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Abstract: We present a revised estimator for the sampling error of local competition variables that builds on the conceptual framework given by Stage & Wykoff (Stage, A. R., and W. R. Wykoff. 1998. Adapting distance-independent forest growth models to represent spatial variability: effects of sampling design on model coefficients. *Forest Science* 44(2): 224–238). Accurate estimation of the sampling error of local competition variables is a requisite for most approaches that correct the effects of measurement error in model fitting and application. Our revision addresses the bias inherent in Stage & Wykoff's estimator due to the overlapping of random samples that are constrained to include a subject tree. We also argue that the adjustment Stage & Wykoff used to account for the absence of treeless plots (zero-truncation) is unnecessary. We illustrate the performance of the new estimator and that of Stage & Wykoff through simulation. For a hypothetical Poisson forest (800 trees/ha and mean dbh 9.8 cm) bias is negligible for the new estimator, and variance is reduced by 92%.

Key words: measurement error, competition index, point basal area, PBA, structural based prediction.

1. Introduction

Competition among neighboring trees for limited resources plays an essential role in forest stand dynamics. Because competition affects each of the basic components of most tree growth models (Begon et al., 1996), it is broadly included in forest growth and yield simulation systems (Hann and Zumrawi, 1991). Competition may be described based on underlying mechanisms. Where trees share a pool of resources, competition may be “diffuse”, and the effect of such competition “symmetric”, because it affects all trees equally or in proportion to their size (Nilsson, 1994). Alternatively, competition may be “sharp”, arising from interference (García, 1992), and the effect described as “asymmetric” because large trees affect the growth of small trees more than they are affected (Nilsson, 1994). Indices or metrics of competition may be used in growth models to represent these mechanisms, and implicit if not explicit in these indices is the definition of the competition neighborhood (Ledermann and Stage, 2001).

Growth and yield models commonly use indices of competition at two spatial scales: at the whole stand level, as an average, or at the local level. Competition metrics, regardless of their scale, are, like most measured variables, subject to measurement error. Stage and Ledermann (2008) provide a concise discussion of the sources of error intrinsic to competition variables. For example, error can arise from a discrepancy between the zone of influence defined in the variable and the actual relationship between a subject tree and its competitors, both diffuse and sharp. Both stand and local level metrics are subject to error arising from the variability inherent to point estimation that originates when competition metrics are estimated through sampling (Stage and Wykoff, 1998).

When competition variables are used as predictors in regression models, the potential effects of measurement error are significant. Under the classical measurement error model, both regression coefficients and model predictions are biased (Fuller, 1987; Canavan and Hann, 2004). The magnitude of the bias is a function of plot size and configuration, and this clearly complicates interpretation of regression parameters based on magnitude, within and among unrelated studies. In addition, bias in model predictions can arise through difference in the configuration of plots used to calibrate a model and plots used to initiate it, or when tree sizes change, trees are removed through management, or through estimation of mortality across

the course of a simulation (Stage and Ledermann, 2008). The recognition of these critical latent risks has motivated sustained attention in forestry literature (Jaakkola, 1967; Hann and Zumrawi, 1991; Stage and Wykoff, 1998; Lappi, 2005). Most of the approaches to correct for measurement error assume some knowledge of the measurement error variance (Fuller, 1987; Carroll et al., 2006). The correct estimation of this variance is one of the most critical steps in all of the most widely used approaches for correcting of measurement error (Fuller, 1987; Carroll et al., 2006).

Estimates of the error variance can be obtained from internal replicates; that is, when several measurements of the competition variable are available one can estimate the variance with the usual sample-based estimator. This approach is straightforward for stand level metrics like stand basal area. Unfortunately, standard forest inventories do not include replicates of local competition variables, like plot basal area. Other options when internal replicates are not available include estimating the variance from external studies or deriving it from the data available in the inventory. Because the variance changes with the spatial structure, diameter distribution and plot size (Stage and Wykoff, 1998) the former option is not feasible in practice. This latter approach involves assumptions about the spatial pattern of tree locations, and about the diameter correlation among neighboring trees. This last approach was selected by Stage and Wykoff (1998), hereafter “S&W”, to implement the only methodology developed until now that corrects for measurement error in competition variables in both the calibration and application phases of diameter increment / basal area models.

