DESIGN OF WIRELESS LINKS USING LARGE ANTENNA ARRAYS WITH CHANNEL CORRELATION

by

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Abstract

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The link where both the transmitter and receiver can use a large antenna array is studied, proposed as a method of antenna offloading and connecting access points in a Two-Tier cellular network. For correlated fading channels, receiver diversity techniques perform worse than the proposed Gram-Schmidt Precoding Algorithm, which successively orthogonalizes the column space of the transmitted streams for a single user. Due to averaging effects from using many antenna elements, filters can be approximated using channel correlation information. A block successive interference cancellation receiver is used to extend the proposed algorithm for the multi-user case. Finally the design of Hybrid Beamformers for limited RF-Chain systems is studied, where phase-only processing is done at the RF-band, followed by digital processing at the baseband. A row combiner, that clusters groups of antenna elements sufficiently correlated, is shown to outperform a random projection scheme that has a Discrete Fourier Transform matrix structure.
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Chapter 1

Introduction

Due to the increasing demand for data and proliferation of wireless devices [1], increasing the capacity of cellular networks continues to be an important objective for operators in the development of 5G systems. For the interference limited cellular network, it is important to develop techniques to ensure users receive a certain quality of service and that providers utilize their network fully. One proposal to address this issue was the idea of femto-cell networks [2] wherein we make the cells smaller and increase the density of cells. While this results in the network capacity being increased, the key issue becomes how to design the right infrastructure to connect the large number of access points. Alternatively we could improve the capacity of the cell itself, which is the approach taken by Massive MIMO [3].

In a Massive MIMO system we can have links with a large number of antenna elements at the base station (BS), and a relatively small number at the user equipment (UE) as a result of size and power constraints. Within the context of LTE rel 10 [4] it has already been proposed to use relays as a method of coverage extension, also proposing increasing the number of antennas at the BS to 8 while considering class of UE with up to 4 antennas. More recently, Rel 13 proposed employing up to 64 antennas at the BS [5]. However we can also significantly improve the link capacity by including a large number of antennas close to the terminal, and then focusing on the first link between the BS and the advanced UE. This is a $M \times N$ MIMO link where both $M$
and $N$ are large and different scenarios for the deployment of this advanced UE are possible. The second link to the UE will not "consume" time/frequency resources because the transmitted power is very low, as depending on the context this link may be several meters in comparison to the first link which may be several 100’s of meters. Within a femto-cell network, one possible application is to form a wireless backhaul[6]-[7] that ensures connectivity for all access points without the need of installing dedicated optical lines. In order to realize this, work must be done on characterizing the performance of devices which have more advanced capabilities than the traditional user equipment that typically use a single antenna.

Figure 1.1: Possible Scenarios for Massive MIMO deployment

Because there are a large number of antenna elements at either the receiver or transmitter (or possibly both), it is important to develop detection schemes which are of low complexity. While linear receivers have been well studied within the context of MIMO (e.g. configurations
of $4 \times 4$ systems), there are new insights to be gained from having a large number of antenna elements (e.g. 100 antennas). When there are a small number of antenna elements, detection schemes try to opportunistically exploit any gains available. On the other hand using a large number acts as averaging both the negative and positive gains, leading to performance gains which are more consistent. Additionally, there needs to changes made to the hardware architecture if we are to implement transceivers with 100 antenna elements since it may not be economically feasible to have an individual RF-chain for each antenna element. The RF-chain is responsible for converting the received modulated signal to the baseband (BB) and sampling it appropriately so that the received analog signal is converted to a digital signal and allows for standard digital signal processing (DSP) methods to be applied for symbol detection. Rather than having a dedicated RF-chain for each antenna element, consider having a system where the number of antenna elements is an order of magnitude greater where it is not economically feasible to have so many RF-chains and the ADC would present a power bottleneck. This train of thought leads to considering the limited RF-chain problem in a Massive MIMO setting, where RF-preprocessing is performed before baseband processing and presents a more realizable version of Massive MIMO that could be deployed for 5G systems. Due to space constraints of packing antenna elements within specific sites, it should be expected that channel correlation will increase which degrades the overall system performance. Hence it is important to consider design parameters such as the number of receive or transmit antennas and characterize the performance under various level of correlation which can be categorized broadly as minimal, moderate and severe.

1.1 Literature Review

The common linear receivers which we will consider as a benchmark are the Matched Filter (MF), Zero-Forcing receiver (ZF), and minimum mean square error estimate receiver (MMSE). The MF maximizes the signal to noise ratio for the single antenna single user (SU) transmitter.
by way of the Cauchy-Schwartz inequality, and is of low complexity since it requires no inversion of a matrix unlike the ZF and MMSE receiver. However when there are multiple streams being transmitted, the MF suffers from an interference floor. In such cases, the ZF receiver would provide significant improvement in terms of figure of merits such as the sum rate. This comes at the cost of added complexity as the ZF receiver can be viewed as the cascade of the matched filter with another matrix that decorrelates any interference. A drawback of the ZF receiver is in the low signal to noise ratio (SNR) regime, noise enhancement results in the MF to performing better. The MMSE receiver balances the interference due to interfering streams and the noise and acts as the MF in the low SNR regime and the ZF receiver in the high SNR regime. When the number of antennas on the receiver and transmitter side become moderately large, the resulting complexity of matrix inversion may prohibit the use of the ZF/MMSE receiver due the channel state information being outdated. This motivated designing low complexity receivers for Massive MIMO by applying approximations to the inverse calculation required for ZF/MMSE receiver such as in [8]. Rather than taking this approach, shortcomings of the ZF/MMSE receiver for symmetric links is highlighted and a precoding algorithm is proposed.

In Marzetta’s paper [3] it was proposed to utilize a very large number of antenna elements at the BS while the UE only had a single antenna element. Under the independent fading assumption, it was shown that the MF would maximize the SNR while at the same time nulling the inter-user interference for an asymptotically large number of receive antenna elements. In practice it is not possible to deploy such a numbers, hence work was done on characterizing the performance the MF achieves as a percentage of the MMSE receiver in [9]. Due to size constraints it is possible that the inter-antenna element spacing would not be sufficient enough to ensure uncorrelated fading at the transmitter and/or receiver. This has led to studies on the effect of channel correlation and the loss of performance as in [10]. However there has not been much work which addresses how to design symmetric links where both the transmitter and receiver employ a considerable number of antenna elements. This then also raises the question of whether anything is gained from employing additional antennas which could incur higher lev-
Chapter 1. Introduction

... correlation \[11\] and how to overcome the severe performance degradation the ZF receiver undergoes. These issues are addressed in more detail in Chapter 3.

The limited RF-chain problem results in the design of a RF-band processing matrix that is in cascade with the baseband matrix. Due to implementation constraints, the RF-band matrix is considered to consist of phase only elements, each with equal magnitude and implemented by variable phase shifters. One approach is to select a specific set of antennas which equals the number of RF-chains available and then proceed to design the digital pre/postcoder. The antenna selection problem results in searching over all possible combinations that maximizes some utility function. When there are a large number of antenna elements to choose from, an exhaustive search is not possible and heuristic methods can be applied, such as looking at the effect of adding an antenna to the channel capacity formula \[12\] and iteratively choosing antennas which gives the highest increase. Within the context of spatial beamforming, adaptive nulling with no amplitude control has been considered in \[13\] and related works. Equal gain combining is a form of phase only processing, which is shown to approach the performance of the MF without the need of amplitude control. For massive MIMO systems, the goal is to benefit from having a large number of antennas despite only utilizing phase-only processing in the RF-band and DSP in the baseband. This problem can be thought of as designing the projection matrices at the transmitter and receiver with the phase only constraint. Several recent papers considered the approach of minimizing the Frobenius norm of the error matrix with respect to the fully digital solution\[14\]. In \[15\] and related works, a Discrete Fourier Transform (DFT) matrix is considered for the RF-processing stage due to its relative ease of implementation and simplifying the precoder/postcoder design problem. While such schemes do not require channel station information and be suboptimal, it also reduces the error that can occur due to phase noise in the analog signal processing stage. Other works has interpreted the limited RF-chain system in relation to an antenna switch architecture\[16\], which allows to have zero elements within the RF-band matrix. Going along this line of reasoning, a simple channel combiner is proposed for the RF-processing stage to take advantage of the channel correlation...
in order to get additional diversity gains and is compared against the DFT matrix. When there is correlation at the transmitter and receiver, the RF stage is responsible for essentially spreading and despreading the transmitted symbol.

1.2 Contributions

This thesis studies the design of wireless links where either the receiver and/or transmitter use a large antenna array. Using the sum rate as a figure of merit, the performance of several linear receivers is considered, and the impact of channel correlation as a result of using many antenna elements is characterized. The limited RF-chain system is considered since it represents a more realizable version of Massive MIMO in practice, and its system performance is evaluated. The main contributions of this thesis are as follows:

- Precoding the transmitted symbol using the $QR$-Decomposition of the channel so that each stream is transmitted along successively orthogonal components of the channel, essentially factorizing the ZF receiver and splitting the processing between the receiver and transmitter.

- When there is a large antenna array at the receiver and transmit side correlation matrix is known, the precoder matrix for the proposed Gram-Schmidt algorithm is shown to be the inverse of the Cholesky decomposition of the transmit side correlation matrix, which relaxes the constraint of having CSI at the transmitter. The ZF and MMSE receiver can be also approximated using the inverse of the transmit side correlation matrix to form a refined matched filter, where having a large number of antenna elements allows the simplification of calculations by using the weak law of large numbers.

- Precoding for the Multi-user channel with no data sharing, the SIC receiver can be used to reduce the CSI overhead to the transmitters. The filter used for suppressing inter-user interference can be interpreted as a block zero-forcing receiver and follows from
calculating an orthonormal basis for the interference

- For the limited RF-chain system, equal gain combining is shown to null the inter-user interference when there is an asymptotically large number of antenna elements

- Proposed a row combiner which relaxes the constraint of phase-only elements at the RF band to allow 0 elements too

- Performance comparison between the row combiner and the static DFT matrix which do not require CSI at the RF stage for pre/post-processing, and the effect of varying the ratio of RF-chains to number of antenna elements when the receiver and/or the transmitter have a limited number of RF-chains in the presence of channel correlation

### 1.3 Organization

The thesis is organized as follows: Chapter 2 goes through the background required by the reader to follow the discussion presented, along with system model and notation that will be used for the remainder of this paper unless stated otherwise. Chapter 3 looks into the precoder/postcoder design problem when a large antenna array is used either at the transmitter or receiver, the effect of channel correlation on the overall system performance when using the Gram-Schmidt precoding algorithm in comparison to the MF/ZF/MMSE receivers. Next the limited RF-chain system which places a phase-only constraint in the precoder design is explored in Chapter 4. Taking cue from the antenna selection algorithms, a simple row combiner is proposed at the RF stage as opposed implementing the phase elements to follow a DFT matrix structure. The linear receivers from Chapter 3 are used to evaluate the different RF pre-processing matrices, varying the correlation levels and ratio of antenna elements to RF-chains. Finally Chapter 5 summarizes the main results and possible directions on future work.
Chapter 2

Background

There are various parameters which are associated with a wireless channel and can affect the structure of the received signal. Relative motion between the receiver and the transmitter causes frequency components of the signal to be shifted due to the Doppler effect. However within the context of having large antenna arrays for the use of wireless backhaul, it can be assumed that the users are fixed in position. Due to the broadcast nature of the wireless channel and reflections in the environment causing multi-path propagation, a simple model for received signal is the tap-delay model\textsuperscript{[17]}. This characterizes the channel with the following impulse response \textsuperscript{2.1}

\[ h(t) = \sum_{i=1}^{P} a_i \delta(t - \tau_i) \]  

