Analytical and Experimental Investigations of Modified Tuned Liquid Dampers (MTLDs)

by

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Abstract

Tuned Liquid Dampers (TLDs), as passive control devices, have been used in high-rise buildings for vibration control. A TLD dissipates energy through the sloshing of the water inside the tank. Modified Tuned Liquid Damper (MTLD) utilizes the rotational spring system at the base, therefore the MTLD experiences both horizontal and rotational motions. In this study, MTLD-structure system subjected to sinusoidal excitation and seismic events has been investigated using Lu’s analytical model. This analytical model has been verified experimentally using Real-Time Hybrid Simulation method. The important parameters of the MTLD including the dimensionless rotational stiffness parameter, mass ratio, frequency ratio and damping ratio were studied in detail; and a preliminary design procedure was outlined.
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Chapter 1

1 Introduction

In the past few decades, a large number of uniquely shaped high-rise buildings have been constructed around the world, especially in the developing countries. In tall buildings, one of the challenges is the control of the structural vibrations to ensure the safety and the comfort of the occupants. Due to wind load and minor earthquakes these tall, flexible high rise building can develop vibrations which might pose serviceability problems. Under more severe dynamic loading, such as in the event of a major earthquake, structural damage and collapse could be observed. In order to avoid these problems several seismic protection systems have been developed and employed in structural systems. These seismic protection systems are discussed in this chapter.

1.1 Seismic Protection Systems

1.1.1 Conventional Systems

The current seismic resistant design is based on the concept of the ductility of structures. The advantage of ductile response is that only some specified elements are allowed to have inelastic deformation and yield, while other members remain elastic. The ductile members can be achieved by yielding in tension or inelastic buckling in braces, or flexural hinging in beams or at the base of the columns.

1.1.2 Seismic Isolation Systems

Seismic isolation systems utilize isolators that are installed between the key supporting points of the structure and the foundation. The isolators are designed to have a much lower lateral stiffness relative to that of the structure. As such they dissipate more seismic energy
and transfer less energy into the structure. In general, seismic isolation systems have three main types: laminated-rubber bearings, lead-rubber bearings and the friction pendulum systems.

1.1.3 Supplemental Damping Systems

Supplemental damping devices are introduced into the structure to dissipate some of the energy introduced during the vibration and thereby mitigate the damage to the structural and non-structural components. They can be classified in three categories as, active, semi-active and passive systems.

1.1.3.1 Active Systems

Active systems monitor the state of structure by using the data acquired from sensors placed on the structure. They determine the action that needs to be taken in order to mitigate the dynamic response and then apply the required forces to the structure to modify its current state. Under dynamic loading all these steps need to be completed in a short time and require a continuous external power source especially to introduce the control forces. However, during a severe earthquake the power source might be lost, and as a result the active systems might stop functioning. For that reason active systems are not preferred as the seismic protective systems for civil structures.

1.1.3.2 Semi-Active Systems

Semi-active systems, unlike the active control systems, only need a small amount of external power. They do not apply the external forces to the structure, but semi-active control systems modify the structural properties, such as the damping as the structure vibrates. Magneto-rheological dampers can be given as an example for semi-active control systems.
1.1.3.3 Passive Systems

Unlike active and semi-active systems, passive systems do not need external power, monitoring systems, sensors and actuators. Properties of passive systems are fixed and they are not get modified during the vibration of the structure. Passive systems are considered to be an effective, robust and economical solution, and they have been already implemented in a variety of civil structures. Passive systems can be divided into three categories: displacement-activated, velocity-activated and motion-activated.

Metallic dampers, friction dampers, self-centering dampers and viscoelastic dampers belong to the group of displacement-activated devices. They absorb seismic energy through the relative displacement between their contact points that are connected to the structure. Metallic dampers absorb seismic energy by their hysteretic behaviour when they deform into the post-elastic range. Friction dampers dissipate energy by friction caused by the sliding motion of two solid surfaces in contact.

Viscous and viscoelastic dampers belong to the group of velocity-activated devices. They dissipate energy through the relative velocity between their connection points. The behaviour of dampers are dependent on velocity and frequency of the motion and are out of phase with the maximum internal forces generated by the peak deformation of structure. Lower design forces might be required for the structural members when this type of dampers are employed.

Tuned Mass Damper (TMD) and Tuned Liquid Damper (TLD) belong to the group of motion-activated devices. These dampers have their own period that need to be tuned to the fundamental structural period. They are typically placed on the top of the structure. The details of TLDs are discussed in the next section.
1.2 Tuned Liquid Damper (TLD)

1.2.1 History

The first application of Tuned Liquid dampers was in space satellites (Bhuta, 1966) and marine vessels (Watanabe, 1969). Around ten years later, the dampers were employed in the offshore platforms and ground structures (Vandiver, 1978 and Lee, 1982). Modi (1987) and Welt (1988) developed parametric study on the annular TLD, and showed that this damper can be effective for civil structures. In the late 1980s, liquid dampers have been used in tall towers in Japan, such as the Haneda Airport Tower, the Narita Airport Tower (Tamura et al., 1992) and Shin Yokohama Prince (SYP) Hotel in Yokohama (Wakahara et al., 1992), and also on bridges such as the Ikuchi Bridge and the Sakitama Bridge in Japan (Kaneko and Ishikawa 1999).

The first nonlinear model of a rectangular TLD was developed by Shimizu and Hayama (1987), where the model combines the shallow water wave theory with the potential flow theory. Later, Sun et al. (1989) improved the model for harmonic motion and by accounting for the wave breaking with the introduction of two empirical parameters. Response to random input was studied in Koh et al.’s extended work (Koh et al., 1994).

Kubo et al. (1989) first developed a preliminary experimental study to investigate the effectiveness of rectangular TLDs in controlling the pitching vibration of structure. Then Kotsubo et al. (1990) produced an equivalent mechanical model of the TLD that predicts the sloshing forces and moments caused by the sloshing water under pitching motion, however, the sloshing damping was not considered. Based on these two papers, Sun et al. (1995) developed a numerical model for rectangular TLDs under purely pitching motion based on the non-linear shallow water wave theory where linear damping was considered. Lu (2001) created a new numerical model by using classical shallow water theory with an improved boundary shear model to simulate the sloshing of a liquid in a rectangular TLD that experiences combined horizontal and rotational motion. Samanta and Banerji (2008) numerically investigated the dynamic behaviour of a structure equipped with Modified
Tuned Liquid Damper (MTLD) under harmonic motions by using Lu’s model. A MTLD can move in both horizontal and rotational motion on the top of structure.

1.2.2 Tuned Liquid Dampers (TLDs)

Tuned Liquid Dampers (TLDs) are passive energy absorbing devices that have been used especially in some flexible, high-rise buildings to control the vibrations. They are typically rectangular tanks partially filled with water that are installed on the roof of the structures. The sloshing TLD absorbs and dissipates energy through boundary layer friction, wave breaking and free-surface contamination (Fujino et al., 1992). Based on the ratio of water depth (i.e., height) to length of tank (h/L), TLD can be divided into two categories, shallow water dampers (h/L < 0.15) and deep water dampers (h/L > 0.15) (Banerji et al., 2000).

An important parameter of TLD is the water sloshing frequency. By tuning the sloshing frequency to the natural frequency of structure, a significant amount of sloshing and wave breaking can be activated. Thereby a considerable amount of vibration energy can be dissipated at resonant frequency (Sun et al., 1992). Malekghasemi (2011) suggested that the TLD can remain effective as long as the ratio of natural frequency of structure to sloshing frequency of TLD is in the range of 1 to 1.2. In addition, the mass ratio is also an important parameter of TLD. Banerji et al. (2000) indicated that the structure vibration control is more effective if the mass ratio is in the range of 1% to 4%.

TLDs have some advantages compared to other passive systems. (i) They are cost-effective and are easy to install and maintain; (ii) It is easy to tune their sloshing frequency by changing the water level or the tank dimensions; (iii) They are effective under small amplitude vibrations and for a wide range of excitation frequencies (Sun et al. 1992); (iv) They can be tuned along two orthogonal directions; (v) They can act as a fire-extinguishing system during fire emergencies.
1.2.3 Modified Tuned Liquid Dampers (MTLDs)

Modified Tuned Liquid Damper (MTLD) is a new type of TLD. The main difference between TLD and MTLD is that MTLD is equipped with a rotational spring system at the base. Instead of connecting the tank rigidly on top of the structure, as is done for the typical application of the TLDs, an MTLD has an appropriately designed set of springs that also provide a rotational degree-of-freedom (DOF) in the connection to the structure. In this study, although linear springs have been introduced with a pivot, the overall base connection of the tank mimics that of a rotational spring. Therefore, the base connection of the MTLD will be referred to as a rotational spring. Once the structure starts to move due to dynamic load the MTLD will move horizontally along with structure. At the same time, due to the special spring connection the tank is also allowed to have rotations with a small angle. As such, it is anticipated that in an MTLD, more of the water will set into the sloshing motion, increasing the effectiveness of TLD in controlling structural vibrations. The angle of rotational motion depends on the rotational spring stiffness $K_\theta$. If the rotational spring is extremely rigid, the MTLD will be identical to the traditional TLD. The previous research showed that there is an optimum value for the rotational spring stiffness that maximizes the effectiveness of MTLD. Therefore, if properly designed, a rotational spring system can make the MTLD perform more effectively than the traditional TLD (Samanta and Banerji, 2008).

