MULTI-OBJECTIVE ENERGY-NOISE WIND FARM LAYOUT OPTIMIZATION UNDER
LAND USE CONSTRAINTS

by

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A thesis submitted in conformity with the requirements
for the degree of Master of Applied Science
Graduate Department of Mechanical and Industrial Engineering
University of Toronto

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Abstract

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University of Toronto

2015

Recently the environmental impact of onshore wind farms is receiving major attention from both governments and wind farm designers. As land is more extensively exploited for wind farms, it is more likely for wind turbines to be in proximity with human dwellings, infrastructure, and natural habitats. This proximity makes significant portions of land unusable for the designers, introducing a set of land-use constraints. In this study, we perform a constrained multi-objective wind farm layout optimization considering energy and noise as objective functions, and considering land use constraints. A stochastic evolutionary algorithm (NSGA-II) solves the optimization problem, while the land-use constraints are handled with penalty functions and a novel hybrid constraint handling approach based on Constraint Programming. Results of this study illustrate the effect of constraint severity on the energy-noise trade-off. In addition, the potential of the new constraint handling approach to outperform existing constraint handling approaches is investigated.

Dedication

*To my family and my friends,
for their love, support, and encouragement.*

Acknowledgements

In this section, it is common to thank the incredibly wonderful people that helped me in the course of my studies. However, first and foremost I am thankful to God, who led me in this path. I feel blessed when I think about how I came such a long way from my country, Iran, to Toronto, Canada and be fortunate to have the opportunity to study in such a lovely place with lovely people. In deed, His guidance during this path led me to this point.

Coming as an international student to a new country with a new culture can definitely be scary for any person. However, the first time that I met with my advisor, Professor Cristina Amon, all this fear changed to hope and happiness. I am grateful that she accepted me to be a member of her research group. During this two year master program, she always supported me by allowing me to pursue my academic interests freely. Meanwhile, Professor Amon has always made constructive comments on my research which helped me to learn the correct way of conducting an academic research. In addition to her academic character, Professor Amon's work ethics and skills to manage hard situations has always been inspirational for me.

My first contact with a member of ATOMS lab was the phone call that I had with Dr. David Romero when I was still in Tehran in Norouz 1392 (March 2013). Although it was a short call, I could feel his friendliness and kindness even from thousands of kilometres away. From my first days in Toronto until the final stages of me thesis defense, David has been like an older brother for me. I learned how to carry out research, how to write academic articles, and how to act in academia from him. He truly is a wonderful supervisor that has had a great impact on my academic progress and life style.

Other than Professor Cristina Amon and Dr. David Romero, attending the classes of Professor Christopher Beck, Dr. Kimia Ghobadi, Dr. Joaquin Moran, Professor Ian G. Currie and Dr. Hanif Montazeri during my master's study has honed my critical thinking and research skills. In fact the research project that was carried out in the Constraint Programming and Local Search course under the supervision of Professor Beck deepened my research in the context of optimization and opened up new horizons to me.

It is clear that the place you work in has a great impact on the performance and productivity. I strongly believe that having the chance to be a member of ATOMS lab was a key point in my master's study. Jim Kuo is definitely the person that had a great impact on my research. From the very first days he kindly shared all his academic knowledge with me and provided unlimited help from the simplest to the hardest problems that I faced in my research. Besides that, he has been the best friend that a person can have in his or her life. I also want to thank all my other friends in ATOMS lab including Carlos Da Silva, Francisco Contreras, Peter Zhang, Sam Huberman, Juan Stockle, Matthew Doyle, Aydin Nabovati, Fernan Saiz, Weiguan Huang, Julia Sborz, Aditya Dhoot, Enrico Antonini, and David Guirguis.

I would like to thank all the people, companies, and institutions, who kindly provided the funding for my research. I would like to thank the Mechanical and Industrial Engineering Department of University of Toronto, Natural Sciences and Engineering Research Council of Canada, and Hatch Ltd for their financial support. Also, special thanks to Hatch Ltd that generously awarded Hatch Graduate Scholarship in

Graduate Studies to me. In deed, this scholarship was a great incentive for me during my master's study.

I was so fortunate to have the chance to be a member of UofT community. I want to thank Helen Ntoukas, the executive assistant to the dean, who kindly helped all the members of ATOMS lab in many circumstances. I want to thank all the lovely people that I met in Graduate House, who made my living place an enjoyable community on campus.

Finally, I owe my deepest appreciation to my parents who have always been the symbols of kindness and ethics in my life. I appreciate the strong support of my younger brother, Ali, who has been not only a wonderful brother, but also a great friend for me. I would like to thank my grandparents, aunts, and uncles, who have always supported me during my studies and without their support I would never be in the place that I am now.

Contents

1	Introduction	1
1.1	Literature Review	2
1.1.1	Problem Modelling and Optimization Approaches	2
1.1.2	Constraint Handling	4
2	Mathematical Formulation	6
2.1	Wind Farm Modelling	6
2.1.1	Wake Modelling	6
2.1.2	Noise Modelling	7
2.1.3	Constraint Modelling	8
2.2	Optimization Model	9
2.2.1	Multi-objective Genetic Algorithm	10
2.2.2	Constraint Handling	11
2.2.3	Spatial Distribution of Non-feasible Land Portions	13
3	Wind Farm Layout Optimization Under Land Use Constraints	15
3.1	Test Cases	15
3.2	Results and Discussion	17
3.2.1	Performance of Constraint Handling Approaches	18
3.2.2	Effects of Constraints on Energy-noise Trade-off	20
4	Constraint Handling via Constraint Programming (CHCP)	24
4.1	Modelling	24
4.2	Test Cases	26
4.2.1	Verification Test Cases	26
4.3	Results and Discussion	28
4.3.1	Verification of CHCP	28
4.3.2	CHCP Performance for WFLO Problem	38
5	Concluding Remarks	50
5.1	Impact of Land-use Constraints	50
5.2	Constraint Handling via Constraint Programming	51
5.3	Future Work	51
	Bibliography	53

List of Tables

3.1	Wind turbine parameters.	16
3.2	Best performing constraint handling approaches with respect to solution quality, quantity, energy generation range, and noise generation range.	19
3.3	Averaged Run-time (hr) and number of converged cases (out of 10 runs) for test cases with 80% feasibility and a maximum objective function evaluation of 80,000.	19
4.1	Average number of infeasible layouts generated per each run by the different constraint handling approaches, for different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.	44
4.2	Average of the CP percentages of each run for different constraint handling approaches and different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.	48
4.3	Number of converged runs (out of 40 runs) for different constraint handling approaches and different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.	48
4.4	Average run-time (hr) per each run by the different constraint handling approaches, for different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.	49

List of Figures

2.1	Uniformity distribution parameter for two sample domains with constant feasibility percentage.	13
3.1	Wind rose showing the distribution of speed-direction probabilities.	16
3.2	Sample wind farm domain with land use constraints. Shaded areas are unavailable for turbine sitting.	17
3.3	Energy-noise trade-off attained by the dynamic penalty approach and 2 different penalty coefficients for 10 turbines. $C_1 = (t/n_{gen})^2 \times 10^4$ and $C_2 = (2t/n_{gen})^2 \times 10^4$	18
3.4	Energy-noise trade-off for different number of turbines and domain feasibilities.	21
3.5	Spatial distribution of non-feasible areas for four different values of the uniformity parameter (UP). Cases (a) to (d) have the same 80% land availability.	22
3.6	Energy-noise trade-off for 10 turbines and 80% of land availability with different distribution uniformities of the non-feasible areas.	23
4.1	CP percentage with different objective targets for CONSTR problem.	29
4.2	Non-dominated hyper volume and maximum crowding distance with different constraint handling approaches for the CONSTR problem after 30 and 40 generations. Note that a CP percentage of 0% corresponds to constraint handling using only the dynamic penalty approach. Notches in each box plot indicate 95% confidence intervals around the median of the distribution.	30
4.3	CP percentage with different objective targets for SRN problem.	31
4.4	Non-dominated hyper volume and maximum crowding distance with different constraint handling approaches for the SRN problem after 4 and 10 generations.	32
4.5	CP percentage with different objective targets for TNK problem.	33
4.6	Crowding distance for different constraint handling approaches for the TNK problem.	34
4.7	CP percentage with different objective targets for WATER problem.	35
4.8	Crowding distance for different constraint handling approaches for the WATER problem.	36
4.9	Crowding distance for different constraint handling approaches for the DTLZ1C3 problem.	37
4.10	Comparison of constraint handling approaches for 5 turbines (x axis is reversed).	39
4.11	Comparison of constraint handling approaches for 10 turbines (x axis is reversed).	40
4.12	Comparison of constraint handling approaches for 15 turbines (x axis is reversed).	41
4.13	Comparison of the all solutions found by the dynamic penalty approach in 40 runs with the Pareto fronts of the different setups of CHCP approach.	43

4.14	Layout comparison for $CP = 0.0\%$, $CP = 76.1\%$, and $CP = 94.6\%$ with same energy generation and different noise production.	43
4.15	CP percentage for different objective targets and 5 turbines (dynamic penalty is represented with an objective target of 0).	45
4.16	CP percentage for different objective targets and 10 turbines (dynamic penalty is represented with an objective target of 0).	46
4.17	CP percentage for different objective targets and 15 turbines (dynamic penalty is represented with an objective target of 0).	47

Nomenclature

Roman Symbols

$\mathbb{P}$	Set of all the feasible and non-feasible polygons
$\mathbb{R}$	Set of pairs representing the coordinates of noise receptors
$\mathbb{S}$	Set of all the non-feasible polygons
$\mathbb{T}$	Set of pairs representing the coordinates of turbines
C	Normalizing constant
D	Turbine diameter, m
L	Sound power
P	Polygon
p	Wind state probability
R	Penalty coefficient
t	Current generation index
u	Downstream wind speed, m/s

Acronyms

AEP	Annual Energy Production
CHCP	Constraint Handling via Constraint Programming
CP	Constraint Programming
EA	Evolutionary Algorithm
GA	Genetic Algorithm
MIP	Mixed Integer Programming
NDHV	Non-dominated Hyper Volume
OT	Objective Target
PSO	Particle Swarm Optimization
SPL	Sound Pressure Level
UP	Uniformity Parameter
WFLO	Wind Farm Layout Optimization

Greek Symbols

ϕ Domain feasibility percentage

Subscripts

d Wind state

f Octave-band frequency

gen Generation

nf Infeasible turbine

off Offspring

pop Population

$prox$ Proximity constraint violation

reg Regulatory constraint violation

t Turbine

w Weighting coefficient

Chapter 1

Introduction

In recent decades, electricity generation from wind energy has shown a sustained growth all over the world. In 2012, 44.8 GW of wind energy capacity was installed in the world, which brought the total installed wind capacity to 282.5 GW [33]. This milestone made wind energy account for 3% of world's electricity demand [25]. In 2013, the wind energy market continued to grow, with the United States of America adding 12 GW of wind power generation capacity (under construction), and with Canada adding 1.6 GW of generation capacity [25]. In the European Union, wind energy represented the largest share of new installed capacity among all energy sources [20] during 2013. These statistics indicate a strong global growth in wind energy generation, increasing the associated market for related products and services [25].

Notwithstanding these trends, wind energy still faces difficulties for wide adoption. Recently, the health and environmental impacts of onshore wind farms have become a matter of concern for governments and wind farm designers. Although it is not proven that the noise production of turbines has negative health impact on human beings, a number of jurisdictions have established regulations that limit noise emissions [5, 46, 45]. Besides the potential health issues, extensive land exploitation for wind farms increases their interference with natural habitats and causes negative environmental impact [54]. This interference together with the noise production of wind farms reduce the available land for turbine siting.

Wind farm design can be a lengthy and iterative process, in which the designer has to maximize the energy generation or revenue, while checking for compliance with environmental and safety regulations or restrictions. Similar to the wind farm designers, most of the researchers have also focused on maximizing energy or revenue of wind farms [49, 28]. However, their studies fail to elucidate the nature of energy-noise trade-off especially under severe land-use constraints. Furthermore, the current approaches are not able to investigate the impact of the extent of land-use constraints on the optimization results. Thus, these approaches fail to generalize case-specific layout optimization.

This work consists of two stages. As the first stage, we study the energy-noise trade-off for the wind farm layout optimization (WFLO) problem while considering a set of land-use constraints. In other words, this stage aims to maximize the energy generation and minimize the noise production while investigating the sensitivity of this trade-off to land-use constraints. To this end, the unconstrained

multi-objective energy-noise optimization carried out by Kwong et al. [41, 42] is extended to include land-use constraints. The optimization is performed using a multi-objective, continuous-variable Genetic Algorithm (GA) [30] based on non-domination sorting (NSGA-II, [12]) and the constraints are handled with penalty functions [9]. The second stage of this study focuses on finding a novel constraint handling approach that outperforms the conventional penalty functions by using Constraint Programming (CP) methods. The novel constraint handling approach combines the global search of the penalty functions with the local search of Constraint Programming method and improves the optimization process by finding wind farm layouts that can generate more energy and produce less noise compared to the layouts found in the first stage of this study.

