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Spinor Field with Polynomial Nonlinearity in LRS Bianchi type-I spacetime

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Within the scope of LRS Bianchi type-I cosmological model the role of spinor field on the evolution of the Universe is investigated. In doing so, we have considered a polynomial type of nonlinearity. It is found that depending on the sign of self-coupling constant the model allows either accelerated mode of expansion or oscillatory mode of evolution. While the non-diagonal components of the energy-momentum tensor of the spinor field in case of a full Bianchi type-I model leading to the vanishing mass and nonlinear term in the spinor field Lagrangian, in case of an LRS Bianchi type-I model neither mass term nor the nonlinear term of spinor field vanish.

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I. INTRODUCTION

Recently, after some remarkable works by different authors [1–15], showing the important role that spinor fields play on the evolution of the Universe, the situation began to change. This change of attitude is directly related to some fundamental questions of modern cosmology: (i) problem of initial singularity; (ii) problem of isotropization and (iii) late time acceleration of the Universe.

Given the role that spinor field can play in the evolution of the Universe, question that naturally pops up is, if the spinor field can redraw the picture of evolution caused by perfect fluid and dark energy, is it possible to simulate perfect fluid and dark energy by means of a spinor field? Affirmative answer to this question was given in the a number of papers [16–20]. In those papers spinor description of matter such as perfect fluid and dark energy was given and the evolution of the Universe given by different Bianchi models was thoroughly studied. In almost all the papers the spinor field was considered to be time-dependent functions and its energy-momentum tensor was given by the diagonal elements only.

Some latest study shows that due to the specific connection with gravitational field the energy-momentum tensor of the spinor field possesses non-trivial non-diagonal components as well and these non-zero non-diagonal components of energy-momentum tensor play decisive role on the character of geometry of spacetime as well as on the components of spinor field [21–24].

In this paper we study the evolution of the Universe filled with spinor field within the scope of a LRS Bianchi type-I cosmological model. It should be noted that a general Bianchi type-I model in presence of a nonlinear spinor field dully evolves into a LRS Bianchi type-I model [22]. In this paper we thoroughly study in role of spinor field in the evolution of the Universe given by a LRS Bianchi type-I model. Here we also consider a more general type of nonlinearity.

II. BASIC EQUATIONS

In this report we plan to study the evolution of the Universe given by a Locally Rotationally Symmetric Bianchi type-I anisotropic cosmological model filled with nonlinear spinor field.

The LRS Bianchi type-I (BI) model is the ordinary Bianchi type-I model with two of the three metric functions being equal to each other and can be given by

$$ds^2 = dt^2 - a_1^2 [dx^2 + dy^2] - a_3^2 dz^2, \quad (2.1)$$

with a_1 and a_3 being the functions of time only.

The nontrivial components of the Einstein tensor corresponding to the metric (2.1) are

$$G_1^1 = G_2^2 = - \left(\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} \right), \quad (2.2a)$$

$$G_3^3 = - \left(2 \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} \right), \quad (2.2b)$$

$$G_0^0 = - \left(\frac{\dot{a}_1^2}{a_1^2} + 2 \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} \right). \quad (2.2c)$$

Keeping this in mind the symmetry between ψ and $\bar{\psi}$ we choose the symmetrized Lagrangian [25] for the spinor field as [5]:

$$L = \frac{i}{2} \left[\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - m_{\text{sp}} \bar{\psi} \psi - F, \quad (2.3)$$

where the nonlinear term F describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. We consider $F = F(K)$, with K taking one of the following expressions $\{I, J, I + J, I - J\}$. It can be shown that such a choice describes the nonlinearity in its most general form.

Here ∇_μ is the covariant derivative of spinor field:

$$\nabla_\mu \psi = \frac{\partial \psi}{\partial x^\mu} - \Gamma_\mu \psi, \quad \nabla_\mu \bar{\psi} = \frac{\partial \bar{\psi}}{\partial x^\mu} + \bar{\psi} \Gamma_\mu, \quad (2.4)$$

with Γ_μ being the spinor affine connection. In (2.3) γ 's are the Dirac matrices in curve space-time and obey the following algebra

