Improved Real-Time Helicopter Flight Dynamics Modelling

by

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University of Toronto
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Abstract

The University of Toronto Institute for Aerospace Studies has a number of previously developed real-time helicopter models for piloted simulations. An area of concern with physics-based helicopter models is that they often have an inaccurate off-axis response to cyclic control inputs compared to flight test data. To explain the cause of this problem, several theories have been put forth in the literature concerning which aspects are modelled incorrectly or not at all, including blade elasticity, rotor wake distortion and curvature, and unsteady aerodynamic effects. In this thesis these modelling improvements were implemented and their effectiveness evaluated. To include blade elasticity, a rotor model was developed using a Ritz expansion approach with constrained elastic modes. The effect of including these features on the on-axis and off-axis response of the UTIAS helicopter models was examined. The various improvements were successful in altering the off-axis response, with notable improvements in some areas, without disrupting the on-axis response. In some conditions, the magnitude of change due to flexibility was greater than differences noted due to dynamic wake distortion or unsteady aerodynamics. The best results were obtained when blade flexibility and wake distortion were used together, which is also the most physically accurate model. The impact of these changes was also evaluated from a pilot-in-the-loop perspective, quantifying the perceived changes using simulation fidelity ratings. Since this is a newly developed metric, the simulator was first evaluated using the original baseline vehicle models. Through this process, experience could be gained in the usage of the fidelity rating scale, while also examining what effect changes to the dynamics had on the overall simulator fidelity rating obtained. While an improved match to flight test data was found to lead to a higher rated fidelity, there was a limit to how high these improvements could increase the
fidelity rating. A number of the possible contributors to the ratings achieved were further explored, which for the UTIAS Flight Research Simulator primarily include the motion system fidelity and the collimated stereoscopic visual environment.
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# Table of Contents

Acknowledgments........................................................................................................ iv

Table of Contents ........................................................................................................ v

List of Tables ................................................................................................................ viii

List of Figures .............................................................................................................. ix

Nomenclature ................................................................................................................ xii

1 Introduction .............................................................................................................. 1

2 Literature Review .................................................................................................... 4

3 Model Overview ....................................................................................................... 9

3.1 Main Rotor ............................................................................................................ 10

3.1.1 Rotor Blade Element Geometry ................................................................. 10

3.1.2 Rotor Blade Kinematics ............................................................................... 11

3.1.3 Ground Effect on Main Rotor ...................................................................... 14

3.1.4 Blade Element Angle of Attack and Mach Number .................................. 14

3.1.5 Blade Pitch ..................................................................................................... 18

3.1.6 Aerodynamic Forces and Moments ............................................................ 19

3.1.7 Lag Damper Forces and Moments ............................................................... 20

3.2 Tail Rotor ............................................................................................................. 22

3.3 Fuselage Aerodynamics ..................................................................................... 23

3.4 Empennage Aerodynamics ............................................................................... 25

3.4.1 Horizontal Tail ............................................................................................. 25

3.4.2 Vertical Tail ................................................................................................ 26

3.5 Helicopter Body Dynamics ............................................................................... 27

4 Dynamic Inflow with Wake Distortion ................................................................. 29
4.1 UH-60 Validation Results .......................................................... 31

5 Flexible Rotor Dynamics ............................................................... 34

5.1 Modal shapes and stiffness matrix ........................................... 37

5.1.1 Sample Normal Mode Results ........................................... 39

5.2 Teetering rotor hub forces ....................................................... 41

5.3 Teetering rotor hub moments .................................................. 41

5.4 Teetering rotor elastic equations ............................................. 43

5.5 Articulated rotor hub forces ...................................................... 44

5.6 Articulated rotor hub moments ................................................ 45

5.7 Articulated rotor elastic equations ......................................... 46

6 Aerodynamic Lag ....................................................................... 48

7 Offline Trials: Simulation Results ................................................ 49

7.1 Teetering Rotor Results ........................................................... 49

7.2 Articulated Rotor Results ......................................................... 51

7.3 Offline Trial Summary .............................................................. 54

8 Piloted Trials: Simulator Fidelity .................................................. 55

8.1 The UTIAS Flight Research Simulator ...................................... 55

8.2 Experimental Setup ................................................................. 59

8.3 Task 1: Hover ....................................................................... 60

8.4 Task 2: Pirouette .................................................................. 61

8.5 Task 3: Lateral Reposition ....................................................... 62

8.6 Task 4: Slalom ....................................................................... 64

8.7 Piloted Trials Summary ............................................................ 65

9 Conclusions .............................................................................. 68

References .................................................................................. 70
List of Tables

Table 1. Simulation Ratings Summary ................................................................. 65
List of Figures

Figure 1: Helicopter Reference Frames ................................................................. 11
Figure 2: Rotating Rotor Reference Frame ............................................................ 12
Figure 3: Articulated Rotor Blade Reference Frames .............................................. 13
Figure 4: Rotor Inflow Reference Frame ............................................................... 13
Figure 5: Teetering Rotor Flapping Geometry and Reference Frames ....................... 15
Figure 6: Articulated Rotor Blade Angles .............................................................. 16
Figure 7: Element Angle of Attack ......................................................................... 18
Figure 8: Cormorant Hub from Ref 41. .................................................................. 20
Figure 9: Lag Damper Geometry ............................................................................ 21
Figure 10: Tail Rotor Axes ...................................................................................... 22
Figure 11: Pitt-Peters Inflow Uniform (r), Lateral (g), and Longitudinal (b) Mode Shapes ...... 29
Figure 12: UH-60 Lateral Cyclic Input ..................................................................... 32
Figure 13: Cyclic Input from Zhao [Ref. 10] ............................................................ 32
Figure 14: UH-60 Lateral On-axis Response ............................................................ 33
Figure 15: UH-60 Longitudinal Off-axis Response ................................................... 33
Figure 16: Lateral On-axis Response from Zhao [Ref. 10] ......................................... 33
Figure 17: Longitudinal Off-axis Response from [Ref. 10] ........................................ 33
Figure 18: Unconstrained Elastic Body .................................................................... 34
Figure 19: First Normal Mode Vertical Deformation ............................................... 40
Figure 20: First Normal Mode Angular Displacement ................................................................. 40
Figure 21: Second Normal Mode Vertical Deformation ............................................................. 40
Figure 22: Second Normal Mode Angular Displacement .......................................................... 40
Figure 23: Teetering Rotor Flapping Angles ................................................................................ 43
Figure 24: Bell 206 Longitudinal and Lateral Cyclic Input .......................................................... 49
Figure 25: Bell 206 Longitudinal Response .................................................................................. 50
Figure 26: Bell 206 Lateral Response ......................................................................................... 50
Figure 27. Combined Effects – Bell 206 Longitudinal ............................................................... 51
Figure 28. Combined Effects – Bell 206 Lateral ......................................................................... 51
Figure 29: CH-124 Longitudinal Cyclic Input ............................................................................. 51
Figure 30: CH-124 Longitudinal On-axis Response .................................................................... 52
Figure 31: CH-124 Lateral Off-axis Response .............................................................................. 52
Figure 32: CH-124 Combined - Longitudinal ............................................................................. 53
Figure 33: CH-124 Combined - Lateral ....................................................................................... 53
Figure 34: CH-149 Cruise Lateral Response ............................................................................... 53
Figure 35: CH-149 Longitudinal Response .................................................................................. 53
Figure 36: UTIAS Flight Research Simulator ............................................................................. 55
Figure 37: FRS Rear Cockpit ..................................................................................................... 56
Figure 38: X-Plane Visual Scenery ............................................................................................. 58
Figure 39: Hover Mission Task Element ................................................................................... 60
Figure 40: Hover MTE Pilot View .......................................................... 61
Figure 41: Pirouette Mission Task Element .............................................. 61
Figure 42: Sidestep Mission Task Element ............................................... 62
Figure 43: Sidestep MTE View ............................................................... 63
Figure 44: Slalom Mission Task Element .................................................. 64
Nomenclature

\( A \) bold font indicates a matrix

\( \mathbf{A} \) bold with underscore is a vector in three dimensional space

\( \mathbf{A}_b \) components of vector \( \mathbf{A} \) in reference frame \( F_b \)

\( \mathbf{A}^T \) transpose of a matrix

\( \mathbf{A}^x \) skew-symmetric operator used to calculate the cross product vectors

\( C_{TA} \) main rotor thrust coefficient

\( C_{LA} \) main rotor aerodynamic rolling moment coefficient

\( C_{MA} \) main rotor aerodynamic pitching moment coefficient

\( c_j, c_{OUT j}, c_{IN j} \) mean, inner, and outer chord of \( j^{th} \) blade element respectively

\( f \) external applied force distribution

\( E_b \) Euler angles of reference frame \( F_b \), equivalent to \( [\phi_b, \theta_b, \psi_b]^T \)

\( EI \) beam bending stiffness \( \text{N} \cdot \text{m}^2 \)

\( F \) rigid body forces \( \text{N} \)

\( G \) rigid body moments \( \text{N} \cdot \text{m} \)

\( i_0, i_1, i_2 \) tail sidewash interference coefficients

\( K_{Re} \) wake curvature parameter

\( L \) augmented inflow gain matrix

\( L_{12} \) rotation matrix from Frame 2 into Frame 1

\( M_A \) apparent mass matrix

\( N_b \) number of rotor blades

\( N_e \) number of elements per blade

\( q_e \) generalized elastic coordinates
$r$ \quad \text{radial position on rotor disk} \quad \text{m}

$r_0$ \quad \text{radial position of inner blade edge} \quad \text{m}

$r_{IN_j, r_m, r_{OUT_j}}$ \quad \text{inner, midpoint, and outer radial position of j\textsuperscript{th} blade element respectively} \quad \text{m}

$r$ \quad \text{inertial position of a body fixed reference point} \quad \text{m}

$R$ \quad \text{main rotor radius} \quad \text{m}

$\bar{R}$ \quad \text{absolute position of a mass element} \quad \text{m}

$S$ \quad \text{main rotor wake spacing}

$S_j$ \quad \text{j\textsuperscript{th} blade element area} \quad \text{m}^2

$u_e$ \quad \text{elastic deformation of a mass element} \quad \text{m}

$U$ \quad \text{total airspeed vector} \quad \text{m/s}

$v$ \quad \text{rigid body linear velocity} \quad \text{m/s}

$V$ \quad \text{mass flow parameter matrix}

$V_0$ \quad \text{mean induced inflow across rotor disk} \quad \text{m/s}

$V_1$ \quad \text{lateral inflow gradient across rotor disk} \quad \text{m/s}

$V_2$ \quad \text{longitudinal inflow gradient across rotor disk} \quad \text{m/s}

$V_{IN}$ \quad \text{local induced inflow velocity} \quad \text{m/s}

$V_m$ \quad \text{mass flow parameter associated with mean inflow}

$V_T$ \quad \text{mass flow parameter associated with inflow harmonics}

$X$ \quad \text{main rotor wake skew}

$\alpha$ \quad \text{angle of attack} \quad \text{rad}

$\beta$ \quad \text{blade flapping angle} \quad \text{rad}

$\beta_w$ \quad \text{helicopter sideslip angle} \quad \text{rad}

$\beta_{ic}$ \quad \text{rotor longitudinal tip path plane tilt angle} \quad \text{rad}
\( \beta_{ls} \) rotor lateral tip path plane tilt angle \( \text{rad} \)

\( \delta \pi \) area of annulus swept by each blade element \( \text{m}^2 \)

\( \Delta_i \) azimuth offset angle of \( i^{th} \) blade \( \text{rad} \)

\( \gamma \) blade lagging angle \( \text{rad} \)

\( \kappa_c \) main rotor longitudinal wake curvature

\( \kappa_s \) main rotor lateral wake curvature

\( \lambda_{pp} \) total airspeed perpendicular to rotor disk \( \text{m/s} \)

\( \mu \) advance ratio

\( \rho \) undeformed position of a mass element within a flexible body \( \text{m} \)

\( \chi_{pp} \) main rotor wake skew angle \( \text{rad} \)

\( \psi \) azimuth angle on rotor disk \( \text{rad} \)

\( \psi_a \) aerodynamic phase lag angle \( \text{rad} \)

\( \psi_\alpha \) basis functions of deformation modes

\( \omega \) rigid body angular velocity \( \text{rad/s} \)

\( \Omega \) main rotor angular velocity about the mast \( \text{rad/s} \)

\( [ \ ]^F \) values pertaining to the fuselage

\( [ \ ]^H \) values pertaining to the blade hinge

\( [ \ ]^h \) values pertaining to the horizontal tail

\( [ \ ]^t \) values pertaining to the tail rotor

\( [ \ ]^V \) values pertaining to the vertical tail
1 Introduction

The Vehicle Simulation group at the University of Toronto Institute for Aerospace Studies (UTIAS) has a number of previously developed real-time helicopter models for piloted simulations. The models represent the CH-124 Sea King,\(^1\) CH-149 Cormorant,\(^2\) and Bell 206 JetRanger.\(^3\) These are physics-based vehicle models, with full non-linear equations of motion, rather than empirical models, with simplifying assumptions and limitations common to real-time pilot-in-the-loop helicopter models, such as rigid rotor blades. An overview of the models which formed the framework for new improvements is presented in Section 3, describing the main and tail rotor, fuselage and empennage aerodynamics, and body dynamics.

An area of concern with physics-based helicopter simulations is that they often have an inaccurate off-axis response to cyclic control inputs compared to the corresponding flight test data,\(^4,5,6\) so this is one aspect targeted for improvement. This means that in the model the initial lateral response to a longitudinal cyclic input, and vice versa, will notably differ from that observed in the actual aircraft, possibly even with the opposite sign. For a training simulation to meet certification requirements, this is typically corrected by way of artificial effects. To explain the cause of this problem, several theories have been put forth in the literature concerning which aspects are modelled incorrectly or not modelled at all, such as blade elasticity,\(^7\) aerodynamic interference effects between the rotor and fuselage,\(^4\) or unsteady aerodynamic effects. The current state of the body of knowledge is introduced through a literature review given in Section 2. The goal of this thesis was to implement and evaluate the effectiveness of a number of possible modelling improvements described below.

One of the more commonly suggested contributing factors is the influence of rotor wake distortion and curvature, with the distorted wake altering the induced inflow distribution through the rotor disk and thereby altering the resulting overall helicopter dynamics.\(^7,8,9\) To account for this effect, methods of modelling the dynamic wake distortion have been developed for inclusion in helicopter flight dynamics models. The dynamic wake distortion model with four states (stretch, skew, and curvature in two directions) and the corresponding augmented Pitt-Peters dynamic inflow model developed by Zhao\(^10\) is computationally compact and therefore suitable
for use in a real-time simulation. Although the model was derived for a generic rotor, only a limited amount of full vehicle simulation results have been presented to date, mostly for the UH-60 Black Hawk, with a four-bladed articulated rotor. This wake model was implemented in the UTIAS real-time helicopter models as discussed in Reference 11, and is presented in Section 4. Thus the effectiveness of this addition to new configurations, specifically the five bladed articulated rotor and the two bladed teetering rotor, is examined and compared against other possible modelling additions.

A further improvement examined was to include the blade elasticity in the rotor dynamics in all of the real-time UTIAS helicopter models. Blade flexibility has been shown to improve off-axis response in a number of non-real-time models, however no real-time implementation of this effect was found in the literature. This feature was therefore realized in this thesis with newly developed rotor dynamics equations presented in Section 5, using a Ritz expansion approach with constrained elastic modes. The flexible rotor dynamics for both a teetering and articulated rotor are presented, and the effects of this inclusion examined.

Finally, an aerodynamic lag was also included to represent the unsteady nature of the rotor blade section lift and drag forces; this is presented in Section 6. This is an empirically determined correction that has previously been applied to models of the AH-64 four-bladed rotor with elastomeric bearings and the Sikorsky Bearingless Main Rotor. As is the case with the wake model, the rotor configurations in the UTIAS helicopter models are a new application for this method, and it has not previously been tested in combination with other effects. The result of this relatively simple addition to the five bladed articulated and teetering rotors is examined and compared against the theoretically derived effects.

Results obtained with the flexibility included are compared to those obtained with a rigid rotor in Section 7, as well as with and without the dynamic wake distortion effects and unsteady section aerodynamics, examining the on-axis and off-axis responses to cyclic inputs. These results are also compared to flight test data where available.

The impact of these changes was also evaluated from a pilot-in-the-loop perspective, presented in Section 8. These tests included quantifying the perceived changes using simulation fidelity ratings. Since this is a newly developed metric for evaluating simulators not previously
applied to this facility, the simulator was first evaluated using the original baseline vehicle models. Through this process, experience could be gained in the usage of the fidelity rating scale, while also examining what effect changes to the off-axis dynamics in particular had on the overall fidelity rating obtained.

Finally, a summary of the work and conclusions is given in Section 9. In addition to this thesis, elements of the work were presented in three conference papers, Reference 11, 12, and 13, as well as a journal paper, Reference 14.
2 Literature Review

The Pitt-Peters dynamic inflow model and Peters-He finite state inflow model\textsuperscript{15} have been widely accepted as robust and efficient models for real-time helicopter simulation\textsuperscript{16}, stability and control investigations\textsuperscript{17}, and handling qualities evaluation\textsuperscript{18}. The work of Zhao\textsuperscript{9,10} represents the most recent developments in improving these simplified analytical rotor inflow models, by adding transient wake distortion through the use of a dynamic vortex tube analysis. The simplified wake models of Pitt-Peters and Peters-He were extended by including the dynamic wake curvature, stretching, and altered skewing that occurs during helicopter manoeuvring. The details of this inflow model are described in Section 4. When the effects of wake curvature on the main rotor inflow, the tail rotor, and the empennage are considered, the off-axis fidelity of the simulation may be greatly improved for certain helicopters. However this has only seen limited use in piloted simulations to date. Murakami\textsuperscript{19} has also analyzed and modified the dynamic inflow model for autorotation, resulting in a slight correction for high descent rates with low forward speed that could be implemented in a full simulation.

