Canadian Journal of Forest Research
Revue canadienne de recherche forestière

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Journal:	<i>Canadian Journal of Forest Research</i>
Manuscript ID	cjfr-2015-0238.R1
Manuscript Type:	Note
Date Submitted by the Author:	01-Aug-2015
Complete List of Authors:	Lynch, Thomas; Oklahoma State University,
Keyword:	increment cores, distance sampling, k tree sampling, Area potentially available, Forest sampling

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VORONOI POLYGONS QUANTIFY BIAS WHEN SAMPLING THE NEAREST PLANT

ABSTRACT: The design-bias in the sample mean obtained from sampling the trees nearest to points randomly and uniformly distributed over a forested area can be exactly quantified in terms of the Voronoi polygons (V-polygons) surrounding each tree in the forest of interest. For this sampling method the V-polygon for a prospective sample tree is its inclusion zone. The sides of such polygons are perpendicular to a line joining adjacent trees and equidistant from these trees. For any individual tree attribute Y , the design-bias in such a sample mean for estimating the population mean of Y will be equal to the covariance between Y and V-polygon area V divided by the mean V-polygon area. The bias as a percent of the population mean of Y is the product of the correlation coefficient between Y and V and the coefficients of variation for Y and V multiplied by 100. This implies that attempts to estimate the means of commonly-measured individual tree variables such as DBH, basal area, and crown diameter or area from sampling trees nearest to randomly-located points will likely be positively biased and the magnitude of that bias will depend on the strength of the linear relationship to V-polygon area as well as the variability among the V-polygon areas and the variable of interest. It is not obvious whether increment core data will be positively or negatively biased because this depends on the characteristics of the forest of interest. The main conclusion of the study is that the bias formula derived for unweighted estimation from sampling the tree nearest to a point indicates that bias in the range of 5-10% or greater can occur in many forest populations.

KEY WORDS: increment cores, distance sampling, k -tree sampling, APA, Area Potentially Available, inclusion zone, forest sampling, forest inventory

INTRODUCTION

Although ecologists and foresters have sometimes used samples obtained from selecting the nearest tree or other plant to a point or plot center in the field to estimate population means for per-tree attributes such as growth from increment cores, Clutter et al. (1983, p. 119) pointed out that this procedure is more likely to sample “relatively open-grown” trees possibly resulting in biased estimators. These remarks were made in a discussion of growth predictions obtained from the “stand table projection” procedure because increment core samples for the procedure had often been obtained by sampling the tree nearest to a sample point or plot center in the field. However, Clutter et al. (1983, p.119) did not provide an exact formula for the magnitude of this bias. Clutter et al. (1983, p. 119) recommended that increment cores be obtained by sampling all trees on small fixed-radius plots.

Barabesi and Fattorini (1998) point out that Overton and Stehman (1995) have suggested a design-based procedure for estimating attributes of plants nearest to randomly points. In their method the probability of selection is proportional to the area of the V-polygon (Voronoi polygon, Wikipedia authors 2015) surrounding the plant, allowing a Horvitz-Thompson estimator to be used. V-polygons can be constructed by employing perpendicular bisectors between adjacent plants. Figure 1 illustrates a typical Voronoi tessellation for a synthetic forest population developed using a spatial inhibition process that had 3m inhibition distance (Venables and Ripley, 2002, p. 434). The package “spatial” was used in R (R Development Core Team, 2012). Figure 1 was then produced by using the “deldir” package (Turner, 2015) in R. The V-

61 polygon associated with a prospective sample tree is its inclusion zone for sampling because
62 whenever a randomly-located point falls in this V-polygon (inclusion zone), it will be closer to
63 that tree than any other in the population and hence will be selected as a sample tree. However
64 Barabesi and Fattorini (1998) conclude that the construction of V-polygons in the field would be
65 “very impractical and time consuming.” Kleinn and Vilkô (2006) give a thorough review of the
66 application of V-polygons to ecological sampling and extend the theory of design-based V-
67 polygon sampling to the situation in which the nearest k trees to a random point are selected and
68 higher-order V-polygons are employed. However, they conclude that the practicality of
69 employing these ideas in the field with current technology is probably limited due to the
70 measurements required to construct higher-order V-polygons. Related sampling concepts that
71 are intended to reduce sampling effort in the field were triangulation methods proposed by
72 Fehrmann et al. (2011). Foresters have long used the concept of the Veronoi polygon under the
73 title of APA (Area Potentially Available, Mercier and Baujard (1997), Brown (1965), Mead
74 (1966), Avery and Burkhart 2002 p. 331-332). APA has been used in the development of forest
75 growth models.

