Canadian Journal of Physics
Revue canadienne de physique

Thermodynamics of rotating black branes in higher dimensional Einstein-nonlinear electrodynamics-dilaton gravity

Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2015-0221.R1
Manuscript Type:	Article
Date Submitted by the Author:	28-Aug-2015
Complete List of Authors:	Sheykhi, A.; Shiraz University, Physics Hendi, S.H.; Shiraz University, Physics
Keyword:	nonlinear , electrodynamics, dilaton, black branes, thermodynamics

SCHOLARONE™
Manuscripts

Only

Thermodynamics of rotating black branes in higher dimensional Einstein-nonlinear electrodynamics-dilaton gravity

A. Sheykhi^{1,2*} and S. H. Hendi^{1,2†}

¹ *Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran*

² *Research Institute for Astrophysics and Astronomy of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran*

In this paper, we propose the n -dimensional action in which gravity is coupled to exponential nonlinear electrodynamics and scalar dilaton field with Liouville-type potential. By varying the action, we obtain the field equations. Then, we construct a new class of charged, rotating black brane solutions, with $k = [(n-1)/2]$ rotation parameters, of this theory. Due to the presence of the Liouville-type dilaton potential, the asymptotic behavior of the obtained solutions is neither flat nor (anti)-de Sitter [(A)dS]. We investigate the causal structure of the spacetime in ample details. We find the suitable counterterm which removes the divergences of the action in the presence of the dilaton field, and calculate the conserved and thermodynamic quantities of the spacetime. Interestingly enough, we find that the conserved quantities crucially depend on the dilaton coupling constant, α , while they are independent of the nonlinear parameter, β . We also check the validity of the first law of thermodynamics on the black brane horizon. Finally, we study thermal stability of the solutions by computing the heat capacity in the canonical ensemble. We disclose the effects of rotation parameter, nonlinearity of electrodynamics and dilaton field on the thermal stability conditions.

PACS numbers:04.20.Jb, 04.40.Nr, 04.70.Bw, 04.70.Dy

I. INTRODUCTION

The extension of the linear Maxwell electrodynamics to nonlinear electrodynamics has arisen a lot of attentions these days. There are several version of nonlinear electrodynamics. The pioneering theory of the nonlinear electromagnetic fields was proposed by Born and Infeld (BI) in 1934 for the purpose of solving various problems of divergence appearing in the Maxwell theory [1]. In recent years, the BI-like Lagrangians have been proposed with various motivations [2, 3]. First, nonlinear field theories are of interest to different branches of mathematical physics because most physical systems are inherently nonlinear in nature. Nonlinear action of abelian gauge theories have been considered in the context of superstring theory [4, 5]. Second, it is also remarkable that, BI-type theories are singled out among the classes of nonlinear theories by their special properties such as the absence of shock waves, birefringence phenomena [6] and enjoying an electric-magnetic duality [7]. From the thermodynamics point of view, nonlinear electrodynamics also leads to some interesting properties for the black hole systems and have been widely studied in the literatures. In addition, from the point of view of AdS/CFT correspondence in hydrodynamic models, it has been shown that, unlike gravitational correction, higher-derivative terms for abelian fields in the form of nonlinear electrodynamics do not affect the ratio of shear viscosity over entropy density [8].

In the present work, we are going to study the black brane solutions of Einstein-dilaton gravity coupled to the exponential nonlinear (EN) electromagnetic field. In the absence of dilaton field, black hole/brane solutions in the presence of EN electrodynamics have been discovered [9]. In addition, Myers-Perry and slowly rotating black hole solutions of this theory have been investigated before [10]. In the presence of the scalar dilaton field, a class of static and spherically symmetric black hole solutions of Einstein-dilaton gravity coupled to the EN electrodynamics in four [11] and higher dimensional spacetime [12] have been constructed. Till now, exact rotating solution of EN electromagnetic coupled to the dilaton field have not been constructed. In this paper, we first introduce the action of EN electromagnetic coupled to the dilaton in the presence of a potential and by varying the action, we find the corresponding field equations. Then, we construct the higher dimensional rotating black brane solution of this theory with cylindrical or toroidal horizons in the presence of Liouville-type potential and investigate the effects of dilaton and EN electrodynamic fields on the stability of the solutions.

The structure of our paper is as follows. In Sec. II we introduce the action of the Einstein-dilaton gravity with EN electrodynamics as the matter source and obtain the field equations. We also give a brief review of the counterterm method in order to calculate the conserved quantities. In Sec. III, we construct the rotating dilaton black branes with a complete set of rotation parameters in arbitrary dimensions and investigate their properties. In Sec. IV, we obtain the conserved and thermodynamical quantities of the n -dimensional black brane solutions and show that these

* asheykhi@mail.uk.ac.ir

† hendi@shirazu.ac.ir

quantities satisfy the first law of thermodynamics. In Sec. V, we perform a stability analysis and show that the dilaton creates an unstable phase for the solutions. We finish our paper with some closing remarks.

II. BASIC EQUATIONS

Our starting point is the following n -dimensional action

$$I = -\frac{1}{16\pi} \int_{\mathcal{M}} d^n x \sqrt{-g} \left(\mathcal{R} - \frac{4}{n-2} (\nabla\Phi)^2 - V(\Phi) + L(F, \Phi) \right) - \frac{1}{8\pi} \int_{\partial\mathcal{M}} d^{n-1} x \sqrt{-h} \Theta(h), \quad (1)$$

where $\mathcal{R}$ is the Ricci scalar curvature, Φ is the dilaton field, and $V(\Phi)$ is a potential for Φ . We choose the Lagrangian of the EN electrodynamics coupled to the dilaton field (ENd), $L(F, \Phi)$, as

$$L(F, \Phi) = 4\beta^2 e^{4\alpha\Phi/(n-2)} \left[\exp\left(-\frac{e^{-8\alpha\Phi/(n-2)} F^2}{4\beta^2}\right) - 1 \right], \quad (2)$$

where α is a constant determining the strength of coupling of the scalar and BI fields, $F^2 = F^{\mu\nu} F_{\mu\nu}$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor field, A_μ is the vector potential, and β is the nonlinear parameter with dimension of mass. The last term in Eq. (1) is the Gibbons-Hawking boundary term which is chosen such that the variational principle is well-defined. The manifold $\mathcal{M}$ has metric $g_{\mu\nu}$ and covariant derivative ∇_μ . Θ is the trace of the extrinsic curvature Θ^{ab} of any boundary(ies) $\partial\mathcal{M}$ of the manifold $\mathcal{M}$, with induced metric(s) h_{ab} . In this paper, we consider the action (1) with a Liouville type potential,

$$V(\Phi) = 2\Lambda e^{4\alpha\Phi/(n-2)}, \quad (3)$$

where Λ is a constant which may be referred to as the cosmological constant, since in the absence of the dilaton field ($\Phi = 0$) the action (1) reduces to the action of Einstein gravity in the presence of nonlinear electrodynamics with cosmological constant. For later convenience, we redefine it as $\Lambda = -(n-1)(n-2)/2l^2$, where l is a constant with dimension of length.

