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Dynamics of three qubit systems in a noisy environment

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Some properties of three-qubit systems interacting with a noisy environment are discussed. The amount of the survival entanglement is quantified for the GHZ and W states. It is shown that the entanglement decay depends on the noise type (correlated or non-correlated), the number of interacting qubits with the environment and the initial state which passes through this noisy environment. In general, the GHZ state is more fragile than the W state. The phenomena of entanglement sudden death appears in GHZ state only for non-correlated noise.

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1 Introduction

Entanglement for two-qubit systems has been investigated extensively in many directions. Quantifying and manipulating entanglement for systems of small dimensions have been covered a wide area of research [1, 2, 3]. However, there are a lot of applications that have been implemented on quantum information and computations by using these systems theoretically and experientially. Quantum teleportation, coding [4] and cryptography represent the more important applications of entanglement. To implement these applications with high efficiency, maximum entangled states (MES) are needed. Even though it is possible to generate maximum entangled states, they turn into partially entangled states (PES) due to their interaction with their own environments [5, 6, 7]. Therefore, another process is needed to recover the entanglement, called quantum purification, which requires infinite number of pairs to get one MES [8]-[12]. Thus, it is important to investigate the behavior of entanglement in the presence of different types of noises. There are some classes of noise states whose teleportation fidelity can be enhanced if one of the two qubits passes through the noisy amplitude damping channel [13].

Recently, Montealegre et. al. [14] have investigated the effect of different types of noise channels on the one-norm geometric discord. They showed that, the effect of the noise channel can be frozen. Metwally [15] showed that the phenomena of the single and double changes and the sudden death of entanglement are reported for identical and non-identical noises. Also, the behavior of multi-qubit systems in the presence of the amplitude damping channel was investigated in [16], where the possibility of performing quantum teleportation with decohered states was discussed. This motivates us to investigate two classes of three-qubit systems, the GHZ and the W states, where the effect of the correlated and non-correlated noisy depolarized channel is considered.

An application of using the GHZ and W states to perform quantum teleportation in the presence of different noises was discussed by Jung et. al [17], where they showed that the robustness of these two states depends on the type of noisy channel. A similar discussion was introduced by M.Liang Hu [18], where the robustness of GHZ and W state was investigated in terms of their teleportation capacity when only one qubit state was under the influence of different surrounding noisy environment. It has been shown that

this robustness depends on the environment's temperature. The current study is different from the above studies from the following points of view:

1. In our treatment, we investigate the robustness of the channel between the users, while in the previous references, the authors investigated the effect of the noise on the fidelity of the teleported state.
2. In the current manuscript, we assume that the qubits pass through the depolarized channel, while in the previous work, the authors consider the thermal effect (zero temperature environment and finite temperature environment).
3. In our work, we consider the case where one, two or three qubits are subjected to the noise, while in the previous work the authors assumed that only one qubit is subjected to the noise.
4. We assume that the noise may be correlated or non-correlated, where this type of study was not considered in the previous works.

The paper is organized as follows: In Sec. 2, we describe the initial state and their evolution in the presence of the depolarized damping channel. The entanglement behavior of the GHZ state is discussed in Sec.3, while Sec.4 is devoted for the W-state. The fate of the initial states is investigated in Sec.5. Finally we conclude our results in Sec. 6.

2 The system

Assume that a source generates a three-qubit state in the form of GHZ or W state that are defined by

$$|\psi_{ini}\rangle = \begin{cases} |\psi_g\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \\ |\psi_w\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle). \end{cases} \quad (1)$$

During the transmission from the source to the users Alice, Bob, and Charlie; it is assumed that one, two or the three qubits pass through the depolarized channel [5]. This channel transforms a single qubit state into a completely mixed state with probability p and leaves it unchanged with probability $1 - p$. For a single qubit noise, the operators are given by

$$\mathcal{D}_1^{(j)} = \sqrt{1-p} I^{(j)}, \quad \mathcal{D}_2^{(j)} = \sqrt{\frac{p}{3}} \sigma_x^{(j)}, \quad \mathcal{D}_3^{(j)} = \sqrt{\frac{p}{3}} \sigma_y^{(j)}, \quad \mathcal{D}_4^{(j)} = \sqrt{\frac{p}{3}} \sigma_z^{(j)}, \quad (2)$$

where $I^{(j)}, \sigma_i^{(j)}, i = x, y, z, j = a, b$ and c are the identity and Pauli operators for the three qubits respectively. In computational basis, these operators can be written as:

$$\begin{aligned} I^{(j)} &= |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_x^{(j)} = |0\rangle\langle 1| + |1\rangle\langle 0|, \\ \sigma_y^{(j)} &= i(|0\rangle\langle 1| - |1\rangle\langle 0|), \quad \sigma_z^{(j)} = |0\rangle\langle 0| - |1\rangle\langle 1|. \end{aligned} \quad (3)$$

We consider that the suggested noise model is of one, two or three sides noisy channel type. For two and three sides noise, we consider that the noise could be correlated or

non-correlated [20]. By correlated noise we mean that the qubits are affected by the same noise at the same time, while for non-correlated noises, the qubits may be affected by different noises at the same time.