A recent, non-real-time modeling effort by Theodore and Celi\textsuperscript{20} has shown that blade flexibility and main rotor wake dynamics can be an important contributor to off-axis handling with a sophisticated non-real-time flight dynamics model. The wake model employed is the Bagai-Leishman free wake model,\textsuperscript{21} which is capable of modeling the wake geometry changes due to maneuvers while making no assumptions regarding the geometry of the wake. The wake is discretized into a number of vortex segments shed from the rotor, which is solved iteratively at each time step and used to determine the local induced velocity at the rotor blade elements. The total number of vortex segments to be tracked is determined by the discretization resolution and the number of rotor revolutions retained in the analysis.

In the Theodore and Celi reference, a coupled flap-lag-torsion finite element formulation is used to model the blade,\textsuperscript{22} combined with a modal coordinate transformation to reduce the number of degrees of freedom by retaining the first seven elastic modes. This approach of using a detailed finite element model to predict the flexible modes and then including only the important modes has been used in other applications as well,\textsuperscript{23} where the full finite element model is not practical to run in real-time. Run times were not given, however the model was not
intended to be solved in real-time, and inclusion of the free wake model is described as requiring
greater than ten times the overall CPU time compared to the same simulation with a linear inflow
model. In addition, a recent paper making use of this model\textsuperscript{24} gave representative run times on
recently updated hardware, requiring 12-15 hours to simulate 2 minutes of flight time.

There have been multiple other aeroelastic rotor models developed over the last few years not
intended for use in a real-time simulation. Researchers at Georgia Institute of Technology and
Politecnico di Milano in Italy including Bachau\textsuperscript{25}, Bottasso\textsuperscript{26}, and Quaranta\textsuperscript{27} have combined
finite element multibody rotor dynamics models with lifting line or computational fluid
dynamics. This approach produces a high fidelity aeroelastic model for detailed analysis of the
blade dynamics including vibrations. The multibody dynamics model is built up using an
element library including rigid bodies, composite capable beams and shell elements, and joint
models. In a typical example, the model included 36 beam elements, 32 joint elements, and 31
rigid body elements, resulting in 910 degrees of freedom. Including the Peters-He dynamic
inflow model, the helicopter dynamics involved a total of 1347 states. These references were not
looking at the off-axis vehicle dynamics, nor do they give representative run times, although it is
clear throughout that the models are far too computationally expensive to be solved in real-time.
In Reference 26, a model closer to those typically used for real-time work, referred to as a flight
mechanics model, is used as the target response and validation for the high fidelity model. In
Reference 27 the intent was adding the computational aerodynamics to the multi-body dynamics,
so the validation consisted of comparing the wake and lift distributions in steady state with
experimental values.

Nitzsche et al\textsuperscript{28} at Carleton University are developing a smart hybrid active rotor control
system, and have modelled the blade dynamics a number of ways in support of this project. One
of these is described in Afagh,\textsuperscript{29} where an aeroelastic stability analysis of the hingeless helicopter
blade with a smart spring in hover was performed. The modelling approach is similar to that used
here, using Galerkin’s method to reduce the full elastic equations to a set of ordinary nonlinear
differential equations, although the eigenfunctions used were those of a non-rotating cantilever
beam. A Eurocopter BO105 helicopter blade with various actuation methods of a “smart spring”
was analyzed, to determine the frequency and damping of the first lead-lag, flap, and torsion
modes. This was not a full vehicle model for piloted tests, rather a blade only analysis for examining stability.

Another example of a model used in the smart hybrid active rotor control project is described by Cheng\textsuperscript{30}, with a time domain structural dynamics model for aeroelastic simulations. A finite element model of the blade using cantilever beam elements is solved in time using second-order backward Euler method to discretize in time and Newton’s method used to solve the nonlinear algebraic equations. This was coupled with an unsteady vortex particle aerodynamic code to form an aeroelastic solution. Cheng is the only one of these papers to give actual run times, using up to 1.8 hours of CPU time for 2 seconds of results, however all of the described models are too complex to currently run in real-time. Even with a simple test condition, comparing the results against those obtained using the model described in Reference 25, required on the order of 20 minutes and 100 seconds of CPU time respectively for 2 seconds of results.

Padfield\textsuperscript{31} describes the reconfigurable real-time flight simulator at the University of Liverpool, similar in many ways to the one at UTIAS. It is capable of both fixed and rotary-wing simulations and employs the commercial software package FLIGHTLAB for developing vehicle models. A recent paper by Manimala et al\textsuperscript{32} describes one of the helicopter models developed in collaboration with the National Research Council in Ottawa for this simulator, NRC’s Bell 412 Advanced Systems Research Aircraft. The base-line model used a rigid articulated blade-element main rotor model with flap and lag degrees of freedom and a Peters-He finite-state inflow model, which yielded reasonable results for the on-axis response. The functional descriptions of the other components of this model also align well with the current UTIAS model, with essentially identical methods for determining tail rotor, fuselage, and tail surface forces. Rotor wake geometry distortion effects were then added to the Peters-He inflow model to improve the off-axis results, although still not completely matching actual flight data. This was achieved in FLIGHTLAB using the approach described in Reference 33, an earlier form of the extended Peters-He inflow model described above. Additionally, although one of the goals of this model is to develop flight envelope protection and structural load alleviation systems such as that described by Voskuiji\textsuperscript{34}, blade bending has not been included. Further improvements were obtained through augmenting the physics-based model with additional parameters using system identification techniques.\textsuperscript{35}
Recent measurements\textsuperscript{36} have shown that effects of unsteady aerodynamics are evident on airfoils, even at relatively low flapping frequencies of 4 Hz. The modeling of flexible rotor blades may result in a wider range of frequencies of airfoil motions being considered, which strongly suggests the need for unsteady aerodynamics. Silva\textsuperscript{37} has suggested using Volterra-Wiener theory to capture the non-linear unsteady effects of airfoils at transonic speeds, where non-linear kernels are identified and used to predict the response of a nonlinear system to an arbitrary input. This method results in a state-space representation of the nonlinear aerodynamic responses, which makes the method a candidate for real-time simulation, and is demonstrated for a transonic pitching rectangular wing in the paper. This method works best when the response is weakly non-linear to keep the number of kernels required small, and also requires significant effort using a time domain analysis of the system impulse response in order to identify the kernels.

An alternative approach to capture the effects of the unsteady airfoil aerodynamics is applying an empirically determined phase lag on the lift and drag coefficients of the rotor blade elements, as suggested by Mansur and Tischler.\textsuperscript{5,38,39} This is accomplished using a simple first-order low-pass filter on the element forces with the lag determined using flight test data, which is then readily implemented in a real-time model. In Reference 5, the method was applied to model of the AH-64 Apache, and responses of the modified model were compared with flight data. A significant improvement in the off-axes in hover was noted, as well as a reasonable improvement in the off-axes responses in cruise. In Reference 39, the phase lag was used to improve the modelling of the off-axis flapping response to a cyclic input of the Sikorsky Bearingless Main Rotor when compared with wind tunnel data.

Throughout the literature, it can be seen that there are a variety of modelling improvements which have been shown to improve the off-axis response. The majority of the papers demonstrate their results using a single helicopter type, and how they relate is not entirely clear. The objective of the current research is to implement a number of the modelling additions, using suitable real-time capable methods, in the UTIAS helicopter models. Once in place, the resulting effect on the vehicle dynamics, particularly in the off-axis, is then examined and compared using the various additions both in isolation and in combination. This also includes testing the same modelling features on three very different sized aircraft with differing main rotor configurations,
rather than only using a single type of helicopter. Finally, the updated model is compared with a previous version in piloted trials using a new developed fidelity rating scale. This allows examining the possible impact of these improvements on the usefulness of the simulation for select training tasks, in addition to the commonly used off-line evaluations.
3 Model Overview

The Sea King, Cormorant, and JetRanger helicopter models, loosely based on Reference 40 combined with first principals derivations, are all programmed in The MathWorks’ Simulink® environment, compiled and run in real-time using OpalRT’s RT-Lab. The main rotor models employ a blade-element formulation, where each rotor blade is subdivided into segments. For each element, the local airspeed and angle of attack is determined, which is then used to determine the aerodynamic forces. The models employ the Pitt-Peters dynamic inflow model in determining the total airflow, augmented with dynamic wake distortion which can be disabled for comparisons. The rotor rotational speed is determined using a powertrain model with second-order engine dynamics. All three helicopters do however have different blade mounting mechanisms, resulting in differences in the main rotor models. The Sea King has an articulated hub, with flap and lag hinges along with a lag damper. The Cormorant uses a cleaner more modern approach to provide the same functionality, with elastomeric bearings in place of the flapping and lagging hinges. To model this, the simulation uses collocated hinges and torsion springs with a suitable stiffness. Finally, the JetRanger has a teetering rotor, so there is only a single teetering hinge.

The remainder of the vehicle modelling is similar across the three models, and the major sections which affect the in-air flight dynamics are described here. Notable exceptions which are not described in this thesis are the landing gear ground reaction forces and the flight control systems. While these are important factors for piloted simulations, they do not impact the dynamics being examined currently. Landing gear is beyond the scope of this study as no ground contact was included. The direct rigging between pilot controls and blade pitch was matched for all three aircraft, and all testing was performed with no stability augmentation active, as this would suppress the natural off-axis response.
3.1 Main Rotor

A blade element model is used to represent the main rotor. The main aspects of the simulation are covered in this section, except for the recently updated inflow and flexible blade dynamics, which are covered in more detail in Sections 4 and 5 respectively. Each blade is divided up into elements and these elements are treated as independent two-dimensional airfoils. The overall forces and moments produced by the main rotor are found by summing up the contributions from all the elements. The main rotor is treated as an independent dynamic system with its angular velocity determined by the Power Train Module. Its forces and moments are transmitted to the helicopter body via the hub, rotor mast, and lag damper where present. The Bell 206 teetering rotor has two rigid body angular degrees-of-freedom; flapping and rotation about the mast, while for the articulated rotors each blade has three degrees-of-freedom; flapping, lagging and rotation about the mast (see Reference 41). The rigid body pitch at the blade root is assumed to be fixed by the swashplate pitch links.

3.1.1 Rotor Blade Element Geometry

In order to apply the blade element theory it is first necessary to specify the geometric properties of each blade and element. Each blade is composed of \(N_e\) elements. The boundaries between blade elements are selected such that each element sweeps out an annulus of equal area \(\delta \pi\) as it completes one revolution about the mast. With a main rotor radius \(R\) and the inner edge of the blade at a radius \(r_o\),

\[
\delta = (R^2 - r_0^2) / N_e
\]  

The mid-point of the \(j\)-th element is defined by the radius \(r_{mj}\) and it is selected so that half the area of the \(j\)-th annulus lies outside the circle swept out by the mid-point. From the above it follows that based on areas (where \(j = 1\) for the element closest to the hub and \(j = N_e\) for the element at the rotor tip):

\[
r_{mj}^2 = r_{m(j-1)}^2 + \delta
\]  

and

\[
r_{m1}^2 = r_0^2 + \delta / 2
\]

The inner (towards the hub) radius \(r_{INj}\) and outer (towards the tip) radius \(r_{OUTj}\) of each element can be found based on annular areas to be
Each element is a trapezoid and thus the area of the j-th element is given by

\[ S_j = c_j(r_{OUTj} - r_{INj}) \]  

where \( c_j \) is the mean chord given by

\[ c_j = \frac{c_{OUTj} + c_{INj}}{2} \]

and \( c_{INj} \) and \( c_{OUTj} \) the inner and outer chords of an element respectively.

### 3.1.2 Rotor Blade Kinematics

The single main rotor for all conventional helicopters is located at the top of the rotor mast. The origin of a rotor shaft reference frame \( F_S \) is located at the teetering hinge point of the main rotor hub, or centre of the hub for an articulated rotor, at the top of the mast with its \( z \)-axis along the mast, specified by the vector \( r_S \) from the origin of the helicopter body frame \( F_B \). These reference frames are depicted in Figure 1. It is assumed that the mast is tilted in pitch by an angle \( \theta_S \) relative to \( F_B \) such that the Euler angle which carries \( F_B \) into \( F_S \) is given by

\[ E_S = [0 \quad \theta_S \quad 0]^T \]

The \( y \)-axis of \( F_S \) is parallel to that of \( F_B \).

\[ r_{INj}^2 = r_{mj}^2 - \delta/2 \]

\[ r_{OUTj}^2 = r_{mj}^2 + \delta/2 \]

Figure 1: Helicopter Reference Frames
A rotating reference frame $F_R$ is defined as shown in Figure 2. $F_R$ has its origin at the origin of $F_S$ and their $z$-axes are coincident. The $y$-axis of $F_R$ is aligned with the projection of Blade 1 in the $x$-$y$ plane of $F_S$ and follows it in rotation. The azimuthal angle $\psi$ of Blade 1 at any instance in time is shown in Figure 2 following the standard helicopter convention.

![Figure 2: Rotating Rotor Reference Frame](image)

Both $\psi$ and main rotor angular velocity relative to the helicopter body ($\Omega$) are positive for counterclockwise rotation of the rotor when viewed from above. This is the assumed direction of rotation in these simulations. $\Omega$ is the time derivative of $\psi$. The Euler angle which carries $F_S$ into $F_R$ is given by

$$E_R = [0 \ 0 \ (\pi/2 - \psi)]^T$$

(9)

For an articulated rotor, a hinge-fixed reference frame for the $i$-th blade $F_{Hi}$ is defined (see Figure 3). The origin of $F_{Hi}$ is located relative to the origin of $F_S$ by the vector $r^{Hi}$. The $y$-axis of $F_{Hi}$ is aligned with $r^{Hi}$ and the $z$-axis of $F_{Hi}$ is parallel to the $z$-axes of $F_R$ and $F_S$. The Euler angles $E_{Hi}$ carry $F_R$ into $F_{Hi}$ where

$$E_{Hi} = [0 \ 0 \ -\Delta_i]^T$$

(10)

and

$$\Delta_i = (i - 1)2\pi/N_b$$

(11)

with

$N_b = \text{number of blades}$.

The azimuthal angle of the $i$-th blade relative to the negative $x$-axis of $F_S$ is $\psi_i$ where

$$\psi_i = \psi + \Delta_i$$

(12)
An additional inflow reference frame $F_{IN}$ is specified as shown in Figure 4, which is required as the Pitt-Peter inflow is calculated based on the free-stream flow direction. The origin of $F_{IN}$ is co-located with the origin of $F_S$ (the hub), with the z-axes of $F_{IN}$ and $F_S$ coincident. The x-axis of $F_{IN}$ is in the direction of the projection of the hub’s instantaneous airspeed vector ($U_S$) on the x-y plane of $F_S$. The helicopter sideslip angle $\beta_W$ is as shown in the figure where

$$\beta_W = \tan^{-1}(U_{yS}/U_{xS})$$

(13)

The Euler angles which carry $F_S$ into $F_{IN}$ include only this slip angle for yaw, and are thus

$$E_{IN} = [0 \ 0 \ \beta_W]^T$$

(14)
3.1.3 Ground Effect on Main Rotor

The effect of the ground on the aerodynamics of the main rotor is predicted by a simple model presented in Reference 40. A scaling factor due to ground effect, $K_{GE}$, is applied to the uniform inflow $V_0$ produced in Section 4. This scale factor reduces the induced inflow as the rotor approaches the ground, resulting in a loss of efficiency and higher power requirements, with a very good match to flight test data. The variation in $K_{GE}$ with height is based on the height of the main rotor hub above the local ground plane $h_{AGL}$ and is given by:

$$K_{GE} = 1 \text{ when } h_{AGL} > 5R$$ (15a)

and

$$K_{GE} = [1 - 0.115(R/h_{AGL})^2U_{2S}/|U_S|]^{-2/3} \text{ when } h_{AGL} \leq 5R$$ (15b)

In Equation 15, $K_{GE}$ approaches unity and ground effect is diminished at higher speeds.

3.1.4 Blade Element Angle of Attack and Mach Number

The aerodynamic forces acting on each blade element depend on the local angle of attack and Mach number. Therefore these quantities must be evaluated at the location of each element at every time step. To start this process, we find the airspeed of the $j$-th element of the $i$-th blade.

For the teetering rotor, let $F_{bi}$ be a blade-fixed reference frame for the $i$-th blade as shown in Figure 5. The origin of $F_{bi}$ is located at the blade root and its $y$-axis lies along the blade towards the tip. The $x$-axes of $F_{b1}$ and $F_{b2}$ are parallel to the $x$-axis of $F_R$, with $F_{b1}$ in the positive $x$ and $F_{b2}$ in the negative $x$ direction of $F_R$. In a teetering rotor the blade motion relative to $F_R$ is a simple flapping about the $x$-axis with the flapping angle $\beta_i$ (see Figure 5) defined to be positive when the blade tip is above the $xy$-plane of $F_R$. Thus the Euler angles which carry $F_R$ into $F_{bi}$ are given by

$$E_{b1} = [-\beta_1 \ 0 \ 0]^T$$ (16a)

and

$$E_{b2} = [-\beta_2 \ 0 \ \pi]^T$$ (16b)

In the above, $\beta_1$ and $\beta_2$ are a function of the fixed pre-coning angle $\beta_0$ and the teetering angle $\beta$, which is determined by the flexible teetering rotor dynamics (shown in Section 5.3).
The local angle of attack and Mach number are based on airspeed components at the mid-point of each blade element expressed in frame $F_{B/i}$. For the $j$-th element of the $i$-th blade the total airspeed is

$$U_{bij} = V_{bij} - W_{bij} \quad (17)$$

where $V_{bij}$ is the inertial velocity of the mid-point and $W_{bij}$ is the wind speed at the location of the mid-point. The inertial velocity is found by applying the method described in Section 5, using the fact that the velocity of the origin of $F_S$ at the teetering hinge point is already known,

$$V_{bij} = V^S_{bi} + \omega^x_{bi}(r^H_{mj} + u^e_{eil}) + \dot{u}^e_{eil} \quad (18)$$

where $V^S_{bi}$ is the inertial velocity of the origin of $F_S$ in $F_{bi}$ coordinates, $\omega_{bi}$ is the angular velocity of $F_{bi}$ relative to the inertial frame. $r^H_{mj}$ is the location of the mid-point relative to the hinge, given by

$$r^H_{mj} = [0 \ r_{mj} \ 0]^T - r^H_{bi} \quad (19)$$

where $r^H_{bi}$ is the location of the teetering hinge in $F_{bi}$. $u^e_{eil}$ and $\dot{u}^e_{eil}$ are the elastic deformation and deformation rate respectively at the midpoint of the blade element.