In order to justify the choice (2) for the Lagrangian of ENd field, let us invoke the BId Lagrangian in n -dimensions which is written as [13, 14]

$$L_{\text{BId}}(F, \Phi) = 4\beta^2 e^{4\alpha\Phi/(n-2)} \left(1 - \sqrt{1 + \frac{e^{-8\alpha\Phi/(n-2)} F^2}{2\beta^2}} \right). \quad (4)$$

It is easy to check that the series expansion of both BId Lagrangian (4) and ENd Lagrangian (2), for large β , have the same behavior

$$L_{\text{BId}}(F, \Phi) = -e^{-4\alpha\Phi/(n-2)} F^2 + \frac{e^{-12\alpha\Phi/(n-2)} F^4}{8\beta^2} - \frac{e^{-20\alpha\Phi/(n-2)} F^6}{32\beta^4} + O\left(\frac{1}{\beta^6}\right), \quad (5)$$

$$L(F, \Phi) = -e^{-4\alpha\Phi/(n-2)} F^2 + \frac{e^{-12\alpha\Phi/(n-2)} F^4}{8\beta^2} - \frac{e^{-20\alpha\Phi/(n-2)} F^6}{96\beta^4} + O\left(\frac{1}{\beta^6}\right). \quad (6)$$

This similarity implies that one can either consider (2) or (4) as the nonlinear electrodynamic Lagrangian coupled to the dilaton field. Here we would like to study the new ENd Lagrangian (2), and investigate the effects of this kind of nonlinear electrodynamics coupled to the dilaton field on the behavior of the solutions. In the absence of the dilaton field ($\alpha = 0$), $L(F, \Phi)$ reduces to EN electrodynamic Lagrangian presented in [9]. On the other hand, in the limiting case where $\beta \rightarrow \infty$, both $L(F, \Phi)$ and $L_{\text{BId}}(F, \Phi)$ recovers the standard linear Maxwell lagrangian coupled to the dilaton field in n -dimensions [15, 16]

$$L(F, \Phi) = L_{\text{BId}}(F, \Phi) = -e^{-4\alpha\Phi/(n-2)} F^2. \quad (7)$$

This is an expected result, since in this case the nonlinear electrodynamics reduces to the linear Maxwell electrodynamics. For latter convenience we rewrite

$$L(F, \Phi) = 4\beta^2 e^{4\alpha\Phi/(n-2)} \mathcal{L}(Y), \quad (8)$$

where

$$\mathcal{L}(Y) = \exp(-Y) - 1, \quad (9)$$

$$Y = \frac{e^{-8\alpha\Phi/(n-2)} F^2}{4\beta^2}. \quad (10)$$

In order to obtain the field equations, we vary action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field Φ and the gauge field A_μ . We find

$$\begin{aligned} \mathcal{R}_{\mu\nu} = & \frac{4}{n-2} \left(\partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi) \right) - 2e^{-4\alpha\Phi/(n-2)} \partial_Y \mathcal{L}(Y) F_{\mu\eta} F_\nu{}^\eta \\ & + \frac{4\beta^2}{n-2} e^{4\alpha\Phi/(n-2)} [2Y \partial_Y \mathcal{L}(Y) - \mathcal{L}(Y)] g_{\mu\nu}, \end{aligned} \quad (11)$$

$$\nabla^2 \Phi = \frac{n-2}{8} \frac{\partial V}{\partial \Phi} + 2\alpha\beta^2 e^{4\alpha\Phi/(n-2)} [2Y \partial_Y \mathcal{L}(Y) - \mathcal{L}(Y)], \quad (12)$$

$$\nabla_\mu \left(e^{-4\alpha\Phi/(n-2)} \partial_Y \mathcal{L}(Y) F^{\mu\nu} \right) = 0. \quad (13)$$

In the limiting case where $\beta \rightarrow \infty$, we have $\mathcal{L}(Y) = -Y$. In this case the system of field equations (11)-(13) restore the well-known equations of Emd gravity [15–19], as expected.

A. FINITE ACTION IN CANONICAL AND GRAND-CANONICAL ENSEMBLES

In general, the total action I given in Eq. (1) is divergent when evaluated on a solution. One way of dealing with the divergences of the action is adding some counterterms to the action (1). The counterterms should contain a part which removes the divergence of the gravity part of the action and a part for dealing with the divergence of the matter action. Since the horizon of our solution is flat, the counterterm which removes the divergence of the gravity part should be proportional to $\sqrt{-h}$. The counterterm for the matter part of the action in the presence of the dilaton is given by

$$I_{\text{ct}} = -\frac{1}{8\pi} \int_{\partial\mathcal{M}} d^{n-1}x \sqrt{-h} \left(\frac{n-2}{l_{\text{eff}}} \right) + I_{\text{deriv}}, \quad (14)$$

where l_{eff} is given by (20) and I_{deriv} is a collection of terms involving derivatives of the boundary fields that could involve the curvature tensor constructed from the boundary metric. Since in our case the boundary is flat so I_{deriv} is zero on the boundary. The variation of the total action ($I_{\text{tot}} = I + I_{\text{ct}}$) about the solutions of the equations of motion is

$$\delta I_{\text{tot}} = \int d^{n-1}x S_{ab} \delta h^{ab} - \frac{1}{16\pi} \int d^{n-1}x \sqrt{-h} e^{-4\alpha\Phi/(n-2)} \partial_Y \mathcal{L}(Y) n^a F_{ab} \delta A^b, \quad (15)$$

where

$$S_{ab} = \frac{\sqrt{-h}}{16\pi} \left\{ \Theta_{ab} - \Theta h_{ab} + \frac{n-2}{l_{\text{eff}}} h_{ab} \right\}. \quad (16)$$

Equation (15) shows that the variation of the total action with respect to A^a will only give the equation of motion of the nonlinear massless field A^a provided the variation is at fixed nonlinear massless gauge potential on the boundary. Thus, the total action, $I_{\text{tot}} = I + I_{\text{ct}}$, given in Eq. (15) is appropriate for the grand-canonical ensemble, where $\delta A^a = 0$ on the boundary. But in the canonical ensemble, where the electric charge $[-e^{-4\alpha\Phi/(n-2)} \partial_Y \mathcal{L}(Y) n^a F_{ab}]$ is fixed on the boundary, the appropriate action is

$$I_{\text{tot}} = I + I_{\text{ct}} + \frac{1}{16\pi} \int_{\partial\mathcal{M}} d^{n-1}x \sqrt{-h} e^{-4\alpha\Phi/(n-2)} \partial_Y \mathcal{L}(Y) n^a F_{ab} \delta A^b. \quad (17)$$

The last term in Eq. (17) is the generalization of the boundary term introduced by Hawking for linear electromagnetic field [20] and the results of [21, 22] for the nonlinear lifshitz black holes to the exponential nonlinear gauge field coupled

to the dilaton field. Thus, both in canonical and grand-canonical ensemble, the variation of total action about the solutions of the field equations is

$$\delta I_{\text{tot}} = \int d^{n-1}x S_{ab} \delta h^{ab}. \quad (18)$$

That is, the nonlinear gauge field is absent in the variation of the total action both in canonical and grand-canonical ensembles.