1.1 Literature Review

In this section, we discuss previous studies that have proposed models or algorithms for the WFLO problem and, due to our focus on constrained wind farm optimization, we also discuss previous work in constraint handling methods for evolutionary algorithms.

1.1.1 Problem Modelling and Optimization Approaches

Regarding the studies on WFLO problem, two main optimization approaches have been applied successfully, namely (i) heuristics and (ii) mathematical programming methods. First, optimization heuristics have been the most commonly applied approach, and both stochastic and deterministic versions have been reported in the literature. Methods such as GA and Particle Swarm Optimization (PSO) [39] are the common stochastic heuristics used for solving WFLO problem [49, 28, 61, 3]. In addition, deterministic heuristics such as Extended Pattern Search (EPS) of Dupont and Cagan [18] are also used in this context; however, they have not ever been as common as stochastic methods. Most of the studies using heuristic methods considered energy or cost as their objective function, while Şişbot et al. [55] carried out a multi-objective energy-cost GA optimization. Kwong et al. [41, 42] considered noise as the second objective function for the first time and solved the unconstrained problem with continuous variable GA. Their study showed that there is a trade-off between energy generation and noise production.

A significant portion of the literature on the WFLO problem has focused on improving the optimization models by including more realistic features of the wind farms. For instance, Kusiak and Song [40] considered minimum turbine proximity constraints, and enforced a closed wind farm boundary, these constraints were converted to a second objective function and handled in a multi-objective fashion. They showed that this multi-objective optimization maximizes energy and satisfies all the constraints by minimizing constraint violation. Réthoré et al. [52] suggested a two stage optimization model that used GA in the first stage and gradient based sequential linear programming in the second stage. The second stage relied on an improved model that considered a comprehensive cost function and a more detailed wind resource distribution including more wind direction and speed bins. Saavedra-Moreno et al. [53] improved their model by considering the spatial distribution of wind speed instead of using a single speed/direction wind distribution for the whole farm terrain. Furthermore, Serrano-González et al. [27, 26] modified their optimization by taking infrastructure costs and wind data uncertainty

into account. Besides all these improvements in wind farm modelling, the most important contribution of the most recent studies with heuristic methods is the switch to continuous-variable formulations [40, 41, 42, 6, 7, 8]. This is an important step in reducing the probability of converging to sub-optimal solutions caused by the coarseness of the discretization approaches typically used in the literature.

Formulations of the WFLO problem amenable for solution with mathematical programming methods, such as mixed-integer programming (MIP), have also been proposed. Donovan [17, 16] introduced MIP for solving the WFLO problem. Fagerfjäll [21] used traditional branch-and-bound together with a heuristic to improve the performance of the optimization algorithm. Although MIP solvers are widely available in operation research software packages, they all have limitations solving non-linear, non-convex problems such as WFLO. Both Donovan and Fagerfjäll tried to address this problem by simplifying their wake model at the expense of losing accuracy. To address this issue, Archer et al. [2] improved the simplified wake model by introducing a wind interference coefficient, while Turner et al. [59] suggested more accurate linear and quadratic wake models that can be solved by MIP solvers. However, the accuracy problem was resolved by Zhang et al. [66], who proposed the first constraint programming (CP) and MIP models that fully incorporated the non-linearity of the problem. Similar to the WFLO work using heuristics, proponents of MIP models have also neglected the land-use constraints associated with practical instances of the WFLO problem. However, the main limitation of MIP models is their dependence on coarsely discretized domains. For example, even in recent MIP studies [59, 66], the wind farm domain is typically discretized into 100-400 potential turbine locations, with memory requirements and solution times increasing exponentially for finer discretizations.

Finally, from the common constraints encountered by wind farm designers, those related with land usage regulations have not received enough attention from researchers. However, there are a few exceptions that considered land-related parameters for their optimization. Chowdhury et al. [6, 7, 8] included the impact of land configuration and turbine selection in their study and used PSO for optimization. They minimized cost of energy and represented it as a function of land orientation and aspect ratio. Chen et al. [4] incorporated the participation rates of land owners in their cost function, which was minimized by GA. They showed that land owners' remittances account for approximately 10 % of the wind farm's operating cost. In spite of these studies, the environmental/regulatory land-use constraints such as setbacks from rivers, lakes, roads and human dwellings, have been neglected in previous work and are a matter of concern in our study.

In this study, a computational approach is proposed for constrained multi-objective, continuous formulation of the WFLO problem. This approach addresses the growing health and environmental concerns of wind farms by not only maximizing energy generation but also minimizing noise production and avoiding natural habitats and human dwellings. To achieve this goal, the unconstrained multi-objective WFLO problem addressed by Kwong et al. [41, 42] is extended to include land-use constraints. The resulting optimization problem is solved with NSGA-II [12] and without linearizing simplifications to retain accuracy. Different constraint handling approaches are incorporated in the optimization algorithm to handle the land-use constraints.

1.1.2 Constraint Handling

Using stochastic meta-heuristics for constrained WFLO problem requires developing a constraint handling approach to drive the search toward high-quality, *feasible* solutions. Penalty functions are perhaps the most widely used constraint handling approaches in evolutionary algorithms due to their simplicity of implementation, general applicability, and strong theoretical basis [9]. The penalty function approaches consist of recasting the constrained problem as an unconstrained one by incorporating a function of the constraint violations as a term in the objective function. Then a certain value is added (or subtracted) from the objective function of the infeasible solutions based on the amount of constraint violation. Hence, penalty functions are generally applicable to constrained optimization problems, regardless of the underlying method used to solve the resulting unconstrained problem. When the penalty functions are used with evolutionary algorithms, there is no need for an initial feasible population, and since in many problems finding an initial feasible population is by itself an NP-hard problem, most researchers consider using penalty functions with evolutionary algorithms for solving constrained optimization problems. However, the main limitation of penalty functions is that their control parameters (i.e., penalty coefficients) are problem-specific. To address this issue, Debchoudhury et al. [14] proposed a modified penalty function, free from scaling parameters that finds the penalty terms based the constraint violation and the fitness function of the infeasible solutions. Datta et al. [11] introduced another penalty function approach, which is able to further improve the best solutions by decreasing the level of constraint violation using a gradient free pattern search method. Montemurro et al. [48] proposed the automatic dynamic penalization method in which all the information needed for tuning the penalty parameters is extracted from the population members of the current generation. In addition, there are some other studies that choose the parameters of the penalty functions adaptively [10, 63, 60]. The main focus of the above mentioned studies is to make the penalty functions independent of any external parameters. Although some of these studies have improved the global search of the penalty functions [11], none of them have suggested a strong local search that can be combined with the penalty functions and thus improve overall search performance of the constraint handling approaches. Here, we shall use the penalty approach; however, we improve its performance in constrained multi-objective optimization by hybridizing it with a powerful local search tool that complements the global search.

In addition to penalty functions, other approaches have also been used for constraint handling with evolutionary algorithms. The most recent general approach to constrained optimization with evolutionary algorithms is proposed by Elsayed et al. [19] where a constrained optimization problem is solved in two stages using different evolutionary algorithms. In the first stage, the decision is made for the optimization methodology and the second stage decides about the search operators. Oh et al. [50] also suggested a general constraint handling approach in which a set of constraints that play a key role in satisfying the feasibility within a certain tolerance are selected and handled before the other constraints. This tolerance is specified by statistics on feasible solutions and several prefixed criteria. The selected constraints are handled first to guide the solution set to the feasible region. Other methods to solve the constrained optimization problems are based on an extended Pareto-dominance criterion, called constrained-domination [23]. For instance, Fonseca et al. [23] modified the binary tournament operation for parent selection to handle the constraints. Typically, in binary tournaments two solutions are selected from the population and the dominated solution is discarded from the parent pool. In Fonseca's approach for multi-objective constrained optimization the solution that constrained-dominates the other

is chosen. In a more recent study by Thakur et al. [58], a similar approach to that of Fonseca's is introduced. In this approach a modified crossover algorithm produces offsprings that are within the upper/lower bounds of each variable. Similar to Fonseca's approach the constraint handling happens during the parent selection and recombination process. Deb et al. [12] also used the same approach as Fonseca et al. for constraint handling with a slight difference in their constraint-domination definition. An alternative approach for constraint treatment is introduced by Ray et al. [51], who defined three different non-domination rankings based on the objective functions, constraints, and combined objectives and constraints on the population. In a new study by Jain et al. [35] the constraint domination principle suggested by Deb is used together with a reference-point based non-domination sorting with the purpose of improving the optimization results, though its reliance on reference points limits its applicability in practical settings. Mohamed et al. [47] modified Deb's constraint handling approach. In addition to the constrained-domination criterion, Mohamed et al. incorporated the sum of constraint violation as a second metric to handle the constraints. All the above mentioned approaches have had an acceptable performance when applied to different test cases; however, they are all based on the penalization paradigm, either by directly penalizing the objective functions or by biasing the selection and/or crossover operators towards the feasible region of the domain by discarding infeasible solutions. In this work, we propose the use of Constraint Programming (CP) to repair infeasible solutions instead of penalizing them.

Although CP is not widely used with evolutionary algorithms, there are some studies that considered taking advantage of CP in improving the performance of evolutionary optimization algorithms. In a study by Wang et al. [62] a CP-based GA is developed to solve the resource portfolio planning of make-to-stock products problem. They formulated the problem as a non-linear MIP and solved it using GA. The infeasible solutions that are generated in the recombination process of the GA are repaired by the CP model. In other words, the CP model finds a feasible solution that is close the infeasible solution. It is mentioned that the use of CP helps solving the problem more efficiently. In a recent study by Di Alesio et al. [15] GA and CP are combined to support stress testing of task deadlines. In this study, after each generation, GA passes the new generation to the CP model. Then, the CP model modifies the solutions by local search, while considering the constraints. Zhu et al. [67] proposed a combination of GA and boolean CP for solving course of action optimization in Influence Nets. The results of all these studies might be affected by negative effects of local search, such as the reduced diversity of the population that results from repairing infeasible solutions instead of allowing them to partake in the evolution process directly. These effects are not investigated in any of the aforementioned studies; thus, it is important to investigate all the potential negative effects pure local search thoroughly.

In this study a multi-objective GA, called NSGA-II [12] is used to solve the constrained WFLO problem. Since GA is not able to handle constraints by its nature, penalty functions are used as the first step to handle the constraints [57]. As the second stage towards improving the performance of constraint handling, a new constraint handling approach is introduced. In the proposed approach, a CP model is hybridized with penalty functions with the purpose of improving the intensification (i.e., local search) of penalty functions and avoiding the negative effects of pure diversification (i.e., global search). This new approach is then verified with test problems and applied to the WFLO problem. The obtained results using this constraint handling approach are finally compared to those of the penalty functions.

Chapter 2

Mathematical Formulation

2.1 Wind Farm Modelling

In this section, we present the mathematical models for the objective functions and constraints of the WFLO problem. We begin with the prediction of annual energy production based on Jensen’s model for wind turbine wake interactions. Then, we describe briefly the ISO-9613-2 calculation method for noise propagation, used in this work to estimate the noise level caused by wind turbines at points of interest within the wind farm domain. Finally, we discuss our approach for modelling the land-use constraints typically encountered on a wind farm project.

2.1.1 Wake Modelling

This work uses the closed-form, analytical wake model suggested by Jensen [36] to quantify the aerodynamic interactions between turbines. Jensen’s model is based on conservation of momentum inside the wake region and its linear expansion in the direction of the main flow. As Betz’s theory [43] indicates, the wind speed right behind the rotor is approximately $\frac{1}{3}$ of the free stream speed. Since it is assumed that the wake region expands linearly, the down stream wind speed can then be calculated as,

$$u = u_0 \left(1 - \frac{2}{3} \left(\frac{r_r}{r_1} \right)^2 \right), \quad (2.1)$$

where $r_1 = r_r + \alpha x$, and α is the wake decay constant, also known as entrainment constant. The sum of kinetic energy deficits from upstream turbines is used to calculate an effective wind speed at the turbines influenced by multiple wakes. Thus, the effective wind speed at a turbine located inside n wake regions can be expressed as,

$$u = u_0 \left[1 - \sqrt{\sum_{i=1}^n \left(1 - \frac{u_i}{u_0} \right)^2} \right]. \quad (2.2)$$

Using the effective wind speed at the turbine’s rotor, the power production of a turbine can be calculated with the manufacturer-supplied power curve. However, without loss of generality, this work

continues the approach of previous work [65, 40, 18, 57] approach and calculates the power output as a cubic function of effective wind speed at hub height. As a result, the annual energy production (AEP), which is the expected value of a random variable, because it is based on the probability distribution of wind speeds and directions, is calculated as

$$AEP = 8766 \sum_{i=1}^k \sum_{d \in \mathcal{D}} \frac{1}{3} u_{id}^3 p_d, \quad (2.3)$$

where u_{id} is the effective wind speed at turbine i at hub height for wind state d , i is an index over the number of turbines k , $d \in \mathcal{D}$ is the set of all possible wind states (i.e., the set of all possible wind speeds and directions), p_d is the probability of wind being at state d , and 8766 is the effective number of hours in a year. The cut-in and cut-off speeds are considered to be 4 m/s and 25 m/s respectively. The rated speed is 15 m/s, for which a power of 1.5 MW is generated.