$\omega_{bi}$ can be found from the sum of the angular velocity of $F_{bi}$ relative to $F_R(\omega_i)$ and the angular velocity of $F_R$ relative to $F_E(\omega_R)$. From Figure 5 it can be seen that the angular motion of $F_{bi}$ relative to $F_R$ is flapping about the $x$-axis of $F_{bi}$. Thus

$$\omega_i = [-\beta_i \ 0 \ 0]^T \quad (20)$$
where $\beta_i$ is derived in Section 5.3. Since $\Omega$ is the angular velocity (about the negative $z$-axis of $F_R$) of $F_R$ relative to $F_B$ and $\omega_B$ is the inertial angular velocity of $F_B$ relative to $F_E$ given in Section 0, it follows that

$$\omega_R = L_{RB} \omega_B + [0 \ 0 \ -\Omega]^T \quad (21)$$

Giving a total blade angular velocity of

$$\omega_{bi} = L_{biR} \omega_R + [-\dot{\beta}_i \ 0 \ 0]^T \quad (22)$$

where $L_{RB}$ is the rotation matrix from $F_B$ to $F_R$ and $L_{biR}$ is the rotation matrix from $F_R$ to $F_{bi}$.

For an articulated rotor, the origin of $F_{bi}$ is located at the hinge (the origin of $F_{Hi}$) and its $y$-axis lies along the blade towards the tip as shown in Figure 6. The Euler angles which carry $F_{Hi}$ into $F_{bi}$ are given by

$$E_{bi} = [-\beta_i \ 0 \ -\gamma_i]^T \quad (23)$$

where $\beta_i$ is the flapping angle and $\gamma_i$ is the lagging angle, which come from Section 5.6.

![Figure 6: Articulated Rotor Blade Angles](image)

The inertial velocity for the articulated rotor blade element is again found by applying the method described in Section 5,

$$V_{bi} = V_{bi}^H + \omega_{bi}^x (r_{mj}^H + u_{eij}) + \vec{u}_{eij} \quad (24)$$

where $V_{bi}^H$ is the inertial velocity of the origin of $F_{Hi}$ which can be found using

$$V_{Hi} = V_{Hi}^S + \omega_{Hi}^x r_{Hi}$$ \quad (25)
and where \( \mathbf{r}_{Hl} \) is the location of the articulating hinge relative to the centre of the hub. In the above, \( \mathbf{\omega}_{Hi} \) is the inertial angular velocity of \( F_{Hi} \), and since \( \mathbf{\omega}_{Hi} = \mathbf{\omega}_R \),

\[
\mathbf{\omega}_{Hi} = L_{HiR} \mathbf{\omega}_R
\]  

The angular velocity of \( F_{bi} \) relative to \( F_E \), \( \mathbf{\omega}_{bi} \), is the sum of the angular velocity \( \mathbf{\omega}_i \) of \( F_{bi} \) relative to \( F_R \) and the angular velocity \( \mathbf{\omega}_R \) of \( F_R \) relative to \( F_E \). From Equation 23 and the Euler angle rate equations (see Reference 47, Equation 4.46) it follows that

\[
\mathbf{\omega}_i = \mathbf{R}_{bi} [\mathbf{\dot{\beta}}_i 0 - \dot{\gamma}_i]^T
\]  

where

\[
\mathbf{R}_{bi} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \beta_i & -\sin \beta_i \\
0 & \sin \beta_i & \cos \beta_i
\end{bmatrix}
\]  

and \( \dot{\beta}_i \) and \( \dot{\gamma}_i \) are derived in Section 5.6. Thus

\[
\mathbf{\omega}_{bi} = \mathbf{\omega}_i + L_{biR} \mathbf{\omega}_R
\]  

and \( \mathbf{\omega}_R \) is given by Equation 21.

\( \mathbf{W}_{bi}\) in Equation 17 is made up of atmospheric effects \( \mathbf{W}^A_{bi} \) and the main rotor inflow.

\[
\mathbf{W}_{bi} = \mathbf{W}^A_{bi} + L_{biS} \mathbf{V}^I_{IN}
\]  

In Equation 30, \( \mathbf{V}^I_{IN} \) is the inflow vector based on Section 4 with \( V_0 \) attenuated by the ground effect parameter \( K_{GE} \) from Section 3.1.3. Let

\[
V_{IN} = K_{GE} V_0 + \frac{r}{R} (V_1 \sin \psi_{IN} + V_2 \cos \psi_{IN})
\]  

Since the inflow is parallel to the \( z \)-axis of \( F_S \) it follows that

\[
\mathbf{V}^I_{Sinj} = \begin{bmatrix} 0 & 0 & V_{IN,ij} \end{bmatrix}^T
\]  

where \( V_{IN,ij} \) is the value of Equation 31 when \( r = r_{mj} \) and (from Figure 4 and Figure 3)

\[
\psi_{INI} = \psi_i + \beta_W
\]
The lift and drag on each blade element is based on the application of simple sweep theory (see Reference 42). In this theory the spanwise component of airspeed is ignored (i.e., set to zero). The angle of attack of the $x$-axis of $F_{bi}$ for the $j$-th element, shown in Figure 7, is given by

$$\alpha'_{ij} = \tan^{-1}(U_{zbij}/U_{xbij})$$

(34)

The zero-lift line ($z\ell\ell$) of this blade element is located relative to the $F_{bi}$ $x$-axis by the pitch angle $\theta_{ij}$. (When $\theta_{ij} = -\alpha'_{ij}$ the lift on the element is zero.) The contributors to $\theta_{ij}$ are described in Section 3.1.5 below. The angle of attack of the zero-lift line, $\alpha_{ij}$, is then

$$\alpha_{ij} = \alpha'_{ij} + \theta_{ij}$$

(35)

The local Mach number is also based on ignoring the spanwise component of airspeed and is given by

$$M_{ij} = (U_{ij}^{2D})^{1/2}/U_{sound}$$

(36)

where

$$U_{ij}^{2D} = U_{xbij}^2 + U_{zbij}^2$$

(37)

and the local speed of sound is determined from the US Standard Atmosphere 1962.\(^{43}\)

### 3.1.5 Blade Pitch

The local blade pitch angle $\theta_{ij}$ is made up of control inputs from the pilot and transmitted by the swashplate and the geometric structure of the hub and blade. Thus

$$\theta_{ij} = \theta_{i}^C + \theta_{j}^{twist} + \theta_{zll} + \theta_{i}^d - \tan\delta_3 \beta_i$$

(38)

where $\delta_3 = 0^\circ$ for the Sea King and $15^\circ$ for the Cormorant to account for the flapping/pitching coupling built into the flapping hinge. $\theta_{i}^C$ is the control input for the $i$-th blade given by

$$\theta_{i}^C = A_0 - A_1 \cos(\psi_i + \Delta) - B_1 \sin(\psi_i + \Delta)$$

(39)

and $\psi_i$ is given by Equation 12.

In the above

- $\theta_{zll}$ is the pitch offset at the root of the main rotor
- $\theta_{j}^{twist}$ is the twist in the blade relative to the root, as measured at the $j$-th element
$\theta_{ij}^e$ is the twist due to elastic deformation at the $j$-th element

$A_0$ is the pilot’s collective input angle

$A_1$ is the pilot’s lateral cyclic input angle; a positive value for $A_1$ produces a positive (right) rolling torque

$B_1$ is the pilot’s longitudinal cyclic input angle; a positive value for $B_1$ produces a negative (nose down) pitching torque

$\Delta$ is a phase shift built into the control rigging intended to reduce response cross-coupling to pilot control inputs

The definitions of $A_1$ and $B_1$ reflect the fact that due to the dynamics of the main rotor, control inputs that produce an aerodynamic rolling torque on the rotor disc tend to produce pitching of the thrust vector. The need for $\Delta$ indicates that this phasing between input and output is not exactly orthogonal. A negative $\theta_j^{twist}$ is normally employed in order to reduce the lift distribution towards the blade tip and thus creates a more uniform inflow with improved rotor aerodynamic efficiency.

3.1.6 Aerodynamic Forces and Moments

The lift and drag on each blade element is found by using $\alpha_{ij}$ and $M_{ij}$ from Section 3.1.4 and a lookup table containing two-dimensional sectional $C_L$ and $C_D$ data for the blade airfoil employed, expressed as a function of $\alpha$ and $M$, where $\alpha$ is the angle of attack of the zero lift line. Due to the large range in angle of attack experienced by a rotor blade, the $\alpha$ range must span $-\pi \leq \alpha \leq \pi$. As suggested in Reference 40, the lift towards the tip of the rotor blade will be reduced in general due to three-dimensional flow effects on the real helicopter. This is modeled by the tip loss factor $K_{TLF}$ which multiplies $L_{ij}$ and varies with radius $r$ along the blade. $K_{TLF}$ only differs from unity at radii very near the tip in practice, and in the present simulation is 1 for all elements except the outermost, where it is given by

$$K_{TLF} = \left(0.97R - r_{INj}\right)/\left(r_{OUTj} - r_{INj}\right)$$

(40)

The $L_{ij}$ and $D_{ij}$ are then found from

$$L_{ij} = K_{TLF} \rho U_{ij}^2 S_j C_{L_{ij}}/2$$

(41)

$$D_{ij} = \rho U_{ij}^2 S_j C_{D_{ij}}/2$$

(42)
These aerodynamic forces can be resolved into $F_b$ components to produce
\[
F_{bij}^A = [L_{ij}\sin\alpha_{ij} - D_{ij}\cos\alpha_{ij} - L_{ij}\cos\alpha_{ij} - D_{ij}\sin\alpha_{ij} 0]^T \tag{43}
\]

The total aerodynamic forces and moments for each rotor blade are found by adding up the contributions from each blade element. Thus the total blade aerodynamic force for the $i$-th blade is given by
\[
F_{bi}^A = \sum_{j=1}^{N_e} F_{bij}^A \tag{44}
\]
The total blade aerodynamic moment about the hinge for the $i$-th blade is given by
\[
G_{bi}^A = \sum_{j=1}^{N_e} [(r_{mj} - r_H)F_{2bij}^A - (r_{mj} - r_H)F_{xbij}^A]^T \tag{45}
\]
The total aerodynamic forces and moments for the complete main rotor are found by adding up the contributions from all blades. For an articulated rotor, this is discussed in Section 5.5. For the teetering rotor, the main rotor aerodynamic force expressed in $F_b$ components is
\[
F_b^A = L_{bb1}F_{b1}^A + L_{bb2}F_{b2}^A \tag{46}
\]
Where $L_{bb1}$ and $L_{bb2}$ are the rotation matrices from the blade reference frames $F_{bi}$ to the body frame. The main rotor aerodynamic moment about the hub (at the teetering hinge location), expressed in $F_b$ components is
\[
G_b^A = L_{bb1}G_{b1}^A + L_{bb2}G_{b2}^A - r_b^{TH}xF_b^A \tag{47}
\]
where $r_b^{TH}$ is the location of the teetering hinge in $F_b$ relative to the origin of $F_{bi}$.

### 3.1.7 Lag Damper Forces and Moments

With articulated rotors, the blade lagging degree of freedom has a lag damper associated with it, as can be seen on the Cormorant main rotor hub in Figure 8 from Reference 44. The corresponding geometry for one blade is shown in Figure 9. The lag damper (represented by the vector $r^P$) has one end attached via a ball joint to the hub and the other end attached via a ball joint to the tension link. As the blade rotates
relative to $F_{Hi}$, the length of the lag damper ($r^D$) changes, causing the damper to generate a force proportional to the rate of change of the length. The damper hub attachment point is located relative to the origin of $F_R$ by the vector $r^L$, the hinge is located relative to the origin of $F_R$ by the vector $r^H$, which are constant geometric parameters, and the link point is given by $r^1$ and $r^2$.

![Figure 9: Lag Damper Geometry](image)

From Figure 9 the length of the damper is given by

$$r^D_b = r^H_b + r^1_b + r^2_b - r^L_b \quad (48)$$

From Figure 3 and Figure 9 it can be seen that

$$r^1_b = [0 \quad r^1 \quad 0]^T \quad (49)$$

and

$$r^2_b = r^2 [-\cos \theta^D \quad 0 \quad \sin \theta^D]^T \quad (50)$$

where

$$\theta^D = \theta^C + \theta^D_0 \quad (51)$$

$\theta^D$ is the pitch angle of the lag damper relative to the x-y plane, $\theta^C$ is given by Equation 39, while $r^1$, $r^2$ and $\theta^D_0$ are constant geometric parameters. The force applied to the blade by the lag damper is then

$$F^D_b = -f(|r^D|, |\dot{r}^D|) \frac{(r^D_b / |r^D_b|)} \quad (52)$$

where the modulus of the force is found from a lookup table as a function of $|r^D|$ and $|\dot{r}^D|$ for the Cormorant and using a constant damping coefficient for the Sea King. The moment applied to the blade about the hinge by the lag damper is then given by

$$G^D_b = (r^1_{bx} + r^2_{bx}) F^D_b \quad (53)$$
3.2 Tail Rotor

The tail rotor loads are determined using a Bailey rotor disc model, which contains a number of assumptions such as uniform inflow and a linear lift curve, and is summarized here. The reference frame $F_t$ is used to determine the forces and moments from the tail rotor system, with the origin located at the tail rotor hub specified by the vector $r_t$ from the origin of the helicopter body frame $F_B$, and the axes assumed to be parallel to those of $F_B$ (i.e., no tail rotor cant is present). This reference frame is shown in Figure 10, along with the sign conventions for the tail rotor inflow, angular velocity, and thrust.

![Figure 10: Tail Rotor Axes](image)

The tail rotor induced inflow $V_{IN}^t$ is assumed to be along the negative y-axis of $F_t$ and is modeled by

$$V_{IN}^t = \left( (W_{yB}^t - V_{yB}^t) t_{3.1} + R^t \Omega^t (\theta^t t_{3.2} + \delta^t t_{3.3}) \right) \left( \frac{4 / a_o s t \sqrt{\mu^t + \lambda^t + t_{3.1}}}{(54)} \right) (54)$$

where $W^t$ is the wind speed at the hub due to atmospheric and interference effects (i.e. excluding tail rotor inflow) and $W_{yB}^t$ is the y-component in $F_B$

$V^t$ is the inertial velocity of the tail rotor hub and $V_{yB}^t$ is the y-component in $F_B$

$R^t$ is tail rotor radius

$\Omega^t$ is tail rotor angular velocity
\( \theta^t \) is tail rotor blade pitch angle
\( \delta \theta^t \) is the tail rotor total blade twist from root to tip
\( a^t \) is the tail rotor blade lift curve slope (per radian)
\( s^t \) is the tail rotor solidity
\( \mu^t \) is the non-dimensional in-plane airspeed of the tail rotor
\( \lambda^t \) is the non-dimensional airspeed through the tail rotor plane

and the \( t_{3.x} \) are given by

\[
t_{3.1} = \frac{K_{T,L,F}^t}{2} + \frac{\mu^t}{4}, \quad t_{3.2} = \frac{K_{T,L,F}^t}{3} + \left( \frac{K_{T,L,F}^t}{2} \right) \mu^t, \quad t_{3.3} = \frac{K_{T,L,F}^t}{4} + \left( \frac{K_{T,L,F}^t}{4} \right) \mu^t^2
\] (55)

where \( K_{T,L,F}^t \) is the tail rotor blade tip loss factor.

With this inflow, the tail rotor thrust also acts along the y-axis of \( F_t \) and is given by

\[
T^t = 2\pi \rho R^t \Omega^t \sqrt{\mu^t + \lambda^t} (\Omega^t / \Omega_{ref}^t)^2 K_B V^\text{INT}^t
\] (56)

where \( \rho \) is the air density and \( K_B \) is the vertical tail blockage factor. The aerodynamic torque applied to the tail rotor about the y-axis of \( F_T \) is given in Reference 46 as

\[
G^t = \pi \rho R^t 5 \Omega^t \left( \frac{U_{yB}^t C_T^t}{R^t \Omega^t} + \frac{\delta^t s^t (1+3 \mu^t)}{8} \right)
\] (57)

where

\[
C_T^t = T^t / (\pi \rho R^t 4 \Omega^t^2)
\] (58)

and

\[
\delta^t = \delta_0^t + \delta_1^t C_T^t + \delta_2^t C_T^t^2
\] (59)

\( \delta^t \) is the average tail rotor blade drag coefficient and \( \delta_0^t, \delta_1^t, \) and \( \delta_2^t \) are fixed parameters in the model. Ignoring the drag on the tail rotor as being small, in frame \( F_B \) the total forces and moments applied to the helicopter body CG are then

\[
\mathbf{F}_B^t = \begin{bmatrix} 0 & T^t & 0 \end{bmatrix}^T
\] (60)

and

\[
\mathbf{G}_B^t = \begin{bmatrix} 0 & -G^t & 0 \end{bmatrix}^T + \mathbf{r}_B^t \times \mathbf{F}_B^t
\] (61)

### 3.3 Fuselage Aerodynamics

Aerodynamic forces and moments for the fuselage and empennage are determined using look-up tables, with slight differences between the different helicopters to make use of the data available. The tables are used to find forces and moments in a fuselage wind frame \( F_{wF} \). The frame has its origin located at the body’s center of pressure, located by the vector \( \mathbf{r}_F^w \) from the
origin of $F_B$, and its $x$-axis along the airspeed vector. An intermediate frame $F_F$ is also used, with the same center of pressure origin as $F_{wF}$, but parallel to $F_B$. The calculations are performed using a total average airspeed at this point, $\mathbf{u}_F$, which includes inertial velocity, atmospheric wind, rotor wash, and interference effects.

