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Deconfining phase transition in a finite volume with massive particles: finite size and finite mass effects

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Abstract.

We study the deconfining phase transition from a hadronic gas phase consisting of massive pions to a Quark-Gluon Plasma phase containing gluons, massless up and down quarks and massive strange quarks. The two phases are supposed to coexist in a finite volume, and the finite size effects are studied, in the two cases of thermally driven and density driven deconfining phase transitions. Finite-mass effects are also examined, then the color-singletness condition for the QGP is taken into account and finite size effects are investigated in this case also.

Résumé: Nous étudions la transition de phase de déconfinement de la phase gaz hadronique, consistant de pions massifs, à la phase plasma de quarks et de gluons, contenant des gluons, des quarks up et down sans masse et des quarks étranges massifs. Les deux phases sont supposées coexister dans un volume fini, et les effets de cette finitude sont étudiés, dans les deux cas où la transition de phase est conduite par la température ou par le potentiel chimique. Les effets de masse finie sont eux aussi examinés, et la condition de "color-singletness" est enfin considérée pour que les effets de volume fini soient examinés dans ce cas aussi.

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1. INTRODUCTION

It is undoubted that the study of the “Quark-Gluon Plasma” (QGP) has gained considerable advances last decades with RHIC and LHC heavy ion collision experiments. Considerable progress has also been made in the study of the Deconfining Phase Transition (DPT) connecting the confined hadronic phase and the deconfined QGP phase, and lattice QCD simulations continue providing results on the equation of state of QCD, the transition temperature, the order of the DPT as well as the QCD phase diagram [1, 2]. Since simulated systems as well as reaction zones in which the formation of the QGP in relativistic heavy ion collisions is assumed to occur are finite, it is of great importance to take finite size effects in consideration. The subject of finite size effects has been deeply studied (see for instance [3–5]). In previous works, one of us has studied the DPT from a hadronic gas (HG) phase consisting of pions to a color-singlet QGP consisting of gluons and u and d quarks, in a finite volume using massless particles in a first approach, and a finite size scaling analysis revealed that the DPT was of first order [6]. In a second paper [7], an additional finite-size effect due to the finite extension of the hadrons, called excluded volume effects, has been taken into account in the study of the deconfining phase transition. In the present work, we take into account massive pions in the hadronic phase and massive strange quarks additionally to gluons and massless u and d quarks in the QGP phase, and calculate several physical quantities allowing to probe the behavior of the system at the phase transition as in [8, 9], but now we examine the finite size effects for the thermal (temperature driven) DPT as well as for the density driven DPT. First, we use infinite matter equations of state, for both HG and QGP phases. We also investigate the finite mass effects on relevant quantities, then we impose the color-singletness requirement for the QGP and re-examine the variations of physical quantities near the transition. We use the phase coexistence model [6, 10], assuming that the mixed

HG-QGP phase system has a finite volume: $V = V_{HG} + V_{QGP}$, and the parameter h representing the fraction of volume occupied by the HG phase is then defined: $V_{HG} = hV$. Thus, the value $h = 1$ corresponds to a total HG phase while $h = 0$ corresponds to a total QGP phase. The mean value of a physical quantity, A , of the system at temperature T , chemical potential μ and volume V , is then given by:

$$\langle A(T, \mu, V) \rangle = \frac{\int_0^1 A(h, T, \mu, V) Z(h) dh}{\int_0^1 Z(h) dh}, \quad (1)$$

where $A(h, T, \mu, V)$ is the total physical quantity in the state h , and $Z(h)$ the total partition function of the system in the state h , which factorizes assuming non-interacting phases as:

$$Z(h) = Z_{QGP}(h)Z_{HG}(h)Z_{Vac}(h), \quad (2)$$

Z_{QGP} being the QGP partition function, Z_{HG} the partition function of the HG phase and Z_{Vac} accounts for the confinement of quarks and gluons by the real vacuum pressure B exerted on the perturbative vacuum of the bag model such that: $Z_{Vac} = e^{-BV/T}$.

