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Forest vertical structure characterization using ground inventory data for the estimation of forest aboveground biomass

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Abstract: A common method for estimating forest biomass is to measure forest height and to apply allometric equations. However, changing forest density or structure heterogeneity increase the variability of the known allometric relationship. Here, we investigated the potential of allometric relationships based on vertical forest structure for biomass inversions with a global potential. First, vertical biomass profiles, which were calculated from ground forest inventory data, were used to model forest vertical structure. Then, a vertical structure ratio based on Legendre polynomials was proposed as a structural descriptor and its sensitivity to biomass was evaluated. Finally, we developed a structure-to-biomass inversion expression which could be extrapolated for aboveground biomass estimations. This is a case study based on inventory data from the Traunstein and Ebersberg test sites, two temperate forests located in the southeast of Germany with different forest structural conditions. Results from the structure-to-biomass inversion algorithm show a clear improvement with respect to traditional height-to-biomass expressions, with increasing correlation factor ($r^2$) from 0.52 to 0.73 for Traunstein and from 0.51 to 0.76 for Ebersberg, and reducing the root mean square errors from 75.32 to 47.56 Mg/ha and from 73.25 to 48.31 Mg/ha, respectively.

Key words: allometry, self-thinning, temperate forest, stand density, Legendre decomposition, ground inventory data
Introduction

Forest biomass is an essential part of the terrestrial carbon pool; therefore, it is a very important contribution in the global carbon cycle (Drake et al. 2002; Houghton et al. 2009). Forest biomass is a main variable for mapping the amount and geographic distribution of forests and its change in time is required to understand the development of carbon fluxes (Brown 2002). However, biomass stock and spatial distributions are still unknown parameters for many forest regions of the world. Thus, the magnitude, location, and cause of terrestrial carbon sinks and sources are not well quantified on a global scale (Houghton et al. 2009). Therefore, methodologies capable of retrieving biomass and forest dynamics at a global scale are highly sought.

Accurate terrestrial biomass measurements are time consuming, expensive and rare (Houghton 2005). Forestry science aims to reduce the effort of measuring biomass with the help of allometric functions which can estimate biomass from easily measurable tree variables such as diameter at breast height (dbh), basal area, top tree height, and form factor (factor between the volume contained in the real-shaped stem and a cylinder which section has a diameter equal to the dbh and same height). However, single-tree measurements are ineffective for large areas and global applications. In contrast, forest stand parameters are an option with great potential as remote sensing (RS) systems can measure them globally. In order to develop methodologies that use forest stand parameter to estimate aboveground biomass (AGB), new allometric relationships need to be derived and investigated.

A first approach in this direction was introduced in Mette (2003). AGB is estimated using allometric relationships based on forest stand height. This allometric approach used a power law equation that relates biomass to height and uses an empirically derived allometric level ($l_a$):
\[ B_a = l_a 1.66H^b \]

where \( B_a \) is the stand (above ground) biomass (Mg/ha), \( H \) is forest dominant height or \( H_{100} \) (height of the 100 thickest trees per hectare (Laar and Akça 2007)), and \( b \) is an allometric exponent. This height-to-biomass allometry allowed biomass estimations from methodologies that are capable of resolving forest height. Several techniques, in addition to ground forest inventory data, can estimate forest height, e.g. photogrammetry (Næsset, 2002; Schneider et al., 2013), light detection and ranging (LIDAR) (Dubayah and Drake, 2000), and polarimetric synthetic aperture radar (SAR) interferometry (Pol-InSAR) (Papathanassiou and Cloude 2001). Height-to-biomass allometric relationships proved to be robust under homogenous forest conditions, and able to accurately estimate AGB from forest dominant height for the predominant temperate European tree species. However, the diversity in forest structure can reduce the precision of this allometric relationship (Toraño et al. 2010), thus reducing its accuracy for AGB estimations in forest stands with a heterogeneous structure. Forest structure is then described, in the most general terms, as the distribution of biomass in space; i.e., as a vertical and horizontal spatial arrangement of tree species, tree sizes, or ages (Zenner and Hibbs 2000) which can be measured by distribution of age classes, arrangement of species into different canopy layers, and distribution of individuals among diameter classes.

Studies based on conventional forest yield-related stand parameters, like diameter at breast height, basal area or height, have tended to neglect three-dimensional (3-D) stand structures (Onaindia et al. 2004) and there is not a clear understanding regarding the allometric relationship that conditions the vertical and horizontal distribution of forest layers and structures (Köhler and Huth, 2010). A study of the horizontal structures, characterizing forest stand density (Pretzsch and Biber, 2005) showed a strong relationship between AGB and horizontal forest structure.
New RS techniques, such as multibaseline Pol-InSAR (Cloude 2006; Dinh et al., 2012; Treuhaft 2010) and full-waveform LIDAR (Harding et al. 2001; Lefsky et al. 2005) enable measurements of variables that in addition to forest height are connected to forest vertical structure. The majority of these methods have attempted to quantify forest vertical structure (Hall et al. 2005; Parker and Russ 2004; Drake et al. 2002), finding good correlations between AGB and height metrics or percentiles that are connected to the vertical organization of the forest stand canopy. However, studies based on structures cannot be directly compared among systems and/or with ground measurements, as it is done in the case of forest height, which is in fact, a physically derived variable. Structure is a variable of complex definition and each RS system is sensitive to different compartments of the vertical biomass distribution (stems, crowns, leaves, etc.).

Due to this difficulty and the limitations of homogenization of methodologies that account for the different system responses, in this study, we have focused on the vertical biomass distribution obtained from ground data from forest inventories. In this way, vertical biomass profiles (independent from the measuring system) can be used to analyze and/or quantify the relationships that exist, at a biophysical level, between forest vertical structure and biomass. These can be interpreted not only as base for RS applications, but also towards silvicultural investigations. Therefore, the allometric relationships between all vertical elements represented by the vertical structure profile and biomass need to be investigated. For this purpose, polynomial series decompositions (Cloude 2006, Toraño 2010) arise as a suitable method that can characterize and preserve these relationships, contrary to single metrics.