The angle of attack of the $x$-axis of $F_F$ is given by
\[
\alpha^F = \tan^{-1}(u_{xF}/u_{xF})
\]
and the sideslip of the $xz$-plane of $F_F$ is given by
\[
\beta^F = \sin^{-1}(u_{yF}/|\mathbf{u}_F|)
\]
These values are used in the linear interpolation tables for aerodynamic coefficients as well as giving the Euler angles which carry $F_{wF}$ into $F_B$,
\[
\mathbf{E}^{wF} = \begin{bmatrix} 0 & \alpha^F & -\beta^F \end{bmatrix}^T
\]
To find the wind frame aerodynamic forces and moments, the non-dimensional coefficients are used along with the dynamic pressure $q^F$, the fuselage planform reference area $S^F$, and the fuselage reference width $w^F$.

\[
D^F = q^F S^F C_D^F(\alpha^F, \beta^F)
\]
\[
Y^F = q^F S^F C_Y^F(\alpha^F, \beta^F)
\]
\[
L^F = q^F S^F C_L^F(\alpha^F, \beta^F)
\]
\[
l^F = q^F S^F w^F C_l^F(\alpha^F, \beta^F)
\]
\[
M^F = q^F S^F w^F C_M(\alpha^F, \beta^F)
\]
\[
N^F = q^F S^F w^F C_N^F(\alpha^F, \beta^F)
\]
where $D^F$ is the drag, $Y^F$ is the side force, $L^F$ is the lift, and $l^F$, $M^F$, and $N^F$ are the aerodynamic roll, pitch, and yaw moments respectively. In frame $F_B$ the total forces and moments applied to the helicopter body CG are then
\[
\mathbf{F}_B^F = L_{BwF}[-D^F \quad Y^F \quad -L^F]^T
\]
and
\[
\mathbf{G}_B^F = L_{BwF}[-l^F \quad M^F \quad N^F]^T + \mathbf{r}_B^F \times \mathbf{F}_B^F
\]
where $L_{BwF}$ the rotation matrix from frame $F_{wF}$ to $F_B$. 
3.4 Empennage Aerodynamics

The derivation of aerodynamic forces acting on the horizontal tail and the vertical tail are similar to those of the fuselage. Wind frames $F_{wh}$ and $F_{wV}$ are again defined for the horizontal and vertical tail, respectively, as well as reference frames $F_h$ and $F_V$ parallel to the body frame. These frames have their origins located at the tail surface center of pressures given by the vectors $\mathbf{r}_h$ and $\mathbf{r}_V$ from the origin of $F_B$. The total average airspeeds at these points are $\mathbf{u}_h$ and $\mathbf{u}_V$, which include inertial velocity, atmospheric wind, main and tail rotor wash, and interference effects.

3.4.1 Horizontal Tail

The angle of attack of the $x$-axis of $F_h$ is given by

$$\alpha^h = \tan^{-1}(u_{zh}/u_{xh})$$

(73)

and the sideslip of the $xz$-plane of $F_h$ is given by

$$\beta^h = \sin^{-1}(u_{yh}/|u_h|)$$

(74)

The bilinear interpolations for aerodynamic coefficients are over the independent variables $\alpha^h$ and $\beta^h$ where

$$\alpha^{hl} = \alpha^h + i^h + \theta^{hc}$$

(75)

where $i^h$ is the rigged angle of incidence of the horizontal tail’s zero lift line relative to the $x$-axis of $F_H$ and $\theta^{hc}$ is the horizontal tail control pitch angle. These angles also give the Euler angles which carry $F_{wh}$ into $F_B$,

$$\mathbf{E}^{wh} = \begin{bmatrix} 0 & \theta^{wh} & \psi^{wh} \end{bmatrix}^T = \begin{bmatrix} 0 & \alpha^h & -\beta^h \end{bmatrix}^T$$

(76)

The aerodynamic forces are determined using aerodynamic coefficients from look-up tables along with the dynamic pressure $q^h$, the surface area of the horizontal tail $S^h$, and a fuselage blockage factor $\eta^h$.

$$D^h = q^h S^h C^h_D(\alpha^{hl}, \beta^{h}) \eta^h$$

(77)

$$L^h = q^h S^h C^h_L(\alpha^{hl}, \beta^{h}) \eta^h$$

(78)

where $D^h$ is the drag and $L^h$ is the lift. In frame $F_B$ the total forces and moments applied to the helicopter body CG are then
\[ F_B^h = L_{Bwh}[-D^h \ 0 \ -L^h]^T \]  
(79)

and

\[ G_B^h = r_B^{h \times} F_B^h \]  
(80)

### 3.4.2 Vertical Tail

Because of the orientation of the vertical tail, the angle of attack of the \( x \)-axis of \( F_V \) is given by

\[ \alpha^V = \tan^{-1}(u_{yV}/u_{xV}) \]  
(81)

and the sideslip of the \( xy \)-plane of \( F_V \) is given by

\[ \beta^V = \sin^{-1}(u_{zV}/|u_V|) \]  
(82)

The bilinear interpolations for aerodynamic coefficients are over the independent variables \( \alpha^{VI} \) and \( \beta^{VI} \) where

\[ \alpha^{VI} = \alpha^V + i^V \]  
(83)

and

\[ \beta^{VI} = \beta^V + \Lambda \]  
(84)

where \( i^V \) is the rigged angle of incidence of the vertical tail’s zero lift line relative to the \( x \)-axis of \( F_V \) and \( \Lambda \) is the sweep angle of the vertical tail. These angles also give the Euler angles which carry \( F_B \) into \( F_{wV} \) (note that this is the opposite reference frame order to (3.5.1) used for the horizontal tail)

\[ E^{wV} = [0 \ \theta^{wV} \ \psi^{wV}]^T = [0 \ -\beta^V \ \alpha^V]^T \]  
(85)

The aerodynamic forces are determined using aerodynamic coefficients from look-up tables along with the dynamic pressure \( q^V \) and the surface area of the vertical tail \( S^V \).

\[ D^V = q^V S^V C_D^V(\alpha^V, \beta^V) \]  
(86)

\[ L^V = q^V S^V C_L^V(\alpha^V, \beta^V) \]  
(87)

where \( D^V \) is the drag and \( L^V \) is the lift. In frame \( F_B^V \) the total forces and moments applied to the helicopter body CG are then

\[ F_B^V = L_{BwV}[-D^V \ -L^V \ 0]^T \]  
(88)

and

\[ G_B^V = r_B^{V \times} F_B^V \]  
(89)
3.5 Helicopter Body Dynamics

In the present development the helicopter is treated as two interconnected dynamic systems; the main rotor and the rest of the helicopter (referred to as the “helicopter body”). The dynamics of the main rotor is described in Section 5, while this section deals with the dynamics of the helicopter body. The rigid-body 6 degrees-of-freedom equations are based on those presented in Reference 47. The force equations (or linear motion equations) come from the application of Newton’s Second Law to the linear acceleration of the CG of the helicopter body. Two axis systems are used, an inertial frame $F_E$ with its $x$-axis pointing north and its $z$-axis downwards and the helicopter body-fixed frame $F_B$, with its origin at the CG of the helicopter body, its $z$-axis downward and its $x$-axis forward. We will assume that a vertical plane of symmetry exists for the helicopter body and that it lies in the $xz$-plane of $F_B$.

In $F_B$ components, Newton’s Second Law has the form

$$ F_B^B + m_B L_{BE} g_E = m_B a_B $$

(90)

where $F_B^B$ is the sum of all the external forces applied to the helicopter body excluding gravity, $m_B$ is the mass of the helicopter body, $g_E$ is the acceleration due to gravity, expressed in $F_E$. $F_B^B$ is made up of the contributions described above, as well as the main rotor forces $F_R^B$ described in Section 5, and the landing gear forces $F_L^B$,

$$ F_B^B = F_R^B + F_L^L + F_B^F + F_B^H + F_B^V + F_B^L $$

(91)

$a_B$ is the inertial acceleration of the CG of the helicopter body, expressed in $F_B$, which given by

$$ a_B = \dot{v}_B + \omega_B x v_B $$

(92)

which together with equation 90 forms the differential equation solved for $v_B$.

As developed in Reference 47, the moment equations (or angular motion equations) are given in $F_E$ components by

$$ G_E^B = \dot{h}_E $$

(93)

where $G_E^B$ is the total external moment applied about the CG of the helicopter body and $h_E$ is the angular momentum of the helicopter body about the CG. When expressed in $F_B$ components, the result is

$$ G_B^B = I_B \omega_B + \omega_B x I_B \omega_B + \sum_i \omega_B x h_{iB} + \sum_i \dot{h}_{iB} $$

(94)
where $I_B$ is the helicopter body inertia matrix expressed in $F_B$ components and $h_{iB}$ is the angular momentum of the $i$-th rotating subsystem (e.g., tail rotor, engines, etc., excluding the main rotor), expressed in $F_B$. In equation 94, the total external moment $G^B_B$ is made up of contributions from the same sources as indicated above for $F^B_B$:

$$
G^B_B = G^R_B + G^L_B + G^F_B + G^h_B + G^V_B + G^L_B
$$

Equation 94 is the differential equation that is solved for $\omega_B$. In order to obtain the Euler angles $E_B$ required in calculating $L_{BE}$, the Euler rate equations (4.4,7) from Reference 47 must be used to determine the Euler rates from the body angular rates $\omega_B$,

$$
\dot{E}_B = T_B \omega_B
$$

where

$$
T_B = \begin{bmatrix}
1 & \sin \phi_B \tan \theta_B & \cos \phi_B \tan \theta_B \\
0 & \cos \phi_B & -\sin \phi_B \\
0 & \sin \phi_B \sec \theta_B & \sec \theta_B
\end{bmatrix}
$$

Equations 90, 94, and 96 are nonlinear differential equations that must be solved simultaneously along with the flexible rotor dynamics of Section 5. To decouple the rotor and body equations, the accelerations of the relatively massive helicopter body are delayed one time step for determining the required rotor forces and moments.
4 Dynamic Inflow with Wake Distortion

In the process of generating lift, a rotor generates an induced flow with velocity \( V_{IN} \) through the rotor disc. In general the induced flow is not uniform over the rotor disc and it responds dynamically to changes in the thrust and moments being generated by the rotor. The Pitt-Peters dynamic inflow model captures these features as described in References 8 and 48, and had previously been included in the UTIAS simulations. This inflow model was augmented to also include the dynamic wake curvature, stretching, and skewing that occurs during helicopter maneuvering using the method described by Zhao.\(^9,10\) The fuselage and empennage interference effects due to wake curvature have also been included. Below is a review of this model as currently implemented, the development of which follows that of Zhao. This is presented as an overview without describing in detail how all of the required input quantities are determined.

The Pitt-Peters dynamic inflow model makes use of three assumed modes, with a uniform inflow \( (V_0) \) combined with linear lateral \( (V_1) \) and longitudinal \( (V_2) \) gradients. These three mode shapes are shown in Figure 11, in red, green, and blue respectively, with the free-stream flow from top left to bottom right. The induced flow velocity through the point at location \((r, \psi)\) on the main rotor disc is therefore given by

\[
V_{IN} = V_0 + \frac{r}{R} (V_1 \sin \psi + V_2 \cos \psi)
\]

with \( \psi \) measured counter-clockwise from the downstream direction in the rotor plane, and \( V_0, V_1, \) and \( V_2 \) coming from the solution to the Pitt-Peters differential equations. With these terms collected into a vector, \( V_{pp} = [V_0 \ V_1 \ V_2]^T \), the differential equations can be expressed as

\[
M_A \begin{bmatrix} \frac{V_{pp}}{\Omega^2 R} \\ \frac{V}{\Omega R} \end{bmatrix} + \begin{bmatrix} \frac{V}{\Omega R} \end{bmatrix} L^{-1} \begin{bmatrix} \frac{V_{pp}}{\Omega R} \end{bmatrix} = \begin{bmatrix} C_{TA} \ -C_{LA} \ -C_{MA} \end{bmatrix}^T
\]

where \( R \) is the main rotor radius, \( \Omega \) is the angular velocity about the mast, and \( C_{TA}, C_{LA}, \) and \( C_{MA} \) are the coefficients of aerodynamic thrust, rolling and pitching moments respectively. \( M_A \) and \( V \)
remains as per the original Pitt-Peters inflow model, while $L$ is an augmented inflow gain matrix which can be written as

$$L = L_0 + \sum_i \Delta L_i \quad i = 1, 2, 3$$

(100)

where $L_0$ is the original Pitt-Peters inflow gain matrix and $\Delta L_i$ are the wake curvature augmentation terms. These augmentation matrices, which depend on a wake curvature parameter $K_{Re}$ and the wake geometric properties of skew $X$, lateral wake curvature $\kappa_s$, and longitudinal wake curvature $\kappa_c$, are given by

$$
\Delta L_1 = K_{Re} \begin{bmatrix} 0 & 0 & 0 \\ 0.5\kappa_s & 0 & 0 \\ 0.5\kappa_c & 0 & 0 \end{bmatrix}, \quad \Delta L_2 = K_{Re} \begin{bmatrix} 0 & 0.75\kappa_s X^2 & 0 \\ 0 & 0 & 0 \\ -0.75\kappa_c X^2 & 0 & 0 \end{bmatrix}
$$

(101, 102)

$$
\Delta L_3 = K_{Re} \begin{bmatrix} 0 & 0 & 0 \\ 1.25\mu\kappa_c X & l_{22} & -2.5\kappa_s X \\ 1.25\mu\kappa_s X & l_{32} & -0.3\kappa_c X \end{bmatrix}
\begin{array}{c}
l_{22} = -2.5\kappa_c X - 1.5\mu\kappa_s (1 + 1.5X^2) \\
l_{32} = -2.5\kappa_s X - 1.5\mu\kappa_c (1 - 1.5X^2) \end{array}
$$

(103)

With the rotor wake geometric properties collected into another state vector, $W = [X S \kappa_c \kappa_s]^T$, they are found by solving the differential equation

$$\tau \dot{W} + W = (W)_{qs}$$

(104)

The diagonal matrix $\tau$ contains the wake distortion time constants, and the quasi-steady wake distortion states are given by

$$
(W)_{qs} = \begin{bmatrix}
tan(\chi_{pp}/2) \\
\frac{2\pi V_m}{(\omega_{yb} - \dot{\beta}_{1c})R} & \frac{\lambda_{pp}}{\alpha_{1c}} \\
\frac{(\omega_{xb} - \dot{\beta}_{1s})R}{\lambda_{pp}} & \frac{(\omega_{xb} - \dot{\beta}_{1s})R}{\lambda_{pp}} \end{bmatrix}
$$

(105)

In these expressions, $\beta_{1c}$ and $\beta_{1s}$ are the rotor longitudinal and lateral tip path plane tilt angles, and their derivatives are therefore the rotor disk flapping rates.

The wake skew and wake curvatures can be treated as similar effects, and from Equations 99 through 103 it is apparent that they produce an equivalent coupling between the inflow and rotor thrust. By equating these effects, an “equivalent wake skew due to wake curvature” can be determined, which for a low advance ratio can be approximated by

$$
\Delta \chi_{eq} = \frac{64}{15\pi} K_{Re} \kappa_c \left(1 - \frac{3}{2}X^2\right) + \frac{64}{3\pi} K_{Re} \kappa_s \lambda_{pp} X^2
$$

(106)
This equivalent wake skew is then added to the previously calculated quasi-steady wake skew angle to model the curvature effect on the helicopter fuselage and empennage.

The final effect currently modelled is the sidewash at the vertical tail and tail rotor due to main rotor wake curvature. However the calculations for determining the lateral induced velocity due to the curved wake vortex tube are quite complex and requires a number of assumptions and simplifications to render them solvable. Continuing to use Zhao’s\(^\text{10}\) approach, only the vortex tube corresponding to the main rotor tip vortex is considered, and the aerodynamic centers of the vertical tail and tail rotor are assumed to be in the plane of the main rotor disk. This eventually leads to an induced velocity given by

\[
\Delta v_y = \kappa_s V_0 (i_0 + i_1 X + i_2 X^2)
\]  

(107)

where \(i_0, i_1, \) and \(i_2\) are interference coefficients, which are a function of the location of the tail aerodynamic centres in the main rotor disk plane. The location is specified using polar coordinates in a wind frame, which has its \(x\)-axis aligned with the in-plane velocity vector, so these coefficients will change over time. Unfortunately the functions that define the interference coefficients are not directly solvable within the Matlab environment used for the simulation. Specifically, they include complete elliptic integrals of the first and second kind

\[
\int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} \, d\theta \quad \text{and} \quad \int_0^{\pi/2} \frac{1}{\sqrt{1 - k^2 \sin^2 \theta}} \, d\theta \ 	ext{respectively},
\]

with an input outside of the domain supported by Matlab. As such, they were solved using the symbolic math package Maple to build a look-up table based on the input angle \(\theta\) for use in the real-time model.

### 4.1 UH-60 Validation Results

To validate the wake distortion implementation, the UTIAS articulated rotor model was set up as a UH-60 Black Hawk simulation using the data from Howlett\(^\text{40}\) to allow direct comparison with Reference 10. The Black Hawk has a fully articulated main rotor, and the overall model can remain much the same as that used for the CH-124 Sea King. The differences lie primarily in the details, such as the Black Hawk having four rotor blades rather than five, a canted rather than vertical tail rotor, plus the differing data sets to capture the appropriate aerodynamic and physical properties.
The control input consists of a series of filtered steps to produce the doublet shown in Figure 12, applied to either the longitudinal or lateral cyclic with no off-axis input, which was set up to approximate the input used in prior evaluations, such as that shown in Figure 13 from Reference 10. A large $K_{Re}$ value of 3.8 was used, matching the value employed in the reference to allow direct comparison.

The sample test case shown is a lateral cyclic input from hover, with the on-axis lateral response shown in Figure 14 and the longitudinal off-axis response shown in Figure 15. To reproduce the results as closely as possible, a large $K_{Re}$ value of 3.8 was used, matching the value employed in Reference 10. The off-axis response has a very definite change due to the dynamic wake distortion, with a sign change compared to the original baseline model. These results are very close to those obtained by Zhao, shown in Figure 16 and Figure 17 for comparison, giving confidence that the implementation is in fact correct.