Taking the Fourier transform of $h(t)$ gives us the transfer function of the channel, and is generally not constant over frequency. Such channels are referred to as frequency selective channels. If the message bandwidth is appropriately small, the effect of the channel in the frequency domain is simply a multiplication by a constant factor. This brings up the idea of a channel gain, and is commonly called small scale fading. In practice orthogonal frequency division multiplexing (OFDM)\textsuperscript{[18]} is used to convert the multi-path channel into a flat fading channel which is characterized by its channel gain. Now suppose that the transmitter and receiver have multiple antenna elements. Each transmitter-receiver antenna element pair would have an associated
channel gain, as illustrated in Fig. 2.1. On top of the flat fading effects introduced by channel,

\[ y = Hx + n \tag{2.2} \]

where \( y \) is the received signal vector where the \( i \)-th component is the signal received at the \( i \)-th antenna element, \( H \) is the matrix which describes the channel gain between each transmitter-receiver antenna element pair, \( x \) is the collection of transmitted signals, and \( n \) is the noise at the receiver. It is generally assumed this noise is Gaussian and uncorrelated, and this model is called additive white Gaussian noise (AWGN). In order to transmit a wireless message the message signal is typically modulated by a carrier. This modulated signal is in the bandpass and can be written as a complex low pass signal. Suppose that there is no line of sight path
between each transmitter and receiver, i.e. rich scattering environment. Each element of $H$ can be modeled to follow a complex normal distribution with mean 0 and unit variance, denoted by $\mathcal{CN}(0, 1)$, and are independently and identically distributed (i.i.d.). For the single transmitter and receiver case, this is the well known Rayleigh fading channel. If $H$ has full rank, then the maximum number of streams that can be sent is $\min(N_R, N_T)$. However in practice channel correlation exists due to the environment, antenna array structure, inter-element spacing and various other reasons. In general correlation is bad since when one antenna element has poor channel gains, it implies that the other antenna elements are under similarly poor conditions as well. Instead it is desirable to have as little as correlation as possible between antennas. This is apparent in the extreme situation where the elements overlap and corresponds to being completely correlated. In such a case the rank of $H$ becomes 1 and represents the number of streams that can be spatially resolved at the receiver. A simple array structure is the uniform linear array, where the inter-antenna element spacing is constant. As a general rule of thumb, the closer the antenna elements are, the higher the correlation is. In the context of massive MIMO, due to having such a large number of antenna elements and physical constraints on the size of the array, it is necessary to consider the effects of channel correlation. Under the assumption that the transmitters and receivers are sufficiently far enough such that the correlation caused on the receiver side has no effect on the transmitter side and vice versa, the following model is employed

$$H = R_t^\frac{1}{2} G R_t^\frac{1}{2} H$$

(2.3)

where now the elements of $G$ follow a $\mathcal{CN}(0, 1)$ and are i.i.d., and $R_t^\frac{1}{2} R_t^\frac{1}{2} H = R$ with the subscripts distinguishing between the receiver and transmitter side correlation matrices. This channel model is called the Kronecker model and is popular for its’ analytical tractability. First assume that the transmitter induces a correlation, while the receiver has been appropriately spaced such that there is no receiver side correlation. This means that the channel gains from the transmit antenna elements to one receive antenna element would be correlated, and it would be the same for all other antenna elements at the receiver. This means that elements within
the row of the channel gain matrix are correlated, which is captured by $GR_i^{1/2H}$. Now when
the receiver also introduces correlation, the elements along the column of the channel gain
matrix will be correlated and the structure of eq. [2.3] follows. For the ensuing work the antenna
correlation model in [19] is adopted, where the correlation matrix is characterized by a single
parameter $\alpha$ and has a Toeplitz structure (correlation between $i$-th and $j$-th antenna is $\alpha^{(i-j)^2}$).
Now consider a single cell where 2 users, each possessing $N_{Ti}$ transmit antennas, are scheduled
to transmit to a single BS which has $N_R$ receive antennas ($N_R \geq N_{T1} + N_{T2}$). The received
signal vector for the multi-user MIMO channel can be written as

$$y = H_1x_1 + H_2x_2 + n$$ (2.4)

Typically the transmitted signal is precoded by some matrix $V_i$ such that $x_i = V_is_i$ and the
precoding matrix columns are normalized to have unit norm to ensure the transmit power con-
straint $\mathbb{E}[|x|^2] \leq P$ is not violated. The users are assumed to be distributed uniformly around
the BS and experience path loss and log-normal shadowing, along with the small-scale fading
mentioned before. There are two cases for the received power from the users. For the unequal
case, we assume each user transmits with the same power $P$, hence the received signal power
for the $i$-th user is given by $P_{r,i}$

$$P_{r,i} = \frac{L_iP}{d_i^\gamma}$$ (2.5)

where $L_i$ is the log-normal shadowing and $d_i^\gamma$ is the path loss experienced by the $i$-th user. In the
equal received power case either the users are co-located so that they undergo similar fading or
are able to employ power control with the BS coordinating the transmit power levels of each UE.
We assume that the BS has perfect CSI from all users, and is therefore able to employ receiver
diversity techniques. On the transmitter side, for precoding global CSI is available but each user
only has access to local data and therefore block diagonalize methods can not be employed. The
case where $N_T = 1$ is a special case where we can’t do any precoding.
Chapter 2. Background

2.1 Linear Receivers for Symbol Detection

We denote the channel gain matrix by \( H \in \mathbb{C}^{N_R \times N_T} \). Linear receivers work by forming some estimate \( \tilde{s} \) of the transmitted vector \( s \) from the received vector signal \( y \) through multiplication of some matrix \( W \) and then using a slicer \( Q() \) that maps the estimated symbol to a symbol from the constellation \( \hat{s} \) which is known a priori. That is

\[
\tilde{s} = W^H y \quad (2.6)
\]

\[
\hat{s} = Q(\tilde{s}) \quad (2.7)
\]

If we take matched received filter (MF) from the single user case and extend it to the multi-user case we have \( W_{MF} = H^H \). While each stream will suffer from more interference, this scheme is of relatively low complexity to implement as it consists of just taking the conjugate transpose of the channel gain matrix.

2.2 Zero-Forcing and Minimum Mean Square Error Receiver

Here we present some well known results from statistics that has been used in communications. Assuming the system model for the single user MIMO channel, the symbol estimate can be formed depending on some criteria. If the criteria were to minimize the norm between the received vector signal and the candidate transmitted vector signal after undergoing the effects of the channel gain matrix, the estimation becomes the well-known least-squares approximation which is given by

\[
\tilde{x} = (H^H H)^{-1} H^H y \quad (2.8)
\]

and is what we refer to as the ZF receiver from nulling the interference due to other transmitted symbols. The solution follows from differentiating the norm squared of \( y - Hx \) equating to zero and solving for \( x \). No assumptions have been made on the distribution of \( n \), however it is important to note that we require \( N_R \geq N_T \). Another criteria could be minimizing the mean
square error, given by \( E[||y - Hx||^2] \). It turns out that this results in the following decoding scheme:

\[
\tilde{x} = \mathbb{E}[x|y]
\]  

(2.9)

If \( x \) is assumed to follow a Gaussian distribution and has a correlation matrix given by \( PI \) and \( n \) is AWGN, then the MMSE estimate of the transmitted symbol is given by

\[
\hat{x} = (H^H H + \frac{1}{\Gamma} I)^{-1} H^H y
\]  

(2.10)

where \( \Gamma \) is the transmit signal to noise (SNR) ratio. In the high SNR regime, the ZF receiver approaches MMSE receiver performance as the regularization constant \( \frac{1}{\Gamma} \) vanishes. The advantage of these methods is that channel state information (CSI) is only required at the receiver side, rather than also at the transmitter.

### 2.3 Successive Interference Cancellation Receiver

Let the received signal be written as follows

\[
y = \sum_i h_i x_i + n
\]  

(2.11)

where \( h_i \) is the \( i \)-th column of \( H \) and \( x_i \) is the symbol transmitted by the \( i \)-th transmit antenna. After a single symbol is detected, then its’ effect is subtracted from the received symbol vector and this procedure is repeated to iteratively decode the transmitted symbols. Letting \( \hat{x}_j \) denote the decoded symbol and \( w_j \) the receive filter to make the decision statistic \( \tilde{y}_j \), the algorithm performs as follows

\[
\tilde{y}_j = w_j^H y
\]  

(2.12)

\[
\hat{x}_j = Q(\tilde{y}_j)
\]  

(2.13)

\[
y' = y - h_j \hat{x}_j
\]  

(2.14)
where $Q()$ is the slicer that uses minimum distance decoder. Foschini et al. proposed the V-BLAST algorithm \[20\] which combines the linear filters from the prior section with the SIC receiver. In this case, $w_j$ is taken to be

$$w_j = \arg \min_i \| (H^H H)^{-1} H^H(i,:) \| \tag{2.15}$$

where $j$ is the row of the ZF with the smallest magnitude. To understand why this works, we look at the estimate of the sent signal as a function of the received signal

$$\tilde{x}_j = (H^H H)^{-1} H^H y \tag{2.16}$$

$$= x_j + (H^H H)^{-1} H^H n \tag{2.17}$$

Over all symbols, the row of the receive filter with the least magnitude will have the highest SNR over all $x_i$’s. This method of decoding is prone to error propagation, since if we make a mistake in decoding at one stage, we use the incorrect symbols to decode the following symbols.

### 2.4 $QR$-Decomposition

The $QR$-decomposition of a matrix takes the columns of a matrix $H$ and forms an orthonormal basis using the Gram-Schmidt algorithm. Since each of the original column vectors of $H$ can be represented in this basis, we can rewrite the matrix as follows

$$H = QR \tag{2.18}$$

where $Q$ satisfies the relationship $Q^H Q = I$ and $R$ is an upper triangular matrix. Because of the structure of $R$, the $QR$-decomposition has been studied in context of applying the SIC receiver and decoding the transmitted symbols bottom up.

For the first column of $Q$ we initialize by a normalized column vector from $H$. Rewriting
the equation for the ZF and MF filter using the QR decomposition, we get $W_{ZF}^H = R^{-1}Q^H$ and $W_{MF}^H = R^HQ^H$. The inverse of an upper triangular matrix is always an upper triangular matrix, and the Hermitian of an upper triangular matrix will result in a lower triangular matrix. Hence the only case in which these two filters could be equivalent is when $R$ is a diagonal matrix (note: the SNR is unaffected by a scaling constant by the receiver side). That is when the columns of $H$ are already orthogonal.

### 2.5 Singular Value Decomposition (SVD)

This linear receiver essentially diagonalizes the channel and represents an upper bound on the performance achievable with respect to the sum rate. Given a matrix $H$, the SVD results in three matrices $U$, $\Sigma$, $V$ such that

$$H = U\Sigma V^H \quad (2.19)$$

where $U$, $V$ are unitary matrices of size $N_R \times N_R$ and $N_T \times N_T$ respectively and $\Sigma$ is a diagonal matrix with entries $\sigma_i$. The maximum number of streams that will be supported is $\min(N_R, N_T)$ and for the $i$-th stream the postcoder is given by the $i$-th column of $U$ and similarly the precoder is the $i$-th column of $V$. Hence the received signal can be written as

$$\tilde{y} = \Sigma s + n \quad (2.20)$$

The main drawback of this method is the complexity of this scheme, which for the large values of $N_R, N_T$ considered for massive MIMO systems requires the consideration of low-complexity receivers.

### 2.6 Massive MIMO System

In a transmitter-receiver pair, employing multiple antennas lead to diversity and multiplexing gains. For a well behaved channel, the multiplexing gain is given by $\min(N_R, N_T)$, so increas-
ing one would only serve to get additional diversity gains. Suppose we have the SU-MIMO channel with no precoding \((V = I)\) and the channel is uncorrelated. Assuming \(N_R\) is very large, multiplying by the MF to decode the \(i\)-th symbol yields

\[
h_i^H y = h_i^H H x + h_i^H n \tag{2.21}
\]

\[
= ||h_i||^2 x_i + \sum_{j \neq i} h_i^H h_j x_j + \tilde{n} \tag{2.22}
\]

\[
\rightarrow ||h_i||^2 x_i + \sum_{j \neq i} N_R E[h_i(l) * h_j(l)]x_j + \tilde{n} \tag{2.23}
\]

\[
= ||h_i||^2 x_i + \tilde{n} \tag{2.24}
\]

where the third line is an approximation from the weak law of large numbers since the inner product between the \(i\)-th and \(j\)-th column of the channel gain matrix \(H\) is the sum of \(N_R\) i.i.d. terms. Therefore increasing the number of receive antenna elements yielded the surprising benefit of simplifying the receive filter. When channel correlation is introduced, this approximation needs to be appropriately modified and is gone into more detail in Chapter 3. The pre/postcoder design problem can be stated as finding matrices \(W^H \in \mathbb{C}^{d \times N_R}\) and \(V \in \mathbb{C}^{N_T \times d}\), where \(d\) is the number of independent streams to be transmitted. Depending on the situation \(N_R \leq N_T\) or \(N_R \geq N_T\), which are referred to as the downlink and uplink respectively and this thesis considers the latter. It also follows that \(d \leq \min(N_R, N_T)\). While designing precoders under the assumption of transmitting \(d = N_T\) streams would be suboptimal when operating with \(d < N_T\), the symmetric link where \(N_R = N_T\) is a case that is considered.

At either the transmitter or receiver, each antenna element has its’ own RF-chain. This is necessary in order to apply digital signal processing techniques on the signal associated with each antenna element. For large values of \(N_R\) and \(N_T\), it may not be feasible to have and equal number of RF-chains. This brings up the notion of a limited RF-chain system where \(L_R\) and \(L_T\) denote the number used by the receiver and transmitter respectively, and the case where they are equal the subscript is dropped.
One approach to the limited RF-chain problem is to select a subset of receive and transmit antennas such that the capacity of the resulting channel is a maximum. The brute force approach would be to go through every possible combination, but is not practical to carry out, especially for moderate values of $N_R, N_T$. Heuristic algorithms such as that presented in [12] were developed to address this issue, but is still relatively computationally intensive due to requiring a selection step. Instead, it would be beneficial to see what the performance is when the selection step is skipped and the remaining antenna elements can be used, at the cost limited signal processing capabilities. Since there are only $L_T$ or $L_R$ RF-chains, this set of signals must be mapped to or from $N_T$ or $N_R$ antenna elements using variable phase shifters with fixed amplitude gain. This leads to the pre/postcoder design to have a digital component that is performed at the baseband and an analog component at the RF-band, and is what is referred to as Hybrid Beamforming in this thesis.

For the case of phase-only processing as is considered for the RF-chain limited system, the MF is replaced by taking only the phase component of each complex element in the corresponding vector and is called equal gain combining (EGC). Let the $l$-th element in the $k$-th column be denoted by $|h_{l,k}|e^{\phi_{l,k}}$, the corresponding element with only a phase component is given by $e^{\phi_{l,k}}$ where $\phi_{l,k}$ is a uniform random variable on the interval $[-\pi, \pi]$. The EGC receiver and its application in designing RF pre/postcoder for the limited RF-chain system is discussed in Chapter 4.
Chapter 3

Pre/postcoder Design using

Gram-Schmidt Algorithm

Assume that full CSI is available at the transmitter side and precoding can be done. The received signal vector can be written as

\[ y = Hx + n \]

where the sum is invariant under the ordering of the channel vectors \( h_i \). Without loss of generality it is assumed the channel vectors are sorted in descending magnitude. Each symbol can be thought of as being associated with the column space corresponding to its channel vector.