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu} \quad (2.5)$$

and are connected with the flat space-time Dirac matrices $\bar{\gamma}$ in the following way

$$g_{\mu\nu}(x) = e_\mu^a(x) e_\nu^b(x) \eta_{ab}, \quad \gamma_\mu(x) = e_\mu^a(x) \bar{\gamma}_a, \quad (2.6)$$

where $\eta_{ab} = \text{diag}(1, -1, -1, -1)$ and e_μ^a is a set of tetrad 4-vectors. The spinor affine connection matrices $\Gamma_\mu(x)$ are uniquely determined up to an additive multiple of the unit matrix by the equation

$$\nabla_\mu \gamma_\nu = \frac{\partial \gamma_\nu}{\partial x^\mu} - \Gamma_{\nu\mu}^\rho \gamma_\rho - \Gamma_\mu \gamma_\nu + \gamma_\nu \Gamma_\mu = 0, \quad (2.7)$$

with the solution

$$\Gamma_\mu = \frac{1}{4} \bar{\gamma}_a \gamma^\nu \partial_\mu e_\nu^{(a)} - \frac{1}{4} \gamma_\rho \gamma^\nu \Gamma_{\mu\nu}^\rho, \quad (2.8)$$

The spin affine connection corresponding to LRS BI metric (2.1) can be written explicitly as

$$\Gamma_0 = 0, \quad \Gamma_1 = \frac{\dot{a}_1}{a_1} \bar{\gamma}^1 \bar{\gamma}^0, \quad \Gamma_2 = \frac{\dot{a}_1}{a_1} \bar{\gamma}^2 \bar{\gamma}^0, \quad \Gamma_3 = \frac{\dot{a}_3}{a_3} \bar{\gamma}^3 \bar{\gamma}^0, \quad (2.9)$$

Varying (2.3) with respect to $\bar{\psi}(\psi)$ one finds the spinor field equations:

$$i\gamma^\mu \nabla_\mu \psi - m_{\text{sp}} \psi - 2F_K (SK_I + iPK_J \gamma^5) \psi = 0, \quad (2.10a)$$

$$i\nabla_\mu \bar{\psi} \gamma^\mu + m_{\text{sp}} \bar{\psi} + 2F_K \bar{\psi} (SK_I + iPK_J \gamma^5) = 0. \quad (2.10b)$$

Here we denote $F_K = dF/dK$, $K_I = dK/dI$ and $K_J = dK/dJ$.

The energy-momentum tensor of the spinor field is given by

$$T_\mu^\rho = \frac{i}{4} g^{\rho\nu} \left(\bar{\psi} \gamma_\mu \nabla_\nu \psi + \bar{\psi} \gamma_\nu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma_\nu \psi - \nabla_\nu \bar{\psi} \gamma_\mu \psi \right) - \delta_\mu^\rho L_{\text{sp}} \quad (2.11)$$

where L_{sp} in view of (2.10a) and (2.10b) can be rewritten as

$$\begin{aligned} L_{\text{sp}} &= \frac{i}{2} [\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi] - m_{\text{sp}} \bar{\psi} \psi - F(K) \\ &= \frac{i}{2} \bar{\psi} [\gamma^\mu \nabla_\mu \psi - m_{\text{sp}} \psi] - \frac{i}{2} [\nabla_\mu \bar{\psi} \gamma^\mu + m_{\text{sp}} \bar{\psi}] \psi - F(K), \\ &= 2F_K (IK_I + JK_J) - F = 2KF_K - F(K). \end{aligned} \quad (2.12)$$

Further inserting (2.4) into (2.11) the energy-momentum tensor of the spinor field can be written as

$$\begin{aligned} T_\mu^\rho &= \frac{i}{4} g^{\rho\nu} (\bar{\psi} \gamma_\mu \partial_\nu \psi + \bar{\psi} \gamma_\nu \partial_\mu \psi - \partial_\mu \bar{\psi} \gamma_\nu \psi - \partial_\nu \bar{\psi} \gamma_\mu \psi) \\ &\quad - \frac{i}{4} g^{\rho\nu} \bar{\psi} (\gamma_\mu \Gamma_\nu + \Gamma_\nu \gamma_\mu + \gamma_\nu \Gamma_\mu + \Gamma_\mu \gamma_\nu) \psi - \delta_\mu^\rho (2KF_K - F(K)). \end{aligned} \quad (2.13)$$