2.1.2 Noise Modelling

For the purposes of WFLO in the present work, the noise receptors are considered as the locations where the sound level has to be measured or calculated. In wind farm layout design, all the residences inside or in the neighbourhood of the wind farm are potential noise receptors. According to the ISO-9613-2 standard [34], the equivalent continuous downwind octave-band sound pressure level (SPL) at each noise receptor is calculated for each sound source and all the eight octave bands with nominal mid-band frequencies from 63 Hz to 8 kHz [34], as $L_f = L_W + D_c - A_f$, where L_W is the octave-band sound power emitted by the source, D_c is the directivity correction for sources that are not omni-directional, A_f is the octave-band attenuation, and f is a subscript indexing each of the eight standard octave-band mid-band frequencies. The attenuation term (A_f) is the sum of attenuation effects caused by geometrical divergence, atmospheric absorption, ground effects, sound barriers, and miscellaneous effects. In the present work, it is assumed that the attenuation effects due to sound barriers and miscellaneous effects are negligible.

The sound pressure level calculated for each octave-band frequency has to be converted to an effective SPL. Among several octave-band weightings available for this conversion, A-weighted sound pressure levels [45] are customarily used in wind farm layout design. The equivalent continuous A-weighted downwind sound pressure level at a specific location is calculated based on the summation of each sound source's contribution at each octave band,

$$L_{avg} = 10 \log \left(\sum_{i=1}^{n_s} \left(\sum_{f=1}^8 10^{0.1(L_f(i,f) + A_w(f))} \right) \right), \quad (2.4)$$

where n_s is the number of sound sources and the $A_w(f)$ are the standard A-weighting coefficients. Further details for the calculation procedure are available in the ISO-9613-2 document [34].

2.1.3 Constraint Modelling

In this study, both proximity and land-use (regulatory) constraints are considered during the optimization. The proximity constraint ensures that the distance between any pair of turbines is at least five times their diameter. This constraint reduces the exposure of the turbines to the strong turbulence and flow-induced vibrations present in the wake regions. On the other hand, the regulatory constraint ensures that the turbines are not allowed to be located inside the non-feasible areas of the domain, such as environmental setbacks, lakes, and private properties of non-participating owners.

The examination of the proximity constraint is performed by calculating Euclidean distances between the turbines in Cartesian coordinates. Hence, turbine i with coordinates (x_{t_i}, y_{t_i}) is violating the proximity constraint if there exists a turbine j with coordinates (x_{t_j}, y_{t_j}) such that,

$$\sqrt{(x_{t_i} - x_{t_j})^2 + (y_{t_i} - y_{t_j})^2} < 5D, \quad (2.5)$$

where D is the turbine rotor diameter. To calculate the amount of constraint violation for an infeasible layout with respect to the proximity constraint, the proximity constraint is considered as the first constraint and a constraint function called g_1 is defined as

$$g_1 = \sum_{i=1}^{n_{prox}-1} \sum_{j=i+1}^{n_{prox}} \left(5D - \sqrt{(x_{t_i} - x_{t_j})^2 + (y_{t_i} - y_{t_j})^2} \right), \quad (2.6)$$

where n_{prox} is the total number of turbines violating the proximity constraint in an infeasible layout and $\{(x_{t_i}, y_{t_i}), (x_{t_j}, y_{t_j})\}$ are the coordinates of each pair of them that violate this constraint.

The regulatory constraint is inspected by assuming that all the non-feasible areas are in the form of convex polygons. This work uses an approach based on the non-feasible polygon's area to determine whether a turbine is located inside it or not. The main idea is to connect the location of the turbine to the polygon's vertices with lines, such that each adjacent pair of vertices creates a triangle with the turbine's location. If the summation of all the triangles' areas is equal to that of the polygon, the turbine is inside the polygon. Based on this approach, turbine i with coordinates (x_{t_i}, y_{t_i}) is violating the regulatory constraint if there exists a non-feasible polygon called P_k such that,

$$A_{i_k} - A_{P_k} = 0, \quad (2.7)$$

where A_P and A_i are the area of the non-feasible polygon and the summation of the areas of the aforementioned triangles, respectively. A_P and A_i are expressed in Eq. 2.8 and Eq. 2.9 using the so-called shoelace formula [69],

$$A_{P_k} = \frac{1}{2} \left[\sum_{j=1}^n |(x_{v_j} y_{v_{j+1}} - y_{v_j} x_{v_{j+1}})| \right] + \frac{1}{2} |(x_{v_n} y_{v_1} - y_{v_n} x_{v_1})| \quad (2.8)$$

$$A_{i_k} = \frac{1}{2} \left[\sum_{j=1}^n |x_{t_i}(y_{v_j} - y_{v_{j+1}}) + x_{v_j}(y_{v_{j+1}} - y_{t_i}) + x_{v_{j+1}}(y_{t_i} - y_{v_j})| \right] \\ + \frac{1}{2} |x_{t_i}(y_{v_n} - y_{v_1}) + x_{v_n}(y_{v_1} - y_{t_i}) + x_{v_1}(y_{t_i} - y_{v_n})| \quad (2.9)$$

where $j \in \{1, 2, \dots, n\}$, n is the number of non-feasible polygon's vertices and (x_{v_j}, y_{v_j}) are the coordinates of each vertex. Similar to the first constraint, a constraint function called g_2 is defined as the sum of the infeasible turbines' minimum distances to the sides of the non-feasible polygon wherein they are located. The distance of turbine i in a polygon with n sides from its j th side is the height of the triangle created by the two vertices of side j and the location point of turbine i . This height can be calculated by dividing the triangle's area by its base, i.e., side j ,

$$d_{i,j} = \frac{|x_{t_i}(y_{v_j} - y_{v_{j+1}}) + x_{v_j}(y_{v_{j+1}} - y_{t_i}) + x_{v_{j+1}}(y_{t_i} - y_{v_j})|}{\sqrt{(x_{v_j} - x_{v_{j+1}})^2 + (y_{v_j} - y_{v_{j+1}})^2}} \quad (2.10)$$

where $j \in \{1, 2, \dots, n\}$. Finally, g_2 can be defined as,

$$g_2 = \sum_{i=1}^{n_{reg}} \min\{d_{i,1}, d_{i,2}, \dots, d_{i,n}\} \quad (2.11)$$

where n_{reg} is the number of turbines violating the regulatory constraint.

2.2 Optimization Model

Before going through the problem's formulation, it is essential to introduce the specific notation used. $\mathbb{T}$ is defined as a set of pairs representing the coordinates of the turbines, i.e.,

$$\mathbb{T} = \{(x_{t_1}, y_{t_1}), (x_{t_2}, y_{t_2}), \dots, (x_{t_{n_T}}, y_{t_{n_T}})\}, \quad (2.12)$$

where n_T is the number of turbines. Similarly, we define $\mathbb{R}$ to show the coordinates of the noise receptors as

$$\mathbb{R} = \{(x_{r_1}, y_{r_1}), (x_{r_2}, y_{r_2}), \dots, (x_{r_{n_R}}, y_{r_{n_R}})\}, \quad (2.13)$$

where n_R is the number of noise receptors.

The regulatory constraint is imposed by dividing the domain into n_p convex polygons, which are the members of $\mathbb{P}$,

$$\mathbb{P} = \{P_1, P_2, \dots, P_{n_p}\}, \quad (2.14)$$

where each P_i is a set of pairs containing the vertices coordinates of polygon i in counter clockwise order,

$$P_i = \{(x_{v_1}, y_{v_1}), (x_{v_2}, y_{v_2}), \dots, (x_{v_n}, y_{v_n})\}. \quad (2.15)$$

Due to the regulatory constraints, some of the above mentioned polygons are identified as non-feasible polygons. All the non-feasible polygons are included in $\mathbb{S}$, $\mathbb{S} \subset \mathbb{P}$, defined as

$$\mathbb{S} = \{P_i | P_i \text{ is non-feasible}\}. \quad (2.16)$$

After identifying the the non-feasible polygons, the feasibility percentage of the wind farm domain is defined as,

$$\phi = \frac{\sum_{i=1}^{n_p} A_{P_i} - \sum_{P_i \in \mathbb{S}} A_{P_i}}{\sum_{i=1}^{n_p} A_{P_i}} \times 100, \quad (2.17)$$

which specifies the available land percentage in the domain for placing turbines.

Now, the WFLO problem can be defined as,

$$\underset{\mathbb{T}}{\text{minimize}} \quad \left\{ -AEP(\mathbb{T}), \max_{\mathbb{R}} (SPL(\mathbb{T}, \mathbb{R})) \right\}, \quad (2.18)$$

subject to,

$$\sqrt{(x_{t_i} - x_{t_j})^2 + (y_{t_i} - y_{t_j})^2} \geq 5D, \quad (2.19)$$

$$\forall \{(x_{t_i}, y_{t_i}), (x_{t_j}, y_{t_j})\} \subset \mathbb{T}, \quad i, j \in \{1, 2, \dots, n_T\}, i \neq j, \text{ and}$$

$$A_{i_k} - A_{P_k} > 0, \quad (2.20)$$

$\forall i \in \{1, 2, \dots, n_T\}, \forall P_k \in \mathbb{S}$, where A_{P_k} and A_{i_k} are calculated using Eq. 2.8 and Eq. 2.9, respectively.

The objective functions of the problem, Annual Energy Production (AEP) and maximum sound pressure level (SPL) are calculated as,

$$AEP(\mathbb{T}) = \sum_{i=1}^{n_T} \sum_{d \in \mathcal{D}} \frac{1}{3} \left(u_{id, \infty} \left(1 - \sqrt{\sum_{j \in \mathcal{U}_{id}} \left(1 - \frac{u_{ijd}}{u_{id, \infty}} \right)} \right) \right)^3 p_d \quad (2.21)$$

and

$$SPL(\mathbb{T}, \mathbb{R}) = 10 \log \left(\sum_{i=1}^{n_T} \sum_{j=1}^8 10^{0.1 \left(L_f^{(i,j)}(\mathbb{T}, \mathbb{R}) + A_w^{(j)} \right)} \right), \quad (2.22)$$

where $d \in \mathcal{D}$ is the set of all possible wind states (i.e., the set of all possible wind speed-direction bins), $\mathcal{U}_{id}$ is the set of upstream turbines with respect to turbine i for wind state d , $u_{id, \infty}$ is the free stream wind speed at turbine i for wind state d , u_{ijd} is the wind speed at turbine i affected by the single wake of the upstream turbine j for wind state d , and p_d is the occurrence probability of wind at state d .

2.2.1 Multi-objective Genetic Algorithm

As mentioned in Sec. 1, evolutionary algorithms have been widely used to solve the non-linear and non-convex problems such as WFLO problem [49, 28, 61, 3]. Following the previous studies, this study solves the multi-objective WFLO problem with a multi-objective Genetic Algorithm, called NSGA-II. This algorithm carries out the fitness assignment by using the objective values to calculate two metrics, namely non-domination rank and crowding distance. First, the non-domination ranking metric assigns an integer rank to each solution according to the Pareto dominance criterion. According to this criterion, the solutions that are part of the Pareto set will receive a rank value of 1, the solutions that become non-dominated after removing the rank 1 solutions are ranked 2, and by repeating the same

process all the solutions will receive a non-domination rank. Second, a crowding distance is given to each solution based on its distance, measured in the objective space, to the closest solution with the same non-domination rank. This metric preserves a certain diversity in the population, which is critical for convergence to the global optima [12]. The following paragraph presents a thorough description of the NSGA-II algorithm in the context of WFLO problem.

In order to solve the WFLO problem, n_{pop} feasible turbine layouts are generated randomly as the initial population, and their corresponding objective functions (energy generation, noise level) are evaluated. Subsequently, each population member is given a rank and crowding distance based on the above mentioned metrics. The parents for the next generation are chosen via binary tournament according to their non-domination rank and crowding distance. Solutions with lower non-domination ranks are preferred and the crowding distance is used as a secondary fitness metric with the purpose of breaking ties in the rank based comparisons. After the parents are selected, an offspring generation of size n_{off} is created by cross-over and mutation operators. Then, the objective function values of this offspring population are calculated, the amount of constraint violation is determined for each individual, and then it is used to penalize the objective functions according to the constraint handling methods. Afterwards, the offspring and parent populations are merged together and re-evaluated with the fitness assignment metrics to select the population that will survive to the next generation. An elitism operator is implemented by maintaining the best n_{pop} members of the population for the next generation. In addition, an off-line archive of the best solutions prevents us from losing any optimal solution during the optimization process. This problem may occur when elitism is implemented on constant population sizes [24]. The readers are referred to [12] for more details about NSGA-II algorithm and its implementation.

