CONTROL OF TRANSIENT GROWTH INDUCED TRANSITION IN A
ZERO–PRESSURE GRADIENT BOUNDARY LAYER USING PLASMA
ACTUATORS

by

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Abstract

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The level of the skin friction drag depends on the boundary layer state, either low, for the laminar case or high for the turbulent case. Flow control is sought to attenuate the streamwise velocity streaks preceding sub-critical bypass transition to turbulence. In addition, the targeted instability is ubiquitous to the self-sustaining wall-bounded turbulence cycle and at the root of the long-term goal of turbulence control. The longer spatial and temporal scales associated with the laminar case make a physical demonstration of a model-based boundary layer flow control more tractable.

The effectiveness of control systems is inherently linked to the ability of the actuator to alter the flow to a desired state. Therefore, actuators are a critical enabling technology component in any active flow control system. Arguably, the most important missing technology in boundary layer flow control is effective and robust actuators, which can be readily integrated with an active flow control system. Plasma actuators fulfil these characteristics and are an ideal candidate for the control of the bypass transition instability.

In this thesis, the receptivity of the boundary layer to forcing by arrays of plasma actuators capable of producing streamwise streaks was characterized. Following, the transient growth instability was targeted in an open-loop framework to identify the physics of the attenuation mechanism, which was shown to be a linear process. The control loop was then closed with feedback from simultaneous spanwise distributed shear stress sensors. A wavenumber specific control objective was used to demonstrate the effectiveness of feedback for steady disturbance attenuation as well as to provide robustness to
off-model conditions. For all cases, the targeted disturbance was reduced by over 94% of its initial energy. The control effectiveness was also validated for quasi-steady forcing by varying the input disturbance level. Ultimately, control of transient growth due to unsteady stochastic excitation is sought, and the dynamic response of the flow to pulsed actuation was studied to support the natural next step in the greater efforts of which this thesis is a critical component.
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Nomenclature

Alphanumeric

\( a \) Length of the leading edge
\( b_h \) Thickness of the boundary layer plate
\( b_u \) Minor axis length of the leading edge upper quarter ellipse
\( C_p \) Coefficient of pressure
\( d \) Roughness element diameter
\( f_a \) Frequency of the sinusoidal signal driving the actuator
\( H_{12} \) Boundary layer shape factor \((\delta_1/\delta_2)\)
\( J \) Objective function for the aerodynamic leading edge optimization
\( k \) Roughness element deployment height
\( V_{pp} \) Peak to peak voltage driving the actuator
\( p_s \) Static pressure
\( p_t \) Total pressure
\( Re_k \) Reynolds number based on roughness element height and the velocity at the apex of the roughness element
\( Re_{\hat{x}} \) Reynolds number based on the location of the virtual leading edge
\( \langle U \rangle_t \) Time-averaged streamwise velocity
\( \langle U \rangle_{tz} \) Time- and spanwise-averaged streamwise velocity
\( U' \) Streamwise disturbance velocity \((U' = \langle U \rangle_t - \langle U \rangle_{tz})\)
\( U_{\infty} \) Free-stream velocity
\( U_k \) Velocity at the height of the roughness element
\( W_{HV} \) Width of the high voltage (HV) electrodes
\( \hat{x} \) Streamwise coordinate from the virtual leading edge of the test plate

\( x \) Streamwise coordinate from the geometric leading edge of the test plate

\( X \) Design variables

\( z \) Spanwise coordinate

**Greek**

\( \alpha_{te} \) Angle of the boundary layer plate flap

\( \beta \) Dimensionless spanwise wavenumber \((2\pi\delta/\lambda)\)

\( \beta_i \) Dimensionless spanwise wavenumber \((2\pi\delta/(\Delta z/i))\)

\( \Delta z \) Spanwise spacing of roughness elements and high-voltage electrodes

\( \delta \) Blasius similarity length scale

\( \delta_1 \) Displacement thickness

\( \delta_2 \) Momentum thickness

\( \eta \) Blasius similarity variable \((y/\delta)\)

\( \lambda \) Spanwise-wavelength

\( \langle \tau \rangle_i \) Time-averaged streamwise shear stress

\( \langle \tau \rangle_{tz} \) Time- and spanwise-averaged streamwise shear stress

\( \nu \) Kinematic viscosity of air

\( \overline{\phi_u} \) Spanwise power spectrum averaged over the thickness of the boundary layer

\( \phi_u \) Spanwise power spectrum

\( \phi_{\tau'} \) Spanwise-wavenumber power spectrum of the streamwise shear stress

\( \rho \) Density of air

\( \sigma \) Standard deviation

\( \tau \) Streamwise shear stress

\( \tau' \) Disturbance streamwise shear stress

**Abbreviations**

AC Alternating Current

AR Aspect Ratio
CTA  Constant Temperature Anemometer
DBD  Dielectric Barrier Discharge
DNS  Direct Numerical Simulation
FPCL Flow Physics and Control Laboratory
LBL  Laminar Boundary Layer
MSE  Modified Super Ellipse
MSU  Michigan State University
PI   Proportional-Integral
PSVG Plasma Streamwise Vortex Generators
REA  Roughness Element Array
TBL  Turbulent Boundary Layer
TS   Tollmien-Schlichting
Chapter 1

Introduction

1.1 Motivation

With increasing environmental awareness and regulation in conjunction with high fuel costs, energy conservation in the aviation industry is a prevalent issue. Meeting this challenge will translate into financial benefits for manufacturers and passengers. It is also toward our social responsibility of energy conservation, emission reduction, and environmental protection. Aerodynamic drag reduction will reduce thrust demands, and consequently, both emissions and fuel consumption. At cruise conditions over half of the aerodynamic drag on civilian aircraft is attributed to skin-friction, see for example Joslin [72], which is due to the presence of the boundary layer.

The boundary layer is the thin region of fluid pulled along by a vehicle as it moves through a stationary fluid (or as moving flow passes a stationary wall). The level of drag depends on the state of the boundary layer, either laminar (low drag) or turbulent (high drag). A two-dimensional laminar boundary layer (LBL) will ultimately transition to turbulence at a critical distance, which depends on the pressure gradient history, however it can transition sub-critically owing to forcing by external disturbances such as surface roughness or free-stream turbulence. The skin friction associated with LBL can be as much as 90 percent less than for a comparable turbulent boundary layer (TBL), which suggests that the laminar case is more desirable [72]. The important caveat to consider is that turbulence is desirable in pressure recovery regions where separation may occur.

Flow control is a key tool that can be used to reduce aerodynamic drag. Two methodologies of boundary layer flow control exist. One methodology is to prevent LBL sub-critical transition to turbulence. The other is to control the turbulent events in the TBL, which cause high drag. Although practical limitations for each remain, the objective is the same; control the boundary layer to reduce drag. Owing to the similar nature of the
Chapter 1. Introduction

Driving instability mechanism in the bypass transition and turbulent cases, this thesis contributes to each of the aforementioned methodologies.

Bypass transition is a form of sub-critical transition to turbulence. It is characterized by the linear process of transient (algebraic) growth of streamwise velocity streaks within the laminar boundary layer. When the streaks reach sufficient amplitude, secondary nonlinear instabilities lead to the generation of turbulence and sub-critical transition. The linear transient growth mechanism is also at the root of the self-sustaining generation of turbulence [79] in the TBL, where streaks are found in the buffer region. In either case, the optimal disturbance causing these streaks consists of a spanwise periodic series of counter-rotating vortices [101]. The larger spatial and longer temporal scales associated with the laminar case are much more tractable from an experimental perspective than those of a comparable turbulent one [41].

The stochastic nature of bypass transition dictates the use of a closed-loop control system. Therefore, three elements have to be available; sensors, actuators, and a controller. Actuator development is considered to be the pacing item of the practical demonstration of boundary layer control [71] and remains an ongoing topic. Recent recommendations suggest that feasible actuators should have low complexity and cost to facilitate this practical demonstration [104]. Single-dielectric-barrier discharge plasma actuators fulfil such a requirement. They are a comparatively new technology [19], and have never before been applied to bypass transition control.

1.2 Objectives, Methodology and Thesis Structure

This thesis focuses on the control of streamwise velocity streaks experiencing transient growth, which is inherent to bypass transition, using plasma actuators. This work is part of a multi-university collaboration that focuses on all aspects of the problem including dynamic disturbance input, sensing and actuation, dynamic estimation, and controller development. A distinction is made on the contributions arising from the author’s work and those of the collaborators. The focus of the author’s contributions are primarily on the issues relating to the actuators and implementation with the architecture closed-loop control system.

The success of the control system ultimately relies on the ability of the actuator to attenuate the target disturbance. For this reason, a systematic approach toward characterizing the attenuation mechanism is sought prior to the more complex active control case. This aspect can be studied in an open-loop framework. In the case of real-world bypass transition, the forcing environment is stochastic in nature, however, the
authority of the actuator can be determined prior for the ideal case of steady disturbance input. This portion of the research is inspired by the work of Jacobson and Reynolds [71] who showed some cancellation of embedded streamwise streaks using a mechanical-type actuator. Specifically, Jacobson and Reynolds [71] embedded a steady velocity streak into the laminar boundary layer and measured the effectiveness of the actuator in this highly tractable environment. This systematic method for characterizing the performance of an actuator enables detailed characterization of the attenuation mechanism prior to the more complex flow field ultimately sought. In the present research, arrays of cylindrical roughness elements were used to generate a steady disturbance of streamwise streaks undergoing transient growth, which has been shown to adequately simulate the transient growth phenomena associated with bypass transition [38; 93; 158; 159].

Following the open-loop control demonstration, the plasma actuator array was integrated with a feedback control system. Issues relating to the sensing of the target disturbance, as well as the design of a suitable control objective, were studied first. Next, the first demonstration of controlling a disturbance undergoing transient growth using feedback control and plasma actuators was accomplished. The effectiveness of feedback control was studied for both the steady case and for time varying changes of the disturbance level. A simple proportional-integral controller was highly effective for establishing and maintaining control in each of these cases.

The structure of this thesis is as follows. The next chapter provides a review of the physical description of bypass transition and transient growth, flow-control, plasma actuators, and experimental generation of transient growth by arrays of roughness elements. Details of the two unique experimental configurations for this research are discussed in Chapter 3 with a description of the experimental measurement techniques. Results of the open-loop control demonstration are shown in Chapter 4 along with the effect of actuator geometry and excitation signal. In Chapter 5, the closed-loop feedback control results are presented. The dynamic assessment of plasma actuation is considered in Chapter 6, which is toward the overarching realtime control objective. A summary of the conclusions, contributions, and recommendations of this work follows in Chapter 7. The appendices include details regarding experimental uncertainty and the design of the boundary layer plate leading edge.

Aspects of this work were published in peer-reviewed journals. The first publication was on the open-loop demonstration of the transient growth instability control using plasma actuators [55]. The second was on the leading edge design for reduced pressure gradient, which manifested as part of the experimental design work [59]. The third is on the closed-loop feedback control of the transient growth instability. Various other aspects
of this work have appeared at international conferences, for example, early work on the open-loop problem [54], as well as the effect of plasma actuator excitation parameters [56; 118], and early stages of the leading edge design [57]. In support of the collaborative work and controller design, experiments were compared with numerical simulations by Belson et al. [8]. Other aspects relating to the dynamic response of the boundary layer to dynamic forcing by plasma actuators [58] as well as early progress toward the closed-loop control problem [60] have appeared at conferences. In addition to the primary objectives, fundamental studies were also aimed at characterizing the robustness of actuators [61] and new manufacturing methods [65].
Chapter 2

Background

2.1 Boundary Layer Theory and Transition

The streamwise location where a boundary layer transitions to turbulence depends on several factors (e.g. free-stream turbulence and surface roughness). In cases where the background disturbance level is low, a classical transition pathway is followed, which is characterized by two-dimensional, exponentially growing, Tollmien-Schlichting (TS) waves followed by secondary instabilities leading to turbulence. At higher disturbance levels, perturbations manifest themselves as three-dimensional instabilities undergoing transient growth that bypass the classical transition pathway and result in transition at sub-critical Reynolds numbers. Further details may be found in the review by Saric et al. [140]. For either pathway the final state is turbulence, which enhances mixing and leads to higher shear stress. A brief presentation of the governing equations relevant to this work are included below. The nomenclature applies only within this subsection.

2.1.1 Governing Equations

The incompressible version of the conservation of mass and momentum equations, viz.

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j},
\]

\[
\frac{\partial u_j}{\partial x_j} = 0,
\]

constitute the incompressible Navier-Stokes equations, which are relevant to the present work. The variable \(u_i\) denotes the velocity components, \(p\) the pressure, \(\rho\) the density, and \(\nu\) the kinematic viscosity. The velocity vector consists of three components \((i = 1, 2,\)
and 3), such that \( \langle u_1, u_2, u_3 \rangle = \langle u, v, w \rangle \), and \( \langle x_1, x_2, x_3 \rangle = \langle x, y, z \rangle \), are the streamwise, wall-normal and spanwise components, respectively. Equations (2.1) & (2.2) can be represented in non-dimensional form by using the relationships,

\[
x_i^* = \frac{x_i}{L}, \quad u_i^* = \frac{u_i}{U_0}, \quad p^* = \frac{p}{p_0}, \quad \text{and} \quad t^* = \frac{tU_0}{L},
\]

The * signifies a non-dimensional variable and \( p_0 = \rho U_0^2 \). The length (\( L \)) and velocity (\( U_0 \)) represent the typical length and velocity scales of the problem. Applying the non-dimensional variables to the incompressible Navier-Stokes equations, (2.1) & (2.2), yields:

\[
\frac{U_0^2}{L} \frac{\partial u_i^*}{\partial t^*} + \frac{U_0^2}{L} \frac{\partial u_j^*}{\partial x_j^*} = -\frac{p_0}{L \rho} \frac{\partial p^*}{\partial x_i^*} + \frac{\nu U_0}{L^2} \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*},
\]

(2.3)

\[
\frac{U_0}{L} \frac{\partial u_i^*}{\partial x_i^*} = 0.
\]

(2.4)

Dropping the * from (2.3) & (2.4) and dividing by \( U_0^2/L \) gives the dimensionless form of the incompressible Navier-Stokes equations, viz.

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j \partial x_j},
\]

(2.5)

\[
\frac{\partial u_i}{\partial x_i} = 0,
\]

(2.6)

where the Reynolds number (\( Re \)) is defined as \( U_0 L/\nu \).

### 2.1.2 Stability Equations

The stability equations are linearized by introducing a small perturbation \( u_i = U_i + u_i' \) and \( p = P + p' \) and subtracting the mean flow, following the method of Schmid and Henningson [141]. The perturbation is assumed to be small such that all quadratic terms are much smaller than the linear terms and can be neglected. The linear stability equations are given by

\[
\frac{\partial u_i'}{\partial t} + u_j \frac{\partial U_i}{\partial x_j} + U_j \frac{\partial u_i'}{\partial x_j} = -\frac{\partial p'}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i'}{\partial x_j \partial x_j},
\]

(2.7)

\[
\frac{\partial u_i'}{\partial x_i} = 0.
\]

(2.8)
For the disturbance analysis, it is assumed that the mean flow is steady, laminar, parallel to the wall, and dependant on only one spatial coordinate i.e. \((U, V, W) = (U(y), 0, 0)\). For a boundary layer this is a reasonable assumption, over a short streamwise region, since the evolution of boundary layer properties with the streamwise coordinate is small compared to the variation along the wall-normal coordinate. Over longer regions there are implications of this assumption, which are addressed at the end of this section. The linearized disturbance equations for \(x, y, z\) coordinates can be expressed as

\[
\begin{align*}
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + v \frac{\partial U}{\partial y} &= -\frac{\partial p}{\partial x} + \frac{1}{Re} \nabla^2 u, \\
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} &= -\frac{\partial p}{\partial y} + \frac{1}{Re} \nabla^2 v, \\
\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} &= -\frac{\partial p}{\partial z} + \frac{1}{Re} \nabla^2 w,
\end{align*}
\]

where the \(\prime\) is omitted. Taking the divergence of (2.9) to (2.11), with the conservation equation, provides an equation for the pressure, \(viz.\)

\[
\nabla^2 p = -2U' \frac{\partial v}{\partial x},
\]

where \((')\) now represents differentiation with respect to \(y\). Using (2.12) to eliminate the pressure term in (2.10) results in the equation for the wall-normal velocity, \(viz.\)

\[
\left[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \nabla^2 - U'' \frac{\partial}{\partial x} - \frac{1}{Re} \nabla^4 \right] v = 0.
\]

To complete the description of the three-dimensional flow field an equation for wall-normal vorticity \((\eta_y)\) is used. From the linearized disturbance equations it can be shown that

\[
\left[ \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} - \frac{1}{Re} \nabla^2 \right] \eta_y = -U' \frac{\partial v}{\partial z}.
\]

Equations (2.13) and (2.14) are subject to the no-slip condition at the wall \((v = \eta_y = 0)\). Wavelike disturbances are introduced into (2.13) and (2.14) to simplify the problem, \(viz.\)

\[
\begin{align*}
v(x, y, z, t) &= \tilde{v}(y)e^{i(\alpha x + \beta z - \omega t)}, \\
\eta_y(x, y, z, t) &= \tilde{\eta}_y(y)e^{i(\alpha x + \beta z - \omega t)},
\end{align*}
\]

where \(\alpha\) and \(\beta\) are the streamwise and spanwise wavenumbers and \(\omega\) is the frequency of the disturbance mode considered [141]. Substituting (2.15) and (2.16) into (2.13) and (2.14) for the wall-normal velocity and vorticity respectively results in the classic
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definition of the Orr-Sommerfeld and Squire equations, \textit{viz.}

\[
\left[ (-i\omega + i\alpha U) \left( D^2 - K^2 \right) - i\alpha U'' - \frac{1}{\text{Re}} \left( D^2 - K^2 \right)^2 \right] \tilde{v} = 0, \tag{2.17}
\]

\[
\left[ (-i\omega + i\alpha U) - \frac{1}{\text{Re}} \left( D^2 - K^2 \right) \right] \eta_y = -i\beta U' \tilde{v}, \tag{2.18}
\]

where differentiation with respect to \( y \) is represented by \( D \) and \( K^2 = \alpha^2 + \beta^2 \). Alternatively, (2.17) and (2.18) can be expressed in state-space (c.f. Kim \cite{77}), \textit{viz.}

\[
\frac{d}{dt} \begin{bmatrix} \tilde{v} \\ \eta_y \end{bmatrix} = \begin{bmatrix} L_{\text{os}} & 0 \\ C & L_s \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \eta_y \end{bmatrix}, \tag{2.19}
\]

where \( L_{\text{os}}, L_{sq}, \) and \( C \) are the Orr-Sommerfeld and Squire operators, and the coupling coefficient, respectively. The character of transient disturbances can be shown using the model problem (see for example Reshotko \cite{130})

\[
\frac{d}{dt} \begin{bmatrix} v \\ \eta_y \end{bmatrix} = \begin{bmatrix} -1/\text{Re} & 0 \\ 1 & -2/\text{Re} \end{bmatrix} \begin{bmatrix} v \\ \eta_y \end{bmatrix}. \tag{2.20}
\]

This system has an exact solution, with eigenvalues \( \{-1/\text{Re}, -2/\text{Re}\} \), which would suggest that the system is stable and decaying. Although this is true for \( v \), consider the equation for \( \eta_y \), which is driven by \( v \) via the coupling term, \textit{viz.}

\[
\eta_y(t) = \eta_{y0} \exp(-2t/\text{Re}) + \text{Re} v_0 \left( \exp(-t/\text{Re}) - \exp(-2t/\text{Re}) \right). \tag{2.21}
\]

A taylor series expansion of the second and third terms from (2.21) for small times gives

\[
v_0 \text{Re} \left( \exp(-t/\text{Re}) - \exp(-2t/\text{Re}) \right) = v_0 \text{Re} \left( -\frac{t}{\text{Re}} + \frac{t^2}{\text{Re}^2} - \frac{2t}{\text{Re}} - \frac{4t^2}{\text{Re}^2} + \ldots \right) = v_0 t - \frac{3v_0}{\text{Re}} t^2 + \ldots \tag{2.22}
\]

This indicates that for small values of \( t \), \( \eta_y \) can experience algebraic growth, which is followed by exponential decay due to the first term of (2.21). The underlying reason that this occurs is due to the nonorthagonality of the coupled eigenvectors of the system.

Although this model describes the growth mechanism to be investigated in this research, it fails to account for continuous forcing by free-stream turbulence and how the initial perturbation enters the boundary layer, a process known as receptivity. Two dif-
ferent approaches for addressing this topic can be found, one method in Zaki and Durbin [162, 163], the other in Goldstein and coworkers [47; 48; 95; 161]. In the first approach, the physics of the coupling of the free-stream vortical disturbances with the boundary layer is interpreted in terms of the continuous Orr-Sommerfeld modes, their boundary-layer penetration depth, and their forcing of the Squire equation. The physical insights gained from this approach were found to be consistent with DNS results; though the absence of non-parallel and non-linear effects in the theory does not allow prediction of proper streak amplitudes and growth rates. The theory developed by Goldstein and coworkers (which was later extended by Ricco and Wu [131], Wu and Choudhari [160], and others) overcomes this limitation by employing the so-called boundary-region equation in which only the streamwise gradients of the pressure and viscous terms in the Navier-Stokes equations are neglected. The streamwise viscous gradients can be neglected since the perturbations governing streak formation are of low frequency and long wavelength [131]. The streamwise pressure gradient can be neglected for Blasius conditions, however this assumption fails near the leading edge [101]. In this manner, the equations remain parabolic in the streamwise direction but retain cross-flow ellipticity, spanwise-diffusion, and non-linear effects. Employing this approach, for the linearized boundary-region equation, Leib et al. [95] found the spanwise ellipticity to have strong influence on the streamwise growth of the streaks, and were able to provide predictions of the associated disturbance energy growth that compared favourably with experiments. Later, Wundrow and Goldstein [161] added non-linear effects, highlighting their significance to the process of streak breakdown.

From a flow control point of view, regardless of the path leading to the formation and growth of the streaks, the latter appears as a spanwise-periodic modulation of the streamwise velocity inside the boundary layer. Such a modulation can be affected (in an opposite sense to that existing in the flow) by introducing counter-rotating streamwise vortex pairs using wall-mounted actuators. This mechanism provides the basis of the actuation design aimed at attenuating the streaks inherent to bypass transition.

2.1.3 Transition and Transient Growth

The instability causing the formation of streamwise oriented and spanwise periodic velocity streaks of low- and high-velocity is described as an inviscid lift-up mechanism [92]. The physical mechanism of the process can be described from inspection of (2.13) and (2.14). A perturbation consisting of pairs of counter-rotating streamwise oriented vortices transports low-velocity fluid away from the wall to the outer region of the boundary layer across the mean-velocity gradient in regions where the wall-normal perturbation
velocity is positive. Simultaneously, high-velocity fluid is transported to the low-velocity near-wall region in regions where the wall-normal perturbation velocity is negative. This mechanism establishes the spanwise variation in the streamwise velocity ($\partial u/\partial z$) known as streaks, which gives rise to wall-normal vorticity ($\eta_z$). This lift-up mechanism was verified both numerically [70] and experimentally [69]. It is an inviscid mechanism commonly referred to as transient growth [17; 63; 67], which is characterized by algebraic kinetic energy growth followed by exponential decay due to viscous damping if of insufficient amplitude to trigger secondary instabilities.

Early experiments by Klebanoff et al. [80] identified the existence of streamwise velocity streaks, which were later visualized by Kendall [76], and termed Klebanoff modes. These modes were believed to be responsible for what was termed bypass transition by Morkovin [111]. Matsubara and Alfredsson [105] introduced smoke along a thin spanwise line in the boundary layer. Visualization of turbulence-induced streamwise streaks in a laminar boundary layer with a free-stream velocity of 6 m/s and turbulence intensity of 1.5% is shown in Figure 2.1. The streaky structure of the smoke layer is apparent and the spanwise spacing of the streaks is approximately 1 cm. A turbulent spot is also observed in this visualization.

Optimal theory applied to the transient growth problem was used to identify characteristics of the perturbations that cause the greatest growth over a specified streamwise extent. The growth factor ($G$) is the maximum ratio of measured output to input energy; the optimal perturbation causes the greatest growth factor. The first attempt to identify the shape of the optimal perturbation was by Butler and Farrell [17] for a computationally simple parallel channel flow. They showed that the optimal perturbation resembled pairs of streamwise oriented, counter-rotating vortices with a wavenumber ($\beta > 0, \alpha = 0$), despite neglecting the spatial growth of a boundary layer.

The linearized optimal-disturbance problem over a streamwise region of a boundary layer was addressed by Andersson et al. [3], Luchini [101], and Tumin and Reshotko [153]. Andersson et al. [3] and Luchini [101] both considered boundary layer growth, whereas

Figure 2.1: Flow visualization of streaky structures in a boundary layer affected by free-stream turbulence. The flow is from the left. Reprinted from Matsubara and Alfredsson [105] with permission from Cambridge University Press.
Tumin and Reshotko [153] restricted their results to a two-dimensional boundary layer (over a rotating object for example). Luchini [101] considered high Reynolds numbers and both stationary and traveling disturbances, whereas Andersson et al. [3] restricted the problem to steady disturbances over a range of low to high Reynolds numbers. The results were discussed in terms of a non-dimensional spanwise wavenumber, \( \beta = 2\pi \delta / \Delta z \), and height at which the peak disturbance energy occurs, \( \eta = y/\delta \), where \( \delta \equiv (x\nu / U_x)^{1/2} \) is the Blasius similarity variable and \( y \) is the wall-normal coordinate. Comparison of the parallel and non-parallel approaches shows only a difference between the streamwise peak disturbance location. The optimal disturbance shapes are consistent, occurring with the maximum disturbance amplitude at \( \eta = 2.2 \) for \( \beta = 0.45 \), and are stationary (\( \omega = 0 \)).

The shape of the optimal perturbation is shown by Figure 2.2(a), which is comprised of \( v \) and \( w \) velocity components. The resulting contours of streamwise velocity are shown in Figure 2.2(b) at \( xf = 1 \). Note that \( x \) is the distance from the leading edge scaled by an arbitrary fixed distance and \( xf \) refers to the final streamwise extent that growth was integrated over. The variation in the growth factor is shown in Figure 2.2(c) as a function of \( \beta \) and for the optimal wavenumber as a function of the streamwise location in Figure 2.2(d). Figure 2.2(c) demonstrates that growth is reduced for sub-optimal conditions (\( \beta \neq 0.45 \)). A similar behaviour occurred for non-stationary disturbances (\( \omega > 0 \)). Figures 2.2(c & d) show Reynolds number independence for \( Re \geq 10^4 \), where \( Re \) is based on the free-stream velocity and distance from the leading edge.

### 2.2 Flow Control of Streaks and Turbulence

The motivation for the control of boundary layer transition and turbulence is centred at the common goal of lowering skin friction drag. Bypass transition can be prevented by controlling the rapid growth of streaks, which trigger this form of sub-critical transition. Streak control is also of interest in the case of wall-bounded turbulence. It has been observed that the linear transient growth mechanism, the root instability of Bypass transition, also occurs in the self-sustaining production of turbulence in the TBL.

The presence of velocity streaks and regions of streamwise vorticity in the TBL has been know for some time [82]. In the case of the TBL, streamwise oriented vortices occurring in the buffer layer cause the formation of streamwise streaks of low- and high-velocity by the same lift-up process described in Section 2.1.3. When sufficient amplitude is reached a non-linear mechanism dominates and leads to further vortex formation of streamwise vortices through a feedback process. This scenario shows a strong resemblance to the bypass transition phenomenon. A review of this process is discussed in detail by
Figure 2.2: Velocity vectors in the $y - z$ plane of the optimal disturbance (a). The $y$ and $z$ coordinates are scaled by the Blasius similarity variable. Contours of constant streamwise velocity representing the downstream response at $xf = 1$ for the optimal disturbance. Solid lines represent positive values and the dashed lines represent negative values (b). Maximum spatial transient growth as a function of $\beta$ (c), and as a function of the downstream position for $Re = 10^3 \circ$, $10^4 \circ$, $10^5 \Delta$, $10^6 +$, $10^9 \circ$, and $Re$ independent — (d). Reprinted with permission from Andersson et al. [3]. Copyright 1999, American Institute of Physics.

Kim [77] and Panton [120]. As discussed by Kim [77], the near-wall turbulence structures as well as near-wall turbulence statistics are almost identical for turbulent channels or TBLs. Therefore, results pertaining to each can be discussed interchangeably.

From the experimental standpoint, the case of bypass transition is more tractable. The reason is related to the larger spatial and temporal scales of the streaks occurring in bypass transition compared to those occurring in the buffer layer of the TBL. Streak spacings in the TBL are over a continuous spectrum of wavelength $\lambda^+ = \lambda u_\tau / \nu$ from 20 - 200, where $u_\tau$ is the friction velocity, and $\lambda$ is the spanwise spacing of the streaks (see for example Brereton and Hwang [12]). Following from the example shown by Carpenter et al. [18], applied to a Bombardier Q400 flying at 650 km/h at an altitude of 7000 m, it can be estimated that the typical spanwise spacing of the streaks would be 0.5 mm and would extend approximately 5 mm in the streamwise direction. Streaks occurring in bypass transition have a spanwise wavelength, $2\pi \delta / \beta$, that is typically one or more orders of magnitude larger than the turbulent case, and are elongated in the streamwise direction by an order of magnitude or more (see for example Matsubara and Alfredsson
Shorter time scales of the TBL also contribute to experimental difficulty. For instance, the averaging bursting frequency of sub-layer streaks can be approximated as $0.0033u^2/\nu$ in the TBL [11], which corresponds with 3.9 kHz using the example case. In bypass transition, a phenomenon known as shear sheltering restricts receptivity to only low-frequency disturbances in the free stream [70]. Therefore, streaks occur at a much lower rate than bursting in the TBL.

