OPTIMIZED DECENTRALIZED CONTROL OF LARGE DYNAMIC STRUCTURES

by

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This thesis has proposed a new optimal decentralized controller design method for solving the tracking and disturbance rejection problems for the class of large scale LTI systems, using only low order decentralized controllers. The objective of the controller is to reject system constant disturbances and track constant reference signals by using an optimal decentralized control procedure. The proposed controller also has the property that if a sensor actuator failure occurs, thereby producing an unstable pole at the origin, that there is time for a repair to be carried out before massive destruction occurs. The proposed optimal controller also has the property that it is strongly robust as compared to the standard centralized LQR-Observer. In particular, the proposed controller can have the property of being some 5 orders of magnitude more robust than the standard LQR-Observer controller.
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Chapter 1 – Introduction

Large flexible space structures (LFSS) are now becoming a way of life in the space industry. For example, the international Space Station, large synthetic aperture telescopes, and space solar power systems are all contained in this category. Two control problems which often occur in this case are attitude control and shape control. The behavior of a LFSS is usually modelled via finite element methods by a set of differential equations whose order may be quite high (say \( n > 100 \)), and in this case the practice of controlling only a subset of elastic body modes can lead to the so called “spillover problem” [1] in which stabilizing a subset of elastic modes may cause instability due to excitation of any uncontrolled elastic modes.

Typical approaches for LFSS control generally have been directed towards centralized control, e.g., using model reduction methods [2], modal control methods [3], [4], output feedback control [1], [5], adaptive control techniques [6] and displacement feedback [7]. However, recent researchers have started to shift their attention to “decentralized control” [7], [8], [9], and in the survey paper [10], it is mentioned that decentralized control is naturally applicable to the LFSS problem. This thesis is directed towards the decentralized control of LFSS which have the property that the resulting controller is “fail-safe”, i.e., for the case of either sensor or/and actuator failure, the resultant system has the property that the failed system remains stable, provided that one can disable the sensors/actuators, and that any un-failed controllers remain fully functional [11].

To obtain the above “fail-safe” controllers, it will be assumed that the LFSS controller is decentralized with \( \nu \) control agents, where \( \nu \geq 1 \). In this case if control agent “\( i \)” has a sensor failure or/and actuator failure, it is said that control agent “\( i \)” has a failure. It is to be noted that more than one control agent may have a failure.

It also turns out that the controller used for a LFSS is also effective in controlling other large scale systems. In particular, it will be shown that one can control the temperature in large buildings, and in preventing buildings taking a severe damage from earthquakes.
A description of the three example systems which will be used throughout the thesis will now be given.
1.1 Modelling of Large Flexible Space Structures (LFSS)

The behavior of an LFSS [12], [13] may be described as follows:

\[ M \ddot{q} + Kq = \bar{B}\bar{u} + \bar{E}\omega \]  

(1.1a)

where \( M \) and \( K \) are the “inertia” and “stiffness” matrices respectively, \( q \in \mathbb{R}^n \) is a vector, \( \bar{u} \) is a vector of control inputs and \( \omega \) is a vector of unmeasurable constant disturbances and/or constant tracking signals. Here \( \bar{B} \) depends on the type and location of the control actuators and similarly \( \bar{E} \) depends on the type of disturbance which occurs in the LFSS.

In general, it will be assumed that the control inputs \( \bar{u} \) are provided for by point-force actuators or by torques.

The well-known point transformation \( q = T\rho \) [12] can now be used to diagonalize \( M \) and \( K \) in (1.1a) to obtain

\[ \dot{\rho} + \Omega^2 \rho = T'\bar{B}\bar{u} + T'\bar{E}\omega \quad \eta \in \mathbb{R}^n \]  

(1.1b)

where \( \Omega^2 \) is a diagonal matrix, and in this model, the outputs to be regulated are given by

\[ y = C\rho + F\omega \]  

(1.1c)

where \( \omega \) is an unmeasurable constant disturbance, and it is assumed that the output \( y \) is measurable.

The resulting equations of motion of the LFSS can then be described by

\[ \dot{x} = \begin{bmatrix} 0 & I_n \\ -\Omega^2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ B^* \end{bmatrix} \bar{u} + \begin{bmatrix} 0 \\ E^* \end{bmatrix} \omega \]  

(1.1d)

\[ y = [C \ 0] x + F\omega \]

where \( y \in \mathbb{R}^\nu, u \in \mathbb{R}^r, \) and \( x \in \mathbb{R}^{2n} \) are given by

\[ x = \begin{bmatrix} \rho \\ \rho \end{bmatrix} \]
where \( B^* = T^T \tilde{B}, \tilde{E} = T^T \tilde{E} \) and where \( C \) depends on the type and location of outputs \( y \) to be regulated, and where \( E \) and \( F \) depend on the types of disturbances associated with the output \( y \).

It now will be assumed that the sensors and actuators are mutually dual (i.e. colocated), which implies that \( CB = 0 \), which [12] implies that (1.1d) may be written as follows.

\[
\dot{x} = \begin{bmatrix} 0 & I_n \\ -\Omega^2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ B \end{bmatrix} u + \begin{bmatrix} 0 \\ \tilde{E} \end{bmatrix} \omega \quad (1.2e)
\]

\[
y = \begin{bmatrix} B^T & 0 \end{bmatrix} x + F \omega
\]

and it will be assumed that the sensors and actuators are located at \( \nu \) control stations, such that only the outputs of a given control station may be used in controlling the inputs of that station, i.e. it will be assumed that a decentralized controller is used, which implies that the model (1.1e) can be written as follows.

**LFSS Model**

\[
\dot{x} = \begin{bmatrix} 0 & I_n \\ -\Omega^2 & 0 \end{bmatrix} x + \sum_{i=1}^{\nu} \begin{bmatrix} 0 \\ B_i \end{bmatrix} u_i + \begin{bmatrix} 0 \\ E \end{bmatrix} \omega \quad (1.1f)
\]

\[
y_i = \begin{bmatrix} B_i \end{bmatrix}^T x + F_i \omega, \quad i = 1,2,...,\nu
\]

\[
e_i = y_i - y_i^{ref}, \quad i = 1,2,...,\nu
\]

where \( x \in \mathbb{R}^{2n} \) is the state, \( y_i \in \mathbb{R} \), \( i = 1,2,...,\nu \) are the outputs to be regulated, \( u_i \in \mathbb{R} \), \( i = 1,2,...,\nu \) are the control inputs, \( \omega \) is an unmeasurable constant disturbance and \( y_i^{ref} \in \mathbb{R}^{m_i}, i = 1,2,...,\nu \) is a constant set point.
**Definition 1.1.1**

In (1.1.e), assume that there are \( \hat{n} \) rigid body modes [12] present and let the rigid body modes and elastic body modes be ordered so that:

\[
\{ -\Omega^2, B, \bar{E} \} = \begin{bmatrix} 0 & 0 \\ 0 & -\Omega^2 \end{bmatrix}, \begin{bmatrix} \bar{B} \\ \bar{E} \end{bmatrix}
\]

where \( -\Omega^2 \in \mathbb{R}^{(n-\hat{n}) \times (n-\hat{n})} \) is a diagonal matrix of strictly negative elements corresponding to the elastic modes of the LFSS. Then the model:

\[
\dot{x} = \begin{bmatrix} 0 & I_n \end{bmatrix} \dot{x} + \begin{bmatrix} 0 \\ \bar{B} \end{bmatrix} u + \begin{bmatrix} 0 \\ \bar{E} \end{bmatrix} \omega
\]

\[
y = \begin{bmatrix} \bar{B}^T & 0 \end{bmatrix} \dot{x}
\]

(1.1g)

is called a rigid body model of the LFSS.

To simplify notation, (1.1f) will be rewritten as:

\[
\dot{x} = Ax + Bu + E\omega
\]

\[
y = Cx + F\omega
\]

(1.1h)

\[
e = y - y^{ref}
\]

where

\[
A = \begin{bmatrix} 0 & I_n \\ -\Omega^2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ B_1 & B_2 & \ldots & B_\nu \end{bmatrix}
\]

\[
E = \begin{bmatrix} 0 \\ \bar{E} \end{bmatrix}, \quad C = \begin{bmatrix} B_1' & 0 \\ B_2' & 0 \\ \vdots & \vdots \\ B_\nu' & 0 \end{bmatrix}, \quad F = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_\nu \end{bmatrix}
\]

Appendix A gives the numerical details of the matrices \( A,B,C,E,F \) obtained by Hablani [13].
1.2 Modelling of a Multistory Building Subject to an Earthquake

This section derives the structural dynamics and the state space form of a six story building under PID control. The method of approach is described by Hart [14], and the present detailed derivation is provided by the author. The derivation starts with deriving the equation of motion, where a description of the building’s structure is provided below.

In this case, the mass of the $i^{th}$ floor is given by $m_i$, the damping factor of the floor is given by $c_i$, the spring coefficient is given by $k_i$, the absolute displacement of each floor is given by $w_i$, the actuator force is given by $u_i$, and the ground acceleration caused by the earthquake is given by $\ddot{u}_g$ where a description of the building’s structure is provided below.

![Figure 1.2 Building structure under earthquake ground acceleration.](image)

See Appendix E for the modelling of an earthquake affecting a multistory building.
1.3 Large Building Temperature Controller

The model of a large multi-room building which requires temperature control is obtained from [15], and will be used in this problem. The heating typically used in most building control uses an “on/off” type of control. However in this thesis, it will be assumed that the heating system is capable of generating continuous heat profile.

The resulting equations of motion of the LFSS can then be described by
\[
\dot{x} = Ax + Bu + (e_{in} + e_{out})
\]
\[
y = [B^T \ 0]x
\]
where \( y \in \mathbb{R}^v, u \in \mathbb{R}^v, \) and \( x \in \mathbb{R}^{2n}, \) and where \( e_{in} \) and \( e_{out} \) are disturbances due to temperature changes from the outside of the building \( (e_{out}) \), and from the inside of the building \( (e_{in}) \).

It will now be assumed that the sensors and heaters are located at \( v \) control stations, such that only the outputs of a given control station may be used in controlling the inputs of that station, i.e. it will be assumed that a decentralized controller is used, which implies that the model (1.3a) can be written as follows.

**LBTC Model**

\[
\dot{x} = Ax + \sum_{i=1}^{v} B_i u_i + (e_{in} + e_{out})
\]
\[
y_i = [B_i^T \ 0]x, \ i = 1, 2, ..., v
\]
\[
e_i = y_i - y_i^{ref}, \ i = 1, 2, ..., v
\]
where \( x \in \mathbb{R}^{2n} \) is the state, \( y_i \in \mathbb{R}, \ i = 1, 2, ..., v \) are the outputs to be regulated, \( u_i \in \mathbb{R}, \ i = 1, 2, ..., v \) are the control inputs, \( e_{in} \) and \( e_{out} \) are constant disturbances and \( y_i^{ref} \in \mathbb{R}, \ i = 1, 2, ..., v \) is a constant set point.
To simplify notation, (1.3c) will be rewritten as

\[
\dot{x} = Ax + Bu + (e_{in} + e_{out})
\]

\[
y = Cx
\]

\[
e = y - y^{ref}
\]

(1.3d)

Appendix C gives the numerical details of the matrices \(A, B, C, e_{in}, e_{out}\).
Chapter 2 – Problem Requirements and Existence Conditions

The three example problems defined in Chapter 1 can be solved by treating the problem as a decentralized servomechanism control problem, which has many advantages over the standard centralized control problem. This chapter begins with the definition of a decentralized servomechanism problem, and it then provides existence conditions for a solution to exist to the decentralized servomechanism problem for a linear time-invariant (LTI) plant.

2.1 General Terminology of a Control System

Definition of centralized fixed modes (CFM) [16]

Consider the following system

\[ \dot{x} = Ax + Bu, \quad A \in \mathbb{R}^{n \times n}, \quad B \in \mathbb{R}^{n \times m} \]

\[ y = Cx, \quad C \in \mathbb{R}^{r \times n} \]  \hspace{1cm} (2.1a)

Suppose now a static gain controller \( u = Ky \) is applied to the above system, where \( K \in \mathbb{R}^{m \times r} \) is a constant gain matrix. Then as \( K \) is changed, some eigenvalues of \( A + BK \) may remain invariant. Those eigenvalues that have such a property are called the centralized fixed modes (CFM). The mathematical expression of centralized fixed modes is given by:

\[ CFM = \bigcap_{K \in \mathbb{R}^{m \times r}} \{ sp(A + BK) \} \]  \hspace{1cm} (2.1b)

Algorithm for finding centralized fixed modes [16]

1. Find the eigenvalues of \( A \).
2. Choose an arbitrary gain matrix \( K \in \mathbb{R}^{m \times r} \) (Can use a random matrix generator).
3. Find the eigenvalues of the closed loop system \( A + BK \).

Repeat step 1) to 3) to ensure that the final results are indeed CFM.
Then for almost all $K$, those eigenvalues contained in both $A$ and $A + BKC$ are CFM, i.e. the elements of $K$ which do not have this property lie on a hypersurface.

Definition of transmission zeros of $(C,A,B,D)$ [16]

Consider the following system
\[\dot{x} = Ax + Bu, \quad x \in R^n, \quad u \in R^m\]
\[y = Cx + Du, \quad y \in R^r\] (2.1c)

The transmission zeros of $(C,A,B,D)$ are defined to be the set of complex numbers $\mu$ which satisfy the following inequality
\[\text{rank} \begin{bmatrix} A - \mu I & B \\ C & D \end{bmatrix} < n + \min(r, m)\] (2.1d)

and a system is said to be degenerate if any point in the complex plane is a transmission zero.

Algorithm for finding transmission zeros [16]

Step 1. Given $(C,A,B,D)$, let $\bar{\mu}_i, i = 1,2, ..., n + 1$ be $n + 1$ distinct real scalars.

Then the system $(C,A,B,D)$ is said to be degenerate iff
\[\text{rank} \begin{bmatrix} A - \bar{\mu}_i I & B \\ C & D \end{bmatrix} < n + r, \quad i = 1,2, ..., n + 1\] (2.1e)

If $(C,A,B,D)$ is degenerate, stop; otherwise continue with step 2.