In the present work, a set of numerical experiments were performed with typical instances of the WFLO problem, aimed at determining the set of NSGA-II control parameters that resulted in the best optimization solutions. As a result of these experiments, we set the cross-over and mutation probabilities as 0.95 and 0.05 respectively. The choice of n_{pop} and n_{off} is discussed in Sec. 3.1. Convergence of NSGA-II is determined by applying the approach of Deb et al. [12], which monitors the changes in crowding distance for a certain number of generations. An optimization process is converged if the variance of rank 1 solutions' crowding distances is less than 0.005 during the last 100 generations. Furthermore, a limit of 80,000 objective function evaluations is introduced as an additional termination criterion; this limit was set as a proxy to restrict the total runtime required to obtain a solution in a way that would be insensitive to the specific hardware used to run the experiments.

2.2.2 Constraint Handling

In this study, three different constraint handling approaches are implemented. The proximity and the regularity constraints are handled with static, dynamic, and death penalty functions. The following paragraphs discuss these approaches and their implementation in the context of the WFLO problem.

Death penalty is the first approach that we use in this work. It discards each infeasible layout as soon as it is generated and produces another layout randomly. As the second approach, static penalty functions [9] are utilized to penalize the objective functions of the infeasible layouts. The penalized

objective functions are defined as,

$$AEP^P(\mathbb{T}) = AEP(\mathbb{T}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 R_{AEP,i} \quad (2.23)$$

and

$$SPL^P(\mathbb{T}, \mathbb{R}) = SPL(\mathbb{T}, \mathbb{R}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 R_{SPL,i}, \quad (2.24)$$

where AEP^P and SPL^P are the penalized objective functions, n_c is the number of constraints, g_i is the i -th constraint function, $R_{AEP,i}$ is the penalty coefficient for the energy objective function for constraint i , and $R_{SPL,i}$ is the penalty coefficient for the noise objective function and constraint i . As mentioned before, we consider two constraints in this study, i.e., proximity and regulatory constraints. Thus, n_c is equal to 2.

Static penalty functions use a constant penalty coefficient for all the generations. This can cause a deviation from convergence and lead to sub-optimal solution sets, especially in the final generations [9]. The dynamic penalty function, on the other hand, avoids this deviation by implementing penalty coefficients that increase gradually as the optimization progresses to the final stages, i.e.,

$$AEP^P(\mathbb{T}) = AEP(\mathbb{T}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 \left(\frac{t}{C_{gen}} \right)^2 R_{AEP,i} \quad (2.25)$$

and

$$SPL^P(\mathbb{T}, \mathbb{R}) = SPL(\mathbb{T}, \mathbb{R}) + \sum_{i=1}^{n_c} (\max(0, g_i))^2 \left(\frac{t}{C_{gen}} \right)^2 R_{SPL,i}, \quad (2.26)$$

where t is the current generation index and C_{gen} is a normalizing constant, defined later. The generation parameter is squared according to the approach suggested in [37]. Due to the presence of this parameter, the average amount of penalization performed by the dynamic penalty function is less than that for static penalties.

A set of computational experiments was carried out to determine an appropriate value for the penalty coefficients. To avoid potential convergence issues resulting from performing our study with a single penalty coefficient, two different sets of penalty coefficients were used when applying the static penalty approach. Based on the computational experiments, the penalty coefficients were selected to be two orders of magnitude larger than the average value of the objective functions. Since the values of AEP (GWhr) and SPL (dBA) are in the same order of magnitude ($\sim 10^2$), the penalty coefficients were then set to 10^4 and 4×10^4 . For the dynamic penalty approach, a single value of the penalty coefficient is used, set to 10^4 . However, two different values are assigned to the C_{gen} parameter, namely $C_{gen} = n_{gen}$ and $C_{gen} = n_{gen}/2$. These parameter choices result in two different trajectories for the penalty coefficients, i.e., $C_1 = (t/n_{gen})^2 \times 10^4$ and $C_2 = (2t/n_{gen})^2 \times 10^4$, in the ranges $0 - 10^4$ and $0 - 4 \times 10^4$, respectively. When appropriate, the solutions achieved by these two formulations for both static and dynamic approaches are combined together and the overall best solutions are reported.

Figure 2.1: Uniformity distribution parameter for two sample domains with constant feasibility percentage.

After discussing the constraint handling methods applied to this work, it is necessary to investigate their behavior in the context of GA. When a layout is penalized with penalty functions, especially with the static or death penalty approaches, the chance for that layout to be chosen by the parent selection, crossover or elitism operators decreases drastically, and GA is compelled to discard that layout and look for a new, random feasible layout in the domain. This characteristic of the penalty functions results in more exploration, or global search. Nevertheless, the situation is slightly different with dynamic penalty functions. The dynamic penalty term increases gradually from a value close to zero in the initial generations to a maximum in the last generation. As a result, the dynamic approach tolerates certain amount of constraint violation. This gives GA the flexibility to use the infeasible layouts as the members of the next generation or as parents. Thus, the GA does not explore the domain to the extent that it does with the static or death penalties, focusing in the areas surrounding the current members of the population during the initial stages. In the final generations, since the penalty term becomes a more dominant value, GA performs a more global search, behaving similarly to the static penalty approach. In other words, the dynamic penalty approaches allows the infeasible solutions to participate in the recombination process and influence the generation of new candidate solutions. However, the static or death penalty functions never provide the opportunity for the infeasible solutions to repair themselves [31, 56, 38].

2.2.3 Spatial Distribution of Non-feasible Land Portions

According to preliminary experiments, the percentage of the wind farm land that constitutes the feasible domain, referred herein as feasibility percentage and represented by the symbol ϕ , is not sufficient to quantify the severity of the land-use constraints on a given WFLO test case. Although this parameter specifies the unavailable areas for turbine sitting, it does not provide any information about their spatial distribution in the domain. For instance, both Fig. 2.1(a) and Fig. 2.1(b) have the same feasibility percentage, i.e., $\phi = 80\%$; however, the spatial distribution of the non-feasible areas are completely different and, as expected, this causes significant differences in the optimization results for the domains in Fig. 2.1(a) and Fig. 2.1(b).

Hence, in this study we propose a uniformity parameter to characterize the spatial distribution of the non-feasible areas in the domain, defined in such a way that the more evenly the non-feasible areas are distributed in the domain, the lower its value is. The uniformity parameter (UP) evaluates the distribution uniformity of the non-feasible polygons based on a modification of the Star Discrepancy method [64] suggested by Hickernell et al. [29], which is known as the Centered Discrepancy. This method considers a rectangular domain and measures the uniformity by calculating the discrepancy of normalized center coordinates of each non-feasible polygon with respect to the closest corner point of the domain. The coordinates are normalized by dividing the abscissa and the ordinate of each center by the domain's length and width respectively. The uniformity parameter, which is known as centered discrepancy in the literature has a formula for computation as follows,

$$\begin{aligned}
 UP^2 = & \left(\frac{13}{12}\right)^2 - \frac{2}{n_p} \sum_{i=1}^{n_p} \left[1 + \frac{1}{2}|x_{c_i} - 0.5| - \frac{1}{2}|x_{c_i} - 0.5|^2 \right] \\
 & \left[1 + \frac{1}{2}|y_{c_i} - 0.5| - \frac{1}{2}|y_{c_i} - 0.5|^2 \right] \\
 & + \frac{1}{n_p^2} \sum_{i=1}^{n_p} \sum_{j=1}^{n_p} \left[1 + \frac{1}{2}|x_{c_i} - 0.5| + \frac{1}{2}|x_{c_j} - 0.5| - \frac{1}{2}|x_{c_i} - x_{c_j}| \right] \\
 & \left[1 + \frac{1}{2}|y_{c_i} - 0.5| + \frac{1}{2}|y_{c_j} - 0.5| - \frac{1}{2}|y_{c_i} - y_{c_j}| \right].
 \end{aligned} \tag{2.27}$$

In this formulation, (x_{c_i}, y_{c_i}) are the normalized coordinates of the i th non-feasible polygon's center. A detailed discussion about discrepancy methods for uniformity measurement is available in [22].

Chapter 3

Wind Farm Layout Optimization Under Land Use Constraints

3.1 Test Cases

Following the standard test cases found in the wind farm optimization literature, a $3 \text{ km} \times 3 \text{ km}$ square wind farm terrain is considered, with turbine and the wind farm terrain characteristics as shown in Table 3.1. For the wind resource, this work implements the distribution defined by Kusiak et al. [40], which utilizes 24 wind directions in 15° intervals and 43 wind speeds from 4 m/s to 25 m/s in 0.5 m/s intervals. Each direction-speed is assigned a probability based on industrial data, which is used for calculating AEP in Eq. 2.21. Figure 3.1 shows the distribution of the direction-speed probabilities.

The main goal of our test cases is to investigate the effect of constraint severity on energy generation and noise production of wind farms. To this end, the wind farm domain is divided into 225 random polygons ($n_P = 225$) with areas of the same order of the magnitude. Based on industrial wind farm design experience, test cases with feasibility percentages (ϕ) of 70%, 80%, and 90% are generated. Fig. 3.2 depicts a sample wind farm test case with $\phi = 80\%$, the noise receptors, represented with (+) in the figure, are placed randomly inside each non-feasible polygon. Thus, the more constrained the domain becomes, the more noise receptors it will have. The optimization is performed for 5, 10, and 15 identical turbines and, in order to account for GA's randomness, each optimization is carried out 5 times for each test case.

The population size and the number of generations for the GA were determined based on a set of computational experiments on sample test cases with the aforementioned land availabilities. Following Kwong et. al [42, 41] populations with 100, 150, and 200 individuals were examined and the corresponding number of generations was determined such that the total number of objective function evaluations remained constant. A population size of 200 resulted in the best solutions for 70% feasible domains, regardless of the number of turbines in the wind farm. In a similar fashion, for 80% and 90% of land availabilities, the population sizes of 150 and 100 performed the best, respectively. It should be noted that the aforementioned population sizes are not necessarily the optimal population sizes, i.e., those which would guarantee the best performance of GA. The main goal here is to find a population size that has an acceptable performance for a given test case, and to use this population size consistently for our

Table 3.1: Wind turbine parameters.

Parameter	Value
Turbine Hub Height (z)	80 m
Terrain Roughness Length (z_0)	0.1 m
Rotor Radius (r_r)	38.5 m
Thrust Coefficient (C_T)	0.8
Power Curve	$0.3u^3$ kW
Cut-in Speed	4 m/s
Cut-off Speed	25 m/s
Rated Speed	15 m/s
Rated Power	1.5 MW
Noise Production (L_w)	100 dB
Noise Receptor Height (h_r)	1.5 m

Figure 3.1: Wind rose showing the distribution of speed-direction probabilities.

experiments. To this end, a small number of experiments was carried out to find the population size and number of generations that resulted in the best performance for a given number of function evaluations. As a result, there might be several other GA parameters affecting the performance of a given population size that were not investigated in these experiments. Thus, the observed interplay between the percentage of land availability of the test case and the best-performing population size cannot be generalized to all wind farm problems, although the existence of this interplay is not unexpected considering that land availability is a measure of constraint severity, i.e., of the size of the search space. In any case, the investigation of optimal GA parameters for the constrained WFLO problem, though an interesting area to explore in the future, is beyond the scope of the present study.

Finally, it should be mentioned that all our tests were performed with an in-house C++ implementation of the NSGA-II algorithm with static, dynamic and death penalty modes, compiled with the

Figure 3.2: Sample wind farm domain with land use constraints. Shaded areas are unavailable for turbine sitting.

TDM-GCC version 4.7.1 compiler under Linux Red Hat version 6.2. This code is run serially on a Dell PowerEdge T420 Tower Server with 2 Intel Xeon E5-2400 processors and 164 GB of RAM.

In the next section, the results of these tests are discussed in detail. First, we discuss how the choice of constraint handling approach affects the optimization results, in terms of the quality of the obtained solutions and the required computational effort. Second, the effect of constraint severity (feasibility percentage) and number of turbines on energy-noise trade-off is investigated. Finally, the impact of different spatial distributions of non-feasible areas, characterized by the proposed uniformity parameter (UP), is studied for a given feasibility percentage and number of turbines.

3.2 Results and Discussion

Before discussing our results regarding the energy-noise trade-off, it is interesting to note the value that optimization methods bring to the wind farm layout design problem. To this end, the results of a constrained optimization using penalty functions as the constraint handling approach are compared to that of an unconstrained optimization combined with manual constraint handling. A test case with 15 turbines and 70% of land availability was optimized using NSGA-II without considering any land-use constraint. Then, the turbines that were violating either the proximity or the regulatory constraints are moved manually to the closest locations such that none of the constraints are violated. Results showed that the use of constrained optimization enabled an increase in energy generation of 0.34% and a reduction in noise levels of 9.09%, compared with manually enforcing the constraints on an unconstrained optimal solution. In general, industrial experience in wind farm design has shown that in most cases, the number of constraints placed on the turbine layout is so significant that a manual approach, without computer assistance, is unlikely to even find a feasible solution, much less an optimal one.