For numerical simulations the entire time resolved flow field is available. Therefore, the effectiveness of an actuator to target specific flow features can be extracted. However, flow control experiments with stochastic forcing typically rely on data from limited spatial locations (see for example Lundell [102] or Rathnasingham and Breuer [127]). Current flow diagnostic tools limit time resolved measurements of control results to discrete positions or time-averaged global flow quantities such as shear stress, which are prone to high uncertainty as discussed by Lundell [102]. Therefore, the physics of disturbance attenuation is often speculated from limited data. For the laminar case it is possible to create a quasi-deterministic forcing environment, which enables detailed flow measurements of the control mechanism. This point is addressed in the following sections.

### 2.2.1 Numerical Flow Control of Transition and Turbulence

From the perspective of active control, the linearity of the instability mechanism causing transient growth in bypass transition (and linked with the self-generation of turbulence) suggests the effectiveness of a linear controller; representing a great reduction in complexity. There has been substantial progress in the use of modern, model-based, linear control theory in computational flow control problems [78], which has been shown to significantly outperform classical proportion-based control schemes [10].

An early numerical example of an active flow control strategy is termed opposition control [21], wherein the wall-normal or spanwise velocity is detected slightly above the wall and a response of equal magnitude and opposite direction is applied. Opposition control prevented streak formation by suppressing streamwise vortices, which reduced turbulence intensities, albeit for a channel flow where the parallel flow assumption is valid. Högberg and Henningson [64] applied linear optimal control theory to a spatially developing boundary layer for both the classic TS and bypass transition scenarios. Control stabilized the flow and decreased the fluctuation energy of single-mode disturbances effectively by forcing them to decay. However, the controller was less successful on multi-mode disturbances growing algebraically, which began to experience significant growth.

---

1Recent evidence suggests that the Taylor microscale is a more appropriate scaling to reduce the Reynolds number dependance [106]
downstream, demonstrating a need for multiple streamwise control locations. The significance of this work was to show the usefulness of the simplified controllers, developed with the parallel flow approach, to spatially evolving flow.

The role of the linear coupling term between the wall-normal velocity and vorticity was investigated by Kim and Lim [79] using a non-linear form of (2.19), viz.

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \tilde{v} \\ \tilde{\eta}_y \end{bmatrix} &= \begin{bmatrix} L_{os} & 0 \\ C & L_s \end{bmatrix} \begin{bmatrix} \tilde{v} \\ \tilde{\eta}_y \end{bmatrix} + \begin{bmatrix} \mathcal{N}_v \\ \mathcal{N}_{\eta_z} \end{bmatrix},
\end{align*}
\]

(2.23)

where \( \mathcal{N}_v \) and \( \mathcal{N}_{\eta_z} \) are the non-linear components of the wall-normal velocity and vorticity, respectively. It was shown that without the coupling term (\( C = 0 \)) the near-wall structures first disappeared. Turbulence intensities were then significantly reduced demonstrating the critical role in the cyclic self-generation of turbulence [77]. The necessity of the coupling term for transition to turbulence and requirement of non-normality for subcritical transition was already well established [62]. However, this result has strong implications toward controller design, showing the importance of the linear sub-problem.

The recent work by Sharma et al. [144] proposed control of turbulent flows based on the knowledge that the non-linear parts of the Navier-Stokes equations are conservative with respect to energy in a closed domain. The basic controller concept is shown in Figure 2.3. This figure describes the non-linear disturbance equations (2.23 from above) in a block diagram with control. The non-linear terms act only as a noise \((n)\) source to the linear sub-problem. A linear controller is applied only to the linear part of the equations, thus controlling the portion responsible for the generation of energy and ensuring stability. The significance of this work was to prove global stability of the non-linear flow, albeit for the first step of a parallel channel flow, never before accomplished. Using this methodology a turbulent channel flow was relaminarised. However, full flow domain sensing and actuation was used.

Figure 2.3: The feedback loop for controlled Navier-Stokes equations. The subsystem inside the dashed box is Q. Figure adapted from Sharma et al. [144].
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The aforementioned examples are benchmark studies for the potential of modern flow control techniques. The controllers are not directly applicable or feasible in practice since the numerical simulations use full flow domain information, which is not available in the laboratory. Nonetheless, the results of numerical control simulations are impressive. For instance, full flow domain information and forcing has led to relaminarised channel flows [144]. In practice, sensors and actuators must be embedded on the wall and are typically restricted to either pressure or shear stress information. In addition, sensor and actuator locations will be discrete and their numbers limited. The practical ramifications will be reduced control effectiveness. For example, Endo et al. [31] showed that using only a finite number of sensors, which were limited to wall information and wall-based actuators, the impressive 30% drag reductions shown by Choi et al. [21] were reduced to approximately 10%.

Controllers using the discretized Navier-Stokes equations are expensive to implement in numerical simulations, owing to the large dimensional model [5; 78]. To facilitate a realtime controller in practice, reduced-order flow models of minimal order to describe disturbances are sought. Also, in practical settings the limited sensing and actuation locations further highlight this need, with extension to accurate estimation techniques. This is currently considered the largest challenge with linear control theory for transition delay [5]. In an attempt to bridge the gap between ideal numerical control simulations and experiment Monokrousos et al. [109] performed control using both full flow domain information and with estimation from only limited wall-based information. Specifically, the wall-parallel shear stress components were observed to be much more effective than pressure-based measurements, which were contaminated by the free-stream turbulence. A visualization of the controlled and uncontrolled flow field is reproduced in Figure 2.4 in a wall-parallel plane, where regions of high- and low-velocity are in greyscale. In the control region the streaks were suppressed however, they continued to grow due to forcing by free-stream turbulence downstream.

The limitation of the study by Monokrousos et al. [109] was the complexity of the controllers, which were designed in Fourier space. It was highlighted by Semeraro et al. [143] that such a design leads to high computational cost. Semeraro et al. [143] measured the input/output relationships between the sensors and actuators. Using balanced proper orthogonal decomposition, they identified the minimal information sufficient for control and reduced the controller complexity by over three orders of magnitude (to a 60 degrees of freedom system). A limitation of this study was that only flow information upstream of the actuators was used. Feedforward control performance often decreases in the presence of non-modelled disturbances, whereas feedback provides robustness [1].
Figure 2.4: Instantaneous streamwise velocity fields, for the uncontrolled and controlled growth of streaks in a LBL from Monokrousos et al. [109]. The streamwise extent is from $Re_x = 3.2 \times 10^4$ to $3.82 \times 10^5$, and the aspect ratio is 1:1. Reprinted from Monokrousos et al. [109] with permission from Elsevier. Relabelled by the author.

The optimal placement of limited sensors and actuators remains an open question [5]. In general, it is accepted that the spanwise placement of actuators and sensors should naturally scale with the targeted structures. However the optimal streamwise locations are not established. The placement of sensors and actuators will have direct consequence to the stability of the controller. For instance, Naguib et al. [112] showed that there is an inherent time delay (at the convective time scale) for the effects of an action on the flow to be measurable at a downstream location from wall-based measurements.

### 2.2.2 Experimental Control of Streamwise Streaks

Passive control of the TBL, aimed at weakening the effect of streamwise vortices, is summarized by Kim [77]. One example of passive control is streamwise oriented riblets on the boundary layer wall. Riblets prevent the direct interaction of streamwise oriented vortices with the wall, leading to drag reductions of around 6% [155]. The drag reduction mechanism is to restrict the streamwise vortices above the wall surface such that only a limited wetted area of the riblets is exposed to the downwash of high-speed fluid induced by the vortices [20]. However, this passive system can also lead to increased drag during off-design flow conditions (for example [94]). An active control system however, can be switched off when not required.
Active control necessitates the use of an actuator. A variety of actuators have been utilized for controlling streaks. Examples include flap-based configurations [71], synthetic jets [127], vertical wall motion [14], suction [39], and blowing [129]. For a further description of actuators for flow control see Cattafesta and Sheplak [19].

Experimental demonstrations of active boundary layer control are rare, and their outcomes are less impressive than comparable numerical simulations. Demonstration of opposition control, based on a previous numerical simulation [21], was made by Rebbeck and Choi for a suction type actuator [129] and a piston type actuator [128]. These studies were limited by the available hardware and did not attempt to implement more than a single sensor/actuator pair, as shown in Figure 2.5. The effectiveness of the actuator was studied through the post-processed results. The streamwise velocity measurements were ensemble averaged based on the detection of a randomly appearing disturbance at the upstream wall-wire. The actuation was shown to inhibit the bursting of the streaks, whereas Choi et al. [21] showed over 15% global reductions in skin friction drag by numerical flow simulation.

Jacobson and Reynolds [71] conducted a systematic study of streak control using a wall-normal cylinder to embed a stationary vortex pair in a LBL. They controlled the high-speed streak using an oscillating cantilever-beam-type synthetic jet actuator mounted flush to the wall. The actuator introduced a counter-rotating vortex pair of the opposite sign to that introduced upstream by the cylinder. Following this they designed an ad hoc linear controller to target the dynamic disturbances introduced with unsteady suction. Figure 2.6 shows the normalized streamwise disturbance velocity caused by the actuator, with suction only, and for both actuation and suction active. The actuation effect is weak and not optimized. Notwithstanding this limitation, they showed these control schemes could decrease disturbance amplitudes.

More recently, Lundell [102] employed distributed suction to attenuate streaks in a laminar boundary layer caused by free-stream turbulence. By varying the threshold level to trigger actuators, the delay time, and suction strength, the control effect was optimized. For a well-tuned system, Lundell [102] found that formal system identification methods gave little improvement. The control results were quantified by the growth of the
fluctuations of the streamwise velocity component downstream, which are an indicator of streaks. These fluctuations were reduced by 15% and 20% for a low and high free-stream turbulence cases, respectively. The effect of control diminished downstream, which was attributed to the stochastic forcing environment and contamination at the spanwise edges of the control region. Rathnasingham and Breuer [127] showed that the zone of controlled flow will decrease downstream, caused by the entrainment of uncontrolled flow at the edge of the controlled region. A numerical flow simulation, which resembled the experimental conditions of Lundell [102], showed that the control effectiveness could be improved with a model-based controller [104], which accounted for the non-monotonic behaviour of the actuator output. Attenuation of the streamwise velocity fluctuations were increased to 55%. In addition, the matched numerical simulations suggest that a full span of sensors and actuators will enable an experimental demonstration of transition delay, which will limit the contamination of the controlled flow associated with a short spanwise extent of sensing and actuation.

The major recurring theme in flow control is the development of sensors and actuators, which is typically done in-house prior to the flow control application. The latter is often considered to be the pacing item to the practical demonstration of boundary layer control [71], and remains an ongoing topic. In the previous examples, actuators are bulky, complex and difficult to extend to arrays. The need for durable, flexible, small, and inexpensive actuators was highlighted as the most important missing technology for boundary layer transition control [104]. This problem is addressed in this thesis.
2.3 Plasma Actuators

The term *plasma actuator* includes a variety of different actuator types such as corona and sliding-discharges [110]. Only dielectric-barrier-discharge (DBD) plasma actuators are considered in this review, and are hereafter referred to as plasma actuators for simplicity. The potential for these actuators as flow control devices was first suggested by Roth *et al.* [134]. Following this introduction to the international community, considerable research has been directed toward the physical analysis of actuator performance and optimization for flow control applications.

2.3.1 Physics of Operation

In a simple form, a plasma actuator consists of two electrodes, one exposed to the surrounding air and the other covered by a dielectric material and insulated, as illustrated in Figure 2.7(a). The grounded electrode is often encapsulated in insulating material to prevent unwanted plasma formation on the underside, which would be a source of inefficiency. The plasma actuator is operated by applying a high potential difference across the electrodes, which is typically between 1 and 50 kV. A gas discharge is created when the electric field is of sufficient amplitude to generate electron-ion pairs through electron impact ionization (see for example [91]), which is referred to as breakdown and initiates an avalanche mechanism followed by streamer formation. Breakdown occurs where the electric field reaches sufficient amplitude first, typically at the edge of the exposed electrode, causing the streamers to propagate toward the dielectric surface, where the potential is lowest. Streamers transfer charge to the dielectric surface, which builds a surface charge causing a self-limiting effect that quenches the plasma. To counteract this effect, the voltage magnitude must continue to increase. Alternatively, the polarity of the field can be reversed to cause a cyclic process of plasma formation. DBD plasma actuators are typically operated with an AC voltage to overcome the self-limiting nature, with the added advantage that the initial breakdown voltage at atmospheric pressures is lower for AC voltages. Typical operating frequencies are between 1-10 kHz. To the eye, the ionized air (plasma) appears blue-violet. However, light emission intensity is low, since air is only weakly ionized in the DBD operating regime. The fraction of ionized to neutral particles is typically in the $10^{-3}$ to $10^{-4}$ range. For a detailed review of the physics of plasma actuator operation see Corke *et al.* [27], or Moreau [110].

The mechanism by which a plasma actuator creates momentum from electrical energy is through a Lorentz force. Ions are accelerated by the electric field. Recalling that the ionization level is low, the forcing is applied to the neutral air by collisions with charged
Figure 2.7: (a) Illustration of a simple plasma actuator. (b) Typical wall-normal profiles of the streamwise velocity in quiescent air at multiple locations downstream of the HV electrode, reproduced from Pons et al. [122].

particles at a high rate, and nearby fluid is entrained by the motion of the weakly ionized air (or electric wind). The result for the simple configuration shown in Figure 2.7(a) is a tangential wall jet, shown in Figure 2.7(b). The spatial distribution of the profile depends on the operating conditions, such as the voltage and frequency, but also key geometric parameters such as the dielectric thickness, electrical properties, electrode arrangement, and atmospheric conditions [36; 90; 122; 133].

The above discussion was on the time-averaged output of plasma actuators. However, the actuators are operated with an alternating voltage causing the electric field to reverse implying an asymmetry between the positive and negative going voltage phases [34]. As noted previously, AC frequencies of 1-10 kHz are commonly used, which would suggest a time varying output. This has been verified to occur (see for example [34; 84]), and the dynamic process is highly dependant on the shape of the excitation waveform [85].

A recent study by Kotsonis and Ghaemi [85] showed that the time-resolved production of momentum by plasma actuators consists of four phases, which occur over one complete voltage cycle, as demonstrated in Figure 2.8. The variation in voltage and current (a) are correlated with variation in the induced velocity (b) and acceleration (c). When a discharge is formed, current spikes are visible, which are known to correlate with visible plasma formation [33]. A strong positive acceleration occurs for the negative going cathode portion (A-B from Figure 2.8), and a weaker acceleration is noted for the anode phase (C-D). When the plasma is extinguished (i.e. no current spikes visible, B-C), a strong negative acceleration occurs at the end of the cathode phase, and only a weak negative acceleration occurs after the anode phase. Light emission measurements of the spatial distribution of the plasma discharge show a more uniform distribution for the cathode phase, compared to localized discharges for the anode phase [34].
Although research on the physics of plasma actuator operation has been highly active for over a decade, significant debate on the temporal nature of the body force remains. This debate stems from the fact that these actuators operate with AC voltages and universal agreement over the force generated during the positive and negative cycles for a sinusoidal input waveforms does not exist. However, it is agreed that the movement of heavy ions in the resulting electric field provides the mechanism to impart momentum to the neutral gas, resulting in a net body force. The two predominant theories are descriptively referred to as push-pull and push-push. For the push-pull theory, a positive and then negative forcing occurs during an AC cycle, and positive-positive for the latter case. Moreover, the relative magnitude of each component is disputed and a further subdivision exists.

The recent experimental work by Enloe et al. [34] suggests that the cathode phase imparts the most significant forcing contribution, 97% versus 3% in the anode phase. Support for either theory is shown by varying the waveform and geometry of the exposed electrodes. For instance, Debien et al. [28], demonstrated support for each theory by modifying the shape of the exposed electrode. Kotsonis and Ghaemi [84] demonstrated that because the waveform shape can play a significant role in the momentum production mechanism, it is possible to optimize the ratio of momentum output to energy consumption [85].

Closure will likely occur with first-principles simulations, which are currently limited due to the high computational costs [146]. In general, empirical and semi-empirical models, which attempt to model the plasma actuator chemistry are inconsistent with various levels of success in comparison to experiments [119]. Models tuned to a calibration...
case can deviate from measured results when the geometry or excitation characteristics change. However, this topic goes beyond the scope of the present work.

2.3.2 Quasi-Steady Behaviour

Many flow control studies using plasma actuators assume that the forcing is steady for constant operating conditions. Although this assumption is not true near the actuator, as described previously, the carrier frequency driving the actuator is high, resulting in an observed steady forcing downstream of the actuators in most cases. For instance, Tollmien-Schlichting waves in a zero pressure-gradient boundary layer are receptive to unsteady forcing for only a range of low frequencies. The maximum reduced frequency \( F = \frac{2\pi \nu f}{U_x^2} \) where unsteady forcing is amplified is typically around \( 250 \times 10^{-6} \) [40]. For a flow velocity of 1 and 100 m/s, this corresponds to a forcing frequency \( f \) of 2.54 and \( 2.54 \times 10^{-4} \) Hz, respectively, which is at least two orders of magnitude less than the carrier frequency required to operate plasma actuators. For airfoil flow separation, the reduced frequency \( F = \frac{fx_{sp}}{U_x} \) where \( x_{sp} \) is the length of the separation region, is typically 1 (see for example [44]). Again this is another example of low-frequency dependence. In the boundary layer the unsteady portion of high-frequency forcing will decay, resulting in an observed steady output further downstream. Owing to this behaviour, the flow output by plasma actuators is often called quasi-steady, since the average output has a mean component and a damped unsteady component. Therefore, to excite a flow instability or to dynamically control separating flows, signal modulation or pulsing at low frequencies is commonly used, see for example Grundmann and Tropea [51] or Little et al. [100].

2.3.3 Utilization in Flow Control

Despite lacking agreement on the exact physics of force production, plasma actuators have become a prevalent device in flow control since introduced by Roth et al. [134]. For instance, plasma actuators have demonstrated ability in various examples, including separation control [136; 126; 121; 73], aircraft noise reduction [66; 97], reducing losses in compressor blades [98], and wake control [152]. The attenuation of TS waves in an adverse pressure gradient laminar boundary layer using a pair of asymmetrical plasma actuators was also recently demonstrated [49; 50].

The plasma actuator geometry can be tailored for a variety of flow control applications and are readily surface mounted to experimental models. For instance, the actuator shown in Figure 2.7(a) can be arranged to inject streamwise momentum. This injection of quasi-steady momentum can rearrange the boundary layer to a stable state [29], causing
TS waves to decay [51]. From an energy perspective however, pulsing the actuator such that the goal is to actively cancel the TS waves is more efficient, requiring only a small fraction of the power used to attenuate TS waves with steady forcing [51]. The control mechanism is then similar to the wave cancellation technique of Milling [107], while the mean boundary layer characteristics remain unaltered.

Alternatively, the actuator can be rotated 90 degrees for spanwise forcing. Arrays of plasma actuators arranged in this manner, with the HV electrodes parallel to the streamwise flow, have been used to induce streamwise vorticity. The first known example of this arrangement for flow control was by Roth et al. [135]. This arrangement of actuators became known as Plasma Streamwise Vortex Generators (PSVG). A simple schematic of a PSVG is shown in Figure 2.9. The first flow visualization example of these actuators generating streamwise vorticity and transition to turbulence is shown in Figure 2.10. For this example the excitation voltage was 5 kV\textsubscript{rms} at 3 kHz, and the flow velocity was 4 m/s. It was shown that the location where the boundary layer transitioned could be modified by varying actuator output.

![Figure 2.9: Schematic of a spanwise array of plasma actuators, adapted from Thomas and Schatzman [151].](image)

![Figure 2.10: Smoke-wire flow visualization over a spanwise array of plasma actuators used to induce transition [135].](image)

Other examples of PSVGs are used for extending the range of attached flow over bluff bodies [86] or for preventing airfoil trailing edge separation [151; 73]. Compared to conventional passive vortex generators mounted on the airfoil surface, the plasma actuators can be switched off when not required. Therefore, the plasma actuators do not have a large drag penalty associated with passive devices during off-design conditions.
Choi et al. [23] used rows of plasma actuators in a turbulent boundary layer with an alternating spanwise force, using a two channel power supply. The control concept was to create a spanwise oscillation of near-wall fluid to reduce skin-friction drag and mimic the effect of a mechanically oscillating wall, which can reduce turbulent drag [22].

Other novel designs include that of Santhanakrishnan and Jacob [137]. They constructed the upper exposed electrode as a concentric ring around a smaller diameter buried ground electrode causing a vertical wall jet in quiescent air. In addition, it was shown that two parallel exposed electrodes, causing opposing wall-jets led to a similar, albeit two-dimensional effect to their circular actuators. By varying the voltage difference between adjacent exposed electrodes, Porter et al. [124] and Benard et al. [9] showed that a simple device with only two exposed electrodes could also be used for jet vectoring. Another type of device is referred to as a serpentine actuator, where the exposed electrode is sinusoidal in shape, which causes both spanwise and streamwise vorticity [30].

The aforementioned examples were experimental demonstrations. Practically, a general model of actuator forcing would be beneficial toward simulating and optimizing plasma actuators prior experimental implementation.

2.4 Arrays of Roughness Elements

Significant progress in transient growth theory led to the prediction of the optimal disturbance shape, frequency, and growth factor. Experimental studies were aimed at validating the transient growth phenomena. For experimental implementation it is not possible to introduce a steady disturbance into the flow at a discrete location with exactly prescribed conditions, as in numerical studies. Spanwise arrays of small, equally-spaced roughness elements are commonly used to produce the initial perturbation. For instance Bakchinov et al. [6] used a spanwise periodic array of rectangular roughness elements, placed at the wall downstream of the leading edge, to generate large amplitude streaks. Alternatively, arrays of cylindrical roughness elements are more commonly utilized [158; 38; 159; 93].

The resulting disturbance caused by arrays of roughness elements is often characterized by its height \( k \) relative to the boundary layer scale \( \delta \) or the roughness element Reynolds number \( \text{Re}_k = u(y = k)k/\nu \). The critical \( \text{Re}_k \) value causing transition is between 300 and 1000 [81]. The disturbances generated from small to moderate values of \( \text{Re}_k \) exhibit distinct differences. The small amplitude disturbances studied by White [158] had \( \text{Re}_k \) values of 45 and 80, whereas those of Fransson et al. [38] reached amplitudes between 5% and 11% of the free-stream velocity with \( \text{Re}_k \) between 180 and 340, respectively. The resulting flow patterns were similar in shape, however, drastically different
in phase. For small values of $Re_k$, low-speed streaks are positioned directly downstream of the roughness element, whereas high-speed streaks occur in this location for moderate to high values of $Re_k$. The streak generation mechanism is arguably different. Fransson et al. [38] conjectured that the spanwise vorticity of the incoming shear flow is wrapped around the cylinder forming a steady horseshoe-shaped vortex with the two streamwise legs pointing downstream, which was later verified with numerical simulations [132]. The resulting flow pattern of the counter-rotating streamwise vortices associated with the horseshoe vortex cause high-speed fluid to be pushed towards the wall in the region behind the roughness element, shouldered by regions of low-speed fluid [68]. This mechanism is depicted in Figure 2.11. Conversely, for small values of $Re_k$ the combined effect of the wake associated with a counter-rotating wall-normal vortex pair (which decays downstream) and the shear-layer over the upper surface of the elements (wake-mode) is not overcome by the spanwise vorticity (roller-mode) wrapping around the element [132], resulting in the observed low-speed region downstream of the roughness element.

![Figure 2.11: Illustration of the roughness element array generating velocity streaks for $Re_k > 180$.](image)

The flow patterns of the experimentally generated streaks are complex downstream of the roughness element array owing to the competing wake and vorticity mechanisms. For example, Fransson et al. [38] showed that near the roughness array, the resulting disturbance is comprised of a fundamental disturbance wavelength defined by the spacing of the roughness elements, as well as higher harmonics. Higher harmonics manifest as a distortion of the fundamental disturbance mode, as shown in Figure 2.12(a). The value $U_N$ is normalized by the half amplitude of the disturbance velocity. It was conjectured
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that the higher harmonics are due to the non-linear generation process. As shown in Figure 2.12(b), the contribution of higher harmonics is weakened downstream and the spatial distribution of the streamwise disturbance velocity resembles that predicted by optimal growth theory, as shown in Figure 2.12(c) and 2.2(b). Sub-optimality of the experimentally generated streaks using roughness element manifests in the peak streamwise disturbance energy, which was located near \( \eta = 2 \), whereas optimal theory predicts a location of \( \eta = 2.2 \). In addition, the experimentally-generated streaks tend to peak in energy further upstream [158]. The majority of research on the experimental generation of streaks has sought to explain this phenomena through differences in the mechanisms of streak generation by arrays of roughness elements, for the purpose of determining an appropriate universal scaling of the sub-optimal streaks.

Figure 2.12: Contours of the streamwise disturbance velocity downstream of the roughness element array (left) and corresponding normalized spanwise profile of the streamwise velocity at the wall-normal location of the peak disturbance energy (right). Three streamwise locations downstream of the array are shown at 55, 100 and 500 mm for a-c respectively. Figure adapted from Fransson et al. [38].

In addition to the deployment height of the roughness element, the diameter of the roughness elements and the factor \( d/\delta \) has a significant role. As the roughness diameter is increased the amount of obstruction to the flow is increased, leading to an increase in the total disturbance energy [159]. The diameter will influence the distance between each paired vortex downstream of each roughness element. Since the net perturbation is
a combination of the decaying wake-mode and the roller-mode associated with the pairs of counter-rotating vortices, larger values of $d/\delta$ may increase the significance of the wake mode [24]. However, there is little evidence to support this claim based only on $d/\delta$. For instance, the persistent low-speed streak of White [158] and White et al. [159] centred about each roughness element had comparable values of $d/\delta$ with Fransson et al. [38]. Furthermore, the likely interaction of the associated duty cycle, the fraction of spanwise spacing of the roughness elements to diameter ($\Delta z/d$) was overlooked. As noted by Lavoie et al. [93] for very small values of $d/\Delta z$ the perturbation from individual roughness elements would have an evolution resembling that of isolated roughness elements, which may evolve differently from a periodic array of interacting flow patterns.

The scaling of the transient growth curves caused by arrays of roughness elements are not yet universal. For example, Fransson et al. [38] suggested scaling the streamwise distance on the standard viscous scaling, such that $\beta = 2\pi\delta/\Delta z$, and normalized the disturbance energy with the associated peak energy. Despite having a good collapse of the data, the authors did not consider the geometry of the array and varied only the velocity. Alternatively, White et al. [159] scaled the disturbance energy with $Re_k^2$ however, only considered the roughness element height. Lavoie et al. [93] conducted a parametric study of the effect of the array spacing ($\Delta z$), cylinder diameter ($d$), deployment height ($k$), and free-stream velocity. The scaling is reproduced in Figure 2.13.

![Figure 2.13](image)

**Figure 2.13:** Normalized averaged disturbance energy at $\beta_f$ for 15 different flow conditions and roughness array arrangements identified by different symbols (based on the measurement and analysis from Lavoie et al. [93]). The dashed line represents the location where the measurements were taken for the MSU experiments.
Plotted in Figure 2.13 is the magnitude of the spanwise wavenumber power spectrum $\phi_u$, which is defined such that

$$\sum \phi_u(\beta) = \sigma (u)^2,$$

where $u$ is the disturbance velocity across the span and $\overline{\phi_u}$ represents the averaged value over the boundary layer thickness. The ordinate is $\beta_f^2 - \beta_{fo}^2$, where $\beta_f$ is the non-dimensional wavenumber at the downstream location, and $\beta_{fo}$ is calculated at the roughness element array. Despite a more physical scaling argument, which included parameters related to the geometry of the array, considerable scatter remains however, the collapse is particularly remarkable given the range of parameter space.
Chapter 3

Experimental Setup and Design

3.1 Overview

The experimental setup consists of several major components: the wind tunnel, boundary layer plate (and mounting hardware), instrumentation and positioning systems, the mechanism to introduce disturbances, plasma actuators, and control sensors. The FCET wind tunnel facility and each of the aforementioned components were not in place when the author began his research. For this reason, the first portion of the experimental program was designed for, and implemented at, Michigan State University (MSU) through an ongoing collaboration with Dr. Ahmed Naguib of the Flow Physics and Control Laboratory (FPCL). The pacing item for the FCET laboratory was the construction of a closed-circuit wind tunnel. The FCET wind tunnel was commissioned by the author nearly two years after the start of this thesis. After commissioning, all of the research for this thesis was performed at UTIAS. Owing to major differences in each facility, there are two distinctly different experimental setups. In addition, the measurement systems varied for both setups. Details pertaining to each of the experimental setups and measurement systems are discussed in this chapter.

3.2 MSU Experimental Setup

The FPCL wind tunnel located at MSU is a suction-type, open loop-return configuration, with a 10.8:1 contraction ratio. A diagram of the wind tunnel test section, traverse and boundary layer plate is shown in Figure 3.1. The working section is 0.35 m × 0.35 m and 2.77 m long and is followed by a diffuser with acoustic treatment. The turbulence intensity of the test section is approximately 0.05% at $U_\infty = 5$ m/s, the free-stream
velocity used in all experiments at MSU. The test section walls diverge by an angle of 0.13 degrees with respect to the centreline to minimize the pressure gradient along the working section. A baseline laminar boundary layer was established on a 0.635 m long and 12.7 mm thick acrylic plate spanning the width of the test section. A schematic of the plate is shown in Figure 3.2. The plate was mounted 0.1 m above the base of the test section, which was between 1/3 and 1/4 of the test section height to minimize the potential effects from contraction-induced secondary flows [138, Chap 12]. A 63.5 mm long sharp leading edge was machined from aluminum to a 15 degree edge such that the measurement side of the plate was flat over the entire length. A 0.152 m long adjustable flap was used to ensure the stagnation point was located on the measurement side of the plate. Small adjustments to the angle of attack, using both the plate mounts and the adjustable flap, were done iteratively until a zero-pressure gradient boundary layer was established.