Step 2. Choose $K \in R^{r \times r}$ so $\text{rank} \ K = r$, and determine the finite closed loop eigenvalues (if any) of:
\[A + \rho BK(I - \rho KD)^{-1}C, \quad \rho \to \infty\] (2.1f)

If the system has no finite closed loop eigenvalues, this implies that $(C,A,B,D)$ has no transmission zeros. The transmission zeros of $(C,A,B,D)$ are then equal to the finite closed loop eigenvalues of the system.
2.2 Definition of Servomechanism Problem for Constant Tracking Signals and Constant Unmeasurable Disturbances

It is assumed that the plant to be controlled can be described by the following model.

\[
\dot{x} = Ax + Bu + E\omega \\
y = Cx + F\omega \\
e = y - y_{\text{ref}}
\]  

(2.2a)

where \( u \), \( x \) and \( y \) are the input, state and output respectively, \( \omega \) are constant unmeasurable disturbances, \( y_{\text{ref}} \) are constant tracking signals and \( e \) are the errors between \( y \) and \( y_{\text{ref}} \).

The size of each vector is as follows:

\[
\begin{align*}
u & \in \mathbb{R}^m \\
x & \in \mathbb{R}^n \\
y & \in \mathbb{R}^r \\
\omega & \in \mathbb{R}^\Omega \\
y_{\text{ref}} & \in \mathbb{R}^r \\
e & \in \mathbb{R}^r
\end{align*}
\]

Problem definition

Robust servomechanism problem [16] for constant disturbances and constant tracking signals.

Find a controller for (2.2a) so that the following three conditions all hold:

a) The closed loop system is asymptotically stable.

b) Asymptotic error regulation occurs in (2.2a), i.e. \( \lim_{t \to \infty} e(t) = 0 \), \( \forall x(0) \in \mathbb{R}^n \), \( \forall \omega \in \mathbb{R}^\Omega \).

c) Property b) holds for all perturbations of the plant which maintain property a).
2.3 Existence Conditions

**Theorem 2.3a** [16]

Given the plant (2.2a) and constant disturbance/tracking signals, there exists a solution to the centralized robust servomechanism problem for (2.2a) if and only if the following two conditions all hold:

a) The system $(C,A,B)$ has no unstable centralized fixed modes (CFM).

b) \( \text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + r \).

The following result gives an existence result for a solution to exist for the decentralized servomechanism problem and is obtained from [17].

**Theorem 2.3b**

Given the system (2.2a), with equal inputs and outputs, there exists a solution to the decentralized robust servomechanism problem [17] for constant tracking signals and constant unmeasurable disturbances if and only if the following two conditions are satisfied:

i) The system (2.2a) has no unstable decentralized fixed modes [17].

ii) \( \text{rank} \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = n + r \).

A list of some of the more recent results in the area of decentralized control is given in [18], [19], [20], [21].

2.4 Controller to be Used in All Three Example Problems

Given a plant with $r$ inputs and outputs, the following decentralized PID controller will be used in all examples:

\[
 u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta \tag{2.4a}
\]

where $\dot{\eta} = y - y_{ref}$, $\beta_1$, $\beta_2$, $\epsilon$ are scalars and $K_1$, $K_2$, $K_3$ are $r \times r$ identity matrices.
Chapter 3 – Derivation of Closed Loop System

Given the three plant models in Chapter 1, it may immediately be verified from Theorem 2.3b that there exists a solution to the decentralized robust servomechanism problem for the case of

i) 5 local controllers for the LFSS
ii) 6 local controllers for the Earthquake Building
iii) 5 local controllers for the Building Temperature Controller

obtained from Theorem 2.3b. This chapter derives the closed loop system equation for all three problems assuming that a decentralized PID controller (2.4a) is used.

3.1 Large Flexible Space Structure

The state space equation of the LFSS defined in (1.2f) where \(\omega\) and \(y_{ref}\) are constant, and the PID controller (2.4a) are given below

\[
\dot{x} = Ax + Bu + E\omega \\
y = Cx + F\omega \\
u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \dot{\eta}
\]

where \(K_1, K_2, K_3\) are \(r \times r\) identity matrices, and \(\beta_1, \beta_2, \epsilon\) are constants associated with the PID controller and

\[
\eta = \int_{0}^{t} (y - y_{ref}) \, dt
\]

On substituting the PID controller into the state space equations and expanding, we obtain:

\[
\dot{x} = Ax + B(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \dot{\eta}) + E\omega \\
\dot{x} = Ax - B\beta_1 K_1 y - B\beta_2 K_2 \dot{y} - B\epsilon K_3 \dot{\eta} + E\omega \\
\dot{x} = Ax - B\beta_1 K_1 C x - B\beta_1 K_1 F\omega - B\beta_2 K_2 \frac{d}{dt}(C x + F\omega) - B\epsilon K_3 \dot{\eta} + E\omega
\]

where \(B\beta_2 K_2 \frac{d}{dt}(F\omega) = 0\)
\[\dot{x} = Ax - B\beta_1 K_1 C x - B\beta_1 K_1 F \omega - B\beta_2 K_2 C \dot{x} - B\epsilon K_3 \eta + E \omega\]

\[(I + B\beta_2 K_2 C) \dot{x} = Ax - B\beta_1 K_1 C x - B\beta_1 K_1 F \omega - B\epsilon K_3 \eta + E \omega\]

\[\dot{x} = (I + B\beta_2 K_2 C)^{-1}(A - B\beta_1 K_1 C)x - (I + B\beta_2 K_2 C)^{-1}B\epsilon K_3 \eta\]

+ \[(I + B\beta_2 K_2 C)^{-1}(E - B\beta_1 K_1 F)\omega\]

and on expanding the controller equation, we obtain

\[u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta\]

\[u = -\beta_1 K_1 (C x + F \omega) - \beta_2 K_2 \frac{d}{dt}(C x + F \omega) - \epsilon K_3 \eta\]

where \(B\beta_2 K_2 \frac{d}{dt}(F \omega) = 0\)

\[u = -\beta_1 K_1 (C x + F \omega) - \beta_2 K_2 C \dot{x} - \epsilon K_3 \eta\]

(3.1c)

\[(I + \beta_2 K_2 C) u = -\beta_1 K_1 (C x + F \omega) - \beta_2 K_2 C A x - \beta_2 K_2 C E \omega - \epsilon K_3 \eta\]

\[u = (I + \beta_2 K_2 C)^{-1}(-\beta_1 K_1 C - \beta_2 K_2 C A) x\]

+ \[(I + \beta_2 K_2 C)^{-1}(-\beta_1 K_1 F - \beta_2 K_2 C E)\omega\]

+ \[(I + \beta_2 K_2 C)^{-1}(-\epsilon K_3) \eta\]

On collecting terms, we obtain the closed loop system:

\[\begin{bmatrix} \dot{x} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} (I + B\beta_2 K_2 C)^{-1}(A - B\beta_1 K_1 C) & -(I + B\beta_2 K_2 C)^{-1}B\epsilon K_3 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} (I + B\beta_2 K_2 C)^{-1}(E - B\beta_1 K_1 F) \\ F \end{bmatrix} \omega - \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_{\text{ref}}\]

(3.1d)

\[y = [C \quad 0] \begin{bmatrix} x \\ \eta \end{bmatrix} + [F] \omega\]

(3.1e)

\[u = [(I + \beta_2 K_2 C B)^{-1}(-\beta_1 K_1 C - \beta_2 K_2 C A) \quad (I + \beta_2 K_2 C B)^{-1}(-\epsilon K_3)] \begin{bmatrix} x \\ \eta \end{bmatrix} + [(I + \beta_2 K_2 C B)^{-1}(-\beta_1 K_1 F - \beta_2 K_2 C E)] \omega\]

(3.1f)

where all inverse matrices generically exist.
3.2 Earthquake Building

The state space equation and PID controller are given below, where $\ddot{u}_g$ is a forcing term associated with the earthquake force on the building’s ground level

$$\dot{x} = Ax + Bu + G\dot{u}_g$$

$$y = Cx$$

$$u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta$$

where $K_1, K_2, K_3$ are $r \times r$ identity matrices, and $\beta_1, \beta_2, \epsilon$ are constants associated with the PID controller and

$$\eta = \int_0^t (y - y_{ref}) \, dt = \int_0^t Cx \, dt$$

Note: $y_{ref} = 0$ for the earthquake control system

On substituting the PID controller into the state space equations and expanding, we obtain:

$$\dot{x} = Ax + B(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) + G\dot{u}_g$$

$$\dot{x} = Ax - B \beta_1 K_1 y - B \beta_2 K_2 \dot{y} - B \epsilon K_3 \eta + G\dot{u}_g$$

$$\dot{x} = Ax - B \beta_1 K_1 Cx - B \beta_2 K_2 \frac{d}{dt}(Cx) - B \epsilon K_3 \eta + G\dot{u}_g$$

$$\dot{x} = Ax - B \beta_1 K_1 Cx - B \beta_2 K_2 C \dot{x} - B \epsilon K_3 \eta + G\dot{u}_g$$

$$\dot{x} = (I + B \beta_2 K_2 C) \dot{x} = Ax - B \beta_1 K_1 Cx - B \epsilon K_3 \eta + G\dot{u}_g$$

$$\dot{x} = (I + B \beta_2 K_2 C)^{-1} (A - B \beta_1 K_1 C)x - (I + B \beta_2 K_2 C)^{-1} B \epsilon K_3 \eta + (I + B \beta_2 K_2 C)^{-1} G\dot{u}_g$$

$$\dot{x} = (I + B \beta_2 K_2 C)^{-1} (A - B \beta_1 K_1 C)x - (I + B \beta_2 K_2 C)^{-1} B \epsilon K_3 \eta + (I + B \beta_2 K_2 C)^{-1} G\dot{u}_g$$
and on expanding the controller, we obtain:

\[ u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta \]
\[ u = -\beta_1 K_1 C x - \beta_2 K_2 \frac{d}{dt} (C x) - \epsilon K_3 \eta \]
\[ u = -\beta_1 K_1 C x - \beta_2 K_2 C \dot{x} - \epsilon K_3 \eta \]
\[ u = -\beta_1 K_1 C x - \beta_2 K_2 C (A x + Bu + G \ddot{u}_g) - \epsilon K_3 \eta \]

(3.2c)

\[(I + \beta_2 K_2 CB)u = -\beta_1 K_1 C x - \beta_2 K_2 CA x - \beta_2 K_2 CG \ddot{u}_g - \epsilon K_3 \eta \]
\[ u = (I + \beta_2 K_2 CB)^{-1} (-\beta_1 K_1 C - \beta_2 K_2 CA) x \]
\[ + (I + \beta_2 K_2 CB)^{-1} (-\beta_2 K_2 CG \ddot{u}_g) \]
\[ + (I + \beta_2 K_2 CB)^{-1} (-\epsilon K_3) \eta \]

On collecting terms we obtain the closed loop system:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
(I + B \beta_2 K_2 C)^{-1} (A - B \beta_1 K_1 C) & -(I + B \beta_2 K_2 C)^{-1} B \epsilon K_3 \\
C & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\eta
\end{bmatrix}
+ (I + B \beta_2 K_2 C)^{-1} (G) \dddot{u}_g
\]

(3.2d)

\[
\gamma = [C \hspace{1cm} 0]
\begin{bmatrix}
x \\
\eta
\end{bmatrix}
\]

(3.2e)

\[ u = [(I + \beta_2 K_2 CB)^{-1} (-\beta_1 K_1 C - \beta_2 K_2 CA)] [(I + \beta_2 K_2 CB)^{-1} (-\epsilon K_3)]
\begin{bmatrix}
x \\
\eta
\end{bmatrix}
+ [(I + \beta_2 K_2 CB)^{-1} (-\beta_2 K_2 CG)] \dddot{u}_g
\]

(3.2f)

where all inverse matrices generically exist.
3.3 Building Temperature Controller

The state space equation and PID controller are given below

\[
\dot{x} = Ax + Bu + e_{in} + e_{out} \\
y = Cx \\
u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta
\]

where \( K_1, K_2, K_3 \) are \( r \times r \) identity matrices, and \( \beta_1, \beta_2, \epsilon \) are constants associated with PID controller

\[
\eta = \int_{0}^{t} (y - y_{ref}) \, dt
\]

and where \( e_{in} \) and \( e_{out} \) are constant unmeasurable disturbance terms, and \( y_{ref} \) is the desired temperature set point.