76 Consider a forest tract having area A tessellated into V-polygons for each tree such that when a
77 randomly located point in area A falls into the V-polygon of area V_i associated with tree i then
78 tree i will be selected as a sample tree. In the latter case tree i will be the nearest tree to the
79 randomly located point in area A . We want to sample attribute Y_i (e.g., increment core data,
80 basal area, diameter at breast height (DBH)) which is an individual tree characteristic associated
81 with tree i within a V-polygon associated with that tree. It is clear that for this procedure the
82 probability of selecting Y_i on a V-polygon having area V_i will be V_i/A . It should be noted that for
83 trees or other plants near the edge of the tract (which could be the tract boundary, pond, etc.) part

of the V-polygon border may include tract edge, and this may not be half the distance to a nearest neighboring plant.

Lappi and Bailey (1987) investigated a very similar situation in which increment cores were selected on angle-count (horizontal point or Bitterlich) sampling trees. They found that the bias in estimating the mean of a variable y on trees selected in a point sample was equal to the covariance between DBH^2 and y divided by the population mean value of DBH^2 . Since angle-count sample trees are selected with probability proportional to DBH^2 the situation is closely analogous to sampling Y_i on V-polygons with probability proportional to V_i . Angle-count sampling is based on probability proportional to size sampling while selection of the plant nearest to a random point samples with probability proportional to the V-polygon of the selected tree. Applying Lappi and Bailey's (1987) methods to the problem of sampling the nearest plant or tree to a randomly-located point leads to a bias similar to that found by those researchers. The bias is equal to the covariance between V_i and Y_i divided by the population mean area of the of V-polygons μ_V . This will be derived below using a somewhat different presentation based on the work of Palley and Horwitz (1961) in point sampling.

We now want to consider the expected value of the average of the individual tree characteristic Y_i associated with the V-polygon of area V_i based on a sample of n points independently and randomly located with a uniform distribution over the tract area A .

METHODS

We will be considering a finite population of trees with associated V-polygons from which a random sample of size n will be drawn. Sample trees will be selected when a randomly-located point on the tract falls within the V-polygon area of a sample tree. Using this model we can

calculate the expected value of the sample mean and quantify the bias in the sample mean estimate of any tree attribute for estimation of the population mean of that attribute. We begin by describing the population and some of its parameters below.

Preliminaries:

We have a finite population of N V-polygons (one for each tree) each with area V_i and so we can define the first and second moments of this finite population as well as the variance as follows (the first moment being the population mean):

$$\begin{aligned} \mu_V &= \frac{1}{N} \sum_{i=1}^N V_i, \text{ the population mean for } V, \\ \mu'_{2V} &= \frac{1}{N} \sum_{i=1}^N V_i^2, \text{ the second moment for } V, \text{ and} \\ \sigma_V^2 &= \mu'_{2V} - \mu_V^2, \text{ the population variance for } V. \end{aligned} \quad (1)$$

And note that the land area A is the sum of the V-polygon areas:

$$A = \sum_{i=1}^N V_i \quad (2)$$

Noting that the area V_i will be the area of the inclusion zone for tree i , we define a Bernoulli random variable Z_{ij} associated with random point location as follows:

$Z_{ij} = 1$ if random point j falls in the V-polygon of area V_i , otherwise

$Z_{ij} = 0$

We then clearly have the inclusion probability (probability of inclusion in the sample with random point i) and the probability of not being in the sample:

$$\begin{aligned} P(Z_{ij} = 1) &= \frac{V_i}{A} \text{ for all } j, \text{ and} \\ P(Z_{ij} = 0) &= 1 - \frac{V_i}{A} \text{ for all } j. \end{aligned} \quad (3)$$