In order to obtain the conserved charges of the spacetime we use the counterterm method [23, 24] inspired by (A)dS/CFT correspondence. For asymptotically AdS solutions, this method works very well [24]. However, in our paper we have the scalar dilaton field with a Liouville potential. It was argued that the presence of Liouville-type dilaton potential, which is regarded as the generalization of the cosmological constant, changes the asymptotic behavior of the solutions to be neither asymptotically flat nor (A)dS. It has been shown that no dilaton dS or AdS black hole solution exists with the presence of only one Liouville-type dilaton potential [25]. But, as in the case of asymptotically AdS spacetimes, according to the domain-wall/QFT (quantum field theory) correspondence [26], there may be a suitable counterterm for the stress-energy tensor which removes the divergences. In this paper, we deal with the spacetimes with zero curvature boundary [$R_{abcd}(h) = 0$], and therefore the counterterm for the stress-energy tensor should be proportional to h^{ab} . We find the finite stress-energy tensor in n -dimensional Einstein-dilaton gravity with Liouville-type in the form [14]

$$T^{ab} = \frac{1}{8\pi} \left[\Theta^{ab} - \Theta h^{ab} + \frac{n-2}{l_{\text{eff}}} h^{ab} \right], \quad (19)$$

where l_{eff} is given by

$$l_{\text{eff}}^2 = \frac{(n-2)(\alpha^2 - n + 1)}{2\Lambda} e^{-4\alpha\Phi/(n-2)}. \quad (20)$$

In the particular case $\alpha = 0$, the effective l_{eff}^2 of Eq. (20) reduces to $l^2 = -(n-1)(n-2)/2\Lambda$ of the AdS spacetimes. The first two terms in Eq. (19) is the variation of the action (1) with respect to h_{ab} , and the last term is the counterterm which removes the divergences. One may note that the counterterm has the same form as in the case of asymptotically AdS solutions with zero curvature boundary, where l is replaced by l_{eff} . If we choose the Killing vector field ξ on spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij} . Then, the quasilocal conserved quantities may be obtained from the following relation [14]

$$Q(\xi) = \int_{\mathcal{B}} d^{n-2}x \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (21)$$

where σ is the determinant of the boundary metric σ_{ij} and n^a is the unit normal vector on the boundary $\mathcal{B}$. In our case, the boundary $\mathcal{B}$ has two Killing vector fields timelike ($\partial/\partial t$) and rotational ($\partial/\partial\varphi$). The corresponding conserved charges are the quasilocal mass and angular momentum may be obtained as

$$M = \int_{\mathcal{B}} d^{n-2}x \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (22)$$

$$J = \int_{\mathcal{B}} d^{n-2}x \sqrt{\sigma} T_{ab} n^a \zeta^b. \quad (23)$$

III. ROTATING DILATON BLACK BRANES IN HIGHER DIMENSIONS

In this section, we would like to construct the rotating black brane solutions of the field equations (11)-(13) with k rotation parameters. The number of independent rotation parameters for an n -dimensional localized object is equal to the number of Casimir operators, which is $[(n-1)/2] \equiv k$, where $[x]$ is the integer part of x [27]. The metric of n -dimensional rotating solution with cylindrical or toroidal horizons and k rotation parameters can be written as [28, 29]

$$ds^2 = -f(r) \left(\Xi dt - \sum_{i=1}^k a_i d\phi_i \right)^2 + \frac{r^2}{l^4} R^2(r) \sum_{i=1}^k (a_i dt - \Xi l^2 d\phi_i)^2$$

$$-\frac{r^2}{l^2}R^2(r)\sum_{i<j}^k(a_id\phi_j-a_jd\phi_i)^2+\frac{dr^2}{f(r)}+\frac{r^2}{l^2}R^2(r)dX^2,$$

$$\Xi^2=1+\sum_{i=1}^k\frac{a_i^2}{l^2}, \quad (24)$$

where a_i 's are k rotation parameters. There are two unknown functions $f(r)$ and $R(r)$ in the above metric which should be determined by solving the field equations. The range of the angular coordinates are $0 \leq \phi_i \leq 2\pi$ and dX^2 is the Euclidean metric on the $(n-k-2)$ -dimensional submanifold with volume Σ_{n-k-2} .

First of all, we integrate the modified Maxwell equation (13) which gives

$$F_{tr} = \frac{q\Xi e^{4\alpha\Phi/(n-2)}}{(rR)^{n-2}} \exp\left[-\frac{1}{2}L_W\left(\frac{q^2}{\beta^2(rR)^{2n-4}}\right)\right], \quad (25)$$

$$F_{\phi_i r} = -\frac{a_i}{\Xi}F_{tr}, \quad (26)$$

where q , is an integration constant related to the electric charge of the brane. Note that in (25), $L_W(x) = \text{Lambert}W(x)$ is the Lambert function which satisfies the identity [30]

$$L_W(x)e^{L_W(x)} = x, \quad (27)$$

and has the following series expansion

$$L_W(x) = x - x^2 + \frac{3}{2}x^3 - \frac{8}{3}x^4 + \dots \quad (28)$$

Clearly, series expansion of Eq. (28) converges for $|x| < 1$. In the absence of the nonlinear dilaton field ($\alpha = 0$) we have $R(r) = 1$, and Eq. (25) reduces to

$$F_{tr} = \frac{q\Xi}{r^{n-2}} \exp\left[-\frac{1}{2}L_W\left(\frac{q^2}{\beta^2 r^{2n-4}}\right)\right], \quad (29)$$

while in the limiting case where $\beta \rightarrow \infty$, it reduces to the electric field of n -dimensional black brane of Einstein-Maxwell-dilaton gravity [27]

$$F_{tr} = \frac{q\Xi e^{4\alpha\Phi/(n-2)}}{(rR)^{n-2}} + O\left(\frac{1}{\beta^2}\right). \quad (30)$$

In order to solve the system of equations (11) and (12) for three unknown functions $f(r)$, $R(r)$ and $\Phi(r)$, we make the ansatz

$$R(r) = e^{2\alpha\Phi/(n-2)}. \quad (31)$$

Substituting (31), the electromagnetic fields (25)- (26) and the metric (24) into the field equations (11) and (12), one can obtain the following solutions

$$f(r) = \frac{2(\alpha^2+1)^2(\Lambda+2\beta^2)b^\gamma}{(n-2)(\alpha^2-n+1)}r^{2-\gamma} - \frac{m}{r^{n-3-(n-2)\gamma/2}} - \frac{4\beta q(\alpha^2+1)b^{(4-n)\gamma/2}}{(n-2)r^{n-3-(n-2)\gamma/2}} \int r^{-\gamma} \left(\sqrt{L_W(\eta)} - \frac{1}{\sqrt{L_W(\eta)}} \right) dr, \quad (32)$$

$$\Phi(r) = \frac{(n-2)\alpha}{2(\alpha^2+1)} \ln\left(\frac{b}{r}\right), \quad (33)$$

where b is an arbitrary constant, $\gamma = 2\alpha^2/(1+\alpha^2)$, and

$$\eta \equiv \frac{q^2 b^{(2-n)\gamma}}{\beta^2 r^{(n-2)(2-\gamma)}}. \quad (34)$$

