PASSIVE ACOUSTIC MONITORING OF DISSOLVING TANK ACTIVITY AND MODELLING OF UNDERWATER VAPOUR EXPLOSIONS

by

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Abstract

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Violent dissolving tank operation can pose a safety hazard, damage equipment, and in extreme cases lead to a dissolving tank explosion. This study investigates the characteristics of smelt-water interaction and the dynamics and acoustics of underwater vapour explosions.

Observations carried out at pulp mills indicate that the dissolving tank soundscape becomes more intense and more variable when the smelt stream is not being shattered. Spectral analysis reveals that the smelt-water interactions carry more energy in low frequencies.

Equations of motion describing a vapour bubble size in time are derived from first principles. The resulting equation is a modified version of the Geers-Hunter doubly asymptotic approximation, which builds upon the Rayleigh-Plesset equation. Using numerical methods, the pressure wave caused by a single smelt droplet explosion is calculated. Droplet statistics simulates the total variation of pressure inside the dissolving tank. Acoustic characteristics of the synthetic pressure waves are consistent with previous observations.
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À Noémie, merci d’être dans ma vie!
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Nomenclature

\( \gamma \)  Ratio of heat gas capacities
\( \mu \)  Dynamic viscosity
\( \nabla T \)  Temperature difference
\( \overrightarrow{dA} \)  Oriented surface area element
\( \phi \)  Velocity potential
\( \rho \)  Mass density
\( \rho_S \)  Molten smelt density
\( \sigma_{ij} \)  Stress tensor
\( \zeta \)  Specific-acoustic-impedance ratio
\( A \)  Area
\( b_i \)  Body force
\( c \)  Speed of sound
\( c_S \)  Smelt heat capacity
\( E_{ij} \)  Strain rate tensor
\( g \)  Gravitational acceleration
\( g_i \)  Gravitational field
\( h \)  Heat transfer coefficient
\( h_v \)  Latent heat of vaporization
\( k \)  Material thermal conductivity
\( L_{fs} \)  Smelt specific latent heat of fusion
\( m_S \)  Mass of smelt droplet
\( P \)  Pressure
\( P_g \)  Partial pressure due to gas
\( P_v \) Partial pressure due to vapour
\( P_{g0} \) Initial gas pressure
\( P_{obs} \) Pressure wave at external observer
\( Q \) Heat
\( Q \) Thermal energy
\( R \) Radius of bubble or fluid volume
\( r \) Radial coordinate outside the fluid volume
\( R_0 \) Initial radius
\( S \) Surface tension
\( V \) Fluid volume element
\( v_i \) Particle velocity
\( z \) Depth from surface (defined positive downwards)
Chapter 1

Introduction

1.1 The Pulp & Paper Industry

Since the 1940s, the Kraft chemical pulping process has been the leading process for producing wood pulp.[6] Chemical pulping is advantageous over other methods such as mechanical or thermal chemical pulping since it can process many varieties of wood species and can extract very fine fibres from them. Approximately half the wood is broken down into cellulose fibres to produce pulp, while the other half is dissolved with spent pulping chemicals to form black liquor, a liquid mixture of organic and inorganic components.[39] Black liquor is burnt in the recovery boiler to generate steam and energy, which can be recycled to power the plant. The combustion of black liquor also produces a molten salt called smelt.[20] Smelt flows out of the boiler through spouts and is dissolved in the dissolving tank to be converted back into pulping agents.

The interaction of hot molten smelt with the water inside the dissolving tank is violent and can result in the water undergoing a rapid phase transition causing a vapour explosion.[32] Vapour explosions are necessary to the rapid and efficient dissolution of smelt since they break the smelt droplets into smaller particles increasing the smelt’s surface area and solubility. Solidified smelt that has not been disintegrated by a vapour explosion is found accumulating at the bottom of the dissolving tank. However, unstable conditions can lead to larger vapour explosions, which pose a safety hazard and can damage equipment or cause an unscheduled boiler shutdown. A total of 34 dissolving tank explosions in North America were reported to the Black Liquor Recovery Boiler Advisory Committee (BLRBAC) in the past 45 years.[17] A timeline of such incidents can be seen in Fig. 1.1. There are most likely more dissolving tank incidents than shown in Fig. 1.1 as not all incidents are reported to the BLRBAC.

There are many different circumstances that can cause the dissolving tank to become unstable. Smelt composition can affect the viscosity of smelt, jellyroll smelt – which is very viscous – can cause the smelt streams to become inconsistent, and improper shattering of the smelt streams can cause the tank behaviour to become unpredictable. Insufficient shattering of the smelt stream causes the dissolving tank to become more violent and its soundscape more intense. This research aims to predict the dissolving tank’s behaviour through the analysis of its soundscape.
Chapter 1. Introduction

1.1.1 The Kraft Process

In addition to being an efficient method for processing various types of wood, the kraft process includes an extremely efficient and cost-effective recovery process that recycles 97% of the pulping chemicals and generates power, which can be used elsewhere in the mill. Figure 1.2 displays a summarized picture of the kraft recovery cycle. At the start of the process, trees are debarked and shredded into wood chips. The chips are then sent to the digester where a mixture of sodium hydroxide (NaOH) and sodium sulfide (Na₂S), called white liquor, converts them into wood pulp. Pulp is separated from the spent chemicals and further processed for paper making. The leftover material called black liquor, a mixture of organic and inorganic components, is mainly composed of lignin and spent pulping chemicals. Black liquor is concentrated in a series of evaporators, which can reduce its water content by 55-70%. The concentrated black liquor is sent into the recovery boiler where all the organic matter is burnt. The remaining inorganic content, which accumulates on the char bed at the bottom of the boiler, forms smelt. Smelt is primarily composed of sodium carbonate (Na₂CO₃), sodium sulfide (Na₂S), and a small amount of sodium sulfate (Na₂SO₄). Molten smelt flows out of the recovery boiler to be dissolved in water inside the dissolving tank. The resulting solution, called green liquor, is sent to a causticizing plant to be treated and converted back into white liquor.

The recovery boiler, one of the largest units in a mill, can be as large as a building. Figure 1.3 shows a schematic for one of the largest recovery boilers in the world at Jinhai Pulp & Paper, Hainan, China. Black liquor is fired into the recovery boiler using multiple spray guns and the organic components of the liquor are burnt in a reducing atmosphere leaving molten smelt to collect at the bottom of the boiler. Molten smelt continuously flows out of the boiler through spouts and falls into the dissolving tank.
Figure 1.2: Diagram of the kraft recovery cycle.[39]

Figure 1.3: Adapted schematic from Metso Power illustrating a recovery boiler.[39] The dissolving tank has been highlighted using a red oval.
1.1.2 The Dissolving Tank

Molten smelt flows out of the boiler at temperatures ranging from 800°C to 850°C through several spouts at a flow rate of 0.7 to 1.3L/s.[20] Smelt is highly fluid when heated to temperatures above its freezing point of approximately 750°C. Below the freezing point, it becomes very viscous and eventually solidifies.[38] Figure 1.4 displays pictures of an uncovered smelt spout in operation. A U-shaped spout as seen in Fig. 1.4 is the most commonly used spout design.

![Figure 1.4: Pictures of uncovered smelt spouts. Panel (a) shows a view from the front of the hatch and panel (b) is a top view of the spout.][18]

In order to minimize the intensity of the violent smelt-water interactions the smelt stream is broken into droplets using impinging high pressure steam jets called shatter jets. The smelt droplets then fall for 2-3 meters before reaching the dissolving tank. Shattering the smelt stream into smaller droplets also facilitates a more rapid and complete dissolution of the smelt in the dissolving tank. A sketch of the shattering process can be seen on Fig. 1.5.

As a molten smelt droplet enters the dissolving tank it transfers heat to the water in the tank, causing the water to vaporize. In some cases, when the amount of heat transferred from the smelt droplet is large, water will vaporize violently. A rapidly expanding vapour bubble forms producing a shock wave that breaks the smelt droplet into finer particles. The violent nature of the vapour explosions are what cause the dissolving tank to rumble. Typical dissolving tanks are very loud and can cause the recovery boiler building to shake.
1.2 Motivation

As mentioned in the previous section, the dissolving tank activity is very violent and in certain situations can pose a safety hazard to the mill workers or cause equipment damage. In less severe cases, upset conditions in the dissolving tank can lead to an unscheduled boiler shutdown, which impacts mill production. The constant flow of molten smelt makes the environment surrounding the dissolving tank too hot and hazardous for most monitoring equipment such as video cameras or flow meters to be set up. However, experienced mill operators claim that they can deduce anomalous tank behaviour simply by listening to its rumble. When the acoustic signature of the dissolving tank activity is recognized by mill workers as sounding atypical they look for the source of the discrepancy. A scientific study is required to have a better understanding of the acoustic behaviour of the dissolving tank.