2. FINITE-SIZE ROUNDING FOR THE THERMALLY DRIVEN AND DENSITY DRIVEN DECONFINING PHASE TRANSITION

To study the effects of volume finiteness, we shall use in a first step the infinite matter equations of state of the hadronic gas phase consisting of pions of mass m_π and the QGP phase consisting of gluons, massless u and d quarks in addition to massive s quarks of mass m_s . We examine the behavior of some thermodynamic quantities with temperature for varying volume, at a vanishing chemical potential $\mu = 0$, but also with chemical potential and volume at fixed temperature, using the common value $B^{1/4} = 145 \text{ MeV}$ for the bag constant. The main quantities of interest are the order parameter,

which is simply in this case the mean value of the hadronic volume fraction $\langle h(T, \mu, V) \rangle$, and the mean values of both energy density $\langle \varepsilon(T, \mu, V) \rangle$ and entropy density $\langle s(T, \mu, V) \rangle$, which are related by the expressions:

$$\langle \varepsilon(T, \mu, V) \rangle = \varepsilon_{QGP} + (\varepsilon_{HG} - \varepsilon_{QGP}) \langle h(T, \mu, V) \rangle, \quad (3)$$

$$\langle s(T, \mu, V) \rangle = s_{QGP} + (s_{HG} - s_{QGP}) \langle h(T, \mu, V) \rangle, \quad (4)$$

with:

$$\langle h(T, \mu, V) \rangle = \frac{(-\frac{V}{T}(\mathfrak{f}_{HG} - \mathfrak{f}_{QGP}) - 1) \exp(-\frac{V}{T}(\mathfrak{f}_{HG} - \mathfrak{f}_{QGP}) - 1) + 1}{(-\frac{V}{T}(\mathfrak{f}_{HG} - \mathfrak{f}_{QGP}) - 1)(\exp(-\frac{V}{T}(\mathfrak{f}_{HG} - \mathfrak{f}_{QGP}) - 1) - 1)}, \quad (5)$$

$$\begin{cases} \mathfrak{f}_{QGP} = -(\frac{37}{90}\pi^2 T^4 + \mu^2 T^2 + \frac{\mu^4}{2\pi^2}) - \frac{6e^{-m_s/T}}{\sqrt{2}} \left(\frac{m_s T}{\pi}\right)^{3/2} (T + \frac{\mu^2}{2!T} + \frac{\mu^4}{4!T^3}) + B \\ \mathfrak{f}_{HG} = \frac{-1}{2\pi^2} \int_0^\infty \frac{k^4 dk}{\sqrt{k^2 + m_\pi^2} (e^{\sqrt{k^2 + m_\pi^2}/T} - 1)} \end{cases}, \quad (6)$$

$$\begin{cases} \varepsilon_{QGP} = \frac{111\pi^2}{90} T^4 + \mu^2 T^2 - \frac{\mu^4}{2\pi^2} + \frac{6e^{-m_s/T}}{\sqrt{2}} \left(\frac{m_s T}{\pi}\right)^{3/2} \left[\left(\frac{1}{2} + \frac{m_s}{T}\right)(T + \frac{\mu^2}{2!T} + \frac{\mu^4}{4!T^3}) + T - \frac{\mu^2}{2!T} - \frac{\mu^4}{4!T^3}\right] + B \\ \varepsilon_{HG} = \frac{-1}{2\pi^2} \int_0^\infty \frac{k^4 dk}{\sqrt{k^2 + m_\pi^2} (e^{\sqrt{k^2 + m_\pi^2}/T} - 1)} + \frac{1}{2\pi^2 T} \int_0^\infty \frac{k^4 dk}{e^{-\sqrt{k^2 + m_\pi^2}/T} (e^{\sqrt{k^2 + m_\pi^2}/T} - 1)^2} \end{cases}, \quad (7)$$

$$\begin{cases} s_{QGP} = \frac{74}{45}\pi^2 T^3 + 2\mu^2 T + \frac{6e^{-m_s/T}}{\sqrt{2}} \left(\frac{m_s T}{\pi}\right)^{3/2} \left[(T + \frac{\mu^2}{2!T} + \frac{\mu^4}{4!T^3})\left(\frac{3}{2T} + \frac{m_s}{T^2}\right) + 1 - \frac{\mu^2}{2!T^2} - \frac{3\mu^4}{4!T^4}\right] \\ s_{HG} = \frac{1}{2\pi^2 T} \int_0^\infty \frac{k^4 dk}{e^{-\sqrt{k^2 + m_\pi^2}/T} (e^{\sqrt{k^2 + m_\pi^2}/T} - 1)^2} \end{cases}, \quad (8)$$

where k is the momentum. Let's note that usual thermodynamic relations are used for the derivation of the free energy densities (eq. (6)), the energy densities (eq. (7)) and the entropy densities (eq. (8)) in both hadronic and QGP phases, while mean values in eqs. (3) to (5) are calculated using eq. (1). The chemical potential has been considered the same for all three quark flavors, in a first approximation (see for example [11]).