We also have a finite population of N individual tree attributes Y_i each associated with tree i located on a V -polygon of area V_i . Parameters associated with this population include:

$$\mu_Y = \frac{1}{N} \sum_{i=1}^N Y_i, \text{ the population mean for } Y,$$

$$\mu'_{2Y} = \frac{1}{N} \sum_{i=1}^N Y_i^2, \text{ the second moment for } Y, \text{ and} \tag{4}$$

$$\sigma_Y^2 = \mu'_{2Y} - \mu_Y^2, \text{ the population variance for } Y.$$

Parameters concerning the relationship between the individual tree attribute Y_i and the area of the V -polygon V_i are:

$$\mu_{YV} = \frac{1}{N} \sum_{i=1}^N Y_i V_i$$

$$\sigma_{YV} = \mu_{YV} - \mu_Y \mu_V, \text{ the population covariance for } Y \text{ and } V, \text{ and} \tag{5}$$

$$\rho_{YV} = \frac{\sigma_{YV}}{\sigma_Y \sigma_V} \text{ the correlation coefficient between } Y \text{ and } V.$$

The expected value of the sample mean of tree attribute Y from point to nearest tree samples:

Following a formulation similar to that used by Palley and Horwitz (1961) for point sampling we can then express the sample mean of Y_i from n independent randomly and uniformly located points over tract area A , intended to be an estimator of μ_Y , as:

$$\bar{Y}_{RP} = \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N Y_i Z_{ij}$$

where $\bar{Y}_{RP}$ is the sample mean based on random point (RP) samples (6)

We now want to find the expected value of this estimator:

$$E(\bar{Y}_{RP}) = E\left(\frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N Y_i Z_{ij}\right) \quad (7)$$

And by using the properties of expected values, since the n random points are independently identically distributed:

$$\begin{aligned} E(\bar{Y}_{RP}) &= E\left(\frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N Y_i Z_{ij}\right) \\ &= \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N Y_i E(Z_{ij}) = \frac{1}{n} \sum_{i=1}^N Y_i E(Z_{i1}) \\ &= \sum_{i=1}^N Y_i E(Z_{i1}) \end{aligned} \quad (8)$$

It is well-known that the expected value of a Bernoulli random variable is equal to the probability of its occurrence, so:

$$\begin{aligned} E(\bar{Y}_{RP}) &= \sum_{i=1}^N Y_i E(Z_{i1}) \\ &= \sum_{i=1}^N Y_i \frac{V_i}{A} \\ &= \frac{1}{A} \sum_{i=1}^N Y_i V_i \end{aligned} \quad (9)$$

and because $A = N\mu_V$ we also have :

$$E(\bar{Y}_{RP}) = \frac{1}{N\mu_V} \sum_{i=1}^N Y_i V_i = \frac{\mu_{YV}}{\mu_V}$$

From the covariance between Y and V we have:

$$\begin{aligned} \sigma_{YV} &= \mu_{YV} - \mu_Y \mu_V, \text{ the population covariance for } Y \text{ and } V, \text{ and} \\ \Rightarrow \mu_{YV} &= \sigma_{YV} + \mu_Y \mu_V \end{aligned} \quad (10)$$

Now combining (9) and (10) yields:

$$E(\bar{Y}_{RP}) = \frac{\mu_{YV}}{\mu_V} = \frac{\sigma_{YV} + \mu_Y \mu_V}{\mu_V} = \mu_Y + \frac{\sigma_{YV}}{\mu_V} \quad (11)$$