**Allometric theory: from the single tree to the stand**

The connection between allometry and growth is outlined in the theory of dynamic
morphology (Bertalanffy 1942). One of the key allometric patterns observed for both plant and animal communities is the inverse relationship between body mass and density. Therefore, the physiological relevance of allometric functions lies in the interpretation that the proportion of the total growth energy received by an organism is proportional to its relative size. This principle can be transferred to the population (stand) using the self-thinning rule, according to which the number of individuals is reduced while they compete for resources, increasing their size until the stand reaches a maximum ecological AGB (carrying capacity) (Luyssaert et al., 2008).

Allometric theory predicts then that the total number of individuals per unit area (N) is proportional to the total body mass (M) per unit area (−3/2 power) (Yoda et al. 1963). Applying these scaling laws to entire communities, organismal interspecific interactions can link larger-scaled allometric properties of communities across different ecosystems (Enquist and Niklas 2001). Accordingly, Pretzsch (2009) describes how the first version of Eichhorn’s rule identified that the statistical relationship between stand mean height and standing volume is valid for more or less untreated stands. Especially, the interpretation of Eichhorn’s rule, according to Assmann, lies on an estimation of total production in relationship with age, height, and yield classes. Extensions of allometric and biomechanical theory predict that the total standing community biomass will not vary with respect to species composition or latitude (Enquist and Niklas 2001).

**Allometric principles for quantifying vertical forest structure**

Recent studies have started to incorporate forest structure heterogeneity variables that are not only considering 1-D variables but a 3-D forest representation with horizontal and vertical components (Lefsky et al. 2009; Zenner and Hibbs 2000). Connections provided by such studies have led to new theories that provide a basis for extrapolation of small-scale and high resolution...
measurements as well as for testing and verifying 3-D measurements (Parker et al. 2004).

In order to understand the mechanistic essence of allometric relationships like the self-thinning rule, a thorough understanding of the relationship between spatial interactions among individuals and spatial arrangements of these individuals is needed (Li et al., 2000). The existing forest structures enable trees to influence factors such as light, temperature, and precipitation, which in turn influence growth and determine the competition among trees for resources and the biomass production (Pretzsch 2009). In this way, 3-D stand structure determines the stability of a forest to a large extent and defines its dynamics (evolution in time) (Onaindia et al. 2004).

The interactions between trees are explained according to Pretzsch (2009) as a process with two feedback loops that proceed at different time scales. In the first feedback loop, the local environmental conditions within the stand are influenced by the structure of the canopy, which provides crucial feedbacks between structure, environment, and growth, drive population dynamics (Pretzsch 2014) and controls the absorption of photosynthetically active radiation (Parker et al. 2004). Since processes modify structures, the resulting structures can assist the interpretation of those processes which are more difficult to measure or observe. The structural relationships of processes and of individual trees also scale-up to generate emergent properties of forests, such as size, structure, and spacing relationships (Enquist 2009). Hence, all of these interactions indicate that forest vertical structure is intrinsically related to forest evolution. In conclusion, the stock of forest biomass is determined by the vertical and horizontal arrangement of trees, it varies over time because of numerous intra-specific stand structural interactions and disturbances and is defined, at the same time, by the site conditions, i.e. climate and soil conditions. An understanding of the relationship between these parameters is essential for improving allometric AGB estimations.
Objectives

The objective of this study is to investigate and describe the relevance of stand structural variables that can be globally connected to estimate AGB. Specifically, this paper is organized as follows: 1) We propose an allometric model to derive a forest vertical biomass function from ground forest inventory data. 2) We propose a new methodology to characterize forest vertical structure. 3) We investigate the allometric relevance of the vertical structure characterization. 4) We develop a structure-to-biomass inversion expression for AGB estimations combining forest height and a vertical structure descriptor.

Materials and methods

Test sites

We investigated inventory data from two different test sites in Bavaria (South of Germany) which represent different stand structures: Bürgerwald Traunstein, and Ebersberger Forst. The forest characteristics are summarized in Table 1.

Bürgerwald Traunstein (from now on Traunstein site) is a temperate, mountainous, managed forest with a maximum biomass of 450 Mg/ha, maximum forest height of approximately 45 m, and mixed species composition. The main tree species are Norway spruce (Picea abies [L.] Karst), European beech (Fagus sylvatica L.), and silver fir (Abies alba Mill). The forest stand is under “close-to-nature” silvicultural treatments that have produced a highly structured and diverse forest structure. Traunstein has a mean tree density of 1960 stems/ha, with 8400 stems/ha in the most densely stocked stands and 20 stems/ha in the least densely stocked stands. The median density for stands with the highest biomass (B > 400 Mg/ha) is 2140 stems/ha, which is above the mean tree density.
Ebersberger Forst (from now on Ebersberg site) is an intensively managed forest site formed by a monoculture of Norway spruce on a very flat terrain and is treated using intensive forest management techniques. The inventory plots are generally very homogenous. The biomass range is 40–350 Mg/ha, with a maximum forest height of approximately 40 m. Ebersberg has a mean tree density of 1530 stems/ha, with 8980 stems/ha in the most densely stocked stands and 20 stems/ha in the least densely stocked stands. The median density for the stands with highest biomass (B > 400 Mg/ha) is 1000 stems/ha, which is below the mean tree density.