Fig. 1 shows the three-dimensional plots of the order parameter (bottom), the mean value of the energy density normalized by T^4 (middle) and that of the entropy density normalized by T^3 (top), vs temperature and volume, at a vanishing chemical potential $\mu = 0$. The first-order character of the transition is showed by the step-like rise of each of the three quantities, when approaching the thermodynamic limit, at a true transition temperature noted $T_c(\infty)$. This sharp discontinuity reflects the existence of a latent heat accompanying the phase transition. The quantities $\langle \varepsilon \rangle / T^4$ and $\langle s \rangle / T^3$ are traditionally interpreted as a measure of the number of effective degrees of freedom; the temperature increase causes then a melting of the constituent degrees of freedom frozen in the hadronic state, making the energy and entropy densities attain their plasma values. In small systems, the phase transition is rounded off, since the probability of presence of the QGP phase below the transition temperature, and of the hadronic phase above it are finite because of the considerable thermodynamical fluctuations. The transition region around the transition temperature is then broadened, acquiring a bigger width δT , smaller is the volume.

Similarly, Fig. 2 shows the 3-D variations of the order parameter (bottom), the mean value of the energy density (middle) and that of the entropy density (top) with chemical potential and system volume, at a fixed temperature $T = 99 \text{ MeV}$. The first-order character of the transition can, also in this case, clearly be seen from the sharp discontinuity of the three quantities at a transition chemical potential noted $\mu_c(\infty)$, at the large volume limit. In small systems, the transition is perfectly smooth over a broadened region of chemical potential of width $\delta \mu(V)$.

Fig. 3 illustrates the 3-D variations of the specific heat density $c(T, V)$ (top) and the susceptibility $\chi(T, V)$ (bottom) with temperature and system volume at $\mu = 0$. These quantities are the derivatives of the energy density and the order parameter respectively, with respect to temperature. It can clearly be seen that the delta function singularity of both quantities occurring in the thermodynamical limit is

smeared out, in a finite volume, into a finite peak of width $\delta T(V)$. For decreasing volume, the widths of the peaks get larger while their heights $c_T^{\max}(V)$ and $\chi_T^{\max}(V)$ decrease. Similar variations are also obtained for the specific heat density $c(\mu, V)$ and the susceptibility $\chi(\mu, V)$ at fixed temperature, which are now obtained respectively by derivating the energy density and the order parameter with respect to chemical potential.

Finite size effects in the case of the thermally driven as well as the density driven DPT, can then be summarized in the following points: the rounding of discontinuities, or equivalently the smearing of singularities at the level of the first derivatives, and the broadening of the transition region. To these effects can be associated three useful characteristic quantities for each of the two transitions, namely, the maxima of the peaks of both thermal susceptibility $\chi_T^{\max}(V)$ and specific heat density $c_T^{\max}(V)$, and the width of the transition region $\delta T(V)$ for the thermal DPT, and for the density driven DPT, the maxima of the peaks of both susceptibility $\chi_\mu^{\max}(V)$ and specific heat density $c_\mu^{\max}(V)$, and the width $\delta\mu(V)$ over which the transition is rounded off. Each of these quantities can be considered as an indicator of the order of the occurring transition, and is expected to exhibit a scaling behavior in the form of a power law of the volume V , characterized by a scaling critical exponent. For a first-order thermal phase transition for example, the set of power laws is: $\chi_T^{\max}(V) \propto V^\gamma$, $c_T^{\max}(V) \propto V^\alpha$, $\delta T(V) \propto V^{-\theta}$, where the scaling exponents have been shown in the finite-size scaling theory to be all equal to unity [12–15].