The three-qubit system under the depolarizing noise channel acting on the first qubit of the quantum state ρ_{ini} is given by

$$\rho^{(f_i)} = \sum_{k=1}^4 \left\{ \mathcal{D}_k^a \otimes I^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes I^{(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (4)$$

where $\rho_{ini} = |\psi\rangle_{ini}\langle\psi|$ and $|\psi_{ini}\rangle$ is one of the initial states (1). However, if the first two qubits are affected by the depolarized channel, then the initial state ρ_{ini} evolves as

$$\rho^{(f_{co2})} = \sum_{k=1}^4 \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_k^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes \mathcal{D}_k^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (5)$$

for correlated noise, while for non-correlated noise, the final state can be written as $\rho^{(f)}$

$$\rho^{(f_{nc2})} = \sum_{k=1}^{k=4} \sum_{\ell=1}^{\ell=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_\ell^{(b)} \otimes I^{(c)} \rho_{ini} I^{(c)} \otimes \mathcal{D}_\ell^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}. \quad (6)$$

Finally, if all three qubits are subjected to the depolarizing channel (2), then for the correlated noise, the final state of the initial three qubits (1) evolves as

$$\rho^{(f_{co3})} = \sum_{k=1}^{k=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_k^{(b)} \otimes \mathcal{D}_k^{(c)} \rho_{ini} \mathcal{D}_k^{\dagger(c)} \otimes \mathcal{D}_k^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (7)$$

while for the non-correlated noise it is given by

$$\rho^{(f_{nc3})} = \sum_{k=1}^{k=4} \sum_{\ell=1}^{\ell=4} \sum_{m=1}^{m=4} \left\{ \mathcal{D}_k^{(a)} \otimes \mathcal{D}_\ell^{(b)} \otimes \mathcal{D}_m^{(c)} \rho_{ini} \mathcal{D}_m^{\dagger(c)} \otimes \mathcal{D}_\ell^{\dagger(b)} \otimes \mathcal{D}_k^{\dagger(a)} \right\}, \quad (8)$$

where $\rho^{(f_{cor})}$ and $\rho^{(f_{ncr})}$ when $r = 2, 3$ stands for the final states for correlated and non-correlated noise if 2 or 3 qubits are affected, respectively.

In the following we shall investigate the dynamics of entanglement which is contained in the initial states (1), where we investigate all the possible noises as described above. For this aim, we use the tripartite negativity as a measure of entanglement. This measure states that if ρ_{abc} represents a tripartite state, then the negativity is defined as [21]

$$\mathcal{N}(\rho_{abc}) = (\mathcal{N}_{a-bc} \mathcal{N}_{b-ac} \mathcal{N}_{c-ab})^{\frac{1}{3}}, \quad (9)$$

where $\mathcal{N}_{i-jk} = -2 \sum_{\ell} \lambda_{\ell}(\rho_{ijk}^{T_i})$, λ_{ℓ} are the negative eigenvalues of the partial transpose of the state ρ_{ijk} with respect to the qubit "i"

3 GHZ-state

1. Correlated Noise:

The initial three-qubit state $\rho_g = |\psi_g\rangle\langle\psi_g|$, $|\psi_g\rangle$ is given from (1) under the depolarizing noise on Alice's qubit is given by

$$\rho_g^{(f)} = (1-p)\rho_g + \frac{p}{3} \left(|111\rangle\langle 100| - |111\rangle\langle 011| \right)$$

Figure 1: The entanglement $\mathcal{N}(\rho_g^{(f)})$ of the GHZ state under the depolarized channel. The dash-dot, dot and solid curves represent the behavior of entanglement when one, two or three qubits are depolarized, respectively.

$$+ \frac{p}{6} (|011\rangle\langle 100| + |100\rangle\langle 011| + |100\rangle\langle 100| + |011\rangle\langle 011|). \quad (10)$$

However, if we assume that the first two qubits are subjected to correlated noise, then the final state (5) is given by

$$\rho_g^{(f_{co2})} = \left(\frac{1}{2}(1-p)^2 + \frac{p^2}{18} \right) \rho_g + \frac{p^2}{9} (|001\rangle\langle 110| + |110\rangle\langle 001| + |001\rangle\langle 001| + |110\rangle\langle 110|). \quad (11)$$

