Effects of sample size and model form on the accuracy of model-based estimators of growing stock volume

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Effects of sample size and model form on the accuracy
of model-based estimators of growing stock volume

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Abstract

In this study we investigate the use of model-based inference in forest surveys where auxiliary data are available as a probability sample. We evaluate the effects of model form and sample size on estimators of growing stock volume, based on different types of remotely sensed auxiliary data. The study was performed through Monte Carlo sampling simulation using a two-phase sampling design within a simulated study area resembling the conditions in mid-western Finland. We show that the choice of model has a minor to moderate effect on the precision of model-based estimators. Similarly, the choice of estimator of the variance-covariance matrix of model parameter estimates, which is at the core of uncertainty assessment in model-based inference, also was found to have a minor to moderate effect on the precision. Regarding sample sizes, the model error contribution to the total variance remains the same regardless of first phase sample size (i.e. the size of the sample of auxiliary data); to reduce the model-error contribution there is a need to increase the second phase sample size (i.e. the size of the sample of field plots for developing regression models). As a baseline for comparisons, model-assisted estimators were applied and were found to be about equally precise as the model-based estimators, in accordance with the theory for the case when models are estimated from the sample data.

Keywords: Airborne LiDAR, heteroskedasticity, Landsat, model-based estimator, sampling simulation.

1 Introduction

The global interest in forests is increasing due to the multiple services provided by this ecosystem (e.g., [Mery et al., 2005](#)). As a consequence, several international agreements, such as the United Nations Framework Convention on Climate Change (UNFCCC) ([UNFCCC, 1997](#)) and the Convention on Biological Diversity (CBD), prescribe measures related to the management of forests and require recurrent reporting of information about forest state and change.

While the general interest in forests is increasing, so is the interest in methods for forest inventories. The UNFCCC is an important driver of this development, since it requires greenhouse gas emissions to be reported annually. For forest ecosystems, many countries apply the stock change method ([Cienciala et al., 2008](#)) which implies that carbon stocks are repeatedly assessed and that emissions of carbon dioxide are calculated from the differences of carbon stocks between successive inventories. The implementation of the REDD+ (Reducing Emissions from Deforestation and Forest Degradation) mechanism is expected to provide further incentives for conducting forest inventories, especially in developing countries in the tropics.

National forest inventories (NFIs) ([Tomppo et al., 2010](#)) are conducted in a large number of countries, typically in regions with good infrastructure conditions. The normal practice is to conduct these inventories through field surveys, facilitating detailed and precise assessments of target variables related to trees, other vegetation, site conditions, and land use. However, following the increased interest also in remote forest areas, e.g. due to the REDD+ mechanism, it has become clear that field surveying alone will not be cost-efficient since remote forest areas in the boreal and tropical zones are difficult to reach and thus expensive to survey with field-based methods. As a consequence, remote sensing assisted forest inventories have been intensively studied during the last decades ([Nelson et al, 2008, 2009](#); [Ståhl et al., 2011](#); [Gregoire et al., 2011](#), etc.). While many authors (e.g., [McRoberts, 2011](#); [Tomppo et al., 2011](#)) point out that remote sensing alone will not be a viable option, it remains to be clarified what combinations of remote sensing and field surveying are appropriate, as well as what estimation techniques should be applied when combining the two data sources.

Today, several types of remotely sensed (RS) data are available from most parts of the world. In principle, the usefulness of the different RS sources depends on what correlations can be obtained between the target forest variables and the RS metrics, the availability of RS

data, the acquisition costs, and the possibility to link the RS metrics to some appropriate source of field data. Often, different RS sources contribute with information of different kinds and thus combinations of several sources of RS auxiliary data may be appropriate (e.g., [Saarela et al., 2015](#)).

The requirements for uncertainty estimates are steadily increasing, not least due to the requirements from the UNFCCC, but often it is unclear how uncertainty assessments should be conducted. For example, [McRoberts \(2011\)](#) demonstrates that the standard type of uncertainty assessment through, so-called, confusion matrices is often misleading. Further, an increasing number of studies (e.g., [Andersen et al., 2012](#); [Gobakken et al., 2012](#)) utilize RS sample data rather than data sets arising from complete enumeration; thus there is a need to account for several sources of errors in the uncertainty analysis, including sampling errors ([Ståhl et al., 2011](#)).

Forest resources assessments in remote forest areas would probably need to be conducted through sample surveys, using a large sample of RS data and a sparse sample of field data (e.g., [Gregoire et al., 2011](#)). Several studies on this topic are reported, (e.g. [Gregoire et al., 2011](#); [Ståhl et al., 2011](#); [Næsset, 2011](#); [Ene et al., 2012](#); [Saarela et al., 2015](#)), but several issues need to be further investigated. Important examples include what field sampling designs to apply, what statistical inferential framework to apply, and how to make the overall procedure straightforward.

From a statistical point of view, two main approaches for combining RS and field data exist. With design-based inference, model-assisted estimators ([Särndal et al., 1992](#)) known to be approximately unbiased can be used. Uncertainty analyses can be conducted based on standard sampling theory. Examples of studies where this estimation framework has been applied include [Gregoire et al. \(2011\)](#), [Andersen et al. \(2012\)](#), [Ene et al. \(2012\)](#) and [Saarela et al. \(2015\)](#). However, model-assisted estimators rely on the availability of a probability sample of field data from the entire study area.

With model-based inference, estimators of target quantities rely on a model ([Gregoire, 1998](#)). The model can be developed from any sample from the target population, and not necessarily a probability sample. Further, contrary to design-based inference, model-based inference assumes that the target quantities for individual population elements are random, and thus population totals and means are also random variables. Model-based estimators have been applied in forest surveys by, e.g., [McRoberts et al. \(2006\)](#) and [Ståhl et al. \(2011\)](#). The availability of data and the choice of model have implications for how the accuracy

assessment in model-based inference can be performed. At the core of this assessment is the covariance matrix of the model parameter estimates, and for estimating this matrix correctly, properties of modelling features such as heteroskedastic error terms (e.g., [White, 1980](#); [Davidson & MacKinnon, 1993](#)) must be accounted for.

The objective of this study was to evaluate the accuracy of model-based estimators of growing stock volume using different model forms and sample sizes for developing the models and for acquiring auxiliary data. The study was conducted through Monte Carlo sampling simulation using an simulated population resembling the forest in Kuortane, western Finland. We describe what estimators of the covariance matrix for the model parameter estimates are linked to different models, and in some cases evaluate different estimators for the same model in order to evaluate effects of heteroskedasticity. As a baseline for comparison, we also evaluated the precision of a model-assisted estimator.

Further, we evaluated the use of two different sources of auxiliary information, Landsat 7 ETM+ and airborne Light Detection And Ranging (LiDAR) scanning data.

2 Material and Methods

2.1 Overview

The study was performed using Monte Carlo sampling simulation (e.g., [Gregoire & Valentine, 2008](#)) within a simulated study area resembling the forests of Kuortane, western Finland. Based on real data from field measurements and LiDAR and Landsat acquisitions, the copula technique ([Ene et al., 2012](#)) was applied to construct the study population.

We focused our study on estimating growing stock volume using model-based inference (e.g., [McRoberts et al., 2006](#)). In this context several issues related to the estimators and the variance estimators arise depending on what models are applied to estimate volumes from the RS data.

It is known (e.g., [Davidson & MacKinnon, 1993](#)) that the ordinary and unweighted non-linear least squares estimators of the covariance matrix for the model parameter estimates are valid only when the observations are independent and the vector of residuals has homogeneous variance. In this study we compared the standard covariance matrix estimator with heteroskedasticity-consistent estimators for linear and nonlinear regression ([White, 1980](#)).

We applied three different model forms in this study. The first was a standard multiple

linear model (Chatterjee & Hadi, 2013), the second was a multiplicative model which was linearised through log-transformation (e.g., Solberg et al., 2010; Næsset, 2011). Thirdly, we used a model that was linearised through a square root transformation (Gregoire et al., 2008).

We applied two-phase sampling (Gregoire & Valentine, 2008), where the first phase sample units were 500 m wide strips and the second phase sample units were grid cells within the strips. The strips were used because LiDAR data are typically acquired in this fashion. RS data were only available for first-phase units (strips) and field data correspondingly only for second-phase units (grid cells). The sampling design was simple random sampling without replacement of strips, and simple random sampling without replacement of grid cells within the strips. Fig.1 provides an overview of Kuortane together with the location of strips.