As it is more interesting and realistic to study the deconfining phase transition to a colorless QGP, we shall extend the study of the finite-size effects to this case in the remainder of this work, but let us first investigate the effects of the mass finiteness on some characteristic quantities in the following, using infinite matter equations of state.

3. FINITE-MASS EFFECTS FOR THE THERMAL DECONFINING PHASE TRANSITION

For the study of the finite-mass effects, it is worth to illustrate the variations of some relevant physical quantities, in the case of massless particles and in the case where massive particles are included.

Fig. 4 shows the variations of the mean values of (bottom) the entropy density normalized by T^3 and (top) the energy density normalized by T^4 , examined previously, but now vs temperature (at $\mu = 0$) at the two volumes $150fm^3$ (dashed line) and $1000fm^3$ (solid line). Three distinct cases are investigated: (a) with three massless " u, d, s " quarks in the QGP phase and massless pions in the HG phase, (b) with massive s quarks and massless " u, d " quarks in the QGP phase and massive pions in the HG phase and (c) with two massless " u, d " quarks in the QGP phase and massless pions in the HG phase. It is well known that the transition point depends on the number of flavors, and this is recovered here since the transition temperature is lowered when including the strange quarks in the QGP phase, and it is clear that the transition temperature is higher with massive strange quarks. Another mass effect appears at the level of the QGP phase ($T > T_c$) when comparing (a) and (b) curves showing that the effective number of degrees of freedom in the two cases where s quarks are considered, in addition to u and d quarks, is lower when the s quarks are massive than when they are massless.

The most striking finite mass effect can be noted at the level of the sound velocity squared, determined by the pressure gradient dP for a given gradient in the energy density $d\varepsilon$, and defined by: $c_s^2 = dP/d\varepsilon$. This quantity is known to have the value $c_s^2 = 1/3$ for an ultra-relativistic ideal gas and to present a vanishing minimum at the transition temperature, for a first order phase transition occurring at the thermodynamic limit. Its three dimensional variations with temperature and volume,

at $\mu = 0$, are illustrated on Fig. 5 for the case of a HG consisting of massive pions of mass m_π and a QGP consisting of gluons, massless u and d quarks in addition to massive s quarks of mass m_s , and show such a behavior. Fig. 6 illustrates the sound velocity squared vs temperature at the volume $V = 1000 fm^3$ in the three cases of Fig. 4, i. e. when considering two massless " u, d " quarks in the QGP phase and massless pions in the HG phase (dotted curves), three massless " u, d, s " quarks in the QGP phase and massless pions in the HG phase (dashed curves) and massive s quarks and massless " u, d " quarks in the QGP phase and massive pions in the HG phase (solid curves). It is obvious from the curves on Fig. 6 that for temperatures well above the transition temperature T_c , when the ratio T/m_π increases, the sound velocity approaches the value $1/3$, but for temperatures below T_c , i. e., for small ratio T/m_π , it is slower and does not attain the value $1/3$, comparatively to the case of massless particles (dashed and dotted curves). Our results are comparable to those found in lattice QCD calculations (see for example [16]).

4. FINITE-SIZE EFFECTS WITH COLOR-SINGLETNESS REQUIREMENT

In the following, because the total color must be neutral in the QGP phase, we impose the color singletness requirement to the partition function Z_{QGP}^{cs} for a free gas of quarks and gluons, consisting of gluons, massless up and down quarks, and massive strange quarks, and contained in a volume V_{QGP} , at temperature T and quark chemical potential μ . We calculate such a partition function using the group theoretical projection method [17] and use it to probe the behavior of some physical quantities characterizing the system by calculating their mean values, using eq. (1), as in the first section.

The obtained expressions of the order parameter and the mean value of the energy density, at

vanishing chemical potential $\mu = 0$, are:

$$\langle \mathfrak{h}(T, V) \rangle = 1 - \frac{\int_0^1 dq \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} d\varphi d\psi M(\varphi, \psi) e^{\mathcal{A}(\varphi, \psi, q, V, T, m_s, m_\pi)}}{\int_0^1 dq \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} d\varphi d\psi M(\varphi, \psi) e^{\mathcal{A}(\varphi, \psi, q, V, T, m_s, m_\pi)}}, \quad (9)$$