Finally, if it is assumed that all three qubits are subjected to a correlated noise, then $\rho^{(f_{co3})}$ can be written as

$$\begin{aligned} \rho_g^{(f_{co3})} = & \left(\frac{(1-p)^3}{2} + \frac{p^2}{18} \right) |000\rangle\langle 000| + \left(\frac{(1-p)^3}{2} - \frac{p^3}{54} \right) |000\rangle\langle 111| \\ & + \frac{(1-p)^3}{2} |111\rangle\langle 000| + \left(\frac{(1-p)^3}{2} + \frac{p^2}{27} \right) |111\rangle\langle 111|. \end{aligned} \quad (12)$$

In Fig.(1), we investigate the behavior of entanglement for the three final states (10 – 12). The general behavior displays that the entanglement decays as p increases. However, these decays depend on the number of the affected qubits by the depolarized channel. For the state $\rho_g^{(f)}$, where only one qubit is depolarized, the entanglement decays smoothly, while for $p = 1$, when the strength of the noise is maximum, the entanglement doesn't vanish. For the state $\rho_g^{(f_{co2})}$, where two qubits are depolarized, the decay rate is larger than that depicted for the state $\rho_g^{(f)}$. Although the entanglement vanishes for some specific value of $p \simeq 0.75$, it re-increases again for farther values of p . Finally, the decay rate of entanglement is the largest if all three qubits are depolarized as shown for the state $\rho_g^{(f_{co3})}$. For this state the entanglement decays gradually until it vanishes completely at $p \simeq 0.78$.

Figure 2: The dash-dot curves represents the entanglement $\mathcal{N}(\rho^{(f)})$, where it is assumed that only two qubits are depolarized, while the dot curve represents $\mathcal{N}(\rho^{(f)})$ when all three-qubits are depolarized.

2. Non-correlated noise:

Now, we assume that two or three qubits are depolarized in a non-correlated way. If only two qubits are depolarized then the final state (6) can be written as

$$\begin{aligned} \rho_g^{(f_{nco2})} = & \mathcal{A}_1(|000\rangle\langle 000| + |111\rangle\langle 111|) + \mathcal{A}_2(|000\rangle\langle 111| + |111\rangle\langle 000|) \\ & + \mathcal{A}_3(|010\rangle\langle 010| + |101\rangle\langle 101| + |011\rangle\langle 011| + |100\rangle\langle 100|) \\ & + \mathcal{A}_4|001\rangle\langle 110| + \mathcal{A}_5|001\rangle\langle 001| + \mathcal{A}_6|110\rangle\langle 110| + \mathcal{A}_7|110\rangle\langle 001| \\ & + \mathcal{A}_8|001\rangle\langle 111| + \mathcal{A}_9(|001\rangle\langle 011| + |110\rangle\langle 100| - |110\rangle\langle 011|), \quad (13) \end{aligned}$$

where

$$\begin{aligned} \mathcal{A}_1 &= \frac{(1-p)^2}{2} + \frac{p}{3}(1-p) + \frac{p^2}{18}, \quad \mathcal{A}_2 = \frac{(1-p)^2}{2} - \frac{p}{3}(1-p) + \frac{p^2}{18}, \\ \mathcal{A}_3 &= \frac{p}{3}(1-p) + \frac{p^2}{9}, \quad \mathcal{A}_4 = \frac{p^2}{9} - \frac{p}{3}(1-p), \quad \mathcal{A}_5 = \frac{p^2}{18}, \quad \mathcal{A}_6 = \frac{p^2}{9} + \frac{p}{6}(1-p), \\ \mathcal{A}_7 &= \frac{p^2}{9} - \frac{p}{6}(1-p), \quad \mathcal{A}_8 = \frac{p}{6}(1-p), \quad \mathcal{A}_9 = \frac{p^2}{18}. \quad (14) \end{aligned}$$