This work could be developed further in the implementation of an automatic monitoring system that would allow a constant real-time tracking of the tank’s activity and alerts could be sent to the control room without requiring the constant presence of an operator in the vicinity of the dissolving tank. Before such a system can be created, an understanding of the different acoustic emissions coming from the dissolving tank and its surroundings is required. The most important sound source to study is the rumble caused by the many vapour explosions. It must be characterized and isolated as much as possible from all other acoustic signals originating from the machinery, pumps, and agitators surrounding the tank. Also, the acoustic profiles associated with normal and dangerous behaviour must be analyzed and discriminated such that they can be automatically identified in the future.
To have a more intuitive understanding of the acoustics produced by the smelt-water interactions inside the dissolving tank, it is important to understand the fundamental dynamics involved in underwater vapour explosions and rapid phase transitions.

1.3 Research Objectives

The first part of this study will be to provide insight on the physical phenomena in effect through observations. Recordings from different pulp & paper mills provide sample acoustic signals whose characteristics can be determined. Phenomenological relations between the dissolving tank activity and smelt characteristics, which will guide the creation of a theoretical model, are to be determined. Observations in the field also explore the possibility of designing a system that could characterize a dissolving tank’s operating conditions based only on the acoustic signals produced within the tank.

The second part is a theoretical study on the dynamics of bubbles under water. Equations of motion relating the bubble radius to time will be derived from first principles. These equations will be solved numerically to model the behaviour of a vapour explosion under water. A special attention is directed to the acoustics involved in the formation, expansion, and collapse of underwater cavities. Combining a single bubble model with previously observed droplet statistics, a synthetic soundscape of many smelt-water interactions will be created.
Chapter 2

Observations

Observations were carried out in several pulp and paper mills as well as in a laboratory setting.

2.1 Pulp & Paper Mills

In order to capture a complete soundscape from an operating dissolving tank and its environment, acoustic recordings were taken at a number of different pulp and paper mills located in Canada. Several recordings were obtained from each mill: Dec. 18 2013 (Mill A), Aug. 26-27 2014 (Mill B) and Aug. 28 2014 (Mill C). Subsequent recordings from Mills B and C were taken on May 5-8 2015.

2.1.1 Equipment

Sound was recorded using a PCB 130E20 pre-polarized, condenser microphone with a frequency range of 20 to 20,000 Hz (±5 dB) and a PCB VO622A01 high frequency accelerometer with a frequency range of 3 to 9000 Hz (±3 dB). Both measuring instruments were connected to a PCB 480B21 3-channel signal conditioner before being stored by a notebook computer. An external PreSonus AudioBox USB sound card handled multiple devices recording simultaneously on separate channels and allowed the adjustment of gain on each channel before being digitized. A Larson Davis SoundTrack LxT1 sound level meter correlated the voltage recordings from the microphone to sound pressure levels. The free open source software Audacity saved all recorded sound signals to files.

2.1.2 Methodology

The first set of measurements from mill A were taken using the microphone and accelerometer described in section 2.1.1, recording various mill operations consecutively. During each recording, the sound level meter provided an independent measurement and a reference for the absolute sound pressure level. For mills B and C, both the microphone and the accelerometer as well as the sound level meter were recording simultaneously with information from each device being saved to a different channel.
Figure 2.1 illustrates the setup put in place during the mill recordings. Measurements were taken with the microphone mounted on a tripod and elevated off the ground and positioned away from any major surface in order to avoid sound reflections and ground effects as much as possible. To ensure that both devices were monitoring the same acoustic signals, the sound level meter was also attached to the tripod. In each mill the mounted microphone and sound level meter were located approximately 2m away from the dissolving tanks and pointed towards its centre.

A magnetic coupler fixed the accelerometer to the exterior wall of the dissolving tank at a height of approximately 1.75m off the ground. This height corresponds to the level of the green liquor inside the dissolving tank and also the elevation at which the vibrations were the most intense. Positioning the accelerometer along the wall of the dissolving tank, away from any extraneous noise sources, allowed for the capture of the strongest signal. The agitators were the main source of noise as they produced a constant large amplitude vibration. At mill C, the non ferromagnetic material used for the dissolving tank walls necessitated the accelerometer be coupled to a protruding pipe that had been sealed with metal plate.

Continuous recordings were taken at each mill to capture the soundscape of a typical active dissolving tank. The conditions for each recording were as described above for all three mills. In order to simulate malfunctioning shatter jets, a mill operator adjusted the steam flow rate of the shatter jets. Insufficient shattering of the smelt streams is a common problem in pulp & paper mills that leads to violent dissolving tank activity and a more intense acoustic signature.

The sound signals were analyzed to capture the intensity and frequency characteristics of the sound signal during both normal operation and operation with inactive shatter jets. Bin widths of one second were chosen when windowing the signal since they are small enough to preserve the time varying characteristics of the signal describing the dissolving tank’s activity while reducing the variance of the signal.
and avoiding the introduction of unnecessary noise. The choice of bin width becomes particularly important for an automatic monitoring system based on sound intensity. A system based on the detection of a signal exceeding a threshold value must manage a tradeoff between the rapid detection of change and the presence of short-time variations that can generate false alarms. Window sizes on the order of one second offer the quickest indication of an increase in dissolving tank activity without triggering false alarms.

### 2.1.3 Results

When a mill operator shuts down some or all of the shatter jets in order to simulate upset conditions, a change in the sounds produced by the dissolving tank can immediately be discerned by the human ear, the most notable feature being a louder rumbling. The intensity of the acoustic signal picked up by both the microphone and the accelerometer is a good primary indicator of changing dissolving tank conditions. Figure 2.2 shows the intensity averaged over 1 second bins for a recording during which all four shatter jets were turned off at mill A. The acoustic measurement presented in Fig. 2.2 was recorded with the microphone.

![Figure 2.2: Mill A: Sound intensity recorded by the microphone while shutting off the shatter jets. The process of turning the jets off by a mill operator began at approximately 40s. Once the shatter jets were completely off, a clear (~3dB) increase in acoustic intensity emerges. The transition period between 40s and 80s is attributed to the time taken by the mill operator to shut down all the shatter jets.](image)

The mill operator began shutting down all four jets at approximately the 40s mark. However, the time it took to complete the task is approximate. The sound signal in Fig. 2.2 has been split into three distinct regions. The left region (blue) represents the time period during which all shatter jets were on and the dissolving tank conditions are considered normal. The middle region (orange) is the estimated
time it took for the mill operator to completely shut down the shatter jets. Since mill observations aim to compare the soundscape of the dissolving tank with fully operational shatter jets to that of the tank without shattering, no discussion is provided for the transition period between 40s and 80s. The use of the steady state cases when the shatter jets are active or inactive offer more consistent means of comparison and lead to a more meaningful analysis. The region on the right (red) is a time period during which the four steam jets were off and the smelt stream was not being shattered properly. As a result, the smelt-water interactions were more violent and the average intensity of the sound signal increased by approximately 3dB, corresponding to a doubling of the acoustic energy.

Similar observations performed at mills B and C yielded the results shown in Figs. 2.3 and 2.4 respectively. In mill B, the operator adjusted the steam flow to all four shatter jets for brief (20-30s) intervals. During the first interval, the steam flow was reduced. During the second interval, the shatter jets were closed completely. In mill C, the operator closed one of four steam jets for a short (20-30s) period, repeating the operation twice using a different jet each time. Both the microphone and the accelerometer were recording simultaneously. For safety reasons, the operator of each mill only shut down the steam jets for short periods of time.

![Figure 2.3: Mill B: Sound intensity recorded by the microphone (top), accelerometer (middle), and the sound level meter (bottom) while shutting off the shatter jets. There is a clear (>3dB) increase in signal intensity during the ≈20s intervals when a mill operator shut down the shatter jets.](image-url)
Figure 2.4: Mill C: Sound intensity recorded by the microphone (top), accelerometer (middle), and the sound level meter (bottom) while shutting off single shatter jets. A change in intensity could not be easily identified so the intervals during which an operator turned off a shatter jet were highlighted in red.

At mill B shown on Fig. 2.3 the effect of turning the jets off can clearly be seen on the signals intensity. During the ∼20s intervals where the shatter jets were turned off, the average acoustic energy increases by more than 3dB for both the microphone and accelerometer recordings. However, this increase in intensity is not as obvious for mill C, as seen in Fig. 2.4. The time intervals where the shatter jets were turned down are highlighted for greater readability. There are many factors that could explain mill C’s signal being less detectable. Not only is mill C the loudest of the three mills, an analysis presented in table 2.1 reveals that the sound signal also has a standard deviation of approximately 2dB. This variability, which is larger than the observed increase in intensity, could easily mask the overall energy change caused by the termination of one of four shatter jets.