1.3 Scope of Study

The main objective of this study is to investigate the dynamic behaviour of a single frame equipped with Modified Tuned Liquid Damper (MTLD) numerically and experimentally. Limited numerical studies have been published on the behaviour of MTLD and the design of optimum rotational spring (Samanta and Banerji, 2008) without any experimental verification. Using Lu’s analytical model (2011), this study focuses on the optimum design of the rotational spring system, and the investigation of the dynamic interaction of the
MTLD-structure system for a wide range of excitation frequencies. It also provides experimental validation using real-time hybrid simulation (RTHS).

Chapter 2 outlines the literature review.

Chapter 3 presents the theory and the analytical model. Lu’s model is utilized in this study. The explanation of Lu’s model is outlined and the details of model are discussed. The effects of rotational spring system on the response of the MTLD are considered.

Chapter 4 introduces the experimental set-up and describes the real-time hybrid simulation method employed. The results of the numerical simulations are provided and the experimental results are presented to verify the numerical results. In addition, a preliminary design procedure is outlined.

Chapter 5 provides summary and conclusions of this study together with some suggestions of future work.
Chapter 2

2 Literature Review

In the past few decades a large number of papers have been published on the study different kinds of TLDs with both numerical analysis and experimental testing components. The different shapes of TLD have been investigated include rectangular and circular tanks. Also, several researchers have established the different mathematical models to describe the behaviour of the water tank, and compared the results with those obtained from experimental tests.

2.1 Mathematical Models of TLD

Shimizu and Hayama (1987) derived the basic equations that describe the nonlinear response of sloshing water in the rectangular tank under horizontal motion using the shallow water wave theory with potential flow theory. They used the Runge-Kutta-Gill Method to solve the ordinary differential equations of time and determined the damping provided by the liquid through experimental testing. Shimizu and Hayama (1989) also investigated the numerical model for circular TLDs.

Similar to the model of Shimizu and Hayama, Sun et al. (1989) developed a mathematical model based on the nonlinear shallow water wave theory to describe the liquid motion in a rectangular tank. But they improved the model by introducing a semi-analytical method for the liquid damping due to the friction of boundary shear. They investigated TLD experimentally by using a shaking table to compare with the simulation results, which showed a good agreement with small oscillation amplitude excitation, up to 10mm. In these tests no wave breaking wave was observed for the tank with the size of 590mm length, 335mm width and 30mm water height.
In order to account for the effect of breaking waves, Sun et al. (1991) improved the mathematical model by introducing two empirical coefficients identified experimentally, one is damping coefficient of liquid $C_{da}$, and another one is the wave phase velocity $C_{fr}$. The model was verified experimentally and it was confirmed that the behaviour of water tank can be predicted, and even the occurrence of wave breaking.

Sun et al. (1995) developed a model with equivalent mass, stiffness, and damping of the TLD using Tuned Mass Damper (TMD) analogy from experimental data of rectangular, circular and annular tanks subjected to harmonic base excitation.

Koh et al. (1994) studied the behaviour of rectangular TLDs under arbitrary excitation. They used an existing model where the energy dissipation was included by liquid viscosity. The results showed that the liquid motion was depended on the sloshing frequency, amplitude and frequency content of the excitation.

Yu et al. (1997) used a different way to model TLD by treating TLD as an equivalent tuned mass damper with nonlinear stiffness and damping. The model can be used to describe the behaviour of TLD under a wide range of excitation amplitude.

Reed et al. (1998) investigated the TLDs by focusing on the large amplitude excitation up to 40mm, with numerical modelling and experimental testing. The authors used the random-choice numerical method to solve the nonlinear shallow water wave equation. In the experimental test, they investigated the TLD in terms of the sloshing force, liquid surface height and the energy dissipation. This paper included the tests for a tank with 590mm length, 335mm width and 30mm height of water, and subjected to excitation amplitude with 10mm, 20mm and 40mm. But in their numerical analyses, they only showed the results for the excitation amplitude of 20mm, which had a good agreement between the experimental and the numerical results. The results also showed that the increased excitation amplitude will increase the response frequency of TLD. Reed et al. suggested that the sloshing frequency of TLD should be tuned to the frequency of structure to have the damper perform better.
Banerji et al. (2000) used Sun’s model to simulate the Single-Degree-of-Freedom (SDOF) structure with fixed-bottom TLD, subjected to real and artificial ground motions. They investigated the parameters of TLD, including the ratio of the sloshing frequency to the fundamental frequency of the structure, the ratio of the water mass to the structural mass and the ratio of the depth of water to the length of tank, which are the important parameters that influence the effectiveness of TLDs in reducing the vibrations. In 2002, Banerji et al. carried out an experimental study to verify the numerical results and found out that Sun’s model could not accurately capture the response of the TLD-structure system. Therefore, in 2006, Samanta and Banerji employed another mathematical model developed by Lu (2001) to simulate the behaviour of TLD-structure system subjected to large amplitude base motion, and obtained better results than those obtained from Sun’s model.

Lee et al. (2007) employed the Real-Time Hybrid Simulation (RTHS) method to investigate the performance of TLD. They tested the TLD physically as experimental substructure and modeled a structure in computer as analytical substructure, then verified the results with those from a shake table test. They indicated that the RTHS method can evaluate the behaviour of TLD-structure system accurately.

Malekghasemi (2011) used real-time hybrid simulation method to experimentally test the TLD-structure system by using shake table. In this study the TLD was physically tested and the structure was analytically modeled in a computer. The whole system was subjected to a sinusoidal force and three ground motions. Malekghasemi (2011) studied the effects of different parameters, including mass ratio, frequency ratio and damping ratio and investigated the accuracy of three simplified mathematical models, namely, Sun’s model, Yu’s model and Xin’s model using the experimental results from RTHS. The results showed that the TLD is most efficient when the frequency ratio is between 1 and 1.2 and the mass ratio is 3%.
2.2 Different types of TLD

Modi et al. (1990) studied the energy dissipation and damping introduced by liquid sloshing motion in torus shaped dampers analytically and experimentally. The authors also investigated the wind induced instability of the damper, including vortex and galloping response and concluded that the torus shaped damper is an effective device to control the vibrations.

Modi and Seto (1997) investigated the rectangular nutation damper for the suppression of wind-excited oscillations of structures. The study included the investigation of system parameters that improve the performance of nutation damper, and the effectiveness of nutation damper introduced to tall buildings by wind tunnel experiments.

Modi and Munshi (1998) improved the rectangular liquid damper by putting two semicircular obstacles at the bottom of the tank to increase the energy dissipation. They investigated the optimum size and location of these obstacles by using wind tunnel tests to evaluate the effectiveness of the improved damper. Compared to the original damper, it was shown that the improved damper had up to 60% more energy dissipation.

Modi and Akinturk (2002) used the same method as Modi and Munshi (1998), to study a new type of liquid damper with wedge-shaped obstacles instead of semicircular obstacles. This new investigation focused on the three different type of wedges, two smooth wedges, one smooth wedge and one wedge with steps and two wedge with holes. Again, the efficiency of the damper and the energy dissipation characteristics were evaluated. The results showed that the optimum configuration can improve the energy dissipation. Furthermore, Modi et al. (2003) investigated the influence of floating particles used in the rectangular damper with wedge-shaped obstacles, 40% improvement was obtained in comparison with the water-only case.

Followed by Gardarsson et al., Olson and Reed (2001) further investigated the TLD with a sloped bottom of 30° by using Yu’s model, and compared the behaviour with that of
box-shaped TLD. They indicated that the sloped-bottom tank can be described by a softening spring. They also suggested that in order to get the maximum effectiveness, the sloshing frequency of tank should be slightly higher than the natural frequency of the structure.

Xin et al. (2009) investigated a density-variable TLD with sloped bottom on a three-story structure. They observed that the density-variable TLD with sloped bottom performs more effectively and more robust than a flat bottom TLD in reducing the story drift and acceleration of structure.

Screens have been used in TLDs to increase the dissipated energy. Tait et al. (2005) used both the linear and nonlinear numerical flow models to predict the behaviour of TLD equipped with screens, subjected to both wind loading and earthquake motion and verified by experimental tests.

Tait et al. (2005) published another paper that studied the behaviour of TLD equipped with damping screen under 2D excitation. The authors employed the same nonlinear model used in 1D-case to simulate the 2D TLD, by decoupling and treating the 2D TLD as two independent 1D cases and super-positioning them together., then compared with experimental test. The experimental tests were carried out using a shake table that can excite the TLD in two independent perpendicular horizontal directions. The findings showed that the response of 2D TLD can be analysed as two 1D TLD independently, and also the 2D TLD can be used in controlling the vibration of a structure in two principle directions.

Tait (2008) introduced an equivalent linear mechanical model which includes the factor of energy dissipated caused by the damping screens, and carried out a verification study experimentally. A preliminary design procedure for initial size of the TLD and the damping screen configuration is included in this paper. Tait et al. (2011) investigated the influence and efficiency of TLD equipped with inclined screens. They developed a
mathematical model and provided experimental validation for this new configuration as well.

### 2.3 Modified Tuned Liquid Damper (MTLD)

Sun *et al.* (1995) investigated the behaviour of rectangular TLDs under pitching motion. They modified the analytical model by replacing the acceleration of gravity $g$ with the vertical acceleration $a_y$, which would change during pitching vibration. The comparison with the experimental results under pitching motion showed that the model predictions were accurate in a small amplitude range.