However, if we assume that all three qubits pass through the depolarized channel (2), then the final state (8) (non-correlated noise case) is given by

$$\begin{aligned} \rho_g^{(f_{nco3})} = & \mathcal{B}_1(|000\rangle\langle 000| + |111\rangle\langle 111|) + \mathcal{B}_2(|000\rangle\langle 111| + |111\rangle\langle 000|) \\ & + \mathcal{B}_3(|110\rangle\langle 110| + |001\rangle\langle 001| + |100\rangle\langle 100| + |011\rangle\langle 011|) \\ & + \mathcal{B}_4(|110\rangle\langle 001| + |010\rangle\langle 101| + |101\rangle\langle 010| + |100\rangle\langle 011| + |011\rangle\langle 100|) \\ & + \mathcal{B}_5(|001\rangle\langle 110| + \mathcal{B}_6(|010\rangle\langle 010| + |101\rangle\langle 101|), \quad (15) \end{aligned}$$

where

$$\begin{aligned}
 \mathcal{B}_1 &= \frac{1}{2} \left[(1-p)^3 + p(1-p) + \frac{5}{9}p^2(1-p) + \frac{p^3}{3} \right], \\
 \mathcal{B}_1 &= \frac{1}{2} \left[(1-p)^3 - p(1-p) + \frac{5}{9}p^2(1-p) - \frac{p^3}{27} \right], \\
 \mathcal{B}_3 &= \frac{1}{3} \left[p(1-p)^2 + \frac{7p^2}{3}(1-p) + \frac{p^3}{3} \right], \\
 \mathcal{B}_4 &= \frac{p^2}{9}(1-p), \quad \mathcal{B}_5 = \frac{p}{2}(1-p)^2 + \frac{p^2}{9}(1-p), \\
 \mathcal{B}_6 &= \frac{1}{3} \left[p(1-p)^2 + \frac{5p^2}{3}(1-p) + \frac{p^3}{3} \right].
 \end{aligned} \tag{16}$$

Fig.(2), shows the behavior of entanglement which is contained in the final states $\rho_g^{(f_{nc03})}$ and $\rho_g^{(f_{nc03})}$, where it is assumed that the noise is non-correlated. The general behavior shows that the entanglement decays hastily as the channel parameter p increases. The decay rate of entanglement depends on the numbers of depolarized qubits. However, the decay rate of entanglement when all three qubits are subjected to the noise is larger than that depicted when only two qubits are depolarized. The phenomena of the sudden-death of entanglement appears clearly when the particles are affected by non-correlated noise.

From Fig.(1) and Fig.(2), we can conclude that the decay of entanglement depends on the type of the noise (correlated or non-correlated) and the number of qubits which are subjected to the noise. For correlated noise, the entanglement decays smoothly to reach its minimum bounds. These bounds depend on the number of the depolarized qubits, where they are non-zero for fewer number of depolarized particles and completely vanish as the number of depolarized qubit increases. The phenomenon of the sudden death appears in the presence of non-correlated noise only. As the number of depolarized qubits increases, the entanglement vanishes for smaller values of the channel strength p .

4 W-state

Now we assume that the users share a three-qubits-state of W-type $\rho_w = |\psi_w\rangle\langle\psi_w|$, where $|\psi_w\rangle$ as defined in (1). If we allow only one qubit to pass through the depolarized channel (3), then the final state is given by

$$\begin{aligned}
 \rho_w^{(f)} &= \frac{3-2p}{9} \left\{ |100\rangle\langle 100| + |010\rangle\langle 010| + |001\rangle\langle 010| + |001\rangle\langle 001| + |010\rangle\langle 001| \right\} \\
 &+ \frac{3-4p}{9} \left\{ |100\rangle\langle 010| + |100\rangle\langle 001| + |010\rangle\langle 100| + |001\rangle\langle 100| \right\}.
 \end{aligned} \tag{17}$$

However, if we assume that only two qubits are forced to pass through the noise channel (3) and that the noise is correlated, then the final state is given by

$$\rho_w^{f_{cor2}} = \tilde{\mathcal{A}}_1 \left\{ |010\rangle\langle 010| + |010\rangle\langle 100| + |100\rangle\langle 010| + |100\rangle\langle 100| \right\}$$

Figure 3: The dash-dot, dot and solid curves represent the entanglement of the final $\rho_w^{(f)}$ when one, two or three qubits pass through the depolarized channel(3) in a correlated way, respectively.

$$\begin{aligned}
 & + \tilde{\mathcal{A}}_2 \left\{ |010\rangle\langle 001| + |001\rangle\langle 100| + |001\rangle\langle 010| \right\} \\
 & + \tilde{\mathcal{A}}_3 \left\{ |100\rangle\langle 011| + |001\rangle\langle 001| \right\} + \tilde{\mathcal{A}}_4 |010\rangle\langle 101| + \tilde{\mathcal{A}}_5 |111\rangle\langle 111|, \quad (18)
 \end{aligned}$$

where

$$\begin{aligned}
 \tilde{\mathcal{A}}_1 &= \frac{(1-p)^2}{3} + \frac{P^2}{9}, & \tilde{\mathcal{A}}_2 &= \frac{(1-p)^2}{3} - \frac{P^2}{27} \\
 \tilde{\mathcal{A}}_3 &= \frac{(1-p)^2}{3} + \frac{P^2}{27}, & \tilde{\mathcal{A}}_4 &= \frac{p^2}{27}, & \tilde{\mathcal{A}}_5 &= \frac{2p^2}{27}. \quad (19)
 \end{aligned}$$