Finally, exploiting the explicit form of spin connection (2.9) after some manipulations one find the following non-trivial components of the energy-momentum tensor of the spinor field

$$T_0^0 = m_{\text{sp}} S + F(K), \quad (2.14a)$$

$$T_1^1 = T_2^2 = T_3^3 = F(K) - 2KF_K, \quad (2.14b)$$

$$T_3^1 = \frac{i}{4} \frac{a_3}{a_1} \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) \bar{\psi} \bar{\gamma}^3 \bar{\gamma}^1 \bar{\gamma}^0 \psi = \frac{1}{4} \frac{a_3}{a_1} \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) A^2, \quad (2.14c)$$

$$T_3^2 = \frac{i}{4} \frac{a_3}{a_1} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) \bar{\psi} \bar{\gamma}^2 \bar{\gamma}^3 \bar{\gamma}^0 \psi = \frac{1}{4} \frac{a_3}{a_1} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) A^1. \quad (2.14d)$$

So the complete set of Einstein equation for BI metric should be

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} = \kappa(F(K) - 2KF_K), \quad (2.15a)$$

$$2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} = \kappa(F(K) - 2KF_K), \quad (2.15b)$$

$$\frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_3}{a_3} \frac{\dot{a}_1}{a_1} = \kappa(m_{\text{sp}}S + F(K)), \quad (2.15c)$$

$$0 = \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) A^2, \quad (2.15d)$$

$$0 = \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) A^1. \quad (2.15e)$$

Before solving the Einstein equations let us first write the equations for the bilinear spinor forms. Recalling there are 16 bilinear spinor forms, namely $S = \bar{\psi}\psi$, $P = \iota\bar{\psi}\gamma^5\psi$, $v^\mu = \bar{\psi}\gamma^\mu\psi$, $A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$ and $Q^{\mu\nu} = \bar{\psi}\sigma^{\mu\nu}\psi$ are the scalar, pseudoscalar, vector, pseudovector and antisymmetric tensor, respectively, for the LRS BI metric one finds the following system of equations:

$$\dot{S}_0 + \mathcal{G}A_0^0 = 0, \quad (2.16a)$$

$$\dot{P}_0 - \Phi A_0^0 = 0, \quad (2.16b)$$

$$\dot{A}_0^0 + \Phi P_0 - \mathcal{G}S_0 = 0, \quad (2.16c)$$

$$\dot{A}_0^3 = 0, \quad (2.16d)$$

$$\dot{v}_0^0 = 0, \quad (2.16e)$$

$$\dot{v}_0^3 + \Phi Q_0^{30} + \mathcal{G}Q_0^{21} = 0, \quad (2.16f)$$

$$\dot{Q}_0^{30} - \Phi v_0^3 = 0, \quad (2.16g)$$

$$\dot{Q}_0^{21} - \mathcal{G}v_0^3 = 0, \quad (2.16h)$$

where we denote $S_0 = SV$, $P_0 = PV$, $A_0^\mu = A_\mu V$, $v_0^\mu = v^\mu V$, $Q_0^{\mu\nu} = Q^{\mu\nu}V$ and $\Phi = m_{\text{sp}} + \mathcal{D}$. We also denote $\mathcal{D} = 2SF_K K_I$ and $\mathcal{G} = 2PF_K K_J$, with $F_K = dF/dK$, $K_I = dK/dI$ and $K_J = dK/dJ$.

Here we also introduce the volume scale

$$V = a_1^2 a_3. \quad (2.17)$$

III. SOLUTION TO THE FIELD EQUATIONS

From (2.16a) - (2.16h) one finds the following relations:

$$(S_0)^2 + (P_0)^2 + (A_0^0)^2 = C_1 = \text{Const}, \quad (3.1a)$$

$$A_0^3 = C_2 = \text{Const}, \quad (3.1b)$$

$$v_0^0 = C_3 = \text{Const}, \quad (3.1c)$$

$$(Q_0^{30})^2 + (Q_0^{21})^2 + (v_0^3)^2 = C_4 = \text{Const}. \quad (3.1d)$$

Let us now go back to Einstein equations. The off-diagonal components of the Einstein equations (2.15d) and (2.15e) impose the following restrictions either on the components of the spinor field or on the metric functions:

$$A^2 = 0, \quad A^1 = 0, \quad (3.2a)$$

$$\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} = 0. \quad (3.2b)$$

The restriction (3.2b) leads to $a_3 = q_0 a_1$ with q_0 being some constant. In this case the system can be described by a FRW model from the very beginning. Here we don't consider this case. We will do it later within the scope of a FRW model.