In addition to the effect of multi-objective constrained optimization, it is advantageous to get an

Figure 3.3: Energy-noise trade-off attained by the dynamic penalty approach and 2 different penalty coefficients for 10 turbines. $C_1 = (t/n_{gen})^2 \times 10^4$ and $C_2 = (2t/n_{gen})^2 \times 10^4$.

understanding about how the constraint severity can affect the solutions of the WFLO problem. For instance, the probability of finding a feasible layout by randomly placing 5 turbines in a domain with 90% of land availability is $0.9^5 \times 100 = 59.05\%$. However, this probability decreases drastically to 16.81%, when placing 5 turbines randomly in a domain with 70% of land available. This reduction demonstrates that the probability of finding a feasible layout decreases exponentially with decreasing the land availability. More so, in industrial wind farms, where the number of turbines in a wind farm is usually more than 5 and it is not unusual to have land availabilities below 50%, the above mentioned probability decreases even further and makes finding feasible layouts more difficult for any stochastic optimization algorithm.

3.2.1 Performance of Constraint Handling Approaches

With this brief introduction about the effect of constraint severity on the energy-noise trade-off, we now compare the performance of the constraint handling approaches in the context of constrained WFLO problem. As was stated before, to avoid convergence problems that may arise from inadequate penalization of infeasible solutions, the experiments were run with two different sets of penalty coefficients, C_1 and C_2 , for both static and dynamic penalty approaches, with values as discussed in Section 2.2.2. The effect of using these sets of penalty coefficients on the results are shown in Fig. 3.3. The intersections and the overlaps of the Pareto sets attained by the first and the second sets of penalty coefficients for 10 turbines indicate that different sets of penalty coefficients have to be implemented in the context of the constrained WFLO problem to make the results reliable. Thus, the comparison of the constraint handling approaches are based on the best solutions attained by merging the solutions from different penalty function sets.

Table 3.2 presents the best performing constraint handling approaches in terms of solution quality,

Table 3.2: Best performing constraint handling approaches with respect to solution quality, quantity, energy generation range, and noise generation range.

n_T	ϕ	Solution Quality	Solution Quantity	AEP Range	SPL Range
5	70%	Dynamic	Static	Static	Dynamic
5	80%	Dynamic	Static	Dynamic	Dynamic
5	90%	Dynamic	Static	Static	Dynamic
10	70%	Dynamic	Static	Dynamic	Dynamic
10	80%	Dynamic	Static	Static	Static
10	90%	Dynamic	Static	Static	Static
15	70%	Dynamic	Static	Dynamic	Static
15	80%	Dynamic	Static	Dynamic	Dynamic
15	90%	Dynamic	Static	Dynamic	Static

Table 3.3: Averaged Run-time (hr) and number of converged cases (out of 10 runs) for test cases with 80% feasibility and a maximum objective function evaluation of 80,000.

n_T	ϕ	Static Penalty		Dynamic Penalty	
		Run-time	Converged Cases	Run-time	Converged Cases
5	80%	19.75	8/10	18.86	9/10
10	80%	70.87	1/10	78.15	1/10
15	80%	149.75	0/10	149.53	0/10

quantity, energy generation range and noise production range for each test case. A given solution (i.e. a given Pareto set) is considered to have a better quality if its maximum energy generation is higher and its minimum noise production is lower, i.e., if its dominated area in the objective space is larger. On the other hand, if the number (cardinality) of feasible/optimal layouts included in a solution (Pareto set) is larger, the solution is considered to have a better quantity. We also utilized two other metrics to characterize the distribution of the optimal layouts in the objective space. A solution is said to have better energy generation range if the difference of maximum and minimum energy generated by its layouts is larger. In a similar way, a Pareto set is better in terms of noise production range, if the noise production of its layouts covers a wider range in the objective space. From Table 3.2, it can be noted that the dynamic penalty approach performs the best in terms of solution quality for all the test cases, presumably due to its balance between local and global searches. Regarding the quantity of solutions, the static penalty approach is capable of finding more feasible layouts for the test cases with 5 and 10 turbines. In these test cases, feasible layouts are found more easily by random sampling as a result of the relatively small number of turbines. Thus, the static penalty approach, which favours domain exploration, seems to find more feasible layouts. By increasing the number of turbines to 15, finding feasible layouts becomes exponentially more difficult, as discussed before. Hence, the number of feasible/optimal layouts found by the dynamic penalty approach becomes very close to the cardinality of the Pareto set found by the static penalty function.

Regarding the computational performance of constraint handling methods, Table 3.3 shows the number of converged cases (out of 10 runs) and average run-time over 10 runs, i.e., 5 random runs with C_1

and 5 random runs with C_2 , for 5, 10, and 15 turbines, and a domain feasibility of 80%. A set of preliminary runs with a termination criteria of 40,000 objective function evaluations (results not shown here) showed that the run-time associated with the death penalty is quite large (30.84 hr for 5 turbines and 80% feasibility), due to the additional objective function evaluations that are required after discarding each infeasible layout. This deficiency, together with the poor performance of death penalty function on the preliminary results, persuaded us to not to use it for the final experiments with 80,000 objective function evaluations, shown in Table 3.3. As Table 3.3 shows, the run-time and convergence of the static and dynamic penalty approaches are rather close to each other. Even though the run-time associated with the dynamic approach is less than that of the static one for some cases, the dynamic penalty does not have a noticeable superiority over the static penalties in this aspect.

3.2.2 Effects of Constraints on Energy-noise Trade-off

After investigating the performance of the constraint handling methods, we move on to discuss the effects of land use constraints on energy-noise trade-off. To achieve this goal, we base our discussion on the results with the best quality according to Table 3.2. Figures 3.4(a), 3.4(b), and 3.4(c) compare the energy-noise trade-off with different land availabilities, but with a specific number of turbines. It should be noted that the x axis in these figures is reversed, so that the utopia point, i.e., the infimum of the objective functions vector, is located in the bottom left corner of the figures. Also, each data point represents a layout and the Pareto front is the set of non-dominated data points. These figures show that as constraint severity increases, maximum energy generation decreases, while minimum attainable noise is increased. The most significant fact depicted by these figures is that the severity of land use constraints is more influential on noise production compared to energy generation. As the land use constraints become more severe, more noise receptors are located in the domain, making it more likely for the turbines to be placed close to them. More importantly, the proximity constraint protects turbines from the wake region of each other and does not let the regulatory constraint affect the energy generation to the same extent to which it affects the noise propagation.

The energy-noise trade-off is also investigated for different number of turbines and a given, fixed value of ϕ . Figures 3.4(d), 3.4(e), and 3.4(f) depict that for all the feasibility percentages both energy generation and minimum attainable noise are strongly dependent on the number of turbines. It should be noted that this comparison is carried out using the solutions with a wider range of noise production according to Table 3.2. As depicted by Fig. 3.4 and due to the proximity constraint, the noise production (SPL) has a wider range compared to energy generation (AEP).

The other parameter that can have an impact on the results of the optimization is the spatial distribution uniformity of the non-feasible areas in the domain. To assess this effect, we created 4 test cases with 80% land availability and different uniformity parameters, optimizing the layout of 10 turbines. Figure 3.5 shows the spatial distribution of non-feasible areas for these test cases. The results for these domains, obtained with the dynamic penalty approach, are depicted in Fig. 3.6. According to Fig. 3.6, domains 3 and 4 produce better solutions in terms of quantity; however, the objective functions' ranges of the feasible/optimal layouts found in these two domains are relatively small compared to that of the other domains, presumably because the uneven distribution of non-feasible areas increases the

Figure 3.4: Energy-noise trade-off for different number of turbines and domain feasibilities.

Figure 3.5: Spatial distribution of non-feasible areas for four different values of the uniformity parameter (UP). Cases (a) to (d) have the same 80% land availability.

Figure 3.6: Energy-noise trade-off for 10 turbines and 80% of land availability with different distribution uniformities of the non-feasible areas.

probability of having large, continuous portions of feasible land. Thus, GA explores this feasible area extensively and finds a large number of layouts that have similar energy generation and noise production, but are not necessarily optimal. On the other hand, with smaller UPs, locating turbines between the narrow spacings of the evenly distributed non-feasible areas becomes difficult; thus the cardinality of the feasible layouts found by the GA decreases. An additional observation is that, for lower values of UP (more uniformly distributed non-feasible areas), the turbine layouts obtained (not shown here) are more dissimilar from each other, and they result in larger ranges of energy generation and noise production (shown in red triangles in Fig. 3.6) than test cases with higher UP (shown in blue squares in Fig. 3.6). Finally, it should be noted that UP calculates the distribution uniformity with respect to the centers of the non-feasible polygons and does not consider their areas. As such, we consider UP an incomplete characterization of the spatial distribution uniformity of the non-feasible areas in the domain. In our ongoing work, we continue to evaluate approaches to characterize the complexity of a WFLO problem case based on the severity and spatial distribution of land use constraints.

In summary, our results show that the severity of land use constraints affects noise production of turbines to a greater extent compared to their energy generation. From the constraint handling point of view, the dynamic penalty function results in wind farm layouts with more energy generation and less noise production compared to the layouts achieved by using other constraint handling approaches. Finally, it is shown that the non-uniform distribution of the non-feasible areas results in finding more feasible solutions while their optimality might not be guaranteed.

Chapter 4

Constraint Handling via Constraint Programming (CHCP)

In this chapter, we describe the proposed Constraint Handling via Constraint Programming (CHCP) approach. However, before doing so, it is necessary to get an insight about the effects of the penalty function approach on the iteration-level behavior of GA. As soon as an infeasible solution is penalized with penalty functions, the chance for that solution to be chosen to participate in the recombination process decreases drastically. The GA is forced to discard that solution and look for new feasible solutions in the domain. This characteristic of the penalty functions is known as exploration, global search, or diversification. For highly constrained problems, the probability of finding feasible solutions is relatively low; thus, the penalty functions can result in finding a small number of solutions (i.e., a non-uniformly distributed approximation of the full Pareto set) and premature convergence due to the inability of the algorithm to generate new feasible solutions [9]. Although the dynamic penalty approach performs a combination of global and local searches due to lower penalizations in the initial stages of the optimization [31, 56, 38], in this chapter we introduce the use of CP to reinforce the local search behavior of the constraint handling approach as an alternative to dynamic penalties.

4.1 Modelling

The idea behind the CP model used in the proposed CHCP approach is to find feasible solutions that are as close as possible to the corresponding infeasible solutions. Since this model only searches the neighbourhood of the infeasible solutions, it can be considered as exploitation, local search or intensification. The advantage of repairing the infeasible solutions is that GA does not have to search for feasible solutions with the small probability that was discussed above, which results in the decrease of the computational cost. However, the drawback is that it prevents GA from exploring the feasible area of the domain and keeps searching in the neighborhood of the infeasible solutions. Our proposed CHCP approach avoids the pure exploration or exploitation by using the CP model together with the penalty functions. When an infeasible layout is generated, it is first passed to the CP model and the CP model searches for a feasible layout which is as close as possible to the infeasible layout. If the CP model cannot find a feasible layout close enough to the infeasible layout in a certain amount of time, the infeasible

layout is penalized using dynamic penalty functions.

Based on the proposed approach, the CP model can be formulated as,

$$\underset{(x_{t_i}^*, y_{t_i}^*)}{\text{minimize}} \quad \sum_{i=1}^{n_{nf}} \left((x_{t_i} - x_{t_i}^*)^2 + (y_{t_i} - y_{t_i}^*)^2 \right), \quad (4.1)$$

subject to,

$$\sqrt{(x_{t_j} - x_{t_i}^*)^2 + (y_{t_j} - y_{t_i}^*)^2} \geq 5D, \quad (4.2)$$

$\forall j \in \{1, 2, \dots, n_T\}, j \neq i$, and

$$A_{i_k}^* - A_{P_k} > 0 \quad \forall P_k \in \mathbb{S}, \quad (4.3)$$

where n_{nf} is the number of infeasible turbines in an infeasible layout (i.e., the number of turbines that violate either the proximity or the regulatory constraint in an infeasible layout) and (x_{t_i}, y_{t_i}) and $(x_{t_i}^*, y_{t_i}^*)$ are the current and repaired coordinates of the i th infeasible turbine respectively. The CP code is developed using IBM ILOG CP Optimizer [32]. Since it is more common to use integer variables in a CP solver, domains of the coordinate variables are discretized for the sole purpose of this optimization sub-problem.

The CP model has three different parameters that can be tuned. The first parameter is the discretization resolution. If we make the discretization finer, the CHCP approach provides a better resolution. However, it is clear that the computational cost will increase. The second parameter is the time limit in which the CP model has to repair an infeasible layout. Longer time limits for the CP model result in feasible solutions that are closer to the infeasible solutions; however, an increase in the overall run time. The last parameter is the maximum objective function value that a solution of the CP model can have in order to be accepted as a close enough feasible layout to the infeasible layout. We refer to this quantity as the objective function target for the CP subproblem or, more simply, as the objective target. In a similar way, decreasing this value results in longer run times and a decrease in the number of infeasible layouts that are repaired with the CP model in each generation of the evolutionary algorithm.