On substituting the PID controller into the state space equations and expanding, we obtain:

\[
\dot{x} = Ax + B(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) + e_{in} + e_{out} \\
\dot{x} = Ax - B\beta_1 K_1 y - B\beta_2 K_2 \dot{y} - B\epsilon K_3 \eta + e_{in} + e_{out} \\
\dot{x} = Ax - B\beta_1 K_1 Cx - B\beta_2 K_2 \frac{d}{dt}(Cx) - B\epsilon K_3 \eta + e_{in} + e_{out} \\
\dot{x} = Ax - B\beta_1 K_1 Cx - B\beta_2 K_2 C\dot{x} - B\epsilon K_3 \eta + e_{in} + e_{out} \tag{3.3b} \\
(I + B\beta_2 K_2 C)\dot{x} = Ax - B\beta_1 K_1 Cx - B\epsilon K_3 \eta + e_{in} + e_{out} \\
\dot{x} = (I + B\beta_2 K_2 C)^{-1}(A - B\beta_1 K_1 C)x - (I + B\beta_2 K_2 C)^{-1}B\epsilon K_3 \eta \\
+ (I + B\beta_2 K_2 C)^{-1}(e_{in} + e_{out})
and on expanding the controller, we obtain:

\[ u = -\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta \]
\[ u = -\beta_1 K_1 C x - \beta_2 K_2 \frac{d}{dt}(C x) - \epsilon K_3 \eta \]

\( u = -\beta_1 K_1 C x - \beta_2 K_2 C \dot{x} - \epsilon K_3 \eta \)

\( (I + \beta_2 K_2 CB)u = -\beta_1 K_1 C x - \beta_2 K_2 CA x - \beta_2 K_2 C(e_{in} + e_{out}) - \epsilon K_3 \eta \)

\[ u = (I + \beta_2 K_2 CB)^{-1}(-\beta_1 K_1 C - \beta_2 K_2 CA)x \]
\[ - (I + \beta_2 K_2 CB)^{-1}\beta_2 K_2 C(e_{in} + e_{out}) \]
\[ - (I + \beta_2 K_2 CB)^{-1}\epsilon K_3 \eta \]

\( u = (I + \beta_2 K_2 CB)^{-1}(-\beta_1 K_1 C - \beta_2 K_2 CA)x \]
\[ + (I + \beta_2 K_2 CB)^{-1}(\beta_1 K_1 F - \beta_2 K_2 CE)\omega \]
\[ + (I + \beta_2 K_2 CB)^{-1}(-\epsilon K_3) \eta \]

On collecting terms, we obtain the closed loop system:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\eta}
\end{bmatrix} =
\begin{bmatrix}
(I + B\beta_2 K_2 C)^{-1}(A - B\beta_1 K_1 C) & -(I + B\beta_2 K_2 C)^{-1}B\epsilon K_3 \\
C & 0
\end{bmatrix}
\begin{bmatrix}
x \\
\eta
\end{bmatrix}
+ \begin{bmatrix}
(I + B\beta_2 K_2 C)^{-1} \\
0
\end{bmatrix}(e_{in} + e_{out}) - \begin{bmatrix}
0 \\
1
\end{bmatrix} y_{ref}
\]

\[ y = [C \quad 0] \begin{bmatrix}
x \\
\eta
\end{bmatrix} \]

\[ u = [(I + \beta_2 K_2 CB)^{-1}(-\beta_1 K_1 C - \beta_2 K_2 CA) \quad (I + \beta_2 K_2 CB)^{-1}(-\epsilon K_3)] \begin{bmatrix}
x \\
\eta
\end{bmatrix} \]
\[ - [(I + \beta_2 K_2 CB)^{-1}\beta_2 K_2 C](e_{in} + e_{out}) \]

where all inverse matrices generically exist.
Chapter 4 – Derivation of Failure Model

Failures in sensors or actuators are also considered in this thesis. There are three types of failures. The first failure is a sensor failure, and in this case, the sensor cannot sense the actual value of the output. Therefore, it is assumed that the failed sensor always has a value of zero for the controller. The second failure is an actuator failure. In this case, the input’s command to the actuator cannot be processed. This chapter will derive the mathematical model of the two types of failure with the capability of simulating the above two failures.

Let $F_u, F_y$ be the failure matrix for the actuator and failure matrix for the sensor respectively.

In the nominal case, $u = F_u[u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5]^T$, $y = F_y[y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5]^T$, and $F_u, F_y$ are just identity matrices.

In the failure case, for example, sensor 3 and actuator 3 failure case, $F_u, F_y$ have change to the following matrices:

$$F_u = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \quad F_y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

As a result, $u_3, y_3$ is forced to be 0.
4.1 Large Flexible Space Structure (LFSS)

The state space equation and PID controller are given below

\[
\dot{x} = Ax + Bu + E\omega \\
y = F_y(Cx + F\omega) \\
u = F_u(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta)
\]

where \(\beta_1, \beta_2, \epsilon, K_1, K_2, K_3\) are constants associated with the PID controller

\[
\eta = \int_0^t (y - y_{ref}) \, dt
\]

On substituting the PID controller into the state space equations and expanding, we obtain:

\[
\dot{x} = Ax + BF_u(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) + E\omega
\]

\[
\dot{x} = Ax - BF_u \beta_1 K_1 y - BF_u \beta_2 K_2 \dot{y} - BF_u \epsilon K_3 \eta + E\omega
\]

\[
\dot{x} = Ax - BF_u \beta_1 K_1 F_y C x - BF_u \beta_1 K_1 F_y F \omega - BF_u \beta_2 K_2 \frac{d}{dt}(F_y C x + F_y F \omega) - BF_u \epsilon K_3 \eta + E\omega
\]

where \(BF_u \beta_2 K_2 \frac{d}{dt}(F_y F \omega) = 0\)
\[ \dot{x} = Ax - BF_u \beta_1 K_1 F_y C x - BF_u \beta_1 K_1 F_y F \omega - BF_u \beta_2 K_2 F_y C \dot{x} - BF_u \epsilon K_3 \eta + E \omega \]

\[
\left( I + BF_u \beta_2 K_2 F_y C \right) \dot{x} = Ax - BF_u \beta_1 K_1 F_y C x - BF_u \beta_1 K_1 F_y F \omega - BF_u \epsilon K_3 \eta + E \omega
\]

\[
\dot{x} = (I + BF_u \beta_2 K_2 F_y C)^{-1} (A - BF_u \beta_1 K_1 F_y C) x - (I + BF_u \beta_2 K_2 F_y C)^{-1} BF_u \epsilon K_3 \eta
\]

\[\quad + (I + BF_u \beta_2 K_2 F_y C)^{-1} \left( E - BF_u \beta_1 K_1 F_y F \right) \omega \]

and on expanding the controller equation, we obtain:

\[ u = F_u (-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) \]

\[ u = -F_u \beta_1 K_1 (F_y C x + F_y F \omega) - F_u \beta_2 K_2 \frac{d}{dt} (F_y C x + F_y F \omega) - F_u \epsilon K_3 \eta \]

where \( F_u \beta_2 K_2 \frac{d}{dt} (F_y F \omega) = 0 \) (4.1c)

\[ u = -F_u \beta_1 K_1 (F_y C x + F_y F \omega) - F_u \beta_2 K_2 F_y C \dot{x} - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 (F_y C x + F_y F \omega) - F_u \beta_2 K_2 F_y C (Ax + Bu + E \omega) - F_u \epsilon K_3 \eta \]

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\begin{align*}
(I + F_u \beta_2 K_2 F_y CB)u &= -F_u \beta_1 K_1 (F_y C x + F_y F \omega) - F_u \beta_2 K_2 F_y CA x - F_u \beta_2 K_2 F_y C E \omega - F_u \varepsilon K_3 \eta \\
u &= (I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \beta_1 K_1 F_y C - SF_u \beta_2 K_2 F_y CA) x + (I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \beta_1 K_1 F_y F - F_u \beta_2 K_2 F_y C E) \omega \\
&\quad+ (I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \varepsilon K_3) \eta
\end{align*}

On collecting terms, we obtain the closed loop system:

\begin{align}
\begin{bmatrix}
\dot{x} \\
\dot{\eta}
\end{bmatrix} &= \begin{bmatrix}
(I + BF_u \beta_2 K_2 F_y C)^{-1} (A - BF_u \beta_1 K_1 F_y C) & -(I + BF_u \beta_2 K_2 F_y C)^{-1} BF_u \varepsilon K_3 \\
F_y C & 0
\end{bmatrix} \begin{bmatrix}
x \\
\eta
\end{bmatrix}
+ \begin{bmatrix}
(I + BF_u \beta_2 K_2 F_y C)^{-1} (E - BF_u \beta_1 K_1 F_y F) \\
F_y F & 0
\end{bmatrix} \omega - \begin{bmatrix}
0 \\
I
\end{bmatrix} y_{ref} \\
\quad(4.1d)
\end{align}

\begin{align}
y &= [F_y C \quad 0] \begin{bmatrix}
x \\
\eta
\end{bmatrix} + [F_y F] \omega \\
\quad(4.1e)
\end{align}

\begin{align}
u &= \begin{bmatrix}
(I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y CA) & (I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \varepsilon K_3) \\
(I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \beta_1 K_1 F_y F - F_u \beta_2 K_2 F_y C E) & 0
\end{bmatrix} \begin{bmatrix}
x \\
\eta
\end{bmatrix}
+ (I + F_u \beta_2 K_2 F_y CB)^{-1} (-F_u \varepsilon K_3) \eta \\
\quad(4.1f)
\end{align}

where all inverse matrices generically exist.
4.2 Earthquake Building

The state space equation and PID controller are given below:

\[
\begin{align*}
\dot{x} &= Ax + Bu + Gu_g \\
y &= F_yCx \\
u &= -F_u(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta)
\end{align*}
\]

where \( \beta_1, \beta_2, \epsilon, K_1, K_2, K_3 \) are constants associated with the PID controller.

\[
\eta = \int_0^t (y - y_{\text{ref}}) dt = \int_0^t F_yCx \, dt
\]

Note: \( y_{\text{ref}} = 0 \) for earthquake control system.

On substituting the PID controller into the state space equations and expanding, we obtain:

\[
\begin{align*}
\dot{x} &= Ax + BF_u(-\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) + Gu_g \\
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= Ax - BF_u \beta_1 K_1 F_y C x - BF_u \beta_2 K_2 \frac{d}{dt} (F_y C x) - BF_u \epsilon K_3 \eta + Gu_g \\
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= Ax - BF_u \beta_1 K_1 F_y C x - BF_u \beta_2 K_2 F_y C \dot{x} - BF_u \epsilon K_3 \eta + Gu_g \\
\end{align*}
\]

\[
\begin{align*}
(I + BF_u \beta_2 K_2 F_y C) \dot{x} &= Ax - BF_u \beta_1 K_1 F_y C x - BF_u \epsilon K_3 \eta + Gu_g \\
\end{align*}
\]

\[
\begin{align*}
\dot{x} &= (I + BF_u \beta_2 K_2 F_y C)^{-1} (A - BF_u \beta_1 K_1 F_y C) x - (I + BF_u \beta_2 K_2 F_y C)^{-1} BF_u \epsilon K_3 \eta + (I + BF_u \beta_2 K_2 F_y C)^{-1} Gu_g
\end{align*}
\]
and on expanding the controller, we obtain:

\[ u = -F_u \beta_1 K_1 y - F_u \beta_2 K_2 \dot{y} - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 \frac{d}{dt} (F_y C x) - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C \dot{x} - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C (Ax + Bu + G \ddot{u}_g) - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C (Ax + Bu) - F_u \epsilon K_3 \eta \]

\[ (I + F_u \beta_2 K_2 F_y CB) u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C Ax - F_u \beta_2 K_2 F_y C G \ddot{u}_g - F_u \epsilon K_3 \eta \]

\[ u = (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y C A x) + (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \beta_2 K_2 F_y C G \ddot{u}_g) \]

\[ + (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \epsilon K_3) \eta \]

On collecting terms we obtain the closed loop system:

\[ \begin{bmatrix} \dot{x} \\ \eta \end{bmatrix} = \begin{bmatrix} (I + BF_u \beta_2 K_2 F_y C)^{-1} (A - BF_u \beta_1 K_1 F_y C) & -(I + BF_u \beta_2 K_2 F_y C)^{-1} B F_u \epsilon K_3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} (I + BF_u \beta_2 K_2 F_y C)^{-1} G \\ 0 \end{bmatrix} \ddot{u}_g \] (4.2d)

\[ y = [F_y C \ 0] \begin{bmatrix} x \\ \eta \end{bmatrix} \] (4.2e)

\[ u = \begin{bmatrix} (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y C A) & (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \epsilon K_3) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} (I + F_u \beta_2 K_2 F_y C B)^{-1} (-F_u \beta_2 K_2 F_y C G) \\ 0 \end{bmatrix} \ddot{u}_g \] (4.2f)

where all inverse matrices generically exist.
4.3 Building Temperature Controller

The state space equation and PID controller are given below

\[ \dot{x} = Ax + Bu + e_{in} + e_{out} \]
\[ y = F_yCx \]
\[ u = -F_u(-\beta_1K_1y - \beta_2K_2\dot{y} - \epsilon K_3\eta) \]

where \( \beta_1, \beta_2, \epsilon, K_1, K_2, K_3 \) are constants associated with the PID controller

\[ \eta = \int_0^t (y - y_{ref}) \, dt \]

On substituting the PID controller into the state space equations and expanding, we obtain:

\[ \dot{x} = Ax + BF_u(-\beta_1K_1y - \beta_2K_2\dot{y} - \epsilon K_3\eta) + e_{in} + e_{out} \]

\[ \dot{x} = Ax - BF_u\beta_1K_1y - BF_u\beta_2K_2\dot{y} - BF_u\epsilon K_3\eta + e_{in} + e_{out} \]

\[ \dot{x} = Ax - BF_u\beta_1K_1F_yCx - BF_u\beta_2K_2\frac{d}{dt}(F_yCx) - BF_u\epsilon K_3\eta + e_{in} + e_{out} \]

\[ (I + BF_u\beta_2K_2F_yC)\dot{x} = Ax - BF_u\beta_1K_1F_yCx - BF_u\epsilon K_3\eta + e_{in} + e_{out} \]

\[ \dot{x} = (I + BF_u\beta_2K_2F_yC)^{-1}(A - BF_u\beta_1K_1F_yC)x - (I + BF_u\beta_2K_2F_yC)^{-1}BF_u\epsilon K_3\eta + (I + BF_u\beta_2K_2F_yC)^{-1}(e_{in} + e_{out}) \]
and on expanding the controller, we obtain:

\[ u = -F_u(\beta_1 K_1 y - \beta_2 K_2 \dot{y} - \epsilon K_3 \eta) \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 \frac{d}{dt}(F_y C x) - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y \dot{C} x - F_u \epsilon K_3 \eta \]

\[ u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C(A x + B u + e_{in} + e_{out}) - F_u \epsilon K_3 \eta \]

\[ (I + F_u \beta_2 K_2 F_y CB) u = -F_u \beta_1 K_1 F_y C x - F_u \beta_2 K_2 F_y C A x - F_u \beta_2 K_2 F_y C(e_{in} + e_{out}) - F_u \epsilon K_3 \eta \]

\[ u = (I + F_u \beta_2 K_2 F_y CB)^{-1}(-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y C A)x - (I + F_u \beta_2 K_2 F_y CB)^{-1}F_u \beta_2 K_2 F_y C(e_{in} + e_{out}) \]

\[ - (I + F_u \beta_2 K_2 F_y CB)^{-1}(-F_u \epsilon K_3) \eta \]

\[ u = (I + F_u \beta_2 K_2 F_y CB)^{-1}(-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y C A)x + (I + F_u \beta_2 K_2 F_y CB)^{-1}(-F_u \beta_1 K_1 F_y C - F_u \beta_2 K_2 F_y C E ) \omega \]

\[ + (I + F_u \beta_2 K_2 F_y CB)^{-1}(-F_u \epsilon K_3) \eta \]
On collecting terms, we obtain the closed loop system:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\eta}
\end{bmatrix} = \begin{bmatrix}
(I + BF_u\beta_2K_2F_yC)^{-1}(A - BF_u\beta_1K_1F_yC) & -(I + BF_u\beta_2K_2F_yC)^{-1}BF_u\epsilon K_3 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x \\
\eta
\end{bmatrix} + \begin{bmatrix}
(I + BF_u\beta_2K_2F_yC)^{-1} \\
0
\end{bmatrix} (e_{in} + e_{out}) - \begin{bmatrix}
0 \\
1
\end{bmatrix} y_{ref}
\]  