![Figure 3.1: Drawing of the FPCL wind tunnel showing the mounted boundary layer plate and traverse system.](image)

### 3.2.1 Flow Measurements

Measurements of the flow velocity were made using a single hot-wire probe, which was manufactured at MSU. The probe had 20 mm long prongs spanned by a 4 mm long, 3.75 μm diameter tungsten wire with a central 1 mm active region shouldered on either side by copper-plated sections. Hot-wire data were taken using a TSI 1750 anemometer.
operated at an overheat ratio of 1.5. The hot-wire was calibrated using a 12-point velocity calibration ranging between 1.5 m/s and 6.5 m/s. The free-stream velocity was obtained from a Pitot-Static tube connected to a Setra Model 239, 0 - 0.5” water-column pressure transducer. King’s Law was fitted to the calibration points. Each velocity measurement consisted of the average of 12 seconds of data sampled at 5 kHz with an analog-to-digital data acquisition card connected to a PC. Temperature corrections based on the method proposed by Abdel-Rahman et al. [2] were applied to the hot-wire data using measurements from a T-type thermocouple located in the free stream to account for possible ambient temperature variations, typically within ±1 degree Celsius. Calibration of the hot-wire was performed immediately before and after each experiment. The uncertainty of the velocity measurements was estimated to be within ±1.4%. Errors were calculated using standard uncertainty analysis methodologies (see for example Moffat [108] or Taylor [150]). A summary of the uncertainty calculations is given in Appendix B.

The hot-wire probe was positioned by a computer-controlled three-axis traversing system using stepper motors (see Figure 3.1). A Motion Group MMC Stepper Controller drove Velmex Bi-Slides in the x- and y-directions and a Velmex A15 Uni-Slide in the z-direction. The y- and x-traverse components were located outside of the test section, and the z-component was inside the test section. A 38 mm thick NACA0012 foam profile was fitted over the rectangular section z traverse to limit potential vortex shedding and blockage effects. The displacement resolution in the y-direction was 2.5 µm and the traverse lead screw has an accuracy of ±3.04 µm over a span of 10 mm. In the spanwise (z) direction, the displacement resolution was 0.625 µm with similar accuracy to the y-traverse.

Two-dimensional contour plots of the velocity disturbance in a y-z plane were obtained from 32 evenly spaced wall-normal profiles of the streamwise velocity. The spanwise
spacing of the profiles was 1.25 mm. In all, the wall-normal profiles covered a spanwise extent of two complete wavelengths (2Δz), where Δz is the spanwise spacing of the roughness elements, which is discussed in Section 3.2.3. The first wall-normal position of the hot-wire was located using a program that searched for a mean velocity less than 20% of $U_\infty$. The physical wall location was then determined by extrapolating a linear-curve fit of, typically, 7 - 8 linearly spaced points between 20 and 30% of $U_\infty$, to the point where $U = 0$. The uncertainty of the wall location was within ±0.02 mm (see Appendix B). Each velocity profile consisted of velocity measurements at approximately 43 wall-normal locations, having a non-uniform spacing for velocities greater than 35% of $U_\infty$ to maintain consistent resolution of the Blasius boundary layer as shown in Figure 3.3. Measuring the streamwise velocity for a $y-z$ plane consisting of 1376 points required approximately 6 hours.

Figure 3.3: (a) Position of measurement locations on the $y-z$ plane, and (b) measurement locations (data marker) with respect to the baseline Blasius boundary layer (solid line).

### 3.2.2 Base Flow

The measurements of the laminar boundary layer, in the absence of wall roughness, agree well with the Blasius solution as illustrated in Figure 3.4(a), where $\langle U \rangle_t$ is the local time-averaged velocity. This point is further exemplified by measurements of the displacement thickness shown in Figure 3.4(b) and the shape factor ($H_{12} = \delta_1/\delta_2$, where $\delta_1$ is the displacement thickness and $\delta_2$ is the momentum thickness) shown in Figure 3.4(c), which remained between 2.59 and 2.64 over the measurement region. For true zero-pressure-gradient conditions $H_{12} = 2.59$ and variations within ±0.05 are acceptably close to zero-pressure-gradient behaviour [138, Chap 12]. The error associated with measurements of $U$, $\delta_1$, and $H_{12}$ were within ±1.4%, ±2.6% and ±3.7%, respectively.
A summary of the uncertainty calculations is given in Appendix B. Error bars are not visible on the data points in Figure 3.4(a, b) since they are smaller than the size of the marker symbols. The virtual leading edge is located 35 mm upstream of the geometric leading edge, as determined from measurements of the displacement thickness [138, Chap 12]. Note that the streamwise location measured relative to the physical leading edge is denoted by \( x \), while that relative to the virtual origin of the boundary layer is given by \( \hat{x} \), where \( \hat{x} = x + 35 \text{ mm} \), which is specific to the experimental conditions.

![Figure 3.4](image)

Figure 3.4: (a) Comparison of a measured boundary layer profile, •, with the Blasius solution, ---. (b) Comparison of the displacement thickness with the Blasius solution. (c) Variation of the shape factor, \( H_{12} \), on the boundary layer plate. The dashed line corresponds to the Blasius solution of \( H_{12} = 2.59 \).

### 3.2.3 Array of Roughness Elements

Transient growth was induced using five cylindrical roughness elements of diameter \( d = 5 \text{ mm} \) and height \( k = 1.29 \text{ mm} \), which were spaced 20 mm apart in the spanwise direction (\( \Delta z \)) and located at 150 mm from the geometric leading edge, as shown in Figure 3.2. The roughness elements were located in an insert that fit into the plate and allowed fine adjustment of the spanwise position to enable alignment with the actuator array. The height of each roughness element was measured with a dial gauge to an accuracy of \( \pm 0.012 \text{ mm} \). The Reynolds number, \( \text{Re}_k = U_k k / \nu \), based on roughness height was 230, where \( U_k \) is the velocity at the apex of the roughness element. The boundary layer thickness, \( 5\delta \), at the roughness elements location was 3.76 mm and 5.96 mm at
the measurement plane ($x = 450$ mm). The ratio of the roughness element height to Blasius length scale ($k/\delta$) is 1.68 at the streamwise location of the roughness array. The fundamental wavenumber of the disturbance induced by an array of roughness elements is associated with the spacing between elements and is denoted by $\beta_f = 2\pi\delta/\Delta z$. The value of $\beta_f$ at the location of the roughness array is denoted as $\beta_{f_0}$ and is equal to 0.24 in these experiments. The geometry of the roughness array was designed using the data of Lavoie et al. [93] such that the disturbance energy at the fundamental wavelength is still growing at the streamwise location of the measurement plane. This is illustrated in Figure 2.13, where the scaling figure of Lavoie et al. [93] is reproduced. Note the difference in the nomenclature ($h = k$). Identified in the figure is $\beta_f^2 - \beta_{f_0}^2$ corresponding to the location of the measurement plane for the present experiments (vertical dash line), which is set by selecting $U_x$, $\Delta z$, and the streamwise location of the measurements. The normalized plot then gives the corresponding ordinate value, which enables selecting $k$ such that the ratio $\bar{\phi}_u/U_x^2$ remains sufficiently low as to avoid transition to turbulence.

### 3.2.4 Plasma Actuator Array

The plasma actuators were arranged such that they generate pairs of impinging wall-jets causing streamwise vortices [135]. The leading edge of the actuator array was located 100 mm downstream of the roughness array, at $x = 250$ mm, and were centred behind the middle roughness element such that each high voltage (HV) electrode was located midway between two roughness elements. A schematic of the actuator is shown in Figure 3.5. The plasma actuator had four surface-mounted HV electrodes that were 40 mm long and spaced 20 mm apart the spanwise direction ($\Delta z$). Three different actuator geometries, differing only in the width of the exposed HV electrodes, were tested. Actuator geometry

![Figure 3.5: Schematic of the plasma actuator array mounted on a wall-plug.](image-url)
A had an exposed electrode width, \( W_{HV} = 5 \text{ mm} \), while for actuator geometry B, \( W_{HV} = 7 \text{ mm} \), and for actuator geometry C, \( W_{HV} = 8 \text{ mm} \). Two layers of Kapton film tape were used as the dielectric, each with a thickness of 90 \( \mu \text{m} \) including the adhesive. Copper foil tape (74 \( \mu \text{m} \) thick including the adhesive) was used for the electrodes. An adjustable wall-plug was used to ensure the actuator array was mounted flush with the boundary layer plate.

### 3.3 UTIAS Experimental Setup

The UTIAS wind tunnel is a closed-loop configuration with a working section that is 1.2 m \( \times \) 0.8 m and 5 m long. The corners of the test section have adjustable fillets, such that the pressure gradient along the test section can be controlled. The free-stream turbulence intensity inside of the test section is less than 0.05% at \( U_\infty = 5 \text{ m/s} \), which is the median free-stream velocity of the experiments performed. A baseline laminar boundary layer was established on a cast aluminum plate, 2.1 m long, 12.7 mm thick, and spanning the 1.2 m width of the 2.5 m long wind tunnel test section it resided in. A drawing of the boundary layer plate in the UTIAS wind tunnel is shown in Figure 3.6. The plate was mounted between 1/4 and 1/3 of the test-section height to minimize potential effects of secondary flows [138, Chap 12]. A machined aluminum insert containing the roughness element assembly and the removable sensor/actuator assembly was flush mounted with the cast-aluminum plate. The boundary layer plate was equipped with an asymmetric leading edge, which was designed to minimize adverse pressure gradients at the leading edge as well as the length of the pressure gradient region [59]. The aerodynamic optimization of the leading edge and the resulting geometry are discussed in Appendix A. The location of the stagnation line was controlled using a 0.4 m long flap at the downstream end of the test plate. The pressure gradient along the plate was controlled by adjusting angle of attack of the plate, the flap angle, and the corner filets of the test section.

A detailed diagram of the boundary layer plate is shown in Figure 3.7. The control region is comprised of a roughness element array, plasma actuator insert, and downstream shear stress sensors. Details of each are discussed within the following sections.

#### 3.3.1 Flow Velocity Measurements

Measurements of the streamwise flow velocity were made by two Auspex body, single hot-wire boundary layer probes, each with a 5 \( \mu \text{m} \) diameter tungsten wire with an active length of approximately 1 mm shouldered by copper-plated regions. The hot-wire probes
were operated using a constant temperature anemometer with an overheat ratio of 1.5. The relationship between the hot-wire data and flow velocity was determined by calibration in the free stream against a Pitot-Static tube connected to an MKS Model 223, 0 - 1 Torr pressure transducer. King’s Law was fitted to 16 calibration velocities between $0.2U_{\infty}$ and $1.25U_{\infty}$. Each velocity measurement consisted of an average of 5 seconds of data sampled at 5 kHz with a 16-bit National Instruments PCI-6259 analog-to-digital data acquisition card, connected to a PC. Temperature corrections were applied to the hot-wire data (c.f. Abdel-Rahman et al. [2]) using measurements from a T-type thermocouple located in the free stream to account for ambient temperature variations, which remained within $\pm 0.5$ degrees Celsius. Calibrations of the hot-wires were performed immediately before and after each experiments to quantify the drift of the hot-wires over the time of the experiment. The uncertainty of the velocity measurements was estimated to be within $\pm 1.1\%$. Discussion of the uncertainty analysis is found in Appendix B.

The two hot-wire probes were separated by 20 mm in the spanwise direction and were positioned locally at the same wall-normal and streamwise coordinate using a custom-
Figure 3.7: Schematic of the experimental arrangement with a detailed view of the control region.

built mount with an integrated 3-axis Newark micro-stage. The hot-wire probes were then positioned globally using a computer-controlled 3-axis traversing system, which was driven by stepper motors. The resolution of this system was at least 2.5 µm and the minimum precision of the traverse, based on the lead screw accuracy, is ±1 µm over a span of 10 mm. Each velocity profile consisted of measurements at 45 wall-normal locations over a region extending into the free stream. The distribution of measurements points for each $y-z$ plane, and the wall-finding routine was discussed in Section 3.2.1.

3.3.2 Shear Stress Measurements

Wall-mounted hot-wire sensors were used to measure surface shear stress in these experiments, which served as a feedback signal for closed-loop control. These sensors provide an in situ measurement of the spanwise-distributed streamwise shear stress, which is altered by the presence of streamwise streaks. Shear stress data were acquired using a near-wall-mounted hot-wire array of nine sensors distributed uniformly over 40 mm ($2\Delta z$) across the span at $x = 500$ mm. This arrangement provided four measurement locations per fundamental disturbance wavelength ($\lambda_z = \Delta z = 20$ mm), resulting in a Nyquist wavenumber corresponding to the wavelength $\Delta z/2$. The sensors were arranged
along the span such that the first, middle, and last sensor was directly in-line with a roughness element as shown in Figure 3.7. This array of sensors was custom built by Mr. Kyle Bade of the FPCL at MSU, through a collaboration with Prof. Ahmed Naguib.

The shear stress sensor array was constructed onto a 65 mm × 20 mm plug that was flush-fit with the flat plate. A picture and schematic of the plug and sensor arrangement is shown in Figure 3.8. The plug was fabricated from electrical-grade fibreglass machined to the above size specifications and milled to include 18 holes for the nine hot-wire sensor stainless-steel support pairs. Each hot-wire was mounted across a pair of support prongs (0.2 mm diameter jewellers broaches) protruding 1 mm from the plug surface, such that each hot-wire was located within the linear Blasius velocity profile range for shear stress calibration. The hot-wire sensing elements consisted of 5 µm diameter tungsten wires shouldered by copper plated regions such that the active-length of each of the wires was approximately 1 mm. The spanwise spacing of the active region of the sensors was 5 mm, which corresponds to a non-dimensional spacing of Δz/4.

![Figure 3.8: Picture of the shear stress sensor array (left) and schematic (right).](image)

Wall-mounted hot-wire data were acquired using constant temperature anemometers operating with an overheat ratio of 1.5 and were connected to the same data acquisition system used for flow measurements. The calibration methodology for wall-mounted hot-wire measurements of shear stress is well established [138; 26; 35]. The relationship between the wall-mounted hot-wire data and shear stress was determined using the free-stream velocity directly above the sensors with the analytical relationship for shear stress, assuming a Blasius boundary layer, which was empirically verified (see Figure 3.12), viz.

\[ \tau = 0.332 \rho U_\infty^2 \left( \frac{Re_x}{Re_x} \right)^{1/2}, \]  

(3.1)

where \( Re_x \) is the Reynolds Number based on the location of the virtual leading edge. The relationship between the hot-wire data and velocity was determined using the method described in Section 3.3.1 for \( U_\infty \pm 0.3 U_\infty \). Shear stress values during experiments were
within the calibrated range and calibrations were performed immediately before and after each experiment to minimize potential drift errors.

### 3.3.3 Roughness Element Array

Transient growth was induced using nine cylindrical roughness elements of diameter $d = 5$ mm, spaced 20 mm apart in the spanwise direction ($\Delta z$) and located at 200 mm from the geometric leading edge. The roughness elements were located in an insert that fit into the plate and allowed fine adjustment of the spanwise position, to enable alignment with the actuator array. The height of each roughness element was measured with a laser displacement meter to an accuracy of $\pm 0.012$ mm. The height of the roughness elements could be adjusted from a wall-flush position ($k = 0$ mm) up to $k = 2$ mm.

### 3.3.4 Plasma Actuator

Plasma actuators in the early MSU experiments had Kapton-tape dielectrics, which enables simple construction to facilitate the initial explorative testing. It was observed that the Kapton layer exposed to the plasma region had visible surface degradation following extended runs. This is demonstrated in Figure 3.9 for the excitation voltage of 6 kV and 4 kHz after 40 hours of continuous operation. Images were acquired by a CCD camera mounted to a stereoscopic microscope. The left part of Figure 3.9 shows the actuator before operation and the right portion of Figure 3.9 shows the actuator following the run. The surface appears affected by the presence of the plasma. A detailed inspection shows that in this region the Kapton surface is completely removed and only the underlying silicone-based adhesive layer remains. As shown by Figure 3.9, under the exposed electrode, the Kapton dielectric remained intact. It was found that a new actuator should be *burned-in* overnight to reduce transients associated with the initial degradation process. Degradation of the Kapton weakened the actuator such that after several days of continuous run the actuators were prone to failure by electrical arcing. These observations were the basis of an ambitious experimental program aimed at health monitoring and characterization of Kapton-based plasma actuator degradation. However, this work is outside the scope of this thesis and further results of this study are discussed by Hanson *et al.* [61].

Pons *et al.* [123] showed that glass dielectrics withstand the intense bombardment of ions, radical species, or ultraviolet radiations, which can be emitted by plasma filaments. The manufacture of these actuators is more difficult since electrodes should be deposited directly on the glass layer. This is required as the copper tape method introduces a
Figure 3.9: Effect of plasma exposure to Kapton dielectrics. The actuator before use (left) and after (right) with a portion of the exposed electrode cut away.

polymer-based adhesive, which is also susceptible to degradation. To increase the robustness of plasma actuators to degradation, glass based dielectrics were utilized for the UTIAS experiments. Glass squares 60 mm by 60 mm were used as the dielectric. These squares could be readily used in typical MEMS manufacturing laboratories as a substrate for electrode deposition.

Figure 3.10: Schematic of spanwise plasma actuator used in the UTIAS experiments.

The leading edge of the plasma actuator array was located 300 mm downstream of the geometric leading edge as shown in Figure 3.7. The actuator tile was comprised of 1 µm thick copper electrodes deposited on a 0.2 mm thick borosilicate glass dielectric layer as shown in Figure 3.10. Two of the 60 mm square actuator tiles, each with three surface mounded high voltage (HV) electrodes, were placed side-by-side to produce an actuator array with six evenly spaced HV electrodes, as shown in Figure 3.7. The grounded electrode spanned the width of the actuator tile and extended 30 mm in the streamwise direction. Electrodes were spaced 20 mm apart (matching the spanwise spacing of the roughness, Δz). The width of the exposed electrodes, $W_{HV}$, was 8 mm. The actuator was
fit flush with the removable sensor/actuator insert, which was situated in the boundary layer plate. The excitation signal was provided by a Agilent 32210A waveform generator, which was amplified by a TREK Model 20/20C High Voltage Amplifier.

**Plasma Actuator Power Consumption**

Measurement of the plasma actuator power consumption was carried out using the probe capacitor method (see for example Kriegseis et al. [89]). A probe capacitor with capacitance, $C_p$, was placed between the lower encapsulated electrode and the ground. The voltage supplied to the exposed electrode, $V_a$, and the voltage across the probe capacitor, $V_p$, were measured simultaneously by a Rigol 1052E digital oscilloscope. The instantaneous charge, $Q(t)$, across across the probe capacitor is the product of the voltage and across the probe capacitor and the capacitance. The energy consumed per cycle, $E_k$, is defined by the area enclosed by a cyclogram (Lissajous figure) of $V_a(t)$ and $Q(t)$, which is identified by the shaded region in Figure 3.11. For DBD actuators the cyclogram is almond-shaped [89]. The average power consumed by the actuator is equal to the average energy consumed per cycle multiplied by the inverse of the driving frequency ($1/f_a$). The cold capacitance (without plasma) is denoted as $C_o$, and an increased capacitance, $C_{eff}$, is caused by the presence of the plasma.

![Figure 3.11: Sample Q-V plot for determining the power consumption of a plasma actuator from Hanson et al. [61]. The solid line shows the averaged data over 800 cycles, and data markers show individual data points over one cycle. The shaded grey area is considered for the energy calculation.](image-url)
3.3.5  Base Flow

The baseline laminar boundary layer was measured with the roughness element array retracted flush to the surface of the boundary-layer plate \((k = 0)\), and with the plasma actuators off. Similar to the measurements of the MSU boundary layer plate, these measurements agree well with the Blasius solution, which is illustrated in Figure 3.12(a) for a typical velocity profile at \(\hat{x} = 450\) mm. The location of the virtual leading edge \((\hat{x})\) is 21 mm downstream of the geometric leading edge location \((x = 0)\), which was determined from measurements of the displacement thickness shown in Figure 3.12(b). The laminar boundary layer closely followed a zero-pressure gradient. This was verified quantitatively using the shape factor, \(H_{12}\), which remained between 2.54 and 2.63 over the measurement region of the plate as shown in Figure 3.12(c) for \(U_\infty = 5\) m/s. A summary of the uncertainty calculations is given in Appendix B.

![Figure 3.12](image)

Figure 3.12: (a) Comparison of a measured boundary layer profile, •, with the Blasius solution, ––. (b) Comparison of the displacement thickness with the Blasius solution. (c) Variation of the shape factor, \(H_{12}\), on the boundary layer plate. The dashed line corresponds to the Blasius solution of \(H_{12} = 2.59\).
Chapter 4

Open-Loop Control of the Transient Growth Instability

4.1 Overview

The effectiveness of a control system is inherently linked to the ability of the actuator to alter the flow to a desired state. Therefore, the actuator is a critical enabling technology component in any active flow control system. In the context of this work, the aim of the control was to negate the disturbance leading to bypass transition. In the first section, the disturbance sought for control was characterized. It consisted of streamwise oriented streaks of spanwise periodic low- and high-velocity. An array of plasma streamwise vortex generators (PSVGs) were investigated for their ability to generate these velocity streaks. Specifically, the receptivity of the boundary layer to actuator geometry and excitation signal were studied for the generation of streaks used to target a specific transient growth mode. In the final section, the attenuation of transient growth modes using plasma actuators was investigated. Details of the experimental setup and measurement techniques relevant to this research are found in Chapter 3, Section 3.2.

4.2 Experimental Generation of Steady Streaks

Streamwise velocity streaks were generated using the array of cylindrical roughness elements described in Section 3.2.3. In summary, the geometry of the roughness array was designed using the data of Lavoie et al. [93] to ensure that the fundamental-wavelength disturbance-energy is still growing at the measurement plane. The roughness elements were spaced $\Delta z = 20$ mm apart along the span and the diameter of each element was
They were deployed to a height of $k = 1.29$ mm and were located 150 mm from the leading edge. The free-stream velocity ($U_\infty$) in these experiments was 5 m/s. At the measurement plane, located 450 mm from the leading edge, the non-dimensional fundamental wavenumber of the disturbance was $\beta_f = 2\pi\delta/\Delta z = 0.38$. This value is based on the local value of the Blasius similarity variable $\delta = (\hat{x}\pi/\nu)^{1/2}$, where $\hat{x}$ is the streamwise location from the virtual leading edge ($\hat{x} = x + 35$ mm). At the location of the roughness array, $\beta_{f_z} = 0.24$. Figure 2.13 shows the scaling of Lavoie et al. [93] with a vertical dashed line at, $\beta_f^2 - \beta_{f_z}^2 = 0.09$, the measurement plane location for this work.

The disturbance velocity is defined as $U' = \langle U \rangle_t - \langle U \rangle_{t_z}$, where $\langle U \rangle_t$ is the time-averaged velocity and $\langle U \rangle_{t_z}$ is the spanwise average of $\langle U \rangle_t$. Two-dimensional contour plots of the disturbance velocity in the $y$-$z$ plane were obtained from 32 evenly spaced wall-normal profiles of the streamwise velocity. Each profile was spaced 1.25 mm ($\Delta z/16$) apart in the spanwise direction, which corresponds to a spanwise extent of two complete wavelengths ($2\Delta z$). A contour plot of the disturbance velocity normalized by the free-stream velocity ($U_\infty$) at $x = 450$ mm is shown in Figure 4.1(a). The wall-normal coordinate $y$ was normalized by the local value of the Blasius similarity variable ($\eta = y/\delta$).

High-speed streaks were centred downstream of each roughness element in the locations $z/\Delta z = -1, 0$ and 1. Low-speed streaks occurred between each high-speed streak, which was similar to the results of Fransson et al. [38]. Figure 4.1(b) shows a spanwise profile of the disturbance velocity at the wall-normal location $\eta = 2.1$ where the disturbance amplitude reached a maximum value. At this location (300 mm downstream of the roughness element array) the spanwise profile of the velocity disturbance was not purely sinusoidal because higher energy modes have not completely decayed. A similar result is shown in Figure 2.12, which was reproduced from Fransson et al. [38], albeit closer to the roughness array. The difference in the development of the streaks with the streamwise coordinate was highly dependant on geometry of the roughness array as well as the flow conditions. Based on the non-dimensional scaling of Lavoie et al. [93] the fundamental mode of wavelength $\Delta z$ was still increasing in energy at the measurement plane.

The spanwise-wavenumber power spectrum of the streamwise disturbance velocity was used to identify the modal content. The spanwise spectrum is defined such that

$$\sum \phi_{U'}(\beta) = \sigma \langle U' \rangle^2,$$

where $\phi_{U'}$ is the one-sided power spectrum. There are 16 measurement points per fundamental wavelength and the power spectrum was computed over two fundamental wavelengths ($2\Delta z$). Therefore, only wavelengths down to $\Delta z/8$ were resolved. The abscissa
Figure 4.1: (a) Contours of $U'/U_\infty$ at $x = 450$ mm and (b) spanwise profile of $U'/U_\infty$ at $\eta = 2$ for the roughness element array deployed at $k = 1.29$ mm.

of the spectrum was more suitably represented in non-dimensional terms, $\beta_\lambda = 2\pi\delta/\lambda$, where $\lambda$ is the spanwise wavelength. In addition, the average energy of the flow can be represented by the average spanwise wavenumber power spectrum, viz.

$$\overline{\phi_{U'}}(\beta) = \frac{\int_0^{5\delta} \phi_{U'}(\beta) d\delta}{5\delta},$$

(4.2)

which was calculated over the thickness of the boundary layer to identify the modal content of the disturbance, shown in Figure 4.2. The disturbance was comprised primarily of a single mode of wavelength $\Delta z$, and non-dimensional wavenumber $\beta_{\Delta z} = 2\pi\delta/\Delta z = 0.38$, containing over 88% of the total energy of the streamwise disturbance velocity. Over 98% of the total energy was contained in the wavelengths $\Delta z$, $\Delta z/2$, and $\Delta z/3$. The energy contained in larger values of $\beta$ caused a distortion of the sine wave (of wavelength $\Delta z$) shown in Figure 4.1(b).

Figure 4.2: Average spanwise wavenumber power spectrum of the streamwise disturbance velocity for the roughness element array.
Band-pass spatial filtering of $U'/U_\infty$ was used to reconstruct the three significant energy containing spanwise modes, which are shown in Figure 4.3(a-c). The purpose of the filtering was to demonstrate the relative phase offset between modes. The spanwise root-mean-square (rms) of the disturbance velocity as a function of the wall-normal position was calculated over the two complete fundamental wavelengths. The profiles for each of the three non-dimensional wavenumbers associated with $\Delta z$, $\Delta z/2$, and $\Delta z/3$, are shown over the boundary layer thickness in Figure 4.3(d). The unfiltered profile shows that the maximum disturbance energy occurred at $\eta \approx 2.1$, which agrees with previous studies [159; 38], and was below the optimal value of $\eta = 2.2$ [3]. The effect of the non-optimal wavenumber $\beta = 0.38$, compared to the calculated optimal value of 0.45, will reduce the maximum energy growth [3]. Non-optimal generation of streaks using roughness element arrays causes additional higher mode energy components, which coincides with the results of Fransson et al. [38]. The authors demonstrated that these higher modes were decaying, resulting in a near sinusoidal modulation of spanwise wavelength $\Delta z$ further downstream.

A double-peaked low-speed streak was observed for the roughness element array disturbance, as shown in Figure 4.1(a) at $\Delta z = -0.5$ and 0.5. This structure was caused by the presence of energy contained in higher wavenumbers. For instance, it is shown
in Figure 4.3(a) that the fundamental mode had regions of high-speed flow at $\Delta z = -1$, 0, and 1, and low-speed regions centred about $\Delta z = -0.5$ and 0.5. However, the next higher wavenumber, with wavelength $\Delta z/2$, had high-speed regions at $\Delta z = -1$, -0.5, 0, 0.5 and 1. Similarly, the disturbance with wavenumber $\Delta z/3$ caused additional positive disturbance at $\Delta z = -0.5$ and 0.5, although the contribution was approximately half of the former. These regions of high velocity superimposed onto the fundamental mode are responsible for the observed double-peaked low-speed streaks.

### 4.3 Plasma Actuator Generated Streaks: Effect of Actuator Geometry

Three different actuator geometries, differing only in the width of the exposed HV electrodes, were tested to assess the sensitivity of boundary layer receptivity to actuator geometry. Actuator geometry A had an exposed electrode width ($W_{HV}$) equal to 5 mm, while for actuator geometry B, $W_{HV} = 7$ mm, and for actuator geometry C, $W_{HV} = 8$ mm. The actuators were located 250 mm downstream of the leading edge, as shown in Figure 3.2. A further description of the actuators is contained in Section 3.2.4.

#### 4.3.1 Actuator Geometry A

Actuator A was driven by an AC wave with amplitude $V_{pp} = 4.4$ kV and frequency $f_a = 3$ kHz. The velocity disturbance produced by this actuator in the boundary layer in the absence of the roughness array is shown in Figure 4.4(a). Figure 4.4(b) shows a spanwise profile of the disturbance velocity at the wall-normal location $\eta \approx 1.9$, where the maximum disturbance amplitude occurred. The maximum disturbance amplitude location was nearer the wall than that observed for the roughness element case at $\eta \approx 2.1$. This was caused by the shorter streamwise development region of the actuators compared to the roughness; the location of maximum disturbance amplitude increases with streamwise distance. In the present experiments, the distance from the roughness array to the $y – z$ measurement plane was 300 mm, whereas the distance was only 200 mm from the actuator to the measurement plane. The spanwise profile of the disturbance velocity was also significantly more distorted than the roughness case and the low-speed streak centred about $z/\Delta z = 0$ was double-peaked.