The S&W approach was motivated by the possibility that many randomly-located plots could contain a single subject tree. Their estimate of measurement error variance aims, therefore, to estimate the variance of local competition over all possible locations of the plot constrained to contain the subject tree. We argue that the variance estimator Stage and Wykoff (1998) derived should account for the overlapping areas of these plots, and therefore the S&W estimator overestimates the variance of local competition. We also argue that the approach used to account for the truncated nature of the variance for the subject tree is unnecessary. In this paper we describe an estimator that accounts for the overlapping area. We illustrate the performance of the new estimator and the S&W estimator through a simulated Poisson forest. Our objective is to present improvements to the S&W estimator and not to address the

assumptions about tree locations or the spatial correlation of tree attributes used in S&W to derive the estimator for the competition variance. We begin with a description of the S&W approach to the estimation of the local competition variance, followed by a rationale for and description of the proposed improved variance estimator. The last part of this paper presents a simulation that illustrate the performance of both the old and new variance estimators.

2. Stage and Wykoff (1998) approach to the estimation of the local competition variance

In this section we outline S&W approach to estimate local competition variance. We follow S&W notation. Population variances are indicated as $Var(.)$, and variance for sample estimates as $var(.)$. S&W's approach is based on two main assumptions: (1) that trees are located following a random (Poisson) pattern and (2) that the diameter of each tree is independent of the locations of the remaining trees. Based on these assumptions the sampling variance of point basal area (PBA) in the neighborhood of the tree is estimated as follows.

In a Poisson forest, the variance of the number of trees per plot (m), $Var(m)$, can be estimated as the mean tree count per plot (μ), that is, $Var(m) = \mu$, and $\mu = \tau \cdot a$, where τ is the mean number of trees per unit area, and a is the area of the plot (e.g., ha).

Changing from statistics for the plot to statistics per unit area $M = m/a$, and $Var(M)$ can be estimated as:

$$Var(M) = Var\left(\frac{m}{a}\right) = \frac{Var(m)}{a^2} = \frac{\mu}{a^2} = \frac{(\tau a)}{a^2} = \frac{\tau}{a} \tag{1}$$

To derive the sampling distribution of basal area in a Poisson forest, divide the population into diameter classes indexed i and sampled with plots of area a_i . Each class is itself a Poisson subpopulation because of the way the Poisson forest is generated. Define BA_i as the basal area in the i^{th} diameter class with a midclass diameter of d , and variance $Var(BA_i)$. The basal area per unit area is the total over the diameter classes, that is, $BA_i = cd_i^2m_i/a_i$, and its variance is:

$$Var(BA_i) = \frac{c^2 d_i^4 Var(m_i)}{a_i^2} = \frac{c^2 d_i^4 \mu_i}{a_i^2} = \frac{c^2 d_i^4 \tau_i}{a_i} \quad (2)$$

Where $c = \pi/[4(10,000\text{cm}^2/\text{m}^2)]$ for diameters in cm and area in hectares.

If we further assume that the diameter of each tree is independent of the locations of the remaining trees, the tree counts within each class are themselves a Poisson-distributed variable. The sum of Poisson-distributed variables is also Poisson-distributed with a mean and variance equal to the sum of the individual parameters (Johnson and Kotz, 1969). Then,

$$Var(PBA) = \sum_i Var(BA_i) = c^2 \sum_i \frac{d_i^4 \tau_i}{a_i} \quad (3)$$

Since any sample of the local competition around a particular tree includes that particular tree, the distribution of the diameter class of the subject tree is not a true Poisson distribution but it is truncated. S&W accounted for this by subtracting d_t^4/a_t from the equation. Therefore, the adjusted estimate of PBA sampling variance, vPBA, in the neighborhood of the t th tree is:

$$var_i(PBA) = c^2 \left(\sum_{j=1}^r \frac{M_j d_j^4}{a_j} \right) - \frac{d_t^4}{a_t} \quad (4)$$

where r is the number of tree records left in the inventory of the sample point.