The numerical methods developed by previous researchers can only predict the behaviour of TLD well under an excitation with amplitude up to 40mm. This is insufficient due to the uncertainty associated with earthquake and wind loading. Therefore, Lu (2001) developed a new numerical model by using the shallow water theory to simulate the liquid sloshing in rectangular TLDs subjected to horizontal excitation combined with rotational motion. He found that the equations in the numerical model is better to be solved by the Lax Finite Difference Scheme method. In this method the parameter $\alpha$ can be chosen empirically. It can be reduced to increase the numerical damping to cut off the high-frequency peak during high excitations that may cause numerical instability. The results showed that the rotation can increase the effectiveness of TLD significantly. The research in this thesis used Lu’s model to simulate the motion of the modified TLD.

Based on Banerji *et al.*’s own previous research and Lu’s model (Lu 2001), Samanta and Banerji (2009) introduced the modified TLD which is equipped with a rotational system at its base. When excited horizontally at its base, the modified TLD exhibits both translational (i.e., horizontal) and rotational motion. As such, An SDOF structure equipped with the modified TLD has two degree of freedoms, one is the horizontal motion of the structure at the roof and the other is the rotation motion of modified TLD. The results showed that there is an optimum value for the rotational spring system which provides the maximum the effectiveness for the modified TLD. Samanta and Banerji also showed that
the modified TLD performs similar to a standard (fixed bottom) TLD if the rotational spring system at the base is rigid. However, Samanta and Banerji only provided the theory and numerical analysis for Modified TLD, they did not perform experimental tests to verify the feasibility of such analytical model. And also there is no other research paper focused on the experimental verification. Therefore, this study is very significant to investigate the behaviour of MTLD numerically and experimentally.

2.4 Practical Implementation

The nutation damper has been used in airport towers at the Haneda and Narita International airport to suppress wind-induced vibration. Tamura et al. (1992) experimentally tested the nutation damper combined with building model by small amplitude excitation. And also they investigated the effectiveness of nutation damper applied to 18 degree-of-freedom analytical model of Haneta Airport tower and 21 degree-of-freedom analytical model of Narita airport tower, where the nutation damper was represented as Tuned Mass Damper (TMD). The results shown the 55% reduction of acceleration response of the tower using the nutation damper with 1% mass ratio and floating particles.

Wakahara et al. (1992) introduced an actual application of the TLD to a high-rise structure, the Shin Yokohama Prince (SYP) Hotel in Yokohama, Japan. They developed a new simulation method to account for the nonlinearity of TLD, which can predict the effects of TLD on high-rise building effectively. In addition, they carried out the experimental test to verify the designed optimum TLD for the tower.

Koh et al. (1994) studied the multiple-mode liquid dampers that applied to the suspension bridge, Golden Gate Bridge, subjected to 3 types of earthquake excitation. The authors employed Sun’s nonlinear numerical model to simulate three modes of TLD corresponding to first three vibration modes of the bridge, which interact with the structure in terms of three vibration modes by applying the multiple-TLD to the different position. The findings showed that the multiple-mode liquid dampers can control the vibration of the bridge with
several frequencies better than one liquid damper could control the vibration with only one frequency, as excited by different types of earthquake.

The first case of TLD application is installation Nagasaki Airport Tower (NAT), Nagasaki, Japan, in 1987 (Tamura et al., 1995). The TLDs were temporarily installed on the NAT in order to verify the effectiveness of TLD in reducing structural vibration. They found that the decrease in amplitude of vibration is 44% while reduction in RMS displacement was about 35% with the installation of 25 vessels of TLD.

The TLD has also been installed on Yokohama Marine Tower (YMT) in Japan (Tamura et al., 1995). It was found that when the TLD was installed, the damping ratio was increased to 4.5% from the original damping ratio of 0.6%. In addition, the maximum acceleration was reduced from 0.27m/s² to 0.1m/s².

The TLDs were also applied on the Shin Yokohama Prince Hotel (SYPH) in Yokohama, Japan (Tamura et al., 1995). The container has the diameter of 2m and the height of 0.22m, and was located in multi-layer stack. Each stack has 9 circular containers and the total height is 2m. It has been noticed that the TLD can reduce 30% of RMS accelerations (from 0.01m/s² to 0.006m/s²) in each direction under wind load with the speed of 20m/s, which satisfied the ISO minimum perception level at 0.31Hz.

Another case of TLD application was on The Tokyo International Airport Tower (TIAT) in 1993 (Tamura et al., 1995). There are 1400 tanks filled with water and floating polyethylene particles in order to enhance the energy dissipation. The container has the diameter of 0.6m and elevation of 0.125m. The mass ratio of the total mass of TLDs and the first modal mass of the tower is around 3.5%. And the sloshing frequency of TLD is lightly lower than natural frequency of the tower. The results showed that the TLD can reduce the 60% of the RMS acceleration response of the tower without TLD.

Novo et al. (2013) employed Finite Element Method to investigate the behaviour of an isolated TLD subjected to sinusoidal force with different amplitudes. They also utilized linear dynamic analysis to evaluate the efficiency of the LTD for modern architecture
buildings in southern European countries. They showed that TLD is efficient and can significantly increase the energy dissipation and the equivalent damping of the structure.
Chapter 3

3 Theory and Analytical Model for Tuned Liquid Damper

3.1 Governing Equation of TLD

A rigid rectangular tank shown in Figure 1 is considered in this study. The length and width of tank are represented by $L$ and $B$, respectively, and $h_0$ represents the initial value of the liquid depth within the tank.

![Figure 1: The dimension of a rectangular TLD](image1)

The tank shown in Figure 2 experiences both horizontal and rotational motions, which are specified by $x_b$ and $\theta_b$, respectively. It should be noted that, the horizontal motion $x_b$ is the absolute displacement at the top of structure, and the rotational motion $\theta_b$ is specified in clockwise direction (Samanta and Banerji, 2009).
Figure 2: TLD under both horizontal and rotational motions

For the tank in Figure 2 assuming that the liquid is not reaching to the top of tank, and by using shallow water wave theory (Stoker, 1992), the governing equations of the liquid’s sloshing motion are listed below (Lu, 2001):

\[
\frac{\partial h}{\partial t} + h \frac{\partial v}{\partial x} + v \frac{\partial h}{\partial x} = 0
\]  

(1)

and

\[
\frac{\partial v}{\partial t} + g \frac{\partial h}{\partial x} + v \frac{\partial v}{\partial x} - g (\theta_b - S) + \frac{\partial^2 x_b}{\partial t^2} = 0
\]

(2)

The equations are solved with the boundary conditions:

\[
v|_{x=0} = v|_{x=L} = 0
\]

(3)

It is also assumed that the liquid is in steady state at beginning \( t = 0 \):

\[
h|_{t=0} = h_0 \text{ and } v|_{t=0} = 0 \quad \forall x \in [0, L]
\]

(4)

In these governing equations, \( h \) represents the sloshing liquid depth at each location \( x \) of tank and at each time instant \( t \), and \( v \) is the velocity of the water relative to the base of tank. Furthermore, \( g \) is the gravity acceleration, and \( S \) describes the slope of the energy grade line (Henderson, 1966) that can be obtained by the equation:

\[
S = \frac{\tau_b}{\rho g h}
\]

(5)
Where, \( \tau_b \) is the liquid’s shear stress at the base of tank and defined by:

\[
\tau_b = \frac{\mu v_{\text{max}}}{h} \quad \text{for } z \leq 0.7
\]  

(6a)

and

\[
\tau_b = \sqrt{\rho \mu \omega v_{\text{max}}} \quad \text{for } z > 0.7
\]  

(6b)

\( \rho \) and \( \mu \) are representing the density and the dynamic viscosity of the liquid, respectively. \( \mu \) is related to the kinetic viscosity \( v \) by the equation:

\[
\mu = \rho v
\]  

(7)

Furthermore, \( \omega \) is the excitation circular frequency, and the liquid’s dimensionless depth \( z \) is introduced by Lu (2001):

\[
z = \frac{\omega g}{2\mu} h
\]  

(8)

An important parameter in equation 6 is \( v_{\text{max}} \), the maximum velocity of liquid at free surface relative to the base of tank. Assuming that the liquid velocity at the base of tank is zero, the velocity \( v \) is distributed uniformly from the base of tank to the free surface due to liquid viscosity. Therefore, the average velocity \( v_{\text{avg}} \) over cross-section is employed to represent the velocity \( v \) in equations (1) and (2). The equation for \( v_{\text{avg}} \) from (Lu, 2001):

\[
v_{\text{avg}} = v_{\text{max}} \left( -0.0011z^6 + 0.0169z^5 - 0.0936z^4 + 0.2093z^3 \right.
\]
\[
\left. -0.1181z^2 + 0.0129z + 0.5012 \right) \quad \text{for } z \leq 5
\]  

(9a)

And

\[
v_{\text{avg}} = v_{\text{max}}[1 - \exp(-0.0853z - 2.2807)] \quad \text{for } z \leq 5
\]  

(9b)

The governing equations of TLD can be numerically solved by using Lax Finite Difference Scheme (Lu, 2001), which is discussed in the next section.
The water sloshing elevation can be calculated by solving the differential equations, and the sloshing force $F$ applied to the walls of tank is given by:

$$ F = \frac{1}{2} \rho g B (h_R^2 - h_L^2) + \rho g B h S \, dx $$

(10)

Also, the moment $M$ applied to the base of tank, given by (Sun et al., 1995):

$$ M = -\frac{1}{6} \rho B a_y (h_R^3 - h_L^3) - \int_0^L \rho B a_y h x \, dx $$

