*

### 4.3 Comparison with Traditional Methods

Using traditional methods, a plant is built based on the growth of expected demand only. As stated in the above section, the expected demand in 25 years will reach 2100 connections. A single stage wastewater treatment plant of that size will require around $10$ million capital
investment on average, while designs following the modular expansion model developed in this thesis are expected to require $6.93 million capital investment. Hence, planning a strategy based on growth of expected demand will cost 66% more in capital investment compared to the methods developed in this thesis.

![Graph](Figure 17 Single stage modeling cost based on Monte Carlo. Parameters used are as in Table 1 and 2. When the Monte-Carlo simulation is applied, a single stage model under the same assumption will lead to similar results. As seen in Figure 17, the optimal strategy is determined to be a plant with 4000 capacity, which leads to an expected total capital requirement of $7.53*10^6, that is still 8.6% more cost expensive than the modular expansion model.)
5 Conclusions and Recommendations

5.1 Conclusions

In this thesis, we developed a RO framework to determine the optimal sizing of a modular wastewater treatment plant expansion project. First, we enhanced the Lawryshyn and Jaimungal’s fixed-date modular expansion model to relax the constraint on the fixed expansion date and developed a numerical model for modular expansion design. In the numerical model, the instantaneous penalty function is assumed to have a payoff form of a call option, which leads to the modeling of the total expected penalty overtime as an Asian option. By utilizing the finite difference scheme, we transformed the underlying Asian real option into a Bermudan real option and hence relaxed the constraint on the timing of the second expansion. However, the applicability of the numerical model depends solely on the assumption that the demand growth follows a GBM. Difficulties arise when the closed form solution cannot be derived for the random process involved in the numerical model. Hence, we developed a more volatile yet easier to understand simulation approach using the Monte-Carlo simulation technique. When combined with the use of the two-dimensional linear interpolation, the simulation method results determined for the optimal timing and sizing for the second expansion were consistent to the numerical method. Unlike the numerical model, the simulation model is not restricted by the stochastic form of the demand growth hence making it more applicable to real world projects. The disadvantage of the simulation approach that it has a much slower optimization speed compared to the numerical method when accuracy is considered. Applying the RO framework we developed in this thesis, we plotted a surface of capital requirements with respect to initial plant design $K_1$ and $K_{2\text{max}}$. Our results suggested that a local minimum exists for the WWTP design project, and the minimum point reveals the optimal expansion strategy that should be
taken. When our results are compared to those obtained using traditional methods, the modular expansion is expected to save more than 30% in capital when compared to design approaches based on demand expectation, and is expected to save more than 8% when compared to design approaches based on Monte-Carlo simulation alone. Therefore, WWTP projects can benefit from the analysis of modular expansion models within the RO framework.

5.2 Recommendations for Future Studies

Future improvements can be made to WWTP modular expansion model. First, for the numerical method, a more general framework for developing closed form solutions for stochastic process shall be developed so that it can be applied to other more sophisticated random processes. Second, a more time-efficient simulation approach should be addressed, studies using the least-squares Monte-Carlo approaches such as works from Longstaff and Schwartz (Longstaff & Schwartz, 2001), Gamba (Gamba, 2002) should be investigated. Third, as the wastewater treatment process is a complicated process that involve many hydraulic, chemical processes, more random variables representing other factors such as effluent stats, treatment performance etc. should be included in the RO framework, thus making the model more applicable in industry. Finally, many wastewater treatment plant designs may allow for multiple expansions. Relaxing the constraint on the number of expansions allowed may further improve the applicability of this model, thus it would allow the government to save millions of dollars that can be allocated in other areas and would be an interesting topic for future work.
6 Bibliography


Hull, J. (2010). In J. Hull, Fundamentals of Futures and Options Markets (7th ed.).


APPENDICES

A. Expected total cost with respect to $K_1$ and $K_{2\text{max}}$

Table 3. Expected total cost with respect to $K_1$ and $K_{2\text{max}}$. Parameters used are shown in Table 1 and Table 2. Units (CAD$)

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B. Results using single stage Monte Carlo optimization

Figure 18 Single stage Monte Carlo optimization results, parameters used are stated in Table 1 and Table 2.

Table 4. Single stage Monte Carlo optimization results

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