Therefore, if we were trying to estimate the mean of Y , μ_Y , by selecting sample trees nearest to a randomly located (uniformly distributed over the tract of area A) point the design-bias is:

$$Bias_{\bar{Y}_{RP}} = \frac{\sigma_{YV}}{\mu_Y} = \frac{\rho_{YV}\sigma_Y\sigma_V}{\mu_Y} = \rho_{YV}\sigma_Y C_V \quad (12)$$

where

$$C_V = \frac{\sigma_V}{\mu_V}, \text{ the coefficient of variation for } V$$

This is the result that we might have expected based on the work of Lappi and Bailey (1987) in the context of point sampling. Indeed this is the bias in any case in which an unweighted average of probability proportional to size samples is obtained, where V would be replaced by any other variable that is proportional to the probability of selection. Equation 11 has also been presented by Gove and Patil (1998, Appendix B) as part of their study of weighted distributions with applications to basal area distributions and as well by Patil and Rao (1978, Theorems 1 & 2) but in neither case was connected to V - polygons or distance sampling as presented here. The Gove and Patil (1998, Appendix B) result is more general than Equation 11 in that it was derived for any weighted distribution. The derivation of Equation 11 given here differs from Gove and Patil (1998), Patil and Rao (1978) and Lappi and Bailey (1987) in that it is done in a design-based framework using the approach of Palley and Horwitz (1961).

This design-bias as a percent of the population mean value of Y , μ_Y , is then:

$$Bias\%_{\bar{Y}_{RP}} = \frac{\rho_{YV}\sigma_Y C_V}{\mu_Y} 100 = \rho_{YV} C_Y C_V 100 \quad (13)$$

where

$$C_Y = \frac{\sigma_Y}{\mu_Y}, \text{ the coefficient of variation for } Y$$

For example if the correlation between Y and the V-polygon area V was 0.8 and both Y and V had correlation coefficients of 30% the bias from selecting sample trees nearest to a randomly – selected point would be $0.8 \times 0.3 \times 0.3 \times 100 = 7.2\%$. Clearly the bias percent in equation (13) will be positive when the correlation between Y and the size of the V-polygon V is positive but negative when the correlation between Y and the size of the V-polygon V is negative. Zero correlation would of course imply no bias. Low bias is also expected if the coefficient of variation for Y or V or both is low. Because a tree V-polygon is equivalent to its APA as used in forest growth simulators (e.g. Burkhart and Avery 2002 p. 331-332) we would often expect correlations between V-polygon size and variables such as DBH, individual tree basal area and crown area to be positive leading to a positive bias for these variables.

The expected size of the V-polygon for sampling the tree nearest to a random point

An interesting special case of estimator 6 occurs if we want to estimate the average size of the V-polygon associated with choosing the tree nearest to a random (uniformly distributed) point in the forest of interest. We then have:

$$\bar{V}_{RP} = \frac{1}{n} \sum_{j=1}^n \sum_{i=1}^N V_i Z_{ij} \quad (14)$$

Because the covariance of a variable with itself is identical to the variance for that variable, from equation 12 the bias in estimating μ_V from this procedure is:

$$Bias_{\bar{V}_{RP}} = \frac{\sigma_V^2}{\mu_V} = \mu_V C_V^2 \quad (15)$$

Equation 15 is equivalent to Patil and Rao (1978) Theorem 1 which they derived for general weighted distributions. As indicated above, they did not connect their result to V- polygons as is done here. And from equation 13 the bias as a percentage of μ_V is then:

$$Bias\%_{\bar{V}_{RP}} = \frac{\sigma_V C_V}{\mu_V} 100 = C_V^2 100 \quad (16)$$

For example a forest for which the coefficient of variation among V-polygons was 30% would have a bias of 9% for $\bar{V}_{RP}$. Naturally this bias would always be greater than or equal to zero.