As a baseline for comparison with the model-based estimators, a model-assisted estimator was applied as well.

2.2 The simulated study population

As reference data for the development of the simulated population we used data collected from the Kuortane study area. The dominating tree species in this area is (middle-aged) Scots pine (*Pinus sylvestris*). Norway spruce (*Picea abies*) and deciduous trees, mainly birches (*Betula pubescens*), occur as mixtures. Because we applied the area-based approach (Næsset, 2002) for the extraction of Landsat and LiDAR features, the entire area was tessellated into 16 m x 16 m squares denoted 'grid cells'. Grid cells on non-forest areas were masked out using digital map data (Tomppo et al., 2008). Hence, our study population comprised 818016 grid cells, which is equivalent to an area of 20941 hectares; it comprised forest areas, only.

Field data were sampled using a modified version of the Finnish NFI, where the sampling density was increased and the number of measured variables was reduced (for details see Saarela et al., 2015). The field survey was conducted during summer 2006. A total of 441 circular plots of radius 9 m (on forest areas) were measured. Growing stock volume per hectare for each sample plot was estimated as described in Laasasenaho (1982) and Tomppo et al. (2008). Several plots had zero growing stock records; they were excluded from our reference data in order to avoid some technical problems in the comparison of models (e.g. taking the logarithm of zeros). Only non-zero observations of growing stock volume were retained for creating the simulated population.

Landsat data were acquired in June 2006 (path 190 and row 16). The raw imagery was downloaded from the [U.S. Geological Survey \(2014\)](#) server. The image was geo-referenced to the ETRS35-FIN metric coordinate system, and the pixel size was re-sampled to 16 m × 16 m using the nearest neighbour re-sampling method. LiDAR data were acquired in July 2006 using an Optech 3100 laser scanning system. The average flying altitude above terrain was 2000 m. The mean footprint diameter was 60 cm and the average pulse density was 0.64 m⁻². Last returns data were used to model the ground surface using the OPALS software ([Kraus & Pfeifer, 2001](#)). First returns data were used to model the vegetation and extracting twenty-six LiDAR metrics for each 16 m × 16 m grid cell and circular field plot with the FUSION software ([McGaughey, 2012](#)). Details about the RS data used in this study are presented in [Saarela et al. \(2015\)](#).

Preliminary regression models were developed in order to evaluate what Landsat and LiDAR features should be made available for all grid cells in the population through the copula technique. For all three model forms, a stepwise selection algorithm ([R Development Core Team, 2013](#)) coupled with Akaike's information criterion ([Bozdogan, 1987](#)) was applied based on the field plots. For Landsat models, the subset of predictors for all model forms comprised Landsat bands 2, 3 and 5, corresponding to the green (*B20*), red (*B30*), and shortwave infra-red band (*B50*). For the LiDAR models the features were maximum height (h_{max}), the 80th percentile of the height distribution (h_{80}), the canopy relief ratio (*CRR*) defined by $(h_{mean} - h_{min}) / (h_{max} - h_{min})$ where h_{max} and h_{min} refer to corresponding canopy height, and the percentage of first returns above 2 m (p_{veg}) as a crown cover estimate.

The simulated study population was developed following the procedure described by [Ene et al. \(2012\)](#), who pioneered the use of copulas for constructing forest populations. The copula technique can be described as a tool for generating a large set of data from a small set of data, where the large set will have the same multivariate distribution of the study variables as the small set. We modelled a relationship between field, LiDAR and Landsat data in order to retrieve a large sample of predictor variables and the corresponding responses. A copula model ([Nelsen, 2006](#)) was fitted to the plot-level data, LiDAR metrics, and Landsat data. The VineCopula package ([Schepsmeier et al., 2013](#)) for the R software ([R Development Core Team, 2013](#)) was used to generate a copula population of 3 000 000 observations. A nearest neighbour technique ([Beygelzimer et al., 2013](#)) was then applied to impute data from the copula population to all grid cells in Kuortane based on LiDAR and Landsat data. The mean value of growing stock volume of this simulated population was 100.98 m³ha⁻¹, which

is thus our ‘true’ population mean. See Table 1.

2.3 Two-phase sampling

Our two-phase inferential approach in this study follows Ståhl et al. (2011). The regression models linking growing stock volume reference and RS data were developed based on the second-phase sample. Thus, in our case the field plots were squares and assumed to be perfectly co-located with the grid cells from which RS data were available.

Our target parameter was the population mean $\mu_Y = \frac{1}{M} \sum_{t=1}^M y_t$, where y_t is the true value of growing stock volume for t^{th} grid cell, and M is the total number of grid cells in the population.

The first-phase sample is a sample of n out of N strips. Thus the sample contains all the grid cells in the n selected strips. The second-phase sample corresponds to sampled grid cells within the selected strips.

We use the notation G for strip totals, i.e.

$$\hat{G}_{MB_i} = \sum_{t=1}^{M_i} g(x_t, \hat{\beta}) = \sum_{t=1}^{M_i} \hat{y}_t \quad (1)$$

is the sum of the $\hat{y}_t = g(x_t, \hat{\beta})$ values for the M_i grid cells within the i^{th} strip, where “MB” denotes “model-based”. The notation $g(x_t, \hat{\beta})$ is used as a generic notation for model predictions of the t^{th} grid cell. Our estimator for the mean value μ_Y is

$$\hat{\mu}_{Y_{MB}} = \frac{\sum_{i=1}^n \hat{G}_{MB_i}}{\sum_{i=1}^n M_i} \quad (2)$$

This ratio estimator was applied since the strips varied considerably in size. The variance of $\hat{\mu}_{Y_{MB}}$ can be estimated as (Ståhl et al., 2011, p. 101, Eq. 15)

$$\begin{aligned} \hat{V}(\hat{\mu}_{Y_{MB}}) \approx & \frac{1}{\bar{M}^2} \left[\frac{(N-n)}{N} \frac{\sum_{i=1}^n (\hat{G}_{MB_i} - \hat{\mu}_Y M_i)^2}{n(n-1)} + \right. \\ & \left. \sum_{k=1}^{p+1} \sum_{l=1}^{p+1} \widehat{Cov}(\hat{\beta}_k, \hat{\beta}_l) \hat{G}'_k \hat{G}_l \right] \end{aligned} \quad (3)$$

where $\overline{M}$ is the first phase sample mean of the M_i , $(p + 1)$ is the number of model parameters (see Section 2.4), $\widehat{Cov}(\hat{\beta}_k, \hat{\beta}_l)$ is the estimated covariance between the parameter estimates $\hat{\beta}_k$ and $\hat{\beta}_l$, $\widehat{G}'_k = \frac{1}{n} \sum_{i=1}^n \sum_{t=1}^{M_i} g'_k(x_{it}, \hat{\beta})$, and $g'_k(x_{it}, \hat{\beta})$ is the derivative of $g(x_{it}, \hat{\beta})$ with respect to the k^{th} model parameter.

From Eq.(3) it can be observed that the variance of the model-based estimator has two terms. The first term corresponds to the sampling error and the second term to the model error. Further, it can be observed the covariance matrix of the model parameter estimates is a core feature of the model error part. As described in Section 2.5 we analysed different estimators of this matrix.

For comparison, a model-assisted estimator was applied as well. We used the model-assisted estimator presented by Särndal et al. (1992, p. 327):

$$\hat{\mu}_{Y_{MA}} = \frac{\sum_{i=1}^n \hat{G}_{MA_i}}{\sum_{i=1}^n M_i} \quad (4)$$

with $\hat{G}_{MA_i}$ given by

$$\hat{G}_{MA_i} = \sum_{t=1}^{M_i} \hat{y}_t + \sum_{t=1}^{m_i} \frac{y_t - \hat{y}_t}{\pi_{t|i}} \quad (5)$$

where $\pi_{t|i}$ is the conditional probability that unit t is included in the second phase sample within the i^{th} strip, m_i is the second phase sample size within the i^{th} strip, and “MA” denotes “model-assisted”.