(4.3d)

\[
y = [C \quad 0] \begin{bmatrix}
x \\
\eta
\end{bmatrix}
\]  

(4.3e)

\[
u = \left[(I + F_u\beta_2K_2F_yCB)^{-1}(-F_u\beta_1K_1C - F_u\beta_2K_2F_yCA) \quad (I + F_u\beta_2K_2F_yCB)^{-1}(-F_u\epsilon K_3)\right] \begin{bmatrix}
x \\
\eta
\end{bmatrix} \\
- \left[(I + F_u\beta_2K_2F_yCB)^{-1}F_u\beta_2K_2F_yC\right] (e_{in} + e_{out})
\]  

(4.3f)

where all inverse matrices generically exist.
Chapter 5 – PID Controller Optimization

The closed loop system for the three sample problems for the nominal and failure case have been developed in Chapter 3 and Chapter 4. However, the choice of the controllers has not been made. This will be carried out by using the Nelder-Mead parameter optimization method, which shall be used to minimize a three variable optimization problem to find the optimal controller parameters.

5.1 Nelder-Mead Parameter Optimization Method

Consider now the open loop asymptotically stable LTI system defined below

\[ \dot{x} = Ax + Bu + E\omega \]
\[ y = Cx + F\omega \]
\[ e = y - y_{ref} \]  

where \( \omega \) and \( y_{ref} \) are constant vectors, and assume that the system is collocated, i.e. \( CB = 0 \), and that the existence result defined in Theorem 2.3b is satisfied. The control objective now is to solve the Decentralized robust servomechanism problem [17] and this will be done by implementing the following 3 term controller.

\[ u_i = -\beta_p e_i - \beta_d \dot{y}_i - \beta_i \eta_i \]
\[ \eta_i = \int_0^t (y_i - y_{ref}) dt \]  

\[ i = 1, 2, ..., \nu \]

where \( u_i, \eta_i, i = 1, 2, ..., \nu \) are scalars, and where \( \beta_p, \beta_d, \beta_i \) are scalar gains yet to be determined.

To simplify the notation, let (5.1b) be rewritten as

\[ u = -\beta_p e - \beta_d \dot{y} - \beta_i \eta \]
\[ \eta = \int_0^t (y - y_{ref}) dt \]  

where \( u \in \mathbb{R}^\nu, y \in \mathbb{R}^\nu, \eta \in \mathbb{R}^\nu, e \in \mathbb{R}^\nu \).
Optimal Controller Parameter Determination

The three scalar parameters $\beta_p, \beta_d, \beta_I$ will now be obtained by generalizing the Nelder-Mead parameter optimization method [22], from a centralized controller problem to a decentralized control problem. Given the plant (5.1a), we introduce the performance index:

$$J_{\beta_p, \beta_d, \beta_I} = \int_0^\infty (e^T e + \epsilon \dot{u}^T \dot{u}) d\tau$$  \hspace{1cm} (5.1d)

where $e = y - y_{\text{ref}}$ is the error signal, and $\dot{u}$ is the input’s rate of change. In this case, a choice of $\epsilon > 0$ must be made. The resultant controller obtained will result in a faster speed of response as $\epsilon \to 0$. The actual choice of $\epsilon$ to be made would depend on engineering considerations.

There are three problems to consider. One is to optimize the system with respect to tracking signals, which is important for the large flexible space structure problem and the large building heat control problem. Another one is to optimize the system with respect to disturbance rejection, which is important for the earthquake building problem, and also for the LFSS problem. To accomplish this objective, we shall minimize the performance index (5.1d).

In particular, the goal is to minimize the performance index (5.1d), which will be done initially by considering the derivative of the plant (5.1a) to obtain:

$$\begin{bmatrix} \ddot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \dot{u}$$  \hspace{1cm} (5.1e)

$$e = [0 \ I] \begin{bmatrix} \dot{x} \\ e \end{bmatrix}.$$
Now substitute the controller equation (5.1c) into (6.1e) to obtain, for $\tilde{x} := \begin{bmatrix} \dot{x} \\ e \end{bmatrix}$, the resultant closed-loop equations:

$$
\begin{align*}
\dot{\tilde{x}} &= \tilde{A}\tilde{x} \\
e &= \tilde{C}\tilde{x} \\
\dot{u} &= \tilde{C}_u\tilde{x}
\end{align*}
$$

(5.1f)

where

$$
\begin{align*}
\tilde{A} &= \begin{bmatrix} A - B(B_pC + B_dCA) & -\beta_tB \\ C & 0 \end{bmatrix} \\
\tilde{C} &= [0 \quad I] \\
\tilde{C}_u &= [-B_pC + B_dCA \quad \beta_t].
\end{align*}
$$

(5.1g)

Recall that $CB = 0$

Next, substitute $e$ and $\dot{u}$ from (5.1f) into (5.1d) to obtain:

$$
\begin{align*}
J_{\beta_p,\beta_d,\beta_t} &= \int_0^\infty (\tilde{C}\tilde{x})^T(\tilde{C}\tilde{x}) + \epsilon(\tilde{C}_u\tilde{x})^T(\tilde{C}_u\tilde{x})d\tau \\
J_{\beta_p,\beta_d,\beta_t} &= \int_0^\infty \tilde{x}^T\tilde{C}^T\tilde{C}\tilde{x} + \epsilon\tilde{x}^T\tilde{C}_u^T\tilde{C}_u\tilde{x}d\tau
\end{align*}
$$

where $\tilde{x} = e^{\tilde{A}T}\tilde{x}(0)$

$$
\begin{align*}
J_{\beta_p,\beta_d,\beta_t} &= \tilde{x}^T(0)\left[ \int_0^\infty e^{\tilde{A}T}\left(\tilde{C}\tilde{C} + \epsilon\tilde{C}_u\tilde{C}^T\tilde{C}_u\right)e^{\tilde{A}T}d\tau \right] \tilde{x}(0) \tag{5.1h}
\end{align*}
$$

or

$$
\begin{align*}
J_{\beta_p,\beta_d,\beta_t} &= \tilde{x}^T(0)\Gamma\tilde{x}(0) \tag{5.1i}
\end{align*}
$$

where $\Gamma > 0$ is the solution of the Lyapunov equation

$$
\tilde{A}^T\Gamma + \Gamma\tilde{A} = -\left(\tilde{C}\tilde{C} + \epsilon\tilde{C}_u\tilde{C}^T\tilde{C}_u\right). \tag{5.1j}
$$

Assume now that the controller is initialized now so that $u(0) = 0$, and choose the performance index:

$$
J_{y_{ref}} = \text{trace}\left( [0 \quad I] \Gamma \begin{bmatrix} 0 \\ I \end{bmatrix} \right). \tag{5.1k}
$$
Using this performance index measures the “average cost” of (5.1d) for all tracking and disturbance signals. In particular, we carry out a 3 dimensional parameter optimization to minimize $J_{y_{ref}}$, using an initial starting point $\beta_p, \beta_d, \beta_t$ which stabilizes the resultant closed loop system. The choice of the Nelder-Mead optimization was used (other parameter optimization methods could also be used).

The results obtained by using the Nelder-Mead parameter optimization method is presented in the following subsections. Subsection 5.2, 5.3 and 5.4 will present the optimal controller for the LFSS Problem, Earthquake Building Problem and Building Temperature Problem respectively.

5.2 Large Flexible Space Structure Results

This subsection presents the optimized controller for the large flexible space structure problem. The list of the open loop eigenvalues of the LFSS is presented below.

Table 5.2a List of the open loop eigenvalues of the large flexible space structure

<table>
<thead>
<tr>
<th>Real Part</th>
<th>Imaginary Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.0000e-07</td>
<td>-3.9327e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>6.6714e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>1.8451e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>5.8482e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>1.1322e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>7.6999e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>9.0464e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>7.3007e-02i</td>
</tr>
<tr>
<td>-5.0000e-07</td>
<td>3.1942e-02i</td>
</tr>
<tr>
<td>0.0000e+00</td>
<td>0.0000e+00i</td>
</tr>
<tr>
<td>0.0000e+00</td>
<td>0.0000e+00i</td>
</tr>
<tr>
<td>-1.0000e-06</td>
<td>0.0000e+00i</td>
</tr>
</tbody>
</table>

In this case, the plant is unstable with 3 rigid body modes located at (0,0,0). However, on examining the rigid body modes of the LFSS, we can observe that the controller
\( u = -ky \), where \( k > 0 \), will stabilize the rigid body modes of the system. In this case, we applied the controller \( u = -1e - 5y \) to stabilize the closed loop system, and the resultant stabilizing controller produced the following closed loop system eigenvalues.

Table 5.2b. List of Closed Loop Eigenvalues Using the Stabilized Controller

<table>
<thead>
<tr>
<th>Eigenvalue 1</th>
<th>Eigenvalue 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5.0000e-07 + 7.6999e-02i)</td>
<td>(-5.0000e-07 + 3.9327e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 7.6999e-02i)</td>
<td>(-5.0000e-07 - 3.9327e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 + 5.8482e-02i)</td>
<td>(-5.0000e-07 + 1.942e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 5.8482e-02i)</td>
<td>(-5.0000e-07 - 1.942e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 + 9.0464e-02i)</td>
<td>(-5.0000e-07 + 1.8451e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 9.0464e-02i)</td>
<td>(-5.0000e-07 - 1.8451e-02i)</td>
</tr>
<tr>
<td>(-5.0000e-07 + 1.1322e-02i)</td>
<td>(-5.0000e-07 + 1.5886e-06i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 1.1322e-02i)</td>
<td>(-5.0000e-07 - 1.5886e-06i)</td>
</tr>
<tr>
<td>(-5.0000e-07 + 7.3007e-02i)</td>
<td>(-5.0000e-07 + 2.6458e-06i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 7.3007e-02i)</td>
<td>(-5.0000e-07 - 2.6458e-06i)</td>
</tr>
<tr>
<td>(-5.0000e-07 + 6.6714e-02i)</td>
<td>(-5.0000e-07 + 2.6458e-06i)</td>
</tr>
<tr>
<td>(-5.0000e-07 - 6.6714e-02i)</td>
<td>(-5.0000e-07 - 2.6458e-06i)</td>
</tr>
</tbody>
</table>

The final controller obtained now on carrying out the parameter optimization algorithm, to minimize \( J_{ref} \) (5.1k), assumed that \( \varepsilon = 1e - 9 \) in (5.1d) was used, and minimized the performance index (5.1k). The final optimal controller obtained is given as follows:

\[
\begin{align*}
  u_i &= -4.86 \times 10^6 e_i - 1.37 \times 10^8 \dot{y}_i - 1.50 \times 10^5 \eta_i \\
  \dot{\eta}_i &= 0\eta_i + (y - y_{ref}), i = 1,2,3,4,5 \\
\end{align*}
\]  

(5.2a)

The controller presented in (5.2a) will be used in the simulation plot. In this case the transient settling time was approximately 120 seconds. Other controllers that were obtained for different values of \( \varepsilon \) are given in Table 5.2c. It may be seen that excellent control has been obtained in all cases and that the controllers are simple to implement.
Table 5.2d shows the rate of progress of the parameter optimization obtained for this example.

Table 5.2c. Optimized $\beta_p, \beta_d, \beta_l$ and $t_{\text{settle}}$ with different $\epsilon$

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$\beta_p$</th>
<th>$\beta_d$</th>
<th>$\beta_l$</th>
<th>$t_{\text{settle}}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.31e3</td>
<td>6.05e5</td>
<td>1.37e1</td>
<td>1250</td>
</tr>
<tr>
<td>1e-2</td>
<td>1.79e4</td>
<td>1.57e6</td>
<td>1.32e2</td>
<td>650</td>
</tr>
<tr>
<td>1e-5</td>
<td>2.04e5</td>
<td>9.64e6</td>
<td>3.42e3</td>
<td>450</td>
</tr>
<tr>
<td>1e-10</td>
<td>1.29e9</td>
<td>9.29e9</td>
<td>8.79e7</td>
<td>70</td>
</tr>
<tr>
<td>1e-15</td>
<td>3.31e8</td>
<td>1.38e8</td>
<td>4.58e8</td>
<td>4</td>
</tr>
</tbody>
</table>

Note that $t_{\text{settle}}$ is the time required to reach steady state ($t_{\text{rise}}$).