In the case when the channel is orthogonal (i.e. \( H^H H = I \)), the matched filter is orthogonal to the other channel vectors. For the more general case, the ZF filter is applied to cancel the inter-stream interference and works by finding the component of each channel vector that is orthogonal to every other channel vector. It should be noted here that it is assumed the number of independent streams \( d \) is equal to the number of transmit antenna elements \( N_T \). One drawback with the ZF receiver is it discards the component of the channel vector that overlaps with other
streams’ channel vectors. Hence it is wasteful in this manner since no stream ends up utilizing this resource, as if it were, as illustrated in Fig. 3.1. A more efficient scheme would be to

![Figure 3.1: Subspace representation for 2 Transmitters](image)

have stream 1 transmit freely while having the second stream transmit along the space that is orthogonal to the first stream. If there is a third stream, then it will transmit in the space along to the first and second stream, and so on for a general number of streams. The Gram-Schmidt algorithm takes a set of vectors and returns an orthonormal basis by successively calculating the orthogonal component of each vector with respect to the prior vectors and is similar to the idea of controlling the space occupied by each stream described previously. Motivated by this, the following section goes into detail on employing the Gram-Schmidt algorithm to design pre/postcoders for the single user MIMO channel.

### 3.1 Single User MIMO

Writing the columns of $\mathbf{H}$ in terms of the constructed orthonormal basis results in the $QR$-decomposition of the channel matrix. Employing the strategy of having each user align their transmission so as to be orthogonal to prior users means the effective channel is now $\mathbf{Q}$. This
is apparent when the transmitted symbols are precoded by $R^{-1}$ as shown in

$$
y = Hx + n$$  \hspace{1cm} (3.3)

$$
y = QRR^{-1}As + n$$  \hspace{1cm} (3.4)

$$
y = QAs + n$$  \hspace{1cm} (3.5)

where $A$ is a diagonal matrix such that the Frobenius norm of $R^{-1}A$ is equal to 1 to ensure normalized precoding so that no transmit power constraints are violated. The diagonal constraint is so that the receiver need simply multiply by $Q^H$ and can decode each stream in parallel by using the minimum distance decoder. One way to determine the matrix $A$ is to cast it as an optimization problem where the sum rate of the link is maximized. The metric taken to characterize the performance is the sum rate which can be expressed as

$$
R_{sum} = \sum_{i=1}^{NT} \log_2 \left( 1 + \gamma_i \right)
$$  \hspace{1cm} (3.6)

where $\gamma_i$ is the post-processing SNR, $s_i$ the $i$-th element in the transmitted signal vector and $\sigma^2$ is the noise power. Writing the $i$-th diagonal element of $A$ as $a_i$ gives

$$
\max_{a_i, P_i} \quad R_{sum} = \sum_{i=1}^{NT} \log_2 \left( 1 + \frac{a_i^2 P_i}{\sigma^2} \right)
$$  \hspace{1cm} (3.7)

$$
\text{s.t } \sum_{i=1}^{NT} ||R^{-1}(; i)||^2 a_i^2 P_i \leq P, \quad a_i^2 P_i \geq 0
$$  \hspace{1cm} (3.8)

This is similar to the weighted sum rate maximization problem, however in this case it is the power constraint which has a weighted form. This can be recast by an appropriate scaling of
variables giving

\[
\max_{\hat{P}_i} \sum_{i=1}^{N_T} \log_2 \left( 1 + \frac{||R^{-1}(.; i)||^{-2} \hat{P}_i}{\sigma^2} \right) \\
\text{s.t} \sum_{i=1}^{N_T} \hat{P}_i \leq P \quad \hat{P}_i \geq 0
\]  

(3.9)

(3.10)

and the solution is given by the water-filling algorithm. When there are a lot of transmit antennas, it may not be feasible to use such methods on the go without CSI becoming outdated. Within this context, the elements of \( A \) are taken to be the inverse of the norm of the columns of \( N_T R^{-1} \) to ensure normalized precoding. For the example of multiple transmitting streams, the first stream does not have to do any alignment and can be decoded using the matched filter. And the second stream has to align itself with the first stream, while the third stream has to align itself with the first and second and so on. The performance of this scheme is expected to be higher than the case where each stream must be orthogonal to every \( N_T - 1 \) other streams, especially in cases where there is high channel correlation.

As previously mentioned, the ordering in which the streams have have to be aligned will impact \( R \) and hence have an impact on the overall performance. An exhaustive search over all permutations of ordering the streams will yield the max sum rate. However, this would be on the order of \( N_T! \) and is prohibitive. Instead, it is proposed to use an ordering based on the channel gains \( ||h_i|| \), where the channel gains of each stream can be viewed as the capability of aligning with other streams while undergoing SNR degradation. The case of maximizing the minimum SNR over all streams would be to order the channel in ascending order, however for the single user case maximizing the overall sum rate at the cost of a few streams is allowable. Therefore, assuming the channel is ordered such that \( ||h_i|| \)'s are in descending order and the precoding matrix \( R^{-1} \) is calculated by applying the \( QR \)-decomposition on the resulting \( H \). It is of interest to note the performance gains as the number of independent streams are changed, and how the number of receive antennas affect the need to orthogonalize the streams. The SNR for \( i \)-th stream for the ZF receiver and Gram-Schmidt precoding can be given in terms of \( R \) as
follows

\[
\gamma_{ZF,i} = \frac{\mathbb{E}[|s_i|^2]}{||R^{-1}(i,:)||^2}
\]

(3.11)

\[
\gamma_{GS,i} = \frac{\mathbb{E}[|s_i|^2]}{||R^{-1}(,:i)||^2}
\]

(3.12)

where \(B(i,:)\) is used to denote the \(i\)-th row and similarly \(B(:,i)\) the \(i\)-th column of a matrix \(B\).

Using these expressions for the post-processing SNR, the sum rate is calculated to compare the performance between the two. For the case of 2 transmit antennas and an arbitrary number of receive antennas, the result is proven analytically while the general case of \(N_T\) transmit antennas is verified through simulations. First it is assumed that the post processing SNR is considerably greater than 1 and the following approximation for the sum rate can be applied

\[
\sum_i \log_2(1 + \gamma_i) \approx \log_2(\gamma_1) + \log_2(\gamma_2)
\]

(3.13)

\[
= \log_2(\gamma_1\gamma_2)
\]

(3.14)

Hence the comparison between the product of the squared norms of the columns and rows of \(R^{-1}\) will tell which scheme has the higher sum rate. The matrix \(R\) is a \(2 \times 2\) upper triangular matrix of the form given in eq. 3.15

\[
R = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}
\]

(3.15)

Since the magnitude of the columns of \(R\) equals the corresponding column in \(H\), it follows that \(|a|^2 \geq |b|^2 + |c|^2\) i.e. \(|a| \geq |b|, |a| \geq |c|\). The inverse of \(R^{-1}\) is straightforward to compute, and comparing the product of squared norms of the columns and rows, all that needs to be done is show the former is less than the latter and the proof is complete. This requires the inequality in
eq. 3.17 to hold

\[
\frac{1}{|a|^2} \left( \frac{1}{|c|^2} + \frac{|b|^2}{|a|^2|c|^2} \right) \leq \frac{1}{|c|^2} \left( \frac{1}{|a|^2} + \frac{|b|^2}{|a|^2|c|^2} \right)
\]

(3.16)

\[
\frac{1}{|a|^2} \leq \frac{1}{|c|^2}
\]

(3.17)

which follows from the initial assumption on the channel magnitudes of \( \mathbf{H} \). Therefore the Gram-Schmidt precoding leads to higher performance than the ZF receiver regardless of the channel with respect to the sum rate in the high SNR regime.

### 3.1.1 Precoder Design with Channel Correlation at Transmitter

From studies in massive MIMO, it is expected that as \( N_R \) grows large, the channels become quasi-orthogonal due to channel hardening\(^2\). Generally is it difficult for the transmitter to obtain CSI, especially so in the case of massive MIMO. Hence a more practical solution would be for the transmitter to utilize channel correlation information while the receiver has access to CSI from the initial pilot training phase. For the case where we have a large antenna array at the receiver with adequate spacing, the received signal model with transmitter side correlation is

\[
y = \mathbf{H} \mathbf{x} + \mathbf{n} \quad \text{(3.18)}
\]

\[
= \mathbf{G} \mathbf{R}_T^{\frac{1}{2}} \mathbf{x} + \mathbf{n}
\]

(3.19)

The precoding vectors for the first and second stream by using Gram-Schmidt precoding for \( N_T = 2 \) are given up to a scaling constant by

\[
\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{v}_2 = c \begin{pmatrix} -\langle \mathbf{h}_1, \mathbf{h}_2 \rangle \\ ||\mathbf{h}_1||^2 \end{pmatrix}
\]

(3.20)
When we have a large number of receive antennas, we can interpret the inner product as the sum of many i.i.d. terms, and apply the weak law of large numbers to approximate a random quantity by its mean. The variance of this quantity bounds the probability by which the r.v. deviates from the approximation by way of the Chebyshev inequality. Taking the quantity $||h_1||^2$, unless this information is fed back from the receiver, is unknown to the transmitter. For large $N_R$, applying the weak law of large numbers results in $||h_1||^2 \to \mathbb{E}[||h_1||^2] = N_R$. For the remainder of this paper, the notation $\to$ will be used as invoking the weak law of large numbers to approximate a r.v. by a deterministic quantity, unless noted otherwise. Similarly we can substitute the inner product $<h_1, h_2>$ by the cross correlation as writing it out in terms of the elements of the columns gives $\sum l h_1(l)^* h_2(l) \to N_R \mathbb{E}[h_1(l)^* h_2(l)] = N_R \alpha$, where $\alpha$ is the $(1, 2)$ element in $R_{Tx}$. Normalizing $v_2$, we get

$$v_2 = \frac{1}{\sqrt{1 + |\alpha|^2}} \begin{pmatrix} -\alpha \\ 1 \end{pmatrix} \quad (3.21)$$

Simulation results show that for a large receiver antenna array, e.g. $N_R = 64, 100$, the precoding vector calculated from using CSI is close to the one obtained by using the method outlined here where we just used the transmitter side channel correlation information (CCI). Let $V$ be the precoding matrix calculated from using CSI while $\tilde{V}$ is the estimate of $V$ by using the CCI. The mismatch between these two is captured by considering the received signal when we use $\tilde{V}$

$$y = H\tilde{V}s + n \quad (3.22)$$
$$= H \left( V + \tilde{V} - V \right) s + n \quad (3.23)$$
$$= HVs + H \left( \tilde{V} - V \right) s + n \quad (3.24)$$
The first column of $V$ requires no CSI for calculation, hence the mismatch only occurs for $v_2$. More specifically from the approximation of

$$\frac{\langle h_1, h_2 \rangle}{||h_1||^2} \approx \alpha$$

(3.25)

This in turn leads to a mismatch in the normalization constant due to the factor of $\sqrt{1 + |\alpha|^2}$. Looking at this plot as a function of $|\alpha|$, it is observed for values close to 0 the second term of $v_2$ holds up better to approximation compared to the first term, while the opposite holds true for values closer to 1. Using CCI designed precoder also facilitates the use of precoded pilot signals, hence the receiver would just need to apply the matched filter of the effective channel to decode the transmitted streams. As before, it is only the second column of $Q$ that is approximated, in this case by

$$h_2 - \frac{\langle h_1, h_2 \rangle}{||h_1||^2} h_1 \approx h_2 - \alpha h_1$$

(3.26)

Recalculating the orthogonal component of the second stream to the first stream at the receiver side will ensure that the approximation of $Q$ is equal to the actual $Q$, but this yields no benefits over just sending regular pilot symbols and calculating $Q$ with CSI on the receiver side. Denoting the error in the approximation by $\epsilon$ gives

$$\epsilon = \left( \alpha - \frac{\langle h_1, h_2 \rangle}{||h_1||^2} \right) h_1$$

(3.27)

Generally taking the expectation of a ratio of random variables is nontrivial, especially when there is some dependency. Under the assumption that $||h_1||^2 \approx N_R$, the expression for the error
can be simplified as follows

\[ \epsilon = \left( \alpha - \frac{\langle h_1, h_2 \rangle}{N_R} \right) h_1 \]

\[ = \left( \alpha - \frac{1}{N_R} \sum_{l=1}^{N_R} h_1^*(l) h_2(l) \right) h_1 \]  

\[ \rightarrow (\alpha - \eta) h_1 \]  

where \( \eta \) can be taken to be a complex normal r.v. from the central limit theorem with mean \( \alpha \) and variance

\[ \sigma_\eta^2 = \frac{1}{N_R} \left\{ \mathbb{E}[|h_1(l)h_2(l)|^2] - \alpha^2 \right\} \]  

\[ \rightarrow 0 \]  

Hence the estimate for \( Q \) using CCI is asymptotically unbiased with respect to the number of receive antennas.