2.4 Investigated models

2.4.1 The linear model

The first model form was a standard linear model, denoted ‘LINEAR’, i.e.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \epsilon \quad (6)$$

This type of model is sometimes (e.g., Nelson et al, 1988) used for modelling biomass and volume based on RS data. For this model the residuals are often heteroskedastic due to the size-dependent variability in the case of biomass or volume.

2.4.2 The log-transformed multiplicative model

Our second model was a multiplicative model of the form

$$y = e^{\beta_0} x_1^{\beta_1} x_2^{\beta_2} \cdots x_p^{\beta_p} e^{\epsilon} \quad (7)$$

This model can be linearised by taking the natural logarithm of both sides, whereby we obtain the linear ‘LOG-LOG’ model

$$\ln y = \beta_0 + \beta_1 \ln x_1 + \beta_2 \ln x_2 + \cdots + \beta_p \ln x_p + \epsilon \quad (8)$$

This model is sometimes used for estimating biomass or volume from RS data. Examples include Nelson et al (1997) and Gobakken et al. (2012). The logarithmic ($\ln$) transformation is often used as a means to obtain homogeneous residual variance (Baskerville, 1972; Clifford et al., 2013).

To use the model in our model-based inferential framework back-transformation by the exponential of the estimated value was required. To avoid biased estimates a correction for back-transformation bias was applied (Baskerville, 1972). There are several approaches available for this (e.g., Baskerville, 1972; Snowdon, 1991; Clifford et al., 2013); in this study we chose to apply the ratio correction $C = \bar{y}/\hat{y}$ proposed by Snowdon (1991). Hence, each estimated value was multiplied by the ratio C , which means that the model intercept is increased by $\ln C$.

2.4.3 The square root transformed model

The third model was

$$y = (\beta_0 + x_1\beta_1 + x_2\beta_2 + \cdots + x_p\beta_p + \epsilon)^2 \quad (9)$$

It can be linearised through a square root transformation of the response variable, i.e.

$$\sqrt{y} = \beta_0 + x_1\beta_1 + x_2\beta_2 + \cdots + x_p\beta_p + \epsilon \quad (10)$$

The model is denoted ‘SQRT’. It has been applied for modelling biomass from RS data by, e.g., Næsset (2011). Like the ‘LOG-LOG’ the square root transformation is often applied to make the residual variance homogeneous (Chatterjee & Hadi, 2013). Also in this case we need

to back-transform the estimated values before we apply them in our model-based inferential framework. In this case correction for back-transformation bias was performed according to the method proposed by [Gregoire et al. \(2008\)](#), which is developed specially for this model. This correction implies that the quantity of the mean square error of the estimated regression model, Eq.(10), is added to back-transformed estimated values of growing stock.

2.5 Covariance matrix estimators

The model-based variance estimator Eq.(3) depends on the estimator of the covariance matrix of the model parameter estimates. The standard estimator under ordinary least squares (OLS) regression is ([Davidson & MacKinnon, 1993](#))

$$\widehat{Cov}(\hat{\beta}) = \hat{\sigma}^2 (\mathbf{X}^T \mathbf{X})^{-1} \quad (11)$$

where $\mathbf{X}$ is a matrix of dimension $m = \sum_{i=1}^n m_i$ times $(p + 1)$, $\hat{\sigma}^2 = \frac{\hat{e}^T \hat{e}}{m - (p + 1)}$ is the residual variance and $\hat{e}^T \hat{e}$ is a sum of squared estimated residuals $\hat{e}$. This estimator assumes independent observations and homogeneous variance of the residuals.

If the residuals variance is heteroskedastic (non-homogeneous), then the estimator Eq.(11) will be biased ([White, 1980](#)). Instead, the heteroskedasticity-consistent (HC) covariance matrix estimator known as the *Eicker-Huber-White* estimator is applied ([White, 1980](#), p. 820-821). The estimator is asymptotically valid when there is a model of heteroskedastic residuals of unknown form ([White, 1980](#); [Davidson & MacKinnon, 1993](#)).

$$\widehat{Cov}(\hat{\beta}) = (\mathbf{X}^T \mathbf{X})^{-1} \left[\sum_{t=1}^m \hat{e}_t^2 X_t X_t^T \right] (\mathbf{X}^T \mathbf{X})^{-1} \quad (12)$$

In the case of back-transformation, the models given by Eq.(7) and Eq.(9) are of nonlinear form. For the nonlinear models the nonlinear least square (NLS) covariance matrix estimator should be applied ([Davidson & MacKinnon, 1993](#)). In general we cannot obtain an exact expression for $Cov(\hat{\beta})$, hence the asymptotic NLS covariance matrix estimator is applied

$$\widehat{Cov}(\hat{\beta}) \cong \hat{\sigma}^2 (\mathbf{F}^T \mathbf{F})^{-1} \quad (13)$$

where $\hat{\sigma}^2$ is the estimated residual variance following back-transformation of the predicted values and correction for back-transformation bias using the ratio correction factor by [Snowdon \(1991\)](#) for ‘LOG-LOG’ models and the factor by [Gregoire et al. \(2008\)](#) for ‘SQRT’. $\mathbf{F}$ is

a matrix of partial derivatives with respect to model parameters of the nonlinear regression model, i.e. $F_{tj} = \frac{\partial g(\hat{\beta}, X_t)}{\partial \beta_j}$ for the m grid cells in the second phase sample.

The subsequent nonlinear heteroskedasticity-consistent (NHC) covariance matrix estimator for the variance-covariance matrix looks as (White, 1980, p. 821)

$$\widehat{Cov}(\hat{\beta}) \cong (\mathbf{F}^T \mathbf{F})^{-1} \left[\sum_{t=1}^m \hat{e}_t^2 \mathbf{F}_t \mathbf{F}_t^T \right] (\mathbf{F}^T \mathbf{F})^{-1} \quad (14)$$

How good these approximations are depends on the model and on the sample size; the approximations are generally more accurate for models that are closer to linearity and for larger sample sizes (Davidson & MacKinnon, 1993).

We applied a test for heteroskedasticity proposed by White (1980, p. 823, Eq.1). To perform the test we used a random sample of 1000 grid cells and estimated model parameters from Table 2 and Table 3 in Section 3.1.

For ‘LINEAR’ models we compared performances of OLS and HC covariance matrix estimators [Eq.(11) and Eq.(12)]. For ‘LOG-LOG’ and ‘SQRT’ models, the performances of OLS, NLS, HC and NHC estimators were compared [(Eq.(11), (13), (12) and (14)].

2.6 Sampling simulation

Monte Carlo sampling simulation with 10 000 repetitions was applied. At each repetition new parameter estimates for the pre-selected variables and their corresponding covariance matrix estimates were computed. Averages of estimated values and estimated variances were recorded.

Empirical variances were computed based on the outcomes from the 10 000 iterations as ($\hat{\mu}_i$ is an estimate of the population mean in an individual iteration)

$$V(\hat{\mu}_Y)_{emp} = \frac{1}{(10000 - 1)} \sum_{i=1}^{10000} (\hat{\mu}_i - \bar{\hat{\mu}})^2 \quad (15)$$

These variances should be good approximations of the true variance for the different sampling strategies (Gregoire & Valentine, 2008) and thus they can be used in ranking the different strategies. Further, they can be used for evaluating the performance of the variance estimators by comparing the average of estimated variances with the empirical variance.

Relative standard errors (‘RSE’) in Table 2 and Table 3 were obtained by dividing the empirical standard errors by the corresponding average parameter estimate. We also presented

in Table 2 and Table 3 absolute difference estimates and their averages between empirical and estimated standard errors for three model forms and two RS auxiliary data types. First phase sample sizes evaluated ranged from 2 to 62 strips (i.e. full cover). Second phase sample sizes ranged from 15 to 1000 grid cells.

In Table 5 and Table 6 RSE was obtained by dividing the empirical standard error by the true population mean. For each estimated variance an absolute difference from the corresponding empirical variance was estimated and presented in Table 5 and Table 6 denoted as ‘diff.’.

Because our strips are of unequal sizes (the smallest strip has 86 grid cells), for the Section 3.3 results we performed the simulation with the second phase sample size of 75 grid cells only, to ensure that the second phase sample size was always smaller than the number of grid cells in the first phase sample of strips. Relative bias of a variance estimator was estimated as

$$RBIAS = \frac{100\% \times [\hat{V}(\hat{\mu}_{Y_{MB}}) - V(\hat{\mu}_{Y_{MB}})_{emp}]}{V(\hat{\mu}_{Y_{MB}})_{emp}} \quad (16)$$

where $\hat{V}(\hat{\mu}_{Y_{MB}})$ is the average of the estimated model-based variances and $V(\hat{\mu}_{Y_{MB}})_{emp}$ is the empirical variance.