$$\langle \varepsilon(T, V) \rangle = X \langle \mathfrak{h}(T, V) \rangle + \frac{T^2}{V} \left[\frac{\int_0^1 dq \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} d\varphi d\psi M(\varphi, \psi) Y e^{\mathcal{A}(\varphi, \psi, q, V, T, m_s, m_\pi)}}{\int_0^1 dq \int_{-\pi}^{+\pi} \int_{-\pi}^{+\pi} d\varphi d\psi M(\varphi, \psi) e^{\mathcal{A}(\varphi, \psi, q, V, T, m_s, m_\pi)}} \right], \quad (10)$$

where:

$$\mathcal{A} = -qV \frac{B}{T} + qVT^3 g_{1(\mu=0)} + (qV)^{\frac{1}{3}} T g_{2(\mu=0)} + g_{3(\mu=0)} - \frac{qV d_\pi}{2\pi^2 T} I_1, \quad (11)$$

$$X = \frac{d_\pi}{2\pi^2} \left(-I_1 + \frac{1}{T} I_2 \right), \quad (12)$$

$$Y = qV \frac{B}{T^2} + 3qVT^2 g_{1(\mu=0)} + (qV)^{\frac{1}{3}} g_{2(\mu=0)} + d_Q e^{-\frac{m_s}{T}} Y_1 \sum_{q=r,g,b} \left(1 - \frac{(\alpha_q)^2}{2!} + \frac{(\alpha_q)^4}{4!} \right), \quad (13)$$

$$Y_1 = \frac{qV}{\sqrt{2}} \left(\frac{m_s}{\pi} \right)^{\frac{3}{2}} \left(\frac{3}{2} \sqrt{T} + \frac{m_s}{\sqrt{T}} \right) + (qV)^{\frac{2}{3}} \left[\left(\frac{3}{4\pi} \right)^{\frac{1}{3}} \sqrt{\frac{2}{\pi}} \sqrt{m_s} \left(\frac{3}{2} \sqrt{T} + \frac{m_s}{\sqrt{T}} \right) - \left(\frac{3}{4\pi} \right)^{\frac{2}{3}} m_s \left(1 + \frac{m_s}{T} \right) \right] \\ + (qV)^{\frac{1}{3}} \left[\frac{2^{\frac{3}{2}}}{4^{\frac{1}{3}} 3^{\frac{2}{3}} \sqrt{\pi}} \sqrt{m_s} \left(\frac{1}{2\sqrt{T}} + \frac{m_s}{T^{\frac{3}{2}}} \right) - \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} \left(1 + \frac{m_s}{T} - \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{m_s}} \left(\frac{3}{2} \sqrt{T} + \frac{m_s}{\sqrt{T}} \right) \right) \right], \quad (14)$$

$$g_{1(\mu=0)} = \frac{\pi^2}{12} \left[2d_Q \sum_{q=r,g,b} \left(\frac{7}{30} - \left(\frac{\alpha_q}{\pi} \right)^2 + \frac{1}{2} \left(\frac{\alpha_q}{\pi} \right)^4 \right) + d_G \sum_{g=1}^4 \left(-\frac{7}{30} + \left(\frac{\alpha_g - \pi}{\pi} \right)^2 - \frac{1}{2} \left(\frac{\alpha_g - \pi}{\pi} \right)^4 \right) \right], \quad (15)$$

$$g_{2(\mu=0)} = \frac{2\pi}{3} \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} \left[\frac{d_Q}{2} \sum_{q=r,g,b} \left(\left(\frac{\alpha_q}{\pi} \right)^2 - \frac{1}{3} \right) - d_G \sum_{g=1}^4 \left(-\frac{1}{3} + \left(\frac{\alpha_g - \pi}{\pi} \right)^2 \right) \right], \quad (16)$$

$$g_{3(\mu=0)} = d_Q e^{-\frac{m_s}{T}} \left[\frac{qV}{\sqrt{2\pi^{\frac{3}{2}}}} (m_s T)^{\frac{3}{2}} + \left(\left(\frac{3qV}{4\pi} \right)^{\frac{2}{3}} + \frac{1}{m_s} \left(\frac{3qV}{4\pi} \right)^{\frac{1}{3}} \right) \left(\sqrt{\frac{2}{\pi}} \frac{(m_s T)^{\frac{3}{2}}}{m_s} - m_s T \right) \right. \\ \left. + \frac{2}{3} \sqrt{\frac{2}{\pi}} \left(\frac{3qV}{4\pi} \right)^{\frac{1}{3}} (m_s T)^{\frac{1}{2}} \right] \sum_{q=r,g,b} \left(1 - \frac{(\alpha_q)^2}{2!} + \frac{(\alpha_q)^4}{4!} \right), \quad (17)$$