Based on a set of experiments with the hybrid CP and static penalty function model, the wind farm terrain is discretized in square cells with size of 20 m $\times$ 20 m. These experiments show that a finer discretization increases the computational cost, while the optimization results do not change significantly. The time limit per call for the CP model is 10 seconds. The experiments on this parameter show that time limit does not have an effect on the effectiveness of the CHCP approach, measured in this context as the percentage of infeasible solutions that the CP model is able to repair, i.e., the percentage of infeasible solutions that do not need to be penalized. However, it is shown that the important parameter in this case is the objective function of the CP model. This objective function is defined as the sum of the squared Euclidean distances of the repaired feasible turbines from their corresponding infeasible turbines. The maximum objective function value (i.e., the maximum value of Eq. 4.1) for which the solution found by the CP solver is accepted is set to 10,000 m². Considering the fact that this value is the sum of squared values, it is assumed as a reasonable value in wind farm with characteristics that are explained in the following section. Specifically, in our WFLO test problem this objective function target for the CP subproblem can be interpreted as accepting feasible solutions for which the total of turbine distances between the feasible solution and its closest infeasible solution is no more than 100 m.

4.2 Test Cases

The CHCP approach is first verified with sample test functions from the optimization literature and then applied to the WFLO problem. The sample test functions used for verification are shown in the next paragraphs. It should be noted that the hardware, software, and the WFLO test cases used herein are the same used to generate the results in Sec. 3.1.

4.2.1 Verification Test Cases

The CHCP approach is verified with five sample constrained multi-objective optimization problems that are previously used by Deb et al. [12, 35] for testing constraint handling approaches with NSGA-II and NSGA-III [35]. The first problem is called CONSTR and is formulated as,

$$\begin{aligned} & \underset{X}{\text{minimize}} && f_1(X) = x_1, \quad f_2(X) = (1 + x_2)/x_1, \\ & \text{subject to} && g_1(x) = x_2 + 9x_1 \geq 6, \quad g_2(x) = -x_2 + 9x_1 \geq 1, \end{aligned}$$

where $X = \{x_1, x_2\}$ and $x_1 \in [0.1, 1.0]$, $x_2 \in [0, 5]$. This problem is a bi-objective optimization with linear and non-linear objective functions and two linear constraints, which can be solved analytically.

The second problem is called SRN and is formulated as,

$$\begin{aligned} & \underset{X}{\text{minimize}} && f_1(X) = (x_1 - 2)^2 + (x_2 - 1)^2 + 2, \\ & && f_2(X) = 9x_1 - (x_2 - 1)^2, \\ & \text{subject to} && g_1(x) = x_1^2 + x_2^2 \leq 225, \quad g_2(x) = x_1 - 3x_2 \leq -10, \end{aligned}$$

where $X = \{x_1, x_2\}$ and $x_1, x_2 \in [-20, 20]$. The SRN problem is a bi-objective optimization with quadratic objective functions, a linear, and a quadratic constraint, which has analytical solution.

The third problem is called TNK. This problem is a bi-objective optimization problem with two linear objective functions, one non-linear, and one quadratic constraint. Deb et al.[12] showed that this problem is different from CONSTR and SRN problems because it exhibits a discontinuous Pareto front. TNK is formulated as,

$$\begin{aligned} & \underset{X}{\text{minimize}} && f_1(X) = x_1, \\ & && f_2(X) = x_2, \\ & \text{subject to} && g_1(x) = -x_1^2 - x_2^2 + 1 + 0.1 \cos(16 \arctan(x_1/x_2)) \leq 0, \\ & && g_2(x) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 \leq 0.5, \end{aligned}$$

where $X = \{x_1, x_2\}$ and $x_1, x_2 \in [0, \pi]$.

The fourth problem that the CHCP approach is tested with is called WATER. This problem has 3 variables, 5 linear and non-linear objective functions, and 7 non-linear constraints. WATER can be

formulated as,

$$\begin{aligned}
& \underset{X}{\text{minimize}} && f_1(X) = 106780.37(x_2 + x_3) + 61704.67, \\
& && f_2(X) = 3000x_1, \\
& && f_3(X) = (305700)2289x_2/(0.06 \times 2289)^{0.65}, \\
& && f_4(X) = (250)2289 \exp(-39.75x_2 + 9.9x_3 + 2.74), \\
& && f_5(X) = 25(1.39/(x_1x_2) + 4940x_3 - 80), \\
& \text{subject to} && g_1(x) = 0.00139(x_1x_2) + 4.94x_3 + 0.08 \leq 1, \\
& && g_2(x) = 0.000306(x_1x_2) + 1.082x_3 + 0.0986 \leq 1, \\
& && g_3(x) = 12.307(x_1x_2) + 49408.24x_3 + 4051.02 \leq 50000, \\
& && g_4(x) = 2.098(x_1x_2) + 8046.33x_3 + 696.71 \leq 16000, \\
& && g_5(x) = 2.138(x_1x_2) + 7883.39x_3 + 705.04 \leq 10000, \\
& && g_6(x) = 0.417(x_1x_2) + 1721.26x_3 + 136.54 \leq 2000, \\
& && g_7(x) = 0.164(x_1x_2) + 631.13x_3 + 54.48 \leq 550,
\end{aligned}$$

where $X = \{x_1, x_2, x_3\}$, $0.01 \leq x_1 \leq 0.45$, $0.01 \leq x_2 \leq 0.10$, and $0.01 \leq x_3 \leq 0.10$.

The last problem that is used in this study to verify the CHCP approach is called DTLZ1C3. DTLZ1 is an unconstrained multi-objective optimization problem with non-linear objective functions that is introduced by Deb et. al [13] to evaluate the performance of multi-objective optimization algorithms. Later, Deb et. al [35] proposed a type of constraints that can be added to this test problem, and referred to them as Type 3 constraints. These constraints do not allow the entire Pareto-optimal front of the unconstrained problem to remain optimal, since portions of the added constraint surfaces constitute the Pareto-optimal front. The unconstrained DTLZ1 problem [13] with the added Type 3 constraints is referred to as DTLZ1C3 [35], and it is formulated as,

$$\begin{aligned}
& \underset{X}{\text{minimize}} && f_1(X) = \frac{1}{2}x_1x_2(1 + g(X_M)), \\
& && f_2(X) = \frac{1}{2}x_1(1 - x_2)(1 + g(X_M)), \\
& && f_3(X) = \frac{1}{2}(1 - x_1)(1 + g(X_M)), \\
& && g(X_M) = 100 \left[|X_M| + \sum_{x_i \in X_M} (x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5)) \right], \\
& \text{subject to} && g_1(x) = f_1(X) + \frac{f_2(X)}{0.5} + \frac{f_3(X)}{0.5} - 1 \geq 0, \\
& && g_2(x) = f_2(X) + \frac{f_1(X)}{0.5} + \frac{f_3(X)}{0.5} - 1 \geq 0, \\
& && g_3(x) = f_3(X) + \frac{f_1(X)}{0.5} + \frac{f_2(X)}{0.5} - 1 \geq 0,
\end{aligned}$$

where $X = \{x_1, \dots, x_7\}$, $X_M = \{x_3, \dots, x_7\}$, and $0 \leq x_i \leq 1 \forall x_i \in X$.

We followed Deb et al. [12] to set the NSGA-II parameters for these problems. Each test case has

a population of 100 and is run for 500 generations. Also, the test case is considered to be converged if the variance of the crowding distances of rank 1 solutions is less than 0.005 over 100 generations. The dynamic penalty approach and the CHCP approach with different setups are tested on these problems to investigate the effect of the parameters of the CHCP approach on the results. To avoid the potential negative impacts of randomness, 20 different random test cases with two different penalty coefficient are solved for each problem and are compared using box plots.

Before presenting our results in the next section, it is important to note that all the test problems discussed above have objective function spaces that are either 2 or 3 dimensional. As mentioned in [12], there is no limitation on increasing the number of objective functions; however, the investigation of the performance of the CHCP approach on constrained optimization problems with more than 3 objectives is beyond the scope of this study.

4.3 Results and Discussion

In this section, the verification results for the test problems are first discussed and then the application of the above mentioned constraint handling approaches in the WFLO problem is investigated.

4.3.1 Verification of CHCP

In this section, the CHCP approach is verified with several test functions. In addition, the setup for which the CHCP approach performs the best is investigated. An important characteristic of the CHCP approach is the percentage of infeasible solutions that are repaired by the CP model. The maximum acceptable objective function for the CP model affects the number of infeasible solutions that are repaired by it. Thus, we run test cases with different maximum acceptable objective function values. For simplicity, hereafter we will refer to the percentage of times that the CP solver successfully returned a feasible solution as “CP percentage”. Similarly, the maximum acceptable objective function value is called “objective target”. The objective target is the maximum squared Euclidean distance of a repaired feasible solution from its corresponding infeasible solution in the input space. It should be noted that the objective target is the independent parameter of the CP model that can be tuned by the user. However, the resulting CP percentage completely depends on the value of the objective target and the optimization problem.

Two different metrics are used to compare the optimization results that are achieved by different objective targets. The first metric is the non-dominated hyper volume (NDHV) that shows how close the Pareto set is to the utopia point (i.e., the infimum of the objective functions vector) and the closer the Pareto set is to the utopia point the smaller the NDHV is. However, this metric may not be sufficient for comparing the optimization results especially at the early stages of the optimization, where it is likely that a Pareto set has a smaller value of NDHV due to incomplete exploration of the objective space. The second metric is the maximum crowding distance in the Pareto set that provides a measure of how well the objective space is explored by the optimization algorithm, in terms of the minimum distance in the objective space between two non-dominated solutions from the current approximation of the Pareto

front. A small value for the maximum crowding distance shows that the optimization algorithm has been successful in having a uniform coverage over all the areas of the Pareto set.

For all these experiments the time limit of the CP model is 10 seconds per call and the decision variable space is discretized into 22500 cells, the same discretization as the WFLO problem. For each CP percentage, the problem is solved with 20 random test cases and two different penalty coefficients for each random test case. The results of the above mentioned 40 experiments for each CP percentage are shown in the format of box plots. The box plot for zero CP percentage represents constraint handling with dynamic penalty functions and the other box plots represent constraint handling with the proposed CHCP approach (which combines CP with dynamic penalties when necessary) and different objective targets. Finally, before going through verification results discussion, it should be mentioned that the performance of the dynamic penalty approach on these test problems is compared to that of the constraint handling approaches that are discussed by Deb et al. [12] and Jain et al. [35]. It is shown that the results obtained using the dynamic penalty functions converge to the optimal Pareto front. In addition, the dynamic penalty approach is capable of achieving Pareto fronts with higher cardinalities.¹

Figure 4.1: CP percentage with different objective targets for CONSTR problem.

We start the discussion with the CONSTR problem. Figure 4.1 shows the variation of CP percentage for different objective targets. As the objective target of the CP model decreases, it is forced to find feasible solutions closer to the infeasible solutions within the same time limit. When the CP model is unable to do so, it passes these solutions to the dynamic penalty operator, thus decreasing the CP percentage. Both constraint handling approaches converge to the analytical Pareto optimal solutions of this problem. Thus, they both have the same performance in finding the Pareto optimal solutions. However, besides converging to the optimal solution, it is important to investigate the convergence speed of different constraint handling approaches. Thus, the iteration-level behavior of the constraint handling approaches and the trajectory of the optimization should be studied. To this end, the intermediate results

¹The figures related to the comparison of the performance of the dynamic penalty approach and the constraint handling approaches proposed by Deb et al. [12] and Jain et al. [35] are not shown in this study as we do not have the copyright to publish their results.

(a) NDHV after 30 generations.

(b) NDHV after 40 generations.

(c) Crowding distance after 30 generations.

(d) Crowding distance after 40 generations.

Figure 4.2: Non-dominated hyper volume and maximum crowding distance with different constraint handling approaches for the CONSTR problem after 30 and 40 generations. Note that a CP percentage of 0% corresponds to constraint handling using only the dynamic penalty approach. Notches in each box plot indicate 95% confidence intervals around the median of the distribution.

at the 30th and 40th generations are presented in Fig. 4.2. The figure shows the comparison of these two metrics for the dynamic penalty approach (CP percentage of 0%) and the CHCP approach with different CP percentages. The penalty function approach has a larger NDHV and maximum crowding distance through the generations compared to the CHCP approach. Both constraint handling approaches are able to find the optimal Pareto set by the 40th generation. Thus, the NDHV does not change significantly from 30th generation to 40th generation especially for the CHCP approach. The maximum crowding distance changes significantly for the CHCP approach from 30th generation to 40th generation, while this change for the dynamic penalty approach is not as large as the CHCP approach. Considering the fact that NDHV has not changed significantly for all the CP percentages, the significant reduction in the maximum crowding distance of the different setups of the CHCP approach can be interpreted as finding more feasible/optimal solutions and improving the uniformity of coverage of the Pareto Front.