However, if all three qubits pass through the non-correlated noise then the final state is given by

$$\begin{aligned}
 \rho_w^{(f_{co3})} &= \tilde{\mathcal{B}}_1 \left\{ |001\rangle\langle 001| + |001\rangle\langle 010| + |001\rangle\langle 100| \right\} + \tilde{\mathcal{B}}_2 |010\rangle\langle 001| + \tilde{\mathcal{B}}_3 |100\rangle\langle 011| \\
 &+ \tilde{\mathcal{B}}_4 \left\{ |100\rangle\langle 100| + |100\rangle\langle 010| + |010\rangle\langle 010| + |010\rangle\langle 100| \right\} \\
 &+ \tilde{\mathcal{B}}_5 \left\{ |010\rangle\langle 101| + |010\rangle\langle 111| + |010\rangle\langle 011| - |100\rangle\langle 111| - |111\rangle\langle 010| \right. \\
 &\quad \left. - |111\rangle\langle 100| - |111\rangle\langle 111| \right\}, \quad (20)
 \end{aligned}$$

where

$$\begin{aligned}
 \tilde{\mathcal{B}}_1 &= \frac{(1-p)^2}{3} + \frac{2p^2}{27}, & \tilde{\mathcal{B}}_2 &= \frac{(1-p)^2}{3} - \frac{p^2}{27}, \\
 \tilde{\mathcal{B}}_3 &= \frac{(1-p)^2}{3} + \frac{p^2}{27}, & \tilde{\mathcal{B}}_4 &= \frac{(1-p)^2}{3} + \frac{2p^2}{9}, & \tilde{\mathcal{B}}_5 &= \frac{p^2}{27}. \quad (21)
 \end{aligned}$$

Fig.(3), describes the behavior of entanglement for the final states (16),(17) and (19), where we assume that one, two or three qubits are subjected to the noise channel (3),

Figure 4: The solid and dash curves represent the entanglement of the final states $\rho_w^{(f_{co2})}$ and $\rho_w^{(f_{co3})}$, respectively.

respectively. The general behavior shows that the entanglement decays as the channel parameter p increases. The decay rate depends on the number of depolarized qubits. It is clear that, if only one qubit is depolarized, then the entanglement decays smoothly to reach its minimum non-zero value. These minimum values decrease as the number of depolarized qubits increases.

Now, we assume that the effect of the noise is non-correlated. If only two qubits are passing through the depolarized channel, then the noise output state is given by

$$\begin{aligned} \rho_w^{f_{nc2}} = & \tilde{\mathcal{A}}_1^{nc} \left\{ |000\rangle\langle 011| + |000\rangle\langle 101| + |011\rangle\langle 00| + |101\rangle\langle 000| \right\} \\ & + \tilde{\mathcal{A}}_2^{nc} \left\{ |010\rangle\langle 010| + |100\rangle\langle 100| + |001\rangle\langle 001| \right\} + \tilde{\mathcal{A}}_3^{nc} |010\rangle\langle 100| \\ & + \tilde{\mathcal{A}}_3^{nc} \left\{ |001\rangle\langle 010| + |001\rangle\langle 100| + |100\rangle\langle 001| + |010\rangle\langle 001| \right\} \\ & + \tilde{\mathcal{A}}_4^{nc} \left\{ |110\rangle\langle 000| + |000\rangle\langle 110| \right\} + \tilde{\mathcal{A}}_5^{nc} |100\rangle\langle 010| + \tilde{\mathcal{A}}_6^{nc} |010\rangle\langle 100| \quad (22) \end{aligned}$$

where

$$\begin{aligned} \tilde{\mathcal{A}}_1^{nc} &= \frac{2p}{27}, \quad \tilde{\mathcal{A}}_2^{nc} = \frac{2}{27}p(1-p) + \frac{p^2}{27} + \frac{1}{3}, \quad \tilde{\mathcal{A}}_3^{nc} = -\frac{2}{27}p(1-p) - \frac{5p^2}{27} + \frac{1}{3}, \\ \tilde{\mathcal{A}}_4^{nc} &= \frac{4p}{27}, \quad \tilde{\mathcal{A}}_5^{nc} = \frac{p^2}{27} + \frac{1}{3}, \quad \tilde{\mathcal{A}}_6^{nc} = -\frac{2}{27}\{3p^2 + p\} + \frac{1}{3}. \quad (23) \end{aligned}$$