In the above expression, m appears as an integration constant and is related to the mass of the black hole. The integration of Eq. (32) can be performed using the Mathematica software. The resulting solution can be written

$$\begin{aligned}
 f(r) = & \frac{2(\Lambda + 2\beta^2)(\alpha^2 + 1)^2 b^\gamma}{(n-2)(\alpha^2 - n + 1)} r^{2-\gamma} - \frac{m}{r^{n-3-(n-2)\gamma/2}} \\
 & + \frac{2\beta q(\alpha^2 + 1)^4 b^{(4-n)\gamma/2}}{(n-2)^2(\alpha^2 - 1)^2} \left(\frac{\beta^2 b^{(n-2)\gamma}}{q^2} \right)^{\frac{1-\gamma}{(n-2)(\gamma-2)}} r^{(n-2)\gamma/2-n+3} \\
 & \times \left(\frac{1-\alpha^2}{2n-4} \right)^{\frac{2-2n+\gamma n}{(\gamma-2)(2n-4)}} \left\{ - (n-2)^2(\gamma-2)^2 \left[\Gamma \left(\frac{\alpha^2 + 3n-7}{2n-4}, \frac{1-\alpha^2}{2n-4} L_W(\eta) \right) - \Gamma \left(\frac{\alpha^2 + 3n-7}{2n-4} \right) \right] \right. \\
 & \left. + (\gamma-1)^2 \left[\Gamma \left(\frac{\alpha^2 - n + 1}{2n-4}, \frac{1-\alpha^2}{2n-4} L_W(\eta) \right) - \Gamma \left(\frac{\alpha^2 - n + 1}{2n-4} \right) \right] \right\}. \quad (35)
 \end{aligned}$$

where $\Gamma(a, z)$ and $\Gamma(a)$ are Gamma functions and they are related to each other via

$$\Gamma(a, z) = \Gamma(a) - \frac{z^a}{a} F(a, 1+a, -z) \quad (36)$$

where $F(a, b, z)$ is hypergeometric function [30]. Using (36), solution (35) can also be reexpressed in terms of hypergeometric function,

$$\begin{aligned}
 f(r) = & \frac{2(\Lambda + 2\beta^2)(\alpha^2 + 1)^2 b^\gamma}{(n-2)(\alpha^2 + 1 - n)} r^{2-\gamma} - \frac{m}{r^{n-3-(n-2)\gamma/2}} \\
 & + \frac{4\beta q(\alpha^2 + 1)^2 b^{(4-n)\gamma/2}}{(n-2)r^{n-3-(n-2)\gamma/2}} \left(\frac{\beta^2 b^{(n-2)\gamma}}{q^2} \right)^{\frac{1-\gamma}{(n-2)(\gamma-2)}} L_W^{\frac{\alpha^2 - n + 1}{2n-4}}(\eta) \\
 & \times \left\{ \frac{L_W^2(\eta)}{\alpha^2 + 3n-7} F \left(\frac{\alpha^2 + 3n-7}{2n-4}, \frac{\alpha^2 + 5n-11}{2n-4}, \frac{\alpha^2 - 1}{2n-4} L_W(\eta) \right) \right. \\
 & \left. - \frac{1}{\alpha^2 - n + 1} F \left(\frac{\alpha^2 - n + 1}{2n-4}, \frac{\alpha^2 + n-3}{2n-4}, \frac{\alpha^2 - 1}{2n-4} L_W(\eta) \right) \right\}. \quad (37)
 \end{aligned}$$

Using the fact that $F(a, b, z)$ has a convergent series expansion for $|z| < 1$ as given in (28), we can expand (37) for large β . The result is

$$\begin{aligned}
 f(r) = & \frac{2\Lambda(\alpha^2 + 1)^2}{(n-2)(\alpha^2 - n + 1)} b^\gamma r^{2-\gamma} - \frac{m}{r^{n-3-(n-2)\gamma/2}} + \frac{2q^2(\alpha^2 + 1)^2 b^{-(n-3)\gamma}}{(n-2)(\alpha^2 + n-3)r^{-(n-3)\gamma+2n-6}} \\
 & - \frac{q^4(\alpha^2 + 1)^2 b^{-(2n-5)\gamma}}{2\beta^2(n-2)(\alpha^2 + 3n-7)r^{(2n-5)(2-\gamma)}} + O\left(\frac{1}{\beta^4}\right). \quad (38)
 \end{aligned}$$

It is worthy to note that as $\beta \rightarrow \infty$ the above solution reduces to the n -dimensional charged rotating dilaton black brane solutions given in Ref. [27]. The last term in the right hand side of the above expression is the leading nonlinear correction to the AdS black brane with dilaton field. In the absence of a nontrivial dilaton ($\alpha = \gamma = 0$), the above solutions reduce to the asymptotically AdS charged rotating black brane solutions of Einstein gravity in the presence of EN electrodynamics [9]. Finally, in the limit $\beta^2 \rightarrow \infty$ and $\alpha = 0 = \gamma$, solution (38) has the form of the asymptotically AdS black brane [28, 31],

$$f(r) = -\frac{2\Lambda}{(n-2)(n-1)} r^2 - \frac{m}{r^{n-3}} + \frac{2q^2}{(n-2)(n-3)r^{2n-6}}.$$

A. Physical properties of the solutions

Next, we study the geometry of this spacetime. For this purpose, we first seek for the curvature singularities in the presence of dilaton and nonlinear electrodynamic fields. It is a matter of calculation to show that the Ricci scalar and

FIG. 1: $f(r)$ versus r for $l = 1$, $b = 1$, $n = 4$, $q = 1$, $\beta = 1.1$, $\alpha = 1.1$, and $m = 7$ (solid line), $m = 8.5$ (bold line) and $m = 10$ (dashed line).

FIG. 2: $f(r)$ versus r for $l = 1$, $b = 1$, $n = 4$, $q = 1$, $\beta = 1.1$, $m = 8.5$, and $\alpha = 1.05$ (solid line), $\alpha = 1.10$ (bold line) and $\alpha = 1.15$ (dashed line).

the Kretschmann invariant behave as

$$\lim_{r \rightarrow 0^+} R = \infty, \quad (39)$$

$$\lim_{r \rightarrow 0^+} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \infty, \quad (40)$$

which indicate that there is an essential singularity at $r = 0$. In order to study the asymptotic behavior of the solutions, we expand the metric function $f(r)$ for $r \rightarrow \infty$ limit. We find

$$\lim_{r \rightarrow \infty} f(r) = \frac{2\Lambda(\alpha^2 + 1)^2}{(n-2)(\alpha^2 - n + 1)} b^\gamma r^{2-\gamma}. \quad (41)$$

FIG. 3: $f(r)$ versus r for $l = 1$, $b = 1$, $n = 4$, $q = 1$, $\alpha = 1.1$, $m = 8.5$, and $\beta = 0.5$ (solid line), $\beta = 1.10$ (bold line) and $\beta = 2$ (dashed line).

FIG. 4: $f(r)$ versus r for $l = 1$, $b = 1$, $q = 1$, $\alpha = 1.1$, $\beta = 1.1$, $m = 8.5$, and $n = 4$ (solid line), $n = 5$ (bold line) and $n = 6$ (dashed line).

Let us note that in the absence of the dilaton field ($\alpha = 0 = \gamma$), the metric function becomes

$$\lim_{r \rightarrow \infty} f(r) = -\frac{2\Lambda r^2}{(n-1)(n-2)}, \quad (42)$$

which describes an asymptotically AdS ($\Lambda < 0$) or dS ($\Lambda > 0$) spacetimes. However, as one can see from Eq. (41), in the presence of the dilaton field the asymptotic behavior is neither flat nor (A)dS. For example, taking $\alpha = \sqrt{2}$, $n = 6$ and $b = 1$, we have

$$\lim_{r \rightarrow \infty} f(r) = -\frac{3\Lambda}{2} r^{2/3}. \quad (43)$$

Clearly, the metric function (43) is neither flat nor (A)dS. This is consistent with the argument given in [25], which states that no dilaton dS or AdS black hole solution exists with the presence of only one or two Liouville-type

FIG. 5: $f(r)$ versus r for $l = 1$, $b = 1$, $n = 4$, $\alpha = 1.1$, $\beta = 1.1$, $m = 8.5$, and $q = 0.7$ (solid line), $q = 1$ (bold line) and $q = 1.2$ (dashed line).