Different CP percentages are tested with the CHCP approach and it is observed that the test case with 97% of constraint handling with the CP model has a better performance in finding the optimal Pareto set. The 97% test case has a smaller NDHV compared to the dynamic penalty approach and other CP percentages, while also having a significantly lower maximum crowding distance. The notch of the corresponding NDHV and crowding distance box plots to the 97% test case does not overlap with any of the other notches of the other box plots. This offers evidence that the median of the NDHV and crowding distance of the 97% test case has a statistically significant difference with the NDHV and crowding distance medians of the other CP percentages [44]. The 23% and 47% test cases also have an acceptable performance over the dynamic penalty approach. Although the NDHV of these two test cases are very close to that of the penalty approach, their maximum crowding distance is slightly lower than that of the dynamic penalty approach. As shown by Deb et al. [12], the Pareto front of the CONSTR problem is located on the borders of the feasible region in the objective space. Thus, the 97% test case that explores the feasible areas close to the infeasible areas extensively using the CP model has a better chance to find the optimal Pareto front in fewer number of generations.

Figure 4.3: CP percentage with different objective targets for SRN problem.

(a) NDHV after 4 generations.

(b) NDHV after 10 generations.

(c) Crowding distance after 4 generations.

(d) Crowding distance after 10 generations.

Figure 4.4: Non-dominated hyper volume and maximum crowding distance with different constraint handling approaches for the SRN problem after 4 and 10 generations.

The second problem to discuss is SRN. Figure 4.3 shows the variation of the CP percentage for different objective targets with SRN problem. A similar trend to the CONSTR problem is observed with the SRN problem and the CP percentage decreases with decreasing the objective target. Figure 4.4 shows the comparison of NDHV and maximum crowding distance for the dynamic penalty approach (CP percentage of 0%) and the CHCP approach with different CP percentages and for two different number of generations. To compare the constraint handling approaches and the effect of different CP percentages with the CHCP approach, the Pareto set found by them at the 4th and 10th generation are evaluated. Based on our experiments, after the 10th generation, there is no further difference between the performance of the two constraint handling approaches, except for those test cases using the CHCP approach that were lagging. The comparison of Fig. 4.4(a) to Fig. 4.4(b) and Fig. 4.4(c) to Fig. 4.4(d) shows that the median NDHV is smaller for the CHCP approach with different CP percentages after only 4 generations, so it can be said that the CHCP approach converges faster. Figure 4.4(b) depicts that the median NDHV is decreasing by increasing the CP percentage. In addition, among different CP percentages for which the CHCP approach is tested, 59.3% and 91.2% have smaller maximum crowding distance. Similar to the CONSTR problem, this can be justified with the fact that the analytical solution of the SRN problem is located on the boundary of the feasible region. Thus, increasing the CP percentage results in a more extensive local search close to the infeasible region and finding the Pareto optimal solutions faster.

The three other problems that are discussed in this section are more complicated due to their non-linearity, constraint severity, and the number of objective functions or variables. The TNK problem has linear objective functions, while its constraints are non-linear. As a result of this non-linearity the Pareto front of the analytical solution becomes discontinuous, which is in contrast with the first two investigated problems, i.e., CONSTR and SRN. Figure 4.5 shows the variation of the CP percentage for different objective targets for TNK problem. In a similar fashion to the CONSTR and SRN problem, the CP percentage decreases with decreasing the objective target.

Figure 4.5: CP percentage with different objective targets for TNK problem.

Our study shows that, unlike the CONSTR and SRN problems in which convergence is observed

(a) Maximum Crowding Distance

(b) Average Crowding Distance

(c) 3rd Quartile Crowding Distance

Figure 4.6: Crowding distance for different constraint handling approaches for the TNK problem.

Figure 4.7: CP percentage with different objective targets for WATER problem.

in the initial generations, the TNK problem exhibits a more gradual and continuous improvement in the optimization results throughout the generations. Thus, to analyze the performance of the CHCP approach on the TNK problem the comparison metrics are evaluated at the final generation. Our results show that bounds of the Pareto front are found in the initial stages of the optimization and the NDHV value converges to its final value after a few generations; thus, the comparison of NDHV might not be helpful for this problem. Figure 4.6(a) shows that all the CP percentages have similar maximum crowding distances. Since the maximum crowding distance is equal to the crowding distances of the solutions in the neighborhood of the discontinuities in the objective space, we can conclude that all the CP percentages have been successful in finding the discontinuities of the Pareto front. To investigate the performance of different CP percentages in providing a uniform coverage of Pareto/optimal solutions on the discontinuous Pareto front, the average and the 3rd quartile of the crowding distances are compared in Fig. 4.6(b) and Fig. 4.6(c) respectively. Both average and 3rd quartile crowding distances have values that are 2 orders of magnitude smaller than that of the maximum crowding distance. The average crowding distance damps the effect of high crowding distances in the neighborhood of the discontinuities by calculating the average of them, while the 3rd quartile crowding distance avoids the effect of high crowding distances by simply withdrawing them. As a result of the damping effect of calculating the average, the medians of the average crowding distances in Fig. 4.6(b) are very close to each other. However, the 3rd quartile of the crowding distances is able to elucidate the success of the constraint handling approaches in finding Pareto/optimal solutions that are uniformly covering the discontinuous Pareto front. The comparison of the 3rd quartile of the crowding distances in Fig. 4.6(c) shows that a moderate use of the CP model, i.e., the test case with 41.5% of repairing the infeasible solutions with the CP model, performs better than the other CP percentages.

WATER is the fourth test problem for which the performance of the CHCP approach is evaluated. This problem has 3 variables, 5 non-linear objective functions and 7 non-linear constraints. Similar to previously evaluated test problems, Fig. 4.7 shows that for WATER problem the CP percentage decreases with decreasing the objective target as well. The WATER problem is similar to TNK in terms

(a) Maximum Crowding Distance

(b) Average Crowding Distance

(c) 3rd Quartile Crowding Distance

Figure 4.8: Crowding distance for different constraint handling approaches for the WATER problem.

(a) Maximum Crowding Distance

(b) Average Crowding Distance

(c) 3rd Quartile Crowding Distance

Figure 4.9: Crowding distance for different constraint handling approaches for the DTLZ1C3 problem.

of its discontinuous Pareto front and convergence behavior. Thus, the same evaluation metrics as TNK problem are applied to this problem. Figure 4.8(a) shows that CHCP approach has maximum crowding distances that are lower than that of the dynamic penalty function. This shows that the CHCP approach is more successful in finding the Pareto/optimal solutions that are located in the neighborhood of the discontinuities of the Pareto front. Fig. 4.8(b) shows that the CHCP approach provides a lower average crowding distance; thus, better optimization results. According to Fig. 4.8(c), the 75.4% test case has the lowest 3rd quartile of the crowding distances and performs better than the other CP percentages. By comparing the average crowding distance to 3rd quartile crowding distance, it is observed that 52.1% test case has a low average crowding distance, but a relatively high 3rd quartile crowding distance. The reason is that the 52.1% test case has a relatively large crowding distance in general, which is not due to the discontinuities, but its effect is damped in Fig. 4.8(b) as a result of calculating the average. However, the effect of this large crowding distance is shown in Fig. 4.8(c), when the 3rd quartile of the crowding distances is calculated.

DTLZ1C3 is the fifth and last test problem for which the performance of the CHCP approach is evaluated in this study. This problem has 7 variables, 3 non-linear objective functions and 3 non-linear constraints. In contrast to the previously studied test problems, the CP percentage does not increase gradually by increasing the objective target. It is observed that after a certain objective target the CP percentage increases from 0% to 100%. The trigonometric functions of this problem are modelled using Taylor series, which can cause an inaccuracy when smaller objective targets are implemented. Further studies are in progress to evaluate the effect of Taylor series on the objective target and CP percentage. Maximum, average and 3rd quartile of the crowding distances are shown in Fig. 4.9 for CP percentages of 0 and 100. As a result of not having discontinuities in the Pareto front of the DTLZ1C3 problem [35], all the three comparison metrics in Fig. 4.9 show similar results. It is observed that dynamic penalty function and pure local search by the CP model have the similar performances. Thus, it is important to evaluate the performance of different CP percentages on this problem.

4.3.2 CHCP Performance for WFLO Problem

In a similar fashion to the verification test problems, the experiments for the WFLO problem are carried out with different objective targets for the CP model and hence different CP percentages. Similar to the test problems, 20 random test cases with two different penalty coefficients are solved for each WFLO problem with a specific number of turbines, land availability, and objective target. Then, the 40 Pareto fronts that result from these experiments are merged and an overall Pareto front is determined, containing the non-dominated solutions across all 40 runs. In this work we have favoured this approach to study the performance of the algorithms, as opposed to obtaining an average or median Pareto front, given that such definitions are not straight forward to implement and interpret in multi-dimensional spaces [1]. More specifically, using an average Pareto front, however calculated, would result in analyzing solutions that are the result of arbitrary operations in the performance space, but that may not correspond to any feasible layout.

Figures 4.10, 4.11, and 4.12 compare the optimal Pareto sets found by the dynamic penalty approach and different setups of the CHCP approach. It should be noted that in all the results presented in this section $CP = 0.0\%$ represents constraint handling with dynamic penalty approach and the x axis is

(a) $n_T = 5, \phi = 70\%$ (b) $n_T = 5, \phi = 80\%$ (c) $n_T = 5, \phi = 90\%$

Figure 4.10: Comparison of constraint handling approaches for 5 turbines (x axis is reversed).

(a) $n_T = 10, \phi = 70\%$ (b) $n_T = 10, \phi = 80\%$ (c) $n_T = 10, \phi = 90\%$

Figure 4.11: Comparison of constraint handling approaches for 10 turbines (x axis is reversed).

(a) $n_T = 15, \phi = 70\%$ (b) $n_T = 15, \phi = 80\%$ (c) $n_T = 15, \phi = 90\%$

Figure 4.12: Comparison of constraint handling approaches for 15 turbines (x axis is reversed).

reversed, so that the utopia point is located in the bottom left corner of each figure. By comparing these figures, it can be claimed that for all the test cases except the test case with 10 turbines and 80% of land availability there are some CHCP setups that are performing better than the dynamic penalty approach (i.e., higher energy generation and lower noise production) and there is one setup that performs the best among all the setups of the proposed CHCP approach. In what follows, we discuss the effect of the spatial distribution of the non-feasible areas, especially for the test case with 10 turbines and 80% of land availability.

For the test case with 10 turbines and 80% of land availability (Fig. 4.11(b)) the Pareto set found by the dynamic penalty approach is slightly better than other CP percentages, i.e., within the same energy generation, the noise production of the dynamic penalty approach is slightly lower than that of different CHCP setups. To investigate the reason of this unexpected phenomenon, the Pareto fronts found for each of the 40 random runs of the dynamic penalty approach are plotted together with the non-dominated Pareto fronts of all the 40 runs of different setups of CHCP approach in Fig. 4.13. Figure 4.13 shows that the Pareto fronts found by different setups of the CHCP approach are better than 38 Pareto fronts out of 40 Pareto fronts that are found during the 40 runs of the dynamic penalty approach. However, there are only 2 runs that make the final non-dominated Pareto of the dynamic penalty approach slightly better than those of the CHCP approach. The corresponding layouts of the points shown in Fig. 4.13 by the arrows, i.e., points with AEP = 48.19 GWhr and noise production of SPL = 41.67 dBA, SPL = 42.35 dBA, and SPL = 43.68 dBA for CP = 0.0%, CP = 76.1%, and CP = 94.6% respectively, are plotted and compared to each other in Fig. 4.14. The red, black, and purple points represent turbine locations for $CP = 0.0\%$, $CP = 76.1\%$, and $CP = 94.6\%$ respectively. Figure 4.14 depicts that the 3 layouts are similar to each other, while their difference is in the turbines residing in $Y \simeq 3000$ and $2000 < X < 3000$ for $CP = 0.0\%$. This part of the domain is far from the non-feasible areas and the CHCP approach is not able to explore it to the extent that the dynamic penalty approach can due to its local search behavior. As a result, none of the different setups of the CHCP approach have been able to explore this area. As Fig. 4.13 shows, only 2 runs out of 40 runs of the dynamic penalty approach have been able to explore this area of the domain. Thus it can be concluded that the non-feasible areas for the domain of this test case are located in such a way that all the constraint handling approaches used in this study have difficulties exploring the above mentioned area. As stated in Sec. 3.2.2, all the test cases have the same spatial distribution uniformity parameters. However, the results of our simulations for 10 turbines and 80% of land availability show that the distribution of the non-feasible areas has an effect on the performance of the constraint handling approaches. In addition, comparing the results of different CP percentages shows that there is no general trend for the best performing CP percentage, although most of them perform better than the dynamic penalty approach. Thus, there is a need to define a parameter that can correlate the spatial distribution of the non-feasible areas to the CP percentage that performs the best.

After comparing the optimization results, it is necessary to compare their performance in terms of total infeasible solutions, percentage of infeasible solutions repaired by the CP model (i.e., CP percentage), convergence, and computational cost. Figures 4.15, 4.16, and 4.17 show the variation of CP percentage with different objective targets for all the test cases. Similar to the verification test functions, box plots are used to show the CP percentages. As the objective target of the CP model decreases, the CHCP approach is forced to find feasible solutions closer to the infeasible solutions within the same time limit.