<table>
<thead>
<tr>
<th></th>
<th>Mean energy</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mill A (Jets ON)</td>
<td>91.1 dB</td>
<td>0.5 dB</td>
</tr>
<tr>
<td>Mill A (Jets OFF)</td>
<td>94.3 dB</td>
<td>0.7 dB</td>
</tr>
<tr>
<td>Mill B (Jets ON)</td>
<td>97.5 dB</td>
<td>0.3 dB</td>
</tr>
<tr>
<td>Mill B (Jets OFF)</td>
<td>101.5 dB</td>
<td>0.8 dB</td>
</tr>
<tr>
<td>Mill C (Jets ON)</td>
<td>106 dB</td>
<td>2 dB</td>
</tr>
<tr>
<td>Mill C (Jets OFF)</td>
<td>107 dB</td>
<td>2 dB</td>
</tr>
</tbody>
</table>

Table 2.1: Numerical comparison of sound intensity produced by the dissolving tank of different mills operating under normal conditions as well as simulated upset conditions.
Comparing a short 5 minute segment of recorded normal operation for each mill reveals the differences between them. Figure 2.5 shows a steady state recording for which the intensity values were normalized using a sound level meter to indicate the differences between the three mills.

![Figure 2.5: Sound intensity recorded by the microphone for an interval of stable mill operation. All intensity values were converted to dB SPL values of the same scale according to an A-weighted curve.](image)

As an added indicator to sound intensity, a frequency spectrum can be used to distinguish between the different operating conditions. When the shatter jets are turned off the acoustic energy increases and the power spectrum shifts. Figure 2.6 shows that when the shatter jets are turned off, energy in the lower frequencies (<1000Hz) increases and energy in the higher frequencies decreases. It is important to note that although the acoustic energy increased for some frequencies and decreased for others, Figs. 2.2 and 2.3 show that the overall energy increased. It is speculated that the decrease in high frequencies is associated to the absence of shatter jet noise when they are turned off. Laboratory experiments discussed in section 2.2 show that the spectrum associated to typical shatter jet nozzles lie primarily in the high frequencies.
Figure 2.6: Power spectrum for mill A’s dissolving tank activity while all shatter jets were turned on (blue) and while all shatter jets were turned off (red). When the steam jets are not operating and shattering the smelt stream, low frequencies (<1000Hz) gain in intensity and higher frequencies (>1000Hz) lose energy.

2.2 Shatter Jets

When monitoring the dissolving tank activity using acoustics, many extraneous sources of noise must be isolated, one in particular coming from the high pressure steam jets produced by the shatter jets. The direct effect the shatter jets have on the smelt-water interaction soundscape poses an additional difficulty in segregating the two sound sources. The dissolving tank activity changes when the shatter jets are turned down or off so the sounds produced by shatter jets are hard to isolate in a mill environment. However, a laboratory setup, made it possible to record the operation of a single shatter jet in free space. To account for design variability between various mills, five different nozzle types were tested at distinct flow rates. A picture of the nozzles used as well as their associated jet profiles can be seen in Fig. 2.7.

All nozzle types were recorded blowing air at a constant volumetric flow rate of 10 standard cubic feet per minute (scfm). The frequency spectrum of each recording was compared to the baseline noise in the lab. A comparison of each nozzle can be seen in Fig. 2.8. Most of the energy produced by the steam jets lie in the high frequencies (>1000Hz). These frequencies can be distinguished from those produced by the smelt-water interactions, which are typically of lower frequency (<1000Hz). See section 2.3. The same experiment was done at a higher volumetric flow rate of 15 scfm for all nozzle types except for the full cone nozzle. For a similar flow rate, the air exit pressure of the full cone nozzle is greater than that of the other nozzles and for safety reasons the experimental apparatus could not support such high pressures. A comparison of the spectra of the air jets compared to ambient noise can be seen in Fig. 2.9.
Figure 2.7: Pictures of the five different shatter jet nozzles that were tested and their projected steam jet profiles. Image adapted from [27].
Figure 2.8: Five different shatter jet nozzles are tested at the same airflow of 10 standard cubic feet per minute (scfm). The results are compared to the ambient noise level in the lab.

Figure 2.9: Four different shatter jet nozzles are tested at the same airflow of 15 standard cubic feet per minute (scfm). The results are compared to the ambient noise level in the lab. For safety reasons, the full cone nozzle could not be used at 15 scfm.
2.3 Cavitation Acoustics

Sound production due to the rapid creation and collapse of underwater cavities can be examined at a smaller scale. A prolonged experiment involving many molten smelt droplets interacting in a small scale water tank is too dangerous to be conducted safely. However, a simple cavitation experiment such as boiling water can be carried out to study the acoustic interaction of several vapour bubbles.

Several types of boiling occur when heating water.[11] A rough boiling curve on Fig. 2.10 separates the different regimes of boiling depending on the temperature difference between the surface of the heating material and the saturation temperature of water.

For excess temperatures of up to approximately 5°C above saturation point, convective currents cause the superheated liquid to flow and evaporate at the free surface. Natural convection boiling (region A) is the first type of boiling where vapour is formed at the surface. After a temperature difference exceeding ~ 5°C, vapour bubbles start to form at nucleation sites underwater. During the first part of nucleate boiling (region B) many small vapour bubbles form and collapse very rapidly while remaining underwater.[3, 41] A further increase in the temperature difference intensifies the heat flux between the heated surface and the liquid to its maximum value. Vapour bubbles in region C are larger and rise to the free surface.

During transition boiling (region D) an unstable vapour film forms around the heated surface and inhibits a further increase of the heat flux.

If the surface is heated even further, the vapour film becomes stable and continuously isolates the heated surface from the liquid. This insulation reduces the heat flux between the two materials until a
Chapter 2. Observations

local minimum called the Leidenfrost point. This regime (region E) is called film boiling, which can be seen when molten smelt droplets first enter a tank of water.

The first type of nucleate boiling (region B) can be studied as an approximation of the vapour explosion phenomenon since vapour cavities are formed, grow, and collapse within the surrounding liquid, as seems to be the case with observations of single and multi droplet smelt-water interactions. This phenomenon can easily be observed in a laboratory environment involving a VWR aluminium hotplate and a 250mL glass beaker half filled with tap water brought to boiling temperature previously to remove the dissolved air in the liquid.[3] Water heated through the multiple boiling regimes and the microphone described in section 2.1.1 provided an audio recording of the process. A spectral analysis presented in Fig. 2.11 reveals that the sounds produced by the rapid creation and collapse of vapour cavities also carries the same frequency components as the resonance of the beaker after being struck. Pressure waves caused by cavitation prompt the container to vibrate. A dissolving tank will therefore have a signature resonance as a consequence of its physical characteristics. Since dissolving tanks may vary in size and material, this signature resonance will not be the same from one mill to another.

![Figure 2.11: Power spectrum of water boiling in a 250mL glass beaker (top). Two outstanding frequency peaks have been identified with red circles. In a separate measurement, the glass beaker was struck and its ringing was recorded. The bottom panel shows a power spectrum of the resulting ringing only. Both peaks, which were identified in the top panel, correspond to the resonance frequencies of the beaker. It is important to note that the beaker was left untouched during the measurement of sound caused by boiling. The presence of resonant peaks is due solely to the action of vapour bubbles.](image)

The environment in which vapour explosions occur plays an important role in the soundscape associated with such events. Sound waves created by vapour explosions are not only transformed by passing through media of varying densities but also added to the waves produced by the vibration of the dissolving tank itself.
2.4 Summary

In this chapter, observations of small and large scale physical phenomena provided empirical correlations that can be used to guide the formulation of a theoretical model describing the sound production in vapour explosions caused by smelt-water interactions.

According to mill observations, poor shattering conditions result in the overall increase of the sound intensity as well as an increase in the variability of the signal. The mill recordings also indicate that a major part of the acoustic energy lies in the low frequencies (< 1000 Hz).

Higher frequency components were found to lose energy when the shatter jets were turned off. These frequencies are hypothesized to correspond to the noise produced by the steam jets. Audio recordings of various types of shatter jet nozzles at different flow rates revealed that the energy of the sounds produced by the jets lies primarily in the frequencies above 1000 Hz.

A laboratory experiment on cavitation using boiling water also indicated that the container plays an important role in the modification of the sound field. Cavitation of water vapour during the creation of bubbles can be a small scale approximation of underwater vapour explosions. Bubbles created in the early stages of boiling collapse before reaching the surface and send a pressure wave through the glass beaker that causes it to resonate. It can be assumed that a similar situation occurs for larger explosions in a dissolving tank.
Chapter 3

Theory & Modelling

In order to reliably distinguish the acoustic characteristics of vapour explosions, a theoretical approach must be considered. An understanding of the various physical phenomena involved in the smelt-water interactions will allow the prediction of a particular acoustic outcome of one or multiple smelt droplets falling into the dissolving tank. This chapter describes the various stages of the interaction between a molten smelt droplet and water.