For this disturbance, the average spanwise wavenumber power spectrum is shown in Figure 4.5. The fundamental mode of wavelength $\Delta z$ and non-dimensional wavenumber $\beta = 2\pi \delta/\Delta z = 0.38$ contained approximately 66% of the total energy. Over 96% of the
total energy was contained in the fundamental wavenumber (first mode) and next two higher harmonics (second and third mode). Fransson et al. [38] demonstrated that energy containing higher harmonics distorted the sinusoidal modulation of the $\Delta z$ fundamental spanwise wavelength nearer the non-optimal disturbance input location prior to their complete decay further downstream. In comparison the roughness case, higher harmonics associated with the actuator disturbance had less downstream extent to decay.

The same methodology from the previous section was used to reconstruct the fundamental wavenumber and next two higher harmonics for actuator A, as shown in Figure 4.6(a-c). It is evident that the first mode generated by the actuator was of opposite phase to that of the roughness. However, the second mode (Figure 4.6b) was in phase with the corresponding roughness generated counterpart (Figure 4.3b). The third mode was in opposite phase with the roughness element counterpart. The second mode will reduce the amplitude of the central region of the fundamental mode low-speed streak centred about $\Delta z = -1, 0, \text{ and } 1$, whereas it will increase the amplitude of the fundamental
mode high-speed streaks centred about $\Delta z = -0.5$ and 0.5. The third mode will reduce the amplitude of the fundamental disturbance centred at $\Delta z = -1$, -0.5, 0, 0.5, and 1. Therefore, the presence of higher harmonics causes the strong double-peaked low-speed streak structure and weak double-peak structure of the high-speed streak of the unfiltered disturbance.

Figure 4.6: (a) Contours of $U'/U_\infty$ at $x = 450$ mm for the fundamental mode, and (b & c) the next two higher harmonics caused by actuator A. (d) Wall-normal disturbance profile for the unfiltered, fundamental mode and next two higher harmonics.

4.3.2 Actuator Geometry B

The central low-speed streak generated by actuator A was wider than the central high-speed streak generated by the roughness array. Similarly, the actuator generated high-speed streaks were narrow compared to the roughness induced low-speed streaks. Ultimately, the goal of the actuator is to attenuate the streaks caused by the roughness element array. A logical actuator geometry modification to compensate for the spatial differences in the streaks was to increase the width of the upper HV electrodes. Actuator geometry B had exposed electrode widths of 7 mm and was driven by an AC wave with amplitude $V_{pp} = 4.4$ kV and frequency $f_a = 3$ kHz. The spatial distribution of the velocity disturbance is shown in Figure 4.7(a). Figure 4.7(b) shows a spanwise profile of the disturbance velocity at the wall-normal location $\eta \approx 1.9$. Actuator B produced a disturbance with a reduced double-peaked low-speed streak structure and wider high-speed streak region.
Figure 4.7: (a) Contours of $U' / U_\infty$ at $x = 450$ mm and (b) spanwise profile of $U' / U_\infty$ at $\eta = 1.9$ for actuator B with $V_{pp} = 4.4$ kV, and $f_a = 3$ kHz.

The average spanwise wavenumber power spectrum for this case is shown in Figure 4.8, which was calculated from the disturbance velocity shown in Figure 4.7(a). The fundamental disturbance, with a wavelength $\Delta z$, contained approximately 67% of the total energy. Over 95% of the total energy was contained in mode 1 and 3, whereas mode 2 contained less than 2% of the total energy. The reconstructed fundamental mode and next two higher harmonics are shown in Figure 4.9(a-c). Again, the fundamental mode was of opposite phase to the corresponding roughness induced mode. However, the second mode, shown in Figure 4.9(b), was of opposite phase to that of the previous actuator geometry (Figure 4.9b), and was weakened. The third mode was also of opposite phase with the roughness induced counterpart. The energy contained in the third mode was approximately 12% of the total energy for this case, significantly greater than the 2% contribution observed for the roughness case. Therefore, the third mode disturbance decreases the amplitude of the central region of the fundamental disturbance centred at $\Delta z = -1, -0.5, 0, 0.5, \text{ and } 1$, resulting in the double-peaked appearance of the low-speed and high-speed streaks of the unfiltered disturbance.
4.3.3 Actuator Geometry C

For the final geometry under consideration the exposed electrode width was increased to 8 mm. Actuator geometry C was driven by an AC wave with amplitude $V_{pp} = 4.4$ kV and frequency $f_a = 3$ kHz. The disturbance velocity produced by the actuator is shown in Figure 4.10(a). Figure 4.10(b) shows the spanwise profile of the disturbance velocity at the wall-normal location $\eta \approx 1.9$. Comparison of Figure 4.4, 4.7, and 4.10, shows that increasing the HV electrode width narrowed the central low-speed streak and increased the width of the high-speed streak.

Figure 4.10: (a) Contours of $U'/U_\infty$ at $x = 450$ mm and (b) spanwise profile of $U'/U_\infty$ at $\eta = 1.9$ for actuator C with $V_{pp} = 4.4$ kV, and $f_a = 3$ kHz.
The average spanwise wavenumber power spectrum is shown in Figure 4.11. The mode 1 ($\Delta z$ disturbance wavelength) contained 70% of the total energy and over 93% of the total energy was contained in modes 1 to 3. Mode 2 and 3 contained 11% and 13.5% of the total energy, respectively. The reconstructed fundamental mode and next two higher harmonics are shown in Figure 4.12(a-c). Again the fundamental mode was in opposite phase to the corresponding roughness induced mode. The next two higher harmonics also were also in opposite phase to the roughness array counterparts. The third mode energy was increased compared to both previous cases. Energy contained in mode 2 and 3 decreased the amplitude of the central region of the fundamental disturbance centred at $\Delta z = -0.5$ and 0.5. The amplitude of the central region of the fundamental disturbance centred at $\Delta z = -1$, 0 and 1 was enhanced by mode 2 however, this was reduced by the third mode. The higher harmonics led to an enhanced double-peaked appearance of the high-speed streaks and subtle double-peaked appearance of the low-speed streaks of the unfiltered disturbance, as shown in Figure 4.10(b).

![Figure 4.11: Average spanwise wavenumber power spectrum of the streamwise disturbance velocity for actuator C.](image)

### 4.3.4 Discussion of Geometry Effects

Increasing the width of the exposed HV electrodes caused a substantial modification of relative energy content and phase of higher harmonics of the fundamental $\Delta z$ disturbance wavelength generated by the actuator. A summary of the relative energy content and phase for the fundamental wavelength and next two higher harmonics is shown in Table 4.1, with the corresponding values for the roughness element array. The energy was evaluated over the thickness of the boundary layer, \( \text{viz.} \)

\[
\frac{E_i}{E_t} = \frac{\phi_u(\beta_{\Delta z/i})}{\int \phi_u(\beta) d\beta},
\]

(4.3)
where $E_i$ represents the energy associated with the wavenumber $\beta_{\Delta z/i}$, and $E_t$ is the total energy over all wavenumbers resolved (to $\Delta z/8$).

Table 4.1: Overall disturbance energy ratio and starting phase of the fundamental and next two disturbance modes for the roughness element array (REA) and actuators.

<table>
<thead>
<tr>
<th>$W_{HV}$ (mm)</th>
<th>$E_1/E_t$</th>
<th>$E_2/E_t$</th>
<th>$E_3/E_t$</th>
<th>Phase (rad)</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>REA</td>
<td>88</td>
<td>7.5</td>
<td>1.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Actuator A</td>
<td>5</td>
<td>66.9</td>
<td>18.8</td>
<td>9.4</td>
<td>$\pi$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Actuator B</td>
<td>7</td>
<td>76.9</td>
<td>1.3</td>
<td>12.6</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>0</td>
</tr>
<tr>
<td>Actuator C</td>
<td>8</td>
<td>69.5</td>
<td>11</td>
<td>13</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>0</td>
</tr>
</tbody>
</table>

A descriptive drawing of the expected streamwise vortex pairs, which could cause the observed velocity streaks, is shown in Figure 4.13. Results from actuator A were used to define the phase. The fundamental mode was spatially fixed by the net motion of fluid pulled down toward the high-voltage electrodes and low-speed fluid in the near-wall region, which was ejected from the wall in the region between the electrodes by the opposing wall jets. Therefore, the spacing of the electrodes ($\Delta z$) fixed the fundamental disturbance wavelength at 20 mm and prescribed the phase, which was opposite to the corresponding roughness generated mode. A similar effect was observed for the third mode ($\Delta z/3$), which retained the same phase for each actuator geometry. Velocity deficit
regions were centred over the exposed electrodes and the relative energy content of the third mode was similar for each actuator (between 9% and 13%). The second mode ($\Delta z/2$) exhibited a remarkably different behaviour. For instance, the relative energy of mode 2 decreased from 19% for actuator A to 1% for actuator B, while retaining the phase. Further increase in width for actuator C caused the relative energy of mode 2 to increase to 11%, however, it occurred opposite in phase to actuator A and B.

![Illustration of vortex pairs](image)

Figure 4.13: Illustration of the expected streamwise vortex pairs causing the observed first three modes relative to the position of the exposed electrodes for actuator A. Regions of high- and low-velocity are identified by red and blue circles, respectively.

Recently, Kriegseis et al. [87] showed that a recirculation zone exists above the exposed electrode for a simple plasma actuator (for example see Figure 2.7) having only a single exposed electrode and embedded ground electrode. The recirculation zone occurred aft of exposed electrode edge and typical wall jet. The recirculation zone was attributed to the downward suction at the edge of the exposed electrode. Santhanakrishnan and Jacob [137] and Thomas and Schatzman [151] show similar behaviour for arrays of actuators, although their results were limited to quiescent conditions and strong forcing conditions. For the present work, the spanwise wavelength of the third mode was $\Delta z/3 = 6.67$ mm, which was within the range of exposed electrode widths considered (from 5 - 8 mm). The strength of mode 3 increased with increasing electrode width. A recirculation zone aft of the edge of each electrode could explain the origin of the mode-3 disturbance shown in Figure 4.13, having a fixed phase.
4.4 Plasma Actuator Generated Streaks: Effect of Actuator Voltage and Frequency

In the previous section it was shown that the width of the exposed electrodes affected the distribution of the relative energy content of the disturbance. In this section, the effect of the driving frequency and voltage on the resulting disturbance was investigated. Only actuator geometry A, with an exposed electrode width of 5 mm, was considered. In the first experiment set, the disturbance caused by the actuator was measured for $V_{pp} = 3.0$ kV and $V_{pp} = 4.25$ kV with various driving frequencies. For $V_{pp} = 3.0$ kV the frequency was varied between 3 and 9 kHz. For $V_{pp} = 4.25$ kV the frequency was varied between 1 and 5 kHz; the maximum frequency was limited by the onset of transition upstream of the measurement plane. In the second experiment set, the effect of voltage amplitude for $3 \text{kV} \leq V_{pp} \leq 4.75$ kV was examined with constant excitation frequency ($f_a = 3$ kHz).

For each excitation frequency the average spanwise power spectrum was computed over the boundary layer using (4.2). Waterfall plots of the average energy spectra are shown in Figure 4.14 for (a) $V_{pp} = 3$ kV and (b) $V_{pp} = 4.25$ kV. Mode 1 and 2 were the most energetic modes containing over 80% of the total disturbance energy.

![Waterfall plots of the average spanwise wavenumber power spectrum of $U'/U_\infty$ for (a) $V_{pp} = 3$ kV and (b) $V_{pp} = 4.25$ kV with increasing frequency.](image-url)

The variation of the total energy, and energy contained in the wavenumbers associated with $\Delta z$, $\Delta z/2$, and $\Delta z/3$, for actuator A are shown in Figure 4.15(a) for $V_{pp} = 3$ kV and (b) for $V_{pp} = 4.25$ kV. The fraction of the energy contained in each mode with respect to the total energy was determined from (4.3). Increasing the frequency resulted in a nearly linear total energy, which is consistent with previous studies on plasma actuators. For example, Porter et al. [125] showed that the plasma actuator body force increases...
linearly with frequency before saturation at high voltages and frequencies. Local non-linear behaviour has been attributed to an optimal operating frequency, which minimized losses associated with the power supply and actuator system [116; 154].

![Graph](image)

Figure 4.15: The variation of the total energy and energy contained in mode 1 to 3 for (a) $V_{pp} = 3$ kV, and (b) for $V_{pp} = 4.25$ kV.

The average energy spectrum was calculated for each voltage amplitude from 3 kV \( \leq V_{pp} \leq 4.75 \) kV and $f_a = 3$ kHz. A waterfall plot of the spectra is shown in Figure 4.16(a). The variation of the total integrated energy and relative energy contained in each mode for actuator A with $f_a = 3$ kHz is shown in Figure 4.16(b). Increasing voltage resulted in a non-linear increase in the disturbance energy. Similar non-linear increases were observed for single DBD actuators. For example, Enloe et al. [32] showed that the maximum wall jet velocity was proportional to $V_{pp}^{3.5}$ and that the power was proportional to $V_{pp}^{3.35}$.

![Graph](image)

Figure 4.16: (a) Waterfall plot of the average spanwise wavenumber power spectrum of $U'/U_{\infty}$ for $V_{pp} = 3 - 4.75$ kV and $f_a = 3$ kHz. (b) The variation of the total energy, and energy contained in modes 1 to 3 with excitation voltage.
4.4.1 Discussion

The relative energy, $E_i/E_t$, for modes 1-3 is shown in Figure 4.17. This plot contains data from the frequency and voltage results found in Figures 4.15 and 4.16. Therefore, the dense distribution of points for $E_t < 0.5 \times 10^{-3}$ was associated with the relatively low energy results from the case with $V_{pp} = 3$ kV, where the frequency was varied between 3 and 9 kHz. The abscissa is the total energy, $E_t$, which increased with both frequency and voltage. Arguably, the power consumed by actuator (a measure of actuator output) would be a more appropriate abscissa. However, this method was not an apparent option prior to the work of Kriegseis et al. [88], which occurred after the present study was performed. It has been shown that the total disturbance energy measured at the downstream plane is correlated with the power consumption, albeit lacking the history of the spatial growth of the disturbance, which can be considered minor for constant geometry (see Osmokrovic [117]). Scaling based on power consumption was shown to reduce scatter, which is apparent in in Figure 4.17, however, the trends remained consistent.

![Figure 4.17: Ratio of the energy contained in mode 1 - 3 with respect to the total energy for actuator A with increasing frequency for $V_{pp} = 4.25$ kV, and $V_{pp} = 3$ kV.](image)

Over the range of values tested, $E_i/E_t$ increased at the downstream plane for mode 1 and 3, while the energy contained in mode 2 decreases. A second order polynomial fit to the measurements was a convenient choice to draw attention to each mode, however, it does not necessarily represent the physics of the process. The observed change in relative modal contribution with excitation frequency and voltage can be explained from current understanding of actuator operation. For example, the characteristic propagation velocity of the plasma and plasma extent has been shown to increase with voltage [33; 88]. Enloe et al. [33] showed experimentally that the extent of the discharge was a function of the excitation voltage amplitude, which was later supported by Orlov [116]. Increasing voltage will increase the spatial extent of the electric field sufficient to support
the ionization of air. However, it was assumed that frequency did not alter the plasma extent (see for example the review by Corke et al. [27]). Recent observations by Kriegseis et al. [88] showed that this is not the case, although the plasma extent was less sensitive to frequency.

The increased forcing region (caused by a great plasma extent) and jet velocity with higher voltage and frequency will alter the spatial signature of the streamwise streaks at the downstream measurement plane. Consider the example with $f_a = 3$ kHz and $V_{pp} = 3.75$ kV, which had a double-peaked low-speed streak located between adjacent electrodes as shown in Figure 4.18(a). As the voltage was increased to 4.25 kV and 4.75 kV the total energy increased by a factor of 2.8 and 7, respectively. For increasing voltage, the double-peaked low-speed streak began to coalesce, as shown in Figure 4.18(b, c). Jukes and Choi [73] showed that the core of the induced streamwise vorticity migrated away from the exposed electrodes with increasing actuator output, which can explain the present observations. A similar observation of the coalescing double-peaked low-speed streak occurs with increased exposed electrode width, as shown for actuators B and C.

![Figure 4.18: Contour plots of $U'/U_\infty$ at $x = 450$ mm operating with $f_a = 3$ kHz for (a) $V_{pp} = 3.75$ kV, (b) $V_{pp} = 4.25$ kV, and (c) $V_{pp} = 4.75$ kV. A dashed line is used to highlight the narrowing of the central low-speed streak with increasing voltage.](image-url)
4.5 Flow Control Results

The three different actuator geometries (A, B, and C) were considered for the open-loop control demonstration, as each produced a unique spanwise wavenumber energy distribution. Each actuator case from Section 4.3 was operated with the roughness element array deployed to investigate the attenuation of velocity streaks. Only a single streamwise location at $x = 450$ mm was considered. This location was just upstream of the location of maximum disturbance energy caused by the transient growth of the streamwise streaks, which were embedded in the boundary layer using an array of roughness elements.

The disturbance velocity produced by the roughness element array is shown in Figure 4.19(a), with the corresponding average spanwise wavenumber spectrum in Figure 4.19(d). Actuator A was initially driven by an AC wave with amplitude $V_{pp} = 4.4$ kV and frequency $f_a = 3$ kHz. The disturbance caused by this actuator, in the absence of the roughness array, is shown in Figure 4.19(b) with the average spectrum in Figure 4.19(d). The value of $\bar{\phi}_{U'}(\beta_{\Delta z})/U_\infty^2$ for the actuator case was approximately 12% greater than the roughness case. The actuator output was decreased by reducing the frequency, which caused a linear reduction in the disturbance energy. The frequency was reduced by 0.25 kHz and the control case was measured as shown in Figure 4.19(c) and (d). With control the energy of the targeted $\Delta z$ mode was reduced by 93%, whereas the energy in mode 2 and 3 increased.

![Figure 4.19](image_url)

Figure 4.19: Contour plots of $U'/U_\infty$ determined at $x = 450$ mm for (a) the roughness disturbance only, (b) actuator A only, and (c) control. (d) The average spanwise wavenumber spectrum for each corresponding case.
The reconstructed the fundamental wavenumber and next two higher harmonics for the roughness element disturbance, the actuator only, and the control case are shown in Figure 4.20. The actuator was positioned such that the $\Delta z$ mode was in opposite phase to the disturbance caused by the roughness element array, within the resolution of the 16 spanwise measurement locations per $\Delta z$ ($\pi/16$). The control case shows that the mode 1 energy was reduced in magnitude, however, it was also phase shifted. Consider the Fourier coefficient of the $\Delta z$ mode at $\eta = 2$, which is represented as $\widehat{U}'_{\Delta z}(\eta = 2)$. For the roughness disturbance only, $\widehat{U}'_{\Delta z}(\eta = 2) = 1.06 + 0.16i$, which has an absolute value of 1.07, and a phase angle of $0.048\pi$. For the actuator only, assuming a reduced magnitude of 92% (for the reduced frequency 2.75/3), the corresponding Fourier coefficient is $\widehat{U}'_{\Delta z}(\eta = 2) = -1.05 - 0.28i$, which has an absolute value of 1.08, and phase angle of $-0.92\pi$. Therefore, the phase shift between the actuator and roughness mode 1 disturbances are offset from the ideal case ($-\pi$). Adding these complex values gives: $\widehat{U}'_{\Delta z}(\eta = 2) = 0.01 - 0.12i$, which has an absolute value of 0.14 and phase angle of $-0.47\pi$. The phase angle is approximately half the difference between the roughness and actuator cases. From the control case, $\widehat{U}'_{\Delta z}(\eta = 2) = -0.006 - 0.18i$, which has an an absolute value of 0.18 and phase $-0.51\pi$ that compares well with the calculated value. Therefore, the observed phase shift was due to misalignment of roughness and actuator generated disturbance. For perfect alignment the incomplete attenuation of mode 1 would retain the phase of the initial disturbance, whereas over-actuation would be of opposite phase.

![Figure 4.20: Contour plots of $U'/U_\infty$ determined at $x = 450$ mm for modes 1 - 3, from left to right, for the roughness disturbance, actuator A only, and control (top to bottom).](image-url)
The mode-2 disturbance caused by the roughness elements only, and actuator only, were in phase, as shown in Figure 4.20. Therefore, the mode-2 disturbances interfered constructively. The resulting mode-2 energy in the control case was the sum of the mode-2 energy for the actuator and roughness, as shown in Figure 4.19(d). The third mode caused by the actuator was of opposite phase with the corresponding roughness element mode causing destructive interference. However, the energy of mode 3 caused by the actuator was a factor of 6.8 times greater than the corresponding roughness case. Therefore, the energy of mode 3 in the control case retained the same phase as the actuator.

It was shown in Section 4.3 that the modal content of the disturbance produced by the plasma actuator array varied depending on the geometry. For example, the central low-speed streak generated by actuator A was wider than the central high-speed streak generated by the roughness array. Comparison of Figure 4.21(a) and 4.21(b) shows a better qualitative match between the disturbance produced by the roughness array and actuator B (operated with $V_{pp} = 4.4$ kV and $f_a = 3$ kHz) since the wider HV electrodes narrowed the central low-speed streak. Despite this observation, remnants of the double-peaked low-speed streak are visible in the control case shown in Figure 4.21(c). It is shown in Figure 4.21(d) that the energy of the target $\Delta z$ mode was reduced by 96%, mode 2 was also attenuated however, mode 3 increased in energy. For the control case, only 8% of the total energy is contained in the $\Delta z$ mode, whereas 40% of the remaining energy is contained in mode 3, as shown by Figure 4.21(d).

The reconstructed the fundamental wavenumber and next two higher harmonics for the roughness element disturbance, the actuator only, and the control case are shown in Figure 4.22. Mode 2 was weakly attenuated by the energy of opposite phase supplied by the actuator. The actuator mode 3 was of opposite phase with the initial disturbance however, the energy was comparatively high, and remained so in the control case.

Increasing the width of the upper exposed electrode caused substantial modification of the higher modes generated by the actuator, which affected the overall control. To further support this idea, results for actuator C are shown in Figure 4.23 at the same operating conditions described in Section 4.3.3. As shown by Figure 4.23(b), the width of the central low-speed streak was further reduced and the width of the high-speed streaks increased comparatively for cases A and B. The disturbance velocity with control is shown in Figure 4.23(c). A nearly complete attenuation of the mode 1 energy is shown by Figure 4.23(d); the energy of the target $\Delta z$ mode was reduced by 97%. Over 35% of the residual energy was contained in mode 3, as shown by Figure 4.23(d).
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Figure 4.21: Contour plots of $U'/U_\infty$ determined at $x = 450$ mm for (a) the roughness disturbance only, (b) actuator B only, and (c) control. (d) The average spanwise wavenumber spectrum for each corresponding case.

Figure 4.22: Contour plots of $U''/U_\infty$ determined at $x = 450$ mm for modes 1 - 3, from left to right, for the roughness disturbance, actuator B only, and control (top to bottom).

For actuator C, the second and third mode attenuated the initial disturbance mode 2, and enhanced mode 3. Both the second and third actuator modes were in opposite phase with that of the initial disturbance, as shown in Figure 4.24. Therefore, each interfered
destructively however, the mode 2 components were of similar amplitude, whereas the actuator mode 3 was comparatively strong, and so remained in the control case. Alternatively, it can be viewed that mode 3 of the actuator was attenuated by that caused by the initial disturbance, since the remaining mode 3 in the control case retained the phase of the actuator disturbance.

4.5.1 Discussion of the Control Mechanism

The overall control effectiveness of each case can be quantified by comparing the average power spectrum of the streamwise velocity disturbance for the roughness element disturbance, actuator only case, and the control case. The averaged power spectrum for both the initial disturbance, actuator only, and the control cases were shown in the previous sections. A summary of the results is given in Table 4.2. For each case, the energy contained in mode 1, 2 and 3, as well as the total integrated energy over all wavenumbers are shown. The effectiveness of the control was quantified using the parameter \( \Omega_{C,i} \), viz.

\[
\Omega_{C,i} = \left( 1 - \frac{\overline{\phi U'(\beta_i)}_{cont}}{\overline{\phi U'(\beta_i)}_{dist}} \right) \times 100\%,
\]  

(4.4)

where \( \overline{\phi U'(\beta_i)} \) is the mode-\( i \) energy from the average spanwise-wavenumber power spectrum of the disturbance velocity data for the un-controlled flow caused by the roughness

Figure 4.23: Contour plots of \( U'/U_\infty \) determined at \( x = 450 \) mm for (a) the roughness disturbance only, (b) actuator C only, and (c) control. (d) The average spanwise wavenumber spectrum for each corresponding case.
Figure 4.24: Contour plots of $U''/U_\infty$ determined at $x = 450$ mm for modes 1 - 3, from left to right, for the roughness disturbance, actuator C only, and control (top to bottom).

Table 4.2: Disturbance energy and summary of the control effect.

<table>
<thead>
<tr>
<th></th>
<th>Mode Energy</th>
<th>Control Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_1$</td>
<td>$E_2$</td>
</tr>
<tr>
<td>REA</td>
<td>0.963</td>
<td>0.083</td>
</tr>
<tr>
<td>Actuator A</td>
<td>1.040</td>
<td>0.278</td>
</tr>
<tr>
<td>Actuator B</td>
<td>0.890</td>
<td>0.013</td>
</tr>
<tr>
<td>Actuator C</td>
<td>1.031</td>
<td>0.163</td>
</tr>
<tr>
<td>Control A</td>
<td>0.067</td>
<td>0.431</td>
</tr>
<tr>
<td>Control B</td>
<td>0.031</td>
<td>0.047</td>
</tr>
<tr>
<td>Control C</td>
<td>0.031</td>
<td>0.025</td>
</tr>
</tbody>
</table>

For each control case, the energy of the target mode 1 for the actuator was within 10% of the corresponding roughness element array disturbance. The predicted mode 1 control from the roughness element disturbance only and actuator disturbance only, was approximately 92, 92, and 93% for the control cases A to C, respectively. The attenuation
of mode 1 measured for each of the control cases was within 4%, which suggests that the control mechanism is linear, i.e. by superposition of the modes. Linearity of the control mechanism is also supported by measurements of the boundary layer shape factor. For instance, the shape factor quantifies the deviation from the Blasius solution. For all cases considered, the spanwise averaged boundary layer had a Blasius shape factor within the error bounds of $2.59 \pm 0.09$, discussed in Appendix B. Therefore, the disturbances were small enough to remain linear. This is also supported by the work of Fransson et al. [38] who showed that streaks of similar amplitude to those considered in this work behave linearly, in comparison to high-amplitude streaks, where amplitudes exceeded 26% and non-linear mechanisms become apparent [4].

4.6 Conclusions

The plasma actuators generated a fundamental spanwise wavelength determined by the spacing between the exposed electrodes. Additional weaker harmonics of the fundamental mode were also generated, which was similar to the roughness array case. The magnitude and phase of the higher harmonics vary with the width of the exposed actuator electrodes, which were 5 mm, 7 mm and 8 mm for cases A, B, and C, respectively. The width of the central low-speed streak and shouldering high-speed streaks were dependent on the superposition of the fundamental mode and higher harmonics. High- and low-speed velocity streaks contained a double-peak that was caused by the higher modes. The spatial character of the higher modes could vary with the actuator geometry. For instance, the amplitude of the first higher harmonic (mode 2) was reduced from actuator A to B however, re-emerged for actuator C with a phase shift of $\pi$ radians. This suggests that variation in the actuator geometry changed the distribution of spatial forcing within the boundary layer, which would affect the excitation of various wavenumbers. The phase of the second higher harmonic (mode 3) remained fixed. It was postulated that this was caused by recirculation over the exposed electrodes, which was of similar spanwise scale to the mode-3 disturbance.

A key result of the effect of excitation characteristics is that the ratio of energetic modes 1 to 3 considered changes with the total integrated energy, for either the frequency or voltage. It is known that both the propagation speed of the plasma as well as the plasma extent will increase with both the frequency or voltage, which in turn increases the total energy delivered to the flow. Recent observations have shown that the vortex core will migrate from the electrode with increased actuator output. Similarly, it was shown that for increasing actuator output, the central double-peaked low-speed streak
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will further coalesce. A similar observation of the coalescing double-peaked low-speed streak occurred with increased exposed electrode width, as shown for actuators B and C. The higher harmonics were shown to cause the double-peaked features. However, the relative energy contained in these higher harmonics depended on the forcing conditions. Increasing either the voltage or frequency for actuator A increased the relative energy of mode 1 and 3, while reducing the mode 2 energy. This is an important result because energy delivered by the actuator to higher modes can enhance existing comparable structures if the corresponding phases are similar. The overall effectiveness of the actuator will be reduced. Therefore, this inefficient use of power should be minimized.

These results have two important implications for the design of actuators used in bypass transition control. The first is that the method to input a pure mode (or at least approximately) may be accomplished by further optimizing the geometry or tuning the disturbance with actuator voltage or frequency. The second is that over a wide range of operating conditions, the output of the actuator will not have a constant ratio of energy in each mode. Therefore a priori knowledge of the disturbance sought for control, and its range of amplitude, is critical to the actuator design to limit inefficient use of the actuator power.