Figure 4.13: Comparison of the all solutions found by the dynamic penalty approach in 40 runs with the Pareto fronts of the different setups of CHCP approach.

Figure 4.14: Layout comparison for $CP = 0.0\%$, $CP = 76.1\%$, and $CP = 94.6\%$ with same energy generation and different noise production.

Thus, CP percentage decreases with decreasing the objective target.

Table 4.1 compares the average infeasible solutions generated in 40 runs using different constraint handling approaches. It is shown that for 5 and 10 turbines, using the CHCP approach results in more infeasible solutions. The CHCP approach replaces the infeasible solutions with feasible solutions that are close the infeasible ones. As a result, there is a high possibility that these feasible solutions become infeasible during the recombination or mutation process of the GA. For 15 turbines, the number of infeasible solutions by the penalty approach increases significantly, while the number of infeasible layouts for the CHCP approach remains in the same order of magnitude as that of 5 and 10 turbines. As the number of turbines increases, the domain becomes more constrained and the probability for which the global search (i.e., penalty approach) can find feasible solutions, decreases drastically. On the other hand, the local search explores the feasible areas of the objective space that are closer the infeasible areas and performs a more accurate search in highly constrained domains. Thus, the CHCP has a more robust performance compared to the dynamic penalty approach from this point of view. However, as the objective target increases, no general pattern for the number of infeasible solutions is observed. Increasing the objective target makes the process of repairing the infeasible solutions easier for the CP model; however, the shape of the feasible space varies for different test cases, i.e. different number of turbines and land availabilities. Thus, there is no guarantee that increasing the objective target results in generating less infeasible solutions in the whole optimization process.

Table 4.1: Average number of infeasible layouts generated per each run by the different constraint handling approaches, for different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.

n_T	ϕ	Dynamic Penalty CP% = 0	CHCP			
			OT = 50	OT = 100	OT = 1000	OT = 10000
5	70%	2189.8	5307.9	5323.8	4672.2	5365.1
5	80%	513.5	1083.5	1199.9	1329.6	1475.3
5	90%	138.6	286.3	325.3	371.6	239.4
10	70%	3056.1	7555.5	5478.3	6202.6	7578.3
10	80%	1869.1	4202.8	5117.1	3459.2	5080.0
10	90%	2662.9	2371.5	3808.6	2922.2	3661.8
15	70%	350575.1	7826.9	8722.8	6808.3	7681.4
15	80%	416098.1	5856.6	5665.1	5551.9	7212.2
15	90%	353616.3	5027.7	5449.8	4934.8	6624.9

Table 4.2 shows the CP percentage for different constraint handling approaches. As expected, within the same objective target, when the number of turbines increases, the CP percentage decreases. An increase in the number of turbines, makes the problem more constrained. Hence, finding feasible solutions that are close to the infeasible solutions becomes harder for the CP model.

Tables 4.3 and 4.4 show the convergence and computational cost of the different constraint handling approaches for the WFLO problem respectively. Table 4.3 shows that in general, using CHCP approach results in better convergence. In addition, Table 4.4 shows that the CHCP approach has the same run-time as the penalty approach. Thus, a better convergence rate within the same run-time can be

(a) $n_T = 5, \phi = 70\%$ (b) $n_T = 5, \phi = 80\%$ (c) $n_T = 5, \phi = 90\%$

Figure 4.15: CP percentage for different objective targets and 5 turbines (dynamic penalty is represented with an objective target of 0).

(a) $n_T = 10, \phi = 70\%$ (b) $n_T = 10, \phi = 80\%$ (c) $n_T = 10, \phi = 90\%$

Figure 4.16: CP percentage for different objective targets and 10 turbines (dynamic penalty is represented with an objective target of 0).

(a) $n_T = 15, \phi = 70\%$ (b) $n_T = 15, \phi = 80\%$ (c) $n_T = 15, \phi = 90\%$

Figure 4.17: CP percentage for different objective targets and 15 turbines (dynamic penalty is represented with an objective target of 0).

Table 4.2: Average of the CP percentages of each run for different constraint handling approaches and different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.

n_T	ϕ	Dynamic Penalty CP% = 0	CHCP			
			OT = 50	OT = 100	OT = 1000	OT = 10000
5	70%	0.0	20.5	41.8	77.8	99.4
5	80%	0.0	22.9	47.8	85.0	99.6
5	90%	0.0	19.1	39.5	84.4	97.2
10	70%	0.0	19.4	42.3	80.1	97.7
10	80%	0.0	19.5	39.1	76.1	96.3
10	90%	0.0	11.0	26.5	69.0	94.6
15	70%	0.0	18.4	31.5	71.4	94.3
15	80%	0.0	16.2	31.8	71.7	94.2
15	90%	0.0	9.8	22.4	67.6	93.9

considered as one of the advantages of the CHCP approach over the penalty approach. Table 4.4 also shows that increasing the objective target increases the run-time. As shown in Table 4.2 increasing the objective target makes solving the problem easier for the CP model and results in a higher CP percentage. As a result, more new feasible solutions are generated that all need their objective functions to be evaluated. As discussed in Sec. 2.2, the objective functions of the WFLO problem are both non-linear and evaluating them for a large number of newly generated feasible solutions increases the computational cost significantly. Thus, increasing the objective target increase the computational cost. In a similar fashion to Table 4.1, as the objective target increases, the convergence and run-time do not show a general trend. As mentioned before, the shape of the feasible objective space is different for different test cases; thus, increasing the objective target might not necessarily result in better run-time or convergence. The total run-time spent on repairing the infeasible solutions is a function of not only the objective target but also the shape of the feasible objective space. Hence, comparing the the run-time and convergence by only considering the objective target might not lead to conclusive results.

Table 4.3: Number of converged runs (out of 40 runs) for different constraint handling approaches and different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.

n_T	ϕ	Dynamic Penalty CP% = 0	CHCP			
			OT = 50	OT = 100	OT = 1000	OT = 10000
5	70%	16	19	17	23	21
5	80%	27	16	19	18	22
5	90%	20	24	16	25	28
10	70%	6	6	12	5	8
10	80%	8	9	7	7	7
10	90%	16	18	19	13	19
15	70%	0	2	1	1	2
15	80%	3	2	4	5	3
15	90%	5	8	9	4	5

In summary, our results show that the CHCP approach has a better overall performance compared to penalty functions when applied to test problems and the WFLO problem. However, the parameters

Table 4.4: Average run-time (hr) per each run by the different constraint handling approaches, for different WFLO test cases. Note that OT denotes the objective target used in the CP model of the proposed CHCP approach.

n_T	ϕ	Dynamic Penalty CP% = 0	CHCP			
			OT = 50	OT = 100	OT = 1000	OT = 10000
5	70%	15.26	14.24	15.70	14.81	13.97
5	80%	15.77	17.02	17.95	16.92	16.36
5	90%	17.59	15.59	19.29	17.33	14.56
10	70%	55.42	48.77	47.80	50.03	58.93
10	80%	61.17	54.04	55.45	54.61	63.47
10	90%	68.56	58.96	60.49	63.22	66.96
15	70%	119.30	106.85	108.82	109.25	129.89
15	80%	124.53	117.53	113.20	116.53	133.04
15	90%	156.82	141.65	138.92	147.02	165.72

of this approach can be tuned in such a way that its performance is optimized. The most important characteristic of the proposed CHCP approach is the percentage of infeasible solutions repaired by the CP model. There is a certain percentage of infeasible solution repair by the CP model for each of the investigated problems for which the proposed CHCP approach performs the best. It is shown in our results that this specific percentage varies for different problems. For the WFLO problem, the uniformity distribution of the non-feasible areas affects the performance of the proposed CHCP approach. In general, it can be said that the parameters of the CHCP approach should be tuned based on the characteristics of the optimization problem.

Chapter 5

Concluding Remarks

This study is based on two stages. In the first stage, the impact of land-use constraints on the energy-noise trade-off for Wind Farm Layout Optimization (WFLO) problem was investigated. The second stage of this work focused on developing a novel constraint handling approach that could outperform conventional penalty approaches on optimization problems of similar nature to those considered in the first part of this study. In what follows the findings of these two stages are delineated and some potential research directions to continue this study are discussed.

5.1 Impact of Land-use Constraints

A multi-objective energy-noise wind farm layout optimization problem including land-use and proximity constraints was investigated. The optimization was carried out with the NSGA-II evolutionary algorithm and the constraints were handled using static, dynamic, and death penalty functions. The energy-noise trade-off was studied for different levels of constraint severity, numbers of turbines, and distribution uniformity of the non-feasible areas within the wind farm domain. In using the penalty functions, we conducted our constraint handling based on different degrees of objective function penalization.

The main purpose of this work was to characterize the impact of the land-use constraints on the energy-noise trade-off. It was shown that a reduction in the percentage of land availability causes a decrease in energy generation and increases the effective noise level at the receptors. However, the most interesting finding in this area was that changes in the severity of land-use constraints do not affect the energy generation to the same extent that they affect noise propagation, because of the dampening effect of the turbine proximity constraints. It should also be noted that increasing the number of turbines results in an increase in both energy generation and noise production.

Regarding the use of constraint handling methods, it was observed that the extensive global search of the static and death penalty approaches resulted in finding more feasible layouts. However, as the number of turbines was increased and the domain became more crowded, static and death penalty approaches were more likely to converge to sub-optimal layouts. In contrast, the dynamic penalty approach converged to layouts with more energy generation and lower noise production compared to the other

methods. Furthermore, our experiments with wind farm domains with higher turbine densities (number of turbines per km^2) showed that the dynamic penalty method found at least as many feasible optimal layouts as the other penalization approaches.

The impact of the distribution uniformity of the non-feasible areas on the energy-noise trade-off was also investigated. The proposed uniformity metric characterized the spatial distribution of the non-feasible areas based on their geometrical center. According to our results, a non-uniform distribution of the non-feasible areas led to finding more feasible/optimal layouts. However, it was observed that these solutions had relatively small energy generation and noise production ranges.

5.2 Constraint Handling via Constraint Programming

The constrained multi-objective energy-noise wind farm layout optimization was solved with a continuous variable Genetic Algorithm, called NSGA-II. Primarily, the dynamic penalty approach was used to handle the constraints. Then, a hybrid approach based on the combination of penalty functions and a Constraint Programming model was introduced to improve the performance of the optimization algorithm.

With the purpose of improving the optimization results, the dynamic penalty approach, which its local search was only confined to smaller penalization in the initial stages of optimization, was hybridized with a powerful local search. To this end, a Constraint Programming model was designed to repair the infeasible solutions by finding the closest feasible solutions to them. The hybridization of the Constraint Programming model with the penalty function approach created a Constraint Handling via Constraint Programming (CHCP) approach that was able to carry out a combination of local and global searches.

The results of the optimization with the test problems showed that the CHCP approach has the potential to perform better than the penalty approach. However, its performance was dependent on the percentage of infeasible solutions repaired by the Constraint Programming model. Our experiments for the WFLO problem also showed that there is a certain percentage of infeasible solution repair that will produce results with higher energy generation and lower noise production compared to the penalty function approach. In addition, the optimization results for the WFLO problem showed that there is a need to define a parameter that can correlate the spatial distribution of the non-feasible areas in the domain to the CHCP setup and result in the best performance of the CHCP approach. Finally, it was shown that the CHCP approach had similar run-time to that of the penalty approach, while providing a better convergence rate; thus, the CHCP approach outperformed the penalty approach from this perspective.

5.3 Future Work

Regarding the impact of land-use constraints on wind farm layout optimization, future work could focus on formulating a uniformity distribution metric that would better characterize the complexity of the constrained optimization problem. Such a metric could find general applicability in constrained optimiza-

tion with evolutionary algorithms and other population-based global optimization approaches, for which both the size and the topology of the feasible region are significant factors affecting their convergence properties. In addition, documenting the energy-noise trade-off for the test cases with larger numbers of turbines on a given terrain area, i.e. larger turbine densities could be an interesting area to explore. To this end, alternative optimization algorithms, or parallelized versions of our current algorithms will be explored for better computational efficiency. Furthermore, the inclusion of a comprehensive cost model that includes land prices, energy prices, participation incentives for landowners, government incentives, among others, has the potential to enrich the discussion in the context of wind farm economics and government policy.

Future work on Constraint Handling via Constraint Programming (CHCP) could focus on expanding the proposed CHCP approach by considering continuous variable Constraint Programming sub-problems, which might require using a different solver. As an option, CP Optimizer [32] can be substituted by a SCIP [68], which is capable of solving continuous variable problems. In addition, the conditions for which the proposed CHCP approach has the best performance should be fully documented. To this end, a larger base of test problems with different number of variables, constraints, and objective functions should be solved using the proposed CHCP approach. Furthermore, the effect of constraint non-linearity on the performance of the proposed CHCP approach needs to be explored.

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