Stage and Wykoff claimed that samples with higher than average counts per point were more frequently overestimating the local mean and variance, and that samples with lower than average counts per point were more frequently underestimating the true mean and variance. To overcome this problem, they added a shrinkage factor (SF) which is a multiplier to the PBA variance. The suggested SF is:

$$SF = \frac{PV \cdot SBA + (1 - PV) PBA}{PBA}$$

Where PV is a constant taking a value between 0 and 1. S&W found a value for PV empirically, based on the criterion of minimizing the error of the estimate of the resulting model. The selected PV was 1, so $SF=SBA/PBA$. The main effect of the shrinkage factor is to shrink the estimated variance from the single sample towards a central value for the stand (Stage and Wykoff, 1998).

3. An improved variance estimator

We argue that the reason behind the discrepancies between observed and estimated variance is that the possible sampling points around a tree have overlapping areas, and this is incongruent with some of the properties of the Poisson distribution.

One of the properties of a Poisson distribution is the memorylessness property (Cox and Isham, 1980). It is derived from this property that the number of events (trees) in an area is independent from the number of events (trees) in another area that is disjoint from the first area. However, if two areas are not disjoint, but are overlapping, the number of trees in one area is not independent of the number of trees in the other area (Figure 1). Since all possible samples including a tree must have an overlapping area that includes the subject tree, the plots are not disjoint, and thus, the number of trees in each plot is not independent of each other. In a Poisson forest with parameter $\lambda = 200$ trees/ha, the expected number of trees in any sample plot, disjoint or not, of size 1 ha is 200 trees. However, the variance in the number of trees is λ when the plots are disjoint, and less than λ if the plots have an overlapping area. If we sample in a Poisson forest with parameter λ in such a way that the expected value of the overlapping area of the plots of size A_p and radius r is A_O , the expected number of trees per plot would still be $\lambda \cdot A_p$ but the variance would be $\lambda \cdot (A_p - A_O)$, because the overlapping area does not contribute to the variance. Since the plot size, A_p , is known and λ can be easily estimated, we focus on estimating A_O .

The expected overlapping area, $E(A_O)$, of two circles that always contain a fixed point can be defined in terms of the position, distances (x_1, x_2) and angles (θ_1, θ_2) , of the centers of the two circles with respect to the fixed point as:

$$E(A_O) = \int_0^r \int_0^{2\pi} \int_0^r \int_0^{2\pi} p(x_1, \theta_1, x_2, \theta_2) A(x_1, \theta_1, x_2, \theta_2) dx_1 d\theta_1 dx_2 d\theta_2 \quad (5)$$

142 where

$$p(x_1, \theta_1, x_2, \theta_2) = \frac{2\pi x_1}{\pi r^2} \cdot \frac{2\pi x_2}{\pi r^2} \cdot \frac{1}{2\pi} \frac{1}{2\pi} \quad (6)$$

143 and $A(x_1, \theta_1, x_2, \theta_2)$ is the overlapping area of two circles with centers at distance x_1 and
144 x_2 and angle θ_1 and θ_2 of the fixed point (Figure 2).

145 To calculate the overlapping area of two intersecting circles of radius r and centers a and
146 b (Figure 2), the angles cad and cbd , where c and d are the two points of intersection, are
147 calculated using the law of cosines. To calculate the angle cad the angle cab , which is half of
148 cad , is first calculated.

149 According to the law of cosines $r^2 = r^2 + x^2 - 2r \cdot x \cdot \cos(cba)$, so $\cos(cba) = x/2r$, and

$$cbd = 2cba = 2 \arccos(x/2r) \quad (7)$$

150 Then, the segment of each of the circles cut off by the chord cd is calculated by taking the
151 area of the sector of the circle bcd and subtracting the area of triangle bcd . Similarly, the area
152 of the sector acd can be found and the area of triangle acd subtracted. That is,

$$Area = A(x_1, \theta_1, x_2, \theta_2) = 2 \left[\frac{1}{2} cbd \cdot r^2 - \frac{1}{2} r^2 \cdot \sin(cbd) \right] = r^2 (cbd - \sin(cbd)) \quad (8)$$

and substituting 7 in 8 we obtain:

$$A(x_1, \theta_1, x_2, \theta_2) = r^2 \left[2 \arccos\left(\frac{x}{2r}\right) - \sin\left(2 \arccos\left(\frac{x}{2r}\right)\right) \right] \quad (9)$$

153 The distance between the centers of the circles, x , can be redefined in terms of $x_1, \theta_1, x_2, \theta_2$
154 from the coordinates for the centers as:

$$x = \sqrt{[a_x - b_x]^2 + [a_y - b_y]^2} = \sqrt{[\cos(\theta_1)x_1 - \cos(\theta_2)x_2]^2 + [\sin(\theta_1)x_1 - \sin(\theta_2)x_2]^2} \tag{10}$$