In this work, control of transient growth in a Blasius boundary layer was demonstrated experimentally for the first time using plasma actuators. Cylindrical roughness elements caused the transient algebraic growth sought for control. Three actuator geometries varying only in the width of the upper exposed electrode were investigated. For each geometry, the plasma actuators were successfully used to attenuate the energy of the disturbance produced by the roughness elements. The overall reduction of the disturbance energy was 41, 67 and 70% for actuators A, B, and C, respectively. However, the target $\Delta z$ mode was reduced by at least 93% in each case. The increase in effectiveness of actuators B and C was attributed to the improved geometry. A key result of this work is that the attenuation mechanism was shown to be linear, such that disturbances and counter disturbances could be linearly superimposed to model the control effect. This is an important simplification with respect to implementing a closed-loop feedback controller as discussed in the next chapter.
Chapter 5

Steady and Quasi-Steady Closed-Loop Feedback Control

5.1 Overview

The previous chapter on open-loop control demonstrated the practical utility of plasma actuators for attenuating the instability mechanism inherent to bypass transition. In this chapter feedback is used to close the control loop. The experimental arrangement for this work is shown in Figure 3.7. Streamwise streaks undergoing transient growth were embedded in the boundary layer upstream of the actuators using an array of roughness elements, which were located 200 mm downstream of the leading edge. The array of plasma actuators were located 100 mm downstream of the roughness array, followed by a spanwise array of shear stress sensors at $x = 500$ mm. Further detail of the experimental setup and measurement techniques are found in Chapter 3, Section 3.3.

This work was part of a collaborative project between the author, Mr. Kyle Bade and Prof. Ahmed Naguib of the FPCL at MSU, and Mr. Brant Belson and Prof. Clarence Rowley of Princeton University. These collaborators addressed issues associated with the development of the sensors and the controller. A distinction will be made wherever appropriate on results arising solely from the collaborators.

This chapter is comprised of four sections. In the first section the control objective, as well as the proportional-integral controller, and tuning, is addressed. An assessment of the practical limitations of the control system is presented in Section 5.3 prior to its application. Experimental results for the steady and quasi-steady control cases are found in Section 5.4, followed by concluding remarks.
5.2 Controller Design and Simulation

Proportional-integral-derivative (PID) controllers are widely used in feedback control for their good performance, robustness properties, and simplicity. The integral term ensures that the system can reach the target value, whereas the derivative term increases performance, although it is highly sensitive to noise and model uncertainty. Since the noise characteristics of the system were not known prior to implementation, a PI controller was chosen. A necessity of basic control techniques is that the output must vary monotonically with the input. For example, a linear function (output $\propto$ input) would always satisfy this, but a quadratic function may not. A monotonically varying control was developed as described in this section. Utilizing this control objective with empirical input/output flow data of the disturbance and actuator counter-disturbance the controller was simulated before implementation to predetermine controller gains such that robustness and performance were balanced.

5.2.1 Control Objective

A suitable control objective, based on measurements of the spanwise distributed streamwise shear stress, was sought. Therefore, it is necessary to first compare the flow field and the shear stress measurements. The disturbance velocity was measured over a $y$-$z$ plane spanning a region of $2\Delta z$, which was centred behind the central roughness element, and over the boundary layer thickness. The plane was located at $x = 490$ mm, which was 10 mm upstream of the shear stress sensor array to limit potential thermal effects of the shear stress sensors on the hot-wire measurements. This distance can be assumed small, such that negligible differences occur. Two sample contour plots of the disturbance velocity are shown in Figure 5.1 for the roughness element array deployed to (a) $k = 1$ mm and (b) $k = 1.5$ mm. High-speed streaks were centred about $z/\Delta z = -1$, 0, and 1, which correspond to locations centred behind the roughness elements, while the low-speed streaks were centred at about $z/\Delta z = -0.5$ and 0.5.

At the approximate off-wall location of the shear stress sensors, $\eta = 0.85$ identified by the dashed lines in Figure 5.1(a, b), the spanwise profile of the normalized streamwise disturbance velocity and normalized disturbance shear stress are shown in Figure 5.1(c, d), respectively. The spanwise resolution of the velocity measurements was $\Delta z/16$, whereas for the shear stress sensors it was $\Delta z/4$. The disturbance shear stress is defined as $\tau' = \langle \tau \rangle_t - \langle \tau \rangle_{tz}$, where $\langle \tau \rangle_t$ is the time-averaged shear stress and $\langle \tau \rangle_{tz}$ is the spanwise average of $\langle \tau \rangle_t$. The disturbance shear stress was normalized by the spanwise average shear stress. As shown in Figure 5.1(c, d), the disturbance shear stress adequately tracked
the disturbance velocity measurement, albeit with lower spanwise resolution.

![Contour plots of the streamwise velocity disturbance for (a) $k = 1$ and (b) 1.5 mm. Spanwise profile of the disturbance velocity at $\eta = 0.85$, and disturbance shear stress for (c) $k = 1$ mm, and (d) $k = 1.5$ mm.](image)

Figure 5.1: Contour plots of the streamwise velocity disturbance for (a) $k = 1$ and (b) 1.5 mm. Spanwise profile of the disturbance velocity at $\eta = 0.85$, and disturbance shear stress for (c) $k = 1$ mm, and (d) $k = 1.5$ mm.

The spanwise wavenumber power spectrum, used previously in Chapter 4, enabled wavenumber specific control. However, energy alone does not indicate the phase of the disturbance. A previous related study addressed this issue by an ad hoc treatment to detect the direction of the controller [60]. Mr. Brant Belson overcame this limitation by considering the full complex Fourier coefficient of the target $\Delta z$-mode, which was used to establish a single monotonic output, since it retains the critical phase information. The Fourier coefficients were determined from the Fast Fourier Transform (FFT) of the equally-spaced measurements of the disturbance quantities along the span. For the target $\Delta z$-mode, only the component of this Fourier coefficient that was controllable by the actuator is of interest. Therefore, the effect of actuation on the $\Delta z$-mode Fourier coefficient of the disturbance velocity, which was linearly related to the $\Delta z$-mode Fourier coefficient of the disturbance shear stress, is the primary interest.

The input to the controller was based on measurements obtained by the spanwise distributed streamwise shear stress sensors. The Fourier coefficient of the target $\Delta z$-mode is denoted as $\tilde{\tau}'_{\Delta z}$. Similarity, the Fourier coefficient for the $\Delta z$-mode of the disturbance velocity is $\tilde{U}'_{\Delta z}$, which was averaged over the boundary layer thickness, i.e. $\langle \tilde{U}'_{\Delta z} \rangle_y$. These complex numbers physically represents the phase and magnitude of the $\Delta z$-mode. The controllable subspace is the direction of $\langle \tilde{U}'_{\Delta z} \rangle_y$, viz.

$$\vec{C}_U = \frac{\langle \tilde{U}'_{\Delta z} \rangle_y}{\| \langle \tilde{U}'_{\Delta z} \rangle_y \|} \quad (5.1)$$

The control objective, which was the output of the flow (or plant), is the magnitude of the projection of the $\Delta z$-Fourier coefficient of the shear stress onto the controllable
subspace. Considering complex numbers as vectors of length 2, the output can be written as a dot product, \( \text{viz.} \)

\[
\text{output} = \varphi_{C\tau} = \text{dot}(\tilde{C}_U, \tilde{\tau}_{\Delta z}) = \text{real}(\tilde{C}_U) \cdot \text{real}(\tilde{\tau}_{\Delta z}) + \text{imag}(\tilde{C}_U) \cdot \text{imag}(\tilde{\tau}_{\Delta z}).
\] (5.2)

This output, \( \varphi_{C\tau} \), is referred to as the \textit{projected shear}. Similarly, the projected velocity is defined as \( \varphi_{CU} = \text{dot}(\tilde{C}_U, \tilde{U'}_{\Delta z}) \). The significance of both these quantities is physically related to the target \( \Delta z \)-mode of the roughness- and actuator-array disturbances, which were approximately in opposite phase, owing to the physical placement of each as shown in Figure 3.7. Following this concept, the \( \Delta z \)-mode disturbance caused by only the roughness element array will have a negative value of \( \varphi_{C\tau} \) or \( \varphi_{CU} \), whereas the one caused only by the plasma actuators will be positive. This provided a monotonic output. The goal of the controller is to drive \( \varphi_{C\tau} \) to as small a value as possible.

### 5.2.2 Empirical Flow Model

The response of the boundary layer to the forcing by the spanwise array of plasma actuators (described in Section 3.3.4) was measured for ten different operating voltages between 3.9 kV and 5.5 kV. Below 3.2 kV, the actuator was not causing any visible plasma formation and a flow response was not detected. For excitation voltages greater than 3.9 kV a monotonic increase in the flow response to forcing was observed. Flow unsteadiness was detected for excitation voltages above 5.5 kV, which indicated early stages of transition events. An example of the flow disturbance caused by the actuator is shown in Figure 5.2(a,b) for two sample conditions with \( V_{pp} = 3.9 \) kV and 5.1 kV, respectively. As discussed in the previous chapter, the counter disturbance caused by the actuator had a greater contribution of higher-mode energy, which caused the double-peaked low- and high-speed streaks.

![Figure 5.2: Contour plots of \( U'/U_\infty \) for the plasma actuator with (a) \( V_{pp} = 3.9 \) kV and (b) 5.1 kV.](image)
Using the methodology described in Section 5.2.1, the relationship between the actuator voltage and the projected velocity, $\varphi_{CU}$, was determined, as shown in Figure 5.3(a). The data was fit with an inverse tangent function, owing to the asymptotic nature of this equation, which is significant for this application. For instance, higher actuator voltages led to transition and consequently, the maximum output voltage should be limited. The inverse tangent provided a simple method to achieve this while retaining a good fit to the measured data, as shown in Figure 5.3(a). Practically, any monotonic function which can fit the observed data would have been acceptable for the purposes of this model.

The relationship between the projected velocity, $\varphi_{CU}$, and shear, $\varphi_{C\tau}$, was determined from measurements of the disturbance caused by the roughness elements, as shown in Figure 5.3(b). The array of roughness elements was deployed from 0.75 to 1.75 mm in increments of 0.25 mm. The linear relationship between the average flow response over the boundary layer thickness and the measurements of the disturbance shear stress was used to determine an appropriate level of actuation for the purpose of modelling the controller. For the roughness element data, each axis has negative values, since this disturbance is of opposite phase with the actuator disturbance. From these empirical fits, the following relationships were derived,

$$
\varphi_{CU} = m \cdot \varphi_{C\tau},
$$

$$
V_{pp} = c_1 \cdot \tan^{-1}(c_2 \cdot \varphi_{CU}) + c_3,
$$

where $m = 1.25$, $c_1 = 2734$, $c_2 = 1.66$, and $c_3 = 3120$.

![Figure 5.3](image-url)

Figure 5.3: (a) The inverse tangent fit for the actuator data and (b) the linear fit for the roughness element data.
5.2.3 Linearization and Control Model

This section and the one following was adapted from the work of Mr. Brant Belson. It is included to explain the methodology employed in the collaborative portion of this research on the development of the controller. Using this methodology, a model of the controller was determined using the input/output flow measurements obtained by the author, which enables tuning the controller prior to implementation.

A model for the plant, $P$, is obtained by inverting (5.3) and (5.4), viz.

$$\varphi_{C_T} = \frac{\varphi_{CU}}{m} = \frac{\tan \left( \frac{V_{pp} - c_3}{c_1} \right)}{m \cdot c_2}. \tag{5.5}$$

Equation 5.5 takes the voltage $V_{pp}$ and outputs $\varphi_{C_T}$. From a control perspective, the nonlinearity makes it difficult to perform control analysis directly [1]. Since the form of the nonlinearity is known, the terms were regrouped into a modified plant which takes a new input, $f$ (for forcing), and outputs $\varphi_{C_T}$ (see for example Åström and Murray [1]).

The nonlinearity may be encapsulated in the model by choosing:

$$f = \tan \left( \frac{V_{pp} - c_3}{c_1} \right), \tag{5.6}$$

such that (5.5) can be rewritten in terms of $f$, viz.

$$\varphi_{C_T} = \frac{f}{m \cdot c_2}, \tag{5.7}$$

which is a linear relationship between the input $f$ and output $\varphi_{C_T}$, as demonstrated in Figure 5.4.

![Figure 5.4: The linear plant, $P'$, takes input $f$ (5.6) and outputs $\varphi_{C_T}$ (5.7).](image)

The disturbance, $d$, caused by the roughness elements also affects the output. As shown in the previous chapter the effect of actuation and the roughness disturbance can
be combined linearly, such that as time evolves the output is given as

$$\varphi_{C\tau}^{i+1} = \frac{f^i + d^i}{c_2 \cdot m},$$

(5.8)

where $i$ is the discrete time step. This can be expressed as a discrete-time state-space system, \textit{viz}.

$$x_{P'}^{i+1} = 0 \cdot x_{P'}^i + \frac{f^i + d^i}{m \cdot c_2}$$

$$\varphi_{C\tau}^i = x_{P'}^i,$$

(5.9)

where the state, $x_{P'}$, is only an intermediate variable. Equation 5.9 comprises the empirical input-output linear model of the plant, $P'$, and facilitates the design of a controller, $K$. This assumes that the disturbance and counter-disturbances are of opposite phase, which is not guaranteed, as shown in the previous chapter. However, for control design only the component of the disturbance that lies in the controllable subspace, in other words that is either in phase, or of opposite phase with the controller, is considered.

In the physical experiment, one input is $V_{pp}$ and the disturbance $d$ is a second input to the plant $P$. In the model however, there is only one input to the plant: an effective $V_{pp}$ that comes from replacing $f$ with $f + d$ in (5.7), which simplifies the analysis. The block diagram in Figure 5.5 shows the entire closed-loop setup. Following along the loop, the actuation $f$ is summed with the unknown disturbance signal $d$, and enters the linear plant which takes $f + d$ and outputs $\varphi_{C\tau}$. This signal is fed to the controller, $K$, and the forcing term $f$ is determined, and the process repeats.

![Figure 5.5: Block diagram of the control scheme.](image)

**5.2.4 PI Controller**

To choose the PI controller gain levels the performance and robustness were analyzed following standard procedures (see for example Skogestad and Postlethwaite [147]). A measure of performance is how quickly the controller drives $\varphi_{C\tau}$ to zero, whereas robustness is the ability of the controller to perform reasonably well in a variety of off-design
conditions. The controller system, \( K \), evolves as
\[
\begin{align*}
 x_{K}^{i+1} & = x_{K}^{i} + \varphi_{C\tau}^{i}, \\
 f^{i} & = -K_I x_{K}^{i} - K_P \varphi_{C\tau}^{i},
\end{align*}
\] (5.10)

where \( x_{K} \) is the time integral of \( \varphi_{C\tau} \) needed for integral feedback. The proportional and integral gains are denoted by \( K_P \) and \( K_I \), respectively. The plant and controller systems are combined to yield the state-space equations of the controlled system, \( \text{viz.} \)
\[
\begin{align*}
 \begin{bmatrix}
 x_{K}^{i+1} \\
 x_{P}^{i+1}
 \end{bmatrix}
 & = \begin{bmatrix}
 -K_P & -K_I \\
 c_2 \cdot m & c_2 \cdot m \\
 1 & 1 \\
 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 x_{P}^{i} \\
 x_{K}^{i}
 \end{bmatrix}
 + \begin{bmatrix}
 \frac{1}{c_2 \cdot m} \\
 0
 \end{bmatrix} d^{i}, \\
 \varphi_{C\tau}^{i} & = \begin{bmatrix}
 \frac{1}{c_2 \cdot m} \\
 0
 \end{bmatrix}
 \begin{bmatrix}
 x_{P}^{i} \\
 x_{K}^{i}
 \end{bmatrix},
\end{align*}
\] (5.12)

For the best performance the state should decay to zero as quickly as possible. For this discrete time system that implies the eigenvalues of the \( 2 \times 2 \) matrix of (5.12) to be as close to zero as possible [147]. It is easily shown that \( K_P = K_I = c_2 \cdot m \) makes both eigenvalues zero.

Robustness was considered by the infinity norm of the sensitivity function \( S(s) \), \( \| S \|_{\infty} \), which is a metric of how well a controller performs in the presence of plant uncertainty (such as a change in free-stream velocity), \( \text{viz.} \)
\[
S(s) = \frac{1}{1 + P \cdot K},
\] (5.14)

where \( s \) is the complex frequency arising from a Laplace transform of time. A general guideline is for the value of \( \| S(s) \|_{\infty} \) to be small, typically in the range of 1.3 to 2 [147]. The best performing \( K_P \) and \( K_I \) (zero eigenvalues) result in \( \| S \|_{\infty} = 2 \). To accommodate a larger degree of uncertainty in the model of the plant, \( \| S \|_{\infty} = 1.33 \), and \( K_P = K_I = G_c \cdot c_2 \cdot m \), where \( G_c \) is the gain coefficient, set to 0.5. The following contour plots in Figure 5.6 show the levels of performance (maximum eigenvalue) and robustness (\( \| S \|_{\infty} \)) for all combinations of PI gains.

An example of the output, using \( K_P = K_I = 0.5 \cdot c_2 \cdot m \), is shown in Figure 5.7. At iteration 0, the system was undisturbed. At the next iteration, a disturbance corresponding with the maximum value considered in the experiments was applied. The controller effectively reduced \( \varphi_{C\tau} \) after only a few iterations. As long as the closed-loop system was stable, a zero steady-state error of zero was guaranteed because of the integral control. The reason for the large spike at iteration 1 was that the initial condition of the plant
Figure 5.6: The maximum eigenvalues of the matrix $A$ with control gains (left), and the value of the infinity norm of the sensitivity function (right).

is zero. Then the disturbance, affects the output at the next iteration. The control is non-zero at iteration 2, after the output is changed by the disturbance.

Figure 5.7: Response of the PI closed-loop controller to a typical steady disturbance.

5.3 Practical Limitations of the Control System

Practical limitations of the feedback control system arising from the physical components and experimental arrangement are discussed in this section.

5.3.1 Hardware

The hardware used to implement the control system operated discretely in time, which justified the discrete form of the control model. A flow chart describing the path of the signal measured at the disturbance plane to the signal sent to the actuator is shown in
Figure 5.8. The analog voltage signals of the shear stress sensors were sampled for 0.05 s and digitized by the data acquisition system. A personal computer time-averaged the signals to minimize noise, and the controller computed the required actuator response. The desired actuator voltage was updated on the function generator via a VISA-USB interface, and the output of the function generator was amplified and supplied to the actuators. The process repeated for the following iterations. The total time for each iteration was 0.5 s.

![Data flow chart](image_url)

Figure 5.8: Data flow chart for the hardware used in the steady and quasi-steady feedback flow control study.

### 5.3.2 Discrete Wall-Normal Sensor Locations

The shear stress sensors were located 1 mm from the plug surface as described in Section 3.3.2, corresponding to $\eta = 0.85$ for $U_\infty = 5$ m/s, which increased the signal-to-noise ratio of the measurements. The significance of the discrete wall-normal location of these sensors was evaluated by considering the ratio of the specific energy contained in the fundamental and first three higher harmonics with respect to the total energy, \emph{viz}.

$$\frac{E_i}{E_t} = \frac{\phi_u(\beta \Delta z/i)}{\int \phi_u(\beta)d\beta}, \quad (5.15)$$

where $E_i$ is the energy in the wavenumber $\beta_i$, and $E_t$ is the integrated energy over all wavenumbers. Recall that the non-dimensional wavenumber is $\beta = 2\pi \delta/\Delta z/i$ and a value $i = 1$ corresponds the fundamental wavelength $\Delta z$. The boundary layer averaged disturbance ratio was given by (4.3). The comparison of the local and average energy ratios for $\beta_{1-4}$, is shown in Figure 5.9(a) for the roughness array disturbance for deployment heights $k = 0.75 - 1.5$ mm, and (b) for the actuator with $V_{pp} = 3.6 - 5.6$ kV. Comparing Figure 5.9(a & b) shows a distinct difference in the trends with increasing disturbance roughness height and voltage, respectively. For example, as the roughness element deployment height was increased the local and average energy ratios converged however, the opposite was true for the actuator with increasing voltage.
Figure 5.9: The variation of the energy contained in $\beta_{1-4}$ from the local flow measurements at $\eta = 0.85$, and the boundary layer averaged measurements. (a) For the roughness element array, $k = 0.75$ to $1.75$ mm, and (b) for the actuator $V_{pp} = 3.9$ to $5.6$ kV.

The spatial distribution of the modal content over the boundary layer thickness is shown in Figure 5.10 for the actuator with $V_{pp} = 5.4$ and $3.9$ kV, with corresponding plots of the disturbance rms to the right. Similarly, the spatial distribution of the modal content over the thickness of the boundary layer is shown in Figure 5.10 for the roughness array deployed to $1.5$ and $1$ mm, with the corresponding rms for each mode. The increased energy at higher wavenumbers $\beta_{2-4}$, was evident for the actuator array. The actuator disturbance was also concentrated nearer the wall due to the reduced streamwise development. In particular, the mode-2 disturbance for the actuator was centred about $\eta = 1.5$, whereas it occurred near the mid region of the boundary layer for the roughness case. Also, as shown in Figure 5.10 as well as in Section 4.4, the actuator disturbance depended strongly on the excitation. These differences in the spatial distribution of energy, as well as evolving actuator energy distribution with voltage observations caused the increased difference observed in the local and averaged measurements shown in Figure 5.9 for the actuator disturbance.

### 5.3.3 Discrete Spanwise Sensor Locations

The goal of this work was to target a spanwise wavenumber of the disturbance having a wavelength $\Delta z$. In the previous Chapter, it was shown that at least 95% the average disturbance energy caused by roughness element array was contained by the first two modes of spanwise wavelength $\Delta z$ and $\Delta z/2$ at the measurement plane. For the plasma actuators, typically 80% the average disturbance energy was contained by the first two modes and over 95% of the energy was contained within the first three modes.
A balance between the hardware demands, i.e. one CTA channel per sensor, and the adequate spanwise resolution of the target mode resulted in four streamwise shear stress measurement locations per fundamental disturbance wavelength. For this arrangement, the Nyquist wavenumber corresponds to a wavelength of $\Delta z/2$, as discussed in Section 3.3.2. The contribution of higher, unresolved, wavenumbers resulted in aliasing to the first two modes. The effect of aliasing can be studied by comparing the flow measurement data at the simulated off-wall position of the shear stress sensors ($\eta = 0.85$). Figure 5.11(a) shows the energy contained by the first four modes calculated using 16 points per $\Delta z$ with the resampled 4 points per $\Delta z$ for the roughness element data and (b) the actuator data. For the roughness array, aliasing was reduced, compared to the actuator array, since the ratio of modal content remained similar with $k$ (see Figure 5.9a). However, as shown by Figure 5.11(b) significant aliasing occurred due to the increased level of unresolved third and fourth mode disturbances for the actuator case.

5.4 Experimental Flow Control Results

Experimental results on the performance, robustness, and the overall effectiveness of the feedback controller designed are discussed in this section. First, the effect of control
5.4.1 Effect of Controller Gain

The gains of the controller, $K_P$ and $K_I$, will affect the performance of a closed-loop control system. Owing to the linearity of the performance and robustness sensitivity, shown in Figure 5.6, the proportional and integral controller gains were maintained equal ($K_P = K_I$). The gain that would optimize performance was $K_P = K_I = c_2 \cdot m$. However, the gain was reduced to $K_P = K_I = G_c \cdot c_2 \cdot m$, where $G_c = 0.5$ is the gain coefficient, to increase robustness to modelled uncertainty. The effect of the gain coefficient on performance was shown experimentally. Figure 5.12(a) shows the control results for the cases having a roughness element deployment height of $k = 1.25$ mm, $U_\infty = 5$ m/s, and gain coefficients set to 3, 1, 0.5, 0.25, and 0.125.

As shown in Figure 5.12(a), the gain that maximizes the performance is $G_c = 1$ and corresponded to the best performance case discussed in Section 5.2.4; the system was critically damped. Further increase of the gain led to a damped oscillation of the control response, as shown for $G_c = 3$. Lowering the gain led to an over-damped response, as shown in Figure 5.12(a), for $G_c < 1$ with reduced performance. It is evident that the measured data points deviated from the model (lines) for $G_c < 0.5$, due to the non-monotonic actuator behaviour at low voltage, which is discussed in the next paragraph.
Figure 5.12: (a) Time variation of the control objective $\varphi_C \tau$ for $k = 1.25$ mm, $U_\infty = 5.0$ m/s, and various gain coefficients $G_c$. Data markers represent the experimental measurements, and lines are from the simulations based on the model data. (b) Corresponding voltage applied to the plasma actuator.

In Figure 5.12(b), the actuator voltage is shown for each case. For the lowest gain coefficient ($G_c = 0.125$), the first four actuator voltages were below 3.9 kV, which was the lowest model development voltage. After the first iteration (at 0.5 s), an increase in actuator voltage caused a reduction of projected shear value. Testing at low actuator voltages with the roughness elements retracted, showed that this behaviour is observable (see Figure 5.13). For voltages less than 3.2 kV, the electric field was insufficient to ionize air and produce a body force; between 3.2 and 3.8 kV a non-monotonic trend occurred. To the authors knowledge there is no evidence in the literature that addresses this range of plasma actuator outputs. Rather, research is focused on plasma actuator output over a wide range of voltage, often at increments of 1 kV or more.

Recently, Osmokrovic [117] showed some sensitivity of the location of maximum streamwise growth of the streaks generated by plasma actuators with voltage. Specifically, lowering the actuator voltage (and hence output) could migrate the location of maximum disturbance amplitude further downstream. The sensitivity of this location was greater for small actuator output, and significantly less for high actuator output. This may cause the non-monotonic...
trend observed in Figure 5.13. However, extensive measurements of the streamwise disturbance development for actuator voltages between 3.2 and 3.8 kV would be required to validate this hypothesis. Nonetheless, the controller remained stable, as shown in Figure 5.12, since low values of the objective caused the controller to increase voltage. Feedback is necessary to account for non-modelled effects such as this and provides increased robustness. However, this type of non-monotonic behaviour would destabilize an ad hoc controller relying on a less sophisticated objective, as shown by Hanson et al. [60].

5.4.2 Effect of Free-Stream Velocity

Robustness to off-design conditions were tested at reduced free-stream velocities. The controller gain in these experiments was $K_P = K_I = 0.5 \cdot c_2 \cdot m$, based on the discussion in Section 5.2.4. Two different roughness element heights, $k = 1.25$ mm and 1.375 mm, were considered. For each, the free-stream velocity was 3, 3.5, 4, and 5 m/s. The results for $k = 1.25$ mm are shown in Figure 5.14 and for $k = 1.375$ mm in Figure 5.15. The control model was developed at $U_\infty = 5$ m/s, whereas $U_\infty = 3, 3.5$ and 4 m/s are off-model cases. At time = 0 s, the controller measured the uncontrolled disturbance due to the presence of the upstream roughness element array.

Reducing the free-stream velocity lowered the magnitude of the initial control objective and consequently, the required actuation voltage to minimize $\varphi_{C_T}$. For $k = 1.25$ mm and $U_\infty = 3.5, 4, and 5$ m/s, $\psi_{C_T}$ was converged to zero within ±0.02 (or ±5% of the maximum value) for time > 2 s, as shown in Figure 5.14(a). The actuator voltages converged to approximately 3.8, 4.5 kV, and 5 kV, respectively, as shown in Figure 5.14(b). For the case at 3 m/s, the first control iteration resulted in a high level of actuation and the controller compensates by reducing the actuator voltage. Below 3.6 kV, as shown in Figure 5.13, a voltage reduction increased the level of forcing at the control plane, which is also shown in Figure 5.14 for $U_\infty = 3$ m/s for time = 0.5 - 1.5 s. The controller compensated by reducing the voltage until the actuator output was extinguished and the control objective reached an uncontrolled value of -0.2. Furthermore, from the empirical data used to construct the flow model shown in Figure 5.3, a value of $\varphi_{C_T} = -0.2$, would correspond to an expected actuation level of 4.5 kV. For free-stream velocities less than the model-development velocity, the actuator voltage required to minimize the control objective was always lower than that predicted by the model. A discussion of this effect follows after the next case.

Results for the second case, with $k = 1.35$ mm, are shown in Figure 5.15. For the free-stream velocities of 3, 3.5, and 4 m/s the controller minimized the objective and
converged on actuator outputs of 4.2, 4.4, and 4.8 kV, respectively. For the case at 5 m/s however, the controller attempts to increase the actuator voltage beyond 5.3 kV. Further increase reduced the mode-1 energy measured by the shear stress sensors, as shown in Figure 5.11(b). The controller compensated by increasing the actuator output (time > 2 s), which resulted in an unstable control case.

Explanation for the observed decrease in the initial control objective and increased actuator authority with reduced free-stream velocity was derived from the results shown in Figure 5.16. For the roughness element array, as the free-stream velocity increased,
so did the velocity at the apex of each element. The thickness of the boundary layer decreases with increasing free-stream velocity and is proportional to \((1/U_\infty)^{0.5}\), which led to a non-linear increase in the disturbance velocity and shear stress, as shown in Figure 5.16(a). An opposite behaviour for the actuator is shown in Figure 5.16(b). The actuator delivers a constant power input to the flow over the actuator for constant input voltage. Therefore, an increase in free-stream velocity will decrease the residence time of the fluid over the actuator, which decreases the total energy delivered. Despite these off-model changes, the steady state error was driven to zero by the controller due to the integral term, which provided robustness to the controller.

![Figure 5.16: Variation in the mode 1 shear stress disturbance and averaged flow disturbance for (a) the roughness element array and (b) the actuator output with free-stream velocity.](image)

5.4.3 Quantification of the Controlled Flow

Measurements of the velocity disturbance, \(U'\), were acquired at \(x = 490\) mm to validate the control over the boundary layer thickness. As shown in Figure 3.7, this location was 10 mm upstream of the feedback sensor array. For the case with \(k = 1.25\) mm, a contour plot of the disturbance velocity is shown in Figure 5.17(a) for the uncontrolled flow, and after the controller minimized the objective, in Figure 5.17(b). The controller converged on the actuator excitation voltage of approximately 5 kV. The corresponding wall-normal energy distribution is shown in Figure 5.17(c & d) for the uncontrolled and controlled flow, respectively. It is evident that the target mode energy was reduced and the remaining energy was located in the near-wall region. The remaining third mode content was similar to that observed in the actuator only case of Figure 5.10.