In Section 3.4 a relative difference between model-based and model-assisted empirical variances was estimated as

$$Relative\ Difference = \frac{100\% \times [V(\hat{\mu}_{Y_{MB}})_{emp} - V(\hat{\mu}_{Y_{MA}})_{emp}]}{V(\hat{\mu}_{Y_{MA}})_{emp}} \quad (17)$$

3 Results

3.1 The estimated models

A sample of 1000 observations from the copula population was drawn to illustrate the performance of the three different model forms in the case of Landsat and LiDAR data, respectively, through Monte Carlo sampling simulation with 10 000 iterations. In Table 2 and Table 3 we present the averages of estimated model parameters and the corresponding standard errors for the three different model forms.

For Landsat models, the average of absolute differences between empirical and estimated

standard errors (SE) was smaller for the OLS estimator compared to the other covariance matrix estimators (see Table 2 and Table 3). For LiDAR models, the HC estimators performed the best. The test for heteroskedasticity revealed that all models had heteroskedastic residuals (see Table 4). Figure 2 supports this as well.

3.2 Performances of the OLS, NLS, HC and NHC covariance matrix estimators in the model-based variance estimator with wall-to-wall first phase data

In this section we present results for model-based variances estimated for different second phase sample sizes but with all strips included in the first phase sample. In this case the sampling error portion of the variance estimator [Eq.(3)] is equal to zero. Model-based variances were estimated based on the different covariance matrix estimators [Eq.(11) – (14)].

Regarding the precision of estimators, the LiDAR models always led to more precise results. Comparing the performances of different models, the square-root transformed models resulted in smallest variances. Studying the performance of the different covariance matrix estimators, similar magnitudes of variances were obtained but the NLS estimators resulted in the smallest variances.

3.3 Precision of the model-based estimators at different first phase sample sizes

Based on previous results in Section 3.2, we used variances based on OLS covariance matrix estimator Eq.(11) in case of ‘LINEAR’ models, and NLS estimator Eq.(13) in case of ‘LOG-LOG’ and ‘SQRT’ models (see also Table 3S and Table 4S in supplementary materials available online).

In Figure 3 we present relative standard errors and their components for the ‘SQRT’ model type. Figure 4 presents relative bias of estimated variance for ‘SQRT’ models. The estimated variance components due to the model error for all three model forms using all covariance matrix estimators – OLS, NLS, HC and NHC – can be found in the supplementary material that is available online (Tables 1S and 2S).

3.4 Comparing model-based and model-assisted estimators

In this section we compare the precision of model-based and model-assisted estimators [Eq. (2) and Eq.(4)] through their empirical variances estimated by Eq.(15). Figure 5 shows how the relative difference[Eq.(17)] varies over different first phase sample sizes. It can be observed that the precision of the two types of estimators was always of similar magnitude. Again, full results are provided in Table 5S and 6S in the supplementary material available online.

4 Discussion

As expected, using LiDAR data in model-based estimation of growing stock volume gave more precise results than using Landsat data. The reason is that LiDAR data are strongly related to variables such as growing stock volume and biomass (Næsset, 2002). On the other hand, Landsat data are less expensive and before any large-scale application of the strategies described in this article it should be evaluated what auxiliary data source is most cost-efficient, i.e. what source of data provides the highest precision at a given inventory budget.

Although no major differences were found between the models evaluated, the ‘SQRT’ model led to slightly more precise results than the other models used in the model-based estimators. This is in line with several other studies (e.g., Gregoire et al., 2008; Næsset, 2011) where this model has been suggested for biomass prediction based on RS data. The ‘SQRT’ model fitted our data well and thus the variances of the model parameter estimates were lower compared to using the other models. Further, our results showed that the choice of covariance matrix estimator affects the model-based variance estimators. The variance of the residuals should be homogeneous in case the standard estimator of the covariance matrix is applied (e.g., Davidson & MacKinnon, 1993). Often this is not the case, and thus we evaluated alternative estimators. In general we obtained the best results, comparing estimated and empirical variances, when we applied the NLS covariance matrix estimator. However, for moderate to large sample sizes no major differences between the different estimators were observed and thus the results of our study suggest that the choice of covariance matrix estimator is not a crucial part of model-based inference of growing stock volume based on RS data. Still, as pointed out by White (1980), in the case of non-homogeneous data and

large sample sizes it might be advisable not to use the standard outputs from regression software but apply HC covariance matrix estimators.

Our results show that the variance estimator of the ratio estimator is negatively biased when a small first phase sample is selected (Figure 3). This is a known property of ratio estimators (e.g., Särndal et al., 1992; Gregoire & Valentine, 2008). Under simple random sampling without replacement the bias is of order $1/n$ (Särndal et al., 1992, Section 5.6). In our study, the bias was large for sample sizes smaller than about 10 (LiDAR) and about 35 (Landsat), respectively. In Särndal et al. (1992) it is stated that the bias should be negligible for sample sizes larger than 20. One way to reduce the bias of the model-based variance estimator is to omit the finite population correction factor $(N-n)/N$ in Eq.(3).

Studying different sample sizes, expected results in terms of decreased standard errors over increasing sample sizes were obtained. However, an interesting feature of model-based estimators is that the magnitude of the model error variance will remain approximately the same regardless of the (first phase) sample size. Thus, at a certain point the variance of estimators can only be negligibly decreased by increasing the sample size. Interestingly, however, we observed a slight tendency of decreased model error variance with increasing (first phase) sample size (see Table 1S and Table 2S in supplementary materials available online). This was most likely due to the second phase sample being more evenly spread across the population and thus the model parameters were more precisely estimated.

Comparing model-based and model-assisted estimators, we found that the difference in precision between these two types of estimators were always small. This is due to how our models were developed in this study. As shown by Särndal et al. (1992, p. 231-232) the sum of the residuals of a regression model is zero under certain fairly common conditions. Thus, if a simple random sample is selected from the population and used for model estimation the sum of the residuals will be zero, and thus the second term of the model-assisted estimator [Eq.(5)] is also zero. Consequently the model-based and the model-assisted estimators would always give the same result and thus their precision is the same. However, in our case the model-based estimators resulted in slightly higher precision compared to the model-assisted estimators, especially at large first phase sample sizes. The reason could be the spatially scattered second phase sample, which implied different inclusion probabilities for grid-cells depending on in what first phase strip they were located. However, the similarity of estimates by model-based and model-assisted estimators cannot be taken for granted in all cases. For example, models may be derived outside the target population and still be useful

for model-assisted estimation; any model bias would be corrected by the model-assisted estimators (Särndal et al., 1992). Further, model-based inference in general does not rely on a probability sample and thus purposive sampling might be applied to estimate the model parameters. In this cases, results from model-assisted and model-based estimators would most likely be very different.

In the simulations we used a fixed population, created with the copula technique. However, in principle the assumptions underlying model-based inference imply that population totals and means are random variables, since the variable linked to each population element is a random variable. Thus, comparing the performance of estimators under model-based inference with estimators under design-based inference (through model-assisted estimators) is not trivial. In principle such comparisons should be based on the generation of multiple random populations over which the average performance of model-based and model-assisted estimation is assessed. This approach was not used in our study. However, since a large population was studied the relative differences between different possible outcomes of random population totals and means would be very small and thus the difference in results between our approach (using a single population for the simulations) and the alternative approach (using multiple random populations) should be small. Instead, the main element contributing to variability for a given sampling strategy is the random selection of sampling units in the two sampling phases.

A conclusion of our study is that the the choice of model form and model parameter covariance matrix estimator has minor to moderate influence on the accuracy of population mean estimators in model-based inference of growing stock volume. However, in case the residual variance is non-homogeneous, the standard covariance matrix estimator of the model parameter estimates might be biased. Further, when models are constructed based on a probability sample from the target population, the difference in precision between model-based and model-assisted estimators is small or negligible in case the same sample is used also for the model-assisted estimation.