FIG. 6: T_+ versus r_+ for $l = 1$, $b = 1$, $n = 4$, $q = 1$, $\Xi = 1.1$, $\beta = 1.1$, and $\alpha = 1.1$ (solid line), $\alpha = 3.4$ (bold line) and $\alpha = 4.5$ (dashed line).

dilaton potential. It is important to note that this asymptotic behavior is not due to the nonlinear nature of the electrodynamic field, since as $r \rightarrow \infty$ the effects of the nonlinearity disappear. This is due to the fact that, $r \rightarrow \infty$ limit corresponds to $\beta^2 \rightarrow \infty$, and in this case the metric functions $f(r)$ restore the result of EMd black brane with unusual asymptotic [27].

Furthermore, from the dilaton field (33) we see that as $r \rightarrow \infty$, the dilaton field does not vanishes, while in case of asymptotic flat or (a)dS we expect to have $\lim_{r \rightarrow \infty} \Phi(r) = 0$. Indeed, by solving the field equation (12) we find

$$\Phi(r) = \frac{(n-2)\alpha}{2(\alpha^2+1)} \ln \left(c + \frac{b}{r} \right), \quad (44)$$

however, the system of equation (11)-(13) will be fully satisfied provided we choose $c = 0$. From the above arguments we conclude that the asymptotic behavior of the obtained solutions is neither flat nor (A)dS.

From (32) we see that the solutions are ill-defined for $\alpha = \sqrt{n-1}$. In order to obtain the location of the horizons

FIG. 7: T_+ versus r_+ for $l = 1$, $b = 1$, $n = 4$, $q = 1$, $\Xi = 1.1$, $\alpha = 3.3$, and $\beta = 0.1$ (solid line), $\beta = 1.1$ (bold line) and $\beta = 3.1$ (dashed line).

FIG. 8: T_+ versus r_+ for $l = 1$, $b = 1$, $q = 1$, $\Xi = 1.1$, $\alpha = 3.3$, $\beta = 1.1$, and $n = 4$ (solid line), $n = 8$ (bold line) and $n = 12$ (dashed line).

of spacetime, we have to find the roots of $f(r_+) = 0$. However, due to the complexity of $f(r)$ given in (35), it is not possible to find the roots of $f(r_+) = 0$, analytically. Nevertheless, we can plot the function $f(r)$ versus r for different model parameters as in figures 1 - 5. These figures show that one may choose an appropriate parameters (m , α , β , n and q) to obtain naked singularity, black brane with an extreme horizon or black brane with an inner and an outer horizons.

IV. THERMODYNAMICS OF BLACK BRANES

As in the case of rotating black hole solutions of the Einstein gravity, the above metric given by (24) and (35) has both Killing and event horizons. The Killing horizon is a null surface whose null generators are tangent to a Killing

FIG. 9: T_+ versus r_+ for $l = 1$, $b = 1$, $n = 4$, $\Xi = 1.1$, $\beta = 1.1$, $\alpha = 3.3$, and $q = 0.5$ (solid line), $q = 1$ (bold line) and $q = 1.5$ (dashed line).

FIG. 10: T_+ versus r_+ for $l = 1$, $b = 1$, $q = 1$, $n = 4$, $\alpha = 3.3$, $\beta = 1.1$, and $\Xi = 1$ (solid line), $\Xi = 1.5$ (bold line) and $\Xi = 5$ (dashed line).

field. It is easy to see that the Killing vector

$$\chi = \partial_t + \sum_{i=1}^k \Omega_i \partial_{\phi_i}, \quad (45)$$

is the null generator of the event horizon, where Ω_i is the i th component of angular velocity of the outer horizon which may be obtained by analytic continuation of the metric.

One can obtain the temperature and angular momentum of the event horizon by analytic continuation of the metric. Setting $t \rightarrow i\tau$ and $a_i \rightarrow ia_i$ yields the Euclidean section of (24), whose regularity at $r = r_+$ requires that we should identify $\tau \sim \tau + \beta_+$ and $\phi_i \sim \phi_i + \beta_+ \Omega_i$, where β_+ and Ω_i 's are the inverse Hawking temperature and the angular

velocities of the outer event horizon. We obtain

$$T_+ = \frac{f'(r_+)}{4\pi\Xi} = -\frac{(\alpha^2 + 1)}{4\pi\Xi} r_+^{1-\gamma} \left\{ \frac{2(\Lambda + 2\beta^2)b^\gamma}{n-2} - \frac{4\beta q}{n-2} r_+^{(n-2)(\gamma-2)/2} b^{(4-n)\gamma/2} \right. \\ \left. \times \left(\frac{1}{\sqrt{L_W(\eta_+)}} - \sqrt{L_W(\eta_+)} \right) \right\} \quad (46)$$

$$\Omega_i = \frac{a_i}{\Xi l^2} \quad (47)$$

where $\eta_+ = \eta(r = r_+)$ and we have used $f(r_+) = 0$ for deleting m . The behavior of T_+ versus r_+ is shown in figures 6 - 10. From these figures we find out that, for large value of r_+ , the temperature tends to a constant independent of the model parameters, while for small values of r_+ , the temperature may be negative ($T < 0$). In this case we encounter a naked singularity. Fig. 6 indicates that there is an α_{min} , in which temperature is positive definite for $\alpha > \alpha_{min}$. Besides, Fig. 7 shows that there exist a maximum nonlinearity parameter, β_{max} , in which T_+ is a positive definite function for $\beta < \beta_{max}$. The charge parameter, q , has the same effects as that of the nonlinearity parameter (see Fig. 9). Figs. 8 and 10 show that both the dimensionality and rotation parameter may affect the temperature. These figures confirm that although dimensionality may change the radius of the zero temperature black branes (root of T_+), rotation parameter does not.

It is known that for an extremal black hole the temperature is zero and the horizon is degenerate. In this case r_{ext} is the positive root of the following equation:

$$\frac{2(\Lambda + 2\beta^2)b^\gamma}{n-2} - \frac{4\beta q_{ext}}{n-2} r_{ext}^{(n-2)(\gamma-2)/2} b^{(4-n)\gamma/2} \left(\frac{1}{\sqrt{L_W(\eta_{ext})}} - \sqrt{L_W(\eta_{ext})} \right) = 0. \quad (48)$$

where

$$\eta_{ext} \equiv \frac{q_{ext}^2 r_{ext}^{(n-2)(\gamma-2)}}{\beta^2 b^{(n-2)\gamma}}. \quad (49)$$

From figures 9 and 10, we see that r_{ext} decreases as α increases, while r_{ext} increases with increasing q . Indeed, the metric of Eqs. (24) and (35) can describe a nonlinear dilaton black hole with inner and outer event horizons located at r_- and r_+ , provided $r > r_{ext}$, an extreme ENd black hole in the case of $r = r_{ext}$, and a naked singularity if $r < r_{ext}$.

Next, we calculate the mass and angular momentum of the black branes. If we denote the volume of the hypersurface boundary at constant t and r by $V_{n-2} = (2\pi)^k \Sigma_{n-k-2}$, the mass and angular momentum per unit volume V_{n-2} of the black branes ($\alpha < \sqrt{n-1}$) can be calculated through the use of Eqs. (22) and (23). We find

$$M = \frac{b^{(n-2)\gamma/2}}{16\pi l^{n-3}} \left(\frac{(n-1-\alpha^2)\Xi^2 + \alpha^2 - 1}{1+\alpha^2} \right) m, \quad (50)$$

$$J_i = \frac{b^{(n-2)\gamma/2}}{16\pi l^{n-3}} \left(\frac{n-1-\alpha^2}{1+\alpha^2} \right) \Xi l m a_i. \quad (51)$$

When $a_i = 0$ ($\Xi = 1$), the angular momentum per unit volume vanishes, and therefore a_i 's are the rotational parameters of the black branes.