However as pointed out in Reference 10, the theoretical maximum value for the wake distortion parameter $K_{Re}$ is 2.0, and for a realistic rotor circulation distribution value should fall between 1.0 and 2.0. When the $K_{Re}$ value is reduced to a mid-range 1.5, the blue line in Figure 14 and Figure 15 is obtained. With this value for the wake distortion parameter, the addition of wake distortion has far less of an effect. More importantly, this value results in no change of direction in the off-axis response compared with baseline.
Figure 14: UH-60 Lateral On-axis Response

Figure 15: UH-60 Longitudinal Off-axis Response

Figure 16: Lateral On-axis Response from Zhao [Ref. 10]

Figure 17: Longitudinal Off-axis Response from [Ref. 10]
5 Flexible Rotor Dynamics

For an arbitrary unconstrained elastic body, the absolute position \( \mathbf{R} \) of a mass element \( dm \) can be expressed as

\[
\mathbf{R} = \mathbf{r} + \mathbf{\rho} + \mathbf{u}_e
\]  

(108)

where \( \mathbf{r} \) is the inertial position of a body fixed reference point and \( \mathbf{\rho} \) is the undeformed position of the mass element within the body. \( \mathbf{u}_e \) is the elastic deformation, with boundary conditions of zero deformation and curl at the reference point to isolate the deformations from rigid body translations and rotations of the reference point. The velocity of \( dm \) is then given by

\[
\mathbf{V} = \mathbf{v} + \mathbf{\omega} \times (\mathbf{\rho} + \mathbf{u}_e) + \dot{\mathbf{u}}_e
\]  

(109)

where \( \mathbf{v} \) and \( \mathbf{\omega} \) are the absolute linear and angular velocity of the body-fixed reference frame. Differentiating with respect to time, the acceleration of \( dm \) is given by

\[
\ddot{\mathbf{V}} = \ddot{\mathbf{v}} + \mathbf{\omega} \times \mathbf{v} + \dot{\mathbf{\omega}} \times (\mathbf{\rho} + \mathbf{u}_e) + \mathbf{\omega} \times (\mathbf{\omega} \times (\mathbf{\rho} + \mathbf{u}_e) + \dot{\mathbf{u}}_e) + \ddot{\mathbf{u}}_e + \mathbf{\omega} \times \dot{\mathbf{u}}_e
\]  

(110)

where the time derivatives on the right side are those seen in the rotating body fixed reference frame. Expressing each term in body frame components leads to

\[
\ddot{\mathbf{V}} = \ddot{\mathbf{v}} - (\mathbf{\rho} + \mathbf{u}_e)^\times \ddot{\mathbf{\omega}} + \mathbf{\omega}^\times \mathbf{v} - \mathbf{\omega}^\times (\mathbf{\rho} + \mathbf{u}_e)^\times \mathbf{\omega} + \ddot{\mathbf{u}}_e + 2 \mathbf{\omega}^\times \dot{\mathbf{u}}_e
\]  

(111)

The equations of motion can then be determined by applying Newton’s Second Law with an applied force distribution \( \mathbf{f} \), pre-multiplying by suitable shape functions for the motion of interest, and then integrating over the body. In performing the integration, a Ritz expansion is assumed for the elastic deformation using generalized elastic coordinates \( q_{e\alpha} \)

\[
\mathbf{u}_e(\mathbf{\rho}, t) = \sum_{\alpha=1}^{N} \mathbf{\psi}_{\alpha}(\mathbf{\rho}) q_{e\alpha}(t)
\]  

(112)

If each shape function \( \mathbf{\psi}_{\alpha} \) satisfies the boundary conditions of the total deformation of no translation or rotation at the origin, the elastic deformation decouples from the rigid body motion. Additionally, a linear stiffness operator is assumed for the elastic deformation, discussed further below.
The “rigid body” translational equations of motion are determined by integrating over the body, with the shape function being the identity matrix, $I$.

$$
\int_V f \, dV = \int_V (\dot{v} - (\rho + u_e)^x \omega + \omega^x v - \omega^x (\rho + u_e)^x \omega + u_e + 2\omega^x u_e) \, dm
$$

Performing the integration leads to

$$
F = m\dot{v} - c^x \omega - \delta c^x \omega + Pq_e + m\omega v - \omega^x c^x \omega - \omega^x \delta c^x \omega + 2\omega^x Pq_e
$$

or

$$
F = [m1 - c^x - \delta c^x] \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} + [m\omega^x - \omega^x c^x - \omega^x \delta c^x] 2\omega^x P \begin{bmatrix} v \\ \omega \end{bmatrix}
$$

For the above equations, the following entities are defined:

$$
F = \int_V f \, dV
$$

$$
c = \int_V \rho \, dm
$$

$$
\delta c = \sum_{\alpha} P_{\alpha} q_{\alpha a}
$$

$$
P = \text{row}(P_{\alpha}), \quad P_{\alpha} = \int_V \psi_{\alpha} \, dm
$$

For “rigid body” rotation, the shape function of interest requires the instantaneous location of the mass element where the force is applied, $(\rho + u_e)^x$, thus

$$
\int_V (\rho + u_e)^x f \, dV = \int_V (\rho^x \dot{v} + u_e^x \dot{v} - \rho^x \rho^x \omega - u_e^x \rho^x \omega - \rho^x u_e^x \omega - u_e^x u_e^x \omega +
\rho^x \omega^x v + u_e^x \omega^x v - \rho^x \omega^x \rho^x \omega - u_e^x \omega^x \rho^x \omega - \rho^x \omega^x u_e^x \omega -
\rho^x \omega^x u_e^x \omega + \rho^x \dot{u}_e + u_e^x \dot{u}_e + 2\rho^x \omega^x \dot{u}_e + 2u_e^x \omega^x \dot{u}_e) \, dm
$$

Once again, performing the integration leads to

$$
G + \delta G = c^x \dot{v} + \delta c^x \dot{v} + l\omega + \delta l \dot{\omega} + Hq_e + \delta Hq_e + c^x \omega^x v +
\delta c^x \omega^x v + \omega^x l\omega + \omega^x \delta l \omega - 2(\tau + \delta \tau)^T \omega q_e
$$

or

$$
G + \delta G = [c^x + \delta c^x \quad I + \delta I \quad H + \delta H] \begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \dot{q}_e \end{bmatrix} +

[c^x \omega^x + \delta c^x \omega^x \quad \omega^x l + \omega^x \delta l \quad -2(\tau + \delta \tau)^T \omega] \begin{bmatrix} v \\ \omega \\ q_e \end{bmatrix}
$$

where the following are defined:

$$
G = \int_V \rho^x f \, dV, \quad \delta G = \sum_{\alpha} \int_V \psi_{\alpha}^x q_{\alpha a} f \, dV
$$

$$
I = -\int_V \rho^x \rho^x \, dm
$$
\[
\delta l = \sum_\alpha \left( - (\tau_\alpha + \tau_\alpha^T) q_{e\alpha} - \sum_\beta \gamma_{\alpha\beta} q_{e\alpha} q_{e\beta} \right)
\]
(125)

\[
H = \text{row}(H_\alpha), \quad H_\alpha = \int_V \rho^x \psi_\alpha dm
\]
(126)

\[
\delta H = \text{row}(\delta H_\alpha), \quad \delta H_\alpha = - \sum_\beta \int_V \psi_\alpha^x \psi_\beta q_{e\beta} dm
\]
(127)

\[
\tau_\alpha = \int_V \psi_\alpha^x \rho^x dm
\]
(128)

\[
\delta \tau_\alpha = \sum_\beta \int_V \psi_\alpha^x \psi_\beta^x q_{e\beta} dm
\]
(129)

Finally, the elastic shape functions \(\psi_\alpha^T\) are used to extract the elastic equations,

\[
\int_V \psi_\alpha^T f dV = \int_V \left( \psi_\alpha^T \dot{v} - \psi_\alpha^T (\rho + u_e)^x \dot{\omega} + \psi_\alpha^T \omega \dot{x} v - \psi_\alpha^T \omega \cdot (\rho + u_e)^x \omega + \psi_\alpha^T \ddot{u}_e + 2 \psi_\alpha^T \omega \dot{u}_e \right) dm
\]
(130)

Integrating these equations leads to

\[
f_e = P^T \dot{v} + H^T \dot{\omega} + \delta H^T \dot{\omega} + M_{ee} \ddot{q}_e + P^T \omega \dot{x} v + \omega \dot{x} \tau \omega + \omega^T \delta \tau \omega - 2 \omega^T \omega q_e + K_{ee} q_e
\]
(131)

or

\[
f_e = \begin{bmatrix} P^T & H^T + \delta H^T & M_{ee} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \ddot{q}_e \end{bmatrix} + \begin{bmatrix} P^T \omega \dot{x} \\ \omega \dot{x} (\tau + \delta \tau) - 2 \omega^T \omega \end{bmatrix} \begin{bmatrix} v \\ \omega \\ q_e \end{bmatrix} + K_{ee} q_e
\]
(132)

making use of the following definitions in addition to those listed above:

\[
f_e = \text{col}\{f_{e\alpha}\}, \quad f_{e\alpha} = \int_V \psi_{e\alpha}^T f dV
\]
(133)

\[
M_{ee} = \int_V \psi_{e\alpha} \psi_{e\beta} dm
\]
(134)

\[
u_{\alpha\beta} = - \int_V \psi_{e\alpha} \psi_{e\beta} dm
\]
(135)

\[
Y_{\alpha\beta} = \int_V \psi_{e\alpha} \psi_{e\beta} dm
\]
(136)

In addition, there is an implied summation in the above notation for simplicity

\[
u^T \omega \dot{q}_e = \sum_\beta \nu_{\beta\alpha}^T \omega \dot{q}_{e\beta}
\]
(137)

Collecting this system of equations back into matrix form results in the following:

\[
\begin{bmatrix} F \\ G + \delta G \\ f_e \end{bmatrix} = \begin{bmatrix} m \mathbf{1} & -c^x - \delta c^x & P \\ c^x + \delta c^x & I + \delta I & H + \delta H \\ P^T & H^T + \delta H^T & M_{ee} \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{\omega} \\ \ddot{q}_e \end{bmatrix} + \begin{bmatrix} m \omega^x \\ -\omega^x c^x - \omega^x \delta c^x \\ \omega^x I + \omega^x \delta I \end{bmatrix} \begin{bmatrix} \nu \\ \omega \\ \dot{q}_e \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ K_{ee} q_e \end{bmatrix}
\]
(138)
The appearance of which can be simplified to

\[
\begin{bmatrix}
\mathbf{F} \\
\mathbf{G} + \delta \mathbf{G} \\
\mathbf{f}_e
\end{bmatrix} = (\mathbf{M} + \delta \mathbf{M}) \begin{bmatrix}
\ddot{\mathbf{v}} \\
\mathbf{0} \\
\ddot{\mathbf{q}}_e
\end{bmatrix} + (\mathbf{C} + \delta \mathbf{C}) \begin{bmatrix}
\mathbf{v} \\
\mathbf{0} \\
0
\end{bmatrix} + \begin{bmatrix}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{K}_{ee} \mathbf{q}_e
\end{bmatrix}
\]  

(139)

where

\[
\mathbf{M} = \begin{bmatrix}
m1 & -c^x & P \\
c^x & I & H \\
P^T & H^T & \mathbf{M}_{ee}
\end{bmatrix}
\]  

(140)

\[
\delta \mathbf{M} = \begin{bmatrix}
0 & -\delta c^x & 0 \\
\delta c^x & \delta I & \delta H \\
0 & \delta H^T & 0
\end{bmatrix}
\]  

(141)

\[
\mathbf{C} = \begin{bmatrix}
m \omega^x & -\omega^x c^x & 2 \omega^x P \\
c^x \omega^x & \omega^x I & -2 \tau T \omega \\
P^T \omega^x & \omega^T \tau & -2 v^T \omega
\end{bmatrix}
\]  

(142)

\[
\delta \mathbf{C} = \begin{bmatrix}
0 & -\omega^x \delta c^x & 0 \\
\delta c^x \omega^x & \omega^x \delta I & -2 \delta \tau T \omega \\
0 & \omega^T \delta \tau & 0
\end{bmatrix}
\]  

(143)

At this point, if using a complete set of shape functions, the above formulation can accurately capture any arbitrary motion, with linear stiffness being the only simplifying approximation. To allow for solving the equations in real-time, the only other approximation required is to exclude the mass matrix corrections (\(\delta \mathbf{M}\)) as well as the associated inertial nonlinearities (\(\delta \mathbf{C}\)), retaining only the first order deformation terms. The deformations involved are small, so these are small corrections, and this approximation is consistent with using a linear stiffness.\(^{49}\) Thus the equations of motion therefore reduce to

\[
\begin{bmatrix}
\mathbf{F} \\
\mathbf{G} + \delta \mathbf{G} \\
\mathbf{f}_e
\end{bmatrix} = \mathbf{M} \begin{bmatrix}
\ddot{\mathbf{v}} \\
\dot{\mathbf{v}} \\
\dot{\mathbf{q}}_e
\end{bmatrix} + \mathbf{C} \begin{bmatrix}
\mathbf{v} \\
\mathbf{0} \\
0
\end{bmatrix} + \begin{bmatrix}
\mathbf{0} \\
\mathbf{0} \\
\mathbf{K}_{ee} \mathbf{q}_e
\end{bmatrix}
\]  

(144)

5.1 Modal shapes and stiffness matrix

While any orthogonal shape functions can be used in the dynamics, using the normal modes of the flexible body should result in a faster convergence to the actual motion. To accomplish this, approximate mode shapes for the shape functions were determined using a finite element analysis based on representative main rotor blade data,\(^{50}\) scaled for each helicopter modelled. The modal integrals over the body presented above then become summations over the elements. The stiffening effect due to axial loading arising from the rotational motion must also be included, as this is a significant contributor to the overall rotor blade stiffness.
Following the approach presented in Bielawa\textsuperscript{51} and Genta,\textsuperscript{52} in the finite element formulation to calculate the beam element stiffness matrix for a rotating beam, axial loading leads to an additional term in the elastic potential energy for vertical deformations. For a beam with stiffness $EI$ under tension $T$ with applied distributed vertical load $f_z$, the basic differential equation governing bending deformation $w$ along its length is

$$\frac{\partial^2}{\partial x^2} (EI \frac{\partial^2 w}{\partial x^2}) - \frac{\partial}{\partial x} (T \frac{\partial w}{\partial x}) = f_z(x)$$

(145)

The axial strain at the beam neutral axis, which is typically given by $\epsilon_x = \partial u / \partial x$, in this case due to the bending becomes

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2$$

(146)

which leads to two terms in the potential energy due to axial strain. The first term leads to the typical axial stiffness matrix, which was not used here as axial deformation was neglected. The second is the additional term for vertical deformation, and can be accounted for using an additional stiffness matrix in the finite element analysis as

$$[K + K_G] \delta = F_n$$

(147)

In Eq. (147), $\delta$ is the nodal displacement vector, $F_n$ is the nodal force vector, $K$ is the standard beam stiffness given by

$$K_{ij} = \frac{EI}{l^3} \int_0^1 \frac{d^2 N_i}{d \xi^2} \frac{d^2 N_j}{d \xi^2} d\xi$$

(148)

and $K_G$ is the beam geometric stiffness matrix given by

$$K_{Gij} = \frac{T}{l} \int_0^1 \frac{d N_i}{d \xi} \frac{d N_j}{d \xi} d\xi$$

(149)

In Eqs. (148) and (149), $l$ is the element length, $\xi$ is the normalized position, and $N_i$ are the Hermitian shape functions for a two noded beam element given by

$$N_1 = 1 - 3\xi^2 + 2\xi^3, \quad N_2 = l(\xi - 2\xi^2 + \xi^3)$$
$$N_3 = 3\xi^2 - 2\xi^3, \quad N_4 = l(-2\xi^2 + \xi^3)$$

(150)

Performing the integration for the matrix entries gives the typical beam stiffness matrix

$$K = \frac{EI}{l^3} \begin{bmatrix}
12 & 6l & -12 & 6l \\
6l & 4l^2 & -6l & 2l^2 \\
-12 & -6l & 12 & -6l \\
6l & 2l^2 & -6l & 4l^2 \\
\end{bmatrix}$$

(151)
and a beam geometric stiffness matrix

\[
K_G = \frac{T}{30l} \begin{bmatrix}
36 & 3l & -36 & 3l \\
3l & 4l^2 & -3l & -l^2 \\
-36 & -3l & 36 & -3l \\
3l & -l^2 & -3l & 4l^2
\end{bmatrix}
\] (152)

For a rotor rotating with constant angular velocity \( \Omega \), the tension at any point along the rotor due to centrifugal force is obtained by integrating the axial loads as

\[
T(r) = \Omega^2 \int_r^R m(r') r' dr'
\] (153)

If the integral in equation 153 (or \( T/\Omega^2 \)) is \( T_i \) at the midpoint of the element, then \( K_\Omega \) is given by

\[
K_\Omega = \frac{T_i}{30l} \begin{bmatrix}
\ldots
\end{bmatrix}
\] (154)

where the matrix terms inside the brackets are the same as those in equation 152.

When combined with the finite element mass properties, an eigenvalue analysis can be performed to identify the normal modes. The normal mode shapes are then used as the shape functions for the elastic deformation in the rotor dynamics. The modal stiffness matrix \( K_{ee} \) is determined using the total stiffness matrix \( [K + K_G] \) projected using these mode shapes, which are the eigenvectors.

### 5.1.1 Sample Normal Mode Results

The first two normal modes of the CH-149 are shown as an example. A number of approximations and simplifications were made, such as considering only the flapping motion, neglecting in-plane bending and torsion of the rotor blade to reduce the number of degrees of freedom. The code was however set up such that it could easily be extended to allow additional degrees of freedom once suitable data was available. Additionally, the blade properties such as stiffness and mass distribution were approximated based on the limited available information. As such, this should not be considered a full analysis of the exact CH-149 rotor, but rather of some similar representative rotor, which could be improved upon with more accurate input information. However for the intended purpose of elastic motion shape functions, which as noted can be any orthogonal set, this approximation would be sufficient to capture the majority of the bending motion.
The first flapping normal mode was determined to have a natural frequency of 24.9 rad/s, or 1.11 per revolution at the nominal rotor speed of 22.4 rad/s, with a blade shape as shown in Figure 19 and Figure 20. The second normal mode, with a natural frequency of 155.1 rad/s or 6.9 per revolution, has a blade shape as shown in Figure 21 and Figure 22. In these plots, the mode shapes have been normalized to a 1 cm maximum deflection. Additionally, as can be seen in Figure 20, the first mode contains a significant rigid body rotation contribution, which is removed from the vertical deformation plot in Figure 19 to make the blade shape visible.