$$I_1 = \int_0^\infty \frac{k^4 dk}{\sqrt{k^2 + m_\pi^2} (e^{\frac{\sqrt{k^2 + m_\pi^2}}{T}} - 1)}, \quad (18)$$

$$I_2 = \int_0^\infty \frac{k^4 e^{\frac{\sqrt{k^2 + m_\pi^2}}{T}} dk}{\left(e^{\frac{\sqrt{k^2 + m_\pi^2}}{T}} - 1 \right)^2}, \quad (19)$$

where k is the momentum, $\mathfrak{q}$ the variable representing the QGP volume fraction, $M(\varphi, \psi)$ the weight function given by:

$$M(\varphi, \psi) = \left(\sin \left(\frac{1}{2} \left(\psi + \frac{\varphi}{2} \right) \right) \sin \left(\frac{\varphi}{2} \right) \sin \left(\frac{1}{2} \left(\psi - \frac{\varphi}{2} \right) \right) \right)^2, \quad (20)$$

d_Q , d_G and d_π are the degeneracy factors of quarks, gluons and pions respectively, and the angles α_q ($q = r, g, b$) and α_g ($g = 1, \dots, 4$) are given by:

$$\alpha_r = \frac{\varphi}{2} + \frac{\psi}{3}, \alpha_b = \frac{-2\psi}{3}, \alpha_g = \frac{-\varphi}{2} + \frac{\psi}{3}, \alpha_1 = \alpha_r - \alpha_g, \alpha_2 = \alpha_g - \alpha_b, \alpha_3 = \alpha_b - \alpha_r, \alpha_4 = 0. \quad (21)$$

The evaluation of these mean values, and others which will follow, is done in a numerical way, at fixed temperature and volume, on a range of temperature and for various volumes, at the common value $B^{1/4} = 145 \text{ MeV}$ for the bag constant.

Fig. 7 illustrates the variations with temperature of the order parameter (bottom), the mean value of the entropy density normalized by T^3 (middle) and that of the energy density normalized by T^4 (top), while Fig. 8 illustrates those of the thermal (bottom) susceptibility and (top) specific heat density, for the displayed volumes. The rounding of discontinuities, and the smearing of singularities at the level of susceptibility and specific heat density, and the broadening of the transition region are also recovered in this case, and the main and striking difference, comparatively with Figs. 1 and 3 showing the same quantities without the color-singletness requirement, is the shift of the transition temperature to higher values for small sizes. This is due to the effect of the color-singletness requirement which

was found to lead to a gradual freezing of the effective number of degrees of freedom in the QGP [18]. In a finite volume, the pressure at a given temperature has a lower value and the equilibrium between the two phases according to the Gibbs criterion is then reached at temperatures greater than $T_c(\infty)$. Thus, additionally to the maxima of the peaks of both thermal susceptibility $\chi_T^{\max}(V)$ and specific heat density $c_T^{\max}(V)$, and to the width of the transition region $\delta T(V)$, the shift of the transition is also expected to exhibit a scaling behavior in the form of a power law of the volume V , characterized by a scaling critical exponent λ , on the form: $\tau(V) = T_c(V) - T_c(\infty) \propto V^{-\lambda}$.

A finite size scaling behavior, similar to that carried out in [6], reveals that the scaling exponents γ , α and θ in this case are equal to unity, and that the shift scaling exponent value is around 0.87, slightly deviating from 1, suggesting the contribution of higher order terms to the expression of $\tau(V)$, as in [6]. The deconfining phase transition is then of first order in this case also where massive particles are contained in the hadronic and QGP phases.

5. CONCLUSION

In the present work, we have studied the deconfining phase transition between a hadronic gas phase consisting of massive pions and a QGP phase containing gluons, massless up and down quarks and massive strange quarks, when the transition is a temperature driven one (at vanishing chemical potential) and when it is a density driven one (at fixed temperature). The two parameters, namely the temperature and the chemical potential play the same role, and finite size effects involve a rounding of the transition on a broadened region of width $\delta T(V)$ or $\delta \mu(V)$, and a smearing of the peaks of the susceptibility and the specific heat density in both cases.