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Figure 1: The Kuortane study area and the delineation of the area into strips.

Table 1: Statistics about the simulated population.

LiDAR variables	max	min	sd	Landsat variables	max	min	sd
h_{max}	26.11	4.96	5.37	green band $B20$	159	24	3.98
h_{80}	22.22	1.50	5.12	red band $B30$	153	13	6.99
CRR	0.71	0.12	0.15	shortwave IR band $B50$	170	9	14.32
p_{veg}	85.83	16.86	17.29	growing stock volume, m^3ha^{-1}	597.08	15.84	90.72

Table 2: Estimated Landsat model parameters. Averages of parameter estimates (β_p) and averages of corresponding standard error estimates (SE) for a sample of 1000 grid cells, for 10 000 Monte Carlo iterations. The subscripts indicate the covariance matrix estimator used: OLS – Eq.(11), NLS – Eq.(13), HC – (12) and NHC – Eq.(14). Under SE_{emp} , the empirical standard errors as the standard deviations of the 10 000 parameter estimates are given.

Variable	$\hat{\beta}_p$	SE_{OLS} , est./ abs.diff.	SE_{NLS} , est./ abs.diff.	SE_{HC} , est./ abs.diff.	SE_{NHC} , est./ abs.diff.	SE_{emp}	RSE, %
‘LINEAR’							
<i>Intercept</i>	220.42	49.41/2.74	—	52.01/0.14	—	52.15	23.66
<i>B20</i>	3.05	1.72/0.08	—	1.79/0.01	—	1.81	59.19
<i>B30</i>	-2.18	0.92/0.02	—	0.94/0.00	—	0.94	-43.29
<i>B50</i>	-2.96	0.40/0.03	—	0.43/0.00	—	0.42	-14.37
Average of absolute differences		0.72	—	0.04	—	—	—
‘LOG-LOG’							
<i>Intercept</i>	11.59	1.81/0.18	1.94/0.06	2.07/0.08	2.75/0.76	1.99	17.19
<i>B20</i>	2.48	0.76/0.06	0.74/0.08	0.87/0.05	1.01/0.19	0.82	32.95
<i>B30</i>	-2.14	0.36/0.04	0.36/0.04	0.43/0.03	0.48/0.08	0.40	18.64
<i>B50</i>	-2.19	0.24/0.02	0.20/0.05	0.27/0.02	0.32/0.06	0.26	11.81
Average of absolute differences		0.08	0.06	0.04	0.27	—	—
‘SQRT’							
<i>Intercept</i>	16.31	2.22/0.13	2.93/0.58	2.35/0.00	3.54/1.19	2.35	14.44
<i>B20</i>	0.16	0.08/0.00	0.10/0.02	0.08/0.00	0.12/0.04	0.08	52.73
<i>B30</i>	-0.15	0.04/0.00	0.06/0.01	0.04/0.00	0.06/0.02	0.04	-28.29
<i>B50</i>	-0.14	0.02/0.00	0.03/0.01	0.02/0.00	0.03/0.01	0.02	-13.23
Average of absolute differences		0.04	0.15	0.00	0.31	—	—

Table 3: Estimated LiDAR model parameters. Averages of parameter estimates (β_p) and averages of corresponding standard error estimates (SE) for a sample of 1000 grid cells, for 10 000 Monte Carlo iterations. The subscripts indicate the covariance matrix estimator used: OLS – Eq.(11), NLS – Eq.(13), HC – (12) and NHC – Eq.(14). Under SE_{emp} , the empirical standard errors as the standard deviations of the 10 000 parameter estimates are given.

Variable	$\hat{\beta}_p$	SE_{OLS} , est./ abs.diff.	SE_{NLS} , est./ abs.diff.	SE_{HC} , est./ abs.diff.	SE_{NHC} , est./ abs.diff.	SE_{emp}	RSE, %
‘LINEAR’							
<i>Intercept</i>	-12.01	4.09/0.14	—	3.87/0.07	—	3.95	32.84
<i>h_{max}</i>	2.97	0.62/0.02	—	0.63/0.01	—	0.64	21.42
<i>h₈₀</i>	-9.40	1.29/0.44	—	1.70/0.03	—	1.73	-18.39
<i>CRR</i>	117.08	18.38/5.07	—	22.90/0.55	—	23.45	20.03
<i>p_{veg}</i>	0.54	0.16/0.05	—	0.21/0.01	—	0.21	39.64
<i>h₈₀²</i>	0.97	0.04/0.03	—	0.07/0.00	—	0.07	7.55
Average of absolute differences		0.96	—	0.11	—	—	—
‘LOG-LOG’							
<i>Intercept</i>	1.68	0.22/0.03	0.32/0.07	0.25/0.01	0.57/0.32	0.25	14.87
<i>h_{max}</i>	0.53	0.06/0.01	0.15/0.07	0.08/0.00	0.23/0.15	0.07	14.22
<i>h₈₀</i>	0.60	0.05/0.01	0.12/0.06	0.06/0.00	0.19/0.13	0.06	10.28
<i>CRR</i>	0.46	0.05/0.01	0.08/0.02	0.06/0.00	0.15/0.09	0.06	13.44
<i>p_{veg}</i>	0.19	0.05/0.01	0.08/0.01	0.06/0.00	0.14/0.08	0.06	32.45
Average of absolute differences		0.02	0.05	0.002	0.15	—	—
‘SQRT’							
<i>Intercept</i>	1.13	0.16/0.00	0.29/0.14	0.15/0.00	0.38/0.23	0.15	13.54
<i>h_{max}</i>	0.07	0.02/0.00	0.04/0.01	0.03/0.00	0.06/0.03	0.03	38.87
<i>h₈₀</i>	0.58	0.03/0.01	0.04/0.01	0.04/0.00	0.07/0.03	0.04	6.26
<i>CRR</i>	4.14	0.71/0.18	0.87/0.02	0.87/0.02	1.42/0.53	0.89	21.58
<i>p_{veg}</i>	0.02	0.01/0.00	0.01/0.00	0.01/0.00	0.01/0.00	0.01	38.35
Average of absolute differences		0.04	0.04	0.004	0.17	—	—

Table 4: White’s test for heteroskedasticity.

Model form	Landsat		LiDAR	
	$\hat{\chi}^2$	p -value	$\hat{\chi}^2$	p -value
‘LINEAR’	39.47	$9.48 \cdot 10^{-06}$	231.58	0
‘LOG-LOG’	54.69	$1.39 \cdot 10^{-08}$	108.37	$1.11 \cdot 10^{-16}$
‘SQRT’	25.90	$2.12 \cdot 10^{-03}$	200.29	0

Table 5: Estimated growing stock volume and model-based variances for Landsat models when using wall-to-wall data, i.e. $n = N$. The subscripts at $\widehat{V}(\hat{\mu}_{Y_{MB}})$ indicate the type of covariance matrix estimator applied in combination with Eq.(3) OLS – Eq.(11); NLS – Eq.(13); HC – Eq.(12); and NHC – Eq.(14).

Plots	$\hat{\mu}_{Y_{MB}},$ m^3ha^{-1}	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{OLS}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{HC}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NHC}$		$V(\hat{\mu}_{Y_{MB}})_{emp}$	RSE, %
		est.	diff.	est.	diff.	est.	diff.	est.	diff.		
‘LINEAR’											
15	96.00	544.54	37.94	—	—	376.29	206.19	—	—	582.48	23.90
25	98.20	280.36	11.41	—	—	229.35	62.43	—	—	291.77	16.92
75	100.44	86.50	0.77	—	—	81.88	5.39	—	—	87.26	9.25
250	100.79	25.15	0.67	—	—	24.77	0.29	—	—	24.48	4.90
1000	100.96	6.25	0.13	—	—	6.23	0.10	—	—	6.13	2.45
‘LOG-LOG’											
15	131.18	$1.21 \cdot 10^8$	$1.17 \cdot 10^8$	$1.28 \cdot 10^8$	$1.24 \cdot 10^8$	$5.92 \cdot 10^7$	$5.49 \cdot 10^7$	$8.76 \cdot 10^7$	$8.33 \cdot 10^7$	$4.27 \cdot 10^6$	$2.05 \cdot 10^5$
25	101.86	433.87	150.55	292.58	9.26	335.00	51.69	299.76	16.44	283.32	16.67
75	101.23	108.11	33.44	72.20	2.47	89.99	15.32	71.04	3.63	74.67	8.56
250	101.02	30.55	9.71	20.60	0.24	26.67	5.83	20.65	0.19	20.84	4.52
1000	101.00	7.54	2.33	5.09	0.11	6.68	1.47	5.15	0.06	5.21	2.26
‘SQRT’											
15	102.27	422.51	68.22	503.32	12.60	334.97	155.76	427.68	63.04	490.73	21.94
25	100.77	202.83	48.52	250.35	1.01	182.12	69.24	226.54	24.82	251.36	15.70
75	100.99	61.78	18.32	79.54	0.55	59.12	20.98	74.64	5.46	80.10	8.86
250	100.93	17.95	5.00	23.31	0.36	17.48	5.47	22.06	0.88	22.95	4.74
1000	100.98	4.46	1.34	5.81	0.00	4.36	1.44	5.50	0.30	5.80	2.39

Figure 2: Predicted values versus residuals for Landsat and LiDAR models.