These investigations relied on inventory data collected in a 10 year cycle by the forest on management services. The inventory data for the two Bavarian test sites has been provided according to the inventory system of the Bavarian state forest (Mette 2003). This system is based in three concentric circles where trees are measured depending on their diameters (Figure 1): in the inner circle (50 m²) all trees are measured, in the second circle (150 m²) trees with a dbh between 10 and 30 cm and in the largest circle (500 m² for Traunstein, 400 m² for Ebersberg) only trees with a dbh larger than 30 cm. Depending on each diameter class expansion factors (x 16, x4, from in to outside, respectively) are applied to extrapolate the measured trees to the inventory area (largest circle).

3D structure modeling: vertical biomass profiles

Forest vertical structure is a 3-D variable; therefore, its characterization requires a method that can express this complexity. The solution considered in this study calculated vertical forest biomass profiles, which modeled for each plot the amount of biomass variation with height. To obtain these profiles on a plot level, first the vertical biomass distribution on a tree level required to be modeled. Adequate models for tree crown and stem forms were required to derive the dry
biomass stored in a single tree. These models are introduced in the following sections.

**Stem model**

Stem radius as function of height was modeled according to the improved Brink’s function from von Gadow (2001) (Appendix A). This function shows good agreement with a theoretical stem shape as the lower section is modeled as a neiolid frustum, the middle section as a paraboloid frustum and the upper section as a cone. Finally, we have calculated the biomass enclosed in this volume using species-specific wood densities (World Agroforestry Centre 2011).

**Crown model**

Crown measurements are very intensive and time consuming; therefore, they are not commonly available in inventory data (Nadkarni et al. 1994). A precise model of the crown layer is required to achieve an accurate representation of the vertical biomass distribution because the forest canopy is a key structural component for understanding and characterizing forest vertical structure, and it has been shown in the past that it is essential for deriving allometric relationships (Lefsky et al. 1999).

We have selected a model that describes the species-specific crown shape based on the parameters dbh and tree height. This crown model is based on the investigations of Pretzsch (2009) and is explained below. The model particularly fits the requirements of this study because it allows the calculation of average species-specific crown shapes in forested conditions. A representation of the crown model with the required parameters is shown in Figure 2.

The biometric reproduction of crown perimeter was performed for different species in a standard calculation procedure describing the change in crown radius $r$ with increasing distance $\text{dist}$ from the tree tip. The crown model splits the crown into a sunlit and a shaded crown. The
sunlit crown is exposed to direct sunlight and its crown radii $r_0$ are calculated according to eq. 2.

$$[2] \quad r_0 = a \times \text{dist}^b,$$

where the species-specific tree parameter is $a$, and the species-specific exponent is $b$. The radii $r_u$ in the shaded crown were calculated using a linear equation with the species-specific parameters $c$ and $d$:

$$[3] \quad r_u = c + \text{dist} \times b$$

Values for the parameters $a$, $b$, $c$, and $d$ are species dependent and can be found for the major European tree species as a function of the maximum crown radius (Pretzsch 2009). Using tree height, crown base height, mean crown radius, and the species-specific crown shape parameter, we estimated the spatial expansion of the crown, crown volume, and crown surface area.

For estimating the biomass enclosed by tree crowns, we used a set of allometric equations that were estimated for the Central and Northern European regions (Ziannis et al. 2005). These allometric equations were species dependent and were chosen to fit the growing conditions found at the test site. They described crown biomass as a function of dbh and/or tree height. The equations used are presented in Table 2.

**Plot representation**

A vertical biomass profile on a plot basis was obtained by adding up the vertical distribution of biomass in stems and crowns for every measured tree in the inventory plot. Biomass distribution along the height gives a characteristic profile for each plot. For this, a vertical biomass profile was subsequently generated by summing up the biomass of all trees within a measured plot area (0.05 ha) (Figure 3). Here, 1 m sampling was chosen, which gave a biomass
value per meter of height.

Vertical structure characterization at plot level

Vertical biomass profiles represent the distribution of biomass with forest height; however, the resulting profile is too complex to be applied in an allometric relationship. To evaluate and understand the vertical biomass profiles and the role of vertical structure on biomass at a stand level, we conducted the following: 1) decomposed each vertical biomass profile into a set of profiles with variable spatial frequency (Legendre series); 2) analyzed the correlation of each vertical component to biomass; 3) normalized each profile in order to study the relative vertical structure independently from the total biomass; and 4) combined the low frequency coefficients into a structure ratio which is sensitive to biomass.

The Legendre decomposition: structure descriptor

The Legendre series were investigated in order to characterize forest biomass profiles. An evaluation was conducted on multiple inventory plots. The Legendre polynomials $P(z)$ are solutions of the Legendre’s differential equation and can be defined as the coefficients in a Taylor series expansion (Arfken, 2005). The Legendre series $B(z)$ are described as:

$$B(z) = \sum \alpha_n P_n(z)$$

$$\alpha_n = \frac{2n+1}{2} \int_{-1}^{1} B(z) P_n(z) dz$$

where $\alpha_n$ is the Legendre coefficient and $P_n(z)$ Legendre polynomial and $n$ stands for the order of the polynomial. Equation 5 shows the Legendre polynomial up to $n = 4$ as a function of height. Figure 4 shows a vertical representation of theses Legendre polynomials.

$$P_0(z) = 1$$
\[ P_1(z) = z \]
\[ P_2(z) = \frac{1}{2} (3z^2 - 1) \]
\[ P_3(z) = \frac{1}{2} (5z^2 - 3z) \]
\[ P_4(z) = \frac{1}{8} (35z^2 - 30z + 3) \]

As profile (forest) height is known, the resulting coefficients for the Legendre decompositions are stretched to their correspondent height during the reconstruction of the profile. An example is shown in Figure 5. A biomass profile was reconstructed using different number of frequencies obtained from the Legendre decomposition (blue).

The number of vertical frequencies which are needed to reconstruct a profile depend on the vertical resolution. The Legendre polynomials tend to reconstruct the original profile with few frequencies and adopt the main features of the profile more easily, particularly in the bottom and the top of the curve.