155 Integrating $E(A_O)$ in [5] results in $0.4596199 \pi r^2$. That is, the expected overlapping area
156 is about 46% of the area of the circle, and only 54% of the area of the plot contributes to the
157 variance. The new estimate (ne) of the vPBA would then be:

$$var_i(PBA) = (1 - 0.4596199) c^2 \left(\sum_{j=1}^r \frac{M_j d_j^4}{a_j} \right) \tag{11}$$

158 The new estimate accounts for the overlapping area of the plots including the subject
159 tree. However, we do not include the deduction for the subject tree proposed by Stage and
160 Wykoff (1998). Since any sample of the local competition around a particular tree includes
161 that particular tree, the distribution of the diameter class of the subject tree is not a “pure
162 Poisson” but rather is zero-truncated Poisson. While the variance of a sum of Poisson variables
163 is the sum of the individual Poisson variances, the variance of the sum of Poisson and a zero-
164 truncated Poisson is not, and some adjustment is required. Intuitively, the certainty that a
165 plot contains at least the subject tree should reduce the sampling error. To address this, Stage
166 and Wykoff (1998) defined the diameter class of the subject tree as “vanishingly narrow”, such
167 that the subject tree would not contribute to the variance; in their formula they subtracted
168 d_t^4/a_t from the equation for $var_i(PBA)$ [4]. However, for a zero-truncated Poisson, as the mean
169 number of trees per plot increases, the effect of the zero truncation has a diminishing effect
170 on the variance. In other words the variance of a truncated Poisson approaches the variance
171 of a “pure Poisson” as the number of trees per plot increases, and thus the fixed correction
172 proposed by Stage and Wykoff (1998) is not appropriate. Unfortunately, there seems to be no
173 way to address this problem in a closed form, but for reasons we argue later the bias due to
174 ignoring the zero-truncation of the class containing the subject tree should be minimal in most

175 applications.

176 4. An illustration of the behaviour of the estimators

177 A straightforward way to test if the suggested variance estimator is an improvement over
178 the S&W variance estimator for Poisson forests is through simulation. We simulate spatially-
179 explicit Poisson forest stands, simulate the measurement error variance, and finally compare
180 the variance to the proposed estimator and the S&W estimator with and without the shrinkage
181 factor.

182 Stands were simulated using spatial point processes (Stoyan and Penttinen, 2000). A thor-
183 ough introduction to point processes can be found in Diggle (2003), and for a more mathemati-
184 cal presentation see Stoyan et al. (1995). To generate the stands we used a spatial point process
185 called homogeneous Poisson process. The homogeneous Poisson process requires only one pa-
186 rameter to be specified; the value we used to simulate the stands is $\lambda=0.08$ (800 trees/ha). 100
187 stands were generated using the same value for λ . Because point processes are stochastic, each
188 realization differs from the rest of the realizations of the stand in tree location, total number of
189 trees and other tree and stand characteristics. All created stands were square with dimension
190 150 x 150 m (i.e., 2.25 ha).

191 We assigned diameters from the truncated Weibull distribution. The Weibull has been
192 commonly used to describe diameter distributions of forest stands (Cao and Coble, 2014).
193 Truncation is necessary because most inventories measure only trees above a minimum di-
194 ameter, which is usually the smallest tree of merchantable size. Equations for the truncated
195 Weibull function are given by Stiff (1979):

$$f_T = \frac{c}{b} \left(\frac{x}{b} \right)^{c-1} e^{\left(\frac{T^c - x^c}{b^c} \right)}$$

$$F_T(x) = 1 - e^{\left(\frac{T}{b} \right)^c} e^{-\left(\frac{x}{b} \right)^c}$$

$$x_T = b \left[\left(\frac{T}{b} \right)^c - \ln (1 - F_T(x)) \right]^{1/c}$$

where x indicates dbh, $f(x)$ the probability density function, and $F(x)$ the cumulative distribution function. We used $T = 5$ cm, $c = 0.87$, and $b = 15$.