Figure 3.1: Hot steel ball experiencing the Leidenfrost effect. In panel (a), the ball is completely isolated from the water by a vapour film. After the steel cools, the vapour film collapses as seen in (b) and the heat transfer between the ball and the water is significantly higher. Reprinted figure with permission from [40].

When the smelt droplet first comes into contact with water, a vapour film forms around it and prevents direct contact with the liquid. This phenomenon, known as the Leidenfrost effect occurs when an extremely hot substance comes into contact with a relatively cool and volatile liquid. If the temperature of the hot material is significantly higher than the boiling point of the cold liquid, the latter
immediately enters the film boiling regime.[4, 5] This effect can be observed when a droplet of water placed on a hot pan floats around for an extended period of time instead of simply boiling. It is also why liquid nitrogen hovers in little beads when spilled on a table or on the floor. The Leidenfrost effect is illustrated in Fig. 3.1, where a steel ball was heated to 250°C and dropped into a fluid primarily composed of perfluorhexane ($C_6F_{14}$), which has a boiling point of 56°C.[40]

The vapour film formed between the two substances insulates them thermally and greatly reduces the heat flux from the droplet.[42] As the droplet loses heat to its surrounding environment, its temperature is no longer high enough to sustain film boiling around it and the vapour film isolating the droplet from the cooler liquid becomes unstable. It can no longer completely isolate the smelt from the surrounding liquid and a contact between the two is made possible. There are different models describing the vapour film collapse mechanism.

A first model developed by Kim and Corradini (1988)[21] suggests that upon the collapse of the vapour film, the water would form jets that impinge upon the hot droplet. The water having penetrated the molten material undergoes a rapid vaporization and the violent expansion of gas causes a vapour explosion. The expanding vapour inside the droplet causes the droplet to break apart and come into contact with more water creating a chain reaction that disintegrates the droplet. The Kim and Corradini model is illustrated in Figure 3.2.

![Figure 3.2: Kim-Corradini model for molten metal droplet in water.][22] 1. A stable vapour film completely isolates the molten material from the surrounding water. 2. As the vapour film collapses, water jets penetrate the droplet. 3. Water trapped inside the droplet is subjected to a rapid heat transfer and becomes vapour. 4. The water vapour expands and fragments the droplet. Image adapted from Koshizuka 1999.[22]

Another model, by Ciccarelli and Frost (1994)[8], argues that an unstable vapour film is no longer uniform and that parts of the film may break to allow a contact between the liquid and the smelt droplet. Nucleate boiling occurs where the film is thin or non-existent and the regions where film boiling is weak are lower in pressure and cause the molten droplet to deform creating spikes protruding from it. The rapid phase transition of the surrounding liquid causes a similar reaction to that in the Kim and Corradini model, breaking the droplet apart as illustrated in Figure 3.3.
For this project, either model can be used since they both result in the rapid expansion of a vapour bubble that annihilates the smelt droplet. In order to predict the acoustic profile of vapour explosion events, it is the large bubble resulting from the rapid phase transition whose expansion and collapse must be characterized.

3.1 Cavitation and Bubble Dynamics

The rapid expansion and collapse of a large vapour bubble creates a pressure wave that propagates from the position of the droplet. In order to predict the dissolving tank’s total pressure profile using sound data, it is important to understand how such a sound originates.

A gas and vapour bubble caused by the rapid phase transition of water that has come in contact with molten smelt can be modelled as a spherical bubble and will obey the laws of underwater bubble dynamics. The Rayleigh-Plesset equation, sometimes referred to as the Lamb equation of motion, is one of the earliest methods for describing the behaviour of a submerged gas bubble.[23, 30] This approach is a simple analytical model that is derived from conservation laws. However, it does not include energy loss under any form and the bubble’s behaviour is not accurately described. The Geers-Hunter approximation, often used to describe the shock propagation of underwater explosions, takes into account the compressibility of both the bubble and the unbounded surrounding liquid. Wave effects are made possible in the compressible fluids and the Geers-Hunter model provides a more realistic description of the bubble’s expansion and collapse.[14] A heat loss function is added to the Geers-Hunter approximation to further specialize the model to the case of smelt induced vapour explosions, where the bubbles rapidly condense back into the surrounding liquid.

3.1.1 Derivation of the Rayleigh-Plesset Equation

Starting from first principles, a system of equations of motion governing a vapour bubble’s behaviour can be derived. The Rayleigh-Plesset equation works under the assumptions that the bubble is irrotational and fixed about its centre such that only radial movement is allowed. The bubble’s gas and vapour contents are held constant and its evolution in time is assumed to be adiabatic. The surrounding fluid is assumed to be incompressible and acts as an infinite domain whose pressure and temperature are kept constant.[7] The liquid is also assumed to be either Newtonian or inviscid. This derivation of the Rayleigh-Plesset equation is loosely based on [12].
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For simplicity, index notation will be used throughout the following derivation. Recall that a vector has the form

\[ \mathbf{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z} = v_i \]  

(3.1a)

for any vector \( \mathbf{v} \) and that the divergence operator is expressed as

\[ \text{div} \equiv \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \equiv \frac{\partial}{\partial x_i} \hat{x}_i \]  

(3.1b)

Starting from the law of conservation of momentum, it follows that, for a given volume of fluid, the rate of change in momentum is equal to the sum of surface forces and the sum of volume forces. The equation

\[ \frac{\partial}{\partial t} \int_V \rho \, dV \, v_i = \int_S \rho v_j \cdot (-\mathbf{n}) \, dA \, v_i + \int_S \mathbf{n}_j \sigma_{ij} \, dA + \int_V b_i \, dV \]  

(3.2)

describes the conservation of momentum of a fluid volume element \( V \) with mass density \( \rho \) and particle velocity \( v_i \). The left hand side of the equation represents the rate of change of momentum for a volume element \( dV \). On the right hand side, the fluid’s mass flux term and the stress tensor \( \sigma_{ij} \) act on the surface element \( dA \). The last term of the right hand side is the sum of body forces \( b_i \) that act on the entire volume of fluid. The vector normal to the fluid’s surface is written as \( \mathbf{n} \).

Applying the divergence theorem to its surface forces, Eq. 3.2 can be rewritten as

\[ \frac{\partial}{\partial t} \int_V \rho \, dV \, v_i = -\int_V \frac{\partial}{\partial x_j} (\rho v_i v_j) \, dV + \int_V \frac{\partial}{\partial x_j} \sigma_{ij} \, dV + \int_V b_i \, dV \]  

(3.3)

Rearranging the terms in Eq. 3.3 gives

\[ \int_V \left( \frac{\partial}{\partial t} (\rho v_i) + \frac{\partial}{\partial x_j} (\rho v_i v_j) - \frac{\partial \sigma_{ij}}{\partial x_j} - b_i \right) \, dV = 0 \]  

(3.4)

No constraints have been put on the size or shape of the fluid volume. Since equation 3.4 must hold for any arbitrarily defined volume element, the integrand must be equal to zero. The chain rule is applied to expand the terms of the integrand in Eq. 3.4 and the negative terms are brought to the right hand side of the equation.

\[ v_i \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} + v_i \frac{\partial (\rho v_j)}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} + b_i \]  

(3.5)

Since the mass of the bubble is assumed to be a conserved quantity, the first and fourth term in Eq. 3.5 are cancelled by the continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]  

(3.6)

which states that if no mass is generated nor removed, the divergence of the flux is equal to the negative rate of change of the mass density. Simply put, mass leaving the volume element is balanced by a decrease in its density and mass entering the volume contributes to an increase in density.
The remaining volumetric forces are defined as

\[ b_i = \rho g_i \]  

(3.7a)

where \( g_i \) is the oriented gravity vector taken to be positive in the downward \( z \) direction. The bubble is assumed to be sufficiently small that the body forces act uniformly on its entire volume making both \( \rho \) and \( g_i \) constants.