A comparison of the average spanwise-wavenumber power spectrum is shown in Figure 5.18 for the disturbance and the controlled flow. The average energy over all
wavenumbers was reduced by 74%. However, the energy of the target fundamental disturbance wavelength was reduced by 94%. A summary of the control results for this case, as well as all others, are shown in Table 5.1. Although the controller reached a converged result of $\psi_{CR} \approx 0$, over 6% of mode-1 energy remained.

Band-pass spatial filtering of $U'/U_\infty$ for the controlled flow (Figure 5.17b) was used to reconstruct the target spanwise mode, as shown in Figure 5.19. Approximately 75% of the residual energy in mode 1 following control was contained in the near-wall region, for $\eta < 2$. The reconstructed mode 1 shows that in the near wall region a central low-speed streak was located at $z/\Delta z = 0$, which indicates over-actuation. However, for $\eta > 2$ the mode 1 energy had a positive velocity at $z/\Delta z = 0$, which indicates a region of under-actuation. This was caused by the difference in the wall-normal distribution of energy for the roughness array disturbance, which existed higher in the boundary layer than that of the actuator (see Figure 5.10). Furthermore, the control objective indicates that mode 1 was completely attenuated. However, the flow measurements show a significant mode 1 energy. Under-prediction of mode 1 by the shear-stress sensors was caused by aliasing, as shown in Figure 5.11. This under-prediction led to a greater actuator output than required, resulting in the near-wall over actuated flow.
5.4.4 Summary of Control Performance

The control effectiveness for the targeted mode 1 is quantified using the parameter $\Omega_1$, viz.

$$\Omega_1 = \left( 1 - \frac{\overline{\phi_U(\beta_1)}/U_\infty^2(U_N)}{\overline{\phi_U(\beta_1)}/U_\infty^2(C)} \right) \times 100\%,$$

(5.16)

where $\overline{\phi_U(\beta_1)}/U_\infty^2$ is the boundary layer averaged mode-1 energy from the spanwise-wavenumber power spectrum of the disturbance velocity data for the un-controlled flow (UC). After the controller has converged to a minimal value of the control objective, $\phi_C \approx 0$, the controlled flow is denoted by $C$. For $G_c = 0.5$, this was always achieved before 4 s (or 8 control iterations). A value of $\Omega_1 = 100\%$ represents complete mode-1 attenuation. A similar definition can be applied to the total average energy over all wavenumbers, represented as $\Omega_0 - 8$. The subscript $0-8$ refers to the integration over all resolved wavenumbers.

The performance of a controller can be measured by the rise time $T_r$, steady state error, settling time $T_s$, and percent overshoot $M_p$ (see for example Åström and Murray [1]). The rise time is typically given as the time required for the signal to go from 10% to 90% of its final value. The settling time is the time required to reach approximately 2% of the final value, and remain within this margin. The overshoot is the percentage that the signal rises above the final value. For the PI controller, the steady state error, i.e. deviation from the desired state was always minimized for all stable cases, and is therefore not discussed further. A summary of the control results for each of the cases in the previous sections are shown in Table 5.1.

From the flow measurements over the thickness of the boundary layer ($5\eta$), and spanning the width of the control region considered ($2\Delta z$), the results demonstrated that the
Table 5.1: Summary of results for the closed-loop feedback flow control experiments, where NA indicates a measurement that was not possible.

<table>
<thead>
<tr>
<th>$k$ (mm)</th>
<th>$U_\infty$ (m/s)</th>
<th>$G_c$</th>
<th>$\varphi_{C_1}(t = 0)$</th>
<th>$T_r$ (s)</th>
<th>$M_p$ %</th>
<th>$T_s$ (s)</th>
<th>$\Omega_1$</th>
<th>$\Omega_{1-8}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>3.0</td>
<td>0.5</td>
<td>-0.20</td>
<td>0.28</td>
<td>56</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>3.5</td>
<td>0.5</td>
<td>-0.26</td>
<td>0.40</td>
<td>&lt; 2</td>
<td>0.5</td>
<td>97</td>
<td>86</td>
<td></td>
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<td>1.25</td>
<td>4.0</td>
<td>0.5</td>
<td>-0.31</td>
<td>1.3</td>
<td>&lt; 2</td>
<td>1.5</td>
<td>95</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>5.0</td>
<td>3</td>
<td>-0.41</td>
<td>0.39</td>
<td>19</td>
<td>7</td>
<td>94</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
<td>5.0</td>
<td>0.5</td>
<td>-0.41</td>
<td>0.42</td>
<td>&lt; 2</td>
<td>1</td>
<td>94</td>
<td>74</td>
<td></td>
</tr>
<tr>
<td>1.25</td>
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<td>0.25</td>
<td>-0.41</td>
<td>3.9</td>
<td>&lt; 2</td>
<td>5.5</td>
<td>94</td>
<td>74</td>
<td></td>
</tr>
<tr>
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<td>5.0</td>
<td>0.125</td>
<td>-0.41</td>
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<td>&lt; 2</td>
<td>12</td>
<td>94</td>
<td>74</td>
<td></td>
</tr>
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<td>0.5</td>
<td>-0.24</td>
<td>1.34</td>
<td>&lt; 2</td>
<td>2</td>
<td>95</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>1.375</td>
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<td>0.5</td>
<td>-0.34</td>
<td>0.90</td>
<td>&lt; 2</td>
<td>1.5</td>
<td>96</td>
<td>82</td>
<td></td>
</tr>
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<td>4.0</td>
<td>0.5</td>
<td>-0.40</td>
<td>0.78</td>
<td>&lt; 2</td>
<td>1</td>
<td>95</td>
<td>81</td>
<td></td>
</tr>
<tr>
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<td>0.5</td>
<td>-0.55</td>
<td>1.14</td>
<td>&lt; 2</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td></td>
</tr>
</tbody>
</table>

controller was capable of attenuating the target mode. Typically, a minimum of 94% mode-1 attenuation was achieved. However, the effect of higher mode energy resulted in a reduced overall effectiveness. Despite not accounting for the energy of these modes, the total energy reduction at the control plane was within 74% to 86%. From the performance metrics, it can be seen that decreasing free-stream velocity reduced the rise and settle times, which was also observed for increasing controller gain. Physically, this was caused by the increased sensitivity of the disturbance caused by the actuator with decreased free-stream velocity, as shown in Figure 5.16.

### 5.4.5 Streamwise Evolution of the Control Effect

In the previous sections, as well as the previous chapter, the control results were for only a single measurement plane. In this section the effect of the control on the transient growth of streaks is examined. The flow conditions coincide with three cases from Section 5.4.2. For the first two cases, the free-stream velocity was 4 and 5 m/s and $k = 1.25$ mm. For the third case, $U_\infty = 4$ m/s and $k = 1.375$ mm. The measurement domain for the uncontrolled disturbance consisted of a region spanning 65 mm downstream of the roughness element array to $x = 700$ mm, over a spanwise region between $z/\Delta z = \pm 1$, and over the thickness of the boundary layer.

For the case with $U_\infty = 5$ m/s and $k = 1.25$ mm, contour plots of $U'/U_\infty$ for the uncontrolled flow are shown in Figure 5.20. High-speed streaks were located downstream
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of each roughness element. As the flow developed, higher mode energy decayed, such that the \( y - z \) planes past \( x = 500 \text{ mm} \) appear to be comprised of a single spanwise wavenumber disturbance of wavelength \( \Delta z \), which corresponded to the spacing of the roughness elements. The voltage supplied to the actuator was determined by the closed-loop controller, based on the signal from the shear stress sensors located at \( x = 500 \text{ mm} \). From Figure 5.14, the actuator voltage is approximately \( 5.05 \text{ kV} \) when \( \phi_{C\tau} \approx 0 \).

For the measurements of the controlled flow the first measurement plane was located downstream of the actuator array at \( x = 400 \text{ mm} \) and extended to \( x = 700 \text{ mm} \), at increments of 50 mm. The contour plots of the controlled disturbance velocity are shown in Figure 5.21. Similar to the results shown in Figure 5.17, at the downstream location of the feedback sensors, the controlled flow was comprised primarily of higher mode energy that was not targeted by the controller.

![Figure 5.20: Streamwise evolution of the disturbance caused by a roughness array with \( \kappa = 1.25 \text{ mm} \) and \( U_\infty = 5 \text{ m/s} \).](image1)

![Figure 5.21: Streamwise evolution of the controlled flow with the actuator operated at 5.05 kV.](image2)

The growth and decay of the disturbance for each of the three cases is shown in Figure 5.22. A function representing the growth and decay of energy was fit to the data points. The equation for this function follows from the work of White et al. [159], viz.

\[
\frac{\phi U''}{U_\infty^2} = A (x - x_0)^2 \cdot \exp \left[ -B (x - x_0) \right], \tag{5.17}
\]

where \( A \) represents the algebraic growth rate, \( B \) represents an exponential decay, and \( x_0 \) is the distance of the roughness element array from the leading edge (200 mm).

The effectiveness of the control over the measurement region was quantified by the spatial evolution of the targeted mode energy, as shown in Figure 5.22 for each of the three cases. For these results, the power spectrum of the normalized disturbance velocity \( U''/U_\infty \) was averaged over the thickness of the boundary layer following (4.2). The actuator attenuated over 90% of the energy contained in the target mode past the control
measurement plane located at $x = 500$ mm however, the absolute level of the control effect upstream was slightly reduced.

### 5.4.6 Discrete Free-Stream Velocity Perturbation

The ability of the controller to respond to a discrete change in the free-stream velocity was investigated with $G_c = 0.5$. Two cases were considered. For the first case, a 20% decrease in the free-stream velocity, from 5 to 4 m/s is shown in Figure 5.23(a-c). For the second case, the free-stream velocity was increased from 4 to 5 m/s, as shown in Figure 5.23(d-f). The discrete change was conducted as follows: (i) the controller was initialized with the first steady flow condition, (ii) after 10 s (corresponding to 20 discrete iterations) the controller was paused (for 30 seconds, and omitted from the plots), and the free-stream velocity was changed, (iii) the controller was permitted to continue iterating. Similar to all other results, the iteration time was 0.5 s.

For the first case shown in Figure 5.23(a-c), the output converged to $\varphi_{C_T} \approx 0$ after 3 iterations, similar to that shown in Figure 5.14 for the same conditions. After the free-stream velocity was decreased at 10.5 s, $\varphi_{C_T} = 0.21$, as shown in Figure 5.14(b). Physically, a positive value of $\varphi_{C_T}$ represents over-actuation. Therefore the controller lowered the applied voltage to the actuator accordingly, as shown in Figure 5.14(c). In the second case, after the free-stream velocity was increased, the $\varphi_{C_T} = -0.17$. A negative value physically represents insufficient disturbance attenuation and the controller raised the actuator output accordingly.

It was shown in the previous section that increasing the flow velocity can also be viewed as decreasing the controller gain. Therefore, the performance of the controller
was higher for a discrete decreased free-stream velocity, than for an increase. For example, it is shown in Figure 5.23(b) that the settle time was 1 second, or 2 control iterations, whereas the settle time was nearly twice as long for an increase in the free-stream velocity, as shown in Figure 5.23(e).

![Figure 5.23: Response of the controller to discrete velocity change with $k = 1.25 \text{ mm}$ and for (a-c) $U_\infty = 5 - 4 \text{ m/s}$ and (d-e) $U_\infty = 4 - 5 \text{ m/s}$. The free-stream velocity is shown in (a, d) with the corresponding control objective $\varphi_{C\tau}$ (b, e) and actuator voltage (c, f).]

### 5.4.7 Continuous Free-Stream Velocity Perturbation

The ability for the controller to track a slowly time varying disturbance was studied. The disturbance was caused by the stationary array of roughness elements subject to a sinusoidal variation of the free-stream velocity at a period of 50 s and an amplitude of 1 m/s. The mean velocity was 4.5 m/s such that the wind tunnel velocity varied from 4 to 5 m/s. A shorter period resulted in a reduced amplitude and mean velocity variation, owing to the fixed time response of the wind-tunnel. Since this period was fixed, the iteration rate of the controller was varied although, as discussed in Section 5.3.1, the minimum iteration time was fixed at 0.5 s. Four controller iteration loop times ($T_d$) were investigated, namely, 0.5 s, 1 s, 2 s, and 4 s. Considering that the period of the wind tunnel was 50 seconds, this resulted in 100, 50, 25 and 12.5 control iterations per free-stream velocity period, respectively.

The experimental results for the cases with $T_d = 0.5, 1, 2,$ and 4 s are shown in Figure 5.24. The effectiveness of the controller to attenuate the time-varying disturbance was
quantified by the standard deviation of the error, normalized with the mean uncontrolled value of the disturbance, \(\text{viz.}\)

\[
e_{\text{std}} = \left| \frac{\text{STD}[\varphi_{C\tau}]}{\varphi_{C\tau}(\text{UC})} \right| \times 100\%,
\]

over two complete cycles. The value of \(\varphi_{C\tau}(\text{UC})\) for a free-stream velocity of 4.5 m/s was \(\approx 0.35\). A summary of the control results are shown in Figure 5.25. As \(T_d\) is decreased,
the ability of the control system to respond to the changing disturbance was increased. As shown in the figure, the trend in the tracking error was approximately linear. Owing to a short measurement duration (0.05 s), the random uncertainty of the measured value of $\varphi_{C_T}$ for steady control conditions was approximately 1.2%, which caused the non-zero $y$-intercept shown in Figure 5.25.

![Figure 5.25: Error of the control result for a sinusoidal velocity change from 4 to 5 m/s with $k = 1.25$ mm and $T_d = 0.5$ to 4 s.](image)

5.5 Conclusions

Control of transient growth in a Blasius boundary layer was demonstrated experimentally using a model-based feedback controller with plasma actuators. An array of cylindrical roughness elements introduced streamwise streaks of spanwise periodic low- and high-velocity undergoing transient algebraic growth. The strength of the disturbance increased with the roughness element deployment height as well as with the free-stream velocity. Control was provided by an array of plasma actuators, which were arranged on the boundary layer plate to generate spanwise periodic streaks of low and high streamwise velocity. In the previous chapter plasma actuators were shown to be an effective means to attenuate transient growth modes, albeit in an open-loop framework. In this chapter an array of wall-mounted hot-wires were employed to sense spanwise periodic shear-stress variations caused by the flow disturbance, which provided feedback to the controller. A novel control scheme was based on a spanwise wavenumber specific objective. An input/output model of the system was developed from empirical relationships that were determined between the flow response to forcing by the roughness element array and that caused by the plasma actuator array. The actuator array was was oriented such that the targeted mode was of opposite phase. This model was used to design and tune a proportional-integral controller prior to implementation in the closed-loop flow control study.
It was shown that for on-model conditions the feedback controller could reduce the energy associated with the targeted mode by at least 94% at the $y$-$z$ plane where the feedback sensors were located. However, the total disturbance energy of the boundary layer was reduced by 74% to 86%, which shows that additional improvement may be possible with optimized actuators or by higher-order, $\beta_2$ & $\beta_3$ control actuators. The effect of control was also considered over a streamwise region both upstream and downstream of the control plane. It was shown that control attenuated the disturbance growth prior to the measurement plane, and that the control affect remained further downstream.

The controller was capable of responding to discrete changes in flow conditions. The decrease in the free-stream velocity, which was not considered in the controller development, caused the apparent gain of the controller to increase. This was attributed to the inverse sensitivity of the generation of streamwise streaks by the plasma actuator arrays with free-stream velocity, which was caused by the change in residence time of the fluid over the actuator for constant actuator output. Despite these off-model conditions, the integral term of the feedback controller prevented steady-state error. Therefore, feedback provided robustness to the non-modelled velocities, which increased the range of controller effectiveness. Further evidence of the effectiveness of the controller was shown for quasi-steady conditions by varying the free-stream velocity. The controller was able to maintain a high level of control authority, which was diminished with a decreasing controller update rate.

These results provide a demonstration of the use of wall-located sensing, disturbance input, and control of the bypass transition instability. Specifically, issues relating to the implementation of wall-located sensing, plasma actuators, and a suitable control objective were addressed. The next logical step is aimed at the development of low-order dynamic models of the pulsed actuator and disturbance input. Previous numerical studies suggest that a combination of feedforward and feedback control with a dynamic model can increase the operational bandwidth of the controller as well as its robustness. In addition, the question of optimal placement of actuators and sensors in a practical framework remains open, which is best suited for numerical simulations.
Chapter 6

Assessment of Dynamic Plasma Actuation

6.1 Overview

The objective of this work was to elucidate the dynamic response of a laminar boundary layer to pulsed excitation by an array of plasma actuators. These actuators were shown to be an effective means for transient growth control, albeit for steady and quasi-steady conditions. To support the long-term goal aimed at implementing a real-time boundary layer flow control demonstration in the laboratory, the dynamic response of the flow to forcing by plasma actuators was studied. The geometry of the actuator was described in Section 3.3.4. This actuator was also used in Chapter 5. The temporal evolution disturbance caused by a short duration pulsed actuator output was determined by phase-averaged hot-wire measurements at multiple $y-z$ planes downstream of the actuator. Using this method, the spatial and temporal character of the disturbance excited by pulsed plasma actuation was reconstructed over the flow field ($z/\Delta z = \pm 1$, $\eta = 0 - 5$). The dynamic response of the flow was considered first at a single streamwise location, corresponding to the feedback control plane discussed in Chapter 5, followed by an analysis of the disturbance evolution along the streamwise direction.

6.2 Measurement Procedures

Measurements were made in the UTIAS wind tunnel on the boundary layer plate described in Section 3.3. Seven downstream disturbance planes, evenly spaced from $x = 400 - 700$ mm at increments of 50 mm were measured. The plane at $x = 500$ mm was
shifted to \( x = 490 \) mm to prevent potential damage of the shear stress sensors. The actuator was operated at \( 5 \) kV\(_{pp} \), and \( 1.5 \) kHz, which corresponds with a disturbance velocity amplitude > 8% of \( U_\infty \) at \( x = 490 \) mm, as shown in Chapter 5. For this condition a significant disturbance persisted at the furthest downstream measurement plane considered. The duration of the excitation signal supplied to the actuator was \( 0.1 \) s followed by a \( 0.9 \) s rest period. A pulse width of \( 0.1 \) s was selected since the transient actuator output appeared to asymptote for this length of time. Each phase-averaged hot-wire velocity measurement is determined from 25 cycles. The velocity signal was time-averaged locally over a window width of \( 0.0005 \) s to reduce sensor noise. Measurement of the phase-averaged power consumption of the actuator were used to assess change in the actuator output momentum, which was used to support observations of the flow response. Measurement procedures, calibration, and uncertainty analysis are discussed in Section 3.3, and Appendix B.

### 6.3 Disturbance Evolution at the Control Plane

The location 10 mm upstream of the feedback sensor array at \( x = 500 \) mm was considered first. A sample of the disturbance velocity measured at a point located at \( z/\Delta z = 0 \), and \( 0.5 \), with \( \eta = 0.85 \) is shown in Figure 6.1 with the input peak-to-peak actuator voltage. As shown in the previous chapter for steady actuation conditions, \( z/\Delta z = 0 \) corresponded to the centre of a low-speed streak, whereas \( z/\Delta z = 0.5 \) corresponded to the centre location of a high-speed streak. Nine characteristic points are labelled in the figure, which are referred to in the following paragraphs.

![Figure 6.1: Phase-averaged disturbance velocity at \( \eta = 0.85 \) for \( z/\Delta z = 0 \), and \( z/\Delta z = 0.5 \) for a wide-pulse actuator input is. A first order response (TF1) is identified.](image-url)
The actuator was turned on at 0 s and off at 0.1 s. A delay of approximately 0.048 s occurred between the time the actuator was turned on and the initial flow response (point-a) due to the convection of the affected air to the downstream measurement location by the boundary layer. Therefore, the time delay will depend on the location of the actuator and sensors as well as the free-stream velocity. To support a dynamic controller, a model relationship between the input and output flow response is required. A first order transfer function of the actuator input to the flow response is shown in Figure 6.1 and is given by

\[ G(s) = \frac{-0.041}{(1 + 0.026s)} \cdot \exp(-0.066s). \]  

(6.1)

The pole is real and negative such that the system is stable within the bandwidth. Only a marginal increase in accuracy was achieved for a second order response. The time delay appears in the exponential term. The significance of this term is to account for the physical time delay between the input voltage and observed flow response, however, it will also limit the bandwidth of the controller. For example, a time delay of 0.066 s has an associated bandwidth of 0 - 15 Hz. This time delay was too small to cause an observable effect on the steady and quasi-steady control results from Chapter 5, which had a minimum update rate of 0.5 s. A limitation of the simple model response given by (6.1) was the inability to reproduce the features at points c, e, and g. In particular, an inverse response to forcing occurs at point-c and g. This response is representative of a non-minimum phase system [1]. Ultimately, the goal of the controller is to target a single spanwise wavenumber rather than a single isolated (unfiltered) point in the flow. The applicability of a first order model for this problem will be discussed following the analysis of the global flow response, and origin of the features at points c, e, and g.

The spatial variation of the boundary layer disturbance over time caused by pulsed excitation of the plasma actuator is shown in Figure 6.2. Contour plots of the disturbance velocity \((U'/U_x)\) at the nine instances in time are shown, which correspond with the points \(a - i\) in Figure 6.1. The wall-normal profile of the disturbance velocity of the fundamental and first two higher harmonics at \(z/\Delta z = 0\) is shown to the right of each contour plot. The disturbance entered in the outer region of the boundary layer, above \(\eta = 2\), as shown in Figure 6.2(a, b). As time increased, the disturbance filled out over the thickness of the boundary layer from the top, where the convecting velocity was the highest, to lower velocity region near the wall. A full disturbance profile was formed after 0.12 s, which was typical of the spatial signature of the actuator under steady conditions, see Figure 6.3. For further increase in time, the disturbance was advected downstream of the measurement plane at a rate that increased with location from the wall in the
boundary layer. At the tail of the cycle only a trace disturbance remained, as is shown in Figure 6.2(i), in the near-wall region. After 0.2 s the disturbance was completely advected downstream of the measurement plane.

Figure 6.2: Contour plots of the streamwise disturbance velocity at instances in time corresponding to points a-i from Figure 6.1. Corresponding wall-normal profiles of the streamwise disturbance energy contained in modes 1-3 (right).
For steady actuation, the point at $z/\Delta z = 0$ and $\eta = 0.85$ had a negative disturbance velocity, while the point at $z/\Delta z = 0.5$ had a positive disturbance velocity. The dynamic response showed an opposite behaviour during the initial disturbance development before filling out over the boundary layer, for example point-c from Figure 6.1. For the present results, the deviation from the steady behaviour can be explained by inspection of the spanwise modal content over time. The wall-normal averaged spanwise power spectrum is shown in Figure 6.3 with the corresponding reconstructed average waveforms at 0.12 s, corresponding with point-f on Figure 6.1. Only the fundamental and next two higher harmonics were considered, since they contained over 92% of the total streamwise disturbance energy. As shown in Figure 6.2(b, c), the disturbance entering the measurement plane first consisted primarily of energy in mode-1 and mode-2.

In the near-wall region ($\eta < 2$) the disturbance velocity profile of mode 1 at $z/\Delta z = 0$ indicates that it is of opposite phase to that shown in Figure 6.3(b). The initial increase in the velocity occurring at point-c from Figure 6.1 for $z/\Delta z = 0$ can be explained by the presence of mode-1. At $z/\Delta z = 0.5$, the perturbation was more pronounced. Mode-2 had a negative disturbance velocity at $z/\Delta z = 0$, which would have a corresponding negative disturbance velocity at $z/\Delta z = 0$, as shown in Figure 6.3(b). Therefore the mode-1 and mode-2 interact constructively to cause a large negative perturbation at near point-d for the point at $z/\Delta z = 0.5$ and $\eta = 0.85$. Similar behaviour was observed for streaks formed by pulsed suction [71], however, this effect was speculated to be caused by a local removal of the low-speed fluid in the boundary layer by suction, prior to the formation of streaks.

![Figure 6.3: (a) Average spanwise wavenumber power spectrum at 0.12 s, and (b) corresponding reconstructed modes 1-3.](image)

The variation in the total average energy and the specific average energy contained in the fundamental and next two higher harmonic modes with time is shown in Figure 6.4. The average disturbance energy increased by 0.5% of the maximum value at 0.05 s. Considering that the distance from the actuator trailing edge to the measurement plane
was 160 mm, the characteristic convection velocity is 3.2 m/s or \( \approx 0.64 U_\infty \). From the Blasius solution, the mean velocity of the boundary layer was calculated to be \( \approx 0.66 U_\infty \), which was 3% greater than the measured value. Lundell and Alfredsson [103] observed a characteristic velocity of \( 0.8 U_\infty \) for turbulence generated streaks, although the streaks had a peak energy at \( \eta = 2.6 \). Breuer and Haritonidis [13] observed wave packets to propagate much slower, at \( 0.34 U_\infty \), however, the peak energy occurred at \( \eta < 1.2 \). These results suggest that the convective time scale will depend on the wall-normal location of the disturbance. For the present results, the peak energy occurred at \( \eta \approx 2 \), where the local velocity is \( 0.63 U_\infty \), which compared well with the measured value.

![Figure 6.4: Phase-averaged measurements of the energy contained in the fundamental mode and first two higher harmonics.](image)

It is shown in Figure 6.4 that the total energy increased sharply after 0.05 s. The mode-2 disturbance energy had a slight overshoot at 0.068 s, identified by line-a, followed by a second overshoot at 0.155 s (line-c) before the disturbance began to advect out of the measurement plane. The fundamental mode had a similar feature at 0.088 s and 0.155 s (lines b and c). This type of velocity overshoot was observed for plasma actuators used in quiescent air on the surface of a cylinder [74], and in the study of a starting vortex formed by a DBD plasma actuator [157]. The reason for the overshoot was attributed to correlation with the transient response of the peak current density of the plasma actuator (see for example [42; 145]). To quantify this effect for the present results, the power consumed by the actuator was measured to identify any transient behaviour in power consumption, which was associated with variation in momentum production. For instance, an increase in power consumption corresponds with an increase in the actuator body force [88; 154]. The power measurement procedure was outlined in Section 3.3.4. Measurements of the phase-averaged power consumption of the pulsed actuator are shown.
in Figure 6.5. For the first frequency cycle the power peaked above the average power, between 0 - 0.1 s, by approximately 30%. Between 0.01 s and 0.096 s the power increased by 4%, and is followed by a power spike of approximately 10% at the end of the pulse.

![Power Consumption](image)

**Figure 6.5:** Phase-averaged measurements of the power consumed by the actuator.

The effect of the power consumption spikes manifested in the flow response to forcing. For instance, the time between the initial and final peak disturbance energy for mode-2 (between lines-a and-c from Figure 6.4) had a time difference of 0.09 s. At these locations a variation in the total energy was also visible in Figure 6.4. For the first location (line-a), the variation in total energy appeared as a slight inflection owing to an average steep increase in the total energy with time. Only the air over the actuator experiences the strong forcing caused by a short duration spike in actuator forcing. Therefore, the width of local deviations were related to the length of the actuator. The width of the grey shaded regions shown in Figure 6.4 is 0.0091 s, and corresponds to the residence time of the fluid over the streamwise length of the actuator (30 mm), given the convection velocity estimated previously.

The spatial distribution of the disturbance velocity shown in Figure 6.2(c) occurred at approximately the centre of the grey shaded region of Figure 6.4 centred about line-a. At this instant in time, the mode-2 disturbance energy peaked at $\eta \approx 2.3$. From Figure 6.2(c), only a weak contribution of mode-1 and 3 was observed. At line-b, the energy of mode-1 reached a local plateau, corresponding with the disturbance shown in Figure 6.2(e), with a peak value occurring at $\eta \approx 2$. Similarly, the mode-3 disturbance peaked at $\eta \approx 2$. Therefore, the mode-2 disturbance is convected at a higher velocity, associated with the peak energy occurring further from the wall than the mode-1 and 3 disturbances.

For closed-loop feedback control the disturbance will be sensed at or near the wall. At $\eta = 0.85$, the location of the near wall shear stress sensors used in the previous chapter, the variation of $\varphi_{CU}$, which was linearly related to the control objective, $\varphi_{C_T}$, is
shown for the wide-pulse response in Figure 6.6. At $\eta = 0.85$, the value of $\varphi_{CU}$ initially went negative, due to the inversion of the mode-1 phase in the near wall, as discussed previously. Similar to the local velocity response shown in Figure 6.1, a first order transfer function was unable to reproduce the more complex response, which was associated with a non-minimum phase system. The second order transfer function of the actuator input to the flow response shown in Figure 6.6 is given by

$$G(s) = 0.14 \cdot \frac{1 - 0.009s}{(1 + 0.01s)^2} \cdot \exp(-0.064s).$$ \hspace{1cm} (6.2)

Including a right-hand plane zero was required to reproduce the inverse response to forcing at approximately 0.07 s and 0.17 s.

Figure 6.6: Phase-averaged response of $\varphi_{CU}$ at $\eta = 0.85$, and averaged over the boundary layer thickness, for a wide-pulse actuator input is. A first order response is identified for each measurement. The boundary layer averaged response of $\varphi_{CU}$ is shown in Figure 6.6. The energy associated with mode 1 phase inversion in the near-wall region did not overcome to the corresponding response in the outer region of the boundary layer, which is evident in Figure 6.2(c), such that $\varphi_{CU}$ was always greater than zero. Contrary to the local response, the boundary layer averaged response was tracked well by a first order transfer function without right-hand-plane zeros, similar to (6.1), as shown in Figure 6.6.