For the general case where \( N_T \) is an arbitrary number, note that the channel is modeled as \( GR_{iq}^{\frac{1}{2}}H \) and compare this to the form of the \( QR \)-decomposition \( QR \). For a large number of receive antennas the columns of \( G \) become quasi-orthogonal, and can be interpreted as being a scaled version of the columns of \( Q \) while the Cholesky decomposition yields a lower triangular matrix \( L \) such that \( LL^H = R_t \). Therefore the matrix \( R_t^{\frac{1}{2}H} \) can be viewed as the expected value of the upper triangular matrix \( R \) with appropriate scaling.

\[ GR_{iq}^{\frac{1}{2}}H \approx \sqrt{N_R}QR_{T_x}^{\frac{1}{2}H} \]

\[ = QR \]  

where the expected norm of the columns of \( G \) is taken to be \( \sqrt{N_R} \). The precoding matrix is the
inverse of $R$ with the columns normalized to have unit norm,

$$R^{-1}B = \frac{1}{\sqrt{N_R}}R_t^{-\frac{1}{2}H}B$$

$$= R_t^{-\frac{1}{2}H}B$$ (3.35)

$$= R_t^{-\frac{1}{2}H}B$$ (3.36)

so the precoding matrix can be calculated by using the channel correlation at the transmitter side. It is known that for maximizing the average SNR when there is channel correlation is to precode along the eigen-vectors of the transmit correlation matrix [22]. Within the context of massive MIMO, a simple proof that makes use of the large number of antennas at the receiver with no correlation to show that this result can be extended to the instantaneous SNR. Letting the precoding vector for a single stream be $v$, the received signal is

$$y = GR_t^{\frac{1}{2}H}v + n$$ (3.37)

and the SNR is

$$\gamma = v^H R_t^{\frac{1}{2}}G^H GR_t^{\frac{1}{2}H}v$$ (3.38)

$$= v^H R_t^{\frac{1}{2}H}N_R I_{N_T \times N_T} R_t^{\frac{1}{2}H}v$$ (3.39)

$$= N_R v^H R_t v$$ (3.40)

For a vector with normalized magnitude, $\gamma$ is maximized by letting $v$ be the eigen-vector of $R_t$ that corresponds to the maximum eigen-value.

### 3.1.2 Postcoder Design with Channel Correlation Information

In similar vein to the prior section, the ZF and MMSE filters can be approximated by using the transmit correlation matrix as follows. This leads to a reduction in the complexity of calculating the ZF and MMSE filter. First note that each filter is the cascade of the matched filter $H^H$ with a symmetric matrix $M$, denoted by $M_{ZF}$ and $M_{MMSE}$ respectively. Each of these matrices
depend on $H^H H$, which can be written out as follows

$$H^H H = R_t^{\frac{1}{2}} H^H G R_t^{\frac{1}{2}}$$

$$\rightarrow R_t^{\frac{1}{2}} N_R I_{N_t \times N_t} R_t^{\frac{1}{2}}$$

$$= N_R R_t$$

Since the elements of $G$ are $CN(0, 1)$ and i.i.d., the diagonal elements of $G^H G$ are the magnitude squared of the columns

$$||h_i||^2 = N_R \frac{1}{N_R} \sum_{l=1}^{N_R} |h_i(l)|^2$$

$$\rightarrow N_R E[|h_i(l)|^2]$$

$$= N_R$$

while the off-diagonal terms is the inner product between the columns of $G$. Writing out this inner product gives

$$h_i^H h_j = N_R \frac{1}{N_R} \sum_{l=1}^{N_R} h_i^*(l) h_j(l)$$

$$\rightarrow N_R E[h_i^*(l) h_j(l)]$$

$$= N_R E[h_i^*(l)] E[h_j(l)]$$

$$= 0$$

Hence approximating $G_{ZF}$ and $G_{MMSE}$ by the transmit side correlation matrix yields

$$\tilde{M}_{ZF} = \frac{1}{N_R} R_t^{-1} \quad \tilde{M}_{MMSE} = \frac{1}{N_R} \left( R_t + \frac{1}{\Gamma} I_{N_T \times N_T} \right)^{-1}$$

When there is no correlation, the ZF filter reduces to the matched filter as is expected with the typical assumptions in Massive MIMO. The MMSE filter uses the matched filter to null the
inter-user interference then multiplies by a scaling constant to balance the effect of noise. In
the presence of receiver side correlation too, the relation in eq. 3.45 and eq. 3.48 no longer
holds. So it can be expected that for scenarios where there is correlation at both the receiver
and transmitter these schemes would fare worse.

For the case of channel correlation only on the receiver side, the channel matrix is modeled
as

$$H = R^\frac{1}{2}G$$

(3.52)

Now the goal is to exploit any structure that can be gleaned from $HH^H$. Under similar assump-
tions for deriving eq. 3.43 for large $N_T$ it will hold that

$$HH^H \rightarrow N_T R_r$$

(3.53)

where $N_T = N_R$. When $N_T \leq N_R$, the rows of $G$ are linearly dependent on the first $N_T$ rows
and the approximations that all rows are asymptotically orthogonal no longer holds. The case
where $N_T \geq N_R$ with correlation at the receiver can be viewed as designing precoders for the
downlink and can be viewed as the dual problem of designing postcoders for the uplink with
correlation at the receiver. By using the matrix inversion lemma, the MMSE filter can re-written
as

$$W_{MMSE}^H = H^H \left( N_T R_r + \frac{1}{N} I_{N_R \times N_R} \right)^{-1}$$

(3.54)

### 3.1.3 Estimating Channel Correlation

The prior section assumed that the receiver and transmitter side correlation ($R_r, R_t$) are known
a priori. In practice, this has to be estimated by the receiver and sent back to the transmitter.
The second order moments are given by

$$R_r = E[HH^H] \quad R_t = E[H^H H]$$

(3.55)
A straightforward choice for estimating the mean is calculating the sample mean. In the context of estimating the correlation matrix, this gives for the receiver side

$$
\tilde{R}_r = \frac{1}{K} \sum_{k=1}^{K} H(k)H(k)^H
$$

(3.56)

$$
= \frac{1}{K} \sum_{k=1}^{K} \sum_{i=1}^{N_T} h_i(k)h_i(k)^H
$$

(3.57)

and similarly for the transmitter side

$$
\tilde{R}_t = \frac{1}{K} \sum_{k=1}^{K} H(k)^H H(k)
$$

(3.58)

where the index $k$ is over independent realizations of the channel $H$ while $i$ is the $i$-th column of $H(k)$. From the prior sections where only either receiver or transmitter correlation was assumed to design the pre/postcoder, the sample mean estimation for $R_r$ and $R_t$ is expected to improve with respect to increasing $N_T$ and $N_R$ respectively. This is because $G^H G \rightarrow N_R I_{N_T \times N_T}$ for large $N_R$ and vice versa for $GG^H$ when $G$ is square. Under the assumption that $H(k)$ are independent, the sample mean is unbiased and the variance of each element is proportional to $\frac{1}{K}$. For the $k$-th sample, writing out the channel in terms of the correlation matrix gives

$$
H^H(k)H(k) = R_{t}^{\frac{1}{2}} G^H(k) R_{r}^{\frac{1}{2}} H_{r} R_{t}^{\frac{1}{2}} G(k) R_{r}^{\frac{1}{2}} H
$$

(3.59)

For no receiver correlation, eq. 3.58 can be simplified to

$$
\tilde{R}_t = R_{t}^{\frac{1}{2}} \left\{ \frac{1}{K} \sum_{k=1}^{K} G^H(k) G(k) \right\} R_{t}^{\frac{1}{2}} H
$$

(3.60)

By the central limit theorem the diagonal elements are given by $||g_i(k)||^2$ which is a normal random variable with mean and variance $N_R$, while the off diagonal terms have mean 0 and variance $N_R$. The number $N_R$ is important in that the larger it is the more accurate and applicable the central limit theorem is. However this comes at the price of high variance in the sample
mean estimate of the correlation and can be mitigated with the tradeoff of having more samples to average over. Similar to how CSI can become outdated, it is also important to ensure that not too many samples are taken over $K$ coherence intervals of the channel such that the channel statistics have changed.

### 3.2 $K$-User MIMO

For simplicity, the received signal for the 2 User case by using the QR-decomposition is first considered.

$$ y = H_1 V_1 s_1 + H_2 V_2 s_2 + n $$

$$ = Q_1 R_1 V_1 s_1 + Q_2 R_2 V_2 s_2 + n $$

Now there is a need to account for the inter-user interference on top of the intra-stream interference. The precoder $V_i$ is in cascade of the full rank matrices $R_i$ and is denoted by $A_i$. Typically any design constraint of the post-coding and pre-coding vectors for one user must also be satisfied for the other user. That is the constraints in eq. 3.63-3.64 must hold for the received streams to be interference free.

$$ \text{rank}(U_i Q_j A_i) \leq N_T $$

$$ U_i Q_j A_j = 0 \quad \forall i, j, i \neq j $$

However, if a block SIC detection scheme is employed (e.g. [23]), then the condition in eq. 3.64 is relaxed to $\forall i, j$ such that $i \leq j$. In the event where a symbol is correctly decoded, the order of the system has been essentially reduced. When an error occurs, the wrong symbol is subtracted which results in error propagation. To combat this, optimal ordering of stream decoding has been studied [24], where metrics such as max SNR is used to determine which stream should be decoded first. At each stage of decoding, a ZF or MMSE filter could be applied to decode
the symbol with the highest received SNR as in the V-BLAST algorithm. When there are \( N \) total streams where \( N \) is a considerable number, it may not be feasible to iteratively calculate these filters despite the reduction in dimension after each stage. One simple method is to use matched filter and successively decode symbols. The performance of this receiver is bounded by the pure nulling schemes (ZF and MMSE) at high SNR regime due to high interference from other users.

Suppose we first detect the user transmitting symbol \( s_1 \). There are two extremes with which we can cancel the interference from user 2. The first is that the precoder matrix \( A_2 \) is designed such that \( Q_2 A_2 \) lies in the null space of \( U_1 = Q_1^H \) and \( A_1 \) is a diagonal matrix.

\[
\tilde{y}_1 = A_1 s_1 + Q_1^H Q_2 A_2 s_2 + Q_1^H n \quad (3.65)
\]

\( Q_1^H \) is a \( N_{T_1} \times N_R \) matrix and has rank \( N_{T_1} \). Therefore its null space has dimension \( N_R - N_{T_1} \). This represents user 1 being able to transmit as if there is no interference, while user 2 is restricted to transmit along the space that is orthogonal to user 1 at the receiver. This the relation in eq. [3.66]

\[
Q_1^H Q_2 A_2 = 0 \quad (3.66)
\]

Note that the term \( Q_1^H Q_2 \) is fixed. In order to have a non-trivial solution for \( A_2 \), the rows/columns of \( Q_1^H Q_2 \) must be linearly dependent. The simplest condition for this is to have a column in \( Q_2 \) orthogonal to every column of \( Q_1 \). Another case is when the channel \( H_1 \) does not have full rank. This is the degenerate case where one of the columns of \( Q_1 \) is 0 and results in nulling the first user too. In general, the columns of \( Q_1, Q_2 \) are linearly independent, hence the columns \( Q_1^H Q_2 \) represent the projection of the basis \( Q_2 \) onto \( Q_1 \). To guarantee linear dependence would require the number of antennas of the second user to be more than the first user. This essentially changes \( Q_1^H Q_2 \) from being a square matrix to a \( N_{T_1} \times N_{T_2} \) matrix which is an under-determined system. This is similar to the idea behind interference alignment, which has gain a lot of interest due to being degree of freedom optimal in the K-User interference channel [25] [26]. For the
purposes of this paper, the context is limited to the symmetric case where both users have the same number of antennas, i.e. null user 2 regardless of what $A_2$ is. This is achieved by using the post-coder $U_1 = (I - Q_2Q_2^H)$ as left multiplying to user 2 yields

$$
(I - Q_2Q_2^H)Q_2A_2s_2 = Q_2A_2s_2 - Q_2Q_2^HQ_2A_2s_2 \tag{3.67}
$$

$$
= Q_2A_2s_2 - Q_2A_2s_2 \tag{3.68}
$$

$$
= 0 \tag{3.69}
$$

Note that the case where $N_R = N_{T_1}$ corresponds to the degenerate case where we only have a single user, since we assumed the number of transmit antennas over all users is less than or equal to the number of receive antennas. The desired signal can now be written as

$$
W_1^Hy = (I - Q_2Q_2^H)Q_1A_1s_1 + (I - Q_2Q_2^H)n \tag{3.70}
$$

Initially it was assumed that the noise is AWGN with correlation matrix given by $\sigma^2I$. Therefore the new noise correlation is given by $(I - Q_2Q_2^H)$ and is not necessarily white. In fact, $(I - Q_2Q_2^H)Q_1A_1$ could possibly have rank 0 if the columns of channel 2 are linearly dependent on the columns of channel 1. If we let $A_1 = I$, we then get that

$$
\text{rank} (I - Q_2Q_2^H)Q_1 = d_1 \tag{3.71}
$$

$$
\leq N_T \tag{3.72}
$$

$d_1$ being the number of streams that can be spatially resolved by the receiver from user 1. Now to decode user 1, we can use the QR decomposition on the augmented channel matrix $(I - Q_2Q_2^H)H_1$, and follow the procedure for the SU-MIMO case as outlined in the previous section. Rather than applying any precoding, it is also possible to either do block decoding via the ZF or MMSE filter [23], or apply the sorted QR-decomposition algorithm with SIC. Now that user 1 has been decoded, its estimated contributions are removed from the original
received signal. Under ideal conditions, the signal will be perfectly decoded, hence the signal with interference subtracted is

\[
y_1 = H_1 V_1 (s_1 - \hat{s}_1) + H_2 V_2 s_2 + n
\]

\[
= H_2 V_2 s_2 + n \tag{3.74}
\]

where eq. [3.74] assumes ideal cancellation. However, it still remains to be addressed in which order the users should be decoded, similar to how the streams were decoded for the single user case. Rather than now ordering by channel magnitude, since there is a matrix, the ordering is done based on the capacity between the user and the BS. Whichever user has the higher channel capacity with the other user being interference is decoded first, as it is equivalent to having a higher SNR on its streams after aligning itself and the goal is to maximize the SNR at each stage to combat error propagation. Additionally, there is also the issue of the inter-user interference nulling postcoder coloring the noise, so an additional step of calculating the noise whitening filter has been avoided. Looking closer at the noise correlation matrix gives an interesting result.