Table 6: Estimated growing stock volume and model-based variances for LiDAR models when using wall-to-wall data, i.e. $n = N$. The subscripts at $\hat{V}(\hat{\mu}_{Y_{MB}})$ indicate the type of covariance matrix estimator applied in combination with Eq.(3) OLS – Eq.(11); NLS – Eq.(13); HC – Eq.(12); and NHC – Eq.(14).

Plots	$\hat{\mu}_{Y_{MB}},$ m^3ha^{-1}	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{OLS}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{HC}$		$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NHC}$		$V(\hat{\mu}_{Y_{MB}})_{emp}$	RSE, %
		est.	diff.	est.	diff.	est.	diff.	est.	diff.		
‘LINEAR’											
15	101.37	78.52	28.18	—	—	42.21	64.50	—	—	106.70	10.23
25	101.15	37.89	9.44	—	—	28.72	18.60	—	—	47.33	6.81
75	101.14	11.52	1.07	—	—	10.84	1.75	—	—	12.59	3.51
250	100.99	3.38	0.08	—	—	3.34	0.12	—	—	3.46	1.84
1000	101.01	0.85	0.00	—	—	0.85	0.01	—	—	0.85	0.91
‘LOG-LOG’											
15	100.62	$6.65 \cdot 10^{10}$	$6.65 \cdot 10^{10}$	$3.49 \cdot 10^{05}$	$3.49 \cdot 10^{05}$	$9.30 \cdot 10^{10}$	$9.30 \cdot 10^{10}$	$7.98 \cdot 10^{03}$	$7.88 \cdot 10^{03}$	101.05	9.96
25	100.51	$4.18 \cdot 10^{04}$	$4.17 \cdot 10^{04}$	62.75	13.74	$3.57 \cdot 10^{04}$	$3.56 \cdot 10^{04}$	54.91	5.89	49.02	6.93
75	100.89	14.70	0.28	14.27	0.70	14.81	0.16	14.29	0.68	14.97	3.83
250	100.92	3.19	1.24	4.06	0.37	3.26	1.16	4.05	0.38	4.42	2.08
1000	100.99	0.74	0.36	1.01	0.10	0.77	0.34	1.00	0.11	1.11	1.04
‘SQRT’											
15	100.90	48.18	32.26	72.42	8.01	39.51	40.93	49.91	30.53	80.44	8.88
25	100.75	24.19	17.14	37.17	4.16	27.63	13.70	31.78	9.55	41.33	6.37
75	100.95	7.29	5.16	11.94	0.51	10.75	1.69	11.03	1.42	12.45	3.49
250	100.94	2.12	1.53	3.53	0.12	3.38	0.27	3.29	0.37	3.65	1.89
1000	100.99	0.53	0.38	0.89	0.03	0.86	0.05	0.82	0.09	0.91	0.95

Figure 3: Relative standard errors and their components for 'SQRT' models.

Figure 4: Relative bias of model-based variance estimators for ‘SQRT’ models. As covariance matrix estimator in Eq.(3), Eq.(13) was used.

Figure 5: Relative difference between empirical model-based and model-assisted variances as a function of strip sample size, for ‘SQRT’ models.

Table 1S: Components of the model-based variance estimator Eq. (3), Landsat models based results; ‘ M_{OLS} ’ – model error component using regression coefficients covariance matrix estimator Eq. (11), ‘ M_{HC} ’ – model error component using Eq. (12), ‘ M_{NLS} ’ – model error component using Eq. (13), ‘ M_{NHC} ’ – model error component using Eq. (14) and ‘S’ – sampling error component; second phase sample size is 75 grid cells.

Strips	‘LINEAR’			‘LOG-LOG’					‘SQRT’				
	M		S	M				S	M				S
	<i>OLS</i>	<i>HC</i>		<i>OLS</i>	<i>NLS</i>	<i>HC</i>	<i>NHC</i>		<i>OLS</i>	<i>NLS</i>	<i>HC</i>	<i>NHC</i>	
2	88.89	83.99	26.73	110.43	74.70	91.38	73.75	49.73	63.33	81.77	61.02	76.94	21.89
4	88.06	83.36	15.32	109.63	73.53	90.99	72.34	29.19	62.71	80.99	60.25	75.94	12.77
6	87.07	82.16	10.32	108.83	72.70	90.44	71.84	19.48	62.20	79.99	59.65	75.20	8.61
8	86.57	81.92	7.75	107.88	72.23	90.02	71.32	14.19	61.84	79.50	59.44	74.83	6.40
10	86.30	81.51	6.16	108.17	72.29	90.35	71.17	11.20	61.77	79.35	59.16	74.46	5.15
12	86.47	81.72	5.16	108.01	72.19	89.79	71.02	9.18	61.91	79.59	59.19	74.63	4.35
14	86.78	82.35	4.30	108.14	72.48	90.25	71.18	7.61	62.01	79.94	59.45	75.01	3.64
16	85.93	81.14	3.66	106.72	71.75	88.87	70.66	6.42	61.25	79.01	58.61	74.14	3.08
18	85.60	80.87	3.12	106.60	71.27	88.73	70.19	5.48	61.10	78.61	58.51	73.74	2.62
20	85.87	81.19	2.68	107.06	71.64	89.11	70.45	4.71	61.40	78.99	58.77	74.00	2.24
22	86.23	81.63	2.29	107.55	71.94	89.56	70.94	4.05	61.66	79.32	59.03	74.62	1.92
24	86.45	82.00	1.99	107.74	72.17	90.30	71.25	3.52	61.76	79.49	59.24	74.85	1.67
26	86.73	81.93	1.76	108.32	72.36	90.52	71.64	3.08	61.91	79.64	59.42	74.92	1.46
28	87.37	82.77	1.50	109.09	73.12	90.69	72.20	2.69	62.42	80.47	59.78	75.85	1.26
30	86.39	81.83	1.32	107.27	72.13	89.51	71.17	2.36	61.54	79.53	59.02	74.69	1.11
32	84.92	80.35	1.15	105.64	70.93	87.92	69.77	2.06	60.67	78.14	57.97	73.18	0.96
34	86.49	81.66	1.01	107.61	72.41	89.90	71.29	1.81	61.82	79.72	59.10	74.73	0.84
36	87.88	82.98	0.89	109.17	73.59	90.85	72.07	1.58	62.72	80.99	59.82	75.78	0.74
38	86.37	81.74	0.78	107.79	72.10	89.94	70.95	1.39	61.73	79.44	59.18	74.54	0.65
40	85.58	80.91	0.68	107.07	71.46	89.60	71.09	1.20	61.13	78.62	58.62	73.97	0.57
42	84.83	80.54	0.59	105.68	70.53	88.18	69.73	1.04	60.48	77.88	57.99	73.19	0.49
44	84.14	79.75	0.51	105.50	70.00	87.60	69.20	0.90	60.05	77.17	57.50	72.49	0.42
46	85.28	80.72	0.44	106.05	70.86	88.44	69.87	0.77	60.77	78.20	58.23	73.35	0.37
48	87.83	82.96	0.38	109.46	73.15	91.06	71.78	0.65	62.79	80.63	59.97	75.46	0.31
50	87.58	82.76	0.30	109.81	73.15	90.93	71.62	0.52	62.90	80.61	59.95	75.30	0.25
52	86.23	81.58	0.25	107.73	71.85	89.60	70.51	0.42	61.69	79.17	59.01	74.09	0.20
54	83.86	79.27	0.18	105.00	70.05	87.78	68.81	0.32	60.12	77.11	57.53	72.30	0.15
56	83.54	79.05	0.13	104.15	69.67	87.21	68.47	0.23	59.77	76.82	57.31	72.17	0.11
58	82.03	77.81	0.09	102.57	68.51	86.22	67.23	0.15	58.78	75.49	56.46	71.04	0.07
60	81.35	77.01	0.04	101.06	67.49	84.77	66.57	0.07	58.06	74.70	55.79	70.17	0.04
62	81.50	77.31	0.00	101.65	67.67	84.48	66.95	0.00	58.19	74.71	55.84	70.51	0.00

Table 2S: Components of the model-based variance estimator Eq. (3), LiDAR models based results; 'M_{OLS}' – model error component using regression coefficients covariance matrix estimator Eq. (11), 'M_{HC}' – model error component using Eq. (12), 'M_{NLS}' – model error component using Eq. (13), 'M_{NHC}' – model error component using Eq. (14) and 'S' – sampling error component; second phase sample size is 75 grid cells.