The Legendre decomposition provides a set of Legendre coefficients \((a_0 - a_n)\) for every biomass profile. A Legendre coefficient represents the degree of adjustment between a characteristic polynomial and the biomass profile. The combination of Legendre polynomials allows reconstruction of the corresponding profile: \(B_n = a_n P_n(z)\) so that a higher number of polynomials results in a better reconstruction of the original profile. The relationship between the Legendre components \((B_n)\) and the results from the single component profile reconstruction and total biomass would be studied in the following sections.

After applying the Legendre decomposition, the Legendre coefficients \((a_1 - a_n)\) depend on the total biomass of the profile. Thus, we normalized the biomass profiles to focus the information.
contained in the biomass profiles to pure structural information. This normalization process allowed a direct comparison between the vertical biomass profiles with other normalized vertical profiles. In accordance with the normalization conducted for the polarization coherence tomography profiles (Cloude 2006), each Legendre coefficient \( (a_n) \) was normalized using \( a_0 \), which contains the total biomass of the profile \( (a_{0n} = H/B) \). The normalization is done using equation 6:

\[
a_{0n} = \frac{a_n}{(1 + a_n)}.
\]

However, the normalization step, using the Legendre coefficient \( a_0 \), decorrelates the structure to a biomass relationship. At this point, we could not use the direct relationship between the Legendre components and the biomass and we were required to find a new descriptor. Therefore, to exploit the relative dependencies between the profile frequencies characterized by the Legendre coefficients, we combined the first four Legendre coefficients into a structure ratio. This ratio, from now on, is called the structure ratio \( S_{rat} \) and was calculated as the fraction between low \((a_{01})\) and high frequency Legendre coefficients \((a_{02}, a_{03}, a_{04},)\):

\[
S_{rat} = \frac{|a_{01}|}{|a_{02}| + |a_{03}| + |a_{04}|}
\]

where \( a_{0n} \) are the normalized coefficients. To demonstrate the potential of the structure ratio for interpreting the allometric relationships between the vertical structure and biomass, we calculated the correlation of the structure ratio with the ratio of stem biomass over tree crown biomass.

**Structure and scale dependency**

Before analyzing the potential of the new structural descriptor, we considered the impact of
the stand size on the structure-to-biomass relationship. A small stand size may present limitations
to characterize the heterogeneity of the forest structures (Shugart 2010). One solution to this
problem was to combine neighboring plots, according to a distance radius criterion, to simulate
larger integration areas. However, if only a distance criterion is used, plots that represent
differently structured stands (mainly because of forest management) may be erroneously
combined. Thus, the relationship of vertical structure to biomass could be lost. Therefore, a
structure and height difference criteria between plots was used.

The integration was conducted in the following manner. First, the structure ratio $S_{rat}$ and
profile height were calculated for every plot. Then, using a moving window of 150 m (to ensure
that the eight closest points were included), $S_{rat}$ and height differences between the central plot
and every other plot in the group were calculated. Those plots with differences below certain
thresholds were averaged. For $S_{rat}$ a threshold $S_{rat} > 0.5$ was used, which corresponded to a
quarter of the structure range. A height (H) < 15 m threshold was selected for the height
difference because this distance would typically avoid the combination of forest stands in
different age classes. Finally, a new profile was generated using the data from the combined
inventory plots, and new heights and $S_{rat}$ were estimated.

**Biomass inversion**

The structure-to-biomass inversion expression was developed from the combined height and
structure ratio based on the following principle. Those inventory plots that followed the height-
to-biomass allometric curve corresponded to forest stands with ideal allometric conditions, which
meant a steady state in terms of resources and demographic conditions (Enquist 2009) and also a
vertical structure that represented those conditions. Using the relationship observed between
forest vertical structure descriptor $S_{rat}$ and the forest biomass, it was possible to compensate for the deviations of the plots that were not representing the expected structural conditions that the height-to-biomass allometric equation predicts. An empirical estimation of the parameters $a, b,$ and $c$ yielded an optimized expression of the structure-to-biomass, thereby determining the structure ratio $S_{rat}$ as a weighting factor (eq. 8):

$$B = aS_{rat}^bH^c$$

where $H$ is the profile height, which is equivalent to the dominant forest height or $H100$.

**Results**

**From height-to-biomass relationships to a structure allometry**

The first application of the Legendre decomposition as a vertical structure indicator of biomass is shown in Figure 6.

The correlation between biomass and the Legendre components of an order $> 4$ ($B_4$) was very low and in all cases $< 0.2$. This shows that the first four frequencies explain the highest proportion of biomass, and for this reason they were the ones considered to be studied. The importance of each frequency is detailed below.

The highest correlation coefficient was observed for component $B_1$ followed by $B_3$. The slope of the fitting line represented the proportion of the total biomass that was explained by each Legendre component. At both sites, the largest proportion of biomass was explained by component $B_3$, followed by component $B_1$, and then component $B_2$. From component $B_4$ and upwards, the explained proportion of biomass was negligible. Moreover, a low number of Legendre polynomials (eq. 5) were sufficient to distinguish between different levels of biomass. The importance of these first four polynomials was also illustrated by examining single profiles.
Profiles of 33-m height, with high biomass (dashed) and low biomass (solid), were displayed for reconstructions with five Legendre polynomials. When four or more polynomials were used, it was possible to identify two levels of biomass: the high biomass profile presented with a smoothed shape, which could be interpreted as a higher proportion of the stem compartment biomass; whereas the low biomass profile was sharper, which could be interpreted as a higher proportion of the crown compartment biomass. In Figure 8, we showed the amplitude of the four Legendre polynomials \(a_0 \ldots a_4\) for the high and low biomass inventory plots to illustrate the effects of frequency change in the structure ratio. In the profile with a lower biomass (left), even if the amplitude of the first Legendre component (blue) was higher than in the high biomass profile (right), the higher frequencies (components 2, 3, and 4) were dominant and in total they were higher than the low frequency (component 1), which resulted in a reduced structure ratio.