(11)

$h_R$ and $h_L$ are the liquid sloshing elevation at right and left walls of the tank, respectively. And $a_y$ is the liquid vertical acceleration given by (Sun et al., 1995):

$$ a_y \approx -(g + \ddot{Z}_0 \cos \theta - \ddot{\theta} x + \dot{X}_0 \sin \theta) $$

(12)

### 3.2 Numerical Solution Procedure

The differential equations are suggested to be solved by Lax Finite Difference Scheme (Lu, 2001). The terms in equations can be discretized as:

$$ \frac{\partial \hat{f}}{\partial t} = \frac{\hat{f}^{j+1}_j - \hat{f}^{j-1}_j}{2 \Delta t} $$

(13)

$$ \hat{f} = \alpha \hat{f}^{(k)}_j + (1 - \alpha) \frac{\hat{f}^{(k)}_{j+1} + \hat{f}^{(k)}_{j-1}}{2} $$

(14)

and

$$ \frac{\partial \hat{f}}{\partial x} = \frac{\hat{f}^{(k)}_{j+1} - \hat{f}^{(k)}_{j-1}}{2 \Delta x} $$

(15)

Here, $\hat{f}$ is the generic variable that can represent other variable $h, v$ or $S$. Superscript $k$ indicates the instant at time $k \Delta t$, where $\Delta t$ is time step. And subscript $j$ represents the node at the location $j \Delta x$ along the length of tank $L$, where $\Delta x$ is the length of one element. Parameter $\alpha$, which is determined empirically, is used to provide numerical stability by
filtering the unrequired and unimportant high-frequency components. The value of \( \alpha \) is taken from 0 to 1, the lower the value, the more damping is provided in the numerical solution. The details of \( \alpha \) are discussed in Lu’s paper (2001), in this study the value is taken to be 0.98.

Furthermore, the value of \( \Delta x \) is obtained by dividing the length of tank with a suitable division number \( n \), which is suggested by an equation (Shimizu and Hayama, 1987):

\[
n = \frac{\pi}{2 \arccos \left( \frac{\tanh(\frac{\pi \varepsilon}{2})}{\sqrt{2 \tanh(\frac{\pi \varepsilon}{2})}} \right)}, \quad (\varepsilon = h_0/(L/2))
\]  

In this study, a value of 35 is used for \( n \). To have a stable solution Lu (2001) suggest \( \Delta t \) satisfy the Courant condition:

\[
\Delta t \leq \frac{\Delta x}{\max(|v|+\sqrt{gh})}
\]

Equation 17 is only used to determine the initial value of \( \Delta t \), and the value is then adjusted to get a stable, convergent solution. In this study, the value of \( \Delta t \) is taken to be 0.001, as suggested by Samanta and Banerji (2009).

The differential equations are solved explicitly to find the new state of liquid elevation and velocity at each discretized location \( j \Delta x \) at time step \((k+1)\Delta t\), \((h_j^{k+1}, v_j^{k+1})\) from the old state at time step \( k\Delta t \), \((h_j^k, v_j^k)\), where \( j \) is taken from 0, 1, 2, ... to \( n \). The details of solution procedure are provided in Appendix A.

### 3.3 TLD-Structure Interaction Configuration

The traditional TLD is rigidly connected to the top of linear single-degree-of-freedom structure. Figure 3 shows the Schematic of TLD-Structure system. The equation of motion for the TLD-Structure system subjected to ground motion is given by:

\[
m_s \ddot{u}_x + c_s \dot{u}_x + k_s u_x = -m_s \ddot{u}_g + F
\]
Where $m_s$, $k_s$ and $c_s$ are total mass, lateral stiffness and damping coefficient of structure, respectively. $u_x$, $\dot{u}_x$ and $\ddot{u}_x$ represent lateral displacement, velocity and acceleration of structure, respectively, and $\ddot{u}_g$ represents the ground acceleration applied to structure. In addition, $F$ denotes the base shear force caused by sloshing water in the tank and can be obtained by equation (10).

![Figure 3: Schematic of Single Frame with TLD](image)

### 3.4 MTLD-Structure Interaction Configuration

The Modified Tuned Liquid Damper (MTLD) equipped with a rotational spring system connected to the top of linear single-degree-of-freedom structure, as shown in Figure 4. In the idealized MTLD-structure system shown in Figure, the connection of the tank with the structure is established with a rotational spring and a rigid rod. The bottom end of rod is connected to structure through the rotational spring, and the top end of rod is connected to the structure. In comparison with TLD-Structure system, MTLD-Structure system is treated as a two-degree-of-freedom system with a horizontal DOF and a rotational DOF. Thus, the equation of motion for MTLD-Structure system subjected to horizontal ground motion is given by (Samanta and Banerji, 2009):
\[
\begin{bmatrix}
(m_s + m_t) & -m_t l \\
m_t & -(m_t l + \frac{J_1 + J_2}{l})
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\dot{\theta}_l
\end{bmatrix}
+ \begin{bmatrix}
0 & C_\theta/l^2 \\
c_s & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\dot{\theta}_l
\end{bmatrix}
+ \begin{bmatrix}
k_s & 0 \\
0 & -K_\theta/l
\end{bmatrix}
\begin{bmatrix}
x \\
\theta_l
\end{bmatrix}
= \begin{cases}
F - \ddot{u}_g(m_s + m_t) \\
F + \frac{M}{l} - \ddot{u}_g m_t
\end{cases}
\]  

(19)

Where \( m_t \) denotes the total lumped mass of the rod and the tank, and \( l \) is the length of the rod. \( J_1 \) and \( J_2 \) represent the mass moment inertial of the rod and the tank, respectively. \( K_\theta \) and \( C_\theta \) are the rotational spring stiffness and the damping coefficient of the rotational spring system, respectively. In addition, \( \theta_l, \dot{\theta}_l \) and \( \ddot{\theta}_l \) express the clockwise rotation angle, clockwise rotation velocity and clockwise rotation acceleration with respect to vertical axis, respectively. Furthermore, \( F \) is the sloshing force at the interface of the MTLD and the structure, and \( M \) is the moment that is applied to the MTLD, given by equation (11).

Figure 4: Schematic of Single Frame with Idealized MTLD
Considering the stability and constructability of the MTLD in real-life, the configuration of rotational spring system is replaced by a set of linear springs with some separation from a rigid pivot as shown in Figure 5. The distance between two springs is denoted by $L_s$, and the stiffness of each spring is presented by $K_r$. The MTLD rotates around the pivot and the length of pivot pin is represented by $l$. The relationship between the idealized rotational spring system from Figure 4 and the practical rotational spring system in Figure 5 is given as the equation below (Samanta and Banerji, 2009),

$$K_\theta = \frac{K_r L_s^2}{2}$$

(20)

3.5 Solution Procedure

The MTLD (TLD)-Structure system can be solved numerically using an explicit scheme. Figure 6 shows the flowchart of numerical solution procedure. At the first time step, the governing equations (1) and (2) of TLD are solved with the initial conditions and the boundary conditions. In this process the Lax Finite Difference Scheme shown in equations
(13) (14) and (15) is used to calculate liquid elevation on two sides of tank for the next time step, and the sloshing force $F$ and the moment $M$ are obtained using equations (10) and (11). Then, $F$ and $M$ are substituted in the equations of motion for MTLD (TLD)-Structure system (18) and (19), and using the Newmark-Beta Average Acceleration Method the dynamic response of the structure at the new time step is calculated. The calculated acceleration is used in the governing equations (1) and (2) to calculate the liquid elevation in the next time step. These calculations are repeated until the final time step. As mentioned before, to obtain a stable and convergent solution the time step size $\Delta t$ is set to be $0.001 \, s$ and the parameter $\alpha$ is taken to be 0.98.

Figure 6: Flowchart of Numerical Solution Procedure
Chapter 4

4 Numerical and Experimental Investigation Results

4.1 Real-Time Hybrid Simulation (RTHS) Testing Method

The hybrid simulation is a variation of the traditional Pseudo-dynamic (PSD) testing method. It has an ability to combine the response from the physically tested critical part of the structure with the remaining structure that is modeled analytically in computer. The physically tested part is called the experimental substructure, while the remaining structure that is modeled in the computer is called the analytical substructure.

When the load-rate dependent vibration characteristics are present, such as in the case of MTLD, to capture the accurate vibration characteristics the hybrid simulation needs to be executed dynamically in real-time. The Real-Time Hybrid Simulation (RTHS) testing method is employed in this thesis to study the dynamic response of the MTLD-structure system under different loading conditions. The MTLD is tested physically as the experimental substructure and the Single-Degree-of-Freedom (SDOF) structure is modeled in computer as the analytical substructure. This way, the parameters of SDOF structure can be easily modified. As shown in Figure 7, the interaction force between MTLD and structure is measured by a load cell that is connected between the MTLD and the shake table. The force is then fed back to the analytical model that is modeled by using Simulink and Real-Time workshop, and the dynamic response of structure is numerically calculated for the next time step. Then, this new command displacement is imposed on shake table, resulting force is measured and fed back and the same procedure is repeated. During the test, the WinCon System and Simulink Interface are employed to implement the communication and synchronization of the outer-loop (command
generation loop) with the inner-loop (control loop for the shake table). The details of implementation of Real-Time Hybrid Simulation method are shown in Ashasi-Sorkhabi et al.’s paper (2013).