We consider the case when the restriction is imposed on the components of the spinor field in details. Subtraction of (2.15b) from (2.15a) gives

$$\frac{\ddot{a}_3}{a_3} - \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1}{a_1} \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right) = 0, \quad (3.3)$$

that leads to [5]

$$a_1 = D_1 V^{1/3} \exp \left(X_1 \int \frac{dt}{V} \right), \quad a_3 = (1/D_1^2) V^{1/3} \exp \left(-2X_1 \int \frac{dt}{V} \right). \quad (3.4)$$

with D_1 and X_1 being the integration constants. Thus we see that the metric functions can be expressed in terms of V .

The solutions to the spinor field equations (2.10a) in this case can be presented as [5]

$$\psi_{1,2}(t) = \frac{C_{1,2}}{\sqrt{V}} \exp \left(-i \int \Phi dt \right), \quad \psi_{3,4}(t) = \frac{C_{3,4}}{\sqrt{V}} \exp \left(i \int \Phi dt \right), \quad (3.5)$$

with C_1, C_2, C_3, C_4 being the integration constants and related to V_0 as

$$C_1^* C_1 + C_2^* C_2 - C_3^* C_3 - C_4^* C_4 = V_0.$$

Here we have assumed that $K = I$, i.e., $F = F(I)$. The reason for this choice is discussed later.

Thus we see that the metric functions, the components of spinor field as well as the invariants constructed from metric functions and spinor fields are some inverse functions of V of some degree. Hence at any spacetime point where $V = 0$ it is a singular point. So it is important to study the behavior of V which we do in the next section.

IV. RESULTS AND DISCUSSIONS

In this section we discuss the results obtained in the previous section. In doing so we pay special attention to the volume scale V .

Let us first see, whether the model becomes asymptotically isotropic. It can be shown that for an expanding Universe, when $V \rightarrow \infty$ as $t \rightarrow \infty$, the isotropization process of the Universe takes place. To prove that we exploit the isotropization condition proposed in [26]

$$\left. \frac{a_i}{a} \right|_{t \rightarrow \infty} \rightarrow \text{const.} \quad (4.1)$$

Then by rescaling some of the coordinates, we can make $a_i/a \rightarrow 1$, and the metric will become manifestly isotropic at large t .

Taking into account that $a = V^{1/3}$ from (3.4) we find

$$\frac{a_1}{a} = D_1 \exp \left(X_1 \int \frac{dt}{V} \right) \rightarrow D_1, \quad \frac{a_3}{a} = (1/D_1^2) \exp \left(-2X_1 \int \frac{dt}{V} \right) \rightarrow 1/D_1^2, \quad (4.2)$$

as $V \rightarrow \infty$. Recall that the isotropic FRW model has same scale factor in all three directions, i.e., $a_1(t) = a_2(t) = a_3(t) = a(t)$. So for the LRS BI universe to evolve into a FRW one we should have $D_1 = 1$. Moreover, the isotropic nature of the present Universe leads to the fact that $|X_1| \ll 1$, so that $\int [V(t)]^{-1} dt \rightarrow 0$ for $t < \infty$ (for $V(t) = t^n$ with $n > 1$ the integral tends to zero as $t \rightarrow \infty$).

Our next step will be to define V . Combining the diagonal Einstein equations (2.15a), (2.15b) and (2.15c) in a certain way for V we find [5]

$$\ddot{V} = \frac{3\kappa}{2} (m_{\text{sp}} S + 2(F(K) - KF_K)) V. \quad (4.3)$$

Now in order to solve (4.3) we have to know the relation between the spinor and the gravitational fields. Using the equations (2.16a) and (2.16b) it can be shown that

$$K = \frac{V_0^2}{V^2}. \quad (4.4)$$

The relation (4.4) holds only for massless spinor field if K takes one of the following expressions $\{J, I+J, I-J\}$, while for $K=I$ it holds both for massless and massive spinor field. In case of $K=I+J$ one can write $S = \sin(V_0/V)$ and $P = \cos(V_0/V)$, whereas for $K=I-J$ one can write $S = \cosh(V_0/V)$ and $P = \sinh(V_0/V)$. In what follows, we will consider the case for $K=I$, as in this case further setting spinor mass $m_{\text{sp}} = 0$ we can revive the results for other cases. Assuming

$$F = \sum_k \lambda_k I^{n_k} = \sum_k \lambda_k S^{2n_k} \quad (4.5)$$

on account of $S = V_0/V$ we find

$$\ddot{V} = \frac{3\kappa}{2} \left[m_{\text{sp}} V_0 + 2 \sum_k \lambda_k (1 - n_k) V_0^{2n_k} V^{1-2n_k} \right], \quad (4.6)$$

with the solution in quadrature

$$\int \frac{dV}{\sqrt{3\kappa \left[m_{\text{sp}} V_0 V + \sum_k \lambda_k V_0^{2n_k} V^{2(1-n_k)} \right] + \bar{C}}} = t + t_0, \quad (4.7)$$

with $\bar{C}$ and t_0 being some arbitrary constants.