![Figure 19: First Normal Mode Vertical Deformation](image1.png)

![Figure 20: First Normal Mode Angular Displacement](image2.png)

![Figure 21: Second Normal Mode Vertical Deformation](image3.png)

![Figure 22: Second Normal Mode Angular Displacement](image4.png)
5.2 Teetering rotor hub forces

Applying this approach to a teetering rotor differs from an articulated blade in that the entire rotor can treated as one continuous body. An overview of the resulting rotor dynamics is presented here, without detailing all of the calculations required to obtain various input quantities. For the teetering rotor, the dynamics equations are expressed in a rotor fixed frame $F_b$, shown in Figure 5, whose origin is at the teetering hinge point where the hub is connected to the rotor mast, with its $x$-axis coincident with the $x$-axis of $F_R$ and its $z$-axis straight down the rotor symmetry axis. The mass is $m = 2m_b$, where $m_b$ is the mass of a single blade, with mass and inertia moments $c = 0$ and $I = I_R$, where $I_R$ is the rotor inertia matrix expressed in $F_b$ which is derived from the inertia matrix of a single blade, $I_b$.

The external forces acting on the main rotor consist of the aerodynamic force $F^A$, $F^G$ due to gravity, and $F^H$ due to the rotor mast. In rotor frame components, the translation equation of Eq. (144) then becomes

$$F^H_b = 2m_b\ddot{a}_b + P\ddot{q}_e + 2\omega_b^xP\dot{q}_e - F^A_b - F^G_b$$ (155)

In Eq. (155), $\dot{a}_b$ is the inertial acceleration of the rotor teetering hinge location, and $\omega_b$ is the inertial angular velocity of frame $F_b$ expressed in $F_b$ components. These are determined using the helicopter body acceleration and rates, which are considered known constraints using one time step delayed values to decouple the rotor and fuselage dynamics, as the helicopter body is far more massive and relatively low bandwidth. By using Newton’s Third Law, $-F^H_b$ is the force applied by the main rotor to the top of the rotor mast at the hub location. The total force acting on the hub in the helicopter body frame $F_B$ from both blades is then given by

$$F^R_B = -L_{bb}F^H_b$$ (156)

5.3 Teetering rotor hub moments

The external moments acting on the rotor at the teetering hinge consist of the aerodynamic moment due to the rotor blades $G^A$, $G^H$ due to the rotor mast, and $G^G$ due to gravity. By using Newton’s Third Law, $-G^H$ is the moment applied by the rotor about the top of the rotor mast at the teetering hinge location. Note that the moments related to the swashplate inputs to blade pitch
and the pitch dynamics of the blade are neglected as they are much smaller than the other contributions.

Using these moments in the rotational flexible dynamics equation from Eq. (144) and expressing in rotor frame components yields

\[ G_b^H + G_b^A + \delta G_b^A + \delta G_b^G = I_R \dot{\omega}_b + H\ddot{q}_e + \omega_b^x I_R \omega_b - 2\tau^T \omega_b \dot{q}_e \]  

(157)

Note that \( G_b^G = 0 \) since the reference point is the undeformed rotor centre of gravity and \( \delta G_b^H = 0 \) since there is no deformation at the hinge, so these terms are dropped. In this analysis, \( F_b \) are principal axes for the rotor system and thus \( I_R \) is diagonal with

\[
I_R = \begin{bmatrix}
I_{xR} & 0 & 0 \\
0 & I_{yR} & 0 \\
0 & 0 & I_{zR}
\end{bmatrix}
\]  

(158)

The angular velocity \( \omega_b \) can be found from the sum of the angular velocity of \( F_b \) relative to \( F_R \) and the angular velocity of \( F_R \) relative to \( F_E (\omega^R) \). The latter velocity is a known quantity, from the rigid body fuselage motion (\( \omega^B \)) plus the engine driven shaft rotation (\( \Omega \)). From Figure 5 it can be seen that the angular motion of \( F_b \) relative to \( F_R \) is flapping about the \( x \)-axis of \( F_b \) and therefore

\[ \omega_b = \omega^R_b + [-\dot{\beta} \ 0 \ 0]^T \]  

(159)

This can be differentiated to give

\[ \dot{\omega}_b = \dot{\omega}^R_b + [-\ddot{\beta} \ 0 \ 0]^T \]  

(160)

Now from Eqs. (157), (158) and (160) express the total moments \( G_b^H \) as

\[ G_b^H = G_b^{H1} - [I_{xR} \dot{\beta} \ 0 \ 0]^T \]  

(161)

where

\[ G_b^{H1} = -G_b^A - \delta G_b^A - \delta G_b^G + I_R \dot{\omega}^R_b + H\ddot{q}_e + \omega_b^x I_R \omega_b - 2\tau^T \omega_b \dot{q}_e \]  

(162)

From the design of the teetering rotor it follows that the moment about the teetering axis is zero,

\[ G_{xb}^H = 0 \]  

(163)

which can be combined with the total moments of Eq. (161) to obtain

\[ \ddot{\beta} = G_{xb}^{H1} / I_{xR} \]  

(164)

which describes the blade flapping motion.
Figure 23 shows the flapping angles $\beta_i$ for a teetering rotor. The angle $\beta_0$ represents a geometrically set coning angle that is preset in the rotor structure. $\beta$ is the teetering parameter and it is seen that the flapping angles are given by

$$
\beta_1 = \beta_0 + \beta, \quad \beta_2 = \beta_0 - \beta
$$

(165, 166)

The differential equation describing the flapping motion of the rotor is given by Eq. (164), which is solved by numerical integration in the simulation.

![Teetering Rotor Flapping Angles](image)

The total moment applied by the main rotor about the top of the mast is the sum of the contributions from both blades. In body components this is

$$
\mathbf{G}^S_B = -\mathbf{L}_{BBb} \mathbf{G}^H_b
$$

(167)

The main rotor applies moments about the helicopter body CG due to $\mathbf{G}^S_B$ (the moment applied about the top of the mast) and due to $\mathbf{F}^R_B$ (the force applied to the top of the mast). Thus the total moment from these two effects is given by

$$
\mathbf{G}^R_B = \mathbf{G}^S_B + \mathbf{r}^S_B \times \mathbf{F}^R_B
$$

(168)

where $\mathbf{r}^S_B$ is the location of the teetering hinge point at the top of the mast relative to the helicopter CG in the body frame.

5.4 Teetering rotor elastic equations

Expanding the final row of the dynamics equations in Eq. (144) gives

$$
f_e = \mathbf{P}^T \dot{\mathbf{v}}_b + \mathbf{H}^T \dot{\mathbf{\omega}}_b + \mathbf{M}_{ee} \ddot{\mathbf{q}}_e + \mathbf{P}^T \mathbf{\omega}_b \times \dot{\mathbf{v}}_b + \mathbf{\omega}_b \times \mathbf{\tau}_b - 2 \mathbf{v}_b^T \mathbf{\omega}_b \dot{\mathbf{q}}_e + \mathbf{K}_{ee} \mathbf{q}_e
$$

(169)

Once again making use of Eq. (160) and solving for the acceleration of the generalized elastic coordinates, this can be written as
\[ M_{ee} \ddot{q}_e = f_e - K_{ee} q_e - P^T a_b - \omega_b^T \tau \omega_b + 2v^T \omega_b \dot{q}_e - H^T \dot{\omega}_b^R + (H^T)_{x} \ddot{\beta} \]  \tag{170}

To eliminate the flapping acceleration \( \ddot{\beta} \), Eqs. (162) and (164) can be solved for \( \ddot{\beta} \) giving

\[ \ddot{\beta} = \dot{\omega}_{xb}^R + (H \dot{q}_e + \omega_b^x I_R \omega_b - 2\tau^T \omega_b \dot{q}_e - G_b)_x / l_{xR} \]  \tag{171}

where \( G_b \) is the sum of the three moment contributions, which can then be substituted into Eq. (170). The acceleration of the elastic coordinates can then be expressed as

\[ M_q \ddot{q}_e = f_e - K_{ee} q_e - P^T a_b - \omega_b^T \tau \omega_b + 2v^T \omega_b \dot{q}_e - H^T \dot{\omega}_q \]  \tag{172}

where

\[ M_q = M_{ee} - (H^T H_x) / l_{xR} \]  \tag{173}

and

\[ \dot{\omega}_q = \begin{bmatrix} \dot{\omega}_y^b \\ \dot{\omega}_z^b \end{bmatrix} \]  \tag{174}

### 5.5 Articulated rotor hub forces

For an articulated rotor, the dynamic equations are expressed in a rotor blade fixed frame \( F_b \), shown in Figure 3, whose origin is at the hinge point where the blade is connected to the hub. The mass is that of a single blade, \( m_b \), with mass moments \( e = [0 \quad r_{CG} m_b \quad 0]^T \) where \( r_{CG} \) is a given parameter specifying the radial location of the blade’s CG from the center of the hub, and \( I = I_b \), which is the individual blade inertia matrix in frame \( F_b \).

The external forces acting on the main rotor consist of the aerodynamic force \( F^A \), \( F^G \) due to gravity, \( F^H \) due to the rotor hinge, and \( F^D \) due to the lag damper. Applying the translational equation in Eq. (144) to the \( i \)-th rotor blade gives

\[ F^H_b + F^A_b + F^D_b + F^G_b = m_b a_b - c^x \dot{\omega}_b - \omega_b^x c^x \omega_b + P \dot{q}_e + 2\omega_b^x P q_e \]  \tag{175}

where \( a_b \) is the known inertial acceleration of the rotor hinge location and \( \omega_b \) is the inertial angular velocity of frame \( F_b \), expressed in \( F_b \) components. As with the teetering rotor, one time step delayed values of the slowly changing helicopter body acceleration and rates are used to decouple the rotor and body dynamics. By using Newton’s Third Law, \( -(F^H + F^D) \) is the force applied by the \( i \)-th rotor blade to the hub. Adding up the contributions from all of the blades, the force applied by the main rotor to the helicopter body in the body frame \( F_B \) is

\[ F^R_B = - \sum_{i=1}^{N_b} L_{Bb}(F^H_b + F^D_b) \]  \tag{176}
5.6 Articulated rotor hub moments

The external moments acting on an articulated rotor about the origin of $F_b$ consist of the aerodynamic moments $G_A$, $G_H$ due to the hinge, $G_D$ due to the lag damper, and $G_G$ due to gravity. From Newton’s Third Law, $-G_H$ is the moment applied by the $i$-th rotor blade to the rest of the helicopter about the hinge. Once again the moments related to the swashplate inputs to blade pitch and the pitch dynamics of the blade are neglected as they are much smaller than the other contributions.

Using these moments and expressing all terms in blade frame components, the rotational flexible dynamics equation from Eq. (144) becomes

$$G_b^H + G_b^A + G_b^D + G_b^G + \delta G_b^A + \delta G_b^D = c^x a^b + I_b \omega_b + H q_e + \omega_b^x I_b \omega_b - 2 \tau^T \omega_b q_e \tag{177}$$

Note $\delta G_b^H = \delta G_b^D = 0$ since there is no deformation at the hinge, therefore these terms are dropped. As with the teetering rotor, $F_b$ are principal axes for the rotor blade and thus $I_b$ is diagonal,

$$I_b = \begin{bmatrix} I_{xb} & 0 & 0 \\ 0 & I_{yb} & 0 \\ 0 & 0 & I_{zb} \end{bmatrix} \tag{178}$$

The angular velocity $\omega_b$ can be found from the sum of the angular velocity of $F_b$ relative to $F_R$, which involves flapping and lagging, and the angular velocity of $F_R$ relative to $F_E (\omega^R)$, which is known from the fuselage motion. As with the teetering rotor, this is differentiated to give an angular acceleration of

$$\dot{\omega}_b = \dot{\omega}_b^R + \begin{bmatrix} -\dot{\beta}_i \\ \dot{\beta}_i \gamma_i \cos \beta_i + \dot{\gamma}_i \sin \beta_i \\ \dot{\beta}_i \gamma_i \sin \beta_i - \dot{\gamma}_i \cos \beta_i \end{bmatrix} \tag{179}$$

where $\beta_i$ is the blade flapping angle and $\gamma_i$ is the lag angle.

From Eqs. (177), (178) and (179) express the hinge moments $G_b^H$ as

$$G_b^{H1} + G_b^{H1} + \begin{bmatrix} -I_{xb} \dot{\beta}_i \\ I_{yb} (\dot{\beta}_i \gamma_i \cos \beta_i + \dot{\gamma}_i \sin \beta_i) \\ I_{zb} (\dot{\beta}_i \gamma_i \sin \beta_i - \dot{\gamma}_i \cos \beta_i) \end{bmatrix} \tag{180}$$
where $G_b^{H1}$ is an intermediate term, defined to be

$$G_b^{H1} = -G_b^{A} - G_b^{D} - G_b^{G} - \delta G_b^{A} - \delta G_b^{G} + c^x a_b + I_b \dot{\omega}_b^R + H \ddot{q}_e + \omega_b^x I_b \omega_b - 2 \tau^T \omega_b \dot{q}_e$$  \hspace{1cm} (181)

For a hub using elastomeric bearings, the bearing can be approximated as a hinge combined with a torsion spring. The $x$-component of $G_b^H$ is then $K_{\beta \beta} \beta_i$ where $K_{\beta \beta}$ is the rotational stiffness parameter, or zero for a pure hinge. Similarly, the $z$-component of $G_b^H$ is either $K_{\gamma \gamma} \gamma_i$ or zero, where $F_{Hi}$ is a hinge fixed reference frame shown in Figure 3. Using these values with Eq. (180), the flapping and lagging acceleration can be found by

$$\ddot{\beta}_i = \left( G_{xb}^{H1} - K_{\beta \beta} \beta_i \right) / I_{xb}$$  \hspace{1cm} (182)

and

$$\ddot{\gamma}_i = \left( G_{zb}^{H1} - G_{yb}^{H1} \tan \beta_i - K_{\gamma \gamma} \gamma_i / \cos \beta_i + (L_{zb} - L_{yb}) \dot{\beta} \dot{\gamma} \sin \beta \right) / \left( I_{yb} \sin \beta \tan \beta_i + L_{zb} \cos \beta_i \right)$$  \hspace{1cm} (183)

As with the teetering rotor equations, $\dot{\beta}_i$, $\beta_i$, $\dot{\gamma}$, and $\gamma$ are found using numerical integration.

For each blade, the moment applied to the mast is made up of three parts, the moment passed through the hinge ($G_b^H$), and the moments produced by the forces acting on the hinge ($F_b^H$) and the lag damper ($F_b^D$). The total moment applied about the top of the mast is the sum of the contributions from all blades, which in body components is

$$G_b^S = \sum_{i=1}^{N_b} \left( L_{Bb} (G_b^H + r_b^{Hx} F_b^H + r_b^{Dx} F_b^D) \right)$$  \hspace{1cm} (184)

The main rotor applies moments about the helicopter body CG due to $G_b^S$ (the moment applied about the top of the mast) and due to $F_b^R$ (the force applied to the top of the mast). Thus the total moment from these two effects is given by

$$G_b^R = G_b^S + r_b^{Sx} F_b^R$$  \hspace{1cm} (185)

where $r_b^S$ is the location of the rotor hub relative to the helicopter CG in the body frame.

### 5.7 Articulated rotor elastic equations

Expanding the final row of Eq. (144) and solving for the acceleration of the generalized elastic coordinates gives

$$M_{ee} \ddot{q}_e = f_e - P^T \dot{v}_b - H^T \dot{\omega}_b - P^T \omega_b^x v_b - \omega_b^x r \dot{\omega}_b + 2 v^T \omega_b \dot{q}_e - K_{ee} q_e$$  \hspace{1cm} (186)

This must be simultaneously solved with the flapping and lagging equations due to their contributions to $\dot{\omega}_b$. To accomplish this, define

$$G_b^q = G_b^{H1} - H \ddot{q}_e - I_b \dot{\omega}_b^R$$  \hspace{1cm} (187)
so that the flapping equation (182) becomes
\[ \ddot{\beta}_i = \ddot{\omega}_{xb}^R + (G_{xb}^q + H\dot{q}_e - K_{\beta}\beta_i)/I_{xb} \] (188)
and the lagging equation (183) can be expressed as
\[ \ddot{\gamma}_i = \ddot{\gamma}_i^q + \left( (H_z - H_y \tan \beta_i)/(I_{yb} \sin \beta_i \tan \beta_i + I_{zb} \cos \beta_i) \right) \ddot{q}_e \] (189)
where
\[ \ddot{\gamma}_i^q = (G_{zb}^q + I_{zb} \dot{\omega}_z^R - (G_{yb}^q + I_{yb} \dot{\omega}_y^R) \tan \beta_i - K_{\gamma} \gamma_i/\cos \beta_i + (I_{zb} - I_{yb}) \dot{\gamma}_i \sin \beta)/(I_{yb} \sin \beta_i \tan \beta_i + I_{zb} \cos \beta_i) \] (190)
Using Eqs. (188) and (189) to eliminate the flapping and lagging accelerations, the acceleration of the elastic coordinates can then be expressed as
\[ M_{ee}^q \ddot{q}_e = f_e - K_{ee} q_e - P^T a_b - H^T \dot{\omega}_b^q - \omega_b^r \tau - 2v^T \omega_b \dot{q}_e \] (191)
where
\[ \dot{\omega}_b^q = \begin{bmatrix} \omega_{yb}^R + \dot{\beta}_i \dot{\gamma}_i \cos \beta_i + \ddot{\gamma}_i^q \sin \beta_i \\ \omega_{zb}^R + \dot{\beta}_i \dot{\gamma}_i \sin \beta_i + \ddot{\gamma}_i^q \cos \beta_i \end{bmatrix} \] (192)
and
\[ M_{ee}^q = M_{ee} + H^T \begin{bmatrix} (H_z - H_y \tan \beta_i) \sin \beta_i/(I_{yb} \sin \beta_i \tan \beta_i + I_{zb} \cos \beta_i) \\ -(H_z - H_y \tan \beta_i) \cos \beta_i/(I_{yb} \sin \beta_i \tan \beta_i + I_{zb} \cos \beta_i) \end{bmatrix} \] (193)
6  Aerodynamic Lag

A further possible method to correct the off-axis response of helicopter models is an empirically determined phase lag for the lift and drag coefficients of the rotor blade elements as suggested by Mansur and Tischler$^5$. This is a simple method easily implemented in a real-time model to capture some of the unsteady aerodynamic effects on the blade element lift and drag calculations.