\[
R_{\text{noise}} = (I - Q_2 Q_2^H) (I - Q_2 Q_2^H)^H
\]

\[
= I - Q_2 Q_2^H - Q_2 Q_2^H + Q_2 Q_2^H Q_2 Q_2^H \tag{3.76}
\]

\[
= I - Q_2 Q_2^H \tag{3.77}
\]

Recall that $Q_2$ is a basis for a subspace of $\mathbb{C}^{N_r \times 1}$ with dimension $N_T$. Then the extended basis can be denoted by $[Q_2 \quad \bar{Q}_2]$ which can be thought of as being constructed by applying the QR-decomposition on the full system given by the block matrix consisting of all the channel matrix of all users. For a square matrix ($A = QR$) which is invertible, it holds that the rows of $Q$ are orthogonal. This implies that

\[
Q_2 Q_2^H + \bar{Q}_2 \bar{Q}_2^H = I \tag{3.78}
\]
and the covariance of the noise can be rewritten as $\bar{Q}_2 \bar{Q}_2^H$. This is also the inter-user interference nulling matrix, and essentially projects the received signal on the dual space of $H_2$. Projecting by just $\bar{Q}_2^H$ would keep the noise white, but requires more calculations ($N_R - N_T$ more orthonormal vectors). The algorithm for the 2-user case is now extended to the general $K$-user case. The received signal now takes on the form of eq. 3.79

$$y = H_1 x_1 + \cdots + H_k x_k + n \quad (3.79)$$

and the rate for each user can be calculated according to the following formula in eq. 3.80

$$R_i = \log_2 |I_{N_R \times N_R} + \hat{R}_i^{-1} H_i H_i^H| \quad (3.80)$$

where $\hat{R}_i$ is given by

$$\hat{R}_i = \sum_{j,j \neq i} H_j H_j^H + I_{N_r \times N_r} \quad (3.81)$$

Ordering the users by their rates in descending order, without loss of generality it is assumed $R_1 > R_2 > \cdots > R_k$. Hence a block matrix $\tilde{H}_2 = [H_2 \ldots H_k]$ can be formed and similarly define $\tilde{x}_2 = [x_2^H \cdots x_k^H]^H$. Then apply the QR-decomposition on $\tilde{H}_2$, which yields an orthonormal basis for the signal space spanned by users 2, $\cdots$, $K$ and proceed as in the 2-user case

$$y = Q_1 R_1 x_1 + \bar{Q}_2 \bar{R}_2 \tilde{x}_2 + n \quad (3.82)$$

Hence the post-coder for the first user is now $I - \bar{Q}_2 \bar{Q}_2^H$, and an estimate of $x_1$ can be formed. This is subtracted from the received signal and repeated to iteratively decode $x_i$.

### 3.3 Performance Analysis and Results

To evaluate the detection scheme proposed, the post-processing SINR for each independent stream is calculated which is used to obtain the sum rate. Following [19], the correlation matrix
on the receiver/transmitter side is modeled as a Toeplitz matrix where the \(i, j\)-th element is given by \(\alpha^{(i-j)^2}\) for a uniform linear antenna array. The parameter \(\alpha\) is the correlation parameter for the MIMO channel, using \(\alpha_R\) and \(\alpha_T\) to distinguish between the receiver and transmitter side values. Typically, antenna spacing is taken to be half wavelength so that correlation between adjacent antennas is minimized. However when there are many antenna elements, due to size constraints on the antenna array either due to placement or location, inter-element spacing is reduced which could lead to high levels of channel correlation. For the purposes of this work, low correlation corresponds to values less than 0.3, medium correlation to values around 0.5 while high correlation is anything greater than 0.8. Hence one question of interest is what is the tradeoff between having more antenna elements vs. higher channel correlation and whether the increase in throughput, if any, is beneficial.

There is interest in massive MIMO systems to be deployed for 5G networks and is an ongoing standardization work. The original idea is to exploit the independence of the channel gains which results in the quasi-orthogonality between channels of different users and leads to the simple matched filter suppressing inter-user interference. This however requires an asymptotically large number of antennas and numbers on the order of a couple magnitudes have been considered in the literature. In practice having antennas on the order of 100’s is difficult to realize, due to issues such as having an individual RF-chain per each antenna element. LTE-A rel 13 proposes using up to 64 antennas at the base station and simulation results are provided for number of receive and transmit antennas in this range.

### 3.3.1 Single User

For the uplink, the receiver is assumed to have at least as many antennas as the transmitter. In the specific case of backhaul links, it is of interest to characterize any gains to be achieved by having a symmetric setup where \(N_R = N_T\). A base case that can be taken as the benchmark against which other configurations should be measured against is when both the transmitter and receiver employ 16 antennas and there is no channel correlation, the ergodic sum rate is plotted for var-
ious receivers in Fig. 3.2. Note that the precoder for the Gram-Schmidt precoding algorithm

and the ZF/MMSE receiver designed using the transmit correlation matrix simplify to the MF
and hence perform the same. When full CSIT is available, there is an approximately constant
gap between the ZF/MMSE receiver and the Gram-Schmidt precoding algorithm, which would
become negligible in the high SNR regime as the overall sum rate increases. Now suppose that
the number of antennas has been doubled and that the transmit power per stream has been nor-
malized with respect to $N_T$ so that total transmit power is the same, the results are plotted in
Fig. 3.3. As a result of doubling the antenna elements, moderate channel correlation is experi-
ence at both the receiver and transmitter. In this case, the regular matched filter outperforms the
transmit channel correlation form of the ZF and MMSE receiver since derivation assumes no
receiver side correlation. Due to the increased correlation, employing purely receiver-side di-
versity techniques leads to performance degradation as seen in the gap between the ZF/MMSE
receiver and the Gram-Schmidt precoding algorithm. For moderately high SNR, the MMSE has a 50% increase in the ergodic sum rate as a result of doubling antennas with an increased channel correlation compared to the numbers in Fig. 3.2 while the ZF receiver requires higher SNR to see a gain. However, precoding yields high throughput gains as it has been approximately doubled. Going one step further, consider doubling the number of antennas again and the channel correlation is now high. As seen in Fig. 3.4 the MMSE receiver performs the best up till around 20 dB, at which point the Gram-Schmidt algorithm overtakes it. The ZF receiver undergoes severe degradation under the high correlation environment, hence requires some transmit side diversity scheme as opposed to sending $N_T$ independent data streams. Compared to the case where $N_R = N_T = 32$, the high correlation has resulted in worse performance despite doubling the total number of antennas employed by the system. One variation then is to con-
Figure 3.4: Ergodic Sum Rate for $64 \times 64$ MIMO with high channel correlation

Consider keeping 64 antennas at the receiver with high channel correlation, but keep 32 antennas at the transmitter that only undergoes moderate correlated fading. In this case, the Gram-Schmidt algorithm provides about a 20% improvement in ergodic sum rate at moderate values of the SNR. The higher correlation at the receiver, the increase in receive antennas lead to improvements of the receiver-side diversity techniques while there is relatively small improvement for Gram-Schmidt precoding in Fig. 3.5 in comparison to Fig. 3.3. Despite the tradeoff of requiring CSIT, the Gram-Schmidt algorithm provides significant gains over purely receiver-side diversity techniques when there is a symmetric link. Under the scenario of high correlation at the transmitter and receiver, it is better to employ less antennas and operate with moderate channel correlation. Increasing the number of receive antennas leads to improvement in the receiver diversity techniques even at the cost of added receiver side channel correlation.
Chapter 3. Pre/postcoder Design using Gram-Schmidt Algorithm

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Figure 3.5: Ergodic Sum Rate for 64×32 MIMO with high channel correlation at receiver and moderate channel correlation at transmitter

3.3.2 Channel Correlation based Pre/postcoder

In this section, values of $N_R$ that are an order of magnitude higher compared to the previous section are considered. It also assumed that the receiver has no correlation as required for the derivation of eq. [3.51]. This can correspond to the case where a BS that is not limited by size communicates with a UE that is portable. As receiver correlation is introduced, the expected form of the ZF receiver would perform worse due there being increased correlation and eq. [3.43] no longer being applicable. However in the case of no transmit correlation, the expected form the ZF receiver simplifies to the MF and the performance is identical. As seen in Fig. 3.6, the refined form of the matched filter which decorrelates the received signal using the transmit CCI performs slightly better than the typical MF even in the presence of high correlation at the
receiver. The gain between using the ZF and Gram-Schmidt precoding scheme is noticeable when there is receiver side correlation, but as this decreases the CCI-based ZF receiver performs better.

![Figure 3.6: Ergodic Sum Rate for transmit CCI-based pre/postcoders with Large number of receive antennas](image)

3.3.3 $K$-User MIMO with SIC Receiver

This can represent the scenario where the BS receives data from 2 points which are not collocated. The SIC receiver structure is employed to improve sum rate by performing block symbol detection per users’ transmitted symbols. In practice the average received power of both users will be different due to shadowing and path loss effects, hence it is assumed power control at the transmitter is available to ensure that average received power of the users is equal. Similar to the single user case, it is interesting to see what is the cost of having $N_T$ antennas per user
where \( N_R \geq 2N_T \). In order to compare with the single user case, each stream is normalized by \( 2N_T \) such that the total transmit power is \( P \). Since the transmitters are not co-located, additional gains can be expected in the form of reduced channel correlation at the transmitter side, while the effective system is a \( N_R \times 2N_T \) MIMO channel. As discussed earlier, data sharing is not done, hence the pre/postcoder for the first user requires global CSI while the SIC structure lets user 2 operate with knowing only its own channel. For systems with a large number of transmit antennas, this reduces the amount of feedback required for precoding techniques that require CSIT. Up till around 15 dB it is seen in Fig. 3.7 that it is preferable to operate with the total number of transmit antennas to be less than \( N_R \) while at high SNR, despite increase channel correlation, there are additional performance gains to be achieved by operating in a symmetric system where the number of total transmit antennas equals \( N_R \). For the more general case of K

![Figure 3.7: Ergodic Sum Rate for 2-User MIMO with high and moderate channel correlation depending on number of antennas](image-url)
users with multiple transmit antennas, the sum rate of the system with $N_R$ receive antennas and $K N_T$ antennas is considered. When $K N_T = N_R$, which corresponds to the symmetric system, higher values of $K$ leads to worse performance as seen in Fig. 3.8. This is due to the same reason why the ZF receiver performs badly, which corresponds to $K = N_R, N_T = 1$. In order to accommodate multiple users, the block SIC mitigates the SNR degradation from finding the component of the desired stream that is orthogonal to the interference. It was observed that having the total number of transmit antennas to be around 75% gave the highest sum rate in the high SNR regime, and without the SIC structure this percentage would decrease for higher values of $K$. Within the low to moderate SNR values ($\approx 18$ dB), for higher values of $K$ the sum rate would be higher when the number of total transmit antennas as a percentage of receive antennas is reduced.

### 3.4 Summary

For systems with large antenna arrays that suffer from channel correlation, a linear precoding algorithm that has the same complexity as the ZF and MMSE receiver was proposed. In a Massive MIMO system where only the receiver employs a very large antenna array and the transmitter has correlation, an approximation for the ZF/MMSE receiver was derived using the channels’ transmit correlation matrix, and the precoding matrix for the Gram-Schmidt algorithm was calculated using the Cholesky decomposition of the transmit correlation matrix. For values of $N_T \ll N_R$, the refined form of the ZF and MMSE where the decorrelator matrix is calculated using the CCI showed performance gains over the typical MF, while the corresponding Gram-Schmidt algorithm only becomes beneficial to use at moderate SNR values and this threshold decreases as correlation is introduced at the receiver. In the general $K$-User case, the inter-user interference suppression matrix can be viewed as a block ZF receiver which projects the channel of the desired user to the space that is orthogonal to every other user. The block SIC structure mitigates the effect of SNR degradation experienced by the ZF receiver in the
symmetric system where the number of transmit antenna elements is equal to the number of receive antenna elements.