Thus we see that the metric functions, the components of spinor field as well as the invariants constructed from metric functions and spinor fields are some inverse functions of V of some degree. Hence at any spacetime point where $V = 0$ it is a singular point. So we consider the initial value of $V(0)$ is small but non-zero. As a result for the nonlinear term to prevail in (4.6) we should have $n_k = n_1 : 1 - 2n_1 < 0$, i.e., $n_1 > 1/2$, whereas for an expanding Universe when $V \rightarrow \infty$ as $t \rightarrow \infty$ one should have $n_k = n_2 : 1 - 2n_2 > 0$, i.e., $n_2 < 1/2$. As is seen from (4.6), $n_k = n_0 : n_0 = 1/2$ leads to a term that can be added to the mass term.

In this case we obtain

$$\begin{aligned} \dot{V} &= \Phi_1(V), \\ \Phi_1(V) &= \frac{3\kappa}{2} \left[(m_{\text{sp}} + \lambda_0) V_0 + 2\lambda_1(1 - n_1) V_0^{2n_1} V^{1-2n_1} + 2\lambda_2(1 - n_2) V_0^{2n_2} V^{1-2n_2} \right]. \end{aligned} \quad (4.8)$$

Equation (4.8) allows the first integral

$$\begin{aligned} \dot{V} &= \Phi_2(V), \\ \Phi_2(V) &= \sqrt{3\kappa \left[(m_{\text{sp}} + \lambda_0) V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C} \right]}. \end{aligned} \quad (4.9)$$

$$(4.10)$$

The solution to the equation (4.8) can be written in quadrature as follows

$$\int \frac{dV}{\Phi_2(V)} = t + t_0. \quad (4.11)$$

To solve the (4.8) we should choose the problem parameters V_0 , m_{sp} , κ , $\bar{C}$, λ_k as well as the initial value of $V(0)$ in such a way that does not lead to

$$(m_{\text{sp}} + \lambda_0) V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C} < 0.$$

For simplicity let us set $V_0 = 1$, $m_{\text{sp}} = 1$, $C_0 = 10$, $\kappa = 1$. In line with our discussion above we consider $n_0 = 1/2$, $n_1 = 2$ and $n_2 = 0$. In our case we set $V(0) = 0.5$. We set $\lambda_0 = 1$, whereas $\lambda_1 = \pm 1$ and $\lambda_2 = \pm 1$ were taken in different combinations. It was found that depending on the sign of λ_2 the model gives principally different types of solutions, namely, in case of positive λ_2 we have an accelerated mode of expansion of the Universe, while for negative λ_2 we have an oscillatory solution.

Defining deceleration parameter

$$q = -\frac{V\ddot{V}}{\dot{V}^2} = -\frac{V\Phi_1(V)}{\Phi_2^2(V)}, \quad (4.12)$$

from (4.8) and (4.9) we have

$$q = -\frac{\frac{3\kappa}{2} \left[(m_{\text{sp}} + \lambda_0) V_0 V + 2\lambda_1 (1 - n_1) V_0^{2n_1} V^{2(1-n_1)} + 2\lambda_2 (1 - n_2) V_0^{2n_2} V^{2(1-n_2)} \right]}{3\kappa \left[(m_{\text{sp}} + \lambda_0) V_0 V + \lambda_1 V_0^{2n_1} V^{2(1-n_1)} + \lambda_2 V_0^{2n_2} V^{2(1-n_2)} + \bar{C} \right]}. \quad (4.13)$$

Taking into account that for an expanding Universe at large t the term V^{1-2n_2} prevails, for deceleration parameter we find

$$\lim_{V \rightarrow \infty} q \rightarrow -(1 - n_2) < 0, \quad \text{since } n_2 < 1/2. \quad (4.14)$$

Thus we see that spinor field nonlinearity generates late time acceleration of the Universe.

In Fig. 1 and Fig. 2 we have plotted the evolution of volume scale V for a positive and negative self-coupling constant λ_2 , respectively. As one sees from Fig. 1, a positive λ_2 gives rise to an accelerated mode of expansion, whereas Fig. 2 with negative λ_2 shows the oscillatory mode of expansion. In Fig. 3 the picture of deceleration parameter q are illustrated for a positive λ_2 .