Figure 7: Schematic Diagram of Real-Time Hybrid Simulation Method

4.2 Experimental Set-Up

Figure 8 shows the Quanser Shake Table used in this study. The shake table consists of a 1 HP servo motor driving a 12.7 mm lead screw. The lead screw drives a circulating ball nut connected to a 457*457 mm table. The table moves on linear bearings with low friction and it has a 76.2 mm stroke. The shake table comes with a real-time control software (WinCon) that executes Simulink models in real-time workshop. The built-in controller of the table is able to impose harmonic or pre-set earthquake ground motion data under displacement control. In this study, a velocity feed forward component was used in addition to the existing feedback controller to improve the tracking of the command displacements by the shake table.
The experimental set-up of MTLD as the experimental substructure is shown in Figure 9. The MTLD is connected to the shake table through a load cell, which has a capacity of 22.2N (5lb) and can carry both compression and tension loads. This load cell is used to measure the interaction force that is transmitted to analytical substructure. The MTLD is placed on 4 blocks that can move in the horizontal direction on two rail guides with low friction. However, the preliminary tests showed that even the low friction existed on rail guides was still a significant amount of force unintentionally introduced to the MTLD-structure system. This friction force can affect the dynamic performance of the MTLD-structure system. In this study, the friction force has been identified both quasi-statically and dynamically. A friction force has been removed from the interaction force that is introduced to the Simulink model. This way the artificial modification of the dynamic response of the MTLD-structure system due to the friction force was avoided. The block has a 65mm stroke on each side of two rail guides, this constitutes the limit of the stroke that can be used in this experimental study.

The MTLD can be considered as made of two parts: the water tank and the rotational spring system. The tank is made of plexiglass with a length of $(L)$ of 464mm and a width $(B)$ of 305mm. The water height $(h_0)$ is set as 40mm. As such, the corresponding weight of the water is 5.66 kg and the sloshing frequency $(f_\omega)$ of the TLD is calculated to be 0.667Hz from:
The rotational spring system includes 4 linear springs and two pivot pins as shown in Figure 10(a). The upper part of pivot pin has the length ($l$) of 100mm. Therefore the tank can rotate through the center of pin in an arc with a 100mm radius. The linear stiffness of each linear spring is 2500N/m. As a result, on one side of the pivot pins, the linear stiffness of the rotational spring system ($K_r$) is 5000N/m.

Figure 9: Schematic Representation of the Experimental Set-up of MTLD

Figure 10: (a) A Close-up View of the Spring and Pivot Connection, (b) Assembled MTLD
Furthermore, the springs are attached to the aluminum angles along slotted lines. This way their locations can be changed. As such, the distance of two springs on two sides of pivot pin denoted as $L_s$, can be modified during the experiments to obtain different values for the rotational spring stiffness of the system. The range of $L_s$ is set to be from 130mm to 450mm. The relationship between linear spring stiffness and rotational spring stiffness is given by equation (19) in Chapter 3. Here, one important parameter of rotational spring system that should be noted is Dimensionless Rotational Stiffness Parameter ($DRSP$), which could affect the dynamic behaviour of MTLD-Structure system directly. It is defined by Samanta and Banerji, (2009):

$$DRSP = K_\theta/(k_s l^2)$$  (22)

It can be seen that $DRSP$ is related to the length of the pivot pin $l$, the stiffness of structure $k_s$ and the rotational spring stiffness $K_\theta$. In addition, based on equation (19) $K_\theta$ can be obtained from linear spring stiffness $K_r$ and the distance between the two sets of springs $L_s$. In this experimental set-up, the length $l$ is fixed as 100mm and the linear spring stiffness $K_r$ is also fixed as 5000N/m on each side of the pivots. Once the structural properties are chosen for analytical substructure in RTHS, the stiffness of structure becomes available as well. In $DRSP$, the distance of two springs $L_s$ can be adjusted to have a value between 130mm to 450mm. The next section presents a study that investigates the effects of $DRSP$ by studying the effects of the different values for $L_s$.

### 4.3 $DRSP (L_s)$ Investigation

This section presents results from numerical simulations. The details of the numerical analysis approach used in this study are shown in Appendix A.

The effects of the Dimensionless Rotational Stiffness Parameter ($DRSP$) are studied using different values for the spring distance $L_s$. The relationship between $L_s$ and $DRSP$ can be found:
\[
DRSP = \frac{K_L L_s^2}{2 k_s l^2}
\]  

(23)

In Equation 23, it can be seen that the value of \( DRSP \) can be adjusted by changing the value of \( L_s \) while other parameters remain constant as was done in this study. When the rotational spring system is very stiff (i.e., a large value for \( L_s \)), since the resulting tank rotations will be negligibly small, the MTLD can be treated as traditional TLD (i.e., as one, that is rigidly connected to the structure). On the other hand, if the spring distance \( L_s \) is short, and the rotational spring stiffness is hence very small, the entire system may become unstable. A previous research paper by Samanta and Banerji (2008) indicated the existence of an optimum value for \( DRSP \), so that the Modified TLD can perform more effectively than the traditional TLD. In this study, considering the properties of the experimental setup, the effects of \( DRSP \) is first numerically studied by specifically considering different values for the spring distance \( L_s \).

The MTLD-Structure System is subjected to a series of sinusoidal excitations with a range of excitation frequencies. The mass ratio (MR) between the MTLD mass (\( m_w \)) to the structural mass (\( m_s \)) is kept to be 2%, and the damping ratio (\( \xi \)) of structure remained as 1.2%. In addition, The MTLD-Structure System is investigated for three frequency ratios \( \alpha \) as 0.8, 1 and 1.2, which represent the ratio of the natural frequency of the structure \( f_s \) to the sloshing frequency \( f_w \). The sloshing frequency \( f_w \) is defined previously and remained unchanged. Therefore, the structural stiffness \( k_s \) should be adjusted to change the natural frequency of structure \( f_s \), to be able to obtain the afore-mentioned frequency ratios (i.e., different values for \( \alpha \)). In the meantime, while changing the structural stiffness values, the excitation amplitude is adjusted in a way to keep the same value of roof displacement for the structure without MTLD. Furthermore, the frequency ratio \( \beta \) between the excitation frequency \( f \) to the natural frequency of structure \( f_s \) is considered to have several different values in a range from 0.8 to 1.2. The properties of the MTLD-Structure system are listed in Table 1.
Table 1: The properties of MTLD-Structure System for DRSP Investigation

<table>
<thead>
<tr>
<th></th>
<th>$m_w$ (kg)</th>
<th>$m_s$ (kg, 2%)</th>
<th>$f_w$ (Hz)</th>
<th>$\alpha$</th>
<th>$f_s$ (Hz)</th>
<th>$k_s$ (N/m)</th>
<th>$c_s$ (1.2%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.66</td>
<td>283.04</td>
<td>0.667</td>
<td>0.8</td>
<td>0.5336</td>
<td>3181.55</td>
<td>22.77</td>
</tr>
<tr>
<td>2</td>
<td>5.66</td>
<td>5.66</td>
<td>0.667</td>
<td>1</td>
<td>0.667</td>
<td>4971.18</td>
<td>28.47</td>
</tr>
<tr>
<td>3</td>
<td>5.66</td>
<td>5.66</td>
<td>0.667</td>
<td>1.2</td>
<td>0.8004</td>
<td>7158.49</td>
<td>34.16</td>
</tr>
</tbody>
</table>

Figure 11: The effects of Spring Distance on structural displacement, alpha=0.8

Figure 12: The effects of Spring Distance on structural displacement, alpha=1
A summary of the effects of the spring distance $L_s$ to the structural response are shown in Figures 11 to 13. The values of spring distance $L_s$ are chosen from 0.13m to 0.45m. The value 0.13m represents a rotational spring stiffness that is flexible and this is the closest value that the two sets of the springs can be located in the experimental set-up. The value 0.45m corresponds to a rotational spring stiffness that is relatively rigid and resulting in a negligibly small rotation at the base of the tank. In this case the MTLD can effectively be treated as a regular fixed-base TLD.

From these three figures it can be seen that, for each value of $\alpha$, the optimum value of $L_s$ that results in the maximum effectiveness of the MTLD under each of the different frequency ratios ($\beta$’s) considered are different. For example, when $\alpha$ is equal to 0.8 (Figure 11), the optimum values of $L_s$ for $\beta$ equal to 0.8 and 1.0 are around 0.15m and 0.19m, respectively. Similarly, when $\alpha$ is equal to 1 (Figure 12), the optimum value of $L_s$ for $\beta$ equal to 0.8, 0.9 and 0.95 are around 0.175m, 0.2m and 0.24m, respectively.

For the frequency ratio $\beta$ greater than 1 (i.e., $\beta = 1.05, 1.1$ and 1.2), the structural displacements either remain the same or increase slightly when the value of $L_s$ reduces from 0.45m to 0.13m. This means that there is no optimum value for $L_s$ for these cases, and the traditional TLD is the more effective option.
Figure 13 showed that for case when $\alpha$ is equal to 1.2, the structural displacement response for all range of frequency ratios $\beta$ are not reduced while the value of $L_s$ is reduced. In other words, there is no optimum value of $L_s$ for $\alpha \beta$. On the contrary the structural displacement response will increase when the MTLD is employed in lieu of a fixed-base TLD.

Although the optimum value of $L_s$ is not unique for a range of frequency ratios $\beta$, in some cases a specific value of $L_s$ can be determined such that with the use of this value of $L_s$ the maximum value of the structural response can be kept below a certain value for the entire range of frequency ratios $\beta$. This structural displacement value is also lower than that of the structure equipped with the traditional TLD, indicating that a properly designed MTLD is more efficient than the regular TLD at all frequency ratios $\beta$.