The aerodynamic phase lag refers to a physical angle in the rotor disk, where the lift and drag coefficients lag behind the static coefficients by an empirically determined angle $\psi_a$. The lag can be implemented using a first order filter on the quasi-steady lift and drag coefficients obtained from look up tables so they are found by solving the differential equation

$$\tau_a \begin{bmatrix} \dot{c}_L \\ \dot{c}_D \end{bmatrix} + \begin{bmatrix} c_L \\ c_D \end{bmatrix} = \begin{bmatrix} c_L \\ c_D \end{bmatrix}_{qs}$$

(194)

with a time constant $\tau_a = \psi_a / \Omega$. The lag angle must be determined empirically, by varying the lag until the best improvement in the simulation off-axis response is achieved. For the UTIAS models, the lag was determined using the Bell 206 simulation compared against flight test data. The lag angle varies with forward speed, and in this manner was set at 36° of lag at hover, 20° at 60 knots, and 16° at 110 knots. In the simulation, the current lag angle is obtained using a linear look up table to interpolate between these speeds. These angles closely agree with the values obtained in Reference 5 for the AH-64 and UH-60, and as such identical values were used for all three models.
7 Offline Trials: Simulation Results

The plots in this section present a sample of results obtained through the addition of dynamic wake distortion, blade flexibility, and unsteady aerodynamics to the rotor dynamics. The figures are presented in pairs, showing both the on-axis and off-axis response of each input/trim condition. In these tests, the first two flexible modes are included in the Ritz expansion, which are obtained from a finite element analysis using representative rotor blade data. As pointed out in Reference 10, the theoretical maximum value for the wake distortion parameter $K_{Re}$ is 2.0, and for a realistic rotor circulation distribution value should fall between 1.0 and 2.0, so a mid-range value of 1.5 was used.

7.1 Teetering Rotor Results

For the teetering rotor Bell 206, the simulation results are compared with actual flight test data collected in an instrumented helicopter by the National Research Council (NRC) Canada. To approximate the pilot inputs from the flight tests, the control time histories were re-sampled to the simulation update rate using linear interpolation. An example for a longitudinal input from a steady out-of-ground-effect hover is shown in Figure 24. To account for the possibility of an off-axis response due to an unintended input, both the longitudinal and lateral cyclic were approximated using this method for all tests. In addition, the controls are fixed for one second of trimmed running in the simulation leading up to the first step input.

Figure 25 and Figure 26 show the response to the cyclic inputs shown in Figure 24. For this particular case, the actual off-axis response
recorded was minimal. In addition to the flight test data, the outputs of four simulation models are shown, the original baseline model, and the result obtained when including dynamic wake distortion, blade flexibility, or aerodynamic lag. In all cases, the on-axis longitudinal response shown in Figure 25 is quite similar, with only minor variations and all matching the flight test data reasonably well.

![Figure 25: Bell 206 Longitudinal Response](image)

The off-axis lateral response in Figure 26 shows definite changes due to the inclusion of the added effects. In this test case including dynamic wake distortion did not notably improve the off-axis response. As previously shown,\textsuperscript{11} the inflow distribution does in fact change when the wake distortion is added, however this does not carry through into a significantly altered off-axis response in this particular case. In comparison, the blade flexibility, even with only two bending modes, did improve the agreement between simulation and flight test. While the aerodynamic lag is not a clear improvement, it did alter the dynamics more than dynamic wake distortion.

The limited change with wake distortion is not entirely unexpected. In Ref. 9, Zhao found that a $K_{Re}$ value of 3.8 was required in order to match flight test data, with lesser changes noted using the nominal value of 1.0. Reference 53 also found that the effects of wake distortion were small compared to what would be needed to improve correlation for an isolated rotor. Further, Ref. 54 presents the effect of hinge offset, with diminishing effects of wake distortion with reduced offset. In that study, with small hinge offset, including the limit of no offset as is the case for a teetering rotor, the added effects did not result in an off-axis response sign change.
In Figure 27 and Figure 28, all of the various possible combinations of effects are plotted with the same test conditions as above. When used in combinations, the effects of dynamic wake distortion and blade flexibility are approximately additive. The aerodynamic lag interaction is more complex, altering the shape of the off-axis response when included. In this case, as with most test conditions, the best overall match for the Bell 206 is obtained with blade flexibility and wake distortion.

7.2 Articulated Rotor Results

For the two articulated rotor helicopter models, the control input consists of a series of filtered steps to produce the doublet shown in Figure 29 and is applied to either the longitudinal or lateral cyclic with no off-axis input, as was done with the UH-60 validating evaluations previously shown. Only the differences due to the inclusion of wake distortion, blade flexibility, and aerodynamic lag are shown, as unfortunately no comparable flight test data is available. While
UTIAS does have a limited amount of Cormorant flight test data used in prior evaluations, it was all collected with stability augmentation active. This notably alters the basic vehicle dynamics and suppresses off-axis motion, and as such is not directly comparable.

For the Sea King simulation, a longitudinal cyclic input from hover is shown as a sample test case. As can be seen in Figure 30, once again only minimal changes are produced in the longitudinal on-axis response with any of the three added effects. Similar to the teetering rotor case, the off-axis lateral response of the articulated rotor shows a definite change due to the inclusion of these effects. As noted, no comparison with flight test data is available, limiting the conclusions that can be drawn in this instance. Most of the differences observed with the wake distortion and aerodynamic lag occur at the control reversal rather than during initial input, and both produce very similar results. However, the blade flexibility has certainly made a larger impact on the vehicle dynamics than that observed with the other effects, particularly in the initial response, and would therefore seem to have potential for obtaining off-axis improvements.

Figure 30: CH-124 Longitudinal On-axis Response

Figure 31: CH-124 Lateral Off-axis Response

Figure 32 and Figure 33 display the results of the various possible combinations of added effects. As seen above, in this test case the changes in the initial response due to wake distortion and aerodynamic lag are slight, but become more significant after the control reversal. This carries on to when used in the various combinations, such that the most significant variation is due to the use of a rigid versus flexible rotor blade model, where including blade flexibility reduces the observed off-axis response. When wake distortion and aerodynamic lag, which both produced similar increases in the off-axis response, are used in combination the effect is doubled.
For the Cormorant, in most test cases a similar pattern to the Bell 206 and Sea King is observed, with typically fairly minor alteration to the on-axis dynamic responses, and more notable off-axis variation, particularly with blade flexibility. Figure 34 and Figure 35 are sample results starting from a 60 knot cruise with a lateral cyclic input as shown in Figure 29. This case is a bit of an exception, as the off-axis is notably changed by dynamic wake distortion with what could be considered a direction change, as well as a much larger response in the off-axis direction than initially observed with the baseline model. However the on-axis response is also altered significantly more than expected, which is not desirable.
7.3 Offline Trial Summary

It is apparent that the effects of adding dynamic wake distortion are configuration dependent. The changes to the four bladed articulated UH-60 model observed here and in many previous publications vary significantly from the results for the two bladed teetering rotor of the Bell 206 or the five bladed articulated rotors of the CH-124 and CH-149.

The blade flexibility may have a greater potential to improve the response than wake distortion, as the former produces a greater change in the off-axis vehicle dynamics than the latter. In comparisons with flight test data for the Bell 206, the best results were obtained when blade flexibility and wake distortion were used together. This is also the most physically accurate model, with theoretically derived representations of as many real world features as possible with the fewest assumptions. For the best possible results, both of these can and should be employed.

The empirical aerodynamic lag produces very similar alterations to the helicopter dynamics as the wake distortion in some tests, making them almost interchangeable. However this effect is not derived from underlying physics but rather is a simple approximation to capture some of the behavior of the unmodelled features, and must be tuned with flight test data, limiting its usefulness in setting up new simulation models.
8  Piloted Trials: Simulator Fidelity

8.1  The UTIAS Flight Research Simulator

The UTIAS Flight Research Simulator, shown in Figure 36, is somewhat atypical in configuration. The most unusual aspect is that the cab actually contains two cockpits, with a front cockpit for fixed wing research and a rotary wing cockpit facing the rear. The cab containing the two cockpits is mounted on a CAE 300 series six degrees-of-freedom synergistic hydraulic motion system with six 36 inch stroke actuators. A number of measurements were previously carried out to define the motion characteristics of the system and are reported in Reference 55.

The motion drive filters are developed at UTIAS, with both classical and adaptive algorithms as described in Reference 56. During these trials, only the classical algorithm was employed, with filter parameters than can be tuned or switched between configurations on-the-fly. By employing on-the-fly switching, the filters do not need to cover all possible pilot inputs and can instead be tailored to the task at hand. This also allows the use of fairly large gains and relatively permissive filtering, particularly in the pitch and roll degrees of freedom which are important for low speed helicopter manoeuvring.

The rear cockpit has been converted for helicopter use by installing a seat, four-point harness, collective and pedals from a Bell 205, which is shown in Figure 37. When operated as a helicopter, a combination of methods is used for force feel characteristics on the primary flight controls. A manually adjustable friction lock is provided for the collective, and a mechanical spring and damper is used to reproduce control forces on the pedals. The cyclic has a digital programmable McFadden hydraulic control loading system to provide force feedback.
The visual scene can be presented on a Kaiser XL100A helmet mounted LCD display system, a CAE projector and fibre-optic rope helmet mounted display system, or a single 25 inch Mitsubishi CRT monitor infinity optics window box display. All three options provide a collimated display, and the helmet displays are stereoscopic. The Kaiser helmet is the most frequently used option, and was employed throughout these trials. The helmet optical system produces a virtual image collimated to appear to be located at approximately 10 m in front of the pilot. The left and right eye images are generated by LCD displays with a resolution of 1024x768 pixels, with a nominal field-of-view of 100° horizontal by 50° vertical and 30° of stereo overlap. There was some optical distortion, which caused problems particularly in the stereo overlap area as the images from each display did not overlap correctly. This was corrected using 3D Perception distortion correction hardware in the image generators.
A Polhemus FASTRAK magnetic head-tracker connected to a Head Tracker Computer from CAE is employed to provide the image generators with the position and attitude of the helmet’s display optics relative to a reference frame located inside the cockpit. This is then used by the image generators to determine the location and attitude of each of the pilot’s eyes in order to generate the appropriate display images. Because the Polhemus head tracker depends on the precision of the magnetic field shape produced by its transmitter (mounted to the cockpit roof above and behind the pilot’s head), any magnetic distortions caused by the presence of metal objects will result in fixed errors. These distortions were measured in the simulator cockpit and a software routine was developed to correct for these errors in real-time.

Helmet mounted displays are sensitive to delays in the image generation process. When continuous head motion is employed in the presence of visual time delay, the world as viewed on the helmet mounted display appears to swim about instead of being fixed in inertial space. Thus it is essential to reduce to a bare minimum any computational delays due to the head-tracker and the image generators. This is achieved by employing high update rates and being careful to properly synchronize all the computers involved in the simulator. In addition lead compensation is used to reduce the effective time delay associated with generating scene motion in response to head rotational motion. The lead compensation subsystem of the CAE Head Tracker Computer employs a three-axis Watson angular rate sensor affixed to a plate secured to the top of the helmet along with the Polhemus head tracker receiver. Its outputs are processed to provide estimates of the future values of the helmet Euler angles before sending them to the image generators.

The scene is rendered using X-Plane 9, modified with a custom plugin to be controlled by the real-time host over a CIGI Ethernet interface. When using the helmet display, the rendered scene includes both the out-the-window view and the cockpit interior complete with flight instruments. Figure 38 shows an example of the rendered scene, with the field of view of a single eye of the Kaiser helmet. In addition, the vertical sync pulse is sent to the real-time host to generate an interrupt for hardware synchronization of the visuals with the flight dynamics.

The central host computer is a Concurrent iHawk 848 with 8 Xeon 2.8GHz processors, running Concurrent’s RedHawk Linux operating system, with a real-time clock and interrupt
module providing system timing. Models are developed in Simulink, then compiled and run in real-time using OpalRT’s RT-Lab. The timing of operations carried out on the simulator is critical to the issue of fidelity, so it is necessary to arrange the sequence of operations carried out on the simulator’s distributed network of computers in such a manner that the associated time delays are minimized. The total time delay between the pilot inputs to the flight controls and the visual display of the outside world scene is approximately 75 ms. With a slightly different path including the use of a head tracker computer, head motions to visual display have a total effective time delay closer to 90 ms. However a significant fraction of this delay is cancelled by the lead compensation for angular motions.

Figure 38: X-Plane Visual Scenery
8.2 Experimental Setup

For each of the various flight dynamics models, four ADS-33 type handling qualities manoeuvres\textsuperscript{57} were selected to be performed, consisting of a hover capture, a pirouette, a lateral reposition, and a 60 knot slalom. These tasks were chosen to require sizeable cyclic inputs at low speeds to highlight off-axis differences. To aid in guiding the ratings, each ADS-33 manoeuvre defines “desired” and “adequate” boundaries on all relevant measures, many of which are reflected in the ground markings used to perform the tasks. The scenery within X-Plane can be expanded with downloaded or custom user created content, and the UTIAS simulator made use of both aspects to provide a representative virtual world to perform the trials. The manoeuvres were all performed at the virtual Ottawa Macdonald-Cartier International Airport, where the NRC maintains an ADS-33 handling qualities testing area in the real world. To improve upon the default X-Plane scenery, a far more detailed representation of the Ottawa Airport was downloaded, over which was placed a reproduction of the NRC ADS-33 area created at UTIAS for this purpose, which can be seen in all of the X-Plane scenery figures.

For each trial the pilot provided a Handling Qualities Rating (HQR) and a Simulation Fidelity Rating (SFR), along with a brief narrative explaining the reason for the ratings. The SFR is a subjective metric akin to the Cooper-Harper HQR scale and evaluated using similar guiding questions, which has been developed at the University of Liverpool\textsuperscript{58} and is included in Appendix A for reference. Since this is a newly developed rating method not previously applied at this facility, or most others, the simulator was first evaluated using the original baseline vehicle models. Through this process, experience could be gained in applying the fidelity rating scale to a real-world example, while also examining what effect changes to the off-axis dynamics in particular had on the overall fidelity rating obtained.

For a variety of reasons including pilot experience, time constraints, and the previously noted flight test data considerations, the current round of piloted trials focused on the Bell 206 JetRanger simulation. Likewise, there were too many possible combinations for testing all of the effects individually without diluting the comparison against the actual aircraft, as the pilot would subconsciously adapt to the simulator with repeated trials. Instead, a single updated model with the latest additions was rated, along with the original rotor disk implementation as described in Reference 3. For the updated model, the additions which led to the best results in offline trials
were used, meaning wake distortion plus blade flexibility, which also corresponds to the most physically accurate model.

The testing was performed by a single highly experienced test pilot, who had previous experience with the SFR scale and was current with recent flight time in a JetRanger. The only training was to familiarize the test pilot with the features of the UTIAS FRS. No training on the specific tasks was performed to minimize unintended adaptation to the simulator. This would allow the pilot the best opportunity to compare against his experience in the simulator with his recollection of the same tasks in the actual aircraft. In keeping with standard HQR methods, tasks were repeated as necessary to reach proficiency before assigning the final rating. The simulation fidelity was assessed as relating to use of the simulator for a type conversion of a qualified pilot in similar environmental conditions of daytime visual flight with light winds. That is to say, how effectively could the simulator allow a helicopter pilot to become proficient in that particular task in the actual JetRanger.

8.3 Task 1: Hover

The first manoeuvre attempted was the precision hover mission task element. This consists of a 45 degree approach to a hover board, with a reasonably aggressive capture of the final capture position, as shown in Figure 39 from the ADS-33E-PRF standard. The pilot’s right eye view at the final location for this task is shown in Figure 40, which includes the white with red border hover board, solid red centre reference marker, and orange diagonal reference cones.

With the old rotor disk based flight dynamics model, an extensive workload was required to stabilize most axes with a tendency for longitudinal pilot-induced-oscillation. While maintaining control was never in question, achieving even adequate performance for the task was a challenge, resulting in an HQR
of 7. This is not reflective of the actual aircraft, so the inability to achieve similar performance with considerable adaptation to the task strategy meant that simulation fidelity was rated an 8.

Switching to the updated model resulted in a drastic change in the overall experience. Pilot control activity and attitude variations were considered representative of a Bell 206. The desired performance was consistently attained with some “minor but annoying deficiencies”, which were mainly due to limitations on head movement while using the helmet display, resulting in an HQR of 4. The performance achieved was equivalent to that expected in the actual aircraft with minimal adaptation required, giving an SFR of 2. This is a level one fidelity, compared to the level three fidelity noted above. The head movement isn’t entirely natural, requiring a small amount of adaptation by the pilot, but skills obtained training for this task would be expected to fully transfer for use in the helicopter.

8.4 Task 2: Pirouette

The second manoeuvre was a pirouette, completing a full 360 degree 100 ft. radius circle with the nose pointed at the centre within 45 seconds, as shown in Figure 41. With the old dynamics model, the same tendency for a longitudinal pilot-induced-oscillation that caused difficulties in the hover capture was noted. This required significant compensation to achieve adequate performance, which means an HQR of 6. The overall performance
achieved was still considered to be similar to what could be expected in the aircraft, even if not an exact reproduction. There was however considerable adaptation to the control inputs, as the level of effort required to maintain stability is not representative of the Bell 206. This equates to an SFR of 6.

The desired performance level was consistently achieved with the new model, and was attained with representative control activity to maintain the desired ground tracking. The pilot compensation was considered moderate, so the HQR was rated a 4. Compared to the aircraft, the adaptation required was rated as moderate, which means that the SFR was also a 4. This adaptation resulted from the simulation task being deemed a bit too easy to establish and maintain the yaw rate compared to the real world. In addition, the test pilot noted challenges in obtaining the visual cueing needed to perform for the task. So although the skills obtained by training for this task in the simulator would be useful in the helicopter, some differences in yaw would likely be observed.

8.5 Task 3: Lateral Reposition

The third manoeuvre was a lateral reposition (or sidestep), as shown in Figure 42. The actual cone layout used for this task by NRC and the pilot’s view of the area in the simulator is shown in Figure 43. This task was most certainly the most challenging with the visual environment employed, as it requires continual information both from 90 degrees to one side and straight ahead in order to effectively maintain heading and longitudinal position while performing the
reposition. In reality, this is achieved through a combination of peripheral vision and rapid head motions to switch between the two sources of information, neither of which is well supported by the helmet mounted display.