Black hole entropy typically satisfies the so called area law of the entropy [32]. This near universal law applies to almost all kinds of black holes and black branes in Einstein gravity [33]. It is easy to show that the entropy per unit volume V_{n-2} of the black brane can be written as

$$S = \frac{\Xi b^{(n-2)\gamma/2} r_+^{(n-2)(1-\gamma/2)}}{4l^{n-3}}, \quad (52)$$

The electric charge per unit volume V_{n-1} can be found by calculating the flux of the electric field at infinity, yielding

$$Q = -\frac{1}{4\pi V_{n-1}} \int_{\Sigma} \nabla_{\mu} (\partial_Y \mathcal{L}(Y) F^{\mu\nu}) dS_{\nu} = -\frac{1}{8\pi V_{n-1}} \oint_{\partial\Sigma} \partial_Y \mathcal{L}(Y) F^{\mu\nu} dS_{\mu\nu} = \frac{\Xi q}{4\pi l^{n-3}}, \quad (53)$$

where the volume is replaced by an arbitrary spacelike hypersurface Σ (partial Cauchy surface) with boundary $\partial\Sigma$. In addition, the volume element on Σ is a non-spacelike co-vector (1-form) dS_{ν} and

$dS_{\mu\nu}$ is the area element of $\partial\Sigma$. We should note that for linear Maxwell case ($\beta \rightarrow \infty$), one obtains $\partial_Y \mathcal{L}(Y) = -1$.

The electric potential U , measured at infinity with respect to the horizon, is defined by

$$U = A_\mu \chi^\mu|_{r \rightarrow \infty} - A_\mu \chi^\mu|_{r=r_+}, \quad (54)$$

where χ is the null generator of the horizon given by Eq. (45). One can easily show that the vector potential A_μ corresponding to the electromagnetic tensor (25) and (26) can be written as

$$\begin{aligned} A_\mu = & (\Xi \delta_\mu^t - a_i \delta_\mu^i) \times b^\gamma \beta (\alpha^2 + 1) \left(\frac{\beta^2 b^{(n-2)\gamma}}{q^2} \right)^{\frac{\alpha^2-1}{2n-4}} \left(\frac{\alpha^2-1}{4-2n} \right)^{\frac{\alpha^2+n-3}{4-2n}} \\ & \times \left\{ -\frac{1}{2(n-2)} \Gamma \left(\frac{\alpha^2+n-3}{2n-4}, \frac{1-\alpha^2}{2n-4} L_W(\eta) \right) \right. \\ & \left. + \frac{1}{\alpha^2-1} \left[\Gamma \left(\frac{\alpha^2+3n-7}{2n-4}, \frac{1-\alpha^2}{(2n-4)} L_W(\eta) \right) - \frac{1}{2} \Gamma \left(\frac{\alpha^2+n-3}{2n-4} \right) \right] \right\}. \end{aligned} \quad (55)$$

Therefore, the electric potential may be obtained as

$$\begin{aligned} U = & \frac{b^\gamma \beta}{\Xi} (\alpha^2 + 1) \left(\frac{\beta^2 b^{(n-2)\gamma}}{q^2} \right)^{\frac{\alpha^2-1}{2n-4}} \left(\frac{\alpha^2-1}{4-2n} \right)^{\frac{\alpha^2+n-3}{4-2n}} \\ & \times \left\{ -\frac{1}{2(n-2)} \Gamma \left(\frac{\alpha^2+n-3}{2n-4}, \frac{1-\alpha^2}{2n-4} L_W(\eta_+) \right) \right. \\ & \left. + \frac{1}{\alpha^2-1} \left[\Gamma \left(\frac{\alpha^2+3n-7}{2n-4}, \frac{1-\alpha^2}{2n-4} L_W(\eta_+) \right) - \frac{1}{2} \Gamma \left(\frac{\alpha^2+n-3}{2n-4} \right) \right] \right\}. \end{aligned} \quad (56)$$

Expanding for large value of r or β , we get

$$U = \frac{q(\alpha^2+1)b^{(4-n)\gamma/2}}{\Xi(\alpha^2+n-3)r_+^{n-3+2\gamma-n\gamma/2}} - \frac{q^3(1+\alpha^2)b^{(8-3n)\gamma/2}}{2\Xi(\alpha^2+3n-7)r_+^{3n-7+4\gamma-3n\gamma/2}\beta^2} + O\left(\frac{1}{r_+^{5n-11+6\gamma-5n\gamma/2}\beta^4}\right).$$

which indicates that for $\beta^2 \rightarrow \infty$, our expression reduces to the electric potential of black branes in EMd gravity [27]. In addition, one can find that the electrical potential goes to zero at spatial infinity ($r \rightarrow \infty$). Now, we are in a position to verify the first law of thermodynamics. In order to do this, we obtain the mass M as a function of extensive quantities S , $\mathbf{J}$ and Q . Using the expression for the mass, the angular momenta, the entropy, and the charge given in Eqs. (50), (51), (52), (53) and the fact that $f(r_+) = 0$, one can obtain a Smarr-type formula as

$$M(S, \mathbf{J}, Q) = \frac{[(n-1-\alpha^2)Z + \alpha^2 - 1] \mathbf{J}}{l(n-1-\alpha^2)\sqrt{Z(Z-1)}}, \quad (57)$$

where $\mathbf{J} = \sqrt{\sum_i J_i^2}$, and $Z = \Xi^2$ is the positive real root of the following equation

$$\begin{aligned} & \mathbf{J} - \frac{1}{2} l b^{\alpha^2} \sqrt{Z-1} \beta Q \left(\frac{Z\beta^2}{16\pi^2 Q^2 l^{2n-6}} \right)^{\frac{\alpha^2-1}{2n-4}} \left(\frac{1-\alpha^2}{2n-4} \right)^{\frac{n-1-\alpha^2}{2n-4}} (\alpha^2+1)(\alpha^2-n+1) \\ & \times \left\{ \frac{4}{(\alpha^2-1)} \left[\Gamma \left(\frac{\alpha^2-7+3n}{2n-4}, -\frac{(\alpha^2-1)}{2n-4} L_W \left(\frac{\pi^2 Q^2}{S^2 \beta^2} \right) \right) - \Gamma \left(\frac{\alpha^2-7+3n}{2n-4} \right) \right] \right. \\ & \left. - \frac{1}{(n-2)^2} \left[\Gamma \left(\frac{\alpha^2-n+1}{2n-4}, -\frac{(\alpha^2-1)}{2n-4} L_W \left(\frac{\pi^2 Q^2}{S^2 \beta^2} \right) \right) - \Gamma \left(\frac{\alpha^2-n+1}{2n-4} \right) \right] \right\} \\ & - \frac{b^{\alpha^2} \sqrt{Z(Z-1)}}{16(n-2)\pi l^{n-2}} \left(\frac{4S l^{n-3}}{\sqrt{Z}} \right)^{\frac{n-1-\alpha^2}{n-2}} (n^2-3n-4\beta^2 l^2+2)(\alpha^2+1) = 0. \end{aligned} \quad (58)$$

FIG. 11: $(\partial^2 M/\partial S^2)_{J,Q}$ (left) and T (right) versus α for $l = 1$, $b = 1$, $\Xi = 1.1$, $n = 4$, $\beta = 1.1$ and $q = 0.5$ (solid line), $q = 1$ (bold line) and $q = 1.5$ (dashed line).