For the case when $\alpha$ is equal to 0.8 as shown in Figures 11 and 14, when the value of $L_s$ is selected as 0.183m, the maximum displacement is less than 10mm, corresponding to a value of $\beta$ equal to 0.9. At all other values of $\beta$ the maximum displacement values are much less than 10mm for this $L_s$ value. If a different $L_s$ value is used, it can be seen that the maximum structural displacement is no longer limited to a value of 10mm for all $\beta$ values, hence the structure can be observed experience larger displacements at some $\beta$ values. As such, for $\alpha$ equal to 0.8, an $L_s$ value of 0.183m can be considered as the optimum design value. Similarly from Figures 12 and 15, it can be concluded that for $\alpha$ equal to 1 the optimum value of $L_s$ is 0.3m. In this case the maximum roof displacement of the structure is less than 13 mm for all $\beta$ values. Unlike before, for $\alpha$ equal to 1.2, it can be seen from Figures 13 and 16 that over the entire range of $\beta$'s the fixed-base TLD is consistently more effective in reducing the displacements in comparison to MTLD. Thus, there is no optimum value for $L_s$ and in the following sections of this study the case where $\alpha$ equal to 1.2 is not further considered.
Figure 14: The Effects of MTLDs with different Spring Distance on Structure, alpha=0.8

Figure 15: The Effects of MTLDs with different Spring Distance on Structure, alpha=1
Figure 16: The Effects of MTLDs with different Spring Distance on Structure, alpha=1.2

Figure 17 and 18 illustrate the displacement time histories of two cases for MTLD-structure system under sinusoidal excitations. The figures show the structural displacements for different values of $L_s$. The case where the frequency ratio $\beta$ is 1 and $\alpha$ is equal to 0.8 is shown in Figure 17, and the case where $\beta$ is 0.95 and $\alpha$ is 1 is shown in Figure 18. In Figure 17, it can be seen that the system with optimum MTLD ($L_s=0.183m$) experiences the lowest vibrations. However, if the value of $L_s$ is reduced to 0.15m, the structural vibration is shown to increase. In Figure 18, the structural vibration for case of $L_s$ equal to 0.2m is lower than that for case of $L_s$ equal to 0.3m as expected from the results of Figure 15. Although $L_s$ equal to 0.3m was selected as the optimum value to reduce the displacement over the entire range of $\beta$ values, a $\beta = 0.95$ considered in Figure 18, an $L_s$ value of 0.2 m. is a bit more effective in reducing the displacements. Nonetheless, in both Figures 17 and 18, the traditional TLD underperforms in comparison to MTLD since values $\alpha$ and $\beta$ less than 1 are considered in the design and loading.
In this section, considering different mass and damping ratio, the efficiency of the MTLD in reducing the structural displacements is investigated and compared with that of the traditional TLD. In this study, three mass ratio (MR) values (1%, 2% and 4%), and three damping ratio values (0.2%, 1.2% and 2%) are investigated. For different mass ratio, the damping ratio ($\xi$) is kept as 1.2% and for different damping ratio, the mass ratio is kept as 2%. In addition, the frequency ratio $\alpha$ are taken to be 0.8 and 1, and the structural mass, stiffness and damping coefficient are adjusted for each case. The displacement reduction for two parameters are presented in Table 2 and Table 3.
Table 2: Structural Displacement Reduction, Damping Ratio = 1.2%

<table>
<thead>
<tr>
<th>MR</th>
<th>alpha</th>
<th>Structure Only</th>
<th>TLD</th>
<th>Optimum MTLD</th>
<th>Re(TLD)</th>
<th>Re(Optimum MTLD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.8</td>
<td>56.3</td>
<td>34.1</td>
<td>16.32</td>
<td>39.4%</td>
<td>71.0%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>33.07</td>
<td>22.07</td>
<td>41.3%</td>
<td>60.8%</td>
</tr>
<tr>
<td>2%</td>
<td>0.8</td>
<td>56.3</td>
<td>40.22</td>
<td>10.38</td>
<td>28.6%</td>
<td>81.6%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>21.78</td>
<td>12.81</td>
<td>61.3%</td>
<td>77.2%</td>
</tr>
<tr>
<td>4%</td>
<td>0.8</td>
<td>56.3</td>
<td>32.64</td>
<td>8.868</td>
<td>42.0%</td>
<td>84.2%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>16.47</td>
<td>7.98</td>
<td>70.7%</td>
<td>85.8%</td>
</tr>
</tbody>
</table>

Table 3: Structural Displacement Reduction, Mass Ratio = 2%

<table>
<thead>
<tr>
<th>DR</th>
<th>alpha</th>
<th>Structure Only</th>
<th>TLD</th>
<th>Optimum MTLD</th>
<th>Re(TLD)</th>
<th>Re(Optimum MTLD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2%</td>
<td>0.8</td>
<td>56.3</td>
<td>35.04</td>
<td>7.169</td>
<td>37.8%</td>
<td>87.3%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>11.19</td>
<td>6.302</td>
<td>80.1%</td>
<td>88.8%</td>
</tr>
<tr>
<td>1.2%</td>
<td>0.8</td>
<td>56.3</td>
<td>40.22</td>
<td>10.38</td>
<td>28.6%</td>
<td>81.6%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>21.78</td>
<td>12.81</td>
<td>61.3%</td>
<td>77.2%</td>
</tr>
<tr>
<td>2.0%</td>
<td>0.8</td>
<td>56.3</td>
<td>44.63</td>
<td>13.66</td>
<td>20.7%</td>
<td>75.7%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>56.3</td>
<td>30.24</td>
<td>18.45</td>
<td>46.3%</td>
<td>67.2%</td>
</tr>
</tbody>
</table>

Considering two tables together, it can be concluded that the displacements of structure equipped with optimum MTLD are lower than those equipped with traditional TLD in all cases. For the structure equipped with traditional TLD, the structural displacements for the case of $\alpha$ equal to 1 is much lower than that for the case of $\alpha$ equal to 0.8, except for the
case of MR equal to 1% and damping ratio equal to 2%. However, if the structure equipped with optimum MTLD, the structural displacements for the case of α equal to 1 is slightly higher than those with α equal to 0.8. This means that the optimum MTLD could provide a higher reduction in displacements than the traditional TLD if is designed with a frequency ratio α that is less than 1.

From the Table 2, it can be seen that for both values of α, the displacement reduction of structure equipped with optimum MTLD increases with the increasing mass ratio. This indicates that an increase of mass ratio from 1% to 4% increases the effectiveness of optimum MTLD.

In table 3, for both values of α, there is less reduction of the displacement of the structure equipped with optimum MTLD as the damping ratio increases from 0.2% to 2%. The results showed that the traditional TLD and optimum MTLD can perform more effectively in a structure that has a low inherent damping ratio.

### 4.5 Experimental verification

In this section, in order to assess the accuracy of the numerical model for the MTLD-structure system, a series of experiments are performed by using the Real-Time Hybrid Simulation method. Here, three cases (shown in Table 4) are presented to investigate the agreement between the numerical results and experimental results. The displacement time histories for these three cases are provided in Figures 19, 20 and 21.

<table>
<thead>
<tr>
<th>Table 4: Cases for Experimental Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass Ratio</strong></td>
</tr>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>Case 2</td>
</tr>
<tr>
<td>Case 3</td>
</tr>
</tbody>
</table>
Figure 19: Displacement Time History, $L_s=0.45\text{m}$, $\beta=1.05$

Figure 20: Displacement Time History, $L_s=0.24\text{m}$, $\beta=1$

Figure 21: Displacement Time History, $L_s=0.2\text{m}$, $\beta=0.95$
In the first case (Figure 19), the oscillations in the experimental response is a little bit higher than the numerical response in the beginning (around 10s) of the test. But after 15s, the experimental result is the same as numerical results. In case 3 (Figure 21), the amplitude of oscillations in the experiment is slightly bigger than the ones in the numerical response both in the beginning and in the end; but still there is a good overall agreement between the two responses. In case 2 (Figure 20), the discrepancy between the experimental and numerical responses is more noticeable, however there is still an acceptable agreement between the two. Additionally, to be able to assess the accuracy of the numerical simulation for responses with no wave breaking, weak wave breaking and wave breaking conditions, excitation amplitudes of 3N, 5N and 8N for the cases shown in Table 5.

Table 5: Experimental Verification for the case of wave breaking

<table>
<thead>
<tr>
<th></th>
<th>Mass Ratio</th>
<th>Damping ratio</th>
<th>Spring distance ($L_s$)</th>
<th>beta (β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>2%</td>
<td>1.20%</td>
<td>0.45m</td>
<td>1.05</td>
</tr>
<tr>
<td>Case 2</td>
<td>2%</td>
<td>1.20%</td>
<td>0.36m</td>
<td>0.95</td>
</tr>
<tr>
<td>Case 3</td>
<td>2%</td>
<td>1.20%</td>
<td>0.2m</td>
<td>1</td>
</tr>
</tbody>
</table>
In Figure 22, it can be seen that the numerical model is able to predict the response for the excitation amplitudes of 3N, and 5N (i.e., no wave-breaking and weak wave breaking cases respectively). However the accuracy of the numerical model starts to suffer when there is strong wave breaking (i.e., with an excitation amplitude of 8N) as the water sloshes inside the tank.
It should be pointed out here that since $L_s = 0.45\text{m}$ is used, the cases in Figure 15 represents the limiting case where MTLD behaves as a fixed-base TLD.