The height-to-biomass equation (eq. 1) was applied to each of the test sites (Figure 9). The results for the two test sites were similar when we used a common allometric exponent \(b\) of 1.58 and an allometric level \(l_a\) of 0.78. They resulted in correlation coefficients of 0.34 for Traunstein and 0.48 for Ebersberg and root mean square errors (RMSEs) of 110.59 Mg/ha and 88.68 Mg/ha, respectively. Traunstein, being a very diverse forest, was characterized by a lower correlation coefficient because a single allometric level could not describe the high variation in stand density. On the other hand, lower stand diversities resulted in slightly higher correlations as seen for the Ebersberg site. The values for biomass were more extreme than expected because the small size of the inventory plot that was particularly visible in the higher biomass range: mean biomass of \(~200\) Mg/ha for Traunstein and \(~151\) Mg/ha for Ebersberg, and a maximum biomass of 685 Mg/ha and 720 Mg/ha, respectively (Figure 9).
The height-to-biomass allometric relationship after the aggregation process is shown in Figure 10 for Traunstein (right) and Ebersberg (left). The correlation coefficients were 0.52 for Traunstein and 0.55 for Ebersberg and the RMSEs were 75.32 Mg/ha and 68.20 Mg/ha, respectively. The deviation was reduced with respect to the non-aggregated case; however, the height value range was retained. The mean values for biomass were now ~190 Mg/ha for Traunstein and ~154 Mg/ha for Ebersberg, with a maximum biomass of 370 Mg/ha and 590 Mg/ha, respectively. A clear stratification of the structure ratio according to height was observed in both Traunstein and Ebersberg.

In Figure 10, we show the relationship between the structure ratio, biomass, and height. Every point is color coded according to the value of the structure ratio $S_{rat}$ (from 0 to 3); hence, blue colors indicate a low ratio (0.5), whereas red indicates a high ratio (2.5). For a fixed height, points with low biomass (dark blue) present a lower value of $S_{rat}$ than points with high biomass (red). Plots with an intermediate ratio value (green) tended to follow the allometric curve. This became more obvious in Figure 11, where biomass was plotted against the structure ratio for plots with the same height, for Traunstein and Ebersberg. A high correlation between the structure ratio and biomass was observed. It was particularly visible in Ebersberg because of the higher number of plots available. The slope of the fitting lines for Ebersberg was lower than for Traunstein.

**Biomass inversion and allometric level correction**

After applying equation 8 to both test sites, we realized that a constant bias was still present for site with respect to one another when using the same parameters $(a, b, and c)$. This bias originated because of the very different forest management plans for Traunstein and Ebersberg.
that resulted in differences in the allometric levels. An approach based on the inventory plots that are placed in the upper side of the height-to-biomass allometry (red plots in Figure 10) was used to correct for these effects (Pretzsch 2006). These plots corresponded to stands that were not treated or were abandoned by management; hence, they present a common structure which is also independent from the management for the same region and growth conditions (characterized by the allometric exponent). The bias in structure between the inventory plots placed in the upper side of the height-to-biomass allometry and the mean structure ($S_{rat}$) of the site could be used to estimate the allometric level $l_a$ and thus to correct for deviations between stands. This was calculated as the ratio between the mean $S_{rat}$ of the site (which was expected to follow the allometric curve) and the mean $S_{rat}$ of the 10% highest value for the stands above 20 m (which represents the extreme cases out of management). Thus $l_a$ is calculated using equation 9:

$$l_a = \frac{S_{rat90}}{S_{rat}}, S_{rat} \in \{S_{rat}, \ H \geq 20 \}, S_{rat90} \in \{S_{rat}, \ S_{rat} \geq 0.9\max(S_{rat}) \}$$

Therefore, keeping the empirical parameters $a, b,$ and $c$ constant for both test sites, the value of $l_a$ was site dependent and estimated directly from the structural information. $l_a$ was a multiplicative term added to equation 8 which resulted in equation 10:

$$B = l_a \cdot aS_{rat}^bH^c$$

The parameter with an optimum value for both sites resulted in: $a = 3.8$, $b = 0.8$, and $c = 0.85$; and the value of $l_a$, calculated using equation 9, for the Traunstein site was 1.75 and 1.89 for the Ebersberg site. The allometric level correction was able to compensate for the slope changes between Traunstein and Ebersberg with comparable levels of performance. The inversion results, shown in Figure 12, presented a correlation coefficient of 0.73 for Traunstein and 0.76 for Ebersberg, with a RMSE of 47.56 Mg/ha and 48.31 Mg/ha, respectively. After the
inversion, the values for the structure ratio, indicated by the color of each point, were now aligned and directly proportional to the biomass. This indicates that deviations from the height-to-biomass allometric curve can be measured using the values of $S_{rat}$. Therefore, keeping the empirical parameters $a$, $b$, and $c$ constant for both test sites, the value of $l_0$ was site dependent and estimated from the structural information.

Discussion

Forest vertical structure was modeled based on vertical biomass profiles using precisely measured ground forest inventory data as opposed to other techniques like LiDAR or multibaseline SAR. This structure was measured and parameterized using a novel technique in forestry: the decomposition of forest vertical biomass profiles in Legendre polynomial series. The Legendre polynomials showed great agreement with the natural distribution of biomass. This approach offered a characterization of the profiles which uses all the vertical forest compartments, without rejecting the lower components as other techniques, (mainly applied on LiDAR) based on profile metrics, may do (Drake et al., 2002).