The Cauchy stress tensor can be written as

\[ \sigma_{ij} = -P \delta_{ij} + 2\mu E_{ij} \]  

(3.7b)

where \( P \) is the hydrostatic pressure, \( \delta_{ij} \) is the kronecker delta, \( \mu \) is the dynamic viscosity of the bubble’s fluid, and \( E_{ij} \) is the strain rate tensor, which is defined as

\[ E_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]  

(3.7c)

The trace of the stress tensor balances the normal forces at the surface of the fluid volume element.

\[ -\sigma_{ii} = P_v + P_g(t) - \frac{2S}{R} \]  

(3.8)

where \( R \) is the radius of the volume element, \( S \) is the surface tension, \( P_v \) is the partial pressure due to vapour, and \( P_g \) stands for the partial pressure due to gas. At equilibrium, the pressure inside the bubble pushing outwards is cancelled by its surface tension. The partial pressure of gas is given by:

\[ P_g(t) = P_{g0} \left( \frac{R_0}{R(t)} \right)^{3\gamma} \]  

(3.9)

where \( P_{g0} \) is the initial gas pressure and \( \gamma \) is the ratio of specific heats for the gas. Since the bubble’s evolution happens at a very short scale and no heat transfer is possible, \( \gamma \) is taken as an adiabatic constant.

Rearranging the terms in equations 3.7b and 3.8 gives rise to the following expression for pressure:

\[ P(R, t) = P_v + P_{g0} \left( \frac{R_0}{R(t)} \right)^{3\gamma} - \frac{2S}{R} + 2\mu E_{ij} \]  

(3.10)

Plugging Eq. 3.7a, Eq. 3.7b, and Eq. 3.10 into 3.5 gives the Navier-Stokes equation with the assumptions that the volume of fluid is compressible and has spherical symmetry.

\[ \rho \frac{\partial v_i}{\partial t} + \rho v_j \frac{\partial v_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \rho g_i \]  

(3.11)

If the fluid volume is taken as a gas bubble submerged in a liquid, a waveless model can be used to find the Rayleigh-Plesset Equation. Since the liquid outside the bubble is assumed to be irrotational and incompressible, an equation of motion can be found for the radial position of a point particle outside the bubble’s radius.
Figure 3.4 illustrates the position and velocity vector of an arbitrary point $r$ outside the bubble radius. Since the exterior liquid is incompressible and mass must be conserved,[24] the flow velocity of a particle at a point $r \geq R(t)$ is given by

$$v(r, t) = \dot{R} \frac{R^2}{r^2}$$  \hfill (3.12)

Figure 3.4: Diagram of an arbitrary point $r$ in the infinite liquid domain outside a bubble of radius $R(t)$. The particle velocity at $r$ is radial due to the irrotational nature of the liquid. Image adapted from [12].

Plugging Eq. 3.12 into Eq. 3.11 and rewriting the terms in polar coordinates gives

$$\ddot{R} \frac{R^2}{r^2} + 2 \dot{R} \frac{R^2}{r^2} \left[ \frac{R^4}{r^2} - \frac{R^4}{r^5} \right] = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_i$$  \hfill (3.13)

Eq. 3.13 is then integrated with respect to $r$ and the pressure at a point far away from the volume’s centre is assumed to be given and is denoted $P_\infty$. The result of this integration is evaluated at $r = R$ to give

$$\ddot{R} R + 3 \frac{2}{2} \dot{R}^2 = \frac{P(R, t) - P_\infty - \rho g z}{\rho}$$  \hfill (3.14)

where $z$ is the bubble depth. For a greater level of realism, the bubble’s vertical position should be expressed as a function of time since the volume of lower density tends to rise towards the surface. However, in this study, since the vapour explosion and the bubble’s collapse happen during a very short time interval (<20 ms) the generated vapour bubble does not have time to rise significantly and $z$ can be held constant as an approximation of the bubble’s position. The initial depth of the bubble is assumed to be 1m below the water surface. This depth allows the underwater vapour explosion to be sufficiently far away from the surface to ignore boundary effects and the possibility of the bubble coming into contact with the outside air. Deeper initial positions only affect the pressure that is exerted on the bubble. A 1m depth is not an unrealistic approach since laboratory experiments have shown that droplets surrounded by a vapour film can last up to a few seconds before the film collapses[19] and research by Vakarelski et al. has shown that through the reduction of the drag coefficient, the Leidenfrost effect increases the terminal velocity of a superheated object dropped in a volatile liquid.[40] With a downward velocity of approximately 1m/s,[19] a smelt droplet could easily travel 1m before a vapour explosion occurs.
Finally, replacing $P(R,t)$ with the expression given in Eq. 3.10 yields the ordinary differential equation known as the Rayleigh-Plesset Equation:

$$\ddot{R}R + \frac{3}{2} \dot{R}^2 + 4\mu \frac{\dot{R}}{R} + \frac{2S}{R} = \frac{P_v + P_{\infty} \left( \frac{R_0}{R(t)} \right)^{3\gamma}}{\rho} - P_\infty - \rho g z$$

(3.15)

Equation 3.15 has been highlighted since it is the equation used to generate the plot in Fig. 3.5.

An underwater rapid expansion can be simulated by increasing the internal bubble pressure. Equation 3.15 was solved numerically to produce a time history of the bubble’s radius. Figure 3.5 illustrates the solved Rayleigh-Plesset equation of motion. The non-linear behaviour of the bubble can easily be observed during the expansion and collapse cycles through which it evolves. The regions of particular interest are the troughs, or the points at which the bubble’s radius is minimal. Although the bubble itself is at its smallest, the acceleration of the bubble’s surface is largest in the cycle. The generated pressure profile is linearly dependent on the bubble’s volume acceleration and is most significant at the bubble’s smallest size. While Eq. 3.15 produces a fair approximation of a bubble’s behaviour at early stages of the rapid pressure increase, the fact that each subsequent oscillation has the same amplitude is not realistic. The pressure waves of a vapour explosion are only released for a short period of time before decreasing in intensity and dying off. A more complete model must be developed to get a better idea of the acoustic energy released during the expansion and collapse cycles.
3.1.2 Derivation of the Geers-Hunter Doubly Asymptotic Approximation

Instead of using a waveless model as derived in section 3.1.1, a doubly asymptotic approximation (DAA) will be used to incorporate acoustic effects in the bubble as well as in the liquid.[30, 15, 16, 14] The Geers-Hunter DAA is often used to model the effects of underwater explosions caused by explosive detonations or torpedoes.[30, 28, 33, 31] Albeit at a much smaller scale, the GH-DAA can be used to describe the explosive nature of the expansion of water vapour following a rapid phase transition. The Geers-Hunter DAA does not assume incompressible fluids either inside out outside the bubble.

For simplicity in performing spatial integrals, the flow velocity is expressed as the gradient of the velocity potential $\phi$. Since in the Geers-Hunter approach the liquid is still considered to be irrotational the curl of the flow velocity is equal to zero

$$\nabla \times \mathbf{v} = 0 \quad (3.16)$$

The flow velocity can therefore be expressed as the gradient of a scalar field:

$$\mathbf{v} = \nabla \phi \quad (3.17)$$

After substituting for $\mathbf{v}$ in Eq. 3.11 using Eq. 3.17, the modified equation can be integrated spatially to give Bernoulli’s equation. Once again, the conditions at infinity for pressure are given. Two domains are used to describe the behaviour of the surface of a bubble. The boundary between the gas bubble and the liquid surrounding is related to both a liquid particle and a gas particle. Therefore, two systems of equations are used:

$$\dot{\phi}_l = \frac{1}{2} (\nabla \phi_l)^2 - \frac{1}{\rho_l} \left[ P_l - (P_\infty + \rho_l g z) \right] \quad (3.18a)$$

$$\dot{\phi}_b = \frac{1}{2} (\nabla \phi_b)^2 - \frac{1}{\rho_b} \left[ P_b - (P_{b0} + \rho_b g (z - z_{COM})) \right] \quad (3.18b)$$

where a subscript $b$ denotes the domain inside the bubble and the subscript $l$ denotes the infinite exterior liquid domain. $P_{b0}$ is the uniform internal pressure of the bubble and $z_{COM}$ indicates the vertical position of the bubble’s centre of mass.