Table 5.2d. Parameter Optimal Control Convergence $\epsilon = 1e - 9$ with Initial Starting Point $\beta_p = 1e - 8, \beta_d = 1e - 4, \beta_l = 1e - 6$

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\beta_p$</th>
<th>$\beta_d$</th>
<th>$\beta_l$</th>
<th>Duration (sec)</th>
<th>Function Count</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1e-8</td>
<td>1e-4</td>
<td>1e-6</td>
<td>0.85</td>
<td>600</td>
<td>2.41e10</td>
</tr>
<tr>
<td>2</td>
<td>-2.43e10</td>
<td>-2.43e7</td>
<td>-2.64e4</td>
<td>0.63</td>
<td>423</td>
<td>3.69e6</td>
</tr>
<tr>
<td>3</td>
<td>-3.45e6</td>
<td>-5.20e7</td>
<td>-5.62e4</td>
<td>0.51</td>
<td>338</td>
<td>1.94e2</td>
</tr>
<tr>
<td>4</td>
<td>-4.86e6</td>
<td>-1.37e8</td>
<td>-1.50e5</td>
<td>0.42</td>
<td>247</td>
<td>1.58e2</td>
</tr>
<tr>
<td>5</td>
<td>-4.81e6</td>
<td>-1.37e8</td>
<td>-1.50e5</td>
<td>0.35</td>
<td>247</td>
<td>1.58e2</td>
</tr>
<tr>
<td>6</td>
<td>-4.81e6</td>
<td>-1.37e8</td>
<td>-1.50e5</td>
<td>0.32</td>
<td>247</td>
<td>1.58e2</td>
</tr>
</tbody>
</table>

The final optimal controller parameters obtained are $\beta_p = 4.86e6, \beta_d = 1.37e8, \beta_l = 1.50e5$, which produced the Cost = 1.58e2.

A list of the closed loop eigenvalues of the system using the controller (5.2a) are given in Table 5.2e.

Table 5.2e. List of Closed Loop Eigenvalues Using Proposed Optimal Controller

| -3.886e+03 + 0.000e+00i | -4.601e-07 + 2.0613e-02i |

33
A comparison of the eigenvalues of the proposed three term controller with the standard optimal LQR observer controller with $\epsilon = 10^{-5}$ is given below.

Table 5.2f. List of Eigenvalues for LQR Observer controller

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.423e+02 + 0.000e+00i</td>
<td>-4.601e-07 - 2.0613e-02i</td>
</tr>
<tr>
<td>-6.015e+03 + 0.000e+00i</td>
<td>-4.556e-07 + 2.0373e-02i</td>
</tr>
<tr>
<td>-5.448e+03 + 0.000e+00i</td>
<td>-4.556e-07 - 2.0373e-02i</td>
</tr>
<tr>
<td>-3.705e+03 + 0.000e+00i</td>
<td>-1.752e-02 + 2.8037e-02i</td>
</tr>
<tr>
<td>-1.244e-06 + 5.4727e-02i</td>
<td>-1.752e-02 - 2.8037e-02i</td>
</tr>
<tr>
<td>-1.244e-06 - 5.4727e-02i</td>
<td>-1.751e-02 + 2.8034e-02i</td>
</tr>
<tr>
<td>-6.260e-07 + 3.4069e-02i</td>
<td>-1.751e-02 - 2.8034e-02i</td>
</tr>
<tr>
<td>-6.260e-07 - 3.4069e-02i</td>
<td>-1.751e-02 + 2.8034e-02i</td>
</tr>
<tr>
<td>-5.012e-07 + 7.3098e-02i</td>
<td>-1.751e-02 - 2.8034e-02i</td>
</tr>
<tr>
<td>-5.012e-07 - 7.3098e-02i</td>
<td>-1.751e-02 + 2.8034e-02i</td>
</tr>
<tr>
<td>-7.608e-07 + 5.6929e-02i</td>
<td>-1.751e-02 - 2.8034e-02i</td>
</tr>
<tr>
<td>-7.608e-07 - 5.6929e-02i</td>
<td>-1.751e-02 + 2.8034e-02i</td>
</tr>
<tr>
<td>-6.042e-07 + 4.5009e-02i</td>
<td>-1.751e-02 - 2.8034e-02i</td>
</tr>
<tr>
<td>-6.042e-07 - 4.5009e-02i</td>
<td>-1.751e-02 + 2.8034e-02i</td>
</tr>
<tr>
<td>-5.7639e-01 + 7.9367e-01i</td>
<td>-2.0931e-05 + 5.4723e-02i</td>
</tr>
<tr>
<td>-5.7639e-01 - 7.9367e-01i</td>
<td>-2.0931e-05 - 5.4723e-02i</td>
</tr>
<tr>
<td>-7.9517e-01 + 5.7168e-01i</td>
<td>-5.0011e-07 + 5.8482e-02i</td>
</tr>
<tr>
<td>-7.9517e-01 - 5.7168e-01i</td>
<td>-5.0011e-07 - 5.8482e-02i</td>
</tr>
<tr>
<td>-3.6165e-01 + 6.2982e-01i</td>
<td>-5.7532e-06 + 4.5009e-02i</td>
</tr>
<tr>
<td>-3.6165e-01 - 6.2982e-01i</td>
<td>-5.7532e-06 - 4.5009e-02i</td>
</tr>
<tr>
<td>-3.4865e-01 + 5.9860e-01i</td>
<td>-3.2579e-05 + 3.4071e-02i</td>
</tr>
<tr>
<td>-3.4865e-01 - 5.9860e-01i</td>
<td>-3.2579e-05 - 3.4071e-02i</td>
</tr>
<tr>
<td>-6.9579e-01 + 0.0000e+00i</td>
<td>-5.0682e-07 + 3.9327e-02i</td>
</tr>
<tr>
<td>-7.2458e-01 + 0.0000e+00i</td>
<td>-5.0682e-07 - 3.9327e-02i</td>
</tr>
<tr>
<td>Real Part</td>
<td>Imaginary Part</td>
</tr>
<tr>
<td>-----------</td>
<td>---------------</td>
</tr>
<tr>
<td>-1.4498e-01 + 2.5507e-01i</td>
<td>-5.0157e-07 + 3.1942e-02i</td>
</tr>
<tr>
<td>-1.4498e-01 - 2.5507e-01i</td>
<td>-5.0157e-07 - 3.1942e-02i</td>
</tr>
<tr>
<td>-2.9004e-01 + 0.0000e+00i</td>
<td>-1.7734e-09 + 2.0613e-02i</td>
</tr>
<tr>
<td>-6.2085e-05 + 1.4991e-01i</td>
<td>-1.7734e-09 - 2.0613e-02i</td>
</tr>
<tr>
<td>-6.2085e-05 - 1.4991e-01i</td>
<td>-2.1535e-06 + 2.0373e-02i</td>
</tr>
<tr>
<td>-5.0296e-07 + 9.0464e-02i</td>
<td>-2.1535e-06 - 2.0373e-02i</td>
</tr>
<tr>
<td>-5.0296e-07 - 9.0464e-02i</td>
<td>-5.1899e-07 + 1.1322e-02i</td>
</tr>
<tr>
<td>-5.0096e-07 + 7.6999e-02i</td>
<td>-5.1899e-07 - 1.1322e-02i</td>
</tr>
<tr>
<td>-5.0096e-07 - 7.6999e-02i</td>
<td>-5.1915e-07 + 1.8451e-02i</td>
</tr>
<tr>
<td>-4.8477e-07 + 7.3098e-02i</td>
<td>-5.1915e-07 - 1.8451e-02i</td>
</tr>
<tr>
<td>-4.8477e-07 - 7.3098e-02i</td>
<td>-2.0946e-05 + 2.0935e-05i</td>
</tr>
<tr>
<td>-5.0316e-07 + 6.6714e-02i</td>
<td>-2.0946e-05 - 2.0935e-05i</td>
</tr>
<tr>
<td>-5.0316e-07 - 6.6714e-02i</td>
<td>-3.3857e-05 + 3.3860e-05i</td>
</tr>
<tr>
<td>-5.0000e-07 + 7.3007e-02i</td>
<td>-3.3857e-05 - 3.3860e-05i</td>
</tr>
<tr>
<td>-5.0000e-07 - 7.3007e-02i</td>
<td>-3.3867e-05 + 3.3850e-05i</td>
</tr>
<tr>
<td>1.8941e-05 + 5.6921e-02i</td>
<td>-3.3867e-05 - 3.3850e-05i</td>
</tr>
<tr>
<td>1.8941e-05 - 5.6921e-02i</td>
<td></td>
</tr>
</tbody>
</table>

It may be noted that the proposed controller has the same performance as the LQR controller in spite of the fact that it has a much smaller order.
5.3 Earthquake Building Control Results

This subsection presents the optimized controller for the earthquake building problem. In this case the plant is open loop stable.

Table 5.3a List of the open loop eigenvalues of the earthquake building controller

<table>
<thead>
<tr>
<th>Eigenvalue 1</th>
<th>Eigenvalue 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.2065e-04 + 1.9845e+01i</td>
<td>-4.1297e-05 + 1.1611e+01i</td>
</tr>
<tr>
<td>-1.2065e-04 - 1.9845e+01i</td>
<td>-4.1297e-05 - 1.1611e+01i</td>
</tr>
<tr>
<td>-1.0034e-04 + 1.8098e+01i</td>
<td>-1.6092e-05 + 7.2479e+00i</td>
</tr>
<tr>
<td>-1.0034e-04 - 1.8098e+01i</td>
<td>-1.6092e-05 - 7.2479e+00i</td>
</tr>
<tr>
<td>-7.1700e-05 + 1.5299e+01i</td>
<td>-1.8594e-06 + 2.4637e+00i</td>
</tr>
<tr>
<td>-7.1700e-05 - 1.5299e+01i</td>
<td>-1.8594e-06 - 2.4637e+00i</td>
</tr>
</tbody>
</table>

The final controller obtained on carrying out the same parameter optimization algorithm, assumed that $\epsilon = 1$ in (5.1d) was used, and minimized the performance index (5.1k). The final optimal controller obtained is given as follows:

$$u_i = -3.87 \times 10^8 e_i - 9.87 \times 10^8 \dot{y}_i - 0.5 \times 10^8 \eta_i$$  \hspace{1cm} (5.3a)

$$\eta_i = 0\eta_i + (y - y_{ref}), i = 1,2,3,4$$

The controller presented in (5.3a) will be used in the simulation plot. Table 5.4b shows the rate of progress of the parameter optimization obtained for this example.

Table 5.3b. Parameter Optimal Control Convergence $\epsilon = 1$ with Initial Starting Point $\beta_p = -1e8, \beta_d = -1e4, \beta_t = -1e7$

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\beta_p$</th>
<th>$\beta_d$</th>
<th>$\beta_t$</th>
<th>Duration (sec)</th>
<th>Function Count</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1e8</td>
<td>-1e4</td>
<td>-1e7</td>
<td>1.20</td>
<td>600</td>
<td>2.715e9</td>
</tr>
<tr>
<td>2</td>
<td>-3.87e8</td>
<td>-9.87e8</td>
<td>-0.5e8</td>
<td>0.83</td>
<td>602</td>
<td>4.878e6</td>
</tr>
</tbody>
</table>
The final optimal controller parameters obtained are $\beta_p = 3.87e3, \beta_d = 9.87e3, \beta_I = 0.5e3$, which produced the Cost = $2.501e3$. In this case, the optimization algorithm converged on the fourth iteration.

A list of the closed loop eigenvalues of the system using the controller (5.3a) are given in Table 5.3c.

<table>
<thead>
<tr>
<th></th>
<th>-5.6395e+03 + 0.0000e+00i</th>
<th>-5.6396e+03 + 0.0000e+00i</th>
<th>-5.6396e+03 + 0.0000e+00i</th>
<th>-5.6396e+03 + 0.0000e+00i</th>
<th>-5.6396e+03 + 0.0000e+00i</th>
<th>-2.8283e-01 + 0.0000e+00i</th>
<th>-1.9660e-01 + 1.0960e-01i</th>
<th>-1.9660e-01 - 1.0960e-01i</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.0072e-01 + 1.0186e-01i</td>
<td>2.0072e-01 - 1.0186e-01i</td>
<td>2.0801e-01 + 8.5987e-02i</td>
<td>2.0801e-01 - 8.5987e-02i</td>
<td>2.1681e-01 + 6.0460e-02i</td>
<td>2.1681e-01 - 6.0460e-02i</td>
<td>1.7913e-01 + 0.0000e+00i</td>
<td>2.2241e-01 + 0.0000e+00i</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.4 Large Building Temperature Control Results

This subsection presents the optimized controller for the large building temperature control problem. In this case the plant is open loop stable.

Table 5.4a List of the open loop eigenvalues of the large building temperature control

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.4034e+00</td>
<td>-3.7500e-02</td>
</tr>
<tr>
<td>-1.6773e-01</td>
<td>-3.7045e-02</td>
</tr>
<tr>
<td>-1.5238e-01</td>
<td>-3.5570e-02</td>
</tr>
<tr>
<td>-1.3355e-01</td>
<td>-3.3818e-02</td>
</tr>
<tr>
<td>-9.2749e-02</td>
<td>-3.2757e-02</td>
</tr>
<tr>
<td>-7.7961e-02</td>
<td>-3.0099e-02</td>
</tr>
<tr>
<td>-6.3597e-02</td>
<td>-1.3680e-02</td>
</tr>
<tr>
<td>-5.7466e-02</td>
<td>-9.4842e-03</td>
</tr>
<tr>
<td>-4.1923e-02</td>
<td>-6.6844e-03</td>
</tr>
<tr>
<td>-3.9453e-02</td>
<td>-3.8170e-02</td>
</tr>
</tbody>
</table>

The final controller obtained on carrying out the same parameter optimization algorithm, assumed that $\epsilon = 500$ in (5.1d) was used, and minimized the performance index (5.1k). The starting point of the optimization is given in Table (5.4c). The final optimal controller obtained is given as follows:

$$u_i = -4.94e_i - 12.3\dot{y}_i - 0.338\eta_i$$
$$\dot{\eta}_i = 0\eta_i + (y - y_{ref}), i = 1,2,3,4$$

(5.4a)

The controller presented in (5.4a) will be used in the simulation plot. In this case the transient settling time was approximately 119 seconds. Other controllers that were obtained for different values of $\epsilon$ are given in Table 5.4b. It may be seen that excellent control has been obtained in all cases and that the controllers are simple to implement.
Table 5.4c shows the rate of progress of the parameter optimization obtained for this example.