Finally we study what happens to shear and anisotropic parameter in this case. In doing so, let us first rewrite the corresponding quantities. The expansion ϑ for LRS BI metric reads

$$\vartheta = 2\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} = \frac{\dot{V}}{V}, \quad (4.15)$$

FIG. 1. Evolution of the Universe for a positive λ_2 FIG. 2. Evolution of the Universe for a negative λ_2

whereas, from

$$\sigma_1^1 = \sigma_2^2 = \frac{\dot{a}_1}{a_1} - \frac{1}{3}\vartheta = \frac{1}{3} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right), \quad (4.16a)$$

$$\sigma_3^3 = \frac{\dot{a}_3}{a_3} - \frac{1}{3}\vartheta = -\frac{2}{3} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right) \quad (4.16b)$$

one finds the expression for shear

$$\sigma^2 = \frac{1}{2} \left[\sum_{i=1}^3 \left(\frac{\dot{a}_i}{a_i} \right)^2 - \frac{1}{3}\vartheta^2 \right] = \frac{1}{3} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right)^2. \quad (4.17)$$

FIG. 3. Plot of deceleration parameter q for a positive λ_2

The anisotropic parameter in this case has the form

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i}{H} - 1 \right)^2 = \frac{1}{3H^2} \left[2 \left(\frac{\dot{a}_1}{a_1} \right)^2 + \left(\frac{\dot{a}_3}{a_3} \right)^2 \right] - 1, \quad (4.18)$$

where $H = \frac{1}{3} \left(2 \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3} \right) = \frac{1}{3} \frac{\dot{V}}{V}$. Further from (3.4) we find that

$$\frac{\dot{a}_1}{a_1} = \frac{1}{3} \frac{\dot{V}}{V} + \frac{X_1}{V}, \quad \frac{\dot{a}_3}{a_3} = \frac{1}{3} \frac{\dot{V}}{V} - 2 \frac{X_1}{V}. \quad (4.19)$$

Now inserting (4.19) into (4.16), (4.17), (4.18) we finally find

$$\sigma_1^1 = \sigma_2^2 = \frac{X_1}{V}, \quad \sigma_3^3 = -2 \frac{X_1}{V}, \quad (4.20)$$

$$\sigma^2 = 3 \frac{X_1^2}{V^2}, \quad (4.21)$$

$$A_m = 18 \frac{X_1^2}{V^2}. \quad (4.22)$$

As it was shown in (4.8) and (4.9) in case of a positive λ_2 that generates late time acceleration, both V and $\dot{V}$ becomes large with the expansion of the Universe leading to $\sigma_i^i \rightarrow 0$, $\sigma^2 \rightarrow 0$ and $A_m \rightarrow 0$. This corresponds to our earlier conclusion regarding isotropization. As far as negative λ_2 is concerned, in this case the model gives rise to a oscillatory mode of expansion. In this case we have both local minima and maxima. The maximum (minimum) value of volume scale V depends on the parameters and the initial condition and might be as large as possible. Hence in case of a negative λ_2 though it is possible to attain solution such that $\sigma_i^i|_{V=V_{\max}} \rightarrow 0$ and $\sigma^2|_{V=V_{\max}} \rightarrow 0$,

but at the same time we have $A_m|_{V=V_{\max}} \rightarrow \infty$ since at any spacetime point where $V = V_{\max(\min)}$ we have $\dot{V}|_{V=V_{\max(\min)}} = 0$. It means at any spacetime point where evolution changes its direction (expansion to contraction and vice versa) the Universe becomes highly anisotropic.

V. CONCLUSIONS

Within the scope of LRS Bianchi type-I cosmological model we have studied the role of spinor field in the evolution of the Universe. The reason for considering the LRS BI model lies on the fact that in case of a full Bianchi type-I model the non-diagonal components of the energy-momentum tensor of the spinor fields imposes severe restrictions on the components of the spinor field resulting in vanishing scalar $S = \bar{\psi}\psi$ and pseudoscalar $P = i\bar{\psi}\gamma^5\psi$ [21, 22]. As a results both the mass term and nonlinear term in the Lagrangian disappears. But as it was shown here, in case of an LRS Bianchi type-I cosmological model neither mass term nor the nonlinear term vanish. Moreover, unlike Bianchi type-VI model the present model leads asymptotic isotropization. It is was also found that depending on the sign of self-coupling constant the model allows either accelerated mode of expansion or oscillatory mode of evolution.

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