We can regard the parameters S , $\mathbf{J}$, and Q as a complete set of extensive parameters for the mass $M(S, \mathbf{J}, Q)$ and define the intensive parameters conjugate to S , $\mathbf{J}$ and Q . These parameters are, respectively, the temperature, the angular velocities, and the electric potential, which are defined as

$$T = \left(\frac{\partial M}{\partial S} \right)_{J,Q}, \quad \Omega_i = \left(\frac{\partial M}{\partial J_i} \right)_{S,Q}, \quad U = \left(\frac{\partial M}{\partial Q} \right)_{S,\mathbf{J}}. \quad (59)$$

Numerical calculations show that the intensive quantities calculated by Eq. (59) coincide with Eqs. (46), (47) and (56). Thus, these thermodynamics quantities satisfy the first law of thermodynamics

$$dM = TdS + \sum_{i=1}^k \Omega_i dJ_i + U dQ. \quad (60)$$

V. THERMAL STABILITY OF THE BLACK BRANES

In order to have a physical black branes, it is necessary to check the stability condition under external perturbation. There are two types of stability: dynamical stability and thermodynamical stability. In this paper, we investigate the thermodynamical stability of dilaton black brane solutions with EN electrodynamics.

In order to investigate the thermal stability of the black branes, one can use the canonical ensemble. In the canonical ensemble, the electric charge and angular momentum are fixed parameters and therefore, the positivity of the heat capacity is the requirements usually referred to stability criterion in canonical ensemble

$$C_{J,Q} = \frac{T}{(\partial^2 M/\partial S^2)_{J,Q}}. \quad (61)$$

Analytical calculation of the heat capacity is too long and hence we leave out the analytical results for reasons of economy. We use the numerical analysis and plot various figures to investigate the stability. Before analyzing the stability, one should restrict the solutions to physical black branes (positive temperature). Therefore, instead of investigation of the Heat capacity, we study the behavior of both $(\partial^2 M/\partial S^2)_{J,Q}$ and T in Figs. 11 - 16. Figs. 11 and 12 show that there is a α_c in which black branes are stable for $\alpha < \alpha_c$. These figures indicate that α_c is an increasing (a decreasing) function of electric charge (dimensions). From Figs. 13 and 14 we find that $(\partial^2 M/\partial S^2)_{J,Q}$ is an increasing function of the nonlinearity parameter and for large values of β they lead to a fixed value (Maxwell case). Fig. 14 shows that for small (large) values of β , increasing d leads to increasing (decreasing) the values of

FIG. 12: $(\partial^2 M / \partial S^2)_{J,Q}$ (left) and T (right) versus α for $l = 1$, $b = 1$, $\Xi = 1.1$, $q = 1$, $\beta = 1.1$ and $n = 4$ (solid line), $n = 5$ (bold line) and $n = 6$ (dashed line).

FIG. 13: $(\partial^2 M / \partial S^2)_{J,Q}$ (left) and T (right) versus β for $l = 1$, $b = 1$, $\Xi = 1.1$, $n = 4$, $\alpha = 1.1$ and $q = 0.5$ (solid line), $q = 1$ (bold line) and $q = 1.5$ (dashed line).

$(\partial^2 M / \partial S^2)_{J,Q}$. From Figs. 15 and 16, one can find the effects of rotation on the stability conditions. Fig. 15 shows that for small values of the electric charge, $(\partial^2 M / \partial S^2)_{J,Q}$ is an increasing function of for slowly rotating black branes and it will be a decreasing function for rapid rotation. For large values of the electric charge, $(\partial^2 M / \partial S^2)_{J,Q}$ is a decreasing function of Ξ . These behaviors are vice versa for dimensions. In other words, $(\partial^2 M / \partial S^2)_{J,Q}$ is a decreasing function of Ξ for low dimensions, and in higher dimensions, $(\partial^2 M / \partial S^2)_{J,Q}$ is an increasing (a decreasing) function of Ξ for small (large) values of rotation parameter.

FIG. 14: $(\partial^2 M / \partial S^2)_{J,Q}$ (left) and T (right) versus β for $l = 1$, $b = 1$, $\Xi = 1.1$, $q = 1$, $\alpha = 1.1$ and $n = 4$ (solid line), $n = 5$ (bold line) and $n = 6$ (dashed line).

FIG. 15: $(\partial^2 M / \partial S^2)_{J,Q}$ (left) and T (right) versus Ξ for $l = 1$, $b = 1$, $\beta = 1.1$, $n = 4$, $\alpha = 1.1$ and $q = 0.5$ (solid line), $q = 1$ (bold line) and $q = 1.5$ (dashed line).

VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we investigated the n -dimensional Einstein-dilaton gravity coupled to the EN electrodynamic field. Then, by varying the action we obtained the equation of motions. In the limiting case of the linear Maxwell electrodynamics with dilaton field we have $L(F, \Phi) = -e^{-4\alpha\Phi/(n-2)}F^2$, and the system of equations reduce to the well-known equations of EMd gravity [15]. Then, we constructed a new class of charged, rotating black brane solutions, with $k = [(n-1)/2]$ rotation parameters, of this theory. Due to the presence of the Liouville-type dilaton potential, the asymptotic behavior of the obtained solutions is neither flat nor (A)dS. We investigated the causal structure of the spacetime in ample details. We found the suitable counterterm which remove the divergences of the action in the presence of the dilaton field and calculated the conserved and thermodynamic quantities of the spacetime. Interestingly enough, we found out that the conserved quantities crucially depend on the dilaton coupling constant, α , while they

FIG. 16: $(\partial^2 M / \partial S^2)_{J,Q}$ (left) and T (right) versus Ξ for $l = 1$, $b = 1$, $\beta = 1.1$, $q = 1$, $\alpha = 1.1$ and $n = 4$ (solid line), $n = 5$ (bold line) and $n = 6$ (dashed line).

are independent of the nonlinear parameter β . In the absence of a non-trivial dilaton ($\alpha = \gamma = 0$), these solutions reduce to the asymptotically AdS charged rotating black brane solutions of Einstein theory in the presence of EN electrodynamics [9]. When $\beta \rightarrow \infty$, these solutions reduce to the charged rotating dilaton black brane solutions given in Ref. [27]. We computed thermodynamic quantities of the higher dimensional rotating charged black brane such as the temperature, mass, angular momentum, entropy, charge and electric potential, and examined the validity of the first law of thermodynamics on the black brane horizon.

Considering the plotted figures with numerical calculations, we investigated the effects of rotation, nonlinearity of electrodynamics and dilaton field with various electric charge and dimensions on the stability conditions. Numerical calculations showed that although nonlinearity and dilaton parameters (β and α) change the stability interval significantly, rotation parameter does not. In other words, one can change β and/or α to affect the stability condition, while changing the rotation parameter does not have significant effect on the roots of $(\partial^2 M / \partial S^2)_{J,Q}$ and T , and therefore the stability interval. Generally, we found that decreasing the dilaton field and the nonlinearity power (increasing β) lead to obtain stable black brane solutions.