Strips	'LINEAR'			'LOG-LOG'					'SQRT'				
	M		S	M				S	M				S
	OLS	HC		OLS	NLS	HC	NHC		OLS	NLS	HC	NHC	
2	12.10	11.36	209.70	15.41	14.93	15.63	14.95	159.60	7.74	12.49	11.28	11.62	188.40
4	11.88	11.20	115.40	14.98	14.74	15.08	14.73	89.91	7.60	12.33	11.17	11.47	104.56
6	11.67	10.94	76.61	14.88	14.43	15.02	14.40	59.93	7.43	12.07	10.91	11.19	69.57
8	11.54	10.90	56.37	14.86	14.30	14.92	14.32	44.18	7.38	11.95	10.84	11.14	51.19
10	11.49	10.83	43.36	14.79	14.18	14.82	14.15	34.02	7.32	11.89	10.76	11.03	39.42
12	11.47	10.78	34.97	14.66	14.23	14.75	14.15	27.49	7.32	11.89	10.73	11.00	31.82
14	11.54	10.83	28.89	14.59	14.29	14.71	14.22	22.76	7.37	11.96	10.82	11.09	26.31
16	11.46	10.85	24.23	14.64	14.19	14.83	14.25	19.10	7.31	11.86	10.79	11.06	22.07
18	11.44	10.79	20.65	14.34	14.22	14.40	14.13	16.26	7.30	11.88	10.76	11.03	18.81
20	11.48	10.76	17.84	14.39	14.13	14.53	14.13	14.07	7.28	11.87	10.72	10.99	16.24
22	11.42	10.79	15.43	14.41	14.19	14.44	14.22	12.14	7.30	11.86	10.80	11.04	14.05
24	11.52	10.85	13.41	14.60	14.23	14.68	14.27	10.55	7.32	11.92	10.78	11.06	12.20
26	11.52	10.85	11.68	14.62	14.35	14.75	14.34	9.20	7.33	11.96	10.83	11.13	10.64
28	11.58	10.80	10.32	14.54	14.41	14.62	14.17	8.13	7.34	12.02	10.77	10.99	9.40
30	11.45	10.72	9.06	14.57	14.32	14.73	14.15	7.13	7.26	11.92	10.74	10.98	8.25
32	11.31	10.69	7.95	15.17	14.03	14.92	14.07	6.25	7.19	11.72	10.62	10.87	7.23
34	11.42	10.65	6.99	14.63	14.21	14.70	14.01	5.50	7.25	11.86	10.64	10.90	6.36
36	11.63	10.76	6.15	14.64	14.40	14.69	14.10	4.84	7.32	12.06	10.72	10.96	5.60
38	11.45	10.75	5.36	14.80	14.23	14.98	14.13	4.22	7.28	11.87	10.70	10.96	4.88
40	11.35	10.81	4.67	14.43	14.09	14.55	14.25	3.67	7.25	11.75	10.74	11.02	4.25
42	11.23	10.66	4.04	14.89	13.96	14.90	13.96	3.18	7.15	11.63	10.58	10.83	3.68
44	11.16	10.51	3.47	14.28	13.89	14.37	13.80	2.73	7.09	11.57	10.47	10.72	3.16
46	11.32	10.56	2.95	14.10	14.01	14.21	13.84	2.33	7.15	11.73	10.50	10.76	2.69
48	11.74	10.78	2.49	15.00	14.50	15.11	14.06	1.95	7.36	12.13	10.69	10.94	2.26
50	11.76	10.73	2.04	14.52	14.39	14.52	13.98	1.60	7.36	12.11	10.62	10.85	1.86
52	11.50	10.65	1.64	14.40	14.17	14.39	13.84	1.29	7.25	11.91	10.56	10.76	1.49
54	11.24	10.72	1.26	14.13	13.79	14.15	13.93	0.99	7.18	11.59	10.58	10.88	1.14
56	11.17	10.75	0.91	14.26	13.77	14.36	14.06	0.71	7.18	11.54	10.65	10.95	0.83
58	10.93	10.66	0.59	14.38	13.54	14.55	14.05	0.46	7.07	11.33	10.60	10.91	0.53
60	10.87	10.64	0.28	13.94	13.43	14.09	13.95	0.22	7.03	11.24	10.55	10.85	0.26
62	10.90	10.88	0.00	14.24	13.52	14.40	14.44	0.00	7.12	11.28	10.79	11.17	0.00

Table 3S: Estimated growing stock volumes and model-based variances, and the empirical variances for Landsat models, the second phase sample size is 75 grid cells.

Strips	‘LINEAR’			‘LOG-LOG’			‘SQRT’		
	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{OLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$
2	104.19	115.63	403.12	104.96	124.43	400.04	104.73	103.66	397.62
4	101.78	103.37	212.42	102.62	102.72	205.45	102.35	93.76	205.85
6	101.16	97.39	161.09	101.98	92.17	152.14	101.73	88.61	154.53
8	101.00	94.32	143.32	101.74	86.42	133.54	101.55	85.91	136.02
10	100.80	92.46	126.52	101.61	83.48	117.13	101.36	84.50	120.37
12	100.85	91.62	116.39	101.57	81.37	106.63	101.37	83.94	110.22
14	100.73	91.08	111.82	101.43	80.10	100.99	101.23	83.58	104.88
16	100.31	89.59	110.16	101.10	78.17	99.19	100.87	82.09	102.97
18	100.44	88.73	100.46	101.23	76.75	90.71	101.02	81.23	93.31
20	100.54	88.54	98.88	101.28	76.35	88.60	101.06	81.24	92.37
22	100.40	88.52	93.95	101.15	76.00	83.07	100.92	81.24	87.79
24	100.31	88.44	96.83	101.05	75.69	84.36	100.83	81.16	89.62
26	100.40	88.49	94.52	101.25	75.45	84.33	100.98	81.10	87.54
28	100.81	88.86	95.93	101.56	75.81	84.31	101.32	81.72	89.00
30	100.73	87.71	90.95	101.42	74.49	79.43	101.23	80.63	84.18
32	100.39	86.07	90.96	101.09	72.99	79.30	100.91	79.10	84.19
34	100.70	87.50	88.98	101.31	74.22	76.81	101.18	80.56	81.76
36	100.87	88.76	88.81	101.50	75.17	76.02	101.39	81.73	81.29
38	100.58	87.15	89.22	101.32	73.49	76.21	101.10	80.09	81.70
40	100.09	86.26	87.26	100.99	72.67	76.45	100.65	79.19	79.70
42	100.06	85.42	86.99	100.98	71.57	74.03	100.66	78.37	79.38
44	100.14	84.65	85.47	101.15	70.90	74.98	100.77	77.59	78.53
46	100.40	85.72	83.55	101.26	71.63	72.23	101.02	78.56	76.32
48	101.00	88.20	85.80	101.68	73.79	73.55	101.58	80.94	78.43
50	101.13	87.89	87.94	101.75	73.67	76.10	101.65	80.86	80.66
52	100.88	86.48	84.17	101.60	72.27	71.94	101.46	79.37	76.82
54	99.99	84.04	83.66	100.81	70.37	71.84	100.54	77.26	76.24
56	99.63	83.67	80.57	100.48	69.91	69.42	100.17	76.93	73.56
58	99.46	82.12	80.49	100.35	68.66	70.24	99.99	75.56	74.25
60	99.22	81.39	79.90	100.09	67.57	69.33	99.78	74.73	73.53
62	98.61	81.50	79.28	99.65	67.67	68.49	99.19	74.71	72.70

Table 4S: Estimated growing stock volumes and model-based variances, and the empirical variances for LiDAR models, the second phase sample size is 75 grid cells.