In Figure 23, the separation between the linear springs of the rotating base is $L_s = 0.36\text{m}$, therefore the MTLD in this case experiences both translation and rotation. It can be seen that for the no wave-breaking case, the agreement between the numerical and experimental
responses is good. However, it suffers as weak and strong wave breakings are observed in the sloshing tank, under 5N, and 8N excitation amplitudes, respectively.

In Figure 24, the separation between the linear springs of the rotating base is $L_s=0.2\text{m}$, therefore increased rotation is experienced by the MTLD under the resonance loading condition. This time, only the no-wave breaking and strong wave breaking cases are presented. An acceptable agreement between the numerical and experimental responses is observed in Figure 24.

Considering the sinusoidal excitation with different frequencies and amplitudes, and also for different rotational stiffness conditions for the rotating base system, it can be concluded that the numerical simulation used in this study can predict the response of the MTLD-structure system with an acceptable accuracy.
4.6 MTLD-Structure Subjected to Ground Motions

To investigate the effectiveness of MTLD-structure system for seismic loads, and also study the accuracy of the numerical model when the input is not sinusoidal but random in nature, Kobe and El Centro earthquake ground motions are considered in this study. Due to the limitation of shake table’s stroke during RTHSs, these two seismic records are scaled down by a factor of 0.1 and 0.3, respectively. To be consistent with the previous cases, here the mass ratio of MTLD-Structure System is kept as 2% and the damping ratio of structure is 1.2%. Figure 25 and 26 show the prediction of numerical model for only Kobe earthquake but with two cases of $\alpha$ equal to 0.8 and 1, respectively.

![Graph showing structural response with and without MTLD](image)

Figure 25: Structural Response (Numerical) with and Without MTLD under Kobe Earthquake, alpha=0.8
Figure 25 shows that changing the spring distance $L_s$ of MTLD affects its effectiveness in controlling the structural vibrations with a decreasing value of $L_s$, the effectiveness of MTLD increases until the optimum value of $L_s$ is achieved. The optimum MTLD is consistently more effective in reducing the structural vibration than the traditional TLD.

In the case when $\alpha$ is 1 (Figure 26), it can be seen that the optimum MTLD does not perform much better than the traditional TLD. On the contrary, the effectiveness of MTLD might be less than the traditional TLD when the value of $L_s$ is not adjusted to an optimum value. It is should be noted in both Figures 25 and 26 the TLD and MTLD do not have effect on the first peak of displacement time history. It takes a while for the sloshing motion in MTLD or traditional TLD to be activated and take effect. Therefore, both MTLD and traditional TLD are ineffective in reducing the vibrations in the first few cycles of oscillation.

Figure 27 and 28 show the comparison between the RTHS results and the numerical results under Kobe and El Centro earthquakes, respectively. For each seismic record, the three different spring distance $L_s$ is considered as 0.45m, 0.36m and 0.2m.
In the case of the Kobe record in Figure 27, the first observation is that the analytical model can predict well independent of the value of $L_s$. And secondly, from the fourth plot which is the comparison of three experimental results, the structural vibrations of two cases ($L_s=0.45m$ and $0.36m$) are almost the same. However, when the spring distance $L_s$ is reduced to 0.2m, the structural vibrations are observed to increase. This experimental result is consistent with the numerical results shown in Figure 26.

For the El Centro record in Figure 28, the comparison results show that the numerical response agrees with the experimental results for the most part except for the ranges around 2.5s and 12.5s. Based on the comparison results, the analytical model can be considered to predict the behaviour of MTLD-Structure acceptable. Similar as fourth graph in Figure 27, the direct comparison of the three experimental results are provided as the last plot in Figure 28. Again, the performance of the MTLD in the cases where $L_s$ is 0.45m and 0.36m is almost same. But when the value of $L_s$ is reduces to 0.2m where the springs are closer to one another, the MTLD losts its effectiveness and performs worse than the traditional TLD.
Figure 27: Structural Response with and Without MTLD under Kobe Earthquake
Figure 28: Structural Response with and Without TLD under El Centro Earthquake
4.7 Preliminary Design Procedure

The results obtained from the numerical analyses and experiments using RTHS is used here to propose a preliminary design procedure for MTLDs. The proposed design procedure of MTLD considers a wide range of frequency ratio $\alpha$ (the ratio of the structural natural frequency to the water sloshing frequency) and $\beta$ (the ratio of the excitation frequency to the structural natural frequency). The details of preliminary design is presented in Figure 29.

Design Procedure

1. Determine the structural vibration reduction percentage that is targeted.
2. If the desired reduction is less than around 60% traditional TLDs can be designed; if more than 60%, proceed with the design of MTLDs.
3. Determine the properties of structure that need to be controlled, including the structural mass, structural stiffness and structural damping ratio.
4. Use an $\alpha$ value of 0.8, and mass ratio in the range from 1% to 4%. For each mass ratio, draw the displacement spectrum as done in Figures 14 and 15 of this chapter, determine the optimum DRSP and mass ratio.
5. Using $\alpha$, mass ratio and DRSP together with the structural properties, calculate the tank dimensions, required water level and the connection details of the rotating base.
Determine the structural vibration reduction percentage that is targeted

If Re<60%
Consider to use Traditional TLD

If Re>60%
Consider to use MTLD

Determine the structural mass, stiffness and damping coefficient

Use an $\alpha$ value of 0.8, and mass ratio in the range from 1% to 4%. For each mass ratio, draw the displacement spectrum, determine the optimum $DRSP$ and mass ratio.

Using $\alpha$, mass ratio and $DRSP$ together with the structural properties, calculate the tank dimensions, required water level and the connection details of the rotating base.

Figure 29: Flowchart or Preliminary Design Procedure
Chapter 5

5 Summary and Conclusion

In this study the Modified Tuned Liquid Damper (MTLD), where the Tuned Liquid damper is attached to the structure with a rotating base to experience both translations and rotations, is investigated numerically and experimentally. For the numerical study of the effectiveness of MTLD-structure system in reducing the structural vibrations the numerical model by Lu (2001) is used. For the experimental study, a state-of-the art testing method, namely Real-time Hybrid Simulation (RTHS) is used. In these RTHSs a MTLD is tested physically in the laboratory, while the structure is modeled in the computer; and the response from these two are coupled in real-time.

Different parameters, such as mass ratio, frequency ratio \( \alpha \), damping ratio of structure and also the excitation conditions, are considered in this study. The main conclusions regarding the MTLD-structure response in comparison to the traditional TLD-structure response are summarized below. Additionally a preliminary design procedure of MTLDs is proposed in order to provide a starting point to the engineers in designing these devices.

\textbf{DRSP Investigation}

The effects of the Dimensionless Rotational Stiffness Parameter (DRSP) that has already been shown to be important in the MTLD-structure response, is considered in this study by changing the distance between the linear springs \( L_s \). It should be noted that the optimum value of \( L_s \), where the MTLD exhibits the most effectiveness in reducing the vibrations, is not unique under different excitation frequency (expressed by frequency ratio, \( \beta \)) for each case of mass ratio and frequency ratio \( \alpha \). It was shown that an optimum value of \( L_s \) is only possible for \( \alpha \) and \( \beta \) values that are less than 1, where the properly designed MTLD
was shown to be more effective than the traditional TLD. In other cases however the traditional TLDs were observed to be more efficient in controlling the vibrations.

**Mass Ratio and Damping Ratio**

By comparing the displacement reduction of MTLD-structure system between the cases of different mass ratio, frequency ratio $\alpha$ and damping ratio, it can be concluded that by using the optimum value of $L_s$ a more effective MTLD can be designed by considering and $\alpha$ value less than 1, a value of 0.8 is suggested in this study. Also, for different values of $\alpha$, the effectiveness of MTLD was shown to increase with an increasing mass ratio and with a decreasing damping ratio.

**Accuracy of Analytical model**

The accuracy of the numerical simulation results were assessed using the experimental results from real-time hybrid simulations. Sinusoidal excitations with different frequencies and amplitudes were considered. The different amplitudes of the input excitation cause the liquid in the sloshing to behave differently where no wave breaking, weak wave breaking and strong wave breaking are observed. It was concluded that the numerical results obtained in this study agree well with the experimental results for the no and weak wave breaking cases. When strong wave breaking occurs the accuracy of the numerical simulations suffer and their agreement with the experimentally obtained responses are still acceptable.

The behaviour of the MTLD-structure system was also studied under two ground motions both numerically and experimentally. The results show that the numerical model can predict the structural vibration well under the Kobe earthquake but the accuracy suffers under the El Centro earthquake. However, it both cases the predictions of the numerical simulations are acceptably accurate. By comparing the RTHS results with each other, it was observed that even when it is optimally designed MTLD does not introduce much additional efficiency in reducing the displacements when compared to the traditional TLD.
On the other hand, for MTLDs where the $L_s$ value was not optimally selected, the MTLD response was worse than the traditional TLD response.

**Future work**

These further investigation and research areas are suggested for MTLDs:

- Instead of a single-degree-of-freedom frame as done in this study, the interaction of MTLDs with multi-degree-of-freedom structures should be studied. This will be significant for practical applications.