It is worth mentioning that the charged rotating dilaton black branes coupled to EN electrodynamics constructed in this paper have flat horizon. Thus, it would be interesting to derive higher dimensional rotating black holes of these field equations with curved horizon. Besides, the solutions obtained here in the presence of the dilaton field with a Liouville type potential (3) is neither flat nor AdS, thus it would be nice if one could construct asymptotically AdS charged rotating dilaton black branes/holes of this theory by taking appropriate expression for the dilaton potential [34]. The study can also be applied for other type of nonlinear electrodynamics such as logarithmic nonlinear electrodynamics with suitable Lagrangian in the presence of dilaton field. It is also interesting to investigate magnetic dilaton string solutions supported by EN electrodynamics. These issues are now under investigations and the results will be appeared elsewhere.

Acknowledgments

A.S grateful to Professor M. H. Dehghani for helpful discussion and useful comments. We also thank the Research Council of Shiraz University. This work has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Iran.

[1] M. Born and L. Infeld, Proc. R. Soc. A **144**, 425 (1934).

- [2] H. H. Soleng, Phys. Rev. D **52**, 6178 (1995).
- [3] S. H. Hendi, JHEP **03**, 065 (2012).
- [4] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **163**, 123 (1985);
R. R. Metsaev, M. A. Rakhmanov and A. A. Tseytlin, Phys. Lett. B **193**, 207 (1987);
E. Bergshoeff, E. Sezgin, C. Pope and P. Townsend, Phys. Lett. B **188**, 70 (1987);
A. A. Tseytlin, Nucl. Phys. B **501**, 41 (1997);
D. Brecher and M. J. Perry, Nucl. Phys. B **527**, 121 (1998).
- [5] C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl. Phys. B **308**, 221 (1988);
O. Andreev and A. A. Tseytlin, Nucl. Phys. B **311**, 221 (1988);
R. Leigh, Mod. Phys. Lett. A **4**, 2767 (1989).
- [6] G. Boillat, J. Math. Phys. **11**, 941 (1970);
G. Boillat, J. Math. Phys. **11**, 1482 (1970).
- [7] G. W. Gibbons and D. A. Rasheed, Nucl. Phys. B **454**, 185 (1995).
- [8] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, Phys. Rev. D **77**, 126006 (2008);
Y. Kats and P. Petrov, JHEP **01**, 044 (2009);
P. Kovtun, D.T. Son and A.O. Starinets, JHEP **10**, 064 (2003);
R. G. Cai and Y. W. Sun, JHEP **09**, 115 (2008).
- [9] S. H. Hendi, Annals of Phys. **333**, 282 (2013);
S. H. Hendi, A. Sheykhi, Phys. Rev D **88**, 044044 (2013);
S. H. Hendi, Annals of Phys. **346**, 42 (2014).
- [10] S. H. Hendi and M. Sepehri Rad, Phys. Rev. D **90**, 084051 (2014);
S. H. Hendi and M. Allahverdizadeh, Adv. High Energy Phys. **2014**, 390101 (2014);
S. H. Hendi and M. Sepehri Rad, Int. J. Mod. Phys. D **23**, 1450095 (2014).
- [11] A. Sheykhi, S. Hajkhalili, Phys. Rev. D **89**, 104019 (2014).
- [12] A. Sheykhi, A. Kazemi, Phys. Rev. D **90**, 044028 (2014).
- [13] A. Sheykhi, N. Riazi, Phys. Rev. D **75**, 024021 (2007);
A. Sheykhi, Int. J. Mod. Phys. D **18**, 25 (2009).
- [14] M. H. Dehghani, S. H. Hendi, A. Sheykhi and H. Rastegar Sedehi, JCAP **02**, 020 (2007);
M. H. Dehghani, A. Sheykhi, S. H. Hendi, Phys. Lett. B **659**, 476 (2008).
- [15] K. C. K. Chan, J. H. Horne and R. B. Mann, Nucl. Phys. B **447**, 441 (1995).
- [16] A. Sheykhi, Phys. Rev. D **76**, 124025 (2007).
- [17] R. G. Cai, J. Y. Ji and K. S. Soh, Phys. Rev D **57**, 6547 (1998);
R. G. Cai and Y. Z. Zhang, Phys. Rev D **64**, 104015 (2001).
- [18] S. S. Yazadjiev, Class. Quantum Gravit. **22**, 3875(2005).
- [19] G. Clement, D. Galtsov and C. Leygnac, Phys. Rev. D **67**, 024012 (2003);
G. Clement and C. Leygnac, Phys. Rev. D **70**, 084018 (2004).
- [20] S. W. Hawking and S. F. Ross, Phys. Rev. D **52**, 5865 (1995).
- [21] M. H. Dehghani, Ch. Shakuri and M. H. Vahidinia, Phys. Rev. D **87**, 084013 (2013).
- [22] M. H. Dehghani, A. Sheykhi, S. E. Sadati, Phys. Rev. D **91**, 124073 (2015), [arXiv:1505.01134v1]
- [23] J. D. Brown and J. W. York, Phys. Rev. D **47**, 1407 (1993).
- [24] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998);
E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998);
O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Phys. Rep. **323**, 183 (2000);
V. Balasubramanian and P. Kraus, Commun. Math. Phys. **208**, 413 (1999).
- [25] S. Mignemi and D. Wiltshire, Class. Quantum Gravit. **6**, 987 (1989);
D. L. Wiltshire, Phys. Rev. D **44**, 1100 (1991);
S. Mignemi and D. L. Wiltshire, Phys. Rev. D **46**, 1475 (1992).
- [26] H. J. Boonstra, K. Skenderis and P. K. Townsend, JHEP **01**, 003 (1999);
K. Behrndt, E. Bergshoeff, R. Hallbersma and J. P. Van der Schaar, Class. Quantum Gravit. **16**, 3517 (1999);
R. G. Cai and N. Ohta, Phys. Rev. D **62**, 024006 (2000).
- [27] A. Sheykhi, M. H. Dehghani, N. Riazi and J. Pakravan, Phys. Rev. D **74**, 084016 (2006).
- [28] A. M. Awad, Class. Quantum Gravit. **20**, 2827 (2003).
- [29] J. P. S. Lemos and V. T. Zanchin, Phys. Rev. D **54**, 3840 (1996).
- [30] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover, New York, (1972);
R. M. Corless, et al., Adv. Computational Math. **5**, 329 (1996).
- [31] M. H. Dehghani, Phys. Rev. D **66**, 044006 (2002);
M. H. Dehghani and A. Khodam-Mohammadi, Phys. Rev. D **67**, 084006 (2003).
- [32] J. D. Beckenstein, Phys. Rev. D **7**, 2333 (1973);
S. W. Hawking, Nature (London) **248**, 30 (1974);
G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
- [33] C. J. Hunter, Phys. Rev. D **59**, 024009 (1999);
S. W. Hawking, C. J. Hunter and D. N. Page, Phys. Rev. D **59**, 044033 (1999);
R. B. Mann Phys. Rev. D **60**, 104047 (1999);
R. B. Mann Phys. Rev. D **61**, 084013 (2000).

- [34] A. Sheykhi, Phys. Rev. D **78**, 064055 (2008);
A. Sheykhi, M. H. Dehghani, and S. H. Hendi, Phys. Rev. D **81**, 084040 (2010);
S. H. Hendi, A. Sheykhi, M. H. Dehghani, Eur. Phys. J. C **70**, 703 (2010).

For Review Only