Weighted estimation

If measurements of the areas of the V-polygons V_i were feasible, the following per-tree estimator which samples are weighted by the inverse probability of selection for tree i , V_i/A could be used to estimate the population mean of Y , μ_Y :

$$\bar{Y}_{RP}^w = \frac{\sum_{j=1}^n \sum_{i=1}^N \frac{AY_i}{V_i} Z_{ij}}{\sum_{j=1}^n \sum_{i=1}^N \frac{A}{V_i} Z_{ij}} \quad (17)$$

$\bar{Y}_{RP}^w$ is the weighted estimator for the mean of Y , μ_Y

This is a ratio of means estimator in which the numerator and denominator are design-unbiased estimates of the sum of tree characteristics Y_i and the total number of tree on the tract N , respectively. For the numerator we have:

$$E\left(\sum_{j=1}^n \sum_{i=1}^N \frac{AY_i}{V_i} Z_{ij}\right) = \sum_{i=1}^N \frac{AY_i}{V_i} E(Z_i) = \sum_{i=1}^N \frac{AY_i}{V_i} \frac{V_i}{A} = \sum_{i=1}^N Y_i \quad (18)$$

and in the denominator we have:

$$E\left(\sum_{j=1}^n \sum_{i=1}^N \frac{A}{V_i} Z_{ij}\right) = \sum_{i=1}^N \frac{A}{V_i} E(Z_{ij}) = \sum_{i=1}^N \frac{A}{V_i} \frac{V_i}{A} = N \quad (19)$$

Clearly the ratio between eq. 18 and eq. 19 is the population mean for Y , μ_Y . The ratio of means estimator is not unbiased but generally has good properties for a sufficiently large sample size (Cochran, 1977 p. 150-169) when the numerator and denominator of the ratio estimator are unbiased. The difficulty with using this estimator in practical situations is that it is very time consuming to measure V-polygon areas V_i for each sample tree in the field.

Because V-polygon areas are difficult to measure we may seek an easily-measured weighting variable that is well correlated with V-polygon area V_i , though this would not typically result in unbiased estimates of the numerator and denominator of a ratio estimator in equation 17. Technically this would make the numerator in equation 17 a mean of ratios estimator (e.g. Avery and Burkhart 2002 p. 52-53) based on mean of ratios Y_i/Z_i between Y_i and a variable Z_i the proxy variable for V_i . One approach to reducing the bias in estimator 6 could be weighting samples inversely proportional to squared sample point-to-plant distance instead of V-polygon area as in equation 17. However, the area of a circle for which the point-to-plant distance is the radius will not generally be equal to the V-polygon area and in all likelihood will be quite different. If we think of this circle as centered around the sample tree or plant we can see that it will very frequently have an area that is smaller than the a “typical” V-polygon area, sometimes (as indicated by Iles 2009) much smaller if the point is close to the sample tree or plant. The area of this circle could also be larger than the V-polygon area if the random plot falls near one of the polygon vertices, depending on how the polygon is shaped. The latter case could be

common for long narrow V-polygons, but this shape is probably not predominant in many forests. In any case, this type of analysis suggests that weighting samples by the inverse of the squared distance from the tree or plant to the random point could be a poor substitute for the V-polygon area V_i in equation 17 above. Iles (2009) suggested that the area of the V-polygon could be calculated by averaging the squared length of radii from the selected tree to the V-polygon edge. This method of area computation has been suggested by Matérn (1956) for tree cross-sectional area and Gregoire and Valentine (1995) for planar areas. Iles' (2009) procedure is unbiased for V-polygon area but precise estimation of the area may take an excessive number of measurements in the field. Still, a sample of two or three of the random radii proposed by Iles (2009) might be more stable than use of the more traditional squared point to plant distance, and there would be a smaller chance of obtaining an extremely short distance. For k -tree sampling with sufficiently large k Iles (2009) has indicated why the relationship between the higher order V-polygons discussed by Kleinn and Vilkô (2006) and the area of the k -tree plot defined by the distance from a random point to the k^{th} closest tree is better for $k > 1$ than for $k = 1$. Even with $k = 2$, Iles (2009) notes this tends to put limits on the minimum length of the radius of the 2-tree plot compare to a 1-tree plot because it has to contain at least two trees. The minimum point-to-plant distance when $k = 2$ is half the distance between any two trees sampled by this plan. Clearly the situation tends to get better as k gets larger. There is no lower limit to the radius of a $k = 1$ -tree plot. Although for a theoretical Poisson distribution there is no limit to how close two trees can be, in actual forests there is always a finite measureable difference between the locations of two trees. Still, situations with clumped distributions can arise in which very small k -tree plots can be problematic from a practical point of view.