Table 5.4b. Optimized $\beta_p, \beta_d, \beta_I$ and $t_{\text{settle}}$ with different $\epsilon$

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$\beta_p$</th>
<th>$\beta_d$</th>
<th>$\beta_I$</th>
<th>$t_{\text{settle}}$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.5762e+01</td>
<td>9.4626e+00</td>
<td>2.0169e+00</td>
<td>15</td>
</tr>
<tr>
<td>100</td>
<td>6.1080e+00</td>
<td>1.3851e+01</td>
<td>6.7290e-01</td>
<td>48</td>
</tr>
<tr>
<td>500</td>
<td>4.9476e+00</td>
<td>1.2324e+01</td>
<td>3.3875e-01</td>
<td>119</td>
</tr>
<tr>
<td>1000</td>
<td>4.5998e+00</td>
<td>1.1523e+01</td>
<td>2.5707e-01</td>
<td>173</td>
</tr>
<tr>
<td>10000</td>
<td>3.7122e+00</td>
<td>7.9597e+00</td>
<td>1.0884e-01</td>
<td>251</td>
</tr>
</tbody>
</table>

Note that $t_{\text{settle}}$ is the time required to reach steady state ($t_{\text{rise}}$).

Table 5.4c. Parameter Optimal Control Convergence $\epsilon = 500$ with Initial Starting Point $\beta_p = 1e - 8, \beta_d = 1e - 4, \beta_I = 1e - 6$

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\beta_p$</th>
<th>$\beta_d$</th>
<th>$\beta_I$</th>
<th>Duration (sec)</th>
<th>Function Count</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1e-8</td>
<td>1e-4</td>
<td>1e-6</td>
<td>0.47</td>
<td>457</td>
<td>7.27e7</td>
</tr>
<tr>
<td>2</td>
<td>-7.56e-2</td>
<td>-1.00</td>
<td>-5.11e-3</td>
<td>0.20</td>
<td>236</td>
<td>56.3</td>
</tr>
<tr>
<td>3</td>
<td>-1.36</td>
<td>-0.548</td>
<td>-1.00</td>
<td>0.065</td>
<td>78</td>
<td>6.25</td>
</tr>
<tr>
<td>4</td>
<td>-1.36</td>
<td>-0.548</td>
<td>-1.00</td>
<td>0.042</td>
<td>78</td>
<td>6.25</td>
</tr>
<tr>
<td>5</td>
<td>-1.36</td>
<td>-0.548</td>
<td>-1.00</td>
<td>0.063</td>
<td>78</td>
<td>6.25</td>
</tr>
<tr>
<td>6</td>
<td>-1.36</td>
<td>-0.548</td>
<td>-1.00</td>
<td>0.51</td>
<td>78</td>
<td>6.25</td>
</tr>
</tbody>
</table>

The final optimal controller parameters obtained are $\beta_p = -1.36, \beta_d = -0.548, \beta_I = -1$, which produced the Cost = 6.25. In this case, the optimization algorithm converged on the third iteration.
A list of the closed loop eigenvalues of the system using the controller (5.4a) are given in Table 5.4d.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.2599e-01 + 0.0000e+00i</td>
<td>-3.0009e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-3.1166e-01 + 0.0000e+00i</td>
<td>-4.2539e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-3.1187e-01 + 0.0000e+00i</td>
<td>-3.2771e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-3.1206e-01 + 0.0000e+00i</td>
<td>-3.4105e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-9.5685e-02 + 0.0000e+00i</td>
<td>-3.5749e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-8.3505e-02 + 0.0000e+00i</td>
<td>-3.5518e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-8.6712e-02 + 1.3226e-03i</td>
<td>-4.0530e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-8.6712e-02 - 1.3226e-03i</td>
<td>-4.0019e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-8.7395e-02 + 0.0000e+00i</td>
<td>-4.0024e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-8.7095e-02 + 0.0000e+00i</td>
<td>-3.7551e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-6.7326e-02 + 0.0000e+00i</td>
<td>-3.7500e-02 + 0.0000e+00i</td>
</tr>
<tr>
<td>-6.0857e-02 + 0.0000e+00i</td>
<td>-2.8900e-02 + 0.0000e+00i</td>
</tr>
</tbody>
</table>
Chapter 6 – Simulation Results

This chapter presents the simulated plots for the three example control problems, where the respective three control parameters for each example problem were obtained in Chapter 5. The simulation was done through Matlab.

6.1 Large Flexible Space Structure Control Problem

This section presents the simulation result of a large flexible space structure. There are 4 different types of cases within this section.

1. Nominal case – No failure
2. Actuator 3 and Sensor 3 both fail
3. Actuator 3 fails
4. Sensor 3 fails

There are 6 plots within each case

a) Output response $y_1 - y_5$ for tracking $y_{ref_1} - y_{ref_5}$

b) Actuator response $u_1 - u_5$ for tracking $y_{ref_1} - y_{ref_5}$

c) Output response $y_1 - y_5$ for rejecting $E$ disturbance ($F = 0$)

d) Actuator response $u_1 - u_5$ for rejecting $E$ disturbance ($F = 0$)

e) Output response $y_1 - y_5$ for rejecting $F$ disturbance ($E = 0$)

f) Actuator response $u_1 - u_5$ for rejecting $F$ disturbance ($E = 0$)
The $y_{ref}$ for each column is as shown below:

<table>
<thead>
<tr>
<th>Column 1 ($y_{ref_1}$)</th>
<th>Column 2 ($y_{ref_2}$)</th>
<th>Column 3 ($y_{ref_3}$)</th>
<th>Column 4 ($y_{ref_4}$)</th>
<th>Column 5 ($y_{ref_5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
</tr>
<tr>
<td>[0]</td>
<td>[1]</td>
<td>[0]</td>
<td>[1]</td>
<td>[0]</td>
</tr>
<tr>
<td>[0]</td>
<td>[0]</td>
<td>[1]</td>
<td>[0]</td>
<td>[1]</td>
</tr>
<tr>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
</tr>
</tbody>
</table>
Large Flexible Space Structure

1A: Output for tracking yref1-yref5

1B: Act for for tracking yref1-yref5

Time (seconds)
Large Flexible Space Structure

1C: Output for rejecting E

1D: Actuator for rejecting E
Large Flexible Space Structure

1E: Output for F with no failure

1F: Act for case of F with no failure
Large Flexible Space Structure

2A: yref1-yref5 with Sen#3 & Act#3 Failure

2B: Act for yref1-yref5 with Sen#3 & Act#3 Failure
Large Flexible Space Structure

2C: Rejecting E with Sen#3 & Act#3 Failure

Time (seconds)

2D: Act for rejecting E with Sen#3 & Act#3 Failure

Time (seconds)

47
Large Flexible Space Structure

2E: Rejecting F with Sen#3 & Act#3 Both fail

2F: Act for rejecting F with Sen#3 & Act#3 Both failure

Time (seconds)
Large Flexible Space Structure

3A: Tracking $y_{ref1}$-$y_{ref5}$ with Act#3 Failure

3B: Act for tracking $y_{ref1}$-$y_{ref5}$ with Act#3 Failure
Large Flexible Space Structure

3C: Rejecting E with Act#3 Failure

Time (seconds)

3D: Act for rejecting E with Act #3 Failure

Time (seconds)
Large Flexible Space Structure

3E: Response for rejecting F dis with Act #3 failure

3F: Act response for rejecting F dis with Act #3 failure
Large Flexible Space Structure

4A: Tracking yref1-yref5 with Sen#3 Failure

4B: Act for tracking yref1-yref5 with Sen#3 Failure
Large Flexible Space Structure

4C: Rejecting E with Sensor #3 Failure

4D: Actuator for rejecting E with Sen#3 Failure
Large Flexible Space Structure

4E: Rejecting F with Sen#3 failure

From: In(1)  From: In(2)  From: In(3)  From: In(4)  From: In(5)

\[ y_1 - y_5 \]

Time (seconds)

4F: Act for rejecting F with Sen#3 failure

From: In(1)  From: In(2)  From: In(3)  From: In(4)  From: In(5)

\[ (u_1 - u_5)/1e5 \]

Time (seconds)
Remarks on LFSS Plots

It is to be noted that for all failure modes, the resulting closed loop system becomes marginally unstable with a pole at the origin.

For case 1 (Figure 1A – Figure 1F), the nominal case, the PID controller performs very well, and the outputs are able to track the reference signals. In addition, the disturbances $E$ and $F$ are perfectly rejected.

For case 2 (Figure 2A – Figure 2F), we assume that there is a simultaneous sensor and actuator failure occurring. Although the system is not able to track the reference value where the failure occurs, it stabilizes the overall performance, so that the remaining stations, except for the failed station, can still track their corresponding reference signals. This occurs because the controller is decentralized, and failure at one station does not have a heavy impact on the performance of the other stations. Moreover the system can still reject disturbances $E$ and $F$ under the sensor and actuator failure. Although there are oscillations in the outputs when the system is rejecting disturbances, the output values decay toward zero as time elapse.

For case 3 (Figure 3A – Figure 3F), we assume that an actuator failure occurs, and thus the system is not able to track a reference signal occurring at the failed actuator, nor reject any disturbance associated with the failed actuator. It is to be noted however that the outputs which do not correspond to the failed actuator can still reach their desired value. It is also to be noticed that if one manually shuts down the sensor which corresponds to the failed actuator, the remaining part of the system can function without error.

For case 4 (Figure 4A – Figure 4F), we assume that a sensor failure case has occurred and in this case various serious problems can arise resulting in instability. In general, the system will be unbounded for tracking the reference signal. However, no major problem
will occur for disturbance rejection. If one shuts down the actuator corresponding to the failed sensor, the remaining control stations however will be operational.

It is to be noted that if a centralized controller was used to control the LFSS, then a failure of any type will results in the overall system having complete failure.
6.2 Earthquake Building Control Problem

This section presents the simulation results of an earthquake building control problem. There are 4 plots in this case. The first plot shows the ground’s acceleration vs time. It corresponds to a decaying sinusoidal wave, which often occurs for the case of earthquake motion. Then a plot of the building’s motion under no controller is shown. Lastly, a plot of the building’s motion under an optimized controller is presented.
Earthquake Building Control Problem

Building Response under Earthquake with Optimized PID Controller

Actuator Response under Earthquake with Optimized PID Controller
Remarks on Earthquake Buildings Plots

It is to be noted that for all failure modes, the resulting closed loop system becomes marginally unstable with a pole at the origin.

Although the ground acceleration caused by an earthquake in real life is not as uniform as what is used in the simulation, it can always be decomposed into a combination of sinusoidal functions by using the Fourier series technique. Thus, for the purpose of simulation, the ground acceleration is modelled by applying a negative exponential function onto a sinusoidal wave.

There are two noticeable improvements in the building response when a controller is applied. First, the displacement of each floor is reduced. The maximum deflection of the building without the controller, and with the optimized controller is 0.35m and 0.00018m respectively. Based on the above evidence, the optimized controller is shown to have significantly reduced the displacement of each floor when subject to a simulated earthquake. Secondly, the building’s oscillation frequency has also been reduced. There is always a warm up period for an actuator before it reaches its desired value. For example, the actuator itself has a controller to keep track of the reference signal, and there is always a period of settling time before the actuator reaches its desired thrust value. In conclusion, the actuator can better track its desired input signal when the oscillation frequency is reduced.

See appendix D to which shows that the controller is robust for different ground accelerations.
6.3 Large Building Temperature Control Problem

This section presents the simulation results of a large building temperature control problem.

There will be 4 different types of cases within this section.

1. Nominal case – No failure
2. Actuator 3 and Sensor 3 both fail
3. Actuator 3 fails
4. Sensor 3 fails

There will be 4 plots within each case

a) Output response $y_1 - y_4$ for tracking $y_{ref_1} - y_{ref_4}$

b) Actuator response $u_1 - u_4$ for tracking $y_{ref_1} - y_{ref_4}$

c) Output response $y_1 - y_4$ for rejecting $E$ disturbance

d) Actuator response $u_1 - u_4$ for rejecting $E$ disturbance

$y_{ref}$ for each column is as shown below:

<table>
<thead>
<tr>
<th>Column 1 ($y_{ref_1}$)</th>
<th>Column 2 ($y_{ref_2}$)</th>
<th>Column 3 ($y_{ref_3}$)</th>
<th>Column 4 ($y_{ref_4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Large Building Temperature Control

1A: Output for tracking yref1-yref4

Time (seconds)

1B: Act for tracking yref1-yref4

Time (seconds)
Large Building Temperature Control

1C: Output for rejecting E

1D: Actuator for rejecting E
Large Building Temperature Control

2A: yref1-yref5 with Sen#3 & Act#3 Failure

2B: Act for yref1-yref5 with Sen#3 & Act#3 Failure
Large Building Temperature Control

2C: Rejecting E with Sen#3 & Act #3 Failure

2D: Act for rejecting E with Sen#3 & Act#3 Failure
Large Building Temperature Control

3A: Tracking yref1-yref5 with Act#3 Failure

3B: Act for tracking yref1-yref5 with Act#3 Failure
Large Building Temperature Control

3C: Rejecting E with Act#3 Failure

Time (seconds)

3D: Act for rejecting E with Act #3 Failure

Time (seconds)
Large Building Temperature Control

4A: Tracking yref1-yref5 with Sen#3 Failure

4B: Act for tracking yref1-yref5 with Sen#3 Failure
Large Building Temperature Control

4C: Rejecting E with Sen#3 Failure

4D: Actuator for rejecting E with Sen#3 Failure
Remarks on Building Temperature Problem

It is to be noted that for all failure modes, the resulting closed loop system becomes marginally unstable with a pole at the origin.

For case 1 (Figure 1A – Figure 1F), the outputs are able to track the reference signals in a reasonable time. Although the optimization algorithm can make the system track reference signals in a much shorter time, this may require an excessive heating resource. It can be noted that heat disturbances from the inside and outside of the building are rejected perfectly.

For case 2 (Figure 2A – Figure 2F), the sensor and actuator failure case, the system under PID control was not able to track the reference signal on the failed station. This is to be expected (also observed in the LFSS simulation) since the failed station does not have a functional heat source anymore.