External pressure terms are introduced as a substitution of variables to reduce the number of terms in the derivation.

$$P_{li} = (P_\infty + \rho_l g z) \quad (3.19a)$$

$$P_{bi} = (P_{b0} + \rho_b g (z - z_{COM})) \quad (3.19b)$$

Because of spherical symmetry the particle velocity is only expressed in the radial direction:

$$\nabla \phi = \dot{\phi} = \dot{R} \quad (3.20)$$

Equation 3.20 is substituted into Eqs. 3.18a and 3.18b and the resulting equations are solved for $P_l$ and $P_b$ respectively. Since the pressure at the surface of the bubble must be balanced on both sides, $P_l$ and $P_b$ must be equal. The surface tension is ignored since its effect is negligible for underwater vapour
Equating both solutions from Eqs. 3.18a and 3.18b gives

\[ P_l + \rho_l \left( \frac{1}{2} \dot{R}^2 - \dot{\phi}_l \right) = P_{bi} + \rho_b \left( \frac{1}{2} \dot{R}^2 - \dot{\phi}_b \right) \]  

(3.21)

Two limit cases are considered for the gradient of the velocity potential. The waveless (incompressible) model gives a relation between the velocity potential and its normal derivative

\[ \phi'_l = -R^{-1} \phi_l \text{ for the exterior fluid and } \phi'_b = 0 \text{ for the internal fluid.} \]

When considering wave effects, Sommerfeld’s radiation condition relates the normal derivative to the partial derivative with respect to time \( \phi' = -c^{-1} \phi_l \) in both the external and internal fluids.[10, 16, 29] The speed of sound \( c \) in each fluid is dependent on the fluid’s pressure and density. Combining these two limits gives an approximation that is asymptotic towards the exact solution both at early and late times:

\[ \phi'_l = -R^{-1} \phi_l - \frac{1}{R} \left( \rho_b \phi_b - \rho_l \phi_l \right) \]  

(3.22a)

\[ \phi'_b = -c^{-1} \phi_b \]  

(3.22b)

Since the particle velocities at the boundary of the bubble must be equal, Eqs. 3.22a and 3.22b are related. Isolating \( \dot{\phi}_l \) and \( \dot{\phi}_b \) in both equations yields:

\[ \dot{\phi}_l = \dot{R}^2 + c_l \left[ -\frac{1}{R} \phi_l + \frac{1}{c_l} \left( \phi_b - \dot{R}^2 \right) \right] \]  

(3.23a)

\[ \dot{\phi}_b = \dot{R}^2 + c_b \left[ -\frac{1}{R} \phi_b + \frac{1}{c_l} \left( \phi_l - \dot{R}^2 \right) \right] \]  

(3.23b)

Substituting Eqs. 3.23a and 3.23b into Eq. 3.21 gives

\[ \dot{\phi}_l = (1 + \zeta)^{-1} \left[ \left( \frac{1}{2} + \zeta + \frac{1}{c_l} \frac{P_{bi} - P_{li}}{\rho_l} \right) \dot{R}^2 - \zeta c_l \phi_l - \frac{P_{bi} - P_{li}}{\rho_l} \right] \]  

(3.24a)

\[ \dot{\phi}_b = (1 + \zeta)^{-1} \left[ \left( 1 + \frac{1}{2} \zeta + \frac{1}{c_l} \frac{P_{bi} - P_{li}}{\rho_l} \right) \dot{R}^2 - \frac{c_b}{R} \phi_l + \left( \frac{c_b}{c_l} \frac{P_{bi} - P_{li}}{\rho_l} \right) \right] \]  

(3.24b)

where the specific-acoustic-impedance ratio is given by \( \zeta = c_b \rho_b / c_l \rho_l \).

Another expression for the material derivative of the velocity potential can be found by introducing Eq. 3.20 into Eq. 3.22b.

\[ \dot{\phi}_b = \dot{R}^2 + \dot{R} c_b \]  

(3.25)

Substituting Eq. 3.25 into Eq. 3.24b and solving for \( \phi_l \) yields:

\[ \phi_l = -R \dot{R} \left[ 1 + \zeta - \frac{1}{2} \left( 1 - \frac{\rho_b}{\rho_l} \right) \frac{\dot{R}}{c_l} \right] + \frac{R}{\rho_l c_l} \left( P_{bi} - P_{li} \right) \]  

(3.26)

Eq. 3.26 could also be derived by introducing Eq. 3.20 into Eq. 3.22a and then substituting the result into Eq. 3.24a. We can replace \( \phi_l \) in Eq. 3.24a by introducing Eq. 3.26.

\[ \dot{\phi}_l = \frac{1}{2} \dot{R}^2 \left( 1 + \frac{\rho_b}{\rho_l} \right) + \zeta c_l \dot{R} - \frac{P_{bi} - P_{li}}{\rho_l} \]  

(3.27)
Finally, the velocity potential in Eq. 3.26 is differentiated with respect to time and the result is set equal to Eq. 3.27 to obtain the equation of motion defining the bubble’s radius:

\[
R \ddot{R} \left[ 1 + \zeta - \left( 1 - \frac{\rho_b}{\rho_l} \right) \frac{\dot{R}}{c_l} \right] + \frac{3}{2} \dot{R}^2 \left[ 1 + \frac{2}{3} \zeta - \frac{1}{3} \left( \frac{\dot{R}}{c_l} \right) + \frac{1}{3} \left( \frac{\rho_b}{\rho_l} \right) \left( 1 + \frac{\dot{R}}{c_l} + \frac{\dot{R}}{c_l} \cdot \frac{\dot{\rho}_b}{\rho_b} \right) \right] + \left( \zeta_{cl} + \dot{\zeta} \dot{R} \right) \dot{R} = \rho_l^{-1} \left[ (P_{bi} - P_{li}) \left( 1 + \frac{\dot{R}}{c_l} \right) + \frac{\dot{R}}{c_l} \dot{P}_{bi} \right] \tag{3.28}
\]

where an overdot specifies the rate of change of a given quantity in time. These rates can be found equal to

\[
\dot{P}_b = -3 \rho_b c_b^2 \frac{\dot{R}}{R}, \quad \dot{\rho}_b = -3 \rho_b \frac{\dot{R}}{R}, \quad \dot{\zeta} = -\frac{3}{2} (\gamma + 1) \frac{\dot{R}}{R} \tag{3.29}
\]

Equation 3.28 has been highlighted since it is the equation of motion used to generate the plot in Figs. 3.6 and 3.7.

Figure 3.6: Numerical solution to the Geers-Hunter DAA (Eq. 3.28). Initial conditions are set for a stable bubble and the internal bubble pressure is increased after 1ms. The moment the pressure increased is identified by a red dashed line.

Again, Eq. 3.28 can be solved numerically to produce a time history of the bubble’s radius following a rapid pressure increase. Figure 3.6 shows the solution to the Geer-Hunter DAA. When compared to the solution of the Rayleigh-Plesset equation (Eq. 3.15) shown in Fig. 3.5, the new equation of motion shows a rapid attenuation (on the order of a few milliseconds) of the bubble radius oscillations.
As in the previous solution, the most intense pressure waves occur at the point where the bubble’s size is smallest during the growth and collapse cycles, where the bubble’s radial acceleration is largest. However, as the oscillatory movement is damped, succeeding pressure waves carry increasingly less energy. This more realistic approach still has certain shortcomings since the damped oscillations will converge towards a stable equilibrium between the bubble’s new internal pressure and the surrounding pressure.

Since the cavity created by the introduction of hot smelt in water is mainly composed of water vapour, it is no longer sustained once the smelt is gone. The lack of a heat source causes the bubble to condense back into the surrounding cooler liquid. Although the exact heat transfer functions are hard to predict, they can be approximated by using Fourier’s heat law.

### 3.1.3 Bubble Heat Loss

Starting from the integral form of Fourier’s heat law, the amount of energy flowing out of the vapour bubble can be described using the following equation:

$$\frac{\partial Q}{\partial t} = -k \int_S \nabla T \cdot dA$$  \hspace{1cm} (3.30)

where $k$ is the material’s thermal conductivity and $\nabla T$ is the temperature gradient between both media at the surface of the bubble. Since the process described is the mass diffusion of vapour back into the surrounding water, the heat flux described in Eq. 3.30 is taken as the latent heat of vaporization for water. As it is assumed the vapour inside the bubble is not superheated any sensible heat flux will be negligible. Taking the bubble to be spherical, Eq. 3.30 can be rewritten to give

$$h_v \rho_b \frac{\partial V}{\partial t} = -4\pi R^2 \cdot \nabla T \cdot h$$  \hspace{1cm} (3.31)

where $h_v$ is the latent heat of vaporization and $h$ is the heat transfer coefficient. Again, taking the bubble to be spherical, its volume can be rewritten as a function of radius

$$\frac{\partial V}{\partial t} = \frac{\partial \left( \frac{4}{3} \pi R^3 \right) }{\partial t} = -4\pi R^2 \cdot \frac{\nabla T \cdot h}{\rho_b h_v}$$  \hspace{1cm} (3.32)

Ignoring effects from the previously derived equations of motion, Eq. 3.32 can be simplified to give

$$\frac{\partial R}{\partial t} = -\frac{\nabla T \cdot h}{\rho_b h_v}$$  \hspace{1cm} (3.33)

Here, $h_v$ is known and $\rho_b$ is found while solving Eq. 3.29. Assuming the vapour bubble is not superheated $\nabla T$ can be predicted since the temperature of the surrounding liquid in a dissolving tank is monitored and typically stable. The truly unknown parameter in Eq. 3.33 is the heat transfer coefficient $h$. Predicting the value of this coefficient is a difficult task since $h$ depends on the volume element geometry and the turbulence in its bulk fluid. Changing the order of magnitude of the heat transfer coefficient has an impact on the bubble’s lifetime. As an attempt to bound $h$ within a certain range of possibilities, various values were tested in conjunction with the previously derived equations of motion. A plot of the bubble’s size and lifetime can be seen in Fig. 3.7.
Figure 3.7: Bubble radius as a function of time using different values for the heat transfer coefficient between water vapour and the surrounding liquid water.