RESULTS

Figure 2 illustrates the bias in the sample mean obtained from sampling the nearest tree to a random point using equation 13. In Figure 2, the coefficient of variation for V and Y is assumed to be equal so that the results can be plotted on a two dimensional graph. This should suffice for providing a general idea of how large the percent bias can be. Figure 2 indicates that even for low levels of correlation between Y and the V -polygon area V the bias can be large if the coefficient of variation for Y and V is substantial. Widely cited forest measurements books present examples in which error levels of 5% (Avery and Burkhart 2002 p.39) and 10% (Bitterlich 1984 p. 141) are used to compute sample sizes. Therefore we conclude that errors of these magnitudes are important in forest sampling and certainly larger errors are to be avoided. While the latter examples actually deal with computing sample size for unbiased estimators, it follows that these levels of bias should be avoided also, all the more so because biases cannot be removed by increasing sample size. Equation 13 indicates that important biases with the methods described above can occur even with low correlations between the individual tree variable Y_i and the areas V_i of V -polygons if the coefficients of variation for Y and V are high. For example, suppose $\rho_{YV}=0.2$, $C_Y=0.5$ and $C_V=0.5$. Then the bias in the mean of Y expected from sampling the tree or plant nearest to a randomly-located point would be $\rho_{YV}C_YC_V100 = 0.2 \times 0.5 \times 0.5 \times 100 = 5\%$.

DISCUSSION

To the extent that in more mature forest populations we might expect variables such as DBH or basal area to be greater for trees having larger V_i , then sampling the nearest trees to randomly located points would be a design-biased procedure for any variable correlated with the V_i . It has been noted that the theory of APA (Avery and Burkhart 2002 p. 331-332) would indicate

correlations between APA and certain tree growth variables commonly occur. In their critique of the “point-centered quarter” method of density estimation, Mark and Esler (1970) found a significant positive relationship between DBH and APA for their data with a slope of 0.648 between DBH (cm) and APA (m^2) where DBH was the independent variable (in the original English units the coefficient was 5.4 with DBH in inches and APA in ft^2). They apparently did not report an R^2 or a correlation coefficient for this relationship, however inspection of the sample points surrounding the regression line in the published figure suggest that although significant the correlation was low. Figure 2 indicates the levels of bias from a range of values of the correlation coefficient and coefficient of variation. A recent study conducted in a 28-year-old even-aged coastal redwood (*Sequoia sempervirens*) forest (Gspalti et al. 2011) found a significant relationship between basal area increment weighted by leaf area (dependent variable) and leaf area divided by weighted APA (independent variable), having an $R^2 = 0.35$ (implying a correlation coefficient $r = 0.59$). These authors also found high levels of variation in APA due to the characteristic sprouting of redwood. APA for tree i is identical to the area of the V-polygon for tree i . This means that to the extent that APA is correlated with individual tree attributes there can be an expectation of bias for several individual tree variables when sample tree are selected as those nearest to randomly-located points in the field.

We can also predict some situations in which the bias would be low. In forests that have recently been thinned the correlation between V-polygons and variables such as crown width, basal area and DBH might be reduced because trees have not yet responded to thinning. Plantations planted with very accurate spacing and having little mortality may have low coefficients of variation among the V_i areas of V-polygons associated with sample trees, leading to low bias.

However, it is hard to imagine a realistic situation in which the coefficient of variation among V-polygons would be zero because some mortality usually occurs and in actual plantations the spacing is never perfect. The coefficient of variation among V-polygons increases with age even in plantations as gaps arise due to mortality and especially when commonly-prescribed silvicultural treatments such as thinning are applied.