The MF was shown to still suffer from an interference floor for the range of receive antennas considered (i.e. ≤ 100) and introducing channel correlation into the model makes dealing with the issue of interference necessary. While increasing the number of transmit antennas would lead to an increase in the sum rate due to increasing the number of independent streams, there is the penalty of increased correlation which would reduce the component of each channel vector that is orthogonal to every other channel vector. Under conditions such as moderate correlation in links with large number of antenna elements at the receiver and transmitter, a substantial increase in sum rate was achieved by using the Gram-Schmidt precoding algorithm. For the
symmetric link where \( N_R = N_T \) and in the presence of high correlation, the MF yields a higher sum rate than the ZF receiver which suffers from severe performance degradation. When the number of receive antenna elements was increased while keeping \( N_T \) fixed, even going from moderate to high channel correlation resulted in the performance of the receiver diversity based techniques to improve while the Gram-Schmidt precoding algorithm was noted to perform the same. Finally the multi-user case was addressed, and it was shown that operating with a greater number of receive antennas has only a constant gap in SNR for the 2-User case. By varying the number of transmit antennas to maximize the sum rate at high SNR values (\( \approx 30 \) dB), a ratio of \( 3/4 \) to the number of receive antennas was used. Compared to the single user system, for a fixed number of total transmit antenna elements, the multiple user case would undergo fading that is less correlated because each user would have a smaller number of antenna elements.
Chapter 4

Hybrid Beamformer Design for Limited RF-chain System

It is typically assumed that each antenna has its own dedicated RF-chain. At the receiver, this is responsible for demodulating the transmitted signal and sampling the pulse using an ADC block. Finally digital signal processing techniques can be employed at the baseband to decode the transmitted symbols. When there are a large number of antenna elements as considered in massive MIMO systems, having a RF-chain for each antenna element is not economically feasible. Therefore a rethinking of the RF structure is required. One approach that has been studied is employing an antenna selection scheme, where heuristic algorithms have been applied to simplify the problem of jointly choosing antennas at the receiver and/or transmitter due to complexity issues. However when the number of RF-chains $L$ is such that $L \ll N_R, N_T$, then there may be additional performance gains to be achieved by considering hybrid beamforming architectures as illustrated in Fig. 4.1.
Figure 4.1: Phase-only signal processing in RF-band followed by down conversion and traditional DSP in baseband

### 4.1 Large antenna array at Receiver

First consider only the receiver employing a large antenna array, the postcoder then takes the following form

\[
W^H = W_{BB}^H W_{RF}^H
\]  

where \( W_{BB} \) is a \( L \times N_T \) matrix and \( W_{RF} \) a \( N_R \times L \) matrix whose elements have the phase-only constraint, \( N_T \) is the number of streams to be decoded and \( L \) is the number of RF-chains available at the receiver. For the typical MIMO scenario where the receiver has an equal number of RF-chains as antenna elements corresponds to the RF-filter being given by \( W_{RF}^H = I_{N_R} \). On the other hand, the traditional approach to limited RF-chain systems is to select a subset of antenna elements and results in the RF filter being all zeros except at the index of the antenna elements selected.

\[
W_{RF}^H = \begin{pmatrix}
    e_1^H \\
    e_5^H \\
    e_7^H
\end{pmatrix}
\]

Eq. 4.2 shows a possible realization of \( W_{RF} \) when the number of RF-chains is 3, \( e_i \) is a column vector which is zero everywhere except at the \( i \)-th index where it contains a 1. For the case
where $W_{RF}^H$ allows phase only elements of the form $e^{j\phi_{m,n}}$, each RF-chain has associated with it $N_R$ variable phase shifters with which to weight the received signal. Meanwhile the precoder need not have any such structure for now (i.e. the transmitter has the same number of antenna elements as RF-chains). The estimated signal is then obtained as follows

$$\tilde{s} = W_{BB}^H W_{RF}^H y$$

$$= W_{BB}^H W_{RF}^H H V_{BB} s + W_{BB}^H W_{RF}^H n$$

(4.3)

(4.4)

For simplicity the single transmit antenna case is first considered, where the matched filter is typically used and there is no correlation at the receiver ($H = G$). Under these assumptions, the received signal is

$$y = h s + n$$

(4.5)

The SNR is an important metric and its expression is derived for various postcoders that are considered. When $L_T = N_T$, which corresponds to having full flexibility in designing the post/precoders, the upper bound is achieved by using the matched filter. This yields the SNR to be

$$\gamma_{MF} = \frac{\sum_{l=1}^{N_R} |h(l)|^2 P}{\sigma^2}$$

(4.6)

$$= N_R \frac{1}{N_R} \sum_{l=1}^{N_R} |h(l)|^2 \Gamma$$

(4.7)

$$\rightarrow N_R \mathbb{E}[|h(l)|^2] \Gamma$$

(4.8)

$$= N_R \Gamma$$

(4.9)

$\Gamma = \frac{P}{\sigma^2}$ is the average SNR and the weak law of large numbers was used to approximate the SNR, which is a random variable, by a deterministic value. For large $N_R$, the central limit theorem states the SNR can be approximated as a complex normal random variable with mean $N_R \Gamma$ and variance $N_R \Gamma^2$ when $h(l)$ are assumed to be i.i.d. and follow a complex normal
distribution with zero mean and unit variance. For the case where \( L < N_t \), a simple antenna selection scheme would be to select the first \( L \) antenna elements (the channel vector is assumed to be ordered in descending order). This corresponds to the RF postcoder matrix to be all 1’s along its main diagonal and 0 everywhere else and results in the following SNR

\[
\gamma_{MF,sel} = \sum_{l=1}^{L} |h(l)|^2 \Gamma 
\]

Equal gain combining has been proposed for systems where full amplitude control of for the postcoder vector is not possible and is phase-only postcoder scheme that essentially utilizes only 1 RF-chain per transmitted stream. Under such a scheme, the SNR is given by

\[
\gamma_{phase} = \frac{1}{N_R} \left( \sum_{l=1}^{N_R} |h(l)| \right)^2 \Gamma 
\]

\[
= \frac{1}{N_R} \left( \frac{N_R}{N_R} \sum_{l=1}^{N_R} |h(l)| \right)^2 \Gamma 
\]

\[
\to \frac{1}{N_R} \left( N_R \mathbb{E}[|h(l)|] \right)^2 \Gamma 
\]

\[
= N_R \mathbb{E}[|h(l)|]^2 \Gamma 
\]

\[
= N_R \left( \mathbb{E}[|h(l)|^2] - \sigma_h^2 \right) \Gamma
\]

The last line guarantees that for any channel where the gains have non-zero variance, i.e. non-deterministic, the matched filter SNR will be greater than the EGC case in proportion to the variance. In the third line the channel dependent quality can be approximated as a complex normal random variable with mean \( N_R \mathbb{E}[|h(l)|] \) and variance \( N_R \sigma_h^2 \). Hence the SNR which is obtained by squaring this channel dependent quantity would follow a non-central chi-squared distribution. Note that each phase component has equal contribution to the transmit power normalization factor. This hints at the fact that it may not be beneficial to transmit across all antennas when employing phase only precoding, depending on the channel gains. To see this, consider the case when there are two antennas. If only 1 is selected, the square root of the SNR
is given by $|h(1)|$, while with 2
\[
\frac{|h(1)| + |h(2)|}{\sqrt{2}} \tag{4.16}
\]
In order to ensure this is greater than just selecting one, it is required that eq. (4.17) holds.
\[
|h(2)| \geq (\sqrt{2} - 1)|h(1)| \tag{4.17}
\]
A similar reasoning can be found in section V in [27], where instead they design phase only vector from trying to minimize the angle with respect to the ideal vector. Therefore one may expect that utilizing all the receive antennas with EGC results in the degradation of overall system performance. A possible strategy to counteract this is outlined next. After selecting the $L$ transmit antennas and calculating $\gamma_{MF,sel}$, an updated SNR is iteratively calculated by having the next antenna combine using just a phase shifter. So for the $L + 1$'th antenna the following comparison is performed
\[
\frac{\sum_{l=1}^{L} |h(l)|^2 + |h(L + 1)|}{\sqrt{1 + \sum_{l=1}^{L} |h(l)|^2}} \geq \sqrt{\sum_{l=1}^{L} |h(l)|^2} \tag{4.18}
\]
\[
|h(L + 1)| \geq \left( \sqrt{1 + \sum_{l=1}^{L} \frac{1}{|h(l)|^2}} - 1 \right) \sum_{l=1}^{L} |h(l)|^2 \tag{4.19}
\]
If the inequality in eq. (4.19) does not hold, then using additional receive antennas while employing phase-only processing signal results in performance degradation. Denoting the postcoding vector without normalization at the $k$-th iteration by
\[
w^{(L+k)} = [h(1 : L)^H e^{-j\phi_{L+1}} \ldots e^{-j\phi_{L+k}}]^H \tag{4.20}
\]
where $h(1 : L)$ is the vector which contains the $L$ largest components of the channel vector per the antenna selection scheme. The magnitude is $\sqrt{||h_L||^2 + k}$, hence the desired SNR is given
by

\[ g_k = \frac{\left( ||h(1 : L)||^2 + \sum_{l=L+1}^{L+k} |h(l)| \right)^2}{||h(1 : L)||^2 + k} P \] (4.21)

At each step, the value of \( g_k \) is compared to the previous value, and if there is an increase then an additional antenna has been selected for phase-only postcoding. The step where \( g_k < g_{k-1} \) occurs, the value of \( k - 1 \) is denoted by \( k^* \) and the algorithm is terminated. The base performance of this scheme is when we operate with just \( L \) transmit antennas, employing a purely antenna selection based algorithm. However, due to the averaging effect of employing a lot of antennas at the receivers, it turns out this performance loss is negligible in the context of massive MIMO as evidenced by simulation results. Now we show that the resulting postcoding vector can be put into the form of a RF processing matrix in cascade with a BB processing matrix. Since the system only has \( L \) RF-chains, it only provide \( L \) different amplitude gains. Hence \( L - 1 \) RF-chains are used as if we select \( L - 1 \) antennas, then use the remaining RF-chain to employ EGC for the remaining antennas. The postcoder can then be written as follows

\[
W_{BB}^H W_{RF}^H = \left( h(1)^* \ h(2)^* \ \cdots \ h(L - 1)^* \ 1 \right) \left( \begin{array}{cc} I_{L-1 \times L-1} & 0_{L-1 \times N_R - L + 1} \\ 0_{1 \times L - 1} & e^{j\phi_{L,N_R}} \end{array} \right) (4.22)
\]

where \( \phi_{L,N_R} \) is the phase component of the \( L \)-th element onwards of the channel vector \( h \). The case of \( L = 1 \) corresponds to EGC. For the general case, the SNR where \( L - 1 \) RF-chains are employed to implement selection-based matched filter while the remaining \( N_R - L + 1 \) perform equal gain combining is given by \( A_{N_R} \) which is

\[ \gamma_{bb} = \frac{\left( ||h(1 : L - 1)||^2 + \sum_{l=L}^{N_R} |h(l)| \right)^2}{||h(1 : L)||^2 + N_R - L + 1} P \] (4.23)

\[ \approx P \sum_{l=1}^{L-1} |h(l)|^2 + (N_R - L + 1)\mathbb{E}[|h(l)|]^2 \] (4.25)
Comparing this expression with the SNR for selection scheme with $L$ length matched filter, forgoing one RF-chain for utilizing phase-only processing for the remaining antennas would lead to better performance if

$$|h(L)|^2 \leq (N_R - L + 1)\mathbb{E}[|h(l)|]^2$$  \hspace{1cm} (4.26)

Under the assumption of Rayleigh fading, $\mathbb{E}[|h(l)|]^2 \approx 0.81$. For large enough $N_R$, the probability of this happening goes to zero. To see this, note that the events $|h(L)|^2 \leq x$ are contained in set of events such that the max of $\{v_i\}_{i=1}^{N_R-L+1} \leq x$ where $v_i$’s are i.i.d. and follow the same distribution as $|h(i)|^2$. This is because $h(i)$ is the $i$–th biggest element and hence can be viewed as the max of the remaining $N_R - i + 1$ elements. Taking the union of a set of $N_R - i + 1$ i.i.d. random variables and an i.i.d. set of r.v.s $\{v\}_{i=N_R-i+1}^{N_R}$, there are 3 cases of interest. The first is when the max of the first set is also the maximum of the second set, while the second case is where the max of the first set is not the max of the union set and the final case is where the max of the first set is less than all the elements from the second set. Therefore the probability is lower bounded by $\Pr(\max\{v_i\}_{i=1}^{N_R-L+1} < x)$. This is the probability that all of the $v_i$’s are less than $x$, which is

$$\Pr \left( \max\{v_i\}_{i=1}^{N_R-L+1} < x \right) = \Pr (v_i < x)^{N_R-L+1}$$

$$= \left(1 - e^{-x}\right)^{N_R-L+1}$$  \hspace{1cm} (4.27)

The quantity in the last line can be evaluated as a limit of a function of $x$ since the comparison would be with respect to $x = (N_R - L + 1)\mathbb{E}[|h(l)|]^2$. $x$ becomes increasingly large the limit
takes the under-determined form of $1^\infty$, so the logarithm of the limit is evaluated

\[
\lim_{x \to \infty} x \ln \left(1 - e^{-x}\right) = \lim_{x \to \infty} \frac{\ln \left(1 - e^{-x}\right)}{\frac{1}{x}}
\]

\[
= \lim_{x \to \infty} \frac{e^{-x}}{\frac{1}{x}}
\]

\[
= \lim_{x \to \infty} -x^2 e^{-x} \left(1 - e^{-x}\right)^{-1}
\]

\[
= 0
\]

(4.29) \hspace{1cm} (4.30) \hspace{1cm} (4.31) \hspace{1cm} (4.32)

hence the original limit is equal to 1 and it is expected that the hybrid beamforming method outperforms the antenna selection method.