4.1. Sampling the stands

The overall approach has two main parts. In the first, plot sampling is simulated on the 100 spatially-explicit stands. For each tree included in each sampling plot the measurement error variance is quantified and compared to alternative variance estimators.

For the first part of the approach four variances are calculated for each tree included in every plot:

1. *true*- the true PBA variance. Estimated by Monte Carlo simulation, by placing 100 random plots around each tree and calculating the PBA for each of these plots. It is assumed that the variance of these 100 replicates is a good approximation to the real variance of the PBA around the subject tree,
2. *SW*- the PBA variance following Stage and Wykoff's methodology without the shrinkage factor,
3. *SW-S*- the PBA variance following Stage and Wykoff's methodology with the shrinkage factor,
4. *ne*- new estimator

Four sampling plots were placed in each stand. The sampling plots were selected in such way that their areas did not intersect with each other and that their centers were at least twice the plot radius from the edge. Around each tree falling inside any of these four plots 100 new randomly placed plots were located. The variance of the PBA of these resampling plots was used to calculate *true* for each tree. These plots were of similar size and shape as the original sampling plots and were located in such a way that all 100 resampling plots included the subject tree.

12

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Root mean square error (RMSE) was used to assess the difference in the precision of the variance estimates (SW , $SW - S$ and ne) and $true$. For example, in the case of SW , the RMSE was calculated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (true_i - SW_i)^2}{n}}$$

Data analysis, simulations and graphics were performed in the R environment (R Development Core Team, 2012) and using the 'spatstat' package (Baddeley and Turner, 2005).

4.2. Comparison of the variance estimators

Both SW and $SW - S$ substantially overestimate $true$ variance (Figure 3). The RMSE for ne (1 463) is more than ten times smaller than the RMSE for SW (21 232), or $SW - S$ (17 605). The SF of S&W does slightly improve the estimates of vPBA, but $SW - S$ still overestimates vPBA. SW and $SW - S$ are not only biased but have also a much wider range of values than $true$ or ne with values nearly twice the largest values of $true$ or ne .

The new estimator is substantially less biased than the S&W estimator. The slight overestimation of the variance originates from not addressing the zero-truncation of the distribution of the diameter class represented by the subject tree. We expect this bias to be higher, relative to the vPBA values, at stands with low densities and smaller at stands with higher densities. In the illustration presented here, for stands with densities of 800 trees/ha, the bias is small compared to the values of vPBA (7%).

5. Discussion

The effective use of techniques to account for the effect of measurement error of competition variables in empirical growth and yield modelling depends on correct specification of the error variance. The estimate derived by Stage and Wykoff (1998) for vPBA provided a useful initial framework for the study of measurement error in local competition variables. However, our simulation study shows that their approach produces a substantial overestimate of vPBA, subject to assumptions about spatial randomness and independence of tree attributes. S&W's

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244 addition of a shrinkage factor only marginally reduces this bias. In contrast, the new estimator
245 we present, which accounts for the expected area of overlap in random samples constrained to
246 include a subject tree, is nearly unbiased, and also has the desirable property of much reduced
247 variance. We draw from the same conceptual framework as S&W, assuming a Poisson forest
248 and dividing the plot into diameter classes, and the new estimator is simpler and compu-
249 tationally less intensive to calculate. Even if the assumptions about spatial randomness and
250 independence do not hold, the issue of overlapping area persists and will need to be addressed.
251 Our demonstration used circular fixed-area plots, but the estimator is equally applicable for
252 Bitterlich plots, because the expected area of overlap is a fixed proportion of the plot size,
253 and though plot size is a function of diameter, each diameter class is treated separately in the
254 calculation of the measurement error.

255 Our new estimator, like the original proposed by S&W, depends on the assumption of
256 a Poisson forest, in which both tree spatial location and essential attributes (i.e., dbh) are
257 uncorrelated. This assumption is obviously unlikely to be strictly met in practice. Numerous
258 studies of spatial patterns have revealed both attraction and repulsion in natural and managed
259 stands at the scale of typical inventory plots, the precise domain of which is a function of initial
260 condition, microsite, thinning, age, and the accumulated and dynamic effects of stand devel-
261 opment (Reed and Burkhart, 1985; García, 1992; Reich and Arvanitis, 1992). When trees or
262 attributes are clustered the Poisson assumption will result in underestimation of the sampling
263 error, and in regular stands (such as plantations) result in overestimation. The asymmetric
264 effect of “sharp” competition is likely antagonistic to the positive correlation induced by mi-
265 crosite, and the resulting temporal interactions are complex (García, 1992). Hence, and in the
266 absence of compelling alternatives, Stage and Wykoff (1998) concluded that “the assumption
267 of independent Poisson subpopulations seems a likely first approximation”. We echo S&W’s
268 recommendation that refinements will benefit from explicit study of mapped stands. In any
269 case, the suitability of the Poisson assumption in any specific case does not diminish the utility
270 of the improvements to the S&W estimator we present here, which was our principal objective.