It is conceivable for some variables that a negative bias could occur from sampling trees nearest to a randomly-located point. Particularly in uneven-aged forests, PAI (periodic annual increment) for DBH (in the most recent years prior to sampling) may be lower for large trees past the point of culmination of PAI for DBH, and if the V-polygons for such trees tend to be larger there could be a negative correlation between PAI for DBH (e.g. as measured on increment cores) and the areas of V-polygons associated with sample trees. According to equation (13) such a negative correlation would lead to negative bias from sampling trees nearest to a randomly located point in the field. In this situation estimates of DBH increment could be too low. On the other hand in plantations or other even-aged forests, large trees with large V-polygons may have greater PAI than other trees of the same current age, leading to positive biases in PAI for the years just prior to and up to the current year of sampling.

There is extensive ongoing research concerning V-polygons due to their importance in many scientific fields. Ferenc and Nédá (2007) have recently published an investigation concerning the size distribution of Poisson Voronoi cells that models this size distribution (for which no exact mathematical form is known in 2 or higher dimensions) with a simple formula. Since the

Poisson distribution has sometimes been used to approximate the spatial distribution for randomly-distributed trees or other plants, this type of result may help us gain intuition into the possible magnitude of the coefficient of variation for the areas of V-polygons, which directly impacts the bias formulae discussed above. However many forest stands have clumped spatial distributions and young plantations have uniform spatial distributions that are not adequately modeled by the Poisson distribution.

Advances in GPS and/or other technology may make it significantly easier to map some tree locations in the field, making the computation of the areas of V-polygons feasible for some sample trees. At first this might lead to a 2-stage sampling procedure in which V-polygons are measured for a small subset of a larger sample of plants or trees located nearest to a random point on in the field. Possibly these data could be used to estimate the bias using equation 12 with estimated values substituted for population values. More likely, ratio or regression estimators could be used to adjust biased estimators from the large sample using paired biased and unbiased estimators in the small sample, reducing bias to more acceptable levels.

CONCLUSIONS

The main conclusion from the study is that the design-bias percentage from using a simple unweighted average to estimate plant attributes when selecting sample trees or other plants nearest to points located randomly and uniformly on a land area has been shown to be equal to $\rho_{YV}C_YC_V100$. Using this formula it has also been concluded that important biases of greater than

5% are possible in many realistic forest conditions. These biases are likely to occur because ecological and silvicultural principles lead to the expectation that many tree attributes are significantly influenced by growing space.. If we think that correlations between V-polygon size and some individual tree variables could range from 0.8 to 0.5 and coefficients of variation for these variables and for V-polygons could range from 20% to 50%, then we can quickly calculate that biases from as low as 2% to as high as 20% may be possible from this procedure depending on the characteristics of the forest of interest. As indicated above, even lower levels of correlation between an individual tree variable of interest Y and the area of the associated V-polygon V_i will imply important levels of bias if the coefficients of variation for Y and V are large (Figure 2). A correlation coefficient of only $\rho_{YV} = 0.1$ if combined with coefficients of variation (in terms of proportion) of 0.70 for Y and V would lead to a bias of almost 5%. Although unbiased estimators are possible they depend on measurement of sample V-polygons which with current technology is difficult in the field. Therefore the procedure of selecting sample plants or trees nearest to a randomly located point in the field should be used with caution if it is used at all. This analysis confirms in a more formal way the observations of Clutter et al. (1983 p. 119) concerning the collection of increment cores from the tree closest to a random point location in the field.

ACKNOWLEDGEMENTS

The author would like to thank Dr. Jeff Gove, Dr. Timothy Gregoire and two anonymous reviewers for valuable comments, and Dr. Steen Magnussen for discussions concerning estimation of per-tree attributes with distance sampling. This article has been approved for publication by the Oklahoma Agricultural Experiment Station and supported by project OKL0-2843.

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437 Figure 1. A Voronoi tessellation for a typical forested area in which circles represent tree
438 locations and the dashed lines are the boundaries of V-polygons enclosing the trees, with the x
439 and y axes measured in meters.

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441 Figure 2. Percent bias in the sample mean of Y versus the coefficient of variation for Y and V-
442 polygon level V assuming that they are equal, for levels of the correlation coefficient r from 0.1
443 to 1.

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