Our investigation has shown that the behavior of the system undergoing a deconfining phase tran-

sition in a finite volume in the presence of massive particles is similar to that in the case of massless particles, meaning that the inclusion of particle masses in our work does not alter the order of the transition, which remains of first-order.

On the other hand, the presence of massive particles has as effects to shift the transition temperature to smaller values, to lower the number of degrees of freedom in the QGP phase as well as to slow down the sound velocity at low temperatures where the ratio T/m_π is small, i. e., in the hadronic phase, preventing it from attaining the value $c_s^2 = 1/3$ for an ultra-relativistic ideal gas. Our results agree very well with those of lattice QCD calculations [16], and with those obtained for example in Ref. [19] studying thermodynamics of the QCD plasma near the phase transition, and in [20].

Finally, let us note that the study carried out here, with the color-singletness requirement for the QGP, has been done at a vanishing chemical potential ($\mu = 0$) in a first approach to the problem with massive particles, while the more complicated case of a finite chemical potential is under consideration in a work in progress.

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Figure Captions

Fig. 1: Three dimensional variations of (bottom) the order parameter, (middle) the mean value of the energy density normalized by T^4 and (top) the mean value of the entropy density normalized by T^3 with temperature and volume at $\mu = 0$.

Fig. 2: Three dimensional variations of (bottom) the order parameter, (middle) the mean value of the energy density and (top) the mean value of the entropy density with chemical potential and volume at $T = 99MeV$.

Fig. 3: Three dimensional plots of (bottom) susceptibility and (top) specific heat density with temperature and volume at $\mu = 0$.

Fig. 4: Variations of the mean values of (bottom) the entropy density normalized by T^3 and (top) the energy density normalized by T^4 with temperature at the two volumes $V = 1000fm^3$ (solid line) and $V = 150fm^3$ (dashed line).

Fig. 5: Three dimensional plot of the sound velocity squared with temperature and volume at $\mu = 0$.

Fig. 6: Sound velocity squared vs temperature at the volume $V = 1000fm^3$ (at $\mu = 0$), in the three displayed cases.

Fig. 7: Variations of (bottom) the order parameter, (middle) the mean value of the entropy density normalized by T^3 and that of (top) the energy density normalized by T^4 with temperature at different volumes at $\mu = 0$, with the color-singletness requirement.

Fig. 8: Variations of (bottom) the susceptibility and (top) the specific heat density with temperature at different volumes at $\mu = 0$, with the color-singletness requirement.

Fig. 1

Three dimensional variations of (bottom) the order parameter, (middle) the mean value of the energy density normalized by T^4 and (top) the mean value of the entropy density normalized by T^3 with temperature and volume at $\mu=0$.
279x361mm (300 x 300 DPI)

Fig. 2

Three dimensional variations of (bottom) the order parameter, (middle) the mean value of the energy density and (top) the mean value of the entropy density with chemical potential and volume at $T=99$ MeV.
279x361mm (300 x 300 DPI)

Fig. 3

Three dimensional plots of (bottom) susceptibility and (top) specific heat density with temperature and volume at $\mu=0$.
279x361mm (300 x 300 DPI)

Fig. 4

Variations of the mean values of (bottom) the entropy density normalized by T^3 and (top) the energy density normalized by T^4 with temperature at the two volumes $V=1000\text{fm}^3$ (solid line) and $V=150\text{fm}^3$ (dashed line).

279x361mm (300 x 300 DPI)

Fig. 5

Three dimensional plot of the sound velocity squared with temperature and volume at $\mu=0$.
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Fig. 6

Sound velocity squared vs temperature at the volume $V=1000\text{fm}^3$ (at $\mu=0$), in the three displayed cases.
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Variations of (bottom) the order parameter, (middle) the mean value of the entropy density normalized by T^3 and that of (top) the energy density normalized by T^4 with temperature at different volumes at $\mu=0$, with the color-singletness requirement.

Fig. 8

Variations of (bottom) the susceptibility and (top) the specific heat density with temperature at different volumes at $\mu=0$, with the color-singletness requirement.
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