Strips	‘LINEAR’			‘LOG-LOG’			‘SQRT’		
	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{OLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$\hat{\mu}_{Y_{MB}}$	$\widehat{V}(\hat{\mu}_{Y_{MB}})_{NLS}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$
2	104.99	221.80	335.13	104.79	174.52	337.13	104.83	200.89	334.33
4	102.65	127.29	141.33	102.41	104.65	143.99	102.49	116.89	141.15
6	101.94	88.28	94.35	101.73	74.36	96.77	101.78	81.63	94.13
8	101.74	67.92	70.02	101.51	58.47	72.69	101.57	63.14	70.04
10	101.61	54.85	56.76	101.35	48.21	58.23	101.43	51.31	55.94
12	101.61	46.44	46.61	101.36	41.73	49.06	101.43	43.72	46.49
14	101.51	40.44	40.52	101.28	37.06	42.69	101.35	38.27	40.38
16	101.39	35.69	36.90	101.09	33.29	39.70	101.20	33.93	37.02
18	101.41	32.09	32.17	101.12	30.48	34.31	101.22	30.69	32.02
20	101.21	29.31	28.86	100.96	28.20	31.20	101.05	28.11	28.78
22	101.24	26.86	26.53	100.96	26.33	28.99	101.06	25.91	26.51
24	101.13	24.93	25.51	100.85	24.77	27.82	100.95	24.12	25.30
26	101.18	23.20	23.74	100.92	23.55	26.24	101.01	22.60	23.55
28	101.25	21.91	22.95	101.05	22.53	25.78	101.10	21.42	23.03
30	101.21	20.51	21.86	101.04	21.45	24.28	101.07	20.17	21.70
32	101.20	19.26	20.26	100.94	20.28	22.89	101.02	18.95	20.24
34	101.21	18.40	19.00	101.03	19.72	21.34	101.06	18.22	19.00
36	101.19	17.77	18.12	101.03	19.25	20.56	101.05	17.66	17.98
38	101.20	16.81	17.42	101.01	18.44	20.33	101.05	16.75	17.54
40	101.09	16.02	17.02	100.82	17.76	19.61	100.91	16.00	17.10
42	101.12	15.27	16.32	100.82	17.14	18.69	100.91	15.32	16.17
44	101.10	14.63	15.56	100.84	16.63	17.90	100.91	14.74	15.44
46	101.05	14.28	15.62	100.83	16.34	17.81	100.87	14.42	15.48
48	101.13	14.23	14.82	100.92	16.45	17.06	100.93	14.39	14.61
50	101.02	13.80	14.75	100.79	16.00	17.13	100.81	13.96	14.58
52	101.14	13.14	14.09	100.88	15.46	16.44	100.92	13.40	14.01
54	101.08	12.50	13.69	100.66	14.77	16.20	100.79	12.73	13.67
56	101.05	12.08	13.44	100.61	14.48	15.70	100.78	12.36	13.29
58	101.10	11.51	13.42	100.61	13.99	15.43	100.81	11.86	13.19
60	101.10	11.15	12.74	100.59	13.65	15.05	100.82	11.50	12.66
62	101.09	10.90	12.91	100.53	13.52	15.23	100.80	11.28	12.72

Table 5S: Empirical model-based and model-assisted variance for Landsat models, the second phase sample size is 75 grid cells.

Strips	‘LINEAR’		‘LOG-LOG’		‘SQRT’	
	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$
2	403.12	403.31	400.04	400.30	397.62	397.83
4	212.42	212.44	205.45	205.43	205.85	205.85
6	161.09	161.23	152.14	152.25	154.53	154.67
8	143.32	143.68	133.54	133.79	136.02	136.33
10	126.52	127.54	117.13	118.14	120.37	121.45
12	116.39	120.58	106.63	110.37	110.22	114.45
14	111.82	118.12	100.99	106.52	104.88	111.13
16	110.16	117.15	99.19	104.76	102.97	109.67
18	100.46	106.72	90.71	95.88	93.31	99.37
20	98.88	105.17	88.60	94.19	92.37	98.39
22	93.95	99.15	83.07	87.67	87.79	92.87
24	96.83	101.69	84.36	88.25	89.62	94.21
26	94.52	98.87	84.33	88.03	87.54	91.84
28	95.93	99.30	84.31	87.24	89.00	92.31
30	90.95	94.71	79.43	82.38	84.18	87.69
32	90.96	94.40	79.30	82.10	84.19	87.53
34	88.98	92.19	76.81	79.27	81.76	84.70
36	88.81	91.46	76.02	78.41	81.29	83.87
38	89.22	92.48	76.21	79.04	81.70	84.81
40	87.26	90.39	76.45	78.69	79.70	82.60
42	86.99	90.79	74.03	76.73	79.38	82.87
44	85.47	89.14	74.98	77.91	78.53	81.97
46	83.55	86.43	72.23	74.24	76.32	78.88
48	85.80	89.11	73.55	76.11	78.43	81.56
50	87.94	90.33	76.10	77.83	80.66	82.96
52	84.17	86.33	71.94	73.75	76.82	78.86
54	83.66	88.16	71.84	75.59	76.24	80.50
56	80.57	86.07	69.42	73.87	73.56	78.70
58	80.49	86.68	70.24	75.24	74.25	80.07
60	79.90	87.28	69.33	75.20	73.53	80.60
62	79.28	86.96	68.49	75.13	72.70	79.80

Table 6S: Empirical model-based and model-assisted variance for LiDAR models, the second phase sample size is 75 grid cells.

Strips	‘LINEAR’		‘LOG-LOG’		‘SQRT’	
	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$	$V(\hat{\mu}_{Y_{MB}})_{emp}$	$V(\hat{\mu}_{Y_{MA}})_{emp}$
2	335.13	335.11	337.13	337.15	334.33	334.32
4	141.33	141.34	143.99	143.97	141.15	141.16
6	94.35	94.33	96.77	96.78	94.13	94.12
8	70.02	70.04	72.69	72.81	70.04	70.11
10	56.76	56.86	58.23	58.37	55.94	56.00
12	46.61	47.18	49.06	49.83	46.49	47.09
14	40.52	41.21	42.69	43.94	40.38	41.34
16	36.90	37.80	39.70	41.30	37.02	38.21
18	32.17	33.01	34.31	35.88	32.02	33.15
20	28.86	29.42	31.20	32.31	28.78	29.57
22	26.53	27.14	28.99	30.10	26.51	27.34
24	25.51	26.21	27.82	29.00	25.30	26.20
26	23.74	24.32	26.24	27.32	23.55	24.32
28	22.95	23.49	25.78	26.64	23.03	23.70
30	21.86	22.50	24.28	25.19	21.70	22.47
32	20.26	20.68	22.89	23.63	20.24	20.81
34	19.00	19.38	21.34	22.03	19.00	19.48
36	18.12	18.43	20.56	21.09	17.98	18.37
38	17.42	17.93	20.33	20.92	17.54	18.07
40	17.02	17.62	19.61	20.34	17.10	17.76
42	16.32	16.96	18.69	19.61	16.17	16.93
44	15.56	16.14	17.90	18.78	15.44	16.17
46	15.62	16.29	17.81	18.73	15.48	16.26
48	14.82	15.61	17.06	18.23	14.61	15.57
50	14.75	15.54	17.13	18.26	14.58	15.51
52	14.09	14.95	16.44	17.59	14.01	14.98
54	13.69	14.47	16.20	17.42	13.67	14.62
56	13.44	14.29	15.70	17.09	13.29	14.32
58	13.42	14.18	15.43	16.88	13.19	14.21
60	12.74	13.65	15.05	16.53	12.66	13.76
62	12.91	13.79	15.23	16.58	12.72	13.74