Finally, if all three qubits of the W state pass through the noise channel, then the output state can be written as

$$\begin{aligned} \rho_w^{(f_{nc3})} = & \tilde{\mathcal{B}}_1^{nc} \left\{ |110\rangle\langle 000| + |000\rangle\langle 110| + |101\rangle\langle 000| \right\} + \tilde{\mathcal{B}}_2^{nc} \left\{ |011\rangle\langle 000| + |000\rangle\langle 101| \right\} \\ & + \tilde{\mathcal{B}}_3^{nc} |111\rangle\langle 100| + \tilde{\mathcal{B}}_4^{nc} |011\rangle\langle 011| + \tilde{\mathcal{B}}_5^{nc} |101\rangle\langle 011| + \tilde{\mathcal{B}}_6^{nc} |000\rangle\langle 011| + \tilde{\mathcal{B}}_7^{nc} |010\rangle\langle 010| \\ & + \tilde{\mathcal{B}}_8^{nc} \left\{ |111\rangle\langle 111| + |110\rangle\langle 011| + |011\rangle\langle 100| + |011\rangle\langle 111| + |101\rangle\langle 101| \right\} \\ & + \tilde{\mathcal{B}}_9^{nc} \left\{ |001\rangle\langle 010| + |100\rangle\langle 010| + |010\rangle\langle 001| + |010\rangle\langle 100| \right\} \end{aligned}$$

$$\begin{aligned}
& + \tilde{\mathcal{B}}_{10}^{nc} \{ |100\rangle\langle 001| + |001\rangle\langle 100| \} + \tilde{\mathcal{B}}_{11}^{nc} \{ |001\rangle\langle 001| + |100\rangle\langle 100| \} \\
& + \tilde{\mathcal{B}}_{12}^{nc} \{ |011\rangle\langle 010| + |111\rangle\langle 010| + |110\rangle\langle 101| \},
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
\tilde{\mathcal{B}}_1^{nc} &= \frac{4}{32}p(1-p)^2 + \frac{1}{32}p^2(1-p) - \frac{4}{34}p^3, & \tilde{\mathcal{B}}_2^{nc} &= \frac{4}{32}p(1-p)^2 - \frac{4}{34}p^3, \\
\tilde{\mathcal{B}}_3^{nc} &= \frac{1}{32}p^2(1-p) + \frac{1}{34}p^3, & \tilde{\mathcal{B}}_4^{nc} &= \frac{2}{34}p^3, & \tilde{\mathcal{B}}_5^{nc} &= -\frac{1}{33}p^2(1-p) - \frac{2}{34}p^3, \\
\tilde{\mathcal{B}}_6^{nc} &= \frac{4}{32}p(1-p)^2 - \frac{2}{34}p^3, & \tilde{\mathcal{B}}_7^{nc} &= (1-p)^3 + \frac{4}{32}p(1-p)^2 + \frac{1}{32}p^2(1-p) + \frac{1}{34}p^3, \\
\tilde{\mathcal{B}}_8^{nc} &= \frac{1}{33}p^2(1-p), & \tilde{\mathcal{B}}_9^{nc} &= \frac{1}{3}(1-p)^3 - \frac{2}{32}p(1-p)^2 + \frac{1}{32}p^2(1-p) + \frac{5}{34}p^3, \\
\tilde{\mathcal{B}}_{10}^{nc} &= \frac{(1-p)^3}{3} + \frac{p^2}{32}(1-p) + \frac{5p^3}{34}, & \tilde{\mathcal{B}}_{11}^{nc} &= \frac{(1-p)^3}{3} + \frac{2p}{32}(1-p)^2 + \frac{p^2}{32}(1-p) + \frac{p^3}{34}, \\
\tilde{\mathcal{B}}_{12}^{nc} &= -\frac{p^2}{33}(1-p).
\end{aligned} \tag{25}$$

The behavior of entanglement for the output states (21) and (23) is described in Fig.(4). In general, the entanglement decreases as the channel's strength increases. However, the decreasing rate is smaller than that depicted for the correlated noise (see Fig.3). Moreover, when all three qubits pass through the depolarized channel, the entanglement decays faster than that depicted for two depolarized qubits. It is clear that for larger values of the channel's strength p , the entanglement increases. However, the upper bound of entanglement is smaller than that in the case of two-depolarized qubit.

On the other hand, comparing the behavior of this entanglement with that for the GHZ state, we can see that the W-state is more robust than the GHZ state, where for the GHZ state the entanglement completely vanishes in a sudden way for smaller values of the channel strength.