For case 3 (Figure 3A – Figure 3F), the actuator failure case, the system is not able to track the reference signal nor reject any disturbances. Although the outputs that do not correspond to the failed actuator can still reach their desired value, output 3 does not follow the desired command. The simplest solution in this case, is to manually shut down the sensor that corresponds to the failed actuator. By doing so, at least part of the system can function without error.

For case 4 (Figure 4A – Figure 4F), the single sensor failure case, the system is able to reject disturbances and track reference signals for those zones without failure. However, in plot 4B, all of the actuators in the 3rd column appear to be unbounded, and so the 3rd
output is also unstable. This did not show up in plot 4A because the sensor has failed, and so it cannot measure the actual temperature correctly.
Chapter 7 – Comparison of Proposed Controller vs. LQR Observer Controller

Measure of Robustness

In the decentralized controller design, there always will be some uncertainty as to mathematical model used in the modelling of the plant model. For example, there will always be high frequency effects which have been ignored, and so it would be useful to obtain some measure of how sensitive the controlled system is to high frequency modes of the plant. This can be done by finding the so called real stability radius $\Delta D$ [23] of the controlled plant.

In particular, consider the asymptotic stable system $\dot{x} = Ax + Bu, y = Cx$ that is subject to the following perturbation.

$$\begin{bmatrix} \dot{x} \\ \epsilon \eta \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \tilde{A} \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix} + \begin{bmatrix} B \\ \tilde{B} \end{bmatrix} u$$

$$y = \begin{bmatrix} C & \tilde{C} \end{bmatrix} \begin{bmatrix} x \\ \eta \end{bmatrix}$$

(7a)

where $\tilde{A}$ is assumed to be asymptotically stable, $\tilde{A}, \tilde{B}, \tilde{C}$ are unknown and $\epsilon > 0$ is a scalar.

Then in the limit as $\epsilon \to 0$, by singular perturbation analysis, the above system simplifies to:

$$\dot{x} = Ax + Bu$$

$$y = Cx + \Delta Du$$

(7b)

where $\Delta D = -\tilde{C} \tilde{A}^{-1} \tilde{B}$. Thus we can now consider $\Delta D$ to be an uncertain matrix, and we can now determine the real stability radius denoted by “rstab” for the system (7.7b) from [23], [24] which has the property that it is the largest bound such that the perturbed closed-loop system is stable for all $\|\Delta D\|_2 < "rstab"$. In particular, the real stability radius “rstab” is obtained by finding the largest value of the norm $\Delta D$ such that the perturbed closed-loop system is stable. For the LFSS example, the perturbed closed-loop system is given by:
\[
\begin{bmatrix}
A - B\beta_1 K_1 C - B\beta_2 K_2 CA & -B\varepsilon K_3 \\
0 & 0
\end{bmatrix} + [0 \ 1] \Delta D [0 \ I]
\]  
\text{(7c)}

and the real stability radius for (7c) can be directly obtained from [23]. It is to be noted that the stability radius “rstab” computed in [23] is exact. In this case, a summary of the real stability radius obtained for the proposed optimal three term controller for various values of \(\varepsilon\) is given in the following table for the proposed controller versus the standard LQR-observer controller.

Table 7a. Comparison of Robustness of Proposed Controller with LQR-Observer Controller

<table>
<thead>
<tr>
<th>(\varepsilon)</th>
<th>Real Stability Radius for Proposed Controller</th>
<th>Real Stability Radius for LQR-Observer Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.542e-3</td>
<td>3.410e-3</td>
</tr>
<tr>
<td>1e-2</td>
<td>2.542e-3</td>
<td>1.295e-3</td>
</tr>
<tr>
<td>1e-5</td>
<td>2.542e-3</td>
<td>5.294e-5</td>
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<tr>
<td>1e-9</td>
<td>2.542e-3</td>
<td>4.024e-6</td>
</tr>
<tr>
<td>1e-12</td>
<td>2.542e-3</td>
<td>2.581e-7</td>
</tr>
<tr>
<td>1e-14</td>
<td>2.542e-3</td>
<td>4.231e-8</td>
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</table>

i.e the real stability radius has the same robustness measure for all values of \(\varepsilon\). It is again to be noted that the real stability radius obtained in the above table is exact. For example, when the proposed controller is used, the system remains stable for all \(\Delta D\) matrices which have norm < 2.542e – 3; but this is not the case if \(\Delta D\) has norm \(\geq 2.542e – 3\). In particular, there always will be a \(\Delta D\) which destabilizes the system. As an example, when the \(\Delta D 5 \times 5\) matrix of Table 7a which has norm \(\Delta D = 2.542e – 3\) is applied to the proposed controller above, the perturbed closed loop system will be unstable, but will be stable if \(\|\Delta D\| < 2.542e – 3\).

It is also to be noted that the proposed controller has a real stability radius of 2.542e-3 for all values of \(\varepsilon\), as compared to the real stability radius of 4.231e-8 for the LQR-observer controller when \(\varepsilon = 1e – 14\), which is some five orders of magnitude worse than the
proposed controller. For completeness, the $\Delta D$ which produces the $rstab = 2.542e - 3$
for the proposed controller is given in Table 7b.

Table 7b. Destabilizing Perturbation $\Delta D$ which has Norm = 2.542e-3

<table>
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<tr>
<th>-1.9099e-2</th>
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Comparison of Closed Loop Eigenvalues for Single Sensor/Actuator Failure

Table 7c. Closed Loop Eigenvalues for LQR Observer Controller with Sensor 1 Failure

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<tr>
<td>-7.1328e-01 - 7.1293e-01i</td>
<td>-3.9399e-07 - 7.3098e-02i</td>
</tr>
<tr>
<td>-2.5272e-01 + 4.4379e-01i</td>
<td>4.7920e-05 + 5.6966e-02i</td>
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<tr>
<td>-2.5272e-01 - 4.4379e-01i</td>
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</tr>
<tr>
<td>-2.2851e-01 + 4.1077e-01i</td>
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<td>-2.2851e-01 - 4.1077e-01i</td>
<td>-1.2791e-03 - 5.6046e-02i</td>
</tr>
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Table 7d. Closed Loop Eigenvalues for LQR Observer Controller with Actuator 1 Failure

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### Table 7e. Closed Loop Eigenvalues for Proposed Controller with Sensor 3 Failure

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### Table 7f. Closed Loop Eigenvalues for Proposed Controller with Actuator 3 Failure

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As shown in Table 7c – Table 7f, the proposed controller only has a pole at zero for single actuator or sensor failure. However, the LQR-Observer controller has 12 positive poles, and one zero pole which can result in severe instability. As a result, the decentralized controller does have the advantage over the centralized controller in failure situations.
Chapter 8 – Conclusion & Future Development

This thesis has proposed a decentralized controller design method for the class of large scale systems, which can be described by the LTI model \( M_2 \ddot{y} + M_1 \dot{y} + M_0 y = Bu + E \omega \), where \( y \) is the output, \( u \) is the input and \( \omega \) is a constant disturbance/tracking set point signal. In particular, to illustrate the proposed design procedure, three examples are considered, namely a large flexible space structure, a building earthquake structure problem, and a building temperature control problem. In all examples, excellent control using a low order controller is obtained. In fact, it is somewhat surprising that the multivariable 3 term controller which is applied and which has only 3 scalar parameters, can produce the same type of high performance as a standard LQR-observer controller. In particular, in this study, the large flexible space structure plant model originally proposed in [13] has 5 inputs and 5 outputs and consists of 100 state variables. Due to space limitations, this plant model was reduced to a 24th order system, and in this case, if a LQR observer controller was used, it would have a 29th order controller, as compared to the 5th order controller which is obtained using our parameter optimization procedure. In addition, if the original 100th order plant model was used instead of the 24th order model, the LQR-observer controller would now consist of order 105, as compared to the present controller, which would still only have a 5th order controller. The same types of results apply also to the earthquake building problem and building temperature control problem.

The simulation results for all example problems contain 4 situations, namely – a Nominal system, a Sensor & Actuator Failure system, a Sensor Failure system, and an Actuator Failure system. In the nominal case, all decentralized control stations were able to track their reference signals and reject disturbances perfectly. In the sensor & actuator failure case, those stations that did not fail were still able to track reference signals and reject disturbances. The actuator failure case has the property that the outputs which do not correspond to the failed actuator will still reach their desired value. The sensor failure case however causes the actuator response to increase until the actuator saturates. One
simple method to solve this problem is to shut off the corresponding actuator once a sensor failure has occurred. It is interesting to note that in contrast to the proposed controller, the LQR Observer controller has the property that if a sensor or actuator fails, the resultant system has 12 unstable eigenvalues which will results in severe instability.

It is clear that this thesis topic can be extended to other large scale system problems such as described (1.1a). A new direction of research in the area of large scale systems can possibly be carried out by extending the present results to the class of unknown systems [25].
References


Appendix A

Appendix A presents the $A,B,C,E,F$ matrix of the LFSS problem.

\[ A(1:24,1:6) = \]

\[
\begin{array}{cccccc}
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0 & 0 & 0 & -3.42e-4 & 0 & 0 \\
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\end{array}
\]
\[ A(1: 24, 7: 12) = \]

\[
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\end{array}
\]
\[ A(1:24,13:18) = \]

\[
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0 & 0 & 0 & 0 & 0 & 0 & 1 \\
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-1.00e-06   0   0   0   0   0   0
0   -1.00e-06   0   0   0   0
0   0   -1.00e-06   0   0   0
0   0   0   -1.00e-06   0   0
0   0   0   0   -1.00e-06   0
0   0   0   0   0   -1.00e-06

84
\[ B = \]

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{cccccc}
0.2355e-3 & 0.2355e-3 & 0.2355e-3 & 0.2355e-3 & 0.2355e-3 \\
0.4257e-3 & -0.4257e-3 & 0 & -0.4257e-3 & 0.4257e-3 \\
0.4257e-3 & 0.4257e-3 & 0 & -0.4257e-3 & -0.4257e-3 \\
0.4824e-3 & 0.4824e-3 & -0.2492e-3 & 0.4824e-3 & 0.4824e-3 \\
0.6381e-3 & -0.6381e-3 & 0 & 0.6381e-3 & -0.6381e-3 \\
0.4460e-3 & 0.4460e-3 & 0 & -0.4460e-3 & -0.4460e-3 \\
-0.7029e-3 & 0.7029e-3 & 0 & 0.7029e-3 & -0.7029e-3 \\
0.2555e-3 & 0.2555e-3 & 0.3485e-3 & 0.2555e-3 & 0.2555e-3 \\
-0.7654e-3 & 0.7654e-3 & 0 & -0.7654e-3 & 0.7654e-3 \\
0.0475e-3 & 0.0475e-3 & 0 & -0.0475e-3 & -0.0475e-3 \\
0.5950e-3 & 0.5950e-3 & -0.1176e-3 & 0.5950e-3 & 0.5950e-3 \\
0.8449e-3 & 0.8449e-3 & 0 & -0.8449e-3 & -0.8449e-3 \\
\end{array}
\]
\[ C(1:5,1:6) = \]

\[
\begin{array}{cccccc}
0.2355e-3 & 0.4257e-3 & 0.4257e-3 & 0.4824e-3 & 0.6381e-3 & 0.4460e-3 \\
0.2355e-3 & -0.4257e-3 & 0.4257e-3 & 0.4824e-3 & -0.6381e-3 & 0.4460e-3 \\
0.2355e-3 & 0 & 0 & -0.2492e-3 & 0 & 0 \\
0.2355e-3 & -0.4257e-3 & -0.4257e-3 & 0.4824e-3 & 0.6381e-3 & -0.4460e-3 \\
0.2355e-3 & 0.4257e-3 & -0.4257e-3 & 0.4824e-3 & -0.6381e-3 & -0.4460e-3 \\
\end{array}
\]

\[ C(1:5,7:12) = \]

\[
\begin{array}{cccccc}
-0.7029e-3 & 0.2555e-3 & -0.7654e-3 & 0.4755e-3 & 0.5950e-3 & 0.8449e-3 \\
0.7029e-3 & 0.2555e-3 & 0.7654e-3 & 0.4755e-3 & 0.5950e-3 & 0.8449e-3 \\
0 & 0.3485e-3 & 0 & 0 & -0.1176e-3 & 0 \\
0.7029e-3 & 0.2555e-3 & -0.7654e-3 & -0.4755e-3 & 0.5950e-3 & -0.8449e-3 \\
-0.7029e-3 & 0.2555e-3 & 0.7654e-3 & -0.4755e-3 & 0.5950e-3 & -0.8449e-3 \\
\end{array}
\]

\[ C(1:5,13:18) = \]

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[ C(1:5,19:24) = \]

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]
\( E = \)

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\( F = \text{rand}(5,5) \)
Appendix B presents the $A, B, C, G$ matrices of the Earthquake Building problem.