The horizontal scale had a big impact on structure characterization and required further consideration. At finer spatial scales, the variability in forest features was large, suggesting that there are forest features visible at a given spatial scale but not at coarser ones (Chave and Levin 2003). Although averaging results from small sized plots reduced the variance due to error in the single-tree biomass-estimation procedure, it also masked biomass variability caused by structural variability (Shugart 2010). Forest structure analyses were also highly affected by the considered spatial scale, whereby the inventory scale influenced the overall forest structure and its dynamics over time. At a large scale (close to 1 ha), a forest stand acquired an equilibrium state with a
characteristic configuration; however, at a small scale (the so-called gap scale), the forest ecosystem state is affected by continuous changes driven by the presence of large trees. The sensitivity to the gaps has been a major factor in the structure characterization and the structure ratio \( S_{rat} \) sensitivity and will influence the further discussion.

The behavior of the Legendre coefficients indicates that AGB was primarily related to low frequency Legendre components, which strongly indicates that Legendre coefficients up to the fourth order were sufficient to describe vertical forest structure for biomass estimations. They followed a linear relationship for both test sites and showed that an average of a third of the total biomass was explained by these structural components and was sufficient for AGB estimations, which is indicated by correlations to biomass \( r^2 > 0.8 \) between the first four Legendre coefficients. However, normalization is necessary to focus only on the structural information contained in the biomass profiles. This process has shown a loss in the direct relationship between the Legendre coefficients and biomass. Nevertheless, the combination of the coefficients in the structure ratio \( S_{rat} \) was able to characterize the allometric relationships between tree structures and biomass, which can be used to improve the biomass inversions.

**Biophysical interpretation of the structure ratio**

The potential of the Legendre decomposition expressed in the structure ratio \( S_{rat} \) to characterize vertical biomass distribution is collected in the trends shown in Figure 10. These are the basis for the structure characterization developed in this study and correspond with basic allometric (biophysical) principles, which are explained in detailed in the following.

High biomass profiles were characterized by low frequency components (Legendre coefficient \( a_{01} \)), since a mature stand approached a homogeneous biomass distribution and a
high crown filling (Pretzsch 2014). However, for the same dominant height (H100), it is expected that for the sites studied in this paper, a stand with less biomass presented more gaps with a higher heterogeneity; therefore, the proportion of higher frequency components increased (Legendre coefficients $a_0$, $a_0$, and $a_0$). Hence, the structure ratio could detect this change in frequency, and it was sensitive to the biomass explained by the vertical structure profile, which already points out the capacity of the ratio to identify biomass-related changes in the gap fraction, as shown in studies like Lefsky et al (1999). The structure ratio $S_{rat}$ trends displayed in Figure 10, where the value of the ratio increased from bottom to top (proportional to biomass) and from left to right (proportional to height), were even more evident when forest height was kept constant. In this case, there was a clear and direct correlation between the values of the ratio and biomass (Figure 11). The correlation appears to be linear, although some residual effects from the relationship between height and biomass may induce some curvilinear trends, especially on the higher biomass range. Both trends are accounted by the parameters $a$ and $b$ in the inversion equation (equation 10).

The next analysis was to interpret the structure ratio, which was based on the allometric relationships that exists between the crown and the stem compartments and their impact on the vertical structure. Canopy structures have been previously studied repeatedly (Bealde et al., 1982, Lefsky et al 1999, Parker and Russ 2004). However, studies like Niinemets et al. (2006) state that up to 70–80% of the AGB is invested in support structures (woody compartments/stems), and the trade-off between the investments in stem and crown-leaf support significantly affects the possible combinations of crown structural design and strategies for light capture and biomass generation. In this study, accounting for the biomass proportion between stems and crowns has therefore introduced a new possibility to estimate the connection between
Accordingly, in this study we have observed that when biomass increased, the ratio of stem/crown increased as the structure ratio increased (Figure 11), showing that the proportion of stem wood increased in relation to foliage and branches (Petersson et al. 2012). In other words, when the stem proportion was larger with respect to the crown proportion, vertical structure profiles presented a smoother surface for a constant profile height. Thus, considering a constant height, a higher proportion of the crown compartment was translated in sharper profiles, which indicated an increase in the lower frequency components of the profile. At both tests sites, Traunstein and Ebersberg, we found that the structure ratio $S_{rat}$ was highly correlated with the proportion of biomass in the stem compartment and the biomass in the crown compartment (Figure 13). Moreover, in accordance with Niklas (2003), this trend was independent to the plot profile species composition. Although some residual variations may be the result of local growth conditions, similar slopes in the fitting lines were observed at both test sites. The same effects as discussed above for Figure 11 are observed here. Nevertheless, the linear trend was chosen as a matter of simplicity to illustrate the direct proportionality observed between the structure ratio and the stem/crown biomass proportion, and to allow a direct comparison between the two sites. The filling of canopy gap structure is still a main driver in the vertical biomass distribution resulting profile and it is also driven by the species composition. Species with complementary light ecology with an interlocking crown enables a more complete light interception of mixed stands compared with that of pure stands (Pretzsch 2014), where species are widely distributed along the canopy (Parker et al. 2004). When comparing different levels of biomass, this was translated into profiles with a higher structure ratio value (smoother vertical biomass profiles) because of a lower gap proportion (Parker and Russ, 2004). Higher biomass can be explained in
terms of higher light interception. The relationship between photosynthetic absorption and crown size is reported in studies like Canham et al. (1990) or Parker et al. (2004), whether it is quantified in terms of leaf area, crown surface and projection area, crown length or width. This is further indicated by studies showing that when trees increase their leaf area they often seem to simultaneously increase crown length or width rather than by increasing leaf area density (Pretzsch 2014).

**Aboveground biomass inversion**

The biomass inversion using the structure information contained in the structure ratio ($S_{rat}$) improved the inversion results from a height-to-biomass allometry by 20% in the correlation factor and significantly reduced the RMSE (ca 25 Mg/ha) without the need for pre-delineating homogenous forest stands.