4.1.1 Transmitter with Multiple Streams

For the transmitter with multiple antennas, there is not an analogous to the hybrid algorithm of antenna selection and phase post-coding that was present for the single antenna case. Rather, the RF stage is viewed as trying to project the $N_t \times N_r$ channel gain matrix into a $L$-dimensional subspace, albeit approximately due to the phase only restriction. The baseband processing matrix would then try to correct any mismatches that happened at the RF-stage. For our purposes, it is assumed $N_T = L_T$ and that the transmitter is not RF-chain limited whereas at the receiver $L_R \ll N_R$. The strategy is then as follows: compute the RF processing matrix, which yields the effective channel to be

\[
H_{\text{eff}} = HV_{\text{RF}}
\]

(4.33)

which due to our assumptions is a square matrix. The baseband precoders are then designed according to traditional MIMO, either employing ZF/MMSE estimates if not channel information is available at the transmitter or more advanced transmission techniques such as singular value decomposition. The ZF receiver results in taking the inverse of the effective channel matrix, which could lead noise enhancement in the low SNR regime. Hence careful consideration needs to be given on how to form the effective channel matrix.
When there are multiple streams to be received, the noise coloring due to the RF processing matrix has to be considered if we are to follow the two stage precoder design outlined in the previous section. One solution that has been proposed in the literature is to use a DFT matrix due to its ease of implementation and rows being orthogonal. This leads to the noise still being white after applying the phase-only processing matrix. Alternatively, an arbitrary phase-only matrix can be chosen, and then the baseband precoder will be cascaded with a noise whitening filter. For the latter approach, the following algorithm is proposed. First, the received signal is written as follows

\[ y = Hx + n \]  

We assume the columns of \( H \) are in descending order of magnitude. Denoting the \( i \)-th column of \( H \) by \( h_i \), we form a new vector by normalizing each element to have unit magnitude and call it \( \phi_i \). Collecting all these vectors and forming the matrix \( \Phi \), the cascade of the BB and RF postcoders is given by

\[ W^H_B W^H_RF = W^H_B \Phi^H \]  

Without any baseband processing, applying the phase-only matched filter (i.e. equal gain combining) for the multi-user case results in inter-user interference. It will now be shown that for an asymptotically large number the interference goes to zero as the case for the matched filter receiver. Each interference term will stem from the result of \( \phi_i^H h_m \) for \( m \neq i \). Writing this out yields

\[ \phi_i^H h_m = \sum_{l=1}^{N_R} e^{-j\phi_{l,i}} |h_m(l)| e^{j\phi_{l,m}} \]  

\[ = \frac{N_R}{N_R} \sum_{l=1}^{N_R} |h_m(l)| e^{j\phi_{l,m}} e^{-j\phi_{l,i}} \]  

\[ \rightarrow N_R \mathbb{E}[|h_m(l)| e^{j\phi_{l,m}} e^{-j\phi_{l,i}}] \]  

\[ = N_R \mathbb{E}[|h_m(l)|] \mathbb{E}[e^{j\phi_{l,m}}] \mathbb{E}[e^{-j\phi_{l,i}}] \]  

\[ = 0 \]
where the fact that $h_m(l)$, the $l$-th element of vector $h_m$, being i.i.d. and following a standard complex normal distribution has been used, hence it is known that $|h_m(l)|$ follows the Rayleigh distribution and $\phi_{l,m}$, the $(l, m)$-th element of matrix $\Phi$, has a phase that is independent of the magnitude and follows a uniform distribution. The variance of the interference quantity is $\mathbb{E}[|\phi_l^H h_j|^2] = N_R$. Recalling that the SNR is not affected by scaling by a constant factor on the received signal, letting the receive filter for the $i$-th stream be $\frac{1}{N_R} \phi_i^H$. This results in the variance of the interference to equal $\frac{1}{N_R}$, which tends to 0 when a large number of receive antenna elements are employed. As noted in the previous chapter, in practice the number of antennas that would be deployed would be around an order of magnitude less (10’s vs 100’s). Hence the BB processing will have to deal with any residual interference and noise whitening. Letting the EGC of the channel be represented as $\text{arg}(H)^H$, the resulting signal after RF-processing is

$$\text{arg}(H)^H y = \text{arg}(H)^H H + \text{arg}(H)^H n$$ (4.41)

Under the assumption that $N_R$ grows asymptotically large, it would hold that $\text{arg}(H)^H \text{arg}(H) \rightarrow I_L$ i.e. the noise is still white. One choice for dealing with the inter-stream interference is simply taking the inverse of the effective channel. This results in the BB and RF postcoder to be given by

$$W_{BB}^H W_{RF}^H = (\text{arg}(H)^H H)^{-1} \text{arg}(H)^H$$ (4.42)

which is essentially the phase-based version of the left pseudo inverse of $H$. Going along this train of thought, one may suggest the RF-processing matrix to be a random matrix to project the channel matrix $H$ to a subspace after which channel inversion can be performed. Instead of this, an alternative scheme that is also blind to the CSI is proposed that takes into account the CCI at the receiver where a large antenna array has been employed. In the traditional antenna selection scheme, the RF-processing matrix consists of purely 1’s and 0’s can be thought of as a switch architecture where each RF-chain is connected to every antenna port and only receives a signal from one antenna port. Now suppose that the receiver has correlation matrix $R_r$, which in turn
induces a correlation along the elements of the columns of $H$, and the degree of correlation can be clustered amongst groups of rows of $H$. In the case where each cluster of rows are perfectly correlated with each other and uncorrelated otherwise, the strategy is to combine the clusters to achieve additional diversity gains. Define the correlation factor $K$ as $K = N_R/L$ and is an integer. The RF-processing matrix is then given by

$$W_{RF}^H = \begin{pmatrix}
1_{1\times K} & 0_{1\times K} & \cdots & 0_{1\times K} \\
0_{1\times K} & 1_{1\times K} & \cdots & 0_{1\times K} \\
\vdots & \vdots & \ddots & \vdots \\
0_{1\times K} & 0_{1\times K} & \cdots & 1_{1\times K}
\end{pmatrix}
$$

(4.43)

which can be viewed as being an analogous to the operation of despreading the received signal in CDMA. Since the rows of $W_{RF}^H$ are trivially orthogonal, it immediately follows that $W_{RF}^H W_{RF} = K I_L$ regardless of the number of receive antennas. Hence there is no need to perform further processing for noise whitening at the baseband. For a cluster of size $K$, the correlation between the middle element and the closest element in the adjacent cluster is $\alpha \frac{K}{2}$. For small values of $\alpha$, this quantity would become negligible for relatively small values of $K$ while higher correlation values will require a bigger value of $K$ for the inter-cluster correlation to be negligible. $K$ is related to the ratio of number of receive antennas to RF-chains. Rather than just seeing the maximum correlation from the middle of one cluster to the adjacent cluster, a more accurate metric is to consider all correlation pairs between the two clusters and average this value. For the case where the correlation between the $i$-th and $j$-th antenna element is given by $\alpha |i-j|$ like that considered in [28] by using the exponential correlation model, starting from
the end of one cluster and listing the correlation values in matrix form gives

\[
B = \begin{pmatrix}
\alpha & \alpha^2 & \cdots & \alpha^K \\
\alpha^2 & \alpha^3 & \cdots & \alpha^{K+1} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha^K & \alpha^{K+1} & \cdots & \alpha^{2K-1}
\end{pmatrix}
\]

(4.44)

Summing over all elements is equivalent to evaluating \(1^T B 1\) and noting that \(B\) is a matrix of rank 1 it can be factored as follows

\[
1^T B 1 = 1^T \alpha \begin{pmatrix}
1 \\
\alpha \\
\vdots \\
\alpha^{K-1}
\end{pmatrix} \begin{pmatrix}
1 & \alpha & \cdots & \alpha^{K-1}
\end{pmatrix} 1
\]

(4.45)

\[
= \alpha \left(\frac{1 - \alpha^K}{1 - \alpha}\right)^2
\]

(4.46)

Averaging over all the \(K^2\) pairs gives

\[
\bar{\alpha} = \alpha \left(\frac{1 - \alpha^K}{K \cdot 1 - \alpha}\right)^2
\]

(4.47)

The number of receive antennas plays into the correlation factor \(\alpha\) since for a fixed physical space the inter-antenna element spacing decreases inversely to the number \(N_R\) causing \(\alpha\) to increase. This interdependency leads to the trade-off between high correlation from having a high number of antennas to requiring a bigger \(K\) to ensure the clusters are sufficiently uncorrelated, which in turn requires a lower number of RF-chains \(L\) and impacts the sum rate since it is the number of independent data streams that are supported.
4.2 Large Antenna Array at Receiver and Transmitter

Let $L_R$ and $L_T$ denote the number of RF-chains at the receiver and transmitter respectively. The goal now is to design the precoder and postcoder as the cascade of a phase-only matrix followed by the baseband digital pre/postcoder.

$$W_{BB}^H W_{RF}^H y = W_{BB}^H W_{RF}^H H x + W_{BB}^H W_{RF}^H n$$

(4.48)

$$= W_{BB}^H W_{RF}^H H V_{RF} V_{BB} s + \tilde{n}$$

(4.49)

$$= \tilde{s}$$

(4.50)

The RF processing on the transmitter side can be viewed as projecting the encoded symbol into a higher dimension space, in essence spreading the signal over space, resulting in diversity gains from utilizing the additional antennas that would not be possible through an antenna selection scheme. The receiver must then operate by despreading this received signal, i.e. project it back to the low dimensional space of the baseband encoded message and decode it accordingly. Viewing a large antenna array as averaging the effect of randomness in the system, the transmitter removes the uncertainty in the channel gain while the receiver also mitigates the effect of the noise. Following the work in the prior section, $W_{RF}^H$ would be given by eq. 4.43 and $V_{RF} = W_{RF}$. Similar to how multiplying the received signal by $W_{RF}^H$ leads to the noise at the baseband stage to be $K I_L$, $V_{RF}$ must be scaled to ensure total transmitted power is $P$, and results in

$$V_{RF} = \frac{1}{\sqrt{K}} W_{RF}$$

(4.51)

4.3 Effect of Phase Noise

The RF processing will be carried out by variable phase shifters which would apply a phase to each signal associated with an antenna element. This is a form of analog signal processing, which in practice requires precise calibration and does not have the flexibility offered by digital
signal processing techniques. Therefore it of interest to study the impact of phase noise in the variable phase shifters due to analog imperfections. Since phase-only processing is used, the error in the phase is modeled as being uniform within the interval $[-c\pi, c\pi]$. Denoting by $\hat{\phi}_i$ the phase vector that is implemented by the system and the error by $\eta_i$ gives the following relationship

$$\hat{\phi}_i = \phi_i + \eta_i$$  \hspace{1cm} (4.52)

Multiplying the received signal by this filter for the single antenna transmitter case, the SNR has decreased due to interference caused by mismatch of the phase components.

$$e^{j\hat{\phi}_i} y = \sum_{l=1}^{N_R} |h(l)| e^{-j\eta(l)} s_1 + \tilde{n}$$  \hspace{1cm} (4.53)

$$\approx \sum_{l=1}^{N_R} |h(l)|(1 - j\eta(l)) s_1 + \tilde{n}$$  \hspace{1cm} (4.54)

$$= \sum_{l=1}^{N_R} |h(l)| s_1 - j \sum_{l=1}^{N_R} |h(l)| \eta(l) s_1 + \tilde{n}$$  \hspace{1cm} (4.55)

where it is assumed the imperfections ($|\eta(l)|$) are small such that the first order Taylor series approximation is applicable and the SINR is given by

$$\tilde{\gamma}_{egc} = \left(\frac{\sum_{l=1}^{N_R} |h(l)|}{N_R \sigma^2_{\eta(l)} + N_R \sigma^2} \right)^2 \frac{P}{P \sigma^2_{\eta(l)}}$$  \hspace{1cm} (4.56)

where $\sigma^2$ is the noise power, $P$ is the average power of the symbol constellation which is assume to be DC balanced and $\sigma^2_{\eta(l)}$ is the variance of the phase noise and the interference due to phase mismatch is approximated as being complex normal by applying the central limit theorem. When there are multiple transmitted streams, in the asymptotic regime the inter-stream interference would go to zero with EGC and eq. 4.56 is the upper limit on the achievable SNR.
4.4 Performance Analysis and Results

4.4.1 Single Antenna Transmitter

Here the performance of varying the number of RF-chains at the receiver while the transmitter is a single antenna user is considered. The optimal solution is to use the traditional matched filter which requires $L = N_R$. For the case where $L \ll N_R$, antenna selection scheme would utilize the $L$ antenna elements with the highest magnitude. Adding more RF-chains yields diminishing returns, as illustrated in Fig. 4.2 where there is a constant gap in the sum rate between the cases where $L < N_R$ and $L = N_R = 64$. For moderate SNR values, using only 16 RF-chains already achieves a significant portion of the throughput achievable. Recall that the performance of using $L - 1$ RF-chains for MF at the baseband and the remaining 1 for EGC at the RF stage (a.k.a...
variable phase shifters) was shown to outperform the antenna selection scheme as the number of receive antennas increased. In fact, as noticed in Fig. 4.3, the EGC has similar performance to the matched filter with 32 RF-chains. This suggests that using the phase-only components of the channel for the multi-user case to project the received signal to the $L$-dimensional subspace would not cause any severe reduction in the available throughput at the RF stage. This is similar to the observation that for a generic channel the matched filter does not affect the capacity of the channel, but for decoding symbols individually further processing is necessary as seen by the structure of the ZF/MMSE receivers.