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-k_1 - k_2 & k_2 & 0 & 0 & 0 & 0 & -c_1 - c_2 & 0 & 0 & 0 & 0 & 0 \\
k_2 & -k_2 - k_3 & k_3 & 0 & 0 & 0 & -c_2 - c_3 & 0 & 0 & 0 & 0 & 0 \\
-k_3 - k_4 & k_4 & 0 & 0 & 0 & -c_3 - c_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_4 & -k_4 - k_5 & k_5 & 0 & 0 & 0 & -c_4 - c_5 & 0 & 0 & 0 & 0 & 0 \\
-k_5 - k_6 & k_6 & 0 & 0 & 0 & -c_5 - c_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
m_i = 1.75e5 \, kg, \quad i = 1, 2, \ldots, 6
\]

\[
k_i = 1.83e7 \frac{kg}{m}, \quad i = 1, 2, \ldots, 6
\]

\[
c_i = 1.12e1 \frac{Ns}{m}, \quad i = 1, 2, \ldots, 6
\]
\[ B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{m_1} & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

\[ C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \]

\[ G = [0 \ 0 \ 0 \ 0 \ 0 \ -1 \ -1 \ -1 \ -1 \ -1 \ -1]^{T} \]
Appendix C

Appendix C presents the $A,B,C,e_{in}, e_{out}$ matrix of the Large Building Temperature Control.

$$A(1: 20, 1: 10) =$$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0.025</th>
<th>0.023</th>
<th>0.028</th>
<th>0.021</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.120</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0.031</td>
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<tr>
<td>0</td>
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<tr>
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<td>-0.135</td>
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<td>0</td>
<td>-0.030</td>
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<td>0</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.055</td>
<td>0</td>
<td>0</td>
<td>0.029</td>
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<tr>
<td>0.028</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.040</td>
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<td>0.021</td>
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<td>-0.060</td>
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<td>0</td>
<td>-0.060</td>
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<td>-0.040</td>
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\[ A(1:20,11:20) = \]

\[
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0.030 & 0.025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.034 & 0.035 & 0.030 & 0.036 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.030 & 0.033 & 0.029 & 0.035 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.031 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.036 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.045 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.017 \\
0 & 0 & -0.041 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.049 & 0 & 0 & 0.013 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.070 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.043 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.013 & 0 & 0 & -0.048 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -0.038 & 0 & 0 & 0 \\
0 & 0.017 & 0 & 0 & 0 & 0 & 0 & -0.055 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.038 & 0
\end{array}
\]
\[ B = \]

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</table>
\[ \mathbf{C}(1:4,1:10) = \]

\[
\begin{array}{cccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[ \mathbf{C}(1:4,11:20) = \]

\[
\begin{array}{cccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

\[
e_{in} =
\begin{array}{c}
0 \\
0 \\
0 \\
0 \\
0 \\
4.00e-3 \\
0 \\
8.50e-3 \\
2.50e-3 \\
0 \\
3.50e-3 \\
0 \\
1.50e-3 \\
9.50e-3 \\
0 \\
5.00e-3 \\
0 \\
9.50e-3 \\
0 \\
2.50e-3 \\
\end{array}
\]

\[
e_{out} =
\begin{array}{c}
0 \\
0 \\
0 \\
0 \\
5.00e-3 \\
0 \\
1.25e-3 \\
0 \\
9.00e-3 \\
5.50e-3 \\
0 \\
6.50e-3 \\
0 \\
6.50e-3 \\
0 \\
5.00e-3 \\
0 \\
2.50e-3 \\
\end{array}
\]
Appendix D

Appendix D presents the optimal controller performance for the earthquake building problem under disturbances with different frequency.

Set 1

Earthquake Ground Acceleration vs Time

Building Response under Earthquake with No controller
Building Response under Earthquake with Optimized PID Controller

Actuator Response under Earthquake with Optimized PID Controller
Set 2

Earthquake Ground Acceleration vs Time

Building Response under Earthquake with No controller

Time (sec)
Set 3

Earthquake Ground Acceleration vs Time

![Graph showing earthquake ground acceleration vs time.]

Building Response under Earthquake with No controller

![Graphs showing building response under earthquake with no controller.]

Time (sec)
Building Response under Earthquake with Optimized PID Controller

Actuator Response under Earthquake with Optimized PID Controller
Appendix E

E.1 Large Flexible Space Structure (LFSS)

Large flexible space structures are now a practical reality, and in this example, a phased array antenna in outer space which is used for receiving signals from earth will be considered. To achieve this, one has to apply shape control in the LFSS, i.e. one has to constantly evolve the LFSS into a desired mirror shape so that it can clearly receive signals sent from earth. [12]

The phased array antenna has the shape of a cuboid with its length and width significantly larger than its height. It is assumed that there are five pairs of sensors and actuators installed on its length-width plane; four on the corners and one in the center. Each actuator is capable of pushing each corner or center in an upward or downward motion. Each sensor can measure the current position of each corner or center.

![Figure 1.1 Large Flexible Space Structure, y and u are the sensors and actuators.](image)

The motion of an LFSS is modelled using the method of finite element analysis by a set of ordinary differential equations. In order to ensure the accuracy of the model, the order of the resulting system is very high. In section II of the paper [12], a complete mathematical model of a LFSS is obtained.
Two control problems which always occur in this case are attitude control and shape control. The behavior of a LFSS is usually modelled via finite element methods by a set of differential equations whose order may be quite high (say $n > 100$), and in this case the practice of controlling only a subset of elastic body modes leads to the “spillover problem” [1] in which ignoring subset of elastic modes may cause instability due to excitation of any uncontrolled elastic modes.

Typical approaches for LFSS control generally have been directed towards centralized control, e.g., using model reduction methods [2], modal control methods [3], [4], output feedback control [1], [5], adaptive control techniques [6] and displacement feedback [7]. However, recent researchers have started to shift their attention to “decentralized control” and in the survey paper [10], it is discussed that decentralized control is a natural application to the LFSS problem.

A fairly complete mathematical model was developed in [13] with 100 dominant elastic modes to study the behavior of a LFSS consisting of 5 inputs and 5 outputs. In this study, we shall use a simplified 24 dominant elastic mode model.
E.2 Earthquake Building Model

An earthquake is one of the most destructive natural disasters. Although researchers have spent many decades in finding methods to predict earthquakes, reliable ways of predicting an earthquake are still absent, which can lead to massive destruction. For example, in September 18th, 2008, an 8.0 magnitude earthquake in SiChuan China caused 69,227 deaths, 374,643 injuries, and 17,923 missing. In March 11th, 2011, an 8.8 magnitude earthquake in Kanto Japan caused 15,875 deaths, 26,992 injuries, and 2725 missing. Tall buildings and skyscrapers in these regions are unavoidable due to massive populations with only limited vacant land available. Thus, these regions must have their buildings designed and structured to resist earthquake.

Aside from improving the material strength of the building, there are many other basic, and proven useful ways to increase a building’s resilience to an earthquake. For example, one method is to install a very heavy damper inside a building and/or mix damping material with the building material. Taipei-101 is a classic example of such method. A 660 tonne tuned mass damper (pendulum) is suspended from the 92th floor to the 87th floor, and the pendulum sways to offset movements in the building caused by disturbances such as strong gusts [26].

In this thesis, the earthquake building problem will be categorized and solved as a large structure control problem. A six story building will be used as an example. Each story will have some spring effects and damping effects modelled by a spring coefficient $k_i$ and a damping coefficient $c_i$. The story mass is denoted by $m_i$. On each story, the relative deflection of the building $w_i$ with respect to the ground movement will be controlled by actuators $u_i$ installed on the side of each story. The acceleration of the ground movement due to earthquake will be denoted as a disturbance $\ddot{u}_g$. 
The \(i^{th}\) story’s absolute displacement satisfies the following system

1\(^{st}\)
\[
m_{1}\ddot{y}_{1} + c_{2}(\dot{w}_{1} - \dot{w}_{2}) + c_{1}(\dot{w}_{1} - \dot{u}_{g}) + k_{2}(w_{1} - w_{2}) + k_{1}(w_{1} - u_{g}) \\
\quad = u_{1}
\]

2\(^{nd}\)
\[
m_{2}\ddot{w}_{2} + c_{3}(\dot{w}_{2} - \dot{w}_{3}) + c_{2}(\dot{w}_{2} - \dot{w}_{1}) + k_{3}(w_{2} - w_{3}) \\
\quad + k_{2}(w_{2} - w_{1}) = u_{2}
\]

3\(^{rd}\)
\[
m_{3}\ddot{w}_{3} + c_{4}(\dot{w}_{3} - \dot{w}_{4}) + c_{3}(\dot{w}_{3} - \dot{w}_{2}) + k_{4}(w_{3} - w_{4}) \\
\quad + k_{3}(w_{3} - w_{2}) = u_{3}
\]

\(\ldots\)

(i-1)\(^{th}\)
\[
m_{i-1}\ddot{w}_{i-1} + c_{i}(\dot{w}_{i-1} - \dot{w}_{i}) + c_{i-1}(\dot{w}_{i-1} - \dot{w}_{i-2}) + k_{i+1}(w_{i-1} - w_{i}) \\
\quad + k_{i-1}(w_{i-1} - w_{i-2}) = u_{i-1}
\]

1\(^{th}\)
\[
m_{i}\ddot{w}_{i} + c_{i}(\dot{w}_{i} - \dot{w}_{i-1}) + k_{i}(w_{i} - w_{i-1}) = u_{i}
\]

Define now the relative displacement \(z\) in terms of absolute displacement and ground displacement

\[
z_{1} = w_{1} - u_{g}, \quad z_{2} = w_{2} - u_{g}, \quad \ldots, \quad z_{i} = w_{i} - u_{g}
\]

and take the derivative of each term to obtain the relative velocity and acceleration

\[
\dot{z}_{1} = \dot{w}_{1} - \dot{u}_{g}, \quad \dot{z}_{2} = \dot{w}_{2} - \dot{u}_{g}, \quad \ldots, \quad \dot{z}_{i} = \dot{w}_{i} - \dot{u}_{g}
\]

\[
\ddot{z}_{1} = \ddot{w}_{1} - \ddot{u}_{g}, \quad \ddot{z}_{2} = \ddot{w}_{2} - \ddot{u}_{g}, \quad \ldots, \quad \ddot{z}_{i} = \ddot{w}_{i} - \ddot{u}_{g}
\]

Replace \(w\) now in the system of ODE with \(z\)

1\(^{st}\)
\[
m_{1}\dddot{z}_{1} + c_{2}(\dot{z}_{1} - \dot{z}_{2}) + c_{1}\ddot{z}_{1} + k_{2}(z_{1} - z_{2}) + k_{1}z_{1} = -m_{1}\dddot{u}_{g} + u_{1}
\]

2\(^{nd}\)
\[
m_{2}\dddot{z}_{2} + c_{3}(\dot{z}_{2} - \dot{z}_{3}) + c_{2}(\dot{z}_{2} - \dot{z}_{1}) + k_{3}(z_{2} - z_{3}) + k_{2}(z_{2} - z_{1}) \\
\quad = -m_{2}\dddot{u}_{g} + u_{2}
\]

3\(^{rd}\)
\[
m_{3}\dddot{z}_{3} + c_{4}(\dot{z}_{3} - \dot{z}_{4}) + c_{3}(\dot{z}_{3} - \dot{z}_{2}) + k_{4}(z_{3} - z_{4}) + k_{3}(z_{3} - z_{2}) \\
\quad = -m_{3}\dddot{u}_{g} + u_{3}
\]

\(\ldots\)
Rearrange the previous system to:

1st

\[ m_1 \ddot{z}_1 + (c_1 + c_2) \dot{z}_1 - c_2 \dot{z}_2 + (k_1 + k_2)z_1 - k_2 z_2 = -m_1 \ddot{u}_g + u_1 \]

2nd

\[ m_2 \ddot{z}_2 - c_2 \dot{z}_1 + (c_2 + c_3) \dot{z}_2 - c_3 \dot{z}_3 - k_2 z_1 + (k_2 + k_3)z_2 - k_3 z_3 = -m_2 \ddot{u}_g + u_2 \]

3rd

\[ m_3 \ddot{z}_3 - c_3 \dot{z}_2 + (c_3 + c_4) \dot{z}_3 - c_4 \dot{z}_4 - k_3 z_2 + (k_3 + k_4)z_3 - k_4 z_4 = -m_3 \ddot{u}_g + u_3 \]

\[ \vdots \]

(i-1)th

\[ m_{i-1} \ddot{z}_{i-1} - c_{i-1} \dot{z}_{i-1} + (c_{i-1} + c_i) \dot{z}_{i-1} - c_i \dot{z}_i - k_{i-1} z_{i-2} + (k_{i-1} + k_i)z_{i-1} - k_i z_i = -m_{i-1} \ddot{u}_g + u_{i-1} \]

i\th

\[ m_i \ddot{z}_i + c_i \dot{z}_i - c_i z_{i-1} + k_i z_i - k_i z_{i-1} = -m_i \ddot{u}_g + u_i \]

and define the states \( x \) and outputs \( y \) (horizontal displacement of each floor) as:

\[ x = \begin{bmatrix} z_1 & \ldots & z_6 & \dot{z}_1 & \ldots & \dot{z}_6 \end{bmatrix}^T \]  

\[ y = \begin{bmatrix} y_1 & \ldots & y_6 \end{bmatrix}^T \]

Equation 1.4d can now be rearranged to the state space form:

\[ \dot{x} = Ax + Bu + G \ddot{u}_g \]  

\[ y = Cx \]  

In this case, \( \ddot{u}_g \) is the disturbance, i.e. the ground acceleration caused by the earthquake.

Appendix B gives the numerical details of the matrices \( A, B, C, G \).
E.3 Building Temperature Controller

A necessary requirement for a building is to provide a comfortable, productive space for the occupants. Air temperature plays a major role in allowing people to be comfortable, which has a direct effect on their productivity. While it is crucial to provide a comfortable and productive environment, it is also important to optimize the energy efficiency. During the winter, a one degree increment to make the building slightly warmer can result in a 3% increase in heating cost. Cooling in the summer can cost even more. One degree to cool things off a bit will add 6% to the cooling cost. In order to find a desirable balance among the optimizing comfort of individuals, productivity and cost, a combination of not only human effort and cooperation, but the help of a good, effective, and efficient controller is also required. [27]

With the above problem comment in mind, it is desired to construct a controller that can automatically tune the temperature of each room to their desired values. Newer buildings are almost always heated with hot water via radiators or fan coil units, which incorporate a fan to push conditioned air (heated or cooled depending upon the season) out of the unit.

This is because fan coil systems are generally zoned such that each room or each fan coil unit has its own thermostat, which will provide for good comfort levels throughout the individual spaces of the building. For simulation purposes, a single floor with five rooms under the control of a fan coil system is used as an example. The space at the center is the work space. The remaining four rooms are the workspace for humans, and the room temperatures need to be controlled.
As shown in figure 1.2, the space is divided into 5 rooms. The central room does not have any heat component, but it is diffusing heat to its surrounding rooms, denoted as $e_{in}$. The outside heat diffusing into the rooms is denoted as $e_{out}$. There is a heat source within each of the remaining four rooms. The temperature and heat input are denoted by $y_i$ and $u_i$ respectively.