Differences between the two sites were induced by their very different management plans; i.e. mixed close-to-natural for Traunstein and intense monoculture in Ebersberg. These can be detected by the estimation of a structure-based allometric level, therefore showing sensitivity to competitive relationships, and disturbances (Latham et al. 1998). An allometric level estimation and subsequent correction could then be conducted using structural information to access the structure-to-biomass relationships of the most densely stocked plots for each site. The considerable changes in the relationships between stand density and biomass was addressed by the structural changes, as seen between pure and mixed stands (Pretzsch 2014). In pure stands such as Ebersberg, a higher correcting value of the allometric level is explained by a narrow saddle in the relationship of maximum growth close to the maximum stand density and a progressive decrease in stand growth with a reduction in stand density. In mixed stands such as
Traunstein, with species that are complementary in light ecology and canopy space occupation, the relationship between stand density and biomass can have a much broader saddle, with higher presence of new recruitment in the lower layers after senescence of the upper layer (Luyssaert et al. 2008), and resulting in a higher carrying capacity. This translates into a lower slope in the allometric level correction.

Conclusions

This paper proposes a methodology using vertical biomass profiles modeled from ground forest inventory data to improve biomass estimations. This has shown potential applications in forests with complex structures and with biomass that cannot be estimated with sufficient precision using forest stand height. The structure-to-biomass relationships proposed here can be applied to any system that is able to profile forest vertical structure. The structure characterization can be used to detect changes in vertical structure for uses other than biomass estimations because it is particularly sensitive to changes in crown filling and changes in the gap structure.

The relationship between vertical structure and biomass as well as the physical interpretation of structure changes would be able to enhance the interpretation of reflectivity profiles obtained from RS systems such as SAR or LiDAR. The usage of ground inventory data to generate the vertical biomass profiles that have been investigated here allowed the characterization of all forest compartments which are potentially detectable by active RS systems, from airborne, ground or spaceborne systems. Since each system, from the highest frequency (e.g. LiDAR, Xband SAR) to the longest (e.g. P-band SAR), detects different biomass compartments, a methodology which is able to connect allometric relationships for a complete vertical structure
with AG biomass, can potentially be adapted and implemented with either RS system.

In this paper, we found that traditional height-to-biomass relationships were not accurate enough to estimate AGB in diverse forest systems which represent a large proportion of the world forest ecosystems (~50%). However, the high variance produced by this allometric relationship can be corrected with the inclusion of second order parameters such as structure as we have seen in two forest stands that represent typical structural conditions in Central Europe.

The structure ratio proposed here shows a direct relationship to biomass, and it can be explained by forest biophysical properties, which were tested using ground forest inventory data and modeled scenarios. The noise introduced because of the small inventory plot size was taken into account and corrected using an aggregation algorithm that discriminated between structural discrepancies based on structure ratio and profile height thresholds. The structural differences present at both test sites due to different management treatments could be accessed using the structure descriptor $S_	ext{rat}$ and corrected with the allometric level $l_a$ to obtain similar biomass inversion performances. We have experienced that areas of at least 0.5 ha are necessary to have a stable and representative estimation of the forest vertical structure (Shugart et al. 2010). The allometric relationship of forest structure to forest biomass becomes representative at this scale. Therefore, forest inventories with full tree measurements in larger areas (0.5–1 ha) would improve the understanding and development of structure allometric relationships.

This method is restricted to the empirical estimation of the exponents for each area. A further extension with more test sites, particularly in different climatic regions, would improve the applicability of the method.
Acknowledgments

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References


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Table 1. Characteristics of the two test sites.

<table>
<thead>
<tr>
<th></th>
<th>Bürgerwald Traunstein</th>
<th>Ebersberger Forst</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Forest type</strong></td>
<td>Temperate forest</td>
<td>Temperate forest</td>
</tr>
<tr>
<td>Biomass range (Mg/ha)</td>
<td>40–450</td>
<td>40–350</td>
</tr>
<tr>
<td>Height range (m)</td>
<td>10–45</td>
<td>5–40</td>
</tr>
<tr>
<td>Stand density (^a)</td>
<td>(stems/ha)</td>
<td></td>
</tr>
<tr>
<td>(min/mean/max)</td>
<td>20/1960/2140</td>
<td>20/1530/8400</td>
</tr>
<tr>
<td>Main species</td>
<td>Norway spruce (29%),</td>
<td>Norway spruce</td>
</tr>
<tr>
<td>(proportion-%)</td>
<td>European beech (28%),</td>
<td></td>
</tr>
<tr>
<td></td>
<td>silver fir (18%)</td>
<td></td>
</tr>
<tr>
<td>Management</td>
<td>Close-to-natural silviculture</td>
<td>Intensively managed for</td>
</tr>
<tr>
<td>Area</td>
<td>230 ha</td>
<td>3400 ha</td>
</tr>
<tr>
<td>Number of Plots</td>
<td>228</td>
<td>3468</td>
</tr>
<tr>
<td>Plot size</td>
<td>500 m(^2)</td>
<td>400 m(^2)</td>
</tr>
<tr>
<td>Spatial Grid (m)</td>
<td>100 × 100</td>
<td>100 × 100</td>
</tr>
<tr>
<td>Average slope</td>
<td>22 %</td>
<td>5 %</td>
</tr>
<tr>
<td>Location</td>
<td>47°51′32.45″N, 12°39′20″E</td>
<td>48°6′56″N, 11°53′31″E</td>
</tr>
</tbody>
</table>

\(^a\) Stand density is calculated without trees with dbh < 7.5 cm (diameter at breast height)
Table 2. Allometric equations selected for deriving crown biomass (Ziannis, et al 2005) as a function of tree diameter.