![Figure 4.3: Comparison of Sum Rate between EGC receiver, Antenna Selection and MF](image)

Figure 4.3: Comparison of Sum Rate between EGC receiver, Antenna Selection and MF
4.4.2 Multiple Transmit Streams

One consideration that needs to be given with using variable phase shifters for RF-processing is that components could suffer from phase noise, modeled as $\eta$ defined by eq. 4.52. Two cases are considered for comparison, one where the noise follows a Gaussian distribution with mean 0 and unit variance while in the other case follows a uniform random variable in the region $[-\pi, \pi]$. The variance of the latter is $\pi^2/3$, which would give an offset of 5.17 dB with respect to the power of the error in the phase component assuming Gaussian noise. For the case of the phase noise following a Gaussian distribution with variance given by $\sigma_e$, there is a constant gap in Fig. 4.4 between the hybrid and the EGC receiver implemented solely in the RF stage. Since there

![Graph](image_url)

Figure 4.4: Effect of Phase noise on implementing purely RF-receiver vs. utilizing digital processing

are many antenna elements, due to manufacturing processes and cost issues it is necessary to
see whether there are any benefits in employing EGC when phase noise has a higher power. As seen in Fig. 4.4 utilizing more RF-chains and making use of the baseband processing which is error-free results in a greater gap with respect to the EGC receiver. For moderate phase noise as in Fig. 4.4 the difference is approximately 10\% which is within an acceptable range. However, when the phase shifters suffer more from phase noise either due to manufacturing processes or scale requiring cheaper components, there is an obvious advantage to utilizing the baseband capabilities. On top of this, the EGC receiver is only able to utilize up to $N_T$ RF-chains, which is the number of independent streams. This is another design consideration that must be taken into account, as the system is essentially underutilized when only employing 16 transmit antennas. $N_R$ was chosen to be 4 times the amount of RF-chains available to see whether there is any improvement over the system considered in Chapter 3. Even with having 256 receive antennas and only a moderate amount of channel correlation the sum rate is affected by an interference floor. This indicates the necessity to employ detection techniques at the baseband to gain the full benefits from employing additional antennas in a RF-chain limited system.

The alternatives that were proposed is the phase-based ZF receiver which performs channel inversion and will be compared to the Gram-Schmidt precoding algorithm that was described in Chapter 3. Unlike the previous receiver design, here $L_R = L_T = N_T$. When $L_R \geq N_T$ the channel inversion becomes taking the left inverse of the effective channel. The purpose is to gauge what the performance is by using the minimum possible number of RF-chains. At least $L_R = N_T$ RF-chains would be required at the receiver to spatially resolve $N_T$ independent streams. When $W_{RF}^H = \arg(H)^H$, the resulting sum rate for these two receivers is given in Fig. 4.5 As expected from Chapter 3, when the number of RF-chains $L$ approaches $N_R$ and becomes a symmetric system, under high correlation the ZF receiver suffers from severe performance degradation. In fact, for the ZF receiver using 16 RF-chains performs better performance than when the number of RF-chains is doubled up till around 30 dB value for SNR when the receiver undergoes high correlation. It should be noted that in this instance, the Gram-Schmidt algorithm requires an additional noise whitening filter to be calculated after phase only projection at the
RF-processing stage. For values of $L$ for which the system can be considered as RF-chain limited (e.g. $L = 8, K = 8$) the ZF receiver performs similar to the Gram-Schmidt precoding algorithm without requiring any CSIT.

Rather than projecting using phase elements of the channel, a simple combiner that groups clusters of correlated rows and performing DSP techniques on the resulting channel is evaluated. In this case no intermediate whitening filter has to be calculated for the Gram-Schmidt precoding algorithm and the effective channel would be a $L_R \times N_T$ matrix which would be significantly smaller than the original $N_R \times N_T$ matrix that would have been required. A caveat of this method is that $N_R$ is assumed to be a multiple of $L$. For channels with large correlation at the receiver side a larger value of $K$ is expected to group together clusters of rows which are highly correlated and vice versa for channels with lower correlation.
Figure 4.6: Row combiner at RF-Processing stage followed by channel inversion or Gram-Schmidt precoding with correlation

In Fig. 4.6, the overall gain of using Gram-Schmidt precoding over channel inversion is greater than when $\arg(\mathbf{H})^H$ was used as the RF-processing matrix. The case of $L = 32$ corresponds to having RF-chains for 50% of the antennas and the QR precoding has a 33% increase in the sum rate compared to when $L = 16$, in fact performing better at high SNR compared to Fig. 4.5. Meanwhile there is a less pronounced difference for the ZF receiver between the $L = 32$ and $L = 16$ cases and at high SNR the former approaches the latter as opposed to performing significantly worse. Unlike when the phase-only projection of the channel was used, the ZF receiver no longer suffers from severe performance degradation as $L_R$ is at most half of $N_R$. For low values of $L$, which is equal to the number of transmit antennas $N_T$, the phase-only projection at the RF stage yields better performance compared to using the row combiner and
reduced the gap between the ZF and GS receivers. Higher values of $L$ which also correspond to higher levels of correlation has the row combiner approach the performance of using the phase only projection. This is because moderate channel correlation strikes a balance between ensuring good behavior for the rank of the channel which is related to the multiplexing gain and ensures that the rows combined are sufficiently correlated such that they add constructively.

Finally the sum rate when a DFT combining matrix as considered in prior work as an alternative to the row combining method proposed earlier is given in Fig. 4.7. As the correlation increases, the Gram-Schmidt receiver that employs 32 RF-chains suffers from slight performance degradation, while for $L = 8, 16$ the sum rate remains relatively close to those in Fig. 4.6. On the other hand, the ZF receiver undergoes severe performance degradation under moderate correlation when $L = 32$. Similar to the Gram-Schmidt receiver, at low and moderate
correlation when a relatively low amount of RF-chains are used \((L = 8, 16)\), the sum rate remains unchanged. Compared to using the row combiner, the DFT matrix combiner performs worse for the ZF receiver as the channel correlation increases for values of \(L\) that are a high percentage of \(N_R\) and overall achieves a lower sum rate.

In Fig. 4.8 and Fig. 4.9, the sum rate using the row and DFT matrix combiner are presented with using a large antenna array at both the transmitter and receiver. Because of this, the channel correlation is set to high, and the effect of varying the number of RF-chains is studied. It is assumed the transmitter and receiver both employ the same number of RF-chains. The same general trends are observed here too as in Fig. 4.6 and Fig. 4.7. By increasing the number of transmit antennas, the SNR threshold for which the Gram-Schmidt receiver performs better
Figure 4.9: Limited RF-chain system at both transmitter and receiver using DFT matrix combiner with correlation

with $L = 32$ compared to $L = 16$ has increased for the row combiner while decreased for the DFT matrix combiner. When a RF phase-only matrix has to be employed at both the transmitter and receiver, the row combiner leads to higher sum rate than the DFT matrix combiner for low values of $L$. As $L$ increases the gains are almost twofold when comparing sum rate for both the Gram-Schmidt and ZF receiver between Fig. 4.8 and Fig. 4.9.

### 4.5 Summary

For massive MIMO systems, it may not be feasible to have the same number of RF-chains as antenna elements at either the transmitter and/or receiver. This leads to the system having reduced flexibility in terms of designing the pre/postcoders. Hybrid beamforming is one approach
that has been proposed in the literature, which requires the design of phase-only analog signal processing at the RF-band and the traditional digital signal processing at the baseband. Rather than proposing an algorithm to design the pre/postcoders, the single transmit antenna case was investigated. A hybrid of the MF and EGC receiver was proposed, and was shown to outperform the MF from employing an antenna selection algorithm. For the multiple transmit streams, it was first considered on what to do when only the receiver has a large antenna array. The problem became how to design a projection matrix with phase only elements. From studying the performance of the EGC receiver for the single antenna transmitter, this receiver was also used when there were multiple transmitted streams. Similar to how the matched filter was shown to suppress inter-stream interference, a similar result was derived for the EGC. Rather than having the RF-band filter be depending on the CSI or have a DFT structure and have no knowledge about the channel, a simple row combiner was proposed that would cluster the antenna elements which are moderately correlated and sum these signals. This brings up the notion of a switch architecture, where rather than having all antenna elements be processed together in one block at the RF-band, they are separated which would allow the relaxation of the phase-only constraint with uniform gain to include zero gain too. One caveat of this method is that it assumes $N_R$ is a multiple of $L_R$. When there is a large antenna array at the transmitter too, a spreading matrix was proposed that would repeat the transmitted symbol amongst antenna elements which are moderately correlated. For the baseband, the filters considered in Chapter 3 were used to remove any residual interference.

The EGC receiver and the effects of phase noise was studied within the context of Massive MIMO. In the presence of analog phase noise/imprecision in the variable phase shifters at the RF-stage, it is beneficial to appropriately factorize the baseband and RF pre/postcoder matrices despite the cascade of the two being equivalent. When correlation is introduced, appropriate linear receivers have to be applied to null residual interference after the RF-stage. For the case of only having a large antenna array at the receiver, the phase component of each element in the channel gain matrix was used to form the phase-only projection. In the baseband the sum rate
after performing channel inversion and Gram-Schmidt algorithm was calculated under various levels of correlation depending on the number of transmit antennas and RF-chains. The row combiner was then proposed as an alternative, dependent on the CCI by combining sufficiently correlated antenna elements, and was compared against the static DFT matrix. When both the transmitter and receiver use a large antenna array and have limited RF-chains, the row combiner was shown to outperform the DFT matrix combiner in terms of sum rate.
Chapter 5

Conclusion

This thesis investigated what the effects of channel correlation would be in a Massive MIMO system, and the subsequent impact on the number of antenna elements to be placed at the transmitter and receiver depending on the linear receiver to be employed. Focus was also given to the symmetric case where $N_R = N_T$ and $N_R$ is large. The trade-off between having more antenna elements, and thereby more independent streams, versus higher correlation was studied. While the matched filter has been shown to suppress inter-user interference under uncorrelated fading, it still suffers from an interference floor when there is correlated fading. Due to the large number of antenna elements, the complexity of the linear receiver to be implemented is an important issue. Therefore the zero forcing and the minimum mean squared error receiver were chosen as benchmarks to gauge overall system performance using the sum rate. It was then proposed to use precoding techniques to take advantage of having multiple transmit antennas, and the Gram-Schmidt precoding algorithm was developed. This essentially performs channel inversion with the help of both the transmitter and receiver, whereas the zero-forcing receiver performs channel inversion on the receiver side.

Under the Kronecker channel model, taking the receiver or transmitter to be a large antenna array, the pre/postcoder can be designed using the channel correlation information respectively. For systems that use a moderate number of antennas at the transmitter, e.g. $\sim 16$, precoders
designed using channel correlation would reduce the feedback signaling required. When there is a large antenna array at the receiver, it was shown that the decorrelator matrix that follows that matched filter matrix in the zero forcing and minimum mean squared error receiver can be approximated by using the transmit side channel correlation matrix. Then pre/postcoders were designed for the multi-user scenario using the $QR$-Decomposition and the successive interference cancellation receiver was proposed to reduce the feedback sent to each transmitting user. Without this structure, the postcoder can be interpreted as a block zero forcing receiver which nulls the signal transmitted from all other users except the desired user. Precoding is then applied on the resulting channel using the Gram-Schmidt algorithm, which outperforms both the zero forcing and minimum mean squared error receiver in terms of sum rate when the receiver suffers from high levels of correlation.

Due to having such a large number of antenna elements, it was discussed how there may be fewer RF-chains and one approach to this problem is to employ an antenna selection algorithm. When there are multiple antennas at the transmitter and receiver, this becomes a difficult problem and typically heuristic algorithms are used. An alternative approach is to use digital signal processing at the baseband and analog signal processing at the RF-band where only the phase can be shifted due to implementation constraints, called hybrid beamforming. This prompted the discussion on phase-only RF processing using variable phase shifters in Chapter 4. For a low numbers of RF-chains with respect to the number of antenna elements, it was observed that the zero forcing receiver performs similar to the Gram-Schmidt algorithm developed in Chapter 3, showing that the gain from precoding is significantly less than when the system has equal number of antenna elements and RF-chains. It was then proposed to use a row combiner at the RF-stage for processing that does not require channel state information and would be robust to phase noise since there is no need to control any variable phase shifters. This operation at the was interpreted as grouping clusters of rows (i.e. receive antenna elements) which are moderately correlated for additional diversity gains. This was compared to the discrete Fourier transform matrix combiner which is blind to the channel, and the zero forcing receiver per-
formed markedly worse compared to the Gram-Schmidt algorithm. When both the receiver and transmitter were RF-chain limited, the row combiner provided better performance than the discrete Fourier transform matrix combiner under high channel correlation.

5.1 Future Work

To study the impact of correlation, the Kronecker channel model was used in this thesis. This model assumes the correlation at the receiver is independent of the correlation at transmitter, and for the uniform linear array the correlation matrix was defined by a single parameter which can be taken to be the correlation coefficient between adjacent antenna elements. One extension would be the consideration of other antenna array structures, such as the planar or circular array, and how the correlation matrix should be modified to fit within the framework of this thesis. In this thesis, the length of the uniform linear array was taken to be fixed, hence increasing the number of antenna elements resulting in an increase in the correlation parameter. Therefore it would be of interest to study whether there is an empirical formula that captures the relationship between inter-antenna element spacing, correlation for a fixed frequency and the impact on the sum rate. Due to the available spectrum in the 60 GHz range, there is growing interest in using millimeter wave technology for telecommunications. Rather than using the Kronecker channel model, it is necessary to consider channel models that take in to account the limited scattering nature of high frequency transmission and the impact of using different array structures.

The Gram-Schmidt precoding algorithm was shown to outperform the zero forcing receiver in terms of sum rate analytically when there are 2 transmit antennas, regardless of the statistics of the channel. While simulation results verified this for the general case, a theoretical result for an arbitrary number of transmit antenna elements is missing. It was also noted that under high correlation for the symmetric link where $N_R = N_T$, the zero-forcing receiver suffers from severe SNR degradation from simulation results on the sum rate. A theoretical framework which can account for this behavior, especially for massive MIMO systems, would be useful.
While generally channel state information can be assumed at the receiver and transmitter, in practice it is hard to realize this. First channel estimation is carried out at the receiver by having the transmitter send pilot symbols, then the receiver must feedback this information to the transmitter. For massive MIMO systems pilot contamination is a bottle neck on the performance that can be achieved due to the pilot re-use from having so many antenna elements, and signaling schemes which reduce the overhead of feedback to the transmitter become important.
Bibliography