Figure 3.7 shows that a difference of two orders of magnitude in the heat transfer coefficient is the difference between a very short lived bubble (\(<1\) ms) and a long lived vapour bubble (\(\gg20\) ms).

Referring to captured video frames presented in Fig. 3.8, a typical vapour bubble caused by a rapid phase transition during the smelt-water interaction lasts on the order of 10 milliseconds. A value of \(h\) on the order of \(10^6\) W/(m\(^2\)K) would therefore be the most realistic choice. A more precise value of the heat transfer coefficient is hard to predict and should not affect the pressure waves generated by the bubble significantly. A gross estimate is sufficient to describe the heat transfer process. See Fig. 3.9 for more details on the exterior pressure profiles of single droplet explosions. The heat transfer coefficient value chosen as the most realistic when compared to laboratory video data also corresponds to the approximate maximal convective heat flux of boiling water under ideal conditions.[26]
Using a formulation by Frost and Harper [13, 14, 7, 34] the bubble’s instantaneous radius can be related to the exterior pressure profile generated by the bubble’s rapid expansion and collapse:

\[ P_{\text{ext}}(r, t) = \left( \frac{\rho_l}{4\pi r} \right) \dot{V}(t) \]  

(3.34)

where \( r \) is the radial distance from the centre of the bubble. Using this formulation, the medium outside the bubble’s surface is assumed to be of constant characteristics and spreading to infinity. The specific damping of certain frequencies has not been considered and there are no interfaces between different materials that the pressure wave must cross. Assuming an observer positioned at a distance of 10m from the bubble’s centre the pressure profiles for vapour explosion including different heat transfer coefficients can be seen illustrated in Fig. 3.9. Initially the hot smelt prevents the vapour film from collapsing regardless of the heat transfer between the film and the water. After 1ms, when the rapid phase transition occurs, the largest pressure wave is released and reaches the same level for all heat transfer coefficients. This level is mainly determined by the mass of the smelt droplet as well as how much smelt disintegrated upon contact with water. The pressure profiles presented in Fig. 3.9 assume a droplet 5mm in radius that completely disintegrates upon the first collapse of the vapour film. The subsequent pressure peaks correspond to the bubble’s collapse in the oscillatory cycle of the bubble size shown in Fig. 3.7. Although different heat transfer coefficients affect the size of the vapour bubble at late times, the initial shock wave and the following pressure peaks remain relatively unchanged.

Using the model derived in this section to predict the pressure waves produced by a single droplet event, the pressure waves for various droplet sizes can be found. The droplet statistics presented in section 3.2 are applied to the single droplet model to generate a synthetic sum of pressure waves inside the dissolving tank when operating at a typical capacity.
Figure 3.9: External pressure wave as calculated from Eq. 3.34 for an observer 10m away form the bubble centre. Again, several values for the heat transfer coefficient were used. It is important to note that at the first pressure peak (immediately after 1ms) all four functions are stacked and reach the same value within a 0.05% difference.

### 3.1.4 Instantaneous Pressure Increase

In section 3.1.2, an equation of motion describing a bubble’s radius as a function of time and various other parameters was derived. To model the underwater explosion, it is assumed that the moment contact between the molten smelt and the surrounding water is made possible, heat will be instantaneously transferred from the hot smelt to the cooler liquid. Since the interactions between the two liquids is violent and results in breaking apart the smelt droplet, the turbulence can be taken to be very large and a large Reynolds number leads to a very fast heat transfer between the two liquids.

<table>
<thead>
<tr>
<th></th>
<th>Molten Smelt</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>1923</td>
<td>1000</td>
</tr>
<tr>
<td>Heat capacity (kJ/kgK)</td>
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<td>4.2</td>
</tr>
<tr>
<td>Heat of fusion (kJ/kg)</td>
<td>142</td>
<td>-</td>
</tr>
<tr>
<td>Heat of vaporization (kJ/kg)</td>
<td>-</td>
<td>2260</td>
</tr>
<tr>
<td>Freezing point (°C)</td>
<td>750</td>
<td>-</td>
</tr>
<tr>
<td>Temperature at typical (°C)</td>
<td>800</td>
<td>95</td>
</tr>
</tbody>
</table>

Table 3.1: Physical and thermal properties of molten smelt and water.[2, 36, 18]
Assuming the thermal properties given in Table 3.1, the total amount of heat transferred can be calculated. From an arbitrary initial droplet radius, the mass of smelt can be found using the following equation:

\[ m_S = \rho_S \cdot \frac{4}{3} \pi r^3 \] (3.35)

where \( m_S \) is the mass of a given smelt droplet and \( \rho_S \) is the density of molten smelt. The total heat that can be transferred from molten smelt to the surrounding water is given by the sum of the sensible heat contained in the smelt droplet and the latent heat due to the molten material freezing.

\[ Q = Q_{\text{sensible}} + Q_{\text{latent}} = m_S c_S \Delta T + m_S L_{fs} \] (3.36)

where \( c_S \) is the smelt heat capacity, \( L_{fs} \) is the smelt specific latent heat of fusion, and \( \Delta T \) is the temperature difference between the droplet’s temperature and smelt freezing temperature, which are given in table 3.1. Even though solidified smelt may still be hotter than the surrounding liquid, its sensible heat is omitted since the time scale at which heat is being transferred is no longer relevant and smelt will have been dispersed in the liquid.

Values for the constants in Eq. 3.36 can be found in Table 3.1. Typical droplet sizes are estimated to range between 1 and 10 mm in diameter. See section 3.2 for more details. The temperature difference \( \Delta T \) is estimated to be approximately 50\(^\circ\)C. Though difficult to accurately know the smelt droplets temperature when the vapour film collapses, since there can be many causes for this collapse, a different smelt temperature will only affect the total heat exchanged minimally and not change its order of magnitude.

The total heat found in equation 3.36 will be transferred to the water that has come into contact with the hot smelt. Using the heat of vaporization of water given in Table 3.1, it is possible to find the amount of water vaporized. The heat used to warm up the water to boiling temperature will take away a small fraction of the total heat available but since dissolving tank temperatures are typically very close to boiling point, the amount of heat required is negligible. From the ideal gas law

\[ PV = nRT \] (3.37)

a relation can be found between the amount of water molecules added to the bubble due to vaporization and the resulting pressure increase. During the instantaneous heat transfer between the molten smelt and the water that has now come into contact with it, the vapour bubble’s volume and temperature are assumed to be held constant. Newly vaporized water will increase the number of vapour particles inside the bubble and from Eq. 3.37 the interior pressure increases proportionally. The initial vapour film bubble caused by the Leidenfrost has a radius slightly larger than the smelt droplet it is surrounding. Typically, vapour films can range between 100 \(\mu\)m and 200 \(\mu\)m.\[40\] The initial pressure of the bubble can be found by calculating the steady state pressure in Eq. 3.28. Assuming atmospheric pressure for the surrounding medium\[25\], the internal pressure of the bubble surrounding a smelt droplet can increase by a factor of up to 500 times depending on its initial dimensions.
3.2 Droplet Statistics

Under normal mill operating conditions, smelt spouts typically have a flow rate of approximately 1L/s. However, individual droplets vary in size. Once the smelt stream is shattered into many smaller droplets, certain formations arise. A small scale shatter jet research conducted by Lin at the University of Toronto has shown that different shattering parameters such as nozzle type, steam flow rate, and the spray’s impingement angle with the smelt stream generate varying droplet size distributions.[27] Although the exact statistics depend on the shattering parameters, the distributions generally resemble a chi-squared distribution. Figure 3.10 shows experimental results for droplet size distributions using different shatter jet nozzle types.

Following these results, the pressure profiles of individual droplets found using the equations of motion derived in section 3.1.2 can be combined following a chi-squared distribution to create a sum of internal pressures. In the event that the smelt stream is not being shattered properly, the mean droplet size of the distribution should increase and the generated pressure profile change. Figure 3.11 illustrates the intensity of a simulated steady state signal for smelt entering the dissolving tank at a rate of 4L/s with varying droplet size distributions. As in the pressure profiles shown in Fig. 3.9, all vapour explosions are assumed to occur 10m away from the observer. Multiple vapour explosions are combined as uncorrelated sound sources.
Similar to the experimental work, fewer larger droplets produce a more intense, as well as a more variable sound signal. Values for the simulated intensities and signal variance can be seen in table 3.2. Simulated data is compared to data taken during normal mill operation with all shatter jets functioning properly and a case where the shatter jets were turned off. It is assumed that when the shatter jets are off, the smelt streams are not broken into smaller droplets and the average droplet size becomes larger. Even though many additional factors contribute to the sound signal produced in a mill, a general behaviour is observed where a distribution of fewer larger droplets produces an acoustic signal with more energy. The signal also has a larger variance when larger droplets are present.