<table>
<thead>
<tr>
<th>Species</th>
<th>Equation</th>
<th>( R^2 )</th>
<th>Dbh(^a) range (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway spruce (<em>Picea abies</em> [L.] Karst.)</td>
<td>[ \text{Ln}(B)(\text{Mg}) = -0.2804 + 8.5242 \times \frac{(0.01 \times \text{dbh})}{(0.01 \times \text{dbh} + 13)} ]</td>
<td>0.945</td>
<td>0–50</td>
</tr>
<tr>
<td>European beech (<em>Fagus sylvatica</em> L.)</td>
<td>[ B(\text{Mg}) = 0.0031 \times \text{dbh}^{3.161} ]</td>
<td>0.924</td>
<td>-</td>
</tr>
<tr>
<td>Silver fir (<em>Abies alba</em> Mill.)</td>
<td>[ B(\text{kg}) = 0.0060722 + 9.58 \times 10^{-6} \times \text{dbh}^{2.2578} ]</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Scots pine (<em>Pinus sylvestris</em> L.)</td>
<td>[ \text{Ln}(B)(\text{Mg}) = -2.8604 + 9.1015 \times \text{dbh} ]</td>
<td>0.922</td>
<td>0–45</td>
</tr>
<tr>
<td>Sessile oak (<em>Quercus petrea</em> [Matt.] Liebl.)</td>
<td>[ B(\text{kg}) = 2.1612 \times 10^{-4} \times \text{dbh}^{2} ]</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\)Diameter at breast height, dbh (cm).
Figure Captions

**Figure 1.** Field inventory plot scheme. For each circle different dbh classes are measured. Inner circle 50m² (dbh<10cm); middle circle, 150m² (10cm<dbh<30cm); outer circle, 500 m², for Traunstein and 400m² for Ebersberg (dbh>30cm).

**Figure 2.** The tree stem is modeled as a cylinder: H = tree height and dbh = diameter at breast height. The radius of the sunlit crown $l_0$ is described by the function $r_0 = a \times \text{dist} b$ with species-specific parameters $a$ and $b$. The shaded crown $l_u$ is modeled by the linear equation $\text{dist} \times d$. The parameters and variables are as follows: $r = \text{crown radius}, l = \text{crown length}, r_0 = \text{sunlit crown radius}, l_0 = \text{length of the sunlit crown}, r_u = \text{shaded crown radius}, l_u = \text{length of the shaded crown}, r_{\text{max}} = \text{largest crown radius}, r_{\text{cb}} = \text{crown radius at crown base}, \text{dist} = \text{distance from the tip of the tree},$ and $a, b, c, d = \text{species-specific shape parameters}$ (Pretzsch, 2009).

**Figure 3.** Vertical biomass profile sample obtained from the sum of the biomass of all modeled trees inside an inventory plot in 1-m height samples.

**Figure 4.** Vertical representation of the Legendre polynomials until order 5 ($P_0$–$P_4 \ [z]$).

**Figure 5.** Biomass profile reconstruction using different number of frequencies obtained from the Legendre decomposition (blue). On the right, four polynomials were used, and on the left, eleven polynomials were used.

**Figure 6.** Correlation between Legendre components ($B_1$–$B_4$, from left to right) and total biomass. Above: Traunstein site; below: Ebersberg site.

**Figure 7.** Maximum (solid) and minimum (dashed) biomass profile reconstruction at 35 m at the test site Traunstein. Four Legendre polynomials are sufficient to detect differences in the profile.

**Figure 8.** Relative amplitude of each of the five first Legendre polynomials for the low biomass
profile (left) and the high biomass profile (right).

**Figure 9.** Height-to-biomass allometry for Traunstein and Ebersberg test sites. Dashed curves correspond to the allometric equation: $b = 1.3H^{0.58}$. Both plots are color-coded according to the value of the structure ratio from 0 (blue) to 3 (red).

**Figure 10.** Height-to-biomass relationship for Traunstein site (left) and Ebersberg site (left) after plot aggregation process. Dashed curves correspond to the allometric equation: $b = 1.3H^{0.58}$. Both plots are color-coded according to the value of the structure ratio from 0 (blue) to 3 (red).

**Figure 11.** Relationship between biomass and the structure ratio for profiles with the same height. Above: Traunstein site; below: Ebersberg site.

**Figure 12.** Height-to-biomass relationship (right) and biomass inversion (left) for Traunstein site (left) and Ebersberg site (left). The values of the coefficients from equation 10 are: $a = 16$; $b = 0.8$; $c = 0.85$. The values of $\rho$ are 0.61 for Traunstein site and 0.43 for Ebersberg site.

**Figure 13.** Relationship between crown/stem biomass ratio and the structure ratio for profiles with the same height. Above: Traunstein site; below: Ebersberg site.
Appendix A

Brink’s function

The Brink functions defined in von Gadow (2001) models the tree stem as following:

\[ r(h) = u + ve^{-ph} - we^{qh} \]

\[ u(h) = u + \frac{1}{1 - e^{q(1.3 - h)}} + (dbh - i) \left( 1 - \frac{1}{1 - e^{p(1.3 - h)}} \right), \]

\[ v = \frac{(dbh - i)e^{1.3p}}{1 - e^{p(1.3 - h)}}, \]

\[ w = \frac{ie^{-qh}}{1 - e^{q(1.3 - h)}} \]

where \( r(h) \) is the tree radius as a function altitude \( h \), \( dbh \) is the diameter at breast height, \( i \) is a species-specific asymptote parameter, \( p \) is a species-specific parameter describing the lower part of the stem, and \( q \) is a species-specific parameter describing the upper part of the stem.
Appendix reference list

Figure 3

Figure 4
Figure 5

Figure 6
Figure 7

Figure 8
Figure 9

Figure 10
Figure 11
Figure 12

Figure 13