Even with the improved model, the pilot simply cannot see enough to consistently reach desired performance, which limits the HQR to a 5. This also results in a task strategy quite different from what would be used in the aircraft, particularly with regards to head movement. Rather than the technique mentioned above, the pilot instead found he had to hold a steady angle, favouring one source of information (forward vs. lateral), and accept the resulting compromised performance (i.e. “hope for the best”) in the other direction. This altered strategy gives an SFR of 7, with both the handling qualities and simulation fidelity in the level two range. The result was an unfortunate situation where much higher fidelity ratings, or lower numerical rating scores, were always just out of reach. The pilot workload was always well within reason, but positional information could not be gathered fast enough in all axes to further improve performance through additional control effort.

Combining the instability of the old model with the demands of this particular task required very large and rapid head movements, which as noted the current visual system simply doesn’t support. As a result, the task cannot be completed consistently within even the adequate bounds. This is clearly not similar performance to what would be expected in the real world, and is combined with a significant adaptation to the task strategy. The technique learned in this
configuration would produce behaviour not suitable for use in the aircraft. This put both the handling qualities and simulation fidelity into level three, with ratings of 7 and 9 respectively.

8.6 Task 4: Slalom

The final task was a 60 knot slalom, similar to that shown in Figure 44, except that without the outer marker cones the runway edge was chosen as the desired gate location. That is to say the task was to pass on alternating sides of the white runway touchdown zone markings, with the outermost position during the turns occurring directly above the runway edge. The approach to this task in the simulation is shown above in Figure 38.

![Figure 44: Slalom Mission Task Element](image_url)

Flying this task in the older model resulted in the largest observed differences versus the real world equivalent. The aggressive manoeuvring required to accomplish the task provoked all of the deficiencies previously noted, with a significant tendency to enter pilot-induced-oscillations. The result was a considerable effort to even maintain control through the process, resulting in an HQR of 8. Clearly this would not result in any useful training for the task, so the SFR is a 10.

Flown with the updated dynamics model, the task was considered almost easier than it would be in an actual Bell 206, with less effort required than expected to maintain airspeed and control yaw. However, when combined with the vague reduced cueing environment of a simulator, the overall workload was judged to be reasonably representative. An HQR of 3 was awarded, mainly due to deficiencies in depth perception requiring some pilot compensation to maintain altitude. Considering that the task may be slightly more challenging in the aircraft, the SFR was rated 4, as the performance is similar but not entirely equivalent.
8.7 Piloted Trials Summary

The handling qualities and simulation fidelity ratings obtained for the various tasks are collected in Table 1, along with a brief note on the driving factor behind the rated simulation fidelity. In the updated form of the dynamics model, it is readily apparent that the main factor limiting simulation fidelity ratings is the visual system, primarily due to the limitations of the helmet display. However, when this is combined with other issues such as less than ideal dynamics, the problems can quickly compound.

With the improved vehicle dynamics, the test pilot felt the overall experience was quite positive, noting that even with the visual limitations it was probably one of the best simulations he had experienced to date for the low level, close contact type of flying being performed in these tasks. Furthermore, he felt it had the potential to be outstanding with improved visual hardware allowing faster head movements. This is clearly a strong statement, worth some consideration of why that is the case.

Table 1. Simulation Ratings Summary

<table>
<thead>
<tr>
<th>Task</th>
<th>Original Model</th>
<th>Updated Model</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>HQR</td>
<td>SFR</td>
</tr>
<tr>
<td>Hover</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Pirouette</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Sidestep</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Slalom</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>
The main issues with the current helmet come from a slow LCD response time, combined with a lack of resolution. The slow response time has a number of implications. It appears that in order to minimize its effect on the scene, the Kaiser helmet display control box applies some processing to the scene and adds a full frame (16.7 ms) of delay in doing so. In addition, the slow LCD transition introduces a notable smearing, which affects all dynamic content within the scene and further increases latency with head motion, impeding free movement.

The advantages of using a helmet display, in addition to the small space required, is that it provides an unlimited field of observation and more importantly a stereoscopic collimated display. Humans rely on a wide range of visual cues to gauge depth, such as relative sizing and convergence. At short ranges however, up to approximately 10 meters, stereopsis is a very strong depth cue, and may be beneficial at distances up to 200 meters. Also of particular note, which depth cues dominate can vary with task demands. Sweet found that binocular vision was dominant when controlling velocity rather than acceleration in a manual control task. Although the dynamics here are more complex than a simple K/s system, pilot control of helicopter angular rates may favour binocular vision. The result is that although certain aspects such as pixel resolutions are degraded compared to other display technologies, a binocular helmet or head mounted display can be very useful for low speed/close contact manoeuvring tasks.

In addition to stereopsis, the helmet also provides collimation. This has been shown to improve pilot control of attitude rates and translational velocities in the type of tasks used, by providing image stability against translational movement of the pilot’s head. As such, it is reasonable to conclude that a binocular collimated display has the potential to greatly aid subjective simulation fidelity ratings with updated display hardware.

Another factor improving the overall experience and therefore increasing subjective simulation fidelity is the UTIAS motion system. Motion has been shown to improve performance and/or reduce pilot workload in unstable aircraft, providing the motion is of reasonable fidelity. For the types of motion produced by a helicopter in the described tasks, even a relatively small motion system such as the one at UTIAS can provide reasonable motion cues. The cues are even more useful due to the considerable amount of effort that has been made to minimize time delays in this cueing path. Compared against the motion requirements
suggested by Schroeder\textsuperscript{65}, the parameter set used places the motion in the “medium fidelity” range, and could possibly have been further improved as the actuators rarely approached their limits.

Finally, it is also important to note that the fidelity ratings alone cannot possibly give a complete picture. The rating can be driven largely by the single worst aspect of the complete system, such as the helmet display in this case, without necessarily being able to say much about the rest of the system. In addition, even for a single simulator, the ratings can vary wildly depending on the task at hand. As such, caution must be used in drawing overly broad conclusions from a fidelity rating alone.
9 Conclusions

Dynamic wake distortion, aerodynamic lag, and main rotor blade elasticity can be included in a real-time helicopter flight dynamics model. The blade elasticity was added to the UTIAS helicopter models using an assumed modes approach to solve the equations of motion, with slightly different implementations for use on a teetering or articulated rotor. As desired, this was successful in altering the off-axis response, with notable changes in some areas, while not disrupting the on-axis response.

Simulation results suggest this blade flexibility may have a greater potential to improve the response than other added effects. The magnitude of the resulting change in the off-axis vehicle dynamics was greater than the differences obtained with the addition of dynamic wake distortion or aerodynamic lag on blade element lift and drag coefficients. In comparisons with flight test data, the best results were obtained when blade flexibility and wake distortion were used together. This is also the most physically accurate model, and for the best possible results, both of these can and should be employed.

These features produce slightly different changes to the overall helicopter dynamics, and could be used in isolation or combination to improve results depending on the data available and the application. The results using the various combinations of dynamic wake distortion, unsteady blade section aerodynamics, and blade flexibility were demonstrated for the UTIAS helicopter models. The changes to vehicle dynamics due to wake distortion and flexibility in particular are approximately additive, and produced the best results when used together for the Bell 206. This is also the most physically accurate model, including as many real features as possible with the fewest assumptions.

In certain conditions, the theoretically derived dynamic wake distortion and empirical aerodynamic lag are almost interchangeable, able to produce similar alterations to the helicopter dynamics with suitable parameters. As such, with sufficient flight test data to tune the time delay, aerodynamic lag can be a useful computationally efficient method to improve the off-axis match if faster processing is required. However this addition lacks the physics-based derivation and predictive quality of dynamic wake distortion or blade flexibility.
For the Bell 206 JetRanger, the updated improved model was evaluated against the original with pilot-in-the-loop tests to gather feedback and comments from a test pilot to back up the results of the generated time histories. Although the effect of each model improvement and their combinations could not be tested with piloted trials, the updated model was very favourably reviewed, and was notably superior to the original rotor disk implementation.

The changes, as well as the overall simulation environment they are running in, were rated using the recently developed Simulator Fidelity Rating scale. While an improved match to flight test data was found to lead to a higher rated fidelity, there was a limit to how high these improvements could push the rating. With an overall rating scheme, the most objectionable aspect will drive the rating, providing limited information about the other aspects of the simulator.

A number of the possible contributors to the ratings achieved were further explored. For the UTIAS Flight Research Simulator, these include primarily the motion system fidelity and the helmet based collimated stereoscopic visual environment.

It is recommended that for the best possible off-axis performance, real-time helicopter dynamics models should include the dynamic wake distortion and blade flexibility effects. Future work could be done to add the parametric data required to examine blade modes other than bending in the flapping direction.
References


Appendix A: Simulator Fidelity Rating Scale

Fidelity Characteristics

Fit For Purpose
- Simulator training sufficient for acquisition or maintenance of skills

Comparative Task Performance
- Equivalent performance

Pilot’s Task Strategy
- Negligible or no adaptation: 1
- Minimal adaptation: 2

Fidelity Level
- 1

Is equivalent performance attainable with minimal adaptation? YES
- Fidelity Warrants Improvement
  - Additional training required for operational performance

Fidelity Level
- 2

Is similar or equivalent performance attainable without excessive adaptation? YES
- Improvement Mandatory
  - Negative training occurs.

Fidelity Level
- 3

Does fidelity permit task execution? YES
- Similar or equivalent performance

Fidelity Level
- 4

Attempt Task

Fidelity Level
- 5

NO

Is similar or equivalent performance attainable without excessive adaptation? NO
- Similar performance

Fidelity Level
- 6

NO

Is equivalent performance attainable with minimal adaptation? NO
- Similar or equivalent performance

Fidelity Level
- 7

NO

Fidelity Level
- 8

An entirely inappropriate strategy is required

Fidelity Level
- 9

NO

Fidelity Level
- 10

NO
Appendix B: Summary of Model Parameters

B.1 Bell 206 JetRanger Parameters

B.1.1 Main Rotor

\( \beta_o = 0.03927 \text{ rad} \)

\( c_{root} = 0.33 \text{ m} \)

\( c_{tip} = 0.33 \text{ m} \)

\[ I_b = \begin{bmatrix} 490.675 & 0 & 0 \\ 0 & 0.461 & 0 \\ 0 & 0 & 490.675 \end{bmatrix} \text{ kg.m}^2 \]

\( m_b = 50 \text{ kg} \)

\( N_b = 2 \)

\( N_e = 5 \)

\( \Delta = 0.17453 \text{ rad} \)

\( R = 5.08 \text{ m} \)

\( r_o = 0.5 \text{ m} \)

\( r_B^s = [0.074,0.0,-1.02]^T \text{ m} \)

\( \theta_s = -0.0873 \text{ rad} \)

\( \theta^{\text{twist}} = -0.19 \text{ rad} \)

\( \theta_{z\ell} = 0 \)

B.1.2 Tail Rotor

\( \alpha_{oT} = 5.73 \text{ rad}^{-1} \)

\( \delta_{oT} = 0.008 \)

\( \delta_{iT} = 0.0 \)

\( \delta_{2T} = 9.5 \)

\( K_B = 0.886 \)

\( R_T = 0.82296 \text{ m} \)

\( r_B^t = [-5.883,0.0,-0.15]^T \text{ m} \)

\( s_T = 0.094314 \)
$TL_T = 0.92$
$\delta \theta_T = 0.0$ rad

B.1.3 Fuselage Aerodynamics

$r_B^F = [0 \ 0 \ 0]^T$ m

B.1.4 Empennage Aerodynamics

$\eta^h = 0.5$
$i_0 = -0.097125$
$i_1 = -0.87495$
$i_2 = -10.638$
$i^h = -0.02$ rad
$i^V = 0$ rad

$\Lambda = 0.35$ rad
$r_B^h = [-3.764, 0, 0.1]^T$ m
$r_B^V = [-5.906, 0, -0.4]^T$ m

$S^h = 1.37$ m$^2$
$S^V = 1.33$ m$^2$

B.1.5 Flight Equations

$I_B = \begin{bmatrix} 1189.8 & 0 & 375 \\ 0 & 4405.2 & 0 \\ 375 & 0 & 3762.3 \end{bmatrix}$ kg.m$^2$

$m_B = 1300$ kg
B.2 CH149 Cormorant Parameters

B.2.1 Main Rotor

\[ c_{root} = 0.68 \text{ m} \]
\[ c_{tip} = 0.92 \text{ m} \]
\[ I_b = \begin{bmatrix} 3298 & 0 & 0 \\ 0 & 3.774 & 0 \\ 0 & 0 & 3298 \end{bmatrix} \text{ kg.m}^2 \]
\[ m_b = 130 \text{ kg} \]
\[ N_b = 5 \]
\[ N_c = 5 \]
\[ \Delta = -0.1745 \text{ rad} \]
\[ R = 9.296 \text{ m} \]
\[ r_o = 1.209 \text{ m} \]
\[ r^1 = 0.887 \text{ m} \]
\[ r^2 = 0.0; \]
\[ r_{r}^{H1} = [0, 0.465, 0]^T \text{ m} \]
\[ r_{r}^{L1} = [-0.4988, 0.5187, 0]^T \text{ m} \]
\[ r_B^s = [0.0, 0.0, -2.98]^T \text{ m} \]
\[ \theta_0^D = 0 \]
\[ \theta_S = -0.0698 \text{ rad} \]
\[ \theta^{twist} = -0.1047 \text{ rad} \]
\[ \theta_{zli} = 0 \]

B.2.2 Tail Rotor

\[ a_{oT} = 5.8 \text{ rad}^{-1} \]
\[ \delta_{oT} = 0.009 \]
\[ \delta_{IT} = -0.1237 \]
\[ \delta_{2T} = 2.29 \]
\[ K_B = 0.925 \]
\[ R_T = 2.0 \text{ m} \]
\( \mathbf{r}_B^T = [-11.5, 1.0, -3.019]^T \) m

\( s_T = 0.2426 \)

\( TL_T = 0.96 \)

\( \delta \theta_T = 0.14 \) rad

### B.2.3 Fuselage Aerodynamics

\( \mathbf{r}_B^F = [0 \ 0 \ 0]^T \) m

### B.2.4 Empennage Aerodynamics

\( \eta^h = 1.0 \)

\( i_0 = -0.1012 \)

\( i_1 = -0.8851 \)

\( i_2 = -11.436 \)

\( i^h = -0.0175 \) rad

\( i^V = 0.0698 \) rad

\( \Lambda = 0.0 \)

\( \mathbf{r}_B^h = [-10.864, 1.544, -0.583]^T \) m

\( \mathbf{r}_B^V = [-10.849, -0.279, -2.251]^T \) m

\( S^h = 1.849 \) m\(^2\)

\( S^V = 3.1 \) m\(^2\)

### B.2.5 Flight Equations

\[
\mathbf{I}_B = \begin{bmatrix}
33720 & -255 & 1875 \\
-255 & 163300 & -50 \\
1875 & -50 & 142320
\end{bmatrix} \text{ kg.m}^2
\]

\( m_B = 14600 \) kg
B.3 CH124 Sea King Parameters

B.3.1 Main Rotor

\( \omega_{\text{root}} = 0.4633 \text{ m} \)
\( \omega_{\text{tip}} = 0.4633 \text{ m} \)

\[
\mathbf{I}_b = \begin{bmatrix}
2278 & 0 & 0 \\
0 & 2.58 & 0 \\
0 & 0 & 2278
\end{bmatrix} \text{ kg.m}^2
\]

\( m_b = 82.03 \text{ kg} \)
\( N_b = 5 \)
\( N_e = 5 \)
\( \Delta = -0.262 \text{ rad} \)
\( R = 9.449 \text{ m} \)
\( r_o = 1.68 \text{ m} \)
\( r^1 = 0.306 \text{ m} \)
\( r^2 = 0.175; \)
\( \mathbf{r}^{H1}_r = [0, 0.320, 0]^T \text{ m} \)
\( \mathbf{r}^{L1}_r = [-0.242, 0.207, 0.0058]^T \text{ m} \)
\( \mathbf{r}^s_B = [0.0, 0.0, -2.39]^T \text{ m} \)
\( \theta^D_0 = 0 \)
\( \theta_S = -0.052 \text{ rad} \)
\( \theta^{\text{twist}} = -0.30 \text{ rad} \)
\( \theta_{\varphi_{i1}} = 0 \)

B.3.2 Tail Rotor

\( \alpha_{oT} = 5.73 \text{ rad}^{-1} \)
\( \delta_{\varphi_{oT}} = 0.008 \)
\( \delta_{\varphi_{iT}} = 0 \)
\( \delta_{\varphi_{i2T}} = 9.5 \)
\( K_B = 0.796 \)
\( R_T = 1.59 \text{ m} \)
\( \mathbf{r}^t_B = [-11.32, 0, -1.58]^T \text{ m} \)
\begin{align*}
s_T &= 0.2232 \\
TL_T &= 0.92 \\
\delta\theta_T &= 0.0 \text{ rad}
\end{align*}

**B.3.3 Fuselage Aerodynamics**

\[ r_B^F = [0 \quad 0 \quad 0]^T \text{ m} \]

**B.3.4 Empennage Aerodynamics**

\[ \eta^h = 1.0 \]

\[ i_0 = -0.1141 \]

\[ i_1 = -0.9533 \]

\[ i_2 = -14.305 \]

\[ i^h = 0.0 \text{ rad} \]

\[ i^V = 0.28 \text{ rad} \]

\[ \Lambda = 0.0 \]

\[ r_B^h = [-11.10, \ 0.91, \ -1.31]^T \text{ m} \]

\[ r_B^V = [-10.65, \ 0.0, \ -1.12]^T \text{ m} \]

\[ S^h = 2.18 \text{ m}^2 \]

\[ S^V = 5.57 \text{ m}^2 \]

**B.3.5 Flight Equations**

\[
I_B = \begin{bmatrix}
14494 & 0 & 0 \\
0 & 62791 & 0 \\
0 & 0 & 51982
\end{bmatrix} \text{ kg.m}^2
\]

\[ m_B = 7528 \text{ kg} \]