The dynamic response of the flow has significant implications for the real-time control of bypass transition. For instance, an increase in actuator strength would not be detected instantly and is related to the downstream location of the feedback sensors and the free-stream velocity of the boundary layer (with zero-pressure gradient). The inherent time delay will reduce system performance. To improve performance feedforward sensing can
provide corrective input action [1] before the disturbed flow reaches the feedback location, however, the success is highly coupled with the accuracy of the modelled response. The shear caused the disturbance to fill in from the top of the boundary layer to the wall, which introduced a transient response to the step input. The local mode-1 response in the near wall is characteristic of a non-minimum phase system. Practically this requires special treatment for the design of a suitable controller [1].

It is interesting to note that although the poles of a system are an intrinsic property, the zeros (i.e. those causing the observed non-minimum phase behaviour) are not [1]. The zeros depend on the coupling of the sensors and actuators to the state of the system. Or said another way, by changing the sensing strategy it may be possible to change the zeros of the system transfer function. Therefore, the right-hand plane zero associated with the transfer function given by (6.2) may be modified or possibly eliminated.

### 6.4 Streamwise Evolution of the Pulsed Disturbance

The streamwise evolution of the pulsed disturbance was studied at $x = 400, 450, 490, 550, 600, 650$ and $700$ mm. A sample of the measured disturbance velocity at a point located at $z/\Delta z = 0$ and $\eta = 0.85$ is shown in Figure 6.7 for two streamwise locations. The first is at $490$ mm and the second is at $700$ mm. At time $= 0$ s, the actuator is turned on, and at $0.1$ s, the actuator is turned off. The delay before the disturbance is measured is twice as long at the furtherest measurement point, since the streamwise distance between the actuator and measurement location is doubled. In addition, the slope of the velocity response between $0.06$ and $0.09$ s is approximately $45\%$ more gradual than at $x = 700$ mm for $0.13$ to $0.17$ s, as identified by lines $s1$ and $s2$ in Figure 6.7.

![Figure 6.7](image-url)  

*Figure 6.7: Phase averaged streamwise disturbance velocity at $\eta = 0.85$ for $z/\Delta z = 0$, and $x = 490$ mm $-$, and $x = 700$ mm, $--$.***
The temporal growth and decay of the boundary layer disturbance caused by pulsed excitation of the plasma actuator is shown in Figure 6.8. Contour plots of the disturbance velocity, $U'/U_c$, at the seven streamwise locations, corresponding to the rows in Figure 6.8, and three instances in time identified in the columns of Figure 6.8 are shown. As time is increased, the disturbance appears in the upstream planes first, and fills out over the extent of the boundary layer from the top, where the convecting velocity is the highest, to lower velocity region near the wall, as shown in Figure 6.2. Further downstream, higher modal content is reduced, as shown in the previous chapter, which causes the double-peaked low-speed streak to coalesce further downstream, as shown for instance from comparison of the disturbance at $x = 400$ and $x = 700$ mm.

Figure 6.8: Contour plots of the streamwise velocity disturbance for $x = 400, 450, 490, 550, 600, 650$ and $700$ mm, from top to bottom. Three different times, corresponding to 0.085, 0.135 and 0.185 s, from left to right.
The spatial-temporal evolution of the disturbance energy of the fundamental mode and first two higher harmonics is shown in Figure 6.9. The dashed-line in Figure 6.9(a) corresponds to central location of the fundamental mode-1 peak, which varies in time and streamwise distance. The slope of the line is 2.78 m/s, which is 15% lower than the average convection velocity of $0.66U_\infty = 3.28$ m/s. For the mode-2 disturbance, the slope of the line is 3.26 m/s, and for mode-3 it is 2.7 m/s. The increased velocity of mode-2 led to the appearance of mode-2 energy at $x = 490$ mm, as shown in Figure 6.4.

![Figure 6.9: Spatial-temporal evolution of the energy contained in (a) the fundamental mode and (b, c) the first two higher harmonics over a streamwise region from $x = 400$ mm to 700 mm.](image)

The mode-2 disturbance retains a similar temporal variation shown in Figure 6.4, overshooting in energy. This variation is observed in Figure 6.9(b), for increasing time. Specifically, the mode 2 first peaks than decays in energy over the time of the actuator pulse. Similarly, for mode-3 the gradual increase in energy with time shown in Figure
6.4 is observed in Figure 6.9(c), for increasing time. Further downstream, however, the mode-3 decays as shown for similar steady conditions in the previous chapter.

6.5 Conclusions

Phased-averaged measurements of the disturbance velocity were used to reconstruct the dynamic response of the flow to pulsed plasma actuation. Owing to a higher convective velocity in the outer region of the boundary layer, the disturbance entered the measurement plane in the outer region and filled out over the thickness of the boundary layer toward the wall. The variation in the average energy for the fundamental mode and the next two higher harmonics showed that the mode-2 energy arrived at the measurement plane prior to the fundamental disturbance wavelength. It was shown that the peak energy of mode-2 occurred higher in the boundary layer than for mode-1. Therefore, the location of the disturbance in the boundary layer can cause a significant change in the rate that the disturbance is advected downstream.

In practical feedback control applications sensors will be located at or near the wall and downstream of the actuators. Therefore, a time delay occurs between the time that actuator is turned on, and that time that the disturbance enters the measurement plane. In the application of unsteady real-time control of bypass transition, an increase in actuator strength would not be detected instantly at the wall and is related to the downstream location of the feedback sensors and the free-stream velocity of the boundary layer. This is known to limit the performance of a feedback controller, and suggests the integration of feedforward control to improve performance. However, the effectiveness of feedforward control relies on an accurate dynamic model of the flow response, which would support the development of a real-time controller.

Following from the control objective developed in the previous chapter, it was shown that the mode-1 dynamics are more complex than a simple first-order response, although the time averaged boundary layer response could be described by a simple first-order equation. In the near-wall location of the feedback control sensors, the mode-1 energy was of opposite phase to the steady state response when it entered at the measurement plane. This behaviour could be modelled by including a right-hand plane zero in the transfer function describing the flow response to forcing. However, it is known that the presence of zeros places limitations on the achievable system performance, and possible measures to avoid this behaviour should be considered.

The pulse-power consumption of the actuator demonstrated temporal variations. Specifically, a spike in power at the start and end of the cycle occurred, as well as a
transient increase in the power over the width of the pulse. In addition, the length of the actuator extending in the streamwise direction affected the dynamic behaviour. This result has implications for dynamic control. Specifically, the results suggest that limiting the streamwise extent of the actuator could decrease the width of local variations in the dynamic behaviour to pulsed actuation, which could not be captured by a first-order model. However, the slow transient increase in power over the width of the pulse could be accounted for.
Chapter 7

Conclusions and Recommendations

7.1 Conclusions

7.1.1 Open-Loop Control of Streamwise Streaks

The effectiveness of control systems is inherently linked to the ability of the actuator to alter the flow to a desired state. Therefore, actuators are a critical enabling technology component in any active flow control system. This work was motivated by the need for flow control actuators that are simple to implement, easy to construct, and readily integrated with a flow control system. Plasma actuators have shown promise in many flow control applications. The growing interest in plasma actuators for flow control stems from the simplicity of their design, ease of construction and integration with controllers.

It is known that plasma actuators can be arranged at the wall of a boundary layer to generate streamwise vorticity, although they have only been used previously for inducing transition or maintaining attached flow. In this work, the response of a Blasius boundary layer to spanwise forcing by an array of plasma actuators for the purpose of controlling the transient growth instability inherent to bypass transition was studied for the first time. The plasma actuators were used to negate the streamwise streaks of alternating low- and high-velocity inherent to bypass transition by introducing comparable amplitude streaks of opposite phase. The significance of this work extends to the more complex case of wall-bounded turbulence control, where streaks occurring in the buffer layer are ubiquitous with the self-sustaining mechanism of turbulent kinetic energy production.

The plasma actuator consisted of an array of streamwise oriented high-voltage electrodes, which were distributed evenly over a spanwise region of the boundary layer sought for control, above a dielectric surface, and over the grounded electrode. This arrangement was shown to be capable of introducing streamwise streaks of spanwise periodic low- and
high-velocity in a controlled manner. The energy imparted to the flow was decomposed into spanwise wavelengths using Fourier analysis. The streaks generated by the plasma actuator were comprised of a fundamental spanwise disturbance of wavelength $\Delta z$, corresponding with the spacing of the high-voltage electrodes, as well as higher harmonics of the fundamental wavelength. The excitation of the higher harmonics was dependent on the actuator geometry, excitation voltage, and frequency, which could alter the spatially distributed actuator forcing.

The energy distribution into spanwise wavenumbers varied with the actuator geometry. It was shown that the energy contained at high harmonics of the fundamental mode could vary in both amplitude and phase. The phase of the fundamental mode was inherently fixed by the geometry. The fundamental mode was spatially fixed by the net motion of fluid pulled down toward the high-voltage electrodes and low-speed fluid in the near-wall region, which was ejected from the wall in the region between the electrodes by the opposing wall jets. The amplitude of the first mode increased with the electrode width, suggesting that the spatial forcing could be optimized for mode-1 excitation. The third mode phase also remained fixed. The spanwise wavelength of the third mode was $\Delta z/3 = 6.67$ mm, which was similar to the width of exposed electrodes considered (from 5 - 8 mm). The strength of mode-3 increased with the width, suggesting the mode-3 excitation was related to this parameter. The variation in energy and phase of the first higher harmonic, mode-2, was strongly coupled with the actuator geometry. As the width was increased from 5 - 8 mm, mode-2 decreased to a minimal energy for a width of 7 mm, then increased in energy for an electrode width of 8 mm. However, mode-2 was phase shifted by 180 degrees.

A key result of the effect of excitation characteristics is that the ratio of the energetic modes changed with the total integrated energy, for either the frequency or the voltage. For the single geometry considered, it was shown that for increasing actuator output the resulting disturbance was comprised of a greater ratio of the fundamental mode, which was required for control. In particular, the central double-peaked low-speed streak caused by the presence of higher harmonics further coalesced. A similar observation of the coalescing double-peaked low-speed streak occurred with increased exposed electrode width, suggesting a variation in the spatial distribution of forcing. This is an important result because energy delivered by the actuator to higher modes can enhance existing comparable structures if the corresponding phases are similar. In this case, the overall effectiveness of the actuator will be reduced. Therefore, this inefficient use of power should be minimized.
Control of transient growth in a Blasius boundary layer was demonstrated experimentally for the first time using plasma actuators. An array of cylindrical roughness elements were used to embed streamwise oriented streaks of spanwise periodic regions of low- and high-velocity, which experience transient algebraic growth. The three actuator geometries varying only in the width of the upper exposed electrode were investigated for attenuating these streaks. For each geometry, the plasma actuators were successfully used to attenuate the energy of the target \( \Delta z \) mode of the disturbance produced by the roughness elements. The target \( \Delta z \) mode was reduced by at least 93% for all cases. However, the overall reduction of the disturbance energy was 41, 67 and 70% for the three geometry cases varying only in the width of the exposed electrodes (5, 7, and 8 mm), respectively. The increase in overall effectiveness was attributed to the improved geometry, which suggests that the actuator design can be optimized to minimize the inefficient energy input to higher (not targeted) modes. The attenuation mechanism was also shown to be linear; disturbances and counter disturbances can be linearly superimposed to model the control effect.

### 7.1.2 Closed-Loop Control of the Streamwise Streaks

Closed-loop feedback control of transient growth in a Blasius boundary layer was demonstrated experimentally using plasma actuators for the first time. The initial disturbance was provided by the same method applied for the open-loop case. To close the control-loop, an array of wall-mounted hot-wires distributed evenly along the span over a region \( 2\Delta z \) wide was constructed by the collaborators at MSU. These sensors provide a measure of the spanwise periodic streamwise shear stress variation caused by the flow disturbance, which was used to estimate the average disturbance energy over the boundary layer thickness. A novel control scheme was based on a spanwise wavenumber specific control objective, which accounted for the amplitude and phase of the target mode. Measurements of the response of the flow to forcing by the plasma actuators were used to build an empirical input/output model of the system. The collaborators on this project, Mr. Brandt Belson, supervised by Prof. Rowley, at Princeton University constructed and tuned a PI-controller based on the measured input/output data gathered by the author.

For steady conditions, the feedback controller provided an effective method to attenuate streamwise velocity streaks without user intervention. It was shown that for on-model conditions the feedback controller could reduce the total energy associated with the targeted mode by 94% at the streamwise location of the feedback sensors. The attenuation of streamwise streaks by the actuators, which was based on a measurements from a single
downstream location, demonstrated effectiveness both upstream and downstream of the feedback sensors.

The controller was capable of responding to discrete changes in flow conditions. Increased free-stream velocity, which was not considered in the controller development, caused the apparent gain of the controller to decrease. This was caused by the inverse sensitivity of the flow response to constant plasma actuator forcing with flow velocity. As the flow velocity was decreased the strength of the disturbance caused by the actuator increased due to the increased residence time of the fluid over the actuator. For quasi-steady conditions the controller maintained a high level of disturbance attenuation (over 90% of the target disturbance), which was diminished with the decreasing controller update rate. The results were limited to slow time-varying disturbances to function within the hardware limitations of the control system.

The results of this work provide the first practical demonstration of closed-loop control of the bypass transition instability using plasma actuators with wall-located sensing. Key issues relating to the implementation of wall-located sensing, plasma actuators, and overall effectiveness of the control objective were addressed.

7.1.3 Assessment of Dynamic Actuation

Phased averaged measurements of the disturbance velocity were used to reconstruct the time-varying dynamic response of the flow to pulsed plasma actuation. Owing to the higher velocity in the outer region of the boundary layer, the disturbance first entered the measurement plane in the upper portion of the boundary layer, and filled-in toward the wall over time. Practical requirements dictate that feasible sensors will be located at, or near, the wall. From the temporal growth and decay of the boundary layer disturbance caused by pulsed excitation of the plasma actuator, it is clear the top-down propagation of disturbance energy will have a delayed detection at the wall. This has significant implications to the real-time control of bypass transition. For instance, the effect of the actuator would not be instantly detected at the wall, although the disturbance would be present at the control plane, albeit higher in the boundary layer. The average time delay is also related to the downstream location of the feedback sensors and the free-stream velocity of the zero-pressure gradient boundary layer.

It was shown that the dynamics of the mode-1 disturbance in the near-wall region are more complex than a simple first-order response, although this could describe the time averaged boundary layer response. In the near-wall location of the feedback control sensors the mode-1 disturbance exhibited an initial inverse response, which was associated
with the start-up and termination of actuation. The inverse response was modelled by including a right-hand plane zero in the transfer function describing the flow response to forcing. Given that the presence of zeros places limitations on the achievable system performance, possible measures to avoid this behaviour should be considered.

Measurements of the power consumed by the actuator for a 0.1 s pulse demonstrated a transient behaviour. The effect of the transient in power consumption during the start-up of the actuator, over the width of the pulse and for the end of actuation was observable in the dynamic response of the boundary layer. For streamwise distributed control and sensing, accounting for these transients may not be feasible from feedback alone. Therefore, the necessity for a dynamic model of the actuator behaviour is critical for the real-time control application.

### 7.2 Recommendations

The design of a practical boundary layer flow control system dictates the use of wall-located sensing and actuation, as well as a suitable controller. Therefore, three inter-related problems exist. Disturbance sensing is of fundamental importance to monitor the flow for feedback control, whereas the effectiveness of the actuator on the target disturbance is a critical enabling component in any active flow control system. From the controller perspective, the real-time constraints necessitate accurate low-order estimates of the flow based only on minimal and discrete flow measurements. Leveraging the capability of numerical simulations and experimental flow measurements is of paramount importance to tackle the problem of bypass transition and turbulence control.

The open-loop control demonstration showed the significance of undesired energy caused by the plasma actuators at modes other than that of the target wavenumber. The long term goal is to design actuators with high performance and minimal power consumption. A parametric study aimed at the optimization of plasma actuators for the boundary layer control is recommended. At an intermediate stage, this recommendation led to a recent MASc project in the FCET lab by Osmokrovic [117]. He expanded the geometric and excitation study of the response of the boundary layer to forcing by PSVGs. However, the optimization of streak generation for the lowest possible power consumption remains an ongoing question.

Optimization of plasma actuators for improved control authority is arguably best suited for numerical studies to efficiently explore the wide range of geometric, electrical and physical variables. The major limitation is that models of the plasma actuator body force are calibrated typically for a particular case (geometry/operating conditions) and
fail with changes in the aforementioned variables. Therefore, a critical missing technology is a universal plasma actuator modelling capability to support the aforementioned goal.

A key result of this work was the observation of a non-minimal phase response of the flow response to the impulsively started actuator. Given that the zeros of the system may be modified or removed by a change in sensing strategy or actuation, this topic should be further expanded to reduce the impact on control performance. In particular, only a single case of the impulsively started actuator was considered in this work. The author recommends that further study should consider the effect of the actuator impulse. For instance, a more gradual ramping of the actuator output may alleviate the problem of the nonminimal phase response. Furthermore, given that only the poles of a system are an intrinsic property and that the zeros depend on the coupling of the sensors to the state of the system, it may be possible to change or possibly eliminate this nonminimum phase response by sensing a different quantity. For example, a control objective based on the spanwise shear stress may prove to be more appropriate.

Owing to limited accuracy of numerical plasma actuator models, measurements of both the streamwise and wall-normal velocity components of the disturbance caused by the PSVG would enable accurate prediction of the actuator body force for flow control simulation. The significance of this is toward the development of controllers suitable for the real-time problem. Specifically, numerical simulations can be used to provide guidelines for the optimal placement of sensors and actuators, as well as for the development of low-order models necessary for real-time control. Numerical simulations can be used to establish benchmark control data. The added benefit of numerical simulations is also to address the optimal location where actuators and sensors should be placed, for which there is no formal method to predetermine.

The results of this work were limited to a quasi-deterministic setting, where disturbances were embedded at specific locations of the flow using roughness elements. Regardless of the forcing mechanism, bypass transition is characterized by the growth of streamwise streaks. Given that the actuators implemented in this work showed a strong control authority over steady and quasi-steady streaks, the next logical extension of this work would be the attenuation of dynamically forced streamwise streaks for a quasi-deterministic system, such as that having activated roughness elements, prior to the more complex case with stochastic excitation, such as free-stream turbulence. The stochastic forcing environment would require several spanwise actuator arrays offset in spanwise phase to target disturbances occurring at random spanwise locations.

With respect to the scalability of these results, various operating conditions should be studied. In particular this work was limited to a small range of velocities and single
streamwise location (and therefore similar boundary layer thicknesses). Scaling the flow control system to smaller dimensions suitable for targeting the sub-layer streaks occurring in the turbulent boundary layer is the ultimate test of scalability.

It was discussed in Chapter 2 that the spatial and temporal scales associated with the turbulent boundary layer place severe constraints on the typical length and time scales that an actuator must operate within. For instance, in the example discussed in Section 2.2 it was shown that at high Reynolds numbers, typical of flight conditions of commercial aircraft, the spanwise spacing of streaks is approximately 0.5 mm, and the streamwise length scales are typically an order of magnitude greater in the turbulent boundary layer. MEMS manufacturing techniques can be used to manufacture actuators on these scales [65]. Furthermore the time scales associated with these streaks would be short (≈ 0.25 ms), which requires an actuator with a fast dynamic response. Plasma actuators possess this capability and the application to turbulent boundary layers is a primary interest toward the overarching objective of this thesis work.
Appendices
Appendix A

Leading Edge Design

A.1 Overview

This project manifested as part of the experimental design work for the UTIAS boundary layer plate discussed in Section 3.3. The author collaborated with Mr. Howard Buckley of the Computational Aerodynamics Group led by Prof. David Zingg at UTIAS. Mr. Buckley trained the author in the use of the flow solver contained in OPTIMA2D, their 2D aerodynamic optimization code, as well as AMBER2D, which was a mesh generation program. In the early stages of this collaboration we designed a leading edge with an optimized asymmetry ratio, which minimized the effect of the leading edge. Later, we extended this collaboration to work on a more general geometric definition of the leading edge (see Hanson et al. [59]). Manual optimization was performed by the author and Mr. Buckley performed the aerodynamic optimization of the more general geometry.

A.2 Introduction

The study of laminar and transitional boundary layers in an experimental framework requires particular attention in the design of the apparatus in order to avoid undesired disturbances. A review of the different elements of concern is given by Saric [138]. When using a flat-plate to study boundary layers a particularly sensitive region is the leading edge, where disturbance first enters the boundary layer. The process where disturbances enter the boundary layer is referred to as receptivity and provides the initial conditions for the transition process [111].

The typical leading edge used on boundary layer plates for the experimental study of laminar boundary layer is elliptical, with sufficient aspect ratio to maintain attached flow
Although a standard elliptical leading edge geometry has a continuous first derivative with respect to the streamwise coordinate (or slope) throughout, a discontinuity in the curvature exists at the juncture between the leading edge and flat section of the plate. The nose of the leading edge, as well as sites of curvature discontinuity provide regions of high receptivity [45; 46]. The standard ellipse therefore has two sites of high receptivity; one at the nose and the other at the leading-edge/flat-plate juncture [99; 139; 156]. The presence of multiple regions of high-receptivity, was not realized in many early receptivity experiments, which made it difficult to interpret the results [53]. Lin et al. [99] proposed a novel leading edge geometry, termed a modified super ellipse (MSE), which eliminates the leading-edge/flat-plate juncture curvature discontinuity. The MSE geometry is described by the following equation,

\[ 1 - x/(AR \times b_h)]^{m(x)} + [y/b_h]^n = 1, \quad 0 < x/b_h < AR, \]  

where \( AR \) is the aspect ratio, \( b_h \) is the half thickness of the ellipse, \( x \) is in the direction of the major axis, and \( y \) is in the direction of the minor axis. The exponents, \( m(x) \) and \( n \) are given by

\[ m(x) = 2 + [x/(AR \times b_h)]^2 \]  

and \( n = 2 \).

In addition to receptivity sites, the pressure gradient caused by the leading edge geometry plays a major historical role in discrepancy between theoretical and measured boundary layer stability calculations [83]. For instance, the presence of adverse pressure gradients destabilizes the boundary layer, resulting in increased disturbance growth rates [163]. Klingmann et al. [83] designed an asymmetrical leading edge and demonstrated that even small deviations from the Blasius profile, originating from non-zero leading edge pressure gradients, contaminates experimental results. Other researchers have utilized asymmetric leading edge geometries to reduce the region of pressure gradient influence. For instance, Li and Gaster [96] used an asymmetric ellipse with a 1:2 thickness ratio on the working side to the non-working side.

In the recent work by Fransson [37], a parametric optimization of the leading edge geometry was performed to minimize the pressure gradient using numerical simulations. It was shown that an asymmetrical leading edge can drastically reduce the non-zero pressure gradient region compared to a Rankine half-body. In this work, only the leading edge was simulated, and the effect of a trailing edge flap, which is used to control the circulation around the plate and thus the position of the stagnation point, was accounted for by varying the mass flow rates above and below the plate. The results were validated with experimental measurements of the velocity over the leading edge, which were con-
Appendix A. Leading Edge Design

(90x744)verted to pressure coefficients using Bernoulli’s equation. In the aforementioned work, two cubic Bezier-curves defined the upper and lower surfaces of the leading edge. For this particular leading edge, a discontinuity in the curvature exists at the junction between the leading edge and flat plate, which can cause higher receptivity at this point.

The goal of this work is to investigate a leading edge with minimal influence in experimental studies of laminar and transitional boundary layers using an aerodynamic optimization framework. The work in this paper can be divided into two main parts. In the first part, the significance of leading edge asymmetry is studied. The resulting pressure gradient region is examined at various levels of asymmetry, while maintaining a constant leading edge length. The effect of asymmetry is quantified using an objective function, which is a measure of the non-zero pressure region over the leading edge. In the next part, the geometry of the leading edge is parameterised to provide a uniquely flexible shaping using a continuous B-spline curve, with continuous curvature at the junction with the flat plate. A novel asymmetric leading edge geometry is thus obtained. It is shown to have much shorter region of non-zero pressure gradient, and eliminated regions of adverse pressure gradient, compared to leading edge geometries defined by ellipses. The predictions from the numerical simulations are validated experimentally on one of the leading edges resulting from this work.

The formulation of the leading edge optimization problem is discussed in Section A.3. In Section A.4 details of the flow simulations including the flow solver and computational grid are discussed. Optimization of the asymmetric ellipse is discussed in Section A.5 followed by experimental validation in Section A.6. In Section A.7, the results of the aerodynamic shape optimization are discussed, followed by concluding remarks in Section A.8.

A.3 Leading-Edge Shape Optimization Problem

A.3.1 Plate Geometry

The boundary layer plate geometry consists of three distinct regions: the leading edge, flat-plate section, and the trailing-edge flap as shown in Figure A.1. The chord length, $c$, of the plate is the distance from the nose of the leading edge to the end of the trailing edge flap, and is set to unity. The flap length was 15% of the chord. The control case, for which all modifications are compared against, is a MSE leading edge geometry, see (A.1) and (A.2), with $AR = 20$, a length corresponding with 10% of the chord and therefore total plate thickness of 1%c. This aspect ratio is of particular interest because it has been
used extensively in the literature, [99; 139; 156]. Using the notation shown in Figure A.1, for a symmetric leading edge, the minor axis length of the upper quarter ellipse, $b_u$, divided by the plate thickness, $2b_h$, is $b_u / 2b_h = 1/2$.

![Figure A.1: Schematic of the flat-plate and coordinate system.](image)

**A.3.2 Problem Formulation**

In general, an aerodynamic shape optimization problem consists of determining values of design variables $X$ such that the objective function $J$ is minimized. The optimization problem can be stated as

$$
\min_X J(Q, X),
$$

subject to the constraint equations $H_j$,

$$
H_j(Q, X) \leq 0 \quad j = 1, \ldots, N_c.
$$

In this case, $Q$ is the vector of conservative flow variables evaluated at the nodes of a computational grid used for simulating the flow characteristics around the boundary-layer plate. $N_c$ is the number of constraints imposed on the optimization problem.

An objective function is formulated to quantify the region of pressure recovery downstream of the leading edge on the side of the boundary-layer plate where experimental measurements will be taken. The objective function is defined as the area bounded by the $C_p$ distribution curve on the measurement side of the boundary-layer plate and a horizontal line passing through the value of the pressure coefficient at mid-chord, $C_{p,\text{mid}}$. This area is represented by the integral, \textit{viz}.

$$
\int_{0.0}^{0.5} C_p \, dS \approx \sum_{j = j_0}^{j_{\text{mid}}} \left( \frac{\tilde{C}_p - C_{p,\text{mid}}}{S} \right)_j.
$$

The integral is approximated as shown above where $\tilde{C}_p$ is the average value of the pressure coefficient between adjacent nodes on the boundary layer plate, $j$ is the index of nodes
along the measurement side of the plate from the leading-edge to mid-chord, and $S_j$ is the distance between adjacent nodes. The objective function to be minimized is stated as

$$J = \sum_{j=1}^{j_{\text{mid}}} \left( |\bar{C}_p - C_{p,\text{mid}}|S + \beta_{\text{adv}} \right),$$

where $\beta_{\text{adv}}$ is a penalty term added to the objective function to discourage leading-edge designs that produce adverse pressure gradients and $\omega$ is a penalty weighting factor, viz.

$$(\beta_{\text{adv}})_j = \max \left\{ 0, \omega \left[ (C_p)_j - (C_p)_{j-1} \right] \right\}.$$  

To prevent flow separation on the measurement side of the boundary-layer plate, a constraint is imposed that requires the stagnation point to coincide with the leading-edge. Given that the largest value of pressure on the boundary-layer plate is observed at the stagnation point, the constraint is expressed as

$$(C_p)_{\text{max}} - (C_p)_{\text{le}} = 0.$$  

### A.4 Flow Simulation

Optima2D is an algorithm for aerodynamic shape optimization developed by Nemec and Zingg [114]. The flow solver contained within Optima2D is used for all flow simulations presented in this work. It is a two-dimensional turbulent flow solver that solves the compressible Reynolds-averaged Navier-Stokes equations with a Newton-Krylov method. The linear system arising at each Newton iteration is solved using the generalized minimal residual method (GMRES) preconditioned with an incomplete lower-upper (ILU) factorization with limited fill. Spatial derivatives in the Navier-Stokes equations are discretized using second-order centered finite differences with added scalar numerical dissipation. Eddy viscosity is computed using the one-equation Spalart-Allmaras turbulence model [148].

The discretized Navier-Stokes equations are solved on a computational grid containing 48985 nodes. The grid is a C-topology with 505 nodes in the streamwise direction and 97 nodes in the normal direction; the off-wall spacing is $2 \times 10^{-6}$ chord. The distances to the far-field boundaries in the streamwise and normal directions are 18 and 10 chord lengths respectively. A representative portion of the mesh is shown in Figure A.2. Given studies of the flow solver accuracy performed by Zingg et al. [164; 165], the grid used can be expected to predict lift coefficients accurate to within 1% and drag coefficients to
within 5% for attached and mildly separated flows, including both numerical and physical model error. A solution obtained on a fine grid with 769x137 nodes compared to one obtained on the grid described above shows negligible difference in the pressure distribution. Therefore it can be assumed that the grid used is sufficiently accurate for the leading-edge design problem. All grids were generated with the multi-block grid-generation tool AMBER2d [113]. Each flow simulation was performed at a Reynolds number based on chord length of $1 \times 10^6$ and with a Mach number of 0.2, inside the incompressible limit, which is similar to the conditions of the experimental case. Additional details specific to the two optimization problems are discussed separately in the following sections.