271 There are several reasons why we should seek an improved model for measurement error
272 variance of local competition variables, like vPBA. One objective of the use of measurement
273 error methods is to correct for the attenuation of regression coefficients (Fuller, 1987; Carroll

et al., 2006); attenuation clearly complicates interpretation of the magnitude of coefficients within and across modelling contexts. This is especially important when the effect of measurement error is a complex function of stand history, inventory design, and management. The original S&W estimate of vPBA overestimates the measurement error, which in application will lead to an overcorrection, and since the overestimate is higher in plots with higher PBA this would lead to overestimation of the effect of competition. Further, when measurement error methods are used in multiple regression models the relative weight attached to alternative competition variables can change due to the correction of the attenuation in variables measured with error. For example, Stage and Wykoff (1998) showed in their example using an individual-tree basal area increment model that correction for measurement error in PBA caused a shift in emphasis away from crown ratio (CR) and towards PBA. In the absence of disturbance, CR at least partially reflects the competitive status of the tree, but is included so that models can generate reasonable predictions when disturbance such as thinning is simulated (Wykoff, 1990). The shift in emphasis implies that the model corrected for measurement error would respond differently to disturbance. Thus, correct specification of measurement error may also be essential to effectively represent tree and stand dynamics. Assumptions about spatial pattern and the correlation of tree attributes will also affect the variance model, and though study of these aspects are outside of the scope of our study they are clearly important for the same reasons and should be investigated in future work.

The adjustment that S&W proposed for zero-truncation in the diameter class that contains the subject tree has the effect of reducing bias, but largely not as intended. It is straightforward to show that the variance of a zero-truncated Poisson rapidly approaches the variance of a “pure Poisson” as the expected count increases. Thus, the reduction in variance due to zero-truncation should be a function of the expected value, not a constant as proposed by S&W. We believe that the small bias that remains in our new estimator is due to ignoring the zero-truncation, especially in plots with few trees. However, we expect that the bias will be small in most conceivable applications. The greatest bias will occur with small plots or in sparse stands, because in both of these cases the expected number of trees per plot will be low. In practice, the size or configuration of forest inventory plots is usually adapted to stand conditions, such that a minimum number of trees are included at each sample location (Avery and Burkhart,

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2002; Gregoire and Valentine, 2008); thus, the presence of plots with very low counts should be uncommon, and if anticipated, the analyst can compensate through altering the sampling design. Furthermore, since basal area is a squared power of diameter, if stands are sparse because trees are small, then we expect both PBA and vPBA will also be small, and the bias from ignoring zero-truncation will be of less practical significance. In the case where local density (e.g., trees per unit area) is used as a local competition variable, then trees are treated equally and the expected measurement error variance can be calculated using a zero-truncated Poisson directly, still following the logic of assuming the parameter of the now zero-truncated Poisson forest varies spatially and the tree counts at each plot are estimates of the parameter of the local population.

The new estimator proposed here can be applied to any growth and yield model that uses local competition variables such as point basal area or number of trees. Implementing the new estimator should enhance the already proven abilities of measurement error techniques to increase the predictive ability of the models.

6. Literature cited

References

- Avery, T.E., Burkhart, H.E., 2002. Forest measurements. 5th ed. McGraw-Hill, New York, NY.
- Baddeley, A., Turner, R., 2005. Spatstat: an r package for analyzing spatial point patterns. Journal of Statistical Software 12, 1–42.
- Begon, M., Harper, J.L., Townsend, C.R., 1996. Ecology: individuals, populations, and communities. Blackwell Science, Oxford, UK.
- Canavan, S.J., Hann, D.W., 2004. The two-stage method for measurement error characterization. For. Sci. 50, 743.
- Cao, Q.V., Coble, D.W., 2014. Deriving a diameter distribution from stand table data. For. Sci. 60.