![Graph showing sound intensity over time for different droplet sizes](image)

**Figure 3.11:** Sound intensity of a simulated sound signal using the droplet size distributions presented in Fig. 3.10. To simulate different shattering conditions, average droplet radii of 1mm, 1.5mm, and 5mm were used. A total quantity of exactly 4 litres of smelt was required for the three simulations to compare equivalent total amounts. The average energy of a droplet size distribution centred about 1mm is used as a reference and set to 0dB. A synthetic signal composed of fewer larger droplets is more intense and more variable than a signal with many smaller droplets.

<table>
<thead>
<tr>
<th>Mean energy</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{R} = 1$mm</td>
<td>0.0 dB</td>
</tr>
<tr>
<td>$\bar{R} = 1.5$mm</td>
<td>1.6 dB</td>
</tr>
<tr>
<td>$\bar{R} = 5$mm</td>
<td>7.0 dB</td>
</tr>
</tbody>
</table>

**Table 3.2:** Numerical comparison of sound intensity produced by soundscape simulation for three different droplet radius distributions. Simulated results are compared to data taken in an operating mill. In both cases larger droplet sizes lead to a more intense as well as a more variable sound signal.
The general shape of the power spectrum is compared between the simulated pressure signal and the recorded measurements. Both signals carry more energy in the low frequencies than in the higher frequencies. Frequencies on the order of 100 Hz and less cannot be compared accurately since the microphone will not be reliable at low frequencies. Figure 3.12 illustrates the power spectrum for the droplet distribution centred around 1mm used to generate Fig. 3.11. When comparing the experimental mill data in Fig. 2.6, it can be seen that the energy in the low frequencies increases for larger droplets. The energy change for high frequencies cannot be compared since the decrease seen in Fig. 2.6 is associated to the closing of the shatter jets, a factor not taken into account for the simulated data.

\[
\text{Slope : -4.2} \\
\text{Jets OFF : -30.8} \\
\text{Jets ON : -25.3}
\]

Figure 3.12: Power spectrum of a droplet distribution with an average droplet radius of 1mm. The energy of the signal decreases steadily as the frequencies increase at a slope one order of magnitude different from observations. The power spectrum for other mean droplet sizes shows the same characteristics.

Many assumptions used to simplify the computation of a synthetic pressure profile could explain the differences between the simulation’s power spectrum and the observed spectrum. The absence of all sound sources other than smelt-water interactions has unpredictable effects on the shape of the frequency spectrum. It is also assumed in the simulation that the vapour explosions occur in an infinite medium with no boundaries, neglecting all possible reflections or transmission losses. Using a simple model for the transmission loss of an acoustic signal increases the loss in energy for higher frequencies. The mass law states that the transmission loss (\(TL\)) of sound passing through a solid panel will increase as a function of the square of the frequency (\(f\)).[35]

\[
TL \propto 20 \cdot \log_{10}(f)
\]  

(3.38)

Figure 3.13 shows a comparison between the shape of a synthetic signal that assumes no frequency dependent attenuation and the same signal with the sound reduction presented in Eq. 3.38.
Figure 3.13: Power spectrum of a droplet distribution with an average droplet radius of 1mm compared to the same distribution that was attenuated using the transmission loss predicted by the mass law. [35]

Lin’s work also showed that shattered smelt droplets are spatially distributed with a high concentration of the liquid’s volume centred about an approximate 2D Gaussian function. [27]

Also observed, the larger droplets tend to remain closer to the centre of the distribution compared to smaller droplets, which extend further. Variations in the spatial position of the vapour explosions relative to the observer affect the pressure wave’s attenuation. This spatial distribution can be included in the radial distance component of Eq. 3.34. However, the spread in droplet positions is small compared to the position of the observer and the effect negligible.

The timing between explosion events is assumed to follow an exponential distribution. After the smelt stream has been shattered into droplets, the droplets enter the water at random instances of time following a uniform distribution. This assumption does not capture the full picture of the temporal distribution of vapour explosion events since it has been observed that one explosion can trigger neighbouring droplets to explode. The model presented above used a total volume of exactly 4L for every second of simulated signal generated. As it required a constant smelt stream to have a reliable comparison between different droplet distributions, long term variability of the smelt stream went beyond consideration. A more realistic approach for describing the dissolving tank’s activity should involve a variable smelt flow rate.
Chapter 4

Conclusion

4.1 Conclusion

A constant stream of molten smelt, an inorganic salt, is shattered into many droplets before falling into the dissolving tank of a kraft recovery boiler. The interaction between the hot smelt droplets and the relatively colder water can lead to a violent heat transfer resulting in a vapour explosion. In severe cases these violent interactions can cause a dissolving tank explosion, which poses a serious safety concern for Kraft mills in the pulp and paper industry.

Sound recordings in various pulp and paper mills as well as in a university laboratory have provided insightful observations on the characteristics of the acoustics of smelt-water interactions. When closing the shatter jets, the vapour explosions inside the dissolving tank become more violent and the intensity of the recorded sound signal increases and becomes more variable. Significant differences in sound intensity have been observed between mills. Spectral analysis shows that smelt-water interactions produce sound waves that carry more energy in the low frequencies.

A theoretical model is proposed to describe the dynamics of a vapour bubble under water. When subjected to a rapid pressure increase, the bubble’s radius oscillates and produces pressure waves that propagate through the water. A system of equations of motion have been derived starting from conservation laws. Solving these equations numerically revealed the behaviour of a vapour bubble caused by a single smelt droplet. Statistical distributions for droplet sizes simulated the total pressure profile inside the dissolving tank. Varying the average droplet size while keeping the total amount of smelt constant, the numerical solutions indicate that fewer larger droplets produce more intense and more variable pressure waves than many smaller droplets. Under the assumption that a smelt stream that has not been shattered produces larger droplets than a shattered stream, the simulation’s predictions agree with mill observations. A spectral analysis of the total pressure profile of the synthetic signal is also consistent with observations.

4.2 Further Work

Future work should aim at gaining an even more intuitive understanding of the production and transformation of the sounds produced in the dissolving tank. The following areas should be expanded upon to improve the theoretical model derived in this thesis and provide the groundwork for its implementation.
Thin vapour film dynamics and collapse

The model formulated in this study is based on the assumption that the vapour film surrounding the molten smelt droplet as it enters the dissolving tank is uniform and invariant in time. At the moment of the vapour film’s collapse, water comes into contact with the molten smelt and the immediate heat transfer adds water vapour to the remaining film. A deeper understanding of how the vapour film changes over time and what happens at the moment of its collapse would lead to a more accurate model of the vapour bubble’s behaviour.

Container characteristics

The properties of the container in which vapour explosions occurred were not considered in this research. In order to obtain a theoretical prediction of the sounds produced by the dissolving tank from the internal pressure waves many acoustic effects must be taken into account. The free surface separating the liquid from air as well as the walls and floor of the tank will contribute to reflections of the pressure waves. Observations revealed that the container itself resonates as a result of events occurring inside.

Furthermore, sound waves are attenuated when travelling through different media and certain frequencies more so than others. A study on the attenuation of sound waves in green liquor should be conducted to determine how each frequency is affected.

Time varying characteristics

Although droplet sizes followed probability distributions and explosion events were randomly distributed in time, larger scale time variations were not accurately represented. Observations have shown that, under constant conditions, the acoustic characteristics of the dissolving tank vary on the scale of seconds or minutes even though smelt-water interactions happen in a much shorter time frame. Fluctuating volumes of smelt flowing from the spouts should contribute significantly to the variance of the acoustic intensity of the dissolving tank.

Observations in a laboratory setting have also revealed that individual vapour explosions are not independent from one another. The pressure wave expanding from one droplet can trigger other neighbouring droplets to explode. Multiple droplet explosions occurring at the same time lead to a more variable pressure profile for the dissolving tank.

Monitoring system

Recording equipment could be placed in a pulp and paper mill for an extended period of time to acquire long term measurements and observe the intensity and frequency variations on scales of hours, days, and weeks. Correlating these recordings with other parameters logged by the mill would help the development and possible implementation of a permanent system monitoring the dissolving tank that would be based on acoustics, thereby safeguarding against accidents, shutdowns, and time loss incidents.
Bibliography