- Only two earthquake ground motions were used in this study. To be able to draw general conclusions a more complete ground motion ensemble should be considered.
References


Appendix A
MatLab Code for Lu’s Analytical model

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L = 0.464; h0 = 0.04; B = 0.305; rho = 1000;
w = ((pi * 9.81 / L) * tanh(pi * h0 / L))^0.5;
g = 9.81; ul = 1.002 * 10^-3;
mw = L * h0 * B * rho;

ms = 283.04; % Mass Ratio = 2%
wns = 1 * w; % Frequency ratio (alpha) = 1
fns = wns / (2 * pi); ks = ms * wns^2;
cs = 2 * ms * wns * 0.012; % Damping Ratio = 1.2%

Ls = 0.2; % Spring Distance
l = 0.1;
kw = 5000; % Linear spring stiffness
ktheta = kw * Ls^2 / 2;
m1 = 9.62; % Weight of tank with rotational spring system
I1 = 1 / 12 * m1 * (L^2) + m1 * l^2;
ctheta = 2 * (ktheta * I1) / 0.5 * 0.0002; cw = 2 * ctheta / L^2;

m = [(ms + m1) m1; m1 (m1 + I1 / l^2)];
k = [ks 0; 0 ktheta / l^2];
c = [cs 0; 0 ctheta / l^2];

dt = 0.001; tfinal = 50; t = 0:dt:tfinal;
Po = 5;
Ff=0;
Bet=0.95;
P=zeros(2,length(t));
xbd=0;
xbb=zeros(1,length(t));
for i=1:length(t)
    if i<=50000
        P(:,i)=Po*sin(Bet*wns*t(i))*[1;0];
    else
        P(:,i)=0;
    end
end

% Seismic Event%

% o_kobe0_001;
P=zeros(2,length(t));
for i=1:length(t)
    P(:,i)=-[ms+m1;m1]*acc(i)*g*0.1;
end

% Lu's model%

N=35;
deltax=L/N;
h=zeros(N+1,length(t));
v=zeros(N+1,length(t));
z=zeros(N+1,length(t));
vmax=zeros(N+1,length(t));
tao=zeros(N+1,length(t));
S=zeros(N+1,length(t));
P=zeros(1,length(t));
M=zeros(1,length(t));
ay=g*zeros(N+1,length(t));
h(:,1)=h0; v(:,1)=0; P(1)=0; M(1)=0;
h(:,2)=h0; v(:,2)=0;
alpha=0.98;
u=zeros(2,length(t));
udot=0;
udot=x=zeros(2,length(t));
udotdot=0;

% Boundary condition
v(1,:)=0; v(N+1,:)=0;

% Initial Conditions
u(:,2)=0; udot(:,2)=0;
udotdot=k*[P(:,2)-c*udot(:,2)-k*udot(:,2)];
udot=k*u(:,2)-dt*udot(:,2)+dt^2/2*udotdot(:,2);

% Integration Constants
khat=c/(2*dt)+m/dt^2;
\[ a = \frac{m}{d^2} - c/(2*dt) \]
\[ b = k - 2*m/d^2 \]
\[ \text{Phat} = \text{zeros}(2, \text{length}(t)) \]

\begin{verbatim}
for i=1:length(t)-1
  n=i
  %Wave height and Wave velocity, Force
  for j=1:N+1
    if j==1
      h(j,i)=alpha*h(j,i)+(1-alpha)*(h(j+1,i)+h(j,i))/2;
      h(j,i+1)=h(j,i)-h(j,i)*v(j+1,i)*dt/deltax;
    elseif j==N+1
      h(j,i)=alpha*h(j,i)+(1-alpha)*(h(j,i)+h(j-1,i))/2;
      h(j,i+1)=h(j,i)+h(j,i)*v(j-1,i)*dt/deltax;
    else
      h(j,i)=alpha*h(j,i)+(1-alpha)*(h(j+1,i)+h(j-1,i))/2;
      v(j,i)=alpha*v(j,i)+(1-alpha)*(v(j+1,i)+v(j-1,i))/2;
      z(j,i)=(w*g/(2*ul))^0.5*h(j,i);
      if z(j,i)<=5
        vmax(j,i)=v(j,i)/(-0.0011*z(j,i)^6+0.0169*z(j,i)^5
                                -0.0936*z(j,i)^4+0.2093*z(j,i)^3
                                -0.1181*z(j,i)^2
                                +0.0129*z(j,i)+0.5012);
      else
        vmax(j,i)=v(j,i)/(-exp(-0.0853*z(j,i)-2.2807));
      end
      if z(j,i)<=0.7
        tao(j,i)=ul*vmax(j,i)/h(j,i);
      else
        tao(j,i)=(rho*ul*w)^0.5*vmax(j,i);
      end
      S(j,i)=tao(j,i)/(rho*g*h(j,i));
      S(j,i)=alpha*S(j,i)+(1-alpha)*(S(j+1,i)+S(j-1,i))/2;
      v(j,i)=v(j,i)+(v(j+1,i)-v(j-1,i))/(2*deltax)*dt;
      v(j,i+1)=v(j,i)+(v(j+1,i)-v(j-1,i))/(2*deltax)*dt;
      udotdot(1,i)=udotdot(1,i)-g*(u(2,i)/s(j,j)-S(j,i))*dt;
    end
  end
  F(i+1)=0.5*rho*B*g*(h(N+1,i+1)^2-h(1,i+1)^2);
  M0=1/6*rho*B*g*(h(N+1,i+1)^3-h(1,i+1)^3);
  for j=1:N
    M(i+1)=M0+rho*((ay(j,i)+ay(j+1,i))/2)*B*((j*L/N-L/2)^2
                        -((j-1)*L/N-L/2)^2)/2*(h(j,i)+h(j+1,i+1))/2);
    M0=M(i+1);
  end
end

%%%%%%%%%%%%%%%%%%Newmark_Beta method%%%%%%%%%%%%%%%%%%%
deltaPhat=(P(:,i+1)-P(:,i))+((M(i+1)-M(i))/l*[0;1]
  +a*udot(:,i)+b*udotdot(:,i));
deltau=deltaPhat/khat;
deltaudot=-(gam/beta/dt*deltau-gam/beta*udot(:,i));
\end{verbatim}
+dt*(1-gam/beta/2)*udotdot(:,i));
deltauddotdot=(1/beta/(dt^2))*deltau -udot(:,i)/beta/dt-udotdot(:,i)/2/beta;
u(:,i+1)=u(:,i)+deltau;
udot(:,i+1)=udot(:,i)+deltaudot;
udotdot(:,i+1)=udotdot(:,i)+deltaudotdot;
e

%%%%%%%%%%%%%%%%%
%Calculating Structural Displacement Without MTLD%
%Using Newmark-Beta Method%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Integration Constants
kshat=ks+gam*cs/(beta*dt)+ms/(beta*dt^2);
as=ms/(beta*dt)+gam*cs/beta;
bs=ms/(2*beta)+dt*(gam/(2*beta)-1)*cs;

%Initial Conditions
uwo=zeros(1,length(t));
udotwo=zeros(1,length(t));
udotdotwo=zeros(1,length(t));
uwo(1)=0; udotwo(1)=0;
udotdotwo(1)=(P(1,1)-cs*udotwo(1)-ks*uwo(1))/ms;
for i=1:length(t)-1
deltaPhatwo=(P(1,i+1)-P(1,i))+as*udotwo(i)+bs*udotdotwo(i);
deltauwo=deltaPhatwo/kshat;
deltaudtwo=(gam/beta/dt*deltauwo-gam/beta*udotwo(i)
+dt*(1-gam/beta/2)*udotdotwo(i));
deltauddotwo=(1/beta/(dt^2))*deltauwo 
-udotwo(i)/beta/dt-udotdotwo(i)/2/beta;
uwo(i+1)=uwo(i)+deltauwo;
udotwo(i+1)=udotwo(i)+deltaudtwo;
udotdotwo(i+1)=udotdotwo(i)+deltauddotwo;
end
Appendix B
List of Symbols

\( h \): Water height in sloshing motion

\( h_0 \): Water Height in steady state

\( L \): The length of water tank

\( B \): The width of water tank

\( t \): Time

\( v \): Velocity of water sloshing

\( x_b \): Horizontal displacement of water tank

\( \theta_b \): Angle of rotation motion

\( g \): Gravity acceleration

\( S \): Slope of the energy grade line

\( \tau_b \): Liquid’s shear stress

\( \rho \): Density of liquid

\( \mu \): Dynamic viscosity of the liquid

\( \omega \): Excitation circular frequency

\( \nu \): Kinetic viscosity of the liquid

\( z \): Liquid’s dimensionless depth

\( F \): Interaction force caused by sloshing water
\( M \): Moment caused by sloshing water

\( a_y \): Vertical acceleration

\( n \): Division number

\( m_s \): Structural mass

\( k_s \): Structural stiffness

\( c_s \): Structural damping coefficient

\( u_x \): Structural displacement

\( \dot{u}_x \): Structural velocity

\( \ddot{u}_x \): Structural acceleration

\( \ddot{u}_g \): Ground acceleration

\( J \): Mass moment inertial of the tank

\( \theta_l \): Rotational angle

\( \dot{\theta}_l \): Rotational velocity

\( \ddot{\theta}_l \): Rotational acceleration

\( K_{\theta} \): Rotational spring stiffness

\( K_r \): Linear spring stiffness

\( L_s \): Linear spring distance

\( f_w \): Water sloshing frequency

\( f_s \): Natural frequency of structure
\( f \): Excitation frequency

\( \alpha \): The ratio between sloshing frequency to structural natural frequency

\( \beta \): The ratio between excitation frequency to structural natural frequency

MR: Mass Ratio between water mass to structural mass

\( \xi \): Damping Ratio