- 330 Carroll, R.J., Ruppert, D., Stefanski, L.A., Crainiceanu, C.M., 2006. Measurement error in
331 nonlinear models. Chapman and Hall/CRC Press, Boca Raton, FL.
- 332 Cox, D.R., Isham, V., 1980. Point processes. volume 12. Chapman & Hall/CRC, London.
- 333 Diggle, P.J., 2003. Statistical analysis of spatial point patterns. Oxford University Press, New
334 York. second edition edition.
- 335 Fuller, W.A., 1987. Measurement error models. John Wiley and Sons, New York, NY.
- 336 García, O., 1992. What is a diameter distribution, in: Proceedings of the Symposium on
337 Integrated Forest Management Information Systems, Tsukuba, Japan. pp. 11–29.
- 338 Gregoire, T.G., Valentine, H.T., 2008. Sampling strategies for natural resources and the envi-
339 ronment. Chapman and Hall/CRC Press, Boca Raton, FL.
- 340 Hann, D.W., Zumrawi, A.A., 1991. Growth model predictions as affected by alternative
341 sampling-unit designs. For. Sci. 37, 1641–1655.
- 342 Jaakkola, S., 1967. On the use of variable size plots for increment research, in: Proceedings of
343 XIV. IUFRO-Congress Sect. 25., DVFFA, Munich. p. 371–378.
- 344 Johnson, N.L., Kotz, S., 1969. Distributions in statistics: discrete distributions. volume 1.
345 Houghton Mifflin, Boston.
- 346 Lappi, J., 2005. Plot size related measurement error bias in tree growth models. Can. J. For.
347 Res. 35, 1031–1040.
- 348 Ledermann, T., Stage, A.R., 2001. Effects of competition spacing in individual-tree indices of
349 competition. Can. J. For. Res. 31, 2143–2150.
- 350 Nilsson, U., 1994. Development of growth and stand structure in *Picea abies* stands planted
351 at different initial densities. Scandinavian Journal of Forest Research 9, 135–142.
- 352 R Development Core Team, 2012. R: A language and environment for statistical computing.
- 353 Reed, D.D., Burkhardt, H.E., 1985. Spatial autocorrelation of individual tree characteristics in
354 loblolly pine stands. For. Sci. 31, 575–594.

Reich, R.M., Arvanitis, L.G., 1992. Sampling Unit, Spatial Distribution of Trees, and Precision. North. J. Appl. For. 9, 3–6.

Stage, A.R., Ledermann, T., 2008. Effects of competitor spacing in a new class of individual-tree indices of competition: semi-distance-independent indices computed for bitterlich versus fixed-area plots. Can. J. For. Res. 38, 890–898.

Stage, A.R., Wykoff, W.R., 1998. Adapting distance-independent forest growth models to represent spatial variability: effects of sampling design on model coefficients. For. Sci. 44, 224–238.

Stiff, C.T., 1979. Modeling the growth dynamics of natural mixed-species Appalachian hardwood stands. Dissertation. Virginia Polytechnic Institute and State University. Blacksburg, VA.

Stoyan, D., Kendall, W.S., Mecke, J., 1995. Stochastic geometry and its applications. volume 2nd ed. Wiley, Chichester.

Stoyan, D., Penttinen, A., 2000. Recent applications of point process methods in forestry statistics. Statistical Science 15, 61–78.

Wykoff, W.R., 1990. A basal increment model for individual conifers in the northern rocky mountains. For. Sci. 36, 1077–1104.

Fig. 1. Any potential plot (grey areas) containing the subject tree (white cross) overlap.

Fig. 2. The expected overlapping area of two circles of radius r and centres a and b , that always contain a fixed point, can be defined in terms of the angles θ_1 and θ_2 and the distances x_1 and x_2 . The distance between the centres is x , and c and d are the intersection points of the circumference of the two circles.

Fig. 3. Violin plots of the sampling variance, $vPBA$, estimated using Monte Carlo simulations (*true*), Stage and Wykoff's methodology with ($SW - S$) and without the shrinkage factor (SW), and the new estimator (*ne*). The solid circle shows the median, and the rectangle the two inner quartiles of the distribution of values. The outer envelope is a kernel density of the entire set.