5 Fate of the initial states

It is important to show how the initial states behave as the noise strength increases. It is clear that the initial states may turn into separable states (SS), partially entangled states (PES) or a superposition of SS and PES depending on the number of depolarized qubits. To investigate this matter further, we consider the behavior of the initial states at $p = 1$.

1. The fate of GHZ state

If only one qubit is polarized, then the state at $p = 1$ can be written as

$$\begin{aligned}
\rho_g^f(p=1) &= \frac{1}{6}(|011\rangle + |100\rangle)(\langle 011| + \langle 100|) + \frac{1}{3}|111\rangle(\langle 100| - \langle 011|) \\
&= \text{PES} + \text{SS}.
\end{aligned} \tag{26}$$

It is clear that the final state is a superposition of partially entangled state and a completely separable state ($\mathcal{N} = 0$). Moreover the fidelity of the initial state with respect to the final state is given by $\mathcal{F} = \text{tr}\{\rho_g(p=0)\rho_g^f(p=1)\} = 0$. This shows that the final state is completely different from the initial state.

Figure 5: The population of (a) GHZ and (b) W-states when only two qubits are subjected to correlated noise.

If the two qubits are depolarized, then the final state is given by,

$$\begin{aligned}\rho_g^{f_{co2}}(p=1) &= \frac{1}{18}\rho_g(0) + \frac{1}{3}|\psi\rangle\langle\psi|, \\ &= PES + PES,\end{aligned}\quad (27)$$

where $|\psi\rangle = |001\rangle + |110\rangle$. Therefore, the final state consists of the initial state with probability $\frac{1}{18}$ and a partially entangled state with probability $\frac{1}{3}$. In this case the, fidelity between the initial state and the final state is $\mathcal{F} = \text{tr}\{\rho_g(0)\rho_g^{f_{co2}}\} = 0.05$ which is very small.

Finally if all three qubits are depolarized, then the final state can be written as

$$\begin{aligned}\rho_g^{f_{cor3}}(p=1) &= \frac{1}{18}|000\rangle\left(\langle 000| - \frac{1}{3}\langle 111|\right) + \frac{1}{2}|111\rangle\left(\langle 000| + \frac{1}{54}\langle 111|\right) \\ &= SS + SS.\end{aligned}\quad (28)$$

This means that, the final state is a superposition of two separable states. On the other hand the fidelity of the initial state with respect to the final state is $\mathcal{F} = 0.22$.

2. The fate of the W- state

If one qubit is subjected to the noise, then the final state at $p=1$ can be written as,

$$\begin{aligned}\rho_w^f(p=1) &= \frac{1}{3}|100\rangle\left(\langle 100| - \frac{1}{3}\langle 010|\right) + \frac{1}{3}(|010\rangle + |001\rangle)(\langle 010| - \frac{1}{3}\langle 100|) \\ &\quad + \frac{1}{3}\left(|001\rangle + |010\rangle - \frac{1}{3}|100\rangle\right)\langle 001|, \\ &= SS + PES + SS.\end{aligned}\quad (29)$$

The degree of closeness of the initial state to the final state is given by the fidelity $\mathcal{F} = \text{tr}\{\rho_w(0)\rho_w^f(p=1)\} = 0.41$.

However, when two qubits are depolarized then, the final state can be written as a sum of a partially entangled state and two separable states. Explicitly it can be written as

$$\begin{aligned}\rho_w^{f_{co2}}(p=1) &= \frac{1}{9}(|010\rangle + |100\rangle)(\langle 010| + \langle 100|) + \frac{1}{27}|001\rangle(\langle 100| + 2\langle 010|) \\ &+ \frac{1}{27}\left[(|001\rangle - |010\rangle)(\langle 001| + |100\rangle\langle 011| + 2|111\rangle\langle 111| + |010\rangle\langle 101|)\right] \\ &= \text{PES} + \text{SS} + \text{SS}\end{aligned}\quad (30)$$

In this case the fidelity of the initial state is given by $\mathcal{F} = \text{tr}\{\rho_w(0)\rho_w^{f_{co2}}(p=1)\} = 0.39$.

Finally, if all the three qubits are depolarized then the final state can be written as a sum of four partially entangled states and one separable state. In this case, the fidelity is given by $\mathcal{F} = \text{tr}\{\rho_w(0)\rho_w^{f_{co2}}(p=1)\} = 0.37$. Also, one can obtain a similar expression in the presence of non-correlated noise.