![Figure A.2: Far and close view of the 505 $\times$ 97-node mesh surrounding the optimized non-elliptical leading-edge geometry.](image)

Turbulent flow is assumed in all flow simulations presented in this work. This is because Optima2D does not have the capability to account for the sensitivity of the pressure distribution with respect to changes in locations of turbulent transition points used by the turbulence model. To properly utilize turbulent transition points in an optimization context, their locations must change corresponding to plate geometry changes during the course of the optimization. It was decided to assume turbulent flow instead of choosing arbitrary fixed locations for turbulent transition points. A study was performed to quantify the effect of turbulence introduced by the turbulence model on the pressure distribution. The study showed that over the region of interest the trend of the pressure gradient was the same and $C_p$ values differed by less than 0.04 when comparing solutions with and without turbulent flow over the leading edge.
A.5 Optimization Results for an Asymmetrical MSE Leading Edge

For a constant length leading edge defined using the well-known MSE geometry, varying the level of asymmetry will cause a coupled variation in the aspect ratio and minor axis length of both the upper and lower ellipses according to (A.2). The effect of independently varying each of these parameters on the resulting pressure distribution is shown relative to the reference symmetric leading edge with AR = 20 with length $a$, defined in Section A.3. As shown in Figure A.3, for the symmetric MSEs with aspect ratios of 20 and 30, increasing only the AR will reduce the suction peak amplitude, while lengthening the total region of non-zero pressure gradient due to the increase leading edge length. A similar result has been shown previously [99; 156]. By reducing the plate thickness, such that the length of the leading edge with AR = 30 coincides with that of the reference case, the region of non-zero pressure gradient migrated upstream as shown in Figure A.3. For a finite thickness plate, with a fixed leading edge length, reduction in suction pressure, and migration of the non-zero pressure gradient region upstream can be achieved by varying the ARs of the upper and lower ellipses, thus defining an asymmetric leading edge.

![Figure A.3: Pressure distribution over the symmetrical leading edge with AR = 20, –, and AR = 30, - - -, for a plate thickness of 1%c, and for AR = 30 with a plate thickness of 0.67%c, – – –.](image)

A systematic study of the leading edge pressure distributions for increasing levels of asymmetry was conducted for a constant leading edge length. The parameter, $b_u$, was reduced for each iteration while the length of the leading edge, $a$, and plate thickness, $2b_h$, remained constant. Therefore, the minor-axis of the measurement side ellipse is reduced while the aspect ratio increases. The parameter, $b_u/2b_h$, is used to define the degree of asymmetry of the leading edge, where a value of 1/2 represents a symmetric leading edge and smaller values represent asymmetrical modifications. The details of the geometries considered are given in Table A.1 and plotted in Figure A.4.
Appendix A. Leading Edge Design

Figure A.4: Schematic of the leading edge geometries of the test cases considered in this study. Geometric details are given in Table A.1.

Table A.1: Geometric parameters for the symmetric and asymmetric leading edges.

<table>
<thead>
<tr>
<th>Name</th>
<th>MSEu AR</th>
<th>MSEl AR</th>
<th>( b_u/2b_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>20</td>
<td>20</td>
<td>1/2</td>
</tr>
<tr>
<td>A1</td>
<td>24</td>
<td>17.14</td>
<td>5/12</td>
</tr>
<tr>
<td>A2</td>
<td>30</td>
<td>15</td>
<td>1/3</td>
</tr>
<tr>
<td>A3</td>
<td>34.3</td>
<td>14.1</td>
<td>7/24</td>
</tr>
<tr>
<td>A4</td>
<td>40</td>
<td>13.3</td>
<td>1/4</td>
</tr>
<tr>
<td>A5</td>
<td>50</td>
<td>12.5</td>
<td>1/5</td>
</tr>
<tr>
<td>A6</td>
<td>60</td>
<td>12</td>
<td>1/6</td>
</tr>
</tbody>
</table>

A symmetrical leading edge geometry with a zero flap angle, in the absence of any additional flow blockages will have a stagnation point located at the nose of the ellipse. For each manual geometry iteration, the location of the nose of the leading edge moves toward the measurement side of the boundary layer plate sought for optimization. For the same flap angle, this geometrical modification causes the stagnation point to migrate to the lower side of the leading edge, often causing flow separation. To facilitate the comparison of the various leading edge geometries given in Table A.1, the flap angle is increased with increasing level of asymmetry to maintain the stagnation point at the tip of the leading edge, and to prevent a separation bubble from forming on the upper (measurement) side. Adjustment of the trailing edge flap is also of practical importance. This is required in physical experiments to control the position of the stagnation point.

The effect of the flap angle on the location of the stagnation point is illustrated in Figure A.5 for leading edge case A2. The corresponding pressure distributions over the measurement side are shown versus chord length in Figure A.6 for each of the four flap angles. It is evident from Figure A.5a that for the 0 degree flap angle, the stagnation point is located on the lower surface, but can be moved to the upper surface by increasing the flap angle (see Figure A.5b-d). A result of increasing the flap angle is a net negative
lift on the plate, owing to the effective negative angle of attack, which therefore causes the mid-chord pressure to increase with flap angle. Additionally, by increasing the flap angle, the difference between the mid-chord pressure and the peak suction pressure is reduced (see Figure A.6). A similar result was also observed by Fransson [37].

![Figure A.5: Pressure contours around the asymmetric leading edge geometry, case A2, for increasing flap angle from (a) 0 degrees, to (d) 2.55 degrees, showing the effect of increasing the flap angle on the stagnation point location.](image)

![Figure A.6: Variation of the $C_p$-distribution on the measurement surface of the plate, for the case A2 leading edge, with increasing flap angle.](image)
Appendix A. Leading Edge Design

The $C_p$-distributions over the leading edge for the symmetrical MSE, and the six asymmetrical cases are shown in Figure A.7a. Since the flap angle was increased for cases A1 to A6, which causes the plate to generate more negative lift, the offset pressure at the mid-chord is subtracted from the $C_p$ data, for comparison purposes. For smaller values of $b_u/2b_h$, the magnitude of the suction peak was decreased, as shown in Figure A.7a. The value of the objective function (A.5) for each of the leading edge geometries is plotted in Figure A.7b. The lowest value of the objective function is obtained when a balance between the magnitude of the suction peak, which decreases with $b_u/2b_h$, and the area under the $C_p$ curve, which becomes more important at low values $b_u/2b_h$, is reached. As $b_u/2b_h$ goes to zero, the resulting geometry would resemble a sharp leading edge. For this case, the stagnation point must be located on the measurement side of the plate, displaced slightly downstream of the tip of the leading edge to prevent flow separation. Schubauer and Skramstad [142] reported the presence of a suction peak extending up to 200 mm from the tip of the sharp leading edge on a plate only 6.35 mm thick, which was suspected by Klingmann et al. [83] to have degraded the corresponding stability measurements. Hence, the benefits of an asymmetrical leading edge cannot change monotonically up to $b_u/2b_h = 0$. The objective function is minimal, with $J = 0.0144$, for the leading edge defined by case A3, with an upper MSE with AR = 34, corresponding to $b_u/2b_h = 7/24$, as shown in Figure A.7(b).

![Figure A.7: (a) Pressure distribution on the top surface of the leading edge (see Table A.1 for geometry) and (b) the value of the objective function, $J$, for the initial symmetric and six asymmetric cases.](image)

Asymmetrical elliptical leading edges have been used for boundary layer studies in the literature. For example, the asymmetric leading edge used by Kachanov et al. [75] had $b_u/2b_h = 1/5$, Fransson [37] had $b_u/2b_h = 1/6$, which has a similar ratio as that of Klingmann et al. [83]. In the two latter examples, the ratio is only illustrative, since
the geometries were not necessarily elliptical. The leading edge used by Li and Gaster [96] had $b_u/2b_h = 1/3$, which is similar to the one found in the present work. The present results provide strong quantitative evidence demonstrating the benefits of an asymmetrical geometry for reducing the extent and potential destabilizing effect of the leading edge pressure recovery region. In particular, there exists an optimal asymmetry ratio which minimizes the pressure recovery region using ellipses.

### A.6 Experimental Validation

Measurements were made in the closed-loop wind tunnel at the University of Toronto, Institute for Aerospace Studies, described in Section 3.3. The free stream velocity was $U_\infty = 5$ m/s. The Reynolds number based on chord length for this setup and flow condition is $8.3 \times 10^5$. Flow velocity measurements were performed using an Auspex, single hot-wire boundary layer probe. Further detail of the experimental measurements is given in Section 3.3.1. The pressure coefficient ($C_p$) was calculated using a form of Bernoulli’s equation, viz.

$$C_p = 1 - \left( \frac{\langle U \rangle_t}{U_\infty} \right)^2,$$

(A.7)

where $\langle U \rangle_t$ is the local time-averaged velocity measured at the edge of the boundary layer, and $U_\infty$ is the free-stream velocity. An automated program was used to position the hot-wire probe at each measurement location. Several measurements were taken at each downstream location to determined the edge of the boundary layer with negligible velocity change. The wind tunnel measurements of the $C_p$-distribution were in good agreement with the results from the numerical flow simulations, as shown by Figure A.8. The computational result, shown in Figure A.8, was recalculated from the flow simulation data using (A.7).

### A.7 Aerodynamic Leading Edge Optimization

Asymmetric modification to the leading edge geometry defined using MSEs were made to minimize the region of non-zero pressure gradient by effectively increasing the aspect ratio of the measurement side ellipse for a constant leading edge length. Assuming the functional geometric description using MSEs, however, limits the potential for higher level of optimization. In this section, the geometry is parameterised to provide a uniquely flexible shaping of the leading edge. Owing to the larger design space, however, manual optimization would be an unrealistic task, and therefore, the gradient-based aerodynamic shape
optimization program, Optima2D, is utilized. Two additional geometric constraints on
the leading-edge geometry are imposed; the leading-edge thickness must be at least 0.1%
chord, and curvature at all points on the leading-edge must be positive. The curvature
constraint is needed because the objective function is not sensitive to the shape of the
bottom part of the leading edge. The combination of these constraints ensure that the
optimal leading-edge design is physically feasible, and supports the manufacture using
typical machining operations.

The boundary layer plate geometry is parameterized using 45 B-spline control points.
The y-coordinates of 25 control points are designated as design variables \( X \). Eleven de-
sign points are located around the leading edge and 14 around the trailing edge. The
14 trailing-edge design variables are linearly coupled such that they act effectively as
one design variable which controls the flap angle. The vertical coordinates of the de-
sign variable control points are considered design variables, thus allowing alterations to
the baseline shape. Figure A.9 illustrates the distribution of control points and design
variables around the B-spline curve representing the boundary layer plate geometry.

The compressible Reynolds-averaged Navier-Stokes equations were solved at each de-
sign iteration with the Newton-Krylov method described in Section A.4. Gradients of
objective and constraint functions that are dependent on the flow solution are calculated
using the discrete-adjoint method; the adjoint equation is solved using preconditioned
GMRES. Function and gradient evaluations are passed to the optimization algorithm,
which determines how to modify the design variables in order to solve the optimization
problem. At each design iteration, the grid around the updated airfoil shape is perturbed
using a simple algebraic grid movement technique. The constrained optimization algo-

Figure A.8: Experimental measurements (symbols) with the computational values (solid line)
of the pressure coefficient \( (C_p) \), determined using Bernoulli’s equation (Eq. A.7), for the
asymmetric leading edge case A3.
Each QP subproblem minimizes a quadratic model of a Lagrangian function which is used to represent the objective function subject to linearized constraints. Iterations are performed until a minimum value of the objective function $J$ defined by (A.5) is achieved.

An improved leading-edge design for the boundary layer plate was achieved after 218 design iterations and the value of the objective function was reduced to $J = 0.0032$. First order optimality was reduced by one order of magnitude, which is a measure of convergence for a constrained optimization problem [115]. The resulting geometry is shown in Figure A.10a with the initial and asymmetrical elliptical optimized geometry. Both of the optimized geometries are asymmetrical. For each case, the $C_p$-distributions are shown in Figure A.10b. The area under the $C_p$ curve for the automated optimized geometry is significantly reduced and regions of adverse pressure gradient have been eliminated - most notably near the tip of the leading edge.

For each leading edge design investigated, a continuous curvature was maintained at the juncture of the leading edge and flat section of the plate. For a plane curve, expressed as $y = f(x)$, the curvature ($\kappa$) is given by the following equation,

$$\kappa = \frac{|y''|}{(1 + y'^2)^{3/2}}. \quad (A.8)$$

The symbol $'$ represents differentiation with respect to the streamwise coordinate $x$. The wall normal coordinate, $y$, and streamwise coordinate, $x$, are normalized by the leading edge length, $a$. The curvature distributions for baseline symmetric leading edges with AR 20 (both a standard and modified-super ellipse) are shown in Figure A.11 with the asymmetric and aerodynamic optimized cases. The modified super ellipse is known to eliminate the discontinuous curvature at the juncture, as discussed in Section A.2. As shown in Figure A.11, increasing the aspect ratio results in a smaller variation in
Appendix A. Leading Edge Design

A.8 Conclusions

The overall goal was to obtain a leading edge that minimizes the region of non-zero pressure gradients while avoiding the potential high receptivity site associated with non-continuous curvature at the juncture of the leading edge and flat-section. The flow over curvature just upstream of the juncture [99; 156]. The aerodynamic optimized case had the smallest curvature variation at this location. For both the asymmetric and aerodynamic optimized case, higher curvature exists further upstream toward the nose of the leading edge. Although this would lead to a higher receptivity coefficient at the nose of the leading edge, the influence of a reduced adverse pressure gradient is known to lead to a lower overall receptivity (see for example [156]).
the complete boundary layer model, including leading edge, flat-section, and the trailing-edge flap was modelled using a two-dimensional CFD code developed at the University of Toronto Institute for Aerospace Studies.

Starting from a leading edge geometry defined by the symmetric MSE with AR = 20, the importance of asymmetrical modifications was studied for a constant leading edge length. It was shown that increasing the degree of asymmetry reduces the magnitude of the suction peak, which is similar to reducing the thickness of the flat plate (while increasing the AR to maintain a constant leading edge length). By means of an objective function, $J$, for which the goal was to minimize the pressure recovery region, while minimizing regions of adverse pressure gradient, a metric to facilitate the comparison of leading edge pressure distributions was established. The objective function is a measure of the integrated pressure distribution over the leading edge, with a penalty added to discourage regions with adverse pressure gradient. The optimized leading edge geometry is one that minimizes the aforementioned objective function. The lowest value of the objective function was obtained when a balance between the magnitude of the suction peak, which decreases with $b_u/2b_h$, and the area under the $C_p$ curve, which becomes more important at low values $b_u/2b_h$, is reached. When $b_u/2b_h$ goes toward zero, the pressure gradient region extends further downstream, and a limit to the level of benefit of increased asymmetry occurred. The objective function was minimized for the leading edge geometry defined by case A3, which corresponded with $b_u/2b_h$ equal to 7/24. Wind tunnel testing was used to validate the quality of the numerical results.

The consequence of restricting the leading edge geometry to only MSEs on the resulting pressure distribution, was investigated using a more general definition of the leading edge geometry. Eleven design variables were used to specify the leading edge geometry with a continuous B-spline, which provides a sufficient amount of geometric flexibility to achieve a greatly improved leading edge geometry. A minimal objective function is found for a novel geometry, which, like the one found using ellipses, was asymmetric, validating the importance of a high-aspect ratio on the measurement side of the plate. The non-elliptical leading edge shape was therefore better suited to minimizing the non-zero pressure gradient region. This novel leading edge provides a great reductionism of the importance of the pressure gradient history for the study of laminar and transitional boundary layer, thus facilitating comparison between experimental results with theoretical or numerical predictions.
Appendix B

Uncertainty Analysis of Measurements

B.1 Overview

The uncertainty of measured flow quantities is discussed in this section. For hot-wire measurements the uncertainty is composed of bias (systematic) errors inherent to the measurements and random errors. The velocity measurements in the boundary layer were affected by low-level free-stream turbulence, minor mean flow variation, and sensor noise. The random velocity error was small. For instance, the measurements outside of the boundary layer for a typical data set indicate that twice the standard deviation ($2\sigma$) of the measured value of $U_x$ was always less than 0.15%. As will be shown in the following sections, the total calibration error was closer to 1%. Calibration error was primarily due to uncertainty in the measured pressure and calculated velocity. However, for similar flows the relative error between measurements was small as discussed for the measurement of the free-stream velocity. The justification of this follows from the typical calibration errors, which were comprised primarily of drift and reference velocity error. Drift can be minimized by linear corrections, whereas the reference velocity error is typically caused by non-linearity and hysteresis inherent to the sensor. These systematic errors, which cause the majority of the absolute error, are negligible for comparable measurements.
B.2 Hot-wire Measurements

How-wires are a type of thermal sensor used in this thesis research. The hot-wires were operated by Constant Temperature Anemometer(s) (CTA). A review of CTA circuits and their characteristics is given by Bruun [15]. The operational physics of the hot-wire are modelled by the calibration coefficients of King’s Law so that only the velocity magnitude and probe signal remain as variables, \( \text{viz.} \)

\[
E^2 = A + BQ^n, \tag{B.1}
\]

where \( E \) is the output voltage of the CTA, \( Q \) is the velocity, \( A \) and \( B \) are calibration coefficients, and \( n \) is a calibration constant (typically). The coefficient \( n \) is often quoted as 0.45 [25] however, other authors have shown that varying the coefficient can improve fitting accuracy [16], which also depends on the flow velocity range [52].

The heat transfer rate from a hot-wire is driven by temperature difference. Therefore, the variation in ambient conditions will cause the calibration to drift. A linear correction formula for small differences in ambient temperature (±5 °C) is typically applied [2], \( \text{viz.} \)

\[
E_{\text{cor}} = E_{\text{meas}} \cdot \zeta, \tag{B.2}
\]

where \( \zeta = ((T_w - T_{\text{ref}})/(T_w - T_a))^1/2 \). The wire temperature is \( T_w \), the calibration reference temperature is \( T_{\text{ref}} \) and the current actual measurement temperature is \( T_a \).

B.2.1 Calibration Uncertainty

The calibration uncertainty is a function of the uncertainty of the measured variables and the fit to the modified King’s Law. For instance, the term \( Q \) was determined using a pitot-static tube, which was connected to a pressure sensor. Therefore, the velocity error depended on error of measured pressure, error caused by the use of a pitot-tube, and fluid density error, which is dependent on atmospheric conditions. Calibration also drifts over time due to un-modelled temperature effects on the CTA circuitry. Calibrations were performed immediately before and after each experiment to compensate this effect.

A sample pre- and post-calibration is shown in Figure B.1. The variation of \( E_{\text{cor}} \) with the measured velocity (\( Q \)) using a pitot-static tube is shown in Figure B.1(a). A close-up view is shown in Figure B.1(b). The calibration curves were close to the data points and drift was minor. The deviation of the data points from the calibration curve is shown in Figure B.1(c and d) for the pre- and post-calibration, respectively.
Appendix B. Uncertainty Analysis of Measurements

Figure B.1: Calibration of hot-wire voltage output to the reference velocity (a, b), and the deviation of the data points to the fitted curve.

The total calibration error $\delta_c$ assumes that each error component is independent and has a normal distribution. Three problems will be solved independently; error associated with the data fit $\delta_f$, reference velocity $\delta_r$ determined by the pitot-static tube, and the drift $\delta_d$, viz.

$$\delta_c = \sqrt{\delta_f^2 + \delta_r^2 + \delta_d^2}. \quad (B.3)$$

The data fitting error $\delta_f$ can be expressed as:

$$\left(\frac{\delta_f}{Q}\right)^2 = \sum_{i=1}^{m} \left(\frac{\partial Q_c}{\partial Q_i} \sigma\right)^2, \quad (B.4)$$

where $m$ is the number of calibration points. Variables $Q_i$ and $E_i$ are the measured velocity and voltage at calibration point $i$. The variable $Q_c$ was determined from King’s Law for the measured voltage $E_i$. The standard deviation of the curve fit to the measured data points can be expressed as:

$$\sigma = \sqrt{\frac{1}{m-3} \sum_{i=1}^{m} (Q_i - Q_c(E_i))^2}. \quad (B.5)$$

The sensitivity as a function of velocity was estimated, viz.

$$\frac{\partial Q_c}{\partial Q_i} \approx \frac{Q(E_i + \epsilon) - Q(E_i)}{\epsilon}. \quad (B.6)$$

The term $m - 3$ was included since there are three fitted variables [150], namely $A$, $B$, and $n$. Using data shown in Figure B.1(a) it was found that $\delta_f/Q = \pm 0.3\%$. 
The error associated with the reference velocity measurement was due to uncertainty in the pressure reading, temperature and ambient pressure and characteristics of the pitot-tube construction and yaw angle. For a pitot-static tube, the velocity $Q$ was calculated using Bernoulli’s equation, viz.

$$Q = C \sqrt{\frac{2(p_t - p_s)}{\rho}}, \quad (B.7)$$

where $\Delta p = p_t - p_s$, the difference between the total and static pressure, and $C$ is the correction coefficient of the pitot-static tube, to account for error caused by pressure variation over the static ports, and $\rho$ is the density of the air. For the pitot-tubes used in this work\textsuperscript{1}, $C$ is quoted as $\approx 1$. A review of pitot-static tube measurements is available by Barlow et al. [7]; Tavoularis [149]. Following the calculations of Barlow et al. [7], the error was estimated within $\pm 0.5\%$ based solely on the probe geometry and pressure-port locations. The error in the yaw angle for the present experiments was $\pm 2^\circ$, corresponding to $\pm 0.15\%$ [7]. Assuming these errors were independent, the error caused by the use of a pitot-static tube $\delta_{pst}$ was $\approx \pm 0.52\%$. Uncertainty of the pressure measurement was $\delta_{\Delta p} = \pm 0.12$ of the reading. The density of the fluid will effect the velocity calculation, which was calculated from measurements of atmospheric pressure and temperature using the ideal gas law. The accuracy of a typical T-type thermocouple used in this work to measure the temperature was approximately $\pm 0.5^\circ C$ and the error of the ambient pressure was $0.2\%$. The uncertainty of the reference velocity $\delta_r$ can be derived using Equation B.7, viz.

$$\delta_r = \left[ \left( \frac{\partial Q}{\partial \Delta p} \delta_{\Delta p} \right)^2 + \left( \frac{\partial Q}{\partial \rho} \delta_{\rho} \right)^2 + \left( \frac{\partial Q}{\partial C} \delta_{C} \right)^2 \right]^{1/2}. \quad (B.8)$$

The density error was calculated using the same approach assuming air is an ideal gas. For the calculation of the density error and total reference velocity error $\delta_r$, error sources were assumed independent, which is a reasonable assumption. Substituting error values into the previous equation gives $\delta_r = 0.0324 \text{ m/s}$ at $100 \text{ kPa}$, $298 \text{ K}$ and $\Delta p = 14.6 \text{ Pa}$, which was for a flow at $5 \text{ m/s}$. The value $\delta_r$ changes linearly with velocity, and the velocity error was $0.65\%$ of $Q$.

The drift error ($\delta_d/Q$) was evaluated by considering the variation in velocity using the pre- and post-calibration data. An example is show in Figure B.1(a). Only a few cases were considered however, which reduces the statistical significance of this result.

\textsuperscript{1}Dwyer Series\textsuperscript{®} 160 Stainless Steel Pitot Tube
The velocity error due to drift was estimated over all velocities in the calibration as 0.6% of $Q$. Pre- and post-calibrations were blended in time, assuming a linear drift to reduce error. This was a reasonable assumption since temperature variations over the run time was approximately linear, such that $\delta_d/Q$ is conservative.

The total calibration error following from Equation B.3 was $\delta_c/Q = 0.93\%$, and will be quoted as 1% herein.

B.2.2 Wall-location Uncertainty

The geometric location of the wall was determined by extrapolating from a linear linear-fit ($y = A + Bu$) to 7 - 8 points between 20 and 30% of $U_\infty$ within a boundary layer profile. The estimated $y$-location is where $u = 0$. The uncertainty of the wall location was due to the uncertainty of the coefficient $A$, the zero intercept. It was caused by the uncertainty of the measured velocity and was a function of the number of points used in the linear-fit, as opposed to the relative error in the $\Delta y$, which was negligible. A review of linear-fit error is discussed by Taylor [150], and the method was used in the following calculations.

The uncertainty of the coefficient $A$ is given as $\sigma_y \sqrt{\sum x^2 / \Delta}$. The value of $\Delta = N \sum x^2 - (\sum x)^2$. To compute the wall fit the abscissa was velocity and the ordinate was the wall-normal position, where $y = 0$ at the starting position when $u/U_\infty \approx 0.2$. A straight line fit was considered such that the velocity error can be converted to an equivalent $y$-error, i.e. $\sigma_y = B \sigma_x$, where $B$ is the slope, and $\sigma_x$ is the error in velocity ($\approx 1\%$). This is an interesting result because it shows that the wall-location error was dependent on the slope $dy/du$, and therefore will be greater closer to the leading edge, where the boundary layer was thinner. For a case where $x = 500$ mm and $U_\infty = 5$ m/s, as shown in Figure B.2, it is found that $A = -0.70 \pm 0.02$ mm. The wall location error was a function of the streamwise coordinate $x$, since the slope, $B$, will change.

Figure B.2: Locating the wall from boundary layer profiles.
B.2.3 Displacement Thickness Uncertainty

In this section the uncertainty in the displacement thickness $\delta_1$ is addressed first, followed by the momentum thickness $\delta_2$, where:

$$\delta_1 = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy.$$  \hfill (B.9)

If $f_d = (1 - u/U_\infty)$, we can compute the error as:

$$\delta f_d = \left[ \left( \frac{\delta u}{u} \right)^2 + \left( \frac{\delta U_\infty}{U_\infty} \right)^2 \right]^{1/2} \left( \frac{u}{U_\infty} \right).$$  \hfill (B.10)

The displacement thickness was calculated from the discretized data points of the wall-normal measurement. The accuracy in the wall-normal direction was high, owing to lead screw accuracy of the traverse\footnote{Based on data provided by Velmex Inc.}, better than 0.07% over a range of 254 mm. Therefore, error caused by the relative motion of the traverse was not considered. However, the accuracy of the wall location is lower. As described in Chapter 3, the geometric wall location was determined by extrapolating a linear-curve fit to, typically, 7 - 8 points between 20 and 30% of $U_\infty$ from boundary layer profiles, and led to an uncertainty in the wall-normal position of $\pm 0.02$ mm. The uncertainty due to location of the wall can be considered independent of the uncertainty on the velocity profile and the resulting error in the $\delta_1$ is given as:

$$\delta(\delta_1)^2 = \sum_{i=1}^{N_u} (\delta f_{di} \Delta y_i)^2 + \left[ \left( \frac{\delta (\Delta y_1)}{\Delta y_1} \right)^2 \right] \left( \frac{u \Delta y_1}{U_\infty} \right)^2.$$  \hfill (B.11)

The first term of Equation B.14 is related to the error associated with each velocity measurement, and the second term includes the wall-normal position error. For an example, consider a flow profile at $x = 500$ mm. Assume approximately 1% uncertainty for velocity measurements, $\pm 0.02$ mm wall-normal position error, and 1%/\sqrt(8) = 0.35% error in $U_\infty$, since $U_\infty$ is calculated from 8 measurements. The displacement thickness should read $\delta_1 = 2.1380 \pm 0.095$ mm, or $\pm 4.5\% \delta_1$.

B.2.4 Momentum Thickness Uncertainty

For the momentum thickness error let $f_m = u/U_\infty (1 - u/U_\infty)$. To solve this problem consider two parts, $\delta f_{mu}$, and $\delta f_{muU_\infty}$, viz.
\[ \delta f_{\text{mu}} = \sqrt{u^2 - 4 \frac{u^3}{U_{\infty}^2}} + 4 \frac{u^4}{U_{\infty}^4}, \quad \text{and} \]  
\[ \delta f_{mU_{\infty}} = \sqrt{\frac{1}{U_{\infty}^2} - 4 \frac{u}{U_{\infty}^3}} + 4 \frac{u^2}{U_{\infty}^4}. \]  

The resulting error in \( \delta_2 \) is then given viz

\[ \delta(\delta_2)^2 = \frac{\sum_{i=1}^{N_y}(\delta f_{\text{mu}}i \Delta y_i)^2 + \sum_{i=1}^{N_y}(\delta f_{mU_{\infty}}i \Delta y_i)^2}{\sum_{i=1}^{N_y}(\delta f_{\text{mu}}i \Delta y_i)^2 + \sum_{i=1}^{N_y}(\delta f_{mU_{\infty}}i \Delta y_i)^2}. \]  

For the same conditions stated previously, this led to \( \delta_2 = 0.81 \pm 0.035 \text{ mm} \), or \( \pm 4.3\% \delta_2 \). The small change in relative error compared to the displacement thickness is not surprising, since the higher-order terms will have a small contribution.

### B.2.5 Shape Factor Uncertainty

The shape factor \( H_{12} \) is the ratio of the displacement thickness and momentum thickness, \( \delta_1/\delta_2 \). Therefore, the uncertainty of \( H_{12} \) is given as,

\[ \delta(H_{12}) = \sqrt{\left( \frac{1}{\delta_2} \delta(\delta_1) \right)^2 + \left( \frac{\delta_1}{1} \delta(\delta_2) \right)^2}. \]  

Using the previously calculated values, the error in the shape factor is 0.09. Therefore a correct representation of the shape factor for a Blasius boundary layer should read \( H_{12} = 2.59 \pm 0.09 \).

### B.2.6 Effect of Spanwise Averaging

For the preceding analysis of the uncertainty of \( \delta_1, \delta_2 \) and \( H_{12} \), only a single boundary layer profile was considered. The uncertainty can be reduced by taking several wall-normal velocity profiles. For instance, in the typical experiments where multiple spanwise locations were measured, the random errors, for instance on the wall-location, can be reduced by \( 1/\sqrt{N_z} \), where \( N_z \) is the number of spanwise locations at the same streamwise position. However, the bias errors caused by calibration, such as non-linearity of the measured pressure to a standard will remain. Therefore, spanwise averaging reduces the error but cannot eliminate it.
References


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