Fig.(5), describes the fate of the initial states when two qubits are depolarized, where we plot the probability of finding the traveling states in the three qubit basis. Fig. (5a) shows the behavior of the GHZ state against the strength of the noise, p . It is clear that at $p=0$ the state can be written as a superposition of $|000\rangle$ and $|111\rangle$ with probability 0.5 and consequently describes the initial state (1). However, as p increases, the probability decreases and reaches its minimum value at $p=1$. On the other hand, another state is generated between the states $|001\rangle$ and $|110\rangle$ with a probability that increases as p increases. At $p \simeq 0.75$, a new state is generated as a superposition of $|000\rangle$, $|111\rangle$, $|001\rangle$ and $|110\rangle$. However, at $p=1$, the total state consists of two partially entangled states.

Fig.(5b) shows the behavior of the population of the W state when two-qubits are polarized. It is clear that, at $p=0$, the state can be written as a superposition of $|100\rangle$, $|010\rangle$ and $|001\rangle$, which is defined by Eq.(1). As p increases, one component is decoupled from the group. This yields a partially entangled state which is a superposition of $|100\rangle$ and $|010\rangle$, in addition to a separable state of the form $|001\rangle\langle 001|$. Moreover, another separable state of the form $|111\rangle\langle 111|$ is generated, whose amplitude increases as p increases. Finally, at $p=1$ the final state is a superposition of a partially entangled state and two separable states.

Finally, to describe the sudden death phenomenon of entanglement, we consider the simple case when two particles are subjected to non-correlated noise. From Eq.(13), it is enough to plot the behavior of the amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$, where it is clear that, when $p=0$, $\mathcal{A}_1 = \mathcal{A}_2 = \frac{1}{2}$ and consequently Eq.(13) is reduced to the initial state (1). However, as p increases, we can write the first and the second terms as a normalized state

$$\psi = \frac{\mathcal{A}_1}{\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}}(|000\rangle\langle 000| + |111\rangle\langle 111|) + \frac{\mathcal{A}_2}{\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}}(|000\rangle\langle 111| + |111\rangle\langle 000|). \quad (31)$$

The behavior of the amplitudes $\frac{\mathcal{A}_1}{\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}}$ and $\frac{\mathcal{A}_2}{\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}}$ are plotted in Fig.(6). It is clear that as soon as p increases, the first amplitude increases while the second amplitude decreases. Therefore, the entangled state turns into a separable state and as

Figure 6: The dash-dot and dash curves are the amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$ (Eq.(13)), respectively. The dot and the solid curves represent the amplitudes $\mathcal{A}_1/\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}$ and $\mathcal{A}_2/\sqrt{\mathcal{A}_1^2 + \mathcal{A}_2^2}$, respectively.

a result the phenomenon of the sudden-death of entanglement appears immediately. Moreover, the remanning components of the state (13) behave as product states.

6 Conclusion

In this contribution, we investigate the behavior of entanglement for two classes of three-qubit states, the GHZ and W states, passing through the depolarized channel. In our treatment, we assume that the effect of the noise channel includes two cases. In the first case we assume that the qubits are subjected to the same noise at the same time (correlated noise). The second case, which is called non-correlated noise, the particles are subjected to different noises at the same time. Analytical expressions are obtained for the final output states. For the correlated noise, we calculate the final output states when one, two or all three qubits are forced to pass through the depolarized channel. However, for the non-correlated noise, we consider the case of two or three qubits passing through the depolarized channel.

The dynamics of the survival amount of entanglement is investigated against the strength of the noisy channel. The general behavior shows that the entanglement decreases as the strength of the channel increases. However, the decreasing rate depends on the number of depolarized qubits, the type of the noise and the type of the initial state. For the *correlated* noise case, the decay rate of entanglement decreases as the number of depolarized qubits increases.

For both states, the entanglement never vanishes when one or two qubits are depolarized. However, when all three qubits are polarized, the entanglement vanishes for GHZ state but doesn't for W state. On the other hand, for the *non-correlated* noise case, the phenomenon of sudden death of entanglement appears when two or all three qubits of GHZ's qubits are depolarized. This phenomenon takes place at larger values of the channel strength when two qubits are depolarized compared to the case of depolarizing all three qubits. For the W state, the phenomenon of the entanglement sudden death

doesn't appear and the lower bounds of entanglement are rather large.

The fate of the initial states as the strength of the noise channel increases is discussed. It is shown that the traveling states can be written as a superposition of separable states and partially entangled states. This structure depends on the number of depolarized qubits.

In conclusion, the robustness or fragility of the initial states which pass through a depolarized channel depends on the type of the noise (correlated or non-correlated). In general the GHZ state is more fragile than the W state. The phenomena of entanglement sudden death appears in one case only, namely when the GHZ state is subjected to non-correlated noise.

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